ESSAYS ON PRICE DISCRIMINATION AND PRICE DISPERSION

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ABSTRACT

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This dissertation consists of three chapters on price discrimination. The first chapter, *"Price discrimination through joint marketing"*, studies joint marketing arrangements by competing firms who engage in price discrimination between consumers who patronize only one firm (single purchasing) and those who purchase from both competitors (bundle purchasers). Two types of joint marketing are considered. Firms either commit to a component-price that applies to bundle-purchasers and then firms set stand-alone prices for single purchasers; or firms commit to a rebate off their stand alone price that will be applied to bundle-purchasers, and then firms set their stand alone prices. Both methods allow firms to raise prices and earn higher profits. However, the effect of price discrimination on social welfare depends on how prices are chosen. The rebate joint marketing scheme increases joint purchasing, whereas bundle pricing diminishes bundle purchases. So if the marginal social value of a bundle over a single purchase is large, the former increases total welfare. However, welfare can also increase with bundle pricing compared to non-discriminatory pricing.

In the second chapter, *"On the strategic choice of add-on pricing policies"*, we use an analytical model to examine the consequences of add-on pricing when firms are both horizontally and vertically differentiated and there is a segment of consumers who are unaware of the add-on fees at the time of initial purchase. We find that consumers who know about the add-on fees can be penalized by the existence of those who do not know. Our consideration of quality differences on base good and add-ons leads to several novel findings regarding firm profits. We then explore the case in which sellers can decide to provide the complementary good for free. It is shown that there exist an incentive not to

charge separate add-on fees that stems from differences in add-on profitability and that is dependent on parameter constellations whether a more profitable or a less profitable firm will want to bundle or not.

In the last chapter, *"Consumer search with price competition"*, we consider model of consumer search with differentiated products, where consumers can readily observe price and an observable characteristic, but have to incur search costs to learn about a hidden characteristic. In this set-up we find that equilibrium prices are decreasing in search costs. As search costs increase, it becomes more important for firms to attract consumers on their first visit, and hence they charge a lower price.

To my beloved wife, Yumi,

and my son, Sihyun

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CHAPTER 1
PRICE DISCRIMINATION THROUGH JOINT MARKETING

1.1 INTRODUCTION

Joint marketing arrangements involving separate firms in which customers are charged differentially when they patronize multiple firms are not new. Thus, the unilateral cancellation of a joint marketing agreement that involved discounted pricing to skiers who bought passes to ski resorts run by separate operators gave rise to claims of illegal refusal to deal in the famed *Aspen Skiing* antitrust suit.[1] In recent years similar joint marketing arrangements involving separate firms have been on the rise. CityPASS, for example, is a package that bundles multiple tourist attractions in nine popular destinations in North America. The package for Chicago allows admission to five attractions: the Shedd Aquarium, the Field Museum, Skydeck Chicago, either of the Adler Planetarium or the Art Institute of Chicago, and either of the John Hancock Observatory or the Museum of Science and Industry. The attractions compete against each other for time-constrained travelers who cannot visit more than a few places. At the same time, the participating venues offer discounted pricing by selling the CityPASS.[2,3]

[1] *Aspen Skiing Co. v. Aspen Highlands Skiing Corp.*, 472 U.S. 585, (1985).

[2] As of April 2014, the total price for visiting five attractions is $187.95, while the discounted price with CityPASS is $93.95. Also, the CityPASS is valid for 9 days following the first use. See `http://www.citypass.com/chicago` for more information.

[3] Other examples cover tied promotions and other joint marketing efforts, including entertainment venues, such as movie theaters, restaurants, or museums, and tour-operators giving reciprocal discounts involving potential competitors; or newspapers jointly marketed under the Newspaper Preservation Act of 1970. Also, a very similar arrangement to the one in the *Aspen* case is currently in place for multiple ski resorts across four countries that are tied together through joint marketing of the epic-pass.

In this paper we investigate the incentives and welfare implications of instances in which companies choose pricing strategies that target consumers who make joint purchases across firms. The firms offer horizontally differentiated products and consumers view the firms' goods as imperfect substitutes. However, each firm's product has unique features and attributes that give a consumer who has already purchased a unit an added utility from buying the competing product as well, and so consumers are endogenously divided into two groups. While some consumers purchase a single product from either firm, others purchase both products.

We show that firms are able to leverage their joint marketing schemes into higher prices and higher profits compared to both uniform pricing and independent price discrimination. However, the mechanism through which prices and profits are raised depend on the nature of the joint marketing scheme used. When firms market to joint purchasers by each setting a price for their contribution to the bundle (bundle pricing), firms commit to a high bundle price, which drives consumers into single-purchasing. This enables the firms to capture more surplus from single-purchasing customers. In contrast, if joint marketing takes the form of each firm setting a rebate offer that applies to the the stand-alone price when a consumer makes a joint purchase, a generous rebate is offered. This draws consumers into joint purchasing. The increased demand for the bundle is reinforced by charging high stand alone prices, which yields higher profits because the fixed rebate then applies to a high price.

Welfare is affected differently in the two cases. If incremental values associated with purchasing a second unit are high, then the rebate scheme which induces more joint purchasing increases welfare; whereas bundle pricing, which reduces joint purchasing, tends to decrease welfare. However, there are also more subtle welfare implications of joint marketing. When a firm lowers its price to bundle-purchasers then this has both a demand-inducing effect in that some new customers are attracted to the firm; as well as a demand-shifting effect, as existing customers of the firm obtain the second good in order to secure an effective price reduction on the first good. An implication of the latter effect is that transportation costs under joint purchasing are partly incurred to obtain a better price on purchases that take place anyway. Because the lower price is welfare neutral (it is merely a transfer between the firm and the consumer) whereas the transportation costs are welfare reducing, there are situations in which joint marketing reduces total welfare regardless of which scheme is used.

The paper is organized as follows. Section 1.2 relates the paper to the literature on joint bundling, joint purchase and patent pools. In Section 1.3 we introduce a variation of the Hotelling location model in which we allow for joint purchasing. We solve for the equilibrium uniform prices for both single and joint purchasing regimes and also study independent price discrimination. Section 1.4 contains the two forms of joint marketing: bundle pricing and bundle rebates; and draws some initial comparisons to uniform pricing and joint price discrimination in terms of prices and consumption patterns. Profits, consumer surplus, and total welfare are presented in Section 1.5, which then also establishes the main findings between the different welfare and consumption implications of the pricing schemes. The final section concludes. Detailed derivations and proofs are relegated to the Appendix .

1.2 LITERATURE REVIEW

There is an extensive literature on bundling for the purpose of price discrimination.[4] Early papers in this literature, however, restrict attention to the case where bundle discounts are offered by a multi-product monopolist (e.g., Adams and Yellen, 1976; McAfee et al., 1989; Armstrong, 1996; Rochet and Choné, 1998), rather than independent firms. Gans and King (2006) are the first to study the situation where bundle discounts are offered by different firms through joint marketing. In contrast to our setting, the two products sold are independent, yet all consumers must purchase both goods so that there are

[4] Armstrong (2006) and Stole (2007) provide excellent surveys.

no single purchasers. They show that the unilateral bundling by the pair of firms against other firms is profitable, whereas bundle rebates by both pairs of firms do not increase their profits. At the same time, mutual joint marketing diminishes social welfare substantially. In the setting we consider price discrimination through joint marketing is always profitable and its impact on social welfare depends on the mechanics of the joint marketing.[5]

The most closely related paper to ours is Armstrong (2013), who studies incentives to offer bundled discounts by separate sellers in a very general setup. The main argument of the paper is that competing firms offer bundle discounts to reduce competition by mitigating the substitutability of products. In the same line of research, our paper looks more closely at the difference between choosing prices vs. choosing rebates; and examines the relationship between total consumption and total welfare for the two cases.

Song et al. (2012) discuss the case of pharmaceutical cocktails. Although the focus of their analysis is on situations in which firms set the same price for a drug, regardless of whether it is used in isolation or as part of a cocktail, they also consider situations where firms can set different prices for the two kinds of uses. They document how a firm selling treatments for HIV/AIDS set different prices for similar chemical depending on whether the drug was part of a cocktail or not. They estimate a demand system for colorectal cancer drugs, where there are at least 12 major drug treatments, 6 of which were cocktails combining drugs from different firms. Gentzkow (2007) estimates the degree of complementarity between print and online newspapers. In his illustrative model in Section 1.A, he supposes that the value of the bundle is the sum of the values of the two individual products plus a constant term (which could be positive or negative).

There is also a nascent literature on the impact of joint purchases to which our paper contributes. Gabszewicz and Wauthy (2003) analyze how joint purchases affect price

[5] Brito and Vasconcelos (Forthcoming) build on the investigation of Gans and King (2006) by considering firms that produce vertically differentiated goods. Their main insight is that bundled discounts may induce a decrease in consumer surplus and always induce a reduction in total welfare.

competition and show that various types of equilibria arise depending on the value of incremental utilities from joint purchasing. Kim and Serfes (2006) and Anderson et al. (2012) extend Gabszewicz and Wauthy (2003) by investigating joint purchasing in a horizontally differentiated market. Kim and Serfes (2006) ask under what conditions the "Principle of Minimum Differentiation" is restored when firms choose their location on the Hotelling line; and Anderson et al. (2012) find a non-monotonic relationship between equilibrium prices and qualities under joint purchasing. There, the additional gain by joint purchasing is valued more by closer consumers so firms have an incentive to sacrifice some sales and set high prices to prevent joint purchases. In contrast to our work, these papers abstract from price discrimination as a motivation for joint marketing.[6]

Somewhat related to joint marketing arrangements are several other recent papers on joint pricing in the context of patents and patent pooling (e.g., Lerner and Tirole, 2004; Cheng and Nahm, 2007; Choi, 2010; Rey and Tirole, 2013; Jeitschko and Zhang, 2014). While this work generally does not consider price discrimination, Lerner and Tirole (2004) and Rey and Tirole (2013) examine how individually set royalty rates interact with pricing in patent pools. Our findings shed further light on this issue and provide additional insights as in our setting we allow for the firms' contributions to the bundle to asymmetric and we consider alternative joint marketing structures.

1.3 A HOTELLING MODEL WITH JOINT PURCHASES

We consider an extension of the Hotelling (1929) model. When consumers make a purchase, the intrinsic utility from consuming the product is given by V—regardless of which firm's product is purchased. We assume that V is sufficiently large so that all consumers buy at least one product.[7] The model is fairly standard except that each firm's product

[6] Gabszewicz et al. (2001) also consider competition between firms that produce complementary goods. There, too, multiple equilibria emerge only for intermediate degrees of complementarity.

[7] This is not a particularly restrictive assumption; indeed if the added values that each firm provides are sufficiently high then even with $V = 0$ the market is covered.

has additional idiosyncratic features that are only obtained when purchasing that particular product. Consumers may choose to purchase from both firms—making a joint purchase—to enjoy the idiosyncratic features of both products. For example, PlayStation offers free online games, whereas Wii is more family friendly and has more games that are suitable for children—families who care about both of these attributes may purchase both systems.

We let v_i denote the consumer's gross added value from using product i's unique features. That is, a consumer who purchases product i obtains the base value of having purchased an item (V) and then the added marginal value of product i's features (v_i) so that the total payoff (gross of prices and transportation costs) is $V + v_i$. In contrast, a consumer who purchases both products receives the base utility and then each product's additional marginal value for a total (gross) payoff of $V + v_i + v_j$.

1.3.1 Joint Purchasing and Uniform Pricing

When a consumer located at $x \in [0, 1]$ purchases product 1 only, her payoff is given by $U_1(x; p_1) \equiv V + v_1 - tx - p_1$, where t denotes the (linear) transportation costs. When she purchases product 2 only, her payoff is given by $U_2(x; p_2) \equiv V + v_2 - t(1 - x) - p_2$. And because the common attributes of the product is captured by V, the payoff from a joint purchase is given by $U_{12}(p_{12}) \equiv V + v_1 + v_2 - t - p_{12}$, where $p_{12} = p_1 + p_2$ is the price paid by a consumer who purchases products 1 and 2 together.

Note that as the idiosyncratic value from either of the two products increases, more consumers undertake a joint purchase—all else equal. This can lead to a corner solution, in which all consumers purchase both products. Also, if the incremental values of the idiosyncratic characteristics are too small, then an equilibrium in which some consumers purchase more than one unit my fail to exist. To rule out these trivial cases we make the following assumption that holds throughout the paper.

Assumption 1. *$v_i \in \left(t - v_j, 2t\right)$, $i, j = 1, 2$ and $i \neq j$.*

Letting $\widehat{x_i}$ denote a consumer who is indifferent between buying from firm i only and buying from both firms (see Figure 1.1), the indifference condition $U_i(\widehat{x_i}; p_i) = U_{12}(p_{12})$ provides $\widehat{x_1} = 1 - (v_2 - p_2)/t$ and $\widehat{x_2} = (v_1 - p_1)/t$. This implies that the mass of consumers who buy good i only is

$$n_i = 1 - \frac{v_j - p_j}{t}, \quad i = 1, 2;$$

and, assuming a positive measure of joint purchasers, the mass of consumers who buy both goods is given by

$$n_{12} = \widehat{x_2} - \widehat{x_1} = \frac{v_1 + v_2 - p_1 - p_2}{t} - 1.$$

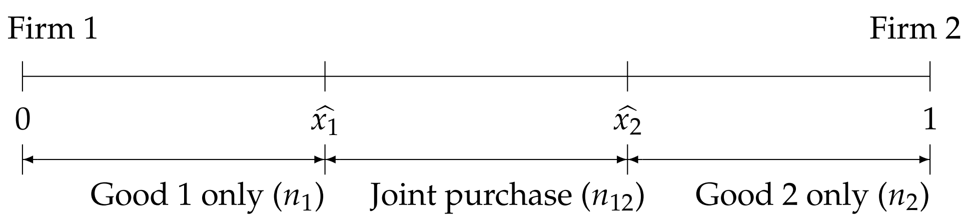


Figure 1.1: The locations of the marginal consumers

This gives each firm's demand function as

$$q_i(p_i, p_j) = \begin{cases} \frac{1}{2} + \frac{1}{2t}(v_i - v_j + p_j - p_i) & \text{if } p_i \geq v_i + v_j - t - p_j, \\ \frac{1}{t}(v_i - p_i) & \text{if } p_i \leq v_i + v_j - t - p_j. \end{cases} \tag{1.1}$$

Note that demand has an inward kink at $v_i + v_j - t - p_j$ (see Figure 1.2). Above the kink prices are so high that there are no joint purchasers, yielding the standard Hotelling model. Below the kink prices are low enough to have joint purchasers. Here consumers are more price sensitive (demand is flatter), because the value-added of a second unit lies

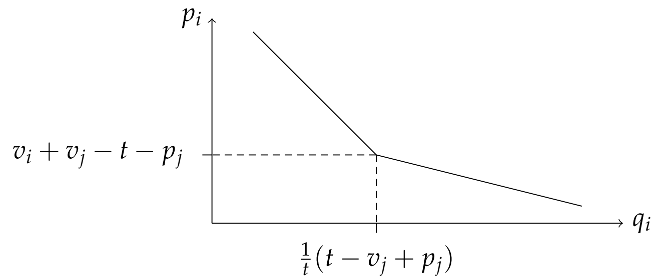


Figure 1.2: Firm i's inverse demand function

below the value of purchasing the first unit. Note also that the other firm's price is irrelevant on this segment and each firm behaves as a monopolist with regard to providing their idiosyncratic value v_i—this is because when pricing to the joint-purchasers on the margin, there is no business stealing from the rival.

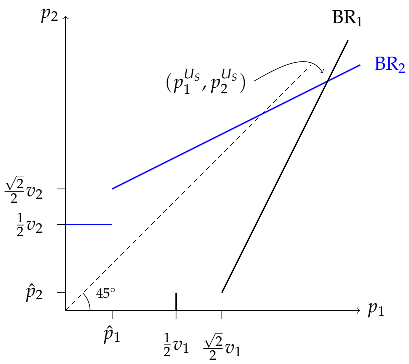
Given demand, firm i's profit function is $\pi_i(p_i, p_j) = p_i q_i(p_i, p_j)$; the Appendix shows that this yields best responses of

$$\phi_i(p_j) = \begin{cases} \frac{1}{2}(t + v_i - v_j + p_j) & \text{if } p_i \geq \hat{p}_j = (\sqrt{2}-1)v_i + v_j - t, \\ \frac{1}{2}v_i & \text{if } p_i \leq \hat{p}_j = (\sqrt{2}-1)v_i + v_j - t. \end{cases}$$

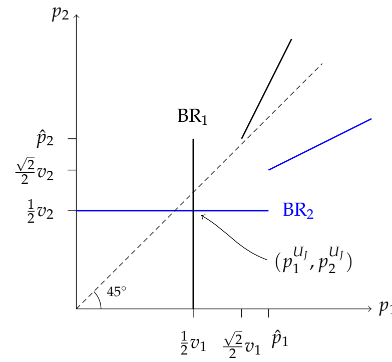
The key implication of the kink in demand is that each firm's marginal revenue is non-monotonic, and therefore the first-order condition may be satisfied twice—possibly permitting two different pricing strategies: pricing high to compete for single purchasers with less elastic demand; or pricing low to attract joint purchasers—whose demand is more elastic. As a result, firms' best response correspondences are not continuous. Figure 1.3 depicts the three possible configurations.

The following proposition gives the equilibrium configurations, the proof is in the Appendix.

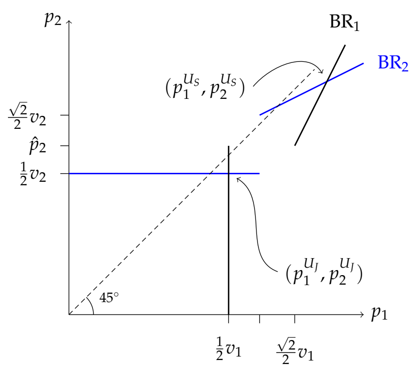
Proposition 1. *Let* $\Phi^S := \left\{(v_1, v_2) \mid (3-\sqrt{2})v_1 + \sqrt{2}v_2 \leq 3\sqrt{2}t \text{ and } (3-\sqrt{2})v_2 + \sqrt{2}v_1 \leq 3\sqrt{2}t\right\}$, *and* $\Phi^J := \left\{(v_1, v_2) \mid (\sqrt{2}+1)(t - \frac{1}{2}v_2) \leq v_1 \leq 2t \text{ and } (\sqrt{2}+1)(t - \frac{1}{2}v_1) \leq v_2 \leq 2t\right\}$; *then*
if



(a) Only Single Purchasing



(b) Some Joint Purchasing



(c) Both Eq. Configurations

Figure 1.3: Best response correspondences (here: $v_1 > v_2$)

(i) $(v_1, v_2) \in \Phi^S$*, there is a single-purchasing regime in which*

$$p_i^{U_S} = \frac{1}{3}(3t + v_i - v_j),$$
$$n_i^{U_S} = \frac{1}{6t}(3t + v_i - v_j) \quad \textit{and} \quad n_{12}^{U_S} = 0,$$
$$\pi_i^{U_S} = \frac{1}{18t}(3t + v_i - v_j)^2;$$

(ii) $(v_1, v_2) \in \Phi^J$, *there is a regime with joint purchases in which*

$$p_i^{U_J} = \frac{1}{2}v_i \quad \textit{and} \quad p_{12}^{U_J} = \frac{1}{2}(v_i + v_j)$$
$$n_i^{U_J} = \frac{1}{2t}(2t - v_j) \quad \textit{and} \quad n_{12}^{U_J} = \frac{1}{2t}(v_1 + v_2 - 2t),$$
$$\pi_i^{U_J} = \frac{v_i^2}{4t}.$$

When the additional gain from a second purchase is small, the firms are better off not attracting joint purchasers and instead charge high prices to single purchasers ($(v_1, v_2) \in \Phi^S$). On the other hand, when a joint purchase adds a large additional gain, secondary customers are an attractive prospect and the firms are willing to lower their prices to capture these consumers ($(v_1, v_2) \in \Phi^J \setminus \Phi^S$).[8] Assuming that both firms can coordinate to achieve an equilibrium that gives higher profits for both of them, we maintain that the single purchasing equilibrium configuration prevails whenever $(v_i, v_j) \in \Phi^S \cap \Phi^J$.

1.3.2 Joint Purchasing and Independent Price Discrimination

Customers who are located close to a firm purchase from that firm; and those far removed from this firm will not purchase from the firm. Thus, the purchase decision reveals something about the consumer's location on the line. Firms can use this information to segment the market and price discriminate even when they do not engage in joint marketing efforts. This scenario yields the second benchmark.

Specifically, suppose that firms provide a rebate ρ_i off their price to consumers who buy the rival's good—i.e., who purchase both goods. The joint purchaser's price is $p_{12} = (p_1 - \rho_1) + (p_2 - \rho_2)$, yielding a net payoff of $U_{12}(p_{12}) = V + v_1 + v_2 - t - (p_1 - \rho_1) - (p_2 - \rho_2)$.

The locations of the marginal consumers follow from $U_i(x_i; p_i) = U_{12}(p_{12})$, implying that $n_i = 1 - (v_j - p_j + \rho_i + \rho_j)/t$ and $n_{12} = (v_1 + v_2 - p_1 - p_2 + 2\rho_1 + 2\rho_2)/t - 1$.

[8]Figure 1.6 in the Appendix illustrates the bounds of the relevant regions.

