

THESIS



This is to certify that the

thesis entitled

A NEW METHOD FOR THE SOLUTION OF ANISOTROPIC THIN PLATE BENDING PROBLEMS

presented by

Benjamin Chin-wen Wu

has been accepted towards fulfillment of the requirements for

Ph.D. degree in Mechanics

Major professor ()Itier

Date February 14, 1980

O-7639



.

OVERDUE FINES: 25¢ per day per item

RETURNING LIBRARY MATERIALS:

Place in book return to remove charge from circulation records .

,

•

·.

. . .

A NEW METHOD FOR THE SOLUTION OF ANISOTROPIC THIN PLATE BENDING PROBLEMS

By

Benjamin Chin-wen Wu

A DISSERTATION

Submitted to Michigan State University in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Department of Metallurgy, Mechanics, and Materials Science

ABSTRACT

A NEW METHOD FOR THE SOLUTION OF ANISOTROPIC THIN PLATE BENDING PROBLEMS

Ву

Benjamin Chin-wen Wu

A numerical method for the solution of thin plate problems is presented. With the conventional assumptions for thin plates implied, plates of arbitrary plan form, subjected to arbitrary loading and boundary conditions, and made of anisotropic material are considered. This method is developed from the concept of the indirect boundary integral method.

The indirect boundary integral method uses the Green's function of a clamped circular plate of isotropic material. To solve an isotropic thin plate problem, the first step is to embed the real plate into the fictitious clamped circular plate for which the Green's function is known. Along the embedded contour, N points are prescribed, at which the boundary conditions for the original problem are specified. The numerical solution of the problem is then to find the magnitude of the set of N line forces and N ring moments imposed along the embedded contour such that the boundary conditions at the N boundary points are satisfied. With this method, problems with clamped and simply supported boundaries can be easily solved. For a free edge, however, due to the logarithmic nature of the Green's function and the fact that fourth order derivatives must be taken for the fictitious ring moments in the boundary condition equations, there are second order singularity difficulties during the numerical integration along the embedded contour.

In this thesis, three major modifications are introduced. These are (1) the set of fictitious moments are replaced by an additional set of fictitious forces, and the entire set of fictitious forces is located outside of the embedded contour, (2) the numerical integration is replaced by a simple summing process, and (3) the Green's function for a clamped circular plate is replaced by the Green's function of an infinite plate. With these modifications, significant improvements in solution accuracy and computing efficiency have been achieved. The second order singularity difficulties associated with free edges are avoided, and due to the simplicity of the new method, the computing costs are reduced by about sixty percent. Since the Green's functions for orthotropic and anisotropic infinite plates are also available, the new method is readily extended to orthotropic and anisotropic thin plate bending problems.

ACKNOWLEDGEMENTS

The author wishes to take this opportunity to express his gratitude and deep appreciation to his major advisor Professor Nicholas J. Altiero for his able guidance and sincere encouragement during the course of this thesis research, and for his spending many hours with the author in the preparation of this thesis. Special thanks are due to other members of the graduate guidance committee, Professor William A. Bradley, Professor George E. Mase, Professor Larry J. Segerlind, Professor David H. Y. Yen, and Professor David L. Sikarskie of the University of Michigan for their helpful advice, suggestions, and discussions. To the Department of Metallurgy, Mechanics, and Materials Science of Michigan State University, the author is grateful for its support on computing costs.

The author also wishes to thank his parents for their continuous encouragement during his studies and research for the past many years. Finally, and most importantly, the author wishes to acknowledge his wife Wei Sheng. Without her genuine understanding, encouragement, and moral support in the past several years, this work could never have been done.

ii

TABLE OF CONTENTS

Page

LIST C LIST C	OF TABL	LES . URES.	•••	•	•	•	•	•	•	•	•	•	•	•	iv v
LIST C	F APPE	NDICES	•••	•	•	•	•	•	•	•	•	•	•	•	vi
INTROD	UCTION	•	•••	•	•	•	•	•	•	•	•	•	•	•	1
CHAPTE	RI	PRELI	MINA	RIES	ON	PL	ATE	THE	EOR	Y.	•	•	•	•	4
		I.1	GOV	ERNI	NG I	EQU	ATIC	ONS	•	•	•	•	•	•	4
		I.2	BOUI	NDAR	Y CO	DND:	ITIC	ONS	•	•	•	•	•	•	6
		I.3	THE	GRE	EN'S	S FI	UNCI	101	N M	ETH	DD	•	•	•	12
CHAPTE	R II	INTEG	RAL 1	EQUA	r i of	I AI	PPRO	DACE	1.	•	•	•	•	•	15
		II.1	THE	BOUI	NDAI	RY :	INTE	EGRA	T I	METI	HOD	•	•	•	15
		II.2	THE	AUX	ILIA	ARY	BOU	JND	ARY	ME	гноі	5.	•	•	19
CHAPTE	R III	A NEW	METI	HOD	•	•	•	•	•	•	•	•	•	•	30
		III.1	ISO	[ROP]	IC H	PLA	re i	PROF	BLE	MS	•	•	•	•	30
		III.2	ANIS	SOTR	OPIC	D PI	LATI	E PH	ROB	LEM	5.	•	•	•	46
			III	.2.1	ORI	CHO.	FROE	PIC	PR	OBLI	EMS	•	•	•	47
			III	.2.2	ANI	[SO	FROE	PIC	PR	OBLI	EMS	•	•	•	54
CHAPTE	R IV	CLOSU	RE .	•	•	•	•	•	•	•	•	•	•	•	64
APPEND	ICES	• •	•••	•	•	•	•	•	•	•	•	•	•	•	67
BIBLIO	GRAPHY	••		•	•	•	•	•	•	•	•	•	•	•	127

.

LIST OF TABLES

•

Table			Pa	ge
1	Comparison of numerical and exact results for a square plate, edge one free, edge two and four simply supported, and edge three clamped	•	•	28
2	Comparison of numerical and exact results for a square plate, edges one and three free, edges two and four simply supported.	•	•	29
3	Comparison of computing costs	•	•	34
4	Comparison of results of a simply supported triangular isotropic plate	•	•	36
5	Comparison of results of a clamped square plate, double-looped fictitious forces at 4m and 6m away from the plate boundary.	•	•	42
6	Comparison of results of a clamped square plate, double-looped fictitious forces at 1m and 3m away from the plate boundary.	•	•	43
7	Comparison of results of a clamped square plate, double-looped fictitious forces at 0.5m and 2.5m away from the plate boundary	•	•	44
8	Comparison of results of a clamped square plate, single-looped fictitious forces at 4m away from the plate boundary	•	•	45
9	Comparison of results of a simply supported orthotropic square plate, $E_x = 2.068 \times 10^5$ MPa, $E_y = E_x/15$, $v_x = 0.3$, $h = 0.01m$, $\rho = 0.1$.	•	•	56
10	Comparison of results of a simply supported orthotropic square plate, $E_x = 2.068 \times 10^5$ MPa, $E_y = E_x/15$, $v_x = 0.3$, $h = 0.01m$, $\rho = 1.0$.	•	•	57
11	Comparison of results of a simply supported orthotropic square plate, $E_x=2.068 \times 10^5$ MPa, $E_y=E_x/15$, $v_x=0.3$, h=0.01m, $\rho=10.0$	•	•	58

LIST OF FIGURES

Figure		Pa	ge
1	Problem of interest	•	16
2	Problem for which analytic solution is known	•	20
3	Fictitious problem	•	20
4	A square plate with an auxiliary integration contour.	•	26
5	A square plate with two sets of fictitious forces	•	32
6	A simply supported equilateral triangular isotropic plate	•	35
7	Comparison of results of a triangular plate.	•	37
8	A simply supported orthotropic plate	•	55
9	Flow chart for the computer programs	•	66

•

LIST OF APPENDICES

A	ppendi	x	Page
	A	Derivatives of the Green's function for a clamped circular plate	67
	В	Computer program for the boundary inte- gral method	80
	С	Computer program for the point-force method	90
	D	Derivatives of the Green's function of an infinite orthotropic plate	95
	Е	Computer program for an orthotropic problem.	100
	F	Derivatives of the Green's function of an infinite anisotropic plate	107
	G	Computer program for an anisotropic problem.	112
	Н	Computer program for the verification of equations used for anisotropic problems	119

.

INTRODUCTION

The thin plate bending problem is one of the most common problems in structural engineering. Design engineers encounter it daily. Plate theories and methods of solution can be traced back to the early eighteenth century. Famous names like Euler, Bernoulli, Lagrange, Navier, and Kirchhoff were all involved in the development of plate theories. In this century, Nadai, Love, Huber, Timoshenko, Lekhnitskii, von Karman, and Reissner, to name a few, are well-known for their work related to plate problems.

Mathematically, the thin plate bending problem is a typical boundary value problem. Since solving the problem reduces to finding a solution satisfying the governing fourth order partial differential equation and all the boundary condition equations, exact solutions are available only for special cases. In addition to many common methods for a wide range of problems shown in [1], the method of complex variables has been successfully applied [2,3,4,5,6] solving many additional problems. However, for a generalized problem, numerical techniques such as finite difference and finite element methods must be employed, [7,8].

In this dissertation, a different numerical method is introduced. Developed from the concept of an indirect boundary integration equation method [9], the new numerical method employs a known Green's function, the scheme of embedding the real plate in a fictitious plate for which the Green's function is known, and the imposition of fictitious forces so that all the boundary conditions are satisfied. This method is very effective because of the simplicity of

its formulation. It can solve constant thickness plate problems with arbitrary plan form, arbitrary loading and boundary conditions, and anisotropic material properties. Since the fictitious forces are located far away from the plate boundary, this method gives more accurate results near the boundary than does the boundary integral method.

The procedure of this method can be summarized in three steps. The first is to find a Green's function of a certain type of plate problem. Many of them are available. Take isotropic problems for example. The Green's functions for a clamped circular plate [1,3,10], and for an infinite plate [1] are two possible choices. The second step is to embed the real plate in the aforementioned fictitious plate for which the Green's function is known, and prescribing a set of N boundary points. Since there are two boundary condition equations associated with each boundary point, a set of 2N fictitious forces are placed around the plate boundary. Solution of the problem, therefore, involves the determination of this set of 2N fictitious forces such that the 2N boundary condition equations are satisfied. The third step is to superimpose these 2N fictitious forces onto the actual loadings of the plate and compute the deflections and bending moments inside the plate.

There are four chapters in this dissertation. The governing fourth order partial differential equations for isotropic, orthotropic, and anisotropic plate problems and their associated boundary condition equations are reviewed in Chapter I. Chapter II introduces the indirect boundary integral method originally derived by Altiero and Sikarskie [9]. Their method is modified by moving the integration contour to the outside of the plate boundary. In so doing, the second order singularity difficulties for the free edge boundary conditions, encountered in their work, are avoided. In the meantime, a significant improve-

ment in the solution accuracy is noticed. Chapter III illustrates the new point-force method for the solution of a generalized thin plate bending problem. The pointforce method contains three major alterations over the boundary integral method, though the basic concept remains. The three changes are (1) the set of fictitious moments are replaced by a second set of fictitious forces, (2) the integration is replaced by an algebraic summing process, and (3) the Green's function for a clamped circular plate is replaced by the Green's function of an infinite plate. These changes have made the original method more effective. A saving of sixty percent for computing costs is realized. More importantly, it is due to the successful use of the Green's function of an isotropic infinite plate and, the Green's functions for orthotropic and anisotropic infinite plates are readily known [11,12,13], the new method can be extended to solve general orthotropic and anisotropic plate problems as well. Chapter IV presents several comments regarding to this new point-force method.

In order to verify the method, several test cases have been solved. The results are tabulated and graphed, and the computer programs are included in the Appendices. Though these computer programs are specifically designed for the example problems, they can be easily revised to accommodate general problems.

CHAPTER I

PRELIMINARIES ON PLATE THEORY

I.1 GOVERNING EQUATIONS

Following the assumptions involved in the well-known Kirchhoff-Love small deflection plate theory, all of the stress components within the plate can be expressed in terms of the vertical deflections, w(x,y). Therefore, for static equilibrium, the governing differential equation can be derived in terms of the deflection function and the two independent coordinate variables x and y. For the three material types, namely isotropic, orthotropic, and anisotropic, the derivation of the governing differential equations can be found in most texts on plate theory [1,7,14].

Assuming constant thickness, for isotropic plates, the governing differential equation is

$$\frac{\partial^4 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^4} + 2 \frac{\partial^4 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^2 \partial \mathbf{y}^2} + \frac{\partial^4 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y}} = \frac{\mathbf{q}(\mathbf{x}, \mathbf{y})}{\mathbf{p}}$$
(1)

where w(x,y) is the vertical deflection of the plate after bending, q(x,y) is the load in the vertical direction, and D is the flexural rigidity of the plate defined by

$$D = \frac{Eh^3}{12(1-v^2)}$$

where E is the Young's modulus, v is the Poisson's ratio, and h is the plate thickness. For an orthotropic plate which has its geometric coordinates aligned with the principal material directions, the governing equation is

$$D_{\mathbf{x}} \frac{\partial^{4} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^{4}} + 2H \frac{\partial^{4} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^{2} \partial \mathbf{y}^{2}} + D_{\mathbf{y}} \frac{\partial^{4} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y}^{4}} = q(\mathbf{x}, \mathbf{y}), \qquad (2)$$

where,

$$D_{\mathbf{x}} = \frac{E_{\mathbf{x}} \cdot h^{3}}{12(1-v_{\mathbf{x}}v_{\mathbf{y}})}; \quad D_{\mathbf{y}} = \frac{E_{\mathbf{y}} \cdot h^{3}}{12(1-v_{\mathbf{x}}v_{\mathbf{y}})}; \text{ and } H = D_{\mathbf{x}}v_{\mathbf{y}} + \frac{Gh^{3}}{12}.$$

The subscripts x and y indicate the principal directions of the material constants, and G is the modulus of rigidity. Finally, the governing equation for an anisotropic plate, with one plane of material symmetry parallel to the middle surface of the plate is

$$D_{11} \frac{\partial^{4} w(x,y)}{\partial x^{4}} + 4D_{16} \frac{\partial^{4} w(x,y)}{\partial x^{3} \partial y} + 2(D_{12} + 2D_{66}) \frac{\partial^{4} w(x,y)}{\partial x^{2} \partial y^{2}} + 4D_{26} \frac{\partial^{4} w(x,y)}{\partial x^{2} \partial y^{3}} + D_{22} \frac{\partial^{4} w(x,y)}{\partial y^{4}} = q(x,y), \qquad (3)$$

where D_{ij} are associated with the material constants, and are determined as follows:

$$D_{11} = \frac{h^3}{12} \frac{(a_{22}a_{66} - a_{26}^2)}{\det}; \quad D_{22} = \frac{h^3}{12} \frac{(a_{11}a_{66} - a_{16}^2)}{\det};$$

$$D_{66} = \frac{h^3}{12} \frac{(a_{11}a_{22} - a_{12}^2)}{\det}; \quad D_{12} = \frac{h^3}{12} \frac{(a_{16}a_{26} - a_{12}a_{66})}{\det};$$

$$D_{16} = \frac{h^3}{12} \frac{(a_{12}a_{26} - a_{22}a_{16})}{\det}; \quad D_{26} = \frac{h^3}{12} \frac{(a_{12}a_{16} - a_{11}a_{26})}{\det};$$

and,

det. =
$$\begin{vmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{vmatrix}$$
;

a_{ij} is the material constant matrix, i.e.,

εx		a ₁₁	^a 12	^a 16	σχ	
ε _y	= <	^a 12	a ₂₂	^a 26	σу	•
(Y _{xy})		^a 16	^a 26	^a 66	т <mark>х</mark> уј	

The coefficient, a_{ij}, are:

$$a_{11} = \frac{1}{E_x}$$
; $a_{12} = -\frac{\nabla x}{E_x}$; $a_{16} = \frac{\eta x, xy}{G_{xy}} = \frac{\eta xy, x}{E_x}$;
 $a_{22} = \frac{1}{E_y}$; $a_{66} = \frac{1}{G_{xy}}$; $a_{26} = \frac{\eta y, xy}{G_{xy}} = \frac{\eta xy, y}{E_y}$;

where $n_{xy,x}$, $n_{xy,y}$, and $n_{x,xy}$, $n_{y,xy}$ are called the coefficients of mutual influence of the first kind and the second kind, respectively, [15,16]. Physically, it is clear that they represent mutual influences between shear strains and normal stresses and between normal strains and shear stresses.

