VIBRATIONAL ANALYSIS OF VERTICAL AXIS WIND TURBINE BLADES

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ABSTRACT

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The goal of this research is to derive a vibration model for a vertical axis wind turbine blade. This model accommodates the affects of varying relative flow angle caused by rotating the blade in the flow field, uses a simple aerodynamic model that assumes constant wind speed and constant rotation rate, and neglects the disturbance of wind due to upstream blade or post. The blade is modeled as elastic Euler-Bernoulli beam under transverse bending and twist deflections. Kinetic and potential energy equations for a rotating blade under deflections are obtained, expressed in terms of assumed modal coordinates and then plugged into Lagrangian equations where the non-conservative forces are the lift and drag forces and moments. An aeroelastic model for lift and drag forces, approximated with third degree polynomials, on the blade are obtained assuming an airfoil under variable angle of attack and airflow magnitudes. A simplified quasi-static airfoil theory is used, in which the lift and drag coefficients are not dependent on the history of the changing angle of attack. Linear terms on the resulting equations of motion will be used to conduct a numerical analysis and simulation, where numeric specifications are modified from the Sandia-17m Darrieus wind turbine by Sandia Laboratories.

I would like to dedicate this work to my mother Gonca Kapucu and my father Ali Kapucu.

ACKNOWLEDGMENTS

I would like to thank my advisor and my master Professor Brian Feeny for his endless technical and mental support and sharing his wisdom generously to influence me to go further. Without his dedication this research would have never existed.

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Introduction

1.1 Background

Our society's increasing demand on energy has reached a critical level where the future projections show us our fossil fuel sources will not be able to supply the demand in near future. Being only dependent on several energy sources makes humankind vulnerable to energy shortage. Moreover, fossil fuel dependence in energy production unnaturally empowers countries with rich fossil sources and increases the tension of international politics.

These facts show us the importance of the search for renewable energy sources. Being aware of the aforementioned facts, most countries are now challenging a productive competition to develop and master the energy sources of the future. Facing the exhaustion of fossil sources, we are spending more effort to exploit renewable sources such as wind, solar energy, geothermal energy.

Among the renewable energy alternatives, wind power is a popular player. Energy Information Administration's Annual Energy Outlook 2014[1] reference case projects 76% increase in electricity production from wind power by 2040, where this increase is more than 300% in the extended policies case. There exist several ways to absorb wind power and convert it into a useful energy form. Wind turbines are the most popular setups to accomplish this purpose. They basically convert the power of the wind into electricity via rotation in the generator. Wind turbines can be distinguished into two main classes depending on the orientation of the rotor: horizontal axis wind turbines (HAWTs) and vertical axis wind turbines (VAWTs). In this notation "axis" represents the axis of rotation of rotor. Axes of rotation of VAWT rotors are perpendicular to the ground, where they are parallel to ground in HAWT blades. See Figure 1.1.

Figure 1.1 A sample HAWT sketch on left[2] and VAWT sketch on right [3]

Horizontal axis wind turbines dominate the current market; however, this might simply be a coincidence [4]. The most common HAWT configuration has its blades facing the wind to generate a rotation as a result of the lift forces on the blades. Being widely used, they can be seen in various sizes from stand-alone installations to offshore fleets. As the energy demand has increased over the time, HAWTs have become larger to produce more energy from a single turbine. As of 2014, tallest commercial HAWT (MHI VESTAS V164) reaches a tower height of 140 meters with a 220 meter tip height and 82 meter blade length and can generate 8MW energy. Yet, as HAWT sizes have increased, problems with installation and durability also have arisen. This might be a limiting factor for the future of the HAWTs [5].

The second most common wind turbine configuration, the VAWT, has its blades rotating around a vertical axis. We can group VAWTs mainly into two: ones using lift-momentum airfoils (i.e. Darrieus wind turbines) and ones using drag cups (Savonius wind turbine)(See Figure 1.2). Among these there exist numerous configurations; such as, the Giromill/H-rotor configurations, which are common types of Darrieus Turbine, due their simplicity. Giromill/H-rotor style VAWTs consist of simply designed airfoils attached parallel to the rotor via several struts or a single strut.

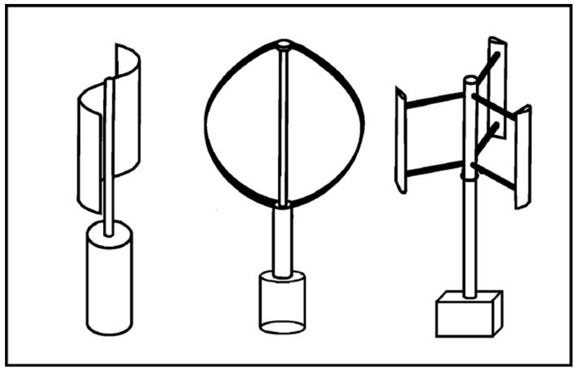


Figure 1.2 A sample sketch for a Savonius WT on the left, Darrieus WT in the middle and H-rotor configuration on the right [5]

Our most significant experience with commercial VAWT installations dates back to the early 80's [5, 6]. Manufacturer FloWind initiated a large operation to install Darrieus type VAWTs in the California Altamont and Tehachipi Wind Farms. The company installed more than 500 turbines and reached a maximum total production of 105,000 megawatt/hour in 1987. However, shortly after this, the turbine fleet began collapsing due to failures in the connection parts of the aluminum blades. By the late 90's, their energy production decreased down to 10,000 megawatt/hour and whole fleet was removed in early 2000's. Nonetheless, a thorough investigation of the blade dynamics was not conducted.

Next is a comparison of HAWTs and VAWTs in terms of efficiency, blade dynamics and installation features.

1.1.1 Efficiency

Betz Law states that a turbine can capture up to 59.2% of the kinetic energy of the flow [7]. When considered stand-alone, HAWTs are more efficient [8, 5, 9, 10] as can be seen from Figure (1.3). However, when located in arrays, as in wind farms, HAWTs heavily affect each other resulting a decrease in the total energy captured. On the other hand, several findings [9, 11, 12] show us that VAWTs may be more effective in capturing wind energy when positioned in wind farms. Moreover, since HAWTs have a fixed axis of rotation, their performance is heavily dependent on wind direction. Yet, VAWT performance is not affected with changing wind direction.

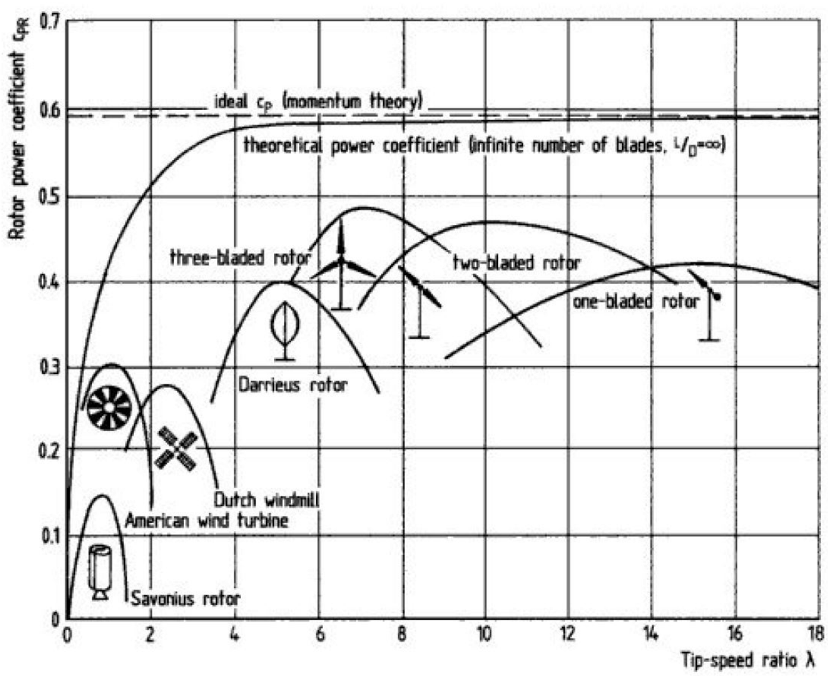


Figure 1.3 Comparison of aerodynamic efficiencies of different wind turbine types [10]

1.1.2 Blade Dynamics

HAWTs encounter cyclic gravitational loads on their blades. In the horizontal blade positions, gravity acts against either the leading or trailing edge. When a blade is in the bottom horizontal position of its rotation, the gravitational load acts in tension, whereas when a blade is in the top horizontal position of its rotation, gravitational load acts in compression. This cyclic change in the direction of load results periodic softening and stiffening of the blade structure, as well as periodic direct forcing [13, 14]. As stated earlier, cyclic gravitational loads may be a limiting factor for HAWTs as we keep increasing the turbine scales. Ongoing research by Acar and Feeny [15] focuses on modeling bend-bend-twist type deformations and resulting vibrations on HAWT blades. Also one can find more information about equations of motion for a rotor blade in the works Kalsøe[16] and nonlinear vibration analysis of the wind turbine blades in the works of Inoue and Ishida [14].

VAWTs don't get affected by cyclic gravitational forces, but they also confront durability problems due the cyclic lift and drag forces and moments. As VAWTs rotate, their blades face the incoming flow with changing angles of attack and varying magnitudes of relative flow velocity. Periodic changes of angle of attack create cyclic changes of lift and drag forces and moments.

HAWT blades also experience cyclic aerodynamic loads due to rotation through a height-dependent wind profile, and due to a pulse effect during tower passes.

1.1.3 Installation

Due their geometry, the center of gravity of a HAWT is higher than the center of gravity of a VAWT of the same size. This requires larger platforms for HAWTs, increasing the material weight and cost. As stated in the Sandia Workshop of 2012, VAWTs might be more advantageous in off-shore installations for this and a variety of other reasons [5].

1.2 Motivation

The following facts motivate the need of a comprehensive research in VAWTs [17]:

1. Not being the primary contributor of its market, our knowledge and experience with VAWT dynamics is limited.

2. Despite all of the efforts and research, large scaled HAWTs suffer failures due cyclic gravitational and aerodynamic effects. Therefore, being heavily dependent on HAWTs might be a bottleneck for the future of wind power. VAWTs might reach larger scales and therefore produce higher stand-alone energy rates in the future. Furthermore, VAWTs may perform better in arrays[9].

3. Known durability problems for VAWT blades are outdated and might be overcome with modern material and production technologies if investigated further.

4. A better understanding of VAWT dynamics might help us to prove and improve better off-shore wind power generation capacities.

5. Efficiency of stand-alone VAWTs might be improved with different blade configurations combining lift-momentum and drag cup.

However, as stated in a recent Sandia Report [5], VAWT technology is still open to development and research in the following areas can be conducted to broaden our knowledge and widen its usage.

1. Aerodynamics: Generally typical NACA blades are used in VAWTs. Improved aerodynamic analysis of turbines will help us find optimized blade solutions which can significantly

improve turbine efficiency. Similarly struts and joints can be optimized to improve performance.

2. Materials, drive train: Failures of VAWT blades in the 80's in California are thought to be related to the aluminum blade and joints. Further analysis of loads and vibrations in VAWTs combined with the improvements in the material science may open the path for durable VAWTs that can be scaled larger than HAWTs. Furthermore, once the loads and vibrations are modeled, optimal drivetrain architecture can be developed.

3. Configurations: Different blade configurations (number of blades, locations, mixed drag-cup, lift momentum usage etc.) can be investigated to improve the efficiency of VAWTs [18]. See Figure 1.4.

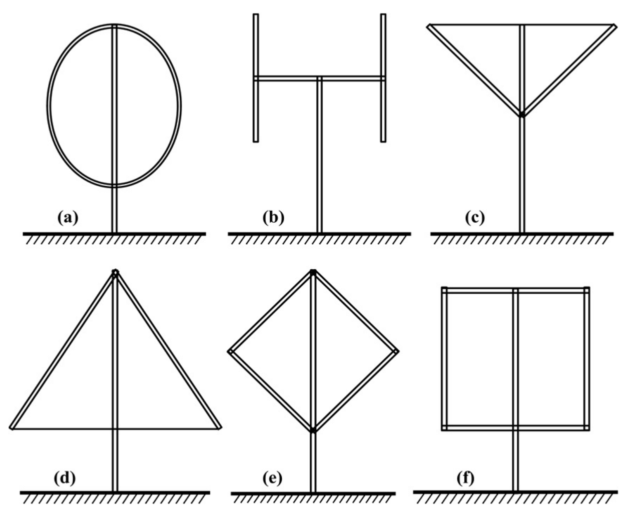


Figure 1.4 Basic VAWT Configurations: (a) Full Darrieus, (b)"H" , (c) "V", (d) "Δ", (e) "Diamond" and (f) "Giromill"[6]
.

4. Off-shore designs: The geometric advantage of VAWT usage offshore is mainly due to the lower center of gravity and zero cyclic gravitational forces. These advantages can be investigated further to contribute widespread usage of VAWTs.

1.3 Objective

This research aims to derive a vibration model that accommodates the affects of varying relative flow angle within a simple aerodynamic model. Throughout the research, a straight vertical airfoil, such as the ones in H-Rotor and Giromill configured VAWTs, will be the target of our interest. It will be supported by two struts located at the ends of the blade; still, it can be modified easily for other locations on the blade.

1.4 Contribution

This research aims to contribute

1. A simple model of VAWT blade vibration, which currently does not seem to exist. This two-mode model will consist of a pair of 2^{nd}-order ODEs that are cyclically forced and parametrically excited.

2. The model will be formulated to include up to cubic nonlinearity.

3. The ODEs will have many terms whose coefficients will be defined in a table of formulas that will be provided.

4. This work serves to set up the follow-up research which will involve analysis and numerical simulations.

1.5 Thesis Outline

In the upcoming chapters:

1. Energies for deriving nonlinear partial differential equations of the vibration of a single blade will be obtained for one-dimensional transverse bending and twist.

2. A simplified aeroelastic model will be introduced based on semi-empirical quasi-static airfoil theory.

3. Single assumed modes for transverse bending and twist will be used to reduce the order of equations in the model. Two ordinary differential equations representing bend and twist will be derived using Lagrange's equations. The equations will be linearized relative to the undeformed state.

4. A numerical evaluation of equation coefficients will be made for a specifically defined turbine blade, resulting in equations that are dependent on few parameters, such as rotation frequency and blade length. Since data for an existing H-rotor/Giromill turbine is not available, we will use data from a Sandia 17m Darrieus turbine as a guide for our H-rotor/Giromill model. The numerical evaluation will include an estimate of the first-mode natural frequency, as a function of blade length, for the non rotating blade. The aeroelastic terms will be based on a NACA0012 symmetric airfoil.

Chapter 2

Formulation and Modeling

We derive a vibration model for an H-rotor/Giromill blade. The blade is treated as a uniform straight beam under bending and twisting deformations.

This chapter can be presented in several parts. First comes the derivation of the energy equations for the bending and twisting blade. It will be followed by a simplified aerodynamic model to obtain the lift and drag force and moment equations as functions of time and spatial location on the blade.

