This is to certify that the

dissertation entitled

TEACHERS' USE OF A PROBLEM-SOLVING ORIENTED
SIXTH-GRADE MATHEMATICS UNIT:
TWO CASE STUDIES

presented by

Anthony Dane Rickard

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Curric., Teaching, & Ed. Pol
Dept. of Teacher Education

Major Professors:

Sandra K. Wilcox — Deborah Loewenberg Ball

Sandra K. Wilcox — Deborah Loewenberg Ball

Date 6/1/93

TEACHERS' USE OF A PROBLEM-SOLVING ORIENTED SIXTH-GRADE MATHEMATICS UNIT: TWO CASE STUDIES

By

Anthony Dane Rickard

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Teacher Education

1993

ABSTRACT

TEACHERS' USE OF A PROBLEM-SOLVING ORIENTED
SIXTH-GRADE MATHEMATICS UNIT:
TWO CASE STUDIES

By

Anthony Dane Rickard

Problem solving is a central issue in current reform initiatives in mathematics education. However, while curriculum developers design problem-solving oriented curricula to help move reforms into K-12 mathematics classrooms, little is known about how teachers actually use problem-solving oriented mathematics curricula to teach.

This study investigates how two sixth-grade mathematics teachers used a problem-solving oriented unit on perimeter and area. A four-dimensional framework is developed and employed to explore how each teacher's knowledge, views, and beliefs shaped her use of the unit. Using data collected through interviews, classroom observations, conversations with teachers and their students, samples of students' work, teachers' lesson plans, and the unit on perimeter and area, two case studies are presented to portray how each teacher used the unit in her classroom.

This study shows that each teacher's use of the unit was consistent with her underlying views and beliefs, and with some aspects of the intentions of the curriculum developers who designed the unit. However, other aspects of the teachers' use of the unit varied from the intentions of the curriculum developers. This study shows further that each teacher's use of the unit was shaped by interplay between her own views, beliefs, and knowledge, and the unit. Therefore, both the

perimeter and area unit and the teachers shaped the teaching which occurred in their classrooms.

This study suggests that while problem-solving oriented curriculum can play a role in shaping mathematics teaching, the views, beliefs, and knowledge of teachers should be addressed in curriculum. This study also points to issues for future research that are connected to teachers' use of problem-solving oriented curricula.

This work is dedicated with love and respect to my father,
who has provided unending support throughout all of my endeavors,
and to the memory of my mother.

ACKNOWLEDGEMENTS

A number of people helped make completing this work easier than it might have otherwise been, and each deserves a note of thanks.

To my father, David Rickard, and my sisters, Tracy and Tiffany: Thank you for your understanding and support throughout my collegiate and graduate career.

To my major professors Deborah Loewenberg Ball and Sandra K. Wilcox: Thank you both for helping me better understand and appreciate myriad issues, ideas, and dilemmas in mathematics, teaching, and doing research in mathematics education. Thank you also for your insights, encouragement, and support throughout work on this dissertation and throughout my doctoral program.

To my advisory and dissertation committee members William Fitzgerald, Bruce Mitchell, and Ralph Putnam: Thank you for guidance, many learning experiences, and valuable advice during my doctoral program at Michigan State University.

To the Connected Mathematics Project directors -- Glenda Lappan, Elizabeth Phillips, and William Fitzgerald: Thank you for the conversations which helped me in conceptualizing this study and for the opportunity to learn about and develop mathematics curriculum.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

The Problem

Over the last 10 years, calls for integrating problem solving into K-12 mathematics have steadily gained momentum. Major initiatives in mathematics education at both state and national levels have provided agendas for reforming K-12 mathematics, envisioning problem solving as perhaps the most central aspect of the curriculum (e.g., California State Department of Education, 1991; National Council of Supervisors of Mathematics [NCSM], 1989; National Council of Teachers of Mathematics [NCTM], 1980, 1989, 1991). Another common feature of these reforms is their acknowledgement that currently in K-12 mathematics many teachers, for a plethora of reasons, do not teach mathematics as problem solving, and therefore students are not learning mathematics as problem solving (Putnam, Lampert, & Peterson, 1990). Instead, reformers argue, current K-12 mathematics instruction tends to concentrate on developing students' computational proficiency and skills in applying algorithms (NCTM, 1989; Stodolsky, 1988). As a consequence, K-12 students have little opportunity to develop higher-order skills in mathematics such as problem solving (Kulm, 1991; NCTM, 1989; Putnam et al., 1990).

Seeking to transform this predominant state of affairs, substantial changes have been proposed for the K-12 mathematics curriculum. The NCTM (1989) *Curriculum and Evaluation Standards for School Mathematics* and the *Mathematics Framework for California Public Schools* (California State Department of Education, 1991) are likely the most ambitious plans describing problem-solving oriented curriculum recommendations for school mathematics. California's *Framework* draws heavily on the NCTM *Standards*, including adoption of the NCTM position that "Problem solving should be the central focus of the mathematics curriculum" (NCTM, 1989, p. 23).

Echoing the problem-solving emphasis of these curriculum agendas are new visions for how students should learn, know, and experience mathematics in school. Reformers highlight the importance of connecting mathematics in concrete ways to the world around us and learning about relationships between mathematical concepts and processes (see Steen, 1990). Advocates of problem-solving centered curriculum reform seek to deemphasize instruction on mechanical and often disconnected algorithms and computation, and to increase instructional emphasis on important mathematical concepts (e.g., measurement, number, shape) via problem solving. Mathematics education reform is calling for sweeping change throughout the K-12 curriculum to move away from rulebound textbook learning. The clear message is that a school mathematics curriculum should help teachers teach mathematics as problem solving and students learn mathematics as problem solving (Greeno, 1991; Lester & Kroll, 1990; NCSM, 1989; NCTM, 1989, 1991; Putnam et al., 1990).

In response to the problem-solving focus of all recent major curriculum reform initiatives in K-12 mathematics education, the last 10 years have also seen a flurry of activity in mathematics curriculum development. For example, the *Used Numbers* project is an elementary mathematics curriculum that centers on students collecting, generating, organizing, representing, and making sense of data (see Friel, Makros, & Russell, 1992). The *Middle Grades Mathematics Project* produced five detailed units incorporating a problem-solving based learning model developed by the authors -- the units focus on measurement, spatial visualization, factors and multiples, probability, and similarity (see Lappan, 1983; Shroyer & Fitzgerald, 1986). The *Computer Intensive Algebra* project is an algebra curriculum that integrates computers and computer software into high school algebra (see Fey & Heid, 1991). The *Connected Mathematics Project*, recently funded by the National Science Foundation, seeks to develop a complete middle school mathematics curriculum by 1996 that emphasizes connections among mathematical concepts and between mathematics and other disciplinary areas (see Fitzgerald, Lappan, & Phillips, 1991). All these curricula assume that problem solving is a central activity in mathematics that K-12 students should be engaged in when studying mathematics. These and other problem-solving oriented curricula assume that the student is not a passive receiver of mathematical facts and procedures. Rather, learners are active in constructing their own understandings of mathematics through problem solving in mathematically rich contexts (e.g., Shroyer & Fitzgerald, 1986).

Problem-solving oriented curricula imply not only new roles for students but also imply new roles for teachers (Cohen & Ball, 1990; NCTM, 1991). If teachers are to teach mathematics as problem solving, instruction cannot be limited to what Jackson (1986) calls the "transmission model" of teaching -- the teacher tells the students information and demonstrates procedures, and students show, by doing what the teacher does, that they have received the information and procedures. In contrast, teachers who teach mathematics as problem solving, reformers argue, employ multiple representations of concepts and relationships, model and engage students in dialogues where conjectures about problem situations are offered, tested, and revised. Teachers help students articulate, represent, and modify their own ideas, and journey with students through a mathematical terrain of important concepts and connections (Ball, 1990a; Lampert, 1990; NCTM, 1989, 1991; Putnam et al., 1990). Teaching mathematics as problem solving may also include teaching students specific problem-solving strategies to solve particular kinds of problems (e.g., Meyer & Sallee, 1983), or teaching students global strategies applicable to varieties of problems (e.g., Charles & Lester, 1982).

Because problem-solving oriented curricula imply new teaching practices for teachers, they are often viewed as vehicles for teacher learning and change. For example, in their National Science Foundation proposal *Connected Mathematics*, the developers of the Connected Mathematics Project note that:

> In order to help teachers make the kinds of changes in instructional thinking and planning implied by the goals of *Connected Mathematics*, the materials developed will take seriously the need to provide instructional strategies and organizational help for teachers so that they can develop new modes and habits of instruction (Fitzgerald et al., 1991, p. 13).

The NCTM (1991) emphasizes that for a teacher to change his or her practice of teaching mathematics to a problem-solving orientation, ongoing effort to implement new practices and analysis of one's own teaching are required.

It is well-known, however, that what curricular materials *imply* for teaching practice or are *intended* to accomplish in classrooms and what *actually* occurs can be quite different. There is substantial evidence that teachers enact curricular materials in many different ways. As persons who work in institutions where curricular and other learning materials are generally imposed by others, teachers tend to shape curriculum to their own immediate situations and available resources (Lipsky, 1980; Lortie, 1975; Sarason, 1982). Teachers also have varying degrees of subject-matter knowledge of mathematics and pedagogical content knowledge about representing and connecting mathematics and problem solving to learners (Ball & McDiarmid, 1990; Wilson, Shulman, & Richert, 1987). Teachers hold different perspectives and beliefs about mathematics, problem solving, and the role of problem solving in mathematics and the mathematics curriculum (Rickard, 1991; Silver, 1985; A. Thompson, 1989; Wilcox, Schram, Lappan, & Lanier, 1991). Teachers can also be constrained or motivated by the context in which they teach (Wilcox, Lanier, Schram, & Lappan, 1992). For example, teachers can feel overwhelmed by perceived time constraints, discouraging them from being open to new ideas about teaching, or be challenged and motivated to change by the learners they encounter in their classrooms (Lortie, 1975; Wilcox et al., 1992). All of these factors -- available resources, subject matter knowledge, pedagogical content knowledge, different perspectives and beliefs, context -- contribute to how teachers use curricular materials to teach. The presence of so many factors suggests that it is uncertain how a teacher will use problem-solving oriented curricula in the classroom.

Despite the uncertainties associated with teachers' use of problem-solving based math curricula, teachers are still being pushed from many directions to use these materials. Yet, how teachers use such materials in the classroom is uncertain, and different materials can imply different perspectives on problem solving (c.f., Meyer & Sallee, 1983; Shroyer & Fitzgerald, 1986). In the current context of problem-solving reform in K-12 mathematics, there is an acute need to study factors that shape how teachers use problem-solving oriented curricular materials. For while such materials are available, with more mathematics curriculum development efforts currently underway, it is not at all clear how teachers actually use these kinds of materials in their classrooms. Studies seeking to investigate teachers' use of problem-solving oriented curricular materials can inform the continued push toward teaching mathematics as problem solving and hold implications for teaching, teacher education, curriculum development, and educational policy.

The Purpose of the Study

The purpose of this study is to investigate how a piece of problem-solving oriented curriculum is used by teachers in classroom settings. The study will help to better identify and understand the issues that need to be considered in trying to conceptualize how teachers use problem-solving oriented curricula in classrooms. The main question central to this study is ***How do teachers use a piece of problem-solving oriented curriculum in their classrooms?***

In this research, I study two sixth-grade teachers, each teaching a unit developed by the Connected Mathematics Project (CMP). The unit, *Covering and Surrounding* (see CMP, 1992a, 1992b), is a geometry unit that focuses on the measurement concepts of perimeter and area, and the relationships between these concepts. I include an argument in Chapter 4 justifying the use of the *Covering and Surrounding* unit in this study. I argue that the unit is congruent with the NCTM *Standards* documents (see NCTM, 1989, 1991) and that it is designed to facilitate teachers' use of problem solving as a context for instruction in their practice (see Fitzgerald et al., 1991). Through problem solving, the unit developers intend to accomplish at least two instructional goals -- to learn mathematical content (i.e., perimeter and area) and to connect concepts (i.e., develop and understand relationships between perimeter and area).

Embedded within the main research question are several research areas related to teachers' use of problem-solving oriented curricula and corresponding sub-questions. These research areas and questions have guided my thinking over the course of the research and have proved useful in framing and conceptualizing the study. Addressing these research areas and how they inform the main question of "How do teachers use a piece of problem-solving oriented curriculum in their classrooms?" is the focus of this study:

- *Conceptualizing problem-solving activity in classrooms*:

  What kinds of issues and challenges do teachers encounter as they teach *Covering and Surrounding*?

  What does problem-solving activity look like in the teachers' classrooms and how is it organized?

  What do teachers believe their students learn about problem solving from the unit?

- *Use of problem-solving oriented curricula*:

  How do teachers use problem solving when teaching the unit?

  How and to what extent do teachers teach/emphasize problem solving?

  How and to what extent does the unit influence teachers' teaching of problem solving and mathematical content?

  How do teachers' perceptions and beliefs about student learning influence their use of the *Covering and Surrounding* unit?

- *Comparisons between the intended and the enacted curriculum*:

  How do the curriculum developers intend mathematical concepts to be taught via problem solving?

How do the intentions of the curriculum developers compare with how the teachers use *Covering and Surrounding* in their classrooms?

- *Teacher change and implications for mathematics education*:

  How and to what extent does teaching the unit cause change in teachers' practice?

  What are the implications of the findings from this study for teacher education, curriculum, and mathematics education reform?

The above guiding research areas and sub-questions are intended to provide a means for thinking broadly about the main research question. My intent has been to focus on the teachers and their use of *Covering and Surrounding* without being blind to factors that influence how teachers use a piece of problem-solving oriented curriculum in their classrooms.

Significance of the Study

This study contributes to research on teachers' use of problem-solving oriented curricula from different perspectives: (a) how teachers conceptualize problem-solving activity; (b) establishing a research-based framework with which to examine and conceptualize teachers' use of problem-solving oriented curricula in classroom settings; (c) identifying, describing, and using the research-based framework to analyze teachers' instructional decisions when using a piece of problem-solving curriculum and examine what they take into account; (d) using findings that inform (a), (b), and (c) to inform implementation of education reforms through problem-solving oriented materials.

Teachers' Conceptions of Problem-Solving Activity

There has been little research on how problem solving looks in classrooms and the factors that shape how problem solving is organized in mathematics classrooms (Greeno, 1991; Silver, 1985). By examining teachers' use of a problem-solving oriented unit, this study can unpack how participating teachers conceptualize problem-solving activity and how their conceptions influence their teaching. This study does not provide a means for framing problem solving in all classrooms, as problem solving can vary significantly between classrooms (see A. Thompson, 1985). In contrast to the extensive work that has already been done on conceptualizing problem-solving for students and small groups (e.g., Schoenfeld, 1985a), this study examines teachers' conceptions of problem solving and how it interacts with other beliefs and knowledge to shape their use of a problem-solving oriented unit.

Teachers' Use of Problem-Solving Oriented Curricula

Teachers do not simply enact curricular materials. Just as a chef modifies a recipe to suit particular tastes, teachers enact curricular materials in different ways based on their own knowledge, beliefs, and the learners they teach. Curricula to a teacher, like a recipe to a chef, is a kind of shorthand for what a lesson might be like. No matter how detailed a lesson plan or description of an activity, curricula cannot specify *everything* a teacher will do. Teaching necessarily involves making decisions and constructing interpretations (see Jackson, 1986).

In seeking to integrate materials to fit their teaching circumstances, teachers may use curricula in particular ways because of availability of time, contextual constraints, negotiations with students, managing tensions internal to teaching like how students will learn from the curriculum (Cohen, 1988; Lipsky, 1980; Lortie, 1975; Sarason, 1982; Wilcox et al., 1991, 1992). I establish a framework based on prior research to conceptualize teachers' use of problem-solving oriented curricula in classroom settings. The purpose of the framework is to better identify and understand the factors which shape teachers' use of problem-solving oriented curricula. The relationships of the teacher participants' own conceptions about problem solving and using problem-solving oriented curricula to knowledge and beliefs defined by the domains of the framework are studied to understand how they use *Covering and Surrounding*.

<u>Instructional Decisions</u>

Directly related to how teachers mold curricular materials to their own practice are the kinds of instructional decisions teachers make when using the materials. For example, faced with time conflicts teachers frequently make judgement calls about what portions of the curriculum are most suitable or inappropriate for their students (e.g., Freeman & Porter, 1989; Lortie, 1975; Wilson, 1990). This study examines teachers' instructional decisions and rationale for making these decisions while teaching *Covering and Surrounding*. Drawing on analysis of the unit and classroom observations and interviews, I explore aspects of the unit perceived by teachers as especially suitable or inappropriate for their teaching situation. I employ the research-based framework to piece together why teachers make the instructional decisions they do and what factors shape these decisions. Understanding teachers' instructional decisions when using problem-solving oriented curricula holds implications for curriculum development, implementing problem-solving reforms, and teacher education (see Fitzgerald et al., 1991; The Holmes Group, 1990; NCTM, 1989, 1991; Shulman, 1987; Wilson et al., 1987).

Problem-Solving Reform

The majority of mathematics teachers, math educators, curriculum developers, policymakers, teacher educators agree that in our increasingly diverse, complex, information-based society, being skilled in problem solving is essential for all citizens (e.g., California State Department of Education, 1991; The Holmes Group, 1990; NCSM, 1989; NCTM, 1989, 1991). But implementing problem-solving approaches to teaching mathematics in school is proving to be a difficult challenge. In many instances, the recognized need to teach mathematics as problem solving in K-12 mathematics is at odds with persistent and seemingly competing demands for computational proficiency, skill in using algorithms to get right answers, and increased performance on standardized assessment tests (NCTM, 1989; Nicholls et al., 1991). Dilemmas of implementing problem solving when many K-12 math curricula are still structured around fragmented behavioral objectives, assessing students' problem-solving performance and understanding, working within the confines of limited time, meeting the needs of diverse students, understanding teachers' beliefs and knowledge of mathematics, are all issues that challenge making problem solving an integral part of K-12 mathematics instruction (Ball & McDiarmid, 1990; The Holmes Group, 1990; Lester & Kroll, 1991; NCTM 1989, 1991; Nicholls et al., 1991; Sarason, 1982; Wilcox et al., 1991, 1992).

A significant facet of this study is that the above issues surrounding problem solving are informed by studying how teachers use the *Covering and Surrounding* unit. The insights provided by this study should be useful to teachers, teacher educators, curriculum developers, policymakers, mathematics educators:

What dilemmas do teachers face when using problem-solving oriented materials?

What do teachers need to know about mathematics and content specific pedagogy (i.e., pedagogical content knowledge) to teach mathematics as problem solving?

What does the previous question imply for preservice and inservice teacher education?

How do teachers use a piece of problem-solving oriented curricula in their classrooms and how does their enactment compare with the intent of curriculum developers?

How do teachers' beliefs about mathematics and problem solving, views and beliefs about student learning, subject-matter knowledge, and conceptions of problem-solving activity interact to shape their use of a piece of problem-solving oriented curricula?

This study seeks to examine these questions and issues in the context of teachers using problem-solving oriented materials in their classrooms. I study the nature of teachers' use of a problem-solving based unit and try to unpack the influences that shape the activity and the rationales that explain it.

Overview of the Dissertation

In this chapter, I have established the central question of this study -- "How do teachers use a piece of problem-solving oriented curriculum?" I have also argued how this study addresses the problem and can also inform other issues within mathematics education (e.g., teaching mathematics as problem solving, teacher education, mathematics education reforms).

In the next chapter, I survey research on problem solving in mathematics and research on teachers' use of curricular materials. I then link these two areas of research by developing a four-dimensional framework to aid in conceptualizing teachers' use of problem-solving oriented curricular materials in teaching mathematics.

In the third chapter, I detail the methodology employed in this study. I describe why constructing case studies is appropriate for addressing the research question and I explain how I selected the teacher participants. I also discuss the interview instruments and protocol I designed for this study. I describe how I observed teacher participants in their classrooms as well as the sources of data and methods of data collection.

The fourth chapter is an analysis of the *Covering and Surrounding* unit. The chapter describes, in terms of problem-solving activity, how the developers of the unit intend problem solving and mathematics to be portrayed in the classroom via the problems and tasks provided in the unit. The chapter unpacks the intentions of the developers of the *Covering and Surrounding* unit to enable comparison between the *intended* curriculum of the unit developers and the curriculum *enacted* by the teacher participants as they teach the unit in their classrooms.

In the fifth and sixth chapters, I present the cases of how teacher participants Karen Knight and Betty Walker (both teachers' names are pseudonyms) used the *Covering and Surrounding* unit in their classrooms. Each chapter provides a profile of the teacher and her instructional setting and describes in detail how she used the unit in her classroom. Using interview data and data collected through classroom observations and conversation, I employ the four-dimensional framework developed in the literature review to conceptualize each teacher participant's use of the unit. The case studies are intended to be non-evaluative and portray teachers' use of *Covering and Surrounding.*

In the last two chapters, I describe how the main research question is informed by this study. The study findings are presented in

Chapter 7 and are supplemented by arguments in Chapter 8 that address issues in mathematics education linked to teachers' use of problem-solving oriented curricular materials. The two final chapters are followed by appendices and references.

CHAPTER 2

REVIEW OF THE LITERATURE

Research on Problem Solving

Teaching and Learning Problem Solving

> Studying the methods of solving problems, we perceive another face of mathematics...Mathematics presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an experimental, inductive science (Polya, 1945, p. vii).

> Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations...problem solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error (National Council of Supervisors of Mathematics [NCSM], 1989, p. 471).

Although problem solving is a central component of recent reform initiatives in mathematics education (e.g., NCSM, 1989; NCTM, 1989, 1991), mathematics educators have been concerned with problem solving for over four decades (Kilpatrick, 1985). In his classic work on problem solving, *How to Solve It*, Polya (1945) presents one of the earliest detailed discussions of problem solving in mathematics and the mathematics curriculum. It was in this work that Polya's now famous heuristic for problem solving was first presented: (1) Understanding the problem; (2) devising a plan; (3) carrying out the plan; (4) looking back (Polya, 1945). Although Polya had contemporaries who investigated problem solving processes (e.g., Bloom and Broder, 1950; Duncker, 1945), *How to Solve It* and other works by Polya (e.g., Polya, 1967, 1968) remain some of the earliest and most influential efforts to formally study problem solving and how problem solving might be taught and learned in school classrooms.

Despite the efforts of Polya and others, however, research on problem solving remained limited for some time. Regarding the status of research on teaching and learning problem solving available around 1960, for example, Kilpatrick (1985, p. 9) notes that:

> [authorities] were suggesting that the best advice the research literature had to give was that problem solving should be taught by giving students lots of problems to solve.

But over the next 10 years, some researchers developed approaches to teaching problem solving where students were taught to classify problems and then select and apply a rule or procedure to obtain a solution (e.g., Dahmus, 1970). For example, in solving word problems elementary or middle school students would typically be taught to first identify keywords like "total" or "difference". Students would then be instructed to classify the problem (e.g., as an addition or subtraction problem) and perform the corresponding arithmetic operation with the data given in the problem to obtain the answer. However, such "algorithmic" approaches to problem solving quickly became controversial. Some researchers argued that problem solving is more complex than simply selecting and following recipes and that teaching students to classify problems by type and then apply a memorized sequence of problem solving steps can be difficult for students and also counterproductive (e.g., Brian, 1967; Kilpatrick & Wirszup, 1972). Researchers began emphasizing that heuristics of problem solving (e.g., Polya's four problem solving steps) should be considered as flexible frameworks and not as iron clad procedures guaranteed to solve any problem.

By the early 1980s, problem solving moved into the limelight of mathematics education. In the *Agenda for Action*, NCTM (1980) pushed for problem solving to become "the focus of school mathematics" (p. 1). During the 1980s, findings by cognitive scientists studying how computer models solve problems suggested that problem solving involves complex information-processing and metacognitive processes (Schoenfeld, 1987; Silver, 1987). Goldin (1992), for example, argues that the findings of cognitive science in the area of problem solving imply that

> ... problem solving is not one thing. It involves a highly complex aggregate of internal psychological processes, which occur to varying degrees and in various combinations (p. 275).

Although some conceptions of problem solving may paint problem solving as a linear process (e.g., Polya's four problem solving steps), findings from cognitive science strongly suggest that problem solving includes an interconnected web of components and processes (Goldin, 1992). For example, researchers have identified pattern recognition, representation, memory schemata, and how individuals interact with technology as interactive components and processes that shape problem solving behavior (see Kaput, 1985; Schoenfeld, 1987; Silver, 1987, 1989; P. Thompson, 1985).

Despite gains in understanding problem solving processes, however, various researchers have argued that cognitive models of problem solving behavior should not be the limit of problem solving instruction. Problem solving instruction should go beyond teaching students set problem solving procedures and models (Greeno, 1991). Instead, problem solving should be pursued in a broader context of mathematical discourse where problems and solutions are negotiated and revised (see Greeno, 1991). In the early 1980s, researchers shifted much of their attention to studying the learning and teaching of problem solving in the context of cooperative groups (e.g., Noddings, 1982, 1985; Schoenfeld, 1982). Much of this research has pointed to the usefulness of cooperative groups and small-group discussion as effective vehicles for teaching and learning problem solving.

Also during the early and mid 1980s, a sharper picture of the components of problem solving performance for individuals and small groups emerged from research. For example, Schoenfeld (1985a) constructed a framework to conceptualize problem solving performance in individuals from his research with college undergraduates. Schoenfeld's (1985a) framework consists of the following components: *resources*, or the mathematical knowledge that an individual has access to in trying to solve a problem (e.g., formulas, concepts, methods of proof); *heuristics*, or the strategies and techniques that can be employed to work toward a solution of a problem (e.g., working backwards, using related problems); *control*, or how resources and heuristics are selected and implemented, and how these judgements are made (e.g., self-monitoring and assessment of progress on a problem); *belief systems*, or individual conceptions and perceptions about mathematics which influence problem solving behavior (e.g., beliefs about self and attitudes toward mathematics) (see Schoenfeld, 1985a). These four components interact and can influence each other (e.g., heuristical knowledge and resources influence control), and also help to map out the territories of problem solving processes (see Schoenfeld, 1985a).

In the current reform climate, the terms "problem solving" and "mathematical problem solving" are being used more frequently and freely s.

le

de

than ever in the mathematics education community. However, there is little shared understanding regarding what problem solving is, how problem solving might be taught, and how it is learned. In particular, there is little research which focuses on problem solving in the setting of mathematics classrooms, and surveying the literature reveals that what problem solving activity in classrooms looks like is poorly conceptualized (Grouws, 1985; Silver, 1985). Most research on problem solving has tended to focus on understanding how individual students or small groups of students, sometimes working from programmed instruction booklets, attempt to solve specific kinds of problems (e.g., Dahmus, 1970; Noddings, 1982; Schoenfeld, 1982, 1985a). While study of individual or small group problem solving has provided insight into the complexity of problem solving (e.g., Schoenfeld, 1985a), future research needs to examine how mathematics as problem solving is organized and orchestrated in classroom settings (Greeno, 1991; Grouws, 1985; Silver, 1985).

Problem Solving in Classroom Settings

An ironic characteristic of most prior research aimed at informing the teaching and learning of problem solving is that the role of the teacher in classroom instruction is almost never addressed (Grouws, 1985). As Silver (1985) notes:

> ... the teacher is truly the "forgotten soul" in research on the teaching and learning of mathematical problem solving. In general, the teacher and teacher-related variables have been systematically controlled (for statistical reasons) or unconsciously ignored by researchers (p. 262).

Silver (1985) also points out that available research on teaching and learning mathematical problem solving is limited by a "lack of good description of *what actually happened* in the classroom when problem solving was taught" (p. 248, emphasis in original). Grouws (1985) echoes Silver's concerns and argues that "What is needed is careful scientific inquiry on the teacher's role in the acquisition by students of problem-solving ability" (p. 301).

Commenting on possible future directions of research on teaching and learning mathematical problem solving, A. Thompson (1989) notes:

> I hope that ten years from now we will be more knowledgeable about effective ways and techniques of teaching problem solving and have a better sense of how to go about preparing teachers in their use. I also hope that by then we will have had an opportunity to examine curricular materials for teaching problem solving and how they are used by teachers in their classrooms (p. 243).

The general lack of research on how problem solving is taught and learned in classroom settings has emphasized that future inquiry should consider the role of the teacher, the nature of curricular materials used to foster problem solving ability in students, how teachers use these materials, rich description of classroom activity, and the implications of findings for teacher education and curriculum development (Shulman, 1985; A. Thompson, 1989, 1992; P. Thompson, 1985).

Researchers investigating problem solving have only recently begun to grapple with the complexity and methodological dilemmas presented by classroom settings, like defining classroom problem solving, creating assessment instruments and criteria for problem solving, and conceptualizing the roles of students and teachers when engaged in problem solving activity (Grouws, 1985; Lampert, 1990; Lester and Kroll, 1991; Silver, 1985). For example, the issue of representation (i.e., how specific concepts are represented and how mathematics is portrayed as a discipline) is a key construct in problem solving research (Kaput, 1985; A. Thompson, 1985). Yet, there is little research which analyzes the disciplinary and pedagogical implications and ramifications of teachers' and students' use of instructional representations in mathematics (see Ball, 1990a for an analysis of representations in teaching fractions). Selection and use of representations in problem solving is also directly related to issues of teacher knowledge. Current research is exploring what kinds of knowledge teachers draw on to choose, design, and use representations in their teaching (Ball, 1990a; Greeno, 1991; Kaput, 1985; Wilson et al., 1987).

In examining current research on problem solving, it is apparent that problem solving in classroom settings is complex and is influenced by an array of factors. Silver (1985) argues that at least ten factors significantly affect teaching and learning mathematical problem solving in the classroom -- these factors include *students' beliefs* and *teachers' beliefs*. Students' beliefs include how they perceive mathematics (e.g., as relevant only to scientists, as an interesting field of inquiry open to all and applicable to wide varieties of situations) and how they perceive themselves in relation to mathematics. Teacher beliefs include how the teacher perceives mathematics, as these beliefs will be implicitly, if not explicitly, communicated to students and will influence learning (Silver, 1985). Teacher beliefs also include how teachers view themselves as learners of mathematics, their conceptions of teacher and student roles in the classroom, and their beliefs about what it means to know mathematics (Wilcox et al., 1991).

Other issues which have emerged from recent inquiry into problem solving include questions about what a problem solving oriented mathematics curriculum looks like (Shulman, 1985), what teaching practice from a problem solving perspective looks like (Grouws, 1985), a

l

i

l

e

i

d

s

r

p

f

Th

no

fr

qu

th

com

The

rule

and how to assess students' problem solving ability and performance (Lester & Kroll, 1991; NCTM, 1989; Romberg, Zarinnia, & Collis, 1991). For example, in a problem solving oriented mathematics curriculum, does *any* kind of problem necessarily engage students in problem solving? Would the problem "Eddie hands the grocery clerk a $20 bill for a $16.38 purchase -- how much change does he get?" belong in a problem solving oriented mathematics curriculum? This kind of example points to an emerging question in problem solving instruction and curriculum design: What *kinds* of problems involve students in problem solving and help develop students' problem-solving skills?

Although the question of what kinds of problems belong in problem solving oriented curricula remains open (see Shulman, 1985), some researchers have posited that some problems are better than others. Polya (1968, p. 139), for example, classified problems according to the following scheme:

1. One rule under your nose
2. Application with some choice
3. Choice of a combination
4. Approaching research level.

The change problem above would be an example of a "one rule under your nose" problem. The second kind of problem requires recall of a rule from among two or more possibilities (e.g., choosing between solving a quadratic equation with the quadratic formula or by factoring), while the third entails combining two or more rules/procedures in some combination (e.g., finding the amount of paint needed to cover a house). The fourth kind of problem not only requires the application of multiple rules/procedures, but also involves making judgements and weighing, formulating, and selecting possible paths to a solution (e.g., what is the best position and synchronization for traffic lights in the downtown area of your city to best facilitate traffic flow?)[1]. Kilpatrick (1985) notes that Polya felt that the "educational value" of problems increases from type 1 to type 4. Unfortunately, however, the kinds of problems researchers know the most about are of the first two types (Kilpatrick, 1985). Other math educators since Polya have tried to refine problem classification for teaching and learning problem solving (e.g., Charles & Lester, 1982), but in general the roles of different kinds of problems in curricula and in teaching and learning problem solving is not well conceptualized (Kilpatrick, 1985; Kulm, 1991; Silver, 1989; P. Thompson, 1985).

Another factor complicating the roles of different kinds of problems in curricula is the interplay between curriculum and assessment. Traditionally, assessment has been separate from mathematics curricula, serving mainly to decide how well students have mastered material and to assign grades. For problem-solving oriented curriculum and instruction, however, assessment can be viewed as an integral component of curriculum, teaching, and learning. Assessment can be employed to shape instruction and provide learning experiences for students (Lester & Kroll, 1991; NCTM, 1989). However, recent emphases on assessment in problem solving oriented mathematics instruction (e.g., Fitzgerald et al., 1991; Kulm, 1991; NCTM, 1989, 1991) are complicated by a lack of understanding and consensus as to

---

[1]Polya's classification scheme for problem types does not address the *relative* nature of problems to the solver. For example, what might be a rule under the nose of an eighth grader could be very problematic for a second grader. The notion that what counts as a problem or problem type is relative to the solver will be examined later in this chapter and revisited in Chapter 4.

what problem solving should look like in classrooms and what the roles and importance of different kinds of problems are in problem solving oriented math curricula (Greeno, 1991; Grouws, 1985; Lester & Kroll, 1991; Silver, 1985, 1989). Problem solving influences assessment because what problem solving entails implies what should be assessed in problem solving (NCTM, 1989). However, assessment influences problem solving because what assessment reveals about how students understand problem solving should shape problem solving instruction (Lester & Kroll, 1991).

Overall, while prior research on mathematical problem solving has proved useful in better understanding the complexity of problem solving processes and behavior, few studies have investigated how teachers teach problem solving in classrooms. Although there is an abundance of problem solving oriented curricular materials available for teachers, little is known about how teachers actually use these materials in their classrooms. Researchers have begun to investigate how teachers' knowledge, beliefs, and perceptions (e.g., knowledge of mathematics, beliefs about mathematics and problem solving, perceptions of student learning) shape their teaching of mathematics and how problem solving is enacted in the mathematics curriculum (e.g., Grouws & Cramer, 1989). Clearly evident from the literature on teaching and learning mathematical problem solving is wide agreement that inquiry needs to be conducted on conceptualizing how teachers enact problem solving oriented curricula and teach problem solving in classroom settings.

Research on Factors Influencing Teachers' Use of Curricular Materials

While there is only sparse understanding of how problem solving plays out in the classroom, there is no shortage of curricular materials to help teachers and students engage in problem solving. The diverse approaches different curricula take toward problem solving is testimony to the scattered perspectives that exist about problem solving in classroom settings. For example, some text books treat problem solving as a discrete topic in the mathematics curriculum (e.g., *Mathematics in Action*, 1992) whereas others seek to integrate problem solving throughout the curriculum (e.g., *Comprehensive School Mathematics Program*, 1978). Other materials, intended to supplement the existing curriculum, pursue problem solving by teaching specific strategies to solve specific kinds of problems (e.g., Meyer & Sallee, 1983), by teaching general heuristic strategies for more global applications to wide varieties of problems (e.g., Charles & Lester, 1982), or weaving problem solving into learning activities without explicitly teaching problem solving strategies (e.g., Shroyer & Fitzgerald, 1986).

But while there is an abundance of curricular problem solving materials available, little is known about what teachers actually do with these materials in their classrooms. Available research suggests that teachers vary considerably in how they use curricular materials. For example, Freeman and Porter (1989) found that teachers in their study did not determine the mathematical content of lessons from their texts, yet Remillard (1991a) studied a math teacher who relied heavily on her text to structure and define the content of her mathematics teaching. Other studies illustrate differences in how math teachers use curricular materials, articulate beliefs about problem solving, and teach problem solving (Bockarie, 1980; Graybeal & Stodolsky, 1987; A. Thompson, 1985). Research has also uncovered discrepancies between how teachers talk about using curricular materials in their classroom and how they actually use the materials in practice (e.g., Stake & Easley, 1978).

There is much research to show that teachers vary in how they use curricular materials, policy guidelines, curriculum objectives in their classrooms (e.g., Cohen & Ball, 1990; Lipsky, 1980; Lortie, 1975; Sarason, 1982). Factors contributing to influencing how teachers shape their practice and use curricular materials include the degree of risk they are willing to take in instruction, their knowledge and beliefs about the subject matter, to what extent the teacher perceives the instructional context to be constraining or empowering, the availability of time (Floden, Porter, Schmidt, Freeman, & Schwille, 1981; Jackson, 1986; Sarason, 1982; Wilcox et al., 1991, 1992). However, despite this variability and complexity, policymakers and curriculum developers interested in moving problem solving reforms (e.g., NCTM, 1989) into classrooms view curricular materials as a vehicle for teacher learning and change. Problem solving oriented curricular materials are often central to implementing problem solving oriented reforms (e.g., California State Department of Education, 1991; Cohen & Ball, 1990). How teachers use curricular materials, and how their practice compares with the intentions of policymakers and curriculum developers, can be difficult to interpret and are unlikely to match exactly (Ball, 1990b; Elmore, 1979; Wiemers, 1990; Wilcox et al., 1992; Wilson, 1990).

Additional reasons for why teachers may vary in their use of curricular materials, or enact a curriculum differently than intended by its developers, can be tied to differing views about the roles of curriculum and problem solving. For example, Stanic and Kilpatrick (1989) argue that students should not only be taught to select and follow the "right strategy" to solve a given problem. Stanic and Kilpatrick (1989) warn that such an approach reduces problem solving to becoming "... a skill, a technique, even, paradoxically, an algorithm" (p. 17). Instead, they argue, problem solving should be taught as "art", where the teacher plays a key role in helping students learn to flexibly and fluently understand and solve problems (Stanic & Kilpatrick, 1989). This view contrasts sharply with the findings of Hembree (1992), who concluded in his meta-analysis of 487 studies on problem solving that:

> Heuristics [problem solving strategies] in middle grades 6-8 seemed mildly better than other approaches and gained a distinctly superior status in high school. A positive impact on students' [problem solving] performance also resulted from teachers especially trained in heuristical methods (p. 242).

This finding points to explicitly teaching selecting and implementing strategies as a good way to teach problem solving (Hembree, 1992). Curriculum developers have produced problem-solving oriented curricula that arguably subscribe to these different perspectives (c.f., Meyer & Sallee, 1983 [i.e., teaching selection and implementation of strategies]; Shroyer & Fitzgerald, 1986 [i.e., problem solving is entwined in doing mathematics]).

Teachers' use of curricular materials can be fairly congruent with the intent of curriculum developers (e.g., Remillard, 1991a), although this is not necessarily (and perhaps generally not) the case (e.g., Wiemers, 1990). Teachers may enact curricular materials differently than intended by curriculum developers because teachers and researchers/curriculum developers can harbor different views about the math curriculum and its use. For example, Prawat, Putnam, and Reineke (1991) assessed the perspectives on elementary math curriculum of seven experts (4 researchers, 3 teachers). The researchers in the sample explicitly addressed curriculum and materials and instruction, whereas the teachers were less explicit about the curriculum and focused on developing in students positive attitudes about mathematics (Prawat et al., 1991). While all of the experts agreed with current, broad problem solving oriented reforms (e.g., NCTM, 1989), an array of views on what ideal math curricula should encompass and what are the most important aspects of classroom teaching practice emerged:

> The problem for curriculum developers and teachers is that underneath a seemingly unified call for major changes in the way mathematics is taught actually lie a number of strikingly different assumptions and images of what good mathematics teaching should look like. We argue that for teachers to make sense of the advice and calls for change that bombard them, they need to realize that they may be based on multiple - and possibly incompatible - assumptions (Prawat et al., 1991, p. 64).

Not only has research confirmed that teachers often have different priorities for instruction than curriculum developers (e.g., Lortie, 1975; Prawat et al., 1991; Sarason, 1982), but teachers can also interpret the goals of a call for reform differently and enact curricula differently based on these different assumptions (c.f., Ball, 1990b; Wiemers, 1990; Wilson, 1990).

As well as teachers' use of curricular materials being shaped by their views on curriculum, and that these views may differ from those of curriculum developers, teachers also vary in their perceptions of mathematics. For example, a teacher may perceive mathematics as an arbitrary collection of rules or as a field of creative inquiry subject to revision and change (Ball & McDiarmid, 1990; A. Thompson, 1992; Wilcox et al., 1991, 1992). But different views of mathematics can also exist within the discipline of mathematics and may translate into different views of teaching and learning mathematics. For example, Hershkowitz (1990) identifies two instructional views of geometry:

> There are two main "classic" aspects of geometry: viewing geometry as the science of space and viewing it as a logical structure, where geometry is the environment in which the learner can get a feeling for mathematical structure (p. 70).

Depending on how a teacher perceives mathematics both in relation to herself and as a discipline, different approaches to teaching mathematics may emerge. For example, geometry perceived as a "logical structure" may translate into teaching students definitions, rules, and two-column proofs -- the "logical structure" of geometry should be mastered. Geometry perceived "as the science of space" may promote engaging students in investigating the geometric patterns and properties of objects and phenomena arising in both mathematics and nature (see Steen, 1990). If curriculum developers and teachers subscribe to different perspectives on learning mathematics, the intent of curriculum developers and teachers' interpretation of the curriculum are unlikely to coincide (Elmore, 1979; Sarason, 1982).

Current research has demonstrated that how teachers perceive mathematics and themselves in relation to mathematics also influences how they use curricular materials, teach problem solving, and choose or design learning experiences for students (A. Thompson, 1985, 1992;

I

c

i

me

th

ab

ha

mat

int

bel

Sara

prob

will

the c

Wilcox et al., 1991, 1992). For example, A. Thompson (1985) studied two teachers whose teaching practices emphasized different positions on the importance of problem solving in school mathematics. One of the teachers, who viewed mathematics as "fixed and predetermined", predominantly taught mathematics by presenting the math content "in a clear, logical, and precise manner" (A. Thompson, 1985, p. 286). In this teacher's classroom, students learned by listening to the teacher and answering the questions posed by the teacher. In contrast, the second teacher, who viewed mathematics as "more a subject of ideas and mental processes than a subject of facts" generally taught mathematics by encouraging students to "guess and conjecture" and by being "receptive to the students' suggestions and ideas" (A. Thompson, 1985, pp. 289-290). While problem solving was an integral part of the latter classroom, it was scarce in the first, especially if problem solving includes making conjectures, connecting ideas, and developing processes more than mastering fixed facts and predetermined procedures. Both of these teachers taught mathematics in a way congruent with their beliefs about mathematics and illustrate the great influence teachers' beliefs have on how they teach mathematics and problem solving.

An implication of findings about teachers' use of curricular materials is that teachers do not simply enact a curriculum, but interpret and filter it through their own perspectives, knowledge, and beliefs (Cohen, 1988; Cohen & Ball, 1990; Lipsky, 1980; Lortie, 1975; Sarason, 1982). In using curricular materials that seek to integrate problem solving (in some way) into mathematics instruction, teachers will also play a role in shaping the curriculum and problem solving in the classroom. Yet, while both the teacher and the curricular materials play roles in molding how problem solving looks in the classroom, it is difficult to assess the roles of the curriculum and teacher and how they interact (see Grouws, 1985; Remillard, 1991a). Further research is needed into how teachers use problem solving oriented materials in math classrooms. Remillard (1991b), for example, argues that

> It is this question of how teachers use and interpret alternative curricula which needs further study. We need to know more about what teachers bring to new and nonstandard mathematics curricula and how they make sense of and use them (p. 60).

Researchers are increasingly acknowledging that in the context of problem solving careful research is needed to better conceptualize the role of curricular materials in problem solving instruction (P. Thompson, 1985), the role of the teacher in problem solving instruction (Grouws, 1985; Silver, 1985), and how teachers and curricula interact in classroom settings (Ball, 1990b; Ball & Cohen, 1990; Grouws, 1985; Remillard, 1991a; Wiemers, 1990; Wilson, 1990).

Constructing a Framework

Teaching and learning problem solving in classroom settings is poorly conceptualized. Despite the proliferation of "problem solving" workshops and inservices for practitioners and math educators, our understandings about problem solving instruction as it occurs among teachers and students in classrooms is sketchy at best. As A. Thompson (1989) notes:

> Reports of instructional studies in problem solving have generally lacked good descriptions of what actually happened in the classroom ... and have often failed to assess the direct effectiveness of instruction. ... As a result, our knowledge of desirable instructional practices in problem solving is mostly of folklore rather than research evidence (p. 232).

Similarly, how teachers use curricular materials in their classrooms, especially to teach "mathematics as problem solving" (see NCTM, 1989), remains an open question. For example, teachers may integrate problem solving into their mathematics teaching or may neglect it almost entirely, viewing open-ended problems, for example, as frustrating to their students (e.g., A. Thompson, 1985). Significant variation is common among the curriculums teachers enact, even when using the same curricular materials. For example, Freeman and Porter (1989) found that teachers may center their mathematics teaching tightly around a textbook to meet district objectives, while other teachers may use the same text only as a convenient resource, focusing instead on the mathematical topics and skills they themselves regard as important. Teachers and math educators can also exhibit diverse -- and sometimes disparate -- assumptions about the role of the curriculum in mathematics instruction. For example, Prawat et al. (1991) found that teachers tended to think of the curriculum "as something that is given to them by the school district" (p. 62), whereas university based math educators tended to view the curriculum as constructed in the classroom by teachers and students and consisting of "rich problem situations, developed around important clusters of mathematical ideas" (p. 63).

However, despite the scattered state of research on problem solving and teachers' use of curricular materials, the literature in these areas and the connections between them can inform the question of how teachers use problem solving oriented curricula in their classrooms. Reviewing the literature reveals that teachers' use of problem solving oriented curricular materials can be influenced and shaped by at least four issues: (a) Teachers' views and beliefs about mathematics and problem solving; (b) Teachers' views and beliefs about problem solving activity in classroom settings; (c) Teachers' subject matter knowledge of mathematics; (d) Teachers' perceptions and beliefs about student learning.

While these four regions overlap (e.g., if a teacher believes problem solving is an individual activity, she probably won't use cooperative groups as a setting for problem solving activity), the literature indicates that each plays a role in shaping how teachers use problem solving oriented curricular materials. In the next four sections, I will survey the literature in each area. Together, the four regions comprise a framework that will be useful in conceptualizing and analyzing teachers' use of the *Covering and Surrounding* unit.

Teachers' Views and Beliefs About Mathematics and Problem Solving

Regarding views and beliefs about problem solving within the mathematics education community, Stanic and Kilpatrick (1989) argue that:

> The term *problem solving* has become a slogan encompassing different views of what education is, of what schooling is, of what mathematics is, and of why we should teach mathematics in general and problem solving in particular (p. 1, emphasis in original).

"Problem solving" and "mathematical problem solving" are loaded terms that call forth myriad images about teaching and learning mathematics. The notion of problem solving in school mathematics is not only open to wide interpretation by mathematics educators in general, but especially among teachers and curriculum developers (see A. Thompson, 1989). Problem solving has become a familiar term to math teachers over the past 10 years, but how teachers address problem solving in their classrooms, and what problem solving means to them, varies considerably and may be at odds with the goals of school mathematics reforms (c.f., Ball, 1990b; A. Thompson, 1985; Wiemers, 1990; Wilson, 1990).

