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SIMULATION AND OPTIMAL CONTROL OF ECONOMIC GROWTH SYSTEM: APPLICATION OF SYSTEM THEORY TO THE DESIGN OF ECONOMIC POLICIES

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SIMULATION AND OPTIMAL CONTROL OF ECONOMIC GROWTH SYSTEM: APPLICATION OF SYSTEM THEORY TO THE DESIGN OF ECONOMIC POLICIES

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ABSTRACT

SIMULATION AND OPTIMAL CONTROL OF ECONOMIC GROWTH SYSTEM: APPLICATION OF SYSTEM THEORY TO THE DESIGN OF ECONOMIC POLICIES

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Application of system theory to the problems of socio-economic systems can be interpreted as an effort to find the analogies between the two different systems--engineering systems and social or economic systems.

The thesis is an attempt to apply the system theoretic approaches to an economic system, an economic growth system in particular, reflecting the increasing concerns of economic growth among LDC's under highly interdependent recent world economic situations.

The economic growth system as explained in the modern theory of economic growth is a closed system with capital being the system state and with the two basic economic processes of capital accumulation and production. Closed two sector (dual) economic growth system has been modeled based on the theory of economic growth and disaggregated into agriculture and nonagriculture which is different from the conventional way of disaggregation for the economic growth model--consumption- and capital-goods sectors.

The dual economic growth system with this dichotomy consists of two system state equations and a measurement equation, and comprises non-linear dynamic system which can be solved numerically.

Sung Joo Park

Optimal control of the dual economic growth system with saving rate being control variable was performed based on Pontryagin's maximum principle. Objective function--social welfare--was defined in terms of the combinations of consumption and capital to conduct the optimal control where constant exponents (weights) on the consumption and capital were used to avoid the possible existence of singularities. The sufficient conditions for the existence of the optimal control also were derived for the case of bounded control and no terminal constraint. Numerical solution of the optimal trajectories could be obtained efficiently by variation of extremals with a simple adjustment scheme--with the constant costate influence function matrix--for the case of the Korean economy.

In the open economic growth system, two components--trade and balance-of-payments--were added to the closed system with further modifications. Income distribution and foreign indebtedness were also considered in the open system along with economic growth. Optimal policies of saving rate and import were formulated and derived by general optimization method with a piecewise quadratic objective function and orthogonal function (Legendre) approximation of the policy paths during a time horizon. The open model was further simulated for the case of alternative objective functions and alternative policies on investment and grain prices.

Considerations on the external food shock has been added to the open economic growth system to investigate the effects of the shock to the internal economic variables and to design feasible policies to mitigate or eliminate the negative effects of the external shock.

Further modifications on the aggregate production function, saving behaviors and investment, price mechanisms, labor (migration), tax policy, trade, foreign capital movements, and inclusion of uncertainties will make the model close to the real situation.

The results of the study (with applications to the case of the Korean economy) demonstrated the practical usefulness of the theory of economic growth for the design of economic policies with respect to the transitory behaviors of an economy during a certain (finite) planning period. In sum, the study showed the possible applications of system theory to an economic system to bridge the gap between the two disciplines, and thus to provide more insights in understanding the dynamics of economic systems.

To my parents and grandma,

for their understanding,

encouragement, and

love.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Recent trends in applying system theory to the problems of socio-economic systems have been rooted to the question: "If system theory can improve the guidance of airplanes and spacecraft, can it also be helpful in devising the policies for solving the problems of an economy or a society?" Obviously, a society or an economy is not a concrete moving object which can be measured precisely, like an airplane or a spacecraft, and thus numerous assumptions and simplifications are needed to find analogies between the two.

Economic systems, as well as other social systems, can be characterized by the three basic properties; uncertainty, dynamics, and the existence of feedbacks. In fact, the classical economic theories have failed to take into account of the three properties: they assume perfect information or full knowledge of the parameters of the economic system, static or comparative static analysis has been dominantly used with the "ceteris paribus" conditions, and the nature of the adaptive economic process, where decision makers increase their knowledge by the cumulative experience of "doing while learning", has not been fully explored in general. With the lack of ability of the traditional methodologies in economics to handle these problems, new methodologies from other disciplines have been sought and applied gradually as a part of diffusion processes among the different disciplines.

Although there were earlier attempts [P4], [S8], [S9], [S15], the pioneering work of applying system theory to economics can be attributed to Arnold Tustin [T16], who viewed economic system as interdependent dynamic systems and analyzed their time responses to exogenous shocks using the transfer function method which was found to be very successful in designing servomechanisms. A. W. Phillips, best known for the Phillips' curve, also applied about the same time the theory of servomechanisms to business cycles and stabilization policies of macroeconomic models [P5]. These earlier works, however, were not successful because of two reasons; the technical reason of computational restrictions, and the circumstantial reason of the inclination of the interests towards the general competitive equilibrium models those days [D1]. With the advent of computers, the first obstacle was removed, and more complex models have been emerged [H6], [M2], [M7].

There are two ways of handling complex and/or large scale systems in general. The first is analytical by aggregation [M1] or decomposition [H5], and the second is by simulation. Each of the two methods has its own merits: by aggregation and decomposition, the model can be simplified enough to be handled by analytical tools and may achieve analytical preciseness which most theoreticians believe to be of foremost importance, while the simulation approach allows one to investigate the interactions and linkages of elements of the system at the detailed level of the real situation with less analytical preciseness. The actual use of any one of the methodologies in modeling depends on the nature of the problem, the use of the model, the availability of information, and the desired level of accuracy.

National economic planning is not only a economic but also a political process: it includes the definition and choice of goals, identifying the economic variables and interrelationships, designing policies, charting the possible paths of economy with respect to the feasible policies, and selecting the best policy to meet the goals. The problem of designing policies for economic growth under different circumstances has been the central core of the thesis; however, the motivation was from the recognition that actual society is too complex to be solved by any one of the methodologies, and there may be certain gains from looking at the problem from different sides using different methodologies, and thus to advance system theory in the analysis of dynamic economic models.

I.1 Background and Needs of the Study

In recent years, the world has experienced increasing interdependency of international economy and a series of shocks--most notably oil crisis and food shortages--which affected virtually every country in the world. Even though the initial motivation of the interdependency originated from the mutual benefits of trade, it created high levels of insecurity as some countries became heavily dependent on others for certain products which are essential for their continuing growth or existence. The immediate concerns for the highly dependent countries, most of the developing countries, are the deepening of foreign indebtedness as a result of the high prices of the essential products in the world market, and their effects to economic growth which may relieve them from the excessive indebtedness.

Economic growth has been one of the principal objectives of the economic policies both in advanced and in less developed countries, believing that growth can be a solution to a variety of other economic problems such as indebtedness of a country, inequitable distribution of income, and so forth.

The theory of economic growth,[1] which tries to explain the primary causes of production and their interrelationships over time periods, has been developed for decades. However, the "state-of-the-art" of the theory is not satisfactory: too many controversies exist over the basic assumptions, heavy reliance of the growth models on the balanced growth, golden-rule paths, or steady-state growth prevents empirical application to a specific economy, thus creating a model of "mythical states"[2] aloof from the reality. If a theory has some value and thus deserves to be called a "theory", it should be able to answer questions raised from the real situation at a certain "admittable level" of preciseness. The existing models of economic growth have failed in this sense by staying a safe distance from reality.

The thesis is an attempt to modify the existing models of economic growth with the following questions in mind:

(1) what are the appropriate descriptions of an economy where the transitions and/or interactions between the primitive and advanced sectors are more significant than those between the consumption- and capital-goods sectors?

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[1]The modern theory of economic growth is meant here instead of other types of growth theory such as the grand theory of economic growth or the theory of economic development [J1], pp. 4-6.

[2]A balanced growth, or golden-rule path, "thus indicating that it represents a mythical state of affairs not likely to obtain in any actual economy," [R5], p. 99.

(2) what is needed in the model to take into account the interactions of an economy with the world in the light of trade and capital flows?

(3) what are the primary linkages between foreign borrowing or foreign capital flows and economic growth?

(4) what are the effects of external shocks to economic growth and other domestic economic variables?

(5) what are desirable and feasible growth policies, and how can the best policy be chosen?

Also, the study was motivated, in part, by a desire to test empirically the basic formulation of the theory of economic growth, its applicability to the LDC's (less developed countries), and its ability to cope with problems relating to external effects.

The empirical study has been done on the economy of Korea for illustrative purposes. Korea, one of the fast growing developing economies among LDC's, has experienced the "take-off" and the "acceleration" stages of development during the past decades. The economic growth has been phenomenal; the average real growth rate during 1965 to 1975 was 10.2 percent, the export has grown about 29 times with an average annual growth rate of 22.6 percent during the same period. High level of average education and technology transfer from the advanced countries enhanced the growth, however, the basic forces for growth came from the increase in investment and the migration of labor from traditionally latent agricultural sector to the industrial sector. Investment, which has risen from 0.151 (gross investment ratio) in 1965 to 0.314 in 1974, has been financed substantially from foreign sources--foreign loans, direct foreign investment, use of international reserves, and net foreign transfers.

As a consequence, the burden of foreign indebtedness, which has accumulated to an estimated total debt standing of six billion dollars at the end of 1975 (from the negligible debt in 1965) with 700 million dollars of debt service payment during 1975,[1] draws heavy attention along with the costs that the Korean economy has to pay as a result of urban migration. Clearly, these costs may play a hindering role in the pursuit of continuing growth. This burden has been acutely, though not devastatingly, experienced in Korea when the oil crisis and food shock struck the whole world. Coupled with the fact that Korea is entirely dependent on overseas for the oil supply, and is chronologically dependent on imports for about a quarter of its total grain requirements.

The main questions, then, become more clear. First, what are the possible ways of investing to further economic growth and to relieve the burden of excessive indebtedness? In other words, what are wise foreign borrowing schedules if the desired investments exceed the domestic savings? Secondly, what are the consequences of the possible paths of growth for other aspects of the economy--foreign trade, level of consumption, distribution of income, etc.? Thirdly, what will be the effects of the probable future shocks to the economy and what policies can dampen or eliminate the negative effects of the shocks? Fourth, is there any policy which is superior to the other available policies and will lead to the higher economic growth, sound indebtedness, more equitable distribution of income, and a higher level of consumption?

Even though the questions are from a specific case, these are not the questions of only Korea but also typical to most of the LDC's.

I.2 Scope and Objectives of the Study

The remaining chapters of the thesis can be divided into two equal parts; the analysis and control of the closed economy, and the analysis and control of the open economy. The high level of aggregation of the closed economy enables one to use analytical tools--state-state representation of the economic system and optimal control of the system model using Pontryagin's maximum principle--while the complexities of the closed model do not allow the application of analytical tools used in the open model.

Chapter II will describe the basic analytical tools. Chapter III starts from the neoclassical model of economic growth which explains the basic causal loops for production in a one sector aggregate economy where the saving (investment) and production mechanisms govern the whole economy. A Meadean type two sector model, which disaggregates economy into consumption- and capital-goods sectors, is also examined and is modified into a two sector model of primitive and advanced sectors, more specifically, agriculture and nonagriculture.

Chapter IV concentrates on finding the optimal policy for a closed economy in terms of the path of saving rate over the planning horizon which maximizes the "alternative" desired social welfare functions. Direct application of the maximum principle has been made, the necessary and the sufficient conditions for the existence of the optimal control have been derived, and a numerical algorithm has been developed for the specific case of an objective functional without terminal conditions.

The closed model has been further modified into the open model in Chapter V. This model includes trade and balance-of-payments components in addtion to the other components of the dual economic growth model.

Designing the economic policies of the open model can be quite different from that of the closed model; it deals not only with the questions of the saving rates and capital formation but also with those of the foreign trade, foreign capital flow and indebtedness, foreign currency reserve, etc. These added features make the model not readily applicable for the analytical tools such as the optimal control theory as in the closed model. "Creative" trial-and-error or rule-of-thumb methods may provide certain insights for the design of better policies whereas an attempt to obtain the "best" policy applying optimization techniques with a given objective function yields a set of optimal solutions of complex system which may be infeasible in strict mathematical sense but may have practical usefulness if the definition of the constraints can be relaxed. In this case, a set of orthogonal functions such as Fourier series or Legendre polynomials can be used to approximate the dynamic paths of the policy variables. Determining objective function is not an easy task and may constitute conceptual difficulties. Piecewise quardratic objective function can be used more generally than quadratic objective function in order to remove some of the difficulties.

One of the prominent advantages of the open model over the closed model is its ability to respond to external shocks and to investigate their effects on internal economic variables. High grain prices in the world market have been used for an external food shock to the open model. Implications for the management of the domestic grain stock to dampen the shock to the internal mechanism have been analyzed in Chapter VII. The final chapter concludes the thesis with a summary, further research areas, and possible gains from applying different methods to the problem of economic growth.

The primary objective of the study is to provide an analytical tool or a model which can be used for the investigation of economic growth, design of policies for the desired growth or sustained growth, both with or without external effects. More specifically, the objectives are to design an analytical framework, to be illustrated by the Korean economy case, which permits one to investigate:

(1) growth and consumption paths, and saving rate of a closed economy where the interactions between agriculture and nonagriculture are significant

(2) the best policy for saving rates which will maximize the desired social welfare functions

(3) the effects of alternative growth strategies on foreign indebtedness, distribution of income, consumption and economic growth in the open model which includes foreign trade and capital movements

(4) the effects of external shocks to internal economic variables such as the growth rate, consumption, foreign debt, food prices and distribution of income

(5) the design of policies for the desired or sustained economic growth with or without external shocks, while satisfying constraints on the control and/or the state variables of the economy.

CHAPTER II

GENERAL CONCEPTS:

BACKGROUND INFORMATION

Any problem solving (in the physical world) necessarily deals with models. By definition, a model is an abstract representation of the actual system. It can be physical, conceptual, verbal, graphical, or mathematical with the purpose to help one to understand the system operation, and to predict its behavior under certain conditions. The model used in the study is based on the theory of economic growth. In this chapter, the basic tools used to deal with the model--system theory, simulation, optimal control--will be discussed in brief.

II.1 System Theory

A system is a set of interconnected entities, conceptual or physical, organized toward a goal or set of goals. This concept is quite general and broad in nature, and thus the word "system" has been used to mean many different things for many different purposes. System theory, which deals with "system", also has been defined in diverse ways.[1]

Among the many possible difinitions, system theory here will be confined to a tool (in engineering sense) to design the "best" system to satisfy a certain purpose. Often, it uses mathematical models in the

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[1] Ludwig von Bertalanffy even goes further by stating that General System Theory (GTS) is the only possible way for the unification of sciences, which encompasses large branches of science such as "classical" system theory, computerization and simulation, compartment theory, set theory, graph theory, net theory, cybernetics, information theory, theory of automata, game theory, decision theory, queueing theory, etc. [B4].

form of algebraic equations, difference equations, ordinary or partial differential equations, and functional equations.

System theory starts from the system equations such as (for the case of ordinary differential equation model),[1]

$$\dot{\bar{x}}(t) = F(\dot{\bar{x}}(t), \bar{u}(t), \bar{w}) \tag{2.1}$$

$$\bar{y}(t) = G(\bar{x}(t), \bar{u}(t), \bar{v}) \tag{2.2}$$

where $\bar{x}$ is the system state vector, $\bar{u}$ is the input vector, $\bar{y}$ is the output vector (or observations), $\bar{w}$ and $\bar{v}$ represent possible vectors of system disturbance and measurement error, and F and G identify the state and measurement equations (or operators). Four basic problems arise in the system theory: modeling, analysis, estimation, and control [S5].

Modeling, developing a model which adequately represents the physical situation, is undoubtedly the most critical step, since if the model is not adequate, the subsequent mathematical or computer manipulations are meaningless. To help to preserve the adequacy of a model, several steps of modeling procedure have been developed which will be discussed in the next section relating to simulation.

Analysis determines the system output $\bar{y}$ given system input $\bar{u}$ and system structure F, G, and $\bar{x}$. Quantitative analysis determines the precise behavior of the output, such as trajectories of the output, and qualitative analysis determines general properties of the behavior of the output such as stability. This is only possible under the assumption that the system structure as given is perfectly known.

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[1]Notation: the upper bar and the lower bar will indicate vector and matrix, respectively, throughout the theses unless otherwise specified.

Estimation raises question about the system structure. It uses the observations of $\overline{y}$ and $\overline{u}$ to estimate the properties of the actual system. Three types of estimation problems can be defined; state estimation, identification, adaptive estimation. State estimation is to estimate $\overline{x}$ using observation $\overline{y}$ with given system structure. Identification refers to the estimation of parameters (or new states in a sense) using the observations, (original) system states, and system structure, and thus completes the estimation of the model. Adaptive estimation is state estimation combined with identification, i.e., the state estimation problem with the combined state of original state and new state (or parameter).

In the control problem one determines the input $\overline{u}$ given the desired output $\overline{y}$ and the system structure. There are three types of control; open-loop, closed-loop, and adaptive. Open-loop control means a control expressed in terms of the initial state of system and independent variable(s) such as time. Closed-loop control is expressed as an explicit function of the observed variables, and thus feeds-back the output to the input. Adaptive control is a more complicated closed-loop control where adaptive estimation is carried out simultaneously.

All the basic problems related to modeling are closely interrelated: any one of input, output or system structure can be determined only by assuming the others are perfectly known. Iterative procedure of analysis, estimation, and control--which will hopefully converge--may be used to complete the system modeling.

II.2 Simulation

Although the concept of simulation can be traced back to long ago, the modern version of simulation, defined as "a numerical technique for conducting experiments on a digital computer which involves certain types of mathematical and logical models,"[1] has its origin in the work of von Neumann and Ulam in the late 1940's when they applied a method based on the random numbers (which later was named as "Monte Carlo Method") to study neutron diffusion and to determine the thickness of reactor shields.[2] The initial motivation of the von Neumann and Ulam's work was to incorporate stochastic uncertainties in a deterministic model, but they also recognized that the method is capable of dealing with problems of a more complicated nature which can't be solved by conventional analytical methods. Uncertainty (randomness) and complexity form a basic motivation for simulation, thus the simulation model can be far more complex than the models that system theory (as defined in the preceding section) usually deals with.

The basic problems of simulation are the same as those of system theory--modeling, analysis, estimation, control. Modeling is the core of the simulation because of its critical importance to the rest of the procedure. To minimize the risk of model being inadequate for the real situation, a conceptual procedure can be followed as a helpful guide to a general problem solving.

The general procedure (or system approach) is an iterative learning process with set of steps (or phases) as following [M3]:

(1) feasibility evaluation

(2) abstract modeling

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[1] [N1], p.3.

[2] Part of the original thoughts of von Neumann suggested by Ulam can be found in a von Neumann's letter to R. D. Richtmyer. R. D. Richtmyer and J. von Neumann, "Statistical Methods in Neutron Diffusion," [T1], pp. 751-764.

(3) implementation design

(4) implementation

(5) system operation

The major phases of systems approach may comprise sub-phases. Feasibility evaluation consists of six sub-phases; needs analysis, system identificaiton, problem formuation, generation of system alternatives, determination of physical, social, and political realizability, and determination of economic and financial feasibility. Abstract modeling may be followed through sub-phases such as; selection of feasible system alternatives emerged from feasibility evaluation, choice of specific type of abstract representation--static, dynamic, micro, macro--, computer implementation, validation, sensitivity analysis, stability analysis, and model application.

The above procedure as suggested in [M3] is, by no means, complete, and the validity of the procedure can only be judged on purely pragmatic grounds. In essence, problem solving (using simulation) is a continuous process of modification and adjustment.

II.3 Optimal Control Theory

Optimal control theory, which is a dynamic extension of static optimization, comprises two major branches; dynamic programming [B2] and Pontryagin's maximum principle [P8]. Dynamic programming is a computational technique which extends the decision making concept to sequences of decisions which together will define an optimal policy and trajectory based on the principle of optimality:[1]

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[1][B2], p. 83.

> An optimal policy has the property that whatever the initial state and initial decision are, the remaining must constitute an optimal policy with regard to the state resulting from the first decision.

While dynamic programming finds many applications in discrete systems, there is one serious drawback; the "curse of dimensionality" which indicates the exponentially-increasing size of computer memory requirements as the number of decision variables increases. On the other hand, Pontryagin's maximum principle was originally developed for continuous systems and will be used here in the closed economic growth system.

Pontryagin's maximum principle, in essence, consists of a set of necessary conditions that must be satisfied by optimal solutions. All of these necessary conditions originate in classical calculus of variations, but are formed in a more or less systematic way by use of a Hamiltonian function. The problem is to find an admissible control $\bar{u}^*$ [1] which satisfies the system

$$\dot{\bar{x}}(t) = \bar{a}(\bar{x}(t), \bar{u}(t), t) \tag{2.3}$$

and to minimize an objective functional (performance index),

$$J(\bar{u}) = h(\bar{x}(t_f), t_f) + \int_{t_0}^{t_f} g(\bar{x}(t), \bar{u}(t), t)\, dt \tag{2.4}$$

with or without additional constraints such as

$$\begin{aligned} \bar{x}^*(t_0) &= \bar{x}_0 \\ \bar{x}^*(t_f) &= \bar{x}_f \end{aligned} \tag{2.5}$$

where $\bar{x}$ is the system state, $\bar{u}$ is the control input, t_0 and t_f are the

---

[1] Asterisk will be used to denote the optimal values.

initial and the final times, respectively.

An augmented scalar function, Hamiltonian, will then be defined as

$$H(\bar{x}(t), \bar{u}(t), \bar{p}(t), t) \equiv g(\bar{x}(t), \bar{u}(t), t) + \bar{p}^T(t)[\bar{a}(\bar{x}(t), \bar{u}(t), t)] \quad (2.6)$$

where $\bar{p}$ is a costate vector or a vector of Lagrange multipliers which corresponds shadow prices for the case of cost minimization. Pontryagin's maximum principle, then, states that an optimal control must minimize the Hamiltonian, i.e., a necessary condition for $\bar{u}^*$ to minimize the functional J is

$$H(\bar{x}^*(t), \bar{u}^*(t), \bar{p}^*(t), t) \leqq H(\bar{x}^*(t), \bar{u}(t), \bar{p}^*(t), t) \quad (2.7)$$

for all $t \varepsilon [t_0, t_f]$ and for all admissible controls.

Thus, the necessary conditions for $\bar{u}^*$ to be an optimal control can be derived as

$$\dot{\bar{x}}^*(t) = \frac{\partial H}{\partial \bar{p}} (\bar{x}^*(t), \bar{u}^*(t), \bar{p}^*(t), t) \quad (2.8)$$

$$\dot{\bar{p}}^*(t) = - \frac{\partial H}{\partial \bar{x}} (\bar{x}^*(t), \bar{u}^*(t), \bar{p}^*(t), t) \quad (2.9)$$

$$H(\bar{x}^*(t), \bar{u}^*(t), \bar{p}^*(t), t) \leqq H(\bar{x}^*(t), \bar{u}(t), \bar{p}^*(t), t) \quad \text{for all admissible } \bar{u}(t) \quad (2.10)$$

for $t \varepsilon [t_0, t_f]$,

and boundary conditions

$$[\frac{\partial H}{\partial \bar{x}} (\bar{x}^*(t_f), t_f) - \bar{p}^*(t_f)]^T \delta \bar{x}_f + [H(\bar{x}^*(t_f), \bar{u}^*(t_f), \bar{p}^*(t_f), t_f) + \frac{\partial H}{\partial t} (\bar{x}^*(t_f), t_f)] \delta t_f = 0 \quad (2.11)$$

where δx_f and δt_f denote the variations of x_f and t_f, respectively.

It should be noted that $\overline{u}^*(t)$ is a control for the Hamiltonian to assume its global minimum, and the given conditions are not, in general, sufficient, and thus further conditions will be needed to guarantee the global minimum.

CHAPTER III

ECONOMIC GROWTH MODEL

The purpose of this chapter is to investigate the basic framework of economic growth models; to survey the one sector neoclassical growth model and (Meade and Uzawa's) two sector model, to modify the two sector model into a dual economic growth model using a different dichotomy, and to apply the modified dual economic growth model to the Korean economy as an illustration to show the ability of the modified growth model for the representation of transitory behaviors of an economy.

III.1 One Sector Growth Model

In the aggregate economic growth model, it is assumed that there are two factors of production, namely, capital and labor, that are combined to produce a single homogeneous output. At any instant in time a fraction of this homogeneous output may be allocated to consumption and investment.

If K(t) and L(t) denote the currently existing stocks of capital and labor, then the current rate of output Y(t) can be expressed by

$$Y(t) = A(t)F[K(t), L(t)] \qquad (3.1)$$

where A(t) is a measure of the current level of technical knowledge and F[.] is the production function exhibiting certain characteristics.

Let C(t) and Z(t) denote the current rates of consumption and investment, then from the national income identities

$$Y(t) = C(t) + Z(t) = [1 - s(t)]Y(t) + s(t)Y(t) \tag{3.2}$$

where s(t) is the fraction of current output that is being saved (saving rate) such that $0 \leq s(t) \leq 1$. If capital is subject to evaporative decay at a constant rate $u > 0$, then the growth of the capital stock can be specified by the differential equation

$$\dot{K}(t) = s(t)A(t)F[K(t), L(t)] - uK(t) \tag{3.3}$$

Assume that N(t) is the current size of the population and that population growth is independent of the economic variable and growing at a constant rate

$$\dot{N}(t) = nN(t) \tag{3.4}$$

where n is the rate of the population growth. Assume further that the number of workers (labor) is a constant fraction $0 < w < 1$ of the total population

$$L(t) = wN(t)$$
$$\dot{L}(t) = nL(t) \tag{3.5}$$

If the technological change can be assumed as an autonomous growth at a fixed rate, a, then

$$\dot{A}(t) = aA(t) \tag{3.6}$$

where a is the rate of the technological growth.

The preceding equations from (3.1) to (3.6) form the basis of the aggregate economic growth system (Figure 3.1). If the neoclassical assumptions on the production function--constant returns to scale in

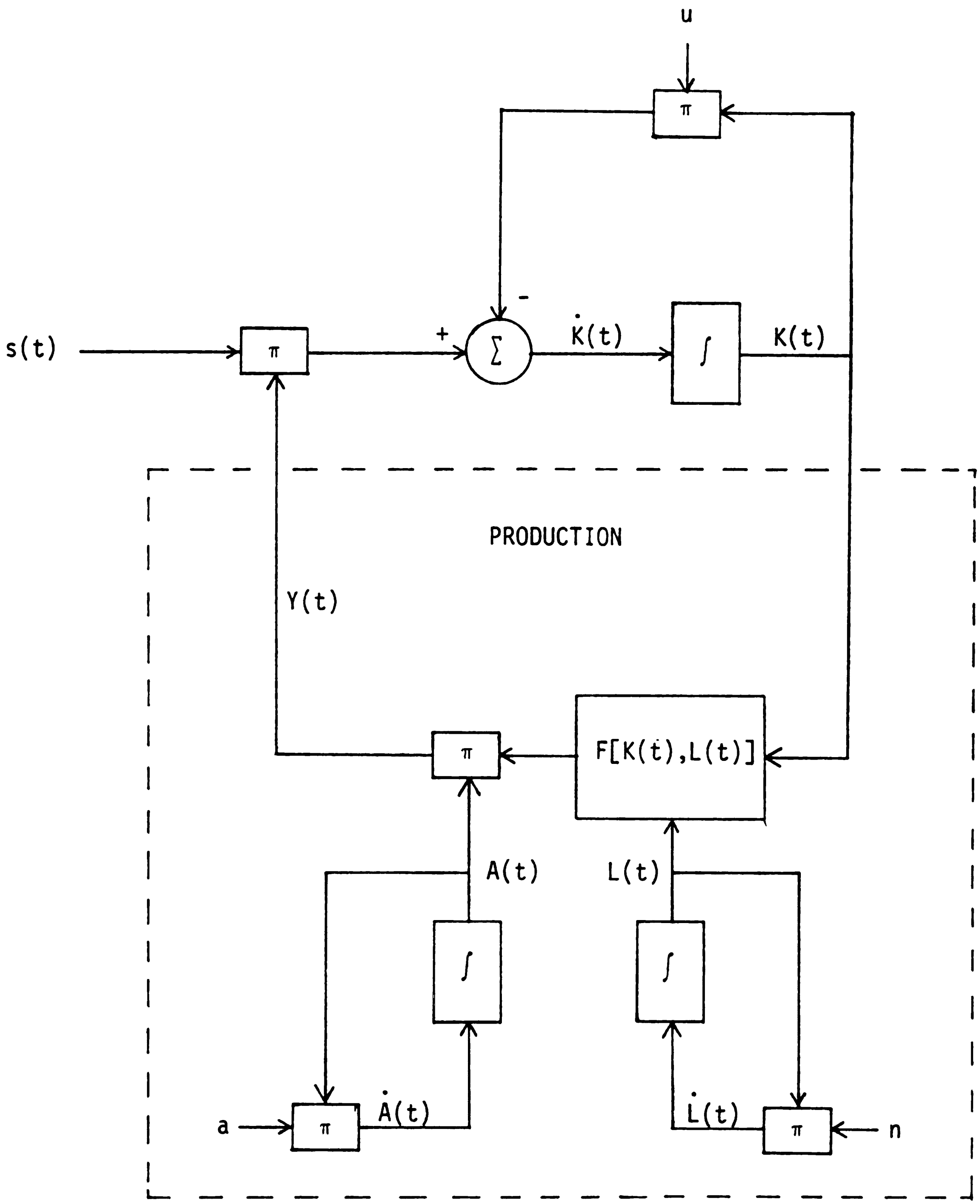

Figure 3.1 Block Diagram of One Sector Economic Growth System

capital and labor, i.e., homogenous of degree one in capital and labor --can be made, the model (neoclassical growth model) can be converted into the equation in the per labor quantities.[1] [S11] Let the variables be defined as:

output per worker : $y(t) = Y(t)/L(t)$

capital per worker : $k(t) = K(t)/L(t)$

consumption per worker : $c(t) = C(t)/L(t)$

investment per worker : $z(t) = Z(t)/L(t)$

Then the system equation and the production function can be reduced to

$$\dot{k}(t) = s(t)y(t) - (n + u)k(t) \tag{3.7}$$

$$y(t) = A_0 \, e^{at} \, f[k(t)] \tag{3.8}$$

and

$$0 \leqq s(t) \leqq 1$$

$$k(0) = k_0$$

where n, u, and a are given constants, k_0 and A_0 are the initial conditions for each variables.

The equations (3.7) and (3.8) form the basis of one sector neoclassical economic growth model--Solovian model--where the equation (3.7) can be thought as a system state equation, and the equation (3.8) can be a measurement equation with k(t) as a state variable, y(t) as a output variable, and s(t) as an input (or control) variable.

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[1]The desirable variables for the analysis should be in terms of per capita value rather than per labor. But in practice, mostly for its convenience, it is assumed that the number of dependents per worker for each sector is constant during the time horizon, and thus eliminating the discrepancies between the two variables.

III.2 Two Sector Growth Model

The Meade and Uzawa's two sector model is basically a straight extension of Solow's neoclassical model by disaggregating an economy into an investment-goods sector and a consumption-goods sector [U1].

The basic assumptions are similar to those of Solow's model such as: perfect foresight, perfect competition, production is subjected to constant returns to scale in both sectors, exponential population growth, gross investment is equal to saving, and all capital goods depreciate at a constant rate. Further assumptions are made for each sectors; The output of the capital-goods sector (net investment) is perfectly malleable, the output of the consumption-goods sector is used only for consumption, and there are no external economies (or diseconomies) and no joint products between the two sectors.

The current production of the investment-goods, $Y_1(t)$, and the consumption-goods, $Y_2(t)$, are dependent upon the current allocation of capital and labor for each sector, i.e.,

$$Y_1(t) = A_1(t)F_1[K_1(t), L_1(t)] \qquad (3.9)$$

$$Y_2(t) = A_2(t)F_2[K_2(t), L_2(t)] \qquad (3.10)$$

where K_i and L_i represent capital and labor, A_i represents a measure of the current level of technical knowledge, and $F_i[.]$ is the production function for each sector.

The factors of production, capial and labor, are constrained by

$$K_1(t) + K_2(t) \leqq K(t) \qquad (3.11)$$

$$L_1(t) + L_2(t) \leqq L(t) \qquad (3.12)$$

where K and L are the current stocks of available capital and labor.

From the assumptions, labor grows exponentially as

$$\dot{L}(t) = nL(t) \tag{3.12}$$

and the growth of the capital stock can be specified by

$$\dot{K}(t) = Y_1(t) - uK(t) \tag{3.14}$$

where n and u represent the (autonomous) population growth rate and the constant rate of capital depreciation.

The system state equation (3.14) along with the production function, which are similar to whose of Solovian model, describe the dynamic behavior of the two sector economic growth model with consumption- and capital-goods sectors.

III.3 Modified Dual Economic Growth Model

Central to developing a formal model of the dualistic (two sector) economy are the criteria employed in bisecting the economy into analytically and empirically meaningful units. One possible framework for sectoral division was represented by the Meade and Uzawa's model which specifies investment-goods and consumption-goods sectors. While this dichotomy may have some value in studying the equilibrium growth path of the industrialized economy, it is less useful for the low-income economy where the transitional phenomena between primitive and advanced sectors dominate the long-run equilibrium paths.

In this section, the Meade and Uzawa's two sector model will be modified using different dichotomy as following:

First, the model economy will be disaggregated into a primitive sector and a modern sector--agriculture and nonagriculture. Thus the total production at time t, Y(t), becomes

$$Y(t) = Y_1(t) + Y_2(t) \tag{3.15}$$

where $Y_1(t)$: agricultural production

$Y_2(t)$: nonagricultural production.

Secondly, it will be assumed that the saving rates (or the marginal propensities to save) for each sector are different

$$\begin{aligned} S(t) &= S_1(t) + S_2(t) \\ &= s_1(t)Y_1(t) + s_2(t)Y_2(t) \end{aligned} \tag{3.16}$$

where S(t) : total saving

$S_i(t)$: gross saving of each sector

$s_i(t)$: saving rates for each sector.

Thirdly, it also will be assumed that the depreciation rates (decay rates) of each capital stocks are different, and finally, to make a link between the saving and investment, the investment decision rule, i(t), will be given as

$$I_1(t) = i(t)S(t) \tag{3.17}$$

$$I_2(t) = [1 - i(t)]S(t) \tag{3.18}$$

where $I_i(t)$: gross investment for each sector

i(t) : investment-allocation parameter, or investment ratio to sector 1 from the total saving

From the above assumptions and equations, the basic system state equations which describe the growth of the economy can be derived as

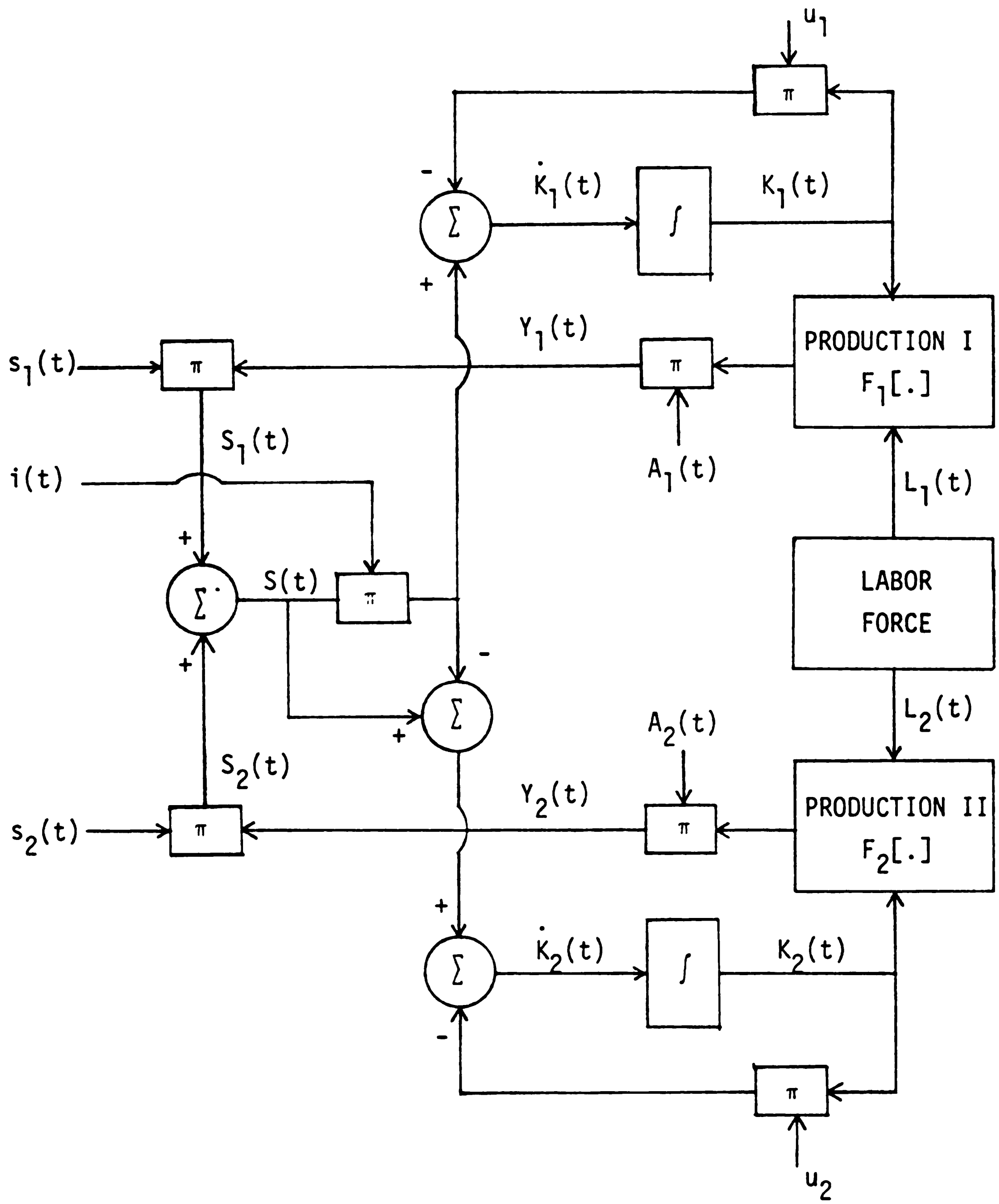

Figure 3.2 Block Diagram of Dual Economic Growth System

$$\dot{K}_1(t) = I_1(t) - u_1K_1(t) \tag{3.19}$$

$$\dot{K}_2(t) = I_2(t) - u_2K_2(t) \tag{3.20}$$

where u_i is the depreciation rate of capital for each sector. These equations can be rewritten using the equations (3.15)-(3.18),

$$\dot{K}_1(t)= i(t)s_1(t)Y_1(t) + i(t)s_2(t)Y_2(t) - u_1K_1(t) \tag{3.21}$$

$$\dot{K}_2(t) = [1-i(t)]s_1(t)Y_1(t) + [1-i(t)]s_2(t)Y_2(t) - u_2K_2(t) \tag{3.22}$$

The block diagram for the above equations, (3.21) and (3.22), is shown in Figure 3.2.

Retaining the assumptions of the Meade and Uzawa's model--neoclassical assumptions--, the system state equations can also be converted into per labor variables. Thus

$$\dot{k}_1(t) = -[u_1+g_1(t)]k_1(t) + i(t)s_1(t)y_1(t)/l_1(t) + i(t)s_2(t)y_2(t)/l_1(t) \tag{3.23}$$

$$\dot{k}_2(t) = -[u_2+g_2(t)]k_2(t) + [1-i(t)]s_1(t)y_1(t)/l_2(t) + [1-i(t)]s_2(t)y_2(t)/l_2(t) \tag{3.24}$$

In matrix form,

$$\begin{bmatrix} \dot{k}_1(t) \\ \dot{k}_2(t) \end{bmatrix} = \begin{bmatrix} -u_1-g_1(t) & 0 \\ 0 & -u_2-g_2(t) \end{bmatrix} \begin{bmatrix} k_1(t) \\ k_2(t) \end{bmatrix} + \begin{bmatrix} i(t)y_1(t)/l_1(t) & i(t)y_2(t)/l_1(t) \\ [1-i(t)]y_1(t)/l_2(t) & [1-i(t)]y_2(t)/l_2(t) \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix}$$

or

$$\dot{\overline{k}}(t) = \underline{A}(t)\overline{k}(t) + \underline{B}(t)\overline{s}(t) \tag{3.25}$$

where

$$y_j(t) = Y_j(t)/L(t)$$

$$k_j(t) = K_j(t)/L_j(t)$$

$$l_j(t) = L_j(t)/L(t)$$

$$g_j(t) = \dot{L}_j(t)/L_j(t) = [\dot{l}_j(t) + l_j(t)\dot{L}(t)/L(t)]/l_j(t)$$

and

$$l_1 + l_2 = 1.$$

For the derivations, the following relationships have been used:

$$\begin{aligned}\dot{k}_j(t) = \tfrac{d}{dt}(K_j/L_j) &= [\dot{K}_j L_j - K_j \dot{L}_j]/L_j^2 \\ &= \dot{K}_j/L_j - k_j \dot{L}_j/L^2\end{aligned} \tag{3.26}$$

$$\begin{aligned}\dot{l}_j = \tfrac{d}{dt}[L_j/L] &= [\dot{L}_j L - L_j \dot{L}]/L^2 \\ &= \dot{L}_j/L - l_j \dot{L}/L\end{aligned}$$

therefore,

$$\dot{L}_j/L_j = (\dot{L}/L)(L/L_j) = [\dot{l}_j + l_j \dot{L}/L]/l_j = g_j$$

Also from the equation (3.26),

$$\dot{K}_j/L_j = \dot{k}_j + k_j g_j.$$

Thus, for the sector 1, it becomes

$$\begin{aligned}\dot{K}_1/L_1 &= is_1Y_1/L_1 + is_2Y_2/L_1 - u_1k_1 \\ &= is_1(Y_1/L)(L/L_1) + is_2(Y_2/L)(L/L_1) - u_1k_1 \\ &= is_1y_1/l_1 + is_2y_2/l_1 - u_1k_1 \\ &= \dot{k}_1 + k_1g_1.\end{aligned}$$

Likewise, for sector 2,

$$\dot{K}_2/L_2 = (1-i)s_1y_1/l_2 + (1-i)s_2y_2/l_2 - u_2k_2.$$

The system state equations (3.25) appear to be a time-varying linear system, however, it is not linear since the output y_j is a function of the system state (capital per labor), thus making the coefficient matrix $\underline{B}$ a function of $\overline{k}$. Therefore, multiplicative terms of state variables and input variables exist which make the analytical solution impossible in general.[1] In essence, the basic difference between the model given as equations (3.25) and the two sector models of Meade and Uzawa lies on the number of system state equations which describe the behaviors of the economic growth.

[1]If the production function is linear, the modified dual economic growth model becomes bilinear system.

III.4 Analysis of Economic Growth of Korea

The dual economic growth model developed in the preceding section will be applied to the case of Korea to look into the behaviors of the economy and to see the validity of the model at the same time.

The production function plays a crucial role in determining the system state equations. The Cobb-Douglas production function will be used for the aggregate production of agriculture and nonagriculture. The estimation of the production function--in the form of Cobb-Douglas production function--using the OLSE (ordinary least square estimation) has been obtained as[1]

$$\begin{aligned} \ln Y_1(t) &= a_1 + b_1 \ln K_1(t) + (1 - b_1) \ln L_1(t) \\ &= -0.989519 + 0.393019 \ln K_1(t) + 0.606981 \ln L_1(t) \\ &\quad (0.158445) \; (0.0646925) \\ &\quad (-6.24521) \; (6.07519) \; ^2 \\ &\quad R^2 = 0.7868 \qquad D.W. = 1.7726 \end{aligned} \tag{3.26}$$

$$\begin{aligned} \ln Y_2(t) &= a_2 + b_2 \ln K_2(t) + (1 - b_2) \ln L_2(t) \\ &= -0.506822 + 0.536155 \ln K_2(t) + 0.463845 \ln L_2(t) \\ &\quad (0.0266334) \; (0.0272827) \\ &\quad (-19.0295) \; (19.6518) \\ &\quad R^2 = 0.9748 \qquad D.W. = 1.3034 \end{aligned} \tag{3.27}$$

where sector 1 and 2 refers to agriculture and nonagriculture.

---

[1]Labor data used for the estimation are listed in Appendix A and capital data have been derived in Appendix B.

[2]The numbers in the parentheses below the estimators are the standard errors of the estimators and the t-statistics.

These production functions satisfy the neoclassical condition and can be rewritten in term of per labor quantity,

$$Y_1(t)/L_1(t) = 0.371755\ k_1(t)^{0.393019}$$
$$Y_2(t)/L_2(t) = 0.602407\ k_2(t)^{0.536155}$$

or

$$y_1(t) = 0.371755\ k_1(t)^{0.393019}\ l_1(t) \tag{3.29}$$
$$y_2(t) = 0.602407\ k_2(t)^{0.536155}\ l_2(t) \tag{3.30}$$

The total labor force (employment) has been grown as

$$L(t) = 7517.49\ e^{0.03819\ t} \qquad R^2 = 0.9963$$

during the periods of 1964 to 1974, and thus the rate of growth of the labor force is

$$\dot{L}(t)/L(t) = 0.03819 \tag{3.31}$$

There has been significant shift of labor force from agriculture to nonagriculture in Korea; the nonagricultural labor share to the total labor was about 41 percent in 1965 and grew to 54 percent in 1975 following linear time trend approximately. Although it is inconceivable to assume that the agricultural labor force would increase beyond a certain percentage of the total labor, the time trend linear function has been fitted to the actual data, and assumed to follow this trend till 1994 when the labor share of nonagriculture would be 75.43 percent. The estimated trend lines are

$$l_1(t) = 0.58504 - 0.01131\ t \qquad R^2 = 0.87103 \tag{3.32}$$
$$l_2(t) = 1 - l_1(t) = 0.41496 + 0.01131\ t$$

where the time periods represent the years from 1965 to 1994, and t takes the values such that $1 \leqq t \leqq 30$.

From the production functions (3.29) and (3.30), and the equations for the labor share (3.31) and (3.32), the system state equations for the two sector growth model for Korea become

$$\begin{bmatrix} \dot{k}_1(t) \\ \dot{k}_2(t) \end{bmatrix} = \begin{bmatrix} -1/25.5-g_1(t) & 0 \\ 0 & -1/13.9-g_2(t) \end{bmatrix} \begin{bmatrix} k_1(t) \\ k_2(t) \end{bmatrix} + \begin{bmatrix} (0.1)(0.371755)k_1(t)^{0.393019} & (0.1)(0.602407)k_2(t)^{0.536155} l_2(t)/l_1(t) \\ (0.9)(0.371755)k_1^{0.393019} l_1(t)/l_2(t) & (0.9)(0.602407)k_2(t)^{0.536155} \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} \tag{3.33}$$

where

$$g_1(t) = [-0.01131 + 0.03819\ l_1(t)]/l_1(t)$$
$$g_2(t) = [0.01131 + 0.03819\ l_2(t)]/l_2(t)$$

the depreciation rates are given as $u_1 = 1/25.5$ and $u_2 = 1/13.9$ taken from Appendix B, and the investment-allocation parameter for agriculture is given as o.1 from the past trend.

