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EFFECT OF SEQUENCE ON LEARNING OF ADDITION AND SUBTRACTION OF INTEGERS

By

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ABSTRACT

EFFECT OF SEQUENCE ON LEARNING OF ADDITION AND SUBTRACTION OF INTEGERS

By

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Recent studies on the learning process in general and programmed instruction specifically have strongly indicated that the traditional methods of information presentation are not necessarily better than reversed or scrambled sequencing of information. But research has only begun to attack the problem. The next step has been viewed as the development and testing of new, experimental ways to present information.

The investigation was specifically designed to help take this step and thus to relate meaningfully to education in general and to mathematics teaching specifically. The central problem of the study was threefold: (1) to investigate the results of using a different definition of subtraction within the traditional sequence of addition-subtraction; (2) to investigate the results of using a different definition of subtraction within an experimental instructional sequence of subtraction-addition; and (3) to investigate the ability of the interference theory to predict proactive or retroactive effects of one learning upon another.

The investigation was specifically designed to test four major hypotheses. All were expressed in the null form. The study's sample population consisted of 68 students. Each study participant engaged in independent learning of specially written units. The sample was divided into the following four groups: SeAp (Experimental Subtract-Add with Pre-Test), ASe (Add-Experimental Subtract), ASt (Add-Traditional Subtract), and SeA (Experimental Subtract-Add).

Data were derived through administration of six test instruments: Test A (Add), Test S (Subtract), Test Fa (Final Add), Test Fs (Final Subtract), and Tests Pa and Ps (Pre-Test Add and Pre-Test Subtract). All were constructed by the researcher.

Statistical analysis provided means, standard deviations, and a computed F value for each of the testings. Analysis of variance (ANOVA) was computed on data derived from Tests A, S, Fa, and Fs. ANOVA was also computed for tests Fa - Pa and Fs - Ps.

Statistical analysis produced few statistically significant results. No significant results were obtained from administration of Tests A, S, and Fs. On the Fa (Final Add) test, the ANOVA and Scheffe Post-hoc calculation indicated that retention of addition dropped when subtraction intervened, regardless of which subtraction method was used. The SeA group performed better than the ASe group, but differences were not statistically significant. Thus, minor indication was given of the existence of proactive inhibition.

Significant values from Fa - Pa and Fs - Ps testing indicated that all groups had significantly greater addition-subtraction knowledge after unit instruction than before.

In conclusion, of the four null hypotheses posited by the study, one was rejected, two were accepted, and one was accepted in part, rejected in part.

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CHAPTER I

INTRODUCTION

It is safe to say that no subjects in the entire field of educational psychology have received more attention than the causes of forgetting and the techniques of learning. The reasons are probably that man is far more dependent on learning than is any other animal and that successful development of problem-solving skills is dependent upon learning experiences that have taken place in the context of all previous learning experiences.

It is not enough, then, for students to learn concepts and skills they are likely to need in out-of-school or future classroom situations. They must also be able to retrieve them from memory storage promptly when a next step in learning or problem solving requires them to do so. Much of what is learned in school is quickly forgotten;[1] yet some skills in mathematics, for example, can be retained throughout life. According to the literature, problem-solving skills in mathematics, once acquired, are, as a rule, well retained.[2]

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[1]Floyd L. Ruch, Psychology and Life: Seventh Edition (Glenview, Ill.: Scott Foresman and Company, 1967), pp. 227-28.

[2]See J.A. McGeoch and A.L. Irion, The Psychology of

The increase in the amount of mathematics deemed desirable for today's studies provides evidence that it is more important than ever before for mathematics teachers to organize learning experiences so that learning from a first experience significantly facilitates learning in a second experience. Until recently it was assumed that the traditional order of presenting mathematics concepts was best. But research findings have seriously challenged that view.[3]

Many have expressed concern regarding the sequencing of basic mathematics concepts for elementary school children and regarding teaching strategies which could more significantly facilitate positive learning transfer.[4] Clearly, meaningful strategies, techniques, and approaches to the ordering of concept learning could have a dramatically positive effect on public school pupils' mathematics achievement levels. It would seem, however, that few have researched the problem area specifically. The literature contains few relevant empirical data. Yet it would seem that such data and findings could significantly impact

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Human Learning (New York: McKay, 1952); J.P. Duncan, "Learning and Measured Abilities," Journal of Educational Psychology 57 (1966): 220-29; also John P. DeCecco, The Psychology of Language, Thought, and Instruction (New York: Holt, Rinehart, & Winston, 1967).

[3]F.C. Niedermeyer et al., "Learning and Varied Sequencing of Ninth-Grade Mathematics Materials," Journal of Experimental Education 37 (1969): 61-66; R.T. Heimer, "Conditions of Learning in Mathematics: Sequence Theory Development," Review of Educational Research 39 (1969): 493-508; also P. Suppes, "Mathematical Concept Formation in Children," American Psychologist 21 (1966): 139-50.

[4]See N.J. Slamecka, ed., Human Learning and Memory:

curriculum and student achievement--not just at the elementary school level but at other levels of education as well.

Statement of the Problem

The central problem of the present study was threefold: (1) to investigate the results of use of a different definition of subtraction within the traditional add-subtract instructional sequence for integers; (2) to investigate the results of use of a different definition of subtraction within an experimental subtract-add instructional sequence, and (3) to investigate the capacity of interference theory adequately to predict proactive or retroactive effects of one learning upon another.

Background of the Problem

Much research in recent years has sought to identify those factors significantly influencing retention and the failure of traditional approaches to maintain mathematics learning at adequate ability levels. According to one authoritative commentary, pupils continue to underachieve in mathematics concepts learning because "the schools lack a specific set of strategies for teaching mathematics . . . To overcome the problem . . . the schools must implement a relatively new view on how students develop mathematical

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Selected Readings (New York: Oxford University Press, 1967).

ideas and abilities."[5]

Everyone knows that the "new math" recently introduced in American public elementary and secondary schools sought to remove the traditional emphasis on the kind of knowledge pupils were expected to acquire, to reduce emphasis on rote learning, and to place new stress on concepts and heuristic problem analysis. That it has failed to achieve its objectives in some areas has become evident; the reasons why it has failed are less clear. Many authorities strongly believe that the crux of the problems lies in ignorance of the ways in which students develop mathematical abilities and ideas:

> . . . . the theory of instruction should specify the most effective sequences in which to present the materials to be learned. Given, for example, that one wishes to teach the structure of modern physical theory, how does one proceed? Does one present concrete materials first in such a way as to elicit questions about recurrent regularities? Or, does one begin with a formalized mathematical notation that makes it simpler to represent regularities later encountered?[6]

A number of studies have attempted to assess in detail the effect that the order of information presentation has on learning.[7] The findings have generally indicated

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[5]A.P. Troutman, "Strategies for Teaching Elementary School Mathematics," The Arithmetic Teacher 20 (October 1973): 526.

[6]J.S. Bruner, "Some Theorems on Instruction Illustrated with Reference to Mathematics," Theories of Learning and Instruction, in Sixty-third Yearbook of the National Society for the Study of Education, pt. 1 (Chicago: National Society for the Study of Education, 1964), p. 308.

[7]See J.L. Brown, "Effects of Logical and Scrambled Sequences in Mathematics Materials on Learning with Programmed Instruction Materials," Journal of Educational

that mixing up the order of information presentation, all other things remaining equal, does not significantly affect achievement and performance adversely. According to Natkin and Moore, for example:

> The usual method of these studies has been to present to one group of subjects some unit of programmed instruction in its logical order, while another group studied the same material but with the frames presented in random or "scrambled" order. The most significant finding of these studies was that scrambling did not adversely affect terminal performance . . . The result . . . implies that our intuitions about cumulative learning processes, and some of the more important learning theories, are seriously in error. The failure of scrambling to significantly reduce performance seems to imply that learners can usually reconstruct their own knowledge structure when given sufficient time and information to do so.[8]

However, research seems to have stopped short of developing better or different methods of presenting instructional learning units. It is widely recognized that problems cannot be remedied if potential solutions do not exist.

Significance of the Study

Since the educational revolution of the 1960s, educators have placed heavy emphasis on theory before

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Psychology 61 (1970): 41-45; also S.A. Bobrow, "Memory for Words in Sentences," Journal of Verbal Learning and Verbal Behavior 9 (1970): 363-72; and Henry S. Kepner, Jr., "An Empirical Investigation of Retroactive Effects on the Retention of Meaningful Mathematical Material" (doctoral dissertation, University of Iowa, 1970).

[8]G.L. Natkin and A.W. Moore, "The Effect of Instructional Sequencing on Learning from a Simple Knowledge Structure," American Education Research Journal 9 (Fall 1972): 599-604.

practice, thereby reducing the need for rote learning. The goal has been to attack the mathematics crisis at its roots.[9] However, much of the concept development was presented at a highly abstract level so that explanations were often more difficult than the idea being explained.

The "new" New Math attempts to make explanations more consistent with the child's level of thinking as expressed by Bruner and Piaget. Analogous efforts to improve the teaching of addition and subtraction of integers can be found; but none has gained widespread acceptance and none has solved the problem of relating the meaning-of subtraction to the method-of subtraction. The present investigation was specifically designed to help take this step and thus to relate meaningfully to American education in general and to mathematics teaching specifically.

Hypotheses

The present investigation was specifically designed to test four major hypotheses. All were expressed in the null form, which asserts that the true mean difference between two testings of the same group, or between two groups, is zero.[10] Rejection of a null hypothesis asserts that

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[9]John H. Lawson, "Is the New Math Doing the Job?" The Education Digest 38 (December 1973): 16-18; S.S. Willoughby, "What Is the New Math?" The Bulletin of the National Association of Secondary School Principals 52 (1968): 4-15; see also T.J. Fletcher, "Secondary Mathematics Today," Trends in Education 35 (October 1974): 11-18.

[10]D.V. Huntsberger, Elements of Statistical Inference (Boston: Allyn & Bacon, 1967), pp. 180, 183; Henry E.

statistical evidence has indicated that the differences obtained between statistics were significant and that "true differences were greater than zero."[11]

The four hypotheses were stated as follows:

1. There is no statistically significant relationship between the order of information presentation when teaching sixth-graders to add and subtract and their terminal performance in terms of learning to add.

2. There is no statistically significant relationship between the order of information presentation when teaching sixth-grade pupils to add and subtract and their terminal performance and achievement in terms of learning to subtract.

3. There is no statistically significant difference between the terminal performance of sixth-grade pupils taught addition and subtraction in the traditional add-subtract sequence, with the traditional definition, and sixth-grade pupils taught addition and subtraction with sequence and definition changed.

4. There is no statistically significant difference in terminal performance of two groups of sixth-graders taught addition and subtraction in subtract-add sequence with the distance and direction definition--where one group was pre-tested and the other was not.

Garrett, Elementary Statistics (New York: Longmans, Green & Co., 1956), pp. 96-98.

[11]J.F. Kenney and E.S. Keeping, Mathematics of Statistics (New York: D. Van Nostrand, 1956), pp. 166-67.

Definition of Terms

A number of terms were defined for the present investigation. For example, three teacher-made instructional units were developed by the researcher specifically for the purposes of the present study (see Appendix C, p. 53). The three definitions are:

Addition Unit: The unit informally introduces integers to describe the results of "finding" and "losing" amounts of money. The result of two such incidents is expressed as an "and then" story. No method for adding is discussed but the child answers the question "Lucky or unlucky?" as a way of looking at the result, for example, of "losing 6 and then finding 4." The next question is "How unlucky?" Answer--2, or "the same as losing 2."

Subtraction--Difference Unit: A traditional treatment of subtraction of integers. The unit does not attempt to model subtraction. Rather, it relates differences to missing addends. For example, $6 - 2 = \boxed{4}$ because $\boxed{4} + 2 = 6$. Adding the opposite is shown to give the same "answer." Pupils are told that adding the opposite always works, and they practice using the method.

The traditional subtraction unit is modern and up-to-date in that it depends on symbolic explanation and logical discovery to justify a rule. It is called "traditional" because it includes the treatment most often used in seventh, eighth, and ninth grade algebra and pre-algebra texts. Symbolic explanation has always been a necessary step in algebraic and pre-algebraic instruction.

