PRESERVICE TEACHERS' USES OF THE INTERNET TO SUPPORT THEIR LEARNING OF MATHEMATICS: THE CASE OF THE PYTHAGOREAN THEOREM

By

Aaron Brakoniecki

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ABSTRACT

PRESERVICE TEACHERS' USES OF THE INTERNET TO SUPPORT THEIR LEARNING OF MATHEMATICS: THE CASE OF THE PYTHAGOREAN THEOREM

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The Internet is a go-to resource for many students, both for academic and recreational purposes. This dissertation explores how preservice elementary teachers use the Internet to support their learning of mathematics. Set in the context of needing to get students to explain a proof of the Pythagorean Theorem and its converse, this study asks preservice teachers to use the web in order to further develop their own understanding of this mathematics. This study seeks to investigate the kinds and quality of the mathematical connections that preservice teachers make after investigating the Pythagorean Theorem online. It also examines the information seeking strategies employed by the preservice teachers while completing their investigations. Lastly this study explores possible connections between the kinds and quality of the connections exhibited by the preservice teachers and the information-seeking strategies employed by the preservice teachers during their investigation.

To my family for their infinite love and support,
my teachers for their infinite wisdom and patience,
and my friends for their infinite kindness and loyalty;
I am thankful these groups are not mutually exclusive.

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There are many people who have had profound influence on my thinking and writing leading up to, and during this dissertation. It is impossible to name them all, but a few deserve special recognition for their work in helping me grow as a researchers and a scholar.

My dissertation co-director and guidance committee chair Dr. Sandra Crespo is responsible for taking this enthusiastic and naïve young graduate student and helping me become a thoughtful and critical researcher and mathematics educator. Saying “I couldn’t have done this without her” is perhaps the biggest understatement I could ever make. I am forever indebted to her for her wisdom, patience, guidance, and love of coffee.

My dissertation was also co-directed by Dr. Ralph Putnam. From my first year in the mathematics education pro-seminar to these final years working on this dissertation (with an independent study in between), my thinking pertaining to the use of technology in mathematics education has become more layered and nuanced because of what I have learned from his expertise. My research has been forever improved because of his work with me.

The other members of my dissertation committee also had their work cut out for them when they agreed to work with me, but they were incredibly successful in improving my work. Dr. Brin Keller is responsible for helping me think through the variety of complex interaction between learners, mathematics, and technology. Dr. Patrick Dickson provided guidance and experience as I situated this study in the context of learning online and outside of the classroom setting. All four of these individuals working together not only made my dissertation better, but also made me a better researcher, and I am forever grateful.

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Lastly, I need to acknowledge my parents (Al and Kris) and brother (Adam) for their support during graduate school. While everyone else listed in these acknowledgements is mostly responsible for supporting my scholarship and my researcher, these three people are responsible for supporting me. I am especially grateful that they didn't ask too many questions annually when I kept hinting I would graduate, ultimately to return for "one more" year. This time, I promise, it's for real.

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CHAPTER 1

Recent research on the preparation of future teachers of mathematics has focused on issues related to the experiences preservice teachers have as part of their teacher preparation programs both in their content and methods courses, and their school field experiences (Cochran-Smith, Feiman-Nemser, McIntyre, & Demers, 2008; Sowder, 2007). Although this has generated important research, the emphasis of learning inside the program has overshadowed opportunities for examining the learning of mathematics that occurs outside of the programs. The learning of mathematics by preservice teachers outside of coursework is an important issue to consider given the limited time teacher preparation programs have with their preservice teachers. Already, the Internet is a resource used by many college students for both academic and non-academic purposes (Wilber, 2008). To this end, the Internet is an emerging resource to support the teaching and learning of mathematics for preservice teachers both inside and outside the mathematics classroom. The use of this resource to learn mathematics is the focus of this dissertation.

Purpose

Teacher preparation programs consist of both coursework and field experiences. They include both content courses and methods courses for the preservice teachers enrolled in the program. The time spent on content and methods in these classes is limited, and cannot prepare teachers for every piece of content, or mathematical scenario they will encounter or be responsible for teaching during their time in schools. In elementary teacher preparation courses, the focus on content is especially limited, as preservice teachers must focus on multiple content areas. It should thus be expected that elementary mathematics teachers may often encounter situations in their classrooms where they are not familiar with the mathematics content that occurs as part of a changing curriculum or that appears based on student inquiries. This poses questions about the approaches used by teachers when they encounter such scenarios. What are the possible strategies employed by preservice teachers when they need to better understand mathematics outside of their content classes?

A possible answer to this question is that they might use the Internet as a learning resource. Most American households now have Internet access (U.S. Census Bureau, 2013) and use the Internet in many difference aspects of their life every day. Users can go online to interact with businesses, government services, communicate in different formats, and receive multiple forms of entertainment. People go to the Internet for its information and services. The characteristic of the Internet as reference is also being used more frequently by students as a tool to help with their studies (Wilber, 2008). As the Internet becomes a more prominent tool in education, it is important to understand the ways in which it is used to support academic learning.

The Internet is filled with diverse content from a variety of sources in a number of different forms. There are numerous ways that mathematical information can be presented to users via the Internet. Oftentimes, a traditional, static presentation (similar to the way it is found in textbooks) has been used to present mathematics content. There are also new, dynamic presentations available (e.g., through animations and web applets) that allow users to see and interact with the content. The communicative features of the Internet can also be utilized for presenting mathematics, which can be seen in YouTube videos, discussion forums, and web blogs.

One of the difficulties that this new resource presents to its users is that it is not always related to or compatible with the curriculum of the classroom. Oftentimes, users have to search in various locations to try to find the information relevant to the mathematical situation they are trying to explore (which is often in a form different from the more structured presentation of relevant information provided by teachers or texts). This ill-defined structure (Spiro & Jehng, 1990) has unique challenges that require information-seeking skills beyond what is required with more traditional informational media used by students (e.g., documents, texts).

Opportunities and Challenges Present with the Internet

The sophistication and omnipresence of the Internet suggest that this venue is an important tool to investigate how it may support the mathematical learning of preservice and in-service teachers. When placed in scenarios where learners are solely responsible for their own understanding of mathematics content, the Internet becomes a valuable resource for solo learners. Not only is there a variety of mathematical content contained within the vast span of the Internet that can cover most any inquiry posed by learners, but that content is also presented in numerous different formats, allowing diverse learners, all with unique learning styles, to not only locate relevant information, but to locate it in a format they prefer (Kuiper & Volman, 2008). Additionally, the ability of the Internet to allow users to communicate and receive feedback with people, groups, and organizations beyond the user's immediate geographic area can also add an interactive aspect to this "solo" learning environment. Physical location is no longer a constraint to collaborative learning.

Yet given these opportunities offered by the Internet, there are also challenges present within this environment. Unique to the Internet is that the burden of determining the quality of a source of information is often left to the learner, whereas in more traditional environments, often the quality of a resource is determined by an instructor before being utilized within a classroom. Users are not often able, or even aware, of a need to check the quality and reliability of the information located (Kafai & Bates, 1997; Schacter, Chung, & Dorr, 1998). Additionally, the location of information online often requires the use of search engines, which necessitate using particular techniques and syntax to use effectively. Users have different skills and techniques for utilizing search engines, producing different kinds of resources within their searching (Fidel et al., 1999; Large & Beheshti, 2000). Also, given the open nature of the Internet, users continuously reflect on their own understanding, controlling their own pace through new content, and also influencing what they know to investigate further (Fidel et al., 1999; Hirsh, 1999; Lawless & Schrader, 2008). In more traditional environments, an instructor would determine the pace of content and help students reflect on their understanding. These are just a few of the demands placed on users of the Internet not usually associated with more traditional resources.

The opportunities and challenges present when one uses the Internet as a resource (especially the opportunities present beyond those of more traditional resources) raises questions about the use of the Internet to support preservice and in-service teachers. We might ask about the contexts under which preservice and in-service teachers go to the Internet for support with content. We might try to better understand the kinds of information they are looking for when they go online. We might investigate how they navigate this new environment for the purposes of their mathematical understanding. Additionally, we may ask about how these preservice teachers learn mathematics from their experiences online, then take that new knowledge and understanding back into other contexts where they can then utilize what they learned.

Goals of This Dissertation

This dissertation will begin to address a small fraction of this large collection of issues. The Internet has the potential to serve as a multipurpose resource to support the lifelong learning of teachers. However, it is not yet clear how preservice teachers might be using this widely available tool as a resource to support their learning of unfamiliar mathematics and mathematics for teaching. It is important to study the ways the Internet is currently being used to learn mathematics as it may suggest strategies that can be leveraged to support learners both inside and outside of the classroom. Additionally, it may also be possible to design experiences that could make using the resources of the Internet more effective for the learning of mathematics. This is what this dissertation begins to investigate.

This research is important as it seeks to better understand the relationships between an evolving educational resource and preservice teachers who frequently use the resource for both academic and non-academic purposes. In particular, it is important to better understand the academic ways this common tool is being used to support subject specific learning (in this case, mathematics). While many people become skilled at exploring the Internet outside of academic purposes, it is not clear how applicable those skills are for academic purposes, nor if users are able to adeptly transfer the skills to new environment for new purposes.

The characteristics unique to the Internet, including its hyperlinked environment, its dynamic and interactive features, and the ability to communicate among diverse communities are new features beyond those that exist within traditional mathematics classrooms and learning environments. Because of these new factors, there could be differences in what is learned from a newer environment like the Internet, and a traditional environment like a classroom or a textbook. It is not clear how some of the big ideas of the mathematics classroom (e.g., reasoning and proving, problem solving) might be supported on in these digital environments. This dissertation seeks to understand the ways in which mathematics is understood when preservice teachers use the Internet to increase their understanding of a mathematical topic.

The Internet is a resource that mathematics educators have at their disposal to support the teaching and learning of mathematics. The availability and presence of the Internet outside of the classroom make it a particularly important and potentially powerful resource that could be harnessed for supporting mathematics learning. Understanding the mathematics that learners are able to understand from engaging with these environments, and the ways they went about exploring these environments to produce that learning would provide insights for teacher educators into how this tool might be leveraged for further support among learners and teachers of mathematics.

CHAPTER 2

This dissertation seeks to investigate the ways in which Internet searching may be a productive means for learning about mathematics. This chapter looks at what is already known about these individual areas, and other related areas related to this study. As one of the key parts of this study involves understanding the ways in which preservice teachers search for information online, the first literature reviewed relates to Internet searching and information seeking strategies of Internet searchers. This study also attempts to better understand how preservice teachers may be learning mathematics via Internet searching. Therefore, the next section of literature reviews frameworks for understanding and researching mathematics content knowledge, and how that knowledge can be represented via concept maps. The last main portion of literature centers on the content that participants will be investigating, the Pythagorean Theorem and its associated proofs. The chapter concludes with a presentation of the three research questions this dissertation addresses.

Information Seeking Online

With the emergence of the Internet as a common phenomenon in peoples' everyday lives, research on how people use the Internet to find information is becoming more prominent. In general, when users go to the Internet to find information, their searches have been described along two different lines: finding or fact-based tasks, and searching or research tasks (Bilal, 2000; Schacter et al., 1998). Finding tasks tend to be situations where a user is looking for a specific piece of information, such as identifying show times for a movie at a local theatre, determining the location of a restaurant, or locating capitals of different countries. Searching tasks, in contrast, are situations where a user needs to explore multiple aspects of a topic in order to identify useful information for their own purposes, such as identifying a mysterious noise your car is making, or looking for information about a new congressional bill. Research reports mixed results on students' success with these types of searches (Bilal, 2001; Schacter et al., 1998).

Research on online learning is clear that users have many challenges with Internet searching tasks. There is agreement that successful searchers tend to have some prior knowledge about the topic that they are investigating (Fidel et al., 1999; Hirsh, 1999; Lawless & Schrader, 2008), making it easier to formulate relevant searches and assess the information that emerges. Students often have difficulties, however, locating relevant information on the web, in assessing the relevance of information, and in exploring the resources of the web beyond the narrow scope of their investigation task (Kuiper, Volman, & Terwel, 2005). One of the most consistent findings reported of Internet searchers is their failure to critically asses the quality of the information presented, and the source of that information (Kafai & Bates, 1997; Large & Beheshti, 2000; Lorenzen, 2001).

There are several different approaches used to quantify and describe the ways that Internet searchers look for information. Some have focused on the initial investigative approaches employed by the searchers (Schacter et al., 1998); others have expanded upon this to look at the patterns of behavior over the course of the investigation (Bilal, 2000, 2001, 2002). Two frameworks in particular have isolated different aspects of the searching process. Juvina and Oostendorp's (2004) framework focuses on the navigation between sources of information from either web searching or web exploration. A second framework (Lawless & Schrader, 2008) narrows in on the skill of locating information within individual resources and how users investigate within a site. These two approaches—locating sites and investigating a site—provided this study a useful way to unpack the complex process of information seeking online. I come back to these two frames in Chapter 4 where I describe how I adapted them for this study.

Frameworks of Knowledge

Mathematics education has a long history of describing ways that learners come to know mathematics. Many of these frameworks attempt to describe deeper and more richly connected understandings of mathematics (Brownell, 1938, 1947; Hiebert & Lefevre, 1986; Skemp, 1976). This deeper understanding of mathematics extends beyond students and has also been found in teachers (Ma, 1999). All these frameworks attempt to describe ways of knowing mathematics beyond just the utility of what to do in mathematical scenarios—to understanding why particular mathematics makes sense in mathematical problems.

There has also been much work in mathematics education to describe not only the mathematical knowledge possessed by students, but also the mathematical knowledge possessed by teachers and how it interacts with teachers' pedagogical knowledge (Ball, Hill, & Bass, 2005; Ball & Hill, 2009; Ball, 2000; Hill, Ball, & Schilling, 2008; Shulman, 1986, 1987). This intersection between content and pedagogy has been expanded to include the knowledge of teaching with technology (Koehler & Mishra, 2009; Mishra & Koehler, 2006). What all these frames, whether pertaining to student or teacher, emphasize is that to effectively do something like mathematics or the teaching of content, one needs knowledge that is richly understood in different ways.

Star (2005) unpacked an underlying issue that existed in many frameworks of mathematical knowledge at the time. Star argued that instead of using the terms *procedural* and *conceptual* to describe two aspects of a single phenomenon, the research literature had instead been overlapping two different phenomena into these labels. Star unpacked the ideas of the kind of knowledge, commonly referred to in the literature as procedural or conceptual, from the quality of that knowledge, which he described as rich versus superficial (Star, 2005). Star argued that in the literature, conceptual knowledge was often used to mean richly connected knowledge, and procedural knowledge was often used to indicate superficial knowledge. Instead he argued, conceptual knowledge (knowledge of a concept) could be richly connected or superficial. Similarly, knowledge of a procedure could be richly connected or superficial. This frame is useful for unpacking both the kind and quality of mathematics understanding possessed by preservice teachers. This framework and how it is used in this study is further discussed in Chapter 5.

Concept Mapping

Concept mapping is a tool used to express connections between concepts via diagram. In a diagram, concepts are connected by words or phrases describing why a connection exists between these two concepts. There can be multiple links to a concept from other concepts, and multiple links emerging from a single concept to other concepts. These concept mappings can vary in size, depth, and detail. The construction of a concept map gives its creator an opportunity to express how they are making connections and interpreting the concepts of a topic. Concept maps have been shown to be an effective way to capture student understanding of content in mathematics (Baroody & Bartels, 2000). Concept maps have also been used to track changes in understanding of mathematics teachers (Hough, O'Rode, Terman, & Weissglass, 2007). Much of the research on concept maps looks at the structures that are used in the concept maps and propose different ways of quantifying these concept maps (Ruiz-Primo, Schultz, Li, & Shavelson, 2001; Wallace & Mintzes, 1990).

Ruiz-Primo, Schultz, Li, and Shavelson (2001) looked at several different kinds of concept maps and how students constructed them. Concept maps varied from having all nodes filled out, leaving students to fill out the connections: having all the connections filled out, leaving students to fill out the nodes; or providing a word bank and letting students construct concept maps from scratch. Providing a word bank resulted in the best representation of students' knowledge structures as compared to responses to multiple-choice assessments given to the same students. This approach for creating concept maps was adapted for this study, as described in Chapter 5.

CCSSM and The Pythagorean Theorem

The Common Core State Standards for Mathematics are the new National Standards that most states will soon begin following. These standards have put forth a list of topics by grade level that will guide what mathematics teachers teach and when (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In the eighth grade level of mathematics, the Pythagorean Theorem is discussed under the content of geometry. Here the authors of the standards focus on two aspects of the Pythagorean Theorem: (a) the proof (and explanation) of the Pythagorean theorem, and (b) applications of the Pythagorean Theorem. The three standards as they appear in the Common Core State Standards are:

- Understand and apply the Pythagorean Theorem.
    - 8.G.6. Explain a proof of the Pythagorean Theorem and its converse.
    - 8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
    - 8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

In a brief description of these standards in the introduction section, the authors of the CCSSM the authors write that in 8th grade:

> Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 52)

Of the three standards, this dissertation focuses only on the first—the meaning of the theorem and explanation of why it works. Preservice teachers will have encountered the Pythagorean Theorem several times previously as students and as teacher preparation students within the mathematics content courses for elementary teachers. Such familiarity of content has been shown to play a role in the success of information queries in online searching (Fidel et al., 1999; Hirsh, 1999). Additionally, this standard contains both content that preservice teachers are likely to be familiar with (the Pythagorean Theorem) as well as content that is likely to be unfamiliar to them (the converse to the theorem), making it well suited for further investigation.

Two textbooks were analyzed for their content related to Pythagorean Theorem: the text used in the content course taken by elementary mathematics majors (Beckmann, 2011) and a common middle-school mathematics curriculum (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2009). This analysis provided a scope of the mathematics content around the Pythagorean Theorem, as well as comparing the content within the context of what teachers might be prepared to teach via the content course text to what they might be required to teach via the middle school text. This analysis revealed little overlap between the content of these two texts. Much of the mathematics content around the Pythagorean Theorem and its proof contained in the middle-school curriculum was not contained in the text used by preservice teachers. This suggests that teachers will need to find a way to be familiar with this content beyond their preparation programs and beyond the curriculum materials they use in and for teaching.

Purpose Revisited

As described above, teacher preparation programs cannot prepare future mathematics teachers for every piece of content they will encounter over their career given the limited nature of the program. It is thus important to study how preservice teachers might be learning mathematics outside their coursework, especially considering that teachers may be asked to teach mathematics that they are unfamiliar with. Because of the Internet's presence as a resource for both academic and non-academic purposes, and its commonplace presence in everyday life, it is important to understand how preservice teachers are using this resource to support their learning of mathematics.

Research Questions

1. What are information-seeking strategies that preservice teachers use when locating mathematical information online?
2. What form and quality of mathematical connections do preservice teachers make around the Pythagorean Theorem after locating information online?
3. What relationships emerge between the information-seeking strategies employed during the research task, and the form and quality of the mathematical connections made by preservice teachers?

The following chapter describes the design and rational of this study including its participants, activities and tasks. Subsequent chapters describe the methods for analysis, findings, results, and what was learned in relation to these research questions.

CHAPTER 3

This dissertation focuses on three aspects of learning mathematics online: (a) the strategies used when searching online, (b) the kind and quality of mathematical connections made when seeking this information, and (c) connections between strategies used and the connections that may be made. For the first of these goals, I asked participants to go online in different tasks with the goal of improving their understanding of a particular piece of mathematical content (in this case, the Pythagorean Theorem). To better understand the mathematical connections that may emerge with these tasks, participants were asked to create concept maps before, during, and after each of the Web tasks. After analyzing the first two research questions individually, the third research question explores relationships between online seeking strategies and the mathematical connections documented. Additionally, data collected during interviews with the participants was used to help provide context for patterns that emerged through the analysis of the concept maps data.

