This is to certify that the

thesis entitled

THE STRUCTURE OF RESERVE REQUIREMENTS

AND MONETARY CONTROL

presented by

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has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Economics

Major professor

Date August 10, 1978

THE STRUCTURE OF RESERVE REQUIREMENTS
AND MONETARY CONTROL

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A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Economics

1978

ABSTRACT

THE STRUCTURE OF RESERVE REQUIREMENTS AND MONETARY CONTROL

By

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This study is an empirical investigation of the extent to which nonmember banks and the structure of Federal Reserve reserve requirements interfere with precise control of the money stock. The theoretical framework employed is a Brunner-Meltzer money supply model in which the money multiplier links the net source base, which the Federal Reserve is assumed to control, to the money stock. Variation in the money multiplier therefore impedes accurate monetary control. One source of its variation is variation in the reserve ratio, base-absorbing reserves divided by the demand-deposit component of the money stock. This study assesses the size, sources, and predictability of variation in the reserve ratio.

Several institutional arrangements built into Federal Reserve reserve requirements introduce variation into the reserve ratio. These include differential reserve requirements which apply different reserve ratios to different banks, the prescription that member banks hold reserves against liabilities that are not money, and the conventions of lagged reserve requirements and excess reserves. Finally, nonmember banks cause the reserve ratio to vary. Since an increasing proportion of commercial banks are nonmembers, the control problem posed by nonmember banks is believed to be of growing severity.

The data used here cover the period January 1, 1961 through December 31, 1974. For member banks, this is weekly averages of actual daily deposit figures; for nonmember banks, the figures are estimated. The sample period encompasses several structural changes in Federal Reserve reserve requirements which have increased the number of categories of deposits to which different reserve ratios are applied. It is frequently claimed that these changes and lagged reserve requirements have introduced greater variability into the reserve ratio.

The reserve ratio is specified as a combination of nine parameters, each of which represents one of the aforementioned institutional aspects of reserve requirements. The historical behavior of each parameter is investigated, as well as the variation in the reserve ratio under the various Federal Reserve reserve schemes. Using the formula for the variation of a linear combination of random variables, the variation in the reserve ratio induced by each parameter is isolated. The effects of the introduction of lagged reserve requirements, the increased number of reserve categories, and the growing proportion of nonmember banks are inferred.

If the parameters in the reserve ratio are variable yet predictable, then the variation they cause is not detrimental to monetary control. To test predictability, two forecasting experiments are performed. First, a naive forecasting model of the reserve ratio is constructed in which each component parameter is assumed in week t to equal its value in week (t-1). The error of this forecast is compared to that for another naive model in which perfect knowledge of one parameter is assumed, while retaining the no-change assumption for all other parameters. This implies the loss, in terms of accurate forecasts of the reserve ratio, associated with a no-change forecast of each parameter. Second, forecast values for each parameter are derived from models based on the time-series analysis of George E. P. Box and Gwilym M. Jenkins.[1] These forecasts are substituted for the naive ones and the resulting errors are compared.

The results indicate that the sources of greatest variation in the reserve ratio are nonmoney deposits and lagged reserve requirements. Nonmember banks are a relatively minor, and not increasing, source of variation. Differential reserve requirements have also not been a serious control problem. The results support the contention that the increased number of reserve categories cause increased variation in the reserve ratio. The Box-Jenkins forecasts of the parameters representing lagged and differential reserve requirements, interbank deposits, and nonmember banks are all quite accurate. The most troublesome sources of unpredictability in the reserve ratio are excess reserves, and time and government deposits.

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[1] George E. P. Box and Gwilym M. Jenkins, Time Series Analysis: Forecasting and Control, Revised Edition (New York: Holden-Day, Inc., 1976).

To my parents

ACKNOWLEDGMENTS

I would like to thank the members of my Committee, especially Robert H. Rasche and Carl M. Gambs, for their help throughout this project. I am particularly grateful to my Chairman, Bob Rasche, for his patience and assistance; without his continued support this dissertation would probably not have been finished.

Thanks are due to Darwin Beck, Neva Van Peski, and their associates in the Banking Section, Division of Research and Statistics, at the Board of Governors of the Federal Reserve System, who compiled much of the data used in this study.

I also thank my mother, who typed most of the first draft, and Mrs. Hope Burd, who typed the final copy, for their fast, accurate typing of a difficult manuscript.

Finally I am indebted to many people who offered their moral and emotional support for the completion of this task. I thank Christine, who has endured a lot and remained a true and enduring friend through it all. I thank Defina for her continuing faith and encouragement. I thank Steve for helping me create the equilibrium in my life that was needed to finish this project. I owe a special thanks to my family, especially my mother, whose quiet support, and unfaltering patience and confidence have been my mainstay.

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CHAPTER 1

INTRODUCTION TO THE PROBLEM

The Federal Reserve's increased emphasis in recent years on monetary aggregates in the formulation of monetary policy is well documented in the literature as well as in Federal Reserve publications.[1] While, according to some spokesmen, the Federal Open Market Committee has always paid attention to the behavior of monetary aggregates, the weight attached to them in the policy process has been growing since 1960. Beginning in 1966 with the inclusion of the "proviso" clause, the Federal Open Market Committee Directive has given explicit consideration to the path of at least one monetary aggregate. Since 1970, the Directive has used the rate of growth of at least one monetary aggregate as a specific policy target. This new policy emphasis has stimulated considerable discussion as to whether the Federal Reserve has the technical capability to control the money stock or some other monetary aggregate, with the desired degree of accuracy.[2] Doubts about the Federal Reserve's capacity to control the money stock arise over

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[1]See, for example, Jack M. Guttentag, "Discussion," in Controlling Monetary Aggregates II: The Implementation (Boston: Federal Reserve Bank of Boston, 1972), pp. 69-72; Alan R. Holmes, "A Day at the Trading Desk," Monthly Review, Federal Reserve Bank of New York 52 (October 1970):234-8; Arthur Burns, "The Role of the Money Supply in the Conduct of Monetary Policy," Monthly Review, Federal Reserve Bank of Richmond (December 1973):2-8; Milton Friedman, "Letter on Monetary Policy," Monthly Review, Federal Reserve Bank of Richmond (May-June 1974):20-23.

[2]See, for example, Alan R. Holmes, "Operational Constraints on the Stabilization of Money Supply Growth," in Controlling Monetary Aggregates (Boston: Federal Reserve Bank of Boston, 1969), pp. 65-78; Thomas Mayer, "A Money Stock Target," in Current Issues in Monetary Theory and Policy, ed. Thomas M. Havrilesky and John T. Boorman (Arlington Heights, Ill.: AHM Publishing Company, 1976), pp. 548-555; and Milton Friedman, "Statement on the Conduct of Monetary Policy," in Current Issues in Monetary Theory and Policy, ibid., pp. 556-565.

a number of issues but one of the most frequently discussed control problems is that posed by the structure of reserve requirements. It is this problem that is the subject of this study.

If the Federal Reserve is to control the money stock (or the demand-deposit component of it) what affords it that control is the basic relationship between bank reserves and deposits. The reserve-deposit relationship is expressed in the demand-deposit multiplier formula found in any introductory economics text. The level of deposits (D) is the product of the reciprocal of the average legal reserve ratio, r, and the level of reserves, R. Thus,

$$(1\text{-}1) \qquad D = \frac{1}{r}(R).$$

Through open market operations and its power to change legal reserve ratios, the Federal Reserve has control (though not complete) over the level of R and the value of r. Therefore, it is through this reserve-deposit linkage that the Federal Reserve can presumably control the level of bank deposits and hence the money stock or some closely-related monetary aggregate.

There are several shortcomings in this avenue of control. One problem lies in the fact that the Federal Reserve does not have complete control over the level of bank reserves. While this issue is a frequently-discussed one, it is not of concern in this study. The problem that this study addresses is variation in the reserve ratio, r, caused by the structure of reserve requirements. Even if precise control of bank reserves is assumed, exact control of the level of deposits is precluded by changes in r. The structure of reserve requirements causes variation in r for a number of reasons.

First, there is no single required reserve ratio, but a number of different reserve ratios that are applicable to different kinds of deposits and different classes of member banks ("differential reserve requirements"). Second, a majority of the commercial banks in this country are not members of the Federal Reserve System and are, therefore, allowed to hold reserves in a form that is not under the control of the Federal Reserve. These two institutional aspects of the reserve requirement system are potential impediments to precise control of the level of deposits, regardless of the Federal Reserve's ability to control the level of reserves. Their existence allows changes in the distribution of a given level of deposits between time and demand deposits, between classes of member banks or between member and nonmember banks to alter the reserve-deposit relationship.

This can be seen explicitly by rewriting equation (1-1) in the form,

(1-2) $r = R/D.$

A shift in the distribution of, for example, a given amount of member bank deposits in favor of small banks will allow a lower level of required reserves to be held against the same level of deposits; the value of r will fall. The result will be the same if a shift in the distribution of deposits occurs toward time deposits or nonmember banks. A change in the distribution of deposits will therefore allow a change in the level of deposits that can be supported by a given quantity of reserves. Changes in the level of deposits may therefore occur even if the Federal Reserve does not alter the level of reserves; changes in distribution of deposits may mitigate intentional changes in the level of reserves.

The level of deposits that may correspond to a given level of reserves will also vary depending on the level of excess reserves that member banks choose to hold. In addition, controlling the level of deposits on a short-run basis may be impeded by the institution of lagged reserve requirements, whereby current required reserves are based on deposit levels two weeks earlier.

If, however, the Federal Reserve had perfect control of the level of deposits, there are additional features of reserve requirements that would interfere with its controlling the money stock with equal precision. This is because there are several kinds of deposits against which reserves must be held but which are not included in the money stock ("nonmoney deposits"). Since only privately-owned demand deposits are included in the money stock, any variation in the ratio of total deposits to privately-owned demand deposits will mitigate precise control of the money stock.

The institutions of differential and lagged reserve requirements, the existence of nonmember banks and excess reserves, and the prescription that member banks must hold reserves against certain nonmoney deposits are all factors that cause slippage in the reserve-deposit linkage; or, in terms of equation (1-2), they cause the value of r to vary. In contrast, if there were one uniform reserve ratio applied contemporaneously only to privately-owned demand deposits at all commercial banks, then the value of r would vary only when the Federal Reserve changes the legal reserve ratio or the level of excess reserves changed. Under such a "uniform" reserve scheme, it is claimed that control of the level of reserves would allow the Federal Reserve more accurate control of the demand-deposit component of the money stock.

Models of the money supply process recognize that the structure of reserve requirements causes variation in the r-ratio and potentially prohibit precise money stock control.[3] Furthermore, discussions of theories and techniques involved in money stock control typically concern themselves with the control problems introduced by the current structure of reserve requirements. Specifically, many authors lament the seeming erosion of control in recent years represented by: 1) a growing proportion of commercial banks that are nonmembers; 2) the introduction of lagged reserve requirements; 3) the increased number of member bank reserve classes; and 4) the increased number of categories of nonmoney deposits.[4]

Early empirical investigations of the importance to the money supply process of variation in r revealed that, relative to other disturbances, changes in the value of r have been a minor source of disruption. Friedman and Schwartz,[5] for example, found that from 1867 to 1960, secular and cyclical changes in the money stock were dominated by changes in high-powered money and institutional changes. They report that both the money multiplier and the reserve ratio have been "remarkably stable," especially since 1902. In a complementary study,

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[3]See Appendix A.

[4]See, for example, Albert E. Burger, The Money Supply Process (Belmont, CA: Wadsworth Publishing Co., Inc., 1971), pp. 50-58; Sherman Maisel, "Controlling Monetary Aggregates," in Controlling Monetary Aggregates (Boston: Federal Reserve Bank of Boston, 1969), p. 160, 165-65; and John H. Kareken, "Discussion," in Controlling Monetary Aggregates II: The Implementation (Boston: Federal Reserve Bank of Boston, 1972), p. 143.

[5]Milton Friedman and Anna Jacobsen Schwartz, A Monetary History of the United States, 1867-1960 (Princeton, N.J.: Princeton University Press, 1971).

Cagan[6] also concluded that variation in the r-ratio has contributed little to either secular or cyclical changes in the money stock; any changes in the value of r have been overwhelmed by changes in high-powered money.

Despite evidence that variation in the r-ratio has been relatively minor, there has been no shortage of proposals to reform reserve requirements in order to reduce the variation in r. According to Arthur Burns, ". . . the present structure of reserve requirements leaves much to be desired. Reforms are needed to increase the precision and the certainty with which the supply of money and credit can be controlled."[7]

Proposals to abolish differential and lagged reserve requirements and reserves against nonmoney deposits have been made by a number of authors as well as official commissions and study groups. Poole and Lieberman, for example, recommend that required reserves against government deposits and time and savings deposits, as well as lagged reserve requirements be eliminated.[8] In the report of the President's Commission on Financial Structure and Regulation (The Hunt Commission), the commission proposed that the Federal Reserve impose a uniform

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[6]Phillip Cagan, Determinants and Effects of Changes in the Stock of Money, 1875-1960 (New York: Columbia University Press, 1965).

[7]Arthur F. Burns, "The Structure of Reserve Requirements," a speech presented to the Governing Council Spring Meeting, American Bankers Association, White Sulphur Springs, West Virginia, April 26, 1973; reprinted in the Federal Reserve Bulletin (May 1973):340.

[8]William Poole and Charles Lieberman, "Improving Monetary Control," Brookings Paper on Economic Activity (2:1972):335.

reserve requirement on all classes of member banks and that required reserves against time and savings deposits be abolished.[9]

By far the most common target for reform, however, has been nonmember banks. Much of the enthusiasm for subjecting nonmember banks to Federal Reserve reserve requirements has been generated by reports of the declining proportion of commercial banks that are member banks.[10] As the portion of the country's demand deposits that are in nonmember banks grows, an ever-increasing part of the nation's money stock falls outside the control of the Federal Reserve. This erosion of the Federal Reserve's monetary control can only be stopped, it is claimed, by placing the reserves of all commercial banks under the Federal Reserve's control. This is the recommendation of a number of authors as well as the position taken by the Commission on Money and Credit, the President's Committee on Financial Institutions, and the Hunt Commission. Every year since 1964, the Federal Reserve itself has requested that Congress put nonmember banks under their control for reserve purposes. The Federal Reserve states that, "Because demand deposits held by an institution are part of the country's money supply just as are those in member banks, applying the same demand-deposit reserve requirements to all

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[9] The Report of the President's Commission on Financial Structure and Regulation (Washington, D.C.: U.S. Government Printing Office, 1971), p. 65.

[10] See, for example, Edward G. Boehne, "Falling Fed Membership and Eroding Monetary Control: What Can Be Done?", Business Review, Federal Reserve Bank of Philadelphia (June 1974):3-15; William Burke, "Primer on Reserve Requirements," Business Review, Federal Reserve Bank of San Francisco (Winter 1974):3-16; Arthur F. Burns, ibid., p. 340-41; Annaul Report of the Board of Governors of the Federal Reserve System, 1972, (Washington, D.C., 1972), pp. 195-96.

such institutions would facilitate the effective implementation of monetary policy."[11]

The fact that various kinds of nonbank financial intermediaries have recently begun to issue deposits that are subject to withdrawal by check has created a control problem analogous to that represented by nonmember banks. Since the deposits of these institutions that are transferable by check function like demand deposits, they constitute another part of the nation's payments system that is not currently controllable by the Federal Reserve. Since the provisions of such services by nonbank financial institutions is becoming more prevalent, it seems apparent that the control problem they represent will be an increasing one in the future. Recognition of this trend has led some to recommend that Federal Reserve reserve requirements be extended, not only to nonmember banks, but to all financial intermediaries that issue deposits subject to checking privileges. The Hunt Commission proposed that Federal Reserve membership "be required of all commercial banks, savings and loan associations and mutual savings banks that offer third party payments services."[12] In the last few years, the Federal Reserve Board has taken the position that, ". . . reserve requirements set by and held with the Federal Reserve be made applicable to all financial institutions that offer money-transfer services in essentially the same manner as do member banks. This would provide the most rational and

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[11] Annual Report of the Board of Governors of the Federal Reserve System, 1971 (Washington, D.C., 1971), p. 212.

[12] The Report of the President's Commission on Financial Structure and Regulation, p. 65.

equitable system of reserve requirements, particularly in view of the evolution toward the use of check-type transfers by thrift institutions."[13]

The desire to extend Federal Reserve reserve requirements to cover nonmember banks is, however, by no means unanimous. Opponents of the proposal claim that the gain in terms of monetary control would be negligible; that so many other factors cause variation in the r-ratio that relatively little would be achieved by placing nonmember banks under the control of the Federal Reserve. In addition, it is usually claimed that such a reform would jeopardize the dual banking system and correspondent banking.[14]

It is easy for those who support retaining the current system of state reserve requirements for nonmember banks to point to the Federal Reserve scheme of lagged and differential reserve requirements and required reserves against nonmoney deposits, all of which also generate variation in r. The fact that the Federal Reserve has only recently increased the number of reserve classes for member banks, fuels the claims that reform could be made within its own reserve requirements system that would also improve monetary control. In a paper written for the Conference of State Bank Supervisors, Robertson and Phillips take the popular position that, "The existence of different reserve requirements for member and nonmember banks does not complicate the

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[13] Annual Report of the Board of Governors of the Federal Reserve System, 1972, ibid., p. 195.

[14] Ira Kaminow, "The Case Against Uniform Reserves: A Lost prespective," Business Review, Federal Reserve Bank of Philadelphia (June 1974): 16-21.

problem of monetary controls to any significant degree; Federal Reserve could make far more important changes for more precise control of the money stock by altering its own reserve rules for member banks."[15]

Despite many claims about the relative sources of variation in the reserve ratio and the proposals for reform, there is little empirical evidence to support or refute them. What empirical evidence there is, which is reviewed in the next chapter, is often not comprehensive or not completely reliable because of data problems. This study investigates the extent to which lagged reserve requirements, differential reserve requirements, nonmember banks, reserves against nonmoney deposits and excess reserves introduce variability and unpredictability into the reserve ratio.

The theoretical basis for this study is a standard Brunner-Meltzer nonlinear money supply model[16] in which the money stock is related to the net source base by the money multiplier. The money-multiplier model presumes that the Federal Reserve attempts to control the money stock by controlling the source base; a prerequisite of precise monetary control is therefore a stable, or at least predictable, money multiplier. The money multiplier is determined by several variable parameters, one of which is the reserve ratio.

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[15]Ross M. Robertson and Almarin Phillips, <u>Optional Affiliation With the Federal Reserve System for Reserve Purposes is Consistent With Effective Monetary Policies</u> (Washington, D.C.: Conference of State Bank Supervisors, 1974), p. 5.

[16]See Appendix A.

Aside from changes in legal reserve ratios, all variation in the reserve ratio comes from one of the aforementioned institutional aspects of reserve requirements or excess reserves. In Chapter 3, the reserve ratio is therefore expressed as the product of nine parameters (or groups of parameters) so that each parameter reflects the impact on the reserve ratio of lagged or differential reserve requirements, nonmember banks, reserved against nonmoney deposits, or excess reserves.

In Chapter 4, the historical behavior of each of the parameters in the reserve ratio is described. The relative variability of each parameter implies the relative severity of the control problem attributably to each institutional aspect of reserve requirements. In some cases, secular or seasonal patterns are discernible in a parameter's behavior; in some cases, a parameter's behavior can be related to some independent economic or institutional occurrence. In Chapter 5, the historical variation in the reserve ratio and its component parameters are assessed; the variation in the reserve ratio attributable to each component parameter is isolated.

Variability in the reserve ratio, whatever its source, is not necessarily troublesome to control if the variation is predictable. To assess the predictability of the reserve ratio and its component parameters, two different forecast experiments are performed. For control purposes, the simplest way to forecast the reserve ratio is to assume no change from the previous week in its component parameters. The first forecast experiment utilizes this naive assumption of no-change in the parameters of the reserve ratio. The error in the reserve ratio that is caused by the naive forecasts of its parameters is calculated.

Using the time series methodology of Box and Jenkins,[17] models are estimated to represent each of the parameters in the reserve ratio. The development of these models is described in Chapter 6. These models are then used to derive another, more sophisticated forecast, of each of the parameters comprising the reserve ratio. These forecast values are then substituted for the naive forecasts and the resulting errors are compared. The relative loss, in terms of accurate forecasts of the reserve ratio, of using the naive or Box-Jenkins forecast of each parameter is determined. These results are reported in Chapter 7.

In all of the empirical results, it is interesting to note the validity of the claims made about the relative severity of the control problems caused by the different institutional aspects of reserve requirements. The following common claims are evaluated:

1) That the growing proportion of banks that are nonmembers is resulting in an increasingly troublesome control problem;

2) That the control problem caused by nonmember banks is not as severe as that caused by lagged reserve requirements, differential reserve requirements, and other structural aspects of Federal Reserve reserve requirements;

3) That the Federal Reserve's introduction of lagged reserve requirements has introduced a source of great variability into the reserve ratio and has created a much more serious control problem than nonmember banks;

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[17] George E. P. Box and Gwilym M. Jenkins, Time Series Analysis: Forecasting and Control, Revised Edition (New York: Holden-Day, Inc., 1976).

4) That the increased number of deposit categories defined by the Federal Reserve has made the monetary control problems of lagged and differential reserve requirements more difficult.

CHAPTER 2

REVIEW OF THE LITERATURE

The variation in the reserve-deposit relationship caused by nonuniform reserve requirement is a control problem which has been recognized by authors since the deposit-expansion process was first detailed by C. A. Phillips[1] in 1920. Beyond recognizing the existence of the problem however, the earliest analysis of the effects of nonuniform reserve requirements is in Laughlin Currie's The Supply and Control of Money in the United States,[2] first published in 1934. While Currie's empirical analysis of the problem is limited by the inadequacies of the data available at the time, he does recognize and discuss the control problems represented by nonmember banks, differential Federal Reserve reserve requirements and reserve requirements against nonmoney deposits, especially time deposits. Currie states, "Indeed it will be found that so many and diverse are the forces causing variations in the reserve ratio against demand deposits that it is quite impossible to predict the magnitude of a change in the volume of money that will result from any given change in the volume of commercial banks."[3]

To assess the control problem represented by time deposits, Currie compares estimates of changes in total required reserves with estimated changes in required reserves against time deposits, using call report data for 1921 through 1933. The results show that a "considerable

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[1] C. A. Phillips, Bank Credit (New York: The MacMillan Company, 1920).

[2] Laughlin Currie, The Supply and Control of Money in the United States (New York: Russell and Russell, 1968).

[3] ibid., p. 71

proportion"[4] of the annual changes in required reserves are due to changes in the level of time deposits. Currie also recognizes the control problem represented by nonmember banks, but minimized its importance. He claims that the two systems are so interdependent that shifts of deposits between member and nonmember banks largely offset each other and their overall effect on the money stock is neutralized. He does recognize that, "Over a period of time, however, the relative volume of member and nonmember banks deposits do change, and this causes changes in the ratio of member bank reserves to total demand deposits."[5] It was however his view that nonmember banks would gradually choose to join the System and therefore that the problem posed by nonmember banks would soon disappear.

To determine the effects of interbank deposits and cash items in process of collection, Currie calculated the ratio of adjusted demand deposits to net demand deposits, using call report data for 1922 through 1932. Since both these asset items are deducted from demand deposits before required reserves are calculated, there is always a discrepancy between net demand deposits (on which reserves are based) and adjusted demand deposits (the money-stock component). Currie finds that the ratio is quite unstable, indicating the variation in the levels of interbank deposits and cash items in process of collection also disrupt the money-reserve ratio.

With regard to differential reserve requirements between classes of member banks, Currie notes that the net expansion or contraction of

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[4]ibid., p. 69.

[5]ibid., p. 74.

money resulting from a redistribution of deposits between classes of member banks will generally be in opposition to intended monetary control. For example, during a business expansion, larger, city banks are apt to lose deposits to smaller, country banks, allowing an expansion of the money stock at a time when discretionary policy would be aimed in the opposite direction.[6]

Currie calculates reserve ratios for classes of member banks, for all member banks, and for all commercial banks. While he finds that these reserve ratios vary little, he claims that it is misleading to conclude that nonuniform reserve requirements are not a control problem; very small changes in the reserve ratios translate into relatively large changes in the money stock. Comparing changes in member bank reserves to: 1) member bank adjusted demand deposits; 2) adjusted demand deposits for all commercial banks; and 3) the money stock, shows that given changes in member bank reserves have historically corresponded to a wide variety of changes in adjusted demand deposits and the money stock. Currie concludes that, "Actually an increase in utilized reserves may correspond with almost any multiple expansion or contraction of money."[7]

In their development of a modern theory of the money supply process, Karl Brunner and Allan H. Meltzer and their followers have incorporated sufficient institutional detail that the models include the effects of nonuniform reserve requirements. A description of Brunner-Meltzer's well-known nonlinear money supply theory as well as Brunner's earlier linear theory are contained in Appendix A. They are discussed

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[6] ibid., pp. 75-6.

[7] ibid., p. 82.

below only briefly and only with reference to the specific issue under consideration in this study. Following the development of modern money supply theories, a number of empirical studies were undertaken to measure the control problem caused by nonuniform reserve requirements. Each of these empirical studies is also discussed below.

Brunner-Meltzer

Brunner's linear theory[8] is based on the notion of surplus reserves, defined as available reserves less desired available reserves. As surplus reserves appear in an individual bank's portfolio, it expands its earning assets, thereby increasing the money stock in the process of ridding itself of surplus reserves. Brunner identified eight ways in which surplus reserves may be generated for an individual bank. Four of those eight sources of surplus reserves are pertinent to this study.

1) Shifts between demand and time deposits, to the extent that either legal or desired reserve ratios on time and demand deposits are not equal;

2) Redistribution of existing deposits or the distribution of newly-created deposits among banks subject to different reserve ratios;

3) Reallocation of a bank's cash assets between interbank deposits and assets that satisfy legal reserve requirements;

4) An individual bank may gain or lose surplus reserves as other banks in the system reallocate their cash assets as described in paragraph (3).

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[8]Karl Brunner, "A Schema for the Supply Theory of Money," International Economic Review 2 (January 1961):79-109.

The expression which explains the generation of aggregate surplus reserves therefore includes four terms that are of interest here. First, the quantity of surplus reserves released or frozen by shifts between time and demand deposits is represented by, (using Brunner's notation),

(2-1) $a_2 = \sum_{i=1}^{n} [(r^{di} + w_1^i)(1 - g_{2i}) - (r^{ti} + w_2^i)]n_2^i$, where (for n banks),

r^{di} = the legal reserve ratio against demand deposits for the i^{th} bank;

w_1^i = the marginal propensity of the i^{th} bank to hold additional reserves against demand deposits;

r^{ti} = the legal reserve ratio against time deposits for the i^{th} bank;

w_2^i = the marginal propensity of the i^{th} bank to hold added reserves against time deposits;

g_{2i} = the marginal propensity of the ith bank to adjust its balances at other banks because of a change in its deposit liabilities;

n_2^i = the deposit flow representing a shift from demand deposits to time deposits at the i^{th} bank.

Second, the term Brunner identifies as ℓ' represents the aggregate loss or gain of surplus reserves resulting from a redistribution of existing deposits among different classes of banks. Specifically,

(2-2) $\ell^1 = \sum_{i=1}^{n} [(1 - g_{3i}) - (r^{di} + w_1^i)g_{1i}]n_3^{1i}$, where,

g_{3i} = the proportion of the i^{th} bank's clearing balance that is settled by the Federal Reserve mechanism;

$g_{\ell i}$ = the proportion of the i^{th} bank's clearing balance that is settled by debiting its deposit liabilities to other banks;

n_3^{1i} = the net deposit inflow to the i^{th} bank resulting from the redistribution of existing deposits among different banks. $(\sum_{i=1}^{n} n_3^{1i} = 0)$.

In order for this term to affect the aggregate level of surplus reserves, the value of the bracketed expression must be different for different banks.

Finally there are two terms in the expression for surplus reserves incorporating the influence of interbank deposits. One deals with the distributional effect of interbank deposits. Dividing banks into four classes (central reserve city, reserve city, country, and nonmember banks), the net interbank position for the s^{th} class of banks is an aggregation over all banks in that class and is represented by,

(2-3) $h_o^s - (1 - g_z^s)d_o^{bs}$ where

h_o^s = interbank deposits that represent assets of banks in the s^{th} class;

d_o^{bs} = demand deposits of the s^{th} class that are owned by other banks.

The average interbank position for all classes of banks, weighted by legal and desired reserve ratios, is therefore defined by

$$(2\text{-}4) \quad \varepsilon_o = \frac{\sum_{s=1}^{4} (r^{ds} + w_1^s)[h_o^s - (1 - g_z^s)d_o^{bs}]}{\sum_{s=1}^{4} (r^{ds} + w_1^s)} .$$

The rate of change in the average distribution of interbank deposits will absorb or release surplus reserves. This is defined by,

$$(2\text{-}5) \quad \sum_{s=1}^{4} (r^{ds} + w_1^s)\dot{\varepsilon}_o,$$

where $\dot{\varepsilon}_o$ is the rate of change in ε_o. The second term dealing with interbank deposits defines the scale effect of the quantity of interbank deposits in the system. The term γ_o is defined,

$$(2\text{-}6) \qquad \gamma_o = \sum_{i=1}^{n} d_o^{bi},$$

summing over all n banks in the system. The rate of change in γ_o therefore determines the surplus released or absorbed by a change in the level of interbank deposits in the system.

Although Brunner has done empirical work in connection with the linear money supply hypothesis, little of it is pertinent to the issues of this study and what is pertinent has been rather sketchily reported by Brunner. Brunner minimizes the effect differential reserve requirements have had in the money supply process and believes that the disturbance from this source has been overrated by Federal Reserve officials. While "there is little doubt that volatile redistribution of deposits among classes of banks could seriously impair the degree of control exercised by the monetary authorities over the money supply, . . . (I) investigation of the variations generated in the average requirement ratio on demand deposits attributable to a shifting distribution of existing deposits, however, yields no support for the contention that volatile shifts in deposit distribution actually impairs the degree of control over the money stock."[9]

In An Alternative Approach to the Monetary Mechanism, Brunner and Meltzer assess the importance of the disturbance represented by the term, ℓ^1. For subperiods when legal reserve ratios remained constant, they computed for each month in the period June, 1945 through September, 1962, the change in required reserves from the same month in the preceding year. These calculations reveal, according to Brunner and Meltzer,

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[9] U.S., Congress, House, Committee on Banking and Currency, Subcommittee on Domestic Finance, An Alternative Approach to the Monetary Mechanism, by Karl Brunner and Allan H. Meltzer, 88th Congress, 2nd Session (Washington, D.C.: Government Printing Office, 1964), p. 18

that the amount of reserves released or absorbed by shifts in the distribution of deposits has been small, regular, and has declined since World War II. In the 41-month subperiod ending in September 1962, the average monthly release of required reserves was $43 million. That same subperiod displays the largest range of monthly values: a maximum value of $141 million and a minimum value of -$111 million. Assuming a money multiplier of 2.5, Brunner and Meltzer infer that changes in reserves attributable to a changing distribution of deposits amounted to an average annual change in the money supply of $87 million in the early 1960's (down from $175 million in the subperiod right after World War II). In percentage terms, the source of reserves was, on the average, responsible for .06% of the growth in the money stock in the early 1960's or for an annual rate of change in the money stock of .2% in the last 15 years. Furthermore, Brunner and Meltzer claim that the average monthly values behave in a regular pattern, implying that the irregular influence of the distribution of deposits on the money stock would be even smaller. Since the size of the disturbance from this source appears smaller than the effects of random forces on the money stock, Brunner and Meltzer conclude that "removal of differential requirement ratios cannot be justified in terms of the degree of control over the money supply."[10] Brunner and Meltzer also conclude that the effect on the money stock of changes in either the level or distribution of interbank deposits is very small. Changes in interbank deposits, even changes that are large relative to observed changes, would have minimal impact on the money stock. For example, a 1% change in the

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[10] ibid.

money stock would require a reallocation of interbank deposits of $8 billion from central reserve city banks to country banks and of $28 billion from reserve city banks to country banks. Since total member interbank deposits were $15 billion at the end of 1962, shifts in deposits of such magnitude seem highly unlikely. Brunner and Meltzer conclude therefore that variations in the money stock due to changes in interbank deposits are so small they are indistinguishable from random variations in the money stock and that omitting their effects on the money stock does not cause significant error.[11]

Benston

Benston's 1969 article[12] is the first systematic attempt to empirically investigate the extent to which the reserve requirement system interferes with monetary control. He considers three sources of variation in the reserve-demand deposit relationship:

(1) differential reserve requirements for different classes of member banks;

(2) the imposition of different reserve ratios on demand and time deposits; and

(3) the tendency of different banks to hold different levels of excess reserves relative to deposits.

The basis of Benston's empirical investigation is the equation,

$$(2\text{-}7) \quad DD_t = TR_t \left[\sum_{i=1}^{n} (rd_i dd_i + re_i\, dd_i) + \sum_{j=1}^{n} rt_j td_j \right]^{-1},$$

where rd_i = the legal reserve ratio against demand deposits for the i^{th} class of banks;

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[11] ibid., p. 27.

[12] George J. Benston, "An Analysis and Evaluation of Alternative Reserve Requirement Plans," Journal of Finance XXIV (December 1969): 849-70.

rt_j = the legal reserve ratio against time deposits for the j^{th} class of banks;

dd_i = the ratio of demand deposits in the i^{th} class of banks to total member banks demand deposits;

re_i = the ratio of excess reserves to demand deposits at the i^{th} class of banks;

td_j = the ratio of time deposits of the j^{th} size to total demand deposits at all member banks.

Benston's analysis centers on the demand deposit multiplier, the expression inside the bracket in equation (2-7). Except for rd_i and rt_j, the values of the terms in the demand deposit multiplier are not known. The values of dd_i, re_i, and td_j must be predicted; they may, however, vary so little or be so predictable, that effects of changes in their values can be easily offset.

Benston attempts to compare the predictability of a change in net demand deposits arising from a change in total reserves, under various reserve requirement systems. He considers the following three systems: 1) the country bank-city bank system; 2) a graduated system, based on bank size; and 3) a uniform reserve requirement scheme. Benston was unable to obtain deposit data distributed by banks size, so he uses the country-city reserve city distinction as a proxy for size for both demand and time deposits. The td_j terms actually reflect two influences on the demand deposit multiplier--the distribution of time deposits among classes of member banks and the ratio of member bank time deposits to member bank demand deposits. Benston does not distinguish between these two effects. The data he uses is semimonthly averages of daily figures for member banks for the period January 1, 1951 to August 1967.

The mean change in dd_i from one period to the next is .021%.[13] Expressed as a percentage of dd_i,[14] the mean change in the ratio is only .061% for country banks and -.033% for city banks. Therefore while the series dd_i exhibits a few large period-to-period changes and extreme values, the overall variation in dd_i has not been large. The Fed may therefore be able to predict total demand deposits successfully while ignoring any shift in deposits between classes of member banks. To test this possibility, Benston "predicts" total demand deposits for each period, using the value of dd_i for the previous period. The overall error in estimating DD_t in this manner proved to be quite small, though there were a few periods for which the errors were substantial. Benston therefore concludes that changes in dd_i are small and predictable.

Benston uses the same simple prediction model to test whether changes in dd_i are offset by differences in re_i. When the dd_i term is lagged, the prediction errors are smaller, implying that unexpected changes in dd_i are offset some by different re_i. Benston concludes that a system of differential reserve ratios may actually be superior to uniform reserve ratios. However, there is little difference in excess reserve behavior between bank classes so the possible advantage of differential reserve ratios is apparently small.

Finally, Benston estimates total demand deposits by the simple model, $DD_t = DD_{t-1}$. The errors encountered with these predictions are large and ten times larger than the errors involved when DD_t were

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[13] $[dd_{i,t} - dd_{i,t-1}] \cdot 100$

[14] $\left\{[dd_{i,t} - dd_{i,t-1}] \div dd_{i,t-1}\right\} \cdot 100$

estimated using $dd_{i,t} = dd_{i,t-1}$. This implies that there are other, larger problems involved in estimating total demand deposits that are not accounted for by shifts in dd_i. Therefore, Benston finds no significant evidence to recommend one reserve requirement system over another.

The td_j terms show much larger changes for all classes of bank than the dd_i terms. But since the ratio of time to demand deposits for all banks grew during the sample period, Benston concludes that the changes in td_j were primarily the result of movements into time deposits rather than the result of shifts in deposits among classes of banks. The td_j ratio as it is constructed does not allow Benston to disentangle the two movements. Assuming $td_{j,t} = td_{j,t-1}$ in his prediction model for DD_t results in errors that are as much as nine times larger than those resulting from assuming $dd_{i,t} = dd_{i,t-1}$. When two or three lagged error terms are added to the prediction model, the prediction errors are reduced by two-thirds. Benston therefore concludes that changes in time deposits need not hamper monetary control either.

Benston sees nonmember banks presenting two control problems. First, is the erosion of control over the total deposits of the nation as more and more banks leave (or fail to join) the System. He considers this problem unimportant, claiming, "the shifts of deposits of this sort are gradual and obviously can be predicted and accounted for easily."[15] The second and, in Bestons's regard, more significant problem is the transitory shifts in deposits between existing member and nonmember banks. Benston compares the Federal Reserve's estimate of nonmember

[15] ibid., p. 859.

bank demand deposits to nonmember bank call data and concludes that estimation between call dates of nonmember bank deposits is a much more serious problem for the Fed than predicting shifts in deposits between classes of member banks.

Poole and Lieberman

In "Improving Monetary Control," William Poole and Charles Lieberman[16] investigate the variation in the ratio of member bank reserves to member bank adjusted demand deposits. They ignore the control problem created by nonmember banks. They calculate the member bank reserve ratio using weekly data for individual member banks for the period October 7, 1970 to November 3, 1971. The ratio proves to be more stable under the new graduated reserve requirement system than under the old city-country system. The ratio has a standard of deviation of .44 and a coefficient of variation of .018 under the city-country bank reserve plan, compared to .37 and .017 for the graduated reserve system.

Poole and Lieberman decompose the member bank reserve-demand deposit ratio into the sum of required reserves against private demand deposits, government demand deposits, interbank deposits, time deposits and nondeposit liabilities (Eurodollars and commercial paper). Total required reserves and each of its components are then divided by total member bank demand deposits. (This ignores excess reserves). The variance of the total required reserve ratio is then the sum of the variance of each of the component required reserve ratios plus twice the covariances of the component ratios.

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[16]William Poole and Charles Lieberman, "Improving Monetary Control," Brookings Papers on Economic Activity (2:1972):293-342.

Their results indicate that differential reserve requirements cause little variation in the total reserve ratio: the variance of the total required reserve ratio is .1403 while that for its demand deposit component is .0021. The greatest source of variation in the total required reserve ratio comes from time deposits (its reserve ratio has a variance of .1101) followed by government deposits (.0308) and interbank deposits (.0209). The reserve ratio for the nondeposit liabilities show even smaller variances. The same calculations for changes in deposits yield the following results: total required reserve ratio, .0931; government deposits, .0306; interbank deposits, .0406; time deposits, .0091; demand deposits, .0020. When first differences are considered then, time deposits cause much less disturbance in the reserve ratio, and the major control problems are changes in government and interbank deposits. Poole and Lieberman therefore conclude that neither graduated reserve requirements nor reserve requirements against nondeposit liabilities causes serious disturbance in the required reserve ratio; changes in the level of time deposits and the reserves needed to back them are substantial, but those changes are apparently more predictable than are changes in either government or interbank deposits.

While Poole and Lieberman present no quantified measure of the disturbance introduced in the reserve ratio by lagged reserve requirements, they do present theoretical arguments to support the view that, "The lagged reserve requirement system in all probability reduces the stability of short run functional relationship and to that extent the precision of week-by-week control."[17] While it is true that lagged

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[17]ibid., p. 311.

reserve requirements provide member banks and the Federal Reserve with perfect certainty about the level of required reserves, they provide no one with information on the amount of reserve adjustment that is necessary. Hence, it is the contention of Poole and Lieberman that some reserve adjustment is necessary with either lagged or contemporaneous reserve requirements, but that lagged reserve requirements preclude any reserve adjustment through manipulation of deposit levels. More of the adjustment is therefore forced into the money market. To assess their impact in the money market, Poole and Lieberman examine three indicators of money market stability (weekly change in the money stock, federal funds rate, and the level of free reserves) for 296 weeks before the introduction of lagged reserve requirements and 197 weeks after their introduction. In each case, they find slightly more instability with lagged reserve requirements.

Starleaf

The most recent investigation of this problem is a study by Dennis Starleaf[18] in which he systematically removes other sources of variation in the reserve ratio in order to assess whether or not nonmember banks present a serious control problem. He defines the reserve ratio,

$$(2\text{-}8)\quad r = \frac{R_m + V_n}{DD},$$

where R_m is member bank reserves, V_n is nonmember bank vault cash and DD is the demand-deposit component of the money supply.

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[18]Dennis R. Starleaf, "Nonmember Banks and Monetary Control," The Journal of Finance XXX (September 1975):955-975.

Initially Starleaf dismisses three of the causes of variation in the r-ratio, assuming that they can be accurately predicted by the Fed. These include changes in legal reserve ratios, changes in the distribution of deposits between classes of member banks,[19] and changes in the level of nondeposit liabilities. Under these assumptions, the artificial reserve ratio r^* is constructed which excludes the effects of nondeposit liabilities and incorporates constant effective reserve ratios for all member banks. Thus,

$$(2\text{-}9) \quad r^* = \frac{.147\ NDD_m + .04T_m + ER_m + V_n}{DDA_m + DDA_n + DD_f + CIP_{m,f} - FLT + FORN}{}^{20},$$

where

NDD_m = member bank net demand deposits;

T_m = member bank time deposits;

ER_m = member bank excess reserves;

DDA_m = member bank demand deposits adjusted;

DDA_n = nonmember bank demand deposits adjusted;

DD_f = demand deposit balances at branches and agencies of foreign banks in New York City plus such balances at international investment corporations in New York City;

$CIP_{m,f}$ = member bank cash items in process of collection associated with foreign agency and branch transfers;

FLT = Federal Reserve Float, and

FORN = deposits at Federal Reserve Banks due to foreigh official institutions.

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[19] He relies on Benston's findings for this conclusion.

[20] After the introduction of lagged reserve requirements in September, 1968, $R^*_{m,t} = .147\ NDD_{m,t-2} + .047_{m,t-2} + ER_{m,t} + (V_{m,t} - V_{m,t-2})$.

He then constructs a reserve ratio, r^*_{-n}, derived from r^* by excluding the influence of nonmember banks. This is accomplished by first, subtracting V_n from the numerator and DDA_n from the denominator of r^*. Second, the reserves that member banks must hold against net interbank deposits due to nonmember banks is deducted from the numerator of r^*. The data needed to make this adjustment however is not available so instead he adds net interbank deposits at member banks due to all commercial banks[21] to the denominator of r^* to obtain,

$$(2\text{-}10) \qquad r^*_{-n} = \frac{.147\,NDD_m + .047T_m + ER_m}{DDA_m + NBD_m + DD_f + CIP_{m,f} - FLT + FORN},$$

where

NBD_m = member bank deposits due other commercial banks less deposits due member banks from other commercial banks.

Starleaf claims that changes in the level of time and government deposits can be foreseen perfectly, because of lagged reserve requirements. Therefore he removes the variation in r^* and r^*_{-n} from these two sources, resulting in the following definitions,

$$(2\text{-}11) \qquad r^*_{-t,-g} = \frac{.147NDD_m - .147DG_m + R_m + V_n}{DDA_m + DDA_n + DD_f + CIP_{m,f} - FLT + FORN},$$

$$(2\text{-}12) \qquad r^*_{-n,-t,-g} = \frac{.147NDD_m - .147DG_m + R_m}{DDA_m + NBD_m + DD_f + CIP_{m,f} - FLT + FORN}.$$

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[21] The rationale for adding NBD_m to the denominator of (2-10) is as follows: if there were no nonmember banks, NDD_m (on which R^*_m is based) would exceed DDA_m (the money supply component) by the amount of government demand deposits; here NDD exceeds (DDA_m + NBD_m) by the same smount. Thus adding NBD_m to the denominator of r^*_{-n} counters the reserves member banks hold against their net balances due nonmember banks. This assumes that member bank balances due to other member banks equal member bank balances due from other member banks (which is in theory true).

Finally Starleaf removes the affects of float and foreign deposits represented by the DD_f, $CIP_{m,f}$, FLT and FORN terms in the denominator of equation (2-12). This results in,

$$(2\text{-}13)\quad r_m^* = \frac{.147NDD_m - .147DG_m + ER_m}{DDA_m + NBD_m}.$$

The only remaining sources of variation in r_m^* are excess reserves and lagged reserve requirements.

The values of these reserve ratios are then calculated for June 30 and December 31 for the years, 1961 through 1973. For nonmember banks, call report data are used; for member banks, weekly data for the week nearest the call data are used. Comparing first r* and r_{-n}^*, both ratios follow the same basic pattern and it is difficult to determine from a plot of them whether one series varies more than the other. The first and second differences of the series r* and r_{-n}^* indicate that r* is slightly more stable than r_{-n}^*; the mean of the absolute values of the second differences of r* is .00332, compared to .00420 for r_{-n}^*. These results imply that the existence of nonmember banks has been stabilizing to a small degree. Using r* and r_{-n}^* to calculate money multipliers, m_b^* and $m_{b,-n}^*$, respectively, Starleaf finds that both m_b^* and $m_{b,-n}^*$ display considerable variability. The $m_{b,-n}^*$ multiplier is slightly more variable than m_b^*; the mean of the absolute values of the second differences is .03097 for $m_{b,-n}^*$ and .02833 for m_b^*. Starleaf concludes that nonmember banks have at least not been a source of instability and may be a minor source of stability.

Comparing $r_{-t,-g}^*$ and $r_{-n,-t,-g}^*$ again shows less variation in the ratio that includes nonmember banks. The mean of the absolute value of the second differences is .00212 for $r^*_{-t,-g}$, compared to .00368 for $r^*_{-n,-t,-g}$. Again the empirical evidence implies that nonmember banks have been a minor source of stability.

Values of r^*_m exhibit considerable variation; the mean of the absolute values of the second differences of r^*_m is .00274. This indicates that excess reserves and lagged reserve requirements alone account for much of the variance in the r-ratio. Finally, Starleaf investigates the stability of the r^*_m ratio with excess reserves also removed ($r^*_{m,-er}$). Since the $r^*_{m,-er}$ ratio displays considerable variation, Starleaf is able to conclude that lagged reserve requirements alone contribute considerable variation to the r-ratio.

Starleaf's study can be criticized on a number of grounds. A primary deficiency is his unavoidable reliance on call report data for nonmember banks. The numerous discontinuities and distortions in call report data are a well-known problem which renders results based on call report data somewhat unreliable. It is possible that the small differences that Starleaf finds between the variation in r* and r^*_{-n} are due to errors and distortions in the data.

Second, Starleaf dismisses as unimportant differential Federal Reserve reserve requirements and requirements against nondeposit liabilities as sources of variation in the reserve ratio. In the case of differential reserve requirements, he relies on Benston's results as a justification. In the second case, he merely assumes that changes in nondeposit liabilities can be perfectly predicted with lagged reserve requirements. This assumption could of course be made about many things that disrupt the reserve ratio: time deposits, government deposits, interbank deposits, and the distribution of deposits among classes of member banks. In actuality, the Federal Reserve does not have the data needed to perfectly predict these items nor the capabilities to perfectly offset their changes.

Starleaf ultimately compares the ratio of reserves to adjusted demand deposits for all commercial banks with that for member banks and concludes that since the latter ratio varies more, nonmember banks are a source of stability. Kopecky[22] has shown that, under certain circumstances, Starleaf's results for r^* and r^*_{-n} are trivial. In a more complete model specifying bank behavior, Kopecky includes the institutional arrangement whereby member banks provide reserve assets to nonmember banks through interbank deposits. Member banks' excess reserves are then taken to be a stochastic function of demand deposit liabilities owed to the public and to nonmember banks. Under these conditions, Kopecky shows that the mean of the absolute value of the second differences of r^* (which is the measure of variability Starleaf uses) is always larger than that for r^*_{-n}.[23]

Summary and Recommendations

The control problems generated by differential reserve requirements appear, according to the empirical work reported to date, to be minimal. Some of the earlier writers, such as Currie, recommended that uniform reserve requirements be placed on all demand deposits. In addition, much of the theoretical work done on the money supply process

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[22] Kenneth J. Kopecky, "Nonmember Banks Revisited: A Comment on Starleaf." Unpublished manuscript, Board of Governors of the Federal Reserve System, Washington, D.C.

[23] ibid., pp. 11-12.

implies that differential reserve requirements introduce troublesome variation in the reserve ratio.[24] Empirical results do not however support this popular contention. Both Benston's and Poole and Liberman's results indicate that the distribution of demand deposits among classes of member banks changes little and, in comparison to other sources of variation in the reserve ratio, have relatively insignificant effects. Comparing a graduated reserve scheme, a uniform reserve plan, and the city-country classification system, Benston concludes that none of the reserve systems can be recommended on the basis of improved monetary control. It should be noted, however, that since Benston uses data on city and country banks as a proxy for data distributed by size of bank, his results with respect to a graduated reserve system are not totally reliable. Poole and Lieberman are also not able to conclude that removing differential reserve ratios on classes of member banks would improve monetary control. Their results also show that the variation in the reserve ratio caused by differential reserve requirements is slightly smaller under the graduated reserve scheme than under the previous city-country system.

Most authors agree that monetary management would be improved if nonmember banks were placed under Federal Reserve reserve requirements, but no one has presented convincing empirical evidence to support that view. Poole and Lieberman include universal reserve requirements in their list of proposed reforms. Benston however claims, "The absence of a Federally controlled, single reserve ratio applied to the demand

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[24]See, for example, Albert E. Burger, The Money Supply Process (Belmont, California: Wadsworth Publishing Company, Inc., 1971), pp. 57-58.

deposits of nonmember banks has resulted in a few fairly large errors of prediction, but these may be a function of discontinuous reporting by nonmember banks rather than their noncontrol."[25] Therefore he is unable to recommend universal membership. Starleaf does not recommend universal membership either, inferring from his empirical results that nonmember banks have been a source of stability, rather than instability.

Removal of reserve requirements against time deposits is also a common recommendation for reform. Currie recommended that this reform be enacted in order to prevent "distortions" of the money supply caused by consumers' shifts in and out of time deposits. On the other hand, some authors have proposed that the distinction between demand and time deposits be removed for reserve purposes, and the same reserve ratio be applied to each. From his empirical results, Benston concludes that changes in time deposits are sufficiently predictable that the question of the best reserve ratio for them cannot be answered on the grounds of monetary control. Poole and Lieberman, however, recommend that reserve requirements on both time deposits and government deposits be eliminated, their empirical results imply that these two nonmoney items introduce significant variation into the reserve ratio.

There is virtual agreement among all authors that lagged reserve requirements should be eliminated. Poole and Lieberman reason that lagged reserve requirements are a major source of disruption and Starleaf's results support this contention.

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[25] Benston, p. 869.

Chapter 3

THE THEORETICAL MODEL

Introduction

The theoretical model employed in this study is a standard money multiplier model like that originally formulated by Karl Brunner and Allan H. Meltzer[1] and modified and refined by Albert E. Burger[2] and others.[3] The underlying assumption of such a model is that the monetary authorities have relatively precise control over the net source base[4] and that they control the net source base in an attempt to control the money stock. The relationship between the net source base, B_a, and the narrowly defined money stock, M_1, is conventionally denoted,

(3-1) $M_1 = mB_a$,

where m represents the money multiplier. The multiplier m is not a

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[1] Karl Brunner and Allan H. Meltzer, "Liquidity Traps for Money, Bank Credit, and Interest Rates," Journal of Political Economy 76 (January/February 1968):1-37.

[2] Albert E. Burger, The Money Supply Process (Belmont, CA: Wadsworth Publishing Company, Inc., 1971).

[3] For a more detailed presentation of Brunner and Meltzer's "nonlinear" money supply model, see Appendix A.

[4] The net source base is defined as the sum of Federal Reserve holdings of U.S. Government securities, the gold stock, treasury currency outstanding, Federal Reserve float, and "other Federal Reserve Assets" minus treasury cash holdings, treasury deposits at Federal Reserve Banks, foreign deposits at Federal Reserve Banks, other deposits at the Federal Reserve, and other Federal Reserve liabilities and capital. For a more detailed description and derivation of the net source base, see Appendix B.

constant; it is dependent on factors representing the behavior of the public, commercial banks, and the federal government.

The monetary authorities supply a specific quantity of "base money" (set the net source base at some prescribed level), but theoretically any size money stock may be generated from that level of base money. The level of the net source base, plus various institutional factors, act as constraints on the size of the money stock, but there is no exact or constant relationship between the net source base and the money stock. The inexactness or variability in the base-money stock relationship is reflected in variation in the money multiplier. Any variation in the value of m causes variation in the money stock, given a constant level of base money. When the monetary authorities determine the level of the net source base, presumably they have some estimated value of m in mind.[5] If the value of m turns out to be larger than predicted, the money stock will be larger, given the controlled level of the net source base. Thus, variation in the value of m hinders control of the money stock.

Variation in the value of m can be best analyzed using a more detailed definition of m. This can be derived as follows. The net source base is completely absorbed by the sum of member bank unborrowed reserves, nonmember bank vault cash, and currency in the hands of the public. This is the channel through which the base acts as a constraint

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[5]Assuming of course that their aim is to control the money stock by controlling the net source base.

on the money stock. The money stock is defined here as currency held by the public plus privately-owned demand deposits.[6] Schematically,

$$(3\text{-}2) \qquad B_a = (R^m - B) + VC^n + C^p = (R - B) + C^p,$$

$$(3\text{-}3) \qquad M = C^p + D^p,$$

where R^m = member bank reserves

B = member bank borrowing

VC^n = nonmember bank vault cash

C^p = currency held by the public

R = total bank reserves that absorb base money

D^p = privately-owned demand deposits.

Substituting the expression for B_a and M into (3-1) gives,

$$C^p + D^p = m\ (R = B) + C^p,$$

$$m = \frac{C^p + D^p}{(R - B) + C^p},$$

The following parameters are defined,

r = R/D^p, bank reserves that absorb base money relative to D^p

k = C^p/D^p, currency holdings of the public, relative to D^p

b = B/D^p, member bank borrowing, relative to D^p

Substitution yields,

$$m = \frac{kD^p + D^p}{rD^p - bD^p + kD^p}.$$

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[6]This "conventional" definition of the money stock will be used throughout this study, unless otherwise indicated. Foreign deposits at Federal Reserve Banks are also included in this definition of the money stock but are ignored here. The rationale for this is first, that the item is quantitatively very small and second, that it has no effect on the issues being investigated in this study.

Dividing the numerator and denominator of the right hand side of the equation by D^p gives

$$(3\text{-}4) \quad m = \frac{1 + k}{r - b + k} .$$

As can be seen in equation (3-4), the value and stability of m is dependent on the value of stability of 1) the public's decision with regard to currency holdings (k); 2) member bank borrowing behavior (b); and 3) various behavorial and institutional factors that govern the level of bank reserves relative to privately-owned demand deposits (r). Variation and unpredictability will occur in m as a result of variation and unpredictability in k, b, or r.

Consider first the parameter k. As equation (3-2) shows, every dollar of the net source base becomes a part of either bank reserves or currency in the hands of the public. Every dollar of the net source base that is held as currency by the public adds exactly one dollar to the money stock. But every dollar of the net source base that is absorbed into bank reserves can potentially support $(\frac{1}{r} \cdot \$1)$ of demand deposits, where r represents the weighted-average reserve requirement. Therefore, how the net source base is apportioned between C^p and $(R - B)$ is one important determinant of the size of the money stock which will ensue from a given amount of base money. Any variation in the portion of the net source base that the public elects to hold as cash will therefore cause variation in m.

The second factor that causes variation in m is member bank borrowing (b). By adding to member bank reserves, borrowing in effect extends the net source base[7] and gives member banks the potential to expand demand deposits above the level that would be possible without borrowing. Therefore an increase in the level of member bank borrowing will increase the size of m (as can be seen in equation (3-4)) and a larger money stock will correspond to a given level of base money.

Finally, m may vary because of variation in the relationship between bank reserves and privately-owned demand deposits (r). The purpose of this study is to investigate the behavior of the parameter r and thereby infer its effect on m. The parameter r summarizes the effects on m of a number of factors determined by bank, public, and federal government behavior in combination with the current reserve requirement systems. The parameter r is examined in more detail in the next section.

Factors that Affect the Value of the r-Ratio

The level of reserves that absorb base money relative to privately-owned demand deposits (r) can, given the current reserve requirement systems, be affected in two basic ways. One way is through changes in the distribution of deposits among classes of banks that are subject to different required reserve ratios. Changes in the distribution of deposits may cause base money to be absorbed or released by increasing or decreasing the amount of reserves that must be held per dollar of

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[7] The net source base plus member bank borrowing is defined as the source base. The monetary authorities exercise some control, though imprecise, over member borrowing through administration of the discount window. Furthermore, it is usually claimed that the Fed can offset member bank borrowing with enough precision to exactly control the source base. In light of this, the source base could be used as the controlled variable as validly as the net source base.

privately-owned demand deposits. The value of r is, therefore, currently affected by the distribution of total deposits between member and nonmember banks; by the distribution of member demand and time deposits among member banks of different reserve categories; by the distribution of nonmember demand and time deposits among nonmember banks in different states;[8] and by the distribution of net interbank deposits and cash items in process of collection between members and nonmembers, members of different reserve categories and nonmembers in different states.

The second way that the value of r is altered is by things that are not part of the money stock, against which member banks must hold base-absorbing reserves. By adding to reserves, these factors reduce the amount of net source base available to back the money stock; they increase the value of r and lower m. Five factors affect the value of r in this manner. They include: excess reserves, government demand deposits, interbank deposits, time deposits, and several categories of nondeposit liabilities against which member banks are required to hold reserves. In addition, the value of r is affected by week-to-week changes in either the level of member bank demand or time deposits, or their distribution among reserve classes. Each of these factors is dealt with separately below.

(i) Nonmember banks: The existence of nonmember banks introduces variation in r in three ways each of which stems from the institutional

[8] Some states subject their nonmember banks to differential reserve requirements, based either on bank size or geographical location. In those states, the distribution of deposits among nonmembers subject to different reserve ratios is also germane.

arrangement which places nonmember banks under different sets of reserve requirements than member banks. Nonmember banks are subject to reserve requirements imposed by the state in which they are chartered. These state-imposed reserve requirements vary widely from state to state; Illinois has no reserve requirements at all, seven states impose reserve ratios that are the same as those of the Federal Reserve System. Table C-1 in Appendix C is a listing of state reserve requirements as of May, 1977.

In general state reserve requirements are quantitatively smaller than those of the Federal Reserve. In addition, there are a number of qualitative differences between state and Federal Reserve reserve requirements. First, while member banks must meet reserve requirements every week, nonmember banks in many states are required to meet their requirements much less frequently. In addition, different assets qualify as legal reserves for nonmember banks than for member banks. As can be seen in Table C-1, at least part of required reserves can be satisfied with holdings of federal or state government securities in 26 states and by cash items in process of collection in 22 states. Nonmember banks' voluntary holdings of vault cash qualify for required reserves in every state; the remainder of required reserves can in every state be held in the form of demand deposits in other commercial banks. In contrast, Federal Reserve reserve requirements require all member banks to hold their required reserves as vault cash or deposits at the Federal Reserve Bank. This means that nonmember banks' reserves can be held in a form that is at least useful to nonmember banks and often, a form which is interest-bearing. Member banks, however, must hold their reserves in noninterest bearing assets.

These quantitative and qualitative differences between Federal Reserve and state reserve requirements have caused discussion and criticism on the grounds of equity. There is no doubt that Federal Reserve reserve requirements put member banks at a cost and competitive disadvantage vis à vis nonmember banks. This is reflected in the diminishing proportion of the nation's commercial banks which choose to be members. The inequity caused by the existence of dual reserve requirements in each state is not, however, an issue here.

The important issue here is the control problem that may be caused by the qualitative differences between Federal Reserve and state reserve requirements. The two assets in which member banks must hold their required reserves are, as can be seen in Appendix B, both base-absorbing. The level of member bank reserves is therefore limited by the size of the net source base and can be controlled by the Federal Reserve. Under the existing state reserve systems, nonmember banks are not specifically required to hold base money as reserves. Only the vault cash portion of nonmember bank reserves absorbs base money directly. The part of nonmember bank reserves made up of interbank deposits may indirectly absorb base money as member banks reserves must be held against those interbank balances. This is one of the ways that nonmember banks affect the value of the r-ratio: by altering the level of interbank deposits in member banks and thereby affecting the level of member bank reserves. This will be discussed more in the section below on interbank deposits. As a whole, however, nonmember bank reserves are not constrained or controlled as are member bank reserves. If bank reserves are defined as those that absorb money, most nonmember bank reserves are actually reserves only in a legal sense; only the vault cash portion of nonmember bank reserves are base-absorbing.

Nonmember bank "reserves" can be expressed as,

$$R^n = VC^n + D^n_m + D^n_n ,$$ [9]

where R^n = nonmember bank "reserves;"

VC^n = nonmember bank vault cash;

D^n_m = demand deposits due nonmember banks from member banks;[10]

D^n_n = demand deposits due nonmember banks from other nonmember banks.

Both member and nonmember banks are required to hold reserves against net demand deposits, defined as total demand deposits minus cash items in process of collection and demand deposits due from other domestic commercial banks. Total nonmember bank "reserves" are then,

$$R^n = \sum_h d_h (D_{n,h} - I^{n,h} - D^{n,h}_n - D^{n,h}_m) + \sum_k t_r T_{n,r} + ER^n;$$

where,

R^n = nonmember bank "reserves;"

d_h = the legally required reserve ratio against nonmember demand deposits in the h^{th} state;[11]

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[9] Security reserve requirements are ignored throughout this analysis, i.e., it is assumed that nonmember bank reserves are reduced by the amount of those reserves that can be held in securities.

[10] The notation D^x_y will be used to denote an interbank deposit that is the asset of bank x and the liability of bank y.