Subtraction--Distance and Direction Unit: An experimental unit which defines subtraction (and integers) as CHANGES having distance and direction. A modified number line and pictures of thermometers are used as learning aids. To use Bruner's terms--the instruction moves from enactive representation to ikonic representation to a very simple symbolic representation. Most importantly, subtraction is defined without reference to addition, so the unit can be taught either before or after addition. One purpose of this study was to determine whether it should be taught before addition.

The other definitions include:

Interference theory: In an educational context, the term "interference" refers, generally, to the competition between old and new responses that results in forgetting. According to DeCecco, the theory is based on two consistent findings for learning: "Learning new associations causes one to forget old associations learned earlier in time and to forget new associations as well. In effect, associations compete, with one association interfering with the retention of the others."[12]

Researchers agree, generally, that of the causes of forgetting, the most active is the interference between competing associations in an individual's storage and

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[12]John P. DeCecco, The Psychology of Learning and Instruction (Englewood Cliffs, N.J.: Prentice-Hall, 1969), p. 351.

reactivation systems.[13] Such interference is common in school subjects. There are, moreover, forward and backward inhibitors of correct retention. Because they work together, investigators have found it quite difficult to isolate their separate effects.

Proactive inhibition: The process by which prior learning interferes with the recall of later, or newer, learning is called "proactive inhibition." It is, in other words, a second source of interference from associations learned earlier in time; retroactive inhibition is the first. Early researchers believed that retroactive inhibition was the major source of forgetting.[14] Later studies, however, provided valid evidence that the major cause of forgetting appears to be, instead, proactive inhibition.[15] Underwood has explained that the fact that proactive inhibition is now assigned the major role as the cause of forgetting is based on the assumption that ". . . during the first ten years of a student's life, he will have acquired more habits that

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[13]D.R. Entwistle and W.H. Huggins, "Interference in Meaningful Learning," Journal of Educational Psychology 55 (1964): 75-78; L.R. Peterson, "Short-Term Memory," Scientific American 215 (1966): 90-95; also Robert M. Goldenson, An Encyclopedia of Human Behavior (New York: Doubleday, 1970), p. 472.

[14]See R.W. Tyler, "Permanence of Learning," Journal of Higher Education 4 (1933): 203-205; J.A. McGeoch and W.T. McDonald, "Meaningful Retention and Retroactive Inhibition," American Journal of Psychology 43 (1931): 579-88.

[15]N.J. Stamecks, "Proactive Inhibition of Connected Discourse," Journal of Experimental Psychology 62 (1961): 295-301; B.J. Underwood, "Interference and Forgetting," Psychological Review 64 (1957): 49-60; also B.J. Underwood

will interfere with the task to be recalled than he will acquire during the one-month interval between learning and the retention test . . ."[16]

Underwood's conclusion has been at odds with the traditional belief that what was learned subsequently--retroactive inhibition--rather than what was learned previously--proactive inhibition--was the primary source of forgetting. DeCecco has noted further that "Present evidence also suggests that extinction is the process underlying interference. Concerning the amount of material, or length of the sequence or list, there is "evidence to indicate that the mind recodes learned information into chunks of information and thereby increases man's capacity to retain bits of information."[17]

Limitations

The study was inherently limited, first, in that there may have been unknown factors not taken into consideration that could have significantly biased the study's results. It should be noted, however, that this is true of all types of research investigations regardless of their design or data treatment methods.[18]

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and R.J. Schultz, Meaningfulness and Verbal Learning (Philadelphia: J.B. Lippincott, 1961).

[16]B.J. Underwood, "Laboratory Studies of Verbal Learning," in Theories of Learning and Instruction, p. 146.

[17]DeCecco, p. 356.

[18]See A.L. Edwards, Experimental Design in

Second, the study was limited to that amount of data that could be obtained from the random sampling of sixth grade pupils comprising the investigation's sample population. In addition, the investigation was also limited to that amount of data that could be derived from the test instruments.

Methodology

The present investigation employed an experimental design that called for a control and three experimental groups and first unit, second unit, and overall post-test administration. Each group studied the addition unit and one of the subtraction units.

Analysis of data involved tabulation of data from administration of six tests. For each group one group was pre-tested. Means, mean deviations, and analysis of variance were calculated. The total variation in the data was measured by the total sum of squares of deviations from the overall mean. One source of variation was the differences among the group means, measured by the sum of squares of the deviation of the group means from the overall means. The only remaining variation was that among the observations within the groups: the variation of the individual values about their group means. This was measured by analysis of

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Psychological Research (New York: Holt, Rinehart, & Winston, 1960); also Arnold J. Lein, Measurement and Evaluation of Learning (Dubuque, Iowa: W.C. Brown, 1971); and M. Ray Loree, Psychology of Education (New York: Ronald Press, 1965).

variance of the pooled sum of squares of deviations of the individual observations from the group means.

Summary

The present chapter has served as an introduction to the investigation. The problem of the study was stated as threefold:

1. To investigate the effects of using a concrete definition of subtraction, unrelated to addition.

2. To examine the effects of reversing the usual addition-subtraction sequence with students using the non-traditional subtraction materials.

3. To investigate the capacity of the interference theory to predict adequately proactive effects of one instructional unit upon another.

It was noted that the study was specifically designed to test four major hypotheses. They were listed in the null form. Limitations of the investigation were noted and definitions of important terms were presented. The research and statistical methodology of the study was explained in detail. The study design called for a control and three experimental groups and the use of three specially developed units of instruction.

The study was divided into five chapters. Chapter II contains a review of the related literature while Chapter III describes methods and procedures. Chapter IV summarizes the data analysis and results; Chapter V contains a summary, conclusions, and recommendations.

CHAPTER II

REVIEW OF RELATED LITERATURE

Forgetting has been variously defined in the literature as failure to recall and as failure to retain previously learned material.[1] The two definitions are probably intended to refer to the same process. At least four major explanations of forgetting have been devised. First, learning has been seen as producing a physiological change in the nervous system, a so-called "memory trace," that can later be revived or reactivated. This theory holds that this trace fades through disuse, just as a muscle atrophies if not used. There is little direct evidence to support this view.

A second theory holds that the memory trace does not decay but becomes distorted through the action of the normal metabolic processes of the brain. These changes are believed to explain why the individual cannot accurately reproduce either verbal information or visual forms even after a short period of time. This has been called the distortion theory.

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[1] A.R. Gilliland, "The Rate of Forgetting," Journal of Educational Psychology 39 (1948): 19-26; P.J. Nicholson, "A Methodological Study of Retroactive Inhibition," unpublished dissertation, University of Iowa, 1966; also N.L. Gagne, Conditions of Learning (New York: Holt, Rinehart & Winston, 1965); and E.R. Hilgard and G.H. Bower, Theories of Learning (New York: Appleton-Century-Crofts, 1966).

Two explanations associated with the idea of distortion have been offered. One holds that the process of remembering and forgetting is influenced by individual interpretation: what we think we see or hear shapes the way we reproduce it. The other holds that motivation is the key to forgetting--that is, we forget because we want to forget.

The fourth theory attributes forgetting to interference of two kinds: proactive and retroactive inhibition. Proactive refers to a condition in which prior learning interferes with new learning.[2] Retroactive inhibition, on the other hand, refers to a condition in which new material interferes with previously learned material.[3] There is extensive evidence to support both concepts.[4]

Basic to the problem of the present investigation are the interference and proactive inhibition theories. The study was designed to focus on retention and forgetting of original learning as dependent on the sequencing of lesson

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[2]Arden N. Frandsen, Educational Psychology (New York: McGraw-Hill, 1967), p. 487.

[3]R.C. Anderson and J.F. Carter, "Retroactive Inhibition of Meaningfully-Learned Sentences," American Educational Research Journal 9 (Summer 1972): 443-48.

[4]See S. Rosenberg, "Retroactive Inhibition in Incidental Learning," American Journal of Psychology 74 (1961): 283-86; R.C. Anderson and D.L. Myrow, "Retroactive Inhibition of Meaningful Discourse," Journal of Educational Psychology Monographs 62 (1971): 81-94; also P.A. Payne et al., "The Effect of Sequenced Programmed Instruction," American Educational Research Journal (1967), pp. 125-32; and R.K. Young, "Retroactive and Proactive Effects Under Varying Conditions of Response Similarity," Journal of Experimental Psychology 50 (1955): 113-19.

information and on the forces exerted by proactive inhibition. Thus the purpose of the review of literature presented in this chapter is twofold: (1) to provide an overall view on the traditional ways subtraction has been defined and the theories supporting such definitions, and (2) to establish an overall perspective on the more important factors related to proactive effects and sequencing order of meaningful learning.

Literature on Subtraction

The work of Bruner and Piaget suggests that the present methods of teaching subtraction of integers are unnecessarily difficult for two major reasons: (1) because most proceed too quickly to purely symbolic expression and (2) because most have a logical dependence upon a student's understanding and skill in addition of integers.[5] Even when students are presented with a "physical" interpretation of a-b, they are not expected to use it in actual computation; they are only expected to use it as a means for making the rule. This denies the student the concrete operational state for the concept.

It is important to explain that the meaning of "concrete" varies from person to person. For the sixth-grade student, for example, a masking tape number line is a

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[5]Jean Piaget, The Origins of Intelligence in Children (New York: International Universities Press, 1952); Jerome S. Bruner et al., A Study of Thinking (New York: Wiley, 1960).

concrete thing and counting spaces on it is a concrete operation. A picture of that number line is iconic. The definition of a-b as the distance and direction from b to a actually takes the student through the enactive phase because he gets 4 as a result of physically putting his index fingers on 3 and 7 and counting the spaces between them. The pupil, then, has a concrete situation and meaning (imagery) for a-b from which he can later verify "properties" such as a-b = a + -b. To teach a child to verbalize a property without giving him the tools with which to verify it can only result in failure to grasp concepts.

Different methods of teaching subtraction of integers to children have been proposed. Most all are mathematically sound and logical. There appears, however, to be much controversy among views as to which is best. Cotter advocated the use of a model involving positive and negative particles as an aid to understanding subtraction of integers.[6] His view considers a field with zero charge but having the same number of positive and negative particles. Removal of two positive charges results in a $^{-}2$ charge on the field. Consider the problem $^{+}3 - {}^{-}2$. An interpretation would be that the field had a $^{+}3$ charge with other neutrally charged particles in the field, which are considered to be made up of one positive and one negative charge. Thus, removal of two negative charges gives the field a $^{+}5$ charge and the

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[6]Stanley Cotter, "Charged Particles: A Model for Integers," The Arithmetic Teacher 16 (May 1969): 349-53.

mathematical sentence would be $^{+}3 - {}^{-}2 = {}^{+}5$.

Cotter's view still suggests a definition for subtraction as "take away," however. Entwhistle proposed another method,[7] one that advocated a definition of subtraction as "take away," or "minus."

The available literature, while advocating different methods, indicates that subtraction is more difficult than addition; that subtraction is the inverse of addition; and that one must use addition skills in order to subtract. In addition, the bulk of the literature suggests that one need only teach children the concept of addition in the best possible way, then teach subtraction completely by rote; that students need only to discriminate between "plus" and "minus" and learn that in subtraction all that is necessary is to change the sign and add. In the final analysis this is how most children have learned to subtract, regardless of the approach used or the theory advocated.

Interference, Proactive Inhibition, and Sequencing Theories

An early study conducted by Jenkins and Dallenbach first offered the theory that forgetting is the result of interference rather than of a mysterious "fading away" of traces in the brain.[8] According to Osgood, this view of

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[7] A. Entwhistle, "Subtracting Signed Numbers," The Mathematics Teacher 48 (March 1955): 174-76.