I.2 BOUNDARY CONDITIONS

Only the three major types of boundary conditions, namely, (1)clamped, (2)simply supported, and (3)free, are considered in this dissertation. Many others such as elastically-supported edges can also be handled with minor changes. The equations associated with these three major types of boundary conditions are

(1) Rigidly clamped edge (B_c):

$$w(x,y) = 0$$
; $\frac{\partial w(x,y)}{\partial n} = 0$. (4a)

(2) Simply supported edge (B_s):

$$w(x, y) = 0$$
; $M_n = 0$. (4b)

(3) Free edge (B_f):

$$M_n = 0$$
 ; $N_n + \frac{\partial H_{tn}}{\partial s} = 0$ (4c)

where $\frac{\partial}{\partial s}$ is the derivative along the contour arc, M_n is the unit edge bending moment, N_n is the unit edge shear force, H_{tn} is the unit edge twisting moment, and the subscript n means acting along the normal direction of the edge. These values can be written in terms of their components in the x and y directions as

$$M_{n} = M_{x} \cdot n_{x}^{2} + M_{y} \cdot n_{y}^{2} + 2H_{xy} \cdot n_{x} \cdot n_{y}$$
(5a)

$$H_{tn} = (M_{y} - M_{x}) \cdot n_{x} \cdot n_{y} + H_{xy} \cdot (n_{x}^{2} - n_{y}^{2})$$
(5b)

$$N_{n} = N_{x} \cdot n_{x} + N_{y} \cdot n_{y}$$
(5c)

where n_x and n_y are direction cosines of an outward normal to the contour arc. For an anisotropic problem, the bending moment, twisting moment, and shear force in the x and y directions can be written in terms of the deflection function w(x,y) as

$$M_{\mathbf{x}} = - \left(D_{11} \frac{\partial^2 w}{\partial x^2} + D_{12} \frac{\partial^2 w}{\partial y^2} + 2 D_{16} \frac{\partial^2 w}{\partial x \partial y} \right)$$
(6a)

$$M_{y} = - \left(D_{12} \frac{\partial^{2} w}{\partial x^{2}} + D_{22} \frac{\partial^{2} w}{\partial y^{2}} + 2 D_{26} \frac{\partial^{2} w}{\partial x \partial y} \right)$$
(6b)

$$H_{xy} = - \left(D_{16} \frac{\partial^2 w}{\partial x^2} + D_{26} \frac{\partial^2 w}{\partial y^2} + 2 D_{66} \frac{\partial^2 w}{\partial x \partial y} \right)$$
(6c)

$$N_{x} = - \left[D_{11} \frac{\partial^{3} w}{\partial x^{3}} + 3 D_{16} \frac{\partial^{3} w}{\partial x^{2} \partial y} + (D_{12} + 2 D_{66}) \frac{\partial^{3} w}{\partial x \partial y^{2}} \right]$$

$$+ D_{26} \frac{\partial^3 w}{\partial y^3}]$$
 (6d)

$$N_{y} = - \left[D_{16} \frac{\partial^{3} w}{\partial x^{3}} + (D_{12} + 2D_{66}) \frac{\partial^{3} w}{\partial x^{2} \partial y} + 3D_{26} \frac{\partial^{3} w}{\partial x \partial y^{2}} \right]$$
$$+ D_{22} \frac{\partial^{3} w}{\partial y^{3}} \right]$$
(6e)

Substituting Eqs. (5) and (6) into Eq. (4), the boundary condition equations for anisotropic plates can be written explicitly as

$$w(x,y) = 0$$
 on $B_{c} + B_{s}$ (7a)

$$\frac{\partial w(x,y)}{\partial x} \cdot n_{x} + \frac{\partial w(x,y)}{\partial y} \cdot n_{y} = 0 \qquad \text{on } B_{c} \qquad (7b)$$

$$\frac{\partial^{2} w(x,y)}{\partial x^{2}} \left[D_{11} \cdot n_{x}^{2} + 2D_{16} \cdot n_{x} \cdot n_{y} + D_{12} \cdot n_{y}^{2} \right] \\ + \frac{\partial^{2} w(x,y)}{\partial x \partial y} \left[2D_{16} \cdot n_{x}^{2} + 4D_{66} \cdot n_{x} \cdot n_{y} + 2D_{26} \cdot n_{y}^{2} \right] +$$

$$+ \frac{\partial^2 w(x,y)}{\partial y^2} [D_{12} \cdot n_x^2 + 2D_{26} \cdot n_x \cdot n_y + D_{22} \cdot n_y^2] = 0 \text{ on } B_s + B_f$$
(7c)

$$\frac{\partial^{3} w(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^{3}} \left[D_{11} \cdot n_{\mathbf{x}} (1+n_{\mathbf{y}}^{2}) + 2D_{16} \cdot n_{\mathbf{y}}^{3} - D_{12} \cdot n_{\mathbf{x}} \cdot n_{\mathbf{y}}^{2} \right] \\ + \frac{\partial^{3} w(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^{2} \partial \mathbf{y}} \left[4D_{16} \cdot n_{\mathbf{x}} + D_{12} \cdot n_{\mathbf{y}} (1+n_{\mathbf{x}}^{2}) + 4D_{66} \cdot n_{\mathbf{y}}^{3} - D_{11} \cdot n_{\mathbf{x}}^{2} \cdot n_{\mathbf{y}} \right] \\ - 2D_{26} \cdot n_{\mathbf{x}} \cdot n_{\mathbf{y}}^{2} \right] + \frac{\partial^{3} w(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x} \partial \mathbf{y}^{2}} \left[4D_{26} \cdot n_{\mathbf{y}} + D_{12} \cdot n_{\mathbf{x}} (1+n_{\mathbf{y}}^{2}) + 4D_{66} \cdot n_{\mathbf{x}}^{3} \right] \\ - D_{22} \cdot n_{\mathbf{x}} \cdot n_{\mathbf{y}}^{2} - 2D_{16} \cdot n_{\mathbf{x}}^{2} \cdot n_{\mathbf{y}} \right] + \frac{\partial^{3} w(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y}^{3}} \left[D_{22} \cdot n_{\mathbf{y}} (1+n_{\mathbf{x}}^{2}) \right] \\ + 2D_{26} \cdot n_{\mathbf{x}}^{3} - D_{12} \cdot n_{\mathbf{x}}^{2} \cdot n_{\mathbf{y}} \right] = 0 \qquad \text{on } B_{\mathbf{f}} \qquad (7d)$$

 B_c is the clamped portion of the boundary B, B_s is the simply supported portion of B, and B_f is the free portion of B. Clearly, $B=B_c+B_s+B_f$. For orthotropic plate problems, with D_{11} , D_{22} , and D_{66} being replaced by D_x , D_y , and D_k , respectively; D_{12} by $v_x D_y$ or $v_y D_x$; and $D_{16}=D_{26}=0$; Eqs.(6) and (7) can be reduced to

$$M_{\mathbf{x}} = -D_{\mathbf{x}} \left[\frac{\partial^2 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^2} + v_{\mathbf{y}} \frac{\partial^2 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y}^2} \right]$$
(8a)

$$M_{y} = -D_{y} \left[\frac{\partial^{2} w(x, y)}{\partial y^{2}} + v_{x} \frac{\partial^{2} w(x, y)}{\partial x^{2}} \right]$$
(8b)

$$H_{xy} = -2D_k \frac{\partial^2 w(x, y)}{\partial x \partial y}$$
(8c)

$$N_{\mathbf{x}} = -\frac{\partial}{\partial \mathbf{x}} \left[D_{\mathbf{x}} \frac{\partial^2 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^2} + H \frac{\partial^2 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y}^2} \right]$$
(8d)

$$N_{y} = -\frac{\partial}{\partial y} \left[H \frac{\partial^{2} w(x, y)}{\partial x^{2}} + D_{y} \frac{\partial^{2} w(x, y)}{\partial y^{2}} \right]$$
(8e)

where,

$$D_{k} = \frac{G \cdot h^{3}}{12}, \text{ and } H = D_{x}v_{y} + 2D_{k}$$

$$w(x,y) = 0 \qquad \text{on } B_{c} + B_{s} \quad (9a)$$

$$\frac{\partial w(x,y)}{\partial x} \cdot n_{x} + \frac{\partial w(x,y)}{\partial y} \cdot n_{y} = 0 \qquad \text{on } B_{c} \quad (9b)$$

$$\frac{\partial^{2}w(x,y)}{\partial x^{2}} (D_{x} \cdot n_{x}^{2} + v_{x}D_{y} \cdot n_{y}^{2}) + \frac{\partial^{2}w(x,y)}{\partial y^{2}} (D_{y} \cdot n_{y}^{2} + v_{y}D_{x} \cdot n_{x}^{2})$$

$$+ \frac{\partial^{2}w(x,y)}{\partial x^{\partial y}} (u_{D_{k}} \cdot n_{x} \cdot n_{y}) = 0 \qquad \text{on } B_{s} + B_{f} \quad (9c)$$

$$\frac{\partial^{3}w(x,y)}{\partial x^{3}} [D_{x} \cdot n_{x}^{3} + D_{x} \cdot n_{x} \cdot n_{y}^{2} (2-v_{y})]$$

$$+ \frac{\partial^{3}w(x,y)}{\partial x^{2} \partial y} [v_{x}D_{y} \cdot n_{y} (1+n_{x}^{2})+4D_{k} \cdot n_{y}^{3}-D_{x} \cdot n_{x}^{2} \cdot n_{y}^{2}]$$

$$+ \frac{\partial^{3}w(x,y)}{\partial x^{2} \partial y} [v_{y}D_{x} \cdot n_{x} (1+n_{y}^{2})+4D_{k} \cdot n_{x}^{3}-D_{y} \cdot n_{x} \cdot n_{y}^{2}]$$

$$+ \frac{\partial^{3}w(x,y)}{\partial x^{3}} [D_{y} \cdot n_{y}^{3} + D_{y} \cdot n_{x}^{2} \cdot n_{y} (2-v_{x})] = 0 \qquad \text{on } B_{f} \quad (9d)$$

For isotropic plate problems, these two sets of equations can be further reduced by having $D_k = \frac{D(1-v)}{2}$, $v_x = v_y = v$, and $D_x = D_y = H = D$:

$$M_{\mathbf{x}} = -D\left[\frac{\partial^2 w(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^2} + v \frac{\partial^2 w(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y}^2}\right]$$
(10a)

$$M_{y} = -D\left[\frac{\partial^{2} w(x, y)}{\partial y^{2}} + v \frac{\partial^{2} w(x, y)}{\partial x^{2}}\right]$$
(10b)

$$H_{xy} = -D(1-v)\frac{\partial^2 w(x,y)}{\partial x \partial y}$$
(10c)

$$N_{\mathbf{x}} = -D\frac{\partial}{\partial \mathbf{x}} \left[\nabla^2 \mathbf{w}(\mathbf{x}, \mathbf{y}) \right]$$
(10d)

$$N_{y} = -D_{\partial y}^{\partial} [\nabla^{2} w(x, y)]$$
(10e)

where,

$$\nabla^{2} \mathbf{w}(\mathbf{x}, \mathbf{y}) = \frac{\partial^{2} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^{2}} + \frac{\partial^{2} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y}^{2}}$$

and;

$$w(x,y) = 0 \qquad \text{on } B_{c} + B_{s} \qquad (11a)$$

$$\frac{\partial w(x,y)}{\partial x} \cdot n_{x} + \frac{\partial w(x,y)}{\partial y} \cdot n_{y} = 0 \qquad \text{on } B_{c} \qquad (11b)$$

$$\frac{\partial^2 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^2} (\mathbf{n_x}^2 + \mathbf{vn_y}^2) + \frac{\partial^2 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y}^2} (\mathbf{n_y}^2 + \mathbf{vn_x}^2) + \frac{\partial^2 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^2} (\mathbf{n_y}^2 + \mathbf{vn_x}^2) + \frac{\partial^2 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y}^2} (\mathbf{n_y}^2 + \mathbf{vn_x}^2) + \frac{\partial^2 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^2} (\mathbf{n_x}^2 + \mathbf{vn_x}^2) + \frac{\partial^2 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^2} (\mathbf{n_x}^2 + \mathbf{vn_x}^2) + \frac{\partial^2 \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^2} (\mathbf{n$$

$$\frac{\partial^{3} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^{3}} n_{\mathbf{x}} [1 + n_{\mathbf{y}}^{2} (1 - \nu)] + \frac{\partial^{3} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y}^{3}} n_{\mathbf{y}} [1 + n_{\mathbf{x}}^{2} (1 - \nu)]$$

$$+ \frac{\partial^{3} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^{2} \partial \mathbf{y}} n_{\mathbf{y}} [(2\nu - 1)n_{\mathbf{x}}^{2} + (2 - \nu) \cdot n_{\mathbf{y}}^{2}]$$

$$+ \frac{\partial^{3} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x} \partial \mathbf{y}^{2}} n_{\mathbf{x}} [(2\nu - 1)n_{\mathbf{y}}^{2} + (2 - \nu) \cdot n_{\mathbf{x}}^{2}] = 0 \quad \text{on } B_{\mathbf{f}} \quad (11d)$$

I.3 THE GREEN'S FUNCTION METHOD

Solving a plate problem is, mathematically, to find the deflection function w(x,y) such that the governing differential equation as well as the prescribed boundary conditions are all satisfied. For the special problem of a concentrated force applied at an arbitrary location, the solution is called the "Green's function" of the problem, and is often written as $G(x,y;\xi,n)$. That is, with the prescribed boundary conditions, a Green's function will provide the deflection at any point (x,y) when there is a concentrated force located at some point (ξ,n) . The deflection function for a distributed load q(x,y) over a region R inside the plate can be written, using superposition, as:

$$\mathbf{w}(\mathbf{x},\mathbf{y}) = \int \int_{\mathbf{R}} \mathbf{G}(\mathbf{x},\mathbf{y};\boldsymbol{\xi},\mathbf{\eta}) \cdot \mathbf{q}(\boldsymbol{\xi},\mathbf{\eta}) d\boldsymbol{\xi} d\boldsymbol{\eta}$$
(12)

This is called the Green's function method or the influence function method, [1]. The Green's function can be either in closed form or in infinite series form, and varies with the problem. For isotropic problems, there are many Green's functions available. Some examples are given here.

The Green's function for a clamped circular plate is [1,3,10],

$$G(\mathbf{x}, \mathbf{y}; \xi, \eta) = \frac{1}{16\pi Da^2} \{ (a^2 - \mathbf{x}^2 - \mathbf{y}^2) (a^2 - \xi^2 - \eta^2)$$

$$(13)$$

$$+ [a^2 (\mathbf{x} - \xi)^2 + a^2 (\mathbf{y} - \eta)^2] \ln \frac{a^2 (\mathbf{x} - \xi)^2 + a^2 (\mathbf{y} - \eta)^2}{(a^2 - \mathbf{x}^2 - \mathbf{y}^2) (a^2 - \xi^2 - \eta^2) + a^2 (\mathbf{x} - \xi)^2 + a^2 (\mathbf{y} - \eta)^2} \}$$

where a is the plate radius.

There are two well known Green's functions for a simply supported rectangular plate, and both of them are of the infinite series type. The one with double trigonometric series is called the Navier's solution, while the single trigonometric series function is named after Levy, [1,7]

$$G(x,y;\xi,\eta) = \frac{4}{D\pi^{4}ab} \sum_{mn} \frac{\sin \frac{m\pi x}{a} \sin \frac{m\pi y}{a} \sin \frac{m\pi \xi}{a} \sin \frac{m\pi \eta}{a}}{(\frac{m^{2}}{a^{2}} + \frac{n^{2}}{a^{2}})^{2}}$$

$$m=1,2,3,4,\ldots\infty; n=1,2,3,4,\ldots\infty$$
(14)

and,

$$G(x,y;\xi,\eta) = \frac{a^{2}}{D\pi^{3}} \sum_{m} (1+\beta_{m} \coth\beta_{m} - \frac{\beta_{m} Y}{b} \coth\frac{\beta_{m} Y}{b} - \frac{\beta_{m} \eta}{b} \coth\frac{\beta_{m} \eta}{b})$$

$$\times \frac{\sinh\frac{\beta_{m} \eta}{b} \sinh\frac{\beta_{m} Y}{b} \sin\frac{m\pi x}{a} \sin\frac{m\pi \xi}{a}}{m^{3} \sinh\beta_{m}}$$
(15)

where, $\beta_m = \frac{m\pi b}{a}$, and $m = 1, 2, 3, 4, ... \infty$; if $y < \eta$, replace η by $b-\eta$; if $y \ge \eta$, replace y by b-y.

There is also a Green's function for an infinite plate, [1]

$$G(\mathbf{x},\mathbf{y};\xi,\eta) = \frac{1}{16\pi D} \left[(\mathbf{x}-\xi)^{2} + (\mathbf{y}-\eta)^{2} \right] \ln \frac{(\mathbf{x}-\xi)^{2} + (\mathbf{y}-\eta)^{2}}{a^{2}}$$
(16)

where a is an arbitrary reference radius.

There are also Green's functions for orthotropic and anisotropic plates [11,12,13,14], though not as many. They will be discussed later.

The major limitation to the Green's function method is that one must know the Green's function of a specific plate shape and boundary conditions before Eq. (12) can be employed.

CHAPTER II

INTEGRAL EQUATIONS APPROACHES

II.1 THE BOUNDARY INTEGRAL METHOD

Several investigators have applied boundary-integral techniques to thin plate problems. Jaswon, et al,[17,18,19], have developed a "direct" approach. Altiero and Sikarskie [9] have developed an "indirect" version of the boundary integral equation. The indirect approach is outlined here. The problem of interest is a thin plate of arbitrary plan form and arbitrary boundary conditions, subjected to an arbitrarily-distributed load, Figure 1. However, due to numerical difficulties in the evaluation of second order singularities for a free edge, plates with free edges were excluded in [9].

A plate problem is solved similar to an elasticity problem [20,21]. The plate of interest is embedded in a fictitious plate of the same material for which the Green's function is known. In order to satisfy the boundary conditions of the original problem, a set of fictitious line forces and a set of fictitious ring moments are introduced along the boundary of the embedded plate. The problem is therefore solved if the magnitude of these fictitious forces and moments can be determined such that the original boundary conditions are satisfied.

Knowing that the influence function for a point moment is simply the derivative of the Green's function for a point force with respect to the direction at which the moment is oriented, the influence function for the moment



Figure 1. Problem of Interest

can be easily derived:

$$H(\mathbf{x},\mathbf{y};\boldsymbol{\xi},\mathbf{\eta}) = -\frac{\partial G(\mathbf{x},\mathbf{y};\boldsymbol{\xi},\boldsymbol{\eta})}{\partial \mathbf{n}} = -\frac{\partial G}{\partial \mathbf{x}} \cdot \mathbf{n}_{\mathbf{x}} - \frac{\partial G}{\partial \mathbf{y}} \cdot \mathbf{n}_{\mathbf{y}}$$
(17)

where $G(x,y;\xi,n)$ is the known Green's function of a point force located at (ξ,n) ; $H(x,y;\xi,n)$ is the derived influence function of a point moment located at the same place (ξ,n) which is oriented along the direction \bar{n} , and n_x and n_y are the direction cosines of the outward normal vector \bar{n} with respect to the x and y coordinates, respectively. The negative sign is for convention only.

Let P* represent a set of fictitious forces, M_n^* represent a set of fictitious moments, and q(x,y) be the given distributed lateral load. By superposition, the deflection at any point (x,y) can therefore be computed from the following deflection equation.

$$w(\mathbf{x}, \mathbf{y}) = \int \int \mathbf{q}(\xi, \eta) G(\mathbf{x}, \mathbf{y}; \xi, \eta) d\xi d\eta + \mathbf{\phi} P^*(\xi, \eta) G(\mathbf{x}, \mathbf{y}; \xi, \eta) ds(\xi, \eta)$$

$$B^* \qquad (18)$$

$$+ \mathbf{\phi} M_n^*(\xi, \eta) \left[-n_{\mathbf{x}}(\xi, \eta) \frac{\partial G(\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \xi} - n_{\mathbf{y}}(\xi, \eta) \frac{\partial G(\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \eta}\right] ds(\xi, \eta)$$

where R is the region over which the distributed load q(x,y)is prescribed, B is the boundary of the embedded plate, and $n_x(\xi,\eta)$ and $n_y(\xi,\eta)$ are the direction cosines of the unit outward normal to the plate boundary at the loading point (ξ,η) . Values of P* and M_n^* are to be determined from the boundary condition equations. The boundary condition equations are shown in Eq. (4), and they can be written in terms of the x and y coordinates as shown in Eq. (11).