[11] When nonmember banks within one state are subject to differential reserve requirements, the expression for nonmember bank reserves will have to be fragmented further to account for the additional reserve categories.

t_k = the legally required reserve ratio against nonmember time deposits in the k^{th} state;

$D_{n,h}$ = gross nonmember bank demand deposits in the h^{th} state;

$T_{n,k}$ = gross nonmember bank time deposits in the k^{th} state;

$I^{n,h}$ = cash items in process of collection for nonmember banks in the h^{th} state;

$D_n^{n,h}$ = interbank balances due to nonmember banks in the h^{th} state from all other nonmember banks in the nation;

$D_m^{n,h}$ = interbank balances due to nonmember banks in the h^{th} state from all member banks in the nation;

ER^n = total nonmember bank "excess reserves."

Member bank reserves may be express as,

$$R^m = VC^m + F^m, \text{ and}$$

$$R^m = \sum_j d_j(D_{m,j} - I^{m,j} - D_m^{m,j} - D_n^{m,j}) + \sum_i t_i T_{m,i} + ER^m,$$

where,

R^m = member bank reserves;

VC^m = member bank vault cash;

F^m = member bank deposits at Federal Reserve Banks;

d_j = the legally required reserve ratio against demand deposits for member banks in the j^{th} reserve category;

t_i = the legally required reserve ratio against time deposits for member banks in the i^{th} reserve category;

$D_{m,j}$ = gross member bank demand deposits in the j^{th} reserve category;

$T_{m,i}$ = gross member bank time deposits in the i^{th} reserve category;

$I^{m,j}$ = cash items in process of collection for member banks in the j^{th} reserve category;

$D_m^{m,j}$ = interbank balances due to member banks subject to d_j from all other member banks;

$D_n^{m,j}$ = interbank balances due to member banks subject to d_j from all nonmember banks in the nation;

ER^m = total member bank excess reserves.

Using the expression $DD_{m,j}$ and $DD_{n,h}$ for net demand deposits gives,

(3-5) $DD_{m,j} = (D_{m,j} - I^{m,j} - D_m^{m,j} - D_n^{m,j})$ for member banks, and

(3-6) $DD_{n,h} = (D_{n,h} - I^{n,h} - D_n^{n,h} - D_m^{n,h})$ for nonmember banks.

Substitution gives,

(3-7) $VC^m + F^m = \sum_j d_j DD_{m,j} + \sum_i t_i T_{m,i} + ER^m$ for member banks, and

(3-8) $VC^n + D_m^n + D_n^n = \sum_h d_h DD_{n,h} + \sum_k t_k T_{n,k} + ER^n$ for nonmember banks.

Equation (3-7) shows how the net source base constrains the deposit expansion of member banks. Member bank net demand deposits, $DD_{m,j}$, can increase only as long as member banks can increase $(VC^m + F^m)$ by d_j $(DD_{m,j})$. Since $VC^m + F^m$ are both uses of the net source base, member banks can only increase their demand deposits if the monetary authorities allow the source base to increase, if banks are willing to reduce excess reserves, or if the public is induced to hold lower levels of currency or time deposits.

Equation (3-8) however demonstrates that nonmember banks are not necessarily constrained in their expansion of the money supply by the monetary base because the quantity $(VC^n + D_m^n + D_n^n)$ is not entirely base money. In the extreme case,[12] if all privately-held demand deposits were held in nonmember banks, the expansion of privately-held demand deposits would be constrained by the base if, and only if, nonmember banks held all their reserves as vault cash or as deposits at member banks. In that case, when the entire base was absorbed by nonmember bank vault cash or member bank reserves against interbank deposits liabilities, privately-owned demand deposits (and the money stock) would be constrained. But since nonmember banks can hold reserves in the form of interbank deposits at other nonmember banks, privately-held demand deposits at nonmember banks (and therefore the money stock) is not theoretically constrained at all by the size of the base. As can be seen by equation (3-8), nonmember banks can increase ($DD_{n,h}$) to infinity by increasing (D_n^n) infinitely. That is, nonmember banks can trade interbank deposits and using the (D_n^n) as legal reserves, increase demand deposits forever. In reality, of course, this does not occur because the public has no such clear-cut preference for demand deposits at nonmember banks over demand deposits at member banks. As nonmember banks expand their deposits, there is leakage of deposits to member banks by the normal process of loss of deposits to other commercial banks. As this leakage of deposits to member banks occurs, privately-held demand deposits are constrained, because member banks are definitely constrained by the level of the base. Nonmember banks

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[12]It is also possible that the level of privately-held demand deposits would be constrained if all base money was absorbed into currency in the hands of the public. That is, as nonmember banks expand demand deposits, the public will also increase their holdings of currency (represented by the k-ratio discussed above). Once currency in the hands of the public grows to absorb all base money, demand deposit expansion would stop due to lack of demand.

are in effect then not constrained at all by legal reserve ratios, but only by the lack of demand for their deposits as opposed to member bank deposits and currency. It is for this reason that many authors have assumed the level of nonmember bank deposits to be some stable function of member bank deposits.[13]

This is the second way in which nonmember banks affect the r-ratio. Since nonmember banks are not required to hold base money in a fixed proportion to the demand deposits they issue, the higher the proportion of the nation's bank deposits that are in nonmember banks, the lower will be the value of r and the larger will be the value of m. More importantly, variation in the proportion of bank deposits held at nonmember banks causes variation in the value of r and therefore in m.

(ii) Differential state reserve requirements: The third way that nonmember banks can affect the value of r is through the impact of differential state reserve requirements. Nonmember banks in different states are, of course, subject to different required reserve ratios. Therefore the level of nonmember bank "reserves" required behind a given level of nonmember bank deposits will depend on how those deposits are distributed among states.[14] However, since not all nonmember "reserves" are base-absorbing reserves, the effect on r of the distribution of nonmember bank deposits among states cannot be translated with precision from its affect on nonmember "reserves."

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[13] See for example, Ronald L. Teigen, "Demand and Supply Functions for Money in the United States: Some Structural Estimates," Econometrica XXXII (October 1964):476-509.

[14] When nonmember banks within one stare are subject to differential reserve requirements, the distribution of nonmember deposits between those groups of banks is also pertinent to this discussion.

If nonmember deposits are concentrated in states with relatively high required reserve ratios, total nonmember "reserves" will be relatively high. Due to the nature of nonmember "reserves" however, it cannot be inferred that the value of r will increase, without knowledge of how the composition of nonmember "reserves" may change as their level grows. If, for all h, the group of nonmember banks subject to d_h holds a constant and uniform ratio of vault cash of total deposits, the distribution of nonmember bank deposits across state lines does not affect the aggregate level of VC^n, reserves that absorb base, r, or m. On the other hand, if each group of nonmembers holds an amount of vault cash to keep the vault cash to required "reserves" ratio constant, the distribution of nonmember deposits across states affects the aggregate level of vault cash, reserves that absorb base money, r and m. In that case, if nonmember deposits are concentrated in states with high (low) d_h, VC^n will be high (low), r will be larger (smaller), and m will be smaller (larger).

The only other channel by which a change in nonmember "reserves" affects r is the indirect one mentioned above whereby nonmember interbank deposits in member banks affect the level of member bank reserves. If, because their required reserves change, nonmembers change the level of their deposits in member banks, member bank (base-absorbing) reserves will change, affecting the value of r. The conclusion then is the same as that for vault cash. If each group of nonmembers holds a uniform and constant ratio of deposits at member banks to total deposits, the distribution of nonmember deposits among states has no effect on r. If each group of nonmembers holds a constant ratio of deposits at members of required "reserves," the distribution of nonmember deposits among states affects r through its effects on member bank reserves against interbank deposit liabilities.

(iii) Differential Federal Reserve reserve requirements: The amount of base money that members banks are required to hold per dollar of deposits is not the same for every bank or for every dollar of deposits. The value of r therefore depends on how a given level of time and demand deposits are distributed among member banks in different reserve categories. More importantly, changes in the distribution of deposits across reserve categories induces changes in the value of r and thereby in the value of m.

During the sample period under consideration in this study (1961-1974), the number of different reserve categories for time and demand deposits has increased from three to nine. This can be seen in Table C-2 of Appendix C. Until 1966, member banks were categorized as reserve city or country banks and the two classes were subject to different required reserve ratios on demand deposits; all member banks were subject to the same single level reserve ratio on time and savings deposits. In 1966,[15] time and savings deposits were divided into three categories for reserve purposes: savings deposits, time deposits less than \$5 million, and time deposits greater than \$5 million. This increased the number of separate Federal Reserve reserve categories to five. In 1968,[16] the number of member bank reserve categories was

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[15]Board of Governors of the Federal Reserve System, Federal Reserve Bulletin (July 1966):979.

[16]Ibid., (January 1968):95-6.

increased to seven as demand deposits less than \$5 million and demand deposits over \$5 million at both reserve city and country banks were each subjected to a different legal reserve ratio. In November, 1972,[17] the reserve city-country distinction was discarded for reserve purposes and a system of graduated reserve requirements was adopted in which required reserve ratios depend on the deposit-size of the bank. Five separate deposit-size groups were defined: less than \$2 million, \$2 million to \$10 million, \$10 million to \$100 million, \$100 million to \$400 million, and over \$400 million. Finally in 1974,[18] the reserve category for time deposits over \$5 million was divided into two reserve categories: time deposits maturing in 30 to 179 days and those maturing in 180 days or more. Thus by the end of the sample period, there were four separate reserve categories relating to time deposits and five relating to demand deposits.

Given some level of member bank deposits, the level of base-absorbing reserves, and therefore the value of r, will vary with the distribution of those deposits among member banks subject to different required reserve ratios. Critics of the Federal Reserve's reserve structure therefore point to the increased number of reserve categories as a potential source of more variation in r. This increased "splintering" of reserve requirements, in light of the Federal Reserve's apparent increased concern over their ability to control the money supply, seems contradictory. "Increased splintering of reserve requirements . . .

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[17]ibid., (November 1972):994.

[18]ibid., (November 1974):799-800.

tended to introduce greater variability in the reserve ratio, therefore making it more difficult for the Federal Reserve to predict the results of any policy action."[19]

(iv) Lagged Federal Reserve reserve requirements: Beginning September 18, 1968,[20] the Federal Reserve introduced a system of lagged reserve requirements under which a member bank's required reserves are based on its average daily close-of-business deposit holdings two weeks earlier. At the same time all member banks were put on a weekly reporting and reserve-settlement schedule. Under the lagged system, a member bank's reserves consist of its average daily close-of-business deposits at the Federal Reserve in the current week plus its average daily close-of-business vault cash holdings two weeks earlier. In addition, any excess or deficiency in required reserves may be "carried over" into the next settlement week to the amount of 2% of required reserves; no excess or deficiency may be carried over more than one week.

Since the introduction of lagged reserve requirements, member banks, at time t, hold reserves based on $DD_{m,t-2}$ and $T_{m,t-2}$, but the money supply existing in the system at time t, is based on current demand deposit levels. If $DD_{m,j,t-2} = DD_{m,j,t}$ for all j, and $T_{m,i,t-2} = T_{m,i,t}$ for all i, the lag in member bank reserve requirements has no effect on the parameter r. If the ratios $T_{m,i,t-2}/T_{m,i,t}$ and $DD_{m,j,t-2}/DD_{m,j,t}$

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[19] Albert E. Burger, The Money Supply Process, p. 57.

[20] Board of Governors of the Federal Reserve System, Federal Reserve Bulletin (May 1968): 437-8.

vary for some i,j, lagged reserve requirements induce variation in r, since R^m depends on $DD_{m,j,t-2}$ and $T_{m,k,t-2}$. For example, if $DD_{m,j,t-2}/DD_{m,j,t} < 1$, for any j, r will be smaller and m will be larger than would be possible without lagged reserve requirements.

Apparently the rationale for instituting lagged reserve requirements was to reduce uncertainty for member banks, provide the Federal Reserve with better information and reduce weekly reserve adjustment pressure. Whether or not lagged reserve requirements have actually served these goals is in question. As burger[21] points out, the fact that the Federal Reserve has more accurate information on required reserves each week is obvious: since required reserves are based on deposit levels two weeks earlier, the Federal Reserve knows with precision the level of required reserves well before each settlement date. They do not, however, know total reserves with any more accuracy; since reserve adjustment pressure is generated by the difference between required and total reserves, there is no evidence that the Federal Reserve is better equipped to ameliorate or exaggerate reserve adjustment pressure now than before lagged reserve requirements existed. Specifically, Burger finds that the Federal Reserve has had more difficulty estimating total reserves since lagged requirements were introduced. In addition, Burger finds that by three different measures, the amount of reserve adjustment experienced in the system has actually increased since the advent of lagged reserve requirements. He therefore concludes, ". . . the evidence indicates that after lagging the Federal Reserve has

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[18]Burger, pp. 55-6.

been less able to accurately determine the extent to which it should intervene in the money market to prevent short-term pressures."[22]

Burger raises other objections to lagged reserve requirements. Lagging procedures have created comparability problems in published bank data relating to reserves and excess reserves. The carry-over convention in itself has, by Burger's findings, introduced greater variation in excess reserves and the excess reserves-deposit ratio. Not only does this create problems with the data series, but more importantly increased variation in the excess reserve ratio introduces additional variation in m and the money supply process.

Whether or not lagged reserve requirements have fulfilled their prescribed functions of reducing uncertainty and reserve adjustment pressure, there is no question that they have introduced an additional element of variation in r and therefore in the base-money supply relationship. "Because $\frac{DD_{m,t-2}}{DD_{m,t}}$ and $\frac{T_{m,t-2}}{T_{m,t}}$ . . . do not remain constant, but exhibit considerable variability, an additional unpredictable source of variation is now included in the reserve ratio. Therefore to this extent the Federal Reserve . . . made the prediction of changes in the money stock more difficult."[23]

(v) Excess reserves: As an addition to bank reserves above those legally required, excess reserves absorb part of the net source base, without supporting or adding to any part of the money stock. Therefore the higher the level of excess reserves member banks choose to hold, the

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[22]Ibid., p. 56.

[23]Ibid., p. 53.

larger the value of r, and the lower the value of m. Variation in the level of excess reserves causes r and therefore m to vary also.

(vi) Government demand deposits: Banks are required to hold base-absorbing reserves against government deposits, but they are not included in the money stock. Therefore a higher level of government deposits means that more of the net source base is absorbed and less base money is left to support money stock items. An increase in the ratio of government demand deposits to privately-owned demand deposits therefore increases the value of r and decreases the value of m.

(vii) The level and distribution of interbank deposits: Letting the superscripts p and g refer to privately-owned and government-owned deposits, respectively, and letting the notation D_y^x refer to interbank deposits as defined above, gross demand deposits can be expressed as,

(3-9) $D_{m,j} = D^p_{m,j} + D^g_{m,j} + D^m_{m,j} + D^n_{m,j}$, for member banks in the j^{th} reserve category, and

(3-10) $D_{n,h} = D^p_{n,h} + D^g_{n,h} + D^n_{n,h} + D^m_{n,h}$, for nonmember banks in the h^{th} state.

Substituting equations (3-9) and (3-10) into the expressions for net demand deposits, equations (3-5) and (3-6) above, gives,

(3-11) $DD_{m,j} = D^p_{m,j} + D^g_{m,j} - I^{m,j} + (D^m_{m,j} - D^{m,j}_m) + (D^n_{m,j} - D^{m,j}_n)$

for member banks;

(3-12) $DD_{n,h} = D^p_{n,h} + D^g_{n,h} - I^{n,h} - (D^n_{n,h} - D^{n,h}_n) + (D^m_{n,h} - D^{n,h}_m)$

for nonmember banks.

In the aggregate all interbank balances cancel out and total net demand deposits equal total privately-owned and government demand deposits, less cash items in process of collection. That is,

$$\sum_j (D^m_{m,j} - D^{m,j}_m) = 0,$$

$$\sum_h (D^n_{n,h} - D^{n,h}_n) = 0, \text{ and}$$

$$\sum_j (D^n_{m,j} - D^{m,j}_n) + \sum_h (D^m_{n,h} - D^{n,h}_m) = 0.$$

In general however, for each individual bank or for the group of banks subject to one required reserve ratio,

$$(D^m_{m,j} - D^{m,j}_m) \neq 0 \text{ for all } j,$$

$$(D^n_{n,h} - D^{n,h}_n) \neq \text{ for all } h, \text{ and}$$

$$(D^n_{m,j} - D^{m,j}_n) + (D^m_{n,h} - D^{n,h}_m) \neq 0 \text{ for all } j,h.$$

Therefore, an individual bank or group of banks subject to the same required reserve ratio may be required to hold reserves against interbank balances. For example, if $(D^m_{m,j} - D^{m,j}_m)>0$ for some j, at least one bank subject to d_j will be required to hold reserves against interbank deposits. Conversely, if $(D^m_{m,j} - D^{m,j}_m)<0$ for some j, then some bank's required reserves will be less than they would be without the presence of interbank deposits. Therefore, when commercial banks are subject to differential required reserve ratios, both the level of interbank deposits and their distribution among banks affect the value of r. The following numerical example illustrates this. For simplicity, the example deals with two member banks, but the results are generalized to include all commercial banks below. Assume banks A and B hold D^p ($D^g = 0$ here) and interbank deposits owned by the other member bank as follows:

EXAMPLE 1

Case 1) No interbank deposits:

	Bank A		Bank B
D^p	\$150.	D^p	\$300.
D^m_m	0.	D^m_m	0.
$(D^B_A - D^A_B)$	0.	$(D^A_B - D^B_A)$	0.
DD_A	150.	DD_B	300.

Case 2) Interbank deposits Allowed:

	Bank A		Bank B
D^p	\$150.	D^p	\$300.
$D^m_m(D^B_A)$	100.	$D^m_m(D^A_B)$	50.
$(D^B_A - D^A_B)$	50.	$(D^A_B - D^B_A)$	-50.
DD_A	200.	DD_B	250.

Case 3) Change in the distribution of interbank deposits:

	Bank A		Bank B
D^p	\$150.	D^p	\$300.
$D^m_m(D^B_A)$	50.	$D^m_m(D^A_B)$	100.
$(D^B_A - D^A_B)$	-50.	$(D^A_B - D^B_A)$	50.
DD_A	100.	DD_B	350.

Case 4) Change in the level of interbank deposits:

	Bank A		Bank B
D^P	\$150.	D^P	\$300.
$D^m_m(D^B_A)$	75.	$D^m_m(D^A_B)$	150.
$(D^B_A - D^A_B)$	-75.	$(D^A_B - D^B_A)$	75.
DD_A	75.	DD_B	375.

	Uniform 10% Reserve Requirement $r = R/D^P$	Graduated Reserve Requirements $r = R/D^P$
Case 1)	\$45/\$450 = .1	\$27.50/\$450. = .061
Case 2)	45/450 = .1	25/450 = .056
Case 3)	45/450 = .1	30/450 = .067
Case 4)	45/450 = .1	31.25/450 = .069

Case 1 shows the system where there are no interbank deposits at all; Case 2 introduces \$150 of interbank deposits; in case 3, the distribution of that \$150 of interbank deposits has been changed and in Case 4, the level of interbank deposits has increased 50%. In all four cases, the money stock (=D^P here) is \$450. The member banks are assumed to be subject first to a uniform reserve requirement of 10%; secondly, to a graduated reserve requirement system in which 5% of the first \$200 of deposits, and 10% of any additional deposits, must be held as legal reserves.

Comparing the four cases, it can be seen that as long as member banks are subject to uniform reserve requirements, neither the level nor the distribution of D^m_m affect the level of reserves relative to privately-owned demand deposits. As the chart shows, r is .1 for all four cases and therefore interbank deposits have no impact on the value of r. If member banks are subject to graduated reserve requirements as they are in reality, a change in either the distribution of D_m^m (Case 3) or a change in the level of D_m^m (Case 4) will alter the value of r. The reason that D_m^m affect the value of r under graduated reserve requirements is that their level and distribution alter the distribution of net demand deposits among banks that are subject to different required reserve ratios. This changes the level of reserves while D^p remains the same, so that the value of r varies.

Comparing the four cases, it can be seen that as long as member banks are subject to uniform reserve requirements, neither the level nor the distribution of D_m^m affect the level of reserves relative to privately-owned demand deposits. As the chart shows, r is .1 for all four cases and therefore interbank deposits have no impact on the value of m. If member banks are subject to graduated reserve requirements as they are in reality, a change in either the distribution of D_m^m (Case 3) or a change in the level of D_m^m (Case 4) will alter the value of r. The reason that D_m^m affect the value of r under graduated reserve requirements is that their level and distribution alter the distribution of net demand deposits among banks that are subject to different required reserve ratios. This changes the level of reserves while D^p remains the same, so that the value of r varies.

The terms $(D_{m,j}^m - D_m^{m,j})$ for all j, measure the amount by which net demand deposits are redistributed by D_m^m among reserve categories. If $(D_{m,j}^m - D_m^{m,j}) = 0$ for all j, then $DD_{m,j} = D_j^p$ for all j, and no redistribution will occur. In that case, D_m^m does not affect the value of r. However, if $(D_{m,j}^m - D_m^{m,j}) \neq 0$ for some j, net demand deposits are redistributed from the group of banks for whom $(D^m_{m,j} - D^{m,j}_m)>0$. This can be seen in the numerical example.

With no D^m_m present (Case 1), the distribution of net demand deposits between banks A and B is identical to the distribution of D^p. By comparison, in the other three cases, the distribution of net demand deposits between the two banks is altered by both the level and distribution of D^m_m and is no longer identical to the distribution of D^p. When the deposits of the two banks are not subject to the same required reserve ratios, the change in the distribution of net demand deposits alters the level of required reserves while total DD_m and D^p remain constant. Therefore the value of r is affected, causing variation in m. Whether r is increased or decreased by a change in D^m_m depends on whether the resulting redistribution of net demand deposits is toward banks subject to larger or smaller required reserve ratios. If $(D^m_{m,j} - D^{m,j}_m)>0$ for banks subject to relatively high d_j, r increases, (Case 3 or 4, compared to Case 1). If $(D^m_{m,j} - D^{m,j}_m)>0$ for banks subject to relatively low d_j, the value of r falls (Case 2, compared to Case 1).

The effects of interbank deposits are the same as those described above whether the banks involved are members or nonmembers or a combination of both, as long as they are subject to different required reserve ratios. The problem is dealing with nonmember "reserves" that are not necessarily reserves in the base-absorbing sense. When nonmember banks are involved, determining the effect of interbank deposits on the distribution among banks of net demand deposits reveals only the levels of legally required reserves for the banks involved.

Just as in example 1, a change in the level or distribution of interbank deposits redistributes net demand deposits and, as long as the banks involved are subject to different required reserve ratios, alters the level of legally required reserves. The effect on base-absorbing reserves (and therefore on r) however, is indeterminate. When nonmember banks are involved, the crucial determinant of reserves that absorb base money is how nonmember banks apportion their reserves between vault cash and interbank deposits. The following numerical example illustrates. Assume bank A is a member bank and B a nonmember and first, that both banks are subject to a uniform reserve requirement of 10% and secondly, that member banks are subject to a 15% required reserve ratio.

EXAMPLE 2

The numerical information used in Example 2 is presented on the following two pages.

EXAMPLE 2

Case 1) No interbank deposits

a) Nonmembers hold all "reserves" in vault cash

	Bank A		Bank B
D^P	\$200.	D^P	\$200.
D^n_m	0	D^m_n	0
$(D^n_m - D^m_n)$	0	$(D^m_n - D^n_m)$	0
DD_m	200.	DD_n	200.

b) Nonmembers hold all "reserves" in interbank deposits

Bank A	Bank B
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c) Nonmembers hold 7% of net demand deposits in interbank deposits, 3% in vault cash.

Bank A	Bank B
--	--
--	--
--	--
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Case 2) Interbank deposits allowed: (Bank A = \$25, Bank B = \$10 + reserves)

a)

	Bank A		Bank B
D^P	\$200.	D^P	\$200.
D^n_m	10.	D^m_n	25.
$(D^n_m - D^m_n)$	-15.	$(D^m_n - D^n_m)$	15.
DD_m	185.	DD_n	215.

b)

	Bank A		Bank B
D^P	\$200.	D^P	\$200.
D^n_m	29.54	D^m_n	25.
$(D^n_m - D^m_n)$	+4.55	$(D^m_n - D^n_m)$	-4.55
DD_m	205.54	DD_n	195.45

c)

	Bank A		Bank B
D^P	\$200.	D^P	\$200.
D^n_m	20.07	D^m_n	25.
$(D^n_m - D^m_n)$	-4.93	$(D^m_n - D^n_m)$	4.93
DD_m	195.07	DD_n	204.93

Case 3) Change in the distribution of interbank deposits:
(Bank A = $10, Bank B = $25 + reserves)

	a) Bank A	a) Bank B	b) Bank A	b) Bank B	c) Bank A	c) Bank B
D^p	$200.	$200.	$200.	$200.	$200.	$200.
D^n_m	0	10.	41.82	10.	37.10	10.
$(D^n_m - D^m_n)$	15.	-15.	31.82	-31.82	27.10	-27.10
DD_m	215.	185.	231.82	168.18	227.10	172.90

Case 4) Change in the level of interbank deposits:

	a) Bank A	a) Bank B	b) Bank A	b) Bank B	c) Bank A	c) Bank B
D^p	$200.	$200.	$200.	$200.	$200.	$200.
D^n_m	20.	50.	40.91	50.	35.05	50.
$(D^n_m - D^m_n)$	-30.	30.	-9.09	9.09	-14.95	14.95
DD_m	170.	230.	190.91	209.09	185.05	214.95

Uniform reserve requirements: 10%

Value of $r = R/D^p$

	a	b	c
Case 1)	\$40/\$400 = .10	--	--
Case 2)	40/400 = .10	\$20.45/\$400 = .051	\$25.66/\$400 = .064
Case 3)	40/400 = .10	23.18/400 = .058	27.90/400 = .070
Case 4)	40/400 = .10	19.09/400 = .048	24.95/400 = .062

Differential reserve requirements,
member bank: 15%, nonmember bank: 10%

Value of $r = R/D^p$

	a	b	c
Case 1)	\$50/400 = .125	--	--
Case 2)	49.25/400 = .123	\$30.67/\$400 = .077	\$35.41/\$400 = .089
Case 3)	50.75/400 = .127	34.77/400 = .087	39.26/400 = .098
Case 4)	48.50/400 = .121	28.64/400 = .072	34.21/400 = .086

Cases 1 - 4 correspond to the cases described in example 1. In addition example 2 divides each case into three subcases each of which corresponds to a different composition of nonmember bank reserves. In subcase a, it is assumed that nonmember banks hold all their legal reserves as vault cash; in subcases b nonmember banks hold all their "reserves" in interbank deposits and in subcases c, they hold 3% of their required reserves as vault cash and the remainder as interbank deposits.

In subcases a, all of nonmember bank "reserves" absorb base money so they are qualitatively identical to member bank reserves. If, in addition, member and nonmember banks are subject to the same required reserve ratio, the presence of interbank deposits will merely redistribute net demand deposits among banks, leaving total reserves unaltered. Neither the level nor distribution of interbank deposits therefore affects r. On the other hand, if the two banks are subject to different required reserve ratios, changes in either the distribution or level of interbank deposits causes variation in r like that caused by D_n^m when member banks are subject to graduated reserve requirements (Example 1). If $(D_m^n - D_n^m)>0$, net demand deposits are redistributed toward the member bank, and reserves rise here, since the member bank is subject to the higher reserve requirement.

In subcases b, nonmembers hold no base-absorbing reserves. Therefore regardless of the level or distribution of interbank deposits, reserves, and therefore r, are lower under reserve option b than any other. Nonmember bank "reserves" affect r only indirectly if by deciding to hold all "reserves" as interbank balances, D_m^n is increased, forcing member banks to hold more reserves (Cases 2b, 2c, and 4b). Under reserve option c, reserves increase above their level under option b, because nonmembers now absorb some base money as vault cash. Under both reserve options b and c, the level of distribution of interbank deposits affect total reserves and therefore the value of r, whether the banks involved are subject to uniform or differential reserve requirements. This is because "uniform" reserve requirements are uniform only in that they subject both banks to the same percentage required reserve ratio; they do not guarantee that all banks hold the same amount of base money per dollar of deposits as reserves. Therefore as long as members and nonmembers are subject to qualitatively different reserve requirements, not only will the level and distribution of interbank deposits affect r, their precise effect on r cannot be determined without knowledge of how nonmembers apportion their required reserves between vault cash and interbank balances. For example, assume deposit levels dictate that the system is represented by case 2-a and the distribution of interbank deposits changes such that it should move to case 3-a. The value of r would be expected to rise. But if non-members simultaneously choose to alter the composition of their required reserves, moving the system to case 3-b or 3-c, the value of r would fall, not rise. Therefore the predictable and determinable effects of a change in the level or distribution of interbank deposits can be completely reversed by a change in nonmember banks' reserve policy.

In summary, unless member and nonmember banks are subject to uniform reserve requirements and nonmembers hold all their required reserves as vault cash, the level and distribution of interbank balances somehow affect the value of r. Without knowledge of how a nonmember might alter its reserve composition in response to a change in the absolute or net level of its balances due to other banks, the effect of D_n^m and D_m^n on base-absorbing reserves and on r is indeterminate.

The effects of D_m^n on nonmember bank required reserves are comparable to the effects of D_n^m on member bank reserves. That is, if non-members are subject to different required reserve ratios and $(D_{n,h}^n - D_n^{n,h}) \neq 0$ for some h, net demand deposits are redistributed and the level of nonmember bank "reserves" is altered accordingly. However, the effect that D_n^n has on reserves that absorb money is again not determinable, without knowledge of the form in which nonmembers hold their reserves. If the level and distribution of D_n^n are such that their presence increases nonmember reserves (i.e., $(D_{n,h}^n - D_n^{n,h})>0$ for relatively large d_n), it could be inferred that base-abosrbing reserves would also rise (nonmembers' vault cash rise and D^n_m rise, increasing members' reserves), but this cannot be determined with certainty or accuracy. Furthermore, since D^n_n themselves act as their legal reserves, nonmembers could theoretically increase deposits infinitely and meet legal reserves with increased D^n_n alone, with no need to absorb additional base money. In that case, the value of r would approach zero and m would approach infinity.

(viii) The level of time deposits: Since the reserve requirements on time deposits are uniformly lower than those on demand deposits, a dollar of time deposits absorbs less base money than a dollar of demand deposits. Therefore, if the money supply is defined to include time deposits, M_2, a given level of source base can support a larger money stock the greater the proportion of bank deposits that are time deposits. On the other hand, if money is defined narrowly, M_1, the presence of time deposits in the system absorbs base money, while adding nothing to the money stock. Throughout this study, the money stock is defined to exclude time deposits, so the more time deposits there are in the system, the larger the value of r and the smaller m is.

(ix) The level and distribution of cash items in process of collection: Both member and nonmember banks are allowed to deduct cash items in process of collection from gross demand deposits before figuring their reserve liabilities. Therefore the amount of such cash items in the bank system and their distribution among different reserve classes affect the value of r. With a given level of privately-owned demand deposits, a change in the level of cash items in process of collection or changes in their distribution between groups of banks subject to different required reserve ratios will cause variation in the value of r.

(x) <u>The level of member bank nondeposit liabilities</u>: Beginning in 1969, the Federal Reserve made various kinds of nondeposit sources of funds subject to reserve requirements. Specifically, the liability items involved are: liabilities arising out of Eurodollar transactions, large certificates of deposit, funds obtained through issuance of debt by affiliate ("bank-related commercial paper"), and funds raised through sales of finance bills (banker's acceptances that are ineligible for Federal Reserve discount).

Liabilities arising out of Eurodollar transactions were originally subject to a 10 percent reserve requirement on October 16, 1969.[24] The original imposition of reserve requirements exempted certain base amounts of Eurodollar-related liabilities from reserve-requirements computation. On June 21, 1973, the definition of this base amount was changed to the following: loans aggregating $100,000 or less to U. S. residents and total loans of a bank to U. S. residents if they do not exceed $1 million.

Before June 21, 1973, certificates of deposits of $100,000 or more and funds obtained through issuance of bank-related commercial paper were subject to the reserve requirement on time deposits over $5 million. Beginning on that date, an 8 percent reserve requirement was imposed on increases in the two liability categories above the level of May 16,

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[24]Board of Governors of the Federal Reserve System, <u>Federal Reserve Bulletin</u> (August 1969):655-6.

1973 or $100 million, whichever is larger.[25] On July 12, 1972,[26] the marginal reserve requirement was extended to cover funds raised through sales of finance bills, which were previously subject to no reserve requirement. Beginning September 19, 1974,[27] large certificates of deposit, bank-related commercial paper, and funds from the sales of finance bills were divided into two subcategories for reserve purposes: those of maturity length less than four months and those maturing in four months or more. The short-term maturity group continued to be subject to the 8 percent marginal reserve requirement, but the long-term category for all three types of liabilities were reverted to the original (5 percent) reserve requirement on time deposits greater than $5 million. The maturity distinction was retained until December 12, 1974,[28] when all time deposits in excess of $5 million were divided into maturity-length subclasses. These structural changes, as well as changes in the required reserve ratios are detailed in Table C-2 in Appendix C.

These four types of nondeposit liabilities (liabilities arising out of Eurodollar transactions, large certificates of deposit, bank-related commercial paper, and sales of finance bills) all represent additional reserve-absorbing liabilities that are not included in the money stock and therefore their level will affect the value of r. The

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[25] ibid., (May 1973):375-7.

[26] ibid., (July 1973):549.

[27] ibid., (September 1974):680.

[28] ibid., (November 1974):799-800.

higher the level of nondeposit liabilities, the higher the value of r and therefore the lower the value of m. In addition, if the different categories of nondeposit liabilities are subject to different reserve ratios, the distribution of a given level of nondeposit liabilities among the various reserve categories will affect the value of r in the same way that differential reserve requirements for demand deposits affect the value of r. The Federal Reserve has for some time periods applied the same reserve ratios to different categories of nondeposit liabilities; this can be seen in Table C-2.

The Model

The parameter r can be expressed so that the impact of each of the factors discussed above can be seen explicitly. First the following sums are defined for period t:

Gross member bank demand deposits in the j^{th} reserve category:

$$D_{m,j,t} = D^{p}_{m,j,t} + D^{g}_{m,j,t} + D^{m}_{m,j,t} + D^{n}_{m,j,t},$$

Gross member bank demand deposits:

$$D_{m,t} = \sum_{j} D_{m,j,t}$$

$$= D^{p}_{m,t} + D^{g}_{m,t} + D^{m}_{m,t} + D^{n}_{m,t},$$

Net member bank demand deposits in the j^{th} reserve category:

$$DD_{m,j,t} = D_{m,j,t} - I^{m,j}_{t} - D^{m,j}_{m,t} - D^{m,j}_{n,t}$$

$$= D^{p}_{m,j,t} + D^{g}_{m,j,t} - I^{m,j}_{t} + (D^{m}_{m,j,t} - D^{m,j}_{m,t}) + (D^{n}_{m,j,t} - D^{m,j}_{n,t}),$$

Net member bank demand deposits:

$$DD_{m,t} = \sum_{j} DD_{m,j,t}$$

$$= D^{p}_{m,j} + D^{g}_{m,t} - I^{m}_{t} + (D^{n}_{m,t} - D^{m}_{n,t}),$$

Member bank time deposits in the i^{th} reserve category: $T_{m,i,t}$,

Member bank time deposits: $T_{m,t} = \sum_i T_{m,i,t}$;

where j = 1 - 5, i = 1 - 4, for the current Federal Reserve reserve requirement system;

Gross nonmember bank demand deposits in the h^{th} state: $D_{n,h,t} = D^p_{n,h,t} + D^g_{n,h,t} + D^n_{n,h,t} + D^m_{n,h,t}$,

Gross nonmember bank demand deposits:

$$D_{n,t} = \sum_h D_{n,h,t}$$

$$= D^p_{n,t} + D^g_{n,t} + D^n_{n,t} + D^m_{n,t},$$

Member bank non-deposit liabilities in the q^{th} reserve category: $ND_{m,q,t}$,

Nonmember bank deposit liabilities: $ND_{m,t} = \sum_q ND_{m,q,t}$,

Net nonmember bank demand deposits in the h^{th} state:

$$DD_{n,h,t} = D_{n,h,t} - I^{n,h}_t - D^{n,h}_{n,t} - D^{n,h}_{m,t}$$

$$= D^p_{n,h,t} + D^g_{n,h,t} - I^{n,h}_t + (D^n_{n,h,t} - D^{n,h}_{n,t})$$

$$+ (D^m_{n,h,t} - D^{n,h}_{m,t}),$$

Net nonmember bank demand deposits:

$$DD_{n,t} = \sum_h DD_{n,h,t}$$

$$D^p_{n,t} + D^g_{n,t} - I^n_t + (D^m_{n,t} - D^n_{m,t}),$$

Nonmember bank time deposits: $T_{n,t} = \sum_k T_{n,k,t}$,

where h = 1 - 100, k = 1 - 56, for the current state reserve requirement systems;

Total gross demand deposits:

$$D_t = D_{m,t} + D_{n,t}$$

$$= D_t^p + D_t^g + D_{m,t}^m + D_{m,t}^n + D_{n,t}^n + D_{n,t}^m$$

$$= D_t^p + D_t^g + D_t^{ib},$$

where $D_t^{ib} = D_{m,t}^m + D_{m,t}^n + D_{n,t}^n + D_{n,t}^m;$

Total net demand deposits:

$$DD_t = DD_{m,t} + DD_{n,t}$$

$$= D_t^p + D_t^g - I_t,$$

Total time deposits:

$$T_t = T_{m,t} + T_{n,t},$$

Total gross deposits:

$$TD_t = D_t + T_t,$$

Total net deposits:

$$NTD_t = DD_t + T_t.$$

Using the definition of r and equations (3-7) and (3-8), r_t can be expressed as,

$$r_t = \frac{\sum\limits_j d_j\, DD_{m,j,t-2} + \sum\limits_i t_i\, T_{m,i,t-2} + \sum\limits_q n_q ND_{m,q,t-2} - ER_t^m + VC_t^n}{D_t^p}$$

$$= \sum_j d_j \frac{DD_{m,j,t-2}}{D_t^p} + \sum_i t_i \frac{T_{m,i,t-2}}{D_t^p} + \sum_q n_q \frac{ND_{m,q,t-2}}{D_t^p}$$

$$+ \frac{ER_t^m}{D_t^p} + \frac{VC_t^n}{D_t^p}.$$

Successively multiplying the numerator and denominator of each term by the same quantities yields,

$$r_t = \sum_j d_j \frac{DD_{m,j,t-2}}{DD_{m,j,t}} \frac{DD_{m,j,t}}{DD_{m,t}} \frac{DD_{m,t}}{DD_t} \frac{DD_t}{D_t} \frac{(D^p_t + D^g_t + D^{ib}_t)}{D^p_t}$$

$$+ \sum_i t_i \frac{T_{m,i,t-2}}{T_{m,i,t}} \frac{T_{m,i,t}}{T_{m,t}} \frac{T_{m,t}}{T_t} \frac{T_t}{D^p_t} + \sum_q n_q \frac{ND_{m,q,t-2}}{ND_{m,q,t}} \frac{ND_{m,q,t}}{ND_{m,t}} \frac{ND_{m,t}}{D^p_t}$$

$$+ \frac{ER^m_t}{TD_{m,t}} \frac{TD_{m,t}}{D^p_t} + \frac{VC^{n,h}_t}{TD_{n,h,t}} \frac{TD_{n,h,t}}{TD_{n,t}} \frac{TD_{n,t}}{D^p_t}.$$

Substitution yields,

$$\text{(3-13)} \quad r_t = \sum_j d_j \lambda^D_{j,t} \delta^D_{j,t} \nu^D_t \xi_t (1 + \gamma_t + \iota_t) + \sum_i t_i \lambda^T_{i,t} \delta^T_{i,t} \nu^T_t \tau_t$$

$$+ \sum_q n_q \lambda^N_{q,t} \delta^N_{q,t} \alpha_t + \varepsilon_t \rho^m_t + \psi_{h,t} \omega_{h,t} \rho^n_t$$

where: $\lambda^D_{j,t} = \frac{DD_{m,j,t-2}}{D_{m,j,t}}$, $\lambda^T_{i,t} = \frac{T_{m,i,t-2}}{T_{m,i,t}}$, $\lambda^N_{q,t} \frac{ND_{m,q,t-2}}{ND_{m,q,t}}$

$\delta^D_{j,t} = \frac{DD_{m,j,t}}{DD_{m,t}}$, $\delta^T_{i,t} = \frac{T_{m,i,t}}{T_{m,t}}$, $\delta^N_{q,t} = \frac{ND_{m,q,t}}{ND_{m,t}}$

$\nu^D_t = \frac{DD_{m,t}}{DD_t}$, $\nu^T_t = \frac{T_{m,t}}{T_t}$

$\xi = \frac{DD_t}{D_t}$

$\gamma_t = \frac{D^g_t}{D^p_t}$

$\iota_t = \frac{D^{ib}_t}{D^p_t}$

$$\tau_t = \frac{T_t}{D_t^p}$$

$$\alpha_t = \frac{ND_{m,t}}{D_t^p}$$

$$\varepsilon_t = \frac{ER_t^m}{TD_{m,t}}$$

$$\rho_t^m = \frac{TD_{m,t}}{D_t^p}, \quad \rho_t^n \frac{TD_{n,t}}{D_t^p}$$

$$\Psi_{h,t} = \frac{VC_t^{n,h}}{TD_{n,h,t}}$$

$$\omega_{h,t} = \frac{TD_{n,h,t}}{TD_{n,t}}.$$

The contribution the the size, variability and predictability of r_t of each of the factors discussed earlier can be seen explicitly in equation (3-13). The terms in equation (3-13) account for the following factors:

ε_t : level of excess reserves;

γ_t : level of government demand deposits;

ι_t : level of interbank deposits;

α_t : level of nondeposit liabilities;

τ_t : level of time deposits;

$\delta_{j,t}^D$, $\delta_{i,t}^T$, and $\delta_{q,t}^N$: distribution of member bank deposits and nondeposit liabilities among reserve categories;

$\lambda^D_{j,t}$, $\lambda^T_{i,t}$, and $\lambda^N_{q,t}$: period-to-period changes in the level or distribution among reserve categories of member bank deposits and nondeposit liabilities;

ν^D_t, and ν^T_t : the proportion of bank deposits held in member banks;

$\Psi_{h,t}$: nonmember bank holdings of vault cash, relative to total deposits, in the h^{th} state;

$\omega_{h,t}$: the distribution of nonmember bank deposits among states.

There are two factors that affect the value and variability of r_t that are not specifically accounted for in equation (3-13), though they were discussed in the preceding text. They are: the distributional effect of interbank deposits and the effect of the level and distribution of cash items in process of collection. They are not explicitly included in equation (3-13) because the data needed to test and isolate their impact on r_t are not available.

CHAPTER 4

BEHAVIOR OF THE PARAMETERS OF r_t

Data, Definitions, and Assumptions

The structure of Federal Reserve reserve requirements was changed several times during the sample period. At the beginning of the sample period member banks were categorized as city and country banks, based on their geographical location; so there were initially two demand deposit reserve categories, and the same required reserve ration was applied to all time and savings deposits. This system of three categories of member bank deposits was in effect until July 14, 1966,[1] when three different reserve ratios were applied to savings deposits and to time deposits less than and greater than \$5 million. Disaggregated data for savings deposits and time deposits are however available only beginning September 7, 1966; they are available for time deposits less than and greater than \$5 million only beginning January 11, 1968.

Beginning January 11, 1968,[2] city and country bank demand deposits, were subdivided into two size categories, less than \$5 million and greater than \$5 million. This change resulted in four demand deposit reserve categories and three categories of savings and time deposits. On November 9, 1972,[3] a graduated reserve requirement system was adopted for demand deposits; this scheme defined five categories of demand

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[1]Board of Governors of the Federal Reserve System, Federal Reserve Bulletin (July 1966):979.

[2]ibid., (January 1968):95-6.

[3]ibid., (November 1972):994.

deposits based on bank size and eliminated the old city-country distinction. Finally, on December 12, 1974,[4] time deposits greater than $5 million were divided into two subcategories based on maturity length, those maturing in thirty to 179 days and those maturing in more than 179 days. In addition a number of kinds of nondeposit liabilities were subjected to reserve requirements during the sample period; these changes are described below in the section on nondeposit liabilities.

On September 18, 1968,[5] a number of the administrative rules governing the calculation of required reserves were changed. Beginning on that date, all member banks were placed on a weekly reserve settlement system under which required reserves are based each Wednesday on a weekly average of daily close-of-business deposit levels, two weeks earlier. Before that date, not all member banks were on a weekly settlement basis and required reserves were based on contemporaneous opening-of-business deposit levels. Required reserves against demand deposits are based on net demand deposits (DD_t above), defined to be total demand deposits less cash items in process of collection and demand deposits due from other domestic commercial banks.

At the same time that lagged reserve requirements were instituted a procedure known as reserve carryover was also introduced. Under reserve carryover, a reserve excess or deficiency, up to 2 percent of required reserves, may be carried ahead to the next reserve computation week. This causes excess reserves to be negative for some weeks after 1968.

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[4]ibid., (November 1974):799-800.

[5]ibid., (May 1968):437-8.

With a few exceptions noted below, the data used in this study are weekly averages of daily figures (Thursday through Wednesday) for the period January 1, 1961 through December 31, 1974. Until September, 1968, the data are "opening of business" figures, and "close of business" figures thereafter; this reflects the change in the definition of deposits used to calculate required reserves. For nonmember banks, the only actual data collected are call report data which occur every June 30 and December 31.[6] The Federal Reserve's weekly estimates of nonmember bank deposits are used whenever possible in this study.

The Federal Reserve makes weekly estimates of three portions of nonmember bank deposits: time deposits, government demand deposits, and "adjusted demand deposits." "Adjusted demand deposits" is the demand-deposit component of the money stock, designated as D_t^p above. Initially these estimates are based on the deposits of a sample of country member banks. When call report data become available, the weekly estimates are benchmarked to these actual deposit figures.[7] The changes in adjusted demand deposits resulting from the benchmark procedure have been substantial in recent years.[8] Since the information from call reports

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[6]For part of the sample period, data from call reports for March 31 and September 30 are also available.

[7]The benchmarking procedure currently followed for adjusted demand deposits consists of calculating from each call report, the ratio R, where R = nonmember bank adjusted demand deposits/country bank adjusted demand deposits. The difference between the actual value of R and its estimated value is then distributed over the 26-week period ending at the call report date.

[8]Darwin Beck and Joseph Sedransk, "Revision of the Money Stock Measures and Member Bank Reserves and Deposits," Federal Reserve Bulletin (February 1974):81-89.

is single-day data, and it is commonly conceded that call report data contain substantial aberrations, the benchmark process may introduce, rather than remove, errors into the nonmember deposit series. Not only is the actual weekly value of nonmember bank deposits never known, given the deficiencies of call report data, there are no reliable figures available to judge the accuracy of the estimates.[9] It is therefore very difficult to assess the direction and extent of errors in the estimation and benchmarking procedures.

The nonmember bank portion of D_t^P in the denominator of γ_t, ι_t, and τ_t is an estimate; the nonmember bank parts of time and savings deposits (T_t), included in the numerator of τ_t and the denominator of ν_t^T is an estimate, as is the nonmember bank part of the numerator of γ_t. In addition, not all of the components of nonmember bank deposits needed to calculate the parameters listed above are estimated.

The parameters γ_t^D, ξ_t, and ι_t require measures of commercial bank deposits for which the nonmember bank portion is not available, except for call report dates. The nonmember bank part of net demand deposits (DD_t), gross demand deposits (D_t) and interbank deposits (D_t^{ib}) are not known or estimated on a weekly basis. These nonmember bank data have been partially constructed by using the definitions given below.

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[9]There is daily deposit data available for a sample of insured commercial banks which was collected by the F.D.I.C. for the Advisory Committee on Monetary Statistics (The "Bach Committee"). The data cover the period from fall, 1974, through spring, 1975. Based on this experience, it appears that revisions in money stock figures could be reduced substantially if the Federal Reserve had better and more frequent information on nonmember bank deposits. The Bach Committee therefore recommended several improvements in the data-collection procedures for nonmember banks. See, Advisory Committee on Monetary Statistics, Improving the Monetary Aggregates (Washington, D.C.: Board of Governors of the Federal Reserve System, 1976).

(4-1) Net demand deposits = Gross demand deposits - demand deposits due from other domestic commercial banks - cash items in process of collection;

(4-2) Adjusted demand deposits = Gross demand deposits - demand deposits due to other domestic commercial banks - cash items in process of collection - government demand deposits - Federal Reserve float.

Federal Reserve float is ignored here because it is quantitatively small. Rearranging the terms in definition (4-2) gives the following expression for gross demand deposits:

(4-3) D_t = Adjusted demand deposits + demand deposits due to other domestic commercial banks + cash items in process of collection + government demand deposits.

Subtracting the definition of adjusted demand deposits from that of net demand deposits and rearranging terms gives the result that net demand deposits is:

(4-4) DD_t = Adjusted demand deposits - demand deposits due from other domestic commercial banks + demand deposits due to other domestic commercial banks + government demand deposits.

Since not all of the items involved in the definitions of net and gross demand deposits are available for nonmember banks, the nonmember bank portions of both net and gross demand deposits are approximated by nonmember bank adjusted demand deposits plus government demand deposits. As can be seen from expression (4-3), this measure of gross demand deposits is less than actual gross demand deposits by the amount of nonmember bank demand deposits due to other domestic commercial banks and nonmember bank cash items in process of collection. As expression (4-4) shows, the figures used here for net demand deposits are greater than actual net demand deposits by the net amount of nonmember bank demand deposits due to other domestic commercial banks less those due from other domestic commercial banks. In addition, since no weekly data are available on nonmember bank holdings of interbank deposits, total commercial bank interbank deposits are not known.

The deficiencies in the data affect two of the parameters that comprise r_t: ν_t^D and ι_t.[10] Both ν_t^D and ι_t were calculated in two ways. First, ν_t^D and ι_t were calculated using call report data for the missing nonmember bank interbank deposits figures. The weekly calculations of ν_t^D and ι_t then include the most recent quarterly or semiannual nonmember bank interbank deposit data. Second, ν_t^D and ι_t were recalculated leaving out the unavailable nonmember bank components of net and gross demand deposits and interbank deposits. The estimated denominator of ν_t^D is therefore larger and the estimated numerator of ι_t is smaller here than the actual values. These alternative calculations of ν_t^D and ι_t are denoted ν_t^{D*} and ι_t^*.[11]

In addition, the ratio of nonmember bank vault cash to total deposits for each state, $\psi_{h,t}$, and the proportion of total nonmember bank deposits in each state, $\omega_{h,t}$, must be calculated using call report data.

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[10] The value of ξ_t is also affected but since ξ_t has no particular economic meaning, the misspecification of the data has no real consequence. An alternative definition of ξ_t, ξ_t^*, may be defined that is analogous to ν_t^{D*} and ι_t^*.

[11] It is, of course, the variation in ν_t^D and ι_t rather than their levels that is most important. If the missing components of the data cause any problem it is not the problem of altering the size of ν_t^D and ι_t but of masking some of their variation. But since the size of the missing components are small relative to their totals, the effects of the deficiencies in the data are probably not very important. The shortcoming of using call report data to calculate ν_t^D and ι_t is that by adding the same call report figures into consecutive weekly observations, some of the actual variation in the parameters may be smoothed out. Again, since the components for which call report data are used are small relative to the totals, the effects are probably minor.

Call report data are the only source of nonmember bank data, distributed by state.

Lagged Reserve Requirements

The effect on r_t of lagged Federal Reserve reserve requirements is reflected by the parameters $\lambda^D_{j,t}$ and $\lambda^T_{i,t}$. A value of λ_t greater than one means that deposit levels in period (t-2) are larger than those in period t; member bank required reserves relative to current deposit levels are therefore higher than legal reserve ratios. Therefore, the value of r_t is larger than if the λ_t parameters were equal to one. If the value of λ_t is less than one, member bank required reserves relative to current deposit levels are lower than legal reserve ratios and the value of r_t is smaller. More importantly, variation in the value of the λ_t parameters above and below one introduces variation in r_t.

a) Demand Deposits

Lagged reserve requirements were introduced on September 18, 1968, but values of the λ-parameters have been calculated for the entire sample period. Table 1 gives statistics for the parameters, $\lambda^D_{j,t}$. The first part of Table 1 is based on the entire sample period and gives results for $\lambda^D_{1,t}$ and $\lambda^D_{2,t}$, referring to demand deposits in city banks and demand deposits in country banks, respectively. The second part of Table 1 is based on the subperiod 1968 - 1974. Part 2A refers to the second reserve requirement scheme for demand deposits and part 2B refers to the graduated reserve requirement scheme.

As can be seen in the first column of Table 1, the mean of all but one λ^D-parameter is less than one, indicating the overall growth in deposits in each reserve category during the sample period. Columns 2

Table 1. Lagged Federal Reserve Reserve Requirements, Demand Deposits (Value of the Parameters $\lambda^D_{j,t}$, $j = 1,11$)

	Mean	Average Deviation From 1.0[1]	Largest Deviation From 1.0[2]	Coefficient of Variation
1. Sample Period, 1961-1974 (730 observations)				
A. Demand Deposits				
City Banks ($\lambda^D_{1,t}$)	.99932	.00088	.09137	.02386
Country Banks ($\lambda^D_{2,t}$)	.99839	.00161	.04836	.01528
2. Sample Period, 1/11/68–12/25/74 (364 observations)				
A. Demand Deposits				
City Banks, less than $5 million ($\lambda^D_{3,t}$)	1.00015	.00015	.01475	.00251
City Banks, greater than $5 million ($\lambda^D_{4,t}$)	.99903	.00097	.09219	.02665
Country Banks, less than $5 million ($\lambda^D_{5,t}$)	.99936	.00064	.01696	.00544
Country Banks, greater than $5 million ($\lambda^D_{6,t}$)	.99812	.00188	.06651	.02494

Table 1. Continued

	Mean	Average Deviation from 1.0[1]	Largest Deviation from 1.0[2]	Coefficient of Variation
B. Demand Deposits				
Deposit Size, less than $2 million ($\lambda_{7,t}$)	.99996	.00004	.00455	.00177
Deposit Size, $2-$10 million ($\lambda^D_{8,t}$)	.99859	.00141	.03767	.01192
Deposit Size, $10-$100 million ($\lambda^D_{9,t}$)	.99877	.00123	.04867	.01745
Deposit Size, $100-$400 million ($\lambda^D_{10,t}$)	.99906	.00094	.06215	.01960
Deposit Size, greater than $400 million ($\lambda^D_{11,t}$)	.99885	.00115	.15404	.04540

[1] 1.0 - mean

[2] |1.0 - maximum value| or |1.0 - minimum value|, whichever is larger.

and 3 of Table 1 give measures of the deviation of the λ^D-parameters from their neutral value of one. Column 2 gives the absolute value of the difference between one and the mean; column 3 gives the largest deviation from one. The last column gives the coefficient of variation of each λ^D-parameter.

While none of these measures of variation in the λ^D-parameters is large on an absolute scale, they are also not zero, indicating that lagged reserve requirements do introduce variation in r_t that would otherwise not be present. As would be expected, the reserve categories corresponding to larger deposit levels show more variation: city banks under the first scheme; both categories of demand deposits greater than \$5 million under the second scheme; and the larger the deposit category, the larger the variation under the graduated scheme.

Table 2 shows statistics for the first differences of each λ^D-parameter. The first two columns of Table 2 show the mean of $(\lambda^D_{j,t} - \lambda^D_{j,t-1})$ and $|\lambda^D_{j,t} - \lambda^D_{j,t-1}|$ for all j. Column 3 is the standard deviation of the first differences of each $\lambda^D_{j,t}$ and the last column shows the largest weekly change in $\lambda^D_{j,t}$ for all j.

The mean of the first differences of $\lambda^D_{j,t}$ is small for all j. The mean of $|\lambda^D_{j,t} - \lambda^D_{j,t-1}|$ is much larger for all j, indicating that each $\lambda^D_{j,t}$ fluctuates considerably from week to week, producing successively positive and negative values of $(\lambda^D_{j,t} - \lambda^D_{j,t-1})$ which cancel each other out.

The standard deviations of $(\lambda^D_{j,t} - \lambda^D_{j,t-1})$ are also not large on an absolute scale; they are, however, slightly larger than the standard deviation of $\lambda^D_{j,t}$ for all j, indicating that the values of $\lambda^D_{j,t}$ deviate considerably from week to week. The largest weekly changes are also

Table 2. **Lagged Federal Reserve Reserve Requirements, Demand Deposits (First Differences of the Parameters, $\lambda^D_{j,t}$, j = 1,11)**

	Mean	Mean of Absolute Value	Standard Deviation	Largest Weekly Change[1]
1. Sample Period, 1961-1974 (729 observations)				
A. Demand Deposits				
City Banks ($\lambda^D_{1,t}$)	.00000	.01914	.02481	.09415
County Banks ($\lambda^D_{2,t}$)	-.00001	.01251	.01562	.05458
2. Sample Period, 1/11/68-12/25/74				
A. Demand Deposits				
City Banks, less than $5 million ($\lambda^D_{3,t}$)	.00000	.00149	.00287	.01801
City Banks, greater than $5 million ($\lambda^D_{4,t}$)	-.00006	.02229	.02864	.09507
Country Banks, less than $5 million ($\lambda^D_{5,t}$)	.00000	.00474	.00599	.01606
Country Banks, greater than $5 million ($\lambda^D_{6,t}$)	-.00003	.02112	.02560	.07414

Table 2. Continued

	Mean	Mean of Absolute Value	Standard Deviation	Largest Weekly Change[1]
B. Demand Deposits				
Deposit Size, less than \$2 million ($\lambda^D_{7,t}$)	.00000	.00141	.00177	.00565
Deposit Size, \$2-\$10 million ($\lambda^D_{8,t}$)	-.00001	.01084	.01356	.03666
Deposit Size, \$10-\$100 million ($\lambda^D_{9,t}$)	-.00002	.01580	.01884	.05200
Deposit Size, \$100-\$400 million ($\lambda^D_{10,t}$)	-.00004	.01378	.01773	.06266
Deposit Size, greater than \$400 million ($\lambda^D_{11,t}$)	-.00010	.03961	.05096	.17098

[1] Largest absolute value of ($\lambda^D_{j,t} - \lambda^D_{j,t-1}$)

not large, but they are much larger than the average weekly changes in $\lambda^D_{j,t}$; apparently most week-to-week changes in $\lambda^D_{j,t}$ are small, but there have occasionally been large weekly jumps in $\lambda^D_{j,t}$. The last two columns of Table 2 again show the tendency for the λ^D-parameters corresponding to larger deposit levels to exhibit more variation and larger weekly changes.

Table 3 gives annual figures for the λ^D-parameters which show that their variation has increased during the sample period. The standard deviations of $\lambda^D_{1,t}$ and $\lambda^D_{2,t}$ have increased quite steadily, especially since 1965. Annual figures for the other nine λ^D-parameters also indicate more variation in the latter years of the sample period; this trend is more pronounced in the reserve categories corresponding to larger deposit levels. The increased variation in $\lambda^D_{j,t}$ implies that the rate of growth (or decline) of deposits in the j^{th} reserve category has increased during the sample period.

b) Time Deposits

Table 4 presents figures $\lambda^T_{i,t}$, i = 1, 5. The first part of Table 4 has statistics for $\lambda^T_{1,t}$, referring to total time and savings deposits for the entire sample period. Part 2 is based on the subperiod beginning September 9, 1966 and contains statistics for $\lambda^T_{2,t}$ and $\lambda^T_{3,t}$, referring to time and savings deposits, respectively. This was the only part of the sample period for which separate data were available for time deposits and savings deposits. Further disaggregation of time deposits was available only after January, 1968, as indicated in Part 3 of Table 4. The division of time deposits by maturity length is only appropriate (and the data only available) after December 12, 1974.

Table 3. Annual[1] Figures for $\lambda^D_{j,t,j} = 1, 11$[2]

	$\lambda^D_{1,t}$ Mean	$\lambda^D_{1,t}$ Standard Deviation	$\lambda^D_{1,t}$ Largest Deviation from 1.0[3]	$\lambda^D_{2,t}$ Mean	$\lambda^D_{2,t}$ Standard Deviation	$\lambda^D_{2,t}$ Largest Deviation from 1.0[3]	$\lambda^D_{3,t}$ Mean	$\lambda^D_{3,t}$ Standard Deviation	$\lambda^D_{3,t}$ Largest Deviation from 1.0[3]
1961	.99906	.00880	.04526	.99838	.01630	.01935			
1962	1.00064	.01001	.05484	.99849	.01702	.01985			
1963	1.00013	.01178	.05394	.99816	.02035	.02572			
1964[4]	.99907	.01197	.04490	.99826	.01993	.02735			
1965	.99968	.01164	.07066	.99796	.02289	.02561			
1966	1.00066	.01303	.06105	.99993	.02445	.02275			
1967	.99778	.01475	.07107	.99763	.02358	.03966			
1968[5]	.99872	.01497	.06137	.99691	.02645	.03645	1.00020	.00480	.01475
1969[4]	.99877	.01693	.07052	.99926	.02628	.04251	1.00062	.00188	.00598
1970	.99870	.01769	.05660	.99893	.02617	.04610	1.00022	.00225	.00639
1971	.99872	.01874	.07835	.99733	.02561	.04538	1.00005	.00159	.00586
1972	.99792	.01656	.07612	.99664	.02416	.03547	.99976	.00167	.00548
1973	1.00018	.02053	.09137	.99884	.02744	.04746	1.00002	.00201	.00593
1974	1.00041	.02118	.09063	1.00068	.03123	.04836	1.00018	.00193	.00676

Table 3. Continued

	Mean	Standard Deviation	Largest Deviation from 1.0[3]	Mean	Standard Deviation	Largest Deviation from 1.0[3]	Mean	Standard Deviation	Largest Deviation from 1.0[3]
		$\lambda^D_{4,t}$			$\lambda^D_{5,t}$			$\lambda^D_{6,t}$	
1968[5]	.99854	.02454	.06206	.99904	.00424	.00124	.99699	.02163	.05476
1969[4]	.99875	.02658	.07135	.99981	.00485	.00931	.99904	.02409	.06101
1970	.99869	.02646	.05723	1.00015	.00613	.01280	.99838	.02479	.06576
1971	.99871	.02588	.07912	.99912	.00621	.01696	.99654	.02561	.06192
1972	.99791	.02440	.07690	.99880	.00582	.01363	.99574	.02233	.04662
1973	1.00018	.02770	.09219	.99861	.00156	.01300	.99902	.02766	.06505
1974	1.00041	.03153	.09141	.99999	.00551	.01057	1.00106	.02840	.06651
		$\lambda^D_{7,t}$			$\lambda^D_{8,t}$			$\lambda^D_{9,t}$	
1968[5]	.99991	.00170	.00416	.99800	.00853	.02035	.99853	.01484	.03363
1969[4]	1.00030	.00170	.00348	.99926	.01062	.02066	.99955	.01774	.04555
1970	1.00056	.00201	.00455	.99954	.01282	.02623	.99911	.01669	.04380
1971	.99988	.00190	.00361	.99803	.01328	.03767	.99771	.01724	.04284
1972	.99967	.00173	.00403	.99753	.01234	.03135	.99660	.01578	.03414
1973	.99942	.00162	.00389	.99767	.01226	.02946	.99937	.01925	.04221
1974	.99993	.00153	.00415	1.00010	.01315	.02505	1.00049	.02048	.04867

Table 3. Continued

	Mean	Standard Deviation	Largest Deviation from 1.0[3]	Mean	Standard Deviation	Largest Deviation from 1.0[3]	Mean	Standard Deviation	Largest Deviation from 1.0[3]
		$\lambda^D_{10,t}$			$\lambda^D_{11,t}$				
1968[5]	.99811	.01792	.04869	.99780	.04572	.10023			
1969[4]	.99955	.01828	.04171	.99802	.04760	.13068			
1970	.99801	.01886	.04390	.99874	.04564	.09073			
1971	.99803	.01907	.05029	.99862	.04336	.12758			
1972	.99751	.01877	.04717	.99775	.04045	.12471			
1973	1.00091	.02181	.06215	1.00019	.04510	.15404			
1974	1.00126	.02263	.06133	1.00085	.05134	.15056			

[1]Based on 52 observations per year unless otherwise indicated.