Firm i's profit maximization problem is given by

$$\max_{p_i,\rho_i} \pi_i = p_i n_i + (p_i - \rho_i) n_{12}, \tag{1.2}$$

and the first-order condition for this problem yields

Proposition 2. *Under independent price discrimination,*

$$p_i^{IPD} = \frac{1}{3}(3t + v_i - v_j), \quad \rho_i^{IPD} = \frac{1}{3}(2t - v_j) \quad \textit{and} \quad p_{12}^{IPD} = \frac{1}{3}(2t + v_1 + v_2)$$
$$n_i^{IPD} = \frac{1}{3t}(2t - v_j) \quad \textit{and} \quad n_{12}^{IPD} = \frac{1}{3t}(v_1 + v_2 - t),$$
$$\pi_i^{IPD} = \frac{1}{9t}(v_i^2 + v_j^2 + 2tv_i - 4tv_j + 5t^2).$$

Note that firm i's choice of the bundle rebate is independent of v_i and is decreasing in v_j, implying that as firm j's unique features are more valuable, firm i can reduce the bundle rebate, as the rival product's attractiveness serves to increase the demand.

Compared to uniform pricing, the stand alone price under independent price discrimination is identical to the equilibrium price in the single purchasing regime. That is, the price charged to the relatively inelastic (i.e., the 'captured') consumers is the same as when only these are targeted. This follows readily, since the margin on which this price operates is the same across the two cases.

On the other hand, the price paid by joint-purchasers under uniform pricing is lower than the price paid by joint-purchasers under independent price discrimination, i.e., $p_i^{U_J} = v_i/2 < (2t + v_1 + v_2)/3 = p_{12}^{IPD}$. This is because when the firm price discriminates and lowers its price to joint purchasers, it cannibalizes its high-profit sales to inelastic consumers who otherwise single-purchase—this limits the amount the firm is willing to lower the price to joint purchasers compared to the case of uniformly low prices (that is, n_i is decreasing in the rebate ρ_i).

An implication of consumers being able to obtain rebates upon purchasing the second product is that despite the bundle price being higher than the uniform price under joint purchasing, the mass of consumers who joint-purchase increases when the firms price discriminate. This can be seen by comparing $n_{12}^{IPD} = (v_1 + v_2 - t)/3t$ to $n_{12}^{U_J} = (v_1 + v_2 - 2t)/2t$ and $n_{12}^{U_S} = 0$. That is, the high stand alone price pushes people into making a joint purchase in order to obtain the price break.

Thus, independent price discrimination leads to (some) joint purchasing and increased profits when otherwise there would only be single purchasing under uniform pricing. Moreover, independent price discrimination also raises profits when under uniform pricing there are some joint purchasers, because both single purchasers and joint-purchasers now face higher prices, and there are more purchases in total as some consumers shift from single to joint purchasing to avoid the higher non-discounted price that single purchasers face.

This result is similar to one in Armstrong (2013, Proposition 3). He shows that if there are separate sellers, then each seller has an unilateral incentive to offer bundle discounts for consumers who buy goods from both sellers whenever the demand for the bundle product is more elastic than total demand for one firm's product.

1.4 JOINT MARKETING AND PRICE DISCRIMINATION

Consider now joint marketing. Firms act non-cooperatively in their pricing decisions so joint marketing does not take the form of price collusion. Joint marketing is done in two stages, with the firms first committing to their pricing strategies vis-á-vis joint purchasers, and then setting their stand alone prices in light of the offer that is made to joint purchasers.

We consider two types of joint marketing schemes. First we suppose that firms non-cooperatively set the price for their product in the bundle, $\widetilde{p}_i$, and then choose stand-alone prices p_i in light of the bundle price of $p_{12}^P \equiv \widetilde{p}_1 + \widetilde{p}_2$. The second scheme we consider is a rebate scheme in which firms first announce a rebate offer ρ_i, and then choose the stand alone price in light of the bundle rebate of $\rho := \rho_i + \rho_j$.[9]

1.4.1 Bundle Price Marketing

With bundle pricing, each firm first chooses the price $\widetilde{p}_i$ for its contribution to the bundle. This yields the bundle price $p_{12}^P \equiv \widetilde{p}_1 + \widetilde{p}_2$ that will be marketed to consumers. The firms' stand-alone prices p_i are then set in light of this bundle price. Given the choice of prices, consumers make their purchasing decisions, with a firm receiving $\widetilde{p}_i$ for each bundle that is sold, and obtaining p_i from consumers that purchase only firm i's product.

Facing a bundle price of $p_{12}^P = \widetilde{p}_1 + \widetilde{p}_2$, a joint purchaser's net payoff is $U_{12}(p_{12}^P) \equiv V + v_1 + v_2 - t - p_{12}^P$, resulting in $n_i^P = 1 - (v_j - p_{12}^P + p_i)/t$ and $n_{12}^P = (v_1 + v_2 + p_1 + p_2 - 2p_{12}^P)/t - 1$. We assume that $n_{12}^P \geq 0$, which yields the candidate pricing structure. We then confirm the conditions on the candidate pricing structure that entails a positive measure of joint purchasers.

Given the bundle price p_{12}^P, each firm chooses p_i to maximize

$$\max_{p_i} \pi_i^P = p_i n_i^P + \widetilde{p}_i n_{12}^P.$$

From the first-order conditions, each firm's stand-alone price is:

$$p_i(\widetilde{p}_i, \widetilde{p}_j) = \frac{(t - v_j) + 2\widetilde{p}_i + \widetilde{p}_j}{2}.$$

Note that p_i is increasing in $\widetilde{p}_i$ and $\widetilde{p}_j$. Thus, commitment to the component price leads to raising the stand-alone price. Moreover, p_i rises with t and falls with v_j, because an increase in t and a decrease in v_j make demand for firm i's product less elastic.

In anticipation of the stand alone prices as a function of the bundle price, the firms

[9] Caminal and Matutes (1990) make a similar distinction between prices and rebates. In their model, however, customers are distinguished by purchasing across periods.

price their individual contribution to the bundle:

$$\max_{\widetilde{p}_i} \pi_i^P = \frac{t - v_j + 2\widetilde{p}_i + \widetilde{p}_j}{2} \cdot \frac{t - v_j + \widetilde{p}_j}{2t} + \widetilde{p}_i \cdot \frac{v_i + v_j - \widetilde{p}_i - \widetilde{p}_j}{2t}.$$

Joint purchases take place provided that the idiosyncratic values of the two firms are sufficiently high, yielding the following equilibrium.

Proposition 3. *When $v_i + v_j \geq 2t$, there exits a joint marketing equilibrium in which firms set bundle component prices with*

$$p_i^P = \frac{1}{4}(5t + 2v_i - v_j), \quad \widetilde{p}_i = \frac{1}{2}(t + v_i) \quad \textit{and} \quad p_{12}^P = \frac{1}{2}(2t + v_1 + v_2),$$
$$n_i^P = \frac{1}{4t}(3t - v_j) \quad \textit{and} \quad n_{12}^P = \frac{1}{4t}(v_1 + v_2 - 2t)$$
$$\pi_i^P = \frac{1}{16t}\Big(11t^2 + 2(2v_i - 3v_j)t + (2v_i^2 + v_j^2)\Big).$$

Compared to the case of uniform pricing with joint purchasers the price paid by joint purchasers is now higher, i.e., $p_{12}^P > p_{12}^{U_J}$; and so is the price paid by single purchasers when compared to the case of uniform pricing with only single purchasing, $p_i^P > p_i^{U_S}$, (which, of course, is higher than the uniform joint purchasing price, i.e., $p_i^{U_S} > p_i^{U_J}$). That is, when jointly marketing the bundle price, both prices (those applied to single purchasers and those applied to joint purchasers) are above the the uniform prices—despite all prices being set non-cooperatively.

The fact that the bundle price is higher under joint marketing suggests that fewer consumers purchase the bundle. However, the fact that the stand alone price is also high, makes the bundle attractive in that it leads to a price discount. The former effect dominates the latter so that fewer joint purchases take place when compared to the case of uniform pricing with joint purchases, i.e., $n_{12}^P = (v_1 + v_2 - 2t)/4t < n_{12}^{U_J} = (v_1 + v_2 - 2t)/2t$, given $v_1 + v_2 \geq 2t$; (naturally, more joint purchases take place when compared to uniform pricing with no joint purchases).

Intuitively, by committing to high bundle component prices the firms push demand towards single purchasing and use this increased demand to charge higher stand alone prices. Overall then, profits are greater compared to either of the uniform pricing cases, i.e., $\pi_i^P > \pi_i^{U_S}, \pi_i^{U_J}$.

Consider now the comparison to independent price discrimination. Writing $\widetilde{p}_i = p_i - \rho_i$, the independent price discrimination optimization problem for firm i, Equation (1.2), is:

$$\max_{\widetilde{p}_i, p_i} \pi_i^{IPD}(p_i, p_j, \widetilde{p}_i, \widetilde{p}_j).$$

The first-order condition with respect to the bundle component price is

$$\frac{\partial \pi_i^{IPD}}{\partial \widetilde{p}_i} = p_i \frac{\partial n_i}{\partial \widetilde{p}_i} + n_{12} + \widetilde{p}_i^{IPD} \frac{\partial n_{12}}{\partial \widetilde{p}_i} = 0, \tag{1.3}$$

where $\widetilde{p}_i^{IPD} = p_i^{IPD} - \rho_i^{IPD}$.

In contrast, with the joint marketing scheme, firm i's choice of its bundle component price is implied by

$$\max_{\widetilde{p}_i} \pi_i^P \left(p_i^P(\widetilde{p}_i, \widetilde{p}_j), p_j^P(\widetilde{p}_i, \widetilde{p}_j), \widetilde{p}_i, \widetilde{p}_j \right).$$

From the envelope theorem, the effect of changes in the stand alone price p_i on π_i^P is of second order ($\partial \pi_i / \partial p_i = 0$), so the optimal bundle price component of firm i, $\widetilde{p}_i$, satisfies the following first-order condition:

$$\begin{aligned} \frac{d\pi_i}{d\widetilde{p}_i} &= \frac{\partial \pi_i}{\partial \widetilde{p}_i} + \overbrace{\frac{\partial \pi_i}{\partial p_i}}^{0} \frac{\partial p_i^P}{\partial \widetilde{p}_i} + \frac{\partial \pi_i}{\partial p_j} \frac{\partial p_j^P}{\partial \widetilde{p}_i} \\ &= \underbrace{\left[p_i \frac{\partial n_i}{\partial \widetilde{p}_i} + n_{12} + \widetilde{p}_i \frac{\partial n_{12}}{\partial \widetilde{p}_i} \right]}_{=\text{Direct Effect}} + \underbrace{\left[p_i \overbrace{\frac{\partial n_i}{\partial p_j}}^{0} + \widetilde{p}_i \frac{\partial n_{12}}{\partial p_j} \right] \frac{\partial p_j^P}{\partial \widetilde{p}_i}}_{=\text{Indirect Effect}} = 0. \end{aligned} \tag{1.4}$$

The first term captures the direct effect that firm i's component price has on profits. It mirrors the first order condition that the firm has in independent price discrimination, namely Equation (1.3). The second term is the indirect effect that firm i's component price has on profits by affecting firm j's stand alone price that is set in light of the bundle prices. The indirect effect is positive, because committing to a bundle component price leads to the rival raising its stand-alone price. Evaluating the first-order condition (1.4) at $\widetilde{p}_i = \widetilde{p}_i^{IPD}$ yields

$$\left.\frac{d\pi_i}{d\widetilde{p}_i}\right|_{\widetilde{p}_i=\widetilde{p}_i^{IPD}} = \widetilde{p}_i \frac{\partial n_{12}}{\partial p_j} \left.\frac{\partial p_j^P}{\partial \widetilde{p}_i}\right|_{\widetilde{p}_i=\widetilde{p}_i^{IPD}} > 0,$$

which implies that $\widetilde{p}_i^P > \widetilde{p}_i^{IPD}$.

Thus, the firms charge a higher bundle price when they commit to the bundle component prices prior to the stand-alone prices. As a consequence, the mass of joint purchaser is smaller in the bundle price commitment case than in the case of independent price discrimination, i.e., $n_{12}^P < n_{12}^{IPD}$. This makes single-purchasing relatively more attractive and allows the firms to raise the stand alone price as well, $p_i^P > p_i^{IPD}$, and so profits on each unit sold exceed those of independent price discrimination, leading to an overall increase in profit, $\pi_i^P > \pi_i^{IPD}$.

1.4.2 Rebate Marketing

In the case of non-cooperative joint rebate marketing, each firm determines a discount or rebate of ρ_i that will be applied to their stand alone price to any consumer who purchases both goods. Together the discounts constitute a rebate offer of $\rho := \rho_i + \rho_j$ that will be jointly marketed to consumers. The firms' stand alone prices are then set in light of the rebate offer. Joint purchasers pay a price of $p_{12}^R \equiv p_i + p_j - \rho$ so that firm i receives a net price of $p_i - \rho_i$ for its own product in the bundle, and obtains p_i from single purchasers.

Given the joint marketing rebate of ρ, a joint purchaser's net payoff is $U_{12}(p_{12}^R) \equiv V + v_1 + v_2 - t - (p_1 + p_2 - \rho)$ implying $n_i^R = 1 - (v_j - p_j + \rho)/t$ and $n_{12}^R = (v_1 + v_2 - p_1 - p_2 + 2\rho)/t - 1$, with joint purchasing increasing as the rebate offer ρ increases.

In light of the joint marketing rebate ρ, firm i solves

$$\max_{p_i} \pi_i^R = p_i n_i^R + (p_i - \rho_i)\, n_{12}^R,$$

where ρ_i can be thought of as the cost of having a product in the bundle. The first-order condition yields the individual firm's stand-alone price

$$p_i(\rho_i, \rho_j) = \frac{v_i + 2\rho_i + \rho_j}{2}.$$

To compare with the benchmark case of uniform pricing with joint purchasing, the price can be written as $p_i(\rho_i, \rho_j) = p_i^{U_J} + \rho_i + \rho_j/2$. This shows the advantage of the joint marketing rebate scheme: each firm raises the stand-alone price to consumers by its rebate and by half the rival's rebate as well. That is, since the firms choose the rebate before choosing their stand-alone prices, their choice of rebate serves as a commitment to raise the stand-alone prices.

Given the optimal stand-alone prices as a function of the joint rebate, each firm selects their partial rebate non-cooperatively:

$$\max_{\rho_i} \pi_i^R = \frac{v_i + 2\rho_i + \rho_j}{2} \cdot \frac{2t - v_j - \rho_i}{2t} + \frac{v_i + \rho_j}{2} \cdot \frac{v_i + v_j + \rho_i + \rho_j - 2t}{2t}.$$

The second-order conditions are satisfied, and the problem has the unique solution.

Proposition 4. *When firms undertake a joint marketing arrangement in which a rebate is given to bundle purchasers,*

$$p_i^R = \frac{1}{4}(6t + v_i - 2v_j), \quad \rho_i^R = \frac{1}{2}(2t - v_j) \quad \textit{and} \quad p_{12}^R = \frac{1}{4}(4t + v_1 + v_2),$$
$$n_i^R = \frac{1}{4t}(2t - v_j) \quad \textit{and} \quad n_{12}^R = \frac{1}{4t}(v_1 + v_2),$$
$$\pi_i^R = \frac{1}{16t}\left(12t^2 + 4(v_i - 2v_j)t + (v_i^2 + 2v_j^2)\right).$$

Compared to the uniform pricing regime with joint purchasing, joint marketing again leads to an increase in the price for joint purchasers. That is, joint purchasers pay $p_{12}^R = (4t + v_1 + v_2)/4 > (v_1 + v_2)/2 = p_{12}^{U_J}$. And, as was also the case for joint marketing with bundle component pricing, with a rebate being jointly marketed, the stand alone prices are higher than they are when a uniform price is charged to single purchasers, $p_i^R > p_i^{U_S}$. In contrast to the case of joint marketing through bundle pricing, in this case the higher stand alone price drives purchasers to buy the bundle in order to obtain the rebate—despite the effective bundle price being higher than before, so $n_{12}^R > n_{12}^{U_J}$. Once again, joint marketing leads to increased profits compared to uniform pricing, that is $\pi_i^R > \pi_i^{U_S}, \pi_i^{U_J}$.

Turning to the comparison with independent price discrimination, without joint marketing of the rebate, firm i solves

$$\max_{\rho_i, p_i} \pi_i^{IPD}(p_i, p_j, \rho_i, \rho_j).$$

The first-order condition with respect to the individual rebate in this case is

$$\frac{\partial \pi_i^{IPD}}{\partial \rho_i} = p_i \frac{\partial n_i}{\partial \rho_i} + (p_i - \rho_i)\frac{\partial n_{12}}{\partial \rho_i} - n_{12} = 0. \tag{1.5}$$

In contrast, when jointly marketing the bundle rebate, the optimal individual rebate is derived by solving

$$\max_{\rho_i} \pi_i^R \left(p_i^R(\rho_i, \rho_j), p_j^R(\rho_i, \rho_j), \rho_i, \rho_j \right),$$

with the following first-order-condition

$$\frac{d\pi_i}{d\rho_i} = \frac{\partial \pi_i}{\partial \rho_i} + \cancelto{0}{\frac{\partial \pi_i}{\partial p_i}}\frac{\partial p_i^R}{\partial \rho_i} + \frac{\partial \pi_i}{\partial p_j}\frac{\partial p_j^R}{\partial \rho_i}$$
$$= \underbrace{\left[p_i \frac{\partial n_i}{\partial \rho_i} - n_{12} + (p_i - \rho_i)\frac{\partial n_{12}}{\partial \rho_i}\right]}_{=\text{Direct Effect}} + \underbrace{\left[p_i \frac{\partial n_i}{\partial p_j} + (p_i - \rho_i)\frac{\partial n_{12}}{\partial p_j}\right]\frac{\partial p_j^R}{\partial \rho_i}}_{=\text{Indirect Effect}} = 0. \tag{1.6}$$

Again, the direct effect reflects the condition obtained in independent price discrimination, Equation (1.5); and the indirect effect captures the effect that one's own rebate has on the stand-alone pricing decision of the rival. Evaluating the first order condition (1.5) at $\rho_i = \rho_i^{IPD}$ yields

$$\left.\frac{d\pi_i}{d\rho_i}\right|_{\rho_i=\rho_i^{IPD}} = \left[p_i \frac{\partial n_i}{\partial p_j} + (p_i - \rho_i)\frac{\partial n_{12}}{\partial p_j}\right]\left.\frac{\partial p_j^R}{\partial \rho_i}\right|_{\rho_i=\rho_i^{IPD}} > 0,$$

implying that $\rho_i^R > \rho_i^{IPD}$.

That is, firm i increases its rebate when it commits to jointly marketing the bundle rebate prior to setting the stand-alone price. The effect of this is to increase the attractiveness of the bundle relative to the stand alone price. For this reason, there are more joint purchasers when rebates are jointly marketed compared to the case of independent price discrimination, i.e., $n_{12}^R = (v_1 + v_2)/4t > (v_1 + v_2 - t)/3t = n_{12}^{IPD}$.

However, because firms set stand alone prices only after the rebates have been fixed, this commitment leads to the ability to increase the stand alone prices, $dp_i^R/d\rho_j > 0$. This commitment is sufficiently strong so that joint purchasers end up paying more with joint marketing then with independent price discrimination, despite the former getting higher rebates, i.e., $p_{12}^R = (4t + v_1 + v_2)/4 > (2t + v_1 + v_2)/3 = p_{12}^{IPD}$.

In sum, when the firms jointly market the bundle rebate they give generous rebates, as this allows very high stand alone prices that drive consumers to joint purchasing, who despite large rebates are very profitable, because the rebate is taken off very high initial (i.e., stand-alone) prices.

1.5 WELFARE ANALYSIS

We now consider the welfare effects of the various pricing schemes derived. We begin with profits, then move to consumer surplus, and finally compare total welfare for the cases analyzed.

1.5.1 Profit Comparisons

Profits are derived in the previous section and are given in the propositions covering the equilibrium configurations. Here, we discuss their relation across the different pricing schemes. Not surprisingly, firms' profits are increasing in their ability to engage in and coordinate price discrimination. Thus, profits are lowest under uniform pricing and firms do better when they independently price discriminate. And regardless of whether the firms choose a bundle price or a rebate scheme when they launch a joint marketing strategy, their profits increase over and above what independent price discrimination achieves. This occurs despite the non-cooperative nature of the agreement, because joint marketing commits the firms to the bundle strategy—a commitment that is leveraged into higher prices and profits.

Given that profit comparisons are quite intuitive when it comes to the degree of commitment and coordination, the intriguing question is whether joint marketing through bundle pricing or through rebates is preferred. While it may be the case that for logistical reasons one method or the other may not be practical or available, whenever firms can engage in either scheme it is worth knowing which yields the greater profit.

Comparing Proposition 3 and Proposition 4 shows that the answer to which scheme dominates is not unambiguously clear. In particular, when firms use a joint marketing scheme in which the bundle is priced, then the effective price for both the bundle and the stand alone products are higher when compared to the case of a rebate scheme, that is, $p_{12}^P = \tilde{p}_1^P + \tilde{p}_2^P = (v_1 + v_2 + 2t)/2 > (v_1 + v_2 + 4t)/4 = p_1^R + p_2^R - \rho = p_{12}^R$, and $p_i^P = (5t + 2v_i - v_j)/4 > (6t + v_i - 2v_j)/4 = p_i^R$.

On the flip side, in the case of rebates the mass of joint purchasers is larger when compared to the bundle price marketing arrangement, $n_{12}^R = (v_1 + v_2)/4t > (v_1 + v_2 - 2t)/4t = n_{12}^P$, so the rebate scheme leads to a greater sales volume when compared to the bundle pricing regime.

