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thesis entitled

SIMPLE MEANS OF FLOW PREDICTION
IN A VANELESS DIFFUSER

presented by

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has been accepted towards fulfillment
of the requirements for

MASTERS degree in MECHANICAL ENGINEERING

Major professor

Date 10-26-94

SIMPLE MEANS OF FLOW PREDICTION IN A VANELESS DIFFUSER

By

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A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

MASTER OF SCIENCE

Department of Mechanical Engineering

1994

AN ABSTRACT OF A THESIS

SIMPLE MEANS OF FLOW PREDICTION IN A VANELESS DIFFUSER

By

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In designing vaneless diffusers, designers need a fast accurate way to predict the flow physics parameters without the need to go to complicated three dimensional computer codes.

The work studies the simple means of predicting the flow properties in a vaneless diffuser, and assessing the validity of the assumptions in trying to solve simultaneously a series of equations relating the properties of the diffuser to the radius by using the computer softwares, Excel and MATLAB.

The study focuses on the change in the properties through a vaneless diffuser with respect to the radius. Relations of all the properties are developed.

The theoretical relations are determined under specified boundary conditions of aerodynamic and thermodynamic conditions.

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NOMENCLATURE

Arabic

A	Flow area
AR	area ratio
C	Absolute velocity
Cr	Velocity component in the radial direction
Cu	Velocity component in the tangential direction
Cz	Velocity component in the z-direction
Cf	Skin-friction coefficient
Cp	Diffuser pressure recovery
c	Local speed of sound
D	Diameter
dQ	Heat transfer rate from the fluid
H	Non dimensional effective passage height, h/hT
h	Effective passage height
h'	Coefficient of heat transfer
K	Loss coefficient
M	Local Mach number
N	Rotational speed parameter
n	Polytropic exponent
P	Pressure ratio, p/po

p Static pressure

r Radius

R^* radius ratio, r/rT

R perfect gas constant

r, θ, z cylindrical coordinates in the r, θ, z directions

T static temperature

T_w Wall temperature

<u>Greek</u>

α flow angle

θ tangential direction, half-divergence angle of conical or channel diffusers

2θ Full divergence angle

δ^* Boundary layer thickness

ρ density

ξ non dimensional friction coefficient , $c_f\left(\frac{r_T}{h_T}\right)$

η efficiency

μ slip factor

τ shear stress due to skin friction

ω Rotational speed

<u>subscripts</u>

1 inlet

2 outlet

i ideal

0 total

t	local
r	radial direction
u	tangential direction

LITERATURE SURVEY

Due to the widespread use of the vaneless diffuser for centrifugal pumps and compressors, computational techniques for the vaneless diffuser emerged early. One of the earliest efforts to predict the performance of the vaneless diffuser was reported by Brown. In this work a modified Bernoulli equation was written in the direction of the streamline for a constant area vaneless diffuser, and a skin friction coefficient was introduced. By comparing the measured wall static pressures with predicted results, a suitable value for skin friction was established. This work showed reasonable agreement between the theoretical model and measured results and could be considered a one-dimensional single parameter model .

In subsequent work, Stanitz(1952), introduced a comprehensive treatment of the vaneless diffuser(Fig.1), and published a correct set of one-dimensional equations for the radial momentum, tangential momentum, conservation of mass, conservation of energy, and the equation of state. In this case a single empirical parameter was still employed but was introduced independently to both the radial and tangential momentum equations. Calculations of this type were reported by Johnston and Dean (1965). A further comparison of this computational approach was reported by Japikse.

Fauldres (1954) reported results that showed that the skin friction coefficient in the inlet (developing) region of a vaneless diffuser is quite high compared to fully developed flow, but the fully developed flow level is

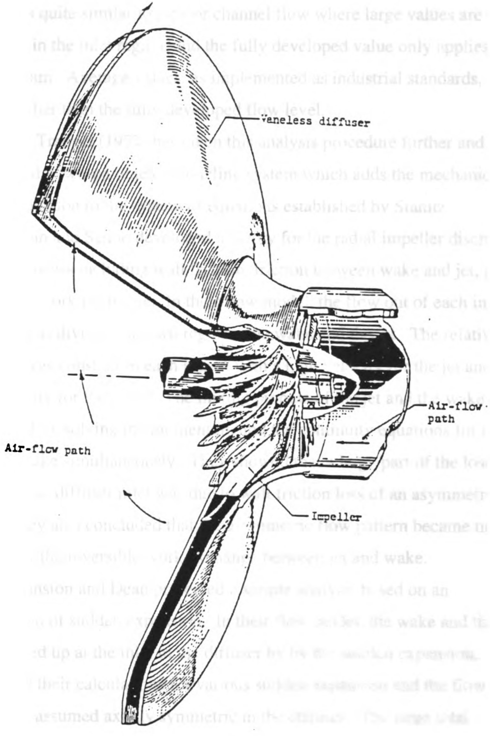


Fig.1

Vaneless Diffuser

approximately correct near the end of the vaneless diffuser. Thus the process is quite similar to pipe or channel flow where large values are to be expected in the inlet region, and the fully developed value only applies well downstream. Average values, as implemented as industrial standards, tend to fall higher than the fully developed flow level.

W. Traupel (1977) has taken this analysis procedure further and has introduced a two-parameter modeling system which adds the mechanical energy equation to the system of equations established by Stanitz.

Dean and Senoo developed a theory for the radial impeller discharge mixing process including wall friction, friction between wake and jet, and reversible work exchange. In their flow model, the flow out of each impeller passage was divided into two regions, the wake and the jet. The relative velocity was constant in each region with a high velocity for the jet and a low velocity for the wake. The mixing process of the jet and the wake was calculated by solving the momentum and the continuity equations for the jet and the wake simultaneously. They showed that a large part of the loss at the vaneless diffuser inlet was due to wall friction loss of an asymmetric flow. They also concluded that the asymmetric flow pattern became uniform rapidly by the reversible work exchange between jet and wake.

Johnston and Dean presented a simple analysis based on an assumption of sudden expansion. In their flow model, the wake and the jet were mixed up at the inlet to the diffuser by by the sudden expansion. They compared their calculations for various sudden expansion and the flow was thereafter assumed axially symmetric in the diffuser. The large total pressure loss at the vaneless diffuser inlet was attributed to the mixing loss centrifugal blower outlet flows with those based on the Dean and Senoo model. Their simple model gave very similar predictions of total pressure loss to those predicted by the more precise method over a wide range of compressor parameters.

Senoo and Ishida found that the flow at the exit of a centrifugal blower is artificially distorted so that decay of the asymmetric flow in the vaneless diffuser is experimentally examined. It is concluded that the shear force between the wake zone and the jet zone does not play the major role for the behavior of flow in the vaneless diffuser, and the behavior is mainly controlled by the reversible work exchange.

Based on the above survey, we will present in this work a documentation of some simple means of flow predictions in a vaneless diffuser. Due to time limitations, we will only present a one-dimensional approach using computer software packages Excel and Matlab.

CHAPTER ONE

Introduction

The design of turbomachinery is dominated by diffusion-the conversion of velocity or dynamic head into stream pressure. Every blade row in a typical axial compressor is a collection of parallel diffusers. In most centrifugal compressors both the rotor and the radial diffuser are limited by the diffusion capabilities of the flow channels.

1.1 The impact of diffusers on turbomachinery performance

In the first decade of this century a strong debate raged in the academic society as to the practical utility of placing an exhaust diffuser downstream of a hydroelectric turbine. Experts argued back and forth whether the exhaust diffuser would, or would not, improve the performance of the turbine. The counter arguments essentially maintained that the fluid had already left the turbine and little good could be done; the proponents recognized the importance of increasing the expansion ratio across the turbine rotor by the reduction in rotor back pressure with the use of a well-designed diffuser.

Today' s arguments and concerns over the role of the diffuser are significantly more advanced. Nonetheless, the details of the diffuser design and performance are in some instances as vague as the early debate on diffuser application for hydroturbines.

Fluid machinery is conveniently divided into positive displacement and turbomachinery categories. The distinction follows directly along the lines of Newton' s Second Law of motion as applied either in Cartesian coordinate system or in cylindrical coordinate system. In the Cartesian coordinate system, Newton' s Second Law indicates that the force applied to an object will be equal to the change in linear momentum which is the basic principle behind positive displacement equipment. In the cylindrical coordinate system, Newton' s Second Law expresses the torque being proportional to the change in angular momentum. This principle leads directly to the Euler turbomachinery equation which expresses the energy transfer through turbomachinery as the change in UC_u. Thus, the inherent function of turbomachinery involves the exchange of significant levels of kinetic energy in order to accomplish the intended purpose. As a consequence, very large levels of kinetic energy frequently accompany the work input and the work extraction processes, often on the order of 10-50% of the total energy transferred. Thus efficient diffusers are absolutely essential for good turbomachinery performance. With kinetic energy intensities of this level at the exit of the impeller; it is not hard to appreciate that the performance of a diffuser influences the overall efficiency level of a turbomachine. Thus the detailed processes that occur in diffusing elements must be carefully understood and thoroughly optimized if good turbomachinery performance is to be achieved.

The range of diffuser performance levels can be appreciated by considering Fig 1.1. This diagram (by Japikse) presents a loss map which plots the loss coefficient K in a typical diffusing element versus the pressure recovery for the same diffuser. Maps of this type serve to focus many important facts concerning diffuser performance in a compact fashion. One can immediately realize the impact of area ratio on diffuser performance, and high level of recovery will only be obtained if the area ratio is sufficiently high. There is also the implicit requirement that the diffuser must be designed so that the effectiveness is quite high.

1.2 Types of diffusers

1.2.1. Overview

In chapter 4, it will be shown that the basic equations of motion reveal the importance of both geometric and aerodynamic parameters on the ultimate performance of a diffuser. The specification of a wide variety of geometric and aerodynamic parameters is essential before the performance of a diffuser is uniquely given. In this section, the various geometric parameters are first reviewed for all classes of diffusers. A general definition of the different aerodynamic parameters is given in the next section.

1.2.2 Geometric Parameters:

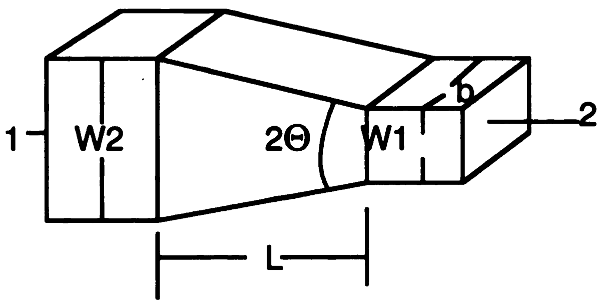
1.2.2.1 Channel diffuser geometric specification

The geometric specification of a channel diffuser is, at first appearance, comparatively straightforward. A simple schematic of a channel diffuser is shown in Fig 1.2.1, and the essential parameters which must be considered are defined in this figure. From these different geometric parameters, dimensionless parameters are formed as follows :

W1=throat width, b =throat depth

L =centerline length, 2θ=divergence angle

At=throat area=bW1, Ae=exit area=bW2



Sketch of Channel Diffuser

Fig. 1.2.1.

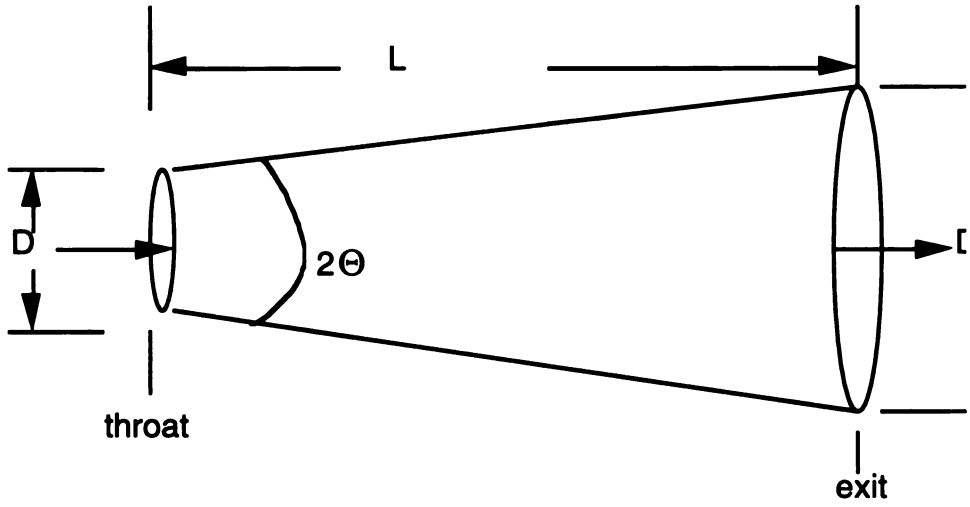
non-dimensional length	L/W_1
aspect ratio	$AS = b_1/W_1$
area ratio	$AR = A_2/A_1$

Not all these geometric parameters are independent. There is a fixed relationship (correlation) between the area ratio and the other geometric parameters as follows :

$$AR = 1 + 2\,(L/W1)\tan\Theta$$

1.2.1.3 Conical diffusers

The definition of the basic geometric parameters for conical diffusers is quite similar to that given previously for a channel diffuser, as illustrated below in Fig 1.2.2. Again, it is possible to define various dimensionless parameters as follows:



Sketch of Conical Diffuser

Fig. 1.2.2.

dimensionless length L/D_1

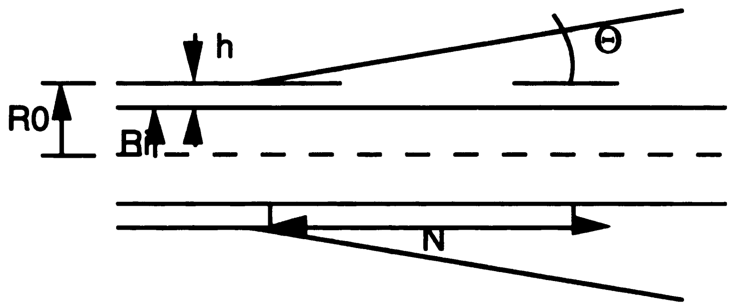
area ratio $AR = A_2/A_1$

The area ratio for a conical diffuser is, of course, dependent on other geometric variables. The dependence is given as follows :

- $AR = (1+2\,(L/D)\tan\Theta)^2$

1.2.1.4 Annular diffusers

It is more difficult to define the essential geometric parameters for annular diffusers since the number of independent variables has increased. Here the essential variables are:



Straight Core Annular

Fig. 1.2.3.

non-dimensional length $L/\Delta r$ or L/h_1

area ratio $AR = A_2/A_1$

Consider Fig 1.2.3, where the types of straight wall annular diffusers are shown. For the equi-angular case, the geometry is specified by:

$AR = 1+2\,(L/h)\sin\Theta$

For the straight core (constant hub) case the equation is more complex and becomes:

$$AR = 1+\frac{2L\sin\Theta}{h\left(1+{}^{R_i}\!/\!{}_{R_o}\right)}+\frac{L^2\sin^2\Theta}{h^2}\left[\frac{\left(1-{}^{R_i}\!/\!{}_{R_o}\right)}{\left(1+{}^{R_i}\!/\!{}_{R_o}\right)}\right]$$

and for more complex, but more common, case of independent changes in Θ_1 and Θ_2 one obtains:

$$AR = 1+2\frac{L}{h}\frac{\left(\sin\Theta_1+{}^{R_i}\!/\!{}_{R_o}\sin\Theta_2\right)}{\left(1+{}^{R_i}\!/\!{}_{R_o}\right)}+\frac{L^2}{h^2}\frac{\left(1-{}^{R_i}\!/\!{}_{R_o}\right)\left(\sin^2\Theta_1-\sin^2\Theta_2\right)}{\left(1+{}^{R_i}\!/\!{}_{R_o}\right)}$$

Unfortunately, many annular diffusers are even more complex and include curved walls. For such cases, the AR to L/h relationship must be derived for each specific case.

1.3 Examples of Common Diffusing Systems for centrifugal pumps or compressors

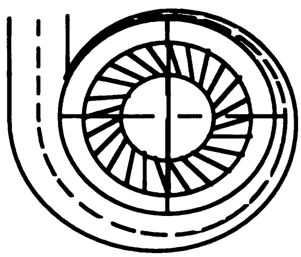


Fig 1.3.1
Volute diffuser

VOLUTE--used for low pressure ratio, mostly for single stage, and has radial thrust at off design flow. Even though widely used, still needs researching.

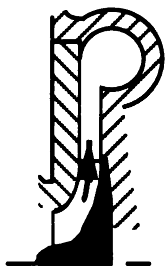


Fig 1.3.2
Vaneless diffuser

VANELESS DIFFUSER--tolerates large range of flow angles (60-80 degrees); simple annular channel, but bulky.

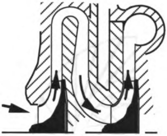


Fig 1.3.3

Return channel

RETURN CHANNEL--employed in multistage applications, yet much is not known about its performance.

Fig 1.3.4

Straight channel

STRAIGHT CHANNEL/WEDGE--has simple geometry, easy to manufacture ; very popular, but large in size.

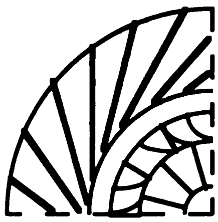


Fig 1.3.5

Straight plate

STRAIGHT PLATE--has large number of vanes, Z>30, and not so good pressure recovery.

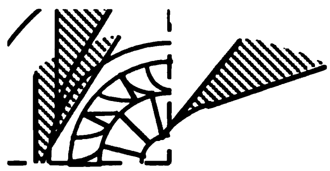


Fig 1.3.6

Vaned island

VANED ISLAND--is a refined straight channel for high pressure ratio and Mc3>1; has good pressure recovery, but again large in size.

Fig 1.3.7
Circular arc

CIRCULAR ARC--has simple geometry, but no outstanding aerodynamic characteristic.

Fig 1.3.8
Cambered diffuser

CAMBERED/AEROFOIL--used for transonic and subsonic applications, small size and good pressure recovery. Its design is based on axial cascade data.

Fig 1.3.9

Twisted diffuser

TWISTED--is a refined cambered vane to produce good efficiency, wide range and high pressure ratio.

Fig 1.3.10

Multiple cascade

MULTIPLE CASCADE--is a cambered van in cascade for higher efficiency with more manufacturing process.

Fig 1.3.11

Conical diffuser

PIPE/CONICAL--used for higher pressure ratio and transonic applications, large in size, but with good pressure recovery.

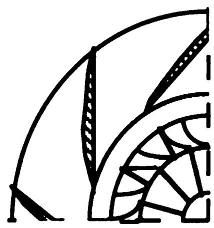


Fig 1.3.12
Low solidity diffuser

LOW SOLIDITY--used for low flow angles, but with good pressure recovery and range. Currently of great interest.

CHAPTER TWO

ESSENTIAL PARAMETERS OF DIFFUSER PERFORMANCE

In this section we will refer to general types of diffusers, Radial diffusers will be discussed in section 2.3.