One of the reasons teachers vary in their views and beliefs about problem solving is that there are aspects of problem solving which are difficult to define. Earlier, I noted that all problems may not be created equal, citing Polya's (1968) scheme for classifying problems as a way of differentiating between different kinds of problems and his view that some problems are of more "educational value" than others. But another fundamental issue in teaching and learning problem solving and in curriculum development is trying to better conceptualize "What *is* a problem?" For example, in surveying both past and current research on teaching problem solving, Kilpatrick (1985) notes that the question of "What is a problem?" is an issue that emerges as being at the heart of teaching problem solving. Polya (1967) asked the question of "What is a problem?" and stated:

> ...to have a problem means: *to search consciously for some action appropriate to attain a clearly conceived but not immediately attainable aim.* To solve a problem means to find such action...Yet some degree of difficulty belongs to the very notion of a problem: where there is no difficulty, there is no problem (p. 117, emphasis in original).

Charles and Lester (1982) answer the same question this way:

> A problem is a task for which:
> 1. The person confronting it *wants or needs to find a solution.*
> 2. The person has *no readily available procedure* for finding the solution.
> 3. The person must *make an attempt* to find a solution (p. 5, emphasis in original).

Despite these formulations of what a problem is, however, there is still a lack of consensus as to what problems are and how they should be used to teach students problem solving (Kilpatrick, 1985). Grouws (1985) and Shulman (1985) argue that research needs to address the issue of how to identify or construct and use "good" problems in teaching problem solving which help meet learning goals for students and inform teaching practice.

Teachers' views and beliefs about problems are directly related to their teaching of problem solving and their views and beliefs about problem solving. In the absence of research findings teachers are largely left to their own devices, or the textbook, to determine the kinds of problems with which their students engage. Even given classification schemes for problems to help determine what are "good" and "not so good" problems (e.g., Polya, 1968), teachers still vary considerably in the kinds of problems and problem solving activity they feel their students need to learn about math content and problem solving (c.f., A. Thompson, 1985; Wiemers, 1990; Wilcox et al., 1991, 1992; Wilson, 1990).

Predictably, the nature of problems teachers develop and/or provide for students is connected to their own views and beliefs about what are "good" problems for their students (see Putnam, Heaton, Prawat, & Remillard, 1992). For example, does a teacher view the problem "What is the perimeter of a rectangle with length = 8 and width = 18?" and "What is the perimeter of a rectangle with area 144?" as essentially the same (e.g., because they both ask about perimeter of a rectangle) or different (e.g., because the first problem has one answer and the second has multiple solutions)? Do students have comparatively limited problem solving opportunity with the first problem than with the more open-ended second problem? This example attempts to illustrate research findings which indicate that teachers' views and beliefs about problem solving are, at least in part, shaped by their beliefs about what kinds of problems with which their students should work (Ball, 1990a; Charles, 1989; Kilpatrick, 1985; Wilcox et al., 1992).

There is considerable research that addresses how teachers' beliefs about mathematics influences their mathematics teaching (see A. Thompson, 1992; Wilcox et al., 1991, 1992). Some of this research has focused on how teachers' beliefs about mathematics is connected to problem solving instruction. Teachers who view mathematics as a fixed set of given rules and procedures to be mastered through practice also tend to approach problem solving as a set of procedures; teachers who see mathematics as a domain of creative inquiry tend to address problem solving as a creative and flexible endeavor inclusive of students' ideas as well as useful heuristics (e.g., A. Thompson, 1985, 1989). With respect to student achievement in problem solving, Hembree's (1992) meta-analysis revealed that "The better that teachers felt toward mathematics, the better their students performed with problems" (p. 254). Hembree's finding corroborates and connects the work of other researchers who argue that teachers' views and beliefs about mathematics influence student learning through their teaching (e.g., Grouws & Cramer, 1989; Silver, 1985) and that an individual's beliefs about mathematics provide a context for his or her problem solving behavior and performance (e.g., Schoenfeld, 1985a).

Teachers' views and beliefs about problem solving seem strongly tied to beliefs about mathematics, including what problems are appropriate for student learning, and may thus shape students' problem solving behavior.

Problem-Solving Activity in Classroom Settings

The scant and scattered state of research on problem solving in classroom settings makes answering the question "What does problem-solving activity in classroom settings look like?" difficult (Grouws, 1985). The research that is available suggests that the kinds of problem-solving activity teachers involve their students in is shaped by their underlying views and beliefs about mathematics and problem solving (e.g., Putnam et al., 1992; A. Thompson, 1985). Rather than being based on research evidence or documented pedagogical approaches (i.e., careful descriptions of how problem solving is taught in different classroom situations and the effects of the instruction on students and teachers -- see Grouws, 1985 for an argument about the need to document teaching problem solving in classrooms), problem-solving instruction emerges from teachers' beliefs and assumptions about mathematics and problem-solving processes (A. Thompson, 1989). Some researchers, such as Schoenfeld (1985a, 1985b) and Lampert (1990) offer descriptions of problem-solving activity in classroom settings that are based on detailed assumptions about mathematics and problem solving.

Schoenfeld (1985a, 1985b) argues that problem solving is complex and that successful problem solving necessitates monitoring one's own progress in solving problems. Problem solving entails being able to *control* a problem by making appropriate decisions about selecting and applying the kinds of *resources* (i.e., mathematical knowledge) and *heuristics* (e.g., problem solving strategies) that will facilitate solving the problem (Schoenfeld, 1985a). In the classroom, problem solving entails students developing control skills so that students can productively select and use heuristics/strategies to implement their mathematical resources. Students must learn how to monitor their progress on a problem and make decisions about how to allocate their mathematical resources and select and implement heuristics (Schoenfeld, 1985b). Learning control is a critical facet of successful problem solving performance. For even if appropriate strategies and heuristics are available to a student, without good control behaviors (e.g., consciously assessing progress, metacognition), a problem solving attempt may be unsuccessful. Schoenfeld (1985a) stresses that:

> ...inefficient control behavior can cause individuals to squander the problem solving resources potentially at their disposal and thus fail to solve problems that are easily within their grasp. The issue [of control] is not just the use of one's heuristic knowledge; it is how all of one's mathematical knowledge is called into play (p. 114).

With respect to his own teaching practice, Schoenfeld (1985b) describes how he attempts to teach college undergraduates *control*. In some cases, Schoenfeld acts as a "manager", where he poses a problem to the class and the students' role is to make suggestions for its solution. Schoenfeld (1985b) says that in this instance the teacher's role "...is not to judge the suggestions", but to "monitor" them, "...raising questions about the efficacy of suggested steps (both when they are useful, and not useful)" (p. 373). Under this kind of "monitoring", students collectively construct a solution to the problem. However, Schoenfeld notes that problem solving in the classroom also entails students working together in small groups (of three or four) where the teacher acts as a "consultant" to the various groups. In this approach, he gives students a problem to solve in their small group. Schoenfeld then circulates around the room to each of the groups helping them productively control the problem by asking questions like "What (exactly) are you doing?", "Why are you doing it?", and "How does it help you?" (see Schoenfeld, 1985b, p. 374). Schoenfeld argues that this kind of pedagogy employing these kinds of questions helps students develop "effective" control which is critical to successful problem solving performance (Schoenfeld, 1985a, 1985b). Teaching problem solving in this manner requires the teacher to not only have a rich and flexible knowledge of mathematics, but also knowledge of how students' and his or her own mathematical knowledge is employed to work towards solutions of problems.

In her teaching practice, Lampert (1990) emphasizes that mathematics is not a collection of fixed facts, rules, and procedures that must be practiced until mastered. Rather, doing mathematics means formulating conjectures and solutions within problem situations and then formulating proofs or counter-examples (i.e., mathematical arguments) to prove or refute potential proofs and solutions. Problem solving, therefore, entails crafting a mathematical argument which provides and justifies a problem solution. Lampert (1990) notes that in the classroom

> ...the teacher must make explicit the knowledge she is using to carry on an argument with them [i.e., the students] about the legitimacy or usefulness of a solution strategy. She needs to follow students' arguments as they wander around in various mathematical terrain and muster evidence as appropriate to support or challenge their assertions, and then support students as they attempt to do the same thing with one another's assertions. As the teacher moves around in mathematical territory in a flexible manner, she is modelling an approach to problem solving (p. 41).

In Lampert's view, the teacher needs to know more than the rules for doing mathematics - she needs to know how to prove mathematical rules and assertions and how to make this knowledge accessible to students.

Problem solving is pursued by the classroom as a community, requiring the teacher to become a facilitator as students formulate their own conjectures and arguments supporting or challenging each others' solutions (Lampert, 1990).

Other research, however, suggests that in many classrooms problem-solving activity looks much different than Schoenfeld and Lampert portray, and is often based on different assumptions about mathematics and problem solving. For example, in discussing four case studies of how fifth-grade teachers approach teaching mathematics for understanding, Putnam et al. (1992) argue that teachers often assume that students need to be told how to do mathematics. This "transmission model" of teaching (see Jackson, 1986) for mathematics means the teacher needs to tell students math facts, demonstrate mathematical procedures, and then the students need to memorize/master the facts/procedures. Mathematics is assumed to consist of a fixed body of facts and procedures, and problem solving means applying the facts and procedures successfully to get the right answer. While teachers may acknowledge that understanding is important, they also may argue that understanding can only come after facts (e.g., how to interpret a fraction) and procedures (e.g., how to divide fractions) have been mastered. Regarding how this view of mathematics and problem solving translates into problem solving activity in the classroom, Putnam et al. (1992) note:

> Problem solving in these classrooms involved applying well-practiced computational skills in particular situations, rather than opportunities to figure out what is reasonable and sensible in these situations. This view of problem solving is consonant with the hierarchial view of learning we discussed earlier - that students must learn the basics before they can understand and apply them.

Schoenfeld (1985a, 1985b), Lampert (1990), and Putnam et al. (1992) describe different kinds of classroom activity that, depending on the underlying assumptions about mathematics and problem solving, could all be described as "problem solving". In trying to conceptualize problem-solving activity in classroom settings, therefore, it is critical to understand how mathematics and problem solving are viewed by the teacher, and that these views and beliefs will play an important role in shaping "problem solving" activity (see Greeno, 1991; Grouws, 1985; Shulman, 1985).

Subject-Matter Knowledge

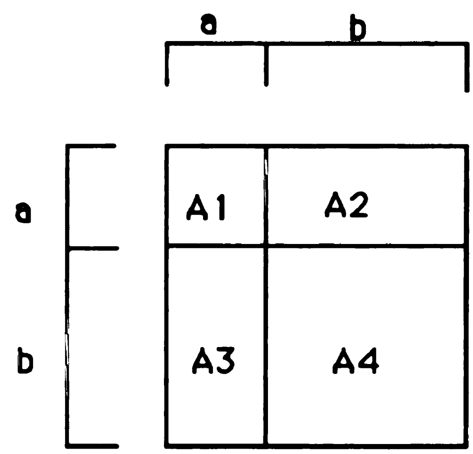
Schoenfeld (1985a) argues that knowledge of mathematics (*resources* in Schoenfeld's terms) has a direct influence on mathematical problem solving. While subject matter knowledge interacts with other factors which together shape problem solving behavior (e.g., selection of appropriate problem solving strategies), knowledge of mathematics is a fundamental component of problem solving performance (Schoenfeld, 1985a). Lampert (1990) argues that teachers need a degree of subject matter knowledge of mathematics when modelling approaches to problem solving because teachers need to make explicit to students the mathematical knowledge they draw on. Schoenfeld (1985b) echoes Lampert's (1990) position, stressing that a teacher should model appropriate problem solving behavior to students and that this includes drawing students' attention to the mathematical "resources" being brought to bear on the problem.

Further evidence that subject matter knowledge shapes how problem solving is taught by teachers in their math classrooms comes from connections among research on problem solving and research on teacher knowledge and teacher education. Shulman (1987), Ball and McDiarmid (1990), and Wilson et al. (1987) demonstrate that teachers' subject-matter knowledge influences how they design, select, and use instructional representations. For example, a teacher may teach his or her students how to square a binomial by the familiar FOIL method (multiply the First terms, then the Outside terms, then the Inside terms, and finally the Last terms):

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$$

However, the teacher whose subject matter knowledge of algebra includes connections between algebra, geometry, and mathematical proof could also employ this representation of squaring a binomial:

Figure 2.1: Geometric representation of squaring a binomial:



> Each side of the square has length a + b, so the area of the square is $(a + b)^2$. However, from the diagram, the subdivided regions of the square A1, A2, A3, and A4 have areas A1 = a^2, A2 = ab, A3 = ba, A4 = b^2. Since the area of the square is also equal to A1 + A2 + A3 + A4 = $a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$, the area of the square has two equivalent forms with the result that $(a + b)^2 = a^2 + 2ab + b^2$.

The first representation is a rule which does not attempt to convey why $(a + b)^2 = a^2 + 2ab + b^2$ but only asserts what to do to square a binomial -- it is a rule, not a proof or a justification. The second representation tries to show, geometrically, how the binomial square can be visualized and proved in terms of an object (i.e., by subdividing the area of a square with edge a + b). Either representation can be used to show students how to square binomials, but they vary in how they represent mathematics (i.e., as a fixed set of rules, as interrelated concepts, respectively). The representations also vary in what they communicate is important for students to know about mathematics -- knowing how to perform rules and procedures versus knowing how to prove relationships and connect ideas. The accessibility of these representations to a teacher will be influenced to a significant degree by his or her subject-matter knowledge of mathematics, including knowledge of relationships between mathematical ideas and knowledge of mathematical proof. Silver (1985) and Kaput (1985) link this finding from teacher knowledge and teacher education research to problem solving by arguing that representations play a key role in problem solving processes, problem solving performance, and teaching and learning problem solving.

While subject-matter knowledge plays a direct role in problem solving performance and in teaching problem solving, Shulman (1987) has argued strongly that subject matter knowledge is important in teaching in general (see also Ball & McDiarmid, 1990). Shulman and his colleagues (e.g., Wilson et al., 1987) have stressed the need to study the role of subject matter knowledge in teaching. Research on teaching has traditionally neglected the specific subject matter being taught, and future inquiry needs to consider the subject matter and the knowledge teachers need about the subject matter to teach it well. As Putnam et al. (1992) comment:

> Commonplace is the claim that it is not just a matter of teachers needing more mathematics courses; they need richer knowledge about mathematics for teaching.

The above passage adds weight to Charles's (1989) argument that a critical question to wrestle with in preparing teachers to teach problem solving is "How good a problem solver must a teacher be to be an effective teacher of problem solving" (p. 263)?

In the above passage, (i.e., Putnam et al., 1992, above), what kind of teacher knowledge is implied when emphasizing a need for "richer knowledge about mathematics for teaching"? Researchers have called for conceptualizing the kind of knowledge teachers need for teaching by linking subject matter knowledge with knowledge of learners (e.g., Shulman, 1987; Wilson et al, 1987). This kind of knowledge, frequently referred to as *pedagogical content knowledge*, is knowledge of subject matter that is powerful for teaching. Wilson (1989) describes that pedagogical content knowledge

> ... consists of understandings and beliefs about the range of alternatives for teaching a particular piece of subject matter to particular students in particular schools, as well as knowledge and beliefs about the ways in which students learn the content in question. This knowledge also enables teachers to generate instructional representations that are justifiable on the basis of the discipline itself, on theories of teaching and learning, on knowledge of the interests and prior knowledge of students, and on educational goals and objectives (p. 1).

In teaching problem solving, then, what does a teacher need to know "to be an effective teacher of problem solving" (Charles, 1989, p. 263)? Knowledge of appropriate instructional representations for teaching problem solving is consistent with Wilson's portrayal of pedagogical content knowledge and research that has emphasized the importance of representations in teaching problem solving (e.g., Hembree, 1992; Silver, 1985). As Kaput (1985) emphasizes, "Whatever the details, most would agree that *some* idea of representation seems to be at the heart of understanding problem-solving processes" (p. 381, emphasis in original).

Issues of subject matter knowledge, pedagogical content knowledge, and problem solving seem to coincide when considering how teachers generate and use instructional representations to teach problem solving (c.f., Kaput, 1985; Schoenfeld, 1985a; Wilson et al., 1987).

Teachers' Perceptions and Beliefs About Student Learning

Research has also indicated that teachers' beliefs and perceptions about student learning interact with other factors which all shape classroom instruction and the nature of learning experiences afforded to students (A. Thompson, 1992). For example, classroom instruction is molded both by teachers' beliefs and perceptions about what is important for students to know about mathematics and by the context of instruction (Wilcox et al., 1992). Although teachers may believe that students should learn mathematics by engaging in structurally rich, problem solving oriented situations, contextual constraints such as limited time, large class size, multiple and demanding responsibilities, the views of other teachers and administrators, can discourage teachers from enacting instruction congruent with these beliefs (Wilcox et al., 1991, 1992). Kohl (1984) has argued that how teachers perceive their students (e.g., as cooperative, as disciplinary problems) directly influences how and what teachers teach.

However, contextual factors that influence teachers' perceptions and beliefs about student learning extend beyond individual classroom situations or building policies. Research has demonstrated that factors such as social class, gender, and race shape perceptions and beliefs about what particular kinds of students should learn in particular schools (e.g., Oakes, 1985). Anyon (1981) found that the predominant social class of a school's community corresponded with different learning opportunities for students. For example, while students in working class schools had their attention largely focused on lessons in all subject areas that were limited to memorizing facts and mastering procedures, students in upper-class (i.e., executive elite) schools engaged in inquiry based learning experiences (Anyon, 1981). For students in the working class school, knowledge is told to them by others and is found in books; for the upper-class students knowledge is created by people and is subject to change (Anyon, 1981). Teachers within both of these respective kinds of schools often supported their students' learning experiences, emphasizing, often in terms of the predominant social class of the school's students, that they were teaching their students what they needed to know. Some teachers in the working-class schools noted that since their students could not do arithmetic well, they needed considerable practice so that they could later, for example, calculate correct change. Upper-class teachers argued that their students needed more open-ended, creative learning experiences to prepare them to be future leaders in society (Anyon, 1981). This example points to the powerful impact students' class status can have on how teachers perceive them as learners and determine what is appropriate for them to learn.

Gender differences in mathematics is an issue which has continued to demand attention from mathematics educators (see Dessart and Suydam, 1983; Fennema, 1989; Swafford, 1980). At least part of the reason for the controversy and uncertainty which pervades debates on the role of gender in mathematics teaching and learning lies in an abundance of inconsistent and often contradictory research findings. For example, some studies have concluded that there is no evidence of gender inequality among math students when the number of mathematics courses taken by both sexes is controlled, whereas other research has shown that under these conditions differences in achievement (in favor of males) persists (Dessart & Suydam, 1983). However, with respect to problem solving, gender differences seem more pronounced. As Fennema (1989) notes:

> Although there has been much research about gender differences in mathematics and many interventions have been developed to alleviate the differences over the last 15 years, there is still powerful evidence that males achieve at higher levels than do females, particularly in tasks of high cognitive complexity, such as true problem solving (p. 211).

Proposed reasons for gender differences in mathematics vary, and include arguments that the factors of class and race affect females differently than males (Hallard & Eisenhart, 1988) and that males tend to be more confident than females about mathematics (McLeod, 1989). Some researchers have argued that teachers, often unconsciously, may treat males and females differently in the classroom (Strauss, 1988). A prevailing view on gender differences in mathematics is that educators need to be aware of gender differences in mathematics and provide equal learning opportunities for students of both sexes (NCTM, 1991; Strauss, 1988).

Not only class and gender, but also race is a portion of the overall context that shapes teachers' perceptions of their students and their beliefs about student learning. Academic tracking in schools, where different students are placed in different subject matter sequences, is a mechanism where racial differences are often quite visible. For example, in her study of 25 different K-12 schools, Oakes (1985) notes that:

> ... it is clear that in our multiracial schools minority students were found in disproportionately small percentages in high-track classes and in disproportionately large percentages in low-track classes. ... this pattern was most consistently found in schools where minority students were also poor. ... In academic tracking, then, poor and minority students are most likely to be placed at the lowest levels of the schools' sorting system (p. 67).

That student class, race, and gender are all important contextual factors in shaping how teachers perceive their learners is evident in current math education reform initiatives. For example, in the *Professional Standards for Teaching Mathematics*, the NCTM (1991) emphasizes the need to educate "every child":

> As a professional organization and as individuals within that organization, the Board of Directors sees the comprehensive mathematics education of every child as its most compelling goal. By "every child" we mean specifically:
>
> - students who have been denied access in any way to educational opportunities as well as those who have not;
> - students who are African American, Hispanic, American Indian, and other minorities as well as those who are considered to be a part of the majority;
> - students who are female as well as those who are male; and
> - students who have not been successful in school and in mathematics as well as those who have been successful (p. 4).

As research and educational reforms both indicate, factors which influence teachers' beliefs and perceptions include not only personal views and perspectives and individual classroom or building situations, but also encompass the broader societal issues of class, gender, and race (Anyon, 1981; Bereiter & Scardamalia, 1987; The Holmes Group, 1990; NCTM, 1989, 1991; Oakes, 1985; Putnam et al., 1992; Strauss, 1988;

Wilcox et al., 1991, 1992). All of these factors (i.e., teachers' own perceptions and beliefs, class, race, gender) contribute to create an overall context in which teaching and learning occurs.

But while teachers' beliefs and perceptions may interact with myriad contextual factors to shape instruction, teachers' beliefs about what their students need to know and how they should learn it remain a critical filter through which instruction must pass (Ball, 1990b; Cohen & Ball, 1990; Silver, 1985; A. Thompson, 1985; Wilson, 1990). One reason for the resilience and powerful influence of teachers' beliefs is that they have generally been established over a long 17 year period in K-16 schooling (Wilcox et al., 1991). As Lortie (1975) argues, however, learning about teaching through the eyes of a student is inherently limited because the motivations, beliefs, rationales, and knowledge behind the teacher behavior a student witnesses are largely hidden and unknown. Teachers' beliefs being powerfully shaped by limited observation and then influencing instruction is especially significant in teaching mathematics. This is because beliefs about the nature of mathematics can be communicated to students through instruction (Lampert, 1990) and because the content and structure of students' learning experiences can lead to confining or empowering learning opportunities and events (e.g., see Anyon, 1981 and Oakes, 1985 for examples of how tracking and student class and race interact with teacher beliefs).

In a synthesis of the research on teachers' beliefs, A. Thompson (1992) notes that teachers do have views and beliefs about how students learn and that these beliefs shape how mathematics gets taught in the classroom. Teachers' beliefs and perceptions about student learning are also connected to their views and beliefs about mathematics and subject matter knowledge of mathematics (Ball & McDiarmid, 1990; Wilcox et al., 1991, 1992). For example, in analyzing case studies of how four different teachers approach teaching fifth-grade mathematics, Putnam et al. (1992) found that the teachers felt basic skills had to come before understanding, that learning only occurs over time, and that students might not be ready for the abstract thinking needed for understanding. Putnam et al. (1992) discuss how these beliefs about student learning shaped the teaching of mathematics in the teachers' classrooms:

> All these beliefs about learning -- that basic skills must be mastered before one can understand, that learning takes time, and that children might not be developmentally ready for abstract thought -- support the traditional emphasis on computational skills and facts, leaving the understanding of mathematics to be dealt with later. Beliefs about learning are also related to what is being learned. What teachers know and believe about mathematics necessarily influences their beliefs about how students learn it.

Putnam et al. (1992) note that in their study the teachers' beliefs about learning taking time is, at least on the surface, congruent with current problem solving oriented reforms (i.e., California State Department of Education, 1991). California state-level reforms argue that learning takes time because students need a variety of rich, problem solving oriented activities to provide multiple opportunities to unpack and connect mathematical ideas (California State Department of Education, 1991). Some teachers in the study, however, felt that learning takes time because students first have to spend time practicing and mastering basic facts, skills, and procedures (Putnam et al., 1992). Collectively, these beliefs reinforced students learning mathematical facts, procedures, and algorithms and little about problem solving.

It is important to acknowledge that teachers' beliefs and perceptions about student learning are not the only factors that shape their teaching of mathematics. For example, teachers' views of what is important for students to learn and how students learn can be influenced by the views of other teachers and administrators (e.g., Wilcox et al., 1992). Standardized assessment and achievement tests at local, state, and national levels are also powerful in shaping how and what teachers teach (Nicholls et al., 1991). As Kulm (1991) emphasizes:

> Basic computational skills have been the focus for competency tests. ... Teachers have been legitimately concerned that if they "fight the system" and teach higher order thinking [e.g., problem solving], their students would suffer on the computationally oriented tests that they are required to pass. Many educators believe that very little change will occur in mathematics curriculum and teaching without a concurrent change in testing, especially in state and national standardized tests that are used to assess and compare school-by-school achievement (p. 72).

Overall, research indicates that teachers' beliefs and perceptions about student learning, which directly influence classroom teaching (e.g., A. Thompson, 1992), are shaped by a variety of factors, including their own experiences in school, the context of instruction, the expressed views and beliefs of others, tests and policies, personal beliefs, perceptions, and knowledge about learners and subject matter (Ball & McDiarmid, 1990; Kulm, 1991; Lampert, 1990; Lortie, 1975; Putnam et al., 1992; Sarason, 1982; Schoenfeld, 1985a, 1985b; Silver, 1985; A. Thompson, 1985, 1989, 1992; Wilcox et al., 1991, 1992).

Summary

Research indicates that while advances have been made in conceptualizing problem solving processes and individual problem solving performance (e.g., Schoenfeld, 1985a), teaching and learning problem solving in classroom settings has not been significantly addressed (Kilpatrick, 1985; Grouws, 1985; Silver, 1985, 1989). Similarly, research on how teachers use problem solving oriented mathematics curricula is scant (A. Thompson, 1989). While an abundance of problem-solving oriented curricular materials exist, these materials take different approaches to teaching problem solving, reflecting a wide array of views about what problem solving is and how it fits into the school mathematics curriculum (Kilpatrick, 1985; c.f., Meyer & Sallee, 1983; Shroyer & Fitzgerald, 1986; see Stanic & Kilpatrick, 1989). Little is known about how teachers actually use curricular materials in their classrooms to shape and guide instruction, despite the importance such findings would have for teacher education, curriculum development, and mathematics education reform (Prawat et al., 1991; Remillard, 1991a).

Research seeking to investigate teachers' use of problem solving oriented curricular materials is informed by four areas of existing research: (a) teachers' views and beliefs about mathematics and problem solving, (b) teachers' views and beliefs about problem solving activity in classroom settings, (c) teachers' subject matter knowledge of mathematics, (d) teachers' perceptions and beliefs about student learning (including the multiple dimensions of student class, race, gender as well as teacher beliefs). The importance and influence of knowledge and beliefs in teaching is well-documented (Ball & McDiarmid, 1990; A. Thompson, 1992), and teachers' knowledge and beliefs interact with external factors (e.g., limited time, class and race of students, standardized tests) to shape classroom practice (Kulm, 1992; Lortie, 1975; Putnam et al., 1992; Sarason, 1982; Wilcox et al., 1991, 1992).

Therefore, research indicates that the nature of curricular materials, teachers' knowledge, beliefs, and the context of instruction are all critical factors to consider in investigating teachers' use of problem-solving oriented curricular materials.

CHAPTER 3

METHODOLOGY

Case Study Research

In prior research, case studies have been used as a means to study how teachers use mathematics curricular materials. Although these studies generally do not focus specifically on how *problem-solving oriented* curriculum is enacted by teachers, the studies in this area do investigate how teachers use curricular materials. For example, Freeman and Porter (1989) studied four elementary teachers to learn how they interacted with their math texts to make decisions about the mathematics content of lessons. Remillard (1991a) crafted a detailed case study which explores and analyzes how one teacher enacted an elementary mathematics curriculum that attempts to engage students in problem-solving situations and discussions about mathematical ideas.

Other researchers have used case methods to study how teachers' knowledge and beliefs interact with policy reform to shape elementary mathematics instruction. This line of research has exposed a variety of similarities, disparities, and tensions between the intentions of state-level mathematics education reform and how teachers enact the reform-based elementary math curriculum. For example, Wilson's (1990) case study analysis demonstrated that teachers' ability to enact mathematics curriculum as outlined by reforms (in this case the California *Framework*, see California State Department of Education, 1991) can be constrained by their knowledge of mathematics and their need for "... help in learning about new [teaching] methods, help in finding time to teach for understanding, resources for evaluating such understanding"

(Wilson, 1990, p. 41). Wiemers's (1990) case study portrays a fifth-grade mathematics teacher whose beliefs about teaching mathematics are at odds with the vision of the California state-level mathematics reforms. The case shows how the teacher filters and transforms the reform policies through his existing beliefs into an interpretation of teaching that he views as somewhat compatible with the reforms and his own beliefs (Wiemers, 1990). Ball's (1990b) case study presents a teacher whose practice reflects some of the California state-level reforms, but is inconsistent with these policies in other respects. Collectively, these three case studies (Ball, 1990b; Wiemers, 1990; Wilson, 1990) are examples of how case study research methods can be used to explore relationships between intended and enacted curricula, teachers' perceptions and understandings about policy and curricula, and teachers' knowledge and beliefs.

While some case study research has focused on how teachers use mathematics curricular materials, or explore connections between intended reforms and curricula and teachers' actual practice, other case study research has focused explicitly on teachers' knowledge. For example, Shulman (1987) used case studies to investigate the kinds of knowledge teachers draw on in their teaching. This line of inquiry has helped to conceptualize the knowledge that seems specific to teaching and teachers (e.g., pedagogical content knowledge -- see Wilson et al., 1987). Research techniques that are used in constructing case studies, such as careful analysis of protocols and interview transcripts, have been used to explore individual and small-group problem-solving performance (e.g., Noddings, 1982; Schoenfeld, 1985a). Such research has led to further insight into the nature of the knowledge of mathematics and heuristics required for successful mathematical problem solving (Schoenfeld, 1985a). Other characteristics of case study research, such as extensive observation, constructing rich descriptions of settings, are congruent with calls for research on problem solving that incorporates detailed accounts of teachers' actions and instruction in classrooms (Grouws, 1985; Silver, 1985, 1989; A. Thompson, 1989).

Overview of Methods

Two teachers participated in this study. The participating teachers teach in different school districts whose students, in general, come from contrasting socioeconomic backgrounds. The two classroom settings studied contrast along various dimensions: size of the schools; building/district policies (e.g., tracking); socioeconomic status and demographics of the school communities. These differences and contrasts between the classroom settings are desirable for this study because factors such as tracking and social class of learners have all been shown to have a significant impact on how teachers teach and on what students have an opportunity to learn (Anyon, 1981; Bereiter & Scardamalia, 1987; Lortie, 1975; Oakes, 1985). My choice of research sites with contrasting characteristics was not intended to provide for generalizeability of the research findings. Rather, the variance in characteristics between the sites has allowed me to study two settings across which a variety of factors and issues are present that can potentially influence teachers' use of the *Covering and Surrounding* unit.

I interviewed each teacher prior to observing their classrooms. Each teacher interview was conducted in two parts (see Interviews: Design and Rationale section). All interviews were audio taped and transcribed for analysis. I then observed each classroom over a period of approximately eight weeks. I spent one class period per week for the first three weeks observing the teachers as they taught and becoming familiar with their classroom settings prior to *Covering and Surrounding* instruction. For the next five weeks I observed each teacher at least one and usually two classes per week teaching the *Covering and Surrounding* unit. During the five weeks of *Covering and Surrounding* instruction, I observed one teacher eight class periods and the other nine class periods. Observational data was collected during classroom observation via field notes and audio taping. Periodically during the observations I had conversations with students and the teacher participants (conversations were audio taped) and collected data from documents (e.g., teachers' lesson plans, samples of students' written work). At the conclusion of the unit, I had a conversation with each teacher participant to reflect on their experience of teaching *Covering and Surrounding.* Confining the study to two classes allowed me to collect rich yet manageable data.

Using the Connected Mathematics Project unit *Covering and Surrounding* in this study makes methodological sense. As a contributing developer of the unit I am familiar with the intentions of the curriculum developers. Throughout the study, however, I have been careful not to let my familiarity with the unit blind me to issues that were not considered or anticipated in its development, but might still be significant factors in the study. The unit is suitable not only because it is problem-solving oriented, but because it also focuses on the challenging middle school mathematics concepts of perimeter and area. Moreover, the unit was designed assuming that the teacher plays a central role in guiding and orchestrating instruction (see Fitzgerald et al., 1991; Shroyer & Fitzgerald, 1986). The unit provides contrasts in how problem solving is used. For example, in some investigations students engage in problem solving to learn content (e.g., the concepts of perimeter and area). In other investigations, students explore problems to learn primarily about mathematical connections and relationships between concepts (e.g., how the perimeter of rectangles can change when area is held constant). While the unit's use of problem solving to teach content and make connections overlaps in each activity, some investigations emphasize one more than the other. The unit is also congruent with curricular recommendations of current problem solving based reform agendas in mathematics education (e.g., NCTM, 1989).

Teacher Participants

One of the first challenges of this study was how to select teacher participants. Both teacher participants needed to teach sixth-grade mathematics, be willing to teach the *Covering and Surrounding* unit, and discuss with me her use of and views of the unit. I also required that the teachers be relatively open to new ideas, curricular materials, and teaching practices in mathematics.

I selected Betty Walker[1] from her school district from a pool of teachers currently affiliated with the Connected Mathematics Project. While I am not acquainted with any of these teachers, they had all made tentative commitments to pilot Connected Mathematics Project units during the 92-93 school year. Betty was interested in mathematics education reform and was willing to use innovative curricular materials

[1]Betty Walker is a pseudonym.

and try new pedagogical practices that could facilitate changes in practice. She also agreed to provide detailed feedback and engage in discussions of her use of Connected Mathematics Project materials.

In selecting Karen Knight[2], who teaches in another school district, I contacted the district's mathematics coordinator and requested recommendations for sixth-grade mathematics teachers who might be interested in participating in this study. The mathematics coordinator is familiar with the CMP materials and serves on the advisory board for the project. After receiving the district coordinator's recommendations, I contacted the teachers to describe the study and gauge each of their interest levels in participating. I then met with the most interested teacher, Karen Knight, to discuss the study in greater detail, and at this meeting she agreed to participate.

Betty Walker and Karen Knight both exhibited the aforementioned characteristics of openness toward trying new materials and practices in teaching mathematics, and both were willing to talk about their views on and use of the *Covering and Surrounding* unit.

Conducting the Study

I employed fieldwork methods to conduct this dissertation study. The nature of my inquiry -- trying to understand how teachers use a problem-solving oriented unit -- is especially suited to the on-site observation and interview techniques of fieldwork research (see Bogdon & Biklen, 1982; Hammersley & Atkinson, 1983). In addition to interviews and observations, I also collected data from relevant documents, such as the *Covering and Surrounding* unit, samples of students' work, teachers'

---

[2]Karen Knight is a pseudonym.

lesson plans, information on the schools and their communities, district curriculum objectives and policies. For example, I have described the *Covering and Surrounding* unit and have drawn on the Connected Mathematics Project NSF proposal *Connected Mathematics* (Fitzgerald et al., 1991) to construct a portrait of the intended curriculum (see Chapter 4). As a researcher, I have triangulated multiple sources of data (i.e., analysis of the curriculum, interview and observational data on participants) to understand how the study participants used *Covering and Surrounding*. The data is used to construct cases that incorporate assertions and supporting arguments to explain how each teacher uses the unit. I use the cases to raise educational issues related to teachers' use of problem-solving oriented curricular materials.

Following are detailed discussions of the two main phases of data collection and then analysis of the data: Interviews -- including design of and rationale for the interview materials; Observation -- how the teacher participants were studied in their classrooms; Analysis -- the predominant areas that emerged from the data which characterize and shape the teacher participants' use of *Covering and Surrounding*, that are employed to construct case studies and obtain findings.

Interviews: Design and Rationale

The first step in data collection was to interview both teacher participants. The purpose of the interview was to acquire general background information on the teacher participants (e.g., number of years teaching, number of years teaching mathematics) and to assess:

(a) Teachers' views and beliefs about mathematics and problem solving.

(b) Teachers' views and beliefs about problem-solving activity in classroom settings.

(c) Teachers' subject-matter knowledge of mathematics.

(d) Teachers' perceptions and beliefs about student learning.

These four areas represent regions which the research literature indicates influence how teachers use curricular materials and how teachers teach, understand, and perceive mathematics and problem solving. My intent in the interviews was not to gain information to predict teacher behavior, but to assess teachers' views and beliefs within these regions to better gauge issues shaping teachers' use of the *Covering and Surrounding* unit over the course of data collection (i.e., as teachers teach the unit).

In each part of the interview, I provided teachers with items (i.e., excerpts from curricular materials or vignettes of classroom situations) to react to. Problem solving is an issue that is difficult to discuss in depth directly, without reference to a particular problem, context, or situation (Grouws, 1985; Kilpatrick, 1985). The excerpts from curricular materials all deal with the concepts of perimeter and area in order to be consistent with the focus of the *Covering and Surrounding* unit. The vignettes depict teachers and students in classroom situations. Using interview items oriented around the same topics as the instructional unit affords more meaningful comparisons between the interview data and classroom observation data than if the interview items dealt with concepts different than perimeter and area. However, to provide for some range of math concepts among the interview items, not all deal with perimeter and area. For example, a brief vignette of a sixth-grade mathematics lesson, used to assess teachers' views and beliefs about problem-solving activity in classroom settings, does not incorporate perimeter or area.

The importance of assessing teachers' knowledge and beliefs by providing them with authentic classroom materials and situations is emphasized by Barnes (1987). Barnes (1987) argues that teaching is very complex and that teachers' knowledge is drawn from a variety of sources (see also Shulman, 1985; Wilson et al., 1987). Moreover, the specific context of a classroom situation or the nature of particular curricular materials has a large influence on how teachers access their knowledge and how their knowledge and beliefs shape their instructional decisions (Barnes, 1987; Wilcox et al., 1991, 1992). Asking or providing teachers with decontextualized questions or situations (e.g., "What are your views on teaching mathematics as problem solving?") often does not yield much about teachers' knowledge or beliefs. As Barnes (1987) notes:

> If teacher assessment is to be valid, it must ... approximate the realities of classroom teaching [and] the assessment ... must allow teachers to create their own meanings from the information provided (p. 9).

By providing teacher participants with a variety of classroom situations and curricular materials to discuss and react to, I have attempted to "approximate the realities of classroom teaching" in the interview assessment. By asking teacher participants open-ended questions throughout the interview, I have also tried to allow them room to construct and articulate their own meanings and knowledge of the curricular materials and classroom vignettes.

The first part of the interview consisted of some preliminary background questions and then three sets of items and questions, each attempting to address the role and use of curricular materials, problem solving, and problem-solving activity in classrooms. In each of these sections, teacher participants were provided with excerpts from various curricular materials (i.e., Hake & Saxon, 1985; *Math in Action*, 1992; Shroyer & Fitzgerald, 1986), and a description of a problem-solving lesson in a sixth-grade mathematics classroom (see Appendix A for the vignette). I expected that using these curricular excerpts and the vignette would provide data for interesting and rich comparisons between their professed views and their actual practice (see also A. Thompson, 1985). This part of my conversation with the teacher participants was structured around open-ended questions so that the teachers would have freedom and latitude in their responses.

The three items used to assess subject-matter knowledge are designed to help understand different facets of teachers' subject-matter knowledge of perimeter and area. Item 1 is intended to assess teachers' knowledge of the relationships between perimeter and area as well as what teachers think counts as proof. Item 2 tries to find out if teachers think conceptually about area (e.g., as the number of square units needed to cover a figure). Item 3 asks teachers to make sense of a student's (hypothetical) response to perimeter problems where correct answers are obtained in a non-standard way and helped me learn how flexible the teachers were in their understanding of perimeter (i.e., does a teacher think that the only way to find perimeter is 2L + 2W) (see Appendix B for items 1, 2, and 3 assessing subject-matter knowledge).

The last section of the interview assesses teachers' beliefs and perceptions about student learning by asking them to react to a vignette that describes a sixth-grade mathematics teacher and her interactions with her students (see Appendix C for this vignette). The vignette describes how the teacher has crafted three different learning situations for three different groups of students, and how she teaches mathematics individually to an individual student from each group. The interviews with both teacher participants were transcribed in their entirety.

Observations: Studying Teacher Participants

After being interviewed, both Karen Knight and Betty Walker were observed in their classrooms once per week for three weeks during the month prior to beginning their instruction of the *Covering and Surrounding* unit. I observed Karen Knight and Betty Walker at least one and usually two classes per week for the next five weeks as they taught the *Covering and Surrounding* unit. Each class was audio taped and I wrote detailed field notes during each class. I also, on occasion, spoke with participating students (i.e., those students who consented to be audio taped and for whom I had also received consenting parental permission forms) about their perceptions of the unit. Conversations with students were also audio taped. I also taped several interactions between the teacher participants and their students.

On most occasions, I was able to expand my field notes the same day as I observed the class. Audio tapes of each class session were reviewed, and selected portions of these tapes were transcribed. I also periodically initiated conversations with both teacher participants about specific lessons on the same day of the lesson to get information on their perceptions while they were still fresh in their minds. In a variety of instances, I was able to compare teacher and student perceptions of the same lesson.

I frequently collected samples of teachers' lesson plans and samples of students' written work. Representative samples of students' writing were collected. At various points during the *Covering and Surrounding* unit, I asked the teachers to react to and discuss specific examples of students' written work with me. I also asked the teacher participants to talk with me about particular events during classroom instruction (e.g., students' conjectures, students' arguments to support their solutions or conjectures).

All of these strategies for data collection were aimed at understanding teachers' use of the *Covering and Surrounding* unit. My goals were to collect data that would help me in answering questions like "How do teachers use the *Covering and Surrounding* unit?", and "How and to what extent does the teachers' teaching address problem solving?" Answering these questions was guided by how each teacher perceived the unit, roles of themselves and their students in the classroom, subject-matter knowledge about mathematics, beliefs about mathematics and problem solving.

Upon finishing the instruction of the unit, I had a reflection session with each teacher. This consisted of an extensive conversation where I tried to assess their overall impressions and reactions to the experience of teaching the unit. I also explicitly asked both Karen Knight and Betty Walker if they felt that teaching the *Covering and Surrounding* unit had motivated any changes in their teaching practice.

Analysis

My first task in data analysis was to carefully analyze the background and assumptions and purposes underlying the *Covering and Surrounding* unit. I developed this analysis prior to observing the teacher participants teaching *Covering and Surrounding* in their classrooms. The result of this analysis appears as the next chapter. My purpose in analyzing *Covering and Surrounding* is to conceptualize the intentions of the developers of the unit. My analysis indicated that the unit developers intend *Covering and Surrounding* to provide a context for interactive roles for students and teachers in the mathematics classroom and engage students with a variety of problem-solving oriented learning experiences. Throughout my analysis of the unit, I draw heavily on the *Covering and Surrounding* student (CMP, 1992a) and teacher (CMP, 1992b) materials, the Connected Mathematics Project NSF proposal (Fitzgerald et al., 1991), and the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and the *Professional Standards for Teaching Mathematics* (NCTM, 1991). Collectively, these documents represent the curricular materials teachers and students use in this study (i.e., CMP, 1992a, 1992b), and the conceptual and ideological foundations for the development of the Connected Mathematics Project curricular materials as represented by the *Covering and Surrounding* unit (i.e., Fitzgerald et al., 1991; NCTM, 1989, 1991).

After conceptualizing the intentions of the developers of the *Covering and Surrounding* unit, I conducted interviews with both teacher participants and then observed them teaching the unit. Analysis of this data was ongoing. For example, I compared my observational data on participants as they taught *Covering and Surrounding* with their responses to the interview items and questions. My intent was not to predict or check the behavior of the teacher participants with respect to their use of the unit, but was to help alert me to issues occurring (or not occurring) in teachers' discussions with me in the interview setting and in their actual teaching practice.

I kept a chronological study log as the participants taught the unit in which I reflected on my observations and conversations with Karen Knight, Betty Walker, and their students. At the conclusion of the unit instruction I carefully examined my data from all sources (i.e., field notes, interviews with the teacher participants prior to teaching the unit, conversations with the teacher participants while teaching the unit, reflections after teaching the unit, samples of teachers' lesson plans, samples of students' work, conversations with students, conversations between students and teacher) to document issues and patterns emerging from the data. I then employed the four-dimensional framework I developed in the literature review to better understand and conceptualize teacher participants' actions and decisions in developing the final case studies.

CHAPTER 4

PROBLEM SOLVING AND THE INSTRUCTIONAL UNIT: BACKGROUND AND ANALYSIS

The Connected Mathematics Project[1]

The CMP and Middle School Mathematics Reform

Responding to calls for reform of K-12 mathematics education (e.g., NCSM, 1989; NCTM, 1989, 1991), the National Science Foundation recently funded four middle school mathematics curriculum development projects. The Connected Mathematics Project was funded for $4.8 million over five years, 1991 through 1996. The principal investigators for the CMP are Glenda Lappan, William Fitzgerald, and Elizabeth Phillips of Michigan State University, Susan Friel of the University of North Carolina at Chapel Hill, and James Fey of the University of Maryland at College Park. The goal of the project is to develop a complete middle school curriculum for grades six, seven, and eight that is congruent with the *Curriculum and Evaluation Standards for School Mathematics* and the *Professional Standards for Teaching Mathematics* of the National Council of Teachers of Mathematics (see NCTM, 1989, 1991). As well as providing students with a curriculum that delivers engaging, problem-solving oriented mathematics, the CMP intends to develop a curriculum that is usable by teachers and will help move teaching and learning school mathematics ahead into the next decade. The CMP curriculum is being designed to help middle school mathematics teachers implement reforms widely advocated in mathematics education (e.g., teaching and learning mathematics as problem solving, making mathematical

[1]For convenience, I will generally refer to the Connected Mathematics Project as the CMP.

connections, teaching *all* students for understanding) (NCSM, 1989; NCTM, 1989, 1991; Putnam et al., 1990).

The CMP is committed to teaching powerful mathematics to all middle school students.

> The proposed Connected Mathematics curriculum will be designed to meet the needs of all students with experiences that are stimulating and challenging to middle school kids with a variety of interests and aptitudes. We believe that students of different interests, abilities, and learning styles can learn with and from each other (Fitzgerald et al., 1991, p. 6).

Teaching connected mathematics for understanding to all students is congruent with other initiatives aimed at reforming teaching and teacher education. For example, The Holmes Group (1990) outlines a reform agenda aimed at restructuring existing K-12 schools into *Professional Development Schools*, or PDSs. PDS sites are schools where school administrators, teachers, university faculty, the private sector, and local, state, and federal government work together to improve teaching and learning for understanding in all subject areas for all students. The Holmes Group (1990) envisions that "The Professional Development School will be a place where everybody's children participate in making knowledge and meaning -- where each child is a valued member of a community of learning" (p. 29).

The title *Connected Mathematics Project* reflects the overall commitment of the project to develop a complete middle school mathematics curriculum that attempts to make connections among mathematical topics, to other school subjects, between the elementary and secondary curricula, and to the real world. As the CMP proposal states:

> The working title of this proposed middle school curriculum, *Connected Mathematics*, has been chosen to suggest the team's interest in developing students' knowledge of mathematics that is rich in connections - connections among the various topic strands of the subject, connections between the planned teaching/learning activities and the special aptitudes and interests of middle school students, and connections between the preparation developed by elementary school mathematics and the goals of secondary school mathematics (Fitzgerald et al., 1991, p. 2).

In contrast to typical curricula that tend to be disjointed, fragmented, and computation driven (NCTM, 1989), the CMP emphasizes interdisciplinary connections between mathematics, social studies, science, and business, and devotes special attention to connections among important mathematical ideas (e.g., relationships between perimeter and area).

<u>The Connected Mathematics Project: Views and Beliefs About Mathematics and Problem Solving</u>.