The Green's function chosen in [9] is the Green's function for a clamped circular plate, Eq. (13). See Figure 2. Substituting this Green's function into the boundary condition equations, one therefore can determine the set of fictitious forces and moments from a set of two boundary conditions. However, since both plan form and boundary conditions are arbitrary, a numerical method must be employed. By modeling the plate with an N-sided polygon, and assuming the fictitious force and moment remain constant along each of the straight edges of this polygon, one can determine a set of N fictitious forces and N fictitious moments such that the boundary conditions at the N mid-points of the N-sided polygon are satisfied. Take the zero deflection boundary condition, for example, which occurs on both clamped and simply supported edges. One will have

$$\sum_{k=1}^{n} P_{k}^{*}(Q_{k}) \left[\int_{S_{k}} G(x,y;\xi,\eta) ds(\xi,\eta) \right]$$

$$+ \sum_{k=1}^{n} M_{nk}^{*}(Q_{k}) \left[-n_{x}(Q_{k}) \int_{S_{k}} \frac{\partial G(x,y;\xi,\eta)}{\partial \xi} ds(\xi,\eta) - n_{y}(Q_{k}) \int_{S_{k}} \frac{\partial G(x,y;\xi,\eta)}{\partial \eta} ds(\xi,\eta) \right] = -\iint_{Q}(\xi,\eta) G(x,y;\xi,\eta) d\xi d\eta$$

$$R$$

$$(19)$$

where Q_k represents the centerpoint of the kth side of the polygon B, and S_k is a coordinate along the kth side, i.e., $0 \stackrel{<}{}_{-}S_k \stackrel{<}{}_{-}\Delta S_k$. Now, if the boundary conditions on B are forced to be satisfied at each of N locations (x_B, y_B) , Eqs. (19) lead to a system of 2N linear algebraic equations for 2N unknowns P_k^* and $M_{n_k}^*$, k=1,2,3,...N. This system of equations can be expressed in matrix form by

$$\begin{bmatrix} RM \\ M \\ M \\ n \end{bmatrix} \left\{ \frac{P^*}{M_n^*} \right\} = \left\{ RL \right\}$$
(20)

where RM is a $2N \times 2N$ matrix, the elements of which are the line integrals of Eq. (19), and RL is a $2N \times 1$ column matrix consisting of the area integrals of Eq. (19). Once Eq. (20) is solved for the unknown fictitious forces and moments, P* and M_n^* , the displacements at any internal point can be found using

$$w(\mathbf{x}, \mathbf{y}) = \int fq(\xi, \eta) G(\mathbf{x}, \mathbf{y}; \xi, \eta) d\xi d\eta + \sum_{k=1}^{n} P_{k}^{*}(Q_{k}) \left[\int G(\mathbf{x}, \mathbf{y}; \xi, \eta) ds(\xi, \eta) \right]$$

$$k = 1 \qquad S_{k}$$

$$+ \sum_{k=1}^{n} M_{nk}^{*}(Q_{k}) \left[-n_{\mathbf{x}}(Q_{k}) \int \frac{\partial G(\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \xi} ds(\xi, \eta) - n_{\mathbf{y}}(Q_{k}) \int \frac{\partial G(\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \eta} ds(\xi, \eta) \right]$$

$$(21)$$

This method works satisfactorily if free boundary conditions are not included. For a free edge, the two boundary condition equations are as shown in Eq.(11). It can be seen that, associated with the fictitious moments, there are eight terms involving fourth order derivatives of the Green's function, Eq.(13). When integrating along the kth side of the polygon B, there will be difficulties in the evaluation of the second order singularities. That is, there will be terms such as

$$\int_{\mathbf{S}_{k}} \frac{1}{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-\eta)^{2}} d\mathbf{s}_{k}(\xi,\eta)$$

which pose difficulties when $\xi + x$ and $\eta + y$. In addition, like other boundary integral methods, the errors in the region near the boundary can be substantial. Due to these two dificiencies, an "auxiliary boundary" method has been developed. This method is presented in the following section.

II.2 THE AUXILIARY BOUNDARY METHOD

The only difference between the current method and the boundary integral method discussed in the previous section is that an integration path, B*, is chosen different from the plate boundary, B; see Figure 3. In so



Figure 2. Problem for which Analytic Solution is Known.



Figure 3. Fictitious Problem

doing, the singularities which arise in the integrand during numerical integration for the free boundary condition are avoided. Since integration is now carried out along the fictitious integration path, B*, there is no need to model the plate with an N-sided polygon. Instead, there are N boundary points prescribed on B where boundary conditions are to be satisfied. This, combined with the fact that the fictitious forces and moments are now located away from the boundary, provide significant improvements in the solution accuracy. For example, at the center point of a clamped rectangular plate, the errors for displacement and bending moments are reduced from 1.8 and 8.0 percent, shown in [9], to 0.04 and 1.6 percent, respectively.

Following the aforementioned procedure but integrating along B* instead of B, plate problems with mixed boundary condition of all the three types can now be solved. Writing the boundary condition equations more explicitly, Eq.(11) become

$$\oint P*(\xi, \eta) G(x, y; \xi, \eta) ds(\xi, \eta)$$

$$B*$$

$$+ \oint M_{n}^{*}(\xi, \eta) [-n_{x}(\xi, \eta) \frac{\partial G(x, y; \xi, \eta)}{\partial \xi} - n_{y}(\xi, \eta) \frac{\partial G(x, y; \xi, \eta)}{\partial \eta}] ds(\xi, \eta)$$

$$= -ffq(\xi, \eta) G(x, y; \xi, \lambda) d\xi d\eta$$

$$(22a)$$

$$if (x, y) is on B_{c} + B_{s};$$

$$\oint P*(\xi, \eta) [n_{x}(x, y) \frac{\partial G(x, y; \xi, \eta)}{\partial x} + n_{y}(x, y) \frac{\partial G(x, y; \xi, \eta)}{\partial y}] ds(\xi, \eta)$$

$$B*$$

$$+ \oint M_{n}^{*}(\xi, \eta) [-n_{x}(\xi, \eta) n_{x}(x, y) \frac{\partial^{2} G(x, y; \xi, \eta)}{\partial y \partial \xi} - n_{y}(\xi, \eta) n_{x}(x, y) \frac{\partial^{2} G(x, y; \xi, \eta)}{\partial x \partial \eta}$$

$$- n_{x}(\xi, \eta) n_{y}(x, y) \frac{\partial^{2} G(x, y; \xi, \eta)}{\partial y \partial \eta} - n_{y}(\xi, \eta) n_{x}(x, y) \frac{\partial^{2} G(x, y; \xi, \eta)}{\partial x \partial \eta}$$

$$(22b)$$

$$= - \int \int g(\xi, \eta) \left[n_{\mathbf{x}}(\mathbf{x}, \mathbf{y}) \frac{\partial G(\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x}} + n_{\mathbf{y}}(\mathbf{x}, \mathbf{y}) \frac{\partial G(\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{y}} \right] d\xi d\eta$$

$$\begin{split} \text{if } (\mathbf{x}, \mathbf{y}) \text{ on } \mathbf{B}_{\mathbf{C}}^{;} \\ & \oint \mathbf{P}^{*} (\xi, \eta) \left[n_{\mathbf{X}}^{2} (\mathbf{x}, \mathbf{y}) \frac{\partial^{2} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x}^{2}} + 2n_{\mathbf{X}} (\mathbf{x}, \mathbf{y}) n_{\mathbf{y}} (\mathbf{x}, \mathbf{y}) \frac{\partial^{2} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x} \partial \mathbf{y}} \right] \\ & + n_{\mathbf{y}}^{2} (\mathbf{x}, \mathbf{y}) \frac{\partial^{2} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{y}^{2}} \right] d\mathbf{s} (\xi, \eta) \\ & + g M_{\mathbf{h}}^{*} (\xi, \eta) \left[-n_{\mathbf{X}} (\xi, \eta) n_{\mathbf{X}}^{2} (\mathbf{x}, \mathbf{y}) \frac{\partial^{3} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x}^{2} \partial \xi} \right] \\ & - 2n_{\mathbf{X}} (\xi, \eta) n_{\mathbf{X}} (\mathbf{x}, \mathbf{y}) n_{\mathbf{y}} (\mathbf{x}, \mathbf{y}) \frac{\partial^{3} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x}^{2} \partial \xi} \\ & - n_{\mathbf{X}} (\xi, \eta) n_{\mathbf{Y}}^{2} (\mathbf{x}, \mathbf{y}) \frac{\partial^{3} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{y}^{2} \partial \xi} - n_{\mathbf{y}} (\xi, \eta) n_{\mathbf{X}}^{2} (\mathbf{x}, \mathbf{y}) \frac{\partial^{3} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x}^{2} \partial \eta} \\ & - 2n_{\mathbf{y}} (\xi, \eta) n_{\mathbf{x}} (\mathbf{x}, \mathbf{y}) n_{\mathbf{y}} (\mathbf{x}, \mathbf{y}) \frac{\partial^{3} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{y}^{2} \partial \xi} - n_{\mathbf{y}} (\xi, \eta) n_{\mathbf{X}}^{2} (\mathbf{x}, \mathbf{y}) \frac{\partial^{3} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x}^{2} \partial \eta} \\ & - 2n_{\mathbf{y}} (\xi, \eta) n_{\mathbf{x}} (\mathbf{x}, \mathbf{y}) n_{\mathbf{y}} (\mathbf{x}, \mathbf{y}) \frac{\partial^{3} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x}^{2} \partial \eta} \\ & - n_{\mathbf{y}} (\xi, \eta) n_{\mathbf{x}} (\mathbf{x}, \mathbf{y}) n_{\mathbf{y}} (\mathbf{x}, \mathbf{y}) \frac{\partial^{3} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{y}^{2} \partial \eta} \\ & = - f f \mathbf{q} (\xi, \eta) \left[n_{\mathbf{x}}^{2} (\mathbf{x}, \mathbf{y}) \frac{\partial^{2} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x}^{2}} + 2n_{\mathbf{x}} (\mathbf{x}, \mathbf{y}) n_{\mathbf{y}} (\mathbf{x}, \mathbf{y}) \frac{\partial^{2} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x} \partial \mathbf{y}} \\ & + n_{\mathbf{y}}^{2} (\mathbf{x}, \mathbf{y}) \frac{\partial^{2} \mathbf{G} (\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{y}^{2}} \right] d\xi d\eta \qquad (22c)$$

if (x,y) is on B_s;

$$\begin{split} & \int_{B^*} P^* (\xi, \eta) \left\{ [n_X^2(x, y) + vn_Y^2(x, y)] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial x^2} \right. \\ & + \left[2 (1 - v) n_X(x, y) n_Y(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial x^2} \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} \right] ds (\xi, \eta) \\ & + \int_{B^*} M_n^* (\xi, \eta) \left\{ -n_X(\xi, \eta) \left[n_X^2(x, y) + vn_Y^2(x, y) \right] \frac{\partial^3 G(x, y; \xi, \eta)}{\partial x^2 \partial \xi} \right] \\ & - n_X(\xi, \eta) \left[2 (1 - v) n_X(x, y) n_Y(x, y) \right] \frac{\partial^3 G(x, y; \xi, \eta)}{\partial x^2 \partial \xi} \\ & - n_X(\xi, \eta) \left[n_X^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^3 G(x, y; \xi, \eta)}{\partial x^2 \partial \eta} \\ & - n_Y(\xi, \eta) \left[n_X^2(x, y) + vn_Y^2(x, y) \right] \frac{\partial^3 G(x, y; \xi, \eta)}{\partial x^2 \partial \eta} \\ & - n_Y(\xi, \eta) \left[2 (1 - v) n_X(x, y) n_Y(x, y) \right] \frac{\partial^3 G(x, y; \xi, \eta)}{\partial x^2 \partial \eta} \\ & - n_Y(\xi, \eta) \left[n_X^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^3 G(x, y; \xi, \eta)}{\partial x^2 \partial \eta} \\ & - n_Y(\xi, \eta) \left[n_X^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^3 G(x, y; \xi, \eta)}{\partial x^2 \partial \eta} \right] ds (\xi, \eta) \\ & = - \int \int q(\xi, \eta) \left\{ \left[n_X^2(x, y) + vn_Y^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial x^2} \right] ds (\xi, \eta) \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial x^2} \right] d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta \\ & + \left[n_Y^2(x, y) + vn_X^2(x, y) \right] \frac{\partial^2 G(x, y; \xi, \eta)}{\partial y^2} d\xi d\eta$$

$$\begin{split} & \oint P^* \left(\zeta, \eta \right) \left[\left[n_X^3 \left(x, y \right) + \left(2 - v \right) n_X \left(x, y \right) n_Y^2 \left(x, y \right) \right] \frac{\partial^3 G \left(x, y; \xi, \eta \right)}{\partial x^3} \right. \\ & + \left[\left(2 v - 1 \right) n_X^2 \left(x, y \right) n_Y \left(x, y \right) + \left(2 - v \right) n_Y^3 \left(x, y \right) \right] \frac{\partial^3 G \left(x, y; \xi, \eta \right)}{\partial x^2 \partial y} \\ & + \left[\left(2 - v \right) n_X^3 \left(x, y \right) + \left(2 v - 1 \right) n_X \left(x, y \right) n_Y^2 \left(x, y \right) \right] \frac{\partial^3 G \left(x, y; \xi, \eta \right)}{\partial x \partial y^2} \\ & + \left[\left(2 - v \right) n_X^2 \left(x, y \right) n_Y \left(x, y \right) + n_Y^3 \left(x, y \right) \right] \frac{\partial^3 G \left(x, y; \xi, \eta \right)}{\partial y^3} \right] ds \left(\xi, \eta \right) \\ & + \oint M_n^* \left(\xi, \eta \right) \left\{ - n_X \left(\xi, \eta \right) \left[n_X^3 \left(x, y \right) + \left(2 - v \right) n_X \left(x, y \right) n_Y^2 \left(x, y \right) \right] \frac{\partial^4 G \left(x, y; \xi, \eta \right)}{\partial x^3 \partial y} \\ & - n_X \left(\xi, \eta \right) \left[\left(2 v - 1 \right) n_X^2 \left(x, y \right) n_Y \left(x, y \right) + \left(2 - v \right) n_X^3 \left(x, y \right) \right] \frac{\partial^4 G \left(x, y; \xi, \eta \right)}{\partial x^2 \partial y \partial \xi} \\ & - n_X \left(\xi, \eta \right) \left[\left(2 - v \right) n_X^3 \left(x, y \right) + \left(2 v - 1 \right) n_X \left(x, y \right) n_Y^2 \left(x, y \right) \right] \frac{\partial^4 G \left(x, y; \xi, \eta \right)}{\partial x \partial y^2 \partial \xi} \\ & - n_X \left(\xi, \eta \right) \left[\left(2 - v \right) n_X^2 \left(x, y \right) n_Y \left(x, y \right) n_Y^2 \left(x, y \right) \right] \frac{\partial^4 G \left(x, y; \xi, \eta \right)}{\partial x \partial y^2 \partial \xi} \\ & - n_X \left(\xi, \eta \right) \left[n_X^3 \left(x, y \right) + \left(2 - v \right) n_X \left(x, y \right) n_Y^2 \left(x, y \right) \right] \frac{\partial^4 G \left(x, y; \xi, \eta \right)}{\partial x^3 \partial \eta} \\ & - n_Y \left(\xi, \eta \right) \left[\left(2 v - 1 \right) n_X^2 \left(x, y \right) n_Y \left(x, y \right) n_Y^2 \left(x, y \right) \right] \frac{\partial^4 G \left(x, y; \xi, \eta \right)}{\partial x^2 \partial y \partial \eta} \\ & - n_Y \left(\xi, \eta \right) \left[\left(2 - v \right) n_X^3 \left(x, y \right) + \left(2 v - 1 \right) n_X \left(x, y \right) n_Y^2 \left(x, y \right) \right] \frac{\partial^4 G \left(x, y; \xi, \eta \right)}{\partial x^2 \partial y \partial \eta} \\ & - n_Y \left(\xi, \eta \right) \left[\left(2 - v \right) n_X^3 \left(x, y \right) + \left(2 v - 1 \right) n_X \left(x, y \right) n_Y^2 \left(x, y \right) \right] \frac{\partial^4 G \left(x, y; \xi, \eta \right)}{\partial x \partial y^2 \partial \eta} \\ & - n_Y \left(\xi, \eta \right) \left[\left(2 - v \right) n_X^3 \left(x, y \right) + \left(2 v - 1 \right) n_X \left(x, y \right) n_Y^2 \left(x, y \right) \right] \frac{\partial^4 G \left(x, y; \xi, \eta \right)}{\partial x \partial y^2 \partial \eta} \\ & - n_Y \left(\xi, \eta \right) \left[\left(2 - v \right) n_X^2 \left(x, y \right) n_Y \left(x, y \right) + n_Y^3 \left(x, y \right) \right] \frac{\partial^4 G \left(x, y; \xi, \eta \right)}{\partial x \partial y^2 \partial \eta} \\ & - n_Y \left(\xi, \eta \right) \left[\left(2 - v \right) n_X^2 \left(x, y \right) n_Y \left(x, y \right) + n_Y^3 \left(x, y \right) \right] \frac{\partial^4 G \left(x, y; \xi, \eta \right)}{\partial x \partial y^2 \partial \eta} \\ \\ & - n_Y$$
$$= -\iint_{R} q(\xi, \eta) \left\{ \left[n_{\mathbf{x}}^{3}(\mathbf{x}, \mathbf{y}) + (2 - \nu) n_{\mathbf{x}}(\mathbf{x}, \mathbf{y}) n_{\mathbf{y}}^{2}(\mathbf{x}, \mathbf{y}) \right] \frac{\partial^{2} G(\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x}^{3}} \right. \\ \left. + \left[(2\nu - 1) n_{\mathbf{x}}^{2}(\mathbf{x}, \mathbf{y}) n_{\mathbf{y}}(\mathbf{x}, \mathbf{y}) + (2 - \nu) n_{\mathbf{y}}^{3}(\mathbf{x}, \mathbf{y}) \right] \frac{\partial^{3} G(\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x}^{2} \partial \mathbf{y}} \right. \\ \left. + \left[(2 - \nu) n_{\mathbf{x}}^{3}(\mathbf{x}, \mathbf{y}) + (2\nu - 1) n_{\mathbf{x}}(\mathbf{x}, \mathbf{y}) n_{\mathbf{y}}^{2}(\mathbf{x}, \mathbf{y}) \right] \frac{\partial^{3} G(\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x} \partial \mathbf{y}^{2}} \right. \\ \left. + \left[(2 - \nu) n_{\mathbf{x}}^{2}(\mathbf{x}, \mathbf{y}) n_{\mathbf{y}}(\mathbf{x}, \mathbf{y}) + n_{\mathbf{y}}^{3}(\mathbf{x}, \mathbf{y}) \right] \frac{\partial^{3} G(\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{x} \partial \mathbf{y}^{2}} \right] \frac{\partial^{3} G(\mathbf{x}, \mathbf{y}; \xi, \eta)}{\partial \mathbf{y}^{3}} d\xi d\eta \quad (22e)$$

if (x,y) is on B_f . The derivatives of the Green's function are listed in Appendix A.