2.1 Overall Performance Parameters

2.1.1 Ideal Pressure Recovery

The pressure recovery of a diffuser (actual or ideal) is most frequently defined as the static pressure rise through the diffuser divided by the inlet dynamic head; in other words:

$$C_p = \left(p_2 - p_1\right) \Big/ \left(p_{02} - p_1\right)$$

which is a very simple way of thinking about the fundamental purpose of a diffuser. An ideal pressure recovery can be set if the flow is assumed to be isentropic and the Bernoulli equation is used both in the numerator and the denominator to reduce the expression to a velocity in and a velocity out. Then, by employing the conservation of mass, the relationship can be converted to an area ratio for incompressible flow. We obtain the following:

- $Cp = 1 - v^2_{exit} / v^2_{inlet} = 1 - 1/AR^2$

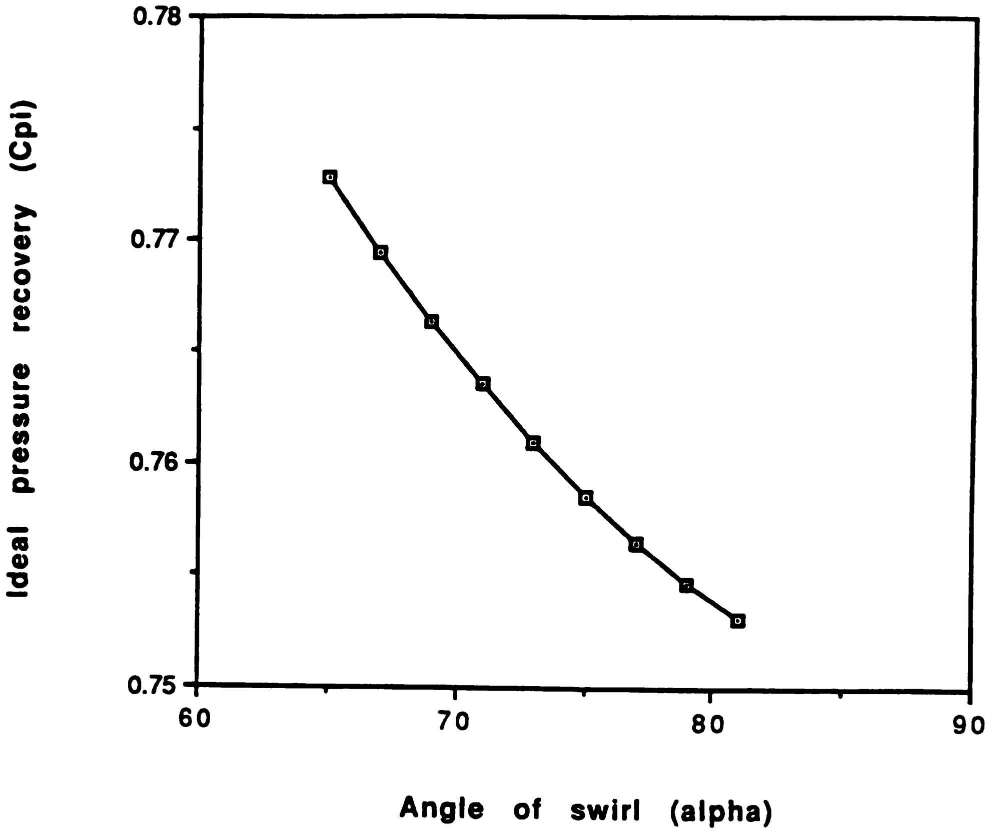
This expression is very well known to most engineers, and it does show the ideal pressure recovery as a useful reference level. However, it can also be used to deduce some very important functional relationships.

For example, in an annular diffuser, a number of different variables can influence the variation of pressure recovery under the conditions of swirling flow. Thus if we write a general expression for the ideal pressure recovery in an annular diffuser, with inlet swirl, one obtains:

$$Cpi = 1-\left[\frac{\bar{r}_1}{r_2}\right]^2\left[\frac{\tan^2\alpha_1 + b_1^2/b_2^2}{\tan^2\alpha_1 + 1}\right]$$

This equation shows that diffuser inlet to diffuser outlet radius ratio is very important if high recovery is to be achieved. It also shows that the inlet to exit passage depth ratio plays a role. The swirl term, in practice, can only be suppressed by designing a diffuser with large radius ratio; another way of saying the same thing is to realize that the swirl component must be recovered in accordance with the law of conservation of angular momentum: rC_θ=constant. The above expression shows maximum recovery with respect to swirl angle $(\partial Cp/\partial\alpha_1 = 0)$ when b1/b2=1; this result is independent of α. In fact when b1/b2<1 (Fig 2.1.1), (common case) the ideal recovery will continue to decrease with increasing swirl. This trend has often been observed in annular diffuser data. For the particular case where b1/b2=1, the equation reduces simply to the form of $Cp = 1-(1/AR^2)$, the same as for the no-swirl problem. The ideal pressure recovery coefficient often illustrates important trends which may be found in actual data.

2.1.2 Static Pressure Recovery and Effectiveness



$$Cpi = 1-\left[\frac{\bar{r}_1}{r_2}\right]^2\left[\frac{\tan^2\alpha_1 + b_1^2/b_2^2}{\tan^2\alpha_1 + 1}\right]$$

Fig 2.1.1

Ideal pressure recovery coefficient vs

swirl angle

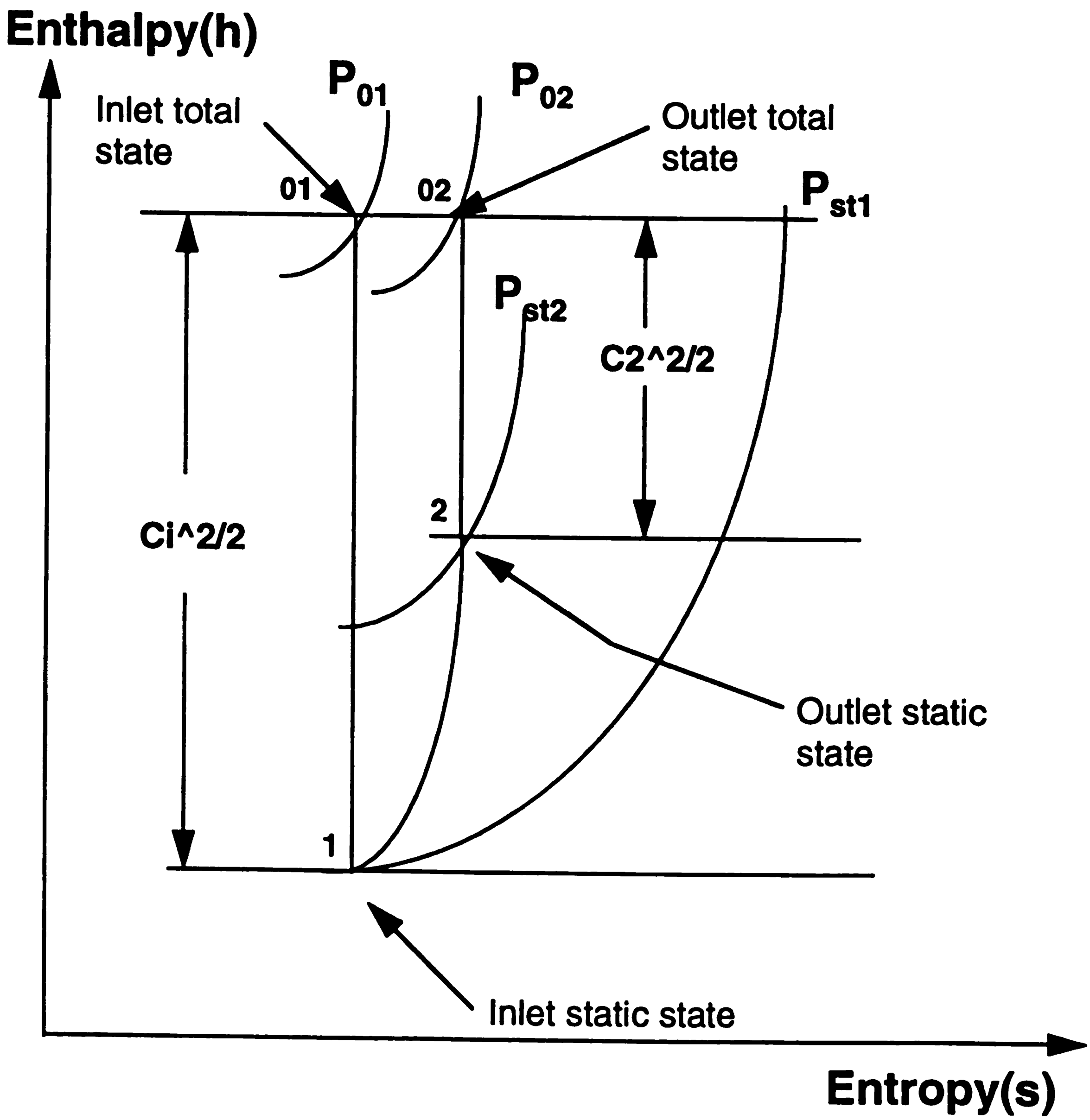


Fig. 2.1.2

Diffusion on the h-s diagram

The static pressure recovery coefficient was defined in simple terms in the preceding section. The most common definition is simply the one given above, namely the percent of inlet kinetic energy which is converted to a static pressure rise through the diffuser. However, other forms have also been employed. Most forms use the static pressure rise through the diffuser as the numerator, but the denominator occasionally varies. In some instances, the difference between the inlet and outlet dynamic heads is used in the denominator instead of simply the inlet dynamic head. In other cases, the denominator may actually be the pressure recovery achieved through a sudden expansion (Borda-Carnot) of the same area ratio. The latter has been used but is not too common. Fortunately , the first convention, described in the preceding section, is by far the most common convention. It is also the easiest to work with and the most revealing for modeling.

The diffuser effectiveness is simply the relationship between the actual recovery and the ideal pressure recovery. One can write:

- $\eta = Cp/Cp_{ideal}$

2.1.3 Total Pressure Loss

In addition to pressure recovery, the designer must be concerned about the loss in total pressure through the diffuser. This loss coefficient, in order to serve any useful and practical purpose, must refer to the entire flow field since the diffuser is a basic fluid dynamic element in some larger system.

Thus, one must be concerned with some integrated value of total pressure loss including all stream tubes through the diffuser. The most common definition of loss coefficient is as follows:

- $K = \left(\bar{p}_{01} - \bar{p}_{02}\right) / \left(\bar{p}_{01} - \bar{p}_{1}\right)$

In this case, integrated mass average parameters are used across the diffuser inlet and outlet. It is important to emphasize that detailed measurements are required across the inlet and outlet, with suitable numerical averaging across the entire flow field, in order to establish an accurate level of the total pressure loss for a diffuser.

2.1.4 Distortion

An additional performance parameter is the distortion level leaving a diffuser. This parameter is essential only if the flow into the combustor is to be well understood, or the flow into any other critical element such as a heat exchanger core or regenerator. Unfortunately, the industry has not established a common standard and some of the techniques used have not been documented. The basic concept is to show how the velocity field or the total pressure distribution departs from some type of norm at the exit of the diffuser.

2.2 Aerodynamic Parameters of Machine Performance

Virtually all of the controlling aerodynamic parameters are manifest at the inlet to the diffuser. In early periods of diffuser research, these parameters were largely ignored but, with time, it was found that different aerodynamic parameters became crucial; to specifying the performance of the diffuser, depending on the type of diffuser involved. In the following subsections, the different parameters are outlined, definitions as used in this text are given and examples where the parameter has been found to be important are presented.

2.2.1 Aerodynamic Blockage

The basic boundary layer equations, reveal the importance of the displacement thickness as a characteristic length scale of the inlet (momentum deficient) boundary layer flow. In the early 1960, Stratford and Tubbs (1965) and Bragg recognized the importance of boundary layer displacement thickness to the diffuser recovery process. From their experiments they deduced that thin inlet boundary layers should be beneficial to high diffuser recovery and that longer and longer diffusers would be required to achieve higher levels of recovery as the inlet boundary thickness increase.

The boundary layer displacement thickness was used informally by various investigators until it was conveniently put into a parameter by Sovran and Klomp called the (aerodynamic) blockage parameter. The blockage is simply the friction, or percentage, of the inlet passage area which is occluded by the boundary layer displacement thickness on all walls. Frequently, the displacement thickness is taken as equal on all surfaces and then the following relationships ensue:

- $B = 2\,\delta^*/W_1$ (for channel diffusers with high aspect ratio, i.e., neglecting end walls)
- $B = 4\,\delta^*/D_1$ (for conical diffusers with uniform inlet boundary layers)
- $B = 2\,\delta^*/h1$ (for annular diffusers with inlet passage height of h_1)

These definitions were proven to be effective and simple to use. However, if complex inlet flows are involved (or lower aspect ratios for channel diffusers), then the assumption of equivalent or equal boundary layers on the different inlet surfaces will fail and a more complex approach and specification is necessary.

In addition to the inlet aerodynamic blockage discussed above, mention is occasionally made in the literature of diffuser exit blockage. Sovran and Klomp developed a relationship that included the pressure recovery, area ratio and inlet blockage, and the exit blockage of the diffuser. In order to derive the equation, it was assumed that an isentropic core passes through the diffuser. The pressure recovery of a diffuser can be computed if the exit blockage, area ratio and inlet blockage are known. Sajben, et al. (1976) were able to deduce these necessary parameters and then compute the pressure recovery as a dependent variable.

2.2.2 Reynolds Number Dependence

Viscosity is clearly recognized as an important parameter in any fluid dynamic process. Typically, diffusers are characterized by a Reynolds number based on an inlet hydraulic diameter. Studies suggest that the Reynolds number is a comparatively weak parameter as long as the flow is in the fully turbulent regime (exception: very low aspect ratio channel diffusers). Very little data is available for the performance of diffusers in laminar or transitional regimes.

2.2.3 Inlet Mach Number

During the early years of diffuser research, the Mach number at the inlet to the diffuser was thought to be important at values of approximately 0.7 and performance was held to fall off past this point. This early belief was erroneous and it was based on incomplete measurements.

Now, it is clearly established by the work established by Dean that one must pass a Throat Mach number of 1.0 before developing any significant dependence on Mach number. Thus for unstalled flows, the Mach number is a comparatively mild parameter. The early error resulted from using a wall static pressure rather than a core static pressure. Hence streamline curvature suppressed the wall static pressure giving a misrepresentative Mach number.

2.2.4 Inlet Turbulence Intensity

The turbulence intensity is most frequently defined for all diffusers as follows:

- $$\left[\frac{1}{3}\left(u'^2 + v'^2 + w'^2\right)\right]^{0.5} / U$$

where the RMS turbulence intensity of all components is considered here. This is the most frequently employed parameter to specify the overall level of inlet turbulence intensity. It will be found, however, that the problem of inlet turbulence is more complex, and under various conditions it will be desirable to have a more detailed description of the turbulence structure (intensity and scale) entering a diffuser.

2.2.5 Inlet Velocity Profiles

No convention has been developed to specify the inlet velocity profile to a diffuser. However, various research programs have shown the effect to be significant. Both simply skewed inlet profiles and highly distorted inlet profiles have been considered and reported. Frequently, an integral scale defined as:

- $$\alpha^\circ = \int_0^A u^3 dA / \left(\bar{u}^3 A\right)$$

is used to define profile shape or distribution.

2.2.6 Inlet Swirl

Swirling flow into the inlet of a diffuser implies that there is a component of velocity in the tangential direction as the flow enters the diffuser. This effect has been considered most often for conical and annular diffusers, but probably is relevant also for channel diffusers as employed in various pumps and compressors. Most frequently, an inlet swirl angle is specified (the symbol α is often employed) to establish the level of inlet swirl.

2.3 Radial Flow (Vaneless) Diffusers

2.3.1 Overview

The radial inflow/ radial outflow (r/r) diffuser, also frequently known as a vaneless diffuser as it is employed for centrifugal compressors and centrifugal pumps, is similar in many regards to the channel, conical and A/A (axial inlet/ axial outlet) annular diffusers. It is also quite different in several important regards. Extensive experimental research has been carried out for the conical, channel and A/A annular diffuser as a discrete element; that is, it has been extensively researched as an individual element, quite apart from its frequent role in turbomachinery performance. The vaneless diffuser, that is the R/R annular diffuser, has received proportionally less attention as a discrete flow element and substantially more attention in the particular context of its turbomachinery application. In addition, engineers have been much more prone to calculate the approximate first-order performance of the vaneless diffuser than to employ empirical data bases. In fact, when the use of the vaneless diffuser is considered from the particular perspective of compressor or pump design, the necessary data bases are considerably weaker.

2.3.2 Passage Divergence and Length (Area Ratio); Zero Swirl

The R/R annular diffuser has been studied at zero swirl by four different groups. The most commonly cited reference is Feiereisen (1971) . Review of Feiereisen's work shows that the inlet region flow was strongly accelerated prior to entering the radial diffuser section. As a consequence, the overall recovery of the vaneless diffuser plus the axial inlet section was negligible and often comprised an accelerating system when both parts were taken together. It was found in this work that when the point of minimum pressure shifted from the radial outflow portion into the inlet bend, then separation was imminent. In fact separation was a complete total separation from the bend side and transitory stall was never observed, as found for many of the previous diffuser types. The mode of separation for this case was steady.

This question of flow separation takes an added significance when other earlier and subsequent investigations are considered. Moller (1965a, 1965b) studied a similar configuration but kept the axial inlet portion under close scrutiny, as well as the radial portion. In his design work, Moller deliberately attempted to limit diffusion in the inlet bend region and spent a considerable amount of time in that sector. Furthermore, he deliberately considered both low inlet aerodynamic blockage and a fully developed inlet profile. For his case, he found peak pressure recovery for the entire A/R system of 0.88 and 0.82 for the low blockage and the high blockage cases respectively. Moller was able to adjust the depth of the diffuser and found an optimum spacing equal to approximately 15% of the inlet pipe diameter. Not surprisingly, his results showed that separation occurred shortly after the inlet flow ceased to be accelerating or constant pressure. Clearly, the existence of diffusion in the bend region, where flow over convex surfaces is involved, becomes most difficult.

In his 1971 paper, de Krasinski evaluated a similar configuration (an axial inlet and a radial parallel diffuser) but allowed the inlet bend contour to rotate at various speeds. Best performance was found in a spacing-to-diameter ratio of 0.15, with appreciable rotation. Rotation helped to control the boundary layer in this critical region and, once again, an accelerating flow in the bend region, or a spin stabilized flow, was necessary to obtain good diffuser performance.

Finally, Yahya and Gupta (1975) conducted the fourth principal study of the radial diffuser at zero swirl. Their case was a bit different in that the diffuser diverged by 10 degrees. Various area ratios were considered and good traverse results were available. The authors carefully studied the traverse data and integrated the profiles to obtain a mass average total pressure at inlet and at outlet. With such data, it is possible to obtain loss coefficients, pressure recovery, and effectiveness data.

2.3.3 Wall Contouring

Virtually every study of the vaneless diffuser has considered parallel walls or, pinched diffusers were employed for enhanced stability. The principal reason is that modifying only the diffuser passage width appreciably changes the meridional (radial) component of velocity which must be retained as a design variable for stability considerations for compressors and pumps. Thus little attention has been given to the deliberate modification of the passage depth for performance enhancement. However, the study by Yahya and Gupta (1975), cited above, did consider a diffuser with a 10 degree divergence.

2.3.4 Aerodynamic Blockage

Aerodynamic blockage played a key role in the performance of the channel, conical and straight centerline annular diffusers. The aerodynamic inlet conditions to the R/R annular diffuser are important, but comparatively little data is available as a systematic guide to understanding the level of inlet aerodynamic blockage for R/R diffuser performance.

