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# MATHEMATICS FOR ALL? EXAMINING ISSUES OF CLASS IN MATHEMATICS TEACHING AND LEARNING

By

Sarah Anne Theule-Lubienski

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ABSTRACT

MATHEMATICS FOR ALL? EXAMINING ISSUES OF CLASS IN MATHEMATICS TEACHING AND LEARNING

By

Sarah Anne Theule-Lubienski

Diversity and equity are popular topics in the mathematics education community today, particularly amidst current reforms intended to "empower all students." Still, little attention is given to socio-economic diversity in relation to mathematics teaching and learning.

In this study, a researcher-teacher explores the ways in which a curriculum and pedagogy aligned with current, mathematics education reforms played out with a socio-economically diverse group of seventh-grade students. Interviews, surveys, teaching journal entries, and daily audio recordings were used to document students' experiences across the 1993-94 school year. Qualitative analyses compared the lower- and higher-SES students' experiences with the whole-class discussions and contextualized, open-ended mathematics problems. The analyses revealed that while the higher-SES students tended to have confidence in their abilities to make sense of the mathematical discussions and problems, the lower-SES students often said the were "confused" by conflicting ideas in the discussions and the open nature of the problems — they desired more specific direction from the teacher and texts. Additionally, while the higher-SES students seemed to approach the problems and discussions with an eye toward the larger, abstract, mathematical ideas, the lower-SES students more often became "stuck" in the contexts of the problems.

The study examines critical links between the current mathematics reforms and literatures on social class, which suggest there might be a mismatch between the culture of lower-SES students and the culture of the mathematics classroom advocated by current reformers. "Cultural confusion" is proposed as an explanation for the struggles the lower- and working-class students faced in the reformed mathematics classroom. The study suggests that a classroom in which taking initiative in solving problems, analyzing and discussing ideas, and abstracting mathematical ideas from contextualized problems, might be more aligned with middle-class students' preferred ways of communicating, thinking and learning.

Dilemmas involved in educating lower- and working-class students are discussed. This study contributes to our understanding of both possibilities and hazards inherent in constructivist-inspired pedagogies and curricula intended to "empower all students," in both mathematics and other fields.

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I feel a bit like Eliza Doolittle, or maybe Cinderella. The past several years have been a time of transformation for me, and I could never have started or finished this process alone. There have been several Professor Higgins and fairy godmothers in my life over the past few years of my intellectual and personal growth. I owe much to many people for their support.

First, I will always appreciate and fondly remember the superb group of students who participated in this study. The seriousness and honesty with which they approached the study were remarkable, and they taught me a great deal. I am also thankful for the "regular" classroom teacher's willingness to share her classroom, observations, and insights with me. I am sorry that these people, so integral to this work, must remain unnamed.

I also want to thank the Connected Mathematics Project (CMP) principal investigators — Bill Fitzgerald, Jim Fey, Susan Friel, Glenda Lappan, and Betty Philips — for allowing me to be a part of writing and piloting the curriculum materials. I learned much from each of them, and I count my work with the project as one of the most rewarding experiences of my graduate program. I also owe much to my fellow staff members of the CMP for their friendship and support. I would particularly like to thank two friends, Kathy Burgis and Angela Krebs, for their generous help in interviewing students.

In addition to teaching me much about curriculum development, Glenda Lappan has been a wonderful mentor who helped me learn about what it means to be a caring teacher and educational leader. Glenda has been a role model of

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graciousness and helpfulness, and I cannot thank her enough for giving me many important opportunities to learn from her in various roles. Being near her has always been an honor and a pleasure, for she constantly reveals how a person can be a very serious educator while also having a wonderful sense of humor and a love of life. I particularly appreciate and admire her openness to my ideas, even when they have conflicted with her own.

Glenda Lappan is one of the five members of my committee, all of who have been supportive of my work and growth in various, important ways.

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David Labaree has given me the encouragement and confidence I needed to tackle a difficult topic I would have otherwise viewed as out of my league. I am deeply grateful to him for the time he has taken to carefully analyze and comment on the writing I did as part of his course, as well as the key sociological pieces of this dissertation. The wonderful sarcastic humor and refreshing honesty about scholarly life that he has provided along the way have helped me maintain some healthy perspective on the process of completing a doctoral program. Still, beneath the humor lies a kind professor who has played a central part in my transformation over the last several years through both what he has taught me about social structures (and, indirectly, myself) and the respect he has given me and my ideas.

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whatever ways she could (including offering advice on what pregnant dissertation-writers should eat, and giving me baby furniture). It has been an incredible honor to work closely with her, and I could never have made it without her tremendous support! As I go off to an academic position of my own, I am a bit fearful that the clock will strike twelve, and I will find that my transformation has been so dependent on her that I will find myself in intellectual rags. Still, I know I have learned too much from her to forget it all, and I am comforted to know she is only an email away.

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me the confidence to take the more risky, graduate school road instead of finding a "real job" after NMU. I only wish all children could have similar "cushions" to fall back on.

Finally, I want to thank my husband, Christopher for his help and friendship. He has listened carefully as I have bounced ideas off him and has contributed many ideas of his own. He has been my toughest proof-reader and my best ready-reference on everything from Marxism to markets. If he were not such a great father to Ella, I would never have been able to finish this in good conscience or in good time (he is bouncing her on his knee and singing — trying, anyway — the Lone Ranger theme song, as I write this). I will always be grateful for his patience and unconditional love. Although I consider him to be the real scholar in the family, he has always been supportive of my work and generously happy for me in my accomplishments.

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TABLE OF CONTENTS

LIST OF TABLES ........ xv

LIST OF FIGURES ........ xvii

CHAPTER 1
INTRODUCTION ........ 1
Why Study Class in the Context of Current Mathematics Education Reforms? ........ 1
Current Mathematics Education Reforms: A Brief Overview ........ 2
Two Keys to Educating "All Students:" Open Problems and Whole-Class Discussions ........ 3
The Promise of the Reforms for All Students ........ 9
Questions About the Reforms in Relation to Equity ........ 11
Class in Mathematics Education: Who Cares? ........ 13
Why is Class Ignored? ........ 16
Definitions of Class ........ 17
Why Study Class and Mathematics Education Reforms Inside a Classroom? ........ 19
Framing Questions ........ 22
Why Do I Ask These Questions? ........ 23

CHAPTER 2
METHODS OF DATA COLLECTION AND ANALYSIS ........ 28
Research Context ........ 28
The School Setting ........ 28
The Classroom and Students ........ 30
The Curriculum ........ 32
The Teacher ........ 39
Challenges and Benefits of the Researcher-Teacher Role ........ 49
Data Collection ........ 54
Student Participation and SES Categorization ........ 56
Interviews ........ 59
Surveys ........ 60
Other Data ........ 61
Data Analysis ........ 61
Initial Data Analysis ........ 64

Sa

CH
PCP
Six
Org

Gui

Sam

Reb

Ros

Sug

Dav

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CH
AL
Wa

Exploring Relevant Literature ........ 65
Further Survey and Interview Analysis ........ 66
Analysis of Students' Participation in Class Discussions ........ 69
Other Analyses ........ 70
Cases of Six Girls ........ 71
The Evolution of Themes ........ 74
So What Did I Find? An Introduction to the Analyses ........ 77
Some Success Stories ........ 78
Initial SES Assumptions ........ 81

CHAPTER 3
PORTRAITS OF SIX GIRLS ........ 85
Six Girls: Guinevere, Samantha, Rebecca, Rose, Sue and Dawn ........ 86
Organization of the Portraits ........ 87
Whole-Class Discussions ........ 89
Curriculum ........ 90
Guinevere ........ 90
Guinevere and Whole-Class Discussions ........ 91
Guinevere and the Curriculum ........ 98
Guinevere: A General Summary ........ 100
Samantha ........ 101
Samantha and Whole-Class Discussions ........ 102
Samantha and the Curriculum ........ 108
Samantha: A General Summary ........ 111
Rebecca ........ 111
Rebecca and Whole-Class Discussions ........ 112
Rebecca and the Curriculum ........ 117
Rebecca: A General Summary ........ 119
Rose ........ 119
Rose and Whole-Class Discussions ........ 120
Rose and the Curriculum ........ 126
Rose: A General Summary ........ 129
Sue ........ 129
Sue and Whole-Class Discussions ........ 130
Sue and the Curriculum ........ 137
Sue: A General Summary ........ 139
Dawn ........ 139
Dawn and Whole-Class Discussions ........ 140
Dawn and the Curriculum ........ 147
Dawn: A General Summary ........ 149
Discussion of the Six Portraits ........ 150

CHAPTER 4
A LARGER LOOK: DATA FROM THE WHOLE CLASS ........ 154
Whole-Class Discussions ........ 155
Students' Views of Their Participation ........ 155

Students' Beliefs About the Purpose of Discussion ..... 156
A Look at Students' Participation in Discussions ..... 160
Making Sense of the Discussion ..... 181
Discussion of Data Regarding Whole-Class Discussions ..... 185
Curriculum ..... 187
General Like / Dislike ..... 189
Confusion / Struggles ..... 189
Direction / Motivation ..... 194
Thinking / Learning More ..... 197
Contextualization ..... 200
Discussion of Data Regarding Curriculum ..... 214
Student "Outcomes" ..... 215
Grades ..... 217
Placements for the Following Year and Feelings About Them ..... 218
Feeling Mathematically Empowered? ..... 221
General Discussion ..... 222

CHAPTER 5
DISCUSSION OF DATA AND LINKS TO LITERATURE:
CULTURAL CONFUSION AS AN EXPLANATORY FRAMEWORK ..... 228
A Survey of Literature on Class Cultures ..... 230
Discussion ..... 243
Summary of Differences in Class Cultures ..... 243
Culture in the Mathematics Classroom ..... 245
One Theory: Cultural Confusion ..... 249
Previous Work on Cultural Congruence ..... 252
Discussion of Cultural Confusion and Congruence ..... 255

CHAPTER 6
OBJECTIONS, QUESTIONS, AND IMPLICATIONS ..... 264
Frequently Raised Objections and Questions About the Research ..... 267
Implications and Further Questions ..... 282

APPENDIX A
SURVEYS AND INTERVIEW PROTOCOL ..... 302

APPENDIX B
EXAMPLE OF GRID FOR CODING EACH CONTRIBUTION TO DISCUSSION ..... 314

BIBLIOGRAPHY ..... 318

Tabl

Tabl

Tabl

Tabl

Tabl
Pro

Tabl

Tabl

Tabl

Tabl
Ne

Tab

Tab

Tab

Tab

Tab

Tab

Tab

Tab

Tab

## LIST OF TABLES

Table 2.1 - Summary of Data Collection ........ 55

Table 2.2 - Participating Students Categorized by Gender and SES ........ 57

Table 2.3 - Separating Achievement from SES in Selecting Target Students ...... 58

Table 2.4 - Data Analysis Timetable ........ 63

Table 2.5 - Example Chart: "Have People Had Their Feelings Hurt in Class Discussions?" ........ 68

Table 2.6 - Data Analysis Themes: How Did My Focus Change? ........ 75

Table 3.1 - Selection of Female Target Students ........ 86

Table 4.1 - Participating Students by SES and Gender ........ 155

Table 4.2 - "Do You Participate Much in Class Discussions? Why or Why Not?" ........ 156

Table 4.3 - Quantity of the Six Girls' Participation ........ 162

Table 4.4 - Quantity of Participation by SES and Gender ........ 163

Table 4.5 - General Type of Contributions by SES and Gender ........ 165

Table 4.6 - Problem Context of Contributions by SES and Gender ........ 167

Table 4.7 - Correctness, Difficulty and Insight of Contributions ........ 172

Table 4.8 - Relationship of Contribution to Past Learning ........ 174

Table 4.9 - Relevancy of Contributions to the Mathematical Agenda ........ 176

Table 4.10 - Reactions when Contribution was Questioned ........ 178

Table 4.11 - Messages for CMP Authors from the Six Girls ........ 190

Tabl

Tabl
Abo

Tabl
Mat

Tab

Tab

Table 4.12 - Students' Average Grades for the Year ........................................... 218

Table 4.13 - Placements for the Following Year and Students' Feelings About Them ........................................................................................ 220

Table 4.14 - Those Mentioned in Response to "Who are the Best Three Math Students?" ........................................................................................ 222

Table 5.1 - Differences in Class Cultures as Discussed in the Literature ............ 244

Table 5.2 - Some Strengths and Weaknesses of Lower-Class Culture ................. 262

# LIST OF FIGURES

Figure 3.1 - Graph of Constant Speed .......... 92

Figure 3.2 - Product Chart for Dice Analysis .......... 143

Figure 4.1 - Bar Graph from Truck Advertisement .......... 202

Figure 4.2 - Problem from Comparing and Scaling, Investigation 1:
Making Comparisons .......... 207

# CHAPTER 1

# INTRODUCTION

This dissertation explores students' experiences in a mathematics classroom aligned with current reforms. It reveals complexities involved with some currently popular ideas and calls us to pause and consider how the current push toward "mathematical power for all" might impact the least powerful among us.

Attention is often given to schools in the midst of reform. Yet, this study is different than most in mathematics education, because it sits at the cross-roads of many lines of inquiry, including those involving socio-economic class, current educational reforms, and mathematics teaching and learning. In this introduction, I discuss why it is important to look at the intersections among these issues.

## Why Study Class in the Context of Current Mathematics Education Reforms?

There are many aspects of this question to consider, including what the reforms mean for mathematics education, the relationship of socio-economic equity issues with the reforms, and the study of socio-economic equity in mathematics education, more generally. I begin this section with a brief overview of the reforms. I then discuss two particular aspects of the reforms that this dissertation focuses on: whole-class discussions and open problems. I also discuss competing views about the reforms in relation to equity, beginning with those that see promise in the reforms for empowering all students, followed by those that are more skeptical. Finally, I discuss the lack of attention given to social class in mathematics education and possible reasons for this void.

Current Mathematics Education Reforms: A Brief Overview

In 1983 the U.S. National Commission on Excellence in Education published A Nation at Risk, which warned against the perils of low educational standards and achievement. In response, the National Research Council (NRC) (1989) and the National Council of Teachers of Mathematics (NCTM) (1989; 1991; 1995) defined new goals for mathematics education. The NRC document, Everybody Counts, emphasizes the importance of "all students" learning mathematics. The NRC's vision of all students "developing mathematical power" (p. 43) is consistent with that put forth by the NCTM Curriculum and Evaluation Standards (1989), as well as the Professional Teaching Standards (1991), which states:

> Congruent with the aims and rhetoric of the current reform movement in mathematics education (e.g., National Research Council 1989, 1990), the Standards is threaded with a commitment to develop the mathematical literacy and power of all students. Being mathematically literate includes having an appreciation of the value and beauty of mathematics as well as being inclined to appraise and use quantitative information. Mathematical power encompasses the ability to 'explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve nonroutine problems' (NCTM, 1989) and the self-confidence and disposition to do so. (p. 19)

Hence, mathematics education reformers are promoting a new vision of mathematics teaching and learning. Their new view of learning emphasizes students valuing mathematics, feeling confident in their abilities to do mathematics, solving mathematical problems, reasoning mathematically, communicating about mathematics, and constructing their own understandings through doing mathematics. The reformers' vision of teaching emphasizes choosing "worthwhile mathematical tasks," facilitating classroom discourse,

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creating a learning environment that fosters students' mathematical sense-making, and assessing students' learning with tools that are consistent with the goals for student learning mentioned above (e.g., tests that assess students' ability to reason and communicate mathematically, as opposed to only measuring computational abilities) (NCTM, 1991, p. 19).

The NCTM Standards have begun to make an impact. By 1993, over half of the states in the U.S. had changed their testing programs or curriculum recommendations in light of the Standards. Furthermore, the National Science Foundation (NSF) funded thirteen curriculum development projects to help implement the reformers' vision (Usiskin, 1993, p. 6). The Standards have even become a model that educators in other subject areas are emulating (McCleary, 1993). Still, evidence indicates that the Standards have had limited impact on actual classroom practice, tending instead to promote changes in surface features of classrooms, such as the use of manipulatives or of textbooks that claim to emphasize problem solving (Ball, 1990; Cohen & Ball, 1991). NCTM calls for much more — a fundamental shift from the teacher as *the* authority for knowledge to a facilitator of students' discourse and discovery.

In this dissertation, I explore students' experiences in a classroom with a teacher committed to making such changes. In doing so, I raise questions about the changes themselves as I focus on two key reform elements: the use of open problems and whole-class discussion in teaching mathematics..

## Two Keys to Educating "All Students:" Open Problems and Whole-Class Discussions

An assumption underlying the current reforms is that a mathematical pedagogy and curricula aligned with the Standards will address the needs of *all* students. In the Curriculum and Evaluation Standards, the authors write, "We are convinced that if students are exposed to the kinds of experiences outlined in

the Standards, they will gain mathematical power" (p. 5). NCTM (1991) explains that changing teaching and curricula is the key to empowering all students.

> To reach the goal of developing mathematical power for all students requires the creation of a curriculum and an environment, in which teaching and learning are to occur, that are very different from much of current practice. (p. 1)

NCTM discusses several aspects to be changed in the curriculum and classroom environment. For example, NCTM (1989; 1991) promotes mathematical changes (e.g., a greater emphasis on understanding probability, statistics, and estimation and less on practicing computation), technological changes (increasing the use of calculators and computers), and changes in teacher and student roles. Although I began this study unsure which components would be my focus, I will concentrate on two key ingredients of NCTM's vision that the students in this study most referred to when asked to discuss how their "reformed" classroom compared with more typical mathematics classes they experienced previously: "open" mathematical problems and whole-class discussions. These two components can be understood in a variety of ways, and, therefore, I discuss my interpretation of what NCTM intends and how the ideas are central to the reforms.

Open problems. NCTM (1989) argues that problem solving should be "the focus" of mathematics in school (p. 6). Instead of traditional, routine exercises that can be solved by simply following a memorized procedure, students should have more opportunities to solve "open" problems that require creativity and allow for a variety of methods. Some of these problems might have no single right answer and could take "hours, days and even weeks to solve" (NCTM, 1989, p. 6). NCTM's rationale for its emphasis on problem solving includes the needs of "today's workplace," as well as the need for students to become "lifelong learners" and "productive citizens" (NCTM, 1989, pp. 4, 6).

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Goals for middle-school students, in particular, include learning to "use problem-solving approaches to investigate and understand mathematical content" and "generalize solutions and strategies to new problem situations" (NCTM, 1989, p. 75). Hence, solving problems is a means to developing an understanding of generalized, mathematical principles that can be applied to a variety of other situations. According to NCTM (1989), "learning should be guided by the search to answer questions — first at an intuitive, empirical level; then by generalizing; and finally by justifying (proving)" (p. 10).

The problems used should be of interest to the students and prompt students' learning of important mathematical skills and ideas. They should also help students connect mathematical ideas with each other, as well as with the world around them.

> Tasks provide the stimulus for students to think about particular concepts and procedures, their connections with other mathematical ideas, and their applications to real-world contexts. Good tasks can help students to develop skills in the context of their usefulness. (NCTM, 1991, p. 24)

Since students should see how mathematics connects to their lives and learn to value the importance of mathematics in the real world, many problems should be set in a real-world context. But although all problems should have some genuine, motivating context, they do not always have to be in real-world settings. Regarding middle-school mathematics, NCTM (1989) states:

> Although concrete and empirical situations remain a focus throughout these grades, a balance should be struck between problems that apply mathematics to the real world and problems that arise from the investigation of mathematical ideas. (p. 75)

I will use the term "open problems" to indicate the type of tasks that NCTM advocates. In summary, these problems arise out of a motivating context of some type (sometimes the real world, sometimes not), have no obvious solution, allow students to approach them in a variety of ways, and create opportunities for students to learn important mathematical concepts and processes that can be generalized to other problem situations. To make this discussion of open problems more concrete, I offer the following example:

> Crystal's Candy Company is going to begin making a box of 64 chocolates. Each chocolate is one cubic inch. The company wants your help in designing a box for the chocolates. What are the dimensions of all possible boxes that will hold the 64 chocolates? Which box will take the least amount of material to construct? Which box would you recommend to the company?

This open problem is constructed in a way that would prompt students' exploration of the relationships among shape, volume and surface area, thereby helping them understand that the more "cube-like" the box, the smaller the surface area for a given volume. Through solving this problem, students would also gain practice in finding the volume and surface area of many boxes, perhaps computing dozens of more typical computational problems, such as the following (that would likely appear after an example showing how to multiply the dimensions to find volume):

> What is the volume of a box with the following dimensions:
>
> a) l = 4 inches, w = 4 inches, h = 4 inches
> b) l = 8 inches, w = 4 inches, h = 2 inches
> c) l = 16 inches, w = 2 inches, h = 2 inches
> d) l = 32 inches, w = 1 inch, h = 1 inch

In a typical text, students would most likely be asked to find the surface area of similar boxes at a different time, when the topic of surface area was discussed. In solving the more open problem, students would likely gain practice in computing volumes and surface areas, as they would in the typical curriculum. Yet, with the more open problem, they could gain so much more: an understanding of the connections among shape, volume and surface area, an appreciation for how these concepts might be useful in the real world, and the development of problem-solving skills.

There are many interpretations of just how students are to learn mathematical ideas and processes through solving rich problems. I have met some teachers who believe that students' learning will occur so naturally from exploring problems that students need not even realize they are learning mathematics. Yet I interpret NCTM as advocating a more explicit role for mathematical ideas, as well as a more active role for the teacher in facilitating discourse about the mathematics involved with students' problem explorations. NCTM (1991) explains that good problems promote classroom discourse about mathematics.

> Good tasks are ones that do not separate mathematical thinking from mathematical concepts or skills, that capture students' curiosity, and that invite them to speculate and to pursue their hunches. Many such tasks can be approached in more than one interesting and legitimate way; some have more than one reasonable solution. These tasks, consequently, facilitate significant classroom discourse, for they require that students reason about different strategies and outcomes, weigh the pros and cons of alternatives, and pursue particular paths. (p. 25)

Hence, students' work on problems is linked with discourse about their mathematical thinking. Three of NCTM's (1991) six standards for teaching mathematics focus on discourse. Although discourse involves various forms of communication, this dissertation focuses on one particular type: whole-class discussion.

<u>Whole-class discussions</u>. NCTM makes clear that whole-class discussions are an important part of its vision of teaching and learning. Central to NCTM's vision of mathematics classrooms as communities of learners, whole-class discussions serve many purposes.

> Whole-class discussions enable students to pool and evaluate ideas, record data, share solution strategies, summarize collected data, invent notations, hypothesize, and construct simple arguments. (NCTM, 1989, p. 79)

The teacher, in facilitating discussions, should ask questions that "provoke students' reasoning about mathematics" (NCTM, 1991, p. 35) and "help students con

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construct connections among concepts, procedures, and approaches" (NCTM, 1989, p. 80). Teachers should also be active listeners, deciding which ideas to pursue and monitoring students' participation.

According to NCTM, the teacher should move away from being the sole authority for knowledge and move toward being one who engages students "in formulating and solving a wide variety of problems, making conjectures and constructing arguments, validating solutions, and evaluating the reasonableness of mathematical claims . . ." (NCTM, 1991, p. 21).

Hence, students should play a very active role in discussions. They need to take risks and publicly share, analyze, defend, and validate ideas.

> Students should engage in making conjectures, proposing approaches and solutions to problems, and arguing about the validity of particular claims. The should learn to verify, revise, and discard claims on the basis of mathematical evidence and use a variety of mathematical tools. Whether working in small or large groups, they should be the audience for one another's comments — that is, they should speak to one another, aiming to convince or to question their peers. (NCTM, 1991, p. 45)

The authors of the Curriculum and Evaluation Standards mention that language-minority students might need extra support in this area (p. 80). Although it might be difficult for some students to communicate their mathematical thinking, the benefits students can reap sound promising:

> Writing and talking about their thinking clarifies students' ideas and gives the teacher valuable information from which to make instructional decisions. Emphasizing communication in a mathematics class helps shift the classroom from an environment in which students are totally dependent on the teacher to one in which students assume more responsibility for validating their own thinking. (NCTM, 1989, pp. 78-79)

In addition to helping students clarify their thinking and become more independent, whole-class discussions promote a view of mathematics as socially constructed.

> When students make public conjectures and reason with others about mathematics, ideas and knowledge are developed collaboratively,

> revealing mathematics as constructed by human beings within an intellectual community. (NCTM, 1991, p. 34)

The Promise of the Reforms for All Students

Both of these elements — whole-class discussion and open problems — fit with NCTM's overall emphasis on active student involvement in mathematical sense-making. NCTM (1991) argues that a classroom environment involving the sharing and respecting of each other's ideas and allowing time for "students to puzzle and to think" will help "all students believe in themselves as successful mathematical thinkers" (p. 57).

When one considers the reputation for mind-numbing, irrelevant boredom that math classes have gained, the ideas advocated by NCTM (and others) seem sensible in many ways. Wanting students to think for themselves and learn to communicate their thinking are powerful goals. Open problems and whole-class discussions seem like sensible means to reach these goals.

Instead of students completing meaningless exercises to practice skills, why not allow students to develop these skills while learning about important mathematical ideas through exploring problems of interest to them? As Janvier (1990) argues, a problem's context heavily affects one's approach to a problem. Practicing skills in isolation does not necessarily help students apply those skills when necessary, since real problems tend to messy and ill-defined. Hence, learning mathematics through exploring problems in interesting contexts seems promising for helping students to learn important mathematical ideas, as well as problem-solving processes that will enable students to apply those ideas.

Also, instead of students memorizing what the teacher tells them, why not actively involve students in the genuine mathematical activities of conjecturing, analyzing, validating and arguing? As NCTM (1991) states, involving students in such discussions helps them come to view mathematical knowledge as

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something they can make sense of, as opposed to a mysterious, arbitrary list of rules handed down from an external authority figure. It seems reasonable to believe that this approach would help students gain confidence in themselves as mathematical sense-makers.

Not only do the ideas seem sensible for "all students," several authors argue that open problems and class discussions are particularly promising strategies for groups of students who have traditionally been under-represented in mathematical careers. Hence, the reforms seem to have the potential to promote equity.

For example, Stiff (1990) argues that the reforms should help African-American students, because research has shown that these students prefer to do math in a relational, more holistic way. Hence, opportunities to actively explore problems set in real contexts and to communicate about mathematics would be helpful for those children. Gilbert and Gay (1985) support Stiff's argument and add that many African-American students prefer to talk, rather than write, and they benefit from having time to really dig in and explore a problem.

Additionally, Damarin (1990) makes a similar argument about how girls prefer to learn, saying that NCTM is moving in the right direction because most girls like to have personal experiences with ideas and to communicate with others. Also, Campbell (1991a) found that seventh- and eighth-grade girls, in particular, prefer working with others on problem-solving activities.

Perhaps one of the most famous educators concerned with the plight of the lower classes[1], Paulo Freire (1970), advocates a "pedagogy of the oppressed" that is similar in many ways to what NCTM advocates. For example, he points out the importance of common people learning to analyze technological

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[1] For the sake of convenience, I will use the term "lower classes" broadly to include both lower and working class people.

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information, and he advocates pedagogical methods in which the teacher and students engage in exploring and discussing real problems.[2]

Also, some have argued that we need to promote the same high expectations for all students, because teachers often hold lower expectations for disadvantaged students, who have typically received more than their share of low-level drill and practice exercises (Means & Knapp, 1991). Anyon (1981) found that students of lower-SES seemed to receive primarily rote instruction, while students of higher-SES were more likely to be actively involved in problem solving. This pattern would seem to perpetuate inequalities, as the higher-SES students are educated to be leaders, while the lower-SES students are trained to be followers.

Lindquist (1993) reported the results of an NCTM survey, saying, "Nearly 90% agreed that NCTM is 'continuing efforts to achieve equity in mathematics education by encouraging teachers to establish and maintain high expectations for all students'" (p. 472). Still, not everyone agrees that uniformly high teacher expectations are the solution for the inequities in our educational system.

Questions About the Reforms in Relation to Equity

Several scholars have raised questions about the mathematics reforms with regard to equity. Secada (1991) expressed concern about the emphasis on higher expectations for all students, saying we need to have meaningful inclusion — not just "sinking or swimming in the rising tide of excellence" (p.

[2] I thank Helene Alpert for pointing out this similarity. Alpert also noted that while Freire expects such methods to radically transform society, NCTM aims, in part, to strengthen the workforce, thereby supporting the current capitalistic society. Frankenstein (1987) points out a possible difference between Freire's methods and those of NCTM. Freire advocates having the teacher and students engage in problem posing together, with the focus on understanding society's problems instead of solving simplified versions of real problems, because today's problems are complex and many cannot be solved.

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40). According to Secada, "We need to ask if reforms will exacerbate, maintain, or diminish previously found disparities" (p. 48).

Others have also questioned the assumption that the reforms offer a solution to equity issues. Meyer (1991) analyzes the ways in which equity has been given attention in the most popular mathematics education reform documents, including those published by NCTM. She concludes:

> Inequities found in mathematics learning were acknowledged, at least in passing . . . (but) few detailed recommendations were made to change the situation. Instead, recommendations were made to benefit all students, with the implicit assumption that what is good for the majority will be good for minorities. The recommendations found . . . were based upon economic and militaristic imperatives, seldom on social justice. (pp. 18-19)

Additionally, Stanic (1991) argues that the reforms are built on false promises of high-tech jobs, even as most jobs are getting more and more low-level. He voices concerns about gender and racial issues, as he says, "focusing on what is apparently good for everybody has, historically, *never* been good for everybody" (p. 60, author's emphasis).

While Secada and Stanic urge math educators to consider more carefully how reforms might impact issues of equity, a few others have argued that the reforms are blatantly classist. For example, Silbert (1991) argues that the NCTM-advocated reforms are being pushed by the vocal, middle-class parents.[3] He draws parallels to Delpit's (1986; 1988) work, as he argues that white, middle-class students are more likely to obtain the mathematical "basics" at home, so disadvantaged students will be at an even greater loss if we do not emphasize

[3] Stories about middle-class parents leading backlashes against the reforms, for example, in California, make me question his argument.

the "basics" in school.[4] He urges the use of *tested* methods that have been proven to work with poor students.

But has the mathematics education community really shown such disregard for equity issues? There is evidence that some in the field are, in fact, hoping for some insight into what can be done about equity (e.g., Ferrini-Mundy, 1993). Also, there has been considerable attention given to issues of gender and race in mathematics education. Much concern has been shown, for example, about boys outperforming girls, and many scholars have tried to understand the exact nature of the gap, including what areas of mathematics seem to be particular strengths and weaknesses for each gender (e.g., Fennema & Sherman, 1978). Assuming the gap is due to nurture (not nature), researchers have searched for ways to help girls more effectively learn mathematics, giving attention to the type of classroom environment and pedagogy that is most helpful for females (Campbell, 1991a; 1991b; Fennema & Leder, 1990; Hart, 1989). Likewise, but to a much lesser extent, some studies have focused on the mathematics performance and learning styles of minority students (Campbell, 1991b; Stiff, 1990). A new, upcoming series on "multiculturalism and gender," edited by Secada contains a volume on gender, and one on each of four ethnicities. Hence, in mathematics education and in education more generally, much concern has been shown for disparities between students of different genders and ethnicities. But what about class?

Class in Mathematics Education: Who Cares?

As with gender and ethnicity, studies have found correlations between social class and achievement in mathematics (e.g., Kohr, Masters, Coldiron, Blust,

[4] Delpit (1986) argues that progressive methods in language arts, in which fluency and self-expression are emphasized more than "basic skills," pose particular problems for disadvantaged black students who are less likely to be taught those "basics" at home.

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& Skiffington, 1991). Yet, after reviewing the literature on "Race, Ethnicity, Social Class, Language and Achievement in Mathematics," Walter Secada (1992) noted the lack of serious attention given to social class in mathematics education.

> It is as if social class differences were inevitable or that, if we find them, the results are somehow explained . . . . Social class differences are not as problematic in the literature as are racial, ethnic, or other disparities. For example, while the research literature and mathematics-education reform documents (for example, NCTM, 1989; NRC, 1989) at least mention women and minorities, issues of poverty and social class are absent from their discussions. Frankly, the literature does not bristle with the same sense of outrage that the poor do not do as well in mathematics as their middle-class peers as it does with similar findings along other groupings. (p. 640)

Perhaps we tend to throw up our hands when confronted with class-based differences in mathematics achievement because many problems lower-class families face seem far beyond the school's control (such as inadequate family resources). Perhaps SES differences will always exist to some degree because of factors outside education's realm. Yet, there are a variety of barriers that lower-SES students need to overcome to break out of the cycle of poverty, and some of these barriers arise within schools, including mathematics classrooms. We need to consider the possibility that changing mathematics curricula and pedagogy can remove or add new barriers for lower-SES students.

Since the reforms are intended to help "all students" gain mathematical power, it makes sense to give attention to how they are likely to impact those in the least powerful positions in our society. It is particularly important for *mathematics* educators to consider class-related equity issues, since mathematics serves as a "critical filter" (Campbell, 1991b), with the potential to reward successful students with high occupational status and pay. For example, the American Association of University Women (AAUW, 1992) report that women still make less than 70 cents for each dollar made by a man with a similar level of education, but that this difference is negated when women take at least eight

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credits of college mathematics. Hence, it is easy to understand why a popular solution is to steer more children, particularly women and other under-represented groups, into more mathematics classes and into mathematical occupations (e.g., AAUW, 1992; NRC, 1989).

Mathematics and related fields have been a "ladder" of mobility for some lower-SES students.[5] Mathematics could be special in this regard because, as mentioned in Everybody Counts, "Among the many subjects taught in school, mathematics is probably the most universal, depending least on a student's background and culture" (NRC, 1989, p. 20). Still, there are scholars who argue that mathematics is culturally relative. For example, Borba (1990) argues that our schools teach a white, middle-class mathematics, and that each culture has its own mathematics. Personally, I can see how mathematical applications, reasoning, teaching and learning can vary by culture, but the fundamental mathematical principles that are used — for example, the concept of addition or volume — are essentially similar. Yet, when we teach mathematics in school, these potential cultural variants (ways of using, reasoning, or learning) are at play. Still, the rules of the typical mathematics classroom culture seem quite clear — for example, look at the example at the top of the page or listen to the teacher's example, then practice the procedure until you have memorized it and can quickly carry it out.[6] [7]

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[5] For example, in a study of social class and freshmen career choices, lower-class students were more likely to choose fields like engineering and accounting, while higher-SES students were more likely to choose fields in the humanities and social sciences (Werts, 1966). Unfortunately, I have not been able to find a more recent study of this relationship.

[6] Contrast this with teaching students to write a critical analysis of the ways in which color is used as a metaphor in Shakespeare's works — there seems to be less of a formula for learning how to think and write in this way.

[7] I do not mean to suggest that white, middle class students do not have advantages in all school activities, including mathematics — just that these advantages have been slightly less than in some other school subjects, such as English or history. To me, it is clearer how a student from a white, middle-class family (that spends time analyzing U.S. politics and stops at historical sites on

Mathematics might have special potential as an equalizer of sorts, both because of its high status in our society and because of its traditional, relative lack of dependence on family background.[8] Yet, social class is rarely focal in current, educational studies. This void is not limited to *mathematics* education. For example, in a survey of all research on science learning and achievement from 1980-1986, out of 73 studies, 4 looked at race, 12 looked at gender, and 7 looked at both. None of the studies examined class (McDowell, 1990).

Why is Class Ignored?

There are many barriers that make social class, in particular, difficult to study and even talk about.

First, as Lucile Duberman (1976) points out, class is difficult to define and measure. Researchers do not agree on how many classes there are, or whether the U.S. class structure is continuous or discrete. We can ask if class is really about money, status, and/or power. Zweig (1991) points out that the top $\frac{1}{2}$ percent of U.S. families own $\frac{1}{3}$ of the wealth — the same amount owned by the bottom 90% of families. He argues that about 2/3 of the U.S. is working class. But in our culture we do not tend to think much about class.

> There is a host of competing definitions and understandings of class. In the popular culture there is a widespread view that class is nonexistent in U.S. society, or irrelevant because of social and economic mobility. (p. 201)

This relates to a second difficulty of studying class — it is a touchy subject. When one studies gender, distinguishing between boys and girls is fairly straightforward and labeling children as such does not seem offensive.[9] When

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vacations), has advantages in a field like history than in mathematics as it is typically taught in schools.

[8] I, like others such as Noddings (1996), have questions about whether math *should* be playing this equalizing role, but I will not explore those here.

[9] Those who emphasize the social construction of gender might disagree, since they view gender as continuous, as opposed to discrete.

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trying to study class, one can easily be attacked for defining class categories in problematic ways and for insulting students by "labeling" them *lower*-class or *dis*advantaged. When exploring differences in class cultures in order to understand how the culture of the school might compare with students' home cultures, one runs the risk of sounding like a proponent of deficit theory — that is, one who views the culture of disadvantaged students as "deficient," or lacking in relation to "mainstream" culture.

Perhaps as a reaction to deficit theory, mathematics and other education communities are tending to limit their talk to the *positive* aspects of diversity. For example, the NCTM 1997 yearbook is entitled "Multicultural and Gender Equity in the Mathematics Classroom: The *Gift* of Diversity." (emphasis added) But it is difficult to view large disparities of wealth and status as a gift of any sort.

In this dissertation I discuss socio-economic class and explore how the culture of a reformed classroom compares with the cultures of students from different class backgrounds. Hence, I walk a fine line between deficit and difference theory, and this will make many people uncomfortable, including myself.[10] But I have become convinced that avoiding the subject is not helpful to lower-SES students.

Definitions of Class

There are several ways in which class is defined in various literatures. In a traditional, Marxist interpretation, class is viewed in terms of discrete categories defined by power and ownership — particularly one's relationship to the means of production. Those who own or control work processes are the bourgeoisie (the "haves"), and those who do not are the proletarians (the "have nots").

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[10] In contrast to deficit theorists, "difference" theorists see various cultures as equally valuable and merely different (e.g., see Connell, R.W., Ashenden, D., Kessler, S., & Dowsett, G., 1982).

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According to Marx, the labels change, but societies generally have two classes in opposition: "Freeman and slave, patrician and plebeian, lord and serf, guild-master and journeyman, in a word, oppressor and oppressed . . . ." (Marx, 1988 p. 55).

Weber moved beyond two discrete categories and viewed classes in terms of culture, politics and lifestyles (Kohl, 1992). These are the various factors that affect one's life chances, including opportunities to become educated, make money, and own property. With this definition, there are several, often overlapping classes, with relatively permeable lines distinguishing them.

Yet another definition was advanced by E.P. Thompson (1963), who focused less on existing structures or categories and more on the process of human interactions. According to Thompson, "class is a relationship, and not a thing" (p. 11).

Although Hogan (1978) embraces a definition much like Thompson's, he notes that his definition differs from that commonly used by historians and sociologists. Hogan writes, "Class is usually used to signify the existence of a group in a stratification system in which different indices of inequality coalesce, for example, educational attainment, occupation, income" (p. 263). In contrast with Marx, this more common definition uses continuous variables to rank individuals in a socio-economic hierarchy — in other words, to indicate one's socio-economic status (SES). For analytic purposes, social scientists place boundaries at various points on this continuous hierarchy to create class categories.

Each of these definitions contributes to the way I think about class in this dissertation. I, like Weber, think about class primarily in terms of groups that share similar "life chances." Yet, this is a difficult definition to operationalize. Hence, when categorizing students in this study, I use common SES indicators —

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occupation, education, income, and reading material in the home — to approximate their class backgrounds. Still, my underlying definition of class is based on "life chances, and I consider SES to be an approximation for class. I think of "class" as going beyond SES, as it connotes more permanence, shared group values and beliefs about roles in society and relationship to power (Secada, 1992).

Although I will talk about classes as existing groups, I also consider ways in which families' positions in society affect their ways of communicating, learning, and knowing and focus on how these ways play out in a particular educational setting. Hence, there are also elements of Thompson's more process-oriented class view in this dissertation.

In this study I tend to focus on two broad categories of class and do not make fine distinctions between lower-and working-classes, or middle- and upper-middle classes. Note that I will use the term "socio-economic status" (SES) when referring to my students' data, but will use "socio-economic class" (or "class" for short) when discussing larger, societal structures.

Why Study Class and Mathematics Education Reforms Inside a Classroom?

Doug McLeod (1992) argues that we need to give attention to students' experiences if the reforms are to succeed. We should avoid a pitfall of the reformers of the 50's and 60's, who gave little attention to possible negative reactions from students. Since the reforms advocate a new pedagogy and curricula aimed at empowering "all students," it seems particularly important to see how the reforms actually play out with students, particularly those from traditionally disadvantaged groups.

One way to see what is happening with students is to look at their test scores, such as the NAEP results. In an NCTM Bulletin article, Lamar Alexander

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(then Secretary of State) credits the reforms for "improved" NAEP results" (NCTM, 1993). The article was optimistically entitled, "NAEP Results Show Improvement." But a closer look at the fine print in the article reveals that, although the overall average rose five points, there was a significant decline in the average proficiency of disadvantaged, urban eighth graders. In fact, the same NAEP data were the basis of an article in the Lansing State Journal entitled, "Test Score Gap Widens for Rich, Poor Kids" (Whitmire, 1994).

But we cannot make sense of what is going on with numbers alone. As Reyes and Stanic (1988) say, we need to look carefully inside classrooms to try and understand the relationship between SES and math performance.

> Before quantifying classroom processes, more qualitative work is necessary to point out categories that may better capture the wholeness and richness of classroom life . . . . Within mathematics education research, relatively little work has been done on race-related differences and almost no work has been done on the relationship between SES and mathematics performance. (pp. 39-40)

Frankenstein (1987) supports these assertions, arguing that issues of mathematics hegemony and anxiety are usually only examined with respect to gender. We need to look at race and class on issues that have been previously examined with the "gender" lens (e.g., attitudes about mathematics, and types of mathematics pedagogy and curricula that are particularly useful or harmful for various groups). Secada (1992) also notes that mathematics education studies have failed to ask whether various teaching behaviors are differentially effective for different students.

Bishop (1994) argues we need further research to uncover hidden cultural assumptions in our mathematics classrooms. Although not talking specifically about class cultures, Bishop offers questions that could be helpful in thinking mo

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more about possible differences between mathematics classroom cultures and the cultures of students who are not part of the white, middle-class mainstream:

> Can mathematical learning activities be usefully characterized as more or less "open" in relation to their cultural framing?
>
> What knowledge about the learners' cultures can help mathematics teachers with their classroom decision making?
>
> Is "cultural distance" of their home mathematical culture from the school mathematical culture a sensible construct? If so, how does it relate to the quality of their mathematical learning in classrooms?
>
> In what sense does bi-cultural mathematical learning differ from bilingual mathematical learning? (p. 19)

It seems particularly important to explore these issues now, in the context of current reforms that advocate changes in not only mathematical content, but the means of teaching and learning mathematics and the general culture of the classroom. Although these reforms promote the idea of "mathematics for all," I wonder about their effects on lower-SES students. While, as discussed previously, the norms of the culture of the typical mathematics classroom are relatively straight forward and easy to follow, current reformers are advocating major changes in these norms. This is potentially problematic if students in society's lower classes — those who are most in need of becoming "empowered" in our society — have more difficulty with the new norms. While we in mathematics education have not given much attention to these issues before, those in some other fields have. For example, there are several scholars who give careful attention to literacy education and social class (often in conjunction with race), and they have raised many important issues about the incongruencies between middle-class discourse norms used in schools and lower-SES norms (e.g., Heath, 1983; Delpit, 1986). But mathematics educators tend to be unaware of these studies and are now urging the use of more classroom discourse than ever before.

Hence, amidst the current rhetoric of "mathematical power for all students," it is worth exploring the experiences of high- and low-SES students as they attempt to learn mathematics in a classroom aligned with current reforms.

## Framing Questions

Most sympathetic but critical examinations of the reforms thus far have been directed toward issues external to the substance of the reforms, such as political realities at the federal or school level (e.g., Apple, 1992). This study seeks to look at reforms from the inside, giving careful attention to the ways in which one version of the reforms played out in a heterogeneous classroom, focusing on possible differences in the needs and experiences of groups that differ by SES.

During the 1993-94 school year, I had the chance to study students' reactions to one version of the reform. I was initially concerned about beginning a study with too broad a focus, and I was particularly interested in students' abilities to analyze mathematical claims in the real world and how these might vary by SES. I was interested in how students' analyses of mathematical claims in the media might be affected by a pedagogy and curriculum designed to encourage students to think for themselves and to analyze and reason about mathematical relationships. Hence, I initially planned to focus on students' understandings of mathematical claims in the media (e.g., newspapers, magazines) and possible SES-related differences in students' understandings and how the curriculum and pedagogy might impact their understandings.

Yet, after spending some months in the classroom, I began to realize that the issues that seemed of most importance were constrained by the media focus. For example, when discussing the pedagogy with students in interviews, I heard many lower-SES students talk about feeling confused in the whole-class

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discussions. Issues such as these seemed too distant from the media focus. After talking with my doctoral committee, I decided to enlarge my focus to study how the curriculum and pedagogy interacted with socio-economic class, more generally. Hence, the questions that frame this study in its ultimate state are as follows:

- Do students of varying SES experience and react differently to the curriculum and pedagogy? If so, in what ways?
- How might any existing differences be explained by the interaction between class cultures and the culture of the classroom and curricula advocated by constructivist-inspired reforms?

As explained previously, I focus on whole-class discussions and open problems as particular elements of the curriculum and pedagogy. Additionally, although socio-economic class is the main focus of this dissertation, I also consider gender as a potentially interactive variable, because several scholars have warned of the dangers of studying class, gender, and race in isolation. For example, Campbell (1991b) urges mathematics education researchers to consider interactions among gender, race and class, because, "dealing with one problem at a time causes research to be incomplete at best, and at worst, to be just plain wrong" (p. 96). Additionally, Gilah Leder (1992) argues that we need qualitative studies to help us see how the variables of class or race can interact with gender.

## Why Do I Ask These Questions?

There are several, diverse factors that have ultimately led to the questions that now frame this study. First, as explained above, several theoretical factors prompted my concerns for how lower class students might fare in reformed classrooms. As argued previously, although SES correlates with mathematics achievement, issues of social class are rarely examined in relation to mathematics education, Additionally, in the context of current reforms aimed at empowering "all students," it seems particularly important to explore how the changes adv

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advocated play out for those with the least power in our society. Reformers are advocating fundamental changes in the culture of the classroom, perhaps removing what has been a traditional ladder of mobility for students from our lower classes.

But my concerns about the possible removal of mathematics as a ladder of mobility are more than theoretical. My own class background is the second source of my questions. Growing up in a lower-SES family, I viewed schooling as an escape. Although I tried hard in all subjects, I felt inadequate compared to my college-bound peers — except in mathematics. There the rules were clear and my hard work seemed to have a direct payoff — there was no hidden background knowledge I lacked.[11] I excelled, and math became my ticket to several scholarships that greatly influenced my career paths. I have reaped benefits from the educational system, and I recognize the crucial role my mathematical proficiency has played. But I also realize that our educational system has a history of reproducing inequalities, and that I am an exception, rather than the rule. As I watch my siblings remain in a cycle of poverty, I tend to worry a great deal about helping lower- and working-class students become empowered through educational means.

My work with the Connected Mathematics Project (CMP) is a third source of my questions. In support of the current reform movement, the National Science Foundation (NSF) funded several mathematics curriculum development projects in an attempt to provide curricular materials aligned with the reforms. The CMP is one such endeavor, beginning in 1991 and funded for five years. The principal investigators (PIs) of the CMP are Glenda Lappan, Elizabeth Phillips,

[11] My mathematics classes stood in contrast to government class, for example, in which I was completely unaware of the difference between a Democrat and a Republican, and I had no family vacations to Washington D.C. or presidents' birth places to draw upon.

and William Fitzgerald from Michigan State University, Susan Friel from the University of North Carolina, and James Fey from the University of Maryland. The PIs chose the name "Connected Mathematics Project" because they believe that students need to see how mathematical ideas connect with each other, as well as to other disciplines and to their own lives (Fitzgerald, W., Lappan, G., Phillips, E., Friel, S., & Fey, J., 1990). The PIs intend to help change mathematics instruction from a rote, drill and practice environment to a more meaningful, problem-centered environment by developing a curriculum aligned with the NCTM Standards.

In the project's proposal, the PIs argue against tracking at the middle school level (Fitzgerald, et. al, 1990). They explain their intentions of writing a curriculum for all students:

> **Mathematics for All**: The middle school is a crucial arena in the development of adolescents. Important mathematical aptitudes are developed and influential attitudes are formed . . . . The proposed Connected Mathematics curriculum will be designed to meet the *needs of all students* with experiences that are stimulating and challenging to middle school kids with a variety of interests and aptitudes. (p. 6, authors' emphasis)

I was fortunate to have enriching experiences working "inside" the reforms as part of the Connected Mathematics Project from its beginning in 1991 until the end of my data collection in 1994. I began working with the project as I began my doctoral studies, and I was optimistic about the potential of the reforms and the curriculum, in particular, for helping all students learn more meaningful mathematics. I was, and continue to be, supportive of the primary goals of the reforms, including the development of all students' competence and confidence in mathematical problem solving.

While working with the CMP, I was involved in writing the materials, and I found it fascinating to be part of key discussions about the curriculum. In these

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discussions, we struggled long and hard with choosing mathematical content, contexts, and wording as we strove to create a curriculum that would be mathematically empowering for all students. For example, we debated about which interpretations of rational numbers we should include in the curriculum and what problems should be used to introduce them. We argued about which students might find contexts involving animals or sports appealing or alienating. We discussed whether the term "problem" sounded too negative. We agonized over how much to tell students in the texts — for example, should we include definitions or formulas? I will discuss the philosophy, form and content of the curriculum in more detail in Chapter Two.

In addition to writing, I also piloted the CMP trial materials and provided feedback to the authors about how the curriculum played out in real classrooms. During the 1992-3 school year, I taught the CMP's sixth-grade trial materials in an ethnically diverse, but rather high-SES middle school near the university. Overall, I was pleased with what my students learned. Yet, I also had some concerns about how my pedagogy and the materials played out for some students, especially those few who seemed to be disadvantaged. For example, Kobie, the only African-American student (and one of the few I suspected was of low-SES), opted to go to the Special Education classroom for mathematics instead of staying in my classroom. He explained that his sister could help him with his more typical mathematics homework for the special education math class, but not for mine. Since I was a guest teacher in the school, only there for an hour each day, I felt rather helpless in this situation. But it prompted me to wonder about how students' backgrounds might influence their learning in a reformed classroom. Were some students less likely to have the resources necessary to thrive in a reformed curriculum? I was curious how the curriculum would play out in a more diverse situation. So when I had the chance to teach in a more socio-economically diverse setting the following year, I decided to explore these issues more systematically.

The fourth and final source of my framing questions is the literature I was led to by my initial data analyses. As I will explain further in the following chapter, my data analyses drew me deeper and deeper into existing literatures on social class. I read several studies of the cultures of middle-class and working-class families. Stanic (1991) argues that math educators need to consider what critical theory can contribute to our understanding of larger, societal inequities, and also how the lens of cultural discontinuity might help us look more closely at what goes on within schools. Hence, I re-framed my initial thinking about ways to examine and interpret differences in students' experiences with the curriculum and pedagogy. The lens of cultural incongruence has become an important part of this dissertation.

My study has had a circular nature. Experiences and literature led me to ask questions, which led me to collect and analyze data, which led me to ask more questions, which led me to seek out more literature, etc. I discuss this process in more detail in the next chapter.

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# CHAPTER 2

# METHODS OF DATA COLLECTION AND ANALYSIS

Chapter 1 outlined the reasons why I chose to study socio-economic class issues in a mathematics classroom. This chapter discusses how I conducted the study, including the research context and methodology of data collection and analyses.

## Research Context

This study was conducted in a middle-school classroom. I played an unusually large role in creating the setting, as I was not only the researcher for this study, but also the teacher and a writer of the curriculum. I explain my roles in the sections that follow and elaborate ways in which I used myself as both a tool and resource in this research.

### The School Setting

The school, which I will call Jones Middle School, was located in a medium-sized city in Michigan. It was not what one would consider an "inner-city" school, in that it was located in a section of the city that had previously been affluent and had seceded from the rest of the city in creating its own school district. Much of the affluence was due to the auto industry, which is no longer a stable source of wealth in this community. Hence, the school had a socio-economic mix of students — a few upper-middle class (e.g., families with professional parents holding graduate degrees), some middle class (e.g., families with college-educated parents with professions, such as teaching or engineering), some working class (e.g., families with parents who work in factories or service jobs

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jobs) and some lower-class (e.g., families whose parents have very limited education, no steady job, and who live below the poverty line). The school's 500 students were primarily white, with roughly 2% Asian, 3% Hispanic, and 11% African-American students. Thirteen percent of students qualified for free or reduced hot lunch.[1] The data regarding students in my study revealed that some working-class families made plenty of money, particularly when both parents worked at the auto factory; yet, some of these parents had only an eighth-grade education.

Jones had seventh and eighth grades, while another school in the district had fifth and sixth grades. During the 1992-93 school year, one of the CMP PIs worked with some of the district's sixth-grade teachers as they piloted the trial curriculum materials. The Jones principal and several seventh-grade teachers were also interested in the CMP curriculum. They decided that all seventh grade math teachers would use the CMP curriculum during the 1993-94 school year. The CMP project leaders met with the teachers involved and made arrangements to provide the trial materials, as well as some implementation support, including hosting a summer institute and ongoing contact with a CMP staff member during the school year.

Since I had piloted the CMP sixth grade trial materials in a more middle-class setting during the 1992-93 school year, and since I wanted the opportunity to teach in a more diverse setting, I was chosen to pilot the seventh-grade trial materials in a Jones classroom. Hence, my role with the CMP during the 1993-94

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[1] These counts came from a State-required "Fourth Friday Count" in April, 1994. Due to SES and possibly racial patterns in absenteeism, it is probable that the actual numbers of those *enrolled* would reflect greater poverty and perhaps ethnic diversity than these numbers, which indicate which students were actually present in school on a given day.

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school year included piloting the curriculum with one Jones class while serving as a liaison and teaching "model" for the other teachers in the school.[2] Although the Jones math teachers were generally willing to try the CMP curriculum, they, along with other teachers in the school, were worried that students would not get enough of the "basics." For example, when some of the CMP project leaders attended a Jones staff meeting, various teachers voiced concerns about students using calculators and not practicing computation enough. Teachers also noted that some parents had similar concerns, and I heard complaints from a few parents at conferences. The math teachers' solution was to use the CMP curriculum daily, while occasionally supplementing with traditional worksheets that emphasized computation practice.

The Classroom and Students

Ms. Mattel, a Jones teacher, generously allowed me to teach one of her classes of about 30 seventh graders.[3] I taught these students each morning for forty-five minutes from September 2, 1993 through the end of April, 1994.[4] When not with me, the students spent most of their academic time with the three teachers on their "team," including Ms. Mattel (who was the math teacher for the other classes on the team as well as the "home base" teacher for my class). This was the second year these students had used the CMP trial materials, so the idea of learning through exploring the CMP's open problems was not foreign to them. Still, I learned from students that their sixth-grade teachers differed from me in their teaching approaches, which included more teacher lecture at the

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2 For example, occasionally other teachers would come and watch me teach, particularly for lessons about which they had questions or fears. Additionally, there were a couple of times when out-of-town guests asked to see the curriculum in action, and they would observe my class.

3 The name Ms. Mattel is a pseudonym.

4 I was absent two weeks in November (for my comprehensive exams) and two weeks in the beginning of April (for AERA and NCTM). During these periods, the regular classroom teacher, Miss Mattel, taught the class.

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chalk board and students quietly working alone. Hence, some elements of my pedagogy, including the emphasis on whole-class discussions, were new for the students.

Ms. Mattel was virtually always present when I taught, and she observed the students and me. She was helpful as another pair of eyes and ears, often informing me of students' behavior and reactions to the curriculum and my pedagogy. She also provided interesting analyses about which students were struggling and why. Although she and I differed on our perspectives about what qualifies as good mathematics teaching and learning, I found her observations helpful in clarifying my own beliefs, as well as in making me more aware of students' experiences.[5]

When I was not present, Ms. Mattel occasionally worked with my students on what she called "basic skills." For example, while we were doing a unit on volume and surface area of objects, she provided worksheets that reviewed how to find areas of two-dimensional shapes, such as rectangles and circles (which was the topic of a CMP unit the previous year). Additionally, during the two, two-week periods when I could not be there, Ms. Mattel taught my students. Her style of teaching placed more emphasis on giving students rules and having them apply them in problem situations, while I tried to promote the discovery of the rules through solving the CMP problems. This difference in emphases worried me initially, since I wanted to have a more "pure" implementation of my intended pedagogy and curriculum. Yet, as a researcher, I came to appreciate the fact that my students experienced these pedagogical contrasts, because I think they enabled students to be clearer about what they liked or did not like about

[5] For example, for Ms. Mattel, students showing signs of confusion and students challenging the teacher's authority were bad things. For me, these could be good or bad, depending on the context.

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elements of my pedagogy. Also, if students had gone two years without completing a practice worksheet (e.g., with fifty similar, one-step, computational exercises), I would have been concerned that some students were remembering the more typical curriculum with rose-colored glasses; but since Ms. Mattel occasionally assigned practice worksheets, I felt more confident that students were making fair comparisons between curricula. Still, the mixture of messages about mathematics learning that parents, teachers, and I gave to students is an important contextual element of the study and deserves consideration.

## The Curriculum

In this study, trial materials from the Connected Mathematics Project provided the mathematical focus of each class, with students working on the problems both in and outside of the classroom (for homework) each day. These materials were full drafts of the seventh-grade curriculum under development and were piloted in a large number of classrooms in many settings, as part of the development process. The descriptions in the following section and throughout the dissertation refer to the materials as they were in an initial draft stage when used in this study, as opposed to the finished product that is now being published. The general format, mathematical content, and philosophy of the draft units used in this study remained primarily the same throughout subsequent revisions. Specific problems and questions were sometimes revised, added, or deleted. Often, the wording of the problems was made clearer with each revision of the materials. This study, however, cannot and does not attempt to make any claims about the final materials. However, in any case, the reader should bear in mind that this study's focus on students' experiences with the problems in the curriculum are intended to illuminate complexities involved with students learning mathematics through solving open, contextualized prob

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problems. In other words, this study is not a test of one curriculum, but an exploration of reform ideas that are being implemented with various curricula.

The CMP curriculum looks very different from typical mathematics textbooks, which tend to consist of two-page lessons that focus on practicing given rules without context. I begin by describing the various components of the curriculum, and then discuss the authors' underlying philosophy.

Description of the curriculum. The CMP trial materials consisted of about eight units for each of the three grade levels (sixth, seventh and eighth). There were teacher materials and student materials, and each of the students' units was produced as an individual, soft-covered book. The student version of each unit contained the following:

- A setting and/or focusing questions for the unit to promote students' thinking and curiosity about the unit's mathematical ideas and their applications.
- Several (generally 4-7) "Investigations," each containing:
  - a short discussion of the theme for the investigation.
  - one to four problems for students to solve and discuss in class. These problems were intended to be the focus of the lessons and were set apart from the rest of the text by a box. After many of the boxed problems, there were follow-up "Questions" that students could think about or respond to in writing. Some of these questions helped students focus on important patterns and ideas, while others helped the teacher assess students' basic understandings (e.g., reading information from a graph). The questions could also serve as an organizer for the discussion that followed students' explorations.
  - "Applications-Connections-Extensions," or homework problems. Each investigation usually contained about a dozen problems that spanned

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several pages. These problems were mathematically similar to the problems in the investigation and offered students another context in which to explore and apply the mathematical ideas.