The basic structure of the sequence of activities can be visualized in Figure 1.

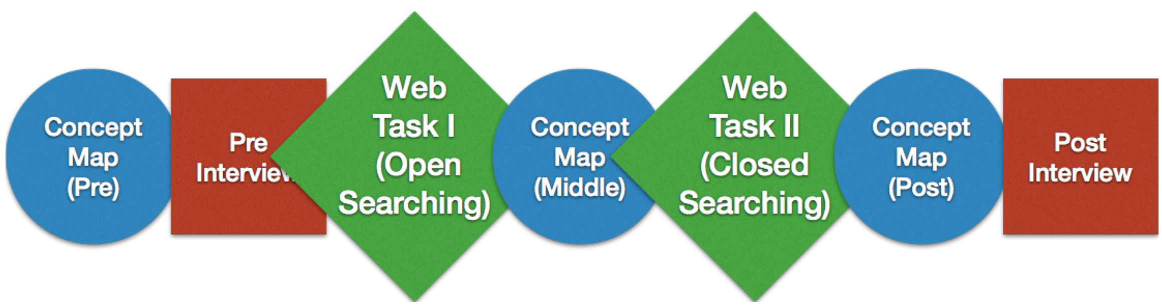


Figure 1 – Representation of the Design of the Interview

The majority of this chapter fleshes out the brief overview I presented here. First, a brief description of the participants is provided. Next, the individual tasks of the study are described, which include the motivation behind many of the tasks as well as how the raw data was cleaned for analysis. Lastly, I end this chapter with a vignette on participant, describing from her point of view, her experience participating in this study.

Participants

For this study I recruited seven participants from a large Mid-Western University. All were enrolled in their teacher preparation programs for elementary school. All of the participants, in addition to becoming certified to teach elementary school, were also enrolled in a math minor program, in which they take additional math courses. These participants were in the senior year of their program, and all happened to be female. They were recruited in one of the math content courses required for their elementary teaching major.

This particular population was selected because, as a course instructor, I was familiar with the mathematics content included in that course, which included the content around the Pythagorean Theorem. This included understanding a statement of the Pythagorean Theorem, using the theorem in mathematical scenarios, and understanding a proof of the theorem. However, in their course work, the theorem was presented as a single statement, not a bi-directional statement with a forward direction and a converse direction. Additionally, the proof covered in this course was only a proof of the "forward" direction of the theorem, not the converse. Thus, the study participants had some working background knowledge of Pythagorean Theorem during their mathematics coursework, but the content of their coursework did not present the material in a way similar to the way that content is discussed in the Common Core standards. This familiarity with the content that they would be investigating has been suggested by research on online information seeking as a crucial criteria for success (Fidel et al., 1999; Hirsh, 1999; Schacter et al., 1998).

Tasks

The preservice teachers engaged in three kinds of activities during the study. The first of these were two tasks that required searching for mathematical information online (an open searching task and a closed searching task). This section focuses on describing these tasks, including what they are, why they were structured in those ways, and how the data collected was cleaned for later analysis. After this section, a description of the other two activities that investigated the effects of these tasks (concept maps and interviews) will be presented.

Web Task I (Open Searching)

The research on information-seeking online has yet to reach a consensus as to whether participants are more successful with open searching tasks or closed searching tasks (Bilal, 2001; Jones, 2002; Schacter et al., 1998). As these two different kinds of searching environments were shown to be successful with students, I designed this study to include opportunities for both kinds of explorations. Task I was an open searching task where participants had free reign to explore the Internet and Task II was a website focused task where participants were directed to explore a limited number of sites.

The preservice teachers in this study first had their attention drawn to standard 8.G.6 of the Common Core State Standards for Mathematics (Explain a proof of the Pythagorean Theorem and its converse). They were informed that their investigations online during these tasks would be focused on their own understanding of the content of this standard. The first task (Task I) asked students to go online and try to locate resources that would help them better understand the content of this standard. As the preservice teachers identified sites, they were asked to describe the specific aspects of the sites that they found beneficial. During this time, the preservice teachers had the opportunity to explore any particular pieces of content they might be unsure of.

In order to make sense of each preservice teachers' searching during these tasks, a database of their web searching was created. The database listed each website that was visited by a participant chronologically. Each site was individually numbered, and the time spent at each site was noted. This database also included descriptions of the content of each of the sites and the participant behavior when exploring each site (gathered from video of the explorations). Additionally, participants' written and verbal comments about these sites was included in the databases. Lastly, any comments and reflections I had in watching the participants explore these resources was also recorded in this database. This database construction was performed for both web searching tasks (open and closed) and served as the main data source for analysis when attempting to uncover the searching strategies used by the preservice teachers.

Web Task II (Closed Searching)

In the second task, the preservice teachers were directed to a series of five sites that each contain different resources related to the Pythagorean Theorem. As participants explored the sites, they were prompted to study the content and describe how the content aligned with their thinking around the Pythagorean Theorem and its converse, and how it pushed their thinking.

The first site, of which a screenshot is presented in figure 2, is a part of the PBS television program titled NOVA. A page on this site displays a 3-4-5 right triangle with squares built off of the legs of sides of length 3 and 4. The applet instructs users to drag one of these squares to the outline of a square off of the side of the triangle with length 5, at which point the square snaps into place. When the remaining square tile is dragged to also fit into the square of side length 5, it is instead broken apart into unit squares, and applet users can fit the individual squares around the existing square. This displays how in a 3-4-5 right triangle, the sum of the areas of the squares off of the legs of this specific right triangle, must be equal in area to the square off the length of the hypotenuse. This applet has users work with one specific triangle, and they drag discrete blocks between the different areas, not following a specific pattern just filling in blank spaces.

Figure 2 – Screenshot of PBS Website

The second website is a GMAT review site and a screenshot of this can be viewed in Figure 3. The GMAT is the Graduate Management Admissions Test and is a test that is often used in determining admissions to management or business schools. The site does not contain and references to the proof of the Pythagorean Theorem. However, it does feature connections to a wide variety of related topics all in one location. The site features pictorial, algebraic, and textual descriptions of what the Pythagorean Theorem actually is. It also lists a couple examples of applying the Pythagorean Theorem. Additionally, the converse to the Pythagorean Theorem is briefly described and an example is presented. There are also connections to Pythagorean Triples on this site as well as special right triangles (45-45-90 and 30-60-90). This site is included as it has a breadth of content and users of this site have a potential to see many connections they might not have made otherwise.

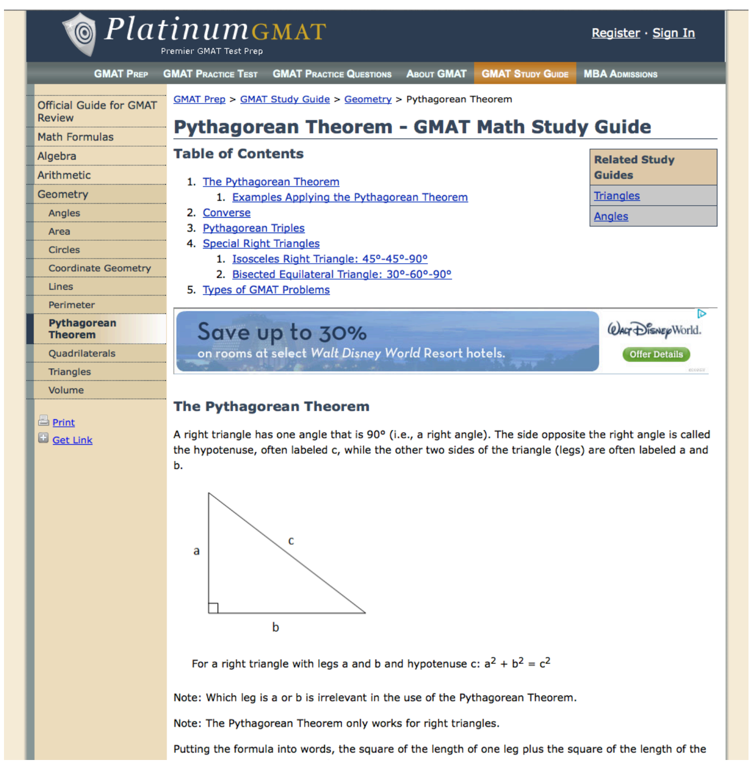


Figure 3 – Screenshot of GMAT Review Site

The third website is a collection of several different aspects related to proving and illustrating the Pythagorean Theorem, and a screenshot of the site can be viewed in Figure 4. The site first defines the Pythagorean Theorem. Then it lists an example of the Pythagorean Theorem with a 3-4-5 right triangle illustrated in a diagram. It presents three scenarios where one of these sides might be unknown, and it illustrates how to find the length of the unknown side in each of these instances. Lastly, the final resource on this page is an applet that demonstrates a proof of the Pythagorean Theorem. The applet is both an animation and text presented simultaneously. This proof calculates the area of a single square in two different ways, sets them equal to each other and determines the Pythagorean Theorem. This applet lets users change the lengths of the starting triangles, and also redefine the labels used in the proof. This page is included as it positions the 3-4-5 triangle as an illustration of the Pythagorean Theorem, while also providing a proof of the Pythagorean Theorem later on the same page.

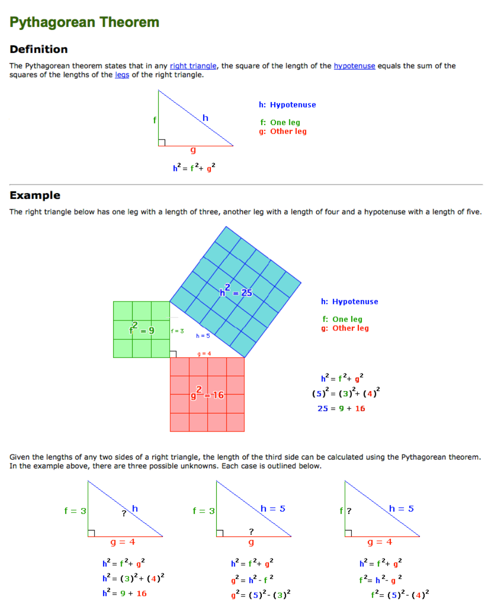


Figure 4 – Screenshot of ronblond.com Site

The fourth website is a YouTube video from the popular Khan Academy series of videos, and a screenshot of this video page can be seen in Figure 5. This video presents a proof of the Pythagorean Theorem very similar to the proof that was presenting as part of the math content course took as part of their teacher preparation program. This video is presented as it features an individual describing the proof to an audience in real time, as opposed to the previous websites where the information was visually on the page for the preservice teachers to put together on their own. This video is presented near the end of all of the websites so there can be a separation between what preservice teachers encounter from the static websites prior to this video. Additionally, the Khan Academy is a popular trend in online education research. The inclusion of it in this study is a small way of beginning to explore the ways that these videos might potentially play a role in preservice teacher preparation.

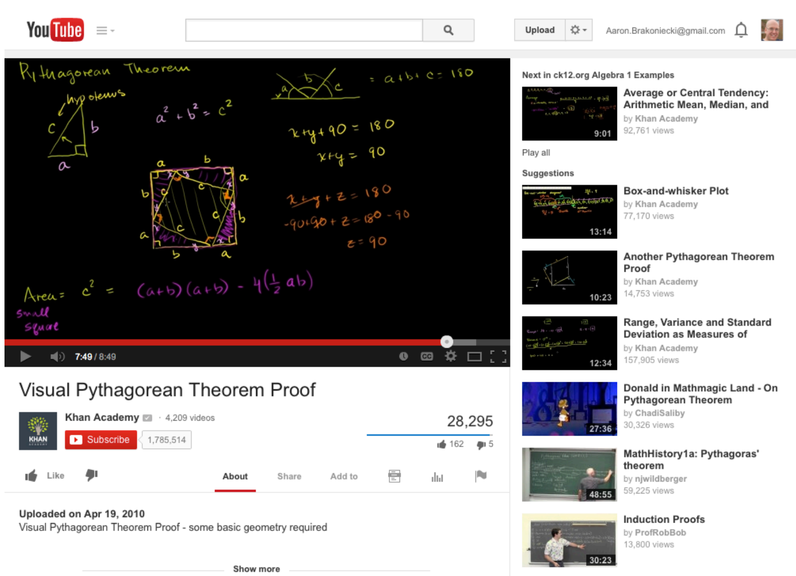


Figure 5 – Screenshot of Khan Academy Video

The last website is a second YouTube video produced by the website TutorVista, and a screenshot of this video can be seen in Figure 6. However, this video (as opposed to the other sites presented in this section) is about the converse to the Pythagorean Theorem, as opposed to the Pythagorean Theorem itself. This video presents a step-by-step proof of the converse to the Pythagorean Theorem. The steps taken are both written on the screen and a voice narrates the steps of the proof simultaneously.

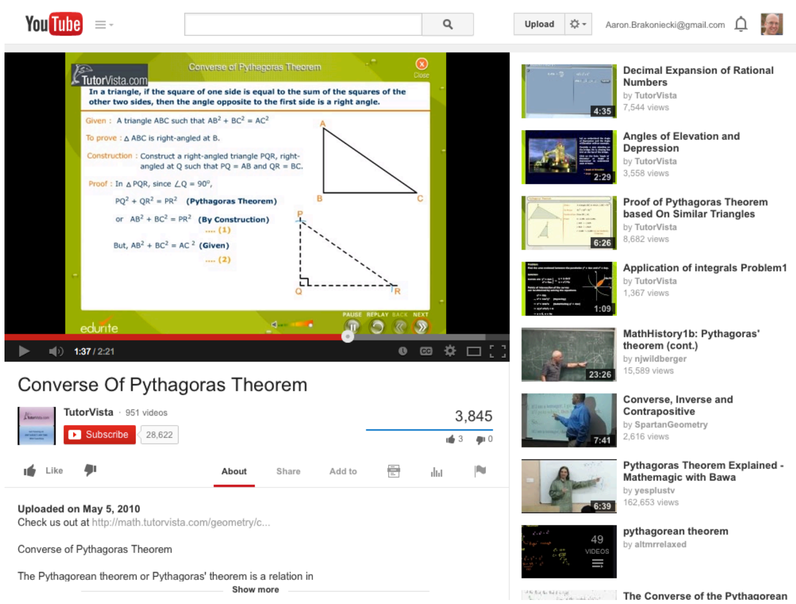


Figure 6 – Screenshot of TutorVista Site

The following table shows a summary of the five different sites used in Task 2 and a very brief description of their characteristics.

Site	Description
PBS	Series of pages with static and dynamic content with an illustrated example of the theorem and applications
GMAT Review	Single page of static content covering the theorem and multiple applications, and special cases of the theorem
Ronblond.com	Single page with statement, application, and a dynamic applet illustrating proof of theorem
YouTube – Khan Academy	Video with narration of proof of Pythagorean Theorem
YouTube - TutorVista	Video with narration of proof of converse to the Theorem

Table 1 – Description of Sites Used in Web Task II

These sites were chosen to incorporate a variety of both dynamic and static presentation styles. They cover sites produced by tutoring companies, educational programming, and independent websites. Some sites were chosen as the content was spread across a series of pages and some were contained within a single page. As with Web Task I (Open Searching), the ways that participants interacted with these five different sites was also described in the same web-searching database.

Investigating the Effect of Tasks

In order to investigate the effects of participating in the above Internet searching tasks, the preservice teachers in this study participated in two other kinds of activities that tried to uncover changes in their thinking. The first of these activities was the creation of concept maps where the preservice teachers were asked to represent how they were thinking about the Pythagorean Theorem. The second kind of activity was responding to interview questions, about the Pythagorean Theorem standard they were investigating, about their participation in this series of tasks, and about their history of information-seeking online. This section tasks each kind of these activities and describes them in detail.

Concept Mapping (Pre, Middle, and Post)

To better understand the connections that the preservice teachers were making among the content of the Pythagorean Theorem, participants were asked to create concept maps representing how they were thinking about that content. Participants were shown example concept maps of different styles around both mathematical and scientific content so that they may be familiar with common features of concept maps. The creation of the concept maps was done before the participants engaged in either searching task, between the two tasks, and again after completing both tasks. The task each time utilized the same prompt and participants were encouraged to reference the example concept maps, or their prior concept maps if they wanted to.

Preservice teachers were asked to create a concept map about the Pythagorean Theorem, using a word bank as a starting guide. As previously discussed, the creation of a concept map from a word bank captures knowledge structures in a similar way to a multiple choice test (Ruiz-Primo et al., 2001). As the preservice teachers created each concept map, they were asked to talk aloud about how they were thinking about the map, why they were making particular connections, and what they were still unsure about. All concept maps are created and recorded with a LiveScribe™ pen, which records, in real time, the creation of the concept map and records the audio that occurs during that time period. The creation of the concept map can then be recreated digitally, in real time with the corresponding audio played simultaneously.

Concept maps contain both nodes and links between the nodes. The nodes describe pieces of mathematical content, and the links describe how those pieces of mathematical content are connected. However, based on piloting attempts with other participants, it was found that the link descriptions between the nodes of content were often left off of concept maps by novice concept map creators. In order to have the most description of how participants were connecting pieces of content in their concept maps, the raw concept maps that were created by the participants were augmented with their descriptions of their concept maps. Using the audio recorded by the Livescribe pen, the text spoken by participants when they created their concept maps was able to be inserted in their concept maps. This allowed both the oral and written descriptions for the links between nodes of content to be analyzed for connections. The below figure (Figure 7) illustrates what one of these concept maps looks like when augmented with the preservice teacher's spoken text.

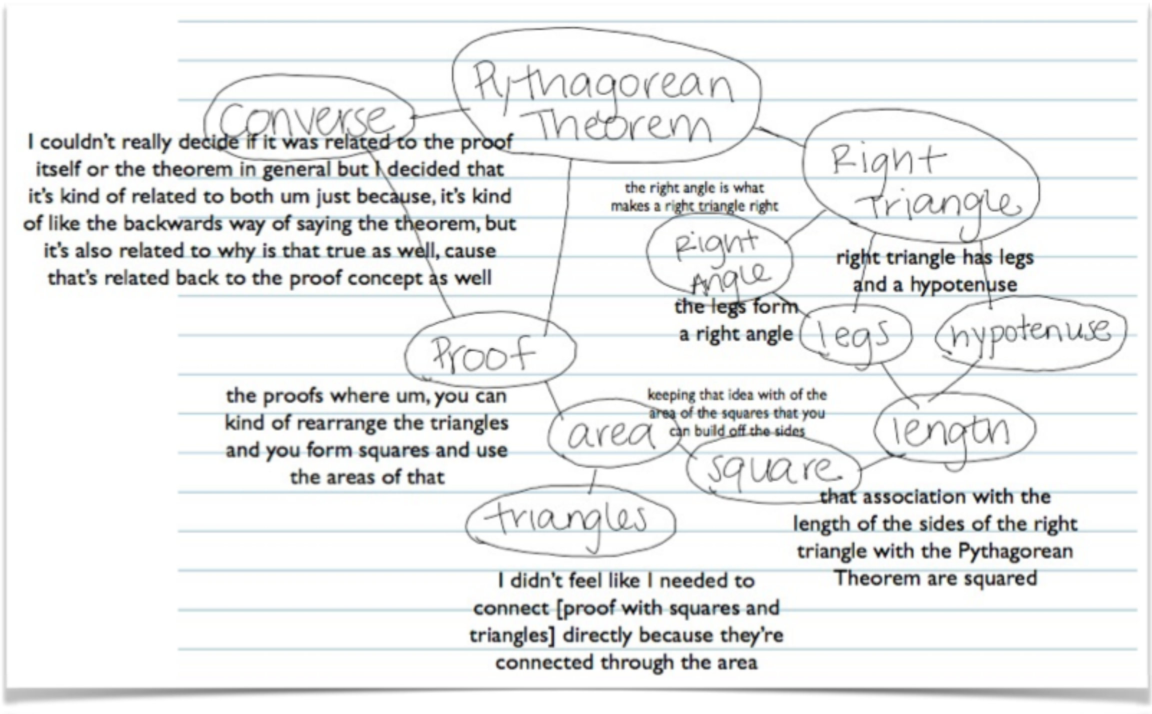


Figure 7 – Example Augmented Concept Map

Pre Interview

Before they engaged in either searching task, and after engaging in both tasks, the preservice teachers were asked about standard 8.G.6 - Explain a proof of the Pythagorean Theorem and its converse. In each iteration participants were asked what they thought this standard meant, what they didn't understand about this standard, why they thought this standard is included in what students need to know about mathematics, and what their goals, as teachers of this standard would be for their own classrooms.

