ABSTRACT

AGGREGATION BIAS IN THE
DEMAND FOR MONEY

By

Edward Elliot Veazey

The primary purpose of this research is to determine whether a single macroequation should be relied upon as an accurate description of the demand for money in the United States. Many authors have assumed that a single equation does adequately describe total U.S. money demand, and they have proceeded on that basis with empirical analysis involving a few arbitrary macrovariables. The rate of return on four to six month commercial paper, for example, is one of the most frequently used interest rate variables and it is usually treated as "the" rate of interest with the implication that it adequately represents all of the various interest rates and thus the opportunity cost of holding money. Likewise, some measure of national income is usually treated as adequately representing all budget constraints. In spite of the fact that the problems associated with using such aggregates in regression analysis have been known for some time, they have been ignored or assumed away by monetary empiricists.

In this research, separate monetary demand equations are estimated for each of the forty-eight continental United States and the District of Columbia using interest rate and income series applicable to each particular state. Then weighted averages of the variables

are calculated for use in a single macroequation. These macrovariables are constructed with fixed weights so that the analysis will fall within the scope of linear aggregation.

The principal conclusions of this research are as follows: (i) Estimation of demand for demand deposits at the state level yields parameter estimates which conform generally with prior expectations based on economic theory. (ii) The system of state demand equations is not consistent with a single macroequation which describes aggregate demand in terms of linearly aggregated macrovariables. (iii) Estimates based on such a misspecified macroequation cannot be assumed to be unbiased; therefore, conclusions based on these estimates are suspect. Calculations based on state estimates indicate a bias in one of the macroparameter estimates equal to 86% of the estimate.

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CHAPTER I

INTRODUCTION

The era of econometric forecasting is currently being dominated by large models which rely on nationally aggregated data. Thousands of man and computer hours are spent each year in attempts to predict the sum total of goods and services which will be produced in the next year. Likewise, major effort is directed toward determining national indices of unemployment, prices, corporate profit, interest rates, etc. In prior years, lack of convenient disaggregated data and a need to limit the scope of early efforts probably dictated the heavy concentration on national aggregates. But now it seems that the marginal gains will be larger if additional effort is applied to collection and analysis of disaggregated data.

The value of a national economic index to most people depends on how well it describes the economic environment relevant for a particular set of individuals, and an estimate of gross national product--even a positively known, correct estimate--would be of no more use to most individual businessmen than a forecast of gross national rainfall would be to an individual farmer. It is possible that some of the national indices describe most of their local counterparts very well and thus are a useful summary of information. It is just as likely, however, that some of the national indices are no more relevant for particular geographic and economic segments of the country than the national forecast of rainfall.

Purpose

The purpose of this research is to determine whether the demand for money in the United States is adequately described by a single macroequation based on national data. Demand equations are estimated for each of the states and the estimates are used to investigate the effects of aggregating to a single marcoequation. Previous authors have estimated the demand for money using every convenient functional form and a wide variety of variables. In each case, however, the estimates have been based on macroequations, and little or no attention has been given to the problems which might be caused by aggregation. This is a serious oversight. If a system of microequations does not meet the conditions for consistent aggregation, then least squares estimates of the macroparameters cannot be assumed to be unbiased and conclusions based on their values are in doubt. Not only are the estimated parameters apt to be biased estimates of the average microparameter, but in the event of large scale inconsistency even unbiased estimates would be of limited value. The greater the inconsistency the less the macroequation tells about the demand in any particular microunit and the less useful it is as a summary of information.

An important indication that aggregation bias might be a problem in demand for money equations comes to light with a close look at the portfolio theory approach to the demand for liquid assets. Reference is often made to portfolio theory as a rationale for regressing quantities of particular assets on a vector of interest rates and other exogenous variables. The portfolio theory approach is developed in

Chapter II and, under certain assumptions, it does yield equations which describe quantities of assets as linear functions of interest rates. However, when the equations are expressed in this linear form, the coefficients of the interest rate terms are implicitly defined as non-linear functions of, among other things, the investor's anticipated second moments of the probability distributions of the future returns to each asset. That is, the investor is assumed to anticipate some probability distribution for the return to each asset, and the coefficients of the interest rates in the linear equations are functions of the second moments of these probability distributions.

One of the standard assumptions of regression analysis is that the coefficients in the regression equation are constant for all observations. This assumption would be met in the portfolio theory equations if, among other conditions, the investor's anticipations regarding the probability distributions of the returns were the same for all observations. Since anticipated distributions are not observable, a direct test of their stability is not possible. However, we might reasonably assume that investors formulate their subjective anticipations in some systematic way according to actual values of past observations. It might be argued, for example, that an investor predicts the variance of the return to time deposits by calculating a sample variance from past observations. Then if all investors calculate the same values for the sample moments, there might be some justification for the assumption that anticipated distribution moments, and thus coefficients in the portfolio equations, are constant for all individuals.

The common usage, in economic literature, of terminology which implies that interest rates are adequately described by a single rate called "the rate of interest" makes it easy to erroneously conclude that each investor faces the same rate of return on, say, time deposits. If this were true, then of course all investors would have the same data available for use in calculating sample moments. However, investors do not receive the same return to such assets as time deposits and savings and loan shares. Even granting the assumption that return to a particular asset is the same throughout each state, there are wide differences in the levels and changes in the return to liquid assets in different states. Likewise, there is large disparity in sample moments calculated from past data in different states.

Recognition of the disparity in interest rates in different states leads naturally into doubts about the appropriateness of aggregating asset and interest rate variables over all states for use in a macro-equation regression. With different investors facing different current and past interest rates, the agrument that the coefficients in the portfolio equations are equal for all investors becomes exceedingly tenuous. It seems unlikely that investors with disparate interest rate experience would, nevertheless, have equal anticipations for the probability distributions of future returns. Thus, it seems unlikely that the coefficients in portfolio equations would be equal for all individuals in every state. Aggregation bias appears a clear possibility.

Even without the specific example afforded by portfolio theory, there are important considerations which suggest that the information gained from a disaggregated approach would justify the extra effort.

States are economically less heterogeneous units than is the nation as a whole. Banking laws differ among various states as do the laws governing savings and loan associations, mutual savings banks, and other financial intermediaries. Usury laws vary among states; the variety of charges applied by national retailers to their credit customers in different states illustrates the variety of state legal environments for financial transactions. The degree and nature of industrialization also varies from state to state. Different states tax and control their corporations differently and, of course, different geographic regions have different comparative advantages.

It seems unlikely that the aggregate demand for money in North Dakota responds to the interest paid on short-term commercial paper in the same proportion as in New York. And if the response is not the same in these two places, then combining their demand deposits in a single macroequation for use in regression analysis may be a serious error. One important purpose of most regressions is to estimate the values of coefficients and draw conclusions or make predictions based on the estimates. When aggregate equations are used in place of a larger number of disaggregated relationships, estimates of the regression coefficients are apt to be biased and conclusions drawn from the estimates are apt to be erroneous.

Summary of Chapters

Chapter II presents several models of demand for liquid assets which have been used in recent empirical research. Two of the models are based on microtheory--one on the theory of demand, and the other on portfolio theory--and the other models are best classified as macromodels in spite of the appeals of the various authors to microtheory in defending various characteristics of their models. This chapter both summarizes the important empirical work in monetary economics and introduces the various model specifications which are used in Chapter IV.

Chapter III deals in a general way with the problem of aggregation associated with using macromodels in place of micromodels. The first part develops the conditions under which the non-stochatic part of the macrorelation is always consistent with the system of microrelations. Since these strigent conditions seem unlikely to hold in general, this leads directly to the questions of what importance the inconsistencies play when estimates are made of the parameters of marcoequations. The second part of this chapter develops answers to this question by investigating the impact of aggregation on the parameter estimates, on the goodness of fit, and on prediction.

Chapter IV contains detailed descriptions of the data and the procedure by which macrovariables are formulated to fall within the scope of linear aggregation analysis. Each of the specifications of Chapter II is estimated using linearly aggregated macrovariables, and the general results are compared with the results obtained by the original author.

In Chapter V, statewide data are used, first to estimate the parameters of the entire system of state demand equations and then to test the hypothesis that the equations may be consistently aggregated to a single macroequation. Equations for both demand deposits and the sum of demand and time deposits are subjected to this estimation and testing.

The final chapter contains a summary of results and the important conclusions and implications.

CHAPTER II

THEORETICAL DERIVATION OF THE DEMAND FOR MONEY

The demand equations for liquid assets are developed in completely different fashion depending on whether the model is purely theoretical or meant to be used in empirical work. Virtually all of the recent empirical studies of demand for money are studies of macrorelations. Typically they involve regressions of total money supply on net national product and some interest rate or index of interest rates. In recent literature, by far the most common approach in estimating monetary relationships is to use aggregate data for the nation as a whole with the underlying implicit assumption that the macrovariables used in the study are related in a stable way according to the proposed macrorelation.

On the other hand, the purely theoretical development of monetary theory has shifted quite heavily to microeconomic analysis. The macrorelations of the classical quantity theory have given way to microrelations based on an individual's maximization of utility. In the classical approach, money was singled out as a unique and separate entity conveniently described and limited by quantity theory equations

$$MV = PT$$

or

$$M = KY.$$

Keynes personalized the demand for money somewhat by focusing on motives for holding money, in particular the speculative and transactions motive.[1] Then Hicks brought monetary theory definitely within the range of microeconomics by referring to it as ". . . standard commodity selected from the rest to serve as standard of value."[2] He emphasized that money, like other commodities, has close substitutes. "The fact that money and securities are close substitutes is absolutely fundamental to dynamic economics."[3]

Hicks also made an important contribution to the early development of portfolio theory.[4] He introduced into monetary theory the concept of the rate of return to a portfolio as having a range of possible values and the idea that the width of this range was indication of the risk incurred by the individual. Markowitz took these notions of expected rather than certain return and dispersion as a measure of risk and developed a comprehensive formulation of portfolio theory.[5] Then Tobin grounded the analysis in utility theory.[6]

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[1] J. M. Keynes, The General Theory of Employment, Interest, and Money, New York: Harcourt Brace and Co., 1936, pp. 195-200.

[2] J. R. Hicks, Value and Capital, 2nd ed., Oxford: Oxford University Press, 1946, p. 170.

[3] Ibid.

[4] J. R. Hicks, "A Suggestion for Simplifying the Theory of Money," Economica, New Series, Vol. 2 (1935), pp. 1-19.

[5] Harry Markowitz, "Portfolio Selection," The Journal of Finance, Vol. 7 (March, 1952), pp. 77-91.

[6] James Tobin, "Liquidity Preference as Behavior Toward Risk," Review of Economic Studies, Vol. 25 (February, 1958), pp. 65-86.

Both portfolio theory and demand theory have gone largely untested and even ignored in empirical work except for an occasional casual reference.[7] They are nevertheless important since they led, through analogy, to the development of estimable aggregate demand equations.

The next two sections of this chapter will present essential elements of the demand theory and portfolio theory approaches to the demand for money. The third section will review the specification of demand for money functions encountered in current empirical research.

Demand Theory

The traditional theory of consumer behavior is stated in terms of flows rather than stocks and so, in order to utilize it in the demand for liquid assets, there must by an assumed relationship between the amount held of a particular asset and the flow of services which it generates. The exact nature of the service flows and the motivation for holding assets received a great deal of attention in the early transition from the quantity theory approach to demand theory. Demand for money, for example, was described in terms of "transactions" and "speculative" demand with the names indicating different motives for holding and different service flows provided by money balances. However, just as in demand theory no attention is paid to the exact nature of utility, it was a natural development in monetary theory to deemphasize the exact nature of the service flows and treat them the same as any other consumer good.

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[7]An important exception to this is Edgar L. Feige, Demand for Liquid Assets: A Temporal Cross-Section Analysis, Englewood Cliffs, N.J.: Prentice-Hall, Inc. 1964. Feige very carefully develops the microtheory on which his model is based. His work is discussed below.

The variables included in theoretical discussions of demand functions are income, tastes, and the prices of the commodity demanded and related commodities. The application of this to demand for financial assets is fairly straight forward except that the concept of price needs to be developed. Edgar Feige suggests the following approach.[8] The price of holding a particular asset and enjoying its service flows is basically an opportunity cost. It is the interest foregone in holding that asset in place of an asset whose return is entirely pecuniary. Feige lets R_o represent the rate of return on the hypothetical asset whose return is entirely pecuniary. This is a troublesome concept because there is no asset which is completely described by its rate of return with no other characteristics to provide satisfaction of "service flows" to its owner. For practical purposes, however, R_o could also be considered as the highest return available among the competing assets. Then if R_i represents the return to the ith asset, the price or opportunity cost P_i can be defined as the difference between R_o and R_i.

$$P_i = R_o - R_i$$

With prices thus defined and with the usual qualifying assumption that tastes and preferences are constant, demand equations for money and other liquid assets can be defined in terms of income and a vector of interest rates on closely related assets.

Any list of assets that should be included among "closely related" monetary assets is necessarily arbitrary. One possibility in limiting

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[8]Feige, pp. 16-18.

the list is to define specific necessary characteristics that the assets must have. For example, it might be reasonable to assume that in order for an owner to consider an asset a close substitute for money it must be stated in a fixed nominal amount and be readily convertible at close to that fixed amount into another of the closely related assets.

Friedman and Schwartz suggest that this limited set includes currency, demand deposits, time deposits, deposits at mutual savings banks, savings and loan shares, cash surrender value of life insurance, and series E government savings bonds.[9] In their empirical tests, however, they considered only four combinations of these seven assets in order to reduce the scope of their investigation. For several practical reasons most empiricists reduce the list even further. The most frequently cited, closely related monetary assets are currency, demand deposits, and time deposits.

Portfolio Theory

The distinguishing characteristic of the portfolio approach is that it explicitly includes risk as an influencing factor in the selection of assets. The investor is assumed to regard the future returns to the assets in his portfolio as random variables and to adjust his portfolio on the basis of their joint probability distribution. Risk is quantified as the variance of the return to the total portfolio and the investor is assumed to minimize risk for the given level of expected return. Markowitz stated this assumption more formally: ". . . the investor

[9] Milton Friedman and Anna J. Schwartz, Monetary Statistics of the United States, New York: National Bureau of Economics Research, 1970.

would want to select one of those portfolios . . . with minimum variance for given expected return or more and maximum expected return for given variance or less."[10]

In looking for a rationale for the investor to regard expected value and variance of a portfolio as the parameters relevant to his investment decision, Tobin discovered the clever device of parametric restrictions on the investors utility of return function. Specifically, Tobin shows that focus on the mean and variance can be justified by the assumption that the utility function is quadratic and the investor acts to maximize the expected value of utility. A full development of this approach leads to the same set of efficient portfolios as does Markowitz's mean variance rule but in more convenient form--a form more apparently akin to the linear regression models in recent empirical studies.

Efficient Portfolios

Let the following variables denote the investor's anticipations regarding the returns to alternative assets:

q_i = the proportion of the total portfolio held in the ith asset i = 1, 2, ... N.

R_i = random variable representing the investor's anticipated return to the ith asset.

$U_i = E(R_i)$ = the mean or expected value, of the return to asset i.

$M_{ij} = E(R_i R_j)$ = the second moment of the joint probability distribution of assets i and j.

From the above parameters associated with the individual assets it is possible to derive the parameters for the total portfolio. Dropping

[10] Markowitz, "Portfolio Selection," p. 81.

the subscript to denote total portfolio rather than a particular asset within it, we have

$$R = \sum_i q_i R_i$$

$$U = E(R) = \sum_i q_i U_i$$

$$M = E(R^2) = \sum_i \sum_j q_i q_j M_{ij}$$

Then if utility of return is represented as the quadratic equation:[11]

$$U(R) = a_o + a_1 R + a_2 R^2$$

$$= a_o + a_1 \sum q_i R_i + a_2 \sum\sum q_i q_j R_i R_j$$

the expected value of that utility is

$$E[U(R)] = a_o + a_1 \sum_i q_i U_i + a_2 \sum\sum q_i q_j M_{ij}$$

Maximizing the expected value of utility subject to the constraint that the total portfolio equal the sum of the individual assets is a straightforward problem readily suited to the Lagrange technique. If λ represents the Langrangean multiplier, the objective function can be written as

$$L = a_o + a_1 \sum_i q_i U_i + a_2 \sum_{ij}\sum q_i q_j M_{ij} + \lambda(\sum_i q_i - 1)$$

Then

$$\frac{\partial L}{\partial q_i} = a_1 U_i + 2a_2 \sum_j q_j M_{ij} + \lambda$$

$$\frac{\partial L}{\partial \lambda} = \sum_i q_i - 1$$

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[11]This is equivalent to the assumption that the utility function is adequately approximated by the terms in the Taylor series expansion which involve only the first two moments of the distribution.

When the first partials are set to zero, they can be represented in matrix notation as $\underline{U} = \underline{MQ}$, where

$$\underline{M} = \begin{bmatrix} 2a_2M_{11} & 2a_2M_{12} & \cdots & 2a_2M_{1N} & 1 \\ 2a_2M_{21} & 2a_2M_{22} & \cdots & 2a_2M_{2N} & 1 \\ \vdots & & & \vdots & \vdots \\ 2a_2M_{N1} & 2a_2M_{N2} & \cdots & 2a_2M_{NN} & 1 \\ 1 & 1 & & 1 & 0 \end{bmatrix}_{N+1xN+1}$$

$$\underline{Q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_N \\ \lambda \end{bmatrix}_{N+1x1}$$

And

$$\underline{U} = \begin{bmatrix} -a_1U_1 \\ -a_1U_2 \\ \vdots \\ -a_1U_N \\ 1 \end{bmatrix}_{N+1x1}$$

By solving for the efficient set q_i $(i = 1, 2, \ldots N)$, we get

$$q_i = \text{Det}(\underline{M}^i)/\text{Det}(\underline{M})$$

where $\underline{M}^i$ indicates the matrix formed by replacing the ith column of $\underline{M}$ with $\underline{U}$ and where $\text{Det}(\underline{M}^i)$ and $\text{Cof}(\underline{M}_{ij})$ indicate the determinant of $\underline{M}^i$ and the cofactor of the i, j element of $\underline{M}$ respectively.

Expanding $\text{Det}(\underline{M}^i)$ on the ith column (which is $\underline{U}$) we get

$$\text{Det}(\underline{M}^i) = a_1U_1\text{Cof}(\underline{M}_{1i}) + a_1U_2\text{Cof}(\underline{M}_{2i}) + \ldots + a_1U_N\text{Cof}(\underline{M}_{Ni})$$

$$+ \text{Cof}(\underline{M}_{N+1,i})$$

By using the last expression, the efficient portfolio can be represented in such a way that the optimum quantity of the ith asset is a linear function of the means of the anticipated returns to all of the assets:

$$(1)\quad q_i = B_{i,N+1} + B_{i1}U_1 + B_{i2}U_2 + \ldots + B_{iN}U_N$$

where

$$B_{ij} = a_1\text{Cof}(\underline{M}_{ji})/\text{Det}(\underline{M}) \quad i = (1 \ldots N+1)$$

Empirical Use

In order to make use of this model in any quantitative way, additional assumptions are required to reduce the number of unknown parameters. Donald Farrar[12] used historical data on rates of return to compute sample moments for large groups of securities and used that information to compare actual with efficient portfolios held by mutual funds under a large number of possible specifications of the utility function parameters.

In the empirical work done on demand for money, the typical use of equation (1) is to support by analogy a macromodel which includes among the regressors one or more interest rates. The most convenient

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[12]Donald Farrar, Investment Decision Under Uncertainty, Englewood Cliffs, New Jersey: Prentice Hall, 1962.

assumptions to limit the unknown parameters are the following: (i) the investor feels the interest rate is no more likely to rise than to fall and thus that

$$U_i = R_i$$

and (ii) the parameters other than U_i in the expression are constant over all observations. Also an assumption must be made which relates the actual amount held to the desired amount and makes a provision for a stochastic disturbance.[13]

Alternative Specifications of Demand Functions

In contrast to the models derived from assumptions regarding consumer behavior and designed to describe the demand function of an individual, the models of this section have been formulated by the authors as aggregate functions. For the most part, the authors make no reference to any specific micromodel. They formulate their models from the start as macromodels and reiterate the common rationale for inclusion of particular variables.

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[13] The partial adjustment model is well suited for this purpose. Letting q_i^* denote the desired quantity of the ith asset, the change in actual quantity from one period to the next is assumed to be proportional to the difference between actual and desired:

$$q_{it} - q_{i,t-1} = k_i(q_i{}^*{}_t - q_{i,t-1})$$

See for example G. Chow, "On the Long-Run and Short-Tun Demand for Money," Journal of Political Economy, Vol. 74 (April, 1966) pp. 111-131; I. Friend, "The Effects of Monetary Policies on Nonmonetary Financial Institutions and Capital Markets," in Commission on Money and Credit, Private Capital Markets, Englewood Cliffs, New Jersey: Prentice Hall, 1963, pp. 165-268; M. Hamburger, "Household Demand for Financial Assets," Econometrica, Vol. 36, No. 1 (January, 1968) pp. 97-118.

The following descriptions are not exhaustive of the models proposed for the demand for money function, but rather represent an arbitrary selection from those models that have attracted considerable attention in economic literature. Their presentation here serves two purposes: In the first place the rationale used in developing these models is typical of that used in almost all empirical monetary economics. Thus a discussion of these models constitutes a general review of a larger body of work than the four or five titles suggest. Secondly, each of the equations discussed will be used in Chapter III as a basis for comparison of results derived by using similar formulations but differently constructed data. The purpose of this comparison will be to show that the macrovariables defined in this study for the purpose of bringing the analysis with the framework of linear aggregation are comparable with the variables commonly used.