Which of the two arrangements is more profitable depends on the size of the idiosyncratic values of the two products. Specifically, note from Proposition 3 and Proposition 4 that $\pi_i^R - \pi_i^P = ((v_j - t)^2 - v_i^2)/16t$, from which it follows that if the idiosyncratic values are similar to one-another (close in size), then the bundle pricing is preferred whenever the idiosyncratic values are sufficiently large.

When the idiosyncratic values differ (are asymmetric), constellations can arise in which the firm with the smaller v would prefer the rebating scheme whereas the other firm would prefer to institute a bundle price. This suggests that firms that are very different may find it difficult to decide on a joint marketing scheme. However, even in these cases it turns out that the overall profit ranking is unambiguous in favor of bundle pricing. Thus, when idiosyncratic values are large, joint profits are higher under bundle pricing, even if firms' individual idiosyncratic values differ substantially. As a consequence, if side payments are permissable, then even under non-cooperative joint marketing, firms can agree on which scheme to employ.

In sum, letting Π denote industry profits, we have the following theorem.

Theorem 1 (Profits under Price Discrimination and Joint Marketing)**.** *Firms' profits are increasing in the extent of price discrimination in that independent price discrimination increases profits above uniform pricing, and price discrimination through joint marketing raises profits even more:*

$$\Pi^{U_S}, \Pi^{U_J} \leq \Pi^{IPD} < \Pi^R, \Pi^P.$$

Moreover, the bundle pricing scheme is preferred to the rebate scheme whenever the idiosyncratic values are large enough to implement the bundle price; otherwise the rebate scheme is chosen:

$$\Pi^P > \Pi^R \iff v_1 + v_2 \geq 2t.$$

1.5.2 Consumer Surplus

Let CS^X be the level of consumer surplus for $X \in \{U_S, U_J, IPD, P, R\}$, defined by

$$\begin{aligned} CS^X &= \int_0^{\widehat{x_1}^X} U_1(x; p_1^X)dx + \int_{\widehat{x_1}^X}^{\widehat{x_2}^X} U_{12}(x; p_{12}^X)dx + \int_{\widehat{x_2}^X}^{1} U_2(x; p_2^X)dx \\ &= \left[V + v_1 n_1^X + (v_1 + v_2)\, n_{12}^X + v_2 n_2^X\right] - t\left[\int_0^{n_1^X} xdx + n_{12}^X + \int_0^{n_2^X} xdx\right] \\ &\quad - \left[p_1^X n_1^X + p_{12}^X n_{12}^X + p_2^X n_2^X\right]. \end{aligned}$$

The first term is the gross utility from consumption, the second term is the transportation costs, and the final term is the total expenditures.[10]

It is natural to conjecture that consumer surplus is directly related to total consumption, because consumers only purchase the second product when the added utility exceeds their transportation costs. In that case consumer surplus would be directly proportional to n_{12}^X. Indeed, this logic applies when comparing uniform pricing with single purchasing to independent price discrimination. The stand alone price is the same for both cases, but under independent price discrimination some obtain the lower price and more consumers are drawn into purchasing each firm's product. Thus, $n_{12}^{IPD} > n_{12}^{U_S}$ and $CS^{IPD} > CS^{U_S}$.

However, comparing uniform pricing with joint purchasing to independent price discrimination reveals that even though there are more purchases under price discrimination, $n_{12}^{IPD} > n_{12}^{U_J}$, consumer surplus is actually lower, $CS^{IPD} < CS^{U_J}$. In this case some

[10]Note that $\widehat{x_1}^{U_S} = \widehat{x_2}^{U_S}$ so $n_{12}^{U_S} = 0$.

consumers make the second purchase even if the additional marginal value from consuming the second unit is smaller than the added transportation costs, simply because the second purchase allows them to take advantage of the lower prices for both the first and the second purchase. And because the stand alone price is higher with price discrimination, consumer surplus is lower, despite more total purchases.

This dynamic, that consumers base their decision on purchasing a second unit in part on how this determines the effective price paid on the first unit, is important in understanding the impact of joint marketing on consumer surplus. In particular, as was shown in Section 1.4, firms use their commitment to the joint marketing scheme to increase overall prices. An unambiguous result of this is that consumer surplus is always lower under a joint marketing scheme compared to independent pricing schemes with or without price discrimination. That is, $CS^{U_S}, CS^{U_J}, CS^{IPD} > CS^P, CS^R$. And yet, because a joint marketing rebate scheme increases joint purchases, total consumption is larger under this scheme compared to any other: $n_{12}^R = \max_X n_{12}^X$.

Finally, the positive relationship between consumption volume and consumer surplus holds again when comparing the two joint marketing schemes. In particular, consumer surplus is always larger under a rebate scheme compared to the bundle pricing arrangement, $CS^R > CS^P$, and so is total consumption: $n_{12}^R > n_{12}^P$.

The main findings concerning consumer surplus are recapped in the following theorem:

Theorem 2 (Consumer Surplus under Price Discrimination and Joint Marketing)**.** *All forms of price discrimination harm consumers, unless firms engage in independent price discrimination and thereby induce joint purchases, when otherwise consumers only purchase a single good given uniform pricing,* $CS^{IPD} > CS^{U_S}$.

Joint marketing raises prices compared to any of the other settings and as a result leads to lower consumer surplus even when more is purchased, $CS^{U_S}, CS^{U_J}, CS^{IPD} > CS^P, CS^R$.

Finally, consumers are better off under a rebate joint marketing scheme compared to the bundle *pricing scheme,* $CS^R > CS^P$*.*

1.5.3 Total Welfare

We take the sum of consumer surplus and industry profits as a measure of social welfare as usual. Because firms' costs are normalized to zero, consumer expenditures are equal to industry profits, so it is immediate that social welfare is the difference between gross utility and transportation costs:

$$TW^X = \left[V + v_1 n_1^X + (v_1 + v_2)\, n_{12}^X + v_2 n_2^X\right] - t\left[\int_0^{n_1^X} x dx + n_{12}^X + \int_0^{n_2^X} x dx\right], \tag{1.7}$$

again with $X \in \{U_S, U_J, IPD, P, R\}$.

There are two opposing effects of joint purchases on social welfare. First, more joint purchases increases social welfare because extra surplus is realized from the consumption of additional features. In the case of the standard Hotelling model with inelastic consumer demand, there is no demand creation effect with pricing, as long as the market is covered. Here, however, the demand for a firm's product increase when it charges a relatively lower price for joint purchasers. The first term in (1.7) captures the base surplus associated with consuming either good, as well as the added gain associated with the idiosyncratic features of the two goods.

The second effect of joint purchasing is that it increases total transportation costs, which is captured in the second term in (1.7). Recall from the discussion on consumer surplus that added surplus through additional consumption need not be enough to offset the added transportation costs, because the consumption decision is based on the price difference between joint and single purchasing due to price discrimination. As a result, the net effect on social welfare depends on the relative size of the surplus creation and transportation cost effects.

While the interests of firms are often directly opposed to those of consumers, assessing overall welfare requires closer scrutiny in all but one case: compared to uniform pricing in which consumers only purchase one good, independent price discrimination increases consumer surplus while also raising firm profits. So total welfare is clearly greater under independent price discrimination compared to uniform pricing when consumers do not engage in joint purchasing, $TW^{IPD} > TW^{U_S}$.

In contrast, independent price discrimination lowers consumer surplus when compared to uniform pricing with joint purchases. The reason for this is that the increased surplus from added consumption is more than offset by the higher prices paid to firms by those who only purchased a single good. This latter effect, however, is a welfare-neutral price transfer from consumers to firms. Therefore total welfare is also greater under independent price discrimination compared to uniform pricing with joint purchases, $TW^{IPD} > TW^{U_J}$.

This result is somewhat reminiscent of welfare effects under third degree price discrimination, in which welfare increases when consumption (output) increases (see Varian, 1985), which is the case here: $n_{12}^{IPD} > n_{12}^{U_J} > n_{12}^{U_S} = 0$. However, note that there is a critical difference in our analysis compared to third degree price discrimination: in our model it is the prices that segment the markets into single and joint purchasers, and consumers self-select into whether they are single or bundle purchasers. Hence the comparison to pricing across distinct markets without an arbitrage possibility is not apt in our setting.

Nevertheless, the association between greater consumption and higher surplus also holds across joint marketing schemes. Thus, total consumption under a rebate scheme is greater than total consumption under bundle pricing, $n_{12}^{R} > n_{12}^{P}$, and total welfare is also greater in the former case, $TW^{R} > TW^{P}$.

A positive association between consumption and total welfare holds more generally, provided that the products' idiosyncratic values are sufficiently large and not too asymmetric. Thus,

Theorem 3 (Total Consumption and Total Welfare)**.** *Whenever the idiosyncratic values are* *not too small* ($23v_1^2 + 23v_2^2 - 90tv_1 - 90tv_2 + 54t^2 + 80v_1v_2 > 0$) *and not too asymmetric,* ($(v_1 - 22t/17)^2 + (v_2 - 22t/17)^2 \leq 288/289$)*, then increases in total equilibrium consumption imply increases in total welfare:*

$$n_{12}^X > n_{12}^Y \Rightarrow TW^X > TW^Y, \text{ with}$$
$$TW^R > TW^{IPD} > TW^{U_J} > TW^P > TW^{U_S}.$$

The area identified in the Theorem 3 is given in Figure 1.4.

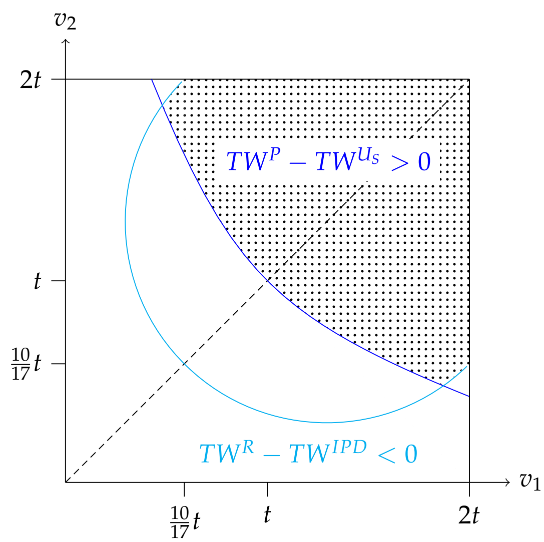


Figure 1.4: Social Welfare

The reason for the existence of a threshold that values must exceed is intuitive. Firms are balancing their pricing decisions across two margins, namely the margin at which they capture new customers who otherwise only purchase from the rival, and the margin at which their own customers decide to become joint purchasers or remain single purchasers. As a result, the firms' pricing decisions induce transportation costs that need not be offset by marginal increases in consumer surplus tied to consumption of the second product. However, greater consumption leads to greater surplus provided that the value from added consumption is sufficiently high to off-set added transportation costs that are incurred by consumers whose second purchase is partly driven by the desire to lower their expenditure on their first purchase.

Two aspects of the theorem are particularly noteworthy. First, Theorem 3 demonstrates that total welfare is greatest when firms engage in joint marketing that sets a rebate for bundle purchasers compared to any of the other schemes, so joint marketing can increase welfare above what firms can do independently. While consumer surplus is lower under this scheme than when compared to independent price discrimination, the increase in profits more than offsets the reduction in consumer surplus. However, there is an important caveat to note here. From the second part of Theorem 1 we know that when idiosyncratic values are high, firms actually prefer the bundle pricing scheme over the rebate scheme.

Nonetheless, the finding points in the direction of which type of joint marketing arrangements should be viewed as beneficial—especially as the case of bundle pricing leads to the lowest mass of joint purchasers, and the lowest welfare of any equilibrium configuration in which there is joint purchasing. Because of this, the comparison between surplus through added consumption and losses through additional transportation is relatively easy for the case of joint marketing: if a joint marketing agreement leads to greater consumption then it likely also increases total welfare.

The second noteworthy finding in Theorem 3 is that compared to uniform pricing with (only) single purchasing, (even) joint marketing through bundle pricing can raise total welfare. And, thus, the welfare implications of joint marketing are not clear ex ante, but whenever idiosyncratic values are high and not too asymmetric, there's a chance that either type of joint marketing may increase total welfare.

There is, of course, an immediate important corollary to Theorem 3. If the idiosyncratic values of the products are not that large, then equilibrium consumption need not be positively correlated with equilibrium welfare. The intuition for this is straightforward: if the added value from consuming a second good is not that large, then pricing schemes that induce the added purchase may generate less additional surplus than the added transportation that the second purchase entails, thus reducing total welfare. In fact, when the idiosyncratic values are very small, then no pricing scheme generates more total welfare than simple uniform pricing with single purchasing.

Related to this is the case where idiosyncratic values are large yet very asymmetric. Adverse welfare effects occur on the margin between joint and single purchasing when it comes to the relatively lower-valued good when switching from independent price discrimination to bundle joint marketing. That is, bundle joint marketing will induce additional purchases of the relatively lower valued good for which the added transportation costs are not offset by the added consumption value.

1.6 CONCLUSION

We have studied the effect of price discrimination through joint marketing by firms producing imperfectly substitutable goods. The main finding is that the impact on overall purchasing and welfare depend on wether firms offer a bundle price or offer rebates off individual prices to consumers who buy from both firms. Firms may want to increase joint purchasing by setting bundle rebates before determining stand-alone prices. Alternatively, they may want to mitigate price competition by pricing the bundle components before setting the individual prices.

Loosely speaking, when a second purchase adds little in terms of incremental utility, firms prefer to offer discounts off their prices. This has the effect of increasing the mass of customers to both firms, which is beneficial because rent-extraction is limited by the added value of the second unit. In contrast, if consumers place a high value on each of the goods and therefore the second unit is relatively valuable, firms prefer to directly price and market the bundle, as this allows them to increase their stand-alone prices and extract more rents from their captured customers. However, we show that the firms interests need not be aligned in this respect if their incremental values to the consumers vary greatly.

With respect to welfare we find that if second purchases add a lot of additional utility, then price discrimination that induces added purchases raises total welfare. However, as firms find bundle pricing more profitable compared to the rebate scheme when incremental values are high, total purchasing can actually decrease with high additional values when both joint marketing options are available to the firms. Nevertheless, there are parameter values for which total welfare increases due to joint marketing compared to uniform pricing regardless of which joint marketing scheme is used. Indeed, whenever both idiosyncratic values are sufficiently high joint marketing raises total welfare; otherwise independent price discrimination is better from a total welfare perspective.

APPENDIX

Appendix for Chapter 1

Derivation of Best Response Correspondences We will derive firm 1's best response correspondence first. Firm 2's problem will be solved in a similar manner. Given the demand function (1.1), the profit function of firm 1 is given by,

$$\pi_1 = \begin{cases} \left(\frac{1}{2} + \frac{v_1 - v_2 + p_2 - p_1}{2t}\right) p_1 & \text{if } p_1 \geq v_1 + v_2 - t - p_2, \\ \frac{1}{t}(v_1 - p_1)p_1 & \text{if } p_1 \leq v_1 + v_2 - t - p_2. \end{cases}$$

Consider first the case in which $p_1 \geq v_1 + v_2 - t - p_2$. Using the first order necessary condition, we identify the candidate candidate best reply $b_1(p_2) = \frac{1}{2}(t + v_1 - v_2 + p_2)$. It yields a payoff $\pi_1^S(p_2) = (t + v_1 - v_2 + p_2)^2/8t$. Notice that this case is valid only when $p_1 \geq v_1 + v_2 - t - p_2$, so we need to have the condition:

$$p_1 \geq v_1 + v_2 - t - p_2 \Longleftrightarrow p_2 \geq \frac{1}{3}(v_1 + 3v_2 - 3t).$$

For the case in which $p_1 \leq v_1 + v_2 - t - p_2$, firm 1 sets the price at $p_1^J = v_1/2$, yielding a payoff $\pi_1^J = v_1^2/4t$. This payoff is valid only when $p_1 \leq v_1 + v_2 - t - p_2$, requiring the following condition:

$$p_1 \leq v_1 + v_2 - t - p_2 \Longleftrightarrow p_2 \leq \frac{v_1}{2} + v_2 - t.$$

Thus, for $p_2 \in \left[\frac{1}{3}(v_1 + 3v_2 - 3t), \frac{1}{2}v_1 + v_2 - t\right]$ both choices satisfy the requirements. In order to identify 'true' best response of firm 1, it remains to compare the payoffs in these two cases. From solving $\pi_1^S(p_2) \geq \pi_1^J$ we have

$$\frac{(t + v_1 - v_2 + p_2)^2}{8t} \geq \frac{v_1^2}{4t} \Longleftrightarrow p_2 \geq \widehat{p}_2 = (\sqrt{2} - 1)v_1 + v_2 - t.$$

Thus, firm 1 switches from the joint-purchasing regime to the single-purchasing regime at $\widehat{p}_2$. A similar argument applies for firm 2. We therefore summarize firm i's best response correspondences ϕ_i as follows

$$\phi_i(p_i, p_j) = \begin{cases} b_i(p_j) = \frac{1}{2}(t + v_i - v_j + p_j) & \text{if } p_j \geq \widehat{p}_j = (\sqrt{2} - 1)v_i + v_j - t, \\ \frac{1}{2}v_i & \text{if } p_j \leq \widehat{p}_j = (\sqrt{2} - 1)v_i + v_j - t. \end{cases}$$

Figure 1.5 illustrates best response correspondence for firm i.

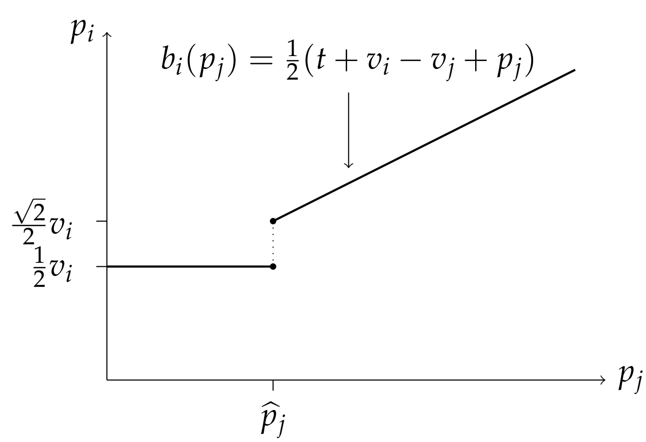


Figure 1.5: Firm i's best response correspondence

Proof of Proposition 1 We first identify two critical regions in the space prices according to their resulting demands and denote each set as $\mathcal{J}$ and $\mathcal{S}$, respectively, i.e.,

$$\mathcal{J} = \{(p_1, p_2) \in \mathbb{R}^2_+ \mid p_1 + p_2 \leq v_1 + v_2 - t\},$$
$$\mathcal{S} = \{(p_1, p_2) \in \mathbb{R}^2_+ \mid p_1 + p_2 \geq v_1 + v_2 - t\}.$$

There are two candidate Nash equilibria according to the region.

$$(p_1^{U_J}, p_2^{U_J}) = \left(\frac{v_1}{2}, \frac{v_2}{2}\right) \qquad \text{if } (p_1, p_2) \in \mathcal{J},$$
$$(p_1^{U_S}, p_2^{U_S}) = \left(t + \frac{v_1 - v_2}{3}, t + \frac{v_2 - v_1}{3}\right) \qquad \text{if } (p_1, p_2) \in \mathcal{S}$$

First, consider $(p_1^{U_J}, p_2^{U_J})$. Suppose one firm, say firm 1, deviates to the region $\mathcal{S}$ then its profits becomes

$$\pi_1^S(p_1, p_2^{U_J}) = p_1 \left(\frac{1}{2} + \frac{v_1 - v_2 + p_2^{U_J} - p_1}{2t} \right),$$

given $p_1 \geq v_1 + v_2 - t - p_2^{U_J} = v_1 + \frac{1}{2}v_2 - t$. Then the optimal deviating price for firm 1 is given by

$$p_1^{dev} = \begin{cases} \frac{1}{2}v_1 - \frac{1}{4}v_2 + \frac{1}{2}t & \text{if } 2v_1 + 3v_2 \leq 6t \text{ and } v_1 + v_2 \geq 2t, \\ v_1 + \frac{1}{2}v_2 - t & \text{if } 2v_1 + 3v_2 \geq 6t. \end{cases}$$

This implies the optimal deviation payoff for firm 1 as follows:

$$\begin{aligned} \pi_1^{dev} &= p_1^{dev} \left(\frac{1}{2} + \frac{v_1 - v_2 + p_2^{U_J} - p_1^{dev}}{2t} \right) \\ &= \begin{cases} \frac{1}{32t}(2t + 2v_1 - v_2)^2 & \text{if } 2v_1 + 3v_2 \leq 6t \text{ and } v_1 + v_2 \geq 2t, \\ \frac{1}{4t}(2t - v_2)(2v_1 + v_2 - 2t) & \text{if } 2v_1 + 3v_2 \geq 6t. \end{cases} \end{aligned}$$

In contrast, the profit in the proposed equilibrium in region $\mathcal{J}$ is $\pi_1^{U_J} = v_1^2/4t$. Thus, the condition for first candidate to be a Nash equilibrium is that $\pi_1^{U_J} \geq \pi_1^{dev}$, and computations show that this is equivalent to $(v_1, v_2) \in \Phi^J$.