This section focuses the preservice teachers towards a specific part of the Pythagorean Theorem, the proof of the theorem and its converse. This is where some of the content the participants encountered as part of their math content course provides a background for their future explorations. This section also begins to ask the participants to begin to imagine this content in the context of teaching to others. This is important as it leads to the next grouping of tasks where the preservice teachers are asked to explore content they might be unsure of as well as locate some resources that might help them teach that content.

Post Interview

The post interview had three themes of questions running throughout the interview: questions asking them about the specific standard they investigated during these Internet searching tasks (the same questions as the pre interview), questions asking the preservice teachers to reflect on the task, and questions asking the preservice teachers about their habits online (see Appendix for detailed protocol). The questions about the activities asked the preservice teachers to reflect on how they viewed their knowledge having changed (if at all) as part of these activities, and reflect on what was reinforced that they already were familiar with. There were also questions asking them to reflect on using the Internet as a tool for exploring mathematics content. The questions asked the participants to discuss what parts of the online resources were the most helpful to them, and what parts of the online resources were the least helpful.

The second theme of questions in this section focused on obtaining information about the preservice teachers relationships with and habits of using the Internet. Participants were asked to describe their time online, what they do task-wise online, and to self rate themselves as Internet users. In asking these questions, a picture began to emerge of how the participants position themselves as Internet searchers. It was also possible to get a sense for the types of activities they typically do, whether they are for academic or recreational purposes. Their skills as an Internet user was important to learn about as it provided context to help think about their searching strategies as a whole.

This section has described the activities and interviews that were conducted during the data collection of this dissertation. Each item was described in detail and the physical prompts can be found in the Appendix of this document. The next section briefly describes how these items were administered, as well as the role of the interviewer during these sessions.

Vignette of Data Collection Experience

So far in this section, I have described the participants, the tasks, and activities that the preservice teachers were asked to complete as part of this study. The seven participants had similar experiences in these series of tasks, so now I present a composite vignette of these experiences. This provides a perspective for what the overall experience was like for the participants as they went through these activities and completed the online searching tasks.

In a content course for elementary mathematics majors, I recruited participants to participate in my dissertation study about learning mathematics online. The interested preservice teachers signed up on a sign-up sheet and eventually received an email if they were selected to participate in the study. Each participant was asked to come to an office in the university mathematics building at a specific time and asked to allot up to two hours for this study, of which they would be compensated with a gift card.

Upon arrival to the office where the data collection would take place, I greeted the preservice teacher and engaged in some brief small talk about their day and their semester. The session began with going over the consent form with the participant. It was at this point that the participant also received the compensation for their participation. Then, the recording devices were turned on and the various activities of the session began.

I told the preservice teacher that the purpose of the study was to better understand how preservice teachers learn math online, and that the activities of the day would be focused on the Pythagorean Theorem. At this point I asked the participant to create a concept map for how they were thinking about content related to the Pythagorean Theorem. The preservice teacher was shown examples of concept maps, and given a task sheet (see Appendix) that listed terms that they may wish to include in their map, but not required to. They were encouraged to talk aloud as they were creating their maps, and also to describe their map after they were done. This activity was done with a LiveScribe™ pen in a special notebook.

After completing the concept mapping, the participant was presented with the three standards from the Common Core State Standards around the Pythagorean Theorem in the eighth grade. Their attention was focused on the first standard, 8.G.6 – Explain a Proof of the Pythagorean Theorem and its converse. The participant was asked a few questions about their understanding of this standard, questions they had about it, why it may be important, and what their goals would be for their own classroom students.

Once this pre-interview was done, the participant was introduced to their first task, the open searching task. The preservice teacher was told that they were to think about their own understanding of the standard, and they were instructed to try to locate resources on the web that would help them better understand the content of that standard.

The participant was provided with a sheet to note any resources that appeared especially beneficial where they could also list reasons why it seemed helpful. I turned on a screen and video recorder and gave the preservice teacher the laptop, showing that they would be using the web browser Firefox, and briefly showing how to use the URL and search bars. As the preservice teacher searched online, I sat off to the side, watching what they were doing. The preservice teacher was given no more than 20 minutes for this activity, though some took less time than that.

After completing the first web task, the participant was again asked to create a concept map reflecting how they were thinking about the Pythagorean Theorem and its related content. The participant was allowed and encouraged to look at their previous concept map, and any notes that they took during the open searching task. Again, the preservice teacher was encouraged to think aloud during this activity and to describe the design of their map after completing it. This was again done with the LiveScribe™ recording pen.

The next activity for the participant was the second Internet task, the closed searching task. The participant was told that the researcher had picked five websites that he wanted them to explore. They were encouraged to explore these sites in whatever way they normally would. The preservice teacher was told to focus on the content of the sites, to see what aligned with what they were thinking and what pushed on their thinking about explaining a proof of the Pythagorean Theorem and its converse. Again, they were given a blank sheet so that they could describe the content they were exploring. I again turned on a screen and video recorder, and directed the preservice teacher to a document on the computer that had the five links for them to explore. The participant clicked on the links, one at a time, and explored each of the sites. After they were done exploring each site, I asked the preservice teacher what they noticed about the site they just explored.

Following the completion of the second task, the participant was again directed to construct a concept map from scratch. Again, like the second time the preservice teacher constructed a concept map, they were encouraged to look at their prior iterations, and any notes they may have taken. Constructed with the LiveScribe™ pen, they again were encouraged to narrate their thinking, and describe their construction.

The preservice teacher then was told that the following two activities would just be interview questions and that they were done with the Internet and constructing concept maps. The first set of questions that were asked of the preservice teacher was the same set of questions that were asked before the first task, focusing on the standard 8.G.6, and how the preservice teacher was thinking about that standard. Following that series of questions, the participant was asked by the interviewer about the activities they just completed. They were asked about the ways that they thought these tasks impacted their thinking, and the ways in which the resources they located also had an effect on their thinking. Lastly, the participant was asked a few questions about their history of using the web (and search engines) for both academic and personal purposes. The participant lastly had the opportunity to ask the researcher any questions that they might have. After this, the recording devices were turned off, and I thanked the participants for their time and their participation in this study.

Summary

This chapter described the participants of this study, the activities that were used with the participants of the study, how I cleaned the raw collected data for later analysis, and a composite vignette of what the entire data collection experience was like for each participant, so as to provide a coherent image of what all of these experiences looked like together. The next three chapters each focus on the three research questions of this study. Each chapter will discuss the frames used to analyze the data, along with examples from the data. These chapters will also present summaries of the analysis performed on the data with the frameworks. The results of these analyses are connected back to the research questions, examining what answers to the questions emerge from the data and what still remains unanswered.

CHAPTER 4

The recruited preservice teachers completed two Internet-searching tasks as part of this study. Before, during, and after these tasks, participants also completed activities that helped illuminate how their understanding of mathematical connections may have been changing. This chapter focuses on the main activities in these sessions, the Internet-searching activities, while the next chapter unpacks what impact these activities had. These Internet-searching activities attempted to capture two different methods used by these participants in order to locate information online. These methods are similar to methods used to locate information in the real world. Consider the following analogy:

When going to the neighborhood library to find information, two different processes occur before obtaining that information. Upon entering the library, the person is confronted with a myriad of books and other resources. The person will need to find a way to narrow in on a specific book or resource that they will want to look at more closely for the desired information. Secondly, once a resource has been selected, the person will then need to examine the resource and attempt to locate the relevant and desired information within the resource they chose.

This process is similar to what users of the Internet experience when they try to locate information online. First, upon opening the web browser (entering the library), the user has access to numerous different resources, although many of these resources may be unfamiliar to the user. The user is then confronted with how to select one of the resources. If the user has some experience, they might know some resources they have used in the past and go directly to these previously used sites. Otherwise, they need a way to identify the specific resources that are located within the web. In the physical analogy of the library, this was traditionally done with a card catalogue. In the digital realm of the Internet, unfamiliar resources are typically located via a search engine.

After locating a specific resource, there are many ways that people can find information within that resource. Users of traditional books might look to a table of contents or glossary to narrow in on information, or they might skim through the pages to try and quickly identify the structure and scope of the resource. This is similar to how users of online resources narrow in on information. Some look for organizing information near the start of web resources, some read lines of text, and some skim over sites to get an overall sense to the information. The ways that we search for and read for information in the digital realm is not separate from the ways we have carried out these similar tasks with more traditional resources, though there are important differences.

This section explores the range of different information seeking strategies that were used by this sample of preservice teachers as they tried to learn more information about one standard from the Common Core State Standards for Mathematics. By looking at the approaches used, both in attempting to locate resources, and in locating information on those resources, we can begin to identify more and less powerful strategies of searching. Ultimately, the goal of this chapter is to help us answer the question, what are information-seeking strategies that preservice teachers use when locating mathematical information online?

Chapter Structure

The Internet activities that the preservice teachers performed were divided into two parts. Following this form, this chapter will also discuss the two activities in turn.

Before presenting the activities and their results, the first part of this chapter will discuss the frames of Internet searching that were used in the analysis of these two activities. In the next part of the chapter, the first task will be discussed where participants were free to use the Internet in whatever way they wanted to locate information they found beneficial. The third part of this chapter will focus on the second task given to the preservice teachers, in which all participants explored the same five resources (three sites and two videos). The final part of this chapter pulls together what was found across the different activities and the information seeking strategies used by the preservice teachers.

Frameworks of Information-Seeking

Within the research on Internet navigation strategies, two different kinds of classifications exist within the literature. One kind focuses on the navigational patterns of Internet searchers. That is, they attempt to describe the ways in which a user goes from page to page, as they look to obtain information. A second kind of classification attempts to describe the intent or motivation or focus of the Internet searchers. These focuses on describing the kinds of sites that users spend time investigating, and looks to classify their navigation by the pattern of behavior while on the site. The first kind of classification scheme of navigational patterns of navigating between sites is more directly observable than the second as it is easier to see changes in navigation between different pages, as compared to potentially just eye movement on a single page. However, both are observable to an extent and both provide ways of answering questions about the information seeking strategies employed by Internet searchers.

To help classify the navigation patterns of the preservice teachers between different sites , this study draws upon a classification system that emerged after researchers analyzed Internet search history of users in a study and categories of behavior of the participants were identified. These categories included the timid navigators, the laborious navigators, and the divergent navigators (Juvina & Oostendorp, 2004). In each category below, I summarize the descriptions of these categories from the research, describe how I adapted their descriptions for the analysis of the navigation patterns in my study, and provide an example of what each behavior looks like within the searching data.

- Timid navigator
    - This group in the original study is described as visiting very few pages and not venturing far from the homepage.
    - For my study, as there was no homepage, I looked at how far my participants went from their search results page, and if they often only clicked one link to take them off of the search results page and then returning back to the results page without exploring their site further.
- Laborious navigators
    - Participants in the original study were said to belong to this group if they had a high amount of uses of the back button and page revisits.
    - For my study, I counted only attempts at revisiting and returning to sites that contained information they were studying. I did not count returning to a search result page as this was not revisiting subject matter information

they had encountered before, but just accessing the results of a previous search. There actually turned out to be no participants in my study that exhibited this particular type of navigation.

- Divergent navigators
    - Participants in the original study were said to belong to this group if they had among the highest number of visits to unique pages and had little or no revisitations to pages already explored.
    - For my study, a participant was said to visit a high number of unique pages if they visited more than 7 and if almost all of their revisitation was contained to search result pages.

A different framework was employed to study the information-seeking strategies employed by the participants while they were searching for information within a single site. In their handbook chapter of research on new literacies, Lawless and Schrader (2008) review and summarize research on navigating complex digital environments. In reviewing research on patterns of navigation behavior, they found several studies (Horney & Anderson-Inman, 1994; Lawless & Kulikowich, 1996; MacGregor, 1999) that, although disconnected from each other, came to similar conclusions about general searching patterns. They found that all the students had the same basic three groups of navigators, though each had their own names for these groups. These were

- Focus on Comprehension - those that had a focus on comprehending the information they were encountering (called “studier profile”, “knowledge seekers” and “concept connectors”, respectively),
- Focus on Static Content - to force a linear structure onto a non-linear environment (called “book-lover profile”, “apathetic hypertext user” and “sequential studier”, respectively).
- Focus on Dynamic Content - those that played with special features embedded in the resources they were using (called “resource junkies”, “feature explorers” and “video viewers”, respectively),

There is one final note that cuts across both of the categorizations presented here. The studies included do not attempt to label any individual as existing wholly within a single category of any categorization. They note that individuals would often use different approaches to searching at different times during their navigation. Because of this, these studies try to describe all the different kinds of navigational patterns that exist within any particular search, and indicate all the different patterns exhibited by an individual throughout their navigational session.

With this in mind, I analyzed the navigation of all seven of my participants and noted which of these strategies participants were using at some point during their completion of the first activity. Again, participants could be using different strategies at different points in time. After focusing on the distinction described by this study, I then turned to the categorization found across all the studies described originally (comprehending information, playing with special features, and forcing a linear structure onto a non-linear environment). The results of this analysis are presented below.

	Part. 1	Part. 2	Part. 3	Part. 4	Part. 5	Part. 6	Part. 7
Timid Navigator	X	X	X	X	X	X	X
Laborious Navigator							
Divergent Navigator	X			X			
Focus on Details	X	X	X	X	X	X	X
Focus on Static Content	X	X		X	X	X	X
Focus on Dynamic Content		X	X	X		X	X

Table 2 – Summary of Information-Seeking Coding of Participants

Task I – Open Searching

The section below will look at each of the categories described above in the context of the first Internet searching task. Each section will first examine what a prototypical example looks like for each category, using examples from the participants wherever possible. Then the category will be examined across all participants to identify what, if any, patterns existed among the participants within each individual category. The following section will then zoom out to look at patterns across all the categories.

The first three groupings of categories focus on how participants move from site to site, looking at the transitions among web searches, the results of the search, and the resulting pages that were clicked on for examination. The different patterns of movement between these various types of pages are difficult to describe via a presentation of the actual data. However, these navigational patterns can be described looking the diagram below, adapted from (Hölscher & Strube, 2000, p. 340). In Figure 8, the common practices of performing and using an Internet search engine are illustrated, including launching a search engine, formulating a query, submitting a query and obtaining results, examining the page of results, selecting and examining a document from the results, and browsing among other pages. There exist numerous arrows (of which only a few are listed) that illustrate the flow of a user as they go from using a search engine to examining results, and back and forth. A few of these included arrows are labeled and the below sections describe what patterns of navigation utilize these arrows for each of the categories.

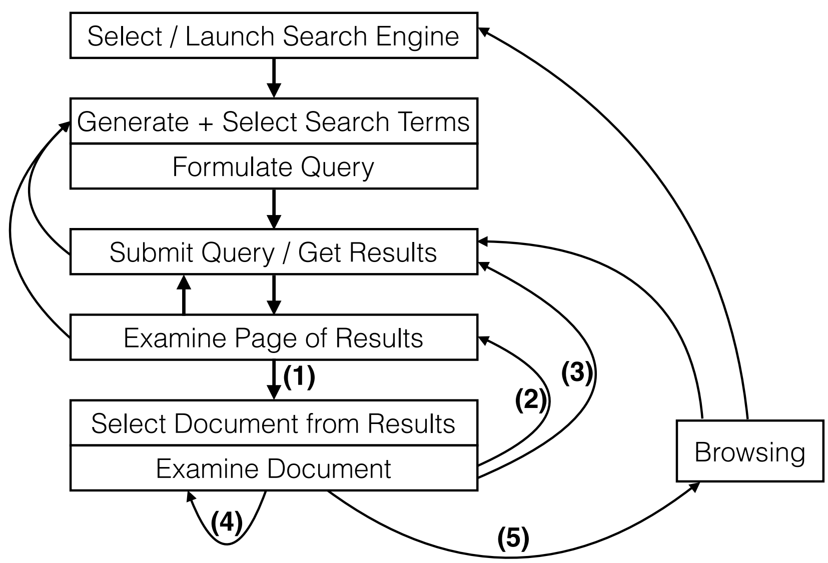


Figure 8 – Diagram of Common Aspects of the Internet Searching Process

Timid Navigator

A timid navigator was one who did not stray far from their search results page. Here they would perform a search, click on a result to examine a single page, and then return to the search results page to either click on an additional result or to perform a new search. In the figure presented above, a timid search would have the majority of their navigation described by arrows one (selecting a document from a page of results), two (going back to the search results page after examining a single document), and three (getting results from a newly submitted query after examining a document).

One excerpt of this kind of searching behavior could be located within the open searching of Participant 7. At the start of her searching, she obtains the results from a page, clicks on one result to examine the site (arrow one), returns to the same search result to locate additional results (arrow two), before clicking on another result to examine (arrow one), and finally returning to submit a new query (arrow three). This same pattern occurs at several other points in her data as well, even when she transitions to looking at video resources.

Across all participants, this category was the most common, with all the participants exhibiting a timid navigator pattern at some point during their entire open searching task. Despite each search result page having at least ten results listed, the most common activity after exploring a result was for participants to go back and perform a new query to obtain results (arrow three), than is was to click an additional result from the same listing of results (arrow two). There were only rare instances that participants used the results that they clicked on as launching points to then browse further links off of those pages they already explored. Most stayed very close to their search results page and the most common behavior across all the participants was to only explore the direct result of a search.

Laborious Navigator

The laborious navigators were those who revisited sites they had already explored, possibly as reminders of content, or to contrast with new content that was encountered. Again, as a reminder, this category includes only revisits to sites or results clicked off of a results page. This does not include revisiting the search result page itself. Distinguishing whether they return to a page of search results or a page of content emphasizes what makes a laborious navigator different from a timid navigator. In the figure presented above, this would be characterized by repeated or frequent navigation along arrow four, where after content in a document was examined, it was then reexamined, at some point in time, after its original examination.

Again, this category of navigation did not appear among the navigational patterns of any participant at any point within their open-searching activity. There are several contributing factors that may have influenced this pattern of behavior. It may that after clicking on a link from the search result page, the color of the link changed, signaling to the participant that they already explored a given resource. Additionally, it may be that participants knew they had a finite amount of time to complete the task and would rather explore new content with that time than revisit content they already encountered. Additionally, it could also be that participants felt they completely understood and remembered all of the content of a site and felt it unnecessary and redundant to revisit any site they already explored. All of these reasons, and others, may have contributed to why there was a lack of the laborious navigator strategy exhibited by the participants in this study.

Divergent Navigator

The pattern of divergent navigator consisted of moments where a participant used the links within one of the pages they had clicked on from the search results, to then go and further explore new sites that may or may not have been included in the search results page. These additional sites are explored after having clicked on a link contained within a page, and not having been clicked from a listing of results from a search. In the diagram described above, this pattern of action would be described by arrow five, where after exploring the results of a page, the participant then transitions to a pattern of browsing, exploring additional sites that are linked off of that document. This is different as compared to those who go back to the search results page for additional sites to explore or a new query (timid navigator), or those who revisit the same page for additional exposure to the same content (laborious navigator).