2.3.5 Swirl

The influence of swirl was demonstrated to be quite significant for the conical diffuser and the A/A annular diffuser. In these preceding diffusers, several different effects were involved. For the conical and the annular diffusers whose centerline is very close to the axial direction, swirl provides essentially a stabilization of the boundary layer region and a very modest variation the core flow conditions. For diffusers with a substantial increase in radius from inlet to outlet, the angular momentum conservation applies and a good deal of recovery of the swirling kinetic energy is obtained. For the R/R annular diffuser, the conservation of angular momentum principle is inherent in the performance of the diffuser and substantial recovery is obtained simply by radius ratio in the recovery of the tangential , swirling velocity component. In this case, the direction of tangential velocity component and the orientation of the surfaces is such that a stabilization of the wall layers does not result; and, instead, the possibility exists that a boundary layer may be skewed in an undesirable direction. Thus the principle effect of swirl is changed so that it works extremely well in the core flow angular momentum exchange, but it is a disadvantage in the wall shear layers.

Wheeler and Johnston (1971) reported the first detailed, and perhaps only, study of the calculation of boundary layers in an R/R annular diffuser using a finite difference calculation method and various eddy viscosity mixing length modeling techniques. The investigation showed that first-order modeling of the basic integral parameters of the boundary layer could be achieved but that the calculation was extremely sensitive to the iteration between the core flow and the boundary layer, that is to the value of core pressure distribution.

2.3.6 Inlet Distortion

Inlet distortions have a pronounced effect on the performance of the vaneless diffuser. Inlet distortions include both distortions in the shape of the steady flow velocity field (either the tangential or meridional velocity components) and also variations with time. From the study by Senoo, et al. (1977), It may be observed that the shape of the initial velocity profile distortion is propagated well into the diffuser and has a significant impact on the onset of back flow or a skewed boundary layer separation.

2.3.7 Reynolds Number Influence

Comparatively little data has been achieved showing the impact of Reynolds number on the R/R radial annular diffuser performance.

2.3.8 Inlet Mach Number Effect

Only limited data showing performance of the R/R annular diffuser at different Mach number levels has been obtained. The data from Faulders suggests that the fall-off of Mach number is mild up to and including modest transonic Mach numbers.

2.3.9 Inlet Turbulence Intensity

No studies have been located on the R/R annular diffuser where the inlet turbulence intensity has been measured and/or systematically varied. Ellis did present a very brief paper arguing that induced vorticity at the inlet to the vaneless diffuser explains the shifting of stall zones from one side of the diffuser to the opposite side. By considering some velocity triangle distributions and the implied inlet vorticity, the author attempted to explain why the stall zones shift from one side to the other.

CHAPTER THREE

One-Dimensional Inviscid method for Flow physics prediction

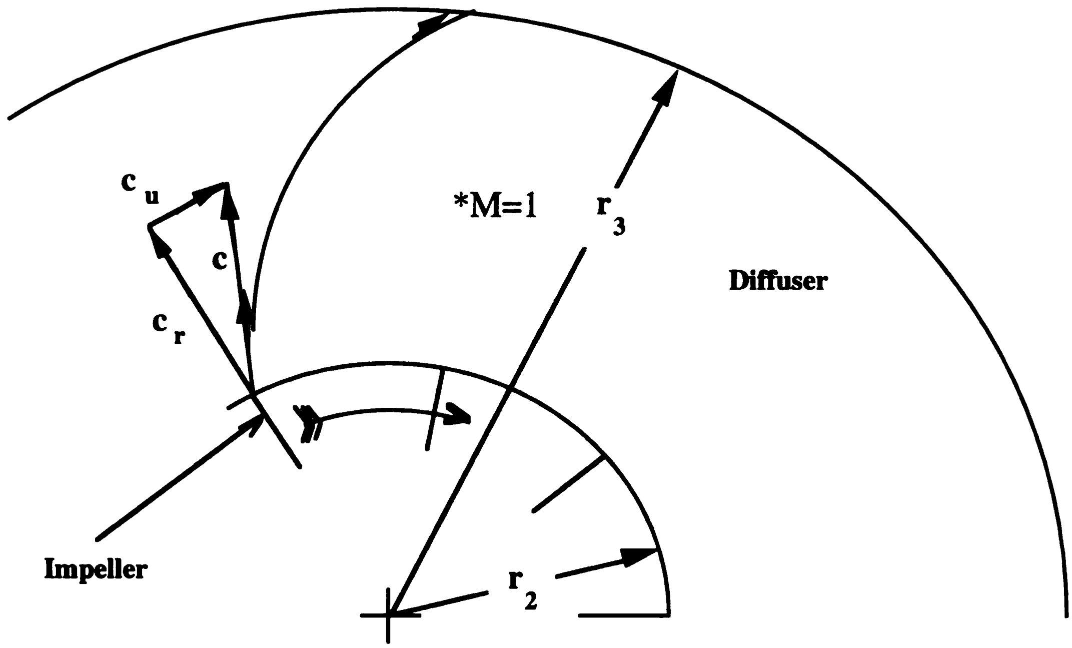


Fig. 3.1

Elementary view of flow velocities in vaneless diffuser

From the above figure, we see that the vaneless diffuser is the space between the compressor rotor exit and the compressor discharge opening. The flow leaving the compressor impeller has a high kinetic energy and since we are interested in the pressure delivered from the compressor, we use the vaneless space to transform the high kinetic energy into an additional pressure.

To estimate the change in the different parameters involved we can use different methods ranging from very simple (classroom) to 3-Dimensional computer modeling. We shall start by presenting one of the simple methods in this chapter and then move on to more complicated methods.

In the simplest approach to one-dimensional methods, the introductory (classroom) method is the most simple and less accurate method.

3.1 Classroom Method

we start with the conservation of angular momentum equation

$$C_u r = \text{cons tan t}$$

As we expect from the compressor behavior, that the tangential velocity decreases as the radius increases and pressure increases respectively from r2 to r3. This equation can be easily plotted given a range of the radius from 1 to 2 and an initial tangential velocity of 80. The result is shown in figure 3.2. To predict the change in the radial velocity, we have from the above figure that

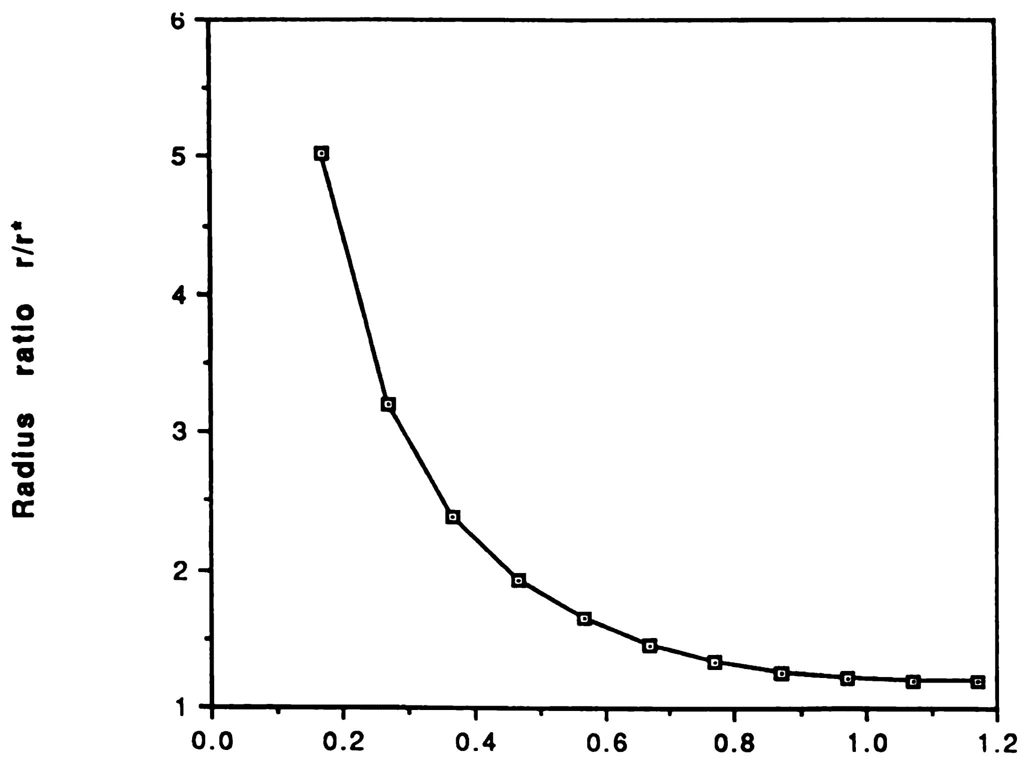
$$\tan\alpha = \frac{C_u}{C_r}$$

or, rearranging

$$C_r = \frac{C_u}{\tan\alpha}$$

a plot for this relation is shown in Figure 2. As shown from Figure 2, from the continuity equation

$$\rho C_r A = \text{cons tan t}$$



$\rho C_r A = \text{constant}$

Figure 3.2

Radial velocity vs. the area

as the area increases and the density increases the radial velocity decreases respectively and the kinetic energy transforms to pressure energy.

3.2 Point (*) approach:

Another approach for the one dimensional method is to specify a point (*) in the vaneless diffuser passage at which place the flow has a sonic velocity i.e. Mach number=1,

We denote the radial position at which M=1 by r* and all properties at this position by (*)

$$C_r = C\cos\alpha$$

continuity equation

$$\rho r C\cos\alpha = \rho^* r^* C^* \cos\alpha^*$$

Angular momentum equation

$$rC\sin = r^* C^* \sin\alpha^*$$

From appendix (B), we obtain

$$\tan\alpha^* = \tan\alpha\left[\frac{2}{\gamma+1}\left(1+\left(\frac{\gamma-1}{2}\right)M^2\right)\right]^{\frac{1}{\gamma-1}} \quad (1)$$

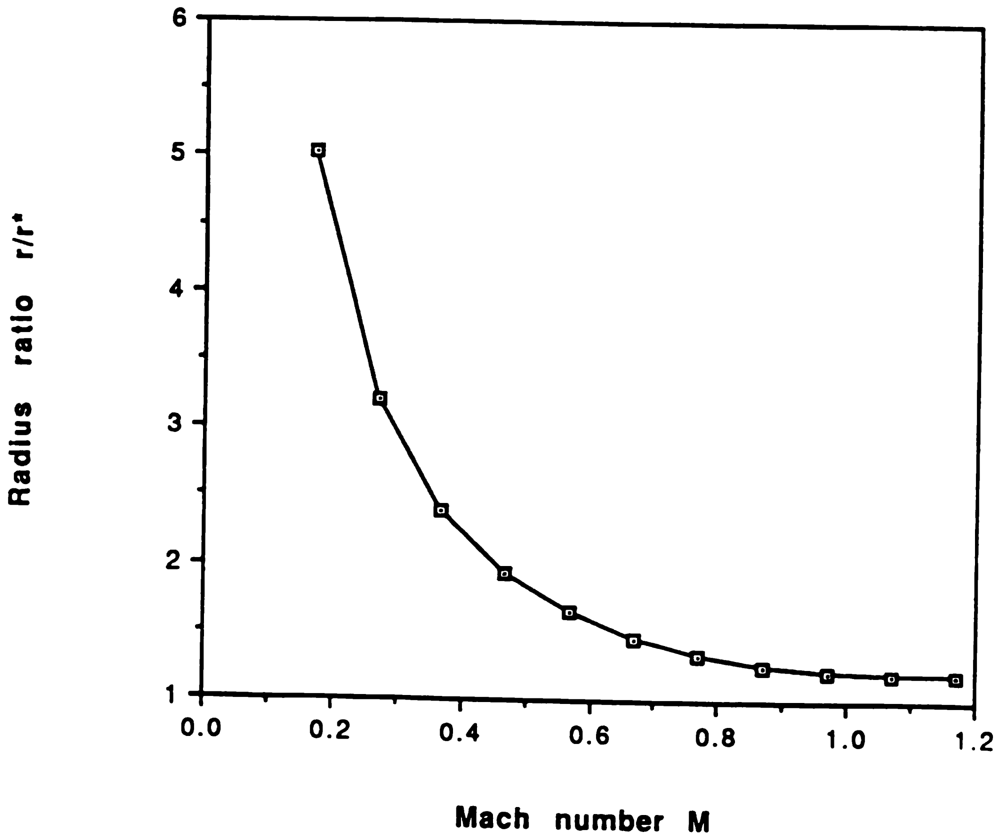
which is a relation between the angle alpha, alpha*, and the Mach number . α^* can be evaluated by substituting $\alpha = \alpha_2$ and $M = M_2$ which are set at design.

Further substitution renders

$$\frac{r^* \sin\alpha^*}{r\sin\alpha} = M\left[\frac{2}{\gamma+1}\left(1+\left(\frac{\gamma-1}{2}\right)M^2\right)\right]^{-\frac{1}{2}} \quad (2)$$

to determine r* substitute r=r2 and M=M2.

In Figure 3.3, we have a relation between r*/r vs. the Mach number. From this figure, we can deduce that as the radius increases over the sonic position



$$\frac{r^* \sin\alpha^*}{r \sin\alpha} = M\left[\frac{2}{\gamma+1}\left(1+\left(\frac{\gamma-1}{2}\right)M^2\right)\right]^{-\frac{1}{2}}$$

Figure 3.3

Radius ratio vs. Mach number

(*) the Mach number drops and the flow becomes subsonic, which results in the conversion of kinetic energy into pressure energy.

also Figure 3.4 shows the change of the angle alpha with the Mach number, we have a change of about 30 degrees over a Mach number range of 1, as we can see from the figure starting from our initial condition that the angle alpha increases drastically as the Mach number decreases that is in the vaneless space.

3.3 Integration of the momentum equation approach :

Another approach is that starting with the equations of momentum in the r and Θ directions and the equation of state

$$\frac{C_r \partial C_r}{\partial r} - \frac{C_u^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}$$

$$C_r \frac{\partial C_u}{\partial r} + C_r \frac{C_u}{r} = 0$$

by combining these equations we obtain

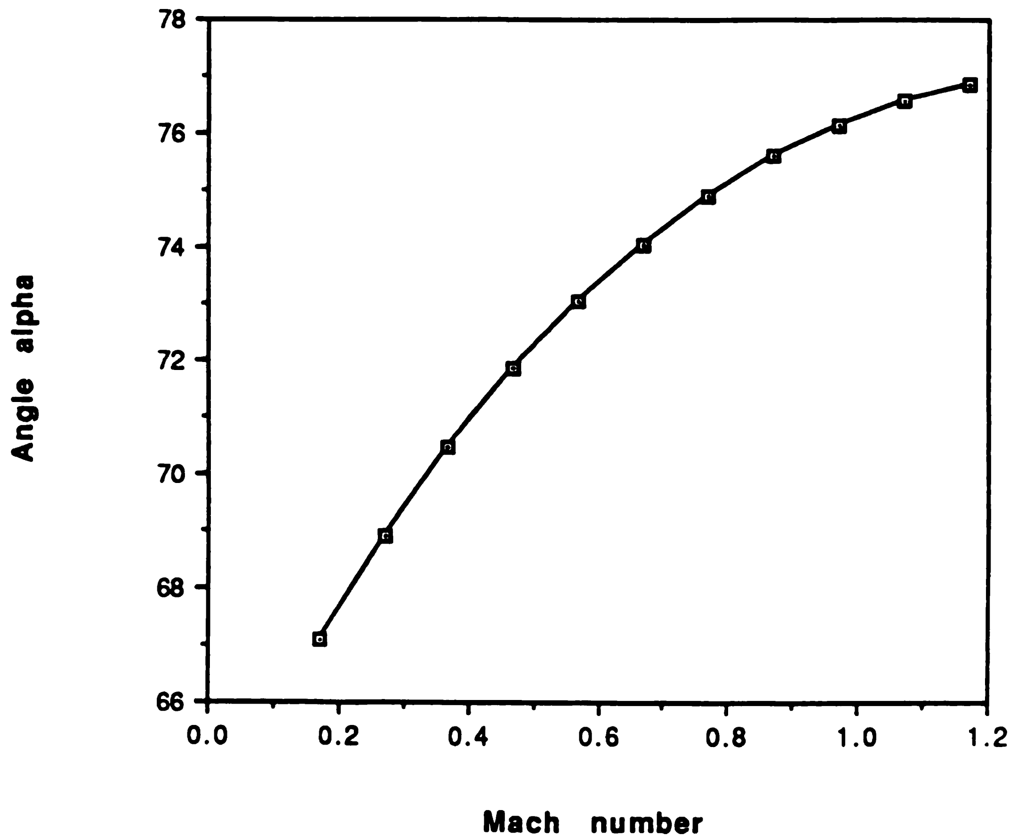
$$C_r \frac{\partial C_r}{\partial r} + C_u \frac{\partial C_u}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}$$

which becomes

$$\frac{\partial P}{\rho} + C_r \partial C_r + C_u \partial C_u = 0$$

Assuming the flow is adiabatic and frictionless we deduce from appendix(1)

$$\left(\frac{P}{P_2}\right)^{-\frac{1}{\gamma}} \frac{\partial P}{P_2} + \frac{C_r \partial C_r + C_u \partial C_u}{RT_2} = 0$$



$$\tan\alpha^* = \tan\alpha\left[\frac{2}{\gamma+1}\left(1+\left(\frac{\gamma-1}{2}\right)M^2\right)\right]^{\frac{1}{\gamma-1}}$$

Figure 3.4

Change in angle alpha vs. Mach number

which by integrating yields the following equation:

$$\left(\frac{P}{P_2}\right)=\left[1+\frac{\gamma-1}{2}M_2^2\left(1-\frac{C^2}{C_2^2}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

Figure 3.5 gives the result of plotting this equation, and as we can see that the pressure drops as the velocity increases or vice versa, which is expected in a diffuser since the main purpose is to increase the pressure at the account of the Kinetic energy.

