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PREDICTION EQUATIONS AND A
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FOR
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TIMOTHY D. REY

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[signature]
Major professor

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# PREDICTION EQUATIONS AND A DETERMINISTIC ALGORITHM FOR CROWN SURFACE AREA

By

Timothy D. Rey

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# ABSTRACT

## PREDICTION EQUATIONS AND A DETERMINISTIC ALGORITHM FOR CROWN SURFACE AREA

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Timothy D. Rey

Crown surface area, defined as the surface area of the geometric solid formed by the extremities of the crown in all dimensions, was evaluated in this study. It was theorized that crown surface area could be predicted from various crown and tree parameters.

A computational algorithm and prediction equations were developed for four different hardwood species. The computational algorithm was very precise and provided a good non-stochastic estimate of crown surface area. The prediction equations all had low standard errors of the estimates (highest equaling 66.200 $m^2$), low standard errors of the regression coefficients and high coefficients of determination (lowest equaling 0.952). It was determined that average crown radius and total crown length were the important variables in accounting for the major portion of the variation in crown surface area.

## ACKNOWLEDGMENTS

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## PROBLEM STATEMENT

Crown surface area, defined as the surface area of the geometric solid formed by the extremities of the crown in all dimensions, is applicable to three specific areas of forestry. First, in many existing individual tree models, such as Arney (1972), Heygi (1974), Ek and Monserud (1974), and Burkhard (1975), crown surface area is used to determine the competition component of the model. Second, the surface area of a tree's crown may be used as an indicator of growth potential. Third, if diameter breast height (dbh.) can be predicted from tree characteristics measureable in aerial photos, such as crown width, the work involved in sampling for estimates of volume per unit area would be greatly reduced.

In forestry, major emphasis is placed on making correct managerial decisions concerning different levels of silvicultural treatments. In the past, most of these decisions were either based on stand averages such as number of trees or volume per unit area, or on measures of competition such as crown competition factors and point density. More recently, researchers have been developing individual tree models to provide managerial information. The reasoning behind this change is based on the idea that a stand is made up of individuals and each individual contributes a given amount to the entire stand.

Applying silvicultural treatments to individual trees is impossible. Therefore, the basic thrust for individual tree modeling lies in understanding the microenvironment around each tree. One of the most important influences on a tree's growth is competition; almost every individual tree model developed has included a competition index. Mitchell (1969), in outlining the major factors of competition, used crown parameters to assess inter-tree competition because the carbohydrates needed for tree growth are produced in the foliage. Other advantages of using crown parameters were that crowns can be measured more readily than root systems, and that the size of the root system is likely reflected in the dimensions of the crown. Therefore, a true assessment of the surface area of a tree's crown, as influenced by surrounding trees, would in fact be of interest in assessing a tree's ability to grow.

In the past researchers developed competition indices that were concerned only with that portion of the crown that received direct sunlight. Beauregard (1975) used an ellipsoid, defined by Equation 1,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \qquad (1)$$

where x, y, and z are variable Cartesian Coordinates, a = maximum crown radius, b = minimum crown radius, and c = 1/2(crown length), to describe the form of a hardwood crown. However, he was concerned only with that portion of the crown that was intercepting direct sunlight. Hatch, Gerrard, and Trappeiner (1975) used cones to describe crown form for coniferous species. Here again, the competition index developed only dealt with the "exposed crown surface area".

Zimmerman and Brown (1977) described the importance of considering the whole crown and its true form:

> "Light, together with gravity, is one of the most important constituents of the environment in determining the course of development in woody plants. The growth and form of trees from the time of seed germination to maturity is directly affected by light intensity, quality and duration. One of the most commonly described effects of light on the direction of growth is the general phenomena of phototropism. Phototropic responses of individual twigs play an important role in the positioning of branches in the much-branched crowns of decurrent species. Light, gravity, and competition for growing space interact to determine the overall size and shape of tree crowns. That light has a direct formative effect on tree crowns is readily seen by unilateral growth of branches into openings in the forest canopy created by partial cuttings or natural causes."

Bormann (1958) and Kramer and Kozlowski (1960) claimed that the considerable variation among photosynthesis rates within a given crown was primarily due to the different stages of phenological development exhibited by any species. Helm (1976) expanded this idea and found that part of this variation was due to both the effects of mutual shading and to differences in environment within tree crowns. These differences in environment led to the idea of sun and shade leaves.

Helm (1976) compiled evidence from other researchers to show that the whole crown should be considered rather than just that portion receiving direct sunlight:

> "Logan (1970) demonstrated that photosynthesis of shade leaves of yellow birch (*Betula alleghaniensis*) was at a higher rate under conditions of low light intensity, also that light saturation was reached at lower light levels and at lower rates of net photosynthesis, than in sun leaves. In natural stands of douglas-fir (*Psuedotsuga menziesii*), Woodman (1971) showed that the most productive

conditions in the upper crown."

Therefore, when evaluating a tree's potential to produce photosynthate, such values may be underestimated if only that portion of the crown receiving direct sunlight is considered. For this reason, the entire tree crown was considered in this study.

In terms of aerial photography, it is easier to measure crown diameter on an aerial photograph than dbh., merchantable height, or volume. If a relationship exists between crown surface area and either dbh. or total height, then the relationship between crown surface area (CSA) and dbh. can in turn be used to predict volumes. Macabeo (1952) determined that stump diameter, merchantable height, and merchantable volume each had significant relationships with crown diameter. The following three equations, accompanied with their respective coefficients of determination ($R^2$) and standard errors of the estimates (SEE), are for white luaun (Pentacme contorta) where X is crown diameter:

$$\text{Stump Diameter} = 28.93 + 4.268X \qquad (2)$$
$$R^2 = 0.961,\ \text{SEE} = \pm 4.98$$

$$\text{Merchantable Length} = 18.216 + 0.233X \qquad (3)$$
$$R^2 = 0.948,\ \text{SEE} = \pm 1.39$$

$$\text{Merchantable Volume} = 1.915 + 0.926X \qquad (4)$$
$$R^2 = 0.967,\ \text{SEE} = \pm 1.03$$

There have been three basic uses for crown surface area outlined in the preceeding pages of this section. It was felt that these three uses show a substantial need for determining whether or not there was a relation between crown surface area and measureable tree parameters.

## OBJECTIVE

Crown surface area, defined as the surface area of the geometric solid formed by the extremities of the crown in all dimensions, was the focal point of this study. Three models were developed with the primary objective of estimating crown surface area from variables readily obtained in the field. Model I, the maximum model, contained crown parameters, dbh., and total height. Model II contained only two variables, dbh. and total height (TOTHT), while Model III contained only one variable, dbh.. Analysis involving Model I was concerned with determining which variables were biologically significant in predicting crown surface area. Models II and III were primarily application models, in that their development was directed at predicting crown surface area with a minimal number of accessible tree characteristics.

## LITERATURE REVIEW

Estimation of various crown parameters has been done in the past. Determining the relationship of these crown parameters to diameter increment was one of the initial reasons for studying crown parameters. Heck (1924) concluded that, with the present state of knowledge, it was not possible to obtain dependable correlations between crown diameter and the width of annual rings. Busse (1930) used graphical representations to correlate tree crown measurements and diameter growth and found no strong relationships. Macon (1939) concluded that no physical dimensions or characteristics of the crown could be employed to express quantitative diameter growth.

Holsoe (1948) successfully regressed basal area growth of red oak (*Quercus rubra*) and white ash (*Fraxinus americana*) on crown diameter (correlation coefficients of 0.927 and 0.867, respectively). Holsoe used a paraboloid of the form in Equation 5,

$$A = \frac{\pi r}{6h^2}((4h^2 + r^3)^{3/2} - r^3) \qquad (5)$$

where r = 1/2(crown diameter) and h = crown length, defined as the length of the crown from maximum crown radius to the top of the tree. This study found that a better relationship existed between crown dimensions and basal area growth than between crown diameter and width of annual rings or diameter growth.

All of the above studies were used as stepping stones to determine basal area growth or some other growth increment. None of the studies contemplated predicting crown surface area directly from stem parameters or from crown parameters.

Berlyn (1962) believed that crown surface area was a good measure of current photosynthetic area and thus theoretically should be closely related to current volume growth. Berlyn developed Equation 6, an equation for crown surface area:

$$CSA = \frac{\pi R^4}{6Z^2} \left(-1 + \left(1 + \frac{4Z^2}{R^2}\right)^{3/2} \qquad (6)$$

where R equals crown radius. This equation was derived by integrating the general formula for a paraboloid with circular base $Z = kR^2$. For *k* large, a narrow paraboloid results; for *k* small a fat paraboloid results; and for *k* = zero, a flat disk results. Other researchers, Beauregard (1975) and Hatch, Gerrard, And Tappeinen (1975) used ellipsoids or cones to describe the form of the crown. Again, none of these studies were involved with predicting CSA from tree or crown parameters.

## RESEARCH HYPOTHESIS AND MODEL SELECTION

### BACKGROUND AND HYPOTHESIS

Since this study is concerned with the estimation of crown surface area, the appropriate parameters had to be chosen to accomplish this task. Many factors influence the diameter and height of a tree: tree height is influenced greatly by site quality and diameter is affected to a large extent by competition and stand density. The first step was to define the relationships of tree height and diameter with crown surface area, both mathematically and biologically.

Krajicek (1961) showed through regression analysis that crown width of an open-grown tree was closely related to its dbh.. Berlyn (1962) found that crown-stem relations were almost completely independent of six soil-site factors: site index; silt plus clay content; available water; foliar nitrogen; nitrifiable soil nitrogen; and available phosphorous. Mitchell (1965), while studying the relation between crown width (CW) and dbh. in coniferous species, found that estimation of dbh. was independent of stand density. He developed the relationship given in Equation 7,

$$DBH = 0.00626(CW)(AGE) + 0.00328(CW)(SI) \qquad (7)$$

where SI = site index. The standard error of the estimate equaled ±2.07 inches, with $R^2 = 0.88$ and a sample size (n) of 400 trees. Since point density was not an important factor in Equation 7 and the $R^2$ was fairly large it appeared that crown width and bole diameter reacted similarly to differences in stand density.

Minckler and Gingrich (1970) and Krajicek (1961) found similar relationships between crown width and tree diameters for both open-grown and forest-grown oak and hickory species. Diameter-crown width relationships were similar for well-stocked, uneven-aged stands, although variations in crown width were greater for forest grown trees. These relationships were also independent of site, crown class, and species.

These studies indicate that crown surface area will react to changes in site, stand density, and competition as do stem and crown parameters. It must be understood that the estimation of crown surface area is based on dbh., total height, average crown width and crown length. It is evident that whatever affects tree diameter or height will affect the relative size of the crown. But if tree diameter and height are used to estimate the crown surface area then the factors such as site and stand density have already been accounted for. For this reason, site parameters and stand density parameters were not included in the predicting equations.

Because crown length, crown diameter, dbh., and tree height react similarly to changes in stand density and site conditions, prediction of crown surface area should be possible using only the tree characteristics.

## ALTERNATIVE MODELS

The following three models will be developed for each species selected for this study. In order to see if the relationships developed between crown surface area and other crown parameters are the same for different species, relative to Minckler and Gingrich's (1970) work, another set of regression models will be developed using combined data.

Model I, the maximum model, will contain a function of dbh., total height, crown length, and a function of tree crown radii. The first reduced model, Model II, will contain dbh. or a function of dbh. and total tree height. This is because dbh. and total height are relatively fast and easy to measure as compared to crown length and crown radius. Model III, the simplest model of the three, will only contain a function of dbh..

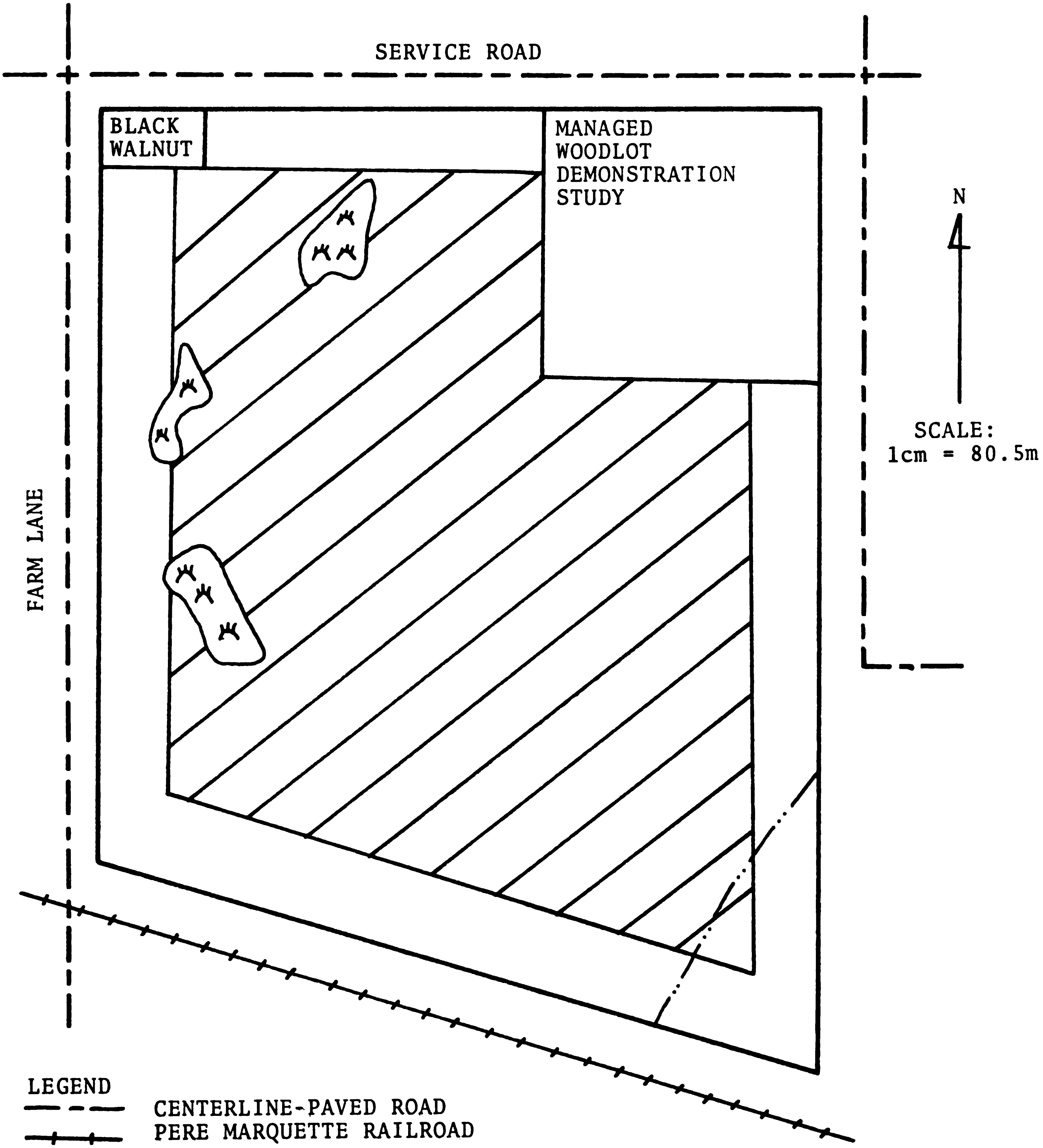
# STUDY AREA AND SPECIES SELECTION

## SITE DESCRIPTION

Baker Woodlot, Michigan State University woodlot #17, formerly known as Farm Lane Woodlot, was chosen for this study. Its close proximity as well as its relatively large species mix made it the most feasible choice. Baker Woodlot is located in the SW1/4 of Sec. 19, T4N R1W, Michigan Meridian, on the Michigan State University Campus. It lies on the SE corner of the intersection of Farm Lane and Service Road and contains 30.76 hectares (Figure 1). Baker Woodlot has an average elevation above sea level of 860 feet ±15 feet. Although there is some hardpan present, Baker mainly consists of a sandy-loam soil, specifically Hillsdale sandy-loam and Connover loam.

## HISTORY

Forest management practices were not considered for Baker Woodlot until December 11, 1894. At this time the State Board of Agriculture decided that all of the University's woodlots should be brought up to "creditable conditions" via professional management strategies. Annual harvests occured from 1894 to approximately the early 1920's; however, records of these removals are few and unreliable. In the late 1920's permanent growth plots were established to help monitor the progress of Baker. Baker Woodlot experienced its heaviest cutting in 1939. After this cutting, a working plan was drawn up by Paul A. Herbert to summarize the present condition of Baker and to establish future management strategies.