Friedman

Friedman does not present the exact specifications of his empirical work and thus it is difficult to find in his voluminous work a specific, concrete formulation of an empirically estimated demand for money function. His contributions to monetary history and the data required for regression analysis are referenced in nearly every important empirical work in the field, yet he has made scant use of regression analysis. Thus we do not attempt in the next chapter to reproduce a particular equation of Friedman but it seems worthwhile to include in this review

one of his early important articles, "The Demand for Money: Some Theoretical and Empirical Results."[14]

Friedman produced this article in an attempt to explain the seemingly contradictory evidence on income velocity from secular and cyclical data. Secularly, changes in the real stock of money per capita are positively correlated with changes in real per capita income. Using measurements at the bottoms of troughs in twenty cycles between 1870 and 1954 Friedman found the following result: "A 1 percent increase in real income per capita has . . . been associated with a 1.8 percent increase in real cash balances per capita and hence with a 0.8 percent decrease in income velocity."[15] This decrease in velocity over the long run is in direct contradiction to the pattern it follows within cycles. Even though the real stock of money expands and contracts in conformance with the short-run cycles, the changes are not enough to leave velocity constant. While income increases 1 percent, real money increases only about a fifth of 1 percent so that velocity tends to rise during cyclical expansions and fall during cyclical contractions.

Friedman's explanation of this phenomenon represents one of the early treatments of money as a durable consumer good yielding a flow of services proportional to the stock. However, his preoccupation with the macrovariable velocity results in the development of a macromodel rather than a micromodel, even though Friedman borrows heavily from the language of microeconomics. The first step is an application of the

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[14] Milton Friedman, "The Demand for Money: Some Theoretical and Empirical Results," The Journal of Political Economy, Vol. 67 (August, 1959), pp. 327-351.

[15] Ibid., pp. 328-329.

permanent income hypothesis which implies that the quantity of money demanded, like the quantity of other goods, is adapted to permanent rather than current income. Under this hypothesis replacing income with permanent income in the velocity formula would provide a more accurate reflection of demand for money, and since permanent income is by construction more stable than current income, the new permanent income velocity is a more stable series than the straight income velocity calculations. In fact Friedman finds that although this permanent income velocity seems to conform positively to the cycles, it would take only small changes in the price index used in reducing income to real terms to convert this positive conformance to the negative conformance implied by the secular results. Thus Friedman proposes that not only income but also the price level has a corresponding "permanent" series, and if velocity were calculated by using permanent income adjusted by this permanent price index, it would yield the same results for both cyclical and secular data.

Friedman's test of this theory is extremely roundabout. First he measures the secular velocity using permanent income and prices. Then he computes cyclical velocities for each cycle in his series by working backwards from secular or permanent velocity to the standard unadjusted velocity formula. He compares these calculated values of velocity with observed values and finds a high correspondence.[16]

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[16]It comes as no surprise but should be pointed out that the money variable which Friedman used throughout the entire analysis includes time deposits as well as demand deposits and cash.

Latane

Like Friedman, H. A. Latane uses some informal empirical procedures in his analysis but he also includes a regression equation which is used in our Chapter IV. In "Cash Balances and the Interest Rate--A Pragmatic Approach,"[17] Latane proposes four hypothetical aggregative equations involving two variables, the money supply as a proportion of income, M/Y, and the interest rate, R. He rejects two of these specifications without the use of regression analysis and retains two closely related specifications because he was unwilling to prespecify one of the variables as independent and the other dependent.

The first equation Latane considers is the crude Cambridge version of the quantity theory of money:

$$M/Y = K$$

He rejects this outright, as do most economists now, as an inaccurate description of reality. "It is apparent from the data for the past 33 years . . . that M/Y is much more stable than either M or Y, but that, even so, it is subject to wide variations."[18]

Latane similarly rejects a second specification

$$M = B_0 + B_1(1/R) + B_2Y$$

as implying relationships which are readily refuted by the data.

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[17]H. A. Latane, "Cash Balances and the Interest Rate--A Pragmatic Approach," Review of Economic and Statistics, (November, 1954), pp. 456-460.

[18]Ibid., p. 458.

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Then he derives two regression lines

$$M/Y = .0074/R + .11$$

and

$$1/R = 95.4\ M/Y - 2.44$$

by using the ordinary least squares method on the two variables M/Y and 1/R treating first one and then the other as the dependent variable. This technique is not very sophisticated by current standards but by using a line which fell between his two estimates, Latane was able to predict other observations to his satisfaction. "The fit seems to indicate that the structural relations established from the 1919-1952 data had some significance both over the longer period and currently."[19]

Chow

The most distinguishing feature of Gregory Chow's "On the Long-Run and Short-Run Demand for Money"[20] is the distinction he makes between long-run or equilibrium demand for money and the short-run or current demand. Much of his analysis of the long-run is based on earlier work in the area of consumer durables. "The prices of services from durable goods depend on the prices of the goods and the interest rate; the price of services from money depends on the rate of interest. The relevant income variable will be some measure of permanent income provided that

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[19] Ibid., p. 459.

[20] Gregory C. Chow, "On the Long-Run and Short-Run Demand for Money," The Journal of Political Economy, Vol. 74 (April, 1966), pp. 111-131.

the economic unit has a fairly long horizon in making its decisions."[21] Thus Chow's first empirical work is with the equation

$$(1)\quad M_t = a_o + a_1 YP_t + a_2 R_t + e_t$$

where YP_t is permanent income at time t and e_t is a stochastic disturbance. He estimates the parameters of this regression with both untransformed data and the natural logarithms of all variables and finds long-run income and interest elasticities of approximately 1 and -.75 respectively. Then, in order to show that permanent income is a better explanatory variable than either wealth or current income, he reestimates the equation twice including first wealth and then income with the other variables previously included. His conclusion is that the permanent income is better than either wealth or current income.[22]

The adjustment mechanism which he uses has two components: one is equivalent to the stock adjustment model and it is this part which Chow refers to as the long-run or equilibrium component; the other, short-run component is simply a constant proportion of the change in the relevant constraint--in this instance permanent income. To test only the relevance of the first component Chow estimates

$$(2)\quad M_t = \xi_o + \xi_1 YP_t + \xi_2 R_t + \xi_3 M_{t-1} + e_t$$

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[21] Ibid., p. 113.

[22] An important consideration at this point which Chow ignores is that if his adjustment specification in his short-run analysis is correct then equation (1) has a specification error.

The equation is derived as an application of the partial adjustment model by substituting

$$M_t - M_{t-1} = k(M_t^* - M_{t-1})$$

into the equation:

$$M_t^* = a_o + a_1 YP_t + a_2 R_t + e_t$$

k in this expression is the partial adjustment coefficient, and M_t^* is desired money balance. All of the estimates are in accordance with Chow's expectations and he takes assurance from the fact that when the logarithms of observations are used, the estimate of the coefficient of adjustment, k, is about .5 and this is consistent with his estimates for ξ_1 and ξ_2 of approximately half the magnitude estimated for a_1 and a_2 in equation (1). If equation (1) is an accurate estimate of desired money, then the partial adjustment relation yields

$$M_t = kM_t^* + (1-k)M_{t-1}$$

$$M_t = k(a_o + a_1 YP_t + a_2 R_t) + (1-k)M_{t-1} + (1+k)e_t$$

Thus, if k were equal to .5, ξ_1 and ξ_2 should be half of a_1 and a_2 since they are related as:

$$\xi_1 = ka_1$$

$$\xi_2 = ka_2$$

The complete specification of Chow's model includes current income among the regressors:

$$(3) \quad M_t = B_o + B_1 YP_t + B_2 R_t + B_3 M_{t-1} + B_4 Y_t + e_t$$

Y enters as part of the expression Y-kYP which represents saving if consumption is assumed to be proportional to permanent income. On this assumption and with the further assumption that a constant portion of new saving is held as money, Chow arrives at specification (3). Clearly, the coefficients are not the same in (3) as in (2) or (1). For one thing, the coefficient of permanent income must now be interpreted as having a component with -k as a factor and we thus expect B_1 in this specification to be less than in the other two specifications. Again Chow's estimates conform to his expectations.

Teigen

The model proposed by Ronald Teigen in "Demand and Supply Functions for Money in the United States: Some Structural Estimates"[23] is a three equation structural model with three endogenous variables--the money stock, the short-term interest rate, and income. Unfortunately a thorough analysis of Teigen's complete model is beyond the scope of the present research and estimation of only Teigen's demand for money equation leaves open the possibility of simultaneous equations bias. However, since the same criticism could be levelled at all of the other demand equations investigated in this work when doubt is cast on the exogeneity of money supply, we include here a description of Teigen's demand equation and estimate it in Chapter III for comparison with other results. Teigen's theoretical development of the demand for money is very similar to

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[23] Ronald Teigen, "Demand and Supply Function for Money in the United States: Some Structural Estimates," Econometrica, Vol. 32, No. 4 (October, 1964), pp. 476-509.

Tobin's transactions demand model.[24] Both make use of the argument that the availability of savings deposits and other equally liquid assets makes money an unlikely choice as a means of holding wealth. Savings deposits are free of risk of capital loss due to interest rate changes and therefore dominate money as a store of wealth. Teigen concludes that " . . . under present institutional arrangements, there should exist only a transactions demand for money."[25]

Relying heavily on Tobin's development of the by now well known square root inventory formula for transactions demand, Teigen first derives the equation for the ith individual

$$(1) \quad M^i = kY^i/2R$$

and then generalizes it to the form

$$(2) \quad M = B_o Y^{B_1} R^{B_2}$$

Rather than follow the usual procedure of estimating this function by taking the logarithms of all the observations, Teigen jumps to the *ad hoc* formulation

$$(3) \quad M_t = B_o + B_1 Y_t + B_2 R_t Y_t + B_3 M_{t-1}$$

This is the equation Teigen uses in his structural estimates but fortunately he was also interested in the question of simultaneous equation bias and estimated (3) by single equation least squares. Although he

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[24]James Tobin, "The Interest Elasticity of Transactions Demand for Cash," *Review of Economics and Statistics*, Vol. 38 (August, 1956), pp. 241-247.

[25]*Ibid.*, p. 483.

does find evidence of serious simultaneous equation bias in estimating the supply equation, his single equation estimation of demand does not differ very much from the structural estimation. The ratios of structural to single equation elasticity estimates are all just slightly greater than one.

Laidler

"The Rate of Interest and the Demand for Money--Some Empirical Evidence"[25] contains eight variations of the basic equation

$$M_t = B_o + B_1 Y_t + B_2 R_t$$

For each of his two definitions of M, one including time deposits and the other including only demand deposits and currency, Laidler estimates the above relationship for both a long (the yield on twenty-year bonds) and a short (the yield on 4-6 month commercial paper) rate of interest. His income variable is real per capita permanent income generously provided by Friedman. All variables are transformed to natural logarithms and regressions are run on these logarithms and on their first differences. Thus Laidler provides a wide variety of the possible combinations for regressions of money on income and interest rate.

In this article Laidler makes no pretense of deriving his model. He simply estimates a regression equation in which money demand is a function of income and an interest rate, and assumes that permanent income works better than other income variables and that the choice of

[25]David Laidler, "The Rate of Interest and the Demand for Money--Some Empirical Evidence," Journal of Political Economy, Vol. 74 (December, 1966), pp. 543-555.

particular short rate and long rate indices is arbitrary and probably of little importance.

Laidler's major conclusions are that there is a stable relationship between the demand for money and the rate of interest and that the short rate performs better than the long one.

CHAPTER III

THE AGGREGATION PROBLEM

The overwhelming majority of empirical studies of demand for money (including all of those reported in the last chapter) use aggregate data for the nation as a whole. There may be several reasons for this but undoubtedly one of the most important is the ready availability of suitable data, especially since the publication of the monumental *A Monetary History of the United States* by Friedman and Schwartz.[1] Another factor which may be of equal importance is the nature of the historical development of monetary theory. At the time when utility theory and other microeconomic concepts were developing, the monetary sector was still being represented by equations such as $MV = PT$. The emphasis of this equation is entirely on aggregates to the complete exclusion of concepts such as a single individual's demand for a particular asset. This conception of the money function as a relationship between aggregates has persisted in current empirical work despite the change in emphasis that monetary theory assumed beginning with Hicks and Keynes.

In defense of the aggregation approach, it must be pointed out that it is still possible to set up monetary equations which right from the

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[1]Milton Friedman and Anna J. Schwartz, *A Monetary History of the United States 1867-1960*. National Bureau of Economic Research, Studies in Business Cycles, No. 12., Princeton, N.J.: Princeton University Press, 1963.

start are specified as macrorelations. This simply requires an assumption that aggregate variables are directly related. However, even in this case it is reasonable to want to go behind the macrorelations to investigate their implications for the corresponding microrelation.

Grunfeld and Griliches have argued that aggregation may actually be beneficial. They argue " . . . that in practice we do not know enough about microbehavior to be able to specify microequations perfectly. Hence, empirically estimated microrelations, whether those of individual consumers or producers, should not be assumed to be perfectly specified either in an economic sense or in a statistical sense. Aggregation of economic variables can, in fact frequently does, reduce these specification errors."[2] Thus, there may be an "aggregation gain" in addition to whatever aggregation error may be present.

The question of whether the overall result of the aggregation is a gain or a loss must be answered in terms of the goal of the research being attempted. If the purpose of the research were to determine information regarding specific microparameters, it would take more than a sweeping generalization about reduced specification error to draw appropriate conclusions from estimates drawn from aggregate data. On the other hand, if the primary research goal were a set of estimates which best predicted the value of a particular aggregate, then it is entirely possible that estimates based on the aggregates would perform better than those based on the disaggregated data.

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[2]Yehuda Grunfeld and Zvi Griliches, "Is Aggregation Necessarily Bad?," The Review of Economics and Statistics, Vol. 42 (February, 1960), p. 1.

There are alternatives, of course, to the two positions discussed. In deciding against the use of observations on single individuals, whether for theoretical reasons or because of data restraints, we do not necessarily have to accept complete aggregation as the alternative. A great deal of data is available for much smaller cross-sectional units than the total United States and, in the particular case of the monetary variables required in this study, data are potentially available by individual bank and publicly available both by states and by Federal Reserve districts.

The question then arises as to what level of aggregation is best. Of course, the answer depends as before on the criteria used for judging but in any event the problems caused by aggregation and the properties of estimates based on aggregated data should be explored. The literature on this subject is meager. There are some early discussions in a series of articles in Econometrica[3] but the first and still the most important systematic treatment is H. Theil's Linear Aggregation of Economic Relations.[4]

Theil's principal contribution is in defining the links between micro and macrorelations. He makes explicit the sources of aggregation bias under several sets of assumptions but his analysis is limited almost entirely to the case of linear aggregation of linear microrelations.

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[3]L. R. Klein, "Macroeconomics and the Theory of Rational Behavior," Econometrica, Vol. 14 (1946), pp. 93-108; K. May, "The Aggregation Problem for a One Industry Model," ibid., pp. 285-298; Shou Shan Pu, "A Note on Macroeconomics," ibid., pp. 299-302; L. R. Klein, "Remarks on the Theory of Aggregation," ibid., pp. 303-312.

[4]Henri Theil, Linear Aggregation of Economic Relations, Amsterdam: North Holland Publishing Company, 1954.

That is, the analysis is based on microrelations which are assumed to be linear. This may be regarded either as an important special case or as an adequate approximation for small changes in the variables. The macrorelations are also assumed to be linear and the variables used in the macrorelations are assumed to be linear aggregates of those used in the microrelations. These conditions are clearly restrictive but Theil's analysis is important both for the special cases which he does cover and for the theoretical framework he provides as point of departure in further analysis.

Consistent Aggregation

Actually there are two distinct problems which are often lumped together under the term "aggregation bias." The first has to do with the consistency of the macrorelation with the microrelations and the second concerns the estimation of the parameters of the macrorelation. In this section we will examine the possibilities of consistency both for aggregation over individuals and for aggregation of various assets.[5]

Aggregation over Individuals

Suppose we assume that the ith individual's demand for demand deposits is a function of income and interest rates and is adequately

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[5]The aggregation discussion is stated in terms of "individuals to conform with existing literature, but "microunit" could be substituted for individual to create the obvious generalization. The empirical work in following chapters deals with states rather than individuals as the basic microunit.

represented by the following equation:[6]

$$(1) \quad A^i_D = B^i_o + B^i_Y Y^i + B^i_T R^i_T + B^i_S R^i_S$$

The quantity of an asset is represented by A. Subscripts D, T, and S indicate demand deposits, time deposits, and savings and loan shares, respectively. Superscript i indicates ith individual or microunit. In addition, suppose that in place of this microequation an analagous macroequation is proposed which, it is hoped, will not contradict the underlying microequations. Theil labels this "the problem of good aggregation" and states the "Rule of Perfection for a Macroequation."

> There is no contradiction between the macroequation and the microequation corresponding to it, for whatever values and changes assumed by the microvariables and at whatever point or period of time.[7]

Aggregation which satisfies this rule is what we have referred to as consistent aggregation.

Convenient Macrovariables

Quantity variables such as the number of dollars held as demand deposits and the amount of income have convenient macrovariables defined as the simple summations of their corresponding microvariables. Dropping the superscript from a microvariable to denote the corresponding

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[6]The stochastic disturbance is dropped from this expression as an alternative to stating all of the arguments which follow in terms of the expected value.

[7]*Ibid.*, p. 140.

macrovariable we define the aggregates

$$(2)\qquad A_D = \sum_i A_D^i \qquad Y = \sum_i Y^i$$

Wealth is another quantity variable which is frequently aggregated in the same way as A_a^i and Y.

Interest rates are ratios rather than quantities and their treatment requires special attention. The macrovariable for interest rate used in empirical work is certainly not a simple summation of the rates applicable to individuals. A simple average of individual rates might realistically be proposed but the more common approach is to define the macrovariable as a weighted average of the microvariables with weights equal to the proportion invested.

$$(3)\qquad R_a = \sum_i w_a^i R_a^i \qquad\qquad w_a^i = A_a^i / \sum_i A_a^i$$

Since w_a^i is defined as the proportion of asset A_a, earning R_a^i this formula is obviously equivalent to dividing total interest paid by total quantity invested. The convenience of this ratio in constructing data undoubtedly explains its wide usage.

<u>Macroequation</u>

In order to investigate the problem of consistency we now postulate a macroequation based on the above macrovariables

$$(4)\qquad A_D = B_o + B_Y Y + B_T R_T + B_S R_S$$

Consistency, or the rule of perfection, requires that A_D, as defined by macroequation 4, be equal to the summation of A_D^i in the microequations for all values and changes of microvariables.

This problem has received considerable attention but from a slightly different point of view. Whereas we have begun with the convenient definitions of macrovariables most often used in empirical work and asked what consistency implies for the parameters, the usual approach is to begin with microvariables and sometimes a relationship between micro and macroparameters and then attempt to develop macrovariables with some set of desirable properties. This latter approach is more often called an index number problem than an aggregation problem and some aspects of the problem have received considerable attention. However, since monetary empiricists have for the most part chosen convenient macrovariables rather than theoretically elegant index numbers it seems appropriate to approach the problem as one of consistency between micro and macrorelationships starting with the variables and relationships actually in use in empirical studies.

With the macrovariables defined by equations (2) and (3) and relationships defined in (1) and (4) it is easy to show that consistency implies the following relationships among parameters:

(5) $B_o = \sum B_o^i$

(6) $B_Y = B_Y^1 = B_Y^2 = \ldots = B_Y^N$

(7) $B_a = B_a^1/w_a^1 = B_a^2/w_a^2 = \ldots = B_a^N/w_a^N \qquad a = T,S$

In a later section we will develop and prove a general case but the above results are illustrative. They follow from the requirement in the rule of perfection that any change in a microvariable has equal effect

on D whether through the microequation (1) or the macroequation (4). Assuming, for example, a change in microvariable R_T^i applicable to individual i's demand for A_D^i, we equate the changes resulting in (1) and (4)

$$(8)\quad \Delta A_D^i = B_T^i \Delta R_T^i = \Delta A_D = B_{RT}\Delta R_T = B_{RT} w_{RT}^i \Delta R_T^i$$

$$B_T^i \Delta R_T^i = B_T w_T^i \Delta R_T^i$$

$$(9)\quad B_T = B_T^i / w_T^i$$

Since asset T and individual i are simply illustrative examples, it is clear that equation (7) is a straightforward generalization of equation (9). It should also be clear that equation (6) could be explained in the same way by substituting Y for R in equation (8) and letting w^i equal 1 for all individuals since each income receives equal weight in the macrovariable Y. Before generalizing these results we will look at two other special cases, aggregation of assets and aggregation of interest rates.

Aggregation of Assets

We rewrite here the equations proposed in the last section which describe the demand for two aggregates, A_D and A_T:

$$(10)\quad A_D = B_{oD} + B_{YD}Y + B_{TD}R_T + B_{SD}R_S$$

$$A_T = B_{oT} + B_{YT}Y + B_{TT}R_T + B_{ST}R_S$$

Suppose now that further aggregation is desired and, as is typical in monetary analysis, a macroequation is desired which describes the demand for a combination of the already aggregated variables, for example, $A_D + A_T$. We write this proposed aggregated equation as linear in the set of macrovariables already used and let * stand for D + T:

$$(11) \quad A_* = B_{o*} + B_{Y*}Y + B_{T*}R_T + B_{S*}R_S$$

The rule of perfection for equation (11) requires that it be consistent for changes in variables with the summation of A_D and A_T described in equation (10). This summation is written

$$(12) \quad \sum_{a=D,T} A_a = \sum_{a=D,T} (B_{oa} + B_{Ya}Y + B_{Ta}R_T + B_{Sa}R_S)$$

It is very easy to show that the rule of perfection imposes the following constraints on the parameters:

$$(13) \quad \sum_{a=D,T} B_{ja} = B_{j*} \qquad\qquad j = O,Y,T,S$$

That is, each parameter in the aggregate equation must equal the summation of the corresponding parameters in the microequations.