The solution is then obtained using the same numerical procedure shown in the previous section. However, the plate is no longer modeled with an N-sided polygon. Instead, N points are assigned along the boundary, and two of the above equations are satisfied at each of these N points. This is done by adjusting the magnitudes of the unknown fictitious forces P* and moments M_n^* at the N meshes along the fictitious integration path B*. Thus, solving a set of $2N \times 2N$ linear algebraic equations for the set of fictitious forces and moments on B*, and substituting into the deflection equation Eq.(21), the problem of Figure 1 is solved.

A plate problem of arbitrary plan form, arbitrary lateral load, and arbitrary boundary conditions has now been solved. The complete computer program is shown in Appendix B.

Two example problems are illustrated here. In both cases uniformly-loaded square plates are considered. The dimensions of the plate and the fictitious contour B* are shown in Figure 4. The plate is embedded in a fic-titious circular plate with a radius of 80m. Other



Figure 4. A Square Plate with an Auxiliary Integration Contour.

constants are E=2.0684×10⁵ MPa, v=0.3, h=0.01m, and g=1N/m². In the first example, edge one is free, edges two and four are simply supported, and edge three is clamped. In the second example, edges one and three are free while the other two are simply supported. The boundary conditions are satisfied at forty points spaced at a distance lm from each other and .5m from the corner; see Figure 4. The contour B* is located 4m away from the plate boundary and is divided into forty intervals. Integration over an interval B* is done by simply multiplying the value of the integrand at the center of an interval by the interval length. Integration over R is accomplished by subdividing R into 100 equal divisions and multiplying the value of the integrand at the centerpoint of each division by the area of that division. This area subdivision is also shown in Figure 4.

Results of displacements and bending moments at each of the fifteen field points shown in Figure 4 are presented in Tables 1 and 2. When compared with the exact solutions from [1], the average errors are less than three percent for both deflections and bending moments. It may be noted that locations one, two, and three are on edge one of the boundary and between two adjacent boundary points where the boundary conditions are forced to be satisfied. Therefore, accuracy of results at these locations are expected to be the worst. Yet, the errors are less than four percent. Thus, the method using an auxiliary integration path is a great improvement over the previous indirect boundary integral method, especially when results near the boundary are needed.

Comparison of Numerical and Exact Results for a Square Plate,

Table 1

Edge One Free, Edges Two and Four Simply Supported, and Edge Three Clamped (Fig. 4)

ч	0	0	0	-	0	9	ŧ	-	5	S	-	0	æ	9	0
XErro	00.00	00.00	0.00	1.96	0.83	0.70	-3.05	-1.80	-1.70	-10.27	-11.28	-13.90	3.66	2.41	2.21
MY(num.) N-m/m	0.000	0.000	0.000	1.040	2.308	2.711	1.035	2.387	2.747	0.339	0.492	0.447	-1.837	-4.918	-6.058
MY(true) N-m/m	0.000	0.000	0.000	1.020	2.289	2.692	1.068	2.387	2.794	0.378	0.555	0.519	-1.772	-4.802	-5.927
SELLOL	4.271	3.091	2.346	1.308	1.673	1.729	0.763	0.967	1.038	-0.5255	-0.1861	0.000	16.937	4.797	3.743
MX(num.) N-m/m	3.845	8.521	9.989	3.183	7.049	8.236	2.378	096.4	5.677	1.325	2.360	2.560	-0.287	-1.360	-1.469
MX(true) N-m/m	3.687	8.266	9.760	3.142	6.933	8.096	2.360	4.912	5.618	1.332	2.364	2.560	-0.247	-1.084	-1.416
ßError	3.281	3.672	3.125	2.542	2.554	2.449	1.937	8 76 . 1	1.930	1.772	1.570	1.563	1.628	1.366	1.345
W (num.) mm.	1.917	t, 966	6.065	1.440	3.710	4.547	0.967	2.473	3.024	0.513	1.291	1.569	0.088	0.214	0.257
W(true) mm.	1.856	067.4	5.881	1.405	3.618	4.438	616.0	2.426	2.966	0.504	1.271	1.545	0.087	0.211	0.254
Loct.	1	2	e	t	5	9	7	8	6	10	11	12	13	14	15

Edges One and Three Free, Edges Two and Four Simply Supported (Fig. 4) Comparison of Numerical and Exact Results for a Square Plate

ßError	0.000	0.000	000.0	1.665	0.218	0.047	-3.621	-2.429	-2.293
MY (num.) N-m/m	0.000	0.000	0.000	0.861	1.841	2.137	0.993	2.249	2.642
MY(true) N-m/m	0.000	0.000	0.000	0.847	1.837	2.136	1.030	2.305	2.704
ßError	3.780	2.925	2.779	1.676	1.958	1.998	1.603	1.807	1.879
MX(num.) N-m∕m	4.915	11.332	13.464	4.550	10.622	12.649	4.500	10.478	12.470
MX(true) N-m/m	4.736	11.010	13.100	4.475	10.418	12.401	4.429	10.292	12.240
%Error	3.086	3.043	3.008	2.542	2.475	2.534	2.243	2.356	2.299
W(num.) Mm.	2.543	6.579	8.092	2.267	5.871	7.225	2.202	5.703	7.010
W(true) mm.	2.467	6.385	7.855	2.211	5.729	7.047	2.154	5.571	6.853
Loct.	1	2	3	t	5	9	7	8	6

CHAPTER III

A NEW METHOD

III.1 ISOTROPIC PLATE PROBLEMS

Though the use of the boundary integral method to solve isotropic plate problems has been proved successful, the boundary condition equations, Eqs.(22) and Appendix A, are quite lengthy, especially for a free edge. To simplify the formulation, it is reasonable to consider replacing the set of fictitious moments by a second set of fictitious forces. In so doing, there is no need to evaluate all the derivatives of the Green's function with respect to ξ and η , since they are associated with the fictitious moment M_n^* only. With this simplification, the length of boundary condition equations, Eqs.(22), can be reduced by about fifty percent.

In practice, there are two simple ways that one can enter twice as many fictitious forces P^* . One can either double the number of meshes along the integration contour, or define a second integration contour. Tests indicate that the latter provides somewhat better results. On the other hand, during numerical integration, the fictitious force is assumed constant along each mesh. It is therefore logical to replace this evenly distributed line force by a concentrated point force and place it at the center of the mesh. With this change, together with the elimination of the fictitious moments, the deflection function w(x,y) of Eq.(21) can be reduced to the following form.

$$w(\mathbf{x}, \mathbf{y}) = \int \int G(\mathbf{x}, \mathbf{y}; \xi, \mathbf{n}) \cdot \mathbf{q}(\xi, \mathbf{n}) d\xi d\mathbf{n}$$

$$R$$

$$+ \sum_{k=1}^{2n} G(\mathbf{x}, \mathbf{y}; \xi, \mathbf{n}) \cdot \mathbf{P}_{k}^{*}(\xi, \mathbf{n})$$

$$k = 1, 2, 3, \dots \qquad (23)$$

Following a numerical procedure similar to that of section II.2, this simplified method was tested using the same square plate. The radius of the fictitious clamped circular plate was kept at 80m. The first set of fictitious forces were located at four meters away from the plate boundary, and the second set were located at two meters away from the first set, Figure 5. The results were almost identical to those obtained using both fictitious forces and moments.

Thus far the boundary integral method has been modified somewhat, in that the boundary integration has been replaced with an algebraic summing process. Though the elimination of the fictitious moments has been successful, the method is still tedious if free boundaries are involved. In order to further simplify the method, the well known Green's function for an infinite plate, [1], is introduced. It is

$$G(x,y;\xi,\eta) = \frac{1}{16\pi D} \left[(x-\xi)^{2} + (y-\eta)^{2} \right] \ln \frac{(x-\xi)^{2} + (y-\eta)^{2}}{a^{2}} .$$
(24)

Mathematically, this is the fundamental solution to the isotropic plate problem with a unit force at (ξ,n) . The beauty of this Green's function is that the denomenator in the logarithmic term is a constant, a^2 , where a is an arbitrary reference radius at which the deflection is zero. When derivatives are evaluated, this new Green's function gives a much shorter form than that obtained from the



Figure 5. A Square Plate with Two Sets of Fictitious Forces.

Green's function of a clamped circular plate. Consider $\frac{\partial^3 w}{\partial x^3}$ for example. The new Green's function gives

$$\frac{\partial^{3} w}{\partial x^{3}} = \frac{1}{4\pi D} \left\{ \int \int \frac{(x-\xi)^{3}+3(y-\eta)^{2}(x-\xi)}{(x-\xi)^{2}+(y-\eta)^{2}} q(\xi,\eta) d\xi d\eta + \frac{2^{N}}{2} \frac{(x-\xi)^{3}+3(y-\eta)^{2}(x-\xi)}{(x-\xi)^{2}+(y-\eta)^{2}} P_{i}^{*}(\xi,\eta) \right\}$$
(25)

while the old Green's function produces a very tedious expression.

In order to assess the advanteges of using this new Green's function, the previously-solved example problems, i.e., square plates, are repeated. The results are unchanged. However, the saving of computing time is large, approximately sixty percent, as shown in Table 3.

As an illustration of the capability for solving plate problems with odd plan forms, a simply supported and uniformly loaded equilateral triangular plate has also been treated; see Figure 6. For this triangular plate, each side is ten meters long and discretized into ten boundary points. There are, therefore, thirty boundary points in all. To simulate the evenly distributed loading condition of one Newton per square meter, one hundred 0.4333 Newton concentrated forces are placed at the centroids of the one hundred little equilateral triangles which form the plate. The sixty fictitious forces are equally spaced along two contours four and six meters away around the plate boundary. The results are compared with the exact solutions obtained from [1] shown in Table 4 and Figure 7. The errors are quite small.

With the new Green's function and all the other simplifications, the formulation of the new method becomes neat and simple. All the boundary condition equations can be written explicitly. Similar to Eqs.(22), we now have

Comparison of Computing Costs

40 Boundary Points, 100 Internal Forces, and 15 Field Points Isotropic Square Plate under Uniformly Distributed Load

Case No. 1: All the four edges are clamped.

Case No. 2: Edge one and three are free; and edges two and four are simply supported.

Savings %	28	60
The New Method	0.361*	0.383*
Boundary Integral Method	0.857*	*6#6.0
	Case No. 1	Case No. 2

*Execution CP seconds on a CDC Cyber-176 computer.



- O Field Point
- △ Loading Point
- Boundary Point

Figure 6. A Simply Support Equilateral Triangular Isotropic Plate.

Comparison of Results of a Simply Supported Triangular Isotropic Plate

(Figure 6)

Loct.	W(true) mm.	. mm M (KError	MX(true) N-m/m	MX (num.) N-m/m	SError	MY(true) N-m/m	MY (num.) M-m/m	KError
1	000.0	638E-4	*	219E-5	0.0188	¥	.219E-5	0.0021	*
2	0.00515	0.00538	4.595	0.612	0.660	7.929	-0.392	-0.409	4.337
3	0.0337	0.0344	2.179	1.125	1.129	0.356	-0.345	-0.366	6.087
4	0.0906	0.0922	1.732	1.526	1.571	2.949	48600.	00356	*
2	0.166	0.169	1.395	1.800	1.799	-0.056	0.540	0.524	-2.963
9	0.242	0.245	1.306	1.934	1.974	2.068	1.111	1.111	-0.450
7	0.294	0.298	1.231	1.913	1.907	-0.314	1.597	1.588	-0.564
8	0.302	0.306	1.228	1.732	1.758	1.501	1.860	1.859	-0.054
6	0.253	0.257	1.282	1.350	1.349	-0.815	1.770	1.766	-0.226
10	0.147	0.149	2.628	0.781	0.811	3.908	1.194	1.197	0.251
11	0.000	298E-4	*	.299E-5	-0.0080	#	.516E-5	.00165	*

*Large percentage error, but small absolute error.



Figure 7. Comparison of Results of a Triangular Plate.

$$\sum_{i=1}^{2N} P_{i}^{*}(\xi, \eta) [(x-\xi)^{2}+(y-\eta)^{2}] \ln \frac{(x-\xi)^{2}+(y-\eta)^{2}}{a^{2}}$$
$$= -\iint_{R} q(\xi, \eta) [(x-\xi)^{2}+(y-\eta)^{2}] \ln \frac{(x-\xi)^{2}+(y-\eta)^{2}}{a^{2}} d\xi d\eta \quad (26a)$$

if
$$(x,y)$$
 is on $B_{c} + B_{s}$;

$$\sum_{i=1}^{2N} P_{i}^{*}(\xi,\eta) \left[1 + \ln \frac{(x-\xi)^{2} + (y-\eta)^{2}}{a^{2}}\right] \left[(x-\xi)n_{x} + (y-\eta)n_{y}\right]$$
(26b)

$$= -\iint_{R} q(\xi,\eta) \left[1 + \ln \frac{(x-\xi)^{2} + (y-\eta)^{2}}{a^{2}}\right] \left[(x-\xi)n_{x} + (y-\eta)n_{y}\right]$$

if
$$(\mathbf{x}, \mathbf{y})$$
 is on \mathbf{B}_{c} ;

$$\frac{2N}{1 = 1} \mathbf{P}_{1}^{*}(\xi, n) \{n_{\mathbf{x}}^{2}[\frac{2(\mathbf{x}-\xi)^{2}}{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}} + 1 + \ln\frac{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}}{\mathbf{a}^{2}}] + 2n_{\mathbf{x}}n_{\mathbf{y}}\frac{2(\mathbf{x}-\xi)(\mathbf{y}-n)}{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}} + 1 + \ln\frac{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}}{\mathbf{a}^{2}}] + n_{\mathbf{y}}^{2}[\frac{2(\mathbf{y}-n)^{2}}{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}} + 1 + \ln\frac{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}}{\mathbf{a}^{2}}] \}$$

$$= -\int \int q(\xi, n) \{n_{\mathbf{x}}^{2}[\frac{2(\mathbf{x}-\xi)^{2}}{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}} + 1 + \ln\frac{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}}{\mathbf{a}^{2}}] + 2n_{\mathbf{x}}n_{\mathbf{y}}\frac{2(\mathbf{x}-\xi)(\mathbf{y}-n)}{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}} + 1 + \ln\frac{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}}{\mathbf{a}^{2}}] + 2n_{\mathbf{x}}n_{\mathbf{y}}\frac{2(\mathbf{x}-\xi)(\mathbf{y}-n)}{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}}$$
(26c)

$$+ n_{\mathbf{y}}^{2}[\frac{2(\mathbf{y}-n)^{2}}{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}} + 1 + \ln\frac{(\mathbf{x}-\xi)^{2}+(\mathbf{y}-n)^{2}}{\mathbf{a}^{2}}] d\xi dn$$

if (x,y) is on B_s;

$$\sum_{i=1}^{2N} P_{i}^{*}(\xi, \eta) \{ [n_{x}^{3} + (2-\nu)n_{x}n_{y}^{2}] \frac{(x-\xi)[(x-\xi)^{2}+3(y-\eta)^{2}]}{[(x-\xi)^{2}+(y-\eta)^{2}]^{2}} + [(2\nu-1)n_{x}^{2}n_{y}^{2} + (2-\nu)n_{y}^{3}] \frac{(y-\eta)[(y-\eta)^{2}-(x-\xi)^{2}]}{[(x-\xi)^{2}+(y-\eta)^{2}]^{2}} + [(2-\nu)n_{x}^{3} + (2\nu-1)n_{x}n_{y}^{2}] \frac{(x-\xi)[(x-\xi)^{2}-(y-\eta)^{2}]}{[(x-\xi)^{2}+(y-\eta)^{2}]^{2}} + [(2-\nu)n_{x}^{2}n_{y}^{2} + n_{y}^{3}] \frac{(y-\eta)[(y-\eta)^{2}+3(x-\xi)^{2}]}{[(x-\xi)^{2}+(y-\eta)^{2}]^{2}} = -\int f_{q}(\xi, \eta) \{ [n_{x}^{3} + (2-\nu)n_{x}n_{y}^{2}] \frac{(x-\xi)[(x-\xi)^{2}+3(y-\eta)^{2}]}{[(x-\xi)^{2}+(y-\eta)^{2}]^{2}} + [(2\nu-1)n_{x}^{2}n_{y}^{2} + (2-\nu)n_{y}^{3}] \frac{(y-\eta)[(y-\eta)^{2}-(x-\xi)^{2}]}{[(x-\xi)^{2}+(y-\eta)^{2}]^{2}} + [(2-\nu)n_{x}^{3} + (2\nu-1)n_{x}n_{y}^{2}] \frac{(x-\xi)[(x-\xi)^{2}-(y-\eta)^{2}]}{[(x-\xi)^{2}+(y-\eta)^{2}]^{2}} + [(2-\nu)n_{x}^{3} + (2\nu-1)n_{x}n_{y}^{2}] \frac{(x-\xi)[(x-\xi)^{2}-(y-\eta)^{2}]}{[(x-\xi)^{2}+(y-\eta)^{2}]^{2}} + [(2-\nu)n_{x}^{3} + (2\nu-1)n_{x}n_{y}^{2}] \frac{(x-\xi)[(x-\xi)^{2}-(y-\eta)^{2}]}{[(x-\xi)^{2}+(y-\eta)^{2}]^{2}}$$

if (x,y) is on B_{f} ; and

$$\sum_{i=1}^{2N} P_{i}^{*}(\xi,\eta) \{ (n_{x}^{2}+\nu n_{y}^{2}) [\frac{2(x-\xi)^{2}}{(x-\xi)^{2}+(y-\eta)^{2}} + 1 + \ln \frac{(x-\xi)^{2}+(y-\eta)^{2}}{a^{2}}]$$

$$+ 2(1-\nu) n_{x}^{n} n_{y} \frac{2(x-\xi)(y-\eta)}{(x-\xi)^{2}+(y-\eta)}$$

$$+ (n_{y}^{2}+\nu n_{x}^{2}) [\frac{2(y-\eta)^{2}}{(x-\xi)^{2}+(y-\eta)^{2}} + 1 + \ln \frac{(x-\xi)^{2}+(y-\eta)^{2}}{a^{2}}] \} =$$

•

$$= -\int \int q(\xi, \eta) \{ (n_{x}^{2} + \nu n_{y}^{2}) [\frac{2(x-\xi)^{2}}{(x-\xi)^{2} + (y-\eta)^{2}} + 1 + \ln \frac{(x-\xi)^{2} + (y-\eta)^{2}}{a^{2}}] \\ + 2(1-\nu)n_{x}n_{y} \frac{2(x-\xi)(y-\eta)}{(x-\xi)^{2} + (y-\eta)^{2}}$$
(26e)
$$+ (n_{y}^{2} + \nu n_{x}^{2}) [\frac{2(y-\eta)^{2}}{(x-\xi)^{2} + (y-\eta)^{2}} + 1 + \ln \frac{(x-\xi)^{2} + (y-\eta)^{2}}{a^{2}}] \} d\xi d\eta$$

if
$$(x,y)$$
 is on B_f. Note that the common constants such as $\frac{1}{16\pi D}$, involved in both sides of the equations have been

deleted.