The system responses of the equation (3.33) to impulse, step, and ramp inputs are shown in Figure 3.3, where the inputs used are:

impulse : $s_1(t) = \delta(t)$

$s_2(t) = \delta(t)$

step : $s_1(t) = 0.05\ \mu(t)$

$s_2(t) = 0.25\ \mu(t)$

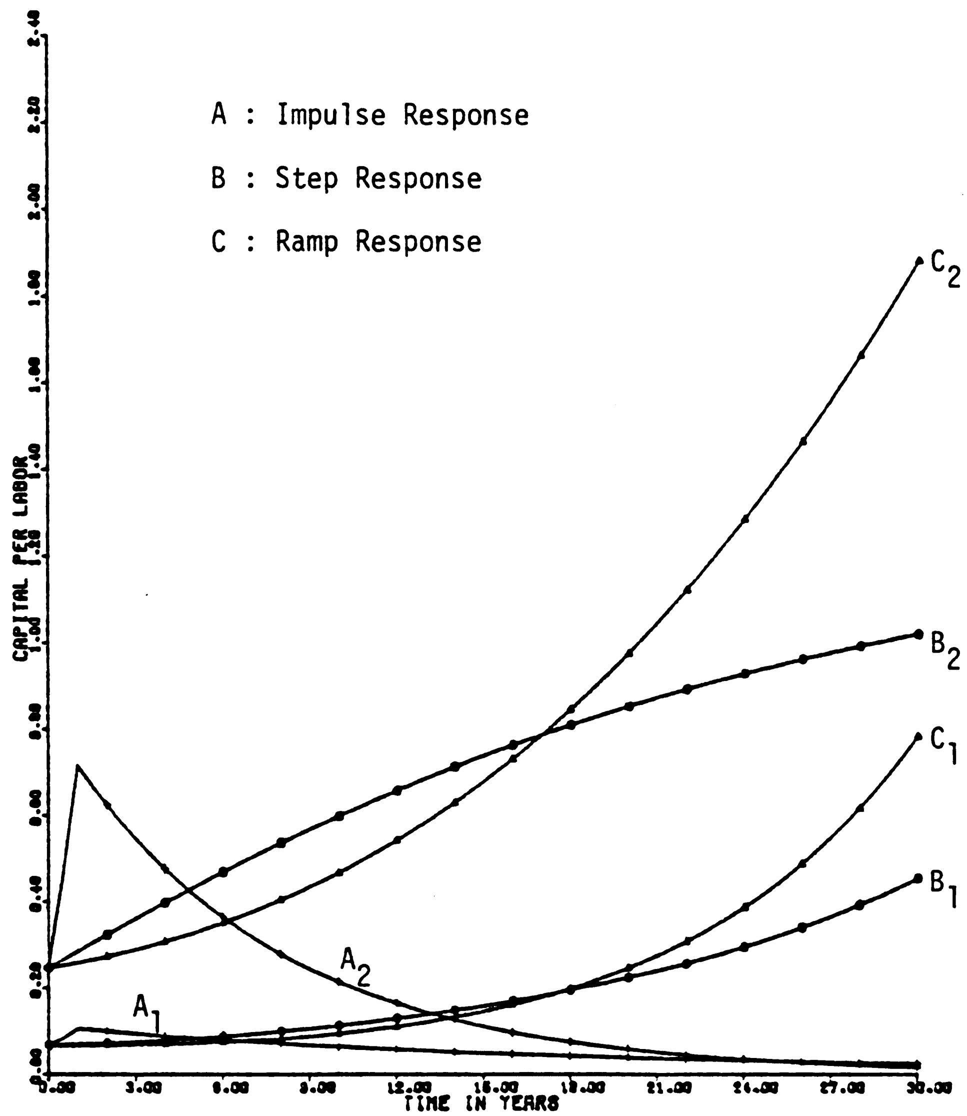

Figure 3.3 Input Responses of Dual Economic Growth System†

†Subscript denotes each sector--1 for agriculture and 2 for nonagriculture.

$$\text{ramp} \quad : \quad s_1(t) = 0.04 + 0.001\ r(t)$$

$$s_2(t) = 0.15 + 0.01\ r(t)$$

The impulse response of the growth system for Korea shows the inherent stability of the system for bounded inputs, which also could be observed from the negative eigenvalues of the diagonal matrix $\underline{A}$ in the equation (3.33).

Average gross fixed investment ratio during 1965 to 1974 in Korea was 22.7 percent of the GNP. Figure 3.4 shows the paths of the state variables with different sets of constant saving rates, i.e., step inputs given as:

$$\text{case B} : \quad s_1(t) = 0.05\ \mu(t)$$

$$s_2(t) = 0.25\ \mu(t)$$

$$\text{case C} : \quad s_1(t) = 0.10\ \mu(t)$$

$$s_2(t) = 0.25\ \mu(t)$$

The case A represents the trajectories of the actual capital per labor during 1965 to 1975. The trajectories generated from the model by the step inputs show closeness to the actual economy; the difference is that the saving rates in actual economy were lower in the early periods of 1965 to 1970 and became higher during the later periods. Thus the satisfactory behavior of the system is expected for the projected values of the future saving rates.

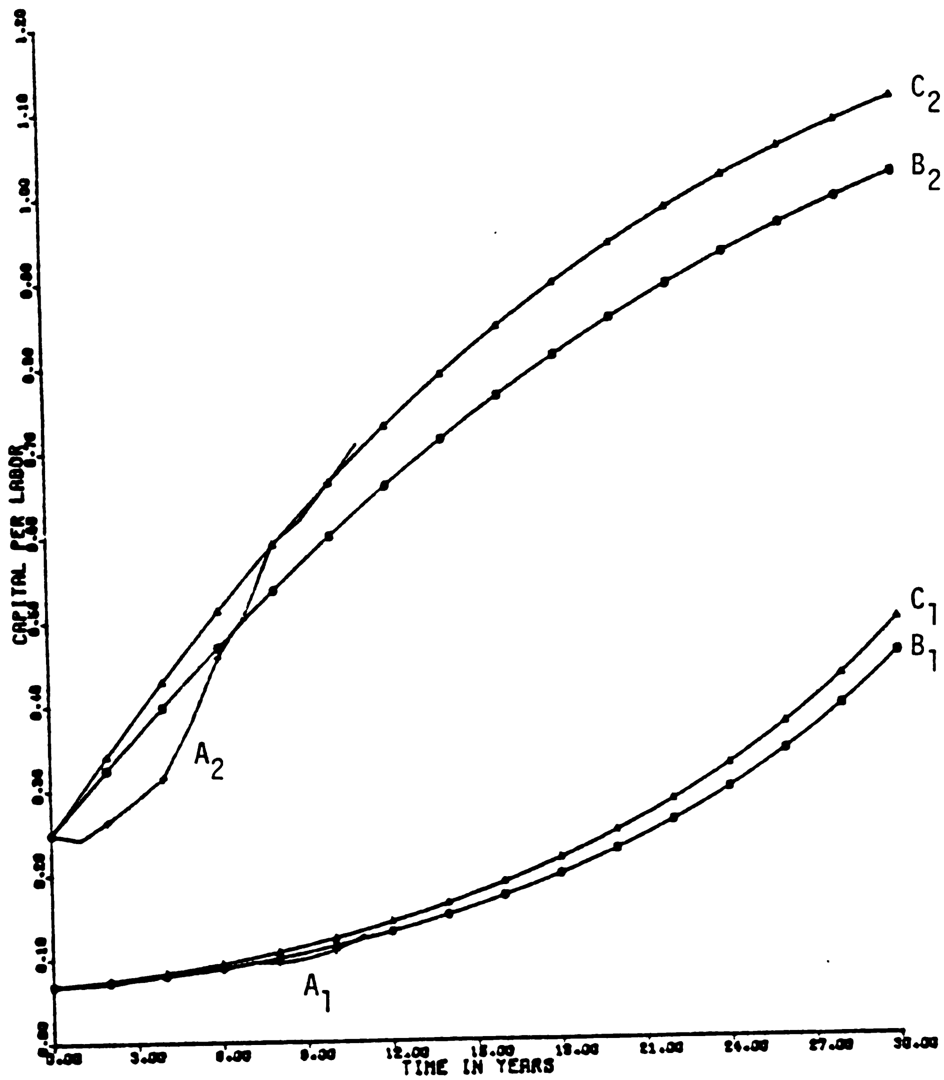

Figure 3.4 State Trajectories of Dual Economic Growth System

CHAPTER IV

OPTIMAL ECONOMIC GROWTH

The considerations of the preceding chapter will further be extended to the optimal growth of an economy by adding social welfare as an objective function. Necessary and sufficient conditions for the optimality will be derived using Pontryagin's maximum principle, and the optimal trajectories--state, costate, control--will be obtained numerically for the case of Korean economy.

IV.1 Economic Objectives of a Society

The usual optimal growth problem of an economy can be posed as follows: "What are the optimal program of capital accumulation or the optimal path of growth among other available paths which is consistent with the growth system equations, and maximizes (or minimizes) some suitable criterion of a society while satisfying additional constraints?" To answer the question, the objective of a society should be defined.

Strictly speaking, there hardly exists a well-defined objective of a society, since there are too many desired outputs which represent the interests of different groups in the society. That is, most of the public decisions are made as the results of collective interactions between the conflicting interests, and moreover, there still exist conflicts between the (commonly agreed) objectives.

This problem of social objective can be handled by defining a more cosmic concept of "social welfare" or "social utility" by assuming the existence of Pareto optimality.[1]

Among the variables which will contribute to the social welfare, consumption has been used as a key variable. Keynes, among other economists, even declared that "consumption is the sole end and object of all economic activity."[2] Following this line, the objective can be to maximize the consumption during a certain time period. That is,

$$U(.) = f[c(t)], \quad 0 \leqq t \leqq T \tag{4.1}$$

where U : social welfare or utility

c : per capita (or per labor) consumption.

Conceptually, there are some desirable conditions for the "well-behaved" utility function.[3] These are:

i) there is no discontinuity in the choices of a society, i.e., the variables in the utility function are continuous

ii) utility function is monotonic increasing with respect to inputs

iii) there exist continuous derivatives up to the third order and the function is strictly concave

iv) the second derivative with respect to input approaches infinity as the input approaches the origin, i.e.,

$$\lim_{c \to 0} U''(c) = \infty$$

---

[1]There still remain paradox and conceptual difficulties with regard to the Pareto optimality. "the so-called Pareto-type welfare function, frequently assumed by economists, is in any case likely to be ethically unacceptable since it (may) reveal as a social improvement some further impoverishment of the poor compensated by some further enrichment of the already rich," [M11], p. 747.

[2][K3], p. 140.

[3][B10], p. 355.

which implies the rate of contribution to the satisfaction is immeasurable at the point of the first infinitesiaml consumption.

The conceptualization of the social welfare function is closely linked with the time horizon to be considered. If the time horizon is infinite, inclusion of only consumption in the welfare function may be correct, whereas, for the case of finite time horizon, which occurs to real planners, intertemporal comparisons of welfare become essential since one has to determine the amount of productive capacities left for the future generations.

This leads one to include capital in the social welfare function as,

$$U(.) = U[c(t), k(t)], \qquad 0 \leqq t \leqq T \tag{4.2}$$

which can be classified into three cases according to the variable(s) used for the social welfare; consumption-oriented, capital-oriented, composite case.

Another consideration related to the time horizon is the weighting schemes for intertemporal or intergenerational comparisons which has been argued on the ethical ground: "Can one generation impose a specific weights to the others just because future generations have no recourse against present generation's insistence to receive the heaviest welfare weight?"[1]

Neither modifications of the social welfare function removes the basic conceptual difficulties, however, the final choice is usually made on the basis of convenience.

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[1]Ramsey maintained that the rate of discount of utilities must be distinguished from the rate of discount of the sums of money, and considered the discount of later enjoyment in comparison with earlier ones is unethical. [R1]

VI.2 One Sector Optimal Growth Model

The simplest form of the utility function has been used in Uzawa's optimal growth model which is given by, [U2]

$$U(c) = c(t) \tag{4.3}$$

Even though the above consumption-oriented utility function violates the last two conditions of the "well-behaved" utility function, it has been used widely for its simplicity. The objective functional, J, will be the integral of the utility over the time horizon, i.e.,

$$\text{Max. } J(.) = \int_{t_0}^{t_f} U[c(t)]\, e^{-rt}\, dt = \int_{t_0}^{t_f} c(t)\, e^{-rt}\, dt \tag{4.4}$$

where t_0 and t_f are the initial and final time, and r is the rate of discount for the future consumption. From the above objective functional and the one sector system equation (3.7), a Hamiltonian can be formulated as

$$H[k(t), s(t), p(t), t] = [1-s(t)]y(t)\, e^{-rt} + p(t)[s(t)y(t) - gk(t)] \tag{4.5}$$

where p is the costate variable--Lagrange multiplier or shadow price--and g is the sum of the depreciation rate and the growth rate of labor, u + n. From the Pontryagin's maximum principle, the necessary conditions for the optimality are[1]

$$\dot{k}(t) = \frac{H}{p} = s(t)y(t) - gk(t) \tag{4.6}$$

$$\dot{p}(t) = -\frac{H}{k} = -[1-s(t)]y'(t)e^{-rt} + p(t)[s(t)y'(t)-g] \tag{4.7}$$

---

[1]The notations will be used as: $\dot{k}(t) \equiv \frac{dk}{dt}$, $y'(.) \equiv \frac{\partial y}{\partial k}$

$$0 = \frac{\partial H}{\partial s} = [p(t) - e^{-rt}]y(t) \tag{4.8}$$

then, $p(t) = e^{-rt}$. (4.9)

Since the Hamiltonian (4.5) is a linear function of input, s(t), this problem constitutes a "bang-bang" control problem. The optimal control for $0 \leq s(t) \leq 1$ can be obtained as,

$$s(t) = \begin{cases} 1 & \text{if } p(t) > e^{-rt} \\ \text{indeterminate} & \text{if } p(t) = e^{-rt} \\ 0 & \text{if } p(t) < e^{-rt} \end{cases} \tag{4.10}$$

Because of the indeterminacy of the control, the optimal control doesn't exist for a time interval $[t_a, t_b]$[1] such that

$$p(t) = e^{-rt} \quad , t \in [t_a, t_b].$$

To avoid the possible existence of singularity, an alternative objective utility function can be suggested as

$$U[c(t)] = c(t)^a \tag{4.11}$$

where a is a constant such that $0 < a < 1$. It can be shown that all the desirable conditions of utility function are satisfied by the above utility function.

Using the utility function (4.11), the Hamiltonian can be

$$H(.) = [1-s(t)]^a y(t)^a e^{-rt} + p(t)[s(t)y(t) - gk(t)] \tag{4.12}$$

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[1]This interval is called singular interval, and the indeterminacy condition is called singular condition.

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and the necessary conditions will become

$$\dot{k}(t) = s(t)y(t) - gk(t) \tag{4.13}$$

$$\dot{p}(t) = [1-s(t)]^a \; ay'(t)^{a-1}e^{-rt} + p(t)[s(t)y'(t) - g] \tag{4.14}$$

$$0 = - a[1-s(t)]^{a-1}y(t)^a e^{-rt} + p(t)y(t) \tag{4.15}$$

thus, $$s(t) = 1 - \frac{1}{y(t)}\,[p(t)e^{rt}/a]^{1/(a-1)} \tag{4.16}$$

The sufficient condition for the maximum can also be obtained from

$$\frac{\partial^2 H}{\partial s^2} = a(a-1)[1-s(t)]^{a-2}y(t)^a e^{-rt} \tag{4.17}$$

which is negative for $0 < a < 1$, and $s(t) < 1$. Since the control has to be constrained--corresponds to the conceptual or practical constraints --to assure the existence of the optimum, the optimal control can be obtained as

$$s(t) = \begin{cases} s_M & \text{if} \quad \frac{1}{y(t)}[p(t)e^{rt}/a]^{1/(a-1)} \leqq 1-s_M \\ 1 - \frac{1}{y(t)}[p(t)e^{rt}/a]^{1/(a-1)} & \text{if } 1-s_M < \quad \text{"} \quad < 1-s_L \\ s_L & \text{if} \quad \text{"} \quad \geqq 1-sL \end{cases} \tag{4.18}$$

where s_L and s_M indicate the lower and the upper boundaries of saving rate such that s_L, $s_M \in (0, 1)$. The sufficient conditions assure the strict concavity of Hamiltonian with respect to s(t), and thus the above optimal control is possible. In essence, the alternative utility function (4.11) removed the possibility of singularity.

IV.3 Two Sector Optimal Growth Model

The optimal economic growth of one sector model will be extended into two sector model in this section. To simplify the total welfare with intertemporal discounts, a modified Hamiltonian will be defined as[1]

$$\tilde{H}(.) \equiv H(.)\, e^{rt} = U(.) + \bar{p}^T e^{rt}(\underline{A}\,\bar{k} + \underline{B}\,\bar{s})$$
$$\equiv U(.) + \tilde{p}^T(\underline{A}\,\bar{k} + \underline{B}\,\bar{s}) \tag{4.19}$$

where the system state equation is from the equation (3.25), and $\tilde{p}$ represents a modified costate variable. Then, the necessary conditions using the modified Hamiltonian can be derived as

$$\dot{\bar{k}} = \frac{\partial \tilde{H}}{\partial \tilde{p}} \tag{4.20}$$

$$\dot{\tilde{p}} = -\frac{\partial \tilde{H}}{\partial k} + r\tilde{p} \tag{4.21}$$

$$\bar{0} = \frac{\partial \tilde{H}}{\partial s} \tag{4.22}$$

The above equations enable one to use Hamiltonian without discount rate, and will simplify the expression and computation.

A natural extension of one sector optimal control problem with consumption-oriented utility function into two sector problem leads to the formulation of utility function

$$U(.) = c_1^{a_1} c_2^{a_2} \tag{4.23}$$

where c_1 is the consumption per labor of agricultural product

c_2 is the consumption per labor of nonagricultural product

a_1 and a_2 are the relative (preference) weights between the consumption of two aggrigate commodities.

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[1]For notational convenience, time t will not be expressed in the variables in this section.

It can be shown that the utility function (4.23) satisfies the conditions for the "well-behaved" utility function.

Using the system state equation for the two sector model, (3.25), and the utility function (4.23), the Hamiltonian can be given by[1]

$$H(.) = c_1^{a_1} c_2^{a_2} + p^T(\underline{A}\,\bar{k} + \underline{B}\,\bar{s})$$

$$= [(1-s_1)y_1]^{a_1}[(1-s_2)y_2]^{a_2} + p_1(v_{11}k_1 + v_{12}y_1s_1 + v_{12}y_2s_2)$$

$$+ p_2(v_{21}k_2 + v_{22}y_1s_1 + v_{22}y_2s_2) \tag{4.24}$$

where
$$v_{11} = -(u_1 + g_1)$$
$$v_{12} = i/l_1$$
$$v_{21} = -(u_2 + g_2)$$
$$v_{22} = (1 - i)/l_2$$

with the definitions as in the equation (3.25).

The necessary conditions are also given by

$$\dot{\bar{k}} = \begin{bmatrix} v_{11}k_1 + v_{12}y_1s_1 + v_{12}y_2s_2 \\ v_{21}k_2 + v_{22}y_1s_1 + v_{22}y_2s_2 \end{bmatrix} \tag{4.25}$$

$$\dot{\bar{p}} = \begin{bmatrix} a_1[(1-s_1)y_1]^{a_1-1}(1-s_1)y_1'[(1-s_2)y_2]^{a_2} + p_1(v_{11}+v_{12}y_1's_1) \\ + p_2v_{22}y_1's_1 \\ a_2[(1-s_1)y_1]^{a_1}[(1-s_2)y_2]^{a_2-1}(1-s_2)y_2' + p_1v_{12}y_2's_2 \\ + p_2(v_{21}+v_{22}y_2's_2) \end{bmatrix} + rp \tag{4.26}$$

[1]The modified Hamiltonian and the modified costate will be used without tilde for the rest of this chapter unless otherwise specifies.

$$0 = \frac{\partial H}{\partial \bar{s}} = \begin{bmatrix} -a_1 y_1[(1-s_1)y_1]^{a_1-1}[(1-s_2)y_2]^{a_2} + p_1 v_{12} y_1 + p_2 v_{22} y_1 \\ -a_2 y_2[(1-s_1)y_1]^{a_1}[(1-s_2)y_2]^{a_2-1} + p_1 v_{12} y_2 + p_2 v_{22} y_2 \end{bmatrix} \quad (4.27)$$

The sufficient condition for the optimality also can be obtained from the Hessian matrix

$$\frac{\partial^2 H}{\partial \bar{s}^2} = a_1 a_2 y_1 y_2 [(1-s_1)y_1]^{a_1-1}[(1-s_2)y_2]^{a_2-1} \cdot \begin{bmatrix} \frac{a_1-1}{a_2}\frac{y_1}{y_2}\frac{(1-s_2)y_2}{(1-s_1)y_1} & 1 \\ 1 & \frac{a_2-1}{a_1}\frac{y_2}{y_1}\frac{(1-s_1)y_1}{(1-s_2)y_2} \end{bmatrix} \quad (4.28)$$

Since $0 < a_1, a_2 < 1$, and $0 < s_1, s_2 < 1$, the scalar term of (4.28) is positive. Furthermore,

$\frac{a_1-1}{a_2}\frac{y_1}{y_2}\frac{(1-s_2)y_2}{(1-s_1)y_1} < 0$, and the determinant of the matrix in (4.28) is

$$\frac{1 - (a_1 + a_2)}{a_1 a_2} .$$

Therefore, the Hessian matrix (4.28) is negative definite if and only if

$$a_1 > 0, \; a_2 > 0, \text{ and } a_1 + a_2 < 1. \quad (4.29)$$

The sufficient condition (4.29)--similar to (4.17) for the one sector case--assures the existence of global optimum. The optimal control, s(t), can be obtained by solving the simultaneous equation (4.27) such as

$$s_1 = 1 - \frac{1}{y_1}\left[\frac{1}{[\frac{1}{a_1y_1}(p_1v_{12}y_1+p_2v_{22}y_1)]^{1-a_2}[\frac{1}{a_2y_2}(p_1v_{12}y_2+p_2v_{22}y_2)]^{a_2}}\right]^{\frac{1}{1-a_1-a_2}} \tag{4.30}$$

$$s_2 = 1 - \frac{1}{y_2}\left[\frac{1}{[\frac{1}{a_1y_1}(p_1v_{12}y_1+p_2v_{22}y_1)]^{a_1}[\frac{1}{a_2y_2}(p_1v_{12}y_2+p_2v_{22}y_2)]^{1-a_1}}\right]^{\frac{1}{1-a_1-a_2}} \tag{4.31}$$

Hence, the optimal saving rates will be given by (4.30) and (4.31) if these values are within the constraints between 0 and 1. Upper limit or lower limit of the constraints will be given whenever the unconstrained optimal saving violates the constraints. This is possible because of the strict concavity of the Hamiltonian with respect to s when the sufficient conditions are satisfied.

Finally, the case of combined consumption-capital utility function will be considered. The utility function can be given as

$$U(.) = c_1^{a_1} c_2^{a_2} k_1^{b_1} k_2^{b_2} \tag{4.32}$$

Then, the Hamiltonian becomes

$$H(.) = [(1-s_1)\, y_1]^{a_1}[(1-s_2)y_2]^{a_2} k_1^{b_1} k_2^{b_2} + p^T \begin{bmatrix} v_{11}k_1+v_{12}y_1s_1+v_{12}y_2s_2 \\ v_{21}k_2+v_{22}y_1s_1+v_{22}y_2s_2 \end{bmatrix} \tag{4.33}$$

and the necessary and sufficient conditions also can be obtained as

$\dot{\bar{k}} = \underline{A}\,\bar{k} + \underline{B}\,\bar{s}$, which is the same with the equation (4.25)

$$\dot{\bar{p}} = - \begin{bmatrix} [(1-s_1)y_1]^{a_1}[(1-s_2)y_2]^{a_2}k_1^{b_1}k_2^{b_2}(ay_1'/y_1+b_1/k_1)+p_1(v_{11}+v_{12}y_1's_1) \\ + p_2v_{22}y_1's_1 \\ [(1-s_1)y_1]^{a_1}[(1-s_2)y_2]^{a_2}k_1^{b_1}k_2^{b_2}(a_2y_2'/y_2+b_2/k_2) + p_1v_{12}y_2's_2 \\ + p_2(v_{21}+v_{22}y_2's_2) \end{bmatrix}$$

$$+ r\bar{p} \qquad (4.34)$$

$$\bar{0} = \frac{\partial H}{\partial \bar{s}} = \begin{bmatrix} -a_1y_1[(1-s_1)y_1]^{a_1-1}[(1-s_2)y_2]^{a_2}k_1^{b_1}k_2^{b_2} + p_1v_{12}y_1 + p_2v_{22}y_1 \\ -a_2y_2[(1-s_1)y_1]^{a_1}[(1-s_2)y_2]^{a_2-1}k_1^{b_1}k_2^{b_2} + p_1v_{12}y_2 + p_2v_{22}y_2 \end{bmatrix} \qquad (4.35)$$

Solving the above simulataneous equation to get the optimal saving rates,

$$s_1 = 1 - \frac{1}{y_1}\left[\frac{k_1^{b_1}k_2^{b_2}}{[\frac{1}{a_1y_1}(p_1v_{12}y_1+p_2v_{22}y_1)]^{1-a_2}[\frac{1}{a_2y_2}(p_1v_{12}y_2+p_2v_{22}y_2)]^{a_2}}\right]^{\frac{1}{1-a_1-a_2}} \qquad (4.36)$$

$$s_2 = 1 - \frac{1}{y_2}\left[\frac{k_1^{b_1}k_2^{b_2}}{[\frac{1}{a_1y_1}(p_1v_{12}y_1+p_2v_{22}y_1)]^{a_1}[\frac{1}{a_2y_2}(p_1v_{12}y_2+p_2v_{22}y_2)]^{1-a_1}}\right]^{\frac{1}{1-a_1-a_2}} \qquad (4.37)$$

Then the sufficient conditions will also become

$$\frac{\partial^2 H}{\partial \bar{s}^2} = a_1 a_2 y_1 y_2 [(1-s_1)y_1]^{a_1-1} [(1-s_2)y_2]^{a_2-1} k_1^{b_1} k_2^{b_2}$$

$$\cdot \begin{vmatrix} \frac{(a_1-1)y_1(1-s_2)y_2}{a_2 \;\; y_2(1-s_1)y_1} & 1 \\ 1 & \frac{(a_2-1)y_2(1-s_1)y_1}{a_1 \;\; y_1(1-s_2)y_2} \end{vmatrix}$$

thus, the Hessian matrix $[\partial^2 H/\partial \bar{s}^2]$ is negative definite if and only if

$a_1 + a_2 < 1$ and $0 < a_1$, $0 < a_2$ (and $s_1, s_2 < 1$)

The above condition guarantees the global maximum at any instant of time, i.e., for each fixed sets of state and costate variables. When $a_1 > 1$ and $a_1 + a_2 > 1$ (and $a_2 > 0$), the Hessian matrix is positive definite; for all the other cases, it becomes indefinite.

IV.4 Numerical Procedures of the Optimal Control

In order to determine the optimal controls, (4.36) and (4.37), explicitly, state and costate equations should be solved which yields a nonlinear two-point boundary problem. Analytical solution of this kind is impossible in general, and thus necessarily rely on numerical procedures. The basic task of the numerical procedure is to find the control so as to satisfy all the necessary conditions of optimal control during the time period.

Since the final time is fixed by the given planning horizon, the necessary conditions can be summarized as following for the case of free final states:

$$\dot{\bar{k}}(t) = \partial H/\partial \bar{p} \tag{4.38}$$

$$\dot{\bar{p}}(t) = -\partial H/\partial \bar{k} + r\bar{p} \tag{4.39}$$

$$\bar{0} = \partial H/\partial \bar{s} \tag{4.40}$$

$$\bar{k}(0) = \bar{k}_0$$

$$0 \leq \bar{s} \leq 1$$

$$\bar{p}(t_f) = \partial h/\partial \bar{k} \,|_{t_f} = \bar{0}$$

Generally, the computational procedure can be given as following:

1) use initial guess on any of the variables to obtain the solution to a problem in which one or more of the necessary conditions is violated
2) adjust the initial guess in an attempt to make the next solution come closer to satisfying all of the necessary conditions
3) repeat the preceding steps until the iterative procedure converges, and thus all the necessary conditions will be satisfied.

Various methods have been used for numerical solutions in practice such as Ritz's method, dynamic programming, the gradient method, the method of steepest descent, variation of extremals, the method of quasi-linearization, and the method of invariant imbedding [S1], [K4]. The method chosen in the study is a variation of extremals which starts from the initial guess of the costate variables and solve the system equations iteratively[1] by changing the initial costate variables without storing the whole paths of the variables until the final costates come closer to zero, since there is no terms of final states appear in the objective functional.

The choice of the method can be justified by the linearity in the

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[1] Simultaneous linear differential equation solver by the method of Bulirsch-Stoer, DREBS, in IMSL (International Mathematical & Statistical Libraries) has been used.

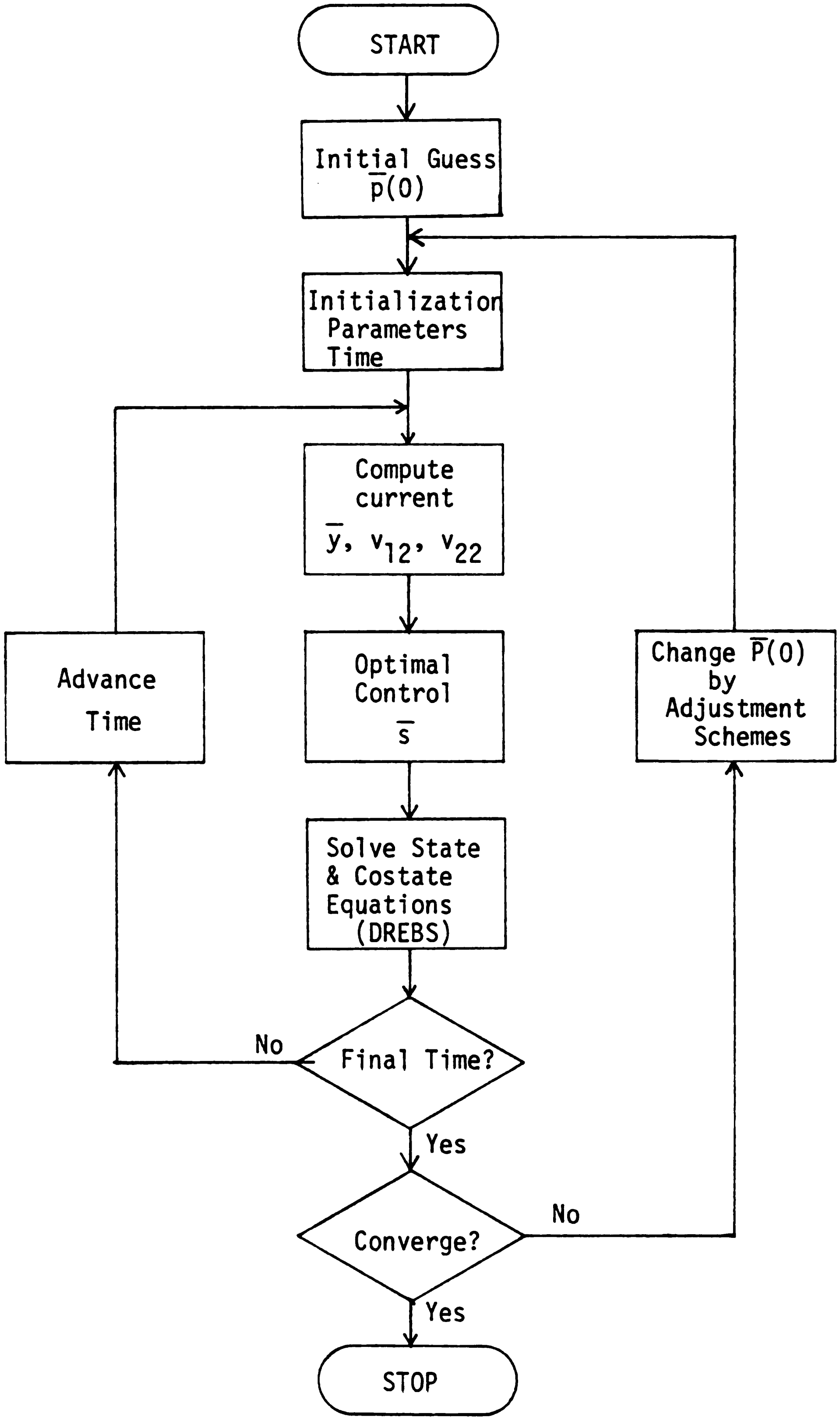

Figure 4.1 Numerical Procedure for the Optimal Trajectory of the Economic Growth System

initial and the final costates, i.e., the changes in the initial costates correspond linearly to the changes in the final costates, which comes from the physical meaning of the costate in the case of economic growth system. The costate implies the social demand price of a unit of investment in terms of a currently foregone unit of consumption, thus the costate will be positive for all normal economy, and will be decreasing monotonically towards the final time as the opportunities for the investment are decreasing as time passes. This concept is just an extention of the Lagrange multipliers--shadow prices in the case of static optimization--along the time scale. With this specific property of the costate variable, a numerical algorithm given in Figure 4.1 can be used efficiently to solve the problem.

The adjustment scheme is the key in this method which can simply be given by (as constant costate influence function matrix),

$$\overline{p}(0)_{new} = \overline{p}(0)_{old} - ALPH \cdot \overline{p}(t_f) \qquad (4.41)$$

where ALPH is a constant adjustment coefficient--diagonal element of costate influence function matrix--such that $0 < ALPH < 1$. For negative final costate, the new initial costate should be increased from the old initial value, and the new initial costate should be decreased for the positive final costate until the norms (or square of Euclidean norms) of the final costate variables are within the given error limit.

IV. 5 Optimal Growth of Korean Economy

In applying the preceding discussions to Korean economy to obtain the optimal growth paths, one has to determine the relative weights in the objective function. This is not an easy task even for the alternative objective functions, because there are virtually innumerable combinations of the ralative weights of each consumptions and capitals--a_1, a_2, b_1, b_2--which satisfy the sufficient conditions of optimality. It also can be inferred from the discussions in chapter IV.1 that the higher the sum of the relative weights of consumptions (or closer to one), the longer the periods of maximum savings waiting for the final consumption spree. Thus, the probability of getting saturated optimal control--bounded optimal saving rates--will be increased for the higher values of the sum.

Although the relative weights between the goods for consumption or capital may change the optimal paths of the state and costate variables not less significantly, the main concern in the theory of optimal growth lies on the consumption-investment decision, i.e., determining the relative weights between the a's and b's. Thus, the relative weights for the alternative objective functions are given in such a way to reflect the alternative decisions between consumption and capital investment by varying the relative weights of these rather than varying the relative weights between the goods of each sector.

Table 4.1 summarizes the values of the relative weights used for the alternative objective functions, initial conditions, parameters, values of the final states, initial values of the costates (the results of the numerical procedure), final costates and its norm, the number of iterations, consumption and output at the final time, and the values of the overall performance index (social welfare).

Because of the definitions of the alternative objective functions, the direct comparisons of the performance indices are meaningless; however, the comparisons of the performance indices for the alternative policies

TABLE 4.1

RESULTS OF THE OPTIMAL CONTROL[†]

	1	2	3	4	5	6
U(.)[‡]	6.800	5.423	4.435	3.675	3.092	2.530
y(10)	0.5168	0.5874	0.6043	0.6116	0.6156	0.6182
c(10)	0.6513	0.7457	0.7685	0.7783	0.7838	0.7873
a_1, a_2	0.2	0.2	0.2	0.2	0.2	0.2
b_1, b_2	0.0	0.2	0.4	0.6	0.8	1.0
$k_1(10)$	0.3204	0.4014	0.4212	0.4297	0.4344	0.4374
$k_2(10)$	1.2040	1.5498	1.6395	1.6787	1.7008	1.7150
$p_1(0)$	1.1573	2.9047	3.7444	4.0659	4.0980	3.9785
$p_2(0)$	0.4127	0.8652	1.1186	1.2376	1.2782	1.2736
$p_1(10)$	-0.0047	0.0159	0.0146	0.0189	0.0184	0.0165
$p_2(10)$	0.0100	-0.0103	-0.0078	-0.0096	-0.0094	-0.0082
$\lVert p(10) \rVert^2$	0.0001	0.0004	0.0003	0.0005	0.0004	0.0003
ITER[§]	7	6	8	8	8	8

[†]Initial Conditions and Parameters:
$p_1(0) = 2.5$, $p_2(0) = 0.8$, ALPH = 0.5, ERR = 0.001

[‡]The value of U(.) is the value of social welfare function using the modified Hamiltonian, i.e., the value of non-discounted welfare.

[§]The number of iterations for the square Euclidean norm to be within the error limit (ERR).

with the identical objective function may have some value as an indicator for the effectiveness of the policies.

As has been discussed, the numerical algorithm in Figure 4.1 shows fast convergency of the final costates to the origin with less than eight iterations from arbitrary initial guesses for the convergency error limit of 0.001 (for square Euclidean norm). This also can be observed in Figure 4.3 which shows the costate trajectories of each goods for alternative objective functions to converge to the zero from the positive initial values.

Figure 4.2 shows the optimal trajectories, $\overline{k}$, of each sector with respect to the different objective functions given in Table 4.1. The corresponding costates and control histories are shown in Figures 4.3 and 4.4. It can be noticed that the change of saving rate from the upper limit to the lower limit (switching for the case of "bang-bang" control) occurs later in the period as the relative weights on capitals are increased since accumulating capital is more important, and the maximum saving will persist till the last minute of impulse consumption for the extreme case of the infinite relative weights on capital.

The inflections in the optimal trajectories occur when the society decides to save less and consume more, which also can be shown by sudden rises of consumption trajectories in Figure 4.5. Figure 4.6 shows the trajectories of the output per labor for corresponding objective functions.

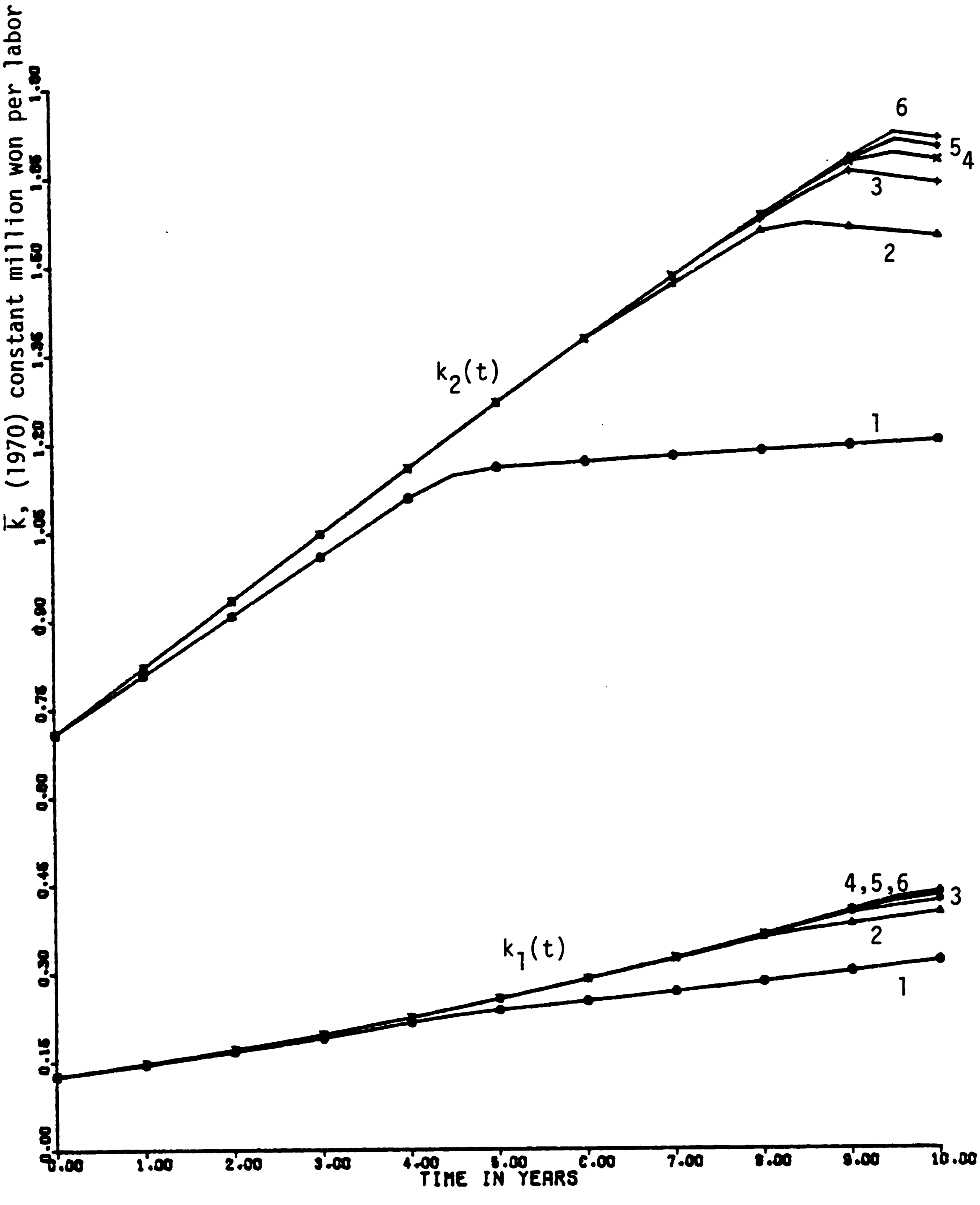

Figure 4.2 Optimal Trajectory, $\overline{k}(t)$

Note: The numbers 1 to 6 correspond to the cases in Table 4.1.