- writing prompts to help students summarize the mathematics in the investigation.[6]

Both the main problems of the investigation and the homework problems were virtually always set in a context of some sort — usually a plausible real-world context involving hypothetical, realistic situations (such as starting a business, designing a house, or eating pizza), and occasionally a genuine real-world context that was based in reality at some level (such as examining actual data about world disasters or about students' lives), or a fantasy context (e.g., a story about stretching and shrinking "Mugwumps") or an abstract, strictly mathematical context (e.g., exploring the relationship between dimensions and volume through finding the dimensions of three-dimensional shapes with a given volume). The problems varied in the amount of time required to solve them from several minutes to several days.

Consider the following example. The following problem appeared in the Around Us unit (pp. 14-15) and was used to introduce the need for exponents:

> **It's a Birthday Party!**
> Yvonne wanted to have a party for her birthday in one week. Her mother agreed, but said that it could not be very large.
>
> **Problem 3.4:** Yvonne had an idea. She would invite two people to her party today, the next day those two would each invite two new people

[6] The draft versions of the units were just beginning to place a greater emphasis on previewing and summarizing the mathematics in the unit. "Mathematical Highlights" (at the start of each unit) and "Mathematical Reflections" (writing prompts at the end of the units) became more prominent in later versions, as authors came to realize that students and teachers often need more explicit support to help them pull out the main mathematical ideas.

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(that's four new guests), the next day those four people would each invite two more, and so on . . . Should Yvonne's mother accept Yvonne's plan?

**Questions**

1) How many new guests are invited on the second day? . . . on the third day? . . . on the fourth day?

2)If everyone invited attends Yvonne's party, how many people would be at Yvonne's party in one week (7 days)?

3) Suppose Yvonne started three weeks before her birthday. Is it possible that on any day there would be at least 1,000,000 new guests invited?

This problem was followed by a "Think About This" box that introduced exponent notation as a shortcut for writing expressions such as 2x2x2x2. The above problem is about average in terms of the amount of specific instruction and focus provided — some problems were more "open," calling for more extended exploration, and other problems were more focused, providing relatively specific guiding instructions or questions.

The teacher materials contained the student pages on the left, and information for the teacher on the right of each two-page layout. A single problem could involve several of these two-page layouts. The teacher information included: a detailed discussion of the mathematics involved; suggestions for teaching the lesson, including ways to help motivate students with the context during the launching of the problem; advice for facilitating students' explorations; key questions to help summarize students' explorations; answers to all problems and questions, with discussion about ways to solve the problems, when appropriate; and assessment, including quizzes, tests, and/or a unit project. The assessment items were similar to the curricular problems. Instead of consisting of 50 computation exercises, the quizzes and tests contained only a few, situated problems, each requiring students to explore and write about their solutions.

In my classroom, a typical class period consisted of a discussion of the homework assigned the previous night, and then exploration and discussion of one of the open problems. (Sometimes discussion of a problem would continue for more than one class period, while at other times we would explore two or more problems in a class period.) The units I used with my class were:

- Around Us (developing quantitative reasoning, particularly with large quantities. Exponents and scientific notation are introduced).
- Variables and Patterns (exploring variables and relationships between them, with an emphasis on tables, graphs and symbols).
- Similarity (studying similarity with 2-dimensional shapes, with an emphasis on ratio and proportion).
- Filling and Wrapping (understanding volume and surface area of 3-dimensional shapes).
- What Do You Expect? (investigating probability, with an emphasis on expected value).
- Accentuate the Negative (understanding integers, including computation with them).
- Comparing and Scaling (proportional reasoning, including rates, ratio, proportion, and percent).[7]

As evident from the names of the units, mathematical themes were the curricular organizers. Each unit was carefully designed to teach specific mathematical ideas. When writing the units, the authors chose the mathematical goals first, and then carefully designed problems to help students reach the

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[7] I stopped teaching in May and did not finish this unit. There was also an eighth unit on linear relationships that was not taught due to time constraints.

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mathematical goals.[8] Hence, the problems in the unit build upon each other, with each investigation helping students understand more about the over-arching mathematical ideas.

Philosophy of the curriculum. The CMP authors assumed that uniformly high expectations and a challenging curriculum are keys to successfully educating all students:

> The Connected Mathematics Project assumes that when all students are held to the same high expectations and given a chance to explore rich problems, all students can succeed in mathematics. (CMP, 1995, p. 74)

The authors view the curriculum as compatible with the NCTM Standards (CMP, 1995). Unlike many texts that claim they are compatible with the Standards , the CMP curriculum does not relegate problem-solving to a few extra pages in the text, isolated from the teaching of mathematical content. The curriculum does not give step-by-step procedures for solving the problems. Instead, the curriculum consists of open problems, and students are to work hard at figuring out how to solve the problems. According to the authors (CMP, 1995, p. 9), the following are features of good problems:

- Students can approach the problem in multiple ways using different solution strategies.
- The problem has important, useful mathematics embedded in it.
- The problem may have different solutions or may allow different decisions or positions to be taken and defended.
- The problem encourages student engagement and discourse.
- The problem requires higher level thinking and problem solving.
- The problem contributes to the conceptual development of students.

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[8] This means of curriculum development stands in contrast to one in which a theme is chosen (e.g., animals or movies) and then a variety of mathematical ideas that relate to the theme are explored through problems involving the theme.

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- The problem promotes the skillful use of mathematics.
- The problem can create an opportunity for the teacher to assess what his or her students are learning and where they have difficulty.

My interpretation of NCTM's view of open problems (as described in the previous chapter) seems consistent with what the authors describe as "good tasks" or "rich problems." Both the CMP authors and NCTM share an emphasis on learning mathematical content and processes through solving problems that might have multiple approaches and solutions and that require students to think creatively to solve the problems. Learning mathematical concepts and skills is not separated from mathematical thinking or applications. While NCTM is rather vague about exactly how students are to learn important mathematical ideas through solving problems, the CMP authors describe how they envision this learning occurring as follows:

> The curriculum is organized around rich problem settings — real situations, whimsical situations, or interesting mathematical situations. Students solve problems and in so doing they observe patterns and relationships; they conjecture, test, discuss, verbalize, and generalize these patterns and relationships. Through this process they discover the salient features of the pattern or relationship; construct understandings of concepts, processes, and relationships; develop a language to talk about the problem; and learn to integrate and discriminate among patterns or relationships. The students engage in making sense of the problems that are posed, and with the aide of the teacher, to abstract powerful mathematical ideas, problem solving strategies, and ways of thinking that are made accessible by the investigations. (CMP, 1995, p. 24)

Hence, the goal is not for students to simply learn about solving the individual problems, but to walk away from the problem exploration having abstracted important mathematical ideas or processes. This abstraction occurs both because problems are carefully designed to require students to think about particular mathematical ideas and relationships, and because the teacher plays an active role in highlighting the intended ideas. Hence, like NCTM, the CMP authors argue that the teacher's pedagogy and the curricular problems should work together to facilitate students' learning.

> The problems are designed to allow students to bump in to the mathematics that is embedded in the situation as they work in pairs or groups to solve the problems. The teacher is expected to pull the class together at the end of each problem and at the end of a whole investigation to help the students explicitly describe the mathematical ideas, patterns and relationships and the strategies that they found and used in the investigation. *The teacher plays a central role in making the mathematics come alive.*" (CMP, 1995, p. 27, authors' emphasis)

Hence, according to both NCTM and the CMP authors, the curriculum should be built around open problems that actively involve students in exploring and learning important mathematical ideas. The teacher has a role in helping students abstract the ideas from the problems. I now turn to discussing my role as the teacher in more detail.

The Teacher

As the teacher, I brought many factors to the site of study. In the previous chapter I discussed my SES background. Here I discuss other relevant factors, including my experiences with learning mathematics, teaching mathematics, and the curriculum. Then I turn to a discussion of my pedagogy, and my role as both researcher and teacher.

I brought a strong mathematics background. Throughout high school, I studied hard and was in the "top track" in mathematics. I stood out from my peers because I was continually worried that I did not understand well enough and that I would do poorly on a test. Hence, I was not satisfied to know *that* something was true; I sought to understand both why it was true and why it mattered.

My efforts paid off. My mathematical knowledge allowed me to earn a score on the American College Test that automatically qualified me for a scho

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scholarship to Northern Michigan University (NMU). In 1989 I graduated summa cum laude in mathematics from NMU, and was given the "Outstanding Graduating Senior" award from the mathematics and computer science department of Northern Michigan University. My comfort and interest in mathematics led me to obtain a Master's in mathematics from Michigan State University in 1991.

But knowing mathematics well or liking it does not automatically supply the ability to teach it to middle school students. Although I had never been a full-time "regular" classroom teacher of middle-school students, in my efforts to teach seventh grade I drew from a variety of mathematics teaching experiences. I had previously taught in both a middle school and high school setting in the student teaching experience that led to my secondary (7-12) mathematics teaching certification. Then, as mentioned previously, I had experience in teaching sixth grade sporadically (piloting specific CMP draft units) during the 1991-92 school year, and then regularly for an hour each day during the 1992-93 school year. I was fortunate to have Glenda Lappan observe and critique my teaching several times during the 1992-93 school year as part of a practicum. During this time, I was not simply trying out the curriculum, I was also trying to learn more about teaching mathematics. Additionally, I had gained a year of experience in working with "gifted" middle- and high- school students as part of my work with a program in which MSU mathematics professors taught the high school mathematics curriculum to those students at an accelerated pace. Finally, I had taught mathematics to beginning college students for two years. I won a teaching award from the MSU Mathematics Department during the semester in which I taught "Mathematics for Elementary Teachers," a course designed to help prospective teachers develop deep and flexible understandings of elementary and middle-school mathematics.

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While I had never taught seventh grade full time, I had gained a variety of mathematics teaching experiences and was successful across the contexts in which I had worked. I was comfortable in my knowledge of middle-school mathematics, and my teaching experiences had allowed me to experiment with finding ways to help students understand and feel comfortable in mathematics. These experiences were coupled with my more theoretical studies about teaching and learning that were part of my doctoral program. As I learned about the history and theory underlying various mathematical pedagogies and their relationships to students' learning, I was able to informally explore these relationships myself as I taught in the various contexts. I was able to further develop my own philosophy about mathematics teaching, which centers on respecting students and their thinking, and making their thinking, along with important mathematical goals, the bases of my lesson planning. I also had opportunities to learn about equity issues in relation to mathematics teaching and learning. For example, I wrote a paper on gender and mathematics, synthesizing the latest research on pedagogical strategies for helping girls succeed.

Another background factor, mentioned already, is my work with the Connected Mathematics Project. I became a graduate assistant on the project when it was first starting. I was part of the many meetings involving the principal investigators, as well as those involving the board of directors (consisting of mathematicians, scientists, educators, and business representatives) and privy to extensive discussions about the goals, content, and format of the entire curriculum. At these meetings, I came to understand the many dilemmas involved with developing the curriculum, as well as the assumptions and aims of the project directors. I witnessed their agonizing over decisions about which mathematical goals to include and exclude, as well as

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which problems would most effectively help students reach those goals. I also took part in writing and piloting several units, and I was part of regular conversations about how to improve the units. Hence, through my experiences with the CMP, I became deeply familiar with the curriculum's intent and content and could envision what good implementation might look like.

My experiences offered some advantages in allowing me to create the type of classroom I intended. Having spent two years thinking about and discussing how the curriculum should play out, I had a clear picture of the type of classroom I wanted to create, and I was able to detect when I was not living up to my vision and could search for ways to address the discrepancy. My mathematics background and deep familiarity with the mathematical goals of the CMP curriculum, including how each problem was intended to contribute to those goals, helped me make decisions as I guided students' explorations of the problems and the discussions afterward. As students raised various mathematical ideas, I felt fairly confident I could discern which ideas were correct, which would lead us into important mathematical terrain, and which would likely take us down a dead-end road.

Because I taught only an hour each day, I had more time to reflect on and plan for my one class than do most teachers. Still, by being in the school only an hour each day, I was not as involved with the students, faculty, and administration as I would have liked. (This led to some of the issues surrounding Ms. Mattel's involvement, which had its own advantages and disadvantages, as discussed above).

Additionally, one could argue that my limited experience as a middle school teacher was a disadvantage. This is probably true in some ways. For example, management issues were not resolved as naturally for me as they seemed to be for Ms. Mattel. While Ms. Mattel immediately demanded and

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earned students' respect and obedience, I struggled with management issues all year. While this difference was most likely due, in part, to our differences in experience, it was also due to our pedagogical differences. For example, I wanted to encourage students to question me and each other about mathematical ideas, and students had difficulty figuring out when and how it was appropriate to challenge my authority. Still, more experience with implementing my teaching philosophy might have helped me communicate my expectations to students more clearly. Additionally, more experience in teaching middle school students probably would have strengthened other important skills, such as predicting when students would have misconceptions, listening to students and understanding the social dynamics of the classroom.

While it feels alternately embarrassing and risky to openly discuss my strengths and weaknesses as a teacher, my ability as a teacher to create my interpretation of a "reformed" classroom is an important contextual factor in the study. My conclusions will raise questions about these reformed practices, and I want the reader to take the questions seriously instead of waive them away by assuming that I was not able to implement the reformed practices due to my inexperience or incompetence as a mathematics teacher. Although I will readily admit that I have more to learn about mathematics and teaching, I hope the personal information I provided will convince the reader that I had at least an average chance of implementing these reform ideas and making them "work" for all her students.

I have discussed my background and its relationship to my ability to implement my intended pedagogy, but I have not yet provided much detail about what my teaching entailed. Hence, I now explain some essential elements of my pedagogy.

First, I tried to adhere closely to the CMP curriculum and teaching philosophy. There were only a few occasions on which I supplemented the CMP trial materials, and this was to make connections between the mathematical ideas under study and current information in the media (primarily newspapers and magazines). As advocated by the CMP, I usually structured my teaching using a "Launch, Explore, Summarize" (LES) model. Sometimes an LES lesson would last more than one class period, and other times I would do more than one such lesson in a class period. I now explain what each of the three parts of the model entails:

Launch. The purpose of the launch is to prepare students for exploring a mathematical problem. During the launch, I would:

- pose the problem, usually by reading (or have the students read) it from the CMP trial materials.
- ask students questions to clarify what the problem might mean and entail and to help them link this problem with previous work (when appropriate) and/or with new mathematical language or ideas.
- prompt students' curiosity about the mathematics by asking them to guess what will happen or what they might find out (when appropriate), or help students focus in some other way on the mathematical challenge in the problem.

One issue I continually struggled with in the launch was how much structure (e.g., detailed instructions, interpretation of the problem, rules, materials) to give students to help them get started. I wanted to launch the tasks "in such a way that the potential of the task is left intact even though students are given a clear picture of what is expected" (CMP, 1995, p. 41).

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Explore. Students' exploration of the problem could occur individually, in groups, or with the whole class and teacher. When we were *not* exploring as a whole class (which usually was the case), I still had an active role. Even if students were not always working with their group, their seats were arranged in groups of three or four. I tried to spend time listening to and talking with the groups. Hence, I would:

- circulate, listen to students' discussions or look at their work to assess them.
- help students understand and encourage them to focus on and persevere with the problem they are exploring when they were having difficulty. This might involve redirecting the students, mediating group conflicts, answering simple logistical questions, affirming their efforts thus far, and/or asking them a question to pique their curiosity about the problem.
- help students recognize and connect the big mathematical ideas they encounter through questioning and/or explaining.
- clarify expectations for communicating the results of their explorations (e.g., writing answers to specific questions, reporting their data on the overhead, etc.).
- provide extra challenges for students who were ready in the form of another problem or just a question to think about.
- help prepare students for the summary (e.g., let them know if there are certain ideas I might want them to talk about).

In the exploration phase of the lessons, I struggled with several issues. First, I wondered to what extent I should push for and how I should structure cooperative group work when I saw much destructive behavior in the groups, for example in terms of gender dynamics (e.g., the boys taking the intellectual lead and putting down the girls), and in terms of individual problems (particularly for a boy named Carl who had severe social difficulties). I tried and felt more successful having single-gender groups. And of course, the amount of direction I would provide students who were having difficulty and wanted help, was always an issue for me. I did not want to be *the* authority for knowledge in the class, so I avoided providing "quick fixes" for students' struggles. When necessary, I would ask students questions to prompt their thinking in productive directions.

I encouraged the use of a variety of tools, and I also wanted students to learn to make their own decisions about which tools would be helpful for exploring various situations. Each day when students arrived, they picked up their TI-82 graphing calculator from a table by the door. Hence, a calculator was always available to them, and they often used them in their problem explorations. Additionally, there was a cart at the front of the room containing a variety of materials, including graph paper, scissors, cubes, and angle rulers. Students could go to the cart at any time during the "explore" phase of the lesson to get the materials they thought would be useful.

Summarize. During the final stage — the summary— the teacher leads a whole-class discussion to help students pull out the big ideas and connect them to previous learning. From my lesson planning and from observing students, I knew which key ideas I hoped to push in the discussion. Yet, at times, students would initiate an additional, interesting direction that we would pursue. During the summary, I would try to:

- ask students to explain how they solved the problems, pushing for clarification or refinement when needed.
- encourage students to make sense of and compare methods, solutions and ideas shared by their classmates.

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- ask questions to help students focus on important mathematical ideas and to make connections to previous learning.

NCTM (1991) suggests questions that teachers can ask in order to facilitate whole class discussions, including:

- What do others think about what Janine said? Do you agree? disagree?
- Does anyone have the same answer but a different way to explain it?
- Can you convince the rest of us that that makes sense?
- Why do you think that? Why is that true? How did you reach that conclusion? Does that make sense?
- Does that always work? Is that true for all cases? Can you think of a counter example? How could you prove that? What assumptions are you making?
- Do you see a pattern?
- How did you think about the problem?
- What is alike and what is different about your method of solution and hers? (pp. 3-4)

It was questions like these that I tried to ask my students during the summarizing discussions. Some days were better than others for allowing me to use a good variety of these questions — schedule constraints, problem types, and students' interests were all variables that shaped the day's discussions. Yet, each day, I did try to involve students in mathematical discussions.

The LES model framed the planning of lessons, and the CMP problems were the focus of the lessons, but my students influenced decisions about our pace. I wanted to take students and their ideas seriously, and this also meant taking their confusion seriously. When students showed confusion about an idea, I would spend more time discussing it than I might have initially planned (unless it was the type of confusion anticipated by the CMP writers who designed several problems to get at the confusing idea). Additionally, my commitment to equity colored my pedagogy. For example, when girls were not contributing to discussions at the beginning of the school year, I took several steps to address the problem, including calling on girls whenever possible, priva

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privately encouraging girls to participate, and discussing with the entire class what research says about middle-school girls' declining participation in mathematics.

I have outlined my view of my intended pedagogy, but the teacher's view might be very different from that of the students. Yet, evidence from surveys and interviews about our classroom indicated that students tended to share my perspective about most major elements of my teaching. For example, I intended to guide students' problem solving, prompting their explorations in fruitful directions when they were stuck, as opposed to giving them the "right answer." When asked a multiple-choice question about my response when they had trouble with a math problem, thirteen students chose "our teacher encourages us to figure it out for ourselves" while six students (all males[9]) chose "our teacher shows us how to do it." No student chose the remaining response, "our teacher tells us the answer."[10] Similarly, I intended to encourage and value the use of different problem-solving methods. When students were asked how often I encouraged them to find different ways to solve the same problem, all but two students marked "always" or "usually." Similarly, students confirmed my belief that students worked in groups and used calculators in math class at least half the time.

The degree of consistency among students on rather straight-forward, relatively objective questions (such as those about calculator usage and time spent in groups) makes me feel more comfortable with the reliability of my students' survey and interview data than I might otherwise be. But there were

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[9] I am curious if this is because I was overcompensating for what traditionally happens — the girls get more help while the boys are challenged. But perhaps the boys and girls interpreted my help differently.

[10] From the end-of-year CMP survey, a general instrument used in pilot sites to gather data on students' perceptions of the curriculum and pedagogies used.

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some surprises in some students' reports. Perhaps the most troubling discrepancy between my intended pedagogy and some students' perspectives related to questions about how fair I was. While I tried to be equitable in my treatment of students, some students thought I was unfair, citing examples such as my allowing a diabetic girl to eat in class and allowing a handicapped girl to arrive from gym late.[11]

Challenges and Benefits of the Researcher-Teacher Role

At this point, I think it would be helpful to talk a bit about my dual role as both teacher and researcher. Many others have made important contributions through research in this capacity, especially on issues relating directly to teaching (Ball, 1993; Ball & Wilson, 1996; Chazan & Ball, 1995; Lampert, 1985). Still, others have used the researcher-teacher role to study students' experiences in their classroom, thereby shedding light on the ways in which certain teaching practices play out with students (e.g., Lensmire, 1993). Similarly, in this study, I occupy the role of both the researcher and teacher, while I focus on students' experiences in my classroom.

Cochran-Smith and Lytle (1990) explain how research questions can come out of the intersection between theory and practice. This was true for me, as it was during my sixth-grade teaching the year prior to this study that questions arose for me out of a disjuncture between what I expected would happen (based on the theories about teaching and learning I had come to believe) and what occurred with some students.

Although important research questions often arise out of real teaching situations, the answers are not always best sought by the teacher involved. My

[11] These examples point out a difference in my interpretation of "fair" and that of some students. While I saw unequal treatment of students as being more fair in this case, some students did not agree.

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questions about socio-economic class and mathematics learning could have been addressed in a variety of ways, but in this study I investigated them in my own classroom as a researcher-teacher. This is an approach in which "the teacher is the principal investigator of the research, and where at least one central goal is to contribute to scholarly discourse communities and to the development of theory" (Ball, in preparation, p. 9).

The researcher-teacher plays a large role in designing the case to be studied. (Ball, in preparation) This was important in my study because I wanted to study students' experiences in a classroom aligned with current reforms (as I interpret them), and these can be difficult to find. For example, I have already described how, although the Jones teachers were using the CMP trial curriculum and perhaps thought they were teaching in ways the authors intended, there were many aspects of their implementation that differed from mine. Hence, if I had chosen to study one of the Jones teachers, I might have felt unable to argue that the site was a reasonable implementation of reform ideas. Even if a teacher agreed with my interpretations of reform ideas and what an implementation should look like, other contextual issues might have become problematic for my purposes, such as the teacher's understanding of mathematics or her sensitivity to lower-SES students. Hence, my role as the teacher allowed me to play a large part in designing the site, which, (ironically) enabled me to focus more on the students' experiences and less on issues about the teaching. An additional advantage to my role as a teacher is that I did not need to worry about hurting another teacher's feelings when analyzing her students' reactions to her pedagogy.[12]

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[12] I was recently part of a writer's support group in which a colleague was struggling with how to handle these issues in writing his dissertation. While he initially chose to study a classroom of a teacher whom he initially thought was excellent, he ended up having many critical thoughts

Another benefit of my role as researcher-teacher is that I had immediate access to data that most "outside" researchers would not have had. For example, when Dawn told me privately that she could not do her homework because she has no ruler at home, or when she "accidentally" (maybe) turned in the wrong paper that contains a story about her cruel, unemployed father, I was sure to know about it immediately, whereas an "outside" researcher might never know about these mundane, yet meaningful, occurrences. I found that the data gathered through informal, logistical interactions with students and their parents provided important information about both the students' backgrounds and their experiences in the class. Additionally, I was aware of virtually all formal and informal *mathematical* interchanges I had with students, including the private "hints" I gave or questions I asked. Thus, there were ways in which I had a more intimate relationship with the students and with both the formal and informal curriculum that students experienced than most "outside" researchers would.

Finally, I, as the teacher, was prompted to do informal data analyses regularly in order to make instructional decisions. For example, as the teacher, I was forced to come into daily contact with student work, as I graded their tests, quizzes, and homework. This responsibility ensured that I was at least conducting an ongoing, informal analysis of students' learning. As another example, I pondered students' mathematical understandings, interaction patterns, and personality dynamics every four or five weeks when I made new seating charts. My need to perform these analyses as a teacher assisted my research in ways that might not have otherwise occurred. For example, through the need to create and re-create seating charts, I came to realize how amicable and on-task certain students were and how easily distracted other students

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about the teaching and was unsure how to avoid offending her as well as what claims he could make about the students' experiences.

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seemed to be, and how certain behaviors were magnified or diffused, depending on which students were seated together. Likewise, my role as a researcher assisted my teaching, as I was collecting and giving attention to data that allowed me to make well-informed instructional decisions.

Of course, there are tradeoffs. First, some might worry that I was not as "objective" as an outsider would be, because I have a personal stake in making claims about my students' learning from my teaching. I do not worry very much about this, because my goal was not to claim that students are or are not learning a great deal, but instead to examine variations between groups of students' experiences and learning. I was more interested in analyzing how my teaching differentially affected students than in making claims about my teaching being uniformly beneficial to all. While I do feel vulnerable as I disclose students' reactions — both positive and negative — to my own teaching, I try and maintain a researcher's curiosity about the causes of these reactions and avoid feeling defensive. The temporal distance I have gained (since the teaching occurred over two years ago) helps me in this regard.

Still, there is the valid point that a researcher in the back of the room, with only research to be concerned about, is able to pay better attention to data collection than the teacher. I certainly believe that an "other" would have been able to focus attention to some things that I, as a teacher-researcher, could not. For example, a researcher in the back of the room might have been better able to record the actions of a student, like Guinevere, who sits in the back of the room and doodles. Yet, it is possible that I, as a teacher who might have a closer relationship and a history with Guinevere and her work than someone from the outside, might better be able to *interpret* her actions. Perhaps not. I think it would depend, in part, on the way in which an "outside" researcher conducted her work. Still, I tried to make up for my lack of ability to take notes during class by

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by tape recording each session and writing notes in my journal immediately after each class. Additionally, as mentioned previously, Ms. Mattel was another helpful set of eyes in the classroom.

My role as the teacher most likely affected the data I was able to gather. Since I had a personal relationship with my students, I was probably able to gather more open, honest answers from some students about some things, while perhaps others were reluctant to give me honest responses, particularly to questions about me and my pedagogy. Although I tried to be a relatively non-authoritarian teacher, I could not expect students to view any teacher as a peer to whom they would reveal all secrets. Despite my less directive role, I was still the person who ultimately judged students' work and determined grades. To try and address this concern, I had colleagues conduct some of the interviews with students, especially about potentially sensitive topics, such as students' opinions about me and their classmates.[13]

My role as a researcher sometimes seemed to be in tension with my role as a teacher. For example, in taking time out to have students complete surveys, I worried about taking too much time away from the "regular" curriculum. Yet, this and similar concerns would also be present if I was studying in another teacher's classroom. In some sense, the dual role enabled me to conduct research in a more humane way — i.e., I had to balance the concerns of my individual students who I personally cared about with my larger research interests. Still, some concerns about role conflicts were unique to my situation as a researcher-

[13] In general, I was impressed by how open the students seemed in interviews, whether I or a colleague was the interviewer. Students did not seem afraid to tell me about their struggles with the curriculum or with my pedagogy, including what they liked better about previous years. Additionally, they seemed to talk openly with me about other students. My colleagues did seem more able to obtain students' opinions about my biases as a teacher. Still, although most students told me things that seemed surprisingly honest, I realize that I cannot assume they were always as open and honest as they could be.

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teacher. For example, I had one student who refused to participate in my study, even though her parents gave permission. I had a difficult time "getting along" with this student all year, and I could not help but wonder if, despite my best intentions, I was treating her differently somehow because of her refusal to participate. Additionally, there were several other students in my class whose parents refused permission to participate, and I worried that these students did not receive as much helpful attention from me as my other students. I tried to compensate by making conscious efforts to give attention to these students.

In weighing the overall situation, I believe that, for my purposes, the costs of my role as a researcher-teacher were worth the benefits. I tried to exploit the advantages and guard against potential problems inherent in my dual role.

## Data Collection

As mentioned in Chapter 1, I began this study with a focus on students' understandings of mathematics in the media. Yet, my focus eventually broadened to examine students' experiences with the curriculum and pedagogy more generally. Hence, the reader should understand that when I mention "media" below, it is the original focus to which I refer.

I collected a variety of data throughout the year. I summarize the data in Table 2.1 and discuss the collection process in more detail in the text that follows.

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**Table 2.1**
**Summary of Data Collection**

| Data | When | Who | Purpose (as related to this research) |
|---|---|---|---|
| Parent Survey & Permission Slip | Beginning of year | Parents | To gain permission and to gather SES data about my students |
| General Surveys | Beginning and end of year | All participating students | To gain information about students' attitudes toward and understandings of mathematics as used in the media, as well as their opinions about the types of educational experiences (including the curriculum and pedagogy) that have influenced these attitudes and understandings |
| CMP Surveys | Beginning, middle and end of year | All students | To gain rather general information about students' attitudes toward the CMP curriculum and their perceptions of their experiences with the curriculum and my pedagogy |
| "Show What You Know" (CMP Survey Instrument) | Two times during the year | All students | To understand students' experiences with the mathematics in the CMP trial units, including their beliefs about the mathematical ideas and the experiences they found most beneficial in attempting to learn the mathematics |
| Student Interviews | Three times per year | Target Students | To gain in-depth information about students' understandings of and attitudes toward mathematics in the media, as well as students' experiences with the curriculum and pedagogy |
| Quizzes, tests, projects, some key homework | Approx. weekly | All participating students | To keep a record of students' mathematical performance and engagement with the curriculum |
| Tape Recordings | Daily | Whole Class | To keep an audio record of the day's events |
| Journal | Daily | I wrote it, giving special attention to the target students | To serve as a guide to the mass of audio tape being collected, to record a summary of my perceptions of the daily curriculum and pedagogy, to record my observations of students' behaviors and contributions in the classroom, and to be a record of the issues I was struggling with as a teacher (as many of these issues involved equity concerns and student experiences) |
| Miscellaneous (e.g., seating charts, notes from interactions with parents) | As needed | All students and parents | Miscellaneous reasons (i.e., seating charts allowed me to keep track of one aspect of students' experiences; parental communication provided information about students' backgrounds, etc.) |

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Student Participation and SES Categorization

At the start of the year, I asked parents for permission to include their child in the study. I ultimately gained permission to include 22 students, but some of these students joined the study late, since their parents initially withheld permission to participate.[14]

To get a sense of my students' socio-economic class backgrounds, I surveyed their parents, asking about occupation, education, income, number of books in the home, and newspapers read regularly.[15] These are commonly used SES indicators (e.g., see Duberman, 1976; Kohr, Masters, Coldiron, Blust & Skiffington, 1991; White, 1982[16]). I used these data to place the students into two, admittedly rough, categories: lower and higher SES. The lower-SES students seemed primarily working class (e.g., the students' parents had little or no college education and worked in factories or service jobs), but a few of them could be considered lower-class (e.g., parents unemployed and living below the poverty level). The higher-SES students were what most Americans would call middle-class, with some families bordering on upper-middle class (e.g., parents with bachelor's or master's degrees who work in professions, such as teaching, engineering, or social work). I categorized the few students who might be

[14] A few students changed their minds and wanted to join the study in the middle of the year, and their parents supplied permission. One boy (or his parents) even changed his mind after the year was over – I was surprised to see his permission slip arrive in the mail. There were several cases in which students seemed to make the decision about parent's responses to the permission slip — i.e., several students whose parents supposedly denied permission in the beginning of the year seemed easily able to reverse the decision later in the year when the student decided that she wanted to participate.

[15] Sending both the SES survey and permission slip home together was probably a mistake. The SES survey touched on sensitive issues for some parents, and some probably denied permission in order to avoid dealing with the survey. I might have ultimately had more participation if the permission slip preceded the survey.

[16] White (1982) conducted a meta-analysis of studies between SES and academic achievement. He found that a correlation existed between SES and achievement when it was defined as income, education and occupation, and this correlation was stronger when home variables, such as reading material, was considered. He was disturbed by the variety of ways in which the term "SES" was tossed around and urged researchers to develop and accept a common definition.

considered lower-middle class as lower-SES (e.g., Mark, who lived with his mother, an administrative assistant with an associate's degree). The parents of two participating students did not supply any SES information. The SES data for two other students — Rodney and Adam — were mixed (e.g., college educated mother who managed a business paired with an unemployed father with no college education), and I did not feel comfortable categorizing them as either lower- or higher-SES. The remaining 18 students were fairly evenly split among four gender/SES categories (see Table 2.2).

**Table 2.2**
**Participating Students Categorized by Gender and SES**

| Higher-SES Males | Higher-SES Females | Lower-SES Males | Lower-SES Females |
|---|---|---|---|
| Benjamin | Samantha | Carl | Rose |
| Timothy | Rebecca | James | Anne |
| Christopher | Guinevere | Nick | Dawn |
| Harrison | Andrea | Mark | Sue |
| Samuel | | | Lynn |

Note: Throughout this dissertation I will use one-syllable pseudonyms to refer to lower-SES students, and three-syllable pseudonyms to refer to higher-SES students. I use two-syllable pseudonyms to refer to students whom I did not categorize because I did not have clear SES data for them.

Categorizing students in this way makes many people — including myself — uncomfortable. But the categories are useful as a starting point for comparing the mathematical learning experiences and needs of lower- and higher-SES students.

Throughout the year, I collected survey and homework data from the participating students. Still, I wanted to follow a smaller group of "target students" more closely, interviewing them in the beginning, middle and end of the year. In selecting the target students, I wanted to separate gender and achievement differences from SES differences (see Table 2.3). I selected a representative for each of these eight categories, although some of the categories were very difficult to fill (e.g., there really were no low-achieving, higher-SES fen

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females, so I took the lowest achieving higher-SES female, who was still quite high achieving):

**Table 2.3**
**Separating Achievement from SES in Selecting Target Students**

| | Lower-SES | | Higher-SES | |
|---|---|---|---|---|
| Low-Achievement | Female | Male | Female | Male |
| High-Achievement | Female | Male | Female | Male |

For a variety of reasons — including fear of attrition, wanting to honor students who asked to be interviewed, and, most importantly, attempting to fill gaps that seemed to be left due to diversity within the categories — I interviewed more than eight students. For example, I added Sue to my pool of interviewees because she requested the opportunity to be interviewed, and she added some dimensions that her fellow lower-SES females did not (she was only mid-achieving, though she tried very hard, and she raised some gender issues that seemed important, because she was often ridiculed by the boys in the class). Hence, while twelve students participated in the first round of interviews at the start of the year, fourteen were interviewed in the second round, and eighteen students were interviewed in the final round.[17]

With the variety of survey instruments used throughout the year, I did not always have full participation from all students, due to students joining the study late, absenteeism (and my occasional slip in getting students to make up everything they missed), as well students' and parents' refusal to answer some of the questions (e.g., students answering questions with evasive responses, such

[17] I didn't anticipate either the changes in permission throughout the year, or students' strong desires to participate in the interviews. Although it might seem haphazard, I felt I needed to include those students who felt they had something to say about the curriculum and pedagogy. This is another case where my role as a teacher might have influenced my role as a researcher — I had a personal relationship with students and did not want to hurt them by implying I did not want to hear what they had to say about the class.

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as, "What does this have to do with math?"). The way in which I address the variation in numbers of students participating in the different instruments is to include all data I have and simply report the numbers I am using. Hence, for example, if I draw from a survey question for which I had sixteen participating students' data, I report the actual numbers, such as "twelve out of sixteen students said . . .".

Interviews

Students were interviewed at the beginning, middle and end of the year. The interviews took place in the school (in the library or an empty room) and tended to last between twenty and thirty minutes.

I conducted the first and last interviews, and colleagues conducted the second interviews. I chose to conduct the first and last interviews so I could be involved with the students and could probe as I saw necessary. Also, I suspected that some of my students might be more comfortable with me as the interviewer. Still, I had concerns that some students might give me, their teacher, different answers than an outsider, particularly on questions that ask them to describe their experiences with the class and my teaching. Additionally, I feared that the way in which I conducted the interviews could be detrimentally influenced by my impressions of the students. Hence, I asked other graduate students who were familiar with mathematics education and my research to conduct the second interviews. I designed the interview protocol, and I talked with the other interviewers about my intentions behind the questions and encouraged them to probe as they saw necessary. Thus, I gathered interview data from students in both situations, giving me two possibly different perspectives on students' thinking.

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The protocols for the three interviews were similar, beginning with questions asking students to interpret mathematical statements in the media, and then moving to questions about students' experiences with the curriculum and pedagogy. For example, questions in the first interview asked students to compare the current math class, including the CMP curriculum and my pedagogy, with previous math learning experiences. Questions in later interviews asked students to consider how their opinions about the class might be changing and why. My ongoing, albeit informal analyses of students' data throughout the year influenced the questions asked in the later interviews. For example, after several students mentioned feeling confused during whole-class discussions, I added a question about this in the final round of interviews. (See Appendix A for interview protocol).

All interviews were tape recorded, and I transcribed the tapes. The interview data complemented the paper-and-pencil surveys, helping me more fully understand my students' thinking and experiences.

Surveys

All participating students completed a variety of paper-and-pencil assessments throughout the year. At the beginning and end of the year, students completed a survey I had designed in order to obtain information about students' analyses of mathematical claims in the media, as well as students' reactions to the class. The survey asked students about the aspects of the curriculum and pedagogy they liked and did not like (and why), what they struggled with, how they viewed our roles in the class, as well as various background information, such as the tools they had at home and their plans for the future.

Students also completed CMP surveys that asked them to describe the pedagogy and environment in our class (such as how often group work and tecl

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technology were used). Additionally, CMP-designed "Show What You Know" surveys asked students to describe their experiences with particular units, including which mathematical ideas were most interesting and important and what activities helped students learn the ideas. These instruments were administered to all students in CMP pilot sites to inform the curriculum development. (See Appendix A for examples of the surveys used.)

### Other Data

Although the surveys and interviews played the largest role in my data analyses, there were several other types of data I collected. Each day I made an audio recording of the class and kept a journal to document my teaching observations, concerns, and intentions. Each journal page began as a template (on my computer) that provided space for me to briefly explain the happenings of the day, to write more detailed notes of any interesting discussions on the tape, to write about each of the "target students," and to discuss my concerns and tensions. This information helped me keep track of what was happening in the class (thereby also serving as an index to the audio tapes), as well as the issues that arose for me and for individual students throughout the year. I also copied participating students' quizzes, tests and selected pieces of homework. Furthermore, I collected various documents across the year, such as seating charts and notes from parents, as well as my notes about interactions I had with parents during parent-teacher conferences.

## Data Analysis

Due to the focus of this dissertation, I concentrate here on my analyses of data relating to students' experiences with the curriculum and pedagogy in my classroom, though I also analyzed students' data in relation to the original media questions (Theule-Lubienski, in press).

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I began the analysis with themes that arose for me as the teacher, such as students' behavior in class (e.g., lower-SES boys goofing off more when given more freedom) and differences in access to material resources (e.g., calculators, rulers) at home. But I tried to put my initial hypotheses aside and more systematically examine the data. In retrospect, I am glad I did not settle for only those themes that struck me while teaching, as I would have missed what I ultimately have come to believe are the most significant themes in this study.

Data analysis has been a long and cyclical process. The questions and themes I thought were important have changed or been re-framed throughout the analysis. The theory I will propose has been developed more inductively than deductively, as it emerged from my classroom data and was later informed by existing studies on social class. Hence, initial data collection and analyses preceded my research of existing literature on socio-economic class. This process is aligned with a "grounded theory" research approach, in which a theory emerges from the data to help us understand important variables and relationships among them. The study and influence of existing literature is important in this approach, but it is delayed until after initial analysis of the data (Becker, 1993; Strauss & Corbin, 1990).

Table 2.4 summarizes my data analysis and related activities, which I discuss in more detail in the text that follows.

**Table 2.4**
**Data Analysis Timetable**

| | |
|---|---|
| 1993-94 School Year | Collected data. |
| Summer, 1994 | Organized data, transcribed interviews, read through journal, displayed interview and survey data in tables, noted recurring themes relating to SES, gender, the pedagogy and curriculum, in interviews, journal, and surveys. |
| Fall, 1994 | Put emerging, general SES themes on hold and conducted media analysis, which I presented at MCTM (11/94) and also wrote about for the 1997 NCTM Yearbook.<br>In December, I returned to the general SES analysis and took stock of emerging interpretations. |
| Spring, 1995 | Data analysis thus far led me to research existing SES literature.<br>Spoke at several graduate classes about my developing analysis and interpretations, thereby learning much about how others will interpret my interpretations. |
| Summer, 1995 | Took stock of my "hunches" thus far. In light of the data and the literature I considered what important themes might be and what evidence I had so far to support or refute them.<br>Took themes/questions I thought might be important and systematically went through all survey and interview data to tally all evidence and counter-evidence I had for each student under each theme. |
| Fall, 1995 | Finished tallying and wrote summaries about what all tallies said — both evidence and counter-evidence for various assertions.<br>Conducted in-depth class discussion analysis by coding many aspects of each turn taken on the audio tapes. Looked at how these results compared to students' interview and survey data.<br>Took stock of student work. Decided not to analyze it further than what I had already done. It did not seem very informative for my purposes at this point.<br>Wrote cases of the six girls to take an in-depth, coherent look at how the various themes or factors were playing out for particular students. |
| Spring, 1996 | Wrote working outline and began writing from it.<br>Interviewed for jobs and, therefore, had interesting conversations with people around the country about my study. |
| Summer, 1996 | Finished first drafts of chapters. |
| Fall, 1996 | Revised chapters, defended dissertation |

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Initial Data Analysis

After finishing data collection, I transcribed all interviews, and created tables containing all survey and interview data organized by question. Using tables enabled me to read across the rows to get a sense of particular students and to read down columns to compare all students' responses to the same question. I organized the rows of students in (rough) order from lowest to highest-SES. I analyzed each question, looking for comparisons between SES and gender groups. I wrote notes about what my analysis of each column of data suggested about possible similarities and differences by SES and gender.

Although my primary focus was SES, I looked for possible interactions with gender, because, as mentioned in Chapter One, several scholars have pointed out the dangers of studying the variables of race, class and gender in isolation (e.g., Campbell, 1991b; Leder, 1992). Still, dealing with three variables simultaneously is difficult. In this study I had primarily Caucasian students, with two African-American boys. One Hispanic female, Rose, had distant relatives living in Mexico, but she spoke English with absolutely no accent, as did her mother; hence, I assumed her family had probably lived here for at least two generations. When looking at the data for these three students, I wondered how their ethnic cultures might be a factor, but I did not do any formal separation for race in my analysis, since the numbers of non-Caucasian students were so low. By having a primarily Caucasian class, I was able to focus primarily on gender and class without much variation in race within those categories.

Using SES- and gender-related themes that arose from the survey and interview data, I looked for related patterns in my journal. I referred to audio recordings, student work, and miscellaneous data when they were relevant to specific questions that arose from my analysis of the surveys, interviews and my journal.

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As I began to draw conclusions about patterns I was seeing, I was cautious. I looked carefully across the variety of data sources I had for confirming and disconfirming evidence. Because of the large variety and quantity of data I had on each child, it was generally easy to find conflicting evidence, not only within data for SES groups, but also within the data for individuals. I tried to understand changes in individual students' responses across time. For example, some students initially expressed much frustration with various aspects of the CMP trial materials, and I have tried to sort out if this frustration was really related to their own experiences with the curriculum or if their attitudes were related to the opinions of others (e.g., classmates, parents, teachers). By asking students to talk about how their feelings changed across the year, I was able to have insight on how they perceived their responses changing. By looking closely at the rationales given for complaints across time, I could see if there was a consistent theme or source of frustration (such as having too many opinions on the floor at one time) or if the student seemed to be inconsistent in his/her complaints, apparently looking for anything to complain about (e.g., saying that we move too fast and too slow, or that more and then less teacher guidance is desired).

### Exploring Relevant Literature

This initial round of data analysis prompted me to read socio-economic class studies conducted from a variety of disciplinary perspectives, including sociology and psychology. These studies helped me consider which patterns I was seeing in the data might be specific to my students and my classroom and which issues might be related to more general factors, such as cultural differences between socio-economic classes. Gradually, I began to focus on two seemingly significant areas: students' experiences with whole-class mathematical discussions and the curriculum's open-ended problems.

Further Survey and Interview Analysis

After taking stock of what my initial analyses of the data and literature suggested, I worked "top-down," logically deducing categories from the themes I thought were important thus far, and "bottom up" from the survey and interview data, as seemingly important data did not always fit in existing categories. I developed roughly sixty questions or mini-themes. Examples include:

- I am afraid to talk in class.
- I think discussing is fun.
- Have people gotten their feelings hurt in class discussions?
- Reactions to being frustrated on problems.
- The CMP problems are like real life.
- The CMP problems are fake.
- I was better at math the old way.
- I am better at math the CMP way.
- General dislike for math this year.
- I think/learn more in CMP.

For each of these themes I created a table in which I recorded all relevant survey or interview data for each student, categorized by SES and gender. I also often recorded key quotes to clarify what students meant, when appropriate. The statements for which I simply recorded agreement (e.g., The CMP problems are like real life) usually had an "opposite" counterpart (e.g., The CMP problems are fake) and this was one way in which I could seek counter-evidence. For other, more open questions, such as the example below, I recorded all evidence relating to it — whether agreement, disagreement or somewhere in-between. Looking at

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the evidence from each student for each data source allowed me to check the student's consistency. Table 2.5 is an example. (You can see that most of the data relating to this question is from the second interview and third survey, but I looked through all the data sources listed on the left for relevant data.)

After creating tables for each of the questions or themes, I wrote summaries of what I was seeing in the data for each one, being careful to note when students were consistent in a particular belief or when they were inconsistent. I also grouped the themes together to examine patterns and conflicting evidence among related topics.

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**Table 2.5**
**Example Chart: "Have People Had Their Feelings Hurt in Class Discussions?"**

| | Higher-SES Males | Higher-SES Females | Lower-SES Males | Lower-SES Females |
|---|---|---|---|---|
| 1st CMP survey 9/27 | | | | |
| 1st media survey 10/28 | | | | |
| 1st interview 11/23 | | | | |
| 1st Show know 11/17 | | | | |
| 2nd CMP survey 2/2 | | | | |
| 2nd Show know 2/14 | | | | |
| 2nd interview 3/21 | Timothy- Sue is kind of slow and we get on her when she acts like she's not listening. | Guinevere - probably<br>Samantha - I don't think so - maybe embarrassed if they voiced the wrong opinion, after they found out what was right, but they get over it and think oh well we were wrong and learn from it.<br>Rebecca - no | James - no<br>Nick - no - they don't talk about people, they just talk about math and what they think is right or wrong.<br>Mark - mostly Sue - class laughs when she gets wrong answer<br>Rodney - they all sigh when this one girl (Sue) tries to ask a question | Dawn - no, they'll usually come back with something<br>Sue - yes - gave ex. of peers making her feel dumb<br>Rose - yes, Sue - when she says something or when she's arguing . . . people roll eyes . . . |
| 3rd survey 4/26 | Adam - probably<br>Benjamin - I feel stupid when I'm wrong<br>Timothy- I don't know<br>Christopher - no | Guinevere - no<br>Samantha - no<br>Andrea - I was embarrassed & mad when others laughed at me<br>Rebecca - yes, when people cut you off and try to disprove | Carl - I don't know<br>James - no, teacher accepts all ideas<br>Nick - no<br>Mark - yes, making mistakes on homework, especially Sue<br>Rodney - yeah, Sue | Dawn - no<br>Sue - Yes, I feel stupid if I get an answer wrong<br>Rose - yes, Sue<br>Lynn - yes, me<br>Anne, no |
| 3rd CMP survey 4/26 | | | | |
| 3rd interview 4/26 | | | | Sue - Yeah, when were in the hallway, guys say oh she's so stupid I can't believe she got that wrong. |

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Up until this point, most of my analyses related to what the students said in surveys, and interviews, as well as what I said in my journal. Because many themes that seemed important were related to what students said about their experiences in whole-class discussion, I decided to examine systematically some of these discussions so that I could compare students' participation with their reported feelings and reactions to the class.

## Analysis of Students' Participation in Class Discussions

To begin my analysis of students' participation in class discussions, I randomly selected one day from each unit (a total of seven days) and listened to the audio tapes for those days. I wanted to develop a coding scheme that would allow me to see what happened in class, including who participated, in what ways, and under what conditions.[18] I began with a few categories that captured the quantity of students' participation and some general attributes of the quality, such as whether the contribution was a question or comment. But I soon became dissatisfied with my categories. By considering the various aspects involved with students' contributions that were not being captured by the coding scheme thus far, I revised and added categories to capture the content, problem context, social context, reasoning, relationship to past learning, visual/tactile references, tone, purpose, correctness, insightfulness, mathematical relevance and difficulty level associated with each contribution. Hence, while coding the randomly selected seven days, I refined the classifications until I became satisfied with a 42x20 grid (in which I would classify each contribution as one of 42 types by

[18] It was extremely difficult to develop a way to capture not only what is said and how often, but the context in which it was said. Edwards (1993) notes the children tend to say different things in different contexts, and that we need to think differently about how to interpret what children say during conceptual discussions. He says that instead of looking at what children say and ask "What does the talk represent?" we need to ask "What is going on?" We need to capture the "interactional features of talk as a sequential unfolding of situated actions." (p. 221)

choosing a row and then work across the columns, assigning a code for each of the twenty categories). I then selected another seven days by taking the class day following each of the initial seven days. When I finished coding the discussions on those days, I looked for patterns in students' participation by gender and SES. I then summarized the main results for the two blocks of seven days and compared them to see how stable the codes seemed. I was satisfied that major patterns in the data for the first seven days also existed in the data for the second seven days. Since I fine-tuned the categories for the second round of coding, I will be drawing only on the data from the second round for some of the more detailed analyses of qualitative aspects of participation.

Some of the codes were quite straight forward, such as what part of the lesson the contribution took place in or whether or not visual aids (such as the overhead) were used. But many of the codes were extremely high-inference, such as "relevance" or "insightfulness." I aimed for consistent definitions of what was considered "insightful" or "relevant." When I discuss the data in Chapter Four, I will provide further explanation of the codes.

### Other Analyses

I initially intended to study students' experiences with the mathematics problems in a similar manner, looking carefully at students' homework and tests, and exploring differences in approaches by gender and SES. For example, many students had reported frustration with understanding what the problems were asking them to do. But I found that looking at students' finished work told me little about the process by which they did the work. For example, when looking at students' homework, I could not be sure if they were receiving a great deal of guidance or help at home, if they had worked on the problem for two minutes or twenty (perhaps doing some work mentally or on another piece of paper, or perhaps pondering what the problem was asking them to do or perhaps understanding immediately), or if the student had simply copied a friend's work at the start of class. Additionally, due to absenteeism and other factors (such as lack of effort), I was missing many more papers from some groups of students than others — for example, the lower-SES males turned in less homework than others. I eventually decided that to try and understand students' experiences with working on the CMP problems was best done by listening to the tapes of the half-dozen group work sessions I recorded by leaving my tape recorder with a group while they were exploring a CMP problem. In retrospect, I wish I had recorded small group interactions more often. Additionally, the surveys, interviews, my journal, and the class discussion coding shed light on students' interactions with the curriculum.

## Cases of Six Girls

In my analyses thus far, I examined data for all participating students for whom I had SES information. I had noticed several trends in the data and decided to select a small subset of the students and write cases about them as a way of exploring how the emerging themes played out when looking at a few students in depth. Looking across the larger data set helped me see trends in SES differences; looking closely at individuals helped me understand the trends and how they interacted to shape the experiences of individual students.

In choosing the students, I decided to focus on six girls for several reasons. First, I considered choosing one girl and boy from each SES and achievement category. But this would mean choosing eight students, with one student representing an entire category. I saw much diversity within categories and I wanted to explore that. Additionally, eight is a large number of students for

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writing in-depth cases. By choosing just one gender, I would be able to more deeply explore SES differences between students of that gender.

But why girls? I chose girls for three reasons. First, because this is a study of *mathematics* education, it seems especially important to understand girls' experiences, because they are under-represented in the field.

Additionally, the major studies done on social class and education thus far, when picking a gender, seem to choose males. For example, Willis (1977) wrote about the "lads," and MacLeod (1987) also focused on boys. Weis (1988) noted this also:

> Most work on class and race has a distinctly male bias .... Volumes on girls and women in the U.S. do not often focus on education per se, but rather, have a chapter or two devoted to schooling. They also exhibit a distinctly middle-class bias with little if any attention being paid to the experiences of working class and/or minority girls. (p. 5)

Perhaps girls are not given much attention because their "resistance" is less apparent. On the surface, the lower-SES girls look more like the middle-class students, sitting in their seats, listening to the teachers, and generally making an effort to get along in the school setting. Hence, maybe these girls do not seem as interesting to study. Or perhaps male researchers are simply more comfortable or interested in studying males. Maybe when girls are studied, they represent their gender first and foremost, because that is the category that takes precedence in the research, but lower-class white males can only represent their class, since that is the only "oppression membership" they have. Whatever the reason, the lack of research on lower-SES girls was a major reason why I decided to focus more on the experiences of girls.

Finally, in my particular data set, the girls' data seemed more rich and reliable. The girls seemed to take the interviews and surveys more seriously than the boys did. The boys would often joke around about their answers, and I often had the sense that they would complain about things for the sake of complaining.

Probably the most severe case of this was Timothy. The person who conducted his second interview, as well as Timothy's other teachers agreed with my assessment: Timothy likes to complain, and his rationale behind his complaints did not seem sincere or consistent. He would claim that the CMP problems were too boring, too easy, too hard, too fake, too directed, and too open. On surveys and in interviews, he would often insert sarcastic comments, such as "What do you think?" or "Why should I care?" When Timothy was asked on a written survey, "What is the most interesting idea you have learned this year?", his complete response was "Interesting? yeah, right!" Timothy was a ring leader of sorts, and there were a few other boys who also seemed to take many of the questions I asked them lightly. In analyzing the data, I have tried to read and use evidence carefully, and I treated with caution statements that appeared to be made in order to look "cool." Although this behavior was not completely absent among the girls, it did not seem to be as prevalent, especially as the year wore on and the girls seemed to have feelings they wanted to express. The girls were quicker to voice strong feelings of fear, anger, frustration, or confusion. I am not sure if this is because girls actually felt different than the boys, or if this is because the girls were more open about their feelings. Perhaps the fact that I am female influenced students' interactions with me. From my observations, I would guess there were at least some differences in their feelings, as students' observable reactions were different. While I saw tears and looks of exasperation from the girls who were really trying hard and cared a great deal about succeeding, the boys were more likely to joke around and not make as much of an effort to complete their homework. Toward the end of the year, four girls (and no boys) approached me individually and asked to be interviewed for my study. (Some of these girls initially did not want to be involved.) Hence, I ended up with more girls than boys involved with the study. It was as if the girls saw the interviews as a chance to have their voices heard.

Thus, for all these reasons, I decided to focus my case work on girls. I chose three higher-SES and three lower-SES girls so that I could examine similarities and differences within those categories. When I initially tried to select only two from each category, there seemed to be gaps. For example, for my two initial lower-SES target students, I chose a low-achieving female who did not make much of an effort in class, and a high-achieving female who made strong efforts to succeed. But most lower-SES girls seemed to fall into a third category — those who were trying very hard, but were not high-achieving. Hence, I chose a third lower-SES girl to fill the gap. By having three students in each category, I was more satisfied that I was representing the major types of diversity that existed.

The Evolution of Themes

The themes I saw emerging from my data analyses shifted and were re-framed several times throughout the analysis process. After letting go of the media focus, I became interested in how students' experiences in and reactions to mathematics class differed by SES. This focus started out quite broadly, embracing anything about students experiences, from which mathematical topics they preferred to what resources they had at home to help with homework. My focus eventually narrowed and became organized around students' experiences with the curriculum and pedagogy, with a focus on the open nature of the mathematics problems and whole-class discussions. This evolution of themes is summarized in Table 2.6.

**Table 2.6**
**Data Analysis Themes: How Did My Focus Change?**

| When? | What was the focus? | Why did this become the focus? |
|---|---|---|
| While I was teaching<br><br>From proposal, dated 3/18/94 | **My focus is on students' understandings of mathematics in the media:** How do students make sense of and respond to mathematical information as presented in the media, and how do efforts to teach mathematics in a way aligned with the current reforms contribute to all students' capacities and dispositions to critically analyze and respond to such information? How might students' understandings differ by SES? | I was interested in studying differences in student's mathematical thinking and learning by SES. I chose math in the media in an attempt to have a well-defined, manageable topic of interest to me. |
| After proposal meeting, just before beginning formal analysis<br><br>(These are excerpts from a 3/30/94 email sent to my committee to clarify my direction after my proposal meeting.) | **My focus includes anything relating to SES, pedagogy and curriculum:**<br>I think my main goal is to raise/illuminate issues involved with trying to implement these math education reforms in a classroom with diverse groups of students, and trying to reach all students. Hence, some issues I might raise include:<br>• Classroom Climate — What happens when I try and share power with students? for example, Who can best handle the freedom of being able to leave their seat to get needed materials?<br>• Discourse issues — WHOSE voices are heard, whose ways of knowing do the reforms value, which kids seem to be marginalized by the type of discourse the reforms have in mind, how might a teacher try to include those who are reluctant to be involved in discussions, how does trying to empower kids to have a voice in the classroom really work?<br>• Math content issues — for example, How does an emphasis on higher level reasoning affect various groups of kids? Which types of content areas and contexts seem to excite which kids, and seem to be important to which kids?<br>• Assumptions about empowerment — for example, that empowering kids within the classroom will somehow transfer outside to the way kids deal with math in their everyday life.<br>• Students' attitudes toward reforms — The attitudes and backgrounds the kids bring to these reforms can affect what they can learn from them. | The issues that struck me most as I was teaching and collecting data involved differences in students' reactions to and experiences with the pedagogy and curriculum. At my proposal meeting, my committee encouraged me to pursue these larger issues, as they seemed more important than the media focus. |

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Table 2.6 (cont'd)

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|---|---|---|
| After first round of analysis<br><br>12/15/94 Document I wrote to share with Deborah | **My focus narrows, but is not well organized. Themes include:**<br>• Discourse/literacy issues<br>Fear of being wrong<br>Confusion in discourse<br>Language difficulties with the curriculum<br>Differences in help at home with curriculum<br>Power dynamics in class discussions<br>• Relationships among frustration, self-esteem, and mathematics learning<br>Dealing with problem ambiguities<br>Enjoying the challenge of problems — Survival versus interest<br>Role of grades<br>Students' focus on fun activities versus big math ideas | After transcribing all surveys and interviews and looking for patterns in them and my journal, I saw these as recurring themes in the data. |
| After reading more class literature and taking stock of data analysis so far<br><br>Analytic memo, 6/16/95 | **Five themes frame my thinking:**<br>• Curriculum<br>Differences in students' interest in the problems<br>Differences in ability or confidence to understand questions<br>• Discourse<br>Differences in views about purpose of discourse<br>Differences in ability/inclination to follow flow of discourse<br>Differences in empowerment resulting from discourse<br>• Knowledge/Skills<br>Differences in everyday and mathematical knowledge<br>• Power<br>Differences in ways students interact when power shared by teacher<br>• Home<br>Differences in support from family, absenteeism<br>Differences in access to materials outside of school | I was striving to narrow the focus and organize the themes in a way that separated various factors involved with students' learning in the classroom and home. |
| After writing cases of the six girls, and as I go into writing this (6/96) | **Themes are organize around two areas — curriculum (particularly open problems) and pedagogy (particularly whole-class discussions). Common issues that relate to these two areas are of particular importance. These include:**<br>• Views and habits relating to literacy<br>• Views and habits relating to knowledge construction, including issues of authority, thinking and reasoning<br>• Motivation/confidence issues | Issues about support and materials at home were dropped to narrow the focus; these issues seemed less pertinent to issues of reforming pedagogy and curriculum than others. I want to speak to mathematics educators, so I use curriculum and pedagogy as main organizing themes. |

The remainder of this chapter serves as an introduction to the data and the following chapters by offering a general discussion of students' experiences with the curriculum and pedagogy, including some struggles that I, as teacher, felt successful in addressing and others I did not. I also describe my initial, often faulty, assumptions about struggles my lower-SES students, in particular, would have. I close with a brief overview of what I *did* find and how I report the data and findings in the remaining chapters.