Within the data collected as part of this study, an instance of the divergent navigation pattern can be identified with participant 1. At least twice during her open searching task, she exhibits the behavior of a divergent navigator. The first instance of this occurs when she clicks on a PBS site from a search results page, which then causes her to explore additional links off of that original PBS site, clicking on two additional pages, before backing back out to the results of the search. Later, in a different search, participant 1 clicked on a Common Core test prep site and browsed on several different subpages off of the original page numbering a total of seven pages in all within that Common Core test prep site. This are both illustrations of arrow five, where after examining a site, a participant then goes to browse further information based on links off of that site, and not based off of the search results.

This pattern of navigation was not commonly observed among the patterns of navigation exhibited by the participants. Only two out of the seven participants exhibited the divergent navigator pattern at some point in time during their open searching activity. Sometimes the additional pages explored were for pages located within the original site clicked (as was the case for participant 1), and sometimes they were for pages outside of the site that was clicked (as was the case for participant 4). It may be the case that these sites just did not have a lot of outside links contained within each page, or at least not a lot of links that the participants identified as having relevant, useful information given their task at hand. It could be that the participants felt that the page that they were directed to in a site from the search results would be the one page in the site that contained the most relevant information. Therefore, other pages within that site would be unlikely to contain more relevant information. This may explain why the majority of the strategies employed by the participants were the timid navigator strategy where additional, pertinent information is located by going back to explore other pages among their results or by performing a new query search, rather than continuing to explore within the site, or off of the site, as with the divergent navigator.

The next section moves away from looking at how participants were navigating from page to page, and site to site in utilizing the results they were obtaining from their search results. Instead, the focus is on the describing how the participants were interacting with the content and information on the individual sites that they were visiting.

Focus on Detail

The focus on detail categorization describes the attention that was being paid to the content by the participants when they were searching for information on the different pages that they visited. Here, it is examined whether, at any point during their explorations, participants spent sustained, focused attention to the content available on the page they were examining. This is, admittedly, a bit more inferential than some of the categories. Participants' eye and head movements were examined to determine whether they were reading text, examining a picture, or intently watching a video.

Within the data of this study, an instance of the focus on detail can be found with participant 4. In her searching, she ended up clicking on the Wikipedia.com site entry that discussed the Pythagorean Theorem, after scrolling and doing some brief exploration of the page, she came to the section focusing on algebraic proofs of the Pythagorean Theorem. It was here that the participant spent an extended amount of time, reading (line-by-line) the algebraic proof and comparing the written proof with the included corresponding diagram to the right of the text. In this section, you can see her eyes, slowly scrolling across the part of the screen where the text is located, they jump of her eyes from the text to the diagram and back, and also the movement of the mouse along the words of the proof that are included on the site. This held her focused attention for over one minute and twenty-one seconds.

Every participant as some point in their searching exhibited this focus on detail. For many of the participants, it occurred at several points during their explorations of the different pages and sites with content. It should also be noted that all of the participants also had moments where they were obviously not focusing on details. In these moments, participants skimmed quickly over portions of pages, or entire pages as well. Here the participants seemed to be attempting to get a broad understanding of the kinds of information located within a part of individual pages. Based on these skimming moments, participants would then later go on to either read more closely, or select different sections or pages to explore.

Focus on Static Content and Focus on Dynamic Content

The final categories of this analysis are described together here as they refer to very similar phenomena. The sites that the participants explored contained two formats of content presentation, static and dynamic. The static content does not change in time while a page is being viewed. This would include things like text and images. Dynamic content does change in time. This would include things like interactive web applets, embedded videos, and animated .gif images. The total amount of time that was spent exploring the content sites was calculated for each participant, noting how much time was spent with static content and how much time was spent on dynamic content. If the amount of time of time spent with static content was over 30% of their total time spent with all content, they were said to have exhibited some focus on static content. Similarly, they were said to have exhibited a focus on dynamic content if they spent over 30% of their total time with dynamic content. The results of this analysis are summarized below (Table 3).

For the focus on static content, this is perhaps best illustrated by participant 5, who spent all of their time exploring content solely within sites that utilized a static presentation of the content. This participant only focused on content that was textual, or that also included diagrams of the content as well. This is contrasted with someone like participant 3 who was categorized as focusing on dynamic content. For this participant, 88.4% of the time she was looking at content, it was dynamic content. For participant 3, the majority of this was online videos about the Pythagorean Theorem, it's proofs, examples, and applications.

	Static Time	Dynamic Time	Total Time
Participant 1	08:04 (89.5%)	00:57 (10.5%)	09:01
Participant 2	08:08 (45.4%)	09:48 (54.6%)	17:56
Participant 3	02:30 (11.6%)	19:08 (88.4%)	21:38
Participant 4	11:39 (55.1%)	09:29 (44.9%)	21:08
Participant 5	04:34 (100%)	00:00 (00.0%)	04:34
Participant 6	05:36 (45.5%)	06:43 (54.5%)	12:19
Participant 7	04:01 (39.1%)	06:15 (60.1%)	10:16

Table 3 – Time (and Percentage) Spent with the Format of Content Presentation

Overall, there tended to be two types of groups within the data, those who spent an almost split amount of time exploring both static content and dynamic content (and hence, those categorized as both "focus on static content" and "focus on dynamic content") and those who spent the majority of their time with one format of the content as opposed to the other (and hence, only was categorized as either "focus on static content" or "focus on dynamic content). Participants 2, 4, and 6 were the most evenly split with the formats of the content that they focused on (around a 45%-55% split for one or the other format). Participants 1, 3, and 5 were heavily focused on either the static content (89.5% & 100% for participants 1 and 5 respectively) or the dynamic content (88.4% for participant 3). Participant 7 was somewhere in between these two categories, showing more of a focus toward the dynamic presentation of content (60.1% of her time), but still spending enough of her time with static content (39.1% of her time), to still be categorized as both. It was interesting that in the interviews with students after the completion of all tasks, most all professed to having strong opinions about whether or not they liked to learn mathematics from web videos. Some of those preferences can be seen in the summary presented here.

This section looked at the different ways of describing how participants were searching for information, whether it be in using web searches to locate resources (such as how one would locate books within a library), or in describing the ways in which they searched for information within individual sites (such as how one would locate information within an individual book). The next section looks across these individual codes to look for patterns across the codes.

Patterns Across Task 1

All of the participants exhibited behavior that would label them as “timid navigators”, that is, they mostly only clicked on one link off of a search result page, explored that single page without clicking any additional links, and then returned to the search result page. Only two participants exhibited behavior at any time that would be considered “divergent navigator”. Looking at how these participants searched for information within individual sites, it’s noted that every participant at some point had a “focus on content”, devoting an extended period of time to particular content on a specific page.

However, there was a notable distinction that emerged when looking at the kinds of resources the participants utilized during their searching. For some of the participants (Participant 1, 3, and 5), tended to focus only on one type of content, either static, or dynamic, but rarely spent time looking at both. For others participants (Participants 2, 4, and 6), they exhibited both a focus on static content and focus on dynamic content, meaning that the participants spent an almost equal amount of time looking at both static and dynamic resources. Only one participant (Participant 7) had a somewhat split focus, though looking at her times, it is clear she still spent more significant time with one type over the other.

Across all of the participants, two commonalities occurred among all the participants. In general, they were all categorized as focusing on the content they were encountering. As all the participants were focusing on completing a specific task designed to push their understanding of familiar content, this finding is not particularly surprising. Also across the participants, I notice that all were categorized as being categorized at some point as “timid navigators”. This indicates that one of the most common traits of participants, no matter the group, was to click on a search result, explore the single page, and then return to the search results to either click a new result or perform another search. There are numerous different reasons that this pattern may be prevalent, from the structure of the websites (mostly self-contained) to the design of the task (clearly identified information was focused upon) to the searching preferences of the participants. This uniformity across participants here does not appear to explain the differences in groupings that appeared in the categorization of the math content.

This section focused on the information seeking behaviors of the participants as they completed activity 1, the open searching task. The next section looks at the behavior of the participants in activity 2, the closed searching task.

Task 2 – Closed Searching

After their open explorations, and the creation of their second concept maps, participants were all asked to explore the same five sites, again with the focus of helping themselves better understand standard 8.G.6., Explain a proof of the Pythagorean Theorem and its converse. This section separates each of the five sites and looks at how participants engaged with the sites separately. At the end of the section, an overall summary of this part of the activity is presented. Throughout this section, the coding scheme described above of how users focused their attention on individual pages has been used.

The nature of this second task directed students to examine specific sites. Only two of the sites (Site 1 and Site 3) contained opportunities to engage with both static content and dynamic content within the same site. Therefore, these are the only two sites that are analyzed with respect to the dichotomy of static vs. dynamic. Also, because this task directed students to examine these sites, all the participants were coded as “focus on details” for the first three sites. A different proxy for “focus on details” was created for the two videos included in this activity, and that scheme is discussed in the description of those sections.

Site 1– PBS (Nova)

This resource is one of two resources is this section whose authors are somewhat known in education. In this case, this site is hosted by the Public Broadcasting Service (PBS) and specifically, their NOVA television program. This site presents a brief, elementary explanation of what the Pythagorean Theorem is, using general language and phrases such as “this side”. Links off of this main page allows users to find more information about who Pythagoras was, see either an interactive, or static visual example of the Pythagorean Theorem relationships holding, and they can also look at several mathematical scenarios that require the Pythagorean Relationships to solve them. This site does not contain information on a general proof of the Pythagorean Theorem, nor does it contain information about the converse to the Theorem.

The below chart shows how much time the participants focused on the static content, and the dynamic content within this overall site.

	Static Time	Dynamic Time	Total Time
Participant 1*	0:42+0:39 (100%)	00:00 (0.00%)	01:21
Participant 2	00:49 (94.2%)	00:03 (05.8%)	00:52
Participant 3	01:29 (69.0%)	00:40 (31.0%)	02:09
Participant 4	01:23 (100%)	00:00 (0.00%)	01:23
Participant 5	00:48 (100%)	00:00 (0.00%)	00:48
Participant 6	02:03 (65.4%)	01:05 (34.6%)	03:08
Participant 7	01:44 (100%)	00:00 (0.00%)	01:44

* This participant originally came across this site in their open exploration. Their time spent in the original exploration has been added to the time spent in the secondary exploration.

Table 4 – Time Spent on Static and Dynamic Content in Site 1

Here we can use the coding scheme from the above section looking at how participants read for content in individual sites. Every participant would be coded as “focus on static content” as every participant had at least 30% (actually at least 65.4%) of her time focusing on the static content contained throughout the PBS site. Only two participants engaged for any significant amount of time with the dynamic content (Participant 3 and participant 6) of this site and thus these two would be coded as “focus on dynamic content” as well. It should be noted that the interactive content was only available on one of the pages within this site. Some participants never clicked on the particular page and, thus never had the opportunity to engage with the dynamic content. Others made it to the web applet, but never clicked on the applet to interact with it.

This site has at least thirteen unique pages directly related to the content of the Pythagorean Theorem immediately linked among each other, and has access to numerous other pages that get further from the mathematical content, and closer to other pages of the television program, and network. However, the majority of the participants only explored less four or fewer of these sites. The only exceptions were participant 6 (who explored eight of the sites) and participant 7 (who explored ten of the sites). Additionally, there were two comments that stood out from those that explored this section. The first was from participant 1 who commented on the authority of the site, recognizing it (PBS) and exclaiming that she thought that a site would have more information and links on it, but that she knew it was a "trusted site". She was the only participant to comment on the reliability of this site.

Site 2 – GMAT Prep Review

The company "Platinum GMAT", which specializes in preparation for the GMAT test, compiles this resource. All the information for the Pythagorean Theorem is contained on this single page, and organized in a table of contents at the beginning of the page. The site begins with a statement of the theorem, and examples of the theorem working. Then a statement to the converse of the Pythagorean Theorem is also presented with an example. The site continues to include sections on Pythagorean Triples, Special Right Triangles (30º-60º-90º, and 45º-45º-90º), and then a final sample problem from the GMAT that requires the Pythagorean Theorem to solve.

The chart below shows just the total time the participants focused on the content of this site (which was all of a static format).

	Time On Site
Participant 1	00:43
Participant 2	03:44
Participant 3	01:37
Participant 4	04:07
Participant 5	02:07
Participant 6	02:56
Participant 7	02:04

Table 5 – Total Time Spent on Site 2

Using the above rubric for coding how participants were engaging with content would result in every participant being coded as “focus on details” (as all participants were instructed to examine the site) and “focus on static content” (as there was no dynamic content located on this site. The interesting pattern to note focuses on the total time spent on this site. Participants 1, 3, 5, and 7 spent around two minutes or less looking at this site. Participants 2, 4, and 6 spent around three minutes or more looking at this site. This distinction falls along very similar lines to the mathematical grouping distinction presented in the previous chapter, and the search strategy grouping distinction presented earlier in this chapter.

Site 3 – RonBlond

This website begins with a statements of the Pythagorean Theorem and utilizes a color-coded diagram illustrating the different sides and their relationships within the formula. After, a diagram of a 3-4-5 right triangle is presented with squares drawn off of each side, continuing the color-coded theme. Below this example are three instances of the formula being used to find the dimension of each of the three sides, if the lengths of the other two sides are given. This site also includes a web applet that contains instructions, a visual display area, and a bottom section containing descriptions of the visual display area. In this applet, users can pick a side of a right triangle to make the “unknown” side, rename sides and angles, show areas of squares off the lengths of the sides, and view a dynamic illustration of a proof that will appear step by step. Directly before this applet is a link to a page with static images that presents a sequential presentation of this same proof.

The below table shows how much time the participants focused on the static content, and the dynamic content within this overall site.

	Static Time	Dynamic Time	Total Time
Participant 1	00:25 (39.7%)	00:38 (60.3%)	01:03
Participant 2	00:33 (34.3%)	01:03 (65.6%)	01:36
Participant 3	00:56 (56.0%)	00:44 (44.0%)	01:40
Participant 4	00:46 (21.3%)	02:50 (78.7%)	03:36
Participant 5	01:28 (55.7%)	01:10 (44.3%)	02:38
Participant 6	00:55 (45.8%)	01:05 (54.2%)	02:00
Participant 7	00:29 (19.7%)	01:58 (80.3%)	02:27

Table 6 – Time Spent on Static and Dynamic Content in Site 3

As opposed to the first website explored by students which had the dynamic content embedded in one of its many pages, this website had almost its entire main content on the main page, including the web applet at the bottom. While most of the static information at the top of the page was basic, the applet at the bottom was more involved and was obvious as the focal point of the page. Coupled with the fact that the applet contained several different tools for the participants to explore, all the participants spent a significant portion of their time with the web applet, and thus all the participants would be coded as “focus on dynamic content” for this site. What was also unique was that many participants divided their time with the static part of this content as well. Only two participants did not also receive a code of “focus on static content”, participant 4 and participant 7.

Site 4 – YouTube Video Khan Academy Proof of PT

The first of two YouTube videos in this collection, this YouTube video is authored as part of the series of math videos put out by Khan Academy. This video runs a total of 8 minutes and 49 seconds. The beginning of the video presents a review of terminology relevant to right triangles. Then a proof of the Pythagorean Theorem is presented, similar to proofs presented in other resources. This video also contains a description/explanation of why a particular shape in the construction must be a square (have right angles), which is often omitted in other proofs. The video is narrated in real time as the drawings and constructions are produced with different color descriptions.

The table below lists the total time spent watching the 8:49 second video and also displays what percentage of the total video was watched.

	Time Watching Video
Participant 1	01:36 (18.1%)
Participant 2	05:02 (57.1%)
Participant 3*	07:37 (86.4%)
Participant 4	05:41 (64.5%)
Participant 5	03:52 (43.9%)
Participant 6	05:05 (57.7%)
Participant 7	03:52 (43.9%)

*This participant originally came across this video in their open exploration. Their time spent in the original exploration has been listed here to capture the time spent in every participants' initial viewing

Table 7 – Time Spent Watching Video in Site 4

As noted for the first three sites that students were directed to as part of this activity, using the coding of "focus on details" presents a challenge as all students paid some attention to parts of these sites and would have all earned that code. However, for the two YouTube videos contained in this study, the only mathematical content in the site is contained within the videos and so we can look at how much focus was paid to the videos by looking at the total time students spent watching the videos.

In the prompt, students were told to interact with the sites however they normally would, which included skipping backwards and forwards in videos if that's what they would do on their own. Every student took liberties to skip ahead for this site with no one watching the entire video. Three of the participants ended up watching less than half of the video (participant 1, 5, and 7). The remaining four participants watched over half of the video, of which participant 3 watched almost the entire video, while the remaining participants (participant 2, 4, and 6) all grouped closer together, centering near the 60% range. It should be noted here that participant 3 originally came across this video during their exploration during task 1 and so the breakdown of time watching this video was taking from that initial viewing of the video. As such, watching this video under the constraints of a different task may have affected how much time she spent watching this video. It may be the case that she would have spent less time watching this video if her initial viewing came during the second activity, further into her total interview session. Though not as dramatic as the other activities, it is possible to note that a similar division appears between those who spent less time watching the video and those who spent more time watching the videos (with participant 3 as an outlier to this pattern for this video). Participants 2, 4, and 6 are once again behaving very similarly.

Site 5 – YouTube Video Proof of Converse

This site is the second of two YouTube videos, this one produced by the company TutorVista. This video also almost always showed an ad before the video that was able to be skipped after 5 seconds. This video contains an slideshow like presentation of a rigorous mathematical proof of the converse to the Pythagorean Theorem. Totaling 2 minutes and 22 seconds, this video is narrated by a woman often using the exact words and phrases displayed in the proof. First a statement of the converse is presented, and then a rigorous proof is presented, with animated diagrams being utilized at each step of the proof. The proof utilizes the Side-Side-Side triangle congruence test to demonstrate how a triangle with side lengths that fall into a Pythagorean relationship, must in fact be a right triangle.

The table below lists the total time spent watching the 2:22 second video and also displays what percentage of the total video was watched.

	Time Watching Video
Participant 1	01:45 (73.9%)
Participant 2	02:22 (100%)
Participant 3	01:49 (76.8%)
Participant 4	04:29 (189%)
Participant 5	01:26 (60.6%)
Participant 6	02:22 (100%)
Participant 7	02:22 (100%)

Table 8 – Time Spent Watching Video in Site 5

As was the case in the last site, we can use the amount of time spent watching this video as a proxy for how focused the participants were on content. Two differences exist between this YouTube video and the prior YouTube video. First, this video is much shorter (almost ¼ the length of time) and so participants did not have as much opportunities to skip. Secondly, the content of this video was less familiar to the participants in this study, as the content in the previous video was of a proof, nearly identical to the proof that was covered as part of their content course.

Within the time spent watching this video, two groups emerge: those that watched the entire video, and those that skipped parts of the video. Participants 2, 4, 6, and 7 watched the entire video, while participants 1, 3, and 5 watched less than the entire video. This grouping is again very similar to those that have been noticed at other points in both the math and the searching strategy analysis. As has happened with other analyses, participant 7 sometimes aligns more closely with participants 1, 3, and 5, and sometimes aligns more closely with participants 2 4, and 6. In this case, she happens to align most closely with the later group.

One unique occurrence happened during the watching of this video that seems important to note. Participant 4 not only watched the entire video, but came very close to watching the video twice. After letting the video play through once from start to finish, this participant skipped backward in the video to rewatch the video almost from the beginning. While we may be tempted to say that those who watched all of the video had a greater “focus on details” than those who skipped parts of the video, it should be noted that the effort put forth by participant 4 to watch the video a second time most certainly shows a focus to the detail of the content.

Patterns Across Task 2

In this second activity, participants were directed to a series of five sites that contained both static and dynamic content (though not necessarily both within a single site). Each individual site was analyzed for how the participants were focusing on the information contained within the sites. With the sites where both dynamic and static content was presented, the attention was centered around whether the attention of the participant was focused on the dynamic content, the static content, or both. For the videos that were included in the list, the focus shifted to describing the percentage of the overall video that was viewed by the participants. This served as a proxy for describing how focused on details the participants were.