Before going forward with modeling several assumptions were made:

1. Steady wind speed
2. Neglect the disturbance of wind due to upstream blade or post
3. Constant (steady-state) rotation rate
4. Inextensible beam
5. Shear center is at the geometric center of the blade

2.1 Elastic Blade Model for Bending

Following the ideas of Feeny [17] we will model our blades as an elastic Euler-Bernoulli beam. A blade that we model could face bend-bend-twist deflections, yet only one-dimensional transverse bending and twist will be taken into consideration for the rest of the formulation and modeling. Here transverse bending is the deflection of the blade to the radial direction, and twist occurs around the axis that passes through the centroid of the blade cross sections.

First, kinetic and potential energy equations for the particles on the beam will be expressed such that the extended Hamilton's principle can be used to obtain the partial differential equations of motion. Alternatively, a Lagrange formulation based on assumed modes can be applied to the energy formulations.

In order to do so, sections along the undeformed beam axis will be located from an origin at the middle center of the rotor using the following position vector:

$$\mathbf{r}(x,t) = (x - s(x,t))\mathbf{i} + (R + y(x,t))\hat{\mathbf{e}}_r \tag{2.1}$$

See Figure 2.1 and Figure 2.2.

In this equation, R is the distance between undeformed beam and the axis of rotation, x is the vertical distance of the blade section to the origin, $y(x,t)$ is the radially transverse displacement of the selected section on beam at time t, $s(x,t)$ is the cumulative foreshortening affecting point x at time t, due deflection. The foreshortening term, s, is formulated to be positive for $x > 0$. Also $\mathbf{i}$ is a unit vector in the direction in the undeformed x axis and $\hat{\mathbf{e}}_r$ is a unit vector in the direction in the y axis, radial to the center of rotation.

We can also formulate the foreshortening for an Euler-Bernoulli beam due to one dimensional transverse bending in the radial direction y to retain up to 4^{th} degree terms, as [13]

$$s(x,t) \cong \int_{-a}^{+a} (\frac{y'^2}{2} + \frac{y'^4}{8})dz, \tag{2.2}$$

which will contribute linear and cubic terms to the equations of motion where a is half the length of the blade. Assuming constant rotational speed and neglecting the effects of transverse deflection in the circumferential direction on the energy, the kinetic energy for

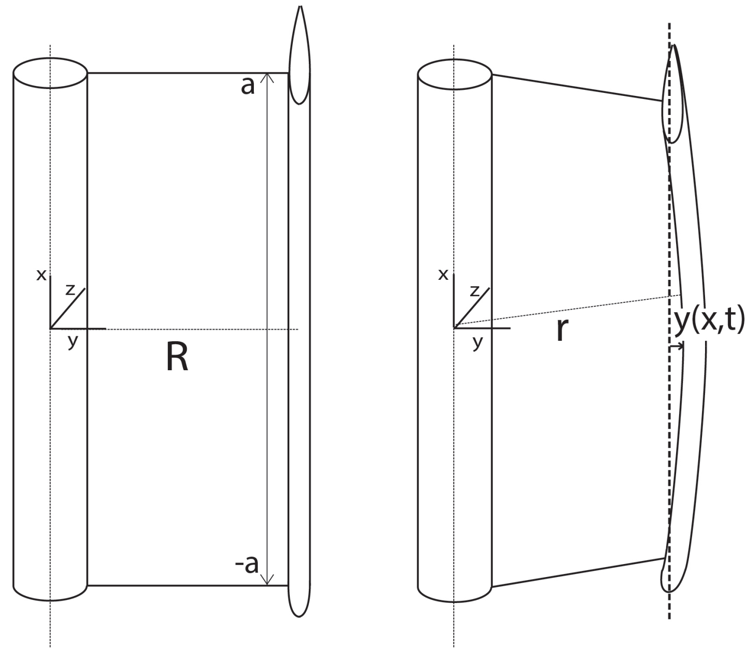


Figure 2.1 A sketch of blade and rotor. Undeformed on left and deformed on right.

the particles on the beam can be formulated with

$$T = \int_{-a}^{+a} \frac{1}{2} m(x) v(x,t)^2 dx + \int_{-a}^{+a} \frac{1}{2} J_{xx}(x)(\dot{\psi} + \Omega)^2 + \frac{1}{2} I_0 \dot{\theta}^2, \tag{2.3}$$

where I_0 is the mass moment of inertia of the rotor, $m(x)$ is the mass per unit length of the beam, and ψ is the angle of twist deformation. Furthermore, $\mathbf{v}$ is its velocity which can be found by taking time derivative of the position function $\mathbf{r}(x,t)$ as

$$\mathbf{v} = \dot{\mathbf{r}} = -\dot{s}\hat{\mathbf{e}}_\theta + \dot{y}\hat{\mathbf{e}}_r + \Omega(y(x,t) + R)\mathbf{k}, \tag{2.4}$$

where Ω is the constant angular speed of the rotating parts.

Similarly, potential energy can also be formulated in the form of integrations. We will express the total potential energy as V which is the summation of the gravitational potential energy, V_{mg}, strain energy due bending and twist, V_s, and the elastic potential energy of the boundaries of the beam, V_b.

As such, neglecting net vertical deflections of the blade, we have

$$V_{mg} = \int_{-a}^{+a} m(x) g s(x,t) dx, \tag{2.5}$$

$$V_s = \int_{-a}^{+a} \frac{1}{2} EI_z(x)[y''^2(1-3y'^2)]dx + \int_{-a}^{+a} \frac{1}{2} GI_x(x)\psi'^2 dx, \tag{2.6}$$

$$V_b = \frac{1}{2}k_1(u_g - s(-a,t))^2 + \frac{1}{2}k_{T_1}(y'(-a,t))^2 + \frac{1}{2}k_2(u_g - s(a,t))^2 + \frac{1}{2}k_{T_2}(y'(a,t))^2, \tag{2.7}$$

where E is the Young's modulus, and $I_z(x)$ is the area moment of inertia of the cross section at x about the neutral axis. V_s includes effects of nonlinear bending curvature [19] approximated to 4^{th} degree. Also one should note that elastic modeling has two parts for each strut. Struts are modeled as superposed linear and torsional springs where linear spring constants are k_1 and k_2, torsional spring constants are k_{T_1} and k_{T_2} for struts 1 and 2. In order to simplify the model, k_1 is assumed to be equal to k_2 and k_{T_1} is assumed to be equal to k_{T_2}.

The extended Hamilton's principle can be used to obtain partial differential equations of motion. Hamilton's principle states that

$$\int_{t_1}^{t_2} (\delta T - \delta V + \delta W_{nc}) dt = 0 \tag{2.8}$$

or

$$\int_{t_1}^{t_2} \left(\int_0^L (\delta \hat{T} - \delta \hat{V} + \delta\ \hat{W}_{nc}) dx + \delta T_b - \delta V_b + \delta W_{nc_b} \right) dt = 0 \tag{2.9}$$

where $\delta \hat{T}$, $\delta \hat{V}$ are energy densities, defined as the integrands in eqns (2.3), (2.5) and (2.6), and δT_b, and δV_b are boundary terms defined as the lumped terms in equations (2.3) and (2.7).

Here W_{nc} is the nonconservative virtual work acting on the airfoil. Since the forces are distributed over the length of the blade,

$$\delta W_{nc} = \int_{-a}^{a} \delta \hat{W}_{nc} dx \tag{2.10}$$

Alternatively, we can apply Lagrange's equations on assumed modal coordinates, as in the next section.

2.2 Reduced Order Modeling

In the previous section, we have introduced our model. As mentioned, a transverse bending and twist will be our sources of deflections on our rotating blade, which is modeled as an Eulerian beam. We will go further to simplify our model by using reduced order modeling. Deflections on the beam will be projected on assumed modes, based on hinged-hinged beam modes to obtain second-order nonlinear ordinary differential equations. Using an assumed mode formulation [20],

$$\tau_k(x,t) \approx \sum_{i=1}^{N} \zeta_i(t)\eta_i(x) \tag{2.11}$$

where τ is the deflection, N is the number of assumed modes, $\eta_i(x)$ are the assumed modal functions, and $\zeta_i(t)$ are the assumed modal coordinates. We can write

$$y(x,t) \cong \sum_{i=1}^{N_b} q_i(t)\xi_i(x) \tag{2.12}$$

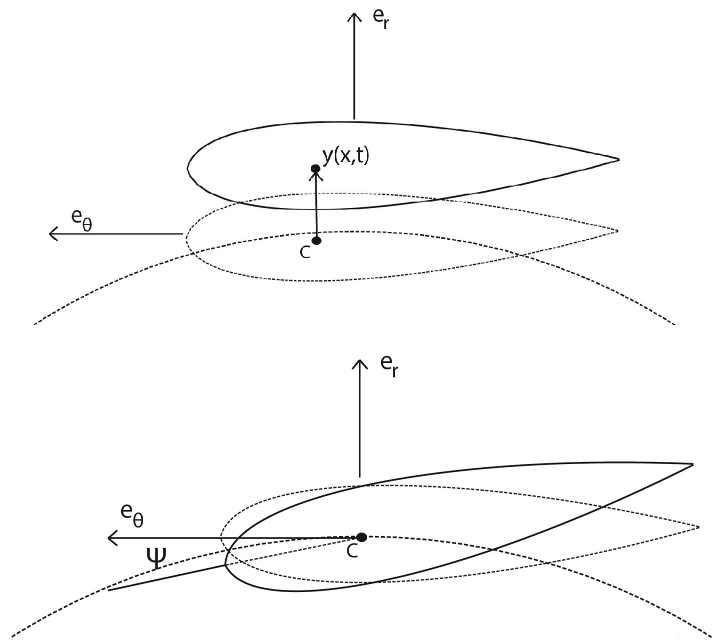


Figure 2.2 Top view representation of transverse deflection on top and twist on bottom.

$$\psi(x,t) \cong \sum_{i=1}^{N_t} b_i(t)\rho_i(x) \tag{2.13}$$

where N_b and N_t are the numbers of bend and twist assumed modes. We will use one assumed mode for both, so will have

$$y(x,t) \approx q(t)\xi(x) \tag{2.14}$$

$$\psi(x,t) \approx b(t)\rho(x) \tag{2.15}$$

As such, $\dot{y}(x,t) \approx \dot{q}(t)\xi(x), \dot{\psi}(x,t) \approx \dot{b}(t)\rho(x), y'(x,t) \approx q(t)\xi'(x), \psi'(x,t) \approx b(t)\rho'(x)$.

Next we can express our energy functions

$$\begin{aligned} T(q,\dot{q},b,\dot{b},t) = \int_{-a}^{+a} \frac{1}{2}m(x)\left((-\dot{s})^2 + \Big(\dot{q}(t)\xi(x)\Big)^2 + (\Omega(y(x,t)+R))^2\right)dx \\ + \int_{-a}^{+a} \frac{1}{2}J_{xx}(x)\left(\dot{b}(t)\rho(x)+\Omega\right)^2 + \frac{1}{2}I_0\dot{\theta}^2, \end{aligned} \tag{2.16}$$

$$V_{mg}(q, \dot{q}, t) = \int_{-a}^{+a} m(x) g s(q, \dot{q}, t) dx, \tag{2.17}$$

$$\begin{aligned} V_s(q, \dot{q}, b, \dot{b}, t) = \int_{-a}^{+a} \frac{1}{2} EI(x) \left[\left(q(t)\xi''(x) \right)^2 \left(1 - 3 \left(q(t)\xi'(x) \right)^2 \right) \right] dx \\ + \int_{-a}^{+a} \frac{1}{2} GJ_{xx}(x) \left(b'(t)\rho(x) \right)^2 dx, \end{aligned} \tag{2.18}$$

$$\begin{aligned} V_b(q, \dot{q}, b, \dot{b}, t) = \frac{1}{2} k_1 \left(u_g - s(-a, t) \right)^2 + \frac{1}{2} k_{T_1} \left(q'(t)\xi(-a) \right)^2 \\ + \frac{1}{2} k_2 \left(u_g - s(a, t) \right)^2 + \frac{1}{2} k_{T_2} \left(q'(t)\xi(a) \right)^2 \end{aligned} \tag{2.19}$$

Now, the energy equations on assumed modes can be plugged into Lagrange's equation:

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{q_k}}\right) - \frac{\partial T}{\partial q_k} + \frac{\partial V}{\partial q_k} = Q_k \tag{2.20}$$

to obtain equations of motion. Here T stands for the kinetic energy, V stands for the potential energy, and q_k is the generalized coordinate (dependent variable) from $k = 1, 2..., n$. Q_k is the generalized force term, and

$$\delta W_{nc} = \sum_{k=1}^{n} Q_k \delta q_k \tag{2.21}$$

Applying eqns. (2.14) and (2.15) for the case of a two assumed mode model

$$\delta W_{nc} = Q_q \delta q + Q_b \delta b \tag{2.22}$$

In our case of bend and twist deflections, application of Lagrange's equations leads to

$$\begin{aligned}
2q\dot{q}\int_{-a}^{a} m(x)\left[\int_{0}^{x}(\xi')^2dx\right]dx + q^2\dot{q}\int_{-a}^{a} m(x)\left[\int_{0}^{x}(\xi')^2dx\right]dx + \ddot{q}\int_{-a}^{a} m(x)\xi^2dx \\
-q\dot{q}^2\int_{-a}^{a} m(x)\left[\int_{0}^{x}(\xi')^2dx\right]dx - q\int_{-a}^{a} m(x)\Omega^2\xi^2dx - \int_{-a}^{a} m(x)\Omega^2R\xi dx \\
+q\int_{-a}^{a} gm(x)\left[\int_{0}^{a}(\xi')^2dx\right]dx + q^3\int_{-a}^{a} g\frac{m(x)}{2}\left[\int_{0}^{x}(\xi')^4dx\right]dx+ \\
q\int_{-a}^{a} EI(x)\xi''^2dx - q^3\int_{-a}^{a} 6EI(x)\xi''^2\xi'^2 \\
+q^3\frac{k_1+k_2}{2}\left[\int_{0}^{\pm a}(\xi')^2\right]^2 + q(k_{T_1}+k_{T_2})(\xi'|_{\pm a})^2 = Q_q
\end{aligned} \tag{2.23}$$

$$\ddot{b}\int_{-a}^{a} I_{xx}\rho^2dx + b\int_{-a}^{a} J_{xx}G\rho'^2dx = Q_b \tag{2.24}$$

where Q_q and Q_b are non-conservative aeroelastic forces to be determined in the next section.

2.3 Aeroelastic Modeling

One of the main problems with VAWTs that requires further investigation is the cyclic forces acting on the blades. Under a constant rotation rate of the rotor and blades, the relative velocity and angle of the wind particles hitting the blade is changing periodically, which creates cyclic lift and drag forces and moments on the blade. These forces are associated with the nonconservative work, W_{nc}, in the Hamilton's principle (2.8).

In order to simplify our aeroelastic model, our assumption is a flow field with constant direction and velocity. This assumption also neglects the effects of the blades on the fluid particles. Yet one should note that blades also have their own velocity due to rotation and deflection. Therefore the relative wind speed has a cyclic and displacement dependent behavior.