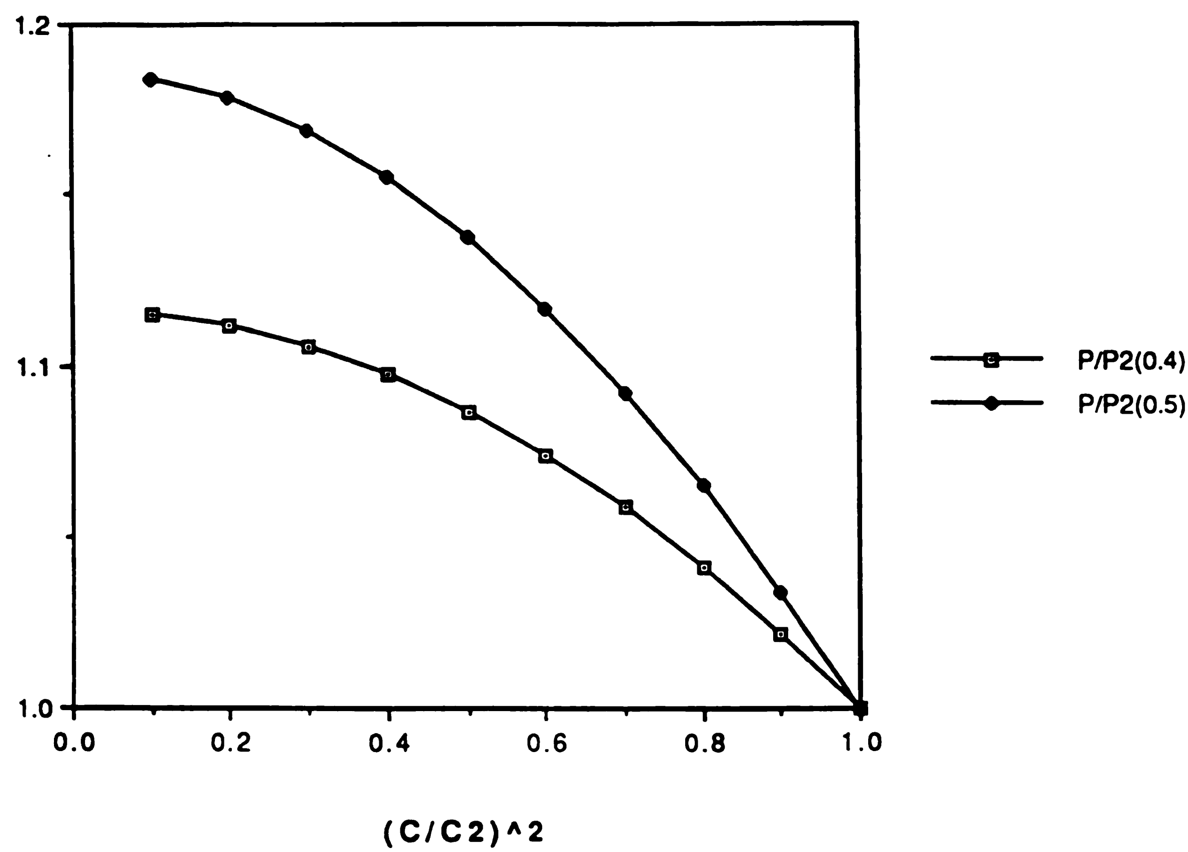
By using the velocity triangle relations and substituting into the above equation we obtain

$$\left(\frac{P}{P_2}\right)^{\frac{\gamma-1}{\gamma}}=1+\frac{\gamma-1}{2}M_2^2\left(1-\tau^2\sin^2\alpha_2-\lambda_D^{-2}\cos^2\alpha_2\right)$$

when plotting this equation we have too many parameters, we can eliminate one by assuming incompressible flow so that the change in density is negligible and Tau=1

we plot this relation for a radius ratios of 1.4 and 1.6.

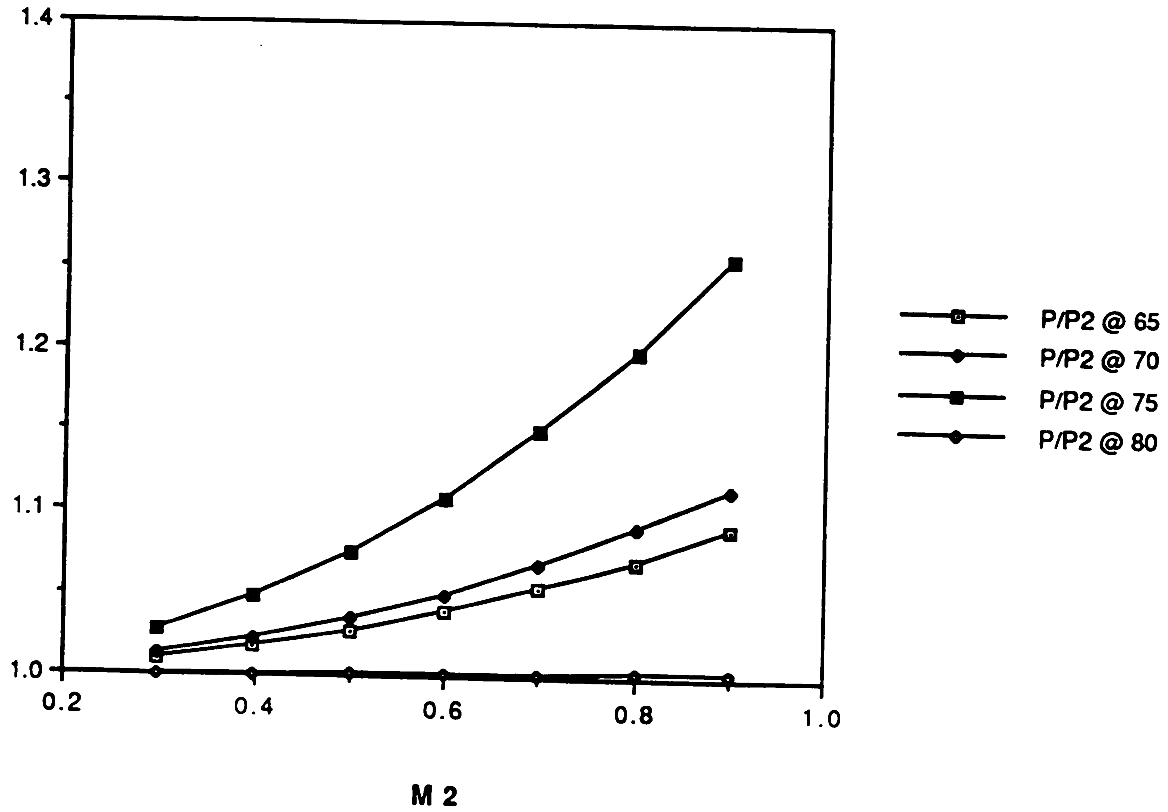
From Figure 6 and 7 we can see that the more space in the vaneless diffuser the more pressure recovery at the diffuser exit for different flow angles. Also, the higher the inlet Mach number (impeller exit) the higher the kinetic energy and hence the higher the conversion into pressure head.



$$\left(\frac{P}{P_2}\right)=\left[1+\frac{\gamma-1}{2}M_2^2\left(1-\frac{C^2}{C_2^2}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

Figure 3.5.

Pressure ratio vs. The velocity in the diffuser

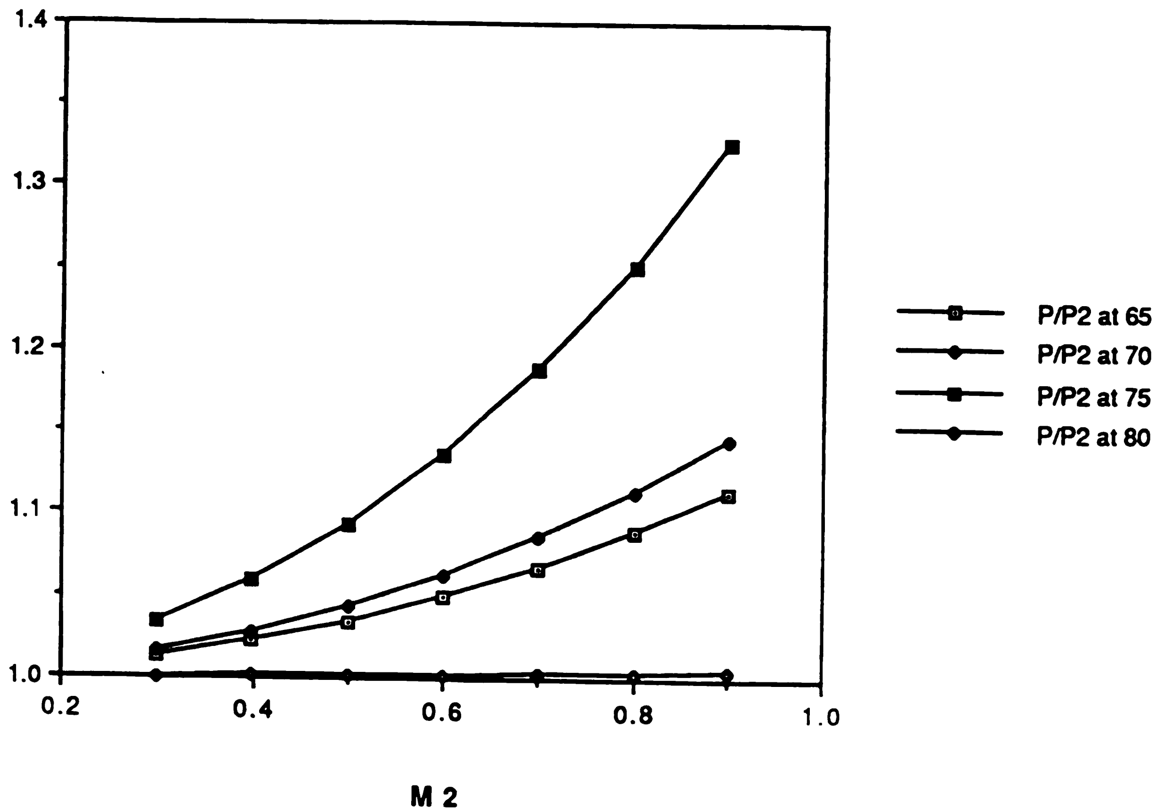


$$\left(\frac{P}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = 1 + \frac{\gamma-1}{2} M_2^2 \left(1 - \tau^2 \sin^2 \alpha_2 - \lambda_D^{-2} \cos^2 \alpha_2\right)$$

Figure 3.6.

Pressure vs. Mach number for a radius

ratio of 1.4



$$\left(\frac{P}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = 1 + \frac{\gamma-1}{2} M_2^2 \left(1 - \tau^2 \sin^2 \alpha_2 - \lambda_D^{-2} \cos^2 \alpha_2\right)$$

Figure 3.7.

Pressure vs Mach number for a radius ratio of 1.6

CHAPTER 4

ONE DIMENSIONAL VISCOUS APPROACH

In radial and mixed-flow centrifugal compressors, the vaneless diffuser is an annulus duct immediately following the impeller and of increasing radius; in the direction of flow. The high tangential velocity of the fluid entering the vaneless diffuser from the impeller decreases with increasing radius and, because the tangential velocity is generally the largest velocity component at the impeller discharge, the vaneless diffuser is an effective means of diffusing the fluid, that is, of converting the velocity head to static pressure. The principle by which this conversion is affected is demonstrated by the case for frictionless flow in the absence of heat transfer. For this case, and assuming that flow conditions are uniform in the tangential direction, the moment of momentum of the fluid is constant so that

$$C_u \, r = \text{constant} \qquad (1)$$

from which as the radius (r) increases the tangential velocity (Cu) decreases and therefore the pressure rises.

Among the advantages of vaneless diffusers is the fact that choke occurs only if C_r (radial velocity component) is sonic. This condition usually corresponds to such high flowrates that choke flow occurs in the impeller, instead of the diffuser as is the usual case for vaned diffusers. The compressor operating range is therefore wider with vaneless diffusers.

Another, and perhaps the most important advantage of the vaneless diffuser is the fact that if the tangential velocity at the impeller discharge is supersonic, the tangential velocity decelerates from supersonic to subsonic velocities without shock losses.

In order to analyze the performance of vaneless diffusers and in order to design these diffusers for optimum performance, it is necessary to have adequate theoretical methods to predict the variation in flow characteristics through the diffusers. These methods should include the effects of diffuser geometry, compressibility, heat transfer.

Differential equations are developed that relate the change in dependent variables with radius to the design and operating characteristics of the vaneless diffuser.

Velocity components:

The velocity (C) at a point on the mean surface of revolution is tangent to the surface and has components Cu and Cr in the r and Θ directions respectively.

$$C = \sqrt{C_u^2 + C_r^2} \tag{2}$$

The flow direction α on the mean surface of revolution is related to C_u and C_r as follows:

$$\tan\alpha = \frac{C_u}{C_r} \tag{3}$$

from which

$$C_u = C\sin\alpha \tag{4a}$$

$$C_r = C\cos\alpha \tag{4b}$$

Fluid particle

A fluid particle on the mean surface of revolution has the dimensions

$rd\Theta$ and dr on the surface of revolution and the height b normal to the surface.

Method of Solving

The state of the fluid at any point (r and Θ) on the mean surface of revolution is described by three thermodynamic properties, by the fluid velocity and the flow direction. These five properties can be determined from five fundamental relations:

Continuity.

Equilibrium in the direction of C_r (radial equilibrium).

Equilibrium in the direction of C_u (tangential equilibrium).

Equation of state.

The heat transfer equation.

In addition to these five fundamental relations certain definitions are required to express the resulting equations in terms of the desired properties. The properties that will be used in this analysis to describe the state of the fluid will be the static pressure p, the static density ρ, the total temperature T_o, the local Mach number M, and the flow direction α.

Mach number

The local Mach number M is defined by

$$M^2 = \frac{C^2}{\gamma R^* T} \tag{5}$$

where γ is the ratio of specific heats, R^* is the gas constant and T is the local static temperature. By differentiating equation (1) we obtain

$$\frac{1}{M^2}\frac{dM^2}{dr} = \frac{1}{C^2}\frac{dC^2}{dr} - \frac{1}{T}\frac{dT}{dr} \tag{5a}$$

Total temperature

The total temperature To is defined by

$$To = T + \frac{C^2}{2Cp} \tag{6}$$

where Cp is the specific heat at constant pressure. By differentiating equation (3), we obtain

$$\frac{1}{To}\frac{dTo}{dr} = \frac{1}{T}\frac{dT}{dr} + \frac{\left(\frac{\gamma-1}{2}\right)M^2}{\left(1+\frac{\gamma-1}{2}M^2\right)}\frac{1}{M^2}\frac{dM^2}{dr} \tag{6a}$$

from equations (2) and (4), we obtain

$$\frac{1}{C^2}\frac{dC^2}{dr} = \left(\frac{1}{1+\frac{\gamma-1}{2}M^2}\right)\frac{1}{M^2}\frac{dM^2}{dr} + \frac{1}{To}\frac{dTo}{dr} \tag{6b}$$

Continuity equation

The continuity equation for compressible flow in vaneless diffuser

$$\rho C_r rb = constant$$

from which

$$\frac{1}{\rho}\frac{d\rho}{dr} + \frac{1}{C_r}\frac{dC_r}{dr} + \frac{1}{r} + \frac{1}{b}\frac{db}{dr} = 0 \tag{7}$$

Equilibrium Equations

Radial equilibrium

The equation for radial equilibrium of a fluid particle in the direction of C_r is obtained from a balance of the pressure forces, shear forces and inertia forces(Appendix 2).

$$\bullet\ \frac{1}{\rho}\frac{dP}{dr} + \frac{c_f C^2 \cos\alpha}{b} = \frac{C_u^2}{r} - C_r\frac{dC_r}{dr} \tag{8}$$

Tangential equilibrium

The equation for equilibrium of a fluid particle in the tangential direction is obtained from a balance of the shear forces with the force required for acceleration(Appendix 2).

$$-\frac{c_f C^2 \sin\alpha}{b} = C_r \frac{dC_u}{dr} + \frac{C_u C_r}{r} \tag{9}$$

Equation of state

The perfect gas equation or the equation of state is as follows:

$$P = \rho RT \tag{10a}$$

from which we get after differentiation

$$\frac{1}{P}\frac{dP}{dr} = \frac{1}{T}\frac{dT}{dr} + \frac{1}{\rho}\frac{d\rho}{dr} \tag{10b}$$

Heat transfer equation

The heat transfer rate to the diffuser must equal the heat transfer rate from the fluid. The heat transfer rate from the diffuser casing is given by

$$dQ = 2h'\left(T_w - T_0\right)2\pi r dr \tag{11a}$$

where h' is the coefficient of heat transfer, T_w is the wall temperature and dQ is the heat transfer rate.

The heat transfer rate from the fluid is given by

$$dQ = \rho C_r 2\pi r b c_p \frac{dT_0}{dr} dr \tag{11b}$$

equating both equations together we obtain

$$\frac{1}{T_0}\frac{dT_0}{dr} = \frac{2h'}{\rho C_r b c_p}\left(\frac{T_w}{T_0} - 1\right) \tag{11c}$$

an approximate value for h' can be obtained from Reynolds' analogy between friction and heat transfer

$$\frac{h'}{c_p \rho C} = \frac{c_f}{2}$$

substituting for h' into equation (11c) we obtain

$$\frac{1}{T_0}\frac{dT_0}{dr}=\frac{c_f \sec\alpha}{b}\left(\frac{T_w}{T_0}-1\right) \tag{12}$$

which gives the change in total temperature with radius as a function of the skin-friction coefficient and the ratio.

A review of the equations up to this point indicates nine unknowns and nine equations for the analysis method.

The unknowns are $P, \rho, T, T_0, M, C, C_u, C_r and \alpha$

We will combine the nine equations to obtain three equations involving three unknowns; T_0, M and α. These three differential equations can be combined to solve for T_0, M and α successively.

Auxiliary differential equation

An auxiliary differential equation for the pressure P in terms of T_0, M and α is obtained from the equilibrium equations

$$\frac{1}{\rho C^2}\frac{dP}{dr}=-\frac{1}{2C^2}\frac{dC^2}{dr}-\frac{c_f \sec\alpha}{b} \tag{13a}$$

but
$$\rho C^2=\frac{P}{RT}C^2\times\frac{\gamma}{\gamma}=\gamma P M^2 \tag{13b}$$

$$\frac{1}{\gamma P M^2}\frac{dP}{dR^*}=-\frac{\gamma M^2}{2}\left[\left(\frac{1}{1+\gamma-1/_2 M^2}\right)\frac{1}{M^2}\frac{dM^2}{dR^*}+\frac{1}{T_0}\frac{dT_0}{dR^*}+\frac{2\xi}{H\cos\alpha}\right] \tag{13c}$$

where

$$\xi=c_f\left(\frac{r_T}{b_T}\right) \tag{13d}$$

$$P=\frac{P}{p_0}$$

$$R^*=\frac{r}{r_T}$$

where P_0 is the compressor inlet total pressure, r_T is the impeller tip radius and b_T is the effective diffuser height at the impeller tip.

Total temperature

$$\frac{1}{T_0}\frac{dT_0}{dR^*}=\frac{2h'}{\rho C_r H c_p}\left(\frac{T_w}{T_0}-1\right)\left(\frac{r_T}{b_T}\right) \tag{14a}$$

using Reynolds' analogy

$$\frac{1}{T_0}\frac{dT_0}{dR^*}=\frac{\xi}{H\cos\alpha}\left(\frac{T_w}{T_0}-1\right) \tag{14b}$$

Mach Number

In order to determine the differential equation for the Mach number squared it is first necessary to express the second term of the continuity equation (7) in terms of known variables. From radial equilibrium equation (8) together with equations (4) and (13b).

$$\therefore \frac{1}{C_r}\frac{dC_r}{dr}=\frac{\tan^2\alpha}{r}-\frac{\sec^2\alpha}{\gamma M^2}\frac{1}{P}\frac{dP}{dr}-\frac{c_f\sec\alpha}{b} \tag{15a}$$

from continuity equation we express the first term as a function of known variables by the equation of state together with equation (6a)

total temperature equation

$$\frac{1}{T_0}\frac{dT_0}{dr}=\frac{1}{T}\frac{dT}{dr}+\left(\frac{\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M^2}\right)\frac{1}{M^2}\frac{dM^2}{dr}$$

Equation of state

$$\frac{1}{P}\frac{dP}{dr}=\frac{1}{\rho}\frac{d\rho}{dr}+\frac{1}{T}\frac{dT}{dr}$$

$$\frac{1}{\rho}\frac{d\rho}{dr}=\frac{1}{P}\frac{dP}{dr}-\frac{1}{T_0}\frac{dT_0}{dr}+\left(\frac{\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M^2}\right)\frac{1}{M^2}\frac{dM^2}{dr} \tag{15b}$$

which after substituting into the continuity equation yields

$$\frac{1}{P}\frac{dP}{dR*}=\frac{-\gamma M^2}{\gamma M^2-\sec^2\alpha}\left[\begin{array}{l}\left(\dfrac{\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M^2}\right)\dfrac{1}{M^2}\dfrac{dM^2}{dR*}-\dfrac{1}{T_0}\dfrac{dT_0}{dR*}\\ -\dfrac{\xi}{H\cos\alpha}+\dfrac{1}{H}\dfrac{dH}{dR*}+\dfrac{\sec^2\alpha}{R*}\end{array}\right]$$

which combined with equation (13) to eliminate $\frac{1}{P}\frac{dP}{dR^*}$

$$\frac{1}{M^2}\frac{dM^2}{dR^*}=\frac{-2\left(1+\gamma-\frac{1}{2}M^2\right)}{M^2-\sec^2\alpha}\left[\begin{array}{l}\left(1+\gamma M^2-\tan^2\alpha\right)\dfrac{1}{2T_0}\dfrac{dT_0}{dR^*}\\ +\left(\gamma M^2-\tan^2\alpha\right)\dfrac{\xi}{H\cos\alpha}-\dfrac{1}{H}\dfrac{dH}{dR^*}-\dfrac{\sec^2\alpha}{R^*}\end{array}\right] \tag{15c}$$

This equation determines the change in M^2 with radius R along the mean surface of revolution in terms of $\frac{1}{T_0}\frac{dT_0}{dr}$, which is known from equations (14a) and (14b).