A central focus of the CMP curriculum is on teaching and learning mathematics as problem solving. In the CMP curriculum, teachers are expected to guide students as they explore rich problem solving situations and contexts. The role of problem solving in the CMP curriculum is congruent with the position of the NCTM:

> Problem solving is the process by which students experience the power and usefulness of mathematics in the world around them. It is also a method of inquiry and application...to provide a consistent context for learning and applying mathematics. Problem situations can establish a "need to know" and foster the motivation for the development of concepts (NCTM, 1989, p. 75).

Unlike many middle school mathematics texts and curricula, the CMP does not include problem solving as an isolated topic in school mathematics. Problem solving is not presented in the CMP curriculum as a few narrow behavioral objectives (e.g., "the student will be able to solve a three-step problem"), nor is problem solving disconnected from mathematical content by appearing only at the end of exercise sets. Rather, the CMP takes the view that problem solving should be integrated throughout school mathematics as a central part of what it means to learn and do mathematics (Lampert, 1990; Lester & Kroll, 1990; NCTM, 1989; Putnam, et al., 1990; Shroyer & Fitzgerald, 1986).

The key role of problem solving in the CMP curriculum is evident in the learning goals for students. Fitzgerald et al. (1991) identify eight process goals:

> COUNT - Determine the number of elements in finite data sets, trees, graphs, networks, permutations, or combinations by application of mental computation, estimation, counting principles, calculators and computers, and formal algorithms.
>
> VISUALIZE - Recognize and describe shape, size, and position of one-, two-, and three-dimensional objects and their images under transformations; interpret graphical representations of functions, relations, and symbolic expressions.
>
> COMPARE - Describe relations among quantities and shapes using concepts such as equal, less than, greater than, more or less likely, congruence, similarity, parallelism, perpendicularity, symmetry, and rates of growth or change.
>
> MEASURE - Assign numbers as measures of geometric objects, probabilities of events, and choices in a decision-making problem. Choose appropriate units or scales and make approximate or exact measurements by successive approximation or application of formal rules.
>
> MODEL - Construct, make inferences from, and interpret concrete, symbolic, graphic, verbal, and algorithmic models of quantitative, visual, statistical, probabilistic, and algebraic relations in problem situations. Translate information from one model form to another.
>
> REASON - Bring to any problem situation the disposition and ability to observe, experiment, analyze, abstract, induce, deduce, extend, generalize, relate, manipulate, and prove interesting and important patterns.
>
> PLAY - Have the disposition and imagination to inquire, investigate, tinker, dream, conjecture, invent, and communicate with others about mathematical ideas.
>
> USE TOOLS - Select and use intelligently calculators, computers, drawing tools, and physical models to represent, simulate, and manipulate patterns and relations in problem settings (pp. 5-6).

Counting, visualizing, comparing, measuring, modelling, reasoning, playing, and using tools are all mathematical processes that are used in problem solving. By developing and connecting these processes in the context of rich problem situations, the CMP intends to provide students with opportunities to learn mathematics as problem solving and build problem solving skills (see Lester & Kroll, 1990; NCTM, 1989; Schoenfeld, 1985a; Silver, 1987).

To conceptualize mathematics as problem solving, the CMP materials are organized around five instructional themes:

> <u>Content and Processes</u> -- The curriculum will be organized around a selected number of important mathematical content and process goals, each to be studied in depth.
>
> <u>Connections</u> -- The curriculum will emphasize significant connections among various mathematical topics that are presented and between mathematics and problems in disciplines that are meaningful to students.
>
> <u>Mathematical Investigations</u> -- Instruction will emphasize inquiry and discovery of mathematical ideas through investigation of structurally rich problem situations.
>
> <u>Representations</u> -- Students will grow in their ability to reason effectively with information represented in graphic numeric, symbolic, and verbal forms and move flexibly among these representations.
>
> <u>Technology</u> -- Selection of mathematical goals and teaching approaches will reflect the information processing capabilities of calculators and computers and the fundamental changes such tools are making in the ways people learn mathematics and apply their knowledge to problem solving tasks (Fitzgerald et al., 1991, p. 3).

These instructional themes are compatible with criteria for teaching and learning mathematics as problem solving as described by the NCTM in the *Professional Standards for Teaching Mathematics* (NCTM, 1991) and in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989):

> ...students should encounter, develop, and use mathematical ideas and skills in the context of genuine problems and situations...they [students] should develop the ability to use a variety of resources and tools, such as calculators

> and computers and concrete, pictorial, and metaphysical models (NCTM, 1991, p. 20).

> ...persistent attention to recognizing and drawing connections among topics will instill in students an expectation that the ideas they learn are useful in solving other problems [outside the mathematics class] and exploring other mathematical ideas... . Curriculum materials can foster an attitude in students that will encourage them to look for connections, but teachers must also look for opportunities to help students make mathematical connections (NCTM, 1989, p. 85).

> The tasks in which students engage must encourage them to reason about mathematical ideas, to make connections, and to formulate, grapple with, and solve problems (NCTM, 1991, p. 32).

The vision of mathematics teaching that is advocated by the CMP is not that teachers "give" students information. Rather, the CMP curriculum is intended to be used with teaching that engages "...students' interest and intellect while providing opportunities to deepen understanding of the mathematics being studied..." (Fitzgerald et al., 1991, p. 12). Since a pedagogy that is committed to teaching mathematics as problem solving is likely to be new to many teachers (see NCTM, 1991), the CMP curricular materials are intended to serve as a lever for teacher change:

> In order to help teachers make the kinds of changes in instructional thinking and planning implied by the goals of *Connected Mathematics*, the materials developed will take seriously the need to provide instructional strategies and organizational help for teachers so that they can develop new modes and habits of instruction (Fitzgerald et al., 1991, p. 13).

The CMP curriculum is intended to help middle school teachers implement problem-solving oriented mathematics teaching.

Problem solving in the CMP curriculum means engaging with and exploring mathematically rich problem situations. The developers describe how students learn mathematics through problem solving processes:

> Starting with a problem situation, students are encouraged to observe patterns, generalize, extend, connect, reflect on their solutions, evaluate, record, and communicate their findings verbally and in writing. Concepts and skills evolve from the explorations of problems (Fitzgerald et al., 1991, p. 13)

The developers do not view problem solving as a set of strategies (e.g., "guess and check", "working backwards", "draw a diagram") that should be presented to, and then practiced by, middle school students. For example, in the *Covering and Surrounding* teacher's edition, the developers note:

> We expect that by the end of this unit students will have gained insight and skill in the following aspects of problem solving involving measurement:
>
> 1) Recognizing situations in which measuring perimeter or area will produce answers to practical problems.
>
> 2) Finding perimeters and areas of regular and irregular figures by using transparent grids, tiles, or other objects to cover the figures and string, straight line segments, rulers, or other objects to surround the figures.
>
> 3) Cutting and rearranging parts of figures to see relationships between kinds of figures, in particular, parallelograms, triangle and rectangles. Then devising strategies for finding areas by using the relationships observed.
>
> 4) Observing patterns in data by organizing tables and graphs to represent the data.
>
> 5) Using multiple representations to understand situations, in particular, physical models, tables, graphs, symbolic and verbal descriptions (CMP, 1992b, pp. 2-3).

In the CMP curriculum, problem solving is represented, in a global sense, as the process of learning and doing mathematics. The *Covering and Surrounding* problem goals also emphasize that the developers view certain kinds of tools (e.g., tiles, grids) and processes (e.g., "cutting and rearranging parts of figures") as especially appropriate to learning about perimeter and area concepts. The *Covering and Surrounding* problem-solving goals represent the developers' pedagogical position that, "It is crucial that we educate our students to think in terms of the 'process' of doing mathematics as much as they do about the 'product' or solution" (Fitzgerald et al., 1991, p. 16).

Problem-Solving Activity in Classroom Settings

New Roles for Students. With its focus on learning mathematics as problem solving, the CMP curriculum implies new roles for students. Students' roles are not oriented around performing numerous and tedious rote computations. Students learn by engaging in mathematical problem solving -- investigating, conducting experiments, modelling, reasoning, measuring, making connections in a variety of problem situations. In *Covering and Surrounding* students will study perimeter and area by taking on the role of an architectural consultant who has to figure out room designs to meet the specific needs of clients. In this scenario, students use tiles to measure and create floor plans subject to various constraints (e.g., a client must have an area of at least 16 square units and wants lots of windows). Students are expected to discuss their designs with the class, establish criteria by which to compare the appropriateness of different designs, and offer explanations for how they would (or would not) alter their designs given different constraints and conditions (see CMP, 1992a).

The CMP envisions students as interactive participants in classrooms. Students are expected to formulate and determine the validity of conjectures, develop mathematical relationships, discuss their ideas about the mathematics in question with peers and teachers in written and oral form (see CMP, 1992a, 1992b; Fitzgerald et al., 1991). This is congruent with the NCTM's (1991) portrayal of students' role in mathematical discourse as outlined in Standard 3 of the *Professional Standards for Teaching Mathematics*:

> *Standard 3:*
>
> *Students' Role in Discourse*
>
> The teacher of mathematics should promote classroom discourse in which students -
>
> • listen to, respond to, and question the teacher and one another;
>
> • use a variety of tools to reason, make connections, solve problems, and communicate;
>
> • initiate problems and questions;
>
> • make conjectures and present solutions;
>
> • explore examples and counterexamples to investigate a conjecture;
>
> • try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
>
> • rely on mathematical evidence and argument to determine validity (p. 45).

<u>New Roles for Teachers</u>. But making changes in how students learn mathematics means helping teachers think about how they teach mathematics and how they might change and develop their teaching practices (Charles, 1989; Cohen, 1988; A. Thompson, 1989; Wilcox et al., 1991, 1992). For students to learn mathematics as problem solving, teachers must take an interactive rather than a transmitive stance in their classrooms (e.g., Lampert, 1990; Schoenfeld, 1985b). The teacher's role is to guide students' learning of mathematics, not tell students about mathematics. The teacher's role should include helping students articulate and refine their own ideas, and revise prior conceptions in light of new findings and insights (Ball, 1990a; Lampert, 1990). Researchers and reformers agree that teaching mathematics should

not be oriented around showing and telling students how to do mathematical procedures (e.g., teaching division by working many long division problems for students, and having students do the same to master the algorithm), but should provide frequent opportunities for students to develop, refine, and revise their understandings of mathematics as they explore problem situations (Cobb, 1989; NCTM, 1991; Noddings, 1990; Von Glaserfeld, 1983).

The CMP views the teacher as an interactive instructor who will encourage and foster mathematical discourse around important mathematical ideas, concepts, and processes and the connections among them (Fitzgerald et al., 1991). The NCTM (1991) describes *discourse* in the mathematics classroom:

> *Discourse* refers to the ways of representing, thinking, talking, and agreeing and disagreeing that teachers and students use to engage in ... tasks [e.g., projects, questions, problems, applications]. The discourse embeds fundamental values about knowledge and authority (p. 20).

The developers of the CMP curriculum see the role of the teacher as a facilitator of discourse and problem solving in the mathematics classroom:

> The image of mathematics teaching to which the team of *Connected Mathematics* is committed parallels that put forward by the NCTM *Curriculum and Evaluation Standards for School Mathematics* and the NCTM *Professional Teaching Standards*. We envision teachers guiding individual, small group and whole group activities; selecting mathematics tasks to engage students' interest and intellect while providing opportunities to deepen understanding of the mathematics being studied; using tools and teaching students to use tools such as calculators, computers, and manipulatives to explore mathematics; and orchestrating mathematical ideas while deliberately seeking connections to previous knowledge. This is in sharp contrast to teachers *giving* students information (Fitzgerald et al., p. 12, emphasis in original).

The vision the CMP developers have of the teacher's role in the curriculum is congruent with the teacher's role in mathematical classroom discourse as described by the NCTM (1991) in Standard 2 of the *Professional Standards for Teaching Mathematics*:

> *Standard 2:*
>
> *The Teacher's Role in Discourse*
>
> The teacher of mathematics should orchestrate discourse by -
>
> • posing questions and tasks that elicit, engage, and challenge each student's thinking;
>
> • listening carefully to students' ideas;
>
> • asking students to clarify and justify their ideas orally and in writing;
>
> • deciding what to pursue in depth from among the ideas that students bring up during discussion;
>
> • deciding when and how to attach mathematical notation and language to students' ideas;
>
> • deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
>
> • monitoring students' participation in discussions and deciding when and how to encourage each student to participate (p. 35).

In *Covering and Surrounding*, students are given the task of designing a room using 1-inch tiles (CMP, 1992a, p. 2):

> **Problem 1:** Mrs. Hide likes the amount of floor space in Mr. Dull's design, but she wants more windows and a more interesting shape for her room. She asks you to help her out.
>
> a) Use 12 square tiles to create a floor plan design for Mrs. Hide. Remember that a window or a door can go into each section of wall space. Mrs. Hide tells you that she wants at least 14 sections of wall space, including windows and doors, in the room you are designing.
>
> b) After you have designed the floor plan for the room make a drawing to show the location of the door and where each window is. Write a paragraph to tell Mrs. Hide why your design is better than Mr. Dull's.

The teacher's edition (CMP, 1992b) stresses that this problem is intended to help students visualize perimeter and area and develop techniques for finding these measures (e.g., counting tiles to find area, counting tile edges to find perimeter):

> It is important that students get a clear picture of the relationship between the physical tiles representing squares of carpet [i.e., area] and the prefabricated wall sections [i.e., perimeter] that are the same width as the edge of a carpet tile (p. 23).

The teacher's edition also specifies problem-solving goals that the designing rooms problem addresses:

> • Developing *awareness* of the differences between area and perimeter.
>
> • Developing concept images (i.e., mental images) that help distinguish between area and perimeter.
>
> • Finding area and perimeter through covering with tiles and counting edges and numbers of tiles (CMP, 1992b, p. 23, emphasis in original).

In this problem, as the NCTM (1991) notes:

> One aspect of the teacher's role is to provoke students' reasoning about mathematics. Teachers must do this through the tasks they provide and the questions they ask. For example, teachers should regularly follow students' statements with, "Why?" or by asking them to explain (p. 35).

In the *Covering and Surrounding* unit, and in the CMP materials in general, the intent is for teachers and students to work together to make sense of mathematical concepts and ideas. The "transmission model" (Jackson, 1986) of teaching -- teachers telling students facts and demonstrating procedures, students absorbing facts and mechanically recalling/practicing the facts/procedures -- is explicitly discouraged in the *Covering and Surrounding* teacher's edition (CMP, 1992b):

> The student materials have been written to support investigative classwork. The expectation is that the investigation will be launched by the teacher and the class working together followed by students working in pairs or

> small groups to make sense of the problem. Then the class returns to whole class format for a class summary and discussion of the findings. The best student learning will occur if students are allowed to do some exploratory work on their own, to discover strategies, and to share their findings with their class, *not* if the teacher launches each class with an explanation of a rigid procedure to do the forthcoming problems (emphasis in original, p.5).

The intent in *Covering and Surrounding* is for the teacher to guide students' learning as they explore rich mathematical problems. The intended roles of students and teachers in the CMP curriculum echoes the position of the NCTM (1991):

> Instead of doing virtually all the talking, modeling, and explaining themselves, teachers must encourage students to do so. Teachers must do more listening, students more reasoning (p. 36).

Subject-Matter Knowledge

The developers of the CMP materials take the position of the NCTM (1991) that

> Knowledge of both the content and discourse of mathematics is an essential component of teachers' preparation for the profession. Teachers' comfort with, and confidence in, their own knowledge of mathematics affects both what they teach and how they teach it. Their conceptions of mathematics shape their choice of worthwhile mathematical tasks, the kinds of learning environments they create, and the discourse in their classrooms (p. 132).

The developers' position that teachers' subject-matter knowledge shapes the entire classroom environment for learning mathematics is why the CMP teacher materials provide suggestions for teaching. While teachers' subject-matter knowledge of mathematics varies (Ball & McDiarmid, 1990), the developers assume that the mathematics and teaching pedagogy required to use the CMP curriculum as intended will be new to many teachers (Fitzgerald et al., 1991). Therefore, as well as to provide answers and solution strategies, the CMP teacher materials are designed, for example, to help the teacher ask students questions to enrich their understanding and to "help the teacher make much better judgements about what mathematics students understand and can use" (Fitzgerald et al., 1991, p. 15).

But no curriculum can provide teachers with teaching suggestions for every classroom situation. Inherent in teaching is managing the myriad dilemmas and uncertainties of practice (Ball, 1990a; Cohen, 1988; Cohen & Ball, 1990; Jackson, 1986). Teachers address dilemmas and uncertainties arising in practice by making instructional decisions based on knowledge and beliefs, the subject matter being taught, and the characteristics of the learners and the instructional context (Jackson, 1986; Lortie, 1975; Wilcox et al., 1991, 1992). Therefore, while the CMP teacher materials are intended to provide assistance in using the curriculum, teachers will necessarily need to draw on their own subject-matter knowledge of mathematics as well as other knowledge and beliefs (see Chapter 2).

Perceptions and Beliefs About Student Learning

Paralleling calls for reforming teaching and learning in schools (e.g., The Holmes Group, 1990; NCTM, 1989, 1991), the CMP developers intend the curriculum to teach powerful mathematics to *all* students (Fitzgerald et al., 1991). To accomplish this goal, the developers craft interesting and appealing instructional situations around topics which are relevant to all middle school students:

> The topics chosen [in the CMP curriculum] provide intriguing and thought-provoking mathematics for all students, and the open-ended problem explorations through which we plan to develop those topics make such a 'mathematics for all' curriculum feasible (Fitzgerald et al., 1991, p 6).

By embedding important mathematical concepts and processes in rich problem situations and contexts that are engaging for middle school students, the developers intend to make mathematics interesting and accessible to *all* students.

As described earlier in this chapter, the developers believe that students learn mathematics by investigating rich problem situations. The view of student learning taken by the developers can be summarized as going through three stages (see Fitzgerald et al., 1991; Lappan, 1983; Shroyer & Fitzgerald, 1986):

- Students engage with a mathematically rich problem.
- As students explore the problem they develop ideas about mathematical concepts and formulate problem-solving strategies.
- Students refine their understandings of the mathematics and problem-solving strategies embedded in the problem by comparing their ideas with others and reflecting on and testing their own ideas.

This conception of how students learn mathematics is congruent with the developers' view that students are active participants in their own learning and that the teacher's role is to guide students as they explore the mathematical terrain of the problem.

Covering and Surrounding

Overview and Description of the Unit

The CMP unit *Covering and Surrounding* (CMP, 1992a, 1992b) is the second of three sequenced geometry units that together constitute the geometry strand of the sixth-grade CMP curriculum[2]. The unit focuses on measurement concepts, specifically perimeter and area, and the relationships between them. As the title of the unit suggests, area is represented as a measure of the number of the square units needed to

[2]The *Covering and Surrounding* student edition (CMP, 1992a) and teacher edition (CMP, 1992b) used in this study are both working drafts of the unit materials. The CMP expects that *Covering and Surrounding*, along with other developed materials, will be revised before final publication (see Fitzgerald, et.al., 1991).

cover a shape; perimeter is a measure of the length or distance needed to go around or *surround* a shape. Maxima and minima are subthemes of the unit. The unit provides problem-solving situations where perimeter and area, and the relationships between them, are used to investigate questions of "What's the biggest?" and "What's the smallest?". Minimizing costs and materials, maximizing area and distances, formulating arguments and recommendations, and estimating measurements are some of the different kinds of activities students engage in.

Covering and Surrounding is divided into seven major activities called *Investigations* (see CMP, 1992a). Each of the seven investigations in the unit begins with one or more problem situations. Labelled *Problems*, they are intended to engage students with mathematical concepts and processes. Problems in *Covering and Surrounding* are frequently supplemented by *Follow-Up Questions*. Follow-up questions are intended to provide additional problem-solving situations to help students refine their understandings about the major concepts and processes embedded in the Problem. These opening activities in each investigation introduce new content and are structured to include class discussion in which students would share ideas and conjectures (see CMP, 1992a, 1992b; see Appendix D for a complete investigation from *Covering and Surrounding*).

Following the problems and follow-up questions is a section called "Applications-Connections-Extensions", or ACE. This section consists of supplementary problems which are divided into three sub-sections -- Application problems, Connections problems, and Extension problems. The teacher's edition explains the purpose of each kind of problem (CMP, 1992b):

> Each of the seven investigations in *Covering and Surrounding* ends with *Applications*, *Connections*, and *Extensions*. Application problems are intended to help students apply what was just learned in the investigation. Connection problems link the current investigation's topics to other mathematical ideas (e.g., to other topics in the same or another unit). Extension problems are challenging exercises that require students to apply the investigation concepts in ways not explicitly covered in the investigation. While the ACE section of an investigation may be a source of homework problems, some problems from this section will warrant classroom time (emphases in original, p. 10).

The table below summarizes the *Covering and Surrounding* unit, including sample Problems and ACE items from each Investigation:

Table 4.1: **Summary of *Covering and Surrounding*** (CMP, 1992a):

Investigation [and representations used]	Central Concept(s) and/or Process(es)	Sample Problem	Sample ACE Item
1 - Measuring and Designing Rooms [1-inch square tiles, grid paper, outlines of rooms that are measured with the tiles]	Understanding & distinguishing between area and perimeter.	"Use 12 square tiles to make a floor plan design for Mrs. Hide. .. Mrs. Hide tells you she wants at least 14 sections of wall space" (p. 2).	"If you walk around the perimeter of a rectangle, through how many degrees have you turned when you get back to where you have started" (p. 12)?
2 - Areas and Perimeters of Figures with Irregular Edges [grid paper and transparent grids]	Developing techniques to estimate area and perimeter.	Students use a grid to estimate the perimeter and area of a marsh in 1985 and again in 1990 (p. 13).	Students outline their shoe on grid paper and find the area of their shoeprint (p. 18).
3 - Going Around in Circles [grid paper, transparent grids, 3-D models]	Estimating the area and circumference of circles; developing an understanding of π.	"Find an estimate for the area of the circle... that .. is *smaller* than the actual area" (p. 21).	Students are asked to locate and describe a circular object in their neighborhood and describe it and its use (p. 28).
4 - Constant Area, Varying Perimeters [1-inch tiles, graphs, grid paper]	Varying the perimeter of rectangles while holding perimeter constant to study relationships between area and perimeter.	Students find all the rectangular rooms that can be made with 24 square tiles and recommend one as the best for office space (p. 31).	Students use 20 tiles to build the rectangles with the largest and smallest perimeter (p. 34).
5 - Constant Perimeter, Varying Area [1-inch tiles, graphs, grid paper]	Extending understandings about relationships between area and perimeter by holding perimeter of rectangular figures constant and varying area.	"Working in your group, find all rectangles that you can make with square tiles that have a perimeter of 18 units" (p. 38).	"Hilda made a rectangle from square unit tiles. It has an area of 16 square units and a perimeter of 16 units. Draw a picture of Hilda's rectangle" (p. 40).
6 - Finding Relationships: Connecting Parallelograms, Triangles, and Rectangles [grid paper, transparent grids, models]	Using knowledge about rectangles to develop ways of finding area and perimeter of triangles & parallelograms.	"Can you find a way to make 1 cut through a parallelogram so that the 2 pieces can always be reassembled into a rectangle" (p. 42)?	"If you know that a rectangle measures 12 cm by 15 cm, how can you use this information to find the area of the rectangle" (p. 47)?
7 - The Bee Problem Revisited [transparent grids, pattern blocks, 1-inch tiles]	Extending area and perimeter concepts to other shapes and developing and verifying conjectures.	"Why do bees make hives from hexagons rather than other tessellating shapes like triangles or squares" (p. 54)?	"As we increase the sides of a regular polygon, what figure does the polygon seem to become" (p. 58)?

The teacher's edition (CMP, 1992b) provides suggestions for teaching as well as answers for the problems, follow-up questions, and ACE items in the student materials. For example, as noted in Table 4.1, Investigation 6 uses understandings of area and perimeter of rectangles to develop strategies for finding area and perimeter of triangles and parallelograms. The teacher's edition (CMP, 1992b) offers this advice to teachers:

> Try not to *tell* students area formulas (e.g., $\text{Area}_{\text{triangle}} = 1/2 \times \text{base} \times \text{height}$) as the investigation is structured to help students develop these or equivalent algorithms. As students work through the problems of finding the relationships between rectangles and parallelograms and parallelograms and triangles, you might instead ask them questions about how they are dividing up their parallelograms or rectangles. For example, when a student discovers that when she cuts a parallelogram along one of the diagonals two congruent triangles are formed, you might ask her how the area of the triangles relates to the area of the original parallelogram. You might ask her further, then, to think about how the area of a triangle could be expressed in terms of the area of the original parallelogram. These kinds of questions help students develop their own problem solving skills and understanding of perimeter and area rather than just learning a rule (emphasis in original, p. 64).

The teacher edition is meant to help the teacher, not to direct teacher action (CMP, 1992b; Fitzgerald et al., 1991)[3]. The developers intend the teacher materials to "provide sufficient help that a well-motivated teacher can teach the materials with little or no inservice" (Fitzgerald et al., 1991, p. 15). The teacher materials are designed to help the teacher in her classroom instruction by highlighting "subtle points of the mathematics and to help the teacher become more reflective and curious about student learning" (Fitzgerald et al., 1991, p. 23).

---

[3]Appendix D provides a complete Investigation from *Covering and Surrounding*, and Appendix E provides the corresponding material from the teacher's edition. Appendices D and E may be consulted to provide an example of the support the teacher edition provides to the teacher in teaching *Covering and Surrounding*.

Problems and Problem Solving in *Covering and Surrounding*

As I argued earlier, teaching and learning problem solving in classroom settings is not well-defined, nor is it clear what kinds of problems are most appropriate for helping students learn about mathematics and problem solving (Charles & Lester, 1982; Grouws, 1985; Kilpatrick, 1985; Stanic & Kilpatrick, 1989). Yet, as reformers emphasize, it is important to carefully consider the kinds of problems students have opportunities to work with, as problems are bridges to mathematical learning goals that go beyond getting the "right answer":

> ... problems that are used in school instruction should be means to more ambitious educational goals, rather than being the goals themselves. We hope that students will be able to reason effectively about mathematical concepts and principles and about the events and systems that they encounter in their lives (Greeno, 1991, p. 75).

Problem solving is one of the "ambitious educational goals" the CMP developers have for students. Problems and problem situations in the CMP curriculum are designed with the intent of helping students learn to reason and work *with* mathematical concepts as well as learning *about* mathematical concepts (CMP, 1992b; Fitzgerald et al., 1991).

But conceptualizing "What is a problem?" and "What is problem solving?" in the *Covering and Surrounding* unit is not a clear-cut task, despite the mathematical content and process goals the CMP curriculum sets for students (see Fitzgerald et al., 1991). While Polya (1967, 1968) and Charles and Lester (1982), for example, have offered "definitions" of what a problem is (see Chapter 2) and heuristic characterizations of problem solving, there is no single means for defining problems and problem solving in *Covering and Surrounding*. Part of the difficulty is because *Covering and Surrounding* arguably includes a variety of *different kinds* of problems. Consider this classification scheme utilized by Hembree (1992):

> ***Standard*** (word or story) problems require the translation of verbal statements into mathematical operations. ***Nonstandard*** (process or open-search) problems encourage the use of flexible methods; the solver possesses no routine procedure for finding an answer. ***Real-world*** problems entail situations where students will need to select and apply the tools of mathematics at their discretion. ***Puzzles*** depend on luck or guessing or a use of unusual strategies toward their solution (p. 249, emphases in original).

The following problems from the *Covering and Surrounding* unit (CMP, 1992a) seem to fit into this framework:

Standard (word or story) Problem

> A rectangle measures 8 cm by 10.5 cm. What is the area? (p. 47).

Nonstandard (process or open-search) Problem

> Using 16 tiles, build the floor plan for a rectangular building with the greatest perimeter. Now rearrange the tiles to build the floor plan with the smallest perimeter. Write a sentence to describe each of the rectangles (p. 34).

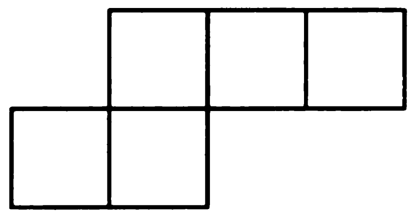
Real-world Problem

> Investigate where measurement is used in the world around you. To do this, you can choose to do one of the following assignments:
>
> a) Interview an adult about how he or she uses measurement on the job. For each kind of measurement that you find, ask how the measurement is made. What units are used? What kinds of instruments are used to make the measurements?
>
> b) Look carefully at a newspaper or magazine and find uses of measurement. You might find examples of measurement in advertisements or articles. Be sure to record what units are used to make the measurements and what the measurements are used for (pp. 19-20).

Puzzle Problem

Figure 4.1: The pentomino problem:

Problem 1: Can You Make 18?
Construct the following pentomino with your square tiles:



Find a way to add tiles to this figure to create a figure with a perimeter of 18 units. When adding tiles to any pentomino, remember that the tiles must touch edge to edge (p. 36):

It is important to also consider the possibility that some kinds of problems may *not* be incorporated into *Covering and Surrounding*. For example, Charles and Lester (1982) include "Drill Exercise" in their categorization of problems as one kind of problem, which they illustrate with the example

$$\begin{array}{r} 346 \\ \underline{\times\ 28} \end{array} \quad \text{(p. 6).}$$

There are no "naked number" drill exercises of the kind above included in the *Covering and Surrounding* unit.

The drill exercise also illustrates a semantic difficulty in trying to determine "What is a problem?" in curricular materials. While some educators and teachers might think of a drill exercise as a legitimate problem, others, such as Greeno (1991) suggest that, depending on the solver, a drill exercise might not be a problem:

> mathematicians regularly work on problems that take many hours -- indeed, if a question can be answered by mechanically applying a known procedure, mathematicians do not think it is a problem at all (p. 83).

Greeno's (1991) remarks address an important issue with respect to teachers' use of problem-solving oriented materials. The conception of problem solving a teacher brings to her use of problem-solving oriented curricula may influence how she perceives problem solving being addressed by the materials. For example, if problem solving is conceived as solving drill exercises, then *Covering and Surrounding* could be perceived as being devoid of problem solving!

Looking Ahead: Teaching Covering and Surrounding

The developers of the CMP curriculum intend *Covering and Surrounding* to engage students with rich, connected mathematics via problem solving. However, university based curriculum developers and practicing teachers can approach mathematics curricula from different perspectives. For example, curriculum developers may intend curricula to develop students' understanding of problem solving and mathematical concepts, while classroom teachers may focus on using the curricula to make mathematics more enjoyable for their students and to help them develop positive attitudes about mathematics (Prawat et al., 1991). This can contribute to teachers emphasizing different content and processes (e.g., computation instead of problem solving) in their use of curricula than those intended by the curriculum developers (e.g., Wilson, 1990).

The next two chapters present the cases of how two different teachers used the *Covering and Surrounding* unit in their classrooms. In each of the cases, a profile of the teacher participant is provided and is followed by an assessment of her knowledge, views, and beliefs in each of domains of the framework developed in Chapter 2. Each teacher's views, knowledge, and beliefs is then used to conceptualize her use of

Covering and Surrounding described in the subsequent section. For each teacher participant, her use of *Covering and Surrounding* is unpacked to address the main research question, "How do teachers use a problem-solving oriented piece of curriculum in their classrooms", and identify issues connected to the main research question.

CHAPTER 5

THE CASE OF KAREN KNIGHT

A Profile of Karen Knight

With her students watching intently, Karen Knight writes "DIVISIBILITY LAWS" on the overhead transparency. She then writes the number 246 and addresses the class:

Karen: Let's review our discussion from yesterday. Is this number divisible by 2?

Marcus[1]: Yea!

Karen: And how do you *know* that this number is divisible by 2?

Marcus: Because each digit can be divided by 2.

Karen: No -- not quite. Think back to what we talked about yesterday.

Alicia: Because the six can be divided by 2?

Karen: Good! You need to remember this [underlines the 6 in 24<u>6</u>] -- a number is divisible by 2 if the *ones* digit is divisible by 2. Now think about some of the other divisibility laws we used yesterday -- what else is this number [points to 246] divisible by?

Ellen: Three!

Karen: And how do you know that?

Ellen: Because the numbers add up to 12!

Karen: Okay -- but *why* does that mean that the number is divisible by 3?

Eric: Because you can divide a number by three if, like, when you add the digits together that number divides by 3.

Karen: Good! Remember, a number is divisible by three if the sum of its digits is divisible by 3.

---

[1]All students' names are pseudonyms. In selecting pseudonyms, I have attempted to represent each student's ethnicity to provide a sense of the diversity of Karen's classroom.

Karen continued the lesson by stating new divisibility laws for 5 and 10. "A number is divisible by 5 if it ends in a 5 or 0," Karen declared. She also emphasized that "A number is divisible by 10 if it ends in a 0." Karen wrote both of these divisibility laws on the overhead and then illustrated how the number 350 is divisible by both 5 and 10 because "it ends in a zero."

Having completed examples illustrating the divisibility laws for 2, 3, 5, and 10, Karen put the number 11930 on the overhead. Addressing her class again she said, "Copy this number down and test it for divisibility by 2, 3, 5, and 10. Now, I don't want 'yes' or 'no' answers either -- you need to explain *why* it is or isn't divisible." Circulating around the room, Karen surveyed her students as they worked silently on the task. She returned to the front of the room after letting the students work for about two-minutes and said, "Okay, what did you find out?" Four different students gave the correct responses for divisibility of 11930 by 2, 3, 5, and 10, explaining their reasoning by reciting or paraphrasing the appropriate divisibility law. Just before the bell sounded, however, Trevor raised his hand and was called on by Karen:

Trevor: I think that you could say 11930 is divisible by 5 because it can be divided by 10.

Karen: Could you explain that again Trevor?

Trevor: Well, if a number is divided by 10 it has to be divisible by 5 too because fives divide tens.

Karen: Very interesting observation Trevor! Did everyone hear that? [nods from the class] Since 5 divides 10, a number that is divisible by 10 has to be divisible by 5 also. Very good Trevor!

Trevor, obviously pleased at the acknowledgement of his contribution, smiled just before the bell rang. Karen dismissed her students by rows after quickly giving a short assignment from the textbook on testing numbers for divisibility.

This excerpt from Karen Knight's 10/14/92 lesson on divisibility laws occurred about three weeks before she began teaching *Covering and Surrounding*. As in the other lessons I observed her teach prior to *Covering and Surrounding*, Karen kept her students focused on the mathematics topic at hand and insisted on participation and correct answers from her class. Karen demands that her students be able to explain how they arrived at their solutions to exercises and encourages them to share their ideas about mathematics. An organized teacher and a strong leader in the classroom, Karen emphasizes practice and proficiency in her mathematics teaching, assigning her students homework several times each week. Karen describes herself as focusing on teaching her students about mathematical procedures, formulas, and how to use them.

The following week I observed Karen leading her students in an analysis of the "Factor Game", an activity from a teacher resource book called *Factors and Multiples*[2]. The activity involved students finding all of the proper factors of the numbers 2 through 30 (i.e., all the factors including 1 but excluding the number itself) and computing the sum of these numbers. Students organized their work on an activity sheet Karen had duplicated from the resource book. After allowing her students some time to work on the task in pairs, Karen placed a transparency of the activity sheet on the overhead that had the answers

---

[2] *Factors and Multiples* (Fitzgerald et. al., 1986) is a unit produced by the Middle Grades Mathematics Project which was a curriculum development project that some of the CMP developers were involved in. The unit features a non-standard teaching guide which provides detailed suggestions for teaching as well as answers. The topics of the unit include factors, multiples, primes, and composite numbers.

filled-in but concealed the answers with a sheet of paper. Karen then had students volunteer answers (i.e., the proper factors and their sum for each number 2 through 30), revealing each answer after it was given to check it. Classtime ran out before the entire sheet was checked and Karen told the class that they would finish checking the answers the next day.

After class I asked Karen about how she liked the *Factors and Multiples* unit. She replied, "I like to use it near the beginning of the year like this because it's good review of number facts and the kids like it." I asked Karen about the suggestions for teaching provided by the unit and how useful she thought they were. After pausing to think about this, she said, "I guess I don't use the teaching suggestions that much because I've taught the unit before. I mainly use it for answers, like I did today to let the students check their work." I then asked Karen about how useful she found her textbook teacher edition compared to *Factors and Multiples*, and she said that

> Well, I usually let the students use the teacher text to check their answers when we work from the text, or I use it to check their answers if I'm grading their assignment.

Karen also noted that the textbook teacher edition "really doesn't have much in it except answers."

Karen's lesson on divisibility laws and her comments about her use of the *Factors and Multiples* unit and textbook teacher edition foreshadow two important features of her practice: (1) Karen's usual mode of teaching mathematics is to focus on mathematical rules, procedures, formulas, and how to use them -- Karen describes this as "abstract teaching"; (2) Karen's use of teacher materials centers on obtaining answers to allow students to check their own work or for Karen herself to check their work.

A teacher with over 25 years of teaching experience, Karen has taught first, third, fourth, and sixth grades in a variety of school settings and has held teaching positions in Alabama, New York, Louisiana, and Michigan. She has taught a wide range of subjects over her career, including mathematics, social studies, music, aerobic dancing, and computer programming. In teaching mathematics, Karen says that she finds working with numbers, making and using graphs and diagrams, and using the language of mathematics to be particularly interesting.

Karen teaches in a large middle school which enrolls about 1,350 students in grades six, seven, and eight. Karen taught *Covering and Surrounding* in her Enriched Mathematics class -- about 25% of the sixth-graders in her school are enrolled in Enriched Mathematics. In most cases, students are placed in Enriched Mathematics by scoring in the top 25% on standardized achievement tests in mathematics near the end of the fifth-grade. The 75% of sixth-graders who are not enrolled in Enriched Mathematics are placed in General Mathematics. According to Karen, the main difference between sixth-grade General Mathematics and Enriched Mathematics is that the Enriched sections use a seventh-grade textbook.

The majority of students enrolled in Karen's school are from working-class socioeconomic backgrounds. For example, some of the students' parents work in heavy industry (e.g., automobile manufacturing). About half of the students at Karen's school qualify for the district's reduced-cost lunch program, and some students live in areas of their community where low-cost housing is provided and crime and drugs are problems. However, the students in Enriched Mathematics do not necessarily share the socioeconomic background of most of the students in the school. Karen commented to me that one of the reasons students in her Enriched Mathematics class tend to be innovative in their thinking is because "Their parents, being professionals, they teach their kids a little bit more than the regular parent who doesn't have the knowledge."

Karen and the Domains of the Framework

During the summer and early fall prior to teaching *Covering and Surrounding*, I interviewed Karen to learn about her views, knowledge, and beliefs within the four domains of the analytic framework developed in Chapter 2 (see Appendices A, B, and C for interview protocol and items). In this section, I explore key aspects of Karen's views, knowledge, and beliefs about mathematics and problem solving, problem-solving activity in classrooms, subject-matter knowledge, and perceptions and beliefs about student learning. Karen's views, knowledge, and beliefs identified in this section will be utilized throughout this chapter to understand her use of *Covering and Surrounding*.

Views and Beliefs About Mathematics and Problem Solving

Problem solving is an important topic within the mathematics curriculum for Karen. Karen believes that problem solving can help her students in real life. She emphasized this to me when I asked her about how important problem solving is for middle school students:

AR: How important would you say problem solving is for middle school kids?

Karen: Oh, very important.

AR: Well, some teachers would say that at the middle school level it's most important to have kids learning their facts, making sure they can --

Karen: All of life's a problem! You see, they have to go to lunch -- that's a problem and they have to solve it. What if, for instance, mom only gives you a dollar this morning because that's all she has? You have a problem to solve to feed yourself at lunch time! It's a problem.

But although she places heavy emphasis on problem solving being a part of real life, Karen sticks closely to the textbook when teaching her students about problem solving. When I asked her about how frequently she teaches problem solving in her classroom, Karen said that she teaches problem solving "At least twice a week, or as it comes up in the textbook." Karen went on to elaborate about how problem solving is integrated into her mathematics teaching:

AR: Ideally, do you think you'd like to spend more time on problem solving, or are you doing about right?

Karen: I'm not doing it about right. We're going more and more toward problem solving, but I'm not spending that much time on it. I use problem solving if it arises in the textbook. But I want to make sure that they have the steps for solving problems, but I have yet to do a whole unit on just problem solving. So problem solving is kind of part of my lessons, but they're not lessons on problem solving.

Karen believes that she addresses problem solving in her teaching, but not to the degree that she should. Problem solving, in Karen's view, is part of her lessons because she does assign exercises from the text which she feels require her students to do problem solving to obtain a solution. Karen thinks that problem solving is part of her lessons because within her usual lessons on other topics students occasionally do problems that require problem solving. While Karen implies that she should teach problem solving more, she believes that her students are learning about "the steps for solving problems."

Because Karen teaches problem solving when "it arises in the textbook", understanding how Karen's textbook addresses problem solving is part of understanding Karen's beliefs about problem solving. Here is an example of a problem from a section labelled "Problem Solving • Applications" in Karen's textbook (Abbott & Wells, 1985) which she said is an example of the kind of problems she typically assigns her students to address problem solving:

> The diameter of Joanna's bicycle wheel is 66 centimeters. How far does the wheel travel in one revolution (p. 345)?

In terms of Hembree's (1992) framework (see Chapter 4), the above problem is a "standard (word or story) problem" that requires "the translation of verbal statements into mathematical operations" (p. 249). In the above problem, the student needs to know the circumference formula $C = 2\pi r$ and then translate the word statement into $C = 2\pi \times 66$ centimeters to get the answer.

Although she believes problem solving is important because it can help her students in real life, word problems of the kind Karen pointed-out in the textbook are generally not the same as problems in real life. Translating word problems into formulas is not the same as solving "real-world" problems "where students will need to select and apply the tools of mathematics at their discretion" (Hembree, 1992, p. 249). Translating a word problem into a formula or operation, for example, is not the same as a real-world problem because the path to the right answer may not be so obvious. Therefore, there are differences between the kind of problem solving embedded in Karen's mathematics teaching and the problem solving she believes students will need to do in real life.

Karen believes that problem solving in the mathematics curriculum is important because it is relevant to real life. However, her views about teaching problem solving are centered on the textbook and solving standard (word or story) problems. While Karen's reasons for *why* problem solving is important for her students are connected to solving real-world problems, her *portrayal* of problem solving in the classroom is focused on solving routine word problems. This comparison shows that how Karen addresses problem solving in her classroom varies from her beliefs about problem solving. Her use of the textbook seems to be at the root of this variance. The "problem solving" sections of Karen's textbook are predominantly sets of standard word problems. Moreover, she believes that problem solving is important for real life, yet also teaches problem solving as it occurs in the text. The product of the interplay between Karen's beliefs and the text is that she maintains her *beliefs* about problem solving while her textbook-oriented *teaching* of problem solving focuses on standard word problems. This interaction creates differences between Karen's beliefs about problem solving and how it is enacted in her teaching.

<u>Problem-Solving Activity in Classroom Settings</u>

AR: What kinds of activities would you say your students do when they are working on problem solving?

Karen: Basically, problems in everyday life. I remember last week we did one with two boys with summer jobs, and we compared how much one had made over the other, and then one went shopping and who had more saved. The guy who had the more saved was the one who made less money. The guy who made more money, I guess he felt free to go shopping. So we made that comparison, but it wasn't a unit, it was just a page in that book. Its got two pages and one page talks about steps of problem solving and then they give them problems to solve. We just solved it and went through there and found out how you would solve it, what steps you would use, eventually, before school is out in June, I will be

> assigning them some problem solving, some more problems to solve.

Karen describes in the above excerpt that students in her class are doing problem solving when they are solving word problems in the text. That Karen conceptualizes problem-solving activity in her classroom as solving textbook word problems is reinforced by her views about the challenges of teaching problem solving. Karen said that "I find the biggest problem with problem solving is reading and understanding the language that the textbook is about." Karen's description of problem-solving activity as solving word problems from the textbook is consistent with how she describes herself teaching problem solving (i.e., teaching problem solving when it arises in the text).

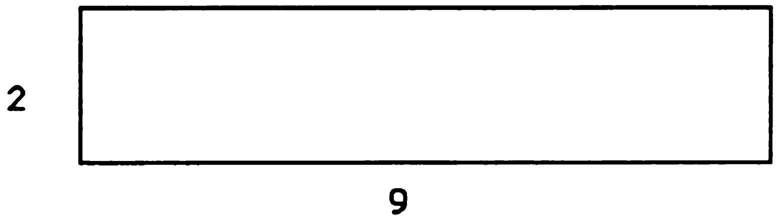
Karen and I also talked about what students *do* when they are doing problem solving in her classroom. Karen described that, in her view, problem solving involves reading a problem "from the textbook", figuring out how to solve it, getting an answer, and then seeing if the answer "makes sense". Karen's description of problem-solving activity is very similar to Polya's (1945) well-known problem-solving heuristic: Understand the problem; formulate a plan; carry-out the plan; look back. When I asked Karen to talk about how she had come to think about problem solving in terms of these steps she replied, "I've seen problem solving stuff on posters before." Karen was referring to the popular kind of posters that display steps for problem solving, Polya's heuristic being one of the most common (see NCTM, 1980). Karen emphasized that understanding the problem and then seeing if the answer "makes sense" are what she stresses the most when her students are doing problem solving.

Just as Karen's views and beliefs about problem solving are tied closely to her textbook, her conceptions about problem-solving activity in classroom settings are also linked to the text. Karen conceptualizes problem-solving activity as occurring in her classroom when students are working on word problems from the text. This claim is reinforced by Karen's citation of the word problem in her textbook in the prior section as an example of what her students work on when doing problem solving. Although Karen referred earlier to problem solving being important in real life, in our discussions about problem-solving activity in her classroom she only talked about doing word problems from the text. These descriptions from Karen about what constitutes problem-solving activity in her classroom reinforce the earlier claim about the differences between her beliefs about problem solving and how she enacts problem solving in her classroom: While Karen believes that problem solving is important because it is useful in real life, problem-solving activity in her classroom focuses on standard word problems because that is how the textbook, which she strongly adheres to, addresses problem solving.

Subject-Matter Knowledge

Karen is not confident of her own subject-matter knowledge of perimeter and area. While she is willing to entertain students' non-standard ideas about perimeter and area even if they are not familiar to her, she prefers to consult the textbook before taking a position as to the validity of students' non-standard ideas. For example, this is how Karen responded to a question in which she is asked to help a colleague make sense of a student's non-standard way of finding perimeter (which is also shown below):

Figure 5.1: Student's non-standard way of finding perimeter.



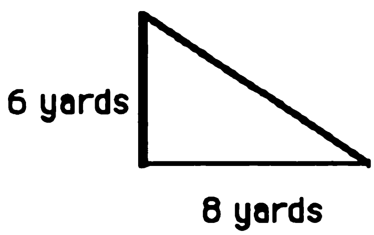
$$P = 2 \times (18/9 + 18/2) = 2 \times (36/18 + 162/18)$$
$$= 2 \times 198/18 = 198/9 = 22$$

> First, I would refer her to the text and we'd look up the perimeter. We would find out what the definition is for finding perimeter. I can't say this is wrong until I find out how do you find perimeter, make sure the child understands that we're finding perimeter, and then see if this makes sense. We would go to the textbook and start there.

Karen is unwilling to either dismiss, accept, or take some other position on the student's work until she refers to the textbook.

Karen also expressed a desire to consult the text in describing how she would respond to a student who had developed a non-standard method for converting between different square units for measuring area:

Figure 5.2: Converting between units for measuring area.



A = 1/2 x B x H = 1/2 x 6 x 8 = 1/2 x 48 = 24 square yards
3 feet = 1 yard so Area = 24 x 3 = 72 square feet

AR: Now, suppose that during your instruction on perimeter and area one of your students raises his hand and is very excited. He says that he has figured out how to convert between different units used to measure area. And he shows you his solution to this area problem [referring to Figure 5.2 above] on this triangle and that he came up with 24 square yards. Now, he says that since there are 3 feet in a

p
te
ma
ab

yard he can convert the area from square yards to square feet How would you respond to this student?