Figure 2.3 shows a sketch of the turbine rotating under the effect of constant wind speed u, such that the wind velocity is $\mathbf{u} = -u\mathbf{j}$, where $\mathbf{j}$ is a unit vector in the y direction in Figure 2.3. Note that in Figure 2.3, the y axis is fixed, and is not to be confused with transverse deflection, $y(x, t)$, which is in the $\mathbf{e}_r$ direction that rotates with the turbine. $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are radial and tangential unit vectors. θ is the coordinate that represents the orientation of the unit vector $\mathbf{e}_\theta$ with respect to the y-axis. Therefore $\mathbf{j} = \mathbf{e}_r \sin\theta + \mathbf{e}_\theta \cos\theta$. Furthermore we let θ, $\dot{\theta} = \Omega$. Therefore, $\dot{\mathbf{e}}_r = \dot{\theta}\mathbf{e}_\theta$ and $\dot{\mathbf{e}}_\theta = -\dot{\theta}\mathbf{e}_r$.

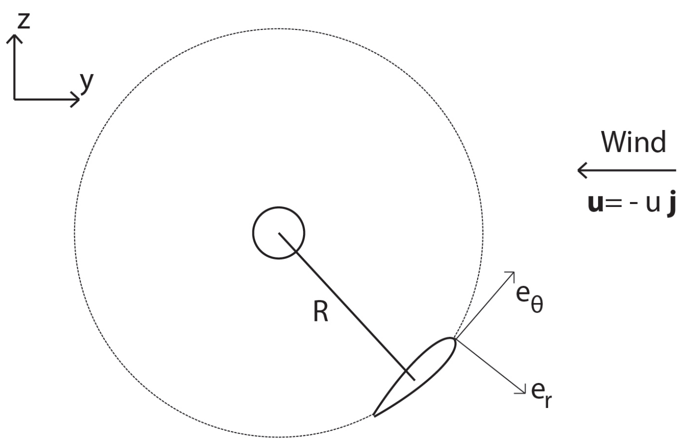


Figure 2.3 Cross section of VAWT showing the coordinate axes

Considering the transverse deflection, the distance $\mathbf{r}$ between the axis of rotation and blade's center of gravity can be notated in radial and tangential coordinates as $\mathbf{r} = (R + y(x, t))\mathbf{e_r}$ and by differentiating $\mathbf{r}$ we will obtain the velocity of the blade as $\mathbf{v}_b = \dot{\mathbf{r}} = \dot{y}\mathbf{e_r} + (R + y)\dot{\theta}\mathbf{e}_\theta$.

The flow velocity relative to the blade will be obtained as $\mathbf{v} = \mathbf{u} - \mathbf{v_b} = -u\mathbf{j} - \mathbf{v_b}$, and hence

$$\mathbf{v} = -(\dot{y} + u\sin\theta)\mathbf{e_r} - [(R + y)\dot{\theta} + u\cos\theta]\mathbf{e}_\theta \tag{2.25}$$

The angle between the relative flow velocity and tangential axis $\mathbf{e}_\theta$ is going to be denoted

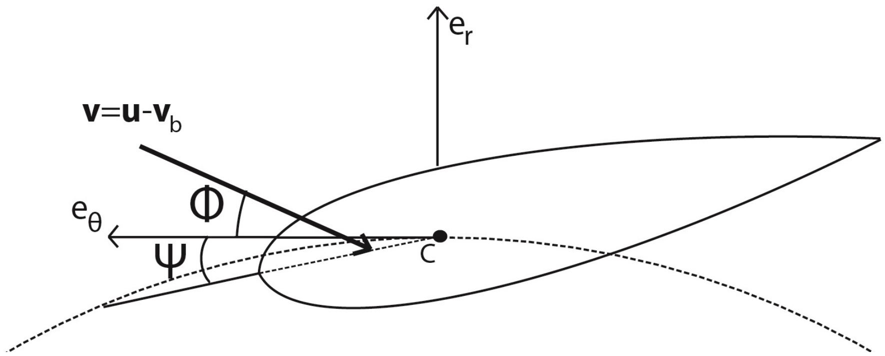


Figure 2.4 Twist angle ψ and relative wind angle ϕ for the case when the pretwist $\beta = 0$

with ϕ, where

$$\tan\phi = \mathbf{v}_r/\mathbf{v}_\theta = \frac{\dot{y} + u\sin\theta}{(R+y)\dot{\theta} + u\cos\theta} \tag{2.26}$$

One should note that angle of attack α is a summation of relative wind velocity angle, ϕ, pre-twist angle, β, and twist deformation angle ψ, i.e.

$$\alpha = \psi + \beta + \phi \tag{2.27}$$

where Figure (2.4) shows a sketch for the case $\beta = 0$.

Using the Taylor series expansion for ϕ for small values of $\mathbf{v}_r/\mathbf{v}_\theta$

$$\phi = \arctan\frac{\dot{y} + u\sin\theta}{(R+y)\dot{\theta} + u\cos\theta} \approx \frac{\dot{y} + u\sin\theta}{(R+y)\dot{\theta} + u\cos\theta} - \frac{1}{3}\left(\frac{\dot{y} + u\sin\theta}{(R+y)\dot{\theta} + u\cos\theta}\right)^3 \tag{2.28}$$

Now, we will expand $\tan\phi$ as a mutivariable power series for small variations of $\mu_k = y, \dot{y}, z, \dot{z}$ and let $\theta = \Omega t$:

$$\tan\phi = \frac{\dot{y} + u\sin\theta}{(R+y)\dot{\theta} + u\cos\theta} = \frac{u\sin\Omega t}{R\Omega + u\cos\Omega t} + \sum_{i=0}^{3}\mu_i\sum_{j=0}^{3}\mu_j\sum_{k=0}^{3}\mu_k d_{ijk} \tag{2.29}$$

where $\mu_0 = 1$ and $d_{000} = 0$. The lift and drag forces per unit length acting on the blade are

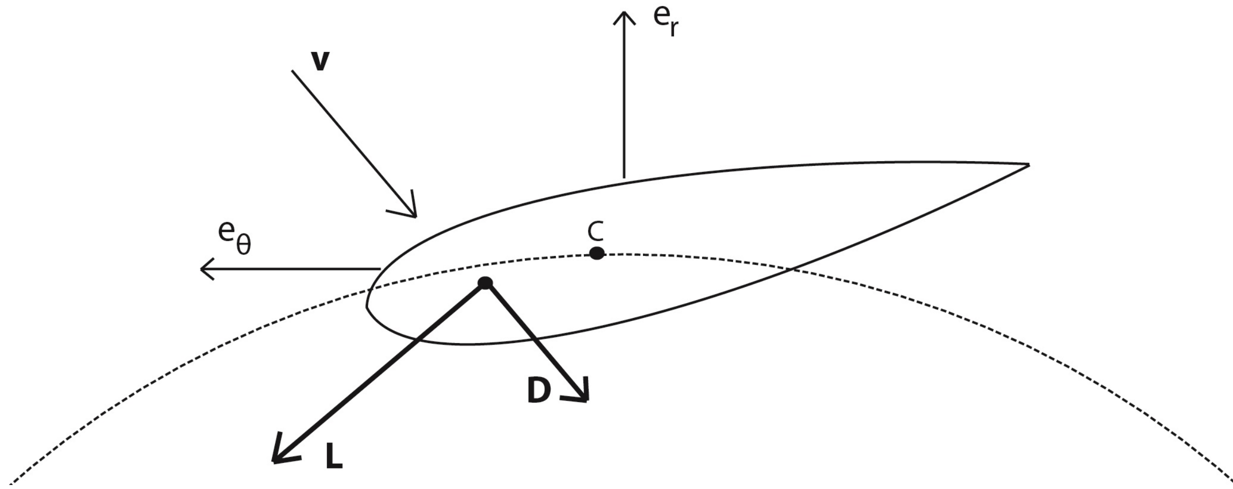


Figure 2.5 Lift and drag force per unit length directions from top view

$$\mathbf{L} = \frac{1}{2} C_L(\alpha) c \rho \nu^2 (-\cos\phi \mathbf{e_r} + \sin\phi \mathbf{e}_\theta) = L_r \mathbf{e_r} + L_\theta \mathbf{e}_\theta \tag{2.30}$$

$$\mathbf{D} = \frac{1}{2} C_D(\alpha) c \rho \nu^2 (-\sin\phi \mathbf{e_r} - \cos\phi \mathbf{e}_\theta) = D_r \mathbf{e_r} + D_\theta \mathbf{e}_\theta \tag{2.31}$$

in the directions that can be seen on Figure (2.5), where C_L and C_D are coefficients of lift and drag respectively, ρ is the density of air and c is the chord length. C_L and C_D are empirically determined functions of α, and for a symmetric airfoil we can assume $C_L(\alpha) \approx c_1\alpha - c_2\alpha^3$ and $C_D(\alpha) \approx d_1 + d_2\alpha^2$, where c_1, c_2, d_1 and d_2 are positive constants to be empirically determined case by case. For example, C_L and C_D for the case of NACA0012 are shown in Figure 2.6 from reference [21].

Since α is dependent on displacement and velocity variables through equations (2.25) and (2.26), C_L and C_D can be considered as functions of $y, \dot{y}$, and ψ, as $C_L(\alpha(y, \dot{y}, \psi))$ and $C_D(\alpha(y, \dot{y}, \psi))$. In addition, the relative flow speed is a function of variables, such that $v(y, \dot{y}))$. Thus,

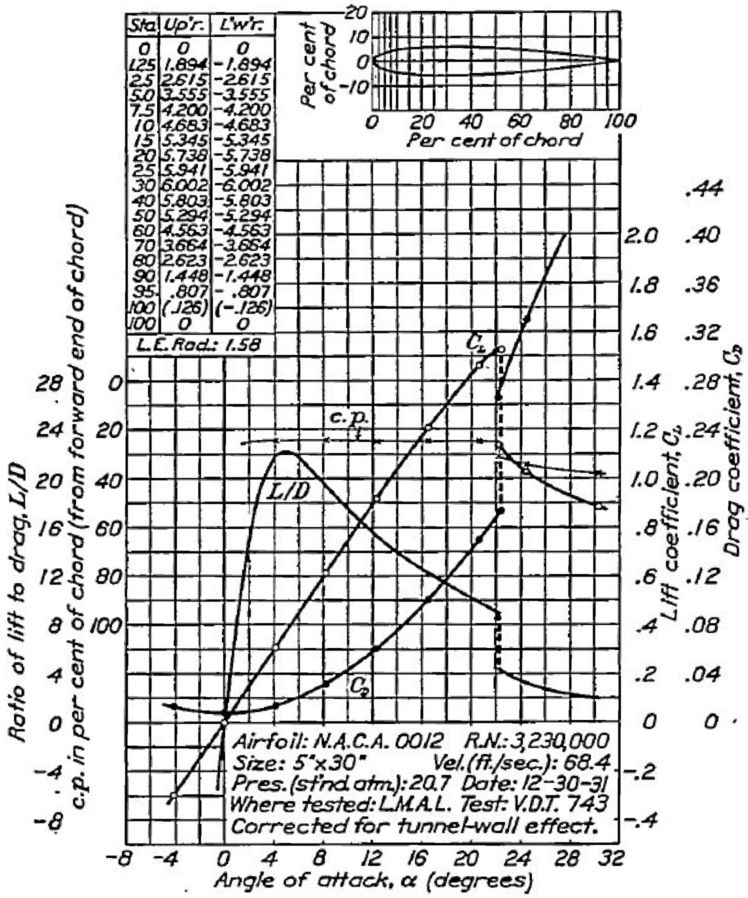


Figure 2.6 NACA0012 C_L and C_D values with respect to angle of attack [21]

$$C_L \approx c_1\alpha(y,\dot{y},\psi)^3 + c_2\alpha(y,\dot{y},\psi) \tag{2.32}$$

$$C_D \approx c_3\alpha(y,\dot{y},\psi)^2 + c_4 \tag{2.33}$$

and

$$v^2 = (\dot{y} + u\sin\theta)^2 + [(R+y)\dot{\theta} + u\cos\theta]^2 \tag{2.34}$$

As such, the aerodynamic loads can be expressed in terms of y,$\dot{y}$, and ψ,$\dot{\psi}$ by inserting equations (2.27) and (2.29) into equations (2.30) and (2.31), and also using equations (2.32), (2.33), and (2.34). This results in

$$\mathbf{L} \approx L_r(y, \dot{y}, \psi)\mathbf{e}_r + L_\theta(y, \dot{y}, \psi)\mathbf{e}_\theta \tag{2.35}$$

$$\mathbf{D} \approx D_r(y, \dot{y}, \psi)\mathbf{e}_r + D_\theta(y, \dot{y}, \psi)\mathbf{e}_\theta \tag{2.36}$$

In our case of transverse deflection and twist, the lift and drag forces and moments contribute to the $\delta\hat{W}_{nc}$ term in equation (2.10) such that

$$\delta\hat{W}_{nc} = \hat{f}(y, \dot{y}, \psi, \dot{\psi}, t)\delta y + \hat{M}(y, \dot{y}, \psi, \dot{\psi}, t)\delta\psi \tag{2.37}$$

where radial forces will define function $\hat{f}$ such as

$$\hat{f}(y, \dot{y}, \psi, t) = L_r(y, \dot{y}, \psi, t) + D_r(y, \dot{y}, \psi, t) \tag{2.38}$$

and a lift moment function $\hat{M}$ as

$$\hat{M}(y, \dot{y}, \psi, t) = (L\cos\alpha + D\sin\alpha)\, l_c \tag{2.39}$$

where l_c is the distance between the aerodynamic center of the airfoil and its center of gravity.