[8] J.G. Jenkins and K. Dallenbach, "Oblivescence During Sleep and Waking," American Journal of Psychology 35 (1924): 605-12.

interference is a behaviorist theory because a memory is simply a response provided by a stimulus.[9] Forgetting, Osgood maintained, is a function of the degree to which substitute responses are associated with original stimuli during the retention interval. Therefore,

> Identity between responses in original and interpolated activities yields facilitation, whereas differences between responses yield interference. . . . The magnitude of either facilitation or interference is a function of the stimulus similarities between original and interpolated activities.[10]

Early investigators believed that a major source of forgetting was retroactive inhibition.[11] An important flaw in this early research was uncovered by Underwood, however.[12] In his studies of retention, Underwood discovered a noticeable, clear relationship between the number of lists learned by his subjects and the amount of forgetting. When a subject learned one list of nonsense syllables, he remembered approximately 80 percent of the list after one hour. If more than fifteen lists were learned, however, rate of recall dropped to about 20 percent after one hour. The researcher concluded that the major cause of forgetting

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[9] Charles E. Osgood, Method and Theory in Experimental Psychology (New York: Oxford University Press, 1957), pp. 550-52.

[10] Ibid., pp. 550-51.

[11] See H. Ebbinghaus, Memory, trans. H.A. Ruger and C. Bussenuis (New York: Teachers College Press, Columbia University, 1913).

[12] Benton J. Underwood, "Interference and Forgetting," Psychological Review 64 (1957): 49-60.

appeared to be proactive inhibition.

> Assume that a student, ten years of age, learns a task and the retention of this task is tested one month later. The fact that proactive inhibition is assigned a major role in the cause of forgetting is based on the assumption that, during the first ten years of the student's life, he will have acquired more habits that will interfere with the task to be recalled than he will acquire during the one month interval between learning and the retention test.[13]

Underwood also suggested that some individuals may be able to resist interference better than others. The stress on the effects of proactive inhibition emphasizes the importance of such inhibition in verbal learning, especially in regard to the young child entering the classroom environment.

The interference theory of forgetting was developed and tested primarily with rote verbal tasks and paired-associate learning, a technique for studying the learning process. Words, syllables, digits, or other items are learned in pairs and a subject is later tested on his or her ability to give the second part of the pair when the first is presented.

The paired-associate learning technique has two important advantages. First, the procedure has been found useful in experimental investigation of many aspects of the learning process, particularly retroactive inhibition, proactive inhibition, and transfer of training. Secondly, the

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[13]Benton J. Underwood, "Laboratory Studies of Verbal Learning," in Theories of Learning and Instruction, in Sixty-third Yearbook of the National Society for the Study of Education, pt. 1 (Chicago: National Society for the Study of Education, 1964), p. 146.

technique is closely related to actual experiences since much of thinking is a chain of associations acquired by "serial learning," in which one response becomes a stimulus for the next.[14]

The interference theory, according to many authorities, occupies an unchallenged position as an explanation of the process of forgetting.[15] There is little doubt among authorities that this view is the most useful for explaining the process of forgetting of materials learned by rote processes. One of the first to question the usefulness of the interference theory in explaining learning that is not acquired by the rote process was Ausubel.[16]

Retention has been found to vary considerably from material to material and from one condition to another.[17] Factors influencing both rote and meaningful learning,

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[14]DeCecco, Psychology of Learning and Instruction, pp. 352-54.

[15]See, for example, E.R. Hilgard and G.H. Bower, Theories of Learning (New York: Appleton-Century-Crofts, 1966); P.J. Nicholson, "A Methodological Study of Retroactive Inhibition" (doctoral dissertation, University of Iowa, 1966); John M. Stephens, The Psychology of Classroom Learning (New York: Holt, Rinehart & Winston, 1966); and B.J. Underwood, "Forgetting," Scientific American 210 (1964): 91-99.

[16]D.P. Ausubel, The Psychology of Meaningful Verbal Learning: An Introduction to School Learning (New York: Grune & Stratton, 1963); D.P. Ausubel, "A Teaching Strategy for Culturally Deprived Pupils: Cognitive and Motivational Considerations," The School Review (Winter 1963), pp. 454-63; and D.P. Ausubel et al., "Retroactive Facilitation in Meaningful Verbal Learning," Journal of Educational Psychology 59 (1968): 250-55.

[17]Stephens, Psychology of Classroom Learning, p. 226.

according to the literature, include speed of learning, motivation, over-learning, information presentation order, and retroactive and proactive inhibition.[18] The effects of both proactive and retroactive inhibition on rote learning have been extensively studied.[19] Almost all the studies employed the paired-associate method, and almost all agreed on the strong influence of both types.

Regarding the learning of materials by the rote process, other studies concluded that stimulus similarity within and between learning materials produced significant interference.[20] Serial position was also found to have a significant influence on retention of rote-learned materials.[21]

Instructional sequencing itself has also been

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[18]See, for example, A.R. Gilliland, "The Rate of Forgetting," Journal of Educational Psychology 39 (1948): 19-26; J.A. McGeoch and A.L. Irion, The Psychology of Human Learning (New York: David McKay, 1961); also M. Stager and A.J.H. Gaite, "Proactive Effects in Meaningful Verbal Learning and Retention," Journal of Educational Psychology 60 (1969): 59-64.

[19]J. Deese and S.H. Hulse, The Psychology of Learning (New York: McGraw-Hill, 1967); N.J. Slamecka, "Retroactive Inhibition of Connected Discourse as a Function of Practice," Journal of Experimental Psychology 59 (1960): 245-49; also N.J. Slamecka, "Proactive Inhibition of Connected Discourse," Journal of Experimental Psychology 62 (1961): 295-301.

[20]I.M. Bilodeau and H. Schlosberg, "Similarity in Stimulus Conditions as a Variable in Retroactive Inhibition," Journal of Experimental Psychology 41 (1951): 199-204; Young, pp. 113-19.

[21]M.E. Franklin and C. Weisiger, "Effect of a Change in Mode of Presentation on Recall in Serial Learning," Psychological Reports 8 (1961): 431-38; G.A. Talland, "Cultural Differences in Serial Reproduction," Journal of Social Psychology 43 (1956): 75-81.

extensively studied.[22] According to Heimer, instructional sequence means "the order in which the learner interacts with units of content."[23] Varied positions and views have been offered in regard to instructional sequencing, but fundamental to each is the assumption that instructional sequence is best formulated and evaluated in conjunction with content structure.

Several major theoretical formulations have been offered. Gagne's work, for example, characterized learning hierarchies as an ordered set of intellectual skills "such that each entity generates a substantial amount of positive transfer in the learning of a not-previously acquired higher-order capability."[24] Gagne saw a connection between a learning hierarchy and the associated presentation sequence. The learning hierarchy concept has been widely accepted as a cornerstone for developing instructional sequenced mathematics at the present time.

Heimer reached the following conclusions from his

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[22]See, for example, L.J. Briggs, Sequencing of Instruction in Relation to Hierarchies of Competence (Pittsburgh: American Institution for Research, 1968); and Joseph H. Scandura, "Prior Learning, Presentation Order, and Prerequisite Practice in Problem Solving," Journal of Experimental Education 34 (1966): 1-6.

[23]Ralph T. Heimer, "Conditions of Learning in Mathematics: Sequence Theory Development," Review of Educational Research 39 (1969): 494.

[24]R.M. Gagne, "Learning Hierarchies," Educational Psychologist 6 (1968): 3-6; R.M. Gagne, The Conditions of Learning (New York: Holt, Rinehart, & Winston, 1965).

careful analysis of the literature on learning hierarchies as put forth by Gagne: (1) there are no self-defined algorithms for producing learning hierarchies; (2) the connection between the logical structure of knowledge and the associated learning hierarchy has not yet been adequately explored; and (3) the role of learning hierarchies in the development of presentation sequences is unclear.[25]

Like Gagne, Suppes upheld the importance of accounting for content structure in the study of learning and sequencing.[26] Suppes' mathematical models dealt with hierarchies potentially important in the design of presentation sequences. His work attempted to conceptualize psychological variables that had previously resisted definition.

Other theoretical formulations and research on sequencing included the work of Ausubel[27] and Pyatte.[28] Ausubel's work closely resembled that of Gagne and presupposed that the preceding step in the learning hierarchy was always clear, stable, and well-organized; if it was not,

[25] Heimer, P. 499.

[26] P. Suppes, "Mathematical Concept Formation in Children," American Psychologist 21 (1961): 139-50; P. Suppes, "Modern Learning Theory and the Elementary School Curriculum," American Educational Research Journal 1 (1964): 79-93.

[27] D.P. Ausubel and M. Youssef, "Role of Discriminability in Meaningful Parallel Learning," Journal of Educational Psychology 54 (1963): 331-36; also D.P. Ausubel et al., "Retroactive Inhibition and Facilitation in the Learning of School Materials," Journal of Educational Psychology 48 (1957): 334-43.

[28] J.A. Pyatte, "Some Effects of Unit Structure of

the learning of all subsequent steps was jeopardized.

Summary

The literature survey served a dual purpose: it established an overall perspective on the important factors related to proactive, inhibitory, and sequencing effects on meaningful learning, and it provided an overall view of the traditional ways in which subtraction has been defined and the theories supporting such definitions. Regarding the former purpose, the chapter reviewed the literature pertaining to sequencing, interference, and retroactive inhibition theories. Early research was found to have used the paired-associate technique and included only rote-learned materials. Varied positions in regard to instructional sequence assumed that instructional sequence was best formulated and evaluated in conjunction with content structure. Ausubel's theories were found to be integral to the rationale of the present study, however.

Regarding the second purpose, the chapter reviewed mainly the literature pertaining to definitions of subtraction and proposed approaches and methods of teaching. Regardless of the proposal, however, it was concluded that all indicated that subtraction was more difficult than addition and that subtraction was the inverse of addition; that addition skills are needed to teach subtraction; and that subtraction implies the definition "take away."

Achievement and Transfer," American Educational Research Journal 6 (1969): 241-61.

CHAPTER III

METHODS AND PROCEDURES

Chapters I and II served to introduce the problem under investigation and to review the literature pertinent to the primary concerns of the study. The purpose of Chapter III is to explain the investigation's methods and procedures for collecting and analyzing the data derived from test instrument administration.

The Sample Population

A total of 68 sixth-grade students from 68 public elementary schools located throughout the state of Wisconsin comprised the study's sample population. The ranking sixth grade teacher in each school was asked to provide one subject for the study (see Appendix A, p. 45, for Letter of Introduction and Appendix B, p. 48, for instructions on tests). Because the selected subject would be required to miss his or her regular mathematics instruction for five or six days and because the study materials were to be done independently by the subject, a necessary requirement was that the subject be above average in math and reading ability, as well as in independence.

Each study participant engaged in independent study of the experimental units. The researcher hoped in this manner to eliminate possible biasing effects attributable to teacher approach.

Procedures for Data Collection

The Wisconsin School Directory was used to provide a systematically random sampling of Wisconsin schools. On each page the first listed school with a sixth-grade class was chosen. The last such school on the page was also included in pages with high densities of listings, such as the pages listing Milwaukee's schools.

The assignment of schools, and thus of students, to treatment groups was also done systematically. Mailing envelopes were prepared for the entire list of schools and then "dealt" into four stacks. The envelopes in each stack were then stuffed with color-coded materials for the students in that treatment group.

The teachers who selected students saw only the material for their own group. No teacher had knowledge of the purpose of the study. All teachers and students considered themselves to be in the "experimental group."

One hundred eighty packets of materials were sent out. Data were tabulated from the first 17 complete packets returned in each group. Actual numbers of packets returned were as follows: SeAp, 17; ASe, 20; ASt, 25; and SeA, 19.

Research Design

As noted, the design of the investigation called for four treatment groups and the administration of six test instruments produced by the experimenter. The intent of the study was to ascertain whether or not the sequence in which students learned addition and subtraction led to significant differences in terminal performance and to significant differences in the amount of learning retained and forgotten. For this reason the subjects of the study were divided into four groups. Treatment Group 1 (SeAp) received a stapled booklet that included the following items, in order: Letter to Student, Addition Pre-Test (Pa), Subtraction Pre-Test (Sa), Experimental Subtraction Unit, Subtraction Test (S), Addition Unit, Addition Test (A), Final Addition Test (Fa), and Final Subtraction Test (Fs). Treatment Group 4 (SeA) was the same as Group 1 but without the pretests. Treatment Groups 2 and 3 (ASe and ASt) received packets having the Addition Unit first followed by Subtraction. But Group 2 studied the Experimental Subtraction Unit (Se) while Group 3 studied the Traditional Subtraction Unit (St). Students were expected to complete each of their two units and the associated test within three math periods (or less). Nearly all students completed their booklets and all tests within a week. The data collected were analyzed for each of the groups according to the types of information desired.