## So What Did I Find? An Introduction to the Analyses

Mathematically empowering students — that is what my teaching and the curriculum were intended to do. Instead of boring students to tears with repetitive worksheets, they would explore, conjecture, discover, discuss, and learn. The contexts of the problems, as well as the sheer intellectual excitement, would engage them in these processes. Students would become confident in their enhanced mathematical knowledge and abilities.

Of course there would be struggles. The students had experienced the curriculum the previous year, but there were still some of what I would call "implementation struggles." When I asked students to compare the previous year with this year, they generally said that I lectured less and our class discussed more and did more group work. A few students also said that because I had helped write the curriculum, I understood what the problems were getting at more than their teachers did the previous year.[19] Hence, although the curriculum was the same, there seemed to be a difference in implementation. I needed to help students learn general norms for our class, such as listening carefully to classmates during discussions or choosing necessary materials from

[19] Apparently, there had been a few CMP problems that their teachers were unsure of the previous year, and this seemed to leave a big impression with some students.

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the cart at the front of the room. One of the most difficult things I tried to teach students was how to work with each other in both small and large group contexts. I never felt satisfied with some of the students' performance in the area. There were some exceptional children in my class, as with most public school classes, and group work was especially difficult for them.[20] Yet, although there were some initial struggles, there were also many success stories.

Some Success Stories

In the beginning of the year, I heard many complaints from students, especially about the curriculum (and indirectly my pedagogy). Many students, especially the girls, voiced negative feelings about their experiences the previous year, and they were not looking forward to another year of CMP math. The rules of the math game had changed on them, and they did not like it. But I was pleased that many of these students' attitudes changed over the course of the year. In the final interview, many students talked about these changes:[21]

> Well, at the beginning of the year I really hated it. I just didn't want to do it at all. But I think I learned a lot and it helped me a lot more than last year, because last year was really boring . . . . It was totally different than what I did 2 years ago, and I guess it was new for me, and I guess I didn't

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[20] Two examples come to mind that seem relevant for demonstrating some of the special students in the class. Carl was emotionally impaired, and had great difficulty interacting with other people. His fellow students did not help this much, as they often picked on him. This came to a head one day in another class when a student threw a piece of paper at him, then the substitute teacher (not knowing Carl and thinking he had thrown the paper) insisted that Carl pick the paper up. Carl became so angry that he started screaming and flailing to the extent that the principal removed the rest of the children from the class while the police came to deal with Carl. A second child had a disease that made her skin covered with scabs, and she was often in a wheelchair. Some students mentioned in interviews and surveys that I showed favoritism to this child by allowing her to arrive to class from gym late. These are both examples of the special needs teachers must address, and how middle school children are often not very charitable when dealing with each other.

[21] When quoting students' survey and interview data, I try to be as true to the student's original statements as possible. Hence, what I report might be grammatically incorrect, including, for example, run-on sentences in the case of interview data or wrong punctuation in the case of survey data.

> really give it a chance . . . . I think it was more clear this year, like what we had to do and why we had to do it than last year. (Anne, 3rd interview)

> At the beginning of the year, I thought that I didn't want to do it, 'cause we already did that before, and I was like, "How boring, another year of this." . . . . [But now] I really don't want to go back to the [old] books because they are just boring . . . . Sometimes they [class discussions] get boring, cause people fight over the silliest thing. Like someone gets 2 and someone else gets 2.5, and who cares? But one time we were talking about getting in a crash, and it was like 4 out of 5 people get hit in the afternoon, and I said you have to take the population into account. So it makes a difference there, not like when we're arguing over little small things . . . . At the beginning, we were kind of just like — they were kind of like, "Oh yeah, I agree," but now they're more comfortable with the class, so they can speak out whatever they want to say. (Andrea, 3rd interview)

Some other students — mostly males — did not have such strong negative reactions initially. Some talked positively throughout the year about the class. Mark said that he learned to think harder in math this year, "because when math is really easy, you don't have to think about it that much, like with regular addition and subtraction you don't have to think that hard." (3rd interview) Additionally, Christopher said:

> I like figuring out the problems because doing, like all the multiplication problems, like 50 problems is really boring . . . . [Through CMP] you know how to use your math skills in life, and not just know what the rules are, but know how to use 'em . . . . I had CMP last year and it was better than it was before, like it's better than the traditional math because you have a lot less homework, well not a lot less, but well homework that's not as boring . . . . [Discussions are good] 'cause it's like if I don't understand something and I was wondering how to do something and they brought it up in class, then I would, like, know how to do it. And it's kind of fun to listen to arguments. (3rd interview)

Hence, what appeared to be initial implementation difficulties were overcome for many students. Helpful factors mentioned by students included time to get used to the problems, having class discussions and working in groups, my explanations of my teaching philosophy and the CMP rationale, and my ability to re-explain what the problems were asking in everyday words.

There were many times when I would come home from teaching very excited about the students' participation and learning. In looking through my journal, I noticed that the days when a wide variety of students were really engaged were most often at the beginning of a unit, when students were on more of an equal mathematical footing and when we would often take some time to informally discuss or actively explore real applications of the unit's focal ideas.

For example, at the beginning of the Similarity Unit, students became actively engaged in making cartoon-like figures of different sizes with a "rubber band stretcher." That day I wrote:

> ... Then I passed out new Similarity books .... They read the first 2 pages, then I recapped and launched into the rubber band stretcher activity. Kids loved it! Nick actually said, "This is fun!" All kids were actively involved — Crystal seemed to need constant attention and approval of her figures — many kids did in fact, especially girls like Sue, Lindsay, & Crystal. I had no problems with them shooting rubber bands at each other — I gave them a pep speech at the beginning, asking them to please let me report back that teachers CAN do these kinds of things with their seventh graders. Having all the kids actively involved today felt great! Yet, we really didn't get into any of the mathematics of it yet, and I hope they will stay with me for that. (Journal, 1/11/94)

Similarly, at the beginning of the probability unit, there was active involvement, even from some that did not regularly participate.

> We began probability! What great fun! We had a lively discussion about what would happen to the pennies — William and Mark argued that it would come up heads 15 out of 30 times, and Sue said that with probability you don't know — anything is possible. Benjamin thought the graph would waver all over with no pattern — I had a fun time asking kids how their graphs compared to Benjamin's theory. I was not happy with Crystal, James, Timothy and others who insisted on throwing their coins all over the room when flipping. But overall, the level of interest was really there! I was chatting with Rose, William, Timothy, Mark, Lynn, Benjamin and perhaps one or two others who had their graphs completed. I told them that I had a computer graph of what happened over 1,500 flips. This seemed to blow their minds, and they wondered about how long it took, what kind of computer it was, etc. Tomorrow will be fun, when we get to discuss this. They are really ready for the law of large numbers idea, I think. They seem to be aware of notions of what should happen

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> with something that is 50/50, and yet they also know that on individual trials anything can happen. (Journal, 2/15/94)

Although there were instances when the majority seemed very engaged in class discussions, I noted many times when this was not the case. Boys dominated discussions at the beginning of the year. My awareness of the latest research on girls and mathematics learning enabled me to feel successful in addressing the situation. Still, inequalities I noticed relating to SES were not so easily addressed.

Initial SES Assumptions

Due to my teaching experiences both during this study and the previous year, I had initial hypotheses about which SES-related issues might be of importance in this study. I have since been surprised several times because issues I had suspected would be problematic, were not, or they turned out to be of less importance in this study than I had anticipated.

For example, I initially thought that, because the CMP authors were primarily middle class, some problem contexts would seem fake or alienating to the lower-SES students. I was wrong. I found no evidence to support the idea that the lower-SES students were more turned off by the contexts chosen, and, in fact, the higher-SES students were quicker to point out contexts they viewed as "fake."

A letter I received from a parent at the beginning of the year supported another of my initial suspicions. A middle-class parent wrote to say she was not granting permission for her daughter to participate in my research because she did not like the "Math Connections" (CMP) program, and she was concerned about students who did not have parental support at home.

> . . . I have spent many hours with Andrea on her Math Connections and making her aware of what the questions actually are asking. I

> can't help but wonder what do the children do that don't have help at home. Please don't waste the whole year on experimentation.[22]

Based on my initial assumptions, bolstered by this letter, I expected the type of help students reported receiving at home to be very different by SES. But again, there was not much evidence to support this. Several students in both SES groups talked about their parents not understanding the new curriculum, and students in both groups talked about receiving help from older siblings and friends, as well as their parents.[23] Still, I could see differences in parents' attitudes at parent-teacher conferences. The higher-SES parents were very assertive and upset about a few missing assignments or low grades, and they stood in stark contrast to most of the lower-SES parents who, if they came, were timid. When Carl's teachers showed concern about his C's and D's, his mom quipped, "He's doing better than I did." But most lower-SES parents showed some concern for their children's education (although their expectations might have been lower in some cases), and the ways in which the students talked about their parents' involvement were often surprisingly similar.

There were some reported differences in students' physical resources at home. For example, some lower-SES students got stuck on their homework because they did not have access to a scientific calculator or a ruler. Initially, I thought this disparity might be a major piece of my research. But as I began to uncover more fundamental differences, I began to think of differences in resources as not terribly illuminating or important. After all, this issue can be relatively simply addressed by letting students take calculators or rulers home,

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[22] I was certainly struck by the irony of this situation -- the mother refusing to participate in my study because, in part, of her concerns for disadvantaged students. Incidentally, the student involved was Andrea, who, as evidenced by her quote above, had grown very fond of me and the class by the end of the year, and she asked to join the study, which she did.

[23] From the limited data I had on this subject, I cannot conclude that the help my lower- and higher-SES students had at home was the same. In students' reports of the help they received at home, there was not enough evidence to conclude there were major differences.

or by making sure homework problems do not require materials that students might not have at home. Such observations do little to plumb the depths of socio-economic class and mathematics learning.

I also noticed some SES-related disparities in what might be considered "common sense" or "every day" knowledge, such as the fact that there are 60 seconds in a minutes. I wondered if these disparities would be a major part of my final story. They are not. The issue arose a few times, and I noticed that Ms. Mattel's way of handling it seemed reasonable — she wrote this type of necessary information at the top of tests or assignments so that all children would have access to it.

Additionally, I noticed that SES was correlated with students' mathematical knowledge coming in to my class. As mentioned previously, it was difficult to find a higher-SES student with low achievement. Luckily, there were some lower-SES students with high achievement, but not many. Might all SES differences in experiences with or reactions to the pedagogy and curriculum be explained by this disparity in initial achievement? I wondered initially but no longer do. In analyzing the data, I paid attention to those rare students, such as Rose, who had high achievement and was of lower SES. As I will discuss later, she was different in many ways from the higher-SES students, and she shared some important attributes with other lower-SES students. I also found the literature on class cultures useful for helping me sort SES differences that were likely cultural from those achievement-related.

What I did find were some fundamental differences in the ways in which students of differing SES experienced and reacted to the CMP trial curriculum materials and my pedagogy — differences rooted, at least in part, in their class cultures. In the following chapters, I tell several stories designed to illustrate and illuminate these differences.

I begin in Chapter 3 by discussing the experiences of a few girls to provide a close look at how the same curriculum and pedagogy felt very different for these girls, and how it might relate to their SES backgrounds. I hope that these six portraits will enable the reader to understand the beliefs and experiences of these students in some depth. But the six students are just that — six students. It is difficult to say much about social class, which is about group membership, from six individual students. In Chapter 4, I will draw from the data about the larger class to further demonstrate differences in my lower- and higher-SES students' experiences with and reactions to the curriculum and pedagogy. This description of trends will hopefully be informative, but it will not give the reader a close, coherent look at any of the students involved. It tends to sweep over the complexities of the issues of gender and class, masking the diversity that exists within each category. Hence, the six portraits in chapter three and the more summative data in chapter four are intended to complement each other. Additionally, in Chapter 5, I draw from the literature on social class cultures to shed light on issues that seem to be at play, using the literature to help build an explanatory theory for the differences in the data. Finally, in Chapter 6, I discuss implications of the study. Through examining these chapters, I hope the reader will be prompted to think more deeply about the current, "mathematics for all" rhetoric and to see new ways of how students' SES backgrounds can influence mathematics learning.

# CHAPTER 3

# PORTRAITS OF SIX GIRLS

This chapter provides portraits of six female students — three of higher-SES and three of lower-SES backgrounds. These portraits are intended to serve three purposes.

First, these portraits allow the reader a close glimpse of the girls' experiences with the pedagogy and curriculum in my classroom, allowing the reader to see that, although there are commonalities among members of these two groups, there are also individual differences. Later summaries and analyses of data will focus much more broadly on SES groups, and will not provide much discussion of individual students or differences within groups. By focusing on individual students first, I hope the reader will develop an empathy for the students in my classroom, thinking of each student as an important, unique child with special talents and temperament.

Second, the portraits are drawn in ways that will allow the reader to begin seeing the themes that will become the focus of later chapters. Thus, a closer look at these six girls enables the reader to take part, at least in a small way, in the inductive processes I have gone through as a teacher and researcher of these students.

Finally, by focusing on individual students first, I hope to provide the reader a context for understanding later discussions of data offering numerical summaries that sweep over complexities to some degree. These portraits draw heavily from students' survey and interview data, as well as from my journal.

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Therefore, through reading the portraits, the reader will become familiar with the types of data I collected in my classroom, including the kinds of information recorded and questions asked of students, and the ways in which students responded to the questions. Hopefully, this familiarity will give more life and richness to discussions of data in later chapters, and allow the reader to understand how the many trends discussed can play out for individual students.

## Six Girls: Guinevere, Samantha, Rebecca, Rose, Sue and Dawn

I explained in the previous chapter why I chose to focus on girls, in particular. Originally, four of the six girls chosen were my initial "target students" (see Table 3.1). As previously mentioned, I attempted to pick a low and high achieving student of each gender and class to follow more closely throughout the year, but, since achievement paralleled class so closely, this was difficult to do. I had an especially difficult time finding a higher-SES female who did not do well.

**Table 3.1**
**Selection of Female Target Students**

| | Low-Achievement | High-Achievement | |
|---|---|---|---|
| Lower-SES | Dawn* | Rose | |
| Higher-SES | Guinevere | Rebecca | |
| | | | |

Both higher-SES girls ultimately proved to be high-achieving students in terms of scores on tests given in class, although Guinevere had a slightly slower start at the beginning of the year (e.g., scoring in the 80's instead of the 90's). Rebecca, Rose and Dawn were good representatives of their categories.[1] As

[1] I should note that I did not choose lower-SES students who made very little effort in the class and attended very sporadically. For the purposes of seeing how the curriculum and pedagogy interact with students of various SES, I thought it best to concentrate most on those students who were present and engaged enough to really experience the curriculum and pedagogy. But it is

mentioned before, I ended up following more than just the eight target students closely. I was afraid of attrition among students, so I decided to add some students who seemed to fill gaps left by the target students. There were not enough participating students who fit nicely into the categories for me to simply "double up." I added Samantha toward the beginning of the year as another case of a higher-SES, high-achieving female. At the time I added her, she was the only other higher-SES female for whom I had permission. She did not, at that time, seem like a leader in her peer group — either mathematically or socially. She was diabetic, and some students were jealous of the special permission she had for eating in school when she felt the need. But unlike the other higher-SES girls, I could group her with anyone in the class and she would be diligent and would help the other group members. I also added Sue in the middle of the year when she asked if she could be interviewed. She had previously returned a permission slip that denied permission to follow her. She struggled throughout the year with being teased by boys in the class, and she seemed to represent a significant faction of lower-SES girls whom I would categorize as "very hard working, but not very high achieving." Sue's case raises interesting gender issues and fills a gap left by the other two girls in the study.

## Organization of the Portraits

The portraits center around students' experiences with class discussions and the CMP problems. For most students, these were the two primary elements that distinguished our math class from previous classes. Additionally, as argued in Chapter 1, whole-class discussions and open problems are also two main elements of the current reform movement.

important to note that there were a couple of lower-SES students, but no higher-SES students, who were chronically absent and disengaged.

Hence, in the following six portraits, there are three parts. First, I introduce the student with general information about her family background, her performance in class, and her plans for the future. The reader will notice that I have more SES-related information for some girls than others, since I had contact with one of the students outside of the school setting, and some parents volunteered more information than others. Second, I look at students' experiences with the pedagogy, focusing on whole-class discussions. I discuss this from two perspectives, first providing data from my journal about my own perceptions of students' behavior in the discussions, and then turning to survey and interview data about how the students, themselves, viewed their experiences in the discussions. Finally, I focus specifically on students' reactions to the curriculum, drawing primarily on what students *said* about their experiences with and opinions about the curriculum, since their observable reactions will have been discussed in various ways in the previous section.

In some sense, it will feel backwards to look first at students' participation in whole-class discussions, and then the curriculum, since the discussions generally grew out of the problems in the curriculum. But the data I have about students' participation in discussions will allow the reader to get to know the students more quickly and hopefully become more engaged in their stories. Additionally, because the whole-class discussions centered around CMP problems, my discussion of the data regarding students' participation in discussions will reveal various aspects of students' experiences with the curriculum. Hence, the sections focusing on whole-class discussions are more extended than those focusing on curriculum, since some of the issues relating to curriculum are discussed as part of the students' participation in discussions..

In writing these portraits, I answered the same questions for each of the girls, trying to give attention to the conflicting evidence that often existed. The

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questions I used arose out of analyses of survey and interview data surrounding the whole class. In those analyses, several themes had emerged, and focusing on these six girls allowed me to explore how these themes played out for individual students. The themes that had emerged thus far included differences in students' interest and participation in the whole-class discussions and the curricular problems, students abilities or confidence to understand the open problems and the flow of discussion, students views of the discussions and curriculum, ways in which other students reacted to the student's participation in discussions, and students beliefs about what they were learning and how they learned best. I used the following questions to guide my writing of the cases:

Whole-Class Discussions

How did I (and others) view the student's quantity & quality of participation?

- How much did she participate?
- Did she offer ideas, ask questions, and/or challenge others?
- How did her ideas contribute to the mathematical agenda?
- What was the reaction of other students to her participation?
- What kind of reasoning did she use?

How did the student view her own participation?

- What influenced the ways in which she participated?
  Was she afraid?
  Did she want to be heard, etc.?
- How did she view/experience discussions?
  What did she think was the purpose of discussions?
  What did she think our roles were?
  How were the discussions helpful or unhelpful? (Did discussions confuse/annoy her? What did they learn? Were discussions fun — did they increase her interest in the class?)
  Did the student prefer to work independently, in groups, with the whole class, or to get help from me?

Curriculum (Particularly, the Open Problems in the CMP Trial Materials)

In what ways did she engage with the curriculum?

- Did she complete assigned problems?
- Did homework efforts enable her to understand the intended mathematical ideas, as assessed on tests?
- Did she show evidence of intrinsic interest in the problems?

Overall, what was the student's opinion about the curriculum?

- What things did she enjoy about the curriculum?
- What things did she dislike about the curriculum?
- Did she feel more competent in this or typical curricula?
- What did she think the CMP helped her learn, as compared with typical texts?

I searched through my teaching journal, as well as all interview and survey data, to find information relating to these questions. Then I organized all relevant data under the above questions and found ways to summarize and present the data. I tried to account for all of the data in some way. All counter-examples were treated carefully — I tried to explore instead of ignore them.

The six portraits will be presented with the higher-SES girls first, followed by the lower-SES girls. Some analysis of the portraits will be provided at the conclusion of this chapter, but more analysis of these girls and their classmates will follow in subsequent chapters.

## Guinevere

Guinevere, a white female, lived with one sibling and her mother, who was a graduate student and a social worker. According to her background survey, Guinevere's household income was around $35,000 and there were over 200 books and a computer in her home. Although Guinevere's father did not live with her, he came to school conferences regularly and seemed very involved in her life. He was an engineer, and Guinevere mentioned that she could call him and ask for help with her math homework. Guinevere said that her plans after high school included getting a master's degree of some kind. She was not worried about getting in or paying for college but was rather private about her rationale for this.

Guinevere seemed extremely confident in her mathematical abilities, saying, "I got a really high grade on my SAT application form," and "My mom says I'm good at Math" (First Survey). At one conference, her dad talked with her teachers about the fact that Guinevere did not take correction well from anyone. He said we should all work on this with her. Although confident, Guinevere did only 81% of her homework across the year, and she tended to be withdrawn at times in class. Her test scores varied, but they averaged 90%. Guinevere is the closest example I had of a female with "higher SES, low achievement," although, calling her "low achievement" is only relative to other higher-SES girls.

Guinevere and Whole-Class Discussions

For Guinevere, class discussions offered the opportunity to share ideas and learn from others, yet they could also be annoying when others did not participate in ways she thought they should. I discuss Guinevere's participation as I observed it, and then turn to what Guinevere, herself, said about the discussions.

Guinevere's Participation in Whole-Class Discussions. Guinevere was inconsistent in her contributions to whole-class discussions. For days or weeks at a time, she would take an active role in the discussions, but then there were strings of days in which she would not participate at all.

During the periods when she did not participate, Guinevere tended to act out in other ways. For example, she would lay her head down on her desk or draw pictures. When I challenged her behavior, she would become angry and defensive — glaring at me, arguing or protesting in other, more subtle ways. However, when Guinevere did participate, she would usually talk several times in one class period, and then continue to talk for several days in a row (e.g., Journal, 12/14, 12/17, 1/7, 1/10).

At various times when I would take stock in my journal, I regarded Guinevere as an "occasional" participant in discussions. Of course, it is important not only to consider *how much* students participate, but also *in what ways* students participate. When students made significant contributions to the discussions, I would remember it and mark it down in my journal immediately after teaching. Guinevere displayed a wonderful ability to view problems in unique ways, coming up with unexpected, intelligent solutions. She was also bold in voicing her disagreement with me and others. Here are some typical examples I recorded about Guinevere's participation:

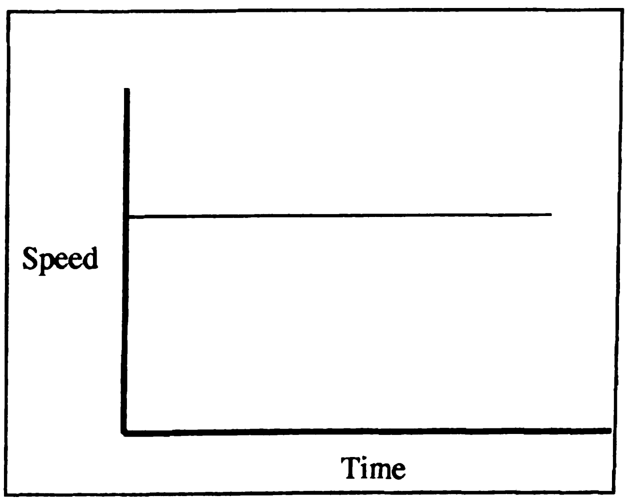


**Figure 3.1**
**Graph of Constant Speed**

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> Again, there were many "ah hah's!" today. During the discussion of homework, Guinevere made a great point — the question was whether a graph with a long horizontal line (meaning speed was constant) was reasonable for a bicyclist (see Figure 3.1). Many kids were arguing no because of hills, etc., and she said that because no scale was given for the graph, we may be looking at a very short time, and then it would be reasonable! (Journal, 10/19)

> . . . Kids were really arguing with each other. Guinevere was very convincing, and William and Mark were arguing with her at first, but then Mark doubled back and started arguing unknowingly for Guinevere, but then I pointed this out to him, and he smiled and conceded . . . . (Later, while discussing a statistic that says the death rate of one car is higher than that of another) At one point, I said something about "what makes this big difference?" and Guinevere caught me and said "We don't know if it's big - it could be teeny weeny." (Journal, 2/25)

As exemplified by the above excerpts, Guinevere did not usually try to answer the "easy" questions. Instead she became involved when she had an idea to share — an insight that would push our mathematical thinking in interesting ways. She also did not hesitate to voice her disagreement with me or other students, and she gave mathematical evidence to support her arguments.

The other students showed respect for Guinevere and her ideas. With the exception of the first time Guinevere was at the overhead (when she and another student were the first students ever to come to the overhead and were learning how to write on it) students listened to her when she talked. Students also remembered Guinevere's contributions. For example, on January 3 (the first day back after winter break) students recalled that Guinevere proposed an important idea.

> Guinevere - very quiet — The kids attributed an old idea to her — I said I thought it was Michelle who said that when the dimensions are closer together that the volume is bigger — I am not sure if the kids are right, but Michelle said "it wasn't me" and then several kids said "It was Guinevere" and Guinevere looked a bit surprised but quietly sat there. (Journal, 1/3)

I checked my journal, and the students were right — it was Guinevere.

At times some students did not understand what Guinevere was talking about and became intimidated or impatient. For example, Lynn once exclaimed "Thank goodness!" after Guinevere said she would give up her attempt to explain how she was doing a complex problem in her head (Journal, 4/21).

Hence, although Guinevere's participation tended to be sporadic, when she did participate, she would usually offer a new idea or share a different perspective on a problem. Although her ideas were often unexpected, they were usually mathematically insightful. So what made Guinevere want to participate at times, and why did she seem so removed at other times? To answer these and related questions, I now turn to Guinevere's view of discussions.

Guinevere's View of Whole-Class Discussions. When asked if she participates much in class discussions, Guinevere said she did because discussions allowed her to learn more and to be heard.

> I like it that we can get into arguments like that because it helps you actually understand what's happening . . . . It helps you know why it's wrong. Not this is wrong and this is right . . . . I like to have my ideas heard, and I need to know if I'm right or wrong . . . [but] sometimes I can learn more by listening to other people. (First Interview)

> Yes, I need to get my point across. (Final Survey)

Guinevere's responses reveal that she valued the discussions as a way to share her ideas, and to learn from having others evaluate them. Discussions helped her understand mathematics better.

In addition to these examples from Guinevere's perspective, there were examples in my journal that give evidence that Guinevere valued the opportunity to share her ideas, as she displayed disappointment when someone would share her idea first or when we would run out of time for discussion (9/30; 12/1).

The positive ways in which Guinevere viewed the discussions help us understand why she actively participated at times. But other evidence suggests reasons why she was not consistently active.

Guinevere made clear that her feelings toward the discussions were not completely positive. As the year progressed, she had developed some negative feelings about my pedagogy. At the end of the year, when asked to rate "math this year" on a scale of 1-10, she ranked it a "4," explaining "It's so boring. I never want to pay attention because the teaching style is SLOW!" She became more certain that she preferred to work alone, as opposed to working in small groups or discussing ideas with the whole class. She had little patience for other people in the class, who were "too noisy" or "goof off too much." In her final interview, she explained her mixture of feelings about discussions.

> *There were many times this year when we had mathematical discussions together. Do you like those discussions?*
>
> Um, sometimes if they're about math. If they're just to argue then I don't really get . . . (faded out)
>
> *Can you give me an example of the difference?*
>
> I know William likes to start arguments just for the heck of it and won't give up the battle even when he knows he's lost. If somebody doesn't understand something and they're like telling you their method, then that's just a mathematical argument.
>
> *So is that a good thing then?*
>
> I think that would be a good thing because that person would either tell you a different way to solve the problem or how you solve it.
>
> . . . .
>
> *Do you learn from those discussions then?*
>
> Sometimes. I don't know, the discussions aren't usually [about] things that bother me.
>
> *Do you feel like you already got the stuff figured out?*

*Yeah.*

Hence, Guinevere appreciated discussions in which good mathematical argument was occurring, and she felt able to discern when the conversation was relevant to the main, mathematical ideas being discussed and when they seemed to get off-track. She thought a good discussion involved people sharing their mathematical methods and learning from each other.

In addition to providing insight into some of the ways in which her fellow students participated in arguments, Guinevere enlightened me with her view of my role. When I tried to ascertain how she thought the discussions moved along when the class was "stuck" as to which ways are right and wrong, she explained that I played an active role in "hinting".

*How does the class or how do we figure out which ways are right and wrong?*

Hints.

*Like what do you mean?*

You say, "Well I don't know if that would work."

*So do I tell you who is right or wrong?*

Not usually, you just hint.

*Does that tell you?*

Not really. It tells you it's wrong, but it doesn't tell you why.

*Why do you think I do that?*

So we don't learn the wrong thing and think it's right.

*Why don't I just tell you she's right and he's wrong?*

So we can figure it out.

Hence, Guinevere seemed to understand my intentions as a facilitator of mathematical discussion: I wanted to help students figure things out for themselves, but I also did not want them to flounder too much and end up learning "the wrong thing."

Although Guinevere sometimes became very involved in mathematical discussions, she claimed that people could not get hurt in discussions because "it's just about numbers, which aren't personal, because people aren't made up of numbers." When pushed a bit and asked if she had ever felt bad because she might have been wrong, she answered, "No. If I'm wrong, I'm wrong, it doesn't mean I'm a bad person" (Second Interview). Hence, although Guinevere said discussions were a helpful forum for sharing and evaluating ideas, she also talked as if she was not particularly invested in those ideas — they involved rather abstract speculations about numbers that were not personally meaningful .

Guinevere and Whole-Class Discussions: A Summary. In summary, Guinevere seemed to view the discussions as a place in which she could voice her opinions and have her ideas be heard. She often enjoyed them, although there were aspects that frustrated her — particularly when the discussions were not moving forward quickly enough. Waiting for other students to catch on or to stop getting side-tracked or goofing around made her frustrated. Guinevere liked to participate in discussions in which she could share her ideas with people who were trying to discern what makes sense and why. She did not try to answer the "easy" questions, but instead became involved when she saw ways she could push the discussion forward. Sometimes this would involve sharing an insight, and at other times this would involve disagreeing with me or another student. She could articulate her mathematical arguments well, and she had the pleasure of having others respect her contributions.

For Guinevere, the purpose of discussions was to share ideas and learn from the group's analysis of the ideas. She felt she could discern when students' ideas were relevant to the mathematical discussion and when they were not. She

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understood that I was trying to facilitate their learning without telling them the answers so that they could learn to figure things out on their own. The discussions were generally helpful and not confusing for her, as she shared my understanding of their intended purpose and our roles.

Guinevere and the Curriculum

Like her participation in class, Guinevere's views of the CMP curriculum were mixed. For example, on the first survey, she said the CMP problems are fake. In her second interview, she said they are like real life. Hence, in discussing Guinevere's opinions, I will try to outline the overall trends that came through in the various interviews and surveys, along with contradictory evidence and changes that seemed to occur in her attitudes over time.

In an interview in the beginning of the year, Guinevere explained that math was different two years ago because "the problems were all the same" and "things were so boring so we [the teacher and the class] made up little games [like Around the World]." When asked if CMP is harder or easier, she replied, "It's just different, it's not really harder. You have to learn new strategies for solving problems. Now you need to find the information you need." In more typical math classes, she thought she learned more "about basic skills we will need for algebra, calculus, trig — we won't always have a calculator" (Second Interview).[2]

In the beginning of the year, Guinevere said that she sometimes got frustrated on CMP problems because they were unclear.

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2 Because of the language and authority with which Guinevere spoke on this issue, I often suspected that Guinevere's opinion about CMP was heavily influenced by other adults — especially her parents — who had obviously shared concerns about CMP with her. For example, in her first survey she wrote, "both my Mother and me think that it is inappropriate to use us as guinea pigs." In her second interview, she explained that her parents did not like CMP and believed it did not teach them everything they needed, so their futures will be affected.

> They don't really tell you what you have to do, and you have to guess. So I'm like, 'Mom, does this show me what I have to do?' And sometimes one word can just throw me off. So like which problem are you supposed to solve or which one are you supposed to graph? So sometimes I do both of them and sometimes I call Rebecca." (First Interview)

In surveys throughout the year, she gave specific suggestions to the CMP for alleviating any confusion, such as putting more specific directions in the book (3/21), writing "instructions on how to do the problems, because if your parent wants to help you and you don't quite remember the process your parent can help" (Second Survey), and including "a glossary for when we forget concepts" (Second Interview).

But aside from offering these suggestions, as the year progressed, she stopped complaining about being confused or frustrated, and instead began to complain that the curriculum and pace of the class were boring. While some students did not seem to see connections between units or ideas, Guinevere indicated she was bored because "we keep doing the same things over and over again — just differently worded" (Second Interview). She reiterated this point on her final survey, when she said we go "too slow," and then in her final interview, when she said, "we keep doing the same things over and over." Her participation in class, as discussed above, reflected this boredom at times.

Throughout the year, she said she preferred to work alone on the problems, as opposed to working with small groups, which she described as "noisy," claiming, "nothing gets done," especially when they contained boys, who like to "rough house" (First Interview, Second Interview, Final Survey).

Of all the higher-SES girls, Guinevere seemed to put forth the least amount of effort in her homework. While the other higher-SES girls consistently turned in over 90% of assigned work, Guinevere's rate of homework completion during the year remained steady at around 81%. Yet, her test average for the year was 90%. Her overall grades for the marking periods were B+, A-, and B+.

Hence, Guinevere's participation in class discussions and her performance on tests revealed that she understood the main ideas under study. For Guinevere, understanding seemed to come without much effort outside of class. She consistently reported that she spent less than 15 minutes a day on her math homework and that she seldom got stuck on homework (Final Survey). Still, she said that when she worked on her homework, she "always" tried to see if an answer makes sense (Final Survey). She also said she "usually" felt like she knew what she was doing. In response to specific questions about where feelings are directed upon receiving a bad grade, she said she "seldom" felt stupid or depressed, but she "always" felt angry at CMP and the teacher.

Guinevere was confident in her abilities to solve the CMP problems. At the end of the year, she ranked herself as one of the top three students in our math class, even though none of her peers mentioned her as one of the top three students (Final Survey). Overall, she said that CMP is "a lot easier" for her. She explained, "I guess our family's just — we are word problem kind of people" (Second Interview). Still, in the first survey, she also wrote that she preferred doing equations with only numbers — "It's a me thing", she said. At the end of the year, she said she was "very happy" the school was not planning to use CMP next year (Final Survey).

Guinevere: A General Summary

Guinevere was a confident student in our math class who was fully engaged at times, but was turned off at other times. For Guinevere, discussions could be an enjoyable, helpful avenue for sharing ideas, but they could also become boring when they did not stay focused on making mathematical progress.

For Guinevere, the CMP curriculum could be confusing occasionally, and she suggested specific ways to make it clearer. Still, overall, she felt confident in her abilities to make sense of the open problems and said CMP math was easier for her than traditional math. Guinevere followed the mathematical connections in discussions and in the curriculum and expressed impatience with revisiting the same ideas over and over again. She also became frustrated with having to wait for others who might be slower or less focused than she was.

Perhaps some light is shed on some of Guinevere's general thinking about our math class by one of her final comments made in an interview when asked if she could tell me if there was something about the people who liked CMP math better than typical math: "People who are sort of smart but they don't really think they are, they would like CMP." Guinevere *did* think she was smart, and she seemed to put herself above some of the mundane activities of the classroom — for example, discussing homework problems or working on problems with groups tended to be boring and frustrating for her. For Guinevere, it seems that one had to be somewhat smart to be able to feel good about learning math in our class, but if you realized you were really smart — smarter than most of your classmates perhaps — then the "slow" pace of the class and having to work with classmates became grating.

## Samantha

Samantha lived with her mother and two other children. They had over 200 books and a computer in their home. They received the local paper daily. Her mom, a substance abuse counselor, had a Bachelor's degree and was working on her Master's. The family income was around $35,000. The following summer, I had the pleasure of teaching Samantha in a Math, Science, & Technology program for "gifted" students, held at Michigan State University.

The fee for the program is over $700, up to half of which can be covered with scholarships. The fact that Samantha was there speaks highly of her mother's dedication to Samantha's education. Her mom regularly attended conferences and took an active interest in Samantha's progress, asking the teachers, for example, why Samantha had not received the "All A's" award the previous semester (the teachers were surprised — they thought she had). Samantha was an "All A" student in our math class, completing virtually all assignments and scoring near 100% on all tests.

Samantha said she planned to go to college and get a Master's degree. She mentioned genealogy and writing as possible career paths. She asserted, "If you don't have a good education, you won't go very far" (Final Survey). When asked if she was worried about how she might pay for college, she said, "Yeah, since my parents don't have money set aside like some others, but I know they will get me there — but I still worry."

Samantha and Whole-Class Discussions

Like Guinevere, Samantha made many substantive contributions to discussions. She found them a helpful, enjoyable, arena in which to share ideas and learn from others. My view of Samantha's emergent participation in the discussions is given in the following section, after which I summarize Samantha's survey and interview data to provide her perspective on the discussions.

Samantha's Participation in Whole-Class Discussions. Samantha did not stand out in the class discussions at the beginning of the year. Yet, as the year progressed, she became one of the most active participants. In January I noted the emergence of Samantha and some other girls in the class discussions.

> I actually feel like girls are dominating this class. Rebecca, Samantha, Michelle, Rose, & Anne have been doing 80% of the talking lately. This is a switch! Yet, I am not sure that boys are not dominating some aspects, i.e. the confronting or arguing. (1/12/94 Journal)

The students also seemed to notice Samantha's increasing participation. For example, while only one student mentioned her as one of the "big arguers" in our class during the first set of interviews, six students mentioned her in the final interview and twelve students on the final survey. In the end, she was mentioned more than any other girl, and surpassed only by one boy.

But, as mentioned above, I initially noticed some differences in the ways the boys and girls participated. On the surface of things, Samantha seemed unlike some of the more vocal boys, who participated in a loud, confrontational way. But when I look through my teaching journal, I notice that whenever I wrote about Samantha's participation, it usually begins with verbs that indicate that Samantha was boldly pushing the discussions forward in interesting ways. She would make her own conjectures, give mathematical evidence for them, and agree or disagree with others' answers, ideas, and arguments. Hence, although she had a still, small voice and presence (being one of the smallest students in the class), she made her ideas known.

> I tried to bring out surface area. We spent 15-20 minutes on this together, and Samantha was completely on the ball with this! She came up and tried to explain it to others. (Journal, 12/9)

> Samantha said, "I disagree!" when Michelle had the wrong answer. Samantha was right. (Journal, 12/15)

> There was much conjecturing about just how you add and subtract these negative numbers. Samantha noticed that when you subtract a negative, it is like adding it. They tried it for a few more examples, and noticed it worked. (Journal, 3/25)

> Nick had a good theory about why the volume gets eight times as big when we double the dimensions — there are three dimensions and two times two times two is eight. Samantha challenged it because she said there are only two dimensions for the cylinder (height and radius) but it comes out to be eight. I decided to make this their assignment for the evening — to try and explain why. I am a bit concerned about most kids feeling clueless about this, and I tried to forewarn them that it is OK if

> they don't feel they can answer it, but they need to show me that they thought about it. (Journal, 1/4/94)

As evidenced by the last example, I sometimes worried that Samantha's ideas or arguments were difficult for other students to grasp. I often saw discussions begin by focusing on a grounded, specific idea set in the context of the problem. Some students were content to leave the discussion at that level, but Samantha pushed the discussion to more abstract levels.

Other students rarely challenged Samantha. Instead, they tended to follow Samantha's lead in mathematical matters, even when she was incorrect.

> Then we did the rice and cone experiment. Kids are feeling freer to jump out of their seats and rush to the front so they can see these kinds of demonstrations. First I had the kids guess — Samantha gave an argument for two cones fitting the cylinder — I took a vote and the whole class agreed. (Journal, 12/13)

Samantha was wrong — it took three cones to fill the cylinder. But she easily convinced the class that she was right. In looking through my journal for examples where students directly challenged Samantha, I found only two such examples — both by higher-SES males.

So far, I have argued that from my perspective and that of her fellow students, Samantha was an important part of our discussions — perhaps too important in that others deferred to her and were perhaps intimidated by her. Now I want to explore Samantha's own view of the discussions. What did she think our roles were? What influenced her participation? What did she gain or lose from it?

<u>Samantha's View of Whole-Class Discussions</u>. In Samantha's first interview, she did not name herself as one of the people who did the most mathematical arguing. She seemed to imply that she had her own ideas, but she did not share them often with the whole class.

*Who do you think does the most arguing?*

> Timothy, and maybe Harrison sometimes. I think everyone has a different idea about something but they don't say it.

After the first interview, she consistently named herself as one of the main arguers. In November, when asked how she contributed to class discussions, she responded, "I contributed by bringing up other ways to solve problems" (First "Show What You Know").

But did she enjoy participating? When asked about her preferred mode of working (alone, small groups, or whole class discussions), Samantha expressed appreciation for both whole group discussions and small group, but her preference shifted from small group to large group as the year progressed. Additionally, Samantha and another higher-SES girl informed me that math was their favorite class. She said, "I love my teacher" on the final survey, and then, "You are a good teacher" during her final interview.

One exception to Samantha's generally positive attitude about the class and the discussions occurred at the beginning of the year, when she told me that she thinks we spend too much time going over homework. But this was the only time I heard her complain about the class. Her words and actions indicated that she fully engaged in the discussions, particularly as the year progressed.

What did Samantha think she gained from discussions? Her explanations of the purpose and nature of the discussions were similar to the current reform rhetoric. In her second interview, she explained that she liked to share her ideas both because she learned from the experience and because others would learn that she is intelligent and involved.

> Because if I have an opinion I like to share it and see if I'm right or wrong. And I want people to know that I'm not stupid and that I think and that I'm paying attention.

At the end of the year, she said she participated because "I want other people to understand my ideas. I like arguing" (Final Survey). In her final interview, she explained:

> I like having discussions in class, cause then you can hear what other people have to say, cause sometimes there's more than one way to figure out a problem, so if you're just thinking one way you can find out another way.
>
> *Do you ever get confused in discussions?*
>
> Not really.
>
> *Because some people have said they aren't sure who is right during discussions*
>
> I guess a few times, like that one time we were doing that thing with Tawanda's toys .... I guess that one kind of confused me. When we had that discussion when Mark was arguing that you should pick the same numbers.

Although she said this discussion with Mark confused her, she later pointed to that exact discussion when asked to recall the discussion she remembered most from the year.

> I remember the one in What are the Chances? where Mark thought you would have a better chance of winning a contest by scratching off two numbers if you picked the same #s all the time. It stuck out in my mind because it was a long and good discussion. Mark was really stubborn; I was arguing against him.

With the exception of Sue (who remembered a specific time she was ridiculed), Samantha was the only one of the six girls to describe a specific mathematical discussion when asked this question. For Samantha, arguing for her ideas in the face of opposition was a good thing, as was being exposed to a variety of ideas. As evidenced above, confusion was rare, but not a bad thing when it occurred. Neither was being wrong:

> *Do you think there have been times when people have gotten their feelings hurt or felt stupid during these arguments?*

> Um, I don't think so, I think they just maybe felt embarrassed if they voiced the wrong opinion, after they found out what was right, but they get over it and think, "Oh well, we were wrong," and learn from it. (Second Interview)

Thus, for Samantha, discussions offered an opportunity to voice her opinions, to have others evaluate her ideas, to prove she was on the ball, and to learn from others' ideas, as well as her own mistakes. Learning how others think about problems was interesting and important to Samantha. Samantha seemed to understand the purpose of class discussions in the ways I intended.

Additionally, Samantha mentioned that, when the class was stuck, I gave "clues," such as "I think this idea would make more sense." Samantha's use of the word "clue" along with Guinevere's use of the word "hint" made me think twice about my role in the discussions. I wondered if my "clues" were only really guiding certain people who understood my discussion norms and were able to pick up on my "clues."

<u>Samantha and Whole-Class Discussions: A Summary</u>. In summary, Samantha was a major player in the discussions. She made conjectures, gave mathematical evidence for them, and analyzed the ideas shared by others. Most students respected and accepted her ideas, and I struggled with what to do when Samantha's thinking and ideas seemed more advanced than those of many other students.

Samantha seemed to view the purpose of the discussions in ways consistent with my intentions. For Samantha, discussions were interesting and helpful, as they offered an opportunity to share her ideas, hear others' views, and show others that she was thinking. Exposure to a variety of ideas and arguing for her ideas in the face of opposition was not intimidating, but rather was viewed in a positive light. Samantha was also not afraid or frustrated by confusion or error.

Samantha and the Curriculum

In general, Samantha thought of the old curriculum as repetitive and boring, with a strict focus on operations with numbers. She said that the CMP is more enjoyable and requires more thinking.

> I think it [CMP] includes more thinking, more fun. Two years ago we had plain books and I didn't like em that much — they were boring and they had lots of, like, hand problems that you just had to write down . . . like Sally bought 15 apples, if she divided them among her friends and there were five friends, how many did each person get? I thought those were dumb, and they didn't involve a lot of thinking . . . . (Second Interview)

Samantha was rare in that she was even willing to publicly defend the CMP curriculum in class. For example, in a class discussion specifically about the curriculum, she was the first to defend it:[3]

> I like these books better because the old books are stupid, their like Jane bought 4 apples . . . they go 1,2,3, and just multiplication problems. And this one explains more and gives you harder problems to challenge you . . . . After you do two problems you don't have to do 50 more to understand it. (Journal, 2/3)

She described the CMP curriculum as "pretty easy, but not so easy that it's boring" (First Interview). Still, Samantha explained on several occasions that initially she found the CMP curriculum frustrating (she recalled one incident when she became so frustrated that she scribbled all over her paper and threw it away), but she became used to it and began enjoying the challenge (Second, Final Interview, Final Survey). In her second interview, she explained:

> [CMP] is more challenging but I'm getting the hang of it more, Last year I was frustrated because they were so different, and I didn't like [that] they took more thinking, so this year I am more used to them, but they still have challenging problems . . . . At the beginning of this year I was always getting frustrated because I think the books that we're using now, the

[3] On this day, I took much of the class period to directly address some of the complaints I had been hearing about the CMP curriculum. I realized that perhaps no one, including myself, had done a thorough job of talking with students about the rationale behind the curriculum.

> problems are worded kind of funny and I couldn't understand them, but I understand it better now because I know a bunch of new terms and stuff.

When she would mention her frustration with problems, she would talk about the use of words she did not understand, usually referring to a specific example. Still, she made clear that she thought that, overall, the CMP curriculum was better. For example, when asked, "What frustrates you?" on the second "Show What You Know" survey, she responded, "Sometimes the textbooks aren't clear enough, like before I knew what 'corresponding' was." On the mid-year CMP survey, when asked what she would like to tell the authors, she responded:

> To explain the questions better* Don't use big words that we kids won't understand.
> *These books are better than the old.

Samantha reiterated on her final survey that the old number problems were easy, and therefore boring. She said she felt better when she tried to understand things instead of just learning rules (Final Survey). In her final interview, she said, "I like to try to understand because then it makes me feel better knowing I can understand it — not just rules given to me. I kind of like learning rules, but I like to understand them." On her mid-CMP Survey, she said, "When I understand, I feel good about myself (I can feel a light bulb!) When I don't, I feel very frustrated." She also said that when she became frustrated, she looked to her mom (11/23 Interview, 3/21 Interview, & 4/26 Interview), friends (4/26) and teachers (4/26) for help.

Although she might have been frustrated at times, Samantha's performance in the curriculum was fantastic! She turned in virtually 100% of her homework assignments throughout the year, and her test scores averaged in the high 90's. Hence, she earned straight A's throughout the three marking periods. She reported spending 15-30 minutes each day on her homework. During class, she seemed to deeply engage with the problems, always staying on task, asking good questions and proposing her ideas. Although one might think that Samantha, like Guinevere, would find working in a group frustrating, the opposite was true.

> I like working with a group, so then there's other people so if you get stuck then you can ask other members for help . . . . And it's not just like one problem that you're getting the answer [to], you're going to understand it better for all the problems like that. (First Interview)

Hence, Samantha valued learning from others' ideas about the problems. Furthermore, she saw how help on one problem was transferable to other problems. Thus, she was able to see connections among the CMP problems, even when the contexts were different.

She indicated feeling intrinsically motivated to work on the problems. For example, on her final survey, when asked what makes her work hard in this class, she responded, "The problems are interesting, I like the teacher, and we're learning things we're going to need to know in the real world."

She indicated in several ways that she felt very competent in mathematics, and that she thought the CMP helped her become even more so. When asked to rank herself in the class at the end of the year, she responded, "#1 (or 2 - I don't mean to sound conceited)." Her classmates also saw her as one of the best math students. In fact, in the final survey, 19 of her classmates ranked her in the top three math students — she was mentioned more often than anyone else in the class. Also, when asked in the second interview if she thought she was better at math now than a couple of years ago, she said, "I think so, I've learned new stuff and I'm able to think more, like probability especially, stuff that will help in real life in the future." Still, when asked what she might learn more about in the typical curriculum, she said that she learned more about "basics, like reciprocals." Yet, she seemed to value the problem-solving abilities that she reported gaining through the CMP curriculum. In her final interview, she explained, "I can

probably figure out a problem now, without having someone just tell me the rule — like if we're doing integers again, I could probably figure out a rule."

In addition to feeling empowered through CMP math, she also reported liking math more by the end of the year. While in her first survey, she indicated feeling rather negative about mathematics before this year, and even math class at the beginning of this year (rating it 4-5, and saying problems were boring and she did not like homework), she quickly changed her tune. At the end of the year, she ranked our math class a "10," explaining, "It is interesting, we have class discussions, the problems make you think (ACE), and I love my teacher!"

Samantha: A General Summary

Overall, Samantha consistently reported enjoying my pedagogy and the CMP curriculum more than those she had experienced previously. Samantha seemed to share my views on the nature and purpose of whole-class discussions and the open problems in the curriculum. Samantha viewed discussions as an interesting and helpful way to share and sort out mathematical ideas. The CMP problems offered a motivating and challenging means of learning mathematics. She enjoyed the contextualized problems, and she could see the mathematical connections among the problems, even when contexts changed. At the end of the year, she reported feeling confident in her abilities to think and solve math problems on her own.

## Rebecca

Rebecca lived with her mother, father, and two siblings. According to Rebecca's background survey, her parents both had graduate degrees, and together they made at least $60,000. Her father was an environmental biologist, and her mother was an elementary teacher. They received the local paper daily and had over 500 books and a computer in their home.

Rebecca studied hard, completed all her homework, and tended to get all A's in school. She said she planned to go to college and probably be an upper elementary teacher and was not worried about how she would pay for college because her mom had started a bank account for her (Final Interview).

Rebecca and Whole-Class Discussions

Rebecca's participation in and views regarding whole-class discussions were similar in many ways to those of Guinevere and Samantha. For Rebecca, discussions offered the opportunity to share ideas and learn from others. Again, I begin by outlining my view of her participation, and then turn to her own views about the discussions.

Rebecca's Participation in Whole-Class Discussions. For the first few weeks of school, I did not hear much from Rebecca (e.g., Journal, 9/14). But she soon began to participate, and throughout the rest of the year, Rebecca was generally active in class discussions (e.g., Journal, 1/10, 2/1).

Unlike Samantha and Guinevere who tended to push the discussion forward in bold, sometimes unexpected ways, Rebecca tended to contribute mathematically correct ideas in answer to my questions. Hence, most of my entries about Rebecca say something general about her many solid contributions, such as "On the ball, participating alot" or "Volunteered some key answers" (Journal, 1/12, 2/28).

Rebecca was not afraid to come to the front of the room and explain her thinking about a problem. For example, on 3/18 she came to the overhead and drew a grid to explain her reasoning on a probability problem. She also had a knack for remembering "basics" from years ago, and thereby helped the class by recalling ideas or processes that most other students had forgotten, such as when she remembered the term "ratio" from fifth grade (Journal, 4/19). In addition to answering questions, Rebecca occasionally asked questions when she needed help (e.g., Journal, 3/29).

Although Rebecca's answers were often the expected, "correct" answer for those problems where a correct answer existed, she sometimes solved problems in interesting ways and shared her unique methods. For example, when comparing popcorn prices among three options, she found the amount of popcorn per dollar instead of the expected price per cubic centimeter of popcorn, and she was able to understand the difference and realized that she was then looking for the largest number instead of the smallest (although this confused most other students). When her way of solving the problem was different than that of others, including me, she did not assume hers was wrong. She knew that her method made sense and was able to explain it to others.

Rebecca showed she could join in a mathematical argument when she desired. For example, on February 17, I noted that Rebecca was talkative and "she was arguing some points." On October 15, I noted that Rebecca listened to another student and built from her idea. Sometimes Rebecca displayed some fantastic, mathematical reasoning that many other students had difficulty understanding.

> Rebecca hit the nail on the head very eloquently when she made the distinction between an *individual* driver not having a greater chance of an accident on the weekend, but overall, there was a greater chance. I really emphasized what she said, pointing out that this is a really tricky idea but important. (Journal, 2/25)

> *We discussed the assignment for last night — to explain why the volume is eight times as big for a cylinder and rectangular prism when the dimensions are doubled. About 1/2 the kids were clueless. Several gave one example of prisms and nothing for cylinders. A few — Adam, Rose, Anne and Samantha drew pictures to show the prism and then approximated for the cylinder. Only a couple — primarily Rebecca (and she referred to Nick's theory) did it algebraically. Her reasoning was great and she wrote, "I don't know what everyone's problem is — there are 3 extra 2's in each case" on her assignment. (Journal, 1/5)*

Hence, Rebecca had little difficulty with making a general, mathematical argument. In fact, she could not understand why some students found this type of reasoning difficult to understand.

Although on the exterior, Rebecca seemed like a sweet girl, she had her moments when she would ridicule other students. For example, Carl was a boy who had great social difficulties and occasionally ended up terribly frustrated and in tears during group work. Instead of being sensitive to this, Rebecca called Carl a "geek" on at least one occasion (Journal, 9/30). On another day, I heard Rebecca rather sharply correct another classmate on his mathematical drawings. Although none of these events occurred during whole class discussion, I think they shed light on some of the power dynamics in the room. It could help explain why people did not interrupt Rebecca or talk over her or make fun of her when she was at the overhead. The fact that Rebecca would stand up for herself meant that she had more space in the whole class discussion, such as on December 10:

> Guinevere did not say anything today, but at one point answered with Rebecca, and Rebecca told her to be quiet. (Rebecca's tone with Guinevere was friendly. I had called on Rebecca, who had her hand up.) (Journal, 1/10)

Hence, Rebecca regularly participated in discussions, with her contributions usually taking the form of providing answers to my questions. Her answers were generally mathematically correct and well-reasoned. Even when her answers or methods differed from others, she seemed confident that her solution made sense and could explain it to the class.

Rebecca's View of Whole-Class Discussions. When asked if she participates much in class discussions, Rebecca said she did. For example, in her second interview, she said, "I try to, cause if I know I can answer something I raise my hand and answer it." Rebecca seemed to like to have the opportunity to have her ideas heard. She once groaned and put her hand down when I called on another student who contributed "her" idea (Journal, 9/30).

Rebecca appreciated the chance to discuss ideas with others in either a small group or whole-class setting, "because then if I don't know the answer then someone can explain it to me" (Second Interview).

As the above quote indicates, Rebecca seemed more oriented toward finding right answers, as opposed to sharing and learning from others' ideas and perspectives when compared with Guinevere and Samantha. Also, according to survey information, Rebecca preferred more teacher involvement and direction in group work and discussions than Guinevere or Samantha did. Perhaps it is not coincidental that Guinevere and Samantha both live with mothers who are graduate students in a social service field, while Rebecca lives with a mother who is an elementary school teacher who was concerned that Rebecca was not getting enough basic computational skills in my class. (Rebecca's father was more supportive of the problem solving orientation of the curriculum.)

Still, Rebecca had much in common with Samantha and Guinevere. For example, she did voice appreciation in her final interview for hearing different opinions. Additionally, she shared the mathematical confidence of Samantha and Guinevere.

> *There were many times this year when we had mathematical discussions together. Did you learn from those discussions?*
>
> Yeah, I think it helps me learn more things instead of just like doing it on your own, I can know everybody's opinions and take it into consideration.
>
> *Do you find it confusing when you have all those different opinions out?*
>
> Not really.
>
> *Why?*
>
> Well, some of 'em aren't true, and some of 'em are, and I can figure out which ones are true and which ones aren't and stuff.

Hence, for Rebecca, class discussions were helpful for learning others' opinions. She had the mathematical confidence to make sense of the contrasting viewpoints, and, therefore, did not find the diverse views confusing.

For most of the year, she consistently said she did not think people were made to feel bad discussions. But on the final survey, in response to the question, "Have there been times when people have gotten their feelings hurt or felt stupid in whole class discussions?" Rebecca wrote:

> Yes, when you are trying to explain something and someone cuts you off or doesn't understand and tries to prove what you weren't saying wrong.

This quote is interesting because she makes it clear that she does not like to have her ideas be misunderstood. It is not having people try to disprove her ideas that bothers her, it is having people misunderstand what she is saying and then attribute ideas to her that they are trying to disprove.

Rebecca and Whole-Class Discussions: A Summary. Rebecca participated often in class discussions. Her participation was usually in the form of providing mathematically correct answers to my questions. But at times she would also explain her ideas and insightful reasoning. She rarely challenged me or other students directly , but she sometimes would build on other students' ideas or offer an alternative opinion. In many ways, her participation was similar to that of Samantha and Guinevere, but Rebecca showed a bit more of an orientation toward finding and giving right answers, as opposed to sharing ideas. Other students showed respect for Rebecca and her ideas.

Rebecca said she liked whole group discussions and she saw them as helpful, since she could learn from others' ideas. She did not think the discussions were confusing, since she was confident in her abilities to understand and judge others' ideas and arguments. She also appreciated the opportunity to voice her own opinions, and she did not like to be cut off or misunderstood.

<u>Rebecca and the Curriculum</u>

Like Samantha, Rebecca turned in virtually 100% of her homework and her test scores were generally in the high 90's. She earned A's throughout the year. Rebecca also talked positively about the CMP curriculum, in contrast with typical curricula. For example, in her first interview, she explained:

> This year we're doing stuff that I like. Two years ago we did 20 x 4, I hated it because I wasn't good at it and I'm good at this.
>
> *Are you afraid that if you stop using this program, you will go back to being bad at math?*
>
> Yeah! . . . . [CMP is] easier. [Before] we just sat there with hundreds of problems on a page.
>
> *. . . Why do you think it's easier for you (when others say it is harder)?*
>
> I don't know it's just my abilities.
>
> . . . .
>
> *Would you rather have the teacher tell you rules, or would you rather figure them out?*
>
> No. Learning about it like exploring how to do things is easier for me than sitting down and learning the rule.
>
> *Why do you think that is?*
>
> I don't know. Some people think it's easier just learning the rule.
>
> *But it's not for you?*
>
> No!

Hence, Rebecca said that the CMP was easier for her. Rebecca reiterated this idea many times, such as in her second interview, when she said, "CMP is easier. I'm not good at like multiplying and stuff, and this year we get to use our calculators." Throughout the year, she said consistently that she thought CMP is easier than "normal math" (First Survey, 10/28, Second CMP Survey, 2/2, and Second Interview, 3/21). She said that in traditional math, she learned more about "how to do multiplication and division problems quicker," while in CMP math, she learned more about "how to use calculators and real work stuff." Rebecca also mentioned that CMP problems are "more fun," and "more like real life," — "We get to do more hands-on stuff." She also described the CMP curriculum as requiring "more thinking" (Second CMP Survey, Second Interview).

Rebecca rarely complained about the CMP curriculum being confusing or frustrating. When asked specifically if she gets frustrated, she responded, "Sometimes, like if I can't figure something out" (Second Interview). When asked what she does when frustrated, she offered a variety of responses, including, "I try to do the problem to the best of my ability," and "I ask someone to help me" ( Second "Show What You Know," Second Interview).

In class, Rebecca rarely indicated that she was confused. As mentioned previously, she generally solved problems correctly, and sometimes her methods were difficult for the other students to follow. She sometimes expressed impatience with other students for not understanding ideas as quickly as she did. As discussed previously, the problems seemed to be fairly easy for her, and she could see the abstract, mathematical ideas in the contextualized problems and make general, algebraic arguments. Her mother also told me at the fall parent/teacher conference, "Rebecca used to not be strong in math." She further explained that Rebecca was doing better in the CMP curriculum and enjoyed it more. Yet, her mom also expressed concern that Rebecca wanted to grab a calculator to add up her Yahtzee scores.