One will find that lift and drag are cyclic functions, and are time periodic in the case where Ω is constant. We will use the multi-variable Taylor series expansion (Appendix) up to cubic terms on our functions $\hat{f}(y, \dot{y}, \psi, t)$ and $\hat{M}(y, \dot{y}, \psi, t)$ around (y,$\dot{y}$,ψ)=(0,0,0). As such,

$$\begin{aligned}\hat{f}(y,\dot{y},\psi,t) = \hat{f}_{000} + \hat{f}_{100}y + \hat{f}_{010}\dot{y} + \hat{f}_{001}\psi + \hat{f}_{200}y^2 + \hat{f}_{020}\dot{y}^2 + \hat{f}_{002}\psi^2 \\ +\hat{f}_{110}y\dot{y} + \hat{f}_{101}y\psi + \hat{f}_{011}\dot{y}\psi + \hat{f}_{300}y^3 + \hat{f}_{030}\dot{y}^3 + \hat{f}_{003}\psi^3 \\ +\hat{f}_{210}y^2\dot{y} + \hat{f}_{201}y^2\psi + \hat{f}_{021}\dot{y}^2\psi + \hat{f}_{120}y\dot{y}^2 \\ +\hat{f}_{102}y\psi^2 + \hat{f}_{012}\dot{y}\psi^2 + f_{111}y\dot{y}\psi\end{aligned} \tag{2.40}$$

$$\begin{aligned}\hat{M}(y,\dot{y},\psi,t) = \hat{M}_{000} + \hat{M}_{100}y + \hat{M}_{010}\dot{y} + \hat{M}_{001}\psi + \hat{M}_{200}y^2 + \hat{M}_{020}\dot{y}^2 + \hat{M}_{002}\psi^2 \\ +\hat{M}_{110}y\dot{y} + \hat{M}_{101}y\psi + \hat{M}_{011}\dot{y}\psi + \hat{M}_{300}y^3 + \hat{M}_{030}\dot{y}^3 + \hat{M}_{003}\psi^3 \\ +\hat{M}_{210}y^2\dot{y} + \hat{M}_{201}y^2\psi + \hat{M}_{021}\dot{y}^2\psi + \hat{M}_{120}y\dot{y}^2 \\ +\hat{M}_{102}y\psi^2 + \hat{M}_{012}\dot{y}\psi^2 + \hat{M}_{111}y\dot{y}\psi\end{aligned} \tag{2.41}$$

where the coefficients $\hat{f}_{ijk}$ and $\hat{M}_{ijk}$ are time periodic. Then we will substitute $y = q(t)\xi(x)$ and $\psi = b(t)\rho(x)$ to write $\hat{f}(y(q,\xi(x)),\dot{y}(\dot{q},\xi(x)),\psi(b,\rho(x)),t) = f(q,\dot{q},b,t;x)$ and $\hat{M}(y(q,\xi(x)),\dot{y}(\dot{q},\xi(x)),\psi(b,\rho(x)),t) = M(q,\dot{q},b,t;x)$, and integrate according to equation (2.10) to write

$$\begin{aligned}\int_{-a}^{a} f(q,\dot{q},b,t;x)\xi(x)dx = f_{000} + f_{100}q + f_{010}\dot{q} + f_{001}b + f_{200}q^2 + f_{020}\dot{q}^2 + f_{002}b^2 \\ +f_{110}q\dot{q} + f_{101}qb + f_{011}\dot{q}b + f_{300}q^3 + f_{030}\dot{q}^3 + f_{003}b^3 \\ +f_{210}q^2\dot{q} + f_{201}q^2b + f_{021}\dot{q}^2b + f_{120}q\dot{q}^2 \\ +f_{102}qb^2 + f_{012}\dot{q}b^2 + f_{111}q\dot{q}b\end{aligned} \tag{2.42}$$

$$\int_{-a}^{a} M(q,\dot{q},b,t;x)\rho(x)dx = M_{000} + M_{100}q + M_{010}\dot{q} + M_{001}b + M_{200}q^2 + M_{020}\dot{q}^2 + M_{002}b^2$$
$$+M_{110}q\dot{q} + M_{101}qb + M_{011}\dot{q}b + M_{300}q^3 + M_{030}\dot{q}^3 + M_{003}b^3$$
$$+M_{210}q^2\dot{q} + M_{201}q^2b + M_{021}\dot{q}^2b + M_{120}q\dot{q}^2$$
$$+M_{102}qb^2 + M_{012}\dot{q}b^2 + M_{111}q\dot{q}b \tag{2.43}$$

where the coefficients f_{ijk} and M_{ijk} are cyclic in time, and can be found in the Appendix. Using equations (2.14) and (2.15), $\delta y = \xi(x)\delta q$ and $\delta\psi = \rho(x)\delta b$. Then the nonconservative work of equation (2.37) can be expressed in terms $\hat{f}(y(q,\xi(x)),\, \dot{y}(\dot{q},\xi(x)), \psi(b,\rho(x)), t)$ $=f(q,\dot{q},b,t;x)$ and $\hat{M}(y(q,\xi(x)), \dot{y}(\dot{q},\xi(x)), \psi(b,\rho(x)), t) = M(q,\dot{q},b,\, t;x)$ as

$$\delta\hat{W}_{nc} = f(y,\dot{y},\psi,\dot{\psi},t;x)\xi(x)\delta q + M(y,\dot{y},\psi,\dot{\psi},t;x)\rho(x)\delta b \tag{2.44}$$

As such, considering equations (2.8) and (2.10) the generalized forces in equations (2.23) and (2.24) have the form

$$Q_q = \int_{-a}^{a} f(q,\dot{q},b,t;x)\xi(x)dx \tag{2.45}$$

$$Q_b = \int_{-a}^{a} M(q,\dot{q},b,t;x)\rho(x)dx \tag{2.46}$$

which are expressed in terms of the expansions in equations (2.42) and (2.43).

Next we will linearize the equations that we obtained.

2.4 Linearization

In order to conduct a simple initial numerical analysis and simulate our model we will decrease the complexity of our model. In order to do so, several linearization methods were discussed.

Since the system has periodic excitation, equilibria do not exist. If we consider finding "unforced" equilibria, we can drop the direct excitation. The resulting equations are nonlinear, with many terms, and with parametric excitation; so equilibria may not exist. If we consider the equations when all cyclic time varying terms are omitted, the resulting equations are still nonlinear with many terms, and so the equilibrium will be difficult to express, and to use as a reference point. As such, we linearize about the equilibrium of the non rotating system, which is zero. Thus, the "linearized" model we consider is obtained by assuming y and ψ are small, hence quadratic and cubic terms in q and b and their derivatives are dropped.

The resulting equations for small deflections of the rotating system within direct and parametric excitations are

$$\ddot{q} + \omega_q^2 q + q a_1(t) + \dot{q} a_2(t) + b a_3(t) = a_0 + f_0(t) \tag{2.47}$$

$$\ddot{b} + \omega_b^2 b + q e_1(t) + \dot{q} e_2(t) + b e_3(t) = e_0 + g_0(t) \tag{2.48}$$

where

$$a_0 = \frac{\int_{-a}^{a} m\Omega^2 \xi R dx}{\int_{-a}^{a} m\xi^2 dx}$$

$$a_1(t) = \frac{-\int_{-a}^{a} f_{100}\xi^2(x)dx}{\int_{-a}^{a} m\xi^2 dx}$$
$$a_2(t) = \frac{-\int_{-a}^{a} f_{010}\xi^2(x)dx}{\int_{-a}^{a} m\xi^2 dx}$$
$$a_3(t) = \frac{-\int_{-a}^{a} f_{001}\xi(x)\rho(x)}{\int_{-a}^{a} m\xi^2 dx}$$
$$f_0(t) = \frac{\int_{-a}^{a} f_{000}\xi(x)dx}{\int_{-a}^{a} m\xi^2 dx}$$
$$\omega_q^2 = \frac{-\int_{-a}^{a} m\Omega^2\xi^2 dx + \int_{-a}^{a} gm\left(\int_0^x \xi'^2 dx\right)dx + \int_{-a}^{a} EI\xi''^2 dx + k_T\left[(\xi'|_{-a})^2 + (\xi'|_a)^2\right]}{\int_{-a}^{a} m\xi^2 dx}$$

and,

$$e_0 = 0$$
$$e_1(t) = \frac{-\int_{-a}^{a} M_{100}\xi^2(x)dx}{\int_{-a}^{a} Jxx\rho^2 dx}$$
$$e_2(t) = \frac{-\int_{-a}^{a} M_{010}\xi^2(x)dx}{\int_{-a}^{a} Jxx\rho^2 dx}$$
$$e_3(t) = \frac{-\int_{-a}^{a} M_{001}\xi(x)\rho(x)}{\int_{-a}^{a} Jxx}$$
$$g_0(t) = \frac{\int_{-a}^{a} M_{000}\xi(x)dx}{\int_{-a}^{a} Jxx\rho^2 dx}$$
$$\omega_b^2 = \frac{\int_{-a}^{a} IxxG(\rho')^2 dx}{\int_{-a}^{a} Jxx\rho^2 dx}$$

Chapter 3

Numerical Analysis and Simulation

3.1 Definition of the Giromill/H-Rotor Blade

After formulating the energy equations and modeling the nonconservative forces to obtain the equations of motion, now we want to make a vibration simulation and numerical analysis for our blade. Instead of focusing on the effects of different variables on VAWT blade vibration, we will focus simulating the behavior of a selected type of blade and accommodate reliability of our models.

As already stated, our knowledge and experience with VAWT is limited. In order to validate our model we need experimental data for reference. For that reason we will focus on the blades of 17 meter diameter Darrieus wind turbine of Sandia (Sandia 17m) [22] which were used by FloWind in California Altamont and Tehachipi wind farms.

Experimental data obtained from Sandia 17m shows us that maximum turbine performance is obtained around a tip ratio, $R\Omega/u$, of 5 [22, 18]. Moreover, Sandia's performance tests are handled for rotor speeds varying from 29.6 rpm to 52.5 rpm, where one can see further power output analysis on rotor speeds of 37 rpm and 48 rpm [22]. Moreover, their addendum for 3 bladed turbine also focused on a rotor speed of 37 rpm [18]. Therefore, for the parts that require a constant rate of rotation, we can assume a constant rate of rotation of 37 revolution per minute or similarly $\Omega = 3.875$ rad/s as a representative example for the numerical study.

The Sandia Darrieus wind turbine uses NACA0012 symmetric airfoil blades of 21 inch, or 0.53 meter, chord length that has a surface equation of [23]

$$\hat{y}(\hat{x}) = \pm\frac{0.12}{0.2}.53\left(0.2969\sqrt{\frac{\hat{x}}{.53}} - 0.1260\left(\frac{\hat{x}}{.53}\right) - .3516\left(\frac{\hat{x}}{.53}\right)^2 + 0.2843\left(\frac{\hat{x}}{.53}\right)^3 - 0.1015\left(\frac{\hat{x}}{.53}\right)^3\right) \tag{3.1}$$

where $\hat{x}$ is the horizontal location along the chord line of the airfoil and $\hat{y}$ defines the surface of the airfoil as a function of $\hat{x}$. This equation will be used with a uniform thickness to estimate the material cross sectional mass center, area moment of inertia and mass moment of inertia per unit length.

Our energy and aeroelastic models are based on Giromill/H-rotor style VAWTs which have straight blades that are parallel to axis of rotation. However, Darrieus VAWT blades have a troposkein curved shape referred as an "egg beater"[6]. Each Sandia 17m blade is constructed from two straight parts and one circular part attached together. See Figure 3.1.

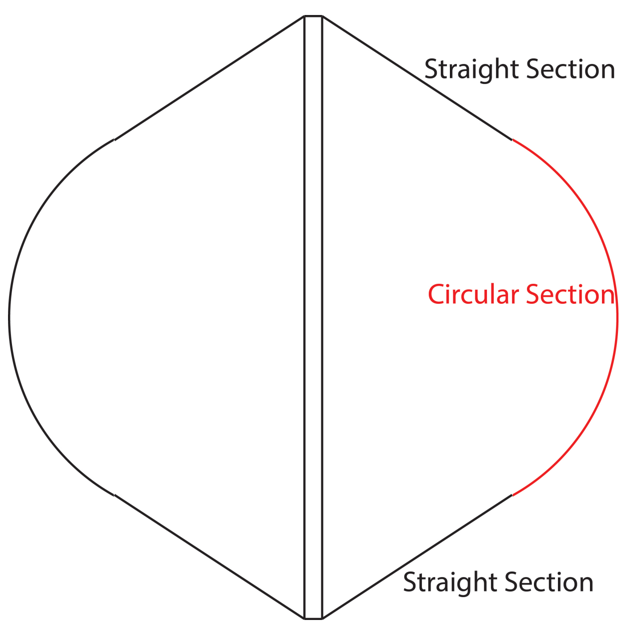


Figure 3.1 Sandia 17m front view

The cross section of a Sandia 17m blade has a joint construction of extruded aluminium, Nomex core, and fiberglass skin. See Figure 3.2.

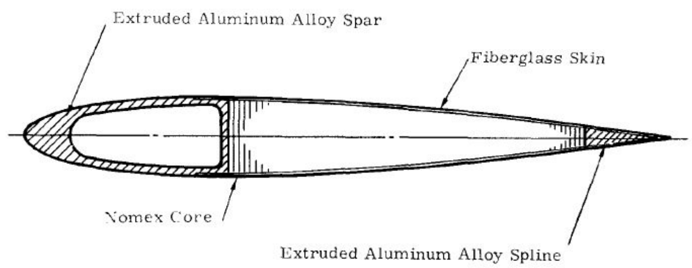


Figure 3.2 Sandia 17m blade cross section showing aluminum spar and spline, Nomex core and fiberglass skin. [22]

We will focus on the straight sections of the Sandia 17m and approximate the blades to be constructed only of extruded aluminum. Straight sections of Sandia 17m weigh 10.137 lb/ft and thus have a mass per unit length of $m = 15.09$ kg/m (Appendix) [22]. In order to have same mass/length ratio, based on equation (3.1) the wall thickness of our extruded aluminum airfoil with density of $\rho_{aluminium} = 2700$ kg/m^3 should be 0.2 inches or rounded to 0.005 meters. See Figure 3.3.

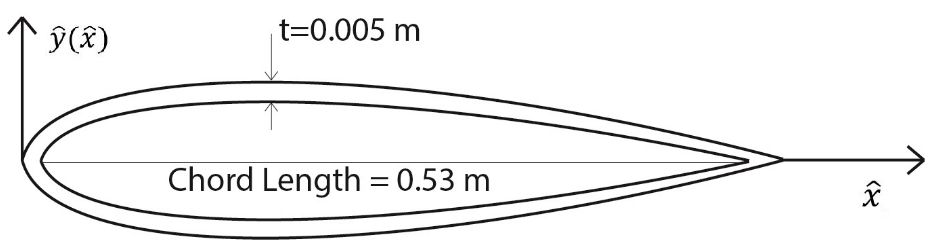


Figure 3.3 Blade dimensions for calculations

Using the NACA0012 with extruded aluminum of thickness 0.005 m and chord length of 0.53 m, see Figure 3.3, we can calculate the centroid of the blade cross section to be located 0.263 m from the front tip and the distance between centroid, and aerodynamic center, 1/4 of chord length for two dimensional incompressible flow [24], of the blade, l_c in Equation

(2.39), is going to be $0.263 - 0.53/4 = 0.13$ meters. Furthermore, we can calculate the mass moment of inertia per unit length around centroid of blade cross section as $J_{\hat{z}} = 0.37$ kg.m, second moment of area around the centroid of the blade as $I_{\hat{z}} = 0.000137$ m^4 and second moment of area around chord line as $I_{\hat{x}} = 3.1034 * 10^{-6}$ m^4. Also we consider an aluminum alloy with a density of 2700 kg/m^3, Young's modulus of $E = 70$ GPa and shear modulus of $G = 26$ GPa [25].

Table 3.1 Selected aluminium alloy's material properties

Property	Value
Density	2700 kg/m^3
Young's Modulus	70 GPa
Shear Modulus	26 Gpa

In order to numerically analyze the equations of motion of an undeformed blade, the radius R should be fixed. We don't have configuration data for a mid-sized or large H-rotor/Giromill turbine, so we will use Sandia's data on a 17m Darrieus blade as a reference and guide for establishing a hypothetical H-rotor/Giromill of about the same size. Sandia 17m has a varying radius from 0 to 27.44 feet at maximum [22]. Yet Giromill/H-rotor type VAWTs have constant undeformed blade radii. Approximating with a half circle shape for Sandia 17m blade, one can calculate an average radius either for the lift force, L, or effective torque by lift on rotor, T. Our calculations gave us an average radius of 19.40 feet in order to obtain the same lift and an average radius of 20.62 feet in order to obtain the same torque on rotor. Referring to these values obtained we will assume a constant radius R of 20 feet, which is 6 meters, for our Giromill/H-rotor turbines. A summary of blade parameters is given in Table (3.2).