Aggregating Interest Rate Variables

Except for the two different interest rate terms, expression (10) is very similar to those actually used in formulating aggregate demand for money functions. The one further step required is a reduction of the number of interest rate variables. Suppose, for example, that the following macroequation is proposed:

$$(14) \quad A_D = B_o + B_YY + B_RR$$

The two interest rates in (10) are replaced by a single new interest rate variable resulting in additional constraints on the parameters if the rule of perfection is to be met.

Two definitions will be considered for the new macrovariable R. The first is a simple extension of the case already described in equation (3). There the aggregation involved individuals whereas here it involves other large aggregates but in both cases the new variable is a weighted average with weights equal to quantities invested. Let R in (14) be defined:

$$(15) \quad R = (\sum_{a=T,S} A_a R_a)/\sum_{a=T,S} A_a$$

or equivalently

$$R = \sum_{a=T,S} v_a R_a$$

where

$$v_a = A_a/\sum_{a=T,S} A_a$$

With this definition of r in (14) and consistency assumed between (14) and (10), the following constraints must be met by the parameters

$$(16) \quad B_o = B_{oD}$$

$$B_Y = B_{YD}$$

$$B_R = B_{aD}/v_a \qquad a = T,S$$

These results are perfectly analagous to those discussed on page 35.

Alternatively, suppose R in (12) is not a weighted average of rates included in (10) but rather some completely unrelated series such as the rate on four- to six-month prime commercial paper or the rate on long-term government bonds. Each of these has been used in demand for money macroequations and been accepted as an important explanatory variable. However, there is no set of constraints on parameters which would allow a specification with either of these among the macrovariables to meet the requirement of the rule of perfection. That is, there is no possibility for perfectly consistent aggregation of the microrelation (1) with macrorelation (14) if R is not related in a definite way to the microvariables in (1).

The General Case

All of the above examples are special cases of the generalization which we develop and prove in this section.[8] We have assumed that there are a number of microrelations which describe for each individual the quantity which he demands of each asset as a linear function of several microparameters.

Generalizing our notation we let X^i be the 1 x K^i row vector of exogenous parameters facing the ith individual (i = 1...N) in each of his demand equations. Let B_a^i be the K x 1 column vector of parameters applicable in the demand for asset A_a. Subscript a in this study will be limited to identifying the assets D, T, and S but in general we can

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[8]The generalization of this section could be derived as an extension of results provided by Theil, ibid., pp. 140-142. Theil did not bother to generalize his results to include weighted rather than simple aggregation of microvariables. Also, the simplification which results from introduction of matrix notation warrants developing the entire proof.

let a = 1,...H, where H is the number of assets under consideration. Then, corresponding to microequation (4), we write

$$(17)\quad A_a^i = X^i B_a^i$$

X^i is written without subscript to indicate that the same variables appear in the demand equations for each asset. The system of microequations which describes the demand for the ath asset by all individuals may now be written.

$$(18)\quad \underset{N\times 1}{\begin{bmatrix} A_a^1 \\ A_a^2 \\ \cdot \\ \cdot \\ \cdot \\ A_a^N \end{bmatrix}} \quad \underset{N\times \Sigma K^i}{\begin{bmatrix} X^1 & 0 & \dots & 0 \\ 0 & X^2 & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & & \cdot\ 0 \\ 0 & \dots & 0 & X^N \end{bmatrix}} \quad \underset{\Sigma K^i \times 1}{\begin{bmatrix} B_a^1 \\ B_a^2 \\ \cdot \\ \cdot \\ \cdot \\ B_a^N \end{bmatrix}}$$

The off-diagonal 0's in the matrix whose diagonal elements are the vectors of exogenous microvariables X^i (i = 1,...N) represent appropriately dimensioned row vectors of zeros. For example, all off-diagonal elements in the ith column are $1 \times K^i$ vectors of zeros to conform with the $1 \times K^i$ dimension of X^i.

Let $A = \begin{bmatrix} A_a^1 \\ A_a^2 \\ \cdot \\ \cdot \\ \cdot \\ A_a^N \end{bmatrix}$, $X = \begin{bmatrix} X^1 & 0 & \cdots & 0 \\ 0 & X^1 & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & & \cdot\ 0 \\ 0 & \cdots & 0 & X^N \end{bmatrix}$, $B_a = \begin{bmatrix} B_a^1 \\ B_a^2 \\ \cdot \\ \cdot \\ \cdot \\ B_a^N \end{bmatrix}$

Now expression (18) may be written

$$A_a = XB_a$$

Again, this expression represents the system of equations which describe each individual's demand for the ath asset. We can expand the system one more step to include each individual's demand for each asset by writing

$$(19) \quad A = XB$$

$$A = \underset{N\text{x}H}{[A_1\ A_2\ \ldots\ A_H]}\ ,\ B = \underset{K^i\text{x}H}{[B_1\ B_2\ \ldots B_H]}$$

Expression (19) contains equations of N individuals for H assets or NH total equations.

Macrovariables

Let A_*, X_*, and B_* denote macrovariables which are assumed to be related according to (20)

$$(20) \quad A_* = X_* B_*$$

Each macrovariable is a linear combination of some set of microvariables. In the case of A we assume specifically that this macrovariable is a simple summation over all i and some subset of the possible values of a. Without loss of generality, we let the included assets be in the first H* columns of matrix A. Then by using a 1 x N row vector of 1's and a 1xH column vector with 1's in the first H* elements and zeros in the remaining $H\text{-}H_*$ elements,

$$(21) \quad i_N = [1\ 1\ 1\ \ldots\ 1]_{1 \times N}$$

$$i_{H*} = [1\ \ldots\ 1_{H*} 0\ \ldots\ ]_{1xH}$$

we can write macrovariable A* as

$$(22) \quad A* = i_N A i'_{H*}$$

The K_* elements of the vector of exogenous macrovariables X_* are more generally defined as linear combinations of their corresponding microvariables. Note that, as in the case when a single interest rate, R*, is the macrovariable under consideration, we include the possibility of more than one "corresponding" microvariable. A macrovariable may be a weighted sum over individuals of more than one of the microvariables in each individual's equation. That is, the jth element of X_* need not necessarily be the weighted sum over N individuals of only the jth element in each individual's vector of microvariables. It might also include, for example, element j + 1 or some other element from each individual's vector.

The diagonal elements of X can be put in row vector form by premultiplying X by the vector i_N defined above

$$(23) \quad i_N X = [X^1\ X^2\ \ldots\ X^N]_{1x\sum_i K^i}$$

Then by suitably defining a matrix of weights W_* the macrovariables X_* may be derived as

$$(24) \quad X_* = i_N X W_*$$

Each column of the matrix W_* contains the weights used in forming a single macrovariable. To form the macrovariable for income Y_* as a simple sum of all Y^i, W_* would contain column vector W_{*_y} with elements w_{jy} where

$w_{jy} = 1$ if the jth element of $i_N X$ is an income variable

$= 0$ otherwise

In general, each column of W_* may be thought of as a column of column vectors say

$$W_{*z} = \begin{bmatrix} W_z^1 \\ W_z^2 \\ \cdot \\ \cdot \\ \cdot \\ W_z^N \end{bmatrix}_{\Sigma K^i x1}$$

where W_z^i has K^i elements corresponding to the elements in X^i. The non-zero elements are the weights attached to the corresponding micro-variables in forming the macrovariable X_{*z}, the zth element of X.

With the notation established it is now exceedingly simple to discover the conditions under which the aggregate of the dependent microvariables is consistent with the macroequation (20), that is, under what conditions (22) is consistent with (20). Rewriting these expressions we have

$$A_* = X_* B_* \text{ and } A_* = i_N A i_{H*}$$

Substituting for X_* and A using (24) and (19), we have

$$(25) \quad i_N X W_* B_* = i_N X B i_{H*}$$

Then by differentiating both sides with respect to the vector of microvariables $i_N X$ we can find the necessary and sufficient condition for consistency:

$$(26) \quad W_* B_* = B i_{H*}$$

In most instances, each microvariable is used in computing only one macrovariable. When this is the case, each row of W_* has only one nonzero element. It may also be that each macrovariable is a simple summation over individuals of a single corresponding microvariable. Then W_* may be written as a column of identity matrices.

$$W_* = \begin{bmatrix} I_K \\ I_K \\ \cdot \\ \cdot \\ \cdot \\ I_K \end{bmatrix}_{NKxK} \qquad K=K^i \quad i=1,2, \ldots N$$

If in addition A_* is a summation over individuals of only a single asset, the ath (i.e., if i_{H*} has a single non-zero element as the ath element) then we have the well known condition[9]

$$B_{*j} = B_j^1 = B_j^2 \ldots = B_j^N \qquad j = 1 \ldots K$$

It is clear that the consistency conditions stated in (26) are much too stringent to be expected to hold in general for equations describing the demand for liquid assets. Even in the simplest case in which the macro and microrelations have the same number of variables and each macrovariable is a weighted average of corresponding microvariables the consistency condition is unlikely to be fulfilled. As we saw in equations (6) and (7), consistency even in this simple case requires that the partial derivative with respect to any change in interest rate in a microequation be exactly proportional to the weight that interest rate has in the corresponding macrovariable. This same condition holds when the macroequation has but a single interest rate variable which is a weighted sum of all the interest rate variables in each microrelation. In the simple case it is at least conceivable that the condition might hold. If, for example, the weighting was based on quantity held, if each macrorelation was homogeneous of degree one, and if all interest rates changed in the same proportion, then the condition would be fulfilled. In the second case, however, with a single macrovariable

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[9]R. G. D. Allen, Mathematical Economics, 2nd ed., New York: St. Martin's Press, 1966, pp. 694-724. All presents these results for a restricted model in the course of his concise and slightly simplified presentation of the principal contributions of Theil's Linear Aggregation, op. cit.

corresponding to several interest rates in each microequation the consistency condition contradicts basic economic precepts. Letting w^i_{ja} be the weight attached to the jth microvariable in the ith individual's demand for asset a, we write out the rows of (26) which correspond to individual i as:

$$w^i_{ya}\, B_{y*} = B^i_{ya}$$

$$w^i_{R_Ta} B_{R*} = B^i_{R_Ta}$$

$$w^i_{R_Sa} B_{R*} = B^i_{R_Sa}$$

Since each of the weights is a positive number and the ratio of each interest rate parameter to its corresponding weight is equal to B_{R*}, these conditions imply that all partials have the same sign whether they are own first partials or cross partials. This means that the direction of change is the same for every asset when any interest rate changes. Few economists would be willing to accept this *a priori* restriction.

General and Specific Inconsistency

Before looking at the effects of using a macrorelation which is not consistent with the system of microrelations it will be useful to distinguish between general and specific consistency and to define corresponding measures of inconsistency. So far, we have dealt only with general consistency which is a property of a system of equations when all of the microvariables are free to take on any values. In this use, consistency is a property of a model and does not depend on particular values of microvariables.

In later sections, however, our primary interest will be in the use of a model with actual observed values of the microvariables. Thus we are not as concerned about general consistency with maximum degrees of freedom for the variables as with the consistency in particular instances when the microvariables are given and thus in a sense have zero degrees of freedom. It is clearly possible for a system of equations to be consistent for a specific set of values and not be consistent in general, so the distinction is useful.

To develop measures of inconsistency corresponding to these two uses we will consider the conditions of equations (26) in the special case when A_* is a simple sum over individuals of only one type of asset, say asset a. Then i_{H*} contains a single non-zero element and (26) may be written

$$(27)\quad W_* B_* = B_a$$

The most obvious measure of the inconsistency of a system of equations would seem to be the difference between $W_* B_*$ and B_a. However, since these are vectors rather than scalars perhaps a more convenient measure would be the inner product of the difference

$$(28)\quad d'd = \text{measure of general inconsistency}$$

where

$$d = W_* B_* - B_a$$

This measure would be zero if and only if all of the consistency conditions of (27) were met and it would, of course, increase in value the larger the absolute value of any deviation from the consistency conditions.

Of greater practical importance to the purpose of this study is a measure of inconsistency for a specific set of values of the micro-variables X and the macrovariables $i_N XW*$. To establish this measure we will assume that the vector d defined in (28) has at least one non-zero element and ask what effect this has on consistency for a particular set of values. We can rewrite (25) substituting $B_a + d$ for $W_* B*$ to allow for general inconsistency. This gives (again assuming i_{H*} has a single unit element):

$$(29) \quad i_N X(B_a + d) = i_N XB_a$$

Then the amount by which quality (29) fails to be satisfied is a measure of the specific inconsistency. Denoting this measure as L we have

$$(30) \quad L = i_N Xd$$

L is a scalar, the dot product of $i_N X$ and d, and it is obvious that general consistency is not a necessary condition for specific consistency.

Aggregation and Estimation

Most of the implications of estimating a macroequation which is not consistent with a system of microequations have been developed in the literature[10] and some of them are well known. However, with a slight

[10] See especially Theil's Theorem 7 in Linear Aggregation, op. cit., pp. 120-121; and H. A. J. Green, Aggregation in Economic Analysis, Princeton: Princeton University Press, 1964, pp. 99-106.

modification of the matrix notation established above it is possible to simplify considerably the presentation of these results.

Suppose we have T observations on each of the N microequations which we now assume contain a stochastic element e_t^i:

$$(31) \quad A_{at}^i = X_t^i B_a^i + e_t^i$$

Now if we generalize our notation to let A_a, X and e_a represent matrices of all observations on the variables

$$(32) \quad A_a = \begin{bmatrix} A_{a1}^1 \\ A_{a2}^1 \\ \cdot \\ \cdot \\ \cdot \\ A_{aT}^1 \\ A_{a1}^2 \\ \cdot \\ \cdot \\ \cdot \\ A_{aT}^N \end{bmatrix}_{NTx1}, \; e_a = \begin{bmatrix} e_{a1}^1 \\ e_{a2}^1 \\ \cdot \\ \cdot \\ \cdot \\ e_{aT}^1 \\ e_{a1}^2 \\ \cdot \\ \cdot \\ \cdot \\ e_{aT}^N \end{bmatrix}_{NTx1}, \; X^i = \begin{bmatrix} X_1^i \\ X_2^i \\ \cdot \\ \cdot \\ \cdot \\ X_T^i \end{bmatrix}_{TxK}, \; X = \begin{bmatrix} X^1 0 & \ldots & 0 \\ 0\; X^2 & & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & & \cdot\; 0 \\ 0 & & 0 X^N \end{bmatrix}_{NTxNK}$$

We can write the entire system of microequations for all observations[11]

$$(33) \quad A_a = XB_a + e_a$$

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[11]We confine our attention at this point to the case in which the dependent macrovariable is the sum of dependent microvariables describing a single asset--for example, all demand deposits or all time deposits but not both.

Now corresponding to the vector i_N which facilitated the formation of the macrovariables we define the matrix i_T to use in creating the T observations on each macrovariable,

$$(34) \quad i_T = [I_T, I_T, I_T, \dots I_T]_{TxNT}$$

where I_T is a T dimensional identity matrix. Then letting A* be the T x 1 matrix of observations on the dependent macrovariable which we continue to define for each time period as the summation of all the dependent microvariables, A_a^i, we have

$$(35) \quad A_* = i_T A_a = i_T X B_a + i_T e_a$$

Similarly we define the matrix of observations on the macrovariables Z* as

$$(36) \quad Z_* = i_T X W_*$$

and introduce the T x 1 vector of macrodisturbances

$$(37) \quad e_* = i_T e_a$$

Then all observations on the proposed macroequation under condition of general consistency may be written as

$$(38) \quad A_* = i_T X W_* B_* + e_*$$

We now consider the OLS estimates of B_*.[12] Theil's approach is to take the expectation of $\hat{B}_*$

$$(39)\quad E[\hat{B}_*] = E[(Z_*'Z_*)^{-1}Z_*'A_*]$$

or, using (35)

$$(40)\quad E[\hat{B}_*] = (Z_*'Z_*)^{-1}Z_*'i_TXB_a$$

Since $(Z_*'Z_*)^{-1}Z_*'i_TX$ describes the matrix of coefficients we would find by regressing each vector of i_TX on Z_*, expression (40) duplicates results achieved by Theil

> "The parameters estimated are sums of weighted averages of microparameters . . .--the weights being equal to the coefficients of those regression equations which are obtained when the statistical method used for the estimation of the microequation is applied to the linear equations that describe the exogenous microvariables as functions of the exogenous macrovariables . . ."[13]

Another way to approach this result is to rewrite (38) making use of d defined in (28) and allowing for general inconsistency

$$(41)\quad A_* = i_TXW_*B_* + i_TXd + e_*$$

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[12]As Theil points out, the following discussion could very easily be generalized to any estimation procedure which is linear in the dependent variable and unbiased under the condition that the expected value of E_* is zero for all observations. Theil, Linear Aggregation, p. 119. Also see Theil's presentation in "Specification Errors and the Estimation of Economic Relationships," Revue Institute Internationale de Statistique, Vol. 25 (1957), pp. 41-51.

[13]Theil, Linear Aggregation, p. 121.

The vector d in this expression may be thought of as another group of unknown regression parameters and their omission from a regression equation is a common type of specification error. It is well known and very easy to show that application of OLS to only part of an equation leads to parameter estimates which are under general conditions both biased and inconsistent.[14] Looking again at the expected value of the OLS estimate of B_* based only on the macrovariables we can rewrite (39) as

$$(42)\quad E[\hat{B}_*] = E[(Z_*'Z_*)^{-1}Z_*'A_*]$$

Then, using (41), we can write

$$(43)\quad E[\hat{B}_*] = B_* + (Z_*'Z_*)^{-1}Z_*Z_T^{i}Xd$$

This expression clearly separates out that part of B_* which is appropriately defined as aggregation bias. The direction and magnitude of the bias quite clearly depends on both the covariances of the macro and microvariables over the sample period and on the elements of d.

One other approach to the relationship between micro and macroparameters will provide useful information. In our last approach we postulated some true set of macroparameters and then analyzed the properties of estimates of these parameters based on our <u>ex ante</u> definition of the "true" model and "true" parameters. In comparison, we now adopt

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[14]See Jan Kmenta, <u>Elements of Econometrics</u>, New York: Macmillan, 1971, pp. 392-395.

an *ex post* definition of the "macroparameters," a definition which is based on our estimates, and then examine the properties of the "parameters" given the nature of the estimates.[15] Specifically, we define the "macroparameters" to be the expected value of the OLS estimates B_*. Then we write the macrorelation.

$$(44) \quad A_* = Z_* E(\hat{B}_*) + u_*$$

A comparison with (35) yields

$$(45) \quad Z_* E(\hat{B}_*) + u_* = i_T X B_a + i_T e_a$$

Then, using (40), we obtain

$$(46) \quad Z_*(Z_*' Z_*')^{-1} Z_*' i_T X B_a + u_* = i_T X B_a + i_T e_a$$

and since $(Z_*' Z_*)^{-1} Z_*' i_T X$ is a matrix of coefficients obtained in the OLS regression of $I_T X$ on Z we can define

$$(47) \quad i_T \hat{X} = Z_*(Z_*' Z_*)^{-1} Z_*' i_T X$$

so that u_* in (44) can be written as

$$(48) \quad u_* = (i_T X - i_T \hat{X}) B_a + i_T e_a$$

and (44) may be rewritten as

$$(49) \quad A_* = Z_* E(\hat{B}_*) + (i_T X - i_T \hat{X}) B_a + i_T e_a$$

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[15]"Parameters" is used with quotation marks because this use contradicts the generally accepted notion of a parameter.

Under this specification, it is clear that if each vector in $i_T X$ is an exact linear function of Z_* so that $i_T\hat{X} = i_T X$, the macrorelation may be written without the $(i_T X - i_T\hat{X})B_a$ term and the error term is the simple sum of errors in the microrelations.

Another implication of specification (49) is that if B_* in (41) is set equal to $E(B_*)$ then

$$(50) \quad i_T Xd = (i_T X - i_T\hat{X})B_a$$

Aggregation and R^2

In most of the demand for liquid asset studies, R^2 is at least as important to the researcher as are the parameter estimates. Unfortunately, Theil and most other students of the aggregation problem have ignored the goodness of fit aspect of the aggregation problem. Grunfeld and Griliches are important exceptions and their article, "Is Aggregation Necessarily Bad?", was important to many of the results in this section.[16] One of the drawbacks of their presentation is lack of distinction between "residuals" and "disturbances."[17] The usual practice, of course, is to let "disturbance" denote the unknown (and usually random) element in a regression equation and let "residual" be the calculated different between the dependent variable and its predicted value with the prediction based on estimates of parameters.

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[16]Grunfeld and Griliches, op. cit.

[17]For example, ibid., p. 6, they explain a quotation from Theil as meaning "...the residual variance of the macroequation must be larger than the variance of the sum of residuals from the microequations." Theil's statement actually applies to disturbances and sums of disturbances.

Variance of u*

Let us assume that the microdisturbances, e_a, all have finite variances. Since the macrodisturbances u_* are seen in (48) to equal the sum of microdisturbances plus the non-stochastic term $(i_T X - i_T\hat{X})B_a$, the variances of the elements of u_* equal the variances of the elements of $i_T e_a$.

$$\text{(51)} \quad \text{Var}\ [u_*] = \text{Var}\ [(i_T X - i_T \hat{X})B_a + i_T e_a] = \text{Var}\ [i_T e_a]$$

This means that in terms of the variance of the disturbance we do not have an *a priori* reason for choosing the aggregate model over the sum of the micromodels.

$E[u_{*t}^2]$

Perhaps a better indication of the predictive power of the models than the variance, or second moment about the mean, is the second moment of the disturbance about zero. For the sum of the micromodels this measure will be the same as the variance if the elements of e_a are assumed to have zero mean. Letting i_t be the tth row of i_T we can write the sum of microdisturbances at period t as $i_t e_a$. Then

$$\text{(52)} \quad E[i_t e_a - E[i_t e_a]]^2 = \text{Var}\ [i_t e_a]$$

For the macrodisturbance, however, the measure is different. Using (51) we have

$$\text{(53)} \quad E[u_{*t}^2] = E[(i_t X - i_t \hat{X})B_a + i_t e_a]^2$$

$$\text{(54)} \quad = \text{Var}\ [i_t e_a] + [(i_t X - i_t \hat{X})B_a]^2 \geq \text{Var}\ [i_t e_a]$$

It is clear from this expression that $E[u_{*t}^2]$ is greater than or equal to $\mathrm{Var}\ [i_t e_a]$ so that in general the second moment about zero of the macrodisturbance is larger than the variance of the sum of the microdisturbances.