Rebecca was different from Samantha and Guinevere in that she seemed less intrinsically interested in solving the problems. Unlike Guinevere and Samantha, she said she "seldom" tries to understand why a method works. She also said she "seldom" enjoys the challenge of solving the problems (Final

Survey). Her enjoyment of the curriculum seemed to center mainly around it being easier for her, and getting better grades. For example, when asked to say how much she liked math before CMP, she ranked it a 5, saying it was hard, while she gave math this year an 8, saying "It's easier than normal math." When asked "How good are you at math? Why do you think that?" she ranked herself a 9, saying that since she switched to CMP last year, she got A's in math, "but before that I struggled." In the end, she was confident about her mathematical abilities. She ranked herself as "#3" in the class on the final survey. She was also mentioned by three others in the class as being one of the "best 3 math students."

Rebecca: A General Summary

Rebecca was generally positive about whole-class discussions and the open problems of the CMP curriculum. She said math was easier for her this year, explaining she was better at figuring out the CMP problems than practicing computation. In discussions, she revealed her ability to make solid, abstract, mathematical arguments. Still, her participation was usually in the form of answering my questions, as opposed to disagreeing with me or another student or sharing an idea that would push us on an unexpected path. Overall, Rebecca seemed to gain affection for and confidence in mathematics through her experiences with the curriculum and pedagogy.

## Rose

Rose is a particularly important case, because she was from a working-class background, yet had high achievement. Hence, her data are helpful for sorting out differences in factors that are possibly linked to achievement, as opposed to SES. She was one of only two non-middle class students to be placed in Honors Algebra for eighth grade. As a student, she seemed to have a mysterious mix of behaviors — attaining very high grades, but sometimes surprising me by exhibiting what mathematicians would consider faulty logic.

Rose was an outgoing and well-liked girl. She was an extremely hard worker who tended to get straight A's in school. All her teachers spoke highly of her and said that she did very well in their classes. Rose has some extended family living in Mexico, although her immediate family had apparently been living in the United States for several generations. She was raised in an all-English-speaking environment and spoke with absolutely no accent. According to the information reported in Rose's background survey, Rose's father had a grade school education and worked in a factory. Rose's mother had completed some college or vocational training and worked as a cashier. Their family income was around $35,000, and they had somewhere between 300-500 books and no computer in the home. The household consisted of three adults and two children.

On the final survey, I asked how important it is to her to get good grades on a scale of 1-10, and she responded "10," saying, "I've always wanted good grades in every class. I feel like I always need to get A's." She said she worries about how she does in school now because "I need good grades to get into a good college." She also said that she is not worried about paying for college because "we have savings bonds, and people say I'll get a scholarship for music or something" (She played the cello). Hence, although her own parents' educations were limited, Rose and her parents seemed to be striving for a more professional path for Rose.

<u>Rose and Whole-Class Discussions</u>

For Rose, discussions were a place for giving right answers to questions posed by the teacher. As before, I begin by describing my view of Rose's participation, and later summarize Rose's own views about the discussions.

Rose's Participation in Whole-Class Discussions. I was intrigued with Rose all year, as she was from a working-class background, but there were many things about her participation that seemed much like that of the higher-SES students.

> Today was a good thing with James, but I still don't hear his voice in the classroom — same with Crystal[4], Dawn, Carl. These are probably my four lowest-SES kids, and they are the kids I NEVER hear from! Yet, there is also Rose, who talks all the time, and she is lower-SES. I really should find out what makes her tick! (Journal, 3/17)

Yet Rose was different than most major participators, as she was the only one to regularly complain that she did not want to have class discussions — particularly about homework problems. I noted several times throughout the year when she would ask if they could just turn the homework in without having to discuss the problems. She would ask this even if she had questions about the homework problems (e.g., Journal 9/23, 1/26).

Although she complained about the discussions, Rose regularly participated in them (Journal, 1/14; 3/15). In fact, she was named by several students as a "big arguer" in the class, although Rose did not see herself this way. She would often make solid contributions — especially direct answers to my questions or asking questions about how to do something — but she rarely provided new insights like other students (e.g., Samantha or Guinevere) did. Hence, on many occasions, I made comments along this line in my journal:

> She participated quite a bit in discussions — as usual, nothing earth-shattering, but solid work at answering questions. (Journal, 12/7)

> She had her hand up on every question today. Again, I always seem to feel that she is not asking or answering the really "meaty" questions —

[4] I did not have permission to include Crystal in my study, and I had no official SES information about her. But from informal measures (language use, sporadic attendance, comments from her neighbor at a parent/teacher conference) I was quite sure she was of very low-SES.

> she seems to stay at a certain surface level — she is usually correct, but not digging deeply into things or having great insights like other kids — e.g., Benjamin or Samantha. (Journal, 2/22)

> Chimed in — again, her ideas always seem rather naive somehow — they are often solid, but then other times she says things that are not quite correct. (Journal, 2/28)

In my journal, I did find one example of Rose sharing a keen insight with us. This involved comparing the volume of a cylinder when the paper is turned vertically and horizontally. We had a discussion just before and after winter break about this.

> Present, made a good observation at the end of the hour about why r = 3 & h = 10 has smaller volume (because r is squared) than r = 10 & h = 3. (Journal, 12/17)

> I had Rose try to explain why she thought they would be different — she repeated what she thought before break — namely that you want the bigger number squared, but she was not that articulate about it. (Journal, 1/3)

Perhaps it is not coincidental that Rose's insight occurred when we were exploring a relatively context-free problem using a formula upon which we previously agreed. Rose often seemed to want to reach for a formula to use, even when she did not understand why she would use it. For example, toward the end of the year, we did a unit on integers in which we tried to use models of elevators or banking to help students understand the operations with integers. Rose seemed to want to just get to the formula without betting bogged down in the models or understanding why the formula would work (Journal, 3/31). Additionally, Rose would sometimes seem to do things for the sole purpose of getting an answer without reasoning about whether or not the method made sense. For example, on September nine, I wrote, "Rose had weird and wrong reasoning about dividing to find the answer" when she explained how she solved a problem asking students to find how many classrooms and schools would be covered by the Exxon Valdez oil spill.

Rose's reasoning was intriguing to me because she was such an intelligent, high-achieving girl, yet she sometimes drew conclusions from one example and did not seem to realize the difference between that and a general proof.

> Rose - On the ball, but didn't seem to distinguish between one example of eight times as big and a more general argument. (Journal, 1/5)

> Benjamin was very insistent about his (correct) theory that the area increases by the square of the scale factor (my language — not his). I was asking for explanations as to *why* this happens. Again, Rose chimed in with, "Well, we know that the area is 18, so it has to be multiplied by 3", and . . . Benjamin said "But WHY does it happen? — If you didn't know what it would be, how could you figure it out, like for Mugwump 4, 5 . . ." Rose has a very difficult time with this — this is the second or third time this exact situation has come up. She just doesn't seem to get the proving stuff idea. (Journal, 1/21)

She also showed some evidence that she had difficulty distinguishing between an opinion-based question versus something that could be mathematically reasoned. For example, on a probability question about where a spinner is most likely to land, she said that since different people have different guesses, it is just an opinion question (Journal, 3/8).

I recorded several examples in which Rose's "real world, common sense" reasoning seemed to take precedence over finding or realizing the usefulness of the intended mathematical solutions to problems. The transcript of one such discussion (on 12/14) about finding the better buy of popcorn reveals Rose was using "common sense" to say that since the prices go roughly in order of size, we only need to choose which size we really need.

> *. . . Now, this costs $3.50, the cone costs $2.50 and the cylinder costs $3.75. Now what do we do? Which one is the better buy? Is there anything we can rule out immediately? . . .*

> I put it depends. Because the one with the less volume is cheapest, and the one with the most volume is the most money. So it depends on how much popcorn you want.

Her reasoning was, in part, mathematically sensible, since she was using the real world context to decide how accurate she needed to be. In the real world, one probably would most likely approach the problem as Rose did. Yet, she did not see the problem as the authors intended, which meant that she did not gain the intended experience of actually finding the volumes of the containers. Similarly, in a discussion about a statistic that says that men are killed in car accidents more often than women, Rose explained, "Maybe men were in the wrong place at the wrong time" (Journal, 2/25).

Rose showed little intrinsic interest in the problems or discussion. When working in her small group, she liked to quickly finish problems and then talk about other things instead of delving into discussions about the mathematics. She also sometimes tuned out the whole class discussions and, instead of participating, talked or argued about other things with neighbors (Journal, 10/22, 11/23, 1/13).

In summary, Rose was generally active in the discussions, but she complained about them and sometimes seemed disinterested. She gave many answers to my questions, and she asked many questions about how to solve problems. Although her contributions were generally mathematically correct, she rarely provided new insights and she sometimes reasoned about the contextualized problems in unexpected, "common sense" ways.

Rose's View of Whole-Class Discussions. In general, although Rose made many contributions to class discussions, she had little enthusiasm for participating. In her first survey, she said she did not like math class much because "we spend too much time correcting things." For her, discussions about problems previously worked on were viewed as "correcting" the work instead of sharing ideas. In her final interview, Rose explained that the discussions were boring when she already understood the topic being discussed, and at other times the discussions were confusing because there was disagreement among people and it was difficult to sort out the conflicting opinions — "Everybody's saying like this is the answer, and that's the answer."

Rose said she would only become involved in discussions "when I know what I'm talking about, but if I'm confused I just listen." She said she participated more in Language Arts class "because that's really easy" (Second Interview, Final Interview). Hence, for Rose, active participation was not viewed as a helpful avenue for learning new things, sharing conjectures or clarifying ideas. She seemed to view discussions in more of a typical math class way, consisting of questions and (hopefully correct) answers.

Although she was not particularly fond of all aspects of my pedagogy, Rose thought I was a fair teacher who did not show favoritism to students — "She seems to act the same way to everyone" (Final Survey). Still, she noted that her classmates occasionally made fun of people during discussions — especially another lower-SES female named Sue.

> *Do you think there have been times when people have gotten their feelings hurt or felt stupid during these arguments?*
>
> Yeah, because Sue Rowley? A lot of people, like when she says something or when she's arguing about something, people roll their eyes or do something that makes her want to feel bad. (Second Interview)

I recorded no instances in my journal when Rose was treated poorly by others in the discussions. Her classmates seemed to treat her and her ideas with respect.

<u>Rose and Whole-Class Discussions: A Summary</u>. Rose participated in, but did not seem to appreciate the whole class discussions. If she understood what was being discussed, then she would answer questions, but she thought

this was rather boring. If she did not understand, then the discussion was confusing. Either way, she did not show interest in sharing her developing ideas, clarifying her thinking, or exploring mathematical relationships through the discussions. She regularly complained to me about the discussions, asking if we could just skip them.

Her participation usually consisted of giving answers to my straight-forward questions, and she was usually correct. This makes sense, given that she only would answer questions when she was sure she understood. She rarely displayed deep thinking or interest in the mathematics. She made solid contributions, but rarely provided keen insights that helped move the discussion forward in interesting ways. Her focus seemed to be on getting correct answers to questions or problems and then getting on with it.

Although other students named her as a "big arguer" in the discussions, Rose did not see herself that way. I, myself, have difficulty classifying Rose as a *mathematical* arguer, since she her reasoning was often more "common sense" than the purely mathematical reasoning I expected. Sometimes her common sense responses made more sense than the traditional mathematical answers, but at other times she seemed to make little sense. She had difficulty distinguishing between proof by one example and a more generalized proof, as well as between a question that requires only an opinion versus one that has a mathematical answer. She often did not seem concerned about the type of reasoning used to arrive at an answer, as long as some answer — hopefully correct — was obtained.

Rose and the Curriculum

Rose diligently turned in over 97% of the homework assigned during the year, but her test scores were lower than her homework average. Her test averages for the three marking periods were 94, 92, and 76%. Hence her efforts in class and on homework usually — but not always — seemed to help her understand the main ideas under study well enough to perform well on tests.

Rose's high achievement makes her seem like the higher-SES girls in many ways. But a major difference was that, although Rose usually seemed to be able to grasp the individual ideas in each problem or unit, she often complained that she did not see connections between the various problems within and between units. For example, on February 3, during a class discussion specifically about the intent of the CMP curriculum, Rose complained that the problems done in class together did not prepare her to do the homework problems. She did not seem to see the mathematical similarities between the problems we did in class and those assigned for homework. Additionally, in her final survey, she said that she liked math better before, "because it was fun and the books explained how to do the problems so it was easy to understand," whereas in CMP math "we go through tons of units and once we get done with one unit we never talk about it again."

More generally, Rose complained throughout the year that she did not like the CMP trial materials because she had difficulty understanding the problems (e.g., First, Second, and Final Interviews, Final Survey). For example, she described the CMP as "harder" because "it doesn't give you enough detail so you have to figure it out on your own" (First Interview). The lack of specific direction seemed to frustrate her at times. When asked directly what frustrated her, she responded, "Not knowing how to do problems when we're given problems that don't explain themselves" (Second "Show What You Know"). In the mid-year CMP Survey, she said, "These books are bad because they are so confusing. We are told to do a page as homework and the page gives directions but it doesn't explain how to do it. These books should be taken off the market. No one likes them. They're boring."

Still, Rose's attitude toward the CMP was not completely negative, and she did not feel constantly frustrated. She said she "seldom" got stuck on homework, but when she did get stuck, she felt very frustrated (Final Survey). She also said that math, in general, was easy for her. Furthermore, she said that CMP is challenging, and that she likes challenges (Second CMP Survey, Final Survey & Interview). She said, "I learn more about real life" in CMP and more about fractions in traditional math (Second Interview). According to Rose, some units were rather interesting, but some were boring (Second Interview).

Rose consistently said that CMP is harder than math was before (e.g., First and Second Interviews). But apparently, this was not a completely negative thing. In her final interview, she said that she had "kind of" changed her opinion about CMP over the course of the year. She felt "kind of glad" they had CMP, but she thought it "should be explained better." She said "CMP is more real life . . . more involving and challenging."

Although CMP was harder for her, she felt confident about her abilities in some ways. For example, in the first survey, when asked, "How good are you at math?" she ranked herself a 9 out of 10, explaining, "I get good grades and most of it is real easy for me." But although many of her peers named her as one of the top three math students at the end of the year, Rose did not name herself.

When asked if there are certain types of people who seem to do well in or like CMP better than traditional math, she talked about Samantha as an example of someone who likes CMP.

> Samantha likes CMP because she's smart — it comes natural. And even when it doesn't explain it, she tries to make it out, so she just does what it says even if it isn't right.

Hence, even though she generally earned high grades and was viewed by many classmates as being a top math student, she differentiated herself from students like Samantha who were "naturally" more able or inclined to work through the ambiguities of the open problems in the CMP trial materials.

Rose: A General Summary

Although Rose liked challenges, the ambiguities of the CMP trial problems were sometimes challenging to the point of too much frustration for Rose. She did not see the purpose of the more open problems, just as she did not see the mathematical connections among them. At times, she approached the contextualized problems in "common sense" ways, with faulty mathematical reasoning, and she seemed to miss the mathematical point of the problems. Still, she worked hard and usually obtained correct answers, which she would share in discussions — only if she was sure they were correct. Discussions were not enjoyable or helpful for Rose, as she was bored when she was sure she was correct, and at other times she became confused by the differences of opinions. She had a more traditional view of the discussions — consisting of a teacher asking questions and students gaining approval by providing correct answers. Her ability to obtain correct answers probably contributed to her classmates ranking her as a top math student in the class. Still, Rose struggled with making sense of many of the problems in the curriculum, as well as class discussions; she saw herself as a fairly good math student, but different from others who were "naturally" better at CMP math.

## Sue

Sue lived with her mother and sister, (although this arrangement was somewhat unstable, since when I talked with her two years later, Sue told me she was moving in with her dad). Her mother had attended a two-year college and was an "administrative assistant." There was no computer in the home. Her mother was private about offering further SES information. Sue said that her mom wanted her to go to college, and that she, herself planned to go to college to be a "V.E.T. or phycyolrist" (Final Survey — I'm not sure if she meant "psychologist" or something else.).

Sue was often ridiculed in class, particularly by the boys. Still, she made a consistent effort in class discussions and in her assigned work, although her test scores did not reflect her strong efforts.

Sue and Whole-Class Discussions

Although treated differently by classmates than Rose, Sue shared Rose's emphasis on providing right answers during discussions.

Sue's Participation in Whole-Class Discussions. At the beginning of the year, Sue would occasionally volunteer answers in class, but she was often wrong, and this frustrated her. For example, on September 28, when classmates tried to help her understand how to do a problem, she said angrily, "I just don't see where you get the numbers!" She wrote "I hate math!" all over her notebook. Even though she attributed her frustrations to ambiguities in the curriculum, I felt like she often took her frustration out on me (Journal, 9/28). I wrote her a note, acknowledging her frustration but asking her to try and have a more positive attitude in the class, and, to my surprise, it actually seemed to help.

> *I gave Sue her paper with my comments about her attitude on it. I heard her show it to Tricia and Lynn saying, "I don't know what I'm supposed to do, because I don't get it." I was worried and wondered if I should go talk with her. I WANT her to ask questions, I just don't want her to ask them in such an inappropriate tone. Is this just a cultural thing? Is it a power thing? I expected her to be very quiet and refuse to participate at all to "punish" me. To my pleasant surprise, she didn't. She contributed to the Japanese discussion, although I was worried because it didn't make much sense . . . . Timothy answered her because he understood the problem with her logic. Additionally, I made a point to check on her group — I am worried because while other groups had answered several questions, they couldn't get the first one, which was very simple — subtract two numbers. Tricia asked if we were ever going back to the*

> other books, and I said, "not this year." Then Sue nicely told me about some of the specific struggles she had . . . . (Journal, 9/30)

Sue did become a much more pleasant part of the class after that point, and she regularly contributed to class discussions. But her participation was usually in the form of asking questions when she was confused about how to do a problem.

> Very interesting issues were being discussed today with regard to the y-axis on the graph being speed and not distance. I had to bring up the word "acceleration" because a straight line slanted upward meant something about constant, but I needed to help them talk about the fact that it is NOT constant speed when the y-axis is speed. Sue twice got confused about this distinction — she thought a horizontal line meant that they were not moving, which is only true if we are looking at distance — not speed. (Journal, 10/19)

> Sue asked about #4 and understood it quickly. She was quite active today. (Journal, 1/6)

Sue was also quick to ask questions about why I graded her papers the way I did, particularly in comparison with other students. For example, on February 14, Sue asked, "How come I got 5 points off and she got 6?" (Journal, 2/14). At other times, Sue volunteered to participate in simple ways, such as reading a paragraph from the book for us or answering simple questions (e.g., Journal, 12/8, 1/4).

Still there were a few times when Sue offered an opinion or made an argument. Most of these instances, as recorded in my journal, revolved around a "real life" situation, with Sue using what might be described as "common sense" reasoning.

> We had a good discussion about the graphs (students brought in from the newspaper) . . . . We also discussed a graph (from an advertisement comparing interest rates for various banks) that had three-dimensional arrows instead of bars. I kept trying to pound on intentionality — WHY would they want to do this (i.e. not start at 0 . . .) Sue noted that the arrows made it look like it was still going up . . . . (Journal, 10/25)

> Then I asked for some of their responses (about what it means to have 10% chance of rain) . . . . Sue said that the weather person was kind of wrong if it did rain, and Michelle said he was 90% wrong and 10% right. Others, i.e. Rebecca and Lindsay, thought that he was not wrong. (Journal, 2/23)

> For the last one (a problem asking if Volvos are safer cars, since their death rate is lower) we had very interesting discussions. Samantha said that people might drive differently if they are in one or the other — Harrison gave a story of how his brother drives crazier in his Beretta. But then Sue said that people with a four-door Volvo probably have families and drive better. (Journal, 2/25)

As in the last example, Sue occasionally revealed that she was capable of reasoning insightfully about mathematics problems set in real-life contexts. Sue certainly showed she was capable of boldly making coherent, logical arguments in situations more real for her than a hypothetical, mathematical problem. For example, she assertively made arguments about why she thought certain problems were not realistic (she once argued, "I don't understand how a girl is supposed to go around the town and collect popcorn boxes and measuring them . . .") (Journal, 2/3).

Still, occasionally it seemed that Sue's lack of basic mathematical knowledge inhibited her abilities to solve problems correctly. For example, when labeling axes on a graph, she was one of a handful of students who did not know what to call the points half-way in between 1, 1.5, 2, et cetera (Journal, 10/19).

A dilemma I struggled with was what to do when Sue's mathematical ideas were wrong, as they often were. I thought that she might be fragile and that one bad experience might silence her permanently. But she showed much more resilience than I expected, as she continued to participate in discussions (e.g., Journal, 12/9).

Quite often other students would try to offer Sue explanations when she asked questions or expressed confusion. Usually it was boys or higher-SES girls who tried to correct and/or help Sue (e.g., Journal, 10/6, 12/10). Even James, a

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lower-SES, African-American male who virtually never spoke in class, chose to speak up and disagree with Sue on one of the few days he opened his mouth.

> James - Today JAMES spoke in class TWICE! AND he actually did the penny graph last night and turned it in today! . . . . The first time he spoke, it was sort of under his breath — he was arguing against Sue, who thought that 1's and 6's don't seem to come up as often on a dice as 2,3,4, & 5's. I asked him to say what he was saying louder . . . . (Journal, 2/16)

Instead of feeling shut out of the discussion, Sue jumped in again on the next question — whether snow or rain are equally likely in Alaska on a December day. She responded, "I don't think they really are, because Alaska is like way up in the north, where all the snow is." A boy named Adam began to disagree with her even before she was finished talking. (Sue was right and Adam was later proved wrong.)

As the year progressed, it became more and more popular to ridicule Sue when she would express her confusion. In the classroom, this initially took the form of quiet eye rolling and sighing whenever Sue would speak (e.g., journal 12/2, 2/7). Sue also explained that she was ridiculed by boys outside the classroom after class, when they would say, "Oh she's so stupid! I can't believe she got that wrong" (Final Interview). By the middle of March, instead of going away, the problem was becoming more overt. Instead of quietly rolling their eyes, students became more vocal when Sue made mistakes publicly. In interviews conducted around this time, some other students specifically mentioned Sue's treatment, in response to the question, "Do you think some people get their feelings hurt or feel stupid during class discussions?"

> Rodney: Yeah . . .like every time she tries to ask a question they all sigh or something.
>
> Timothy: Yeah. Well, Sue is kind of slow, and like we get kind of at her when she acts kind of like she's not listening anymore. (Second Interview, 3/21)

Through the ways in which I addressed the problem (e.g., publicly and privately talking with students about this), Sue learned that I took the issue seriously and saw sexism as one underlying issue.[5] At about this time, Sue, who in the beginning of the year wanted to have nothing to do with me, the curriculum, or my research, asked if she could be interviewed as part of the study (Journal, 3/18).

Why was Sue picked on? As I mentioned in my teaching journal, Sue tended to be inarticulate, unsure of herself, and willing to express her confusion in order to get help. Sue focused on getting right answers, and she seemed panicked when she was not able to complete assignments correctly. Therefore, she would ask questions when she had them.

Another factor might involve Sue's interactions with the boys in the class. She would occasionally misbehave (e.g., Journal, 1/4, 2/24, 3/1) or get into petty fights with them (e.g., Journal, 12/14, 12/16). Some of Sue's behavior could be interpreted as flirtatious. Perhaps some class members interpreted her "I'm so confused" helplessness as flirtatious. Perhaps this behavior made boys want to "flirt back" in the form of ridiculing her. Or perhaps people did not respect her because of her lack of mathematical understanding and because of her flirtatious behavior, and, therefore, they felt free to ridicule her. This is one arena where the age of these children seems to be particularly important.

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[5] Sue was not the only student ridiculed in discussions. For example, I recorded a similar episode with another lower-SES girl named Lynn:

> I asked for a teacher to come up to the overhead and explain their group's solution . . . Lynn seemed flustered, even though she did volunteer, and the kids were really bad while she was talking. They laughed at her when she asked her group to remind her of what they did for one part. Then they ignored part of the problem, and kids were jumping on her. I told them afterwards that although I heard good reasoning on the problem, I didn't like the way they treated Lynn . . . If Samantha of Guinevere or Andrea or Rebecca (all higher-SES girls) are up, then this crap doesn't happen. I think part of why girls like Sue and Lynn are prey is that they are inarticulate, unsure of themselves, and actually tend to be flirtatious in other arenas (but not in class, really). (Journal, 4/21)

In summary, Sue participated often in class discussions, but her contributions were usually in the form of answers to simple questions. Sue showed she was capable of making a coherent argument, but she tended to apply these skills to real-world, non-mathematical topics. Her mathematical contributions were often wrong, and other students tended to ridicule her in subtle and not-so-subtle ways. As will be discussed in the next section, Sue's interviews and surveys revealed that she was aware of the ridicule and persevered in spite of it.

Sue's View of Whole-Class Discussions. Through Sue's interviews, I learned that she felt silenced because of the treatment she received from others. In her final survey, she said, "I feel stupid when I get an answer wrong during class." When asked which class discussion she remembered most, she said:

> I remember one time I went to answer an answer to a homework question. I got the wrong answer and all of the boys said I was stupid and a dumb blond. Ever since I haven't really answered many questions.

Sue told the person who conducted her second interview that other students made her feel bad, but I, the teacher, never did. Still, she seemed hurt by the ways in which I distributed my attention. While most students said I did not show favoritism to any students, Sue said I "babied" Samantha by allowing her to eat in class because she has diabetes (Second Interview). In her final survey, she reiterated the remark about Samantha, and she added that I favor Benjamin because he is "really smart."

Hence, Sue was quite jealous of the teacher's affections and attention. Sue seemed to prefer a strong, directive teacher role. In her second interview, she said I was a good teacher because I can explain the questions and answers well. In the beginning of the year, she said she preferred working in her group and having the teacher solve problems with the class and give examples of how to do problems. As the year progressed, she expressed less appreciation for any whole

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group interactions. In her second interview, she explained that she preferred working in smaller groups because in the whole group "everyone is talking at one time and you can't get anything across that well." In her final interview, she said she preferred learning from just the teacher because she becomes confused when there are many opinions offered:

*There were many times this year when we had mathematical discussions together. Did you learn from those discussions?*

Yeah, kind of. I learn better from just like the teacher instead of the whole group . . . . When everyone is there they give their opinions and stuff it may not be right, and I mix those two up, and it just confuses me.

In her final survey, she reiterated these feelings by saying she preferred to work alone and in small groups because "I understand more without all the people."

One might wonder why Sue participated in discussions at all. When asked about this in her first interview, she said she participates "Cause I want to understand 'em better." But then in her final survey, she, like Rose, said she only participates if she was sure she "got the right answer," and that she participates more in discussions in other classes because she was "better in those classes."

<u>Sue and Whole-Class Discussions: A Summary</u>. Throughout her comments, Sue spoke of her role in the discussions as a contributor of right or wrong answers, as opposed to one who makes conjectures, shares ideas, or helps others revise their thinking. She tried to get help for her confusion by participating in discussions, and perhaps she also wanted to get "credit" for contributing right answers, since she said she would only give an answer if she knew the right one. She felt she needed to know the right answer before participating — discussions were not for learning from others, unless she had a specific question she wanted answered, and then she generally directed that

question toward me, the teacher. When she did not have the right answer, or sometimes when she would ask a simple question, she was subject to ridicule from other students. Although I thought it was good to try and get other students to answer the questions, now I am not so sure of the wisdom of that philosophy, since the power relations among students became so unequal, with Sue near the bottom of the heap.

Sue was capable of making logical arguments, but she rarely used these skills in our mathematical discussions. As the year progressed, she expressed less appreciation for whole class discussions, saying that they were confusing, she had trouble speaking in them, and people made fun of her. She preferred working in smaller groups or having only the teacher give her help with problems.

Sue and the Curriculum

Sue made a consistent effort, turning in over 90% of the assigned homework. But her test scores were never in the 90's. Her test averages for the three marking periods were 75, 83, and 73%. Her overall grades were in the B-C range. Hence, Sue was like several other lower-SES girls who made consistent efforts that did not seem to pay off in terms of really understanding the content in a way that allowed them to do well on tests.

Sue expressed much frustration with the CMP trial curriculum throughout the year. In both informal discussions and in surveys and interviews, Sue's attitude about the curriculum was generally negative, with the dominant theme being that the books are too hard because they are confusing.

> I hate the math books. They are confusing and very hard. They are not very accurate with my level of math. (First "Show What You Know")

> [The authors should] explain better, books confusing, questions are too long and complicated . . . . (Mid-year CMP Survey)

> The questions are not explained very well, so I don't understand them. (Second Interview)

She said that before she had CMP math, she liked math and "understood it," which enabled her to "get good grades," whereas this year she thought math books were "*way* too confusing." She also said, "I'm not too smart and I don't understand it most of the time" (Final Survey). On many other occasions, Sue reported feeling mathematically incompetent, and she attributed this, directly and indirectly, to her experiences with the CMP trial curriculum. The following is an excerpt from her final interview.

> I like [learning math] the other way better. I used to do really good in math, but now I'm getting C's and stuff. I'm not doing too good now.
>
> *Do you think that the grading scale was easier before or is it really different?*
>
> Yeah, I used to really understand it and stuff. Like some of the questions in like the books for homework, I don't understand at all, they are really confusing and are too long . . . .
>
> *The words are too long or the questions themselves are too long?*
>
> The questions themselves, they have too many words in a sentence, so I get confused.

In her final survey, when asked, "Do you feel like you know what you are doing?" she circled "never." More than most other students, she seemed to internalize failure. She was one of three students (all lower-SES) who said that when they get a bad grade, they "always" feel stupid. At the end of the year, she ranked herself 17th out of the 27 students. Instead of feeling empowered as a problem solver, she seemed to "shut down" when faced with difficulty. When asked in her final interview how her opinion about CMP might have changed over the year, she replied, "I totally hated it at first. Now I just don't do the problems if I don't understand them" (Final Interview, 4/26).

Sue's complaints about being frustrated by the curriculum were consistent with her in-class behavior, as discussed above. It seemed that Sue struggled just to keep her head above water in the class, and perhaps this inhibited her ability to enjoy the challenge of the problems or be intrinsically interested in them. On her final survey, she said she "never" enjoyed the challenge of solving problems. In her final interview, she said she prefers memorizing rules to figuring things out for herself, "cause it takes to much time to figure it all out and everything."

Sue said that although the Jones School teachers thought that the CMP helped them do better on the state MEAP tests, she did not think she was learning more in the curriculum. In addition to her frustrations with not understanding the problems in the curriculum, Sue expressed some concern that "CMP doesn't teach us the basics that we need to know" (Second Interview).

Sue: A General Summary

Sue struggled with the open problems in the CMP trial curriculum and with the class discussions. She felt lost and frustrated when approaching the open problems, saying that the words confused her and she did not know how to proceed. Because she was often confused, she saw whole-class discussions as an opportunity to ask questions, as well as and to provide answers when she was sure she was right. Still, she was often wrong, and other students leaped at any opportunity to ridicule her. Sue seemed to prefer a more typical style of mathematics learning — one in which the teacher gives students the rules to memorize, and the problems to be solved are straight-forward. She was willing to work hard, but she wanted a clear direction in which to put forth her efforts.

Dawn

Dawn tended to be quiet and insecure. According to Dawn's background survey, completed by her mother, Dawn's father had a high school diploma and was unemployed. Her mother did not finish high school and cleaned houses for a living. Their household income was less than $11,000 and there were less than

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40 books in their home. There were three children living in their household. Dawn, like the other lower-SES females, said she planned to attend college and be a veterinarian.

Dawn was the student who best fit the "lower-SES, low-achievement" category. Dawn made some effort to complete her homework regularly but usually did quite poorly on tests and quizzes.

### Dawn and Whole-Class Discussions

For Dawn, class discussions were a spectator sport. Although the discussions were interesting to watch, Dawn (like Rose and Sue) found them confusing at times. In the sections below, I discuss Dawn's participation in discussions as I observed it, and then turn to what Dawn, herself, said about the discussions.

Dawn's Participation in Whole-Class Discussions. Dawn almost never talked in whole class discussions. I was concerned about her lack of participation throughout the year. Toward the end of the year, I noted that Dawn was one of four of my lowest-SES students, and these were also the same four who virtually never joined class discussions (Journal, 1/14, 3/17).

A nagging dilemma for me was how to handle students who did not seem to want to be involved in discussions. Since there were usually plenty of volunteers to talk in discussions, I generally chose volunteers to participate, selecting the volunteer who had been talking the least recently. But occasionally I would call on students who did not volunteer to talk.

I made some attempts to include Dawn, even though she did not volunteer to become involved in discussions. For example, on October 26 I asked Dawn to read information from a graph, and on December 12, I asked Dawn to make a guess about how many given cans of water would fit into a two-liter bot

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bottle. Two months later, when Dawn raised her hand to "vote" on whether or not she thought events of a pop can landing on its side or on an end were equally likely, I used that as an opportunity to ask her to explain why she guessed the way she did. I also occasionally asked Dawn to read information from the book for us to help us launch a problem exploration (e.g., journal, 1/14).

Another simple way I occasionally tried to get the entire class more involved was to bring them all to the front of the room for an informal time of whole-class, teacher-directed experimentation. Most students eagerly came to the front to see what was happening and to talk with each other and with me about it. But Dawn was different.

> While the rest of the class huddled around the front table to see the water experiments, she (Dawn) sat back, and I asked her to "come and play" with the rest of us. (Journal, 1/5)

The only instance recorded in my journal of Dawn volunteering a contribution occurred on October 27.

> Dawn actually volunteered an answer today — I asked what kind of scale we should use, and she said we should go up by 5's — a good answer! (Journal, 10/27)

In retrospect, I realize now that this was the day after I called on her (noted above) to answer a question, and I now wish I had done that more often with Dawn.[6]

There is some evidence that Dawn did not always fully listen to the discussions. In my journal I recorded a few instances in which Dawn was talking

[6] As I analyze the various ways I tried to include the quieter students, I notice that I tried to involve them in rather low-risk, low-pressure ways. I wonder, in retrospect, what might have happened if I tried to involve them in more challenging ways, such as asking them to evaluate other students' ideas. My initial experience in trying to get substantive participation from some of the quieter students was that it made the conversation come to a painful and screeching halt, and I was afraid it was causing more harm than good. But I can't help but wonder what would have happened if I had been more persistent in my expectations that they would all participate fully. On the other hand, I have some respect for my gut instinct about what these adolescent students could handle and how they might react or feel in more high-pressure, public situations.

with friends instead of focusing on the whole-class discussion. But generally speaking, she was quiet. So much so, in fact, that I was happy to hear her talking at all, even if she was simply asking me a question privately or talking when she should not have been (Journal, 12/15, 1/12).

Since I was concerned about Dawn's progress, I carefully chose her small group, often putting her with all females, and usually with Samantha, who was able and willing to patiently help Dawn without making her feel stupid although Dawn continued to be quiet in whole group discussions, by late November I began noticing that she was participating constructively in her small group.

> Dawn - She seems to be working fabulously in her group . . . . She was finding volumes of the various cubes and keeping up with Shelley and Julie! (Journal, 11/23)

> Dawn - She worked with Shelley today, and helped her catch up because Shelley wasn't in class yesterday. (Journal, 11/30)

> Dawn - present, I heard her doing really good reasoning about the genetics. (Journal, 3/15)

Occasionally, it seemed that Dawn's group served as an indirect pathway into the whole class discussions. For example on March 9, we were analyzing a game involving dice products, and Dawn noticed that a multiplication chart on the wall (that I had never noticed before) was redundant with a dice product chart we were creating (see figure 3.2). She told her group members about her discovery, which they related during the whole-class discussion (Journal, 3/9).

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 |

**Figure 3.2**
**Product Chart for Dice Analysis**

Hence, Dawn's observable participation was primarily limited to sitting quietly during whole-class discussions, although she did engage with her small group. Still, from observations alone, one cannot tell if Dawn was totally disengaged during whole-class discussions, or if she was listening carefully and learning a great deal. Hence, I turn now to Dawn's perspective of the discussions.

Dawn's View of Whole-Class Discussions. Despite her lack of participation in whole-class discussions, Dawn actually had a rather positive attitude toward me and my pedagogy. She said she liked math class this year more than last, "Becaus you arnt as boring and you don't write everything on the bored" (First Survey), and "Because we have group discussions and we get in our group of 4 and talk" (Final Survey). In her second interview, she reiterated that she liked having more talking and variety in class.

> I like it more, it's more interesting because you talk about more things in class . . . . You get shown different things, like you bring in articles and stuff like that.[7]

[7] By "you," she was referring to the students in the class, since the person doing the interview was not part of the class.

Yet, Dawn's enthusiasm was inconsistent. For example, one morning (February 28) she came dragging in, asking "Why do we have to have math?" However, for the most part, Dawn was positive about my pedagogy, although she never became active in class discussions.

Still, as the year progressed, she showed progress in her ability to participate actively in a small group. Throughout the year, Dawn voiced (in surveys and interviews) consistent preference for small group work, as well as heavy teacher guidance and direction. She said she received help in small groups and preferred not to work alone or have whole-class discussions. In her final interview she said she preferred working with girls because they give help without ridicule.

Some guys will say "uh, your not as smart as us"

*And girls don't do that?*

No, they pretty much just help you out and don't say nothing about it.

Why was Dawn so withdrawn in whole-class discussions? Dawn said it was because she was shy — just like her mother.

*How come you never join in?*

I don't know (laughs). I'm kind of shy; It kind of runs in my family; My mom never likes to speak up. I speak up more than she does or she did when she was in school.

*Do you get anything out of it when the other people are arguing? Or would you like to just get on with working in your group?*

Well, kind of, when they argue they kind of show you what they are talking about, but I think group. I like group!

In her second interview, Dawn explained in more detail why she found small groups more helpful than whole class discussions.

*What way do you learn the most — working on problems in small groups, alone, or having whole-groups discussions?*

Hm, small groups I think.

*Why?*

Cause, um, you get, um, you can hear what like your group thinks better cause you always get different people saying different things out in the whole class, kind of hard to think well, maybe I should go this way, or maybe I should go this way. Usually in a (small) group you can almost all agree on something.

*In a small group?*

Yeah.

*And you don't find that in the big group?*

No, usually somebody's disagreeing.

*And you don't always find that helpful?*

No.

In her final interview, Dawn reiterated that when people disagreed, she became "confused cause you don't know if this is right or this is right cause they don't agree." I asked her what happens when people cannot agree on an idea:

*When different people have these different ideas, how do we figure out who is right and who is wrong?*

Um, (laughs nervously) they like explain how their reasoning is, and usually you say, this reasoning is OK.

*Do I flat out say it, or—*

No, you say like, I think this reasoning is more likely to be in the real world or something.

*Why do you think I don't just say you're right and you're wrong?*

Because it might hurt their feelings. Like if you say you're wrong.

Because this was a line of questioning that did not occur to me until half way through the final interviews, I did not ask everyone these questions. Still, the two higher-SES girls I asked — Guinevere and Samantha — showed a better sense of my desire to help them figure things out for themselves, as opposed to

Dawn's idea that I was cryptic for the sake of sparing feelings. Still, Dawn was correct in that I struggled with how to help students like Sue feel comfortable about being wrong in front of the whole class.

Additionally, Dawn explained that she was afraid of being wrong in front of the group. In her final survey, in response to the question, "Do you participate much in class discussions?", she wrote, "no because I dont like to be wrong in front of a whole group." In response to a question asking her to compare her participation in this and other classes, she wrote, "I don't really raise my hand unless Im positive or if I have to answer I do."

Dawn revealed a different orientation to the nature and purpose of our mathematical discussions than most higher-SES students. She thought of being wrong in front of the class as a scary thing, and this kept her from participating in the discussions. She also tended to think of disagreement as not only confusing, but also hurtful, as opposed to something interesting from which she could learn. Hence, in the first interview, she said there were two ways that people could get their feelings hurt in mathematical arguments.

> 'Cause people say, like they say things like, "No, they do this and this", and 'cause they could just, like, after class they could say something.

The first way that Dawn mentioned reveals that she views disagreement as hurtful. In her second interview, she seems to say that if people are able to stand up to a person disagreeing with them, then that makes the disagreement less hurtful.

> *Do you think there have been times when people have gotten their feelings hurt or felt stupid during these arguments?*
>
> No, no, cause usually they'll come back with something.

<u>Dawn and Whole-Class Discussions: A Summary</u>. Dawn's understanding of mathematical argument seems to be one of potentially hurtful combat. She considered herself a shy person who perhaps did not have what it takes to stand

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up to critics. Her lack of confidence in her own mathematical knowledge compounded this problem. She did not feel able to discern who was right or wrong when there was disagreement. So Dawn was usually silent during class discussions, and she did not feel confident that she could make sense of other students' contributions to discussions. I tried to involve Dawn in various ways throughout the year, but my efforts often seemed futile. Dawn did become a constructive participator in her small group, and at times this would serve as an indirect link to the whole class discussions. Overall, Dawn had a positive attitude toward me and the class, yet she found working in her small group most helpful.

Dawn and the Curriculum

Dawn made a fair, but inconsistent, effort to complete her homework. She completed about 80% of the assigned work. Her test scores were far worse than her homework average — 63, 65, and 61% for the three marking periods. She received C's on her report card.

In her first interview, I asked Dawn to compare math this year to math two years ago.

> They give you like problems that you would like read and solve them. But a couple years ago you would just read a short thing of directions that say "add, subtract," . . . and you would just do the problems.
>
> *Is math easier or harder this year?*
>
> I think the ones when I was in fourth and fifth grade were easier. These are hard to figure these out.
>
> *Why do you think they [the authors] made it harder?*
>
> So you learn more?
>
> *Do you think that's true? Do you learn more?*
>
> Yeah (sounds hesitant), like when I was there you just learned the basics, the multiplication, division. Here you learn how to put them together.

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When asked about her experiences with the curriculum, Dawn complained about having difficulty understanding the CMP trial problems. She said the CMP was more frustrating than "regular math" because "the books are hard to understand" (First "Show What You Know"). When asked what she would like to tell the CMP authors, she replied, "I don't like this math book because it doesn't explain EXACTLY!" (Mid-Year CMP Survey). She also told me, "The tests help me a lot. I get more out of the tests than the book .... When I'm taking it I understand it better — a lot of the questions they ask." In talking with her further about this, she said the tests helped her understand what it was that she was supposed to be learning in the unit (First Interview).[8]

At the end of the year, Dawn summarized her view of the CMP by saying, "I like this math project but I tend to like the other ones better because they aren't as confusing." (Final Survey) Dawn did not seem to enjoy the challenge of solving the problems. She said she prefers to just practice rules "'cause I just get flustered over hard problems" (Final Interview).

When asked, Dawn talked about being frustrated, but she was unlike Sue and Rose in that she was not a constantly active complainer about the curriculum. As mentioned previously, she actually liked many things about our math class, particularly the small group work. She also said that the CMP curriculum "shows you how you're going to use math in life" because "it's not just problems, just like 3x6 or something like that, it's more story problems" (Second Interview).

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[8] The CMP curriculum has suggested assessment items, and I used some of these. I also made up some of the items or used unassigned ACE problems. In general, tests usually contained some problems or questions that were rather straight-forward, allowing me to see if students understood the main ideas of the unit, and then there were a few more lengthy, contextualized problems, that allowed me to assess students' abilities to apply the key mathematical ideas to new situations.

In comparing her with Sue, their overall grades were not much different, but perhaps because Dawn did not put in the type of effort that Sue did, she did not seem to be as frustrated with the curriculum. For better or worse, Sue was more typical of the lower-SES girls in the class. While Dawn seemed relatively relaxed about not being able to do some of the problems, Sue (and others like her) tended to become extremely upset and feel like a failure. When asked how she feels when she gets a really bad grade, Dawn responded (Final Interview):

> Not REALLY depressed about it, but you kind of want to get it up there. You're kind of disappointed but not really disappointed.
>
> *Do you get angry?*
>
> No.

It was not that Dawn considered math unimportant, but she thought she did not need to worry much about grades yet.

> I have like a long time, well not a long time — starting next year I gotta get my grades up and keep em up so I can get into college . . . . A lot of times I hear that math is really important, cause no matter what you do you gotta know math . . . cause even being a veterinarian, you gotta know math. (Final Interview)

When asked, Dawn said she feared she might not be smart enough for college. Still, she viewed herself as a slightly above-average math student in the class, ranking herself #10. She ranked Rose, Samantha and Rebecca as the top three because "they usually get into the discussions and get answers right" (Final Survey).

### Dawn: A General Summary

Dawn preferred learning math the "old" way, where she just practiced rules the teacher gave her. The open problems frustrated Dawn, and she did not enjoy the challenge. Still, perhaps because Dawn made less of an effort or perhaps because she was more accustomed to doing poorly in school, Dawn seemed less upset about the change in curriculum and pedagogy than other lov

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lower-SES girls, such as Sue or Rose. Dawn was shy and withdrawn during class discussions, so she was not ridiculed like Sue was. Still, instead of being a helpful opportunity to learn from others' ideas, the mix of opinions confused her because she did not feel capable of discerning which ideas were sensible. Not only did the curriculum and discussions fail to help Dawn understand the main ideas under study, they did not even enable her to realize what, in particular, she was supposed to be learning.

## Discussion of the Six Portraits

Detailed summaries and analyses of various trends in the data for the six girls and their fellow students are provided in the following chapters. Here, I provide a rough sketch of some of the trends that emerge from the portraits.

The quantity of the girls' participation appeared to be more related to individual differences than SES differences. For example, Guinevere's moods seemed to influence her participation, and Dawn's shyness tended to keep her quiet. But the *ways* in which the girls participated, as well as the ways in which they thought about the purposes of discussions did vary by class. While Guinevere, Samantha and Rebecca valued the opportunity to share and debate opinions in the discussions, Rose, Sue and Dawn focused on giving right answers, but only when certain they were right. Hence, the higher-SES girls occasionally pushed the mathematical discussions forward in bold, insightful ways, while the lower-SES girls' contributions were generally confined to providing answers to my more straight-forward questions. The lower-SES girls revealed in various ways that they were capable of making coherent, logical arguments, but they did not use these talents much in the mathematical discussions.

Hence, the lower-SES girls seemed to focus primarily on contributing right answers, fearing the social consequences if they were incorrect. The higher-SES girls seemed to focus less on right and wrong answers, and they appeared less concerned about being "wrong." The higher-SES girls seemed to better understand the role I was intending to play as a facilitator of discovery and discussion, and they understood my use of "hints." Additionally, when various opinions conflicted, the lower-SES girls did not feel able to discern which ideas made sense, while the higher-SES girls felt more confident they could. While the higher-SES girls thought discussions could be helpful for clarifying their ideas and learning from others' ideas, the lower-SES girls preferred more direct teacher guidance or work in a smaller group.

All six girls complained about the CMP trial curriculum at one time or another, but their complaints differed. While the higher-SES girls made specific complaints about not understanding certain words or particular problems, the lower-SES girls consistently complained about general confusion and frustration — the open problems did not give enough direction, and they wanted to be told how to solve the problems. While Guinevere and Rebecca both said that the open, "word problems" were easier for them than regular number problems, the lower-SES girls said that the CMP trial problems were much harder for them.

Homework completion rates did not differ by SES. Samantha, Rebecca, Rose and Sue all completed over 90% of their homework, while Guinevere and Dawn completed about 80%. But the ways in which their efforts translated into test scores did differ by SES. The higher-SES girls' test scores tended to be in the 90's, while the lower-SES girls' scores were lower. While the lower-SES students would engage with the contextualized problems, they did not always interpret or solve the problem in the ways intended, thereby missing the mathematical point of some problems. For example, although Rose was a very bright student overall, she sometimes used what might be termed "common-sense" reasoning in approaching the problems, thereby missing the abstract, mathematical ideas intended. Hence, while the higher-SES girls said the CMP trial curriculum was merely "different" or even easier than typical mathematics texts, the lower-SES girls agreed they struggled more in the CMP trial curriculum, and they did not enjoy the struggle. Both Rose and Sue were extremely concerned about getting good grades, and it was frustrating for them when their efforts did not consistently result in high test scores and feelings of accomplishment, as they did in a more "drill-and-practice" environment.

Overall, the pedagogy and curriculum seemed to combine to enable the higher-SES girls to feel confident in their abilities to solve mathematics problems and create and share mathematical ideas, with each of them ultimately naming themselves among the top three math students in the class. Meanwhile, the lower-SES girls said the discussions and the curriculum were confusing for them. Even Rose, who was considered by others to be a top math student, and Sue, who said she used to be really good at math, did not seem to feel mathematically empowered.

As mentioned previously, the following chapter contains further discussion of these trends in relation to the data for the whole class. Hopefully the portraits provided in this chapter have prompted the reader's curiosity and concern for how the pedagogy and curriculum played out with the larger class. The next chapter contains more summative data, giving more attention to the overarching trends in the data for all participating students and much less attention to individual students within SES and gender categories. Then Chapter 5 will offer further analyses of the trends, making the argument that the patterns in my data are not likely an isolated case, but arise, at least in part, from the ways in which the class-based cultures of my students align with the culture of the pedagogy and curriculum in my classroom.

# CHAPTER 4

# A LARGER LOOK: DATA FROM THE WHOLE CLASS

The previous chapter provided a look at the experiences and reactions of six girls. I hope I have shown that, although this dissertation's focus tends to highlight differences between SES groups and downplay differences within SES groups, there is obviously great variation within groups as well. In this chapter, I provide some analysis of similarities and differences between the two groups of girls. Additionally, I present analyses of the data involving the larger class. I argue that my analyses of these data suggest that key elements of the curriculum and pedagogy in my classroom were more aligned with my middle-class students' beliefs and preferred ways of thinking, communicating and working. The key elements I focus on are:

- whole-class discussion with teacher as facilitator and students creating, analyzing, defending and validating ideas.
- contextualized, (relatively) open-ended problems students are to explore as a means to learning important mathematical ideas and processes.

Hence, the structure this chapter takes is as follows. I begin by presenting some analysis of the data provided in the previous chapter, and then discuss the broader data of the eighteen students for whom I had permission and SES data. Recall from Chapter Two that I categorized the eighteen students by SES and gender (see Table 4.1).

**Table 4.1**
**Participating Students by SES and Gender**

| Five Higher-SES Males | Four Higher-SES Females | Four Lower-SES Males | Five Lower-SES Females |
|---|---|---|---|
| Benjamin | Samantha | Carl | Rose |
| Timothy | Rebecca | James | Anne |
| Christopher | Guinevere | Nick | Dawn |
| Harrison | Andrea | Mark | Sue |
| Samuel | | | Lynn |

As in the previous chapter, the discussion is organized around students' experiences with and reactions to the whole-class discussions and the problems in the curriculum.

## Whole-Class Discussions

In my teaching, I strove to create a discourse community like that called for in current reform documents. Each day, I tried to involve students in mathematical discussions. NCTM states that opportunities for mathematical communication allow students to clarify their math thinking and more deeply understand ideas. Yet, my analyses of the data raise questions about which students found mathematical discussions empowering.

### Students' Views of Their Participation

When making comparisons across students on specific questions, survey data seem especially useful, since questions are asked of students in the same order and in the same manner. On the final survey I asked students about a variety of their experiences during the year. The six girls' responses to a question about their participation in class discussions are provided in Table 4.2.

**Table 4.2**
**"Do You Participate Much in Class Discussions? Why or Why Not?"**

| Higher-SES Girls | Lower-SES Girls |
|---|---|
| Guinevere: Yes, because I need to get my point across. | Rose: Yes. If I know what I'm talking about. But if I'm confused I just listen. |
| Samantha: Yes, because I want other people to understand my ideas. I like arguing. | Sue: Sometimes, only if I know I've got the right answer. |
| Rebecca: Yes because I do | Dawn: No because I don't like to be wrong in front of a whole group |

This chart serves as somewhat of a summary, albeit incomplete, of these girls' orientations to discussions, and raises some comparisons between the two socio-economic classes of students. While Guinevere, Samantha and Rebecca all said they contribute to the discussions, Rose, Sue and Dawn did not give the same unqualified "yes" to the question. Rose and Sue said they only participate if they are confident in what they have to say. Dawn said she was so afraid of being wrong in front of the whole class that she did not participate much.

The patterns in the data for the six girls are consistent with the larger class. In looking across my students' interview and survey data, I found that seven students consistently said that their lack of confidence in their abilities kept them from wanting to participate in whole group discussions. All of these seven were lower-SES. This included two males and every one of the five lower-SES females who participated in the study. The males explained they "felt awkward" or thought they were not smart enough to participate. The girls said they were afraid of being wrong or that math was too hard. Most of these students did not participate much in discussions, but those who did, such as Rose, said that they only did if they were sure they were right.

Students' Beliefs About the Purpose of Discussion

One possible reason why lower-SES students seemed more fearful of participating in discussions could be that they were treated with less respect by their peers. As discussed previously, Sue had valid reasons for feeling like participating in discussions was risky. When asked if students got their feelings hurt in discussions, six students specifically mentioned Sue. Five of these six students were lower-SES, and the remaining student was Timothy, who took "credit" for doing the ridiculing, blaming Sue for being slow. Even if classmates did not overtly criticize her in class, they would find ways to make her feel less than smart in or outside of class. Examining excerpts from my journal, I noticed a pattern in my classroom in which the lower-SES students, especially the girls, tended to be ridiculed in various ways by other, more powerful players, especially the boys. It was as if, by trying to give power and freedom to students within the classroom, social inequities were reproduced in my classroom, with the least powerful group, the lower-SES females, landing on the bottom. "Why wouldn't this occur?" should be the question, perhaps.

Still, my analyses of the data suggest another possible reason why the lower- and higher-SES students differed in their participation in and their fears about contributing to discussions: Lower- and higher-SES students' views of the purpose of the discussions seemed to differ. Recall that the three higher-SES girls, Guinevere, Samantha, and Rebecca, tended to talk about sharing ideas or making a point, while Rose, Sue, and Dawn tended to talk about being right or wrong. In the larger class, the four students who most strongly said that they like to have their ideas heard in discussions were higher-SES: Guinevere, Samantha, Benjamin and Andrea. Other higher-SES students, even quiet students like Christopher, who was rarely vocal in discussions, seemed to share the view that discussions offered the opportunity to be exposed to different ideas, and that part of their role was to analyze the ideas. Christopher, explained, "I think I learn from them [discussions] because I hear other people's ideas, see how other people think, and compare other ideas and stuff." (Christopher, First Interview)

In analyzing and categorizing students' interview and survey responses, I found that the lower-SES students talked about their role in the discussion as obtaining or giving right answers. When others disagreed with their contributions, the lower-SES students tended to assume their answer was wrong, as opposed to correct and/or simply different.

It took me much analysis to uncover these patterns in my data, and I often found that my students were way ahead of me in terms of having important insights about our class. For example, although I would guess that she was unaware of the class dynamic in what she was saying, Anne made an interesting distinction between who "argues" and who "discusses". By mid-year I had become aware of baggage the word "argue" carried, and in Anne's final interview, I tried to understand her interpretation of the term. I had begun to suspect that some students — especially lower-SES students — thought someone who argued was confrontational, or even mean and upset. For the sake of consistency, I used the word "argue" throughout the year when asking students about participation patterns. Yet, I also probed a bit into students' understandings of this term.

*Who are the people who do the most discussing?*

Samantha, Benjamin, Guinevere, um, sometimes Rose, maybe Sue

*If I ask you "Who are the big arguers?" would you change your answer?*

Yeah.

*So tell me who are the big arguers in the class.*

Benjamin, Samantha, maybe Guinevere.

*Why is that different?*

> Well, some of the people if they get an idea they'll tell it, but they won't really argue about it, they'll just say, oh yeah that's wrong, kind of like me.
>
> *So what does it take to be an arguer in your mind?*
>
> Guts. Cause if I know something is wrong, I don't think there's any use to argue about it.
>
> *What do Benjamin and Samantha do that make you think they argue?*
>
> They, they'll be like no this is wrong, that's wrong because on mine it's so and so . . . .

So for some lower-SES students, such as Anne, "arguing" was something that other, higher-SES students did, because they had the "guts" to believe they were correct and defend their opinions instead of backing down at the first sign of disagreement. As a facilitator, I tried to avoid being *the* authority for knowledge in the classroom. But an issue that students' experiences in my classroom raise is how having my authority be replaced by that of Benjamin and Samantha was any more empowering for my lower-SES students. I will return to this issue later.

Samantha's attitude that being wrong was not a big deal and can be a learning experience was more prevalent among the higher-SES students. Recall, that when she was asked if people get their feelings hurt or feel stupid during discussions, she said that embarrassment, but no permanent damage, might occur.

> Um, I don't think so, I think they just maybe felt embarrassed if they voiced the wrong opinion, after they found out what was right, but they get over it and think, "Oh well, we were wrong" and learn from it. (Second Interview)

So again, Samantha's response exemplifies the trend in students' survey and interview responses: The higher-SES students saw the discussions as an opportunity to try out ideas and to learn from mistakes. Hence, any participation in the discussions can be a positive learning experience, as learning from a variety of perspectives is valuable. But for lower-SES students, if being right indicates intelligence and implies positive participation in the discussions, and being wrong indicates stupidity and implies negative participation, then becoming involved without confident in one's knowledge or understandings is a scary thing. These factors might help us understand why lower-SES students tended to say that their fears of being wrong kept them from participating in discussions.

This is not to say that higher-SES students enjoyed being wrong or were as nonchalant as Samantha about this issue. For example, Benjamin said, "I feel stupid when I'm wrong," and Andrea said, "I get embarrassed and mad when others laugh at my ideas" (Third Survey). Guinevere and Rebecca expressed annoyance at being cut off when trying to say something. Still, none of these students seemed to be silenced by these drawbacks. For example, Benjamin was one of the most active discussants.

So an important difference seems to be that the lower-SES students said their fears kept them from participating in the discussions, while that was not true for the higher-SES students' feelings of embarrassment, anger or annoyance. While my lower-SES students' comments referred to their own inadequacies, my higher-SES students' comments often referred to specific situations and/or others' actions.

From the above comments, we might expect to see drastic differences in the quantity of students' participation by SES. But it is not that simple.

A Look at Students' Participation in Discussions

Thus far, I have discussed students' views about their participation in discussions. Yet, I also compared what students said about their participation with what I observed on audio recordings of those discussions. Therefore, as explained in Chapter Two, in addition to drawing from survey, interview, and journal data to examine students' reactions to discussions, I sampled fourteen days throughout the year — randomly selecting a day from each of the seven units taught, and then also using the day that followed the selected day. I refer to these two sets of seven days as the "first seven days" and the "second seven days." Some of the categories below were developed during the coding of the first seven days, and, therefore, the data reported here are based on only the second seven days of coding for those categories (as is noted in the footnotes). I analyzed each discussion contribution made on those days, rating each one on a number of factors, including the quantity, substance, and tone of students' contributions to discussions.

Quantity of participation. The tallies of the simple numbers of students' contributions revealed several things.[1] Table 4.3 shows the data for the six girls. The data support Dawn's claim that she did not contribute much throughout the year. She spoke only three times across the 14 days, while the average for the other five girls was 26 times, or about two contributions per day. Additionally, we can see that the girls' participation was slow in the beginning, but then increased substantially. The participation for the lower-SES girls increased by

[1] I considered anything a student would say that was part of the mathematical conversation to be a "contribution." When more than one "turn" at speaking was taken because a student was providing further explanation or rationale of his/her idea, I would count the entire exchange as one contribution for that student. For example, if I asked a question and Sue responded, and then another student or I pushed for more of a rationale, and Sue responded again, this would be one contribution for Sue. In order to be considered a contribution to the mathematical conversation, the comment had to be related to what was being discussed, although it did not have to be strictly about the mathematics. For example, when we were discussing statistics of disasters, and Andrea supported a classmate's interpretation of the statistics by sharing a story about her Grandma surviving a hurricane, this would count as a contribution to the discussion. I did tally what I would call "completely irrelevant" comments as well — such as students' questions about my new haircut or what the class in the hallway was doing — but these were not counted as contributions to the discussion.

almost fifty percent, and the participation for the higher-SES girls more than doubled. Additionally, contrary to what one might expect from their comments above, Sue and Rose made almost twice as many contributions as the higher-SES girls (an average of 37 versus 19 across the 14 days).