Table 3.2 Specifications of the model created

Blade	NACA0012
Material and Construction	Extruded Aluminum Shell
Radius	6 m
Cross sectional weight	15.09 kg/m
Mass moment of inertia around centroid per length	0.37 kg.m
Second moment of area around chord line	3.1034x10^{-6} m^4
Second moment of area around centroid	0.000137 m^4
Tip Speed Ratio	5

3.2 Modal Frequencies

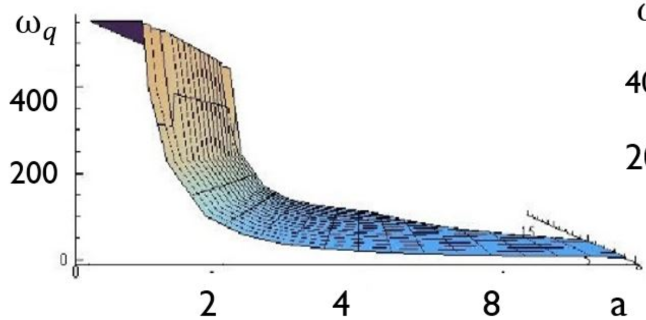
Now we would like to see the effects of varying blade length and rotation rate on the natural frequencies of the blade. For this purpose we assume that the blade is rotating under no aeroelastic forces, such that Q_q and Q_b are equal to zero. Assuming zero torsional stiffness on struts, i.e. the strut connections are ideal pin connections, Equations (2.47) and (2.48) will be

$$\ddot{q}\int_{-a}^{a} m\xi^2 dx - q\int_{-a}^{a} m\Omega^2\xi^2 dx - \int_{-a}^{a} m\Omega^2 \xi R dx + q\int_{-a}^{a} gm\left(\int_0^x \xi'^2 dx\right)dx + q\int_{-a}^{a} EI\xi''^2 dx = 0 \tag{3.2}$$

$$\ddot{b}\int_{-a}^{a} J_{xx}\rho^2 dx - b\int_{-a}^{a} I_{xx}G(\rho')^2 dx = 0 \tag{3.3}$$

where ξ and ρ are approximated by $\cos \pi x/2a$ for the first mode shapes. Note that the linear conservative model is decoupled, since the shear center was assumed to be at the geometric center of the cross section of the blade.

Plots of bending natural frequency for varying values of Ω and a are given in Figure 3.4 for varying half length a and rotor speed Ω.



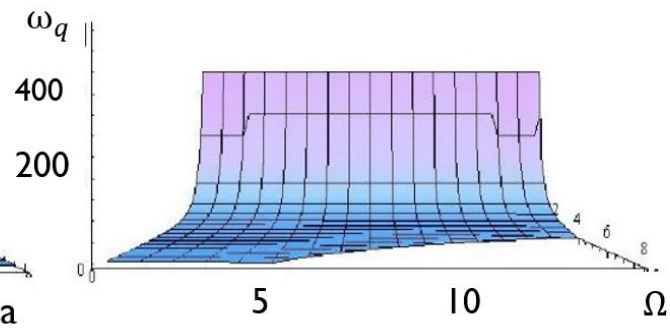


Figure 3.4 Variation of w_{n_q} with Ω on left and blade length a on right

The approximate model frequencies for bend and twist for a stationary blade (rotor speed $\Omega = 0$)) are shown in Figure 3.5 as functions of half length a. Note that the torsional frequency is much higher than the transverse bending frequency. This is consistent with individual models of bending torsion [20].

3.3 Blade with Quasistatic Aeroelastic Airfoil Model

It is mentioned that lift and drag coefficients with varying angle of attack differ with the type of blade used. Furthermore, we already stated that $C_L(\alpha) \approx c_2\alpha + c_1\alpha^3$ and $C_D(\alpha) \approx c_4 + c_3\alpha^2$. Using Figure 2.6 and curve fitting the data points we will obtain

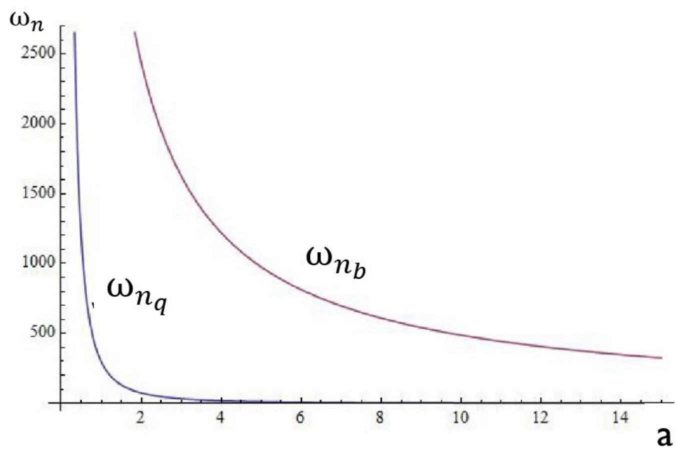


Figure 3.5 Variation of w_{n_q} and w_{n_b} with a when $\Omega = 0$

following equations for lift and drag coefficients:

$$C_L(\alpha) \approx 4.4287\alpha - 2.9916\alpha^3 \tag{3.4}$$

$$C_D(\alpha) \approx 0.0094 + 1.185\alpha^2 \tag{3.5}$$

These values contribute to load coefficients f_{ijk} in equations (2.42) and (2.43) through equations (2.25), (2.27), and (2.28)-(2.34), and the formulas listed in the appendix. The offset of the lift and drag forces is used to express the lift moment, based on the approximation that the offset is constant.

As such, the coefficients shown in equations (2.47) and (2.48) are listed in the appendix equations for specified values of parameters, but for arbitrary Ω, u, and a.

The key point to note is that the coefficients $a_1(t)$ through $a_3(t)$ and $e_1(t)$ through $e_3(t)$ are complicated functions of time, each with fundamental frequency Ω and harmonics, representing parametric excitation terms. The terms a_0 and $f_0(t)$ and e_0 and $g_0(t)$ represent constant plus periodic direct excitations.

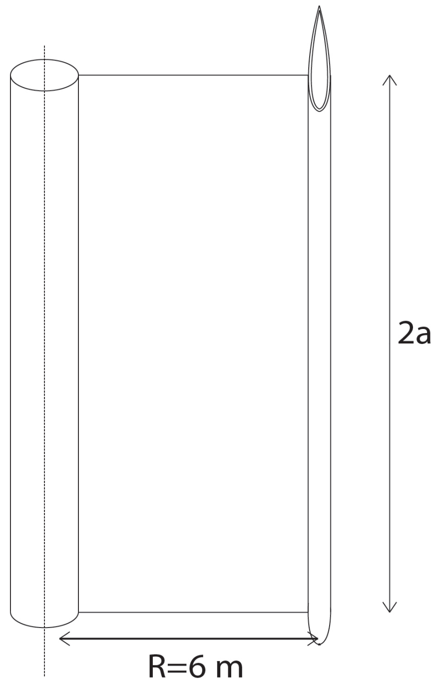


Figure 3.6 Rotor dimensions used in calculations

Thus, for the bend-twist VAWT blade vibration model, we derived an approximate lower-order set of linearized equations that can now be verified, analyzed and simulated for resonances and instabilities. This sets up the model for future analysis.

Chapter 4

Conclusion and Comments

4.1 Summary of Results

As a result of the research, a blade vibration model for an H-rotor/Girmoill type VAWT for bending and twisting deflections was formulated. In addition, energy equations for an Eulerian beam under transverse bend and twist deflections was obtained. The system was discretized using reduced order modeling on first assumed modes on each independent variable in energy equations. Lagrange's method was applied to resulting equations to obtain two equations of motion where generalized force terms were due aeroelastic forces on blades. Next, an aeroelastic model was defined based on quasistatic airfoil theory. Lift and drag forces and moments were formulated for an airfoil for changing angle of attack, where stall effects were neglected. Resulting complicated formulas were simplified to cubic order using Taylor series expansion. One should note that, resulting system had parametric and direct excitation due to varying flow magnitude and direction relative to blade. Very complicated terms were tabulated based on how to generate them with computer algebra.

In order to conduct a simple numerical analysis the system was linearized assuming small deflections for bend and twist. Linearized equations of motion were derived for a specific blade. Referring to Sandia 17m VAWT, hypothetical H-rotor/Giromill blade was defined for numerical analysis where natural frequencies of the blade for a non rotating system were found.

Furthermore, initial simulations showed complicated dynamics and propensity for instability.

4.2 Contribution

In a large absence of VAWT blade vibration models, this work has provided the initial steps at modeling VAWT blade vibrations to the research literature.

The model presented here can be used as a foundation for subsequent vibration modeling research on VAWTs. Research to be done while building this model is described in the next section on Future Work.

4.3 Future Work

This work sets up follow-up work on a systematic study of vibrations and how responses depend on parameters. To this end, future work will include

- verification of the terms which are very complicated
- perturbation analysis and numerical simulations to determine the potential resonances and instabilities, and the effects of parameters on these phenomena
- higher degree of freedom reduced model, where more than one assumed mode will be used in the calculations
- improvement of the aeroelastic model, i.e. including the effects of stall and the hysteretic effects caused by the influence of the changing angle of attack on the local flow field.

- improvement of the structural model, for example in separating the shear center from the geometric center, and including bend-bend-twist deflections.
- nonlinearity back into the model and perform nonlinear analyses and simulations
- modeling of the curved blades of Darrius-type turbines

APPENDIX

The following terms are obtained from Taylor Series expansion of radial force component f and moment M in equations (2.42) and (2.43) where c_1, c_2, c_3 and c_4 are constants from lift and drag coefficient formulations and dependent on the geometry of the airfoil.

$$\mathbf{f_{000}} = 0.5\rho_{air}c(\Omega R + u\cos\theta)\sqrt{(\Omega R + u\cos\theta)^2 + u^2\sin^2\theta}$$
$$\left(c_1\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^3 + c_2\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right)$$
$$+0.5\rho_{air}cu\sin\theta\sqrt{(\Omega R + u\cos\theta)^2 + u^2\sin^2\theta}\left(c_3\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2 + c_4\right)$$

$$\mathbf{f_{100}} = \frac{0.5\rho_{air}c\Omega(\Omega R+u\cos\theta)^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$
$$\left(c_1\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^3 + c_2\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right)$$
$$+ 0.5\rho_{air}c(\Omega R + u\cos\theta)\sqrt{(\Omega R + u\cos\theta)^2 + u^2\sin^2\theta}$$
$$\left(3c_1\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4} - \frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2\right.$$
$$\left.+c_2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4} - \frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right) + 0.5\rho_{air}c\Omega\sqrt{(\Omega R + u\cos\theta)^2 + u^2\sin^2\theta}$$
$$\left(c_1\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^3 + c_2\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right)$$
$$+ \frac{0.5\rho_{air}c\Omega u\sin\theta(\Omega R+u\cos\theta)\left(c_3\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+c_4\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$
$$+ \rho_{air}c_3cu\sin\theta\sqrt{(\Omega R + u\cos\theta)^2 + u^2\sin^2\theta}\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4} - \frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$
$$\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$

$$\mathbf{f_{010}} = \frac{0.5\rho_{air}cu\sin\theta(\Omega R+u\cos\theta)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$
$$\left(c_1\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^3 + c_2\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right)$$
$$+ 0.5\rho_{air}c(\Omega R + u\cos\theta)\sqrt{(\Omega R + u\cos\theta)^2 + u^2\sin^2\theta}$$

$$\left(3c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2\right.$$
$$\left.+c_2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)$$
$$+\frac{0.5\rho_{air}cu^2\sin^2\theta\left(c_3\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+c_4\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$
$$+0.5\rho_{air}c\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(c_3\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+c_4\right)$$
$$+\rho_{air}c_3cu\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$
$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$

$$\mathbf{f_{001}}=0.5\rho_{air}c(\Omega R+u\cos\theta)\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}$$
$$\left(3c_1\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+c_2\right)$$
$$+\rho_{air}c_3cu\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$

$$\mathbf{f_{200}}=0.25\rho_{air}c$$
$$\left(\frac{4c_3\Omega u(\Omega R+u\cos\theta)\sin\theta\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\right.$$
$$+c_3u\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}$$
$$\left(2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2+2\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\right.$$
$$\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)$$
$$+u\sin\theta\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$
$$\left.\left(c_3\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_4\right)\right)$$
$$+0.5\rho_{air}c\left(2\left(\frac{\Omega(\Omega R+u\cos\theta)^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+\Omega\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right)\right.$$

$$
\left(3c_1\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.
$$

$$
\left.+c_2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right)
$$

$$
+\left(c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^3\right.
$$

$$
\left.+c_2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)
$$

$$
\left(\frac{2(\Omega R+u\cos\theta)\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+(\Omega R+u\cos\theta)\right.
$$

$$
\left.\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\right)
$$

$$
+(\Omega R+u\cos\theta)\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(c_2\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\right.
$$

$$
+c_1\left(6\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2+\right.
$$

$$
\left.\left.\left.3\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right)\right)\right)
$$

$$
\mathbf{f_{020}}=0.25\rho_{air}c\left(4c_3\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right.
$$

$$
\left(\frac{u^2\sin^2\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right)
$$

$$
+c_3u\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2\right.
$$

$$
\left.-\frac{4u\sin\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)}{(\Omega R+u\cos\theta)^3}\right)+\left(c_3\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.
$$

$$
\left.+c_4\right)
$$

$$
\left(u\left(\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{u^2\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\sin\theta\right.
$$

$$
\left.\left.+\frac{2u\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\right)\right)
$$

$$
+0.5\rho_{air}c\left(\frac{2u(\Omega R+u\cos\theta)\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\right.
$$

$$\left(\left(3c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.\right.$$
$$\left.\left.+c_2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)\right)$$
$$+(\Omega R+u\cos\theta)\left(\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{u^2\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$
$$\left(c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^3+c_2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)$$
$$+(\Omega R+u\cos\theta)\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(c_1\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2\right.\right.$$
$$\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)-\frac{6u\sin\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2}{(\Omega R+u\cos\theta)^3}\right)$$
$$\left.\left.-\frac{2c_2u\sin\theta}{(\Omega R+u\cos\theta)^3}\right)\right)$$

$$\mathbf{f_{002}}=0.5\left(3.\rho_{air}c_1c(\Omega R+u\cos\theta)\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right.$$
$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$
$$\left.+\rho_{air}c_3cu\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}+0.\right)$$

$$\mathbf{f_{110}}=\frac{0.5\rho_{air}c\Omega}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\left(3c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.$$
$$\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)(\Omega R+u\cos\theta)^2$$
$$-\frac{0.5\rho_{air}c\Omega u\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}$$
$$\left(c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^3+c_2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)$$
$$(\Omega R+u\cos\theta)^2$$
$$+\frac{\rho_{air}c_3c\Omega u\sin\theta\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)(\Omega R+u\cos\theta)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+\frac{0.5\rho_{air}c\Omega\left(c_3\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_4\right)(\Omega R+u\cos\theta)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$-\frac{0.5\rho_{air}c\Omega u^2\sin^2\theta\left(c_3\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_4\right)(\Omega R+u\cos\theta)}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}$$

$$+0.5\rho_{air}c\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(3c_1\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\right.$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2$$

$$+6c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$

$$\left.+c_2\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\right)(\Omega R+u\cos\theta)$$

$$+\frac{0.5\rho_{air}cu\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\left(3c_1\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right.$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2$$

$$\left.+c_2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right)(\Omega R+u\cos\theta)$$

$$+\rho_{air}c_3cu\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$

$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$+\rho_{air}c_3cu\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)+\rho_{air}c_3c\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}$$