Across the sites, participants 2, 4, and 6 often behaved very similar to each other. Among this behavior includes spending the most time exploring many of the sites. They spent the most time examining the second site as out of all participants, were in the four highest percentages of the video watched in fourth site, and also were three of the four participants who watched all of the video in site five. This is in contrast to participants 1, 3, and 5 who typically spent the least time engaging with any of the five sites. Participant 7 swung back and forth, sometimes behaving in ways similar to the first group and sometimes behaving in ways similar to the other.

Summary

Overall, this section looked at the searching behavior of the participants in this study as they searched online for information. The analysis was broken into two parts, first focusing on the open searching activity, and then the second activity where sites were provided to the participants. In this analysis, where appropriate, I examined both the ways in which participants used Internet search engines to find resources (similar to examining how someone finds books in a library), and also the focus of attention when searching for information within an individual site (similar to examining how someone finds information within a book). A couple patterns of behavior were uncovered as part of this study.

In general, the participants all had very similar information seeking behaviors when they were searching for information between pages and content sites. There did not appear to be many noticeable distinctions in participants based on this type of analysis. When focusing on how participants searched for information within individual pages, it was found that all participants again spent at least some time focusing intently on the content of the pages.

A unique distinction appeared when looking at the kinds of content these participants spent time investigating. One group (made up of participants 2, 4, and 6) examined a diverse format of content, splitting their time almost equally among static and dynamic content (identified during their searching within Task 1). This same group was also found to spend, on average, the most time engaging with the content of a site, and watching online videos (identified during their searching within Task 2). Contrasting with this first group was a second group (made up of participants 1, 3, and 5) that showed a preference for one type of format of content, either static, or dynamic (identified during their searching within Task 1). This second group also was more likely to spend the least amount of time exploring sites (identified during their searching within Task 2). Participant 7 fluctuated in behavior between these two groups.

This chapter investigated the question, “what are information-seeking strategies that preservice teachers use when locating mathematical information online?” Based on the performance of these seven preservice teachers, we know that there tended to be a strong pattern of timid navigation when navigating between search engine results and individual websites. Additionally, when looking at information in individual sites, there was a strong pattern among all the participants of focusing on the content within the sites. However, two different behavior groups emerged when examining the resources available online. One group tended to spend longer examining individual resources, while also seeking a balance between static and dynamic content. A different group tended to spend less time with resources while also preferring a single format of resource, either static or dynamic. It was particularly interesting that there seemed to be a strong correlation between the amount of time spent exploring resources, and whether there was a preference for a single format of resource, or a balance of resources. This work shows that there appear to be information-seeking strategies common to the preservice teachers in this study, as well as different patterns of information-seeking strategies.

In the next chapter, the results of the prior chapter (which looked at the mathematical connections that participants were able to make in their concept maps), and this chapter are compared and contrasted. Patterns of similarity and divergence are discussed, as well as what might be underlying influences that lead to these correlations.

CHAPTER 5

To assess the impact of searching for mathematical information online, participants constructed concept maps before, during, and after their Internet searching tasks. These concept maps depicted how they were structuring their understanding of the content around the Pythagorean Theorem. The ultimate goal of this analysis is to better understand the connections that the preservice teachers were making between pieces of mathematical content and how these connections changed as participants progressed through the two searching tasks. To do this, the concept maps (and specifically the links between the content included in the concept maps) is analyzed in this chapter.

The following figures are two concept maps created by a single participant in this study. The first was constructed at the start of the interview session before either activity was administered (Figure 9), and the second was at the end of the session after the completion of both activities (Figure 10).

Figure 9 – Pre Concept Map from Participant 5

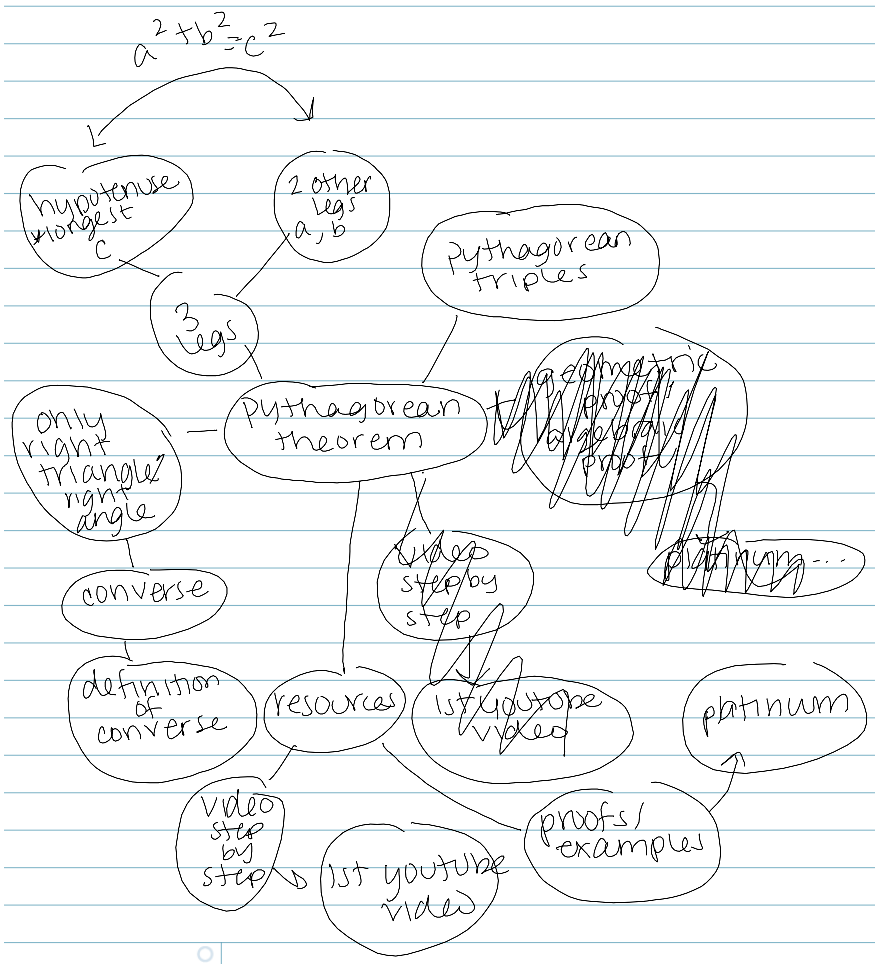


Figure 10 – Post Concept Map from Participant 5

Although these two maps seem very similar with their general structure and form, there are important differences to note, both with the content of the maps and the links that this preservice teacher included in each of the maps. This chapter describes the work I did to unpack these maps, and all of the maps produced in this study. I will return to these two maps in each section of this study to help illustrate some of the concept that I describe.

Chapter Structure

Although the main goal of this study is to investigate the connections included in the concept maps, this chapter first begins with an analysis of the mathematical content of the concept maps. Next the framework used to analyze the links in the concept maps is presented before continuing to the results of the analysis. Next, a brief look at the overall structure of the concept maps is presented. Finally, the chapter ends with a summary of the main findings, as well as how these findings help answer the question of what form and quality of mathematical connections do preservice teachers make around the Pythagorean Theorem after locating information online?

Mathematical Content of Created Concept Maps

As described in the methods section, during each construction of the concept map, preservice teachers were provided with a list of 12 terms that will sometimes be brought up in conjunction with the Pythagorean Theorem. Participants were instructed that this was only a partial list and they were free to use some, none, or all of this list when they were creating their own concept maps that reflected their current thinking around the Pythagorean Theorem. This section examines the content that participants included in each of their concept maps including how much of it came from the provided list and how much was content beyond what was provided.

As stated in the literature review and methods section, this study use the "construct-a-map" technique where preservice teachers were provided a list of content that they may wish to include in their maps (Ruiz-Primo et al., 2001). Table 1 below shows the content that preservice teachers used in each of their mappings. Each grouping of three columns shows the content in each participant's Pre (Pr), Middle (Mi), and Post (Po) mapping. The first twelve rows of the table are the content that was included in the activity prompt. The subsequent rows are content that the preservice teachers included without prompting.

We can use the two concept maps (Pre and Post) from Participant 5 presented at the start of this chapter to help illustrate the idea of content. This preservice teacher included in her pre map six pieces of content. Five of these pieces were from the original list provided as part of the prompt (right angle, right triangle, side, hypotenuse, Pythagorean Theorem) and one was content not included in the original list (the formula). Looking at the concept map this participant created after completing the activities (her post map) we see that she included eleven pieces of content. Seven pieces of content were from the list provided: right angle, right triangle, hypotenuse, and Pythagorean Theorem from before, and now adding leg, proof and converse. Four pieces of math content were not included as part of the list provided: the formula from before, and not adding Pythagorean triples, the definition, and applications of the theorem. A listing of all the different content included in each of the maps created is contained in Table 1.

	Participant 1			Participant 2			Participant 3			Participant 4			Participant 5			Participant 6			Participant 7		
Content	Pr	Mi	Po	Pr	Mi	Po	Pr	Mi	Po	Pr	Mi	Po	Pr	Mi	Po	Pr	Mi	Po	Pr	Mi	Po
Triangle		X	X		X	X	X	X	X							X	X	X			
Angle					X													X			
Right Angle	X			X			X	X	X		X		X	X	X		X		X	X	X
Right Triangle	X			X	X	X	X	X	X	X		X	X	X	X	X	X	X	X	X	X
Side				X	X	X				X			X			X	X	X	X	X	X
Hypotenuse	X	X		X		X	X	X	X	X			X	X	X	X	X		X	X	X
Leg	X						X	X	X	X				X	X				X	X	X
Length	X	X		X		X	X	X	X							X	X	X			
Square		X	X	X	X	X	X	X	X								X	X			
Pythagorean Theorem	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Proof	X	X	X	X	X	X	X	X	X	X	X	X			X		X	X	X	X	X
Converse		X	X			X			X		X	X		X	X		X		X	X	X
Formula	X	X	X	X	X	X				X	X		X	X	X	X		X	X	X	X
Triples	X				X	X									X	X	X	X			X
Definition		X	X												X						
Perimeter		X																			
Area		X			X			X	X												
Application			X	X	X	X									X		X	X			
Pythagoreans				X						X											
Algebra					X	X															
Congruency					X													X			
Coordinate System					X							X				X	X	X			
Diagonals						X															
Rectangles						X															
Construction						X															
Base										X											
Wheel of Theodorus										X											
SSS																		X			
Special Cases																		X			X

Table 9 – Content Included in Concept Mappings

The content described in Table 9 is summarized in Table 10 and visualized in Figure 11 below. This new table shows the counts of tallies broken apart by how much content included in the concept map came from the list provided as part of the task, and how much was from outside the list provided.

		Content Given	Content Not Given	Total Content
Participant 1	Map – Pre	7	2	9
	Map – Middle	7	4	11
	Map - Post	5	3	8
Participant 2	Map – Pre	8	3	11
	Map – Middle	7	7	14
	Map – Post	9	7	16
Participant 3	Map – Pre	9	0	9
	Map – Middle	9	1	10
	Map – Post	10	1	11
Participant 4	Map – Pre	6	4	10
	Map – Middle	4	1	5
	Map – Post	4	1	5
Participant 5	Map – Pre	5	1	6
	Map – Middle	6	1	7
	Map – Post	7	4	11
Participant 6	Map – Pre	6	3	9
	Map – Middle	10	3	13
	Map – Post	8	7	15
Participant 7	Map – Pre	8	1	9
	Map – Middle	8	1	9
	Map – Post	8	3	11

Table 10 – Summary Counts of Content in Each Concept Map

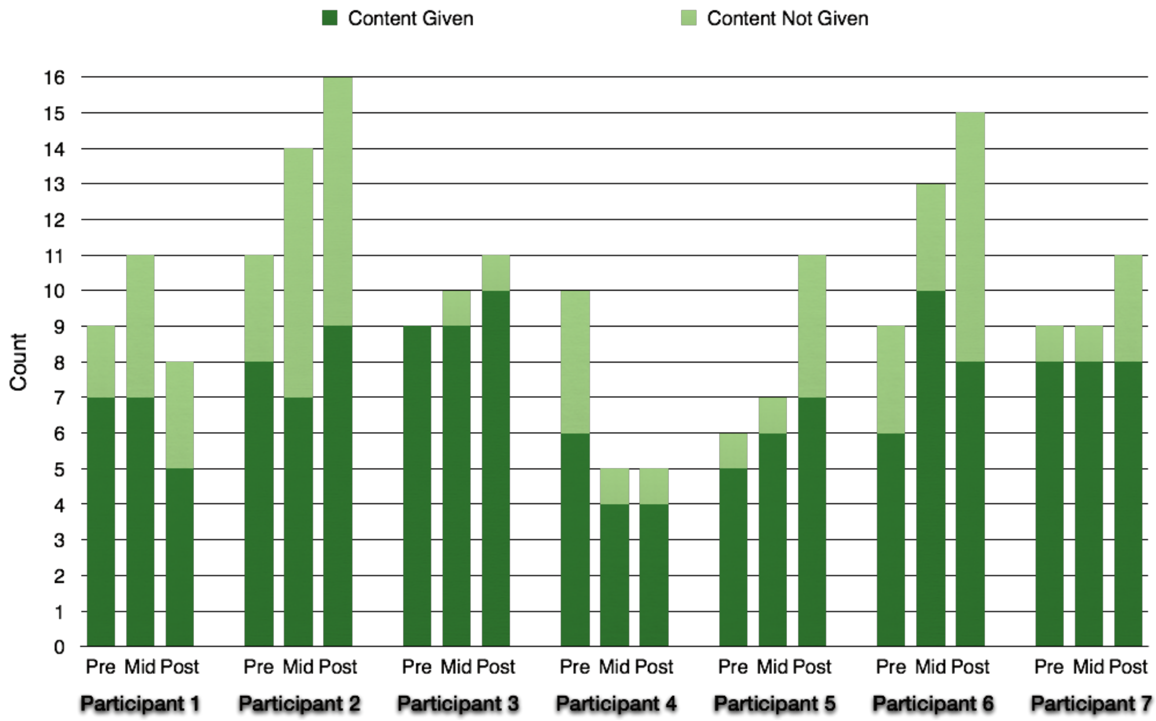


Figure 11 – Counts of Content in Each Concept Map

Unpacking the Counts of Content

When looking at these two tables and figure, a couple patterns begin to emerge. For most all of the preservice teachers, they increased the amount of content included in their concept maps with each successive mapping (5 of the 7 participants). The decrease in the variety of content for Participant 4 is a result of her increasing focus solely on the proof of the Pythagorean Theorem. The early peak of content in Participant 1 seems to be due to her choice of consolidating information in her final map, as opposed to a continued expansion that was typical of many others.

In general it is important to note that, as a whole, the amount of content increased with each successive generation of concept maps produced by participants. This suggests that as the preservice teachers engaged with the two tasks, they chose to represent how they were thinking about the Pythagorean Theorem with more content, which then allows for these additional ideas to be incorporated and connected with other content in the later mappings.

This section has taken a look at the content that was present in the mappings produced by the participants. The next section will explore the different forms of links that participants described between the content that they included in their concept maps. From this we will be able to get a snapshot of the forms and quality of mathematical connections that the preservice teachers had among the content.

Mathematical Links Among Content

In the prototypical concept map, nodes (content) are connected with words and phrases that help explain the relationships between two different pieces of content. Because the preservice teachers in this study were novice concept map makers and might not connect their nodes with phrases, the concept maps created during the activities were later augmented with the text spoken by the preservice teachers. This text may have been spoken as they were creating the concept maps, or later when they were asked to provide an overview for the concept maps they created. This text was included in an effort to provide additional information that would help make visible the connections participants had between the pieces of content they were joining.

The augmented concept maps were broken into smaller chunks, isolating the links that participants were including. These chunks were coded for the mathematical links that were contained in each chunk. Each mathematical link was given a code based on two aspects of that mathematical relationship, the form and the quality of the mathematical relationship. This section begins by defining the forms and qualities of mathematical links utilized in the analysis. Later, results from the coding of the concept maps are presented with an eye towards identifying patterns that emerged out of the coding.

Framework for Forms and Quality of Links

In his 2005 article, Jon Star unpacked an underlying issue that existed in some frameworks of mathematical knowledge. Instead of using the terms procedural and conceptual to describe two aspects of a single phenomenon, Star argued that some research literature had been overlapping two different phenomena into these labels. Star unpacked the ideas of the forms (kinds) of knowledge, commonly referred to in the literature as procedural or conceptual, from the quality of that knowledge, which he described as rich vs. superficial (Star, 2005). Star argued that in the literature, conceptual knowledge was often times used to mean richly connected knowledge, and procedural knowledge was often times used to indicated superficial knowledge. Instead, it was argued that conceptual knowledge (knowledge of a concept) could be richly or superficially connected. Similarly, knowledge of a procedure could just as well be richly or superficially connected. This could also be illustrated with a table as in Table 11 below.

		Quality of Knowledge	
		Rich	Superficial
Forms of Knowledge	Conceptual		
	Procedural		

Table 11 – Forms and Quality of Mathematical Knowledge

Although this distinction was elaborated by later researchers (Baroody, Feil, & Johnson, 2007), the fundamental distinction of paying attention to the form of mathematical knowledge separate from the quality of that mathematical knowledge remains a crucial frame for describing mathematical content. Just as this lens allows us to help understand the form and quality of the mathematical connections students are making as they engage with the mathematical content, we can also see the form and quality of the mathematical connections preservice teachers are making.

The distinction between conceptual knowledge and procedural knowledge seems at the surface to be a fairly easy distinction to make based upon how it is discussed, either the piece of knowledge is about an underlying mathematical concept, or it is about a procedure, an application of that concept. However, when trying to classify pieces of content in practice, the distinctions between concepts and procedures become much more nuanced. This is complexified when analyzing concept maps as these representation will often times be comprised of brief words or phrases, which don't make clearly visible some of the meanings involved. In these cases, the augmenting of the concept maps with spoken text helps contextualize these phrases and provides more information around how the content is being described.

Definitions and Examples of Forms and Qualities

From these augmented concept maps, the two forms of knowledge that were coded in each of the concept maps are described according to the following descriptions:

- Conceptual – Link describes an underlying mathematical phenomena (i.e. defining it, listing properties of it), or extension of mathematical phenomena to related contexts (i.e. real world examples of phenomena, special cases of phenomena)

- Procedural – Link describes a mathematical process (i.e. describing a step or series of steps) or represents a phenomenon (i.e. drawing a picture, writing a formula).

While the process of augmenting the concept maps provided some new insights into the reasoning preservice teachers were using to structure their concept maps in particular ways, the text they spoke didn't always make every decision transparent. In particular, it was common for participants to explain why they included particular pieces of content, but it was less common for them to explicitly say what they chose to link two pieces of content, thus allowing for a clear picture of the math connection they were making. Sometimes the reasoning for these links were embedded within their descriptions of the content, which made coding the forms of math connections more challenging.

In addition to the codes around the forms of mathematical links, there were also codes around the quality of that mathematical links as it was described in the augmented concept maps. The below distinction was used in categorizing the quality of the included content contained in each map as either rich or superficial:

- Rich – The link contained a description of a mathematical phenomena (i.e. mathematically accurate definition) or described connections between mathematical phenomena.
- Superficial – The default quality of mathematical content coded. Described phenomena was either incomplete or mathematically incorrect (i.e. an incomplete definition) or possibly contained a mathematical error.

The brief text included in these maps created by these novice concept map makers presented some difficulties in assigning a quality code to the mathematical content contained in each map. The default code for quality of these mathematical links was superficial unless there was present enough description (either in the actual or augmented map) that allowed for me to see some rich links of mathematical content. By having the default code of quality be superficial, the results obtained as part of this coding are, by nature, more conservative than having the default code of rich.