**Table 4.3**
**Quantity of the Six Girls' Participation**

| Lower-SES Females | Total for 6 days in the 1st part of year | Total for 8 days in the 2nd part of year | Total for the 14 days |
|---|---|---|---|
| *Dawn* | 2 | 1 (1)* | 3 |
| *Sue* | 11 | 20 (15) | 31 |
| *Rose* | 13 | 30 (23) | 43 |
| Total | 26 | 51 (38) | 77 |
| | | | |
| Higher-SES Females | | | |
| *Rebecca* | 6 | 11 (8) | 17 |
| *Samantha* | 6 | 17 (13) | 23 |
| *Guinevere* | 3 | 14 (11) | 17 |
| Total | 15 | 42 (32) | 57 |

*To make the numbers for the six days (before the mid-winter break) and the eight days (after the break) more comparable, I multiplied the number of contributions during the eight days by .75 and placed the results in parentheses.

Table 4.4 summarizes the data for the 18 students. It shows that the total numbers of contributions for the year were remarkably equal among the groups (averaging about 73 contributions, or about 1.5 contributions per student per day) except for lower-SES males, who contributed about half as many times as the other groups. When looking at data for the eighteen students from the first part to the second part of the year, we see that the participation of the Higher-SES males decreased slightly from 40 to 32 contributions. Meanwhile, the participation of the other three groups increased, more than doubling for the higher-SES females and lower-SES males, and increasing by 50% for the lower-SES females.

**Table 4.4**
**Quantity of Participation by SES and Gender**

| Quantity of Participation | Higher-SES Males (5) | Higher-SES Females (4) | Lower-SES Males (4) | Lower-SES Females (5) |
|---|---|---|---|---|
| Total Contributions - Six days before winter break (9/30, 10/1, 10/25, 10/26, 12/3, 12/7)* | 40 | 19 | 9 | 29 |
| Total Contributions - Eight days after winter break (1/17, 1/18, 3/1, 3/2, 4/1, 4/18, 4/19, 4/20)* | 32 | 47 | 23 | 43 |
| Total Participation - Note: To arrive at these totals, I did not use the data adjusted for the difference between the six and eight days. Hence, the totals might appear a bit higher than if one adds the totals for the two charts above. The numbers were still adjusted to represent four students in each category. | 74 | 71 | 34 | 74 |

* The numbers in the columns are adjusted to represent four students for each column for comparison purposes. (I multiplied the actual numbers for the higher-SES males and the lower-SES females by .8, as there were five in these categories and four in the other two categories.) I adjusted down to four students per column instead of adjusting up to five, so as to be conservative and not exaggerate differences. Then, to make the coding of the eight days in the later part of the year comparable with the first six days coded, I multiplied the numbers for part 2 by .75. (Again, I choose to multiply down instead of up to be conservative in making comparisons.)

Hence, these numbers reveal a success story of sorts, as the higher-SES males who seemed to dominate conversations in the beginning of the year were the only group whose participation decreased, while the females and lower-SES males' participation greatly increased.[2] On the other hand, it is crucial to consider the quality of participation and how it varied by group.

Substance of Contributions. Although my analyses tended to focus on differences among gender and SES groups, there were many similarities among groups in terms of what participation usually looked like. The majority of students' participation consisted of giving answers to my (or less often other students') questions, usually in relation to solving problems. (These "answers" to my questions were not always, or even usually, simple answers to a problem — they were often explaining their thinking, etc.) The vast majority (92%) of students' contributions were voluntary. Although I rarely forced students to make contributions, about half of the times I did do so was with lower-SES girls,

[2] I think this reflects my desire to intervene for the sake of equity. It also appears that I was doing less talking and/or was allowing more time for whole-class discussion.

often because I was pushing them to elaborate on a one-word answer they voluntarily provided.

But there were some notable SES and gender differences in the quality of participation, both in the substance and the tone of students' contributions. The following sections focus on the substance of students' contributions including: the general type of contribution — whether asking or answering a question or offering an idea; the problem context; type of language used and proof given; relationship to past learning; level of correctness, difficulty, and insightfulness; and relevancy to the mathematical agenda. Later sections will focus on the tone of students contributions.

Some of these categories, such as if the contribution was a question or assertion or the type of problem context involved, were part of my initial categorization scheme designed to characterize the general substance of the contributions. Yet, as I coded, I continued to refine the categories. and I often noticed that there was something fundamentally different about two students' contributions that I was not capturing. For example, Samantha and Sue might answer a question in a plausible real world context, but the two contributions were very different, and I sought ways in which to capture the distinction — for example, sometimes the differences were due to the type of question they were answering, and other times the distinction was in the form or content of their answers. Hence, to fill the gaps, I added categories, such as the difficulty level of the question being discussed, or the degree of "relevancy" or mathematical "correctness" of the contribution.

*General type of contribution*. Table 4.5 outlines students' contributions by their general type — whether students asked a question, answered one of my questions, or took the initiative to offer their own idea (not in response to a question).[3] The table gives the number of contributions (again, adjusted so that the data in each column represent four students), as well as percents of the total number of contributions for each category. Although I looked for changes between the first part and the second part of the year for this and subsequent coded categories, I saw no strong patterns. Hence, I will summarize the data for the entire year in this and later tables.

**Table 4.5**
**General Type of Contributions by SES and Gender**

| | Higher-SES Males | Higher-SES Females | Lower-SES Males | Lower-SES Females |
|---|---|---|---|---|
| Number of questions asked | 3 - 4% | 8 - 11% | 1 - 3% | 9 - 13% |
| Number of answers to a question | 58 - 78% | 43 - 60% | 32 - 94% | 56 - 74% |
| Number of ideas offered | 13 - 18% | 20 - 29% | 1 - 3% | 9 - 13% |
| TOTAL PARTICIPATION | 74 | 71 | 34 | 74 |

Note: Again, the numbers in the columns were adjusted to represent four students in each category.

As mentioned previously, most contributions in each SES and gender category were answers to a question. This was least true for the higher-SES females (with only 60% of their contributions in the form of an answer to a question) and most true of the lower-SES males' (with answers to a question comprising 94% of their contributions). The females asked more questions than the males, with the females asking 17 of the 21 questions posed. The higher-SES females contributed more ideas than others (20 of the 43 ideas offered), with the higher-SES males in second place (offering 13 of the 43 ideas). The lower-SES students offered less than one-fourth of the ideas.

Additionally, during the second round of coding, I created several categories that allowed me to characterize the general content of students' initial, voluntary contributions (as opposed to what they ended up contributing after

[3] Even when a student would answer another student's question, it would generally also be an answer to my question, since I usually would ask for a response to the question.

follow-up prompting).[4] The categories included whether a student's contribution was primarily about an answer to a problem or a method of solving a problem. The lower-SES students were more likely to contribute around answers they had obtained — one third of their contributions were of this sort, in contrast with 12% for the higher-SES students. Meanwhile, the higher-SES students were twice as likely to share their method of solving a problem (35 versus 18%). (There were several additional categories as well, such as whether contributions related to concepts or to answers or procedures in other ways, but the numbers in these categories were quite small).

*Problem context.* I developed four categories to describe that various problem contexts around which our discussions usually centered: genuine real world, plausible real world, abstract, and fantasy. These categories were discussed briefly in Chapter Two. A "genuine real-world" context is one that involves the real world in some way (such as having students collect water they use when they brush their teeth or asking students to analyze real data about world disasters)[5], as opposed to "plausible real world", which generally involved problems set in hypothetical, realistic situations (such as a problem about students sharing pizzas or starting a bicycle touring business)[6]. If I coded a contribution as referring to an "abstract" context, this means that either the problem was in a purely mathematical context (such as asking students to find

[4] This is the first of several categories that involves data from only the second seven days. On these days, the contributions were distributed as follows:

| Contributions During the Second Set of Seven Days | High-SES Males (5) | High-SES Females (4) | Low-SES Males (4) | Low-SES Females (5) |
|---|---|---|---|---|
| TOTAL CONTRIBUTIONS The numbers shown here for the high-SES males and low-SES females were adjusted to represent only four students. | 32 | 42 | 19 | 48 |

[5] From the unit, Around Us.

[6] From Bits and Pieces, Part II and Variables and Patterns, respectively.

the dimensions of a cylinder with a volume of 1,000 cubic units)[7] or the discussion was focusing on just the mathematics without any context (even if originally the discussion grew out of a contextualized problem). An example of a "fantasy" context would be students drawing enlargements of fictional characters called "Mugwumps."[8]

**Table 4.6**
**Problem Context of Contributions by SES and Gender**

| | Higher-SES Males | Higher-SES Females | Lower-SES Males | Lower-SES Females |
|---|---|---|---|---|
| TOTAL RELEVANT CONTRIBUTIONS | 73 | 70 | 33 | 72 |
| Genuine real world | 6<br>8% | 1<br>1% | 3<br>9% | 5<br>7% |
| Plausible real world | 46<br>63% | 49<br>70% | 21<br>63% | 47<br>65% |
| Abstract context | 18<br>25% | 20<br>29% | 8<br>24% | 12<br>17% |
| Fantasy | 3<br>4% | 0<br>0% | 1<br>3% | 8<br>11% |

As indicated in Table 4.6, there was remarkable consistency in the data tallies in that about 2/3 of the contributions by each category of student related to an plausible real-world context. This reflects the fact that the majority of problems in the CMP curriculum fall into this category. In looking at the percent of contributions relating to abstract contexts, it appears there were no major differences, but the lower-SES females seem to lag behind the others with only 17% of their contributions being in an abstract context, while the other groups ranged from 24% to 29%. What this masks, though, is that 19 of the 20 lower-SES students' contributions in this category were made in the latter part of the year, when we studied integers and operations with integers. The regular classroom teacher taught much of this unit while I was absent, and she did so in a more

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7 From Filling and Wrapping.
8 From the Similarity unit.

typical, rule-based way with the help of the old texts.[9] So even when I returned to the classroom to finish up the unit, students still talked about the rules for computing with integers in the abstract. In other units, when problems began in contexts, and then discussions moved to the abstract mathematical ideas, the lower-SES students generally did not participate in those parts of the discussions. In fantasy contexts, the lower-SES girls contributed the most (still only 11% of their contributions), but the higher-SES females made no contributions.

*Language and proof.*[10] I attempted to learn more about students' mathematical thinking behind their contributions, but this is difficult. I studied the type of language used and rationale or proof given as a rough means of shedding some light on my students' thinking. But, of course, it is difficult to know how a person's thoughts are connected to his/her language.

I coded whether students used generalized or contextualized language. For example, if given a problem about differences in prices between stores, I would record the language of someone who constantly refers to the objects, the dollars, and the stores as contextualized, and the language of one who only refers to the numbers without any contextual attachment as generalized. This is different than rating their reasoning or proof — they could give solid mathematical reasoning in the context or without the context.

While coding, I found that many contributions were not solidly categorizable as generalized or contextualized — instead, they were somewhere in between. In order to avoid exaggerating or masking differences, I put contributions that were not clearly generalized or contextualized in the "in-between" category.

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[9] I missed two weeks of teaching in the spring because of AERA and NCTM.
[10] Again, these data were based on the second seven days coded.

According to the data, roughly two-thirds of each group's contributions were coded as "in-between." Still, there were patterns in how the remaining contributions fell. The lower-SES males (16% of their contributions) and females (23%) used contextualized language more than the higher-SES males (3%) and females (10%). The higher-SES males (38%) and females (24%) used more generalized language than the lower-SES males (11%) and females (6%).

I developed six categories of proof: general proof, proof by pattern, proof by one example, common sense, deference to rule, and what I termed as "normal," again a type of "in between" category . A general proof was what might be considered a typical, deductive proof — an abstract argument that logically derives a general (i.e., not tied to the problem at hand) conclusion from previous conclusions. A proof by pattern involved arguing that something is true because it follows a predictable sequence, while proof by one example was an argument that a general statement is true because it held in one case. A common sense proof involved offering a non-mathematical (at least in the traditional sense) rationale that relates to every-day living, while deference to rules involved arguing that something is true because it was a rule learned previously. A "normal" proof involved giving a general explanation of some sort using the problem context at hand — hence, a proof that was somewhere in between a general proof and a proof by one example.[11] It was often difficult to

[11] To give a specific example, a proof that the volume of a 3-dimensional object, such as a rectangular prism, increases by 8 when the dimensions are doubled could vary considerably. A general proof would be that the volume increases by 8 because there are 3 dimensions doubling and since you multiply the dimensions to get the volume, you have three extra factors of 2, and 2x2x2=8. A proof by pattern might involve looking at how the dimensions of a 1-dimensional object is affected (length is twice as large), and then a 2-dimensional object (area is 2x2=4 times as large), and then predicting what would happen for a 3-dimensional object. Proof by one example would be arguing that the volume increases 8 times because it is true for this particular cube-shaped dog house in the problem at hand. A common sense proof is more difficult to describe in this case, but an example could be someone recalling that the dimensions of her regular Rubic's cube are about double that of her brother's key-chain version, but it seems a lot bigger — eight times as big seems about right. (A better example is when we were finding the "best buy" of

determine if students were using a proof by one example or they were at least attempting to argue more generally. When in doubt, I coded the contribution as "normal." I should note that I did not keep careful track of how many times no argument was given at all. I usually did not let kids "get away" with contributing something without a rationale for it. If other students did not push for an explanation from the contributing student, I would generally do so.

The majority of students' contributions were coded as "normal," with the percentage of each group's contributions in that category ranging from 52% (lower-SES females) to 76% (higher-SES females). The higher-SES students contributed eight of the ten general proofs offered. Proof by pattern was used only once in each category. Proof by one example was used two times, both by lower-SES girls. The lower-SES girls gave seven of the nine common sense proofs. The lower-SES students deferred to rules more often than higher-SES students, providing sixteen of the twenty four instances (about one-fourth of the lower-SES students' contributions were in this category — twice the average of the higher-SES students).

*Correctness, difficulty, and insight.* Although it was difficult, I wanted to capture how "good" the contribution was. If the contribution provided a new

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three popcorn boxes, and Rose said we don't need to bother because it just depends on how much popcorn we want, since the smallest is cheapest and the largest is most expensive.) Deference to a rule would involve someone saying that it is true because they learned it last year. And the type of proof students would normally use would involve talking about how it makes sense that volume increases 8 times because if we explore how it works with this cube-shaped dog house, we see that the sides went from 3, 3, 3, to 6, 6, 6, and this made the volume increase from 3x3x3=27 to 6x6x6=216, and that is 8 times as big; other examples would work the same way, so it should always be true. Others have broken down categories of proof in different ways. For example, as explained by Chazan (1993), Balacheff (1988) distinguishes between "pragmatic proofs" and "conceptual proofs," with the former based on actual examples and the latter involving solely abstract properties. Pragmatic proofs could take the form of either "naive empiricism," or "crucial experiment." What I am referring to as proof by one example would fall under Balacheff's "naive empiricism" category, while a statistical survey or a careful scientific experiment involving many trials would fall under his "crucial experiment" category. Balacheff also noted the existence of a category much like what I call "normal" — that is, his "generic example proof," in which an example is used to more generally represent all objects in its class.

math insight, then it was good, but since so few contributions fit in that category, and because many that did not were still very insightful, I needed to approach the evaluation of the contributions in several ways. First, I could simply judge the "correctness" of each contribution. If an answer was completely mathematically correct, it received a "five." If it was totally wrong, it was rated a "one."

Still, contributions could be completely correct, but that did not mean they were a particularly good contribution — perhaps they were only stating that 1 + 1 = 2. So I also rated the difficulty involved. If the student made a contribution of average difficulty level, it was rated a "three." This meant I would expect every seventh grader to be able to understand the contribution offered, perhaps with some effort. A difficulty level of one meant that I would have expected the contribution offered to be obvious to most seventh graders (i.e., 90% of them). A difficulty level of "five" meant that the contribution involved ideas beyond what I would expect from most seventh graders. A contribution could rate high on the difficulty scale but still be mathematically erroneous.

I wanted to include a measure that would more holistically characterize the insightfulness of a student's contribution. Hence, I included a more subjective "Insight" category. If a student proposed an incorrect, but clever solution to a difficult problem, then the contribution would rank high on the insight scale. On the other hand, if a student offered a correct, but virtually obvious answer to an easy question, the contribution would be ranked rather low on the insight scale. Again, I tried to think of an insightfulness score of "three" as average — a contribution that I would expect most seventh graders to be able to make with some thought, with a "one" indicating virtually no insight (e.g., an incorrect contribution to something that should be well within reach of all seventh graders) and a "five" indicating an incredibly insightful response (e.g., a correct contribution to a really difficult question). Although I did not use the correctness and difficulty numbers in any type of formula to arrive at the insight number, I probably could have developed such a formula. Instead, I ranked it independently, but when I look across columns, the numbers loosely reinforce each other.

**Table 4.7**
**Correctness, Difficulty and Insight of Contributions**

| Degree of correctness | | | | How difficult is problem/idea? | | | | Degree of insightfulness | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Higher-SES Males (5) | Higher-SES Females (4) | Lower-SES Males (4) | Lower-SES Females (5) | Higher-SES Males (5) | Higher-SES Females (4) | Lower-SES Males (4) | Lower-SES Females (5) | Higher-SES Males (5) | Higher-SES Females (4) | Lower-SES Males (4) | Lower-SES Females (5) |
| 4.4 | 4.4 | 4.3 | 3.6 | 3.4 | 3.3 | 2.7 | 2.6 | 3.2 | 3.5 | 2.9 | 2.5 |
| 58 | 64 | 30 | 83 | 39 | 42 | 19 | 61 | 58 | 64 | 30 | 83 |
| Scale: 1 = extremely low 2 = below average 3 = average 4 = above average 5 = extremely high | | | | | | | | | | | |

Note: The categories of correctness and insightfulness developed half-way through the Round One coding. I used eleven days for which I had data in these categories. The difficulty category was developed between Round One and Round Two, so that data are based on seven days of coding.

Table 4.7 indicates that the lower-SES girls lagged behind the other three groups in terms of correctness — an average of 3.6, as opposed to the 4.3 of the lower-SES males, and the 4.4 of the higher-SES males and females. When one considers the infrequency with which lower-SES males participated, perhaps it should not be surprising that they were correct more often than the lower-SES females. The SES differences are more pronounced in the difficulty and insightfulness categories. The higher-SES students scored .3 (females) to .4 (males) points above average, for the difficulty level, while the lower-SES students scored .3 (males) to .4 (females) points below average. Similarly, the higher-SES students scored higher on the insightfulness category, with the higher-SES females being the most insightful (3.5) and the lower-SES females being the least (2.5).[12]

[12] In thinking about "How big are these differences?" I should note that I found myself rarely using some of the ends of the scales. In the "correctness" category, I often used the five but I rarely used numbers lower than a two or three, as I could usually construe something correct in

As I coded the second set of seven days, I looked at these categories in relation to the general content of contribution, so that I could explore possible interactions. Through this analysis, I found that the lower-SES females were really very correct (4.6) when giving answers they had gotten for problems — it was in the more meaning-oriented categories, such as discussing methods or concepts, that they scored low on the correctness scale.

A relatively high percentage of the lower-SES students' responses involved giving answers they had gotten for problems, and this was where I tended to rank the contributions as below average on the difficulty and insightfulness scales.

*Relationship to past learning.*[13] I used three categories to characterize how a student's contribution was related to what was learned in the past. First, the student might simply recall something that was learned in the past. In order for me to code a contribution in this way, I would have to be certain the student actually had learned it in the past (not just that it was taught). An example would be a student saying that last year she or he learned that the median of a data set is the middle number. Second, students might put some ideas together in a new way and offer what I termed a "new mathematical insight" — a generalized (as opposed to being particular to the problem at hand), substantial, mathematical idea that was new for the class.[14] For example, when several problems were geared toward helping students discover how to find the volume

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what the child was saying. For the difficulty and insightfulness categories, I used the one and the five only a few times. Hence, most of the answers were concentrated in the two to four range, and, therefore, a difference between 2.5 and 3.4 is substantial. I kept thinking that I should have been more easily convinced of a one or a five in order to have differences be clearer, but I could not bring myself to call things one's or five's very often. The same can be said for insightfulness — I very rarely used one's or five's.

[13] Again, these data were based on the second seven days coded.

[14] Of course, what is "substantial" is a judgment call. Each of the CMP units were designed to teach several mathematical ideas and processes, and I used these as a guide to making the judgment.

of a cylinder, and a student would finally break through with the formula, I would call that a new mathematical insight. Finally, most contributions took the form of applying ideas learned previously to the problem at hand.[15]

**Table 4.8**
**Relationship of Contribution to Past Learning**

| **Round 2** | High-SES Males (5) | High-SES Females (4) | Low-SES Males (4) | Low-SES Females (5) |
|---|---|---|---|---|
| TOTAL RELEVANT CONTRIBUTIONS | 32 | 42 | 19 | 48 |
| Recall what was learned in past | 2<br>6% | 3<br>7% | 5<br>26% | 5<br>10% |
| Applying to world/lives | 0 | 0 | 0 | 0 |
| New mathematical insight | 2<br>6% | 2<br>5% | 0 | 0 |
| Normal - applying past to new problem | 28<br>88% | 37<br>88% | 14<br>74% | 43<br>90% |

According to the data presented in Table 4.8, most contributions (74% - 90% for each group) came from students applying previous knowledge to a new problem under study. Ten of the fifteen instances of recalling something from the past involved lower-SES students, with the lower-SES males contributing half of the ten, which was a much larger portion (26%) of their total contributions than the lower-SES girls (only 10%, but still higher than the higher-SES students). There were only four contributions that I coded as a "new mathematical insight," and all were made by the higher-SES students (divided evenly between the males and females).

*Relevancy to mathematical agenda.*[16] I recorded a relevancy code for a contribution only if it was less than what I could construe as relevant to a mathematical agenda and the contribution was made during a mathematical

[15] I realize that there are other possibilities, but the contributions seemed to fall rather nicely into these three categories. For example, I thought some contributions might take the form of applying a previously learned idea to one's own life, but none of the contributions I coded seemed to fit this category.

[16] Again, these data were only collected during the second seven days of coding.

discussion (and made in such a way that I thought there was any reason to believe the student was actually trying to participate in the discussion, as opposed to making an under-the-breath joke, for example). I coded the responses using "one" through "four," assuming "five" was completely relevant. I tried to conceive of "agenda" broadly. Hence, even if it was not relevant to *my* agenda, if it was mathematically relevant, I would count it as relevant to some extent. When contributions were not made in the context of a mathematical discussion (e.g., a question about what the homework assignment is at the end of class), or when contributions were made during discussions that were clearly not intended to be part of the mathematical discussion (e.g., a joke about what is written on someone's shirt), then I coded this as "Non-math participation."

As examples, a question about a picture in the book (not a mathematical question and not a picture directly related to the problem at hand) would be coded as totally irrelevant (a "one" on the scale). A story about Grandma's house being hit by a tornado when discussing a math problem about a tornado would be ranked as a "two" (at least the contribution shows she/he is engaging in the problem at hand to some extent, but the question of whether there is mathematical thinking there is open). Contributions that were somehow mathematical in nature but unrelated to the mathematics at hand would generally be ranked a "three" or "four" (e.g., a comment about bike prices in a story problem being unrealistic when the mathematics at hand is graphing speed and distance). Whether a contribution was mathematically correct was irrelevant to its relevancy to the mathematical agenda.

**Table 4.9**
**Relevancy of Contributions to the Mathematical Agenda**

| | High-SES Males (5) | High-SES Females (4) | Low-SES Males (4) | Low-SES Females (5) |
|---|---|---|---|---|
| TOTAL CONTRIBUTIONS | 32 | 42 | 19 | 48 |
| Relevancy to math agenda (mark if <5) | 5 - 3s<br>1 - 2s | 1 - 4s<br>2 - 3s<br>1 - 2 | 1 - 4<br>1 - 3<br>1 - 2<br>1 - 1 | 8 - 4s<br>5 - 3s<br>3 - 2s<br>1 - 1 |
| Total contributions irrelevant to some degree | 6<br>19% | 4<br>10% | 4<br>21% | 17<br>35% |
| Scale: 1 = Totally irrelevant to discussion; 2 = Primarily mathematically irrelevant; 3 = Somewhat mathematically relevant; 4 = Primarily mathematically relevant; 5 = Totally relevant | | | | |

As Table 4.9 indicates, I recorded 31 of the 141 contributions made as irrelevant to some degree. For all groups, the irrelevant contributions tended to be coded as three or four, with only two "ones" recorded (both for lower-SES students). Only 10% of the higher-SES female contributions were coded as irrelevant to some extent, in contrast with 35% of the lower-SES females' contributions. The males' hovered in between the two groups of females, with about 20% of their contributions coded as irrelevant.

As far as non-math participation, the lower-SES girls tried to clarify expectations more (e.g., procedural questions related to homework assignments), and the lower-SES students asked more grade-related questions (e.g., "Why did I get six points for this and he got seven?"). But the numbers were very low. The males made ten out of eleven jokes.

Mode of contribution. In addition to examining SES-related differences in the quantity and substance of students' participation, I also considered data regarding the way in which contributions were given, including the confidence with which a contribution was given, how well it was articulated, and how the student reacted to challenges.

*Confidence and articulation.* The confidence with which a contribution is given and how articulate the student is while giving it are difficult to code. In order to avoid reading more into a student's tone than I should, I recorded only extreme cases in which confidence and articulation stood out as being clearly above or below average.

As clues for how confident students were in making the contribution, I listened for tone of voice, hesitancy in responding, relation to social context (e.g., trailing off when challenged versus interrupting others to correct them), and words used (such as "I'm not sure" or "obviously").

I recorded ten contributions as being made in a very confident manner. All ten of these contributions were from higher-SES students, split evenly between the males and females. The four contributions that I coded as being made with a clear lack of confidence were from lower-SES students — three from females and one from a male.

In terms of articulation, a student was able to state a complex idea clearly and concisely, the contribution was ranked as positive on the articulation scale. If a student had difficulty putting her/his idea into words clearly and therefore stumbled through making the contribution, that was an indication of negative articulation. Again, I only recorded contributions that were very clear examples of above- or below-average articulation.

In comparison to the confidence data, the data for articulation were slightly more mixed, and the numbers were small (only fourteen instances of clearly positive or negative articulation recorded). The five instances of negative articulation I recorded were fairly evenly divided among the lower-SES girls, the higher-SES males, and the higher-SES females. Yet eight of the nine instances of positive articulation involved higher-SES students — four males and four females.

*Reaction to challenges.* When a student's contribution was questioned or challenged, I coded the student's reaction to the challenge — whether the student backed down, clarified, or defended his or her contribution. Backing down means that a student, when challenged, would not defend her answer, but instead give in to the challenger. A typical scenario of backing down went like this:

Anne: I got 6, because I multiplied the 2 and the 3.

Benjamin: It's 8, because it's 2x2x2, or 2 to the third power, since you multiply the three twos . . . .

STL: Anne, what do you think about that?

Anne I don't know, I guess he's right.

The difference between "clarify" and "defend" is difficult in some cases, but "clarify" indicates the student tried to explain what was meant more clearly, and "defend" means the student argued that her or his interpretation was sensible.

**Table 4.10**
**Reactions when Contribution was Questioned**

| | Higher-SES Males (5) | Higher-SES Females (4) | Lower-SES Males (4) | Lower-SES Females (5) |
|---|---|---|---|---|
| TOTAL RELEVANT CONTRIBUTIONS | 32 | 42 | 19 | 48 |
| Back down when student questions | 0 | 0 | 2 | 2 |
| Clarify when student questions | 1 | 1 | 0 | 0 |
| Defend when student questions | 1 | 2 | 0 | 0 |
| Back down when teacher questions | 1 | 2 | 2 | 9 |
| Clarify when teacher questions | 5 | 9 | 0 | 4 |
| Defend when teacher questions | 3 | 1 | 0 | 0 |

Table 4.10 indicates that the lower-SES students were more likely to back down when questioned or challenged, and the higher-SES students were more likely to clarify or defend their views, even when challenged by me. More

specifically, the lower-SES students tended to back down when questioned by other students (four times) or me (eleven times); they never defended their answers, but clarified four times in response to my question or challenge. The higher-SES students never backed down when questioned by other students, and rarely backed down (three times) when I questioned them. They tended to clarify their ideas (two times) or defend them when a student argued against them (three times), and, likewise, to clarify (fourteen times) or defend (four times) when I questioned them. The higher-SES boys defended more and the higher-SES girls clarified more when I would question them.

I also looked at disagreements among students, examining who disagreed with whom. Most of the disagreement came from higher-SES students, but the numbers were small. Additionally, the lower-SES females were the only group to be "helped" by other students.

Summary of the participation coding. This compilation of data from my analysis of students' contributions is, perhaps, a bit overwhelming in the multitude of categories and findings. The main trends can be summarized as follows:

- The higher-SES males dominated the class in the beginning of the year. In the second half of the year, both female groups made more contributions than the higher-SES males. Overall, the higher-SES males and females and the lower-SES females made roughly the same number of contributions over the fourteen days. The Lower-SES males contributed the least — less than half as much as the other groups.

- The higher-SES students gave more method-oriented contributions, while the lower-SES students gave more answer-oriented contributions — especially reporting the answers they had gotten for problems.

- The lower-SES students were more likely to use contextualized language, common sense reasoning, and to defer to rules as proof. The higher-SES students were more likely to contribute in relation to abstract contexts and to use generalized language and proof. Still, most contributions consisted of something in between.

- The lower-SES females' contributions were least correct, least insightful, and least relevant. Lower-SES students contributed around the easiest questions. The higher-SES students were most correct, confident. The higher-SES females' contributions were the most insightful and most relevant.

- The Lower-SES students tended to be less confident, and they were more likely to back down when questioned or challenged. The higher-SES students were more likely to clarify (especially females) or defend (especially males) their views. The lower-SES females were "helped" more by other students.

Each individual finding is not, in itself, terribly convincing of anything, since the numbers are low in many cases, and because with the multitude of categories involved, one would expect to find some SES and gender differences. But taken altogether, the individual findings convey messages consistent with those that emerged from my analyses of the interview/survey data with respect to fears of being wrong in discussions and students' beliefs about the purpose of discussions: The lower-SES students appear to be less confident and tend to participate in ways that maximize their "rightness" and minimize confrontational situations. They contribute more by offering answers to problems they have already worked out, instead of sharing ideas about methods or concepts. The higher-SES students were more likely to share their ideas and methods of solving problems. They were also more likely to talk and reason in a generalized way about the mathematical ideas. Issues about generalization and contextualization will be discussed further in relation to students' experiences with the curriculum.

Up to this point, we have looked at data about students' active participation in the discussion. Yet, students can participate in discussions without ever saying a word. And although the participation between two groups of students might be different, it is possible they both learn what they need to learn, but in different ways. So instead of solely focusing on the question, "What are students contributing to discussions?" it is probably more important to explore the question, "What are students taking away from discussion?" While one can code and tally contributions to get at the former question, it is more difficult to get at the latter. To address it, I will rely heavily on students' survey and interview responses to questions about the sense they made of the discussions.

## Making Sense of the Discussion

Understanding my role in the discussion. I tried to facilitate discussions in a non-authoritarian way. Hence, I tried to avoid being *the* person who always decides if answers are right or wrong or if methods are sensible. I continually faced dilemmas about how much authority to assert and in what ways.

The lower-SES students seemed to prefer a more teacher-directed style: The five lower-SES girls and one lower-SES boy consistently voiced a strong preference to learn with teacher direction (as opposed to learning alone or through whole class or group discussions without strong teacher direction — most said at some point that they wished I would just "show how to do it" or "tell the answer"). Samantha and Rebecca also voiced their appreciation for teacher direction at one time or another, but evidence was more contradictory for them.

Toward the end of the year, I came to realize that I had been assuming that students understood why I facilitate classroom discussions the way I do. I had talked with them at times about wanting them to learn to think for themselves and figure things out on their own. But I do not think I ever specifically explained how the type of discussions I tried to establish contributed to that agenda — I thought it was obvious. But I found that it was more obvious to the higher-SES students than the lower-SES students.

Some lower-SES students thought I refrained from being the judge of students' ideas because I "didn't want to hurt their feelings," as Dawn stated. Some of the higher-SES students — such as Samantha and Guinevere — offered insight into my role in the discussions that helped me realize that one way I resolved dilemmas about how to guide discussion (in situations when the students were not going in fruitful directions on their own) without becoming the main authority for knowledge in the classroom was to give "clues" or "hints." Since no lower-SES students mentioned my use of "hints" or "clues," I worry that these "hints" were only guiding those students — mostly middle-class — who understood what I was trying to do. Still, one lower-SES girl, Lynn, gave evidence that she understood my use of questioning strategies:

> *How does the class, or how do we figure out which ways are right and wrong?*
>
> Well, you don't really say who is wrong, but you put all the ideas up there, and then you work with them, and I think you try to ask people questions and they figure out that they're wrong or something like that, like you just keep asking questions - you don't come out and say that's right and that's not right
>
> *Why do you think I don't just say you're right and you're wrong?*
>
> So we think about it more.
>
> *But does that add to the confusion?*
>
> *(I asked her this because she had been talking about her confusion due to her perceived inability to decide which ideas are right or wrong.)*
>
> Yeah, a little bit, cause you have all these different ideas to think about and stuff.

But Lynn was the only lower-SES student who gave strong evidence that she understood what I was trying to do in the discussions (although I did not have the opportunity to ask every lower-SES student these questions). Although

Lynn seemed to understand my role and intent, she was still confused. This seems especially important, because it shows that a different understanding about the purposes and structure of the discussions is not the only factor contributing to students' confusion. Students must also have the confidence needed to participate in the discussions in the ways the reformers and I intend. They need confidence to make contributions to the discussions, as well as confidence to make sense of others' ideas.

Confusion in discussions. As Lynn indicated, some students found the discussions confusing. Seven lower-SES (five girls and two boys) students consistently said that having so many ideas being tossed around confused them. In general, the confusion centered around hearing so many ideas and not feeling able to discern which ideas made sense. Recall Sue saying that she learns better from the teacher instead of the whole group because the variety of opinions confused her. Other lower-SES students said similar things.

> Sometimes they confuse me, if I think I know something and somebody else says something, then I totally forget what I was really supposed to do. We just talk about why theirs was wrong — that confused me.
> (Anne, Final Interview)

> We're like OK this is a rule, and the rule I got I think this works, but it might work on one problem but not on all the other ones, so I get mixed up. I want to use MY rule. . . sometimes people say things that aren't true, like wrong ideas, and I get those stuck in my head, and I have all these different ideas going and it's confusing because I don't know which one is right until at the end and I think that's sort of confusing.
> (Lynn, Final Interview)

> Sometimes I don't know what they're talking about, like Benjamin will say something to me and I wouldn't know what he's talking about.
> (Nick, Final Interview)

This stands in stark contrast to many of the higher-SES students, who felt confident in their abilities to decide who is right. Recall Rebecca's confident claim that through discussions, she "can know everybody's opinions and take it into consideration . . . and I can figure out which ones are true and which ones aren't" (Final Interview).

I must note that two higher-SES girls and one higher-SES boy said on one occasion that *sometimes* the discussions confused them. These statements were heavily qualified. For example, Samantha was one of these two, and she pointed to the discussion with Mark as her example, which she also called a memorable, "good discussion." So for Samantha, confusion was not necessarily a bad thing — for her, it seemed to make a discussion interesting.

Enjoyment of discussions. I cannot say that my lower-SES students did not like to participate in the discussions, and my higher-SES students did. It is not that simple. When I would ask students which mode of working they preferred — alone, small group, or whole group discussions — students' answers often varied throughout the year, and there were no strong SES patterns.

Only one (higher-SES girl) used the word "fun" to describe discussions, along with four males (two higher-SES and two-lower-SES). For the males, especially, there was often an element of mathematical "machismo" associated with saying that "arguing" was fun. For example, Mark said, "It's fun trying to prove others wrong," (Final Survey). All five of these students were what I would call "high ability" students. Yet, it is curious that the high-ability lower-SES girls did not think discussions (or arguing, in particular) were fun.

Also, there were a few students, such as Guinevere, who referred to being bored in discussions, but this did not seem to have as much to do with SES as with achievement.

Helpfulness of discussions. But for the majority of lower-SES students, the confusion seemed to make the discussions disempowering, and the seven students who explained that discussions were generally helpful to them were higher-SES (with the exception of Rodney, an African-American student whom I had trouble categorizing). I should note that in looking closely at these students' responses, the lowest-achieving higher-SES male seemed to view it less in this light than others. Additionally, although these seven included every one of the six higher-SES students (three boys and three girls) I followed closely all year, not all of these six said they generally enjoyed the discussions — two of the higher-SES females expressed negative feelings about the discussions. But these seven students seemed to share a view of what the discussions were about, as well as confidence to participate constructively in ways I intended. For them, the discussions in our class allowed them to share their ideas and to learn from others, and they found this opportunity helpful.

Discussion of Data Regarding Whole-Class Discussions

Whole-class discussions were a major part of our class and my pedagogy. I intended to facilitate these discussions in a way that encouraged students to create, share, analyze and validate mathematical ideas.

My intentions seemed to be more aligned with the expectations of higher-SES students, such as Samantha, who had the beliefs, habits and skills that allowed her to fully participate in and benefit from the discussions. These higher-SES students tended to view the discussions as a place to share interesting ideas and learn from each other as mathematical ideas were discussed and debated, more often using generalized, mathematical language and arguments. The lower-SES students seemed to hold a different view of discussions. Instead of a place for sharing and understanding ideas, they talked primarily of right and wrong answers. These students' fears of being wrong seemed to restrict their participation to answering "easier" questions when they were sure they had "the right answer." If opposition appeared, they would back down, assuming they were wrong, as opposed to thinking they might be right or that there might be more to the issue than right/wrong.

The higher-SES students seemed to share my beliefs about discussions as a place to share, play with and debate ideas. These students also seemed to have skills and habits in relation to discussing ideas that allowed them to delve into discussions as both listeners and contributors. In the mathematical discussions, the lower-SES students were more likely to stay close to the context in discussing the ideas, using more contextualized language and reasoning. The higher-SES students' seemed more oriented toward pulling back from the context in both language and thought, and abstracting the intended, key mathematical ideas from the contextualized problems. (These issues of contextualization will be discussed further in the following section.)

Because the higher-SES students were more likely to share my beliefs about the purpose and our roles in the discussion, and they had the knowledge and skills needed to fulfill their role, they were more likely to make relevant contributions that pushed our learning of important mathematical ideas forward. While more lower-SES students became confused by the variety of ideas offered in the discussions, the higher-SES students had knowledge, skills and beliefs that gave them confidence in discerning which ideas were sensible. Therefore, the variety of ideas offered in the discussions could be considered interesting and helpful, as opposed to confusing and overwhelming.

Gender seemed to interact in some ways with SES. While the higher-SES males dominated the class, particularly in the beginning of the year, the lower-SES males contributed only half as much as the other groups. But on measures other than quantity of participation, the females were sometimes on the extremes of the scales, with the males in the middle. For example, higher-SES females' contributions were the most relevant and the most insightful. The lower-SES females' contributions were the least correct, the least relevant, the least insightful. Yet, the females shared some things in common. For example, both groups came out of their shells and participated more often as the year unfolded. They also asked more questions than the males and participated in less confrontational ways (recall that the higher-SES females would clarify their ideas, while the higher-SES males defended them). Hence, there are ways in which the females occupied traditional roles, wanting to please and not anger others. The lower-SES females, particularly, seemed to participate in order to get "credit" for giving the right answer. The males seemed to be more relaxed or apathetic about their participation in class in some ways, making more jokes and the lower-SES males not participating much at all.

These SES and gender trends in students' discussion participation are supported in some ways by the trends in students' experiences with and reactions to the curriculum. I now discuss the trends I found in the data regarding curriculum, after which I discuss what they, when combined with the trends discussed above, convey about my students' experiences in my classroom.

## Curriculum

During the year, I enjoyed watching some successes of the CMP trial curriculum unfold, such as seeing even the most seemingly apathetic students become actively involved in various problem explorations. But still, SES patterns in the various data relating to students' experiences with the curriculum convey some stories that seem less successful.

The students had the CMP trial curriculum in sixth grade, so they were somewhat accustomed to it when I began teaching them. But it was still a huge change from previous years, so there was general complaining about all the reading, the lack of specific directions, etc. Certainly, doing the CMP problems was quite different than doing a worksheet of 50 problems that all basically follow the example at the top.

In the beginning of the year, complaining about the curriculum seemed to be the cool thing to do. It was sometimes difficult to sort out the complaints for the sake of getting attention from me or peers versus genuine struggles with the curriculum. But by the end of the year, I could look across many forms of data for each student and try to make sense of the patterns. In looking closely at students' complaints about the curriculum, I saw SES patterns in who preferred the CMP curriculum and who preferred the "old books." In examining students' explanations for their preferences, I noticed many lower-SES students said they often struggled with the curriculum because it was "too confusing." In delving into students explanations' further, I saw evidence relating to which students seemed to want or need more external direction, and which students seemed more internally motivated. Still, despite students' feelings of frustration, if they were all learning more important mathematics things equally effectively with the CMP trial curriculum than with a more typical one, then the concerns about students' feelings would seem less important. Hence, I also sought to understand what students thought they were learning. Additionally, through my analysis of data, I uncovered something the students had not discussed with me directly, but I have come to view as important: differences in students thinking and learning through contextualized problems. In the following sections, I discuss SES-related trends related to who liked the curriculum, who struggled with it, who was motivated by it, what students said they learned from it, and what I thought they might be learning from it.

General Like/Dislike

The survey and interview evidence relating to students' curricular preferences indicated that higher-SES students were more likely to prefer the CMP trial materials over the "big textbooks" they had before CMP. The four students who quite consistently expressed preference for CMP math (as used this year) over typical math were all higher-SES: three girls (including Samantha and Rebecca) and one boy. One higher-SES male and one-lower-SES male also seemed rather positive about CMP, especially in the beginning of the year. Four of the six people who consistently said they preferred typical math to CMP were lower-SES, including Sue and Dawn and two males. Guinevere was one of two higher-SES students who preferred the old curriculum.

But whether or not a student likes a curriculum does not tell us why the feeling exists or what a student learns from it. The CMP asks students to think harder and differently than before, so it is not surprising that some students reacted negatively to the new expectations. But if there are SES patterns in how students react and how they benefit from the curriculum, then this might give cause for concern. Hence, in the following sections, I attempt to further explore students' experiences with the curriculum.

Confusion/Struggles

As a brief recap of how the six girls viewed the CMP trial curriculum, Table 4.11 presents their responses when asked, "What else would you like to tell the (CMP) authors?" (from the mid-year CMP Survey).

**Table 4.11**
**Messages for CMP Authors from the Six Girls**

| NAME | What else would you like to tell the authors? |
|---|---|
| Guinevere | You should not just give examples, but write instructions on how to do the problems, because if your parent wants to help you and you don't quite remember the process your parent can help. |
| Rebecca | This kind of math is more fun and easier than the kind of math we used to do. Take the bike tours out! |
| Samantha | To explain the questions better* Don't use big words that we kids won't understand.<br>*These books are better than the old.<br>*(Samantha made this asterisked note on her survey)* |
| Rose | These books are bad because they are so confusing. We are told to do a page as homework and the page gives directions but it doesn't explain how to do it. These books should be taken off the market. No one likes them. They're boring. |
| Sue | Explain better, books confusing, questions are too long and complicated, fake |
| Dawn | I don't like this math book because it doesn't explain EXACTLY! |

Rebecca and Samantha gave specific suggestions for improvement, but made clear they preferred the CMP trial curriculum, with Rebecca even saying that CMP math is easier. Guinevere suggested giving more explanation of the process for solving problems, in case it was forgotten and parents wanted to help. The three lower-SES girls made their negative feelings quite clear, and attributed their feelings to confusion about not understanding or knowing what to do with the problems. These responses are consistent with the data from the larger class.

At one time or another, most students complained about various aspects of the curriculum, but their complaints were not all the same. The predominant theme among the lower-SES students was that the curriculum was too confusing or hard (this was a theme for nine out of the ten lower-SES students). Moreover, six lower-SES students specifically said they were better at math the old way. For example, recall Sue's comment "I used to do really good in math" (Final Survey). Dawn said she was better at math the old way because it was "not as confusing." Rose consistently said the CMP is harder because of the lack of specific direction given for completing the problems (although, unlike most of her lower-SES peers, she also expressed some positive feelings about the curriculum, especially toward the end of the year).

The idea that the CMP was harder and they were not as good at CMP math was reiterated many times by other lower-SES students, such as Lynn, who said, "I'm better at number problems than problem solving," and James, who said, "I'm worse (now), 'cause I used to could do the work, but here I don't understand it."

When pushed to explain what exactly was hard or confusing, the lower-SES students would primarily talk about not being able to figure out what they are supposed to do with the problems in the CMP trial materials, and they blamed this on the words and general sentence structures used, as well as the lack of specific directions for how to solve the problems. For example, in her third interview, Lynn said, "Why don't they word it like you say it? . . . . When I figure out what I'm supposed to do, I can do it, it just takes me longer."

Only one higher-SES student — Timothy — said he was better at math the old way. Timothy said "I was better in fifth grade. You're packing it all in too fast, we start things and drop it . . . . " Yet, Timothy was one of the most inconsistent in his complaints and was known by his teachers as a professional complainer. At other times he complained that CMP math was too boring, that nothing was new to him, and there was too much homework.

When higher-SES students would complain about the books being confusing, it was usually toward the beginning of the year (when it was cool to complain about this) and usually not as passionate or personalized, often offering a specific suggestion or pointing to one specific problem or word that was unclear (e.g., Guinevere's suggestion to write a glossary or Samantha saying that before she knew what "corresponding" meant, that she was confused). Hence, their complaints seem quite different from the lower-SES students, who conveyed the feeling of having no idea how to proceed on the problems. While no lower-SES students said that the CMP curriculum was easier for them than typical curricula, several higher-SES students made comments about CMP problems being easier for them than computation. Recall Guinevere's explanation of why the CMP problems were "a lot easier" for her: "I guess our family's just — we are word problem kind of people" (Second Interview). Also, Rebecca stated, "CMP is easier. I'm not good at like multiplying and stuff, and this year we get to use our calculators" (Second Interview). Two higher-SES boys said similar things. Christopher explained why CMP is easier for him: "I'm pretty good at problem solving" (Second Interview). Benjamin encouraged the CMP authors to make their problems "more challenging" (Second CMP Survey).

The higher-SES students who did not say CMP is easier tended to say that comparing the difficulty level of CMP and typical math is like comparing apples and oranges — they are just different. Again, many higher-SES students made comments about the need to make some wording in the books clearer, but this did not seem to translate into overall confusion or frustration with the curriculum.

By the end of the year, no higher-SES students were complaining about confusion (although some were complaining about other things, such as being tired of particular contexts), but seven of the nine lower-SES students still said they were confused: Sue, Dawn, Rose, Lynn, James, Rodney and Nick (these students showed quite a bit of consistency across the year).

I suspected that part of the students' frustration might be due to feeling less than confident with the mathematical ideas assumed by the curriculum. But to my surprise, on a written survey, most students said that the material covered in class is *not* mostly new to them. James, who had just transferred in from another school, said that all of the material was new. Nick, Lynn, and Adam said that most of it was new. Everyone else said that at least half of it was familiar. So it seems that from most students' point of view, if they were struggling, it was not the *mathematics* they felt they were struggling with, but instead they felt confused by the wording of the problems or the in-class discussions. These comments surprised me, since I assumed the students thought they were struggling with the mathematics much of the time. Still, there is some evidence to suggest that the lower-SES students might have had difficulty in judging how much of the content was new, since they were sometimes unsure which ideas we were focusing on. Also, they might have believed they understood some things, such as "fractions," since they were successful with learning about them in other contexts; yet, they might have only learned how to reduce fractions with a rote procedure instead of having a conceptual understanding of "fractions." Hence, students' responses to the question of how much material was new, indicate their perceptions of what they struggled with, but their comments do not necessarily accurately reflect what their actual struggles were. This is particularly true, because part of the intended "content" of the CMP curriculum (and my pedagogy) was learning to take initiative in solving open problems. Hence, the lower-SES students struggled with processes that the CMP considered part of the mathematical material. Therefore, the students were struggling with some of the mathematical material without realizing what they struggled to learn was part of the intended goals of the curriculum.

Direction/Motivation

As one might guess from their comments about wanting clearer directions in the CMP trial materials, the lower-SES students seemed to want more direction from me about how to solve the problems. This was evident in surveys, when I asked students to rank various modes of working on problems — the lower-SES students, especially the girls, ranked having specific teacher direction higher. Those students who said they prefer to have the teacher "tell them the rules" were all lower-SES (with the exception of Rodney, an African-American boy with mixed-SES). Three of these lower-SES students said it confused them to try and explore things, and two of them said that being told rules allowed them more time to work with them and understand them. Also, I noted in my fieldnotes on several occasions that lower-SES students, especially the girls, wanted to know "the rule," but they did not strive to understand why the rule worked. For example, on March 31, I wrote:

> We discussed #2 for almost 1/2 hour, and the kids were really confused and hostile. I would say that only Samantha and Benjamin were getting it. Rose asked questions like — will that always work? — i.e., for +7 - + 5 = + 7 + -5. She really seems to still seek algorithms — I THINK THIS IS IMPORTANT — some kids, like Rose and Lynn, and Sue, want to find an algorithm for everything because they really want to do well. They don't necessarily want to understand it! . . . . I wonder if culturally, there is a bit of pragmatism or efficiency or survival that plays into this mentality.

When the lower-SES students talked about what made me a good teacher, they focused on my ability to explain things well, particularly the CMP questions. The higher-SES students tended to make other comments, such as Andrea, who said that I was not too strict, and Benjamin, who said, "She doesn't give answers, but helps and is nice." The lower-SES students — especially girls — tended to ask me, "Is this right?" more often.

Additionally, the higher-SES students tended to show more intrinsic motivation to delve into the problems. The four students who said that they like to figure things out and really understand ideas were higher-SES. While there are several examples in my fieldnotes of higher-SES students showing intellectual curiosity and excitement about challenging mathematical problems, there are no such examples for lower-SES students. For example, Benjamin once became totally engaged in a context-free problem that asked him to find the dimensions of a cylinder with a volume of 1,000 cubic units. Although he asked me a few questions as he excitedly used various methods to get closer and closer to 1,000, he made it clear he did not want me to tell him too much. When finished, he proudly said that this was the hardest thing he had ever done. Christopher also became very engaged in that problem and created an extension of the problem on his own, which he showed me the following day (Journal, 12/16). This is not to say that the lower-SES students did not ever engage with or enjoy the problems, but that they seemed to be motivated more by the activities involving fun, games, and contexts of interest to them (such as sports for Nick or dream houses for Rose and Sue). But in these cases, it was not the mathematics that tended to draw them in or receive focus (I write more about this later under the heading "contextualization").

The lower-SES students seemed to be more concerned about getting their work done and arriving at an answer than really understanding the ideas underlying the problems. For example, there were several examples in my journal of Rose doing nonsensical computations with numbers to arrive at an answer (e.g., Journal, 4/20; 4/21). Also, tapes of her during group work show that she and the lower-SES students in her group tried to hurry through problems to get them over with but did not want to appear to be finished, for fear I might push them to think more or that I would begin discussions sooner (e.g., Journal, 12/14). When I asked them to stop working before they were finished (because they had been discussing other things), they would scramble for an answer, saying "What are we going to put for this one?" (e.g., Journal, 11/24).

When stuck, the lower-SES students seemed more passive. On the various surveys and interviews, lower-SES students tended to say they would become frustrated and give up when stuck, or they would wait to ask a teacher, friend, or family member. The higher-SES students also mentioned asking friends and family members, but they were more likely to say they would think harder about the problem or just interpret it in a sensible way and get on with it. For example, Christopher (First Interview) explained his reaction to confusing problems as follows:

> I just, like, try to figure out what I was trying to do. And just go and do what I think it's trying to say.
>
> *Do you get afraid you are doing the wrong thing? How come you have enough confidence to do what you think makes sense.*
>
> Well, I just try my best to figure out the directions, so If I get it wrong it's just because of directions and not because I did the problem wrong.

More higher-SES students seemed to have entered my class with important skills, such as interpreting the open problems in a reasonable way, believing their interpretations were valid, and following their instincts in finding a solution. The lower-SES students seemed more concerned with having clear direction and were less apt to creatively venture toward a solution; they seemed to become stuck in the uncertainty of relatively open problems. These problem-solving skills possessed by more higher-SES students were, again, both a means to learning the curriculum, as well as part of the curriculum, itself. Students' views of what they learned in the CMP curriculum are discussed in the following section.

Thinking/Learning More

I asked students to talk about what they learned in CMP versus typical mathematics. On the surface of the responses, there were no SES patterns. Students —both lower- and higher-SES — were quick to say that in more typical math classes they learned more basic skills, such as fractions or multiplication and division.[17] Even students like Samantha who were extremely positive about the CMP curriculum talked as though they thought they would be missing some basics if they never had typical curricula. There is much informal evidence to suggest that this view was espoused by other teachers and some parents, and I suspect they influenced the students' thinking about the issue.

Many students said that what they learned in the two curricula was simply different — it was not that they learned more or less in one. (Recall Guinevere's comment that CMP and typical math are really two different types of math that are difficult to compare.) Four students — two of low- and two of higher-SES, said that the CMP made them think more. Additionally, a mix of nine higher and lower-SES students talked about the CMP helping them see how math is connected to real life.

But one difference I notice in looking over the way students talked about the benefits of CMP was that the higher-SES students seemed to more clearly state that *they* were really helped by the CMP, while the lower-SES students

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[17] The accuracy with which students were able to relate the general intent of the CMP authors, and the seriousness with which they answered the questions was impressive to me. It encourages me that the students who generally wanted to convey a message of frustration with the curriculum could also admit that they changed their minds about some aspects of the curriculum and that they learned some things through the curriculum. I think this speaks to the seriousness with which students responded to my questions (particularly at the end of the year).

seemed to talk about it more externally — either saying what they thought they *should* be learning or what *others* say they should be learning. For example, recall Samantha saying that she thinks she can figure out problems on her own now. Additionally, Andrea explained in her final interview that, in her former math classes, she would forget everything over the summer, but that CMP math "sticks with you, it just stays with you."

As with the higher-SES students, most lower-SES students could articulate (when asked in their final interviews) much of what the CMP was intending. For example, Rose explained, "CMP is kind of like real life stuff, so I think that maybe they want you to extend your brain. They aren't going to say here are the rules, they want you to figure it out yourself." But when pushed to say whether the CMP worked for them like the authors intended, Timothy and Guinevere gave mixed reviews, while the remaining higher-SES students said yes. But the lower-SES students, especially the girls, tended to be much more hesitant, saying "I don't know" (Anne), or "kind of" (Rose). The majority of lower-SES students talked about difficulties with the curriculum getting in the way of their learning as the authors intended. For example, Sue responded, "Not really. I don't know, they're just confusing and stuff," and James said, "No, cause it's hard." Lynn was quite passionate in her interview, and I will include a lengthy excerpt here because she articulates views that seemed to be shared by many of her lower-SES (especially female) peers, as indicated by their survey and interviews throughout the year.

> *Do you prefer learning math by figuring things out as you explore challenging problems or by learning rules and practicing them?*
>
> .... I usually just try to remember it [rules] and do the problems. I don't ask how or why it works. I think sometimes it confuses me more, and sometimes I'm not interested, I just want to know how to do the problems or something .... Like when we were doing the, um, the negatives and positives, adding and all that stuff. For a while there you tried to have us

figure out the rules, and then when Miss Mattel taught us, she just told us the division and multiplication laws, and I liked it when she told us so we could just do 'em, and I didn't have to sit and try to figure them out. 'Cause I think when I didn't know the rules, I got em wrong, and now that I know the rules I get most of em right now.

*. . .Why do you think we [the CMP and I] want you to figure out the rules?*

So you can, like, learn how they do it. Or try to, um, so you can just like, so you can, um, I don't know why really, but I know it's a good reason.

*Why do you think it's a good reason if you don't know it?*

'Cause if you can figure em out, then maybe you're learning more; if you can figure out how they do it or how the rules go, maybe you understand it more or something.

*But you prefer not to do that?*

Yeah, cause it confuses me . . . 'cause when I try to figure things out, I like the rule I get, and I stick to even though its not right and when we get the real one it confuses me.

*How do you get the real one?*

Like, you tell us or someone else figures it out and were like OK this is a rule, and the rule I got I think this works, but it might work on one problem but not on all the other ones, so I get mixed up. I want to use *my* rule.

*Has your opinion about the CMP changed at all this year?*

I used to hate it. But now I think I've learned a little bit from it. But like last year and the beginning of this year - oh god! I couldn't stand it . . . . I didn't' see the point of it . . . . [But now] I remember things that were doing and its like I'm learning things, so I think its better. Cause at first I didn't think I was learning anything from it. Cause like last year I don't think I learned anything from it. Our teacher was sort of like (laughs) she was hilarious — I don't think she knew what she was doing.

Lynn raises many of the issues I have discussed so far in relation to both the pedagogy and curriculum. The lower-SES students (especially the females) tended to prefer having the teacher tell them "the rule" so they could get "the right answer" to problems. The lower-SES students were more likely to not care why things work — Lynn and many other lower-SES students, again especially the females, were very concerned about obtaining a rule and a right answer. Trying to figure things out got in the way of understanding what was really important — the rules for solving the problems. Hence, Lynn did not seem to view learning to work through ambiguities of problems as an important curricular goal — the ambiguities were simply confusing obstacles to knowing the right rule to use to obtain the correct solution.

Like many other lower-SES students, Lynn talked in the third person about the curriculum being helpful in learning how to figure things out or gaining a deeper understanding. She thought maybe the CMP and I had a "good reason" or a decent theory about students learning to figure out the rules themselves, but it did not work very well for her, because the lack of clear direction and rules was too confusing for her. She felt lost in the variety of ideas posed in discussions — she felt unable to discern what was right or sensible. In the end, Lynn stated that she thought that her experience with CMP had improved, and she attributed this (at other times) to having a teacher who understood the problems and could explain things (or simply restate the problem) in plain English. She is the student who asked me when I launched a problem during class, "Why don't they write it like you say it?"

Much of what Lynn expressed difficulty with was the open nature of the problems in the CMP trial materials. That is, the problems did not have one clear path to "the right answer." One final issue to be discussed is how the contextualized nature of the problems might have affected students.

Contextualization

I touched on some issues relating to contextualization above in relation to students' contributions to whole-class discussions. The lower-SES students seemed to talk and reason about the mathematics in more contextualized ways, while the higher-SES students focused more on the abstract, mathematical ideas. I discuss these trends further in this section.

Because my original plan was to confine my focus to differences in students' thinking about mathematics in the media, the most in-depth data I have on students' actual mathematical thinking surrounds questions related to this topic. I asked students to interpret several advertisements and news stories in interviews and surveys across the year. My analysis of these data will be published elsewhere (Theule-Lubienski, in press). In a nutshell, I found that, while both groups of students were more skeptical of the data than I anticipated, their skepticism was different. The lower-SES students were more likely to ignore the data provided, and to reason about the questions I asked by referring to pictures provided in the media, personal stories about friends and family that tried various products, and/or other "common sense" means of reasoning. Higher-SES tended to make use of the mathematical information in their interpretations and to scrutinize it carefully to find a "loophole" (although it is important to note that this did not always mean they came up with the "best" mathematical explanation). Additionally, higher-SES students would often combine their scrutiny with "common sense" reasoning, especially on items that seemed of personal interest to them.