$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$

$$+\frac{\rho_{air}c_3cu^2\sin^2\theta\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+0.5\rho_{air}c\Omega\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(3c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.$$

$$\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)$$

$$+\frac{0.5\rho_{air}c\Omega u\sin\theta\left(c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^3+c_2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$\mathbf{f_{101}}=\frac{0.5\rho_{air}c\Omega(\Omega R+u\cos\theta)^2\left(3c_1\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+c_2\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$
$$+0.5\rho_{air}c\Omega\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(3c_1\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+c_2\right)$$
$$+3.\rho_{air}c_1c(\Omega R+u\cos\theta)\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$
$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$
$$+\frac{\rho_{air}c_3c\Omega u\sin\theta(\Omega R+u\cos\theta)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$
$$+\rho_{air}c_3cu\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$\mathbf{f_{011}}=\frac{0.5\rho_{air}cu\sin\theta(\Omega R+u\cos\theta)\left(3c_1\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+c_2\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$
$$+3.\rho_{air}c_1c(\Omega R+u\cos\theta)\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$
$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)+\frac{\rho_{air}c_3cu^2\sin^2\theta\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$
$$+\rho_{air}c_3c\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$
$$+\rho_{air}c_3cu\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$

$$\mathbf{f_{300}}=\frac{1}{6}\left(0.5\rho_{air}c\left(6c_3u\sin\theta\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right.\right.$$
$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$

$$\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$+c_3 u\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}$$

$$\left(6\left(\frac{2\Omega^2 u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right.$$

$$+2\left(\frac{20\Omega^3u^3\sin^3\theta}{(\Omega R+u\cos\theta)^6}-\frac{6\Omega^3u\sin\theta}{(\Omega R+u\cos\theta)^4}\right)$$

$$\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)+\frac{3c_3\Omega u(\Omega R+u\cos\theta)\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$\left(2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2+2\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\right.$$

$$\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)$$

$$+u\sin\theta\left(\frac{3\Omega^3(\Omega R+u\cos\theta)^3}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{5/2}}-\frac{3\Omega^3(\Omega R+u\cos\theta)}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$\left.\left(c_3\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_4\right)\right)$$

$$+0.5\rho_{air}c\left(\left(c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^3\right.\right.$$

$$\left.+c_2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)$$

$$\left((\Omega R+u\cos\theta)\left(\frac{3\Omega^3(\Omega R+u\cos\theta)^3}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{5/2}}-\frac{3\Omega^3(\Omega R+u\cos\theta)}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\right.$$

$$\left.\left.+3\Omega\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\right)\right)$$

$$+3\left(3c_1\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.$$

$$\left.+c_2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right)\left(\frac{2(\Omega R+u\cos\theta)\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+(\Omega R+u\cos\theta)\right.$$

$$\left.\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\right)$$

$$+(\Omega R+u\cos\theta)\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(c_2\left(\frac{20\Omega^3u^3\sin^3\theta}{(\Omega R+u\cos\theta)^6}-\frac{6\Omega^3u\sin\theta}{(\Omega R+u\cos\theta)^4}\right)\right.$$

$$+c_1\left(6\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^3+18\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\right.$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$+3\left(\frac{20\Omega^3u^3\sin^3\theta}{(\Omega R+u\cos\theta)^6}-\frac{6\Omega^3u\sin\theta}{(\Omega R+u\cos\theta)^4}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\Bigg)\Bigg)$$

$$+3\left(\frac{\Omega(\Omega R+u\cos\theta)^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+\Omega\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right)$$

$$\Bigg(c_2\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)$$

$$+c_1\Bigg(6\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2$$

$$+3\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\Bigg)\Bigg)\Bigg)\Bigg)$$

$$\mathbf{f_{030}}=\frac{1}{6}\Bigg(0.5\rho_{air}c\Bigg(c_3u\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}$$

$$\left(-\frac{12u\sin\theta\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)}{(\Omega R+u\cos\theta)^3}-\frac{4\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)}{(\Omega R+u\cos\theta)^3}\right)$$

$$+3c_3\left(\frac{u^2\sin^2\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right)$$

$$\left(2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2-\frac{4u\sin\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)}{(\Omega R+u\cos\theta)^3}\right)$$

$$+\left(c_3\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_4\right)\Bigg(u\sin\theta$$

$$\left(\frac{3u^3\sin^3\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{5/2}}-\frac{3u\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$+3\left(\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{u^2\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\Bigg)$$

$$+6c_3\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$

$$\Bigg(u\left(\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{u^2\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\sin\theta+$$

$$\frac{2u\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\Bigg)\Bigg)+0.5\rho_{air}c\,(3(\Omega R+u\cos\theta)$$

$$\left(\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{u^2\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$
$$\left(3c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.$$
$$\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)$$
$$+(\Omega R+u\cos\theta)$$
$$\left(\frac{3u^3\sin^3\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{5/2}}-\frac{3u\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$
$$\left(c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^3\right.$$
$$\left.+c_2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)+(\Omega R+u\cos\theta)\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}$$
$$\left(c_1\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^3-\frac{36u\sin\theta}{(\Omega R+u\cos\theta)^3}\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right.\right.$$
$$\left.\left.\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)-\frac{6\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2}{(\Omega R+u\cos\theta)^3}\right)-\frac{2c_2}{(\Omega R+u\cos\theta)^3}\right)$$
$$+\frac{3u(\Omega R+u\cos\theta)\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\left(c_1\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2\right.\right.$$
$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$
$$\left.\left.\left.\left.-\frac{6u\sin\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2}{(\Omega R+u\cos\theta)^3}\right)-\frac{2c_2u\sin\theta}{(\Omega R+u\cos\theta)^3}\right)\right)\right)$$

$$\mathbf{f_{003}}=\frac{1}{2}\left(0.5\rho_{air}c\left(-\frac{4c_3\Omega u^2(\Omega R+u\cos\theta)}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right.\right.$$
$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\sin^2\theta+\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$
$$\left(c_3u^2\left(2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2+2\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\right.\right.$$
$$\left.\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)\sin^2\theta\right)+$$

$$\frac{4c_3\Omega u(\Omega R+u\cos\theta)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+\frac{4c_3\Omega u(\Omega R+u\cos\theta)\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+2c_3u\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$

$$\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\sin\theta$$

$$+c_3u\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.$$

$$\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)+4\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)$$

$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)+2\left(\frac{2\Omega^2}{(\Omega R+u\cos\theta)^3}-\frac{12\Omega^2u^2\sin^2\theta}{(\Omega R+u\cos\theta)^5}\right)$$

$$\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)\sin\theta$$

$$+u\left(\frac{3\Omega^2u(\Omega R+u\cos\theta)^2\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{5/2}}-\frac{\Omega^2u\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$\left(c_3\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_4\right)\sin\theta$$

$$+\frac{4c_3\Omega(\Omega R+u\cos\theta)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+c_3\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}$$

$$\left(2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2\right.$$

$$\left.+2\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)$$

$$+\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$\left(c_3\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.$$

$$+c_4))+0.5\rho_{air}c\left(2\left(\frac{\Omega(\Omega R+u\cos\theta)^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+\Omega\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right)\right.$$

$$
\left(3c_1\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+\right.
$$

$$
6c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)
$$

$$
\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)
$$

$$
\left.+c_2\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\right)
$$

$$
+2\left(\frac{\Omega u\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega u(\Omega R+u\cos\theta)^2\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)
$$

$$
\left(3c_1\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.
$$

$$
\left.+c_2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right)+\left(c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^3\right.
$$

$$
\left.+c_2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)\left((\Omega R+u\cos\theta)\left(\frac{3\Omega^2u(\Omega R+u\cos\theta)^2\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{5/2}}\right.\right.
$$

$$
\left.\left.-\frac{\Omega^2u\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)-\frac{2\Omega^2u(\Omega R+u\cos\theta)\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)
$$

$$
+\left(3c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.
$$

$$
\left.+c_2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)\left(\frac{2(\Omega R+u\cos\theta)\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+(\Omega R+u\cos\theta)\right.
$$

$$
\left.\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\right)+(\Omega R+u\cos\theta)
$$

$$
\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(c_2\left(\frac{2\Omega^2}{(\Omega R+u\cos\theta)^3}-\frac{12\Omega^2u^2\sin^2\theta}{(\Omega R+u\cos\theta)^5}\right)\right.
$$

$$
+c_1\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.
$$

$$
\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2+12\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)
$$

$$
\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)
$$

$$
+3\left(\frac{2\Omega^2}{(\Omega R+u\cos\theta)^3}-\frac{12\Omega^2u^2\sin^2\theta}{(\Omega R+u\cos\theta)^5}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2
$$

$$
+6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)
$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\Bigg)\Bigg)\Bigg)$$

$$+\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$\Bigg(u(\Omega R+u\cos\theta)\sin\theta\Bigg(c_2\left(\frac{2\Omega^2 u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)$$

$$+c_1\Bigg(6\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2$$

$$+3\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)$$

$$\mathbf{f_{210}}=\frac{1}{2}\Bigg(0.5\rho_{air}c\Bigg(-\frac{4c_3\Omega u^2(\Omega R+u\cos\theta)}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}$$

$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\sin^2\theta c_3u^2$$

$$\Bigg(\frac{2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+2\left(\frac{2\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\sin^2\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\right)\Bigg)$$

$$+\frac{4c_3\Omega u(\Omega R+u\cos\theta)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+\frac{4c_3\Omega u(\Omega R+u\cos\theta)\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+2c_3u\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$

$$\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\sin\theta$$

$$+c_3u\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\Bigg(2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$

$$\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)$$

$$+4\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$+2\left(\frac{2\Omega^2}{(\Omega R+u\cos\theta)^3}-\frac{12\Omega^2u^2\sin^2\theta}{(\Omega R+u\cos\theta)^5}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\Bigg)\sin\theta$$

$$+u\left(\frac{3\Omega^2u(\Omega R+u\cos\theta)^2\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{5/2}}-\frac{\Omega^2u\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$\left(c_3\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_4\right)\sin\theta$$

$$+\frac{4c_3\Omega(\Omega R+u\cos\theta)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+c_3\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\Bigg(2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2$$

$$+2\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\Bigg)$$

$$+\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$\left(c_3\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.$$

$$\left.+c_4\right)\Bigg)+0.5\rho_{air}c\left(2\left(\frac{\Omega(\Omega R+u\cos\theta)^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+\Omega\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right)\right.$$

$$\left(3c_1\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.$$

$$+6c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$

$$\left.+c_2\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\right)$$

$$+2\left(\frac{\Omega u\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega u(\Omega R+u\cos\theta)^2\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$\left(3c_1\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.$$

$$\left.+c_2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right)+\left(c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^3\right.$$

$$\left.+c_2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)((\Omega R+u\cos\theta)$$

$$\left(\frac{3\Omega^2 u(\Omega R+u\cos\theta)^2\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{5/2}}-\frac{\Omega^2 u\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$\left.-\frac{2\Omega^2 u(\Omega R+u\cos\theta)\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$+\left(3c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.$$

$$\left.+c_2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)\left(\frac{2(\Omega R+u\cos\theta)\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+(\Omega R+u\cos\theta)\right.$$

$$\left.\left.\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\right)\right)+(\Omega R+u\cos\theta)$$

$$\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(c_2\left(\frac{2\Omega^2}{(\Omega R+u\cos\theta)^3}-\frac{12\Omega^2u^2\sin^2\theta}{(\Omega R+u\cos\theta)^5}\right)\right.$$

$$+c_1\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.$$

$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2+12\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$+3\left(\frac{2\Omega^2}{(\Omega R+u\cos\theta)^3}-\frac{12\Omega^2u^2\sin^2\theta}{(\Omega R+u\cos\theta)^5}\right)$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$

$$\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)$$

$$\left.\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)\right)$$

$$+\frac{u(\Omega R+u\cos\theta)\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$\left(c_2\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)+c_1\left(6\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right.\right.$$

$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2+3\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)$$

$$\left.\left.\left.\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right)\right)\right)\right)$$

$$\mathbf{f_{201}} = \frac{1}{2}\left(0.5\rho_{air}c\left(\left(\left(\frac{2\Omega^2(\Omega R+u\cos\theta)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}} + (\Omega R+u\cos\theta)\right.\right.\right.\right.$$

$$\left.\left.\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}} - \frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)\right)\right)(3c_1$$

$$\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2$$

$$+c_2) + c_1(\Omega R+u\cos\theta)\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(6\left(\frac{2\Omega^2 u\sin\theta}{(\Omega R+u\cos\theta)^3} - \frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\right.$$

$$\left.\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right) + 6\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4} - \frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2\right)$$

$$+12c_1\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4} - \frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$\left(\frac{\Omega(\Omega R+u\cos\theta)^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}} + \Omega\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right)$$

$$\left.\left.\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right)\right) + 0.5\rho_{air}c\left(2c_3u\sin\theta\right.$$

$$\left(\frac{\Omega^2}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}} - \frac{\Omega^2(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$

$$+2c_3u\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3} - \frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)$$

$$\left.\left.+\frac{4c_3\Omega u\sin\theta(\Omega R+u\cos\theta)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4} - \frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\right)\right)$$

$$\mathbf{f_{021}} = \frac{1}{2}\left(0.5\rho_{air}c\left((\Omega R+u\cos\theta)\left(\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\right.\right.\right.$$

$$\left.-\frac{u^2\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$\left(3c_1\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2 + c_2\right)$$

$$+\frac{12c_1u\sin\theta(\Omega R+u\cos\theta)\left(\frac{1}{\Omega R+u\cos\theta} - \frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\beta - \frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3} + \frac{u\sin\theta}{\Omega R+u\cos\theta}\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+c_1(\Omega R+u\cos\theta)\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2-\right.$$

$$\left.\left.\frac{12u\sin\theta\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)}{(\Omega R+u\cos\theta)^3}\right)\right)$$

$$+0.5\rho_{air}c\left(2c_3\left(u\sin\theta\left(\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\right.\right.\right.$$

$$\left.\left.-\frac{u^2\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)+\frac{2u\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\right)$$