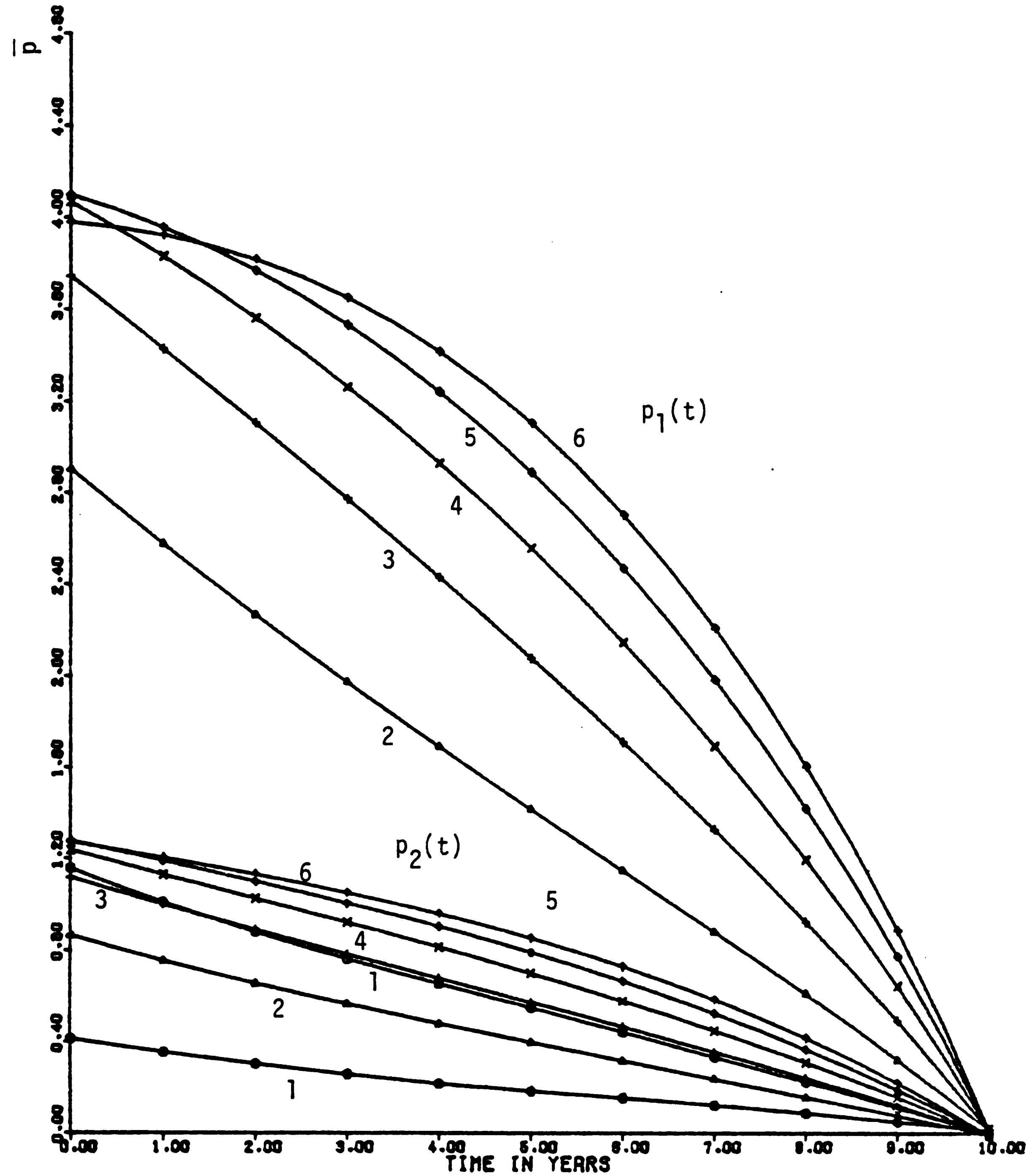

Figure 4.3 Costate Trajectory, $\overline{p}(t)$

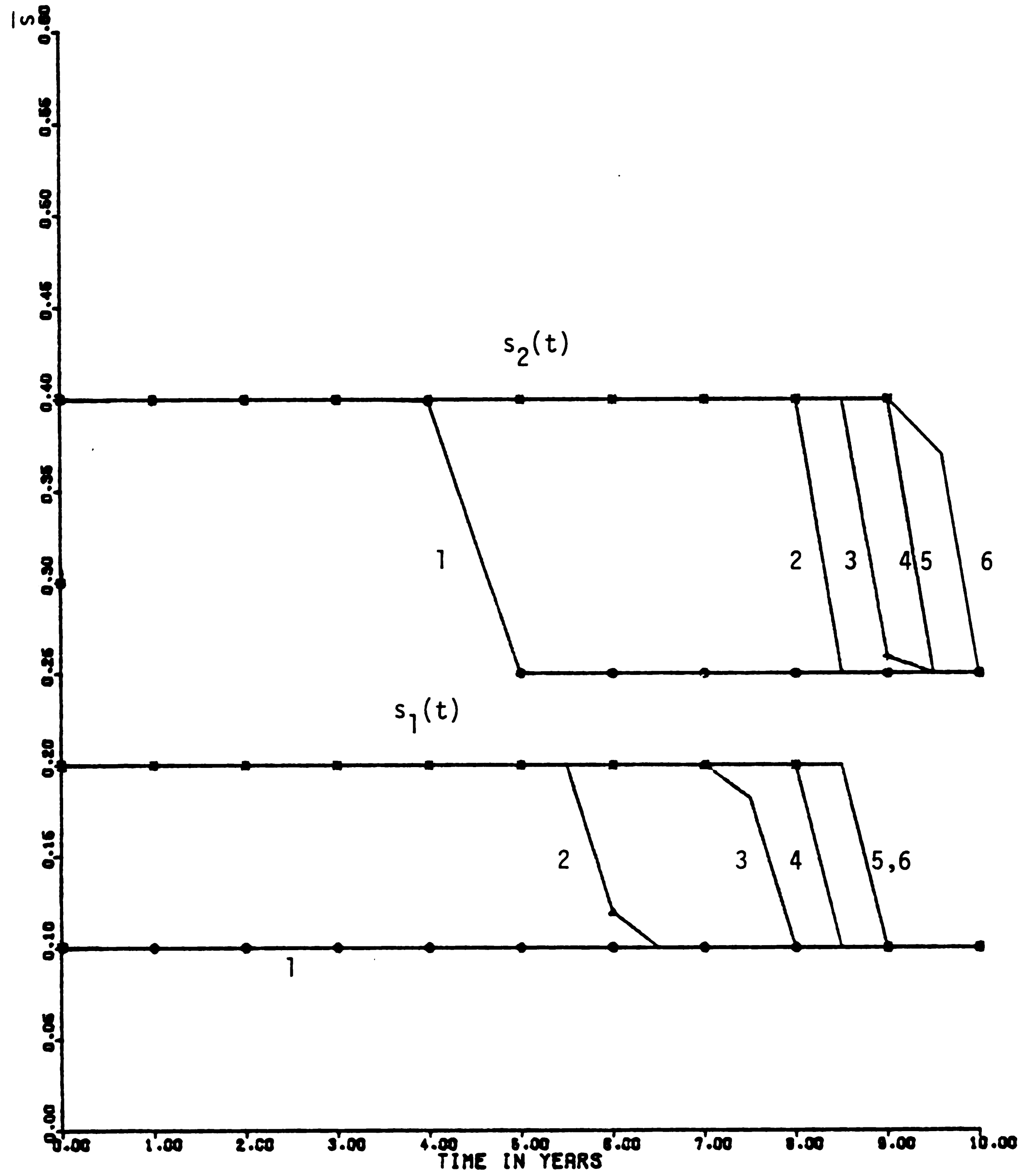

Figure 4.4 Optimal Control, $\overline{s}(t)$

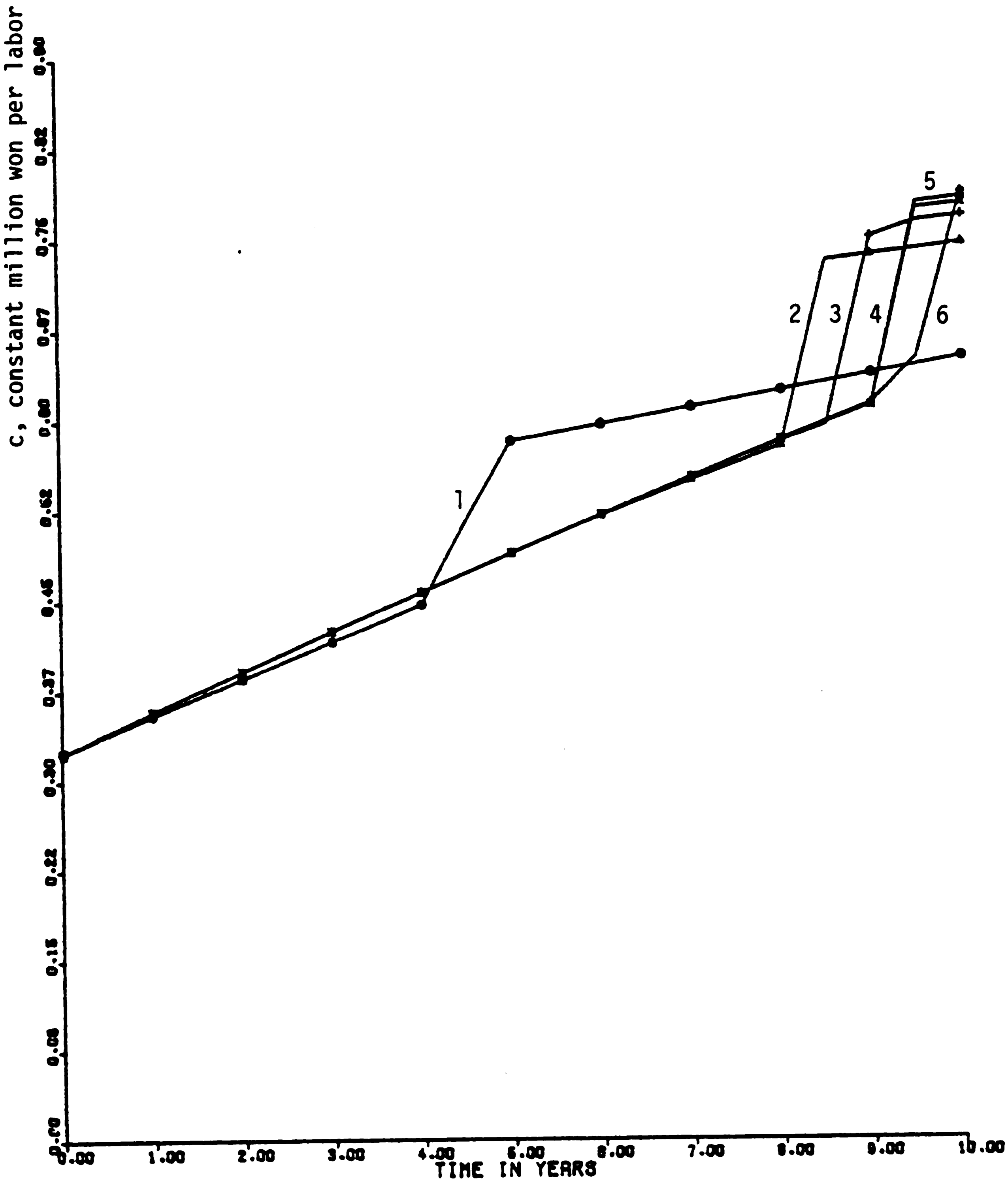

Figure 4.5 Trajectory of Consumption per Labor, c(t)

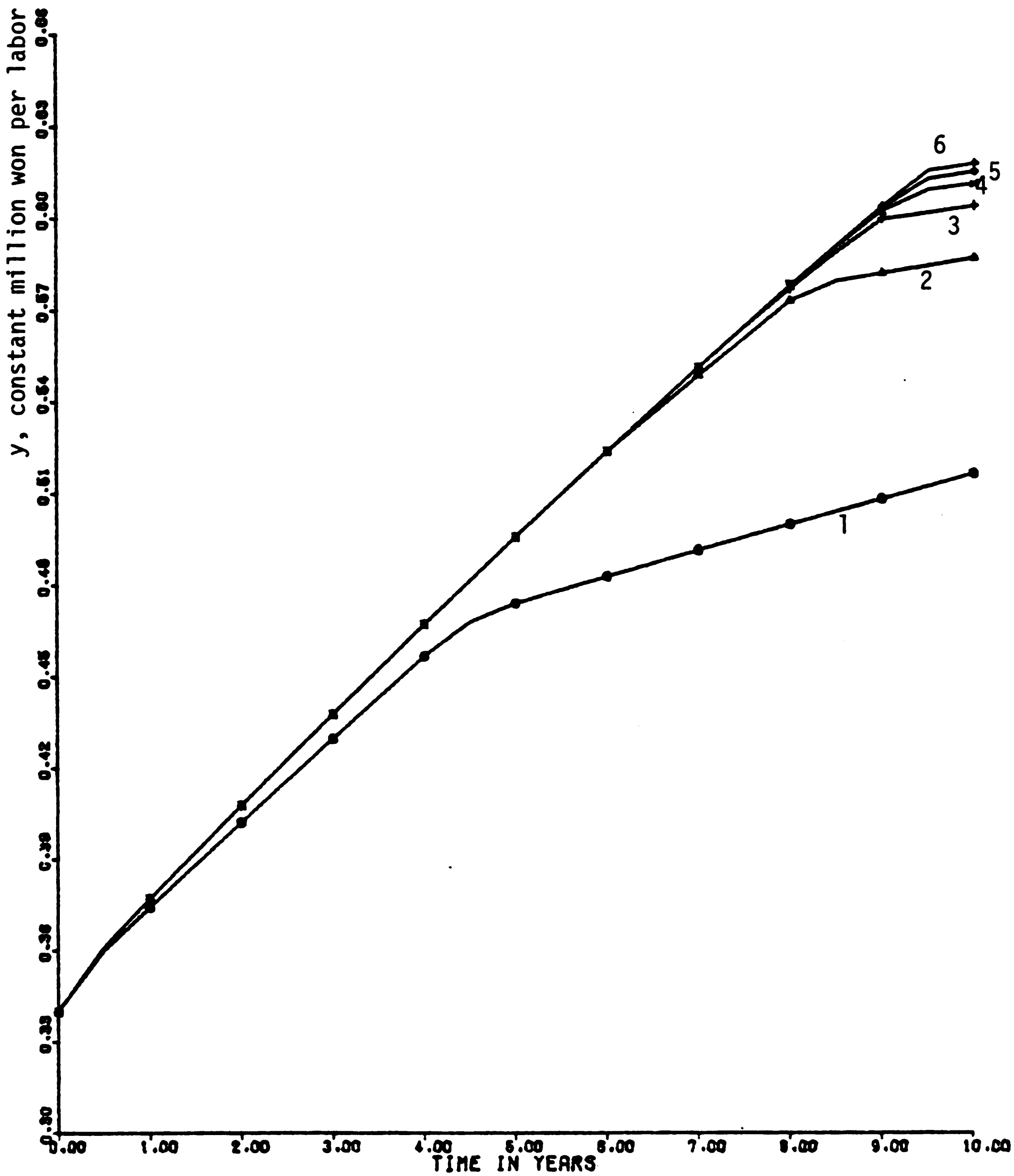

Figure 4.6 Trajectory of Output per Labor, y(t)

CHAPTER V

GROWTH MODEL OF THE OPEN ECONOMY

V.1. Need for the Open Economic Model

One of the major weaknesses of the model which has been developed in the preceding chpater is its closedness, which neglects foreign trade and its effects on the other economic variables.

Since World War II, the whole world has experienced fast growing world trade and increasing dependency of total production on foreign trade. With the growing importance of foreign trade, it is essential to open the economy to include foreign transactions--not only products but also services, foreign transfer, and capital flows--to look into the whole picture of the behavior and interactions of the economic variables.

Nevertheless, not much has been known of the forces and interactions within the variables for the foreign trades and in relation to the domestic economic variables. One obvious reason for this is the complexities of the international economic situation--many of the economic problems among nations can be solved via political means rather than economic policies--and the decreased emphasis on laissez faire in international markets adds to the above complexity. Consequently, the approach which has been taken in this chapter is more or less normative, i.e., stress is more on the behavioral linkages than the estimation of functional forms in strict sense.

Basically, two components were added to the closed economic model: these are foreign trade and balance-of-payments. While the foreign trade component is for transactions of products and services, the balance-of-payments component is for the movement of foreign capital and foreign currency.

The production component is the same except for a few modifications. It has been disaggregated into non-agricultural, non-grain agricultural, and grain production--anticipating the further analysis in a later chapter to investigate the effects of the world food situation. Also, saving and consumption components were extended to include the different saving ratios for the wage share and profit share of the non-agricultural production, and different savings from the non-grain agricultural production and grain production.

Income distribution was considered in terms of the ratio of the per capita farm income to the per capita non-farm (urban) income. This is another important indicator of the total welfare of the economy and should be included in investigating overall economic policies. Grain market mechanisms--price mechanism, grain stock policy, grain import policy--were also included for the case of world food problems (food shocks).

Figure 5.1 is an overall block diagram for the open model.

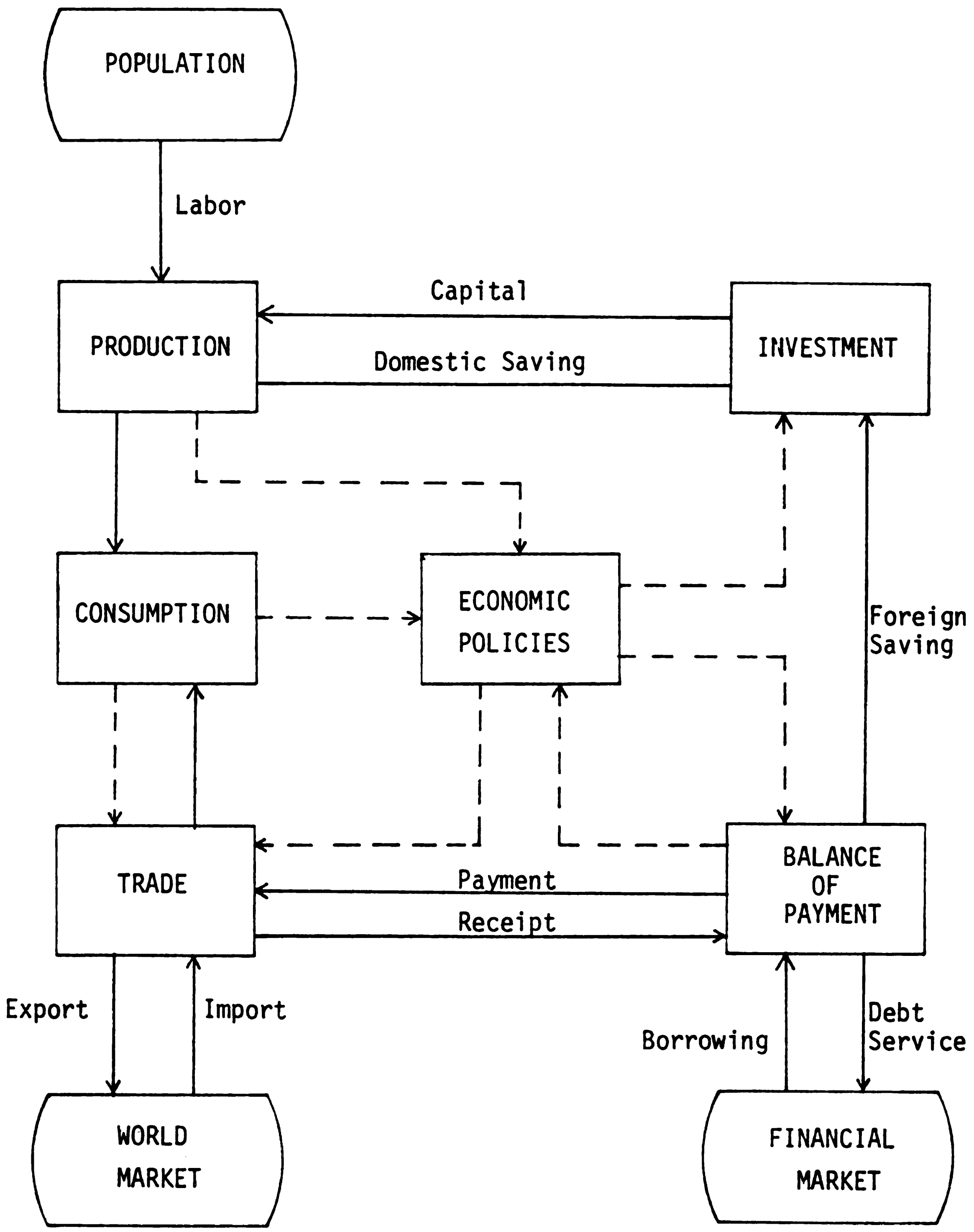

Figure 5.1 Major Components of the Open Economic Growth System

V.2. Description of the Components

Production

The production component computes the aggregate outputs of agriculture and non-agriculture, total GNP, and its growth rates. The use of the aggregate production function with capital and labor as the factors of production has been used widely for empirical studies regardless of its conceptual difficulties. The basic difficulty--measuring the heterogeneous outputs and inputs in one unit or the validity of the assumed homogeneity--has been the subject of unending disputes.[1] However, it is clear that the quantity of output produced (regardless of whether it's homogeneous or heterogeneous) by any economy is constrained by the available supplies of capital and labor.

Thus, an aggregate production function may be expressed by:

$$Y = F[K, L] \tag{5.1}$$

where Y is the aggregate output, K and L are the amounts of capital and labor, respectively. Specific forms and characteristics of the production function commonly used are summarized in Table 5.1. Although the Constant Elasticity of Substitution (CES) production function [A5] and the Transcendental Logarithmic (TRANSLOG) production function [C4] are more general, the Cogg-Douglas production function has been used most frequently because of its

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[1] J. Robinson pointed out the difficulties inherent in measuring the quantity of capital by a single number (or index), and suggested to measure capital in labor units [R4]. Against Robinson's view, D. G. Champernowne showed that a chain index of the capital stock can be formed when output per labor decreases as the rate of interest increases [C1], and R. Solow discussed further necessary and sufficient conditions for the aggregation. [S10]"Of course, it's not true that only one kind of capital good exists, but then there's also more than one kind of labor," Ibid., p. 101.

TABLE 5.1

AGGREGATE PRODUCTION FUNCTIONS

Function	Characteristics: MPK	MPL	Returns to Scale	Elasticity of Substitution
1. Fixed Proportion PF (Input-output or Leontief)				
$Y = \text{Min}\,[aK, bL]$	0 or a	0 or b	constant	0
2. Linear PF				
$Y = aK + bL$	a	b	constant	∞
3. Cobb-Douglas PF			a+b=1: constant < : decreasing > : increasing	
$Y = A\,K^a\,L^b$	$a\,Y/K$	$b\,Y/L$		1
4. CES PF			k=1: constant < : decreasing > : increasing	
$Y=r[dK^{-p}+(1-d)L^{-p}]^{-k/p}$	$\frac{dkY^{1+p/k}}{r^{p/k}K^{1+p}}$	$\frac{(1-d)kY^{1+p/k}}{r^{p/k}L^{1+p}}$		$\frac{1}{1+p}$
5. TRANSLOG PF			depends on the values of the coeffiecients, lnK, and lnL. constant, iff $a_3+a_4+a_5=0$ & $a_1+a_2+[2a_3+a_5]\ln K$ $+[2a_4+a_5]\ln L=1$	
$\ln Y=a_0+a_1\ln K+a_2\ln L+a_3[\ln K]^2$ $+a_4[\ln L]^2+a_5[\ln K][\ln L]$	$\frac{Y}{K}u$ $u=a_1+2a_3\ln K+a_5\ln L$	$\frac{Y}{L}v$ $v=a_2+2a_4\ln L+a_5\ln K$		$\frac{uv[u+v]}{uv[u+v]+2[a_5uv-a_3v^2-a_4u^2]}$

simplicities in parameter estimation and analytical conveniences.[1]

Using the Cobb-Douglas production function, the aggregate production of agriculture and non-agriculture are calculated as

$$PROD_i(t) = A_i \; K_i(t)^{a_i} \; L_i(t)^{b_i} \tag{5.2}$$

where

$PROD_i$: Production of the i-th sector

$a_i + b_i = 1$, for i=1,2, for the case of constant returns to scale

i = 1 : non-agriculture

2 : agriculture

Then, the gross national product, GNP, the GNP growth rates, and the output-labor ratios at time t can be

$$GNP(t) = \textstyle\sum_{i=1}^{2} PROD_i(t). \tag{5.3}$$

$$RGNP(t) = [GNP(t) - GNP(t-DT)]/GNP(t-DT)$$

$$GLR_i(t) = PROD_i(t)/L_i(t)$$

where

RGNP : GNP growth rate for a time period of DT

GLR : output-labor ratio

Income Distribution

Unlike the usual definition of functional income distribution in the theory of economic growth, the concept of regional income distribution will be adopted because of the significance of the income transfer between the urban and the rural areas in the LDC's. Inevitably, there exist

---

[1]One of the weaknesses of the CES PF is with the estimation of parameters. Kmenta's approximation method of CES PF yields the TRANSLOG PF with constrained coefficients; $a_3 = a_4 = -a_5/2$.

inconsistencies between the sectoral data and the regional data, i.e., between the time series and the cross-sectional data.

To overcome these difficulties, further assumptions are needed. First, incomes generated from each sector--agriculture and non-agriculture--are the net of fixed rates which are the sums of the rates of indirect and direct taxes, capital consumption allowances, transfer payments, etc. Secondly, it will be assumed that the income generated from the agricultural sector goes entirely to rural income, and it also will be assumed that there is a channel for the income transfer through non-agricultural production of which a certain portion is shared by the rural people as a result of rural industrialization.[1]

Thus the rural and urban incomes and the per capita incomes will be given by

$$FINCM(t) = PROD_1(t)[1 - TX_1]PAR(t) + PROD_2(t)[1 - TX_2]$$

$$UINCM(t) = PROD_1(t)[1 - TX_1][1 - PAR(t)]$$

$$FINCMP(t) - FINCM(t)/POPL_2(t)$$

$$UINCMP(t) = UINCM(t)/POPL_1(t)$$

$$DIM(t) = FINCMP(t)/UINCMP(t)$$

where

FINCM : rural (or farm) income

UINCM : urban income

FINCMP : rural income per capita

UINCMP : urban income per capita

DIM : income distribution ratio

TX_i : tax rate for the i-th sector

---

[1]Strictly speaking, the rural labor newly employed to industry located in rural areas should be interpreted as the migration of labor from agriculture to non-agriculture. Yet, there exists the increasing chances of income earnings from side jobs.

$POPL_i$: population in the i-th region (urban or rural)

PAR : proportion of non-agricultural production transferred to the rural income

PAR is a behavioral parameter reflecting the effects of migration and income distribution. If the gap between desired income distribution and actual widens, PAR will be increased as a result of migration, and decreased otherwise. People migrate not only because of the discrepancy in income level but also for living atmosphere, opportunities for better education, easy access to transportation and communication facilities, and so forth. Moreover, PAR will also be determined by government policy or regulation on zoning, pollution, etc., which will force firms to relocate (or locate initially) their plants in rural areas. Retaining the effects of the income gap as the most important factor among others, a simple feedback loop shown in Figure 5.2 will be given to approximate the changing mechanisms of PAR. Thus, using Euler's formula, it can be computed by

$$PAR(t+DT) = PAR(t) + DT*PCF*[DDIM(t) - DIM(t)] \quad (5.4)$$

where

DDIM : desired income distribution ratio which is normally 1.0 for balanced income distribution

PCF : coefficient of the gain in feedback

Saving and Consumption

GNP at time t can be written as the sum of three components: (1) consumption expenditure, (2) saving or investment, and (3) net trade.

$$GNP(t) = CONSMT(t) + SAVE(t) + NTRD(t) \quad (5.5)$$

where

CONSMT : aggregate total consumption

SAVE : saving

NTRD : net trade, export minus import.

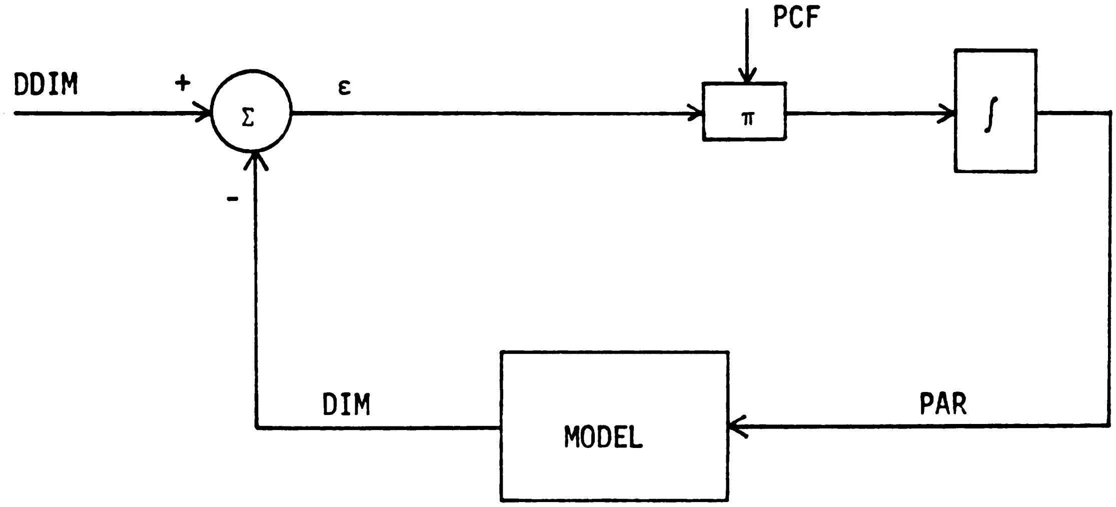

Figure 5.2 Simple Feedback Loop for the Changing Proportion of Income Transfer

If GNP and NTRD are given--GNP has been determined through production link--determining consumption or saving automatically determines the other at time t. A normal behavioral assumption may be that people consume first and then save the rest given the amount of income during a certain period of time. Nevertheless, in the theory of economic growth, the assumption has been reversed; determine the saving first and then the rest goes for the consumption. This is because the main interest in the theory is on the production side of the economy--given saving rate, labor, and other factors of production, how much will the total output be--and much of the effort is given to design policies which will lead to saving schedules for the desirable output (growth) paths.[1]

---

[1]The concern for consumption has been added later in the optimal growth model, where consumption is included in the objective function which is to be maximized with respect to the saving schedules.

Savings behavior, like other economic behaviors, has been explained in diverse ways with different assumptions. The most important factors determining savings are, in general, the level of income, profit rate, and income distribution among others. The classical saving function assumes the profit rate as the only factor of saving. Harrod-Domar used a fixed rate of income, and Kaldor used the different rates of savings for the profit and wage shares. Generally, it can be differentiated between the saving functions of the Cambridge economists and those of the neoclassical economists; the difference is that the Cambridge economists believe the aggregate savings rate depends on the distribution of income while the neoclassicists maintain that the aggregate savings rate is independent of the distribution of income but would argue that individuals save to maximize their intertemporal utility. It comes from the fundamental differences in philosophy between the two schools, however, the neoclassicists are committed to an economic theory derived from some kind of rational behavior, while the Cambridge economists believe that individuals are not so calculating and that rules of thumb are often used in a world of imperfect competition and uncertainty.

Since the thoughts of Cambridge school have some relevance in the LDC's, income distribution effects--functional and regional--will be included in the saving function.

$$SAVE(t) = \sum_{i=1}^{2} SAVE_i(t) \tag{5.6}$$

$$SAVE_1(t) = RSAVE_1(t)*WAGE(t) + RSAVE_2(t)*PROFT(t) \tag{5.7}$$

$$SAVE_2(t) = RSAVE_3(t)*PROD_2(t) \tag{5.8}$$

where

$SAVE_i$: saving from non-agricultural production and agricultural production for i=1,2, respectively

WAGE : wage of non-agricultural production

PROFT : profit of non-agricultural production

$RSAVE_1$: saving rate (or marginal propensity to save) for the aggregate saving out of wage

$RSAVE_2$: saving rate for the aggregate saving out of profit

$RSAVE_3$: saving rate for the aggregate saving out of agricultural production

The saving rates, $RSAVE_i$, and wage and profit will be determined by

$$WAGE(t) = WSHR*PROD_1(t) \tag{5.9}$$

$$PROFT(t) = PSHR*PROD_2(t) \tag{5.10}$$

$$RSAVE_i(t) = RSAVEX_i(t)*[DMN(t)/DIM(t)]^{ESI_i} \tag{5.11}$$

where

WSHR : wage share of non-agricultural production

PSHR : profit share of non-agricultural production

$RSAVEX_i$: expected saving rate

DMN : income distribution ratio on which the expected saving rates are based

DIMN : income distribution ratio

ESI_i : constant for the effect of regional income distribution

The last term of equation (5.11) represents the effects of regional income distribution on total saving from the belief that the saving rate in rural area is low, thus the more income transfer from urban to rural, the less total aggregate saving. The agricultural production was not disaggregated in terms of wage and profit shares because the farmer, a laborer, is also the owner of the capital, and it's hard to expect to have different behaviors (of saving) for one person with the income from the different sources. (Moreover, there's no distinction for the sources--wage and profit--other than the functional conception.)

Foreign saving or investment of foreign capital is another source of capital formation. However, not much is known of the forces that motivate international movement of investment capital except that it certainly responds to relative profit opportunities at home and abroad, being attracted to high-profit locations. In the real world, especially in LDC's, many of those opportunities are subject to governmental policies. Thus, if the government can be assumed to have control (or plan) the desired level of foreign savings, it can be given by

$$DRFSV(t) = TDSR(t) - DSAVR(t) \quad (5.12)$$

where

DRFSV : desired foreign saving rate

TDSR : total desired saving rate

DSAVR : domestic saving rate.

Total desired saving rate, TDSR, can be determined as an instrumental variable to achieve certain objectives which will be discussed later. Domestic saving rate, DSAVR, can be determined by

$$DSAVR(t) = SAVE(t)/GNP(t)$$

The actual foreign saving may be influenced by other factors, like the state of the economy and ability of repayment, thus, it will be calculated by

$$RFSV(t) = DRFSV(t)*SCURVE(5.0,0.5,0.5,DSR,2) \quad (5.13)$$

where

RFSV : foreign saving rate

DSR : debt service ratio

SCURVE : subroutine for a S-shaped curve

Then, the actual total saving, TSAVE, can be given by

$$TSAVE(t) = SAVE(t) + FSAVE(t) \quad (5.14)$$

$$FSAVE(t) = RFSV(t)*GNP(t) \quad (5.15)$$

Investment for each sector is determined by a parameter which will allocate the total investment (or total saving) into each sector.

$$INVT_1(t) = CI(t)*TSAVE(t) \quad (5.16)$$

$$INVT_2(t) = [1 - CI(t)]*TSAVE(t) \quad (5.17)$$

where

$INVT_i$: investment for non-agriculture and agriculture for i=1,2 respectively

CI(t) : proportion of total investment to non-agriculture, i.e., investment-allocation parameter

CI, the investment-allocation parameter, is dependent on the induced private investment and government spendings on investment, which will be determined by the investment program.[1]

The capital formation process, which forms the central dynamic system equation, is identical with those in Chapter IV.

$$K_i(t+DT) = K_i(t) + DT*[INVT_i(t) - DEPR_i(t)*K_i(t)] \quad (5.18)$$

where

K_i : capital stock of the i-th sector

$DEPR_i$: depreciation rate for the i-th sector

Using the linear depreciation rule, the depreciation rate, DEPR, can be obtained by

$$DEPR_i(t) = 1/ALIFE_i(t) \quad (5.19)$$

where ALIFE is the average life span for the capital of the i-th sector.

---

[1]The investment-allocation parameter is the key variable to be determined in the classical economic optimization for the allocation of resources.

Other variables, capital-labor ratio, CLR, and capital-output ratio, COR, can be calculated

$$CLR_i(t) = K_i(t)/L_i(t) \tag{5.20}$$

$$COR_i(t) = K_i(t)/GNP_i(t).$$

Aggregate consumption, which is net of saving and trade from the total output, can also be calculated by

$$CONSMT(t) = GNP(t) - SAVE(t) - NVT(t) \tag{5.21}$$

$$CONPL(t) = CONSMT(t)/LABR(t)$$

$$CONR(t) = [CONPL(t) - CONPL(t-DT)]/[CONPL(t-DT)*DT]$$

where

CONPL : aggregate consumption per labor

CONR : rate of change of the consumption per labor

NVT : net visible trade, i.e., export minus import

LABR : total labor employed

Trade

A comprehensive analysis of export and import growth could be quite complex--examining factor availability, technology, market structure, demand patterns, and government policies in the focus country, its customers, and its competitors.

Constant-Market-Shares (CMS) analysis is a simplified method for examining a country's export growth. It basically ascribes favorable or unfavorable export growth either to a country's export structure or to its "competitiveness." Thus, if the export shares of a given country are a function of that country's relative "competitiveness,"

$$s = \frac{q}{Q} = f\left(\frac{c}{C}\right), \qquad f'(.) > 0 \tag{5.22}$$

where

s : the export share of the focus country

q, Q : total exports of the focus country and the world, respectively

c, C : "competitiveness" of the focus country and the world, respectively.

The equation (5.22) can be rewritten as

$$q = s\,Q$$

then,

$$\dot{q} = s\,\dot{Q} + Q\,\dot{s} = s\,\dot{Q} + Q\,f'(.) \tag{5.23}$$

where

$\dot{q}$: total export growth of the focus country

$s\,\dot{Q}$: world export growth effect

$Q\,\dot{s}$: competitiveness effect

The appropriate measures of relative competitiveness may be relative prices, quantity improvements, improvements in servicing, changes in trade policy, etc. Export shares have to be expressed in terms of quantity in order to satisfy the condition that export shares vary directly with relative competitiveness, since an increase in relative competitiveness could lead to a decrease in export shares if export value shares are used when an elasticity of substitution is less than one in absolute value.[1]

In practice, however, export value shares have been used largely because of the absence of reliable quantity data and measure. The basic assumption of the CMS analysis--a country's export share in world markets

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[1]Elasticity of substitution is defined as the percentage change in relative quantities demanded divided by the percentage change in relative prices.

should remain unchanged over time--has been set for retrospective purposes; to explain the causes of the export changes in the past. It will be modified for the projective purpose that the export (value) share is assumed to follow the logistics curve with the belief that there may exist upper limit for the growth of export share (or competitiveness) of a country.[1]

The logistics curve, which has been used for the population growth, can be given by

$$Y = 1/[ab^x + g] \tag{5.24}$$

where a, b, g are constants and $|b| < 1$.

As the independent variable x goes to infinity, Y approaches to 1/g. Thus, 1/g is the upper limit of Y, Y_M. If g or Y_M is given, other constant parameters can be obtained by using OLSE (ordinary least square estimation) such as

$$\frac{1}{Y} - g = ab^x \tag{5.25}$$

$$\ln[1/Y - g] = \ln a + x \ln b$$

or

$$Y^* = a^* + b^*x.$$ [2]

Often g or Y_M is not given, thus, it can be varied to obtain one which yields the best fit--not only in terms of R^2 but also the significance of the t-test--since Y can be viewed as a function of Y_M.

---

[1]When all the countries reach to their (asymptotic) limits, or reach to the steady state in world trade, the assumption which the CMS analysis is based on holds true.

[2]The gompertz curve can be used for the same purpose:

$$y = ga^{b^x} \quad , \qquad |b| < 1$$

or $\ln y = \ln g + b^x \ln a = g^* + a^{*b^x}$.

The logistics curve can be generalized further as

$$Y = 1/[a_0 a_1^{x} a_2^{x^2} a_3^{x^3} \ldots a_n^{x^n} + g], \qquad |a_n| < 1 \qquad (5.26)$$

as $x \to \infty$, $Y \to 1/g$.

The equation also can be rewritten

$$\ln[1/Y - g] = \ln a_0 + x \ln a_1 + x^2 \ln a_2 + \ldots + x^n \ln a_n$$

or

$$Y^* = a_0^* + a_1^* x + a_2^* x^2 + \ldots + a_n^* x^n.$$

If the total world export grows exponentially, and the export share of the focus country grows along the (generalized) logistics curve, the export growth of the country can be given by

$$q = s\ Q = [b_0 e^{b_1 t}]/[a_0 a_1^{t} a_2^{t^2} \ldots a_n^{t^n} + g]$$

Retaining to the second order of the generalized logistics curve,

$$EXPO(t) = [B_0{*}EXP(B_1{*}T)]/(A_0{*}A_1^{T}{*}A_2^{T^2} + G] \qquad (5.27)$$

where

EXPO	: export of the focus country
B_0, B_1, A_0, A_1, A_2, G	: constant parameters to be determined
T	: time in year

Import will be divided into two parts, import content of export and the others. Import content of export represents the direct and indirect import needs of foreign natural resources, raw materials, or machineries to produce the goods which are to be exported. The other imports will generally compete with the domestic goods, and will be labeled as compressible import since these are subject to the government import control or policy. Thus, the total import can be

$$TIMPO(t) = EMCONT(t) + CMPO(t) \qquad (5.28)$$

where

TIMPO : total import of goods

EMCONT : import content of export

CMPO : compressible import.

The import content of export and the compressible import can be determined by

$$EMCONT(t) = CONMX(t)*EXPO(t) \tag{5.29}$$

$$CMPO(t) = TMPI(t)*GNP(t) + C \tag{5.30}$$

where

CONMX : import content of export ratio

TMPI : the marginal propensity to import (for compressible import)

C : a constant

There is invisible trade, other than the visible trades of goods, which includes foreign travel, transportation, insurance, investment income, government transactions, donations, and miscellaneous services. Considering the relative insignificance of the net gap, it will be assumed that the net gap is bounded by the lower and upper boundaries such as

$$NIVT(t) = \begin{cases} \text{Max } [NIT(t), LINVT], \text{ if } NIT(t) < LINVT \\ \text{Min } [NIT(t), UINVT], \text{ if } NIT(t) > LINVT \end{cases} \tag{5.31}$$

$$NIT(t) = RECIT(t) - PAY(t)$$

where

NIVT : net invisible trade

NIT : expected invisible trade gap

RECIT : receipts of invisible trade

PAY : payments of invisible trade

LINVT, UINVT : lower and upper limits of invisible trade gap.

The receipts and the payments of the invisible trade will be given using the trend lines.

Balance-of-Payments

The balance-of-payments component determines the foreign currency reserve, desired and actual foreign borrowings, total foreign debt, and debt service ratio.

From the basic foreign currency equation,

$$BOP(t) = \int_0^T [NTRD(t) + NCI(t)]\, dt \tag{5.32}$$

or,

$$BOP(t+DT) = BOP(t) + DT*[NTRD(t) + NCI(t)]$$

where

BOP : foreign currency (or its equivalences such as gold, SDR's, reserve position in the IMF, etc.) reserve

NCI : net capital inflow

and

$$NTRD(t) = NVT(t) + NIVT(t) \tag{5.33}$$

$$NCI(t) = FBR(t) - DBS(t) \tag{5.34}$$

where

FBR : foreign borrowing at time t

DBS : debt service at time t

The foreign borrowing is determined by the amount of the desired foreign borrowing and the world financial market situation.

$$FBR(t) = MFLA(t)*DBR(t) \tag{5.34}$$

where

DBR : desired foreign borrowing

MFLA : multiplier for the availability of foreign loans.

The multiplier for the availability of foreign loan, which reflects the world financial loaning situation, will be derived from the monotonic decreasing or downward sloping S-shaped curve believing that countries facing critical foreign indebtedness may not be able to obtain the sources of foreign loan as much as they want, i.e., they have to cut down the consumption level and economic growth to overcome the vicious circle of deepening indebtedness.

Frequently, S-shaped curves are given by the TABLE (table look-up) functions [F2], [M3]. Table function, which assigns the values to the points of interest by interpolating or extrapolating the observed data points, is not restricted to specific shape of the function. For the conceptual or hypothetical functions without observed data, the flexibility to test the sensitivity is essential. The use of table function is cumbersome, if not inflexible, in this respect. To remove this inconvenience, the combined negative exponential function will be used for the S-shaped curves in general.

For ID = 1,

$$\text{SCURVE(A,B,C,T,ID)} = \begin{cases} 1 - B*EXP[A*(T-C)], & \text{if } T < C \\ 1 - B*[2 - EXP[-A*(t-C)]], & \text{if } T > C \end{cases} \qquad (5.36)$$

where

A,B,C : parameters for S curve

T : independent variable

ID : index indicating

ID = 1 : monotonic increasing

2 : monotonic decreasing

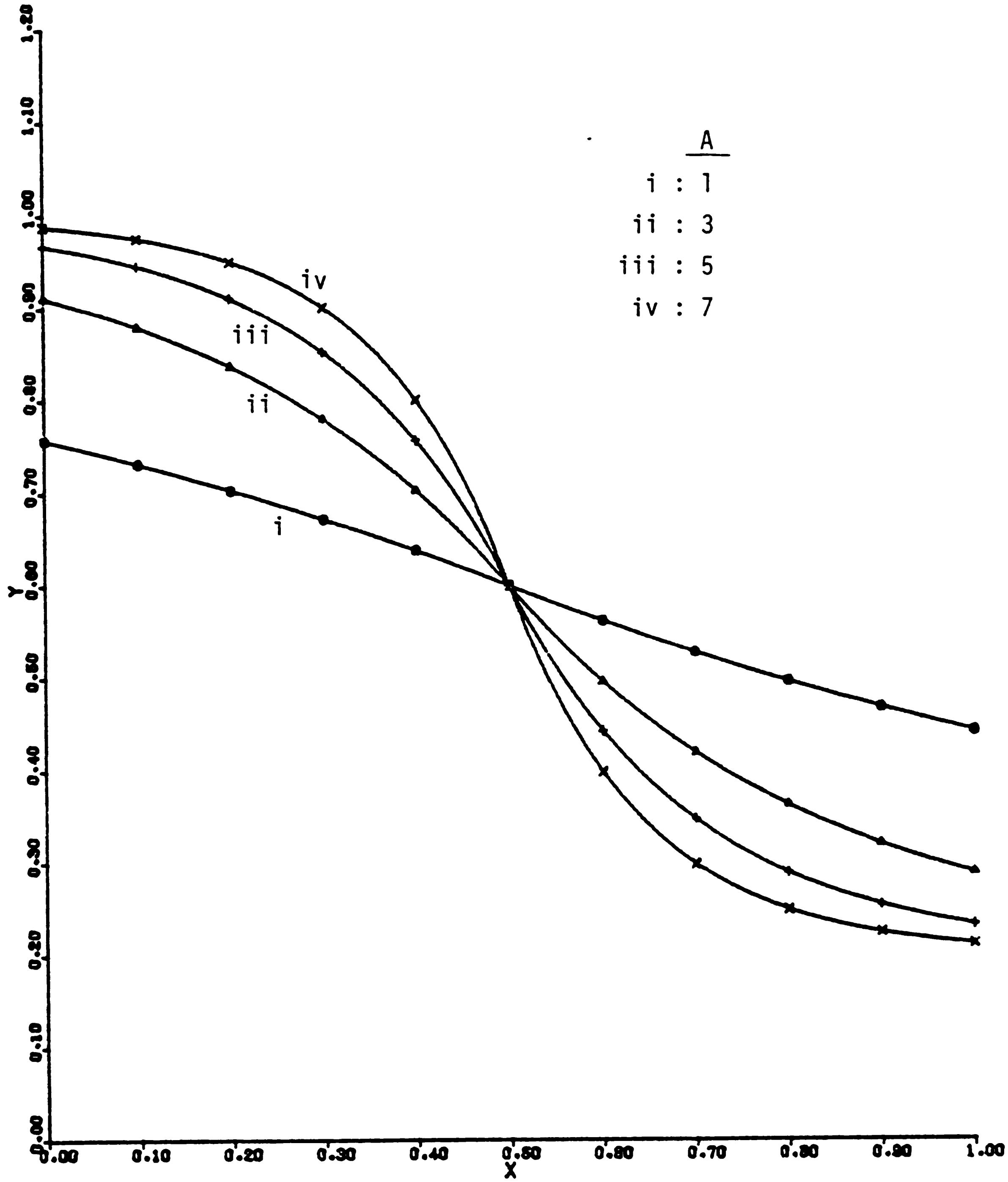

Figure 5.3 Monotonic Decreasing SCURVE for B=0.2, C=0.5, and ID=2

For ID = 2,

$$SCURVE(A,B,C,T,2) = 2*[1 - B] - SCURVE(A,B,C,T,1)$$

The SCURVE for B = 0.2, C = 0.5, ID = 1, and for different values of A are shown in Figure 5.3.

Thus, MFLA will be calculated by

$$MFLA(t) = SCURVE(AMF,BMF,CMF,DSR,2) \tag{5.37}$$

where AMF,BMF,CMF are the parameters reflecting the world financial situation.

The desired foreign borrowing is

$$DBR(t) = Max\ [0,\ SLK(t)] \tag{5.38}$$

$$SLK(t) = DBOP(t) - BOP(t)$$

where DBOP is the desired foreign currency reserve.

The desired foreign currency reserve increases as the national economy or the total amount of trade grows

$$DBOP(t) = K_1*[EXPO(t) + TIMPO(t)] \tag{5.39}$$

where

K_1 : constant parameter for the proportion of desired foreign currency reserve to the total trade

The process of the accumulation of debt is a delay process with borrowing as input, debt service as output, and the loaning period as average delay, which may be approximately described by the distributed delay [F2], [M3]. One of the basic properties of the distributed delay process is the conservation of flows--every single entity which enters into the delay process should come out eventually without loss or gain. In practical problems like the maturation or population growth models, the distributed delay process with loss (or attrition) has been used.

[V] [A] However, the basic feature of the debt accumulation process is the gain in the delay, the accrual from the interest on capital, which can be formulated by using accrual rate opposite of the attrition rate in sign in the model of the distributed delay with attrition as shown in Figure 5.4.

Thus, it can be formulated as

$$\frac{dQ_i(t)}{dt} = r_{i-1}(t) - r_i(t) + G_i(t), \qquad i = 1,\dots,k \tag{5.40}$$

and

$$Q(t) = \sum_{i=1}^{k} Q_i(t). \tag{5.41}$$

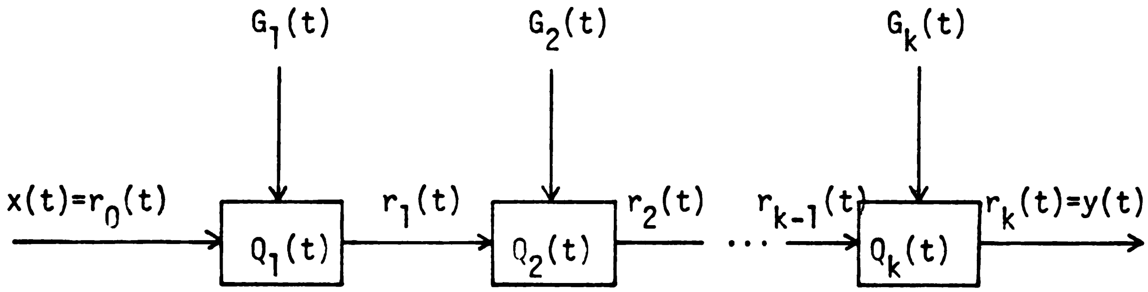

Figure 5.4 The k-th order Distributed Delay with Gain

where

Q_i : the i-th storage in the cascaded stages

r_i : the flow rate out of the i-th storage

G_i : gain at the i-th stage

Q : storage in the k-th order delay process.

The basic assumptions relating the storage, flow rate, and gain of the delay process are

$$r_i(t) = \frac{k}{DEL} Q_i(t), \qquad i = 1,2,\ldots,k \tag{5.42}$$

$$G_i(t) = AC(t)\ Q(t) = AC\ Q(t) \tag{5.43}$$

where

DEL : average delay

AC : accrual rate.