Karen: Converting square yards to square feet?

AR: Yes. He's multiplying the square yards by 3 since there are 3 feet in a yard.

Karen: OK, into square feet Hmm.. (long pause) Well, I don't know. I could try it, I'd let him try his theory. And I'd like him to do some experimentation, again, to see if it makes sense, to see if it's mathematically correct. ... I'd ask him which book did he use and let me see it -- if he's used his text I'd like to see what they've [i.e., the textbook authors] have done there -- and it might be that if he's really excited about this I'd let him share this with his classmates. I'd make a transparency of this. So this would be okay.

Karen is uncertain about the validity of the student's faulty conjecture, but she eventually accepts it as she indicates in the above transcript. In understanding Karen's subject-matter knowledge, an important point about the above response is her tentativeness in accepting the student's theory and her desire to consult the textbook.

As well as showing how Karen refers to consulting the textbook in making sense of students' non-standard work, the two excerpts also demonstrate that she does not cite the textbook as the final authority for judging students' ideas. In the perimeter question, Karen stresses that she "can't say this is wrong" until she investigates what is meant by perimeter and how the student understands perimeter. In converting area units above, Karen is careful to allow the student an opportunity to explain and "let him try his theory."

While the textbook clearly plays an important role in Karen's pedagogical reasoning, it is not the central authority guiding her teaching. Karen's uncertainty about her own subject-matter knowledge is manifested in her desire to consult the textbook before making decisions about the validity of students' ideas. However, she is adventurous enough to entertain students' unfamiliar conjectures and allow them opportunities to develop their ideas. Karen's strong tendency to consult the textbook is the result of her uncertainty about her own subject-matter knowledge of perimeter and area combined with her willingness to give students' ideas a chance.

Perceptions and Beliefs About Student Learning

Karen believes that middle school students increasingly need to do "hands-on" activities to understand mathematics. By "hands-on" activities Karen means mathematics lessons or tasks in which students use manipulatives or tools (e.g., tiles, grids, cubes) to "concretely understand and work with the concepts." For Karen, the intent of hands-on activities is to give students opportunities to physically work with mathematical concepts so they can understand them.

Prior to teaching the *Covering and Surrounding*, I asked Karen to share her impressions of several different pieces of curriculum (e.g., Shroyer & Fitzgerald, 1986). Some of the curricula involved activities where students use square tiles to measure perimeter and area of plane figures. Reacting to the materials, Karen said "My usual material doesn't really include exercises where they use the tiles, and that is my goal this year -- to get them to do more hands-on, use more manipulatives." Karen noted that she is "not a tile person" but that since fewer kids are able to think abstractly she is trying to change her own style of teaching:

> I find out that as we go further and further into the future we have fewer and fewer kids that can think abstractly -- they need more hands-on. So I have to change my own style of teaching.

In subsequent conversations, Karen said that the *Covering and Surrounding* unit, with its use of a variety of manipulatives (e.g., tiles, transparent grids) is "really hands-on." Noting that "hands-on is the wave of the future", Karen said that since her students need hands-on activities to understand, she must change her teaching to include more hands-on to meet their needs. Karen views the *Covering and Surrounding* unit as something that will provide her students with the hands-on experiences using manipulatives that they need to understand perimeter and area.

By "thinking abstractly", Karen means being able to understand concepts, like perimeter and area, by knowing and using formulas. Karen described to me that her usual mode of teaching is "teaching abstractly"[3], meaning that she would tell her students what the concepts were and then teach formulas or procedures:

> After I had told them the idea of area and perimeter, then we'd go into formulas rather than hands-on. I think most middle school teachers are similar, that we think a lot of the hands-on is done in elementary school, so we don't have to do hands-on and we treated them like junior-high students when they got here. Now we're finding that even some of the junior-high kids still need hands-on.

Karen believes that sixth-graders need hands-on activities because they can't think abstractly -- as Karen said, "They cannot really understand the concepts of perimeter and area by having me explain it to them, they need to do hands-on activities to understand." Karen also noted that after doing hands-on students "are more able to think abstractly about the concepts, like using the formulas correctly." Karen has assumed in

---

[3] "Teaching abstractly" and "think(ing) abstractly" are phrases that Karen used frequently throughout the study to describe her own teaching. I use these terms with quotations in the text to emphasize when Karen used the term(s) to describe or explain aspects of her own practice.

the past that sixth-graders don't need hands-on, and therefore she hasn't done hands-on in her teaching. Now, however, she feels that her students need to do hands-on before she can teach abstractly.

Karen's lesson on divisibility laws at the beginning of this chapter is an illustration of her tendency to emphasize the abstract. Karen presented her students with the divisibility rules for different numbers and demonstrated them. She then had her students practice the rules by finding whether the number 11930 was divisible[4] by 2, 3, 5, and 10. While Trevor made a contribution to the class discussion that went beyond the four divisibility laws Karen had presented, she restated his conjecture to the class in the form of a rule. None of the rules in Karen's lesson, with the possible exception of Trevor's explanation of his own conjecture, were conceptually motivated or concretely illustrated. Karen taught the divisibility laws "abstractly" -- she told her students the rules, demonstrated their use, and then had them practice using the rules.

With respect to teaching and learning perimeter and area, Karen wants her students to learn the same mathematics from hands-on activities that she would ordinarily tell them about in her abstract teaching. When I asked her about what she would like her students to know about perimeter and area, Karen said she wants her students to "understand the basic concepts -- what perimeter and area are and how to find them and how to use them to solve simple problems." She described how her students have difficulty in "doing the basic concepts, like

---

[4]As emphasized in Chapters 2 and 4, an important aspect of problem solving is that it is relative to the solver -- a problem for one individual is not necessarily a problem for somebody else. While Karen's problem to her students of deciding if 11930 is divisible by 2, 3, 5, and 10 might be considered a problem for sixth-graders, the way Karen used it was as a drill exercise, i.e., to practice using divisibility laws.

finding the correct areas of shapes" and that she thinks "hands-on is going to help them with that." Karen noted that the textbook doesn't do hands-on but just has formulas which the students are supposed to learn. However, Karen said that, "The kids really can't think abstract like that, just learning formulas cold, so I need to do hands-on so that they can understand them." Karen elaborated on this point, noting that to understand perimeter and area her students need to know "when to use which formula to get the right answer -- the hands-on should help them know to, like, when to multiply for area and add for perimeter."

While Karen has decided to teach perimeter and area using a hands-on approach, her comments suggest that this change in her mode of teaching has not been accompanied by a change in what she believes students should know about perimeter and area. Karen had surveyed both the student and teacher materials for *Covering and Surrounding* prior to our first discussion. However, she only cited topics from the textbook in talking about what she would like her students to know about perimeter and area. The learning goals *Covering and Surrounding* posits for students (see Chapter 4), for example, include understanding relationships between perimeter and area, developing estimation strategies, and finding area and perimeter of irregular figures to which formulas may not apply. However, Karen's expectations of what her students should know/understand about perimeter and area focus on "doing the basic concepts" which are oriented around computation and the kinds of standard word problems found in the textbook. Using formulas correctly is at the heart of what Karen wants her students to know about perimeter and area.

Karen also believes that maintaining students' interest in lessons is critical to successful student learning. For example, in discussing the quizzes included in the teacher materials for the unit (see CMP, 1992b), Karen talked about how she would decide whether or not to use them:

> I will determine whether I'm going to use them based on their interest. You have to deal with sixth-graders, they're kind of flighty on some days, and if they're not interested in the thing it starts going downhill after the first two or three assignments. Even though they're [i.e., Karen's Enriched Mathematics students for the upcoming 92-93 year] an enriched group, they could be a much slower group for interest. Again, it all depends on the interest of the students I have.

Karen believes that students need to be interested in the mathematics they're doing or the lesson will inevitably start "going downhill." Karen pointed out to me on other occasions that what she would be able to accomplish in the *Covering and Surrounding* unit with her students would all depend on their level of interest. As I explain next, student interest, for Karen, is a necessary condition for successful student learning.

Karen noted that *Covering and Surrounding* would be a piece of curriculum that would probably maintain students' interest. For example, Karen commented prior to teaching the unit that the last investigation, "The Bee Problem Revisited", would likely be interesting to her students -- "The bee project looks fun and I bet they would like it, and if they like it they'll be more interested." However, Karen also commented that all the work with tiles in the first investigation might not be as interesting because it "might get boring after awhile for them [i.e., her students] and they might just start multiplying." Karen's comments provide insight into what counts as student interest for her -- if students are bored with an activity they won't be interested, whereas if they like an activity they will be interested.

Karen's conception of student interest focuses on whether students like the *activity* involved in a lesson, as opposed to whether or not they find the *mathematics* involved in a lesson engaging. The activity part of a lesson is what students are doing (e.g., cutting out shapes, using manipulatives, using a calculator, multiplying numbers by hand); the mathematics component of a lesson is the mathematical concept(s) embedded within students' activity. For example, *Covering and Surrounding* has students use tiles to measure rooms (the activity) to learn about perimeter and area (the mathematical concepts) in the first investigation. It is conceivable that in instructional situations students might find the mathematics interesting (e.g., perimeter and area concepts and relationships) and find the activity not interesting (e.g., pulling numbers from rectangle diagrams to find area via rote multiplication). Karen's attention to student interest centers on the *activity* component of lessons and not on the *mathematics* involved.

<u>Summary</u>

Karen believes problem solving is an important component of middle school mathematics because it can help her students in real life. However, Karen teaches problem solving from the textbook, which focuses on solving standard word problems, not real-world problems. Problem solving for Karen *means* solving word problems from the textbook. Karen believes that problem-solving *activity* occurs in her classroom when students are solving textbook word problems and following a linear process of problem solving steps -- reading and understanding the problem, figuring out a plan, getting the answer, and seeing if the answer makes sense.

Karen is uncertain about her own subject-matter knowledge about perimeter and area. She consistently refers to the textbook as a resource in making sense of students' non-standard work because she is hesitant about her own knowledge, yet willing to entertain students' conjectures. Providing students with hands-on learning experiences and maintaining their interest are two central issues in Karen's perceptions and beliefs about student learning. However, while Karen wants to change her mode of teaching perimeter and area from abstract to hands-on activities, her beliefs about what students should know about perimeter and area remain focused on learning how to use formulas. Moreover, Karen's conception of student interest is attentive to whether students like the activity of a lesson but does not address whether they find the mathematics engaging.

Karen's Use of Covering and Surrounding

This section begins by providing a summary of Karen's coverage of *Covering and Surrounding*. I give a synopsis and brief analysis of the exercises Karen assigned from each investigation in order to illustrate how she used the unit. The subsequent sections each describe and analyze a specific aspect of Karen's use of *Covering and Surrounding*. I focus on four key aspects: (1) how Karen's sequencing of investigations and perceptions of problem solving compare with the intentions of the curriculum developers, (2) the coexistence of hands-on activities and skill maintenance while teaching *Covering and Surrounding*, (3) her use of teacher materials, and (4) the influence of *Covering and Surrounding* on her practice. Throughout these sections, the assessment of Karen's views, knowledge, and beliefs in the four domains of the framework is employed to unpack and understand her use of *Covering and Surrounding*.

Summary of Unit Coverage

Karen's instructional decisions about sequencing investigations and perceptions of problem solving in teaching *Covering and Surrounding* were shaped predominantly by her own beliefs, not by the teacher materials. Karen selected and taught investigations in *Covering and Surrounding* in a different order than intended by the curriculum developers because of the power of her preexisting beliefs about student learning. Moreover, Karen did not believe her students were learning about problem solving in *Covering and Surrounding* because her beliefs about problem solving are different than the problem solving occurring in the unit.

Karen began teaching *Covering and Surrounding* in her Enriched Mathematics class on 11/3/92 and concluded the unit on 12/3/92. After teaching the first investigation of *Covering and Surrounding*, Karen allowed her students to determine the subsequent investigations studied by majority vote. To portray Karen's use of *Covering and Surrounding*, it is helpful to first show which investigations she covered in the unit and the Problems, Follow-Up Questions, and ACE items she assigned. Table 5.1 below summarizes the sequence in which Karen taught investigations and the Problems, Follow-Up Questions, and ACE items she assigned:

Table 5.1: Summary of Problems, Follow-Up Questions, and ACE items assigned by Karen.

Investigation Title	Assigned Problems	Assigned Follow-Up Questions	Assigned ACE Items
1 - Measuring and Designing Rooms	1, 2, and 3	1	none - extra credit
6 - Finding Relationships - Connecting Triangles, Parallelograms and Rectangles	1 and 2	Follow-Up Question to Problem 1 and the Follow-Up Question to Problem 2.	1 and 2
7 - The Bee Problem Revisited	1, 2, and 3	Follow-Up Question to Problem 3.	1
5 - Constant Perimeter, Varying Area	1, 2, and 3	Follow-Up Question to Problem 1.	none - extra credit

Referring to Appendix D and noting the exercises Karen assigned in Investigation 1 from Table 5.1 provides a particular example of Karen's coverage of an investigation in *Covering and Surrounding*. To provide a sense of how Karen's coverage of the investigations she taught compares to the number of problems in the unit, Table 5.2 below shows the number of Problems, Follow-Up Questions, and ACE items that Karen assigned together with the total number (in parentheses):

Table 5.2: Karen: Assigned number of exercises compared to total.

Number of Invest.	# of Problems Assigned and (total)	# of Follow-Up Assigned and (total)	# of ACE assigned and (total)
1	3 (3)	1 (1)	0 (10)
6	2 (2)	2 (2)	2 (6)
7	3 (3)	1 (1)	1 (4)
5	3 (4)	1 (4)	0 (8)

Table 5.3 below gives the percentage of Problems, Follow-Up Questions, and ACE items Karen assigned from each investigation she taught in *Covering and Surrounding*. Comparing the proportions shows that Karen

emphasized the Problems and Follow-Up Questions, but did not assign many ACE items:

Table 5.3: Karen: Percentage of exercises assigned in each covered investigation.

Number of Investigation	% Problems Assigned	% Follow-Up Assigned	% ACE Assigned
1	100%	100%	0%
6	100%	100%	33%
7	100%	100%	25%
5	75%	25%	0%

Sequencing Investigations and Perceptions of Problem Solving: The Power of Beliefs

Table 5.1 shows that Karen taught the investigations in *Covering and Surrounding* in a different sequence than they appear in the unit. Karen taught the four investigations she covered in the sequence 1, 6, 7, and 5. She taught the investigations in that order because this is the sequence in which her students *chose* to study the investigations. After teaching the first investigation, "Measuring and Designing Rooms", Karen allowed her students to vote on which investigation to study next. Her students voted to study Investigation 6 as the second investigation. After Investigation 6, Karen's students voted for Investigation 7 and then for Investigation 5 (in that order).

Karen decided to allow her students to choose the sequence of the *Covering and Surrounding* investigations by majority vote because she wanted to maintain their interest. Observing Karen's class at the beginning of the second week of *Covering and Surrounding*, I was puzzled to find that, having just finished Investigation 1, the students were now working in Investigation 6. I asked Karen about her decision to allow her class to study Investigation 6 having just finished Investigation 1:

AR: Now, the students are doing Investigation 6?

Karen: Yes, as the second investigation. And they chose number 6 because of all the shapes in there -- they want to get into it. And especially the parallelograms, they really like those.

AR: When you decided to let the class choose what investigation to do next, how were you thinking about that?

Karen: Well, we looked at all of them. And again, it has to do with their interest. If they're not interested in it, I didn't want to dwell on it -- you know, OK, we go through Investigation 1, then Investigation 2, and so on -- that's just like a textbook. So I threw it out to them and said "What should we do next?" And their interest was to do number 6. It was practically unanimous to do 6, although someone wanted to do the beehive [i.e., Investigation 7].

AR: So by letting them choose the investigations --

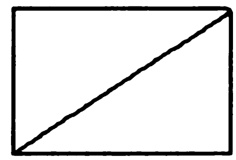
Karen: It seems to keep their interest up on the project, rather than me saying to them that we're going to this one and then the next one, and the next.

During her instruction of Investigation 6 in *Covering and Surrounding*, Karen did not address the class as a whole. As in the other investigations she taught in *Covering and Surrounding*, Karen spent most of the class time circulating around the room and working with students individually or in small groups. On the third day of working on Investigation 6, a group of three students, Matt, Trevor, and Aaron, were eager to share something with Karen. They were very excited about a discovery they had made while working on Problem 2 in Investigation 6 (CMP, 1992a, p. 44):

> **Problem 2:** Orlando wonders whether there is a relationship between triangles and parallelograms. Look back at the parallelogram in Problem 1. Can you find a way to cut a parallelogram into 2 triangles that match each other? Be sure to show how you know the 2 triangles are the same.
>
> Now draw a triangle. Cut out 2 copies of your triangle. What figures can you make by putting your triangles together so that an edge matches?
>
> How does the area of the original triangle compare to the figures you made?
>
> Describe as many ways to find the area of a triangle as you can. Draw any diagrams that help explain your methods.

Matt, Trevor, and Aaron showed Karen two congruent right triangles which were pushed together to form a rectangle:

Figure 5.3: Matt, Trevor, and Aaron's rectangle for proving triangle area.



The three students told Karen that they had discovered why the area of a triangle was "one-half the base times the height." Trevor said that his mother had showed him the area formula for a triangle the night before, and he and his two classmates were puzzled about "why it worked." Karen listened closely as the students explained their reasoning to her:

Matt: Basically, you can make a rectangle from two equal triangles. The base times the height gives you, like, the area of the rectangle.

Trevor: But we want the area of the triangle, not the area of the whole thing -- I mean the rectangle. So you just take half of the base times the height because the triangle is half of the rectangle.

Aaron: Yea. You can see that it's [indicating the triangle] half of it [indicating the rectangle] so you only need half of it, like half of the area of the rectangle.

Karen: This sounds very interesting -- good work! What I would like each of you to do is sit down and write your own explanation -- in writing and showing me this diagram [i.e., the rectangle constructed from two congruent triangles] -- of this discovery. Maybe tomorrow I'll have you share this with the class.

Clearly excited at having their discovery praised[5], Matt, Trevor, and

---

[5]Matt, Trevor, and Aaron's discovery is a special case of a more general justification for the area formula for triangles. For *any* triangle (not just right triangles), two congruent copies can be put together to form a parallelogram (e.g., a rectangle in the case of right triangles as in Matt, Trevor, and Aaron's discovery). The area of the parallelogram is the product of the base and height of the triangle since the parallelogram can always be transformed into a rectangle with dimensions equal to the base and height of the triangle. Since two copies of the triangle form the parallelogram, the area of the triangle is one-half the product of its base and height, i.e., half of the area of the parallelogram.

Aaron went back to their desks and began writing descriptions of the reasoning they had just shared with Karen.

A few minutes after Matt, Trevor, and Aaron had shared their discovery with Karen, I drew her aside and asked her about what she thought the three boys were learning about problem solving:

AR: What did you think of Matt, Trevor, and Aaron's discovery?

Karen: Oh, very good! They're really into it and I liked the way they figured that out about the area of a triangle. I'll have them share it with the class tomorrow.

AR: Karen, what would you say they learned about problem solving in making their discovery?

Karen: Well, really nothing significant.

AR: [pause] Hmm... What would you say they did learn about?

Karen: Well, they're doing hands-on. I mean, they're doing stuff with the triangles and rectangles, cutting and moving them around, and now they understand the area formula. I'd say that they're doing hands-on work with the shapes rather than problem solving.

Karen's decision to allow her students to determine the sequence of the *Covering and Surrounding* investigations and her perception that students were not learning problem solving are not consistent with the teacher materials. The *Covering and Surrounding* teacher materials (CMP, 1992b) describe how the sequenced investigations presented in the unit are intended to "...build a deep understanding of what it means to measure area and what it means to measure perimeter" (p. 1). While the materials do not explicitly say that the investigations must be taught in the presented sequence, the teacher edition does outline how the developers expect students' understanding of perimeter and area to develop through study of the sequenced investigations (see Chapter 4; CMP, 1992b). The teacher materials also state that it is expected that students will gain "...insight and skill in ... problem solving" (p. 2).

In fact, the teacher edition explicitly addresses the kind of discovery made by Matt, Trevor, and Aaron as a specific problem-solving strategy that students should develop in *Covering and Surrounding*:

> Cutting and rearranging parts of figures to see relationships between kinds of figures, in particular, parallelograms, triangles, rectangles, then devising strategies for finding areas by using the relationships observed (CMP, 1992b, p. 2).

All five of the problem-solving strategies in the teacher materials (see Chapter 4) were highlighted by Karen in her copy of the teacher edition. Although Karen did not perceive Matt, Trevor, and Aaron's discovery as problem solving, from the perspective of the curriculum developers their discovery is an excellent example of the kind of problem-solving skill that the developers intend students to develop in *Covering and Surrounding*. The triangle area discovery illustrates differences between Karen's beliefs about problem-solving activity and the conception of problem-solving activity underlying the *Covering and Surrounding* materials.

How Karen chose to sequence the unit investigations illustrates the power of her beliefs about student learning. While Karen's decision about sequencing investigations by letting students vote on which investigations they find the most interesting or appealing is not consistent with the *Covering and Surrounding* unit, it is consistent with her beliefs. Recall that Karen places heavy emphasis on student learning being directly tied to whether or not students are interested in the lessons or activities. This belief completely overshadowed teaching the investigations in the existing sequence intended by the curriculum developers. Karen's concern with maintaining students' interest was apparently not influenced by other factors, such as how the enacted investigation sequence would affect student learning.

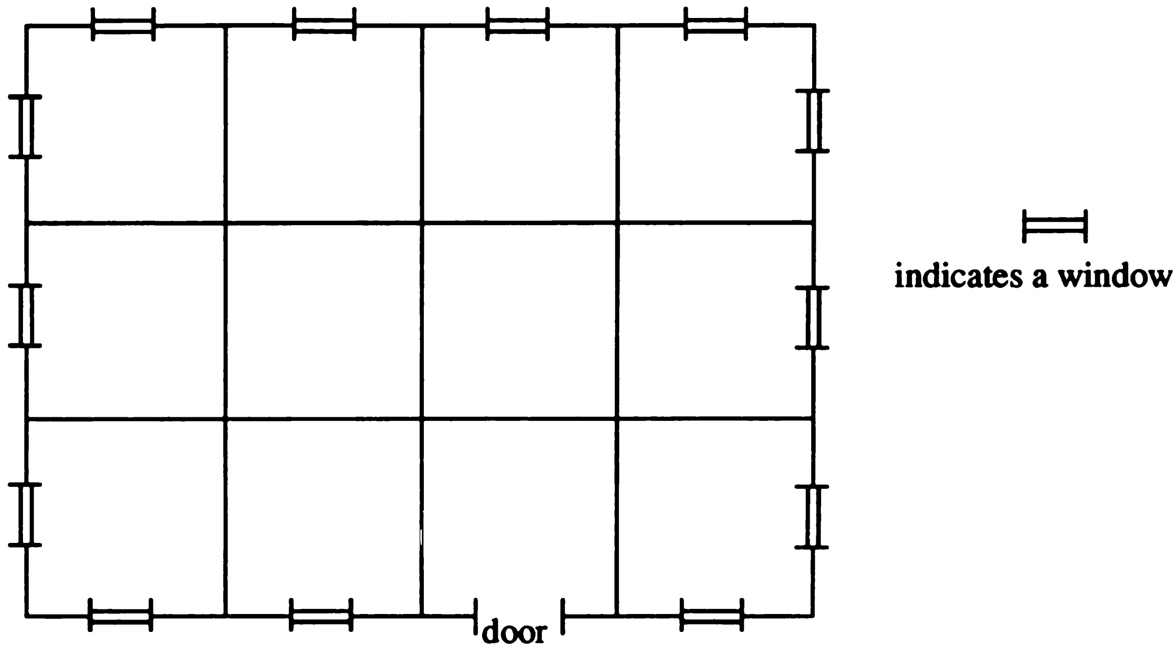
Karen's beliefs about problem solving also clearly overrode the teacher materials in shaping her perception of what her students were learning about problem solving in *Covering and Surrounding*. Karen's beliefs about problem solving are oriented around standard word problems like in the textbook, not the non-standard or puzzle problems that appear in *Covering and Surrounding*. Karen's perception of Matt, Trevor, and Aaron's discovery as hands-on is consistent with her prior beliefs about *Covering and Surrounding* being hands-on. Karen's decision about sequencing investigations and her perceptions about students' (not) learning about problem solving sharply illustrate the power of her beliefs in shaping her use of *Covering and Surrounding*. The teacher materials had little or no impact on these decisions and perceptions.

<u>The Room Design Problem and Skill Maintenance</u>

Karen used *Covering and Surrounding* to provide her students with hands-on activities, but she did not feel that the unit was sufficient to maintain their computational skills. However, Karen also taught problems from *Covering and Surrounding* in ways that gave her students hands-on opportunities to wrestle with perimeter and area.

Karen's teaching of Problem 1 in Investigation 1 is a representative example of how she used *Covering and Surrounding* to give her students open-ended learning experiences with perimeter and area. Karen began Investigation 1 by having her class read the first part of of the investigation that precedes Problem 1 (see Appendix D for Investigation 1 in its entirety):

Figure 5.4: Mr. Dull's room.



> As an architect, Mr. Dull uses grid paper to design his plans, like in his design of Mrs. Hide's room. Architects also use models in their work. Physical or pictorial models are used to represent and learn about other things that are too big or small to study easily. In designing rooms it is much easier to build models of rooms to make different designs than to build all of the actual rooms.
> Take 12 tiles, which will be used to model carpet sections, and arrange them in the same design as Mr. Dull's (CMP, 1992a, pp. 1-2).

As a student read the above excerpt from the unit aloud to the class, Karen circulated around the room giving each student about 15 1-inch square tiles.

After distributing tiles to each student, Karen discussed the perimeter and area of the room Mr. Really Dull designed for Mrs. Hide:

Karen: How many of your tiles does it take to *cover* Mr. Dull's design? [As Karen asks this question all the students count the tiles they have used to arrange in the shape of Mr. Dull's room.]

Wendy: Twelve!

Karen: And how many tile edges does it take to *surround* the room? [As Karen asks this question every student begins quietly counting around the edge of the room.]

Marcus: Fourteen!

Karen: OK. Now, according to the unit [Karen begins reading from the text (CMP, 1992a, p. 2)], 'the number of square tiles needed to *cover* the floor is a measure of the *area* and the number of wall sections around *the edge* of the room is a measure of the *perimeter*' [emphasis in the text and in Karen's recitation.] So, the area of Mr. Dull's room design is 12 and the perimeter is 14.

At this point in the lesson, Karen paused as she noticed several students still counting the number of tile edges surrounding the room. After a momentary pause she reiterated the analogy of covering with tiles to measure area and counting the tile edges surrounding the room to measure perimeter:

Karen: Okay. I want all of you to place your tiles exactly on top of Mr. Dull's room design in your unit -- put the tiles right on top of it. [pause while students place their tiles over the above illustration of Mr. Dull's room design in their unit] Now, what did you just do with your tiles?

Wendy: We covered up the room with the tiles.

Karen: Right! You covered the room with the tiles. And the number of tiles it takes to cover the room gives you what measurement?

Class: Area!

Karen: Good! And how many wall sections are there around the edge of the room and what's that a measure of?

Trevor: There are 14 wall sections.

Karen: Okay, so that's a measure of what?

Trevor: The perimeter!

Karen: Right! Now does everyone understand this? You *cover* with tiles to find area and *surround* with wall sections to find perimeter.

Karen's intent in the above excerpt was to make sure that her students understood the carpet tile and wall section analogy for area and perimeter. To this point in the lesson, Karen's teaching somewhat reflects her "abstract" teaching mode -- she is trying to illustrate a procedure. Although Karen is not telling her students how to use a formula, she is *telling* them how to use the tiles to measure perimeter and area. But Karen's teaching more sharply reflects the intentions of the curriculum developers than her mode of teaching abstractly. For example, Karen was clearly concerned that her students understand the representation of carpet tiles and wall sections and how it connects to measuring perimeter and area -- the developers also stress that understanding the representation is very important (see Chapter 4). Karen's emphasis on hands-on experiences with the tiles for her students is also reflected in her teaching, especially in directing them to physically cover Mr. Dull's room with the tiles.

After the exchange with her class to clarify the analogy for area and perimeter, Karen had a student distribute 1-inch grid paper (the squares on the grid paper and the manipulative tiles are both 1-inch square) and told the class that Problems 1, 2, and 3, and Follow-Up Question 1 on page 9 would be due at the end of the week (i.e., four days later). Karen stressed to the class that each problem should be read carefully and that they should trace their room designs made with tiles on the grid paper for Problem 1. Karen's students went quickly to work for the rest of the class time.

Problem 1 in Investigation 1 asks students, in the context of designing a room, to make a rectangular figure with 1-inch tiles that has an area of 12 and a perimeter of at least 14:

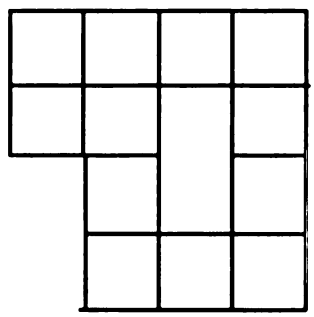
Problem 1: Mrs. Hide likes the amount of floor space in Mr. Dull's design, but she wants more windows and a more interesting shape for her room. She asks you to help her out.

a) Use 12 square tiles to create a floor plan design for Mrs. Hide. Remember that a window or a door can go into each section of wall space. Mrs. Hide tells you that she wants at least 14 sections of wall space, including windows and doors, in the room you are designing.

b) After you have designed the floor plan for the room make a drawing to show the location of the door and where each window is. Write a paragraph to tell Mrs. Hide why your design is better than Mr. Dull's (CMP, 1992a, p. 2).

At this point in the lesson, students began using the tiles to explore perimeter and area. For example, as Shakaya was working on Problem 1 she had a question for Karen. She had designed this room using 12 tiles:

Figure 5.5: Shakaya's room.



Shakaya was confused about how to find the perimeter of her room. Specifically, she wanted to know if she should count the perimeter of the inner "courtyard" as well as the distance around the figure. Karen and Shakaya had this conversation:

Shakaya: Should I count the (pause) perimeter around the inside?

Karen: Would this room satisfy the requirement of 12 tiles?

Shakaya: Yea, cause I did use (pause) yea, I used all 12 tiles.

Karen: Do you have 14 walls?

Shakaya: [Long pause as the student counts the exterior tile edges, then pausing on, but not counting, the courtyard perimeter] Yea. The tiles are the shape of the room.

Karen: What's this [indicates the courtyard] -- a hallway?

Shakaya: Yea. Sort of a hallway, would that be OK for the perimeter?

Karen: [Hesitates] I don't know. [long pause] This might be OK, but would you want a room like that [indicating the courtyard]?

Shakaya: Hmm. (pause) I don't know.

Karen: OK. [looks at clock and sees that the class period is almost over] Well, let's look at that again tomorrow.

The next class period, by which time students had finished designing their rooms, Karen had each member of the class present his or her room to the rest of the class. Each student showed his or her room design traced on grid paper and shared their answer for part b) of the problem, i.e., their explanation of why their room was better than Mr. Dull's. For example, Joe made his room with 12 tiles and a perimeter of 17, including three doors. Joe said that he included three doors because "The more doors I put in would be more easy to get from place to place, and is safer in case of fire." Shakaya had changed her design to a rectangular shape without a courtyard. When I asked her why she had eliminated the courtyard, she replied that the room looked "more complete" without the "space (i.e., the courtyard) in it". Shakaya's new room was a 2 x 6 rectangle. Presenting her room to the class, she explained:

> My room requires 12 squares of carpet to cover the floor and 16 sections for the wall. Since my room has more wall space Mrs. Hide can hang more pictures than in Mr. Dull's room.

Karen complemented Shakaya on her explanation but did not ask her to talk about why she had changed her room design by eliminating the courtyard.

The responses given by Joe and Shakaya to Problem 1 illustrate how students developed varying room designs and explanations which Karen felt were legitimate. Karen's class collectively produced a wide range of rooms and explanations for them. The students clearly enjoyed the problem and Karen was very pleased at their "high level of interest."

Karen used Problem 1 in Investigation 1 to give her students a hands-on activity with perimeter and area. Moreover, Karen's use of the problem is open-ended because students were given the freedom to *create* a solution rather than trying to find *the answer*. This view is reinforced by Karen's encouragement to her students to think about their questions to resolve them for themselves. Karen's perception, however, was not that her students were doing open-ended investigations with perimeter and area concepts, but that they were doing hands-on activity with the tiles to find perimeter and area of their rooms. Karen considered the most important facet of the room design problem not to be the open endedness of the activity, but the hands-on work with tiles to correctly find perimeter and area:

AR: How would you say the students are doing with this problem?

Karen: Oh, very well! They have a high level of interest in this problem and are really into it.

AR: What do you think is most important for them to get out of this lesson?

Karen: To do the hands-on work with the tiles to find the perimeter and area of their rooms. And they're doing a great job -- all their rooms that I've seen have the correct perimeter and area.

AR: How about the room design part of the problem and their explanations -- how do you see that fitting with what they're learning?

Karen: Well (pause), I want them to tell me why they designed their rooms the way they did, but the most important thing for me is for them to use those tiles. I think that the hands-on of doing that is important. Yes I want them to design a room they like, but I really want them to know how to find the right perimeter and area of whatever room they come up with.

Problem 1 in Investigation 1 in *Covering and Surrounding* unit did influence Karen's teaching to the extent that, in her terms, she did hands-on instead of "teaching abstractly". Her use of the room design problem also may have provided students with more opportunity to develop conceptual understanding about perimeter and area than working with area and perimeter formulas, which is consistent with the intentions of the curriculum developers. But Karen's perception of what was most important in this lesson, that students find the correct perimeter and area of their room, is consistent with her prior emphasis on doing hands-on activities to develop students' proficiency at getting right answers.

Over the next two class periods Karen's students completed their assignment in Investigation 1 and then studied Investigation 6, which they worked on for a week. At the beginning of the third week of *Covering and Surrounding*, students were working in Investigation 7. On the first day of Investigation 7, students were spending part of their class time working from the textbook. As well as the assignment from *Covering and Surrounding*, students were also doing whole number multiplication and division exercises. The exercises were "naked number" drill exercises (see Chapter 4) which do not appear in *Covering and Surrounding*. I asked Karen why she had decided to have her students working out of the textbook. She replied:

> Skill maintenance. They're doing exercises in computing with whole numbers and using the calculator to keep their skills up. Some of the parents were happy to know [i.e., at recent parent-teacher conferences] that I'm still holding them to a textbook. The parents like the unit [i.e., *Covering and Surrounding*] but we don't want to be so far out here with this project that they've [i.e., the students] forgotten basic concepts.

Karen noted that while her students were learning about perimeter and area in *Covering and Surrounding*, she felt that "these kinds of materials don't give students practice with computation skills." Karen said that her students need to maintain skills because they will be tested on them when they take standardized tests later in the year. Moreover, Karen said that she felt skill maintenance in mathematics was as important as hands-on experiences. Karen continued to supplement *Covering and Surrounding*, both during classwork and in homework assignments, with computational practice from the textbook for the duration of the unit.

Karen's remarks highlight an important facet of her use of *Covering and Surrounding*. While she does use the unit to provide hands-on activities for her students, as shown in her teaching of Problem 1 and her students' reasoning about triangle area, she does not believe that the unit adequately helps her students maintain computational skills. Karen's dual focus on *Covering and Surrounding* hands-on activities and skill maintenance did not emerge until halfway through the unit, but it remained constant after beginning Investigation 7.

Recall that Karen's views and beliefs about mathematics and problem solving, and her views about problem-solving activity, are oriented around the textbook. Karen makes extensive use of the textbook, including using it to address problem solving. Noting how Karen's practice of teaching mathematics is linked to the text helps to understand why she began bringing it back into her teaching after two weeks of *Covering and Surrounding*. In particular, Karen's use of the textbook along with *Covering and Surrounding* was more than just a relapse into old habits. Even before she began teaching the unit, in a conversation during a summer interview, Karen revealed that using the textbook along with *Covering and Surrounding* was on her mind:

Karen: Ah, will the *Covering and Surrounding* unit be related to the textbook they're [i.e., the students] using at all?

AR: If you feel that need to bring the textbook in at some point, or if there is something that the unit doesn't do very well, feel free.

Karen: Well, that's what I meant, because when I'm teaching *Factors and Multiples* [Fitzgerald et al., 1986], sometimes I have to go back to the textbook and lay the groundwork for what I'm about to introduce to them.

AR: This is something that I'm very interested in.

Karen: Good, and I sometimes use the book as homework and they get back to me to make sure that they understand what's there.

Karen's above comments from the summer prior to teaching *Covering and Surrounding* emphasize her orientation around the textbook in her mathematics teaching. Karen views the material in the textbook, particularly on skill maintenance, as important for her students. Since she does not believe *Covering and Surrounding* addresses skill maintenance like the textbook does, she brought the textbook into her use of the unit. For Karen, the textbook helps students maintain their computational skills while the *Covering and Surrounding* unit is hands-on. Since Karen believes that students need hands-on experience to understand mathematical concepts like perimeter and area, and she also believes that it is important for her students to maintain computational skills, it is perfectly reasonable for her to use the textbook and *Covering and Surrounding* simultaneously in her mathematics classroom.

Different Curriculum, Similar Use of Teacher Materials

Recall that Karen's use of her textbook teacher edition and the *Factors and Multiples* unit teacher materials was mainly to obtain answers to check students' work. Karen's predominant use of the

Covering and Surrounding teacher materials was similar to her use of the textbook teacher edition and *Factors and Multiples*. Karen used the *Covering and Surrounding* teacher materials mainly to obtain answers to problems, although her teaching sometimes reflected the spirit of the materials in asking her students to explain their reasoning.

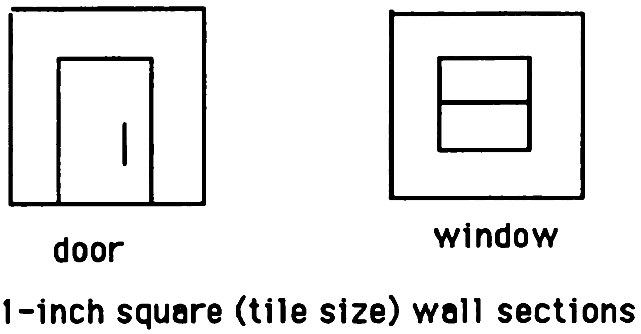
The intention of the developers of *Covering and Surrounding* is that the teacher materials are not just a source of answers to problems, but are a resource for teachers to develop and refine modes of teaching practice that are consistent with the problem-solving orientation of the unit (see Chapter 4). Suggestions for teaching are presented frequently in the *Covering and Surrounding* teacher materials and are intended to provide the teacher with opportunities to plan for or rethink how a lesson might be taught or organized.

Karen, in general, did not make use of the suggestions for teaching provided in the teacher's edition of *Covering and Surrounding* in her classroom practice. Although the unit provides suggestions for how to represent concepts and processes and how to question students to further develop their understandings of concepts, Karen did not use these ideas in her teaching. For example, the *Covering and Surrounding* teacher edition suggests to the teacher that she might want to make a set of illustrations to make the wall section and carpet tiles analogy for perimeter and area, respectively, more explicit to students (CMP, 1992b, pp. 23-24):

> The story of designing rooms in the student text can be used to launch the investigation. It is important that students get a clear picture of the relationship between the physical tiles representing squares of carpet and the prefabricated wall sections that are the same width as the edge of a carpet tile. The prefabricated sections can each contain either a window or a door, but may also be blank wallspace.

> You may want to prepare a few cut outs from cardboard with a door or a window drawn on to show students a floor plan:

Figure 5.6: Diagrams of door and window wall sections.



In her teaching of this investigation, as discussed earlier, Karen was clearly concerned that her students understand the analogy but she did not make use of any such illustrations in her teaching.

Another example is Karen's interaction with Matt, Trevor, and Aaron about their discovery justifying the area formula for triangles in Investigation 6. Recall that Karen listened carefully to their explanation, complimented them, and then had them return to their seats to express their reasoning in writing. Karen did not ask Matt, Trevor, or Aaron probing questions about their discovery. However, in Investigation 6, the *Covering and Surrounding* teacher edition (CMP, 1992b) suggests that:

> As students work through the problems of finding the relationships between rectangles and parallelograms and parallelograms and triangles, you might ask them questions about how they are dividing up their parallelograms or rectangles. For example, when a student discovers that when she cuts a parallelogram along one of the diagonals two congruent triangles are formed, you might ask her how the area of the triangles relates to the area of the original parallelogram. You might ask her further, then, to think about how the area of a triangle could be expressed in terms of the area of the original parallelogram (p. 64).

While the teacher edition cites instructional events like Matt, Trevor, and Aaron's discovery as an opportunity to help students develop their understandings of perimeter, area, and relationships between shapes, Karen did not pursue this with the three boys in-depth. She did not probe their understanding, for example, to help them extend their conjecture to parallelograms that are not rectangles and thus triangles that are not right triangles. This does not mean that Karen's use of the triangle area problem with Matt, Trevor, and Aaron was ineffective. She did ask them, for example, for a detailed explanation of their conjecture both in words and in writing. To this extent, Karen used the problem in the spirit suggested by the *Covering and Surrounding* teacher edition. But counter to the suggestions of the teacher materials, Karen did not take this opportunity to push her students' understanding toward a more general statement about general relationships between areas of triangles and parallelograms, not just right triangles and rectangles.

One explanation for Karen's omission of the wall section illustrations, or why she did not question her students' discovery in Investigation 6 further, is that she was not aware of the suggestions in the teacher materials. This, however, was not the case. Throughout her teaching of *Covering and Surrounding*, Karen read the teacher materials carefully. She remarked to me during Investigation 6 that "This material is new for me, so I've been putting in some extra time with the teacher edition." That Karen was reviewing the teacher notes carefully was evident from looking at her copy of the teacher materials in which she highlighted, as she said, "important points and answers." For example, Karen had highlighted in her copy of the teacher edition portions of the above passage from Investigation 6 in the teacher materials[6]:

> As students work through the problems of finding the relationships between rectangles and parallelograms and parallelograms and triangles, you might ask them questions about how they are dividing up their parallelograms or rectangles. For example, when a student discovers that when she cuts a parallelogram along one of the diagonals two congruent triangles are formed, you might ask her how the area of the triangles relates to the area of the original parallelogram. You might ask her further, then, to think about how the area of a triangle could be expressed in terms of the area of the original parallelogram. These kinds of questions help students develop their own problem-solving skills and understanding of perimeter and area ... (p. 64).

So Karen *was* aware of these suggestions for teaching Investigation 6 and she emphasized them in her own copy of the teacher materials. Another explanation for why Karen did not make use of suggestions in the teacher materials emerges from a conversation with Karen after her interaction with Matt, Trevor, and Aaron.

After class, I asked Karen about Matt, Trevor, and Aaron's explanation for triangle area and if she thought it might be helpful to ask them more questions about their discovery. She replied, "Not really -- they know that the area of a triangle is half base times height." When I pressed Karen further about asking the three boys about what problem solving skills they used to make their discovery and relating parallelograms and other kinds of triangles besides right triangles she said

> I don't know about doing any problem solving yet What I want them to know is one-half base times height -- *that's* (i.e., the formula) what they'll use to do problem solving because now they have the formula to use to solve problems.

---

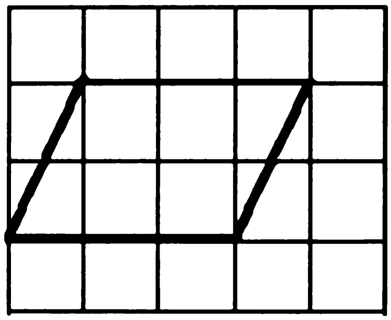
[6]The underlined portions of the passage indicate what Karen highlighted in her copy of the teacher edition (CMP, 1992b) in yellow highliner.

As Karen's response indicates, she didn't ask her students about problem solving because she didn't perceive that problem solving was happening. From her perspective, the point of the problem was for students to learn one-half base times height as a formula to use for solving other problems, not developing problem solving skills or a more general relationship between triangles and parallelograms. Karen did not pursue suggestions from the *Covering and Surrounding* teacher materials because the materials suggested pursuing goals that she did not have for her students. Knowing the area formula for triangles was what Karen wanted her students to know. When Matt, Trevor, and Aaron demonstrated this, Karen asked them to explain their reasoning but did not dig deeper because the formula had been discovered.

During the same conversation with Karen after class, I asked her about the wall section analogy for perimeter and area and the illustrations provided in the teacher edition. Karen said that she didn't feel the wall section illustrations were "really necessary" because her students "got the idea of using the tiles on the rooms" with little difficulty. So Karen was also aware of the *Covering and Surrounding* teacher edition suggestion about using the illustrations of the wall sections (see Appendix E) but chose not to use it. Karen's decision not to use the wall section illustrations is congruent with the suggestion of the teacher materials which suggest that the illustrations be used if students are having difficulty understanding the analogy (see Appendix E).

Karen *did* use the *Covering and Surrounding* teacher materials to see if her students had found the correct areas and perimeters of figures. For example, Problem 1 in Investigation 6 presents students with different parallelograms on a grid and students are to develop and describe a strategy to find the area of each parallelogram. An example is given below:

Figure 5.7: Parallelogram on a grid (CMP, 1992b, p. 43).



In the teacher materials (CMP, 1992b), Karen had marked the answers to the parallelogram area problems and used them in checking her students' work. As I observed from looking over students' corrected papers on the above problem, their answers were graded by Karen as right or wrong, depending on whether or not their answer matched the answers in the *Covering and Surrounding* teacher materials. Karen did not comment on any of the students' papers about their strategies for finding area.

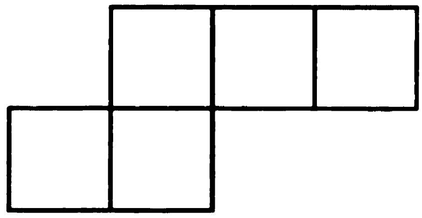
Overall, Karen's use of the *Covering and Surrounding* teacher materials was similar to her prior use of other teacher materials (e.g., her textbook teacher edition, *Factors and Multiples*) -- she used the *Covering and Surrounding* teacher edition primarily as a source of answers to problems. However, she also read the teacher edition more carefully than just to obtain answers, even highlighting some passages. But while Karen was aware of suggestions for teaching provided in the teacher edition, these suggestions usually did not appear in her practice. Sometimes this was the result of Karen filtering the suggestions through her own beliefs (e.g., the triangle area problem) and at other times was based on decisions she made that were consistent with the teacher materials (e.g., the wall section analogy).

Karen and *Covering and Surrounding*: "I'm Trying to Change"

Karen felt that her teaching using *Covering and Surrounding* was different from her usual mode of practice -- she believed that she had taught perimeter and area doing hands-on activities instead of teaching abstractly. Contrasted with her usual mode of teaching perimeter and area "abstractly", Karen did teach differently while using *Covering and Surrounding*. However, while she felt that the unit had pushed her to teach differently, Karen expressed and exhibited a tendency to teach abstractly after teaching *Covering and Surrounding*.