The six test instruments of the study were designed and developed by the researcher specifically for the present investigation. They included: Addition Pre-Test (P), Subtraction Pre-Test (Sa), Subtraction Test (S), Addition Test (A), Final Addition Test (Fa), and Final Subtraction Test

(Fs). Each was designed in such a way as to test learning for the related module unit of instruction only.

Collection and evaluation of study data were accomplished in five sequential steps. These were as follows:

1. Administration of the pretests and data collection for group SeAp.

2. Administration of first unit and second unit tests and data collection from all groups.

3. Final (retention) tests (addition and subtraction) and data collection from all groups.

4. Comparison, analysis, and statistical computation of first, second, and posttest data for the study's four groups, taking into concern two major areas of assessment: the effects of reversed order sequencing and the effects of the experimental definition of subtraction.

5. Evaluation and determination of the significance of the results, with subsequent application of the findings to test the four null hypotheses of the study.

Summary

The sample population included 68 sixth-grade students attending an equal number of Wisconsin elementary schools. Six test instruments were used. The design of the investigation called for four groups with pretests, unit tests, and retention tests in addition and subtraction.

CHAPTER IV

DATA ANALYSIS AND DISCUSSION

The purpose of this portion of the study is to present and analyze the data that resulted from the investigation. This chapter has been divided into two major parts: (1) presentation and analysis of the data, and (2) discussion of the results of statistical computations. The first subsection explains and presents the computation tables. A delineation of the important values obtained and their meanings is the subject matter of the second subsection.

Data Analysis

Statistical analysis, as noted, used data obtained through administration of six test instruments, all of which were constructed by the researcher (see Appendix B, p. 47). These included: Test A (Addition), Test S (Subtraction), Test Fa (Final Add), Test Fs (Final Subtract), Test Pa (Pre-Test Add), and Test Ps (Pre-Test Subtract).

The study's sample population consisted of four groups of 17 students each. It is important to explain that data packets were numbered on their return to the researcher. Groups were randomly reduced to the lowest number responding for any one group to obtain groups of equal size as required by the statistical analysis. This total was 17.

Thus the study's total sample population consisted of 68 subjects. There were 17 students in each of the following groups: SeAp (Experimental Subtract-Add with Pre-Test), ASe (Add-Experimental Subtract), ASt (Add-Traditional Subtract), and SeA (Experimental Subtract-Add).

Table 1 presents the mean scores and standard deviations obtained from administration of Test A for four groups: SeAp, ASe, ASt, and SeA. Tables 2-4 present similar data for the same four groups; but these data were derived through administration of Tests S, Fa, and Fs, respectively. The same information is provided in Tables 5 and 6, but for administration of Tests Fa - Pa and Fs - Ps and for five groups: SeAp, ASe, ASt, SeA, and Pa. It should be noted that Ps and Pa scores derived from the SeAp Group.

TABLE 1

MEANS, AND STANDARD DEVIATIONS OBTAINED FROM ADMINISTRATION OF TEST A FOR GROUPS: SeAp, ASe, ASt, SeA

Group	$\overline{X}$	σ^2
SeAp	19.294	1.766
ASe	19.059	4.996
ASt	18.589	4.242
SeA	19.294	1.031

Analysis of variance (ANOVA) was computed for Tests A, S, Fa, and Fs for groups SeAp, ASe, ASt, and SeA. Tables 7 - 10 report the findings. Degrees of freedom and

TABLE 2

MEANS, AND STANDARD DEVIATIONS OBTAINED FROM ADMINISTRATION OF TEST S FOR GROUPS: SeAp, ASe, ASt, SeA

Group	$\overline{X}$	σ^2
SeAp	16.941	5.114
ASe	16.000	8.706
ASt	17.588	2.830
SeA	18.423	1.345

TABLE 3

MEANS, AND STANDARD DEVIATIONS OBTAINED FROM ADMINISTRATION OF TEST Fa FOR GROUPS: SeAp, ASe, ASt, SeA

Group	$\overline{X}$	σ^2
SeAp	19.352	.699
AsE	15.705	12.796
ASt	16.647	6.111
SeA	19.412	6.301

TABLE 4

MEANS, AND STANDARD DEVIATIONS OBTAINED FROM ADMINISTRATION OF TEST Fs FOR GROUPS: SeAp, ASe, ASt, SeA

Group	$\overline{X}$	σ^2
SeAp	13.706	8.678
ASe	15.588	6.595
ASt	14.824	4.851
SeA	14.176	8.381

TABLE 5

MEANS, AND STANDARD DEVIATIONS FROM ADMINISTRATION OF TEST Fa-Pa FOR GROUPS: SeAp, ASe, ASt, SeA, and Pa

Group	$\overline{X}$	σ^2
SeAp	19.352	0.699
ASe	15.705	12.796
ASt	16.647	6.111
SeA	19.412	6.301
Pa	11.059	24.173

TABLE 6

MEANS, AND STANDARD DEVIATIONS FROM ADMINISTRATION OF TEST Fs-Ps FOR GROUPS: SeAp, ASe, ASt, SeA, and P

Group	$\overline{X}$	σ^2
SeAp	13.706	8.678
ASe	15.588	6.595
ASt	14.824	4.851
SeA	14.176	8.381
Ps	4.529	2.249

TABLE 7

ANOVA FOR TEST A FOR GROUPS: SeAp, ASe, ASt, SeA

Source of Variance	df	MS	F Value
Within groups	3	3.06	0.614
Between groups	64	1.88	
Totals	67		

TABLE 8

ANOVA FOR TEST S FOR GROUPS: SeAp, ASe, ASt, SeA

Source of Variance	df	MS	F Value
Within groups	3	4.676	1.970
Between groups	64	9.230	
Totals	67		

TABLE 9

ANOVA FOR TEST Fa FOR GROUPS: SeAp, ASe, ASt, SeA

Source of Variance	df	MS*	F Value
Within groups	3	5.55	10.941**
Between groups	64	60.750	
Totals	67		

* $p < .05$
** $p < .01$

TABLE 10

ANOVA FOR TEST Fs FOR GROUPS: SeAp, ASe, ASt, SeA

Source of Variance	df	MS	F Value
Within groups	3	7.572	1.499
Between groups	64	11.348	
Totals	67		

mean squares within and among groups as well as the derived F values are presented in each tabulation. Significance is identified by an asterisk immediately following an appropriate F value.

Analysis of variance was also computed for Tests Fa - Pa and Fs - Ps. Tables 11 and 12 present degrees of freedom and mean squares within and among the five groups. A computed value for F is included. Again, significance is identified by an asterisk following the appropriate value. Significance was determined at both the .05 and .01 levels of critical probability. Computed F values were compared to table values in appropriate texts for determination of significance (Glass and Stanley, 1970; Hays, 1963; Kerlinger, 1973; Thorndike, 1971).

TABLE 11

ANOVA FOR TEST Fa-Pa FOR GROUPS: SeAp, ASe, ASt, SeA, Pa

Source of Variance	df	MS*	F Value
Within groups	4	3.970	20.788**
Between groups	80	113.821	
Totals	84		

* $p < .05$
** $p < .01$

TABLE 12

ANOVA FOR TEST Fs-Ps FOR GROUPS: SeAp, ASe, ASt, SeA, Ps

Source of Variance	df	MS*	F Value
Within groups	4	5.623	81.937**
Between groups	80	460.745	
Totals	84		

*$p < .05$
**$p < .01$

The Scheffe Post-hoc procedure applied to ANOVA results for the Fa test indicated that nearly all of the between groups variation could be attributed to sequence of instruction.

Discussion of Results

Statistical analysis produced few statistically significant results. No significant results were obtained from administration of Tests A and Fs. Some effect was found on addition when subtraction intervened--that is, through administration of the Fa test. The SeA group performed at a slightly higher level than the ASe group; but the differences between the two groups were not found to be statistically significant. The researcher thus assumes that slight indication was given of the existence of proactive inhibition.

Significant values were derived for F from Fa - Pa and Fs - Ps testing. Significance was found at the .05 and .01 critical probability levels. These results indicated that all groups had significantly greater addition and subtraction knowledge after unit instruction than before.

Results of the data analysis also showed that subtraction scores dropped if the addition unit was learned first. But again, these reductions were not statistically significant. The unusually low variance values should also be noted. As previously explained, the sample population was limited to the "brighter" students who teachers felt could miss a few days of class instruction and still keep pace with the regular class. It would seem that this limitation explains the low variances obtained.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Much research has been devoted in recent years to identifying factors significantly influencing retention of learning and the failure of traditional mathematics approaches to maintain learning at adequate ability levels. A number of recent studies concerned with the development and evaluation of programmed instruction have attempted to assess the effect that the order of information presentation has on learning. Programmed instruction studies have indicated that reversed and scrambled sequencing of materials has little effect on terminal performance. The studies strongly suggest that traditional methods of information presentation are not necessarily better than reversed or scrambled sequencing.

Research has only just begun to attack the problem. It has been widely noted that the next step must be the development and testing of new, experimental ways to present information, particularly in the area of mathematics. The present study was specifically designed to help take this step. The study was seen as relating meaningfully to education in general and to mathematics learning specifically.

Summary of the Research

The problem of the present investigation was threefold: (1) to investigate the results encountered when a different definition of subtraction is used within the traditional instructional sequence of addition-subtraction; (2) to investigate the results of using a different definition of subtraction within an experimental instructional sequence of subtraction-addition; and (3) to investigate the ability of the interference theory adequately to predict proactive effects on retention of original learning.

The sample population of 68 subjects was divided into four groups: Experimental Subtract-Add Group with Pre-Test Subtract (SeAp), Add-Experimental Subtract (ASe), Add-Traditional Subtract (ASt), and Experimental Subtract-Add (SeA). Each study participant engaged in independent learning of teacher-made units. Statistical analysis consisted of means and standard deviation computations and analysis of variance (ANOVA) for groups from six test instrument administrations.

The investigation posited four major hypotheses expressed in null form. These were stated as follows: (1) There is no statistically significant relationship between the order of information presentation when teaching sixth-graders to add and subtract and their terminal performance in terms of learning to add; (2) There is no statistically significant relationship between the order of information presentation when teaching sixth-grade pupils to add and subtract and their terminal performance and achievement in terms of learning to subtract; (3) There is no statistically significant difference between the terminal performance of sixth-grade pupils taught addition and subtraction in the traditional add-subtract sequence, with the traditional definition, and sixth-grade pupils taught addition and subtraction with sequence and definition changed; and (4) There is no statistically significant difference in terminal performance of two groups of sixth-graders taught addition and subtraction in subtract-add sequence with the distance and direction definition--where one group was pretested and the other was not.

Regarding the first hypothesis, a significant difference was established for addition retention when subtraction intervened. Therefore the null form of Hypothesis 1 was rejected.

Results of the subtraction unit test showed that the experimental subtract-add (SeA) group performed slightly better than the add-experimental subtract (ASe) group. Still, differences were not statistically significant. Hypothesis 2 was accepted.

As regards Hypothesis 3, various findings emerged. First, on the addition unit test an insignificant difference was found between the performance levels of the subtract-add sequence group (SeA) and of the addition-subtraction (traditional) group (ASt). Second, a minor difference was found between the performance levels of the same two groups on the subtraction unit test. Third, a statistically significant difference was found between the performance levels of the same groups on the final addition test. Fourth, no significant difference was found between the performance levels of the same two groups on the final subtraction test.

As regards the fourth hypothesis, results of the data analysis indicated no statistical significance. Therefore the null form of Hypothesis 4 was also accepted.

Conclusions

Based on the results and findings, the researcher concluded the following:

1. There was a statistically significant relationship between the order of information presentation when teaching sixth-graders to add and subtract and their terminal performance in terms of learning to add.

2. As regards subtraction skill, there was no statistically significant difference between the terminal performance of sixth-graders taught addition and subtraction in the add-subtract sequence with traditional definition and pupils taught addition and subtraction with sequence and definition changed.