Two conditions are immediately obvious which change the weak inequality in (54) to an equality: (1). If $i_t X$ is a linear function of Z_{*t} so that $i_t\hat{X}$ equals $i_t X$: (2). If $W_* B_*$ equals B_a so that d is a vector of zeros by (50) we have

$$(55) \quad i_t Xd = (i_t X - i_t \hat{X})B_a = 0$$

While it is true that these are only sufficient and not necessary conditions for the equality, it is also true that they are not apt to be met in the demand for liquid asset specifications under consideration. Thus on the basis of second moments about zero the macrorelation suffers some disadvantage.

Correlation Coefficients

R^2 can be represented as 1 minus the ratio of the sample second moment about zero of the residuals to the sample second moment about the mean of the dependent variable. Thus for the macroequation we have

$$(56) \quad R_*^2 = 1-(S_{\hat{u}*}/S_{A*})$$

where $S_{\hat{u}*}$ and S_{A*} represent sample second moments about the mean of the calculated residuals $\hat{u}_*$ and dependent variables A_*. Then we follow Grunfeld and Griliches in defining a "composite R^2" which measures the percentage of the aggregate dependent variable which is explained by summing the predicted values of the dependent variables from each microrelation. We do this by finding the residuals for each microequation and adding together all the residuals for a given year to form a composite residual for that year. The vector of these composite residuals may be written $\hat{e}_+ = i_T\hat{e}$ and we let $S_{\hat{e}+}$ be the second moment of these residuals about zero. Corresponding to these composite disturbances we have a composite dependent variable which at each time period is calculated as the sum of all the dependent microvariables for that period. Since this is exactly the same as the definition of the dependent macrovariable we can write second moments about the mean for each of them as S_{A*}. Now we define a composite R_+^2 as

$$(57) \quad R_+^2 = 1 - (S_{\hat{e}+}/S_{A*})$$

A comparison of R_*^2 and R_+^2 shows that the relationship between these two measures of goodness of fit depends on the relationship between $S_{\hat{u}*}$ and $S_{\hat{e}+}$.

In light of the result implied by (54) that $S_{u*} \geq S_{e+}$ it is tempting to think that perhaps $R_+^2 \geq R_*^2$. Of course, this is not the case. The moments in R^2 are moments of residuals not of disturbances and when that distinction is kept in mind it is not surprising to find that R_*^2 <u>can be</u> greater than R_+^2. Nevertheless, it is well known that under certain assumptions commonly made in model specifications $S_{\hat{u}*}$ and $S_{\hat{e}+}$ when adjusted for degrees of freedom are unbiased estimates of S_{u*} and S_{e+} so that $E[S_{\hat{u}*}] \geq E[S_{\hat{e}+}]$ and on this basis we might expect the composite correlation coefficient R_+^2 to be greater than R_*^2. In two separate cases, however, Grunfeld and Griliches found that the aggregate correlation coefficient was at least as large as the composite correlation coefficient which led them to the conclusion that "...aggregation is not necessarily bad if one is interested in the aggregates."[18]

In view of the importance attached to the R^2 statistic in the literature, this conclusion of Grunfeld and Griliches deserves further attention. In the empirical studies of demand for liquid assets the coefficient of determination has probably been used to justify more conclusions than any other single statistic. Almost every researcher in the monetary field has used R^2 in his defense of selecting one reported equation specification over another, to say nothing of the perhaps thousands more specifications which are never reported. Friedman carries it one step further. He has used R^2 to define money! His wording is a little less direct but nonetheless he uses R^2 to select the correct dependent variable in the demand for money equation. "The criterion was that the correlation between the total called money and national income be higher than between each of the individual components of the total and national income."[19]

Specification Error in Microrelations

Before calculating and comparing in the next chapter the actual values of R_*^2 for the macroequation and the analagous microequation composite coefficient of determination R_+^2, we will generalize a rationale given by Grunfeld and Griliches to explain their experience in consistently finding $R_*^2 > R_+^2$. We have assumed throughout the previous

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[18]*Ibid.*, p. 10.

[19]Friedman and Schwartz, *Monetary History*, p. 177.

analysis that the microrelations were exactly and correctly specified. This assumption is useful under many circumstances but it clearly is impossible to expect it to always be true in practice. Now if the microrelations are not specified correctly then their residuals contain a term which is a result of the specification error, and the second moments which we calculate from the residuals will be larger on the average than the actual second moment of the disturbances in the true model. Under these conditions, it is impossible to specify a priori that $E[S_{\hat{u}*}] \geq E[S_{\hat{e}+}]$ because the relationship depends not only on the effect of aggregation but on the effect of the specification error in both the microrelations and in the macrorelations.

Suppose that instead of estimating with the true model A = XB + e we use the incorrect model A = VB + disturbance. It will be important to the later analysis that V is defined similarly to X as a block diagonal matrix of exogenous variables. Now if we estimate B by OLS we have:

$$(58) \quad \hat{B} = (V'V)^{-1}V'A$$

Then if we use these estimates of B to calculate residuals $\hat{e}$ we get

$$(59) \quad \hat{e} = A - V\hat{B}$$
$$= e + XB - V\hat{B}$$
$$= (I - V(V'V)^{-1}V')(XB + e)$$

and as long as V is non-stochastic and all of the true disturbances have zero mean we can write the expected value of the sum of squared

residuals:

$$(60) \quad E(\hat{e}'\hat{e}) = E(e'M_V e) + B'X'M_V XB$$

where

$$M_V = (I - V(V'V)^{-1}V')$$

Similarly we find the expected value of the sum of squared composite residuals

$$(61) \quad E(\hat{e}'i_T'i_T\hat{e}) = E(e'M_V i_T' i_T M_V e) + B'X'M_V i_T' i_T M_V XB$$

Equation (61) is the expression we wish to compare with the sum of squared residuals in the macroequation $A_* = V_* B_* + u_*$. Here V_* is assumed to be a T x K matrix of observations on a set of exogenous macrovariables. If we estimate B_* by OLS we have a situation very similar to one just presented and we can write

$$(62) \quad \begin{aligned} \hat{u}_* &= i_T A - V_* \hat{B}_* \\ &= i_T e + i_T XB - V_* \hat{B}_* \\ &= M_{V_*}(i_T XB + i_T e) \end{aligned}$$

M_{V*} is defined similarly to M_V with the substitution of V_* for V and appropriate changes made in the dimension of the identity matrix.

Now we can write the expected value of the sum of squared residuals of the macroequation:

$$(63) \quad E[\hat{u}'\hat{u}] = e'i_T' M_{V*} M_{V*} i_T e + B'X' i_T M_{V*} M_{V*} i_T XB$$

In comparing the expressions in (61) and (63) it will simplify the analysis a great deal if we make a restrictive assumption regarding the distribution of the microdisturbance e. Specifically, we assume that

(64) $E[ee'] = \sigma^2 I$

where σ^2 is a positive real number and I is an appropriately dimensioned identity matrix. While this assumption is clearly restrictive there are many circumstances, especially with cross-section and time series data combined, when observations can be transformed so that the new disturbance term conforms at least asymptotically to the above specification.

The first term on the right hand side of (61) is $E[e'M_V i_T' i_T M_V e]$. Now because V can be partitioned as a block diagonal matrix, M_V is also block diagonal. M_V^i, the diagonal component corresponding to the ith microequation is $I_T - V^i(V^{i\prime}V^i)^{-1}V^{i\prime}$ where I_T is a T dimensional identity matrix. Now since the matrix $i_T' i_T$ may be partitioned into NN submatrices all equal to I_T, the matrix product $M_V i_T' i_T M_V$ can be partitioned with submatrices on the diagonal equal to $M_V^i M_V^i$. Now since M_V is idempotent the product $M_V i_T' i_T M_V$ has diagonal elements identical to M_V.

(65) $$E\,[e'M_V i_T' i_T M_V e] = \sigma^2 \text{ Trace } [M_V i_T' i_T M]$$
$$= \sigma^2 \text{ Trace } M_V = (NT - NK)J^2$$

where K is the number of exogenous variables in each microequation. We assume that the observations are linearly independent so that $V'(V'V)^{-1}V$ has rank and trace equal to NK.

The first term on the right hand side of (63) is $e'i_T' M_{V*} M_{V*} i_T e$.

We assume that V_* has full column rank equal to K_* and it is easy to show

$$(66) \quad E[e'i'M_{V*}M_{V*}i_T e] = N(T - K_*)\sigma^2$$

A comparison of (65) and (66) shows that if each expression were corrected for degrees of freedom, as it would be if $\bar{R}^2$ were used in place of R^2 as goodness of fit criterion, the results would be equal.

The relationship between (61) and (63) thus depends on the other terms $B'X'M_V i_T' i_T M_V XB$ and $B'X' i_T' M_{V*} M_{V*} i_T XB$. These are easily reduced to $B'X'M_V XB$ and $B'X' i_T' M_{V*} i_T XB$ and then $B'X'B - B'X'V(V'V')^{-1}V'XB$ and $B'X' i_T' i_T XB - B'X' i_T' V_*(V_* V_*)^{-1} V_* i_T XB$. Without more specific information on the nature of V and V_* not much more can be said. It is readily apparent, however, that the relationships depend on how closely related the variables used in the regression are to the true variables. The first expression depends on the correlation of X and V and the second on $i_T X$ with V_*.

Disaggregation

The arguments in the previous sections have been based on the assumption that each individuals demand for a particular asset could be written as a linear function of a particular set of variables. This is the microtheory approach to the demand for money and there are clearly defined problems when aggregation of the microequation is attempted. However, the majority of the specifications described in Chapter I have to be classified as macroequations and none of them pretends to be a linear aggregation of all microequations. It is tempting to think that in these cases there are no aggregation problems. However, unless the author of a particular macromodel is willing to place absurd restrictions on the applicability of his model, the problems of aggregation or, perhaps more appropriately, the problems of disaggregation still require investigation.

To take an extreme example, it seems most unlikely that an author would deny the applicability of his micromodel if a single individual whose balances had been included in the specification happened to emmigrate. A little more realistically it is doubtful that any of the authors in Chapter I would admit that total demand could no longer be written as a linear function of macrovariables if data for Hawaii and Alaska were excluded. Since the exact boundary between macro and microeconomics is not defined we are left wondering at what point in the process of excluding individuals one by one from the macromodel does the model cease to apply.

In this research we investigate the problems of aggregation when the macromodel is assumed to apply to units as small as single states. When this is the case we can write an equation for each state which is linear in some set of variables and the problems associated with aggregating over all states are exactly analagous to those associated with aggregating over individuals.

Actually, even the question of aggregating over individuals will be explored in this paper by using statewide data. Essentially, we ask in both micro and macro tests whether summing only over individuals within a single state is better than aggregating all individuals in all states.

CHAPTER IV

ESTIMATION I

The purpose of this chapter is to explore the practical effects of aggregation on the estimation of demand for money equations. In order to do this, we first estimate with aggregate data the equations developed by the several authors discussed in Chapter I. It would have been desirable to repeat estimation techniques of those authors with their data and thus hopefully reaffirm exactly their results, but several circumstances dictate against this technique. Probably most important is the fact that virtually none of the data series used by the original authors are available on a statewide basis, so that even if the aggregate data could be reproduced (or borrowed), the state data would still be lacking. Further, in order for the estimation procedures and data to conform to the theoretical framework established in the last chapter, the macrovariables must be fixed weight aggregates of microvariables, and the data used by the original authors were not constructed in this way. Convenience often dictates choice of variables and with all of the published series of corporate, government, and bank interest rates it is hardly surprising that the previous authors selected readily available series rather than constructing weighted averages. Thus the first step in investigating the effects of aggregation on empirical work is the formulation of variables which conform to the theoretical framework established in the last chapter.

Definitions of State Variables

There has been a great deal of controversy in the monetary field as to the appropriate definition of money but the area of disagreement has been over what, if anything, should be included in money besides currency and demand deposits. Yet this basic accepted quantity, the sum of currency and demand deposits, is an aggregate of quite dissimilar quantities and it is entirely possible that these quantities are most accurately described with separate demand equations.[1] Unfortunately, figures for currency in circulation are not available at the state level so that this aspect of aggregation cannot be explored.

Demand and Time Deposits

The finest available breakdown of deposits by ownership is the sum of deposits held by individuals, partnerships and corporations. It could easily be argued that these IPC demand and time deposits are also heterogeneous aggregates and ought to be broken down further. In this study, however, IPC demand and time deposits at insured commercial banks by state are the finest breakdown attempted of the asset variables. There is more justification for this than the availability of data. The purpose of the empirical work in this chapter is to discover the effects of aggregation to national totals on estimating demand functions for money. The deposits used in those aggregate equations by previous investigators have been IPC deposits. Any division of assets into quantities accurately defined by a single equation is arbitrary

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[1]Philip Cagan has explored this topic with national data in "The Demand for Currency Relative to the Total Money Supply," *Journal of Political Economy*, Vol. 66 (August, 1958), pp. 303-328.

and the IPC figures deserve attention if for no other reason than because they have been used and were useful in the past. The specific figures used in this study are IPC demand and time deposits at all insured commercial banks, by state, taken from the Reports of Call For 1949-68.

Interest Rates

The following quotation by Laidler typifies the rationale for selecting an interest rate to use in empirical work. Laidler explained his selection of the rate on 4-6 month commercial paper as ". . . prompted by the fact that a short rate seemed a more appropriate proxy for the opportunity cost of holding money than a long rate, and partly because the variable performed slightly better than the yield to maturity on 20-year bonds in a series of preliminary tests."[2] Friedman uses this same sort of argument based on opportunity cost to explain his use of yield on corporate bonds.[3] The fact is that almost any return could be rationalized in one way or another and the high correlation among rates would probably yield unimportant differences when one was substituted for another. In this reasarch, the rates of return on time deposits and on savings and loan shares are assumed to be the relevant opportunity costs of holding demand deposits or, alternatively, they are assumed to be directly related to the prices of the service flows from the closely related assets. All rates are calculated in the same way, namely by dividing total interest payments to each asset in a

[2]Laidler, "Some Evidence," p. 55.

[3]Friedman, "The Demand for Money," p. 345.

given state by the average quantity of the asset held for that period. Then for each state a third interest rate is calculated which is a weighted average of the other two rates. Although a number of reasonable alternatives exist for the selection of weights there is little practical difference in the resulting numbers and with no precedent in the empirical work the choice is in any event completely arbitrary. The central purpose of exploring the effects of linear aggregation eliminates from consideration the immediately apparent index based on weights which change each year in proportion to quantity invested. If the analysis is to conform to the framework of linear aggregation the weights must remain constant for the entire period under consideration.

Roy Gilbert proposes that an appropriate index could be constructed with the use of principal component analysis and he includes the following as a desirable property of such an index: "In the case of both the Paasche and Lespeyres indices the selection of the weights depends upon the arbitrary choice of the base year . . . . Thus the choice of base year affects the characteristics of the index. With the principal component method the estimates of the weights are objectively determined by the data."[4] Gilbert also describes other desirable properties of the principal component method but for the present purpose the extra computational effort of the procedure does not seem merited. Of course the choice of base year in the Lespeyres index is arbitrary but the choice of data to use in the component method is also arbitrary and the weights depend entirely on the data selected.

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[4]Roy F. Gilbert, "The Demand for Money: An Analysis of Specification Error," Unpublished Ph.D. dissertation, Michigan State University, 1969, pp. 46-47.

A preliminary investigation of the changes in a Lespeyres type index resulting from different base year selection reveals very small differences. The rates of return to time and savings deposits for the United States as a whole in 1949, calculated as total interest paid divided by the average asset value, were .913% and 2.518% respectively. This is the year of both the largest absolute and the largest percentage difference in the two rates for the twenty years covered in this study. Yet the choice of base year weights has a very small effect on the resulting average. The 1968 base weights of .5284 for time deposits and .4716 for savings and loan shares yield a 1949 index rate of 1.669%. The 1959 weights yield 1.613%. The largest difference that choice of base years can possibly make is a little over .3% between the unusual year of 1949 when time deposits heavily outweighed savings and loan shares (2.7 to 1) and 1961 when the two quantities of assets were almost equal. As the interest rates move closer together over time, it is clear that choice of weights becomes even less important so that in 1968 (when the rates are nearly equal) the choice of base year weights can change the index by no more than .05%.

The base year 1968 was selected to construct the index. It is the latest year for which data were collected and the weights are between the extremes of 1949 and 1961.

Income

Total personal income and per capita personal income are available on a state basis. The particular series used in this research is a Department of Commerce series uniformly constructed for the years

1948-1968 inclusive.[5]

The permanent income data had to be constructed. In spite of the fact that Friedman's permanent income work has come under severe attack ever since its initial publication,[6] many researchers continue to use figures supplied by Friedman in their calculations which involve permanent income. This practice has several shortcomings. In 1969 Colin Wright tried unsuccessfully to reproduce Friedman's figures even though he used essentially the same data and original formulation of permanent income's definition. He states: "My estimates of the weight current income had in determining permanent income differed from those obtained by Friedman and did so in a systematic and interesting manner . . . . If my results are correct, then not only are the original calculations made by Friedman suspect but the use to which they have been put by Friedman and others needs modification."[7]

Gilbert also sharply criticizes Friedman. After pointing out an obvious case of omitted variables, Gilbert subjected Friedman's consumption function to four tests for specification error which ". . . resulted in rejection at better than the 5% confidence level by all four tests."[8] By estimating much more inclusive consumption functions, Gilbert was able to find one which passed the specification error tests

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[5]United States Department of Commerce, Office of Business Economics, <u>Survey of Current Business</u>, Vol. 49, No. 8 (August, 1969), pp. 14-15.

[6]Milton Friedman, <u>A Theory of the Consumption Function</u>, Princeton: Princeton University Press, 1957.

[7]Colin Wright, "Estimating Permanent Income: A Note," <u>Journal of Political Economy</u>, Vol. 77 (September/October, 1969), p. 846.

[8]Gilbert, "Demand," p. 50.

and gave reasonable estimates of other parameters.[9] From his estimates he calculated that the weight which current income receives in determining permanent income is .52. This figure is right in between the .3 to .4 range of Friedman and the .7 to .8 range which Wright calculates so it was considered a reasonable choice in creating the state permanent income data.

Since actual estimation of the series for each state was not possible, it was necessary to make an assumption about the initial values of the permanent income series. Real per capita personal income in 1947 and 1948 are equal and approximately 1.5% above the 1949 figure. In 1950, income jumps by almost 7%. The years prior to 1947 are distorted by the war.

In our calculations, we use 1948, the middle year of the three year period of almost constant income, as a base in calculating the permanent income series. We assume that in this year permanent income is equal to the actual measured value of current income and although this choice is arbitrary it is certainly no more arbitrary than Friedman's creation of past income data by extrapolating backward using an assumed 2 per cent growth rate. Friedman creates data both to begin his series and to replace war year data. While 2% may be a good overall index of growth rate it is more than twice the actual rate for the war years of the 1940's. The growth in per capita real income in 1948 prices from $1,304 in 1942 to $1,365 in 1947 implies an exponential growth rate of a little less than 1 per cent per year. Thus using a 2% rate to span these war years or create the additional data would

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[9]Gilbert, "Demand," p. 62.

overstate considerably the actual growth. It is hard to imagine that people's expectations for these early post war years were 5% higher than the acutal incomes when the actual income levels were at an all time high.

Fortunately, as with the interest rate variables, the selection of a particular weighting scheme and an initial value for the series seems to have little practical effect on the results. In a preliminary check on the data a simple linear regression was run on data for the state of Michigan. The series based on a weight of .52 for current income with an assumed initial value equal to 1948 current income was regressed on the values developed by Feige for use in his estimation of the demand for liquid assets. Feige bases his weighting scheme on Friedman's work but makes no mention of how he develops initial values for his series. The correlation between Feige's figures and those used in this research is .9958 and it is well known that if the correlation were 1 the effect of substituting one for the other in another regression would be exactly the same as scaling the original variable with a linear transformation. Such a scaling has no effect on the usual t tests of significance of the variable.

Friedman and Wright both use real per capita income in constructing their permanent income series. The original data used in this study are similarly scaled so that with Y_t interpreted as per capita real personal income the formula used in constructing permanent income for each state was

$$Y^*_t = .52Y_t + .48Y^*_{t-1}$$

As pointed out above, the series was begun in 1948 with the assumption that current real per capita income for that year was equal to permanent or expected income.

Adjustment Variables

The population data were calculated as the value implied by the per capita and total income figures described above. However, the figures obtained in that way were subjected to a preliminary comparison with an independently formulated set of population estimates[10] and all twenty of the figures checked fell with .05% of each other.

Asset values and income series were adjusted to real terms by dividing by the consumer price index for all items with 1969 equal to 1. The original series from United States Department of Labor, Bureau of Labor Statistics, Handbook of Labor Statistics 1971 was transformed to make 1969 the base year.

Macrovariables

The macrovariables used in this study are fixed weight linear aggregates of the microvariables just described. The unscaled quantity variables, demand and time deposits and current nominal income, are simple summations over all states of the corresponding microvariables. The real variables corresponding to these nominal ones are similarly constructed. Since the consumer price index is used to adjust the nominal data in all states, the real macrovariables are computed equivalently as either the simple summation of real microvariables or the

[10]United States Department of Commerce, "Population Estimates and Projections," Current Population Reports, Series P-25, No. 436 (January, 1970), p. 13.

summation of the nominal microvariables with the summation then adjusted by the consumer price index.