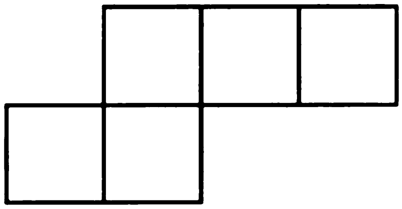
During the fourth and final week of teaching *Covering and Surrounding*, Karen's students studied Investigation 5 which they had chosen as the last investigation. Investigation 5 explores the relationship between perimeter and area of rectangular figures when perimeter is held constant and area is varied. Karen said that she was glad the class picked Investigation 5 because it would give them more hands-on work with the tiles. For example, Karen assigned her class these two Problems from Investigation 37 (CMP, 1992a, p. 37):

Problem 2: What's the Smallest? Construct the original pentomino again.



What is the smallest number of tiles you can add and have a perimeter of 18 units? Explain and illustrate your answer.

Problem 3: What's the Largest? Construct the original pentomino again.



What is the largest number of tiles you can add and have a perimeter of 18 units? Explain and illustrate your answer.

Students worked through Investigation 5, including the two problems above, at their own pace.

During this final week of *Covering and Surrounding*, I asked Karen to share her thoughts and impressions about teaching the unit. In summarizing her experience teaching *Covering and Surrounding*, Karen said:

> It forced me to use tiles, and I'm not a tile person -- I think abstractly and I memorize formulas and things ... it made me use the tiles and grids to give kids the hands-on they need to understand. I would actually prefer to use formulas and teach abstractly, but I'm doing more hands-on. I'm trying to change but it's tough!

In her above reflection on teaching *Covering and Surrounding*, Karen provided confirming evidence about what she believed to be perhaps the most important feature of the unit for her students -- doing hands-on activities. Moreover, while Karen felt that teaching *Covering and Surrounding* had pushed her to teach differently, she expressed a preference for teaching abstractly.

Karen's remarks are especially interesting given her use of *Covering and Surrounding*. Karen believes that she should do more hands-on activities rather than teach abstractly. The fact that she taught *Covering and Surrounding* for a month is evidence of the strength of this belief. Yet, Karen also claims she would rather teach abstractly, and this claim was reinforced by a lesson on circumference of circles which she taught after most of the class had finished Investigation 5. Beginning with this lesson, Karen resumed working from the textbook while students who were not quite finished with Investigation 5 completed it. Karen's lesson was on circumference of circles and was taught on 12/2/92. The following excerpt is from the beginning of the lesson:

Karen: [addressing the class] Do you know the reason why we're using π? [some confused looks from the class] When you're looking for circumference, according to your author [i.e., Abbott & Wells, 1985], your supposed to use π, to the measurement of --

Class: Three point one-four.

Karen: Or --

Class: Twenty-two sevenths!

Karen: OK. Now when you start this off, they usually will give you a formula. And you will see both of the formulas at the bottom of page 344. Can anybody remember why there are two different formulas?

Sheila: There's one for the diameter and one for the radius.

Karen: [to the class] Hmm. He said that there's one for the diameter and one for the radius. Everyone look at it and see if you agree. Do you think that's what that is? [nods from the class] Well, what's the C for then?

Class: Circumference!

Karen: Right. So we're saying that if the diameter is given, which formula do you use? (pause) Give me the formula if the diameter is given in the problem. Which formula would you use?

Carlos: Two r?

Karen: If the diameter is given? Look again now on 344, don't go to 345 yet, that's a whole different story there -- stay on 344. Suppose a problem is given and they said "The diameter of this circle is 14". Which of these formulas do you think you would use? (pause) There are two formulas -- they're right down there at the bottom of 344 where it says "step 1 and step 2". Which formula would you use?

Carlos: Step one?

Karen: Give me the formula. Give me the whole formula -- read what you see.

Darren: $C = \pi d$.

Karen: OK. Now what does πd mean to you?

Darren: Three point one-four diameters.

Karen: Three point one-four diameters? I disagree with you. What is πd? There's something implied there but there's something left out.

Becky: Three point one-four *times* the diameter!

Karen: Alright! So when the diameter is given you must understand that that π is represented by three point one-four. Now, in some cases, it gives you a reason at the top of page 344 as to when you're to use the fractional equivalent. Now you would use this one for π [writes 3.14 on the chalk board] if you're given a whole number or a decimal. But suppose you were given that the radius is, say, 2 1/2. What would you have to do with this is?

Melissa: You would have to multiply that by twice and then multiply it again by three point one-four.

Karen: Alright, I wouldn't use 3.14 with this one. I could if I changed 2 1/2 over to a decimal, but if it's like this I'd use the other one [writes 22/7 on the chalkboard]. First I'd multiply 2 1/2 by 2 because you need two what?

Melissa: Two radiuses make a diameter!

Karen: Alright. So you have to have two radiuses to equal one diameter. Now if the diameter is given such that it's say, 26, I would use this [points to 3.14] $C = \pi d$, this being three point one-four times whatever number is given. You have to know which one [i.e., which formula] to use. Now suppose you're given a radius of, say, 6. Do you know what to do? What would you're formula be if you're just given the radius?

Melissa: $C = 2\pi r$.

Karen: $C = 2\pi r$ [writes on the board]. Which means that you take this two, multiply it by 6, and then you have to still multiply by 3.14.

The above transcript provides a good example of what Karen means when she describes herself as an "abstract" teacher -- she focused on teaching her students formulas and how to use them. The formulas Karen taught were not conceptually motivated nor concretely illustrated, they were simply presented to the students and then demonstrated. But the lesson also illustrates how different teaching *Covering and Surrounding* was from Karen's "abstract" teaching. Compare the circle circumference lesson above with, for example, Karen's use of the room design problem with Shakaya or her interaction with Matt, Trevor, and Aaron about triangle area. Comparing these three snapshots of Karen's teaching yields striking differences between her "abstract" mode of teaching and her "hands-on" mode using *Covering and Surrounding*.

What is striking about Karen's use of *Covering and Surrounding* when compared to the circumference of circle lesson above is the coexistence of her "hands-on" and "abstract" modes of teaching perimeter and area. For example, the *Covering and Surrounding* unit does contain an investigation about the area and perimeter of circles (see Chapter 4). Karen's "abstract" lesson on circumference of circles, rather than teaching the "hands-on" investigation from *Covering and Surrounding*, coupled with her remarks about preferring to teach abstractly illustrates the strength of her beliefs about teaching abstractly. Her use of *Covering and Surrounding* simultaneously exhibits her belief that teaching "hands-on" is important. Karen's comment that "I'm trying to change but it's tough!" may reflect a struggle to move away from "teaching abstractly" to "hands-on" teaching, or perhaps that she is trying to somehow balance these two modes of practice.

Summary

Karen's use of *Covering and Surrounding* reflected her views and beliefs. Her views and beliefs were predominant in shaping her teaching and overshadowed the intended curriculum when her views and beliefs did not correspond with the intended curriculum. Karen's sequencing of investigations to maintain her students' interest, her concerns about skill maintenance and computation, and her perception that her students were not learning about problem solving illustrate this. These beliefs were constant throughout Karen's use of *Covering and Surrounding* and did not seem to be challenged or change while teaching the unit.

Karen's mode of teaching perimeter and area while using *Covering and Surrounding* did change. Karen used *Covering and Surrounding* to teach perimeter and area using hands-on activities rather than teaching abstractly. Congruent with the intentions of the curriculum developers, Karen had her students using tiles, grids, and cutting up and rearranging shapes to help them solve perimeter and area problems. Karen's use of the *Covering and Surrounding* teacher materials was similar to her use of prior materials, but her teaching did reflect the spirit of the *Covering and Surrounding* materials in terms of having students explain their reasoning and allowing students to explore problems. Karen's perceptions about what her students were learning in the unit, however (e.g., about problem solving), contrast sharply with the intentions the curriculum developers have for students' learning. Karen perceived her own experience teaching *Covering and Surrounding* as pressing her to teach differently, although she still expressed views and taught perimeter and area in ways that matched her abstract mode of teaching.

CHAPTER 6

THE CASE OF BETTY WALKER

A Profile of Betty Walker

As Betty turned on the overhead projector to begin class, her students quickly found their seats and opened their math journals. Everyone in the class was writing the date, 11/5/92, on a clean sheet of paper in their spiral-bound notebooks as Betty drew their attention to the problem on the overhead projector:

Betty: Alright everyone, now listen closely. The problem we're going to start with today is to [reading the problem on the overhead] 'Find the prime factorization of the number represented by the year of your birth.' Now, not everyone is going to have been born in the same year. Who was born in 1981? [most of the class raises hands] Who was *not* born in 1981? [the remainder of the class raises their hands]. OK, so not everyone was born in the same year so you may be working with different numbers in this problem.

Stuart[1]: What's the prime factorization? Should we --

Betty: Remember the clue yesterday? The end of the word sounds like what? Let's see the hands of who can remember what factor<u>ization</u> sounds like. Does anyone remember the trick I told you yesterday to remember? [About half of the class raises their hands]

Andrea: Factor<u>ization</u> sounds like multi<u>plication</u>!

Betty: Right! Factor<u>ization</u> sounds like multi<u>plication</u>, so that means we're going to have a multi<u>plication</u> problem. Now, how do we find a prime factor<u>ization</u> -- what were we doing yesterday?

Dwayne: Making factor trees of the prime numbers that divide into the number.

Betty: Right! Very good! So what everyone needs to do is find the prime factorization, by making a prime factor tree, of their birth year.

---

[1]All students' names are pseudonyms. In choosing pseudonyms, I have attempted to represent each student's ethnicity to provide a sense of the diversity of Betty's class.

Directing her students to work with a partner if they would like, Betty began to circulate around the room as her students grouped themselves and started working on the problem. Betty's students were obviously used to working together. As Betty moved around the room she answered students' questions about getting started on the problem:

Stephan: How do we know if our birthday, I mean our birth year, is prime or not?

Betty: Stephan, use your calculator and what you know about prime factors to figure that out. Remember? Start out by finding the prime whose square is bigger than your birth year[2]. Does that help?

Stephan: Oh yea. Okay, I think I got it now [presses some keys on his calculator]. So, like, 47 times 47 is (pause) 2209, so I check the numbers --

Betty: Not the numbers, the prime numbers.

Stephan: Yea, I mean the prime numbers, less than 47?

Betty: That's it -- got it? [Stephan nods] Compare your strategy with someone else and see if they're finding the prime factorization like you are. [Betty continues circulating].

After the class had worked on the problem for about 15 minutes, Betty called for everyone's attention back up at the overhead. Betty asked if anyone wanted to share their solution. Raising her hand, Angela said, "I think that the prime factorization of 1981 is 7 times 283." There were nods of agreement from about one-third of the class. Writing 1981 = 7 x 283 on the overhead, Betty asked the class, "What do other people think? How do we know that this is the prime factorization?" Stephan raised his hand and says, "Well, cause 7 x 283 is 1981 and both of those numbers are primes, so they work." Betty

---

[2]Prior to this lesson, Betty's students had completed an activity in finding prime factorizations of numbers from *Factors and Multiples*. One of the intentions of the activity was for students to understand that to find the prime factorization of a given number, only the primes whose squares are smaller than the given number need to be tested (see Fitzgerald et. al., 1986).

replied, "Does everyone agree with that? Is 283 really prime?" Some students punch buttons on their calculators and the rest of the class nods. Betty then said, "Since both of these numbers are prime, yes, this has to be the prime factorization, because, remember, a number only has one prime factorization."

The preceding vignette is from a lesson on factors, multiples, and primes that Betty taught four days before beginning to teach *Covering and Surrounding*. Betty often poses problems to her students at the beginning of the class period, which they then work on for the remainder of class time. As her students work on a problem, Betty moves briskly about the classroom, answering students' questions, challenging them to come-up with another solution or strategy, and encouraging them to compare answers and ideas with one another.

An experienced and dedicated middle school mathematics teacher, Betty brings tremendous enthusiasm and energy to her sixth-grade mathematics instruction. Betty has taught middle school for 20 years. She taught seventh and eighth grade as a substitute teacher for 4 years, and has been teaching sixth-grade full-time for the last 16 years. Although she has also taught English, social studies, and reading during her career, Betty especially loves to teach mathematics. Betty feels that while students seem to struggle more with mathematics than any other subject, helping students understand and enjoy mathematics is challenging and rewarding.

Betty traces her own interest in mathematics to her father who is a mechanical engineer. Originally majoring in mechanical engineering in college, she switched her major to elementary education by the end of her first year. Difficulty with chemistry, as well as with adjustment being a woman in the engineering school, and an increasing interest in teaching, were the factors that eventually led Betty to switch her career goal to teaching. Betty completed her master's degree in education in 1976.

Betty teaches in a middle school that enrolls about 550 students in grades six, seven, and eight. The class periods at Betty's school are 41 minutes long. The short class periods are an ongoing concern for Betty, who emphasized on a number of occasions that 41 minutes "just isn't enough time to do mathematics!" Betty teaches mathematics two class periods per day and she taught *Covering and Surrounding* in both of her mathematics classes.

Betty's school district does not have a tracking program for mathematics in grade six. In grades seven through twelve, however, the district does track students in mathematics. Betty describes her students as a "heterogeneous group" and does not believe that tracking is beneficial to students' learning.

Betty and the Domains of the Framework

During the summer prior to teaching *Covering and Surrounding*, I interviewed Betty to learn about her views, knowledge, and beliefs within the four domains of the analytic framework developed in Chapter 2 (see Appendices A, B, and C for interview protocol and items). Some observational data of Betty prior to teaching *Covering and Surrounding* is also used in this section. Important landmarks which help define the terrain of Betty's views, knowledge, and beliefs are explored in detail in this section with respect to each of the domains of the framework. Key conceptions within Betty's views, knowledge, and beliefs about mathematics and problem solving, problem-solving activity in classrooms, subject-matter knowledge, and perceptions and beliefs about student learning are identified.

Views and Beliefs About Mathematics and Problem Solving

Betty views problem solving as central to teaching and learning mathematics and computation as only a minor part of school mathematics:

> We try to solve problems every day! That's what mathematics is all about! Why bother to learn all this stuff if you're not going to use it to solve problems? It's a waste of time just to do a bunch of drill and computation, other than the fact that, I guess, you learn, you know, your times tables that way and things like that, but what good do the times tables do you if you never use them? We really do try to do lots of problem solving, and I think, gosh, how often do we do it? I'm trying to think -- I mean, you're asking them questions all the time, asking them to solve problems!

Betty believes that she teaches problem solving as an integral part of her practice and that problem solving is routine in her mathematics teaching.

Betty describes herself as trying to teach mathematics so that her students don't just memorize formulas but understand how to solve problems. Betty says that wants her students to learn mathematics so that

> mathematics is interesting. It's intriguing, and there's more there than just a right answer. And it's OK to not always use paper and pencil in mathematics. A lot of exploration and experimenting is done without a paper and pencil. To me, you don't need to memorize formulas and practice them. Mathematics is exploratory and creative -- it's not just a bag of tricks and rules! If you just memorize formulas then you really don't even know when to use them or what they mean - they're just there! This way [i.e., exploring and experimenting], if they don't remember what the formula is, they can figure it out.

Betty believes that learning mathematics in school should include opportunities for students to explore and experiment with mathematics and to formulate and test their own ideas.

The snapshot of Betty's lesson on factors, multiples, and primes at the beginning of this chapter, when compared with her expressed beliefs about mathematics above, is somewhat paradoxical. While Betty believes that mathematics is not "just a bag of tricks and rules", she did give students a mnemonic device in this lesson. The mnemonic was not connected to the concepts of factors or multiples but was intended to help her students remember that factor<u>ization</u> means a multi<u>plication</u> problem. Still, this does not mean that she teaches rulebound mathematics despite her stated beliefs. Rather, Betty's own views and beliefs about mathematics may be enacted to varying degrees in her mathematics teaching. While Betty's expressed views and beliefs about mathematics and problem solving are oriented around learning mathematics as "exploratory and creative", her practice includes problem solving as solving both standard word problems and non-standard, more open-ended problems.

Betty's use of the mnemonic in the birth year problem illustrates problem solving as solving standard word problems -- Betty gave her students the problem and then helped them translate it into an operation (i.e., check for divisibility by all primes whose square is less than the birth year) to solve it (see Chapter 4). In contrast, two days after the birth year problem, Betty assigned this "mystery number" problem to the class:

> I'm thinking of a number that's between 2000 and 3000. This number has the prime factors 2, 3, 7, and 17. What are all the possibilities that this number could be?

Betty supplied her students with no hints or explicit guidance as she had in the birth year problem. She assigned the problem as part of a homework assignment and instructed her students to work on the problem independently and record their strategy for solving it in their math journals along with their solution. The mystery problem was, for Betty's students, what Hembree (1992) would classify as a "non-standard (process or open-search) problem" which "encourage(s) the use of flexible methods; the solver possesses no routine procedure for finding an answer" (p. 249). This kind of "open-search" problem is more congruent with Betty's beliefs about mathematics being exploratory and creative.

Betty's expressed views and beliefs about mathematics and problem solving place emphasis on representing mathematics as a domain of inquiry that is more than "just a bag of tricks." However, in Betty's teaching, she represents problem solving as both implementing rules and procedures to solve standard word problems (e.g., the birth year problem) and as solving more open-ended, non-standard problems (e.g., the mystery number problem). Therefore, while Betty's expressed views and beliefs about mathematics center on teaching mathematics and problem solving as exploratory, creative, and open-ended, her practice includes a range of representations. These representations vary from mnemonics and rules to more open-ended problems.

Views and Beliefs About Problem-Solving Activity

Betty conceptualizes problem-solving activity in her classroom as "exploring and experimenting." By exploring and experimenting, Betty means investigating problems unencumbered by rigid guidelines or rules to learn about mathematical concepts and processes. Exploring and experimenting involves formulating strategies and approaches to solve problems and also making sense of and interpreting data:

AR: What kinds of activities would you say you and your students do when you're working on problem solving?

Betty: They may be experimenting, measuring -- they may be manipulating things such as tiles and stuff like that to try to find out, say, how do you find the area of this table, what's one way they could do it? One group might decide to use tiles, another group might decide to use a ruler, you know, depending on what they're doing. (pause) They do a lot of work with graphs, reporting the data -- so you found this data, what form can you put it in, and then once you do that what does it tell you? They do some writing, trying to explain things, like corresponding with other groups of people to help them learn how to do something. I've sent them home to do problem solving -- for example, finding out the height of the people in their family. First of all they had to figure out what kinds of units they were going to use and then what kinds of tools they would need.

Betty's conception of problem-solving activity as exploring and experimenting is further illustrated by her comments about her students' work on the mystery number problem discussed earlier:

> I'm thinking of a number that's between 2000 and 3000. This number has the prime factors 2, 3, 7, and 17. What are all the possibilities that this number could be?

Betty said that in solving the mystery number problem

> The students experimented with different approaches. Like one kid, he tried to list all the numbers between 2000 and 3000 divisible by 17 and others tried to work backwards. Like, another student, she said that 2 x 3 x 7 x 17 = 714 so you can't use the 7 or the 17 again because the number would be too big. They were trying different things and explaining their approaches -- that's what I mean by exploring -- they come up with a strategy and then see if they can solve the problem with it. Doing stuff like that is how kids should be learning math!

Betty's comments emphasize that she believes exploring and experimenting is open-ended, allowing students to develop problem-solving processes as well as understandings about mathematical concepts.

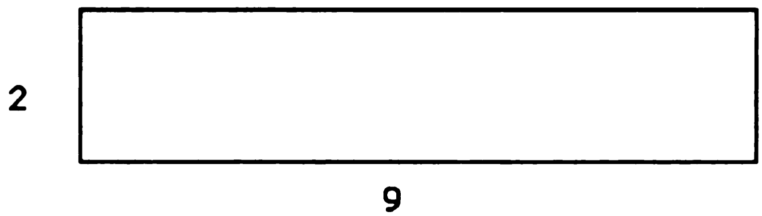
While Betty characterizes problem-solving activity as exploring and experimenting, she also describes classroom activity that was not as open-ended as problem solving. For example, Betty described the mystery number problem as an example of problem solving and she also referred to the lesson on the birth year problem as problem solving. However, as the vignette at the beginning of this chapter illustrates, the birth year problem was less open-ended than the mystery number problem. In the birth year problem, students were instructed to solve the problem using factor trees and were reminded that factoring involves multiplying. The mystery number problem, by comparison, is more open-ended. The mystery number problem leaves students to explore and experiment more broadly than in the birth year problem where firm guidelines were established at the outset (e.g., make a factor tree).

Comparison between the birth year and mystery number problems does not imply that the latter was problem solving and the former was not. Rather, the contrast shows that while Betty describes problem-solving activity in terms of exploring and experimenting, her teaching practice includes more structured experiences with problems that she characterizes as problem solving. As with her views and beliefs about mathematics and problem solving, Betty's teaching demonstrates a wider range of conceptions of problem-solving activity than her expressed views and beliefs. Along with problem-solving activity in Betty's classroom including non-standard (process or open-search) problems like the mystery number problem, standard word problems like the birth year problem play a role also. As the birth year and mystery number problems illustrate, some of the problems Betty provides for her students involve more "exploring and experimenting" than others. However, this variation in practice is not completely represented in her expressed views and beliefs about problem-solving activity.

Subject-Matter Knowledge

Betty's subject-matter knowledge of perimeter and area is rich and connected. She is able to use both symbolic representations and conceptual relationships to understand and work with students' non-standard algorithms and conjectures. For example, Betty's subject-matter knowledge helped her to unpack a non-standard technique for finding perimeter of rectangles. This was demonstrated in her response to a question about a student's non-standard way of calculating perimeters of rectangles prior to teaching *Covering and Surrounding* (see Appendix B). In making sense of the student's work from two examples provided, Betty derived an expression that was a generalization of the student's method for finding perimeter. Below is one of the two examples of student work and Betty's comments as she constructed a mathematical explanation of what the student was doing:

Figure 6.1: Student's work on finding perimeter of a rectangle:



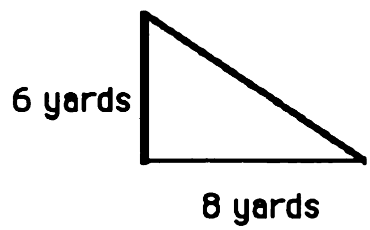
$$P = 2 \times (18/9 + 18/2) = 2 \times (36/18 + 162/18)$$
$$= 2 \times 198/18 = 198/9 = 22$$

> Well, 2 times the quantity -- and this would be A divided by b, the base, A being area -- plus area again [begins writing a formula out on paper] divided by the height and we can get rid of the times sign [continues to write and produces the formula P = 2 x (A/b + A/h) where A = b x h. Betty then writes the formula as P = 2[(Ah + Ab)/bh]. Well, that would be the rule. That's good enough!

While Betty did not *prove* that the student's rule was valid[3], she was able to obtain a general expression for how the student was computing perimeter. In this case, Betty's subject-matter knowledge enabled her to work flexibly with symbols to uncover how the student was most likely thinking about finding perimeter of rectangles.

Directly related to her subject-matter knowledge, Betty demonstrated a conceptual understanding of relationships between units for measuring area. Moreover, she drew on pedagogical content knowledge to craft a concrete learning experience to address a student's understanding of relationships between square units. Below is an interview excerpt (see Appendix B) where Betty addressed a student's conjecture about converting from square yards to square feet in measuring the area of a triangle:

Figure 6.2: Student's conjecture about square units for area:



A = 1/2 x B x H = 1/2 x 6 x 8 = 1/2 x 48 = 24 square yards
3 feet = 1 yard so Area = 24 x 3 = 72 square feet

AR: Now, suppose that during your instruction on perimeter and area one of your students raises his hand and is very excited. He says that he has figured out how to convert between different units used to measure area. And he shows you his solution to this area problem [referring to Item 2 above] on this triangle and that he came up with 24 square yards. Now, he says that since there are 3 feet in a yard he can convert the area from square yards to square feet How would you respond to this student?

---

[3]Replacing area in Betty's expression with bh yields: P = 2 x [(Ah + Ab)/bh] = 2 x [(bhh + bhb)/bh] = 2 x [bh(h + b)/bh] = 2 x (h +b) = 2h + 2b = h + b + h + b, which is the perimeter of a rectangle of height h and base b. So Betty's expression is a valid formula for finding the perimeter of a rectangle of dimensions b and h.

Betty: Well, first of all, I'd congratulate him for putting those two things together. Then I think I would ask him if he could describe what a square foot was -- could he draw a picture for me of a square foot. If he could not I'd help him draw, you know, a square and we could measure it one foot by one foot. And then we would talk about what would be the area of that -- that's one square foot. And then I'd say, well if that works for that, does it stand then that if I take a square yard and do some the same thing, have it one yard on each of the sides, what would it's area be -- well, one square yard. And then I would ask him to look -- I might even have him cut it out. Some times if you have them cut out a square foot and then (pause) cut out a square yard, it's much easier to see that, very obviously, it's going to take more than three of these [i.e., square feet] to fill-up this [i.e., the square yard]. Well, how is that possible? Sometimes the touching, the feeling of it makes it - no matter what you would say -- that makes it so much simpler. My guess is I usually have somebody who says 'But wait a minute, you have to measure along the side and find it's length first in the same units.' That means if you're going to talk about feet, well that one over there, you should say instead of being one yard long that it's three feet long on a side. Oh, well then you could say, that's like three times three -- so 1 square yard is really 9 square feet!

Betty drew on her pedagogical content knowledge in describing how she would represent the student's conjecture in a way that would allow him to explore the relationship between square feet and square yards in greater depth. Rather than telling the student he is wrong and showing him the correct conversion factor (i.e., multiply square yards by 9 to convert to square feet), Betty stressed the need to assess the student's current understanding and then connect him concretely with the two different area units. The instructional experience Betty describes draws on her knowledge of area concepts, relationships, and beliefs about how students learn to connect the student with the relationship between square yards and square feet This blend of subject-matter knowledge and pedagogy is what some researchers refer to as pedagogical content knowledge (see Chapter 2).

Taken together, the two excerpts suggest that Betty's knowledge of perimeter and area concepts facilitates her ability to unpack students' understandings about perimeter and area as well as inform her pedagogical reasoning. Betty's ability to move between symbolic (e.g., deriving the perimeter formula) and pictorial representations (e.g., her description of comparing actual square yards and square feet) of perimeter and area concepts further emphasizes the depth of her pedagogical content knowledge about perimeter and area.

Perceptions and Beliefs About Student Learning

Betty believes that students, particularly sixth-graders, best learn about mathematical concepts through concrete learning experiences that utilize physical objects (e.g., manipulatives, like tiles). For example, Betty described how important she thought it was for middle school students to learn about perimeter and area by physically placing square tiles on figures to measure perimeter and area. Betty argued that most curricular materials merely *describe* how area and perimeter of figures might be found by using square tiles and that

> a lot of kids at sixth-grade are not ready for this just being described. They have to have the kinesthetic, they have to pick it up, touch it, set it down, touch each tile as they count. And then after they have some time with that, to see it, then it's a piece of cake, because now they understand what perimeter and area are.

Elaborating on how she believes sixth-graders learn mathematics for understanding, Betty maintained that students must be given "a chance to discover and experiment around, and try to pull out some concepts, some patterns, looking for those kinds of things." Betty believes that this is what students have opportunities to do by using manipulatives, like tiles, to learn about perimeter and area.

Betty's belief that students learn about perimeter and area concepts by working with manipulatives is supported further by her prior teaching. For example, although Betty had not taught *Covering and Surrounding* before, she had previously taught a unit called *Mouse and Elephant: Measuring Growth* (Shroyer & Fitzgerald, 1986) several times. The *Mouse and Elephant* unit is an activity-oriented unit that makes use of a variety of manipulatives (e.g., 1-inch square tiles, cubes, grids) and representations (e.g., tables, graphs) to teach students about the concepts of area, perimeter, surface area, and volume and the relationships among them[4] (Lappan, 1983; Shroyer & Fitzgerald, 1986). Betty liked the fact that *Covering and Surrounding* also makes use of tiles to teach perimeter and area concepts and was interested in how the unit explores perimeter and area of irregular figures and circles, which the *Mouse and Elephant* unit does not address. Betty's past use of curricula that use manipulatives reinforce her stated belief that students learn mathematics by having concrete learning experiences where they can work with physical objects.

Another facet of Betty Walker's perceptions and beliefs about student learning is that students learn mathematics by working together. Betty says that cooperative groups are an ideal setting for students to "experiment with mathematical ideas" because "they share those ideas, they talk over those ideas." Betty emphasized that "I believe kids

---

[4]The *Mouse and Elephant* unit was developed by the Middle Grades Mathematics Project (MGMP). The MGMP was a middle school curriculum development project at Michigan State University that developed a series of problem-solving oriented units in the early 1980s (see Lappan, 1983). Among the MGMP developers were William Fitzgerald, Glenda Lappan, and Elizabeth Phillips. These individuals are also co-directors of the Connected Mathematics Project and contributed to the development of *Covering and Surrounding*.

learn so much from one another -- it doesn't bother me when there is a whole lot of good talking going on in my room."

It was obvious from watching Betty's students prior to *Covering and Surrounding* that they routinely work together in groups. For example, during her preparation period immediately preceding mathematics class on 10/12/92, Betty moved all of the desks in her classroom together into clusters of four. On one of the desks in each cluster she taped a different color index card. As her students entered the room, they reached into a paper bag Betty was holding and (randomly) drew out an index card. Each student then sat down at the desk cluster with the same color as the card they had drawn. Betty explained that this was "a non-biased way of grouping the kids so they can really learn from one another in class and not just their friends." One of Betty's students remarked that they chose groups like this "about once a week" and that "we work together in groups all the time."

Students needing concrete learning experiences with manipulatives to learn and understand mathematics, and students learning from one another, are two predominant themes in Betty's perceptions and beliefs about student learning. These perceptions and beliefs are evident in Betty's talk about her own teaching and are clearly enacted in how she teaches mathematics in the classroom. Betty's past use of curricular materials that use a wide range of manipulatives and her routine use of cooperative groups are congruent with her expressed beliefs about how students learn. Betty's comments suggest that she is predisposed to teaching a unit like *Covering and Surrounding* because it does make extensive use of manipulatives to teach perimeter and area concepts.

Summary

Betty believes that mathematics is an interesting and intriguing subject and that problem solving is "what mathematics is all about!" Betty's expressed views about problem solving and problem-solving activity center on students developing strategies and approaches to learn problem-solving skills as well as understanding mathematical concepts. However, in her teaching Betty demonstrated use of standard problems (i.e., implementing a specific procedure) as well as open-ended problems. Problem-solving activity in Betty's classroom is more variable than her characterization of "exploring and experimenting" would indicate. This is because some of the problems are more (or less) open-ended than others (e.g., the birth year problem as compared to the mystery number problem), entailing different degrees of exploring and experimenting.

Betty's subject-matter knowledge of perimeter and area is rich and flexible. Betty's pedagogical content knowledge allows her to address students' understandings about perimeter and area concepts consistent with her emphasis on exploring and experimenting. Betty's perceptions and beliefs about student learning are oriented around students using manipulatives to understand concepts like perimeter and area, and the belief that students learn mathematics from one another. Both of these perceptions/beliefs about student learning appear in Betty's classroom instruction, shaping the way she structures her class (e.g., into cooperative groups) and in her choice of curricula (e.g., prior teaching of the *Mouse and Elephant* unit).

Betty's Use of Covering and Surrounding

This section begins by providing a summary of Betty's coverage of *Covering and Surrounding*. A synopsis and brief analysis of the exercises Betty assigned is given to convey which problems she emphasized most in her teaching of the unit. The subsequent sections each describe and analyze a specific characteristic of Betty's use of *Covering and Surrounding*. I focus on three aspects: (1) Issues surrounding Betty's characterization of problem-solving activity as exploring and experimenting; (2) use of teacher materials, and; (3) changes in practice while teaching *Covering and Surrounding*. Throughout these sections, Betty's views, knowledge, and beliefs in the four domains of the framework are employed to unpack and understand her use of *Covering and Surrounding*.

Summary of Unit Coverage

Betty began *Covering and Surrounding* on 11/9/92 and finished teaching the unit on 12/14/92. Betty began with Investigation 1 and sequentially covered the first five investigations of the unit. Throughout her instruction, she emphasized the Problems and Follow-Up Questions in each investigation, with ACE items being assigned if she felt that time permitted. Betty believed that her students were "exploring and experimenting" (i.e., doing problem solving) throughout *Covering and Surrounding*, but the degree to which students had opportunities for open-ended exploration and experimentation varied. Moreover, Betty did not use the *Covering and Surrounding* teacher materials much and this influenced the degree to which the teacher materials had the potential to shape her teaching. After teaching the unit, Betty's perception was that teaching *Covering and Surrounding* did not influence her to teach perimeter and area differently, nor did she perceive a need to change her teaching. For Betty, teaching *Covering and Surrounding* involved nothing unfamiliar and was perceived by her to be congruent with her prior teaching of perimeter and area.

Table 6.1 below summarizes the sequence in which Betty taught the investigations in *Covering and Surrounding* and the Problems, Follow-Up Questions and ACE items she assigned from each:

Table 6.1: Betty: Summary of Problems, Follow-Up Questions, and ACE items assigned.

Investigation Title	Assigned Problems	Assigned Follow-Up Questions	Assigned ACE Items
1 - Measuring and Designing Rooms	1, 2, and 3	1	1 thru 6
2 - Areas and Perimeters of Figures with Irregular Edges	1 and 2	1-3 (Problem 1) 1-4 (Problem 2)	1 thru 3
3 - Going Around in Circles	1, 2, and 3	1, 2, and 6 through 11	none
4 - Constant Area, Varying Perimeters	1	1 thru 5 and 7	1 thru 5, 9 and 10
5 - Constant Perimeter, Varying Area	1, 2, 3, and 4	1 thru 4	none

Referring to Appendix D and noting the exercises Betty assigned in Investigation 1 from Table 5.1 gives a particular example of her coverage of an investigation in *Covering and Surrounding*. To provide a sense of how Betty's coverage compares to the number of exercises in the unit, Table 6.2 below shows the number of Problems, Follow-Up Questions, and ACE items she assigned in each investigation she covered and the total number (given in parenthesis):

Table 6.2: Betty: Assigned number of exercises compared to total.

Number of Investigation	# Problems Assigned and (total)	# Follow-Up Assigned and (total)	# ACE items Assigned and (total)
1	3 (3)	1 (1)	6 (10)
2	2 (2)	7 (7)	3 (4)
3	3 (3)	8 (11)	0 (6)
4	1 (1)	6 (7)	7 (14)
5	4 (4)	4 (4)	0 (8)

Table 6.2 shows that Betty assigned all of the Problems in each of the investigations she covered and most of the Follow-Up Questions as well. There is, however, variance in the proportion of ACE items Betty assigned her students. Table 6.3 below illustrates the proportion of Problems, Follow-Up Questions, and ACE items Betty assigned her students in each investigation in percentages to more clearly illustrate her emphasis on Problems and Follow-Up Questions:

Table 6.3: Betty: Percentage of exercises assigned in each covered investigation.

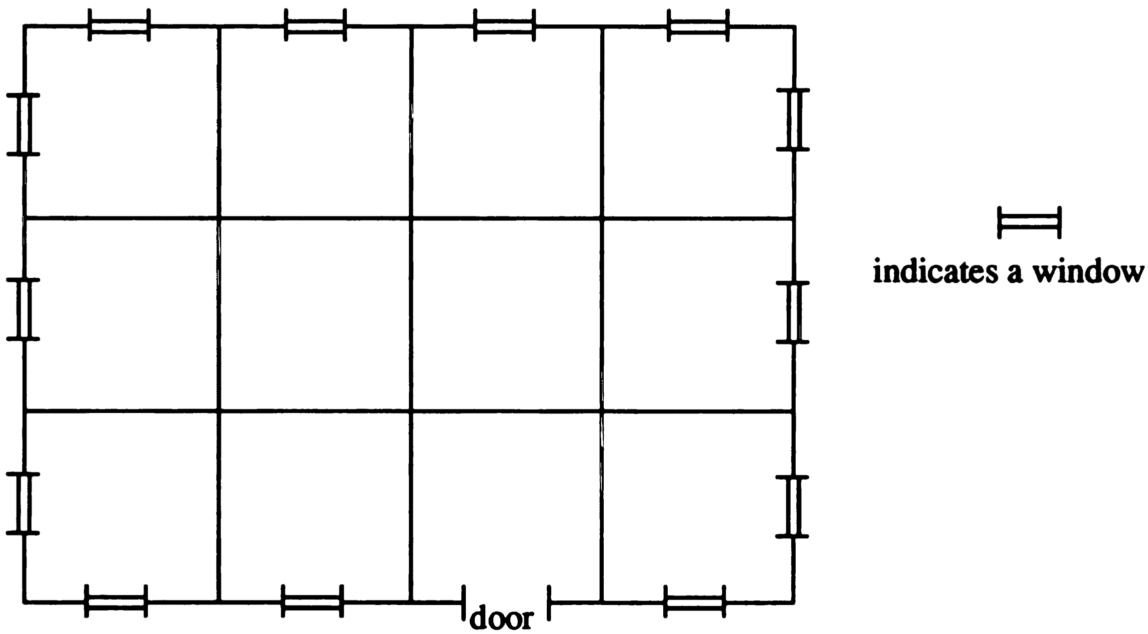
Number of Investigation	% Problems Assigned	% Follow-Up Assigned	% ACE Assigned
1	100%	100%	60%
2	100%	100%	75%
3	100%	72%	0%
4	100%	85%	50%
5	100%	100%	0%

Problem Solving: Different Degrees of Exploring and Experimenting

Betty believed she emphasized "exploring and experimenting" in *Covering and Surrounding*, but the degree to which students had opportunities to explore and experiment varied. Betty was directive in her teaching of some of the investigations, explicitly telling students what she wanted them to do. In other investigations, however, Betty allowed her students to wrestle with problems, guided them, and helped them reach their own conclusions. Regardless of the the opportunities for exploring and experimenting in lessons, Betty believed her students were learning about problem solving. Two lessons from Betty's teaching of *Covering and Surrounding* are illustrative of these points.

Betty began *Covering and Surrounding* with Problem 1 in Investigation 1. To begin the unit, Betty asked the class to open up their copies of *Covering and Surrounding* to the first page and look carefully at the room Mr. Dull designed for Mrs. I Wanna Hide (see also Appendix D):

Figure 6.3: Mr. Dull's design (CMP, 1992a, p. 1).



As her students were looking at Mr. Dull's design, Betty clarified the representation by explaining how doors and windows are indicated on the room design:

> When it [i.e., Mr. Dull's design above] shows a window, if you were actually seeing the window itself, you'd be looking at the wall with the window in it. If you were looking at the door wall, the section of the wall that had a door in it, you would actually have a door. What you are seeing on the plan is like the edge -- the *edge*. You're actually seeing like this line right here on the floor [points to a section of the base board of the classroom where it meets the floor]. That's what you're seeing on the plan.

> Sometimes walls can be blank. In this particular case, Mr. Dull has designed this room so it can get lots of sunshine. So there are lots of windows.

Betty then distributed about 15 one-inch tiles and a sheet of grid paper to each student and instructed the class to cover Mr. Dull's design with tiles. Betty emphasized to her students as they began placing the tiles, "Remember that the tiles must touch all the way." Betty quickly held up two tiles and showed how they should touch fully edge-to-edge, like the grid squares in Mr. Dull's design. After the students placed their tiles, Betty and her class had this conversation:

Betty: All right, now how many tiles did it take to *cover* Mr. Dull's room?

Jaime: Twelve!

Betty: Good. And how many tile edges does it take to *surround* Mr. Dull's room?

Class: [pause while students count] Fourteen.

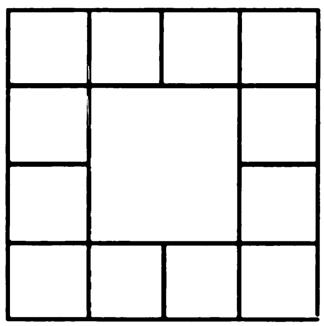
Betty: So it takes 12 tiles to cover Mr. Dull's room and 14 tile edges to surround his room. Now, another name for covering is *area*, and another name for surrounding is *perimeter*. So Mr. Dull's room has an area of 12 and a perimeter of 14. Does everyone see that? [about half of the class nods.] Now in this first problem, you'll be designing your own room. Read the problem carefully before you get started and remember to make your room with tiles first and then trace it on grid paper like Mr. Dull traced his -- okay? [more nods from the class].

Betty's students begin reading Problem 1 (CMP, 1992a) from the *Covering and Surrounding* unit:

> **Problem 1:** Mrs. Hide likes the amount of floor space in Mr. Dull's design, but she wants more windows and a more interesting shape for her room. She asks you to help her out.
>
> a) Use 12 square tiles to create a floor plan design for Mrs. Hide. Remember that a window or a door can go into each section of wall space. Mrs. Hide tells you that she wants at least 14 sections of wall space, including windows and doors, in the room you are designing.
>
> b) After you have designed the floor plan for the room make a drawing to show the location of the door and where each window is. Write a paragraph to tell Mrs. Hide why your design is better than Mr. Dull's (p. 2).

Within two minutes, most of the students in Betty's class were moving tiles around on their desks forming designs for rooms. As her students began Problem 1, Betty circulated around the room, watching what students were doing and answering their questions. Max was one of the first students to ask Betty a question about a room he had designed. Max showed Betty his room which had a "courtyard" in it. He was confused about whether he should count the perimeter of the courtyard as part of the perimeter of his room. Betty looked carefully at Max's room before responding:

Figure 6.4: Max's room:



Betty: If there can be window or a door, than that's included in the perimeter [i.e., each exterior tile edge of the 'room' can have a 'door' or 'window' built into it]. But now, be careful. You're talking about two different kinds of perimeter. This looks to me like if you do this you have an inside courtyard, am I correct? You have plants, and trees, and things like that?

Max: Yea, that's what I meant.

Betty: Let's look at the outside perimeter and let's compare the outside perimeter of the room with the courtyard. When we do these [flipping back to rooms A - J in Problem 2, Investigation 1 -- see Appendix D] we'll compare the outside perimeters. Would that be fair? Remember, perimeter is *surrounding* and are you surrounding the room if you count around the courtyard?

Max: Oh! OK, so we shouldn't include courtyards.

Betty: Right. Because, if we did, we might not always be consistent in how we count perimeter.

Max seemed satisfied that his question had been answered, deciding not to count the perimeter of the courtyard. He did not change his design, but crossed out "perimeter is 24" on his paper and wrote "perimeter is 16" instead. After talking with Max, Betty continued to circulate around to individual students for the remaining five minutes of class, during which she also collected tiles. Betty then announced to her students that they would be finishing their room designs the next day in class. After class, Betty commented on what she thought her students had learned about problem solving:

> Well, they got to work with the tiles and began to design their rooms. They're exploring and experimenting with different shapes to be creative and make a room with 12 tiles that has a perimeter of at least 14. I'd say they're doing a lot of problem solving as they're experimenting with the tile rooms.

In her use of the designing room problem, Betty began the lesson by trying to make sure her students understood the wall sections and carpet tiles analogy of perimeter and area, respectively. Betty was then very explicit with her students about what perimeter and area are and connected the concepts of perimeter and area back to the wall sections and carpet tiles analogy by measuring Mr. Dull's room with the tiles. In her teaching to this point in the lesson, Betty had provided little opportunity for her students to do any "exploring and experimenting." For example, Betty did not have her students cover Mr. Dull's room with tiles, ask them to interpret what perimeter and area are, and then discuss students' conjectures. Instead, Betty was clearly concerned that her students understand how perimeter and area are represented with the tiles and how they are used to measure the perimeter and area of rooms. While this is a defensible instructional approach (e.g., students need to understand how to use the tiles before measuring perimeter and area with them), Betty telling her students what to do with the tiles leaves little room for exploring and experimenting.

As students began working on Problem 1, however, they did begin to explore and experiment. The problem of designing your own room with 12 tiles such that the perimeter is at least 14 is a problem with multiple solutions and students need to decide, at some point, if their room design met the problem conditions. This is precisely what Max was struggling with when he asked Betty about whether he should count the perimeter of the courtyard in his room as part of the perimeter. Max's question had the potential to provide an interesting context for exploring and experimenting. Although Betty simply told Max that he should not count the perimeter of the courtyard, this position could be challenged. For example, if Max had to purchase baseboard for his room design he *would* need to count the wall sections enclosing the courtyard as part of the perimeter. Investigating Max's question actually seems well-suited to the kind of exploring and experimenting activity that Betty emphasized in her interviews prior to *Covering and Surrounding.* However, in this case, she did not pursue the opportunity.

On 11/30/92 Betty began Investigation 4 in *Covering and Surrounding* with her students. Betty's use of the Problem and its Follow-Up Questions in Investigation 4 (there is only one Problem in Investigation 4), relative to her use of Problem 1 in Investigation 1, provided greater opportunity for students to explore and experiment. Betty said before she started Investigation 4 that she was going to have her students do the investigation in cooperative groups. In the Problem and Follow-Up Questions, the students are to assist Jennifer in making a recommendation to the City Council of Clean City, Michigan about what kind of 24 square meter room should be added on to the city administration building (CMP, 1992a, pp. 31-32):

> **Problem:** Jennifer has decided that the room must be a rectangle to keep costs down. Using 24 square tiles, build every possible rectangular room that Jennifer could suggest. Cut a rectangle out of grid paper to show each room that you found. Label the bottom edge and side edge of each.
>
> Make a table that shows the data from all of the rooms.

Follow-Up Questions:

1. Order your rectangles from the largest perimeter of the smallest perimeter on your desk. How are they changing? What patterns do you see? How does the rectangle with the largest perimeter and the rectangle with the smallest perimeter compare in shape?

2. Working with your group, figure out how to make a graph that shows how the side edge of the rectangle changes as the bottom edge gets larger.

3. Make a graph that show how the perimeter changes as the bottom edge gets larger.

4. Look carefully at your two graphs and describe all the patterns that you see. Look back at you data table. What patterns do you see in the data?

5. What do you think Jennifer should recommend to the City Council and why?

7. Describe a way to find the area of a rectangle. Now try to describe a different way to find the area of a rectangle.

Betty explained how each group would represent their findings in answering the Problem and its Follow-Up Questions above:

> I'm going to have each group create a poster. And on that poster they're going to have to include all the rectangles that they cut out. They're going to have to include their tables to keep their data. And it also has to include their written recommendation to the City Council. "What should Jennifer recommend to the Council" -- so they're actually going to make a poster that represents their group's findings.

After going over the problem with the class, Betty emphasized that

> The purpose of doing this problem in groups is so that you can explore and learn about ideas together with your classmates -- there is not one right answer, what is important is that you need to work together to formulate a recommendation that makes sense.

All of Betty's other comments to the students to start the Problem focused on organizing their groups, not on the Problem itself.

During their second day on Investigation 4, one of the groups in Betty's class encountered a problem in deciding if they had found all of the possible rectangular rooms that could be made with 24 tiles. One of the members of the group, Hillary, asked Betty about this:

Betty: You aren't sure about how many rooms there are?

Hillary: Yea. Because we need to find all the rooms with a (pause) with 24 tiles and put them on the table. But we aren't sure if we have them all.

Betty: Hmm. Well, let me ask you this -- think about the rooms you have found -- what rooms have you found so far?

Hillary: Well, we've got one with a bottom edge of 12 and a side edge of 2, and another with a bottom edge of 8 and a side edge of 3 -- I don't know the others.

Betty: Okay. Well, why don't you go back to your group and look carefully at the rectangles you do have and see if you can find any relationships. You just talked about the bottom edge and side edge -- work in your group to see if the bottom edge and side edge might help you decide how many rooms there are with 24 tiles.