3. As regards addition skill, there was a statistically significant difference between the terminal performance of sixth-graders taught addition and subtraction in the add-subtract sequence with traditional definition and pupils taught addition and subtraction with sequence and

definition changed. In brief, when subtraction intervened, retroactive inhibition was noted and the test groups scored lower on the final addition test. This conclusion applies in the cases of conclusions 2 and 3.

4. Results of the study indicated the existence of proactive inhibition.

Notably, a form of proactive inhibition occurs if a child's previous knowledge of subtraction as "take-away" interferes with his or her learning either a different meaning of subtraction or how to subtract integers. This could be expected to happen to the pretested and the addition-first groups. At the same time, the pretest could be considered an Ausubel-type "organizer" that could contribute to learning. Still, results of the data analysis in one particular case led the researcher to conclude that the interference theory was able to a limited extent to predict retroactive and proactive effects of one learning upon another.

5. The researcher concluded, finally, that all groups learned significantly in terms of addition and subtraction of integers. This conclusion was based on ANOVA computations with four groups on the Final Add and Final Subtract tests and one group on the Pre-Add and Pre-Subtract tests.

Recommendations

The investigation indicates justification for the following recommendations based on the study's findings and conclusions:

1. That similar investigations should be undertaken to support and validate, or refute, the findings and conclusions of the present investigation and the contentions of other researchers cited in this study.

2. That similar studies of a wider scope should be undertaken in the future. More subjects could be examined and the time for learning teacher-made units could be extended. Some results were found in the present study but few were significant. The present study suggested that the findings of no significance resulted, in part, from the limited sample size.

3. That further investigation be conducted as regards the use of a more concrete definition for the operation of subtraction with integers.

4. That additional research should be undertaken to develop and test new, experimental ways to present information. Clearly, new and meaningful strategies and approaches to the ordering of concept learning could have a dramatic and positive impact on mathematics achievement levels of public elementary school students. The increased amount of mathematics necessary for today's children provides clear evidence that the need for such research is greater now than ever before.

APPENDIX A

COVER LETTER

Dear Sixth Grade Teacher:

Do you have at least one student who could afford to miss the usual beginning-of-the-year review of whole numbers? If so, would you be willing to have one such student use some new instructional material as a part of the research design for my doctoral thesis?

If you cannot participate, you need not do anything - just throw this whole package in the waste basket and forget it.

But, in case you are saying "yes", I have enclosed the necessary materials and instructions and I hope that you will continue.

Since I am studying the effectiveness of the materials, there are only three things for you to do.

1. Choose a student and give him/her the equivalent of six 30-45 minute periods to work independently on the enclosed material.

2. Check 3 or 4 short tests with the student.

3. "Guard" the tests and mail them back to me in the enclosed envelope by September 23.

INSTRUCTIONS

Choosing the student -- The ideal student for this experiment will be independent, highly motivated and a good reader, but not so mathematically brilliant that he/she has already "figured out" how to add and subtract with positive and negative numbers.

Administering the tests -- The tests are stapled in the student's packet of material at the point where they should be used. You need not time the tests. Most students will finish quickly. The answer key for the tests is included at the end of this letter.
Note: The tests will probably not be in the same order as they appear on the key, but each test is readily identifiable. Please allow the student a few minutes to look over the test before you put it in the envelope to mail to me.
IMPORTANT: The student should not take the FINAL TEST immediately after the preceding test. A period of 3-24 hours should intervene.

Returning results -- I am sending out more packets than I need for a good study, to allow for "shrinkage". If, in your opinion, something has happened - the student was ill, or you decide that you made a poor choice of student, or whatever - please indicate that you consider the data from your school to be invalid. A note in your envelope with the tests will do that without "insulting" the student. Of course, low test scores, by themselves, would not be invalid.

I am working under a nearly impossible deadline, which explains the ridiculous deadline that I have given you. Your efforts in meeting it will be most appreciated!

If you would like to have a summary of the data and an abstract of the thesis, please enclose a self-addressed envelope. You may wish to note the color of your tests and the scores for comparison with the summary when it arrives.

Thank you very much for reading this far! If you can participate, I feel sure the experience will be interesting for your student and both you and your student will be making a contribution to the improvement of math education! The world may not thank you, but I will!

Sincerely,

Shirley M. Davis

APPENDIX B

1. INSTRUCTION SHEET
2. TEST A (ADDITION)
3. TEST S (SUBTRACTION)
4. TEST F (FINAL)
5. TEST P (PRETEST)

Congratulations!

Your teacher has chosen you to take part in a small experiment. About 200 other 6th graders all over Wisconsin were also chosen. The experiment is to find out whether these materials can be read and understood by 6th grade students working independently. For that reason it is important that you only ask the teacher for help when it is absolutely necessary.

HOW TO USE THIS BOOKLET

You may write in the booklet and keep it. The booklet is stapled in the upper corner to make it easy to fold back pages after you finish them. Answers to each set of problems are on the back of the page before.

Example: the answers to page 7 are on the back of page 6. You will just unfold your booklet to check a set of problems.

HOW TO USE THE ANSWERS

Answers are provided to help you learn. It is important that you use them correctly.

Step 1. Do the first 2 or 3 problems in the set.

Step 2. Check these problems.

- A. If correct - continue.
- B. If wrong - read the lesson and instructions again and try the problems again. Check again.
  1. If correct - continue.
  2. If wrong - ask the teacher for help.

Step 3. Finish the set of problems and check. Study the problems you missed. Ask your teacher if you don't know why they are wrong.

HOW TO USE THE TESTS

When you come to a TEST, do the test, ask your teacher to check it and let you look it over. Then your teacher will keep it to mail back to the experimenter.

Thank you for taking part in the experiment and Good Luck!

TEST A

Remove this page and do the test when you are sure that you are ready.
Then ask your teacher to check it and let you see what you missed.
Your teacher will keep the test to send to the experimenter.

1. +4 + +6 = ___	11. +8 + +3 = ___
2. +5 + -1 = ___	12. -5 + -1 = ___
3. -8 + +3 = ___	13. -4 + +6 = ___
4. -2 + -5 = ___	14. -10 + -10 = ___
5. +10 + +10 = ___	15. + 2 + - 5 = ___
6. + 4 + - 6 = ___	16. -10 + +10 = ___
7. - 8 + - 3 = ___	17. - 2 + - 5 = ___
8. +10 + -10 = ___	18. + 8 + - 3 = ___
9. - 5 + + 1 = ___	19. + 5 + + 1 = ___
10. + 2 + + 5 = ___	20. - 4 + - 6 = ___

OPINION QUESTIONS

These pages were:	easy	just right	difficult	for me.
My regular math work is usually:	easy	just right	difficult	for me.
These pages were:	interesting	O.K.	boring.	
My regular math work is usually:	interesting	O.K.	boring.	

NAME____________________ NUMBER CORRECT______ DATE________

SCHOOL____________________ TEACHER'S SIGNATURE____________

TEST S

Remove this page and do the test when you are sure that you are ready. Then ask your teacher to check it and let you see what you missed. Your teacher will keep the test to send to the experimenter.

1. +4 - +6 = ___			11. +8 - +3 = ___	
2. +5 - +1 = ___			12. -5 - -1 = ___	
3. -8 - +3 = ___			13. -4 - +6 = ___	
4. -2 - -5 = ___			14. -10 - -10 = ___	
5. +10 - +10 = ___			15. +2 - -5 = ___	
6. +4 - +6 = ___			16. -10 - +10 = ___	
7. -8 - -3 = ___			17. -2 - -5 = ___	
8. +10 - -10 = ___			18. +8 - -3 = ___	
9. -5 - +1 = ___			19. +5 - +1 = ___	
10. +2 - +5 = ___			20. -4 - -6 = ___	

<u>OPINION QUESTIONS</u>

These pages were:	easy	just right	difficult	for me.
My regular math work is usually:	easy	just right	difficult	for me.
These pages were:	interesting	O.K.	boring.	
My regular math work is usually:	interesting	O.K.	boring.	

NAME______________________________ NUMBER CORRECT____ DATE______

SCHOOL__________________________ TEACHER'S SIGNATURE________________

TEST F - FINAL

Part A - Please allow from 3-24 hours between your other test and this one. Ask your teacher to check it and let you see what you missed. Your teacher will keep the test to send to the experimenter. You may do either part first.

1. -4 + -6 = ___	11. +2 + +5 = ___
2. +5 + +1 = ___	12. -5 + +1 = ___
3. +8 + -3 = ___	13. +10 + -10 = ___
4. -2 + -5 = ___	14. -8 + -3 = ___
5. -10 + +10 = ___	15. +4 + -6 = ___
6. +2 + -5 = ___	16. +10 + +10 = ___
7. -10 + -10 = ___	17. -2 + -5 = ___
8. -4 + +6 = ___	18. -8 + +3 = ___
9. -5 + -1 = ___	19. +5 + -1 = ___
10. +8 + +3 = ___	20. +4 + +6 = ___

Part B.

1. -4 - -6 = ___	11. +2 - +5 = ___
2. +5 - +1 = ___	12. -5 - +1 = ___
3. +8 - -3 = ___	13. +10 - -10 = ___
4. -2 - -5 = ___	14. -8 - -3 = ___
5. -10 - +10 = ___	15. +4 - -6 = ___
6. +2 - -5 = ___	16. +10 - +10 = ___
7. -10 - -10 = ___	17. -2 - -5 = ___
8. -4 - +6 = ___	18. -8 - +3 = ___
9. -5 - -1 = ___	19. +5 - +1 = ___
10. +8 - +3 = ___	20. +4 - +6 = ___

NAME________________________ NUMBER CORRECT______ ______
Part A Part B

SCHOOL________________________ TEACHER'S SIGNATURE________________

TEST P - PRE-TEST

Directions: You are not expected to know very many of these answers, but you are welcome to make a "good guess" on each question.

1. +8 + +3 = ____
2. -5 + -1 = ____
3. -4 + +6 = ____
4. -10 + -10 = ____
5. +2 + -5 = ____
6. -10 + +10 = ____
7. -2 + -5 = ____
8. +8 + -3 = ____
9. +5 + +1 = ____
10. -4 + -6 = ____
11. +4 + +6 = ____
12. +5 + -1 = ____
13. -8 + +3 = ____
14. -2 + -5 = ____
15. +10 + +10 = ____
16. +4 + -6 = ____
17. -8 + -3 = ____
18. +10 + -10 = ____
19. -5 + +1 = ____
20. +2 + +5 = ____

21. +8 - +3 = ____
22. -5 - -1 = ____
23. -4 - +6 = ____
24. -10 - -10 = ____
25. +2 - -5 = ____
26. -10 - +10 = ____
27. -2 - -5 = ____
28. +8 - -3 = ____
29. +5 - +1 = ____
30. -4 - -6 = ____
31. +4 - +6 = ____
32. +5 - +1 = ____
33. -8 - +3 = ____
34. -2 - -5 = ____
35. +10 - +10 = ____
36. +4 - -6 = ____
37. -8 - -3 = ____
38. +10 - -10 = ____
39. -5 - +1 = ____
40. +2 - +5 = ____

NAME______________________________ NUMBER CORRECT________

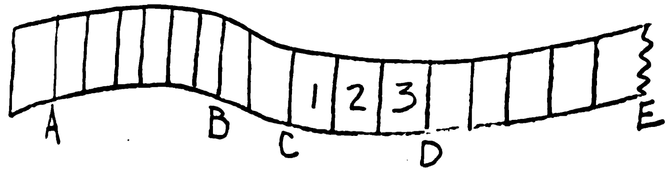
SCHOOL______________________________ TEACHER'S SIGNATURE________________

APPENDIX C

1. DISTANCE AND DIRECTION UNIT
2. ADDITION UNIT
3. SUBTRACTION UNIT

DISTANCE AND DIRECTION

LESSON 1A <u>DISTANCE</u>



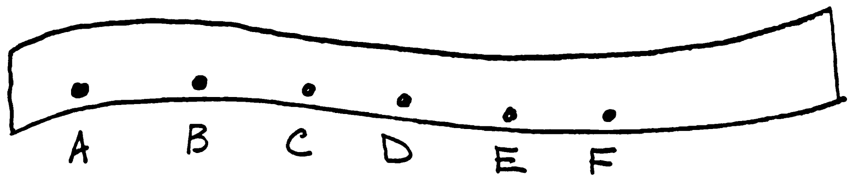
Here are some easy questions about SPACES on a track or tape.