The equations can be rewritten in terms of the flow rates which is more convenient in simulation,

$$\frac{dr_i(t)}{dt} = \frac{k}{DEL} [r_{i-1}(t) - [1 - AC(t) \frac{DEL}{k}]\ r_i(t)], \qquad i = 1, \ldots, k \tag{5.44}$$

The number of stages, k, and the average delay can be determined by the grace period and maturity of a loan. The accrual rate equals the interest rate, the output of the delay process is the debt service, and the sum of the storages is the total debt.

$$TDBT(t) = \sum_{i=1}^{k} Q_i(t) \tag{5.45}$$

$$DBS(t) = R_k(t) = Y(t). \tag{5.46}$$

The overall block diagram of the balance-of-payments component is shown in Figure 5.5.

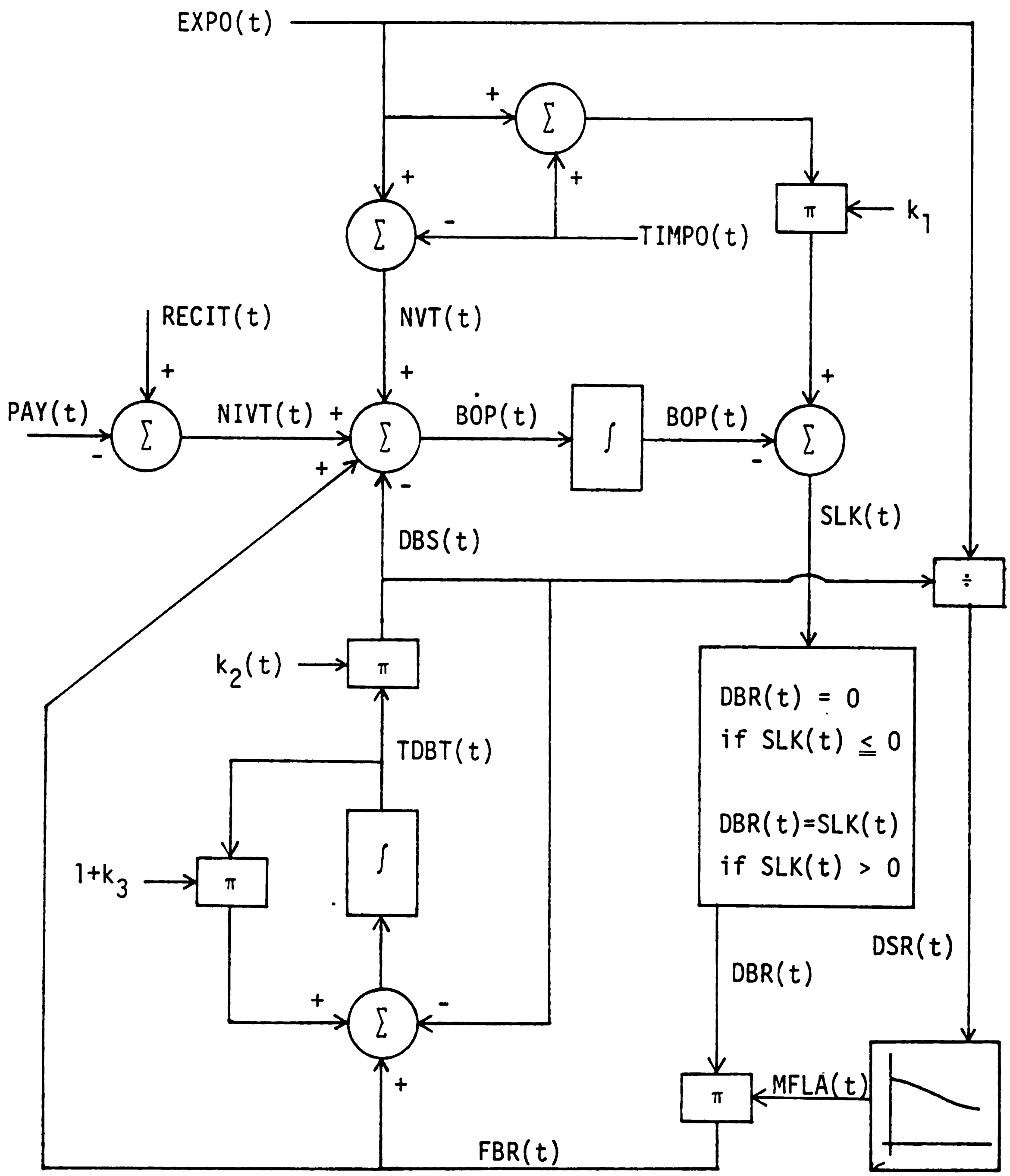

Figure 5.5   Balance-of-Payments Component

<u>Grain Component</u>

Grain component has been added to examine the external and internal effects of the grain policies to the total economy. The agricultural production has been disaggregated into non-grain and grain agricultural production, grain consumption demand and price mechanisms, and grain storage subcomponents are added.

The grain production in physical quantity can be given by

$$QG_i(t) = AREA_i(t)*YLD_i(t) \tag{5.47}$$

where

OG_i : grain production in MT

$AREA_i$: area planted for the i-th grain

YLD_i : yield of the i-th grain

i=1 : rice

2 : barley

3 : wheat.

Generally, the planted area is a function of the expected price (and capital investment for the case of the land reclamation, land development), and the yield is a function of capital investment, labor, and technological progress such as new variety, fertilizers, herbicides, etc. Retaining the Cobb-Douglas form as in the equation (5.2),

$$QG_i(t) = A_i*K_i(t)^{a_i}*L_i(t)^{b_i}*TT_i(t-1)^{c_i}, \quad i = 1,2,3 \tag{5.48}$$

where

TT_i : one year lagged terms of trade which is the one year lagged i-th grain price divided by the overall price index.

In the grain component, it is assumed that the government has complete control over the supply or the prices of grains. It should be noticed that the demand of grains cannot be directly controlled, i.e., the government cannot set the supply and prices of the grains at the same time, and thus the individual preferences not directly controlled by government policy. This is the case with the perfectly inelastic supply which is shown in Figure 5.6.

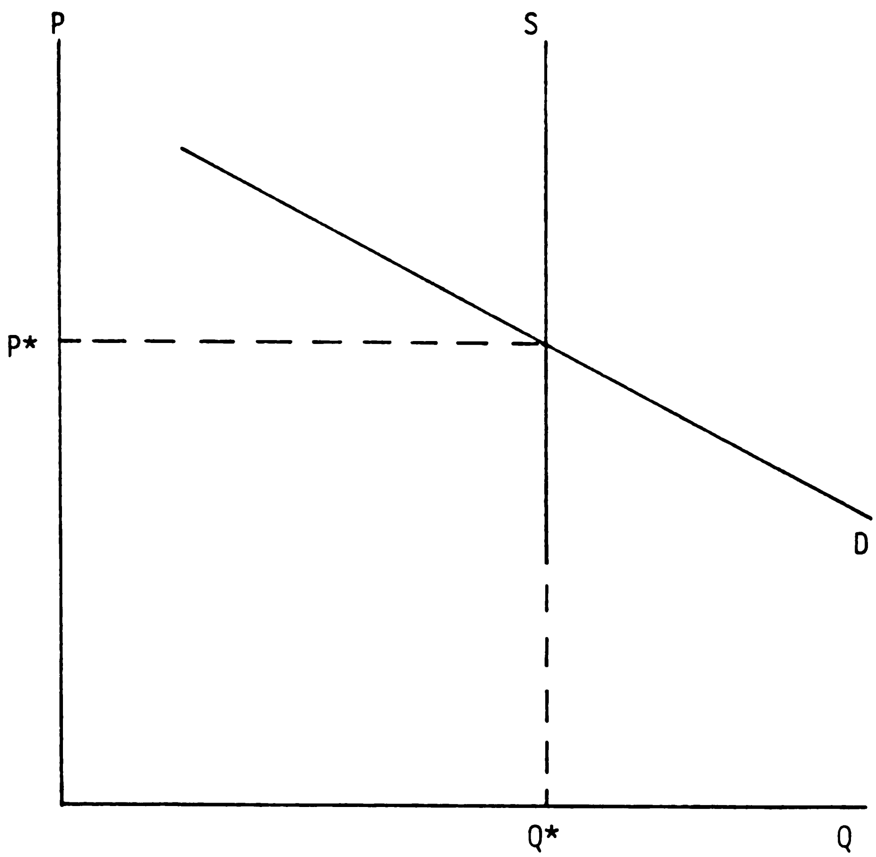

Figure 5.6 Price-Quantity Determination with Perfectly Inelastic Supply

The demand functions for grain can be given in different forms; the linear-expenditure system, the system of double logarithmic functions, the Rotterdam differential model, and the indirect addilog system [Y1], [P1][1]. Though there are several shortcomings,[2] the system of double logarithmic function will be used because of the relevance in the case of market-clearing situation.

$$\ln[\overline{D}(t)/POPL(t)] = \underline{A} + \underline{B} \ln \overline{P}(t) + \overline{C} \ln I(t) \qquad (5.49)$$

where

$\overline{D}$: demand per capita

$\overline{P}$: price of grains

I : income or GNP per capita

$\underline{A}$: constant vector

$\underline{B}$: matrix for the price elasticity of demand

$\overline{C}$: vector for the income elasticity of demand.

Equation (5.49) can be used in two ways for the market-clearing situation: first to determine the amount of demand given prices, secondly to determine the (market clearing) prices of grains if the supply fails to meet the demand. For the case of determining the prices of grains given supply, SUP (which is not equal to the demand), equations can be inverted such that

---

[1]The linear-expenditure system (or Stone-Geary expenditure system) has been pointed out to be superior to others in terms of the desired properties of the demand system such as income constraint, invariance with respect to increasing transformation, Slutsky condition, and Ville's condition [Y1]. However, this system has a serious weakness in the estimation procedure since the additive disturbance terms are implicitly assumed to be mutually dependent [P1].

[2]Problems relating to the use of this form has been discussed in pp. 83-84 of [A2].

$$\ln \overline{P}(t) = \underline{B}^{-1} [\ln SUP(t)/POPL(t) - \underline{A} - \overline{C} \ln I(t)] \quad (5.50)$$

given the elasticity matrix $\underline{B}$ is non-singular.

Grain storage in terms of annual carry-over stock is determined by the grain production, demand, storage capacity, and desired stock levels. The desired grain stock will be given by the proportion of total demand

$$DGSTK_i(t) = CSTK_i(t)*DMGRN_i(t) \quad (5.51)$$

where

DGSTK : desired grain stock level

CSTK : desired proportion of grain stock to the total demand

DMGRN : demand of grains

The change in grain stock and the demand-supply gap can be

$$DSTK_i(t) = DGSTK_i(t) - GSTK_i(t) \quad (5.52)$$

$$DSGAP_i(t) = DMGRN_i(t) - QG_i(t)$$

where

DSTK : changes in the grain stock

GSTK : existing grain stock

DSGAP : demand-supply gap of grains.

Grain import requirements (or increases in the stock level) are determined by the net shortages (or surpluses)

$$GQIMPT_i(t) = DSGAP_i(t) + DSTK_i(t) \quad (5.53)$$

$$GSTK_i(t) = GSTK_i(t) + DT*DSTK_i(t)$$

where

GQIMPT : grain import requirement to meet the demand (consumption demand plus storage demand)

If net surplus occurs, i.e., DSGAP plus DSTK is negative, the stock will be piled up.

$$GSTK_i(t) = GSTK_i(t) - DT*DSGAP_i(t) \tag{5.54}$$

$$GQIMPT_i(t) = 0.$$

The actual import is determined by the import requirements and grain import policy parameter.

$$GQIMP_i(t) = CGM_i(t)*GQIMPT_i(t) \tag{5.55}$$

where

GQIMP : actual grain import

CGM : grain import policy parameter which may be a function of domestic and/or world grain prices.

The total supply of grain is given by the production, import (or export), and changes in stock.

$$SUPGRN_i(t) = QG_i(t) + GQIMP_i(t) - DSTK_i(t) \tag{5.56}$$

where SUPGRN is the total annual supply of grain in year t.

The dependency on the grain import can be calculated by

$$DPGIMP(t) = \frac{\sum_{i=1}^{3} GQIMP_i(t)}{\sum_{i=1}^{3} SUPGRN_i(t)} \tag{5.57}$$

where DPGIMP is the dependency on the grain import.

Other Components

Population and labor, price indices, and the world market conditions are given exogenously by fitted trend line in the model. Total labor force is given by the trend with the constant growth rate, and the proportions of the agricultural and non-agricultural labor are given by a linear function of time. Urban and rural populations are calculated in the similar way except the proportions are given by the monotonic increasing negative exponential function.

Price indices are given by the trend projections except the grain prices for the case of supply shortage or surplus.

The overall price indices are

$$PIG(t) = \frac{\sum_{i=1}^{3} PG_i(t)*QG_i(t)}{\sum_{i=1}^{3} QG_i(t)} \tag{5.58}$$

$$PIA(t) = w_{a1}*PIG(t) + w_{a2}*PING(t)$$

$$PI(t) = w_1*PIA(t) + w_2*PINA(t)$$

where

PIG : price index of grains

PIA : price index of agricultural product

PING : price index of non-grain agricultural product

PINA : price index of non-agricultural product

PI : overall price index

w_i^1 : the weight of price index for each product.

Two kinds of world market conditions are considered. The first is the financial situations which are taken into account in the multiplier for the availability of foreign loans, parameters in the debt accumulation process--grace period, maturity,and interest rate. The second is the world grain prices, which are not known generally, but will be assumed to follow a set of scenario.

CHAPTER VI

SIMULATION OF THE OPEN MODEL

VI.1. Design of Economic Policies

The need for macroeconomic policies arises because of the inadequate self-adjustments of the economic system to the shocks it is constantly subjected. Thus, economic policy consists of the deliberate manipulation of a number of means external to it in order to attain certain aims. Taxes may be lowered to stimulate employment; social security may be introduced to further an equitable distribution of the national product, and so forth. In a strict sense, the policies can be discriminated into three different kinds according to the nature of the means used; reforms, qualitative policy, and quantitative policy. Reforms, the most far-reaching types of policy, refers to changes in foundation or introduction of new systems of policies, qualitative policy means changes in structure such as a change in the number of taxes,and quantitative policy is the changes that can be brought about in the values of the instruments of economic policy.[1] Most of the policies experimented with in the models are of the quantitative type because they are particularly used to quickly adapt the position of the economy to variations in the frequently changing environment.

In general, designing policies can be hindered by the complexities and the high degree of uncertainties in societal systems not only with regard to the structures but also to the aims or objectives of the society.

[1] [T3], p. 7.

Because of these obstacles and the interdependence between most economic phenomena, the policy and the desired states of the economy at a certain time can't be solely determined; it is necessary to consider them as a (coherent) whole. Examples of the goals of an economy might be: (1) high, stable, and growing level of real income, (2) stable prices or low inflation rate, (3) a balance-of-payment surplus, (5) equitable or balanced distribution of income over various groups of the population--social, industrial, regional--etc. The instruments or instrument variables may be tax rates, public expenditures, the rate of discount, reserve ratios, foreign exchange rate, wage rates, etc. In fact, all these goals and instruments are interrelated in such a way to produce both "goods" and "bads."

Among other possible instruments, the desired total saving rate and the import of compressible goods, i.e., controllable import, are chosen to investigate the behavior of the model in a search for optimal policies. Higher total saving rates will increase the total production and the growth rate, while it may raise the foreign indebtedness if the country fails to provide with domestic savings, or, at the same time, higher domestic saving can translate into lower consumption levels. Import of goods has a similar effect on the economy; higher imports mean higher consumption but an unsound balance-of-payments and probably increase in the foreign debt. What a country wants is conflicting; high production and economic growth, low foreign indebtedness, and high consumption level. This is only possible for a country with abundant sources of production. The matter of deciding the social goals is not simple; it involves both material and spiritual well-being and is complicated by political factors.

However, interactions between decision makers and models may help a great deal in determining goals and once the (conflicting) goals are given, a feasible policy can be defined as one which can trace a path along the time scale that satisfies the "desirable" or reasonable boundaries (constraints) of the state variables and achieves the goals.

Normally, this has been done by the analysis of a system which starts from a set of policies and examine their consequences--paths of variables and certain performance indices which the policy makers may be interested in. One inherent weakness of this approach is that it may be too costly to find a satisfactory dynamic path of policy which satisfies all the desirable requirements, since one doesn't know in advance how the changes in input will be translated into the changes in the paths of other economic variables.

Conceptually, this problem can be handled inversely, i.e., by the control of a system which starts from the desired performance indices and constraints to determine the inputs. However, technical difficulties in solving the optimization for control in a complex system, and the conceptual difficulties in constructing the objective function, which usually is not known initially, have been the barrier for the control concept to be applied in complex systems.

Since the conceptual difficulties can by no means be completely removed so as those in constructing the policies for the analysis, the concept of "alternative objective functions" can be used to lessen the difficulty where the policies can be determined using a set of possible objectives.

The remaining portion of this section will be devoted to the discussion of technical difficulties in practice, and the ways of removing these difficulties which can lead to the design of dynamic policies in an open economy.

Optimization of a Complex System

Finding the best economic policy of a complex system, i.e., determining the values of instrument variables that render a certain welfare function a maximum, is quite complicated in mathematical nature and impossible in general if not only the instrument (input) variables but also the state variables are subject to certain boundary conditions.

The usual constrained optimization problem can be stated as follows:

$$\text{Optimize} \qquad f(\overline{X}) \tag{6.1}$$

$$\text{subject to} \qquad l_i \leq g_i(\overline{X}) \leq u_i, \qquad i=1, \ldots, n$$

where $f(\overline{X})$ represents a certain objective (or welfare) function, $g_i(\overline{X})$'s are the functions of control (instrument) and state variables that have to be constrained, and $\overline{X}$ is a vector of instruments such as saving rates, tax rates, public expenditures, etc.,for the case of an economic system. For special cases of $f(\overline{X})$ and $g_i(\overline{X})$, linear and nonlinear programming techniques can be used to solve the problem. If the constraints are on the independent variables which can be transformed in such a way not to alter the objective function, the unconstrained optimization routines such as search techniques and gradient methods can be applied depending on the nature of the model. When constraints exist on the state variables which cannot be controlled directly but derived by the control variables, solving the optimization is impossible in a mathematical sense where only feasible solutions are qualified to optimize the objective function. But in practice, the constraints may not be strict as in a mathematical sense, i.e., some of the constraints may be relaxed to get the optimum solutions. Certain feasible boundaries for the income distribution ratio, the debt service ratio, and the level of consumption may be desired to ensure the total economy working without drastic chaos, but one may not be sure of the exact boundaries.

Due to these complex implicit constraints, search or gradient methods cannot be directly used to solve the problem; however, one possible way may be the method of peanlty function which includes the constraints as a penalty function in the objective criterion.

The objective function can be:

$$\text{Optimize } Z = f(\overline{X}) + P[g(\overline{X})] \qquad (6.2)$$

where $f(\overline{X})$ is the constrained objective function, $P[g(\overline{X})]$ is the penalty function which is the function of the constraints, and Z is the objective function of unconstrained optimization, so that the general unconstrained optimization routines can be applied.

The penalty is given in such a way as to make the value of Z higher (or lower) for violating the constraints for the case of minimization (or maximization). One of the commonly used penalty functions is in the form of the Heaviside unit step function which gives penalty or no-penalty according to whether the constraints are violated or satisfied. One weakness of using the Heaviside unit step penalty function is its rigidness, i.e., the values of the objective function may not be sensitive to the changes in the constrained state variables which are improved from the previous states, especially when the boundaries are not strict in a mathematical sense but rather can be considered as "desired boundaries" which can be violated by a certain amount.

This is shown in Figure 6.1 for illustration, with the case of the error-minimization and one-sided constraints given as

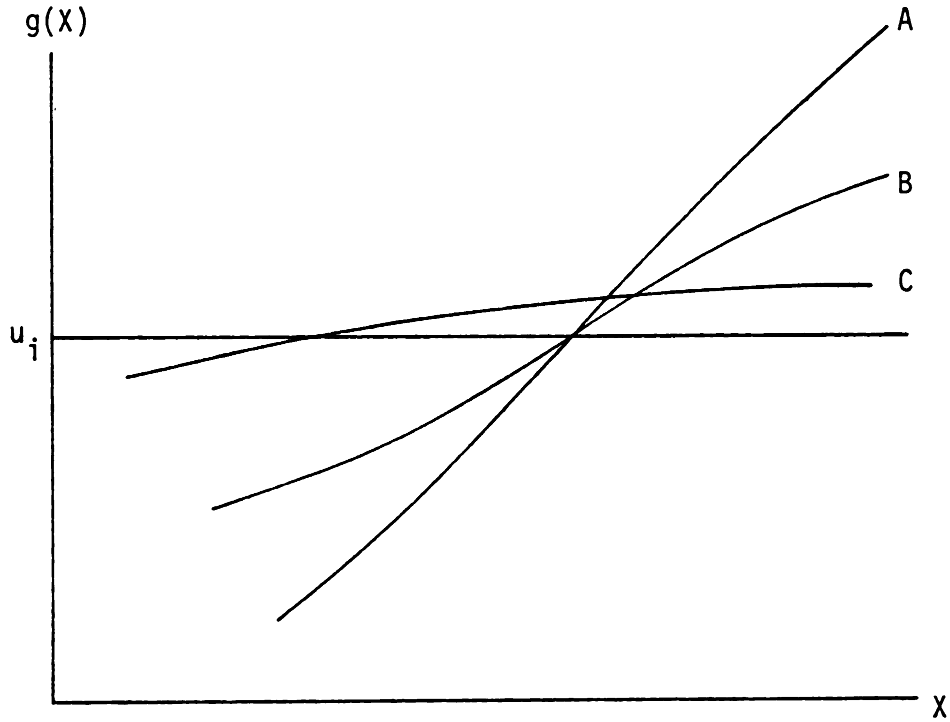

Figure 6.1 Possible Paths of Constraints for the Case of Heaviside Unit Step Penalty Function

$$g_i(\overline{X}) \leqq u_i, \quad i = 1, \ldots, n. \tag{6.3}$$

The cases of A and B yield the same value of the penalty function regardless of the closeness of B to the boundary. Moreover, the state C may be the best among the three while it yields the highest value of the objective function if the Heaviside unit step penalty function is used.

The preceding illustration leads to the formulation of penalty functions in such a way as to reflect the sensitivity of closeness to the desired boundaries. This can be achieved by using a penalty function as follows instead of the Heaviside unit step function.

Thus, the penalty function can be

$$P[g(X)] = \begin{cases} m_i \cdot G[g_i(\overline{X}) - l_i], & \text{if } l_i \geq g_i(\overline{X}) \\ 0 \quad , & \text{if } l_i \leq g_i(\overline{X}) \leq u_i \\ M_i \cdot G[g_i(\overline{X}) - u_i], & \text{if } \quad g_i(\overline{X}) \geq u_i \end{cases} \tag{6.4}$$

where G(.) is a function of the deviations from the desired boundaries, m_i and M_i are the penalty weights for the violation of the lower and upper boundaries, l_i and u_i, respectively. The distinction from the former, in essence, lies in the choice of the functional form of G; linear, quadratic, etc.

Another distinction that can be made between the normal optimization problem and that of a complex system is the formulation of the objective function. The most commonly used form is the quadratic objective function with the error terms to be minimized which implies underachieving a desired level or path, and overachieving are penalized equally. This has to be modified in many real economic problems, since overachieving and underachieving may have different values. For example, the potential undesirable implications of any given surplus of balance of payments may differ substantially from those of a deficit of equal magnitude. The undesirability of underachieving and overachieving the target growth rates will be significantly different. It is in this sense that the piecewise quadratic function offers a more general framework for policy optimization.[1]

$$\begin{aligned} \text{Optimize } Z &= f(\overline{X}) + P[\overline{g}(\overline{X})] \\ &= a'\overline{X} + b'[\overline{g}(\overline{X})] + 1/2[\overline{X}'\underline{A}\overline{X} + \overline{g}(\overline{X})'\ \underline{B}\overline{g}(\overline{X})] \\ &= c'\overline{Y} + 1/2\ \overline{Y}'\underline{C}\overline{Y} \end{aligned} \tag{6.5}$$

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[1][F], pp. 183-196.

where

$$\overline{Y} = [\overline{X}, \overline{g}(\overline{X})]', \qquad c = [a, b], \qquad \underline{c} = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$$

and the elements of the matrix C, c_{ii}, can be given by

$$c_{ii} = \begin{cases} c_{ii}^{u} & \text{if } y_i \in U(y_i) = \{y_i | y_i > y_i^u\} \\ 0 & \text{if } y_i \in M(y_i) = \{y_i | y_i^l \leq y_i \leq y_i^u\} \\ c_{ii}^{l} & \text{if } y_i \in L(y_i) = \{y_i | y_i < y_i^l\} \end{cases}$$

where

$U(y_i)$: the upper extreme set for y_i

$L(y_i)$: the lower extreme set for y_i

$M(y_i)$: the set of y_i which satisfies the lower and the upper boundaries

c_{ii}^u, y_i^u : the upper boundary value and the point

c_{ii}^l, y_i^l : the lower boundary value and the point

The preceeding considerations on the optimization of complex systems--redefining the strictness of the constraints and the use of a piece wise objective function--allows one to apply control concepts to the complex system, and general optimization routines such as search techniques can be used for this purpose.

Orthogonal Representation of Policy Variables

Policy making is dynamic in nature, i.e., one has to decide policies at every specific point in time to produce the overall desired goals during a certain time period.

In the closed model, a policy could be expressed as a function of time which was obtained by solving the optimal control problem. With the complexities in the open model, optimal control cannot be directly applied.[1] Even in the complicated model, however, there are two ways to fulfill this purpose: first is the determination of the discretized points of the paths of policy variables as a whole, and the second is to find or fit a specific function which will trace the path satisfactorily with the unknown coefficients to be determined.

Within reasonable numbers of the discretized points to be determined, the general optimization routines can be used for the first,however,as the number of variables (points) increases, it is impossible to get solutions in general at reasonable cost. The second way can be applied more generally in this sense. It can be observed that the basic idea of this is the same with the function representation of signals in engineering. The analogy is not only that the signal forces the system to take the action necessary for accomplishing the desired objective but also with the generally very complicated realized shapes.

Because of complexity which can be caused by enormous numbers of sources, the signal is represented as a function of time, and thus any quantitative representation of a real signal is necessarily an approximation.

The usual way of representing an arbitrary time function is to use a linear combination of a set of elementary time functions which form a basis in the functional space of a certain dimension. Among other properties, one important desirable property of the basis functions is

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[1]If the system state equations can be maintained as in the closed model, the approximation of function can also be used for optimal control where the problem is to find the polynomial of degree n (or others) which has the least mean square deviation from the given function on a certain interval, i.e., the classical problem of finding the Fourier coefficients in the expansion of the function in Legendre polynomials [P8], pp. 197-213.

orthogonality which will allow one to determine any given coefficient without the need for knowing any other coefficient--the finality of coefficients--which can be illustrated as follows:

Let f(t) be an arbitrary function of time to be represented by the linear combination of orthogonal (basis) functions

$$f(t) = \sum_{i=0}^{n} a_i \; x_i(t) \tag{6.6}$$

where x_i's are the orthogonal functions, i.e.,

$$\int_{t_1}^{t_2} x_i(t) \; x_j(t) \; dt = 0 \,, \quad \text{for } i \neq j \tag{6.7}$$

$$= \lambda_j, \quad \text{for } i = j$$

Multiply both sides of (6.6) by $x_j(t)$,

$$x_j(t) \; f(t) = x_j(t) \sum_{i=0}^{n} a_i \; x_i(t) \; dt \qquad (6.8$$

then,

$$\int_{t_1}^{t_2} x_j(t) \; f(t) \; dt = \sum_{i=0}^{n} a_i \int_{t_1}^{t_2} x_j(t) \; x_i(t) \; dt \tag{6.9}$$

$$= a_j \; \lambda_j$$

Thus,

$$a_j = 1/\lambda_j \int_{t_1}^{t_2} x_j(t) \; f(t) \; dt \tag{6.10}$$

This shows the finality of coefficients as described before.

The possible forms of arbitrary functions can vary. The Fourier series, which uses sinusoidal elementary functions as the basis, and the polynomial function with the Legendre polynomials as the basis are commonly used. Any one of the two functions can be more efficient in a specific case in the sense of better approximation to the real values and better convergence in carrying out optimization. Legendre polynomials which are a particular solution of the Legendre equation, will be used here.

If an arbitrary function is f(X), then

$$f(X) = \sum_{i=0}^{\infty} a_i \; P_i(X) \tag{6.11}$$

where P_i's are the Legendre polynomials such as

$$P_0(X) = 1 \tag{6.12}$$

$$P_1(X) = X$$

$$P_2(X) = 1/2\ [3X^2 - 1]$$

$$P_3(X) = 1/2\ [5X^3 - 3X]$$

$$\cdots\cdots$$

$$P_{k+1}(t) = 1/(k+1)\ [(2k+1)\ X\ P_k(X) - k\ P_{k-1}(X)], \quad k=2, 3, 4, \ldots$$

It can also be shown that the Legendre polynomials satisfy the orthogonality relation within $X \varepsilon [-1, 1]$ such that

$$\int_{-1}^{1} P_m(X)\ P_n(X)\ dX = \begin{cases} 0, & \text{for } m \neq n \\ 2/(2n+1), & \text{for } m = n \end{cases} \tag{6.13}$$

In the actual application of designing policies, the domain is the planning period of time, t_1 to t_2. Thus, by changing the variable as

$$X = [2(t - t_1)]/(t_2 - t_1) - 1 \tag{6.14}$$

or

$$t = [(t_2 - t_1)/2](X + 1) + t_1$$

then,

$$\int_{-1}^{1} P_m(X)\ P_n(X)\ dX \tag{6.15}$$

$$= 2/(t_2-t_1) \int_{t_1}^{t_2} P_m(t)\ P_n(t)\ dt = \begin{cases} 0, & \text{for } m \neq n \\ 2/(2n+1), & \text{for } m = n \end{cases}$$

hence,

$$\int_{t_1}^{t_2} P_m(t)\ P_n(t)\ dt = \begin{cases} 0, & \text{for } m \neq n \\ (t_2-t_1)/(2n+1), & \text{for } m = n. \end{cases} \tag{6.16}$$

The equation (6.16) ensures the orthogonality of the basis (Legendre polynomials) for $t\ \varepsilon[t_1,t_2]$, and thus the arbitrary function (6.11) can be used to approximate the policy paths during the planning period from t_1 to t_2.

VI.2 Empirical Application to the Korean Economy

The considerations of the preceding section, design of dynamic policy using optimization of a complex system, will be applied to the case of Korea with the open model. More specifically, the problem can be stated as following: find the coefficients of the arbitrary time functions of Legendre polynomials for the total desired saving rate (TDSR) and the marginal propensity to import of compressible goods (TMPI) in such a way to maximize the economic growth subject to the desired boundaries for foreign indebtedness and consumption level.

As already been observed, the use of the same functional form for the objective and the penalties for the constraints allow one to handle them in a unified fashion. Thus, using the piecewise quadratic form, the overall objective function (performance criterion) is given by

$$\text{Minimize } U = \int_{t_1}^{t_2} \{[CW_1(RGNY(t)-DRGNY(t))]^2 + [CW_2(DSR(t)-DDSR(t))]^2 + [CW_3(CONR(t)-DCONR(t))]^2\} \qquad (6.17)$$

where

RGNY : real GNP growth rate

DRGNY : desired real GNP growth rate

DSR : debt service ratio,[1] i.e., debt payment divided by export

DDSR : desired debt service ratio

CONR : rate of change of aggregate consumption

DCONR : desired rate of change of aggregate consumption

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[1]Although the debt service ratio has been commonly used to represent the foreign indebtedness as an indicator of debt servicing capacity of a country, other indicators such as the ratio of non-compressible import to total imports, export fluctuation index, growth rate of export, etc. can also be used. [F3]

CW_i's are the relative weights of the objective function and the penalty functions such that

$$CW_1 = W_1 \ \mu[DRGNY(t) - RGNY(t)] \tag{6.18}$$

$$CW_2 = W_2 \ \mu[DSR(t) - DDSR(t)] \tag{6.19}$$

$$CW_3 = W_3 \ \mu[DCONR(t) - CONR(t)] \tag{6.20}$$

where $\mu(.)$ is the Heaviside unit step function and W_i's are constants.

The policy variables, TDSR and TMPI, are expressed in the linear combination of the Legendre polynomials up to the second order, thus

$$TDSR(t) = SRA + SRB\ X(t) + [SRC/2][3X(t)^2 - 1] \tag{6.21}$$

$$TMPI(t) = TMPA + TMPB\ X(t) + [TMPC/2][3X(t)^2 - 1] \tag{6.22}$$

where

$$X(t) = [2(t - t_1)]/(t_2 - t_1) - 1 \tag{6.24}$$

and SRA, SRB, SRC, TMPA, TMPB, TMPC are the coefficients of the polynomial to be determined.

Optimization routine COMPLEX,[1] which searches the minimum (or maximum) of the multivariable, nonlinear objective function sequentially with nonlinear inequality constraints, has been used to find the optimum values of the coefficients, i.e., the values of the coefficients which minimizes the objective function (6.17) satisfactorily with a certain convergence criteria.

Different optimization results have been obtained using the alternative objective functions with different sets of relative weights

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[1] [K9], pp. 368-385.

TABLE 6.1

RESULTS OF THE OPTIMIZATIONS

	I	II	III	IV	V	VI
Parameters†						
CW2	1	10	10	1	1	1
DRGNY	0.1	0.1	0.05	0.1	0.1	0.1
DDSR	0.2	0.1	0.1	0.2	0.1	0.1
B. Condns‡						
TMPA	0.15	0.15	0.15	0.15	0.15	0.15
	0.25	0.25	0.25	0.25	0.20	0.19
TMPB	-0.05	-0.05	-0.05	-0.05	-0.015	-0.015
	0.05	0.05	0.05	0.01	-0.003	-0.003
TMPC	-0.05	-0.05	-0.05	-0.05	0.001	0.001
	0.05	0.05	0.05	0.05	0.005	0.005
SRA	0.2	0.2	0.2	0.2	0.1	0.1
	0.4	0.4	0.4	0.4	0.35	0.3
SRB	-0.1	-0.1	-0.1	-0.1	-0.125	-0.125
	0.1	0.1	0.1	0.01	-0.015	-0.03
SRC	-0.1	-0.1	-0.1	-0.1	0.005	0.01
	0.1	0.1	0.1	0.1	0.025	0.025
TMPA+	0.15	0.15	0.15	0.17		
TMPC	0.25	0.25	0.25	0.23		
SRA+	0.2	0.2	0.2	0.245		
SRB	0.4	0.4	0.4	0.352		
TMPB/					-6.0	-6.0
TMPC					-3.0	-3.0
SRB/					-6.0	-6.0
SRC					-3.0	-3.0
I. Condns						
TMPA	0.2	0.2	0.2	0.2	0.187	0.187
TMPB	0.0	0.0	0.0	0.0	-0.01	-0.01
TMPC	0.0	0.0	0.0	0.0	0.0033	0.0033
SRA	0.3	0.3	0.3	0.3	0.284	0.284
SRB	0.0	0.0	0.0	0.0	-0.05	-0.05
SRC	0.0	0.0	0.0	0.0	0.016	0.016
Results						
TMPA	0.24965	0.22308	0.21801	0.21718	0.19991	0.18995
TMPB	0.01335	0.00877	0.00021	0.00977	-0.01490	-0.01068
TMPC	0.00033	0.02509	0.03180	0.01266	0.00490	0.00306
SRA	0.39917	0.39990	0.39871	0.39988	0.34953	0.29990
SRB	0.06617	0.07330	0.09941	-0.00250	-0.01575	-0.03028
SRC	-0.00179	-0.00578	-0.04457	-0.09983	0.00509	0.01006
U	369.759	404.819	356.271	435.618	670.794	919.208
Iterations	143	118	89	73	96	68

†Other parameters: CW1 = 1, CW3 = 1, DCONR = 0.1, DT = 1.0
ALPHA = 1.3, BETA = 1.0, GAMMA = 5, DELTA = 0.001

‡Boundary conditions. The first number is the lower boundary and the second number is the upper boundary.

and desired values, and using the alternative implicit and explicit boundaries which will determine the maximum and minimum of TDSR and TMPI. Table 6.1 summarizes the different parameters used for the alternative objective functions, the different boundary conditions and initial conditions, the results, and the number of iterations to meet the convergence criteria of the COMPLEX algorithm specified by the parameters, ALPHA, BETA, GAMMA, and DELTA.

The constraints on the coefficients of the polynomials (which are the variables in the COMPLEX algorithm to be searched) are given to limit the values of TDSR and TMPI within a reasonable boundaries. These have been constrained by the intervals [0.25, 0.48] and [0.18, 0.26] in general for TDSR and TMPI, respectively. For the last two cases, V and VI, the shape of the functions also has been restricted to reflect a certain pattern of path (or policy) which a country may prefer; such as a policy to decrease its dependency on the foreign capital and the import of compressible goods. The monotonic decreasing values of TDSR and TMPI can be attained by imposing additional constraints on the coefficients as follows:

$$\begin{aligned} f(X) &= a_0 + a_1 X + (a_2/2)[3X^2 - 1] \\ &= [3a_2/2][X + a_1/3a_2]^2 + a_0 - a_2/2 - a_1^2/6a_2 \end{aligned} \qquad (6.25)$$

then, the conditions will be

$$a_2 > 0, \text{ and } X = -a_1/3a_2 \geqq 1, \text{ or equivalently, } a_1/a_2 \leqq -3. \qquad (6.26)$$

The optimal paths of TDSR and TMPI with respect to alternative objective functions and constraints are shown in Figures 6.2 and 6.3, respectively. Policy path I shows increases in both total saving and imp-

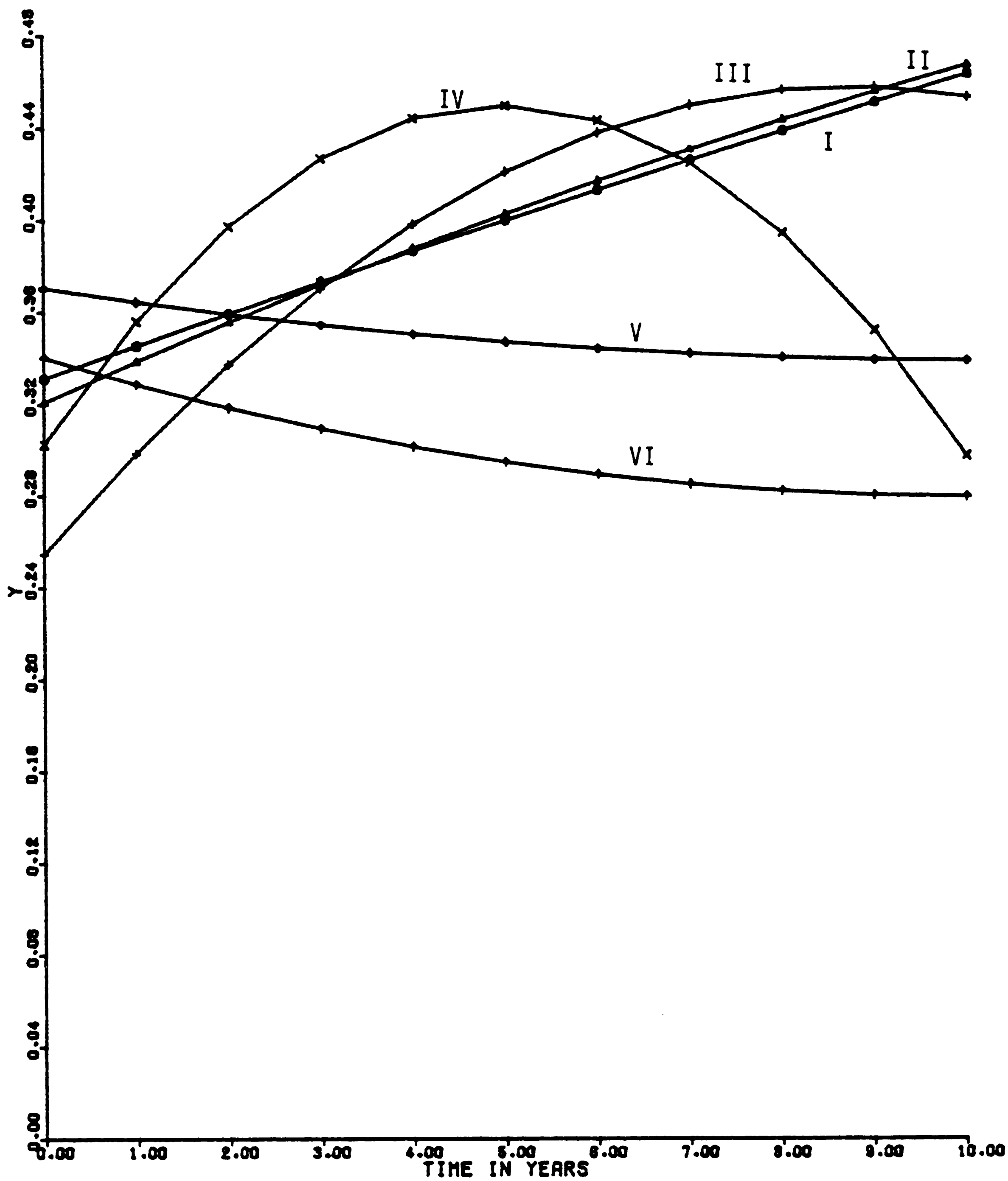

Figure 6.2 Optimal TDSR (total desired saving rate)

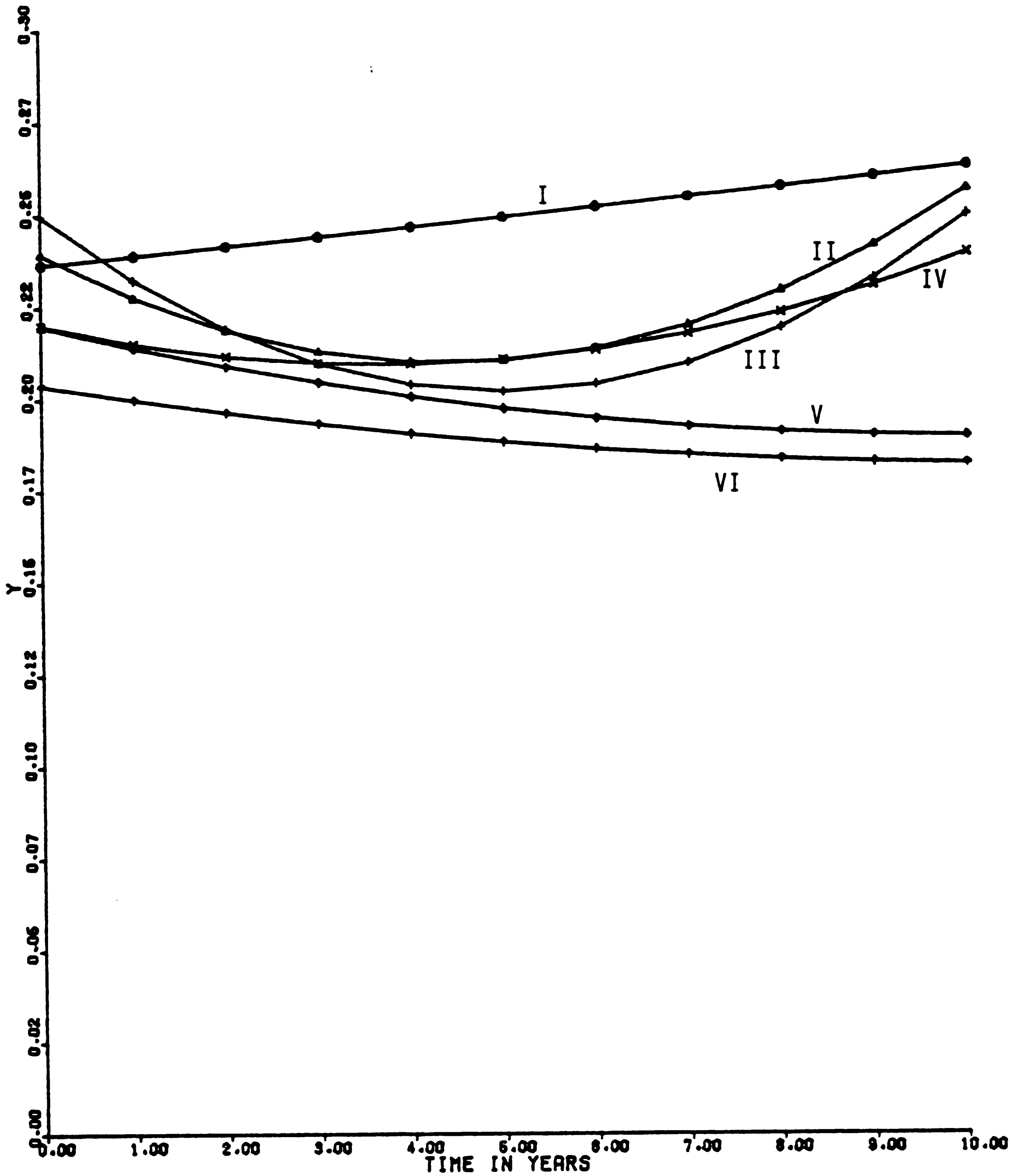

Figure 6.3 Optimal TMPI (the marginal propensity to import)

ort thus implying the high economic growth and high foreign indebtedness since it is unrealistic for a country (like Korea) to achieve the saving rate of 0.46 from only domestic sources.

Policy II shows an increasing path similar to I for TDSR but a significantly different path for TMPI. This comes from the higher weights on the foreign indebtedness relative to the others for policy II, i.e., heavier penalty on foreign indebtedness by changing the weight of CW_2 from 1 to 10 and lowering the desired debt service ratio to 0.1 from 0.2. Thus, policy II exhibits less import of compressible goods while maintaining about the same level of foreign capital borrowing. This is conceptually correct, in other words, if a country wants to lower the foreign indebtedness, it has to restrict the import of compressible goods first of all rather than to restrict the production capacity by lowering the foreign capital borrowings. Policy paths, V and VI, reflect the policy towards self-sufficiency in capital formulation and reduction in the import of compressible goods, which, in turn, will lower the consumption level. Other paths, III and IV, show policies in between the above two groups.

The preceding considerations of the shape of the policy paths, TDSR and TMPI, can be more clearly shown by the paths of economic variables of interest--GNP, growth rate, debt service ratio, annual foreign borrowings, total debt standing, aggregate consumption per labor, income distribution ratio, domestic and foreign saving ratio. These are plotted in Figures 6.4 to 6.13.