[2]The subscript j refers to the following demand deposit categories: 1 = city banks; 2 = country banks; 3 = demand deposits less than $5 million in city banks; 4 = demand deposits greater than $5 million in city banks; 5 = demand deposits less than $5 million in country banks; 6 = demand deposits greater than $5 million in country banks; 7 = demand deposits less than $2 million; 8 = demand deposits greater than $2 million and less than $10 million; 9 = demand deposits greater than $10 million and less than $100 million; 10 = demand deposits greater than $100 million and less than $400 million; 11 = demand deposits greater than $400 million.

[3]|1.0 - maximum value| or |1.0 - minimum value|, whichever is larger.

[4]Based on 53 observations.

[5]Based on 51 observations.

Table 4. Lagged Federal Reserve Reserve Requirements, Time Deposits (Values of the Parameters, $\lambda^D_{i,t}$, i = 1,5)

	Mean	Average Deviation from 1.0[1]	Largest Deviation from 1.0[2]	Coefficient of Variation
1. Sample Period, 1961-1974 (730 observations)				
Total Time and Savings Deposits ($\lambda^T_{1,t}$)	.99536	.00464	.02077	.00483
2. Sample Period, 9/7/66-12/25/74 (434 observations)				
Time Deposits ($\lambda^T_{2,t}$)	.99387	.00613	.04827	.00982
Savings Deposits ($\lambda^T_{3,t}$)	.99867	.00133	.01854	.00448
3. Sample Period, 1/11/68-12/25/74 (364 observations)				
Time Deposits, less than $5 million ($\lambda^T_{4,t}$)	.99742	.00258	.01170	.00186
Maturities of 30-179 days[3]	.99959	.00041	.00170	.00182
Maturities of more than 179 days[3]	.99906	.00094	.00141	.00067
Time Deposits, greater than $5 million ($\lambda^T_{5,t}$)	.99333	.00667	.04219	.01069
Maturities of 30-179 days[3]	.97179	.02821	.02873	.00075
Maturities of more than 179 days[3]	.98874	.01126	.01184	.00083

[1] 1.0 - mean [2] |1.0 - maximum value| or |1.0 - minimum value|, whichever is greater.

[3] Includes two observations for the period, 12/12/74-12/25/74.

The results for $\lambda^T_{i,t}$ are similar to those for $\lambda^D_{j,t}$. The means of $\lambda^T_{i,t}$ are less than one, indicating the overall growth in all categories of time deposits during the sample period. Neither the coefficient of variation nor the measures of deviation from one are large on an absolute scale; on the other hand, they are not zero either. Like those for $\lambda^D_{j,t}$ the coefficients of variation for $\lambda^T_{i,t}$ are in general larger for larger deposit-size categories. That is, the variation in time deposits ($\lambda^T_{2,t}$) is much larger than that in savings deposits ($\lambda^T_{3,t}$) and the λ^T-parameter for time deposits greater than \$5 million is much larger than that for time deposits less than \$5 million. The exception to this pattern is the coefficient of variation for total time and savings deposits, $\lambda^T_{1,t}$, which is relatively low.

In comparison to $\lambda^D_{j,t}$, the variation in $\lambda^T_{1,t}$ is small; the largest coefficient of variation ($\lambda^T_{5,t}$) is less than all but three of those for $\lambda^D_{j,t}$, and those three correspond to the smallest demand deposit reserve categories. The largest weekly deviations from one for $\lambda^T_{i,t}$ are also in general smaller than those for $\lambda^D_{j,t}$ except for the smallest categories of demand deposits. The average weekly deviations from one are larger for $\lambda^T_{i,t}$ than for most $\lambda^D_{j,t}$; this is because the means of $\lambda^T_{i,t}$ are in general smaller, reflecting the more rapid rate of growth in time deposits.

Table 5 presents statistics on the first differences of the λ^T-parameters. Like the λ^D-parameters, the mean of the first differences of $\lambda^T_{i,t}$ is small for all i. Again the means of the absolute value of the first differences are much larger than the means of the first differences, reflecting sizable weekly fluctuation in the λ^T-parameters.

Table 5. Lagged Federal Reserve Reserve Requirements, Time Deposits (First Differences of the Parameters, $\lambda^T_{i,t}$, i = 1,5)

	Mean	Mean of Absolute Value	Standard Deviation	Largest Weekly Change[1]
1. Sample Period, 1961-1974 (729 observations)				
Total Time and Savings Deposits ($\lambda^T_{1,t}$)	-.00002	.00206	.00276	.01132
2. Sample Period, 9/7/66-12/25/74 (433 observations)				
Time Deposits ($\lambda^T_{2,t}$)	-.00005	.00380	.00576	.04827
Savings Deposits ($\lambda^T_{3,t}$)	.00000	.00222	.00320	-.01472
3. Sample Period, 1/11/68-12/25/74 (363 observations)				
Time Deposits, less than $5 million ($\lambda^T_{4,t}$)	.00000	.00070	.00110	.00170
Time Deposits, greater than $5 million ($\lambda^T_{5,t}$)	-.00006	.00408	.00543	.02431

[1]Largest absolute value of $(\lambda^T_{i,t} - \lambda^T_{i,t-1})$.

The standard deviations and largest weekly values for $(\lambda^T_{i,t} - \lambda^T_{i,t-1})$ are all small; the $\lambda^T_{i,t}$ corresponding to larger deposit categories again show greater variation.

The results for the first differences of the λ^T-parameters again imply that they vary less and may therefore be more predictable than the λ^D-parameters. The means of the absolute value of the first differences and the largest weekly changes are all larger for the λ^D-parameters than for the λ^T-parameters, except for the smallest demand-deposit categories. The standard deviations of the first differences also show that $(\lambda^T_{i,t} - \lambda^T_{i,t-1})$ in general vary less than do $(\lambda^D_{j,t} - \lambda^D_{j,t-1})$.

Annual figures for $\lambda^T_{i,t}$ show no important secular trends that can be identified by simple observation. The only discernible pattern is in the λ^T-parameter for time deposits less than \$5 million; the standard deviation of $\lambda^T_{4,t}$ declines during the sample period. Since a decrease in the variation of a λ-parameter indicates a fall in the rate of deposit growth (or decline), the drop in the variation of $\lambda^T_{4,t}$ occurs because most of the possible growth of deposits in that category is achieved by 1968. After 1968, the increasing stability of $\lambda^T_{4,t}$ reflects the overall slow-down in the growth of that deposit category. Variability in the other λ^T-parameters also show changes over time but those patterns are probably traceable to changes in Regulation Q interest rate ceilings and market interest rates; these influences are difficult to isolate by observation and will therefore be dealt with in Chapter 6. The λ^T-parameters corresponding to the larger deposit-level categories tend to vary more at the end of the sample period, but this may well be due to the effects of interest rates. Annual figures for $\lambda^T_{i,t}$ are reported in Table D-1 of Appendix D.

Differential Reserve Requirements

The effect on r_t of differential Federal Reserve reserve requirements is summarized in the parameters $\delta^D_{j,t}$ and $\delta^T_{i,t}$. The parameters $\delta^D_{j,t}$ and $\delta^T_{i,t}$ represent the proportion of member bank demand and time deposits in the j^{th} and i^{th} reserve category, respectively. If the distribution of member bank deposits shifts in favor of banks or deposit categories subject to relatively high (low) legal reserve ratios, the value of r_t will rise (fall). More importantly, variation in the distribution of deposits, represented by variation in the δ-parameters, will induce variation in r_t.

a) Demand Deposits

Table 6 gives the results for $\delta^D_{j,t}$. Neither the standard deviation nor the coefficient of variation of $\delta^D_{j,t}$ for any j are large on an absolute scale. The coefficient of variation of $\delta^D_{j,t}$ is in general larger than that for $\lambda^D_{j,t}$ for all j, except for $\delta^D_{11,t}$. There is a tendency for the standard deviation of $\delta^D_{j,t}$ to increase as the size of the j^{th} reserve category increases, although the relationship is not as consistent as it was for the λ^D-parameters.

Table 7 gives statistics for the first differences of $\delta^D_{j,t}$. The mean of the first difference of $\delta^D_{j,t}$ is not large for any j, but in general is larger than that for the λ^D-parameters. Like the λ^D-parameters, the mean of the absolute value of the first difference is larger than the mean of the first difference for all $\delta^D_{j,t}$; this indicates that the $\delta^D_{j,t}$ fluctuate considerably from week to week. Neither

Table 6. Differential Federal Reserve Reserve Requirements, Demand Deposits (Values of the Parameters $\delta^D_{j,t}$, j = 1,11)

	Mean	Standard Deviation	Coefficient of Variation
1. Sample Period, 1961-1974 (730 observations)			
A. Demand Deposits			
City Banks ($\delta^D_{1,t}$)	.59001	.02071	.03510
Country Banks ($\delta^D_{2,t}$)	.40999	.02071	.05051
2. Sample Period, 1/11/68-12/25/74 (364 observations)			
A. Demand Deposits			
City Banks, less than $5 million ($\delta^D_{3,t}$)	.00617	.00061	.09887
City Banks, more than $5 million ($\delta^D_{4,t}$)	.56823	.01046	.01841
Country Banks, less than $5 million ($\delta^D_{5,t}$)	.14256	.00792	.05556
Country Banks, more than $5 million ($\delta^D_{6,t}$)	.28303	.01671	.05904
B. Demand Deposits			
Deposit Size, less than $2 million ($\delta^D_{7,t}$)	.07452	.00681	.09138
Deposit Size, $2 to $10 million ($\delta^D_{8,t}$)	.14381	.00357	.02482
Deposit Size, $10 to $100 million ($\delta^D_{9,t}$)	.27583	.00426	.01544
Deposit Size, $100 to $400 million ($\delta^D_{10,t}$)	.21687	.00386	.01780
Deposit Size, more than $400 million ($\delta^D_{11,t}$)	.28898	.01028	.03557

Table 7. Differential Federal Reserve Reserve Requirements, Demand Deposits (First Differences of the Parameters $\delta^D_{j,t}$, j = 1,11)

	Mean	Mean of Absolute Value	Standard Deviation	Largest Weekly Change[1]
1. Sample Period, 1961-1974 (729 observations)				
A. Demand Deposits				
City Banks ($\delta^D_{1,t}$)	-.00010	.00359	.00466	.01606
Country Banks ($\delta^D_{2,t}$)	.00010	.00359	.00466	-.01606
2. Sample Period, 1/11/68-12/25/74 (363 observations)				
A. Demand Deposits				
City Banks, less than \$5 million ($\delta^D_{3,t}$)	.00000	.00007	.00008	-.00024
City Banks, more than \$5 million ($\delta^D_{4,t}$)	-.0005	.00446	.00554	.01628
Country Banks, less than \$5 million ($\delta^D_{5,t}$)	-.00005	.00157	.00195	.00588
Country Banks, more than \$5 million ($\delta^D_{6,t}$)	.00011	.00344	.00415	-.01085

Table 7. Continued

	Mean	Mean of Absolute Value	Standard Deviation	Largest Weekly Change[1]
B. Demand Deposits				
Deposit Size, less than \$2 million ($\delta^D_{7,t}$)	-.00005	.00083	.00101	.00297
Deposit Size, \$2-\$10 million ($\delta^D_{8,t}$)	.00001	.00165	.00212	.00705
Deposit Size, \$10-\$100 million ($\delta^D_{9,t}$)	.00000	.00315	.00393	-.01203
Deposit Size, \$100-\$400 million ($\delta^D_{10,t}$)	-.00004	.00107	.00138	-.00441
Deposit Size, more than \$400 million ($\delta^D_{11,t}$)	.00008	.00593	.00747	.02365

[1] Largest absolute value of $(\delta^D_{j,t} - \delta^D_{j,t-1})$.

the standard deviation nor the largest weekly change in $\delta^D_{j,t}$ indicate any weekly fluctuation that is large on an absolute scale. By all three of these measures of variation, the variation in $(\delta^D_{j,t} - \delta^D_{j,t-1})$ is much smaller than that in $(\lambda^D_{j,t} - \lambda^D_{j,t-1})$ for all j. In addition, unlike the λ^D-parameters, the standard deviation of $(\delta^D_{j,t} - \delta^D_{j,t-1})$ is much smaller for all j than the standard deviation of $\delta_{j,t}$; the behavior of the δ^D-parameters may therefore be relatively easy to predict. The first differences of $\delta^D_{j,t}$ show the same tendency for variation to rise with the size of the deposit category.

Annual figures for $\delta^D_{j,t}$ are presented in Table 8. The standard deviations of $\delta^D_{1,t}$ and $\delta^D_{2,t}$ have risen slightly during the same period, although this has not been a consistent or strong tendency. The proportion of member bank demand deposits in city banks has fallen; this is reflected in the decline in the mean of $\delta^D_{1,t}$, $\delta^D_{3,t}$, and $\delta^D_{4,t}$. This is no doubt the result of the growth in the size and number of country banks. As would be expected, the behavior of the δ^D-parameters in the graduated reserve scheme reflects the overall growth of deposits during the same sample period. The mean of the δ^D-parameter for the smallest deposit-size category falls during the sample period and that for the largest deposit-size category rises.

b) Time Deposits

Table 9 gives figures for the parameters $\delta^T_{i,t}$, i = 2, 5. Of the δ^T-parameters for the four major time deposit categories, that for time deposits greater than \$5 million shows the largest standard deviation; that for savings deposits has the largest coefficient of variation, although all four coefficients of variation are close in size. The standard deviation and coefficient of variation for all four

Table 8. Annual[1] Figures for $\delta^D_{j,t}$, j = 1,11[2]

	$\delta^D_{1,t}$ Mean	$\delta^D_{1,t}$ Standard Deviation	$\delta^D_{1,t}$ Coefficient of Variation	$\delta^D_{2,t}$ Mean	$\delta^D_{2,t}$ Standard Deviation	$\delta^D_{2,t}$ Coefficient of Variation	$\delta^D_{3,t}$ Mean	$\delta^D_{3,t}$ Standard Deviation	$\delta^D_{3,t}$ Coefficient of Variation
1961	.63031	.00280	.00444	.36969	.00280	.00757			
1962	.62219	.00539	.00866	.37781	.00539	.01426			
1963	.61113	.00529	.00866	.38887	.00529	.01360			
1964[3]	.60335	.00338	.00560	.39665	.00338	.00852			
1965	.59601	.00562	.00943	.40399	.00562	.01391			
1966	.58726	.00446	.00759	.41274	.00446	.01081			
1967	.58863	.00455	.00773	.41137	.00455	.01106			
1968	.58383	.00592	.01014	.41617	.00592	.01422	.00710[4]	.00018[4]	.02535[4]
1969[3]	.58021	.00498	.00857	.41979	.00497	.00184	.00676	.00018	.02663
1970	.58378	.00446	.00764	.41622	.00446	.01072	.00650	.00016	.02462
1971	.57958	.00729	.01258	.42042	.00729	.01734	.00614	.00014	.02280
1972	.57163	.00605	.01058	.42837	.00605	.01412	.00579	.00016	.02763
1973	.56106	.00661	.01178	.43894	.00661	.01506	.00551	.00013	.02359
1974	.56109	.00536	.00955	.43891	.00536	.01221	.00542	.00013	.02399

Table 8. Continued

	Mean	Standard Deviation	Coefficient of Variation	Mean	Standard Deviation	Coefficient of Variation	Mean	Standard Deviation	Coefficient of Variation
		$\delta^D_{4,t}$			$\delta^D_{5,t}$			$\delta^D_{6,t}$	
1968[4]	.57647	.00561	.00973	.15461	.00219	.01416	.26182	.00583	.02227
1969[3]	.57345	.00505	.00881	.15083	.00266	.01764	.26896	.00380	.01413
1970	.57728	.00451	.00781	.14630	.00279	.01907	.26992	.00391	.01452
1971	.57344	.00726	.01266	.14013	.00224	.01599	.28029	.00646	.02305
1972	.56584	.00603	.01066	.13615	.00269	.01976	.29223	.00576	.01971
1973	.55555	.00667	.01201	.13480	.00336	.02493	.30414	.00441	.01450
1974	.55567	.00546	.00983	.13521	.00299	.02211	.30371	.00375	.01235
		$\delta^D_{7,t}$			$\delta^D_{8,t}$			$\delta^D_{9,t}$	
1968[4]	.08500	.00177	.02082	.14511	.00830	.05720	.27684	.00320	.01156
1969[3]	.08134	.00174	.02139	.14534	.00230	.01583	.27593	.00393	.01424
1970	.07806	.00185	.02370	.14251	.00222	.01558	.27235	.00323	.01186
1971	.07356	.00141	.01917	.14044	.00302	.02150	.27313	.00341	.01248
1972	.06969	.00173	.02482	.14140	.00287	.02030	.27537	.00367	.01333
1973	.06721	.00158	.02351	.14490	.00399	.02754	.27941	.00399	.01428
1974	.06682	.00160	.02394	.14698	.00282	.01919	.27778	.00370	.01332

Table 8. Continued

	Mean	Standard Deviation	Coefficient of Variation	Mean	Standard Deviation	Coefficient of Variation	Mean	Standard Deviation	Coefficient of Variation
		$\delta^D_{10,t}$			$\delta^D_{11,t}$				
1968[4]	.21799	.00148	.00679	.27506	.00627	.00280			
1969[3]	.21627	.00198	.00916	.28112	.00774	.02753			
1970	.21721	.00141	.00649	.28987	.00645	.02225			
1971	.22060	.00108	.00490	.29227	.00697	.02385			
1972	.22053	.00154	.00698	.29302	.00657	.02242			
1973	.21555	.00318	.01475	.29293	.00762	.02601			
1974	.20996	.00212	.01010	.29846	.00821	.02751			

[1] Based on 52 observations per year unless otherwise indicated.

[2] The subscript j refers to the following demand deposit categories: 1 = city banks; 2 = country banks; 3 = demand deposits less than \$5 million at city banks; 4 = demand deposits greater than \$5 million at city banks; 5 = demand deposits less than \$5 million at country banks; 6 = demand deposits greater than \$5 million at country banks; 7 = less than \$2 million; 8 = more than \$2 million and less than \$10 million; 9 = more than \$10 million and less than \$100 million; 10 = more than \$100 million and less than \$400 million; 11 = more than \$400 million.

[3] Based on 53 observations.

[4] Based on 51 observations.

Table 9. Differential Federal Reserve Reserve Requirements, Time Deposits (Values of the Parameters $\delta^T_{i,t}$, i = 2, 5)

	Mean	Standard Deviation	Coefficient of Variation
1. Sample Period, 9/7/66-12/25/74 (434 observations)			
Time Deposits ($\delta^T_{2,t}$)	.56395	.07147	.12702
Savings Deposits ($\delta^T_{3,t}$)	.43605	.07147	.16390
2. Sample Period, 1/11/68-12/25/74 (363 observations)			
Time Deposits, less than \$5 million ($\delta^T_{4,t}$)	.09704	.01348	.13891
Maturities of 30-179 days[1]	.30062	.00155	.00516
Maturities of more than 179 days[1]	.07248	.00032	.00442
Time Deposits, greater than \$5 million ($\delta^T_{5,t}$)	.48574	.07390	.15214
Maturities of 30-179 days[1]	.40082	.00181	.00452
Maturities of more than 179 days[1]	.19345	.00020	.00103

[1]Includes 2 observations for the period, 12/12/74-12/25/74.

δ^T-parameters are much larger than for any δ^D-parameter or λ^T-parameter. The smallest coefficient of variation for the δ^T-parameters is .12702 (for $\delta^T_{2,t}$) which is more than three times the largest coefficient of variation for the δ^D-parameters (.03557 for $\delta^D_{11,t}$); the largest coefficient of variation for the δ^T-parameters (.16390 for $\delta^T_{3,t}$) is more than four times the size of the largest δ^D-parameter coefficient of variation. The coefficient of variation for $\delta^T_{i,t}$, i = 2, 5, is at least 12 times the size of that for each corresponding λ^T-parameter; the coefficient of variation for $\delta^T_{4,t}$ is seventy-four times the size of that for $\lambda^T_{4,t}$.

Table 10 gives the results for the first differences of $\delta^T_{i,t}$, i = 2, 5. The mean of the first differences of $\delta^T_{i,t}$ is large for all i, relative to that for the λ^T-parameters or the δ^D-parameters. The mean of the absolute value for the first differences of $\delta^T_{i,t}$ is larger than the mean of the first differences for all i; this is the same result reported for the parameters discussed above, but the difference between the two means is smaller for $\delta^T_{i,t}$. Therefore, while the weekly fluctuations present in the behavior of $\lambda^T_{i,t}$ and $\delta^D_{j,t}$ are also present in $\delta^T_{i,t}$, they are apparently much smaller. Even though the mean of $(\delta^T_{i,t} - \delta^T_{i,t-1})$ is in general larger than that for $\lambda^T_{i,t}$ or $\delta^D_{j,t}$, the mean of $|\delta^T_{i,t} - \delta^T_{i,t-1}|$ is smaller than it is for $\lambda^T_{i,t}$ or $\delta^D_{j,t}$.

The standard deviation of $(\delta^T_{i,t} - \delta^T_{i,t-1})$ is much smaller than the standard deviation of $\delta^T_{i,t}$ for all i. The standard deviation and the largest weekly value of $(\delta^T_{i,t} - \delta^T_{i,t-1})$ are small for all i and are smaller than the same measures of variation in $\lambda^T_{i,t}$ and $\delta^D_{j,t}$. Comparing $\delta^T_{i,t}$ with $\lambda^T_{i,t}$ or $\delta^D_{j,t}$ therefore yields the following

Table 10. Differential Federal Reserve Reserve Requirements, Time Deposits (First Differences of the Parameters $\delta^T_{i,t}$, i = 2, 5)

	Mean	Mean of Absolute Value	Standard Deviation	Largest Weekly Value[1]
1. <u>Sample Period, 9/7/66-12/25/74</u> (433 observations)				
Time Deposits ($\delta^T_{2,t}$)	.00060	.00116	.00146	-.00899
Savings Deposits ($\delta^T_{3,t}$)	.00060	.00116	.00146	.00899
2. <u>Sample Period, 1/11/68-12/25/74</u> (363 observations)				
Time Deposits, less than $5 million ($\delta^T_{4,t}$)	-.00007	.00026	.00032	.00124
Time Deposits, greater than $5 million ($\delta^T_{5,t}$)	.00064	.00129	.00152	.00381

[1]Largest absolute value of ($\delta^T_{i,t} - \delta^T_{i,t-1}$).

conclusions: while the levels of $\delta^T_{i,t}$ vary more and the mean change of $\delta^T_{i,t}$ is in general larger, the standard deviation, mean of the absolute value of the first differences, and the largest weekly change in $\delta^T_{i,t}$ are smaller than the corresponding measures of changes in $\lambda^T_{i,t}$ and $\delta^D_{j,t}$. Thus it would appear that predicting the value of $\delta^T_{i,t}$ is an easier task.

Table 11 gives annual figures for $\delta^T_{i,t}$. The overall growth in time deposits that occurred during the sample period is reflected in the behavior of the mean of $\delta^T_{i,t}$ for all i. As would be expected, the means of $\delta^T_{3,t}$ and $\delta^T_{5,t}$ both decline and that for $\delta^T_{4,t}$ rises. Furthermore the decline in $\delta^T_{3,t}$ and $\delta^T_{4,t}$ and the increase in $\delta^T_{5,t}$ is well-explained by changes in the level of time deposits (τ_t). That is, years of large (small) decreases in $\delta^T_{3,t}$ and $\delta^T_{5,t}$ and increases in $\delta^T_{4,t}$ are also years of large (small) increases in τ_t. The year for which the δ^T-parameters have consistently high coefficients of variation is 1970; 1970 is also a year of unusually high variation in τ_t. Therefore it is inferred that the behavior of $\delta^T_{i,t}$ reflects influences such as market interest rates and Regulation Q ceilings which affect the overall rate of growth of time deposits. The impact of these variables will be dealt with in Chapter 6. In addition, quarterly figures indicate that seasonal factors strongly affect the behavior of $\delta^T_{i,t}$; these also will be considered in detail in Chapter 6.

Nonmember Banks

a) The Level of Nonmember Bank Deposits

The parameters ν^D_t and ν^T_t represent the proportion of the nation's demand and time deposits, respectively, that are in member banks and are therefore under the direct control of the Federal Reserve. Changes in

Table 11. Annual[1] Figures for $\delta^T_{i,t}$, i = 3, 5[2]

	Mean	Standard Deviation	Coeffecient of Variation	Mean	Standard Deviation	Coefficient of Variation	Mean	Standard Deviation	Coefficient of Variation
		$\delta^T_{3,t}$			$\delta^T_{4,t}$			$\delta^T_{5,t}$	
1966[3]	.56620	.00288	.00509	6	6	6	6	6	6
1967	.52382	.01263	.02411	6	6	6	6	6	6
1968[4]	.49020	.01288	.02627	.10018	.00110	.01098	.40961	.01275	.03113
1969[5]	.47896	.00851	.01777	.11097	.00589	.05308	.41007	.01430	.03487
1970	.45854	.02245	.04896	.11475	.00505	.04401	.42671	.02747	.06438
1971	.41965	.00601	.01432	.10081	.00247	.02450	.47954	.00768	.01602
1972	.40381	.00864	.02140	.09297	.00240	.02581	.50322	.01088	.02162
1973	.35377	.01717	.04853	.08315	.00286	.03440	.56308	.01993	.03539
1974	.31585	.01115	.03530	.07620	.00289	.03793	.60795	.01401	.02304

[1]Based on 52 observations per year unless otherwise indicated.

[2]The subscript i refers to the following time deposit categories: 3 = savings deposits; 4 = time deposits, less than \$5 million; 5 = time deposits, greater than \$5 million.

[3]Based on 17 observations.

[4]Based on 51 observations.

[5]Based on 53 observations.

[6]Data not available.

the distribution of deposits between member and nonmember banks will cause ν_t^D and ν_t^T to vary which will induce variation in r_t. Table 12 gives the results for the levels and first differences of ν_t^D and ν_t^T as well as the alternative specification of ν_t^D, ν_t^{D*}, described above.

Consider first the behavior of ν_t^D. As can be seen from Table 12, the mean of ν_t^D for the entire sample period is .82484 and its standard deviation is .02653. The standard deviation and coefficient of variation for ν_t^D are not large on an absolute scale. As would be expected, the alternative definition of ν_t^{D*}, which leaves out nonmember bank interbank deposits (call report data), shows slightly more variation than ν_t^D but the difference is small. Presumably it makes little difference which definition of ν_t^D is used.

The mean of the week-to-week change in ν^D is -.00014 and the standard deviation is .00202. Therefore the average disturbance caused by ν_t^D from one week to the next is small, as is the variability of the disturbance. The mean of the absolute value of the first differences of ν_t^D is much larger than the mean of the first differences, implying that ν_t^D fluctuates considerably from week to week, although the size of those fluctuations is apparently not large. The standard deviation of the changes in ν_t^D is much less (.00202) than the standard deviation of the levels of ν_t^D (.02653); therefore, the variation in the first difference of ν_t^D is small. The largest weekly change in ν_t^D occurring during the sample is also small.

The mean of ν_t^D has, of course, fallen during the sample period, as can be seen in Table 13, which gives annual figures of ν_t^D. The mean of ν_t^D has decreased steadily each year from a high of .86252 in 1961 to

Table 12. Level of Nonmember Banks Deposits (1961-1974)

	Mean	Mean of Absolute Value	Standard Deviation	Coefficient of Variation	Largest Weekly Change[1]
1. Values of the Parameters ν_t^D and ν_t^T (730 observations)					
Demand Deposits (ν_t^D)	.82484		.02653	.03216	
ν_t^{D*}	.79641		.02840	.03566	
Time Deposits (ν_t^T)	.79442		.02416	.03041	
2. First Differences of the Parameters ν_t^D and ν_t^T (729 observations)					
Demand Deposits (ν_t^D)	-.00014	.00162	.00202		.00927
ν_t^{D*}	-.00014	.00151	.00188		-.00577
Time Deposits (ν_t^T)	-.00006	.00029	.00040		.00315

[1]Largest value of $(\nu_t^D - \nu_{t-1}^D)$ or $(\nu_t^T - \nu_{t-1}^T)$.

Table 13. Annual[1] Figures for ν_t^D

	Mean	Standard Deviation	Percentage Change from Previous Year: Mean	Percentage Change from Previous Year: Standard Deviation
1961	.86252	.00302	--	--
1962	.85682	.00310	-.661%	2.648%
1963	.85072	.00291	-.712%	- 6.129%
1964[2]	.84567	.00316	-.594%	8.591%
1965	.84140	.00552	-.505%	74.684%
1966	.83509	.00442	-.750%	-19.928%
1967	.83360	.00349	-.178%	-21.041%
1968	.82788	.00610	-.686%	74.785%
1969[2]	.81805	.00519	-1.187%	-14.918%
1970	.81389	.00384	-.509%	-26.012%
1971	.80746	.00684	-.790%	78.125%
1972	.79520	.00659	-1.518%	-3.655%
1973	.78444	.00724	-1.353%	9.863%
1974	.77469	.00618	-1.243%	-14.641%

[1]Based on 52 observations per year unless otherwise indicated.

[2]Based on 53 observations.

a low of .77469 in 1974. Over the same period the standard deviation of ν_t^D has more than doubled from its 1961 value of .00302 to its 1974 value of .00618. Therefore, the variability of ν_t^D, though small, is rising.

Neither the overall decline in ν_t^D nor the increase in its standard deviation occurs smoothly during the sample period. The largest declines in the mean of ν_t^D occur in 1969 and in every year after 1971. A plot of ν_t^D indicates that it has declined more and varied more since 1965. Particularly large increases in the standard deviation of ν_t^D occurred in 1965, 1968, and 1971; since 1971 the standard deviation of ν_t^D has remained at or near that higher level.

Presumably much of the behavior of ν_t^D should be attributable to changes in the number of member banks relative to the total number of commercial banks in the country. Information on this variable is contained in Table D-2 in Appendix D. As can be seen in the last column of Table D-2 the largest decline in the proportion of banks that are members occurred during 1969 and 1970. While 1969 was a year when ν_t^D fell a lot, 1970 was not a year of unusual decline in ν_t^D. Overall the decline in the proportion of commercial banks that are members is greater in the later years of the sample and this mirrors the decline in ν_t^D and the increase in its variation after 1965. The behavior of ν_t^D however does not consistently reflect the member banks-commercial banks ratio; there are years of exceptional decline in ν_t^D which are not matched by years of unusual exit of banks from the System and vice versa. Apparently shifts of deposits between member and nonmember banks occur that are independent of the attrition of banks from the System.

Much more of the behavior of ν_t^D is explained by movements and levels of interest rates.[12] The analysis here will be based on the interest rate on newly-issued three-month Treasury bills, used as an indicator of the short-term interest rate. Information on this variable is contained in Table D-3, in Appendix D. The years of the largest declines in ν_t^D, 1969, 1972, 1973, and 1974, are each also years of historically high (1969, 1973, and 1974) or rapidly rising (1972) interest rates or both, as can be seen in Table D-3.

During the early years of the sample period, while interest rates rose considerably each year, they were not at historically high levels; during the same years, ν_t^D fell but not at record rates. As rates continued to rise in the mid-sixties, the decline in ν_t^D gained momentum. The only two years since 1965 that ν_t^D did not drop at a substantial rate were 1967 and 1970; these were also the only two years in the sample period when interest rates were not either rising rapidly or already at record-high levels.

A relationship between high and/or rising interest rates and declining values of ν_t^D is not surprising considering the cost differentials between member and nonmember bank reserve requirements described above. It is puzzling, however, that the effects of high or rising interest rates do not show up as clearly in the member banks-commercial banks ratio. During years of marked decline in the relative number of member banks which were not also years of an unusual fall in ν_t^D, one of two other factors had to be at work: either some independent shift of

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[12]The relative number of member banks is no doubt related to the behavior of interest rates too, so there is probably considerable interrelation between both factors and ν_t^D. This will be dealt with in Chapter 6.

deposits toward remaining member banks was occurring or the banks that were leaving (not joining) the System were of below-average size. By the same reasoning during years of exceptional decline in ν_t^D not matched by a fall in the relative number of member banks, either an independent shift of deposits away from member banks was at work or the banks leaving the System were of above-average size. The latter situation would be consistent with the close correspondence between falling ν_t^D and high interest rates if it is assumed that larger banks are more responsive to high interest rates. Therefore when interest rates rise, large banks are more apt to react by leaving the System, causing a proportionally larger fall in ν_t^D than in the relative number of member banks.[13] It may be that this is the case and that in the years of large decline in the relative number of member banks not matched by any exceptional drop in ν_t^D, some independent (or lagged interest rate) effect was causing a disproportionate number of small banks to leave the System.

The two years in the sample period that interest rates were not high or rising (1967 and 1970) were also years when the standard deviation of ν_t^D fell and the year after each of those years was a year of exceptional increase in the standard deviation of ν_t^D. It may be that the years of relative interest rate ease (1967 and 1970) caused bankers to ease up in changing their membership-status (and the standard deviation of ν_t^D fell), while the return to high and rising rates in each of

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[13]While it is rational to believe that large banks would be more apt to recognize and react to the cost of membership during periods of high interest rates, to the extent that large banks are also national banks, large banks should find it costlier and should require more time to exit the System, since charter conversion would be necessary.

the next years reinforced bankers' expectations and increased their propensity to change membership status (thereby causing the variance of ν_t^D to rise). It cannot be concluded here that this is a very strong relationship however since there are other years when the standard deviation of ν_t^D rose or fell, unaccompanied by any particular interest rate behavior. In 1965, for example, the standard deviation of ν_t^D rose substantially and it fell in 1966, while interest rates behaved similarly in both years.

The values of ν_t^D display a discernible seasonal pattern. Despite the secular decline in ν_t^D, a plot of ν_t^D shows that its value increases during the first quarter of every year in the sample period. During the second quarter of each year, the increase in ν_t^D slows, flattens out, and the overall decline in ν_t^D is accomplished during the third and fourth quarters of each year. This pattern is more pronounced during the years since 1964. This seasonal pattern can be seen in Table D-4 in Appendix D, which gives quarterly figures for ν_t^D.

Referring to Table 12, the mean of ν_t^T is .79442 and its standard deviation is .02416. The mean of ν_t^T is less than that of ν_t^D, indicating that on the average over the sample period, member banks hold a larger proportion of the nation's demand deposits than time deposits. This is probably due to the tendencies of large, metropolitan banks to be member banks and have deposit structures where demand deposits are relatively important.

Part 2 of Table 12 also indicates that ν_t^T varies less than ν_t^D. The mean of $(\nu_t^T - \nu_{t-1}^T)$ is -.00006 and its standard deviation is .00040, each of which is small on an absolute scale and is less than one-half the magnitude of comparable figures for ν_t^D. The mean of $|\nu_t^T - \nu_{t-1}^T|$

is .00029 and the largest value of $(\nu_t^T - \nu_{t-1}^T)$ is .00315, both of which are much smaller than comparable figures for ν_t^D. A plot of $(\nu_t^T - \nu_{t-1}^T)$ is virtually flat; the absolute value of the change in ν_t^T equals or exceeds .0005 only 126 weeks out of the 729 weeks in the sample period and exceeds .001 only four times.

Like ν_t^D, the value of ν_t^T falls overall during the sample period but the decline in ν_t^T is smaller. The value of ν_t^D drops 11.99% during the sample period, compared to 8.66% for ν_t^T. Furthermore, the decline of ν_t^T does not occur as regularly and smoothly as does that of ν_t^D. As the annual figures in Table 14 show, the value of ν_t^T rises during the first five years of the sample period and does not begin to fall until 1966. It continues to decline in 1967 and 1968, and then drops drastically during 1969; nearly half of the overall decline of ν_t^T is accomplished in 1969. The mean of ν_t^T falls again in 1970 but during that year, the value of ν_t^T actually increases; this shows up the next year as the 1971 mean rises. The decline in ν_t^T is resumed and it falls slightly during 1971 and 1972. During the last two years of the sample period, the behavior of ν_t^T becomes more erratic. It shows its characteristic relative stability during the first half of 1973, but rises during the third quarter, then falls during the fourth quarter. This decline continues into the first quarter of 1974, followed by an increase in the second quarter, a leveling off and resumed decline during the third and fourth quarters.

While the standard deviation of ν_t^T does not rise as steadily and consistently as did that of ν_t^D, the overall result is that it increases more than it decreases, and the later years of the sample period display consistently larger variability in ν_t^T than do the early years.

Table 14. Annual[1] Figures for ν_t^T

	Mean	Standard Deviation	Percentage Change from Previous Year	
			Mean	Standard Deviation
1961	.81070	.00381	--	--
1962	.81376	.00138	.377%	-63.780%
1963	.81680	.00099	.374%	-28.261%
1964[2]	.82009	.00088	.403%	-11.111%
1965	.82046	.00054	.045%	-38.636%
1966	.81643	.00275	-.491%	409.259%
1967	.81139	.00121	-.617%	-56.000%
1968	.80365	.00241	-.954%	99.174%
1969[2]	.78379	.00885	-2.471%	267.220%
1970	.77132	.00281	-1.591%	-68.249%
1971	.77280	.00198	.192%	-29.537%
1972	.76514	.00202	-.991%	2.020%
1973	.76046	.00201	-.612%	.495%
1974	.75478	.00217	-.747%	7.960%

[1]Based on 52 observations per year unless otherwise indicated.

[2]Based on 53 observations.

As can be seen in Table 14, most of the increase in the standard deviation of ν_t^T occurs in 1966 and 1969.

By the same reasoning applied to ν_t^D, the behavior of ν_t^T over time should be at least partially attributable to the relative number of member banks and interest rates. In addition, the effectiveness of Regulation Q interest-rate ceilings may influence ν_t^T via their impact on the growth of total time deposits. As Regulation Q ceilings encourage or impede the growth of time deposits, ν_t^T will also be affected, to the extent that the change in the level of time deposits is unevenly divided between member and nonmember banks. Since member banks dominate the market for large, negotiable certificates of deposit and this category of time deposits is apt to be most interest-elastic, the effects of Regulation Q ceilings will likely be felt most heavily by member banks.

The Regulation Q ceiling relative to the Treasury bill rate appears to have an important impact on ν_t^T. The increase in ν_t^T at the beginning of the sample period occurred while the Regulation Q ceiling was not effective; this was the period of time when the market for large certificates of deposit grew rapidly so the member bank portion of time deposits also grew. The decline in ν_t^T began in 1966 when Regulation Q ceilings became effective, and accelerated as the Treasury bill rate rose further above the ceiling. The decline in ν_t^T and the increase in its variation is consistently larger the larger the gap between market and ceiling rates and abates (or ν_t^T grows) when the gap is smaller or the ceiling ineffective.

While the number of member banks and cost-of-membership issue represented by high interest rates no doubt exert some influence on ν_t^T, the effect of interest rates relative to Regulation Q ceilings appears to be dominant. The relationship between ν_t^T and interest rates indicates that member banks lose time deposits more rapidly than nonmembers when Regulation Q ceilings are effective. This is no doubt because of the member bank dominance of the interest-elastic market for certificates of deposit. The complex relationships between ν_t^T, membership attrition and market interest rates will be analyzed with more sophisticated techniques in Chapter 6.

b) Differential State Reserve Requirements

As discussed in Chapter 3, the existence of nonmember banks can also affect the value of r_t through the impact of differential state reserve requirements. Since nonmember banks in different states are subject to different required reserve ratios, the ratio of nonmember bank reserves to total deposits will vary depending on how a given level of deposits are distributed among states. If nonmember bank total deposits are concentrated in states with relatively high (low) required reserve ratios, the level of reserves per dollar of total deposits held by nonmember banks will be relatively large (small). Hence as the distribution of nonmember bank deposits among states changes, the reserves-to-total deposits ratio for nonmember banks will vary. This phenomenon is analogous to the effect of differential Federal Reserve reserve requirements.

Whatever the effect of differential state reserve requirements on the ratio of nonmember bank reserves-to-total deposits, the impact on the value of r_t cannot be inferred directly. Since nonmember bank vault cash is the only part of nonmember bank reserves that is base-absorbing, it is nonmember bank vault cash relative to total deposits that is relevant to the value of r_t; it is changes in the ratio of vault cash to total deposits, rather than the reserve-deposit ratio, that will cause variation of r_t. If the behavior of nonmember banks is such that the vault cash-total deposits ratio is relatively constant in every state, then differential state reserve requirements have no effect on r_t. On the other hand, if the vault cash-total deposit ratio is not approximately equal for nonmember banks in different states, then the distribution of nonmember bank deposits among states will effect the value of r_t. In this latter case, any variation in either the ratio of vault cash to total deposits for each state or in the distribution of nonmember bank deposits among states will cause variation in r_t.

In the equation for r_t, the nonmember bank reserves portion of the expression for r_t is given by,

$$\sum_{h=1}^{51} \psi_{h,t}\omega_{h,t}\rho_t^m ,$$

where $\psi_{h,t} = \dfrac{\text{nonmember bank vault cash in the } h^{th} \text{ state:}}{\text{nonmember bank total deposits in the } h^{th} \text{ state}}$;

$\omega_{h,t} = \dfrac{\text{nonmember bank total deposits in the } h^{th} \text{ state:}}{\text{nonmember bank total deposits}}$;

$\rho_t^m = \dfrac{\text{nonmember bank total deposits}}{\text{privately-owned demand deposits}}$.

The effects of changes in the level of vault cash relative to total deposits in the h^{th} state is represented by $\psi_{h,t}$ and $\omega_{h,t}$ represents the proportion of total nonmember bank deposits in the h^{th} state.

The only data available from which to calculate $\psi_{h,t}$ and $\omega_{h,t}$ are Federal Deposit Insurance Corporation call report data. Until June, 1963, call report data are available on a quarterly basis; for the last eleven years of the sample period, data are available on a semiannual basis only. Table 15 gives the mean and standard deviation for $\psi_{h,t}$ between states for each of the thirty-four call report dates in the period December 31, 1960 through December 31, 1974. Table 16 presents the mean and standard deviation of $\psi_{h,t}$ and $\omega_{h,t}$ for each state based on all thirty-four call report dates.

As can be seen from Table 15, the mean of $\psi_{h,t}$ has declined during the sample period and does vary from one call date to the next. Thus, the average ratio of vault cash to total deposits for all nonmember banks is not constant. The standard deviation of $\psi_{h,t}$ reported in Table 15 implies that the variation in $\psi_{h,t}$ between states is not consistently high, but it is relatively high for some call dates. Furthermore, Table 16 indicates that the vault cash-total deposit ratio is not uniform between states, since the mean of $\psi_{h,t}$ for the thirty-four call dates ranges from .01144 for California to .03352 for Arkansas. The standard deviation of $\psi_{h,t}$ reported in Table 16 measures the variation in $\psi_{h,t}$ that has occurred through time for each state. The standard deviation of $\psi_{h,t}$ is small for most states but varies considerably between states; it ranges from .00151 for Vermont to .04584 for Arkansas. All of this implies that the ratio of nonmember bank vault cash to total deposits is not particularly stable; it apparently has varied considerably over time and is not especially uniform between states.

A number of factors may cause the variation in $\psi_{h,t}$ between states. It may be that nonmember banks choose to hold a relatively constant proportion of their legal reserves in the form of vault cash. If this is so, differential state reserve requirements would account for different values of $\psi_{h,t}$ for different states. Variation in the ratio between states is no doubt influenced by state institutional and demographic

Table 15. Nonmember Bank Holdings of Vault Cash

$\psi_{h,t} = \frac{VC_{h,n,t}}{TD_{h,n,t}}$			$\psi_{h,t}$		
Call Date	Mean	Standard Deviation	Call Date	Mean	Standard Deviation
12/31/60	.02135	.00652	12/31/66	.02042	.00500
4/12/61	.02479	.00686	6/30/67	.01876	.00421
6/30/61	.02220	.00581	12/31/67	.01953	.00479
9/30/61	.02646	.00702	6/30/68	.01805	.00412
12/31/61	.02329	.00621	12/31/68	.02103	.00598
3/26/62	.02395	.00563	6/30/69	.01929	.00605
6/30/62	.02112	.00512	12/30/69	.02073	.00818
9/28/62	.02201	.00578	6/30/70	.02022	.00675
12/31/62	.02389	.00640	12/31/70	.01785	.00600
3/18/63	.02858	.03109	6/30/71	.02046	.01528
6/29/63	.02603	.03055	12/31/71	.01680	.00487
12/31/63	.02051	.00522	6/30/72	.01610	.00775
6/30/64	.02326	.00605	12/31/72	.01629	.00557
12/31/64	.02424	.02128	6/30/73	.01455	.00512
6/30/65	.02291	.00583	12/31/73	.01735	.00625
12/31/65	.02295	.02007	6/30/74	.01364	.00341
6/30/66	.02142	.00512	12/31/74	.01680	.00500

Data Source: Federal Deposit Insurance Corporation, Assets, Liabilities and Capital Accounts, Commercial Banks and Mutual Savings Banks, Report of Call Nos. 54-84, December 31, 1960-December 31, 1968; and Federal Deposit Insurance Corporation, Board of Governors of the Federal Reserve System, and Office of the Comptroller of the Currency, Assets, Liabilities and Capital Accounts, Commercial Banks and Mutual Savings Banks, June 30, 1969-December 31, 1974.

Table 16. Effects of Nonmember Bank Holding of Vault Cash December 31, 1960-December 31, 1974 (34 observations)

State	$\psi_{h,t} = \frac{VC_{h,n,t}}{TD_{h,n,t}}$ Mean	$\psi_{h,t}$ Standard Deviation	$\omega_{h,t} = \frac{TD_{h,n,t}}{TD_{n,t}}$ Mean	$\omega_{h,t}$ Standard Deviation
Alabama	.02372	.00423	.01379	.00109
Alaska	.02047	.00871	.00126	.00077
Arizona	.02200	.02723	.01008	.00075
Arkansas	.03352	.04548	.01186	.00050
California	.01144	.00155	.04473	.00294
Colorado	.01828	.00291	.00903	.00058
Connecticut	.02558	.02541	.01280	.00219
Delaware	.01908	.00206	.01537	.00273
District of Columbia	.01542	.00321	.00627	.00356
Florida	.02142	.00419	.04467	.00498
Georgia	.02501	.00396	.02614	.00199
Hawaii	.02543	.00653	.01305	.00258
Idaho	.01741	.00379	.00206	.00023
Illinois	.01519	.00241	.07014	.00411
Indiana	.01959	.00392	.03240	.00214
Iowa	.01511	.00282	.03744	.00239
Kansas	.01396	.00229	.02421	.00123
Kentucky	.02213	.00364	.02325	.00065
Louisiana	.02473	.00464	.02442	.00150
Maine	.02751	.00561	.00375	.00046
Maryland	.02495	.00364	.02220	.00097

Table 16. Continued

State	$\psi_{h,t} = \frac{VC_{h,n,t}}{TD_{h,n,t}}$ Mean	Standard Deviation	$\omega_{h,t} = \frac{TD_{h,n,t}}{TD_{n,t}}$ Mean	Standard Deviation
Massachusetts	.02607	.00457	.01409	.00101
Michigan	.02028	.00344	.02849	.00217
Minnesota	.01504	.00300	.03228	.00220
Mississippi	.02511	.00316	.01817	.00194
Missouri	.01739	.00285	.05065	.01975
Montana	.01551	.00279	.00297	.00015
Nebraska	.01873	.02501	.01257	.00042
Nevada	.02852	.00925	.00350	.00120
New Hampshire	.01355	.00178	.00323	.00040
New Jersey	.02114	.00268	.01911	.00072
New Mexico	.02228	.00491	.00471	.00020
New York	.01370	.00249	.02905	.00494
North Carolina	.03014	.00440	.02979	.00271
North Dakota	.01189	.00234	.00738	.00074
Ohio	.02151	.00270	.02875	.00432
Oklahoma	.02001	.00353	.01227	.00172
Oregon	.01783	.00238	.00662	.00048
Pennsylvania	.02110	.00365	.05437	.00444
Rhode Island	.02888	.01133	.00676	.00365
South Carolina	.03081	.00475	.00964	.00052
South Dakota	.01223	.00241	.00578	.00107
Tennessee	.02242	.00306	.02284	.00093

Table 16. Continued

State	$\psi_{h,t} = \frac{VC_{h,n,t}}{TD_{h,n,t}}$ Mean	Standard Deviation	$\omega_{h,t} = \frac{TD_{h,n,t}}{TD_{n,t}}$ Mean	Standard Deviation
Texas	.01971	.00398	.06234	.00593
Utah	.02012	.00406	.00435	.00086
Vermont	.01532	.00151	.00520	.00051
Virginia	.02475	.00438	.01640	.00095
Washington	.02450	.00422	.00520	.00045
West Virginia	.02526	.00515	.00844	.00020
Wisconsin	.01727	.00266	.04465	.00233
Wyoming	.01781	.00340	.00148	.00014

Data Source: Federal Deposit Insurance Corporation, Assets, Liabilities and Capital Accounts, Commercial Banks and Mutual Savings Banks, Report of Calls Nos. 54-84, December 31, 1960-December 31, 1968; and Federal Deposit Insurance Corporation, Board of Governors of the Federal Reserve System, and Office of the Comptroller of the Currency, Assets, Liabilities and Capital Accounts, Commercial Banks and Mutual Savings Banks, June 30, 1969-December 31, 1974.

features such as branching laws, average bank size and urban-rural mix. Variation in $\psi_{h,t}$ through time is partially attributable to the effects of high and rising interest rates during the sample period. Whatever the cause of variation in $\psi_{h,t}$, the fact that nonmember banks do not hold a constant amount of vault cash per dollar of deposits means that changes in the distribution of nonmember bank deposits among states affects r_t. Consequently variation in either $\psi_{h,t}$ or $\omega_{h,t}$ will generate variation in r_t.

Table 16 also gives the results for $\omega_{h,t}$ which indicate the extent to which the distribution of nonmember bank deposits between states has changed historically. The standard deviations of $\omega_{h,t}$ are all small; thus shifts in the distribution of deposits among states has apparently been minimal.

The infrequency of available nonmember bank data seriously limits analyzing the impact of state reserve requirements on r_t.[14] To the extent that nonmember bank deposits within a state are subdivided into categories for reserve purposes, the distribution of deposits among these categories will also affect r_t. State deposit data that are disaggregated into these additional reserve categories are not available at all. The lack of weekly nonmember bank deposit data makes it impossible to analyze the nonmember bank reserves portion of r_t in a way

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[14]In addition, some authors have questioned the reliability of the data that are available. This study however is concerned with the amount and source of variation in measured r_t, which is by necessity calculated with available data. The only available measure of r_t therefore incorporates whatever errors or inadequacies are in the deposit data. The deficiencies of the data, while possibly troublesome, are therefore not at issue here.

that is comparable to the analysis of the member bank portion. Therefore, the analysis in the following chapters does not include the nonmember bank part of r_t.

Nonmoney Deposits

Since member banks are required to hold base-absorbing reserves against deposits that are not included in the money stock, the base money supplied by the monetary authorities will support a smaller money stock, the higher the levels of nonmoney deposits. The nonmoney deposits include deposits of the U. S. Government, interbank deposits, and time deposits. Their influence is reflected here in the parameters, γ_t, ι_t, and τ_t, respectively, where each category of nonmoney deposits is expressed as a ratio to privately-owned demand deposits.

Table 17 gives statistics for the parameters γ_t, ι_t, and τ_t and their first differences. Of the three types of nonmoney deposits, τ_t has the largest standard deviation but γ_t has the highest coefficient of variation. The standard deviations and coefficients of variation for γ_t, ι_t, and τ_t are all large relative to the parameters discussed above. For example, the coefficients of variation of ν_t^D and ν_t^T are less than one-third the level of the lowest coefficient of variation of the three nonmoney items (ι_t).

The mean of the first differences for γ_t and ι_t are small, -.00003 and .00002, but as expected τ_t shows a considerably larger average weekly change (.00193). The standard deviation and largest weekly values of $(\tau_t - \tau_{t-1})$ are also larger than for either γ_t or ι_t. The mean of the absolute value of the first differences is, for all three parameters, much larger than the mean of the first differences; thus

Table 17. Nonmoney Deposits (Values and First Differences of the Parameters, γ_t, ι_t, τ_t) (1961-1974)

	Mean	Mean of Absolute Value	Standard Deviation	Coefficient of Variation	Largest Weekly Change
1. Values of the Parameters γ_t, ι_t, and τ_t (730 observations)					
Government Deposits (γ_t)	.04149		.01455	.35069	
Interbank Deposits (ι_t)	.13886		.01645	.11846	
ι*	.13341		.01572	.11782	
Time Deposits (τ_t)	1.28061		.36787	.28726	
2. Period-to-Period Changes in γ_t, ι_t, and τ_t (729 observations)					
Government Deposits (γ_t)	-.00003	.00866	.01088		.03996
Interbank Deposits (ι_t)	.00002	.00476	.00648		-.03590
ι*	.00002	.00475	.00646		-.03567
Time Deposits (τ_t)	.00193	.01399	.01852		.07472

γ_t, ι_t, and τ_t apparently all fluctuate considerably. The mean of $|\tau_t - \tau_{t-1}|$ is much larger than that for γ_t or ι_t, indicating that τ_t indeed varies considerably more. The statistics on the first differences also indicate that the size of, and variation in, the weekly change of γ_t, ι_t, and τ_t are larger than that for the parameters discussed earlier.

a) Government Deposits

A plot shows that the behavior of γ_t is dominated by its continual fluctuations rather than any discernible systematic pattern. The standard deviation of $(\gamma_t - \gamma_{t-1})$ is nearly as large as the standard deviation of γ_t itself and the plot of the first differences exhibits as many and as large fluctuations as the one of the levels of γ_t. Table 18 gives annual figures for γ_t. During the first five years of the sample period, γ_t rose slightly, reaching its high in 1965, and declined thereafter, especially in 1966, 1973, and 1974. The standard deviation has also declined some; the early years of the sample in general show more variation for γ_t.

While γ_t displays large week-to-week variation, a plot of γ_t indicates that it follows the same pattern of fluctuation within the same quarter of nearly every year of the sample. Given the regularity of this seasonal pattern, it appears that the behavior of γ_t may reflect institutional factors such as tax payment dates. If this is the case, the behavior of γ_t should be predictable even though it displays a lot of variability. This issue will be considered in the regression analysis in Chapter 6.

Table 18. Annual[1] Figures for γ_t

	Mean	Standard Deviation	Percentage Change from Previous Year: Mean	Percentage Change from Previous Year: Standard Deviation
1961	.04263	.01169		--
1962	.05174	.01513	21.370	29.427
1963	.05006	.01567	-3.247	3.569
1964[2]	.04799	.01484	-4.135	-5.297
1965	.05061	.01906	5.460	28.437
1966	.03760	.01533	-25.706	-19.570
1967	.03749	.00944	-.293	-38.421
1968	.03987	.01128	6.348	19.492
1969[2]	.03716	.01243	-6.797	10.195
1970	.04145	.00941	11.545	-24.296
1971	.03961	.01141	-4.439	21.254
1972	.04025	.01118	1.616	-2.016
1973	.03617	.01363	-10.137	21.914
1974	.02822	.01036	-21.980	-23.991

[1]Based on 52 observations per year unless othe wise indicated.

[2]Based on 53 observations.

b) Interbank Deposits

As can be seen in Table 17, there is little difference between the behavior of ι_t and its alternative definition ι_t^*, which excludes call report data on nonmember bank interbank deposits. By all measures, the variation in ι_t^* is slightly smaller than in ι_t. The behavior of ι_t is dominated by large weekly changes. The standard deviation of the first differences of ι_t is much smaller than that of its level, so $(\iota_t - \iota_{t-1})$ displays much smaller weekly fluctuations than ι_t.

The value of ι_t has increased overall during the sample years. Table 19 gives annual figures for ι_t and shows that the mean of ι_t has grown slightly each year since 1964 except in 1966, 1972, and 1973. A plot shows that most of the rise in the value of ι_t was accomplished during 1968, 1969, 1970, and 1973. As can be seen in Table 19 there is also a tendency for the variation in ι_t to rise during the sample period; since 1967 the standard deviation of ι_t has been larger. The largest increases in the standard deviation occurred in 1968 and 1972.

There have been two occurrences during the sample period that would logically influence ι_t. One is the decline in the relative number of member banks. Since nonmember banks can hold legal reserves in the form of interbank balances, ι_t might be expected to rise as the relative number of nonmembers grows. Gilbert[15] however has found that member banks hold more interbank deposits than nonmembers of comparable size. Consequently ι_t may fall as relatively more banks are nonmembers.

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[15] Alton Gilbert, "Utilization of Federal Reserve Bank Services by Member Banks: Implications for the Costs and Benefits of Membership," Review, Federal Reserve Bank of St. Louis 59 (August 1977):12.

Table 19. Annual[1] Figures for ι_t

	Mean	Standard Deviation	Percentage Change from Previous Year: Mean	Percentage Change from Previous Year: Standard Deviation
1961	.12961	.00460	--	--
1962	.12756	.00581	-1.582	26.304
1963	.12428	.00483	-2.571	-16.867
1964[2]	.12272	.00455	-1.255	-5.797
1965	.12457	.00415	1.507	-8.791
1966	.12452	.00442	-.040	6.506
1967	.12847	.00432	3.172	-2.262
1968	.13335	.00652	3.799	50.926
1969[2]	.14565	.00733	9.224	12.423
1970	.15529	.00733	6.619	4.366
1971	.16792	.00779	8.133	1.830
1972	.15441	.01154	-8.045	48.139
1973	.14433	.00756	-6.528	-34.489
1974	.16156	.00744	11.938	-1.587

[1]Based on 52 observations per year unless otherwise indicated.

[2]Based on 53 observations.

The second phenomenon that should be at work on ι_t is the record of high and/or rising interest rates during the sample years. Since interbank deposits are noninterest bearing, high and/or rising interest rates should cause banks to economize on their holdings of interbank deposits and the level of ι_t should fall.[16]

Although the results are not clear cut, Table 19 and Table 13 indicate that in general the increase in ι_t mirrors the decline of ν_t^D and the variation in both parameters rises. The years when ι_t is high or rises (1968 through 1974) are, except 1970, also years when ν_t^D declines substantially. The first year of unusual decline in ν_t^D was 1969 and that was also the year of the first large increase in ι_t. There are, however, years when ν_t^D falls substantially and ι_t also falls (1963, 1966, 1972, and 1973, for example), so the relationship between ι_t and the relative decline in member banks is not clear. Any impact of market interest rates on ι_t is difficult to discern from observation (comparing Tables 19 and D-3). The general tendency of interest rates to rise or be high during the sample period and the increase in ι_t contradicts the expected relationship between ι_t and interest rates. The influence of both of these independent variables on ι_t will be considered further in the regression analysis in Chapter 6. In addition, the analysis in the next chapter will deal with the seasonal behavior of ι_t which appears to be strong.

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[16]Banks, of course, earn a "return" on interbank balances in the form of services rendered. Banks typically hold a level of interbank balances necessary to "pay" for the services needed from a correspondent. To determine the required level of interbank balances, correspondents often apply a market interest rate to the level of interbank balances and compare this to the cost of services provided. Therefore, higher market interest rates allow lower interbank balances to pay for a given level of correspondent services.

c) Time Deposits

The value of τ_t, of course, rises continually throughout the sample period. Table 20 gives annual figures for τ_t and shows that the annual mean rises every year but 1969; the mean of τ_t more than triples its value between 1961 and 1974. As expected, τ_t also displays considerable variation; as can be seen in Table 17, its standard deviation is .36787 which is higher than that for γ_t, ι_t, ν_t^D, or ν_t^T. The coefficient of variation of τ_t is .28726, exceeded only by the coefficient of variation of γ_t. The standard deviation of $(\tau_t - \tau_{t-1})$ is only six percent of the standard deviation of τ_t, indicating that a large portion of the variation in τ_t is due to its upward trend. A plot of $(\tau_t - \tau_{t-1})$ still displays considerable weekly fluctuations however; the mean of $(\tau_t - \tau_{t-1})$ is .00193 and the mean of $|\tau_t - \tau_{t-1}|$ is .01399, both of which are much higher than comparable statistics for γ_t, ι_t, ν_t^D or γ_t^T. As can be seen in Table 20, the variation in τ_t has also increased; the standard deviation of τ_t is larger after 1968, except for 1972.