For example, when asked if a toothbrush advertisement from a magazine would convince them to buy the toothbrush, Rose was one of the three students (two of lower-SES and Rodney, the mixed-SES student who was also African American), to base her answer on the picture of the toothbrush provided ("it bends like the one I have, so it's better"). Dawn and Mark were the only lower-SES students to respond like the higher-SES students, who tended to scrutinize the wording and statistics given, and point out the asterisked small print at the bottom of the page. Guinevere said that advertisers "try to fool you," and

Timothy explained, "There's almost always a way in math to make it sound better." This was a theme that ran throughout the media data I collected: The higher-SES students assumed that the information might be skewed, so they had to read the fine print, while the lower-SES students were more likely to think that business people simply made up the numbers, so there was no point in taking them seriously. (For example, when looking at a graph of water usage over several decades, some lower-SES students said they did not think the company would remember that far back so they must have just made up the numbers — "unless they keep records or something, but I don't think so," said Lynn.)

As a more extended example, on a quiz I asked students three questions about a Chevy advertisement in which a 2.5% difference is made to look very large because the y-axis begins at 95% (see Figure 4.1).

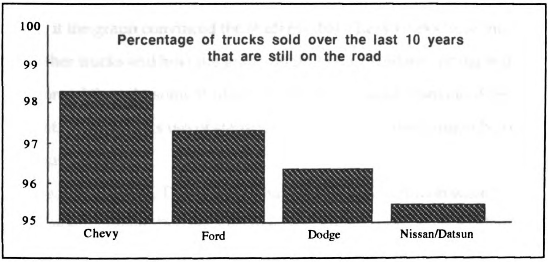


**Figure 4.1**
**Bar Graph from Truck Advertisement**

First, I asked the students to show they could read basic information from the bar graph, by asking them to say what percent of Chevy trucks made within the previous ten years were still on the road (essentially, reading the "y-axis" for the Chevy bar). All but James successfully answered the question. Then I asked students the following two questions:

- The bar for Chevy is about six times as tall as the bar for Nissan/Datsun. Does this mean that the chances of one of Chevy's trucks lasting 10 years are about six times as great as the chances for a Nissan/Datsun truck?
- If you wanted to buy a truck, would this graph convince you to buy a truck from Chevy?

I asked the first question to see if students would realize how the scale on the y-axis affected the meaning of the graph. If students did not look carefully at the graph and think carefully about what the data meant, they would probably believe that Chevy trucks would be much more dependable than Nissan trucks since a glance at the bars would send that message. If students looked at the graph carefully, they would realize that the differences between the companies was less than three percent, or that the data indicates only what percent of trucks made over the last ten years are still on the road, which might not totally correlate with the quality of the trucks. I asked the second question to see, in another way, if the graph convinced the students that Chevy trucks were much better than other trucks and how the graph might impact students' actual real-world decisions. I thought some students might be completely convinced by the advertisement, because of its use of statistics, while other students might be very skeptical about the data.

For the first question, Dawn used what I would call "common sense" information, as she replied, "No because maybe the other company is just having a bad year, so there they cant keep the rate up." The other "common sense" response to the question came from Rodney, who said it depends on the engine. Three lower-SES students (Nick, Rose and Lynn) said "yes," referring to, but not scrutinizing the information, perhaps only using the information in the question and not on the graph. For example, Rose responded, "yes Because the graph shows how many cars are made and last for 10 years and a large percent of Chevy trucks last but a small percent of Nissan/Datsun last." Mark and Anne responded to the questions more like the higher-SES students, who all (except Guinevere) said "no," and scrutinized the data to offer a mathematical rationale for their answer. For example, Samantha responded, "No. It means that about 3% of all of Chevy's trucks that were bought ten years ago are still on the road today, not six times more." Even Rebecca and Timothy, who did not mention the scale on the y-axis, offered an alternative explanation that showed their attempts to think critically about the statistics involved, even if the response was not mathematically correct. Rebecca insightfully pointed out that we do not know when in the previous ten years the trucks were made. For example, Chevy might have made more trucks in the latter part of the ten years, and Nissan could have produced most of theirs at the beginning of the decade. Also, Timothy wrote:

> No, because Chevy could have only sold say 10 trucks and 8 of them are still on the road then the percent is 80 percent but then Toyota sells 100 and 80 of them are still on the road. That's 80 percent again, but Toyota sold more trucks and at the rate Chevy was going at 100 cars theyed have a higher percent.

But, to my amazement, no matter what their answer to the first question, the lower-SES students all answered "Yes" to the second question, indicating they would be convinced to buy a Chevy truck. In contrast, all higher-SES students answered "No."

Hence, in comparison with the lower-SES students, the higher-SES students more often scrutinized the mathematical basis for ads and said they would use the information cautiously in decision-making. In other examples some higher-SES students said they would look to Consumer Reports for more objective information, or that the advertisement would help them decide what to look at or try. Meanwhile, lower-SES students tended to display more outright skepticism of the information, often saying (in other examples) that they thought data were often just made up for the sake of convenience and/or profit, yet they seemed more easily persuaded by the overall presentation.[18]

What is the point of these media-related examples? I struggled all year to figure out why bright, lower-SES students such as Rose, often seemed so different in their mathematical reasoning than what I expected. Through analyzing the media examples, I saw evidence that the lower- and higher-SES students thought differently about mathematics, especially concerning examples from the real world. While the higher-SES students seemed to be attentive to the mathematics in the examples (although this did not mean their interpretations were always correct), the lower-SES students seemed to ignore the data provided and reason in more contextualized, "common sense" ways.

In looking more closely where I noted in my journal that students seemed to be talking past each other or using very different types of reasoning, I began to see a pattern I came to think of as lower-SES students "getting stuck" in the context of the problems. I realize that "getting stuck" has a negative connotation, and one could argue that their reasoning is not deficient and/or they are just not "playing school" (and telling the teacher what she expects) as effectively as the middle-class students. But I use the term because I saw evidence that the contextualized problems, while allowing lots of different approaches and means of thinking about them, tended to also allow some students to miss the intended

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[18] Perhaps the lower-SES students did not see any contradiction between thinking that ads are often lies and being influenced by them. In an interview, James was asked to give examples of dishonest news stories, and he responded, "How would I know if it wasn't true?" Perhaps, as Crossen (1994) argues, when people feel powerless to judge the truthfulness of the information, they do not attempt to make rational decisions based on the information. Additionally, Bruner (1975, p. 36) argues that poverty produces "a sense of powerlessness [that] alters goal striving and problem solving in those it affects." Hence, the lower-SES students might have been more easily persuaded by information in the media, since they may not have felt they had the power to scrutinize and judge it, and all possible influences on their opinion were equally suspect.

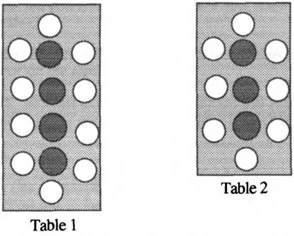
mathematical point of the problem, and that these students were too often the lower-SES students.

As an example, I offer the following data surrounding a problem relating to dividing up pizza. I discovered this example while conducting the discussion coding described above. While teaching, I considered the encounter so mundane that I mentioned nothing about it in my journal. Yet, in listening to the recording of the discussion two years after the teaching occurred, I noticed differences in the way some lower- and higher-SES students approached the problem. It is especially convenient that the reader is already familiar with the primary actors in this episode, since it involves some of the girls discussed in Chapter Three.

This is an example in which both lower- and higher- SES students were actively engaged with the curriculum and discussion. But in looking beyond that similarity, we can see some differences between the thinking and reasoning of the higher- and lower-SES students.

The authors intended the problem presented in Figure 4.2 to help students learn to create and compare ratios. One way students could learn to do this was to compare four pizzas for eleven people versus three pizzas for nine people, seeing that 4/11, or 36% of a pizza was greater than 3/9, or 33% of a pizza. This was one of three problems that made up an "investigation" intended to teach students to compare numbers using rates, division, ratios, and percents.

Imagine that you entered a pizza parlor and saw pizzas being served at two tables of your friends, one table with 10 people and the other with 8 people. Friends at each of the tables call out to you to pull up a chair and share their pizzas. You could choose the table with fewer friends or the one with more friends. Or you might choose the table with the most pizza. But is this the best way to make the decision? Are there other more helpful ways that we can use mathematics to *compare* the two situations?




Problem 1.2: If you like the two groups of friends equally well, which table would you join and why?

**Figure 4.2**
**Problem from *Comparing and Scaling,* Investigation 1: Making Comparisons**

As I launched this problem, I clarified that students should assume that they want to go where they can eat the most pizza. Students worked on this problem in groups for about fifteen minutes, and then Rose began our whole-class discussion by sharing her work she had done in her group.

Rose: Um, you'd think there would be equal if they were divided up into 4, I don't — anyway, if they were divided up into 4 . . . at Table 1 there would be 16 pieces, and on table 2 there'd be 12? . . . . And so, um, there'd be 4 people who wouldn't have seconds on table 2? (pause) because that would be 8 and 8 is 16? And then the same on Table 1.

*(While she spoke, I recorded her information on the board.)*

*STL:* *Does anyone want to ask them anything . . . ? (pause) Sue.*

Sue: OK um, you don't know how much the, the, the people at the table who already ate, and (pause)

*(She tended to mumble toward the ends of her contributions, and I had difficulty understanding her.)*

*STL:* *You don't know — I'm sorry—*

Sue: You don't know, like say you came five minutes after they served everyone and everything, and say like two pieces were gone at like both tables, and then there's only two on both of them.

*STL:* *OK, so if, so you don't, OK, you don't know how much is left at each table, is that—?*

Sue: Yeah! You don't, you — I don't know! *(She mumbled something I couldn't hear on the tape.)*

*STL:* *(Pause) OK, let's just assume — they don't say this explicitly — but let's assume they just got the pizzas and they haven't started eating yet.*

A Student: It says that.

*STL:* *It does say that? (I checked — the problem did say that.) OK . . . Um, Samantha you've had your hand up a long time.*

Samantha: Um, I want to take the people — um, there's four people without seconds at each thing, but the Table One has more people, so, um, um, those people without seconds on Table Two that's half the people, but the people without seconds on Table One, that's um, less than one half, so you have a *better chance* of getting seconds at Table One than at Table Two.

*(We had recently finished a probability unit, and it seemed that Samantha was able to use those ideas here.)*

*STL:* *OK, so you would say table one is better. So you like this idea of getting seconds —*

Samantha: —I have another idea.

*STL:* *Go ahead.*

Samantha: OK, I got the answer by dividing by um, dividing how many pizzas there are by the number of people, and so I took four divided by eleven because you have to add more people if you're going to join the table.

*(Getting seconds was not the original way she solved it. But she had listened to their argument and understood it's mathematical flaw and tried to address it. Now she offered her original way of solving it.)*

*STL:* *Because you're going to be there too, so you're saying there's eleven people and —*

*(I'm clarifying as I'm writing on the board.)*

Samantha: And you get 36% of the pizza.

At this point, I pushed a bit on what the 36% was a portion of — one pizza or all the pizza. While many students seemed to understand Samantha's approach and answer, we ran out of time to really pursue the differences in Rose's and Samantha's approaches. As the bell was about to ring, Lynn asked:

Lynn: Well, I don't see where she got nine.

*STL: OK, the reason she got nine. Samantha, tell her.*

Samantha: Because OK, there's eight of your friends are already at Table Two, and if you're going to join, then you —

Lynn: Oh, I'm at Table Two? OK.

*STL: Yeah. OK, I'm going to give you your homework now . . . .*

Rose: Mrs. T.L. — um, A it says like which one is most crowded? How would you figure that out?

*(Rose asked this question about the first homework problem as the bell was ringing. While most students were quickly packing up and leaving, she was seeking help with how to begin her homework.)*

*STL: That's what you're going to have to do. Rose, one way might be to figure out how much space there is for each kid . . . .*

Instead of exploring the problem in a way that allowed them to learn more about generalizable methods of comparing numbers using division or ratios, Sue and Rose's approaches to the problem seemed heavily influenced by the real-world concerns, such as getting seconds on pizza or arriving late to dinner. Hence, these students had difficulty obtaining a "correct" answer in this case. But, more importantly, they did not solve the problem in a way that helped them learn what was intended regarding the powerful, generalizable methods of comparing using of division or ratios, as was intended. As Samantha proved, Rose's approach with "getting seconds" could have allowed her to reach the right answer, in which case she might have left class confident that she understood what was intended. But she would not have learned general principles that would have helped her solve the mathematically similar homework problem she had that night.[19] It asked which park was most crowded, given the areas and number of children playing in each park:

> In the city of Canton there are three parks in which many children are playing. The names and areas of the parks are: Flyaway Park — 5,000m$^2$; Golden Park — 7,235m$^2$; and Pine Park — 3060m$^2$.
>
> a) If the number of children playing in these parks respectively is 400, 630, and 255, which park is the most crowded?
>
> b) Which is the next most crowded?
>
> c) Explain your method of solution.
>
> d) Suppose that there is another park, Oak Park, that has 5240m$^2$ of area and 462 children. Now which is the most crowded and why?

In looking closely at their homework from that night, the SES patterns were striking. The five lower-SES girls tried to do their homework, but they all received "√'s," indicating that they did not do the problem correctly. Both of the two lower-SES boys who turned in their work also had difficulty. Well over half the higher-SES students made good sense of the problem and got perfect scores on their assignment, including Timothy, who often struggled more with the problems than his higher-SES peers. In looking more closely at their work, the higher-SES students made good sense of the problem and their solutions to it. Their answers were things like ".087 kids per square meter," or "11.4 square meters per person," and this seemed to be sensible to them, even though fractions of children allocated to each square meter or fractions of square meters allocated to each child are quite abstract ideas. The lower-SES students generally did some type of computation with the numbers, but could not make sense of their

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19 These problems both involve dividing things (pizza or square meters of playing space) among a certain number of people and comparing quotients obtained to answer the questions posed.

answers when they finished. For example, here are my descriptions of the responses of five of the six girls:[20]

> Rose — She divided numbers, but she couldn't interpret what they meant. She wrote, "I divided the area by the number of children playing. this found the percent of people on each playground." This seems to make no sense, although she had some idea that she needed to divide. She showed her division of each set of numbers. What she actually found was the number of square meters that each child had to play in.
>
> Dawn — She also divided all the areas by the numbers of kids, except that she squared the areas, perhaps because it said "$m^2$." She then took the largest number, which gave her the correct answer, but then when she answered some of the other parts, she had wrong answers. For part "c" she wrote, "I took 5,000^2 ÷ 400 = 62500 and did the same thing for each one and the one that had the biggest # was the biggest." For part "d" she wrote "Still Golden Park because there are still more kids per meter."
>
> Sue — She had several problems. She divided the areas by the people, and said she had found "people square per meter square," so she had it backwards. She figured out that Flyaway had 12.5 and Pine had 12, but then she chose Pine as the most crowded, explaining, "The one with the most people per meter$^2$, has the most people. She then said Flyaway was the second most crowded, which contradicted what she had said.
>
> Guinevere — She made an organized chart, and she divided the area of each park by the number of children playing in the park. She correctly labeled all her answers with "$m^2$ per child." She also explained, "I take the number of meters$^2$ for each park and divide the number of children into it. That gives you the number of meters$^2$ for each child in the park. The highest number of meters$^2$ for each child is the least crowded. The lowest number of meters$^2$ for each child is the most crowded park."
>
> Samantha — She got the right answers, and explained, "You take the number of children in the park and divide it by the area (meters squared), and you get how many children are playing in one square meter (children per square meter) in the park." She consistently applied this logic to all parts of the problem. For part "d" she wrote "Oak Park, because it has .088 of a kid per square meter, and Golden Park has .087 of a kid per square meter."

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[20] I did not have a copy of Rebecca's work saved.

There were strikingly consistent SES patterns in this homework assignment.[21] It seems reasonable to say that these are due, in part, to previous achievement differences, which are correlated with SES. But my students' experiences raise the possibility of other factors. When lower-SES students, such as Lynn, were known for being very bright perfectionists in their other classes (as I learned from her other teachers and her parents during parent-teacher conferences), why were they struggling in my class? Why did the lower-SES students tend to say they used to understand math better before CMP?[22] Additionally, Rose generally did do well in my class, and Timothy did not do very well in my class — he struggled. But on this problem, Rose's reasoning seemed faulty, and Timothy's response to this problem was similar to Samantha's. So what was happening in this particular case?

One thing that strikes me is how odd it might seem to talk about ".087 kids per square meter." When we are doing mathematics, we want students to eventually be able to deal with that type of abstraction. As argued in Chapter One, it sounds promising to allow students to explore math through familiar contexts. But in learning mathematics, the goal is not to stay tied to the contexts, but to abstract the mathematical principles involved so they can be utilized elsewhere. Although the CMP authors (including me) were changing the way in

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[21] I should note that in this example, I have not gone into any depth about what my role was or could have been in helping the students "pull out" the intended mathematics. The summarizing discussion was cut short, and I do not consider this one of my finest teaching episodes. But my focus here is on how the students made sense of the contextualized problems, and, although stronger teacher intervention might have helped the matter, the fact still remained that it was the low-SES students who had the most difficulty with learning the intended mathematical ideas from the problem.

[22] Unfortunately, I do not have more "objective" evidence to support students' perceptions of whether they "did better" or "worse" in the CMP curriculum. The students' perceptions were sometimes confirmed by other students who would talk about certain higher-SES students doing better in CMP and lower-SES students doing better previously. Rebecca's mother supported Rebecca's claims that she found CMP math easier and more enjoyable than typical mathematics curricula.

which students learned important mathematical ideas and processes, they still valued the power of understanding abstract principles that can then be applied in the concrete again. (These issues involving abstraction, the nature of mathematics, and the goals of school mathematics curricula will be discussed in Chapters Five and Six.)

It seemed as if the lower-SES students tended to get stuck in the contexts, and it was harder for them to pull back and see the mathematical ideas that were really the goal of the lesson. As mentioned in the previous chapter, Dawn said that she could never figure out what she was supposed to be learning until she took the test. Also, recall Rose complaining that she had difficulty seeing how the problems we did in class related to the assigned homework problems — she did not see our in-class work as helpful to understanding homework. Additionally, while the higher-SES girls said that we hit the same mathematical ideas over and over, Rose said she could not see any connections between the units — once we finished with a unit or idea, we never revisited it. The contextualized problems seemed to grab many students' interest, but the mathematical benefits often seemed to be inequitably distributed. While some students seemed to easily be able to pull back from the context and see the mathematical point, other students seemed to approach the problems in ways that allowed them to miss the mathematical point altogether. Again, these differences fell along SES lines.

Twice during the year, the students were asked to talk about the most important and most interesting thing learned in past units. I categorized which students talked about processes and which talked about ideas, which students talked about the heart of the unit versus a peripheral concept in the unit. While there was no pattern in terms of the processes versus ideas, the higher-SES students seemed better able to articulate the abstract, mathematical heart of the units more clearly for the "most important" question. Additionally, while the higher-SES students never talked about the "most *important* things" in terms of the context in which they were learned, many of them named ideas or processes in contexts when asked about the "most *interesting* thing" learned. Also, when asked what specific thing they are still struggling to learn, none of the higher-SES responses were in context, but several of the lower-SES students' responses were, including Rose's.

Discussion of Data Regarding Curriculum

There are two aspects of the problems in the CMP trial materials that seemed to play out differently for my lower- and higher-SES students. First, the more open nature of the problems required students to read and make sense of the problems, and to take initiative and explore the problem, as opposed to following the step-by-step rules given at the top of the page. The higher-SES students were generally able to explore the open problems in this way, without becoming flustered. In fact, many higher-SES students voiced appreciation for their increased confidence and abilities in mathematical problem solving due to the CMP curriculum. But the lower-SES students, especially the females, more consistently complained of feeling completely confused about what to do with the problems, and asked (often passionately) for more teacher direction and a return to typical drill and practice problems.

Second, the contextualized nature of some of the problems seemed to involve the low- and higher-SES students differently. The CMP problems were not all set in real-world contexts, but many of them were, to varying degrees. While these contexts often seemed to be powerful motivators, drawing the lower-SES students into solving the problems, the contexts also were powerful influences on how these students approached solving the problems. While the higher-SES students seemed more able to pull back from the context and see the intended mathematical ideas involved, the lower-SES students seemed to think more about the real-world constraints involved with the contexts as they solved the problems, thereby allowing them to miss the intended mathematical ideas. Usually the lower-SES students' thinking about these real world constraints was very sensible, and I (and the CMP authors) did want students to be able to think about mathematical ideas in the messy, real-world contexts in which they are often embedded. But these contextualized problems were also supposed to be a means to learning more general mathematical ideas that could then be applied in other contexts. This seemed to be the case more often for the higher-SES students.

But now, one might ask, "What actually happened in terms of student outcomes?" Up to this point, I have been discussing patterns in how students experienced and reacted to the pedagogy and curriculum to address the overall question of how my pedagogy and the curriculum played out with students of differing SES. The final area I will address is students' outcomes in terms of their mathematical achievement and feelings of mathematical confidence.

## Student "Outcomes"

Questions regarding student outcomes are difficult to answer. I realize that the entire dissertation could have been just about what students learned, with pre- and post- tests carefully given to gauge their mathematical growth. While I think this data would have been interesting and should be included in other studies, I did not collect this type of data for my students.[23] Most mathematics education studies focus primarily on the issue of mathematical

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[23] Since my focus was only on media-related, mathematical thinking and reasoning, I did not collect general, mathematical achievement information for my students at the beginning of the year.

achievement (Noddings, 1996). In this study, I concentrate more on students' experiences in the mathematics classroom. Yet, I realize that the "bottom line" for judging the effectiveness of a pedagogy and curriculum is generally considered to be students' learning. Hence, I draw from data I do have about students' achievement, which address the issue of student outcomes in three cursory, but illuminating ways.

First, I discuss their achievement in terms of students' homework and tests grades for the year, as these indicators provide information about students' efforts and their understanding of the main mathematics ideas under study. Second, I relate what happened at the end of the year, when they were tested to be put in tracks for the following year. This information conveys, in a sense, a type of ultimate outcome for students once they leave my classroom and re-enter the "real world" of testing and filling "slots." These measures indicate very general information about what mathematical knowledge students take away from my class. But the goals of current reforms are about "empowerment." How mathematically powerful did my students feel after two years of CMP math and one year of my pedagogy? I try to answer this question by exploring how students felt about their placements for the following year, as well as where they saw themselves in the class in relation to other students.

Still, I am troubled that I do not have concrete evidence of students' growth over the year. While the data I do have provide evidence of the effectiveness of the pedagogy and curriculum from the various perspectives, I realize this study would be strengthened if I could provide data on students' mathematics learning from more "objective," pre- and post-tests.

Grades

Many of the differences I described might seem attributable to simple motivational differences, perhaps due to parental influences. But my analyses of the data do not support that hypothesis, particularly when examining gender and SES together.

Table 4.12 shows the students' average grades for the year. The first number shown for each student is the percent of assignments (homework and classwork) completed. This is a rough measure of their effort. The second number is their quizzes/test average. This is a rough measure of their understanding of the intended mathematics.

Notice that work and test averages are very close for the higher-SES males (84 W - 85 Q&T) and females (92 W - 92 Q&T), as well as for the lower-SES males (76W - 77 Q&T or 63/69 including the outlier James who only completed 21% of assignments). But the story is quite different for the lower-SES females, who showed the most frustration with the class throughout the year. These girls made an effort, most turning in over 90% of their homework, but they still did not understand the mathematics in a way that allowed them to do really well on the tests. In other words, their efforts did not pay off in the same way that they did for the higher-SES girls, who completed about the same percentage of homework and were rewarded for their efforts on the quizzes and tests.

Both groups of boys were less diligent about doing their homework. Hence, there seemed to be stronger gender differences than SES differences in terms of effort put forth on homework.

**Table 4.12**
**Students' Average Grades for the Year**

| Higher-SES Males | Higher-SES Females | Lower-SES Males | Lower-SES Females |
|---|---|---|---|
| Christopher: 96 W 95 Q&T | Rebecca: 99 W 95 Q&T | Mark: 82 W 84 Q&T | Rose: 97 W 87 Q&T |
| Samuel: 89 W 90 Q&T | Samantha: 98 W 96 Q&T | Nick: 78 W 87 Q&T | Anne: 94 W 88 Q&T |
| Timothy: 89 W 81 Q&T | Andrea: 91 W 86 Q&T | Carl: 69 W 61 Q&T | Lynn: 94 W 86 Q&T |
| Benjamin: 77 W 92 Q&T | Guinevere: 81 W 90 Q&T | James: 21 W 42 Q&T | Sue: 90 W 77 Q&T |
| Harrison: 70 W 69 Q&T | | | Dawn: 80 W 63 Q&T |
| 84 W - 85 Q&T | 92 W - 92 Q&T | 76 W - 77 Q&T (w/o James) 63 W - 69 Q&T (w/ James)[24] | 91 W - 80 Q&T |

Placements for the Following Year and Feelings About Them

Near the end of the year, the students were asked to take the school's traditional "algebra placement test." Partially on the basis of this test, and partially on the basis of other factors such as students' behavior and parental pressure, the regular classroom teacher placed the students into one of three tracks for the following year. (I had very little to do with this process, except that the teacher asked my advice about a couple of students.) On the final survey, I asked students where they were placed and how they felt about it.

Table 4.13 shows the results, ordered roughly from lowest to highest SES with Dawn and James being what I would call "lower class" and the other lower-

24 Since James is an outlier for the low-SES males, I calculated the overall average both with and without his grades.

SES students being working class.[25] From my experiences with students over the course of the year, I expected to see some patterns in the data, but I certainly did not expect them to be this strong. Every one of the higher-SES students was chosen to be in honor's algebra the following year. The students of lowest SES were placed in pre-algebra — a euphemism for remedial basic mathematics. The working-class students were placed in algebra, with the exception of Rose and Anne — two very smart young ladies who displayed exceptional dedication to their schooling.[26]

The pattern we saw in the data for grades is reinforced, with the higher-SES students out-performing the lower-SES students in test situations. But what about a student like Timothy, who did not display great effort or understanding of the curriculum? His placement in honors algebra was due to pressure exerted by his higher-SES parents. Timothy's case reminds us that changing the curriculum and pedagogy to better educate all students has its limits — there are still factors outside of school that will intervene to ensure that higher-SES students will maintain their class advantages.

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[25] Both Dawn's and James' fathers (or step-father) were unemployed. Dawn's mom and James' step-father had not graduated from high school. Dawn's mother cleaned houses for a living, and James' mom said her occupation was "industrial." Their incomes were well below the poverty level. In the families I considered working class, the fathers (who were present in the home) generally worked in the local auto factory, and the mothers either stayed home or worked in factories, offices, or as cashiers. While some of the working-class parents had not graduated from high school, most had, and some mothers had taken some college or vocational courses. The working-class families generally had enough money to live on, and some even made over $60,000 per year (such as a family where both parents worked in the auto factory).

[26] As it turned out, the school district decided to continue using CMP the following year, so the placements were not carried out.

**Table 4.13**
**Placements for the Following Year and Students' Feelings About Them**

| Name | What math will you be taking next year? | How do you feel about it? |
|---|---|---|
| Dawn | Pre-algebra | OK, because I'm not the best student so I think I will learn what I need to learn next year. |
| James | Pre-Algebra | I feel very good because I think that's a good start for me. |
| Carl | Pre-Algebra | Okay 'cause I need to get better at certain math problems. |
| Sue | Algebra | OK. I don't know. |
| Nick | Algebra | Good, because I will be in a class where everybody is pretty equal. |
| Lynn | Algebra | Good, because that's where I wanted to go. |
| Rose | Honors algebra | I'm happy, but I've heard it was hard. So I'll have to work extra hard. |
| Mark | Algebra | I'm happy, it is easier than honors. |
| Anne | Honors Algebra | I think it will be challenging because it is something new. |
| | | |
| Rodney | Algebra | Kind of happy |
| | | |
| Guinevere | Honors Algebra | I feel that it suits my level because anything lower would be boring. |
| Samantha | Honors algebra | I think it's the right one because I feel I understand algebra and I can handle it (and the homework). |
| Benjamin | Honors Algebra | Fine, we took an algebra placement test and that told us where. |
| Andrea | Honors Algebra | I like it, but I might think its too hard, because I'm OK in math. Honors is good though. |
| Rebecca | Honors Algebra | Good, it is an honor to be in honors. |
| Samuel | Honors Algebra | Happy, it's the best |
| Timothy | Honors Algebra | Nice. I'm smart. |
| Christopher | Honors Algebra | (Drew a happy face) |

Note: I am missing information for Harrison — he skipped these questions on his survey.

Table 4.13 raises another, perhaps more important issue than the placements themselves — that is, students' acceptance of their placements. In general, the students said they were satisfied with their placements — they thought they were placed where they deserved to be. The lower-SES students were complacent in their lower placements. But Rose & Anne, the two working-class students who were placed in honors, said that they thought it would be challenging for them. With the exception of Andrea who thought honors algebra might be difficult for her, the higher-SES students were positive and confident in their higher placements. Even Timothy felt confident that he was in the right place — "I'm smart," he asserted.

Hence, the higher-SES students seemed to leave my class feeling more confident in their abilities to know and do mathematics than the lower-SES students. But were students just saying that they were satisfied to "save face"? That is, did the lower-SES students think that seeming satisfied with their placement would alleviate further embarrassment?

Feeling Mathematically Empowered?

To explore feelings of empowerment in another way, I asked students to name the best three math students in the class (Final Survey). Table 4.14 contains the tallies of those who were named. Those with an asterisk ranked themselves in the top three. Notice that NONE of the lower-SES students ranked themselves in the top three — even Rose, who was considered by nine other students to be in the top three. Meanwhile, every one of the higher-SES students who were mentioned at all, also mentioned themselves. Even Samuel and Guinevere, who were not mentioned by anyone else, named themselves.

**Table 4.14**
**Those Mentioned in Response to "Who are the Best Three Math Students?"**

| Higher-SES Males | Higher-SES Females | Lower-SES Males | Lower-SES Females |
|---|---|---|---|
| Benjamin* - 18 | Samantha* - 20 | Carl - | Rose - 9 |
| Timothy* - 2 | Rebecca* - 4 | James - | Anne - 2 |
| Christopher* - 2 | Guinevere* - 1 | Nick - | Dawn - |
| Samuel* - 1 | Andrea - | Mark - | Sue - |
| Harrison - | | | Lynn - |

* Students who included themselves among the top three students are denoted with an asterisk.
Note: Two other females were mentioned by one person. I do not have SES information for them.

According to their homework and test grades, Rose and Anne (and perhaps even Lynn) had as much cause to feel mathematically confident as Timothy, Guinevere, or Samuel. And yet they did not.

## General Discussion

At the end of the year, I asked students about the type of student who might prefer traditional math or math this year. Several lower-SES girls indicated that those who were smart favored CMP math, implying that they did not count themselves in the "smart" category (since they said they preferred "traditional" math). So a reasonable question to ask is, "Is it simply a difference in previous achievement, or is there something more that seems to make the higher-SES students more comfortable with my pedagogy and the CMP trial curriculum?"

My sense is that there is more to it than previous achievement differences. Through informal conversations with parents and teachers at parent-teacher conferences, I learned that some of the lower-SES students who were not doing well in my class were at the top of their other, more typical classes. These are the same students who talked about the traditional curriculum being easier for them. Additionally, I heard some parents, such as Rebecca's mom, explain that their middle-class child had not been good at math previously, but she/he was doing better or enjoying math more with CMP. Also, recall Guinevere's comments that she and her family were just "word problem kind of people," and other higher-SES students' comments about being better at story problems than number problems. Hence, it does not appear to be a simple case of the lower-SES students being dumb or slow, and the higher-SES students being more advanced. Recall that overall, Rose did very well in the class, but she still shared many of the same reactions to the curriculum and pedagogy with her lower-SES peers. Similarly Timothy did not do as well in the class as his higher-SES peers, but he still had the confidence to rank himself at the top of the class. There seemed to be something about my pedagogy and curriculum that was playing out differently for my lower- and higher-SES students, and some of the differences did not seem completely attributable to achievement differences.

Thus far, I have summarized several trends I saw in relation to whole class discussions and the curriculum. I now discuss two themes that seem to cut across these two areas.

The first theme involves students taking the initiative to make sense of things themselves, as opposed to depending on an authority figure to give them more direction. Both the discussions and the open nature of the CMP trial problems called for students to take intellectual risks as they explored, shared, and analyzed mathematical ideas.

In discussions, the lower-SES students seemed to minimize their risk of involvement, tending to wait for the teacher to ask a question to which they were sure of the answer before contributing. The lower-SES students seemed more fearful of participating in discussions, and when their ideas were challenged, they would quickly back down. They seemed to become overwhelmed in discussions when ideas conflicted; instead of making sense of the ideas and determining which ones were reasonable and which were not, the lower-SES students said they felt confused and unable to discern which ideas were correct. The higher-SES students seemed more comfortable with ambiguity, feeling confident to make sense of ideas being debated, and defending their own ideas in the face of opposition. They were more willing to take risks and share and analyze ideas, not waiting to be told what to think or which ideas were correct.

The lower-SES students also consistently complained about their difficulties with making sense of the problems. Many of them said they preferred being told what to do and how to do it, as opposed to figuring it out themselves. Many lower-SES students viewed having to figure things out for themselves as an extra obstacle to learning what was really important — the mathematical "rule" that would tell them how to obtain the correct answer. From my perspective and that of the CMP authors, learning how to solve open mathematics problems without constant teacher direction was an important mathematical process to be learned. Yet, the lower-SES students seemed to view this process as a frustrating annoyance that got in the way of learning mathematics. The higher-SES students seemed to enter my classroom with more problem-solving skills that helped them deal with the ambiguities of the curriculum. Although some higher-SES students occasionally complained about the difficulty of specific aspects of the curriculum, they generally seemed more confident in interpreting the problems in a way that was sensible to them and getting on with the business of solving them.

A second theme is more difficult to articulate, but involves students' ways of knowing and communicating that either helped or hindered their abilities to enjoy and abstract the mathematical ideas from the discussions and problems.

The students seemed to have different beliefs about the purpose of our discussions. The lower-SES students viewed them as a means to finding right answers. They showed little motivation to discuss or understand abstracted, mathematical ideas for their own sake. Many higher-SES students seemed more intellectually curious and viewed the discussions as a helpful forum for sharing and learning ideas. Instead of seeing the discussions and CMP problems strictly in terms of right and wrong answers, they seemed more able and inclined to pull back and analyze various mathematical ideas. Hence, the higher-SES students seemed to have beliefs about discussion, analytic skills and ways of communicating about ideas that enabled them to participate in discussions in ways I intended. The lower-SES students seemed to hold a more traditional, authoritarian view of class discussions, with the teacher being the asker of questions and validator of answers. Many of these students made an effort in discussions, but their orientation toward gaining approval and correct methods from the teacher seemed to limit their contributions to asking questions and supplying answers when they were sure they were right.

When problems were posed in real world contexts, many lower-SES students seemed to stick closer to the contexts, in an attempt to find a particular answer to the specific situation. The lower-SES students would often find creative, very reasonable solutions to the particular problems, but these were often tied so tightly to the contexts that the solution was not generalizable to other situations. Hence, the Lower-SES students had more difficulty in understanding which mathematical ideas were the focus of our problem-solving efforts, and, consequently, learning the intended ideas. In contrast, the higher-SES students seemed more inclined to pull back from the situation and to use abstract ways of talking about and proving mathematical statements. The higher-SES students seemed to understand that the CMP authors and I expected them to focus on the mathematical essence of problems, and they had the analytic skills that allowed them to do that. The higher-SES students seemed more aware of the mathematical point of problems and units. The higher-SES students more often walked away from a problem and discussion having learned the intended, generalized mathematical ideas.

It is not surprising that the lower-SES students tended to use the word "confusing" so often in relation to both the discussions and the curriculum. The lower-SES students seemed to focus on obtaining immediate, correct answers to individual problems, and they preferred having the teacher tell them what to do, and what is right. They less often looked for and analyzed the larger, abstract, mathematical ideas underlying the problems or discussions. The higher-SES students had the beliefs and preferred ways of knowing, communicating, and working to fulfill their expected roles. The pedagogy and curriculum in my class seemed to combine to enable the higher-SES students to leave my class feeling mathematically empowered, while the lower-SES students seemed to find their experiences in my classroom disempowering.

Hence, there were patterns in my classroom for my small group of students that fell along SES lines. One could ask why I see class as an explanatory factor for differences that existed between SES groups. Perhaps it is just a fluke of a small sample — what we are seeing is simply individual differences between kids, that just happen to fall along SES lines in this case. Or perhaps the lower-SES students could not be expected to do well in any classroom environment or curriculum because they have so many problems outside of school over which educators have no control.

I did not follow students home to see first-hand how their home environments might have supported or hindered students experiences in my classroom. But my analyses of the data led me to relevant literature about social class cultures in order to address some of these issues. What I found provides insight into, and possible explanations for, differences I saw in my students. I will discuss these issues further in Chapter 5.

## CHAPTER 5

## DISCUSSION OF DATA AND LINKS TO LITERATURE: CULTURAL CONFUSION AS AN EXPLANATORY FRAMEWORK

Chapters 3 and 4 presented analyses of my students' experiences with and reactions to my pedagogy and the CMP trial materials. In these analyses I focused specifically on whole-class discussions and open-ended problems (some of which were set in real-world contexts) in which students were asked to take initiative and explore, abstract, analyze and communicate about mathematical ideas. The patterns I saw in the data suggested that key aspects of my pedagogy and the CMP trial curriculum were more aligned with my higher-SES students' beliefs and ways of knowing, communicating and working. This chapter sets out to explore explanations for these SES-related patterns.

As discussed in Chapter 1, the use of whole-class discussion and open-ended problems are currently popular ideas in the current push to mathematically empower all students. In other words, these elements of pedagogy and the curriculum are not unique to my classroom. They are being implemented in many classrooms nationwide. Thus, we should give serious consideration to questions about their potential for promoting or hindering equity.

In thinking about possible explanations for the patterns in my students' experiences in my classroom, several questions arise. Might the patterns in the data be due primarily to individual differences that happen to fall along SES lines in my small sample? Perhaps differences in previous achievement might be the sole explanatory factor. How might the trends be related to students' socio-economic class backgrounds?[1]

The patterns in the data from my classroom raise issues relating to students' ways of communicating, reasoning, learning, and knowing. These aspects are central to what Erickson(1986) defines as culture: "learned and shared standards for perceiving, believing, acting and evaluating the actions of others" (p. 129). Culture can be defined in other, similar ways, including "what people do, what people know, and things that people make and use," or in terms of "shared meaning" (Bogdan & Biklen, 1992, pp. 38-39). Definitions of culture vary, but most would consider fundamental beliefs about how one comes to know something, what counts as valuable knowledge, as well as means of communicating about what is known, to be central aspects of culture. Hence, in considering the nature of the patterns in my students' data, I was prompted to explore possible cultural differences between my lower- and higher-SES students. For my purposes here, I view culture as including beliefs about and ways of reasoning, knowing, and communicating — hence, this view includes both values and actions. My conception of how class relates to culture is that class can affect one's beliefs and ways of acting, and, therefore, one's culture. Hence, I do not view socio-economic class *as* culture, but think of culture as arising out of one's class position.

I sought literature that connected the issues raised by my analyses of the data with socio-economic class cultures. My hope was to find literature that would help me understand the differences I was seeing and help me sort out which differences might be related to class and which differences might due to

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[1] As explained previously, I primarily use the term SES when discussing my students, as I base my categorizations on some rough indicators. I will be using "class" as I discuss issues that are about class cultures and structures.

other factors, such as individual differences in temperament, beliefs, or previous achievement of the students in my small sample.

The literature I found prompted me to frame my analyses of the data in terms of class cultures. The first part of this chapter summarizes relevant literature regarding socio-economic class cultures.[2] Based on the literature, I argue that the variation in students' experiences in my classroom is due to a kind of "cultural confusion" experienced by the lower-SES students in my class.

## Survey of Literature on Class Cultures[3]

Most of the literature on class contrasts middle-class with working-class cultures. Most of my students fell into one of those two categories, with a couple of students seeming more lower-class than working class, and a couple of students falling in between. Some studies also considered race, but I tended to restrict my focus to contrasts in white cultures, since these more closely matched the demographics of my students.

The literature on class and culture is difficult to organize by topic, since the issues discussed by various authors overlap considerably. The literature I discuss below gives attention to beliefs about knowledge, discourse patterns, ways of thinking and reasoning, and views of authority in relation to self. I begin with studies that discuss, at least to some extent, why being part of a certain socio-economic class would be likely to produce specific cultural elements.

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[2] According to Banks (1988), we know little more about differences between social classes now than we did in the 1960's. I, too, found that studies of class differences seemed to fade out at the end of the 1960's. Still, I was able to find some more recent literature on the subject, and I drew from that when possible. Some of the older literature mentioned dated assumptions, such as that middle-class mothers stayed home with their children. This raises the point that more recent work on class cultures is needed.

[3] I should note that were some very interesting findings that might be indirectly relevant, but I do not include them here. For example, findings about how working- and middle-class parents are involved with schools were interesting, but did not seem directly relevant to helping us understand the trends in my data.

From there I move to more empirical studies that give less attention to reasons for links between classes and specific cultures, and instead focus on describing elements found in working-class and middle-class cultures. After summarizing the various pieces of literature, I discuss recurring themes related to class cultures.

As explained in Chapter 1, I am using occupation, education, and income as primary determinants of socio-economic class. Obviously, these three factors are heavily intertwined. Education is a major factor in determining occupation, and occupation determines income. Hence, occupations, alone, indicate much about a person's education and income. Hence, it is not surprising that occupations are often linked directly with socio-economic class.

For example, Kohn (1963, 1983) argues that middle-class occupations involve handling ideas and working with people about ideas. In relation to working-class jobs, middle-class occupations allow more autonomy and self-expression and require more self-direction and initiative. Working-class occupations tend to involve handling objects, as opposed to ideas or inter-personal relationships. Working-class employees are supervised as they conform to rigid routines established by others in authority.

To make Kohn's claims more concrete, I offer a few examples. Consider a factory worker on an assembly line, a hotel maid, or a convenience store clerk. These working-class workers would likely begin and end their work at the assigned time and follow routines established by those in charge. Their work is more physical than intellectual, offering little opportunity for creativity or autonomy. Then consider more middle-class occupations, such as teaching or middle-level management. Persons in these occupations would likely have duties assigned to them, but fulfilling those duties would allow more personal creativity and require initiative, as well as thinking about ideas and discussing ideas with other people. Upper-middle class occupations, such as medicine, law or upper-level management involve even more autonomy and intellectual work than middle-class occupations.

Anyon (1981) makes the same argument as she introduces her study on social class and school knowledge. She argues that professionals have more decision-making power and their jobs involve "more creativity, conceptualization, and autonomy" than lower-middle class or working-class people (p. 28). In her study, Anyon found striking patterns in the type of knowledge being taught in working-class, middle-class, and "affluent professional" (upper-middle class) schools.[4] The working-class schools taught students to follow orders and do rote activities, while the middle-class and professional schools taught students to be creative problem solvers — leaders instead of followers. For example, although the schools used the same mathematics textbooks (mandated by the state) the working-class schools focused solely on "the procedures or steps to be followed in order to add, subtract, multiply, or divide" and the purposes of such procedures were "unexplained" and were "seemingly unconnected to thought processes or decision making of their [students'] own" (pp. 7-8). Anyon describes an example in which the teacher gives students step-by-step directions with no rationale for making a grid on their paper, giving commands, such as "Do it this way, or it's wrong," and "Don't cut until I check." The working-class teachers described the extra, problem-solving pages in the texts as "the thinking pages" that are "too hard" and simply "extra" (pp. 7-8). Anyon noted that in interviews with the working-class children, students talked about knowledge as coming from others in authority, such as the teacher, scientists, the Board of Education or books.

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[4] Anyon also visited an "executive elite" school, but I do not discuss that here, because I am concentrating on working- and middle-classes, not upper classes.

When Anyon visited middle-class schools, she noticed "more flexibility" in the mathematics curriculum, with more of a focus on understanding processes involved with procedures. In the "affluent, professional" school, mathematics teachers emphasized discovery and direct experience in their classrooms, and students talked about knowledge as coming from their own thinking.

Anyon's study is often interpreted as a condemnation of our educational system, including teachers and administrators who might hold different expectations for different students. Yet, another possible interpretation suggested by the literature discussed below is that the system is simply being responsive to students' cultures. Granted, the end result seems to be the same — children are being educated to remain in their classes.

Basil Bernstein's (1975) theory of "elaborated" versus "restricted" codes is also relevant. He argues that linguistic codes (or the underlying principles of speech, as opposed to surface features, such as dialect) are transmitted from the class society to bodies such as families and schools, and then to individuals. He theorizes that the class system has affected knowledge distribution. According to Bernstein, the more privileged classes, who have power and tend to be individualistic, are socialized in a way that develops high-status knowledge and the language of control and innovation. Because of the emphasis on the individual, meanings are not assumed to be shared with others. Hence, middle-class families use what he calls "elaborated codes," or language with meaning that is more explicit and less tied to local contexts. This is the language of mainstream society, including school where meanings are not assumed to be shared. This, more context-independent form of language allows communication with those who do not share the same background knowledge.

Meanwhile, according to Bernstein, lower-status families use "restricted codes," or language with implicit and context-dependent meanings. This language makes sense in contexts in which an emphasis is placed on community and in which common knowledge and values are shared. According to Bernstein, individuals in lower-status families assume that the listener will understand their intended meanings, and therefore they use language that is tied to shared assumptions and experiences, instead of making the meanings explicit and, therefore, independent of common contexts.

Bernstein theorizes that working-class language is more contextualized than middle-class language, and much is theorized about differences in thinking behind the language. There have also been some empirical studies of class differences in language and reasoning, although many of these studies concentrate more on the cultural differences and less on their origins in the class structure.

One such study that can help bring to life Bernstein's distinction between elaborated and restricted codes is Heath's (1983) famous study of middle-class and black and white working-class communities, in which she found differences in family discourse patterns. The middle-class parents emphasized reasoning and discussing, as they tended to ask their children many questions, beginning with "what explanations" before moving on to "reason explanations" or affective commentaries of books, objects, or events. Through these interactions with their parents, the children "developed ways of decontextualizing and surrounding with explanatory prose the knowledge gained from selective attention to objects" (p. 56). The children learned interaction styles for orally displaying their knowledge that seemed to match the styles of their middle-class teachers who also asked questions and allowed for students to discover and display knowledge.

Meanwhile, the white, working-class parents emphasized conformity, giving their children follow-the-number coloring books and other materials that send the message, "begin at the beginning, stay in the lines for coloring, draw straight lines to link one item to another, write your answers on lines, keep your letters straight, match the cutout letter to diagrams of letter shapes . . . . " (p. 62).[5] In contrast to the middle-class parents, these parents tended to tell their children things instead of asking questions to prompt their children's thinking and explaining. They had their children sit quietly while books were read to them. Parents told their children, "Do it like this," while demonstrating a skill (such as swinging a baseball bat), instead of discussing or explaining the features of the skill or the principles behind it. Parents did not ask their children questions, except those that were "directive or scolding in nature." The children tried to mimic the action and then ask, "You want me to do it like this?", as opposed to asking, "What is that?" or "I don't understand." When frustrated, they often tried to "find a way of diverting attention" from the task" (p. 62). These children learned to be passive knowledge receivers, and they did not learn to decontextualize knowledge and then shift it into other contexts or frames. These students did well in early grades, but when they hit more advanced activities that require more creativity and independence, they frequently asked the teacher, "Do you want me to do this?" or "What do I do here?"[6]

Bruner (1975) synthesized the work of others on this subject, including Hess and Shipman (1965), whose findings were similar to Heath. In studies in

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[5] In contrast, the black, working-class families emphasize creative story telling. Like Delpit (1986), Heath describes these children as linguistically fluent and argues that they need to learn more decontextualized, factual and explanatory means of reasoning and communicating for success in school.

[6] One particularly telling example of how these two cultures clashed in my classroom involved Samantha and James working in a small group. When Samantha's groupmates were stuck on finding the volume of a cone after they had found that of the cylinder, Samantha tried to encourage them to remember what we had learned about the relationship between those two volumes (when we poured rice from one to the other):

Samantha: Just think about it. It's really easy.
James: Will you shut up and tell us!

which they observed mothers helping their children perform tasks, Hess and Shipman found that the middle-class mothers asked questions that helped focus children's attention to key features of the problem, which taught them generalizable problem-solving strategies. Working-class mothers tended to explicitly tell the child how to solve the problem, often solving it for them.

Bruner adds that middle-class mothers tend to react more to children's achievement — offering encouragement and praise, while working-class mothers react more to children's errors — attempting to correct them. In general, middle-class parents reward achievement, while lower-class parents tend to punish poor behavior. Bruner also noted a connection between these findings and that of Hawkins (1968) — lower-class children ask fewer questions and show less doubt in the presence of adults.

After reviewing the literature relating to class and language use, Bruner noted two trends. The first is using language to analyze and synthesize information during problem solving, "wherein the analytic power of language aid in abstraction or feature extraction, and the generative, transformational powers of language are used in recognizing and synthesizing the features thus abstracted." The second trend is decontextualization, or "learning to use language without dependence upon shared precepts or actions" (pp. 40-41). Decontextualization allows communication between people without common assumptions or experiences.

Why do these trends exist? Bruner proposes that the culture of poverty has a "paraochializing effect" (p. 40) that keeps language tied to context and common experiences.

> Both trends seem to reflect the kind of goal striving and problem solving characteristic of those who without protest have accepted occupancy of the bottom roles and statuses in the society that roughly constitute the position of poverty. (p. 41)

Hence, perhaps those who are not part of the more powerful mainstream tend to limit their interactions to getting by in the here and now. This seems to be the thesis of Warren Haggstrom's (1964) summary of the literature on poverty's effects, quoted by Bruner. Haggstrom writes:

> The fact of being powerless, but with needs that must be met, leads the poor to be dependent on the organizations, persons, and institutions which can meet these needs. The situations of dependency and powerlessness, through internal personality characteristics as well as through social position, leads to apathy, hopelessness, conviction of the inability to act successfully, failure to develop skills, and so on. (p. 215)

It is important to note that not all working-class people are in poverty. Yet, those in the working class do share important features with the lower-class poor in that they are in positions of low status with little power. Lack of money is not the only factor in hopelessness about one's role in society. Working a dead-end job in which others constantly have power over you would seem to produce an element of hopelessness, as well.

Bruner (1975) concludes that it is not that lower-class children can not think and talk like the middle-class children, but that they are not in the habit of doing so. He writes:

> It would seem to be the case, though I am aware of how very insufficient the data still are, that middle-class upbringing has the tendency to push the child toward a habitual use of formal categories and strategies appropriate to such categorizing — featural analysis of tasks, consideration of alternative possibilities, questioning and hypothesizing, and elaborating. (p. 39)

I only found one empirical study that focused specifically on class differences in thinking and reasoning. Holland (1981) asked children from middle- and working-class backgrounds to sort pictures of various objects. She found that middle-class children tended to categorize them in terms of trans-situational or abstract properties of the objects. For example, they would group foods together that were made from milk or came from the sea. The working-class children tended to categorize the objects in terms of context-dependent meanings, such as grouping foods that they ate for dinner the previous night. She, like Bruner, concluded that middle- and working-class children have different *orientations toward meaning,* making clear that she was not saying that children *could* not think differently, but that they had been raised with a particular orientation.

A few studies focused specifically on class differences in beliefs about what knowledge is most important. Lutrell (1989) concentrated on women's beliefs about what knowledge is valuable, and differences between races and classes. She found that for middle-class women, valuable knowledge is similar to "school knowledge." Meanwhile, for working-class women, common sense knowledge is what really matters, and this can be ruined by too much school knowledge. Common sense means relying on family and friends; it is relevant to everyday life.

Studies have found class differences in the what parents believe should be taught in school. For example, Donovan (1990) in an Australian study, found that working-class parents emphasized "basic knowledge." such as the importance of learning to accept authority, follow a moral code, and learn basic skills. They also believed that there is a division between work and play, just as there is a clear division between work and play for working-class workers. For the working class, knowledge was more black and white, including the belief that there is one correct answer to problems. Middle-class parents thought that learning should be exciting, and they placed a higher emphasis on their children having good socializing experiences in school. Just as professionals often have less of a division between their work and leisure time (e.g., they take work home to do in their spare time or read trade journals on the weekends), the middle-class parents believed in less of a division between work and play for their children. The Australian teachers talked openly about their students' social classes, including how they felt they must adapt their instruction to meet the different desires and expectations of the different classes of parents.

Also, in a study of parents' relationships with schools, Lareau (1987) found that lower-class parents thought of the teacher as a professional whose role is to give students that which they cannot, just as a doctor must treat their children. The teacher's job is to teach their children the "school-like" knowledge they need. Hence, for these parents there was a division between knowledge learned in school and knowledge learned at home. In contrast, middle-class parents thought of teachers as their social peers, and they saw themselves as equally qualified to teach their children necessary knowledge and skills. For these parents, learning at home and school was more continuous.

Differences in what parents want from schools are aligned with differences Bratlinger (1993) found in what students value in a teacher. According to Bratlinger's study of high school students, the middle-class students were more likely to view a teacher as good if she knew the subject well and was creative in her pedagogy. In contrast, the working-class students worried much more about understanding the material; for them, a good teacher is one who explains very clearly and is willing to help them understand.

As many of the studies discussed thus far suggest, there seem to be class differences in how authority is viewed. Much has been written on issues involving authority and locus of control in relation to class. The dominant theme in the literature is that middle-class people feel they have more control over their lives, while working- and lower-classes feel subject to the wills and whims of others. Banks (1988) distinguishes between an "internal" and "external locus of control." According to Banks, the middle class tends to believe that consequences result directly from their actions — this is an internal locus of control. But lower classes have more of an external locus of control — they do not see a direct connection between their own actions and consequences. Instead, they believe external forces, such as more powerful others, or just luck or chance, are at play. Banks suggests that teachers need to help these students see the relationship between their effort and their academic performance.

Duberman (1976) links issues of authority and control with child-rearing practices. Like Heath, Duberman describes middle-class families as more egalitarian, with parents who appeal to children's sense of guilt for discipline. They tend to stress motivation, self-control, initiative, curiosity, reasoning and consideration. In this way, middle-class children are more likely to be taught to think about their actions and consequences. Working-class families tend to emphasize obedience to authority, conformity, order, cleanliness, and respect. Hence, working-class children are often not taught to take initiative or to reason about their actions, but instead to obey specific rules established by others. Middle-class people tend to find pleasure in variety, believing the world is generally good and people can manipulate it. But working class people tend to prefer the "safety of sameness," believing the world is cruel and people must submit to fate (p. 120).

Duberman's findings provide a context for understanding the results of a study by Zigler and DeLabry (1962), which were similar to those of Terrell, Durken, and Wiesley (1959). They had students perform various tasks under different reward conditions. They found that intangible rewards (such as praise, a flash of light) were most effective for middle-class children, and that tangible rewards (such as candy) were more effective for lower-class children. Terrell, Durken, and Wiesley wrote:

> There is evidence to indicate that parents of middle-class children place a greater emphasis on learning for learning's sake than do parents of lower-class children (Davis, 1944, Erickson, 1947) . . . . It is possible that the

> lower-class child is too preoccupied with obtaining the material, day-to-day necessities of life to have the opportunity to learn the value of less material, symbolic incentives. (p. 271)

Motivation/Direction

Although I primarily sought out literature that specifically addresses social class differences, I also found related literature on motivation to be helpful. The literature suggests an interpretation of differences in my students' reactions to the more open CMP trial problems, including possible interactions between class and gender.

First, Condry and Chambers (1978), found that adolescent males who were given external, monetary rewards for solving problems chose to do easier tasks than those who solved problems without a reward. They found that those who were intrinsically motivated focused more on the problem solving process than the answer and developed better strategies for solving problems.

> Intrinsically motivated subjects attend to and utilize a wider array of information; they are focused on the way to solve the problem rather than the solution. They are, in general, more careful, logical, and coherent in their problem-solving strategies than comparable subjects offered a reward to solve the same problems. (p. 69)

This seems relevant, as several studies (discussed above) suggest that middle-class children tend to be more intrinsically motivated.

Covington & Omelich (1979) conducted research in support of their popular theory that effort serves as a "double-edged sword" in school achievement. They build on self-worth theory, which says that students want to maintain a self-concept of high ability. To avoid the appearance of failure, students sometimes set goals that are so easily obtained that no risk is involved. Appearing to expend effort is a threat to self-esteem. But because teachers punish the lack of effort, it becomes a double-edged sword for many students, and they must invent excuses for their lack of effort in order to avoid punishment. In their study of college students (although they cite work that suggests similar results for middle school), they found an interaction with gender: Females were more likely than males to interpret failure as evidence that they have low ability, regardless of the circumstances of the failure (with or without excuse or with high or low effort). But this was not equally true of all females — those who hold themselves in high academic self-regard were less likely than low-esteem women to perceive failure as evidence of incompetence. This pattern was <u>not</u> significant for males.

Others have written about self-esteem and gender in relation to mathematics. The AAUW (1992) reports that girls' mathematical performance and confidence tend to decline in middle school, and the drop in confidence precedes the drop in achievement. Furthermore, boys are more likely to attribute their successes in math to ability, while girls are more likely to attribute their failures to a lack of ability.

In thinking about my own students, it was the lower-SES students in general, who seemed to have lower self-esteem in the area of their math performance (as discussed previously — recall the rankings and comments about deserving their placements). Additionally, Kohr, Coldiron, Skiffington, Masters & Blust (1988) found that SES correlates with students' general self-esteem in school (more so than race or gender). In conjunction with Covington & Omelich's work, this might explain why my lower-SES students preferred drill and practice — there is little risk involved. Additionally, it was my lower-SES females who seemed to get the most personally frustrated with not doing well in the class, and this makes sense, according to Covington & Omelich and the AAUW, because these are the students who are more likely to internalize their "failure."

## Discussion

I begin a discussion of the literature by offering a summary of differences found in working- and middle-class cultures. I then discuss the culture of my classroom, and how it was similar to the culture of mathematics classrooms intended by NCTM. I then discuss differences between the culture of lower-SES students and the culture of my classroom. I propose that incongruities between the culture of my pedagogy and curriculum and the culture of my lower-SES students created cultural confusion for those students. After a brief summary of previous work on cultural incongruities, I offer an analysis of how this study fits with that work and resulting questions this study raises.

### Summary of Differences in Class Cultures

Before beginning this discussion, I want to make clear that I do not assume that individual members of a class will exhibit all traits associated in the literature with that class. Again, I am not equating class with culture, but view culture as arising out of one's class, and, therefore, people in the same class tend to share some common cultural traits. I will discuss broad generalizations about class cultures that will not hold true for all, or even any, individuals within a class. There are many other factors, besides class, that affect the behaviors and beliefs of individuals (including gender, ethnicity, age, temperament, and the particular context in which a person is acting).

At this time, I pull together the various work on class differences discussed above. As a summary of the literature, I offer Table 5.1, which I have designed specifically to highlight contrasts between working- and middle-class cultures. Some of the literature from which I drew blurred the distinction between class categories (such as the lower and working classes), and I do not try to clarify the lines or make fine class distinctions for the purposes of this study.