We can apply same logic for candidate $(p_1^{U_S}, p_2^{U_S})$. Suppose firm 1 deviates to the region $\mathcal{J}$. Then, its profit becomes

$$\pi_1^{U_J}(p_1) = p_1 \left(\frac{v_1 - p_1}{t} \right),$$

given $p_1 \leq v_1 + v_2 - t - p_2^{U_S} = \frac{4}{3}v_1 + \frac{2}{3}v_2 - 2t$. Then, the optimal deviating price for firm

1 is given by

$$p_1^{dev} = \begin{cases} \frac{4}{3}v_1 + \frac{2}{3}v_2 - 2t & \text{if } 5v_1 + 4v_2 \leq 12t, \\ \frac{1}{2}v_1 & \text{if } 5v_1 + 4v_2 \geq 12t \text{ and } v_1 + v_2 \leq 3t. \end{cases}$$

Thus, the optimal deviation payoff of firm 1 is

$$\begin{aligned} \pi_1^{dev} &= p_1^{dev}\left(\frac{v_1 - p_1^{dev}}{t}\right) \\ &= \begin{cases} \frac{2}{t}\left(\frac{2}{3}v_1 + \frac{1}{3}v_2 - t\right)\left(2t - \frac{1}{3}v_1 - \frac{2}{3}v_2\right) & \text{if } 5v_1 + 4v_2 \leq 12t, \\ \frac{1}{4t}v_1^2 & \text{if } 5v_1 + 4v_2 \geq 12t \text{ and } v_1 + v_2 \leq 3t. \end{cases} \end{aligned}$$

In contrast, the profit in the candidate equilibrium in region $\mathcal{S}$ is given by $\pi_1^{U_S} = (3t + v_1 - v_2)^2/18t$, so the condition for $(p_1^{U_S}, p_2^{U_S})$ is that $\pi_1^{U_S} \geq \pi_1^{dev}$. Computations show that this is satisfied whenever $(v_1, v_2) \in \Phi^S$.

Figure 1.6 illustrates the equilibrium pattern depending on the value of each firm's unique features v_i and v_j. If v_i and v_j are sufficiently small ($\Phi^S \setminus \Phi^J$), (only) single purchasing arises in equilibrium; and if v_i and v_j are sufficiently large ($\Phi^J \setminus \Phi^S$), joint purchasing occurs in equilibrium. For the case of intermediate values of v_i and v_j ($\Phi^S \cap \Phi^J$), both configurations are possible.

□

Proof of Theorem 1 Recall that we have derived the firm's profits under various scenarios (see Proposition 1 – Proposition 4). From the information in each propositions, we have following industry profits: $\Pi^{U_S} = \frac{1}{9t}(v_1^2 + v_2^2 - 2v_1v_2 + 9t^2)$, $\Pi^{U_J} = \frac{1}{4t}(v_1^2 + v_2^2)$, $\Pi^{IPD} = \frac{1}{9t}(2v_1^2 + 2v_2^2 - 2tv_1 - 2tv_2 + 10t^2)$, $\Pi^P = \frac{1}{16t}(3v_1^2 + 3v_2^2 - 2tv_1 - 2tv_2 + 22t^2)$ and $\Pi^R = \frac{1}{16t}(3v_1^2 + 3v_2^2 - 4tv_1 - 4tv_2 + 24t^2)$.

Independent price discrimination increases profits above uniform pricing, which is

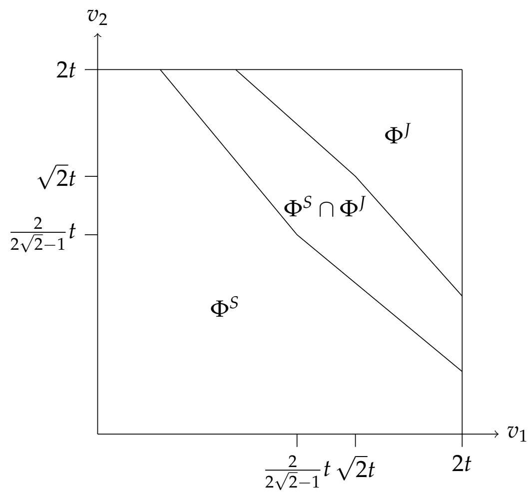


Figure 1.6: Equilibrium pattern under uniform pricing

simply shown as

$$\Pi^{IPD} - \Pi^{U_S} = \frac{1}{9t}(v_1^2 + v_2^2 + 2v_1v_2 - 2tv_1 - 2tv_2 + t^2) = \frac{1}{9t}(v_1 + v_2 - t)^2 > 0$$

and

$$\Pi^{IPD} - \Pi^{U_J} = -\frac{1}{36t}(v_1^2 + v_2^2 + 8tv_1 + 8tv_2 - 40t^2) \geq 0.$$

The last inequality above comes from the fact that $\Pi^{IPD} - \Pi^{U_J}$ is decreasing in v_i and it takes the value of 0 when $v_1 = v_2 = 2t$.

The comparison between joint marketing regime and independent price discrimination shows

$$\Pi^{IPD} - \Pi^{R} = \frac{1}{144t}(5v_1^2 + 5v_2^2 + 4tv_1 + 4tv_2 - 56t^2) < 0,$$
$$\Pi^{IPD} - \Pi^{P} = \frac{1}{144t}(5v_1^2 + 5v_2^2 - 14tv_1 - 14tv_2 - 38t^2) < 0.$$

Finally, we check whether the bundle pricing scheme is better for the firms when $v_1 + v_2 \geq 2t$. The difference between the bundle pricing scheme and the rebate scheme is given by

$$\Pi^P - \Pi^R = \frac{1}{8}\left(v_1 + v_2 - t\right) > 0$$

which holds provided that $v_1 + v_2 \geq 2t$. □

Proof of Theorem 2 Computation shows that

$$\begin{aligned}
CS^{IPD} - CS^{U_S} &= \frac{(v_1 + v_2 - t)^2}{36t} > 0, \\
CS^R - CS^{U_S} &= \frac{1}{288t}\left((v_1 + v_2)^2 + 36(v_1 + v_2)t + 14v_1 v_2 - 144t^2\right) < 0, \\
CS^R - CS^{U_J} &= -\frac{1}{32t}\left(3v_1^2 + 3v_2^2 - 20tv_1 - 20tv_2 + 56t^2\right) < 0, \\
CS^R - CS^P &= \frac{5}{16}(v_1 + v_2 - t) > 0.
\end{aligned}$$

□

Proof of Theorem 3 From the results in Proposition 1 – Proposition 4, we can obtain the measure of joint purchasing consumers in each case:

$$\begin{aligned}
&n_{12}^{U_S} = 0 && n_{12}^{U_J} = \frac{1}{2t}(v_1 + v_2 - 2t) && n_{12}^{IPD} = \frac{1}{3t}(v_1 + v_2 - t) \\
&n_{12}^R = \frac{1}{4t}(v_1 + v_2) && n_{12}^P = \frac{1}{4t}(v_1 + v_2 - 2t)
\end{aligned}$$

Provided that $v_1 + v_2 > 2t$, the rank of measure is

$$n_{12}^R > n_{12}^{IPD} > n_{12}^{U_J} > n_{12}^P > n_{12}^{U_S} = 0.$$

First, consider the case in which $(v_1, v_2) \in \Phi^J$. Computation shows the following relationship of total welfare in each regime:

$$TW^R - TW^{IPD} = -\frac{1}{288t}(17v_1^2 + 17v_2^2 - 44tv_1 - 44tv_2 + 40t^2) > 0,$$
$$TW^{IPD} - TW^{U_J} = -\frac{1}{72t}(7v_1^2 + 7v_2^2 - 16tv_1 - 16tv_2 + 8t^2) > 0,$$
$$TW^{U_J} - TW^P = \frac{1}{32t}(5v_1^2 + 5v_2^2 - 6tv_1 - 6tv_2 + 2t^2) > 0.$$

Thus, we have

$$TW^R > TW^{IPD} > TW^{U_J} > TW^P.$$

Next, consider the case in which $(v_1, v_2) \in \Phi^S$. Define the subsets

$$\Phi_1^S := \{(v_1, v_2) \mid TW^P - TW^{U_S} \geq 0\},$$
$$\Phi_2^S := \{(v_1, v_2) \mid TW^R - TW^{IPD} \geq 0\}.$$

After some algebra one can see that $TW^{IPD} - TW^P > 0$ for any (v_1, v_2). So if $(v_1, v_2) \in \Phi_1^s \cap \Phi_2^S$, we obtain the following relationship:

$$TW^R > TW^{IPD} > TW^P > TW^{U_S}$$

□

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CHAPTER 2
ON THE STRATEGIC CHOICE OF ADD-ON PRICE POLICIES

2.1 INTRODUCTION

Consumers are often faced with fees for add-ons not included in the price of a base product (or service). For example, hotel guests, after room rates are already paid, often have to pay additional charges for services such as Internet access, safe access, parking, a fitness center, or pool use. Banks charge fees for consumers who opt to cancel a check, use a foreign ATM, or use a debit card. Air travelers frequently find themselves paying for pets, snacks, checked luggage, and even blankets. Cell phone service providers charge subscribers to send text messages and download data.

The central focus of this paper is in determining the implications of competing firms charging separately for the add-ons. Whereas the previous add-on pricing literature assumes quality symmetry between the horizontally differentiated firms, we account for the fact that competing firms may actually differ in quality. As a consequence, the implications of add-on pricing may be different for a superior firm than for an inferior firm.

In addition to quality asymmetry, this paper uniquely incorporates market realities regarding consumer segmentation. Specifically, there is often heterogeneity in the willingness to pay for the add-on and heterogeneity in the anticipation of the add-on fee. To illustrate heterogeneity in willingness to pay for the add-on, suppose that some consumers may need to park a car (or use Internet service), whereas a consumer without a car (computer) does not value parking (Internet service). With regard to heterogeneity in anticipating the add-on, we allow for some consumers to be knowledgeable about the add-on price yet still purchase the add-on, whereas other consumers do not take the add-on price into consideration at the time of initial purchase. Moreover, they are boundedly rational in that they do not form rational expectations.

In the following, we enrich the model of add-on pricing by assuming that firms can decide whether to charge separate prices for add-ons before they set their prices. This allows us to examine when the add-on price policy is optimal for each firm and why add-on pricing arises as an equilibrium.

Two reasons to explain the existence of boundedly rational consumers in add-on pricing. First, consumers are sometimes not informed of add-on fees when purchasing a base product—a recurring theme in the add-on pricing literature that is attributed to the lack of advertisement of add-on fees and even deliberate information hiding (Verboven, 1999; Ellison, 2005; Gabaix and Laibson, 2006). Second, although many consumers may know these add-on fees at the time of their initial purchase decision, there is strong anecdotal evidence that many other consumers are surprised by the charges for services they thought were included in the posted price. Internet chat rooms are filled with complaints from hotel consumers who were faced with fees they had not anticipated. Sullivan (2007) describes consumers who were surprised to learn how much it costs to send each text message. Our consideration of boundedly rational consumers is also consistent with the increasing attention on bounded rationality in the broader business research context.

This paper addresses three questions.

1. Who will gain from the advent of add-on pricing?

2. What happens to prices and consumer surplus when more consumers are boundedly rational and do not anticipate the add-on prices?

3. Who will charge for add-ons and when does add-on pricing occur?

The result of the research can be used to determine the implications of a newly realized ability to charge separately for an add-on previously included in the posted price (either through innovative thinking or technological advances). They also help show the implications of government intervention intended to limit the ability to charge for an add-on (for instance, President Obama has taken aim at bank fees[1] and senator Charles Schumer has proposed prohibiting certain airline fees[2]). The answers to theses questions are not straightforward because they each depend critically on the mix of consumer segments and/or the nature of asymmetry between firms.

In the popular press, add-on pricing is often viewed as a means for firms to boost profit via locking consumers into decisions that are more costly than anticipated. There is ample evidence that consumers make a purchase with the inaccurate expectation that the add-on is included in the posted price. After such a purchase, as one traveler opined, "It's like the people are at the mercy of the [companies]".[3] Although the intuition would suggest that add-on pricing benefits firms, a well-known result in the unadvertised prices and add-on pricing literature is that charging for add-ons results in equivalent profits as an all-inclusive price. We aim to resolve this discrepancy by showing when the profit irrelevance result will hold and when firms will benefit from add-on pricing. We also aim to identify conditions for when add-on pricing will diminish a firm's profitability.

Intuitively, the effect on prices of firms serving a greater number of boundedly rational consumers is unclear. On the one hand, the higher add-on revenue from exploiting boundedly rational consumers incentivizes firms to charge lower posted base prices to attract consumers. With the add-on price capped by the customer's valuation of the add-on, the influx of boundedly rational consumers increases competitive intensity in base prices, thereby lowering the total price and thus increasing surplus. This logic is consistent with the predictions of the add-on literature (e.g., Gabaix and Laibson, 2006). On the other hand, boundedly rational consumers expect a lower total price (i.e., base price plus

[1]See "Obama blasts Bank of America debit card fee," *Washington Post*, October 3, 2011.

[2]See "Schumer asks airlines to reverse flight change fee," *USA Today*, May 20, 2013.

[3]See "Fees of flying airlines charge travelers to supplement base fares, raise revenue in tough economic time," *Pittsburgh Post-Gazette*, November 28, 2010.

add-on fee) than observed by consumers who are knowledgeable about add-on fees and thus may have a higher willingness to pay. The higher willingness to pay may increase total prices and hurt all consumers. This latter effect has not been found in the current add-on pricing literature. We aim to resolve these two viewpoints and identify when total prices will (and will not) increase in the number of boundedly rational consumers.

In the previous literature, firms are symmetric in their add-on prices. In reality, however, firms vary in the add-on prices charged.[4] Our model proposes asymmetry in firm quality as a driver of variance in add-on prices. The question becomes whether the superior firm or inferior firm is more likely to charge add-on prices. Because add-on prices are a form of price discrimination (Ellison, 2005), applying predictions from another form of price discrimination, such as rebates, would suggest the inferior firm is more likely to price discriminate than the superior firm. We examine whether this finding holds true for add-on pricing.

2.2 RELATED LITERATURE

A well-known result in the unadvertised prices add-on pricing literature is that charging for add-ons results in equivalent profit as when add-on pricing is prohibited or infeasible. Any gains from the unadvertised price are competed away through the posted prices. Lal and Matutes (1994) modeled a retailer's use of loss-leader products coupled with unadvertised prices of other products that are not discounted. Verboven (1999) looked at add-on pricing for products in which the price of the base product (i.e., the car) is advertised and the firms can choose whether to advertise the prices of premium upgrade add-ons. Gabaix and Laibson (2006) produced the profit-irrelevancy result of add-on prices with a segment of boundedly rational consumers who inaccurately believe the add-on is included in the posted price. This paper differs from the above papers in that we

[4]See e.g. "Hotels piling on hidden fees," *CNNMoney*, October 31, 2011.

show add-on pricing can affect firm profits.[5]

Ellison (2005) identified how add-on pricing can actually lead to improved profit for the firms if consumers who are more sensitive to interfirm price differences are less likely to purchase costly add-ons. The correlation between consumer price sensitivity and add-on valuation creates an adverse selection problem that softens competition in base prices. Our paper complements Ellison's (2005) in that we find alternative venues in which add-on pricing can influence firm profits. Furthermore, we show that add-on pricing can have opposing profit implications for the two competing firms. Ellison and Wolitzky (2012) extended the sequential search model of Stahl (1989) and showed that concealing prices—which increases search costs—reduces the fractions of consumers who search and thus benefits firms.[6] Ellison and Ellison (2009) provided empirical support for the explanations of Ellison (2005) and Ellison and Wolitzky (2012).

Because add-on pricing is a form of price discrimination, our paper relates to a body of research on targeted pricing. The ability to price discriminate between old and new customers is shown to diminish profits for competing firms (Villas-Boas, 1999) and a durable-good monopolist (Villas-Boas, 2004). Corts (1998) modeled vertically differentiated firms and showed that price discrimination between customers who value quality and those who do not value quality will result in lower profit for both the high-quality firm and the low-quality firm. Chen (2008) modeled firms of differentiated quality and found that the inferior firm earns less profit when price discrimination based on consumer purchase history is feasible than when uniform pricing is adopted.

This paper also belongs to the broader literature on competitive price discrimination.[7] Much of this literature examines third-degree price discrimination, i.e., models in which firms can identify whether consumer is "high" or "low" type and charge different prices

[5]Chen and Rey (2012) also found that being able to charge different prices to one-stop shoppers from multistop shoppers may increase the firm's profit and result in socially harmful effect.

[6]The add-on pricing literature assumes two products: base and add-on products. In contrast, Ellison and Wolitzky (2012) only consider a single product in a search model.

[7]Stole (2007) provides an excellent survey.

accordingly. Borenstein (1985) and Holmes (1989) provide some of the most basic results. Borenstein notes that differences across groups in the interfirm price sensitivity are needed to generate price discrimination. Holmes notes that banning price discrimination will lower prices in one market and raise them in the other; the net effect on profits is ambiguous. Within this literature few papers analyzes models with both vertical and horizontal differentiation, no doubt because it is difficult to construct models that are sufficiently tractable to allow closed-form solutions. (Borenstein (1985) presented his results using numerical simulations.) Two notable exceptions are Armstrong and Vickers (2001) and Rochet and Stole (2002). Among other contributions, each of these papers derives a non-discrimination theorem. They show that when brand preferences are of the type generally assumed in discre-choice models and brand preferences are independent of consumers' valuations for quality, then the outcome of the competitive second-degree price discrimination model is that firms do not use quality levels to discriminate.

Establishing conditions that define when equilibrium leads to charging separately for add-ons is also new to the bundling literature. There are a variety of papers that theoretically and empirically identify various reason to offer bundles rather than price each element separately (e.g., Adams and Yellen, 1976; Schmalensee, 1984; McAfee et al., 1989; Fang and Norman, 2006). Specifically, Whinston (1990), Chen (1997), Nalebuff (2004) analyzed bundling in an oligopolistic setup. Whinston (1990) showed that a firm can leverage its monopolistic power in one market into another oligopolistin market via product tying. Chen (1997), from a different research angle of ex ante homogeneous firms, showed that bundling can serve as a tool for product differentiation. Our research also differs from existing works in that the bundling literature treats all prices as common knowledge, whereas our paper allows for the possibility that add-on fees are not observable to at least some consumers. Such unobservability can result in a high magnitude of add-on fees—a signature dynamic in the add-on literature—that is absent from bundling research.

This paper also belongs to the broader literature on switching costs.[8] Although the early switching cost papers stress applications where consumers buy the same product in multiple periods, many arguments are equally applicable to situations where the product purchased in the second period is different from the product bought in the first period. For example, Klemperer's (1987a) discussion of situations where profits with infinite switching costs are identical to profits with no switching costs is essentially the same as Lal and Matutes' irrelevance result, and a number of papers have used similar frameworks to discuss market power in aftermarket service, e.g., Shapiro (1995) and Borenstein et al. (2000). The most basic result in the switching cost literature is that switching costs can increase or decrease profits because they usually make first-period prices (think base goods) lower and second-period prices (add-ons) higher. The literature contains several well-known arguments about why switching costs may tend to raise profits, for example, Farrell and Shapiro (1988), Klemperer (1987b), and Beggs and Klemperer (1992). The argument in the above papers are inapplicable to add-ons, however, because they require an assumption that firms cannot differentiate between new and old customers; i.e., that the firm cannot choose an add-on price different from the price for good L.

This paper is also loosely related to all papers discussing a strategic investment that softens competition. Chapter 8 of Tirole (1988) reviews a number of such papers. A classic example is Thisse and Vives (1988), which notes that firms are better off competing in FOP prices than in delivered prices, because when they choose separate delivered prices for each location they end up being in Bertrand competition for the consumers at each location. As in this paper, they also note that FOB pricing in not individually rational in an extended game in which firms first choose pricing policies, and then compete in prices.

Gabaix and Laibson (2006) were among the first in the add-on literature to allow for boundedly rational consumers.[9] As noted by Spiegler (2006), consumers may find it dif-

[8]Farrell and Klempere (2007) provide an excellent survey.

[9]Ellison (2006) and Spiegler (2011) provide excellent survey on the issue with boundedly rational consumers.

ficult to think and account for all possible dimensions (e.g., all possible services that may or may not be included in the posted price) and thus resort to simplifying heuristics (e.g., posted price) to save cognitive resources. The concept of bounded rationality has been around for decades (Simon, 1955), though relatively recently it has gained more acceptance in economics (Rubinstein, 1998). Conlisk (1996) reviewed a large body of evidence supporting bounds on rationality and provided a discussion of when and why they should be incorporated in economic models. In our setting, there is a large body of anecdotal evidence supporting bounded rationality in terms of expectations surrounding hidden add-on fees. As described by Conlisk (1996), human cognition is a scarce resource, and thus a consumer's inability to accurately forecast the price for each possible add-on is consistent with a process by which cognitive resources are allocated elsewhere.[10]

2.3 THE MODEL

2.3.1 Firms

The model is one of imperfect competition. There are two competing firms, 1 and 2, that are horizontally differentiated in that they are located at $x_1 = 0$ and $x_2 = 1$ on a linear city (Hotelling, 1929). We also allow for asymmetry in the quality of each firm's offerings, which we will specify later when we discuss consumer reservation utilities. As in Liu and Serfes (2005), we assume the cost of developing a certain quality to be a fixed cost (i.e., the cost of developing a product or establishing a brand name), which has no effect on the marginal cost of production (see also Motta, 1993; Ikeda and Toshimitsu, 2010).[11] Each firm offers a *base good* with marginal cost $c = 0$. Each firm also offers

[10] See also Armstrong and Vickers (2012); Grubb (2009, Forthcoming).

[11] Alternatively, asymmetric quality could be modeled as having asymmetric marginal costs of production (e.g., Moorthy, 1988; Kuksov and Lin, 2010; Dahremöller, 2013). Incorporating this assumption would complicate the model, yet logic suggests the current findings are preserved when the difference in costs is sufficiently less than the difference in qualities. The basis for this logic is that our results hold when the difference in costs is zero, and thus hold for local deviations.

an *add-on*, with marginal cost $\hat{c} = 0$.[12] For example, a base good can be a hotel room, checking account, wireless plan, or flight seat, and the associated add-on can be pool access, overdraft protection, data usage, or pillow usage, respectively.