With the new Green's function and boundary condition equations, and following the same numerical procedure shown previously to solve a set of 2N×2N linear algebraic equations for the unknowns of 2N fictitious forces, a general isotropic plate problem with arbitrary plar form, loading and boundary conditions can be solved. Although there are many improvements from the original boundary integral method [9], there are added numerical questions to be studied. In the original method, the radius of the fictitious plate involved in the Green's function is the only value to be chosen before analysis. An improper selection of this radius will result in poor solution accuracy. Fortunately, it has been found that good results can be obtained for a wide range of values of this radius. Take a 10m square plate for example. No change in solution has been noticed for values of this radius selected between 80m and 8000m.

For the new point-force method, on the other hand, in addition to the reference radius a in Eq.(24), the locations of the fictitious forces must also be determined. It has been observed from the numerical tests of this 10m plate problem that the fictitious forces must be placed within

a narrow band 1m to 10m away from the plate boundary. Less accurate solutions will result if they are placed within 1m from the boundary, and no solution can be obtained if they are located farther than 10m away. It is conceivable that, like the boundary integral method, when the fictitious forces are too close to the boundary, it is impossible to get good results for those field points near the boundary. This is simply due to the fact that the boundary conditions are not satisfied everywhere along the boundary, but at those discretized boundary points only. On the other hand, when these fictitious forces are placed too far from the plate boundary, the influence due to each individual fictitious force is so weak that together with computing truncation errors, the RM matrix may become ill-conditioned.

Some results for a 10m clamped square plate are shown in Tables 5 through 8 to illustrate the change of solutions when fictitious forces are placed at different locations. Tables 5, 6, and 7 are for double-looped fictitious forces, and the double loops are at 4m and 6m, 1m and 3m, and 0.5m and 2.5m away from the plate boundary, respectively; see Figure 5 for reference. Table 8 is for a single-looped approach. That is, all the 2N fictitious forces are distributed along a single contour surrounding the plate. This contour is 4m away from the plate boundary; see Figure 4 for reference. The results are compared with the exact solution published in [1]. The five locations indicated in these tables correspond to locations 1,6,10,13 and 15 shown in Figure 5. The discrepencies between M_X and M_V of the exact solution were due to the fact that they were obtained from truncated infinite series solutions.

The computer program for isotropic plate problems with arbitrary plan form, loading, and boundary conditions using this simple point-force method is shown in Appendix C. It is believed that for a plate of any size and shape, good solution accuracy can be achieved if the locations of fictitious forces are selected properly. The determination of

Double-Looped Fictitious Forces at 4m and 6m Away from the Plate Boundary Comparison of Results of a Clamped Square Plate

%Error	10.221	-19.900	-3.127	-1.823	-1.572
MY (num.) N-m/m	-0.371	0.145	1.115	1.939	2.254
MY(true) N-m/m	-0.336	0.181	1.151	1.975	2.291
ßError	0 + 7 + 0	-19.500	-3.070	-1.790	-1.550
MX(num.) N-m/m	-0.371	0.145	1.115	1.939	2.254
MX(true) N-m/m	-0.337	0.181	1.151	1.974	2.290
%Error	000.0	-0.150	-0.073	0.000	000.0
.mm.)W	0.016	0.137	0.361	0.576	0.664
W(true) mm.	0.016	0.137	0.361	0.576	0.664
Loct.	٢	2	3	4	5
	Loct.W(true)W(num.)KerrorMX(true)MX(num.)KerrorMY(true)MY(num.)Kerrormm.mm.N-m/mN-m/mN-m/mN-m/mN-m/m	Loct. W(true) W(num.) Kerror MX(true) MX(num.) Kerror MY(num.) Kerror MY(num.)	Loct. W(true) W(num.) Kerror MX(true) MX(num.) Kerror N=m/m N=m/m	Loct.W(true)W(num.)KerrorMX(true)MY(true)MY(num.)Kerrornm.mm.mm.N-m/mN-m/mN-m/mN-m/mNem/m10.0160.0160.000-0.337-0.3719.740-0.336-0.37110.22120.1370.137-0.1500.1810.145-19.5000.1810.145-19.500-19.90030.3610.361-0.0731.1511.115-3.0701.1511.115-3.127	Loct.W(true)W(num.)KerrorMX(true)MX(num.)KerrorMY(num.)Kerrornm.mm.mm.mm.N-m/mN-m/mN-m/mN-m/m10.0160.0160.000-0.337-0.3719.740-0.336-0.37110.22120.1370.137-0.1500.1810.145-19.5000.1810.145-19.90030.3610.361-0.0731.1511.115-3.0701.1511.115-3.12740.5760.5760.0001.9741.939-1.7901.939-1.823

Double-Looped Fictitious Forces at 1m and 3m Away from the Plate Boundary Comparison of Results of a Clamped Square Plate

%Error	11.296	-17.970	-3.113	-1.890	-1.635
MY (num.) M−N/m-N	-0.373	0.149	1.116	1.938	2.253
MY(true) N-m/m	-0.336	0.181	1.151	1.975	2.291
%Error	10.514	-17.562	-3.056	-1.860	-1.61
MX (num.) N-m/m	-0.373	0.149	1.116	1.938	2.253
MX(true) N-m/m	-0.337	0.181	1.151	1.974	2.290
ßError	0.000	-0.08	-0.146	-0.091	-0.870
W (num.) mm.	0.016	0.137	0.360	0.576	0.664
W(true) mm.	0.016	0.137	0.361	0.576	0.664
Loct.	l	2	3	t	2

Double-Looped Fictitious Forces at 0.5m and 2.5m Away from the Plate Boundary Comparison of Results of a Clamped Square Plate

ßError	9.104	-17.823	-3.808	-2.397	-2.078
MY (num.) N-m/m	-0.367	0.149	1.108	1.928	2.243
MY(true) N-m/m	-0.336	0.181	1.151	1.975	2.291
ßError	8.630	-17.414	-3.750	-2.368	-2.054
MX (num.) N-m/m	-0.367	0.149	1.108	1.928	2.243
MX(true) N-m/m	-0.337	0.181	1.151	1.974	2.290
ßError	-2.300	-1.188	-1.091	-0.912	-0.870
W(num.) mm.	0.016	0.135	0•360	0.570	0.658
W(true) mm.	0.016	0.137	0.361	0.576	0.664
Loct.	٢	2	3	ħ	2

Single-Looped Fictitious Forces at 4m Away from the Plate Boundary Comparison of Results of a Clamped Square Plate

%Error	18.523	-19.598	-3.088	-1.806	-1.558
MY(num.) N−m/m	-0.398	0.146	1.116	1.940	2.255
MY(true) N-m/m	-0.336	0.181	1.151	1.975	2.291
%Error	18.005	-19.190	-3.030	-1.776	-1.533
MX (num.) N-m/m	-0.398	0.146	1.116	1.940	2.255
MX(true) N-m/m	-0.337	0.181	1.151	1.974	2.290
ßError	-6.868	-0.192	-0.058	0.000	000.0
W(num.) mm.	0.015	0.137	0.361	0.576	0.664
W(true) mm.	0.016	0.137	0.361	0.576	0.664
Loct.	1	2	3	tı	5

of these locations may not be an easy task, and this numerical question requires further study.

On the other hand, for multiply connected plates, seemingly there will be difficulties if the "holes" are small and many boundary points are prescribed at the holes, since placing many fictitious forces in a small area inside a hole will certainly lead to numerical problems. Further study is needed in the search for optimum locations for fictitious forces.

III.2 ANISOTROPIC PLATE PROBLEMS

Due to the increased use of composite and multilayered plates for strength and weight reduction, anisotropic plate problems are becoming more and more important. With midplane symmetry of the material properties, the governing equation is Eq.(3), and the boundary conditions are given by Eqs.(5) and (7). They are far more complicated than when the plate is made from an isotropic material. Finite difference and finite element methods are generally used to obtain a solution. In this dissertation, a new numerical method is introduced. Using the Green's function for an anisotropic infinite plate and the same point-force technique shown previously, solution of an anisotropic plate problem with arbitrary plan form, loading, and boundary conditions is obtainable.

Since general anisotropic problems are very difficult to solve, they are often reduced to orthotropic problems through coordinate transformation or approximation, whenever possible. Therefore, orthotropic problems will be discussed first. For an orthotropic material, there are three mutually perpendicular planes of symmetry with respect to the elastic properties of the material, and the problems are greatly simplified compared with general anisotropic problems. In practice, it appears that orthotropic problems are more common than the general anisotropic plate problems. Reinforced decks in civil, marine, and aerospace engineering, and plates made of layered composite materials are typical examples.

III.2.1 ORTHOTROPIC PROBLEMS

For an orthotropic problem, the governing differential equation, Eq.(2), is a special form of Eq.(3), the equation for anisotropic problems, with $D_{16}=D_{26}=0$. The boundary condition equations are also simpler than those for their anisotropic counterparts; see Eqs.(4) and (8). It is due to these simplifications that solutions are obtainable for many problems. Bares and Massonet [22] used a beam and grid analogy, Vinson and Brull [23] used a power series expansion, and Rajappa [24] tried a Maclaurin's series. In addition, the application of finite difference method is clearly presented by Szilard [7], and the theory of finite element method is explicitly shown in Zienkiewicz's text [8]. For classic approaches, texts [14,26] of Lekhnitskii and Huber, respectively, are probably the most important.

For an orthotropic material, if the geometric coordinates are aligned with the principal material directions, the governing differential equation for equilibrium can be shown in Eq.(2). There are four material constants, namely E_x , E_y , v_x , and G_{xy} , where E_x and E_y are the two Young's moduli evaluated along the x and y directions, respectively; v_x is the Poisson's ratio in the x direction due to normal stress in the y direction; G_{xy} is the shear modulus. The other Poisson's ratio v_y is related to v_x by Betti's reciprocal theorem

 $v_{\mathbf{y}} = \frac{\mathbf{E}_{\mathbf{y}} v_{\mathbf{x}}}{\mathbf{E}_{\mathbf{x}}}$

and therefore is not an independent material constant.

In order to apply the new point-force method, the first requirement is to find the Green's function of some appropriate problem. There are two Green's functions readily available for a simply supported rectangular plate, namely Navier's double series solution and Levy's single series solution:

$$G(x,y;\xi,\eta) = \frac{4b^{3}}{\pi^{4}a} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin\frac{m\pi\xi}{a}}{D_{x}(\frac{bm}{a})^{4} + 2H(n\frac{bm}{a})^{2} + D_{y}n^{4}}$$

$$m=1,2,3,\ldots, n=1,2,3,\ldots$$
(27)

and

$$G(\mathbf{x},\mathbf{y};\xi,\eta) = \frac{2b^{2}}{\pi^{3}D_{\mathbf{x}}D_{\mathbf{y}}} \cdot \frac{1}{(\beta^{2}-\lambda^{2})} \sum_{n}^{\infty} \frac{\sin \frac{n\pi\eta}{b} \sin \frac{n\pi y}{b}}{n^{3}} \times$$

$$\times \left[\frac{\beta \sinh \frac{n\pi\lambda(\mathbf{a}-\xi)}{b} \sinh \frac{n\pi\lambda x}{b}}{\sinh \frac{n\pi\lambda a}{b}} - \frac{\lambda \sinh \frac{n\pi\eta(\mathbf{a}-\xi)}{b} \sinh \frac{n\pi\beta x}{b}}{\sinh \frac{n\pi\beta a}{b}}\right]$$

$$(28)$$

$$n = 1, 2, 3, \dots$$

for $0 \le x < \xi$; and substitute x by (a-x) and (a- ξ) by ξ for $\xi \le x \le a$, where β and λ are the roots of the characteristic equation (which will be discussed later), a and b are the dimensions of the rectangular plate, and ξ and η are the location of the point force. It is known that the Navier's double series solution converges slowly. However, due to its simplicity in higher order derivatives, it was also tested along with Levy's single series solution. Before the full development for orthotropic problems, these two Green's function were evaluated for their efficiency in isotropic problems. For a square plate under uniformly distributed load, the results were disappointing for both approaches. For a solution accuracy greater than ninety percent, more than one hundred terms of Levy's series were needed, and the number is even higher for Navier's series. The computing

costs were formidable. Therefore, the idea of using either approach was abandoned.

Since the fast converging Levy's series failed to yield satisfactory results in the application of the point-force method, it was clear that the Green's function to be adopted for the method must be in closed form. One of the currently existing Green's function in closed form is given in [12] for an infinite plate. Depending on inter-relationships among the material constants, this Green's function contains a group of three independent equations. These equations are derived in terms of several new material parameters. Therefore, it is necessary to introduce these new material parameters prior to the presentation of the governing equations. Let

$$\rho = \frac{H}{\sqrt{D_{\mathbf{x}}D_{\mathbf{y}}}}$$
, and $\varepsilon^{4} = \frac{D_{\mathbf{x}}}{D_{\mathbf{y}}}$ (29)

then, the governing partial differential equation, Eq.(2), can be re-written in the form

$$\frac{\partial^{4} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial^{4} \mathbf{y}} + 2\rho \varepsilon^{2} \frac{\partial^{4} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^{2} \partial \mathbf{y}^{2}} + \varepsilon^{4} \frac{\partial^{4} \mathbf{w}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}^{4}} = \frac{\mathbf{q}(\mathbf{x}, \mathbf{y})}{\mathbf{D}_{\mathbf{y}}}.$$
 (30)

Since the Green's function is the solution for the Dirac delta loading function of this equation, and this equation can be integrated in its homogeneous form, i.e., q(x,y)=0 for $(x,y)\neq(\xi,\eta)$, we can write the Green's function symbolically as

$$D_{1}D_{2}D_{3}D_{4}G(x,y;\xi,\eta) = 0, \qquad (31)$$

where the D's are linear differential operators, in the form

$$D_{i} = \frac{\partial}{\partial y} - r_{i} \cdot \frac{\partial}{\partial x}$$
(32)

and r_i are determined as the roots of the characteristic equation

$$\mathbf{r}^{4} + 2\rho\varepsilon^{2}\mathbf{r}^{2} + \varepsilon^{4} = 0$$
 (33)

These roots are either complex or pure-imaginary as shown by Lekhnitskii [27]. That is, the roots are in the form of

$$r_{1,2} = \pm i\beta$$
; $r_{3,4} = \pm i\lambda$. (34)

Depending on the value of ρ , either greater than, equal to, or less than unity, the values β and λ can be easily determined by using either one of the following three equations,

$$\beta = \varepsilon \sqrt{\rho + \sqrt{\rho^2 - 1}}, \quad \lambda = \varepsilon \sqrt{\rho - \sqrt{\rho^2 - 1}}$$
 (35a)

for $\rho > 1$;

$$\beta = \epsilon$$
 , $\lambda = \epsilon$ (35b)

for $\rho=1$; and

$$\beta = \mu_1 + i\mu_2$$
, $\lambda = \mu_1 - i\mu_2$ (35c)

for $\rho < 1$, where

$$\mu_{1,2} = \epsilon \sqrt{\frac{1 \pm \rho}{2}} .$$

It is clear that, depending on the material constants, each of the three criteria must be considered. For an infinite plate, Mossakowski [12] has derived three different Green's functions for these three different material types. With

$$D_{o} = \sqrt{D_{x}D_{y}} = \varepsilon^{2}D_{y},$$

we have

$$G(\mathbf{x}, \mathbf{y}; \xi, \eta) = \frac{1}{8\pi D_{O}(\beta^{2} - \lambda^{2})} \{\beta [(\mathbf{x} - \xi)^{2} - \lambda^{2} (\mathbf{y} - \eta)^{2}] \ln \frac{(\mathbf{x} - \xi)^{2} + \lambda^{2} (\mathbf{y} - \eta)^{2}}{\mathbf{a}^{2}}$$

-4\lambda\beta (\mathbf{x} - \xi) (\mathbf{y} - \eta) [\mathbf{arc} \text{tg} \frac{\lambda (\mathbf{y} - \eta)}{(\mathbf{x} - \xi)} - \text{arc} \text{tg} \frac{\beta (\mathbf{y} - \eta)}{(\mathbf{x} - \xi)}]
-\lambda [(\mathbf{x} - \xi)^{2} - \beta^{2} (\mathbf{y} - \eta)^{2}] \lambda \frac{\lambda (\mathbf{y} - \eta)}{(\mathbf{x} - \xi)} - \text{arc} \text{tg} \frac{\beta (\mathbf{y} - \eta)}{(\mathbf{x} - \xi)}]
-\lambda [(\mathbf{x} - \xi)^{2} - \beta^{2} (\mathbf{y} - \eta)^{2}] \lambda \frac{(\mathbf{x} - \xi)^{2} + \beta^{2} (\mathbf{y} - \eta)^{2}}{\textbf{a}^{2}}
-\lambda [(\mathbf{x} - \xi)^{2} - \beta^{2} (\mathbf{y} - \eta)^{2}] \lambda \frac{(\mathbf{x} - \xi)^{2} + \beta^{2} (\mathbf{y} - \eta)^{2}}{\textbf{a}^{2}}

$$-3(\beta-\lambda)[(x-\xi)^{2}+\lambda\beta(y-\eta)^{2}]$$
 (36a)

.

for $\rho > 1;$

$$G(\mathbf{x}, \mathbf{y}; \xi, \eta) = \frac{1}{32\pi D_{O}} \{ \frac{(\mathbf{x}-\xi)^{2} + \varepsilon^{2} (\mathbf{y}-\eta)^{2}}{\mu_{1}} \frac{(\mathbf{x}-\xi)^{4} + 2\rho\varepsilon^{2}(\mathbf{x}-\xi)^{2}(\mathbf{y}-\eta)^{2} + \varepsilon^{4}(\mathbf{y}-\eta)^{4}}{\mathbf{a}^{4}} - \frac{2\left[(\mathbf{x}-\xi)^{2} - \varepsilon^{2} (\mathbf{y}-\eta)^{2}\right]}{\mu_{2}} + \operatorname{arc} \operatorname{tg} \frac{2\mu_{1}\mu_{2} (\mathbf{y}-\eta)^{2}}{(\mathbf{x}-\xi)^{2} + \rho\varepsilon^{2} (\mathbf{y}-\eta)^{2}} - \frac{2\varepsilon^{2} (\mathbf{x}-\xi) (\mathbf{y}-\eta)}{\mu_{1}\mu_{2}} \ln \frac{\mu_{1}^{2}(\mathbf{y}-\eta)^{2} + \left[(\mathbf{x}-\xi) - \mu_{2} (\mathbf{y}-\eta)\right]^{2}}{\mu_{1}^{2}(\mathbf{y}-\eta)^{2} + \left[(\mathbf{x}-\xi) + \mu_{2} (\mathbf{y}-\eta)\right]^{2}} - \frac{6\left[(\mathbf{x}-\xi)^{2} + \varepsilon^{2} (\mathbf{y}-\eta)^{2}\right]}{\mu_{1}} \right\}$$
(36b)

for $\rho < 1$; and

$$G(x,y;\xi,\eta) = \frac{1}{16\pi\epsilon D_{O}} \{ [(x-\xi)^{2} + \epsilon^{2} (y-\eta)^{2}] ln \frac{(x-\xi)^{2} + \epsilon^{2} (y-\eta)^{2}}{a^{2}} - [3(x-\xi)^{2} + \epsilon^{2} (y-\eta)^{2}] \}$$
(36c)

for $\rho=1$.