The ratio variables require a little more attention. It was necessary to construct fixed weight indices for both real income per capita and the interest rates on time deposits and savings and loan shares. As with the index of the two interest rates within each state, these also were constructed with weights from the base year 1968. The real income per capita macrovariable is the weighted average of the corresponding figures of each state with the weights proportional to the 1968 population for each state. Each macro-interest rate index is similarly a weighted average of the state data with weights proportional to the 1968 quantities of the assets held.

Perhaps a more common procedure than the one just described for constructing indices is to use a weighting scheme which changes each year in proportion to the change in the selected weighting variable, i.e., population or asset value. Although this procedure could not be used and still keep the research within the scope of fixed weight linear aggregation, the indices produced by the two methods are not significantly different. This is not a surprising result. There are two circumstances in which a variable weight index corresponds exactly to the fixed weight index, and both of the conditions are very nearly met by many economic variables. The two indices are the same if all of the variables used for weighting change in the same proportion in each state each year. Either of these conditions is sufficient and both are often approximated by economic data.

Equation Estimates

In this section we take up the problems of estimating the parameters of the regression equations. In general, we will want to estimate each specification using data both from total United States and from each individual state. The regressions using total U.S. data are fairly straightforward. They are based on assumptions which the original author made either explicitly or implicitly in his original specifications of the model. In general, these are classical assumptions and are so well known they need almost no explanation.

The regressions using state data, on the other hand, lend themselves to an estimation procedure developed by Zellner.[11] In this procedure, information about the error terms is used to provide estimators asymptotically more efficient than single equation estimators.

Macroequation Estimates

Table 1 presents the results of ordinary least squares estimation of all of the demand for money specifications developed in Chapter I using in each case the macrovariables just defined. It is obvious that all of these equations cannot be correctly specified at the same time, even with respect to which variables are included, let alone with respect to functional form or the scaling and transforming of these variables. A more subtle inconsistency of these equations appears in the nature of the disturbance terms. If, for example, the demand for money is written in one instance as linear in income and interest rate

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[11] Arnold Zellner, "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias," Journal of American Statistical Association, Vol. 57 (June, 1962), pp. 348-368.

with the assumption that e is a normally distributed random disturbance which has zero mean and constant variance, then the same assumption cannot consistently be made for the disturbance term when the original equation is rewritten in the logs of the original variables. That is, if

$$e_1 = A - XB$$

and

$$e_2 = \ln(A) - \ln(X)B$$

then e_1 and e_2 cannot both be distributed the same.

Unfortunately this argument, although correct, is not very useful. Economic theory does not pretend to specify the distribution of the stochastic disturbance in demand equations. As a matter of fact it is only recently that a stochastic element has figured in economic analysis at all. Thus the nature of the disturbance must be assumed and the most convenient assumptions are those which, if they were true, would yield estimates with desirable properties. In this respect economists have, by default, allowed their tools to dictate their assumptions.

All of the results in Table 1 are derived from least squares estimation. This is the technique which the original authors applied to each of the equations with the implicit or explicit assumptions that it would yield unbiased estimates of both the coefficients and their variances. When the results in Table 1 are interpreted in conformance with the same assumptions most of the conclusions are in general agreement with those previously drawn. Certainly the results support the

TABLE 1

COMPARISON OF MACROEQUATION ESTIMATES

No.	Original Author	Dep. Var.	Independent Variables				R^2
			B_o	YP	R		
1		D	54193 (1572)	.190 (.022)	-5078 (2690)		.991
	Chow	M_1	29424	.7382 (.0129)	-8845 (985)		
			B_o	YP	R	Y	
2		D	52740 (2023)	.3432 (.1380)	-6877 (3109)	-1328 (.1180)	.992
	Chow	M_2	29424	1.021 (.187)	-9297 (980)	-.1356 (.0657)	

Notes:

For each equation, the estimates obtained by the original author, using his variables, are given directly below the estimates obtained in this study using linearly aggregated macrovariables. Chow's variables are GNP in current dollars, Friedman's permanent income and R_L. Laidler uses Friedman's permanent income, and both R_C and R_L. Teigen uses current GNP and R_C. Latane uses current GNP and R_L. R_L = yield on long-term corporates. R_C = yield on 4-6 month commercial paper. M_1 and M_2 are money defined respectively to exclude and include time deposits. Figures in parentheses are standard deviations.

*The quantity variables in these equations are in real per capita terms.

TABLE 1 (cont'd.)

No.	Original Author	Dep. Var.	Independent Variables					R^2
			B_o	YP	R	D_{t-1}		
3		D	28367 (5866)	.1338 (.0193)	-6273 (1806)	.4679 (.1046)		.996
	Chow	M_1	8838	.1004 (.0336)	-2358 (467)	.8688 (.0456)		.999
			B_o	YP	R	D_{t-1}	Y	
4		D	25660 (6521)	.020 (.120)	-5204 (2125)	.535 (.126)	.092 (.095)	.996
	Chow	M_1	7926	-.0382 (.0659)	-1891 (485)	.8973 (.0451)	.0564 (.0235)	.999
			B_o	ln(YP)	ln(R)			
5		ln(D)	1.923 (.620)	.776 (.052)	-.244 (.046)			.996
	Chow	$\ln(M_1)$	-.1387	1.055 (.010)	-.7448 (.0447)			.997

TABLE 1 (cont'd)

No.	Original Author	Dep. Var.	Independent Variables					R^2
			B_o	ln(YP)	ln(R)	ln(Y)		
6		ln(D)	2.006 (.721)	.705 (.288)	-.239 (.053)	.064 (.255)		.996
	Chow	$\ln(M_1)$	-.1365	1.069 (.148)	-.7476 (.0540)	-.0132 (.1390)		.997
			B_o	ln(YP)	ln(R)	$\ln(D_{t-1})$		
7		ln(D)	1.031 (.746)	.656 (.080)	-.232 (.043)	.208 (.111)		.997
	Chow	$\ln(M_1)$	.3180	.4987 (.0758)	-.4644 (.0512)	.5056 (.0697)		.999
			B_o	ln(YP)	ln(R)	$\ln(D_{t-1})$	ln(Y)	
8		ln(D)	1.120 (.716)	.175 (.328)	-.192 (.049)	.304 (.124)	.384 (.255)	.997
	Chow	$\ln(M_1)$	.3067	.0616 (.1428)	-.3325 (.0597)	.5878 (.0669)	.3274 (.0940)	.999

TABLE 1 (cont'd)

No.	Original Arthur	Dep. Var.	Independent Variables				R^2
			B_o	Y	D_{t-1}	Y×R	
9		D	26418 (5400)	.226 (.052)	.168 (.134)	-.017 (.006)	.996
	Teigen	M_1	-1.7941 (.8966)	.2035 (.0389)	.5361 (.1161)	-.0142 (.0023)	.99
			B_o	ln(YP)	ln(R)		
10*		ln(D)	1.549 (.662)	.692 (.088)	-.319 (.031)		.920
	Laidler	ln(M_1)	6.030	.044 (.333)	-.176 (.030)		.844
	Laidler	ln(M_2)	3.145	.523 (.484)	-.516 (.111)		.787
			B_o	ln(YP)	ln(R)		
11*		ln(D+T)	-6.252 (2.284)	1.739 (.302)	-.311 (.107)		.903
	Laidler	ln(M_2)	2.018	.665 (.214)	-.142 (.019)		.836
	Laidler	ln(M_2)	1.508	.777 (.450)	-.338 (.103)		.530

TABLE 1 (cont'd)

No.	Original Arthur	Dep. Var.	Independent Variables			R^2
			B_o	$\Delta \ln(YP)$	$\Delta \ln(R)$	
12*		$\Delta \ln(D)$	-.009 (.010)	1.01 (.306)	-.296 (.103)	.587
	Laidler	$\ln(M_1)$	-.024	.820	-.031	.318
	Laidler	$\ln(M_2)$	-.024	.896 (.382)	-.119 (.087)	.338
			B_o	$\Delta \ln(YP)$	$\Delta \ln(R)$	
13*		$\Delta \ln(D+T)$	.006 (.016)	1.020 (.473)	-.223 (.159)	.328
	Laidler	$\Delta \ln(M_2)$	.012	.694 (.400)	-.039 (.026)	.269
	Laidler	$\Delta \ln(M_2)$	.013	.675 (.442)	-.064 (.101)	.164
			B_o	(1/R)		
14		(D/Y)	.176 (.006)	.319 (.014)		.968
	Latane	(M/Y)	.109	.007		.911

general conclusion that most of the variation in the quantity of money can be explained by an interest rate and an income variable.

The general purpose of estimating all of these macroequations was to establish that the macrovariables defined above produce results which are comparable to those produced by more conventional variables. The results in Table 1 substantiate the comparability. In reestimating the wide variety of functions originally estimated by Teigen, Laidler, Chow, and Latane, we derived parameter estimates which agree in sign with theirs with only two exceptions.

Theil wrote Linear Aggregation of Economic Relations nearly twenty years ago and yet the problems of aggregation have been ignored in all but a few special areas. The areas which have been explored universally possess the distinguishing characteristic that all of the variables involved are quantity variables whose conventional aggregates are simple sums of the microvariables. This means that all specifications which include prices, income per capita, and other ratio variables have been ignored.

In this research we have sought to circumvent the difficulties of conventionally calculated ratio indices by creating a new set of macrovariables which fall within the scope of linear aggregation.

The reestimation of macroequations and comparison of results then serves to establish that the newly created macrovariables are not wildly divergent from the more conventional ones but rather directly comparable. These results make it seem likely that the results of tests for aggregation bias are more generally applicable than to only the variables created in this research.

Macrovariables as Microvariables

In an attempt to explain why a single macroequation might yield closer estimates of the dependent macrovariable than the summation of estimates of dependent microvariables, Grunfeld and Griliches consider a model of micro behavior which includes the assumption that the aggregate independent variable contains some information relevant to the macro equations. Then, if the microequations are estimated without including the aggregate variables among the regressors, the specification error in the microequations might outweigh the aggregation error in the macroequation.[12]

Table 2 contains OLS estimates of coefficients for each state when demand deposits are estimated as a function of both the microvariables for that state and the set of linearly aggregated macrovariables for the total United States. Specifically, Table 2 contains for each state i the OLS estimates of the function

$$D^i = B_0^i + B_1^i R_T^i + B_2^i R_S^i + B_3^i YP^i + B_4^i R_{TS*} + B_5^i YP_* + e^i$$

where R_{TS*} and YP_* are macrovariables. The estimates and their standard errors are given in the first two lines for each state.

The third line contains the results of reestimating the equation but restricting the parameters of the macrovariables to zero. If R_{TS*} and YP_* are relevant in explaining the variation of D then we should be able to reject the null hypothesis that $B_4 = B_5 = 0$. The value of the F statistic used to test this hypothesis is given in one column and the significance level (conditional on the disturbance term being normally

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[12] Y. Grunfeld, and Z. Griliches, "Is Aggregation Necessarily Bad?", p. 7.

TABLE 2

ESTIMATES AND RESTRICTED ESTIMATES:
$D = f(B_o, R_T, R_S, YP, R_{TS*}, YP_*)$

State	B_o	R_T	R_S	YP	R_{TS*}	YP_*	F	Sig.
20	33.8 (33.8)	7.7 (13.6)	12.4 (14.1)	.103 (.033)	-15.6 (14.8)	.0001 (.0001)		
	36.5	-.228	7.50	.126			.737	.498
	237.6				90.8	-.0006	802.2	<.0005
30	58.1 (21.3)	10.9 (7.3)	-7.4 (9.9)	.101 (.024)	4.8 (9.9)	-.00004 (.00007)		
	56.9	10.6	-7.9	.102			1.1	.377
	153.97				96.5	-.0006	1262.3	<.0005
46	29.1 (11.9)	8.3 (6.6)	-2.6 (7.4)	.119 (.019)	-11.0 (8.1)	.00007 (.00006)		
	37.3	1.2	-6.7	.148			1.5	.262
	106.5				36.5	-.0002	764.4	<.0005
22	1429.6 (229.2)	-177.1 (75.0)	-71.99 (120.8)	.217 (.031)	65.6 (86.3)	-.0008 (.0006)		
	1340.2	-241.4	-138.8	.247			3.9	.046
	3137.9				902.6	-.0061	709.8	<.0005
40	232 (30.9)	-24.3 (11.5)	17.7 (23.3)	.104 (.030)	-61.4 (31.2)	.0004 (.0002)		
	261.4	-29.1	-25.6	.161			2.2	.148
	407.3				-16.7	..0001	110.8	<.0005
7	449.7 (72.1)	-169.4 (86.1)	20.2 (13.6)	.193 (.035)	24.2 (107.2)	-.0002 (.0007)		
	434.7	-171.7	22.6	.194			.155	.858
	1295				402.7	.003	428	<.0005
33	14303.5 (3263.7)	323.8 (1513.7)	4665.9 (2650.9)	-.099 (.276)	-4959.8 (1760.7)	.029 (.011)		
	8386.1	-3394.1	-3227.1	.766			5.75	.016
	23145.8				2073.5	-.015	176.6	<.0005
31	1155.7 (351.9)	32.0 (165.3)	-80.1 (257.9)	.141 (.039)	-23.1 (292.8)	.000008 (.002)		
	1131.5	-5.8	-118.2	.152			.932	.418
	2751.3				1157.3	-.007	966.5	<.0005
39	2871.7 (741.5)	-873.98 (281.3)	-162.7 (221.1)	.281 (.037)	451.6 (337.7)	.003 (.002)		
	3350.9	-605.5	-177.8	.241			1.1	.362
	7027.7				397.3	-.002	578.7	<.0005
8	242.9 (37.4)	31.02 (22.8)	-1.3 (5.2)	.133 (.053)	12.6 (25.99)	-.0001 (.0002)		
	225.2	27.5	-.189	.138			1.2	.338
	396.6				153.5	.001	148.2	<.0005

TABLE 2 (cont'd.)

State	B_o	R_T	R_S	YP	R_{TS*}	YP*	F	Sig.
21	679.98 (115.9)	14.8 (52.3)	-85.4 (43.9)	.121 (.014)	-119.98 (85.4)	.0009 (.0006)		
	632.7	-44.8	-68.5	.136			1.6	.237
	1084.3				308.6	-.002	2076.3	<.0005
9	283.9 (73.6)	35.8 (14.2)	23.9 (36.8)	.242 (.034)	-77.5 (32.5)	.0006 (.0002)		
	341.8	34.05	-16.02	.283			3.3	.069
	893.7				212.6	.001	726.02	<.0005
23	1338.6 (332.7)	-142.1 (117.3)	-454.95 (254.8)	.215 (.022)	682.8 (304.8)	-.005 (.002)		
	1043.3	-168.97	4.91	.164			3.8	.051
	3226.99				296.6	-.0009	710.99	<.0005
36	1336.7 (394.8)	-733.5 (196.2)	358.6 (258.04)	.193 (.035)	379.9 (447.2)	-.003 (.003)		
	1332.4	-624.8	509.5	.164			.429	.660
	4858.9				740.4	.004	367.03	<.0005
15	649.5 (204.2)	-186.7 (84.7)	97.7 (108.8)	.177 (.024)	-26.9 (125.5)	.0002 (.0009)		
	643.7	-198.6	97.6	.181			.069	.934
	2074.3				378.7	.002	525.08	<.0005
14	3896.5 (613.1)	-467.02 (210.9)	298.08 (467.7)	.202 (.053)	-773.3 (769.2)	.005 (.006)		
	4197.1	-588.3	-188.2	.257			1.02	.388
	8339.98				664.7	-.003	344.4	<.0005
50	-237.9 (172.7)	-149.4 (52.3)	412.6 (86.9)	.131 (.028)	-133.4 (77.5)	.0009 (.0005)		
	-115.2	-211.6	288.03	.182			2.4	.132
	1752.9				503.4	-.003	1391.7	<.0005
24	715.5 (157.8)	-45.5 (131.2)	59.9 (65.3)	.157 (.068)	-106.9 (108.8)	.0006 (.0007)		
	555.9	-219.3	14.01	.253			2.02	.172
	1690.7				455.5	-.003	454.6	<.0005
16	870.1 (170.7)	117.5 (64.0003)	-123.3 (87.4)	.161 (.033)	-7.3 (90.9)	.00009 (.0007)		
	892.4	124.7	-128.02	.159			.139	.871
	1426.3				322.5	-.002	361.02	<.0005
26	1062.7 (242.2)	-201.05 109.97	225.1 (172.8)	.206 (.072)	-106.8 (206.3)	.0008 (.001)		
	1081.5	-239.1	162.2	.237			.146	.865
	2984.1				497.4	-.003	373.3	<.0005

TABLE 2 (cont'd.)

State	B_o	R_T	R_S	YP	R_{TS*}	YP*	F	Sig.
35	276.2 (21.6)	39.3 (10.8)	-22.3 (11.3	.054 (.026)	-10.4 (12.99)	.00007 (.00009)		
	276.6	38 .1	-26.1	.065			.611	.558
	328.3				71.3	-.0005	180.9	<.0005
42	231.1 (55.7)	14.2 (22.9)	2.2 (34.8)	.099 (.046)	-9.04 (24.4)	.00007 (.0002)		
	236.8	14.4	-2.9	.108			.069	.934
	357.8				84.3	-.0006	64.03	<.0005
28	670.03 (160.4)	18.1 (47.2)	43.2 (62.5)	.078 (.064)	-112.9 (64.7)	.0007 (.0004)		
	637.97	-16.7	-3.04	.158			2.9	.091
	998.3				109.9	-.0007	57.1	<.0005
17	648.7 (99.3)	74.99 (42.6)	29.4 (70.4)	.069 (.043)	-51.9 (57.9)	.0003 (.0004)		
	700.3	57.6	-28.1	.108			.609	.559
	1112.3				302.1	-.002	542.9	<.0005
47	559.1 (132.004)	111.7 (121.1)	-48.3 (67.4)	.114 (.045)	-73.9 (143.9)	.0005 (.001)		
	594 .8	76.1	-70.7	.1303			.202	819
	1240.9				499.4	-.003	744.8	<.0005
49	172.5 (61.2)	19.03 (23.03)	-4.2 (19.1)	.1496 (.026)	-26.8 (25.1)	.0002 (.0002)		
	179.1	14.9	-11.1	.1595			.9096	.427
	573.7				121.4	- .0007	561.5	<.0005
18	759.6 (138.6)	207.6 (103.7)	22.9 (52.7)	.033 (.079)	-238.7 (117.3)	.002 (.0007)		
	647.8	22.6	-51.5	.182			2.1	.165
	1223.1				331.5	-.002	826.5	<.0005
43	481.5 (109.7)	1.1 (40.5)	-55.3 (65.6)	.209 (.026)	-15.4 (108.6)	.0002 (.0008)		
	517.2	19.8	-64.8	.204			1.2	.320
	1239.2				419.99	-.002	1272.9	<.0005
34	347.2 (167.6)	39.4 (104.4)	24.99 (77.3)	.136 (.047)	-84.02 (104.1)	.0006 (.0007)		
	418.8	15.5	-13.2	.152			.572	.578
	1245.9				498.4	-.003	790.7	<.0005
41	301.01 (54.6)	-41.04 (10.8)	-81.4 (24.7)	.2101 (.011)	83.8 (24.03)	-.0006 (.0001)		
	148.6	-25.7	-2.7	.171			10.5	.002
	550.96				177.2	-.001	2452.99	<.0005

TABLE 2 (cont'd.)

State	B_o	R_T	R_S	YP	R_{TS*}	YP*	F	Sig.
11	418.3 (244.6)	-24.01 (36.9)	-20.6 (112.2)	.194 (.021)	-80.1 (80.5)	.0006 (.0006)		
	603.3	-14.6	-102.3	.208			.625	.551
	1288.1				449.97	-.003	1716.3	<.0005
10	-496.7 (367.7)	239.8 (163.6)	615.5 (192.6)	.046 (.046)	-757.7 (277.2)	.005 (.002)		
	203.99	-109.5	232.5	.178			5.8	.016
	1904.9				1567.1	-.0103	563.2	<.0005
1	518.3 (203.5)	-14.2 (48.6)	-124.02 (123.3)	.218 (.064)	70.3 (227.6)	-.0005 (.002)		
	489.7	8.1	-93.7	.194			.581	.573
	907.002				277.6	-.002	823.3	<.0005
25	-14.4 (113.5)	8.2 (16.9)	65.8 (52.2)	.171 (.032)	-204.2 (73.4)	.002 (.0006)		
	251.9	-3.1	-57.9	.246			4.6	.031
	567.2				166.99	-.0009	926.97	<.0005
19	1125.2 (259.8)	115.4 (71.3)	-324.3 (114.9)	.229 (.041)	11.5 (120.3)	.0001 (.0009)		
	1181.2	174.3	-312.8	.197			4.8	.027
	1215.2				372.1	-.002	855.7	<.0005
4	237.5 (83.1)	50.6 (16.3)	23.5 (37.5)	.133 (.024)	-110.3 (41.3)	.0008 (.0003)		
	348.8	25.2	-36.8	.189			4.99	.025
	642.8				212.6	-.001	629.1	<.0005
37	792.8 (263.6)	78.2 (68.1)	-10.5 (182.3)	.132 (.115)	-95.8 (202.6)	.0007 (.001)		
	895.9	66.8	-75.3	.1701			.3303	.725
	1345.5				342.4	-.002	218.1	<.0005
44	2993.5 (1182.9)	319.7 (406.5)	114.6 (704.5)	.143 (.107)	-312.4 (790.2)	.002 (.005)		
	3316.1	183.2	-214.4	.199			.309	.74
	5908.1				2066.3	-.013	242.7	<.0005
32	114.3 (25.9)	-11.5 (9.8)	-3.9 (10.4)	.1701 (.019)	14.7 (11.4)	-.0001 (.00008)		
	114.5	-11.3	-5.5	.173			.844	.452
	280.8				214.3	-.001	631.2	<.0005
3	70.8 (70.9)	-52.1 (55.5)	57.9 (45.2)	.172 (.037)	137.1 (57.4)	-.001 (.0004)		
	27.6	-21.6	81.8	.128			2.9	.09
	438.4				531.03	-.004	250.2	<.0005

TABLE 2 (cont'd.)