The next day, Hillary's group had resolved the question of finding all of the rectangular rooms that can be made with 24 tiles. The group had determined that the connection that the rectangles with area 24 have with the bottom edge and side edge is that the rectangles were really "the areas of factor pairs of 24" (Hillary's words). Hillary said that, after "thinking hard" with her group, they decided that, "When we got all the factor pairs of 24 we got all the rectangles.[5]"

---

[5]For example, 1 and 24, 2 and 12, and 3 and 8 are all factor pairs of 24 because their products are 24. One interpretation of the product of two numbers is the area of a rectangle that has length and width (i.e., bottom edge and side edge) corresponding to the two numbers in the factor pair. Hillary and her group recognized that since factor pairs correspond to rectangles, one way of finding the bottom edge and side edge of all the rectangles with area 24 is to find all the factor pairs of 24.

After the lesson, Betty noted that the poster project was an example of "group problem solving" and that along with problem solving through using tiles and graphs, the students learned to problem solve together:

> One of the things is that they're learning interdependence. They don't have to rely on only themselves for every step of the problem. That they do have a responsibility to one another in their group, in order to have the group solve the problem. It isn't just one person doing it, it's the group's responsibility to explore and experiment around and that everyone in the group knows what's going on and that they understand too.

Betty's reflections show that she believed her students were "exploring and experimenting" as they investigated both the problem of designing their own room and the problem of helping Jennifer decide which room to recommend to the City Council. Betty's characterization of students' activities in the two lessons as "exploring and experimenting" is congruent with her belief that students learn about problem solving as they explore and experiment with problems, learning about mathematical concepts and processes. But although Betty's perceptions of the two lessons share these commonalties, the degree to which students had opportunities to explore and experiment varied between the two lessons. This variation is consistent with her teaching prior to *Covering and Surrounding*, as the different degrees of exploring and experimenting parallel differences between the birth year problem and the mystery number problem.

Betty's use of the Problem in Investigation 4 contrasts sharply with the room design problem. Betty explicitly framed the problem of helping Jennifer with her recommendation to the City Council as open-ended and not having just "one right answer." Perhaps more significant, however, was how Betty interacted with Hillary when she asked her question about determining all the different rectangles that could be made with 24 tiles. Rather than telling Hillary to find all the rectangles corresponding to the factor pairs of 24, Betty suggested to her that she and her group try to find a relationship between the bottom edge and side edge of the rectangles they had already constructed. In contrast to Max's question where exploring and experimenting was not pursued, Betty used Hillary's question as a means to motivate exploring and experimenting. Both the room design problem and the group poster project are, for most sixth-graders, non-standard problems that are potentially open-ended. The variance in how Betty used the problems in her teaching led to different degrees of openness.

The varying opportunities in Betty's teaching for exploring and experimenting were similar in that each experience provided students with opportunities for problem solving, albeit different *kinds* of problem solving. For example, Betty's teaching motivated Hillary and others in her group to construct relationships between factor pairs and the rectangles connected to the factor pairs. P. Thompson (1985), for example, notes that "creating relationships is the hallmark of mathematical problem solving" (p. 190). In her interactions with Max, Betty provided him with an explanation to not count the perimeter of the courtyard and he changed his answer accordingly. Betty's teaching can be viewed as giving Max an explanation for finding perimeter so he could use a "guess and check" strategy (see Charles & Lester, 1982) to see if his room met the conditions of the problem. After designing his room (i.e., a "guess" at a suitable room design), Max used Betty's way of finding perimeter (i.e., don't count the perimeter of the courtyard) to "check" to see if the room met the problem conditions (i.e., area of 12 and perimeter of at least 14). Betty's teaching illustrates that varying degrees of exploring and experimenting can create different kinds of problem-solving opportunities for students -- in these examples constructing relationships for Hillary and using a guess and check strategy for Max (c.f., Charles & Lester, 1982; P. Thompson, 1985).

<u>Use of Teacher Materials: "I Haven't Ever Looked at That!"</u>

Betty's use of the *Covering and Surrounding* teacher materials corresponded to her prior use of teacher materials -- she hardly used them at all. Betty did not believe she had the time to use the *Covering and Surrounding* teacher edition. Moreover, she did not perceive a need to use the teacher materials. Not using the teacher edition shaped Betty's encounters with *Covering and Surrounding*, and the potential of the material to influence how she taught problem solving with perimeter and area.

After she had taught the first investigation in *Covering and Surrounding* and was midway through the second investigation, I asked Betty about how often she used the teacher edition. Betty quickly replied, "I haven't ever looked at that!" I then asked her about why she hadn't looked at the teacher edition so far in her teaching. Betty went on to talk about how the constraints she was under in teaching mathematics don't allow her the time needed to consult the *Covering and Surrounding* teacher materials:

> To be totally honest, I haven't used the teacher materials at all so far, except to get the masters to xerox grid paper and the table for the kids to record their room data [see Labsheet 1.2 in Investigation 1 teacher materials in Appendix E]. I'm always frazzled for time! I've pretty much gotten a feel for the unit as I've been teaching it. With almost 30 kids in each of my two math classes, and only

> a 41-minute period -- not to mention spelling, social studies, assemblies and activities, correcting papers, and everything else -- I really don't have the time to sit down and look at the teacher's guide!

Betty also cited that there had been two assemblies in the last two weeks which had taken away from her time for teaching and preparation.

About four days after Betty made her above remarks, I asked her how the *Covering and Surrounding* teacher materials compared to other teacher materials she had used. One of my intentions in this conversation was to discover whether Betty didn't have time to use the *Covering and Surrounding* teacher materials in particular, or whether she found little time to use teacher materials of any kind:

AR: How do the *Covering and Surrounding* teacher materials compare, in terms of how helpful they are, with other teacher materials you've used?

Betty: Well, actually they really aren't that helpful. Now I know that the current stuff is just a working draft, and I hope that you'll eventually go back to the three-column MGMP format[6] like in the *Mouse and Elephant*. But I don't even use the MGMP teacher stuff anymore really.

AR: Why not? I mean, are the MGMP teacher materials not useful anymore or --

Betty: Oh, they're fine, and the MGMP materials were useful when I first started with them. But now I'm pretty familiar with it, having taught MGMP a number of times, and I don't need the teacher stuff anymore.

AR: What about the adopted textbook you use -- is the teacher edition for the text something you use or --

Betty: [laughs] I didn't use the textbook 20 times last year! I use different kinds of materials during the year and meet our curriculum goals, but I generally do it with a variety of outside materials.

---

[6]The Middle Grades Mathematics Project materials provide the teacher with a three-column "script" which describes in detail how each lesson might be taught. The columns, "teacher action", "teacher talk", and "expected response", each simultaneously represent a different aspect of the lesson. The teacher action column specifies materials needed, how to display data, and suggestions for illustrating concepts. The teacher talk column provides important questions for the teacher to ask students to help develop understandings of concepts and problem-solving strategies. The expected response column gives the teacher correct answers to the questions in the teacher talk column, as well as anticipated common errors students might suggest (see Shroyer & Fitzgerald [1986] and Lappan [1983]).

AR: And how are the teacher materials for these outside materials helpful to you -- I mean MGMP stuff and the other things you use too?

Betty: Well, I generally don't need to use them. (pause) I mean, I might glance at them once in a while for an answer or to get a blackline master to xerox, but that's about all I need from the teacher guides. It isn't the teacher guides that are most important anyway. Now to me -- everything that I do -- it isn't the materials so much as how I do it that makes it effective or not effective.

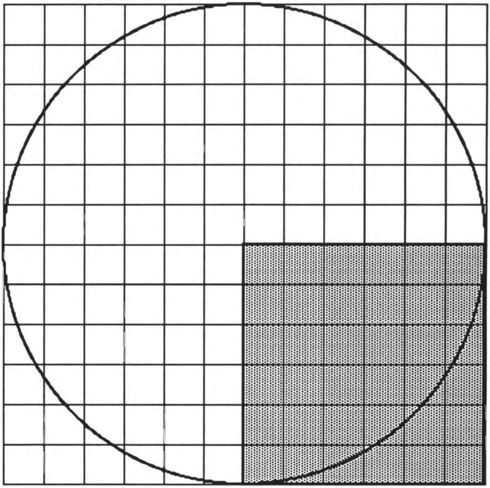
The above conversation reveals that as well as not having the *time* to use the *Covering and Surrounding* teacher materials, Betty doesn't perceive the *need* to use teacher materials in general. Betty's use of the *Covering and Surrounding* teacher materials is congruent with her use of other teacher materials -- she doesn't use them much.

Not using the *Covering and Surrounding* teacher materials shaped the opportunities Betty had to learn from the material. There were occasions where suggestions for teaching provided in the teacher materials might have been useful to Betty in teaching *Covering and Surrounding*. However, because she did not use the teacher materials, Betty was not always aware of these suggestions and was therefore not able to take advantage of them in the classroom. An example of this kind of situation occurred when Betty taught her students about areas of circles and π in Investigation 3.

On 11/23/92 Betty and her class worked on Problem 2 in Investigation 3. This problem presents students with a circle and a square that has edge length equal to the radius of the circle (CMP, 1992a, p. 24):

Problem 2: How many of the large shaded squares would it take to cover the circle? You may want to make copies of the square on grid paper and cut them out to see how much you can cover or trace the square out on your transparent centimeter grid.

Figure 6.5: Shaded square and circle diagram:



Betty began this problem by asking the class to predict how many of the shaded squares it would take to cover the circle[7]. Betty had reproduced the circle and the shaded square on separate transparencies so that the shaded square could actually be moved around on top of the circle:

Betty: So, how many of these squares do you think it would take to cover the circle?

---

[7] $A = \pi r^2$ is the well-known formula for finding the area of a circle where r is the radius of the circle. One way to interpret the formula is that π squares of area r^2 (i.e., squares with edge length equal to the radius of the circle) are needed to cover the circle (i.e., to measure the area of the circle). The intent of the problem is to help students begin to develop an understanding of π by seeing that there are always more than 3, but less than 4, squares needed to cover the corresponding circle.

John: Four -- see, you can cover the circle all the way with four squares.

Betty: But aren't we, then, also covering this extra part of the grid outside the circle [Betty indicates a corner of the grid that is outside the circle]?

Dana: I think you only need 3 1/2 squares to cover the circle [murmurs of agreement and nods can be seen around the class].

Betty: OK. Well, would three squares be enough, do you think, to cover the circle? [class seems uncertain] Well, if 4 squares is too much, than lets see if 3 is enough. How should we do that? (pause) Well, it so happens that I have 4 squares [Betty produces a total of four squares on transparency material]. We just said that 4 squares would be too much [Betty places the four 6 x 6 squares so that they cover up the entire 12 x 12 grid] and we can see that [Betty points to where the squares cover the parts of the grid outside the circle and then removes one of the four squares]. Now, how could we figure out if these 3 are enough? (pause) Well, how about this -- let's just trim those pieces off the three squares!

At this point, Betty produced a pair of scissors and trimmed off the parts of the three squares that lied outside the circle; she then replaced them so that exactly 3/4 of the circle was covered with the 3 pie-shaped wedges. Betty then put the pieces she had cut off back on the overhead next to the circle:

Betty: Now let's see. (pause) If I use these pieces to cover up the remaining part [i.e., the uncovered 1/4] of the circle and there is some area I can't cover, than that means I'll need more than three squares -- right? [the class is quiet and some students look confused] Because everything I've got here came from just three squares, so whatever I can't cover I'll need to get from the fourth square [Betty holds up the whole fourth square on transparency material]. Now let's see if I can do this.

Betty began cutting and arranging, with some difficulty, the cut-off pieces from the three squares to cover the remaining 1/4 of the circle. During this time, which lasted several minutes, the class was restless and the students didn't seem to grasp what Betty was doing -- one student remarked to another, "Why's she doing that?" Betty eventually managed to place all the pieces on the uncovered portion of the circle, with an area of about 5 square centimeters left uncovered.

Betty: Okay! I used all of the area of three of the squares to cover the circle and look! There's some area of the circle that I couldn't cover -- that means that we do need more than three squares to cover the circle. But not too much, just (pause) about 5 more square centimeters. [the class looks confused] Does everyone see that? [some students nod]. Okay -- well [Betty glances at the clock] we're almost out of time so we'll do more with this tomorrow. What we're doing here has to do with finding the number π! Has anyone heard of that? [a few hands go up] Well, a few of you have -- π turns out to be a very important number in mathematics and we'll explore it more tomorrow.

Within two minutes the bell rang and the students left. I asked Betty how she thought the lesson had gone. She replied:

> They need more stuff with the tiles. I'm not sure that the unit gives them enough practice with the tiles -- they need to play with the tiles and put them on the rooms more, I think. ... The circle stuff is in the wrong place. We should work through squares and rectangles, and maybe even triangles before we get to the circle stuff. ... While it might seem nice that they right away realize that not everything is perfect and has nice neat corners and stuff, I think that for some kids it was very confusing[8].

Betty perceived that her students had difficulty understanding her lesson. She believed that the source of their difficulty was in not being able to connect their understandings of perimeter and area developed from using the tiles to the circle because they hadn't had enough experience using the tiles yet While this is congruent with Betty's belief that students learn about perimeter and area through

---

[8] *Covering and Surrounding* is sequenced to introduce students to measuring perimeter and area (Investigation 1), then measuring perimeter and area of irregular figures (Investigation 2) and circles (Investigation 3) to immediately reinforce that the concepts of perimeter and area apply to figures with irregular or curved edges as well as straight edges. The unit then returns to rectangles to examine relationships between perimeter and area (Investigations 4 and 5), connects perimeter and area of rectangles to parallelograms and triangles (Investigation 6), and then links these ideas to a real-world context (Investigation 7). See Table 4.1 in Chapter 4 for a detailed summary of *Covering and Surrounding*.

using tiles, it doesn't address the possibility that her use of the transparency materials was what her students found confusing.

The *Covering and Surrounding* teacher edition provides two suggestions to teachers for teaching the circle problem. One suggestion is similar to Betty's use of the transparencies, but doesn't specify cutting up and rearranging the squares like Betty did. The alternate strategy suggested in the *Covering and Surrounding* teacher edition is for students to estimate the area of the circle in the manner they learned to estimate area in the second investigation[9] and then compare it to the area of the shaded square which is 36 square centimeters:

> note that the area of the square is 36 sq. cm and the [estimated] area of the circle is ... about 112 sq. cm. 112 divided by 36 = 3.11 ... a good estimation of π. Another way to get at this is to note that since the square has area 36, three squares have area 36 + 36 + 36 = 108, which is 4 less than the estimated area of the circle, so three squares is not quite enough, but four squares, with total area 144, would be too much (CMP, 1992b, pp. 40-41).

So the *Covering and Surrounding* teacher's edition suggests that another way to get at the problem is to estimate the area of the circle and numerically compare it to the area of the square.

The alternate strategy for teaching the circle problem outlined in the teacher edition may or may not have helped Betty's students. What is clear, however, is that Betty didn't have the opportunity to use the strategy in her teaching because she was not aware of it. She was not aware of it because she didn't use the *Covering and Surrounding* teacher

[9]Investigation 2 in *Covering and Surrounding* examines perimeter and area of irregular figures. Problems include developing strategies for estimating perimeter and area of irregular figures like marshes and shoeprints (CMP, 1992a). In Investigation 2 Betty's students developed a strategy of "piecing together" area -- first count the "whole square units" of area that cover the figure and then "piece together other whole units" from the partial square units covering the figure; the sum of the whole units and the "pieced together" units is the area of the figure (phrases in quotes were used by Betty's students to describe their strategy).

materials to any significant extent. Had Betty used the teacher edition in thinking about teaching the circle problem, she might have had another option open to her in teaching the lesson. Had she still chosen to use the transparencies, she would still have had another representation readily available. The central issue Betty's teaching of the circle problem raises is that her use of the *Covering and Surrounding* teacher materials shaped the opportunities she had to change her teaching through awareness of other approaches to teaching the unit.

Same Teaching, Unchallenged Beliefs

Betty perceived that her prior education and experience with other curricula (e.g., *Mouse and Elephant*) had already prepared her to use curricular materials like *Covering and Surrounding*. This belief supported her non-use of the *Covering and Surrounding* teacher materials. Additionally, Betty did not believe that teaching *Covering and Surrounding* would lead to significant changes in her future practice.

After concluding her teaching of *Covering and Surrounding*, I asked Betty to reflect on her experience of teaching the unit:

AR: Has teaching the unit led you to reexamine or reaffirm any of your thinking or assumptions about teaching mathematics or area and perimeter concepts?

Betty: I think that maybe I'm a bit biased since I've done activity-based kinds of things like this before. Maybe for me it didn't really cause change like it would for some other people who might not have done a lot of this kind of thing before, but I've done this kind of stuff before. And, in fact, I've been trying to do work like this all along when I look back at my own experience and what I had when I was in undergraduate school.

AR: So you feel that your experience is congruent all along with this kind of curricular material?

Betty: When I look back at what my math methods courses, we did a lot of stuff with Cuisenaire rods, we played around with pattern blocks and geoboards. What you're doing now, we did then. I can see where stuff like this [i.e., the *Covering*

> *and Surrounding* unit] developed from. So I think that, and I'll put this in quotes, that I was already "trained". So I think that I'm kind of biased because I've already done a lot of this stuff. And I've been looking for more of this kind of activity-based, hands-on kinds of things, compared to somebody who came from a different kind of background.

Teaching *Covering and Surrounding* did not motivate Betty to reexamine her beliefs or assumptions about teaching because she perceives that her past education and experiences have already prepared her to teach mathematics with materials like *Covering and Surrounding*. For example, recall that Betty had taught the *Mouse and Elephant* unit several times. Betty noted that *Covering and Surrounding* is "a lot like *Mouse and Elephant* -- using tiles and grids and stuff like that." Because she perceived the two units to be similar[10], Betty's prior use of *Mouse and Elephant* supported her belief that *Covering and Surrounding* incorporated little that was new for her teaching. For Betty, teaching *Covering and Surrounding* was essentially business as usual.

Betty's beliefs about what *Covering and Surrounding* has to offer her teaching are consistent with her use of the unit's teacher materials. Since she perceives *Covering and Surrounding* as bringing little, if anything, new into her teaching or classroom, there seems to be no reason for Betty to spend already scarce time reviewing the teacher's edition. What this perpetuated, however, is that there were few instances for *Covering and Surrounding* to provide Betty with opportunities to reflect on her approach to teaching this content. It is unlikely she would have a chance to seriously consider any suggestions for teaching or reflect on instructional problems or

---

[10]Of the seven investigations in *Covering and Surrounding*, three (i.e., 1, 4, and 5) are similar to activities in the *Mouse and Elephant*. But while *Mouse and Elephant* also focuses on surface area and volume of rectangular solids, four of the investigations in *Covering and Surrounding* examine perimeter and area of irregular figures, circles, parallelograms, and triangles.

situations to supply motivations for change from the teacher materials if she doesn't use them. The *Covering and Surrounding* unit, screened through Betty's beliefs that she had already "done a lot of this stuff" led directly to her lack of using the teacher materials.

Summary

Betty's use of *Covering and Surrounding* paralleled her prior teaching and use of curricular materials. Different degrees of exploring and experimenting were present in her teaching both prior to and during *Covering and Surrounding*. These different degrees of exploring and experimenting corresponded to different opportunities for students to learn about problem solving. Moreover, Betty does not, in general, make extensive use of teacher materials and this was also the case with *Covering and Surrounding*. Betty's beliefs about teaching were not challenged by *Covering and Surrounding* as she perceived the unit as congruent with her prior teaching and experience. Betty's beliefs not being challenged by *Covering and Surrounding* are linked to her use of the teacher materials -- since she did not use them, there were really no instances for the unit to shape her practice. Betty would have, of course, been doing different problems and activities with her students had she not used *Covering and Surrounding*. But, from her perspective, she still would have taught perimeter and area by having her students explore and experiment with situations similar to those found in *Covering and Surrounding*.

CHAPTER 7

KAREN AND BETTY: THE INTERPLAY BETWEEN TEACHER AND CURRICULUM

The cases of Karen's and Betty's uses of *Covering and Surrounding* depict the *interplay* between each teacher and a piece of problem-solving oriented curriculum. Both cases describe how Karen and Betty shaped *Covering and Surrounding* and the extent to which the unit materials influenced their teaching. In this chapter, I first unpack the notion of interplay and how this concept describes Karen's and Betty's uses of *Covering and Surrounding*. I then analyze Karen's and Betty's uses of *Covering and Surrounding* from the perspective of the curriculum developers by carefully examining how the enacted curriculum was similar to, and also different from, the intended curriculum. Finally, I revisit *Covering and Surrounding* and use the four-dimensional framework developed in Chapter 2 to analyze the unit from Karen's and Betty's perspectives. The chapter concludes with a synopsis of study findings.

Interplay: Reciprocal Influence Between Teacher and Curriculum

Some research has shown that curricular materials can have little influence on teachers' practice (e.g., Porter & Freeman, 1989) while other studies reveal that curriculum can play a powerful role in shaping how teachers teach (e.g., Remillard, 1991a). In the cases of Karen and Betty, their uses of *Covering and Surrounding* were shaped by a mixture of influences from both the materials and their own views, beliefs, and knowledge. For example, Karen's teaching was shaped by *Covering and Surrounding* -- the unit "forced" her to use tiles and teach perimeter and area using "hands-on" activities instead of "teaching abstractly."

But Karen's use of *Covering and Surrounding* was also shaped by her own beliefs. Because she believes it is important to maintain her students' interest, she allowed them to determine the sequence of investigations as a way of keeping them interested in the unit.

Betty's *nonuse* of the teacher edition represents a facet of her use of *Covering and Surrounding* that is a component of interplay between her and the unit. Since she did not use the teacher edition, Betty's use of the unit was based primarily on her own views and beliefs, not the recommendations of the developers. For example, Betty had her students complete Investigation 4 as a group poster project because she believes students learn from one another, not because this was suggested in the teacher edition. But not using the teacher materials also shaped Betty's teaching of *Covering and Surrounding*. The circle problem investigating π, for example, put Betty in a teaching situation where her own approach left students confused. Had she used the teacher materials, Betty would at least have had the alternate numerical strategy available to her and would have had the opportunity to teach the lesson differently. Karen's and Betty's teaching was thus shaped by both the unit and their own beliefs -- their enactments of the unit was the product of the interplay between teacher and curriculum.

To further unpack the notion of interplay in Karen's and Betty's uses of *Covering and Surrounding*, consider the room design problem. Recall that Shakaya's question to Karen and Max's question to Betty were the same -- whether or not to count the courtyard perimeter as part of the perimeter of their rooms. For both teachers, the unit shaped their teaching with these students. *Covering and Surrounding*, by putting students in the open-ended situation of designing a room without direction whether or not to include courtyards, provided the context for Shakaya's and Max's question to emerge. The unit shaped Karen's and Betty's teaching by placing them in the situation where they needed to address the question of whether or not to include the perimeter of courtyards as part of the perimeter of the rooms. But Karen's subject-matter knowledge about perimeter and area and Betty's prior experience with other curricular materials also shaped their teaching with Shakaya and Max.

Karen's uncertainty about her own subject-matter knowledge about perimeter and area shaped her decision to respond to Shakaya by asking her to think about the perimeter of her room further. Recall that Karen was hesitant and uncertain in her responses to perimeter and area items on the interview prior to teaching *Covering and Surrounding*. Karen also remarked in her conversation with Shakaya "I don't know" when Shakaya asked her if she should count the perimeter of the courtyard as part of the perimeter of her room. Since Karen herself was not sure whether or not to count the perimeter of Shakaya's room, she was not prescriptive in her answer, but instead left the question open. Karen's use of the room design problem, shaped by uncertainty in her own subject-matter knowledge[1], resulted in a learning experience that gave Shakaya the opportunity to explore the concept of perimeter further.

[1]Karen saying "I don't know" could indicate other kinds of uncertainty besides uncertainty in her subject-matter knowledge about perimeter and area. For example, Karen's comment could also reflect unfamiliarity with the materials, uncertainty as to where different interpretations of measuring perimeter (i.e., whether or not to count the courtyard perimeter) might lead, or perhaps she was hesitant because she had not anticipated the situation. In any case, the instruction with Shakaya was a product of interplay -- the materials put Karen in an open-ended situation that was unfamiliar and Karen's use of the room design problem was shaped by her uncertainty.

Betty's decision to tell Max not to include the perimeter of courtyards when finding the perimeter of his room was shaped by her prior use of other curricular materials. As Betty noted on multiple occasions, she had previously used the *Mouse and Elephant* unit (Shroyer & Fitzgerald, 1986) to teach perimeter and area. *Mouse and Elephant* uses a different representation to help students learn about perimeter and area[2]. In using the representation, the materials explicitly direct teachers to have their students construct figures without "holes" in them[3], i.e., the rectangular figures designed by students in *Mouse and Elephant* are specified to not have "courtyards" (see Shroyer & Fitzgerald, 1986). Betty's use of the room design problem was consistent with *Mouse and Elephant* to the extent that she directed Max not to count the perimeter of his courtyard. In her comments about that lesson, Betty noted that in the *Mouse and Elephant*, "holes aren't part of the perimeter." While this decision is consistent with the *Mouse and Elephant* representation, it is not necessarily valid in the *Covering and Surrounding* representation. For example, if perimeter is interpreted as the number of wall sections in the room, then it is arguable that Max *should* have counted the perimeter of the courtyard in his room.

The room design problem shows how Karen's and Betty's teaching is shaped both by *Covering and Surrounding* and by their own knowledge and prior experiences. But as well as illustrating how Karen's and Betty's

---

[2]In *Mouse and Elephant*, banquet tables are used as a representation for perimeter and area. Each 1-inch tile is interpreted as a square table that can seat one person per edge. Banquet tables are formed by pushing together square tables. The area of a rectangular figure, or "banquet table", is the number of square tables. The perimeter is the number of people who can be seated around it.

[3]In *Mouse and Elephant* relationships between perimeter and area of rectangular figures are explored. The unit focuses students on rectangles (without holes/courtyards) to prepare them for study of perimeter and area relationships in rectangles (see Shroyer & Fitzgerald, 1986).

instruction is the product of the interplay between teacher and curriculum, this study reveals that interplay can result in potentially unexpected teaching. For example, in the room design problem Karen was open-ended in her response to Shakaya and Betty was more directive in her interaction with Max. Given Karen's prior teaching of perimeter and area focusing on formulas and "teaching abstractly", and Betty's continued emphasis on open-ended "exploring and experimenting", one might expect the situation to be reversed. While Karen's and Betty's instruction with the room design problem may be surprising at first, studying the interplay between them and *Covering and Surrounding* sheds light on how their teaching with Shakaya and Max unfolded.

In the next two sections I take a closer look at the interplay between Karen and Betty and *Covering and Surrounding*. I first examine Karen's and Betty's uses of the unit from the perspective of curriculum development. I then turn the tables and survey Karen's and Betty's uses of *Covering and Surrounding* from their perspectives. These contrasting viewpoints on Karen's and Betty's teaching of *Covering and Surrounding* provide findings from the dual perspectives of curriculum developers and teachers.

Unpacking Karen's and Betty's Uses of Covering and Surrounding: A Perspective from Curriculum Development

Through their uses of *Covering and Surrounding*, Karen and Betty both involved their students in learning about perimeter and area in ways that reflected multiple goals of the unit developers. At the same time, however, each teacher exhibited practices or perceptions and beliefs that, to some extent, go against the grain of the developers' goals or intentions.

Karen and Betty both used *Covering and Surrounding* to engage their students in problem solving as they investigated and learned about perimeter and area. Karen's use of the triangle area problem with Matt, Aaron, and Trevor, and Betty's use of the problem of recommending a room to the city council with Hillary are examples of each teacher involving her students in problem-solving activity rich in mathematical connections and content. These instructional snapshots of Karen and Betty are consistent with learning goals the curriculum developers outline for students in *Covering and Surrounding* (see Chapter 4). The two lessons are also congruent with the underlying philosophy of the developers that students learn mathematics and problem solving skills by investigating problems and wrestling with the mathematical ideas and patterns embedded in problems (see Fitzgerald et al., 1991).

At least in the case of Karen, *Covering and Surrounding* was essential in changing her mode of instruction from teaching perimeter and area "abstractly" to teaching "hands-on". For example, she noted on multiple occasions that the unit made her use tiles to do hands-on activities. Not only her comments, but also her actual practice provide evidence of this change in Karen's teaching. Prior to *Covering and Surrounding* Karen taught lessons that focused on rules and procedures (e.g., the lesson on divisibility laws). After *Covering and Surrounding* she taught a lesson on the perimeter of circles that was formula-oriented and exemplified her mode of "teaching abstractly." When contrasted with these two lessons, Karen's *Covering and Surrounding* instruction that occurred between them is clearly different. In fact, while Karen wanted her students to learn formulas in *Covering and Surrounding*, she resisted telling them to her students, waiting for the students to develop them (see Chapter 5). Karen's lesson on triangle area with Matt, Trevor, and Aaron is an example of this. These changes in Karen's practice while teaching *Covering and Surrounding* sharply and clearly reflect the kinds of shifts in practice the unit developers would like to see teachers make through using the materials (see Fitzgerald et al., 1991).

Another feature of both Karen's and Betty's uses of *Covering and Surrounding* is their heavy use of concrete materials. Both teachers, for example, used tiles and transparent grids extensively as they taught the unit. While using manipulatives does not guarantee that students will learn about problem solving or make connections among mathematical ideas, manipulatives can be used effectively as tools to help accomplish these kinds of learning goals with students (c.f., Ball, 1990b; NCTM, 1989, 1991; Putnam et al., 1992). The extensive and routine use of mathematical "tools" such as tiles and transparent grids is one of the skills the developers of *Covering and Surrounding* intend students to develop (Fitzgerald et al., 1991). While this study did not assess students' learning, Karen's and Betty's use of concrete materials while teaching *Covering and Surrounding* seems to be on the trajectory of the developers' for students to use tools (e.g., transparent grids) and models (e.g., tile models of rooms) to "... represent, simulate, and manipulate patterns and relations in problem settings" (Fitzgerald et al., 1991, p. 6).

While Karen's and Betty's uses of concrete materials, their engagement of students in investigating mathematically rich problems, and Karen's change from abstract to hands-on teaching while using *Covering and Surrounding* all reflect intentions of the curriculum developers, other aspects of their teaching do not. Karen's integration of arithmetic drill exercises from the textbook while teaching *Covering and Surrounding*, for example, clearly does not match the developers' vision of how middle school students should learn mathematics. In their proposal for the Connected Mathematics Project, *Connected Mathematics*, the developers state:

> The predominant method of mathematics instruction asks students to learn facts and skills by listening, imitating, and practicing routine exercises. This skill-oriented curriculum and the drill-and-practice instructional style with which it is commonly "delivered" are particularly inappropriate for middle school students (Fitzgerald et al., 1991, p. 2).

Another aspect of Karen's use of *Covering and Surrounding* that varies from the philosophy of the developers is her perception of problem solving. While Karen did not believe that her students were learning about problem solving in *Covering and Surrounding*, the developers definitely view the unit as problem-solving oriented (see CMP, 1992b). While it is not clear in this study to what extent Karen's perception of her students' learning about problem solving influenced her teaching, problem solving is a component of her views and beliefs that contrasts sharply with the perspective and intentions of the developers.

How both Karen and Betty used the *Covering and Surrounding* teacher materials is another aspect of their practice that varied from the intentions of the developers. While the developers intend the teacher materials (i.e., CMP, 1992b) to be a rich resource for teaching *Covering and Surrounding* (see Chapter 4), Betty hardly used the teacher materials at all. Although Karen highlighted passages, she seemed to use the teacher materials in practice only for answers to problems. The developers view the *Covering and Surrounding* materials as a sharp departure from the usual curricula predominantly used in middle school mathematics (Fitzgerald et al., 1991). *Covering and Surrounding* was also a new unit for both Karen and Betty. Despite this, Karen and Betty appear to use the *Covering and Surrounding* teacher materials as they have used other curricular materials.

The aspects of Karen's and Betty's uses of *Covering and Surrounding* that reflect intentions of the curriculum developers may be encouraging news to curriculum developers. The features of their practice that vary from the developers' perspectives, however, warrant closer examination. I now look at Karen's and Betty's uses of *Covering and Surrounding* from their viewpoints to shed more light on the aspects of their practice that varied from the intentions of the curriculum developers.

Unpacking Covering and Surrounding: Karen's and Betty's Perspectives

The cases suggest that two mechanisms were particularly powerful in shaping Karen's and Betty's uses of *Covering and Surrounding*: (1) Filtering the unit through powerful views, beliefs, and knowledge; and (2) use of the teacher materials in a manner consistent with prior use of teacher materials. But as well as examining mechanisms present in their practices, it is also critical to unpack what the *Covering and Surrounding* unit did or did not provide for Karen and Betty to use in their teaching *from their perspectives*. This is because the interplay between *Covering and Surrounding* and Karen and Betty is influenced by how the unit addresses their beliefs and the issues *they* feel are important in teaching. Unpacking *Covering and Surrounding* through the four-dimensional framework (see Chapter 2) sheds more light on Karen's and Betty's uses of the unit and their role in the interplay between teacher and curriculum.

Views and Beliefs About Problem Solving

Even though Karen had read the five problem-solving goals in the *Covering and Surrounding* teacher edition, one of which was congruent with an event in her classroom, she perceived her students as doing "hands-on" activities rather than problem solving. Why did *Covering and Surrounding* not affect Karen's perception that her students were not learning about problem solving? A strong possibility is that neither the student materials nor the teacher edition addressed Karen's beliefs about problem solving.

Problems in *Covering and Surrounding* are almost all non-standard (process or open-search) problems, real-world problems, or puzzle problems (see Chapter 4). Standard word problems are a rarity in *Covering and Surrounding*, yet solving these kinds of problems is how Karen conceptualizes problem solving. Nowhere in the *Covering and Surrounding* materials is the range of problems, or the absence of standard word problems, made explicit to the teacher. In fact, the five problem-solving goals supplied in the teacher edition (CMP, 1992b) are the only explicit statements about problem solving in the materials. The five goals specify problem-solving strategies and skills that are not descriptive of solving standard word problems (see Chapter 4). As discussed in Chapter 4, the range of types of problems in *Covering and Surrounding* imply different kinds of problem-solving for students. While some problems (e.g., standard word problems) require translating information from words into symbols or computations, other kinds of problems (e.g., non-standard, open-search problems) may involve cutting up or manipulating objects as Karen's students did in exploring the area relationships between rectangles and triangles.

Since the perspective on problem solving of the curriculum developers was not clear and the kinds of problems in *Covering and Surrounding* are not standard word problems, from Karen's perspective problem solving was not happening. No scheme or framework was supplied in the *Covering and Surrounding* teacher edition for classifying problems and associated problem-solving processes in the unit (as in Chapter 4). Such a framework could conceivably illustrate that problem solving can occur for students in *Covering and Surrounding* even though students encounter few standard word problems. Providing an explicit, more detailed portrait of different kinds of problems and problem solving is no guarantee that the teacher materials would have changed Karen's perception about her students' learning about problem solving. However, at least the opportunity for change would have existed for Karen to have a clearer picture of what kinds of problem-solving opportunities the curriculum developers were trying to provide.

Problem-Solving Activity in Classroom Settings

In Betty's use of *Covering and Surrounding*, she structured problem-solving activities with different degrees of "exploring and experimenting" for students. However, she perceived that all the problem-solving activity in her teaching of *Covering and Surrounding* was very open-ended. While problems in *Covering and Surrounding* may have different degrees of openness (see Chapter 4), this is another issue that the teacher edition does not explicitly address. The teacher materials do state that the problems in *Covering and Surrounding* are intended to provide students with rich and engaging mathematics (see

CMP, 1992b). However, comparisons or suggestions addressing the relative openness of problems are not given.

The situation for Betty with respect to the degree of openness of problems is analogous to beliefs about problem solving for Karen -- the teacher edition does not address Betty's beliefs about "exploring and experimenting" in problem-solving activity. Betty may have relied on her own beliefs about problem solving being universally open-ended because this aspect of problem solving is not discussed in the teacher edition[4].

Subject-Matter Knowledge

As detailed earlier, both Karen and Betty bumped-up against the situation of whether or not to count the perimeter of a courtyard as part of the perimeter of a room. While the situation of distinguishing between the "inner" (i.e., courtyard) perimeter and the "outer" (i.e., exterior) perimeter emerged for both teachers, the *Covering and Surrounding* teacher edition does not address it. The teacher edition provides suggestions to help students understand the wall section and carpet tile analogy for perimeter and area (see Appendix E), but does not help teachers in addressing the courtyard dilemma.

Karen's and Betty's use of the room design problem and responses to their students illustrates their reliance on their own subject-matter knowledge of perimeter and area to address students' questions whether or not they were open to using suggestions from the teacher edition. If

---

[4]Since Betty did not use the *Covering and Surrounding* teacher edition it is questionable whether the teacher materials would have had an impact on her conceptions of openness in problem solving activity even if they were addressed. However, for Betty or other teachers with similar beliefs about problem-solving always being open-ended, the opportunity to bring this issue to the surface in the teacher materials does not exist.

either teacher looked to the *Covering and Surrounding* teacher edition for suggestions on how to deal with courtyard perimeter, none would be found. It is perhaps this kind of situation that led Karen to remark, when I asked her how the *Covering and Surrounding* teacher materials could be made more useful, "By doing more about giving suggestions and examples for what counts as good student answers and responses." Karen went on to connect this comment to her own subject-matter knowledge by explaining that "The kids came up with a lot of interesting ideas in the unit [i.e., *Covering and Surrounding*], but I don't always know whether or not they're [i.e., the students] right!"

Perceptions and Beliefs About Student Learning

The issues of computation and skill maintenance contrast sharply in Karen's and Betty's uses of *Covering and Surrounding*. Karen believed that computation is an important part of what her students should be learning in mathematics and felt that the unit was not sufficient for her students to build and maintain computational skills. Betty did not assign computation exercises in her use of *Covering and Surrounding* whereas Karen had her students do computation practice from the textbook.

The *Covering and Surrounding* teacher materials do not address computation or make the perspective of the curriculum developers on computation explicit (see CMP, 1992b). While the CMP funding proposal makes clear that rote computation practice (e.g., "naked number" drill exercises -- see Chapter 4) will not be included in the CMP materials and provides a rationale, this stance is not communicated to teachers in *Covering and Surrounding* (c.f., CMP, 1992b; Fitzgerald et al., 1991). From Karen's perspective, the unit did not address an important component of her students' learning. She therefore brought in the textbook which did address her belief that students need computational practice and skill maintenance. From Betty's perspective, that the *Covering and Surrounding* unit did not explicitly address computation was not a factor in her teaching because she either did not view computation as important for her students or felt that computation was sufficiently addressed. For example, Betty noted that she felt her students were doing computation practice in Investigation 4 finding factor pairs of 24 and calculating the perimeter of all the rectangles of area 24 for their graphs. However, she also stressed prior to teaching *Covering and Surrounding* that doing "naked number" computation exercises (like Karen assigned) were really "not useful" for students.

Commentary

One possible implication for the *Covering and Surrounding* teacher materials that could emerge from the above discussion is to simply make them more explicit in suggesting to teachers how they should teach. The situation, however, is more complex than that. Researchers have argued persuasively that teachers necessarily make instructional decisions based on their own views, beliefs, knowledge, and context (Ball, 1990a, 1990b; Ball & McDiarmid, 1990; NCTM, 1991; Putnam et al., 1992; A. Thompson, 1989, 1992; Wilcox et al., 1991, 1992). This growing body of research implies that providing more structured, explicit, or directive materials for teachers won't necessarily result in teaching that is more closely aligned with the intentions of the curriculum. No matter what the curriculum, in a classroom situation a teacher will still teach it.

A major finding of this study that emerges from Karen's and Betty's uses of *Covering and Surrounding* is that curriculum developers may not be able to put teachers on the trajectory of the intended curriculum by ignoring their perspectives or developing more explicit or directive materials. As this study shows, Karen and Betty are necessarily partners *with* the curriculum in the interplay from which the enacted curriculum emerges.

Synopsis of Findings

Karen's and Betty's uses of *Covering and Surrounding* were not simply a matter of each teacher implementing the unit in her classroom. Both teachers' uses of the unit involved an interplay between teacher and curriculum which shaped the instruction I observed in their classrooms. As the cases of Karen and Betty show, this interplay resulted in some teaching that was perhaps unexpected (e.g., the room design problem) but is understandable when the interplay between teacher and curriculum is examined.

Karen and Betty provide a contrast in the degree to which teaching *Covering and Surrounding* corresponded to changes in their practices. Karen changed her mode of teaching perimeter and area from abstract to hands-on. Betty, however, maintained what she felt was her teaching mode of "exploring and experimenting", perceiving no reason to change her practice during *Covering and Surrounding*. This comparison between Karen and Betty readily demonstrates the importance of both sides of the interplay between teacher and curriculum: Karen's use of the unit provides evidence for how the curriculum shapes classroom teaching; Betty's use of *Covering and Surrounding* shows how teachers' views and beliefs can influence how instruction plays out in the classroom.

Moreover, as the cases suggest (see Chapters 5 and 6), teaching *Covering and Surrounding* did not appear to change Karen's or Betty's views or beliefs even when their views and beliefs ran counter to the intentions of the curriculum developers. There are at least two plausible explanations for the resilience of Karen's and Betty's views and beliefs. One is that teaching *Covering and Surrounding* was simply not a long enough or sufficiently extensive intervention to challenge and/or change Karen's and Betty's views or beliefs. Some researchers, for example, have argued that it takes time for teachers to change their views and beliefs, especially those related to problem solving (e.g., A. Thompson, 1989, 1992). Another explanation is that the beliefs Karen and Betty maintained that are, at least to some extent, counter to intentions of the curriculum developers were not addressed by the teacher materials. Since teaching *Covering and Surrounding* is an interplay between Karen and Betty and the unit, if the unit materials do not address aspects of their knowledge and beliefs, it is not surprising that they may remain intact.

Having discussed the conclusions that can be drawn about Karen's and Betty's uses of *Covering and Surrounding*, I next turn to the broader implications of this study. In the next chapter I will provide arguments for what this study holds for curriculum development and, to a lesser extent, teacher education. I will also indicate some possible directions for future research this study points to.

CHAPTER 8

WHAT LIES AHEAD?: CHALLENGES AND DILEMMAS FOR CURRICULUM DEVELOPMENT

The *Covering and Surrounding* unit influenced the teaching which occurred in Karen's and Betty's classrooms. This study shows that Karen's and Betty's uses of *Covering and Surrounding* were shaped by *interplay* between their respective views, beliefs, and knowledge, and the unit. As a result, each teacher's instruction reflected her views, beliefs, and knowledge, and some of the intentions of the curriculum developers. This study shows how teachers' views, beliefs, and knowledge can interact with a piece of problem-solving oriented curriculum to shape teaching.

Studying how teachers like Karen and Betty use curricular materials like *Covering and Surrounding* may hold implications for teacher education as well as curriculum development. For example, a relevant issue for teacher education is to conceptualize how experiences such as teaching a unit like *Covering and Surrounding* can be used to assist mathematics teachers in changing their beliefs (see Dewey 1938/1963; Paley, 1989) and teaching to align with reforms. A. Thompson (1992) suggests that such research could examine relationships and interactions between teachers and teacher educators that can be crucial in supporting teachers and helping them change their practices and beliefs to implement mathematics education reforms in their classrooms.

But interaction with a teacher educator and Karen or Betty did not occur in this study. As discussed in Chapter 3, my role was that of an interviewer and an observer. My intent was to study teachers' use of a problem-solving oriented piece of curriculum, not to intervene or show Karen or Betty how they should be using the *Covering and Surrounding* unit. Since no interventions were present besides the *Covering and Surrounding* student and teacher materials, the implications of this study are limited for teacher education and are more suited to illuminating issues in curriculum development.

However, while this study does not imply specific practices or recommendations for teacher education, the implications I will discuss for curriculum development in this chapter do contribute to teacher learning and change. As I argued earlier, problem-solving oriented curricula are viewed by reformers as a lever for helping change how teachers predominantly think about and teach mathematics. Since this study does inform the role such curricula can play in shaping how teachers teach, implications for curriculum development of this study are also relevant to teacher education. Karen's and Betty's uses of *Covering and Surrounding* being the product of interplay between teacher and curriculum further suggests that the arenas of curriculum development and teacher education can both be informed by examining issues connected to teachers' use of problem-solving oriented curricula.

In the first section of this chapter, I outline a fundamental dilemma facing curriculum developers and how Karen's and Betty's uses of *Covering and Surrounding* reflect the dilemma. The next section digs deeper into specific challenges and dilemmas facing curriculum developers by using snapshots of Karen's and Betty's teaching with *Covering and Surrounding* to illustrate the complexity of four major issues. For each of the issues I discuss, anomalies embedded in them point to areas for future research that are connected to teachers' use of problem-solving oriented mathematics curriculum.

A Dilemma for Curriculum Developers

Designing mathematics curriculum that helps teachers bring new roles for themselves and their students into classrooms (see Chapter 4) involves balancing a dilemma between reform and practice. On the one hand, curriculum developers view mathematics curricula as not only providing teachers and their students with opportunities to engage in rich mathematics (NCTM, 1989), but also to push teachers to develop their teaching along the trajectory of reforms (Cohen & Ball, 1990). However, curriculum developers and reformers also argue that teachers are professionals who necessarily shape their teaching and make decisions based on their own context and beliefs; therefore, it is not possible to prescribe to teachers how to teach (The Holmes Group, 1990; NCTM, 1991). Mathematics curriculum developers are confronted with the dilemma of how to design mathematics curriculum that will move reforms into classrooms while simultaneously respecting the teacher's role which includes shaping curriculum to meet the needs of his or her students.

Traces of the above dilemma can be found in Karen's and Betty's uses of *Covering and Surrounding*. As discussed in the last chapter, engaging students in problem solving and using concrete materials were two characteristics of both teachers' uses of *Covering and Surrounding* that are consistent with the intentions of the curriculum developers and reformers. Karen's use of the unit to change her mode of teaching perimeter and area from "abstract" to "hands-on" is another example of using *Covering and Surrounding* in a manner that is in line with the vision of the developers and reforms. But Karen's use of drill exercises from the textbook along with teaching *Covering and Surrounding* and her perception that her students were not learning problem solving in the unit clearly vary from the intentions of the developers. Similarly, Betty's belief that the materials had nothing new to offer her teaching are in tension with the intentions of the developers and reforms. In the case of these facets of Karen's and Betty's uses of *Covering and Surrounding*, how might the developers balance the dilemma of pressing Karen and Betty to reexamine their beliefs and use of the unit and yet respect their role in shaping curriculum in their classrooms?

Managing the dilemma of pushing teachers to reexamine their beliefs and practice while still respecting their role in shaping curriculum is complicated further because it is not clear if the vision of teaching and learning mathematics of curriculum developers is necessarily "right" in all cases or circumstances. After all, it seems possible that teachers could be justified in their use of curricula like *Covering and Surrounding* even when their use of the materials varies from the intentions of the curriculum developers. For example, could Karen be justified in her insistence on supplementary work and practice on computational skills while teaching *Covering and Surrounding*? Are *both* the curriculum developers and Karen somehow justified in their different stances on computation and skill maintenance?

From the perspective of the developers, students will gain more by engaging with rich problems in *Covering and Surrounding* than the rote computation drill Karen assigned. But, at the same time, students do not all have the same learning needs; it is conceivable that at least some students in Karen's class did need explicit computation practice which *Covering and Surrounding* does not provide. Moreover, the concerns of teachers, their students' parents or guardians, and their schools and communities may vary from the intentions of curriculum developers -- e.g., some teachers, parents, schools, and communities may view rote computation drill as important to students' learning. Given this variance and the necessary role teachers play in shaping curriculum, it seems inevitable that some teachers, like Karen, will assign computation drill. From Karen's perspective, since computation is important for her students to learn, it seems reasonable to spend some instructional time on computation practice and skill maintenance.