The below table shows the four different combinations of forms and qualities of mathematical links and provides an illustrative example of each.

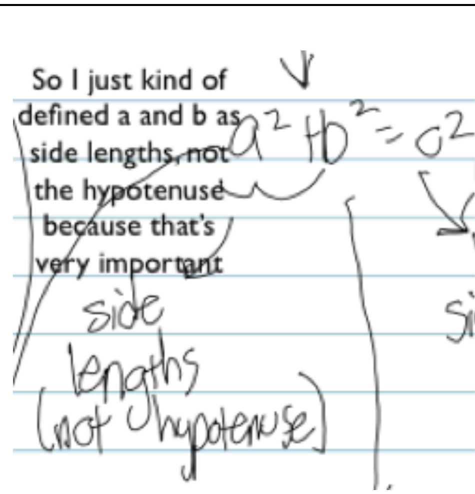
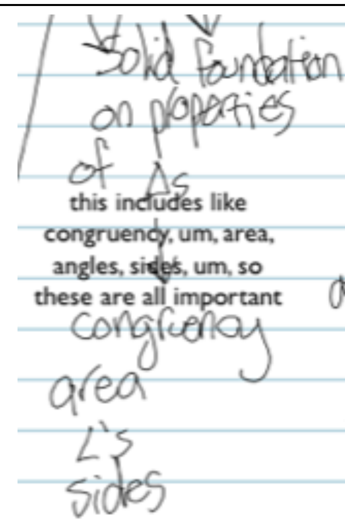
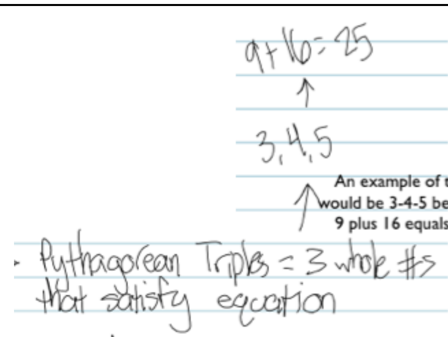
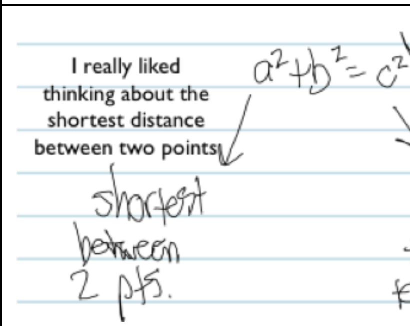
		Quality	
		Rich	Superficial
Form	Concept		
	Procedure		

Table 12 – Examples of Forms and Qualities of Mathematical Links

My understanding of the different aspects of these codes became greater and more nuanced as I went through more and more coding experiences with the data of this study. In order to ensure that the coding that occurred during data analysis was as uniform as possible, the data from the beginning of the analysis process was recoded a second time after a complete pass through all the data. This was an attempt to ensure that the codes given to early participants were assigned through the same understanding of the rubric as those later participants.

Illustration of Mathematical Link Coding

Before beginning this analysis, in an effort to focus on understanding what mathematical connections were closely related to the Pythagorean Theorem, two texts that describe the Pythagorean Theorem, one used in the preservice teaching elementary mathematics content course (Beckmann, 2011), and a textbook commonly used in middle school classrooms (Lappan et al., 2009) were analyzed. From this analysis, a list was created of mathematical connections related to the Pythagorean Theorem. This served as the original starting list of connections for the identification and coding of mathematical connections in the augmented concept maps.

Below are the same two concept maps created by Participant 5 both before the beginning of the activities and after completing the activities. However, these two concept maps, the pre (Figure 3) and post (Figure 4) have also been augmented with the text spoken by the participant when describing their concept maps. After this augmentation, the maps were chunked into the different links that the participants were making to make the entire map easier to analyze.

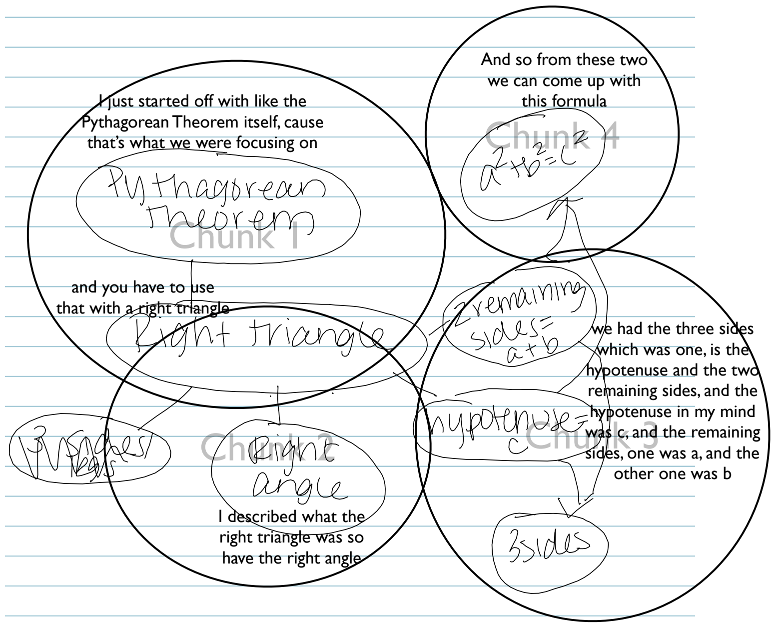


Figure 12 – Augmented and Chunked Concept Map from Participant 5 (Pre)

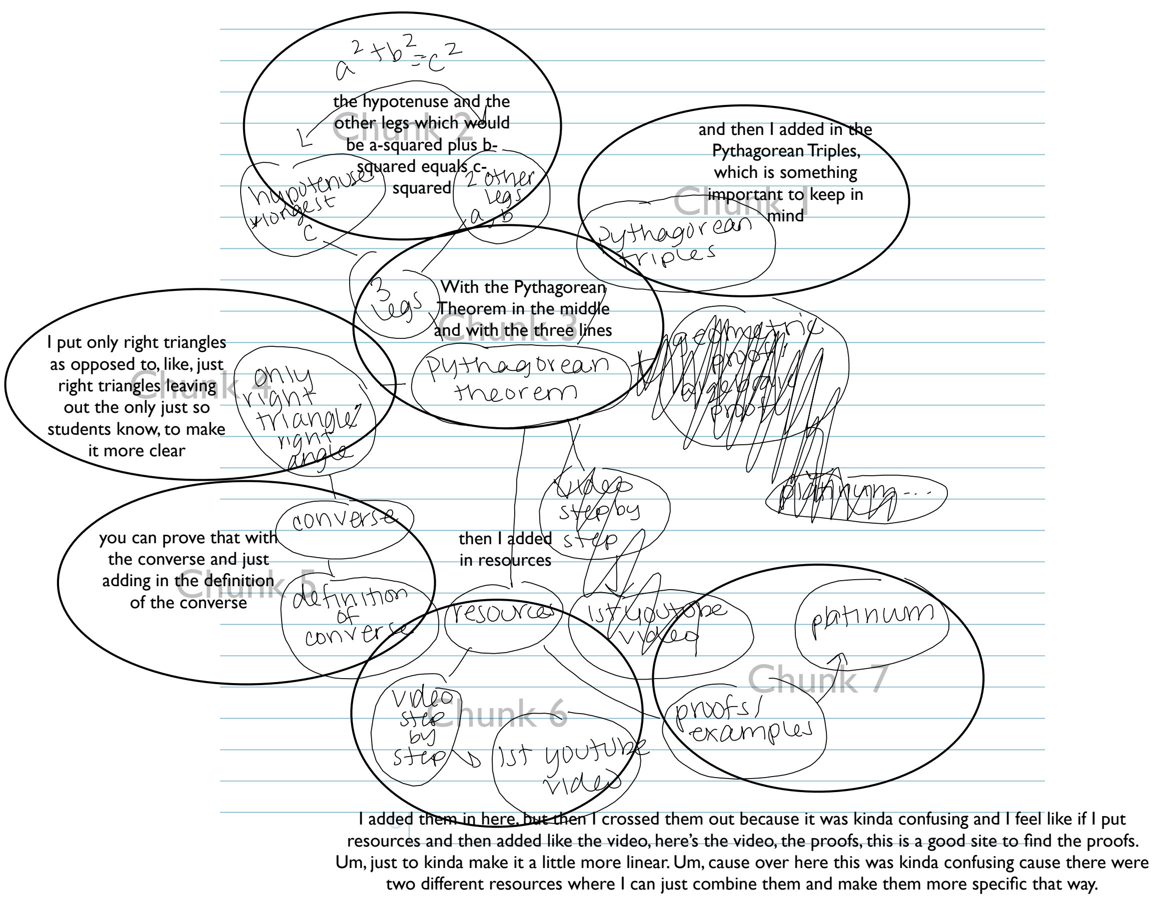


Figure 13 – Augmented and Chunked Concept Map from Participant 5 (Post)

For Participant 5's pre concept map (Figure 12) we can see the links that the participant included in their concept map around the vocabulary of and properties of shapes (Chunk 2 and 3), the stating and meaning of the Pythagorean theorem (Chunk 4) and the Conditions and Generality of the Pythagorean Theorem (Chunk 1). All of these links were of the conceptual form, and all were superficial.

When looking at this participant's post concept map (Figure 13), we see that there are more links among the mathematical content included, 8 in total. This time the links still included the vocabulary and properties of shapes (Chunk 2 and 3), the stating and meaning of the Pythagorean Theorem (Chunk 2) and the conditions and generality of the Pythagorean Theorem (Chunk 4). However, this post concept map also includes linkss around the proof of the Pythagorean Theorem (Chunk 6), applications of the Pythagorean Theorem (Chunk 7), stating and meaning of the converse to the theorem (Chunk 5), and applications to the converse (Chunk 1). Again, most of the links made by this participant appear to be of the conceptual form and superficial quality. However, the links made around the proof and the applications of the Pythagorean Theorem were procedural links, and the information included when discussing the conditions and generality of the theorem suggested a rich understanding of the links. From this we can see links in the post mapping that weren't included in the pre mapping for Participant 5. Not only were there more links included in this mapping, but those links were of a greater variety (both concepts and procedures) and of different qualities (evidence of a rich link). The following section contains a summary of this analysis for all of the concept maps produced as part of this study.

Summary of Form and Quality of Mathematic Links in Concept Maps

The table and figure below (Table 5 and Figure 2) are a representation of the forms of mathematical knowledge (procedural or conceptual) contained in each of the concept maps based on the categories of links. In the table, the three concept maps of each participant are presented sequentially, with first their pre map, middle map, then post map.

		Procedure	Concept	Total Form
Participant 1	Map – Pre	1	5	6
	Map – Middle	2	5	7
	Map - Post	5	6	11
Participant 2	Map – Pre	3	6	9
	Map – Middle	2	6	8
	Map – Post	2	8	10
Participant 3	Map – Pre	1	6	7
	Map – Middle	1	5	6
	Map – Post	4	6	10
Participant 4	Map – Pre	4	5	9
	Map – Middle	4	4	8
	Map – Post	5	5	10
Participant 5	Map – Pre	0	4	4
	Map – Middle	0	5	5
	Map – Post	2	6	8
Participant 6	Map – Pre	5	3	8
	Map – Middle	4	4	8
	Map – Post	6	5	11
Participant 7	Map – Pre	1	5	6
	Map – Middle	1	4	5
	Map – Post	1	7	8

Table 13 – Form of Mathematical Link Contained In Concept Map

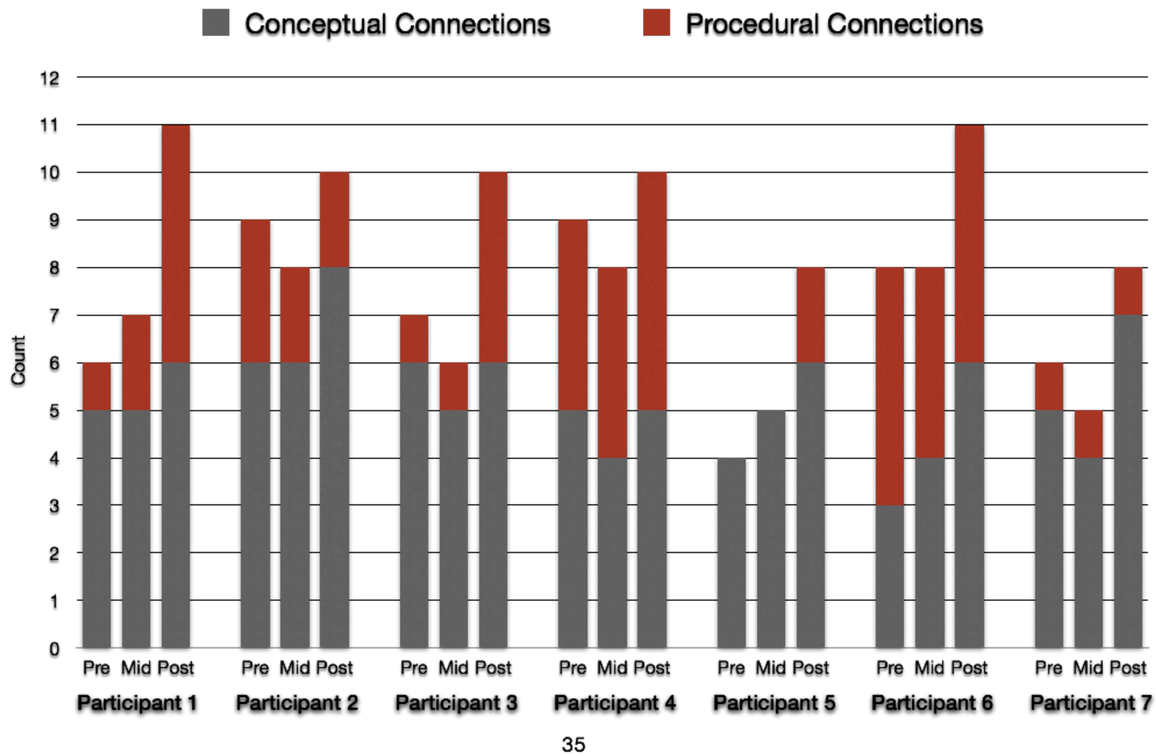


Figure 14 – Forms of Mathematical Links Contained in Concept Maps

Noticed that for every participant, the amount of mathematical links included in the concept mappings is highest in the final, post mapping created during each administration. The smallest amount of links included in each mapping is contained either the pre or mid mapping. The implication here is that after doing these tasks of searching for mathematical information online, the preservice teachers included more mathematical links in their concept maps than they did before they completed both tasks.

Focusing specifically on the form of mathematical links included within each concept map (Table 13 and Figure 14), it is noticed that in almost every mapping, the amount of conceptual links included is greater than the amount of procedural links. This may be due to the nature of the activity which asked the preservice teachers to create a "concept map", which suggests that it may be more likely to include conceptual links rather than procedural ones. Additionally, when looking at the procedural links included in concept maps, two different patterns emerged. For some preservice teachers, the amount of procedural links remained fairly constant during subsequent iterations of their maps (Participants 2, 4, 6, and 7). Contrasting this consistency were the participants whose procedural links went up in their post mapping, as compared to their first two pre and mid mappings (Participants 1, 3, and 5). For all three of the participants in this group, this was due to the inclusion of links related to the proof of the Pythagorean theorem, proof of the converse to the Pythagorean theorem, or both.

		Superficial	Rich	Total Quality
Participant 1	Map – Pre	4	2	6
	Map – Middle	6	1	7
	Map - Post	11	0	11
Participant 2	Map – Pre	7	2	9
	Map – Middle	7	1	8
	Map – Post	7	3	10
Participant 3	Map – Pre	7	0	7
	Map – Middle	6	0	6
	Map – Post	10	0	10
Participant 4	Map – Pre	5	4	9
	Map – Middle	6	2	8
	Map – Post	7	3	10
Participant 5	Map – Pre	4	0	4
	Map – Middle	5	0	5
	Map – Post	7	1	8
Participant 6	Map – Pre	5	3	8
	Map – Middle	3	5	8
	Map – Post	10	1	11
Participant 7	Map – Pre	6	0	6
	Map – Middle	5	0	5
	Map – Post	8	0	8

Table 14 – Quality of Mathematical Links Contained In Concept Map

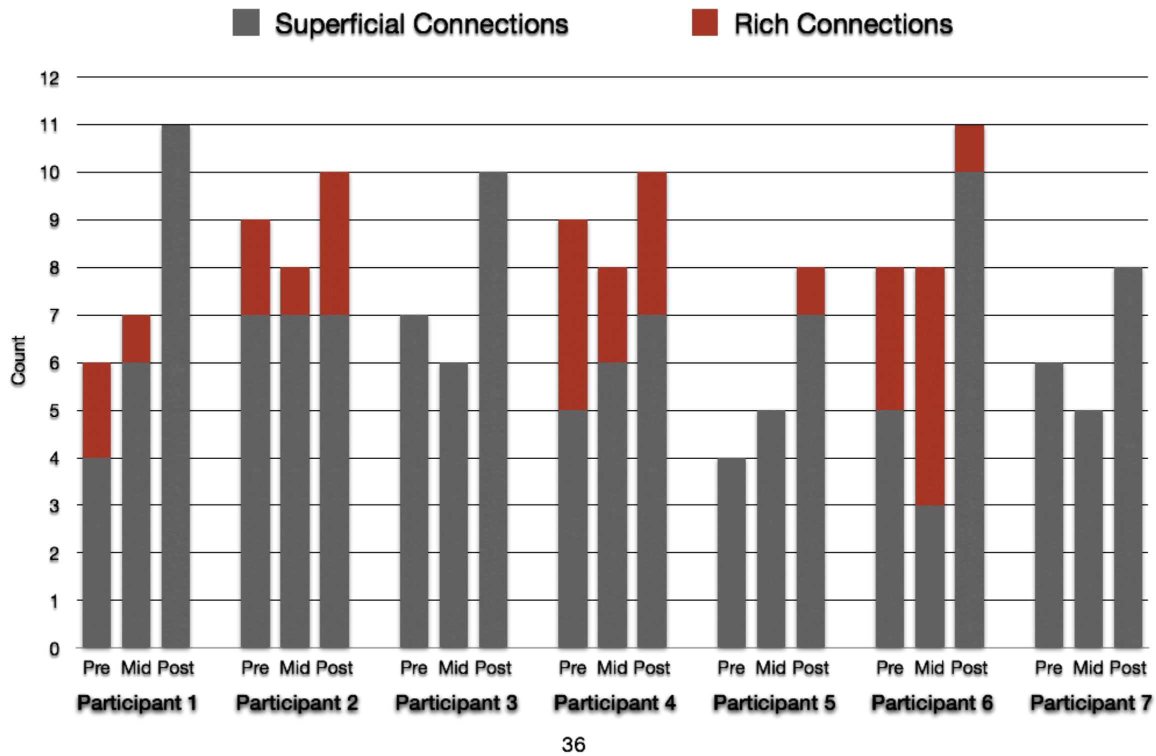


Figure 15 – Quality of Mathematical Links Contained in Concept Map

When focusing on the quality of the mathematical links included in the concept maps (Table 14 and Figure 15), one general pattern that emerges across all of the concept maps is that the amount of mathematical links that are richly described is almost always less than the amount of links that are superficially described. As stated before, the default code for this category was "superficial". The brief phrases that many preservice teachers utilized during the creation of their concept maps, or in describing their maps, did not always provide explicit evidence that there were rich links between pieces of content. This might help explain why the superficial understanding codes seem to outweigh the rich understanding codes.