LEGEND
CENTERLINE-PAVED ROAD
PERE MARQUETTE RAILROAD
LOWLAND
DITCH

FIGURE 1. Baker Woodlot, Sec. 19, T4N R1W, Michigan Meridian.

Historically, Baker Woodlot has been used for instructional purposes. However, its main purpose has been the demonstration of timber production, using silvicultural treatments to improve species composition, growth, age class distribution, stocking, and health. As Baker Woodlot is an uneven-aged mixed hardwood stand, selection cuttings have been done for harvest and reproduction. Intermediate cuttings have been limited to weeding, thinning, and improvement. A rotation age was not selected in Herbert's plan but the cutting cycle was set at one year. This plan was followed until 1958, when a similar management plan was developed and implemented.

Since 1940 the most evident change due to managerial practices has been the change in species composition. By means of the intermediate improvement cuttings, Baker Woodlot has been approaching an American Beech-Sugar Maple climax forest. Selection cutting was again chosen for the 1958 plan, and again no rotation age was selected.

## DETERMINATION OF AREA TO BE SAMPLED

Baker Woodlot is still used primarily for instructional purposes, mainly forestry and wildlife, although recreational use also occurs. There are designated research areas within Baker, but only two of these had any effect on the present study.

In the NW corner there is a Black Walnut plantation occupying approximately 0.39 hectares (Figure 1), while in the NE corner there is a 4.05 hectare Managed Woodlot Study. These two areas were omitted from the study area due to a significant change in species composition. A fifty meter buffer zone from surrounding roads and open fields was also taped off in order to decrease edge effect on the crowns. The shaded area in Figure 1 represents the study area as described above.

## SPECIES SELECTION

As mentioned earlier, Baker Woodlot was chosen partly because of the variety of species present. This study was concerned with four different species: sugar maple (*Acer saccarhum*), basswood (*Tilia americana*), red oak (*Quercus rubra*), and american beech (*Fagus grandifolia*). These four species were chosen for two reasons. First, they are the four most abundant species in the woodlot. This was determined by Rudolph (1973), Gammon (1958), and Herbert (1940). Second, these four species represent a wide variety of crown forms (Avery 1967). This variety was important in showing the flexibility of the deterministic model used to calculate the dependent variable, crown surface area.

It was felt that Baker Woodlot provided both a good species mix and an adequate number of sample points for each species. These factors, coupled with close proximity and easy accessibility, made Baker Woodlot an ideal study area.

## COMPUTATIONAL ALGORITHM FOR COMPUTING CROWN SURFACE AREA

Crown surface area has been defined as the surface area of the geometric solid formed by the extremities of the crown in all dimensions. For a tree, measuring or estimating surface area is not as easy as it is for cubes, spheres, or prolate spheroids. Deterministic equations (i.e., nonstochastic) have been developed for cubes, spheres, and prolate spheroids. Hardwood crowns are quite irregular but there have been previous attempts to describe the form of individual crowns.

Busgen and Munch (1929) discovered that the deliquescent branching habit of hardwoods can produce virtually any regular shape by varying the angle of branching and the degree of terminal dominance. Utilizing this concept, Horn (1971) developed the following equation to predict the polymorphic shapes of tree crowns:

$$X^a = (bY)^a = C^a \qquad (8)$$

The values a, b, and C are constants and X and Y are variable Cartesian Coordinates. If $a = 1$, the equation defines a straight line; if $a = 2$, the equation defines an ellipse. As $a$ approaches infinity the shape becomes more convex and eventually becomes rectangular. The ratio of height to width is given by $b$ and the absolute size of the final shape is given by $C$.

As this study was only concerned with hardwoods, there was no need for such a flexible equation. However, this equation is very useful if the researcher is willing to assume that the perimeter of the crown is defined by an ellipse. In this study this assumption was made in the following fashion.

In three dimensional space the concept of an ellipse translates into an ellipsoid (Equation 1). Rather than assuming the total crown was an ellipsoid the crown was partitioned into 16 sections as shown in Figure 2. It was decided to use 16 sections because 8 or less may not have given the wanted precision and 32 or more would have taken too much time. The equation of an ellipsoid is used to define the extremities of the crown for any one section (Figure 3).

The crown was divided into the following sections defined by the parameters given. The upper portion of the crown, above maximum crown radius (MCR), had as its parameters for the ellipsoid, crown radius (CR) and upper crown length (UCL), as shown in Figure 4. Lower crown length (LCL) and CR were used to describe the lower portion of the tree (Figure 4).

Each portion of the tree, upper and lower, was divided into eight forty-five degree sections (Figure 2). Given the assumption that the extremities of the crown for one section are defined by an ellipsoid over the specified forty-five degree section, then the surface area can be obtained by calculus integration techniques.

Define the shaded region k, in Figure 5, to be the 45° section over which the integration is to be carried out. Fuller (1964) gave Equation 9 as the equation for the surface area of a function in the form $Z = f(x,y)$.

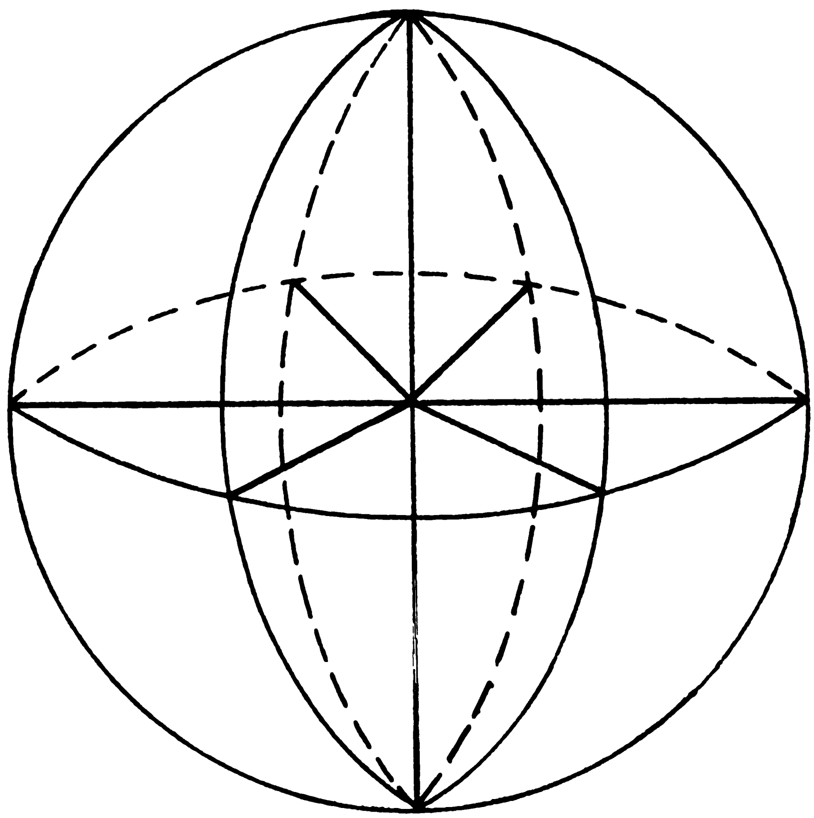


FIGURE 2. Partitioning of the crown.

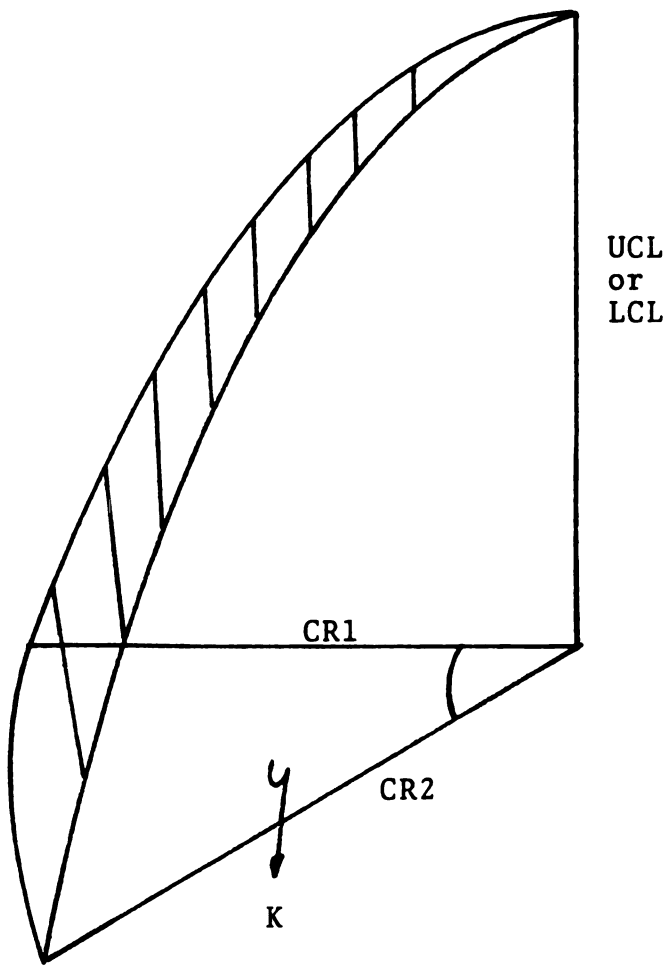



FIGURE 3. One $45^{o}$ section of the crown.

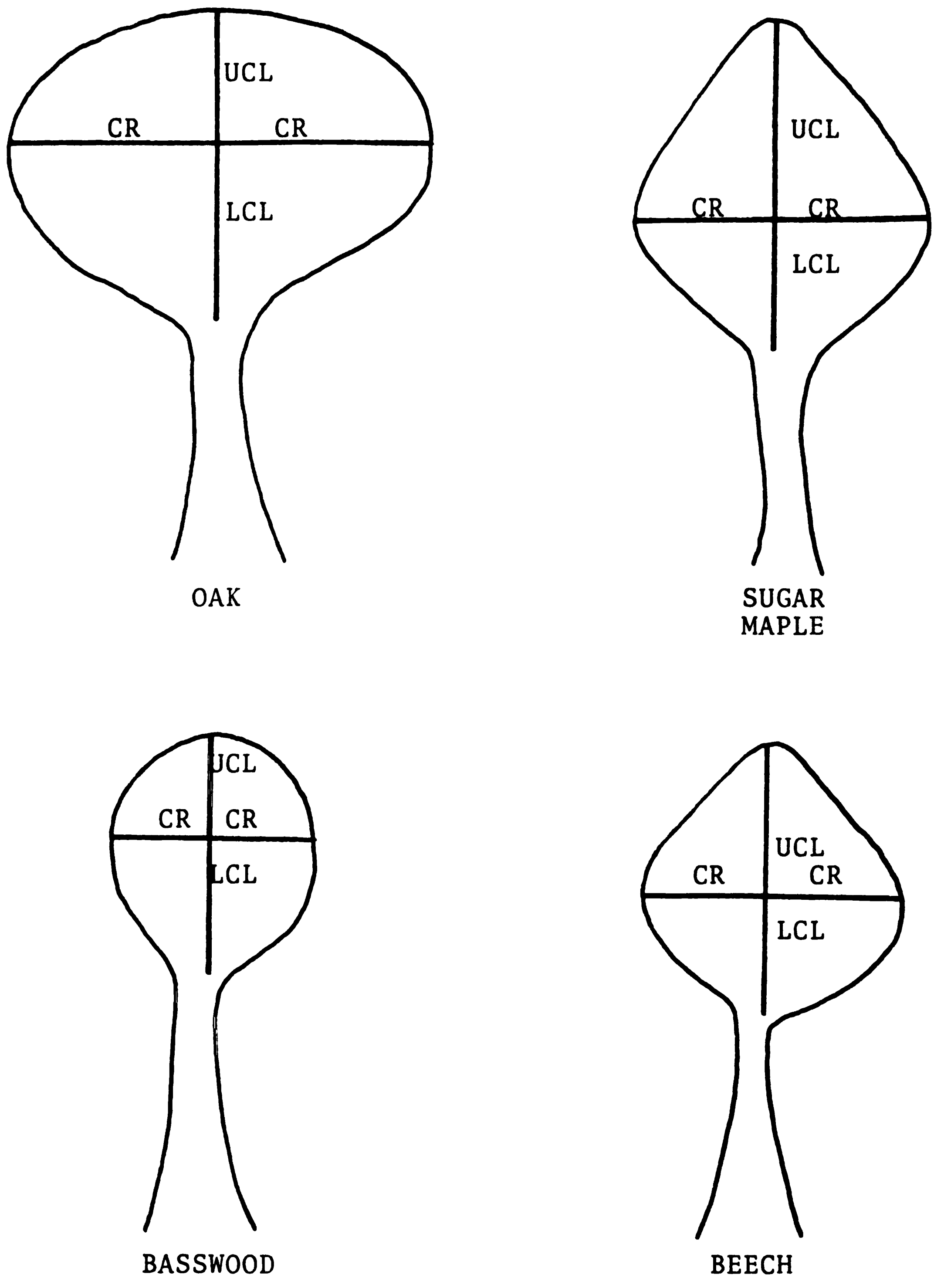



FIGURE 4. Establishment of upper crown length (UCL), lower crown length (LCL), and crown radius (CR) for the study species.

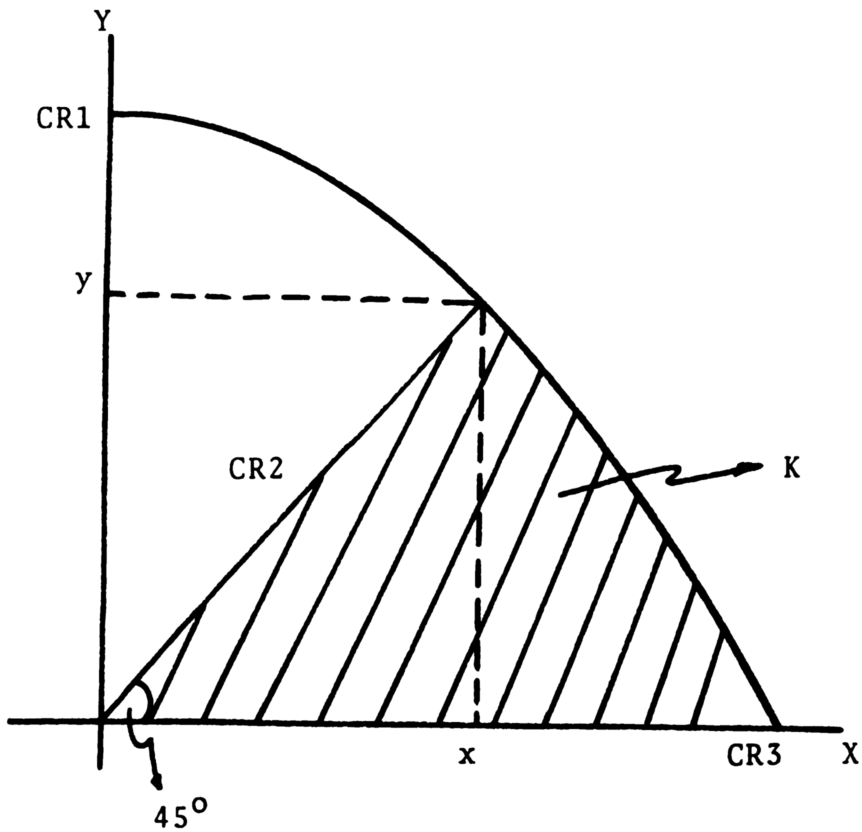



FIGURE 5. Integration base for a 45° section of the crown.

$$\iint_{x\,y}\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dydx \tag{9}$$

Given that $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \tag{1}$$

then $$z = C\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}} \tag{10}$$

producing $$\frac{\partial z}{\partial x} = \frac{(-xc)}{a^2\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{1/2}} \tag{11}$$

and $$\frac{\partial z}{\partial y} = \frac{(-yc)}{b^2\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{1/2}} \tag{12}$$

Therefore Equation 9 becomes

$$\iint_{x\,y}\sqrt{1 + \frac{(x^2c^2)}{a^4\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)} + \frac{(y^2c^2)}{b^4\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)}}\ dydx \tag{13}$$

which simplifies to

$$\iint_{x\,y}\sqrt{\frac{a^4b^4 - a^2b^4x^2 - a^4b^2y^2 + c^2(b^4x^2 + a^4y^2)}{a^4b^4 - a^2b^4x^2 - a^4b^2y^2}}\ dydx \tag{14}$$