This example of Karen's use of computation drill along with *Covering and Surrounding* shows that determining the rightness or wrongness of a particular use of curriculum is highly problematic. In the next section, I will unpack some specific issues that demonstrate the difficulty of unraveling teachers' use of problem-solving oriented curricula and illustrate them with snapshots of Karen's and Betty's uses of *Covering and Surrounding*. My analysis is based on the premise that Karen's and Betty's uses of *Covering and Surrounding* are shaped by interplay between the teacher and the unit.

Karen and Betty: Teachers Who Provide Snapshots of Dilemmas for Curriculum Developers

To what issues must curriculum developers attend as they design problem-solving oriented curricula intended to help teachers teach mathematics as problem solving? What do curriculum developers need to know, or know more about, to design curricular materials that are more effective in helping teachers reexamine their beliefs and practice as they use them? Schwab (1978), for example, notes that in designing curriculum the developers must have

> ...knowledge of what ... teachers are likely to know and how flexible and ready they are likely to be to learn new materials and new ways of teaching (p. 367).

But researchers have provided detailed portraits of what teachers know about mathematics (e.g., Ball, 1988; Ball & McDiarmid, 1990), what teachers believe about teaching mathematics and how they are prepared to teach problem solving and mathematics for understanding (e.g., A. Thompson, 1992; Wilcox et al., 1991, 1992), and the match between teachers' expressed and enacted conceptions of problem solving and how these conceptions can change (e.g., Charles, 1989; A. Thompson, 1985, 1989). Curriculum developers must now turn to the challenge of using this extensive knowledge of teachers to design mathematics curriculum that will help teachers reach new goals for teaching school mathematics and address the dilemmas of attaining these goals (e.g., NCTM, 1989, 1991). This study suggests that conceptualizing teachers' use of curriculum as interplay between teacher and curriculum sheds light on a number of dilemmas that illustrate challenges facing curriculum developers in the context of mathematics education reform.

Computation and Skill Maintenance

Although Karen used *Covering and Surrounding* to provide her students with open-ended, "hands-on" problems to learn conceptually about perimeter and area, she also divided her students' time between the CMP unit and computational practice with calculators. Recall that Karen assigned whole number and decimal multiplication drill exercises to be done with paper and pencil and then checked with a calculator during the second half of *Covering and Surrounding* (see Chapter 5).

Even though rote drill of the kind Karen assigned clearly varies from the intentions of the curriculum developers, skill maintenance is an important part of what Karen believes her students should be doing in her mathematics class. Betty more clearly represented the vision of the curriculum developers with respect to computation. She did not assign drill exercises in computation, but maintained her focus on *Covering and Surrounding*. Betty believed that her students were getting sufficient computation practice through working on problems in the unit. Who is "right" in her approach to computation along with teaching *Covering and Surrounding* -- Karen or Betty?

It may be tempting to view Betty as more faithfully representing the "right" stance on computation of the curriculum developers and dismiss Karen as subscribing to the focus of the traditional, procedure-oriented mathematics curriculum. However, the contrast in stances on computation and skill maintenance of teachers like Karen and curriculum developers designing materials like *Covering and Surrounding* is more complex than determining who is "right."

From the perspective of a classroom teacher like Karen using materials like *Covering and Surrounding*, how to handle computational practice may not be clear. While curriculum developers and reformers clearly deemphasize computation, they have not given it up entirely. Reformers and curriculum developers often refer to, for example, "the recent emergence of calculators" (Fitzgerald et al., 1991, p. 8) as an argument against incorporating computation practice into problem-solving oriented materials. By emphasizing the wide availability of calculators, curriculum developers and reformers argue that less time needs to be spent on developing students' computational skills. This argument also aims to ease fears that students studying the materials will be unable to perform computations when they need to because students will be able to use calculators (see also NCTM, 1989). In fact, reformers advocate building students' computational skills with calculators (e.g., Fitzgerald et al., 1991; NCTM, 1989).

Since *Covering and Surrounding* does not include computation drill exercises *nor* explicit use of calculators or calculator exercises (see CMP, 1992a, 1992b), Karen's use of calculators as part of computation practice can be interpreted as a reasonable instructional decision that is arguably aligned with reforms. Karen could conceivably view having her students check their answers with a calculator as building their skills in calculator use. As noted above, reforms *do* imply that computation, especially with calculators, is worth spending *some* time on. From this analysis, it is more apparent that deciding who -- Karen or Betty and the curriculum developers -- has the "right" stance on computation and skill maintenance is problematic. Either Karen or Betty can interpret her approach to computation as aligned with reforms.

Karen's use of computation along with *Covering and Surrounding* is also significant because her perspective is common among teachers. For example, while standardized tests are slowly changing to decrease emphasis on computational proficiency, such proficiency is still a major evaluative criterion. Some teachers, therefore, feel a need to focus on computation in their teaching (Kulm, 1991). Karen, for instance, did not believe that computation embedded in problems in *Covering and Surrounding* was sufficient for her students; she believed that explicit work with computation (i.e., drill) was needed (see Chapter 5).

If the *Covering and Surrounding* developers wish to see teachers like Karen not spend instructional time on computation practice, then perhaps computation should be directly addressed in the materials. For example, should the developers include computation practice that is somehow related to the content to address Karen's beliefs about computation? Or, on the other hand, should the developers include arguments in the teacher materials to convince teachers like Karen that it is not worth spending time doing supplementary computational drill? These are difficult questions, especially considering teachers like Betty who may not believe that they need to use the teacher materials and teachers like Karen who may use the materials mainly for answers.

The dilemma of computation for curriculum developers thus leads to another challenge -- how can curriculum developers address computation in the curricula they design when teachers might not use the materials or not use them as intended? To explore this in the context of this study, I next unpack the difficulty Karen's use and Betty's nonuse of the *Covering and Surrounding* teacher edition presents to curriculum developers.

The Sufficiency of Curriculum Materials for Supporting Teachers

Karen's and Betty's uses of *Covering and Surrounding* reveal contrasts in how two teachers used the same piece of problem-solving oriented curriculum (c.f., computation, sequencing of investigations). Given the variance in views, beliefs, and use of a single unit between just two teachers, trying to develop materials that are usable and address the needs of teachers is clearly difficult. In this sense, interplay between teacher and curriculum complicates the charge of curriculum developers to move reforms into classrooms. Teachers have a wide range of views and beliefs and trying to address them all in a set of curricular materials is a formidable undertaking. However, confronting this challenge seems to be part of the agenda of at least some curriculum developers. The developers of *Covering and Surrounding*, for example, note that while inservice and professional development with the CMP curriculum is desirable and strongly recommended, they intend to develop the materials so that "a well motivated teacher can teach the materials with little or no inservice" (Fitzgerald et al., 1991, p. 15).

It is understandable why the CMP or other curriculum developers would undertake designing a curriculum that could, if need be, stand on its own in supporting teachers. Other forms of teacher support, like inservices or workshops, are often too infrequent to have significant or lasting meaning for teachers, providing only scant glimpses of complex issues (A. Thompson, 1992). Some teacher support efforts are ongoing and involve teacher collaboration and sustained interaction with teacher educators and other teachers (e.g., The Holmes Group, 1990; A. Thompson, 1989). But, in most cases, teachers work in isolation and continued professional development is rare due to, for example, institutional and fiscal constraints (Lipsky, 1980; Lortie, 1975; Sarason, 1982). Given the limitations and constraints of teacher support mechanisms like peer collaboration, contact with support staff, workshops and inservices, it seems reasonable to expect curriculum materials to provide support to teachers. Yet Karen's, and particularly Betty's, uses of *Covering and Surrounding* raise questions about whether curriculum materials alone are sufficient to help teachers align their practices with the intentions of curriculum developers and the reforms embedded in the materials.

Betty reflected many of the intentions of the curriculum developers in her use of *Covering and Surrounding*. However, she did not use the teacher materials much and did not believe that she needed to. Yet, as her lesson on circumference of circles illustrates, the *Covering and Surrounding* teacher materials had potential to offer Betty support in her practice. How can the curriculum support Betty if she does not use the teacher materials and does not believe she needs support? Consider Karen's use of the *Covering and Surrounding* teacher materials as she taught the unit. While Karen did use the *Covering and Surrounding* teacher edition, she used it mainly for obtaining answers to problems and not as a resource for her practice as the developers intend.

The *Covering and Surrounding* teacher edition was a form of support available to Karen and Betty, whereas other potential means of support were not at hand. For example, I did not support them as they taught the unit, I only observed. Neither Karen nor Betty attended inservices or workshops on teaching *Covering and Surrounding*. None of the other teachers in either Karen's or Betty's building taught *Covering and Surrounding*, so neither of them had other teachers to talk to about teaching the unit. Yet, Betty did not use the teacher edition at all and Karen used it almost exclusively for answers. Although Karen's and Betty's uses of *Covering and Surrounding* reflected many of the intentions of the developers and teaching with the materials helped Karen change her teaching of perimeter and area, the point here is that the teacher materials did not seem influential or supportive for either teacher. Therefore, while this study provides evidence that using curricula like *Covering and Surrounding* can help teachers change their teaching, it also raises questions about the extent to which the materials actually supported Karen and Betty in their teaching.

Limitations of this study should be considered before inferring that teacher materials would be ineffective in supporting teachers in their use of the CMP curriculum or similar curricula. For example, this study examined Karen's and Betty's uses of only one unit over a period of about one month. *Covering and Surrounding*, in actuality, is not an isolated unit as it was for Karen and Betty in this study, but is part of a complete sixth-grade curriculum. Might Karen's or Betty's uses of CMP teacher materials change over the course of teaching the entire CMP sixth-grade curriculum? During a full year of CMP instruction would Betty begin to use the teacher materials? Would Karen continue to allow her students to vote to determine the sequence of investigations in each CMP unit and use the teacher materials mainly for answers? These limitations are emphasized further by the context of Karen's and Betty's uses of *Covering and Surrounding*. For both teachers, the unit was an isolated piece of their curriculum that did not represent a long-term investment to change their practice. Since Karen's and Betty's teaching of *Covering and Surrounding* entailed only a four to five week commitment, it is not surprising that both maintained beliefs and patterns of practice that have been part of their prior teaching.

Another factor to consider is that the *Covering and Surrounding* teacher's edition, along with the student and teacher materials for all other CMP units, are still in the preliminary stages of development. Later versions of the *Covering and Surrounding* teacher materials might prove more useful to Karen and Betty than the version they used in this study. For example, the *Covering and Surrounding* student and teacher editions used by Karen and Betty are formatted separately. While neither teacher mentioned the two editions being separate as problematic, it is possible that the separate format shaped how they each used the teacher materials[1]. It is not clear in this study, however, to what extent reformatting the teacher materials (e.g., to include the student text) would help support Karen and Betty in teaching with the CMP curriculum.

The limitations of this study preclude dismissing teacher materials as a form of support for Karen and Betty in using the CMP materials. At the same time, Karen's and Betty's uses of the *Covering and Surrounding* teacher edition put into question the adequacy of curricula alone as a teacher's sole source of support.

Teaching and Learning Subject Matter

Ball and McDiarmid (1990) emphasize that little is known about how teachers' subject-matter knowledge is shaped from using curricula, or how teachers learn subject matter from curriculum. Since teachers' subject-matter knowledge of mathematics can be fragmented and rulebound, using problem-solving oriented curricula will push at the boundaries of some teachers' subject-matter knowledge. This was also the case with the "New Math", a national K-12 mathematics curriculum reform effort during the late 1950s and early 1960s post-Sputnik era. The new math curriculum was designed to increase the mathematics preparation and achievement of K-12 students and was based largely on the axioms of set

[1]Trials of the CMP materials in other sites have indicated that at least some teachers have found juggling both versions to be cumbersome. Some teachers report reading the teacher materials once, making notes in a student copy, and then teaching from the student book. These teachers would prefer a teacher edition that includes the student text. Currently, the CMP is reformatting the teacher editions for all units so that the teacher edition displays for the teacher the corresponding text in the student edition.

theory and the field properties of real numbers. Although the new math was intended to provide students with a solid foundation to study higher mathematics in college, the movement floundered. The failure of the new math is usually attributed to unrealistic expectations for teachers to teach new kinds of mathematics with little or no subject-matter preparation (Sarason, 1982).

The new math reform era showed that it may be quixotic to expect teachers to teach mathematics that is unfamiliar or not well-understood. In the present context of problem-solving oriented reforms, the failure of the new math suggests that curriculum developers may need to be careful to provide teachers with subject-matter support. Curricula like *Covering and Surrounding* have the potential to put teachers in mathematical situations that are unfamiliar. In retrospect, the new math may suggest that curriculum developers need to supply teachers with extensive subject-matter support. For example, curriculum developers might provide detailed solutions to problems that have the potential to be problematic for teachers. In light of the previous discussion of the possible limitations of teacher support through curricula, perhaps video tape segments or even teaching consultants should be supplied to give teachers examples of how to handle students' ambiguous questions, misconceptions, or conjectures that may be unanticipated.

However, this study also suggests that the issue of subject-matter knowledge need not be a matter of providing lots of comprehensive support to teachers. For example, it may be possible that subject-matter knowledge is not a major issue for some teachers in teaching with problem-solving oriented curricula. Karen's use of *Covering and Surrounding* suggests that some teachers may be able to use problem solving oriented materials effectively with students even if the subject matter is unfamiliar and subject-matter support is scarce. Betty's use of the unit shows that providing detailed subject-matter support in teacher materials, such as described above, may still not address teachers' instructional problems, even for teachers with deep and connected subject-matter knowledge.

Karen exhibited limited subject-matter knowledge about perimeter and area in preliminary interviews. Moreover, in using *Covering and Surrounding*, she was hesitant in responding to Shakaya's question about whether or not to count the perimeter of the courtyard in a room she had designed as part of the room's perimeter. Karen's interaction with Shakaya, coupled with the interview data, raises questions about whether her subject-matter knowledge was sufficient for her to unravel the perimeter situation. However, the outcome of this lesson (i.e., Shakaya deciding for herself to redesign the room without a courtyard) is actually congruent with at least some aspects of problem-solving oriented reforms[2]. Moreover, there is no discussion in the *Covering and Surrounding* teacher edition about Shakaya's question. Although the subject-matter appeared to be unfamiliar to Karen and the teacher materials offered no assistance, Shakaya's question was resolved and Shakaya appeared to have a conceptual understanding of perimeter and area upon completion of designing her room.

Curricula like *Covering and Surrounding*, by providing students with open-ended problems like the room design problem, do have the

---

[2]For example, students addressing their own questions and determining the validity of their own mathematical ideas, as Shakaya did with her question about perimeter, is part of "mathematical power" which reformers advocate students should develop in K-12 mathematics (see NCTM, 1989, 1991).

potential to provide students with opportunities to explore their own ideas about mathematical concepts. Karen's use of the room design problem shows how a teacher's use of an open-ended problem can contribute to providing an opportunity for students to pursue their own questions. However, when coupled with her thin subject-matter knowledge about perimeter and area, Karen's use of the room design problem raises the issue of how curriculum can support teachers in encouraging student exploration when the teacher herself is not necessarily familiar with the mathematical terrain students may wander into.

In contrast to Karen, Betty demonstrated rich and connected subject-matter knowledge of perimeter and area in the interviews prior to teaching *Covering and Surrounding*. However, a growing body of research argues that subject-matter knowledge is a necessary but not sufficient component of the knowledge teachers need to teach mathematics for understanding; teachers should also possess knowledge of how to represent and make content accessible to learners (Shulman, 1987; Wilson et al., 1987). This point is illustrated in Betty's use of the problem on circumference of circles. She attempted to get at the concept of π by using transparencies to show that slightly more than three squares of edge length equal to the radius of the circle are needed to exactly cover the circle. While Betty appeared to understand the content of the problem, her way of representing it was confusing to at least some of her students. What is also significant is that the *Covering and Surrounding* teacher edition *did* provide another approach for teaching the circle circumference problem of which Betty was not aware. This example shows that subject-matter knowledge alone may not be sufficient for teaching with problem-solving oriented materials and, perhaps more alarming from a curriculum developers' perspective, even if needed pedagogical support is provided, teachers may not use it.

Betty's use of the circle problem also points to the importance of teachers' pedagogical content knowledge, i.e., the knowledge teachers draw on to represent and connect content to learners (Shulman, 1987; Wilson et al., 1987; Wilson, 1989). Curricula like *Covering and Surrounding* employ representations extensively. Teaching with the representations provided in problem-solving oriented curricula like *Covering and Surrounding* will require teachers to know about how to connect representations with learners as well as understanding subject matter (Fitzgerald et al., 1991; Shulman, 1987; Wilson et al., 1987). For example, *Covering and Surrounding* uses the circle problem as a representation to connect students with the concept of π . As Betty's use of the circle problem shows, however, it may be difficult for some teachers to use the representation effectively. While Betty understood the subject matter embedded in the circle problem, she struggled in her teaching because she was not able at the time to use the representation to connect her students with the content she herself understood.

Betty's use of the circle problem also speaks to how curricula like *Covering and Surrounding* may influence teachers' pedagogical content knowledge. Although Betty did not take advantage of it, the teacher edition does provide suggestions for how to teach the circle problem and Betty was unfamiliar with at least one of the suggested approaches. Even though Betty did not learn about the alternate approach to the circle problem from the *Covering and Surrounding* materials, the fact that the approach was new to her and that it is detailed in the teacher edition is significant. It suggests that curricula like *Covering and Surrounding* have the potential to help teachers who do use the teacher materials develop their pedagogical content knowledge. For example, if Betty had used the teacher materials in planning her lesson on the circle problem, she would at least have had at her disposal another approach to using the representation.

Karen's and Betty's uses of *Covering and Surrounding* illustrate two facets of subject-matter knowledge for curriculum developers. One is the perhaps encouraging finding that even when on potentially unfamiliar mathematical ground, teachers might use problem-solving oriented materials in ways that lead to conceptual understanding in students. Raising a thorny dilemma, however, is the possibility that some teachers who are in familiar mathematical territory may still have difficulties teaching with problem-solving materials and may not use the materials for assistance even when it is provided.

The dilemma of Betty's nonuse of teacher materials may also be connected to subject-matter knowledge. *Covering and Surrounding* addresses perimeter and area, concepts that are both part of the standard sixth-grade mathematics curriculum and which Betty had taught before with other nonstandard curricula. Betty's familiarity with teaching perimeter and area with other materials may have contributed to her belief that she did not need to consult the *Covering and Surrounding* teacher materials. From Betty's perspective, she may have perceived *Covering and Surrounding* as mapping out familiar mathematical territory she had traversed with other students before, and therefore did not feel the need to consult the teacher materials. However, this might not be the case for Betty with other curricular materials that address less familiar content. For example, if the *content* of *Covering and*

Surrounding had been unfamiliar to Betty -- e.g., if she had taught a CMP unit on statistics, probability, or spatial visualization -- she might have been more inclined to use the teacher materials to get ideas for teaching problems or for help in answering students' questions.

Covering and Surrounding is part of a complete curriculum. The varying (and perhaps unfamiliar) subject matter of the other sixth-grade CMP units might contribute to Betty changing her use of CMP teacher materials. Of course, Betty could conceivably maintain her nonuse of teacher materials even when teaching unfamiliar content. However, Betty's familiarity with the content of *Covering and Surrounding* nonetheless raises the issue to curriculum developers of how teachers may change their use of teacher materials when confronted with teaching *unfamiliar* subject matter. Another open research question connected to teaching unfamiliar content is to examine how teachers *learn* from mathematics curricula[3]. For example, what might Karen have learned about perimeter through her use of *Covering and Surrounding* and her interaction with Shakaya? What might Betty have learned about teaching circumference of circles? Materials like *Covering and Surrounding* seem to hold potential to help teachers learn about mathematics and teaching mathematics. But how this learning might occur, and to what extent it shapes subsequent teaching, are areas needing further investigation.

Researchers have argued persuasively that subject-matter knowledge and pedagogical content knowledge shape practitioners' teaching (e.g., Ball & McDiarmid, 1990; Shulman, 1987; Wilson et al., 1987), and curriculum developers seek to support teachers in these areas (e.g.,

[3]Ball and McDiarmid (1990) point to this same line of inquiry as an area for future research in teacher education.

Fitzgerald et al., 1991). However, snapshots of Karen's and Betty's uses of *Covering and Surrounding* show that anomalies can exist between teachers' knowledge and their use of curricula. Karen shows that teachers can use problem-solving oriented curricula in ways powerful for students even if subject-matter knowledge is thin. Betty shows that teachers can experience instructional difficulties even if the content is familiar and knowledge of the content is rich and connected. These snapshots emphasize the complex role subject-matter knowledge and pedagogical content knowledge can play, or not play, in using problem-solving oriented curriculum.

Teachers' Perceptions and Beliefs About Learners

Finally, this study suggests that teachers' perceptions and beliefs about student learning may not only shape how teachers use problem-solving oriented curriculum, but also what they teach to learners. In agreeing to participate in this study, Karen specified that she would teach *Covering and Surrounding* to her Enriched Mathematics class, but not her General Mathematics classes (see Chapter 5). Karen's reason for this decision was that she perceived her Enriched Mathematics students as being more likely to be interested in the unit and succeed with it. She also felt that she had to know how her "best students could handle the unit" before teaching it to the General Mathematics classes. After teaching *Covering and Surrounding*, Karen noted that she would probably teach parts of the unit (e.g., the first investigation) to the General Mathematics classes.

Although the developers of *Covering and Surrounding* clearly intend the materials to be a curriculum for *all* students (see Fitzgerald et al., 1991), Karen did not initially perceive the unit as suitable for all of her students. While Karen's Enriched Mathematics class had over four weeks of studying *Covering and Surrounding*, her General Mathematics students most likely received far less. Karen's perception of student learning between her Enriched and General students leads to an instructional decision (i.e., what parts of the unit should be taught to whom) that is inconsistent with the intent of the curriculum developers and illustrates contemporary arguments about inequity in learning opportunities in school (e.g., Anyon, 1981; The Holmes Group, 1990; NCTM, 1989, 1991; Oakes, 1985).

Another facet of Karen's use of *Covering and Surrounding* related to perceptions and beliefs about learners is that throughout teaching the unit she maintained that her students were not learning about problem solving. Although there are different ways that problem solving can be conceptualized for teaching and curriculum, it seems surprising that Karen did not, for example, view Matt, Trevor, and Aaron's argument about triangle area as involving problem solving. Should curriculum developers address Karen -- e.g., try to convince her that students can learn about problem solving in *Covering and Surrounding*? If so, how? The problem solving objectives for students in the unit (which Karen highlighted) did not convince her that problem solving occurred in her classroom. This suggests that Karen's beliefs about problem solving may not be easily challenged.

One approach could be to try and change Karen's perceptions about her students' learning about problem solving -- e.g., engage her in thinking more about problem solving by, for example, including a framework in the teacher materials discussing a range of problems. Karen would then have a resource to fit *her* conception of problem solving into a spectrum of problems and processes (e.g., see Hembree, 1992; Chapter 4). These could range from standard word problems which fit her conceptions of problem solving and problem-solving activity, to non-standard, real-world, and puzzle problems which constitute most of the kinds of problems in *Covering and Surrounding* (see Chapter 4). Such a revision of the teacher materials would perhaps help Karen see more happening in *Covering and Surrounding* than just "hands-on" activities. But another approach that curriculum developers could take would be to do nothing, i.e., in a sense accommodate Karen's beliefs by not challenging them. After all, whether Karen believes her students are learning about problem solving or not is perhaps not so important to the curriculum developers if her students are doing what they envision as problem solving.

It is unlikely that there is a single "right" way to address Karen's perception that her students were not learning about problem solving in *Covering and Surrounding* (see Chapter 2). Before either of the above approaches -- change or tacit accommodation -- should be considered, however, an issue embedded in both warrants further scrutiny. To what extent does how the *teacher* conceptualizes problem solving shape what her students *learn* about problem solving when using problem solving oriented curricula? This is a critical research issue to consider, especially since reforms put problem solving at the core of what students should be learning about in school mathematics. If teachers' conceptions of problem solving do shape students' learning about problem solving, then perhaps curriculum developers should be more concerned about how teachers conceptualize problem solving. For example, a framework such as suggested above might be included in problem-solving oriented curricula to help teachers understand how problem solving is envisioned and embedded in the materials.

Another perspective on the extent to which teachers' perceptions of what their students are learning about problem solving shapes students' learning would be to consider what curriculum developers have to learn from teachers like Karen. For example, perhaps the developers should incorporate more standard word problems into *Covering and Surrounding*. Since it is not clear what role standard word problems play in students' learning about problem solving, the developers might be overlooking a type of problem in *Covering and Surrounding* that is key to students' learning about problem solving. While these multiple perspectives do not provide answers to how teachers' perceptions about student learning *should* be addressed, research in this area could help guide curriculum developers' future decisions about the substance and design of problem-solving oriented curricula.

Final Comments

This chapter has raised a host of issues and dilemmas that are related to teachers' use of problem-solving oriented curricula. Furthermore, the four major areas discussed in this section are by no means exhaustive. As a researcher, I feel that this study has raised more questions than it answers. In retrospect, however, I think that this is desirable. Amid the current flurry of mathematics education reform and curriculum development, it would be a mistake to lose sight of the complexity of these endeavors. Most importantly, I hope that those who are involved in reforming K-12 mathematics education, especially through developing problem-solving oriented curriculum, continually keep teachers like Karen and Betty in mind -- for teachers, some like Karen or Betty, will be the final arbiters of reform.

APPENDICES

APPENDIX A

Below is the interview protocol used for the vignette about a lesson on problem solving. Following the protocol is the vignette that was provided for the teacher participants:

Provide teacher participants with the "Mr. Fern" vignette. Allow the teacher adequate time to read the vignette and then ask the following questions:

What stands out to you about this lesson?

What are the strengths and weaknesses that you see in this lesson?

Do you think that Mr. Fern chose a good problem for his students to solve? Why or why not?

How would you structure a lesson in your math class, using the same or a different problem, to teach the problem solving strategy of "guess and check"?

Altogether, this lesson took about two-thirds of the class period. Do you think that the lesson was a good use of classroom time? Why or why not?

Vignette:

The following passage is a vignette of a sixth-grade mathematics lesson. Mr. Fern, the teacher, was observed by a researcher who wrote this description of Mr. Fern's class:

An example of how Mr. Fern teaches specific problem solving strategies can be drawn from a lesson he taught at the beginning of the year. He told the researcher before the lesson that his students were going to learn about the problem solving strategy "guess and check". This particular strategy, where a potential solution to the problem is guessed and then checked for validity against the conditions of the problem, is a problem solving tool that is often incorporated into classroom problem solving materials. Mr. Fern presented students with the following problem:

> *If you have 9 coins that are together worth 58 cents, what are the kinds of coins you can have?*

Mr. Fern illustrated the "guess and check" strategy to the students by asking them how they might solve the problem if one of the coins was a half-dollar. Many students quickly volunteered that 1 half-dollar and 8 pennies would be the solution. After noting the students' "guess", he "checked" the solution by seeing if it met the conditions of the problem - does 1 half-dollar and 8 pennies make 9 coins and 58 cents? After noting how the solution was validated by the check, Mr. Fern imposed an additional condition on the problem - find a solution to the problem <u>without</u> using a half-dollar. He instructed the class, working in their cluster groups (the students' desks are arranged in clusters of 3 or 4) to find two solutions to the problem.

The students quickly began working on the coin problem. Mr. Fern said later to the researcher that although none of the groups used a half-dollar in their solutions, at first most attended to only one of the other two conditions. Students generally either used 9 coins and neglected their value (e.g., one group's initial "guess" solution used 2 quarters, 1 nickel, and 6 pennies) or had a number of coins different than 9 with a value of 58 cents (e.g., another group's first "guess" at a solution used 1 quarter, 3 dimes, and 3 pennies). Mr. Fern circulated among all the groups and in each case asked if the group's "guess" at a solution "checked" with all the conditions of the problem. Groups whose solution was revised and validated by meeting all the problems conditions were challenged to find another solution. Groups whose solutions did not meet all conditions were encouraged by Mr. Fern to revise their solutions and check again. After 15 minutes of working on the coin problem, all the groups had at least one valid solution and most groups had two.

Mr. Fern brought the class back together through a class discussion where two groups were each asked to give a solution and describe why it was valid. Mr. Fern emphasized to the class that this was an example of a mathematics problem which has more than one solution and could be solved using the problem solving strategy of "guess and check". He also mentioned that the "guess and check" strategy would be useful for many other kinds of problems.

After class, Mr. Fern said that he had several goals for students in the lesson. First, he wanted his students to understand and successfully implement the "guess and check" problem solving strategy. This includes formulating a solution and then checking the solution against the conditions of the problem; a solution is valid if it meets all of the problem conditions. Mr. Fern also wanted students to work with a math problem that has more than one answer, can be investigated cooperatively with peers, and draws on a variety of prior knowledge (e.g., knowledge of money as well as knowledge of multiplication and addition).

APPENDIX B

Below are the interview protocols used for Items 1, 2, and 3 in the teacher interview to assess subject-matter knowledge. Following the protocols are the individual items provided for teachers to react to:

<u>Item 1</u>:

Suppose that while you are teaching your class about perimeter and area, one of your students comes up to you. She says that she has discovered a new theory that you never told the class. She says that she has found that as the perimeter of a figure increases its area also increases. She shows you these pictures she has made, and says that they prove her theory [Show item 1 sheet (i.e., Figure A.B1) to the teacher participant]. *How would you respond to this student?*

<u>Item 2</u>:

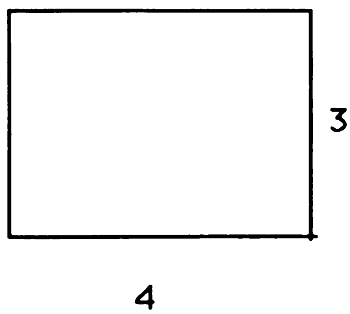
Suppose that during your instruction on perimeter and area a student raises his hand and is very excited. He says that he has figured-out how to convert between units used to measure area. He shows you his solution to this area problem [Show item 2 sheet (i.e., Figure A.B2) to the teacher participant]. *He says that the area of the triangle is 24 square yards. He also says that since there are 3 feet in a yard, he can convert the area of the triangle to square feet* [indicate students' calculations on the interview item]. *How would you respond to this student?*

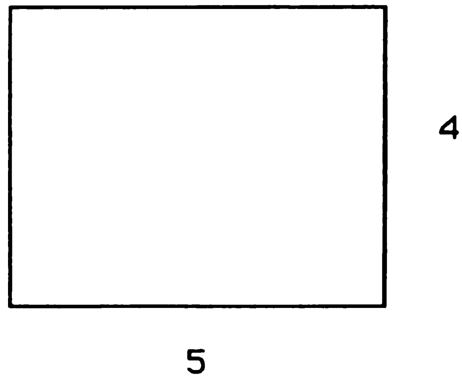
<u>Item 3</u>:

Suppose that one day after school a fellow sixth-grade teacher stops by your room. She has a student's assignment on perimeter with her and shows it to you [Show item 3 (i.e., Figure A.B3) to the teacher participant]. *She says that she isn't sure if the student's method is valid, or what the student's method is, even though he got right answers. She would like you to help her make sense of the student's work. How would you try and explain the student's work to her?*

Probe: *Can you generalize the student's rule?* and/or *Can you show whether the student's method is valid or invalid for finding the perimeter of rectangles?*

Item 1



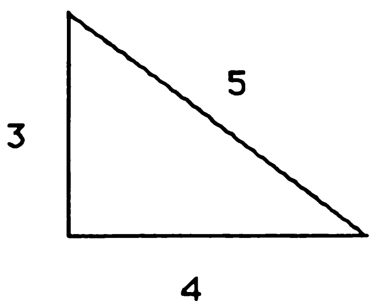


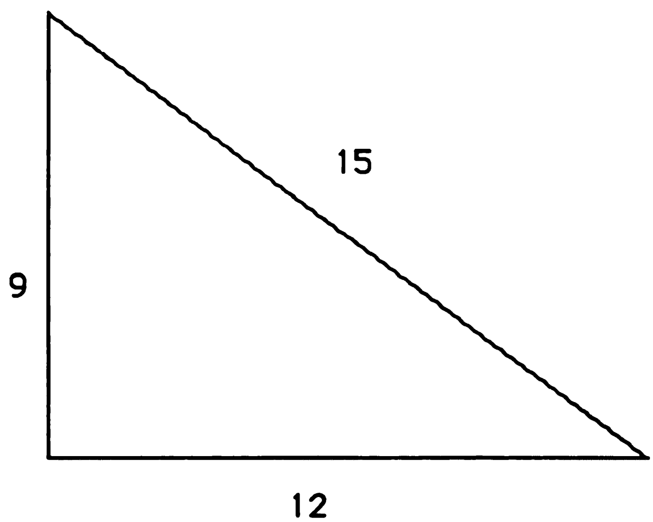
Perimeter = 14

Area = 12

Perimeter = 18

Area = 20





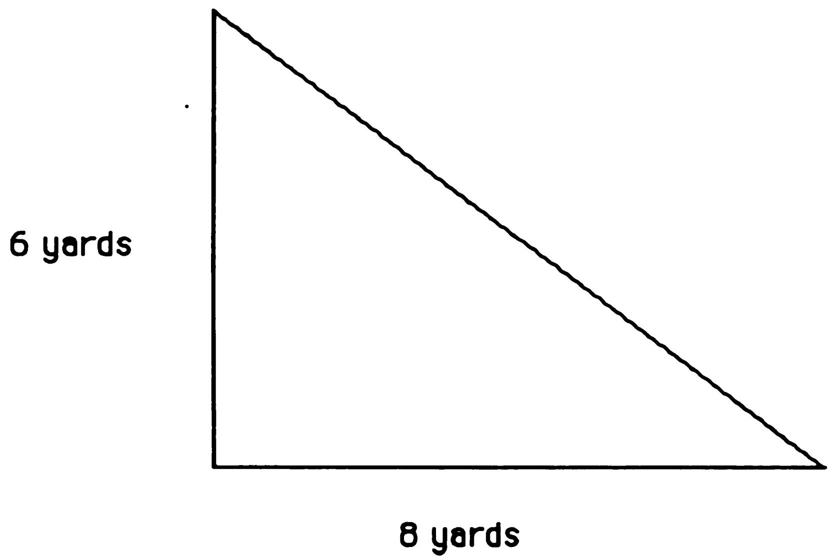
Perimeter = 12

Area = 6

Perimeter = 36

Area = 54

Item 2

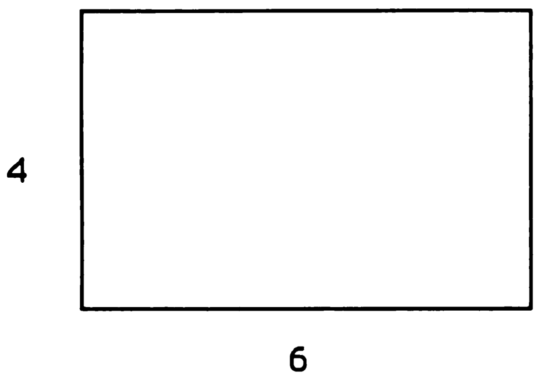


A = 1/2 x B x H = 1/2 x 6 x 8 = 1/2 x 48 = 24 square yards

3 feet = 1 yard

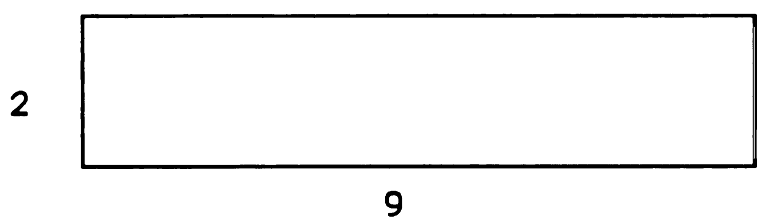
Area = 24 x 3 = 72 square feet

Item 3



Perimeter = 2 x (24/4 + 24/6) = 2 x (72/12 + 48/12) = 2 x 120/12

= 120/6 = 20



Perimeter = 2 x (18/9 + 18/2) = 2 x (36/18 + 162/18) = 2 x 198/18

= 198/9 = 22

APPENDIX C

Below is the interview protocol used for the vignette about structuring problem-solving activity in classroom settings. Following the protocol is the vignette that was provided for the teacher participants:

Provide the teacher participant with the "Mrs. Beacham" vignette. Allow the teacher participant adequate time to read the vignette. Read the following passage as the vignette is provided:

> *Karen Beacham is a sixth-grade mathematics teacher. This is a description of a lesson from one of her math classes and some of her views on teaching. Please take a few minutes to read this vignette, and then I'd like to ask you some questions about what you think about Mrs. Beacham's teaching* [Provide the teacher participant with the vignette].

After the teacher participant has read the vignette, ask these questions, probing if necessary:

Do you have any first reactions to Mrs. Beacham and her classroom teaching? [Probe for reasons behind the teacher participant's views]

What do you think about how Mrs. Beacham is structuring the lesson (i.e., the three students illustrating how the class is divided up into three different groups) for Dana - Jessie - Pat? [Probe for reasons underlying the teacher participant's views]

What would you say Dana - Jessie - Pat is learning from this lesson?

What beliefs about mathematics do you think Dana's - Jessie's - Pat's assignments promote? Why?

Would you structure your class lessons like Mrs. Beacham? Why? If not, *Why not* and *How would/do you structure your lessons differently?*

Vignette:

Karen Beacham is a sixth-grade middle school teacher. Her schedule includes teaching two math classes. Mrs. Beacham emphasizes the diversity of her students' mathematical abilities in her two mathematics classes. She notes that some students catch on to math very easily while others have math skills that are below grade level. Mrs. Beacham says that to accommodate the diverse needs of her students, she organizes her classes around themes like geometry, number concepts, or statistics. Structuring the class around themes, Mrs. Beacham notes, allows her to provide students with different learning tasks on the same topic that address their specific mathematical strengths and weaknesses. Mrs. Beacham says she generally divides the class into three groups. What follows are excerpts from one of Mrs. Beacham's lessons where number concepts is the common topic for the entire class.

After answering questions at the beginning of the period, Mrs. Beacham gives out three assignments and the students quickly go to work. A few minutes later, Dana, who is busily working on a drill sheet of decimal multiplication exercises, is confused about where to put the decimal point in the product of 2.63 x 4.8. "Remember Dana," Mrs. Beacham says, "count up all the places behind the two decimals and make sure that that is how many places you have behind the decimal point in your answer." Dana says "OK" and goes back to work. Looking over Dana's shoulder, Mrs. Beacham says "Be sure to work all of the problems Dana, because practice makes perfect."

Mrs. Beacham next walks over to Jessie, who has constructed 1x1, 2x2, 3x3, and 4x4 squares with 1-inch tiles. Jessie's assignment was to find numeric and geometric representations of the number sequence 1, 4, 9, 16, ... Mrs. Beacham asks Jessie what the squares mean. Jessie says that the 1x1 square, which has area 1, corresponds to the number 1, the 2x2 square, which has area 4, corresponds to the number 4, and so on. Mrs. Beacham asks Jessie "What would you make for the eighth term in the sequence?" "An 8x8 square" Jessie replies. "Why?" asks Mrs. Beacham. "Because an 8x8 square has area 64 and that's the eighth number in the sequence 1x1, 2x2, 3x3, 4x4, and on and on." Mrs. Beacham asks Jessie if there are other connections between the squares and the numbers in the sequence besides area. Jessie says "The sides of the square are factors of the numbers. Like in the 8x8 square, 8 times 8 is 64." Mrs. Beacham smiles and says "You've provided some good justifications, Jessie." She then asks Jessie to next try using cubes to represent the number sequence 1, 8, 27, ...

Near the end of the class period, Mrs. Beacham is circulating around the room and notices an error on Pat's paper. Pat is working on an assignment which consists of story problems that involve fractions, decimals, and percents. Pat is using a calculator to check answers. One problem asks for a solution in percent form, and Pat found the answer to be 13/20. Mrs. Beacham notices that in converting 13/20 from fraction form to decimal form, Pat put .75 down as an answer. "How many 20s are there in 130?" Mrs. Beacham asks Pat, pointing to the paper where Pat is dividing 20 by 13. "Six with 10 left over" answers Pat. "Then how can you have .75?" Mrs. Beacham says, emphasizing the number 7. Mrs. Beacham asks Pat to check the answer on the calculator. Pat punches in 20 divided by 13 and says "Oh, I messed up! I should have .65, not .75." Mrs. Beacham tells Pat to be careful so as not to make arithmetic mistakes, and Pat continues using the calculator to check other problems.

After working with Pat, there are about two minutes left before the end of the period. Mrs. Beacham asks the class to clean-up and finish their assignments for homework.

APPENDIX D

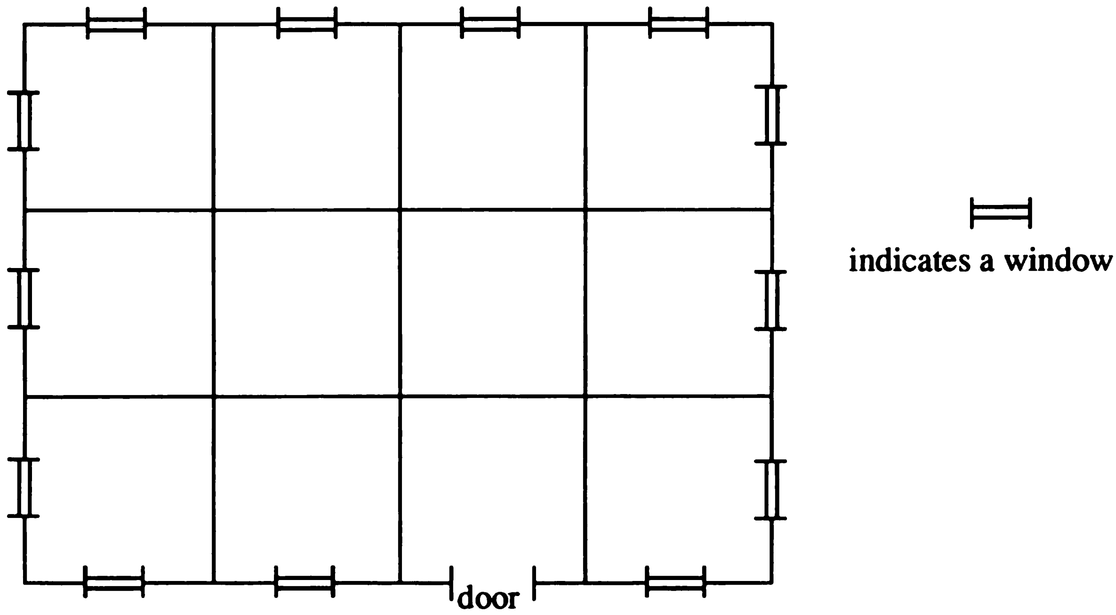
This appendix is a complete investigation from the student edition of *Covering and Surrounding* (CMP, 1992a, pp. 1-12). This investigation is representative of the format and kinds of questions/tasks present in all of the investigations in *Covering and Surrounding*:

Investigation 1: Measuring and Designing Rooms

Mathematics is used in the world around us in many ways. Some of these important uses of mathematics involve measuring things. This unit is about measurement. The questions "How big is it?" "How small is it?" "What is the biggest or most?" and "What is the smallest or least?" will be asked again and again in this unit. You are going to be asked to think about what it means to measure something and to plan how you can actually measure properties of objects or events.

Architects and builders deal with measurement in all aspects of their work. In this first investigation we are going to help an architectural firm measure and design interesting rooms.

Mrs. I. Wana Hide is a friend of yours. She has asked an architect, Mr. Really Dull, to design a special room to add onto a covered walkway attached to her house. Mr. Dull has divided the plan of the room up into squares because the carpeting needed to cover the floor of the room comes in large squares. The sections of wall that are used to build the room may have either a door or a window made into them. In Mr. Dull's design, each wall section has a door or window. This is the design Mr. Dull produces for Mrs. Hide:



As an architect, Mr. Dull uses grid paper to design his plans, like in his design of Mrs. Hide's room. Architects also use models in their work. Physical or pictorial models are used to represent and learn about other things that are too big or small to study easily. In designing rooms it is much easier to build models of rooms to make different designs than to build all of the actual rooms.

Take 12 tiles, which will be used to model carpet sections, and arrange them in the same design as Mr. Dull's. Note that it takes 12 carpet tiles to cover the room and there are 14 wall sections, or tile edges, in the room. In Mr. Dull's design each wall section has a door or window, but this may not be the case for every room. The names that mathematicians use for the kinds of measures you have computed for each room are the area and perimeter. You have probably already realized that the number of square carpet tiles needed to *cover* the floor is a measure of *area* and the number of wall sections around *the edge* of the room is a measure of the *perimeter*. Because units which measure perimeter are straight, like the tile edges, units used to measure perimeter are also called *linear* units. Because units which are used to measure area are square, like the tiles, units used to measure area are also called *square* units. Therefore, the room Mr. Dull designed has an area of 12 tiles or square units, and a perimeter of 14 tile edges or linear units.

In the following problems you will use the tiles to make models to help you to measure and design rooms.

Problem 1: Mrs. Hide likes the amount of floor space in Mr. Dull's design, but she wants more windows and a more interesting shape for her room. She asks you to help her out.

a) Use 12 square tiles to create a floor plan design for Mrs. Hide. Remember that a window or a door can go into each section of wall space. Mrs. Hide tells you that she wants at least 14 sections of wall space, including windows and doors, in the room you are designing.

b) After you have designed the floor plan for the room make a drawing to show the location of the door and where each window is. Write a paragraph to tell Mrs. Hide why your design is better than Mr. Dull's.

Problem 2: The Add-a-Room Architectural Firm works with its customers to meet their special needs when designing a room. Some customers want rooms that have a great deal of window space to let in lots of light. Others need a room with less sunlight. The floors of all the rooms the architectural firm produces are covered with large square carpet tiles. The wall sections are as long as the edge of a carpet tile; a window or a door may be put into each wall section.

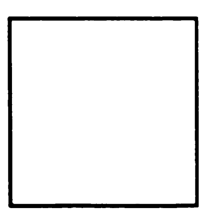
Julio and Julianna work for the firm. They drew the following floor plans for rooms labeled A-J.

a) A customer needs at least 10 square units of floor space, which rooms are possible?

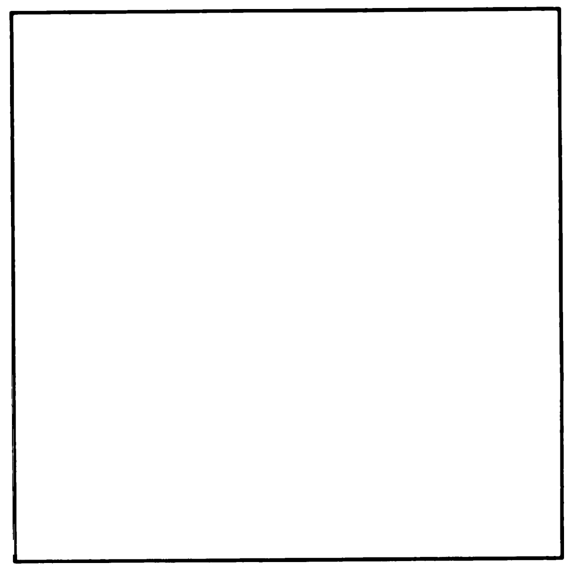
b) A customer wants at least 15 windows and one door (16 sections of wall) in the room. Which plans are possible?

c) Examine each floor plan and find the area and perimeter. Organize your answers so that for each room you record the letter of the room design, the number of carpet tiles needed to cover the floor (the area), and the number of wall sections in the room (the perimeter). One way of organizing your data would be to make a table.