The answers will help you understand the questions.

QUESTIONS			ANSWERS
How many spaces between C and D			3
spaces between A and C	= ?		8
|between D and C|	= ?		3
|A and D|	= ?	Say to yourself- "how many spaces between A and D?"	11
|D——A|	= ?	between D and A	11
|D—B|	= ?		5
|C – C|	= ?		0
|B-D|	= ?		5

LESSON 1B <u>DIRECTION</u>

Look at the tape below. Put your finger on B and move toward A.



From B GOING TOWARD → A the direction is LEFT.

Now, move toward D

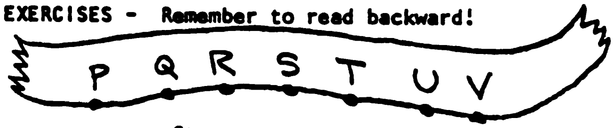
from B GOING TOWARD → D - - - - - - - is RIGHT.

D ← DRAWOT GNIOG from B = Right — Watch the arrow. It still starts with B and goes toward D, but you have to read it backward.

A ← C = Left — Start with C! Say to yourself, "From C toward A is left."

F ← D = Right — Say to yourself, "from D toward F is right."

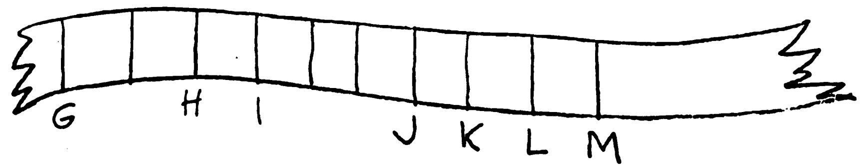
EXERCISES - <u>Remember to read backward!</u>



1. R ← P (Start) = ___	6. D ← P = ___
2. S ← Q = ___	7. P ← O = ___
3. R ← U = ___	8. Y ← X = ___
4. U ← R = ___	9. K ← L = ___
5. Z ← V = ___	10. U ← Z = ___

NOW CHECK!

Practice Page



DISTANCE EXERCISES

Notice that you can start at either letter and get the same answer.

1. |G - J| = ___
2. |I - G| = ___
3. |M - K| = ___
4. |J - L| = ___
5. |K - K| = ___
6. |I - K| = ___
7. |L - J| = ___
8. |K - M| = ___
9. |H - I| = ___
10. |K - G| = ___

Now check your work!

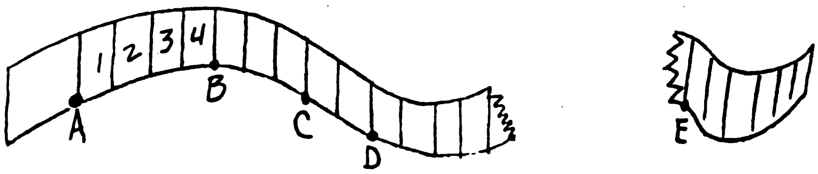
DIRECTION EXERCISES

Remember to always start at the second letter.

1. G ← J = ___ (Start)
2. I ← G = ___
3. M ← K = ___
4. J ← L = ___
5. K ← K = ___
6. I ← K = ___
7. L ← J = ___
8. K ← M = ___
9. H ← I = ___
10. K ← G = ___

Now check your work!

LESSON 2 DISTANCE and DIRECTION



Now we will do Distance and Direction <u>together</u>

From A —to→ B is Right 4 (Direction is Right. Distance is 4.)

Read backward again! B ←— From A is Right 4 (Say to yourself - "From A to B is Right 4.)

D ←— C = R2 (Say to yourself - from C to D is Right 2)

B ←— C = <u>?</u> (L3)

A ←— D = <u>?</u> (L9)

D —— B = <u>?</u> (R5)

D —— E = <u>?</u> (L10)

EXERCISES - Remember —— means (which direction? how far?)

Always start at the second letter.

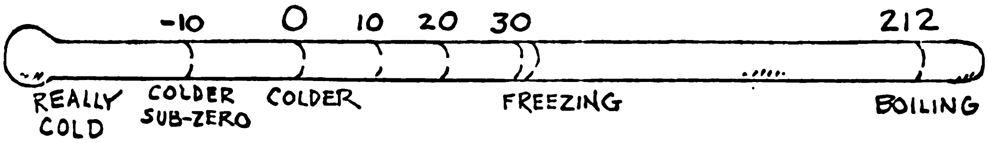
1. A—B = ____
2. C—B = ____
3. C—D = ____
4. B—D = ____
5. C—E = ____
6. E—A = ____
7. A—E = ____
8. E—C = ____

LESSON 3 CHANGES

Think of an ordinary thermometer "lying down"

with COLD - BELOW ZERO to the left

and HOT - ABOVE ZERO to the right.



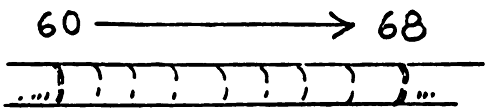
Most people use the symbol (+10) to mean 10 degrees above 0. They say: plus ten or positive ten

They use (-10) to mean 10 degrees below 0. They say: minus ten or negative ten

We can use Distance and Direction to describe CHANGES in temperature.

Here is a story: This morning the temperature was +60. By noon it was +68.

Here is a picture of the story:



How big was the CHANGE — 8 spaces - 8 degrees

What direction was the CHANGE — warmer - to the Right, toward the positive

(R8 or +8)

Another CHANGE in temperature.

Story:

Yesterday the temperature was 18 above 0. Today it is 2 above

How much did it CHANGE?

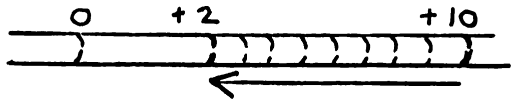
Which direction?

Symbol Sentence

$^{+}2 \longleftarrow {}^{+}10 = {}^{-}8$

Read this:
"from 10 above to 2 above is negative 8."

Picture:

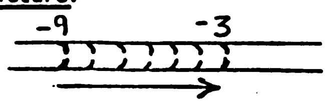


8 spaces
Left or
toward negative — (-8)

When we write ($^{+}2 \longleftarrow {}^{+}10$)
we really don't need the arrow, because we will <u>always</u> start at the <u>second number</u>.

Another CHANGE in temperature:

Picture:



Symbol sentence

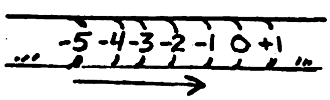
-3 (start) -9 = [?]

Can you tell the story?

From -9 to -3 is (positive 6)

Another Example:

Picture:



Notice that the change is <u>positive</u> even though it starts and ends with negative numbers.

Symbols:

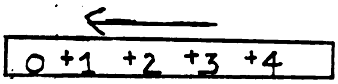
-1 ——— -5 = [?]

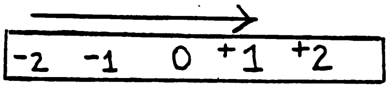
Say:

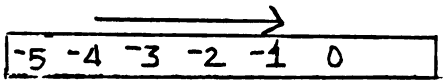
"From negative 5 to negative 1 is (positive 4)"

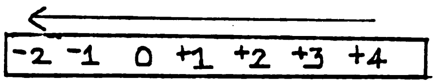
EXERCISES

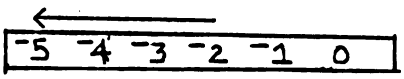
Show the symbols for these number line pictures.

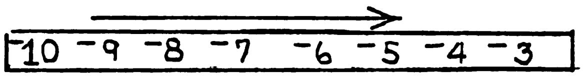
1. 

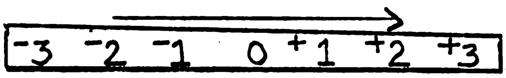
2. 

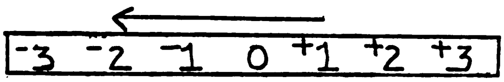
3. 

4. 

5. 

6. 

7. 

8. 

Use positive and negative numbers to tell the Distance and Direction of the changes. You may wish to draw a number line or make one on your desk with masking tape.

9. ${}^{+}3 - {}^{-}6 = \square$

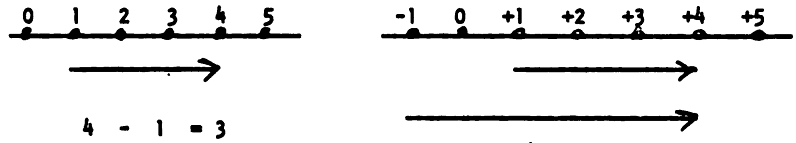
10. ${}^{-}7 - {}^{-}4 = \square$

11. ${}^{-}3 - {}^{+}8 = \square$

12. ${}^{+}12 - {}^{+}4 = \square$

LESSON 4

Compare:



4 - 1 = 3

+4 — +1 = +3

+4 — -1 = +5

Can you see why what you have been doing is usually called SUBTRACTION - even though sometimes you used adding to get the answer. The answer to a <u>DISTANCE</u> and <u>DIRECTION</u> <u>SUBTRACTION</u> is also called the DIFFERENCE.

Practice using DISTANCE and DIRECTION to find the DIFFERENCE. Check each set and find out <u>why</u> you made mistakes before you continue.

A.

1. $^{+}8 - {}^{+}3 =$ ___
2. $^{-}8 - {}^{+}3 =$ ___
3. $^{+}4 - {}^{-}6 =$ ___
4. $^{-}6 - {}^{+}4 =$ ___
5. $^{-}7 - {}^{+}4 =$ ___
6. $^{-}7 - {}^{-}4 =$ ___
7. $^{+}5 - {}^{-}3 =$ ___
8. $^{+}5 - {}^{+}3 =$ ___
9. $^{+}6 - {}^{-}8 =$ ___
10. $^{+}6 - {}^{+}8 =$ ___

B.

1. $^{-}11 - {}^{-}3 =$ ___
2. $^{-}11 - {}^{+}3 =$ ___
3. $^{-}7 - {}^{+}9 =$ ___
4. $^{-}7 - {}^{-}9 =$ ___
5. $^{+}7 - {}^{-}9 =$ ___
6. $^{+}7 - {}^{+}9 =$ ___
7. $^{-}9 - {}^{+}7 =$ ___
8. $^{-}9 - {}^{-}7 =$ ___
9. $^{+}9 - {}^{-}7 =$ ___
10. $^{+}9 - {}^{+}7 =$ ___

C

1. $^{+}10 - {}^{-}3 =$ ___
2. $^{+}10 - {}^{+}3 =$ ___
3. $^{+}3 - {}^{+}10 =$ ___
4. $^{+}3 - {}^{-}10 =$ ___
5. $^{-}7 - {}^{-}8 =$ ___
6. $^{-}8 - {}^{-}7 =$ ___
7. $^{-}3 - {}^{+}2 =$ ___
8. $^{+}2 - {}^{-}3 =$ ___
9. $^{+}8 - {}^{-}3 =$ ___
10. $^{-}3 - {}^{+}8 =$ ___

D.

1. $^{+}10 - 0 =$ ___
2. $0 - {}^{+}10 =$ ___
3. $0 - {}^{-}3 =$ ___
4. $0 - {}^{+}4 =$ ___
5. $^{+}10 - {}^{-}10 =$ ___
6. $^{+}10 - {}^{+}10 =$ ___
7. $^{-}10 - {}^{+}10 =$ ___
8. $^{-}10 - {}^{-}10 =$ ___
9. $^{+}8 - {}^{-}8 =$ ___
10. $^{-}6 - {}^{+}6 =$ ___

When you are ready, take Test S.

ADDITION UNIT

Suppose:

On the way to school today you lost $1.

On the way to school your friend lost $2.

Both of you were UNLUCKY.

Who was more unlucky?

Suppose:

On the way to school you lost $3,

and then found $5. Altogether, were

you lucky or unlucky? How lucky? $2

Suppose:

You lost $3 and then you lost $7.

Lucky or unlucky? very unlucky

How unlucky? $10

EXERCISES A. Answer these. A few are done for you.