Let $x = ar\cos\theta$ and $y = br\sin\theta$. This substitution simplifies the limits of integration (Figure 6). Then by the definition of the Jacobian, $\frac{\partial(x,y)}{\partial(r,\theta)}$ becomes $abrdrd\theta$ and Equation 14 is now

$$\iint_{r\,\theta}\sqrt{\frac{a^4b^4 - a^4b^4r^2\cos^2\theta - a^4b^4r^2\sin^2\theta + c^2(b^4a^2r^2\cos^2\theta + a^4b^2r^2\sin^2\theta)}{(a^4b^4 - a^4b^4r^2\cos^2\theta - a^4b^4r^2\sin^2\theta)}}\ abrdrd\theta \tag{15}$$

which simplifies to

$$\int_0^{\tan^{-1}a/b}\int_0^1\sqrt{\frac{a^2b^2(1-r^2) + b^2c^2r^2\cos^2\theta + a^2c^2r^2\sin^2\theta}{(1-r^2)}}\ rdrd\theta \tag{16}$$

Let $x = r^2$, then

$$dx = 2rdr$$
$$(dx/2) = rdr$$

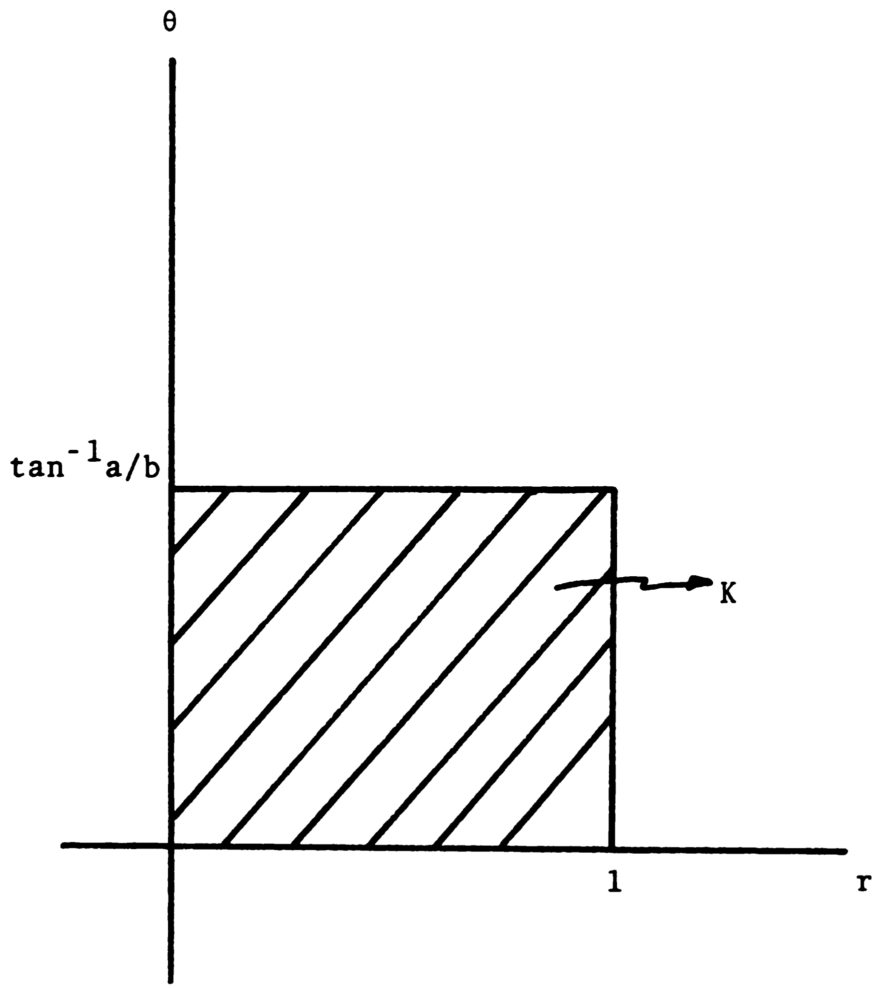



FIGURE 6. New limits of integration for the 45° section.

From this, Equation 16 becomes

$$\frac{1}{2}\int_0^{\tan^{-1}a/b}\int_0^1\sqrt{\frac{a^2b^2+x(b^2c^2\cos^2\theta+a^2c^2\sin^2\theta-a^2b^2)}{(1-x)}}\,dxd\theta \qquad (17)$$

The form of the portion of the integral in x is similar to that found in Dwight's (1949) table of integrals in that

$$\int_0^1\frac{U^{1/2}}{V^{1/2}}dx = \frac{U^{1/2}V^{1/2}}{A'}\Big|_0^1 - \frac{K}{2B'}\int_0^1\frac{dx}{V^{1/2}U^{1/2}} \qquad (18)$$

where

$$U^{1/2} = \sqrt{(F + Gx)}\ ;\quad F = a^2b^2;$$
$$G = b^2c^2\cos^2\theta + a^2c^2\sin^2\theta - a^2b^2$$

and $V^{1/2} = \sqrt{(A' + B'x)}$ ; $A' = 1$; $B' = (-1)$.

Also, $k = A'G - B'F = b^2c^2\cos^2\theta + a^2c^2\sin^2\theta$.

Therefore,

$$\int_0^1\frac{U^{1/2}}{V^{1/2}}dx = \frac{\sqrt{(1-x)(a^2b^2+x(b^2a^2\cos^2\theta+a^2c^2\sin^2\theta-a^2b^2))}}{(-1)}\Big|_0^1 + \qquad (19)$$
$$\frac{b^2c^2\cos^2\theta+a^2c^2\sin^2\theta}{2}\int_0^1\frac{dx}{V^{1/2}U^{1/2}}$$

Again from Dwight's (1949) table of integrals,

$$\int_0^1\frac{dx}{V^{1/2}U^{1/2}} = \frac{2}{\sqrt{-B'G}}\tan^{-1}\sqrt{\frac{-GV}{B'U}} \quad \text{for } B'G < 0 \qquad (20)$$

$$= \frac{2}{\sqrt{B'G}}\log|\sqrt{B'GV} + B\sqrt{U}| \quad \text{for } B'G > 0 \qquad (21)$$

Since B' always equals -1 the first case to be considered is when $G > 0$.

$$\int_0^1\frac{dx}{V^{1/2}U^{1/2}} = \frac{2}{\sqrt{G}}\tan^{-1}\sqrt{\frac{GV}{U}}\Big|_0^1 = + \frac{2}{\sqrt{G}}\tan^{-1}\sqrt{\frac{G}{a^2b^2}} \qquad (22)$$

Thus, the whole equation for a positive G is

$$\frac{1}{2}\int_0^{\tan^{-1}a/b} ab + \frac{b^2c^2\cos^2\theta+a^2c^2\sin^2\theta}{2}\left(-\frac{2\tan^{-1}}{\sqrt{G}}\sqrt{\frac{G}{a^2b^2}}\right)d\theta \quad (23)$$

expanding,

$$\frac{1}{2}\int_0^{\tan^{-1}a/b} ab + \frac{b^2c^2\cos^2\theta+a^2c^2\sin^2\theta}{\sqrt{b^2c^2\cos^2\theta+a^2c^2\sin^2\theta-a^2b^2}}\left(\tan^{-1}\sqrt{\frac{b^2c^2\cos^2\theta+a^2c^2\sin^2\theta-a^2b^2}{a^2b^2}}\right)d\theta \quad (24)$$

For $G < 0$

$$\int_0^1 \frac{U^{1/2}}{V^{1/2}}dx = \frac{2}{\sqrt{B'G}}\log\left|\sqrt{B'GV} + B'\sqrt{U}\right|\Big|_0^1 \quad (25)$$

expanding,

$$\frac{2}{\sqrt{-G}}\left(\log\left|\sqrt{a^2b^2+G}\right| - \log\left|\sqrt{-G} - ab\right|\right) \quad (26)$$

Thus, the whole equation for a negative G is

$$\frac{1}{2}\int_0^{\tan^{-1}a/b} ab + \frac{b^2c^2\cos^2\theta+a^2c^2\sin^2\theta}{\sqrt{-(b^2c^2\cos^2\theta+a^2c^2\sin^2\theta-a^2b^2)}}\ \cdot$$

$$\left(\log\left|\sqrt{b^2c^2\cos^2\theta+a^2c^2\sin^2\theta}\right| - \log\left|\sqrt{b^2c^2\cos^2\theta+a^2c^2\sin^2\theta-a^2b^2} - ab\right|\right)d\theta \quad (27)$$

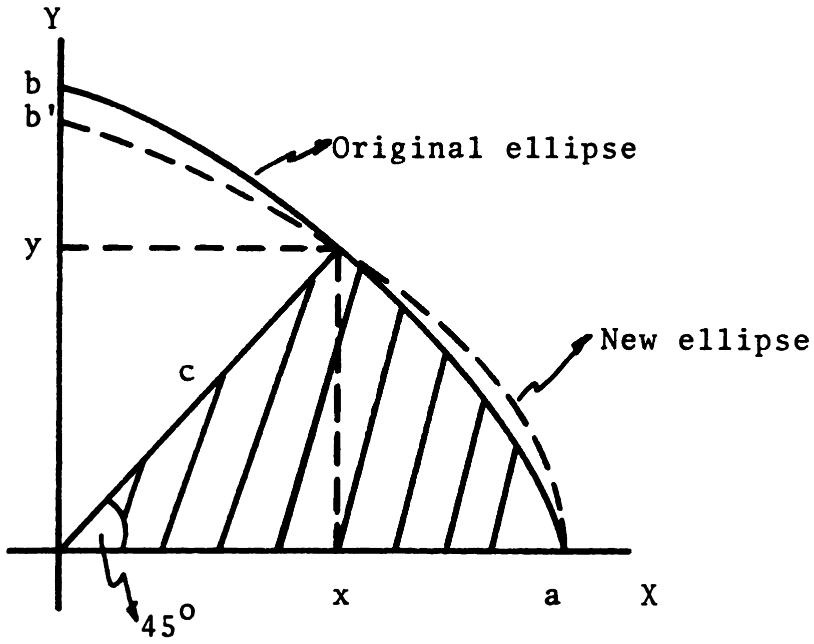
Equations 24 and 27 were then used as the primary equations in the computer program (Appendix A) to solve for the surface area of one section.

So that each forty-five degree section contributed as a separate entity to the total surface area, rather than as a portion of the ninety-degree section, the following method was used to determine exactly what ellipsoid was to be used. Three crown radius measurements a, b, and c are taken for any one ninety-degree section (Figure 7). But, in order to define the forty-five degree section as a separate entity from the ninety-degree section, the b' that belongs to a and c for the shaded region in Figure 7 alone, was computed.

Since the equation for an ellipse is

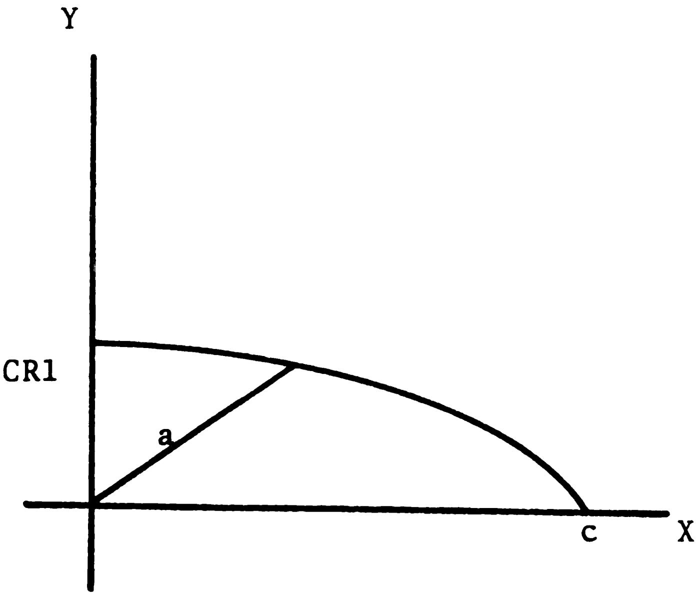
$$\frac{x^2}{a^2} + \frac{y^2}{b'^2} = 1 \quad (1)$$

for the line $x = y$, this becomes




b=CR1, b'=CR1', c=CR2, a=CR3

FIGURE 7. Defining the ellipsoid over the 45° section rather than over the 90° section.




a=CR2, c=CR3

FIGURE 8. Transposing the vectors for redefinition.

$$x^2\left(\frac{1}{a^2} + \frac{1}{b'^2}\right) = 1 \tag{29}$$

Now solving for b'

$$b' = \frac{ax}{\sqrt{a^2 - x^2}} \tag{30}$$

where $x\sqrt{2} = c$

This procedure will fit a separate ellipsoid through each forty-five degree section.

In order for these procedures to work for all possible situations, several assumptions were made. If, in Equation 30,

$$a^2 < \frac{c^2}{2} \tag{31}$$

then the square root of a negative number results. To alleviate this problem the a and c vectors were transposed as in Figure 8 and the procedure continued as before. Second, in Equations 24 and 27, if

$$b^2c^2\cos^2\theta + a^2c^2\sin^2\theta = a^2b^2 \tag{32}$$

there is a division by zero. In both cases, $G > 0$ and $G < 0$, if this situation arose then the difference

$$b^2c^2\cos^2\theta + a^2c^2\sin^2\theta - a^2b^2 \tag{33}$$

was set equal to 0.001. This substitution can be substantiated by inserting values ranging from $10^{-12}$ to 1 in Equations 24 and 27, without the integrals, and verifying that the final results change very little from one value to the other.

One final problem was encountered whenever a given defining vector of the ellipsoid was zero. This occurred when two trees were very close to each other, and the crown from one tree dominated the area between the two trees. Since crown radius measurements were taken to the nearest 0.25 meters, radii from 0.000 to 0.124 meters were considered 0.000. To alleviate the problem of an undefined ellipsoid, those crown radii less than 0.125 meters were rounded into the 0.25 meter bracket.

In order to verify that this computational algorithm would produce accurate results, several tests were conducted. Surface areas for two different prolate spheroids, the geometric solid formed by rotating an ellipse about its major axis, were estimated via the developed computer program. The resulting estimated surface areas were correct to the fourth decimal place. A prolate spheroid was chosen because there is a deterministic equation to solve for the surface area (SA):

$$SA = 2\pi b^2 + 2\pi ab(\sin^{-1}\varepsilon) \qquad (34)$$

where $\varepsilon = c/a$, $c = \sqrt{a^2 - b^2}$, and a and b are as defined above.

It was felt that, since the prolate spheroid was computed accurately (i.e., a given forty-five degree section was done correctly), then a given section of a tree's crown would be done accurately. This would produce a better estimate of the true surface area than if the whole crown was assumed to be an ellipsoid.

# RESEARCH METHODS

## DESIGN OF SAMPLE SURVEY

### Sample Size

Determination of sample size for multiple regression analysis is not a precise science. In order to determine sample size for most univariate problems, an estimate of the variance or the true variance is needed. When considering multiple regression, sample size determination becomes complex because of the increase in dimensionality due to the presence of additional independent variables.

For this study, the sample size was chosen to be fifty trees per species. It was felt that fifty trees would provide an accurate estimate of the mean square error. It also kept the cost and time required for sampling within reasonable limits.

### Plot Assignment

In order to determine approximately how many plots were needed to obtain a sample size close to fifty, an estimate of the number of stems/hectare was needed. Since beech was the least prevalant species, a sample size based on the number of stems/hectare for beech would be adequate for all species. The average number of stems/hectare equaled 46 (Gammon 1958).

Plot size was chosen to be one-twentieth of a hectare. A plot size of one-twentieth of a hectare would produce an adequate number of sample trees from a given area in Baker.

This was determined from the densities calculated by Gammon (1958). This led to an initial estimate of twenty-two plots. The estimate of forty-six stems/hectare was not a recent estimate, and Baker Woodlot has since lost part of its beech population. Therefore, sample size was increased to thirty one-twentieth hectare circular plots (plot radius was 12.62 meters).

Both the X and Y coordinates for each plot center were selected at random. If the plot overlapped the buffer zone, an area under water, or another plot, it was discarded and another set of coordinates were chosen. This plot allocation scheme is referred to as area sampling.

## TWO PHASE DATA COLLECTION

Since the plot assignment procedure called for going to each plot in the same order that the random points were selected, it was decided that the data collection would be divided into two phases in order to save time. Phase one consisted of establishing the perimeter of the study area and then locating and establishing the plot centers. During phase one, tree diameters and species were recorded for all trees on the plot. Phase two consisted of returning to the plots and measuring tree heights, crown radii, and basal area/hectare.