It is also noted that for four of the preservice teachers (Participants 1, 3, 5, and 7) each produced at least one iteration of their concept map that contained no rich mathematical links (Participants 3 and 7 never included any descriptions in any iteration of their maps that were coded as rich). This contrasts with Participants 2, 4, and 6 which had some rich mathematical links contained within each iteration of their concept maps.

Putting together the patterns noticed in both the forms of mathematical links coding and the quality of mathematical links coding, I notice a correlation. For Participants 1, 3, and 5, they included few procedural forms of links in their pre and middle maps, but noticeable more in their post map. These participants also had at least one iteration (usually more) of their concept map that contained no richly described mathematical links. This is in contrast to Participants 2, 4, and 6, where the amount of procedural links included in all iterations of their map remained about the same while also producing maps that always contained some rich links. Participant 7, though displaying behavior that could put her in each of these groups, seems to have produced maps that are much more similar to Participants 1, 3, and 5.

Characteristics of Forms of Links	• Heavy reliance on conceptual links • More procedural links appear in later mappings	• More balance between conceptual and procedural links • Consistent amount of procedural links in each mapping
Characteristics of Qualities of Links	• At least one mapping lacking any rich links	• Every map contains at least one rich link
Participant	• Participants 1, 3, 5, & 7	• Participants 2, 4, 6

Table 15 – Categories of Behavior of Participants

This section has sought to uncover some patterns that emerged among the preservice teachers concept maps when looking at the forms and quality of mathematical links they were including in their maps. The next section looks at the structure of the concept maps as a whole.

Structures and Purpose of Created Concept Maps

The previous sections looked at the mathematical content and links among the content in the pre, mid and post mappings of the participants. These were analyzed for each individual mapping, and then the results were compared across mappings to see what changes or alterations occurred. This was done as a change in the iterations of the concept map might represent that a change of understanding has occurred within the individual. On a larger scale, all of the content and links in these concept maps are represented with overall structures. Large changes in understanding may be made visible through changes to the overall structures of the maps. This section looks at the structures utilized by the participants when creating their maps, and how these structures may have changed over the course of the activity.

Structure

There are several different forms that concept maps can take. Originally, maps were said to always require a hierarchal structure, with concepts and content arranged in an ascending or descending order. However, others have also accepted radial structure maps with an idea is located at the center of a map and branches (and sub branches) radiate outward from the original idea (Williams, 1998). The below table lists the general structure used in each of the concept maps created by participants.

Participant	Pre Map	Mid Map	Post Map
1	Tree	Tree	Tree
2	Tree	Tree	Tree
3	Radial	Radial	Radial
4	Radial	Radial	Radial
5	Radial	Radial	Radial
6	Radial	Radial	Tree
7	Radial	Radial	Radial

Table 16 – Structure in Each Concept Map

One observation from this table is noticing that almost every participant chose to utilize the same structure for their concept map in every iteration of construction. Despite constructing each new concept map from scratch there was a strong tendency for the structure a participant used during their first construction to be utilized during each subsequent construction. This suggests that overall, these tasks did not produce large changes in how participants were understanding the content of the Pythagorean Theorem, and therefore, there was no need to significantly alter how they were representing their understanding.

Purpose

During this activity, I noticed that the participants who utilized a tree hierarchy for their concept maps seemed to have a different purpose for their concept maps, compared to the diagrams that utilized a radial hierarchy. Those that made us of the tree hierarchy seemed to have an instructional or pedagogical purpose for the maps that were constructed. Here, the preservice teachers commented or wrote that particular pieces of content, or particular links they were making between pieces of content, were being made of the benefit of students, or it was something they wanted to teach in a particular way. The suggestion, either explicit or implicit of an instructional sequence was absent from those who utilized the radial sequence. This may be because the nature of the tree structure lends itself well to a sequential presentation of ideas. This is in contrast to the radial structure which does not lend itself well to an order of first, second, third, etc.

The nature of the task and the background of the participants make it unsurprising that many of these maps would incorporate not only the content, but also the teaching and learning of that content. However, the attention paid to these pedagogical themes occurred only with the tree hierarchy structure and not the radial hierarchy structure.

Summary of Results

This section focused on the mathematical content included by preservice teachers in concept maps they created before, during, and after completing Internet searching tasks. The main analysis of this chapter focused on describing the form and quality of the links between pieces of content in these maps. Additional analysis also examined the content itself, and the overall structure of these mappings.

When analyzing the links among the content that was included in the concept maps, two patterns of behavior appeared. There was one group of participants (Participants 2, 4, and 6) who included both procedural and conceptual links in their concept maps, maintaining roughly the same amount of procedural links in all of their mappings. This group also included rich links among their content in every one of their mappings that they produced. This pattern of behavior is in contrast to that of a different group of participants (Participants 1, 3, 5, and 7). This group created concept maps that had a heavy reliance on conceptual links with almost no procedural links (until their final mappings, which did include some procedural links). Additionally, this second group always had at least one of their three mappings (and usually multiple mappings) contain no rich links.

The analysis of the content in the mappings of the preservice teachers, showed that the participants tended to include more content after completing each activity, as compared to the map they previously constructed, suggesting that these tasks may have had an impact on the additional content. There was a significant amount of content not contained in the provided list that which suggests that the preservice teachers were incorporating content they knew or that they picked up from the explorations into their representations. In the investigation of the overall structure of the concept maps, I found that the participants overall did not change the structure of their concept maps each time they reconstructed it. Additionally, the participants who used a tree hierarchal structure for their maps also included pedagogical comments or descriptions in their mappings.

There is one note to make about the results of this analysis. This chapter examined the mappings of the preservice teachers in this study, who created their own representations of how they were thinking about the content and links among the content. I do not claim that these concept maps are perfect representation of how these preservice teachers are thinking about the content. Additionally, I do not claim that the absence of content, or a superficially described link among content means that the preservice teacher doesn't know the content or have a rich understanding of the content itself. All that is being analyzed is how these preservice teachers are choosing to represent their understanding of the content via these concept maps.

This chapter investigated the research question of, what form and quality of mathematical connections do preservice teachers make around the Pythagorean Theorem after locating information online. In general, it was found that the preservice teachers tended to represent their connections with conceptual links among content maps that tended to be superficially described as a whole. However, while this was generally true of all the participants, two patterns of behavior emerged, one very similar to the overall pattern, and a different group that had an increase in the amount of procedural links in their mappings and also an increase in the amount of richly described links in their mappings too. Overall, there is evidence that after locating information online, participants represent their understanding of the content with a greater amount of content and more links among that content. Additionally, there is also evidence that some participants have a more varied understanding of the content (both concepts and procedures), and choose to represent that understanding in visibly rich ways too. This shows when completing the tasks of this study there was an increase in both the range (amount) and depth of understanding (quality) represented in the concept maps.

The next chapter looks at the results of the previous searching strategies chapter, and this math task chapter and looks for patterns in the results. This chapter will connect the ways in which the searching strategy behavior of participants might also be related to the forms and quality of mathematical connections the preservice teachers included in their concept maps.

CHAPTER 6

The previous two chapters explored two aspects of investigating mathematical information online: the information-seeking strategies that preservice teachers employed when completing Internet-searching tasks, and the mathematical content and links that preservice teachers displayed in concept maps when exploring this environment and content. This chapter explores what connections exist between these two different aspects of the investigation. Throughout this chapter, I will be reviewing the findings from the analyses performed in this dissertation and how they help answer the research questions of this study: As these results are presented, I fill in the table below (Table 17) to help organize the information and the patterns that emerge from these results.

<table>
<tr><td rowspan="2">RQ1</td><td>Format of Accessed Resource</td><td rowspan="2">• What are <u>information-seeking strategies</u> that preservice teachers use when locating mathematical information online?</td></tr>
<tr><td>Time Spent Exploring</td></tr>
<tr><td rowspan="2">RQ2</td><td>Form of Links</td><td rowspan="2">• What <u>form and quality of mathematical</u> connections do preservice teachers make around the Pythagorean Theorem after locating information online?</td></tr>
<tr><td>Quality of Links</td></tr>
<tr><td rowspan="2">RQ3</td><td>Related Patterns of Behavior</td><td rowspan="2">• What <u>relationships</u> emerge between the form and quality of the mathematical connections made by preservice teachers, and the information-seeking strategies employed during the research task?</td></tr>
<tr><td>Initial Goals at Task Launch</td></tr>
</table>

Table 17 – Table of Categories from Findings (Empty)

Review of Search Strategy Findings

In chapter four, I presented the findings of this dissertation as they related to the information seeking strategies utilized by the preservice teachers when completing two tasks. That chapter was broken into two sections, describing the information seeking strategies employed during the open-searching task, and those employed during the closed-searching task. During the open-searching task, I found that all participants used a timid-navigator approach and only two participants, at any point in time during their exploration, used a divergent navigator approach. When exploring they ways that participants explored information within a particular results page, it was found that all participants, at some point, focused on the content, giving sustained, close attention to some aspect of content on a page. It was also discovered that participants fell into two groups with regards to the kind of content that they were focusing on (either static content or dynamic content). One group of participants clearly favored one format over the other format while the other group had more balance with the forms they explored.

When analyzing the results of the information seeking that occurred during the closed searching activity, many similar trends were found. Again, at some point during this activity, every participant focused on some content, providing a sustained, close read of the information within the pages. Additionally, those who accessed a balance of formats in their first task were also the ones who had longer exploration times with sites during the second task. Those who spent the majority of their time with one format of information during the first activity were also the participants who had shorter exploration times during the second task.

The differences in the two different groups found when looking at the information seeking strategies can be displayed in the updated table below (Table 18). The different groupings had two characteristics, one pertaining to the formats of the resources they accessed, and the second with the amount of time they spent exploring the accessed resources. These groupings were one of the main results found when answering the first research question about the searching strategies used by the preservice teachers. Next I review the results obtained when looking at the second research question.

<table>
<tr><td rowspan="2">RQ1</td><td>Format of Accessed Resource</td><td>• Preference for one format (either static or dynamic) of content</td><td>• Preference for both formats (static and dynamic) of content</td></tr>
<tr><td>Time Spent Exploring</td><td>• Shorter exploration times</td><td>• Longer exploration times</td></tr>
<tr><td rowspan="2">RQ2</td><td>Form of Links</td><td colspan="2" rowspan="2">• What <u>form and quality of mathematical</u> connections do preservice teachers make around the Pythagorean Theorem after locating information online?</td></tr>
<tr><td>Quality of Links</td></tr>
<tr><td rowspan="2">RQ3</td><td>Related Patterns of Behavior</td><td colspan="2" rowspan="2">• What <u>relationships</u> emerge between the form and quality of the mathematical connections made by preservice teachers, and the information-seeking strategies employed during the research task?</td></tr>
<tr><td>Initial Goals at Task Launch</td></tr>
</table>

Table 18 – Table of Categories from Findings (Searching)

Review of Math Content and Connections Findings

In chapter five, I presented the findings of this dissertation as they related to the mathematical content and connections that preservice teachers included in their concept maps they constructed during their experiences exploring the Internet for this study. With regards to the content that they included in the concept maps, the main finding was that the content increased in frequency as participants created successive iterations of their maps after exploring more content.

When looking at the connections between the content that was included in the concept maps, two different aspects were investigated, the form of links that was included (procedural vs. conceptual) and the quality of those links (rich or superficial). I found that the majority of the links that were included in the concept maps were of the conceptual form and the majority of all the connections were of a superficial quality. Like the exploration of the content, I also found that the links increased in quantity as the activity progressed.

From this exploration among the links, two groups emerged in the classifications. One group of participants included most almost all conceptual links in their concept mappings. Additionally, these participants had at least one iteration of their mapping (and frequently multiple iterations of their mapping) that contained no evidence of any rich links. A second group of participants had more of a balance between the conceptual and procedural links included in their maps. These participants always included enough detail in each iteration of their concept maps to provide evidence of at least one rich link.

Lastly, when exploring the overall structure of the concept maps, it was found that participants utilized two different kinds of concept map structures: tree (hierarchal) and radial. Despite constructing each iteration of the concept map from scratch, only one participant changed the overall structure of their concept map, while most everyone decided to make minor alterations to the content and links of previous concept maps. Additionally, it was found that those who chose to represent their understanding of their concept maps with a tree structure also indicated some attention to pedagogical issues when describing their map.

The differences in the two different groups found when looking at the links among the content in the concept maps can be displayed in the updated table below (Table 19). The different groupings had two characteristics, one pertaining to the forms of the links included in the concept maps, and the second pertaining to the quality of those links. These groupings were one of the main results found when answering the second research question about the form and quality of mathematical connections that preservice teachers included in their concept mappings.

<table>
<tr><td rowspan="2">RQ1</td><td>Format of Accessed Resource</td><td>• Preference for one format (either static or dynamic) of content</td><td>• Preference for both formats (static and dynamic) of content</td></tr>
<tr><td>Time Spent Exploring</td><td>• Shorter exploration times</td><td>• Longer exploration times</td></tr>
<tr><td rowspan="2">RQ2</td><td>Form of Links</td><td>• Heavy reliance on conceptual connections
• More procedural connections appear in later mappings</td><td>• More balance between conceptual and procedural connections
• Consistent amount of procedural connections in each mapping</td></tr>
<tr><td>Quality of Links</td><td>• At least one mapping lacking any rich connections</td><td>• Every map contains at least one rich connection</td></tr>
<tr><td rowspan="2">RQ3</td><td>Related Patterns of Behavior</td><td colspan="2" rowspan="2">• What <u>relationships</u> emerge between the form and quality of the mathematical connections made by preservice teachers, and the information-seeking strategies employed during the research task?</td></tr>
<tr><td>Initial Goals at Task Launch</td></tr>
</table>

Table 19 – Table of Categories from Findings (Searching & Math Connections)

The next section takes the results of these first two results chapters that focused on information-seeking strategies and mathematical connections and looks for patterns that emerge out of the results. This is done with the ultimate goal of answering the third and final research question, what relationships emerge between the form and quality of the mathematical connections made by preservice teachers, and the information-seeking strategies employed during the research task?

Relationships Between Search Strategies and Content

In the searching strategy analysis, the group that tended to prefer a single format of site, and spent shorter amounts of time with the sites they explored was made up of Participants 1, 3, 5, and 7. The group that preferred a balance of formats in their searching and spent longer amounts of time with the sites they explored was made up of Participants 2, 4, and 6. In the math content and connections analysis, the group that included almost exclusively conceptual links, and always had at least one of their mappings contain no rich links was made up of Participants 1, 3, 5, and 7. The group that had more of a balance between their conceptual and procedural links and also always included at least one rich connection in their mappings was made up of Participants 2, 4, and 6. There was a strong relation between the pattern of behavior identified when analyzing the searching strategies, and the patterns of behavior identified when looking at the concept maps. Across both of these analyses, two groups emerged. These two group are unpacked briefly here.

One of the groups identified (made up of participants 1, 3, 5, and 7) seemed to be oriented to the big ideas around the Pythagorean Theorem. These participants exhibited behavior during their searching that suggested a particular preference for one format of information presentation, and also a more brief engagement with the content presented. During the first open-searching task, these were the participants who decided to engage with mostly only static content, or mostly only dynamic content, and very rarely engaged with content that was not the format of their preference. During the second, closed searching task, these were the participants who spent the least amount of time engaging with the static website, and the least amount of time with the videos. When representing how they were thinking about the Pythagorean Theorem, they primarily only described concepts, rarely procedures in their maps, and frequently had maps lacking the detail to see rich connections.

One possible explanation for this interaction between the searching strategy and the concept mapping may be that the focus on only one type of presentation format, and the more limited engagement with the content didn't allow for a variety of mathematical connections, nor a rich understanding of those connections, to form. It may be the case that this single format, brief search approach is beneficial for providing Internet searchers with broader ideas of the connections and interactions of mathematical phenomena, allowing Internet searchers to gain a high-level overview of the mathematical terrain. In this way, this quicker and narrower searching can be beneficial for those who are looking to get a survey of the content and begin to understand its connections.

The second group identified (made up of participants 2, 4, and 6) seemed to be oriented to details around the Pythagorean Theorem. These participants exhibited behavior during their searching that suggested a balanced approach to the format of the information presented, and also a more extended engagement with the content presented. During the first open-searching task, these were the participants who split their time across two different formats of presentation (dynamic and static) spending almost equal amounts of time with each format. During the second, closed searching task, these were the participants who spent the most amount of time engaging with the static website, and the most amount of time with the videos. When representing how they were thinking about the Pythagorean Theorem, they described (though still heavy on concept) more of a balance between concepts and procedures, and always produced maps that had enough detail to provide evidence of a rich understanding of the mathematical connections.

One possible explanation for this interaction between the approach taken during the information-seeking activities and the concept map production may be that the exploration of different formats of content, and the extended amount of time spent with that content allowed for a more diverse amount of mathematical connections to form, and allowed for some of that content and those connections to be understood in a rich way. It may be the case that not only does an extended amount of time with content help produce richer understandings of diverse kinds of content, but the time exposed with the content could also be spent looking at a variety of presentation styles of that content. In this way, those looking to understand specifics of content in rich ways need to not only devote time to their investigation, but also explore multiple ways of presenting information.

Classroom Goals of the Participants

From the earlier chapters, there appeared to be two different approaches taken by the participants of this study, a more zoomed-out (big idea) orientation, and a more zoomed-in (detail) orientation. After discovering the existence of these two groupings, I wondered if there might be anything that may have been contributing to the distinctions observed. I wondered if the participants were going into the searching tasks with different goals and I looked to the pre-interviews to try and identify the goals of these preservice teachers.

Before the Internet searching tasks began, the participants were asked four questions about the standard (8.G.6) that they were investigating. These questions focused on what they understood already about the standard, what they were unsure about with regards to the standard, why they thought this standard might be important for students, and what their goals would be for their classroom. When looking at the first three questions, all the participants gave the same, or nearly identical answers. They all thought the standard was centered around understanding and proving the Pythagorean Theorem and its converse. They all had questions about what exactly was meant by the converse to the Pythagorean Theorem, or what converse actually meant. They all thought that this standard was included as it was important for students to understand proofs. With regard to these three questions, the participants seemed to be entering the Internet searching tasks with very similar orientations.

The last of the four questions asked the preservice teachers what their own goals would be as a teacher in their own classrooms as they had students work toward this standard of "explain a proof of the Pythagorean Theorem and its converse". As I looked at the responses that the participants gave to this question, I noticed differences in how they talked about their own classroom goals. The table below (Table 20) shows the descriptions two types of responses I saw, as well as examples of those types of responses.

	General	Specific
Descriptions	• Goal of understanding • Common aspects to all proofs • Student work before teacher	• Mathematics unique to the Pythagorean Theorem • Acknowledgement of multiple representations/proofs of the theorem
Examples	"just, to get 8th graders to understand why a proof work and how it works, it would be cool" "I would want the children to work it out first before I show, demonstrate it for them"	"although the proofs may be different they're, still end up with a-squared plus b-squared equals c-squared" "I think that there's so many visual ways to think about the Pythagorean Theorem"

Table 20 – Descriptions and Examples of Classroom Goals

The responses of both of the groups had an overarching goal of students understanding proof as the goal of the classroom. However, there was a difference in the focus of the responses that existed in the two groups. In the general orientation, these participants spoke in general terms of the understanding they wanted students to walk away with including highlighted “higher thinking” (Participant 1) and “just getting 8th graders to understand why a proof works and how it works” (not necessarily the Pythagorean Theorem proof) would be “cool” (Participant 5). While most of these responses did have a thread of student understanding running through them, there was a lack of detail included in their responses about what parts of the Pythagorean Theorem that they wanted students to talk away with. The participants in this group were Participants 1, 3, 5, and 7.