The second order derivatives of these equations are also given in [12]. Since they are needed not only in the boundary condition equations for a simply supported or free edge, but also in the determination of bending moments after fictitious point forces are computed, they are worth including in the following. Other derivatives are listed in Appendix D. There are three sets of equations, one set for each Green's function.

$$\frac{\partial^{2} G}{\partial x^{2}} = \frac{1}{4\pi D_{O}(\beta^{2} - \lambda^{2})} \left[\beta \ln \frac{(x-\xi)^{2} + \lambda^{2} (y-\eta)^{2}}{a^{2}} - \lambda \ln \frac{(x-\xi)^{2} + \beta^{2} (y-\eta)^{2}}{a^{2}}\right]$$

$$\frac{\partial^{2} G}{\partial y^{2}} = \frac{\varepsilon^{2}}{4\pi D_{O}(\beta^{2} - \lambda^{2})} \left[\beta \ln \frac{(x-\xi)^{2} + \beta^{2} (y-\eta)^{2}}{a^{2}} - \lambda \ln \frac{(x-\xi)^{2} + \lambda^{2} (y-\eta)^{2}}{a^{2}}\right]$$

$$\frac{\partial^{2} G}{\partial x \partial y} = \frac{\varepsilon^{2}}{2\pi D_{O}(\beta^{2} - \lambda^{2})} \left[\operatorname{arc} tg \frac{\beta (y-\eta)}{(x-\xi)} - \operatorname{arc} tg \frac{\lambda (y-\eta)}{(x-\xi)}\right] \qquad (37a)$$
for $\rho > 1$;
$$\frac{\partial^{2} G}{\partial x^{2}} = \frac{1}{4\pi D_{O}(\beta^{2} - \lambda^{2})} \left[\operatorname{arc} tg \frac{\beta (y-\eta)}{(x-\xi)} - \operatorname{arc} tg \frac{\lambda (y-\eta)}{(x-\xi)}\right] \qquad (37a)$$

$$\frac{\partial^{2} G}{\partial x^{2}} = \frac{1}{16\pi D_{O}} \left[\frac{1}{\mu_{1}} n \frac{(x-\xi)^{2} + 2\rho\epsilon^{2} (x-\xi)^{2} (y-\eta)^{2} + (y-\eta)^{2}}{a^{4}} - \frac{2}{\mu_{2}} \operatorname{arc} tg \frac{2\mu_{1}\mu_{2} (y-\eta)^{2}}{(x-\xi)^{2} + \rho\epsilon^{2} (y-\eta)^{2}} \right]$$

$$\frac{\partial^{2} G}{\partial y^{2}} = \frac{\epsilon^{2}}{16\pi D_{O}} \left[\frac{1}{\mu_{1}} n \frac{(x-\xi)^{4} + 2\rho\epsilon^{2} (x-\xi)^{2} (y-\eta)^{2} + \epsilon^{4} (y-\eta)^{4}}{a^{4}} + \frac{1}{2} \right]$$

$$+ \frac{2}{\mu_{2}} \operatorname{arc} \operatorname{tg} \frac{2\mu_{1}\mu_{2}(y-\eta)^{2}}{(x-\xi)^{2}+\rho\varepsilon^{2}(y-\eta)^{2}}$$

$$\frac{\partial^{2}G}{\partial x\partial y} = \frac{-\varepsilon^{2}}{16\pi D_{0}\mu_{1}\mu_{2}} \ln \frac{\mu_{1}^{2}(y-\eta)^{2}+[(x-\xi)-\mu_{2}(y-\eta)]^{2}}{\mu_{1}^{2}(y-\eta)^{2}+[(x-\xi)-\mu_{2}(y-\eta)]^{2}}$$
(37b)

for $\rho < 1$; and

$$\frac{\partial^2 G}{\partial x^2} = \frac{1}{8\pi\epsilon D_0} \left[\ln \frac{(x-\xi)^2 + \epsilon^2 (y-\eta)^2}{a^2} - \frac{2\epsilon^2 (y-\eta)^2}{(x-\xi)^2 + \epsilon^2 (y-\eta)} \right]$$

$$\frac{\partial^2 G}{\partial y^2} = \frac{\epsilon}{8\pi D_0} \left[\ln \frac{(x-\xi)^2 + \epsilon^2 (y-\eta)^2}{a^2} + \frac{2\epsilon^2 (y-\eta)^2}{(x-\xi)^2 + \epsilon^2 (y-\eta)} \right]$$

$$\frac{\partial^2 G}{\partial x \partial y} = \frac{1}{4\pi D_0} \left[\frac{\epsilon (x-\xi) (y-\eta)}{(x-\xi)^2 + \epsilon^2 (y-\eta)^2} \right]$$
(37c)

for $\rho=1$.

Following the same numerical procedure as in isotropic problems, orthotropic plate problems of arbitrary plan form, loading, and boundary conditions can now be solved. The added complexity is that depending on $\rho_{<}^{>1}$, there are three sets of equations to be concerned with. In order to verify the results for all the three possibilities, three sample cases have been solved. Consider a simply supported 10m square plate, and let $E_x=2.068\times10^5$ MPa, $E_y=E_x/15$, $v_x=0.3$, and h=0.01m. Varying ρ from 0.1 to 1.0 and 10.0, the accuracy of all theses three sets of results of deflections and bending moments are excellent when compared with the double series solution shown in [14], taking 400 terms. It is due to the fact that the changes are minimal when more than 100 terms are taken in the double series solution, the 400-term double series solution is believed very close to the exact one. The comparisons are tabulated in Tables 9 through 11. The nine locations of field points are shown in Figure 8.

The large errors for the bending moments in the y direction of Table 11 are somewhat misleading, because their magnitudes are small in comparison with the bending moments in the x direction. The computer program for these examples is shown in Appendix E.

III.2.2 ANISOTROPIC PROBLEMS

An anisotropic thin plate is considered as a plate made with a material which has the mid-plane of the plate as the only plane of material symmetry. It is due to the complexity of its governing and boundary condition equations, Eqs.(3) and (7), that efforts are always made to reduce anisotropic problems to orthotropic problems. That is, methods such as coordinate transformation are often tried to eliminate the two material constants a_{16} and a_{26} in Eq.(3). This is, however, not always possible. In general, neglecting these constants often times will lead to large errors, [28]. Therefore, though techniques to the solution of orthotropic problems are more important than that of general anisotropic problems, methods for the latter must also be developed.

Since a large number of anisotropic problems are related to man-made layered composite materials, it is wise to review a few references that will provide a better understanding of composite anisotropic materials: [28] presents basic concepts, fundamental equations, and many interesting illustrations; [29] introduces many exotic materials, their mechanical properties and applications; [30] illustrates many matrix systems and their characters; and [31] gives many studies of the applications and their significant contributions to the aerospace industry.

The solution of anisotropic plate problems is again



Figure 8. A Simply Supported Orthotropic Plate.

6	
le	
ab	
E-	

Comparison of Results of a Simply Supported Orthotropic Square Plate

$$E_{x} = 2.068 \times 10^{5} \text{ MPa, } E_{y} = E_{x} / 15, v_{x} = 0.3, h = 0.01 \text{m}$$

p=0.1

%Error	-8.967	-1.252	-0.831	-11.749	-3.762	-2.330	-15.297	-5.799	-4.664
MY (num.) N-m/m	0.300	0.763	0.907	0.303	0.842	1.048	0.237	0.666	0.824
MY(true) N-m/m	0.329	0.775	0.915	0.343	0.875	1.073	0.280	0.707	0.864
ßError	4.716	1.794	1.335	2.198	1.137	1.008	1.947	1.018	0.918
MX (num.) N-m/m	2.296	4.596	5.236	4.418	10.140	12.030	4.974	11.613	13.858
MX(true) N-m/m	2.192	4.515	5.167	4.323	10.026	11.910	4.879	11.496	13.732
ßError	1.996	1.821	1.763	1.347	1.268	1.243	8 17 6 * 0	1.111	1.109
W(num.) mm.	0.996	2.543	3.107	2.235	5.780	7.103	2.571	6.674	8.210
W(true) mm.	0.976	2.497	3.053	2.205	5.707	7.016	2.546	6.601	8.120
Loct.	1	2	3	ħ	5	9	7	8	6

Comparison of Results of a Simply Supported Orthotropic Square Plate

 $E_{x} = 2.068 \times 10^{5} \text{ MPa}, E_{y} = E_{x} / 15, v_{x} = 0.3, h = 0.01 \text{m}$

p=1.0

ßError	-13.233	-4.049	-2.208	-14.938	-6.001	-4.833	-17.123	-7.246	-5.949
MY (num.) N-m/m	0.195	0.510	0.611	0.205	0.567	0.699	0.179	0.499	0.617
MY(true) N-m/m	0.225	0.531	0.625	0.241	0.603	0.735	0.216	0.538	0.656
ßError	2.307	1.145	0.889	1.444	0.791	0.689	1.297	0.694	0.610
MX (num.) N-m/m	1.614	3.182	3.610	3.232	7.128	8.329	3.671	8.266	9.728
MX(true) N-m/m	1.578	3.146	3.578	3.186	7.072	8.272	3.624	8.209	9.669
ßError	1.670	1.598	1.527	1.070	1.039	4.556	3,340	0.928	006.0
W(num.) Mm.	0.695	1.770	2.160	1.578	4.063	4.983	1.837	4.743	5.823
W(true) mm.	0.684	1.742	2.127	1.561	4.021	4.766	1.778	4.699	5.771
Loct.	1	2	3	t	5	9	7	8	6

Comparison of Results of a Simply Supported Orthotropic Square Plate

 $E_x = 2.068 \times 10^5 \text{ MPa}, E_y = E_x / 15, v_x = 0.3, h = 0.01 \text{m}$ $\rho = 10.0$

KELLOL	-73.766	-32.780	-27.331	-65.801	-30.951	-26.367	-96.285	-30.464	-25.591
MY (num.) N-m/m	0.015	0.089	0.114	0.022	0.102	0.129	0.025	0.105	0.1323
MY(true) N-m/m	0.058	0.132	0.157	0.066	0.148	0.175	0.668	0.151	0.178
%Error	-5.914	-4.547	-4.333	-2.107	-1.891	-1.858	-1.734	-1.599	-1.566
MX (num.) N-m/m	0.460	0.754	0.808	1.022	1.764	1.902	1.190	2.093	2.263
MX(true) N-m/m	0.489	0.790	0.845	1.044	1.798	1.938	1.211	2.127	2.299
%Error	0.158	0.869	0.912	0.365	0.546	0.752	0.443	0.537	0.506
W(num.) mm.	0.176	0.441	0.534	0.408	1.025	1.244	0.484	1.216	1.475
W(true) mm.	0.176	0.437	0.529	0.407	1.020	1.234	0.481	1.209	1.467
Loct.	1	2	3	Ħ	5	. 9	7	8	6

obtained using a known Green's function for an infinite anisotropic plate [11,13] and applying a set of fictitious forces surrounding the plate boundary such that all the boundary conditions are satisfied. Similar to the previous problems, the numerical procedure is to solve the 2N×2N algebraic boundary condition equations for the unknown magnitude of fictitious forces. The only added work is in the determination of the complex roots of the characteristic polynomial equation. IMSL computer subroutine ZPOLR has been conveniently employed for this purpose.

The characteristic equation for the homogeneous solution of Eq.(3) is [13,14],

$$r^{4} + 4\frac{D_{26}}{D_{22}}r^{3} + 2\frac{D_{12}^{+2D}66}{D_{22}}r^{2} + 4\frac{D_{16}}{D_{22}}r + \frac{D_{11}}{D_{22}} = 0$$
 (38)

where the roots r_i are involved in the four linear differential operators $\frac{\partial}{\partial y} - r_i \cdot \frac{\partial}{\partial x}$, the same as in the orthotropic formulation. Solving this fourth order algebraic equation, the roots can be determined in the form of

$$r_{1,2} = \alpha \pm i\beta$$
; $r_{3,4} = \gamma \pm i\lambda$

They are all complex values as proved in [27].

For an infinite plate, the Green's function shown in [13] is

$$G(\mathbf{x},\mathbf{y};\xi,\eta) = \frac{1}{8\pi D_{22}\phi_{1}\phi_{2}} \{ \frac{(\alpha-\gamma)^{2} - (\beta^{2} - \lambda^{2})}{\beta} R_{1}(\mathbf{x},\mathbf{y};\xi,\eta) + \frac{(\alpha-\gamma)^{2} + (\beta^{2} - \lambda^{2})}{\lambda} R_{3}(\mathbf{x},\mathbf{y};\xi,\eta) + 4(\alpha-\gamma) [S_{1}(\mathbf{x},\mathbf{y};\xi,\eta) - S_{3}(\mathbf{x},\mathbf{y};\xi,\eta)] \}$$
(39)

where,

$$\phi_{1} = (\alpha - \gamma)^{2} + (\beta - \lambda)^{2} ; \quad \phi_{2} = (\alpha - \gamma)^{2} + (\beta + \lambda)^{2} ;$$

$$R_{1}(x, y; \xi, n) = \{ [(x - \xi) + \alpha (y - n)]^{2} - \beta^{2} (y - n)^{2} \}$$

$$\times \{ ln \frac{[(x - \xi) + \alpha (y - n)]^{2} + \beta^{2} (y - n)^{2}}{a^{2}} - 3 \}$$

$$- 4\beta (y - n) [(x - \xi) + \alpha (y - n)] arc tg \frac{\beta (y - n)}{(x - \xi) + \alpha (y - n)} ;$$

$$S_{1}(x, y; \xi, n) = \beta (y - n) [(x - \xi) + \alpha (y - n)]$$

$$\times \{ ln \frac{[(x - \xi) + \alpha (y - n)]^{2} + \beta^{2} (y - n)^{2}}{a^{2}} - 3 \}$$

+{
$$[(x-\xi)+\alpha(y-\eta)]^2-\beta^2(y-\eta)^2$$
} arc tg $\frac{\beta(y-\eta)}{(x-\xi)+\alpha(y-\eta)}$;

and $R_3(x,y;\xi,\eta)$ and $S_3(x,y;\xi,\eta)$ are obtained by replacing α and β by γ and λ , respectively.

As with the orthotropic Green's function, the first order derivatives are quite lengthy. The second order derivatives, however, can be reduced to very compact forms. Since they are the most important derivatives, they are listed here. Others are shown in Appendix F.

$$\frac{\partial^{2} G}{\partial x^{2}} = \frac{1}{4\pi D_{22}^{\phi_{1}\phi_{2}}} \left\{ \frac{(\alpha - \gamma)^{2} - (\beta^{2} - \lambda^{2})}{\beta} L_{1}(x, y; \xi, \eta) + \frac{(\alpha - \gamma)^{2} + (\beta^{2} - \lambda^{2})}{\lambda} L_{3}(x, y; \xi, \eta) + 4(\alpha - \gamma) \left[N_{1}(x, y; \xi, \eta) - N_{3}(x, y; \xi, \eta) \right] \right\}$$

$$(40a)$$
$$\frac{\partial^{2} G}{\partial y^{2}} = \frac{1}{4\pi D_{22} \phi_{1} \phi_{2}} \left\{ \frac{(\alpha^{2} + \beta^{2} - 2\alpha\gamma) (\alpha^{2} + \beta^{2}) + (\alpha^{2} - \beta^{2}) (\gamma^{2} + \lambda^{2})}{\beta} L_{1}(x, y; \xi, n) + \frac{(\gamma^{2} + \lambda^{2} - 2\alpha\gamma) (\gamma^{2} + \lambda^{2}) + (\gamma^{2} - \lambda^{2}) (\alpha^{2} + \beta^{2})}{\lambda} L_{3}(x, y; \xi, n) - N_{3}(x, y; \xi, n) - 4 \left[\alpha (\gamma^{2} + \lambda^{2}) - \gamma (\alpha^{2} + \beta^{2}) \right] \left[N_{1}(x, y; \xi, n) - N_{3}(x, y; \xi, n) \right] \right\}$$
(40b)
$$\frac{\partial^{2} G}{\partial x \partial y} = \frac{1}{4\pi D_{22} \phi_{1} \phi_{2}} \left\{ \frac{(\alpha - 2\gamma) (\alpha^{2} - \beta^{2}) + \alpha (\gamma^{2} + \lambda^{2})}{\beta} L_{1}(x, y; \xi, n) + \frac{(\gamma - 2\alpha) (\gamma^{2} + \lambda^{2}) + \gamma (\alpha^{2} + \beta^{2})}{\lambda} L_{3}(x, y; \xi, n) + 2 (\alpha^{2} + \beta^{2} - \gamma^{2} - \lambda^{2}) \left[N_{1}(x, y; \xi, n) - N_{3}(x, y; \xi, n) \right] \right\}$$
(40c)

where,

$$L_{1}(x,y;\xi,\eta) = ln \frac{[(x-\xi)+\alpha(y-\eta)]^{2}+\beta^{2}(y-\eta)^{2}}{a^{2}}$$

$$N_{1}(x,y;\xi,n) = \operatorname{arc} tg \frac{\beta(y-n)}{(x-\xi)+\alpha(y-n)}$$

and $L_3(x,y;\xi,n)$ and $N_3(x,y;\xi,n)$ are obtained by replacing α and β by γ and λ , respectively. It is worth noting that Eqs.(39) and (40) can be easily reduced to Eqs.(36) and (37) by making $\alpha=\gamma=0$ for $\rho>1.0$; $\alpha=\mu_2$, $\gamma=-\mu_2$, $\beta=\lambda=\mu_1$ for $\rho<1.0$; and $\alpha=\gamma\neq0$, $\beta=\lambda\neq\varepsilon$ for $\rho=1.0$.

Following the same numerical procedure shown in the previous two sections, using the new point-force method, the solution of an anisotropic thin plate problem with arbitrary plan form, loading, and boundary conditions can be obtained. For the verification of results, however, due to lack of exact solutions available for anisotropic problems to be compared with, a different approach must be taken. An orthotropic plate problem will become apparently anisotropic if the geometric coordinates are made different from the principal material directions. Therefore, solutions of orthotropic plate problems can be used to validate the equations for general anisotropic problems. This approach can be summarized in four steps as shown in the following. First, an angle of rotation for the geometric coordinates is chosen arbitrarily, and corresponding to the new coordinate system, locations of the boundary points, the fictitious forces, the field points, and the unit outward normals of the boundary points are determined. The second step is to compute the six flexural rigidity constants D_{ij} used in Eq.(3), [14]. The next step is to employ the Green's function of the infinite anisotropic plate to solve the pseudo-anisotropic problem. The final step is to determine the displacements and bending moments at the prescribed field points in the original coordinate system using coordinate transformation, and then make comparison with the orthotropic solutions.