	Lucky or Unlucky	How Much	Same As
Lost 3 and then found 4	lucky	1	found 1
Found 5 and then lost 6			
Found 7 and then found 8		15	found 15
Found 6 and then found 10			
Lost 7 and then lost 6			
Found 8 and then found 9			
Found 10 and then lost 4			
Found 4 and then lost 8			

EXERCISES B. Try these problems.

<u>+4</u> means <u>found 4</u>.

<u>-2</u> means <u>lost 2</u>.

	Lucky or Unlucky	How Much	Same As
+4 and then -3	lucky	1	+1
-7 and then -4			
-8 and then +3			
+10 and then +7			

Now we'll use **+** to mean "and then."

EXERCISES C. Try these.

$^{+}4$ **+** $^{+}2$	$^{+}6$
$^{+}5$ **+** $^{-}3$	
$^{+}1$ **+** $^{-}8$	
$^{-}7$ **+** $^{-}2$	
$^{-}7$ **+** $^{+}3$	

> Say the problem to yourself -
>
> Found 4 and then found 2
>
> is the same as found 6.

Have you noticed that even though we write "and then" with a plus sign "+" sometimes you add and sometimes you subtract to get the answer.

Now write it this way, using " = " for "is the same as"

$^{+}4$ **+** $^{+}2$ = $^{+}6$

It still means "<u>found 4</u> and then <u>found 2</u> is the same as <u>found 6</u>."

EXERCISES D. Do these.

1. $^{+}5 + ^{-}2 =$ ____
2. $^{+}6 + ^{+}4 =$ ____
3. $^{-}7 + ^{-}3 =$ ____
4. $^{-}3 + ^{+}7 =$ ____
5. $^{+}10 + ^{-}10 =$ ____
6. $^{-}8 + ^{+}1 =$ ____
7. $^{+}8 + ^{+}2 =$ ____
8. $^{-}5 + ^{-}4 =$ ____
9. $^{-}10 + ^{+}2 =$ ____
10. $^{-}6 + ^{-}1 =$ ____
11. $^{-}6 + ^{+}3 =$ ____
12. $^{-}3 + ^{+}7 =$ ____

The answers to "and then" problems are called sums - even though you sometimes use subtraction to get the answer.

EXERCISES E. Find these "and then" sums.

1. $^{+}10 + ^{-}16 =$ ____
2. $^{-}8 + ^{-}20 =$ ____
3. $^{-}15 + ^{+}10 =$ ____
4. $^{+}11 + ^{-}7 =$ ____
5. $^{-}17 + ^{+}5 =$ ____
6. $^{-}17 + ^{-}10 =$ ____
7. $^{-}6 + ^{+}14 =$ ____
8. $^{+}8 + ^{+}25 =$ ____
9. $^{-}8 + ^{+}25 =$ ____
10. $^{+}8 + ^{-}25 =$ ____

Check yourself. If you missed more than 2 problems, ask your teacher for help.

EXERCISES F. More sums.

1. $^{+}23 + ^{-}10 =$ ____
2. $^{+}25 + ^{+}17 =$ ____
3. $^{-}16 + ^{+}17 =$ ____
4. $^{-}16 + ^{-}6 =$ ____
5. $^{+}26 + ^{-}10 =$ ____
6. $^{-}40 + ^{+}25 =$ ____
7. $^{+}36 + ^{-}20 =$ ____
8. $^{-}18 + ^{-}9 =$ ____
9. $^{-}21 + ^{+}3 =$ ____
10. $^{-}100 + ^{-}300 =$ ____
11. $^{+}400 + ^{-}100 =$ ____
12. $^{+}956 + ^{-}4 =$ ____

When you think you are ready, do the A test.

Dear Sixth Grade Teacher:

Do you have at least one student who could afford to miss the usual beginning-of-the-year review of whole numbers? If so, would you be willing to have one such student use some new instructional material as a part of the research design for my doctoral thesis?

If you cannot participate, you need not do anything - just throw this whole package in the waste basket and forget it.

But, in case you are saying "yes", I have enclosed the necessary materials and instructions and I hope that you will continue.

Since I am studying the effectiveness of the materials, there are only three things for you to do.

1. Choose a student and give him/her the equivalent of six 30-45 minute periods to work independently on the enclosed material.

2. Check 3 or 4 short tests with the student.

3. "Guard" the tests and mail them back to me in the enclosed envelope by September 23.

INSTRUCTIONS

Choosing the student -- The ideal student for this experiment will be independent, highly motivated and a good reader, but not so mathematically brilliant that he/she has already "figured out" how to add and subtract with positive and negative numbers.

Administering the tests -- The tests are stapled in the student's packet of material at the point where they should be used. You need not time the tests. Most students will finish quickly. The answer key for the tests is included at the end of this letter.
Note: The tests will probably not be in the same order as they appear on the key, but each test is readily identifiable. Please allow the student a few minutes to look over the test before you put it in the envelope to mail to me.
IMPORTANT: The student should not take the FINAL TEST immediately after the preceding test. A period of 3-24 hours should intervene.

Returning results -- I am sending out more packets than I need for a good study, to allow for "shrinkage". If, in your opinion, something has happened - the student was ill, or you decide that you made a poor choice of student, or whatever - please indicate that you consider the data from your school to be invalid. A note in your envelope with the tests will do that without "insulting" the student. Of course, low test scores, by themselves, would not be invalid.

I am working under a nearly impossible deadline, which explains the ridiculous deadline that I have given you. Your efforts in meeting it will be most appreciated!

If you would like to have a summary of the data and an abstract of the thesis, please enclose a self-addressed envelope. You may wish to note the color of your tests and the scores for comparison with the summary when it arrives.

Thank you very much for reading this far! If you can participate, I feel sure the experience will be interesting for your student and both you and your student will be making a contribution to the improvement of math education! The world may not thank you, but I will!

Sincerely,

Shirley M. Davis

Congratulations!

Your teacher has chosen you to take part in a small experiment. About 200 other 6th graders all over Wisconsin were also chosen. The experiment is to find out whether these materials can be read and understood by 6th grade students working independently. For that reason it is important that you only ask the teacher for help when it is absolutely necessary.

HOW TO USE THIS BOOKLET

You may write in the booklet and keep it. The booklet is stapled in the upper corner to make it easy to fold back pages after you finish them. Answers to each set of problems are on the back of the page before.

Example: the answers to page 7 are on the back of page 6. You will just unfold your booklet to check a set of problems.

HOW TO USE THE ANSWERS

Answers are provided to help you learn. It is important that you use them correctly.

Step 1. Do the first 2 or 3 problems in the set.

Step 2. Check these problems.

A. If correct - continue.

B. If wrong - read the lesson and instructions again and try the problems again. Check again.

1. If correct - continue.

2. If wrong - ask the teacher for help.

Step 3. Finish the set of problems and check. Study the problems you missed. Ask your teacher if you don't know why they are wrong.

HOW TO USE THE TESTS

When you come to a TEST, do the test, ask your teacher to check it and let you look it over. Then your teacher will keep it to mail back to the experimenter.

Thank you for taking part in the experiment and Good Luck!

TEST A

Remove this page and do the test when you are sure that you are ready. Then ask your teacher to check it and let you see what you missed. Your teacher will keep the test to send to the experimenter.

1. $^{+}4 + {}^{+}6 =$ ___

2. $^{+}5 + {}^{-}1 =$ ___

3. $^{-}8 + {}^{+}3 =$ ___

4. $^{-}2 + {}^{-}5 =$ ___

5. $^{+}10 + {}^{+}10 =$ ___

6. $^{+}4 + {}^{-}6 =$ ___

7. $^{-}8 + {}^{-}3 =$ ___

8. $^{+}10 + {}^{-}10 =$ ___

9. $^{-}5 + {}^{+}1 =$ ___

10. $^{+}2 + {}^{+}5 =$ ___

11. $^{+}8 + {}^{+}3 =$ ___

12. $^{-}5 + {}^{-}1 =$ ___

13. $^{-}4 + {}^{+}6 =$ ___

14. $^{-}10 + {}^{-}10 =$ ___

15. $^{+}2 + {}^{-}5 =$ ___

16. $^{-}10 + {}^{+}10 =$ ___

17. $^{-}2 + {}^{-}5 =$ ___

18. $^{+}8 + {}^{-}3 =$ ___

19. $^{+}5 + {}^{+}1 =$ ___

20. $^{-}4 + {}^{-}6 =$ ___

OPINION QUESTIONS

These pages were: easy just right difficult for me.

My regular math work is usually: easy just right difficult for me.

These pages were: interesting O.K. boring.

My regular math work is usually: interesting O.K. boring.

NAME________________ NUMBER CORRECT______ DATE______

SCHOOL________________ TEACHER'S SIGNATURE__________

TEST S

Remove this page and do the test when you are sure that you are ready. Then ask your teacher to check it and let you see what you missed. Your teacher will keep the test to send to the experimenter.

1. $^{+}4 - {}^{+}6 =$ ___
2. $^{+}5 - {}^{+}1 =$ ___
3. $^{-}8 - {}^{+}3 =$ ___
4. $^{-}2 - {}^{-}5 =$ ___
5. $^{+}10 - {}^{+}10 =$ ___
6. $^{+}4 - {}^{-}6 =$ ___
7. $^{-}8 - {}^{-}3 =$ ___
8. $^{+}10 - {}^{-}10 =$ ___
9. $^{-}5 - {}^{+}1 =$ ___
10. $^{+}2 - {}^{+}5 =$ ___
11. $^{+}8 - {}^{+}3 =$ ___
12. $^{-}5 - {}^{-}1 =$ ___
13. $^{-}4 - {}^{+}6 =$ ___
14. $^{-}10 - {}^{-}10 =$ ___
15. $^{+}2 - {}^{-}5 =$ ___
16. $^{-}10 - {}^{+}10 =$ ___
17. $^{-}2 - {}^{-}5 =$ ___
18. $^{+}8 - {}^{-}3 =$ ___
19. $^{+}5 - {}^{+}1 =$ ___
20. $^{-}4 - {}^{-}6 =$ ___

OPINION QUESTIONS

These pages were:	easy	just right	difficult	for me.
My regular math work is usually:	easy	just right	difficult	for me.
These pages were:	interesting	O.K.	boring.	
My regular math work is usually:	interesting	O.K.	boring.	

NAME______________________ NUMBER CORRECT____ DATE_____

SCHOOL______________________ TEACHER'S SIGNATURE______________

TEST F - FINAL

Part A - Please allow from 3-24 hours between your other test and this one. Ask your teacher to check it and let you see what you missed. Your teacher will keep the test to send to the experimenter. You may do either part first.

1. ${}^{-}4 + {}^{-}6 =$ ___
2. ${}^{+}5 + {}^{+}1 =$ ___
3. ${}^{+}8 + {}^{-}3 =$ ___
4. ${}^{-}2 + {}^{-}5 =$ ___
5. ${}^{-}10 + {}^{+}10 =$ ___
6. ${}^{+}2 + {}^{-}5 =$ ___
7. ${}^{-}10 + {}^{-}10 =$ ___
8. ${}^{-}4 + {}^{+}6 =$ ___
9. ${}^{-}5 + {}^{-}1 =$ ___
10. ${}^{+}8 + {}^{+}3 =$ ___
11. ${}^{+}2 + {}^{+}5 =$ ___
12. ${}^{-}5 + {}^{+}1 =$ ___
13. ${}^{+}10 + {}^{-}10 =$ ___
14. ${}^{-}8 + {}^{-}3 =$ ___
15. ${}^{+}4 + {}^{-}6 =$ ___
16. ${}^{+}10 + {}^{+}10 =$ ___
17. ${}^{-}2 + {}^{-}5 =$ ___
18. ${}^{-}8 + {}^{+}3 =$ ___
19. ${}^{+}5 + {}^{-}1 =$ ___
20. ${}^{+}4 + {}^{+}6 =$ ___

Part B.