Baker Woodlot has very obvious boundaries and corner posts. Both the black walnut plantation and the Woodlot Management Study also have obvious boundaries and corner posts. This simplified establishment of corners and reference posts in the study area, using a chain and compass to help define the buffer zone.

After corners and reference posts were established in the field (Figure 9), distances and bearings between sequential plots were calculated. With these calculations available, individual plot centers were then located (Figure 9). Plot centers were established using a yellow stake with a tag connected to it containing the plot number. A string, 12.62 meters long, was brought into the field to facilitate establishing whether questionable trees were in or out of the plot. In order to keep confusion to a minimum, trees were visited in a clockwise direction from true north. To simplify species identification, this phase was completed before leaf abscission occurred.

It was decided that all measurements would be recorded using the International System of Units (metric), in keeping with the conversion to the metric system. Tree diameters were measured at 1.37 meters up from the ground on the uphill side of all trees. A metric diameter tape was used and all diameters were recorded to the nearest centimeter. If a fork or abnormal swell was encountered during diameter measurement, the measurement was taken above or below the abnormality, whichever was closest to 1.37 meters. A tag with the tree number and plot number was nailed to ea ch tree at the point where the diameter was measured. When the cumulative total number of trees/species reached fifty for a given species, that species was not tallied for the remaining plots.

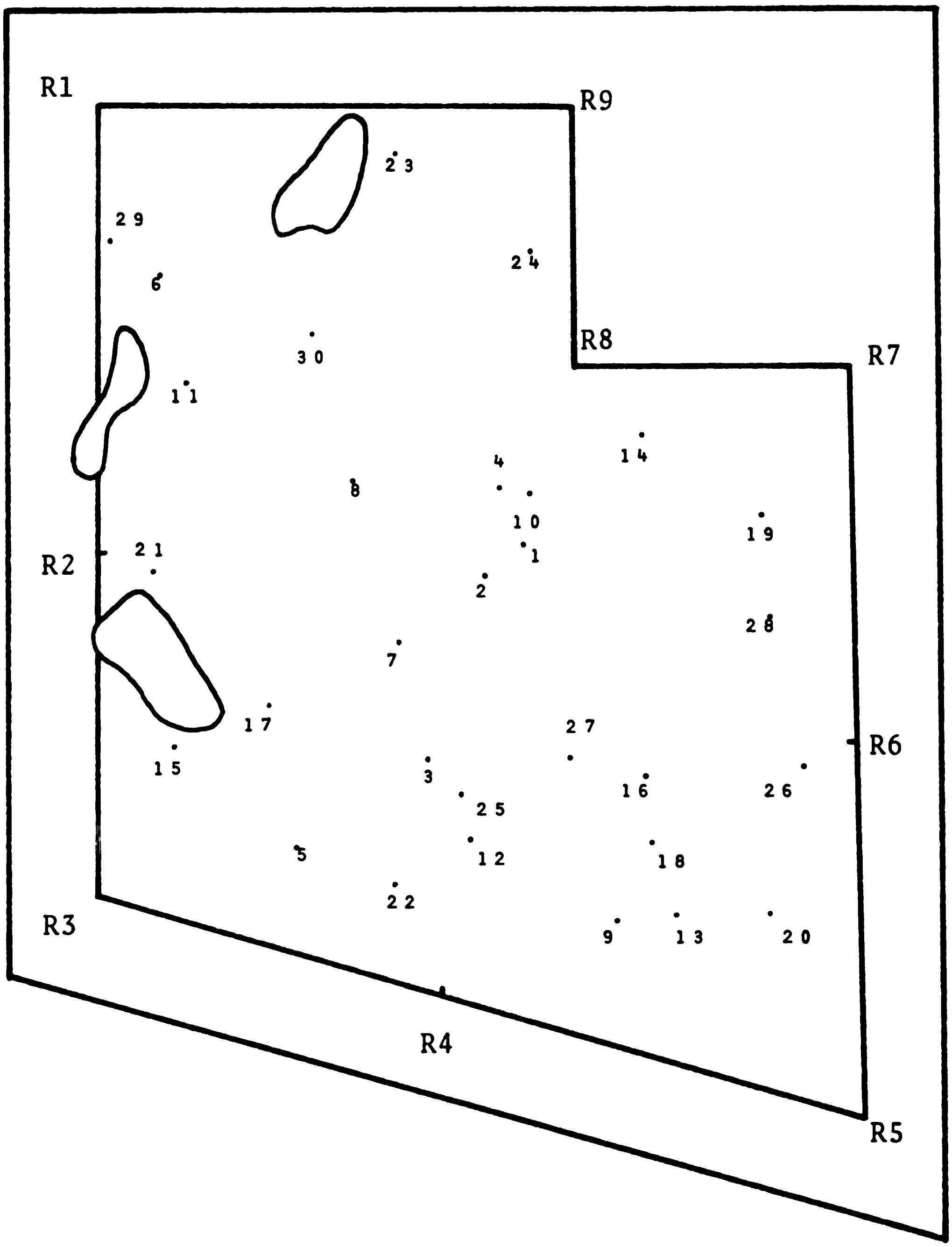



FIGURE 9. Reference corners (R(I)), and plot centers (ij).

The second phase of data collection consisted of taking three additional measurements, height, crown radius, and basal area/hectare. Again, each of these measurements were taken in meters.

Height to first contributing live branch (HTFCB), height to maximum crown radius (HTMCR), and total height shots were estimated using a clinometer. These measurements were recorded to the nearest 0.25 meters.

In order to measure upper crown length (UCL) and lower crown length (LCL) (Figure 10), two measurement points had to be defined. First, the lower extremities of the crown were determined by the first live contributing branch (FCB). This was measured at the bottom of the fork where this first branch occurred. The use of the first "live" branch has its obvious reasons. As far as first contributing branch is concerned, the major emphasis was to alleviate problems with the epicormic branching habit of oak and to a lesser extent of beech. The second measurement necessary was height to maximum crown radius (MCR). These measurements were both recorded to the nearest 0.25 meters as was total height.

There are many techniques in the literature for measuring crown radius, such as Beauregard (1975), Sheppard (1974), Hetherington (1967), Kiss (1966), Walters and Soos (1963), and Stienhilb (1962). Stienhilb (1962) used a mirror system assembled in a small box to measure crown extent. Dunn (1977) mounted a right-angle prism and rod level on a rod that was approximately two meters long. All of these methods were not only time consuming but presented an equipment transport problem

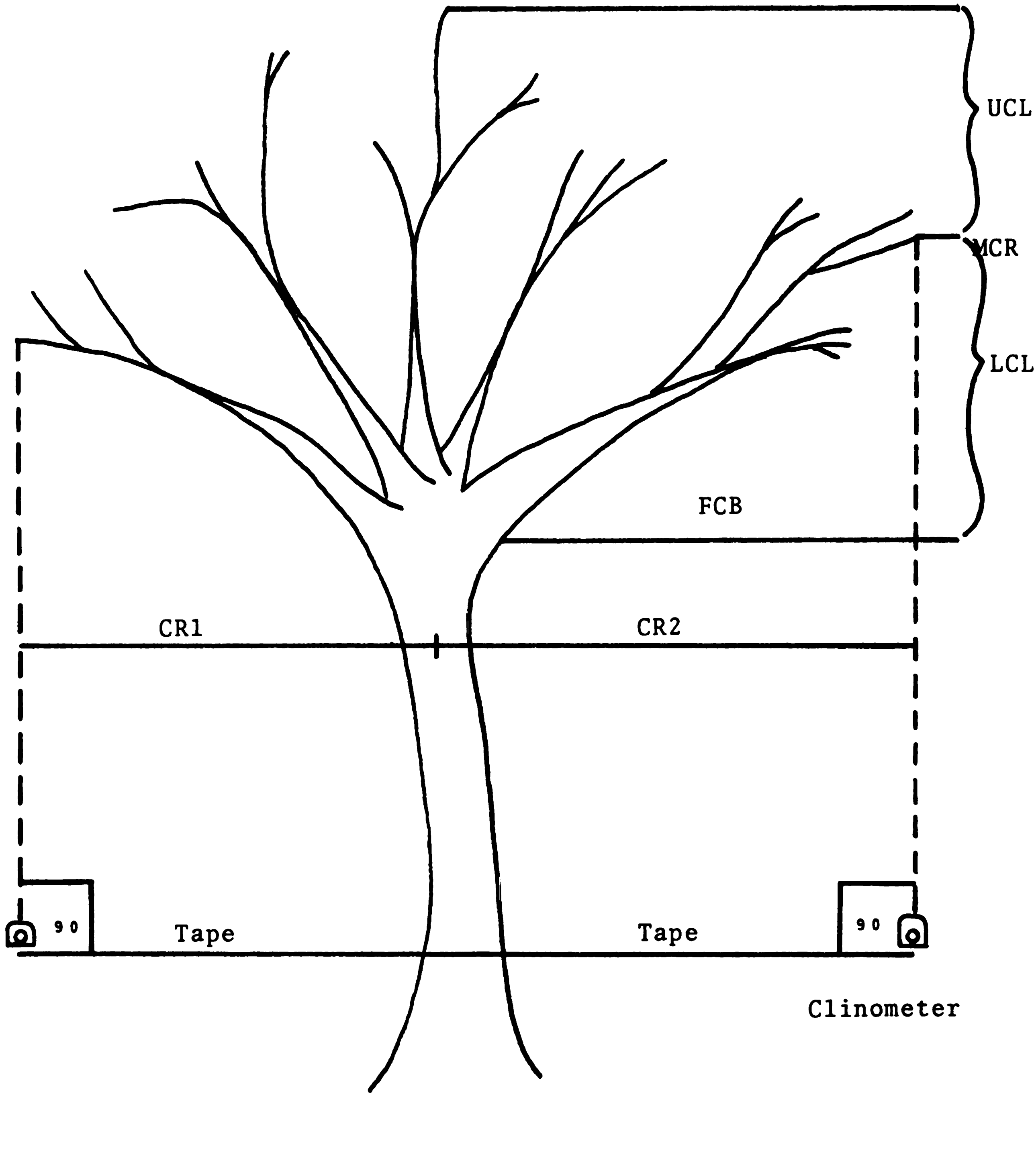



FIGURE 10. Defining upper crown length (UCL), lower crown length (LCL), and maximum crown radius (MCR).

for a one-person crew. Therefore, it was decided to measure crown radius using a tape and a Suunto clinometer. Eight crown radius measurement were taken, each at 45° intervals, with the first measurement taken at due north. A due north bearing was taken and the plot tape was extended from the trunk to the approximate edge of the crown. This was accomplished by watching the edge of the crown while walking away from the bole of the tree. Next, to insure direct alignment under the edge of the crown, a clinometer was used to determine the 90° angle shown in Figure 10. This procedure was followed for all eight radii, and measurements were recorded to the nearest 0.25 meters.

## STATISTICAL ANALYSIS

### Linear Regression Analysis

Many researchers assume, without testing, that the biological population they are working with is normally distributed. Conover (1971) presented the   Goodness of Fit statistic, to verify the normality of the underlying distribution. This test is important, as a basic assumption for inference from ordinary least squares is that the dependent variable is distributed normally (Neter and Wasserman 1974).

Ordinary least squares regression techniques as presented by Draper and Smith (1966), were used to determine if there was a relationship between the dependent variable, crown surface area, and the four "independent" variables, dbh., total height, total crown length (TOTCL), and average crown radius (AVCR). Independent is emphasized because another critical requirement, one too readily assumed, is that the variables used for the prediction of the dependent variable are not highly correlated among themselves. First order correlations, supplied by most statistical packages in the form of a correlation matrix, can be inspected to see if there is a problem with multicolinearity.

One final requirement for linear regression concerns the distribution of the error terms. Draper and Smith (1966) shoed that the error terms should be normally distributed with mean zero and homogeneous variance to accurately predict mean squared error (MSE). This assumption can be checked by examination of residual plots as well as tests such as those outlined by Anscombe and Tukey (1963). If all of these assumptions are valid, then ordinary least squares regression techniques can be utilized.

Three models were developed for each species. Model I was the maximum model, containing dbh., TOTHT, TOTCL, and AVCR. Model contained dbh. and TOTHT. Model III contained only dbh.. In order to determine the best form of the maximum model to use, and also to choose those variables most important in predicting crown surface area, a stepwise regression procedure was implemented. Stepwise analysis determines which variables are the best predictors out of the total number of variables in the maximum model. Deletion and addition F-ratios were set at the same value to avoid the possibility of cyclic entry and deletion of the same variable. A value of 5.00 was chosen because it provides a more stringent test than an $F_{(1,60)} = 4.00$ at $\alpha = 0.05$.

It was expected that Model I would have multicolinearity problems. Hoerl and Kennard (1970) discussed three basic problems associated with high correlations between independent variables. First, the estimate of mean square error of the regression coefficients is inflated; secondly, the regression coefficient vector is not stable, especially outside of the region where the study was conducted. And finally, it is most difficult to untangle the correlated variables with a "stepwise" regression routine to determine what variables are most important.

Because these three problems would occur in Model I for individual species, two techniques were used to investigate their effect. Ridge regression, as defined by Hoerl (1962), helps deliniate the inflated MSE for the regression coefficients and also stabilized the regression vector. Principal component analysis (Marriott 1974), helps determine which variables are accounting for the largest portion of the variation in the data. Since Models II and III are relatively simple application models, rather than biological models, problems with multicolinearity are minor.

## Ridge Regression

Teekens and DeBoer (1977) presented a modification of Hoerl's (1962) solution to the generalized ridge regression problem. They defined their ridge estimator, $\hat{\alpha}_i^*$, as:

$$\hat{\alpha}_i^* = \hat{\alpha}_i \quad \text{if } |\hat{\alpha}_i| < 2\sigma_i$$
$$\hat{\alpha}_i^* = \frac{1}{2}\,\hat{\alpha}_i + \left(1 + \sqrt{1 - 4\hat{\sigma}_i^{\,2}/\hat{\alpha}_i^{\,2}}\right) \quad \text{if } |\hat{\alpha}_i| > 2\hat{\sigma}_i \tag{35}$$

where $\hat{\sigma}_i$= MSE from regression, divided by $\lambda_i$. If the ridge estimator is different from the OLS estimator, via Equation 35, then there is a preliminary test that can be conducted to see if the ridge estimator is better than the ordinary least squares estimator. If

$$\left|\frac{\hat{\alpha}_i\sqrt{\lambda_i}}{\sigma}\right| < 2.59668 \quad \text{for all } \hat{\alpha}_i \text{ and } \lambda_i$$

then the ridge estimator is believed to be better than the ordinary least squares estimator (i.e., produce a smaller MSE of the regression coefficients). Here $\hat{\alpha}_i$ is an estimator of a regression coefficient of the orthogonal data set and $\lambda_i$ is the associated eigen value from the data matrix multiplied by its transpose.

## Principal Component Analysis

Variable selection was cited by Hoerl and Kennard (1970) as one of the problems encountered when there are large correlations among the independent variables. Principal component analysis is one way to overcome this problem. Marriott (1974) stated that the purpose of principal component analysis is to express the main content of the data in fewer dimensions in order to make it easier to understand and handle mathematically. There is a problem with principal component analysis in that the first two or three principal components are fairly easy to understand, whereas the remaining components are not.

Principal component analysis was only carried out on the maximum model for all data combined. Again the reasoning for this will be explained in the results.

# RESULTS

All of the computations were carried out on the CDC65000 at the computer center on the Michigan State University campus. Fortran IV was used to develope the program to compute crown surface area (Appendix A). Various canned routines were used during the analysis, including EZLS (Myers 1978), MATRIX (Myers 1978), DCADRE (deBoor 1971), and ZBRENT (Brent 1971). Descriptions of the last two routines are included in Appendix B.

## DATA

During data collection, a total of 193 trees were measured on 30 plots. Plot 18 could not be located during the second phase of data collection due to vandalism of plot stakes. Therefore, the total number of sample trees was reduced to 191. The cumulative totals for each species were: 50 for sugar maple after plot 12; 49 for basswood after plot 10; 49 for red oak after plot 30; and 43 for beech after plot 30. Species distribution by plot are presented in Table 1. Table 2 contains the means and standard deviations of all species combined for the variables measured.