With this validation method, orthotropic plate example problems shown earlier with all the three types of ρ , i.e., greater than, equal to, and less than unity have been tested against four coordinate rotation angles, namely 15, 30, 45, and 60 degrees. The discrepencies of results were within one percent and were believed to be due to truncation errors during the added numerical processes. The computer program for this validation is shown in Appendix H, while the program for a general anisotropic plate problem is shown in Appendix G.

It must be noted that the two flexural rigidity constants D_{16} and D_{26} are based on the two material constants a_{16} and a_{26} of Eq.(3). In order to investigate the influence due to these two material constants, several sample problems using a simply supported square plate have been tested. Since the

the material constant matrix shown in Eq.(3) must be positive definite, a_{16} and a_{26} were selected to be less than a_{11} and a_{22} , respectively. Under this condition, take four typical cases with $a_{16}=0.1/E_x$, $a_{26}=0.1/E_y$; $a_{16}=0.9/E_x$, $a_{26}=0.9/E_y$; $a_{16}=0.1/E_x$, $a_{26}=0.9/E_y$; and $a_{16}=0.9/E_x$, $a_{26}=0.1/E_y$ for example, it has been found that the differences in displacements and bending moments were smaller than one percent. However, when the shear modulus G_{xy} is small in comparison with E_x and E_y , a_{16} and a_{26} can be made greater than a_{11} and a_{22} ; their contribution to the solution may become significant.

CHAPTER IV

CLOSURE

Starting from a boundary integral equation method for an isotropic thin plate problem with boundary clamped or/and simply supported, a very efficient numerical solution to problems with arbitrary plan form, arbitrary loading and boundary conditions, and anisotropic material, has been developed. The method uses the known Green's functions of isotropic, orthotropic, and anisotropic infinite plates. The problem is solved after the real plate is embedded in the fictitious infinite plate, and the boundary conditions at the N prescribed boundary points are forced to be satisfied with an imposed set of 2N calculated fictitious forces located somewhere outside the plate boundary. Though no efforts have been made to compare with the two leading numerical methods, the finite element and the finite difference methods, it is believed that the new method has the following two advantages: (1) since the Green's function is the exact solution to a point force problem, and there are no assumed polynomials for results, high solution accuracy is expected; (2) due to the fact that the equations are simple, and the modeling is for the plate boundary only, the current method is easier to use.

Large percentage errors indicated in all tables are somewhat misleading. Take the simply supported triangular plate problem for example. Percentage errors shown in Table 4 are huge at certain locations. However, the real errors are small as shown in Figure 7.

During the development of the current method, it was found that though series type Green's functions were easy for

formulation, they were not suitable for the current method. This was due to the fact that large number of terms of the series were needed to provide acceptable solution accuracy, and this would lead to formidable computing costs.

Though the current method is efficient, numerical questions remain. An improper choice of the locations for the fictitious forces may result in poor solution accuracy or no solution at all. Therefore, in order to take full advantage of this method, some further studies should be made so that the locations of fictitious forces chosen will bring optimum results. In the meantime, due to the involvement of many "looped" summing processes, Eqs. (19), (21), (22), and (26), it is also necessary to do sensitivity studies to minimize the numbers of boundary points, fictitious forces, and internal forces for least computing cost.

All the five computer programs developed for this thesis research are shown in Appendices B, C, E, G, and H. The first is for the boundary integral method. It uses the Green's function of a clamped circular plate. The second is for the new point-force method for isotropic problems. The third and the fourth are for orthotropic and anisotropic problems, respectively. The fifth is a method employed to validate the equations for general anisotropic problems, using exact solutions for orthotropic plate problems. All these computer programs are coded in FORTRAN, and their flow chart is shown on the next page, Figure 9.



Figure 9

The flow chart for the programs shown in Appendices B, C, E, G, and F. APPENDICES

.

•

APPENDIX A

DERIVATIVES OF THE GREEN'S FUNCTION FOR A CLAMPED CIRCULAR PLATE

APPENDIX A

DERIVATIVES OF THE GREEN'S FUNCTION FOR A CLAMPED CIRCULAR PLATE

The Green's function shown in Eq.(10) can be written as

$$G(\mathbf{x},\mathbf{y};\xi,\eta) = \frac{a}{16\pi D} \{ (1-r_1^2) (1-r_2^2) + r_{12}^2 \ln \frac{r_{12}^2}{(1-r_1^2) (1-r_2^2) + r_{12}^2} \}$$

where,
$$r_1^2 = \frac{x^2 + y^2}{a^2}$$
, $r_2^2 = \frac{\xi^2 + \eta^2}{a^2}$, and $r_{12} = \frac{(x-\xi)^2 + (y-\eta)^2}{a^2}$.

For simplicity, from now on the variables x, y, ξ , and n are all made non-dimensional. That is, the variables x, y, ξ , and n shown in the following equations are actually the ratios of $\frac{x}{a}$, $\frac{y}{a}$, $\frac{\xi}{a}$, and $\frac{n}{a}$, respectively.

$$\frac{\partial G}{\partial x} = \frac{a^2}{8\pi D} \{x \cdot r_2^2 - \xi - \frac{r_{12}^2 (x \cdot r_2^2 - \xi)}{(1 - r_1^2) (1 - r_2^2) + r_{12}^2}$$

+
$$(x-\xi) l_n \frac{r_{12}^2}{(1-r_1^2)(1-r_2^2)+r_{12}^2}$$

$$\frac{\partial G}{\partial y} = \frac{a^2}{8\pi D} \{y \cdot r_2^2 - \eta - \frac{r_{12}^2 (y \cdot r_2^2 - \eta)}{(1 - r_1^2) (1 - r_2^2) + r_{12}^2}$$

$$\begin{aligned} &+(y-\eta) \ln \frac{r_{12}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} \} \\ &+(y-\eta) \ln \frac{r_{12}^{2}(\xi\cdot r_{1}^{2}-x)}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} \\ &+(\xi-x) \ln \frac{r_{12}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} \\ &+(\xi-x) \ln \frac{r_{12}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} \} \\ &\frac{\partial G}{\partial \eta} = -\frac{a^{2}}{8\pi D} (\eta \cdot r_{2} - y - \frac{r_{12}^{2}(\eta \cdot r_{2}^{2}-y)}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} \\ &+(\eta-y) \ln \frac{r_{12}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} \} \\ &\frac{\partial^{2} G}{\partial^{2} x} = \frac{a^{2}}{8\pi D} (r_{2}^{2} + \frac{2(x-\xi)}{r_{12}^{2}})^{2} + \frac{4(x-\xi)(\xi-x\cdot r_{2}^{2})-r_{12}^{2}\cdot r_{2}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} \\ &+ \frac{2r_{12}^{2}(\xi-x\cdot r_{2}^{2})^{2}}{(1-r_{2}^{2})(1-r_{2}^{2})+r_{12}^{2}} + \frac{\ln \frac{r_{12}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} \\ &\frac{\partial^{2} G}{\partial x \partial y} = \frac{a^{2}}{8\pi D} (\frac{2(x-\xi)(y-\eta)}{r_{12}^{2}} + \frac{2(y-\eta)(\xi-x\cdot r_{2}^{2})+(x-\xi)(\eta-y\cdot r_{2}^{2})}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} \end{bmatrix}$$

$$\frac{\partial^2 G}{\partial y^2} = \frac{a^2}{8\pi D} \{r_2^2 + \frac{2(y-\eta)^2}{r_{12}} + \frac{4(y-\eta)(\eta-y\cdot r_2^2) - r_{12}^2 r_2^2}{(1-r_1^2)(1-r_2^2) + r_{12}^2} \}$$

+
$$\frac{(1-2yn)(1-r_1^2)(1-r_2^2)-2(y-n)(y-n\cdot r_1^2)-2(y-n)(y\cdot r_2^2-n)}{(1-r_1^2)(1-r_2^2)+r_{12}^2}$$

$$- \ln \left[(1 - r_1^2) (1 - r_2^2) + r_{12}^2 \right]$$

$$\frac{3^3 G}{3x^2 \vartheta\xi} - \frac{a^2}{4\pi D} \left\{ \frac{3(x - \xi)}{r_{12}^2} - \frac{2(x - \xi)^3}{r_{12}^2} - \frac{x(1 - 2x + \xi) (1 - r_2^2) + \xi(1 - r_1^2) (1 - r_2^2)}{(1 - r_1^2) (1 - r_2^2) + r_{12}^2} \right]$$

$$- \frac{(x - \xi) (1 - 2x\xi) - 2(\xi - x + r_2^2) + (x - \xi) r_2^2}{(1 - r_1^2) (1 - r_2^2) + r_{12}^2} - \frac{4r_{12}^2 (x - \xi r_1^2) (\xi - x + r_2^2)^2}{((1 - r_1^2) (1 - r_2^2) + r_{12}^2)^3}$$

$$+ \frac{(1 - 2x\xi) (\xi - x + r_2^2) (1 - r_1^2) (1 - r_2^2) - 4(x - \xi) (x - \xi r_1^2) (\xi - x + r_2^2)}{[(1 - r_1^2) (1 - r_2^2) + r_{12}^2]^2}$$

$$+ \frac{2(x - \xi) (\xi - x + r_2^2)^2 + r_{12}^2 r_2^2 (x - \xi r_1^2) - r_{12}^2 (1 - 2x\xi) (\xi - x + r_2^2)}{[(1 - r_1^2) (1 - r_2^2) + r_{12}^2]^2}$$

$$+ \frac{2(x - \xi) (\xi - x + r_2^2)^2 + r_{12}^2 r_2^2 (x - \xi r_1^2) - r_{12}^2 (1 - 2x\xi) (\xi - x + r_2^2)}{[(1 - r_1^2) (1 - r_2^2) + r_{12}^2]^2}$$

$$+ \frac{2(x - \xi) (\xi - x + r_2^2) + r_{12}^2 r_2^2 (x - \xi r_1^2) - r_{12}^2 (1 - 2x\xi) (\xi - x + r_2^2)}{[(1 - r_1^2) (1 - r_2^2) + r_{12}^2]^2}$$

$$+ \frac{2(x - \xi) (\xi - x + r_2^2) (1 - r_1^2) (1 - r_2^2) + r_{12}^2}{r_2^2} + r_{12}^2 r_2^2 (x - \xi r_1^2) - r_{12}^2 (1 - r_2^2) + r_{12}^2 r_2^2 r_2^2 (x - \xi r_1^2) - r_{12}^2 r_2^2 r_2^$$

$$\frac{\partial^{3}G}{\partial y^{2}\partial \xi} = \frac{a^{2}}{4\pi D} \left\{ \frac{(x-\xi)}{r_{12}^{2}} - \frac{2(y-\eta)^{2}(x-\xi)}{r_{12}^{4}} - \frac{4r_{12}^{2}(x-\xi r_{1}^{2})(\eta-y\cdot r_{2}^{2})^{2}}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{3}} - \frac{y(-2y\xi)(1-r_{2}^{2})+\xi(1-r_{1}^{2})(1-r_{2}^{2})+(x-\xi r_{1}^{2})+(y-\eta)(-2y\xi)+(x-\xi)r_{2}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} \right]$$

+
$$\frac{(-2y\xi)(\eta-y\cdot r_{2}^{2})(1-r_{1}^{2})(1-r_{2}^{2})-4(y-\eta)(x-\xi r_{1}^{2})(\eta-y\cdot r_{2}^{2})+2(x-\xi)(\eta-y\cdot r_{2}^{2})^{2}}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{2}}$$

$$\frac{+r_{12}^{2}r_{2}^{2}(x-\xi r_{1}^{2})+r_{12}^{2}(-2y\xi)(\eta-y\cdot r_{2}^{2})}{12}$$

$$\frac{\partial^{3}G}{\partial y^{2}\partial \eta} = -\frac{a^{2}}{4\pi D} \left\{ \frac{3(y-\eta)}{r_{12}^{2}} - \frac{2(y-\eta)^{3}}{r_{12}^{4}} - \frac{4r_{12}^{2}(y-\eta r_{1}^{2})(\eta-y\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{3}} - \frac{y(1-2y\eta)(1-r_{2}^{2})+\eta(1-r_{1}^{2})(1-r_{2}^{2})+(y-\eta r_{1}^{2})+(y-\eta)(1-2y\eta)-2(\eta-y\cdot r_{2}^{2})+(y-\eta)r_{2}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} + \frac{(1-2y\eta)(\eta-y\cdot r_{2}^{2})(1-r_{1}^{2})(1-r_{2}^{2})-4(y-\eta)(y-\eta r_{1}^{2})(\eta-y\cdot r_{2}^{2})+2(y-\eta)(\eta-y\cdot r_{2}^{2})^{2}}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{2}} + \frac{(1-2y\eta)(\eta-y\cdot r_{2}^{2})(1-r_{1}^{2})(1-r_{2}^{2})-4(y-\eta)(y-\eta r_{1}^{2})(\eta-y\cdot r_{2}^{2})+2(y-\eta)(\eta-y\cdot r_{2}^{2})^{2}}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{2}} + \frac{4r_{12}^{2}r_{2}^{2}(y-\eta r_{1}^{2})-r_{12}^{2}(1-2y\eta)(\eta-y\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{2}} \right\}$$

$$\frac{\partial^{3}G}{\partial x \partial y \partial \xi} = -\frac{a^{2}}{4\pi D} \left\{ \frac{y-\eta}{r_{12}^{2}} - \frac{2(x-\xi)^{2}(y-\eta)}{r_{12}^{4}} - \frac{4r_{12}^{2}(x-\xi r_{1}^{2})(\xi-x-r_{2}^{2})(\eta-y-r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{3}} \right\}$$

$$+ \frac{(\xi - \mathbf{x} \cdot \mathbf{r_2}^2) (1 - 2y\eta) (1 - \mathbf{r_1}^2) (1 - \mathbf{r_2}^2) - 2(\xi - \mathbf{x} \cdot \mathbf{r_2}^2) (y - \eta) (y - \eta \mathbf{r_1}^2) + 2(\xi - \mathbf{x} \cdot \mathbf{r_2}^2) \cdot (1 - \mathbf{r_1}^2) (1 - \mathbf{r_2}^2) + \mathbf{r_{12}}^2 [(1 - \mathbf{r_1}^2) (1 - \mathbf{r_2}^2) + \mathbf{r_{12}}^2]^2}{[(1 - \mathbf{r_1}^2) (\eta - y \cdot \mathbf{r_2}^2) - 2(x - \xi) (y - \eta \mathbf{r_1}^2) (\eta - y \cdot \mathbf{r_2}^2) + 2(\eta - y \cdot \mathbf{r_2}^2) \eta \mathbf{x} \cdot \mathbf{r_{12}}^2}$$

+
$$\frac{x(r_2^2-1)(1-2yn)+2xn(y-n)+(\xi-x\cdot r_2^2)}{(1-r_1^2)(1-r_2^2)+r_{12}^2}$$
 }

$$\frac{\partial^{3}G}{\partial x^{3}} = \frac{a^{2}}{4\pi D} \left\{ \frac{3(x-\xi)}{r_{12}^{2}} - \frac{2(x-\xi)^{2}}{r_{12}^{4}} + \frac{3(\xi-x\cdot r_{2}^{2})-3r_{2}^{2}(x-\xi)}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} + \frac{4r_{12}^{2}(\xi-x\cdot r_{2}^{2})^{3}}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{3}} \right\}$$

+
$$\frac{6(\xi - \mathbf{x} \cdot \mathbf{r}_{2}^{2})^{2}(\mathbf{x} - \xi) - 3r_{12}^{2}r_{2}^{2}(\xi - \mathbf{x} \cdot \mathbf{r}_{2}^{2})}{[(1 - r_{1}^{2})(1 - r_{2}^{2}) + r_{12}^{2}]^{2}} \}$$

$$\frac{\partial^{3}G}{\partial y^{3}} = \frac{a^{2}}{4\pi D} \left\{ \frac{3(y-\eta)}{r_{12}^{2}} - \frac{2(y-\eta)^{2}}{r_{12}^{4}} + \frac{3(\eta-y\cdot r_{2}^{2})-3r_{2}^{2}(y-\eta)}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} + \frac{4r_{12}^{2}(\eta-y\cdot r_{2}^{2})^{3}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} \right\}$$

+
$$\frac{6(y-n)(n-y\cdot r_2^2)^2 - 3r_{12}^2 r_2^2(n-y\cdot r_2^2)}{[(1-r_1^2)(1-r_2^2) + r_{12}^2]^2} \}$$

$$\frac{\partial^{3}G}{\partial x^{2} \partial y} = \frac{a^{2}}{4\pi D} \left\{ \frac{y-\eta}{r_{12}^{2}} - \frac{2(x-\xi)^{2}(y-\eta)}{r_{12}^{4}} + \frac{\eta-y\cdot r_{2}^{2}-r_{2}^{2}(y-\eta)}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} + \frac{4(x-\xi)(\xi-x\cdot r_{2}^{2})(\eta-y\cdot r_{2}^{2})-r_{12}^{2}r_{2}^{2}(\eta-y\cdot r_{2}^{2})+2(y-\eta)(\xi-x\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{2}} \right\}$$

+
$$\frac{4r_{12}^{2}(\xi-x\cdot r_{2}^{2})^{2}(\eta-y\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{3}} \}$$

$$\frac{\partial^{3}G}{\partial x \partial y^{2}} = \frac{a^{2}}{4\pi D} \left\{ \frac{x-\xi}{r_{12}^{2}} - \frac{2(y-\eta)^{2}(x-\xi)}{r_{12}^{4}} + \frac{\xi-x\cdot r_{2}^{2}-r_{2}^{2}(x-\xi)}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} + \frac{4r_{12}^{2}(\eta-y\cdot r_{2}^{2})^{2}(\xi-x\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{3}} + \frac{4(y-\eta)(\eta-y\cdot r_{2}^{2})(\xi-x\cdot r_{2}^{2})-r_{12}^{2}r_{2}^{2}(\xi-x\cdot r_{2}^{2})+2(x-\xi)(\eta-y\cdot r_{2}^{2})^{2}}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{2}} \right\}$$

$$\frac{\partial^{4}G}{\partial x^{3}\partial \xi} = -\frac{a^{2}}{4\pi D} \left\{ \frac{3}{r_{12}^{2}} + \frac{8(x-\xi)^{4}}{r_{12}^{6}} - \frac{12(x-\xi)^{2}}{r_{12}^{4}} - \frac{24r_{12}(x+\xi)r_{1}^{2}/(1-r_{2}^{2}) + r_{12}^{2}}{[(1-r_{1}^{2})(1-r_{2}^{2}) + r_{12}^{2}]^{4}} + \frac{-(1-r_{2}^{2})(1-2x\xi) + 4x\xi(1-r_{2}^{2}) - 2(1-2x\xi) + 2\xi(x-\xi) - 3r_{2}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2}) + r_{12}^{2}} \right\}$$

$$+ \frac{-4x(1-r_2^2)(\xi-x\cdot r_2^2)(1-2x\xi) -4\xi(1-r_1^2)(1-r_2^2)(\xi-x\cdot r_2^2)-6(\xi-x\cdot r_2^2)}{2}$$

$$\frac{(x-\xi r_1^2)-8(\xi-x\cdot r_2^2)(x-\xi)(1-2x\xi)+6(\xi-x\cdot r_2^2)-12r_2^2(x-\xi)(\xi-x\cdot r_2^2)}{[(1-r_1^2)(1-r_2^2)+r_{12}^2]^2}$$

$$-r_{2}^{2}(1-r_{1}^{2})(1-r_{2}^{2})(1-2x\xi)+6r_{2}^{2}(x-\xi)(x-\xi r_{1}^{2})+2r_{12}^{2}r_{2}^{2}(1-2x\xi)+2r_{12}^{2}\xi(\xi-x\cdot r_{2}^{2})$$