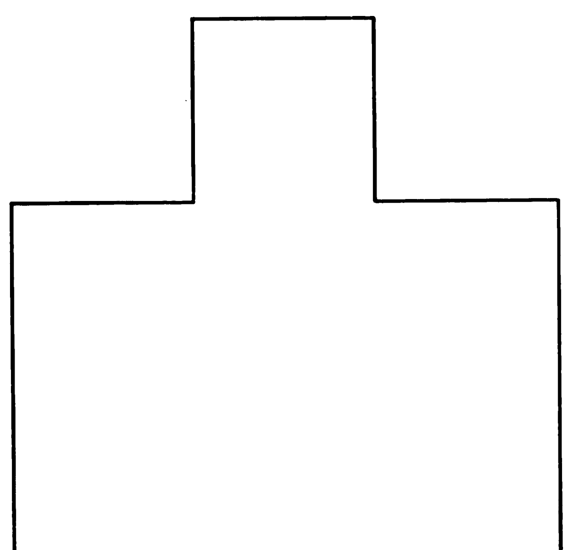
These are the floor plans for Rooms A-J. Here is the picture of one of the large square carpet tiles. As you can see, your small square tiles can be used to represent the carpet tiles to cover the floor plans of the rooms.



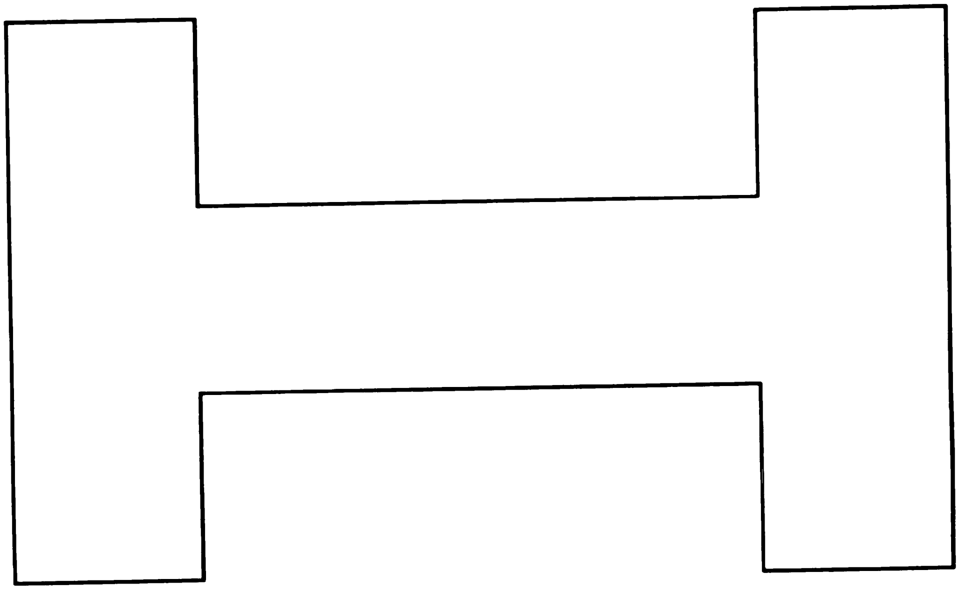
Room A



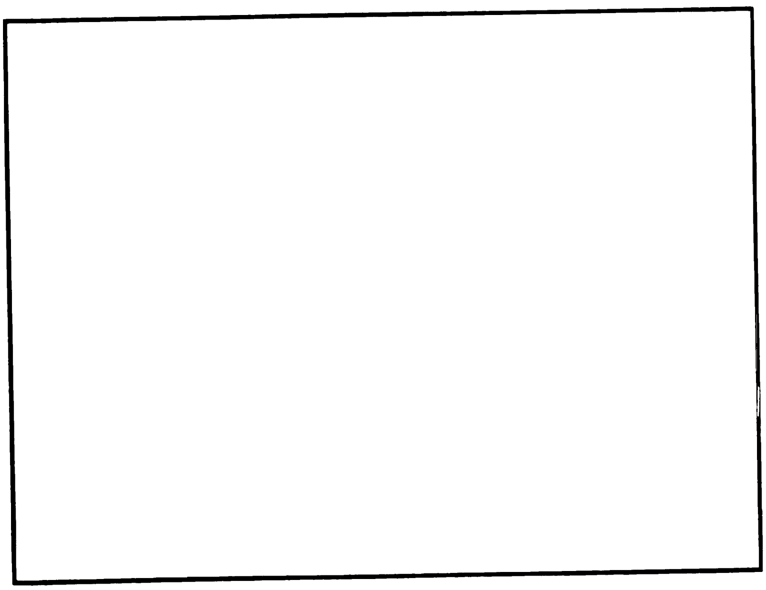
Room B



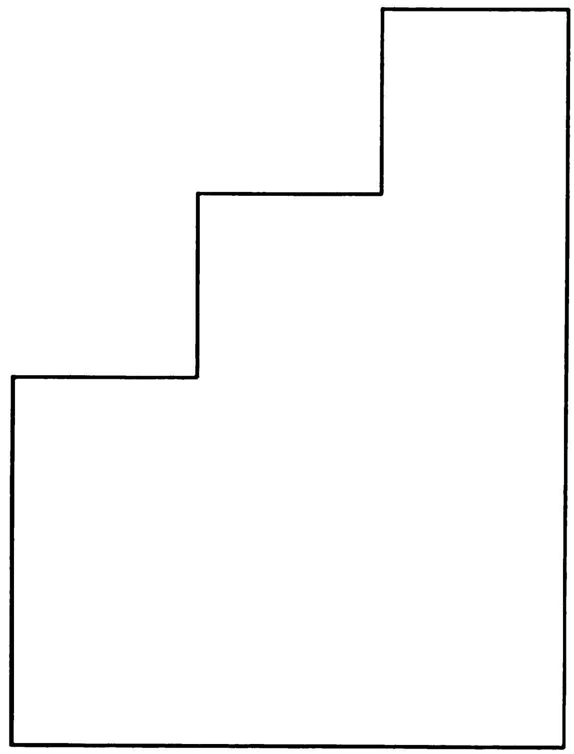
Room C



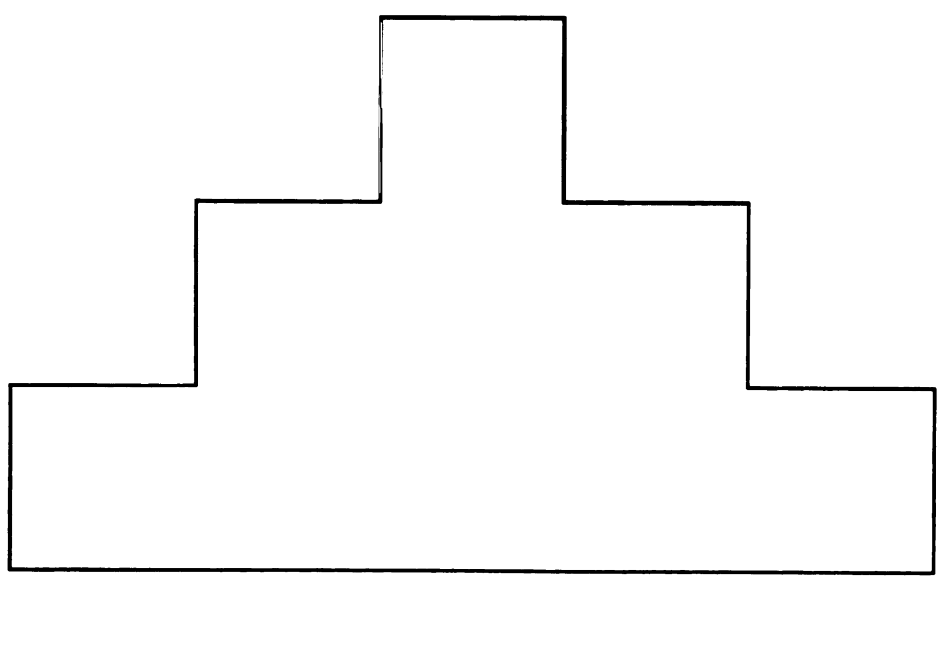
Room D



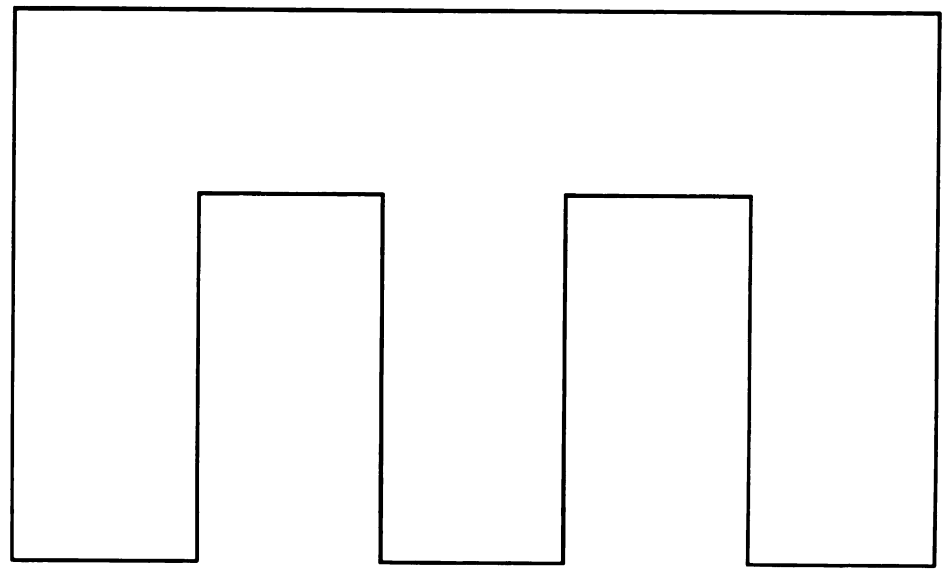
Room E



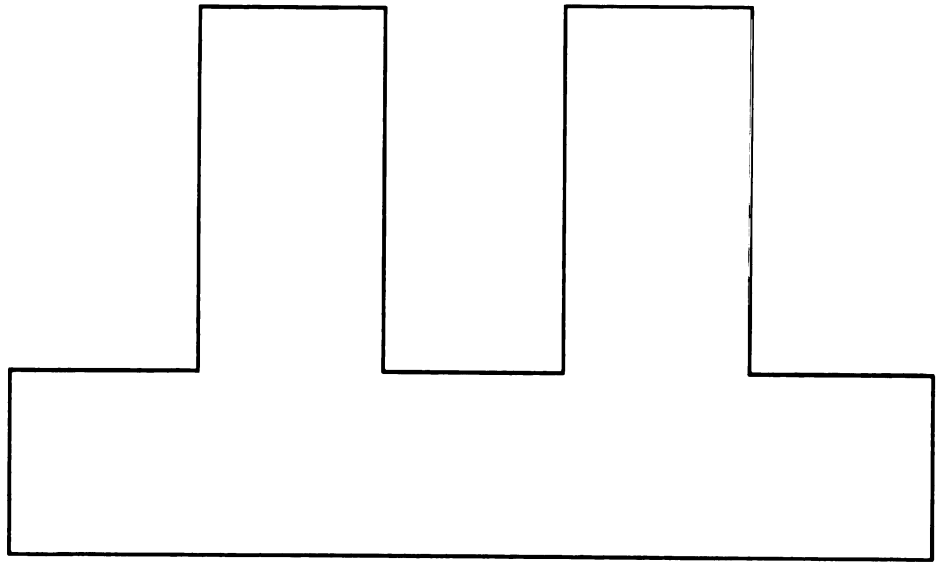
Room F



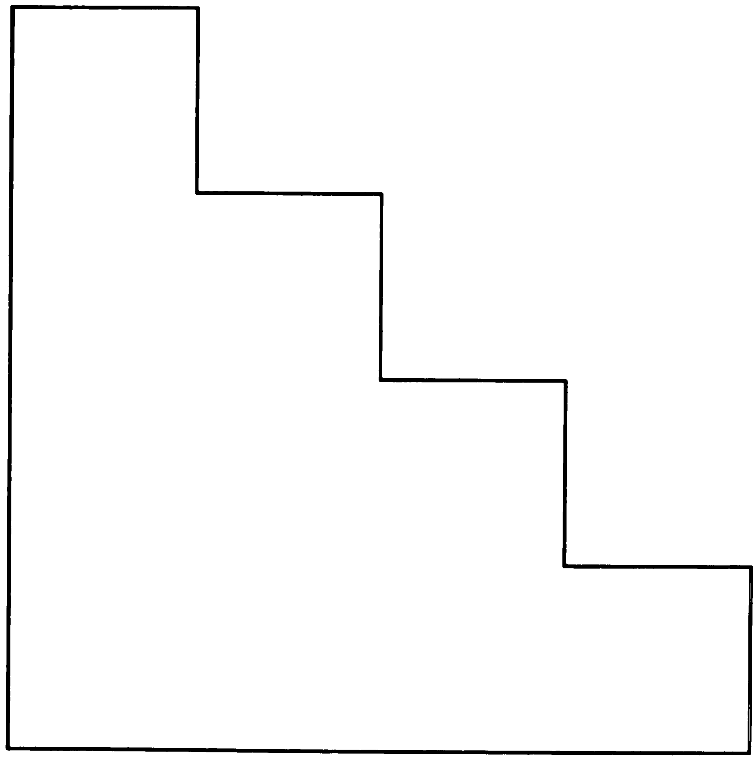
Room G



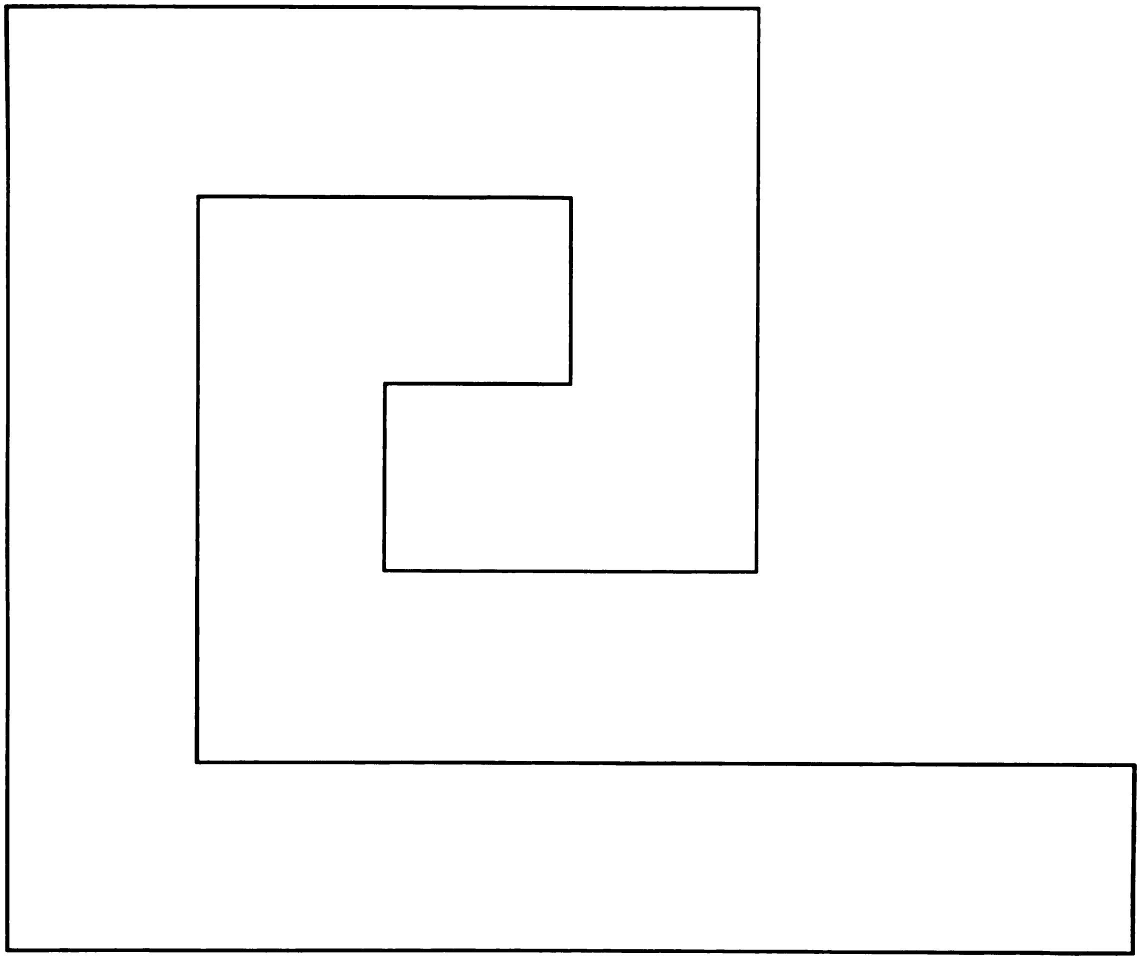
Room H



Room I



Room J



Follow-Up Question:

1. Use the data in your table to decide which three designs you think are the "best". You may want to consider such things as what you think each room is to be used for as you make your decisions. Be sure you can give a convincing argument to support your choices.

Problem 3: The Add-a-Room Architectural Firm is always looking for clever designers. The following questions give you a chance to be a designer. Build your answers using the tiles, but be sure to make a record of your response in your journal. One easy way to record your work is to draw a picture on a piece of grid paper to represent what you built with the tiles. Each question refers to one of Julio and Julianna's rooms.

1. a) Can you make a room from tiles with the same area as Room A, but with a smaller perimeter? Why or why not?

   b) Can you make a room from the tiles with the same area as Room C, but with a smaller perimeter? Why or why not?

2. Room E can be made from Room D by removing 3 tiles. This makes the area of E three less than the area of D. How does the perimeter of D compare to E? Explain why.

3. Look at Room G and Room H. Without using your tiles, find the area and perimeter of the new figure that can be made by pushing Room G and H together to form a rectangle.

4. Rooms F and I have the same perimeter. Cover F with your tiles. Can you rearrange the tiles to match the shape of Room I? Why or why not?

5. Make a tile pattern to match Room B. Moving only one of the tiles make a new figure that has a perimeter of 14.

Applications - Connections - Extensions

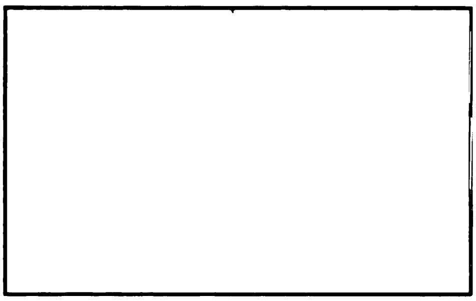
Applications

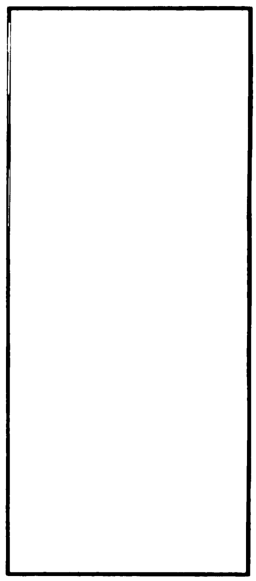
1. Stephanie claims that she can add tiles to the plan for Room J to create a new plan with an area of 6 square units more than Room J, but with a perimeter of 22 units. Using your tiles, make Stephanie's figure. Why are the perimeters of Room J and Stephanie's room so different?

2. Look back at Room B. Arrange tiles that match B. Move exactly 2 tiles and make another figure with perimeter 14. Record your figure.

3. Arrange your tiles to match Room B. Can you rearrange tiles to make a figure with a perimeter of 30? Why or why not?

4. In tile units what is the area and the perimeter of the rooms below:

a) 

b) 

5. Copy rooms a) and b) above on a sheet of paper. Use a centimeter ruler to measure each edge of the rectangles a) and b). What is the perimeter of each room in centimeters?

6. Use a centimeter ruler as a guide to draw in the square centimeters to show the area of the rooms in problem 4 in square centimeters.

Connections

7. Suppose you have 18 square tiles. The edge of a tile is one unit long. What are the perimeters of the rectangles you can make with an area of 18 square tiles? Draw a picture on grid paper of each rectangle you found. How do you know you have found all possible rectangles?

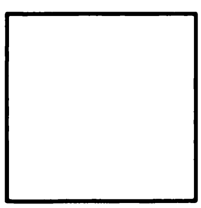
8. If you walk around the perimeter of a rectangle, through how many degrees have you turned when you get back to where you started?

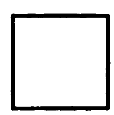
Extensions

9. Using 12 tiles build the floor plan for a rectangular room that will provide space for the most wall sections. Now rearrange the tiles to make a floor plan that will provide space for the fewest wall sections. Record your solutions.

10. Look back at Room I.

   a) Find the area and the perimeter of the room. Use the square tile below as the square unit to measure the area and perimeter of Room I:



   b) How would your answers to part a) change if this  is the square unit for measuring area?

APPENDIX E

This is the supporting material provided for teaching Investigation 1 in *Covering and Surrounding*. The material is from the *Covering and Surrounding Teacher's Edition* (CMP, 1992b, 22-29):

Investigation 1 - Measuring and Designing Rooms

The context for this investigation is measuring and designing rooms. The **mathematical and problem solving goals** for students are:

* Developing *awareness* of the differences between area and perimeter.

* Developing concept images (i.e., mental images) that help distinguish between area and perimeter.

* Finding area and perimeter through covering with tiles and counting edges and numbers of tiles.

* Understanding that two figures with the same area may have different perimeters and two figures with the same perimeter may have different areas.

* Visualizing what changes occur when tiles forming a figure are rearranged, added, or subtracted.

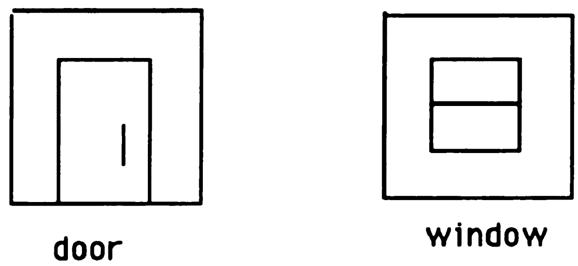
* Organizing information in a table.

Materials Needed

* Square 1-inch tiles, 24 per student (if you have different size tiles, the figures will need to be redrawn to match the tile)

* 1-inch grid paper (Labsheet 1.1).

The story of designing rooms in the student text can be used to launch the investigation. It is important that students get a clear picture of the relationship between the physical tiles representing squares of carpet and the prefabricated wall sections that are the same width as the edge of a carpet tile. The prefabricated sections can each contain either a window or a door, but may also be blank wallspace. You may want to prepare a few cut outs from cardboard with a door or a window drawn on to show students a floor plan:



1-inch square (tile size) wall sections

Once the students understand how the floor plans are modeled by the tiles, then they are ready for the first problem, designing a floor plan with square units of floor space but with more windows and doors. This part of the investigation can be done individually or in pairs.

The second and third problems involve the set of floor plans A-J, given in the student text. These problems provide an excellent opportunity to help students **begin** to build flexible understandings of perimeter and area and to help students see the power of organizing information in a table. The questions in problem three also involve visual reasoning as the students explore the changes that occur when tiles are moved, added or removed.

In the ACE, problem 4, 5, and 6 are designed to help students begin to connect measuring dimensions and finding perimeter and area. Some students may quickly see short cuts or create formulas for rectangles. Other students will need to continue to cover and count (i.e., cover figures with tiles and count them to find area) or surround and count (i.e., count the edges of the tiles that surround figures to find perimeter).

Problem 1: Room designs with 12 square tiles will vary; the perimeter should be at least 14 tile edges long.

Problem 2

a) Rooms D(a=12), G(a=11), I(a=10), and J(a=16) are possible rooms with at least 10 square units of floor space.

b) Rooms C(p=20), F(p=16), G(p=24), H(p=20), I(p=16), and J(p=34) are possible rooms with at least 15 windows and one door.

c)

Room	Area	Perimeter
A	9	12
B	7	12
C	9	20
D	12	14
E	9	14
F	9	16
G	11	24
H	9	20
I	10	16
J	16	34

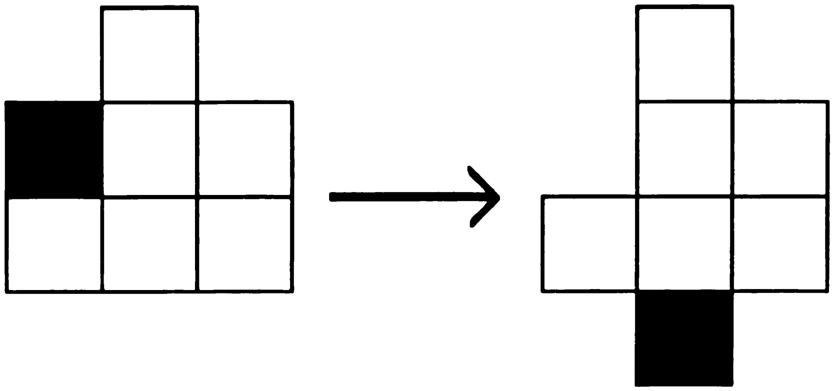
Follow-Up: Students' answers will vary - what is important is they have defensible arguments for selecting the three designs that they think are best. For example, a student might choose buildings G, I, and J because they are constructed with at least 10 tiles and have 16 windows so they are roomy, bright working environments. Other students might go with the three buildings using the least number of tiles because they are the least expensive to construct.

Problem 3

1. a) No--Room A is a square, with the largest area possible for its given perimeter.

   b) Yes--Examples are given in Room A, E, and F.
2. The perimeter will still be the same, because removing the tiles does not affect the perimeter.
3. The new rectangle Room will have an area of 20 and a perimeter of 18.
4. No--The area of I = 10, the area of F = 9. An area of 9 can't be arranged into an area of 10.

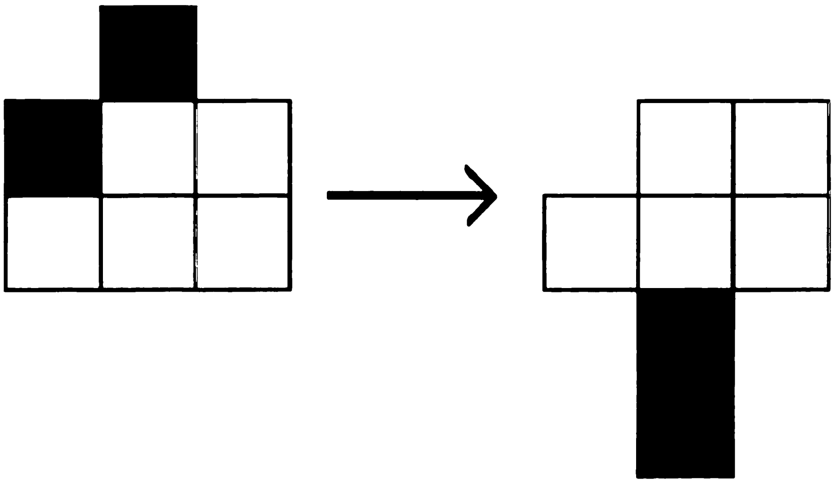
5. Here is one solution; others are possible.



Applications

1. The perimeters are so different because the 6 tiles are used to "fill-up" the middle of the spiral. Although the area has increased by 6 square units, the perimeter decreases because the interior of the figure has no "inside" perimeter to count.

2. Here is one solution; others are possible.

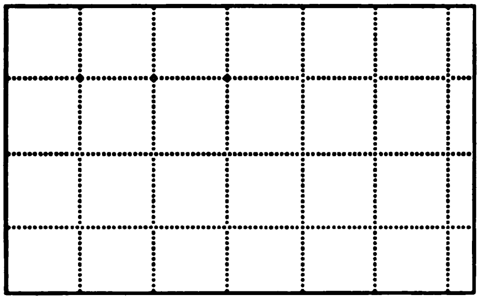


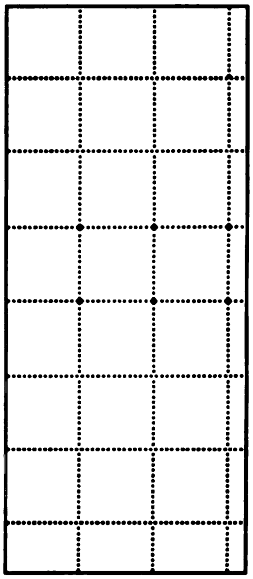
3. No--Since B has an area of 7 square units, the greatest perimeter would have to be less than 7 x 4 = 28. In other words, the perimeter of each individual tile is 4, so a figure made up of 7 tiles could never have a perimeter larger than 28.
4. a) area = 3.75 square units, perimeter = 8 units

   b) area = 3.75 square units, perimeter = 8.5 units
5. a) perimeter = 21 cm

b) perimeter = 21.5 cm

6.

a) 

b) 

Students do not need to label a) and b) precisely - what is most important is that they are able to draw a centimeter grid over each rectangle.

Connections

7. There are six different rectangles possible: 1x18, 2x9, 3x6, 6x3, 9x2, 18x1. These rectangles have perimeters of 38, 22, 18, 18, 22, and 38, respectively. Some students may only find three rectangles, because each of the six rectangles has a reversal, e.g., 1x18 and 18x1. However, the rectangles can still be considered different because they have different bottom and side edges, i.e., they are interchanged. In future investigations in this unit students will need to record reversal rectangles separately. Students should know that they have found all of the possible rectangles because they have used all the different factor pairs of 18 as edges of the rectangles.
8. You turn through 360 degrees. This is because, if a student traces his or her path, he or she will turn through a complete circle. Alternately, students might reason that four 90-degree turns are

made walking around the rectangle for a total of 4x90 = 360 degrees.

Extensions

9. A 1x12 or a 12x1 rectangle room--with a perimeter of 26--would provide space for the most windows(25) and one door. A 3x4 or a 4x3 rectangle room--with a perimeter of 14--would provide space for the fewest windows(13) and one door.

10. a) The area of the building is 10, the perimeter is 16.

    b) With the new unit, the area is 40 and the perimeter is 32. This is because there are four of the smaller units for every one of the larger units, so the area with the smaller unit is 10 x 4 = 40. Using the new unit, there are 2 units of perimeter for every unit of perimeter with the old unit, so the perimeter with the new unit equals 16 x 2 = 32.

Labsheet 1.1 - Tile Grid Paper

Labsheet 1.2 - Room Table

Room	Perimeter	Area
A		
B		
C		
D		
E		
F		
G		
H		
I		
J		

LIST OF REFERENCES

REFERENCES

Abbott, J.S. and Wells, D.W. (1985). *Mathematics today* (Grade Seven). Chicago: Harcourt, Brace Javanovich, Publishers.

Anyon, J. (1981). Social class and school knowledge. *Curriculum Inquiry, 11,* 3-42.

Ball, D.L. (1988). *Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education.* Unpublished doctoral dissertation, Michigan State University, East Lansing, MI.

Ball, D.L. (1990a). *Halves, pieces, and twoths: Constructing representational contexts in teaching fractions* (Craft Paper 90-2). East Lansing, MI: National Center for Research on Teacher Education, Michigan State University.

Ball, D.L. (1990b). Reflections and deflections of policy: The case of Carol Turner. *Educational Evaluations and Policy Analysis, 12,* 263-276.

Ball, D.L., and McDiarmid, G.W. (1990). The subject-matter preparation of teachers. In W.R. Houston (Ed.), *Handbook of research on teacher education.* New York: Macmillan.

Barnes, H.L. (1987). *Intentions, problems, and dilemmas: Assessing teacher knowledge through a case methods system* (Issue Paper 87-3). East Lansing, MI: Michigan State University, National Center for Research on Teacher Education.

Bereiter, C. and Scardamalia, M. (1987). An attainable version of high literacy: Approaches to teacher higher order skills in reading and writing. *Curriculum Inquiry, 17,* 9-30.

Bloom, B.S., and Broder, L.J. (1950). *Problem solving processes of college students.* Chicago: University of Chicago Press.

Bockarie, A. (1980). *The effectiveness of a unit in teaching and learning growth relations in sixth and seventh grades.* Unpublished doctoral dissertation, Michigan State University.

Bogdan, R.C. and Biklen, S.K. (1982). *Qualitative research for education - An introduction to theory and methods.* Boston: Allyn and Bacon

Brian, R.B. (1967). Processes of mathematics: A definitional development and an experimental investigation of their relationship to mathematical problem solving behavior. *Dissertation Abstracts*, 28, 1202A (University Microfilms No. 67-11,815).

California State Department of Education. (1991). *Mathematics framework for California public schools - Kindergarten through grade twelve*. Sacramento: Author.

Charles, R.I. (1989). Teacher education and mathematical problem solving: Some issues and directions. In R.I. Charles and E.A. Silver (Eds.), *The teaching and assessing of mathematical problem solving*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Charles, R., and Lester, F. (1982). *Teaching problem solving - What, why, & how*. Palo Alto, CA: Dale Seymour Publications.

Cobb, P. (1989). Experimental, cognitive, and anthropological perspectives in mathematics education. *For the Learning of Mathematics*, *9* (2), 32-42.

Cohen, D.K. (1988). *Teaching practice: Plus ca change...* (Issue Paper 88-3). East Lansing, MI: National Center for Research on Teacher Education, Michigan State University.

Cohen, D.K. and Ball, D.L. (1990). Policy and practice: An overview; Relations between policy and practice: A commentary. In Cohen, D.K., Peterson, P.L., Wilson, S., Ball, D.L., Putnam, R., Prawat, R., Heaton, R., Remillard, J., and Wiemers, N. *Effects of state-level reform of elementary school mathematics curriculum on classroom practice* (Research Report 90-14). East Lansing, MI: National Center for Research on Teacher Education, Michigan State University.

Comprehensive School Mathematics Program. (1978). Kansas City, MO: Mid-Continent Regional Educational Laboratory.

Connected Mathematics Project, The. (1992a). *Covering and surrounding* (Student Edition - working draft). East Lansing, MI: Department of Mathematics, Michigan State University.

Connected Mathematics Project, The. (1992b). *Covering and surrounding* (Teacher Edition - working draft). East Lansing, MI: Department of Mathematics, Michigan State University.

Dahmus, M.E. (1970). How to teach verbal problems. *School Science and Mathematics*, *70*, 121-138.

Dessart, D.J. and Suydam, M.N. (1983). *Classroom ideas from research in secondary school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Dewey, J. (1938/1963). *Experience and education*. New York: Macmillan Publishing Company.

Duncker, K. (1945). On problem solving. *Psychological Monographs, 58,* (5, Serial No. 270).

Elmore, R.E. (1979). Backward mapping: Implementation research and policy decisions. *Political Science Quarterly, 94,* 601-616.

Fennema, E. (1989). The study of affect and mathematics: A proposed generic model for research. In D.B. McLeod and V.M. Adams (Eds.), *Affect and mathematical problem solving - A new perspective.* New York: Springer-Verlag.

Fey, J. and Heid, M.K. (1991). *Computer intensive algebra.* College Park, Maryland: University of Maryland and Pennsylvania State University.

Fitzgerald, W., Lappan, G., and Phillips, E. (1991). *Connected Mathematics* (Proposal to the National Science Foundation). Department of Mathematics, Michigan State University.

Fitzgerald, W.M., Winter, M.J., Lappan, G., and Phillips, E. (1986). *Factors and multiples.* Middle Grades Mathematics Project series. Reading, MA: Addison-Wesley Publishing Company.

Floden, R.E., Porter, A.C., Schmidt, W.H., Freeman, D., and Schwille, J. (1981). Responses to curriculum pressures: A policy-capturing study of teacher decisions about content. *Journal of Educational Psychology, 73,* 129-141.

Freeman, D. and Porter, A. (1989). Do textbooks dictate the content of mathematics instruction in elementary schools? *American Educational Research Journal, 26,* 403-421.

Friel, S.N., Makros, J.R., and Russell, S.J. (1992). *Used numbers.* Palo Alto, CA: Dale Seymour Publications.

Goldin, G.A. (1992). Meta-analysis of problem solving studies: A critical response. *Journal for Research in Mathematics Education, 23*(3), 274-283.

Graybeal, S.S. and Stodolsky, S.S. (1987). *Where's all the "good stuff"?: An analysis of fifth grade math and social studies teacher's guides.* Paper presented at the annual meeting of the American Educational Research Association, Washington DC.

Greeno, J.G. (1991). A view of mathematical problem solving in school. In M.U. Smith (Ed.), *Toward a unified theory of problem solving - Views from the content domains.* Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Grouws, D.A. (1985). The teacher and classroom instruction: Neglected themes in problem-solving research. In E. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives.* Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Grouws, D.A. and Cramer, K. (1989). Teaching practices and student affect in problem-solving lessons of select junior-high mathematics teachers. In D.B. McLeod and V.M. Adams (Eds.), *Affect and mathematical problem solving - A new perspective*. New York: Springer-Verlag.

Hake, S. and Saxon, J. (1985). *Math 76: An incremental development*. Norman, OK: Grassdale Publishers, Inc.

Hallard, D.C. and Eisenhart, M.A. (1988). Women's ways of going to school: Cultural reproduction of women's identities as workers. In L. Weis (Ed.), *Class, race, and gender in American education*. Albany, NY: State University of New York Press.

Hammersley, M. and Atkinson, P. (1983). *Ethnography - Principles in practice*. New York: Routledge.

Hawkins, D. (1974). *The informed vision: Essays on learning and human nature*. New York: Agathon Press.

Hembree, R. (1992). Experiments and relational studies in problem solving: A meta-analysis. *Journal for Research in Mathematics Education, 23*(3), 242-273.

Hershkowitz, R. (1990). Psychological aspects of learning geometry. In P. Nesher and J. Kilpatrick (Eds.), *Mathematics and cognition: A research synthesis by the International Group for the Psychology of Mathematics Education*. New York: Cambridge University Press.

Holmes Group, The. (1990). *Tomorrow's schools - Principles for the design of professional development schools*. East Lansing, MI: Author.

Jackson, P. (1986). *The practice of teaching*. New York: Teachers College Press.

Kaput, J.J. (1985). Representation and problem solving: Methodological issues related to modeling. In E. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Kilpatrick, J. (1985). A retrospective account of the past 25 years of research on teaching mathematical problem solving. In E. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Kilpatrick, J. and Wirszup, I. (Eds.). (1972). *Instruction in problem solving* (Soviet studies in the psychology of learning and teaching mathematics, Vol, 6). Stanford, CA: School Mathematics Study Group.

Kohl, H. (1984). *Growing minds: On becoming a teacher*. New York: Harper Calophon.

Kulm, G. (1991). New directions for mathematics assessment. In G. Kulm (Ed.), *Assessing higher order thinking in mathematics.* Washington D.C.: American Association for the Advancement of Science.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29-63.

Lappan, G. (1983). *Middle grades mathematics project.* Final Report NSF SED 80 - 18025.

Lester, F.K., and Kroll, D.L. (1990). Assessing student growth in mathematical problem solving. In G. Kulm (Ed.), *Assessing higher order thinking in mathematics.* Washington D.C.: American Association for the Advancement of Science.

Lipsky, M. (1980). *Street-level bureaucracy: Dilemmas of the individual in public services.* New York: Russell Sage Foundation.

Lortie, D.C. (1975). *Schoolteacher: A sociological study.* Chicago: University of Chicago Press.

Mathematics in Action. (1992). Chicago: MacMillan/McGraw-Hill School Publishing Company.

McLeod, D.B. (1989). Beliefs, attitudes, and emotions: New views of affect in mathematics education. In D.B. McLeod and V.M. Adams (Eds.), *Affect and mathematical problem solving - A new perspective.* New York: Springer-Verlag.

Meyer, C. and Sallee, T. (1983). *Make it simpler - A practical guide to problem solving in mathematics.* Reading, MA: Addison-Wesley Publishing Company.

National Council of Supervisors of Mathematics. (1989). Essential mathematics for the twenty-first century: The position of the National Council of Supervisors of Mathematics. *Mathematics Teacher, 81*(1), 16-21.

National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics for the 1980s.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

Nicholls, J.G., Cobb, P., Yackel, E., Wood, T., and Wheatley, G. (1991). Students' theories about mathematics and their mathematical knowledge: Multiple dimensions of assessment. In G. Kulm (Ed.), *Assessing higher order thinking in mathematics.* Washington D.C.: American Association for the Advancement of Science.

Noddings, N. (1990). Constructivism in mathematics education. In R.B. Davis, C.A. Maher, and N. Noddings (Eds.), *Journal for Research in Mathematics Education monograph number 4 - Constructivist views on the teaching and learning of mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Noddings, N. (1982). On the analysis of four-person problem-solving protocols. In M.J. Shaughnessy (Chair), *Investigations of children's thinking as they go about solving mathematical word problems.* Symposium presented at the annual meeting of the American Educational Research Association, New York.

Noddings, N. (1985). Small groups as a setting for research on mathematical problem solving. In E. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives.* Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Oakes, J. (1985). *Keeping track: How schools structure inequality.* New Haven: Yale University Press.

Paley, V.G. (1989). *White teacher.* Cambridge, MA: Harvard University Press.

Polya, G. (1945). *How to solve it.* Princeton, NJ: Princeton University Press.

Polya, G. (1967). *Mathematical discovery: On understanding, learning, and teaching problem solving* (Vol. 1). New York: John Wiley & Sons, Inc.

Polya, G. (1968). *Mathematical discovery: On understanding, learning, and teaching problem solving* (Vol. 2). New York: John Wiley & Sons, Inc.

Prawat, R.S., Putnam, R.T., and Reineke, J.W. (1991). *Experts' views on the elementary mathematics curriculum: Views of the ideal and critique of current practice* (Elementary Subject Center Series No. 44). East Lansing, MI: Michigan State University, Center for the Learning and Teaching of Elementary Subjects.

Putnam, R.L., Heaton, R.M., Prawat, R.S., and Remillard, J. (in press). Teaching mathematics for understanding: Discussing case studies of four fifth-grade teachers. *Elementary School Journal.*

Putnam, R., Lampert, M., and Peterson, P. (1990). Alternative perspectives on knowing mathematics in elementary schools. *Review of Research in Education, 16*, 57-150.

Remillard, J. (1991a). *Abdicating authority for knowing: A teachers' use of an innovative mathematics curriculum* (Elementary Subjects Center Series No. 42). East Lansing, MI: Center for the Teaching and Learning of Elementary Subjects, Michigan State University.

Remillard, J. (1991b). *Is there an alternative? An analysis of commonly used and distinctive elementary mathematics curricula* (Elementary Subjects Center Series No. 31). East Lansing, MI: Center for the Teaching and Learning of Elementary Subjects, Michigan State University.

Rickard, A. (1991). *Challenges, change, and problem solving: A study of collaborative mathematics instruction.* Unpublished manuscript, Department of Teacher Education, Michigan State University.

Romberg, T.A., Zarinnia, A., and Collis, K. (1990). A new world view of assessment in mathematics. In G. Kulm (Ed.), *Assessing higher order thinking in mathematics*. Washington D.C.: American Association for the Advancement of Science.

Sarason, S.B. (1982). *The culture of the school and the problem of change*, second edition. Boston: Allyn and Bacon.

Schwab, J.J. (1978). *Science, curriculum, and liberal education -- selected essays*. I. Westbury and N.J. Wilkof (Eds.). Chicago: The University of Chicago Press.

Schoenfeld, A.H. (1982). On the assessment of two-person problem solving protocols. In M.J. Shaughnessy (Chair), *Investigations of children's thinking as they go about solving mathematical word problems*. Symposium presented at the annual meeting of the American Educational Research Association, New York.

Schoenfeld, A.H. (1985a). *Mathematical problem solving.* New York: Academic Press, Inc.

Schoenfeld, A.H. (1985b). Metacognitive and epistemological issues in mathematical understanding. In E. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Schoenfeld, A.H. (1987). Cognitive science and mathematics education: An overview. In A. Schoenfeld (Ed.), *Cognitive science and mathematics education*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Shroyer, J.L., and Fitzgerald, W.M. (1986). *Mouse and elephant: Measuring growth*. Middle Grades Mathematics Project series. Reading, MA: Addison Wesley Publishing Company.

Shulman, L.S. (1985). On teaching problem solving and the problems of teaching. In E. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Shulman, L.S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*, 1-22.

Silver, E.A. (1985). Research on teaching mathematical problem solving: Some underrepresented themes and needed directions. In E. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Silver, E.A. (1987). Foundations of cognitive theory and research for mathematics problem solving. In A. Schoenfeld (Ed.), *Cognitive science and mathematics education*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Silver, E.A. (1989). Teaching and assessing mathematical problem solving: Toward a research agenda. In R.I. Charles and E.A. Silver (Eds.), *The teaching and assessing of mathematical problem solving*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Stake, R. and Easley, J. (1978). *Case studies in science education*. Urbana, IL: University of Illinois.

Stanic, G.M.A. and Kilpatrick, J. (1989). Historical perspectives on problem solving in the mathematics curriculum. In R.I. Charles and E.A. Silver (Eds.), *The teaching and assessing of mathematical problem solving*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Steen, L.A., (Ed.). (1990). *On the shoulders of giants - New approaches to numeracy*. Washington DC: National Academy Press.

Stodolsky, S.S. (1988). *The subject matters: Classroom activity in math and social studies*. Chicago: University of Chicago Press.

Strauss, S.M. (1988). Girls in the mathematics classroom: What's happening to our best and brightest? *Mathematics Teacher, 81*(7), 533-537.

Swafford, J.O. (1980). Sex differences in first-year algebra. *Journal for Research in Mathematics Education, 11*(11), 335-346.

Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws, (Ed.), *Handbook of research on mathematics teaching and learning*. New York: NCTM, MacMillan Publishing Company.

Thompson, A.G. (1985). Teachers' conceptions of mathematics and teaching problem solving. In E. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Thompson, A.G. (1989). Learning to teach mathematical problem solving: Changes in teachers' conceptions and beliefs. In R.I. Charles and E.A. Silver (Eds.), *The teaching and assessing of mathematical problem solving*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Thompson, P.W. (1985). Experience, problem solving, and learning mathematics: Considerations in developing mathematics curricula. In E. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives.* Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Von Glaserfeld, E. (1983). Learning as a constructive activity. In J. Bergeron and N. Herscovics (Eds.), *Proceedings of the fifth annual meeting of the psychology of mathematics education - North American group.* Montreal.

Wiemers, N.J. (1990). Transformation and accommodation: A case study of Joe Scott. *Educational Evaluation and Policy Analysis, 12,* 297-308.

Wilcox, S.K., Schram, P., Lappan, G., and Lanier, P. (1991). *The role of a learning community in changing preservice teachers' knowledge and beliefs about mathematics education* (Research Report 91-1). East Lansing, MI: Michigan State University, National Center for Research on Teacher Education.

Wilcox, S.K., Lanier, P., Schram, P. and Lappan, G. (1992). *Influencing beginning teachers' practice in mathematics education: Confronting constraints of knowledge, beliefs, and context* (Research Report 92-1). East Lansing, MI: Michigan State University, National Center for Research on Teacher Education.

Wilson, S.M. (1989). *A case concerning content: Using case studies to teach subject matter* (Craft Paper 89-1). East Lansing, MI: Michigan State University, National Center for Research on Teacher Education.

Wilson, S.M., Shulman, L.S., and Richert, A. (1987). "150 ways of knowing": Representations of knowledge in teaching. In J. Calderhead (Ed.), *Exploring teacher thinking* (pp. 104-124). Sussex: Holt, Rinehart & Winston.

Wilson, S.M. (1990). A conflict of interests: The case of Mark Black. In Cohen, D.K., Peterson, P.L., Wilson, S., Ball, D.L., Putnam, R., Prawat, R., Heaton, R., Remillard, J., and Wiemers, N. *Effects of state-level reform of elementary school mathematics curriculum on classroom practice* (Research Report 90- 14). East Lansing, MI: National Center for Research on Teacher Education, Michigan State University.