TABLE 1. Species Frequency by Plot.

| Plot Number | Number of Sugar Maple | Number of Bass-wood | Number of Red Oak | Number of Beech |
|---|---|---|---|---|
| 1 | 11 | 8 | 6 | 1 |
| 2 | 6 | 4 | 3 | 1 |
| 3 | 8 | 9 | 1 | 1 |
| 4 | 2 | 2 | 0 | 1 |
| 5 | 2 | 2 | 9 | 0 |
| 6 | 2 | 0 | 1 | 2 |
| 7 | 1 | 7 | 7 | 0 |
| 8 | 7 | 3 | 1 | 4 |
| 9 | 5 | 7 | 6 | 1 |
| 10 | 3 | 9 | 1 | 0 |
| 11 | 0 | NM* | 1 | 2 |
| 12 | 3 | NM | 1 | 2 |
| 13 | NM | NM | 0 | 0 |
| 14 | NM | NM | 0 | 0 |
| 15 | NM | NM | 1 | 0 |
| 16 | NM | NM | 2 | 5 |
| 17 | NM | NM | 8 | 1 |
| 18 | NM | NM | -** | - |
| 19 | NM | NM | 1 | 0 |
| 20 | NM | NM | 0 | 0 |
| 21 | NM | NM | 0 | 5 |
| 22 | NM | NM | 2 | 0 |
| 23 | NM | NM | 0 | 2 |
| 24 | NM | NM | 0 | 0 |
| 25 | NM | NM | 5 | 4 |
| 26 | NM | NM | 2 | 0 |
| 27 | NM | NM | 0 | 1 |
| 28 | NM | NM | 0 | 4 |
| 29 | NM | NM | 0 | 7 |
| 30 | NM | NM | 1 | 1 |

*NM = not measured
**- = lost plot

TABLE 2. General Statistics for all species combined.

| Variable Name | Mean | Standard Deviation |
|---|---|---|
| DBH | 29.607 cm | 16.448 |
| TOTHT | 23.628 m | 6.606 |
| UCL | 7.598 m | 4.657 |
| LCL | 6.006 m | 22.752 |
| BA/HECTARE | 34.740 $m^2$ | 11.924 |
| R1 | 3.007 m | 2.074 |
| R2 | 3.300 m | 2.154 |
| R3 | 3.268 m | 1.973 |
| R4 | 3.205 m | 2.037 |
| R5 | 3.389 m | 2.025 |
| R6 | 3.270 m | 2.048 |
| R7 | 3.127 m | 2.008 |
| R8 | 2.847 m | 1.866 |
| TOTCL | 13.605 m | 5.828 |
| AV(R(I)) | 3.177 m | 1.570 |
| $DBH^2$ | 1145.702 $cm^2$ | 1384.282 |
| $LOG_{10}DBH$ | 1.416 | 0.214 |
| CSA | 293.116 $m^2$ | 247.685 |

Table 3 contains the means and standard deviations, by species, of those variables used in the regression analysis.

Since an inexperienced crew was used to measure the tree heights, a 10% data check was conducted on the height measurements. Results showed that height measurements were within acceptable limits, ±2.5% (Loetsch and Haller 1972), for this study.

The correlation matrix was computed to begin investigation of the relationships between all initial variables, including the dependent variable CSA. This correlation matrix is presented in Table 4. Several significant relationships were apparant. First, average crown radius had a higher correlation with the

TABLE 3. Descriptive statistics, by species, of the variables used in the analysis.

| Species | n* | Variable | Mean | Standard Deviation |
|---|---|---|---|---|
| Sugar Maple | 50 | DBH | 21.660 | 7.199 |
| | | TOTHT | 21.735 | 5.007 |
| | | TOTCL | 12.965 | 4.501 |
| | | AVCR | 2.666 | 0.798 |
| | | CSA | 216.857 | 112.715 |
| Basswood | 49 | DBH | 23.327 | 7.891 |
| | | TOTHT | 21.505 | 3.918 |
| | | TOTCL | 10.643 | 3.331 |
| | | AVCR | 2.451 | 1.024 |
| | | CSA | 173.725 | 115.620 |
| Red Oak | 49 | DBH | 43.204 | 18.024 |
| | | TOTHT | 28.505 | 5.846 |
| | | TOTCL | 16.382 | 6.574 |
| | | AVCR | 4.035 | 1.814 |
| | | CSA | 437.134 | 295.556 |
| Beech | 43 | DBH | 30.511 | 19.574 |
| | | TOTHT | 22.691 | 8.568 |
| | | TOTCL | 14.558 | 6.876 |
| | | AVCR | 3.620 | 1.859 |
| | | CSA | 353.727 | 309.970 |

*n = Sample Size

TABLE 4. Correlation matrix for all variables measured.

| | X1 | X2 | X3 | X4 | X5 | X6 | X7 | X8 | X9 | X10 | X11 | X12 | X13 | X14 | X15 | X16 | X17 | X18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| X1 | 1.00 | .82 | .54 | .55 | .01 | .63 | .67 | .68 | .67 | .66 | .59 | .62 | .62 | .69 | .83 | .97 | .97 | .86 |
| X2 | | 1.00 | .72 | .57 | -.07 | .49 | .54 | .54 | .51 | .50 | .51 | .53 | .49 | .85 | .66 | .75 | .86 | .81 |
| X3 | | | 1.00 | .18 | .10 | .35 | .34 | .38 | .32 | .32 | .45 | .50 | .41 | .89 | .49 | .51 | .53 | .75 |
| X4 | | | | 1.00 | -.20 | .40 | .42 | .38 | .37 | .43 | .32 | .30 | .33 | .62 | .47 | .51 | .55 | .56 |
| X5 | | | | | 1.00 | -.01 | -.09 | -.20 | -.09 | -.12 | -.06 | -.09 | -.07 | -.02 | -.11 | -.03 | -.00 | -.08 |
| X6 | | | | | | 1.00 | .80 | .54 | .39 | .37 | .31 | .54 | .73 | .47 | .76 | .62 | .60 | .68 |
| X7 | | | | | | | 1.00 | .71 | .47 | .39 | .26 | .45 | .62 | .47 | .76 | .66 | .64 | .69 |
| X8 | | | | | | | | 1.00 | .77 | .57 | .36 | .43 | .49 | .48 | .78 | .68 | .63 | .72 |
| X9 | | | | | | | | | 1.00 | .77 | .52 | .51 | .44 | .43 | .79 | .66 | .64 | .68 |
| X10 | | | | | | | | | | 1.00 | .76 | .64 | .45 | .45 | .80 | .65 | .64 | .70 |
| X11 | | | | | | | | | | | 1.00 | .79 | .50 | .51 | .72 | .56 | .57 | .67 |
| X12 | | | | | | | | | | | | 1.00 | .72 | .54 | .81 | .59 | .61 | .74 |
| X13 | | | | | | | | | | | | | 1.00 | .48 | .79 | .60 | .58 | .68 |
| X14 | | | | | | | | | | | | | | 1.00 | .62 | .65 | .69 | .86 |
| X15 | | | | | | | | | | | | | | | 1.00 | .81 | .79 | .89 |
| X16 | | | | | | | | | | | | | | | | 1.00 | .89 | .85 |
| X17 | | | | | | | | | | | | | | | | | 1.00 | .81 |
| X18 | | | | | | | | | | | | | | | | | | 1.00 |

X1 = DBH
X2 = TOTHT
X3 = UCL
X4 = LCL
X5 = BA/HEC.
X6 = R1
X7 = R2
X8 = R3
X9 = R4
X10 = R5
X11 = R6
X12 = R7
X13 = R8
X14 = TOTCL
X15 = AVCR
X16 = $DBH^2$
X17 = $LOG_{10}DBH$
X18 = CSA

dependent variable than any individual crown radius measurement. Second, TOTCL was more highly correlated to crown surface area than were either UCL or LCL. Dropping UCL, LCL, and the individual radius measurements from the set of independent variables reduced the total number of variables from 18 to 8. Also, dbh. had a higher correlation coefficient with CSA than did either of its transformations, (dbh.) or $\log_{10}$(dbh.). This indicated a linear relationship between crown surface area and dbh., therefore these two variables were also discarded. Finally, as basal area/hectare was only weakly correlated with CSA, it too was discarded.

Most of these results were expected. An average crown radius measurement would contribute more to the explanation of CSA than an individual radius measurement. The same logic would hold for the contributions of TOTCL to CSA versus the individual contributions of either UCL or LCL. In general, basal area/hectare for a given plot would have a definite effect on the crown surface area of an individual tree, but since it was not significant in this case it was discarded. On the other hand, the relationship between CSA and dbh., or a function of dbh., was not quite as obvious. Since polynomial regression techniques were not used, the relation was assumed closer to linear rather than logarithmic or quadratic.

Through such assumptions, the total number of independent variables was reduced from 18 to 4. The correlation matrix is shown in Table 5 for these four variables.

TABLE 5. Correlation matrix for the variables used in the analysis for all species combined.

| | DBH | TOTHT | TOTCL | AVCR | CSA |
|---|---|---|---|---|---|
| DBH | 1.000 | .823 | .687 | .826 | .856 |
| TOTHT | | 1.000 | .845 | .661 | .813 |
| TOTCL | | | 1.000 | .618 | .864 |
| AVCR | | | | 1.000 | .892 |
| CSA | | | | | 1.000 |

These variables were highly correlated amongst themselves as well as with the dependent variable.

The distribution of crown surface area was found to be approximately normal; this was verified using the $\chi^2$ Goodness of Fit at a level of significance equal to 0.05.

## LINEAR REGRESSION ANALYSIS

Relationships for crown surface area were individually developed for each species using ordinary least squares techniques. The maximum model, Model I, was analyzed first, followed by stepwise procedures to determine if any variables could be deleted. The two reduced models, Model II and Model III, were then developed. After all models for individual species were developed the data was then combined and all three models were again developed using this combined data set.

A review of the models developed for individual species and for the combined data set indicated that all models showed some basic commonalities. First, all stepwise models found AVCR and TOTCL to be the variables that explained the major portion of the variation. Also, when TOTHT and/or dbh. were used as predicting variables the $R^2$'s dropped significantly and the SEE's rose significantly. The equations developed for all models for all species shall be presented but only the results for sugar maple will be discussed in detail.

Before analyzing the models themselves, a correlation matrix for each species and all species combined was computed (Tables 6, 7, 8, 9, and 10).

TABLE 6. Correlation matrix for the variables used in the maximum model for sugar maple.

| | DBH | TOTHT | TOTCL | AVCR | CSA |
|---|---|---|---|---|---|
| DBH | 1.000 | .805 | .719 | .534 | .709 |
| TOTHT | | 1.000 | .897 | .563 | .831 |
| TOTCL | | | 1.000 | .589 | .888 |
| AVCR | | | | 1.000 | .869 |
| CSA | | | | | 1.000 |

TABLE 7. Correlation matrix for the variables used in the maximum model for basswood.

| | DBH | TOTHT | TOTCL | AVCR | CSA |
|---|---|---|---|---|---|
| DBH | 1.000 | .711 | .558 | .654 | .657 |
| TOTHT | | 1.000 | .726 | .696 | .752 |
| TOTCL | | | 1.000 | .606 | .816 |
| AVCR | | | | 1.000 | .921 |
| CSA | | | | | 1.000 |

TABLE 8. Correlation matrix for the variables used in the maximum model for red oak.

| | DBH | TOTHT | TOTCL | AVCR | CSA |
|---|---|---|---|---|---|
| DBH | 1.000 | .676 | .580 | .802 | .788 |
| TOTHT | | 1.000 | .875 | .479 | .744 |
| TOTCL | | | 1.000 | .524 | .844 |
| AVCR | | | | 1.000 | .858 |
| CSA | | | | | 1.000 |

TABLE 9. Correlation matrix for the variables used in the maximum model for beech.

| | DBH | TOTHT | TOTCL | AVCR | CSA |
|---|---|---|---|---|---|
| DBH | 1.000 | .920 | .762 | .900 | .936 |
| TOTHT | | 1.000 | .861 | .754 | .896 |
| TOTCL | | | 1.000 | .586 | .885 |
| AVCR | | | | 1.000 | .882 |
| CSA | | | | | 1.000 |

TABLE 10. Correlation matrix for the variables used in the maximum model for all species.

| | DBH | TOTHT | TOTCL | AVCR | CSA |
|---|---|---|---|---|---|
| DBH | 1.000 | .823 | .687 | .826 | .856 |
| TOTHT | | 1.000 | .845 | .662 | .813 |
| TOTCL | | | 1.000 | .618 | .864 |
| AVCR | | | | 1.000 | .892 |
| CSA | | | | | 1.000 |

The general high correlation between independent variables for the maximum model may have caused problems (Table 6). However, in the course of the analysis, it became evident that multicolinearity was not a major problem. TOTCL and AVCR had the highest correlation coefficients, $r = 0.888$ and $r = 0.869$, respectively. These relationships were used in the next step of the analysis.

The models are given in the order of their development (Tables 11, 12, 13, 14, and 15). Model I had quite a high coefficient of determination, $R^2 = 0.973$, and a low standard error of the estimate, SEE = $\pm 19.281$ $m^2$ (Table 11). These statistics indicate a significant relationship between CSA and the four independent variables. The standard errors of the regression coefficients of Model I were also low (Table 11). Theoretically, stepwise regression will not identify the most important variables in a regression equation if multicolinearity is present. However, the stepwise regression gave the best

TABLE 11. Comparison of models developed to predict CSA for sugar maple

| MODEL | $R^{2\sim}$ | SEE† | F†† | VARIABLE | REGRESSION COEFFICIENT | SE§ | t§§ |
|---|---|---|---|---|---|---|---|
| MODEL I | .973 | 19.281 | 407.390 | Y-intercept | -185.322 | 14.853 | |
| | | | | DBH | -0.272 | 0.654 | -0.420 |
| | | | | TOTHT | 1.900 | 1.454 | 1.307 |
| | | | | TOTCL | 12.603 | 1.423 | 8.858 |
| | | | | AVCR | 74.303 | 4.354 | 17.064 |
| STEPWISE | .972 | 19.330 | 809.510 | Y-intercept | -169.558 | 10.112 | |
| | | | | TOTCL | 14.405 | 0.759 | 18.974 |
| | | | | AVCR | 74.898 | 4.281 | 17.496 |
| MODEL II | .695 | 63.568 | 53.486 | Y-intercept | -183.276 | 41.115 | |
| | | | | DBH | 1.812 | 2.125 | 0.853 |
| | | | | TOTHT | 16.604 | 3.055 | 5.434 |
| MODEL III | .503 | 80.290 | 48.560 | Y-intercept | -236.505 | 36.334 | |
| | | | | DBH | 11.104 | 1.104 | 6.969 |

~$R^2$ = coefficient of determination
†SEE = standard error of the estimate
††F = F-statistic
§SE = standard error of the regression coefficient
§§t = t-statistic

TABLE 12. Comparison of models developed to predict CSA for basswood.

| MODEL | $R^2$~ | SEE† | F†† | VARIABLE | REGRESSION COEFFICIENT | SE§ | t§§ |
|---|---|---|---|---|---|---|---|
| MODEL I | .954 | 25.942 | 227.366 | Y-intercept | -150.099 | 22.531 | |
| | | | | DBH | -0.131 | 0.711 | -0.184 |
| | | | | TOTHT | -0.983 | 1.718 | -0.572 |
| | | | | TOTCL | 14.806 | 1.669 | 8.871 |
| | | | | AVCR | 78.062 | 5.468 | 14.277 |
| STEPWISE | .953 | 25.524 | 469.459 | Y-intercept | -163.455 | 12.501 | |
| | | | | TOTCL | 14.172 | 1.390 | 10.195 |
| | | | | AVCR | 76.032 | 4.525 | 16.805 |
| MODEL II | .596 | 75.103 | 33.881 | Y-intercept | -275.967 | 62.200 | |
| | | | | DBH | 3.657 | 1.953 | 1.872 |
| | | | | TOTHT | 16.945 | 3.934 | 4.307 |
| MODEL III | .433 | 88.012 | 35.836 | Y-intercept | -51.081 | 39.602 | |
| | | | | DBH | 9.637 | 1.610 | 5.936 |

~$R^2$ = coefficient of determination
†SEE = standard error of the estimate
††F = F-statistic
§SE = standard error of the regression coefficient
§§t = t-statistic

TABLE 13. Comparison of models developed to predict CSA for red oak.