**Table 5.1**
**Differences in Class Cultures as Discussed in the Literature**

| Middle Class | Working Class |
|---|---|
| Jobs traditionally involve: Creativity, autonomy, control of people & ideas, intellectual work. | Jobs traditionally involve: Obedience, conformity to rigid routines, physical work. |
| In child-rearing, parents tend to: Emphasize reasoning and discussing, as well as intellectual curiosity and initiative. | In child-rearing, parents tend to: Emphasize obedience to authority, conforming with rules. |
| Guide problem-solving with questions that help focus attention to structure & details of the problem, encouraging children to solve it and learn strategies for the future. | Show or tell how to solve problems with emphasis on finding the one right solution. |
| Encourage the use of language as an instrument of analysis and synthesis in problem solving. | Encourage communicating and reasoning in a more contextualized manner. |
| Emphasize learning general, "mainstream" knowledge, including school knowledge. | Emphasize learning to follow a moral code, as well as "common sense" knowledge needed in everyday life in the immediate community. School knowledge is taught in school. |
| Help children develop an internal locus of control, believing they have control over their environment. | Help children develop an external locus of control, feeling their actions do not result in desired consequences. |
| Reward achievement with praise. | Punish poor behavior. |
| Emphasize pleasure in variety. | Emphasize safety in sameness. |
| Treat work as intertwined with play; learning is fun. | Treat work as separate from play. |
| Studies have found that students: Are oriented to abstract meanings. | Studies have found that students: Are oriented to context-dependent meanings. |
| Ask more questions and show more doubt in the presence of adults. | Ask fewer questions and show less doubt in the presence of adults. |
| Are more motivated by intangible rewards. | Are more motivated by tangible rewards. |
| Think a good teacher is one who knows the subject well and is creative in teaching it. | Think a good teacher is one who shows she cares by explaining clearly and helping them understand. |

Culture in the Mathematics Classroom

There appear to be some fundamental cultural differences relating to communicating, learning and knowing in working-class and middle-class families. These differences are important for mathematics educators to consider, because culture is central to the changes reformers are calling for in the mathematics classroom .

NCTM (1989; 1991) promotes fundamental shifts in both the culture of the classroom and the skills and dispositions required on the part of the students. NCTM (1991) writes:

> To reach the goal of developing mathematical power for all students requires the creation of a curriculum and an environment, in which teaching and learning are to occur, that are very different from much of current practice. (p. 1)

The skills students need to fulfill their roles in an NCTM-like environment involve many attributes that are culture-based. For example, NCTM (1991) states that "students' flexibility, perseverance, interest, curiosity, and inventiveness also affect the realization of mathematical power" (p. 1). As discussed in Chapter 1, NCTM calls for students to take initiative in making sense of mathematics and solving problems, instead of being told what to do by an outside authority, such as the book or the teacher. NCTM envisions students constructing their own knowledge of mathematics through their explorations of problems, instead of memorizing rules given to them by the teacher. How interested and able students are to persevere through difficulties, take initiative in their own learning, and openly acknowledge their uncertainty, are factors that can vary by culture.

In addition to learning through problem solving, students are expected to learn about ideas through mathematical discourse. As NCTM states, classroom discourse involves core cultural values about ways of communicating, learning and knowing:

> Discourse refers to the ways of representing, thinking, talking and agreeing and disagreeing that teachers and students use . . . . The discourse embeds *fundamental values about knowledge and authority*. Its nature is reflected in what makes an answer right and what counts as legitimate mathematical activity. Teachers, through the ways in which they orchestrate discourse, convey *messages about whose knowledge and ways of thinking and knowing are valued,* who is considered able to contribute and who has status in the group. (p. 20, emphasis added)

The discourse NCTM envisions conveys a view of knowledge as socially constructed. NCTM (1991) calls for a classroom in which students make sense of ideas for themselves, and in the process "acknowledge their confusions openly" (p. 49). The focus of discourse is not on finding the single correct answer or procedure, but on exploring and valuing a variety of mathematical ideas. Hence, the changes in both the curriculum and classroom discourse that NCTM (1991) advocates are, in their essence, cultural, as they involve central beliefs and norms regarding ways of knowing and communicating.

NCTM (1991) acknowledges that their vision involves fundamental changes in students' roles. They urge teachers to help students learn mathematics in these new ways by creating "*an environment in which everyone's thinking is respected and in which reasoning and arguing about mathematical meanings is the norm*" (p. 35, emphasis added). Yet, this expectation raises two difficulties.

First, when we open up our classrooms to invite students' ideas and opinions, it is seems idealistic to think that the teacher can create an environment in which all ideas are valued and differences are always respected. In his study of writer's workshop, Lensmire (1993) found that, despite his best intentions, his pedagogy allowed the cruel sides of students to be exposed when he opened up his classroom to involve students in sharing their ideas. Societal power struggles moved into his classroom and were exposed as part of the classroom discourse.

In any heterogeneous classroom, there are issues of power relating to race, class, gender, beauty, etc., that the teacher can sweep under the rug in a more traditional pedagogy, but cannot always control or suppress in a more open pedagogy.

This is particularly true in a heterogeneous classroom in which students from different cultures can have conflicting norms about how ideas should be shared and what type of knowledge is valuable. Furthermore, difficulties are especially likely to arise in mathematics classrooms, in which some ideas are definitely more mathematically correct than others. When certain students often share ideas that are, in essence, wrong, and other students are almost always correct, most students realize the difference. In fact, NCTM would want the students to realize the difference, and to scrutinize it, understand it and discuss it openly. The students in my class understood the difference: There was tremendous agreement in my classroom that Benjamin and Samantha were top mathematics students, and there was total agreement that most lower-SES students, such as Dawn and Sue, were not at the top of the class. Just as there are inequities in wider society, we reproduce similar status structures in our classrooms when we encourage all students to share their ideas openly and discuss how the ideas are or are not mathematically sensible. In a sense, we expose students' ideas and bring out differences in students' thinking and ways of knowing that are culture-based. I have no doubt that some very skillful teachers can create a culture of "niceness." But even when a teacher manages to create an environment of apparent respect within a classroom, she generally has little control over what students really think and what happens in the hallway after class.

There is a second, related difficulty raised by the above charge for the teacher. Perhaps an environment of respect for all students' thinking is contradictory to an environment in which "arguing about mathematical meanings is the norm." That is, perhaps requiring students to learn through arguing about mathematical meanings is not equally congruent with different students' preferred ways of thinking and learning.

I wanted my classroom to be one in which all students' ideas and ways of thinking were respected. It seemed reasonable to believe that open discussions (in which a variety of methods and ideas are considered) and more open-ended problems (that can be solved a variety of ways, including drawing from one's own experiences in the case of contextualized problems) would communicate to all of my students that their ways of thinking and communicating were valued. This was the message I read in much of the current reform literature. For example, NCTM's (1991) "learning environment" standard states:

> This standard focuses on key dimensions of a learning environment in which serious mathematical thinking can take place: a genuine respect for others' ideas, a valuing of reason and sense-making, pacing and timing that allow students to puzzle and to think, and the forging of a social and intellectual community. Such a learning environment should help all students believe in themselves as successful mathematical thinkers. (p. 57)

But my analyses of my classroom data and the literature discussed above suggest that the very nature of a classroom culture that expects students to share, puzzle over and make sense of mathematical ideas, conflicted with valuing some of the cultural beliefs and norms my lower-SES students brought to the classroom. In other words, by trying to maintain an open environment in which diversity of students' methods and ideas are valued equally, I might have placed a higher value on my middle-class students' beliefs and preferred ways of communicating, learning, and knowing.

In my classroom I tried to create the type of classroom envisioned by NCTM. I used a pedagogy and curriculum that reflected key, cultural changes advocated in the Standards. As the teacher in my classroom, I attempted to facilitate students' learning by asking guiding questions instead of being an authority figure who tells students exactly what to think and do. The curriculum offered variety and required taking initiative to find solutions, instead of giving many of the same problems that could be solved by following step-by-step directions. The problems and discussions required taking intellectual risks — they were most effective for students who were internally motivated and had the confidence and will to persist in problem solving instead of those who waited to be told what to think and do. Although many problems were set in a real world context, the goal was not to stay in the context but to abstract the mathematics from the contexts.

Hence, in my classroom, I tried to reflect NCTM's vision for mathematics teaching and learning in the classroom. Their vision involves cultural assumptions about ways of communicating, learning, and knowing. According to my data analyses, my lower-SES students had difficulty operating in this new environment. The literature on class cultures suggests that these students likely entered my classroom with cultural beliefs and norms that conflicted with the cultural expectations in my classroom. The culture of my classroom seemed to create "cultural confusion" for my lower-SES students.

## One Theory: Cultural Confusion

We must consider the possibility that a pedagogy and curriculum in which students are to actively explore, abstract, analyze and communicate about mathematical ideas, are more aligned with the middle class students' cultures. Up to this point, I have discussed differences in class cultures, as well as the culture of my classroom as advocated by NCTM. I agree with NCTM that the shifts being called for in the classroom are fundamental, cultural changes.

Some might suspect that the patterns in my students' experiences can be explained primarily by differences in their knowledge and skills, such as those involving "higher-order" thinking. Although this seems like a reasonable hypothesis for explaining some of the trends I saw in the data, I propose another theory that helps explain not only these trends, but why some differences in students' relevant knowledge and skills might have existed in the first place. The literature suggests that the differences I saw in my students' reactions to the curriculum and pedagogy in my classroom were due, at least in part, to differences in class cultures.

While many of my higher-SES students described how their experiences with the curriculum and pedagogy enabled them to feel more confident that they could solve new mathematical problems, my lower-SES students did not gain similar mathematical confidence. Instead, the lower-SES students often used the word "confused" to describe how the pedagogy and curriculum made them feel. Based on the data and the literature discussed above, it seems reasonable to hypothesize that my lower-SES students were suffering from cultural confusion in my classroom. Although I do not assume that cultural confusion is a complete explanation for SES-related trends in my students' data, I propose it as one plausible and helpful theory.

According to this theory, my students' class-related cultures influenced their beliefs about ways of knowing and communicating that were relevant in our mathematics class, including the curriculum and pedagogy. The higher-SES students seemed to come into my classroom with the beliefs and discursive skills necessary to succeed in the culture of my classroom. They were more confident in themselves as mathematical problem-solvers and sense-makers. The literature on class cultures suggests that these middle-class students were more likely to come from homes where they were routinely asked questions to prompt and guide discovery, as well as to promote the analysis and articulation of ideas and relationships. Hence, my and the curriculum's roles as facilitators of discovery were not entirely unfamiliar (even though it might have been different than previous math classes). The higher-SES students, praised by parents for their accomplishments while growing up, more readily took risks in discussions as well as problem solving. Instead of a singular focus on finding "the right answer" to contextualized problems, they were more likely to view the endeavor as I intended — as learning and sharing mathematical ideas and processes. Other students tended to respect their contributions in discussions. In the end, many higher-SES students found the curriculum and pedagogy mathematically empowering. Others did not.

In contrast, my lower-SES students were less confident in themselves as problem solvers and mathematical sense-makers. They were more externally motivated, and more reliant on the authority of others for knowledge construction and validation. They were accustomed to learning by listening to authority figures who told them "the right way" to think and do things. These lower-SES beliefs and habits seemed more aligned with a typical mathematics classroom environment, in which the students give and receive the right answers to problems, with the teacher as the authority for providing rules and judging right and wrong answers. In their discourse, the lower-SES students tended to be contextualized, with more of an emphasis on common sense knowledge and proof, with a black/white orientation toward knowledge. While the lower-SES students often engaged with solving the CMP trial problems set in real-world contexts, the "common sense" ways in which they approached the problems seemed to leave them missing the intended, more abstract mathematical point of the problem. Hence, although the lower-SES students would often attempt to solve the problems, they seemed to get "stuck" in the contexts, approaching the problems in ways that — although sensible — did not promote engagement with, and abstraction of, the intended mathematical ideas. These students became frustrated because they did not see how work on one contextualized problem prepared them for later problems. The lower-SES students were more often afraid of being wrong in discussions and tended to restrict their classroom participation to answering straight-forward questions for which they were sure they had the right answer. This restriction was compounded by other students' disrespectful reactions to their participation. Lower-SES students more often did not feel able to judge their classmates' contributions, and, therefore, the discussions felt confusing, as they were not sure which ideas were right. They did not feel able or motivated to make good sense of the problems and take the intellectual risks involved with delving into solving the problems. Instead of sharing my assumptions about our roles in the learning process, the lower-SES students found the curriculum and pedagogy confusing and frustrating — *dis*empowering instead of empowering.[7]

Previous Work on Cultural Congruence

I am not the first to write about cultural differences between a classroom environment and students' backgrounds. Erickson (1986), in a chapter on "Qualitative Methods in Research on Teaching," emphasizes the importance of using qualitative methods to study what he terms "cultural incongruence"

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[7] Due to my focus here on SES, I am giving much less attention to gender. But from the interactions involving SES and gender in my data, and from the literature on motivation discussed above, it seems reasonable to believe that gender would also affect students' reactions to a curriculum and pedagogy aligned with current reforms. More specifically, the work of Covington and Omelich (1979) would suggest that, when lower-SES children are immersed in a situation in which they are culturally confused and have difficulties as a result, the girls are more likely to internalize their failure and feel incompetent, while the boys are more likely to view the problem as external. In my classroom the lower-SES girls seemed much more concerned about earning high grades, and this seemed to compound their frustration when confused, as well as feelings of failure when they did not understand the intended mathematical ideas.

between students' home cultures and those in various classrooms. In summarizing the differences identified in studies thus far, he writes:

> Taken together, cultural differences between home and school that have been identified at the level of basic structural properties in the organization of interaction, and at the level of global differences in assumptions about appropriate role relationships between adults and children, involve fundamental building blocks, as it were, of the conduct of classroom interaction as a medium for subject matter instruction and for the inculcation of culturally specific values — definitions of honesty, seriousness of purpose, respect, initiative, achievement, kindliness, reasonableness. (p. 135)

Hence, according to Erickson, cultural elements are central to classroom interaction patterns. Since classroom interaction is a means to learning subject matter, we must give attention to possible incongruencies between cultures.

Erickson argues that these cultural incongruencies in classroom settings can become problematic in two ways. First, they pose extra challenges to students who must learn how to operate within the expected culture in addition to learning the subject matter under study. Those whose cultures are incongruent with that of the classroom must cope with "interactional interference" and learn extra things (the expected cultural norms) in addition to the intended subject matter (p. 136). Those students whose cultures are congruent with that of the classroom have an advantage because "the social organization is clear and familiar," and they can "concentrate more fully on the subject matter content" (p. 136).

Second, cultural incongruencies between home and school can cause students to resent the teacher and to resist learning. Erickson suggests that culture clashes can set off "a contest of wills between teacher and students in which the students refuse to learn what the teacher intends to teach" (p. 137). Erickson refers to the work of Giroux and others in arguing that this clash of wills is most likely to occur in cases where there are political dimensions to the differences in cultures, such as those involving race and class. In these situations, students are likely to resent the need to conform to the hegemonic culture, and "withhold learning as a form of resistance to teachers" (p. 138).

Erickson points to some success stories as evidence that cultural congruence between home and school environments can advance students' learning of subject matter. He refers to studies conducted in Alaska (Barnhardt, 1982) and Hawaii (Au & Mason, 1981) in which students' achievement improved when culturally congruent interaction patterns were implemented in classrooms. Erickson also discusses findings regarding highly ritualized instruction, such as DISTAR, improving the academic performance of cultural minority students, even those students whose cultures did not have similar interaction patterns (Stallings and Kaskowitz, 1974). Instead of interpreting DISTAR's success as evidence of the superiority of direct instruction, Erickson suggests that the results may be due to the ritualized quality of classroom interactions, which allows students to easily master the routine and concentrate on learning what is intended. Erickson concludes from this example that culturally incongruent teaching might be effective if the rules of the culture are very clear.

> If clarity is of the essence, and if clarity can be achieved by instructional means that are culture-specific and culturally congruent, as well as by means that are culturally incongruent, then a wider range of policy options becomes available for improving the academic performance of cultural minority students. (p. 136)

The push for culturally congruent teaching has been criticized by some. Indeed, many would claim that the mission of our public schools should be the promotion of a common culture. Furthermore, Floden, Buchmann and Schwille (1987) argue that the aim of education is to help students see beyond their own experiences. Cultural congruence, they claim, prohibits this.

> We argue that emphasizing continuity conflicts with two central goals of schooling: promoting equality of opportunity and developing

> disciplinary understanding. For, unless students can break with their everyday experience in though, they cannot see the extraordinary range of options for living and thinking; and unless students give up many commonsense beliefs, they may find it impossible to learn disciplinary concepts that describe the world in reliable, often surprising ways. (p. 485)

They go on to explain that culturally congruent teaching reinforces inequality because it reinforces the outlook and beliefs of disadvantaged students, who are raised to accept low status in society. Additionally, students often have naive assumptions about how the world operates, and these can inhibit their learning of disciplinary knowledge (e.g., scientific principles).

Discussion of Cultural Confusion and Congruence

In analyzing this debate over cultural congruence, I notice that proponents and opponents of cultural congruence seem to focus on different things. On one hand, those who advocate cultural congruence between students' home and school culture seem to focus primarily on *methods* of teaching and learning. For example, Erickson concentrated on norms for classroom interaction, such as whether students are allowed to talk at the same time or the ways in which teachers discipline students. In the examples Erickson provides, the means of communicating in the classrooms were primarily irrelevant to the disciplinary content under study. He expressed concern about classroom interaction patterns getting in the way of what teachers were intending to teach. From his perspective, it makes sense to closely match students' preferred ways of communicating, so they do not have extra, irrelevant things (communication norms) to learn that get in the way of the content being taught. Hence, it makes sense that he would promote a more culturally appropriate, and, therefore, presumably more effective means of teaching the content.

On the other hand, critics of cultural congruence focus more on the disciplinary substance being taught. These scholars argue that the purposes of education include learning new ideas and perspectives. They remind us that our *common* schools began to teach a common base of knowledge to all students. Yet, some cultural minority groups want their children to learn their own history and culture in schools. For example, some Native Americans have argued that schools should emphasize their history of struggle in the United States, as opposed to emphasizing the general history of the United States, which is often portrayed from a white, middle-class perspective with a positive spin. Critics of cultural congruence fear that if schools do not emphasize the knowledge needed to succeed in mainstream society, disadvantaged students are shortchanged, since they are least likely to learn that knowledge at home.

It seems a reasonable compromise could be struck between these two parties if culturally congruent interaction patterns are the most effective way to help students learn material that can ultimately help them break away from their limited experiences and outlook. That is, perhaps classroom interactions consistent with a student's culture will best help the student learn new ideas.

Yet, in the case of the current mathematics reforms, the situation is murkier. The changes being advocated involve more than simple interaction patterns or disciplinary concepts. As argued above, the shifts occurring in classrooms involve fundamental beliefs about learning and knowing that seem to be more aligned with middle-class culture. These shifts are more subtle, yet crucial for students' abilities to make sense of what is going on in a mathematics classroom in which open problems and class discussion are primary elements.

Additionally, the cultural elements of the curriculum and pedagogy advocated by NCTM might not be simply means to learning mathematics, but mathematical in their own right. Can we separate our intended mathematical goals from the currently advocated, culture-based means to reach the goals? Or are the means also the ends?

NCTM (1989) claims that many of the elements the class cultures literature describes as culturally biased are actually the essence of mathematical activity.

> As students progress from grade 5 to grade 8, their ability to reason abstractly matures greatly. Concurrent with this enhanced ability to abstract common elements from situations, to conjecture, and to generalize — in short, to *do* mathematics — should come an increasing sophistication in the ability to *communicate* mathematics. (p. 78, authors' emphasis)

Abstracting relationships between variables involved in real situations, exploring and analyzing mathematical relationships, making conjectures, solving elusive problems, using mathematical reasoning and proof to create new mathematical knowledge — these are all elements of what I would call "genuine mathematical activity." These are also key elements of what NCTM calls for in mathematics classrooms. These elements also seem to conflict with beliefs and behaviors that tend to be part of working- and lower-class cultures, such as following rules instead of being intellectually curious and taking initiative to explore and solve problems, or keeping language and reasoning tied to contexts instead of focusing on abstracting relationships from contexts and analyzing those relationships.

Mathematics as typically taught in schools involves little of what I would call "genuine mathematical activity." Instead of learning how to "do mathematics," students typically learn how to perform computations. One way to talk about this distinction is that in a typical classroom, instruction is geared toward helping students learn to *use* mathematics instead of *do* mathematics. Do all students need the beliefs and skills that will enable them to *do* mathematics? Or is simply learning to use mathematics enough?

There seem to be intrinsically empowering aspects of learning to *do* mathematics, even if students are not destined to be professional mathematicians. Learning to take initiative in problem solving, to believe in one's abilities to work through difficult problems, to reason mathematically and to communicate clearly about ideas, seem like inherently useful skills and values. Yet, this study suggests that bringing these elements into our classrooms can create cultural incongruencies for lower-SES students.

Erickson discusses two problems that can arise from cultural incongruencies. First, students from cultural minority groups can resent having to learn in ways more aligned with the cultural majority, and therefore "resist" learning. Second, the incongruencies can create extra, irrelevant things for students to learn, such as interaction norms.

The idea that culture-minority students resist the dominant culture and thereby contribute to the perpetuation of their low status in society is known as resistance theory (Willis, 1977). This perspective is generally viewed as an improvement on previous theories that place blame for the perpetuation of the cycle of poverty solely with either the disadvantaged groups themselves (such as genetic deficit theory or cultural deficit theory) or with the school and society (as in social reproduction theory). While resistance theory seems a sensible explanation for many cases reported in the literature, this study raises questions about how complete an explanation it is. The theory was advanced by Willis' (1977) study of working-class, secondary-school-aged, British "lads." The fact that most research on class, like Willis' study, focuses on males who are somewhat class-conscious (due to their age and/or their regional setting) is important. With my students, especially the girls, I did not see much resistance. I was not aware of any year-long "contest of wills" between any of the lower-SES girls and me.[8] My Midwestern, middle-school students seemed quite unaware of

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[8] Still, because students might want to hide their resistance from me, I do wonder if a researcher in the back of the room might have noticed resistance that I did not. Additionally, the one female who I would describe as most resistant to my teaching, Tricia, did not participate in this study. Although I do not know if Tricia was of lower-SES, she raises the issue that the students involved in this study were the students who were compliant enough to participate. Hence, the sample might be biased in this way. Still, most students did participate in the study, and those few

the socio-economic class dynamics in our classroom; in fact, the lower-SES girls internalized their struggles (assuming they were not good in math) and talked about the higher-SES students as "smart" (instead of "rich" or "advantaged"). These girls seemed to make honest efforts in my classroom as evidenced, for example, by their high rate of homework completion and the quantity of their contributions in class discussions. My study may raise some questions about the limits of resistance theory. It might not explain the poor performance of younger students, especially females, in contexts in which there are apparently low degrees of class consciousness.

Erickson's second point — that cultural incongruencies create extra, irrelevant things for students to learn — might be more significant in this case. In order for my students to learn the intended mathematics, they needed certain cultural beliefs and behaviors.

In my classroom, there was evidence that my higher-SES students were more comfortable with the new, culture-based norms for learning mathematics. My lower-SES students held beliefs about knowing, learning and communicating that seemed to inhibit their ability to learn what I intended. It was not that my lower-SES students were simply uninterested in the content or had to learn some straight-forward communication norms. The culture of my pedagogy and the curriculum called for radically different skills, beliefs, and values on the part of my students. The lower-SES students who seemed to have different, conflicting orientations and skills in relation to learning and knowing were not simply inconvenienced or annoyed. Many of these students complained often of confusion, and they had difficulty functioning and learning in the environment as I intended. My lower-SES students tended to be culturally confused in my

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students, like Sue, who showed initial anger about the CMP curriculum and my pedagogy, still made a consistent effort to succeed and eventually asked to be involved in the study.

classroom in which central cultural assumptions relating to learning conflicted with the beliefs and skills they brought to the classroom.

I have chosen to use the term "cultural confusion" to describe the result of the cultural differences in my classroom, because the term seems to more accurately portray the state of affairs for my lower-SES students. While cultural incongruencies can involve surface differences in interaction patterns or differences in the disciplinary content being taught (e.g., national history versus a local history), I see cultural confusion as resulting from a specific type of incongruency — that which involves core aspects of culture, such as beliefs about power and knowledge.

While one can argue that some cultural incongruencies in content to be learned are beneficial for students, the term "cultural confusion" implies a state of distress that needs to be addressed, particularly if the students who are most confused are those who are already disadvantaged. Cultural confusion can create roadblocks for those students who already tend to be underserved in the system. Instead of "opening up" our classrooms to a variety of ideas and ways of learning, we might be shutting down opportunities for disadvantaged students to learn.

NCTM views the changes they advocate in the classroom culture as an equitable means to mathematically empowering all students. NCTM (1991) also states, "What students learn is fundamentally connected with how they learn it" (p. 21). This statement seems sensible, but also worrisome. While "mathematical empowerment for all students" sounds like a good idea, if the required means are more aligned with middle-class culture, might our attempts to empower everybody further disempower the lower classes? Might the resulting cultural confusion for lower-SES might result in disadvantaging them in new ways? Since the goal of current reforms is to empower all students, and since mathematics achievement is a key gatekeeper for obtaining positions of power, we need to consider the reforms' potential to perpetuate disparities between lower- and higher-SES students.

In considering how to address the possibility of cultural confusion in reformed classrooms, two questions arise.

First, although simple interaction patterns can be learned, can fundamental, culture-based beliefs about authority and knowledge be re-learned in the school setting? This question is an empirical one. Even if research reveals that we *can* successfully teach these beliefs to students, we need to consider the second question: *Should* we teach them to all students?

This second question raises further issues about what is worth teaching. Are the cultural elements embedded in the reforms valuable ends in themselves that are worth teaching? If elements aligned more with middle-class culture should be taught to lower- and working-class students, then the implication seems to be that their cultures are lacking. Hence, this question about what is worth teaching raises issues related to deficit theory, or the idea that disadvantaged students do not perform well in school because their culture is deficient, as opposed to merely different. Deficit theory places a higher value on the mainstream, middle-class culture of schools. In contrast with deficit theory, I maintain that there are strengths and weaknesses in the cultures of both the middle and lower classes. For example, Corwin (1965, p. 177) summarizes some strengths and weaknesses that tend to be part of lower-class culture (see Table 5.2).

**Table 5.2**
**Some Strengths and Weaknesses of Lower-Class Culture**

| *Strengths* | *Weaknesses* |
| --- | --- |
| Cooperativeness and mutual aid that mark the extended family | Narrowness of traditionalism, pragmatism, and anti-intellectualism |
| Avoidance of the strain accompanying competitiveness and individualism | Limited development of individualism, self-expression and creativity |
| Informality | Frustrations and alienation |
| Freedom from self-blame and parental over-protection | Political apathy |
| Kids' enjoyment of each other's company | Boring occupational tasks |
| Lessened sibling rivalry | |

A similar list could be written about middle-class culture. For example, as one who has lived inside both working- and middle-class cultures, I have found middle-class people to be much more indirect and careful in their communication styles. This can be a strength in situations calling for tact, but a weakness in situations calling for candid honesty. Similarly, Bruner (1975), after a survey of literature on class differences concluded that "it's not a simple matter of deficit" (p. 41). Still, although he could see strengths and weaknesses in the culture of both the middle and lower classes, Bruner, as an educator, was particularly concerned about feelings of helplessness and hopelessness that tend to be more pervasive in the culture of the lower classes.

> Let me, in closing this section, make one thing clear. I am *not* arguing that middle-class culture is good for all or even good for the middle-class. Indeed, its denial of the problems of dispossession, poverty, and privilege make it contemptible in the eyes of even compassionate critics. Nor do I argue that the culture of the dispossessed is not rich and varied within its limits . . . . But, in effect, insofar as a subculture represents a reaction to defeat and insofar as it is caught by a sense of powerlessness, it suppresses the potential of those who grow up under its sway by discouraging problem solving. The source of powerlessness that such a subculture generates, no matter how moving its by-products produces instability in the society and unfulfilled promise in human beings. For poverty in economic life affects family structure, affects one's symbolic sense of worth, one's feeling of control. (p. 42)

Although Bruner's statement lies too dangerously close to deficit theory to be accepted by most, I tend to agree with him. There are elements assumed by my culturally biased pedagogy and curriculum that, although not strictly mathematical, were still intrinsically valuable things to learn, such as the belief in one's own ability to solve problems and the ability to work and communicate with other people. Yet, if these elements are more aligned with middle-class culture, then making these elements an important part of the mathematics pedagogy and curriculum could give the already privileged middle-class students advantages in new ways, since they would have key beliefs and skills necessary for success in our classrooms.

If, in fact, the essence of genuine mathematical activity, as well as related skills and beliefs we want to teach students, are more closely aligned with middle-class culture, then what are we to do? Can we find equitable means of reaching the desired ends, or are inequitable ends and means inextricably intertwined? Might the ends be worth pursuing despite the possibility of giving middle-class students new, different advantages in our classrooms?

Although this study does not provide answers to the above questions, the remaining chapter will further explore issues that need to be considered in addressing these questions.

# CHAPTER 6

# OBJECTIONS, QUESTIONS, AND IMPLICATIONS

I hope the previous chapters have prompted the reader to think harder about the current "mathematics for all" rhetoric, particularly as it relates to social class differences. Although inviting all students to explore, discover, create and discuss mathematical ideas might sound equitable, such an "open" pedagogy and curriculum is not necessarily aligned equally with all students' preferred ways of learning.

My analyses of the data from my classroom indicated that, in comparison with the higher-SES students, the lower-SES students had more difficulty with several cultural expectations in my classroom. While the open nature of the pedagogy and curriculum required students to confidently take initiative in solving problems and making sense of mathematics, the lower-SES students seemed to prefer being told what to think and exactly what to do. The higher-SES students seemed more comfortable with ambiguity and were more willing to take intellectual risks. The higher-SES students tended to believe in their ability to interpret open problems sensibly and decide which ideas posed in discussions were reasonable. The lower-SES students seemed to become overwhelmed by the lack of specific direction in the problems and were confused by conflicting ideas in discussions. They became frustrated, as they felt these aspects of the curriculum and pedagogy were roadblocks to learning what really mattered — the right rules and the right answers.

The contextualized problems in the CMP trial materials often engaged the lower-SES students, and they tended to delve into the context and often considered a complex variety of contextual, real-world variables in solving the problem. Yet, in doing so, these students sometimes approached the problems in ways that allowed them to miss the intended, generalized mathematical point. While the higher-SES students seemed to approach the problems and our discussions with an eye toward the larger, mathematical ideas, the lower-SES students seemed to "get stuck in the context." In our discussions, while the lower-SES students focused on finding a solution to the immediate problem at hand, the higher-SES students seemed to view our discussions as a forum for sharing and analyzing mathematical ideas. The higher-SES students seemed to have the assumed values and skills that allowed them to abstract the key mathematical ideas from the situated problems and discussions.

In general, the higher-SES students seemed to enter my classroom with more of the beliefs and skills necessary to succeed in the new environment advocated by NCTM. The lower-SES students seemed more confused by many elements of their expected roles.

The previous chapter drew from literature on social class cultures to propose that cultural incongruencies created confusion for my lower-SES students. In other words, the differences I noticed in my data were due to differences between working-class and middle-class cultures. My analyses of the literature and my data suggest that key elements of my pedagogy and the curriculum were more aligned with middle-class culture. The literature also suggests that these incongruencies are likely not limited to my classroom, since they involve general differences in social class cultures and currently popular ideas about teaching and learning mathematics.

NCTM suggests that opening up the pedagogy and curriculum to encourage all students to think about mathematical problems in ways that are sensible to them is a means of promoting equity. That is, according to NCTM, we are valuing all students' ways of thinking and knowing by using problems that can be approached in a variety of ways and discussion in which students' diverse ideas are shared and analyzed.

Theoretically, this sounds sensible. Yet, this study suggests that by asking students to take initiative in solving problems and abstracting mathematical ideas from contexts, and by emphasizing the discussion and analysis of ideas, we are valuing middle-class students' preferred ways of thinking and knowing in new ways.

But the goals of the current reforms seem beneficial, as they move away from some of the inequities of traditional schooling (e.g., tracking beginning in early grades) and move toward helping students develop the knowledge and skills needed to solve mathematical problems, critically analyze ideas, and discuss differences of opinions calmly, rationally. The previous chapter closed with several questions that are now important to consider. These questions include whether the means advocated by NCTM (and other current reformers) are the only or best way to achieve the desired ends, as well as if the ends are worth pursuing even if the result could be to perpetuate, in new and different ways, the disparity in mathematical achievement between socio-economic classes.

This study's implications for action are neither simple nor self-evident. Instead of giving clear directions for action, this chapter offers ideas and raises further questions for consideration. Therefore, instead of a straightforward presentation of implications, the chapter is organized as a discussion of questions relating to the study. Many of the questions grow out of my conversations with various education communities. I begin by discussing frequently raised objections and other questions about the study itself, including its research context and the generalizability of results. Then, assuming the validity of the study and its conclusions, I discuss a variety of questions that could be raised by those interested in making sense of what this study implies for mathematics curriculum, pedagogy, teacher education, and further research.

## Frequently Raised Objections and Questions About the Research

*In this study, you draw conclusions about class cultures, in general, while the study only involved about twenty students. You did not follow the students home and study their families' cultures. We don't know if these students were typical of lower- and higher-SES students in the United States. How can you generalize about how key aspects of the reforms are or are not aligned with various class cultures from such a small sample of students?*

If my argument about the reforms being more aligned with middle-class culture were based solely on the data from my classroom, I would worry about how representative my students were or how small the sample was. I would then say that what the study showed is one way that reforms can play out, but I would not generalize beyond my classroom. Yet, while I used my data to look carefully at students' reactions to one version of a reformed curriculum and pedagogy, I drew from the class cultures literature to help me make sense of the patterns I saw, and to argue that these patterns are likely not limited to my students alone.

There are variations around the country (e.g., urban versus rural) and around the world in terms of the cultures of different social classes. The class cultures literature from which I drew involved studies conducted in various countries and settings. From this literature, I tried to pull out the common themes that seem to be inter-woven with the very definition of being lower- and higher-SES. For example, whether parents have power in their occupations and general societal positions would be constant within an SES group, since the occupation, education and income were used as SES indicators. Hence, the data allows us see inside a classroom to view up-close what issues arise, and my analyses of these data led me to the class cultures literature, which helps us see how these issues might relate to students' socio-economic class backgrounds.

*You were one teacher with one particular pedagogy, using one curriculum. How can you draw conclusions about ideas, like those advocated by NCTM, from the case of one classroom?*

As noted in this question, my conclusions are not just about social class cultures, but their fit with some current beliefs about teaching mathematics. In earlier chapters I explained how I see the pedagogy and curriculum I used in my classroom as aligned with currently popular ideas in mathematics education, such as the use of open, contextualized problems, and the emphasis on the teacher being a facilitator of whole-class discussions about mathematical ideas instead of *the* giver of mathematical knowledge. I also argued that, although I had limited teaching experience, with the blend of knowledge and skills I brought to the teaching context, I had at least an average chance of implementing the pedagogy and curriculum in ways that would be helpful to all students. Hence, difficulties my lower-SES students faced in my classroom are unlikely unique to my implementation of the pedagogy and curriculum. Furthermore, I indicated in the previous chapter that the very ideas advocated by reformers, such as students taking initiative to be active problem solvers, creators and validators of knowledge, seem to be inherently culture-ridden. While my classroom data allow us an up-close view of how the cultural incongruencies can play out, the literature about the reforms and class cultures helps make the case that the incongruencies are not unique to my implementation of reform ideas or my students.

*For your students, this was the first year they experienced your pedagogy, and only the second year they had the CMP trial curriculum. Of course the students struggled with the implementation of these new ideas. How could you expect these methods to prove their empowering potential in such a short time? What would happen if we started teaching this way in kindergarten? Additionally, what would happen if the entire school participated in using constructivist-inspired methods for all subject areas, instead of only isolated mathematics classes?*

NCTM (1991) acknowledges that establishing classroom norms consistent with the Standards takes "hard work, especially with older students who have become accustomed to a different set of standards for school thinking and talking" (p. 45). The Standards state that good classroom discourse "does not occur spontaneously in most classrooms. It requires an environment in which everyone's thinking is respected and in which reasoning and arguing about mathematical meanings is the norm" (NCTM, 1991, p. 35). But if the struggles I saw in my classroom were just a matter of initial implementation obstacles, then these problems should have impacted my students equally. But the issue I am raising is that it might be more difficult for some children — exactly those children who have so many hurdles already — to adapt to these norms. I hope I have raised questions about possible contradictions between an environment in which, for example, "students acknowledge their confusions openly and . . . build on one another's ideas" (NCTM, 1991, p. 49) and an environment in which everyone's cultures, including their preferred ways of learning, knowing and communicating, are equally respected. The new roles for the teacher and students embed fundamental cultural norms that the literature surveyed indicates are more aligned with middle-class students' ways of operating in a learning environment.

Hence, even if all students can eventually be taught to accept new beliefs and adapt to new ways of knowing, learning and communicating, we need to consider both the normative question, "Are these norms worth teaching to all students?" and a more critical question, "What happens to the lower-SES students when these middle-class norms become a more integral part of the mathematics curriculum?" Even if we have a school-wide effort to use a more discussion-oriented pedagogy and open curriculum beginning in kindergarten, if the lower-SES students struggle more with learning through these methods, the ultimate result could be the growth of the already existing disparity between higher- and lower-SES students in their academic achievement. Certainly, if my students had begun in kindergarten, my data might have looked much different. But how it would be different is not entirely clear. Perhaps the lower-SES students would have been much more comfortable learning in my classroom, and the patterns I saw in my data would not have existed. But there is also the possibility that the lower-SES students would have begun the school year even further behind than their middle-class peers, and there might have been more students who seemed "shut down" to school altogether. This is certainly an area where more research is needed to help us understand how this might play out.

*Some of the difficulties your lower-SES students faced might be related to the socio-economic diversity in your classroom. What might happen in a more homogeneous, lower-SES classroom?*

One might not see the disparities I saw in a classroom with only lower-SES students. Perhaps such a setting would force more lower-SES students to take leadership roles in the discussions and perhaps there would be less ridiculing of low-status students like Sue (although with students this age, class is only one of many subtle variables associated with the students who are "picked on"). Still, if the methods used are more aligned with middle-class culture, disparities are likely to exist across schools. Hence, inequities might only be masked by looking in more homogeneous classrooms. Again, this is an area in which more research would be helpful.

*Now that you have addressed several questions about the research site, particularly those about your students and you as the teacher, what about you as the researcher-teacher? As you said in Chapter Two, you had a large role in creating the research site as both the teacher and the researcher in the classroom. You, being from a lower-class background, probably had ideas about what you were looking for and were able to find it because of the powerful role you played in this study.*

It is true that I had a sympathy for lower-class students because of my background, and it is true that my background was a factor in my concern about how lower-class students would fare in a reformed mathematics classroom. In fact, I previously thought that I was particularly good at relating to lower-class students, and I was surprised to realize that my pedagogy could be so alienating.

As mentioned in Chapter One, my experiences in piloting the CMP trial materials the previous year prompted me to wonder about how students' backgrounds might influence their learning in a reformed classroom. For example, I worried about Kobie, whose sister had difficulty helping him with homework. I wondered if students from lower class families were less likely to have the necessary resources at home to help with the problems in a reformed curriculum.

Additionally, when I began teaching as part of this study, there were a few episodes in which lower-SES students did not have the material resources at home necessary to complete their homework. For example, Sue and Dawn said they had no rulers at home, and Sue and Lynn said they had no calculator at home that could handle large numbers or exponents.

Furthermore, I initially wondered about the contexts we were using in the CMP curriculum. Were we, a group of primarily middle-class academics, choosing contexts that were more appealing to middle-class students? I was so concerned about this that I conducted a study about the interests of middle-school students, and how these varied by class and gender (Theule-Lubienski, Burgis, & Keiser, submitted for publication).

Thus, my initial hypotheses about class differences were related primarily to students' resources at home (such as help with homework) and students' interests in the contexts chosen. Yet, as the year progressed, I saw little evidence of differences in lower- and higher-SES students' interest in the contexts. Additionally, I saw conflicting evidence related to my hypothesis that the lower-SES students had less help at home on homework. As mentioned in Chapter 2, some of the higher-SES students talked about their parents having difficulty with some of the CMP trial problems, and some lower-SES students received help from older siblings and parents. There were clear SES patterns regarding which students did not have material resources at home, but the occasions on which this was an issue were few, and this SES disparity is not particularly unexpected or profound (yet, it is something for educators to consider). As the year progressed, I became less concerned about students' human and physical resources at home or the motivating nature of the contexts, and became more concerned about fundamental differences I noticed in the ways in which students experienced the curriculum and pedagogy. These differences led me to read more about class cultural differences.

While one might suspect I knew about working-class culture all along — after all, it is my culture, right? — I actually found my study of the literature to be incredibly enlightening about my own background. I never before realized that my mother's favorite phrase, "There ain't nothing I can do about it," might be a reflection of her socio-economic situation, as opposed to her particular personality. Likewise, I came to see my family's class reflected in many other areas, such as a focus on physical strength, as opposed to intellectual prowess (especially for males — e.g., my step-dad picking up the front end of the car to show off for my mother), authoritarian behavior (e.g., yelling and using physical force instead of discussion), and reliance on extended family members for help (e.g., waiting two years for Uncle Bill to come and fix the washing machine — by that time, the dryer was broken too). Hence, although most of what I read about lower- and working-class culture now rings true to me, I was previously unaware of it. Hence, my conclusions about incongruencies between the culture of my classroom and that of lower- and working-class cultures grew out of my analyses and were not the result of my initial hypotheses.

Once the cultural incongruency interpretation began to emerge, I tried to resist the urge to ignore all counter-evidence and proceed full-speed ahead. There was plenty of conflicting evidence in my students' data on individual students. I tended to look at overall trends, but also tried to treat counter-evidence carefully. In looking carefully at counter-examples, I often found them enlightening. For example, I noticed that the lower-SES students seemed to complain more than higher-SES students about the curriculum, but I noticed that some higher-SES students also complained. In exploring the complaints of the higher-SES students, I realized that they were qualitatively different. The higher-SES students tended to complain about very specific things, such as a particular word with which they were unfamiliar or particular problem with which they struggled, while the lower-SES students talked about general confusion and overwhelming frustration. Hence, exploring this counter-example pushed me to think beyond whether students liked or did not like the curriculum and pedagogy, and to consider the quality of students' experiences more carefully.

Thus, like every researcher, my background influenced my initial questions and concerns. Still, I was open to having my initial hypotheses challenged and changed by the data.

*If the literature was so powerful in helping you draw and generalize conclusions, then what role do your data play? Couldn't this have been a strictly theoretical study?*

If I had initially headed straight for the literature, I could have drawn many of the same conclusions I draw in this dissertation. But the data have played an important role in three ways. First, my analyses of the data led me to the literature — without the data, I would not have known to look at class-cultural differences. Second, the data allowed me to see how some of these cultural differences play out. For example, if I had only read about lower-class students' preference for reasoning in contexts, my conclusion might have been that using contextualized problems in mathematics classrooms would help these students learn mathematics. But Rose's case, for example, helped me realize the potential danger of using contextualized problems to teach mathematical ideas — namely, that lower-SES students might be more likely to approach these problems in "common sense" ways, which can allow them to miss the intended mathematical point of the problems. Finally, the students in this study bring the conclusions to life, both for me and hopefully the reader. They prompt us to care about what otherwise might have seemed like strictly theoretical mind exercises.

*You seem to see SES as the explanation for patterns in your data. But since achievement correlated with SES, couldn't students' previous achievement be the real explanatory variable?*

As discussed in Chapter 4, I have no doubt that students' previous achievement was *a* factor, but not the only factor influencing the ways in which my students experienced the curriculum and pedagogy. Anticipating that SES and achievement would likely be confounding variables, I chose my target students in a way that would help me sort out these factors as much as possible. Achievement does not explain why Rose, a diligent, intelligent student with a generally good grasp of mathematical knowledge, shared some of the same patterns of reasoning and preferred ways of working with other lower-SES students. Additionally, achievement does not explain why almost all higher-SES students, even Timothy, who was not a high achiever, viewed themselves as a top math student in the class, while none of the lower-SES students, even those viewed by others in that way, did not name themselves as a top math student. Achievement does not explain why those students who said CMP math was easier for them were all higher-SES, while those students who said math was easier for them before CMP were primarily lower-SES. Additionally, the literature suggests that what some might view as achievement differences — such as "higher order thinking" abilities — might be attributable to a difference in class cultures, as opposed to "intelligence" or "achievement." Hence, class and achievement might be confounded, but class cultures might help us understand achievement differences in a new light.

As argued in Chapter 1, work is rarely done on socio-economic class. Thus, although class is not the only variable, I have chosen to focus on it in this study. Further studies might help us better sort out which differences might be attributable to class cultures, and which might be more directly related to students' previous achievement.

*The pedagogical and curricular ideas you describe as being more aligned with middle-class culture seem to go beyond mathematics. Is what you describe applicable to all disciplines? It sounds like the problem is constructivist-inspired curricula and pedagogies. But there is much evidence to support the theory that students construct their own learning.*

I believe my findings do raise questions about constructivist-inspired methods, more generally. McDowell (1990) notes that when popular educational methods do not work well with certain race, class, or gender groups, we tend to think it is an implementation problem instead of a problem with the theoretical research base. We rarely acknowledge "the possibility that the theories that underlie the practice may not have considered relevant population characteristics" (p. 285).

Many of the aspects of my pedagogy and curriculum that I have described as aligned with middle-class culture could be considered constructivist-inspired notions. For example, the emphasis on students exploring and making sense of ideas, and the teacher as a facilitator of students' discoveries, are based on assumptions about students constructing their own knowledge.

Some others have raised concerns about these methods in other disciplines. For example, as mentioned in Chapter Five, Heath (1983) and Delpit (1986) raise concerns about indirect speaking styles in the teaching of literacy to disadvantaged students. Furthermore, Anderson (1993) studied students in a science classroom that was also taught in a constructivist-inspired manner, with students exploring and discussing scientific ideas. He found that middle and upper class students of European or Asian origins benefited the most. He wrote, "The same students who had always been left out were still being left out by our new approaches" (p. 8). After conducting case studies of students, he concluded that one of the fundamental problems that separated the students who were successful in this environment from those who were not was whether they understood the "language game of developing theoretical explanations for natural phenomena" (p. 9). The students who were not good at it were excluded from playing it, and thus from improving their performance.

Still, some aspects seem particular to mathematics. In fact, as mentioned in the previous chapter, NCTM argues that many of the processes that this study suggests are more aligned with middle-class culture, are actually part of the essence of genuine mathematical activity. Making conjectures about relationships between general (as opposed to situated in a particular, real-world context) ideas and then proving those ideas seems, indeed, to be at the heart of what professional mathematicians do. Still, many other disciplines might claim these processes as central to work in their field as well. For example, scientists explore and attempt to prove the existence of relationships, as well.

Yet, unlike most other disciplines, there is a deductive, abstract nature to mathematics. When using contextualized, open problems to help students come to understand ideas, perhaps we run the risk of lower-SES students missing the intended ideas more so in mathematics than in other disciplines. In science, if students' are working with heat and water, they are often coming to understand something about those substances. In history, if the Civil War is being discussed, students are supposed to be learning something about the Civil War. Granted, students might also be learning about the processes involved with science or history, but the real-world examples being explored are also part of the content. Yet, in my classroom, when we were discussing a problem about sharing pizzas, I did not intend for my students to learn about pizzas, and I really was not concerned about whether they could divide up a pizza in a real situation or not.

Additionally, since some evidence suggests that mathematics has traditionally been a ladder of mobility for students (e.g., AAUW, 1992; Werts, 1966), it is especially worrisome if SES disparities in mathematics achievement grow.

*It sure sounds like you are returning to a deficit model. Are you saying that lower-class students are not capable of abstract, higher-order thinking or taking initiative in solving complex problems?*

As argued in Chapter Five, I see strengths and weaknesses in working-class (and lower-class) and middle-class cultures. I am arguing that some aspects of working-class culture are not aligned with the culture of the classroom being advocated by current reforms. In that sense, the working-class culture has weaknesses *in relation to* helping students thrive in that classroom culture. I am not saying that lower-class students cannot learn to do higher-order thinking or to take initiative in problem-solving. I am saying that my analyses of the classroom data and the literature suggest that children are raised to be comfortable working and learning in ways consistent with their culture, and that the lower-SES students prefer to work and learn in ways that are incongruous with the culture of my classroom.

Whether these differences in cultures are viewed as deficits involves value judgments about what means of learning and knowing are better. As mentioned in Chapter Five, I, like Bruner (1975), am concerned about the existence of a lower-class culture of survival and hopelessness. But this hopelessness is more a poor reflection of our inequitable society, as opposed to the lower- or working-class people, themselves. Although it is popular to speak only positively about differences between cultures, I find this emphasis on the *gift* of diversity problematic. The Swedish social scientist Gunnar Myrdal (1974) writes that the American glorification of diversity is a product of "upper-class intellectual romanticism" and only serves conservative interests, as it "does not raise the crucial problems of power and money" (p. 28, Quoted in Havighurst, 1976, pp. 63-64).

*You make it sound as if the culture of lower-SES students is more aligned with typical mathematics teaching, with the teacher telling students exactly what to do, and students practicing rote computation. If this is the case, then why has SES correlated with mathematics achievement? Shouldn't the lower-SES students have an advantage in typical mathematics classes?*

There are aspects of the culture of typical mathematics classes that seem more aligned with lower-SES culture, such as the role students play in receiving "the rules" from the teacher. Still, there are a variety of factors, both inside and outside the classroom, that can affect the correlation that exists between SES and school achievement. For example, higher-SES students have parents with high expectations who are living proof of the value of putting effort into education. Lower-SES students see less evidence that their efforts have a direct pay-off. Berliner (1993, p. 6) draws from evidence provided by Reich (1991), in arguing that "higher social-class standing allows parents to buy high quality day care, preschool, and k-12 schooling; permits the purchase of instructional toys, encyclopedias and computers; and ensures first-rate health care . . . in a society that is witnessing a reduction in the standard of living for eighty percent of its people." Hence, lower-SES students also struggle in typical mathematics classrooms, when compared with higher-SES students. Still, the curriculum and pedagogy can be a help or a hindrance for lower-SES students, and this study suggests that inequities might not be helped in the way reformers intend in classrooms with a non-authoritarian pedagogy and a heavy emphasis on class discussion and students exploring open, contextualized problems.

*You have said that you see value in the goals of the current mathematics reforms. You have pointed out some of the shortcomings of typical mathematics instruction, including students being taught to follow orders mindlessly instead of make sense of mathematical problems themselves. But what is the problem? Since current reforms are promoting valuable goals for all students, then everyone wins, right? Even if some students might reach those goals sooner than others, as long as we finally have everyone on the right mathematical path, we are serving all students better, aren't we?*

This argument would be compelling if students only learned mathematics for its intrinsic usefulness and schools were not sorting mechanisms for positions in society. The problem with the argument is that it ignores the larger social context. Achievement in school mathematics can make a powerful impact on a student's future status (e.g., the AAUW reports that earning eight credits of college mathematics had an equalizing effect on men's and women's salaries).

Hence, we need to consider the possibility that methods of teaching mathematics that favor those students who are already in positions of relatively high status could perhaps exacerbate already existing inequalities. We might have to decide if it is worth heading in a more fruitful direction even if it has the potential to be more inequitable — that is, raising both the floor and the ceiling even though the distance between them grows.

Still, it is worth considering the argument that if we bring middle-class, cultural elements into our classrooms and make them part of the curriculum and pedagogy, then we are helping lower-status students gain what is needed to succeed in main-stream society. That is, perhaps the reforms can allow lower- and working-class students access to the "culture of power" (Delpit, 1986). Yet, if this is to occur, we cannot assume that students enter our classrooms with equal access to the norms of this culture. We need to give lower-SES students extra support in learning the norms. (This issue is discussed further later in this chapter.) Still, this would not counter the preceding argument that if middle-class students already have more access to the culture assumed in our classrooms, then they would seem to have an advantage in learning mathematical content that is likely to be a part of later gatekeeping.

*Are you concluding from this study that reform-minded teaching and curricula are worse for lower-SES students than traditional practices?*

No. The reader needs to remember that this dissertation is not a comparative study of traditional and reform-minded practices. I studied how some particular reform-minded practices — the use of open-ended, contextualized problems and whole-class discussion — played out with a small group of students, and how what I found contrasts with the aims and rhetoric of the mathematics reforms. This study is not a test of any particular curriculum or any particular teacher. The data I collected does not allow me to make claims about the overall narrowing or widening of gaps between lower- and higher-SES students' achievement.

Much has been written about how schools have traditionally given advantages to middle-class students through various means, such as within-school tracking (Oakes, 1985), between-school tracking of sorts (Anyon, 1981), and the general values and norms assumed by middle-class teachers and administrators (e.g., Heath, 1983; Lareau, 1987; Willis, 1977). This study suggests that key aspects of the reforms seem incongruent with lower- and working-class culture in important, yet unanticipated ways. I did not focus on other aspects of the reforms, such as small group work, that might be particularly congruent with lower- and working-class cultures. I did not focus on the possibly more equitable effects that moving away from middle-school tracking could have. Hence, while the particular practices studied in this dissertation seem to be more aligned with middle-class culture, there might be other aspects of the reforms that hold particular promise for promoting equity.

Still, that said, the results from my classroom raise concerns about key reform ideas not living up to the "mathematical power for all" rhetoric surrounding them. The study helps us understand some ways in which lower- and working-class students might struggle with learning mathematics in the ways reformers intend. This study suggests that we consider the possibility that the reforms could contribute, in new, fundamental ways, to perpetuating inequalities between classes — or at least not eliminate them as much of the reform rhetoric seems to suggest.

*Aren't you just rearranging the deck chairs on the Titanic? That is, our entire educational system only serves to perpetuate inequities in our society, so quibbling about changing pedagogical methods or curricula is a waste of time, isn't it?*

I am aware of the limitations of classroom methods for promoting equity. But does that mean we should not be concerned if methods are promoting inequity?

Coleman found in 1966 that a student's sense of control over his own fate was more related to achievement than any other background factor. Although I would like to think that we can work in classrooms to help students develop this sense of control, I know there are plenty of factors external to schools, such as whether or not students see any real evidence that schooling can pay off for them, that have a more powerful impact on students' beliefs. Berliner (1993) writes, "Education is irrelevant to those without hope, and succeeds, remarkably well, for those who have it" (p. 32). He reminds us that schools are too often blamed for the poor performance of disadvantaged students. When disparities of wealth in our society continue to increase and our educational system serves to legitimize these disparities, on a theoretical level it can seem senseless to worry about what form instruction takes in our classrooms. But on a practical level, how will society change? In working within the realities and confines of our current situation, it seems sensible to strive to educate lower-status students in a way that will allow them to use the system as effectively as possible to gain power. We can then hope that these students might see the inequities and work to create a more equitable system.

## Implications and Further Questions

*So what are we to do? If reformed means of teaching are more aligned with middle-class culture, is the solution to throw the reforms out and return to drill and kill?*

Unfortunately, this study's implications for teaching are unclear. As discussed previously, we need to consider if the goals of the reforms can only be reached by the advocated means. If the means and ends can not be separated, then we should think about whether the goals are worth pursuing regardless of the possibility of exacerbating disparities between classes. The first question has implications for further research, while the second question involves a value judgment. Hence, I cannot offer definitive answers from this study alone.

*But what is your hunch? What is your personal opinion about the goals of the reform? Are they worth pursuing?*

The over-arching goal discussed in the NCTM Standards is mathematical empowerment for all students. NCTM (1991) defines this as involving:

> the ability to explore, conjecture, and reason logically; to solve nonroutine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spatial information in solving problems and in making decisions. (p. 1)

These sound like valuable goals. Certainly, the reformers are wise to question what students have been learning through the practice of isolated computational skills in typical classrooms. I offer some questions about the goals of the reforms.

First, some of the rhetoric underlying the reformers' goals involve the development of "mathematically literate workers" (NCTM, 1991, p. 3). I tend to be skeptical of claims about businesses needing better educated workers — are these the same companies that are moving jobs to Mexico, which has much lower educational standards than the U.S.? There is contradictory evidence about the need for a more skilled workforce. While NCTM (1989) draws from evidence indicating that the need is real, Berliner and Biddle (1995) argue that the need for more technologically literate workers is a myth perpetuated by businesses who profit from an over-supply of highly educated people and who deflect any blame for their mismanagement onto the schools. In one survey they discuss, mathematical knowledge was named as one of the five least important attributes by employers, with values such as "honesty" or "follows directions" at the top of their list.

Although NCTM seems to consider the importance of businesses gaining well-educated individuals and of individuals gaining profitable employment, it seems to give little attention to the larger social and political structures at play, such as the limited number of high-status positions in our society and the tendency for middle-class students to retain their advantages through our educational system. NCTM's business-oriented goals would seem to promote the preservation of existing, inequitable structures in our society.

Still, there might be other, more compelling reasons for promoting "mathematical power for all students." But Noddings (1993) offers a different perspective. She argues that the push for high expectations and mathematics for all is "morally wrong and pedagogical disastrous" (p. 159). Noddings says we must develop caring individuals who pursue what they are interested in:

> We should not be so concerned with motivating everyone to do well in mathematics, but, rather, with giving everyone a chance to find out whether he or she is interested in doing mathematics. (p. 156)

While Noddings' ideas are refreshing, I still have concerns about how we help disadvantaged students see beyond their current situation and help them gain the skills and knowledge needed to break the cycle of poverty. Again, mathematics plays a large gate-keeping role in our society, and mathematical information is used to persuade people of many things, such as to part with their money or cast their votes. Therefore, I agree with reformers that we need to help students develop the abilities, confidence and dispositions needed to critically analyze and use mathematical information in decision-making. Hence, many of the norms this study implies are more aligned with middle-class culture seem intrinsically valuable. Additionally, as discussed above, these norms are part of the "culture of power," and therefore, from a practical standpoint, are beneficial for lower- and working-class students to learn.

*But you were arguing that these elements — the abilities, confidence and dispositions needed to use mathematics critically in solving real problems — seem to be more aligned with middle-class culture. So are you saying these things are inherently worth teaching anyway?*

Yes, I am saying these goals seem worth pursuing. Still, I worry about possible incongruencies between these elements and the culture of lower- and working-class students. Again, more research is needed on exactly what disparities exist and if the means advocated by current reformers are the most equitable means of reaching these goals.

*Well, OK, more research is needed, but you must have some opinion about what is to be done in the mean time, don't you? What would you do if you were going to teach seventh grade next year?*

In terms of pedagogy, there are a couple of options I would explore. The first entails a more careful implementation of a reformed pedagogy, and the second involves adaptations of reformed pedagogy.

If I were to use a pedagogy and curriculum like I used previously, I would attempt to address possible cultural incongruencies through explicit, cross-cultural training. As Delpit (1988) writes, making the rules of the "culture of power" explicit can help students gain power. Hence, as a teacher, I would strive to clearly explain the rationale for the classroom culture I was intending to create and make the norms for operating within that culture explicit. I would show students videos of other classrooms to model these cultural norms. In an ironic sense, I would use very direct instruction in order to get away from such direct instruction. Yet, as mentioned previously, I am not sure that the culture-based beliefs and ways of learning, knowing and communicating that are central in reformed classroom can be taught to students who live with conflicting orientations and norms at home.

I would also consider ways I might adapt my pedagogy to more closely match the expectations of lower- and working-class students, without giving up the goals discussed above. There has been a tendency to dichotomize "traditional" teaching with "reformed" teaching (Chazan & Ball, 1995). What is often called "traditional" teaching is not the only other option. Certainly, I would not advocate returning to a pedagogy that teaches students only *how* to carry out computations, with no conceptual understanding of *why* the methods work, as well as a curriculum that gives little attention to other areas of mathematics (such as geometry, statistics and probability). But we might consider other ways we can teach students to make sense of mathematics.