We denote the price charged for the base good by firm $i = 1, 2$ as p_i while the price for add-on good as $\hat{p}_i$. Posted prices are easily accessible by consumers at negligible costs, such as through popular Internet-based price comparison site like Expedia.com, MyRatePlan.com, and Google Product Search. As a result, consumers have full information over posted prices. In contrast $\hat{p}_i$ is not easily discoverable by consumers. For example, on popular price comparison sites such as Expedia.com, hotels post their room rates yet often keep silent on their Internet access, pool usage, or parking fees, and airlines show their ticket prices but not their charges for check-in baggage, pet fees, or Internet access. Although some consumers may track down this information, others may not know to do so. Outside the travel industry, marketing letters from credit card companies highlight APRs, yet keep details on how default rates are triggered in fine print.[13] Wireless companies like Verizon and AT&T clearly announce their access fees for voice plans, but per-unit text messaging rates are in their fine print.

2.3.2 Consumers

A consumer type θ's utility from buying the base product from firm i is equal to

$$R_i - t|x_i - \theta| - p_i, \tag{2.1}$$

where transportation cost t is a positive constant, consumer location θ are uniformly distributed along the linear city with a mass normalized to one, and the reservation value for

[12] For example, we can think that the marginal cost of the hotels to provide amenities such as wireless internet services, telephone calls would be significantly small so that we can ignore that costs. But in some industries, it could not be the case. Providing meal in flight and overdraft fee in banking industries may be such examples.

[13] One controversial yet popular trigger is called "universal default," where a credit card's default rate is triggered if a consumer defaults on credit cards issued by any other financial institute.

the base good from firm i, R_i, depends on the quality of the firm. Without loss of generality, let firm 1 be the qualitatively superior firm with quality normalized to 1 (i.e., $R_1 = R$). Firm 2 is weakly inferior with quality of the base good ($q \leq 1$) (i.e., $R_2 = qR$). Similarly to Villas-Boas (1999) and Shin and Sudhir (2010), we assume that qR is large enough to ensure full market coverage.[14] A consumer type θ's utility from buying the add-on from firm i is equal to

$$\hat{R}_i - \hat{p}_i, \tag{2.2}$$

where $\hat{R}_1 = \hat{R}$ and $\hat{R}_2 = g\hat{R}$ with firm 1's quality of the add-on normalized to 1 and firm 2's quality of the add-on equal to $g \leq 1$.

We allow for consumer heterogeneity in willingness to pay for the add-on service as well as in bounded rationality.[15] As such we have three segments that we describe and provide justification for.

We allow a proportion $(1 - \alpha)$ of consumers do not derive utility from the add-on (i.e., $\hat{R} = 0$). Consumers may be in the base segment because they do not value the add-on or because they have found reasonable substitutes from the outside market. Examples include a wireless customer who does not want to text message, a checking account holder who balances the books and does not require overdraft protection or cancel-check services, and a flyer who packs light for a trip or carries their own pillow and thus doesn't check a bag or pay for use of pillow. In ancillary analyses in Appendix, we confirm our results when an additional segment exists who will consume the add-on unless it carries a charge.

Whereas base consumers have zero value for the add-on, all remaining consumers have a positive value for the add-on (i.e., $\hat{R} > 0$ for all θ). We assume $g\hat{R} > \hat{c}$, i.e., the reservation value of the add-on for either firm is higher than the marginal cost of the add-on. As in Gabaix and Laibson (2006), we allow for consumers to both value the

[14]In Section 2.5 we provide the explicit condition for full market coverage in equilibrium.

[15]In Appendix, we examine a model in which consumers know that they will be charged add-on fees, but do not know what those are before committing to purchase the base product.

add-on and inaccurately believe that the add-on is provided for free. Let β represent the proportion of nonbase consumers who have this form of bounded rationality (i.e., consumers who believe $\hat{p}_i = 0$). In other words, the proportion of the entire consumer population who fall in this segment is equal to $\alpha\beta$. As described by Gabaix and Laibson (2006), these consumers incompletely analyze the future game tree and do not anticipate the add-on fee at the time of base purchase. There are several reasons why consumers may be boundedly rational in this sense. It has been documented that even experienced travelers are surprised to learn that there are charges for services that had at a previous time been free.[16] Alternatively, consumers may not have enough knowledge about the product to know what services will or will not be included in the price. Consistent with Conlisk (1996), boundedly rational consumers do not allocate cognitive resources toward finding the actual add-on fee or toward forming accurate expectations.

The remaining $\alpha(1-\beta)$ consumers have $\hat{R} > 0$ and know the add-on fees. Their knowledge of add-on fees may come from their experience or search. As such, knowledgeable consumers explicitly account for the add-on fee in their base purchase decision.

2.3.3 Timing of the Model

The sequence of game is as follows. In Stage 1, firms firms decide whether to charge separately for add-ons. Given the pricing policy chosen in Stage 1, the firms simultaneously choose their posted and add-on prices in Stage 2. In Stage 3, consumers choose from which firm to buy. In Stage 4, all consumers are aware of add-on fee and only nonbase consumers purchase the add-on if the add-on fee is less than or equal to the utility it provides. Consumers in Stage 4 are locked in to their Stage 3 purchase and cannot switch firms. Figure 2.1 depicts the sequence of events.

[16] See, for example, "As hotels struggle for business, some guests find an upside", *USA Today*, February 5, 2009.

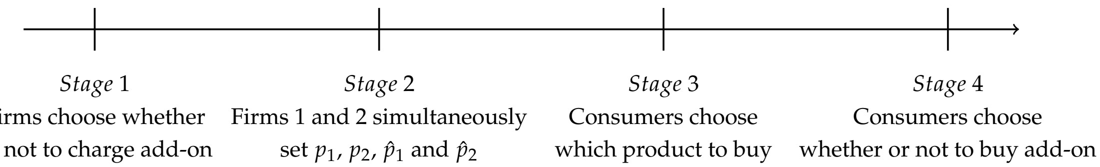

Figure 2.1: Sequence of Events

2.4 ANALYSIS

We analyze the model using backward induction. We have three different consumer segments to examine: $(1-\alpha)$ base consumers, $\alpha\beta$ myopic consumers, and $\alpha(1-\beta)$ sophisticated consumers. For the add-on to be purchased by consumers in Stage 4, it must be that $\hat{p}_1 \leq \hat{R}$ and $\hat{p}_2 \leq g\hat{R}$. Otherwise, no consumer will consume the add-on.

To the best of our knowledge this is the first research on add-on pricing that considers all of the aforementioned three segments of consumers. It turns out, as the results will demonstrate, that insights new to the literature arise when both firms serve *all three* segments of consumers in equilibrium. We therefore will focus our attention on the scenario where all three segments of consumers purchase in equilibrium in the body of the paper, and we will provide the corresponding regularity conditions.

To derive the quantity of consumers who purchase each base product and each add-on in Stage 3 and Stage 4, we identify the marginal consumer in each segment who is indifferent between purchasing product 1 and purchasing product 2. Given firm prices $(p_i, \hat{p}_i)$ for $i = 1, 2$, the marginal consumer in the sophisticated segment satisfies

$$R + \hat{R} - t|0 - \theta_s| - p_1 - \hat{p}_1 = qR + g\hat{R} - t|1 - \theta_s| - p_2 - \hat{p}_2,$$

and thus she is located at

$$\theta_s = \frac{1}{2} + \frac{(1-q)R + (1-g)\hat{R}}{2t} + \frac{p_2 + \hat{p}_2 - p_1 - \hat{p}_1}{2t}.$$

All consumers in the sophisticated segment with $\theta \leq \theta_s$ will purchase both the base product and the add-on service from firm 1, whereas those with $\theta > \theta_s$ will purchase from firm 2.

Similarly the locations of the marginal base consumer and the marginal myopic consumer are $\theta_b = 1/2 + [(1-q)R + p_2 - p_1]/2t$ and $\theta_m = 1/2 + [(1-q)R + (1-g)\hat{R} + p_2 - p_1]/2t$, respectively. Demand of each firm's base product, D_i, and add-on, $\hat{D}_i$, are then

$$D_i = \frac{1}{2} - \eta(i)\frac{\alpha(1-\beta)(\hat{p}_1 - \hat{p}_2) + (p_1 - p_2) - \alpha(1-g)\hat{R} - (1-q)R}{2t}, \tag{2.3}$$

$$\hat{D}_i = \frac{\alpha}{2} - \alpha\eta(i)\frac{(1-\beta)(\hat{p}_1 - \hat{p}_2) + (p_1 - p_2) - (1-g)\hat{R} - (1-q)R}{2t}, \tag{2.4}$$

where $\eta(i) = 1$ if $i = 1$, and $\eta(i) = -1$ if $i = 2$. Firm profits are

$$\pi_i = (p_i - c)D_i + (\hat{p}_i - \hat{c})\hat{D}_i. \tag{2.5}$$

In this model with three consumer segments and two firms that are vertically differentiated along two dimensions as well as horizontally differentiated, the Stage 2 pricing analysis is relatively involved. Similarly to Shin and Sudhir (2010), we proceed by first analyzing the symmetric case. Examining the benchmark allows us to isolate the impacts of having three distinct consumer segments served by the firms and asymmetry in quality of the base products.

Before analyzing equilibrium prices in the general framework, we focus on the special case in which two firms are symmetric in that $q = 1$. Thus, there is no quality differences between the products of each firm. This case will provide the key to the more general analysis in the next section. Suppose that each firm charges add-on fees separately. To ensure that firms choose to serve all three consumer segments we impose the following assumption regarding the degree of product differentiation relative to add-on value.

2.4.1 Symmetric Firms and Boundedly Rational Consumers

The first special case is when two firms are symmetric, i.e., $q = 1$ and $g = 1$. We impose the following assumption regarding the level of horizontal differentiation relative to the add-on value to ensure that firms choose to serve all three consumer segments.

Assumption 2. *$t > \hat{R} - \hat{c}$.*

Applying $q = 1$ and $g = 1$ to equations (2.3) and (2.4), in this special case we have $D_i = 1/2 - \eta(i)[\alpha(1-\beta)(\hat{p}_1 - \hat{p}_2) + (p_1 - p_2)]/(2t)$ and $\hat{D}_i = \alpha/2 - \alpha\eta(i)[(1-\beta)(\hat{p}_1 - \hat{p}_2) + (p_1 - p_2)]/(2t)$. We can derive the equilibrium posted prices and add-on fees from the first-order conditions of profit expressions in (2.5). It turns out that there are two mutually exclusive cases.

Lemma 1. *Define $\bar{\beta} \equiv (\hat{R} - \hat{c})(1-\alpha)/\big(t + (\hat{R} - \hat{c})(1-\alpha)\big)$. Given symmetric firms, i.e., $q = 1$ and $g = 1$:*

(i) if $\beta < \bar{\beta}$, the unique equilibrium is an interior equilibrium, in which firms charge posted prices $p_1 = p_2 = c + t - [\alpha\beta/\big((1-\alpha)(1-\beta)\big)]t$ and add-on fees $\hat{p}_1 = \hat{p}_2 = \hat{c} + [\beta/\big((1-\alpha)(1-\beta)\big)]t$;

(ii) if $\beta \geq \bar{\beta}$, the unique equilibrium is a corner equilibrium, in which firms charge posted prices $p_1 = p_2 = c + t - \alpha(R - c)$ and add-on fees $\hat{p}_1 = \hat{p}_2 = \hat{R}$.

See Appendix for all proofs. In contrast to the Gabaix and Laibson's (2006) prediction that firms will charge the maximum possible add-on price, Lemma 1 shows that this result is conditional on the relative magnitude of two ratios $\beta/\big((1-\alpha)(1-\beta)\big)$ and $(\hat{R} - \hat{c})/t$. Although part (ii) of Lemma 1 is consistent with Gabaix and Laibson's (2006) shrouded prices equilibrium, part (i) indicates when the equilibrium add-on price may be less than the consumer valuation of the add-on.[17] Proposition 5 highlights a key insight gained

[17] Note that combined with Assumption 2, the interior equilibrium holds for $\beta/[(1-\alpha)(1-\beta)] < (\hat{R} - \hat{c})/t < 1$. This inequality is true for any $\hat{R} - \hat{c}$ and t that satisfy Assumption 2 if β is small enough and $\alpha < 1$.

from considering rational consumers who buy the add-on.

Proposition 5. *Given symmetric firms, the total price for buying the base product and the add-on is increasing in β if and only if $\beta < \bar{\beta}$. As such, consumer welfare for rational consumers and myopic consumers is decreasing in the number of myopic consumers (β) if and only if $\beta < \bar{\beta}$ holds.*

Proposition 5 identifies when rational consumers who value the add-on are penalized—and increasingly so—by the existence of myopic consumers. To our knowledge this result is new to the add-on pricing literature and contrasts with the results of Gabaix and Laibson (2006, p. 517), who found that consumers who account for the add-on during the base purchase earn greater surplus with more myopic consumers.[18] Proposition 5 also shows that an inattentive consumer is worse off when there are a greater number of peers who are also inattentive. Gabaix and Laibson (2006) found the opposite result when prices are hidden.

The intuition behind the contrasting results lies in how the sizes of the consumer segments affect posted prices and add-on fees. When a greater number of consumers will purchase the add-on, firms engage in increasingly intensified competition over posted prices to lock in additional profit on the add-on. This effect is present in our model and that of Gabaix and Laibson (2006). In the Gabaix and Laibson (2006) paper, all consumers who buy the add-on when its price is hidden are boundedly rational and the add-on prices are set at their maximum (further increasing them will trigger consumers to stop buying the add-on). Therefore, the only effect of increasing the number of boundedly rational consumers is to lower posted prices, which in turn means lower total prices. However, when there is a sufficient number of sophisticated consumers and base consumers (i.e., $\beta < \bar{\beta}$), add-on prices are not set at their maximum. As such, in this parameter range there is a second effect of increasing the number of boundedly rational consumers. In this

[18]Gabaix and Laibson (2006) consider both a shrouded prices equilibrium in which firm hide the add-on fee from myopic consumers and an unshrouded prices equilibrium in which some myopic consumers know about the add-on fee. In their shrouded prices equilibrium, the surplus of consumers who know about the fee increases in the number of myopic consumers; in the unshrouded prices equilibrium, this surplus is a constant that is lower than the surplus under the shrouded prices equilibrium.

scenario, firms soften their competition over the add-on—and thus increase their add-on fees—to take advantage of locked-in boundedly rational consumers. The increased add-on fee dominate the reduced posted prices, resulting in higher total prices that sophisticated and myopic consumers pay.

As the intuition suggests, Proposition 5 is driven by the fact that the add-on price is set below its maximum level. We refer to this case as the interior equilibrium. The new result in Proposition 5 regarding total price and consumer welfare requires participation in the marketplace by each of the three consumer segments. To see this, the condition from Proposition 5 can be rewritten as follows:

$$\begin{aligned} \beta < \bar{\beta} &\Longleftrightarrow \frac{\alpha\beta}{(1-\alpha)\cdot[\alpha(1-\beta)]} < \frac{\hat{R}-\hat{c}}{t} \\ &\Longleftrightarrow \frac{\text{number of boundedly rational consumers}}{[\text{number of base consumers}]\cdot[\text{number of sophiscated consumers}]} < \frac{\hat{R}-\hat{c}}{t} \end{aligned} \tag{2.6}$$

Equation (2.6) highlights that the equilibrium is the interior equilibrium when there is a sufficient number of *both* base consumer and sophisticated consumers. Notice that the condition holds trivially when there are no boundedly rational consumers, and the condition never holds when all consumers who buy the add-on are boundedly rational (i.e., there are no sophisticated consumers). Intuitively, the inclusion of enough base consumers dampens the firms' incentive to slash posted prices in an attempt to exploit boundedly rational consumers. Furthermore, when there are enough sophisticated consumers, firms will not set total prices too high, because otherwise they will drive away sophisticated consumers. the above two dynamics together driver down the add-on price. Hereafter, we refer to (2.6) as the "interior-equilibrium condition" for ease of exposition. Its essence that there should be enough numbers of both base and sophisticated consumers to ensure interior add-on fees will hold across the more general case, though the condition will be relatively more involved.

When (2.6) does not hold, Lemma 1 shows that firms will charge the maximum possible add-on fees, under which total price is independent of β. The equilibrium prices are analogous to the shrouded prices equilibrium in Gabaix and Laibson (2006) and thus do not add new insight.[19] For the rest of this paper, we focus attention on the more novel case in which equilibrium add-on fees are not corner solutions.

The inclusion of consumers who do not know about the add-on price before the base purchase but still buy the add-on has a notable impact on the welfare of the consumer segments. However, this model feature has no effect on the result of profit irrelevancy of add-on prices. Firm profit in either case of Lemma 1 is equal to $t/2$. In contrast, when no firms charge add-on fees (i.e., $\hat{p}_1 = \hat{p}_2 = 0$), the equilibrium prices are

$$p_1^{N,N} = p_2^{N,N} = t + \hat{c} + \alpha\hat{c}$$

and profit is also $t/2$.

2.4.2 Solving the Model When Only One Firm Charges an Add-On Price

Consider now an asymmetric pricing policy of the following type. Assume that firm 1 charges an add-on fee for its add-on consumer, whereas firm 2 does not charge the add-on fee, i.e., $\hat{p}_2 = 0$. Intuitively, we would expect that firm 1 charges a higher base price than firm 2. This intuition is confirmed in the following lemma.

Lemma 2. *For $\beta < \bar{\beta}$, in the pure strategy Nash equilibrium of games in which firm 1 charges an*

[19] Clarifying $\alpha(1-\alpha)$ as the number of consumers who buy (do not buy) the add-on, the prices in part (ii) of Lemma 1 are equivalent to the prices in Equations in (3) and (4) of Gabaix and Laibson (2006).

add-on price, the equilibrium prices are given as follows:

$$p_1^{A,N} = \frac{6\omega - 3\alpha\beta}{6\omega - \alpha\beta^2}t, \quad p_2^{A,N} = \frac{6\omega - 2\alpha\beta^2}{6\omega - \alpha\beta^2}t,$$
$$\hat{p}_1^{A,N} = \frac{3\beta}{6\omega - \alpha\beta^2}t,$$

where $\omega = (1-\alpha)(1-\beta)$. Also, in equilibrium, we have $p_2^{A,N} > p_1^{A,N} > 0$ and $\hat{p}_1^{A,N} > 0$.

Since firm 1 and firm 2 are symmetric, the Nash equilibrium in the case where only firm 2 uses an add-on pricing policy is analogous to Lemma 2. This lemma implies that each firm makes strictly positive sales to consumers in each segment in equilibrium. Also, the fact that the firm who can price discriminate charges a lower base price is not surprising. If a consumer who is ignorant of add-on fees purchases a product from firm 1, she is locked into its aftermarket. Thus it creates an incentive for the firm to charge the maximum possible price for the add-on and a lower price for the base to induce more consumers into its aftermarket.[20]

Indeed we can observe that firm 1's add-on fee is increasing in β. Due to the presence of the sophisticated consumers, firm 1 cannot charge the maximum possible add-on fee, but when there are more myopic consumers the firm will sacrifice some profit gain from sophisticated consumers and increase the add-on fee. Also increasing the number of boundedly rational consumers increases the competitive pressure in the base product market, thus the equilibrium posted price decreases with an increasing of β. However a price reduction of the base product causes more profit loss for firm 2 compared to the firm 1, since firm 2 does not have any other way to compensate for this loss whereas firm 1 can make it up from the sales of add-ons. That is $|\frac{\partial p_1}{\partial \beta}| > |\frac{\partial p_2}{\partial \beta}|$.

[20]When $\beta > \bar{\beta}$, we also have another equilibrium in which $p_1 < 0 < p_2$. In order to make it easier to compare the results with the one in Lemma 1, we restrict the parametric space of β.

Lemma 2 allows us to obtain the following equilibrium profits:

$$\pi_1^{A,N} = \frac{9\omega(4\omega - \alpha\beta^2)}{2(6\omega - \alpha\beta^2)^2}t, \qquad \pi_2^{A,N} = \frac{6\omega - 2\alpha\beta^2}{2(6\omega - \alpha\beta^2)^2}t.$$

Table 2.1 summarizes the payoffs for the firms.

Table 2.1: Summary of Firm's payoffs

Firm 1 \Firm 2	A	NA
A	$\frac{t}{2}, \frac{t}{2}$	$\frac{9\omega(4\omega - \alpha\beta^2)}{2(6\omega - \alpha\beta^2)^2}t, \frac{6\omega - 2\alpha\beta^2}{2(6\omega - \alpha\beta^2)^2}t$
NA	$\frac{6\omega - 2\alpha\beta^2}{2(6\omega - \alpha\beta^2)^2}t, \frac{9\omega(4\omega - \alpha\beta^2)}{2(6\omega - \alpha\beta^2)^2}t$	$\frac{t}{2}, \frac{t}{2}$

The analysis so far has compared the equilibrium profits associated with the four different outcomes of firm's add-on pricing decisions. The equilibrium profits, summarized in Table 2.1, define a normal-form game in which each firm chooses whether to charge separate prices for the base product and add-on. It can be verified that firm 1 will charge an add-on price no matter what pricing policy is adopted by firm 2. Likewise, firm 2 charges an add-on price without regard to firm 1's price policy. This proves that add-on pricing is a dominant strategy for each firm. Proposition 6 summarizes the result.