Generally, more than twice the real GNP of 1975 (4200 billion 1970 won) can be achieved in 1084 by any of the six policies. A high level of GNP during the planning period (1975 to 1984), could be attained by policy IV which borrows more than the other policies for the first half of the period. But its growth rate drops significantly below 6 percent during the last period because of the early fast growth and the decreased foreign borrowings in the latter period. While policies I, II, and III show the similar high level of GNP and GNP growth rates (about 10 percent throughout), policies V and VI show the lower growth rates due to the lower levels of total desired saving rates. However, these curves still exhibit 8 to 9 percent average growth rates and about 7 percent in 1984.

Debt service ratio, current foreign borrowings, and total debt combined can show a clear picture of foreign indebtedness. With the high borrowings and imports, policy I yields the highest and fast increasing foreign indebtedness. Policies II, III, and IV show similar paths of debt service ratio and total debt while the sharp decrease in the growth rate of the foreign borrowings can be attributed to the lower total desired savings for the second half of the planning period. Policy V maintains a current debt service ratio of 0.11 to 0.12 throughout, and increase the foreign borrowings and total debt by relatively moderate rates. Policy VI decrease the debt service ratio substantially to about 0.07, maintains almost current level of foreign borrowings, and shows slowly increasing total debt in current terms, thus providing a sound basis of economy from a possible vicious circle of indebtedness.

Annual foreign borrowings in 1984 can be varied from the current level of 3500 million dollars in 1975 to more than ten times, and the total debt may increase two to nine times the 1975 debt standing of about six billion dollars.

Aggregate consumption per labor which is one of the most important

Measure for the social welfare is increasing about one and half to two times the 1975 level in 1984 with different increasing rates. For higher production and import, like policies I and IV, the consumption level during the period is high. For the policies with lower foreign indebtedness, like V and VI, the consumption level is relatively low. Paths of the consumption growth rates show more clearly the characteristics of the consumption behavior during the planning period with respect to the different policies. The real consumption growth rate will be about 6 to 7 percent annually with policy I, and about 4 to 3 percent with policies V and VI, respectively. Other policies show significant changes in consumption behavior: policy IV reaches about 8 percent then drops to 4 percent, policies II and III show rapid increase from 2 and 4 percent to about 9 percent annual consumption growth.

Income distribution is another important consideration. It is measured by the income distribution ratio, defined as the per capita farm real income divided by the per capita urban real income. The improved income distribution ratio (Fig. 6.11) is mainly because of the internal feedback loop through migration and transfer of income from urban to rural via the nonagricultural sources. The distribution ratio ranges from 0.9 to 0.95 by 1984, and the best policy for equitable distribution is VI which shows higher ratios most of the time. Policy V shows slightly less values of distribution ratio than VI, and policy IV shows relatively low income distribution ratio throughout the planning period.

As has been mentioned, there are two sources of savings--domestic and foreign savings. If domestic saving is given, then the foreign saving can be determined by the difference from the total savings which can be given by a specific policy. Domestic saving rate has been given at about 24 percent during the planning perod. The least equitable income distribution policy IV can achive the highest domestic savings, and the most equitable distribution policy VI yields the lowest domestic savings.

Foreign saving, implicitly assumed to be under the government regulation or policies which can control the flow of the foreign savings, can be determined as the net of the domestic savings from the total savings needed. The overall paths of the foreign savings will therefore become similar to those of the total savings given the steady domestic savings.

Each of the policies represents the optimal policies which optimize the "alternative" objective functions and constraints. Nevertheless, each policy may have substantially different implications. Policy I shows high and increasing debt service ratio, high foreign borrowing and total debt, lower income distribution ratio, and high economic growth and consumption. Policy II represents higher growth, foreign indebtedness, and consumption but a lower income distribution ratio. Policy III exhibits higher growth and foreign borrowing, medium debt service ratio and consumption, and lower income distribution ratio. Policy IV displays high economic growth and consumption, higher foreign indebtedness, and low income distribution ratio. Policy V presents lower growth, foreign indebtedness, consumption, and the higher income distribution ratio. Policy VI results in low economic growth, foreign indebtedness, and high income distribution ratio. In sum, policy I represents a program which focuses on enjoying affluence with foreign borrowings, but results in the least equitable distribution of income. Policy VI represents an austerity program on consumption towards self-sufficiency in capital formation, and thus provides a sound basis of foreign indebtedness and

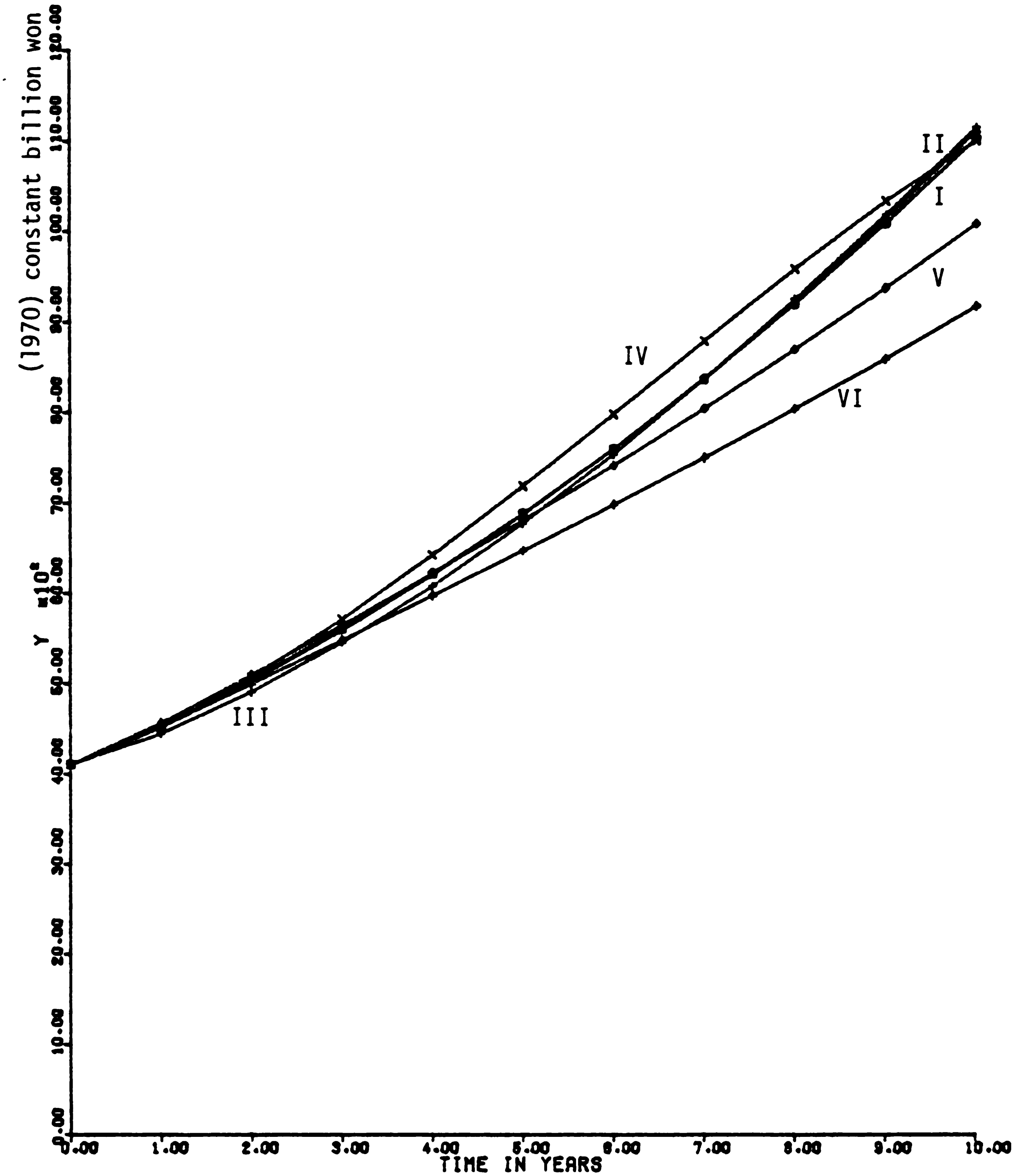

Figure 6.4 Gross National Product

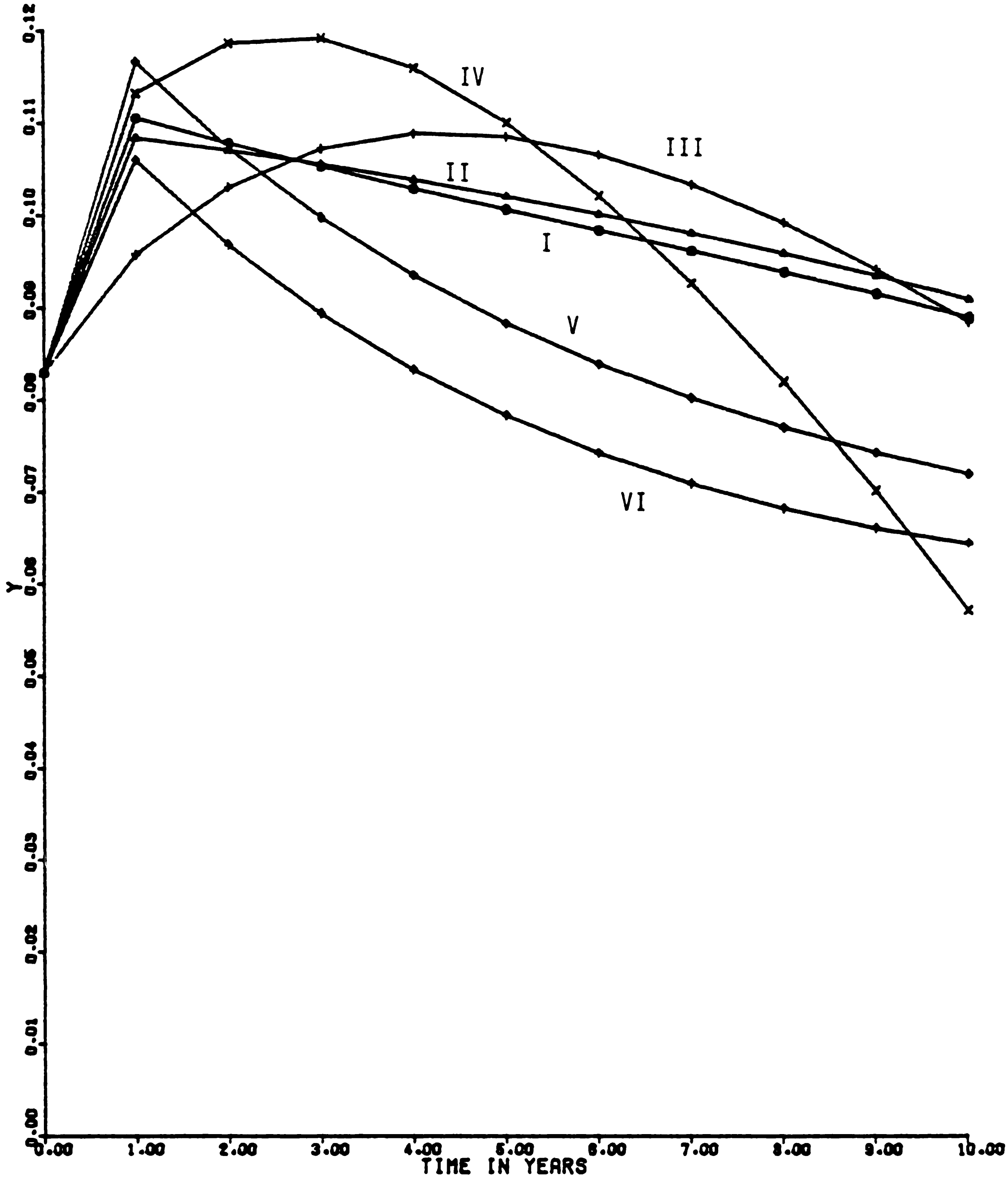

Figure 6.5 Real GNP Growth Rate

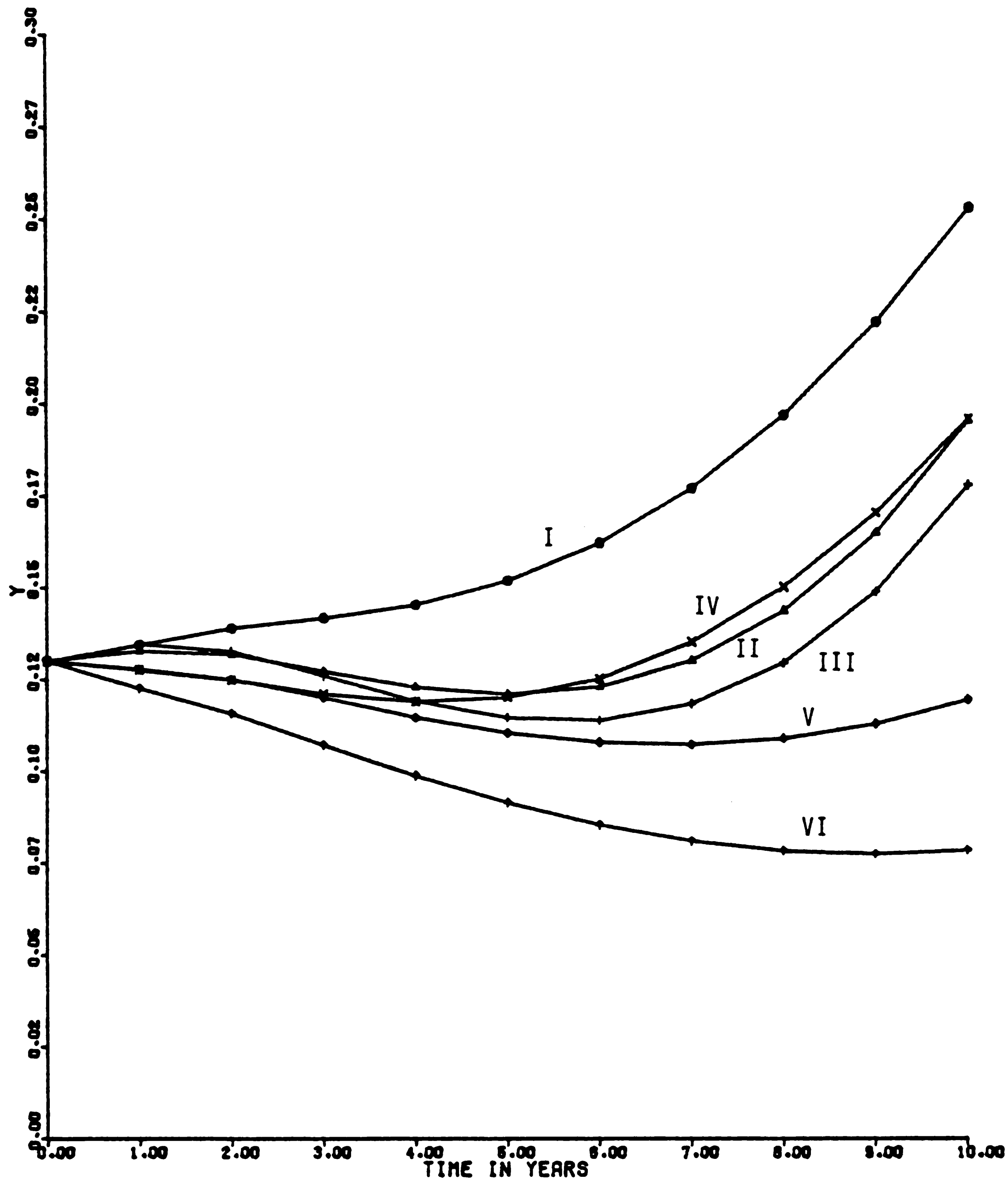

Figure 6.6 Debt Service Ratio

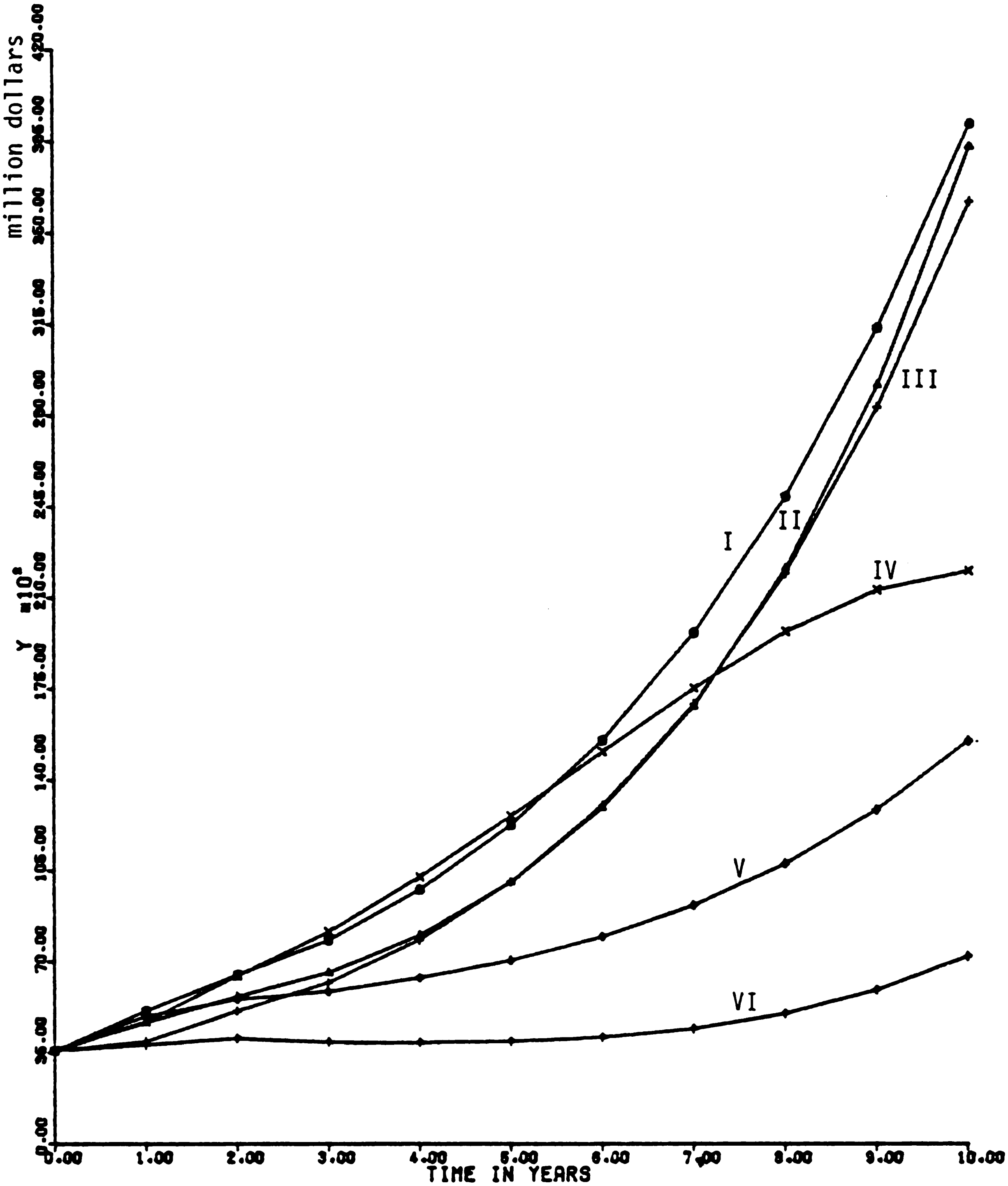

Figure 6.7 Foreign Borrowing

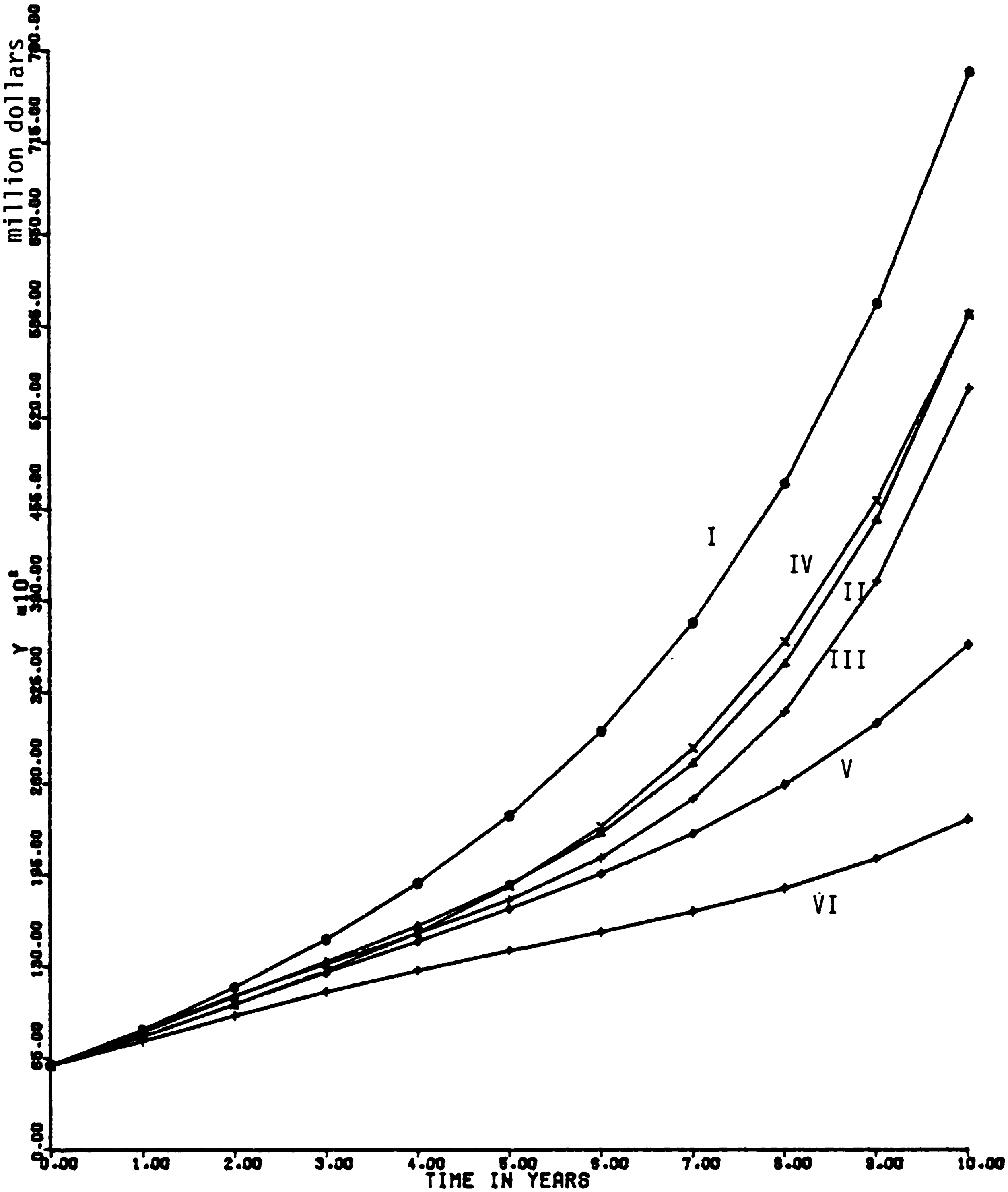

Figure 6.8 Total Debt

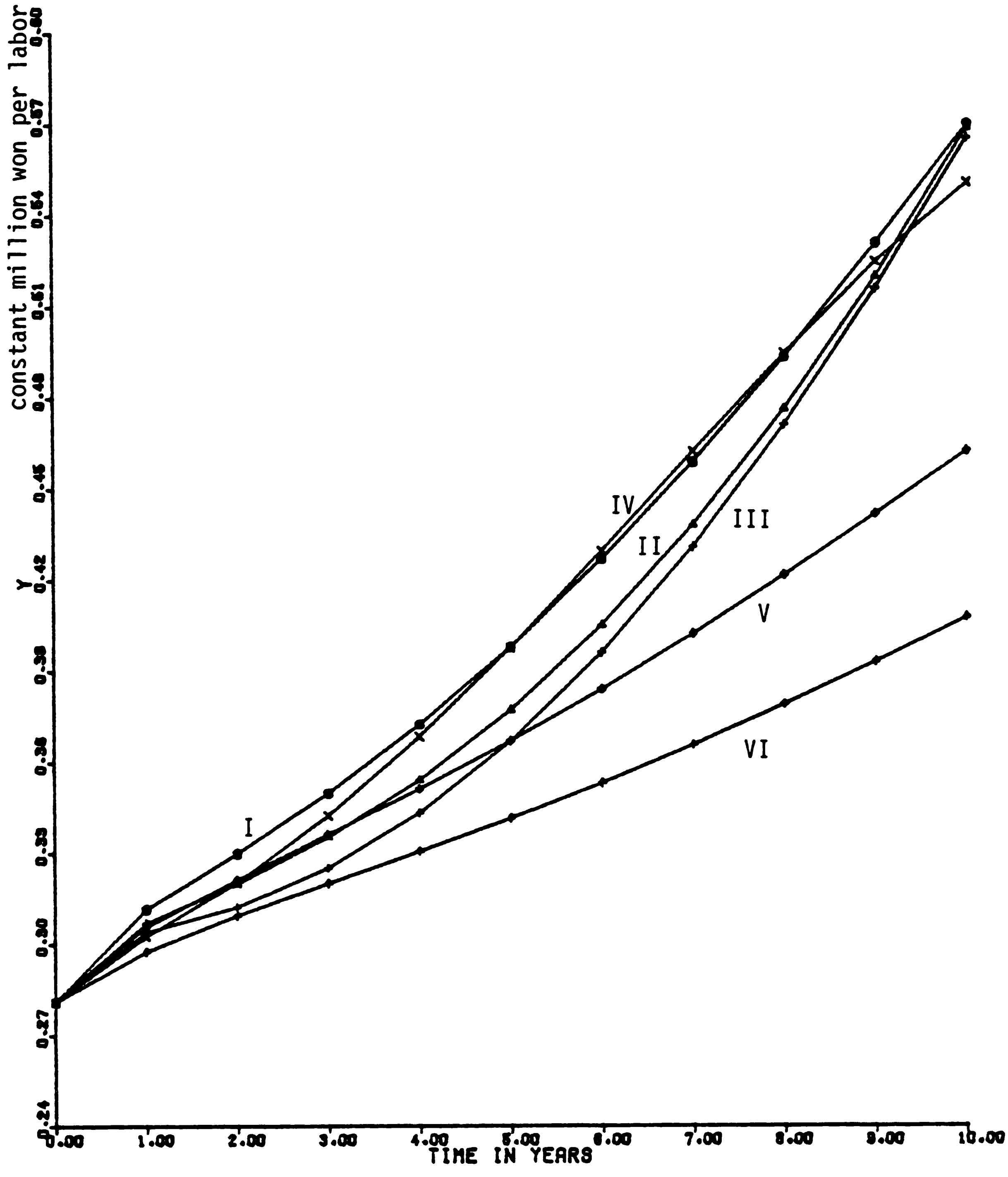

Figure 6.9 Aggregate Consumption per Labor

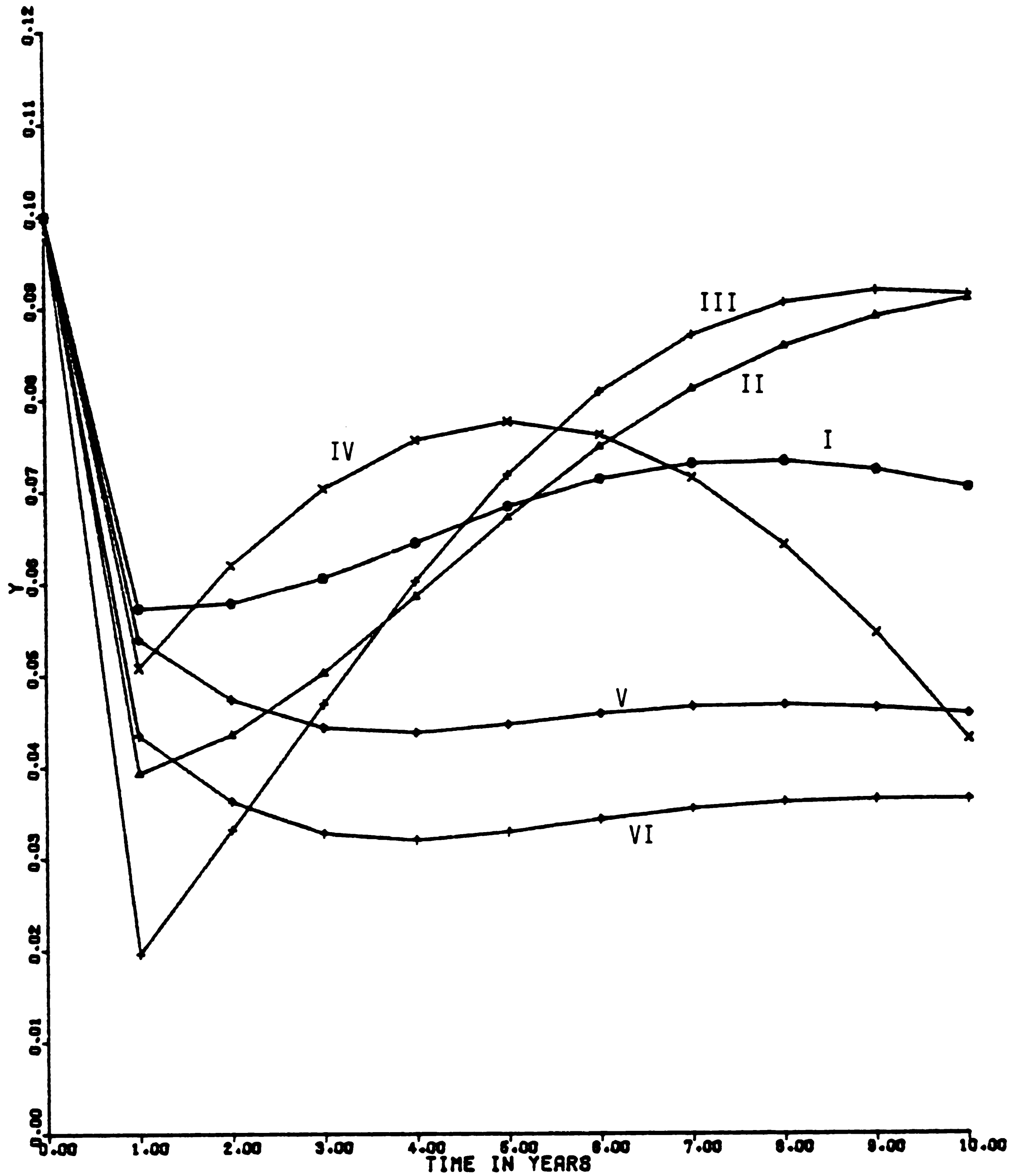

Figure 6.10 Rate of Change of Consumption per Labor

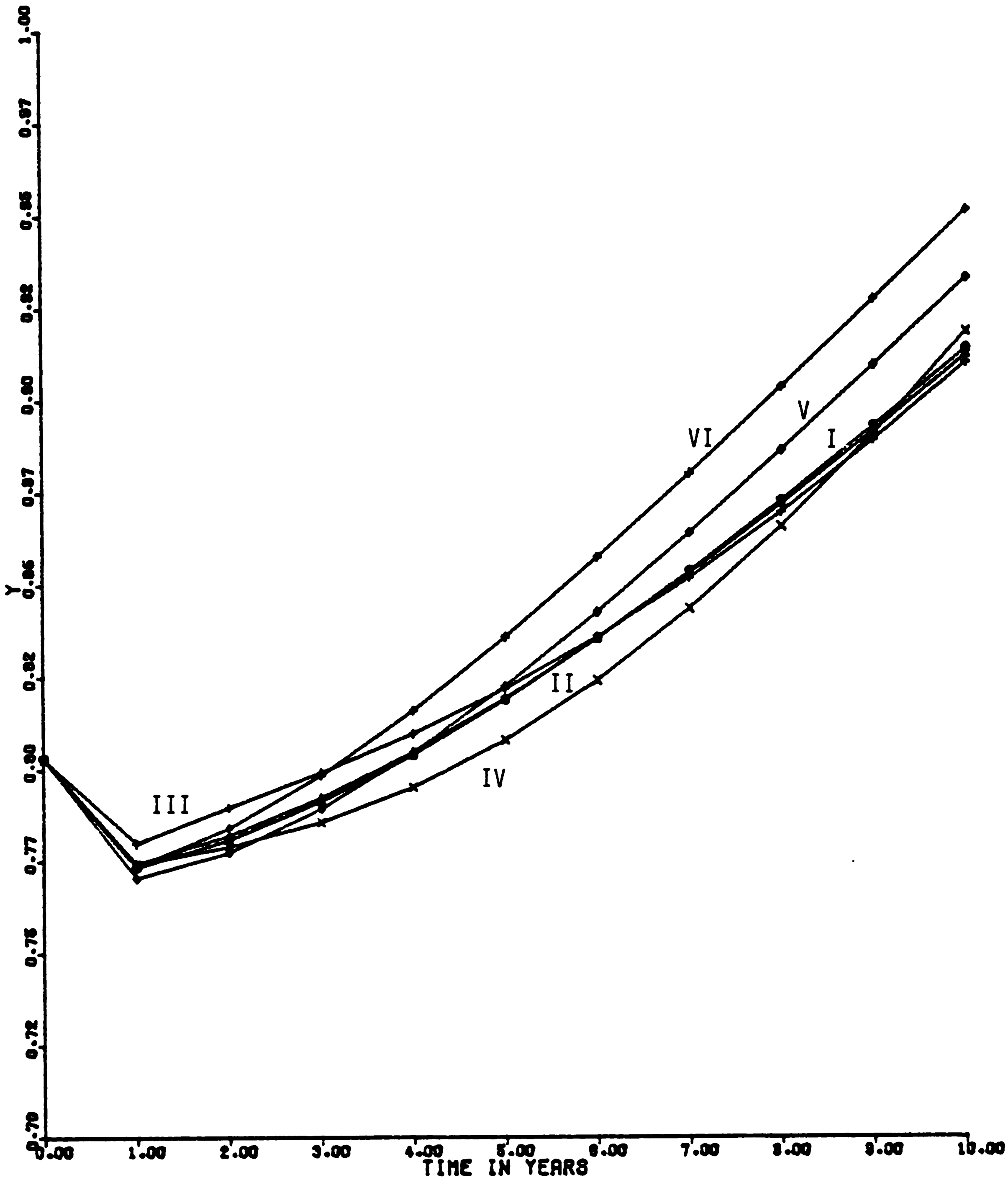

Figure 6.11 Income Distribution Ratio

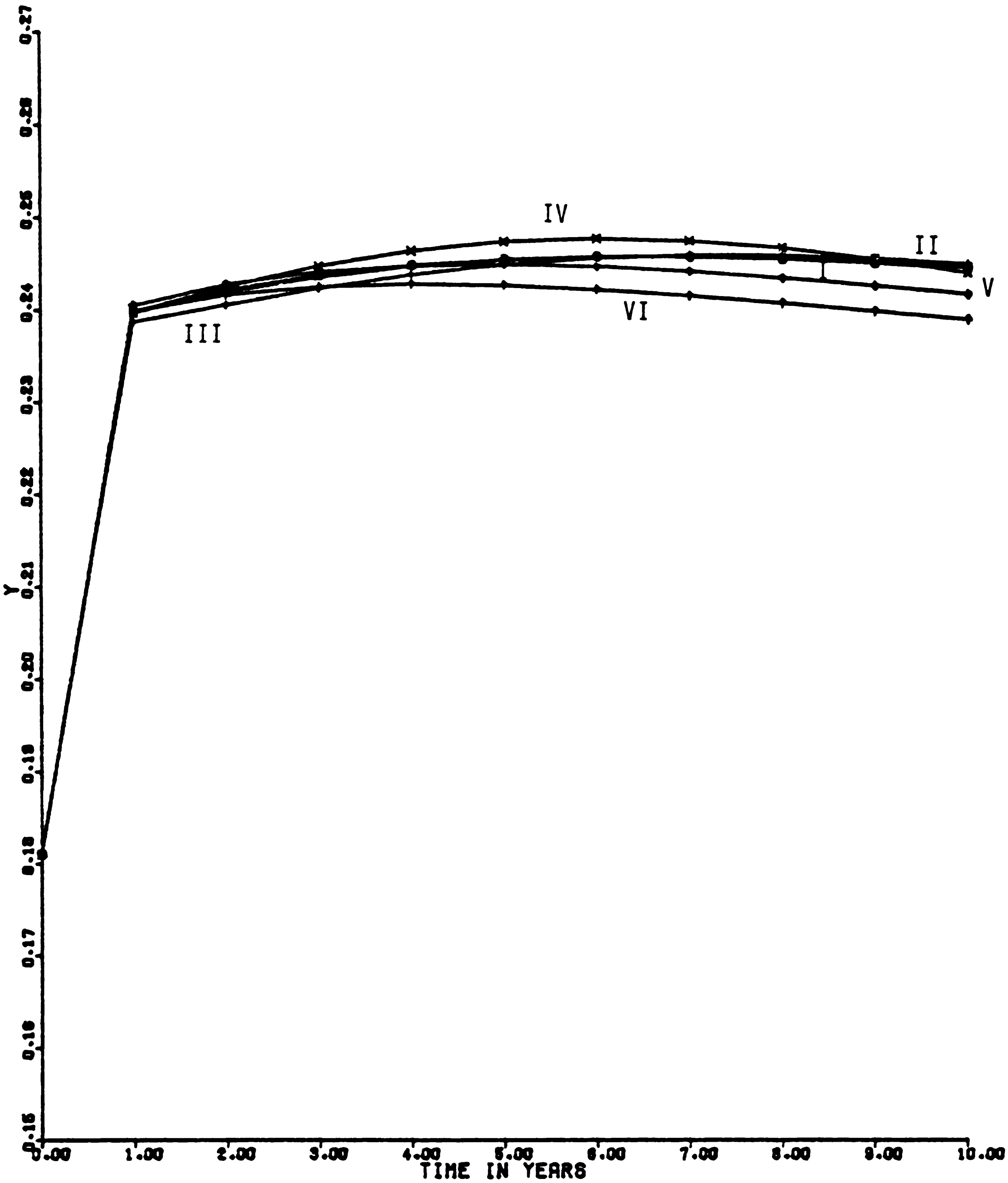

Figure 6.12 Domestic Saving Rate

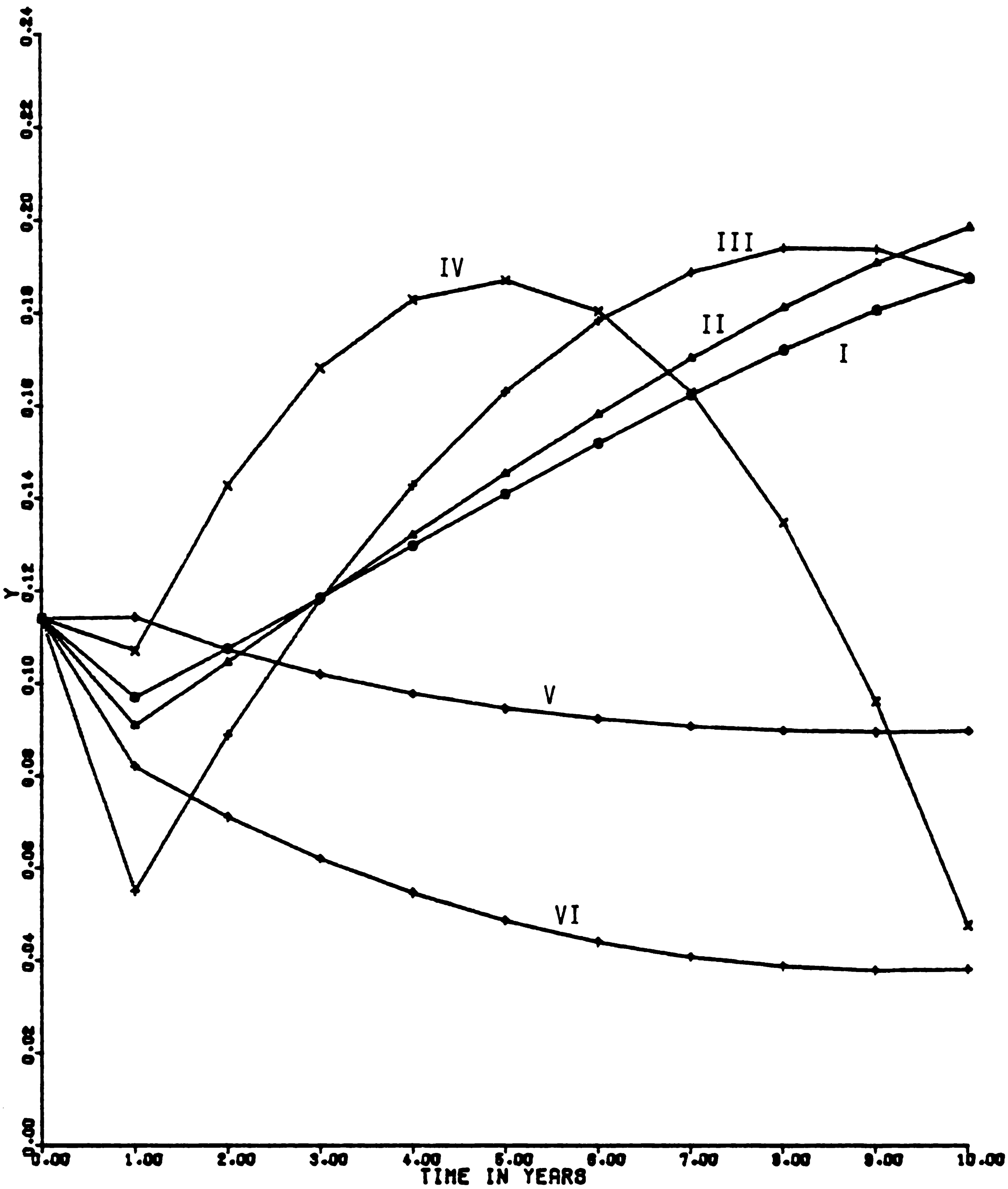

Figure 6.13 Foreign Saving Rate

equitable distribution of income. Other policies exhibit results between the affluence and austerity.

Although it has not been included in the objective function (6.17) explicitly, the distribution of income is an importan objective of a society, and thus will be investigated under the different policies which will gear to the equitable distribution.

There are two channels of income transfer from urban to rural in general: first, increase in quantity and/or value of agricultural production, secondly, farm income from nonagricultural sources which are the earnings of farm labor participating in the production of nonagriculture. More investment to agriculture for land developement, irrigation, researches for new varieties, etc., will increase the physical production of agriculture, while price increase (or control) of agricultural product will increase the values of the agricultural production relative to nonagricultural production. The second channel of income transfer, which involves the complex interactions of socio-economic policies, is possible as a result of the rural industrialization.

Assuming the policy VI in the preceding discussion for the total desired saving and the marginal propensity to import reflecting the foreign indebtedness, seven policies will be designed in addition to look into the effects of income transfer.

Policy 1 : normal trend of grain prices and other variables

Policy 2 : linearly increasing agricultural investment from 0.1 in 1975 to 0.15 in 1984

Policy 3 : lower grain prices, i.e., 10 percent lower than the normal increasing rate of grain prices

Policy 4 : higher grain prices, i.e., 10 percent higher than the normal increasing rate of grain prices

Policy 5 : encouragement of rural industrialization by making the feedback gain for farm labor participation into nonagricultural production higher, thus making it respond more sensitively to inequitable income distribution

Policy 6 : combined policy of 3 and 5

Policy 7 : combined policy of 2, 3, and 5.

The last two policies are added to investigate the possibility of maintaining the income distribution ratio by agricultural investment and rural industrialization. The paths of the key economic variables are plotted in Figures from 6.14 to 6.16. Effects of the policies on GNP are essentially the same except for possible inflationary and deflationary effects.

Grain price control is the most significant factor for the transfer of incomes: however, in the long run, other policies--agricultural investment and rural industrialization--may be more effective, i.e., may provide the sound basis for the continuing equitable distribution of income. High grain price policy, policy 4, leads to the transfer of income from urban to farm, but it also leads to low domestic savings since more income has to be spent on food for the urban people which will yield higher foreign borrowings to meet the total desired savings.

Consumption paths, like those of GNP, are the same except for certain discrepancies due to inflationary or deflationary effects. For the policy of low grain prices (which is likely to be pursued in Korea), the greatest concern is how to provide farmers with certain channels which will force the income transfer to keep a level of normal income distribution ratio. Policy 6, which specifies low grain prices and rural industrialization, more than restores the level of income distribution to the normal trend by 1984. Policy 7, which includes more agricultural investment than policy 6, recovers faster than policy 6, and it reaches the balanced state of income distribution. Without any additional actions for the case of low grain price policy (policy 3), the distribution of income is far less than the normal path.

No one policy dominates the others at every point in time, i.e., none can be ranked in a strict preference over others for all the elements of the performance criteria at every point in time. Some policy may be strictly superior to the others for a certain objective but may be inferior for the other objectives or a certain policy may dominate the others during some period of time but not during the rest of the planning period.

There is no gain without cost. One has to decide what to sacrifice in order to obtain the others, in other words, one has to decide a preference set (or weights of the objectives) from the possible sets of preferences of time and objectives. Trade-offs are essential in this sense to determine a policy which will actually be selected and implemented: the decision making processes will enter the scene for this purpose.