Flow direction

The differential of the flow direction is obtained from equation (4a)

$$\frac{1}{\tan\alpha}\frac{d\tan\alpha}{dr}=\frac{1}{C_u}\frac{dC_u}{dr}-\frac{1}{C_r}\frac{dC_r}{dr}$$

which from the tangential equilibrium equation (9) and equation (15a) becomes

$$\therefore \frac{1}{\tan\alpha}\frac{d\tan\alpha}{dR^*}=\frac{\sec^2\alpha}{\gamma M^2}\frac{1}{P}\frac{dP}{dR^*}-\frac{\sec^2\alpha}{R^*}$$

and from equations (13c) and (15c)

$$\frac{1}{\tan\alpha}\frac{d\tan\alpha}{dR^*}=\frac{\sec^2\alpha}{M^2-\sec^2\alpha}\left\{\begin{array}{l}\left(1+\frac{\gamma-1}{2}M^2\right)\frac{1}{T_0}\frac{dT_0}{dR^*}+\\ \left(1+(\gamma-1)M^2\right)\frac{\xi}{H\cos\alpha}-\frac{1}{H}\frac{dH}{dR^*}-\frac{M^2}{R^*}\end{array}\right\}$$

Equations (14a), (14b) and(15c) are three differential equations that can be solved

simultaneously for T_0, M^2 and α.

Pressure

After the variations in T_0, M^2 and α with radius R* are known, the pressure P can be obtained from the continuity equation

$$\rho_1 C_1 \cos\alpha_1 r_T b_T = \rho C\cos\alpha r b$$

where the subscript 1 refers to known conditions at the diffuser inlet. From the equation of state and from the definition of Mach number

$$\frac{P_1}{T_1}M_1\sqrt{T_1}\cos\alpha_1 r_T b_T=\frac{P}{T}M\sqrt{T}\cos\alpha r b$$

finally from equations (6) and (13e)

$$\frac{P}{P_1}=\frac{1}{R^*H}\frac{\cos\alpha_1}{\cos\alpha}\frac{M_1}{M}\sqrt{\frac{T_0\left(1+\frac{\gamma-1}{2}M_1^2\right)}{T_{01}\left(1+\frac{\gamma-1}{2}M^2\right)}} \qquad (17b)$$

Equation (17b) determines P from the known conditions at the diffuser inlet and from the known values of T_0, M^2 and α determined by the simultaneous solution of equations (14a), (14b), (15c) and (16).

Flow Path

The flow path on the mean surface of revolution in the vaneless diffuser can be obtained from the known variation in $\tan\alpha$ with R* given by the solution of equation (16).

$$\tan\alpha = \frac{R^{*}d\Theta}{dR^{*}}$$

or
$$\frac{d\Theta}{dR^{*}} = \frac{\tan\alpha}{R^{*}} \tag{18}$$

because the angle alpha is a known function of R, equation (18) determines the flow path.

<u>Influence coefficient</u>

In some analysis problems it may be convenient or desirable to solve directly for one or more of the other dependent quantities rather than To, M^2, and alpha. Also, in the design problem, it may be desired to specify one of these quantities as a function of R and solve for the required value of 1/H*dH/dR. For these cases the change in the dependent variables P, Rho, T, C, Cr and Cu with radius R along the mean surface of revolution, as well as the change in To, M^2, and alpha, must be expressed in terms of the known quantities 1/T0*dT0/dR, $\frac{\xi}{H\cos\alpha}$, $\frac{1}{H}\frac{dH}{dR}$, $\frac{1}{R}$ which quantities are multiplied by influence coefficients. Thus, if X is any one of the dependent variables,

$$\left(M^{2} - \sec^{2}\alpha\right)\frac{1}{X}\frac{dX}{dR} = I_{1}\left(1 + \frac{\gamma-1}{2}M^{2}\right)\frac{1}{To}\frac{dTo}{dR} + I_{2}\frac{\xi}{H\cos\alpha}$$
$$+ I_{3}\frac{1}{H}\frac{dH}{dR} + I_{4}\frac{1}{R}$$

where I_1 through I_4 are influence coefficients that are determined in the same way that equations (15c) and (16) were developed. The influence coefficients for various dependent variables X are given in the following table:

Tabble 1-Influence coefficients

x	Influence	Coefficients		
	I_1	I_2	I_3	I_4
P	γM^2	$[1 + (\gamma - 1)M^2]\gamma M$	$-\gamma M^2$	$-\gamma M^2 \sec^2\alpha$
ρ	$\sec^2\alpha$	$M^2\left(\gamma\sec^2\alpha - \tan^2\alpha\right)$	$-M^2$	$-M^2\sec^2\alpha$
T	γM^2 $-\sec^2\alpha$	$(\gamma - 1)M^2(\gamma M^2 - \tan^2\alpha)$	$-(\gamma - 1)M^2$	$-(\gamma - 1)M^2\sec^2\alpha$
M^2	$\tan^2\alpha - 1$ $-\gamma M^2$	$2\left(1 + \frac{(\gamma - 1)}{2}M^2\right)$ $\left(\tan^2\alpha - \gamma M^2\right)$	$2\left(1 + \frac{(\gamma - 1)}{2}M^2\right)$	$2\left(1 + \frac{(\gamma - 1)}{2}M^2\right)$ $\sec^2\alpha$
C^2	-2	$2\left(\tan^2\alpha - \gamma M^2\right)$	2	$2\sec^2\alpha$
C_r	$-\sec^2\alpha$	$M^2\left(\tan^2\alpha - \gamma\sec^2\alpha\right)$	$\sec^2\alpha$	$\sec^2\alpha + M^2\tan^2\alpha$
C_u	0	$\sec^2\alpha - M^2$	0	$\sec^2\alpha - M^2$
$\tan\alpha$	$\sec^2\alpha$	$\sec^2\alpha[1 + (\gamma - 1)M^2]$	$-\sec^2\alpha$	$-M^2\sec^2\alpha$

Small stage efficiency

The small stage or polytropic efficiency at a given radius R on the mean surface of revolution in a vaneless diffuser is defined as the ratio of the ideal (ignoring friction and heat transfer) to the actual differential change in static enthalpy with radius required to accomplish the actual differential change in static pressure with radius. This definition leads to the following expression for the small-stage efficiency η.

$$\eta = \frac{\frac{1}{P}\frac{dP}{dR}}{\frac{1}{P}\frac{dP}{dR} + \frac{\gamma}{\gamma - 1}\left(1 + \frac{\gamma - 1}{2}M^2\right)\frac{1}{To}\frac{dTo}{dR} + \frac{\gamma M^2 \xi}{H\cos\alpha}} \tag{20a}$$

Equation (20a) indicates that in the absence of heat transfer (dTo/dR=0) and friction (ξ=0), the small-stage efficiency is 100 percent. Also, for heat transfer from the fluid to the diffuser walls, (1/T0*dT0/dR) is negative and therefore results in an apparent increase in the small-stage efficiency. Thus, in the presence of heat transfer, the small-stage efficiency, as just defined, is not a good measure of the performance of vaneless diffusers in that it is not a measure of the magnitude of the losses involved. In the absence of heat transfer (dT0/dR=0) and equation (20a) reduces to

$$\eta = 1 - \frac{\xi\left(M^2 - \sec^2\alpha\right)}{\xi\left(\gamma M^2 - \tan^2\alpha\right) - \cos\alpha\left(\frac{dH}{dR} + \frac{H\sec^2\alpha}{R}\right)} \tag{20b}$$

Numerical Procedure

In the analysis problem , the variation in fluid properties with R are determined for a specified geometry of the vaneless diffuser. In the design problem, the variation with R in one of the fluid properties is prescribed and the remaining fluid properties together with the variation in diffuser height H with radius R are determined.

In both approaches we shall use MATLAB to solve for the different parameters, given the three unknown equations (14a) or (14b), (15c) and (16). We shall first solve for the three unknowns To, M^2 and the angle alpha.

For this numerical example R varies from a value of 1 to a value of 2.

After the distribution of To, M^2 and the angle alpha with R Have been determined the distribution of P, ρ, T, C, Cu and Cr can be determined from equation (17b) and (4), (5), (6) and the equation of state (10a).

Flow Path The flow path on the mean surface of revolution in the vaneless diffuser is given by Θ as a function of R along the surface. Because $\tan\alpha$ is a known function of R, the flow path ($\Theta=\Theta(R)$) can be determined by the integration of equation (18) assuming $\Theta=0$ at R=1.0.

Design Problem

In the design method, the variation in effective diffuser wall spacing with radius is determined for a prescribed variation in one fluid property. For efficient diffuser designs the selection of the one fluid property and its optimum prescribed variation will depend on viscous flow effects that are considered in boundary-layer studies.

In the design problem the variation in H with R is unknown and must be determined to satisfy a specified variation in one characteristic of the flow (Cr, for example) with R.

For this specified variation in one characteristic of the flow $\frac{1}{H}\frac{dH}{dR}$ can be determined. Again using Matlab, we solve for H as a function of R, with H=1 at R=1.

Numerical Examples

The numerical examples are divided into two groups:

1- effects of some operating conditions

2- vaneless diffuser design problem

The first group of numerical examples shows the effects of heat transfer and friction on the flow in vaneless diffusers. Three numerical examples are given:

- Isentropic compressible flow
- Compressible flow with friction
- Compressible flow with friction and heat transfer.

Inlet conditions:

For the first group of numerical examples the flow conditions at the diffuser inlet (R=1.0) are:

$P_1 = 3.022$

$M_1^2 = 1.37$

$(To)_1 = 941\ ^\circ R$

$(\tan\alpha)_1 = 3.829$

These conditions were estimated for the following design and operating conditions of the impeller :

Compressor flow coefficient, φ	0.75
Impeller tip Mach number, M_T	1.5
Impeller slip factor, μ	0.9
Impeller polytropic efficiency, η	0.9
Compressor stagnation inlet temperature, To, $^\circ$R	520

Diffuser design:

The design characteristic of the diffuser are :

Passage height	1/R
Wall temperature, Tw	750
Friction parameter, ξ	0.03

Results

The results of the first group of three numerical examples are given in figure (4.1). In Figure (a) the change in M^2 with R is shown for the three numerical examples. The effect of friction is to reduce M^2 at each R, and the effect of heat transfer from the fluid is to increase M^2 slightly (primarily because of the reduced speed of sound at the lower temperature) for the magnitudes of To and Tw involved in these examples.

In Figure (b) the change in P with R is shown. As expected the effect of friction is to reduce P at each radius (primarily because of the decreased values of Cu, which require a smaller pressure gradient for equilibrium). The effect of heat transfer from the fluid is to raise P slightly or the magnitudes of To and Tw involved in these examples.

In Figure (c) the change in flow direction α with R is shown. The effect of friction is to reduce α because Cu is reduced and Cr is increased to satisfy continuity with lower density due to lower P. The effect of heat transfer from the fluid is to increase α slightly because of the reduced value of Cr resulting from the increased value of ρ.

In Figure (d) the flow path in the vaneless diffuser is shown. The effect of friction is to shorten the flow path because α is decreased (figure (c)) . The effect of heat transfer is to lengthen the path slightly.

A Vaneless Diffuser Design Problem

The second part of the section on numerical examples is a simple vaneless diffuser design problem. The design variable in a vaneless diffuser is H= H(R), and the design problem will be to determine H(R) for a prescribed variation in Cr.

For purposes of demonstrating the design method it is assumed that the deceleration of Cr, is the criterion for boundary-layer separation in a vaneless diffuser , so that a safe rate of deceleration is