Proposition 6. *For $\beta < \bar{\beta}$, choosing the add-on pricing policy is a dominant strategy for each firm and, consequently, (A, A) with the market prices is the unique subgame-perfect equilibrium.*

Proposition 6 allows us to answer the question of whether the firm should charge a separate price for the add-on or not. We observe that the firms' optimal strategy is committing to the add-on pricing policy, which explains why the firms use this pricing scheme even though they earn the same profits compared to the case where they do not engage in this form of price discrimination.

Add-on pricing has no impact on total welfare since all consumers have unit demand for the base good, and they purchase the product from the closer firm with either pricing policy. However, it has different implications on consumer welfare according to the type of consumers. The total price for add-on users (sophisticated and boundedly rational consumers) are, from the results in Lemma 1, given by

$$p^{A,A} + \hat{p}^{A,A} = t + \frac{\beta}{(1-\beta)}t > t = p^{N,N}, \tag{2.7}$$

which implies that when both firms use an add-on pricing policy, the add-on users are worse off compared to the no add-on equilibrium. On the other hand, those who do not purchase an add-on can be better off. Also, direct comparison between $p^{A,A}$ and $p^{N,N}$ shows that add-on pricing policies benefit base consumers as compared to the case without add-on pricing. Notice that if there are no boundedly rational consumers in the model (i.e., $\beta = 0$), we can see that there are no effects in consumer welfare, as in standard add-on pricing literature.

2.5 ASYMMETRIC QUALITIES

We now consider the case with quality differentiation on base goods, i.e., $q < 1$. For example, hotels can differ in their star ratings. For ease of exposition, we frequently refer to firm 1 with base good quality 1 as the superior firm and firm 2 with base good quality $q < 1$ as the inferior firm. To isolate the consequences of quality differentiation over base goods, in this special case we consider symmetric quality of the add-on which is represented by $g = 1$. Add-ons such as Internet access, check cancellation, and text messaging likely fall this category because their add-on quality is likely irrespective of the quality of base offering. We impose the following assumption to ensure that each firm will serve all three consumer segments:

Assumption 3. *$t > \hat{R} - \hat{c} + (1-q)R/3$.*

After applying $g = 1$ to Equations (2.3) and (2.4), we can derive the equilibrium posted prices and add-on fees from the first-order conditions of profit expressions in (2.5). Similar to Lemma 1, we again get mutually exclusive cases and present the conditions leading to the interior equilibrium below.

Lemma 3. *Consider symmetric add-on quality, i.e., $g = 1$. If*

$$\frac{\beta}{\omega} < \frac{1}{t}\Big[\hat{R} - \hat{c} - \frac{\beta}{9\omega - 2\alpha\beta^2}(1-q)R\Big], \tag{2.8}$$

the firms charge the following posted prices and add-on fees:

$$p_1^{A,A} = c + t - \frac{\alpha\beta}{\omega}t + \frac{3\omega - \alpha\beta}{9\omega - 2\alpha\beta^2}(1-q)R \quad ; \quad \hat{p}_1^{A,A} = \hat{c} + \frac{\beta}{\omega}t + \frac{\beta}{9\omega - 2\alpha\beta^2}(1-q)R \tag{2.9}$$

$$p_2^{A,A} = c + t - \frac{\alpha\beta}{\omega}t - \frac{3\omega - \alpha\beta}{9\omega - 2\alpha\beta^2}(1-q)R \quad ; \quad \hat{p}_2^{A,A} = \hat{c} + \frac{\beta}{\omega}t - \frac{\beta}{9\omega - 2\alpha\beta^2}(1-q)R \tag{2.10}$$

The above parametric condition (2.8) which ensures $\hat{p}_i < \hat{R}$ in equilibrium for each firm i is analogous to the interior equilibrium condition in the symmetric case, yet more involved because of asymmetric base qualities.[21] This condition, combined with Assumption 3, ensures that the indifferent consumers in each consumer segment are located in the interior. Also note that full market coverage again requires a high enough qR.[22] We again limit our discussion to the more novel case where both firms charge interior add-on fees. Notice that the interior-equilibrium condition holds trivially as the number of boundedly rational consumers approaches zero (because the left-hand side of the inequality approaches zero and the right-hand side is strictly positive by fact that $R > c$). As such, both the interior-equilibrium condition and Assumption 3 (which does not depend on β) can hold simultaneously.

[21]Note that firm 1's (firm 2's) prices are increasing (decreasing) in R because, as we show in the proof of Lemma 3, conditions $9\omega - 2\alpha\beta^2 > 0$ and $3\omega - \alpha\beta > 0$ hold under the interior equilibrium.

[22]We provide the proof, in the appendix, that it is possible to have all regularity conditions for the interior equilibrium to hold at the same time.

Below we discuss the implications of this general model on the firm's pricing. An examination of the prices reveals that firm 1's total price always increases in β when condition (2.8) holds, i.e., when the equilibrium is an interior equilibrium. Firm 2's total price under this condition, however, increases in β if and only if t is above a threshold:

$$t > \tilde{t} \equiv \frac{\omega(1-\beta)\big(9-\alpha(3-2\beta)^2\big)}{(9\omega-2\alpha\beta^2)^2}(1-q)R. \tag{2.11}$$

Therefore, when (2.11) holds, the findings of Proposition 5 are replicated in the general model.[23] Nevertheless, when $t < \tilde{t}$, the effect of β on firm 2's total price is negative. This is because the inferior firm experiences an additional dynamic that differs from the symmetric case. Because the superior firm increases its total price to get a higher margin out of the boundedly rational consumers (while experiencing a greater market share due to its quality superiority), the inferior firm has an incentive to actually cut its total price because the marginal increment in market share is now more significant. The second dynamic dominates the first dynamic when t is small enough: when t is small, consumers are more price sensitive, and thus the inferior firm can be more effective in acquiring market share by lowering price.

When both firms use add-on pricing, Lemma 3 leads us to have the following equilibrium profits:

$$\pi_1^{A,A} = \frac{t}{2} + \frac{(1-\alpha)\alpha}{2t(9\omega-2\alpha\beta^2)}\big((1-q)R\big)^2 + \frac{1}{3}\Big[1-\frac{\alpha\beta^2}{9\omega-2\alpha\beta^2}\Big](1-q)R \tag{2.12}$$

and

$$\pi_2^{A,A} = \frac{t}{2} + \frac{(1-\alpha)\alpha}{2t(9\omega-2\alpha\beta^2)}\big((1-q)R\big)^2 - \frac{1}{3}\Big[1-\frac{\alpha\beta^2}{9\omega-2\alpha\beta^2}\Big](1-q)R. \tag{2.13}$$

[23] Analysis of the impact of β on total prices is included at the end of the proof of Lemma 3. To see that the condition in equation (2.11) can be satisfied at the interior equilibrium, observe that the right-hand side of the inequality is bounded as β approaches zero. Thus, the interior-equilibrium condition and the equality of equation (2.11) can be satisfied for low value of β.

We now solve the model for the game in which neither firm can charge add-on prices to the consumers. Thus, we solve for the Nash equilibrium in which $\hat{p}_1 = \hat{p}_2 = 0$ is imposed in expressions (2.3), (2.4), and (2.5). When both firms do not charge separate add-on fees, we obtain the following Nash equilibrium prices and corresponding profits:

$$p_1^{N,N} = t + \frac{1}{3}(1-q)R, \qquad p_2^{N,N} = t - \frac{1}{3}(1-q)R, \tag{2.14}$$

$$\pi_1^{N,N} = \frac{1}{2t}\Big(t + \frac{1}{3}(1-q)R\Big)^2, \qquad \pi_2^{N,N} = \frac{1}{2t}\Big(t - \frac{1}{3}(1-q)R\Big)^2. \tag{2.15}$$

When all consumers know the add-on prices, the add-on prices are set to marginal cost, the add-on pricing does not affect profit for either firm (i.e., when $\beta = 0$, the above $\pi_i^{A,A}$ is equal to $\pi_i^{N,N}$ for $i = 1, 2$). This result, which holds even for firms who are asymmetric in quality, is consistent with the bulk of the add-on pricing research (see Lal and Matutes, 1994; Verboven, 1999; Gabaix and Laibson, 2006). However, the existence of boundedly rational consumers in the context of asymmetric base qualities has unique profit implications.

Proposition 7. *Given the presence of myopic consumers and horizontally differentiated firms that are asymmetric in the quality of the base product but symmetric in the quality of the add-on (i.e.,* $\beta > 0$*,* $q < 1$*,* $g = 1$*), add-on pricing by both firms will diminish profits for the superior firm and increase profits for the inferior firm in the interior equilibrium.*

Proposition 7 is the first finding in the add-on pricing literature to identify opposing effects of add-on pricing on profitability for a superior firm and for an inferior firm. It also demonstrates how add-on pricing with boundedly rational consumers differ from other forms of price discrimination. the finding that the superior firm earns less profit when both firms practice price discrimination via add-on pricing is counter to the finding of Shaffer and Zhang (2000) that the firm with larger loyal following earns greater profit with preference-based price discrimination. The finding that the inferior firm can earn greater profit when both firms practice price discrimination via add-on pricing is counter to Chen

(2008), who found that the inferior firm is always worse off with price discrimination based on purchase histories. It also contrasts with the finding that price discrimination between consumers who value quality and consumer who do not value quality will lead to lower profits for both the high- and low-quality firms (Corts, 1998).

To understand this contrasting result, first consider the fact that $g = q$ implies the reservation value differential between firms is the same for each type of consumer. Specifically, base consumers who do not buy the add-on experience the same reservation value differential, absent transportation cost, $(1-q)R$ as both sophisticated and myopic consumers because $(R+\hat{R}) - (qR+\hat{R}) = (1-q)R$. In the absence of the boundedly rational consumer segment, both firms will earn all profit on the base consumers and give the add-on at marginal cost.[24] However, when the myopic consumer segment exits, both firms have an incentive to boost revenue by charging higher add-on fees, which does not affect the base purchase decision by the boundedly rational consumers. As a result, there is greater competitive intensity in posted prices that will attract consumers who will generate further income via add-on purchases. This diminishes profitability on the nonbase consumers. However, the superior firm has a larger number of base consumers to which this diminished profit margin applies. For the superior firm, the loss in profit margin on base consumers outweighs the profit gain from selling the add-on above cost. The opposite is true for the inferior firm. As a consequence, the superior firm loses profit and the inferior firm gains profit with the advent of add-on pricing.

2.5.1 Equilibrium Pricing Strategies

We now solve the model in which only one firm charges an add-on price. Because we assumed asymmetry in qualities, we have two cases according to which firm adopts the add-on pricing policy.

Lemma 4. *Suppose that the interior-equilibrium condition in* (2.8) *holds.*

[24] This follows straightforwardly from simplifying the prices of Lemma 3 with $\beta = 0$

(i) When only superior firm charges an add-on price, then the equilibrium prices are

$$p_1^{A,N} = \frac{6\omega - 3\alpha\beta}{6\omega - \alpha\beta^2}t + \frac{2\omega - \alpha\beta}{6\omega - \alpha\beta^2}(1-q)R, \quad p_2^{A,N} = t - \frac{\alpha\beta^2}{6\omega - \alpha\beta^2}t - \frac{2\omega}{6\omega - \alpha\beta^2}(1-q)R,$$
$$\hat{p}_1^{A,N} = \frac{3\beta}{6\omega - \alpha\beta^2}t + \frac{\beta}{6\omega - \alpha\beta^2}(1-q)R,$$

and we have $p_2^{A,N} > p_1^{A,N} > 0$ *and* $\hat{p}_1^{A,N} > 0$.

(ii) When only inferior firm charges an add-on price, the equilibrium prices are

$$p_1^{N,A} = t - \frac{\alpha\beta^2}{6\omega - \alpha\beta^2}t + \frac{2\omega}{6\omega - \alpha\beta^2}(1-q)R, \quad p_2^{N,A} = \frac{6\omega - 3\alpha\beta}{6\omega - \alpha\beta^2}t - \frac{2\omega - \alpha\beta}{6\omega - \alpha\beta^2}(1-q)R,$$
$$\hat{p}_2^{N,A} = \frac{3\beta}{6\omega - \alpha\beta^2}t - \frac{\beta}{6\omega - \alpha\beta^2}(1-q)R$$

and $p_1^{N,A} > p_2^{N,A} > 0$ *and* $\hat{p}_2^{N,A} > 0$.

The idea behind the equilibrium prices in Lemma 4 is follows. First, the firm who price discriminates charges a lower base price and the intuition is same with the one in Lemma 2. Second, as the quality differences, $(1-q)R$, are larger, the base price for superior firm increases while decreases for inferior firm. When the quality difference is large, firm 1 can attract more consumers in base good market and consequently its incentive to higher base product price increases as well. Firm 2 on the other hand, need to lower its base price in order to make up for the quality disadvantage. Third, the superior firm charges a higher price to the consumer group in which it correctly anticipates the add-on fees. In doing so, it can generate high margin per consumer in the sophisticated group, and this marginal benefit exceeds the marginal loss from the decreased sales because the superior firm also can generate profits from the myopic consumers through the increased add-on fees.

The equilibrium profits for the first case are

$$\pi_1^{A,N} = \frac{\omega(4\omega - \alpha\beta^2)}{2t(6\omega - \alpha\beta^2)^2}((1-q)R + 3t)^2, \quad \pi_2^{A,N} = \frac{2\left[\omega(1-q)R - (3\omega - \alpha\beta^2)t\right]^2}{t(6\omega - \alpha\beta^2)^2} \quad (2.16)$$

and the profits for the second case are

$$\pi_1^{N,A} = \frac{2\left[\omega(1-q)R + (3\omega - \alpha\beta^2)t\right]^2}{t(6\omega - \alpha\beta^2)^2}, \quad \pi_2^{N,A} = \frac{\omega(4\omega - \alpha\beta^2)}{2t(6\omega - \alpha\beta^2)^2}((1-q)R - 3t)^2 \quad (2.17)$$

Now we can make use of the profit levels to characterize the Nash equilibrium with respect to the add-on pricing polices with quality differences. First consider the superior firm. When the rival firm adopts an add-on pricing, the profit for superior firm is larger when it also charges a separate add-on price (i.e., $\pi_1^{A,A} > \pi_1^{N,A}$), provided that the interior equilibrium condition (2.8) is satisfied). The fact that the superior firm can get higher profits from add-on pricing holds even when the rival firm does not charge separate add-on fees. For the inferior firm, we can observe same results, i.e., the inferior should charge a separate price for the add-on no matter which pricing policy is used by the superior firm.

Proposition 8. *Given the presence of myopic consumers and asymmetric firms, add-on pricing policy arises as the unique Nash equilibrium when the interior equilibrium condition* (2.8) *holds.*

This finding is consistent with the well-known results in price discrimination literature that the use of more instrument enables the industry to better extract consumer surplus, which causes profits to rise. As a result, the firms wish to use the more ornate pricing policy even if its rival does not. But as we observed in Proposition 7, the firms might be worse off when they practice price discrimination at the same time. In other words, the firm is always better off if it can price discriminate, for give pricing policies offered by its rivals. However, once account is taken of what rivals too will do, firms in equilibrium can be worse off when discrimination is used. Firms then find themselves in a classic prisoner's dilemma.

2.6 CONCLUSION

This paper extends prior work on add-on pricing in two dimensions. First, it allows for asymmetry in the quality of the base good. Second, it accommodates three segments of consumers: base consumers who do not derive utility from the add-on, sophisticated consumers who value the add-on and correctly anticipate the add-on fee, and boundedly rational consumers who value the add-on yet do not take the add-on price into consideration at the time of initial purchase. Each of these model considerations has substantive implications and reverses predictions from prior research.

Specifically, previous research in add-on pricing predicts that consumers who account for the add-on price in their base purchase decision will enjoy greater surplus when there are more boundedly rational consumers in the marketplace. However, we show that this is not the case when there are enough base and sophisticated consumers served by the firms serve. In this case, an increasing number of myopic consumers results in lower base prices and higher add-on fees. We find the add-on fee increase dominates the base price reduction for both firms when the quality differential between the firms is not too large. Consequently, a greater number of boundedly rational consumers can lead to increased total prices and lower consumer surplus for the boundedly rational consumers and the sophisticated consumers.

Our consideration of quality asymmetry (along with the inclusion of boundedly rational consumers) leads to new findings regarding firm profits. Two specific features are discussed: quality asymmetry in the base good, and the presence of boundedly rational consumers. Strikingly, the standard profit-irrelevancy result of add-on pricing is robust to any one of these two features, but is challenged in the presence of both features.

We also find when both firms are engaged in price discrimination, they are worse off compared to the case in which neither firm price discriminates. This is consistent with the price discrimination literature on endogenous price discrimination model.

This research can be extended in several directions. First, an avenue for future research is to nest the current model in a customer lifetime value framework where customers are repeat buyers. Such a model could allow boundedly rational consumers to punish firms that charge add-on prices by switching to competitors. The add-on prices by each firm, and the feasibility of equilibrium add-on pricing in general, would depend on the firm's discount rate on future payoffs, the size of each consumer segment, and the manner in which consumer beliefs are based on purchase history.

APPENDIX

Appendix for Chapter 2

Proof of Lemma 1 First consider the case where $\beta < \bar{\beta}$, i.e., the interior-equilibrium condition holds. Plug $(1-q)R = 0$ into equations (2.3) and (2.4), then solve the following first-order conditions simultaneously: $\partial\pi_i/\partial p_i = 0$ and $\partial\pi_i/\partial\hat{p}_i = 0$ for $i = 1, 2$. We have base good prices

$$p_1^* = p_2^* = t - \frac{\alpha\beta}{(1-\alpha)(1-\beta)}t \tag{2.18}$$

and add-on fees

$$\hat{p}_1^* = \hat{p}_2^* = \frac{\beta}{(1-\alpha)(1-\beta)}t. \tag{2.19}$$

For these prices to constitute an equilibrium, we need all following four conditions to hold: (i) $\hat{p}_i < \hat{R}$ for any i, (ii) second-order conditions hold at the equilibrium prices, (iii) no firm will deviate by abandoning sophisticated consumers, and (iv) full market coverage. Notice that (i) is apparently true under the condition $\beta < \bar{\beta}$, and (iv) holds if $\hat{R}$ is sufficiently large (as we assumed). We will prove that (ii) and (iii) hold in the proof of the general case and thus they also hold in this specific case.

Next consider the case where $\beta \geq \bar{\beta}$. The above interior equilibrium (2.18) and (2.19) cannot hold now because the add-on fee would surpass $\hat{R}$, which implies no consumer would buy the add-on. To show that parametric conditions can lead to the corner equilibrium as in part (ii) of Lemma 1, we need the following two conditions to hold at equilibrium prices: (i) $\partial\pi_i/\partial p_i = 0$ and (ii) $\partial\pi_i/\partial\hat{p}_i > 0$ for $i = 1, 2$. Without loss of generality, we only consider $i = 1$. Plugging $p_2 = t - \alpha\hat{R}$ and $\hat{p}_2 = \hat{R}$ into (2.5), we have

$$\pi_1 = \frac{1}{2t}\Big[p_1\big(2t - \alpha\hat{R} - p_1 + \alpha(1-\beta)(\hat{R} - \hat{p}_1)\big) + \alpha\hat{p}_1\big(2t - \alpha\hat{R} - p_1 + (1-\beta)(\hat{R} - \hat{p}_1)\big)\Big]. \tag{2.20}$$

It is straightforward to verify that, at $p_1 = t - \alpha\hat{R}$ and $\hat{p}_1 = \hat{R}$,

$$\frac{\partial \pi_1}{\partial p_1} = 0$$

and

$$\frac{\partial \pi_1}{\partial \hat{p}_1} = \frac{\alpha\big(t\beta - (1-\alpha)(1-\beta)\hat{R}\big)}{2t} > 0$$

if and only if $\beta \geq \bar{\beta}$. □

Proof of Lemma 3 The flow of this prof is analogous to that of Lemma 1, albeit more involved because of quality asymmetry. Recall that we need the following condition from Assumption 3,

$$t > \hat{R} + \frac{1}{3}(1-q)R. \tag{2.21}$$

Suppose the interior equilibrium exists. Given profit function in (2.5), we solve first-order conditions and get equilibrium prices as presented in Lemma 3. For these prices to constitute an interior equilibrium, we need all of the following conditions to hold: (i) $\hat{p}_i < \hat{R}$ for any i, (ii) second-order conditions hold, (iii) no firm deviates by abandoning sophisticated consumers, (iv) full market coverage, and (v) marginal consumers between 0 and 1. We will also show that all regularity conditions can hold simultaneously.

First, from (2.9) we get the interior-equilibrium condition that guarantees $\hat{p}_1 < \hat{R}$ in equilibrium:

$$\frac{\beta}{\omega} < \frac{1}{t}\Big[\hat{R} - \frac{\beta}{9\omega - 2\alpha\beta^2}(1-q)R\Big]. \tag{2.22}$$

In the subsequent analysis we will frequently use the fact that $\beta < \omega = (1-\alpha)(1-\beta)$, which directly follows the interior-equilibrium condition and Assumption 3. We will also frequently use the fact, $9\omega - 2\alpha\beta^2 > 9\beta - 2\alpha\beta^2 > 0$.

For (ii) to be true, we need the determinant of Hessian matrix of profit fuction π_i

$$\begin{aligned} H &= \frac{\partial^2 \pi_i}{\partial p_i^2} \frac{\partial^2 \pi_i}{\partial \hat{p}_i^2} - \frac{\partial^2 \pi_i}{\partial p_i \partial \hat{p}_i} \frac{\partial^2 \pi_i}{\partial \hat{p}_i \partial p_i} \\ &= 4\alpha(1-\beta) - \alpha^2(2-\beta)^2 \\ &= \alpha(4\omega - \alpha\beta^2) > 0, \end{aligned}$$

and this inequality always hold because $\omega > \beta$.