Table 20 shows that the largest increases in the mean of τ_t occurred during 1962, 1963, 1965, 1970, 1971, 1973, and 1974. The best explanation of the behavior of τ_t ought to be market interest rates and interest rates relative to Regulation Q ceilings. The relationship between the Treasury bill rate as an indication of short-term interest rates and τ_t is, however, ambiguous. If an increase in the Treasury bill rate represents a general increase in all rates including that paid on time deposits, τ_t should be expected to rise as consumers shift out of cash and demand deposits into all interest-bearing instruments, including time deposits. If, on the other hand, an increase in the Treasury bill rate represents more attractive terms being paid on an

Table 20. Annual[1] Figures for τ_t

	Mean	Standard Deviation	Percentage Change from Previous Year: Mean	Percentage Change from Previous Year: Standard Deviation
1961	.69941	.02543	--	--
1962	.79322	.03792	13.413	49.115
1963	.89351	.03379	12.643	-10.891
1964[2]	.98015	.03157	9.697	-6.570
1965	1.09300	.04096	11.514	29.743
1966	1.17717	.04142	7.701	1.123
1967	1.28536	.04187	9.191	1.086
1968	1.34193	.03330	4.401	-20.468
1969[2]	1.32260	.04932	-1.440	48.108
1970	1.35303	.07401	2.301	50.061
1971	1.56403	.05120	15.595	-30.820
1972	1.64276	.03842	5.034	-24.961
1973	1.78432	.07819	8.617	103.514
1974	2.00300	.09062	12.256	15.897

[1]Based on 52 observations per year unless otherwise indicated.

[2]Based on 53 observations per year.

instrument that is an alternative in time deposits, τ_t would be expected to fall. Whenever market interest rates are below the Regulation Q ceiling, τ_t should rise more rapidly; when Regulation Q ceilings are effective, τ_t would be expected to fall (or rise more slowly).

The limited analysis here indicates no particular relationship between τ_t and the Treasury bill rate. While five of the years of exceptional increase in τ_t (1962, 1963, 1965, 1973, and 1974) were also years of high or rising interest rates, the other two years during which τ_t increased a lot, 1970 and 1971, were years of falling and low (respectively) rates. In addition, many other years in the sample periods recorded equally high or rising rates, when τ_t did not rise exceptionally, including 1969 when τ_t actually fell.

The behavior of τ_t is more closely related to the effectiveness of Regulation Q ceilings. The rapid growth in τ_t in the early years of the sample occurred when Regulation Q ceilings were not effective. The slow-down in its growth in the late 1960's corresponds to effective Regulation Q ceilings, the decline in τ_t in 1969 occurred when the difference between interest rates and ceilings was large. A more sophisticated analysis of the effects of interest rates and ceilings on τ_t is included in the regression analysis in Chapter 6.

The values of τ_t display a distinct seasonal pattern of behavior; Table D-5 gives quarterly figures for τ_t. Substantial increases occur in τ_t during the first quarter of every year; this is reflected by the fact that the first quarter of every year (except 1969) has the largest quarterly standard deviation and the lowest quarterly mean. During the second quarter, the rise in τ_t continues but at a slower pace; in the last seven years of the sample period, τ_t has fallen first during the second quarter, then rises again, above the level reached at the end of the first quarter. During the third quarter, the level of τ_t stabilizes at a level above that of the first and second quarters and fluctuates around that level but its level does not change much. During the fourth quarter of all but one year, τ_t drops drastically; but, except 1969, τ_t never drops back to the level of the first quarter of the same year, so that the overall increase in τ_t continues unabated from year to year.

Nondeposit Liabilities

The data available for the various categories of nondeposit liabilities are deficient in a number of ways which hamper analyzing the effects of lagged and differential reserve requirements against nondeposit liabilities. In addition, the structure of reserve requirements against these liabilities has been changed so many times that it is difficult to isolate the effects of lagged and differential reserve requirements for any reasonably long period of time. What follows is a specification of the nondeposit liability portion ($\sum_q n_q \lambda_q^N \delta_q^N \alpha_q^N$) of r_t, as it should be analyzed (i.e., assuming perfect data). This specification will then be compared with what analysis is possible, given the available data.

As was indicated in Chapter 3, the first nondeposit reserve requirement was a marginal reserve requirement imposed on liabilities arising out of Eurodollar transactions which began on October 16, 1969. At the time this study began no data on this liability were available and therefore the "Eurodollar" category of nondeposit liabilities is ignored below.

Beginning June 21, 1973,[17] a marginal reserve requirement was applied to certificates of deposit in excess of \$100,000 (CD_t) and funds obtained through issuance of commercial paper (CP_t). Beginning on that date then the nondeposit liability portion of r_t would be represented by:

$n_{1,t}\lambda^N_{1,t}\alpha^N_{1,t}$, where

$n_{1,t}$ = the marginal required reserve ratio against $(CD + CP)_t$,

$$\lambda^N_{1,t} = \frac{(CD + CP)_{t-2}}{(CD + CP)_t},$$

$$\alpha^N_{1,t} = \frac{(CD + CP)_t}{D^P_t}.$$

Beginning July 12, 1973, the marginal reserve requirement on CD_t and CP_t was extended to cover funds from the sale of sales finance bills (SF_t) and therefore nondeposit liabilities should be represented in the equation for r_t by:

$n_{2,t}\lambda^N_{2,t}\alpha^N_{2,t}$, where

$n_{2,t}$ = the marginal required reserve ratio against $(CD + CP + SF)_t$,

$$\lambda^N_{2,t} = \frac{(CD + CP + SF)_{t-2}}{(CD + CP + SF)_t},$$

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[17] Since all reserve requirements against nondeposit liabilities were introduced after lagged reserve requirements were instigated, changes in reserve requirements are imposed on one date, but applied to nondeposit liabilities two weeks earlier. In each case, the dates referred to here are the dates when the reserve requirement was changed. For example, the reserve requirement which became effective June 21, 1973, was first applied to nondeposit liability levels as of June 7, 1973.

$$\alpha^N_{2,t} = \frac{(CD + CP + SF)_t}{D^P_t}.$$

Beginning September 19, 1974, all three liability categories (CD_t, CP_t, and SF_t) were divided into two maturity-length categories: those maturing in less than four months and those maturing in more than four months. The shorter-maturity group continued to be subject to the marginal reserve requirement against nondeposit liabilities, but the longer-maturity group was reverted to the original reserve requirement against time deposits in excess of \$5 million ($T_{t,>\$5}$). After this change then the nondeposit liability portion of r_t should be revised to:

$$n_{3,t}\lambda^N_{3,t}\delta^N_{1,t}\alpha^N_{3,t} + n_{4,t}\lambda^N_{4,t}\delta^N_{2,t}\alpha^N_{4,t}, \text{ where}$$

$n_{3,t}$ = marginal required reserve ratio against nondeposit liabilities maturing in less than 4 months;

$n_{4,t}$ = required reserve ratio against time deposits in excess of \$5 million plus nondeposit liabilities, maturing in more than 4 months;

$$\lambda^N_{3,t} = \frac{(CD + CP + SF)_{t-2,\ <4 \text{ months}}}{(CD + CP + SF)_{t,\ <4 \text{ months}}};$$

$$\delta^N_{1,t} = \frac{(CD + CP + SF)_{t,\ <4 \text{ months}}}{(CD + CP + SF)_t};$$

$$\alpha^N_{3,t} = \frac{(CD + CP + SF)_t}{D^P_t};$$

$$\lambda^N_{4,t} = \frac{T_{t-2,\ >\$5m} + (CD + CP + SF)_{t-2,\ >4 \text{ months}}}{T_{t,\ >\$5m} + (CD + CP + SF)_{t,\ >4 \text{ months}}};$$

$$\delta^N_{2,t} = \frac{T_{t,\ >\$5m} + (CD + CP + SF)_{t,>4 \text{ months}}}{(T_{>\$5m} + CD + CP + SF)_t};$$

$$\alpha^N_4 = \frac{(T_{>\$5m} + CD + CP + SF)_t}{D^P_t}.$$

The special reserve requirements against nondeposit liabilities were dropped on December 12, 1974, and were combined with time deposits in excess of $5 million. At that time the reserve category, time deposits over $5 million plus the three categories of nondeposit liabilities, was divided into two subcategories based on maturity-length: 30 to 179 days and over 179 days. For this scheme, the nondeposit liability specification is represented by:

$$(n_{5,t}\lambda^N_{5,t}\delta^N_{3,t} + n_{6,t}\lambda^N_{6,t}\delta^N_{4,t})\alpha^N_{5,t}, \text{ where}$$

$n_{5,t}$ = the required reserve ratio against $(T_{>\$5m} + CD + CP + SF)_t$, maturing in 30-179 days;

$n_{6,t}$ = the required reserve ratio against $(T_{>\$5m} + CD + CP + SF)_t$, maturing in more than 179 days;

$$\lambda^N_{5,t} = \frac{(T_{>\$5m} + CD + CP + SF)_{t-2,\ 30\text{-}179\ \text{days}}}{(T_{>\$5m} + CD + CP + SF)_{t,\ 30\text{-}179\ \text{days}}};$$

$$\delta^N_{5,t} = \frac{(T_{>\$5m} + CD + CP + SF)_{t,\ 30\text{-}179\ \text{days}}}{(T_{>\$5m} + CD + CP + SF)_t};$$

$$\lambda^N_{6,t} = \frac{(T_{>\$5m} + CD + CP + SF)_{t-2,\ >179\ \text{days}}}{(T_{>\$5m} + CD + CP + SF)_{t,\ >179\ \text{days}}};$$

$$\delta^N_{6,t} = \frac{(T_{>\$5m} + CD + CP + SF)_{t,\ >179\ \text{days}}}{(T_{>\$5m} + CD + CP + SF)_t};$$

$$\alpha^N_{5,t} = \frac{(T_{>\$5m} + CD + CP + SF)_t}{D^P_t}.$$

Table 21 is a summary of the four different reserve schemes that have applied to nondeposit liabilities.

The first deficiency in the data available for nondeposit liabilities arises from the fact that many of the reserve requirements imposed on them are marginal. That is, the required reserve ratios are applied

Table 21. Summary of the Specification of the Nondeposit Liability Portion of the Equation for r_t

Effective Dates	Specification of the $\sum_q n_q \lambda_q^N \delta_q^N \alpha_q^N$ Portion of r_t	Description of Parameters
Reserve Scheme 1 6/21/73 - 7/12/73	$n_1 \lambda_1^{N*} \alpha_1^{N*}$	$\lambda_1^{N*} = \frac{(CD + CP)_{t-2}}{(CD + CP)_t}$ $\alpha_1^{N*} = \frac{(CD + CP)_t}{D_t^p}$
Reserve Scheme 2 7/12/73 - 9/19/74	$n_2 \lambda_2^{N*} \alpha_2^{N*}$	$\lambda_2^{N*} = \frac{(CD + CP + SF)_{t-2}}{(CD + CP + SF)_t}$ $\alpha_2^{N*} = \frac{(CD + CP + SF)_t}{D_t^p}$
Reserve Scheme 3 9/19/74 - 12/12/74	$n_3 \lambda_3^{N*} \delta_1^{N*} \alpha_2^{N*} + n_4 \lambda_4^N \delta_2^N \alpha_3^N$	$\lambda_3^{N*} = \frac{(CD + CP + SF)_{t-2,\ <4\ \text{months}}}{(CD + CP + SF)_{t,\ <4\ \text{months}}}$ $\delta_1^{N*} = \frac{(CD + CP + SF)_{t,\ <4\ \text{months}}}{(CD + CP + SF)_t}$ $\alpha_2^{N*} =$ (defined above)

Table 21. Continued

Effective Dates	Specification of the $\Sigma_q n_q \lambda^N_q \delta^N_q \alpha^N_q$ Portion of r_t	Definition of Parameters
Reserve Scheme 3 - cont'd.		$\lambda^N_4 = \frac{(T_{>\$5m} + CD + CP + SF)_{t-2,\ >4\ months}}{(T_{>\$5m} + CD + CP + SF)_{t,\ >4\ months}}$
		$\delta^N_2 = \frac{(T_{>\$5m} + CD + CP + SF)_{t,\ >4\ months}}{(T_{>\$5m} + CD + CP + SF)_{t}}$
		$\alpha^N_3 = \frac{(T_{>\$5m} + CD + CP + SF)_{t}}{D^P_t}$
Reserve Scheme 4 12/12/74 - 12/25/74	$(n_5\lambda^N_5\delta^N_3 + n_6\lambda^N_6\delta^N_4)\alpha^N_4$	$\lambda^N_5 = \frac{(T_{>\$5m} + CD + CP + SF)_{t-2,\ 30\text{-}179\ days}}{(T_{>\$5m} + CD + CP + SF)_{t,\ 30\text{-}179\ days}}$
		$\delta^N_3 = \frac{(T_{>\$5m} + CD + CP + SF)_{t,\ 30\text{-}179\ days}}{(T + CD + CP + SF)_{t}}$
		$\lambda^N_6 = \frac{(T_{>\$5m} + CD + CP + SF)_{t-2,\ >179\ days}}{(T_{>\$5m} + CD + CP + SF)_{t,\ >179\ days}}$

Table 21. Continued

Effective Dates	Specification of the $\Sigma_q n_q \lambda_q^N \delta_q^N \alpha_q^N$ Portion of r_t	Definition of Parameters
Reserve Scheme 4 - cont'd.		
		$\delta_4^N = \frac{(T_{>\$5m} + CD + CP + SF)_{t,>179\ days}}{(T + CD + CP + SF)_t}$
		$\alpha_4^N = \frac{(T + CD + CP + SF)_t}{D_t^P}$

to changes in the liability items rather than their levels; or, prescribed base amounts of the liabilities are exempted from reserve requirements. In each case, the data that are available reflects levels of nondeposit liabilities. Therefore the data cannot be used to show changes in, or increases above a base level of nondeposit liabilities for an individual bank, which is the relevant figure for the determination of required reserves. This problem arises in calculating the parameters for nondeposit liability Reserve Schemes 1, 2, and three of the parameters in Scheme 3. The marginal aspect of these reserve requirements is denoted by the asterisks attached to the appropriate parameters in Table 21. In all of the empirical work that follows, the marginality of these reserve requirements is ignored due to the lack of data.

The empirical results for the λ_q^N, δ_q^N, and α_q^N-parameters are summarized in Table 22. The two parameters in Reserve Scheme 1 cannot be calculated because the data do not distinguish between funds arising from issuance of commercial paper and those from sales finance bills; Schemes 1 and 2 therefore are combined into the same Reserve Scheme. In Scheme 3, the breakdown of $(CP + SF)_t$ into those maturing in less than four months and those maturing in more than four months is not available. The only data available for $(CP + SF)_t$ are for those maturing in more than thirty days; the needed maturity-length breakdown is available for CD_t only. In calculating the λ^N- and δ^N-parameters, the only data available for $(CP + SF)_t$, those maturing in more than thirty days, are used in both maturity-groups. In general, dates quoted in Table 22 do not match those defining the reserve schemes

Table 22. Effects on r_t of Nondeposit Liabilities (Values of the Parameters of λ_q^n, δ_q^n, α_q^n)

	Mean	Average Deviation from 1.0	Coefficient of Variation	First Differences: Mean	First Differences: Mean of Absolute Value	First Differences: Standard Deviation
1. Lagged Reserve Requirements (λ_q^N)						
Reserve Schemes 1&2 6/13/73 - 11/27/74[1] (75 observations)						
$\lambda_2^{N*}\left[\lambda_1^{N*}\right] = \frac{(CD + CP + SF)_{t-2}}{(CD + CP + SF)_t}$	.98887	.01113	.01956	-.00013	.00855	.01509
Reserve Scheme 3 9/11/74 - 11/27/74 (10 observations)						
$\lambda_3^{N*} = \frac{(CD + CP + SF)_{t-2,\ <4\ months}}{(CD + CP + SF)_{t,\ <4\ months}}$[2]	1.0463	.04630	.00705	-.00067	.00524	.00684
$\lambda_4^{N} = \frac{(T_{>\$5m} + CD + CP + SF)_{t-2,\ >4\ months}}{(T_{>\$5m} + CD + CP + SF)_{t,\ >4\ months}}$[2]	.98163	.01837	.00756	.00171	.00439	.00497
Reserve Scheme 4 12/04/74 - 12/25/74 (2 observations)						
$\lambda_5^{N} = \frac{(T_{>\$5m} + CD + CP + SF)_{t-2,\ 30-179\ days}}{(T_{>\$5m} + CD + CP + SF)_{t,\ 30-179\ days}}$	.97224	.02776	.00061	.00087	.00087	*
$\lambda_6^{N} = \frac{(T_{>\$5m} + CD + CP + SF)_{t-2,\ >179\ days}}{(T_{>\$5m} + CD + CP + SF)_{t,\ >179\ days}}$	.98871	.01129	.00093	-.00134	.00134	*

Table 22. Continued

		Mean	Average Deviation from 1.0	Coefficient of Variation	First Differences: Mean	First Differences: Mean of Absolute Value	First Differences: Standard Deviation
Other Nondeposit Liability Categories							
$\frac{(CD)_{t-2}}{(CD)_t}$	6/13/74 - 11/27/74 (75 observations)	.98862	.01138	.01897	-.00013	.00877	.01453
$\frac{(CP + SF)_{t-2}}{(CP + SF)_t}$	10/22/69 - 12/25/74 (269 observations)	1.00821	.00821	.12363	.00021	.06035	.12016
2. Differential Reserve Requirements (δ_q^N)							
Reserve Scheme 3 9/11/74 - 11/27/74 (12 observations)			Standard Deviation				
$\delta_1^{N*} = \frac{(CD + CP + SF)_{t,\ <4\ months}}{(CD + CP + SF)_t}$[2]		.76012	.05882	.07739	-.00570	.00570	.00980
$\delta_2^N = \frac{(T_{>\$5m} + CD + CP + SF)_{t,\ >4\ months}}{(T_{>\$5m} + CD + CP + SF)_t}$[2]		.69738	.02407	.03452	.00572	.00572	.00308
Reserve Scheme 4 12/04/74 - 12/25/74 (4 observations)							
$\delta_3^N = \frac{(T_{>\$5m} + CD + CP + SF)_{t,\ 30\text{-}179\ days}}{(T + CD + CP + SF)_t}$		.40032	.00363	.00906	.00277	.00277	.00034

Table 22. Continued

	Mean	Standard Deviation	Coefficient of Variation	First Differences: Mean	First Differences: Mean of Absolute Value	First Differences: Standard Deviation
Reserve Scheme 4 - cont'd.						
$\delta_4^N = \frac{(T_{>\$5m} + CD + CP + SF)_{t,\ >179\ days}}{(T + CD + CP + SF)_t}$	.19305	.00030	.00155	-.00013	.00032	.00042
3. Nondeposit Liabilities (α_q^N)						
Reserve Schemes 1&2 6/13/73 - 11/27/74[‡] (77 observations)						
$\alpha_2^N\ (\alpha_1^{N*}) = \frac{(CD + CP + SF)_t}{D_t^P}$	.56670	.07101	.12509	.00268	.00936	.01193
Reserve Scheme 3 9/11/74 - 11/27/74 (12 observations)						
$\alpha_3^N = \frac{(T_{>\$5m} + CD + CP + SF)_t}{D_t^P}$	1.63796	.02493	.01522	.00231	.01329	.02220
Reserve Scheme 4 10/22/69 - 12/25/74 (271 observations)						
$\alpha_4^N = \frac{(T + CD + CP + SF)_t}{D_t^P}$ [3]	1.27025	.17421	.13714	.00208	.01530	.01936

Table 22. Continued

					First Differences		
		Mean	Standard Deviation	Coefficient of Variation	Mean	Mean of Absolute Value	Standard Deviation
Other Nondeposit Liability Categories							
$\frac{(CD)_t}{D^p_t}$,	6/13/73 - 11/27/74 (77 observations)	.55709	.07229	.12976	.00270	.00913	.01162
$\frac{(CP + SF)_t}{D^p_t}$,	10/22/69 - 12/25/74 (271 observations)	.00707	.00475	.67103	.00001	.00047	.00182

[1]The structural change in reserve requirements actually occurred on September 19, 1974, but the values of the parameters are based on all available observations.

[2]The data for the disaggregation of commercial paper and sales finance bills into those maturing in less than four months and in more than four months are not available; therefore the data used in calculating all parameters are for (CP + SF), maturing in more than thirty days.

[3]This parameter α^n_4 is actually only relevant beginning December 4, 1974, but its value is based on all available observations, beginning October 22, 1969.

*Not sufficient observations to calculate.

in Table 21. In some instances this is merely because of lagged reserve requirements; in other cases, it is because the dates for which data are available do not match the dates for which reserve schemes were in force. In each case, all available observations of data were used to calculate the parameters. It should be noted that all of the parameters calculated for Reserve Scheme 3 and the λ^N- and δ^N-parameters in Reserve Scheme 4 are based on very few observations of data.

Some authors[18] have asserted that the various categories of nondeposit liabilities are quite volatile and therefore that application of reserve requirements in general to nondeposit liabilities, and specifically lagged and differential reserve requirements, introduces highly variable parameters into the reserve ratio. Given the many definitional and data problems described above, it is difficult to present any conclusive evidence on the behavior of $\lambda^N_{q,t}$, $\delta^N_{q,t}$, and $\alpha^N_{q,t}$. It is however possible to compare the results for lagged and differential reserve requirements against nondeposit liabilities to similar parameters for demand and time deposits to see whether λ^N_q and δ^N_q are highly variable.

Consider first the λ^N-parameters which are reported in the first part of Table 22. The results in Table 22 indicate that the variation in $\lambda^N_{q,t}$ has not been large on an absolute scale. The coefficient of variation is largest for $\lambda^N_{2,t}$, corresponding to all three categories of liabilities; the coefficients of variation for $\lambda^N_{3,t}$ and $\lambda^N_{4,t}$ in

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[18]See, for example, Burger, p. 57.

Reserve Scheme 3 are both smaller and when nondeposit liabilities are combined with time deposits over \$5 million in Reserve Scheme 4 ($\lambda^N_{5,t}$ and $\lambda^N_{6,t}$) the λ^N-parameters are even more stable. The mean of $\lambda^N_{q,t}$ is considerably below one for all but q = 3 (corresponding to nondeposit liabilities maturing in less than four months); this reflects the overall growth in nondeposit liabilities. Thus the average deviation of $\lambda^N_{q,t}$ from its neutral value of 1.0 has been relatively large for all but q = 3.

The average deviation from one of $\lambda^N_{q,t}$ is many times larger than that for any λ^D- or λ^T-parameter (see Tables 1 and 4) for all q. This is due to the unusually high rate of growth in nondeposit liabilities in comparison to any category of deposits. The coefficients of variation for $\lambda^N_{q,t}$ are however not large relative to those for $\lambda^D_{j,t}$ or $\lambda^T_{i,t}$. Except for $\lambda^N_{1,t}$, the coefficients of variation for $\lambda^N_{q,t}$ are of the same general size as those for $\lambda^D_{j,t}$ and $\lambda^T_{i,t}$; even the coefficient of variation for $\lambda^N_{1,t}$ is exceeded by that for five of the λ^D-parameters.

The mean of the first difference of $\lambda^N_{q,t}$ indicates sizable weekly changes in $\lambda^N_{q,t}$ for all q. The mean of ($\lambda^N_{q,t}$ - $\lambda^N_{q,t-1}$) is much larger for all q than the mean of the first difference of any λ^D- or λ^T-parameter (see Tables 2 and 4). The mean of the absolute value and the standard deviation of the first differences of $\lambda^N_{q,t}$ are both small for all q relative to $\lambda^D_{j,t}$. It appears that the rate of growth (and changes in the rate of growth) in nondeposit liabilities has been high relative to any deposit category and therefore the average deviation of $\lambda^N_{q,t}$ from one and the mean of ($\lambda^N_{q,t}$ - $\lambda^N_{q,t-1}$) are large relative to other λ-parameters. The weekly fluctuations that are characteristic of $\lambda^D_{j,t}$ and $\lambda^T_{i,t}$ however are smaller for $\lambda^N_{q,t}$ so that the variation in the level and first differences of $\lambda^N_{q,t}$ is not large relative to other λ-parameters.

The end of part one of Table 22 gives statistics for two additional categories of nondeposit liabilities, certificates of deposit and the sum of commercial paper and sales finance bills. As would be expected, these two categories show a lot of variation relative to the other $\lambda^N_{q,t}$ categories. This is true for both levels and first differences and is especially true of commercial paper and sales finance bills.

The results for $\delta^N_{q,t}$, the parameters representing the application of differential reserve requirements to nondeposit liabilities, are presented in part two of Table 22. The δ^N-parameters for Reserve Scheme 3, vary more than those for Reserve Scheme 4. This is apparent from the coefficient of variation for the levels of $\delta^N_{q,t}$, as well as from the size and standard deviation of the first differences of $\delta^N_{q,t}$. Of course the small number of observations involved in both reserve schemes makes this a tenuous result. Comparing part two of Table 22 to part one indicates that the $\delta^N_{q,t}$ vary more than $\lambda^N_{q,t}$ for all q; this is, in general, true for levels as well as first differences.

The coefficient of variation for $\delta^N_{q,t}$, for all q, is not large on an absolute scale or relative to the coefficients of variation of $\delta^D_{j,t}$ or $\delta^T_{i,t}$. The mean of the first differences of $\delta^N_{q,t}$ is however quite large and is much larger than the mean of the first differences of $\delta^D_{j,t}$ or $\delta^T_{i,t}$. Thus the weekly changes in $\delta^N_{q,t}$ are large but the standard deviation of these changes is not particularly large relative to that for $\delta^D_{j,t}$. The mean of $|\delta^N_{q,t} - \delta^N_{q,t-1}|$ is the same as the mean of $(\delta^N_{q,t} - \delta^N_{q,t-1})$ so that weekly fluctuations characteristic of the other δ-parameters do not occur in $\delta^N_{q,t}$.

Part 3 of Table 22 gives the results for $\alpha^N_{q,t}$ which denote the ratio of each category of nondeposit liabilities to privately-owned demand deposits. The $\alpha^N_{q,t}$ show considerable variation on an absolute scale as well as relative to the nonmoney deposits parameters, γ_t, ι_t, τ_t (see Table 17). Only the standard deviation of τ_t exceeds that for $\alpha^N_{q,t}$ for all q. The mean of the first differences of $\alpha^N_{q,t}$ is also large for all q, larger than that for any of the nonmoney deposit parameters. The mean of the absolute value and the standard deviation of the first differences of $\alpha^N_{q,t}$ are also large; they consistently exceed comparable measures of variation for γ_t and ι_t and are close to or larger than those for τ_t except under Reserve Scheme 1.

It is apparent that the categories of nondeposit liabilities relative to privately-owned demand deposits vary considerably relative to other nonmoney items and therefore the application of reserve requirements may well have introduced highly variable parameters into the reserve ratio. It does not appear however that the λ^N- and δ^N-parameters are any more variable than comparable parameters for deposit categories. The limited analysis here however does not represent conclusive evidence, especially in light of the data problems discussed above. Unfortunately the lack of comprehensive, consistent data on these liabilities makes further analysis difficult; consequently in the empirical work that follows reserve requirements against nondeposit liabilities are ignored.

Excess Reserves

Since member bank excess reserves absorb base money like required reserves without "supporting" any part of the money stock, changes in the level of excess reserves will cause variation in r_t. A given level of base money will correspond, all other things equal, to a smaller money stock, the higher the level of excess reserves member banks choose to hold. The control problem presented by excess reserves is reflected here in the parameter ε_t.

A plot of ε_t shows it is extremely stable relative to the other parameters. As can be seen in Table 23, ε_t and its variation are both very small. Its standard deviation is only .00093, the mean and standard deviation of $(\varepsilon_t - \varepsilon_{t-1})$ are small, -.00001, and .00061, and the mean of $|\varepsilon_t - \varepsilon_{t-1}|$ is only .00047. The relative stability of ε_t is demonstrated by the fact that $(\varepsilon_t - \varepsilon_{t-1})$ equalled or exceeded .00015 only 21 times in the 729 observations and never exceeded .0027. Thus ε_t and its first differences vary much less than any of the other parameters in r_t. Due to the small mean of ε_t, its coefficient of variation is high relative to the other parameters.

The trend during the sample period has been for ε_t to decline. This can be seen in Table 24 which gives annual statistics for ε_t; the mean of ε_t dropped every year except 1973. A plot of ε_t displays a tendency for the standard deviation of ε_t to be larger after 1968; the average deviation of ε_t from its mean is no larger in the latter years of the sample period, but the frequency of such deviations is much higher.

The best explanation of the fall in ε_t is no doubt the overall increase in interest rates during the sample period. With the exception

Table 23. Excess Reserves (Values and First Differences of the Parameter, ε_t) (1961 - 1974)

	Mean	Mean of Absolute Value	Standard Deviation	Coefficient of Variation	Largest Weekly Change
1. Values of the Parameter, ε_t (730 observations)	.00127		.00093	.73228	
2. First Differences of the Parameter, ε_t (729 observations)	-.00001	.00047	.00061		-.00268

Table 24. Annual[1] Figures for ε_t

	Mean	Standard Deviation	Percentage Change from Previous Year	
			Mean	Standard Deviation
1961	.00320	.00061	--	--
1962	.00254	.00033	-20.625	-45.902
1963	.00206	.00050	-18.898	51.515
1964[2]	.00172	.00043	-16.505	-14.000
1965	.00149	.00036	-13.372	-16.279
1966	.00132	.00046	-11.409	27.778
1967	.00127	.00040	-3.788	-13.043
1968	.00107	.00043	-15.748	7.500
1969[2]	.00075	.00043	-29.907	0.000
1970	.00056	.00030	-25.333	-30.233
1971	.00052	.00035	-7.143	16.667
1972	.00047	.00038	-9.615	8.571
1973	.00048	.00040	2.128	5.263
1974	.00034	.00025	-29.167	-37.500

[1]Based on 52 observations per year unless otherwise indicated.

[2]Based on 53 observations per year.

of 1970, each of the years showing the largest drop in ε_t was also a year of high and/or rising interest rates. The two years in the sample period during which interest rates were low and did not rise, 1967 and 1971, were also years of relatively small declines in ε_t. This relationship will be explored further in Chapter 6.

Both the decline in ε_t and its increased variation may however be artificially caused by the carry-over procedure introduced in the reserve requirement system in 1968. Since the carry-over practice has been allowed, the weekly figures for excess reserves are often negative which would exaggerate both the fall in ε_t and the increase in its variation. The fact that the mean of ε_t fell drastically in 1969 and 1970 seems to support that hypothesis. On the other hand, annual increases in either the standard deviation or coefficient of variation display no pattern to indicate that their increases were distorted in 1968 by the introduction of the carry-over procedure. In addition, the procedures used in 1972 to ease member banks into the new graduated reserve requirement structure also caused distortion in the data on excess reserves.

The values of ε_t follow a discernible seasonal pattern, as can be seen from the quarterly figures given in Table D-6. The largest quarterly mean occurs in the first quarter of nine years and the smallest quarterly mean occurs in the fourth quarter. The first and fourth quarters show the most fluctuation in ε_t; those two quarters record the highest standard deviation and coefficient of variation in all but three and four years, respectively. This seasonal behavior will be further analyzed in Chapter 6.

CHAPTER 5

THE VARIATION IN r_t

Historical Variation in r_t

Five different reserve requirement schemes were imposed on member bank deposits during the sample period; these reserve systems will be referred to below as Reserve Schemes A through E. In Reserve Scheme A, demand deposits are divided into two categories, those at city banks and at country banks, and all time and savings deposits are included in one reserve category. Reserve Scheme B uses the same two reserve categories for demand deposits, but introduces different reserve ratios for time and for savings deposits.[1] Under Reserve Scheme C, demand deposits at city banks and at country banks are each divided into those less than \$5 million and greater than \$5 million; there are three time and savings deposits categories, saving deposits, and time deposits less than and greater than \$5 million. Reserve Scheme D employs the same seven reserve categories as Scheme C, but includes lagged reserve requirements. Reserve Scheme E, consists of the graduated reserve scheme for demand deposits.[2]

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[1] This does not correspond to the structural changes in reserve requirements. (See Table C-2, Appendix C). Time and savings deposits were divided into three categories (savings deposits, time deposits less than \$5 million, and time deposits greater than \$5 million) on July 14, 1966, but data for these categories are only available beginning January 10, 1968. Separate data on time deposits and savings deposits are available beginning September 7, 1966. Hereafter Scheme B is defined to correspond to available data rather than actual reserve categories.

[2] On December 12, 1974, time deposits were divided further, based on maturity length. But since the sample period only includes four observations after this change, it is ignored.

The dates defining the subperiods covered by each reserve scheme and the appropriate expression for r_t are summarized in Table 25. Each of these five subperiods represents a period of time during which no structural changes in reserve requirements occurred. It is, therefore, interesting to compare the amount of variation that has occurred in r_t under the various reserve schemes.

Table 26 gives the mean, standard deviation and coefficient of variation for r_t and each of the parameters in the expression for r_t for each reserve scheme. Comparing the standard deviation of r_t for each of the five reserve schemes shows variation in r_t fell when the first two structural changes were enacted. Upon the introduction of lagged reserve requirements in Reserve Scheme D, however, the variation in r_t increased and it increased again under Reserve Scheme E, after graduated reserve requirements were instituted. This result indicates that the latest structural changes in Federal Reserve reserve requirements, especially lagged requirements, have generated more variation in r_t.[3]

As can be seen in Table 26, neither the standard deviations nor the coefficients of variation for ν_t^D and ν_t^T are large relative to those for the other parameters in r_t. This indicates that the existence of non-member banks has not been a major source of variation in r_t for any of

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[3]The comparability of the variation in r_t between subperiods may be questioned because changes in legal reserve ratios cause variation in r_t too and of course the number of magnitude of those changes is not the same in each subperiod. No changes in legal reserve ratios occurred in subperiods A or C. In the other three subperiods however, the variance of $d_{i,t}$ is at most .000006 and variance of $t_{i,t}$ is at most .000002. Furthermore, it is not clear whether more or less variation in r_t for a particular reserve scheme can be attributed to the structural aspects of the reserve scheme alone, or to the particular time period it covers.

Table 25. Historical Specification of the Expression for r_t

Reserve Scheme	Dates (Number of Observations)	Espression for r_t
A	1/1/61-9/7/66 (296)	$r_{A,t} = \sum_{j=1,2} d_{j,t}\delta_{j,t}\nu_t^D\xi_t(1+\gamma_t+\iota_t) + t_{1,t}\nu_t^T\tau_t + \varepsilon_t\rho_t^m + \sum_{h=1}^{51}\psi_{h,t}\omega_{h,t}\rho_t^n$
B	9/7/66-1/11/68 (70)	$r_{B,t} = \sum_{j=1,2} d_{j,t}\delta_{j,t}^D\nu_t^D\xi_t(1+\gamma_t+\iota_t) + \sum_{i=2,3} t_{i,t}\delta_{i,t}^T\nu_t^T\tau_t + \varepsilon_t\rho_t^m + \sum_{h=1}^{51}\psi_{h,t}\omega_{h,t}\rho_t^n$
C	1/11/68-9/18/68 (36)	$r_{C,t} = \sum_{j=3}^{6} d_{j,t}\delta_{j,t}^D\nu_t^D\xi_t(1+\gamma_t+\iota_t)$ $\sum_{i=3}^{5} t_{i,t}\delta_{i,t}\nu_t^T\tau_t + \varepsilon_t\rho_t^m + \sum_{h=1}^{51}\psi_{h,t}\omega_{h,t}\rho_t^n$
D	9/18/68-11/9/72 (217)	$r_{D,t} = \sum_{j=3}^{6} d_{j,t}\lambda_{j,t}^D\delta_{j,t}^D\nu_t^D\xi_t(1+\gamma_t+\iota_t) + \sum_{i=3}^{5} t_{i,t}\lambda_{i,t}^T\delta_{i,t}^T\nu_t^T\tau_t + \varepsilon_t\rho_t^m + \sum_{h=1}^{51}\psi_{h,t}\omega_{h,t}\rho_t^n$
E	11/9/72-12/25/74 (111)	$r_{E,t} = \sum_{j=7}^{11} d_{j,t}\lambda_{j,t}^D\delta_{j,t}^D\nu_t^D\xi_t(1+\gamma_t+\iota_t) + \sum_{i=3}^{5} t_{i,t}\lambda_{i,t}^T\delta_{i,t}^T\nu_t^T\tau_t + \varepsilon_t\rho_t^m + \sum_{h=1}^{51}\psi_{h,t}\omega_{h,t}\rho_t^n$

Definitions of Symbols for Table 25

Subscripts j and i		Reserve Category	Reserve Scheme
j = 1,11;	j = 1	city banks	A & B
	j = 2	country banks	
	j = 3	city banks, <$5 million	C & D
	j = 4	city banks, >$5 million	
	j = 5	country banks, <$5 million	
	j = 6	country banks, >$5 million	
	j = 7	<$2 million	E
	j = 8	$2 million - $10 million	
	j = 9	$10 million - $100 million	
	j = 10	$100 million - $400 million	
	j = 11	>$400 million	
i = 1,5;	i = 1	total time and savings deposits	A
	i = 2	total time deposits	B
	i = 3	total savings deposits	B, C, D & E
	i = 4	time deposits, <$5 million	C, D & E
	i = 5	time deposits, >$5 million	C, D & E

$d_{j,t}$ = required reserve ratio against the j^{th} demand deposit category;

$t_{i,t}$ = required reserve ratio against the i^{th} time deposit category;

$$\lambda^D_{j,t} = \frac{\text{member bank net demand deposits in the } j^{th} \text{ reserve category, lagged two periods}}{\text{member bank net demand deposits in the } j^{th} \text{ reserve category, current period}};$$

$$\lambda^T_{i,t} = \frac{\text{member bank time deposits in the } i^{th} \text{ reserve category, lagged two periods}}{\text{member bank time deposits in the } i^{th} \text{ reserve category, current period}};$$

$$\delta^D_{j,t} = \frac{\text{member bank net demand deposits in the } j^{th} \text{ reserve category}}{\text{member bank net demand deposits}};$$

$$\delta^T_{i,t} = \frac{\text{member bank time deposits in the } i^{th} \text{ reserve category}}{\text{member bank time deposits}};$$

$$\nu^D_t = \frac{\text{member bank net demand deposits}}{\text{net demand deposits in all commercial banks}};$$

$$\nu^T_t = \frac{\text{member bank time deposits}}{\text{time deposits in all commercial banks}};$$

$$\xi_t = \frac{\text{net demand deposits in all commercial banks}}{\text{gross demand deposits in all commercial banks}};$$

Definitions of Symbols for Table 25 - continued

$$\gamma_t = \frac{\text{U. S. government demand deposits}}{\text{privately-owned demand deposits}};$$

$$\iota_t = \frac{\text{interbank demand deposits}}{\text{privately-owned demand deposits}};$$

$$\tau_t = \frac{\text{time deposits in all commercial banks}}{\text{privately-owned demand deposits}};$$

$$\varepsilon_t = \frac{\text{member bank excess reserves}}{\text{member bank total deposits}};$$

$$\rho_t^m = \frac{\text{member bank total deposits}}{\text{privately-owned demand deposits}};$$

$$\psi_{h,t} = \frac{\text{nonmember bank vault cash in the } h^{th} \text{ state}}{\text{nonmember bank total deposits in the } h^{th} \text{ state}};$$

$$\omega_{h,t} = \frac{\text{nonmember bank total deposits in the } h^{th} \text{ state}}{\text{nonmember bank total deposits}};$$

$$\rho_t^n = \frac{\text{nonmember bank total deposits}}{\text{privately-owned demand deposits}}.$$

Table 26. Historical Record of the Variation in r_t

	Mean	Standard Deviation	Coefficient of Variation		Mean	Standard Deviation	Coefficient of Variation
Reserve Scheme A: 1/1/61-9/7/66 (296)				Reserve Scheme B: 9/7/66-1/11/68 (70)			
$r_{A,t}$	.16474	.00345	.02094	$r_{B,t}$	.16532	.00296	.01790
$\delta^D_{1,t}$	.60970	.01475*	.02419**	$\delta^D_{1,t}$	.58788	.00495	.00842**
$\delta^D_{2,t}$	.39030	.01475*	.03779	$\delta^D_{2,t}$	.41212	.00495	.01201
				$\delta^T_{2,t}$	.53392	.02148*	.04023
				$\delta^T_{3,t}$	.46608	.02148*	.04609
ν^D_t	.84976	.00914	.01076**	ν^D_t	.83278	.00353	.00424
ν^{D*}_t	.82323	.00887	.01077	ν^{D*}_t	.80582	.00204**	.00253**
ν^T_t	.81659	.00399**	.00489**	ν^T_t	.81170	.00151**	.00186**
γ_t	.04771	.01591*	.33347*	γ_t	.03579	.00993	.27745*
ι_t	.12546	.00534**	.04256	ι_t	.12825	.00438	.03415
ι^*_t	.12089	.00553	.04576	ι^*_t	.12339	.00436	.03535

Table 26. Continued

	Mean	Standard Deviation	Coefficient of Variation		Mean	Standard Deviation	Coefficient of Variation
τ_t	.95520	.16188*	.17497*	τ_t	1.26122	.05612*	.04450*
ε_t	.00210	.00078**	.37143*	ε_t	.00129	.00045**	.34884*
ψ_t^1	.02354	.01314	.55817	ψ_t^1	.00940	.00994	1.05804
ω_t^1	.01961	.01625	.82864	ω_t^1	.00980	.01556	1.58688
Reserve Scheme C: 1/11/68–9/18/68 (36)				Reserve Scheme D: 9/18/68–11/9/72 (217)			
$r_{C,t}$	.17182	.00270	.01571	$r_{D,t}$	.17102	.00554	.03239
				$\lambda_{3,t}^D$	1.00013	.00188	.00188**
				$\lambda_{4,t}^D$	.99850	.02600	.02604
				$\lambda_{5,t}^D$	.99926	.00573	.00573
				$\lambda_{6,t}^D$	.99717	.02418	.02425
$\delta_{3,t}^D$	.00718	.00012**	.01671	$\delta_{3,t}^D$	.00637	.00040**	.06279
$\delta_{4,t}^D$	.57863	.00421	.00728**	$\delta_{4,t}^D$	.57277	.00676	.01180

Table 26. Continued

	Mean	Standard Deviation	Coefficient of Variation		Mean	Standard Deviation	Coefficient of Variation
$\delta^D_{5,t}$	.15520	.00172	.01108	$\delta^D_{5,t}$	.14442	.00625	.04328
$\delta^D_{6,t}$	.25900	.00371	.01432	$\delta^D_{6,t}$	.27644	.00994	.03596
				$\lambda^T_{3,t}$	.99816	.00493	.00494**
				$\lambda^T_{4,t}$	.99730	.00168**	.00168**
				$\lambda^T_{5,t}$	.99442	.01121	.01127
$\delta^T_{3,t}$	.49730	.00692	.01392	$\delta^T_{3,t}$	.44436	.03174*	.07143
$\delta^T_{4,t}$	.10017	.00129**	.01288	$\delta^T_{4,t}$	.10507	.00905	.08613
$\delta^T_{5,t}$	.40253	.00652	.01620	$\delta^T_{5,t}$	.45057	.03946*	.08750
ν^D_t	.83081	.00412	.00496	ν^D_t	.81020	.00979	.01208
ν^{D*}_t	.80229	.00260	.00324**	ν^{D*}_t	.78105	.01000	.01280
ν^T_t	.80426	.00245	.00305**	ν^T_t	.77564	.01068	.01377
γ_t	.04064	.01111*	.27338*	γ_t	.03970	.01132	.28514*

Table 26. Continued

	Mean	Standard Deviation	Coefficient of Variation		Mean	Standard Deviation	Coefficient of Variation
ι_t	.13050	.00533	.04084	ι_t	.15517	.01198	.07721
ι_t^*	.12506	.00556	.04443	ι_t^*	.14903	.01174	.07877
τ_t	1.33865	.03149*	.02352	τ_t	1.45761	.14454*	.09916*
ε_t	.00112	.00037**	.33036*	ε_t	.00059	.00049**	.66102*
ψ_t^1	.01879	.00451	.24000	ψ_t^1	.01872	.00653	.34856
ω_t^1	.01961	.01710*	.87199*	ω_t^1	.01961	.01718	.87630
Reserve Scheme E: 11/9/72–12/25/74 (111)							
$r_{E,t}$	.16669	.00622	.03731				
$\lambda_{7,t}^D$	.99971	.00155**	.00155**				
$\lambda_{8,t}^D$	.99888	.01238	.01239				
$\lambda_{9,t}^D$	.99904	.01960	.01962				
$\lambda_{10,t}^D$	.99973	.02211	.02212				

Table 26. Continued

	Mean	Standard Deviation	Coefficient of Variation		Mean	Standard Deviation	Coefficient of Variation
$\lambda^{D}_{11,t}$	.99847	.04793*	.04800				
$\delta^{D}_{7,t}$	.06699	.00160	.02388				
$\delta^{D}_{8,t}$	.14560	.00379	.02603				
$\delta^{D}_{9,t}$	.27850	.00388	.01393				
$\delta^{D}_{10,t}$	.21322	.00418	.01960				
$\delta^{D}_{11,t}$	.29569	.00832	.02814				
$\lambda^{T}_{3,t}$	.99928	.00400	.00400**				
$\lambda^{T}_{4,t}$	.99835	.00123**	.00123**				
$\lambda^{T}_{5,t}$	.99102	.00931	.00939				
$\delta^{T}_{3,t}$	.33821	.02662	.07871				
$\delta^{T}_{4,t}$	.08030	.00500	.06227				
$\delta^{T}_{5,t}$	.58148	.03155*	.05426				

Table 26. Continued

	Mean	Standard Deviation	Coefficient of Variation
ν_t^D	.78003	.00824	.01056
ν_t^{D*}	.74756	.00756	.01011
ν_t^T	.75793	.00362	.00478
γ_t	.03228	.01238	.38352*
ι_t	.15211	.01155	.07593
ι_t^*	.14524	.01148	.07903
τ_t	1.87574	.15095*	.08047*
ε_t	.00043	.00036**	.83721*
ψ_t^1	.01535	.00593	.38656
ω_t^1	.01961	.01761	.89818

[1]Based on 17 observations on 51 states for Reserve Scheme A; 4 observations on 51 states for Reserve Scheme B; 2 observations on 51 states for Reserve Scheme C. Based on 9 observations on 51 states for Reserve Scheme D; 6 observations on 51 states for Reserve Scheme E.

*Indicates three largest values.

**Indicates three smallest values.

the four reserve schemes. In the first three reserve schemes, ν_t^D and ν_t^T have lower coefficients of variation than any of the other parameters in the expression for r_t. In Reserve Schemes D and E, the coefficients of variation for ν_t^D and ν_t^T are smaller than those of all other parameters except some of the λ-parameters and one δ-parameter in Reserve Scheme D. The coefficients of variation for ν_t^D and ν_t^T are however exceeded by those for two λ-parameters in Reserve Scheme D and by those for four λ-parameters in Scheme E. The variation of the λ- and δ-parameters for individual reserve categories understates the total variation generated in r_t by lagged and differential reserve requirements. Therefore the above results show that in each of the reserve schemes, ν_t^D and ν_t^T display less variation than do the other parameters in r_t. There is also no support here for the claim that nonmember banks have become a more serious problem in recent years. The standard deviations of ν_t^D and ν_t^T are largest under Scheme D; their standard deviations in the time period covered by Scheme E are however smaller than during Scheme A. The parameters that show the consistently largest coefficients of variation for all reserve schemes are τ_t, γ_t, and ε_t, representing time and government deposits and excess reserves, respectively. In the first three reserve schemes, the variation of ω_t, representing the distribution of nonmember bank deposits between states, is also large.

While the variation of the parameters in the expression for r_t provides a way of comparing their relative variation, it does not measure the variation that each parameter causes in r_t. This is because each of the parameters is combined multiplicatively with the other parameters in the expression for r_t so that any change in a given parameter is first multiplied by several other parameters before it is translated into variation in r_t. That is, the variation in any given parameter causes variation in r_t that is first weighted by the parameters by which it is multiplied in the expression for r_t. In the next section a method is devised which deals with this problem.

"Partial Variation" of r_t

Variation in r_t is caused by simultaneous variation in all parameters in the expression for r_t. To isolate the effects on r_t of variation in each of the parameters alone, the values of all other parameters are held constant at their means, so that only the variation in one parameter causes variation in r_t. This will be referred to as the partial variance of r_t.

The following result for the variance of a linear combination of random variables is employed in deriving the partial variance of r_t:

If g is a function of random variables such that,

$$g(x_1, x_2, \ldots x_n) = \sum_{i=1}^{n} a_i x_i + c, \text{ then}$$

$$\text{(5-1)} \quad \text{var}(g) = \sum_{i=1}^{n} a_i^2 \text{ var}(x_i) + \sum_{\substack{i,j=1 \\ i \neq j}} a_i\, a_j \text{ cov}(x_i, x_j)^4.$$

Consider the following expression, taken from Table 25 above, for r_t for Reserve Scheme A with the λ-parameters included:

$$\text{(5-2)} \quad r_{A,t} = \sum_{j=1,2} d_{j,t} \lambda^D_{j,t} \delta^D_{j,t} \nu^D_t \xi_t \, (1 + \gamma_t + \iota_t) + t_{1,t} \lambda^T_{1,t} \nu^T_{1,t} \tau_t + \epsilon_t \rho^m_t + \sum_{h=1}^{51} \psi_{h,t} \omega_{h,t} \rho^n_t \,.$$

[4]Maurice G. Kendall and Alan Stuart, The Advanced Theory of Statistics, Volume 1 (London: Charles Griffin and Company Limited, 1963), p.231-3.

To derive the partial variance of $r_{A,t}$ due to lagged reserve requirements, denoted by var$_{\lambda}$ $(r_{A,t})$, the values of all parameters in $r_{A,t}$ but $\lambda^D_{1,t}$, $\lambda^D_{2,t}$ and $\lambda^T_{1,t}$ are held constant at their means. Representing the means of variables by a (-) over them, equation (5-2) for $r_{A,t}$ then becomes,

$$(5\text{-}3)\quad r_{A,t} = \sum_{j=1,2} \bar{d}_{j,t}\, \lambda^D_{j,t} \bar{\delta}^D_{j,t} \bar{\nu}^D_t \bar{\xi}_t \,(1 + \bar{\gamma}_t + \bar{\iota}_t) + \bar{t}_{1,t} \lambda_{1,t} \bar{\nu}^T_t\, \bar{\tau}_t + \bar{\varepsilon}_t \bar{\rho}^m_t + \sum_{h=1}^{51} \bar{\psi}_{h,t} \bar{\omega}_{h,t} \bar{\rho}^n_t\,.$$

Employing the result represented by equation (5-1), the partial variance of $r_{A,t}$ due to lagged reserve requirements is then,

$$\begin{aligned}(5\text{-}4)\quad \text{var}_{\lambda}\,(r_{A,t}) = {} & \left[\bar{\nu}^D_t \bar{\xi}_t (1 + \bar{\gamma}_t + \bar{\iota}_t)\right]^2 \left[\sum_{j=1,2} \bar{d}_{j,t} (\bar{\delta}_{j,t})^2 \text{ var } (\lambda^D_{j,t})\right. \\ & \left. + 2(\bar{d}_{1,t} \bar{d}_{2,t} \bar{\delta}^D_{1,t} \bar{\delta}^D_{2,t} \text{ cov } (\lambda^D_{1,t}, \lambda^D_{2,t})\right] \\ & + (\bar{\nu}^T\, \bar{\tau})^2 \left[\bar{t}^2_{1,t} (\bar{\delta}^T_{1,t})^2 \text{ var } (\lambda^T_{1,t})\right] \\ & + \left[2\bar{\nu}^D_t \bar{\xi}_t (1 + \bar{\gamma}_t + \bar{\iota}_t)\; \bar{\nu}^T_t\, \bar{\tau}_t\right] \\ & \left[\sum_{j=1,2} \bar{d}_{j,t}\; \bar{t}_{i,t} \bar{\delta}^D_{j,t} \text{ cov } (\lambda^D_{j,t}, \lambda^T_{1,t})\right].\end{aligned}$$

The partial variance of $r_{A,t}$ due to differential reserve requirements is analogously represented by:

$$\begin{aligned}(5\text{-}5)\quad \text{var}_{\delta}\,(r_{a,t}) = {} & \left[\bar{\nu}^D_t \bar{\xi}_t (1 + \bar{\gamma}_t + \bar{\iota}_t)\right]^2 \left[\sum_{j=1,2} \bar{d}^2_{j,t} \bar{\lambda}^{D^2}_{j,t} \text{ var } (\delta^D_{j,t})\right. \\ & \left. + \; 2(\bar{d}_{1,t} \bar{d}_{2,t} \bar{\lambda}^D_{1,t} \bar{\lambda}^D_{2,t} \text{ cov } (\delta^D_{1,t}, \delta^D_{2,t})\right].\end{aligned}$$

Table 27 shows the expressions for $var_\lambda(r_t)$, $var_\delta(r_t)$, $var_\nu(r_t)$, $var_\gamma(r_t)$, $var_\iota(r_t)$, $var_\tau(r_t)$, and $var_\varepsilon(r_t)$, the derivations for which are presented in Appendix E. The expressions for the partial variance of r_t are different for different reserve schemes as the definitions of j and i vary. This is detailed in the bottom part of Table 25.

Table 28 gives the results for the partial variance of r_t for each reserve scheme. The partial variance due to lagged reserve requirements is calculated for all reserve schemes even though lagged reserve requirements were not instituted until September 18, 1968. Schemes C and D are therefore combined into one subperiod, labelled Scheme D', containing 253 observations. Using the alternative definitions of ν_t^{D*}, ι_t^*, and ξ_t^* described above, the figures for $var_{\nu^*}(r_t)$ and $var_{\iota^*}(r_t)$ are also given in Table 28; using the values of ν_t^{D*}, ι_t^* and ξ_t^* however makes very little difference in the calculation of the other partial variances.

As can be seen in Table 28, the variation in r_t caused by lagged reserve requirements has increased steadily with each new reserve system.[5] Differential reserve requirements however caused the greatest variation in r_t under Reserve Scheme D' although the partial variance is larger under the current graduated reserve system than under either scheme based only on the city-country distinction for demand deposits.[6] The partial variance in r_t contributable to nonmember banks

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[5]This is true on an absolute scale, but not if $var_\lambda(r_t)$ is expressed relative to $var(r_t)$. The ratio $var_\lambda(r_t)/var(r_t)$ is largest for Scheme B and smallest for Scheme E. This is because $var(r_t)$ is larger for the periods of time covered by Schemes D' and E.

[6]The ordering between reserve schemes is again changed if $var_\delta(r_t)$ is expressed relative to $var(r_t)$. $Var_\delta(r_t)/var(r_t)$ is largest for Reserve Scheme B and smallest for E.

Table 27. Specification of Equations for the Partial Variance of r_t

Partial Variance of r_t	Due to
$\mathrm{var}_\lambda(r_t) = \left[\bar{\nu}^D_t \bar{\xi}_t (1 + \bar{\gamma}_t + \bar{\iota}_t)\right]^2 \Big[\sum_j (\bar{d}_{j,t})^2 (\bar{\delta}^D_{j,t})^2 \mathrm{var}(\lambda^D_{j,t})$ $+ \sum_j \sum_{j'}{}_{j \neq j'} \bar{d}_{j,t} \bar{d}_{j',t} \bar{\delta}^D_{j,t} \bar{\delta}^D_{j',t} \mathrm{cov}(\lambda^D_{j,t}\ \lambda^D_{j',t}) \Big]$ $+ \left[\bar{\nu}^T_t \bar{\tau}_t\right]^2 \Big[\sum_i (\bar{t}_{i,t})^2 (\bar{\delta}^T_{i,t})^2\ \mathrm{var}(\lambda^T_{i,t})$ $+ \sum_i \sum_{i'}{}_{i \neq i'} \bar{t}_{i,t} \bar{t}_{i',t} \bar{\delta}^T_{i,t} \bar{\delta}^T_{i',t} \mathrm{cov}(\lambda^T_{i,t}, \lambda^T_{i',t}) \Big]$ $+ \bar{\nu}^D_t \bar{\xi}_t (1 + \bar{\gamma}_t + \bar{\iota}_t) \bar{\nu}^T_t \bar{\tau} \Big[\sum_j \sum_i \bar{d}_{j,t} \bar{t}_{i,t} \bar{\delta}^D_{j,t} \bar{\delta}^T_{i,t}$ $\mathrm{cov}(\lambda^D_{j,t} \lambda^T_{i,t}) \Big]$	Lagged Reserve Requirements
$\mathrm{var}_\delta(r_t) = \left[\bar{\nu}^D_t \bar{\xi}_t (1 + \bar{\gamma}_t + \bar{\iota}_t)\right]^2 \Big[\sum_j (\bar{d}_{j,t})^2 (\bar{\lambda}^D_{j,t})^2$ $\mathrm{var}(\delta^D_{j,t}) + \sum_j \sum_{j'}{}_{j \neq j'} \bar{d}_{j,t} \bar{d}_{j',t} \bar{\lambda}^D_{j,t} \bar{\lambda}^D_{j',t}\ \mathrm{cov}(\delta^D_{j,t}, \delta^D_{j',t}) \Big]$	Differential Reserve Requirements

Table 27. Continued

Partial Variance of r_t	Due to
$+\left[\bar{\nu}^T_t\bar{\tau}_t\right]^2\left[\sum_i(\bar{t}_{i,t})^2\ (\bar{\lambda}^T_{i,t})^2\ \mathrm{var}(\delta^T_{i,t})\right.$	
$\left.+\sum_i\sum_{i'}{}_{i\neq i'}\ \bar{t}_{i,t}\bar{t}_{i',t}\bar{\lambda}^T_{i,t}\bar{\lambda}^T_{i',t}\ \mathrm{cov}(\delta^T_{i,t},\delta^T_{i',t})\right]$	
$+\left[\bar{\nu}^D_t\bar{\xi}_t(1+\bar{\gamma}_t+\bar{\iota}_t)\ \bar{\nu}^T_t\bar{\tau}_t\right]\left[\sum_j\sum_i\bar{d}_{j,t}\bar{t}_{i,t}\bar{\lambda}^D_{j,t}\bar{\lambda}^T_{i,t}\right.$	
$\left.\mathrm{cov}(\delta^D_{j,t},\delta^T_{i,t})\right]$	
$\mathrm{var}_\nu(r_t) = \left[\Sigma_j\bar{d}_{j,t}\bar{\lambda}^D_{j,t}\bar{\delta}^D_{j,t}\bar{\xi}_t(1+\bar{\gamma}_t+\bar{\iota}_t)\right]^2\mathrm{var}(\nu^D_t)$	Nonmember Banks
$+\left[\sum_i\bar{t}_{i,t}\bar{\lambda}^T_{i,t}\bar{\delta}^T_{i,t}\bar{\tau}_t\right]^2\ \mathrm{var}(\nu^T_t)$	
$+\left[2\sum_j d_{j,t}\bar{\lambda}_{j,t}\bar{\delta}_{j,t}\bar{\xi}_t(1+\bar{\gamma}_t+\bar{\iota}_t)\sum_i\bar{t}_{i,t}\bar{\lambda}^T_{i,t}\bar{\delta}^T_{i,t}\bar{\tau}_t\right]$	
$\mathrm{cov}(\nu^D_T,\nu^T_t)$	
$\mathrm{var}_\gamma(r_t) = \left[\sum_j\bar{d}_{j,t}\bar{\lambda}^D_{j,t}\bar{\delta}^D_{j,t}\bar{\nu}^D_t\bar{\xi}_t(1+\bar{\iota}_t)\right]^2\ \mathrm{var}(\gamma_t)$	Government Deposits

Table 27.

Partial Variance of r_t	Due to
$var_\iota(r_t) = \left[\sum_j \bar{d}_{j,t} \bar{\lambda}^D_{j,t} \bar{\delta}^D_{j,t} \bar{\nu}^D_t \bar{\xi}_t (1 + \bar{\gamma}_t)\right]^2 \; var(\iota_t)$	Interbank Deposits
$var_\tau(r_t) = \left[\sum_i \bar{t}_{i,t} \bar{\lambda}^T_{i,t} \bar{\delta}^T_{i,t} \bar{\nu}^T_t\right]^2 \; var(\tau_t)$	Time Deposits
$var_\varepsilon(r_t) = (\bar{\rho}^m_t)^2 \; var(\varepsilon_t)$	Excess Reserves

Table 28. Partial Variance of r_t
(All amounts are expressed in the form E -05)

	Reserve Scheme (Number of Observations)					
	A and B (366)			D' and E (364)		
	A (296)		B (70)	D' (253)		E (111)
$var_{\lambda}(r_t)$[1]	.3396		.4066	.4593		.5560
$var_{\delta}(r_t)$	.0299		.0668	.1000		.0880
$var_{\nu}(r_t)$	.1391		.0260	.5646		.1374
$var_{\nu *}(r_t)$	.1416		.0602	.6221		.1354
$var_{\gamma}(r_t)$		.2538		.1181		.1003
$var_{\iota}(r_t)$		.0289		.1873		.0874
$var_{\iota *}(r_t)$		.3156		1.9411		1.1106
$var_{\tau}(r_t)$	3.1964		.4377		6.1066	
$var_{\varepsilon}(r_t)$		.2312		.0935		.0819
$var(r_t)$[2]	1.298	1.310	1.212	2.793	3.533	3.864

[1]The sample period for each reserve scheme contains the indicated number of observations, minus two.

[2]The calculations of r_t here includes the λ-parameters for every reserve scheme, so the variance of r_t given here does not match that in Table 26 above.

was also highest under Reserve Scheme D'; this was a period during which the variance and covariance of ν_t^D and ν_t^T were especially large. Since 1972, however, the variance of ν_t^D and ν_t^T has declined so that $var_{\nu}(r_t)$ under Reserve Schemes A and E are nearly identical. The same pattern of results occurs for $var_{\nu*}(r_t)$ of r_t when the alternative definition of ν_t^{D*} is used.