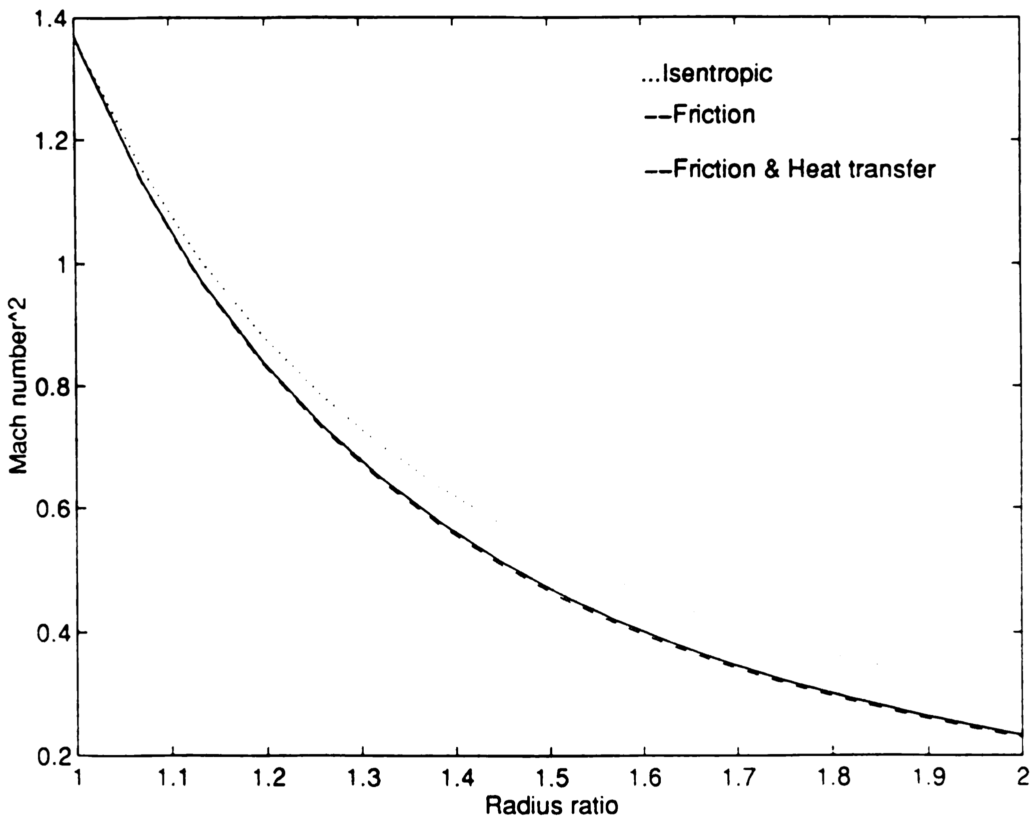


Figure 4.1.a

Change in Mach number^2 vs. the radius ratio

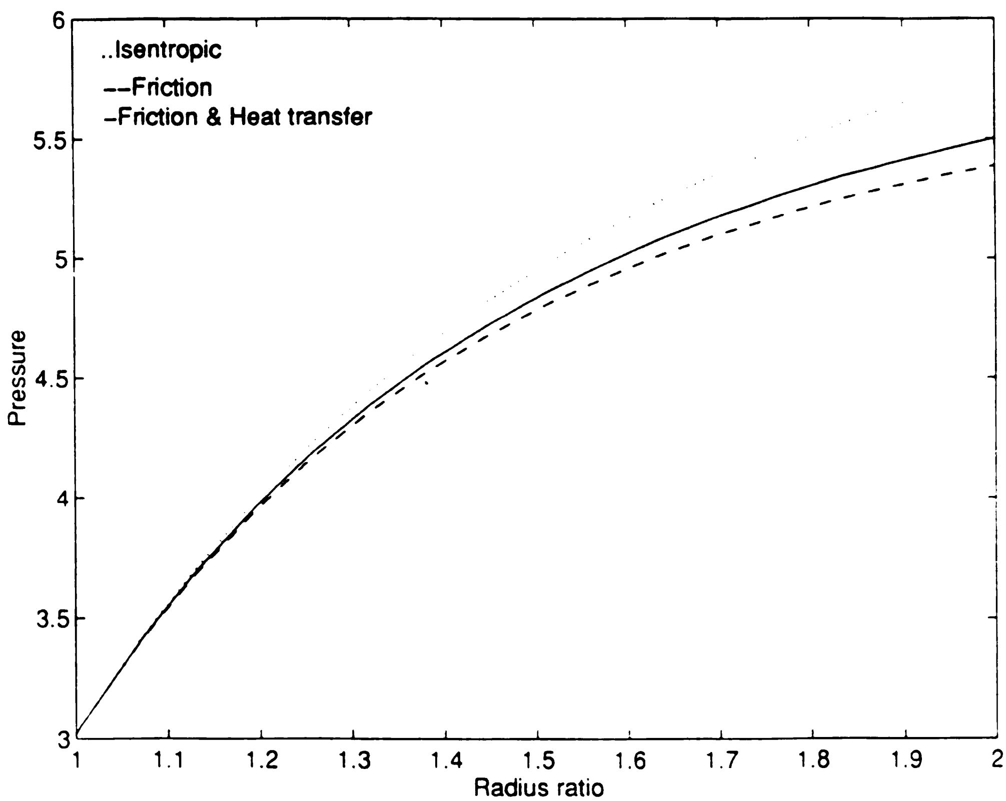


Figure 4.1.b

Change in pressure vs the radius ratio

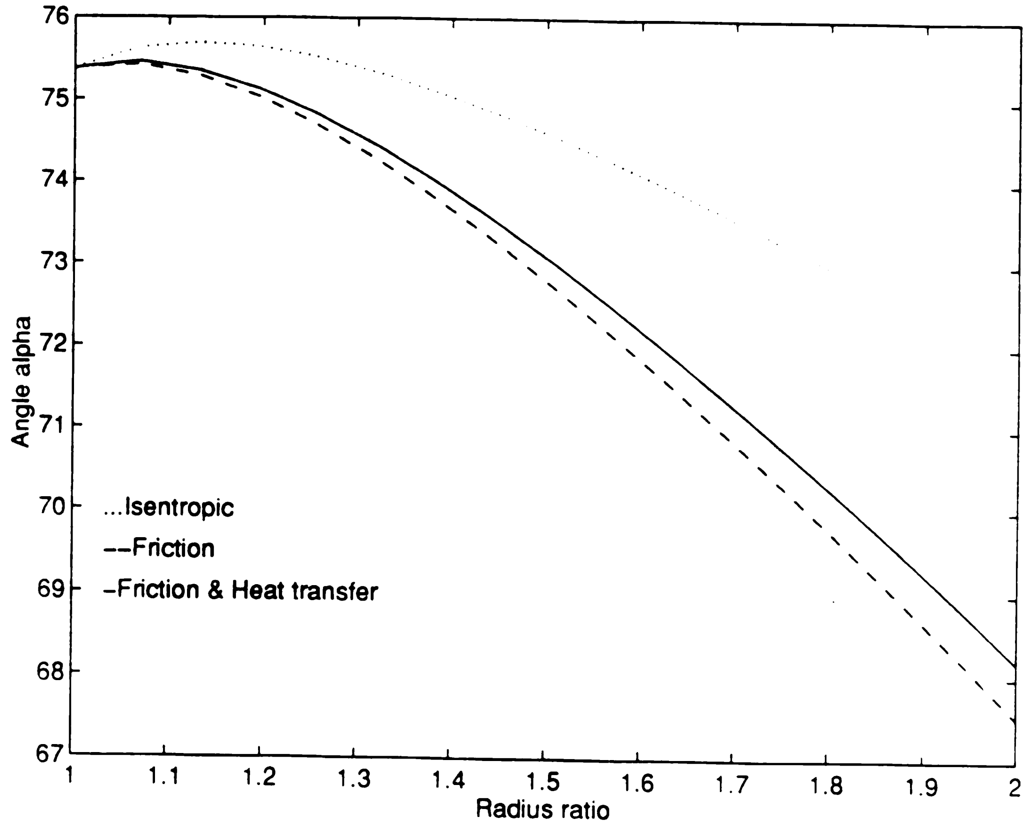


Figure 4.1.c

Change in angle of swirl alpha vs. the radius ratio

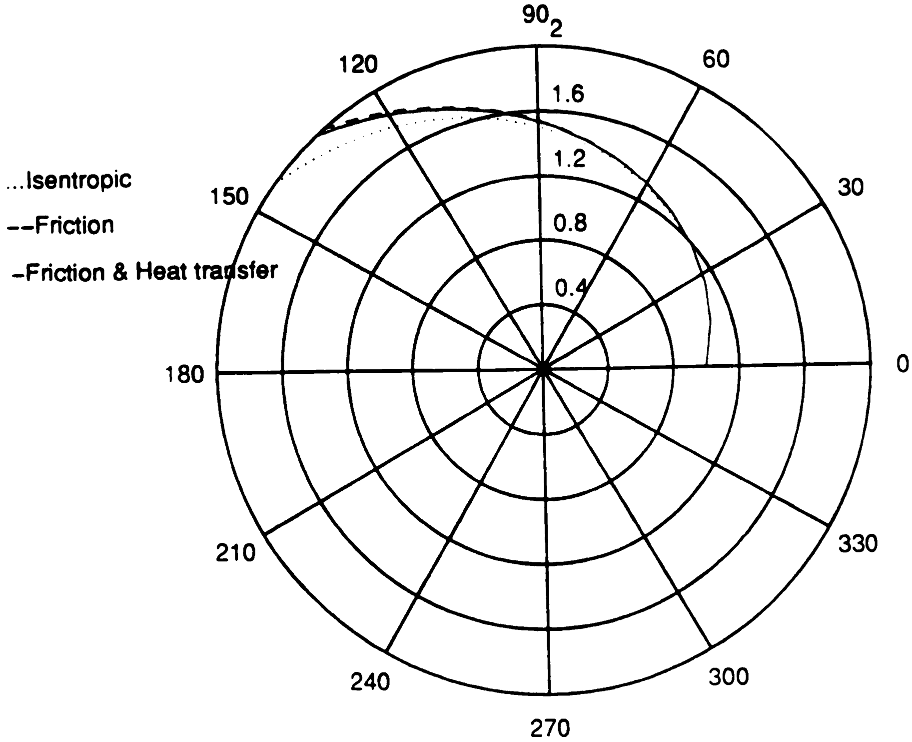


Figure 4.1.d

Change in flow path vs. radius ratio

$$\frac{\delta}{C_r}\frac{dC_r}{dr} = -0.05$$

where δ is proportional to the boundary-layer thickness. For purposes of the design example we assume δ is equal to b/2, which is the effective thickness of a fully developed boundary layer in the vaneless diffuser. Thus,

$$\frac{\left(H/2\right)}{C_r}\frac{dC_r}{dR}\left(\frac{b_T}{r_T}\right) = -0.05$$

and

$$\frac{1}{C_r}\frac{dC_r}{dR} = -\frac{1}{H}$$

if $\frac{r_T}{b_T}$ is equal to 10. Because of the assumption involved, this specified variation in $\frac{dC_r}{dR}$ with H may have no practical significance with regard to vaneless diffuser performance and has been selected only to demonstrate an application of the design method. It should be pointed out that design variations in H affect primarily the velocity component Cr and through this component the flow direction α.

Inlet conditions

The impeller design and operating conditions are the same as for the first group of numerical examples and so the diffuser inlet conditions are the same

$P_1 = 3.022$

$M_1^2 = 1.37$

$(To)_1 = 941\ °R$

$(\tan\alpha)_1 = 3.829$

Diffuser design

The variation in H with R is to be determined. Heat transfer effects are neglected, and the value of the friction parameter is the same as for the first group of numerical examples(0.03).

Results

The results of the design problem are given in Figure (4.2). In the figures is shown the variations in H, $\frac{1}{C_r}\frac{dC_r}{dR}$, M^2, P, α and η with radius R. as specified , $\frac{1}{C_r}\frac{dC_r}{dR}$ is equal to -1/H. In order to accomplish this variation, H at first decreases with increasing R and then increases to approximately its initial value at R equal 2. The variation in α with R was slightly more than 3 degrees so that the flow path is approximately a logarithmic spiral.

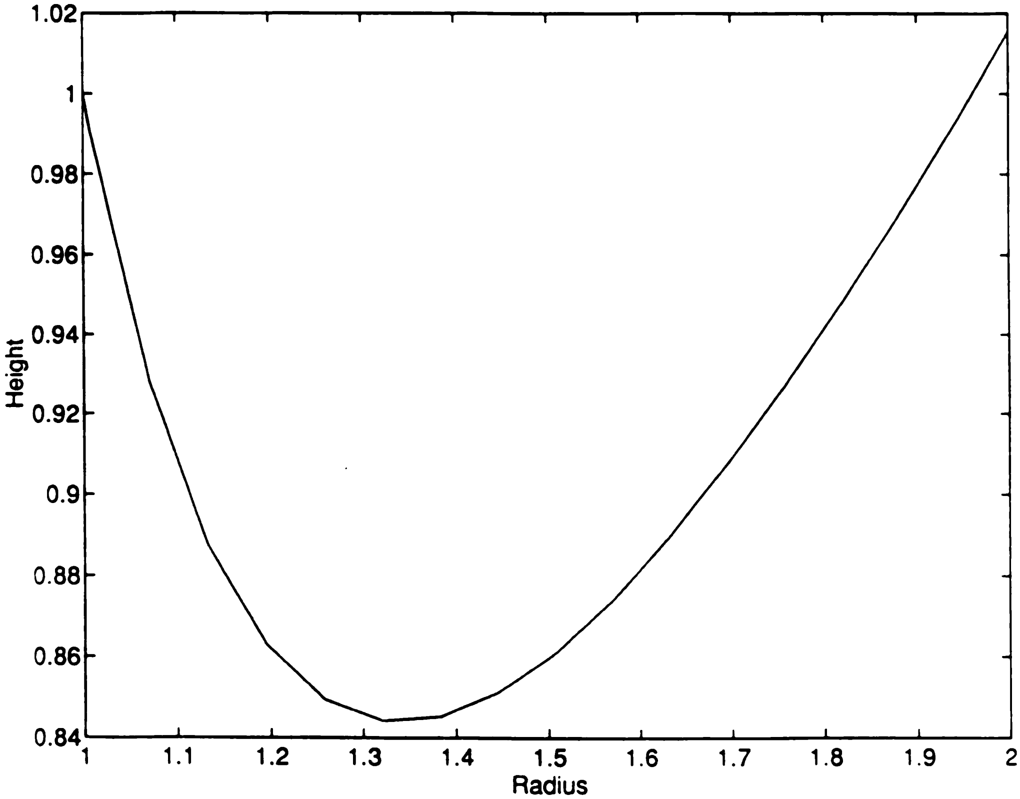


Figure 4.2.a

Change in diffuser height vs. radius ratio

for a design problem

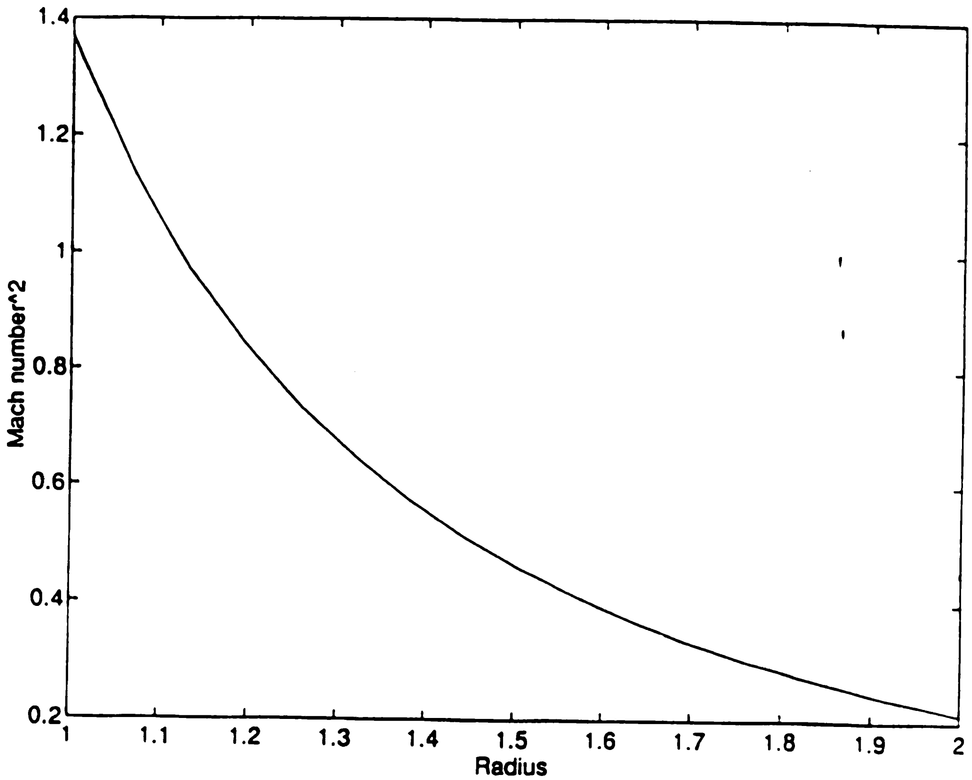


Figure 4.2.b

Change in Mach number^2 vs. radius ratio

for a design problem

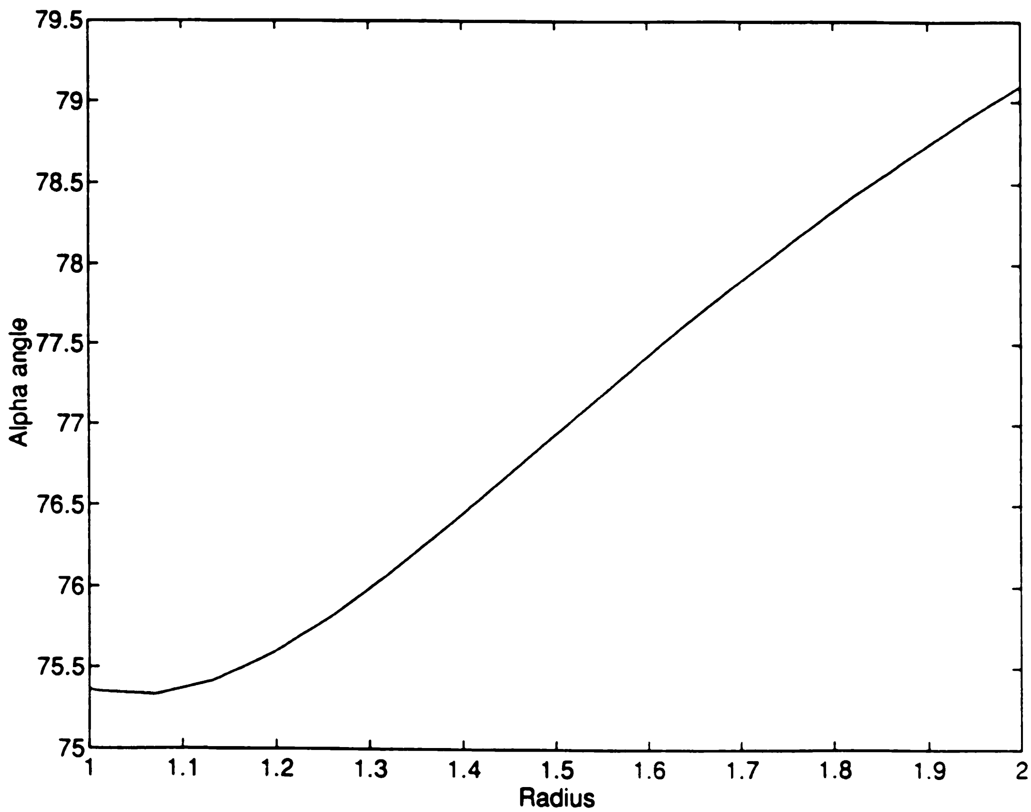


Figure 4.2.c

Change in angle alpha vs. radius ratio

for a design problem

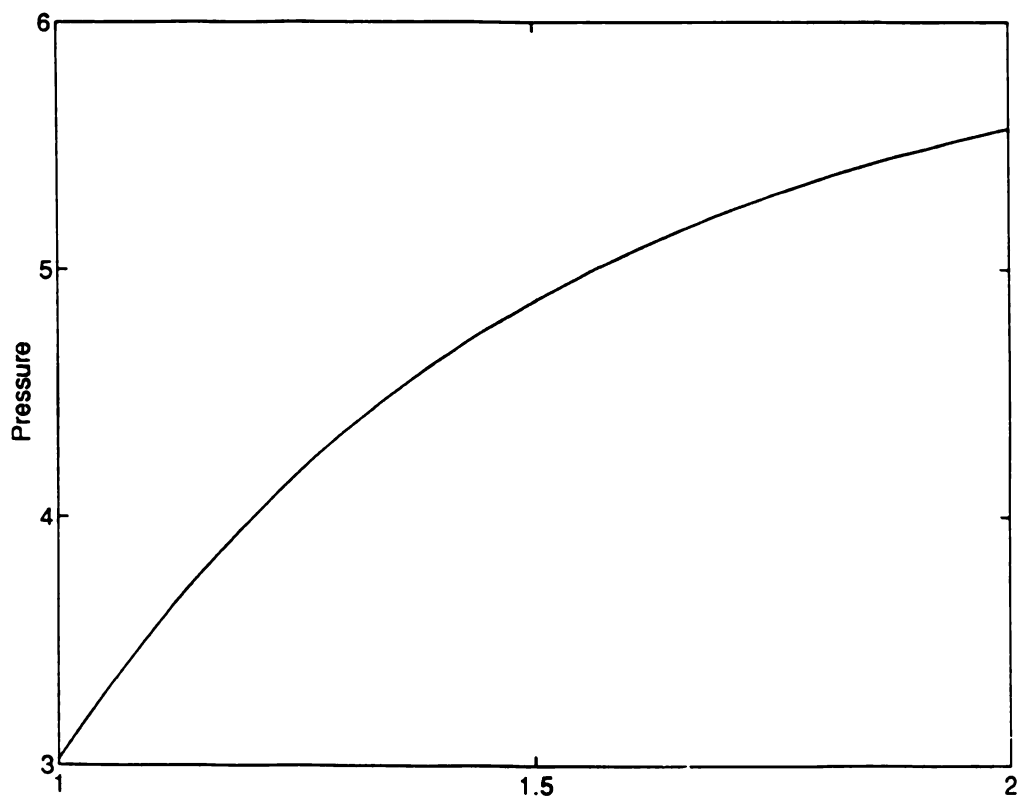


Figure 4.2.d

Change in pressure vs. radius ratio

for a design problem

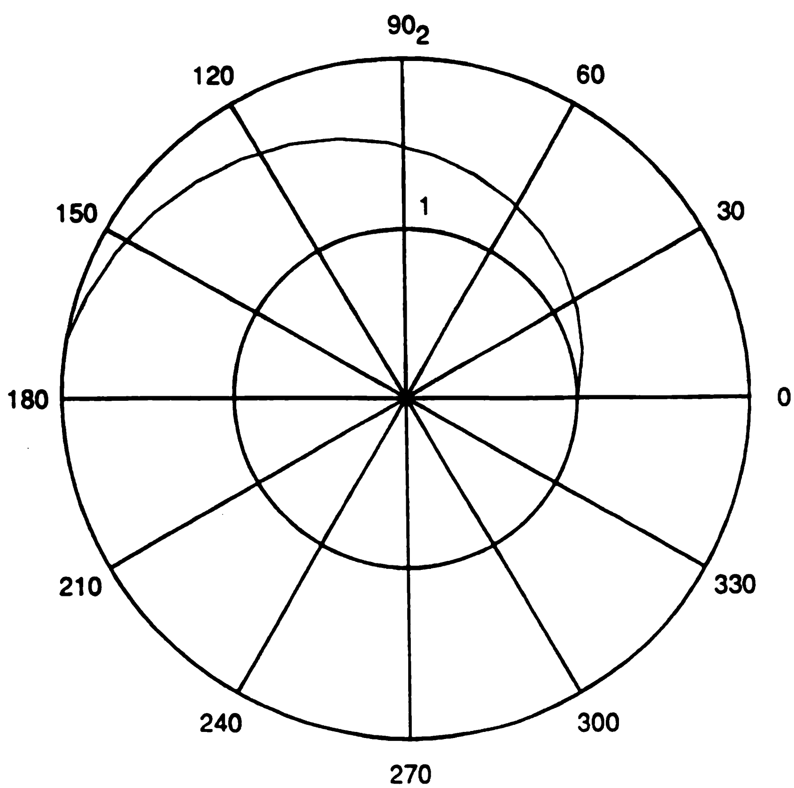


Figure 4.2.e
Change in flow path vs. radius ratio
for a design problem

CHAPTER FIVE

Order of Magnitude analysis
&
Validity of the one-dimensional model

In this chapter, we shall present the full set of equations and conduct an order of magnitude analysis, to determine whether the terms canceled based on our previous assumptions could affect the solution.