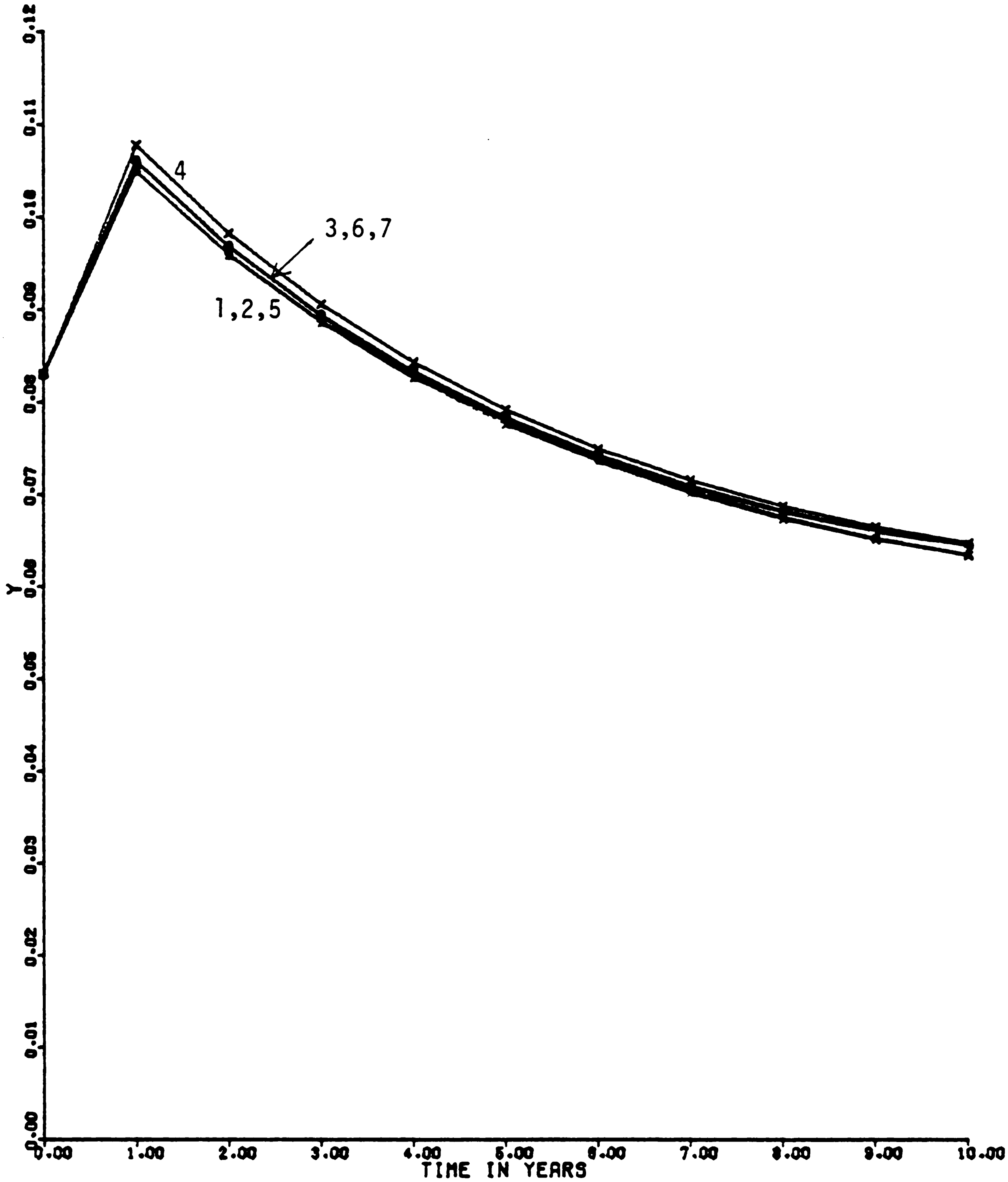

Figure 6.14 Real GNP Growth Rate

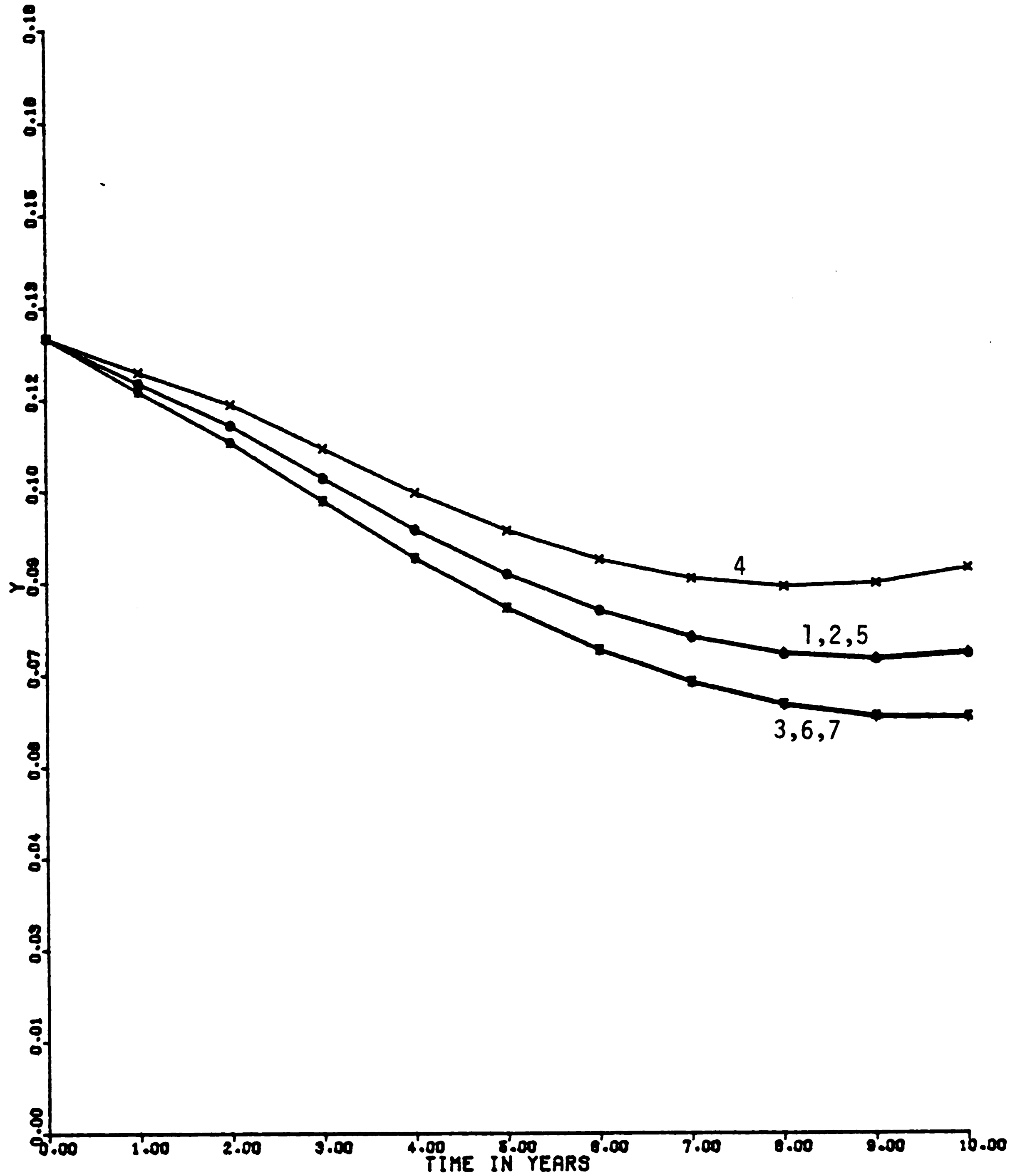

Figure 6.15 Debt Service Ratio

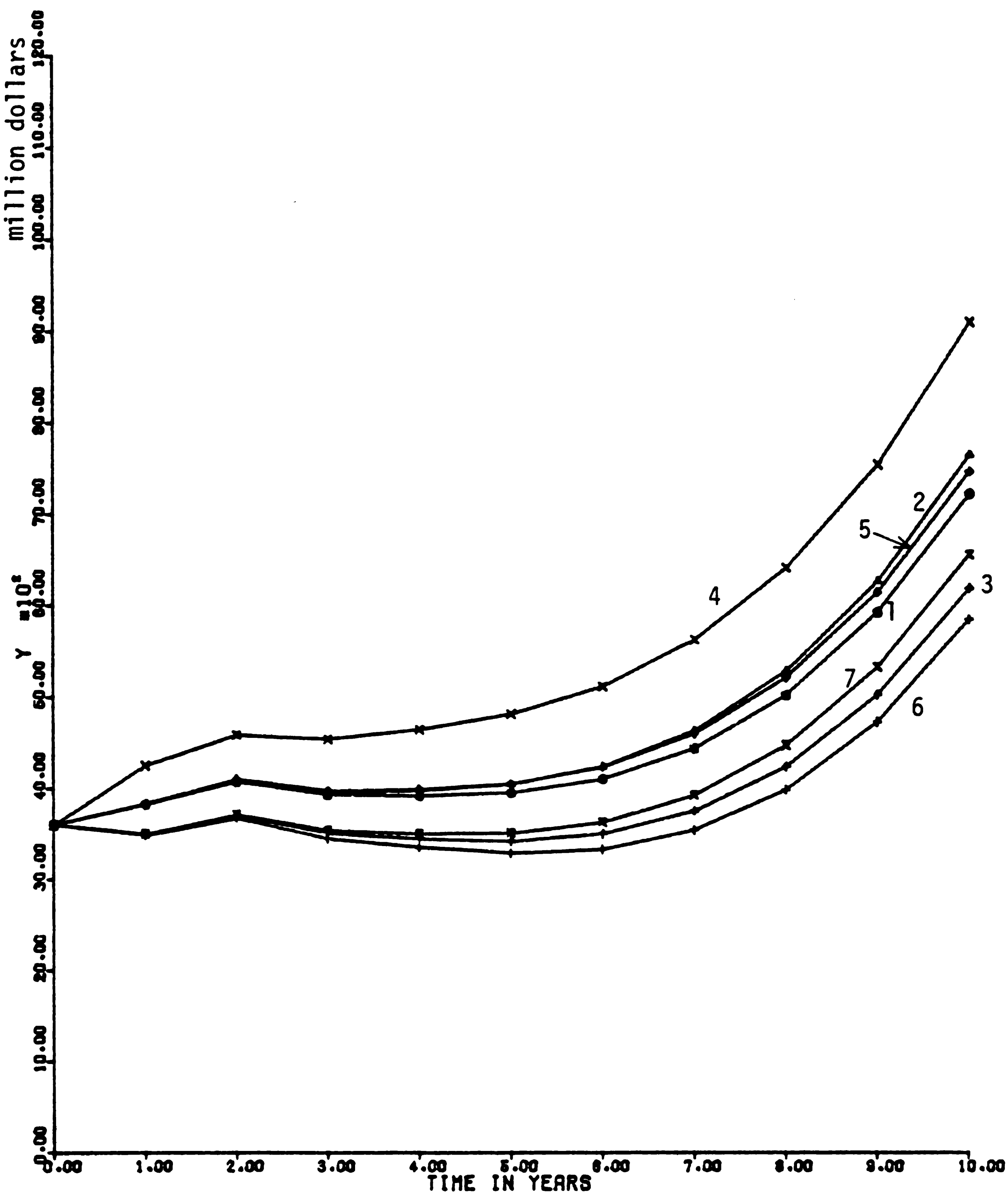

Figure 6.16 Foreign Borrowing

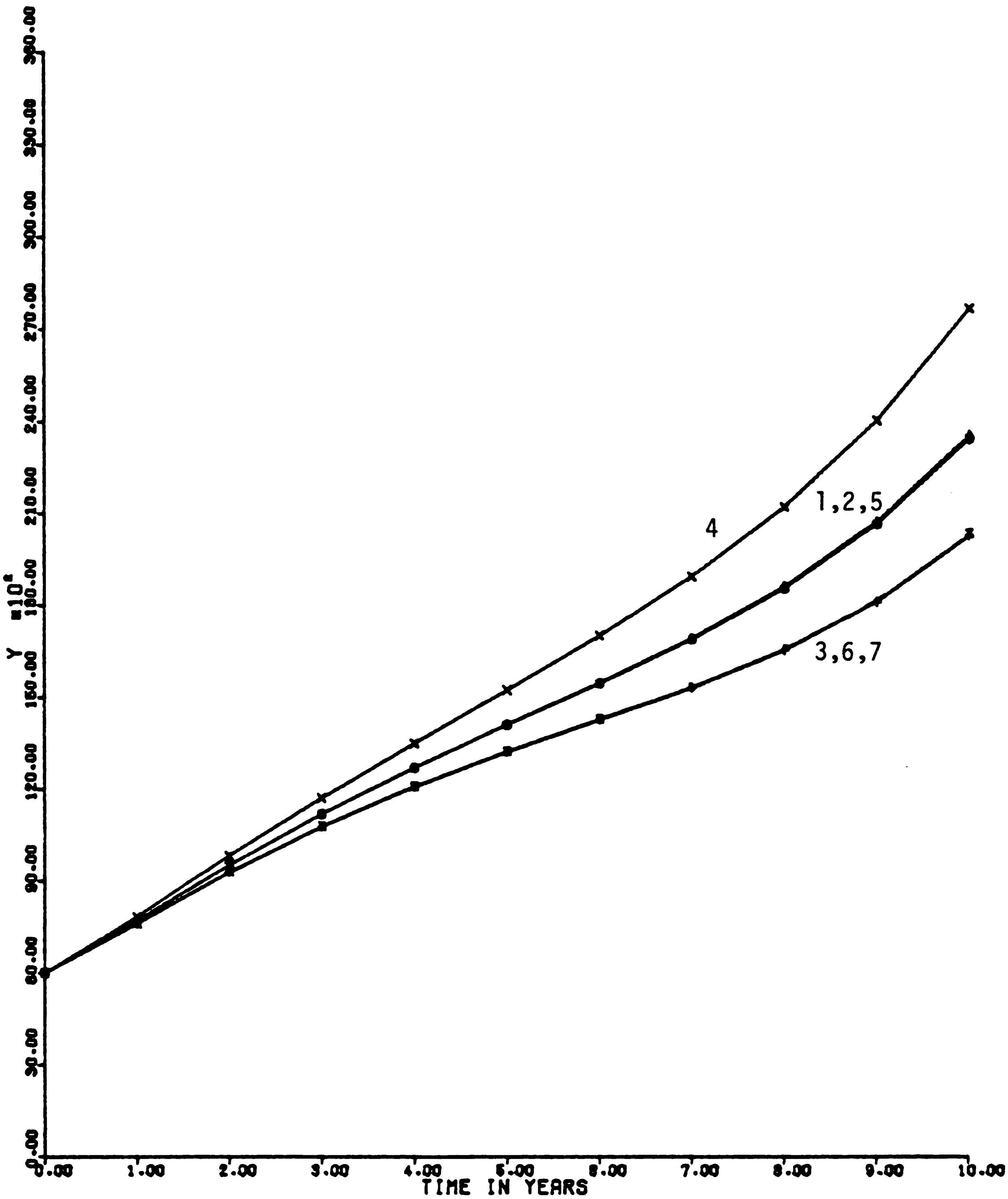

Figure 6.17 Total Debt

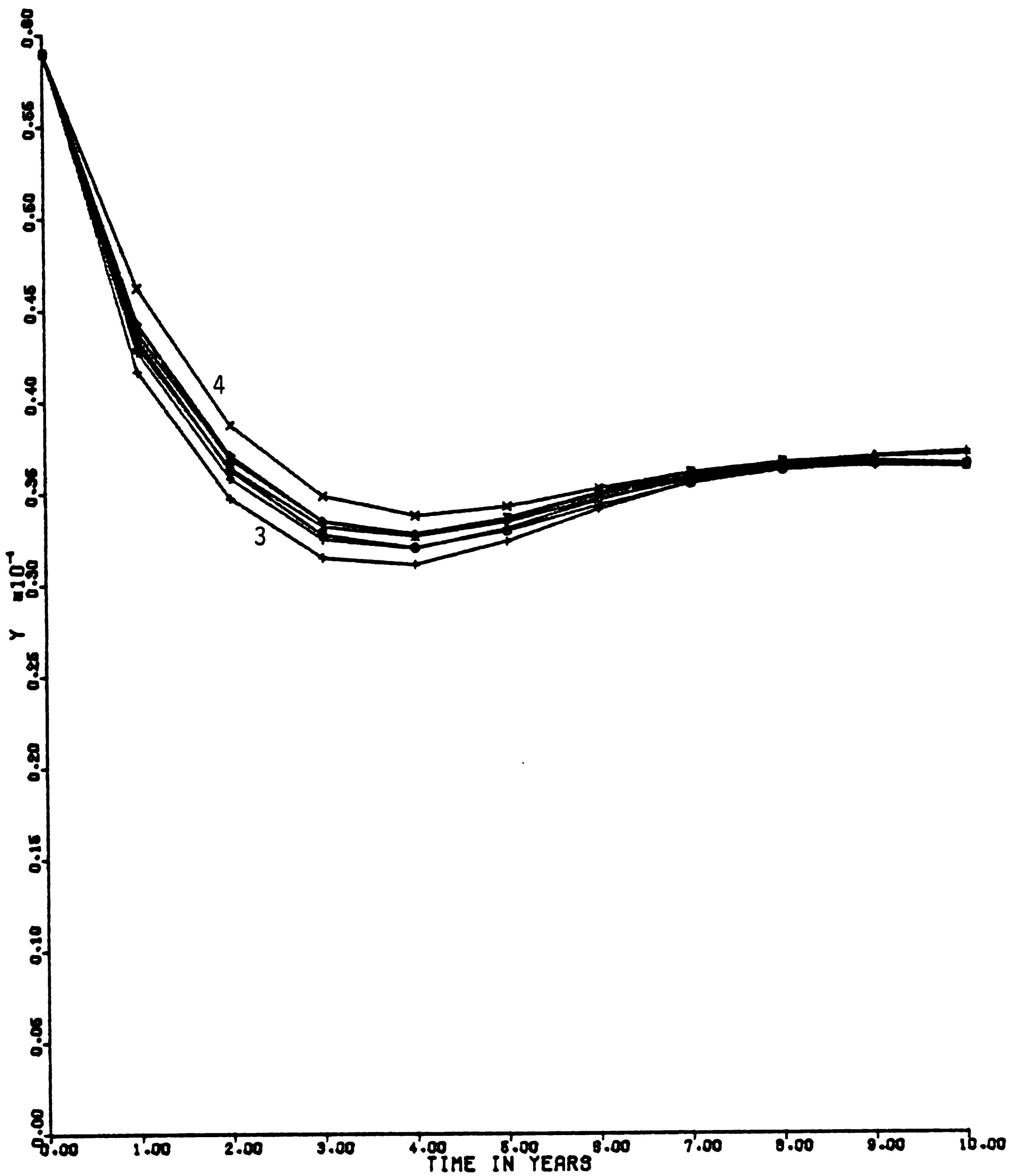

Figure 6.18 Rate of Change of Consumption per Labor

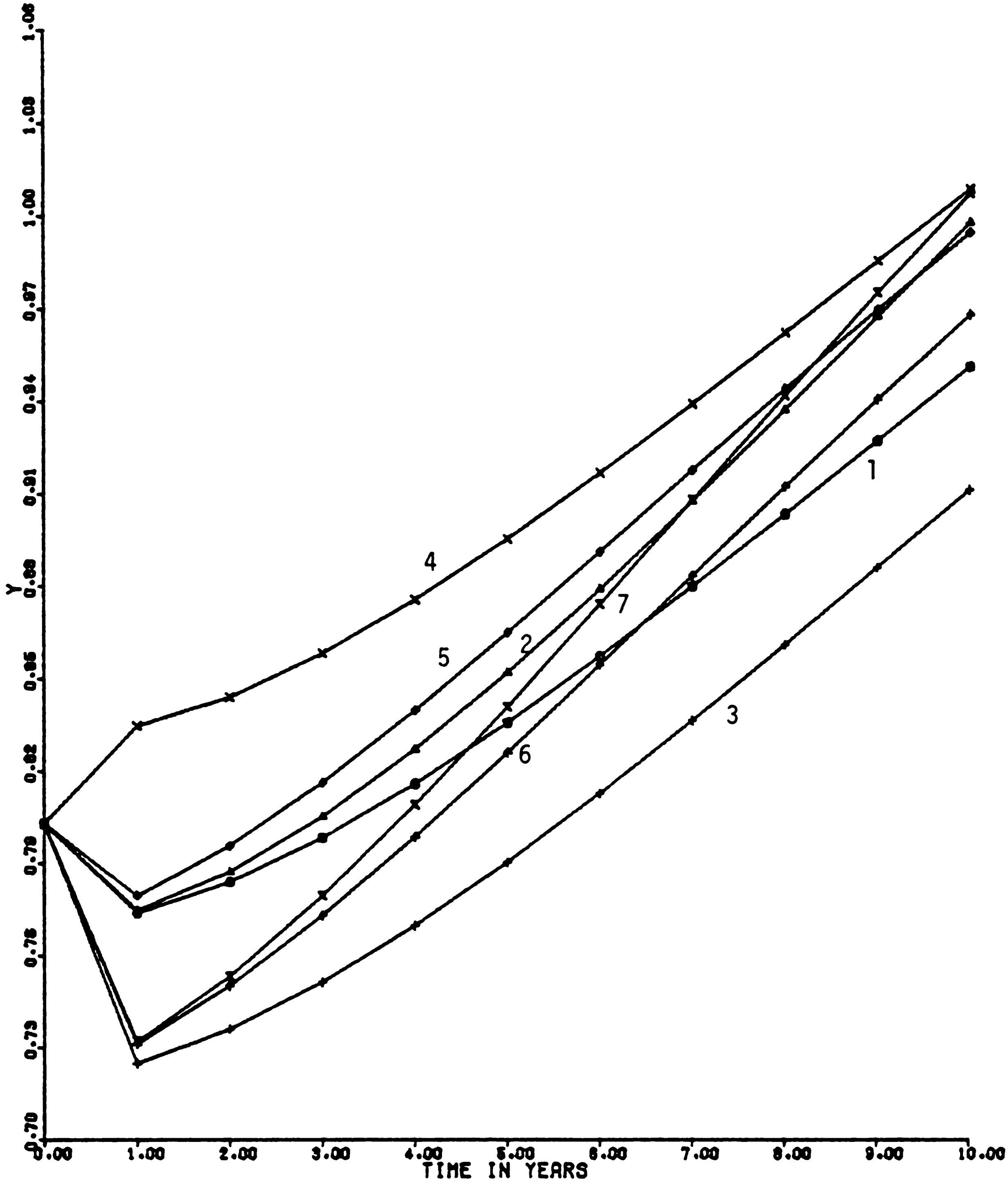

Figure 6.19 Income Distribution Ratio

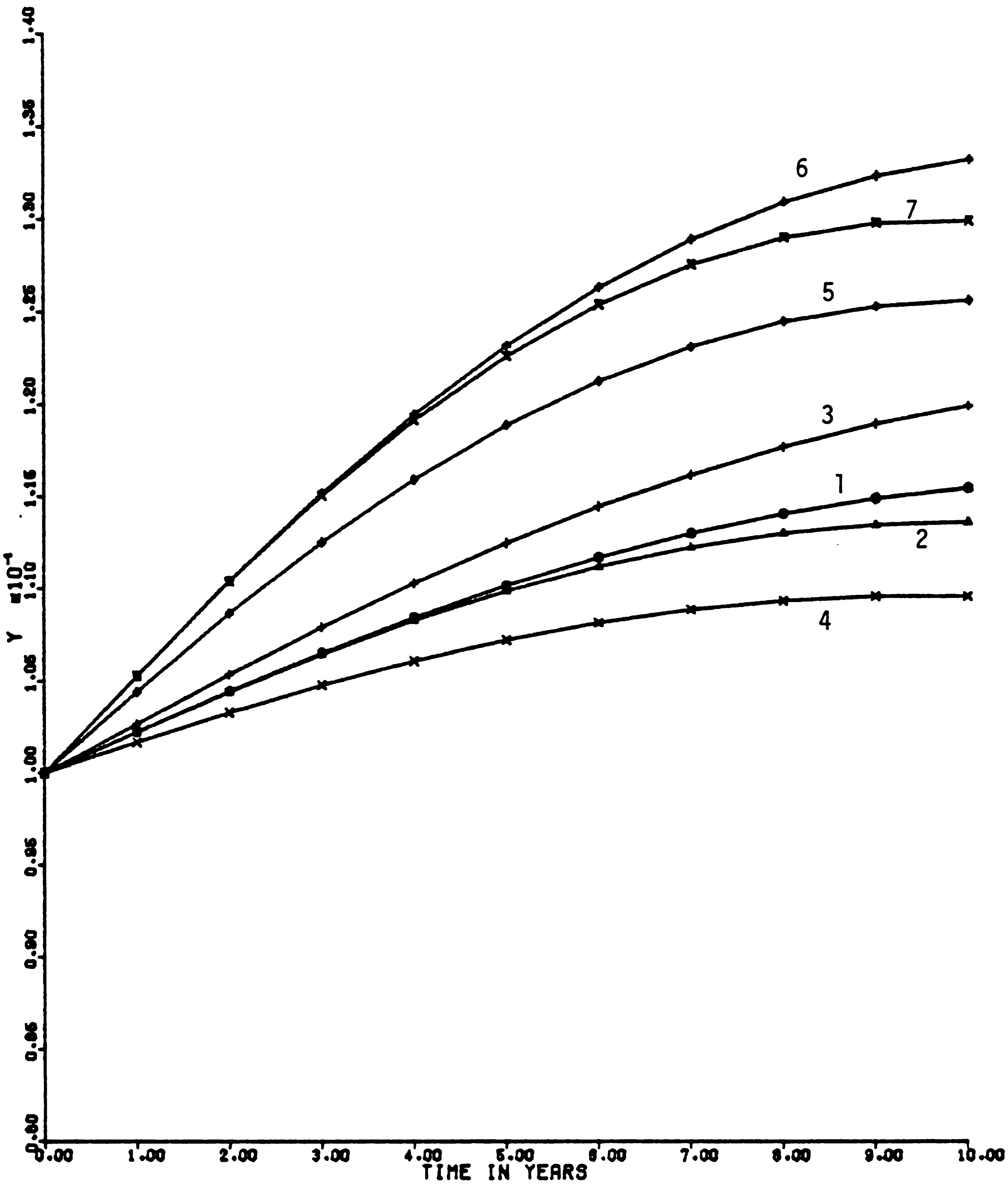

Figure 6.20 Participation Ratio

CHAPTER VII

FURTHER ANALYSIS OF THE OPEN ECONOMY UNDER THE EXISTENCE OF EXTERNAL FOOD SHOCK

VII.1. Introduction

For most of the decades before 1972, world grain markets were soft with real prices slowly declining, (Figure 7.1) and the grain stocks in a certain part of the world were accumulated to be burdensome. The trend was reversed in 1972 when severe weather struck some parts of the world causing bad harvest and massive grain purchases by the Soviet Union (which had been a major exporter before) creating highly unstable and insecure world grain situations. The buffer of the remaining North American reserve was inadequate to absorb all the pressures, and the world grain reserve stocks declined far below the minimum contingency levels, only 31 days amount remaining by 1975-1976. Thus in fact the entire world was "living hand to mouth." [B8] Food aid was reduced and some exporting countries placed restrictions on commercial grain sales abroad.

The waves of the shock were immense--wide spread malnutrition, famine, and starvation--and consequently, it awoke people from the myth that "the world is capable of feeding itself forever."

For the countries who depend on the outside for much of their food needs, the effects of the world food shortage can be very costly; it directly affects foreign exchange and affects the development programs by forcing reductions in the volume of nonfood imports. Economic stability is affected by way of heightened inflationary pressures and the levels of food consumption through cutbacks in cereal imports, which in turn may

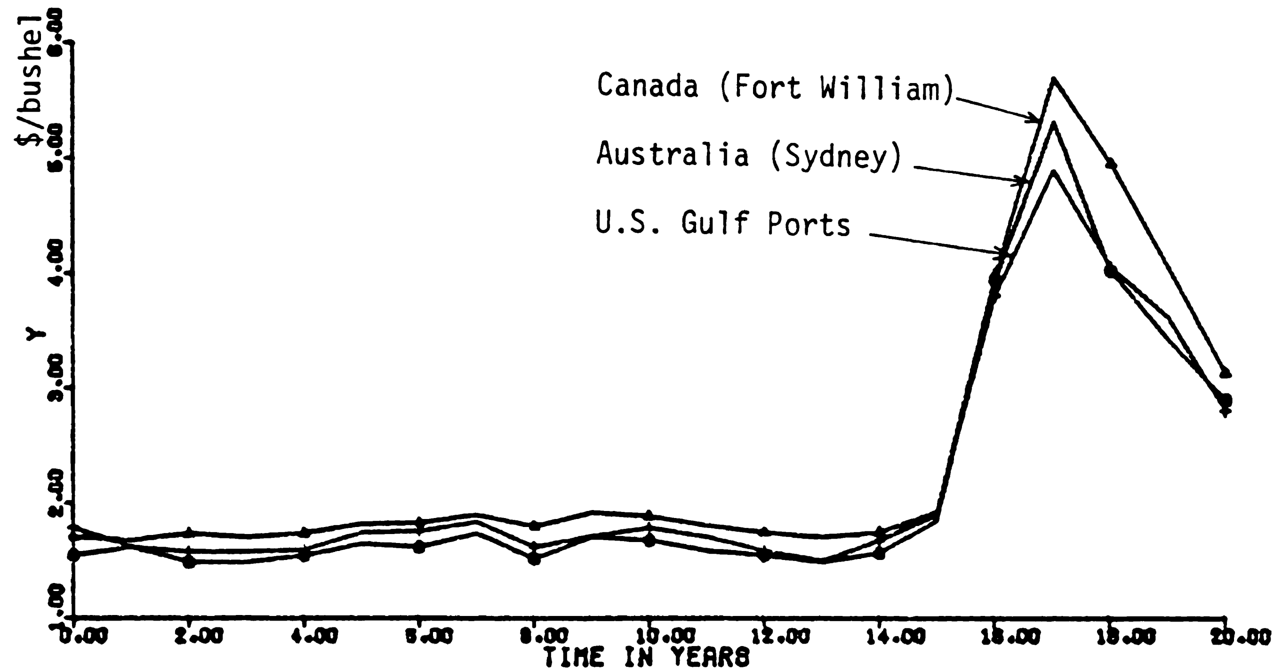

Figure 7.1 World Price of Wheat

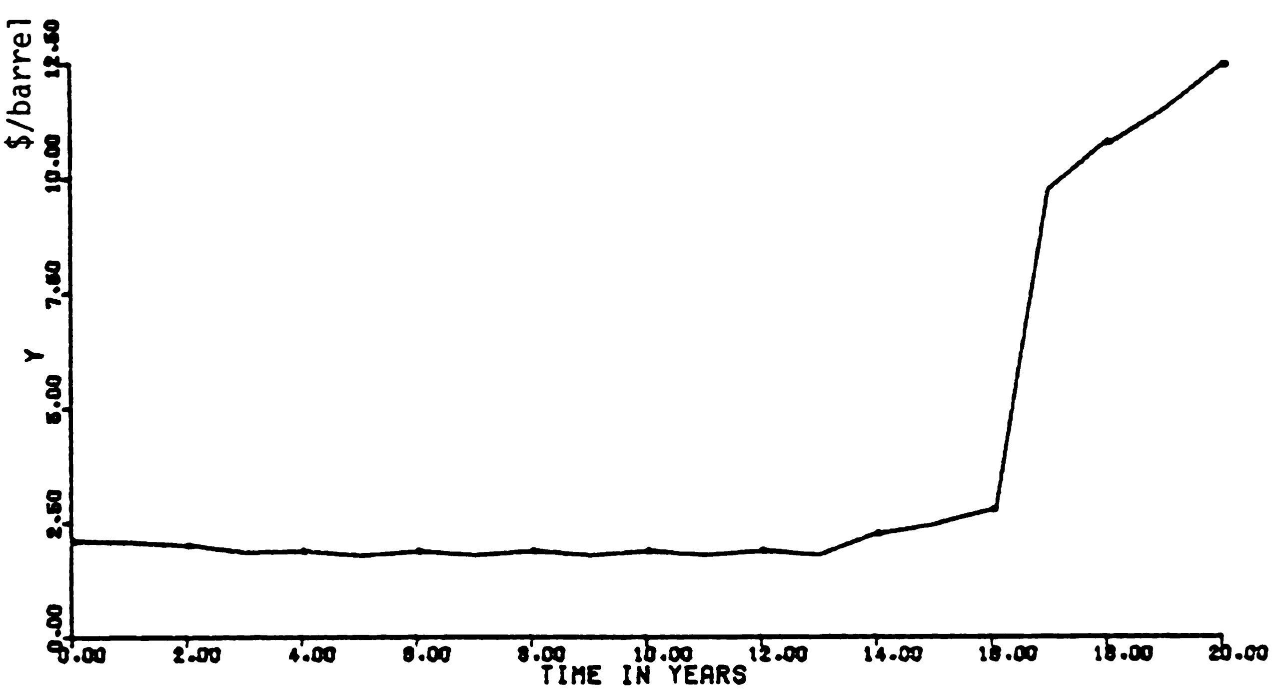

Figure 7.2 Petroleum Price: Saudi Arabia (Ras Tanura)

TABLE 7.1

DEPENDENCY ON THE GRAIN IMPORT†

Year	Rice	Barley	Wheat	Total Grain‡
1965	0.0	0.041	0.622	0.088
1966	0.009	0.0	0.596	0.079
1967	0.029	0.0	0.819	0.143
1968	0.057	0.05	0.695	0.182
1969	0.191	0.031	0.845	0.284
1970	0.123	0.0	0.804	0.229
1971	0.19	0.0	0.784	0.28
1972	0.134	0.122	0.877	0.317
1973	0.102	0.169	0.854	0.308
1974	0.044	0.143	1.024	0.253
1975	0.089	0.157	0.931	0.266

†Source: MAF, Food Bureau Grain Balance Sheets. The dependency is in terms of the ratio of import to the total utilization which includes seed, feed, industrial consumption, loss, and human consumption.
Dependency = 1 - self sufficiency.

‡Total grain is the sum of rice, barley, and wheat in metric ton.

result in a spreading and worsening of malnutrition, famine, and starvation.

For some of the LDC's who can afford the foreign currency needed for the import of foods and have been pursuing sustained economic growth, like Korea, the main concern will be the effects of the external obstacle to the continued growth and effects to other internal economic variables.

During the course of economic development in Korea, the structure of the economy has changed drastically from a traditional agrarian economy to a semi-industrialized economy, and thus the nation became highly dependent on the imports for its grain needs.

Table 7.1 shows the trends of import dependency of grains during the past decade. While the imports of rice and barley fluctuated but stabilized at low levels, the import of wheat increased almost to the total consumption level and thus contributed to make the total dependency from about 9 percent in 1965 to 27 percent in 1975.

The main purpose of this chapter is to analyze the effects of the uncertain world grain market and corresponding grain prices on economic growth and various internal economic variables using the open growth model, and to design possible grain reserve rules to lessen the effects of the shock on domestic prices and the total economy.

VII.2 Analysis of Shocks

Although there are qualitative aspects such as psychological effects, the shock (of food or oil) can immediately be observed in the form of sharp price increases as long as the markets are still functioning. (Figures 7.1 - 7.2)

Generally, there are three patterns of price increases, namely, stable, unstable, and asymptotic stable shocks. "Stable shock" implies a price increase which will eventually settle down to a constant price apart from the original price, "unstable" shock means the price increase without bound, and "asymptotically stable" shock indicates a price increase which will return to the initial price eventually. (Figure 7.3)

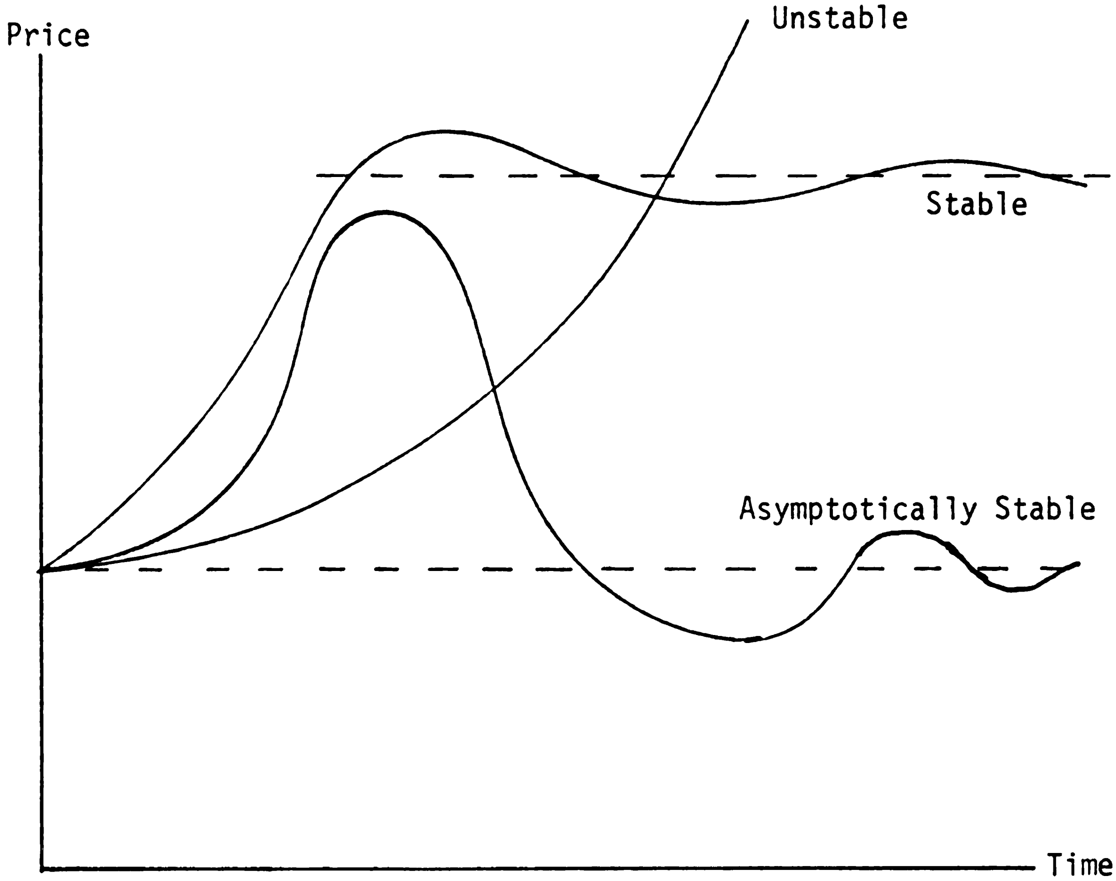

Figure 7.3 Different Patterns of Shock

As was seen in Figures 7.1 and 7.2, the oil and food price increases were stable (almost unit step) and asymptotically stable shocks, respectively.

There may be various ways of expressing the shocks; using differential equations as in the dynamic system theory, TABLE look up function, or a certain form of algebraic function which can approximate the shape of a shock. One possible use of an algebraic function for an asymptotic stable shock of grain price is squared sinc function. The sinc function, performs ideal low-pass filtering when it enters into convolution, i.e., removes all components above cutoff frequency and leaves all below unaltered (since the Fourier transform of the function can be expressed in the form of box-car, II(x)). It can be given as follows:[1]

$$\text{sinc } x = \frac{\sin \pi x}{\pi x} \tag{7.1}$$

with the properties that

$\text{sinc } 0 = 1$

$\text{sinc } n = 0$, for all nonzero integer n

$\int_{-\infty}^{\infty} \text{sinc } x \, dx = 1.$

The square of sinc function is

$$\text{sinc}^2 x = \left(\frac{\sin \pi x}{\pi x}\right)^2 \tag{7.2}$$

which represents the power radiation pattern of a uniformly excited antenna, or the intensity of light in the Fraunhofer diffraction pattern of a slit. Also the properties are

$\text{sinc}^2 0 = 0$

---

[1] [B6], pp. 62-67.

$\mathrm{sinc}^2 n = 0$ for all nonzero integer n

$\int_{\infty}^{\infty} \mathrm{sinc}^2 x \, dx = 1.$

For the world grain price shock, the equation (7.2) has been modified as

$$A\left[\frac{\sin\,[2/T\ \pi(x-1)]}{2/T\ \pi(x-1)}\right]^2 \qquad (7.3)$$

where A represents the magnitude of shock, and T is the period of the shock. It has been shown in Figure 7.15, for the world wheat price shock, with A and T equal 2 and 4 years, respectively.

VII.3 Storage Rules

The idea of a food reserve for the buffer between uncertain production and demand has been thousands of years old. Reserve stocks--working stocks and contingency reserve--may serve the purposes to: (1) reduce the danger of food shortages, (2) reduce price variations and protect producers and consumers from unstable markets,[1] (3) stabilize farmers' income and the general economy, and (4) assist economic growth.

A storage rule, which defines how reserve stocks will be achieved in order to achieve a specified objective, should be precise to show how much will be added to or taken from reserve stocks in a given period. The more common storage rules used are to make the level of reserve stocks equal to or a function of:

(1) a constant target quantity

(2) production level

(3) price, loan rate, target stocks

(4) price with upper and lower price bounds

(5) supply, i.e., beginning stocks plus production

---

[1]The benefits of price stability to consumers and/or producers have been a moot subject [W4], [S3].

All the above rules focus on the stabilization of domestic market with no attention to the foreign market. This is because protecting farmers from low prices and reducing government farm subsidy burden have been the main concern in a country like the United States. However, for a food deficit country like Korea, one of the main concerns is to protect the domestic market and economic growth from the fluctuations of the foreign market. Thus the storage rules should reflect the effects of foreign markets. This leads to the consideration of world price level as a major factor in determining the reserve stock level.

Two possible rules were tried in the study: first is a storage rule which accumulates and releases the stock proportional to the amount of price increase above a given normal level, and the second is a storage rule which uses a constant level of price increase above the normal level as a signal to release the below the constant level as a signal to accumulate to specified lower and upper limits. To be more specific, the first storage rule can be obtained by changing the desired grain stock level according to the world grain price such as

$$CSTK_i(t) = WSTK*[1. - (WPR_i(t) - 1.)/2.] + WSTKL \qquad (7.4)$$

$$DGSTK_i(t) = CSTK_i(t)*DMGRN_i(t) \qquad (7.5)$$

$$WPR_i(t) = WPG_i(t)/WPGR_i(t) \qquad (7.6)$$

where

CSTK : desired proportion of grain stock to the total demand

WSTK : maximum proportion of grain stock to the total demand

WPR : ratio of world price to (average) normal price

WSTKL : minimum proportion of grain stock (contingency level) to the total demand

DGSTK : desired grain stock level

DMGRN : demand of grains

WPG : world grain prices

WPGR : normal level of world grain prices

The second storage rule is given by

$$CSTK_i(t) = \begin{cases} WSTK & , \text{ for } WPR_i(t) < CWPR \\ WSTKL & , \text{ for } WPR_i(t) \geq CWPR \end{cases} \quad (7.7)$$

where CWPR is a policy decision parameter to switch the desired stock level to the maximum or minimum limit, and the equations (7.5) and (7.6) are the same as for the first rule. The actual values used for the application to Korean economy will be given in Table 7.2, and the implications of the storage rules will be discussed in the next section.

VII.4 Application to the Open Model of Korea

A scenario of food shock has been used to show the effects of the shock, which is almost identical to the one experienced during 1972 to 1976, with the peak around 1980. This is a hypothetical setting just to illustrate the mechanisms, however, some studies show that this can be realized regarding the global demand and supply trends. [I]

Six cases of major economic variables have been generated and plotted for the normal case; no storage rule with food shock and different storage rules with the food shock. Table 7.2 summarizes the cases with different parameters.

In carrying out the experiment, further assumptions are needed: first, there exist some limitations on the availability of grains in the world market (at whatever price!) which reflect the difficulties to meet the

TABLE 7.2

PARAMETERS FOR THE GRAIN STORAGE RULES

	1	2	3	4	5	6
Food Shock	No	Yes	Yes	Yes	Yes	Yes
WSTK	0.0	0.0	1.0	2.0	1.0	2.0
WSTKL	0.15	0.15	0.05	0.05	0.05	0.05
CWPR	-	-	-	-	1.1	1.1
WPGR	-	120	120	120	120	120

import requirement as long as the world grain price remains high, e.g., government restrictions of grain exporting countries on the commercial grain sales abroad. Secondly, since the importance of wheat imports relative to the imports of rice and barley is dominantly increasing in Korea (Table 7.1), it will be assumed that the only restriction on grain import is on the import of wheat. In other words, only wheat is imported and only its supply is directly influenced by the fluctuations of the world grain market.

Using the previous parameters and assumptions, the paths of GNP, GNP growth rate, debt service ratio, foreign borrowings, domestic grain prices, stock levels of wheat, wheat import, dependency on the grain import, and world price of wheat are obtained as shown in Figures 7.4-7.15.

The real GNP shows no substantial change except slight zigzags in the growth rate with respect to the world price shock. This is partly because of the faster growth the total economy relative to the portion of grain production to the total economy, and partly because of the assumed relatively weak effects (linkages) between the saving rate and income transfer which create insignificant changes in the accumulation of capital.

Froeign indebtedness, expressed in terms of debt service ratio and foreign borrowings, doesn't show significant change between the cases with no additional reserve and with one or two years' reserve. The results therefore imply that no substantial increase in the burden of indebtedness will occur as a result of increasing the reserve level of grain stock.

More clear effects of the shock appear on the domestic grain prices. Due to the nonavailability of wheat in the world market, up to 10 percent shortages of the total wheat needs, the domestic rice, barley, and wheat prices rose substantially to about 1.6 to 1.8 times of the trend levels which are quite high but may be realistic considering the tripling world price shock. All of the grain reserve rules show damping effects of the price increases, however, the higher prices of the rule 3 and 4 in the after-the-peak period shows the inappropriateness of shortage accumulation when the price is still high in the world market even though it is decreasing.

Although rule 6 (two years' wheat consumption reserve with a constant policy decision parameter which will signal the release and accumulation of the stocks) appears to be superior to the others, the lowest price level of storage rule 4 at the peak shock period implies that further damping could be possible by the combination of the two, i.e., release the stocks according to the rule 4 when world price increases, but don't start to accumulate the stock until the world price is lower than a certain level.

Figure 7.11 shows the actual storage levels of wheat which follow the specified storage rules. According to the different desired level of wheat reserve for each storage rule, the import of wheat will change widely from zero to more than twice of the current import level, thus causing the grain import dependency from near zero to 50 percent of total grain needs (utilization) since the most of the grain imports are wheat.

In sum, the model does not show significant effects of the food shock to the economic growth of Korea even without any reserve policies, however, its main effects can be realized as higher domestic grain prices which will affect and spread throughout the total economy with changes in income distribution and higher inflation rates.

Considering the relatively small increase in burden of foreign indebtedness needed to operate a grain reserve and the substantial damping of domestic price increase possible, increasing grain reserve stocks when the world price is low is essential to the stabilization of the total economy. The ideal maximum desired level of grain (wheat) reserve seems to be higher than the two years' total consumption, however, the actual level should be determined by considering the total costs of operating the stocks, cost of building storage, losses, etc., in addition to the benefits. Obviously, the costs of removing the risks for each country will not be necessary if there exist world grain reserve stocks which are enough to dampen any sudden changes in the world supply of grains, thus stabilizing the world grain prices.