$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$

$$-\frac{4c_3u^2\sin^2\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}{(\Omega R+u\cos\theta)^3}+4c_3\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$

$$\left.\left.\left.\left(\frac{u^2\sin^2\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right)\right)\right)\right)$$

$$\mathbf{f_{120}}=\frac{1}{2}\left(0.5\rho_{air}c\left(4c_3\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.\right.$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$

$$\left(\frac{\Omega(\Omega R+u\cos\theta)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{\Omega u^2(\Omega R+u\cos\theta)\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$+4c_3\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$

$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(\frac{u^2\sin^2\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}+\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right)$$

$$+4c_3\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$

$$\left(\frac{u^2\sin^2\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\right.$$

$$\left.+\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right)+c_3u\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}$$

$$\left(4\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.$$

$$-\frac{4u\sin\theta\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)}{(\Omega R+u\cos\theta)^3}+\frac{12\Omega u\sin\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)}{(\Omega R+u\cos\theta)^4}\Bigg)$$

$$+\frac{c_3\Omega u(\Omega R+u\cos\theta)\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$\left(2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2-\frac{4u\sin\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)}{(\Omega R+u\cos\theta)^3}\right)$$

$$+\left(c_3\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_4\right)\left(u\sin\theta\left(\frac{3\Omega u^2(\Omega R+u\cos\theta)\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{5/2}}\right.\right.$$

$$\left.\left.-\frac{\Omega(\Omega R+u\cos\theta)}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)-\frac{2\Omega u(\Omega R+u\cos\theta)\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$+2c_3\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)(u$$

$$\left(\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{u^2\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$

$$\sin\theta+\frac{2u\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\Bigg)\Bigg)+0.5\rho_{air}c\left(-\frac{2\Omega u\sin\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right.$$

$$\left(3c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.$$

$$\left.+c_2\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)(\Omega R+u\cos\theta)^2$$

$$+\frac{\Omega}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$\left(c_1\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right.\right.$$

$$\left.\left.-\frac{6u\sin\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2}{(\Omega R+u\cos\theta)^3}\right)-\frac{2c_2u\sin\theta}{(\Omega R+u\cos\theta)^3}\right)(\Omega R+u\cos\theta)^2$$

$$+\frac{2u\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$\left(3c_1\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right.$$

$$+6c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$
$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$
$$+c_2\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\Bigg)\Bigg)(\Omega R+u\cos\theta)+\Bigg(\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$
$$-\frac{u^2\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\Bigg)\Bigg(3c_1\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$
$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_2\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\Bigg)(\Omega R+u\cos\theta)$$
$$+\left(\frac{3\Omega u^2(\Omega R+u\cos\theta)\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{5/2}}-\frac{\Omega(\Omega R+u\cos\theta)}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$
$$\Bigg(c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^3$$
$$+c_2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\Bigg)(\Omega R+u\cos\theta)+\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}$$
$$\Bigg(\frac{6c_2\Omega u\sin\theta}{(\Omega R+u\cos\theta)^4}+c_1\Bigg(6\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2$$
$$+12\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$
$$\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$
$$+\frac{18\Omega u\sin\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2}{(\Omega R+u\cos\theta)^4}$$
$$-\frac{12u\sin\theta\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)}{(\Omega R+u\cos\theta)^3}\Bigg)\Bigg)(\Omega R+u\cos\theta)$$
$$+\frac{2\Omega u\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$
$$+\Omega\left(\frac{1}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}-\frac{u^2\sin^2\theta}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}\right)$$
$$\Bigg(c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^3$$
$$+c_2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\Bigg)+\Omega\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\,(c_1\,(6$$
$$\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$

$$-\frac{6u\sin\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2}{(\Omega R+u\cos\theta)^3}\Bigg) - \frac{2c_2 u\sin\theta}{(\Omega R+u\cos\theta)^3}\Bigg)\Bigg)\Bigg)\Bigg)$$

$$\mathbf{f_{102}} = \frac{1}{2}\left(\frac{3.\rho_{air}c_1 c\Omega(\Omega R+u\cos\theta)^2\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}} + 3.\rho_{air}c_1 c\Omega\right.$$

$$\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right) + 3.\rho_{air}c_1 c(\Omega R+u\cos\theta)$$

$$\left.\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right) + \frac{\rho_{air}c_3 c\Omega u\sin\theta(\Omega R+u\cos\theta)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\right)$$

$$\mathbf{f_{012}} = \frac{1}{2}\left(\frac{3.\rho_{air}c_1 cu\sin\theta(\Omega R+u\cos\theta)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}\right.$$

$$+3.\rho_{air}c_1 c(\Omega R+u\cos\theta)$$

$$\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right) + \frac{\rho_{air}c_3 cu^2\sin^2\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$\left.+\rho_{air}c_3 c\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\right)$$

$$\mathbf{f_{111}} = \frac{3.\rho_{air}c_1 c\Omega}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)(\Omega R+u\cos\theta)^2$$

$$-\frac{0.5\rho_{air}c\Omega u\sin\theta\left(3c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_2\right)(\Omega R+u\cos\theta)^2}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}$$

$$+\frac{\rho_{air}c_3 c\Omega u\sin\theta\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)(\Omega R+u\cos\theta)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+\frac{3.\rho_{air}c_1 cu\sin\theta}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)(\Omega R+u\cos\theta)$$

$$+\frac{\rho_{air}c_3c\Omega\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)(\Omega R+u\cos\theta)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$-\frac{\rho_{air}c_3c\Omega u^2\sin^2\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)(\Omega R+u\cos\theta)}{\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)^{3/2}}$$

$$+0.5\rho_{air}c\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(6c_1\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.$$

$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$\left.+6c_1\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)$$

$$(\Omega R+u\cos\theta)+\rho_{air}c_3cu\sin\theta\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)$$

$$+\rho_{air}c_3c$$

$$\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$+\frac{\rho_{air}c_3cu^2\sin^2\theta\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$+3.\rho_{air}c_1c\Omega\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)$$

$$+\frac{0.5\rho_{air}c\Omega u\sin\theta\left(3c_1\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+c_2\right)}{\sqrt{(\Omega R+u\cos\theta)^2+u^2\sin^2\theta}}$$

$$\mathbf{M_{000}}=0.5\rho_{air}cl_c\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)$$

$$\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^3\right.$$

$$\left.+(c_2+c_4)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right)$$

$$\mathbf{M_{100}}=\rho_{air}cl_c\Omega(\Omega R+u\cos\theta)\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^3\right.$$

$$\left.+(c_2+c_4)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right)$$

$$+0.5\rho_{air}cl_c\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)$$

$$\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2\right.$$
$$\left.+(c_2+c_4)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right)$$

$$\mathbf{M_{010}}=\rho_{air}cl_c u\sin\theta\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^3\right.$$
$$\left.+(c_2+c_4)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right)+0.5\rho_{air}cl_c\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)$$
$$\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2\right.$$
$$\left.+(c_2+c_4)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)$$

$$\mathbf{M_{001}}=0.5\rho_{air}cl_c\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\right.$$
$$\left.\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+c_2+c_4\right)$$

$$\mathbf{M_{200}}=0.5\left(0.5\rho_{air}c\left(2l_c\Omega^2\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^3\right.\right.\right.$$
$$\left.+(c_2+c_4)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right)+l_c\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)$$
$$\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\right.$$
$$\left(3\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+6\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\right.\right.$$
$$\left.\left.\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right)+(c_2+c_4)$$
$$\left.\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\right)$$
$$+4l_c\Omega(\Omega R+u\cos\theta)\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right.$$
$$\left.\left.\left.\left.\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+(c_2+c_4)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right)\right)\right)\right)$$

$$\mathbf{M_{020}} = 0.5\left(0.5\rho_{air}c\left(2l_c\left(\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^3\right.\right.\right.\right.$$
$$+(c_2+c_4)$$
$$\left.\left.\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right)\right)+4l_cu\sin\theta\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\right.$$
$$\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$
$$\left.\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+(c_2+c_4)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)$$
$$+l_c\left((\Omega R+u\cos\theta)^2\right.$$
$$\left.+u^2\sin^2\theta\right)\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2\right.\right.$$
$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$
$$\left.\left.\left.\left.\left.-\frac{6u\sin\theta\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2}{(\Omega R+u\cos\theta)^3}\right)-\frac{2u(c_2+c_4)\sin\theta}{(\Omega R+u\cos\theta)^3}\right)\right)\right)\right)$$

$$\mathbf{M_{002}} = 0.5\left(3.\rho_{air}cl_c\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)\right.$$
$$\left.\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)+0.\right)$$

$$\mathbf{M_{110}} = \rho_{air}cl_cu\sin\theta\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right.$$
$$\left.\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+(c_2+c_4)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right)$$
$$+0.5\rho_{air}cl_c\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\right.$$
$$\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)$$
$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+6\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$
$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)+(c_2+c_4)$$
$$\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}\right.$$
$$\left.\left.-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\right)+\rho_{air}cl_c\Omega(\Omega R+u\cos\theta)\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{1}{\Omega R+u\cos\theta}\right.\right.$$

$$-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\Big)$$

$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+(c_2+c_4)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\Bigg)$$

$$\mathbf{M_{101}}=\rho_{air}cl_c\Omega(\Omega R+u\cos\theta)\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\right.$$

$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2$$

$$+c_2+c_4)$$

$$+3.\rho_{air}cl_c\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)$$

$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$

$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$

$$\mathbf{M_{011}}=\rho_{air}cl_cu\sin\theta\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2\right.$$

$$+c_2+c_4)$$

$$+3.\rho_{air}cl_c\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$

$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$

$$\mathbf{M_{300}}=\frac{1}{6}\left(0.5\rho_{air}c\left(6l_c\Omega^2\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right.\right.\right.$$

$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+(c_2+c_4)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\Bigg)$$

$$+6l_c\Omega(\Omega R+u\cos\theta)\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(3\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\right.\right.$$

$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+6\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2$$

$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\Bigg)+(c_2+c_4)\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\Bigg)$$

$$+l_c\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(3\left(\frac{20\Omega^3u^3\sin^3\theta}{(\Omega R+u\cos\theta)^6}-\frac{6\Omega^3u\sin\theta}{(\Omega R+u\cos\theta)^4}\right)\right.\right.$$

$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2$$

$$+18\left(\frac{2\Omega^2 u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$
$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$
$$\left.\left.\left.\left.+6\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^3\right)+(c_2+c_4)\left(\frac{20\Omega^3u^3\sin^3\theta}{(\Omega R+u\cos\theta)^6}-\frac{6\Omega^3u\sin\theta}{(\Omega R+u\cos\theta)^4}\right)\right)\right)\right)$$

$$\mathbf{M_{030}}=\frac{1}{6}\left(0.5\rho_{air}c\left(6l_c\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.\right.\right.$$
$$\left.\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2+(c_2+c_4)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)$$
$$+l_c\left((\Omega R+u\cos\theta)^2\right.$$
$$\left.+u^2\sin^2\theta\right)\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(-\frac{6\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2}{(\Omega R+u\cos\theta)^3}\right.\right.$$
$$-\frac{36u\sin\theta\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)}{(\Omega R+u\cos\theta)^3}$$
$$\left.+6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^3\right)$$
$$\left.-\frac{2(c_2+c_4)}{(\Omega R+u\cos\theta)^3}\right)+6l_cu\sin\theta\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2\right.\right.$$
$$\left.\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)-\frac{6u\sin\theta\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2}{(\Omega R+u\cos\theta)^3}\right)$$
$$\left.\left.\left.-\frac{2u(c_2+c_4)\sin\theta}{(\Omega R+u\cos\theta)^3}\right)\right)\right)$$

$$\mathbf{M_{003}}=\frac{1}{6}\left(3.\rho_{air}cl_c\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)+0.\right)$$

$$\mathbf{M_{210}}=0.25\rho_{air}c\left(2l_c\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.\right.$$
$$\left.\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+(c_2+c_4)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)\Omega^2$$

$$+4l_c(\Omega R+u\cos\theta)\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\right.$$
$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+6\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)$$
$$\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$
$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)+(c_2+c_4)$$
$$\left.\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\right)\Omega+l_c\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)$$
$$\left((c_2+c_4)\left(\frac{2\Omega^2}{(\Omega R+u\cos\theta)^3}-\frac{12\Omega^2u^2\sin^2\theta}{(\Omega R+u\cos\theta)^5}\right)+\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\right.$$
$$\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right.$$
$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2+12\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)$$
$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$
$$+3\left(\frac{2\Omega^2}{(\Omega R+u\cos\theta)^3}-\frac{12\Omega^2u^2\sin^2\theta}{(\Omega R+u\cos\theta)^5}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2$$
$$+6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)$$
$$\left.\left.\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\right)\right)\right)$$
$$+2l_cu\sin\theta\left((c_2+c_4)\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)+\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\right.(6$$
$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2$$
$$\left.\left.\left.+3\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2\right)\right)\right)$$

$$\mathbf{M_{201}}=0.25\rho_{air}c\left(2l_c\Omega^2\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2\right.\right.$$
$$\left.+c_2+c_4\right)$$
$$+l_c\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)\left(6\left(\frac{2\Omega^2u\sin\theta}{(\Omega R+u\cos\theta)^3}-\frac{4\Omega^2u^3\sin^3\theta}{(\Omega R+u\cos\theta)^5}\right)\right.$$
$$\left.\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)+6\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)^2\right)+24l_c\Omega$$
$$\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)$$

$$(\Omega R+u\cos\theta)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\Bigg)\Bigg)$$

$$\mathbf{M_{021}}=0.25\rho_{air}c\left(2l_c\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)^2\right.\right.$$
$$+c_2+c_4)$$
$$+24l_cu\sin\theta\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$
$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$
$$+l_c\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2\right.$$
$$\left.\left.-\frac{12u\sin\theta\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)}{(\Omega R+u\cos\theta)^3}\right)\right)$$

$$\mathbf{M_{120}}=0.25\rho_{air}c\left(4l_cu\sin\theta\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\right.\right.$$
$$\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)$$
$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+6\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)$$
$$\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$
$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)+(c_2+c_4)$$
$$\left.\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)\right)+2l_c\left(3\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\right.$$
$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$
$$\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2+(c_2+c_4)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right)$$
$$+l_c\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)\left(\frac{6(c_2+c_4)\Omega u\sin\theta}{(\Omega R+u\cos\theta)^4}+\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\right.$$
$$\left(6\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2\right.$$
$$+12\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)$$

$$\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$

$$+\frac{18\Omega u\sin\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2}{(\Omega R+u\cos\theta)^4}$$

$$-\frac{12u\sin\theta\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)}{(\Omega R+u\cos\theta)^3}\Bigg)\Bigg)$$

$$+2l_c\Omega(\Omega R+u\cos\theta)\left(\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(6\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)^2\right.\right.$$

$$\left.\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)-\frac{6u\sin\theta\left(-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}+\beta\right)^2}{(\Omega R+u\cos\theta)^3}\right)$$

$$\left.\left.-\frac{2(c_2+c_4)u\sin\theta}{(\Omega R+u\cos\theta)^3}\right)\right)$$

$$\mathbf{M_{102}}=\frac{1}{2}\left(6.\rho_{air}cl_c\Omega\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)(\Omega R+u\cos\theta)\right.$$
$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$
$$+3.\rho_{air}cl_c\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)$$
$$\left.\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\right)$$

$$\mathbf{M_{012}}=\frac{1}{2}\left(6.\rho_{air}cl_cu\sin\theta\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)\right.$$
$$+3.\rho_{air}cl_c\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)\left(\frac{1}{\Omega R+u\cos\theta}\right.$$
$$\left.\left.-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\right)$$

$$\mathbf{M_{111}}=6.\rho_{air}cl_cu\sin\theta\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)$$
$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)+6.\rho_{air}cl_c\Omega\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)(\Omega R+u\cos\theta)$$
$$\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)$$
$$+0.5\rho_{air}cl_c\left((\Omega R+u\cos\theta)^2+u^2\sin^2\theta\right)\left(6\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\right.$$

$$\left(\frac{3\Omega u^2\sin^2\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega}{(\Omega R+u\cos\theta)^2}\right)$$
$$\left(\beta-\frac{u^3\sin^3\theta}{3(\Omega R+u\cos\theta)^3}+\frac{u\sin\theta}{\Omega R+u\cos\theta}\right)+6\left(c_1-\frac{c_2}{2}+c_3-\frac{c_4}{6}\right)\left(\frac{1}{\Omega R+u\cos\theta}-\frac{u^2\sin^2\theta}{(\Omega R+u\cos\theta)^3}\right)$$
$$\left(\frac{\Omega u^3\sin^3\theta}{(\Omega R+u\cos\theta)^4}-\frac{\Omega u\sin\theta}{(\Omega R+u\cos\theta)^2}\right)\Bigg)$$