We shall first start by presenting the continuity, r-momentum and θ-momentum including the unsteady term due to the start-up:

Mass equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rC_r\right)+\frac{1}{r}\frac{\partial}{\partial \theta}\left(C_u\right)=0.0$$

r-momentum

$$\left[\frac{\partial C_r}{\partial t}+C_r\frac{\partial C_r}{\partial r}+\frac{C_u}{r}\frac{\partial C_r}{\partial \theta}-\frac{C_u^2}{r}\right]=-\frac{\partial P}{\partial r}+\mu\left[\begin{array}{l}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(rC_r\right)\right)+\frac{1}{r^2}\frac{\partial^2 C_r}{\partial \theta^2}\\ -\frac{2}{r^2}\frac{\partial C_u}{\partial \theta}\end{array}\right]$$

θ-momentum

$$\left[\frac{\partial C_u}{\partial t}+C_r\frac{\partial C_u}{\partial r}+\frac{C_u}{r}\frac{\partial C_u}{\partial \theta}+\frac{C_r C_u}{r}\right]=-\frac{1}{\rho}\frac{1}{r}\frac{\partial P}{\partial \theta}+\nu\left[\begin{array}{l}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(rC_u\right)\right)\\ +\frac{1}{r^2}\frac{\partial^2 C_u}{\partial \theta^2}\\ +\frac{2}{r^2}\frac{\partial C_r}{\partial \theta}\end{array}\right]$$

we non-dimensionalize the equations with the following scaling

R=r/b	θ*= θ/2 π	Cu*=Cu/2 π a ω
Cr*=Cr/Cs	P*=P/Ps	t*=t/ts

we need to use the equation to tell us Cs,Ps

Substituting into mass we obtain

$$\frac{C_s}{b}\frac{1}{R}\frac{\partial}{\partial R}(Rv)+\frac{1}{b}\frac{2\pi a\omega}{2\pi}\frac{1}{R}\frac{\partial u}{\partial \theta^*}=0$$

or

$$\frac{C_s}{a\omega}\frac{1}{R}\frac{\partial}{\partial R}(Rv^*)+\frac{1}{R}\frac{\partial u^*}{\partial \theta^*}=0$$

to keep both terms we must have

$\frac{C_s}{a\omega}$ as 0(1)

which gives

$C_s = a\omega$

Now substituting into r-momentum

$$\frac{a\omega}{t_s}\frac{\partial v}{\partial t^*}+\frac{a^2\omega^2}{b}v\frac{\partial v}{\partial R}+\frac{2\pi a^2\omega^2}{2\pi b}\frac{u}{R}\frac{\partial v}{\partial \theta}-\frac{4\pi^2a^2\omega 2}{b}\frac{u^2}{R}=-\frac{1}{\rho}\frac{P_s}{b}\frac{\partial P^*}{\partial R}$$

$$+\nu\left[\frac{a\omega}{b^2}\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}(Rv)\right)+\frac{a\omega}{4\pi^2b^2}\frac{\partial^2 v}{\partial \theta^2}-\frac{2a\omega 2\pi}{2\pi b^2}\frac{1}{R^2}\frac{\partial u}{\partial \theta^*}\right]$$

we now divide by $4\pi^2a^2\omega^2\,/\,b$

$$\frac{b}{4\pi^2a\omega t_s}\frac{\partial v}{\partial t^*}+\frac{1}{4\pi^2}v\frac{\partial v}{\partial R}+\frac{1}{4\pi^2}\frac{u}{R}\frac{\partial v}{\partial \theta}-\frac{u^2}{R}=-\frac{P_s}{4\pi^2a^2\rho\omega^2}\frac{\partial P^*}{\partial R}$$

$$+\frac{\nu}{4\pi^2\omega ab}\left[\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}(Rv)\right)+\frac{1}{4\pi^2}\frac{\partial^2 v}{\partial \theta^2}-\frac{1}{R^2}\frac{\partial u}{\partial \theta^*}\right]$$

Now let us look at the θ-momentum

$$\frac{2\pi a\omega}{t_s}\frac{\partial u}{\partial t^*}+\frac{2\pi a^2\omega^2}{b}v\frac{\partial v}{\partial R}+\frac{4\pi^2a^2\omega^2}{2\pi b}\frac{u}{R}\frac{\partial u}{\partial \theta}$$

$$+\frac{2\pi a^2\omega^2}{b}\frac{vu}{R}=-\frac{P_s}{2\pi b\rho R}\frac{\partial P^*}{\partial \theta}+\nu\left[\begin{array}{l}\frac{2\pi a\omega}{b}\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}(Ru)\right)+\\ \frac{2\pi a\omega}{4\pi^2b^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{2a\omega}{2R\pi b^2}\frac{\partial v}{\partial \theta}\end{array}\right]$$

Now divide this equation by $\dfrac{2\pi a^2\omega^2}{b}$

$$\frac{b}{a\omega t_s}\frac{\partial u}{\partial t^*}+v\frac{\partial u}{\partial R}+\frac{u}{R}\frac{\partial u}{\partial \theta}+\frac{vu}{R}=-\frac{P_s}{2\pi a^2\rho\omega^2R}\frac{\partial P^*}{\partial \theta}+$$

$$\frac{\nu}{\omega ab}\left[\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}(Ru)\right)+\frac{1}{4\pi^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{1}{2R\pi^2}\frac{\partial v}{\partial \theta}\right]$$

Now to decide on which equation to use to scale the pressure and the time. First note that each momentum equation would give us a different expression for ts and Ps if we simply set the coefficients equal to one.

In the θ-direction we would expect that the time rate of change of momentum would be balanced with inertia and viscous forces, but not necessarily the pressure. Since the inertia terms are of order one we make $\frac{b}{a\omega t_s}$ is of order one and $t_s = b/a\omega$

We note that the last two terms of the viscous term are of order $\frac{1}{4\pi^2}$, and can be neglected.

For Ps, we consider the r-momentum equation, and note that inertia can be balanced with the pressure term. Then

$\frac{P_s}{4\pi^2 a^2 \rho\omega}$ is of order one and $P_s = 4\pi^2 a^2 \rho\omega$

note that the transient term becomes of order $\frac{1}{4\pi^2}$, and all the viscous terms of order $1/2\pi$, $1/Re$ so they can all be neglected.

Our scaled equations become :

continuity:

$$\frac{1}{R}\frac{\partial}{\partial R}(Rv) + \frac{1}{R}\frac{\partial u}{\partial \theta} = 0$$

r-momentum:

$$\frac{u^2}{R} = \frac{\partial P}{\partial R}$$

θ-momentum :

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial R} + \frac{u}{R}\frac{\partial u}{\partial \theta} + \frac{vu}{R} = \frac{1}{Re}\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}(Ru)\right)$$

From these scaled equations, we can see that our previous assumption that there is no variation of any of the properties with time(steady state), proved to be partially wrong and that there is a time dependency of the tangential velocity which will affect our results to a certain degree. Also our assumption of asymmetric flow, there is a variation of the tangential velocity with the θ direction which also adds some inaccuracy in our model.

Further study in that area is recommended to investigate to what extent our one dimensional model' s results vary from that of a two dimensional that includes all the time and tangential variations that were not previously included in our model.

CHAPTER SIX

Summary of Results and Conclusions

This chapter concludes the work with a summary of the findings and a statement of the conclusions.

Summary

The work shows some simple means for flow predictions, some of the methods presented earlier in this work are very simple because most of the affecting terms were eliminated by assumptions and are only good for classroom purposes.

In later chapters,analysis methods have been developed for one dimensional model that takes into account the compressibility, friction, heat transfer, and area changes in vaneless diffusers. In the analysis method, the variation in fluid properties, including the velocity and flow direction can be determined as a function of radius for a prescribed variation in diffuser height with radius. In the design method, the variation in diffuser height and all fluid properties except one can be determined as a function of radius for a prescribed variation in the one fluid property. For efficient diffuser designs the selection of the one fluid property and its optimum prescribed variation will depend on viscous flow effects that are considered in boundary-layer studies.

Three groups of numerical examples are presented in which the effects of friction, heat transfer, and diffuser height are investigated; and a simple design problem is presented. As a result of these examples it is concluded that:

1- Heat transfer from the fluid has the opposite effect of friction on pressure rise in vaneless diffusers and is therefore to be desired. On the other hand, heat transfer to the fluid has the same effect as friction and is to be avoided.

2- If the friction coefficient is unaffected by the diffuser height, and if flow separation does not occur, the diffuser efficiency is slightly improved by increasing the diffuser height.

3- With relatively low friction coefficients and neglecting mixing losses at the impeller tip, the friction losses in most vaneless diffuser designs are considerable, as indicated by computed diffuser efficiencies in the low 80's , and these losses result from the usually large ratio of wetted surface to flow area in vaneless diffusers.

APPENDICES

APPENDIX A

Momentum in the r-direction

$$\frac{C_r \partial C_r}{\partial r} - \frac{C_u^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}$$

Momentum in the Θ-direction

$$C_r \frac{\partial C_u}{\partial r} + C_r \frac{C_u}{r} = 0$$

$$\frac{C_u}{r} = -\frac{\partial C_u}{\partial r}$$

reduces to

$$\frac{C_u}{r} = -\frac{\partial C_u}{\partial r}$$

Substitute (1) into (2)

$$C_r \frac{\partial C_r}{\partial r} + C_u \frac{\partial C_u}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}$$

which becomes

$$\frac{\partial P}{\rho} + C_r \partial C_r + C_u \partial C_u = 0$$

the flow is adiabatic and frictionless then ds=0, and

$$\frac{\rho}{\rho_2} = \left(\frac{P}{P_2}\right)^{\frac{1}{\gamma}}$$

or

$$\rho_2 = \frac{1}{RT_2} P_2$$

Substitute into (3) to obtain

$$\left(\frac{P}{P_2}\right)^{-\frac{1}{\gamma}} \frac{\partial P}{P_2} + \frac{C_r \partial C_r + C_u \partial C_u}{RT_2} = 0$$

but we have

$$C_r \partial C_r + C_u \partial C_u = \frac{1}{2} d\left(C_r^2 + C_u^2\right) = \frac{1}{2} d\left(C^2\right) = CdC$$

Integrate from P/P2=1 to P/P2, and from C2 to C

$$\int_{\frac{P}{P_2}=1}^{\frac{P}{P_2}} \left(\frac{P}{P_2}\right)^{-\frac{1}{\gamma}} d\left(\frac{P}{P_2}\right) = -\int_{C_2}^{C} \frac{CdC}{RT_2}$$

evaluate at the end points

$$\frac{\gamma}{\gamma - 1}\left[\left(\frac{P}{P_2}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right] = -\frac{1}{2RT_2}\left(C^2 - C_2^2\right)$$

Further reduced

$$\left(\frac{P}{P_2}\right)^{\frac{\gamma - 1}{\gamma}} = \frac{\gamma - 1}{\gamma} \frac{C_2^2 - C^2}{2RT_2} + 1$$

with

$$M = \frac{C}{\sqrt{\gamma RT}}$$

finally becomes

$$\left(\frac{P}{P_2}\right) = \left[1 + \frac{\gamma - 1}{2} M_2^2 \left(1 - \frac{C^2}{C_2^2}\right)\right]^{\frac{\gamma}{\gamma - 1}} \qquad (1)$$

Recall from velocity triangle

- $C^2 = C_r^2 + C_u^2$
- $C_2^2 = C_{r2}^2 + C_{u2}^2$

also from continuity equation with the diffuser thickness (b) as a constant

- $C_r = \left(\frac{\rho_2}{\rho}\right) C_{r2} = \tau C_{r2}$

also from conservation of momentum

- $C_u = \left(\frac{r_2}{r}\right) C_{u2} = \frac{C_{u2}}{\lambda_D}$

where $\lambda_D = \frac{r}{r_2}$

from the velocity triangle at the impeller exit (diffuser inlet)

- $C_{r2} = C_2 \sin\alpha_2$
- $C_{u2} = C_2 \cos\alpha_2$

substituting into equation (8) we get

$$\left(\frac{P}{P_2}\right)^{\frac{\gamma - 1}{\gamma}} = 1 + \frac{\gamma - 1}{2} M_2^2 \left(1 - \tau^2 \sin^2 \alpha_2 - \lambda_D^{-2} \cos^2 \alpha_2\right)$$

Consider the vaneless space between r2 and r3, with a constant b from Continuity

$$\rho r C_r = \text{const}$$

Angular momentum conservation

$$C_u r = \text{const}$$

Usually the flow leaving the impeller is supersonic M2>1, and the flow leaving the vaneless diffuser is subsonic M3<1.

Denote the radial position at which M=1 by r* and all properties at this position by (*)

$$C_r = C\cos\alpha$$

continuity equation

$$\rho r C\cos\alpha = \rho^{*} r^{*} C^{*} \cos\alpha^{*}$$

Angular momentum equation

$$rC\sin\alpha = r^{*}C^{*}\sin$$

Dividing (3) by (2)

$$\frac{\tan\alpha}{\rho} = \frac{\tan\alpha^{*}}{\rho^{*}}$$

Assuming frictionless adiabatic flow ds=0

$$\frac{T^{*}}{T} = \left(\frac{\rho^{*}}{\rho}\right)^{\gamma-1}$$

From the energy equation

$$T = \frac{T_0}{1+\frac{1}{2}(\gamma-1)M^2}$$

for the case of M=1

$$T = \frac{2T_0}{\gamma + 1}$$

back to

$$\frac{T^*}{T} = \left(\frac{\rho^*}{\rho}\right)^{\gamma-1}$$

rearranging

$$\frac{\rho^*}{\rho} = \left(\frac{T^*}{T}\right)^{\frac{1}{\gamma-1}}$$

substituting (4) and (5) for T and T* in (6)

$$\frac{\rho^*}{\rho} = \left[\frac{2T_0}{\gamma+1} \bullet \frac{1+\frac{1}{2}(\gamma-1)M^2}{T_0}\right]^{\frac{1}{\gamma-1}} = \left[\frac{2}{\gamma+1}\left(1+\left(\frac{\gamma-1}{2}\right)M^2\right)\right]^{\frac{1}{\gamma-1}} \quad (7)$$

Recall

$$\frac{\tan\alpha}{\rho} = \frac{\tan\alpha^*}{\rho^*}$$

substituting (3) into (7) we get

$$\tan\alpha = \tan\alpha^*\left[\frac{2}{\gamma+1}\left(1+\left(\frac{\gamma-1}{2}\right)M^2\right)\right]^{\frac{1}{\gamma-1}} \quad (8)$$

α^* can be evaluated by substituting $\alpha = \alpha_2$ and $M = M_2$ which are set at design

also from (3)

$$\frac{r^*\sin\alpha^*}{r\sin\alpha} = \frac{C}{C^*} = \frac{C}{a}\frac{a}{a^*} = M\left(\frac{T}{T^*}\right)^{\frac{1}{2}}$$

substituting for T/T*

$$\frac{r^{*}\sin\alpha^{*}}{r\sin\alpha}=M\left[\frac{2}{\gamma+1}\left(1+\left(\frac{\gamma-1}{2}\right)M^{2}\right)\right]^{-\frac{1}{2}} \tag{9}$$

to determine r* substitute $r=r_2$ and $M=M_2$
to determine α_3 from a known M_3 use equation (8)
to determine r_3 from a known M_3 and α_3 use equation (9)

APPENDIX B

By definition of the Mach number

$$M^2 = \frac{C^2}{\gamma R^* T} \tag{5}$$

Differentiating both sides of the equation with respect to r

$$\frac{dM^2}{dr} = \frac{\gamma R^* T \frac{dC^2}{dr} - \gamma R^* C^2 \frac{dT}{dr}}{\left(\gamma R^* T\right)^2}$$

Dividing both sides by C^2

$$\frac{\left(\gamma R^* T\right)^2}{C^2} \frac{dM^2}{dr} = \frac{\gamma R^* T}{C^2} \frac{dC^2}{dr} - \frac{\gamma R^* T}{T} \frac{dT}{dr}$$

we get

$$\frac{1}{M^2} \frac{dM^2}{dr} = \frac{1}{C^2} \frac{dC^2}{dr} - \frac{1}{T} \frac{dT}{dr} \tag{5a}$$

By definition of the total temperature

$$To = T\left(1 + \frac{\gamma - 1}{2} M^2\right) \tag{6}$$

Differentiating both sides of the equation with respect to r

$$\frac{dTo}{dr} = \left(1 + \frac{\gamma - 1}{2} M^2\right)\frac{dT}{dr} + T\left(\frac{\gamma - 1}{2}\right)\frac{1}{M^2} \frac{dM^2}{dr}$$

Dividing both sides by $T\left(1+\frac{\gamma-1}{2}M^2\right)$

$$\frac{1}{To}\frac{dTo}{dr}=\frac{1}{T}\frac{dT}{dr}+\frac{\left(\frac{\gamma-1}{2}\right)M^2}{\left(1+\frac{\gamma-1}{2}M^2\right)}\frac{1}{M^2}\frac{dM^2}{dr} \tag{6a}$$

from 5

$$\frac{1}{C^2}\frac{dC^2}{dr}=\frac{1}{M^2}\frac{dM^2}{dr}+\frac{1}{T}\frac{dT}{dr}$$

from 6a

$$\frac{1}{T}\frac{dT}{dr}=\frac{1}{To}\frac{dTo}{dr}-\frac{\left(\frac{\gamma-1}{2}\right)M^2}{\left(1+\frac{\gamma-1}{2}M^2\right)}\frac{1}{M^2}\frac{dM^2}{dr}$$

from 5, 6a

$$\frac{1}{C^2}\frac{dC^2}{dr}=\frac{1}{M^2}\frac{dM^2}{dr}+\frac{1}{To}\frac{dTo}{dr}-\frac{\left(\frac{\gamma-1}{2}\right)M^2}{\left(1+\frac{\gamma-1}{2}M^2\right)}\frac{1}{M^2}\frac{dM^2}{dr}$$

rearranging

$$\frac{1}{C^2}\frac{dC^2}{dr}=\frac{1}{M^2}\frac{dM^2}{dr}\left(1-\frac{\left(\frac{\gamma-1}{2}\right)M^2}{\left(1+\frac{\gamma-1}{2}M^2\right)}\right)+\frac{1}{To}\frac{dTo}{dr}$$

simplifying

$$\frac{1}{C^2}\frac{dC^2}{dr}=\left(\frac{1}{1+\frac{\gamma-1}{2}M^2}\right)\frac{1}{M^2}\frac{dM^2}{dr}+\frac{1}{To}\frac{dTo}{dr} \tag{6b}$$

From continuity equation we have

$\rho C_r rb = \text{cons}\tan\text{t}$

Differentiating both sides of the equation with respect to r

$$d(\rho C_r rb) = 0$$

$$\frac{d\rho}{dr}(rC_r b) + \frac{dC_r}{dr}(\rho rb) + \frac{db}{dr}(\rho C_r r) + \rho C_r b = 0$$

Dividing both sides by $\rho C_r rb$ we obtain

$$\frac{1}{\rho}\frac{d\rho}{dr} + \frac{1}{C_r}\frac{dC_r}{dr} + \frac{1}{r} + \frac{1}{b}\frac{db}{dr} = 0 \tag{7}$$

The differential pressure forces (opposed to the direction of C_r) are equal to the differential change on the end forces on the particle minus the component of the differential side forces on the particle in the direction of C_r.

Differential pressure forces $= \left(P + \frac{dP}{dr}dr\right)rd\Theta b - \mathrm{Pr}\, d\Theta b$

$$= \frac{dP}{dr} rdrd\Theta b$$

where the component of the differential side forces in the direction of C_r (last term in the equation) is equal to the pressure P multiplied by the projected area (in the direction of C_r) of the side surface of the particle.

The differential shear stress (τ) on a diffuser wall is opposed to the direction of C and is given by

$$\tau = c_f \frac{\rho C^2}{2}$$

where c_f is the skin friction coefficient. The differential shear forces in the radial direction on the fluid particle in Fig. are opposed to the direction of C_r and act on both walls of the diffuser.