The variance of r_t attributable to government deposits is largest for the first half of the sample period (for Reserves Schemes A and B), and is only half as large after January, 1968. The disturbance in r_t caused by interbank deposits is greatest during the latter part of the sample period, especially during the 1968-1972 period. The same pattern shows up in the results for $var_{\iota*}(r_t)$, but the variation in r_t caused by interbank deposits is much greater when the alternative definition, ι_t^* is used. The variation in r_t caused by time deposits is also greater for the latter years of the sample period. The disturbance attributable to excess reserves, however, is smaller during the end of the sample period, reflecting the relative low levels of (and therefore little variation in) excess reserves after 1967-68.

It can be seen from Table 28 that under every reserve scheme the major source of variation in r_t is variation in τ_t. In addition, differential reserve requirements are a minor source of disturbance in r_t under all four reserve schemes. Under both Reserve Schemes A and B, the second largest source of variation in r_t is lagged reserve reuirements, followed by government demand deposits (or interbank deposits, if the alternative definition, ι_t^*, is used). Under both of those schemes, the partial variance in r_t caused by nonmember banks is relatively small.

Under Reserve Scheme D', the disturbance in r_t caused by nonmember banks is much larger: $var_{\nu}(r_t)$ is larger than either $var_{\lambda}(r_t)$ or $var_{\delta}(r_t)$ and is only exceeded by the partial variance of r_t due to time deposits or to interbank deposits if the ι_t^* definition is used. Under Reserve Scheme E, $var_{\nu}(r_t)$ is again smaller and is again exceeded by $var_{\lambda}(r_t)$ as well as $var_{\tau}(r_t)$ and $var_{\iota}*(r_t)$.

In summary the partial variance of r_t caused by nonmember banks is of major importance only under Reserve Scheme D', during a period when ν_t^D and ν_t^T were particularly unstable. In each of the other reserve schemes the disturbance caused in r_t by nonmember banks is smaller than that caused by lagged reserve requirements and time deposits. On the other hand, $var_{\nu}(r_t)$ is not zero; the existence of nonmember banks clearly introduces some variation in r_t. The variation in r_t caused by differential reserve requirements is small for all reserve schemes; nonmember banks cause more disturbance than differential reserve requirements except during the very short life of Reserve Scheme B. The partial variance of r_t caused by both government deposits and excess reserves is much less during the latter years of the sample period, increasing the relative importance of $var_{\nu}(r_t)$.

A Naive Forecasting Model

It is possible that while the parameters in r_t vary, any one of them may vary in a sufficiently predictable fashion to pose no serious control problem. To investigate the predictability of the parameters in r_t, two different forecast models are employed. First, a naive forecasting model is used to calculate the error in r_t that results from assuming there is no change in each of the parameters. Second, in the next chapter a model is estimated for each of the parameters in r_t, which then provides a more sophisticated forecast for each parameter. The two forecasting experiments are then compared, using the loss in terms of accurate predictions of r_t as a criterion.

In a control situation, the simplest solution in week t to the lack of knowledge of the actual values of the parameters that comprise r_t, is to make a no-change assumption. This naive model provides a forecast value of r_t, denoted $\hat{r}_{1,t}$,

$$(5\text{-}6) \quad \hat{r}_{1,t} = \sum_j d_{j,t}\lambda^D_{j,t-1}\delta^D_{j,t-1}\nu^D_{t-1}\xi_{t-1}(1 + \gamma_{t-1} + \iota_{t-1}) + \sum_i t_{i,t}\lambda^T_{i,t-1}\delta^T_{i,t-1}\nu^T_{t-1}\tau_{t-1} + \varepsilon_{t-1}\rho^m_{t-1} + \sum_h \psi_{h,t-1}\omega_{h,t-1}\rho^n_{t-1}.$$ [7]

The average error in predicting r_t by this method is represented by the mean square error defined as,

$$(5\text{-}7) \quad MSE_1 = \frac{1}{N}\sum_{t=1}^{N}(r_t - \hat{r}_{1,t})^2,$$

where N is the number of observations. The value of MSE_1 for each reserve scheme is given in the first part of Table 29.

The loss, in terms of accurate forecasts of r_t, attributable to the naive forecasts of one parameter (or set of parameters) is then calculated by assuming perfect knowledge of it (them), while retaining the no-change assumption for the other parameters. For example, for lagged

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[7]The calculation of r_t, $\hat{r}_{1,t}$ and $\hat{r}_{2,t}$ (defined below) includes the λ-parameters for all reserve schemes, even though lagged reserve requirements were not in effect until Reserve Scheme D.

Table 29. Errors Resulting from the Naive Forecasting Model (Results of the Calculation of E_1)

	Reserve Scheme (Number of Observations)			
	A (100)	B (69)	D' (100)	E (111)
MSE_1	.7235 E-05	.7837 E-05	.7767 E-05	.9376 E-05
E_1^{λ}	-.0086	.2534	.2014	.3826
E_1^{δ}	-.0333	.0110	.0199	.0294
E_1^{ν}	-.0077	.0245	.0527	.0214
E_1^{γ}	.1858	.1266	.0577	.0930
E_1^{ι}	-.0672	-.0510	-.1808	-.0666
E_1^{τ}	.2666	.3117	.2677	.2903
E_1^{ε}	.2612	.1643	.1246	.2103

reserve requirements it is assumed that the current value of each λ-parameter is known, but that the best available estimate for the other parameters is their value in the previous week. Thus, $\hat{r}^{\lambda}_{2,t}$ is defined as

$$(5\text{-}8) \quad \hat{r}^{\lambda}_{2,t} = \sum_j d_{j,t}\lambda^D_{j,t}\delta^D_{j,t-1}\nu^D_{t-1}\xi_{t-1}(1 + \gamma_{t-1} + \iota_{t-1})$$

$$+ \sum_i t_{i,t}\lambda^T_{i,t}\delta^T_{i,t-1}\nu^T_{t-1}\tau_{t-1} + \varepsilon_{t-1}\rho^m_{t-1}$$

$$+ \sum_h \psi_{h,t-1}\omega_{h,t-1}\rho^n_{t-1}.$$

The error associated with this forecast of r_t is given by

$$(5\text{-}9) \quad MSE^{\lambda}_2 = \frac{1}{N}\sum_{t=1}^{N}(r_t - \hat{r}^{\lambda}_{2,t})^2.$$

Comparing MSE^{λ}_2 with MSE_1 shows the benefit (in terms of more accurate predictions of r_t) accruing from perfect knowledge of the λ-parameters. The value of MSE^{λ}_2 relative to MSE_1 also implies the error introduced into the forecast of r_t by the naive forecasts of the λ's. To facilitate this comparison, the error-coefficient E^{λ}_1 is defined as

$$(5\text{-}10) \quad E^{\lambda}_1 = \frac{MSE_1 - MSE^{\lambda}_2}{MSE_1}$$

$$= 1 - \frac{MSE^{\lambda}_2}{MSE_1} \quad .$$

A value of E^{λ}_1 close to one implies that the damage (in terms of poor forecasts of r_t) done by assuming no change in the λ's is large; or that the gain from perfect knowledge of the λ-parameters is large. A near-zero value of E^{λ}_1 indicates that a no-change assumption for the λ-parameters contributes only small errors to the forecast of r_t, or that the benefits of forecasting r_t from perfect knowledge of the λ's are small. A negative value of E_1 will occur if $MSE_2 > MSE_1$, which implies that the error in r_t is smaller with a no-change assumption for the λ's than with perfect knowledge of them. The concepts MSE_2 and E_1 are defined analogously for the other parameters in r_t.

The value of E_1 for each of the parameters in r_t for each reserve scheme is given in Table 29. For Reserve Schemes A and D', only the last 100 observations are used. Because of programming limitations, only 100 of the observations in Reserve Scheme A and D' could be used in the forecasting model described in succeeding chapters; the same limitation is therefore imposed here to make the results of the two forecasting experiments comparable.

None of the error-coefficients are especially close to one so apparently the error resulting from the naive forecasts is not large on an absolute scale for any of the parameters. The value of E_1^{λ} is relatively large for Schemes B and E; for Reserve Scheme E, the error-coefficient for the λ-parameters is larger than that for any other parameter. The partial variance of r_t also showed that lagged reserve requirements caused more disturbance in Reserve Scheme E than the others.

The value of E_1^{δ} is very small for all reserve schemes, although it is larger for each successive reserve scheme. The value of E_1^{ν} is also uniformly low for every scheme; it is largest under Scheme D', as is $var_{\nu}(r_t)$. These results indicate that the distribution of deposits between classes of member banks and between member and nonmember banks is relatively stable; therefore assuming no change in the δ-parameters and ν_t^D and ν_t^T introduces very little error in the forecast of r_t and even perfect knowledge of them provides little net benefit.

Several of the E-coefficients are negative under Scheme A and E_1^{ι} is consistently negative for all schemes. In these cases, perfect knowledge of the parameter(s) in question results in poorer forecasts of r_t than assuming no-change. The error-coefficient for the other nonmoney deposits and excess reserves are relatively large for all reserve schemes. The value of E_1^{γ} is smaller for each successive reserve scheme, as is $var_{\gamma}(r_t)$. Except for Reserve Scheme A, the largest error-coefficients are consistently E_1^{λ}, E_1^{τ}, and E_1^{ε}. This implies that the variation in time deposits, excess reserves, and the λ-parameters causes the largest errors in forecasting r_t by a no-change procedure; therefore the most benefit can be gained from more accurate forecasts of the λ's, τ_t and ε_t.

Summary of Findings

Comparing the three measures described above for the variation and predictability of the parameters in r_t, the results are not always consistent, but some general conclusions may be drawn. It is frequently claimed that the increase by the Federal Reserve in the number of classes of member banks has introduced increases variability into r_t. The results reported in this Chapter support this claim.

The value of $var_{\lambda}(r_t)$ is consistently larger for each successive reserve scheme; $var_{\delta}(r_t)$ in general rises as more reserve categories are added, although $var_{\delta}(r_t)$ for Scheme E is smaller than for D'. In addition E_1^{λ} is largest under the graduated reserve scheme and E_1^{δ} is consistently larger with more deposit categories. This means that the naive forecasts of the λ- and δ-parameters result in the largest errors under the graduated reserve scheme. Thus under the graduated scheme, more variation is introduced in r_t via the λ- and δ-parameters and easy forecasts of these parameters are less successful than under previous schemes.

It is also commonly claimed that nonmember banks cause less disturbance in r_t than do lagged and differential Federal Reserve reserve requirements. The results given in the first section of this Chapter support this contention; the standard deviations of ν_t^D and ν_t^T are in general smaller in all reserve schemes than those of the λ- and δ-parameters. The partial variance of r_t is also smaller for nonmember banks than for lagged reserve requirements, except under Reserve Scheme D'. The value of E_1^{ν} is also much smaller than E_1^{λ} for all reserve schemes; the naive forecasts of ν_t^D and ν_t^T are therefore much more successful than for the λ-parameters and much more can be gained from better forecasts of the λ's. It is concluded that lagged reserve requirements have in general caused more variation and unpredictability in r_t than nonmember banks.

For differential reserve requirements the results are less consistent. The $\text{var}_{\nu}(r_t)$ is larger than $\text{var}_{\delta}(r_t)$ for every reserve scheme except B. The naive forecasts of the δ-parameters also result in smaller errors than those for ν_t^D and ν_t^T except under Reserve Scheme E when E_1^{δ} and E_1^{ν} are nearly equal. In general the disturbance in r_t caused by differential reserve requirements is small and there is no convincing evidence here that they represent a more serious control problem than nonmember banks.

These results do not support the view that the disturbance caused by nonmember banks has increased in recent years. The standard deviations of ν_t^D and ν_t^T are smaller for the period covered by Reserve Scheme

A. The value of $var_{\nu}(r_t)$ is nearly equal for Schemes A and E. The value of E_1^{ν} is largest under Reserve Scheme D', but is nearly the same for Schemes B and E.

All three measures of variation show that nonmoney deposits, especially time deposits, and excess reserves are consistently serious sources of disruption in r_t. All three measures also imply that lagged reserve requirements are a more serious control problem than differential reserve requirements.

CHAPTER 6

ESTIMATION OF ARIMA MODELS FOR THE PARAMETERS IN r_t

Introduction

In this chapter a more sophisticated forecast equation is estimated for each parameter; these forecast results are then compared to the results in the last section of Chapter 5. The technique employed here is the time series analysis developed by George E. P. Box and Gwilym M. Jenkins.[1]

The first step in developing a time series model is to achieve stationarity in the series to be described. Stationarity requires that the series remain in equilibrium about a constant mean; that is, displacement through time has no effect on the joint probability distribution of any set of observed values. Consider, for example, a time series of observed values of y_t. The series y_t is said to be stationary if the joint probability distribution for m observations, y_1, $y_2 \ldots y_{t_m}$, is the same as that for the m observations, y_{1+k}, $y_{2+k}, \ldots y_{t_{m+k}}$, where k is some integer. In practice, stationarity is achieved by differencing the data. Thus even though the original series y_t may not be stationary, some degree, d, of ordinary differencing will in general make the series $\Delta^d y_t$ stationary. The differenced series $\Delta^d y_t$ may also be represented by the backward shift operator, B. The operator B is defined such that

$$(1 - B)_y = y_t - y_{t-1}.$$

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[1]George E. P. Box and Gwilym M. Jenkins, <u>Time Series Analysis: Forecasting and Control</u> (San Francisco: Holden - Day, Inc., 1976).

Therefore the d^{th} ordinary difference of y_t can also be represented by $(1 - B)^d y_t$. Most of the series analyzed in this study show pronounced seasonal behavior and therefore seasonal differences of the data also must usually be taken to ensure stationarity. The degree s of seasonal differences of y_t is denoted $\Delta^s y_t$; here the seasonal period is 52 so that,

$$\Delta^s y_t = y_t - y_{t-52}.$$

Henceforth, z_t will be used to represent the differenced series $\Delta^d \Delta^s y_t$ where d and s are of necessary degree to achieve stationarity in y_t. In practice, d and s are usually less than or equal to two.

The Box-Jenkins methodology consists of representing z_t by either a moving average model or an autoregressive model or a combination of both (referred to as an ARIMA model). In a moving average model the stationary process z_t is represented by the weighted sum of current and past disturbances, denoted here by a_t. That is, the moving average process is written,

$$(6\text{-}1) \quad z_t = a_y - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots.,$$

where θ_1 are called the moving average parameters from z_t. Typically it is assumed that $\theta_i = 0$ for $i > q$ where q is known as the order of the moving average process. Equation (6-1) then reduces to,

$$(6\text{-}2) \quad z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \ldots. \theta_q a_{t-q}.$$

A moving average process of order q is often denoted MA(q). In terms of the backward shift operator, an MA(q) process may also be written

$$(6\text{-}3) \quad z_t = (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q) a_t.$$

The invertibility condition for a MA(q) process is that the roots of equation,

$$(6\text{-}4) \qquad (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q) = 0$$

lie outside the unit circle.[2] There are no restrictions on the θ_i parameters needed to insure stationarity for the MA(q) process.

The autoregressive process is one in which z_t is represented by the current disturbance and a weighted sum of past observations of z_t. That is,

$$(6\text{-}5) \qquad z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \phi_3 z_{t-3} \ldots. + a_t.$$

The autoregressive parameters ϕ_i are assumed to be equal to zero for $i > p$, where p is the order of the autoregressive process. Thus the autoregressive process of order p, AR(p), is,

$$(6\text{-}6) \qquad z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \ldots. + \phi_p z_{t-p} + a_t.$$

The AR(p) process may also be written using the backward shift operator,

$$(6\text{-}7) \qquad (1 - \phi_1 B - \phi_2 B - \ldots. - \phi_p B^p)\, z_t = a_t.$$

In order for an AR(p) process to be stationary the roots of the equation,

$$(6\text{-}8) \qquad 1 - \phi_1 B - \phi_2 B^2 \ldots. - \phi_p B^p = 0,$$

B_i must satisfy the condition $|B_i| < 1$. No restrictions are required on the parameters ϕ_i to ensure invertibility.[3]

The stationary series z_t may also be represented by a mixed ARIMA process which is a combination of MA(q) and AR(p) processes so that,

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[2] ibid., p. 67. [3] ibid., pp. 53-54.

$$(6\text{-}9) \quad z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \ldots\ldots \theta_p z_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \ldots\ldots \phi_q a_{t-q}.$$

Equation (6-9) may also be written in terms of the backshift operative B so that,

$$(6\text{-}10) \quad (1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p) z_t = (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q) a_t.$$

To ensure stationarity and invertibility of a mixed process, the roots of equations (6-4) and (6-8) above must all lie outside the unit circle.[4]

Moving average, autoregressive or mixed processes will be denoted here by (p,d,q)(d,s), where p and q represent the orders of autoregressive and moving average processes, respectively; d is the degree of ordinary differing and s is the degree of seasonal differing needed to achieve stationarity.

Identification of the orders p and q is made by examining the autocorrelation and partical autocorrelation functions of z_t. The autocorrelation of z_t a lag k is defined to be

$$(6\text{-}11) \quad \rho_k = \frac{E[(z_t - \mu)\ (z_{t\ k} - \mu)]}{\sqrt{E}[(z_t - \mu)^2]E\ [(z_{t+k} - \mu)^2]}$$

$$= \frac{E[(z_t - \mu)\ (z_{t+k} - \mu)]}{\sigma_z},$$

where μ is the mean and σ_z is the standard deviation of the series z_t. The estimated value of ρ_k is given by,

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[4]ibid., pp. 73-74.

$$(6\text{-}12) \quad r_k = \frac{c_k}{c_o}, \text{ where}$$

$$c_k = \frac{1}{N} \sum_{t=1}^{N-k} (z_t - \bar{z})(z_{t+k} - \bar{z}) \quad \text{for } k = 0, 1, 2 \ldots K,$$

and $\bar{z}$ is the sample mean of z_t and N is the number of observations on z_t.

The partial autocorrelation function comes from expressing an AR(P) process as p nonzero functions of the autocorrelations.[5] These functions are the Yule-Walker equations which may be written,

$$(6\text{-}13) \quad \rho_1 = \phi_1 + \phi_2\rho_1 + \cdots \_ \phi_p\rho_{p-1}$$

$$\rho = \phi_1\rho_1 + \phi_2 + \cdots + \phi_p\rho_{p-2}$$

$$\vdots$$

$$\rho_p = \phi_1\rho_{p1]} + \phi_2\rho_{p-2} + \cdots + \phi_p.$$

The partial autocorrelations are then estimated by substituting the estimated values r_k for ρ_k and solving recursively for ϕ_i.

The values of p and q are implied by the autocorrelations and partial autocorrelations. In general the values of the autocorrelations will be large for $k < q$ only, so a standard for "largeness" is needed. It can be shown[6] that the variance of the estimated autocorrelation r_k can be approximated by,

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[5]For a more detailed derivation of the partial autocorrelation function see Box and Jenkins, ibid., pp. 54-56, 64-66.

[6]ibid., pp. 34-36.

$$\text{(6-14)} \quad \operatorname{var}(r_k) \simeq \frac{1}{N}\left[1 + 2 \sum_{i=1}^{q} r_i^2\right] \quad k > q.$$

The standard error of the estimated partial autocorrelation is approximated by $\frac{1}{\sqrt{N}}$.[7]

The size of the autocorrelations and partial autocorrelations, relative to their standard errors, are then used to determine the size of p, q, d, and s by following these general rules:

1) If both the autocorrelations and partial autocorrelations fail to tail off (remain large relative to their standard errors even at large values of k), the series needs further differencing.

2) A moving average process of order q is implied if the autocorrelations are large at lag 1, 2...q only and the partial autocorrelations tail off.

3) An autoregressive process of order p is implied if the autocorrelations decay exponentially and the partial autocorrelations are large at lags 1, 2...p only.

4) A mixed process of order p, q is implied if the autocorrelations decay exponentially after lag q and the partial autocorrelations tail off after lag p.

Once the values of p, q, d, and s are identified the values of ϕ_i and θ_i are estimated by an iterative nonlinear least squares procedure. The estimation procedure required initial values for the parameters which represents a starting point for the estimation process. The

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[7]Ibid., pp. 65-66.

preliminary parameter estimates used here were obtained by solving the equations provided by Box-Jenkins.[8]

The estimation results for an ARIMA model are evaluated like those for any regression model. The overall goal of the ARIMA process however is to specify the model so that all systematic behavior in the series, z_t, is removed and that the residuals of the ARIMA process, a_t, be "white noise." White noise is a series of random drawings from a fixed Normal distribution with zero mean and variance σ_a^2. The adequacy of an ARIMA process is therefore tested by determining whether or not its residuals are indeed white noise. This is done by examining the autocorrelations of the residuals of the model, denoted here by $r_{aa}(k)$.

In general it is desirable that the $r_{aa}(k)$ be "small," relative to their estimated standard error which is approximated by $\frac{1}{\sqrt{N}}$. It has been shown however that this approximation of the standard error is not very reliable,[9] especially at short lags, and should be considered an upper bound only. A more reliable and comprehensive test of whether a_t is white noise is to examine the first K autocorrelation $r_{aa}(k)$ $(k = 1, 2....K)$. The quantity, Q, is defined as,

$$Q = n \sum_{k=1}^{K} r_{aa}^2(k), \quad n = N - d - s.$$

It has been shown[10] that Q is distributed as $\chi^2(K-p-q)$. The value of Q is usually calculated for lags k = 1 - 10, k = 11 - 20, and k = 21 - 30. If the values of Z are less than the $\chi^2(K - p - q)$ values, the a_t

[8]ibid., pp. 176-77, 187-93, and Charts B, C, and D, pp. 518-20.

[9]ibid., p. 290. [10]ibid., p. 291.

can be considered white noise. If, however, the values of Q are large relative to the critical χ^2 values, there is reason to suspect that the model is not appropriately specified.

If there is reason to suspect the adequacy of the model, one way to improve it is through overfitting. Overfitting consists of refitting the data to a more elaborate model and comparing its results to the original results. Thus an autoregressive or moving average variable is added to the model, or the degree of ordinary differencing is changed to see if the model can be improved.

Multivariate ARIMA models consist of a noise model like that described above, which hopefully converts the dependent variable z_t to white noise, plus relevant independent variables. The independent variables are introduced in the form of a "transfer function" which hopefully converts the independent variable, $x_{i,t}$ to white noise while including in the process of impact of $x_{i,t}$ on z_t. Each transfer function may contain autoregressive and/or moving average variables of its own.

The procedure is to use the autocorrelations and partial autocorrelations of each independent variable to identify an ARIMA process that transforms $x_{i,t}$ to white noise (or as close to white noise as is reasonably possible). This ARIMA transformation is then applied to both $x_{i,t}$ and the dependent variable z_t, and the cross correlations between z_t and $x_{i,t}$ are calculated and used to identify the nature of the transfer function for $x_{i,t}$. Applying the same whitening transformation to $x_{i,t}$ and z_t is known as "pre-whitening" the input $x_{i,t}$.

The cross correlations between $x_{i,t}$ and z_t are defined by

$$(6\text{-}15)\quad \rho_{x,y}(k) = \frac{E[(x_t - \mu_x)(y_{t+k} - \mu_y)]}{\sigma_x \sigma_y}, \text{ for } k = 0, \pm 1, \pm 2 \ldots$$

where the μ's denote the means of the series and $\sigma_x \sigma_y$ is the covariance between x and y. Unlike the autocorrelations of a series, the cross correlations are not symmetric about zero and must therefore be calculated for both positive and negative lags. The estimated cross correlations are then calculated by,

$$(6\text{-}16)\quad r_{xy}(k) = \frac{c_{xy}(k)}{s_x s_y}, \quad k = 0, \pm 1, \pm 2, \ldots, \text{ where}$$

$$c_{xy}(k) = \frac{1}{n} \sum_{t=1}^{n-k} (x_t - \bar{x})(y_{t+k} - \bar{y}), \quad k = 0, 1, 2 \ldots$$

$$\frac{1}{n} \sum_{t=1}^{n+k} (x_{t-k} - \bar{x})(y_t - \bar{y}), \quad k = 0, -1, -2, \ldots,$$

where $\bar{x}$ and $\bar{y}$ are the sample means and $s_x = \sqrt{c_{xx}(0)}$, $s_y = \sqrt{c_{yy}(0)}$.

The estimated cross correlations between z_t and the "prewhitened" input $x_{i,t}$ give clues as to the form of the transfer function for $x_{i,t}$. Consider the general form of a transfer function model with one independent variable. It can be written in the form,

$$(6\text{-}17)\quad z_t = \delta^{-1}(B)\omega(B)x_{t-b} + N_t, \text{ where } N_t \text{ is the noise model for } z_t \text{ and}$$

$$\delta(B) = 1 - \delta_1 B - \delta_2 B^2 - \ldots - \delta_r B^r$$

$$\omega(B) = \omega_o - \omega_1 B - \ldots - \omega_u B^u.$$

The parameter b is a delay parameter representing the delay in the effect of x_t on z_t. The transfer function for x_t can therefore be represented by v(B) which is the ratio of two polynomials, in B,

$$(6\text{-}18)\quad v(B) = \left[\frac{\omega_o - \omega_1 B - \omega_2 B^2 - \ldots - \omega_u B^u}{1 - \delta_1 B - \delta_2 B^2 - \ldots - \delta_r B^r}\right] x_{t-B}$$

$$= (v_o + v_1 B + v_2 B^2 + \ldots) x_{t-b}.$$

It can be shown that the values of v_j are proportional to the cross correlations between z_t and the prewhitened input x_t.[11] Using the cross correlations, estimates of the v_j, $\hat{v}_j$ for j = 0, 1, 2... can be obtained and used to identify the values of r, u, and b in the transfer function. The series $\hat{v}_j$ will behave according to the following general rules:[12]

1) v_j will have b zero values, v_o, v_1, ... v_{b-1};
2) followed by $u - r + 1$ nonzero values, v_b, v_{b+1}, ... v_{b+u-r}, which follow no particular pattern (no such values occur if $u < r$);
3) v_j for $j \geq b + u - r + 1$ which follow the pattern dictated by an r^{th} order difference equation which has r starting values v_{b+u}, ... $v_{b+u-r+1}$.

The series $\hat{v}_j$ are also used to calculate initial values for the parameters δ_1, δ_2, ... δ_r and ω_o, ω_1, ... ω_u to be used in the estimation process. The transfer function will be denoted here by (r,u,b)(d,s) where r and u are the orders of the polynomials in the numerator and denominator of the transfer function, b is the delay parameter and d and s are again the orders of regular and seasonal differencing. The delay parameter is zero for all the series described here.

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[11] ibid., pp. 379-80.

[12] ibid., p. 378.

Evaluation of the multivariate results follows that for the univariate situation plus some standard for the adequacy of the transfer functions is needed. The cross correlations between the residuals of the noise model and the prewhitened x_t have the approximate standard error $\frac{1}{\sqrt{n}}$ which can be compared to the value of the cross correlations to identify those that are significantly different from zero. The estimated cross correlations between a_t and the prewhitened x_t can be used to calculate

$$S = n \sum_{k=0}^{K} - r^2_{a\alpha(x)}(k).$$

It has been shown[13] that S is distributed χ^2 with $K + 1 - (r + u + 1)$ degrees of freedom. Therefore if the values of S are small relative to the critical value of χ^2, there is no reason to question the adequacy of the transfer function.

A time series model of this type is now estimated to represent each of the parameters in r_t. These estimated models are described in the rest of this chapter. For some of the parameters in r_t, a noise model alone ("univariate" model) is sufficient; for some parameters, relevant independent variables are also introduced in the form of transfer functions ("multivariate" model).[14]

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[13]ibid., p. 395.

[14]The models are identified and estimated with computer programs written by David A. Pierce, Board of Governors of the Federal Reserve System.

Lagged Reserve Requirements ($\lambda^D_{j,t}$ and $\lambda^T_{i,t}$)

a) Demand Deposits

i) $\lambda^D_{i,t}$: There are 730 available observations for $\lambda^D_{i,t}$ and $\lambda^D_{2,t}$, but the computer program used to estimate ARIMA models is limited to 500 data observations. Since $\lambda^D_{1,t}$ and $\lambda^D_{2,t}$ enter the equation for r_t only in Reserve Schemes A and B, their ARIMA models are identified and estimated using the first half of the sample period, January 1, 1961 through January 10, 1968 (366 observations). This latter date corresponds to one of the major structural changes in reserve requirements described in Chapter 4.

Examination of the autocorrelations and partial autocorrelations for $\lambda^D_{1,t}$ indicate that the following ARIMA models are potentially useful in accounting for the behavior of $\lambda^D_{1,t}$:

(0,0,3)(0,1)
(0,0,3)(0,2)
(0,1,3)(1,1)
(0,2,3)(2,1).

The second seasonal difference of the data does not perform as well as the first in the model (0,0,3). In addition, the model (0,1,3)(1,1) performs better than (0,2,3)(2,1). The best results therefore occur with (0,0,3)(0,1) and (0,1,3)(1,1); both models have very low residual variance but they transform $\lambda^D_{1,t}$ to white noise at short lags only. The two models have therefore been overfitted with the models (1,0,3)(0,1) and (1,1,3)(1,1). In both cases, these models are slight improvements over the original ones; the augmented models however still do not result in white noise except at short lags.

For all four models (0,0,3)(0,1), (1,0,3)(0,1), (0,1,3)(1,1) and (1,1,3)(1,1), the residual autocorrelations are large at or near the seasonal lags 13 and 26. This implies that seasonal factors are affecting the behavior of $\lambda^D_{1,t}$ that are not removed by the seasonal differencing of the series. Therefore each of these models was rerun including a moving average variable at the appropriate seasonal lags. Adding the seasonal variables does not improve the performance of either (0,1,3)(1,1) or (1,1,3)(1,1); in each case the additional variables have either an adverse or a negligible affect on the R^2 and residual autocorrelations. For both (0,0,3)(0,1) and (1,0,3)(0,1) adding seasonal variables improves the model somewhat. The best result is obtained with (1,0,3)(0,1) plus seasonal variables at lags 13 and 23; the resulting model converts $\lambda^D_{1,t}$ to white noise at all lags and is therefore the best model for the series. The model's estimated coefficients satisfy the conditions for invertibility and stationarity; its results are given in Table 30.

ii) $\lambda^D_{2,t}$: The autocorrelation and partial autocorrelation functions for $\lambda^D_{2,t}$ indicate that the following models are of interest:

(0,0,3)(0,1)
(0,0,3)(0,2)
(0,1,2)(1,1)
(0,2,2)(2,1)
(0,2,2)(2,2).

Comparing these five models, the best is (0,0,3)(0,1). It has a high R^2, low residual variance and it transforms $\lambda^D_{2,t}$ to white noise at all lags; the estimated coefficients are significant and they satisfy invertibility conditions. Its results are given in Table 30.

iii) $\lambda^D_{3,t}$: The ARIMA models for $\lambda^D_{j,t}$, j = 3,11 are based on all the available observations of data (364) which cover the last half of

Table 30. ARIMA Results for $\lambda^D_{j,t}$, j=1,11 (Regression Results for the Best ARIMA Model Tested)

$\lambda^D_{1,t}$ (1,0,3)(0,1) plus seasonal variables at lags 13 and 23 (1/1/61-1/10/68)

$$\lambda^D_{1,t} = \underset{(-.9983)}{-.21487}\lambda^D_{1,t} + a_t - \underset{(-1.830)}{.38122}a_{t-1} + \underset{(4.580)}{.55898}a_{t-2} + \underset{(.9960)}{.09246}a_{t-3} - \underset{(-4.415)}{.17322}a_{t-13} + \underset{(3.156)}{.11730}a_{t-23}$$

R^2 = .739   Variance of Residuals = .1175 E-03   Durbin-Watson Statistic = 1.979

k	$n\Sigma r^2_{aa}(k)$
1-10	4.749
1-20	21.109
1-30	30.794

$\lambda^D_{2,t}$ (0,0,3)(0,1) (1/1/61-1/10/68

$$\lambda^D_{2,t} = a_t - \underset{(-16.34)}{.92341}a_{t-1} + \underset{(2.579)}{.19645}a_{t-2} + \underset{(2.516)}{.14387}a_{t-3}$$

k	$n\Sigma r^2_{aa}(k)$
1-10	6.780
1-20	22.259
1-30	31.480

R^2 = .812   Variance of Residuals = .2969 E-04   Durbin-Watson Statistic = 1.963

$\lambda^D_{3,t}$ (0,0,3)(0,1) (1/10/68-12/31/74)

$$\lambda^D_{3,t} = a_t - \underset{(-17.05)}{.72617}a_{t-1} + \underset{(5.226)}{.35731}a_{t-2} + \underset{(1.671)}{.07478}a_{t-3}$$

k	$n\Sigma r^2_{aa}(k)$
1-10	19.623
1-20	38.507
1-30	47.590

R^2 = .828   Variance of Residuals = .6573 E-05   Durbin-Watson Statistic = 1.999

Table 30. Continued

$\lambda^D_{4,t}$ (1,0,3)(0,1) plus a seasonal variable at lag 13 (1/10/68–12/31/74)	k	$n\Sigma r^2_{aa}(k)$
$\lambda^D_{4,t} = -.39818\ \lambda^D_{4,t-1} + a_t - .15067a_{t-1} + .62822a_{t-2} + .08525a_{t-3} - .18427a_{t-13}$	1–10	10.303
(−2.502) (−.9550) (8.942) (.9198) (−4.366)	1–20	22.833
	1–30	39.046
R^2 = .654 Variance of Residuals = .2467 E-03 Durbin-Watson Statistic = 1.990		

$\lambda^D_{5,t}$ (0,0,3)(0,1) (1/10/68–12/31/74)	k	$n\Sigma r^2_{aa}(k)$
$\lambda^D_{5,t} = a_t - .78990a_{t-1} + .34200a_{t-2} + .13868a_{t-3}$	1–10	8.747
(−14.36) (4.892) (2.425)	1–20	21.234
	1–30	33.080
R^2 = .807 Variance of Residuals = .1231 E-04 Durbin-Watson Statistic = 1.980		

$\lambda^D_{6,t}$ (0,1,2)(1,1) (1/10/68–12/31/74)	k	$n\Sigma r^2_{aa}(k)$
$\Delta\lambda^D_{6,t} = a_t - .00525a_{t-1} + .93714a_{t-2}$	1–10	12.996
(−.2525) (44.98)	1–20	23.752
	1–30	33.759
R^2 = .851 Variance of Residuals = .9295 E-04 Durbin-Watson Statistic = 2.031		

$\lambda^D_{7,t}$ (0,0,2)(0,1) (1/10/68–12/31/74)	k	$n\Sigma r^2_{aa}(k)$
$\lambda^D_{7,t} = a_t - .85956a_{t-1} + .13850a_{t-2}$	1–10	16.980
(−17.37) (2.567)	1–20	33.676
	1–30	42.369
R^2 = .947 Variance of Residuals = .1549 E-05 Durbin-Watson Statistic = 1.981		

Table 30. Continued

	k	$n\Sigma r^2_{aa}(k)$
$\lambda^D_{8,t}$ (0,0,3)(0,1) plus a seasonal variable at lag 14 (1/10/68–12/31/74)		
$\lambda^D_{8,t} = a_t - .79244a_{t-1} + .38308a_{t-2} + .22305a_{t-3} + .03181a_{t-14}$	1-10	8.379
(-14.28) (5.613) (3.877) (1.507)	1-20	20.790
	1-30	31.210
$R^2 = .732$ Variance of Residuals = .4471 E-04 Durbin-Watson Statistic = 2.004		
$\lambda^D_{9,t}$ (0,0,3)(0,1) (1/10/68–12/31/74)	k	$n\Sigma r^2_{aa}(k)$
$\lambda^D_{9,t} = a_t - .82088a_{t-1} + .28508a_{t-2} + .20992a_{t-3}$	1-10	7.385
(-14.70) (4.014) (3.732)	1-20	20.257
	1-30	27.570
$R^2 = .867$ Variance of Residuals = .4151 E-04 Durbin-Watson Statistic = 1.963		
$\lambda^D_{10,t}$ (0,1,2)(1,1) plus a seasonal variable at lag 13 (1/10/68–12/31/74)	k	$n\Sigma r^2_{aa}(k)$
$\Delta\lambda^D_{10,t} = a_t + .06321a_{t-1} + .92131a_{t-2} + .11223a_{t-13}$	1-10	10.093
(2.302) (43.61) (-4.017)	1-20	22.505
	1-30	34.217
$R^2 = .673$ Variance of Residuals = .1251 E-03 Durbin-Watson Statistic = 2.063		
$\lambda^D_{11,t}$ (1,0,3)(0,1) plus a seasonal variable at lag 13 (1/10/68–12/31/74)	k	$n\Sigma r^2_{aa}(k)$
$\lambda^D_{11,t} = -.62022\,\lambda^D_{11,t-1} + a_t + .14186a_{t-1} + .77080a_{t-2} - .03918a_{t-3} - .10846a_{t-13}$	1-10	7.156
	1-20	30.244
(-4.968) (1.039) (17.28) (-.3831) (-2.903)	1-30	46.760
$R^2 = .622$ Variance of Residuals = .7661 E-03 Durbin-Watson Statistic = 2.003		

Note: For simplicity, the degree of seasonal differencing (Δ^s) is omitted from the regressive equations. The degree of seasonal differencing is denoted in the (p,d,q)(d,s) description for each parameter.

the sample period, from January 10, 1968 through December 31, 1974. The autocorrelation and partial autocorrelation functions for $\lambda^D_{3,t}$ imply that the following models are potentially useful in describing the behavior of $\lambda^D_{3,t}$:

(0,0,3)(0,0)
(0,0,3)(0,1)
(0,0,2)(0,2)
(0,1,2)(1,0)
(0,1,2)(1,1)
(0,1,2)(1,2)
(0,2,3)(2,0)
(0,2,3)(2,1)
(0,2,3)(2,2).

Each of the above models was fitted to $\lambda^D_{3,t}$; the best results occur with (0,0,3)(0,1), (0,1,2)(1,0) and (0,2,3)(2,0). Each of the models have a relatively high R^2, significant coefficients, and the variance of residuals is small, but none of them transform $\lambda^D_{3,t}$ to white noise. Each of the three models have therefore been overfitted by the following models:

<u>(0,0,3)(0,1)</u>:
(0,1,3)(1,1)
(1,1,3)(1,1)
(1,0,3)(0,1);

<u>(0,1,2)(1,0)</u>:
(0,2,2)(2,0)
(0,0,2)(0,0)
(1,1,2)(1,0)
(0,1,3)(1,0)
(1,1,3)(1,0)
(2,1,3)(1,0);

<u>(0,2,3)(2,0)</u>:
(1,2,3)(2,0)
(2,2,3)(2,0)
(3,2,3)(2,0).

None of the models overfitted to (0,0,3)(0,1) give better results than it does, even when variables are added at the seasonal lags indicated by the residual autocorrelations. The autocorrelations of residuals for (0,0,3)(0,1) are large at lags 3 and 12 so the process (3,0,3)(0,1) was fitted, as well as (0,0,3)(0,1) plus a variable at lag 12; neither of these models is however at all successful.

Consider now the models overfitted to (0,1,2)(1,0); (1,1,2)(1,0), (0,1,3)(1,0), and (1,1,3)(1,0) are all improvements in that their residuals are closer to white noise, but the estimated coefficients of all three models violate the invertibility condition. Even if relevant seasonal variables are added, the invertibility condition is violated. The estimated coefficients of the models overfitted to (0,2,3)(2,0) also violate invertibility conditions.

Of all the processes tested for $\lambda^D_{3,t}$ only (0,0,3)(0,1), (0,1,2)(2,0), and (0,2,3)(2,0) satisfy invertibility conditions. The best model among these three is (0,0,3)(0,1); its results are given in Table 30. Although it does not result in white noise, this is apparently the best invertible model than can be constructed for $\lambda^D_{3,t}$.

iv) $\lambda^D_{4,t}$: The autocorrelation and partial autocorrelation functions for $\lambda^D_{4,t}$ imply that the following ARIMA models may describe the behavior of λ^D_4,t:

(0,0,3)(0,1)
(2,0,3)(0,1)
(0,1,2)(1,1)
(0,1,2)(1,2)
(0,2,3)(2,1)
(0,2,3)(2,2).

Each of the above models were fitted to the series $\lambda^D_{4,t}$; the best results occur with the first seasonal difference of the series so the other two models are not pursued further. The R^2 for all models is however quite small and none transforms $\lambda^D_{4,t}$ to white noise at long lags.

Each of the four models that uses the first seasonal difference has therefore been overfitted by the following models:

<u>(0,0,3)(0,1)</u>:
(0,1,3)(1,1)
(1,0,3)(0,1)
(1,1,3)(1,1);

<u>(2,0,3)(0,1)</u>:
(2,1.3)(1,1)
(3,0,3)(0,1);

<u>(0,1,2)(1,1)</u>:
(0,0,2)(0,1)
(0,2,2)(2,1)
(1,1,2)(1,1)
(1,0,2)(0,1);

<u>(0,2,3)(2,1)</u>:
(1,2,3)(2,1).

Of all these processes, the best results are obtained with (1,0,3)(0,1), (2,0,3)(0,1), and (1,0,2)(0,1). The results for these three models are quite similar; none converts $\lambda^D_{4,t}$ to white noise except at short lags. The autocorrelations of the residuals are large at or near seasonal lags such as 13 and 26, so all three models were refitted including moving average variables at the appropriate seasonal lags. Adding the seasonal variables improves the performance of each model slightly. The best model is (1,0,3)(0,1) plus a variable at lag 13; it converts $\lambda^D_{4,t}$ to white noise at the 5% level of significance for all lags and its estimated coefficients satisfy the invertibility condition. Its results are given in Table 30.

v) $\lambda^D_{5,t}$: The autocorrelation and partial autocorrelation functions for $\lambda^D_{5,t}$ reveal that the following models are of interest:

(0,0,2)(0,1)
(0,0,2)(0,2)
(0,1,3)(1,1)
(0,1,2)(1,2)
(0,2,3)(2,1)
(0,2,3)(2,2).

The second seasonal differencing of $\lambda^D_{5,t}$ in general yields poor results; of the remaining three models, the best results are obtained with (0,1,3)(1,1) and (0,0,2)(0,1). They both have significant coefficients and low residual variances, but neither converts $\lambda^D_{5,t}$ to white noise. These two processes have therefore been overfitted in the following ways:

<u>(0,0,2)(0,1)</u>:
(0,1,2)(1,1)
(1,0,2)(0,1)
(0,0,3)(0,1)
(1,1,2)(1,1);

<u>(0,1,3)(1,1)</u>:
(1,1,3)(1,1)
(1,0,3)(0,1).

Of these models, (1,0,2)(0,1), (0,0,3)(0,1), (1,0,3)(0,1), and (1,1,3)(1,1) all convert $\lambda^D_{5,t}$ to white noise. The model with the highest R^2 and lowest residual variance is (0,0,3)(0,1) so it is judged superior to the other three; the estimated process is invertible. Its results are given in Table 30.

vi) $\lambda^D_{6,t}$: Examination of the autocorrelation and partial autocorrelation functions for $\lambda^D_{6,t}$ indicates that the following ARIMA models may be appropriate:

(0,0,3)(0,1)
(0,1,2)(1,1)
(0,1,2)(1,2)
(0,2,3)(2,1)
(0,2,3)(2,2).

Of these five models, the best results occur with (0,1,2)(1,1) and (0,2,3)(2,1). Both models have a relatively high R^2 and both convert $\lambda_{6,t}^{D}$ to white noise. The estimated coefficients of (0,2,3)(2,1) however violate the invertibility condition; those of (0,1,2)(1,1) satisfy the condition, so (0,1,2)(1,1) is chosen as the model to represent $\lambda_{6,t}^{D}$. Its results are given in Table 30. Its first moving average parameter is not significantly different from zero, but this result is not surprising because the autocorrelation at lag 1 for the (1,1) differenced data is also not significantly different from zero.

vii) $\lambda_{7,t}^{D}$: The autocorrelations and partial autocorrelations indicate that the following models may describe the behavior of $\lambda_{7,t}^{D}$:

(0,0,1)(0,1)
(0,0,1)(0,2)
(0,1,2)(1,1)
(0,1,2)(1,2)
(0,2,3)(2,1)
(0,2,3)(2,2).

Each of the three process performs better with the first season difference of the data. Each of these models has a high R^2 and reduces the variance of residuals to a very low level. None of the models however transforms $\lambda_{7,t}^{D}$ to white noise so there is reason to suspect inadequacies in the models. Therefore each of the models has been overfitted with the following processes:

<u>(0,0,1)(0,1)</u>:
(1,0,1)(0,1)
(0,0,2)(0,1)
(1,0,2)(0,1)
(0,0,3)(0,1);

<u>(0,1,2)(1,1)</u>:
(0,2,2)(2,1)
(1,1,2)(1,1)
(0,1,3)(1,1)
(1,1,3)(1,1);

(0,2,3)(2,1):
(1,2,3)(2,1).

Of the models overfitted to (0,0,1)(0,1), only (0,0,2)(0,1) is an improvement, but it still does not result in white noise. The autocorrelations of residuals is large at lag 10 but adding a variable at that lag causes $n\Sigma r_{aa}^2(k)$ to rise.

Considering the models overfitted to (0,1,2)(1,1), both (1,1,2)(1,1) and (1,1,3)(1,1) give better results, but the estimated coefficients of the latter violate the invertibility condition. The estimated coefficients of (1,1,2)(1,1) satisfy the invertibility condition but the process does not convert $\lambda_{7,t}^D$ to white noise. The autocorrelation of residuals is large at lag 10 but adding a variable at that lag increases $n\Sigma r_{aa}^2(k)$. The coefficients estimated for (0,2,3)(2,1) and (1,2,3)(2,1) violate the invertibility condition; even when appropriate seasonal variables are included, the estimated models are not invertible.

Thus of all the models tested for $\lambda_{7,t}^D$, those that give the best results do not satisfy the invertibility condition. Of the few that are invertible, the best is (0,0,2)(0,1); its results are given in Table 30. The model has as good R^2 and low variance of residuals but does not result in white noise.

viii) $\lambda_{8,t}^D$: The behavior of the autocorrelation and partial autocorrelation functions for $\lambda_{8,t}^D$ imply that the following models need to be investigated:

(0,0,2)(0,1)
(0,0,2)(0,2)
(0,1,3)(1,1)
(0,1,2)(1,2)
(0,2,3)(2,1)
(0,2,3)(2,2).

Each of the models listed above were fitted to the series $\lambda^D_{8,t}$. The best results occur with (0,0,2)(0,1) and (0,1,3)(1,1). Each of the models has significant coefficients and very low residual variance, although neither process results in white noise or has a very large R^2. Consequently, the specification of the models is in question and they have been overfitted with the models listed below:

<u>(0,0,2)(0,1)</u>:
(0,1,2)(1,1)
(1,0,2)(0,1)
(0,0,3)(0,1)
(1,1,2)(1,1);

<u>(0,1,3)(1,1)</u>:
(1,1,3)(1,1)
(1,0,3)(0,1).

Comparing the results for these models (0,0,3)(0,1), (1,1,3)(1,1), and (1,0,3)(0,1), all convert $\lambda^D_{8,t}$ to white noise, but the R^2 for each model remains low. The autocorrelations of the residuals of each model are large at lag 14, so they are rerun including a moving average variable at that lag. Adding this variable improves only the results for (0,0,3)(0,1) which is chosen as the best model for $\lambda^D_{8,t}$. Its estimated coefficients satisfy the invertibility condition; the results are reported in Table 30.

ix) $\lambda^D_{9,t}$: Its autocorrelations and partial autocorrelations indicate that the following models will be useful in explaining the behavior of $\lambda^D_{9,t}$:

(0,0,3)(0,1)
(0,0,3)(0,2)
(0,1,2)(1,1)
(0,1,2)(1,2)
(0,2,3)(2,1)
(0,2,3)(2,2).

Both (0,0,3)(0,1) and (0,2,3)(2,1) have high R^2's and low residual variance; in addition, each process transforms $\lambda^D_{9,t}$ to white noise. The (0,0,3)(0,1) process is chosen since it has a slightly higher R^2 and lower residual variance; its results are given in Table 30. The estimated coefficients of the process satisfy the invertibility condition.

x) $\lambda^D_{10,t}$: Autocorrelations and partial autocorrelations for the parameter $\lambda^D_{10,t}$ show that the following ARIMA processes are important:

(0,1,2)(1,0)
(0,1,2)(1,1)
(0,1,2)(1,2)
(0,2,2)(2,0)
(0,2,3)(2,1)
(0,2,3)(2,2).

The processes (0,1,2)(1,1) and (0,2,3)(2,1) give the best results. The model (0,1,2)(1,1) converts $\lambda^D_{10,t}$ to white noise at short lags and (0,2,3)(2,1) converts $\lambda^D_{10,t}$ to white noise at all lags. Neither model however has an especially high R^2. To attempt to account for more of the variation in $\lambda^D_{10,t}$, the following models were overfitted:

<u>(0,1,2)(1,1)</u>:
(1,1,2)(1,1)
(1,2,2)(2,1)
(0,1,3)(1,1)
(1,1,3)(1,1);

<u>(0,2,3)(2,1)</u>:
(1,2,3)(2,1).

Of all these models, (1,1,3)(1,1) appears to be the best; it results in white noise at all lags and has a higher R^2 than (0,1,2)(1,1) and (0,2,3)(2,1). The estimated coefficients of (1,1,3)(1,1) however do not satisfy the invertibility condition; even when variables at pertinent seasonal lags are added, the invertibility condition is violated. The coefficients of (0,2,3)(2,1) also violate the invertibility condition;

with or without relevant seasonal variables. The best model that can be devised for $\lambda^D_{10,t}$ is therefore (0,1,2)(1,1); it converts $\lambda^D_{10,t}$ to white noise at all lags when a variable is included at the seasonal lag 13. Its estimated coefficients satisfy the invertibility condition, but the R^2 for the model is not very high. The results are given in Table 30.

xi) $\lambda^D_{11,t}$: From the autocorrelation and partial autocorrelation functions for $\lambda^D_{11,t}$, it appears that the following models are pertinent:

(0,0,3)(0,1)
(0,1,2)(1,1)
(0,1,2)(1,2)
(0,2,3)(2,1)
(0,2,3)(2,2).

The (0,0,3)(0,1), (0,1,2)(1,1), and (0,2,3)(2,1) processes perform best but none of the models has a very high R^2 or result in white noise. Therefore each of the models is apparently inadequate and has been overfitted with the models listed below:

<u>(0,0,3)(0,1)</u>:
(0,1,3)(1,1)
(1,0,3)(0,1)
(1,1,3)(1,1);

<u>(0,1,2)(0,1)</u>:
(0,0,2)(0,1)
(0,2,2)(2,1)
(1,1,2)(1,1)
(1,0,2)(0,1);

<u>(0,2,3)(2,1)</u>:
(1,2,3)(2,1).

The model that performs best of those listed above is (1,0,3)(0,1), but it converts $\lambda^D_{11,t}$ to white noise at short lags only. The autocorrelation of the residuals is large at the seasonal lag 13 and adding a moving average variable at that seasonal lag improves the model's performance somewhat. It still only results in white noise at short lags and does not have a very high R^2, but it is apparently the best model that can be devised for $\lambda^D_{11,t}$. The estimated model is invertible. Its results are presented in Table 30.

b) Time Deposits

The λ-parameters for time deposit categories have each been fitted first to an ARIMA noise model and second, to a multivariate ARIMA model that also includes transfer functions for three input variables. The input variables are designed to capture the effects of market interest rates and interest rate ceilings on the rate of growth in the various categories of time and savings deposits. They include the three-month Treasury bill rate[15] (denoted TB_t), the percentage change in the Treasury bill rate ($\%TB_t$), and the difference between the Treasury bill rate and the Regulation Q interest rate ceiling[16] (Q_t).

The relationship between $\lambda^T_{i,t}$ and TB_t and $\%TB_t$ is not totally clear. When market interest rates are high or rising, savings and time deposits should be attractive to the public relative to cash and demand deposits and, since $\lambda^T_{i,t}$ is inversely related to the rate of growth of the i^{th} category of time deposits, $\lambda^T_{i,t}$ would be expected to fall (reflecting an increased growth in time and savings deposits). Thus,

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[15]The rate on new issue of three-month U.S. Government Securities; data are weekly. Source: Board of Governors of the Federal Reserve System, Federal Reserve Bulletin, various dates, p. A-33.

[16]For parameters relating to savings deposits, Q_t is the Treasury bill rate minus the Regulation Q ceiling for savings deposits; when that quantity is negative, $Q_t = 0$ is used. For parameters relating to other categories of time deposits, Q_t is the Treasury bill rate minus the Regulation Q ceiling for time deposits maturing in one year or more until July 20, 1966, and for time deposits less than $100,000 maturing in two years or more thereafter; when the quantity is negative, $Q_t = 0$ is used. Source: Board of Governors of the Federal Reserve System, Federal Reserve Bulletin, various dates, p. A-10.

TB_t and $\%TB_t$ would be expected to vary inversely with $\lambda^T_{i,t}$. On the other hand, if high or rising market interest rates imply that other interest-bearing assets are more attractive than time and savings deposits, large values of TB_t and $\%TB_t$ would be coupled with relatively high values of $\lambda^T_{i,t}$ (indicating a decline, or slowdown in the rate of growth, of time and savings deposits). A positive value for the input variable Q_t indicates that market interest rates are above the allowable rates on time and savings deposits so Q_t and $\lambda^T_{i,t}$ should be directly related.

The estimation program used here does not allow different degrees of seasonal differencing for the input and dependent variables in a multivariate model. The parameters in r_t all require some degree of seasonal differencing to achieve stationarity. This necessitates, in the multivariate processes described here and in the following sections, that the input variables be subjected to the same degree of seasonal differencing as the dependent variable, regardless of what their identification procedures imply. In what follows, the description of the transfer functions will indicate the degree of seasonal differencing desired for each transfer function but in the estimation process all variables are seasonally differenced to the degree required for the noise model.

i) $\lambda^T_{1,t}$: Since $\lambda^T_{1,t}$ is used in the equation for r_t only under Reserve Scheme A, its ARIMA model is based on the first half of the sample period only. The autocorrelations and partial autocorrelations imply that the following noise models are relevant for $\lambda^T_{1,t}$:

(0,1,2)(1,1)
(0,2,3)(2,1)
(0,1,3)(1,2)
(0,0,2)(2,2).

Each of these four models were fitted to the series $\lambda_{1,t}^{T}$ and the best results were obtained with (0,1,2)(1,1) and (0,1,3)(1,2). The model (0,1,3)(1,2) results in white noise at all lags but its R^2 is not very high. On the other hand, the process (0,1,2)(1,1) has a healthy R^2 but does not result in white noise. The (0,1,2)(1,1) model seems promising so it has been overfitted with the models (1,1,2)(1,1) and (0,1,3)(1,1). Both models have a high R^2 and both convert $\lambda_{1,t}^{T}$ to white noise at all lags but (0,1,3)(1,1) is chosen as the best model.

The noise model (0,1,3)(1,1) was therefore used in a multivariate ARIMA model with transfer functions for the three independent variables described above. The cross correlation functions between $\lambda_{1,t}^{T}$ and the independent variables imply the following transfer functions:

TB_t : (0,1,0)(1,0);
$\%TB_t$: (0,2,0)(0,0);
Q_t : (0,3,0)(0,0).

Adding the input variables increases the model's R^2 and reduces its residual variance somewhat, but Q_t is the only input variable whose coefficients are statistically significant. The other two independent variables were therefore excluded and the model was reestimated; the results are given in Table 31.

The model has a high R^2 and low residual variance and the values of $n\Sigma r_{aa}^{2}(k)$ indicate that it converts $\lambda_{1,t}^{T}$ to white noise. The cross correlations of residuals are also small so the transfer function appears to be adequate. Three of the four coefficients in the transfer function are not of the expected sign, but the one positive coefficient

Table 31. ARIMA Results for $\lambda^T_{i,t}$, i = 1, 5 (Regression Results for the Best ARIMA Model Tested)

$\lambda^T_{1,t}$: (0,1,3)(1,1); Q_t: (0,3,0)(0,1) (1/1/61-1/10/68

$$\Delta\lambda^T_{1,t} = a_t - \underset{(-7.783)}{.44096a_{t-1}} + \underset{(16.84)}{.75880a_{t-2}} + \underset{(4.038)}{.23230a_{t-3}} - \underset{(-1.189)}{.00281Q_t} - \underset{(-2.588)}{.00652Q_{t-1}} - \underset{(-.1468)}{.00036Q_{t-2}} + \underset{(3.060)}{.00773Q_{t-3}}$$

Lag k	Autocorrelations of Residuals $n\Sigma r^2_{aa}(k)$	Cross Correlations of Residuals $n\Sigma r^2_{a\alpha(Q)}(k)$
1-10	4.445	2.49
1-20	16.003	14.71
1-30	25.964	18.71

R^2 = .918 Variance of Residuals = .3670 E-05 Durbin-Watson Statistic = 1.967

$\lambda^T_{1,t}$: (1,1,4)(1,1); TB_t: (0,1,0)(1,1) plus a variable at lag 10; $\%TB_t$: (1,0,0)(1,1) plus variables at lags 10 and 20; Q_t: (0,4,0)(0,1) (1/10/68-12/25/74)

$$\Delta\lambda^T_{1,t} = \left[\frac{1 + \underset{(1.268)}{.14518B} + \underset{(8.625)}{.67655B^2} - \underset{(-1.840)}{.16792B^3} + \underset{(3.375)}{.23059B^4}}{1 + \underset{(5.753)}{.62989B}}\right] a_t + \underset{(2.334)}{.00220\Delta TB_t} + \underset{(1.595)}{.00084\Delta TB_{t-1}} - \underset{(-.8819)}{.0091\Delta TB_{t-10}}$$

$$+ \left[\frac{\underset{(-1.349)}{-.00007} + \underset{(1.929)}{.00011B^{10}} - \underset{(-.8396)}{.00001B^{20}}}{1 + \underset{(71.60)}{.97498B}}\right]\Delta\%TB_t + \underset{(2.971)}{.00187Q_t} \; \underset{(2.318)}{.00161Q_{t-1}} - \underset{(-.0369)}{.00002Q_{t-2}} + \underset{(.9407)}{.00054Q_{t-3}}$$

$$- \underset{(-.1158)}{.00006Q_{t-4}}$$

Table 31. Continued

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals		
k	$n\Sigma r^2_{aa}(k)$	$n\Sigma r^2_{a\alpha(TB)}(k)$	$n\Sigma r^2_{a\alpha(\%TB)}(k)$	$n\Sigma r^2_{a\alpha(Q)}(k)$
1-10	7.879	8.70	14.31	1.55
1-20	13.417	17.07	29.92	3.74
1-30	26.140	28.23	36.93	12.02

$R^2 = .891$ Variance of Residuals = .6777 E-05 Durbin-Watson Statistic = 2.016

$\lambda^T_{2,t}$: (2,1,2)(1,0); TB_t: (0,1,0)(1,0); $\%TB_t$: (0,2,0)(0,0); Q_t: (0,4,0)(0,0) (7/9/66-12/31/74

$$\Delta\lambda^T_{2,t} = \left[\frac{1 + \underset{(2.151)}{.20355}B + \underset{(6.242)}{.59474}B^2}{1 + \underset{(3.719)}{.42494}B + \underset{(2.023)}{.23314}B^2}\right] a_t + \underset{(2.054)}{.00201}\Delta TB_t + \underset{(.3349)}{.00034}\Delta TB_{t-1} + \underset{(2.323)}{.00020}\%TB_t + \underset{(3.601)}{.00027}\%TB_{t-1}$$

$$+ \underset{(.2479)}{.00002}\%TB_{t-2} - \underset{(-1.110)}{.00165}Q_t - \underset{(-3.875)}{.00713}Q_{t-1} + \underset{(2.674)}{.00499}Q_{t-2} - \underset{(-.0490)}{.00008}Q_{t-3} + \underset{(.5423)}{.00058}Q_{t-4}$$

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals		
k	$n\Sigma r^2_{aa}(k)$	$n\Sigma r^2_{a\alpha(TB)}(k)$	$n\Sigma r^2_{a\alpha(\%TB)}(k)$	$n\Sigma r^2_{a\alpha(Q)}(k)$
1-10	3.735	3.84	3.83	0.20
1-20	11.713	13.61	15.41	1.98
1-30	19.876	17.33	18.55	6.90

$R^2 = .840$ Variance of Residuals = .2377 E-04 Durbin-Watson Statistic = 2.007

Table 31. Continued

$\lambda^T_{3,t}$ = (0,1,3)(1,1); TB_t: (0,1,0)(1,1); $\%TB_t$: (0,3,0)(0,1); Q_t: (0,2,0)(0,1) (7/9/66-12/31/74)

$$\Delta\lambda^T_{3,t} = a_t - \underset{(-.3225)}{.01671a_{t-1}} + \underset{(24.38)}{.81174a_{t-2}} - \underset{(-3.061)}{.15982a_{t-3}} + \underset{(2.473)}{.00091\Delta TB_t} - \underset{(-1.988)}{.00078\Delta TB_{t-1}} + \underset{(1.183)}{.00003\%TB_t}$$

$$- \underset{(-.6444)}{.00001\%TB_{t-2}}^{*} - \underset{(-1.422)}{.00003\%TB_{t-3}} - \underset{(-1.575)}{.00054Q_t} - \underset{(-.8033)}{.00022Q_{t-1}} - \underset{(-.8161)}{.00029Q_{t-2}}$$

Lag k	Autocorrelations of Residuals $n\Sigma r^2_{aa}(k)$	Cross Correlations of Residuals $n\Sigma r^2_{a\alpha(TB)}(k)$	$n\Sigma r^2_{a\alpha(\%TB)}(k)$	$n\Sigma r^2_{a\alpha(Q)}(k)$
1-10	6.532	12.23	12.71	0.28
1-20	27.115	17.15	16.38	3.51
1-30	30.102	22.12	21.48	4.37

R^2 = .941 Variance of Residuals = .3901 E-05 Durbin-Watson Statistic = 2.028

*The estimated coefficient of $\%TB_{t-1}$ is .0000.