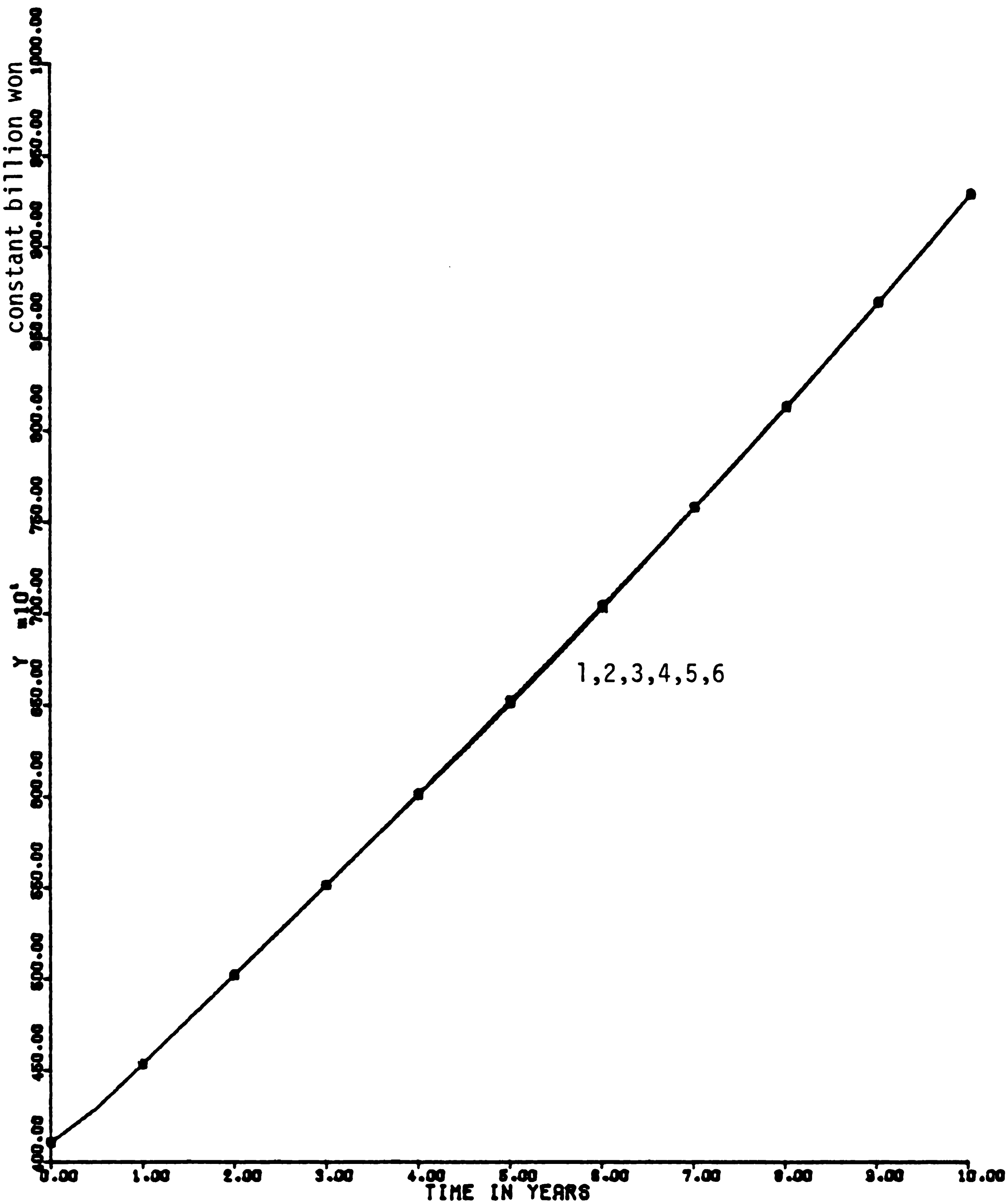

Figure 7.4 Real GNP

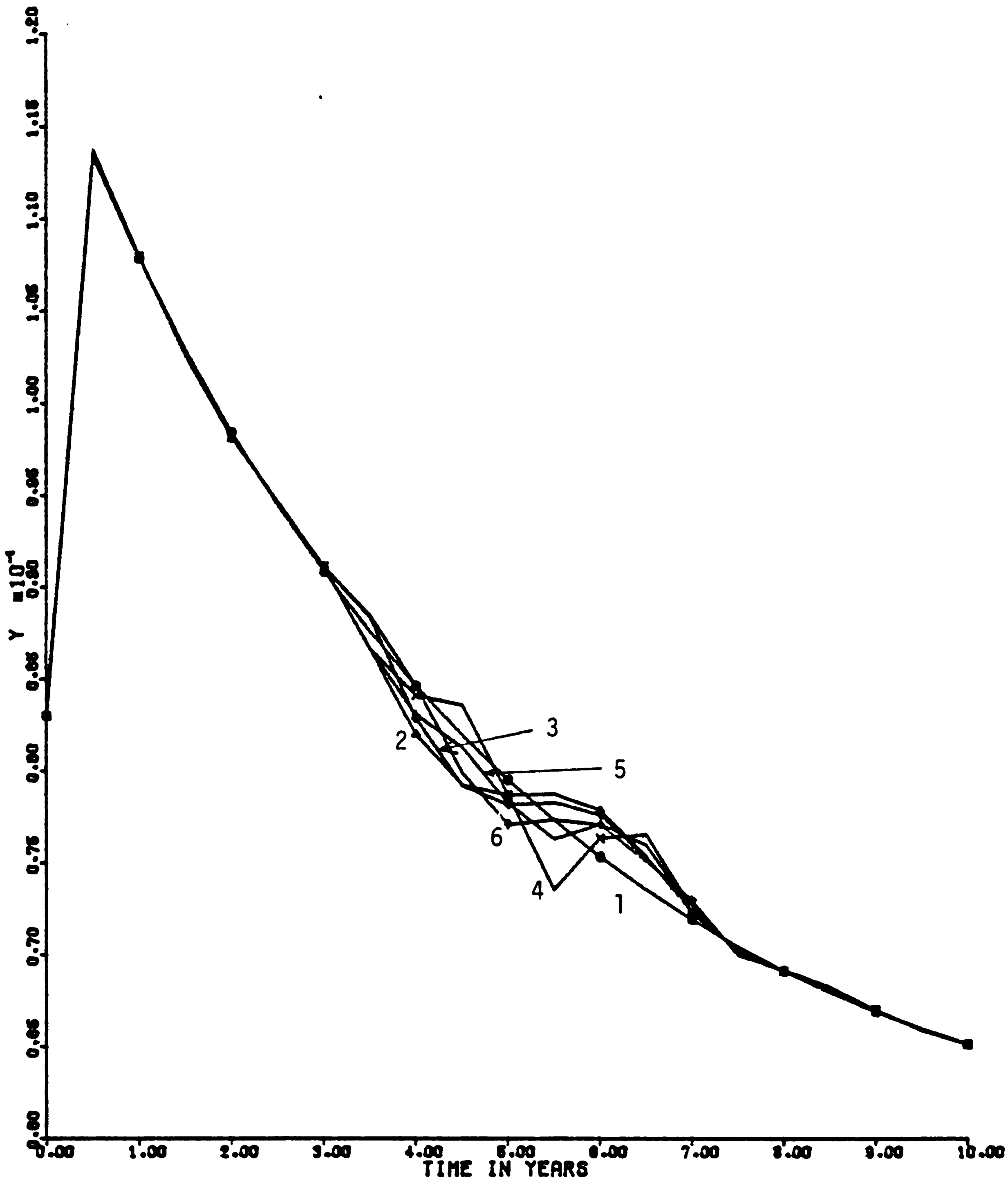

Figure 7.5 Real GNP Growth Rate

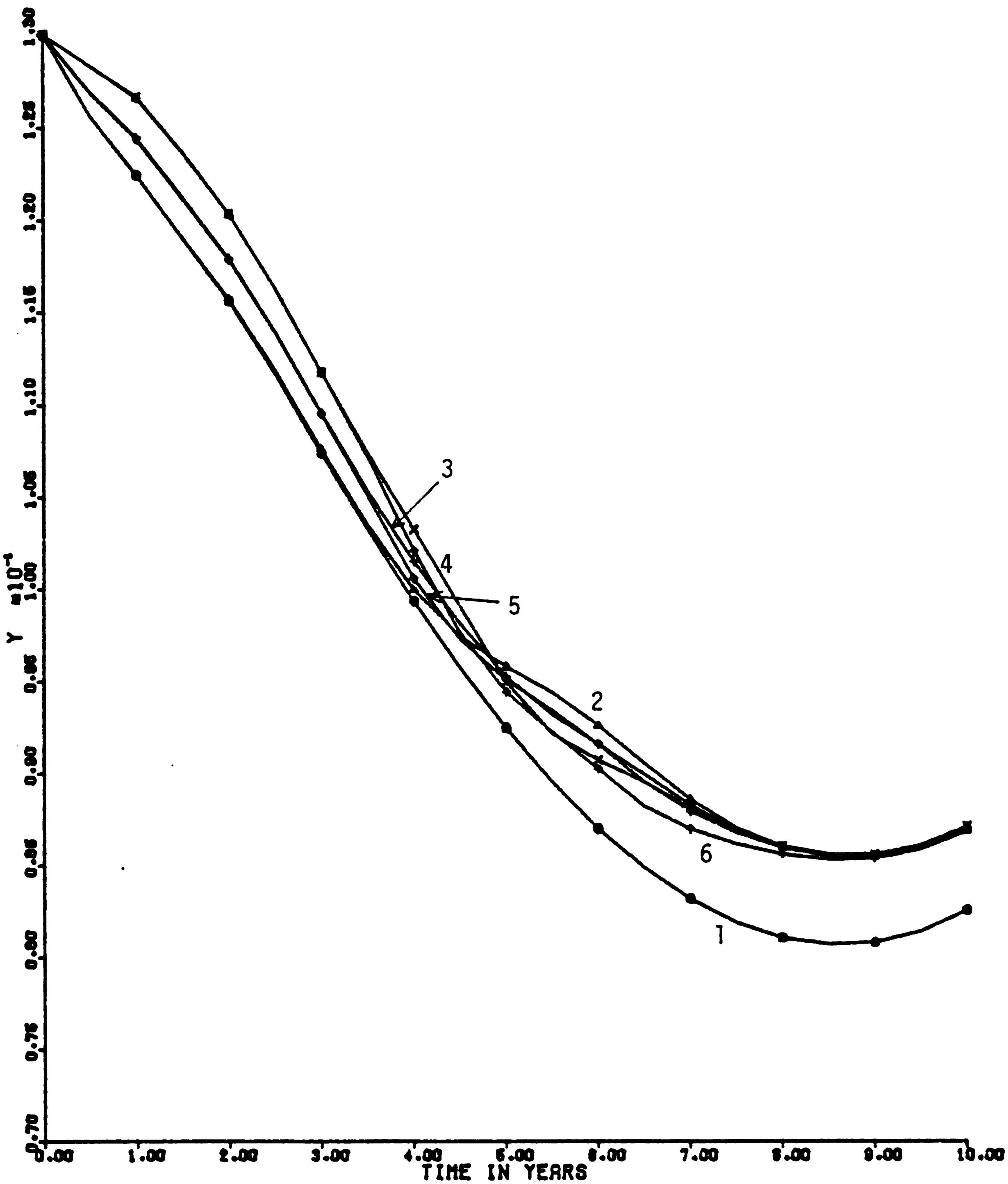

Figure 7.6 Debt Service Ratio

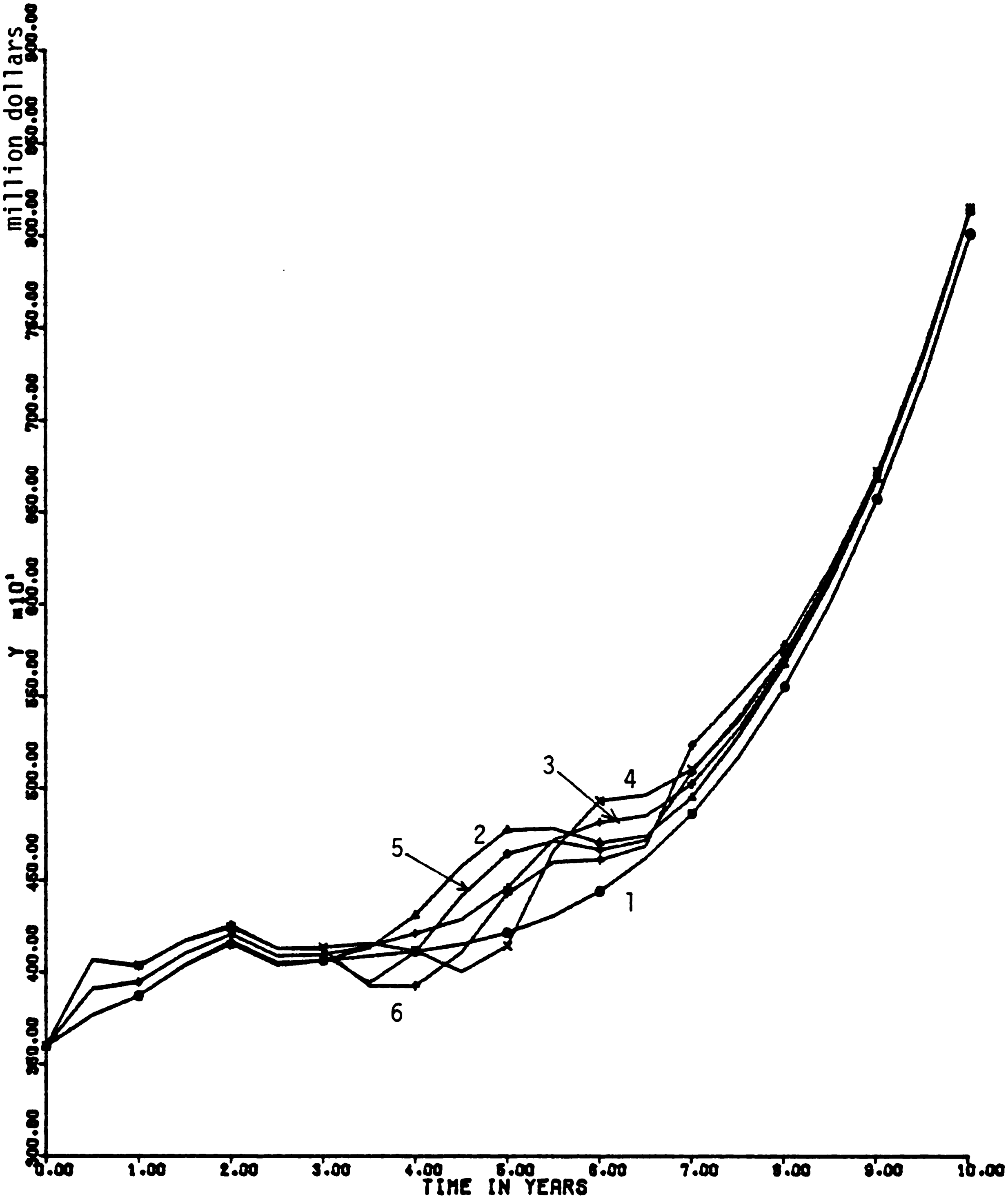

Figure 7.7 Foreign Borrowing

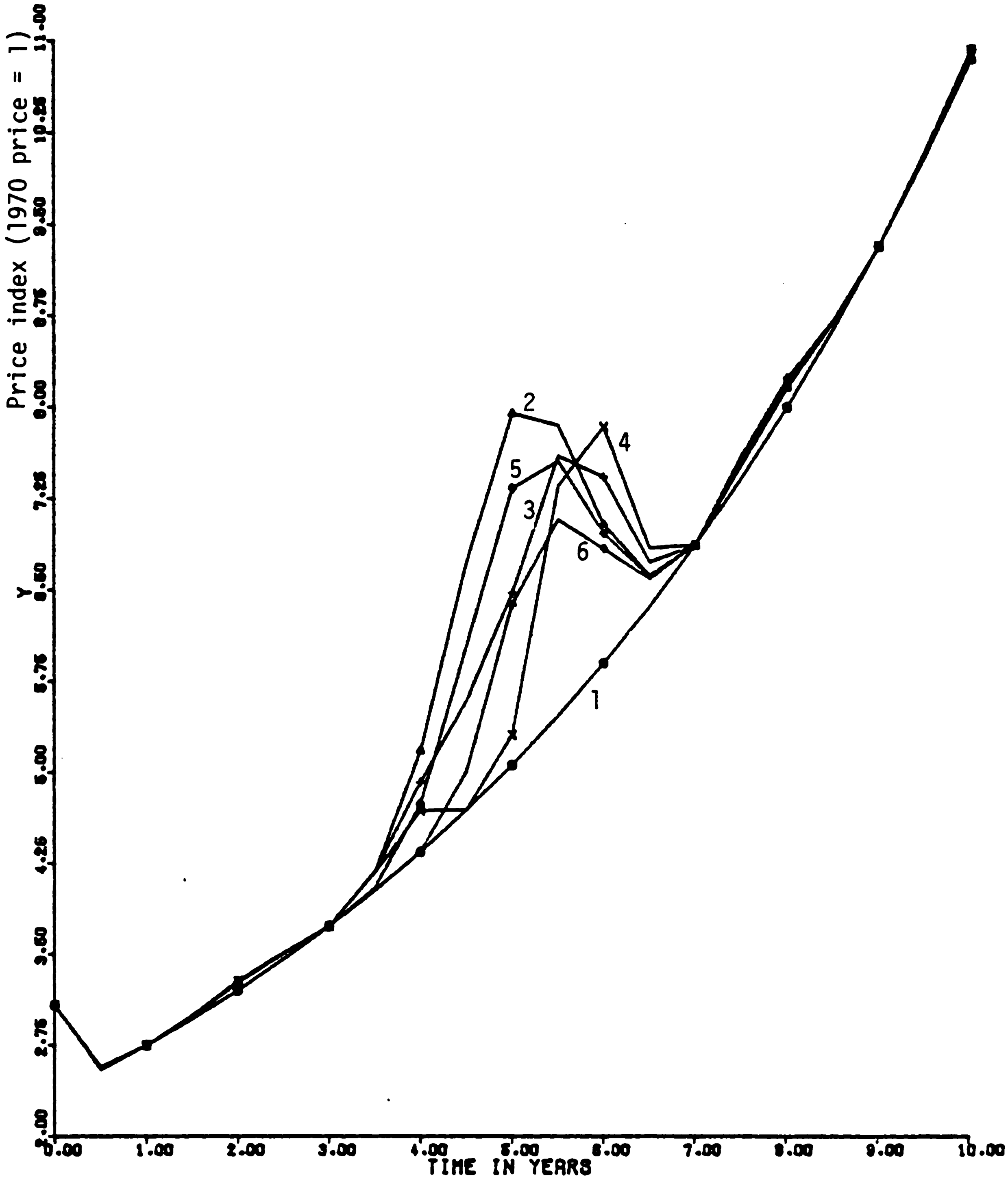

Figure 7.8 Domestic Rice Price

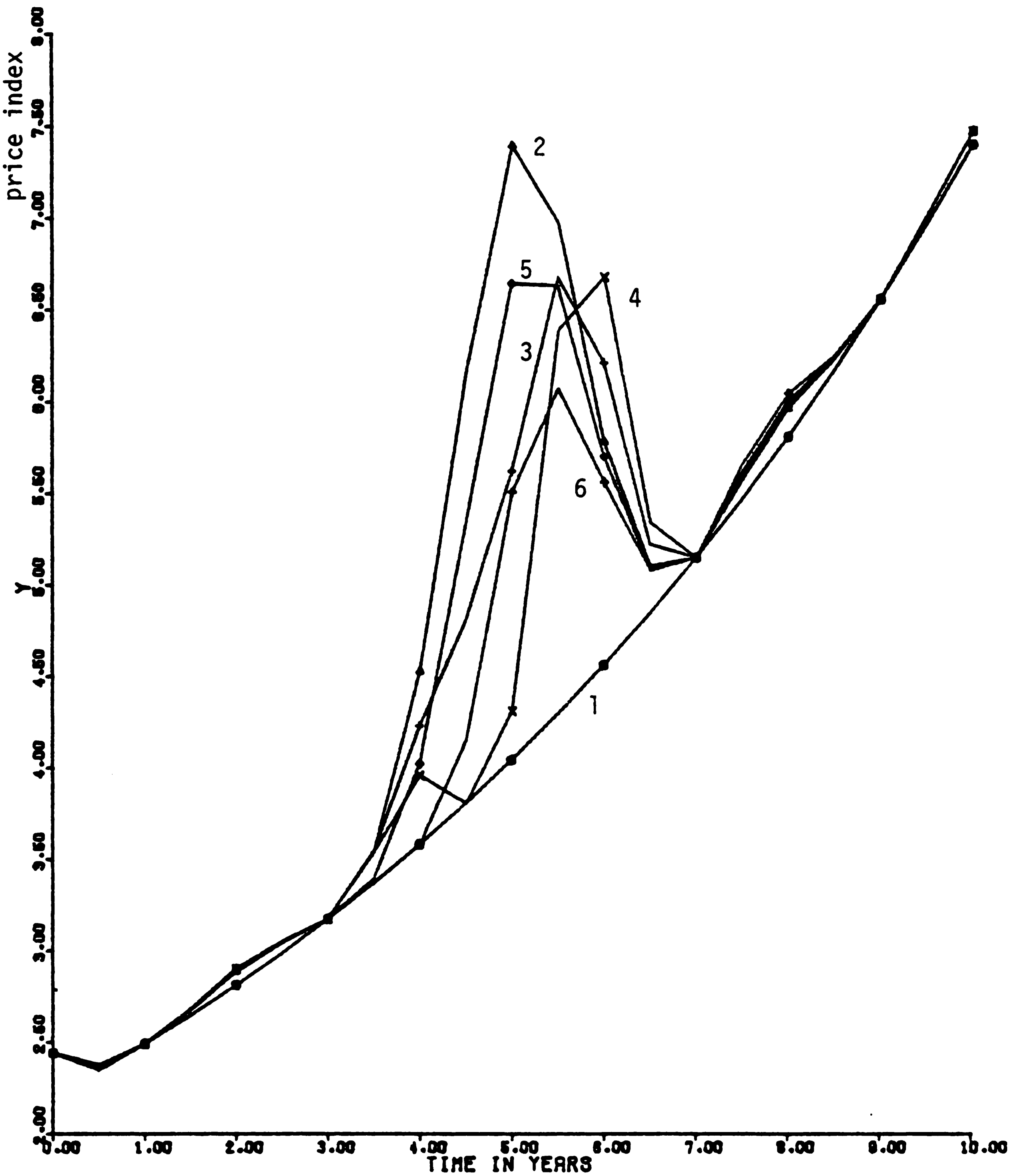

Figure 7.9 Domestic Barley Price

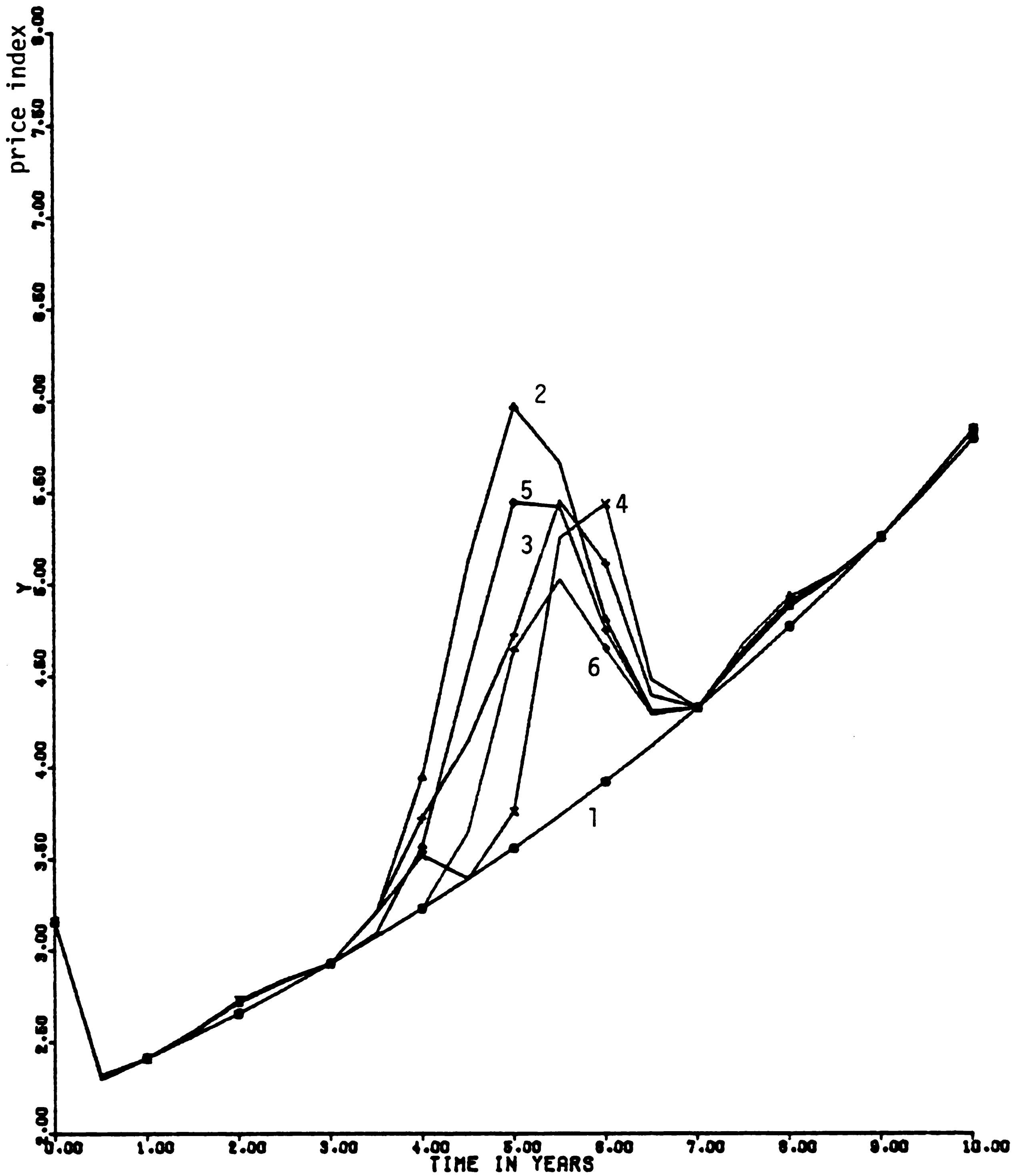

Figure 7.10 Domestic Wheat Price

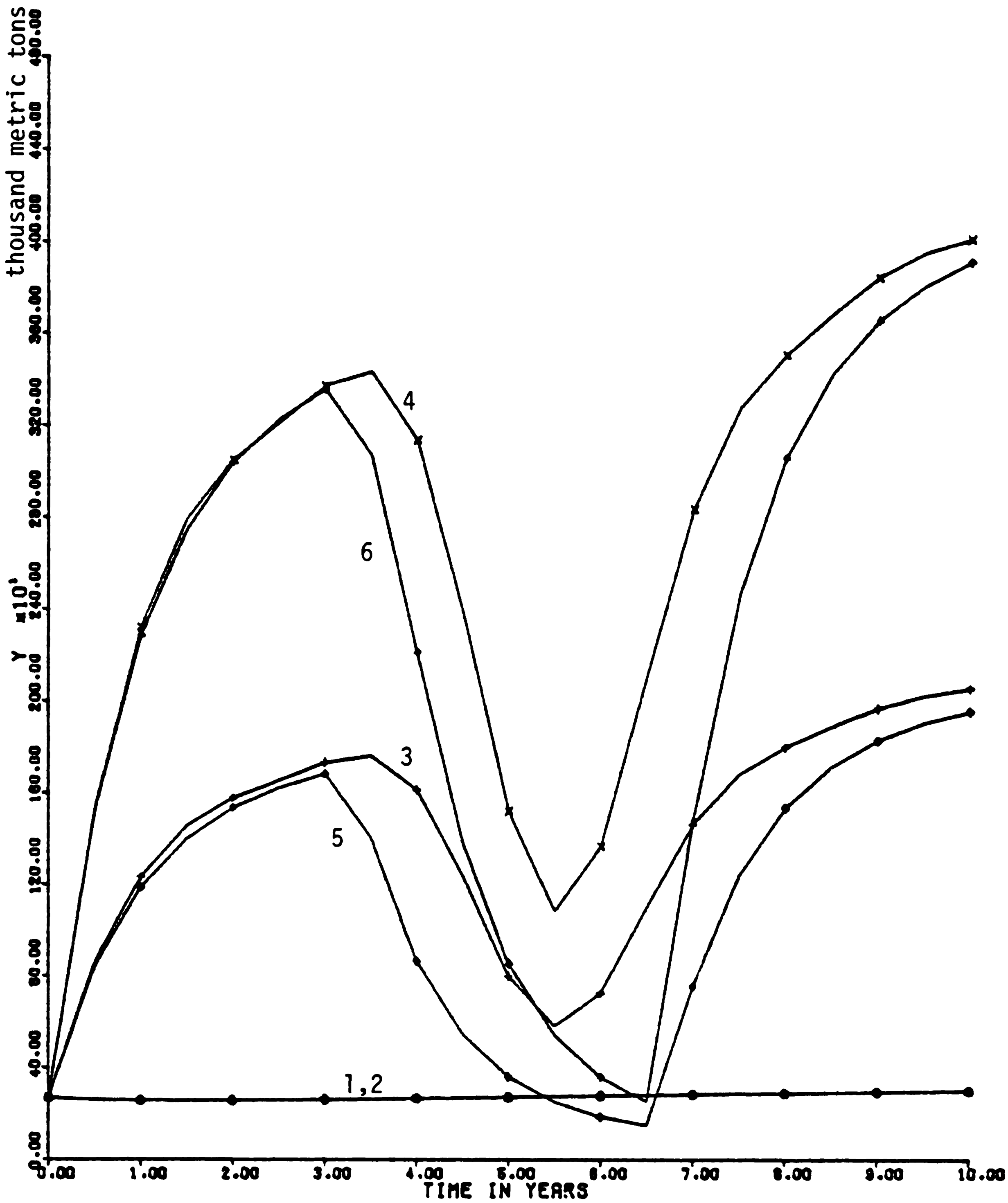

Figure 7.11 Reserve Stock Level of Wheat

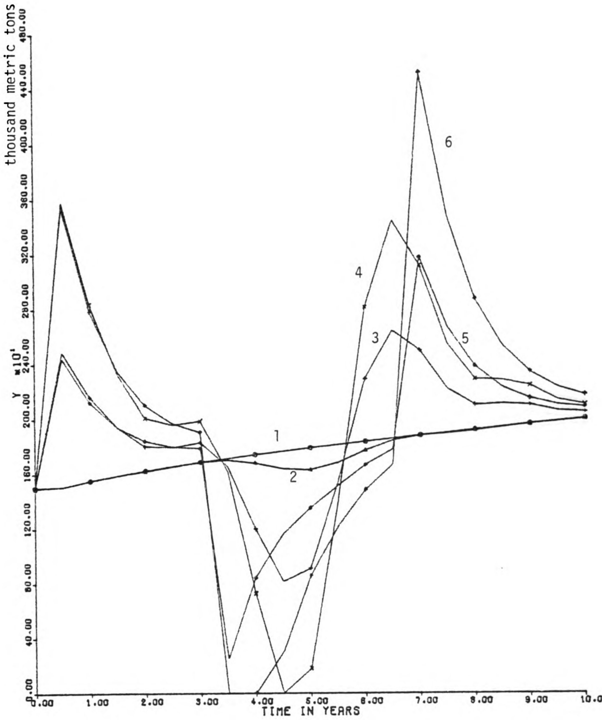

Figure 7.12   Amount of Wheat Import

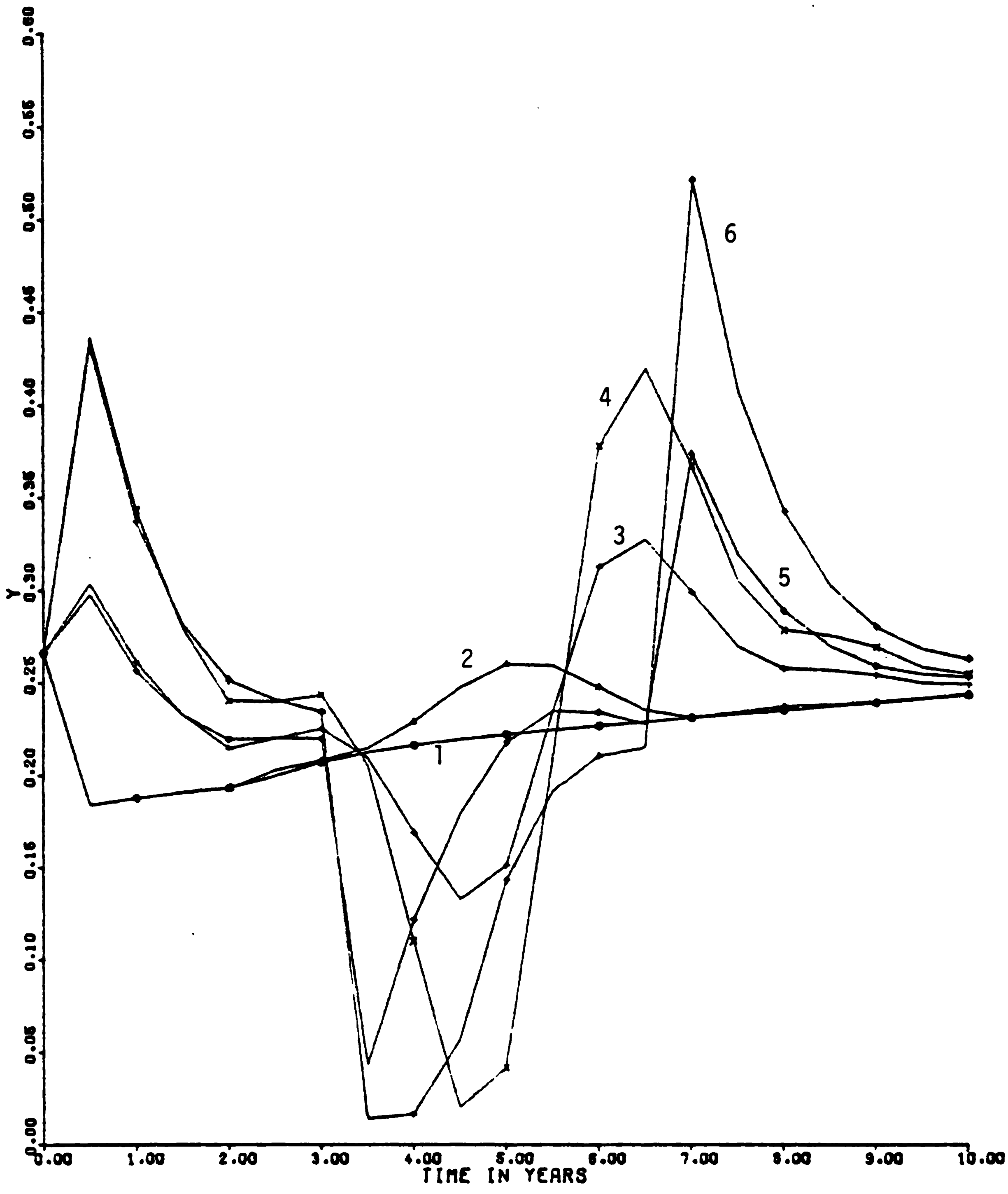

Figure 7.13 Dependency on the Grain Import

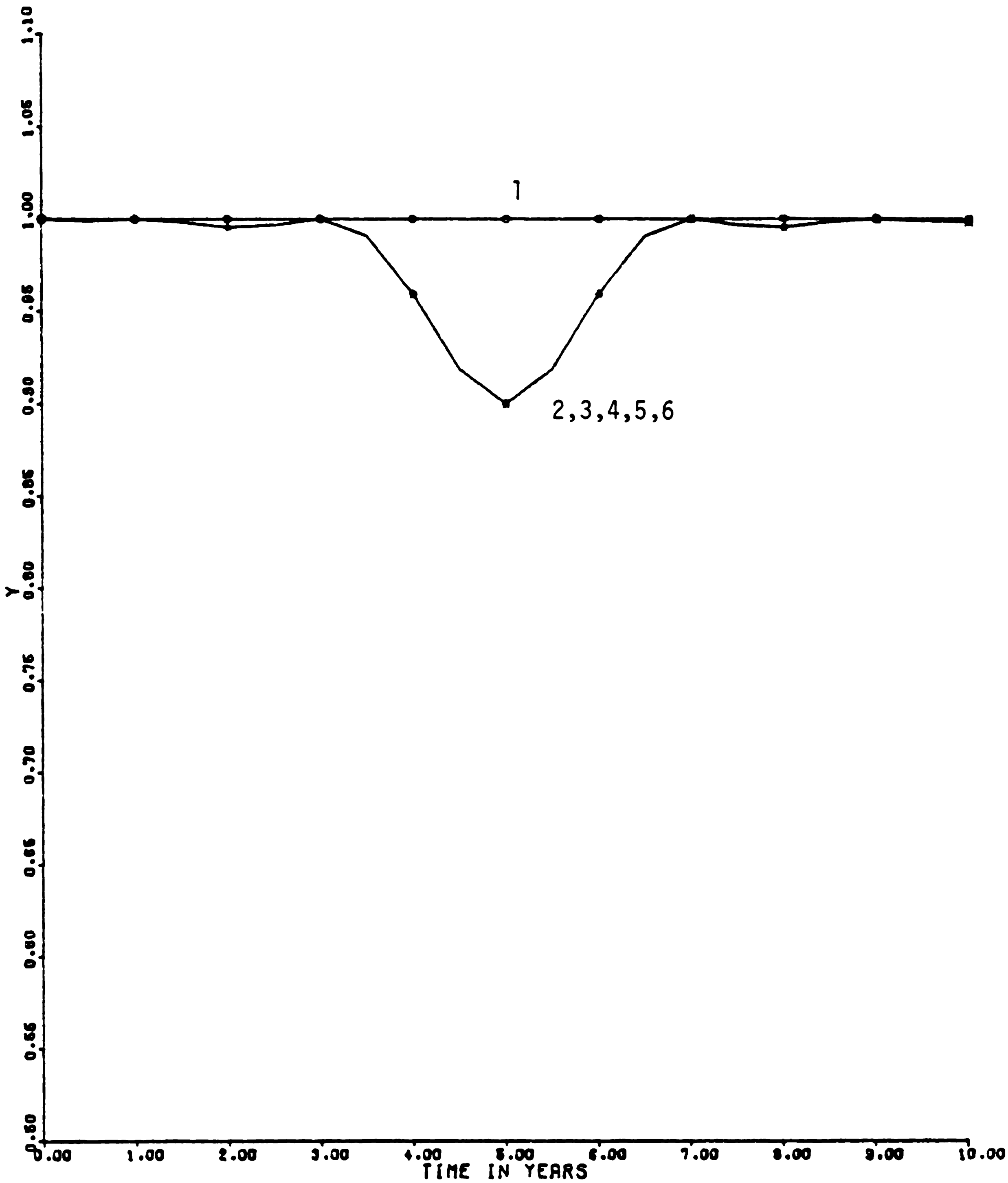

Figure 7.14 Availability of Wheat in World Market

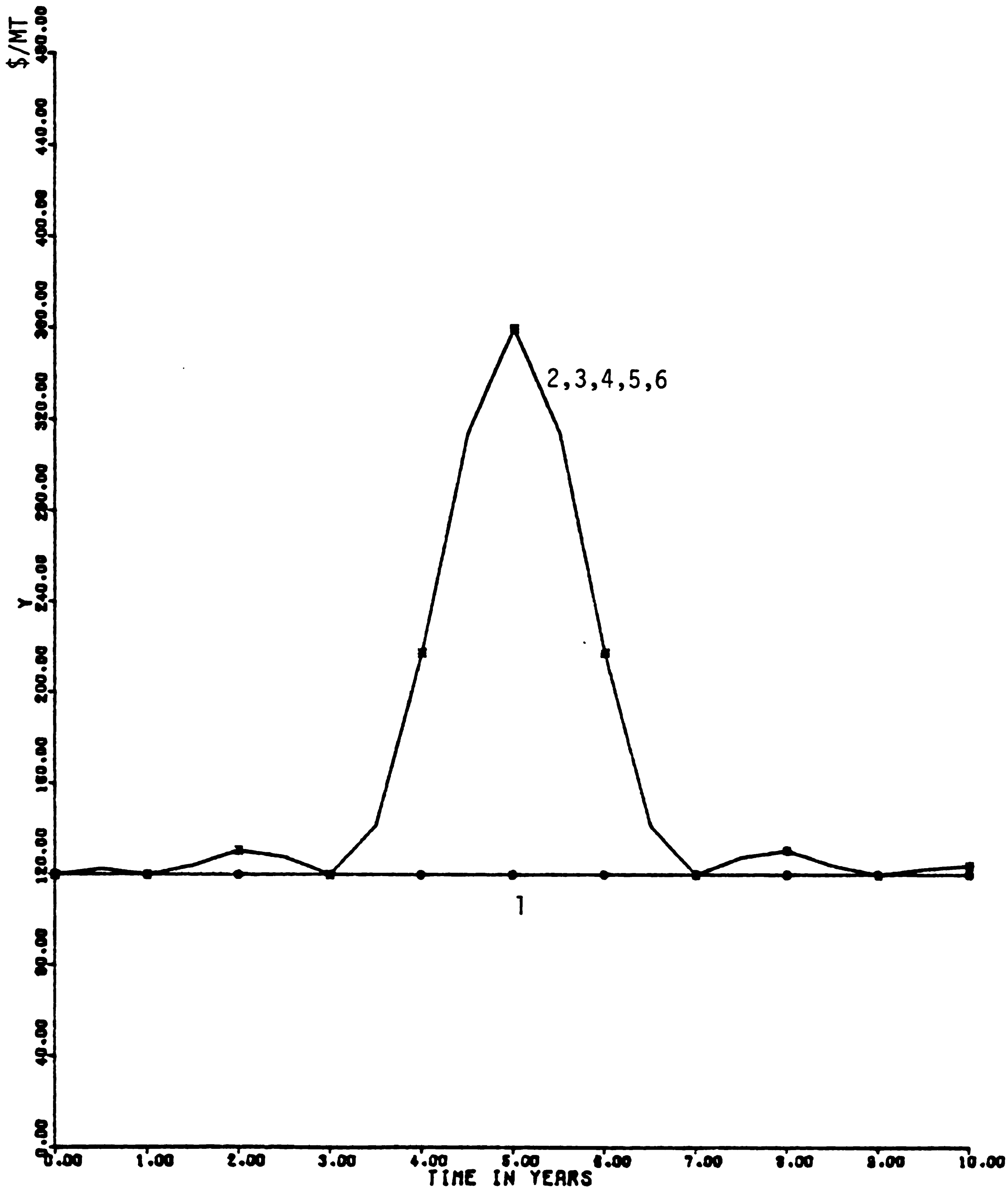

Figure 7.15 World Price of Wheat

CHAPTER VIII

SUMMARY AND CONCLUSIONS

VIII.1 Summary of the Results

The overall process of economic growth as explained in the modern theory of economic growth can be viewed as a closed system of a single system state equation or a set of system state equations with capital per labor being the system state.

Following Meade and Uzawa, the traditional way of treating the two sector growth model is to disaggregate an economy into consumption- and capital-goods sectors. From the technical point of view, it can be seen as a scheme to keep the model simple as possible, i.e., the whole model has only one system state equation with the single capital accumulation process of the capital-goods sector. Hence the basic framework of the one sector model can directly be used with only minor changes. The dual economic growth model dichotomized into agricultural and nonagricultural sectors comprises simultaneous differential equations with different saving rates, different depreciation rates of capital, and different production processes. The resulting system state equations are nonlinear depending on the form of production function used, and thus the analytical solutions are impossible in general.

To find the optimal capital accumulation programs or paths, social welfare function should be explicitly defined. While there is no way to remove the conceptual difficulties regarding social welfare completely, three types of welfare functions (which satisfy desirable properties of a "well-behaved" utility function)--consumption-oriented, capital-oriented, composite--have been tried. The Hamiltonian obtained using a simple linear consumption,being a utility function as in the Meade and Uzawa's model, is linear with respect to the control variables (saving rates) and constitutes a "bang-bang" control problem, and it may contain a singular interval during which the optimal control is indeterminate. To avoid this problem, a utility function with constant exponent (weight) can be used, where the sufficient condition is that (the sum of) constant weight(s) should be less than one with each weight being nonnegative.

For a finite planning horizon as in practical policy making, capital should also be included in the welfare function. The sufficient conditions for the existence of optimal control (saving rate) for the dual economic growth with a composite welfare function are: the constant weights of consumption are nonnegative and the sum of the weights of consumption is less than one regardless of the forms of production function used. (There is no condition on the weights of capital for the existence of optimal control except the conceptual constraints of nonnegativity).

The optimal control problem for the optimal capital accumulation yields a nonlinear two-point boundary problem for which it is generally difficult to obtain even numerical solutions. A special property of the costate variables in the case of the economic growth model, namely, the linearity in the changes between the initial values and the terminal values of the costate variables, enables one to use an efficient iterative method based on the variation of extremals with a simple adjustment scheme. The property comes from the physical meaning of the costate variable in this case, that is, the costate means the social demand price of a unit of investment in terms of a currently foregone unit of consumption (or opportunity cost over the remaining time horizon).

Results of the application of the closed model to the Korean economy for both analysis and control can be summarized as follows: First, the basic structure of economic growth model as given can be used to keep track the transitory paths of economic variables with further modifications for proper (projected) values of saving rates.

Secondly, the special form of the dual economic growth model allows one to investigate the stability properties of the nonlinear system. In other words, the impulse response shows the stability with respect to the impulse input, and thus the overall stability of the system depends mainly on the form of production function. In this regard, the diminishing marginal productivity of the production function is the key to ensure the existence of the stability (or steady state) of the economic growth model.

Thirdly, the changes (switchings) of saving rate from the upper boundary to the lower boundary occur in most cases of the optimal accumulation of capital. The switchings imply the changes in the social decision on the relative importance of the consumption and investment, i.e., whether to consume more and save less or to save more and consume less at a specific time.

Fourth, the heavier the weight of capital relative to consumption, the later the switching occurs. This reflects that the society will build up the productive capacity (capital) as long as possible until everyone agrees to consume more in return for their earlier labours.

Fifth, the relative weights of capital and consumption in the social welfare function for the case of optimal capital accumulation affect the control and state variables substantially, and thus the main decision for the policy maker(s) lies in the determining the relative weights of the social welfare function, i.e., the priorities of the social choices.

The closed model can be converted into the open model by introducing two additional sectors, namely, trade and balance-of-payments. Export is given by exogenous projections using a generalized logistics curve, and import is divided into two parts--import proportional to export and compressible import determined by the marginal propensity to import which can be controlled by import policy. The balance-of-payments component includes total foreign debt and foreign borrowings along with the level of foreign currency reserve. A distributed delay process with accrual (or gain) has been used to model the debt accumulation process. These added complexities do not allow one to use the analytical tools as in the closed model, and thus one has to rely on more general methodologies to handle complex models such as general optimization and simulation.

Obtaining optimal policies in a complex model is often impossible by the normal optimization techniques. A penalty function method which relaxes the constraints as "desired" constraints can be used for this purpose. To lessen the conceptual difficulties in the objective function, alternative objective functions in piecewise quadratic form can also be used which offer a more general framework for policy optimization. Orthogonal representation of policy variables using Fourier series or Legendre polynomials will further simplify the efforts to obtain the dynamic paths of policy.

The results of the application of the open model to the case of Korean economy can be summarized as follows: First, the paths of optimal controls can be generated using optimization routines with respect to alternative functions and constraints which represent the alternative economic policies.

Secondly, different paths of the two instrumental variables, saving rate and import (total desired saving rate and the marginal propensity of import of compressible goods), can result in substantially different paths of economic variables which can be grouped into three categories; abstinence policies, prodigality policies, and policies in between.

Thirdly, examining the foreign indebtedness paths--paths of debt service ratio, foreign borrowing, total debt--suggests that the indebtedness can be handled and reduced to more a sound level while retaining higher economic growth at the same time by the abstinence policy which decreases both the foreign saving rate and import of the compressible goods.