| MODEL | $R^2$~ | SEE† | F†† | VARIABLE | REGRESSION COEFFICIENT | SE§ | t§§ |
|---|---|---|---|---|---|---|---|
| MODEL I | .953 | 66.861 | 223.487 | Y-intercept | -289.053 | 66.791 | |
| | | | | DBH | 1.123 | 1.139 | 0.986 |
| | | | | TOTHT | -3.173 | 4.106 | -0.904 |
| | | | | TOTCL | 26.610 | 3.295 | 8.801 |
| | | | | AVCR | 86.147 | 9.789 | 8.801 |
| STEPWISE | .952 | 66.244 | 454.743 | Y-intercept | -340.580 | 27.634 | |
| | | | | TOTCL | 24.461 | 1.707 | 14.329 |
| | | | | AVCR | 93.439 | 6.185 | 15.108 |
| MODEL II | .703 | 164.631 | 54.351 | Y-intercept | -495.712 | 121.756 | |
| | | | | DBH | 8.592 | 1.290 | 4.800 |
| | | | | TOTHT | 19.203 | 5.519 | 3.570 |
| MODEL III | .620 | 184.056 | 76.771 | Y-intercept | -120.818 | 68.694 | |
| | | | | DBH | 12.914 | 1.474 | 8.762 |

~$R^2$ = coefficient of determination
†SEE = standard error of the estimate
††F = F-statistic
§SE = standard error of the regression coefficient
§§t = t-statistic

TABLE 14. Comparison of models developed to predict CSA for beech.

| MODEL | $R^2$~ | SEE† | F†† | VARIABLE | REGRESSION COEFFICIENT | SE§ | t§§ |
|---|---|---|---|---|---|---|---|
| MODEL I | .988 | 35.518 | 780.220 | Y-intercept | -280.124 | 25.206 | |
| | | | | DBH | 3.857 | 1.185 | 3.254 |
| | | | | TOTHT | -5.749 | 2.186 | -2.630 |
| | | | | TOTCL | 25.952 | 1.593 | 16.290 |
| | | | | AVCR | 74.259 | 7.515 | 9.881 |
| STEPWISE | .985 | 39.255 | 1289.389 | Y-intercept | -348.465 | 15.106 | |
| | | | | TOTCL | 25.305 | 1.087 | 23.272 |
| | | | | AVCR | 92/211 | 4.022 | 22.925 |
| MODEL II | .885 | 107.750 | 153.780 | Y-intercept | -184.604 | 59.514 | |
| | | | | DBH | 11.507 | 2.164 | 5.318 |
| | | | | TOTHT | 8.251 | 4.944 | 1.669 |
| MODEL III | .877 | 110.074 | 292.057 | Y-intercept | -98.722 | 31.348 | |
| | | | | DBH | 14.829 | 0.868 | 17.090 |

~$R^2$
†SEE = standard error of the estimate
††F = F-statistic
§SE = standard error of the regression coefficient
§§t = t-statistic

TABLE 15. Comparison of models used to predict CSA for combined data.

| MODEL | $R^{2\sim}$ | SEE† | F†† | VARIABLE | REGRESSION COEFFICIENT | SE§ | t§§ |
|---|---|---|---|---|---|---|---|
| MODEL I | .959 | 50.622 | 1090.623 | Y-intercept | -249.710 | 17.184 | |
| | | | | DBH | 2.415 | 0.532 | 4.542 |
| | | | | TOTHT | -3.404 | 1.356 | -2.510 |
| | | | | TOTCL | 22.284 | 1.209 | 18.438 |
| | | | | AVCR | 78.251 | 4.266 | 18.344 |
| STEPWISE -- (SAME AS MODEL I) | | | | | | | |
| MODEL II | .769 | 119.778 | 312.229 | Y-intercept | -262.532 | 36.711 | |
| | | | | DBH | 8.727 | 0.930 | 9.380 |
| | | | | TOTHT | 12.580 | 2.317 | 5.430 |
| MODEL III | .732 | 128.488 | 517.032 | Y-intercept | -88.426 | 19.183 | |
| | | | | DBH | 12.887 | 0.567 | 22.738 |

~$R^2$ = coefficient of determination
†SEE = standard error of the estimate
††F = F-statistic
§SE = standard error of the regression coefficient
§§t = t-statistic

model as that model containing only TOTCL and AVCR. This was due to the low t-values of the regression coefficients for dbh. and TOTHT, -0.042 and 1.307 respectively (Table 11). The $R^2$ for the stepwise model, $R^2 = 0.972$, is not significantly lower than that of Model I, $R^2 = 0.973$ (Table 11). Also, the standard error of the estimate is not significantly higher for the stepwise model, SEE = ±19.330, than for Model I, SEE = ±19.281 (Table 11). Therefore, the stepwise model was the better model because only two variables were needed to get the same predicting capabilities as with four variables.

Since one of the problems with multicolinearity is the stability of the regression coefficients, the coefficients for Model I and the stepwise model were compared. For Model I the regression coefficient for TOTCL was 12.603 and for the stepwise model it was 14.405. Since these two values did not differ greatly, it was felt that multicolinearity did not cause any instability. As for AVCR, the regression coefficients were even more stable, 74.303 for Model I and 74.898 for the stepwise model. The standard errors for these regression coefficients were also quite low, 4.354 and 4.281 respectively (Table 11). These relationships provide evidence that multicolinearity was not present to any great extent.

Model II was developed by including only dbh. and TOTHT in the regression equation. The coefficient of determination dropped significantly, $R^2 = 0.695$, and the standard error of the estimate rose significantly, SEE = ±63.568 $m^2$. The standard errors for the regression coefficients of these variables rose slightly (Table 11).

Model III, developed with dbh. alone, had the lowest coefficient of determination, $R^2 = 0.503$, and the highest standard error of the estimate, SEE = ±80.290. Each species followed these basic patterns though the specific values of the $R^2$'s, SEE's, regression coefficients, SE's and t-values did change. Because Model II and Model III had significantly lower $R^2$'s and significantly higher SEE's, it was felt that these two prediction equations were not comparable to Model I.

For all data combined, AVCR and TOTCL were also the most important variables in the relationship between CSA and the four independent variables: AVCR, TOTCL, TOTHT, and dbh.. TOTCL and AVCR were added to the stepwise model first, indicating that their partial F-values were the most significant (Table 15). This order of importance coincides with the models developed for the individual species. The commonalities between the individual species and the combined data set gave strong evidence that TOTCL and AVCR were very important in describing the biological relationship to crown surface area. Also, dbh. was shown to be a poor predictor of crown surface area by itself.

## RESIDUAL ANALYSIS

Residuals are defined as the difference between the observed values of the dependent variable and the predicted values. Assumptions about the regression error terms can be checked by plotting these values against the predicted values. Therefore, Y(observed) - $\hat{Y}$(predicted), was plotted against $\hat{Y}$. None of the residual plots (Figures 10-14),

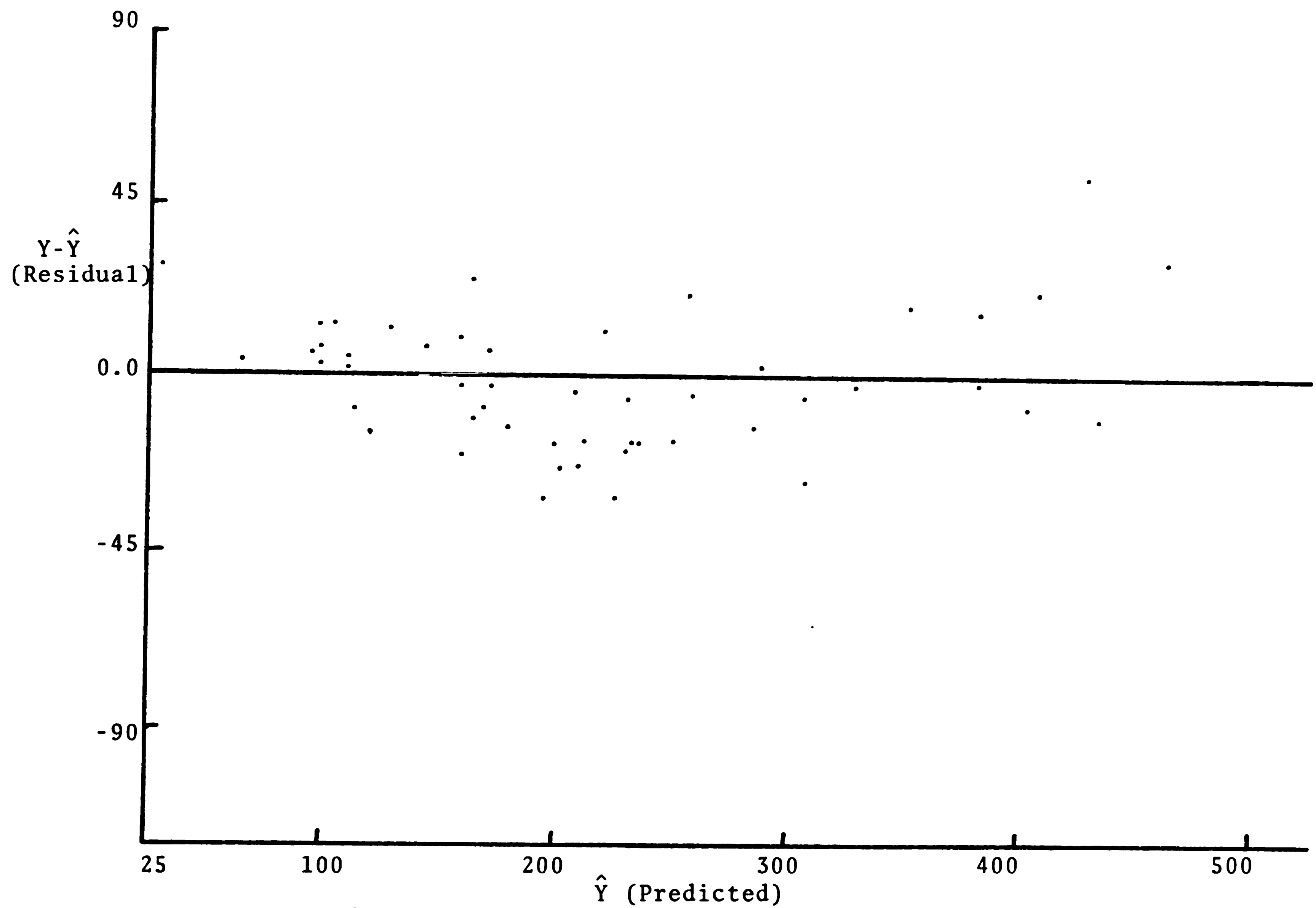



FIGURE 11. Residual plot for the stepwise model for sugar maple.

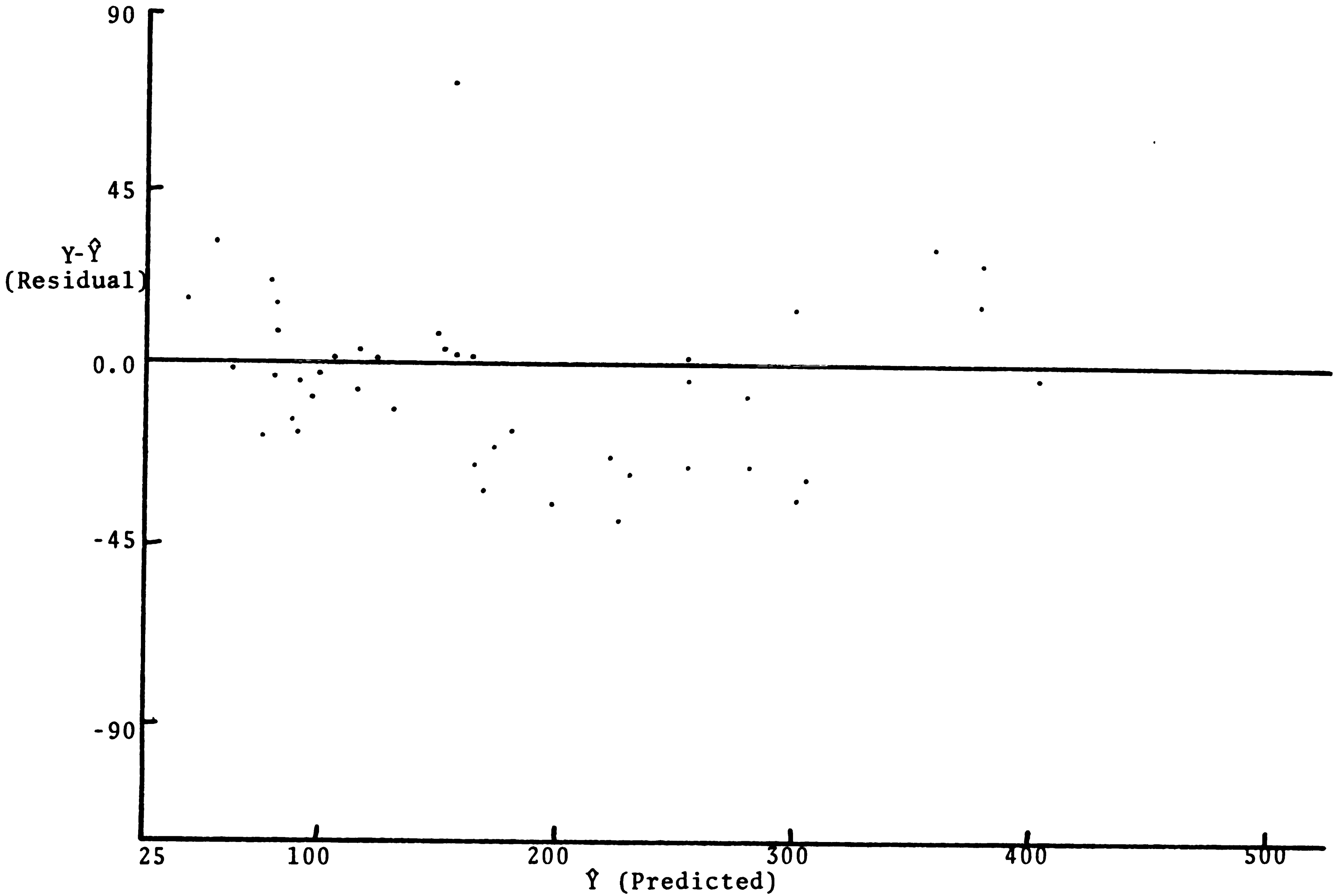



FIGURE 12. Residual plot for the stepwise model for basswood.

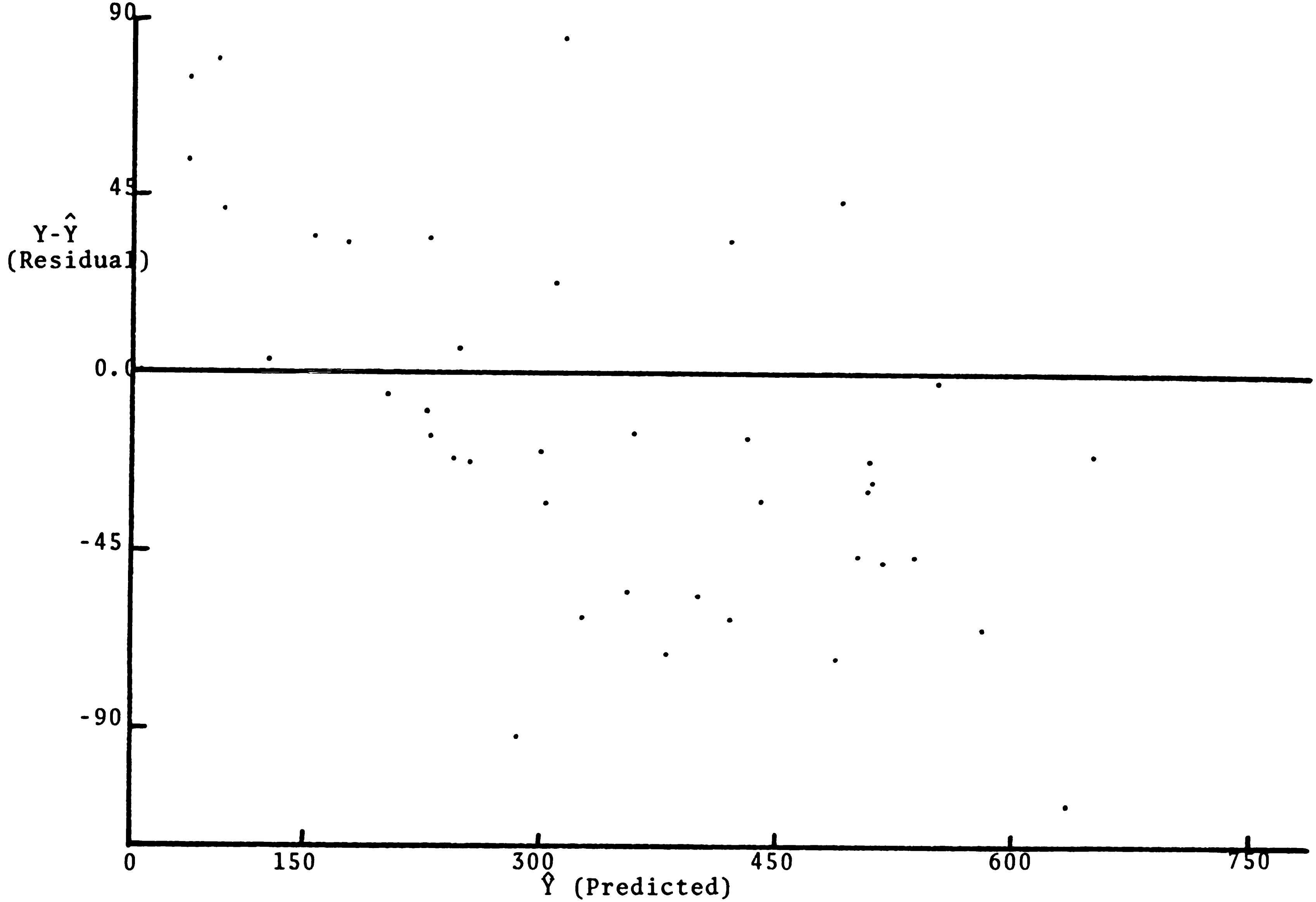



FIGURE 13. Residual plot for the stepwise model for red oak.