$\Delta\lambda^T_{4,t}$: (0,1,3)(1,1); TB_t: (1,1,0)(1,1) (1/10/68-12/31/74)*

$$\Delta\lambda^T_{4,t} = a_t - \underset{(\pm.08058)}{.23197a_{t-1}} + \underset{(\pm.00867)}{.85406a_{t-2}} + \underset{(\pm.07963)}{.04268a_{t-3}} + \left[\frac{\underset{(\pm.00003)}{.00046} + \underset{(\pm.00003)}{.00049B}}{1 + \underset{(\pm.00399)}{.99077B}}\right]\Delta TB_t$$

Table 31. Continued

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals
k	$n\Sigma r_{aa}^2(k)$	$n\Sigma r_{a\alpha(TB)}^2(k)$
1-10	38.099	24.905
1-20	53.931	45.009
1-30	58.174	54.626

$R^2 = .999$ Variance of Residuals = .67343 E-06

$\lambda_{5,t}^T$: (0,1,2)(1,1) with a variable at lag 4; Q_t: (0,2,0)(0,1) (1/10/68-12/31/74)*

$$\Delta\lambda_{5,t}^T = a_t - \underset{(\pm.04610)}{.52439a_{t-1}} + \underset{(\pm.11770)}{.34322a_{t-2}} + \underset{(\pm.08506)}{.16703a_{t-4}} + \underset{(\pm.00186)}{.00378Q_t} + \underset{(\pm.00078)}{.00173Q_{t-1}} + \underset{(\pm.00196)}{.00220Q_{t-2}}$$

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals
k	$n\Sigma r_{aa}^2(k)$	$n\Sigma r_{a\alpha(Q)}^2(k)$
1-10	10.657	8.250
1-20	19.558	15.642
1-30	31.570	20.817

$R^2 = .999$ Variance of Residuals = .28119 E-04

*There is different information provided for $\lambda_{4,t}^T$ and $\lambda_{5,t}^T$. All ARIMA models were identified and estimated with one program and forecasted with another. For $\lambda_{4,t}^T$ and $\lambda_{5,t}^T$ the forecast results are more successful if the model was reestimated with the second program. This program, however, does not supply the same information on the regression results. The numbers in parentheses below the coefficients are upper and lower limits for the coefficients, at the 95% significance level.

Note: For simplicity, the degree of seasonal differencing (Δ^s) is omitted from the regression equations. The degree of seasonal differencing is denoted in the (p,d,q)(d,s) description for each parameter.

is also statistically significant. Furthermore, there are very few non-zero observations of Q_t in the time period used here so the measured relationship between $\lambda^T_{1,t}$ and Q_t is probably not very reliable.

ii) $\lambda^T_{2,t}$: The parameter $\lambda^T_{2,t}$ (corresponding to total time deposits) enters the equation for r_t only under Reserve Scheme B. That scheme covers only 70 weekly observations of data, but with so few observations, the ARIMA identification and estimation procedures fail. The ARIMA model for $\lambda^T_{2,t}$ is therefore based on all available observations, which cover the period July 9, 1966, through December 31, 1974.

The autocorrelations and partial autocorrelations for $\lambda^T_{2,t}$ suggest the following ARIMA noise models:

(2,1,2)(1,0)
(0,2,2)(2,0)
(0,1,4)(1,1)
(0,1,4)(1,2)
(0,2,4)(2,1)
(0,2,4)(2,2).

Several of these models convert $\lambda^T_{2,t}$ to white noise but (2,1,2)(1,0) appears to be the best since it has the highest R^2 and lowest residual variance. The noise model is used in a multivariate model along with the input variables TB_t, $\%TB_t$, and Q_t.

After prewhitening each input variable, their cross correlations with $\lambda^T_{2,t}$ suggest the following transfer functions:

TB_t : (0,1,0)(1,0);
$\%TB_t$: (0,2,0)(0,0);
MB_t : (0,4,0)(0,0).

Each of the transfer functions has at least one statistically significant coefficient; the model including all three input variables results in white noise and the cross correlations of residuals imply that all three transfer functions are adequate. The signs of the coefficients of TB_t and $\%TB_t$ show that $\lambda^T_{2,t}$ is directly related to interest rates; the nature of its relationship with Q_t is not clear from these results.

iii) $\lambda^T_{3,t}$: The parameter $\lambda^T_{3,t}$ (savings deposits) is included in r_t under Reserve Schemes B, C, and D' so its ARIMA model is based on all available observations of data for the period, July 9, 1966 to December 31, 1974. The autocorrelation and partial autocorrelation functions for $\lambda^T_{3,t}$ imply that the following processes are important:

(0,1,2)(1,0)
(0,1,3)(1,1)
(0,1,3)(1,2)
(0,2,3)(2,0)
(0,2,3)(2,1)
(0,2,3)(2,2).

Of these six models, the best results occur for (0,1,3)(1,1) and (0,2,3)(2,0); each model has a high R^2 and low residual variance. The best model appears to be (0,1,3)(1,1) because it converts $\lambda^T_{3,t}$ to white noise at lags 1-10 and 1-30; the value of $n\Sigma r^2_{aa}(k)$ is large at lag 13, but adding a variable at that lag does not improve the results.

The noise model (0,1,3)(1,1) was therefore used, along with the three independent variables given above, in a multivariate ARIMA model. The cross correlations between $\lambda^T_{3,t}$ and the independent variables imply the following transfer functions:

TB_t : (0,1,0)(1,0);
$\%TB_t$: (0,3,0)(0,0);
Q_t : (0,2,0)(0,0).

Adding the input variables to the (0,1,3)(1,1) process increases the R^2 and lowers the residual variance. Only the transfer function for TB_t has a statistically significant coefficient, but when Q_t and $\%TB_t$ are excluded the results are not as good, so all three input variables are retained. The values of $n\Sigma r^2_{aa}(k)$ indicate that the model converts

$\lambda^T_{3,t}$ to white noise and the cross correlations for each independent variable are less than the critical χ^2 values, implying that each transfer function is adequately specified. The results are reported in Table 31.

The signs of the coefficients in the transfer functions for TB_t are not consistent, although the statistically significant coefficient implies a direct relationship. The signs of the coefficients of $\%TB_t$ are mixed, but they are not statistically significant. None of the coefficients of Q_t are of the expected sign but they are also not statistically significant.

iv) $\lambda^T_{4,t}$: The parameters $\lambda^T_{4,t}$ and $\lambda^T_{5,t}$ are included in r_t under Reserve Schemes C and D' so their ARIMA models are based on the corresponding time period, January 11, 1968, to December 31, 1974, 364 observations. The autocorrelation and partial autocorrelation functions for $\lambda^T_{4,5}$ (time deposits less than \$5 million) indicate that the following models may represent the noise function for $\lambda^T_{4,t}$:

(0,1,3)(1,1)
(0,1,3)(1,2)
(0,2,2)(2,1)
(0,2,2)(2,2).

For both processes, the first seasonal difference of $\lambda^T_{4,t}$ gives better results than the second. Both (0,1,3)(1,1) and (0,2,2)(2,1) have high R^2's and low residual variances; (0,1,3)(1,1) appears to be the best model since it converts $\lambda^T_{4,t}$ to white noise at all lags.

The noise model (0,1,3)(1,1) is therefore included in a multivariate ARIMA model along with the three input variables mentioned above. The cross correlation functions of $\lambda^T_{4,t}$ and the independent variables imply the following transfer functions:

TB_t : (1,1,0)(1,0);
$\%TB_t$: (0,2,0)(1,1);
Q_t : (0,2,0)(0,0).

Only the transfer function for TB_t has statistically significant coefficients so the other two input variables were eliminated and the model reestimated; the results are presented in Table 31.

When the transfer functions are added the process does not result in white noise; the cross correlations of residuals for the transfer function are large, indicating that the function is not adequately specified. The cross correlation is large at lag 13 but adding a variable at that lag does not improve the model's performance. The model was also refitted with several modifications of the transfer function but in no case are the cross correlations lower.[17] This is apparently the best model that can be constructed for $\lambda^T_{4,t}$. The signs of the coefficients of TB_t consistently show a direct relationship between it and $\lambda^T_{4,t}$ implying that at high interest rates, time deposits in this reserve category grow at a slower rate.

v) $\lambda^T_{5,t}$: The autocorrelations and partial autocorrelations for $\lambda^T_{5,t}$ (time deposits greater than \$5 million) indicate that the following noise models should be considered:

(0,1,2)(1,1)
(0,1,2)(1,2)
(0,2,3)(2,0)
(0,2,3)(2,2).

The second seasonal difference of $\lambda^T_{5,t}$ gives very poor results in both of the above models; each of the other three models has a respectable

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[17]The transfer functions (0,1,0)(1,0) and (1,2,0)(1,0) were also tried.

R^2 and low residual variance, but neither results in white noise. The model (0,1,2)(1,1) converts $\lambda^T_{5,t}$ to white noise at lags 11-30, but not at short lags because the autocorrelation of residuals is large at lag 4. For (0,2,3)(2,0), the autocorrelation of residuals is large at lag 13. Adding the seasonal variable to (0,2,3)(2,0) improves the performance but it still does not result in white noise. The model (0,1,2)(1,1) plus a variable at lag 4 does result in white noise so it appears to be the best noise model.

This noise model was then included in a multivariate process with the same three independent variables. The cross correlations functions for $\lambda^T_{5,t}$ and the prewhitened input variables indicate the following transfer functions:

$$\begin{aligned} TB_t &: (1,1,0)(1,0); \\ \%TB_t &: (0,2,0)(1,1); \\ Q_t &: (0,2,0)(0,0). \end{aligned}$$

Q_t is the only input variable that has statistically significant coefficients so the model was rerun excluding the other two transfer functions. The results are reported in Table 31.

The process converts $\lambda^T_{5,t}$ to white noise and the transfer function for Q_t appears to be adequate. The coefficients of Q_t imply the expected direct relationship between Q_t and $\lambda^T_{5,t}$.

<u>Differential Reserve Requirements</u> ($\delta^D_{j,t}$ and $\delta^T_{i,t}$)

a) Demand Deposits

By definition, the sum of the $\delta^D_{j,t}$ for each reserve scheme is one. Therefore an ARIMA model is not needed for one $\delta^D_{j,t}$ in each reserve scheme; its value can be derived directly from the values of the other $\delta^D_{j,t}$. For the first demand deposit reserve scheme, the ARIMA models for $\delta^D_{1,t}$

and $\delta^D_{2,t}$ are identical. For the last two demand deposit schemes, ARIMA models are not estimated for $\delta^D_{6,t}$ and $\delta^D_{11,t}$.

i) $\delta^D_{1,t}$ and $\delta^D_{2,t}$: Like $\lambda^D_{1,t}$ and $\lambda^D_{2,t}$, $\delta^D_{1,t}$ and $\delta^D_{2,t}$ appear in r_t in Reserve Schemes A and B, so their ARIMA models are based on the first part of the sample only. The autocorrelation and partial autocorrelation autocorrelation functions indicate that the following models should be considered for $\delta^D_{1,t}$ and $\delta^D_{2,t}$:

(3,1,0)(1,1)
(0,2,2)(2,1)
(0,2,1)(2,2)
(2,1,2)(1,2).

When these four models are fitted to $\delta^D_{1,t}$, the (3,1,0)(1,1) process performs best, but it does not transform the series to white noise except at short lags. Examination of the autocorrelations of the residuals shows high correlations at the seasonal lags 13 and 26. The model was therefore rerun including moving average variables at these lags and this modified model does convert $\delta^D_{1,t}$ to white noise at all lags. All five coefficients are statistically significant and they satisfy invertibility and stationarity conditions; the model has a high R^2 and low residual variance. The results are given in Table 32.

ii) $\delta^D_{3,t}$: The ARIMA models for $\delta^D_{j,t}$, j = 3,11 are based on all the available observations; they cover the last half of the sample, January 10, 1968 through December 31, 1974. Examining the autocorrelations and partial autocorrelations for $\delta^D_{3,t}$ indicates that the following ARIMA models may be useful in explaining its behavior:

Table 32. ARIMA Results for $\delta^{D}_{j,t}$, j = 1, 11 (Regression Results for the Best ARIMA Model Tested)

$\delta^{D}_{1,t}$ (3,1,0)(1,1) plus seasonal variables at lags 13 and 26 (1/1/61-1/10/68)

$$\begin{array}{llllll} \Delta\delta^{D}_{1,t} = .37419\Delta\delta^{D}_{1,t-1} & + .21569\Delta\delta^{D}_{1,t-2} & + .14832\Delta\delta^{D}_{1,t-3} & + a_t & -.23747a_{t-13} & - .20096a_{t-26} \\ \quad (6.596) & \quad (3.648) & \quad (2.599) & & (-3.933) & (-3.185) \end{array}$$

R^2 = .982 Variance of Residuals = .4306 E-05 Durbin-Watson Statistic = 2.006

k	$n\Sigma r^2_a(k)$
1-10	8.351
1-20	20.452
1-30	29.849

$\delta^{D}_{3,t}$ (3,1,1)(1,1) plus a seasonal variable at lag 13 (1/10/68-12/31/74)

$$\begin{array}{llllll} \Delta\delta^{D}_{3,t} = -.62948\Delta\delta^{D}_{3,t-1} & - .14154\Delta\delta^{D}_{3,t-2} & + .08075\Delta\delta_{3,t-3} & + a_t & + .93263a_{t-1} & - .02250a_{t-13} \\ \quad (-8.593) & \quad (-2.038) & \quad (1.277) & & (20.02) & (-.8732) \end{array}$$

R^2 = .987 Variance of Residuals = .4107 E-08 Durbin-Watson Statistic = 1.976

k	$n\Sigma r^2_a(k)$
1-10	3.239
1-20	25.474
1-30	37.254

Table 32. Continued

$\delta^D_{4,t}$ (0,1,3)(1,1) plus seasonal variables at lags 13 and 14 (1/10/68-12/31/74)

$$\Delta\delta^D_{4,t} = a_t + \underset{(8.009)}{.44380a_{t-1}} + \underset{(1.299)}{.07590a_{t-2}} + \underset{(2.387)}{.12942a_{t-3}} - \underset{(-5.987)}{.33042a_{t-13}} + \underset{(4.656)}{.26303a_{t-14}}$$

r^2 = .885 Variance of Residuals = .1088 E-04 Durbin Watson Statistic = 1.937

k	$n\Sigma r^2_a(k)$
1-10	4.985
1-20	15.399
1-30	29.141

$\delta^D_{5,t}$ (3,1,1)(1,1) plus a seasonal variable at lag 13 (1/10/68-12/31/74)

$$\Delta\delta^D_{5,t} = \underset{(-5.276)}{-.45800\Delta\delta^D_{5,t-1}} - \underset{(-1.632)}{.11104\Delta\delta^D_{5,t-2}} + \underset{(1.994)}{.12734\Delta\delta^D_{5,t-3}} + a_t + \underset{(11.94)}{.81158a_{t-1}} - \underset{(-1.794)}{.05850a_{t-13}}$$

R^2 = .967 Variance of Residuals = .2039 E-05 Durbin-Watson Statistic = 1.983

k	$n\Sigma r^2_a(k)$
1-10	4.479
1-20	22.959
1-30	33.235

$\delta^D_{7,t}$ (3,1,3)(1,1) (1/10/68-12/31/74)

$$\Delta\delta^D_{7,t} = \underset{(4.287)}{.58591\Delta\delta^D_{7,t-1}} + \underset{(.6233)}{.08682\Delta\delta^D_{7,t-2}} - \underset{(-4.182)}{.45719\Delta\delta^D_{7,t-3}} + a_t - \underset{(-2.244)}{.26393a_{t-1}} + \underset{(1.471)}{.15576a_{t-2}} + \underset{(8.014)}{.73202a_{t-3}}$$

R^2 = .987 Variance of Residuals = .5560 E-06 Durbin-Watson Statistic = 1.986

k	$n\Sigma r^2_a(k)$
1-10	3.195
1-20	19.650
1-30	28.150

Table 32. Continued

$\delta^D_{8,t}$ (1,2,1)(2,1) plus a seasonal variable at lag 13 (1/10/68-12/31/74)

$$\Delta^2\delta^D_{8,t} = \underset{(3.898)}{.23024}\Delta^2\delta^D_{8,t-1} + a_t + \underset{(41.74)}{.94626}a_{t-1} - \underset{(-2.604)}{.05703}a_{t-13}$$

k	$n\Sigma r^2_a(k)$
1-10	5.619
1-20	26.795
1-30	40.199

$R^2 = .776$ Variance of Residuals = .2861 E-05 Durbin-Watson Statistic = 1.978

$\delta^D_{9,t}$ (3,1,3)(1,1) plus a seasonal variable at lag 13 (1/10/68-12/31/74)

$$\Delta\delta^D_{9,t} = \underset{(4.610)}{.76566}\Delta\delta^D_{9,t-1} - \underset{(-.8434)}{.11719}\Delta\delta^D_{9,t-2} - \underset{(-.9881)}{.10583}\Delta\delta^D_{9,t-3} + a_t - \underset{(-2.189)}{.33799}a_{t-1}$$

$$+ \underset{(6.464)}{.53135}a_{t-2} + \underset{(2.139)}{.27245}a_{t-3} - \underset{(-5.467)}{.26906}a_{t-13}$$

k	$n\Sigma r^2_a(k)$
1-10	5.742
1-20	17.593
1-30	28.022

$R^2 = .703$ Variance of Residuals = .5698 E-05 Durbin-Watson Statistic = 1.944

$\delta^D_{10,t}$ (3,1,3)(1,1) plus a seasonal variable at lag 13 (1/10/68-12/31/74)

k	$n\Sigma r^2_a(k)$

$$\Delta\delta^D_{10,t} = \underset{(1.767)}{.41833}\Delta\delta^D_{10,t-1} - \underset{(-2.004)}{.29902}\Delta\delta^D_{10,t-2} + \underset{(1.674)}{.22639}\Delta\delta^D_{10,t-3} + a_t - \underset{(-.3706)}{.08733}a_{t-1}$$

$$+ \underset{(8.860)}{.66178}a_{t-2} - \underset{(-.3777)}{.07674}a_{t-3} - \underset{(-3.786)}{.17115}a_{t-13}$$

$R^2 = .816$ Variance of Residuals = .1763 E-05 Durbin-Watson Statistic = 1.977

Note: For simplicity, the degree of seasonal differencing (Δ^s) is omitted from the regression equations. The degree of seasonal differencing is denoted in the (p,d,q)(d,s) description for each parameter.

(0,1,1)(1,1)
(0,1,1)(1,2)
(3,1,1)(1,1)
(1,1,1)(1,1)
(1,1,1)(1,2)
(0,2,2)(2,1)
(0,2,1)(2,2).

The best results occur with the models (3,1,1)(1,1), (1,1,1)(1,1), and (0,2,2)(2,1). All three models result in a high R^2 and very low residual variance and they all convert $\delta^D_{3,t}$ to white noise, but at lags 1-10 only. The last two models have therefore been overfitted in the following ways:

<u>(1,1,1)(1,1)</u>:
(1,2,1)(1,1)
(2,1,1)(1,1)
(1,1,2)(1,1);

<u>(0,2,2)(2,1)</u>:
(0,1,2)(1,1)
(1,2,2)(2,1).

None of the overfitted models however perform better than their original formulations or (3,1,1)(1,1).

The autocorrelations of the residuals for all three models are large at the seasonal lag 13 so the three models were all refitted including a moving average variable at that lag. Adding the seasonal variables improves each model slightly. The best model appears to be (3,1,1)(1,1), the results of which are presented in Table 32. The seasonable variable in (3,1,1)(1,1) does not have a significant coefficient but the augmented model does convert $\delta^D_{3,t}$ to white noise at all lags at the 5% level of significance and its estimated coefficients satisfy the conditions for stationarity and invertibility.

iii) $\delta^D_{4,t}$: The autocorrelations and partial autocorrelations for $\delta^D_{4,t}$ imply the following processes:

(0,1,3)(1,1)
(1,1,1)(1,1)
(0,2,2)(2,1)
(0,2,2)(2,2)
(0,1,1)(1,2)
(1,1,1)(1,2).

Each of the six models listed above was fitted to the series $\delta^D_{4,t}$. The second seasonal difference of the data is consistently less successful than the first so those three models are not pursued further. Each of the remaining three models has a relatively high R^2 and transforms $\delta^D_{4,t}$ to white noise for lags 1-10. Since none of the three models transforms $\delta^D_{4,t}$ to white noise at long lags each was overfitted with the following models:

<u>(0,1,3)(1,1)</u>:
(1,1,3)(1,1);

<u>(1,1,1)(1,1)</u>:
(1,1,2)(1,1)
(2,1,1)(1,1);

<u>(0,2,2)(2,1)</u>:
(0,2,3)(2,1)
(1,2,2)(2,1).

None of the overfitted models however is an improvement over the original formulation.

For each of the original models, the autocorrelations of the residuals are large at or near the seasonal lags 13 and 26. Each model was therefore refitted including moving average variables at these seasonal lags. Adding the seasonal variables to (1,1,1)(1,1) and (0,2,2)(2,1) does not reduce $n\Sigma r^2_{aa}(k)$ to the level required for white noise. For (0,1,3)(1,1), adding variables at lags 13 and 14 reduces its residual autocorrelations so that the model results in white noise at all lags. The estimated process is also invertible, so it is chosen as the best model for $\delta^D_{4,t}$. Its results are given in Table 32.

iv) $\delta^D_{5,t}$: From the autocorrelation and partial autocorrelation functions for $\delta^D_{5,t}$, the following ARIMA processes appear to be relevant:

(0,1,1)(1,1)
(0,1,1)(1,2)
(1,1,1)(1,1)
(1,1,1)(1,2)
(0,2,2)(2,1)
(0,2,1)(2,2).

The models (1,1,1)(1,1), (0,2,2)(2,1), and (0,2,1)(2,2) give better results than the other three listed above. Each of these three models have a respectable R^2 and significant coefficients, but transforms $\delta^D_{5,t}$ to white noise at short lags only. Therefore these three models have been overfitted in the following way:

<u>(1,1,1)(1,1)</u>:
(2,1,1)(1,1)
(3,1,1)(1,1)
(1,1,2)(1,1)
(1,2,1)(2,1);

<u>(0,2,2)(2,1)</u>:
(1,2,2)(2,1)
(0,2,3)(2,1);

<u>(0,2,1)(2,2)</u>:
(1,2,1)(2,2)
(0.2.2)(2,2).

Of all these models, the best results occur with (3,1,1)(1,1) and (0,2,2)(2,1); both models still only convert $\delta^D_{5,t}$ to white noise at short lags.

The autocorrelations of the residuals for both are large at the quarterly lag 13, so both models were rerun including a moving average variable at that lag. Adding the seasonal variable reduces the residual autocorrelations of both models. With the variable at lag 13, (3,1,1)(1,1) results in white noise at all lags and its estimated coefficients satisfy the stationarity and invertibility conditions.

It is therefore chosen as the best model to describe $\delta^D_{5,t}$; its results are given in Table 32.

v) $\delta^D_{7,t}$: The autocorrelation and partial autocorrelation functions for $\delta^D_{7,t}$ show that the models listed below are pertinent:

(0,1,3)(1,1)
(3,1,3)(1,1)
(0,2,2)(2,1)
(0,1,1)(1,2)
(1,1,1)(1,2)
(0,2,1)(2,2).

Each of the models was fitted to $\delta^D_{7,t}$. Other than (0,1,1)(1,2), which gives very poor results, all of the models above perform well. Each of the five models has a very low residual variance and a high R^2. The model (3,1,3)(1,1) however is the best because it converts $\delta^D_{7,t}$ to white noise at all lags; the other four models only result in white noise at lags 1-10. The estimated process (3,1,3)(1,1) also satisfies the stationarity and invertibility conditions; its results are given in Table 32.

vi) $\delta^D_{8,t}$: The autocorrelations and partial autocorrelations of $\delta^D_{8,t}$ imply that the models listed below should be considered:

(0,1,1)(1,1)
(0,1,1)(1,2)
(1,1,1)(1,1)
(0,2,1)(2,1)
(0,2,1)(2,2).

These five models were fitted to appropriate differences of $\delta^D_{8,t}$. The (1,1,1)(1,1) and (0,2,1)(2,1) processes out-perform the other models. Each one results in a reasonably good R^2 and a low residual variance, but (1,1,1)(1,1) results in white noise at lags 1-10 only and (0,2,1)(1,1) does not result in white noise at all. Both models have therefore been overfitted by the following models:

<u>(1,1,1)(1,1)</u>:
(2,1,1)(1,1)
(1,1,2)(1,1);

<u>(0,2,1)(2,1)</u>:
(1,2,1)(2,1)
(0,2,2)(2,1)
(1,2,2)(2,1).

Each of the overfitted models is an improvement over the original; the best results are obtained with (2,1,1)(1,1), (1,1,2)(1,1), (1,2,1)(2,1), and (0,2,2)(2,1), but they all only result in white noise at short lags. The autocorrelations of the residuals for all four models are consistently large at the quarterly lags 13 and 26, so each of the models was rerun including moving average variables at lag 13 and at lags 13 and 26. The best result occurs with (1,2,1)(2,1), plus a variable at lag 13; it transforms $\delta^D_{8,t}$ to white noise at all lags at the 5% level of significance; the other three models give white noise only at the 2.5% level. The estimated coefficients of (1,2,1)(2,1) plus a variable at lag 13 satisfy the stationarity and invertibility conditions. Its results are given in Table 32.

vii) $\delta^D_{9,t}$: The characteristics of the autocorrelation and partial autocorrelation functions for $\delta^D_{9,t}$ indicate that the models listed below are relevant for $\delta^D_{9,t}$:

(0,1,3)(1,1)
(3,1,3)(1,1)
(0,2,3)(2,1)
(1,1,1)(1,2)
(0,2,2)(2,2).

Each of these models was estimated for $\delta^D_{9,t}$; (0,1,3)(1,1) and (3,1,3)(1,1) are the most promising models. Neither model converts $\delta^D_{9,t}$ to white noise except at short lags and neither model has an especially high R^2. The (0,1,3)(1,1) process was therefore overfitted with the following models:

<u>(0,1,3)(1,1)</u>:
(1,1,3)(1,1)
(2,1,3)(1,1);

neither model however gives better results than (0,1,3)(1,1).

The autocorrelations of the residuals of both (0,1,3)(1,1) and (3,1,3)(1,1) are large at the seasonal lags 13 and 26, so the models were rerun including a moving average variable(s) at lag(s) 13 and 13 and 26. Adding the seasonal variable(s) reduces the autocorrelations of residuals for both models. The best model is (3,1,3)(1,1) plus a variable at lag 13; these results are given in Table 32. This augmented model results in white noise at all lags at all levels of significance and it is stationary and invertible.

viii) $\delta^D_{10,t}$: Examination of its autocorrelations and partial autocorrelations imply the following models to describe $\delta^D_{10,t}$:

(0,2,1)(2,0)
(0,2,1)(2,1)
(0,2,1)(2,2)
(0,1,3)(1,1)
(3,1,0)(1,1)
(3,1,3)(1,1)
(0,1,3)(1,2).

Of all the models, (0,1,3)(1,1) or (3,1,3)(1,1) appear to be most promising; both (0,1,3)(1,1) and (3,1,3)(1,1) convert $\delta^D_{10,t}$ to white noise at lags 1-10. Since many of the coefficients in (3,1,3)(1,1) are not statistically significant, the models listed below have also been tested:

(1,1,3)(1,1)
(2,1,3)(1,1)
(3,1,1)(1,1)
(3,1,2)(1,1).

Of the above four models (3,1,1)(1,1) and (2,1,3)(1,1) perform best; along with (3,1,3)(1,1), these models all have very similar R^2's and they all transform $\delta^D_{10,t}$ to white noise at lags 1-10 only. The autocorrelations of the residuals are uniformly large at lag 13 so each of these three models was refitted including a moving average variable at lag 13. Adding the seasonal variable results in a more successful model for each one; the seasonal variable has a significant coefficient in all three models and all three models transform $\delta^D_{10,t}$ to white noise. Since its R^2 is highest and residual variance is lowest, (3,1,3)(1,1) is chosen as the best model. Its estimated coefficients satisfy the stationarity and invertibility conditions; its results are given in Table 32.

b) Time Deposits

Like $\lambda^T_{1,t}$, $\delta^T_{1,t}$ have been fitted first to an ARIMA noise model, which is then included in a multivariate ARIMA model with the three input variables, TB_t, $\%TB_t$ and Q_t. Hopefully the independent variables account for the effects of interest rates and interest rate ceilings on the relative growth of the various categories of time and savings deposits. Since $\delta^T_{2,t} + \delta^T_{3,t} = 1$ for Reserve Scheme B, and $\delta^T_{3,t} + \delta^T_{4,t} + \delta^T_{5,t} = 1$ for Reserve Schemes D' and E, an ARIMA model is not fitted for $\delta^T_{2,t}$ or $\delta^T_{4,t}$.[18]

i) $\delta^T_{3,t}$: The ARIMA model for $\delta^T_{3,t}$ (savings deposits) is based on the period July 9, 1966, through December 31, 1974, which corresponds

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[18]Estimation of an ARIMA model was more difficult, and the results poorer, for $\delta^T_{4,t}$ than for $\delta^T_{5,t}$, so $\delta^T_{4,t}$ was chosen as the one to be derived from the values of the other two.

to the period covered by Reserve Schemes B, D', and E. The autocorrelation and partial autocorrelations for $\delta^T_{3,t}$ indicate the following noise models:

(2,2,2)(2,0)
(3,1,0)(1,1)
(3,1,0)(1,2)
(2,2,1)(2,1)
(2,2,1)(2,2).

Each of the five models above performs well but the best results occur with (3,1,0)(1,1) which has a high R^2, significant coefficients and it converts $\delta^T_{3,t}$ to white noise at all lags.

The noise model (3,1,0)(1,1) was therefore combined with a transfer function for each of the three input variables discussed above. The cross correlation functions between $\delta^T_{3,t}$ and the prewhitened independent variables indicate the following transfer functions:

TB_t : (0,2,0)(1,0);
$\%TB_t$: (0,2,0)(0,0);
Q_t : (0,2,0)(0,0).

None of the coefficients in the transfer function for Q_t is statistically significant so the model was rerun with only TB_t and $\%TB_t$ as input variables. Its results are presented in Table 33. The combined noise-transfer function model has a high R^2 and converts $\delta^T_{3,t}$ to white noise, although only at the 5% level of significance for long lags. The cross correlations of residual at positive lags for both input variables are much smaller than the critical values of χ^2 so the transfer functions are apparently adequate.

Since $\delta^T_{3,t}$ measures the proportion of member bank time and savings deposits that are savings deposits, it is not clear what the relationship should be between $\delta^T_{3,t}$ and market interest rates. The regression results indicate a direct relationship between $\delta^T_{3,t}$ and the Treasury

Table 33. ARIMA Results for $\delta^T_{i,t}$, i = 3, 5 (Regression Results for the Best ARIMA Model Tested)

$\delta^T_{3,t}$: (3,1,0)(1,1); TB_t: (0,2,0)(1,1); $\%TB_t$: (0,2,0)(0,1) (7/9/66–12/31/74)

$$\Delta\delta^T_{3,t} = \left[\frac{1}{1 + \underset{(5.207)}{.26701B} + \underset{(6.632)}{.33387B^2} + \underset{(3.197)}{.16425B^3}}\right] a_t + \underset{(2.020)}{.0004\Delta TB_t} + \underset{(-.0780)}{.00005\Delta TB_{t-1}} + \underset{(1.060)}{.00068\Delta TB_{t-2}}$$

$$+ \underset{(.5378)}{.00002\%TB_t} - \underset{(-2.003)}{.00008\%TB_{t-1}} - \underset{(-2.523)}{.00003\%TB_{t-2}}$$

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals	
k	$n\Sigma r^2_{aa}(k)$	$n\Sigma r^2_{a\alpha(TB)}(k)$	$n\Sigma r^2_{a\alpha(\%TB)}(k)$
1-10	5.386	4.95	3.87
1-20	20.353	8.49	8.71
1-30	37.675	12.02	12.48

R^2 = .999 Variance of Residuals = .1806 E-05 Durbin-Watson Statistic = 1.981

$\delta^T_{5,t}$: (2,1,0)(1,1); TB_t: (1,1,0)(1,1); $\%TB_t$: (1,0,0)(1,1); Q_t: (0,3,0,)(0,0) (1/10/68–12/31/74)

$$\Delta\delta^T_{5,t} = \left[\frac{1}{1 + \underset{(2.752)}{.19721B} + \underset{(2.690)}{.18540B^2}}\right] a_t + \left[\frac{\underset{(-19.14)}{-.14350} + \underset{(19.15)}{.14571B}}{1 - \underset{(-272.1)}{1.01523B}}\right] \Delta TB_t + \left[\frac{\underset{(17.86)}{.00827}}{1 + \underset{(1244.)}{1.0035B}}\right] \Delta\%TB_t$$

$$-\underset{(-2.715)}{.01021Q_t} + \underset{(.6207)}{.00286Q_{t-1}} - \underset{(-1.772)}{.00710Q_{t-2}} - \underset{(-.9568)}{.00296Q_{t-3}}$$

Table 33. Continued

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals		
k	$n\Sigma r^2_{aa}(k)$	$n\Sigma r^2_{a\alpha(TB)}(k)$	$n\Sigma r^2_{a\alpha(\%TB)}(k)$	$n\Sigma r^2_{a\alpha(Q)}(K)$
1-10	10.817	4.76	5.17	0.29
1-20	16.623	14.98	14.38	1.28
1-30	24.154	27.20	24.60	2.51

$R^2 = .934$ Variance of Residuals = .2313 E-03 Durbin-Watson Statistic = 2.000

Note: For simplicity, the degree of seasonal differencing (Δ^s) is omitted from the regression equations. The degree of seasonal differencing is denoted in the (p,d,q)(d,s) description for each parameter.

bill rate; the significant coefficients of $\%TB_t$ imply an inverse relationship between $\delta^T_{3,t}$ and changes in the Treasury bill rate.

ii) $\delta^T_{5,t}$: The ARIMA model for $\delta^T_{5,t}$ (time deposits greater than \$5 million) is based on the period January 10, 1968, through December 31, 1974, which corresponds to Reserve Schemes D' and E. The autocorrelations and partial autocorrelations for $\delta^T_{5,t}$ imply the following noise models:

(2,1,0)(1,0)
(2,1,0)(1,1)
(2,1,0)(1,2)
(3,2,1)(2,0)
(2,2,1)(2,1)
(2,2,1)(2,2).

For both (2,1,0) and (2,2,1), the best results are obtained with the first seasonal difference of $\delta^T_{5,t}$. The models (2,1,0)(1,1), (2,2,1)(2,1), and (3,2,1)(2,0) all have a very high R^2 and a low variance of residuals and all three models transform $\delta^T_{5,t}$ to white noise at all lags. Since it has the fewest coefficients to estimate, (2,1,0)(1,1) is chosen as the best model.

The noise model (2,1,0)(1,1) is included with the input variables in a multivariate ARIMA process. The cross correlation function for $\delta^T_{5,t}$ and the prewhitened independent variables imply the following transfer functions:

TB_t : (1,1,0)(1,0);
$\%TB_t$: (1,0,0)(1,0);
Q_t : (0,3,0)(0,0).

Combining (2,1,0)(1,1) with the input variables does not improve its R^2 or residual variance, but at least one coefficient in every transfer function is statistically significant. The autocorrelations of residuals show that the model results in white noise and the cross correlations of residuals for each input variable are small relative to the critical values of χ^2 so there is no reason to question their specification. Again the coefficients in the transfer functions do not have consistent signs and therefore do not provide much insight into the relationship between $\delta^T_{5,t}$ and the input variables.

<u>Nonmember Banks (ν^D_t and ν^T_t)</u>

The rest of the parameters are used in the equation for r_t during all four reserve schemes. There are 730 observations available but since the estimation program cannot accept that many observations, the sample is divided into two parts and separate ARIMA models are estimated for each subperiod. For all the remaining parameters except ε_t, the sample is divided at January 10, 1968; this date almost divides the sample in half and also corresponds to one of the major structural changes in Federal Reserve reserve requirements described earlier.

a) Demand Deposits

For ν^D_t, the autocorrelation and partial autocorrelation functions for the first part of the sample imply that the following ARIMA models may be useful:

(3,1,0)(1,1)
(3,1,2)(1,1)
(3,1,1)(1,2)
(0,2,1)(2,1)
(0,2,1)(2,2).

The first three models all give very good results; each has a very high R^2, low residual variance and each converts ν^D_t to white noise at all lags. The (3,1,0)(1,1) process is judged superior however because it has the highest R^2 and the fewest coefficients to estimate.

It is hypothesized that the behavior of ν_t^D is affected by the number of member banks and by interest rates. The number of member banks is represented here by the number of member banks divided by the number of commercial banks (MB_t);[19] the behavior of interest rates is again summarized by the level and percentage change in the Treasury bill rate (TB_t and $\%TB_t$). There should of course be a direct relationship between ν_t^D and MB_t and it is expected that both TB_t and $\%TB_t$ are negatively related to ν_t^D. This latter hypothesis is based on the reasoning that during times of high or rising interest rates, membership is costlier and the tendency will be greater for banks to leave the System, and for ν_t^D to therefore fall.

Transfer functions have been fitted for these three input variables and are combined with the noise model for ν_t^D, (3,1,0)(1,1). After prewhitening each input variable, the cross correlation functions between them and ν_t^D imply the following transfer functions:

MB_t : (1,4,0)(0,0);
TB_t : (0,1,0)(1,0);
$\%TB_t$: (0,2,0)(1,0).

In the combined transfer function-noise model, none of the variables in the transfer function for $\%TB_t$ has a significant coefficient, so the model was refitted excluding $\%TB_t$. In that version, the transfer function for TB_t has no statistically significant coefficient and the coefficients are of the wrong sign; the transfer function for TB_t has therefore also been excluded. Excluding these two input variables has no harmful effects on the model's R^2 or residual variance; the

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[19] The variable MB_t is based on monthly data (last Wednesday of each month) on the number of member banks and commercial banks. Source: Board of Governors of the Federal Reserve System, Federal Reserve Bulletin, various dates, p. A-18.

results for the model with MB_t as the sole input variable are presented in Table 34. The coefficients in the transfer function for MB_t are in general of the expected sign although only one is statistically significant.[20] The model converts ν_t^D to white noise at all lags and the cross correlations are small relative to the critical χ^2 values, so the transfer function appears to be adequately specified.

For the second part of the sample period, the autocorrelation and partial autocorrelation functions indicate the following ARIMA processes to describe the behavior of ν_t^D:

(3,1,0)(1,1)
(0,2,1)(2,1)
(0,2,1)(2,2)
(1,1,1)(1,2).

In the (0,2,1) process, the second seasonal difference of ν_t^D performs better than the first. The remaining three models all reduce the residual variance to a low level, have significant coefficients, and high R^2's; each however results in white noise at short lags only. They have therefore been overfitted with the following models:

<u>(3,1,0)(1,1)</u>:
(3,1,1)(1,1);

<u>(0,2,1)(2,2)</u>:
(0,2,2)(2,2)
(1,2,1)(2,2)
(1,2,2)(2,2);

<u>(1,1,1)(1,2)</u>:
(1,1,2)(1,2)
(2,1,1)(1,2)
(2,1,2)(1,2).

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[20]Since the coefficients in the numerator of the transfer function for MB_t are not statistically significant, the model was also estimated using lower-order formulations of the transfer function, but the results are not as successful.

Table 34. ARIMA Results for ν_t^D and ν_t^T (Regression Results for the Best ARIMA Model Tested)

ν_t^D: (3,1,0)(1,1); MB_t: (1,4,0)(0,1) (1/1/61–1/10/68)

$$\Delta\nu_t^D = \left[\frac{1}{1 - \underset{(-4.578)}{.26125B} - \underset{(-4.085)}{.23488B^2} - \underset{(-1.943)}{.11107B^3}}\right] a_t + \left[\frac{\underset{(.4671)}{.09237} - \underset{(-1.039)}{.21060B} + \underset{(.0708)}{.02051B^2} + \underset{(.2924)}{.05970B^3} + \underset{(1.136)}{.22618B^4}}{1 - \underset{(-141.5)}{.99614B}}\right] MB_t$$

Lag k	Autocorrelations of Residuals $n\Sigma r^2_{aa}(k)$	Cross Correlations of Residuals $n\Sigma r^2_{a\alpha(MB)}(k)$
1-10	2.271	0.06
1-20	16.709	0.42
1-30	24.250	0.57

$R^2 = .988$ Variance of Residuals = .1369 E-05 Durbin-Watson Statistic = 2.004

ν_t^D: (3,1,0)(1,1) plus variables at lags 11, 13, and 27; MB_t: (1,4,0)(0,1); TB_t: (1,1,0)(1,1); $\%TB_t$: (1,0,0)(1,1) (1/10/68–12/31/74)

$$\Delta\nu_t^D = \left[\frac{1 + \underset{(3.167)}{.19391B^{11}} - \underset{(-3.391)}{.21155B^{13}} - \underset{(-2.259)}{.13918B^{27}}}{1 - \underset{(-4.253)}{.27226B} - \underset{(-1.289)}{.08470B^2} - \underset{(-1.020)}{.06264B^3}}\right] a_t + \left[\frac{\underset{(1.849)}{.65931} + \underset{(1.679)}{1.31336B} - \underset{(-1.756)}{1.03153B^2} + \underset{(2.654)}{.72843B^3} - \underset{(-3.854)}{.35126B^4}}{1 + \underset{(89.61)}{1.00299B}}\right] MB_t$$

$$+ \left[\frac{\underset{(-1.990)}{-.00146} + \underset{(2.130)}{.00157B}}{1 - \underset{(-57.52)}{.99225B}}\right] \Delta TB_t + \left[\frac{\underset{(1.133)}{.00005}}{1 + \underset{(23.26)}{1.00332B}}\right] \Delta\%TB_t$$

Table 34. Continued

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals		
k	$n\Sigma r^2_{aa}(K)$	$n\Sigma r^2_{a\alpha(MB)}(k)$	$n\Sigma r^2_{a\alpha(TB)}(k)$	$n\Sigma r^2_{a\alpha(\%TB)}(k)$
1-10	16.787	2.49	7.23	5.93
1-20	27.731	4.69	11.80	14.55
1-30	43.656	6.55	20.61	21.17

$R^2 = .992$ Variance of Residuals = .2120 E-05 Durbin-Watson Statistic = 1.962

ν^T_t: (3,2,3)(2,1); MB_t: (1,4,0)(0,1); Q_t: (0,3,0)(0,1) (1/1/61-1/10/68)

$$\Delta^2\nu^T_t = \left[\frac{\underset{}{1} - \underset{(-.4675)}{.31212B} + \underset{(3.889)}{.49566B^2} - \underset{(-.0380)}{.01273B^3}}{1 - \underset{(-1.460)}{.97101B} - \underset{(-.6600)}{.31659B^2} - \underset{(-1.294)}{.17165B^3}}\right]a_t + \left[\frac{\underset{(.5515)}{.03380} + \underset{(.0681)}{.00485B} - \underset{(-.1748)}{.01514B^2} - \underset{(-.0861)}{.00644B^3} + \underset{(.8045)}{.05117B^4}}{1 - \underset{(-24.04)}{1.01377B}}\right]MB_t$$

$$- \underset{(-2.031)}{.00093Q_t} - \underset{(-2.732)}{.00249Q_{t-1}} + \underset{(2.707)}{.00250Q_{t-2}} - \underset{(-2.319)}{.00112Q_{t-3}}$$

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals	
k	$n\Sigma r^2_{aa}(k)$	$n\Sigma r^2_{a\alpha(MB)}(k)$	$n\Sigma r^2_{a\alpha(Q)}(k)$
1-10	5.295	0.09	2.93
1-20	13.753	0.32	16.33
1-30	19.159	2.39	18.96

$R^2 = .994$ Variance of Residuals = .1296 E-06 Durbin-Watson Statistic = 1.979

Table 34. Continued

ν_t^T: (2,2,1)(2,1) plus a seasonal variable at lag 13; MB_t: (1,6,0)(0,1); TB_t: (1,1,0)(1,1); $\%TB_t$: (1,0,0)(1,1); Q_t: (0,4,0)(0,1) (1/10/68-12/31/74)

$$\Delta^2\nu_t^T = \left[\frac{1 + \underset{(10.59)}{1.00620B} - \underset{(-1.946)}{.07371B^{13}}}{1 + \underset{(4.103)}{.46614B} + \underset{(3.759)}{.31795B^2}}\right] a_t + \left[\frac{\underset{(2.515)}{.00052} - \underset{(-1.791)}{.00049B}}{1 - \underset{(-1.614)}{.73609B}}\right] \Delta TB_t + \left[\frac{\underset{(-2.003)}{-.00002}}{1 + \underset{(92.51)}{.99297B}}\right] \Delta\%TB_t$$

$$- \underset{(-1.615)}{.00025Q_t} - \underset{(-2.289)}{.00057Q_{t-1}} + \underset{(2.977)}{.00060Q_{t-2}} - \underset{(-2.215)}{.00038Q_{t-3}} + \underset{(1.010)}{.00009Q_{t-4}}$$

$$+ \left[\frac{\underset{(1.594)}{.09813} + \underset{(1.879)}{.12119B} + \underset{(.8852)}{.07302B^2} - \underset{(-1.546)}{.13318B^3} - \underset{(-.2220)}{.01672^4} + \underset{(.6928)}{.04649B^5} + \underset{(.0118)}{.00067B^6}}{1 - \underset{(-38.04)}{.99253B}}\right] MB_t$$

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals			
k	$n\Sigma r_{aa}^2(k)$	$n\Sigma r_{a\alpha(MB)}^2(k)$	$n\Sigma r_{a\alpha(TB)}^2(k)$	$n\Sigma r_{a\alpha(\%TB)}^2(k)$	$n\Sigma r_{a\alpha(Q)}^2(k)$
1-10	9.071	0.13	9.71	13.30	2.97
1-20	13.663	7.73	21.48	30.08	10.00
1-30	22.120	19.80	29.38	38.07	43.19

R^2 = .999 Variance of Residuals = .1808 E-06 Durbin-Watson Statistic = 1.967

Note: For simplicity, the degree of seasonal differencing (Δ^s) is omitted from the regression equations. The degree of seasonal differencing is denoted in the (p,d,q)(d,s) description for each parameter.

The model (3,1,1)(1,1) is not an improvement over (3,1,0)(1,1); the coefficient of the moving average variable is not significant. The model (0,2,2)(2,2) gives better results than (0,2,1)(2,2) and the other two models overfitted to it. Of the models overfitted to (1,1,1)(1,2), the best is (2,1,1)(1,2). The most promising models are therefore (3,1,0)(1,1), (0,2,2)(2,2), and (2,1,1)(1,2); each one has a high R^2 but results in white noise at lags 1-10 only. The autocorrelations of the residuals of all three models are large at lags, 11, 13, and 27, so each was rerun including moving average variable(s) at those lag(s). With the seasonal variables added, the results for all three processes are very similar. The (3,1,0)(1,1) process plus variables at lags 11, 13, and 27 is chosen as the best since it has the highest R^2; it transforms ν_t^D to white noise except at short lags.

This noise model plus transfer functions for MB_t, TB_t, and $\%TB_t$ were included in a multivariate model. After prewhitening each input variable, their cross correlations with ν_t^D imply the following transfer functions:

$$\begin{aligned} MB_t &: (1,4,0)(0,0); \\ TB_t &: (1,1,0)(1,0); \\ \%TB_t &: (1,0,0)(1,1). \end{aligned}$$

Each independent variable has at least one statistically significant coefficient and the cross correlations of residuals for each transfer function are small, implying that they are adequately specified. The multivariate process does not however result in white noise. The autocorrelations of residuals are large at lags 7, 9, and 15 but when variables at those lags are added, the model's results are poorer. The other two successful noise models described above, (0,2,2)(2,2) and (2,1,1)(1,2) were also used in the multivariate process, but they both perform more poorly than (3,1,0)(1,1). This noise model plus the three transfer functions therefore appears to be the best model that can be constructed for ν_t^D; its results are given in Table 34. The signs of coefficients for MB_t are not consistently positive but the majority of them do imply the expected direct relation between MB_t and ν_t^D. The coefficients of TB_t are also of mixed sign and those for $\%TB_t$ are not of the expected sign.

b) Time Deposits

The autocorrelations and partial autocorrelations for ν_t^T for the first part of the sample period indicate that the following ARIMA models are relevant:

(3,2,0)(2,0)
(3,2,3)(2,1)
(3,2,2)(2,2).

All three models give very good results; both (3,2,0)(2,0) and (3,2,3)(2,1) result in white noise at all lags. The (3,2,0)(2,0) process appears to be preferable since it has fewer coefficients to estimate and its R^2 is slightly higher; both (3,2,0)(2,0) and (3,2,3)(2,1) were however used in the multivariate process described below.

The behavior of ν_t^T is presumably also affected by MB_t, TB_t, and $\%TB_t$. In addition, Q_t, the difference between the Treasury bill rate and the Regulation Q ceiling for time deposits is relevant. Transfer functions for these four independent variables are therefore fitted and included in a multivariate process with the noise model (3,2,0)(2,0) and with (3,2,3)(2,1). After prewhitening each independent variable, their cross correlations with ν_t^T indicate the following transfer functions:

$$
\begin{aligned}
MB_t &: (1,4,0)(0,0); \\
TB_t &: (0,1,0)(1,0); \\
\%TB_t &: (0,2,0)(0,0); \\
Q_t &: (0,3,0)(0,0).
\end{aligned}
$$

The best results are obtained using (3,2,3)(2,1) as the noise model. The transfer functions for TB_t and $\%TB_t$ contain no statistically significant coefficients so the model was refitted excluding these input variables. These results are reported in Table 34. The model has a high R^2 and low residual variance. The model converts ν_t^T to white noise at all lags and the cross correlations of residuals for both transfer functions are small, indicating that the functions are adequately specified.

The transfer function for MB_t would be expected to have positive coefficients, so two of the coefficients in its numerator are of the wrong sign. The _a priori_ relationship between ν_t^T and Q_t is not clear. When Q_t is large, time deposits in commercial banks will decline (or not grow as fast). The impact on ν_t^T of this situation will depend on the relative ability of member and nonmember banks to attract (or fail to lose) time deposits as their overall level grows more slowly or falls. The coefficients in the transfer function for Q_t, all of which are statistically significant, indicate that ν_t^T and Q_t are inversely related; this implies that when market conditions make time deposits relatively unattractive, member banks fail to effectively compete and their share of the nation's time deposits falls. Since member banks dominate the interest-elastic market for large certificates of deposit, this result seems appropriate. The negative coefficients for Q_t may also be the result of contemporaneous rise in Q_t and fall in ν_t^T during the sample period, without the existence of any causal link.

For the second part of the sample period, the autocorrelation and partial autocorrelation functions for ν_t^T show that these ARIMA models should be considered:

(3,2,0)(2,0)
(2,2,1)(2,1)
(2,2,1)(2,2).

The (2,2,1)(2,1) process performs best; it has a very high R^2, low residual variance, significant coefficients and converts ν_t^T to white noise at all lags. The noise model (2,2,1)(2,1) is combined with transfer functions representing MB_t, TB_t, $\%TB_t$, and Q_t. After prewhitening each input variable, the following transfer functions are implied by their cross correlations with ν_t^T:

MB_t : (1,6,0)(0,0);
TB_t : (1,1,0)(1,0);
$\%TB_t$: (1,0,0)(1,0);
Q_t : (0,4,0)(0,0).

There are statistically significant coefficients in all four transfer functions so all of the input variables are retained; the results for the model are given in Table 34. The noise model does not result in white noise; the autocorrelation of residuals is large at lag 13 so a moving average variable is added at that lag. With this addition, the model does transform ν_t^T to white noise at all lags.

The cross correlations of residuals for MB_t are small so the transfer function is apparently adequate. None of the seven coefficients in the numerator are statistically significant and would therefore appear to be redundant, but a transfer function of lower order results in much larger cross correlations. The signs of the coefficients for MB_t in general reflect the expected direct relationship between MB_t and ν_t^T, although two are negative.

The relationship between ν_t^T and TB_t or $\%TB_t$ is complicated and there is no a priori way to characterize it. As discussed earlier, large values of TB_t or $\%TB_t$ may correspond to either increases or decreases in the level of time deposits; furthermore, whatever the relationship between the Treasury bill rate and the rate of growth in time deposits, the relationship between the growth of time deposits and ν_t^T is also not clear. In addition, there may be some tendency for high or rising interest rates to encourage banks to leave the System, implying an inverse relationship between ν_t^T and TB_t and $\%TB_t$. The results here reflect an inverse relationship between $\%TB_t$ and ν_t^T; the relationship between TB_t and ν_t^T is not clear, although the statistically significant coefficient implies a direct relationship. Both transfer functions result in white noise, although for $\%TB_t$, at the 2.5% level of significance only.

The coefficients of Q_t are also mixed; the majority are negative, implying the inverse relationship between Q_t and ν_t^T also reported for the first part of the sample period. The cross correlations of residuals are large, implying that the transfer function for Q_t is not adequate, but no reasonable modification of it gives any better results.[21]

Nonmoney Deposits (γ_t, ι_t, τ_t)

a) Government Deposits

The autocorrelation and partial autocorrelation functions for the parameter γ_t indicate that for the first part of the sample period the

[21]The transfer function used here results in white noise except at lags 21-30. The cross correlation is large at and near lag 28, but including variables at these lags does not remedy the situation.

following models should be considered:

(3,1,0)(1,1)
(3,1,0)(1,2)
(0,2,3)(2,1)
(0,2,3)(2,2).

For both models, the first seasonal difference of the series gives better results than the second. However neither (3,1,0)(1,1) nor (0,2,3)(1,1) converts γ_t to white noise. These two models have therefore been overfitted in the following ways:

(3,1,0)(1,1):
(3,1,1)(1,1)
(3,1,2)(1,1);

(0,2,3)(2,1):
(1,2,3)(2,1)
(2,2,3)(2,1).

Both models overfitted to (3,1,0)(1,1) give poorer results than it does. The models overfitted to (0,2,3)(2,1) are both slight improvements but neither results in white noise, even at short lags. The most promising model appears to be (3,1,0)(1,1) since it does convert γ_t to white noise at short lags. Its residual autocorrelation is large at the seasonal lag 13 and when a moving average variable is added at that lag, the resulting model transforms the series to white noise at all lags at the 5% level of significance. This is apparently the best model that can be devised.

It is presumed that the value of γ_t is strongly affected by the schedule of payments dates for federal taxes. A dummy variable (G_t) was constructed to represent tax payment dates[22] and is included as an

[22] The dummy variable is defined as $G_t = 1$ for the first Wednesday after every tax payment date and $G_t = 0$ otherwise. The tax payment dates used are January 15, March 15, April 15, June 15, September 15, and December 15.

input variable for γ_t, along with the noise model (3,1,0)(1,1) plus a variable at lag 13. Since the series G_t varies so little from week to week, the process for prewhitening and identification of its transfer function fails. Therefore the simple transfer function (1,0,0) is used.

The coefficients in the transfer function for G_t have the proper signs but they are not statistically significant. In addition, inclusion of the transfer function has no effect on the model's performance. Since these results show no important relationship between G_t and γ_t the input variable is dropped. It may be that G_t is not a good representation of the tax-payment date cycle or it may be that the seasonal variable at lag 13 in the noise model (which is statistically significant) is related to that cycle and therefore the introduction of G_t does not add to the explanatory power of the model. The univariate process (3,1,0)(1,1) plus a variable at lag 13 is apparently the best model that can be constructed for γ_t for this time period. The estimated coefficients satisfy stationarity and invertibility conditions. The results for the model are given in Table 35.

For the second part of the sample period, the following models appear to be promising:

(3,1,3)(1,1)
(0,2,1)(2,1)
(0,2,1)(2,2)
(3,0,3)(0,2).

Of these models, the best results are obtained with (3,1,3)(1,1) and (0,2,1)(2,1), but neither model results in white noise at long lags. The latter model has therefore been overfitted with the following models:

Table 35. ARIMA Results for γ_t, ι_t, τ_t and ε_t (Regression Results for the Best ARIMA Model Tested)

γ_t: (3,1,0,)(1,1) plus a seasonal variable at lag 13 (1/1/61-1/10/68)

$$\Delta\gamma_t = \underset{(3.934)}{.21762\Delta\gamma_{t-1}} + \underset{(3.186)}{.17864\Delta\gamma_{t-2}} + \underset{(4.447)}{.24727\Delta\gamma_{t-3}} + a_t - \underset{(-5.219)}{.29344a_{t-13}}$$

Lag k	Autocorrelations of Residuals $n\Sigma r^2_{aa}(k)$	
		R^2 = .777
		Variance of Residuals = .5855 E-04
1-10	9.964	Durbin-Watson Statistic = 1.999
1-20	27.552	
1-30	37.915	

γ_t: (2,1,3)(1,1) plus seasonal variables at lags 13 and 26 (1/10/68-12/31/74)

$$\Delta\gamma_t = \underset{(1.379)}{.21075\Delta\gamma_{t-1}} + \underset{(.8600)}{.11726\Delta\gamma_{t-2}} + a_t + \underset{(.3088)}{.04362a_{t-1}} + \underset{(.9071)}{.11605a_{t-2}} + \underset{(4.024)}{.27732a_{t-3}} + \underset{(-4.590)}{.25452a_{t-13}}$$
$$+ \underset{(4.777)}{.27156a_{t-26}}$$

Lag k	Autocorrelations of Residuals $n\Sigma r^2_{aa}(k)$	
		R^2 = .787
1-10	7.715	Variance of Residuals = .5645 E-04
1-20	19.630	
1-30	30.179	Durbin-Watson Statistic = 1.996

Table 35. Continued

ι_t: (0,1,3)(1,1) plus variables at lags 13 and 21; MB_t: (1,1,0)(0,1); TB_t: (0,1,0)(1,1); ; $\%TB_t$:(0,2,0)(0,1) (1/1/61-1/10/68)

$$\Delta\iota_t = a_t + \underset{(10.49)}{.59486}a_{t-1} + \underset{(.2667)}{.01781}a_{t-2} + \underset{(3.096)}{.21983}a_{t-3} - \underset{(-2.810)}{.11108}a_{t-13} + \underset{(3.307)}{.13627}a_{t-21} + \left[\frac{\underset{(1.124)}{.25322} + \underset{(.9810)}{.22167}B}{1 - \underset{(-79.31)}{.99704}B}\right] MB_t$$

$$+ \underset{(.3492)}{.00045}\Delta TB_t + \underset{(.2734)}{.00096}\Delta TB_{t-1} + \underset{(.4179)}{.00002}\%TB^{*}_{t-1} + \underset{(1.426)}{.00007}\%TB_{t-2}$$

Lag k	Autocorrelations of Residuals $n\Sigma r^2_{aa}(k)$	Cross Correlations of Residuals $n\Sigma r^2_{a\alpha(MB)}(k)$	$n\Sigma r^2_{a\alpha(TB)}(k)$	$n\Sigma r^2_{a\alpha(\%TB)}(k)$
1-10	6.732	0.13	1.91	0.68
1-20	24.044	0.77	10.89	12.27
1-30	36.185	1.20	15.78	17.98

R^2 = .805 Variance of Residuals = .5782 E-05 Durbin-Watson Statistic = 2.050

*The estimated coefficient of $\%TB_t$ is .0000.