State	B_o	R_T	R_S	YP	R_{TS*}	YP*	F	Sig.
27	318.6 (61.9)	6.5 (23.7)	-3.9 (26.6)	.082 (.079)	-62.6 (33.6)	.0005 (.0003)		
	266.7	-20.1	-19.8	.198			1.8	.203
	412.7				9.3	.0001	23.9	<.0005
13	142.1 (14.4)	7.6 (11.7)	-4.1 (10.6)	.135 (.035)	-24.1 (16.1)	.0002 (.0001)		
	151.4	1.4	-14.9	.1703			1.2	.341
	275.6				64.6	-.0004	262.3	<.0005
51	121.9 (18.6)	.739 (7.4	-20.9 (9.2)	.214 (.074)	-12.5 (7.6)	.00009 (.00006)		
	129.1	1.5	-25.2	.2198			1.7	.223
	189.3				30.4	-.0002	82.2	<.0005
6	365.3 (54.2)	-24.7 (31.3)	78.1 (27.2)	.131 (.026)	-11.3 (34.4)	-.00003 (.0003)		
	358.5	-66.8	59.3	.169			4.001	.044
	945.3				423.3	-.003	628.7	<.0005
45	146.2 (30.2)	8.7 (22.3)	44.7 (22.9)	.049 (.043)	-59.5 (23.6)	.0004 (.0002)		
	191.2	-9.2	10.4	.116			3.7	.055
	350.1				106.3	-.0007	146.8	<.0005
48	681.7 (65.7)	24.96 (40.9)	116.3 (48.6)	.052 (.024)	-242.5 (74.9)	.002 (.0005)		
	683.8	-68.3	30.1	.121			5.3	.02
	1333.98				214.03	-.001	494.1	<.0005
38	557.4 (49.3)	8.7 (46.1)	-27.8 (31.01)	.118 (.032)	-52.3 (56.5)	.0003 (.0004)		
	529.1	-29.97	-39.1	.153			1.4	.293
	876.5				80.2	-.0005	192.99	<.0005
29	28.7 (16.8)	-16.7 (4.4)	12.7 (7.1)	.185 (.015)	32.7 (11.04)	-.0003 (.00008)		
	8.3	-15.9	16.5	.177			9.8	.002
	139.6				173.6	-.001	1025.9	<.0005
5	2531.2 (1411.98)	-1040.2 (456.6)	533.2 (642.1)	.195 (.037)	1511.3 (822.5)	-.012 (.006)		
	900.04	-883.6	1168.9	.161			2.6	.114
	8247.9				5818.5	-.039	356.8	<.0005

TABLE 3

ESTIMATES AND RESTRICTED ESTIMATES
(D+T) = F (B_o, R_T, R_S, YP, R_{TS*}, YP*)

State	B_o	R_T	R_S	YP	R_{TS*}	YP*	F	Sig.
20	-47.7	32.3	22.2	.254	-55.9	.0004		
	(95.5)	(38.3)	(39.6)	(.094)	(41.8)	(.0004)		
	-4.2	14.2	-6.7	.322			1.8	.212
	461.6				236.1	-.001	713.7	<.0005
30	-195.4	-5.9	65.8	.254	-25.6	.0002		
	(44.2)	(15.2)	(20.6)	(.049)	(20.4)	(.0001)		
	-146.8	-28.1	41.8	.324			1.5	.261
	256.1				206.3	.001	2307.5	<.0005
46	-44.2	12.3	23.6	.429	10.1	-.00009		
	(45.5)	(25.1)	(28.2)	(.074)	(31.0)	(.0002)		
	-39.4	16.98	17.96	.427			.226	.801
	295.7				184.2	-.0012	735.1	<.0005
22	1594.3	734.7	461.5	.015	-1279.2	.008		
	612.8	200.6	322.9	.082	230.8	.002		
	2607.8	171.9	-1057	.411			30.7	<.0005
	4039.8				960.2	-.0061	477.6	<.0005
40	90.7	-27.5	-43.0	.479	-24.5	.0002		
	(54.6)	(20.2)	(44.7)	(.053)	(55.1)	(.0004)		
	110.1	-23.8	-47.8	.478			.838	.455
	703.2				143.2	-.0008	796.4	<.0005
7	344.7	207.8	38.8	.198	-825.7	.006		
	(91.8)	(109.7)	(17.4)	(.045)	(136.5)	(.0009)		
	-63.2	-424.8	4.2	.460			23.4	<.0005
	1841.5				572.9	.003	1370.7	<.0005
33	7742.2	1433.2	-7263.5	1.007	-4541.5	.036		
	(9553.5)	(4431)	(7759.7)	(.808)	(5153.9)	(.033)		
	8031	692.8	-9294.1	1.204			1.6	.23
	27716.3				9080.6	-.053	236.4	<.0005
31	-2869.99	-344.1	2553.2	.143	-2136.98	.015		
	(1414.4)	(664.7)	(1036.9)	(.156)	(1176.9)	(.008)		
	-2279.96	-1196.2	1337.7	.433			2.1	.164
	5225.7				2918	-.018	483.5	<.0005
39	2930.7	2902.3	1167.5	.029	-6504.5	.046		
	(4023.6)	(1526.7)	(1199.6)	(.199)	(1832.3)	(.013)		
	-6743.7	-1854.1	1862.3	.700			6.5	.011
	10878.6				3363.9	-.020	352.9	<.0005
8	237.3	104.6	-2.4	.166	-53.3	.0003		
	(44.98)	(27.4)	(6.2)	(.064)	(31.3)	(.0002)		
	148.5	60.9	.247				7.6	.006
	505.02				244.2	-.0016	412.1	<.0005

TABLE 3 (cont'd.)

State	B_o	R_T	R_S	YP	R_{TS*}	YP*	F	Sig.
21	730.6 (313.3)	-383.9 (141.4)	-256.5 (118.8)	.411 (.037)	540.9 (230.99)	-.004 (.002)		
	1019.004	-64.5	-335.8	.328			2.9	.091
	1630.04				763.1	.005	1499.1	<.0005
9	-168.2 (149.96)	104.2 (28.95)	-79.1 (74.98)	.717 (.068)	-252.9 (66.3)	.002 (.0004)		
	14.1	95.6	-214.08	.8604			7.97	.006
	1161.6				396.7	-.002	1105.1	<.0005
23	462.3 1242.9	1535.98 438.1	260.06 952.05	.232 .083	-4420.3 1138.8	.031 .008		
	2435.9	1717.4	-2623.6	.552			9.1	.003
	6357.04				2302.2	-.012	932.9	<.0005
36	4831.8 (938.96)	2860.97 (466.7)	-1475.9 (611.3)	.204 (.083)	-3695.3 (1063.6)	.026 (.008)		
	4964.2	1849.4	-2904.2	.481			6.4	.012
	8125.5				2906.5	-.017	939.09	<.0005
15	1142.7 492.05	520.9 204.05	-581.8 262.09	.389 .057	-656.5 302.3	.004 .002		
	1747.7	523.6	-1105.8	.483			4.8	.027
	3401.5				956.7	-.005	937.5	<.0005
14	7938.2 (1102.6)	2871.9 (379.2)	-2661.01 (841.1)	.383 (.096)	-3502.4 (1383.2)	.026 (.0099)		
	9189.95	2337.4	-4409.8	.601			3.4	.065
	12342.8				4072.9	.023	1549.7	<.0005
50	1680.2 (856.2)	150.1 (259.3)	-914.1 (430.8)	.651 (.140)	-131.6 (384.1)	.0009 (.0028)		
	1805.6	88.8	-1040.1	.701			.099	.906
	3342.4				1533.1	-.009	623.6	<.0005
24	532.99 370.5	92.6 308.1	-673.2 153.4	.735 .159	-211.03 255.6	.001 .002		
	331.2	-162.8	-752.01	.878			.571	.578
	2896.9				1336.3	-.008	1177.1	<.0005
16	-188.8 (361.1)	127.9 (135.4)	-131.3 (184.8)	.562 (.0703)	-183.2 (192.2)	.001 (.001)		
	-220.5	42.6	-215.8	.639			1.9	.176
	2270.9				860.99	-.005	817.3	<.0005
26	1432.6 (480.5)	968.6 (218.2)	15.6 (341.6)	.168 (.143)	-1649.2 (409.3)	.011 (.003)		
	1637.7	300.2	-1071.9	.707			8.27	.005
	4034.2				1501.5	-.009	1052.7	<.0005

TABLE 3 (cont'd.)

State	B_o	R_T	R_S	YP	R_{TS*}	YP*	F	Sig.
35	21.2 (53.8)	76.8 (26.8)	-12.3 (28.07)	.450 (.063)	-139.01 (32.3)	.0009 (.0002)		
	26.3	61.2	-62.7	.5902			17.1	<.0005
	498.9				261.4	-.002	699.7	<.0005
42	-42.8 (111.8)	92.1 (46.1)	-12.9 (69.95)	.442 (.092)	-162.5 (49.03)	.0012 (.0004)		
	43.6	88.3	-103.3	.622			5.9	.015
	507.7				283.7	-.002	358.5	<.0005
28	660.97 (303.9)	229.2 (89.4)	-137.2 (118.4)	.304 (.122)	-545.99 (122.6)	.004 (.0009)		
	522.9	66.4	-357.1	.680			17.5	<.0005
	1234.2				251.5	-.001	244.6	<.0005
17	246.2 (507.1)	316.9 (217.7)	-253.5 (359.3)	.391 (.218)	-264.2 (295.6)	.002 (.002)		
	509.1	239.8	-526.7	.574			.452	.646
	1444.6				796.4	-.005	273.2	<.0005
47	-.918 (297.7)	19.2 (273.02)	-180.3 (152.04)	.471 (.101)	-187.6 (324.5)	.001 (.002)		
	-62.8	-189.97	-201.96	.545			.503	.616
	2182.8				1258.1	-.008	1411.3	<.0005
49	-9.9 (148.7)	322.5 (55.9)	60.4 (46.4)	.112 (.064)	-451.1 (60.9)	.003 (.0004)		
	-296.5	87.8	-59.9	.483			33.02	<.0005
	923.4				340.9	-.002	1077.5	<.0005
18	701.6 (276.3)	245.1 (206.7)	-321.5 (105.1)	.417 (.156)	-261.5 (233.8)	.002 (.002)		
	449.8	-22.2	-392.6	.619			1.1	.354
	1628.5				656.2	-.004	1449.5	<.0005
43	463.9 (403.7)	-45.4 (149.3)	-593.1 (241.4)	.695 (.094)	616.99 (399.7)	-.004 (.003)		
	116.1	186.9	-276.1	.526			3.1	.078
	1853.1				1223.7	-.007	677.9	<.0005
34	274.7 (280.02)	-95.7 (174.4)	-356.8 (129.2)	.495 (.079)	-218.1 (173.9)	.002 (.001)		
	411.4	-202.002	-456.5	.557			.941	.415
	1826.6				817.4	-.005	1752.2	<.0005
41	307.9 (86.7)	-5.7 (17.1)	-124.8 (39.3)	.30006 (.0179)	80.6 (38.2)	-.0007 (.0003)		
	198.2	-2.7	-82.2	.281			5.3	.021
	710.2				298.8	-.002	2662.5	<.0005

TABLE 3 (cont'd.)

State	B_o	R_T	R_S	YP	R_{TS*}	YP*	F	Sig.
11	489.7 (594.1)	-65.1 (89.6)	-376.3 (272.5)	.533 (.051)	-110.2 (195.6)	.0007 (.001)		
	806.6	-74.9	-540.2	.571			.594	.566
	1864.4				899.7	-.005	1629.6	<.0005
10	969.7 (364.8)	34.5 (158.5)	-542.8 (186.5)	.527 (.045)	-269.04 (268.37)	.001 (.002)		
	1288.02	-169.4	-769.1	.603			6.4	.012
	2809.1				2923.4	-.019	3445.3	<.0005
1	-1371.2 (558.98)	-63.4 (133.6)	599.1 (338.8)	.266 (.177)	-949.3 (625.4)	.007 (.005)		
	-675.3	-239.1	130.1	.537			1.3	.294
	1334.7				790.2	-.005	783.6	<.0005
25	-436.8 (219.5)	-33.8 (32.6)	53.5 (100.9)	.515 (.061)	-182.1 (142.01)	.001 (.001)		
	-185.6	-36.7	-55.8	.575			1.6	.234
	772.5				478.6	-.003	1389.6	<.0005
19	-696.7 (511.2)	-247.7 (140.3)	150.3 (226.1)	.540 (.081)	-97.3 (236.7)	.0008 (.002)		
	-533.7	-236.2	90.9	.552			.599	.564
	1700.4				945.9	-.006	1143.5	<.0005
4	-481.6 (204.3)	58.04 (40.1)	175.9 (92.3)	.334 (.059)	-285.09 (101.5)	.002 (.0008)		
	-178.6	.727	19.2	.473			4.7	.029
	816.7				528.6	-.003	652.3	<.0005
37	-11.7 (387.4)	16.8 (100.1)	-186.4 (267.8)	.648 (.168)	-116.97 (297.7)	.0009 (.002)		
	113.6	14.5	-249.1	.679			.667	.53
	1662.3				884.4	-.005	962.2	<.0005
44	-1308.01 (2396.1)	1334.1 (823.4)	973.9 (1427.1)	.243 (.216)	-2545.4 (1600.7)	.017 (.011)		
	534.2	485.8	-711.7	.554			1.3	.315
	7142.005				4909.1	-.031	436.5	<.0005
32	62.7 (59.4)	88.5 (22.5)	-35.1 (23.9)	.243 (.044)	-118.6 (26.1)	.0008 (.0002)		
	72.99	88.4	-33.7	.219			14.2	.001
	376.1				387.1	-.003	763.5	<.0005
3	41.8 (160.8)	148.6 (125.9)	-163.2 (102.4)	.513 (.085)	-266.7 (130.2)	.002 (.0095)		
	12.7	82.4	-202.8	.5996			1.5	.252
	648.2				977.5	-.006	536.97	<.0005

TABLE 3 (cont'd.)

State	B_o	R_T	R_S	YP	R_{TS*}	YP*	F	Sig.
27	-366.9	-65.2	77.5	.708	-87.7	.0006		
	(107.4)	(41.2)	(46.1)	(.138)	(58.4)	(.0004)		
	-454.5	-106.5	49.8	.888			1.9	.185
	570.95				230.7	-.001	371.7	<.0005
13	-75.01	17.4	1.4	.509	-138.8	.0009		
	37 .7	30.6	27.9	.093	42.3	.0003		
	-38.6	-33.5	-68.1	.758			10.8	.002
	456.1				210.9	-.001	550.2	<.0005
51	-74.1	42.9	56.2	.1799	-113.6	.0008		
	(64.9)	(26.002)	(32.2)	(.257)	(26.7)	(.0002)		
	-22.6	43.7	8.89	.305			13.03	.001
	267.7				137.99	-.0009	208.5	<.0005
6	380.3	125.8	-191.6	.453	-296.5	.002		
	(164.8)	(95.1)	(82.8)	(.079)	(104.7)	(.0008)		
	547.1	14.7	-297.6	.577			4.1	.041
	1300.7				878.3	-.006	643.05	<.0005
45	49.9	44.9	8.1	.353	-181.6	.001		
	(42.3)	(31.3)	(32.1)	(.061)	(33.02)	(.0002)		
	178.7	.661	-81.3	.5203			15.4	<.0005
	603.7				350.2	-.002	983.4	<.0005
48	-944.7	-840.6	-699.7	1.1	265.8	.0009		
	(500.4)	(311.3)	(369.6)	(.1808)	(569.4)	(.004)		
	-446.7	-257.4	-97.8	.598			20.1	<.0005
	1629.1				-39.95	.003	164.7	<.0005
38	76.2	17.9	-31.1	.445	-112.6	.0008		
	(88.1)	(82.5)	(55.5)	(.058)	(101.0001)	(.0007)		
	48.02	-41.9	-52.8	.503			.762	.486
	1448.3				516.9	-.003	1008.2	<.0005
29	28.1	19.9	-7.5	.391	-12.7	.000007		
	(52.9)	(13.97)	(22.4)	(.048)	(34.7)	(.0003)		
	74.2	15.3	-34.6	.437			2.8	.096
	243.1				318.5	-.002	630.5	<.0005
5	4542.3	-273.9	-1298.5	.522	273.95	-.001		
	(2456.02)	(794.2)	(1116.9)	(.064)	(1430.6)	(.0101)		
	3868.2	-203.6	-925.6	.504			.187	.832
	15312.7				12038.6	-.076	1088.7	<.0005

and independently distributed) is given in another. The hypothesis is rejected at the 5% level of significance in only 10 states and at the 10% level in only 15 states. Thus the Grunfeld and Griliches assumption of omitted macrovariables is not very well supported by these results. In the majority of states, macrovariables do not contribute significantly to the variation in the quantity of demand deposits when microvariables are included.

We also test the converse hypothesis that the state variables are not significant when macrovariables are included. That is, we again use an F statistic to test the null hypothesis that $B_1 = B_2 = B_3 = 0$. If this hypothesis were true, so that the demand equation in each state could be written with only macrovariables as independent variables, then the system of state equations would be consistent with a macroequation which described total demand as a function of the same macrovariables. The parameters in the macroequation would just have to be interpreted as the sum of the corresponding microparameters and there would be no questions of aggregation bias.

The results of restricting the state parameters to zero and reestimating the equation are given in line 4 for each state. The hypothesis that $B_1 = B_2 = B_3 = 0$ is rejected at less than a 1% level of significance in every single state, so that the necessary conclusion is that aggregation problems may not be assumed away on the basis that the demand in each state is accurately described as a function of only macrovariables.

Table 3 contains results of repeating the calculations of Table 2 with D + T substituted for D as the independent variable in each state. About half of the states show a rejection of the null hypothesis

$B_4 = B_5 = 0$ at the 10% level of significance suggesting that in those states the macrovariables do contribute to the explanation of variation in the quantity D + T. Since the hypothesis is not rejected in the other states, these results are somewhat inconclusive. However, with respect to the relevance of state variables, the results are overwhelming. The null hypothesis that $B_1 = B_2 = B_3 = 0$ is again rejected in every state at less than the 1% level of significance implying again that macrovariables are not an adequate substitute for state variables in the state demand for money equations.

CHAPTER V

ESTIMATION II

In estimating the demand for money within each state and in calculating test statistics for aggregation bias we make use of the model developed by Chow and described in detail in Chapter I. The only change which we make is a substitution of the two interest rates, R_T and R_S, in place of Chow's yield on twenty year corporate bonds. Both time deposits and savings and loan shares are plausible substitutes for demand deposits and it seems reasonable to include both yields in the demand equation for money rather than dropping one arbitrarily or using an index of them. The relative importance of the rates can then be determined empirically.

State Demand Equations

The demand for demand deposits in the ith state at time t is written:

$$D_t^i = B_o^i + B_1^i R_{Tt}^i + B_2^i R_{St}^i + B_3^i Y_t^i + B_4^i YP_t^i + B_5^i D_{t-1}^i + e_t^i$$

Letting D^i, X^i, and e^i represent the matrices of observations on the variables and B^i represent the vector of coefficients, this equation may be rewritten

$$D^i = X^i B^i + e^i$$

Then the complete system of equations may be written as in Chapter II:

$$D = XB + e$$

where

$$D = \begin{bmatrix} D_1^1 \\ D^2 \\ \cdot \\ \cdot \\ \cdot \\ D^N \end{bmatrix}, \quad X = \begin{bmatrix} X^1 & 0 & \dots & 0 \\ 0 & X^2 & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & 0 \\ 0 & & \dots & 0X^N \end{bmatrix}, \quad B = \begin{bmatrix} B^1 \\ B^2 \\ \cdot \\ \cdot \\ \cdot \\ B^N \end{bmatrix}, \quad e = \begin{bmatrix} e^1 \\ e^2 \\ \cdot \\ \cdot \\ \cdot \\ e^N \end{bmatrix}$$

This notation conforms well to that used by Zellner in his presentation of the two stage procedure for efficient estimation of seemingly unrelated regressions. In order to use Zellner's estimator, we have divided the states into nine geographic regions with N_g states (usually 6) in the gth region and then, following Zellner, we assume that the $N_g T \times 1$ vector of disturbances has zero mean and the following variance-covariance matrix:

$$\sum_g = \begin{bmatrix} \sigma^{11} & \sigma^{12} \dots & \sigma^{1Ng} \\ \sigma^{21} & \sigma^{22} & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \sigma^{Ng1} & \dots & \sigma^{NgNg} \end{bmatrix} \otimes I_{TxT}$$

$$\sum_g = \sum_{cg} \otimes I_{TxT}$$

$$\sigma^{ij} = E(e_{it}, e_{jt}) \qquad \begin{matrix} t = 1,2 \dots T \\ i,j = 1 \dots Ng \end{matrix}$$

Dropping the group subscript to simplify notation we can write the ZA estimator for the parameters in each group of states as

$$\tilde{B} = (X' \hat{\Sigma}^{-1} X)^{-1} X' \hat{\Sigma}^{-1} D$$

where $\hat{\Sigma}$ is the estimate of Σ based on OLS estimates of the regression coefficients for each state individually. The ij element of $\hat{\Sigma}_c$ is calculated by using vectors of residuals from states i and j in the formula

$$\tilde{\sigma}^{ij} = \frac{1}{T-K} \tilde{e}_i' \tilde{e}_j$$

and $\hat{\Sigma}$ is then written as the Kroeneger product

$$\hat{\Sigma} = \hat{\Sigma}_c \otimes I_{TxT}$$

The estimator $\tilde{B}$ has the property of being asymptotically more efficient than single equation OLS estimators because it takes account of the correlation of disturbances between states.