1. ${}^{-}4 - {}^{-}6 =$ ___
2. ${}^{+}5 - {}^{+}1 =$ ___
3. ${}^{+}8 - {}^{-}3 =$ ___
4. ${}^{-}2 - {}^{-}5 =$ ___
5. ${}^{-}10 - {}^{+}10 =$ ___
6. ${}^{+}2 - {}^{-}5 =$ ___
7. ${}^{-}10 - {}^{-}10 =$ ___
8. ${}^{-}4 - {}^{+}6 =$ ___
9. ${}^{-}5 - {}^{-}1 =$ ___
10. ${}^{+}8 - {}^{+}3 =$ ___
11. ${}^{+}2 - {}^{+}5 =$ ___
12. ${}^{-}5 - {}^{+}1 =$ ___
13. ${}^{+}10 - {}^{-}10 =$ ___
14. ${}^{-}8 - {}^{-}3 =$ ___
15. ${}^{+}4 - {}^{-}6 =$ ___
16. ${}^{+}10 - {}^{+}10 =$ ___
17. ${}^{-}2 - {}^{-}5 =$ ___
18. ${}^{-}8 - {}^{+}3 =$ ___
19. ${}^{+}5 - {}^{+}1 =$ ___
20. ${}^{+}4 - {}^{+}6 =$ ___

NAME________________________________ NUMBER CORRECT ______ ______
Part A Part B

SCHOOL____________________________ TEACHER'S SIGNATURE____________________

TEST P - PRE-TEST

Directions: You are not expected to know very many of these answers, but you are welcome to make a "good guess" on each question.

1. $^{+}8 + ^{+}3 =$ ____
2. $^{-}5 + ^{-}1 =$ ____
3. $^{-}4 + ^{+}6 =$ ____
4. $^{-}10 + ^{-}10 =$ ____
5. $^{+}2 + ^{-}5 =$ ____
6. $^{-}10 + ^{+}10 =$ ____
7. $^{-}2 + ^{-}5 =$ ____
8. $^{+}8 + ^{-}3 =$ ____
9. $^{+}5 + ^{+}1 =$ ____
10. $^{-}4 + ^{-}6 =$ ____
11. $^{+}4 + ^{+}6 =$ ____
12. $^{+}5 + ^{-}1 =$ ____
13. $^{-}8 + ^{+}3 =$ ____
14. $^{-}2 + ^{-}5 =$ ____
15. $^{+}10 + ^{+}10 =$ ____
16. $^{+}4 + ^{-}6 =$ ____
17. $^{-}8 + ^{-}3 =$ ____
18. $^{+}10 + ^{-}10 =$ ____
19. $^{-}5 + ^{+}1 =$ ____
20. $^{+}2 + ^{+}5 =$ ____
21. $^{+}8 - ^{+}3 =$ ____
22. $^{-}5 - ^{-}1 =$ ____
23. $^{-}4 - ^{+}6 =$ ____
24. $^{-}10 - ^{-}10 =$ ____
25. $^{+}2 - ^{-}5 =$ ____
26. $^{-}10 - ^{+}10 =$ ____
27. $^{-}2 - ^{-}5 =$ ____
28. $^{+}8 - ^{-}3 =$ ____
29. $^{+}5 - ^{+}1 =$ ____
30. $^{-}4 - ^{-}6 =$ ____
31. $^{+}4 - ^{+}6 =$ ____
32. $^{+}5 - ^{+}1 =$ ____
33. $^{-}8 - ^{+}3 =$ ____
34. $^{-}2 - ^{-}5 =$ ____
35. $^{+}10 - ^{+}10 =$ ____
36. $^{+}4 - ^{-}6 =$ ____
37. $^{-}8 - ^{-}3 =$ ____
38. $^{+}10 - ^{-}10 =$ ____
39. $^{-}5 - ^{+}1 =$ ____
40. $^{+}2 - ^{+}5 =$ ____

NAME________________________________ NUMBER CORRECT________

SCHOOL______________________________ TEACHER'S SIGNATURE____________________

SUBTRACTION

Now that you know how to find SUMS of positive and negative numbers, the next job is to find DIFFERENCES.

Here is a way to think about SUMS and DIFFERENCES.

Jill is playing a game. On each turn she has 2 chances to gain points.

Turn	1st try	2nd try	Total Change	Math Sentence
1	6	3	9	6 + 3 = 9
2	0	1	1	0 + 1 = 1
3	8	2	?	8 + 2 = ?
4	10	0	?	?
5	5	4	?	?

Answers: 10   8 + 2 = 10
10   10 + 0 = 10
9   5 + 4 = 9

Simple addition shows the total changes in her score for each turn.

Joe is playing a different game. On each turn he has 2 chances to gain or lose points.

Turn	1st try	2nd try	Total Change	Math Sentence
1	+2	+3	+5	+2 + +3 = +5
2	-3	+4	+1	-3 + +4 = +1
3	-3	-2	?	-3 + -2 = ?
4	+1	-8	?	?
5	0	-2	?	?

Answers: -5   -3 + -2 = -5
-7   +1 + -8 = -7
-2   0 + -2 = -2

"And Then" addition shows the total change in his score for each turn.

Simple "takeaway" subtraction shows what Jill gets on the second try when we know the first try and the total change.

Turn	1st Try	2nd Try	Total Change	Math Sentence: Add	Math Sentence: Subtraction
1	6	☐	10	6 + ☐ = 10	10 - 6 = ☐
2	3	☐	8	3 + ☐ = 8	8 - 3 = ☐
3	0	☐	9	0 + ☐ = 9	9 - 0 = ☐
4	2	☐	3	2 + ☐ = 3	3 - 2 = ☐

If we try the same thing with "and then addition", it isn't so easy. Sometimes it looks like "Takeaway" and sometimes it does not. But we will still call the answer the Difference and we will still use a subtraction (-) sign.

Turn	1st Try	2nd Try	Total Change	Math Sentence: And then	Math Sentence: Difference
1	+6	☐	+10	+6 + ☐ = +10	+10 - +6 = ☐
2	+6	☐	+4	+6 + ☐ = +4	+4 - +6 = ☐
3	-7	☐	+3	-7 + ☐ = +3	+3 - -7 = ☐
4	-8	☐	-7	-8 + ☐ = -7	-7 - -8 = ☐

The problem is -

How do you get the answer if you can't use "takeaway?"

It is possible but difficult to use the missing addend sentence to get the answer.

Instead

We will use the missing addend sentence to develope an easier way to find the answers.

Remember, that in whole number subtraction, the answer to a subtraction problem is a missing addend.

For example:

the answer to this problem — $8 - 6 = \square$

must also fit this problem — $\square + 6 = 8$

Now use the same idea with positive and negative numbers.
Try these. Read them out loud (quietly).

	Difference		Subtraction Sentence		Addition Sentence
1.	$\boxed{^{+}2}$	is the answer to	$^{+}8 - {}^{+}6 = \square$	because it fits	$\square + 6 = 8$
2.	$\square$	is the answer to	$^{+}9 - {}^{+}3 = \square$	because it fits	$\square + 3 = 9$
3.	$\square$	is the answer to	$^{-}7 - {}^{-}6 = \square$	because it fits	$\square + 6 = 7$
4.	$\boxed{^{+}12}$	is the answer to	$^{+}10 - {}^{-}2 = \square$	because it fits	$\square + 2 = 10$

Look at the same four subtraction problems with their answers.

Match each subtraction to a new addition sentence. These additions don't have missing addends

$^{+}8 - {}^{+}6 = {}^{+}2$ ———————— $^{+}8 + {}^{-}6 = {}^{+}2$

$^{+}9 - {}^{+}3 = {}^{+}6$ ———————— $^{+}9 + {}^{-}3 = {}^{+}6$

$^{-}7 - {}^{-}6 = {}^{-}1$ ———————— $^{-}7 + {}^{+}6 = {}^{-}1$

$^{+}10 - {}^{-}2 = {}^{+}12$ ———————— $^{+}10 + {}^{+}2 = {}^{+}12$

In each pair, subtracting a number gives the same answer as adding the opposite of the number. If this would always work, you wouldn't need to learn how to subtract. You could just add the opposite instead.

IT DOES ALWAYS WORK! TRY IT!

EXERCISES: Check <u>each</u> <u>group</u> of 5 problems before you do the next.

Subtraction Problem — To get the answer, <u>add the opposite</u>

Example: $^{+}6 - \enclose{circle}{+}2 = \boxed{+4}$ answer $\qquad ^{+}6 + \enclose{circle}{-}2 = \boxed{+4}$ answer

A.

$^{+}6 - {}^{+}2 = \square \qquad ^{+}6 + {}^{-}2 = \square$

$^{+}8 - {}^{+}3 = \square \qquad ^{+}8 + {}^{-}3 = \square$

$^{+}4 - {}^{+}12 = \square \qquad ^{+}4 + {}^{-}12 = \boxed{-8}$

$^{+}3 - {}^{+}7 = \square \qquad ^{+}3 + {}^{-}7 = \square$

$^{+}2 - {}^{+}8 = \square \qquad \triangle(^{+}2) + \bigcirc = \square$

B

$^{+}3 - {}^{+}10 = \square \qquad \triangle + \bigcirc = \square$

$^{+}7 - {}^{+}4 = \square \qquad \triangle + \bigcirc = \square$

$^{-}12 - {}^{-}3 = \square \qquad ^{-}12 + {}^{+}3 = \square$

$^{-}8 - {}^{-}2 = \square \qquad ^{-}8 + \bigcirc = \square$

$^{-}11 + {}^{-}6 = \square \qquad ^{-}11 + \bigcirc = \square$

C

$^{-}9 - {}^{-}10 = \square \qquad ^{-}9 + \enclose{circle}{^{+}10} = \square$

$^{-}7 - {}^{-}9 = \square \qquad ^{-}7 + \bigcirc = \square$

$^{-}3 - {}^{-}8 = \square \qquad \triangle + \bigcirc = \square$

$^{-}2 - {}^{-}9 = \square \qquad \triangle + \bigcirc = \square$

$^{+}6 - {}^{+}2 = \square \qquad \triangle + \bigcirc = \square$

D.

$^{+}6 - {}^{+}1 = \square \qquad ^{+}6 + {}^{-}1 = \square$

$^{+}6 - 0 = \square \qquad ^{+}6 + 0 = \square$

$^{+}6 - \enclose{circle}{^{-}1} = \square \qquad ^{+}6 + {}^{+}1 = \boxed{+7}$

$^{+}6 - {}^{-}2 = \square \qquad ^{+}6 + \bigcirc = \square$

$6 - 3 = \square \qquad ^{+}6 + \bigcirc = \square$

E.

$^{-}7 - {}^{+}3 = \square \qquad -7 + {}^{-}3 = \square$

$-7 - {}^{+}6 = \square \qquad -7 + \bigcirc = \square$

$^{-}7 - {}^{+}2 = \square \qquad -7 + \bigcirc = \square$

$-7 - {}^{+}4 = \square \qquad \triangle + \bigcirc = \square$

$-7 - {}^{+}8 = \square \qquad \triangle + \bigcirc = \square$

F.

$^{+}6 - {}^{+}4 = \square \qquad \triangle + \bigcirc = \square$

$^{-}6 - {}^{+}4 = \square \qquad \triangle + \bigcirc = \square$

$^{+}3 - {}^{-}3 = \square \qquad \triangle + \bigcirc = \square$

$^{-}3 - {}^{-}2 = \square \qquad \triangle + \bigcirc = \square$

$^{+}1 - {}^{+}8 = \square \qquad \triangle + \bigcirc = \square$

PRACTICE

A.

1. +8 —— +12 =
2. -8 —— -12 =
3. +3 —— +1 =
4. +1 —— -3 =
5. -5 —— +6 =
6. +5 —— -6 =
7. +34 —— +41 =
8. -34 —— -41 =
9. -18 —— -11 =
10. +18 —— -11 =
11. +25 —— -10 =
12. +25 —— +10 =
13. -25 —— +10 =
14. -25 —— +25 =
15. -10 —— +25 =

B.

1. +4 − -6 =
2. +7 − +2 =
3. +5 − +11 =
4. +8 − -1 =
5. -4 − +6 =
6. -7 − -2 =
7. -5 − -11 =
8. -8 − +1 =
9. +9 − +17 =
10. -9 − -17 =

C.

1. +9 − -17 =
2. -9 − +17 =
3. +21 − +13 =
4. -4 − -8 =
5. +3 − -9 =
6. -16 − -10 =
7. -38 − +6 =
8. -1 − -7 =
9. +12 − -13 =
10. -6 − +15 =

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