Table 35. Continued

ι_t: (3,1,1)(1,1) plus a seasonal variable at lag 13; MB_t: (1,4,0)(0,1); $\%TB_t$: (1,1,0)(1,1) plus a variable at lag 12 (1/10/68-12/31/74)

$$\Delta\iota_t = \left[\frac{1 + \underset{(6.630)}{.63634B} - \underset{(-2.833)}{.14188B^{13}}}{1 - \underset{(-.0897)}{.00971B} - \underset{(-.9332)}{.07630B^2} - \underset{(-1.395)}{.10230B^3}}\right] a_t$$

$$+ \left[\frac{\underset{(1.751)}{2.11270} + \underset{(.3870)}{3.67169B} - \underset{(-.1998)}{2.21974B^2} + \underset{(.2038)}{.95575B^2} - \underset{(-.1628)}{.30347B^4}}{1 + \underset{(.1472)}{.53634B}}\right] MB_t$$

$$+ \left[\frac{\underset{(.3723)}{.00003} - \underset{(-2.464)}{.00019B} - \underset{(-3.028)}{.00024B^{12}}}{1 - \underset{(-.0138)}{.00371B}}\right] \Delta\%TB_t$$

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals	
k	$n\Sigma r^2_{aa}(k)$	$n\Sigma r^2_{a\alpha(MB)}(k)$	$n\Sigma r^2_{a\alpha(\%TB)}(k)$
1-10	5.498	0.57	2.46
1-20	18.520	4.74	17.20
1-30	27.594	5.41	27.77

$R^2 = .723$ Variance of Residuals = .5339 E-04 Durbin-Watson Statistic = 1.966

Table 35. Continued

τ_t: (3,1,3)(1,2) plus a seasonal variable at lag 13; TB_t: (0,2,0)(1,2) plus a variable at lag 14; $\%TB_t$: (0,2,0)(0,2) plus a variable at lag 13; Q_t: (0,3,0)(0,2) (1/1/61-1/10/68)

$$\Delta\tau_t = \left[\frac{1 - \underset{(-.4583)}{.08865B} - \underset{(-1.105)}{.22671B^2} + \underset{(1.981)}{.30591B^3} - \underset{(-1.787)}{.11423B^{13}}}{1 - \underset{(-1.930)}{.38608B} - \underset{(-.7601)}{.18151B^2} - \underset{(-.3693)}{.06856B^3}}\right] a_t + \underset{(.1357)}{.00047\Delta TB_t} + \underset{(2.893)}{.03404\Delta TB_{t-1}}$$

$$+ \underset{(2.239)}{.02703\Delta TB_{t-2}} + \underset{(1.194)}{.01517\Delta TB_{t-14}} + \underset{(1.966)}{.00082\%TB_t} - \underset{(-1.941)}{.00082\%TB_{t-1}} + \underset{(1.622)}{.00021\%TB_{t-2}}$$

$$- \underset{(-.6629)}{.00029\%TB_{t-13}} + \underset{(2.604)}{.01677Q_t} + \underset{(4.330)}{.04024Q_{t-1}} - \underset{(-2.246)}{.02039Q_{t-2}} + \underset{(.5682)}{.00351Q_{t-3}}$$

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals		
k	$n\Sigma r^2_{aa}(k)$	$n\Sigma r^2_{a\alpha(TB)}(k)$	$n\Sigma r^2_{a\alpha(\%TB)}(k)$	$n\Sigma r^2_{a\alpha(Q)}(k)$
1-10	11.853	6.15	10.18	4.16
1-20	18.257	23.42	23.46	11.76
1-30	25.319	35.02	31.68	18.00

$R^2 = .997$ Variance of Residuals = .6838 E-04 Durbin-Watson Statistic = 1.995

Table 35. Continued

τ_t: (2,1,3)(1,2); TB_t: (1,0,0)(1,2); $\%TB_t$: (1,0,0)(1,2) (1/10/68-12/31/74)

$$\Delta\tau_t = \left[\frac{\underset{(1.788)}{1 + .77435B} + \underset{(.1110)}{.05361B^2} + \underset{(.3019)}{.04022B^3}}{1 + \underset{(1.315)}{.56904B} + \underset{(.9067)}{.35189B^2}}\right] a_t + \left[\frac{\underset{(-.8942)}{-.00310}}{1 - \underset{(-2.529)}{.88030B}}\right]\Delta TB_t + \left[\frac{\underset{(-1.785)}{-.00041}}{1 + \underset{(69.73)}{1.00112B}}\right]\Delta\%TB_t$$

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals	
k	$n\Sigma r^2_{aa}(k)$	$n\Sigma r^2_{a\alpha(TB)}(k)$	$n\Sigma r^2_{a\alpha(\%TB)}(k)$
1-10	6.541	9.89	6.98
1-20	17.207	17.64	21.77
1-30	30.059	30.51	40.18

R^2 = .971 Variance of Residuals = .6002 E-03 Durbin-Watson Statistic = 1.995

ε_t: (0,1,3)(1,1); TB_t: (0,1,0)(1,1) (1/1/61-9/18/68)

$$\Delta\varepsilon_t = a_t + \underset{(20.11)}{1.08805a_{t-1}} - \underset{(-1.980)}{.15719a_{t-2}} - \underset{(-1.260)}{.06824a_{t-3}} + \underset{(2.117)}{.00039\Delta TB_t} - \underset{(2.471)}{.00045\Delta TB_{t-1}}$$

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals
k	$n\Sigma r^2_{aa}(k)$	$n\Sigma r^2_{a\alpha(TB)}(k)$
1-10	8.224	5.45
1-20	18.328	13.74
1-30	24.228	25.43

R^2 = .729 Variance of Residuals = .1718 E-06 Durbin-Watson Statistic = 1.931

Table 35. Continued

ε_t: (1,0,4)(0,1); TB_t: (1,0,0)(1,1); $\%TB_t$: (1,0,0)(0,1) (9/18/68–12/31/74)

$$\varepsilon_t = \left[\frac{\underset{(35.49)}{1 + .96346B}}{1 + \underset{(22.27)}{1.50635B} - \underset{(-5.528)}{.61917B^2} + \underset{(.7978)}{.08906B^3} - \underset{(-.8453)}{.05414B^4}}\right] a_t + \left[\frac{\underset{(1.663)}{.00008}}{1 - \underset{(-51.76)}{.98917B}}\right] \Delta TB_t + \left[\frac{\underset{(2.480)}{.00001}}{1 - \underset{(-17.69)}{.93807B}}\right] \%TB_t$$

Lag	Autocorrelations of Residuals	Cross Correlations of Residuals	
k	$n\Sigma r^2_{aa}(k)$	$n\Sigma r^2_{a\alpha(TB)}(k)$	$n\Sigma r^2_{a\alpha(\%TB)}(k)$
1-10	7.456	12.35	11.52
1-20	15.998	23.14	30.17
1-30	27.478	30.75	38.40

$R^2 = .197$ Variance of Residuals = .1303 E-06 Durbin-Watson Statistic = 2.001

Note: For simplicity, the degree of seasonal differencing (Δ^s) is omitted from the regression equations. The degree of seasonal differencing is denoted in the (p,d,q)(d,s) description for each parameter.

(0,2,1)(2,1):
- (0,2,2)(2,1)
- (1,2,1)(2,1)
- (1,2,2)(2,1)
- (0,2,3)(2,1).

None of these models are any better than (0,2,1)(2,1); in each case, the coefficients of the added variables are not statistically significant and the impact on R^2 and $n\Sigma r^2_{aa}(k)$ is negligible.

The most promising model appears to be (3,1,3)(1,1) because it has a higher R^2 than (0,2,1)(2,1) and it results in white noise at short lags. The autocorrelations of its residuals are large at lags 16 and 18; the best results occur with an added moving average variable at lag 18 only. The resulting model converts γ_t to white noise at the 5% level of significance, except at lags 11-20.

When the (1,0,0)(0,0) transfer function for G_t is combined with the noise model, the results are poorer than those for the univariate model. The coefficients in the transfer function are not statistically significant and one has the wrong sign. In addition, inclusion of the transfer function for G_t has a slightly harmful effect on the model's R^2, residual variance and on the autocorrelations of residuals of the noise model. Thus the univariate model is apparently the best model that can be devised for γ_t for this time period.

The noise model (3,1,3)(1,1) however violates stationarity and invertibility conditions so the lower order models (2,1,3)(1,1) and (3,1,2)(1,1) were estimated. The best results are obtained with (2,1,3)(1,1) plus seasonal variables at lags 13 and 26. This model results in white noise at all lags and satisfied invertibility and stationarity conditions. Its results are given in Table 35.

b) Interbank Deposits

For the first part of the sample, the autocorrelation and partial autocorrelation functions for ι_t imply that the following ARIMA processes are useful:

(0,1,3)(1,1)
(0,1,3)(1,2)
(0,2,2)(2,1)
(0,2,2)(2,2).

In the second-order moving average process, the first seasonal difference of the data gives better results than the second; (0,2,2)(2,1) has a good R^2 but does not result in white noise. It has therefore been overfitted with the models (1,2,2)(2,1) and (0,2,3)(2,1), but neither gives better results than (0,2,2)(2,1). Neither (0,2,2)(2,1) nor the overfitted models converts ι_t to white noise, even at short lags.

The (0,1,3)(1,2) model transforms ι_t to white noise at all lags but has a very low R^2. On the other hand, (0,1,3)(1,1) has a good R^2 but results in white noise at short lags only. The residual autocorrelations for (0,1,3)(1,1) are large at lags 13, 21, and 29, so the model was refitted with moving average variables at those lag(s) included. The best result occurs with variables at lags 13 and 21; it results in white noise at all lags.

It is hypothesized that ι_t is influenced by the relative number of member banks (MB_t) and by interest rates, represented by TB_t and $\%TB_t$. Since nonmember banks hold interbank deposits for required reserves, it may be that ι_t and MB_t vary inversely. Gilbert however has found that member banks tend to hold levels of interbank deposits higher than what would be required for reserve purposes if they are nonmembers.[23] It is

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[23] Alton Gilbert, "Utilization of Federal Reserve Bank Services by Member Banks: Implications for the Cost and Benefits of Membership," Review, Federal Reserve Bank of St. Louis 59 (August 1977):12.

expected that ι_t is inversely related to TB_t and $\%TB_t$ since high or rising interest rates should be an incentive to economize on noninterest bearing assets such as interbank balances.[24]

For the first part of the sample period the noise model (0,1,3)(1,1) plus variables at lags 13 and 21 is used to fit a multivariate process which also includes MB_t, TB_t, and $\%TB_t$. After prewhitening the input variables, their cross correlations with ι_t imply the following transfer functions:

$$\begin{aligned} MB_t &: (1,1,0)(0,0); \\ TB_t &: (0,1,0)(1,0); \\ \%TB_t &: (0,2,0)(0,0). \end{aligned}$$

Only the transfer function for MB_t contains statistically significant coefficients but when the other two input variables are left out, the model gives poorer results. The results of the model including all three independent variables are given in Table 35.

The cross correlations of residuals for all three input variables are small, so the transfer functions appear to be appropriate. The coefficients of MB_t indicate that ι_t and MB_t are directly related, supporting Gilbert's result that member banks hold more interbank balances than nonmembers. The coefficients of TB_t and $\%TB_t$ are not of the expected sign, but they are of course also not statistically significant. The model results in white noise at the 5% level of significance.

The autocorrelations and partial autocorrelations of ι_t for the second subperiod indicate that the following models are promising:

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[24]In addition, if high or rising interest rates encourage banks to leave the System and nonmember banks do hold fewer interbank balances then TB_t and $\%TB_t$ would again be expected to be inversely related to ι_t.

(0,1,3)(1,1)
(3,1,0)(1,1)
(3,1,3)(1,1)
(3,1,1)(1,2)
(0,2,2)(2,1)
(0,2,2)(2,2).

Both of the models that employ the second seasonal difference of ι_t give relatively poor results, but none of the models result in white noise. Consequently the following overfitted models were also estimated with the first seasonal difference of the data:

<u>(0,1,3)(1,1)</u>:
    (1,1,3)(1,1)
    (2,1,3)(1,1);

<u>(3,1,0)(1,1)</u>:
    (3,1,1)(1,1)
    (3,1,2)(1,1);

<u>(0,2,2)(2,1)</u>:
    (1,2,2)(2,1)
    (0,2,3)(2,1).

Of all these models the most promising are (2,1,3)(1,1), (3,1,1)(1,1), (3,1,2)(1,1), and (3,1,3)(1,1), each of which has a respectable R^2 and results in white noise at lags 1-10. They do not result in white noise after that because for each model the autocorrelations of residuals are large at lags 13 and 17. All four models were therefore refitted including moving average variables at lag 13 and at lags 13 and 17. The best results are obtained with the process (3,1,1)(1,1) plus a seasonal variable at lag 13; it transforms ι_t to white noise at all lags.

This noise model is then combined with transfer functions for MB_t, TB_t, and $\%TB_t$. After prewhitening each input variable, their cross correlations with ι_t lead to the following transfer functions:

MB_t : (1,4,0)(0,0);
TB_t : (1,1,0)(1,0);
$\%TB_t$: (1,1,0)(1,1).

None of the coefficients in the transfer function for TB_t is statistically significant, so it is eliminated from the multivariate process. The cross correlation of residuals for the transfer function for $\%TB_t$ is large at lag 12, so the transfer function was modified to include a variable at that lag.

This model results in white noise at all lags and the cross correlations for both transfer functions are small; the results are given in Table 35. When the variable at lag 12 is added to the transfer function for $\%TB_t$, the coefficients of MB_t lose their statistical significance. The coefficients of MB_t however reflect a direct relationship with ι_t as was the case for the first part of the sample. The significant coefficients of $\%TB_t$ show that ι_t and $\%TB_t$ are inversely related, as would be expected.

c) Time Deposits

The autocorrelation and partial autocorrelation functions for τ_t indicate that the following processes are relevant for the first subperiod:

(3,1,3)(1,0)
(3,1,3)(1,1)
(3,1,3)(1,2)
(0,2,3)(2,1)
(0,2,3)(2,2).

The best results occur with the (3,1,3)(1,2) process which has a high R^2 and converts τ_t to white noise at all lags. This noise model is therefore used to estimate a multivariate process using TB_t, $\%TB_t$, and Q_t as input variables. After prewhitening, the cross correlations between τ_t and the independent variables imply the following transfer functions:

TB_t : (0,2,0)(1,0);
$\%TB_t$: (0,2,0)(0,0);
Q_t : (0,3,0)(0,0).

All three transfer functions have at least one statistically significant coefficient, but the cross correlations of residuals for TB_t and $\%TB_t$ imply that their transfer functions are not adequate. Furthermore, the model converts τ_t to white noise at the 2.5% level of significance only. Both transfer functions and the noise model are therefore modified as indicated by their cross and autocorrelations of residuals. For TB_t, this entails adding a moving average variable at lag 14; for $\%TB_t$ and the noise model, variables at lag 13 are included. This modified model converts τ_t to white noise except at lags 1-10 and all the transfer functions have cross correlations smaller than the criitcal χ^2 values.

The nature of the relationship between τ_t and TB_t and $\%TB_t$ is again not a priori clear. As TB_t or $\%TB_t$ increase, τ_t may also grow as time deposits become more attractive relative to noninterest bearing assets; but if large values of TB_t and $\%TB_t$ represent a situation where other interest-bearing assets are more attractive than time deposits, τ_t may not rise (or may rise more slowly). The coefficients for TB_t are all positive, implying that the former relationship dominates. The signs of the coefficients for $\%TB_t$ are mixed so the relationship between $\%TB_t$ and τ_t remains unclear; the coefficients of $\%TB_t$ also lose their statistical significance when the variable at lag 13 is added. Q_t and τ_t are expected to vary inversely since large values of Q_t mean that time deposits are an unattractive investment relative to other market instruments. Three of the four coefficients of Q_t are however positive. This result probably occurs because whatever causal relationhip there is between Q_t and τ_t is weak, and the appearance of a direct relationship occurs because of the contemporaneous rise in τ_t and Q_t during the sample period.

For the second part of the sample the autocorrelations and partial autocorrelations show that these ARIMA models may describe the behavior of τ_t:

(1,1,2)(1,1)
(2,1,2)(1,2)
(0,0,2)(2,1)
(0,2,2)(2,2).

The best results are obtained with the (2,1,2)(1,2) process; it has a high R^2 and low residual variance and it transforms τ_t to white noise at all lags.

This noise model plus the input variables TB_t, $\%TB_t$, and Q_t are combined in a noise-transfer function model. After prewhitening each input variable, their cross correlations with τ_t suggest the following transfer functions:

TB_t : (1,0,0)(1,0);
$\%TB_t$: (1,0,0)(1,0);
Q_t ; (1,5,0)(0,0).

None of the coefficients in the transfer function for Q_t are statistically significant so it is dropped.[25] With the remaining two input variables included, the process does not convert τ_t to white noise. The autocorrelations of residuals suggest including an additional moving average variable in the noise model. With this noise model, the multivariate process transforms τ_t to white noise at all lags and the cross correlations of residuals for both transfer functions are small. The results for this model are reported in Table 35.

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[25]Several alternative formulations of the transfer function were also tried but the results are the same.

The signs of the coefficients for TB_t and $\%TB_t$ show an inverse relationship between τ_t and market interest rates. This implies that high and rising interest rates slow down the growth in τ_t as other interest-bearing assets become more attractive than time deposits. While this contradicts the result for the earlier part of the sample, there is a possible explanation for the contradiction. Given the overall rise in interest rates and τ_t during the sample, it seems plausible that initially high and rising rates enticed consumers to hold more time deposits and fewer noninterest bearing assets. As interest rates continued to rise however, the movement into time deposits tempered as other interest-bearing assets became more attractive. This would account for a direct relationship between τ_t and interest rates during the first part of the sample and an inverse one thereafter. The lack of a significant relationship between Q_t and τ_t supports the conclusion drawn for the first part of the sample of a weak causal link between Q_t and τ_t.

Excess Reserves (ε_t)

For ε_t, separate ARIMA models are estimated for the part of the sample period before September 18, 1968 (402 observations) and after that date (328 observations). This is the date when the reserve carry-over procedure, described in Chapter 4, was introduced and it is hypothesized that this procedure altered the behavior of ε_t. For the first subperiod, the autocorrelation and partial autocorrelation functions for ε_t imply that the following processes are pertinent:

(0,1,1)(1,1)
(0,2,2)(2,1)
(0,2,2)(2,2)
(1,0,1)(0,2)
(0,1,2)(1,2).

Of these five models, those that perform best are (0,1,1)(1,1), (0,1,2)(1,2), and (0,2,2)(2,1). The last two models both result in white noise, but neither has a very high R^2. The (0,1,1)(1,1) process however does not result in white noise although it has a reasonably high R^2. The models have therefore been overfitted with the following models:

<u>(0,1,1)(1,1)</u>:
(0,1,2)(1,1)
(0,1,3)(1,1)
(1,1,1)(1,1);

<u>(0,1,2)(1,2)</u>:
(0,1,3)(1,2)
(1,1,2)(1,2);

<u>(0,2,2)(2,1)</u>:
(0,2,3)(2,1)
(1,2,2)(2,1).

Both the second and third order moving average processes are improvements over (0,1,1)(1,1); both models result in white noise at all lags, and each has a reasonably high R^2. The other models do not perform better than the original formulations. There are three models then which convert ε_t to white noise, (0,1,2)(1,1), (0,1,3)(1,1), and (0,2,2)(2,1). The model (0,1,3)(1,1) is however the best since it has the highest R^2.

The noise model (0,1,3)(1,1) is used in a multivariate model which also includes the effects of interest rates, represented by TB_t and $\%TB_t$. The presumption is that ε_t will be inversely related to both input variables. After prewhitening the input variables, the cross correlations between them and ε_t indicate the following transfer functions:

TB_t : (0,1,0)(1,0);
$\%TB_t$: (0,2,0)(0,0).

The coefficients in the transfer function for $\%TB_t$ are not statistically significant so the model was refitted excluding that independent variable. The results of the model are given in Table 35. The coefficients in the transfer function for TB_t are both significant but not of the expected sign. The autocorrelations and cross correlations of residuals are small, so the model transforms ε_t to white noise and the transfer function appears to be adequate.

For the second subperiod, from the autocorrelations and partial autocorrelations of ε_t, the following models appear to be important:

(0,1,1)(1,0)
(1,0,2)(0,1)
(0,1,5)(1,1).

Each of these three models has a very low R^2 even though the last two convert ε_t to white noise at all lags. In an attempt to increase the R^2, each model has been overfitted with the following models:

<u>(0,1,1)(1,0)</u>:
(1,1,1)(1,0)
(0,1,2)(1,0)
(1,1,2)(1,0);

<u>(1,0,2)(0,1)</u>:
(1,0,3)(0,1)
(1,0,4)(0,1)
(2,0,2)(0,1);

<u>(0,1,5)(1,1)</u>:
(1,1,5)(1,1).

Adding a moving average to (0,1,1)(1,0) improves the model's performance some but adding autoregressive variables does not. The (0,1,2)(1,0) process however still does not have a very high R^2 or result in white noise. Therefore additional moving average variables were successively included up to a fifth-order process; each added variable improves the model slightly but $n\Sigma r_{aa}^2(k)$ is not reduced to the level required for white noise. All of the models overfitted to (1,0,2)(0,1), as well as (0,1,5)(1,1), result in white noise but none has a very high R^2. The best model tested is (1,0,4)(0,1) since it has the highest R^2, which is however only .178.

The process (1,0,4)(0,1) plus transfer functions for TB_t and $\%TB_t$ are used in a multivariate model. The cross correlations of ε_t and each prewhitened input variable imply the following transfer functions:

$$\begin{array}{rcl} TB_t & : & (1,0,0)(1,0); \\ \%TB_t & : & (1,0,0)(0,0). \end{array}$$

Both transfer functions contain at least one statistically significant coefficient, but the coefficients are again not of the expected sign. Inclusion of the independent variables improves the model's R^2 some, but it is still not very high; the process transforms ε_t to white noise at all lags. The cross correlations of residuals for both transfer functions are small relative to the χ^2 values, implying that they convert the respective input variables to white noise. The result of the transfer function for $\%TB_t$ is however white noise at the 2.5% level of significance only, but no modification of the function reduces its cross correlations further.[26]

The model's low R^2 in the second part of the sample may be attributable to the reserve-carryover rules. It is possible that this procedure causes ε_t to behave in a (perhaps systematic) way that cannot be captured by this type of model. Or, it may be that reserve carryover plus an efficiently functioning federal funds market removes most

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[26]The model was also fitted using (1,0,0)(1,1), (0,2,0)(0,1) (1,1,0)(0,1), and (0,1,0)(0,1) as a transfer function for $\%TB_t$ but the results are not better.

systematic behavior in ε_t. That is, the amount of excess reserves left in the system is just random and the series ε_t has little systematic behavior left to be explained.

Summary

Table 36 provides a summary of the ARIMA models chosen for the parameters in r_t. For the univariate processes, it identifies the order of the process, its R^2, and the results of the white noise test on the model's residuals. In addition, for the multivariate processes, the order of each transfer function is also given. A model that results in white noise at all lags at all levels of significance is identified by "all lags" in the column for the white noise test; some models result in white noise only for certain lags or at lower significance levels and they are so identified. A model that does not result in white noise at all, "fails" the white noise test.

In general the estimated ARIMA models are quite successful. Except for the second model for ε_t, the R^2 for every model is good and it is very high for some models. As can be seen from Tables 30 through 35, the variance of residuals for every model is extremely small. Only four models fail the white noise test and most result in white noise at all lags and levels of significance. One of the models that does not result in white noise however is the second model for ν_t^D.

When the appropriate independent variables are added to the noise models, the multivariate models perform better, so the transfer functions apparently add to the explanatory power of the models. Independent variables that would be expected to be important to a parameter's behavior however do not always have statistically significant

Table 36. Summary of Estimated ARIMA Models

Parameter	Estimated Model: Noise Model (p,d,q)(d,s)	Estimated Model: Transfer Function (r,u,b)(d,s)	R^2	White Noise Test
$\lambda^D_{1,t}$	(1,0,3)(0,1) + lags 13 and 23		.739	all lags
$\lambda^D_{2,t}$	(0,0,3)(0,1)		.812	all lags
$\lambda^D_{3,t}$	(0,0,3)(0,1)		.828	fails
$\lambda^D_{4,t}$	(1,0,3)(0,1) + lag 13		.654	all lags - 5%
$\lambda^D_{5,t}$	(0,0,3)(0,1)		.807	all lags
$\lambda^D_{6,t}$	(0,1,2)(1,1)		.851	all lags
$\lambda^D_{7,t}$	(0,0,2)(0,1)		.947	fails
$\lambda^D_{8,t}$	(0,0,3)(0,1) + lag 14		.732	all lags
$\lambda^D_{9,t}$	(0,0,3)(0,1)		.867	all lags
$\lambda^D_{10,t}$	(0,1,2)(1,1) + lag 13		.673	all lags
$\lambda^D_{11,t}$	(1,0,3)(0,1) + lag 13		.622	lags 1-10 only
$\lambda^T_{1,t}$	(0,1,3)(1,1)	Q_t: (0,3,0)(0,1)	.918	all lags
$\lambda^T_{2,t}$	(2,1,2)(1,0)	TB_t: (0,1,0)(1,0) $\%TB_t$: (0,2,0)(0,0) Q_t: (0,4,0)(0,0)	.840	all lags

Table 36. Continued

Parameter	Estimated Model: Noise Model (p,d,q)(d,s)	Estimated Model: Transfer Function (r,u,b)(d,s)	R^2	White Noise Test
$\lambda^T_{3,t}$	(0,1,3)(1,1)	TB_t: (0,1,0)(1,1) $\%TB_t$: (0,3,0)(0,1) Q_t: (0,2,0)(0,1)	.941	all lags
$\lambda^T_{4,t}$	(0,1,3)(1,1)	TB_t: (1,1,0)(1,1)	.999	fails
$\lambda^T_{5,t}$	(0,1,2)(1,1) + lag 4	Q_t: (0,2,0)(0,1)	.999	all lags
$\delta^D_{1,t}$	(3,1,0)(1,1) + lags 13 and 26		.982	all lags
$\delta^D_{3,t}$	(3,1,1)(1,1) + lag 13		.987	all lags - 5%
$\delta^D_{4,t}$	(0,1,3)(1,1) + lags 13 and 14		.885	all lags
$\delta^D_{5,t}$	(3,1,1)(1,1) + lag 13		.967	all lags
$\delta^D_{7,t}$	(3,1,3)(1,1)		.987	all lags
$\delta^D_{8,t}$	(1,2,1)(2,1) + lag 13		.776	all lags - 5%
$\delta^D_{9,t}$	(3,1,3)(1,1) + lag 13		.703	all lags
$\delta^D_{10,t}$	(3,1,3)(1,1) + lag 13		.816	all lags
$\delta^T_{3,t}$	(3,1,0)(1,1)	TB_t: (0,2,0)(1,1) $\%TB_t$: (0,2,0)(0,1)	.999	all lags, but 5% for lags 21-30

Table 36. Continued

Parameter	Estimated Model: Noise Model (p,d,q)(d,s)	Estimated Model: Transfer Function (r,u,b)(d,s)	R^2	White Noise Test
$\delta^T_{5,t}$	(2,1,0)(1,1)	TB_t: (1,1,0)(1,1) $\%TB_t$: (1,0,0)(1,1) Q_t: (0,3,0)(0,0)	.934	all lags
ν^D_t	1) (3,1,0)(1,1)	MB_t: (1,4,0)(0,1)	.988	all lags
	2) (3,1,0)(1,1) + lags 11, 13 and 27	MB_t: (1,4,0)(0,1) TB_t: (1,1,0)(1,1) $\%TB_t$: (1,0,0)(1,1)	.992	fails
ν^T_t	1) (3,2,3)(2,1)	MB_t: (1,4,0)(0,1) Q_t: (0,3,0)(0,1)	.994	all lags
	2) (2,2,1)(2,1) + lag 13	MB_t: (1,6,0)(0,1) TB_t: (1,1,0)(1,1) $\%TB_t$: (1,0,0)(1,1) Q_t: (0,4,0)(0,1)	.999	all lags
γ_t	1) (3,1,0)(1,1) + lag 13		.777	all lags - 5%
	2) (2,1,3)(1,1) + lags 13 and 16		.787	lags 1-10 and 21-30 only - 5%
ι_t	1) (0,1,3)(1,1) + lags 13 and 21	MB_t: (1,1,0)(0,1) TB_t: (0,1,0)(1,1) $\%TB_t$: (0,2,0)(0,1)	.805	all lags - 5%
	2) (3,1,1)(1,1) + lag 13	MB_t: (1,4,0)(0,1) $\%TB_t$: (1,1,0)(1,1) + lag 12	.723	all lags

Table 36. Continued

	Estimated Model			
Parameter	Noise Model (p,d,q)(d,s)	Transfer Function (r,u,b)(d,s)	R^2	White Noise Test
τ_t	1) (3,1,3)(1,2) + lag 13	TB_t: (0,2,0)(1,2) + lag 14 $\%TB_t$: (0,2,0)(0,2) + lag 13 Q_t: (0,3,0)(0,2)	.997	lags 11-30 only
	2) (2,1,3)(1,2)	TB_t: (1,0,0)(1,2) $\%TB_t$: (1,0,0)(1,2)	.971	all lags
ε_t	1) (0,1,3)(1,1)	TB_t: (0,1,0)(1,1)	.729	all lags
	2) (1,0,4)(0,1)	TB_t: (1,0,0)(1,1) $\%TB_t$: (1,0,0)(0,1)	.197	all lags

coefficients and those that are significant do not always have the expected signs.

Market interest rates would logically be a major determinant of the growth rate of the various categories of time and savings deposits, but the direction of their influence is not clear. The regression results presented here show that TB_t is a statistically significant independent variable for three of the λ^T-parameters and $\%TB_t$ is significant for $\lambda^T_{2,t}$ only (total time deposits). The results show that $\lambda^T_{i,t}$ and TB_t vary directly, so as interest rates rise the rate of growth in the various categories of time and savings deposits falls; the rate of change in interest rates is apparently not an important influence on rates of growth in time and savings deposits. The effectiveness of Regulation Q interest rate ceilings is also a statistically significant variable for three λ^T-parameters; as expected, $\lambda^T_{1,t}$ and $\lambda^T_{5,t}$ are directly related to Q_t, but the relationship between $\lambda^T_{2,t}$ and Q_t is not clear. For $\delta^T_{3,t}$ and $\delta^T_{5,t}$, both TB_t and $\%TB_t$ are statistically significant, but Q_t is significant for $\delta^T_{5,t}$ only. The rate of growth in savings deposits ($\lambda^T_{3,t}$ and $\delta^T_{3,t}$) therefore does not appear to be affected by Q_t, but is directly related to interest rates.

It was hypothesized that ν^D_t and ν^T_t are directly related to the relative number of member banks and inversely related to interest rates. In the regression results, MB_t is statistically significant for both ν^D_t and ν^T_t and in general the expected direct relationship is verified. Interest rates are not significant variables for either ν^D_t or ν^T_t in the first half of the sample; they are significant for both parameters in the second part of the sample, but the signs of the coefficients imply no consistent relationship between interest rates and attrition from the Federal Reserve System. In addition, Q_t is a significant variable for ν_t^T for both halves of the sample period. The signs of the coefficients consistently show ν_t^T and Q_t are inversely related; this is attributed to the member bank dominance of the highly interest-elastic market for large certificates of deposit.

The ARIMA models for ι_t indicate that it is directly related to the relative number of member banks, but in general the behavior of market interest rates is not a significant determinant of ι_t. This implies that member banks hold fewer interbank balances as assets; this supports the result reported by Gilbert.[27]

Regulation Q ceilings and interest rates would be expected to influence τ_t, although the direction of influence for the latter is not *a priori* clear. Both TB_t and $\%TB_t$ are statistically significant in both models for τ_t; for the first half of the sample the signs are mixed, but for the second half they are consistently negative. The latter result indicates that high and rising interest rates cause the growth of time deposits to slow, as other assets become more attractive. Q_t ought to be inversely related to τ_t, but it is concluded here that there is no strong causal link between the two. Q_t is statistically significant in the first model for τ_t only, the majority of its coefficients are not of the expected sign; furthermore, this result is based on very few nonzero observations of Q_t.

It is hypothesized that ε_t and interest rates are inversely related but the results reported here show the opposite to be true. As

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[27]Gilbert, p. 12.

discussed above however, the model for ε_t for the last half of the sample is probably not very reliable.

CHAPTER 7

AN ARIMA FORECASTING EXPERIMENT FOR r_t

The problem of nonuniform reserve requirements arises in a week-to-week control situation when the Federal Reserve does not know the value of r_t or its component parameters. The simplest solution to this lack of knowledge is to assume each week that there is no change in the value of each parameter. This is the basis of the naive forecasting model described in Chapter 5. The ARIMA models described in Chapter 6 are now used to forecast values for each of the parameters in r_t. These more sophisticated forecasts are used as an alternative to the naive forecasts to predict values of r_t, and the resulting error is calculated.

Following the techniques developed by Box and Jenkins,[1] forecast values for the parameters in r_t are derived from their estimated ARIMA models.[2] The ARIMA forecast for each parameter is then substituted for the naive forecasts in the first forecast equation for r_t (equation 5-6). Denoting the ARIMA forecasts with an asterisk, this model provides a forecast value for r_t, defined as,

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[1]George E. P. Box and Gwilym M. Jenkins, Chapter 5.

[2]Forecasts of the univariate models were calculated using Computer Programs for the Analysis of Univariate Time Series Using the Methods of Box and Jenkins, Supplementary Program Series No. 517 (Madison, WI: The University of Wisconsin Computer Center, revised May, 1975); the multivariate processes were forecast with the computer program, Analysis of Time Series Models Using the Box-Jenkins Philosophy by David J. Pack (Columbus, Ohio: The Ohio State University, revised January, 1978).

$$(7\text{-}1)\qquad \hat{r}_{3,t} = \sum_j d_{j,t}\,\lambda^{D*}_{j,t}\,\delta^{D*}_{j,t}\,\nu^{D*}_t\,\xi_{t-1}\,(1+\gamma^*_t+\iota^*_t) + \sum_i t_{i,t}\,\lambda^{T*}_{i,t}\,\delta^{T*}_{i,t}\,\nu^{T*}_t\,\tau^*_t + \varepsilon^*_t\,\rho^m_{t-1} + \sum_h \psi_{h,t}\,\omega_{h,t-1}\,\rho^n_{t-1}.^{3}$$

The average error in r_t caused by the ARIMA forecasts is given by,

$$(7\text{-}2)\qquad MSE_3 = \frac{1}{N}\sum_{t=1}^{N}(r_t - \hat{r}_{3,t})^2,$$

where N is the number of observations. The value of MSE_3 for each reserve scheme is given in the first part of Table 37; it measures the error in r_t associated with the ARIMA forecasting experiment.

Following the procedure used for the naive forecasting model, the ARIMA forecasts for each parameter (or set of parameters) in r_t is (are) in turn replaced by its (their) actual value(s) in week t. Consider, for example, the parameters representing lagged reserve requirements. The actual current values of $\lambda^D_{j,t}$ and $\lambda^T_{i,t}$ are substituted for their ARIMA forecasts in equation (7-1); this yields another forecast of r_t,

$$(7\text{-}3)\qquad \hat{r}^{\lambda}_{4,t} = \sum_j d_{j,t}\,\lambda^{D}_{j,t}\,\delta^{D*}_{j,t}\,\nu^{D*}_t\,\xi_{t-1}\,(1+\gamma^*_t+\iota^*_t) + \sum_i t_{i,t}\,\lambda^{T}_{i,t}\,\delta^{T*}_{i,t}\,\nu^{T*}_t\,\tau^*_t + \varepsilon^*_t\,\rho^m_{t-1} + \sum_h \psi_{h,t-1}\,\omega_{h,t-1}\,\rho^n_{t-1}.$$

The difference between this and the actual value of r_t is measured by,

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[3] The definitions of r_t, $\hat{r}_{3,t}$, and $\hat{r}_{4,t}$ (defined below) all include the λ-parameters for all reserve schemes.

Table 37. Error Resulting from the ARIMA Forecasting Model (Results of the Calculation of E_2)

	Reserve Scheme (Number of Observations)			
	A (100)	B (52)	D' (97)	E (111)
MSE_3	.55152E-05	.13263E-04	.11961E-02	.11015E-02
E_2^{λ}	-.03351	.60452	.01237	.01562
E_2^{δ}	-.01770	.62686	-.01655	.01253
E_2^{ν}	-.01777	.61659	.01597	.00690
E_2^{γ}	.38091	.66564	-.00150	.00209
E_2^{ι}	.02486	.64610	.01129	.00527
$E_2^{\tau 1}$	.03574	.66410	.35685	.00236
E_2^{ε}	.21559	.64444	.36265	.99445

[1]Based on a univariate forecasting model.

(7-4) $$MSE_4^\lambda = \frac{1}{N}\Sigma(r_t - \hat{r}_{4,t}^\lambda)^2,$$

where N is the number of observations.

Comparing MSE_4^λ and MSE_3 indicates the loss, in terms of accurate forecasts of r_t, from using the ARIMA forecasts of the λ-parameters. To facilitate this comparison, the error-coefficient E_2^λ is defined as,

(7-5) $$E_2^\lambda = \frac{MSE_3 - MSE_4^\lambda}{MSE_3}$$

$$= 1 - \frac{MSE_4^\lambda}{MSE_3} .$$

If the ARIMA forecasts of the λ-parameters are very poor, then replacing them with their actual values will reduce the error in the forecasted r_t; MSE_4^λ will be smaller than MSE_3 and E_2^λ will be close to one. Therefore the closer the value of E_2^λ to one, the poorer the ARIMA forecasts of the λ-parameters and the larger the error they cause in forecasting r_t. On the other hand, if the ARIMA forecasts of the λ-parameters are good, then using their actual values will reduce the error in r_t very little; MSE_4^λ and MSE_3 will be similar and E_2^λ will be close to zero. Therefore the smaller the value of E_2^λ, the smaller the error in r_t caused by the ARIMA forecasts of $\lambda_{j,t}^D$ and $\lambda_{i,t}^T$. It is possible for E_4^λ to be negative; this occurs when $MSE_4^\lambda > MSE_3$. This indicates that the error in r_t is smaller with the ARIMA forecasts for the λ-parameters than when their actual values are used. The concepts of MSE_4 and E_2 are defined analogously for the other parameters in r_t.

The results for E_2 for each parameter in r_t for all four reserve schemes are given in Table 37. Because of programming limitations, the forecasting experiment is limited to the last 100 observations of Schemes A and D'.[4] Some of the values forecasted for $\lambda^T_{i,t}$ and $\delta^T_{i,t}$ under Scheme B, and for $\lambda^T_{i,6}$ under Scheme D', had to be deleted from the calculation of E_2 because the forecast equation gives nonsensical results. For Reserve Scheme B, the erratic results probably occur because there are no observations available for $\lambda^T_{i,t}$ and $\delta^T_{i,t}$ before the forecast origin. Under Scheme D' the erratic forecasts of $\lambda^T_{i,t}$ are traceable to erratic behavior in the independent variables.

As can be seen in Table 37, the E_2-coefficients are consistently large under Scheme B; the E_2-coefficients for B are also about the same size for all parameters. This indicates that the mean square error in r_t falls considerably when the actual value of each (set of) parameter(s) is substituted for its (their) ARIMA forecasts. The loss therefore from using the ARIMA forecasts is substantial for each parameter and the loss is approximately equal for each one. This result is probably due to the small number of observations left for Scheme B.

For the period covered by Reserve Scheme A the poorest ARIMA forecasts are for γ_t and ε_t, since the error in r_t falls most when they are replaced by the actual values of γ_t and ε_t. The value of E_2^λ, E_2^δ, and E_2^ν are all negative, implying that the error in r_t is smaller with the ARIMA forecasts than when the actual values of the parameters are used. The values of E_1^λ, E_1^δ, and E_1^ν are also all negative (see Table 29, Chapter 5). None of the E_2-coefficients, including E_2^γ and E_2^ε, are large on an absolute level so the error resulting from using the ARIMA forecasts is not large for any parameter.

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[4]The forecasting programs used here allow at most five different forecast origins per run.

For Reserve Scheme D', the E_2-coefficients are largest for τ_t and ε_t and very small or negative for the other parameters. Even E_e^{τ} and E_2^{ε} are not very close to one. Again, using the ARIMA forecasts apparently does not cause very large errors in r_t and perfect knowledge of the parameters yields little gain.

Under Reserve Scheme E, E_2^{ε} is very close to one so the mean square error in r_t falls considerably when the ARIMA forecast of ε_t is replaced by its actual value. Thus the ARIMA forecasts of ε_t are apparently very poor for this period of time and the loss from using ε_t^* is significant. The E_2-coefficients for the other parameters are all very small so their ARIMA forecasts are apparently quite good.

In the naive forecasting experiment described in Chapter 5, E_1^{λ} is larger under the later reserve schemes, especially the graduated scheme. Ignoring Scheme B, E_2^{λ} is small for all schemes and is not appreciably larger for Scheme E. Therefore while the naive forecasting model is less successful with more reserve categories, this is not true for the ARIMA model. In the last two reserve schemes, E_2^{λ} is much smaller than E_1^{λ}, so the error associated with the ARIMA forecasts of the λ-parameters is smaller than with the no-change forecasts.

The errors resulting from the naive forecasts of the δ-parameters also increase slightly with the number of reserve categories. Ignoring again Reserve Scheme B, E_2^{δ} also increases for successive reserve schemes, although E_2^{δ} is negative for Schemes A and D'. Except for Scheme B, E_2^{δ} is less than E_1^{δ} so the ARIMA forecasts result in smaller losses than the naive forecasts for the δ-parameters.

Both the naive and ARIMA forecasts of ν_t^D and ν_t^T result in very small errors. The largest value of E_1^{ν} is for Reserve Scheme D' and, except for Scheme B, E_2^{ν} is also largest during that period. Apparently ν_t^D and ν_t^T vary most during Scheme D' and are therefore difficult to successfully predict during that period, regardless of the methodology. Except for Schemes B, E_2^{ν} is smaller than E_1^{ν}, so the ARIMA forecasts of ν_t^D and ν_t^T are better than their naive forecasts.

Except for Scheme B, E_2^{λ}, E_2^{δ}, and E_2^{ν} are all small so the ARIMA forecasts introduce only small errors in r_t. For the last reserve scheme, E_2^{ν} is however smaller than E_2^{λ} and E_2^{δ}, indicating that lagged and differential reserve requirements cause more unpredictable variation in r_t than do nonmember banks. This is not true under Scheme D', when E_2^{ν} is relatively large, or under Scheme A, when all three coefficients are negative.

The gain from perfect knowledge of γ_t over its ARIMA forecasts is large for the first half of the sample but falls drastically for the last half. The values of E_1^{γ} show the same pattern so apparently γ_1 is more predictable in the first half of the sample. In addition, ARIMA forecasts for γ_t in the first half of the sample are relatively poor, since E_2^{γ} is larger than E_1^{γ}. For the second part of the sample, the ARIMA forecasts are more successful and result in smaller losses than the naive forecasts of γ_t.

Except for Scheme B, the ARIMA forecasts of ι_t yield small errors, although E_2^{ι} is consistently larger than E_1^{ι} which is negative for all schemes. The ARIMA experiment for τ_t is very successful during Schemes A and E; E_2^{τ} is small and much smaller than E_1^{τ} so the ARIMA experiment results in smaller errors than the naive model. The ARIMA model for τ_t is less successful during Scheme D' however; E_2^{τ} is relatively large and larger than E_1^{τ}. Due to programming limitations,[5] the ARIMA forecasts of τ_t had to be derived from the noise model alone, excluding the transfer functions. The poor results in Reserve Scheme D' probably occur because the effects of interest rates and Regulation Q on τ_t have been excluded and Reserve Scheme D' corresponds to the time period when the behavior of these independent variables were most crucial to the behavior of τ_t. Considering the small losses during Schemes A and E from using the ARIMA forecasts even though the transfer functions are excluded, it appears that τ_t could be very successfully forecasted with a multivariate ARIMA model.

Using the ARIMA forecast for ε_t causes relatively large losses in r_t for all reserve schemes and this is especially true for Reserve Scheme E. Furthermore, E_2^{ε} is larger than E_1^{ε} except for Scheme A. The ARIMA forecasts are apparently very poor and, except during Scheme A, the $\varepsilon_t = \varepsilon_{t-1}$ assumption represents a better forecast. This is not a surprising result, especially for the last part of the sample, considering the very low R^2 of that ARIMA process. As discussed earlier, it appears that after reserve-carryover was introduced, the behavior of ε_t is erratic (especially, relative to its low level) and difficult to successfully forecast with an ARIMA model.

In summary, the ARIMA methodology causes the largest losses in accuracy of r_t when used to forecast ε_t for all schemes, τ_t during Reserve Scheme D', and γ_t during Scheme A (this ignores the result for Scheme B). The ARIMA forecasting experiment for the λ- and δ-parameters,

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[5]For both parts of the sample period, the second seasonal difference of τ_t was required for stationarity. The program used here to forecast the multivariate models would not allow for the second seasonal difference of order 52.

ν_t^D and ν_t^T, and ι_t, τ_t for Schemes A and E, and γ_t for the last half of the sample yields small errors in r_t; the ARIMA forecasts for these parameters cause smaller losses than the naive forecasts.

Comparing MSE_3 to MSE_1 (Table 29, Chapter 5) shows whether the ARIMA forecast experiment is more successful than the simple no-change model. For Reserve Scheme A, $MSE_3 < MSE_1$ so the ARIMA forecasts of the parameters in r_t provides a better forecast for r_t than the naive forecasting model. For the other reserve schemes however, this is not true. Especially during Schemes D' and E, the error in $\hat{r}_{3,t}$ estimated with the ARIMA models, is much larger than the error in $\hat{r}_{1,t}$, estimated with the naive model. Considering the values of the E_2-coefficients for the individual parameters, it seems likely that this poor ARIMA performance is due almost entirely to bad forecasts of τ_t and ε_t in Scheme D' and ε_t alone in Scheme E.

To test this proposition, MSE_3 is recalculated for Schemes D' and E, using ε_{t-1} instead of ε_t^*. The results are .80908E-03 and .80425E-05 for each scheme, respectively. The naive forecasts of ε_t therefore result in smaller errors than ε_t^* for both schemes. For Scheme E, the error falls below MSE_1 so when the ARIMA forecasts for ε_t are excluded, the ARIMA forecast experiment is more successful than the naive one. For Scheme D', the error associated with the ARIMA forecasts is still larger than MSE_1, even when the ε_t^* values are not used.

CHAPTER 8

SUMMARY AND RECOMMENDATIONS

The behavior of the individual parameters in r_t shows that those representing the categories of nonmoney deposits have varied most during the sample period; both the levels and first differences of γ_t, ι_t, and τ_t show more variation than the other parameters. Of these three parameters, τ_t has varied most. In addition, the variation in τ_t and ι_t have increased during the sample period; the variance of γ_t however has decreased over time.

While the parameters representing lagged reserve requirements do not vary much on an absolute scale, they all display considerable week-to-week fluctuations about one; in some instances these weekly fluctuations are large. This is indicated by the fact that in general the standard deviation of the first differences is larger than that of the levels, and the mean of the absolute value of the first differences of the λ-parameters is larger than the mean of the first differences. These patterns occur for $\lambda^T_{i,t}$ as well as $\lambda^D_{j,t}$ although, by all measures of variation, $\lambda^D_{j,t}$ vary more than $\lambda^T_{i,t}$.

The parameters corresponding to differential reserve requirements behave much like the λ-parameters: the variation in the δ-parameters is not large on an absolute scale, but they do fluctuate considerably from week-to-week. The levels of the δ-parameters vary more, and the mean of the first differences is larger than for the λ-parameters, but variation in the first differences is smaller for the δ-parameters; this pattern holds for both demand and time deposits. Therefore, while the δ-parameters vary more, they appear to be more easily predicted than the λ-parameters. The δ^T-parameters also vary more than the δ^D-parameters, but the average size and variation in their weekly changes is smaller; this implies that while differential reserve requirements applied to time deposits causes considerable variation in r_t, it is more predictable variation than that caused by the δ^D-parameters or λ^T-parameters.

The variances of ν_t^D and ν_t^T are small on an absolute scale and the variation in their first differences is even smaller. They do not therefore show the sizable weekly fluctuations characteristic of the λ- and δ-parameters. This implies that the distribution of deposits between member and nonmember bank does not vary much from week-to-week and that ν_t^D and ν_t^T do not vary in an unpredictable fashion. Of the two parameters, ν_t^T is the most stable. Both ν_t^D and ν_t^T have, of course, declined during the sample period; the decline is more pronounced in ν_t^D and in addition, the standard deviation of ν_t^D more than doubled between 1961 and 1974. The changes in ν_t^T appear to be closely tied to the effectiveness of Regulation Q ceilings.

The statistics for $\psi_{h,t}$ indicate that the base-aborbing money per dollar of deposits held by nonmember banks varies considerably from state to state; this implies that differential state reserve requirements are potentially a serious control problem. On the other hand, $\omega_{h,t}$ shows that shifts in the distribution of deposits between states have been minimal. Further investigation of these parameters is hampered by lack of data.

Data problems also seriously limit analysis of the variation caused by reserve requirements against various categories of nondeposit liabilities. The parameters representing the application of lagged reserve requirements to nondeposit liabilities do not fluctuate from week to week as much as the other λ-parameters, but the erratic rates of growth in the categories of nondeposit liabilities result in relatively large changes in the λ^N-parameters. The δ^N-parameters also show large weekly changes but the variation in these parameters is not large relative to the other δ-parameters. Therefore the application of lagged and differential reserve requirements do not appear to introduce particularly variable parameters into r_t. The α^N-parameters, representing the ratio of nondeposit liabilities to privately-owned demand deposit do vary a great deal; their variance is only exceeded by τ_t. The application of reserve requirements themselves to nondeposit liabilities therefore does introduce additional, highly variable, parameters into r_t.

The level of excess reserves has of course declined during the sample and the variation in ε_t has increased, especially since 1968, when the reserve-carryover procedure began. The coefficient of variation of ε_t is large, but by other measures of variation, ε_t is very stable relative to the other parameters.

The sample period is divided into five subperiods corresponding to the various reserve plans that have been in effect. The variation in r_t is larger during later reserve schemes. Successive changes in Federal Reserve reserve requirements that have introduced more categories of deposits and lagged reserve requirements into the reserve structure have apparently also introduced increased variation in r_t. In some reserve schemes, the standard deviation of some λ- or δ-parameters for individual deposit categories are among the largest of all the parameters. The parameters that have consistently varied most are γ_t, τ_t, ε_t and in one reserve scheme, ω_t. In no reserve scheme is the variation in ν_t^D and ν_t^T large relative to other parameters, and in some schemes the standard deviations of ν_t^D and ν_t^T are among the smallest of all the parameters. Thus the historical record of r_t under the various reserve plans shows that the largest sources of disturbance have been time deposits, government deposits, and excess reserves; lagged and differential reserve requirements have been a moderate, but increasing, source of variation in r_t. Changes in the distribution of deposits between member and nonmember banks have been only a minor source of variation.

The partial variance of r_t is used here as a device to decompose the variation in r_t into its sources. The partial variance due to τ_t is consistently the largest for all reserve schemes and $var_\tau(r_t)$ increases during the sample period. The partial variance shows that lagged reserve requirements have also been a major source of variation in r_t; $var_\lambda(r_t)$ is larger for each successive reserve scheme so as more reserve classes have been added, lagged reserve requirements have caused more variation in r_t. $Var_\gamma(r_t)$ and $var_\varepsilon(r_t)$ are also large for the first half of the sample, but both decline in the second half.

$Var_\delta(r_t)$ increases only slightly with the number of reserve classes. The value of $var_\delta(r_t)$ is small for all reserve classes so shifts of deposits between classes of member banks have not been a major source of variation in r_t. According to $var_\nu(r_t)$, shifts of deposits between member and nonmember banks have also been only a minor source of variation in r_t and there is no convincing evidence that nonmember banks are an increasing source of disruption. $Var_\nu(r_t)$ is largest for Scheme D', but nearly equal for Schemes A and E.

To test the relative predictability of each parameter, two forecasting experiments on r_t are performed. One is a simple model which assumes no change in the value of each parameter; the second uses forecasts of each parameter derived from models estimated using the time series methodology of Box and Jenkins.

Results of the naive forecasting experiment support the general results described above. The naive forecasts of τ_t, ε_t, and the λ-parameters cause the largest forecast error in r_t; the naive forecasts of ν_t^D, ν_t^T, and the δ-parameters result in small errors in r_t only. In addition the $\gamma_t = \gamma_{t-1}$ forecasts cause large errors in r_t in the first half of the sample. Thus τ_t, ε_t, the λ-parameters, and γ_t for the first part of the sample, are the parameters that are the most difficult to forecast using this simple technique; ν_t^D, ν_t^T, and the δ-parameters can be quite successfully forecasted, even with this simple no-change procedure. The errors associated with the naive forecasts of the λ-parameters and the δ-parameters increase with the number of reserve classes; the error caused by using no-change forecasts for ν_t^D ν_t^T does not increase through time.

The ARIMA forecasting experiment provides more sophisticated forecast values for each of the parameters. Substituting the ARIMA forecasts for the naive ones, however, reduces the overall forecast error in r_t only during Reserve Scheme A. The results reported for the E_2-coefficients show that the ARIMA forecasts are most successful for the λ-parameters and the δ-parameters, ν_t^D and ν_t^T, and ι_t. The error in r_t caused by the ARIMA forecasts of these parameters is very small. In addition, the ARIMA forecasts of γ_t are very good for the last half of the sample and those for τ_t are successful except during Scheme D'.

The ARIMA forecasts of ε_t result in large error in r_t for all reserve schemes. The ARIMA forecasts on most of the parameters are therefore quite successful. The fact that the error in r_t is larger for the ARIMA forecasts than for the naive forecasts in most reserve schemes is attributable to the poor ARIMA forecasts of ε_t, γ_t during the first half of the sample, and τ_t during Scheme D'. When the naive forecasts are used for ε_t instead of its ARIMA forecasts, the error in r_t falls substantially. Thus the major impediment to successfully forecasting the parameters of r_t with the ARIMA methodology appears to be the difficulty of describing ε_t with an ARIMA process.

None of the evidence presented here indicates that declining membership in the Federal Reserve System has caused increased variation or unpredictability in r_t. Relative to the other parameters in r_t, ν_t^D and ν_t^T vary little and are easily predicted. The variation in ν_t^D and ν_t^T are of course not zero, so presumably the variability in r_t could be reduced if nonmember banks were subjected to Federal Reserve reserve requirements. In addition, ν_t^D and ν_t^T do not reflect the total impact of nonmember banks on r_t; $\psi_{h,t}$ and $\omega_{h,t}$ also represent the influence of nonmember banks.

It is often contended that the Federal Reserve could make structural changes within their own reserve requirements that would remove more troublesome sources of variation and/or unpredictability in r_t than nonmember banks; this claim is in some cases supported here. According to the results of this study, removal of lagged reserve requirements and reserves against some categories of nonmoney deposits, especially time deposits, would remove more variable and unpredictable parammeters from r_t than ν_t^D and ν_t^T. The distribution of deposits among classes of member banks however, apparently does not shift much, so removal of differential reserve requirements would have little effect on the stability or predictability of r_t; there is no evidence that removing differential Federal Reserve reserve requirements would be a more fruitful institutional change that universal membership.

The claim is also supported here that as the Federal Reserve has increased the number of categories of deposits, more variation has been introduced into r_t through lagged and differential reserve requirements. In general, the variation and unpredictability in r_t has increased with the number of Federal Reserve reserve categories, especially because of the increased number of λ-parameters.

The greatest gain in terms of reducing the variation in r_t would accrue from removing reserves against time deposits, government deposits, and interbank deposits, and lagged reserve requirements. In addition, it appears that the introduction of reserve requirements against nondeposit liabilities and the reserve-carryover procedure have increased the variability of r_t. Since ι_t and the λ-parameters are successfully forecasted with the ARIMA procedure, the greatest gain in terms of easy predictability of r_t would come from removing reserves against time deposits, government deposits, and the reserve-carryover procedure.

A P P E N D I C E S

APPENDIX A

SUMMARY OF MODELS OF THE MONEY SUPPLY PROCESS

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Brunner

Brunner's first formulation of a theory of the money supply process is his linear money supply theory.[1] It is based on the notion that a commercial bank will adjust its assets to absorb any surplus reserves that emerge in its portfolio. Surplus reserves, s, are defined as the excess of actual available reserves over desired available reserves where available reserves are total cash assets less required reserves. This gives,

$$s = v - v^d, \text{ where}$$

$$v = \text{total cash assets} - \text{required reserves},$$

$$v^d = w_o + w_1 d_1 + w_2 d_2,$$

and w_o is a part of desired reserves that a bank will hold independent of its level of deposits and depends on an index of loan and bond yields, borrowing from the Fed, and the bank's loan-investment ratio; w_1 and w_2 are the bank's marginal propensities to hold reserves against demand and time deposits, d_1 and d_2, respectively.

As surplus reserves appear in a bank's portfolio, the bank is induced to expand its earning assets and as it does so, its surplus reserves are drained away for four reasons. First, part of the asset expansion will be held by borrowers as cash and therefore as earning

[1]Karl Brunner, "A Schema for the Supply Theory of Money," *International Economic Review* 2 (January 1961):79-109.

assets are expanded, surplus reserves are reduced by a currency drain. Second, part of the asset expansion will become deposits at other commercial banks. Third, part of the asset expansion will be held by the public in time deposits, thereby increasing required reserves and reducing surplus reserves. Fourth, part of the expansion will become demand deposits at the expanding bank; again, this increases required reserves thereby diminishing surplus reserves.

The function describing the generation of funds by a bank is of the form,

(A-1) $a^s = \frac{1}{\lambda}(s)$, where a^s denotes the supply of bank funds, and

$$\lambda = [c + (r^t + w_2)t + (1 - f_3)p_3 + (r^d + w_1)$$
$$(1 - c - t - f_1p_3)].$$

Each term in the definition of λ corresponds to one of the channels listed above by which surplus reserves are dissipated. The currency drain is accounted for by c, which represents the portion of the bank's asset expansion that the borrower holds as cash. The time deposit drain is represented by $(r^t + w_2)t$, where r^t is the legal reserve ratio and t is the fraction of the asset expansion the borrower holds in time deposits; $(r^t + w_2)t$ then is the amount by which the bank's required and desired reserves rise, as its asset expansion induces an increase in time deposits.

The last two terms in λ account for the amount of s absorbed by increased demand deposits at other banks and at the expanding bank. In accounting for these factors, Brunner's scheme includes explicit consideration of the institutional procedure by which interbank claims are settled. Let

p_1 = the amount of demand deposits the borrower must hold as a compensating balance at the expanding bank;

p_2 = additional demand deposits redeposited in the expanding bank;

p_3 = demand deposits held at other banks;

f_1 = the proportion of p_3 collected from the expanding bank via Federal Reserve Funds;

f_2 = the proportion of p_3 collected by debiting the expanding bank's balances at other banks;

f_3 = the proportion of p_3 collected by crediting other banks' balances at the expanding bank.

The surplus reserves that are lost to increased demand deposits at other banks then amounts to p_3, minus the part (f_3) of p_3 that are collected by increasing other banks' deposits at the expanding bank. If $f_3 = 1$, the expanding bank will have traded publicly-owned demand deposits for other bank-owned demand deposits so that no surplus reserves are lost at all. The net amount of surplus reserves lost to deposits at other banks is therefore $(1 - f_3)p_3$.

Finally, the expanding banks' own demand deposits will increase because of the asset expansion, thereby increasing its required reserves and further depleting its surplus reserves. The amount by which their own demand deposits will increase can be derived algebraically. For every dollar of asset expansion, the expanding bank's own demand deposits must increase by

$$1 - c - t - (1 - f_3)p_3,$$

plus any change in its balance at other banks, which is equal to f_2p_3 per dollar of asset expansion. Thus the net increase in the expanding banks' demand deposits is,

$$1 - c - t - (1 - f_3)p_3 + f_2p_3 \text{ or,}$$

$$(1 - c - t - f_1p_3) \text{ since } f_1 + f_2 + f_3 = 1.$$

This term must be multiplied by $(r^d + w_1)$, the legal and desired level of required reserves, to determine the increase in reserves (depletion of s) the bank will hold against the new demand deposits.

Having completely specified the form of λ, the $a^s = 1/_\lambda$ s function describes the process by which a bank generates money as it adjusts its portfolio to eliminate any surplus reserves (or acquire reserves if $s < 0$).

The other major relationship of Brunner's linear money supply hypothesis explains the process by which surplus reserves are generated. Brunner discusses eight ways in which the surplus reserves of an individual bank may be generated.

1) A bank may experience a net current inflow, designated by n_1, which will increase surplus reserves by the amount of the inflow less the increase in required reserves caused by the inflow. This currency inflow is independent of any portfolio changes by the bank. Surplus reserves generated from this source is therefore denoted by,

$$[1 - (r^d + w_1)(1 - g_2)]n_1,$$

where g_2 represents the marginal propensity of the bank to adjust its balances at other banks because of the change in its deposit liabilities represented by the cash inflow, n_1.

2) A bank may experience shifts between demand and time deposits that are independent of its own asset expansion discussed above. Such a shift will generate surplus reserves for the bank to the extent that

different levels of reserves are held against time and demand deposits. We have,

$$[(r^d + w_1)(1 - g_2) - (r^t + w_2)]n_2,$$

denoting the surplus reserves generated by a demand deposit-time deposit shift.

3) A bank may acquire or lose surplus reserves as existing deposits are redistributed among banks or as a result of the distribution among banks of new deposits generated by an asset expansion by some bank in the system. Brunner designates n_3 as the net inflow to a bank resulting from the redistribution of existing deposits and the distribution of newly generated deposits. The surplus reserves acquired by that bank is then given by,

$$[(1 - g_3) - (r^d + w_1)g_1]n_3, \text{ where}$$

g_1 = the proportion of the bank's clearing balance settled by debiting its deposit liabilities to other banks;

g_3 = the proportion of the bank's clearing balance settled by the Federal Reserve mechanism.

4) A bank may experience an increase in surplus reserves as a result of its dealings with the Federal Reserve. Brunner divides this source of surplus reserves into two parts. The first, denoted by n_4, is the net increase of cash assets coming from transactions with the Fed that involve a banks nondeposit liabilities (such as borrowing from the Federal Reserve or the bank's earning assets). The second part, represented by n_5, is the net accrual of cash assets resulting from transactions with the Federal Reserve having to do with the bank's

deposit liabilities (such as open market operations). Surplus reserves resulting from transactions with the Fed are then represented by,

$$n_4 + (1 - \nu)n_5,$$

where ν involves an assumption regarding the bank's behavior toward deposits generated in dealings with the Fed (i.e., perhaps $\lambda = \nu$, but perhaps not).

5) A bank will gain or lose surplus reserves when legal reserve ratios are changed. This is measured by,

$$-\Delta r^d d_1 - \Delta r^t d_2.$$

6) A bank may reallocate its cash assets between those that satisfy legal reserve requirements and interbank deposits. Because of the definition of "net demand deposits" used in determining legally required reserves, this reallocation will generate or absorb surplus reserves. This is measured by

$$(r^d + w_1)\ \Delta h_o,$$

where Δh_o represents the reallocation of the bank's cash assets.

7) A bank may also be the recipient of surplus reserves as other banks reallocate their cash assets as described in (6). Letting d_o^b be the portion of d_1 that is owned by other banks, the change in surplus reserves involved here is represented by,

$$[1 - (r^d + w_1)(1 - g_2)]\ \Delta d_o^b.$$

8) A bank may gain or lose surplus reserves because of a change in the level of reserves it desires to have on hand, independent of the level of its deposits. This factor is represented by changes in w_o.

These eight sources of surplus reserves together determine s,

$$s = [1 - (r^d + w_1)(1 - g_2)]n_1 + [(r^d + w_1)(1 - g_2) - (r^t + w_2)]n_2 + [(1 - g_3) + (r^d + w_1)g_1]n_3 + n_4 + (1 - \nu)n_5 - \Delta r^d d_1 - \Delta r^t d_2 + (r^d + w_1)\Delta h_o + [1 - (r^d + w_1)(1 - g_2)]\Delta d_o{}^b - \Delta w_o. \tag{A-2}$$

Equations (A-1) and (A-2) completely explain the process by which an individual bank generates money. Function (A-2) describes a bank's sources or surplus reserves and function (A-1) describes the way in which a bank will operate to purge its portfolio of surplus reserves, thereby supplying money.

To construct a macro-theory of his linear money supply process, Brunner derives a system expansion coefficient as the sum of an infinite series of individual banks' responses. He assumes that all surplus reserves in the system, s_o, are at one given point in time held at one bank and that that bank will then expand its assets by ks_o. The cash assets of some second bank will thereby increase by $(1 - f_2)p_3ks_o$ and the public's deposits at the second bank grow by p_3ks_o. Surplus reserves present in the system after the first "round" of expansion is the increase in cash assets experienced by the second bank less its cash and reserve drain:

$$s_1 = [(1 - f_2) - c_2 - t_2(r^t + w_2) - (r^d - w_1)(f_1 - c_2 - t_2)]p_3ks_o.$$

The change in the money stock (including time deposits) in any round is ks_1. Therefore the total increase in the money supply after all rounds of the asset expansion is,

$$\Delta m = k[1 + \sum_{t=1}^{\infty} (\mu k)^t]s_o,$$

where $\mu = [(1 - f_2) - c_2 - t_2(r^t + w_2) - (r^d + w_1)(f_1 - c_2 - t_2)]p_3$.