We prove (iii) by contradiction. Suppose that firm 2 prefers to deviate from this equilibrium by abandoning sophisticated consumers (and thus will charge the maximum possible add-on fee). Given p_1 and $\hat{p}_1$ as specified in equation (2.9) and $\hat{p}_2 = \hat{R}$, firm 2's profit over base and myopic consumer groups would be

$$\pi_2 = p_1(1 - \theta_m + 1 - \theta_b) + \hat{R}(1 - \theta_m),$$

which implies that optimal deviating posted price for firm 2 as follows:

$$p_2 = \frac{2\omega - \alpha\beta}{2\omega} t - \frac{6\omega + \alpha\beta(1-2\beta)}{2(9\omega - 2\alpha\beta^2)}(1-q)R - \frac{\alpha\beta}{2(1-\alpha(1-\beta))}\hat{R}.$$

A contradiction arises if, given the above prices, the marginal sophisticated consumer is located within the linear city. We have $\theta_s < 1$ when plugging the above prices into $\theta_s = 1/2 + ((1-q)R + p_2 - p_1 + \hat{p}_2 - \hat{p}_1)/(2t)$ (thus the contradiction) if and only if

$$t > \frac{\omega}{1-\alpha(1-\beta)}\hat{R} + \frac{\omega\big(6 - 8\beta - \alpha(2-\beta)(3-2\beta)\big)}{\big(2-\alpha(2-\beta)\big)\big(9(1-\beta) - \alpha(3-\beta)(3-2\beta)\big)}(1-q)R. \tag{2.23}$$

Each of the two terms on the right-hand side are decreasing function of β. The first term is apparently decreasing in β. The second, $\big(6 - 8\beta - \alpha(2-\beta)(3-2\beta)\big)/\big(9(1-\beta) - \alpha(3-\beta)(3-2\beta)\big)$, or equivalently, $1 - (3(1-\alpha) - \beta + 2\alpha\beta)/\big(9(1-\beta) - \alpha(3-\beta)(3-$

$2\beta)\big)$, is decreasing in β because

$$\partial\left(\frac{3(1-\alpha)-\beta+2\alpha\beta}{9(1-\beta)-\alpha(3-\beta)(3-2\beta)}\right)\bigg/\partial\beta=\frac{9(1-\alpha)(2-\alpha)+2\alpha\beta\big((6-\beta)(1-\alpha)+\alpha\beta\big)}{\big(9(1-\beta)-\alpha(3-\beta)(3-2\beta)\big)^2}>0.$$

Therefore, if the above inequality (2.23) holds at $\beta = 0$, it will hold for any positive β. At $\beta = 0$, (2.23) simplifies to $t > \hat{R} + (1/3)(1-q)R$, which is our Assumption 3. Thus the contradiction. For $(1-q)R = 0$, the inequality (2.23) at $\beta = 0$ simplifies to Assumption 2. The analysis of firm 1 abandoning sophisticated consumers is analogous and thus omitted.

For (iv), we now derive the full-market-coverage condition. The market is fully covered in equilibrium if the marginal consumer in each segment who is indifferent between purchasing product 1 and 2 derives a non-negative utility, i.e., the following three inequalities hold:

$$R - p_1 - t\theta_b > 0,$$
$$R + \hat{R} - p_1 - t\theta_m > 0,$$
$$R + \hat{R} - p_1 - \hat{p}_1 - t\theta_s > 0.$$

Plugging equilibrium prices into these inequalities (and afterward it is straightforward that the second inequality will hold if the first one holds), we can simplify the first and the third inequalities to the following, respectively:

$$u(\theta_b) \triangleq R - \frac{1}{2}(1-q)R - \frac{3\omega - 2\alpha\beta}{2\omega}t \geq 0,$$
$$u(\theta_s) \triangleq R + \hat{R} - \frac{1}{2}(1-q)R - \frac{3-\beta}{2(1-\beta)}t \geq 0.$$

Therefore, full market coverage is true if

$$\min\{u(\theta_b), u(\theta_s)\} \geq 0. \tag{2.24}$$

We refer to this as the full-market-coverage condition.

For (v), we simplify the location of the marginal consumer who is indifferent between firm 1 and firm 2 at the equilibrium prices for each segment:

$$\begin{aligned}
\theta_b &= \frac{1}{2} + \frac{(1-\beta)(3-\alpha(3-2\beta))}{2t(9\omega - 2\alpha\beta^2)}(1-q)R, \\
\theta_m &= \frac{1}{2} + \frac{(1-\beta)(3-\alpha(3-2\beta))}{2t(9\omega - 2\alpha\beta^2)}(1-q)R, \\
\theta_s &= \frac{1}{2} + \frac{3-5\beta-\alpha(1-\beta)(3-2\beta)}{2t(9\omega - 2\alpha\beta^2)}(1-q)R.
\end{aligned}$$

It can be shown that these values are each between 0 and 1 given Assumption 3 and the interior-equilibrium condition.

We next show that all regularity conditions for the interior equilibrium are feasible to hold simultaneously: Assumption 3, the interior-equilibrium condition, and the full-market-coverage-condition,. We simplify this feasibility proof using the following fact: all sides of all the above conditions are continuous functions of $(1-q)R$. Therefore, if all three conditions can hold strictly at $(1-q)R = 0$, it must be true that they will also hold when $(1-q)R$ is close enough to zero. Applying $(1-q)R = 0$ to the above three regularity conditions, i.e. into equations (2.21), (2.22) and (2.24), they are simplified to

$$t > \hat{R}, \tag{2.25}$$

$$\frac{\beta}{\omega} < \frac{1}{t}\hat{R}, \tag{2.26}$$

$$\min\left\{R - \frac{3\omega - 2\alpha\beta}{2\omega}t,\ R + \hat{R} - \frac{3-\beta}{2(1-\beta)}t\right\} \geq 0. \tag{2.27}$$

When $(1-q)R = 0$, the left-hand side of (2.25) goes to zero, thus this first condition holds if β is low enough. When $(1-q)R = 0$, (2.26) holds if $\hat{R}$ is sufficiently small relative to t. Only the third condition (2.27) involves R. When $(1-q)R = 0$ and for any given values of all other model parameters, (2.27) holds if R is large enough.

Given that all sides of all three regularity conditions for this general model are continuous function of $(1-q)R$, we can now conclude that all regularity conditions for the existence of the interior equilibrium will hold simultaneously when (i) $(1-q)R$ is small enough, (ii) β is low enough, (iii) $\hat{R}$ is sufficiently small relative to t, and (iv) R is large enough.

Finally, we show how each firm's total price changes in β:

$$\begin{aligned}
\frac{\partial(p_1+\hat{p}_1)}{\partial\beta} &= \frac{t}{(1-\beta)^2} + \frac{(1-\alpha)\big(9-\alpha(3-2\beta)^2\big)}{(9\omega-2\alpha\beta^2)^2}(1-q)R > 0, \\
\frac{\partial(p_2+\hat{p}_2)}{\partial\beta} &= \frac{t}{(1-\beta)^2} - \frac{(1-\alpha)\big(9-\alpha(3-2\beta)^2\big)}{(9\omega-2\alpha\beta^2)^2}(1-q)R
\end{aligned}$$

This last expression is positive if and only if condition (2.11) holds. □

Proof of Proposition 7 From the equilibrium profit in (2.12), (2.13), and (2.15), the firm's profit differentials between when add-on pricing is adopted and when add-on pricing is not adopted are

$$\pi_1^{A,A} - \pi_1^{N,N} = -\frac{\alpha\beta^2\big(3t-(1-q)R\big)}{9t(9\omega-2\alpha\beta^2)}(1-q)R < 0 \tag{2.28}$$

and

$$\pi_2^{A,A} - \pi_2^{N,N} = \frac{\alpha\beta^2\big(3t+(1-q)R\big)}{9t(9\omega-2\alpha\beta^2)}(1-q)R > 0. \tag{2.29}$$

It should be noted that, from (2.14), the pricing equilibrium without add-on fees results in $p_2^{N,N} = t-(1-q)R/3$. Thus, $(1-q)R < 3t$ is a necessary condition for both firms to sell to consumers when add-on pricing is not adopted or prohibited. □

Add-on Usage Dependent on Price The base consumers never use the add-on regardless of whether it is free or not. In practice, however, there might be scenarios where consumers who do not need a product or service asks for one if it is offered for free. In this section, we show that our results are robust to such consumer behavior. Consider a fraction, ρ, of the base consumers who will consume the add-on if it is provided for free and will not consume the add-on if $p_1 > 0$. Thus, we now have four consumer segments: sophisticated consumers, boundedly rational consumers, base consumers who never use add-ons regardless of whether it is free or not, and base consumers who will use add-ons only if it is free.

Firm i's profit can now be written as

$$\pi_i = (p_i - c)D_i + (\hat{p}_i - \hat{c})\hat{D}_i - \mathbf{1}_{\hat{p}_i=0} \cdot \rho(D_i - \hat{D}_i)\hat{c},$$

where $\mathbf{1}_{\hat{p}_i=0}$ is an indicator function that takes the value 1 if $\hat{p}_i = 0$ and 0 otherwise. Notice the term $\mathbf{1}_{\hat{p}_i=0} \cdot \rho(D_i - \hat{D}_i)\hat{c}$ captures the fact that a fraction ρ of the consumers who do not buy the add-on (which is equal to $D_i - \hat{D}_i$) will use the add-on if and only if $\hat{p}_i = 0$. Without add-on pricing, each firm's optimization problem $\max_{p_i} \pi_i|_{\hat{p}_i=0}$ is concave ($\partial^2\pi_i/\partial p_i^2 = -1/t < 0$), and simultaneously solving the first-order conditions leads to equilibrium prices of

$$p_1 = t + c + (\alpha + \rho - \alpha\rho)\hat{c} + \frac{1}{3}[\alpha(1-g)\hat{R} + (1-q)R],$$
$$p_2 = t + c + (\alpha + \rho - \alpha\rho)\hat{c} - \frac{1}{3}[\alpha(1-g)\hat{R} + (1-q)R].$$

The price effect due to the existence of price-dependent add-on usage is $\partial p_i/\partial\rho = (1 - \alpha)\hat{c} \geq 0$, i.e., by increasing and charging a higher price to consumers, the firms try to compensate the loss from the 'cherry-picking' consumers who use add-ons only if it is provided for free. Notice that for the case of $\hat{c} = 0$, the prices are identical with the result in the benchmark. Intuitively, if the firms do not incur any cost to produce or provide add-on then consumer's behavior on add-on good does not affect firm's profit, so their optimization problems remain unchanged.

When $g = 1$, equilibrium profits for the firms implied by these prices are

$$\pi_1^{N,N} = \frac{1}{18t}\left[(3t + (1-q)R)^2\right]; \qquad \pi_2^{N,N} = \frac{1}{18t}\left[(3t - (1-q)R)^2\right],$$

which give the same profits with the case of $\alpha = 0$, (see (2.15)). If we assume that α is common knowledge, each firm is able to set the base good price based on this information and indeed the firms' optimal prices are given as the function of α. Thus, the firms can easily attain the original equilibrium profits by adjusting the equilibrium prices and we don't have any other reason to have different level of equilibrium profits. Thus, we can conclude that Proposition 5 and Proposition 7 are trivially preserved with an introduction of 'cherry-picking' consumers.

Inattention to Add-on Price In the basic model, we assumed that myopic consumers mistakenly anticipate that they will not be charged for add-ons. However, as more firms adopt add-on pricing schemes, consumers are now well aware of the extra fees charged for an add-on. With such learning, one may argue that having boundedly rational consumers who think that any extra goods or services can be utilized for free is not a realistic assumption. To investigate this issue, the model needs to be modified to allow the possibility of different specifications on boundedly rational consumers. More specifically, let us assume consumers are rational and know they will be charged for the add-on, but do not know the add-on price before deciding to purchase the base good. As such, there are two consumer segments: $(1-\alpha)$ consumers for whom $\hat{R} = 0$ and α consumers for whom $\hat{R} > 0$.

Claim 1. *Consumers will expect that after they have committed to a firm by buying its base product, the add-on price will be at the maximum level.*

Proof. For any consumer belief, firms optimally charge their maximum possible add-on fees (i.e., $\hat{R}$ for firm 1 and $g\hat{R}$ for firm 2). If firms charge above this price, no consumers will buy the add-on in Stage 4. If firms charge below this price, then the add-on margin will be lower without an effect on sales. Thus, the only rational belief is that the add-on fees will be at their maximum. □

Claim 2. *Profit can be expressed as* $\pi_1 = (p_1 + \alpha\hat{R} - \alpha\hat{c} - c)(t - p_1 + p_2 + (1-q)R)/2t$ *and* $\pi_2 = (p_2 + \alpha g\hat{R} - \alpha\hat{c} - c)(t - p_2 + p_1 - (1-q)R)/2t$ *for superior and inferior firms, respectively.*

Claim 2 follows from the analysis of marginal base consumer and the marginal consumer who values the add-on and rationally expects $\hat{p}_1 = \hat{R}$ and $\hat{p}_2 = g\hat{R}$.

The first-order conditions are uniquely satisfied at $p_1^* = t - (3\alpha\hat{R} - (1-q)R)/3$ and $p_2^* = t - (3\alpha\hat{R} + (1-q)R)/3$ leading to equilibrium profits of

$$\pi_1^* = \frac{1}{18t}(3t + (1-q)R)^2, \tag{2.30}$$
$$\pi_2^* = \frac{1}{18t}(3t - (1-q)R)^2, \tag{2.31}$$

provided differentiation between firms is such that both firms compete for both consumer segments.

In comparison, when add-on pricing is not used, the superior and inferior firms maximizes

$$\pi_1 = \frac{1}{2t}\Big[p_1(t - p_1 + p_2 + (1-q)R) - \alpha(t - p_1 + p_2 + (1-q)R)\Big],$$
$$\pi_2 = \frac{1}{2t}\Big[p_2(t + p_1 - p_2 - (1-q)R) - \alpha(t + p_1 - p_2 - (1-q)R)\Big].$$

The first-order conditions are uniquely satisfied at $p_1^* = t + \alpha\hat{c} + c - (2\alpha\hat{R} + g\alpha\hat{R} - (1-q)R)/3$ and $p_2^* = t + \alpha\hat{c} + c - (\alpha\hat{R} + 2g\alpha\hat{R} - (1-q)R)/3$, leading to equilibrium profits of $\pi_1^* = (3t + \alpha(1-g)\hat{R} + (1-q)R)^2/18t$ and $\pi_2^* = (3t - \alpha(1-g)\hat{R} - (1-q)R)^2/18t$ provided differentiation between firms is such that both firms compete for both consumer segments.

The difference in profit from add-on pricing relative to when add-on pricing is not used are $\pi_1^* - \pi_1^N = -3\alpha\hat{R}(3t + (1-q)R) < 0$ and $\pi_2^* - \pi_2^N = -3\alpha\hat{R}(3t - (1-q)R) < 0$. Thus, add-on pricing decreases profits for both the superior firm and the inferior firm, and we can conclude that the qualitative results derived with the consumers who are ignorant of add-on fees are robust to the relaxation of this assumption.

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CHAPTER 3
CONSUMER SEARCH WITH PRICE COMPETITION

3.1 INTRODUCTION

Models of search costs routinely assume that consumers are completely uninformed about a firm's offer before they visited that firm. Only after incurring some search costs and visiting a firm does the consumer learn the prices that the firm charges and, in the case of differentiated products, how much it likes the product that the firm has on offer. Yet, in the real world, things are often different. With the advent of the Internet, for example, it becomes very easy to compare prices. There are many search engines that list all prices without requiring any effort on behalf of the consumer. Also, it is very easy to find some characteristics of a product online, without having to make a costly visit to the store. Yet, some product characteristics are impossible to digitize. A consumer that wants to buy a car can easily find all technical specifications and prices online, yet still has to make a costly visit to the dealer to be able to kick the tires and take it for a test drive. Similarly, the consumer that wants to buy a new jeans may find many details and pictures of this product online, which allows her to check whether she likes the design, but she still has to make a costly visit to the store in order to try them on.

In this paper, we try to model these cases. We consider products that feature two dimensions of horizontal product differentiation. One characteristic is observable for free, but the other requires a costly visit to the store. We refer to the first characteristic as observable, and to second as hidden. Prices are readily observable as well. We thus build on models of consumer search with differentiated products by adding a characteristics that can be observed for free, but by also assuming that prices can be observed at zero costs.

We find that this has profound implications. Rather than having prices increasing in search costs, which is true for all consumer search models that we are aware of, we find that prices *decrease* in search costs. The intuition is as follows. Different from most other search models, the order of search in our model is *not* random. Rational consumers will first visit the firm where they expect the best deal, that is the firm that offers the best combinations of price and observable characteristic for this consumer. As search costs increase, however, consumers are less likely to walk away from a firm that they have visited. Hence, as search costs increase, it becomes more important for a firm to attract consumers at their first search. The only way to do so is by charging a lower price.

This intuition suggests that it is crucial for our results that prices are readily observable. We show that this is indeed the case. If consumers have to search for prices as well as for the hidden characteristic, prices increase in search costs.

Our model builds on the literature on search with differentiated products, pioneered by Wolinsky (1986) and Anderson and Renault (1999).[1] Yet, different from those papers, we assume that firms are not visited at random. Other papers also drop that assumption. In Chen and He (2011) and Athey and Ellison (2011) firms pay for placing advertisements on a search engine. A firm has private information about its quality, that is, the probability that consumers like its product. In equilibrium, higher placed advertisements represent higher quality products and consumers rationally search firms in the order in which their ads are placed. In Arbatskaya (2007), the search order is exogenously given. Prices fall in the order of search: a consumer that walks away from a firm reveals that she has low search costs, giving the next firm an incentive to charge a lower price. Zhou (2011) studies a similar model with differentiated products but finds the opposite effects on price: a con-

[1] Weitzman (1979) is an earlier paper which studies the general optimal search among options with stochastic match values. But there is no supply side model.

sumer who walks away has fewer options left and is thus price sensitie. In Wilson (2010) a firm can choose the size of search cost consumers have to incur to visit it. Consumers are then more likely to first visit firms with low search cost, and prices fall in the order of search. Armstrong et al. (2009) study a search market with differentiated products where one firm is always visited first, while the other firms are sampled randomly if a consumer decides not to buy from the pioneered firm. Haan and Moraga-Gonzalez (2011) study a model of search with differentiated products where the order of search is affected by the amount of advertising.

In the literature on search in labor markets, recent papers have analyzed "directed search", where employers advertise their wages and workers choose their search order accordingly. An important additional feature in the labor market relative to a typical consumer market is that a job vacancy can be filled, so that workers do not necessarily visit the highest-wage employers first if they anticipate that many other workers will apply for the same post.[2]

The remainder of this paper is structured as follows. In the next section, we set up the model. We solve for the equilibrium in Section 3.3, and show that prices are decreasing in search costs. Section 3.4 considers the case in which prices are hidden as well. In that case, prices are increasing in search costs. Section 3.5 considers an extension in which consumers have to pay a small search costs to observe prices and the unobservable characteristic. We show that, if this cost is low enough, our main result still holds. Section 3.6 concludes.

3.2 THE MODEL

Consider a market where two firms sell horizontally differentiated products. Marginal costs are constant and normalized to zero. For simplicity, we assume that there is one

[2]See Rogerson et al. (2005, section 5) for discussion of these models.

consumer. She derives utility

$$u^i(p_i) = v + \epsilon_i + \eta_i - p_i,$$

if she buys product i at price p_i, $i = 1, 2$. The term v is the gross utility of buying the worst possible product. We assume that v is high enough, so the market is always covered. The term $\epsilon_i + \eta_i$ can be interpreted as the match value between this consumer and the product of firm i. This match value consists of two components: the observable component η_i reflects characteristics that can be observed without visiting firm i, the hidden component ϵ_i can only be observed upon visiting firm i. We assume that ϵ_i is the realization of a random variable that is uniformly distributed on $[0, 1]$, while η_i is the realization of a random variable that uniformly distributed on $[0, h]$, with h some parameter.