Differential shear forces $= 2\tau rd\Theta dr\cos\alpha$

$$= c_f \frac{\rho C^2}{2} 2rd\Theta dr\cos\alpha$$

$$= c_f \rho C^2 rd\Theta dr\cos\alpha$$

The acceleration of the fluid particle in the direction opposed to C_r is made up of:

- the component of the centripetal acceleration $\frac{C_u^2}{r}$
- the negative of the acceleration $\frac{dC_r}{dt}$

differentiating with respect to r instead of t

- $$\frac{dC_r}{dt} = \frac{dC_r}{dr}\frac{dr}{dt} = C_r\frac{dC_r}{dr}$$

The differential force required for acceleration of the fluid particle becomes

- $= m \times a$
- $= \rho V \times a$
- $= \rho b r d\Theta dr\left(\frac{C_u^2}{r} - C_r\frac{dC_r}{dr}\right)$ (differential force required for acceleration in a direction opposed to C_r).

the sum of the differential pressure force and the shear forces must equal the force required for acceleration

- $$\frac{1}{\rho}\frac{dP}{dr} brd\Theta dr + c_f \rho C^2 rd\Theta dr \cos\alpha = \rho brd\Theta dr\left(\frac{C_u^2}{r} - C_r\frac{dC_r}{dr}\right)$$

dividing both sides by $(brd\Theta dr)$

we get the equation of radial equilibrium

- $$\frac{1}{\rho}\frac{dP}{dr} + \frac{c_f C^2\cos\alpha}{b} = \frac{C_u^2}{r} - C_r\frac{dC_r}{dr} \qquad (8)$$

To get the tangential equilibrium equation

the differential shear forces = $2\tau rd\Theta dr\sin\alpha$

substituting for the shear coefficient

$$c_f\frac{\rho C^2}{2} 2 rd\Theta dr\sin\alpha$$

which gives

$$c_f \rho C^2 r d\Theta dr \sin\alpha$$

the tangential acceleration of the fluid particle opposed to the direction of Cu consists of

• the negative of the tangential acceleration

$$r\frac{d}{dt}\left(\frac{C_u}{r}\right)$$

• the negative of the Coriolis equation

$$\frac{2C_u C_r}{r}$$

But

$$\frac{d}{dt}\left(\frac{C_u}{r}\right) = \frac{1}{r}\frac{dC_u}{dt} - \frac{C_u}{r^2}\frac{dr}{dt}$$

changing the dependency of the right hand side of the equation to r instead of t

$$\frac{d}{dt}\left(\frac{C_u}{r}\right) = \frac{1}{r}\frac{dC_u}{dr}\frac{dr}{dt} - \frac{C_u}{r^2}\frac{dr}{dt}$$

rearranging

$$\frac{d}{dt}\left(\frac{C_u}{r}\right) = \frac{C_r}{r}\left(\frac{dC_u}{dr} - \frac{C_u}{r}\right)$$

substituting into the equation

$$-r\frac{d}{dt}\left(\frac{C_u}{r}\right) - \frac{2C_u C_r}{r} = -r\left(\frac{C_r}{r}\frac{dC_u}{dr} - \frac{C_r C_u}{r^2}\right) - \frac{2C_u C_r}{r}$$

rearranging

$$-r\frac{d}{dt}\left(\frac{C_u}{r}\right) - \frac{2C_u C_r}{r} = -C_r\frac{dC_u}{dr} - \frac{C_u C_r}{r}$$

Acceleration is defined as

$$m \times a$$

or $\rho V \times a$

substituting for each term

$$-\rho b r d\Theta dr\left(C_r \frac{dC_u}{dr} + \frac{C_u C_r}{r}\right)$$

equating the acceleration to the differential shear force

$$-\rho b r d\Theta dr\left(C_r \frac{dC_u}{dr} + \frac{C_u C_r}{r}\right) = c_f \rho C^2 r d\Theta dr \sin\alpha$$

we finally get

$$-\frac{c_f C^2 \sin\alpha}{b} = C_r \frac{dC_u}{dr} + \frac{C_u C_r}{r} \tag{9}$$

Equation of state

$$P = \rho R T \tag{10a}$$

differentiating with respect to r

$$\frac{dP}{dr} = \rho R \frac{dT}{dr} + TR \frac{d\rho}{dr}$$

Dividing both sides by ρRT

we get the differential form of the equation of state

$$\frac{1}{P}\frac{dP}{dr} = \frac{1}{T}\frac{dT}{dr} + \frac{1}{\rho}\frac{d\rho}{dr} \tag{10b}$$

Heat transfer equation:

The hate Transfer rate to the diffuser casing is given by:

$$dQ = 2h'\left(T_w - T_0\right) 2\pi r dr \tag{11a}$$

where (h') is the coefficient of heat transfer, Tw is the wall or diffuser casing temperature, and dQ is the heat transfer rate.

The heat transfer rate from the fluid is given by:

$$dQ = \rho C_r 2\pi r b c_p \frac{dT_0}{dr} dr \tag{11b}$$

equating both equations together

$$2h'\left(T_w - T_0\right)2\pi r dr = \rho C_r 2\pi r b c_p \frac{dT_0}{dr} dr$$

rearranging terms

$$\frac{1}{T_0}\frac{dT_0}{dr} = \frac{2h'}{\rho C_r b c_p}\left(\frac{T_w}{T_0} - 1\right) \qquad (11c)$$

substituting (h') from Reynolds's analogy

$$\frac{h'}{c_p \rho C} = \frac{c_f}{2}$$

from which the equation becomes

$$\frac{1}{T_0}\frac{dT_0}{dr} = \frac{c_f \sec\alpha}{b}\left(\frac{T_w}{T_0} - 1\right) \qquad (12)$$

developing an auxiliary equation

from radial equilibrium equation

$$\frac{1}{\rho}\frac{dP}{dr} + \frac{c_f C^2 \cos\alpha}{b} = \frac{C_u^2}{r} - C_r \frac{dC_r}{dr}$$

we have

$$\frac{C_r}{C} = \cos\alpha$$

rearrange

$$C_r = C\cos\alpha$$

we have

$$\frac{C_u}{C} = \sin\alpha$$

rearrange

$$C_u = C\sin\alpha$$

substitute into the equation

$$\therefore \frac{1}{\rho}\frac{dP}{dr} + \frac{c_f C^2 \cos\alpha}{b} = \frac{C^2 \sin^2\alpha}{r} - C\cos\alpha \frac{d(C\cos\alpha)}{dr}$$

from tangential equilibrium

$$-\frac{c_f C^2 \sin\alpha}{b} = C_r \frac{dC_u}{dr} + \frac{C_u C_r}{r}$$

substituting for the velocities

$$-\frac{c_f C^2 \sin\alpha}{b} = C\cos\alpha \frac{d(C\sin\alpha)}{dr} + \frac{C^2 \sin\alpha\cos\alpha}{r}$$

Dividing by : $\sin\alpha\cos\alpha$

we get

$$-\frac{c_f C^2}{b\cos\alpha} = \frac{1}{2}\frac{dC^2}{dr} + \frac{C^2}{r}$$

from the radial equilibrium equation

$$\frac{1}{\rho}\frac{dP}{dr} + \frac{c_f C^2 \cos\alpha}{b} = \frac{C^2 \sin^2\alpha}{r} - \frac{1}{2}\cos^2\alpha \frac{dC^2}{dr}$$

dividing by : $\sin^2\alpha$

$$\frac{1}{\rho\sin^2\alpha}\frac{dP}{dr} + \frac{c_f C^2 \cos\alpha}{b\sin^2\alpha} = \frac{C^2}{r} - \frac{1}{2}\cot^2\alpha \frac{dC^2}{dr}$$

substituting for $\dfrac{C^2}{r}$ from the radial equilibrium equation

$$\frac{1}{\rho\sin^2\alpha}\frac{dP}{dr} + \frac{c_f C^2 \cos\alpha}{b\sin^2\alpha} = -\frac{c_f C^2}{b\cos\alpha} - \frac{1}{2}\frac{dC^2}{dr} - \frac{1}{2}\cot^2\alpha \frac{dC^2}{dr}$$

$\times \sin^2\alpha$ we get

$$\frac{1}{\rho}\frac{dP}{dr} + \frac{c_f C^2 \cos\alpha}{b} = -\frac{c_f C^2 \sin^2\alpha}{b\cos\alpha} - \frac{1}{2}\frac{dC^2 \sin^2\alpha}{dr} - \frac{1}{2}\cos^2\alpha \frac{dC^2}{dr}$$

rearranging

$$\frac{1}{\rho}\frac{dP}{dr}=-\frac{c_f C^2\cos\alpha}{b}-\frac{c_f C^2\sin^2\alpha}{b\cos\alpha}-\frac{1}{2}\frac{dC^2}{dr}\left(\sin^2\alpha+\cos^2\alpha\right)$$

simplifying

$$\therefore\frac{1}{\rho}\frac{dP}{dr}=-\frac{c_f C^2}{b}\left(\frac{1}{\cos\alpha}\right)-\frac{1}{2}\frac{dC^2}{dr}$$

dividing by C^2

$$\frac{1}{\rho C^2}\frac{dP}{dr}=-\frac{1}{2C^2}\frac{dC^2}{dr}-\frac{c_f\sec\alpha}{b} \tag{13a}$$

but

$$\rho C^2=\frac{P}{RT}C^2\times\frac{\gamma}{\gamma}=\gamma PM^2 \tag{13b}$$

$$\therefore\frac{1}{\gamma PM^2}\frac{dP}{dr}=-\frac{1}{2C^2}\frac{dC^2}{dr}-\frac{c_f\sec\alpha}{b}$$

from equation (3)

$$\frac{1}{C^2}\frac{dC^2}{dr}=\left(\frac{1}{1+\gamma-\frac{1}{2}M^2}\right)\frac{1}{M^2}\frac{dM^2}{dr}+\frac{1}{T_0}\frac{dT_0}{dr}$$

substituting

$$\frac{1}{\gamma PM^2}\frac{dP}{dr}=-\frac{1}{2}\left[\left(\frac{1}{1+\gamma-\frac{1}{2}M^2}\right)\frac{1}{M^2}\frac{dM^2}{dr}+\frac{1}{T_0}\frac{dT_0}{dr}\right]-\frac{c_f\sec\alpha}{b}$$

rearranging ,we obtain

$$\frac{1}{P}\frac{dP}{dR^*}=-\frac{\gamma M^2}{2}\left[\left(\frac{1}{1+\gamma-\frac{1}{2}M^2}\right)\frac{1}{M^2}\frac{dM^2}{dR^*}+\frac{1}{T_0}\frac{dT_0}{dR^*}+\frac{2\xi}{H\cos\alpha}\right] \tag{13c}$$

where

$$\xi=c_f\left(\frac{r_T}{b_T}\right) \tag{13d}$$

$$P = \frac{P}{p_0}$$

$$R^* = \frac{r}{r_T} \tag{13e}$$

Total temperature

$$\frac{1}{T_0}\frac{dT_0}{dR^*} = \frac{2h'}{\rho C_r H c_p}\left(\frac{T_w}{T_0} - 1\right)\left(\frac{r_T}{b_T}\right) \tag{14a}$$

using Reynolds' analogy

$$\frac{h'}{c_p \rho C} = \frac{c_f}{2}$$

$$\frac{1}{T_0}\frac{dT_0}{dR^*} = \frac{\xi}{H\cos\alpha}\left(\frac{T_w}{T_0} - 1\right) \tag{14b}$$

Mach number

from the continuity equation

$$C_r \frac{dC_r}{dr} = \frac{C_u^2}{r} - \frac{1}{\rho}\frac{dP}{dr} - \frac{c_f C^2 \cos\alpha}{b}$$

substituting for the tangential velocity

$$C_r \frac{dC_r}{dr} = \frac{C_r^2 \tan^2\alpha}{r} - \frac{1}{\rho}\frac{dP}{dr} - \frac{c_f C_r^2 \cos\alpha}{b\cos^2\alpha}$$

dividing by Cr

$$\frac{1}{C_r}\frac{dC_r}{dr} = \frac{\tan^2\alpha}{r} - \frac{1}{\rho C_r^2}\frac{dP}{dr} - \frac{c_f \sec\alpha}{b}$$

but, from the equation of state

$$\rho = \frac{P}{RT}$$

and expressing $\rho C_r^2 = \rho C^2 \cos^2\alpha = \gamma P M^2 \cos^2\alpha$

we get

$$\therefore \frac{1}{C_r}\frac{dC_r}{dr} = \frac{\tan^2\alpha}{r} - \frac{\sec^2\alpha}{\gamma M^2}\frac{1}{P}\frac{dP}{dr} - \frac{c_f \sec\alpha}{b} \tag{15a}$$

Total temperature equation

$$\frac{1}{T_0}\frac{dT_0}{dr}=\frac{1}{T}\frac{dT}{dr}+\left(\frac{\gamma-1/_2 M^2}{1+\gamma-1/_2 M^2}\right)\frac{1}{M^2}\frac{dM^2}{dr}$$

equation of state

$$\frac{1}{P}\frac{dP}{dr}=\frac{1}{\rho}\frac{d\rho}{dr}+\frac{1}{T}\frac{dT}{dr}$$

$$\therefore \frac{1}{T}\frac{dT}{dr}=\frac{1}{P}\frac{dP}{dr}-\frac{1}{\rho}\frac{d\rho}{dr}$$

substituting into the total temperature equation

$$\frac{1}{T_0}\frac{dT_0}{dr}=\frac{1}{P}\frac{dP}{dr}-\frac{1}{\rho}\frac{d\rho}{dr}+\left(\frac{\gamma-1/_2 M^2}{1+\gamma-1/_2 M^2}\right)\frac{1}{M^2}\frac{dM^2}{dr}$$

rearranging

$$\frac{1}{\rho}\frac{d\rho}{dr}=\frac{1}{P}\frac{dP}{dr}-\frac{1}{T_0}\frac{dT_0}{dr}+\left(\frac{\gamma-1/_2 M^2}{1+\gamma-1/_2 M^2}\right)\frac{1}{M^2}\frac{dM^2}{dr} \qquad (15b)$$

continuity equation

$$\frac{1}{\rho}\frac{d\rho}{dr}+\frac{1}{C_r}\frac{dC_r}{dr}+\frac{1}{b}\frac{db}{dr}+\frac{1}{r}=0$$

substituting for $\frac{1}{\rho}\frac{d\rho}{dr}$ and $\frac{1}{C_r}\frac{dC_r}{dr}$ into the continuity equation

$$\frac{1}{P}\frac{dP}{dr}-\frac{1}{T_0}\frac{dT_0}{dr}+\left(\frac{\gamma-1/_2 M^2}{1+\gamma-1/_2 M^2}\right)\frac{1}{M^2}\frac{dM^2}{dr}+\frac{\tan^2\alpha}{r}-\frac{\sec^2\alpha}{\gamma M^2}\frac{1}{P}\frac{dP}{dr}-$$

$$\frac{c_f \sec\alpha}{b}+\frac{1}{b}\frac{db}{dr}+\frac{1}{r}=0$$

grouping terms together

$$\left(\frac{\gamma M^2 - \sec^2\alpha}{\gamma M^2}\right)\frac{1}{P}\frac{dP}{dr} - \frac{1}{T_0}\frac{dT_0}{dr} + \left(\frac{\gamma - 1/2\, M^2}{1 + \gamma - 1/2\, M^2}\right)\frac{1}{M^2}\frac{dM^2}{dr} + \frac{\tan^2\alpha}{r}$$

$$-\frac{c_f \sec\alpha}{b} + \frac{1}{b}\frac{db}{dr} + \frac{1}{r} = 0$$

to get the final result

$$\therefore \frac{1}{P}\frac{dP}{dR^*} = \frac{-\gamma M^2}{\gamma M^2 - \sec^2\alpha}\left[\begin{array}{l}\left(\frac{\gamma - 1/2\, M^2}{1 + \gamma - 1/2\, M^2}\right)\frac{1}{M^2}\frac{dM^2}{dR^*} - \frac{1}{T_0}\frac{dT_0}{dR^*} \\ -\frac{\xi}{H\cos\alpha} + \frac{1}{H}\frac{dH}{dR^*} + \frac{\sec^2\alpha}{R^*}\end{array}\right]$$

which combined with equation (13) to eliminate $\frac{1}{P}\frac{dP}{dR^*}$, finally gives

$$\frac{1}{M^2}\frac{dM^2}{dR^*} = \frac{-2\left(1 + \gamma - 1/2\, M^2\right)}{M^2 - \sec^2\alpha}\left[\begin{array}{l}\left(1 + \gamma M^2 - \tan^2\alpha\right)\frac{1}{2T_0}\frac{dT_0}{dR^*} \\ +\left(\gamma M^2 - \tan^2\alpha\right)\frac{\xi}{H\cos\alpha} - \frac{1}{H}\frac{dH}{dR^*} - \frac{\sec^2\alpha}{R^*}\end{array}\right] \tag{15c}$$

Flow direction

$$\tan\alpha = \frac{C_u}{C_r}$$

$$\frac{d\tan\alpha}{dr} = \frac{C_r\frac{dC_u}{dr} - C_u\frac{dC_r}{dr}}{C_r^2} \times \frac{C_r}{C_r}$$

$$\frac{1}{\tan\alpha}\frac{d\tan\alpha}{dr} = \frac{1}{C_u}\frac{dC_u}{dr} - \frac{1}{C_r}\frac{dC_r}{dr}$$

from equation $$-\frac{c_f C^2 \sin\alpha}{b} = C_r \frac{dC_u}{dr} + \frac{C_r C_u}{r}$$

$$\frac{dC_u}{dr} = -\frac{1}{C_r}\left(\frac{c_f C^2 \sin\alpha}{b} + \frac{C_r C_u}{r}\right)$$

$$\therefore \frac{1}{\tan\alpha}\frac{d\tan\alpha}{dR^*} = \frac{\sec^2\alpha}{\gamma M^2}\frac{1}{P}\frac{dP}{dR^*} - \frac{\sec^2\alpha}{R^*}$$

$$\frac{1}{\tan\alpha}\frac{d\tan\alpha}{dR^*} = \frac{\sec^2\alpha}{M^2 - \sec^2\alpha}\left[\begin{array}{l}\left(1+\frac{\gamma-1}{2}M^2\right)\frac{1}{T_0}\frac{dT_0}{dR^*} \\ +\left(1+(\gamma-1)M^2\right)\frac{\xi}{H\cos\alpha} - \frac{1}{H}\frac{dH}{dR^*} - \frac{M^2}{R^*}\end{array}\right] \tag{16}$$

To find an expression for pressure

$$\rho_1 C_1 \cos\alpha_1 r_T b_T = \rho C \cos\alpha r b$$

substituting

$$\frac{P_1}{T_1} M_1 \sqrt{T_1} \cos\alpha_1 r_T b_T = \frac{P}{T} M \sqrt{T} \cos\alpha r b$$

we finally get

$$\frac{P}{P_1} = \frac{1}{R^* H}\frac{\cos\alpha_1}{\cos\alpha}\frac{M_1}{M}\sqrt{\frac{T_0\left(1+\frac{\gamma-1}{2}M_1^2\right)}{T_{01}\left(1+\frac{\gamma-1}{2}M^2\right)}} \tag{17b}$$

Flow Path

$$\tan\alpha = \frac{R^* d\Theta}{dR^*}$$

$$\frac{d\Theta}{dR^*} = \frac{\tan\alpha}{R^*} \tag{18}$$

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