Brunner assumes that $0 < \mu k < 1$, so the above expression for Δm can be evaluated as the limit of an infinite series. The aggregate change in the money stock that emerges as banks respond to surplus reserves is therefore

$$\text{(A-3)} \qquad \Delta M = \frac{k}{1-\mu k}\,\sigma$$

where σ is aggregate surplus reserves. This aggregation assumes that all banks have identical response coefficients, k and μ.

Aggregate surplus reserves, σ, emanate from the same eight sources as do surplus reserves of the individual bank. Brunner's expression for σ is therefore the summation over the banking system of the cash and deposit flows, n_1, n_2, n_3, n_4, and n_5 enumerated above; plus terms representing the gain or loss of aggregate surplus reserves due to: a change in legal reserve ratios, a change in the interbank deposit structure, or a change in banks' propensities to hold reserves that are independent of changes in deposits or interest rates. The expression for σ is given by,

$$\text{(A-4)} \qquad \sigma = -a_1\dot{c}_o + a_2\dot{t}_0 + \ell^1 + a_3\dot{B} + \ell^2 + a_4\dot{\varepsilon}_o + \gamma_o - \omega_o.$$

The components of the equation for σ represent the following sources of surplus reserves:

currency flow: $a_1 = - \sum_{i=1}^{n} \frac{1-(r^{di} + w_1^i)(1 - g_{2i})}{\dot{c}_o} n_1^i$, where

$$-\dot{C}_o = \sum_{i=1}^{n} n_1^i;$$

time deposit shifts: $a_2 = \sum_{i=1}^{n} \frac{[(r^{di} + w_1^i)(1 - g_{2i}) - (r^{ti} + w_2^i)]}{t_o} n_2^i$,

where $\dot{t}_o = \sum_{i=1}^{n} n_2^i$;

redistribution of existing deposits: $\ell^1 = \sum_{i=1}^{n} [(1 - g_{3i}) - (r^{di} + w_1^i)g_{1i}]n_3^{1i}$, and

$\sum_{i=1}^{n} n_3^{1i} = 0$ by construction;

transactions with Federal Reserve: $\dot{B} = \sum_{i=1}^{n} (\Delta n_4^i + \Delta n_5^i)$,

$a_3 = v + (1 - v)(1 - \nu)$, where

$$v = \frac{\sum_i n_4^i}{\sum_i (n_4^i + n_5^i)} ;$$

changes in legal reserve ratios: $\ell^2 = - \sum_{i=1}^{n} (\Delta r^{di} d_1^i + \Delta r^{ti} d_2^i)$;

changes in the distribution of interbank deposits: $a_4 = \sum_{s=1}^{4} (r^{ds} + w_1^s)$, where s refers to class of member bank, and

$\dot{\varepsilon}_o$ = the average rate of change in the four classes of banks' net interbank deposit positions;

level of interbank deposits: $\dot{\gamma}$ = rate of change in the level of interbank deposits;

excess reserves: $\dot{\omega}_o$ = the rate of change in desired reserves, independent of the level of deposits.

Equation (A-4) defines the process by which surplus reserves are generated in the banking system from sources independent of the banks' portfolio adjustments. Equation (A-3) defines the process by which those surplus reserves are converted to money as banks adjust their portfolios to get rid of surplus reserves. An expression for changes in the money stock consists of the changes represented by equation (A-3) plus transactions that cause changes in the money stock, but transactions that are independent of portfolio adjustment of banks. This latter contribution to the money stock is $(1 - v)\dot{B}$. Therefore,

$$\text{(A-5)} \qquad \dot{M} = \frac{1}{\lambda - \mu}\sigma + (1 - v)\dot{B},$$

in which σ may be replaced by (A-4). The above function can be integrated to obtain an espression for the money stock.

Interbank deposits are dealt with explicitly and have both a distributional effect and a scale effect in Brunner's model. Due to differential legal reserve requirements and different propensities to hold reserves among different classes of banks, the distribution of a given level of interbank deposits will affect the money supply process. This effect is represented by the h_o^i and d_o^{bi} terms. Brunner believes the distributional effect of interbank to be very small.[2] The scale effect of interbank deposits is measured by $a_4\varepsilon_o + \gamma_o$. The degree to which interbank deposits affect the money supply is aggravated by the Federal Reserve's definition of "net demand deposits" against which legal reserves must be held.

Friedman-Schwartz and Cagan

In A Monetary History of the United States,[3] Friedman and Schwartz (F-S) utilize a theory of the money supply process that is based on three "proximate determinants" of the money stock. According to their theory, the money stock is determined by: 1) the level of high-powered money outside the monetary authorities, designated by H; 2) the ratio of deposits to bank reserves (D/R), and 3) the ratio of deposits to currency in the hands of the public (D/C). The level of H is the sum of banks' vault cash, commercial bank deposits at the Federal Reserve, and currency in the hands of the public. Their high-powered money concept differs from the monetary base because of their inclusion of nonmember bank deposits at Federal Reserve Banks. The amount of high-powered money depends mainly on the behavior of the monetary authorities.

The ratio D/R is under the control of the banking system and is a function of legal required reserves ratios, expectations about currency

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[2]It would require very large shifts of interbank deposits to have any appreciable effect on the money supply. Furthermore, evidence seems to indicate that r^{di} and w_{1i} are inversely related, thereby counteracting each other. ibid., pp. 96-7.

[3]Milton Friedman and Anna Jacobsen Schwartz, A Monetary History of the United States, 1867-1960 (Princeton, N.J.: Princeton University Press, 1971).

flows, and interest rates. Specifically, the D/R-ratio will reflect banks' decisions with respect to holdings of excess reserves. Changes in D/R will also reflect changes in the distribution of deposits between classes of banks, shifts in deposits between demand and time deposits, and shifts in deposits between privately-owned demand deposits and other demand deposits that are not included in the money stock (defined to include time deposits of commercial banks). The D/C ratio is determined by the behavior of the public and is a function of the relative usefulness of the two assets, the cost of holding them, and income.

The money stock, M, and H are defined by:

$M = C + D$,

$H = C + R$, where

C = currency in the hands of the public;

D = privately-owned demand deposits plus time deposits at commercial banks;

R = commercial bank vault cash plus deposits at the Federal Reserve.

The money supply process is then represented by,

$$\text{(A-6)} \qquad M = H\left[\frac{D/R(1 + D/C)}{D/R + D/C}\right].$$

F-S find that the money supply process summarized by equation (A-6) has been dominated historically by changes in the level of H. When the Federal Reserve alters the level of high-powered money, the banking community responds to the resulting change in their actual excess reserves relative to desired excess reserves. If, for example, the level of H rises, actual excess reserves exceed their desired level and banks respond by acquiring earning assets, thereby expanding deposits and the money stock. The resulting rise in deposit levels causes required reserves to rise (actual excess reserves fall) and also causes the public's desired level of currency holdings to increase. These two forces work to absorb the addition to high-powered money. Either the D/R-ratio or the D/C-ratio can change for the reasons discussed above, all of which are independent of any change in H. A change in either of these ratios will of course also cause changes in the money stock.

In his study of the determinants of changes in the money stock, Phillip Cagan[4] uses a model of the money supply process that is very similar to that employed by Friedman and Schwartz. Cagan also relates the money stock (defined to include commercial bank time deposits) to the same three determinants: the level of high-powered money, a currency ratio, and a reserve ratio. He defines the currency ratio as currency in the hands of the public divided by the money stock. His version of the money supply process is then represented by:

$$\text{(A-7)} \qquad M = \frac{H}{\frac{C}{M} + \frac{R}{D} - \frac{C}{M}\frac{R}{C}} .$$

Teigen

Teigen[5] also specified a money supply function and tests it empirically, but his money supply function does not acknowledge the importance

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[4]Phillip Cagan, Determinants and Effects of Changes in the Stock of Money, 1875-1960 (New York: Columbia University Press, 1965).

[5]Ronald L. Teigen, "Demand and Supply Function for Money in the United States: Some Structural Estimates," Econometrica XXXII (October 1964):476-509.

of the institutional factors on which this study focuses. Teigen differentiates between exogenous and endogenous portions of the money stock. The former is that part of the money stock that is generated from the reserves supplied by the Fed and is therefore directly controlled by the Fed. The endogenous portion is money that arises from member bank borrowing and therefore not under the direct control of the Fed. Currency held by the public and nonmember bank demand deposits are both assumed to be constant fractions of the total money stock. Though Teigen acknowledges that these fractions may vary over time, he claims that changes in their values are sufficiently known or predictable to be of no concern for control purposes. His complete model can be outlined as follows:

(A-8) $M = Cp + D'p + D''p;$

(A-9) $Cp = cM \quad (0<c<1);$

(A-10) $D''p = hM \quad (0<h<1);$

(A-11) $D'p = kR^r$ or $(Dp' = k(R^r - \frac{1}{k} D'g);$

(A-12) $R^t = R^r + R^e = R^s + B$, where

M = the money supply, conventially defined;

Cp = currency in the hands of the public;

C = a behavioral parameter representing the proportion of the money supply held as currency;

$D'p$ = privately-owned demand deposits at member banks;

$D''p$ = privately-owned demand deposits at nonmember banks;

h = the proportion of the money supply in nonmember banks;

$D'g$ = government demand deposits at member banks;

R^r = required reserves against demand deposits at member banks;

R^e = member bank excess reserves;

k = the reciprocal of the weighted average reserve ratio;

R^t = member bank total reserves;

R^s = nonborrowed member bank reserves;

B = member-bank borrowing at the Federal Reserve.

An expression for the money stock can be solved for by substitution,

$$\text{(A-13)} \qquad M = \frac{k}{1 - c - h} (R^s - R^e) + \frac{k}{1 - c - h} (B),$$

where the first term represents the exogenous money stock and the second is the endogenous part of the money stock. The maximum potential money stock, M^{**}, occurs when excess reserves are zero; maximum potential money stock is then,

$$\text{(A-14)} \qquad M^{**} = \frac{k}{1 - c - h} (R^s) + \frac{k}{1 - c - h} (B).$$

The part of the maximum money stock that is under the control of the Federal Reserve, M^*, is given by,

$$\text{(A-15)} \qquad M^* = \frac{k}{1 - c - h} (R^s).$$

It is the ratio M/M^* on which Teigen focuses and he proposes the following money supply hypothesis:

$$\text{(A-16)} \qquad \frac{M}{M^*} = X(r, r_c),$$

where r represents the return to banks from making loans and r_c is the cost for commercial banks of lending. When, for example r rises, banks will be enticed to extend relatively more loans, causing M to rise relative to M^*.

The obvious criticism of Teigen's model is the artificial distinction between exogenous and endogenous parts of the money stock. Member bank borrowing is of course not entirely outside the control of the Federal Reserve, nor is the portion of the money stock he labels exogenous completely under their control. The important implication of Teigen's work for this study is his assumption that nonmember bank deposits are a constant, predictable fraction of the total money stock and his lack of consideration for the behavior of and components of the average required reserve ratio, k. Teigen's treatment of these parameters of the money supply process is deficient; it is important in that succeeding authors have often followed him and adopted the same assumptions.

Brunner-Meltzer

Brunner and Meltzer's (B-M) nonlinear money supply theory[6] incorporates less institutional detail than Brunner's original theory, but it places greater emphasis on the role of interest rates in the money supply process. Brunner and Meltzer's original version of their nonlinear money supply process has been refined and expanded by Albert E. Burger.[7] The model consists of a function describing banks' supply of assets to the public and a function accounting for the public's supply of assets to the banking system. In equilibrium, the two determine an

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[6]Karl Brunner and Alan H. Meltzer, "Some Further Investigations of Demand and Supply Function for Money," Journal of Finance XIX (May 1964): 240-283; and "Liquidity Traps for Money, Bank Credit, and Interest Rates," Journal of Political Economy 76 (January/February 1968):1-37.

[7]Albert E. Burger, "An Analysis and Development of the Brunner-Meltzer Non-Linear Money Supply Hypothesis," Project for Basic Monetary Studies, Working Paper No. 7, Federal Reserve Bank of St. Louis, St. Louis, Mo., 1969; and The Money Supply Process (Belmont, CA: Wadsworth Publishing Company, Inc., 1971).

interest rate and the money stock. The nonlinear money supply hypothesis is strictly a macro-theory and therefore sidesteps the complications of moving from a micro- to a macro-theory encountered with Brunner's linear theory.

In the B-M system, the banking system demands earning assets as it adjusts its portfolio to rid it of surplus reserves. The rate of portfolio adjustment is therefore a function of the divergence between desired and actual reserves. The level of desired reserves is taken to be a function of the level of deposits, a vector of interest rates and the discount rate. Required reserves are of course determined by legal reserve requirements and member bank deposits. Excess reserves are a function of interest rates, the discount rate, and the level of deposits. Schematically this is represented by:

$$\dot{E}^s = h(R - R^d), \text{ with} \tag{A-17}$$

$$R^d = f(D,T,i,\rho), \tag{A-18}$$

$$R = R^r + R^e + V^n, \tag{A-19}$$

$$R^r = r^d\delta D + r^t\tau T, \text{ and} \tag{A-20}$$

$$R^e = e(i,\rho,\pi)(D + T), \text{ where} \tag{A-21}$$

$\dot{E}^s$ = banks' demand for earning assets;

R = actual reserves;

R^d = desired reserves;

R^r = required reserves;

R^e = excess reserves;

D = demand deposits;

T = time deposits;

i = vector of interest rates;

ρ = discount rate;

v^n = nonmember bank vault cash;

r^d = required reserve ratio against demand deposits;

r^t = required reserve ratio against time deposits;

δ = the portion of total demand deposits in member banks;

τ = the portion of total time deposits in member banks;

π_1 = composite variable representing interest rate variability, deposit flows, and anticipated reserve requirement changes.

Member banks determine two behavioral coefficients. The first defines the ratio of excess reserves to total deposits, e, and is described by equation (A-21) above. The level of excess reserves is taken to be a function of interest rates, the discount rate and a composite variable, π_1, which represents interest rate variability, deposit flows, and anticipated reserve requirement changes. The second behavioral coefficient, b, defines borrowing from the Fed, A, relative to total deposits, TD, represented by

$$b = \frac{A}{TD} = f(i,\rho,\pi_2),$$

where π_2 indicates the Fed's administrative policies with regard to the discount window.

The public's supply of assets to banks, $\dot{E}^d$, is given by

(A-22) $\dot{E}^d = f(i,W,E)$, where

E = earning assets of commercial banks;

W = wealth.

The public also determines two behavioral coefficients, k and t. The amount of currency the public wishes to hold relative to demand deposits is represented by k which obeys the function,

$$k = \frac{K^p}{DD^p} = f(q, Y/Y_p, Tx, S, W), \text{ where}$$

K^p = currency in the hands of the public;

DD^p = privately-owned demand deposits;

q = service charges on demand deposits;

Y/Y_p = the ratio of current to permanent income;

Tx = the public's tax liability;

S = a variable representing population mobility and seasonal factors.

B-M do not make currency holdings a function of interest rates, though the influence of interest rates on k could easily be incorporated into the analysis. The public also determines the distribution of bank deposits between time deposits and demand deposits, represented by the coefficient t, where

$$\text{(A-23)} \quad t = \frac{T^p}{DD^p} = f(i^f, i^t, \frac{W}{Pa}, \frac{Y}{Y^p}), \text{ where}$$

T^p = privately-owned time deposits;

i^f = index of interest rates paid on financial assets;

i^t = index of interest rates paid on time deposits;

$\frac{W}{Pa}$ = real value of nonhuman wealth.

The rate paid on time deposits, i^t, is determined by banks and is postulated to be dependent on i^f and Regulation Q restrictions.

Finally, U. S. Treasury policy determines the ratio, d, of government deposits, DD^g, relative to publicly-owned deposits,

$$d = \frac{DD^t}{DD^p}.$$

The d ratio has been known to fluctuate considerably, however it is determined by Treasury policy and therefore is exogenous to the model.

The following identities and definitions complete the system:

(A-24) $M^1 = DD^p + K^p$, where M^1 represents the money supply;

(A-25) $B = B^a + A$, where B represents the monetary base and B^a is the adjusted base;

(A-26) $B = R^m + K^p + V^n$, where R^m is member bank reserves;

(A-27) $R^m = R^r + R^e$;

(A-28) $r = \frac{R}{TD}$;

(A-29) $v = \frac{V^n}{TD}$;

(A-30) $DD = DD^p + DD^t$.

Assuming B^a and W to be exogenous, the money multiplier can be derived. Substituting equations (A-26) and A-27) into equation (A-25) yields:

$$B^a = R^m - A + K^p + V^n,$$

$$B^a = R^r + R^e - A + K^p + V^n,$$

$$B^a = R - A + K^p.$$

Dividing both sides by privately-owned demand deposits gives,

$$\frac{B^a}{DD^p} = \frac{R}{DD^p} - \frac{A}{DD^p} + \frac{K^p}{DD^p}.$$

Substituting for the definitions of r, b, and k yields:

$$\frac{B^a}{DD^p} = \frac{rTD}{DD^p} - \frac{bTD}{DD^p} + k.$$

Since, by definition, TD is comprised of privately-owned deposits plus government deposits plus time deposits, the above equation can be rewritten.

$$\frac{B^a}{DD^p} = \frac{r(DD^p + DD^t + T)}{DD^p} - \frac{b(DD^p + DD^t + T)}{DD^p} + k.$$

Substituting for the definitions of d and t, and factoring yields:

$$\text{(A-31)} \quad \frac{B^a}{DD^p} = (r - b)(1 + d + t) + k.$$

Dividing both sides of equation A-24) by DD^p and substituting k for $\frac{K^p}{DD^p}$ gives,

$$\frac{M^1}{DD^p} = 1 + k; \text{ hence,}$$

$$\frac{M^1}{1 + k} = DD^p.$$

Substituting this result for DD^p in equation (A-31) yields;

$$\frac{Ba}{\frac{M^1}{1 + k}} = (r - b)(1 + d + t) + k.$$

Manipulation gives,

$$\text{(A-32)} \quad M^1 = \frac{1 + k}{(r = b)(1 + d + t) + k} \text{ Ba.}$$

Equation (A-32) describes the relationship, m_1, between Ba and M^1. The multiplier, m_1, is dependent on interest rates because of the dependence of the t and b ratios and the level of excess reserves on interest rates. When the bank credit market is in equilibrium ($\dot{E}^s = E^d$), an interest rate is determined. This equilibrium interest rate then determines values t, b, and the level of excess reserves which, along with the values of r and d, determine the equilibrium value of m_1. A value for the multiplier m_1, or for any of the parameters on which it depends, can of course be calculated at any point in time. But an equilibrium value of m_1 and its parameters implies complete portfolio adjustment by both the banks and the public. That is, equilibrium requires that the public holds the desired levels of currency, demand and time deposits and earning assets, given its wealth, income, and interest rates; banks hold the desired levels of earning assets, borrowed and nonborrowed reserves, given the interest rate, discount rate, and deposit levels.

The value of the money multiplier,

$$\text{(A-33)} \qquad m_1 = \frac{1 + k}{(r - b)(1 + t + d) + k},$$

is of course dependent on the size of the r-ratio. The dependence of r on the distribution of deposits between member and nonmember banks, nonmember bank vault cash and member bank excess reserves can be seen by writing r explicitly as:

$$r = \frac{r^d \delta(1 + d) + r^t \tau\, t + e + v}{1 + d + t}, \text{ where}$$

v = ratio of nonmember banks vault cash to DD^p;

e = ratio of member bank excess reserves to DD^p.

APPENDIX B

DERIVATION OF THE SOURCE BASE

APPENDIX B

DERIVATION OF THE SOURCE BASE

The source base is defined as the net monetary liabilities of the Federal Reserve System and the U. S. Treasury and can be derived from their consolidated balance sheet. The net source base is the source base less member bank borrowing.

Table B-1 shows the calculation of the source base for December, 1977. The right side of the table computes the source base as the sum of the net liabilities of the Federal Reserve and the Treasury; the source base is therefore the sum of member bank deposits at the Federal Reserve plus all currency and coin held by commercial banks and the public. This defines the source base in terms of its uses; all base money is absorbed by member bank reserves, nonmember bank vault cash, and currency in the hands of the public.

The left column of Table B-1 calculates the source base by summing all the sources of base money. This derivation is simply a rearrangement of the balance sheet of the Federal Reserve System, plus three modifications to allow for the monetary activities of the U. S. Treasury. The items involved in these modifications are identified with an asterisk.

First, the currency issued by the Treasury must be added into the source base ("Treasury Currency Outstanding") since it circulates and can be used as bank reserves just as currency issued by the Federal Reserve. On the other hand, the Treasury holds currency (its own or Federal Reserve notes) in its vaults, thereby holding base money out of the system; Treasury cash holdings are therefore subtracted in the

Table B-1. Calculation of the Source Base, December, 1977
(Monthly Averages of Daily Figures, Millions of Dollars)

Sources of Base		Uses of Base	
Gold Stock*	$ 11,696.	Member Bank Reserve	
S.D.R.[1] Certificates	1,208.	Deposits at the Federal Reserve	$ 27,057.
Federal Reserve Credit		Currency and Coin	9,351.
U. S. Government Securities[2]	107,948.		
Acceptances	326.	Currency in Circulation[3]	93,511.
Loans	558.		
Float	5,308.		
Other Federal Reserve Assets	2,242.		
Treasury Currency Outstanding*	11,354.		
Less:			
Treasury Cash Holdings*	408.		
Treasury Deposits at Federal Reserve	5,640.		
Other Deposits at Federal Reserve	956.		
Other F. R. Liabilities and Capital	3,718.		
Source Base	$129,919.	Source Base	$129.919

[1]Special Drawing Rights; [2]Includes $1450. of securities held under repurchase agreements.

[3]Includes nonmember bank vault cash.

Source: Board of Governors of the Federal Reserve System, Federal Reserve Bulletin (February 1978):A4-A5.

calculation of the source base. Included in the item Treasury cash holdings is also the part of the nation's gold against which gold certificates have not been issued. The net result of adding the gold stock and subtracting the gold portion of Treasury cost holdings is therefore the amount of gold certificates owned by the Federal Reserve.

With these modifications, the left column merely sums the assets of the Federal Reserve System and deducts its liabilities and capital accounts to isolate the items that comprise the source base. The Federal Reserve's control of the base comes from complete control of its portfolio of government securities, which is the dominant source of base money.

APPENDIX C

STATE AND FEDERAL RESERVE RESERVE REQUIREMENTS

APPENDIX C

Table C-1. State Reserve Requirements for Nonmember Banks, May, 1977

State	Percent of Deposits: Demand	Percent of Deposits: Time	Reserves Held Only as Vault Cash or Balances Due from Banks Unless Otherwise Indicated Below
Alabama	10	3	
Alaska	20	8	
Arizona	10	4	U.S. Government Securities maturing in not more than 6 months; CIPC, Negotiable C.D.'s maturing within 6 months.*
Arkansas	7-≤$2 million; 9.5-$2-$10 million 11.75-$10-$100 million; 12.75->$100 million.	3-S 3-T, ≤$5 million; 5-T, >$5 million.	CIPC, drawn on banks in same or adjoining municipalities.*
California	FR	5	U.S. Government Securities, up to 20% of reserves against T & S.*
Colorado	15-total deposits		U.S. Government Securities.*
Connecticut	12-<$5 million 12.5->$5 million.	0	U.S. Government, up to 16.67% of reserves.
Delaware	7-<$100 million 9->$100 million.	3	CIPC.*
Florida	20-total deposits		U.S. Government (or agency) Securities.
Georgia	15	5	U.S. Government Securities, maturing within 1 year, plus unpledged C.D.'s can be used for up to 50% of reserves against DD;** U.S. Government Securities maturing within 1 year, other Public Securities and unpledged C.D.'s can be used for reserves against T & S.**

Table C-1. Continued

State	Percent of Deposits: Demand	Percent of Deposits: Time	Reserves Held Only as Vault Cash or Balances Due from Banks Unless Otherwise Indicated Below
Hawaii	12	5	Balances due from other banks, up to 50% of reserves against DD.*
Idaho	15	5	*
Illinois	no legal reserves required		
Indiana	7	3	CIPC.
Iowa	7	3	*
Kansas	7.5-<$2 million; 10- $2-$10 million; 12-$10-$100 million; 13->$100 million.	3-S FR -T	
Kentucky	7	3	U.S. Government (or agency) Securities, maturing within 1 year, plus other public securities, plus CIPC and C.D.'s issued by Kentucky banks can be used for up to 25% of reserves.
Louisiana	8-<$2 million; 10-$2-$10 million; 12-$10-$100 million; 13-$100-$400 million; 14->$400 million.	3	U.S. Government Securities, up to 50% of reserves.
Maine	10	2-S 4-T	U.S. Government Securities, FF Sold.*
Maryland	15	3	U.S. Government Securities, Other Public Securities.
Massachusetts	20-Boston banks 15-other banks	Total Deposits	U.S. Government Securities, plus Other Public Securities (less than 20% of reserves) can be used for up to 80% of reserves.

Table C-1. Continued

State	Percent of Deposits: Demand	Percent of Deposits: Time	Reserves Held Only as Vault Cash or Balances Due from Banks Unless Otherwise Indicated Below
Michigan	11	6	U.S. Government (or Agency) Securities plus CIPC, can be used for up to 90% of reserves.*
Minnesota	7	2	U.S. Government Securities, maturing within 1 year, plus CIPC can be used for up to 30% of reserves.
Mississippi	FR	FR	U.S. Government (or Agency) Securities, maturing within 1 year, plus CIPC plus C.D.'s issued by Mississippi banks (up to 15% of reserves) can be used for up to 30% of reserves.
Missouri	7.5-≤$2 million; 10-$2-$10 million; 12-$10-$100 million; 13->$100 million.	3	CIPC.* **
Montana	7.52-≤$2 million; 10.03≥$2 million.	3	*
Nebraska	15	4	U.S. Government Securities, up to 50% of reserves.
Nevada	10	5	CIPC.*
New Hampshire	12	5	U.S. Government Securities, maturing within 2 years, up to 40% of reserves.
New Jersey	FR	FR	CIPC.*
New Mexico	12	5	U.S. Government Securities, maturing within 100 days, up to 50% of reserves.
New York	FR	FR	CIPC.

Table C-1. Continued

State	Percent of Deposits: Demand	Percent of Deposits: Time	Reserves Held Only as Vault Cash or Balances Due from Banks Unless Otherwise Indicated Below
North Carolina	8-≤$2 million; 10-$2-$10 million; 12-$10-$100 million; 13-$100-$400 million; 15->$400 million.	6-T,>$5 million, maturing in less than 180 days. 3-all other.	CIPC.*
North Dakota	8	2	*
Ohio	7	6-T,>$5 million, maturing in less than 180 days; 3-all other.	U.S. Government Securities can be used for up to 60% of reserves against T & S.
Oklahoma	FR	3-S 6-T,>$5 million, maturing in less than 180 days 3-all other.	CIPC, drawn on banks in the same city.*
Oregon	12	4	CIPC for reserves against DD; U.S. Government Securities, maturing within 1 year, plus CIPC for reserves against T & S.
Pennsylvania	12	3-S 3-T,≤$5 million; 5-T,>$5 million.	U.S. Government (or Agency) Securities plus CIPC can be used for up to 50% of reserves.
Rhode Island	15	0	U.S. Government Securities, maturing in not more than 91 days.
South Carolina	7	3	CIPC, collectable within 10 days.
South Dakota	17.5-Total Deposits		U.S. Government (or Agency) Securities plus CIPC can be used for up to 60% of reserves.

Table C-1. Continued

State	Percent of Deposits		Reserves Held Only as Vault Cash or Balances Due from Banks Unless Otherwise Indicated Below
	Demand	Time	
Tennessee	10	3	
Texas	15	5	CIPC.*
Utah	FR	FR	
Vermont	27	6.5	U.S. Government (or Agency) Securities, maturing within 1 year, other Public Securities.
Virginia	10	3	CIPC for reserves against DD; U.S. Government Securities, maturing within 1 year, plus CIPC for reserves against T & S.
Washington	FR	FR	CIPC drawn on banks in same city.*
West Virginia	7	3	CIPC (20% of reserves must be held as vault cash).
Wisconsin	20	12	U.S. Government Securities, maturing within 18 months plus other Public Securities can be used for up to 33% of reserves against DD and 58% of reserves against T & S.
Wyoming	20	10	U.S. Government Securities, can be used for up to 50% of reserve against DD; at least 40% of total reserves must be in vault cash and balances due from other banks.

Table C-1. Continued

<u>Key</u>

*Deposits due from other banks must be held in approved depository banks.

**Eligible deposits due from other banks are net of reciprocal deposits only.

DD = Demand Deposits.
T & S = Time and Savings Deposits.
T = Time Deposits.
S = Savings Deposits.
CIPC = Cash Items in Process of Collection.
C.D.'s = Certificates of Deposit.
F.R. = Same as Federal Reserve.
F.F. = Federal Funds.

Source: Board of Governors of the Federal Reserve System, "The Burden of Federal Reserve Membership, N.O.W. Accounts, and the Payment of Interest on Reserves," Washington, D.C., June, 1977, Appendix A, Table A-5.

Table C-2

Summary of Changes in Federal Reserve Reserve Requirements
1961 - 1974

I. Structural Changes

1. Beginning July 14, 1966 (reserve city banks) and July 21, 1966 (country banks), the following reserve categories for time deposits became effective:

   a) savings deposits;
   b) time deposits, less than $5 million;
   c) time deposits, greater than $5 million.

2. Beginning January 1, 1967, time deposits such as Christmas and vacation club accounts were defined as "savings deposits" for reserve purposes.

3. Beginning January 11, 1968 (reserve city banks) and January 18, 1968 (country banks), the following reserve categories for net demand deposits became effective:

   a) reserve city banks, less than $5 million;
   b) reserve city banks, greater than $5 million;
   c) country banks, less than $5 million;
   d) country banks, greater than $5 million.

4. Beginning September 18, 1968, lagged reserve requirements became effective whereby a bank's required reserves are based on its average deposit holdings two weeks earlier. Average deposits figures are based on a seven day (Thursday through Wednesday) average of daily close-of-business figures, where Friday's figures are used for Saturday and Sunday if the bank is not open over the weekend. With this change, all National Banks were put on a weekly reporting and reserve settlement schedule. A bank's reserve balance is the sum of its average daily close-of-business deposit balance with its FRB during the current settlement week plus its average daily close-of-business holdings of currency and coin during the week two weeks earlier. Under this new scheme, a member bank is allowed to carry over any excess or deficiency of required reserves into the next settlement week, provided the carry-over does not exceed 2 percent of required reserves. No part of an excess or deficiency not offset in the next week may be carried over into additional settlement weeks. Prior to this change, required reserves were based on contemporaneous beginnings-of-day figures every Wednesday for banks designated as weekly reporting member banks and every other Wednesday for all other member banks. Reserve balances were also measured by contemporaneous levels of deposits at Federal Reserve Banks and vault cash.

5. Beginning July 31, 1969, the definition of gross demand deposits was changed to include outstanding checks or drafts arising from Euro-dollar transactions.

6. Beginning October 16, 1969, member banks were required to hold reserves amounting to 10 percent against their net balances due from domestic offices to their foreign branches, foreign branch loans to U. S. residents, and borrowings from foreign banks by domestic member banks. On January 7, 1971, the reserve ratio was increased to 20 percent. On June 21, 1973, the reserve ratio was reduced to 8 percent and the following loans were exempted from required-reserve calculations: loans aggregating $100,000 or less to U. S. residents and total loans of a bank to U. S. residents if they do not exceed $1 million. Originally certain base amounts were exempted from required-reserve computation but these items were gradually eliminated beginning July 5, 1973 and were completely eliminated by March 14, 1974.

7. Effective November 9, 1972, reserve requirements on net demand deposits were restructured to be based on a bank's deposit-size. The following five deposit size categories were defined for the calculation of required reserves: less than $2 million; more than $2 million and less than $10 million; more than $10 million and less than $100 million; more than $100 million and less than $400 million; and greater than $400 million. Each deposit interval applies to that designated portion of a bank's net demand deposits.

8. Beginning June 21, 1973 (based on deposits as of June 7, 1973), a marginal reserve requirement of 8 percent was imposed on increases in a) certificates of deposit of $100,000 or more, b) outstanding funds obtained through issuance by an affiliate of obligations subject to the existing reserve requirements on time deposits ("bank related commercial paper"). The reserve ratio was applicable to increases in the total of both types of funds above the level held during the week of May 16, 1973 or $10 million, whichever is larger. Since both categories of funds were previously subject to a 5 percent reserve requirement on time deposits, the marginal reserve requirement represents a 3 percent increase in reserve requirements on additions to such funds.

9. Beginning July 12, 1973 (based on deposits of June 28, 1973), the reserve requirement described in (8) above was extended to include funds raised through sales of finance bills. Funds subject to this reserve requirement are those used in the institution's banking business which were obtained through banker's acceptances that are ineligible for Federal Reserve discount. Since there was previously no reserve requirement on such funds, this change placed all outstanding financial bills under the basic 5 percent requirement on time deposits, plus the additional 3 percent requirement on increases in such funds above the level held during the week of May 16 or $10 million, whichever is greater.

10. Effective October 4, 1973 (based on deposits of September 20, 1973), the marginal reserve requirement on large certificates of deposit, bank-related commercial paper, and funds raised in the sale of finance bills described in (7) and (8) above was raised to 11 percent. The same reserve ratio was reduced to 8 percent, effective December 27, 1973 (based on deposits of December 13, 1973).

11. Effective September 19, 1974 (based on deposits of September 5, 1974), large certificates of deposit, bank-related commercial paper, and funds from sales of financial bills were each divided into two categories for reserve purposes: those of maturity length less than four months and those of maturity length of four months, or more. The shorter-term categories continued to be subject to the marginal 8 percent reserve requirement, but the long-term categories for all three types of funds were reverted to the regular 5 percent reserve requirement on time deposits exceeding $5 million.

12. Effective December 12, 1974 (based on deposits of November 28, 1974), the marginal reserve requirement of 8 percent on large certificates of deposit, bank-related commercial paper and sales of finance bills, all with maturities of less than four months was removed and these funds were reverted to the 5 percent reserve requirements on time deposits in excess of $5 million. At the same time, all time deposits (including the three types of liabilities named above) with maturities of six months or more were placed under a 3 percent reserve requirement; all such funds maturing in less than six months were placed under a 6 percent reserve requirement, except the first $5 million which are subject to a 3 percent requirement.

Source: Board of Governors of the Federal Reserve System, <u>Federal Reserve Bulletin</u>, various dates.

II. Changes in Required Reserve Ratios (Dollars are in millions)

1. **Net Demand Deposits**	Reserve City		Country		
Januray 1, 1961 - January 10 (17), 1968[1]	16%		12%		
	0-$5	Over $5	0-$5	Over $5	
January 11 (18),[1] 1968 - April 16, 1969	16.5%	17%	12%	12.5%	
April 17, 1969 - November 8, 1972	17 %	17.5%	12.5%	13%	
	0-$2	$2-$10	$10-$100	$100-$400	Over $400
November 9, 1972 - November 15, 1972	8%	10%	12%	16.5%	17.5%
November 16, 1972 - July 18, 1973	8%	10%	12%	13%	17.5%
July 19, 1973 - December 31, 1974	8%	10%	12.5%	13.5%	18%

2. **Time Deposits**	Savings Deposits	Time Deposits		
		0-$5	Over $5	
July 14 (21),[1] 1966 - September 7 (14),[1] 1966	4%	4%	5%	
September 8 (15),[1] 1966 - March 1, 1967	4%	4%	6%	
March 2, 1967 - March 15, 1967	3.5%	3.5%	6%	
March 16, 1967 - September 30, 1970	3%	3%	6%	
October 1, 1970 - December 11, 1974	3%	3%	5%	
		Maturity 30-179 days		Maturity Over 179 days
December 12, 1974 - December 31, 1974	3%	3%	6%	3%

[1]Dates in parenthesis are applicable to country banks.

Source: Board of Governors of the Federal Reserve System, Federal Reserve Bulletin, December, 1974, p.A9.

APPENDIX D

SUPPLEMENTAL TABLES TO CHAPTER 4

Table D-1. Annual[1] Figures for $\lambda^T_{i,t}$, i = 1,5[2]

	Mean	Standard Deviation	Largest Deviation from 1.0[3]	Mean	Standard Deviation	Largest Deviation from 1.0[3]	Mean	Standard Deviation	Largest Deviation from 1.0[3]
	$\lambda^T_{1,t}$			$\lambda^T_{2,t}$			$\lambda^T_{3,t}$		
1961	.99502	.00387	.01666						
1962	.99352	.00370	.01448						
1963	.99460	.00257	.01132						
1964[4]	.99540	.00279	.01359						
1965	.99430	.00378	.01676	1.00671[5]	.01720[5]	.04827[5]			
1966	.99744	.00449	.00981	.98981[5]	.00991[5]	.04128[5]	1.00018[5]	.00104[5]	.00251[5]
1967	.99446	.00392	.01488				.99852	.00427	.01467
1968	.99610	.00366	.01098				.99960[6]	.00340[6]	.00727[6]
1969[4]	1.00342	.00297	.00180				1.00172	.00349	.00984
1970	.99315	.00604	.02077				.99863	.00382	.00956
1971	.99384	.00439	.01613				.99522	.00565	.01854
1972	.99473	.00298	.01077				.99686	.00432	.01286
1973	.99450	.00614	.01761				1.00011	.00407	.00742
1974	.99439	.00496	.01686				.99830	.00393	.01071

Table D-1. Continued

	Mean	Standard Deviation	Largest Deviation from 1.0[3]	Mean	Standard Deviation	Largest Deviation from 1.0[3]	Mean	Standard Deviation	Largest Deviation from 1.0[3]
		$\lambda^T_{4,t}$			$\lambda^T_{5,t}$				
1966	++	++	++	++	++	++			
1967	++	++	++	++	++	++			
1968[6]	.99547	.00215	.01170	.99286	.00893	.02889			
1969[4]	.99696	.00232	.01041	1.00711	.00605	.02150			
1970	.99723	.00111	.00516	.98641	.01246	.04219			
1971	.99741	.00169	.00749	.99192	.00664	.02074			
1972	.99801	.00101	.00449	.99251	.00668	.02000			
1973	.99806	.00100	.00410	.99037	.01084	.03068			
1974	.99874	.00138	.00482	.99183	.00809	.02352			

[1]Based on 52 observations per year unless otherwise indicated. [2]The subscript i refers to the following categories of time deposits: 1 = total time and savings deposits; 2 = time deposits; 3 = savings deposits; 4 = time deposits less than \$5 million; 5 = time deposits greater than \$5 million.

[3]|1.0 - maximum value| or |1.0 - minimum value|, whichever is greater. [4]Based on 53 observations per year. [5]Based on 17 observations per year. [6]Based on 51 observations per year. [+]Data not available.

Table D-2. Annual[1] Figures for the Proportion of Commercial Banks that are Member Banks

	Number of Member Banks/Number of Commercial Banks				
	Mean	Percentage Change from Previous Year	High Value	Low Value	Percentage Change[2] from High to Low Value
1961	.4564	---	.458	.455	-.655%
1962	.4523	-.898%	.455	.450	-1.099
1963	.4496	-.597	.450	.449	-.222
1964	.4516	.445	.453	.450	.667
1965	.4517	.022	.452	.451	-.221
1966	.4483	-.753	.450	.447	-.667
1967	.4441	-.937	.446	.442	-.897
1968	.4396	-1.013	.442	.437	-1.131
1969	.4337	-1.342	.437	.430	-1.602
1970	.4245	-2.121	.428	.421	-1.636
1971	.4178	-1.578	.421	.416	-1.188
1972	.4118	-1.436	.415	.409	-1.446
1973	.4063	-1.336	.408	.405	-.735
1974	.4022	-1.009	.405	.400	-1.235

[1] Annual figures based on monthly data, last Wednesday of each month. Source: Board of Governors of the Federal Reserve System, Federal Reserve Bulletin, various dates, p. A-18.

[2] Percentages were figured using the high value as the base year, since the direction of change was from high to low in each year except 1964 for which the low value was used as the base year.

Table D-3. Annual[1] Figures for 3-Month Treasury Bill Rate

	3-Month Treasury Bill Rate				
	Mean	Percentage Change from Previous Year	High Value	Low Value	Percentage[2] Change from Low to High Value
1961	2.376	---	2.62	2.27	15.419%
1962	2.780	17.003%	2.95	2.69	9.655
1963	3.158	13.597	3.52	2.90	21.379
1964	3.553	12.508	3.86	3.48	10.920
1965	3.949	11.146	4.36	3.81	14.436
1966	4.882	23.626	5.39	4.54	18.722
1967	4.331	-11.286	5.01	3.48	43.966(-30.539)
1968	5.345	23.413	5.92	4.97	19.115
1969	6.686	25.089	7.72	6.08	26.974
1970	6.437	-3.724	7.91	4.86	-38.559
1971	4.338	-32.608	5.40	3.32	62.651
1972	4.068	-6.224	5.06	3.18	59.119
1973	7.026	72.714	8.67	5.31	63.277
1974	7.873	12.055	8.74	7.06	23.796

[1]Annual figures based on monthly rates on new issue. Source: Board of Governors of the Federal Reserve System, Federal Reserve Bulletin, various dates, p. A-33.

[2]Percentages were figured using the low value as the base year, since the rate rose in every year except 1967 and 1970. For 1967, the figure in parenthesis uses the high value as the base. For 1970, the figure presented uses the high value as the base, since the rate fell during that year.

Table D-4. Quarterly[1] Figures for ν_t^D

	Mean	Standard Deviation		Mean	Standard Deviation
1961 - i	.86530	.00183	1968 - i	.83431	.00149
ii	.86400	.00166	ii	.93224	.00144
iii	.86227	.00143	iii	.82493	.00243
iv	.85852	.00154	iv	.82005	.00252
1962 - i	.86067	.00171	1969 - i	.82425	.00184
ii	.85599	.00188	ii	.82090	.00262
iii	.85708	.00172	iii	.81474	.00192
iv	.85358	.00173	iv[2]	.81272	.00264
1963 - i	.85389	.00116	1970 - i[3]	.81689	.00211
ii	.85161	.00109	ii	.81715	.00198
iii	.85045	.00152	iii[2]	.81269	.00209
iv	.84693	.00191	iv	.80915	.00187
1964 - i	.84695	.00111	1971 - i	.81493	.00161
ii	.84832	.00129	ii	.81229	.00227
iii[2]	.84637	.00200	iii	.80408	.00187
iv	.84099	.00158	iv	.79854	.00186
1965 - i	.84512	.00161	1972 - i	.80243	.00147
ii	.84695	.00140	ii	.79975	.00219
iii	.83975	.00253	iii	.79167	.00284
iv	.83377	.00201	iv	.78696	.00188
1966 - i	.83808	.00151	1973 - i	.79398	.00181
ii	.83969	.00167	ii	.78779	.00254
iii	.83278	.00198	iii	.77921	.00159
iv	.82980	.00227	iv	.77676	.00254
1967 - i	.83567	.00209	1974 - i	.78093	.00197
ii	.83712	.00127	ii	.77967	.00211
iii	.83189	.00205	iii	.77058	.00266
iv	.82973	.00201	iv	.76759	.00222

[1]Based on 13 observations per quarter unless otherwise indicated.

[2]Based on 14 observations.

[3]Based on 12 observations.

Table D-5. Quarterly[1] Figures for τ_t

	Mean	Standard Deviation	Coefficient of Variation
1961 - i	.66507**	.01975*	.02970*
ii	.70013	.01395	.01992
iii	.72208	.00639**	.00884**
iv	.71036	.01257	.01770
1962 - i	.73987**	.03149*	.04256*
ii	.79229	.01654	.02088
iii	.82166*	.00920	.01120
iv	.81904	.00893**	.01090**
1963 - i	.84734**	.03028*	.03574*
ii	.89608	.01810	.02020
iii	.91920*	.01057**	.01150**
iv	.91142	.01084	.01189
1964 - i	.93878**	.03179*	.03386*
ii	.98701	.02062	.02089
iii[2]	1.00400*	.00986**	.09821**
iv	.98896	.01116	.01128
1965 - i	1.04240**	.04160*	.03991*
ii	1.09757	.02667	.02430
iii	1.12476*	.01599**	.01422**
iv	1.10727	.01802	.01627
1966 - i	1.12945**	.03729*	.03302*
ii	1.17962	.03209	.02720
iii	1.21588*	.01618**	.01330**
iv	1.18372	.02159	.01824
1967 - i	1.23321**	.04573*	.03708*
ii	1.29624	.02405	.01855
iii	1.31778*	.01328**	.01008**
iv	1.29420	.01756	.01357
1968 - i	1.31460**	.04672*	.03554*
ii	1.34048	.02177	.01624
iii*	1.35683	.02007	.01479
iv	1.35580	.01983**	.01463**
1969 - i	1.34606	.03989*	.02963*
ii	1.36500*	.02281	.01671**
iii	1.32289	.02279**	.01723
iv[2]	1.26116**	.02980	.02363

Table D-5. Continued

	Mean	Standard Deviation	Coefficient of Variation
1970 - i[3]	1.26316**	.05117*	.04051*
ii	1.31196	.03207	.02444
iii[2]	1.39825	.03231	.02312
iv	1.42835*	.01695**	.01187**
1971 - i	1.51049**	.06635*	.04409*
ii	1.56377	.02938	.01879
iii	1.58884	.02046**	.01288**
iv	1.59302*	.02786	.01749
1972 - i	1.63195**	.05321*	.03261*
ii	1.64025	.03393	.02069
iii	1.66072*	.02076**	.01250**
iv	1.63813	.03678	.02245
1973 - i	1.68849**	.08191*	.04851*
ii	1.78567	.03355**	.01879**
iii	1.83553*	.04476	.02439
iv	1.82760	.03643	.01993
1974 - i	1.88549**	.07729*	.04099*
ii	1.99584	.05876	.02944
iii	2.06534*	.03477	.01684
iv	2.06532	.02884**	.01396**

[1]Based on 13 observations per quarter unless otherwise indicated.

[2]Based on 14 observations per quarter.

[3]Based on 12 observations per quarter.

*Indicates largest quarterly values in each year.

**Indicated lowest quarterly values in each year.

Table D-6. Quarterly Figures for ε_t

	Mean	Standard Deviation		Mean	Standard Deviation
1961 - i	.00364	.00093	1970 - i[3]	.00051	.00020
ii	.00317	.00040	ii	.00051	.00031
iii	.00315	.00031	iii[2]	.00058	.00034
iv	.00285	.00031	iv	.00062	.00035
1962 - i	.00274	.00043	1971 - i	.00061	.00038
ii	.00247	.00028	ii	.00051	.00024
iii	.00257	.00025	iii	.00048	.00038
iv	.00240	.00029	iv	.00047	.00042
1963 - i	.00233	.00074	1972 - i	.00044	.00033
ii	.00203	.00029	ii	.00040	.00026
iii	.00204	.00033	iii	.00046	.00049
iv	.00185	.00042	iv	.00060	.00039
1964 - i	.00194	.00057	1973 - i	.00049	.00042
ii	.00153	.00029	ii	.00043	.00039
iii[2]	.00175	.00035	iii	.00047	.00045
iv	.00167	.00038	iv	.00052	.00038
1965 - i	.00161	.00041	1974 - i	.00033	.00024
ii	.00138	.00024	ii	.00034	.00013
iii	.00151	.00032	iii	.00032	.00020
iv	.00145	.00043	iv	.00034	.00039
1966 - i	.00133	.00041			
ii	.00129	.00028			
iii	.00136	.00062			
iv	.00127	.00051			
1967 - i	.00140	.00054			
ii	.00129	.00025			
iii	.00127	.00039			
iv	.00113	.00037			
1968 - i	.00123	.00044			
ii	.00112	.00039			
iii	.00105	.00037			
iv	.00087	.00048			
1969 - i	.00085	.00057			
ii	.00069	.00040			
iii	.00077	.00040			
iv[2]	.00070	.00037			

[1]Based on 13 observations per quarter unless otherwise indicated.

[2]Based on 14 observations per quarter. [3]Based on 12 observations per quarter.

APPENDIX E

DERIVATION OF THE EQUATIONS FOR THE PARTIAL VARIANCE OF r_t

APPENDIX E

DERIVATION OF THE EQUATIONS FOR THE PARTIAL VARIANCE OF r_t

Consider the general expression for r_t,

$$(E-1)\quad r_t = \sum_j d_{jt}\lambda^D_{j,t}\delta_{j,t}\nu^D_t\xi_t(1 + \gamma_t + \iota_t) + \sum_i t_{i,t}\lambda^T_{i,t}\delta^T_{i,t}\nu^T_t\tau_t$$

$$+ \varepsilon_t\rho^m_t + \sum_{h=1}^{51} \psi_{h,t}\omega_{h,t}\rho^n_t,$$

where j and i are defined appropriately for each reserve scheme. The partial variance of r_t is defined here as the variance caused in r_t by one parameter while all other parameters in r_t are held constant at their means. The expression for r_t is therefore a function of at most (j + i) random variables. The partial variance of r_t can then be derived using the following result:[1]

If g is a function of random variables such that,

$$g(x_1, x_2, \ldots x_n) = \sum_{i=1}^{n} a_i x_i + c, \text{ then}$$

$$(E-2)\quad \text{var}(g) = \sum_{i=1}^{n} a_i^2 \text{ var}(x_i) + \sum_{\substack{i,j=1 \\ i \neq j}}^{n} a_i a_j \text{ cov}(x_i, x_j).$$

i) $\text{var}_\lambda(r_t)$:

Using the definition of partial variance given above,

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[1] Maurice G. Stuart and Alan Stuart, <u>The Advanced Theory of Statistics</u>, Volume I, (London: Charles Griffin and Company, Limited, 1963):231-3.

$$\mathrm{var}_{\lambda}(r_t) = \mathrm{var}\Big[\sum_j \bar{d}_{j,t}\lambda^D_{j,t}\bar{\delta}^D_{j,t}\bar{\nu}^D_t\bar{\xi}_t(1+\bar{\gamma}_t+\bar{\iota}_t)$$

$$+\sum_i \bar{t}_{i,t}\lambda^T_{i,t}\bar{\delta}^T_{i,t}\bar{\nu}^T_t\bar{\tau}_t + \bar{\varepsilon}_t\bar{\rho}_t + \sum_{h=1}^{51}\bar{\psi}_{h,t}\bar{\omega}_{h,t}\bar{\rho}^n_t\Big]$$

where (-) over a variable denotes its mean. Since the last two terms in the expression are entirely constants, their variance is zero. Using equation (E-2) above, $\mathrm{var}_{\lambda}(r_t)$ can be rewritten as,

$$\mathrm{var}_{\lambda}(r_t) = [\bar{\nu}^D_t\,\bar{\xi}_t\,(1+\bar{\gamma}_t+\bar{\iota}_t)]^2\Big[\sum_j(\bar{d}_{j,t}\,\bar{\delta}^D_{j,t})^2\mathrm{var}(\lambda^D_{j,t})$$

$$+\sum_j\sum_{\substack{j' \\ j\neq j'}}\bar{d}_{j,t}\bar{d}_{j',t}\bar{\delta}^D_{j,t}\bar{\delta}^D_{j',t}\,\mathrm{cov}\,(\lambda^D_{j,t},\,\lambda^D_{j',t})\Big]$$

$$+[\bar{\nu}^T_t\,\bar{\tau}_t]^2\Big[\sum_i(\bar{t}_{i,t}\bar{\delta}^T_{i,t})^2\,\mathrm{var}\,(\lambda^T_{i,t})$$

$$+\sum_i\sum_{\substack{i' \\ i\neq i'}}\bar{t}_{i,t}\bar{t}_{i',t}\bar{\delta}^T_{i,t}\bar{\delta}^T_{i',t}\,\mathrm{cov}\,(\lambda^T_{i,t},\lambda^T_{i',t})\Big]$$

$$+[\bar{\nu}^D_t\,\bar{\xi}_t(1+\bar{\gamma}_t+\bar{\iota}_t)\bar{\nu}^T_t\,\bar{\tau}_t]\Big[\sum_j\sum_i\bar{d}_{j,t}\bar{t}_{i,t}\,\bar{\delta}^D_{j,t}\,\bar{\delta}_{i,t}$$

$$\mathrm{cov}\,(\lambda^D_{j,t},\,\lambda^T_{i,t})\Big].$$

ii) $\mathrm{var}_{\delta}(r_t)$:

The partial variance of r_t due to differential reserve requirements is defined as,

$$\mathrm{var}_{\delta}(r_t) = \mathrm{var}\Big[\sum_j \bar{d}_{j,t}\,\bar{\lambda}^D_{j,t}\,\delta^D_{j,t}\,\bar{\nu}^D_t\bar{\xi}_t(1+\bar{\gamma}_t+\bar{\iota}_t)$$

$$+\sum_i \bar{t}_{i,t}\,\bar{\lambda}^T_{i,t}\,\delta^T_{i,t}\,\bar{\nu}^T_t\bar{\tau}_t + \bar{\varepsilon}_t\bar{\rho}^m_t + \sum_{h=1}^{51}\bar{\psi}_{h,t}\,\bar{\omega}_{h,t}\,\bar{\rho}^n_t\Big].$$

The last two terms are again zero. Employing the result in equation (E-2),

$$\mathrm{var}_{\delta}(r_t) = [\bar{\nu}_t^D \bar{\xi}_t(1 + \bar{\gamma}_t + \bar{\iota}_t)]^2[\sum_j (\bar{d}_{j,t}\, \bar{\lambda}_{j,t}^D)^2\ \mathrm{var}\,(\delta_{j,t}^D)$$

$$+ \sum_j \sum_{\substack{j' \\ j \neq j'}} \bar{d}_{j,t}\bar{d}_{j',t}\bar{\lambda}_{j,t}^D\bar{\lambda}_{j',t}^D\ \mathrm{cov}\,(\delta_{j,t}^D, \delta_{j',t}^D)]$$

$$+ [\bar{\nu}_t^T\, \bar{\tau}_t]^2[\sum_i (\bar{t}_{i,t}\, \bar{\lambda}_{i,t}^T)^2\ \mathrm{var}\,(\delta_{i,t}^T)$$

$$+ \sum_i \sum_{\substack{i' \\ i \neq i'}} \bar{t}_{i,t}\, \bar{t}_{i',t}\, \bar{\lambda}_{i,t}^T\bar{\lambda}_{i',t}^T\ \mathrm{cov}\,(\delta_{i,t}^T, \delta_{1',t}^T)]$$

$$+ [\bar{\nu}_t^D\, \bar{\xi}_t(1 + \bar{\gamma}_t + \bar{\iota}_t)\bar{\nu}_t^T\, \bar{\tau}_t][\sum_j \sum_i \bar{d}_{j,t}\bar{t}_{i,t}\, \bar{\lambda}_{j,t}^D\bar{\lambda}_{i,t}^T$$

$$\mathrm{cov}\,(\delta_{j,t}^D,\ \delta_{i,t}^T)].$$

iii) $\mathrm{var}_{\nu}(r_t)$:

Using the definition of partial variance, $\mathrm{var}_{\nu}(r_t)$ is defined,

$$\mathrm{var}_{\nu}(r_t) = \mathrm{var}\ [\sum_j \bar{d}_{j,t}\, \bar{\lambda}_{j,t}^D\, \bar{\delta}_{j,t}^D\, \nu_t^D\, \bar{\xi}_t(1 + \bar{\gamma}_t + \bar{\iota}_t)$$

$$+ \sum_i \bar{t}_{i,t}\, \bar{\lambda}_{i,t}^T\, \bar{\delta}_{i,t}^T\, \nu_t^T\, \bar{\tau}_t + \bar{\varepsilon}_t\, \bar{\rho}_t^m$$

$$+ \sum_{h=1}^{51} \bar{\psi}_{h,t}\, \bar{\omega}_{h,t}\, \bar{\rho}_t^n].$$

The variance of the last two terms is again zero and $\mathrm{var}_{\nu}(r_t)$ is rewritten,

$$\mathrm{var}_{\nu}(r_t) = [\sum_j \bar{d}_{j,t}\bar{\lambda}_{j,t}^D\bar{\delta}_{j,t}^D\bar{\xi}_t\ (1 + \bar{\gamma}_t + \bar{\iota}_t)]^2\ \mathrm{var}\,(\nu_t^D)$$

$$+ [\sum_i \bar{t}_{i,t}\ \bar{\lambda}^T_{i,t}\ \bar{\delta}^T_{i,t}\ \bar{\tau}_t]^2 \ \mathrm{var}\ (\nu^T_t)$$

$$+ 2\sum_j \bar{d}_{j,t}\ \bar{\lambda}^D_{j,t}\ \bar{\delta}^D_{j,t}\ \bar{\xi}_t\ (1 + \bar{\gamma}_t + \bar{\iota}_t)\sum t_{i,t}\bar{\lambda}^T_{i,t}\bar{\delta}^T_{i,t}\ \bar{\tau}_t$$

$$\mathrm{cov}\ (\nu^D_t, \nu^T_t)].$$

iv) $\mathrm{var}_{\gamma}\ (r_t)$:

The expression for $\mathrm{var}_{\gamma}\ (r_t)$ is defined by,

$$\mathrm{var}_{\gamma}\ (r_t) = \mathrm{var}\ [\sum_j \bar{d}_{j,t}\bar{\lambda}^D_{j,t}\bar{\delta}_{j,t}\bar{\nu}^D_t\bar{\xi}_t\ (1 + \gamma_t + \bar{\iota}_t)$$

$$+ \sum_i \bar{t}_{i,t}\bar{\lambda}^T_{i,t}\bar{\delta}^T_{i,t}\bar{\nu}^T_t\ \bar{\tau}_t + \bar{\varepsilon}_t\ \bar{\rho}^m_t$$

$$+ \sum_{h=1}^{51} \bar{\psi}_{h,t}\ \bar{\omega}_{h,t}\ \bar{\rho}^n_t.$$

Since they are constants, the variance of the last three terms in $\mathrm{var}_{\gamma}\ (r_t)$ is zero. Using equation (E-2), $\mathrm{var}_{\gamma}\ (r_t)$ is,

$$\mathrm{var}_{\gamma}\ (r_t) = [\sum_j \bar{d}_{j,t}\ \bar{\lambda}^D_{j,t}\ \bar{\delta}^D_{j,t}\ \bar{\nu}^D_t\ \bar{\xi}_t\ (1 + \bar{\iota}_t)]^2\ \mathrm{var}\ (\gamma_t).$$

v) $\mathrm{var}_{\iota}\ (r_t)$:

The expression for the $\mathrm{var}_{\iota}\ (r_t)$ is defined by,

$$\mathrm{var}_{\iota}\ (r_t) = \mathrm{var}\ [\sum_j \bar{d}_{j,t}\ \bar{\lambda}^D_{j,t}\ \bar{\delta}^D_{j,t}\ \bar{\nu}^D_t\ \bar{\xi}_t\ (1 + \bar{\gamma}_t + \iota_t)$$

$$+ \sum_i \bar{t}_{i,t}\ \bar{\lambda}^T_{i,t}\ \bar{\delta}^T_{i,t}\ \bar{\nu}^T_t\ \bar{\tau}^T_t + \bar{\varepsilon}_t\ \bar{\rho}^m_t + \sum_{h=1}^{51} \bar{\psi}_{h,t}\ \bar{\omega}_{h,t}\ \bar{\rho}^n_t].$$

The variance of the last three terms is again zero, so $\mathrm{var}_{\iota}\ (r_t)$ can be written,

$$\text{var}_{\iota}\,(r_t) = [\sum_j \bar{d}_{j,t}\,\bar{\lambda}^D_{j,t}\bar{\delta}^D_{j,t}\bar{\nu}^D_t\,\bar{\xi}_t(1 + \bar{\gamma}_t)]^2\ \text{var}\,(\iota_t).$$

vi) $\text{var}_{\tau}\,(r_t)$:

Using the definition of the partial variance of r_t, $\text{var}_{\tau}\,(r_t)$ is given by,

$$\text{var}_{\tau}\,(r_t) = \text{var}\,[\sum_j \bar{d}_{j,t}\,\bar{\lambda}^D_{j,t}\,\bar{\delta}^D_{j,t}\,\bar{\nu}^D_t\,\bar{\xi}_t\,(1 + \bar{\gamma}_t + \bar{\iota}_t)$$

$$+ \sum_i \bar{t}_{i,t}\,\bar{\lambda}^T_{i,t}\,\bar{\delta}^T_{i,t}\,\bar{\nu}^T_t\,\tau_t + \bar{\varepsilon}_t\,\bar{\rho}^m_t$$

$$+ \sum_{h=1}^{51} \bar{\psi}_{h,t}\,\bar{\omega}_{h,t}\,\bar{\rho}^m_t].$$

The first, third, and fourth terms in $\text{var}_{\tau}\,(r_t)$ are constants so the expression can be rewritten,

$$\text{var}_{\tau}\,(r_t) = [\sum_i \bar{t}_{i,t}\,\bar{\lambda}_{i,t}\,\bar{\delta}^T_{i,t}\,\bar{\nu}^T_t]^2\ \text{var}\,(\tau_t).$$

vii) $\text{var}\ \varepsilon\ (r_t)$:

The equation for $\text{var}_{\varepsilon}\,(r_t)$ is defined,

$$\text{var}_{\varepsilon}\,(r_t) = \text{var}\,[\sum_j \bar{d}_{j,t}\,\bar{\lambda}^D_{j,t}\,\bar{\delta}^D_{j,t}\,\bar{\nu}^D_t\,\bar{\xi}_t\,(1 + \bar{\gamma}_t + \bar{\iota}_t)$$

$$+ \sum_i \bar{t}_{i,t}\,\bar{\lambda}_{i,t}\,\bar{\delta}^T_{i,t}\,\bar{\nu}^T_t\,\bar{\tau}_t + \varepsilon_t\,\bar{\rho}^m_t$$

$$+ \sum_{h=1}^{51} \bar{\psi}_{h,t}\,\bar{\omega}_{h,t}\,\bar{\rho}^n_t].$$

Only the third term in $\text{var}_{\varepsilon}\,(r_t)$ is nonconstant so the above expression reduces to,

$$\text{var}_{\varepsilon}\,(r_t) = (\rho^m_t)^2\ \text{var}\,(\varepsilon_t).$$

For each of the expressions given above for the partial variances of r_t, the indices j, j', i, and i' are defined to conform to each of the reserve schemes.

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