The consumer must incur search costs s if she want to visit a firm to learn the value of ϵ_i. We assume that the search process is without replacement and there is costless recall (i.e., the consumer can return to any firm she has visited without extra cost). We assume that $h < 1 - \sqrt{2s}$. As we will show in our analysis, this assumption assures that the observable characteristics η_i and η_j can never be such that the consumer knows in advance that she will buy from firm i for sure, regardless of the value of ϵ_i that she encounters. In other words, we assume that the hidden characteristic can always at least potentially influence the consumer's buying behavior. Hence, we require the observable characteristic to be more noisy than the hidden characteristic. Firms can never observe either ϵ_i or η_i, so practicing price discrimination is not feasible. Crucially, and different from most of the search literature, we assume that prices are observable as well. Having the observable characteristic then becomes crucial to assure pure strategy equilibria.[3]

For ease of exposition, we define Δ_η as the difference in the value of the observable

[3]Suppose that we would not have an observable characteristic. In a symmetric pure strategy equilibrium, both firms would charge the same price and attract half of consumers on their first visit. Yet firms strictly benefit from being visited first. Slightly undercutting the tentative equilibrium price would then lead to a discrete increase in first-visits and hence sales, which implies that a firm would benefit from such a defection, and hence a symmetric pure strategy equilibrium.

characteristic for both firms, and Δ_p as the difference in prices:

$$\Delta_\eta \equiv \eta_2 - \eta_1,$$
$$\Delta_p \equiv p_2 - p_1.$$

Also, we define Δ as the net advantage that firm 1 has over firm 2, so

$$\Delta \equiv \Delta_p - \Delta_\eta.$$

For the purpose of the analysis, it is also useful to define the probability density of Δ_η. Note that

$$\Gamma(z) \equiv \Pr(\Delta_\eta < z) = \int_0^{\max\{\min\{z+\eta_1,h\},0\}} \int_0^h dG(\eta_1)dG(\eta_2).$$

With a uniform distribution of G on $[0,h]$ this implies

$$\Gamma(z) = \begin{cases} \frac{1}{2h^2}(h+z)^2 & \text{if} \quad z \leq 0 \\ 1 - \frac{1}{2h^2}(h-z)^2 & \text{if} \quad z \geq 0 \end{cases}$$

which in turn implies

$$\gamma(z) = \begin{cases} \frac{1}{h^2}(h+z) & \text{if} \quad z \leq 0 \\ \frac{1}{h^2}(h-z) & \text{if} \quad z \geq 0 \end{cases}$$

3.3 ANALYSIS

Consider the best reply of firm 1 to some price p_2. Suppose that firm 1 chooses to set $p_1 < p_2$. A consumer first goes to firm 1 whenever it expects a better deal there, thus if

$$v + \eta_1 + \mathbb{E}[\epsilon_1] - p_1 > v + \eta_2 + \mathbb{E}[\epsilon_2] - p_2,$$

which simplifies to

$$\Delta > 0.$$

Total sales of firm 1 will consist of three groups of consumers. First, there will be consumers who have visited firm 1 first, and sufficiently like the hidden characteristic to buy from 1. Second, there are consumers that visit firm 2 first, but find the hidden characteristic at that firm so disappointing that they are willing to incur additional search costs s to go to firm 1 and buy there. Third, there are consumers that first visit firm 1, find the hidden characteristic at that firm so disappointing that they are willing to incur additional search costs s and go to firm 2, only to find out that the hidden characteristic at firm 2 is so adverse that they prefer to buy from firm 1 anyway. We will consider these three groups in turn.

Let $\omega \in \{1, 2, 12, 21\}$ denote which firms a particular consumer visits, and in what order. Thus $\omega = 12$ implies that the consumer has first visited firm 1, and then firm 2. Let q_i^ω denote total demand for firm i from such consumers. Thus q_1^{12} denotes demand for firm 1 from consumers that visit firm 1 and 2 in that order, while q_1^1 denotes demand for firm 1 from consumers that only visit firm 1. Similarly, we define D_i^ω as the set of consumers that buy from firm i after having visited the firms in ω in the given order.

We first consider consumers that visit firm 1 first and decide to buy there right away. Given that they prefer to come firm 1 first, these consumers have $\Delta > 0$. They will immediately buy if the expected gains from search are lower than search costs. The expected benefits of sampling one more firm equal

$$\int_{\epsilon_1+\Delta}^{1} (\epsilon - \epsilon_1 - \Delta) d\epsilon = \frac{[1 - (\epsilon_1 + \Delta)]^2}{2}. \tag{3.1}$$

A consumer stays at this firm if this is smaller than search costs

$$\frac{[1 - (\epsilon_1 + \Delta)]^2}{2} < s \tag{3.2}$$

or[4]

$$\epsilon_1 > 1 - \sqrt{2s} - \Delta. \tag{3.3}$$

Note that necessarily $\Delta_\eta > -h > -(1-\sqrt{2s})$, so for small deviations in price we indeed have that the right-hand side of this inequality is strictly positive, hence that there is some probability that the consumer will walk away. Also not that $\Delta > 0$ implies that the right-hand side is strictly smaller that 1, which implies that the crucial value of ϵ_1 is well-defined.

Let's define

$$c \equiv \sqrt{2s},$$

so the consumer buys from firm 1 if

$$\epsilon_1 > 1 - c - \Delta.$$

Define

$$D_1^1 \equiv \{\eta_1, \eta_2, \epsilon_1, \epsilon_2 : \Delta > 0 \text{ and } \epsilon_1 > 1 - \Delta - c\}.$$

The mass in this set is demand of group 1:

$$q_1^1 = \int_{D_1} d\Gamma(\Delta_\eta) dF(\epsilon_1) \tag{3.4}$$

so

$$q_1^1 = \int_{-h}^{\Delta_p} \int_{1-c-\Delta_p+\Delta_\eta}^{1} f(\epsilon_1)\gamma(\Delta_\eta) d\epsilon_1 d\Delta_\eta \tag{3.5}$$

As the probability density function of Δ_η has a kink at $\Delta_\eta = 0$, we have to split this

[4]Note that we implicitly assume that, for the cut-off consumer, $\epsilon_1 + \Delta < 1$ so $\epsilon_1 < 1 - \Delta$. Plugging in the cut-off value yields $1 - \sqrt{2s} - \Delta < 1 - \Delta$.

expression to obtain

$$\begin{aligned} q_1^1 &= \int_{-h}^{0}\int_{1-c-\Delta_p+\Delta_\eta}^{1}\left(\frac{1}{h^2}(h+\Delta_\eta)\right)d\epsilon_1 d\Delta_\eta + \int_{0}^{\Delta_p}\int_{1-c-\Delta_p+\Delta_\eta}^{1}\left(\frac{1}{h^2}(h-\Delta_\eta)\right)d\epsilon_1 d\Delta_\eta \\ &= \frac{1}{2}c + \frac{1}{6}h + \frac{1}{2}\Delta_p - \frac{1}{6h^2}\Delta_p(3c\Delta_p - 3h\Delta_p + \Delta_p^2 - 6ch). \end{aligned}$$

Now consider the consumers that go to the other firm first, walk away, then buy from firm 1. A consumer goes first to firm 2 if $\Delta < 0$. Similar to the analysis above, she decided not to buy right away if $\epsilon_2 < 1 + \Delta - c$. After having visited firm 1 as well, this consumer has observed all relevant parameters, and hence decides to buy from firm 1 as if $\epsilon_2 < \Delta + \epsilon_1$. We thus have

$$D_1^{21} \equiv \{\eta_1, \eta_2, \epsilon_1, \epsilon_2 : \Delta < 0 \text{ and } \epsilon_2 < 1 + \Delta - c \text{ and } \epsilon_2 < \Delta + \epsilon_1\}. \tag{3.6}$$

For analytic convenience, it is useful to partition this set into two subsets, depending on whether or not $\epsilon_1 < 1 - c$:

$$D_1^{21} = D_{1a}^{21} \cup D_{1b}^{21}$$

with

$$\begin{aligned} D_{1a}^{21} &\equiv \{\eta_1, \eta_2, \epsilon_1, \epsilon_2 : \Delta < 0 \text{ and } \epsilon_1 < 1 - c \text{ and } \epsilon_2 < \Delta + \epsilon_1\} \\ D_{1b}^{21} &\equiv \{\eta_1, \eta_2, \epsilon_1, \epsilon_2 : \Delta < 0 \text{ and } \epsilon_1 \geq 1 - c \text{ and } \epsilon_2 < 1 + \Delta - c\} \end{aligned}$$

Total demand from these consumers equals $q_1^{21} = q_{1a}^{21} + q_{1b}^{21}$, with

$$\begin{aligned} q_{1a}^{21} &= \int_{\Delta_p}^{h}\int_{-\Delta}^{1-c}\int_{0}^{\Delta+\epsilon_1} dF(\epsilon_2)dF(\epsilon_1)d\Gamma(\Delta_\eta) \\ &= \int_{\Delta_p}^{h}\int_{-\Delta_p+\Delta_\eta}^{1-c}\int_{0}^{\Delta_p-\Delta_\eta+\epsilon_1}\left(\frac{1}{h^2}(h-\Delta_\eta)\right)d\epsilon_2 d\epsilon_1 d\Delta_\eta \\ &= \frac{1}{24h^2}(h-\Delta_p)^2(6c^2 + 4ch - 4c\Delta_p - 12c + h^2 - 2h\Delta_p - 4h + \Delta_p^2 + 4\Delta_p + 6) \end{aligned}$$

and

$$q_{1b}^{21} = \int_{\Delta_p}^{h} \int_{1-c}^{1} \int_{0}^{1+\Delta_p-\Delta_\eta-c} \left(\frac{1}{h^2}(h-\Delta_\eta) \right) d\epsilon_2 d\epsilon_1 d\Delta_\eta$$
$$= \frac{1}{6}\frac{c}{h^2}(h-\Delta_p)^2 \left(3(1-c)+\Delta_p-h\right).$$

Finally, there will be consumers that start out at firm 1, then walk away, but do come back. For these people, to have them start out at firm 1, we require $\Delta > 0$. For them to then walk away requires $\epsilon_1 < 1-\Delta-c$. For them to prefer firm 1 anyway, after having observed the offer of firm 2, requires $\epsilon_2 < \Delta+\epsilon_1$. Hence

$$D_1^{12} = \{\eta_1, \eta_2, \epsilon_1, \epsilon_2 : \Delta > 0 \text{ and } \epsilon_1 < 1-\Delta-c \text{ and } \epsilon_2 < \Delta+\epsilon_1\}.$$

We thus have

$$q_1^{12} = \int_{-h}^{\Delta_p} \int_{0}^{1-\Delta_p+\Delta_\eta-c} \int_{0}^{\Delta_p-\Delta_\eta+\epsilon_1} dF(\epsilon_2)dF(\epsilon_1)d\Gamma(\Delta_\eta)$$

As the probability density function of Δ_η has a kink at $\Delta_\eta = 0$, we have to split this expression to obtain

$$q_1^{12} = \int_{-h}^{0} \int_{0}^{1-\Delta_p+\Delta_\eta-c} \int_{0}^{\Delta_p-\Delta_\eta+\epsilon_1} \left(\frac{1}{h^2}(h+\Delta_\eta) \right) d\epsilon_2 d\epsilon_1 d\Delta_\eta$$
$$+ \int_{0}^{\Delta_p} \int_{0}^{1-\Delta_p-\Delta_\eta-c} \int_{0}^{\Delta_p-\Delta_\eta+\epsilon_1} \left(\frac{1}{h^2}(h-\Delta_\eta) \right) d\epsilon_2 d\epsilon_1 d\Delta_\eta$$
$$= \frac{1}{4}(1-c)^2 - \frac{1}{24}h^2 - \frac{1}{6}h\Delta_p - \frac{1}{4}\Delta_p^2 + \frac{1}{24h^2}\left(6(1-c)^2(2h-\Delta_p)+\Delta_p^2(\Delta_p-4h)\right).$$

Total demand for firm 1 consists of consumers that visit firm 1 first and buy immediately, those that visit firm 2 first, then walk away and buy from firm 1, and consumers that visit firm 1 first, then go to firm 2, but end up buying from firm 1 anyway. Thus

$$q_1 = q_1^1 + q_1^{21} + q_1^{12}.$$

Adding these expression yields

$$q_1 = \frac{1}{3}\Delta_p(3-h) - \frac{1}{3h}\Delta_p^3 + \frac{1}{12h^2}\Delta_p^4 + \frac{c^2}{h}\Delta_p - \frac{c^2}{2h^2}\Delta_p^2 + \frac{1}{2}.$$

Profits equal

$$\Pi_1 = p_1 q_1.$$

Taking the first-order condition and imposing symmetry, we have

$$\frac{1}{3}hp^* - p^* - \frac{c^2}{h}p^* + \frac{1}{2} = 0$$

which yields equilibrium price

$$p^* = \frac{3h}{6c^2 + 6h - 2h^2}. \tag{3.7}$$

From this, we immediately have

Proposition 9. *When prices are observable, equilibrium prices are strictly decreasing in the search costs of the hidden characteristic. In other words,*

$$\frac{\partial p^*}{\partial c} < 0. \tag{3.8}$$

The intuition is as follows. Rational consumers will first visit the firm where they expect the better deal, that is the firm that offers, for this consumer, the best combination of price and observable characteristic. As search costs increase, however, consumers are less likely to walk away from a firm that they have visited. Hence, as search costs increase, it becomes more important for a firm to attract consumers at their first search. The only way to do so is by charging a lower price.

To determine the effect of a change in h, note that

$$\frac{\partial p^*}{\partial h} = \frac{3(3c^2 + h^2)}{2(h^2 - 3c^2 - 3h)^2} > 0.$$

Thus, as the hidden characteristic becomes more important, prices are higher.

3.4 THE CASE OF HIDDEN PRICES

Suppose that we are in a world with one observable and one hidden characteristic, as we had above, but where prices are also hidden. Hence, in this case, to learn the price consumers also have to fist visit a firm. The decision which firm to visit first now only depends on the observable characteristic: a consumer will visit firm 1 first if $\Delta_\eta \equiv \eta_2 - \eta_1 < 0$. We look for the equilibrium price p^* that is such that, if firm 2 charges p^*, it is a best reply for firm 1 to do the same.

As in the standard search literature, we impose that consumers have "passive beliefs" about the equilibrium price, and the price p a consumer is offered does not alter her anticipated p^*. This restriction implies that a consumer's search and purchase behavior can be described by an optimal stopping rule: she buys the current product if the utility she obtains is more than or equal to a threshold value, and continues searching otherwise.

When the consumer faces the decision whether to buy from firm 1 immediately, however, she faces a decision similar to that above: she will walk away if she expects that also visiting firm 2 is not worth her while, hence if

$$\epsilon_1 > 1 - \Delta - c,$$

where again $\Delta \equiv \Delta_p - \Delta_\eta$, but now $\Delta_p \equiv p^* - p_1$. Hence Δ_p is not the true difference between firm 1 and firm 2, but the price difference that the consumer expects after having observed p_1 and assuming that firm 2 charges p^*. Using the same notation as in the previous section, we now have

$$D_1^1 \equiv \{\eta_1, \eta_2, \epsilon_1, \epsilon_2 : \Delta_\eta < 0 \text{ and } \epsilon_1 > 1 - \Delta - c\}.$$

This implies

$$\begin{aligned} q_1^1 &= \int_{-h}^{0} \int_{1-c-\Delta_p+\Delta+\eta}^{1} dF(\epsilon_1) d\Gamma(\Delta_\eta) \\ &= \int_{-h}^{0} \int_{1-c-\Delta_p+\Delta+\eta}^{1} \left(\frac{1}{h^2}(h + \Delta_\eta) \right) d\epsilon_1 d\Delta_\eta \\ &= \frac{1}{2}c + \frac{1}{6}h + \frac{1}{2}\Delta_p. \end{aligned}$$

Consider consumers that visit firm 2 first. These consumers have $\Delta_\eta > 0$. They decide not to buy at firm 2 if $\epsilon_2 < 1 - \Delta_\eta - c$, as these consumers assume that firm 1 also charges the equilibrium price. They buy from firm 1 if it turns out that $\epsilon_2 < \epsilon_1 + \Delta$. Hence

$$D_1^{21} \equiv \{\eta_1, \eta_2, \epsilon_1, \epsilon_2 : \Delta_\eta > 0 \text{ and } \epsilon_2 < 1 + \Delta_\eta - c \text{ and } \epsilon_2 < \Delta + \epsilon_1\}.$$

It is convenient to partition this set into two subsets, depending on whether or not $\epsilon_1 < 1 - c - \Delta_p$:

$$D_1^{21} = D_{1a}^{21} \cup D_{1b}^{21}$$

with

$$\begin{aligned} D_{1a}^{21} &\equiv \{\eta_1, \eta_2, \epsilon_1, \epsilon_2 : \Delta_\eta > 0 \text{ and } \epsilon_1 < 1 - c - \Delta_p \text{ and } \epsilon_2 < \Delta + \epsilon_1\}, \\ D_{1b}^{21} &\equiv \{\eta_1, \eta_2, \epsilon_1, \epsilon_2 : \Delta_\eta > 0 \text{ and } \epsilon_1 \geq 1 - c - \Delta_p \text{ and } \epsilon_2 < 1 - \Delta_\eta - c\}. \end{aligned}$$

Total demand from these consumers equals $q_1^{21} = q_{1a}^{21} + q_{1b}^{21}$, with

$$
\begin{aligned}
q_{1a}^{21} &= \int_0^h \int_{-\Delta}^{1-c-\Delta_p} \int_0^{\Delta+\epsilon_1} dF(\epsilon_2)dF(\epsilon_1)d\Gamma(\Delta_\eta) \\
&= \int_0^h \int_{-\Delta_p+\Delta_\eta}^{1-c-\Delta_p} \int_0^{\Delta_p-\Delta_\eta+\epsilon_1} \left(\frac{1}{h^2}(h-\Delta_\eta)\right) d\epsilon_2 d\epsilon_1 d\Delta_\eta \\
&= \frac{1}{4}c^2 + \frac{1}{6}ch - \frac{1}{2}c + \frac{1}{24}h^2 - \frac{1}{6}h + \frac{1}{4}
\end{aligned}
$$

and

$$
\begin{aligned}
q_{1b}^{21} &= \int_0^h \int_{1-c-\Delta_p}^{1} \int_0^{1-\Delta_\eta-c} \left(\frac{1}{h^2}(h-\Delta_\eta)\right) d\epsilon_2 d\epsilon_1 d\Delta_\eta \\
&= \frac{1}{6}(c+\Delta_p)(3(1-c)-h).
\end{aligned}
$$

Finally, there will be consumers that start out at firm 1, then walk away, but do come back. For these people, to have them start out at firm 1, we require $\Delta_\eta < 0$. For them to then walk away requires $\epsilon_1 < 1 - \Delta - c$. For them to prefer firm 1 anyway, after having observed the offer of firm 2, requires $\epsilon_2 < \Delta + \epsilon_1$. Hence

$$
D_1^{12} = \{\eta_1, \eta_2, \epsilon_1, \epsilon_2 : \Delta_\eta < 0 \text{ and } \epsilon_1 < 1 - \Delta - c \text{ and } \epsilon_2 < \Delta + \epsilon_1\}.
$$

We thus have

$$
\begin{aligned}
q_1^{12} &= \int_{-h}^0 \int_0^{1-\Delta_p+\Delta_\eta-c} \int_0^{\Delta_p-\Delta_\eta+\epsilon_1} dF(\epsilon_2)dF(\epsilon_1)d\Gamma(\Delta_\eta) \\
&= \int_{-h}^0 \int_0^{1-\Delta_p+\Delta_\eta-c} \int_0^{\Delta_p-\Delta_\eta+\epsilon_1} \left(\frac{1}{h^2}(h+\Delta_\eta)\right) d\epsilon_2 d\epsilon_1 d\Delta_\eta \\
&= \frac{1}{4}c^2 - \frac{1}{2}c - \frac{1}{24}h^2 - \frac{1}{6}h\Delta_p - \frac{1}{4}\Delta_p^2 + \frac{1}{4}
\end{aligned}
$$

Total demand again equals

$$
q_1 = q_1^1 + q_1^{21} + q_1^{12}
$$

Adding these expressions yields

$$q_1 = \Delta_p - \frac{1}{2}c\Delta_p - \frac{1}{3}h\Delta_p - \frac{1}{4}\Delta_p^2 + \frac{1}{2}$$

Profits equal

$$\Pi_1 = p_1 q_1.$$

Taking the first-order condition and imposing symmetry yields:

$$\frac{1}{2} - p^* \left(1 - \frac{1}{2}c - \frac{1}{3}h\right) = 0,$$

so

$$p^* = \frac{3}{6 - 3c - 2h}. \tag{3.9}$$

Proposition 10. *When prices are hidden, equilibrium prices are strictly increasing in search costs. That is,*

$$\frac{\partial p^*}{\partial c} > 0. \tag{3.10}$$

Note that this is in line with the intuition we gave for our main result. With observable prices, an increase in search costs implies that firms are more eager to attract consumers on their first visit, and hence to lower prices. Once prices are also hidden, as we consider here, this effect is no longer present and we are back to the standard result that prices increase in search costs.

The effect of a change in h is qualitatively the same as that in the case of observable prices: as the hidden characteristic becomes more important, prices increase.

3.5 EXTENSION

So far, we assumed that although hidden characteristics are costly to observe, information about prices and observable characteristics come for free. Of course, this is a simplification: in the real world there may also be some costs involved from observing these, although they are relatively small compared to the costs of observing the hidden characteristics. Still, ever since Diamond (1971) it is well known that even infinitesimally small costs may fundamentally change the nature of equilibrium. In this section we argue that in our model, that is not the case.

Suppose that it is also costly to observe price and 'observable' characteristic, and these cost equal s' per firm, where $s' \ll s$. An individual consumer now picks one firm at random, and observes price and the observable characteristic at a price s'. She now has two options: She either also inspects the hidden characteristic of firm 1, or she first inspects price and the observable characteristic at firm 2, such that she can decide on the basis of that which firm's hidden characteristic to observe. In the latter case, this consumer thus uses the approach described in this paper.

In the case that $s' = 0$, it is easy to see that this consumer strictly prefers to first do a check of the observable characteristics at firm 2. By obtaining this information, she will be able to make a better choice regarding which firm's hidden characteristic to inspect first, and hence she will be less likely to have to make additional costly searches, and hence also be likely to find a better match on average.

But if this is true for the case that $s' = 0$, it will also be true for very small s'. Also if prices and observable characteristics of each firm are costly to observe, it will still be worth her while for her to do so, rather than to go look for the hidden characteristic of a firm right away. In turn that implies that also for small s', our analysis still holds true in equilibrium, and hence we still have that equilibrium price are decreasing in search costs of the hidden characteristic.

3.6 CONCLUSION

In this paper we considered a model in which consumers can readily observe prices and some product characteristics, but have to incur search costs to be able to observe other crucial product characteristics. In such an environment, we showed that an increase in search costs decrease equilibrium prices, as higher search costs imply that consumers are less likely to walk away from a firm, which implies that firms are more eager to attract them. We also showed that this result is still valid if there are small search costs involved to observe both the price and the product characteristics that we assume to be readily observable.

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