Tables 4 and 5 present the ZA estimates by state of the demand equations for D and D + T, respectively. The results are considered in more detail below but in general they conform well with our prior expectations.

Tests for General Consistency

A macroequation corresponding to the state demand equations has already been estimated using the linearly aggregated macrovariables D_t, R_{TS}, t, and Y_t. (The results are listed in Table 1, equation 4.)

TABLE 4

ZA ESTIMATES BY STATES

$$D_t = B_o + B_1R_{Tt} + B_2R_{St} + B_3Y_t + B_4YP_t + B_5D_{t-1} + E_t$$

State	B_o	R_T	R_S	Y	YP	D_{t-1}
20	35.6 (14.9)	-2.2 (2.3)	-3.2 (7.7)	.088 (.028)	-.0008 (.037)	.407 (.134)
30	43.1 (12.3)	10.6 (2.8)	-6.8 (5.1)	.092 (.056)	-.028 (.068)	.307 (.1496)
46	21.2 (10.06)	-1.4 (3.6)	-3.02 (5.5)	-.026 (.053)	.142 (.066)	.293 (.166)
22	1126.6 (268.8)	-132.5 (72.9)	-242.5 (96.03)	.259 (.096)	-.088 (.109)	.365 (.1904)
40	202.2 (58.6)	-18.6 (9.5)	-27.2 (12.7)	-.019 (.126)	.148 (.154)	.245 (.211)
7	284.3 (50.7)	-94.2 (29.1)	3.8 (6.4)	.1005 (.052)	.013 (.063)	.412 (.089)
33	-470.5 (3244.7)	-2354.99 (855.3)	-2738.9 (1043.99)	.217 (.278)	.291 (.3496)	.702 (.169)
31	1139.2 (329.7)	31.6 (99.96)	-150.01 (156.9)	.074 (.071)	.064 (.083)	.069 (.1695)
39	2572.2 (478.5)	-422.9 (84.9)	-259.9 (132.5)	.076 (.048)	.105 (.063)	.2903 (.082)
8	162.1 (52.04)	19.8 (14.8)	.259 (3.2)	.235 (.106)	-.131 (.113)	.262 (.203)
21	490.5 (134.6)	-28.2 (14.5)	-87.4 (45.1)	.031 (.057)	.062 (.071)	.396 (.124)
9	117.7 (53.9)	.041 (9.5)	-.022 (17.9)	.312 (.101)	-.156 (.118)	.534 (.095)
23	523.6 (234.7)	-115.2 (92.9)	6.5 (134.1)	.158 (.0397)	-.075 (.052)	.515 (.100)

TABLE 4 (cont'd.)

State	B_o	R_T	R_S	Y	YP	D_{t-1}
36	1007.8 (266.1)	-166.03 (103.1)	-24.9 (131.9)	.121 (.049)	-.043 (.061)	.577 (.103)
15	379.4 (120.3)	-60.7 (50.9)	-25.95 (53.4)	.133 (.040)	-.054 (.054)	.602 (.121)
14	2854.7 (512.3)	-21.2 (157.7)	-629.3 (196.2)	.093 (.089)	.036 (.0996)	.561 (.113)
50	136.6 (110.3)	-129.6 (30.9)	82.4 (58.8)	-.034 (.045)	.183 (.061)	.315 (.112)
24	91.4 (148.0)	-190.02 (46.2)	-95.5 (36.3)	-.266 (.093)	.474 (.106)	.639 (.166)
16	670.5 (138.3)	60.6 (46.2)	-55.02 (55.6)	.076 (.045)	.0695 (.0599)	.118 (.138)
26	663.2 (128.98)	-145.96 (37.4)	59.3 (60.2)	.1495 (.079)	-.003 (.090)	.432 (.078)
35	282.5 (28.8)	39.9 (6.9)	-29.1 (6.6)	-.085 (.023)	.163 (.036)	-.036 (.097)
42	160.3 (36.1)	2.5 (11.2)	1.6 (15.4)	-.174 (.036)	.3096 (.056)	.165 (.115)
28	290.3 (148.5)	-10.1 (32.8)	-16.3 (44.7)	.057 (.0995)	.036 (.112)	.551 (.168)
17	424.9 (175.9)	7.4 (40.8)	.6802 (34.7)	-.031 (.059)	.124 (.073)	.306 (.205)
47	336.8 (75.9)	26.2 (27.9)	-22.7 (26.7)	.026 (.049)	.064 (.063)	.355 (.114)

TABLE 4 (cont'd.)

State	B_o	R_T	R_S	Y	YP	D_{t-1}
49	81.1 (35.3)	4.6 (10.1)	-7.03 (9.9)	.066 (.027)	.029 (.039)	.465 (.112)
18	491.2 (98.6)	15.9 (24.2)	-61.4 (30.8)	-.049 (.072)	.196 (.0897)	.286 (.132)
43	311.5 (75.9)	-12.7 (19.9)	17.6 (33.0)	.264 (.072)	-.128 (.079)	.241 (.159)
34	121.6 (89.6)	-42.4 (27.8)	57.3 (30.0)	.139 (.047)	-.034 (.060)	.349 (.111)
41	118.99 (42.9)	5.6 (11.5)	-4.5 (15.4)	.1801 (.042)	-.097 (.062)	.397 (.154)
11	249.97 (69.4)	-41.5 (18.3)	-9.1 (27.9)	.122 (.038)	.0098 (.048)	.377 (.103)
10	384.9 (235.2)	-60.5 (93.3)	-127.4 (139.8)	.114 (.200)	-.046 (.240)	.872 (.203)
1	113.5 (72.6)	-23.7 (15.4)	7.7 (24.5)	.1799 (.039)	-.094 (.052)	.552 (.120)
25	63.4 (63.9)	-13.6 (12.0)	-13.9 (22.1)	.047 (.080)	.083 (.100)	.546 (.146)
19	484.98 (160.0)	62.5 (40.9)	-93.96 (55.8)	.242 (.067)	-.151 (.085)	.481 (.1296)
4	-13.9 (47.5)	-15.1 (9.5)	44.5 (17.5)	.338 (.063)	-.292 (.069)	.689 (.109)
37	338.03 (130.6)	-11.95 (29.6)	6.6 (51.2)	.329 (.128)	-.253 (.150)	.563 (.113)

TABLE 4 (cont'd.)

State	B_o	R_T	R_S	Y	YP	D_{t-1}
44	1475.5 (604.95)	-113.4 (151.0)	-256.8 (232.5)	.039 (.126)	.1124 (.152)	.567 (.108)
32	103.1 (19.0)	.878 (7.3)	14.2 (6.9)	.141 (.088)	-.042 (.110)	.394 (.175)
3	42.1 (46.8)	-16.1 (38.3)	-4.96 (31.1)	.3097 (.224)	-.285 (.264)	.9504 (.203)
27	8.95 (32.0)	-35.6 (6.96)	1.8 (8.9)	-.098 (.048)	.253 (.064)	.669 (.076)
13	55.8 (34.1)	4.99 (9.4)	-4.7 (7.2)	.118 (.069)	-.064 (.097)	.628 (.219)
51	76.3 (21.97)	-1.3 (5.1)	-19.1 (5.7)	.165 (.095)	-.006 (.104)	.409 (.135)
6	182.1 (92.6)	-100.5 (25.4)	70.7 (33.0)	.164 (.122)	-.021 (.139)	.301 (.222)
45	21.1 (24.4)	-29.8 (12.4)	-9.5 (10.3)	.305 (.132)	-.242 (.135)	.971 (.119)
48	308.8 (123.9)	-8.6 (26.1)	-57.5 (39.0)	.178 (.076)	-.121 (.091)	.681 (.181)
38	488.2 (79.2)	-6.6 (23.9)	-62.7 (20.2)	.208 (.083)	-.083 (.095)	.174 (.140)
29	6.99 (15.9)	-7.5 (5.7)	12.2 (6.0)	.207 (.164)	-.075 (.207)	.197 (.275)
5	1438.9 (666.2)	-8.74 (347.1)	34.6 (328.7)	.137 (.171)	-.079 (.192)	.619 (.167)

TABLE 5

ZA ESTIMATES BY STATES

$$(D+T) = B_o + B_1R_{Tt} + B_2R_{St} + B_3Y_t + B_4YP_t + B_5(D+T)_{t-1} + E_t$$

State	B_o	R_{Tt}	R_{St}	Y_t	YP_t	$(D+T)_{t-1}$
20	48.5 (27.2)	7.9 (4.0)	-34.5 (13.3)	.185 (.047)	-.100 (.064)	.863 (.106)
30	-21.4 (48.7)	-2.5 (9.2)	-1.6 (18.3)	.331 (.126)	-.253 (.194)	.836 (.308)
46	-11.7 (25.2)	14.4 (9.7)	-6.5 (15.3)	.013 (.135)	.125 (.154)	.766 (.146)
22	-177.4 (372.1)	124.0 (102.4)	-266.9 (148.5)	.225 (.180)	-.244 (.164)	1.3 (.117)
40	132.2 (48.5)	-6.8 (15.9)	-45.0 (20.6)	.382 (.129)	-.011 (.175)	.146 (.223)
7	208.8 (60.0)	-137.8 (34.9)	18.3 (8.4)	.143 (.063)	-.022 (.175)	.464 (.223)
33	3000.2 (3146.9)	1663.5 (1065.6)	-2090.9 (1374.2)	.392 (.386)	-.355 (.417)	.970 (.126)
31	366.5 (383.4)	286.8 (158.9)	-532.4 (252.8)	.018 (.125)	-.037 (.110)	1.23 (.135)
39	51.9 (534.8)	446.6 (155.9)	-395.6 (212.7)	.063 (.072)	-.264 (.061)	1.5 (.012)
8	45.2 (69.1)	21.7 (28.8)	2.8 (5.4)	.408 (.181)	-.269 (.190)	.566 (.314)
21	23.6 (197.2)	10.0 (29.5)	-44.6 (76.8)	.168 (.086)	-.145 (.098)	1.0 (.151)
9	68.1 (41.4)	-5.2 (7.0)	8.3 (13.9)	.430 (.072)	-.325 (.083)	.680 (.071)
23	323.7 (389.5)	238.9 (157.7)	-308.3 (235.4)	.394 (.053)	-.472 (.068)	1.3 (.079)

TABLE 5 (cont'd.)

State	B_o	R_{Tt}	R_{St}	Y_t	YP_t	$(D+T)_{t-1}$
36	432.6 (406.6)	358.5 (150.0)	-448.6 (208.5)	.381 (.062)	-.358 (.069)	1.0 (.097)
15	571.1 (328.2)	314.7 (123.9)	-441.7 (156.1)	.309 (.101)	-.285 (.142)	1.0 (.153)
14	2723.4 (1048.5)	1225.9 (246.0)	-1205.6 (490.0)	.197 (.145)	-.224 (.165)	1.0 (.142)
50	225.1 (492.9)	63.0 (93.3)	-242.7 (226.4)	.112 (.155)	-.013 (.233)	.974 (.176)
24	63.2 (120.1)	-183.1 (43.4)	-77.7 (35.3)	-.244 (.091)	.438 (.104)	.663 (.134)
16	66.6 (240.6)	122.3 (91.5)	-195.6 (115.2)	.102 (.125)	.065 (.199)	.815 (.180)
26	240.0 (296.8)	150.0 (81.8)	-204.1 (136.3)	.426 (.222)	-.400 (.205)	1.0 (.189)
35	35.5 (42.8)	68.0 (20.9)	-61.6 (21.5)	-.130 (.094)	.382 (.180)	.582 (.138)
42	29.3 (55.1)	60.8 (23.1)	-79.7 (33.6)	-.212 (.091)	.455 (.148)	.744 (.108)
28	-3.2 (153.9)	45.1 (42.5)	-59.5 (58.8)	.040 (.124)	.031 (.138)	.982 (.094)
17	488.6 (207.2)	33.4 (49.6)	14.8 (39.6)	.020 (.070)	.053 (.085)	.226 (.241)
47	64.6 (109.6)	-43.7 (52.9)	-167.5 (52.1)	.098 (.083)	.050 (.103)	.890 (.104)

TABLE 5 (cont'd.)

State	B_o	R_{Tt}	R_{St}	Y_t	YP_t	$(D+T)_{t-1}$
49	-115.3 (65.9)	10.4 (21.5)	-7.4 (21.2)	.175 (.046)	-.142 (.061)	1.07 (.066)
18	238.5 (140.7)	25.8 (42.2)	-231.4 (66.5)	.010 (.134)	.203 (.199)	.804 (.153)
43	119.8 (143.9)	155.8 (53.8)	-125.5 (89.7)	.094 (.221)	-.111 (.211)	1.1 (.159)
34	57.3 (152.7)	-49.6 (51.3)	-148.2 (72.6)	.291 (.088)	-.138 (.115)	.810 (.127)
41	132.5 (45.4)	10.7 (12.2)	-11.5 (16.9)	.171 (.044)	-.091 (.065)	.412 (.157)
11	225.9 (158.9)	29.6 (38.5)	-198.0 (73.9)	.251 (.091)	-.161 (.126)	.984 (.111)
10	1040.2 (333.5)	11.4 (138.5)	-612.7 (183.6)	.218 (.237)	.131 (.333)	.456 (.312)
1	99.4 (101.1)	1.5 (32.9)	-13.7 (42.9)	.214 (.067)	-.181 (.086)	1.0 (.104)
25	-101.4 (80.0)	3.5 (18.6)	2.7 (33.8)	.153 (.124)	-.065 (.173)	.910 (.156)
19	-83.1 (186.4)	-29.2 (58.6)	-13.2 (74.2)	.412 (.090)	-.301 (.122)	.851 (.154)
4	62.4 (43.3)	-4.6 (8.7)	19.2 (15.7)	.323 (.060)	.260 (.065)	.619 (.093)
37	34.8 (146.6)	3.5 (45.0)	-50.1 (88.1)	.434 (.180)	-.241 (.237)	.735 (.186)

TABLE 5 (cont'd.)

State	B_o	R_{Tt}	R_{St}	Y_t	YP_t	$(D+T)_{t-1}$
44	7.0 (1057.8)	-85.1 (290.3)	-307.1 (515.2)	-.053 (.309)	.115 (.304)	1.1 (.204)
32	50.6 (35.9)	27.1 (17.2)	-20.1 (14.0)	.353 (.154)	-.351 (.167)	.958 (.115)
3	43.9 (50.6)	-7.4 (43.2)	-12.3 (36.1)	.189 (.257)	-.173 (.304)	1.0 (.231)
27	-143.3 (49.5)	-29.9 (15.1)	13.3 (17.8)	.061 (.091)	.097 (.145)	.974 (.104)
13	2.34 (18.6)	42.0 (15.9)	-14.9 (11.2)	.253 (.114)	-.449 (.190)	1.4 (.144)
51	-17.4 (20.0)	2.4 (8.4)	-3.3 (10.3)	.248 (.144)	-.198 (.165)	1.0 (.083)
6	-21.0 (109.5)	-44.3 (39.7)	-39.5 (56.3)	-.063 (.210)	.177 (.265)	.975 (.135)
45	36.6 (29.5)	-25.3 (14.3)	-15.2 (13.2)	.200 (.170)	-.126 (.176)	.931 (.148)
48	-305.8 (736.3)	-177.0 (385.1)	329.1 (455.0)	2.1 (.827)	-2.0 (1.1)	.372 (.435)
38	111.6 (69.3)	66.1 (63.4)	-110.8 (46.3)	.515 (.186)	-.367 (.251)	.734 (.233)
29	2.7 (20.2)	6.9 (7.1)	-5.1 (8.3)	.553 (.137)	-.517 (.153)	.979 (.163)
5	539.3 (713.7)	-444.8 (403.5)	531.5 (394.3)	.219 (.191)	-.148 (.216)	.561 (.197)

We rewrite that equation letting the absence of superscripts denote the macroequation variables and parameters:

$$D_t = B_o + B_{R_{TS}} R_{TS,t} + B_Y Y_t + B_{YP} YP_t + B_{D_{t-1}} D_{t-1} + e_t$$

It is now possible to test empirically for aggregation bias in this macroequation. Equation (26) in Chapter III gives the necessary and sufficient conditions for consistent aggregation. Using the notation established in Chapter III, we let X be the block diagonal matrix of observations for all states and W_* be the matrix of weights attached to these observations in forming the matrix of observations on the macrovariables. Then

$$Z_* = XW_*$$

and the set of observations on the macroequation may be written as

$$D_* = Z_* B_* + e \qquad \text{or} \qquad D_* = XW_* B_* + e$$

The necessary and sufficient condition for general consistency of the micro and macroequations is

$$W_* B_* = B$$

where B is the column vector of all state parameters.

In the context of our demand for money equations, this consistency condition implies the simultaneous satisfaction of the following equations for i = 1 ... N:

$$B_{R_{TS}} = (1/W^i_{R_T})\, B^i_{R_T} = (1/W^i_{R_S})\, B^i_{R_S}$$

$$B_Y = B_Y^i$$

$$B_{YP} = B_{YP}^i$$

$$B_{D_{t-1}} = B_{D_{t-1}}^i$$

Since each of these equations must be satisfied for general consistency, it is possible now to formulate several hypotheses whose rejection would imply inconsistent aggregation. Each of the hypotheses are tested on 6 New England States using the $\tilde{F}$ statistic described by Zellner:[1]

$$\tilde{F}_{q,N(T-K)} = \frac{N(T-K)}{q} \times$$

$$\frac{D'\Sigma^{-1}X(X'\Sigma^{-1}X)^{-1}C'[C(X'\Sigma^{-1}X)^{-1}C']^{-1} \quad C(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}D}{D'\Sigma^{-1}D-D'\Sigma^{-1}X(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}D}$$

In this formula q is the number of restrictions on the system and C is the matrix of restrictions in the null hypothesis CB = 0.

The hypotheses and the test results are listed in Table 6. The last hypothesis, H_o6, is formulated to test the simultaneous satisfaction of H_o1 through H_o5. Actually the test of H_o6 would be sufficient for determining inconsistent aggregation, but the first five tests shed light on the particular areas responsible for the inconsistency.

Zellner has shown that $\tilde{F}$ has the same asymptotic distribution as F, but in order to interpret the significance level of F given in the table as applying to the value of $\tilde{F}$, we have to assume, as Zellner suggests, that $\tilde{F}$'s distribution is closely approximated by that of $F_{q,N(T-K)}$. Under this assumption, the null hypothesis of consistent

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[1]Zellner, "Seemingly Unrelated Regressions," p. 355.

TABLE 6

TESTS OF CONSISTENT AGGREGATION FOR 6 NEW ENGLAND STATES

Null Hypothesis (ij = 1, 2 ... 6)	$\tilde{F}$	Significance of F
$H_0 1$: $(1/W^i_{R_S})\ B^i_{R_S} = (1/W^j_{R_S})\ B^j_{R_S}$	3.359	.008
$H_0 2$: $(1/W^i_{R_T})\ B^i_{R_T} = (1/W^j_{R_T})\ B^j_{R_T}$	9.184	<.0005
$H_0 3$: $B^i_Y = B^j_Y$	2.125	.071
$H_0 4$: $B^i_{YP} = B^j_{YP}$	1.74	.135
$H_0 5$: $B^i_{D_{t-1}} = B^j_{D_{t-1}}$	.308	.907
$H_0 6$: $(H_0 1, H_0 2, \ldots H_0 5)$	8.438	<.0005

aggregation is rejected at a significance level of less than 1 percent. This strong rejection comes in spite of the fact that only six states, which are all New England States and might easily have equal parameters, were included in the test.

To confirm the results achieved using ZA and restricted ZA estimation another test was designed for a group of nine states--one state selected randomly from each of nine geographic regions--whose disturbance terms were assumed not to be correlated. Exactly the same set of hypotheses could have been tested but a linear transformation of the X matrix produces a more convenient set of equivalent hypotheses.