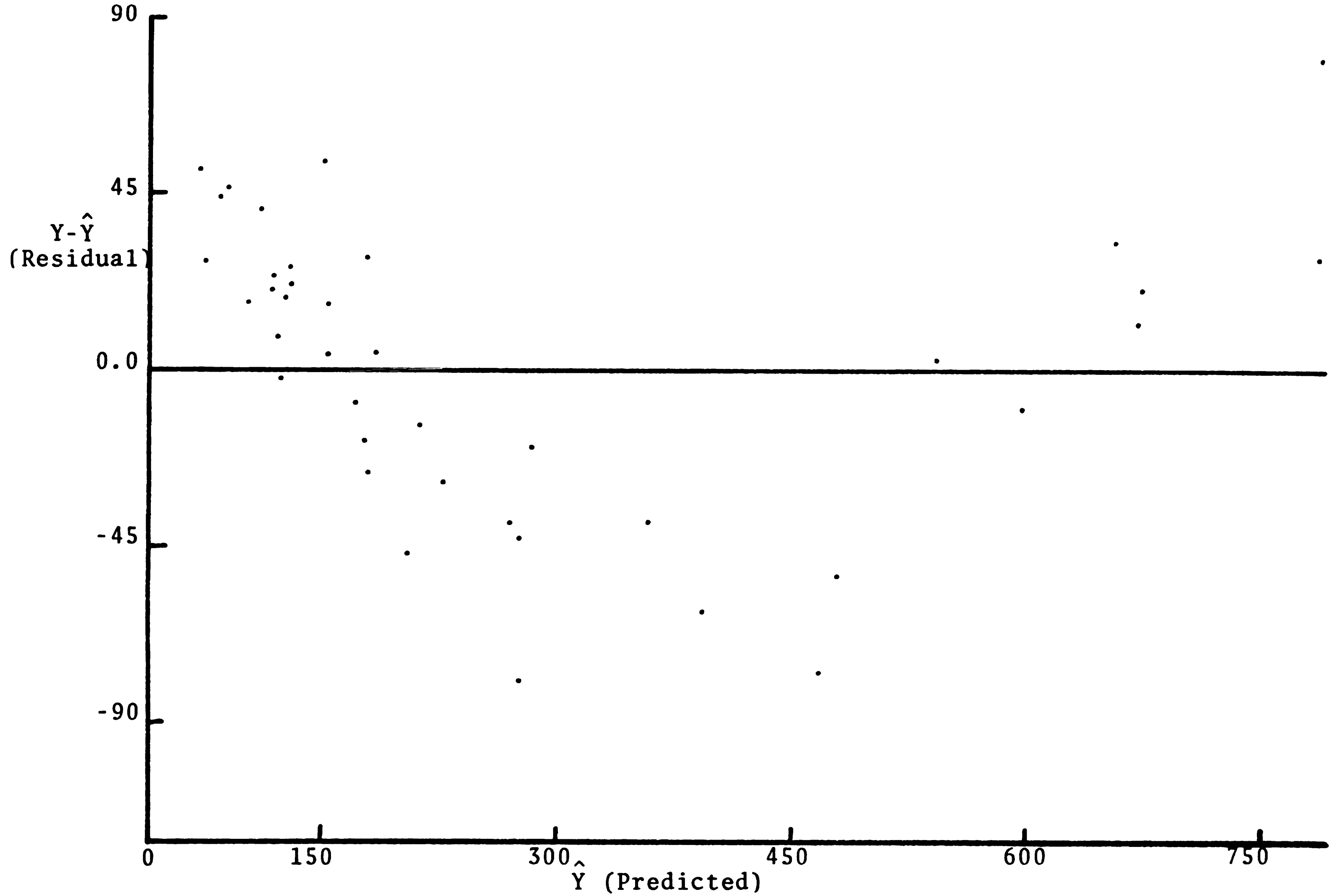



FIGURE 14. Residual plot for the stepwise model for beech.

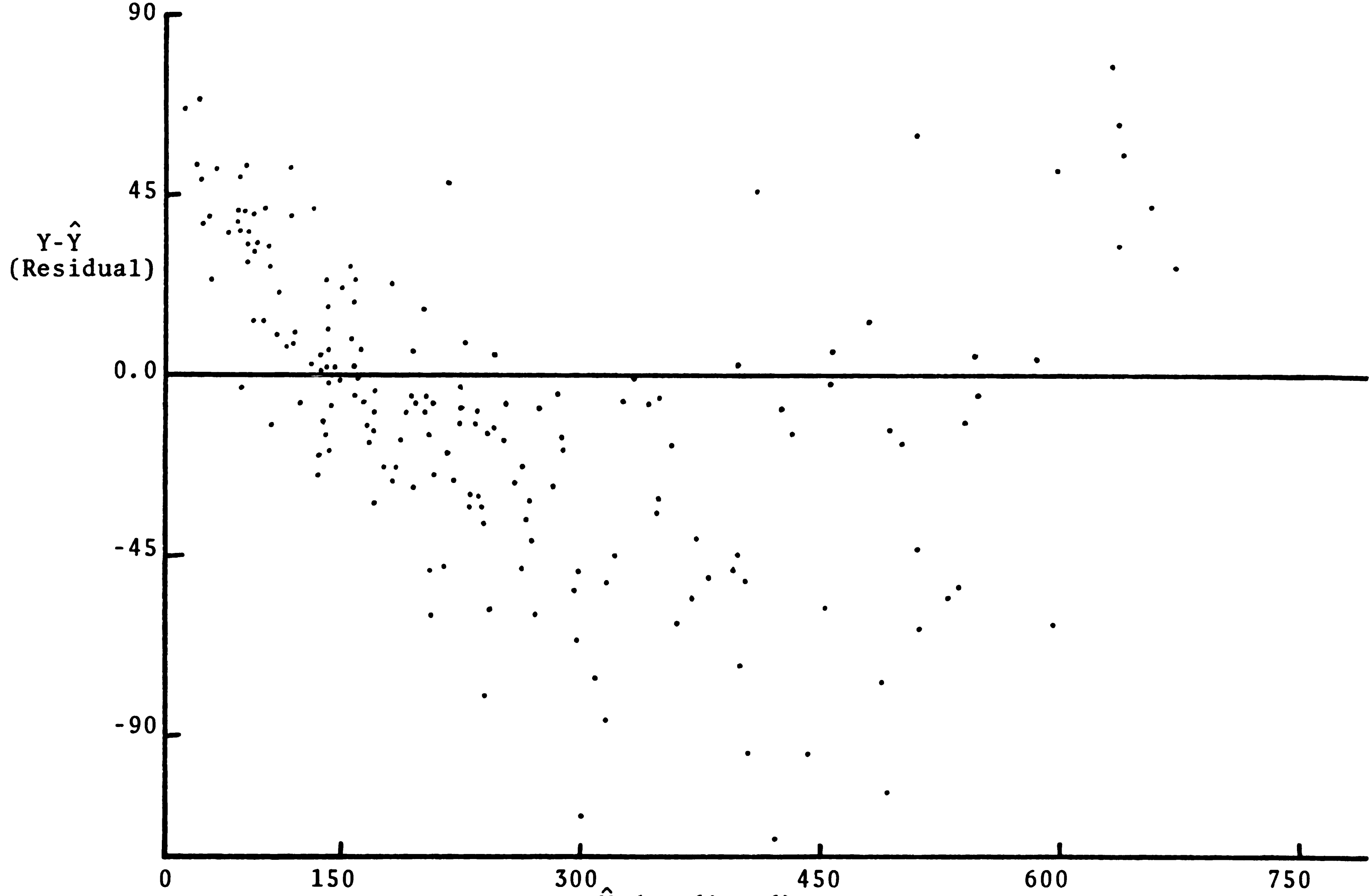



FIGURE 15. Residual plot for the maximum model for all species combined.

showed that any of the assumptions for the error terms were violated. However, Figures 13 and 14 suggest that a quadratic or cubic interaction term should be included in the model. Draper and Smith (1966), along with Neter and Wasserman (1974), claimed that models exhibiting these types of residual plots probably have poor fits. On the contrary, in this study the $R^2$'s were high and the SEE's were low for each of the models analyzed by a residual plot. Therefore, the point to consider is that a nonlinear interaction term will reduce the SEE's even more.

The need for this nonlinear term is more explicit for the smaller CSA values, specifically those less than 150 $m^2$ and especially those less than 25 $m^2$. Smaller values are currently underestimated by a large margin. This was because smaller values were in the region of the confidence band where it diverges from the central equation the most. These values, being negative, were not included on the residual plots.

## COMPARISON OF REGRESSION MODELS

It was previously mentioned that Minckler and Gingrich (1970) found that the relationship between crown width and tree diameter was independent of species. The maximum model for all species combined had an $R^2$ of 0.959 and SEE = 50.622 $m^2$ (Table 15). This, along with the low standard errors for the regression coefficients, indicated that CSA may also be independent of species. To test and verify this result, Bartlett's (1932) $\chi^2$ test for homogeneity of variances was conducted (Table 16). This test is the first step in

TABLE 16. Comparison of regression equations.

| DATA SOURCE | SSQ[†] | DF[§] | MSQ[∿] | LOG(MSQ) | DF*LOG(MSQ) | 1/DF |
|---|---|---|---|---|---|---|
| ALL | 476,694.119 | 186 | 2562.630 | 3.409 | 634.018 | .005 |
| SM | 16,729.149 | 45 | 371.759 | 2.570 | 115.665 | .022 |
| BD | 29,691.221 | 44 | 672.982 | 2.828 | 124.432 | .023 |
| RO | 196,695.753 | 44 | 4470.358 | 3.651 | 160.644 | .023 |
| BH | 47,937.427 | 38 | 1261.511 | 3.101 | 117.834 | .026 |
| TOTAL | 767,622.669 | 357 | | | 1152.593 | .099 |

M = (2.303)(357(3.333) - 1152.593) = 85.448
C = 1 + 1/12(.099 - (1/357)) = 1.008
M/C = 84.768 > $\chi^2_4(.05)$ = 9.488

[†]SSQ = Sum of Squares
[§]DF = Degrees of Freedom
[∿]MSQ = Mean Square = SSQ/DF

comparison of multivariate regression equations for similarities (Snedecor and Cochran 1968). The test statistic was significant at the 10 percent confidence level indicating that the variances were heterogeneous (i.e., that the regression equations were not the same). This is not to say that the overall model, with all species combined, could not be implemented. This test only showed that in fact the relationship was not totally independent of species. The model can still be used for all species combined but the SEE will be higher than for an individual species.

## RIDGE REGRESSION

Ridge regression was only carried out for the model using the combined data. This was mainly because of time and funding constraints. Also, the model for all species would most likely be the one implemented in practice. This would be the case with a mixed hardwood stand since equations were not developed for every species. The first step in implementing Teekens and DeBoer's (1977) ridge estimator was to see if in fact the ridge estimators were different from the OLS estimators. The $\hat{\alpha}_i$, regression coefficients of the orthogonal data set, were: $\hat{\alpha}_1 = 2.649$, $\hat{\alpha}_2 = -.404$, $\hat{\alpha}_3 = 0.940$, and $\hat{\alpha}_4 = 20.110$. The calculations for determining whether or not the ridge estimators were different than the OLS estimators for this problem are given in Table 17.

TABLE 17. Calculations to see if the ridge estimator is different from the ordinary least squares estimator.

| $\hat{\lambda}_i$ † | $\hat{\sigma}_i^2$ | IS $\lvert\hat{\alpha}_i\rvert$ § $< 2\hat{\sigma}$ |
|---|---|---|
| $\lambda_1 = 321.290$ | $\hat{\sigma}_1^2 = 186.915$ | YES |
| $\lambda_2 = 23.334$ | $\hat{\sigma}_2^2 = 2573.575$ | YES |
| $\lambda_3 = 5.076$ | $\hat{\sigma}_3^2 = 11831.645$ | YES |
| $\lambda_4 = 0.729$ | $\hat{\sigma}_4^2 = 82383.420$ | YES |

† $\hat{\lambda}_i$ = Eigen values from the variance-covariance matrix.

§ $\lvert\hat{\alpha}_i\rvert$ = Regression coefficients from the orthogonal data set.

From Table 17, all of the $\hat{\alpha}_i$ were less than $2\bar{\hat{\sigma}}_i$, therefore the ridge estimators were the same as the OLS estimators for this problem.

This result, combined with the low SE's for the regression coefficients for the stepwise models, indicated that there were not any major problems with inflated mean square errors of the beta vectors and that the regression coefficients were fairly stable.

## PRINCIPAL COMPONENT ANALYSIS

Principal component analysis, like ridge regression, was only carried out for the model using the combined data. The main function for principal component analysis is to transform a correlated data matrix into an uncorrelated data matrix. Each transformed variable accounts for decreasing amounts of variation in the data, with the first component absorbing the most. The first two transformed vectors accounted for 92.16% of the variation in the data. A look at the transformation matrix provided insight to what combination of the original variables produced the first two variables of the transformed data set. The first eigen vector was comprised of equal amounts of information from all of the original vectors (Table 18).

TABLE 18. Transformation matrix.

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | .517* | .321 | -.526 | -.594 |
| 2 | .517 | -.361 | -.438 | .641 |
| 3 | .486 | -.593 | .528 | -.365 |
| 4 | .479 | .644 | .503 | .320 |

* = the ij*th* weight to transform the original data.

This first vector is considered a general size vector if the weights are distributed equally (Marriott 1974). The second eigen vector indicated that TOTCL and AVCR explained the major portion of the remaining variance. Since the first two components accounted for 92.16% of the variation in the data, then the other two components can be discarded (Marriott 1974).

This information reinforces the results from the stepwise models, that TOTCL and AVCR are the two most important variables in Model I. If a researcher was using principal component analysis before regression analysis, then TOTCL and AVCR would be the only two variables that would be considered for the model.

## SUMMARY

All phases of the study were thought to have gone fairly well. The survey design carried through well, the sampling intensity was sufficient, the computational algorithm provided good results, and the analysis worked out favorably.

Area sampling proved to be a very efficient method for choosing the study trees. From the analysis it was evident that there were enough degrees of freedom (i.e., sufficient sample size) for an accurate estimate of mean squared error.

Even though the algorithm for computing the dependent variable, CSA, was not exact, it was felt that it was much more accurate than procedures using ellipsoids to estimate the CSA. For symmetrical forms the algorithm was very accurate.

As mentioned earlier, before using a given statistical technique, such as ordinary least squares, it is important to determine if the underlying assumptions are satisfied. The $\chi^2$ Goodness of Fit statistic showed that crown surface was distributed approximately normal. Also, residual analysis showed that the error terms were distributed normally with mean zero and homogeneous variance. With these two points covered, the remainder of the results can be summarized.

All of the stepwise regression results pointed to AVCR and TOTCL as the two major variables defining crown surface area. When the maximum models for each species were compared to the respective stepwise models, it appeared that the stepwise models were "sufficient" for predicting CSA.