These responses are contrasted with the responses produced by the specific orientation. The responses here still carried the thread of student understanding, but also included either an attention to the mathematics behind the proof or details specific to the Pythagorean Theorem that they might not have included when discussing other proofs. Here, participants talked about multiple proofs for the Pythagorean Theorem and wanting students to see “difference between the proofs and also make connections on how they’re the same” (Participant 4) or that “there’s so many visual ways to think about the Pythagorean Theorem” (Participant 6). Additionally these participants sometimes made comments against the process of just memorizing individual steps in a procedure. The participants in this group were Participants 2, 4, and 6.

In asking these preservice teachers to consider their own classrooms, and to interpret this standard (8.G.6) for their own students, two different kinds of considerations emerge. Across all the participants, there existed this uniform goal of understanding the proof of the Pythagorean Theorem. However, there were also those that listed goals that were unique to the Pythagorean Theorem and paid attention to the specific mathematics that went into understanding the proof of that theorem. These participants seemed have a different entry point to the task than those that gave general descriptions. Those participants in that grouping talked about bigger ideas of understanding proof in general and understanding why something worked. The participants in these groupings were divided in exactly the same way that they were in the groupings of the previous two chapters. The different starting points of these two groups may have had an impact on the ways in which the preservice teachers performed their information-seeking tasks and constructing their concept maps. With this discovery, I now complete the table used in this chapter (Table 21).

RQ1	Format of Accessed Resource	• Preference for one format (either static or dynamic) of content	• Preference for both formats (static and dynamic) of content
	Time Spent Exploring	• Shorter exploration times	• Longer exploration times
RQ2	Form of Links	• Heavy reliance on conceptual connections • More procedural connections appear in later mappings	• More balance between conceptual and procedural connections • Consistent amount of procedural connections in each mapping
	Quality of Links	• At least one mapping lacking any rich connections	• Every map contains at least one rich connection
RQ3	Related Patterns of Behavior	• Big Idea (Zoomed Out) Orientation	• Detail (Zoomed In) Orientation
	Initial Goals at Task Launch	• Attention to understanding proof in general	• Attention to specific mathematics unique to Pythagorean Theorem proof

Table 21– Table of Categories from Findings (Filled)

It should be noted that the research question of this chapter (what relationships emerge between the form and quality of the mathematical connections made by preservice teachers, and the information-seeking strategies employed during the research tasks) was a question of identifying the existence of a relationship. It only looked to gain insight if different patterns of behavior happened at the same time. There was no assumption of causality. However, with the description of the initial goals of the participants at the launch of the task, I introduce what might be a causal variable. It might be the case that the different goals of the preservice teachers produce different behaviors during the tasks and activities in this study. Further study would be needed to confirm whether this was a causal effect for the other variables, or simply just appeared to be related to them.

Connecting to the Literature

Beyond the seven participants in this study, there are a numerous ways in which the findings of this study speak back to the literature and inform mathematics education more generally. As was stated before, the characteristics of tasks and the personal intentions of the users performing the tasks affect the ways that participants behave during the tasks (Lawless & Schrader, 2008). This aligns with the finding presented in the previous section, namely that the goals that the preservice teachers had for their classrooms affected the ways that these teachers then performed their searching tasks and then represented their knowledge.

This is important as it suggests that the knowledge of preservice teachers does not solely determine the behavior of teachers in the classroom. In other words, though two teachers might have the same content knowledge and understanding of mathematics, this does not mean they will enact lessons in the same way, nor have the same goals for their classes. The goals that these preservice teachers have for their students seems to be influential in affecting the ways that they think about content which then influences how they look for information and then represent their understanding.

This study found that the distinctions in categories of information-seeking behavior found in previous studies (Horney & Anderson-Inman, 1994; Juvina & Oostendorp, 2004; Lawless & Kulikowich, 1996; MacGregor, 1999) also seemed to be productive ways of categorizing the information seeking behavior in this study, with a different population, and newer technological resources. This study also extends these findings to a new content area of mathematics.

Of particular interest is the extension of the work around dynamic and static content, and whether users focused on one, or the other, or both. Mathematics education research has often described the impact that technology can have on the teaching and learning of mathematics. The advantages presented by technology often include the ability to dynamically present material (Kilpatrick, Swafford, Findell, National Research Council (U. S.), & Mathematics Learning Study Committee, 2001; National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Given the ability of the Internet to present content dynamically, and the advantages of dynamic presentation of mathematics, it is important to understand the ways that Internet searchers might prefer, or not prefer, to interact with this dynamic content.

Lastly, in terms of the mathematical content, one of the most important findings was that given only a quick exploration of online resources, participants were able to increase the amount of content they included when describing how they were thinking about the Pythagorean Theorem, and they were also able to increase the amount of connections between that content. We know that practicing teachers have limited time to learn any mathematics they are unfamiliar with and it is unreasonable to expect that drastic changes in understanding would occur in short periods of time. However, when participants had a familiarity with the content initially, similar to what was shown in prior research (Fidel et al., 1999; Hirsh, 1999), they were then able to increase the content and connections in the concept maps they were able to use to represent their thinking. Again, this shows that for preservice teachers, given their limited available time, the Internet may be able to be leveraged as a resource to support their learning.

Summary

This chapter looked at the main results from the first two findings chapters that examined the searching strategies used when locating mathematics information online, and the forms and qualities of mathematical connections made when locating information online. The patterns of behavior that were found in each of these analyses were found to be highly related. The participants that performed one way in their information seeking, also behaved in similar ways when they constructed their concept maps. The two groupings tended to focus on big ideas and have a zoomed-out orientation, or were more focused on details and had a zoomed-in orientation. This chapter also looked at the ways that the participants described their goals around this standard of proving the Pythagorean

Theorem and its converse as they imagined their classrooms. I discovered that the participants in the detail oriented group seemed to also have classroom goals that were unique to the proof of the Pythagorean Theorem, while those in the big idea group seemed to have classroom goals about proof in general.

The next chapter will offer a discussion of the study as a whole, including limitations of this study, and possible future directions for research based on the findings of this study.

CHAPTER 7

In this dissertation, I investigated the information-seeking strategies of preservice teachers when looking for information online about a proof of the Pythagorean Theorem and its converse. I also examined how these preservice teachers represented their thinking around this topic in the form of concept maps. Lastly, I looked for patterns of behavior between the information seeking strategies used, and the ways that these participants were representing their understanding of the content. Two patterns of behavior emerged: one pattern of behavior seemed to have a big idea (zoomed out) orientation, while the other seemed to have a detail (zoomed in) orientation.

Returning back to the beginning of this dissertation, this study was posed within the larger context of teachers learning online. The Internet is a potential resource for teachers to learn mathematics content. However, there are multiple challenges to learning from and with online resources as opposed to learning with and from more traditional educational resources. Included in this list of challenges is the Internet's non-linearity, its inclusion of multiple formats, and the burden of critically evaluating the reliability of information being placed on the Internet user. This dissertation explored how preservice teachers (those about to embark on their careers as teachers) were navigating these challenges, among others, to make sense of mathematical information they encountered online.

This chapter contextualizes results from the previous chapters in the broader field of mathematics education. The following section highlights implications of these findings in relation to current research in mathematics education and educational technology. I also discuss the limitations of the study, describing the extent to which findings may be generalized. Lastly, this chapter considers how this study might be continued, or expanded to learn more about how people learn mathematics online.

Implications

This dissertation focused on how preservice teachers may be using the Internet to learn mathematics content on their own. Previous research had explored patterns of information-seeking behavior of learners, but it was unknown what behaviors would appear in college-aged, preservice teachers and if they would be any different from younger Internet users. This study is an initial exploration into connecting questions about *what* preservice teachers are learning about mathematics online, with questions about *how* preservice teachers are learning mathematics online. The results of this study have implications that help mathematics educators better understand what processes future teachers of mathematics naturally use as they engage with content through new, digital environments.

One of the main findings was revealing two patterns of behavior among the participants—a big-idea orientation, and a detail orientation. I do not claim that either pattern of behavior is better. Both orientations are necessary at different times. While the detail orientation might be thought to be preferred (as it seems synonymous with a depth of knowledge), the big idea orientation is also important in mathematics education. Shulman identified curricular knowledge (Shulman, 1986), Ball and colleagues identified Horizon Content Knowledge (Ball, Thames, & Phelps, 2008), and Ma identified Profound Understanding of Fundamental Mathematics (Ma, 1999) as big idea types of knowledge critical for effective mathematics teaching. This is knowledge of content and how it fits in with the big ideas found in mathematics, as well as prior and future content.

Additionally, this study adds to the conversation that teacher knowledge of mathematics is not sufficient to predict how mathematics will be enacted in the classroom. During the preinterview, when participants were first asked about the content of standard 8.G.6, all participants gave nearly identical answers when asked about the content of the standard. However, when asked to imagine their practice and their classroom goals for achieving mastery of this standard, participants showed different ways of how explaining the Pythagorean Theorem and its converse would be enacted in their classrooms. While teacher content knowledge is considered to be important to the mathematics that occurs in the classroom, this kind of knowledge does not account for all the variance that is observed in classroom practice (Hill, Blunk, et al., 2008). Teachers' pedagogical goals and how they enact this content also shapes practice.

Teachers do not have many opportunities to learn new content within the hours of the regular school day. Additionally, teachers predominately work alone with little time to collaborate with their fellow teachers, or learn new content from others with expertise (Stigler & Hiebert, 1999). This study shows that even in a brief amount of time for exploration, the tasks used here seemed to lead the preservice teachers to represent their knowledge with more content and more connections. This suggests that the Internet may indeed be a productive place to study learning.

Lastly, it is clear that there are possible connections between the goals of the preservice teachers thinking about this content, the information seeking strategies they used while searching for mathematical information online, and the ways in which they represented their knowledge. There are still many questions left unanswered regarding the relationship among these factors and how they may be interacting or affecting each other. A better understanding of these factors could provide mathematics teachers with a diverse set of tools they can use when reflecting on their understanding of content and the strategies they could use to strengthen their content knowledge or learn new content.

Mathematics educators (and to an extent, any subject-specific educator) can also benefit from the findings of this study. Knowing that preservice teachers tend to have two different orientations to the mathematics content of their classroom is a potential tool that can be leveraged during teacher preparation programs. Lesson planning in particular is one moment when preservice teachers are asked to think critically about not just the content within a particular lesson, but how that content fits in with prior and future content of the course. Here, mathematics educators can have discussions about detail orientation and big idea orientations with preservice teachers, as these mentalities are already present in their thinking about content. Additionally, as mathematics educators try to provide strategies that will enable preservice teachers to become lifelong learners, they can also have discussions about different efficient and effective strategies they can use when using the Internet to learn content. Mathematics educators who study teacher preparation programs can benefit from the discovery of these two orientations as well. It is important for researchers who examine the content knowledge of mathematics teachers, and the content of mathematics classrooms to be aware of mathematics that connects across a breadth of content, as well as within a depth of content.

This differentiation between big idea orientation and detail orientation also has implications for those who work with educational technology. Technological Pedagogical Content Knowledge (TPACK), a popular framework that examines the knowledge required for teachers' uses of technology, attempts to identify the interactions between content knowledge, pedagogical knowledge, and technology knowledge (Koehler & Mishra, 2009; Mishra & Koehler, 2006). However, this framework lumps all knowledge within any category together, no matter the grain size. This research shows that the different orientations of learners have an impact on what is learned. Within content, pedagogical, and technological knowledge (and all of their intersections), learners can focus on better understanding small grain size details, or putting those details together in larger grain size big ideas, still within a single kind of knowledge. This study has provided a finer grain lens that can be used to identify the kinds of knowledge important to the use of technology in teaching.

Limitations

This study contributed to understanding how educators may be learning mathematics in contexts other than inside the classroom, in particular by researching content in online educational materials. Yet, despite furthering our understanding in this area, this research was still a small slice of a much larger pie. This section looks at the limitations of this research, clarifying the ways in which many important questions have still yet to be investigated.

By studying preservice teachers, this study looks at a starting point of where educators begin their careers and the ways in which they use online resources to learn content. However, this study does not assume (nor would it be expected) that experienced in-service teachers would use the resource of the Internet in identical ways as preservice teachers. It is not yet understood how experienced teachers may be using this tool in productive ways to learn unfamiliar content. Understanding these different approaches is crucial for helping preservice teachers transition to possibly more effective information seeking strategies used by practicing teachers.

In addition to the population limitation of the study described above, there are questions associated with the gender differences in terms of their learning styles and interactions with technology. Research has shown already that male and female Internet users exhibit different information seeking strategies (Hirsh, 1999; Large & Beheshti, 2000; Schacter et al., 1998). The participants in this study all happened to be female, despite attempts to involve male participants. It is not clear what, if any, differences may have emerged if males had participated in this research as well.

The content of the tasks utilized in this study also has limitations. The content of the Pythagorean Theorem was selected intentionally as it draws upon numerous areas of mathematics including algebra, geometry, and reasoning and proof (among others). Additionally there are numerous ways of proving this theorem, and many different ways of illustrating/representing this relationship visually. Because of the broad reach of this content, I knew that there would be many resources available online that the preservice teachers in this study could draw upon. However, it is not clear if the tasks used here would be as successful if alternate content was used that may be harder to find online. It may be the case that the results seen here are dependent upon the mathematics content chosen for the task. Different patterns of behavior may have been observed had other content within the same grade, or even different grades, been used.

One final limitation of this study involves the assessment technique used to examine the mathematical understanding of the participants. This study had the preservice teachers construct concept maps to represent their own thinking. With this technique, I was only able to get a glimpse of what the participants chose to represent. It is very likely that participants had much more content and connections than they represented in their maps. Additionally, this study assumes that they participants are able to accurately represent their thinking with this technique. The absence of content or connections in the concept maps produced does not mean that they participants do not understand that content, just that they didn't represent it.

Future Directions

Based on the implications and limitations described so far, there are several opportunities for future study. This study showed a correlation between the orientation of participants beginning these tasks, the information seeking strategies they used during the tasks, and how they chose to represent their thinking in the concept maps they produced. Future studies might attempt to control for the individual factors to examine the extent to which any factor influences the others. Additionally, future studies might also explore other potential influences that might also help explain the correlation of these factors.

As described in the limitations section, the focus on preservice teachers in this study is the first step of a much larger plan. Performing this series of activities with practicing teachers would illuminate how this, more experienced population is learning from online resources. Once these patterns of behavior were understood, it would allow researchers and practitioners to examine any transitions that may exist in patterns of behavior, as preservice teachers become in-service teachers. This would provide mathematics educators with opportunities to give preservice strategies that may enhance how they learn from online resources.

The mathematical content chosen for this study – The Pythagorean Theorem – was selected because it is a very well connected piece of mathematics content with a very rich history. It is not clear if the patterns of behavior found here (both in the information seeking, and in the concept map construction) would also be visible with the exploration of other content. Future studies could look at different content within the same level, or at earlier and later levels, to better understand the impact of specific content on this task.

Lastly, though the participants in this task were asked to locate information that helped them better understand the content of this standard, it was visible the some participants extended the task to also attempt to find ways that they could teach this content. Indeed, teacher understanding of the content is just one step in the process of teaching the content to students. Future studies might also examine how preservice or in-service teachers are using information seeking strategies to learn not just the content they are teaching, but the different ways in which this content might be taught.

Conclusion

This study raises new questions about learning mathematics online. The results when analyzing information seeking patterns of behavior and the content and connections of concept maps suggest a relationship between how people search and what people learn. This study serves as a first step to help understand the ways in which teachers may be taking control of their learning outside of teacher preparation programs. As mathematics teacher educators, the more we understand about this process, the better equipped we become to help our beginning teachers become life-long learners after they leave our institutions.

This study extends the research on teacher learning beyond the contexts of teacher preparation programs and their school field experiences. While examination of the learning in these environments is important, it is also important to recognize that these are not the only environments where learning takes place. The Internet currently is, and will continue to be, a source of learning for many preservice and practicing teachers. The discovery of different orientations towards the exploration of mathematics content uncovered study serves as a starting point towards understanding both what can be learned online by mathematics teachers and how it can be learned.

APPENDIX

Thank you for helping out with this study.

We know that there is no way any amount of math classes can prepare teachers for every piece of mathematics they will encounter over their career. Teachers need to have support that they can use when they leave their teacher preparation program and encounter unfamiliar mathematics

The purpose of this study is to help us better understand what kind of supports teacher might benefit from as they expand their math knowledge on their own. To that end, this study is not to evaluate what you know and don't know. It's to help us understand how teachers might be using the Internet to support their learning of, and knowledge of mathematics for teaching.

This study will have you complete several tasks and use the Internet as a resource to complete several of the tasks.

Imagine yourself teaching an 8th grade geometry class about the Pythagorean Theorem (this is usually when the Pythagorean Theorem is taught to students). What do you imagine you would be responsible for teaching students about the Pythagorean Theorem?

A concept map is a visual representation that connects different concepts together with connecting phrases around a central theme. Create a concept map around the theme of "Pythagorean Theorem". You may wish to use some, all, or none of the terms listed below:

- Triangle
- Angle
- Right Angle
- Right Triangle
- Side
- Hypotenuse
- Leg
- Length
- Square
- Pythagorean Theorem
- Proof
- Converse

Below are the Common Core State Standards related to the Pythagorean Theorem that 8th grade students are expected to know and do.

8.G.6. Explain a proof of the Pythagorean Theorem and its converse.

8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Imagine that some day, you are expected to teach your class about the Pythagorean Theorem and are using these standards as a guide.

Focusing on standard 8.G.6:

- What do you think is meant by this standard? Can you rephrase this standard in your own words?
- What are some questions you have regarding what is meant by this standard?
- Why do you think this standard might be included in what students need to know about mathematics?
- What do you think your goals might be for this standard?

With the ultimate goal of having students be able to **explain a proof of the Pythagorean Theorem and its converse** (Standard 8.G.6.), use the Internet to help further develop your own abilities in explaining a proof of the Pythagorean Theorem and its converse. (~About 20 minutes)
For each site you find useful, identify what about the site and its contents you find helpful.

Site:

Description:

Site:

Description:

Site:

Description:

Site:

Description:

Site:

Description:

Explore each of the links below and study the math content on its pages. How does it align with what you are currently thinking about proving the Pythagorean Theorem and its converse, and how does it push your thinking so far? (~About 25 minutes)

http://www.pbs.org/wgbh/nova/proof/puzzle/

Describe:

http://www.platinumgmat.com/gmat_study_guide/pythagorean_theorem

Describe:

http://ronblond.com/MathGlossary/Division03/Pythagorean%20Theorem/index.html

Describe:

http://www.youtube.com/watch?v=r382kfkqAF4

Describe:

http://www.youtube.com/watch?v=wxUnodhYGEQ

Describe:

Revisit your concept map from before. Think about what additions/modifications/deletions would you make to your map after having engaged with this task. Using the theme of "Pythagorean Theorem", create a concept map that contains the following terms:

- Triangle
- Angle
- Right Angle
- Right Triangle
- Side
- Hypotenuse
- Leg
- Length
- Square
- Pythagorean Theorem
- Proof
- Converse

Below are the Common Core State Standards related to the Pythagorean Theorem that 8th grade students are expected to know and do.

8.G.6. Explain a proof of the Pythagorean Theorem and its converse.

8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Imagine that some day, you are expected to teach your class about the Pythagorean Theorem and are using these standards as a guide.

Focusing on standard 8.G.6:

- What do you think is meant by this standard? Can you rephrase this standard in your own words?
- What are some questions you have regarding what is meant by this standard?
- Why do you think this standard might be included in what students need to know about mathematics?
- What do you think your goals might be for this standard?

Post-Interview Questions

In what ways do you think this task has impacted what you know about the Pythagorean Theorem?

What are some questions you still have around the Pythagorean Theorem after doing this task?

In what ways did you find the different Internet resources beneficial?

In what ways could the Internet resources have better helped you?

About how often do you go online for academic use? For personal use?

What kinds of tasks do you do tend to do online?

How would you rate your skills as an Internet user?

How would you rate your skills in using Internet search engines?

What year and major are you?

What math classes have you taken?

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