Transformation of Variables

Again using notation established in Chapter II, we let W^i be the matrix applied to variables in the ith state in forming the linearly aggregated macrovariables. For our particular demand model, W^i may be written as:

$$W^i = \begin{bmatrix} 1 & 0 & \cdots & & 0 \\ 0 & W^i_{R_T} & 0 & \cdots & \cdot \\ 0 & W^i_{R_S} & 0 & & \cdot \\ \cdot & 0 & 1 & 0 & \cdot \\ \vdots & & 0 & 1 & 0 \\ 0 & \cdots & & 0 & 1 \end{bmatrix}_{6x5}$$

Using the non-zero elements in each row of W^i, we form the diagonal transformation matrix for each state

$$Q^i = \begin{bmatrix} 1 & 0 & \cdots & & & 0 \\ 0 & W^i_{RT} & 0 & \cdots & & \\ 0 & W^i_{RS} & W^i_{RS} & 0 & \cdots & \\ 0 & 0 & 0 & 1 & 0 & \cdot \\ 0 & \cdots & & 0 & 1 & 0 \\ 0 & \cdots & & & 0 & 1 \end{bmatrix}$$

Then the matrix Q, which we use to transform X, is written:

$$Q = \begin{bmatrix} Q^1 & 0 & \cdots & & 0 \\ Q^2 & Q^2 & 0 & & \cdot \\ Q^3 & 0 & Q^3 & 0 & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \vdots & \vdots & & \ddots & 0 \\ Q^N & 0 & \cdots & 0 & Q^N \end{bmatrix} \text{ and } Q^{-1} = \begin{bmatrix} Q^{1^{-1}} & 0 & \cdots & & 0 \\ -Q^{1^{-1}} & Q^{2^{-1}} & & & \cdot \\ -Q^{1^{-1}} & 0 & Q^{3^{-1}} & & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \vdots & \vdots & & \ddots & 0 \\ -Q^{1^{-1}} & 0 & \cdots & 0 & Q^{N^{-1}} \end{bmatrix}$$

If we let ξ represent $Q^{-1}B$ the complete system of demand equations for the nine states may be written either as

$$D = XB + e$$

or

$$D = XQ\xi + e$$

Since the variance of the disturbance is assumed to vary from state to state, the disturbance vector of the system of equations is heteroskedastic. To adjust for this, the estimation of $\hat{\xi}$ is carried out in two stages. First OLS estimates are used to calculate residuals for each state and these residuals are used to estimate the variance:

$$\hat{\sigma} = \frac{1}{T-K^i} \sum \hat{e}_t^{i2}$$

Then each state's observations are scaled by the value of $1/\hat{\sigma}^{ii}$. This scaling makes the disturbance of the transformed observations asymptotically homoskedastic with variance equal to 1. In the second stage, we find estimates of the parameters of $D = XQ\xi$ by applying OLS to the scaled variables. It is well known that the least squares estimates $\hat{\xi}$ and $\hat{B}$ are linear transmations of one another. Thus all of the hypotheses listed above in terms of B's have counterparts in terms of ξ's and may be tested using $\hat{\xi}$. Specifically,

$$(W_*B_* = B) \Longleftrightarrow (W_*B_* = Q\xi)$$

$$Q^{-1}W_*B_* = \xi$$

Each of the first six rows of the 6N x 5 matrix $Q^{-1}W_*$ has a 1 as the only non-zero element. The last 6N-6 rows consist entirely of zeros and we have the following null hypotheses:

$$H_o1^*: \quad \xi_1^1 = \xi_1^2 = \xi_1^3 = \ldots \xi_1^N = 0$$

$$H_o2^*: \quad \xi_{R_T}^2 = \xi_{R_T}^3 = \ldots \xi_{R_T}^N = 0$$

$$H_o3^*: \quad \xi_Y^2 = \xi_Y^3 = \ldots \xi_Y^N = 0$$

$$H_o4^*: \quad \xi_{YP}^2 = \xi_{YP}^3 = \ldots \xi_{YP}^N = 0$$

$$H_o5^*: \quad \xi_{D_{t-1}}^2 = \xi_{D_{t-1}}^3 = \ldots \xi_{D_{t-1}}^N = 0$$

Each of these hypotheses conforms to the standard format of the usual F test for the relevance of a group of regression variables. In the last three hypotheses ξ_a^i is equivalent to $(B_a^1 - B_a^i)$. Similarly

$\xi^i_{R_T} = (1/W^1_{R_T})B^1_{R_T} - (1/W^i_{R_T})B^i_{R_T}$ so that H_o2 through H_o5 are exactly equivalent to H_o2* through H_o5*. The nature of ξ^i_1 and the presence of ξ^1_1 in H_o1* permit testing one final hypothesis which is implied by the consistency condition. If R_T and R_S are to be aggregated to the variable R_{TS} in a macroequation, consistency implies that $(1/W^i_{R_T})B^i_{R_T}$ must equal $(1/W^i_{R_S})B^i_{R_S}$ for all states. ξ^i_1 is equal to the difference between these weighted parameters in the ith state and thus H_o1* is the hypothesis that this consistency condition is met in all states. Together H_o1* and H_o2* imply H_o1 and H_o2 but the converse is not true.

Results in Table 7 confirm earlier results for the six New England States. Again the null hypothesis of consistent aggregation is rejected at a significance level less than 1 percent.

H_o1* stands out by virtue of its low F statistic. If there is a difference in the weighted parameters of the interest rates R_T and R_S, the sample contains too little information to uncover it. Thus, this particular test cannot be used in argument against the use of R_{TS} as an adequate index of interest rates. This result is not very surprising. Aside from the fact that R_T and R_S may well affect demand deposits in proportion to the quantity held of T and S, R_T is highly correlated with R_S and high correlation between two independent variables in a regression makes it difficult to sort out the contribution of either variable alone.

A test similar to that in H_o1* was performed on each of the nine states independently to test whether the index $(W^i_{R_T})R^i_T + (W^i_{RS})R^i_S$ was a consistent aggregate of R^i_T and R^i_S in the demand deposit equation for each state. The weights $W^i_{R_T}$ and $W^i_{R_S}$ were obtained analagously to those used in the national index with each weight equal to the proportion held in the corresponding asset in 1968. When the null hypothesis

TABLE 7

TESTS OF CONSISTENT AGGREGATION OF D^i FOR 9 UNRELATED STATES

Null Hypothesis	$\tilde{F}$	Significance of F
$H_0 1^*$: $\xi_1^1 = \xi_1^2 = \xi_1^3 = \ldots = \xi_1^N = 0$	.8057	.612
$H_0 2^*$: $\xi_{R_T}^2 = \xi_{R_T}^3 = \ldots = \xi_{R_T}^N = 0$	3.0523	.004
$H_0 3^*$: $\xi_Y^2 = \xi_Y^3 = \ldots = \xi_Y^N = 0$	4.4684	<.0005
$H_0 4^*$: $\xi_{YP}^2 = \xi_{YP}^3 = \ldots = \xi_{YP}^N = 0$	3.2997	.002
$H_0 5^*$: $\xi_{D_{t-1}}^2 = \xi_{D_{t-1}}^3 = \ldots = \xi_{D_{t-1}}^N = 0$	1.4724	.175
$H_0 6^*$: $(H_0 1^*, H_0 2^*, \ldots H_0 5^*)$	4.6517	<.0005

$(1/W^i_{R_T})B^i_{R_T} = (1/W^i_{R_S})B^i_{R_S}$ was tested in 9 different states, the highest F statistic obtained was 1.56 with a significance level of only .233. The average F value was much lower and thus even less significant.

Our conclusion from these tests is that inconsistent aggregation cannot be demonstrated to result from the use of the index R_{TS} in place of R_T and R_S in either the aggregate demand equation or those of the individual states. However, other null hypotheses implied by the necessary condition for consistent aggregation seem extremely unlikely to hold so that, taken as a group, they point very strongly to the rejection of the consistent aggregation hypothesis.

Demand and Time Deposits

When the aggregation to a single macroequation involves two dependent microequations which have the same independent microvariables, an additional degree of freedom is created for the microparameters. Thus suppose we assume that the demand in the ith state for demand deposits plus time deposits can be written:

$$(D+T)^i_t = \alpha^i_o + \alpha^i_T R^i_{Tt} + \alpha^i_S R^i_{St} + \alpha^i_Y Y^i_t + \alpha^i_{YP} YP^i_t + \alpha^i_{D+T}(D+T)^i_{t-1}$$

General consistency of the macroequation (written without superscripts),

$$(D+T)_t = \alpha_o + \alpha_{R_{TS}} R_{TS,t} + \alpha_Y Y_t + \alpha_{YP} YP_t + \alpha_{D+T}(D+T)_{t-1} + e_t$$

with the system of microequations for all states implies a set of conditions in α^i_a equivalent to those implied for B^i_a in the aggregation of D^i, i.e., H_o1^* through H_o6^*. However, if D^i and T^i are each linear functions of the same variables given in the demand for their sum, $D^i + T^i$, then α^i_a (a=T, S, YP, Y, A_{t-1}), may be interpreted as the sum of the corresponding parameters in the equations for D^i and T^i alone. Then the null hypotheses tested for consistency in aggregating over states pertain to these sums rather than to individual parameters in the equation for either D^i or T^i alone.

Again a transformation of variables simplifies the analysis. Table 8 gives the results of testing H_o1*+ through H_o6*+ with exactly the same procedure used in testing H_o1* through H_o6* in Table 7 for the aggregation of D^i alone. Each of the five null hypotheses necessary for consistent aggregation over states is rejected at a level of significance less than 10 percent and the hypothesis that all conditions are simultaneously met is again rejected at a significance level of less than 1 percent. Including time deposits in the definition of money does not seem to reduce the inconsistency of aggregating over states.

The tests of the preceeding chapter establish with very high probability that state demand equations cannot be consistently aggregated to a single macroequation whether the dependent variable is demand deposits or the sum of demand and time deposits. Conditions for general consistency imply restrictions on the parameters of every state and yet null hypotheses which postulate the existence of the conditions for as few as six states are very strongly rejected.

In the following sections we present estimates of several measurements which describe the extent of the inconsistency and its importance in estimation.

General Inconsistency

The vector d was defined in Chapter III as the difference between W*B* and B and its inner product d'd was suggested as one measure of the

TABLE 8

TESTS OF CONSISTENT AGGREGATION OF D^i+T^i FOR 9 UNRELATED STATES

Null Hypothesis	$\tilde{F}$	Significance of F
$H_0 1^{*+}$: $\xi_1^1 = \xi_1^2 = \xi_1^3 = \ldots = \xi_1^N = 0$	3.9657	<.0005
$H_0 2^{*+}$: $\xi_{R_T}^2 = \xi_{R_T}^3 = \ldots = \xi_{R_T}^N = 0$	1.8075	.082
$H_0 3^{*+}$: $\xi_Y^2 = \xi_Y^3 = \ldots = \xi_Y^N = 0$	2.7190	.009
$H_0 4^{*+}$: $\xi_{YP}^2 = \xi_{YP}^3 = \ldots = \xi_{YP}^N = 0$	4.7515	<.0005
$H_0 5^{*+}$: $\xi_{D_{t-1}}^2 = \xi_{D_{t-1}}^3 = \ldots = \xi_{D_{t-1}}^N = 0$	6.0678	<.0005
$H_0 6^{*+}$: $(H_0 1^*, H_0 2^*, \ldots H_0 5^*)$	4.9788	<.0005

amount by which the system of equations fails to meet the conditions for general consistency It is clear that without knowing $B*$ and B we cannot calculate d. However, by using the vector of unbiased ZA estimates, $\tilde{B}$, we can calculate an estimate of B_* which, if $\tilde{B}$ were equal to B, would give us a lower bound on d'd. We simply utilize the well known property of the least squares estimator, write

$$B = W_* B_* + d$$

and calculate $\tilde{B}*$ as the vector which minimizes the sum of squared terms in d.

$$\bar{B}_* = (W_*'W)^{-1} W_*'\tilde{B}$$

In the case at hand, $(W_*'W)^{-1}$ is a diagonal matrix and such that

$$\bar{B}*_a = \frac{1}{N} \sum_i \tilde{B}_a^i \qquad (a = Y,\ YP,\ D_{t-1})$$

and

$$\bar{B}*_{RTS} = \frac{1}{\sum_i (W_{R_T}^{i2} + W_{R_S}^{i2})} \sum_i (W_{R_T}^i \tilde{B}_{R_T}^i + W_{R_S}^i \tilde{B}_{R_S}^i)$$

If $\tilde{B}$ were equal to B, then d'd could not be less than $\bar{d}'\bar{d}$. Using $\tilde{d}'\tilde{d}$ in a manner analagous to the sum of squared errors and then treating the inner product of the vector of state estimates, $\tilde{B}'\tilde{B}$, as total sum of squares, we can separate $\tilde{B}'\tilde{B}$ into the proportion which is explained by $W_*\bar{B}_*$ and that which is due to general inconsistency. The proportion of $\tilde{B}'\tilde{B}$ explained by $W_*\bar{B}$ for the estimates of B, by state, given in Table 4, is .208. Since $\tilde{d}'\tilde{d}$ is a lower bound under the conditions stated, inconsistency accounts for at least 80% of the total sum of squares.

Specific Inconsistency

General consistency permits maximum degrees of freedom for the microvariables. For specific consistency, all that is required is that B_* and B satisfy the following relationship for the particular values of the variables observed at a specific time:

$$iXW_*B_* = iXB$$

When this equality fails to hold, the vector of differences from all observations can be written as iXd. Again using $\tilde{d}$ to approximate the true vector, we calculate $iX\tilde{d}$ and then proceed, as with the general inconsistency measurement, to find the proportion of B'X'i'iXB which is due to specific inconsistency.

The nature of the microvariables in X is such that specific inconsistency seems to be considerably less than general inconsistency. d'X'i'iXd is slightly more than 5% of total sum of squares.

Aggregation Bias

Aggregation bias was defined in Chapter II as the amount by which the expected value of the least squares estimate of B_* differed from the actual value, i.e., $E(\hat{B}_*) - B_*$. From (43) in Chapter III this difference equals $(Z_*'Z_*)^{-1}Z_*' \, i_T Xd$ where Z_* is the T x K_* matrix of observations on the macrovariables. Again using $\tilde{d}$ as an estimate of d we can get an idea of the size of aggregation bias by performing least squares regression of the vector of estimated values of specific inconsistency, $i_T X\tilde{d}$, on the matrix of macrovariables.[2]

The estimated amounts of aggregation bias are given in Table 9.

TABLE 9

MACROPARAMETER AND AGGREGATION BIAS ESTIMATES

	$\hat{B}_{R_{TS}}$	$\hat{B}_Y$	$\hat{B}_{YP}$	$\hat{B}_{D_{t-1}}$
Estimates	-5203.7	.092	.020	.535
Std. Error	2125.4	.095	.120	.126
Est. Agg. Bias	4492.6	-.011	.007	.077
Bias/Estimate	.86	.12	.35	.14

Aggregation bias in the interest rate parameter stands out immediately, but all of the calculated biases are greater than 10% of the parameter estimates.

[2]Theil calculates an estimate of aggregation bias by the formally equivalent procedure of first calculating a matrix $G = (Z_*'Z_*)^{-1}Z_*'i_T X$ and then multiplying to get the vector of biases, $G\tilde{d}$. Since we already have the vector of specific inconsistency measures, $i_T X\tilde{d}$, Theil's approach would be an unnecessary duplication of effort in this instance. See H. Theil, "Principals of Econometrics, New York: John Wiley and Sons, 1971. pp. 562-566.

CHAPTER VI

CONCLUSIONS

The principal conclusions of this research are as follows. i) Estimation of demand for demand deposits at the state level yields parameter estimates which conform generally with prior expectations based on economic theory. ii) The system of state demand equations is not consistent with a single macroequation which attempts to describe aggregate demand in terms of linearly aggregated macrovariables. iii) Estimates based on such a misspecified macroequation cannot be assumed to be unbiased; therefore, conclusions based on these estimates are suspect.

Of course these conclusions have only been firmly established for the variables and functional form used in this study. However, the reestimation of macroequations, which were estimated by other authors using different variables, establishes that the variables of this study are not widely divergent from those used previously, and in fact, produce comparable results in estimation. This result makes it seem likely that problems of inconsistency and aggregation bias occur in other specifications and with other variables.

One of the implications of inconsistency is that the parameter estimates of the macroequation provide no information about the demand for money in any particular location. This arouses some curiosity as to why aggregate demand is of any interest at all. Of course state demand is also aggregate demand and perhaps the next step in disaggregating should be at the SMSA level. However, states are less heterogeneous economic units than is the country as a whole and it may be that inconsistency and aggregation bias are insignificant problems within a state and thus that state demand equations provide a useful summary of information. This would be a fruitful area for further research.

The primary purpose of this research was to determine whether a single macroequation should be relied upon as an accurate description of the demand for money in the United States. Many authors have assumed that a single equation does adequately describe total U.S. money demand and they have proceeded on that basis with empirical analysis involving a few arbitrary macrovariables. The rate of return on four to six month commercial paper, for example, is one of the most frequently used interest rate variables and it is usually treated as "the" rate of interest with the implication that it adequately represents all of the various interest rates. However, in spite of the use of macrovariables in the equations and in the related empirical analysis, most studies nevertheless appeal to microeconomic concepts in developing a theoretical framework. The interest rate in most studies is described as an opportunity cost, and national income or wealth is used as a budget constraint.

It is this appeal to the language of microtheory which suggested that consideration of state demand for money equations might prove worthwhile. It would be possible to propose a linear relationship between macrovariables which did not rest on microtheory. However, once an author formulates the tempting rationale that interest rate is analagous to opportunity cost and income serves as budget constraint, it is difficult to argue at the same time that the demand relationship is not applicable to large subgroups of the population. The rates of return to time deposits and savings and loan shares within each state are, for most people, much more realistic indications of the opportunity cost of holding demand deposits than the return on four to six month commercial paper which is so frequently used in empirical work.

Justification for the disaggregation of demand for money was also derived from portfolio theory. Interest rates in different locations exhibit considerable variation both in their absolute levels and in their patterns of change, and in view of the disparity of past historical data in the various states, it seems highly unlikely that investors within each state would hold similar expectations regarding the probability distribution of future returns. Thus there is considerable doubt whether a single macrovariable can adequately represent the expected value of the future return on financial assets in all states. Moreover, since the coefficients of interest rates in the linear equation derived from portfolio theory also depend on the nature of the investors' anticipated probability distribution, it seems unlikely, on theoretical grounds, that the coefficients would be the same in all states.

This anticipation of disparity in state coefficients and the superiority of state versus national variables in explaining variation in demand for money was overwhelmingly supported by the empirical work in this study. State interest rate and income variables were calculated and demand equations were estimated for each of the states.

The coefficients of interest rate and income variables differed widely among the states and the national indices of interest rate and income could not be shown to be significant explanatory variables in most of the states. In every state, the hypothesis that the state income and interest variables were not important in explaining variation in the demand for money was rejected at a 1% level of significance. On the other hand, when national income and interest rate variables were used as regressors in the state demand equations, in most states they did not contribute significantly to the variation in quantity of money demanded. These results implied that the problems of aggregation in demand for money could not be assumed away on the basis that the national variables adequately represented the opportunity cost and budget constraint for each of the states separately. If the converse had been true, if the state interest and income variables had proven insignificant and the national macrovariables had explained the variation in demand for money in each state, then many of the problems associated with aggregation would have disappeared. The macroparameters could have been interpreted as the sum of the corresponding parameters in all the states. Estimates of macroparameters likewise would have been equal to the sum of corresponding estimates in all the states and there would have been no possibility of aggregation bias. Of course the macroparameter estimates would give no indication of the disparity in parameters between states, but all summary statistics result in some information loss--a loss which is offset by convenience or some other consideration.

The inadequacy of the national variables to explain state demand for money, and the significance, at the same time, of the state interest rate and income variables indicated that there might be serious problems associated with estimating the demand for money as a function of national macrovariables. Of course the same sort of problem might exist with the state demand functions, and it might prove worthwhile in further research to reestimate for, say, the standard metropolitan statistical areas to determine whether variables calculated for the state as a whole adequately explain the demand for money in the individual SMSA's. But for the present study, equations for each state were a convenient level of disaggregation and, as regression models, they were at least as acceptable, under such standard criteria as R^2 and the F test for significance of variables, as macromodels for the total United States have been. Thus, if there is a single national macroequation, there are also fifty, equally valid state demand equations; there do not seem to be any good reasons for rejecting the state equations which would not be equally valid in rejecting the national equation.

Thus the problems of aggregation had to be confronted. The conditions under which a single national equation is consistent with a set of underlying state equations are highly restrictive. In order to bring the analysis within the scope of linear aggregation, macrovariables had to be constructed as fixed weight linear aggregates of the corresponding variables from each state. Although a number of reasonable alternatives existed for the selection of weights, there was little practical difference in the resulting indices. The quantity variables, money and income, were simply added over all states to give their corresponding macrovariables. National interest rate indices were constructed as weighted averages of the state interest rates with the weights proportional to the amount invested in each asset in the base year, 1968. For further verification of the reasonableness of these variables they were used to reestimate (by least squares) the demand for money specifications previously estimated by other authors. The reestimation confirmed that the new variables produced results in regression analysis which the original authors probably would have accepted as comparable to their own results. Thus, it seems likely that the results of tests for consistent aggregation and aggregation bias are more generally applicable than to just the variables created in this research.

The tests for consistent aggregation left little doubt that the system of state demand for money equations is inconsistent with a single macroequation for the nation as a whole. Essentially what this inconsistency means is that the quantity of money implied by demand equations in each of the states does not necessarily add up to the quantity implied by the demand equation for the nation as a whole. This need not have been the case. It is perfectly feasible for a system of microequations to be consistent with a macroequation and, in fact, several authors have assumed that this condition was met. But in this study, when the necessary conditions for consistent aggregation were stated as a null hypothesis, the hypothesis was strongly rejected in each of two tests. The conditions were not met, even in the two small groups of states selected for the tests.

In order to determine the extent of the inconsistency, the vector of estimates from all of the states was split into two parts--a vector of state parameters which would be consistent with a single macroequation, and a residual vector whose non-zero elements indicate inconsistency. In case of general consistency of the state parameter estimates with a single macroequation, this residual vector would consist entirely of zeros and, of course, would have an inner product equal to zero. But in this study, the inner product of the vector of residuals was 80% of the total inner product of the vector of state parameter estimates. This result overwhelmingly reaffirmed the results of the earlier tests which indicated that the system of state demand equations was not consistent with a single macroequation.

With inconsistency thus firmly established, two sets of calculations were made to estimate the effect of this inconsistency when regressions are run under the assumption of a single macroequation. It is possible for a set of state demand equations to be consistent with a single national equation for a specific set of exogenous variables even though the conditions for general consistency do not hold. If, for example, all of the state income variables moved proportionally then a single aggregate variable could be consistent with all of the state income variables. While it is true that the state variables used in this study are highly correlated with one another they nevertheless do not move in direct proportion and we have a positive value for the measure of specific inconsistency for the twenty year period of this study. This measure was constructed by forming two vectors of estimates of total money demanded for each of the years in the study. One set of estimates was obtained by adding together the estimates from each of the states; the other set was calculated by using the estimated parameters of the macroequation and the national exogenous variables. The difference between these vectors would be a zero vector in the absence of specific inconsistency. Instead, in this study that vector of differences had an inner product equal to 5% of the inner product of the vector of estimates based on state data. This indicates considerable disparity between state and nationally based estimates.

Probably the most significant effect of this specific inconsistency is the bias created in the estimates of the national parameters when total money is regressed on national interest rate and income variables. Based on the results of this study, the estimate of the interest rate parameter derived from national variables might be biased by as much as 86%, and all of the other parameter estimates have biases of at least 10%. Due to the widespread practice of dropping insignificant variables from regression equations, aggregation bias may have already resulted in some specifications being eliminated which, in the absence of aggregation bias, might describe money demand quite well. In the specifications which are reported, the biases can lead to erroneous conclusions about the importance of particular variables.

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