Secada (1992) suggests that instead of focusing on what works for "regular" students and adapting those practices for disadvantaged students, we should look first to what practices work for disadvantaged students. When examining projects known to be "successful" with disadvantaged students, I found some interesting pedagogical variations that make sense in light of what I discussed previously regarding lower- and working-class culture.

For example, Bob Moses' "Algebra Project" (1989) is known to be successful with helping disadvantaged middle-school students learn algebra. Moses found that the students felt vulnerable being put "on the spot" and having to expose their uncertainty publicly. Hence, the pedagogy used does not emphasize whole-class discussion. Instead, the teacher is a coach who answers students' questions privately, as they work through problems in their book alone and with peers. Moses also found that the main difficulty the students had in moving from arithmetic to algebra was "failure to make the generalization" of asking the concrete question, "how many" to more abstract, algebraic questions

(p. 422). Moses and he developed a five-step plan to help students avert frustration and help students move "from physical events to a symbolic representation of those events," (p. 433). The five-steps are:

1. Physical event
2. Picture or model of this event
3. Intuitive (idiomatic) language description of this event
4. A description of this event in regimented English
5. Symbolic representation of the event.

Moses advocates teaching students' problem-solving skills explicitly and helping them learn, in non-threatening ways, how to be more self-reliant learners of mathematics, including how to generalize from concrete situations.

In another interesting twist, Project SEED (Phillips & Ebrahimi, 1993) uses group discovery to help low-income and minority elementary and middle-school students learn abstract mathematics in order to promote the study of more advanced mathematics later. The students are actively involved in mathematical thinking and problem exploration, but there are several differences between SEED's methods and those advocated by NCTM. First, students do not explore open problems independently, but instead the teacher leads the entire class through the exploration, using focusing questions. Second, the students do not discuss ideas with each other, but instead offer guesses to the teacher who tells the class if the guess is right or wrong. The teacher requires students to constantly use hand signals indicating their agreement or disagreement with proposed ideas, which allows the teacher to motivate and continually assess students' participation. Finally, the problems being explored are not contextualized — abstract ideas are taught in the abstract. Project SEED 's methods have been found to be successful in improving disadvantaged students' computational skills, attitude about mathematics, and conceptual understanding.

Finally, probably the most famous program for teaching mathematics to disadvantaged students is Jaime Escalante's initiative for helping high school students prepare for the Advanced Placement calculus examinations (Escalante & Dirmann, 1990). Like NCTM, he advocates having very high expectations for all students. But his methods are very unlike those proposed by NCTM — he has tightly organized lessons that teach one concept completely before proceeding. The teacher is the complete authority in the classroom, using simple, direct instruction. The textbooks used have "a tremendous number of practice problems because practice, practice, and more practice is demanded from each student" (p. 411). He takes students on motivational field trips to see past graduates working in technological professions.

Hence, there is certainly diversity in methods known to be "successful" with disadvantaged students. Part of this diversity probably stems from differences in how "successful" is defined. Moses and SEED are concerned about students' conceptual understanding of the material and their ability to think about mathematics abstractly in order to be successful in further mathematical study. In contrast, Escalante focuses on clearing a single, standardized test hurdle, and, therefore encourages his students to complete many practice problems.

But the similarities in these examples, as well as my classroom data, suggest possible pedagogical methods for helping disadvantaged students make sense of mathematics.

For example, some evidence suggests that lower-SES students are most motivated by the potential to be affirmed by an authority figure. In my classroom, it seemed more hurtful to be corrected or proven wrong by a classmate than by me, the teacher. Hence, perhaps there is merit in maintaining a stronger teacher role. Maybe more student explorations should be teacher-directed. The few times I facilitated whole-class exploration and discovery seemed to be generally successful at maintaining students' interests and promoting understanding of intended ideas. For example, in teaching students the relationship between the volume of a cone and a cylinder, I led the class in exploring this by standing at the front of the room and demonstrating that a cylinder holds three times as much water as a cone with equal height. I involved the students in guessing what the relationship would be at the beginning of the lesson, and closed the lesson by comparing their guesses to what we found. In addition to teacher-led explorations, perhaps we need to consider possible merits of teachers giving conceptual, coherent, and interesting explanations of mathematical ideas and relationships that are both conceptual and interesting. A teacher could begin such a lesson by posing an interesting question for students to think about and then proceeding to answer the question through an explanation that would involve the students in thinking and would demonstrate, probably with the use of visual aids, the key mathematical ideas. Hence, instead of using a question and answer format and encouraging the sharing of a variety of ideas in discussions, a teacher-led exploration or explanation could maintain a tighter focus on the ideas that we ultimately want students to understand from the lesson. If the lesson begins by posing an interesting question or problem situated in a real-world context, the teacher could carefully help students focus on the abstract ideas to be generalized from the problem by explaining what the ideas mean and how they transcend the context. Such teacher-directed lessons could be powerful when they are well designed to help students intellectually dig in and understand mathematical ideas.

Still, I struggle with this issue of teacher direction, since I would ultimately like my students to move away from feeling the need to be instructed and validated by authority figures. NCTM's vision of students exploring mathematics and making sense of ideas themselves sounds more appealing in many ways. Yet, this study raises questions about who is most likely to learn most effectively in such an environment.

*You have talked at length about possible pedagogical variations. What about the curriculum?*

This study raises several issues relating to the curriculum. First, we need to consider whether one curriculum for all students is the most equitable or if tailoring the curriculum for different students' needs is the most equitable. Traditionally, "tailoring" has meant tracking, with disadvantaged students being disproportionately left in the dust in the lowest track. In this context, one curriculum for all students appears to be a progressive move toward promoting equity. Still, another proposal Stodolsky and Lesser (1967) offered three decades ago amidst their research on learning patterns in disadvantaged students of different ethnic groups, involves tailoring the curriculum to build on the strengths and weaknesses of various groups. In this way, the curriculum could give disadvantaged students the knowledge and skills necessary to cope with and change their environment. On a theoretical level, this sounds promising, but on a practical level, it poses difficulties. Gaining general agreement about strengths and weaknesses of various groups, finding those with the mathematical and cultural expertise necessary for building such a curriculum for each group, and educating teachers to teach each group would be an overwhelming enterprise. Additionally, some might raise philosophical arguments against such an undertaking because it is contrary to the purpose of the common school (Floden, Buchmann & Schwille, 1987).

Hence, although I do not advocate developing a separate curriculum for each group in our society, I think it is especially important to consider ways in which we might make curricula more helpful for lower- and working-class students, as these students, by definition, are the least powerful in our society. Here I discuss several particular issues relating to the mathematics curriculum and lower- and working-class students, including the content and the types of problems we should include in the curriculum.

My classroom data and the surveyed literature suggest that support in understanding how to reason and communicate mathematically might be particularly helpful for students from lower- and working-class backgrounds. As a specific example, some of my students needed explicit help to understand phrases used in the CMP trial materials, such as "look for patterns" and "explain your reasoning." The content of the mathematics curriculum could be expanded to include explicit attention to proof and reasoning, beginning in early grades. In the traditional mathematics curriculum, proof is "done" in the tenth grade geometry class. In a mathematics curriculum aligned with the NCTM Standards, students are expected to reason mathematically, but little is said about how we might explicitly teach students to do so. After researching students' views of evidence and proof, Chazan (1993) concluded:

> By juxtaposing different ways of knowing and by trying to help our students better understand why the mathematical community places such a value on deductive proof, we can help our students realize some of the ways in which mathematics claims to be a unique and important human endeavor, different from other human activities. (p. 385)

Since studies have suggested that many people from lower- and working-class backgrounds prefer to reason in ways that are more contextualized than generalized, giving explicit attention to general "mathematical" reasoning might be helpful for these students.

We might also consider which pieces of the mathematics curriculum might be particularly helpful for empowering lower- and working-class students. Apple (1992) writes:

> Thinking critically is not necessarily a natural occurrence. It doesn't automatically arise simply because one is told to look for problems. Rather, such an awareness is built through concentrated efforts at a relational understanding of how gender, class, and race power actually work in our daily practices and in the institutional structures we now inhabit. (p. 418)

Apple, along with Frankenstein (1990) argues that our curriculum needs to include socially critical material, such as mathematics problems that ask students to analyze the inequitable proportions of white males in positions of power.

Yet, this leads me to consider issues of curricular form. One question this study raises is the extent to which curricular problems should be set in real-world contexts. While Apple and Frankenstein advocate building the curriculum around real problems involving equity, my study suggests that this could be problematic. While I want students to be able to critically analyze real-world problems, particularly those involving inequities, I question the extent to which those problems are equitable means of building the mathematical understandings necessary for the analysis of those problems. In other words, perhaps other means should to be used to help students learn the mathematics necessary for critically analyzing real-world situations. Although it sounds efficient to use real-world problems to develop students' abilities to analyze those problems, evidence from my classroom raises the possibility that lower-SES students might have more difficulty learning the abstract, mathematical principles when taught in real-world contexts. My lower-SES students more often approached these problems in a "common sense" way and get "stuck" in the contexts, thereby missing the intended mathematical point of the problem. Hence, when real world contexts are used, perhaps the lower- and working-class students are less likely to learn the abstract, mathematical ideas that will allow them to analyze similar problems in different contexts.

Additionally, Ball (1995) suggests that teaching mathematics through real-world problems can pose difficulties because of the differences in ways in which students interpret and approach the problems, and because of the uneven access to relevant knowledge that some children have. Regarding her use of real-world problems in teaching mathematics, she writes, "The children were distracted, or confused, or the differences among them were accentuated in ways that diminished the sense of collective purpose and joint work" (p. 672) In her classroom experience, abstract mathematical contexts often seemed more inclusive of all students. Abstract contexts often seemed to give her students more of a sense of common understanding and purpose. She writes, "What seems like more abstract mathematics, unconnected to the real world, may be one step toward the reconstruction of mathematics as common property and pursuit" (p. 677). Hence, the ways in which contextualized problems can be helpful or a hindrance for teaching lower-SES students mathematics and, in general, promoting equity in our classrooms, is another important area for further research.

More fundamentally, my analyses of the classroom data, as well as the programs for disadvantaged students discussed above, raise questions about the extent to which the curriculum should be built around open problems of any sort. Does the lack of specific direction create too much frustration for those students who prefer to learn in a more teacher-directed style? Are lower- and working-class students more likely to flounder and give up, instead of learning problem-solving skills and gaining the intended feelings of empowerment? Yet, since students taking initiative in mathematical problem solving is not only a means to learning mathematics, but also a valid mathematical goal itself, resorting to simple, one-step problems does not seem to be an appealing solution.

Again, using the test, "What would I do next time?" I would likely continue to use open problems, but I would give explicit attention to helping students learn to work in such an environment. I would perhaps use an adaptation of Moses' five-step plan for averting frustration to help my students work through their difficulties in solving open problems.

*You have discussed implications and questions this study raises in terms of mathematical pedagogy and curriculum. You seem to raise many questions and offer few definitive answers. Can you say anything about implications for teacher education?*

Although I am not prepared to advocate a new pedagogy for teaching mathematics and, therefore, a new direction for mathematics teacher education, there are some implications I can discuss. Again, I draw from my own experience in educating mathematics teachers the last few years — I have struggled with, on the one hand trying to help prospective teachers teach in new ways and on the other hand, having reservations about these new ways. I can discuss ways I have found to live with this tension as well as several further questions I have.

One particular question has been moving to the forefront of my thinking over the past year, not only because of this study but because of other research I have been involved with on mathematics professional development. That is, how do we help teachers move beyond the "cute problem" syndrome and learn to help students abstract important mathematical ideas and skills from their problem explorations? In some workshops I have observed, it seemed that teachers believed that a variety of mathematical ideas would automatically be learned if students just worked on interesting problems that are in some way related to those ideas — for example, that if students made an origami swan, they would learn about angles, parallel lines, and fractions. In light of this study, this issue seems particularly important, as the lower-SES students seemed to need extra support in abstracting and connecting the intended mathematical ideas when problem solving. While I do not believe that NCTM advocates such casual notions about learning mathematics, I do see how its emphasis on problem solving can be misunderstood in this way. Hence, this study highlights the need to push prospective teachers to read the Standards carefully and to consider ways in which the teacher's role can be very active in ensuring that students learn specific ideas, even though the teacher is not always "handing down" the ideas to be learned.

Another teacher education issue I struggle with is how to have conversations with teachers about my research without validating or promoting lower expectations for lower-SES students. My argument is not that lower-SES students cannot learn important ideas, or that we should perpetuate what Anyon (1981) found in her study — that lower class students do mindless, rote tasks in school, while upper-class students learn how to critically think and manage people and ideas. One way I have found to talk about issues of class is to explain Anyon's research and to discuss strengths and weaknesses in each culture. Also, I stress that we need to help students reach important mathematical goals while not ignoring what children bring to the classroom. As discussed previously, it is important to emphasize that if a teacher suspects incongruencies between the culture of her students and the culture she is trying to establish in her classroom, she needs to work hard at helping her students learn the norms needed to thrive in her classroom.

Likewise, we have a parallel issue as teachers of college students who have some diversity of backgrounds. In our attempts to educate teachers to be reflective and critical about pedagogical possibilities, class cultures might come into play. I have certainly been frustrated by my experiences in teaching prospective teachers who tend to reason from what seems like personalized common sense and who have trouble analyzing abstract arguments. For example, I have had many students who, after reading a meta-analysis of research that indicates that calculators are not harmful, go on to conclude from one personal example that calculators are, of course, harmful without any acknowledgment of (and certainly not any indication of possible flaws in) the research with which they are disagreeing. When I teach mathematics, I am certain that I want to teach about the use of statistics and issues involved with reasoning and proof. But when I teach a methods course, I am less certain of my role in challenging students' ways of knowing.

As teacher educators try to help teachers understand and teach mathematics in non-traditional ways, their class backgrounds could help or hinder this process. NCTM calls for teachers to teach in adventurous ways and to be able to discern which students' ideas are sensible or most fruitful for further exploration and which are not. This new role for teachers implies that teachers need to know mathematics in deep and flexible ways. Yet, this study suggests that teachers from working-class backgrounds might have conceptions of knowledge and learning that conflict with that of NCTM. For example, some prospective teachers from working-class backgrounds might have more difficulty in learning approaches to mathematics teaching that de-emphasize rule-based approaches to mathematics learning, since working-class conceptions of knowledge and learning seem to be more rule-based. Also, some teachers' background cultures might place less of an emphasis on the analysis and discussion of ideas, and this could impact their ability to facilitate this in their classrooms. Hence, teachers' views of valuable mathematical knowledge and activity might hinder their ability to understand and create NCTM's vision of valuable mathematical learning experiences for students.

*NCTM is in the process of creating a second edition of the* Standards*. Given the results of this study, what changes should be made to these reform documents?*

I see this study as raising many questions that need further research before advocating major changes in NCTM's vision of mathematics teaching and learning. Yet, this study poses questions about the reforms, and some of these questions could be reflected in revisions of the Standards. The current documents tend to sound very sure of their vision and oversimplify the complexity of making the vision work for "all students." For example, NCTM (1989) states, "We are convinced that if students are exposed to the kinds of experiences outlined in the Standards, they will gain mathematical power" (p. 5).

Similarly, NCTM claims that a learning environment in which students have their ideas respected and are given time to "puzzle and think" about problems "should help all students believe in themselves as successful mathematical thinkers" (1991, p. 57). This is one of several statements made in the documents in which the term "all students" is used in a way that might sound inspiring, but probably raises more questions than it answers for many readers. Virtually every experienced teacher can think of a student who would seem to be a very likely counter-example to the statements made about what "all students" can and will do in a reformed classroom. Since the documents were written to promote a unified vision for mathematics education and create radical change in classrooms, it makes sense that little attention was given to possible difficulties of the vision for some groups of students.

If, in the next version of the Standards, NCTM would acknowledge complexities and uncertainties, the documents might promote more lasting, beneficial change for mathematics education in the long run. As the Standards are currently written, if a teacher finds that the rosy colored depictions in the documents do not ring true with her experiences, she might feel vindicated in both her conclusion that the entire vision is too idealistic and in her decision to return to "drill and kill." I have seen this type of reaction from some prospective teachers I have taught in mathematics methods courses.

This study is concerned particularly with equity issues relating to class. Noddings (1996) writes about the issues involved in trying to draw conclusions about reforms from a study of this type.

> Careful advocates of equity are often caught in a real dilemma. On the one hand, they properly wish to raise questions at the level of philosophy and culture; almost always, the questions are new to the discussion underway in math education. On the other hand, they do not want to be seen as advocating total abandonment of the program under discussion . . . . Consideration of the philosophical and cultural aspects of equity need not lead to paralysis or cynicism in mathematics reform. Things may indeed move more slowly, but more reflective movement may avoid debilitating swings of the pendulum and link mathematics education more securely to the larger social problems of the education. (p. 614)

The idea of a slower pendulum swing seems appealing in this case. This study does raise concerns that NCTM's vision could serve to further the tendency of our education system to give advantages to those students who are already advantaged. Although I have difficulty drawing concrete conclusions for changing the curriculum and pedagogy advocated by NCTM, I have some suggestions that relate to socio-economic class equity.

First, issues of class are virtually ignored in the current documents. When the Standards briefly discuss issues involving equity or opportunity for all students, they mention gender, ethnicity, and students for whom English is a second language. Class differences are virtually never mentioned.[1] In revised versions of the reform documents, socio-economic class differences should no longer be ignored.

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[1] An exception is the mentioning of differences in students' "social background" in the Assessment Standards (1995, p. 15).

Second, a revision of the documents could at least consider the issues raised in this study. Culture, including class cultures, can influence students' beliefs, values and preferred ways of learning and knowing. As mentioned before, the current documents express too much confidence that all students will thrive in the environment they advocate. Some discussion of well-known and accessible studies, such as Heath's, could help teachers be more aware of how their students' class cultures might interact with their efforts to create the type of learning environment advocated by the Standards.

Third, since the documents are tools intended to help teachers learn to teach in a new way, the issues raised above regarding teacher education should be kept in mind. A new version of the teaching standards should discuss the possibility that some students might have great difficulty abstracting what is intended from contextualized problems and that the teacher should consider taking a different, possibly more directive role to help these students.

Finally, NCTM has made some efforts to track the implementation and effects of the reforms. These studies should include attention to equity issues, particularly those involving the learning of lower-SES students, as these are the students who most need the type of empowerment envisioned by reformers.

*You have alluded often to areas that need further research. You mentioned that research would be helpful for exploring the effect of a reformed pedagogy and curriculum in early grades and in a more homogeneous classroom. You mentioned the need for research on various pedagogical and curricular means for helping students, particularly those from lower classes, achieve mathematical competence and confidence. You also noted the need for further research on possible disparities between the cultures of disadvantaged students and the culture of the classrooms we are attempting to create. Do you have anything else to add?*

As mentioned previously, we need to think critically about the current emphasis on only the positive aspects of diversity. Has cultural pluralism become an "opiate for the minorities," as Myrdal (1974) has suggested? This seems particularly relevant to issues of differences between classes, since large disparities in wealth and power between groups of people are not strictly positive. Although it is risky in the current political climate, we need to take the risk of walking closely to deficit theory in order to raise issues of class cultural strengths, weaknesses, and incongruencies.

Class needs more attention in our ongoing research. For example, the U.S. Department of Education annually publishes educational statistics in various forms. These documents give attention to students' academic performance by race and gender, but class is often ignored (e.g., National Center for Education Statistics, 1993; 1996). Similarly, initiatives for tracking the progress of mathematics education reforms should include explicit attention to the experiences of lower- and working-class students (Corbett, 1995).

Additionally, we need to study how to make use of what lower-SES students bring to school and how to help them become critical thinkers and actors. We need to understand how what students bring to school can interact with our intended goals and means of reaching those goals. In exploring possible incongruencies between the cultures of our classrooms and students' homes, we need to better understand both the classrooms' culture and that of our students. Some of the research on class cultures from which I drew was rather dated, based on the questionable assumption, for example, that middle-class mothers are full-time homemakers. It seemed popular in the 1960's to study the way in which cultural norms for thinking, learning, and communicating varied by class, but few studies have been conducted recently. Hence, we need updated research on class cultures. We must also give attention to interactions among class, race and gender.

A final thought with which to close this dissertation: Through my research, I learned that, unlike women and ethnic groups, lower-class students have no rights as a minority group. Hence, if pedagogy, curriculum or assessment methods are culturally biased toward the middle classes, there is no legal recourse for the lower classes (Pullin, 1993). Hence, it seems particularly important for the educational community to give attention to issues of class in teaching and learning. In the context of current reforms intended to empower all students, it is especially important to look critically at how the advocated methods play out with our society's least powerful students.

# APPENDIX A

# APPENDIX A

## Interviews and Surveys

(Note: Several questions were also asked about statistics-based claims media, but I do not include these here because they are not directly relevant to this study.)

First Interview:

1a. How is mathematics class different this year than 2 years ago? Than last year?

b. Do you find it easier/harder? how?

c. Do you get frustrated sometimes?

d. What do you do when you are frustrated? (Do people at home help?, etc.)

2. What way do you learn the most -- working on problems in small groups, alone, or having whole-groups discussions? Why?

3a. Do you think there are times that we have mathematical arguments in class?

b. If so, who do you think do the most arguing?

4. Do you think people can get their feelings hurt in these arguments? Why/Why not?

# APPENDIX A

## Second Interview

1a. How is mathematics class different this year than 2 years ago? Than last year?

b. Do you find it easier/harder? how?

c. Do you get frustrated sometimes?

d. What do you do when you are frustrated? (Do people at home help?, etc.)

e. Do you think you are better at math now than you were a couple years ago (when you had "regular, big books for math") Why?

f. Do you like math more or less now than before? Why?

g. Do you think you are learning more in math class this year than you did in "regular" math classes?

h. What are some things about CMP math that you think are better for you in the long run than "regular" math? Why?

i. What are some things about regular math that might be better for you in the long run than CMP math? Why?

2a. What do your parents think of CMP math? (Do they think it is better/worse than "regular math"?)

b. What do your other teachers think of CMP math? Why?

3a. What way do you learn the most -- working on problems in small groups alone, of having whole-groups discussions? Why?

b. Do you think there are times that we have mathematical arguments in class? Who does the most arguing?

c. Do you think there have been times when people have gotten their feelings hurt or felt stupid during these arguments? (If yes, press for examples)

4. Does Mrs. T.L. show favoritism to any kids or groups of kids in the class? Give examples.

5a. Do you participate much in class discussions? Why or why not?

b. Do you participate more or less in your other classes? Why or why not?

6. Do you usually do your homework for class? Why or why not? How much time do you spend on it per night?

# APPENDIX A

## Final Interview

1a. Would you say that you are the type of math student who tries to memorize formulas and procedures, or who tries to understand what you are doing (or both)? Explain why you think that. (e.g., Once you learn how to do something (for example, finding the area of a rectangle), do you try to understand why your method works? Why or why not?)

2. Do you prefer learning math by figuring things out as you explore challenging problems or by learning rules and practicing them? Explain why.

3a. Which way do the CMP Math books and I want you to learn math -- by exploring problems or by learning rules and practicing them? Why do you we want you to learn that way? (If don't know, say that we hope that by figuring rules out on your own, that you will understand them and remember them better, and also feel better about yourself as someone who really knows and can use math.)

b. Do you think it worked for you like we wanted it to? Why or why not?

4a. There were many times this year when we had mathematical discussions together. Do you like those discussions? Why or why not?

b. Did you learn from those discussions?

5a. What are you hoping to do for a career when you get older?

b. Are you planning to go to college?

c. Are you worried about how you do in school now might affect your plans?

d. Are you worried about how you might pay for college?

e. Getting in to college?

f. Are you worried you are not smart enough for college?

6. How important is the math you are learning for your later life?

7. How important is it to you that you get good grades? Why?

8. If you get a really bad grade how do you feel?

9. What percent of your time do you spend worrying about getting good grades? worrying about school in general?

# APPENDIX A

## CMP Student Questionnaire
## (Versions of this were administered at all CMP pilot sites at the beginning, middle and end of the year)

Answer the following based on **this year's math** class.

Use the following choices for 1-7:

**A=always, B=usually, C=half the time, D=seldom, and E=never**

1. We use calculators in our math class.
2. We write about our ideas in math class.
3. We use things like blocks, spinners, graph paper, or rulers in math class.
4. We spend most of our class period practicing computation.
5. My math teacher encourages us to find different ways to solve the same problem.
6. We talk about how different ideas in math are connected to each other.
7. The material covered in math class is new to me.

Use the following choices for 8-11:

**A=strongly agree, B=agree, C=not sure, D=disagree, E=strongly disagree**

8. My math book helps me understand the problems.
9. I feel confident that I can solve math problems.
10. I like discussing math problems in groups.
11. Math problems can only be worked one way.

For items 12-14, choose the best answer.

12. When we have trouble with a math problem, our teacher
    a. tells us the answer
    b. shows us how to do it
    c. encourages us to figure it out for ourselves.

13. When we work problems in math class, we
    a. usually work in groups
    b. work in groups about half of time
    c. usually work by ourselves

14. On an average day, I spend about ____ minutes a say on math homework.

    a. less than 15  b. 15 to 30  c. 30  d. 45  e. more than 45

## APPENDIX A

15. What is the most interesting idea you have learned in math this year?

16. How do you know when you understand a math idea and when you don't.

17. What else would you like to tell the writers of the Connected Mathematics Project?

# APPENDIX A

## Show What You Know
(A CMP survey adapted for my classroom -- This is one of the two "Show What You Know's I used with my students)

Since Winter Break, we have completed two units — Filling and Wrapping and Similarity. I would like you to write about the ideas you have learned in these units and what things helped you learn these ideas.

1. Think about the problems in these units. What are the most important mathematical ideas that the units tried to teach?

Filling and Wrapping:

______________________________

______________________________

______________________________

Similarity:

______________________________

______________________________

______________________________

2. What are the most interesting mathematical ideas have you learned about in these units? Explain what things that we did in class or that you did on your own that helped you learned the most about these ideas. (Explain in detail!— Give examples of what you mean.)

Filling and Wrapping:

______________________________

______________________________

______________________________

Similarity:

______________________________

______________________________

______________________________

3. What things are you still struggling to learn? (Be specific! Give examples of what you mean.)

______________________________

______________________________

______________________________

## APPENDIX A

4. What ways did you contribute to classroom discussions about mathematics? Do you think you need to work more on this? Why or why not?

______________________________________________________________

______________________________________________________________

______________________________________________________________

5. On a scale of 1 to 10 (with 10 being the highest) explain how much the following activities help you learn new mathematical ideas: (BE HONEST)

Working on problems with my classmates
1 2 3 4 5 6 7 8 9 10

Working on problems by myself
1 2 3 4 5 6 7 8 9 10

Working on problems with my teacher and the whole class
1 2 3 4 5 6 7 8 9 10

Having class discussions about problems that I have already worked on
1 2 3 4 5 6 7 8 9 10

Having the teacher give examples of how to do problems
1 2 3 4 5 6 7 8 9 10

Having a whole-class discussion about mathematical ideas (with my teacher)
1 2 3 4 5 6 7 8 9 10

Discussing (or arguing about) mathematical ideas with my classmates (without my teacher)
1 2 3 4 5 6 7 8 9 10

6. What things about mathematics class (including the textbooks, the teaching, the class activities...) are the most frustrating for you? Explain and give examples!

______________________________________________________________

______________________________________________________________

What do you do when you get frustrated?

______________________________________________________________

______________________________________________________________

# APPENDIX A

## Final Survey

1. Here are the names of the units we did this year. Tell me how much you liked each unit by rating each one on a scale of 1 to 10 (with 10 the highest).

______ *Around Us (big numbers -- rounding, exponents, scientific notation)*

______ *Variables and Patterns (graphing, algebra)*

______ *Filling and Wrapping (volume and surface area of shapes)*

______ *Similarity (similar shapes, scale factors)*

______ *What are the Chances? (probability)*

______ *Accentuate the Negative (negative numbers)*

______ *Comparing and Scaling (ratios, fractions, etc.)*

Which unit was your favorite? Why? (be specific)

Which unit did you dislike the most? Why? (be specific)

In which unit did you learn the most important ideas? Explain why you think those ideas are important.

In which unit did you learn the least? Why?

We had many class discussions this year. What class discussion do you remember the most? Why do you remember that particular discussion? Explain.

2a. On a scale of 1 to 10 (with 10 highest) how much did you like math before this year?
1 2 3 4 5 6 7 8 9 10
Why (be specific)?

b. On a scale of 1 to 10 (with 10 highest) how much do you like math this year?
1 2 3 4 5 6 7 8 9 10
Why (be specific)?

c. On a scale of 1 to 10 (with 10 highest), how good are you at math?
1 2 3 4 5 6 7 8 9 10
Why do you think that?

## APPENDIX A

3. What way do you learn the most -- working on problems alone, in small groups, or having whole-class discussions? Why?

4. Do you think there are times that we have mathematical arguments in class? If so, which students do you think have done the most arguing?

5. Have there been times when you or other people have gotten their feelings hurt or have felt stupid during whole class discussions? If so, give one or two examples.

6. Have there been times when you or other people have gotten their feelings hurt or felt stupid during small group discussions? If so, give one or two examples.

7. Does Mrs. T.L. show favoritism to any kids or groups of kids in the class? Explain.

8a. Do you participate much in class discussions? Why or why not?

b. Do you participate more or less in discussions in your other classes? Why?

9. What are your average grades this year in:
Science___ Grammar____ Language Arts____ Social Studies____ Math____

10. What is the average amount of time you spend on your math homework each night?

11. What is the average amount of time you spend on your homework for other classes each night?

12a. Do you have a computer at your house?

b. Do you have a graphing calculator at your house?

c. Do you have a regular calculator at your house?

d. When you work on your math homework at home, can you find whatever materials you need quite easily? (For example, paper, pencils, rulers, a quiet place to work, etc.)
If not, explain the types of things you have had any trouble with.

## APPENDIX A

13. Answer these questions about math class this year by circling the best number.
**0 = Never 1 = Seldom 2 = Half the time 3 = Usually 4 = Always**

| | | | | | |
|---|---|---|---|---|---|
| a. When you get an answer to a math problem, do you try to figure out if the answer makes sense? | 0 | 1 | 2 | 3 | 4 |
| b. When you work on a math problem, do you feel like you know what you are doing? | 0 | 1 | 2 | 3 | 4 |
| c. When you work on a tough problem, do you feel like you enjoy the challenge of solving it? | 0 | 1 | 2 | 3 | 4 |
| d. When you work on a tough problem, do you feel like giving up? | 0 | 1 | 2 | 3 | 4 |
| e. Once you learn how to do something (for example, if you learn how to find the area of a rectangle), do you try to understand why your method works? | 0 | 1 | 2 | 3 | 4 |
| f. When you get a bad math grade, do you feel like you are stupid? | 0 | 1 | 2 | 3 | 4 |
| g. When you get a bad math grade, do you feel angry at your teacher? | 0 | 1 | 2 | 3 | 4 |
| h. When you get a bad math grade, do you feel angry at the CMP books (or authors)? | 0 | 1 | 2 | 3 | 4 |
| i. When you get a bad math grade, do you feel really depressed? | 0 | 1 | 2 | 3 | 4 |
| j. When you work on your math homework, how often do you get stuck? | 0 | 1 | 2 | 3 | 4 |
| k. When you get stuck on a math problem, do you feel angry at your teacher? | 0 | 1 | 2 | 3 | 4 |
| l. When you get stuck on a math problem, do you feel angry at the CMP books (or authors)? | 0 | 1 | 2 | 3 | 4 |
| m. When you get stuck on a math problem, do you feel really depressed? | 0 | 1 | 2 | 3 | 4 |
| n. When you get stuck on a math problem, do you feel really frustrated? | 0 | 1 | 2 | 3 | 4 |
| o. When you get stuck on a math homework problem, do you ask a parent (or guardian) for help? | 0 | 1 | 2 | 3 | 4 |
| p. When you get stuck on a math homework problem, do you ask a brother or sister for help? | 0 | 1 | 2 | 3 | 4 |

## APPENDIX A

q. When you get stuck on a math homework problem, do you ask your friends for help? 0 1 2 3 4

r. When you get stuck on a math homework problem, do you ask Miss Mattel for help? 0 1 2 3 4

s. When you get stuck on a math homework problem, do you ask Mrs. T.L. for help? 0 1 2 3 4

14. Has the way that you have dealt with getting stuck on math problems changed over the course of the year? If yes, how has it changed and what made you change?

15. Would you say that you are the type of math student who tries to memorize formulas and procedures, or who tries to understand what you are doing (or both)? Explain why you think that.

16. Do you prefer learning math by figuring things out as you explore challenging problems or by learning rules and practicing them? Explain why.

17. What makes you work hard (or not work hard) in this math class?

18. What do you plan to do after high school? Do you plan to go to college? What kind of career do you want to have? Why?

19. <u>On a scale of 1 to 10, with 10 the highest</u>:

How important is getting good grades to <u>you</u>? 1 2 3 4 5 6 7 8 9 10
Explain why:

How important is getting good grades to your <u>parents</u>? 1 2 3 4 5 6 7 8 9 10
Explain why:

How important is getting good grades to your <u>friends</u>? 1 2 3 4 5 6 7 8 9 10
Explain why:

How important is the math you are learning for your life later on?
1 2 3 4 5 6 7 8 9 10
Explain why:

## APPENDIX A

20a. There are 27 students in our class. If you numbered all the students from best in math to worst in math, with #1 being the best, and #27 being the worst, which number would you be? (For example, if you think you are the third best math student in the class, you would be #3.)

____________________

b. Who do you think are the 3 best math students in our class?

#1_______________ #2___________________ #3__________________

What makes you think they are the best math students?

21a. What math will you be taking next year?

b. How do you feel about the math class you were placed in for next year? Why?

c. How do you feel about <u>not</u> having CMP math next year? (Circle one.)

VERY SAD A LITTLE SAD SORT OF GLAD VERY HAPPY

Explain why. Be sure to tell me what you like about CMP math and what you like better about regular math:

# APPENDIX B

# APPENDIX B

## EXAMPLE OF GRID FOR CODING EACH CONTRIBUTION TO DISCUSSION

| General Content | Name Of Student | Refer To Others | | Context | | | | |
|---|---|---|---|---|---|---|---|---|
| | | AG= agree DIS= disagree<br>B= build on ideas<br>R= revise own ideas based on other's<br>CL= ask for clarification<br>CO= compare own and other's idea<br>P= push for rationale<br>ANS= answer student's question<br>H= help in other ways N= nice-compliment M= mean-insult | Who is being referred to? | ASK=ask qstn<br>Blank= answer qstn<br>I= offer idea | Volunteer?<br>Y = yell out<br>VV= volunteer out of blue<br>Blank= volunteer when I ask<br>NV - not volunteer | RG = report group's work<br>Blank = speak for self | What part of the lesson?<br>L = launch<br>S = summary<br>H = homework<br>U = unit wrap up<br>Blank = none of the above | The context is:<br>Blank = attempted real world<br>RRW= really real world<br>AB = abstract ideas<br>F= fantasy |
| ANSWERS | | | | | | | | |
| About answer they got for a problem | Rod | | | | | | h | ab |
| | Mark | h | Sue | | | | h | ab |
| | Rod | | | | | | h | ab |
| | Rod | | | | | | h | ab |
| | Nick | | | | | | | ab |
| | James | | | | y | | | |

| Name Of Student | Tone | | | Medium | Lang. | Proof | Rel. To Past | Math Value | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Degree of confidence<br>Blank= avg<br>1 = very hesitant<br>5 = very confident | Articulate?<br>Blank = avg.<br>1= problems getting idea across<br>5= remarkably articulate | When questioned/challenged, do they:<br>Blank = not challenged<br>BD-S# or DEF-S# or CL-S# back down, defend (with argument), or clarify when challenged by:1=low-SES female, ...4=high-SES male<br>BD, DEF or CL-T when I qstn them | Use pictures or symbols when making contribution?<br>RP = refer to a picture<br>DP = draw picture<br>RM - refer to manipulative<br>S= symbol (e.g., write #'s on board)<br>Bank = words only | G= Generalized<br>C = contextualized<br>Blank = in between or unclear | Proof<br>G = gnl proof<br>P = proof by pattern<br>E = one ex.<br>CS= common sense<br>R= deference to rule<br>Blank = "normal" - giving gnl expln using the context<br>NA= no arg. or exp. | Where it came from<br>R= Recall what was learned in past<br>AW= applyi to world/their lives<br>N= new math insight<br>Blank= apply what was learned in past to new problem | Degree of relevancy to math agenda<br>1 = low<br>5 = high | Degree of correctness<br>1 = Totally wrong<br>5= Totally correct<br>Blank = unknown | How tough is the question being answered or idea being discussed<br>1 = very easy--all st. Should be able to do<br>5= tough - few able to do | How insightful --<br>1 = not at all<br>5 = very |
| Rod | | | | | | r | r | 5 | 5 | 1 | 2 |
| Mark | | | bd-s4 | | | r | | 5 | 3 | 2 | 2 |
| Rod | | | | | | r | | 5 | 5 | 2 | 2 |
| Rod | | | | | | r | | 5 | 5 | 2 | 2 |
| Nick | | | | | | r | r | 5 | 5 | 1 | 2 |
| James | | | | | | | | 5 | 1 | 3 | 2 |

# APPENDIX B

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| About if an answer is correct or makes sense | Tim | dis | Mark | | y | | h | ab | 4 | | | | g | r | | 5 | 5 | 3 | 3 |
| | Harriso | | | | | | h | ab | | 2 | | | | r | | 5 | 2 | 4 | 2 |
| About why an answer is wrong | Will | dis | me | I | | | s | | 5 | | | | | | | 5 | 5 | 4 | 4 |
| About how to get an answer (focus on answer, not method) | | | | | | | | | | | | | | | | | | | |
| About how to interpret an answer - what does it mean? | Andrea | r | Willia | I | vv | | s | | | | | | | | | 5 | 5 | 3 | 3 |
| | Saman | b | Willia | | | | s | | | | | | | | | 5 | 5 | 3 | 3 |
| | Andrea | b | Sama | I | vv | | s | | | | | | c | cs | | 2 | 5 | 3 | 3 |
| Giving a quick calculation from head or calculator | | | | | | | | | | | | | | | | | | | |
| Guessing about a possible answer to a problem | | | | | | | | | | | | | | | | | | | |
| PROCEDURES/FORMULAS | | | | | | | | | | | | | | | | | | | |
| About their method of solving a problem (focus on method, not answer) | Sue | | | | NV | | h | ab | | | | | | | | 5 | 1 | 2 | 2 |
| | Nick | | | | | | h | ab | | | | | g | r | r | 5 | 4 | 3 | 2 |
| | Guinev | dis | Jame | | | | | ab | | | cl-t | | | | | 5 | 5 | 3 | 3 |
| | Anne | | | | | | | | | | | | | | | 4 | 5 | 3 | 3 |
| | Benjam | co | Anne | | | | | ab | | | cl-t | | | | | 5 | 5 | 4 | 4 |
| | Benjam | | | | | | | | | | | | | | | 5 | 5 | 3 | 3 |
| About if a method is correct/wrong | | | | | | | | | | | | | | | | | | | |
| About if/how a procedure /method makes sense | Rose | | | I | vv | | h | ab | | | | | g | r | | 4 | 2 | 4 | 2 |
| | Guinev | | | I | | | | ab | | | | | g | g | n | 5 | 5 | 5 | 5 |
| A factor or info. that must be considered when solving problem | Sue | | | ask | | | I | | | | | | c | cs | | 5 | 3 | 3 | 3 |
| | Andrea | | | ask | | | I | | | | | | | | | 5 | 4 | 3 | 4 |
| About interpreting math info. (not directly part of problem solving) | | | | | | | | | | | | | | | | | | | |
| About how to do a more general procedure (e.g. general formula or rule) | Harriso | | | | NV | | s | ab | | | def-s4 | | | | | | 1 | | |
| | Will | | | | | | s | ab | | | | | | | | 5 | 5 | 4 | 4 |
| About where a procedure came from - e.g., how derived | | | | | | | | | | | | | | | | | | | |
| Asking where in mathematics this procedure/formula applies (e.g., will this always work?) | | | | | | | | | | | | | | | | | | | |

# APPENDIX B

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| About where the procedure is used in the world | | | | | | | | | | | | | | | | | | | |
| EXPLORING PROBLEMS | | | | | | | | | | | | | | | | | | | |
| About an interpretation of a problem | | | | | | | | | | | | | | | | | | | |
| About how to explore a problem situation | | | | | | | | | | | | | | | | | | | |
| About why something happens in an exploration/experiment | | | | | | | | | | | | | | | | | | | |
| About existence of a pattern | | | | | | | | | | | | | | | | | | | |
| About explanation of a pattern | | | | | | | | | | | | | | | | | | | |
| Offering numbers to use in a problem as an example | | | | | | | | | | | | | | | | | | | |
| CONCEPTS | | | | | | | | | | | | | | | | | | | |
| About what a concept means or is about (inc. what is defn.?) | Guinev<br>Sam<br>Rose | dis | Rose | | | | h<br>h<br>h | ab<br>ab<br>ab | 5 | | | | g | g | R | 5<br>5<br>5 | 4<br>5<br>4 | 4<br>3<br>4 | 4<br>4<br>4 |
| About what a definition means | | | | | | | | | | | | | | | | | | | |
| If a definition makes sense | | | | | | | | | | | | | | | | | | | |
| About where a concept came from - who/how discovered? | | | | | | | | | | | | | | | | | | | |
| About where a definition came from - who/how defined? | | | | | | | | | | | | | | | | | | | |
| About where in mathematics the concept applies/connects | | | | | | | | | | | | | | | | | | | |
| About where the concept is used in the world/their lives | Rose | | | | | | h | | | | bd-s3 | | | | | 4 | 2 | 3 | 2 |
| MISCELLANEOUS | | | | | | | | | | | | | | | | | | | |
| Very general "I'm confused" | | | | | | | | | | | | | | | | | | | |
| Recalling past info. obtained upon by the class | | | | | | | | | | | | | | | | | | | |

# APPENDIX B

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Making comments about class or discussion, but not about the *math* | Andrea Lynn | | | ask | | | | | | | | | | | | | | | |
| Making comments about doing or *not* doing their work or homework | | | | | | | | | | | | | | | | | | | |
| Clarifying expectations | Rose | | | | | | | | | | | | | | | | | | |
| About the workings of the class (e.g., answering the question "Who reported last time?") | | | | | | | | | | | | | | | | | | | |
| Questions about how to *grade* something | Sue | | | | | | | | | | | | | | | | | | |
| Complimenting another person (*not* their math ideas) | | | | | | | | | | | | | | | | | | | |
| Ridiculing another person (not their math ideas) | | | | | | | | | | | | | | | | | | | |
| Reading from book for the class | Dawn | | | | | | | | | | | | | | | | | | |
| Making jokes | David | | | | | | | | | | | | | | | | | | |
| Miscellaneous comments | Nick | | | | | | | | | | | | | | | | | | |
| Miscellaneous questions | | | | | | | | | | | | | | | | | | | |

# BIBLIOGRAPHY

## Bibliography

American Association of University Women (1992). How school's shortchange girls. Wellesley, MA: Wellesley College Center for Research on Women.

Anderson, (1993). Teaching Content in a Multicultural Milieu. Paper presented at the Symposium: Didactics and/or curriculum, Institut für die Pädagogic der Naturwissenschaften (IPN), Kiel, Germany.

Anyon, J. (1981). Social class and school knowledge. Curriculum Inquiry, 13, 3-42.

Apple, M.W. (1992). Do the Standards go far enough? Power, policy, and practice in mathematics education. Journal for Research in Mathematics Education, 23(5), 412-431.

Au, K.H., & Mason, J. (1981). Social organizational factors in learning to read: The balance of rights hypothesis. Reading Research Quarterly, 17(1), 115-152.

Balacheff, N. (1988). Aspects of proof in pupils' practice of school mathematics. In D. Pimm (Ed.) Mathematics, teachers and children (pp. 216-238). London: Hodder and Stoughton.

Ball, D.L. (1990). Reflections and deflections of the framework: The case of Carol Turner. Educational Evaluation and Policy Analysis, 12(3), 247-260.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. Elementary School Journal, 93(4), 373-397.

Ball, D. L. (1995). Transforming pedagogy: Classrooms as mathematical communities. A response to Timothy Lensmire and John Pryor. Harvard Educational Review, 65, 670-677.

Ball, D. L. (in preparation). Working on the inside: Designing to use one's own practice as a site for studying mathematics teaching and learning. Manuscript in preparation, University of Michigan, Ann Arbor.

Ball, D. L. & Wilson, S. W. (1996). Integrity in teaching: Recognizing the fusion of the moral and the intellectual. American Educational Research Journal, 33, 155-192.

Banks, J. A. (1988). Ethnicity, class, cognitive, and motivation styles: Research and teaching implications. Journal of Negro Education, 57, 452-466.

Barnhardt, C. (1982). "Tuning-in": Athabaskan teachers and Athabaskan students. In R. Barnhardt (Ed.), Cross-cultural issues in Alaskan education (Vol. 2). Fairbanks: University of Alaska, Center for Cross-Cultural Studies.

Becker, P. H. (1993). Common pitfalls in published grounded theory research. Qualitative Health Research, 3, 254-260.

Berliner, D. (1993). Educational reform in an era of disinformation. Education Policy Analysis Archives, Tempe, Arizona.

Berliner, D.C. & Biddle, B.J. (1995). The Manufactured Crisis: Myths, Fraud, and the Attack on America's Public Schools. New York: Addison-Wesley.

Bernstein, B. (1975). Class, codes and control (Vol. 3). Boston: Routledge & Kegan Paul.

Bishop, A. (1994). Cultural conflicts in mathematics education: Developing a research agenda. For the Learning of Mathematics, 14(2), 15-18.

Bogdan & Biklen (1992). Qualitative research for education: An introduction to theory and methods. Boston: Allyn and Bacon.

Borba, M.C., (1990). Ethnomathematics and Education, For the Learning of Mathematics, 10(1), 39-43.

Bratlinger, E.A. (1993). Politics of class in secondary schools: Views of affluent and impoverished youth. New York: Teachers College Press.

Bruner, J. S. (1975). Poverty and childhood. Oxford Review of Education, 1, 31-50.

Campbell, P. B. (1991a). Girls and math: Enough is known for action. Women's Educational Equity Act Publishing Center Digest, June, 1-3.

Campbell, P. B. (1991b). So what do we do with the poor, non-white female? Issues of gender, race, and social class in mathematics and equity. Peabody Journal of Education, 66, 95-112.

Chazan, D. (1993). High school geometry students' justification for their views of empirical evidence and mathematical proof. Educational Studies in Mathematics, 24, 359-387.

Chazan, D. & Ball, D. L. (1995). Beyond exhortations not to tell: The teacher's role in discussion-intensive pedagogy. (Craft Paper 95 - 2). East Lansing: Michigan State University, National Center for Research on Teacher Learning.

Cochran-Smith, M., & Lytle, S. L. (1990). Research on teaching and teacher research: The issues that divide. Educational Researcher, 19(2), 2-11.

Cohen, D.K., & Ball, D.L. (1990). Relations between policy and practice: A commentary. Educational Evaluation and Policy Analysis, 12(3), 331-338.

Condry, J.C. and Chambers, J.C. (1978). Intrinsic motivation and the process of learning. In M. R. Leper & D. Greene ( Eds.), The hidden costs of reward: new perspectives on the psychology of human motivation. Hillsdale, New Jersey, Lawrence Erlbaum Associates. 61-82

Connected Mathematics Project, (1975). Getting to Know CMP: An Introduction to the Connected Mathematics Project. East Lansing: Michigan State University.

Connell, R.W., Ashenden, D., Kessler, S., & Dowsett, G. (1982). Making the Difference. George Allen and Unwin, Boston.

Corbett, D. & Wilson, B. (1995). Make a difference with, not for, students: A plea to researchers and reformers. Educational Researcher, 24(5), 12-17.

Corwin, R.G. (1965). A Sociology of education: Emerging patterns of class, status and power in public schools. New York: Meredith Publishing.

Covington, M.V. & Omelich, C.L. (1979). Effort: The double-edged sword in school achievement. Journal of Educational Psychology, 71(2), 169-182.

Crossen, C (1994). Tainted Truth: The Manipulation of Fact in America. New York: Simon and Schuster, 1994.

Damarin, S. (1990). Teaching mathematics: A feminist perspective. Teaching and Learning Mathematics in the 1990s (Yearbook of the National Council of Teachers of Mathematics, pp. 144-151), Reston, VA: NCTM.

Davis, A. (1944). Socialization and adolescent personality. Adolescence, (43rd yearbook of the National Society for the Study of Education, Part I, pp. 198-216), Chicago: National Society for the Study of Education.

Delpit, L. D. (1986). Skills and other dilemmas of a progressive black educator. Harvard Educational Review, 56(4), 379-385.

Delpit, L. D. (1988). The silenced dialogue: Power and pedagogy in educating other people's children. Harvard Educational Review, 58(3), 280-298.

Donovan, B.F. (1990). Cultural power and the defining of school mathematics. Teaching and Learning Mathematics in the 1990's (Yearbook of the National Council of Teachers of Mathematics, pp. 166-173), Reston, VA: NCTM.

Duberman, L. (1976). Social inequality: Class and caste in America. Philadelphia, J.B. Lippincott.

Edwards, D. (1993). But what do children really think?: Discourse analysis and conceptual content in children's talk., Cognition and Instruction, 11(3) 207-225.

Erickson, F. (1986). Qualitative research on teaching. In M.C. Wittrock (Ed.), Handbook of research on teaching (3rd ed.) (119-161). New York: MacMillan.

Erickson, M.S. (1947). Social status and child rearing practices. In T.M. Newcomb & E.L. Hartley (Eds.), Readings in social psychology. (pp. 494-501). New York: Holt.

Escalante, J, & Dirmann, J (1990). The Jaime Escalante Math Program. Journal of Negro Education, 59 407-423.

Fennema, E. & Leder, G. (1990). Mathematics and gender. New York: Teacher's College Press.

Fennema, E. & Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization and socio cultural factors. American Educational Research Journal, 14(1), 51-71.

Fennema, E. & Sherman, J. (1978). Sex-related differences in mathematics achievement and related factors: A further study. Journal for Research in Mathematics Education, 9(3), 189-203.

Ferrini-Mundy, J. (1993). School mathematics and multiple realities: new paradigms for assimilation. Journal for Research in Mathematics Education, 5(24), 477-483.

Fitzgerald, W., Lappan, G., Phillips, E., Friel, S., & Fey, J. (1990). Connected Mathematics (Proposal Submitted to the National Science Foundation). East Lansing: Michigan State University.

Floden, R.E., Buchmann, M., and Schwille, J.R. (1987). Breaking with everyday experience. Teacher's College Record, 88(4), 485-506.

Frankenstein, M. (1987). Critical mathematics education. In I. Shor & Paulo Freire (Eds.). Freire for the classroom: A Sourcebook for Liberatory Teaching (180-210). Portsmouth: Boynton/Cook, Heinemann.

Frankenstein, M. (1990). Incorporating Race, Gender, and Class issues into a critical mathematical literacy curriculum. Journal of Negro Education, 59(3), 336-347.

Freire, (1970). Pedagogy of the oppressed. New York: Seabury.

Gilbert, S.E., & Gay, G. (1985). Improving the success in school of poor black children. Phi Delta Kappan, 67, 133-137.

Haggstrom, W. (1964). The power of the poor. In F. Riessman, J. Cohen & A. Pearl (Eds.) , Mental health of the poor. New York: Free Press.

Hart, L. (1989). Classroom processes, sex of student, and confidence in learning mathematics. Journal for Research in Mathematics Education, 20, 242-260.

Havighurst, R.J. (1976). The relative importance of social class and ethnicity in human development. Human Development, 19, 56-64.

Hawkins, P.R. (1968). Social class, the nominal group, and reference. London: Sociological Research Unit, University of London, Institute of Education.

Heath, S.B. (1983). Ways with words: Language, life, and work in communities and classrooms. Cambridge: Cambridge University Press.

Hess, R.D. & Shipman, V. (1965). Early experience and socialization of cognitive modes in children. Child Development, 36, 869-886.

Holland, J. (1981). Social class and changes in orientation to meaning. Sociology, 15, 1-18.

Janvier, C. (1990). Contextualization and mathematics for all. Teaching and Learning Mathematics in the 1990s (Yearbook of the National Council of Teachers of Mathematics, pp. 183-193), Reston, VA: NCTM.

Kohl, H. (1992). From Archetype to zeitgeist: Powerful ideas for powerful thinking. New York: Little, Brown & Company.

Kohn, M.L. (1963). Social class and parent-child relationships: an interpretation," American Journal of Sociology, 68, 471-480.

Kohn, M.L. (1983). On the transmission of values in the family: A preliminary formulation. Research in Sociology of Education and Socialization, 4, 1-12.

Kohr, R. L., Coldiron, J. R., Skiffington, E.W., Masters, J.R., and Blust, R.S. (1988). The influence of race, class, and gender on self-esteem for fifth, eighth, and eleventh grade students in Pennsylvania schools. Journal of Negro Education, 57(4), 467-481.

Kohr, R. L., Masters, J., Coldiron, J. R., & Skiffington, E.W. (1991). The relationship of race, class, and gender with mathematics achievement for fifth, eighth, and eleventh grade students in Pennsylvania schools. Peabody Journal of Education, 66, 147-171.

Lampert, M. (1985). How do teachers manage to teach? Perspectives on problems in practice. Harvard Educational Review, 55, 178-194.

Lareau, A. (1987). Social class differences in family-school relationships: The importance of cultural capital. Sociology of Education, 60, 73-85.

Leder, G.C. (1992). Mathematics and Gender: Changing Perspectives. In D. Grouws (Ed.), Handbook of research on mathematics teaching and learning. New York: MacMillan. 623-660.

Lensmire, T. (1993). Following the child, socioanalysis, and threats to community: Teacher response to children's texts. Curriculum Inquiry, 23(3), 265-299.

Lindquist, M. M. (1993). President's report: Tides of change — Teachers at the helm. Journal for Research in Mathematics Education, 24(5), 467-476.

Lutrell, W. (1989). Working-class women's ways of knowing: Effects of gender, race and class. Sociology of Education, 62, 33-46.

MacLeod, J. (1987). Ain't no makin it: Leveled aspirations in a low-income neighborhood. Boulder: Westview Press.

Marx, K. (1988/1872). Manifesto of the Communist party. In F.L. Bender (Ed.), Karl Marx: The Communist manifesto (pp. 54-86). New York: W.W. Norton.

McCleary, B. (1993). Writing in math class still iffy, latest NAEP figures show. Composition Chronicle, 6, 1-4.

McLeod, D.B. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.), Handbook of research on mathematics teaching and learning (575-596). New York: MacMillan.

McDowell, C.L. (1990). The unseen world: Race, class, and gender analysis in science education research. Journal of Negro Education, 59(3), 273-291.

Means, B. & Knapp, M.S. (1991). Cognitive approaches to teaching advanced skills to educationally disadvantaged students. Phi Delta Kappan, December, 282-289.

Meyer, M.R. (1991). Equity: The missing element in recent agendas for mathematics education. Peabody Journal of Education, 66, 6-21.

Moses, R., Kamii, M., Swap, S.M., Howard, J. (1989). The Algebra Project: Organizing in the spirit of Ella. Harvard Educational Review, 59(4), 423-443.

Myrdal, G. (1974). Ethnicity, The Center Magazine, 7(4), Center for the Study of Democratic Institutions, Santa Barbara, California, 26-30.

National Center for Education Statistics (1993). Mini-Digest of Education Statistics. Washington, DC: U.S. Department of Education.

National Center for Education Statistics (1996). The Condition of Education. Washington, DC: U.S. Department of Education.

National Council of Teachers of Mathematics, (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics, (1991). Professional standards for teaching mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics, (1995). Assessment standards for school mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics, (1993). NAEP results show improvement. News Bulletin, 12, 1&12.

National Research Council, (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington D.C.: National Academy Press.

Noddings, N (1993). Politicizing the mathematics classroom. In S. Restivo, J.P. Van Bendegem, & R. Fischer (Eds.), Math Worlds: Philosophical and social studies of mathematics and mathematics education. Albany: State University of New York Press.

Noddings, N. (1996). Equity and mathematics: Not a simple issue. Journal for Research in Mathematics Education, 27(5), 609-615.

Oakes, J. (1985). Keeping track: How schools structure inequality. New Haven: Yale University Press.

Phillips, S., & Ebrahimi, H. (1993). Equation for success: Project SEED. In G. Cuevas & M. Driscoll (Eds.) Reaching all Students with Mathematics, (pp. 59-74). Reston, VA: National Council of Teachers of Mathematics.

Pullin, D.C. (1993). Legal and ethical issues in mathematics assessment. in Mathematical Sciences Education Board and National Research Council, Measuring what counts: A conceptual guide for mathematics assessment (201-223). Washington D.C.: National Academy Press.

Reich, R.B. (1991). The work of nations: Preparing ourselves for the 21st century. New York: Knopf.

Reyes, L. H. & Stanic, G.M.A. (1988). Race, sex, socioeconomic status, and mathematics. Journal for Research in Mathematics Education, 19(1), 26-43.

Secada, W. (1991). Agenda setting, enlightened self-interest, and equity in mathematics education. Peabody Journal of Education. 66, 22-56.

Secada, W. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. Grouws (Ed.), Handbook of research on mathematics teaching and learning. New York: MacMillan. 623-660.

Silbert, J. (1993). A new direction in the fight against educational discrimination. Effective School Practices, Summer, 66-74.

Stallings, J.A., & Kaskowitz, D. (1974). Follow-through classroom observation evaluation 1972-1973, Menlo Park, CA: Stanford Research Institute.

Stanic, G.M.A. (1991). Social inequality, cultural discontinuity, and equity in school mathematics. Peabody Journal of Education. 66, 57-71.

Stiff, L. (1990). African-American students and the promise of the Curriculum and Evaluation Standards, Teaching and Learning Mathematics in the 1990s (Yearbook of the National Council of Teachers of Mathematics, pp. 152-157), Reston, VA: NCTM.

Stodolsky, S.S. & Lesser, G.S. (1967). Learning patterns in the disadvantaged. Harvard Educational Review, 37(4) 546-593.

Strauss, A. & Corbin, J. (1990). Basics of qualitative research: Grounded theory procedures and techniques. Newbury Park: Sage.

Terrell, G., Durken, K., & Wiesley, M. (1959). Social class and the nature of the incentive in discrimination learning, Journal of abnormal social psychology, 59, 270-272.

Theule-Lubienski, S. (in press). Class matters: A preliminary exploration. Multicultural and gender equity in the mathematics classroom: The gift of diversity (The 1997 yearbook of the National Council of Teachers of Mathematics).

Theule-Lubienski, S., Burgis, K., and Keiser, J. (submitted for publication). What are middle-school students interested in? School Science and Mathematics.

Thompson, E.P. (1963). The making of the English working class. New York: Vintage Books.

Usiskin, Z. (1993). What changes should be made for the second edition of the NCTM Standards? University of Chicago School Mathematics Project Newsletter. Winter, 6-11.

Weis, L. (Ed.) Class, Race, & Gender in American Education, State University of New York Press, Albany, 1988.

Werts, C.E. (1966). Social class and initial career choice of college freshmen. Sociology of Education, 39(1). 74-85.

White, K.R. (1982). The relation between socioeconomic status and academic achievement. Psychological Bulletin, 91(3), 461-481.

Whitmire, (1994). Test score gap widens for rich, poor kids. Lansing State Journal, December 4, 9A.

Willis, P. (1977). Learning to labor. How working class kids get working class jobs. New York: Teachers College Press.

Zigler, E. & DeLabry, J. (1962). Concept-switching in middle-class, lower-class, and retarded children, Journal of Abnormal and Social Psychology, 65(4), 267-273.

Zweig, M. (1991). Class and poverty in the U.S. economy, in Zweig (Ed.), Religion and Economic Justice (196-218), Philadelphia: Temple University Press.