Sufficiency was determined by three criteria: the SE's of the regression coefficients; standard error of the estimates; and the coefficient of determination. In every case, the standard errors for the regression coefficients were somewhat lower for the stepwise models than for the maximum models. The largest increase in the SEE from the maximum model to the stepwise model was 3.74 $m^2$. The largest decrease in $R^2$ from the maximum model to the stepwise model was 0.004. None of the changes in $R^2$ and standard error of the estimates were significant, therefore it appeared that the stepwise models should be used rather than the maximum models because there are fewer variables in the stepwise models. Models incorporating AVCR and TOTCL were sufficient to predict CSA accurately, as shown by the high $R^2$'s (lowest equaling 0.952 for red oak) and low SEE's (highest equaling 66.200 $m^2$ for red oak).

The principal component analysis also gave evidence to show that TOTCL and AVCR accounted for the major portion of the variation in CSA. This fact, combined with the evidence from stepwise regression, erased any doubt concerning which variables were the important variables in the equations with multicolinearity present.

The application equations, Models II and III, were not quite as fruitful as the biological equations (i.e., stepwise models). Dbh. alone was a poor predictor in all models but one, the model that contained all four species combined, but this model had an unacceptable standard error of the estimate, SEE = $\pm 128.50$ $m^2$. Therefore, dbh. should not be considered as a univariate predictor of CSA.

Equations containing TOTHT and dbh. did have higher $R^2$'s, but the SEE's were still quite high. These two variables were not considered adequate for predicting CSA for the combined data set or for individual species.

Ridge regression techniques were not defined for this study under the primary restriction that Teekens and DeBoer (1977) derived. This was not a problem because the analysis showed that the multicolinearity in the variables did not bring about the consequences normally encountered. The SE's of the beta's were not extremely large, and both the stepwise regression procedures and the principal component analysis showed that AVCR and TOTCL explained the major portion of the variation in the data.

## CONCLUSIONS

Species specific equations for predicting CSA were characterized by AVCR and TOTCL. These prediction equations all had low SEE's, low SE's for the regression coefficients, and high $R^2$'s. No significant problems associated with multicolinearity were encountered. Both stepwise regression and principal component analysis produced the same two variables, AVCR and TOTCL, for all models. The ridge estimator as derived by Teekens and DeBoer (1977) was not defined for this study.

Equations using AVCR and TOTCL were good predicitng equations for the area studied. DBH. and TOTHT alone had acceptable $R^2$'s in some cases ($R^2 > 0.70$ for red oak, beech, and all species combined), but the SEE's were far too high. The stepwise equation developed for the entire data set would be applicable in an area where there is a large species mix.

It was evident that AVCR and TOTCL are very important in defining the underlying relationships between CSA and the crown and tree parameters studied: dbh., dbh.$^2$, $\log_{10}$(dbh.), TOTHT, TOTCL, AVCR, and basal area/hectare.

## SUGGESTIONS

As mentioned, dbh. by itself was not a good predictor of CSA. There are other techniques that could be used in order to implement such an easily obtainable parameter as dbh. (i.e., polynomial regression for unequally-spaced independent variables). Secondly, upon inspection of the correlation matrix in Table 5, dbh. was highly correlated to both AVCR and TOTCL. A two-stage procedure could be implemented where dbh. is used to predict AVCR and TOTCL, and then the estimated AVCR and TOTCL values would be used to predict CSA.

It would be quite useful if the equations were developed for an average crown diameter with fewer parameters than eight (i.e., (maximum crown diameter + minimum crown diameter)/2 or just two measurements at right angles to each other). In their present form the equations can be used with these two measurements but the same predicting capabilities may not be obtained.

In the residual analysis it was found that crown surface areas less than 25 $m^2$ were underestimated. Also, the residual plots showed a need for a non-linear term in the models. These two circumstances can be investigated by adding a non-linear term to the models similar to $(AVCR)^2TOTCL$ or $AVCR^3$.

A final important consideration is that, before the equations are used, they should be tested on an independent data set. Then the actual and relative errors can be analyzed to provide a final check on the predicting capabilities of the models.

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# APPENDICES

APPENDIX A

Program SURAR, Surface Area, was developed to solve for the crown surface area by utilizing computer techniques. Eight crown radius measurements, along with upper and lower crown length, are the major inputs to the program. Other parameters have to be defined internally as specified by the canned routines that were used.

The following is a listing of the program accompanied by comment cards to explain the general flow of the program.

```
      PROGRAM SURAR(INPUT,OUTPUT,TAPE60=INPUT,TAPE61=OUTPUT,
     1PUNCH)
      EXTERNAL FP,FN,FC
      COMMON/VAR/A,B,C
      DIMENSION CL(2),R(8),THA(5),FNC(5)
C VARIABLE LIST
C CL(1) = UPPER CROWN LENGTH
C CL(2) = LOWER CROWN LENGTH
C R(I) = ITH CROWN RADIUS MEASUREMENT
C RSCM = TOTAL SURFACE AREA
C HSUM = SURFACE AREA FOR LOWER AND UPPER PORTION OF THE CROWN
C A = THE X VECTOR FOR THE ELLIPSOID
C B = 45° VECTOR FOR THE ELLIPSOID
C THETA = TAN- (A/B)
C THA(K) = K/4 PORTION OF THE THETA FOR K = 1.5
C FNC(K) = G, SECTION VI
C QSUM = SURFACE AREA FOR A 45° SECTION
C UPPER CROWN LENGTH, CL(1), AND LOWER CROWN LENGTH, CL(2),
C ARE READ IN ALONG WITH THE 8, R(I), I=1, . . , 8, CROWN
C RADIUS MEASUREMENTS
   10 READ(60,50) IPLT,ISP,ITREE,IDBH,TOTHT,HTCMR,HTFCB,(CL(I),
     1I=1,2),B
   50 FORMAT(1X,I2,1X,I1,1X,I2,1X,I2,1X,6(F5.2,1X))
      IF(IPLT.EQ.50) GO TO 3000
      READ(60,75) (R(J),J=1,8),IPLT2,ITR2
   75 FORMAT(1X,8(F5.2,1X),I2,2X,I2)
C THE CROWN RADIUS MEASUREMENTS ARE REDEFINED IF THEY ARE
C BETWEEN 0.000 AND 0.124, SECTION VI
      DO 25 I=1,8
      IF(R(I).EQ.0.0) R(I)=0.25
   25 CONTINUE
      TSUM=0.0
C LOOP TO SUM OVER THE TOP AND BOTTOM PORTION OF THE CROWN
      DO 500 J=1,2
      C=CL(J)
      HSUM=0.0
C LOOP TO SUM OVER THE EIGHT SECTIONS OF THE UPPER OR LOWER
C CROWN
```

```
      DO 1000 I=1,8
      IF(I.EQ.8) GO TO 65
      A=R(I+1)
      GO TO 70
   65 A=R(1)
   70 C1=2.0*(A((2.0)
      C2=R(I)**2.0
C THE TWO VECTORS THAT DEFINE THE ELLIPSOID ARE CHECKED TO SEE
C IF THEY HAVE TO BE INTERCHANGED
      IF(C1.LT.C2) GO TO 90
      B=SQRT(ABS(((R(I)**2)*(A**2))/((2.0*(A**2))-(R(I)**2))))
      GO TO 110
   90 A=R(I)
      B=SQRT(ABS(((R(I+1)**2)*(A**2))/((2.0*(A**2))-(R(I+1)**
     12))))
  110 D=A/B
      THETA=ATAN(D)
      DO 1500 K=1,5
      Q=K
C A GIVEN 45° ARC IS BROKEN UP INTO FOUR PARTS TO SEE IF G,
C SECTION  VI, CHANGES SIGN
      THA(K)=THETA*((5.0-Q)/4.0)
      FNC(K)=(B**2)*(C**2)*((COS(THA(K)))**2)+(A**2)*(C**2)
     1*((SIN(THA(K)))**2)-(A**2)*(B**2)
 1500 CONTINUE
      QSUM=0.0
      DO 2000 L=1,4
C DECISION STATEMENTS TO DETERMINE WHAT INTEGRATION EQUATION
C TO USE
      IF(A.EQ.B.AND.B.EQ.C) GO TO 1600
      IF(FNC(L).EQ.0.0.AND.FNC(L+1).EQ.0.0) GO TO 2000
      IF(FNC(L).GE.0.0.AND.FNC(L+1).GE.0.0) CALL INTPOS(THETA,
     1L,AREA)
      IF(FNC(L).LE.0.0.AND.FNC(L+1).LE.0.0) CALL INTNEG(THETA,
     1 L,AREA)
      IF(FNC(L).LT.0.0.AND.FNC(L+1).GT.0.0) CALL ZEROS(THETA,
     1L,AREA,FNC(L))
      IF(FNC(L).GT.0.0.AND.FNC(L+1).LT.0.0) CALL ZEROS(THETA,
     1L,AREA,FNC(L))
      GO TO 1750
 1600 AREA=3.141592654*(A**2.0)/16.0
 1750 QSUM=AREA+QSUM
 2000 CONTINUE
      HSUM=HSUM+QSUM
 1000 CONTINUE
      TSUM=HSUM+TSUM
  500 CONTINUE
      WRITE(61,4000) ITREE,IPLT,TSUM
 4000 FORMAT(1H,"THE TOTAL SURFACE AREA FOR TREE NO.",I2,2X,
     1"PLOT NO."1,I2,1X,"IS",F25.4)
      PUNCH 3250,IPLT,ISP,ITREE,IDBH,TOTHT,HTMCR,HTFCB,(CL(I),
     1I=1,2),BA,TSUM
 3250 FORMAT(1X,I2,1X,I1,1X,I2,1X,I2,1X,6(F5.2,1X),F25.4)
      PUNCH 3500,(R(I),I=1,8),IPLT2,ITR2
 3500 FORMAT(1X,8(F5.2,1X),I2,1X,I2)
```

```
      GO TO 10
 3000 STOP
      END

      SUBROUTINE INTPOS(THETA,L,AREA)
      EXTERNAL FP
C CALCULATES THE SURFACE AREA FOR A 45° SECTION WHEN G IS ALWAYS
C POSITIVE
      S=L
      B1=((5.0-S)/4.0)*THETA
      A1=((4.0-S)/4.0)*THETA
      AERR=0.0
      RERR=.001
      AREA=DCADRE(FP,A1,B1,AERR,RERR,ERROR,IER)
      RETURN
      END

      SUBROUTINE INTNEG(THETA,L,AREA)
      EXTERNAL FN
C CALCULATES THE SURFACE AREA FOR A 45° SECTION WHEN G IS
C ALWAYS NEGATIVE
      S=L
      B2=((5.0-S)/4.0)*THETA
      A2=((4.0-S)/4.0)*THETA
      AERR=0.0
      RERR=.001
      AREA=DCADRE(FN,A2,B2,AERR,RERR,ERROR,IER)
      RETURN
      END

      SUBROUTINE ZEROS(THETA,L,AREA,FNC)
      EXTERNAL FC
      DIMENSION FNC(5)
C CALCULATES THE ZEROS OF G IG F CHANGES SIGN IN A GIVEN 45°
C SECTION.  THE THE APPROPRIATE, POSITIVE OR NEGATIVE, SUBROUTINE
C IS CALLED TO CALCULATE THE SURFACE AREA OVER THE PORTION THAT
C HAS A POSITIVE OR NEGATIVE G
      S=L
      B3=((5.0-S)/4.0)*THETA
      A3=((4.0-S)/4.0)*THETA
      BU=B3
      BL=A3
      EPS=0.0
      NSIG=3
      MAXFN=100
      CALL ZBRENT(FC,EPS,NSIG,A3,B3,MAXFN,IER)
      Z0=B3
      IF(FNC(L).LT.0.0) CALL ZINTNEG(THETA,L,Z0,BL,BU,ZAREA)
      IF(FNC(L).GE.0.0) CALL ZINTPOS(THETA,L,Z0,BL,BU,ZAREA)
      AREA=ZAREA
      RETURN
      END
```

```
      SUBROUTINE ZINTNEG(THETA,L,Z0,BL,BU,ZAREA)
      EXTERNAL FN,FP
C CALCULATES THE SURFACE AREA WHEN THE FIRST PORTION OF THE
C 45° SECTION HAS A NEGATIVE G
      B4=BU
      A4=Z0
      AERR=0.0
      RERR=.001
      AREA1=DCADRE(FN,A4,B4,AERR,RERR,ERROR,IER)
      B5=Z0
      A5=B1
      AREA2=DCADRE(FP,A5,B5,AERR,RERR,ERROR,IER)
      ZAREA=AREA1+AREA2
      RETURN
      END

      SUBROUTINE ZINTPOS(THETA,L,Z0,BL,BU,ZAREA)
      EXTERNAL FN,FP
C CALCULATES THE SURFACE AREA WHEN THE FIRST PORTION OF THE
C 45° SECTION HAS A POSITIVE G
      B6=BU
      A6=Z0
      AERR=0.0
      RERR=.001
      AREA1=DCADRE(FP,A6,B6,AERR,RERR,ERROR,IER)
      B7=Z0
      A7=BL
      AREA2=DCADRE(FN,A7,B7,AERR,RERR,ERROR,IER)
      ZAREA=AREA1+AREA2
      RETURN
      END

      FUNCTION FP(THETA)
      COMMON/VAR/A,B,C
C THE EQUATION FROM SECTION VI USED TO CALCULATE THE SURFACE
C AREA WHEN G >0
      FP=0.50*((A*B)+(((B**2)*(C**2)*((COS(THETA))**2)+(A**2)*
     1(C**2)*((SIN(THETA))**2))/(SQRT(ABS(TFNC(THETA))))*ATAN(
     1SQRT(ABS(TFNC(THETA))/((A**2)*(B**2))))))
      RETURN
      END

      FUNCTION FN(THETA)
      COMMON/VAR/A,B,C
C THE EQUATION FROM SECTION VI USED TO CALCULATE THE SURFACE
C AREA WHEN G < 0
      FN=0.50*((A*B)+ ((B**2)*(C**2)*((COS(THETA))**2)+(A**2)*
     1(C**2)*((SIN(THETA))**2))/(SQRT(ABS(TFNC(THETA))))*(ALOG10
     1(ABS(SQRT((B**2)*(C**2)*((COS(THETA))**2)+(A**2)*(C**2)*
     1((SIN(THETA))**2))))-ALOG10(ABS(SQRT(ABS(TFNC(THETA)))-A*B)
     1)))
      RETURN
      END
```

```
      FUNCTION TFNC(THETA)
      COMMON/VAR/A,B,C
C THE CHECK EQUATION USED TO ADJUST FOR A DIVISION BY ZERO,
C SECTION VI
      TFNC=((B**2)*(C**2)*((COS(THETA))**2)+(A**2)*(C**2)*
     1((SIN(THETA))**2-(A**2)*(B**2))
      IF(TFNC.LT.0.01.AND.TFNC.GE.0.0) TFNC=0.01
      IF(TFNC.LT.0.0.AND.TFNC.GE.-0.01) TFNC=-0.01
      RETURN
      END

      FUNCTION FC(THETA)
      COMMON/VAR/A,B,C
C THE FUNCTION G, SECTION VI
      FC=       (B**2)*(C**2)*((COS(THETA))**2)+(A**2)*(C**2)
     1*((SIN(THETA)**2)-(A**2)*(B**2)
      RETURN
      END
```

# APPENDIX B

FUNCTION DCADRE(F,A,B,AERR,RERR,ERROR,IER)

Purpose

DCADRE attempts to solve the following problem: Given the name F of a real function subprogram, two real numbers A and B, and two non-negative numbers AERR and RERR, find a number DCADRE such that

$$\left|\int F(x)dx - \text{DCADRE}\right| \leq \text{MAX}(\text{AERR}, \text{RERR}^* \left|\int F(x)dx\right|)$$

Algorithm

This routine uses a scheme whereby DCADRE is computed as the sum of estimates for the integral of F(x) over suitable chosen subintervals of the given interval of integration. Starting with the interval of integration itself as the first such subinterval, cautious Romberg extrapolation is used to find an acceptable estimate on a given subinterval. If this attempt fails, the subinterval is divided into two subintervals of equal length, each of which is considered separately.

CALL ZBRENT(F,EPS,NSIG,A,B,MAXFN,IER)

Purpose

ZBRENT finds a zero of a continuous function which changes sign in a given interval.

Algorithm

The algorithm combines linear interpolation and inverse quadratic interpolation with bisection. Convergence is usually superlinear, and is never much slower than for bisection.