Mathematica code for computing f_{ijk},

```
num[yd, z] = (yd - z Om + u Sin[th]);
den[y, zd] = (R + y) Om + zd + u Cos[th];
vsquared[y, yd, z, zd] = num[yd, z]^2 + den[y, zd]^2;
ratio[y, yd, z, zd] = num[yd, z]/den[y, zd];
mf[y, yd] = (ratio[y, yd, z, zd] /. {z -> 0} /. {zd -> 0});

Expand[Normal[Series[mf[y, yd], {y, 0, 3}, {yd, 0, 3}]]]

mfunc[y, yd] = (Om^2 y^2 yd)/(Om R + u Cos[th])^3 - (
   Om y yd)/(Om R + u Cos[th])^2 + yd/(Om R + u Cos[th]) - (
   Om^3 u y^3 Sin[th])/(Om R + u Cos[th])^4 + (
   Om^2 u y^2 Sin[th])/(Om R + u Cos[th])^3 - (
   Om u y Sin[th])/(Om R + u Cos[th])^2 + (u Sin[th])/(
   Om R + u Cos[th]);

alp[y, yd, psi] =
  bet + psi + Expand[((mfunc[y, yd]) - (1/3) (mfunc[y, yd])^3) + bet + psi];

Cl = c1*alp[y, yd, psi]*alp[y, yd, psi]*alp[y, yd, psi] +
   c2*alp[y, yd, psi];
Cd = c3*alp[y, yd, psi]*alp[y, yd, psi] + c4;
z = 0;
zd = 0;
num[yd, 0] = (yd - 0 Om + u Sin[th]);
den[y, 0] = (R + y) Om + 0 + u Cos[th];

vsquared[y, yd, 0, 0] = num[yd, 0]^2 + den[y, 0]^2;

liftr[y, yd, psi] =
  den[y, zd] Sqrt[vsquared[y, yd, 0, 0]] Cl*airdens*chord*0.5;
dragr[y, yd, psi] =
  num[yd, z] Sqrt[vsquared[y, yd, 0, 0]] Cd*airdens*chord*0.5;

radial[y,yd,psi]= liftr[y,yd,psi]+dragr[y,yd,psi]
```

```
fhatexpansion[y,yd,psi] =
Normal[Series[radial[y,yd,psi], {y, 0, 3}, {yd, 0, 3}, {psi, 0, 3}]]
```

From fhatexpansion, the constant and linear coefficients are retained. In the example below $TERM0 = \hat{f}_{000}$, $TERM1 = \hat{f}_{100}$, $TERM2 = \hat{f}_{010}$, $TERM3 = \hat{f}001$.

```
c1 = -2.9916;
c2 = 4.4287;
c3 = 1.185;
c4 = 0.0094;
chord = 0.533;
airdens = 1.292;
lc = 0.1302360914550899;
th = Om*t;
bet = 0;

TERM0 = (airdens*chord*0.5
    (Om R + u Cos[th]) Sqrt[(Om R + u Cos[th])^2 +
     u^2 Sin[th]^2] (c2 (bet + (u Sin[th])/(Om R + u Cos[th]) - (
         u^3 Sin[th]^3)/(3 (Om R + u Cos[th])^3)) +
      c1 (bet + (u Sin[th])/(Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
         3 (Om R + u Cos[th])^3))^3) +
    u Sin[th] Sqrt[(Om R + u Cos[th])^2 +
     u^2 Sin[th]^2] (c4 +
      c3 (bet + (u Sin[th])/(Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
         3 (Om R + u Cos[th])^3))^2))

TERM1 = (airdens*chord*0.5
    (Om R + u Cos[th]) Sqrt[(Om R + u Cos[th])^2 +
     u^2 Sin[th]^2] (c2 (-((Om u Sin[th])/(Om R + u Cos[th])^2) + (
         Om u^3 Sin[th]^3)/(Om R + u Cos[th])^4) +
      3 c1 (-((Om u Sin[th])/(Om R + u Cos[th])^2) + (
         Om u^3 Sin[th]^3)/(Om R + u Cos[th])^4) (bet + (u Sin[th])/(
         Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
         3 (Om R + u Cos[th])^3))^2) + (
   Om (Om R +
      u Cos[th])^2 (c2 (bet + (u Sin[th])/(Om R + u Cos[th]) - (
         u^3 Sin[th]^3)/(3 (Om R + u Cos[th])^3)) +
      c1 (bet + (u Sin[th])/(Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
         3 (Om R + u Cos[th])^3))^3))/
   Sqrt[(Om R + u Cos[th])^2 + u^2 Sin[th]^2] +
   Om Sqrt[(Om R + u Cos[th])^2 +
     u^2 Sin[th]^2] (c2 (bet + (u Sin[th])/(Om R + u Cos[th]) - (
         u^3 Sin[th]^3)/(3 (Om R + u Cos[th])^3)) +
      c1 (bet + (u Sin[th])/(Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
```

```
        3 (Om R + u Cos[th])^3))^3) +
  (2 c3 u Sin[th] Sqrt[(Om R + u Cos[th])^2 +
      u^2 Sin[th]^2] (-((Om u Sin[th])/(Om R + u Cos[th])^2) + (
       Om u^3 Sin[th]^3)/(Om R + u Cos[th])^4) (bet + (u Sin[th])/(
       Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
       3 (Om R + u Cos[th])^3)) + (
     Om u (Om R + u Cos[th]) Sin[
       th] (c4 +
        c3 (bet + (u Sin[th])/(Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
           3 (Om R + u Cos[th])^3))^2))/
     Sqrt[(Om R + u Cos[th])^2 + u^2 Sin[th]^2]))

TERM2 = (airdens*chord*0.5
    (Om R + u Cos[th]) Sqrt[(Om R + u Cos[th])^2 +
     u^2 Sin[th]^2] (c2 (1/(Om R + u Cos[th]) - (
        u^2 Sin[th]^2)/(Om R + u Cos[th])^3) +
      3 c1 (1/(Om R + u Cos[th]) - (
        u^2 Sin[th]^2)/(Om R + u Cos[th])^3) (bet + (u Sin[th])/(
        Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
        3 (Om R + u Cos[th])^3))^2) + (
  u (Om R + u Cos[th]) Sin[
     th] (c2 (bet + (u Sin[th])/(Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
        3 (Om R + u Cos[th])^3)) +
      c1 (bet + (u Sin[th])/(Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
        3 (Om R + u Cos[th])^3))^3))/
  Sqrt[(Om R + u Cos[th])^2 + u^2 Sin[th]^2] +
  (2 c3 u Sin[th] Sqrt[(Om R + u Cos[th])^2 +
      u^2 Sin[th]^2] (1/(Om R + u Cos[th]) - (
       u^2 Sin[th]^2)/(Om R + u Cos[th])^3) (bet + (u Sin[th])/(
       Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
       3 (Om R + u Cos[th])^3)) + (
     u^2 Sin[th]^2 (c4 +
        c3 (bet + (u Sin[th])/(Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
           3 (Om R + u Cos[th])^3))^2))/
     Sqrt[(Om R + u Cos[th])^2 + u^2 Sin[th]^2] +
     Sqrt[(Om R + u Cos[th])^2 +
       u^2 Sin[th]^2] (c4 +
        c3 (bet + (u Sin[th])/(Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
           3 (Om R + u Cos[th])^3))^2)))

TERM3 = (airdens*chord*0.5
    (Om R + u Cos[th]) Sqrt[(Om R + u Cos[th])^2 +
     u^2 Sin[th]^2] (c2 +
      3 c1 (bet + (u Sin[th])/(Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
        3 (Om R + u Cos[th])^3))^2) +
```

```
(2 c3 u Sin[th] Sqrt[(Om R + u Cos[th])^2 +
   u^2 Sin[th]^2] (bet + (u Sin[th])/(Om R + u Cos[th]) - (
    u^3 Sin[th]^3)/(3 (Om R + u Cos[th])^3))))
```

Mathematica code for computing M_{ijk},

```
num[yd, z] = (yd - z Om + u Sin[th]);
den[y, zd] = (R + y) Om + zd + u Cos[th];
vsquared[y, yd, z, zd] = num[yd, z]^2 + den[y, zd]^2;
ratio[y, yd, z, zd] = num[yd, z]/den[y, zd];
mf[y, yd] = (ratio[y, yd, z, zd] /. {z -> 0} /. {zd -> 0});
Expand[Normal[Series[mf[y, yd], {y, 0, 3}, {yd, 0, 3}]]]
mfunc[y, yd] = (Om^2 y^2 yd)/(Om R + u Cos[th])^3 - (
   Om y yd)/(Om R + u Cos[th])^2 + yd/(Om R + u Cos[th]) - (
   Om^3 u y^3 Sin[th])/(Om R + u Cos[th])^4 + (
   Om^2 u y^2 Sin[th])/(Om R + u Cos[th])^3 - (
   Om u y Sin[th])/(Om R + u Cos[th])^2 + (u Sin[th])/(
   Om R + u Cos[th]);

alp[y, yd, psi] = ((mfunc[y, yd]) - (1/3) (mfunc[y, yd])^3) + bet +
   psi;
Momentcof[y, yd, psi] =
  alp[y, yd, psi]*(c4 + c2) +
   alp[y, yd, psi]*alp[y, yd, psi]*
    alp[y, yd, psi]*(c3 + c1 - (c2/2) - (c4/6));

z = 0;
zd = 0;
num[yd, 0] = (yd - 0 Om + u Sin[th]);
den[y, 0] = (R + y) Om + 0 + u Cos[th];
vsquared[y, yd, 0, 0] = num[yd, 0]^2 + den[y, 0]^2;

moment[y, yd, psi] =
Momentcof[y, yd, psi] vsquared[y, yd, 0, 0]*chord*lc*airdens*0.5;
Mhatexpansion[y,yd,psi] =
Normal[Series[moment[y,yd,psi], {y, 0, 3}, {yd, 0, 3}, {psi, 0, 3}]]
```

From Mhatexpansion, the constant and linear coefficients are retained. In the example below $TERM0 = \hat{M}_{000}$, $TERM1 = \hat{M}_{100}$, $TERM2 = \hat{M}_{010}$, $TERM3 = \hat{M}001$.

```
c1 = -2.9916;
c2 = 4.4287;
c3 = 1.185;
```

```
c4 = 0.0094;
chord = 0.533;
airdens = 1292;
lc = 0.1302360914550899;
u = 4.72;
th = Om*t;
bet = 0;
R = 6;

TERM0 = 0.5` airdens chord lc ((Om R + u Cos[th])^2 +
    u^2 Sin[th]^2) ((c2 + c4) (bet + (u Sin[th])/(
       Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
       3 (Om R + u Cos[th])^3)) + (c1 - c2/2 + c3 - c4/6) (bet + (
       u Sin[th])/(Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
       3 (Om R + u Cos[th])^3))^3)

TERM1 = 0.5` airdens chord lc ((Om R + u Cos[th])^2 +
     u^2 Sin[th]^2) ((c2 +
        c4) (-((Om u Sin[th])/(Om R + u Cos[th])^2) + (
        Om u^3 Sin[th]^3)/(Om R + u Cos[th])^4) +
     3 (c1 - c2/2 + c3 - c4/
        6) (-((Om u Sin[th])/(Om R + u Cos[th])^2) + (
        Om u^3 Sin[th]^3)/(Om R + u Cos[th])^4) (bet + (u Sin[th])/(
        Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
        3 (Om R + u Cos[th])^3))^2) +
  1.` airdens chord lc Om (Om R +
     u Cos[th]) ((c2 + c4) (bet + (u Sin[th])/(Om R + u Cos[th]) - (
        u^3 Sin[th]^3)/(3 (Om R + u Cos[th])^3)) + (c1 - c2/2 + c3 -
        c4/6) (bet + (u Sin[th])/(Om R + u Cos[th]) - (
        u^3 Sin[th]^3)/(3 (Om R + u Cos[th])^3))^3)

TERM2 = 0.5` airdens chord lc ((Om R + u Cos[th])^2 +
     u^2 Sin[th]^2) ((c2 + c4) (1/(Om R + u Cos[th]) - (
        u^2 Sin[th]^2)/(Om R + u Cos[th])^3) +
     3 (c1 - c2/2 + c3 - c4/6) (1/(Om R + u Cos[th]) - (
        u^2 Sin[th]^2)/(Om R + u Cos[th])^3) (bet + (u Sin[th])/(
        Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
        3 (Om R + u Cos[th])^3))^2) +
  1.` airdens chord lc u Sin[
    th] ((c2 + c4) (bet + (u Sin[th])/(Om R + u Cos[th]) - (
        u^3 Sin[th]^3)/(3 (Om R + u Cos[th])^3)) + (c1 - c2/2 + c3 -
        c4/6) (bet + (u Sin[th])/(Om R + u Cos[th]) - (
        u^3 Sin[th]^3)/(3 (Om R + u Cos[th])^3))^3)

TERM3 = 0.5` airdens chord lc ((Om R + u Cos[th])^2 +
```

```
u^2 Sin[th]^2) (c2 + c4 +
3 (c1 - c2/2 + c3 - c4/6) (bet + (u Sin[th])/(
   Om R + u Cos[th]) - (u^3 Sin[th]^3)/(
   3 (Om R + u Cos[th])^3))^2)
```

These values of $\hat{f}_{ijk}$ and $\hat{M}_{ijk}$ are to be used in equation (2.37) ultimately to express equation (2.44) and finally (2.45) and (2.46).

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