Fourth, it can be observed that grain price control is the most significant factor for the transfer of income from urban to farm in the model with the grain sector, however, more agricultural investment and rural industrialization may provide the sound basis for the equitable distribution of income without substantial changes in the growth rates and foreign indebtedness.

Fifth, with the policy of low grain prices, rural industrialization expressed as a participation ratio should be increased in substantial level to maintain a good income distribution ratio. Increase in agricultural investment will aid in maintaining the level of income distribution without substantial increase in the foreign indebtedness.

To show external effects on the open model, a scenario of food shock--sudden increase in the world grain prices--, similar to the one during 1972 to 1976--has been given to the open model.

The food shock, given as an asymptotically stable shock, showed negligible effect to the real GNP paths because of the decreasing importance of the grain production relative to the total GNP, however, it created high increases in the domestic grain prices and may have a critical effect on the inflation of the whole economy.

As can be expected, a grain reserve policy linked to the world grain prices contributes to dampening the effect of the food shock on the domestic price increases without further increases in foreign indebtedness. The actual level of grain reserve should be determined by considering other factors also such as storage building costs, stock operating costs, losses of storage, etc.

All the above results are based on the assumptions and formulations of the model used. It obviously needs further verification and modeling efforts for application in practical policy making.

VIII.2 Further Research and Recommendations

Being specific instead of broad and general always increases the risk of making untrue statements. So does showing the specific paths of economic variables. (And even the optimal paths of an economy!) On the other hand, being broad and general loses practicality which is important in the design of economic policies. Further efforts on the modifications, extensions, and refinements of the model should be interpreted as an effort to bring the model away from the two extremes to preserve both rationality and practicality.

The Production Function

Aggregate production functions are not conceptually justifiable but have been used for practical convenience.[1] Recent research and disputes among economists on the concepts of capital, labor, and technical progress reflect, in essence, the needs for a more refined and justifiable production function.

The production potential can be determined by (1) shifts along a given production surface (or an isoquant for the case of constant returns to scale) and (2) shifts of the production surface. The former implies input substitutability and the latter implies technical progress.

The original Solow's model assumed that input proportions can vary at any time. It was later termed a "putty-putty"[2] model. If equipment

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[1] R. Solow even said that, "I have never thought of the macroeconomic production function as a rigorously justifiable concept. In my mind it is either an illuminating parable, or else a mere device for handling data, to be used as long as it gives good empirical results, and to be abandoned as soon as it doesn't, or as soon as something better comes along," [S12].

[2] "putty" stands for capital in a malleable state which can be made into equipment requiring variable capital labor ratios; "clay" stands for capital in a "hardened" state with a constant capital labor ratio.

can have various input proportions (ex ante) and become immutable once the equipment has been set up (ex post), the production process is said to be putty-clay. Another possible case of input substitutability is clay-clay where one technique will always be chosen irrespective of factor prices.

Technical progress shifts isoquants inwardly, i.e., the same amount of output can be produced with less inputs. It can be exogenous or endogenous according to the way the technical progress enters into the production function, and can be embodied or disembodied according to whether the technical progress is due to any changes in the factor inputs or not. Neutrality and nonneutrality of technical progress was defined to determine the contribution of specific factor of production to total technical progress, i.e., to determine the direction of the shifts of isoquants. Hicks neutrality assumes equal contributions of capital and labor to the total progress--output-augmenting. Harrod neutrality means technical progress with only contribution from labor (labor-augmenting), and Solow neutrality is for the shift of isoquants along labor axis (capital-augmenting).

These considerations on the refinements of aggregate production function either by redefinition of capital and/or labor or by introducing "pure" technical progress together with further disaggregation of an economy (multi-sector model) will undoubtedly give more insights on the overall economic growth.

Saving and Investment

Another area of further research relates to the investment decisions. Saving behavior reflecting the distribution of income as given in the open model were largely determined by preassigned values of expected saving rates and parameters for the effects of income distribution. Saving rates out of wages and profits of nonagricultural saving were also given as predetermined. Efforts to refine the basic framework and to gather more information about these are needed for better representation of real behavior.

Investment to agriculture and nonagriculture has been determined by an investment-allocation parameter. As mentioned earlier, the classical economic optimization deals mainly with determining the investment-allocation parameter, i.e., determining "how to allocate the limited resources to different activities in order to achieve a certain objective." In the growth model, it usually is given as a constant to determine the optimum saving rates. Including the allocation parameter as another control variable (along with the saving rates) in the closed model will complicate the system state equations of (3.25) as follows:

$$\begin{bmatrix} \dot{k}_1(t) \\ \dot{k}_2(t) \end{bmatrix} = \begin{bmatrix} -u_1-g_1(t) & 0 \\ 0 & -u_2-g_2(t) \end{bmatrix} \begin{bmatrix} k_1(t) \\ k_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ y_1(t)/l_2(t) & y_2(t)/l_2(t) \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} + \begin{bmatrix} y_1(t)/l_1(t) & y_2(t)/l_1(t) \\ -y_1(t)/l_2(t) & -y_2(t)/l_2(t) \end{bmatrix} \begin{bmatrix} i(t)\ s_1(t) \\ i(t)\ s_2(t) \end{bmatrix} \tag{8.1}$$

where the last term includes multiplicative terms of control (input) variables, which prevents obtaining the optimal solution in general/ Further modifications of the model or theoretical developments may lead to obtaining the optimum saving rates and the optimum investment-allocation simultaneously.

Modifications of Open Model

Most of the modifications which aided to convert the closed model into the open model can be the subjects of further refinements and verifications. Trade and foreign capital movements are highly uncertain and may be modified to include internal effects of other economic variables once more convincing relationships are identified. Labor migration and the closed loop mechanism of income distribution are too simplified to describe complexities in the actual processes. Equitable distribution of income does not necessarily mean equitable distribution of utility; people move for many reasons other than money--educational opportunities, pollution and ecological problems, conveniences in transportation and other facilities, tradition and cultural influences, etc.--which make a model far more complicated, and will need behavioral assumptions for further refinements. Tax policy have not been explored in detail and will deserve further work to explain the consequences of alternative tax policies and to design the feasible tax policies. Exogenously given price mechanisms by time trend may also be refined to reflect inflation effects.

Uncertainty

Both the closed and the open models as given are deterministic; there are no uncertainties and disturbances (noise) introduced in the models. Considering the highly uncertain economic activities and their interrelationships--such as savings behavior, foreign capital flows, labor migration, world financial market conditions, world grain prices, income distribution and its effect to other economic variables, etc.--, it will be a logical step for further study to include uncertainties in the form of specific probability distribution functions.

Stochastic models, which introduce disturbances, can not be solved analytically in general unless drastic simplifications are made. Stochastic control may be applied to a highly simplified model for the solution of optimal growth. Generally, however, Monte Carlo techniques will provide insights into the behavior of models with statistical disturbances.

External Shocks

The food shock given in the model is a hypothetical scenario. Additional modificaitons will certainly be needed to investigate more rigorously the effects of food shocks on the domestic economy. The areas which need more work are: domestic grain demand and production functions, world grain production, consumption, and market conditions, foreign aid trends and prospects of a world grain reserve, and effects of grain prices on the inflation of the total economy.

Investigation of the effects of other shocks, such as an energy crisis, may also be possible by adding relevant linkages using a framework similar to the one used in the case of food shock.

VIII.3 Conclusions

The research attempted to describe the transitory (dynamic) behavior of economic growth and to design economic policies to fulfill the economic objectives during a certain time period using system theory. It has demonstrated the practical usefulness of the modern theory of economic growth by showing the trajectories of economic state and control variables, and optimal trajectories for the case of the Korean economy.

No theory or model is perfect. This is especially true for the social systems where no precise information is available unlike physical or engineering systems where relatively precise information can be gathered by experimentation. Due to this imperfection, a model or models can only provide decision makers with a limited set of information that they must weigh against many other (quantifiable or unquantifiable) factors. The point will be illustrated further by the payoff matrix[1] of Korean economy with respect to alternative objective functions (performance indices) given in Table 8.1. The payoffs indicate the values of the social welfare expressed as discounted sum of weighted capital and consumption over the planning horizon with alternative objective functions and alternative policies (and models). Selecting the largest value (the highest social welfare) is meaningless, since the value may be realized if both the (uncertain) objective and the model are true. In other words, one still has to weigh the uncertainties and thus needs a certain bases--decision criteron such as Laplace

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[1]The basic idea of the objective payoff matrix is in [C3]. However, the alternative objective functions are also considered in this case.

TABLE 8.1

COMPARISONS OF PERFORMANCE INDICES:†
UTILITY PAYOFF MATRIX

	CLOSED MODEL	OPEN MODEL I	II	III	IV	V	VI
$U_1(t)$	5.2489	3.7488	3.6973	3.6718	3.7255	3.6206	3.5289
$U_2(t)$	4.1079	2.9753	2.9347	2.8944	3.0177	2.8112	2.6318
$U_3(t)$	3.3035	2.4035	2.3727	2.3285	2.4914	2.2090	1.9784
$U_4(t)$	2.6949	1.9750	1.9528	1.9107	2.0939	1.7557	1.4986
$U_5(t)$	2.2334	1.6493	1.6347	1.5974	1.7887	1.4108	1.1435
$U_6(t)$	1.8129	1.3983	1.3902	1.3588	1.5507	1.1452	0.8788

†The performance indices are:

$$U = \int_0^{10} [c_1(t)^{a_1}\, c_2(t)^{a_2}\, k_1(t)^{b_1}\, k_2(t)^{b_2}]\, e^{-rt}\, dt$$

and

$U_1(t)$: $a_1 = a_2 = 0.2$, $b_1 = b_2 = 0.0$ $U_2(t)$: $a_1 = a_2 = 0.2$, $b_1 = b_2 = 0.2$

$U_3(t)$: " " $= 0.4$ $U_4(t)$: " " $= 0.6$

$U_5(t)$: " " $= 0.8$ $U_6(t)$: " " $= 1.0$

$r = 0.05$

criterion, max-min (pessimist) criterion, max-max (optimist) criterion, min-max regret criterion, Hurwicz criterion, Savage criterion, ect. [W5], [S2]-- for the final choice of policies. This reflects incompleteness of decision making which may require the unending process of modification and refinement.

With these and other possible limitations in mind, the contributions of the thesis will be summarized as follows:

(1) the development of the modified dual economic growth model with practical relevance, and determining transitory behaviors of economic growth

(2) illustrating the framework for designing (optimal) policies and the improvement of optimization in complex systems

(3) providing a framework for the analysis of open economy, and for investigating the external effects on overall economic growth

(4) most of all, the attempt can be interpreted as an effort to bridge the gap between the system theory and the economics, thus to provide more insights in understanding the dynamics of economic systems.

APPENDICES

APPENDIX A

BASIC DATA

TABLE A.1

GROSS NATIONAL PRODUCTION

Year	CURRENT GNP	CURRENT AG	CURRENT NONAG	CONSTANT GNP	CONSTANT AG	CONSTANT NONAG
1960	246.34	90.70	155.64	1129.72	466.57	663.15
1961	297.08	119.49	177.59	1184.48	522.20	662.28
1962	348.89	127.71	221.18	1220.98	492.17	728.81
1963	488.54	206.02	282.52	1328.31	532.05	796.26
1964	700.20	321.00	379.20	1441.99	614.59	827.40
1965	805.32	309.12	496.20	1529.70	602.65	927.05
1966	1032.45	365.15	667.30	1719.18	667.91	1051.27
1967	1269.95	399.26	870.69	1853.01	634.78	1281.23
1968	1598.04	455.18	1142.86	2087.12	650.08	1437.04
1969	2081.52	597.46	1484.06	2400.49	731.48	1669.01
1970	2589.26	724.59	1864.67	2589.26	724.59	1864.67
1971	3151.55	910.74	2240.81	2826.82	748.46	2078.36
1972	3860.00	1094.62	2765.38	3023.63	760.93	2262.70
1973	4928.67	1280.15	3648.52	3522.72	802.95	2716.77
1974	6779.11	1717.69	5061.42	3825.50	847.56	2977.94
1975	9051.78	2325.21	6726.57	4107.71	899.95	3207.76

Source: Bank of Korea, Economic Statistics Yearbook.
Unit : Billion Won.

TABLE A.2

POPULATION

Year	Total	Total Employed	Employed Agriculture	Employed Non-Agriculture	Farm Population	Urban Population
1964	27678	7799	4825	2974	15553	12125
1965	28327	8206	4810	3396	15812	12515
1966	29160	8423	4876	3547	15781	16078
1967	29541	8717	4811	3906	16078	13463
1968	30171	9155	4801	4354	15908	14263
1969	30738	9414	4825	4589	15589	15149
1970	31435	9745	4916	4829	14422	17013
1971	31828	10066	4876	5190	14712	17116
1972	32360	10559	5346	5213	14677	17683
1973	32905	11139	5569	5570	14645	18260
1974	33459	11586	5584	6002	13459	20000

Source: Economic Planning Board, Korean Statistical Yearbook.
KASS projections on the ratio of farm and non-farm populations is used in calculating the farm and urban populations [R6].

Unit: in thousand persons.

TABLE A.3
CAPITAL FORMATION

Year	Gross Domestic Capital Formation	Fixed Capital Formation	Agriculture	Non-Agriculture	Domestic Saving	Foreign Saving
1960	26.80	97.01	11.10	85.91	3.54	20.99
1961	38.79	104.45	14.27	90.18	11.58	25.29
1962	45.47	133.38	11.24	122.14	5.48	37.95
1963	90.26	167.79	17.44	150.35	30.49	52.36
1964	102.24	155.12	19.25	135.87	51.94	49.13
1965	121.98	195.40	23.74	171.66	60.5	51.53
1966	224.48	294.28	35.08	259.20	122.45	87.63
1967	280.97	358.63	31.48	327.15	151.81	112.86
1968	427.87	498.30	35.30	463.00	218.32	184.33
1969	620.70	639.23	39.91	599.32	365.18	229.02
1970	704.66	650.20	52.37	597.83	423.20	249.31
1971	805.35	680.63	55.79	624.84	458.27	354.00
1972	805.48	659.14	71.97	587.17	577.31	215.03
1973	1292.29	851.89	75.54	776.35	1087.77	198.92
1974	2125.88	939.07	98.42	840.65	1302.88	917.72
1975	2459.78	1020.52	123.22	897.30	1636.18	1028.49

Source: Bank of Korea, Economic Statistics Yearbook
Unit: in current billion won for gross capital formation, domestic and foreign savings, and in 1970 constant billion won for others.

APPENDIX B

ESTIMATION OF CAPITAL STOCK

Since very little information has been known about the gross capital stock in Korea, the capital stock was estimated assuming specific rules of capital depreciation.[1]

Using the available data--capital consumption (provisions for the consumption of fixed capital) and fixed capital investments for each sectors--, the problem was formulated with the objective to estimate the time series of the capital stocks in such a way to minimize the errors between the estimated capital consumption and the actual capital consumption.

Assuming linear depreciation rule with constant life spans of capital stocks of each sectors--agriculture and nonagriculture--, the capital consumption for the t-th year can be expressed as

$$C_{it} = \frac{1}{L_i} S_{it} \qquad \text{(B.1)}$$

$$S_{it} = [S_{it-1}(1 - \frac{1}{L_i}) + I_{it-1}] \, DF_{it,t-1} \qquad \text{(B.2)}$$

where

C_{it} : consumption of i-th sector at t-th year in current won

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[1]This is similar to Kuznets' method to estimate the capital stock of U.S.A. Kuznets used the life spans of 13 years for producer's durables and 50 years for construction [K14], p. 66.

L_i : average life span

S_{it} : capital stock

I_{it} : investment of fixed capital

$DF_{it,t-1}$: capital deflator of the i-th sector from year t-1 to t

Then, the total fixed capital consumption, C_t, becomes

$$C_t = C_{1t} + C_{2t} \tag{B.3}$$

Available data are C_t, I_{it}, $DF_{it,t-1}$, thus the problem is to find L_i and the series of S_{it} which give the best fit to C_t. Optimization routine COMPLEX was used to minimize the objective function

$$U = \sum_{t=1960}^{1975} [(S_{1t}/L_1 + S_{2t}/L_2 - C_t)/C_t]^2 \tag{B.4}$$

which reflects the proportional error of the total capital consumption for each year. The explicit (and implicit) constraints can be set large enough to cover the intervals of conceptual maximum and minimum for each variables.

The results obtained were;[1]

$S_1 = 99.2$, $S_2 = 118.67$, $L_1 = 25.48$, $L_2 = 13.9$

$U = 0.090773028$.

Figure B.1 shows the estimated capital consumption and the actual capital consumption. Time series of the capital stock are given in Table B.1.

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[1]Even though the disaggregation criteria for the sectors are different, the estimated average life span for the nonagricultural capital--13.9 years--is close to the 13 years used by Kuznets for the producer's duarables and the 14 years used by Behrens, et al. for the average life span of industrial capital [M7], pp. 221-222.

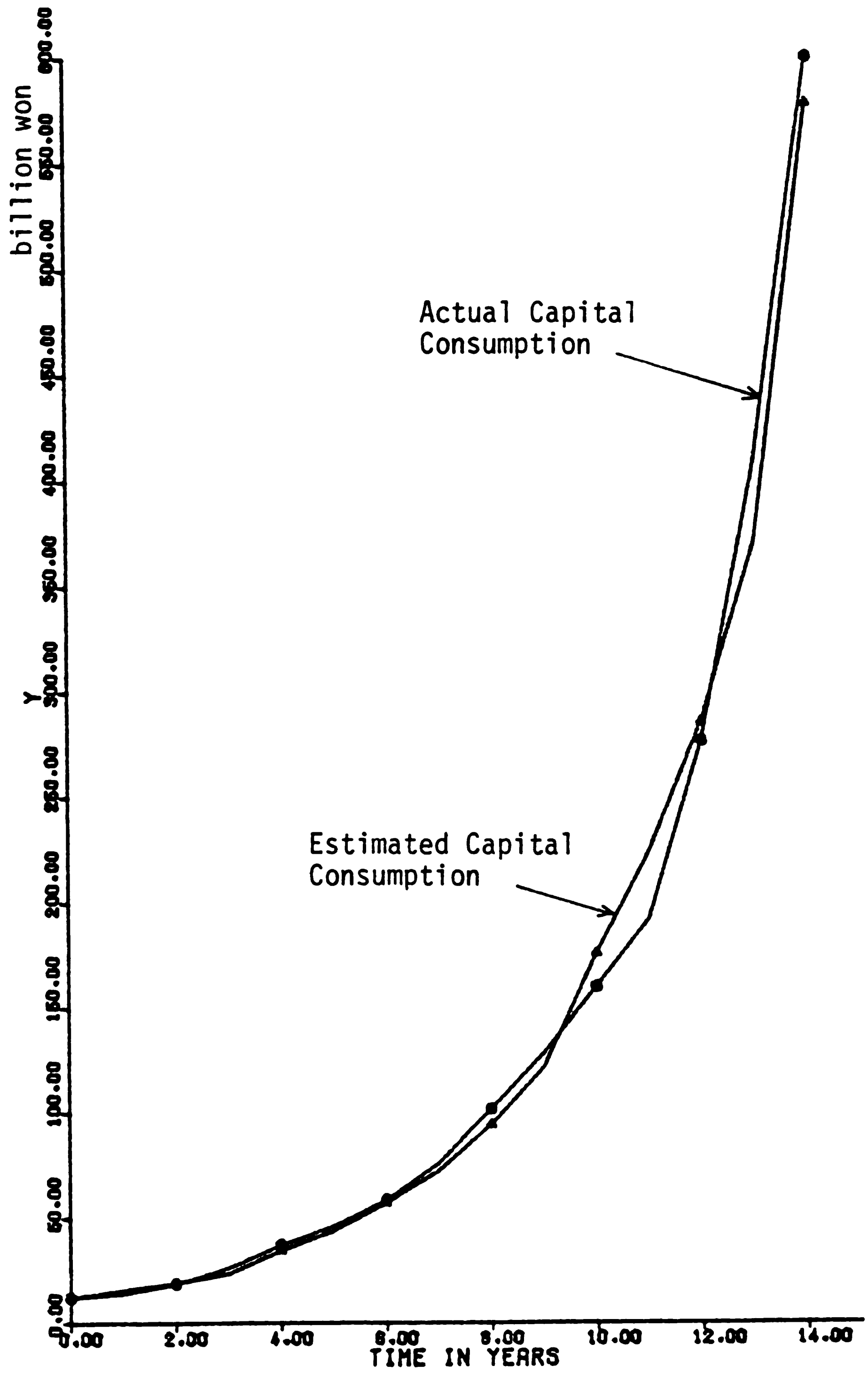

Figure B.1 Fixed Capital Consumption

TABLE B.1

CAPITAL STOCK

Year	CURRENT AG	CURRENT NONAG	CONSTANT AG	CONSTANT NONAG
1960	99.92	118.67	326.54	441.15
1961	109.83	162.00	324.94	495.41
1962	117.21	200.69	326.49	549.84
1963	130.61	256.77	324.90	632.44
1964	165.13	389.28	329.60	737.27
1965	193.49	504.42	335.92	820.20
1966	245.30	661.31	346.47	932.74
1967	271.92	858.34	367.96	1124.95
1968	335.34	1128.57	385.01	1371.29
1969	354.95	1499.47	405.19	1735.50
1970	429.21	2210.15	429.21	2210.15
1971	496.35	2839.71	464.75	2648.98
1972	592.76	3650.87	502.34	3083.51
1973	737.11	4749.13	554.64	3448.90
1974	1173.05	7405.41	608.43	3977.13
1975	1551.74	10209.72	682.98	4531.61

Unit: Billion Won.

APPENDIX C

COMPUTER PROGRAM†

```
      PROGRAM GROWTH(INPUT,OUTPUT)
      REAL K,L,LABR,NVT,NIVT
      COMMON/MAIN/T,DT,K(2),L(2),LABR,PROD(3),GG(3),PG(3),PI,DEF(2),
     1            GNP,GNPP(2),RGNY,BOD,SAVEE(3),FSAVED,DMGRN(3),
     2            PGX(3),GGIMP(3),GGIMPT(3),EXPO,NVT,NIVT,TIMPB,DCAP
     3            ,REXCH,DEFF,CONPL,SAVE,DIM,PINA,SUPGRN(3)
     4            ,PIG,WPGR(3),WPG(3),POP,DP(3),TDF,A(3),B(3,3),BI(3,3)
     5            ,C(3),IDCRL,POPL(2)
      COMMON/PRDBLC/GNPC,GNPCC(2),GLR(2),GNPRT,RGNP,GNPX,PAR
      COMMON/SAVBLC/RSAVE(3),DSAVR,TDSR,RFSV,TSAVE,CLR(2),COR(2)
     1            ,SRA,SRB,SRC
      COMMON/CSMBLC/CONS(2),CL(2),CONR,CONPX
      COMMON/GRNBLC/GSTK(3),DPGIMP,CGM(3)
      COMMON/TRDBLC/TMPA,TMPB,TMPC
      COMMON/BALBLC/TDBT,BOT,DSR,FBR,BOP
      DIMENSION U(6),A1(6),A2(6),B1(6),B2(6)
      DATA A/-2.19,-2.62,-3.85/
      DATA B/-0.17,0.09,0.11,
     1       0.05,-0.17,0.09,
     2       0.09,0.12,-0.4/
      DATA BI/-11.78897301,-10.14014639,-5.523495466,
     1        -5.791426215,-11.97444353,-4.286892003,
     2        -4.385942292,-5.873866447,-5.028854081/
      DATA C/0.03,-0.04,0.2/
      DATA TMPA,TMPB,TMPC/0.18995,-0.01068,0.00306/
      DATA SRA,SRB,SRC/0.2999,-0.03028,0.01006/
      DATA A1/0.2,0.2,0.2,0.2,0.2,0.2/
      DATA A2/0.2,0.2,0.2,0.2,0.2,0.2/
      DATA B1/0.0,0.2,0.4,0.6,0.8,1.0/
      DATA B2/0.0,0.2,0.4,0.6,0.8,1.0/
C --- INITIALIZATION
      PRINT 149
  149 FORMAT(≠1≠///50X,≠* OUTPUT *≠/)
      DT=0.1
      IEND=10.
      CONPX=0.281
C --- INITIAL CONDITIONS
      DMGRN(1)=4423.
      DMGRN(2)=1719.
      DMGRN(3)=1770.
      GSTK(1)=992.
      GSTK(2)=1466.
      GSTK(3)=270.
      PG(1)=3.05
      PG(2)=2.44
      PG(3)=3.16
      PGX(1)=2.415
      PGX(2)=2.294
      PGX(3)=2.677
      PIG=3.098
      WPG(1)=250.
      WPG(2)=130.
      WPG(3)=120.
      WPGR(1)=250.
      WPGR(2)=130.
      WPGR(3)=120.
      DEF(1)=2.097
      DEF(2)=2.584
      DCAP=2.271
      PING=2.492
      PINA=2.283
      PI=2.38
      K(1)=4531.61
      K(2)=682.98
      GNPX=4107.71
      TDBT=6000.0
      BOP=1550.1
      BOT=1700.
      BOD=1.0
      DSR=0.13
      REXCH=484.0
```

†This program listing is for the open economic growth model. Outputs should be modified for other specific cases.

```
      DEFF=2.785
      NVT=2200.
      PAR=0.1
      ITEND=IFIX(1./DT+0.00001)
      T=11.
      YEAR=1975.
      GNP=GNPY
      RGNY=0.083
      DSAVR=0.181
      RFSV=0.114
      DPGIMP=0.266
      NVT=-2256.4
      FER=3600.
      CONPL=0.281
      CLR(1)=0.7075
      CLR(2)=0.1259
      DIM=0.803
      POP=34681.
      CGM(1)=0.
      CGM(2)=1.0
      CONR=0.059
      DTPRNT=0.5
      TPRNT=T+DTPRNT
      DO 1 I=1,6
    1 U(I)=0.
C
C --- YEARLY LOOP
C
      DO 100 IYEAR=1,IEND
C
C --- TIME LOOP
C
      DO 99 ITIME=1,ITEND
      T=T+DT
      CALL LABOR
      CALL PRODN
      CALL SAVING
      CALL CONSM
      CALL GRAIN
      CALL TRADE
      CALL BALANS
      CALL PRICE
C --- OUTPUTS
      IF(ABS(T-TPRNT).GT.0.00001) GO TO 99
      GNPL=GNP/LABF
      C1=CONPL*CL(1)/(CL(1)+CL(2))
      C2=CONPL-C1
      DO 2 I=1,6
    2 U(I)=U(I)+DTPRNT*(C1**A1(I)*C2**A2(I)*CLR(1)**B1(I)*CLR(2)**B2(I))
      PRINT 21,T,GNPL,CONPL,(U(I),I=1,6)
      TPRNT=T+DTPRNT
   99 CONTINUE
      YEAR=YEAR+1.
  100 CONTINUE
   21 FORMAT(5X,F5.1,9F10.4)
      END
```

```
      SUBROUTINE LABOR
C --- LABOR
      REAL K,L,LABR,NVT,NIVT
      REAL LS(2),LTLB
      COMMON/MAIN/T,DT,K(2),L(2),LABR,PROD(3),GG(3),PG(3),PI,DEF(2),
     1                GNP,GNPP(2),RGNY,BOC,SAVEE(3),FSAVED,DMGRN(3),
     2                PGX(3),GGIMP(3),GGIMPT(3),EXPO,NVT,NIVT,TIMPO,DCAP
     3                ,REXCH,DEFF,CONPL,SAVE,DIM,PINA,SUPGRN(3)
     4                ,PIG,WPGR(3),WPG(3),POP,DP(3),TDP,A(3),B(3,3),BI(3,3)
     5                ,C(3),IDCRL,POPL(2)
      DATA LTLB/0.2/
      DATA POPGR/0.01978/
      TLB=T+1.
      LABR=7517.49*EXP(0.03819*TLB)
      LS(2)=0.58504-0.01131*T
      IF(LS(2).GE.LTLB) GO TO 15
      LS(2)=LTLB
   15 LS(1)=1.-LS(2)
      L(1)=LS(1)*LABR
      L(2)=LABR-L(1)
      POP=POP*(1.+DT*POPGR)
      POPR=SCURVE(0.03525,0.5548,0.,T,1)
      POPL(1)=POPR*POP
      POPL(2)=POP-POPL(1)
      RETURN
      END


      SUBROUTINE PRODN
      REAL K,L,LABR,NVT,NIVT
      COMMON/MAIN/T,DT,K(2),L(2),LABR,PROD(3),GG(3),PG(3),PI,DEF(2),
     1                GNP,GNPP(2),RGNY,BOC,SAVEE(3),FSAVED,DMGRN(3),
     2                PGX(3),GGIMP(3),GGIMPT(3),EXPO,NVT,NIVT,TIMPO,DCAP
     3                ,REXCH,DEFF,CONPL,SAVE,DIM,PINA,SUPGRN(3)
     4                ,PIG,WPGR(3),WPG(3),POP,DP(3),TDP,A(3),B(3,3),BI(3,3)
     5                ,C(3),IDCRL,POPL(2)
      COMMON/PRDFLC/GNPC,GNPCC(2),GLR(2),GNPRT,RGNP,GNPX,PAR
      DATA COMER/0.74/
      DATA TX1,TX2/0.35,0.25/
      DATA PCF/0.01/
C --- PRODUCTION
      PROD(1)=0.602407*K(1)**0.536155*L(1)**0.463845
      PROD(2)=0.236978*K(2)**0.497321*L(2)**0.502679
C --- GRAIN PRODUCTION
      AGKL=K(2)/L(2)
      GG(1)=0.976226*(PG(1)/PI)**0.0412178*AGKL**0.0912563*L(2)
      GG(2)=0.446642*(PG(2)/PI)**(-0.425207)*AGKL**0.0561454*L(2)
      GG(3)=0.000851*(PG(3)/PI)**(-0.639943)*AGKL**(-1.75902)*L(2)
C --- MONETIZATION OF PRODUCTION (NET OF PRODUCTION COSTS)
      PROD(3)=COMER*(GG(1)*PG(1)*101767.+GG(2)*PG(2)*54050.+GG(3)*PG(3)*
     $30383.)/(DEF(2)*1.0E+6)
      GNP=PROD(1)+PROD(2)+PROD(3)
      GNPP(1)=PROD(1)
      GNPP(2)=PROD(2)+PROD(3)
      GNPCC(1)=GNPP(1)*DEF(1)
      GNPCC(2)=GNPP(2)*DEF(2)
      GNPC=GNPCC(1)+GNPCC(2)
C --- CONSTANT GNP PER LABOR AND THE RATIO OF AGRICULTURE AND NON-AGRICULTURE
      DO 20 I=1,2
   20 GLR(I)=GNPP(I)/L(I)
      GNPRT=GLR(1)/GLR(2)
C --- GNP GROWTH RATE
      RGNP=(GNP-GNPX)/GNPX
      GNPX=GNP
      RGNY=RGNP/DT
C --- INCOME DISTRIBUTION
      FINCM=GNPP(1)*(1.-TX1)*PAR+GNPP(2)*(1.-TX2)
      UINCM=GNPP(1)*(1.-TX1)*(1.-PAR)
      FINCMP=FINCM/POPL(2)
      UINCMP=UINCM/POPL(1)
      DIM=FINCMP/UINCMP
      PAR=PAR+DT*PCF*(1.-DIM)
      RETURN
      END
```

```
      SUBROUTINE SAVING
      REAL K,L,LABR,NVT,NIVT
      REAL RSAVEX(3),ELSF(3),ELSG(3),DEPR(2),INVT(2),ESI(3)
      COMMON/MAIN/T,DT,K(2),L(2),LABR,PROD(3),GG(3),PG(3),FI,DEF(2),
     1               GNP,GNPP(2),RGNY,BOD,SAVEE(3),FSAVED,DMGRN(3),
     2               PGX(3),GQIMP(3),GGIMPT(3),EXPO,NVT,NIVT,TIMPO,DCAP
     3               ,REXCH,DEFF,CONPL,SAVE,DIM,PINA,SUPGRN(3)
     4               ,PIG,WPGR(3),WPG(3),POP,DP(3),TDP,A(3),B(3,3),BI(3,3)
     5               ,C(3),ICCRL,PCPL(2)
      COMMON/SAVBLC/RSAVE(3),DSAVR,TDSR,RFSV,TSAVE,CLR(2),COR(2)
     1               ,SRA,SRB,SRC
      COMMON/BALBLC/TDPT,BOT,DSR,FBP,BCP
      DATA DEPR/0.07194,0.03922/
      DATA RSAVEX/0.15,0.4,0.1/
      DATA WSHRX/0.463844/
      DATA ESI/0.5,0.3,0.1/
      DATA DMN/0.8/
      DATA CWSHR/0.2/
      DATA CSAVE/0.1/
      DATA PILMT/1.35/
      DATA CIA,CIB/0.1,0.1/
C --- SAVING AND CAPITAL FORMATION
      DMEFT=DMN/DIM
      WSHR=WSHRX
      DO 30 I=1,3
      RSAVE(I)=RSAVEX(I)*DMEFT**ESI(I)
   30 CONTINUE
      WAGE=WSHR*PROD(1)
      PROFT=PROD(1)-WAGE
      SAVEE(1)=RSAVE(1)*WAGE+RSAVE(2)*PROFT
      SAVEE(2)=RSAVE(3)*PROD(2)
      SAVEE(3)=0.*PROD(3)
      SAVE=SAVEE(1)+SAVEE(2)+SAVEE(3)
C --- DOMESTIC SAVING RATE
      DSAVR=SAVE/GNP
C --- FOREIGN SAVING
      XS=2.*(T-11.)/10.-1.
      TDSR=SRA+SRB*XS+SRC*(3.*XS**2-1.)/2.
      RFSV=TDSR-DSAVR
      RFSV=RFSV*SCURVE(5.0,0.5,0.5,DSR,2)
      IF(RFSV.GE.0.) GO TO 31
      RFSV=0.
   31 FSAVE=RFSV*GNP
      FSAVED=FSAVE*1000./REXCH
C --- TOTAL SAVING
      TSAVE=SAVE+FSAVE
C --- INVESTMENT
      CI=1.-(CIA+(CIB-CIA)*(T-11.)/10.)
      INVT(1)=CI*TSAVE
      INVT(2)=TSAVE-INVT(1)
C --- CAPITAL FORMATION
      DO 32 I=1,2
      K(I)=K(I)+DT*(INVT(I)-DEPR(I)*K(I))
C --- CAPITAL-LABOR RATIO
      CLR(I)=K(I)/L(I)
C --- CAPITAL-OUTPUT RATIO
      COR(I)=K(I)/GNPP(I)
   32 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CONSM
      REAL K,L,LABR,NVT,NIVT
      COMMON/MAIN/T,DT,K(2),L(2),LABR,PROD(3),GG(3),PG(3),PI,DEF(2),
     1            GNP,GNPP(2),RGNY,EOC,SAVEE(3),FSAVED,DMGRN(3),
     2            PGX(3),GGIMP(3),GGIMPT(3),EXPO,NVT,NIVT,TIMPO,DCAP
     3            ,REXCH,DEFF,CONPL,SAVE,DIM,PINA,SUPGRN(3)
     4            ,PIG,WPGR(3),WPG(3),POP,DP(3),TDP,A(3),B(3,3),BI(3,3)
     5            ,C(3),IDCRL,POPL(2)
      COMMON/CSMBLC/CONS(2),CL(2),CONR,CONPX
      DIMENSION PGCF(3),DL(3),PL(3),G(3)
      DATA PLCY/1.0/
C --- CONSUMPTION
      PGCF(1)=0.152625
      PGCF(2)=0.121028
      PGCF(3)=0.097455
      DO 40 I=1,2
      CONS(I)=GNPP(I)-SAVEE(I)
   40 CL(I)=CONS(I)/LABR
C --- AGGREGATE CONSUMPTION PER LABOR
      CONSMT=GNP-SAVE-(NVT)*REXCH/(1000.*DEFF)
      CONPL=CONSMT/LABR
      CONR=(CONPL/CONPX-1.)/DT
      CONPX=CONPL
C --- DEMAND FOR GRAINS
      PGNP=GNP*1000./POP
      PGNPL=ALOG(PGNP)
      DO 43 I=1,3
   43 PGCF(I)=PGCF(I)*PLCY
      TPG=T+3.
      PG(1)=0.278659*EXP(PGCF(1)*TPG)
      PG(2)=0.405708*EXP(PGCF(2)*TPG)
      PG(3)=0.559126*EXP(PGCF(3)*TPG)
      DO 1 I=1,3
    1 PL(I)=ALOG(PG(I))
      CALL MATMPY(B,3,3,PL,G)
      TDP=0.
      DO 2 I=1,3
      DL(I)=A(I)+G(I)+C(I)*PGNPL
      DP(I)=EXP(DL(I))
      TDP=TDP+DP(I)
    2 DMGRN(I)=DP(I)*POP
      RETURN
      END

      SUBROUTINE TRADE
      REAL K,L,LABR,NVT,NIVT
      COMMON/MAIN/T,DT,K(2),L(2),LABR,PROD(3),GG(3),PG(3),PI,DEF(2),
     1            GNP,GNPP(2),RGNY,EOC,SAVEE(3),FSAVED,DMGRN(3),
     2            PGX(3),GGIMP(3),GGIMPT(3),EXPO,NVT,NIVT,TIMPO,DCAP
     3            ,REXCH,DEFF,CONPL,SAVE,DIM,PINA,SUPGRN(3)
     4            ,PIG,WPGR(3),WPG(3),POP,DP(3),TDP,A(3),B(3,3),BI(3,3)
     5            ,C(3),IDCRL,POPL(2)
      COMMON/TRDBLC/TMPA,TMPB,TMPC
      DATA CONMX,UINVT,ELEB,ELCB/0.45,200.,-0.1,0.5/
C --- TRADE
      TEX=T+13.
      SHREXP=1./(3544.07*1.03751**TEX*0.99172**(TEX**2)+50.)
      EXPOW=50091.3381*EXP(0.099981*TEX)
      EXPOT=SHREXP*EXPOW
      EXPO=EXPOT
C --- IMPORT CONTENT OF EXPORT
      EMCONT=CONMX*EXPO
C --- IMPORT OF GRAINS
C --- NET FOREIGN CURRENCY REQUIREMENT
      GIMPT=0.
      DO 53 I=1,3
      GIMPT=GIMPT+WPG(I)*GGIMP(I)
   53 CONTINUE
      GIMPT=GIMPT/1000.
C --- COMPETITIVE IMPORT
      GNPC=GNPP(1)*DEF(1)+GNPP(2)*DEF(2)
      XI=2.*(T-11.)/10.-1.
      TMPI=TMPA+TMPB*XI+TMPC*(3.*XI**2-1.)/2.
      CMPORT=(TMPI*GNPC-46.555)*1000./REXCH
      CMPO=CMPORT
```

```
C --- TOTAL IMPORT
      TIMPO=EMCONT+CMPO+GIMPT
C --- NET VISIBLE TRADE
      NVT=EXPO-TIMPO
C --- INVISIBLE TRADE
      TIN=T+2.
      RECIT=91.03699*EXP(0.19866*TIN)
      PAY=19.67382*EXP(0.29979*TIN)
      NIVT=RECIT-PAY
C --- CONTROL OF INVISIBLE TRADE
      IF(NIVT.LT.0.) GO TO 55
      IF(NIVT.LT.UINVT) GO TO 56
   55 NIVT=UINVT
   56 CONTINUE
      RETURN
      END

      SUBROUTINE GRAIN
      REAL K,L,LABR,NVT,NIVT
      COMMON/MAIN/T,DT,K(2),L(2),LABR,PROD(3),QG(3),PG(3),PI,DEF(2),
     1             GNP,GNPP(2),RGNY,BOC,SAVCE(3),FSAVEC,DMGRN(3),
     2             PGX(3),GGIMP(3),GGIMPT(3),EXPO,NVT,NIVT,TIMPO,DCAP
     3             ,REXCH,CEFF,CONPL,SAVE,DIM,PINA,SUPGRN(3)
     4             ,PIG,WPGR(3),WPG(3),POP,DP(3),TDP,A(3),B(3,3),BI(3,3)
     5             ,C(3),IDCRL,POPL(2)
      COMMON/GRNBLC/GSTK(3),DPGIMP,CGM(3)
      DIMENSION DGSTK(3),CSTK(3),SUPLG(3),UPGS(3),DELSTK(3)
      DATA CSTK/0.25,0.5,0.15/
      DATA UPGS/10.,10.,10./
      DATA CGMAG/0.1/
      DO 50 I=1,3
      DSGAP=DMGRN(I)-QG(I)
C --- GRAIN STOCK POLICY (MINIMUM GRAIN STOCK)
      DGSTK(I)=CSTK(I)*DMGRN(I)
      DSTK=DGSTK(I)-GSTK(I)
      CHKST=DSGAP+DSTK
      IF (CHKST.LE.0.) GO TO 51
      GGIMPT(I)=CHKST
      DELSTK(I)=DSTK
      GSTK(I)=DGSTK(I)
      GO TO 50
   51 GGIMPT(I)=0.
      DELSTK(I)=-DSGAP
      GSTK(I)=DGSTK(I)-DT*CHKST
C --- GRAIN STORAGE CAPACITY OR MAXIMUM DESIRABLE GRAIN STOCK
      UPSTK=UPGS(I)*DMGRN(I)
      IF(GSTK(I).LE.UPSTK) GO TO 50
      DELSTK(I)=UPSTK-GSTK(I)
      GSTK(I)=UPSTK
   50 CONTINUE
C --- GRAIN IMPORT POLICY
      DO 56 I=1,3
      WPG(I)=PRW(I,T)
   56 CONTINUE
      CGM(3)=1.-CGMAG*(WPG(3)/WPGR(3)-1.0)
      DO 52 I=1,3
      GGIMP(I)=CGM(I)*GGIMPT(I)
   52 CONTINUE
      DO 60 I=1,3
      DELG=GGIMP(I)-DELSTK(I)
   60 SUPGRN(I)=QG(I)+DELG
      IDCRL=0
      DO 61 I=1,3
      IF(ABS(SUPGRN(I)-DMGRN(I)).GT.0.00001) GO TO 63
   61 CONTINUE
      GO TO 64
   63 IDCRL=1
   64 CONTINUE
C --- DEPENDENCY ON THE GRAIN IMPORT
      TGIMP=0.
      TGDMD=0.
      DO 54 I=1,3
      TGDMD=TGDMD+SUPGRN(I)
   54 TGIMP=TGIMP+GGIMP(I)
      DPGIMP=TGIMP/TGDMD
      RETURN
      END
```

```
      SUBROUTINE BALANS
      REAL K,L,LABR,NVT,NIVT
      REAL MFLA,K1
      COMMON/MAIN/T,DT,K(2),L(2),LABR,PROD(3),GG(3),PG(3),PI,DEF(2),
     1            GNP,GNPP(2),RGNY,BCD,SAVEE(3),FSAVED,DMGRN(3),
     2            PGX(3),GGIMP(3),GGIMPT(3),EXPO,NVT,NIVT,TIMPO,DCAP
     3            ,REXCH,DEFF,CONPL,SAVE,DIM,PINA,SUPGRN(3)
     4            ,PIG,WPGR(3),WPG(3),POP,DP(3),TDP,A(3),B(3,3),BI(3,3)
     5            ,C(3),IDCRL,POPL(2)
      COMMON/BALBLC/TDBT,BOT,DSR,FBR,BCP
      DATA K1,TLOAN,RL/0.19,10.,0.05/
      DATA AMF,BMF,CMF/5.,0.2,0.5/
C --- BALANCE-OF-PAYMENTS
      DBOP=K1*(EXPO+TIMPO)
      DFS=FSAVED*DCAP
      DBS=TDBT/TLOAN
      SBOP=NVT+NIVT-DBS
      SLK=DBOP+DFS-BOT-SBOP
      DSR=DBS/EXPO
      IF(SLK.LE.DFS) GO TO 61
      DBR=SLK-DFS
C --- WORLD FINANCIAL MARKET CONDITIONS
      MFLA=SCURVE(AMF,BMF,CMF,DSR,2)
      FBR=DBR*MFLA
      GO TO 62
   61 DBR=FBR=0.
   62 CONTINUE
      TDBT=TDBT+DT*(RL*TDBT+FBR-DBS)
C --- FOREIGN BORROWING TO THE BALANCE-OF-PAYMENTS
      FBRB=FBR
      FBR=FBRB+DFS
      BOT=BOP+NVT+NIVT+FBRB-DBS
      BOP=BOP+DT*(NVT+NIVT+FBRB-DBS)
      BCD=BOT/DBOP
      RETURN
      END


      SUBROUTINE PRICE
      REAL K,L,LABR,NVT,NIVT
      COMMON/MAIN/T,DT,K(2),L(2),LABR,PROD(3),GG(3),PG(3),PI,DEF(2),
     1            GNP,GNPP(2),RGNY,BCD,SAVEE(3),FSAVED,DMGRN(3),
     2            PGX(3),GGIMP(3),GGIMPT(3),EXPO,NVT,NIVT,TIMPO,DCAP
     3            ,REXCH,DEFF,CONPL,SAVE,DIM,PINA,SUPGRN(3)
     4            ,PIG,WPGR(3),WPG(3),POP,DP(3),TDP,A(3),B(3,3),BI(3,3)
     5            ,C(3),IDCRL,POPL(2)
      DIMENSION DL(3),PL(3),Q(3)
C --- DEFLATORS AND PRICE INDICES
      TPI=T+4.
      TDF=T+5.
      DEF(1)=0.2164*EXP(0.138*TDF)
      DEF(2)=0.1783*EXP(0.161*TDF)
      DEFF=0.607*EXP(0.115*T)
      REXCH=231.*EXP(0.06*T)
      DCAP=0.26323*EXP(0.125*TDF)
      PING=0.21561*EXP(0.15528*TPI)
      PINA=0.35402*EXP(0.11138*TPI)
C --- PRICES OF GRAINS
      PGNPL=ALOG(GNP*1000./POP)
      IF(IDCRL.EQ.1) GO TO 74
      GO TO 75
C --- DETERMINE PRICES GIVEN DEMAND OR SUPPLY
   74 TDP=0.
      DO 5 I=1,3
      IF(SUPGRN(I).LT.0.) SUPGRN(I)=0.
      DP(I)=SUPGRN(I)/POP
      TDP=TDP+DP(I)
      DL(I)=ALOG(DP(I))
    5 Q(I)=DL(I)-A(I)-C(I)*PGNPL
      CALL MATMPY(BI,3,3,Q,PL)
      DO 6 I=1,3
    6 PG(I)=EXP(PL(I))
   75 CONTINUE
C --- PRICE INDICES
      PIG=(PG(1)*QG(1)+PG(2)*QG(2)+PG(3)*QG(3))/(QG(1)+QG(2)+QG(3))
      PIA=(99.9*PIG+79.5*PING)/179.4
      PI=0.1794*PIA+0.8206*PINA
      RETURN
      END
```

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