+
$$\frac{4(1-2x\xi)(\xi-x\cdot r_2^2)(1-r_1^2)(1-r_2^2)-24(x-\xi)(x-\xi r_1^2)(\xi-x\cdot r_2^2)}{[(1-r_1^2)(1-r_2^2)+r_{12}^2]^3}$$

$$\frac{+8(x-\xi)(\xi-x\cdot r_2^2)^3+12r_1^2r_2^2(x-\xi r_1^2)(\xi-x\cdot r_2^2)-8r_{12}^2(1-2x\xi)(\xi-x\cdot r_2^2)^2}{2}$$

-

$$\begin{split} \frac{\partial^{4}G}{\partial x^{3}\partial \eta} &= -\frac{a^{2}}{4\pi D} \left(\frac{-6(x-\xi)(y-\eta)}{r_{12}^{4}} + \frac{8(x-\xi)^{3}(y-\eta)}{r_{12}^{6}} - \frac{2^{4}r_{2}^{2}(y-r_{1}^{2}\eta)(\xi-x+r_{2}^{2})}{((1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2})^{4}} \right. \\ &+ \frac{8x^{2}\eta(1-r_{2}^{2})(\xi-x+r_{2}^{2})-4\eta((1-r_{1}^{2})(1-r_{2}^{2})(\eta-x+r_{2}^{2})-6(y-\eta r_{1})(\xi-x+r_{2}^{2})+4}{((1-r_{1}^{2})(1-r_{2}^{2})+6r_{2}^{2}(x-\xi)(y-\eta r_{1}^{2})-6r_{2}^{2}(y-\eta)}{((1-r_{1}^{2})(1-r_{2}^{2})+6r_{2}^{2}(x-\xi)(y-\eta r_{1}^{2})-6r_{2}^{2}(y-\eta)} \\ &+ \frac{412x\eta(x-\xi)(\xi-x+r_{2}^{2})+2r_{2}^{2}x\eta(1-r_{1}^{2})(1-r_{2}^{2})+6r_{2}^{2}(x-\xi)(y-\eta r_{1}^{2})-6r_{2}^{2}(y-\eta)}{((1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2})^{2}} \\ &+ \frac{-6x\eta(\xi-x+r_{2}^{2})-4x\eta r_{12}^{2}r_{2}^{2}+4x\eta(x-\xi)(\xi-x+r_{2}^{2})+2\eta r_{12}^{2}(\xi-x+r_{2}^{2})}{((1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2})^{3}} \\ &+ \frac{-6x\eta(\xi-x+r_{2}^{2})^{2}(1-r_{1}^{2})(1-r_{2}^{2})-24(x-\xi)(y-\eta r_{1}^{2})(\xi-x+r_{2}^{2})+4r_{12}^{2}}{((1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2})^{3}} \\ &+ \frac{-6x\eta(\xi-x+r_{2}^{2})^{3}+12r_{12}^{2}r_{2}^{2}(y-\eta r_{1}^{2})(\xi-x+r_{2}^{2})+16r_{12}^{2}x\eta(\xi-x+r_{2}^{2})}{((1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2})^{3}} \\ &+ \frac{-6x\eta(\xi-x+r_{2}^{2})^{3}+12r_{12}^{2}r_{2}^{2}r_{2}^{2}(y-\eta r_{1}^{2})(\xi-x+r_{2}^{2})+16r_{12}^{2}x\eta(\xi-x+r_{2}^{2})}{(1-r_{2}^{2})+r_{12}^{2}} \right) \\ &+ \frac{2^{4}G}(y-\eta)(\xi-x+r_{2}^{2})^{3}+12r_{12}^{2}r_{2}^{2}r_{2}^{2}(y-\eta r_{1}^{2})(\xi-x+r_{2}^{2})+16r_{12}^{2}x\eta(\xi-x+r_{2}^{2})}{(1-r_{2}^{2})+r_{12}^{2}} \\ &+ \frac{8y^{2}\xi(1-r_{2}^{2})(\eta-y+r_{2}^{2})-4\xi(1-r_{1}^{2})(1-r_{2}^{2})(\eta-y+r_{2}^{2})-6(x-\xi r_{1}^{2})(\eta-y+r_{2}^{2})+4}{((1-r_{1}^{2})(1-r_{2}^{2})+6r_{2}^{2}(y-\eta)(x-\xi r_{1}^{2})} - (1-r_{1}^{2})(1-r_{2}^{2})+6r_{2}^{2}(y-\eta)(x-\xi r_{1}^{2}) - (1-r_{1}^{2})(1-r_{2}^{2})+6r_{2}^{2}(y-\eta)(x-\xi r_{1}^{2}) - (1-r_{1}^{2})(1-r_{2}^{2})+6r_{2}^{2}(y-\eta)(x-\xi r_{1}^{2})} - (1-r_{1}^{2})(1-r_{2}^{2})-4y\xi(y-\eta)(\eta-y+r_{2}^{2})+2\xi r_{12}^{2}(\eta-y+r_{2}^{2}) - (1-r_{2}^{2})+6r_{2}^{2}(y-\eta)(x-\xi r_{1}^{2}) - (1-r_{2}^{2})+6r_{2}^{2}(y-\eta)(x-\xi r_{1}^{2}) - (1-r_{1}^{2})(1-r_{2}^{2})+2\xi r_{12}^{2}(\eta-y+r_{2}^{2})} - (1-r_{1}^{2})(1-r_{2}^{2})+2\xi r_{$$

$$+ \frac{-8y\xi(1-r_{1}^{2})(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})^{2}-24(y-\eta)(x-\xi r_{1}^{2})(\eta-y\cdot r_{2}^{2})+}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{3}}$$

$$+ 8(x-\xi)(\eta-y\cdot r_{2}^{2})^{3}+12r_{12}^{2}r_{2}^{2}(x-\xi r_{1}^{2})(\eta-y\cdot r_{2}^{2})+16r_{12}^{2}y\xi(\eta-y\cdot r_{2}^{2})$$

$$- \frac{24r_{12}^{2}(x-\xi r_{1}^{2})(\eta-y\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{4}} \}$$

$$\frac{\partial^{4}G}{\partial x^{2} \partial y \partial \xi} = -\frac{a^{2}}{4\pi D} \left\{ -\frac{6(x-\xi)(y-\eta)}{r_{12}^{4}} + \frac{8(x-\xi)^{3}(y-\eta)}{r_{12}^{6}} + \frac{2y\xi(1-r_{2}^{2})+2y\xi}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} + \frac{-2x(1-2x\xi)(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})-2\xi(1-r_{1}^{2})(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})}{(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})-2\xi(1-r_{1}^{2})(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})} - \frac{1}{2} + \frac{-2x(1-2x\xi)(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})-2\xi(1-r_{1}^{2})(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})}{(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})-2\xi(1-r_{1}^{2})(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})} - \frac{1}{2} + \frac{1}{2$$

$$\frac{-2 (x-\xi r_{1}^{2}) (\eta-y \cdot r_{2}^{2})-2 (x-\xi) (1-2x\xi) (\eta-y \cdot r_{2}^{2})+4 (\xi-x \cdot r_{1}^{2}) (\eta-y \cdot r_{2}^{2})}{[(1-r_{1}^{2}) (1-r_{2}^{2})+r_{12}^{2}]^{2}}$$

$$\frac{-2 (x-\xi) r_{2}^{2} (\eta-y \cdot r_{2}^{2})-2y (1-2x\xi) (\xi-x \cdot r_{2}^{2}) (1-r_{2}^{2})+8y\xi (x-\xi) (\xi-x \cdot r_{2}^{2})+8y\xi (x-\xi) (\xi-x \cdot r_{2}^{2}) (1-r_{1}^{2}) (1-r_{2}^{2}) (\eta-y \cdot r_{2}^{2})-16 (x-\xi) (x-\xi r_{1}^{2}) (\xi-x \cdot r_{2}^{2})+8y\xi (x-\xi) (\xi-x \cdot r_{2}^{2}) (\xi-x \cdot$$

$$\cdot (\eta - y \cdot r_2^2) + 8 (x - \xi) (\xi - x \cdot r_2^2)^2 (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (x - \xi r_1^2) (\eta - y \cdot r_2^2) - \frac{1}{2} (\eta - y \cdot r_2^2) + \frac{$$

$$\frac{-4r_{12}^{2}(1-2x\xi)(\xi-x\cdot r_{2}^{2})(\eta-y\cdot r_{2}^{2})-8(y-\eta)(x-\xi r_{1}^{2})(\xi-x\cdot r_{2}^{2})+}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{3}}$$

$$\frac{+8r_{12}^{2}y\xi(\xi-x\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{2}(\eta-y\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{4}}$$

$$\frac{\partial^{4}G}{\partial y^{3}\partial \eta} = -\frac{a^{2}}{4\pi D} \{\frac{3}{r_{12}^{2}} - \frac{12(y-\eta)^{2}}{r_{12}^{4}} + \frac{8(y-\eta)^{4}}{r_{12}^{6}} - \frac{24r_{12}^{2}(y-\eta r_{1}^{2})(\eta-y\cdot r_{2}^{2})^{3}}{[(1-r_{1}^{2})(1-r_{2}^{2}) + r_{12}^{2}]^{4}} \\ + \frac{-(1-r_{2}^{2})(1-2y\eta) + 4y\eta(1-r_{2}^{2}) - 2(1-2y\eta) + 2\eta(y-\eta) - 3r_{2}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2}) + r_{12}^{2}} \\ + \frac{-4y(1-2y\eta)(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2}) - 4\eta(1-r_{1}^{2})(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2}) - 6(\eta-y\cdot r_{2}^{2}) - 6(\eta-y\cdot r_{2}^{2}) - (\eta-y\cdot r_{2}^{2}) - (\eta-y\cdot r_{2}^{2}) - 6(\eta-y\cdot r_{2}^{2}) - (\eta-y\cdot r_{2}^{2}) - (\eta-y\cdot r_{2}^{2}) - 6r_{2}^{2}(y-\eta)(\eta-y\cdot r_{2}^{2}) - 6(\eta-y\cdot r_{2}^{2}) - (1-r_{1}^{2})(1-r_{2}^{2}) + r_{12}^{2} + r_{12}^{2}]^{2}} \\ \frac{-r_{2}^{2}(1-2y\eta)(1-r_{1}^{2})(1-r_{2}^{2}) + 6r_{2}^{2}(y-\eta)(y-\eta r_{1}^{2}) + 2r_{12}^{2}r_{2}^{2}(1-2y\eta) + 2r_{12}^{2}(\eta-y\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2}) + 6r_{2}^{2}(y-\eta)(y-\eta r_{1}^{2}) + 2r_{12}^{2}r_{2}^{2}(1-2y\eta) + 2r_{12}^{2}(\eta-y\cdot r_{2}^{2})} \\ + \frac{4(1-2y\eta)(1-r_{1}^{2})(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})^{2} - 24(y-\eta)(y-\eta r_{1}^{2})(\eta-y\cdot r_{2}^{2})^{2} + 8(y-\eta) \cdot [(1-r_{1}^{2})(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})^{2} - 24(y-\eta)(y-\eta r_{1}^{2})(\eta-y\cdot r_{2}^{2})^{2} + 8(y-\eta) \cdot [(1-r_{1}^{2})(1-r_{2}^{2})^{2} + r_{12}^{2}]^{3}} \\ - \frac{(\eta-y\cdot r_{2}^{2})^{3} + 12r_{12}^{2}r_{2}^{2}(y-\eta r_{1}^{2})(\eta-y\cdot r_{2}^{2}) - 8r_{12}^{2}(1-2y\eta)(\eta-y\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})^{3} + 12r_{12}^{2}r_{2}^{2}(y-\eta r_{1}^{2})(\eta-y\cdot r_{2}^{2}) - 8r_{12}^{2}(1-2y\eta)(\eta-y\cdot r_{2}^{2})}}{[(1-r_{1}^{2})(1-r_{2}^{2})^{3} + 12r_{12}^{2}r_{2}^{2}(y-\eta r_{1}^{2})(\eta-y\cdot r_{2}^{2}) - 8r_{12}^{2}(1-2y\eta)(\eta-y\cdot r_{2}^{2})}]$$

.

-

$$\frac{\partial^{4}G}{\partial x^{2} \partial y \partial \eta} = -\frac{a^{2}}{4\pi D} \left\{ \frac{-1}{r_{12}^{2}} + \frac{8(x-\xi)^{2}(y-\eta)^{2}}{r_{12}^{6}} + \frac{2y\eta(1-r_{2}^{2})-(1-2y\eta)-r_{2}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} + \frac{4x^{2}\eta(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})-2\eta(1-r_{1}^{2})(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})+r_{12}^{2}}{(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})-2(y-\eta r_{1}^{2})(\eta-y\cdot r_{2}^{2})+r_{12}^{2}} + \frac{4x^{2}\eta(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})-2\eta(1-r_{1}^{2})(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})+r_{12}^{2}}{(1-r_{2}^{2})(\eta-y\cdot r_{2}^{2})-2(y-\eta r_{1}^{2})(\eta-y\cdot r_{2}^{2})+r_{12}^{2}}$$

$$\frac{+4x\eta (x-\xi) (\eta-y\cdot r_2^{\ 2})-2r_2^{\ 2} (y-\eta) (\eta-y\cdot r_2^{\ 2})+4xy\eta (\xi-x\cdot r_2^{\ 2}) (1-r_2^{\ 2})-(1-r_2^{\ 2})-(1-r_2^{\ 2})+r_1^{\ 2})^2}{[(1-r_1^{\ 2}) (1-r_2^{\ 2})+r_1^{\ 2}]^2}$$

$$\frac{-4 (x-\xi) (\xi-x\cdot r_2^{\ 2}) (1-2y\eta)+2 (\xi-x\cdot r_2^{\ 2})^2+2 (y-\eta) r_2^{\ 2} (y-\eta r_1^{\ 2})+(1-2y\eta)^2}{[(1-r_2^{\ 2})+r_1^{\ 2})^2+2 (y-\eta) r_2^{\ 2} (y-\eta r_1^{\ 2})+(1-2y\eta)^2+2 (\xi-x\cdot r_2^{\ 2})^2+2 (y-\eta) r_2^{\ 2} (y-\eta r_1^{\ 2})+(1-2y\eta)^2}$$

$$\frac{+r_{12}^{2}r_{2}^{2}(1-2y\eta)+4x\eta(y-\eta)(\xi-x\cdot r_{2}^{2})}{4}+\frac{-8x\eta(\xi-x\cdot r_{2}^{2})(1-r_{1}^{2})(1-r_{2}^{2})\cdot}{4}$$

$$\frac{\cdot (\eta - y \cdot r_2^{\ 2}) - 16 (x - \xi) (y - \eta r_1^{\ 2}) (\xi - x \cdot r_2^{\ 2}) (\eta - y \cdot r_2^{\ 2}) + 8 (y - \eta) (\xi - x \cdot r_2^{\ 2})^2 \cdot (1 - r_1^{\ 2}) (1 - r_2^{\ 2}) + r_{12}^{\ 2}]^3}{[(1 - r_1^{\ 2}) (1 - r_2^{\ 2}) + r_{12}^{\ 2}]^3}$$

 $\frac{(\eta - y \cdot r_2^2) + 4r_{12}^2 r_2^2 (y - \eta r_1^2) (\eta - y \cdot r_2^2) + 8r_{12}^2 x \eta (\xi - x \cdot r_2^2) (\eta - y \cdot r_2^2)}{(\eta - y \cdot r_2^2) - (\eta - y \cdot r_2^2) + 8r_{12}^2 x \eta (\xi - x \cdot r_2^2) (\eta - y \cdot r_2^2)}$

$$-8(y-\eta)(y-\eta r_1^2)(\xi-x\cdot r_2^2)^2 - 4r_{12}^2(1-2y\eta)(\xi-x\cdot r_2^2)^2$$

$$-\frac{24r_{12}^{2}(y-\eta r_{1}^{2})(\xi-x\cdot r_{2}^{2})^{2}(\eta-y\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{4}}$$

$$\frac{\partial^{4}G}{\partial x \partial y^{2} \partial \xi} = -\frac{a^{2}}{4\pi D} \left\{ -\frac{1}{r_{12}^{2}} + \frac{8(x-\xi)^{2}(y-\eta)^{2}}{r_{12}^{2}} + \frac{2x\xi(1-r_{2}^{2})-(1-2x\xi)-r_{2}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} + \frac{4y^{2}\xi(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\xi(1-r_{1}^{2})(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2(x-\xi r_{1}^{2})(\xi-x\cdot r_{2}^{2})+}{(1-r_{1}^{2})(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2(x-\xi r_{1}^{2})(\xi-x\cdot r_{2}^{2})+} + \frac{4y\xi(y-\eta)(\xi-x\cdot r_{2}^{2})-2\xi(1-r_{1}^{2})(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2(x-\xi r_{1}^{2})(\xi-y\cdot r_{2}^{2})-}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{2}}$$

$$- 4(y-\eta) (1-2\xi x) (\eta-y \cdot r_2^2) + 2(\eta-y \cdot r_2^2)^2 + 2(x-\xi)r_2^2(x-\xi r_1^2) +$$

$$+ r_{12}^{2} r_{2}^{2} (1-2x\xi) + 4y\xi (x-\xi) (\eta-y \cdot r_{2}^{2}) + \frac{-8y\xi (\eta-y \cdot r_{2}^{2}) (1-r_{1}^{2}) (1-r_{2}^{2}) \cdot (1-r_{$$

$$\frac{(\xi - \mathbf{x} \cdot \mathbf{r}_{2}^{2}) - 16(\mathbf{y} - \eta)(\mathbf{x} - \xi \mathbf{r}_{1}^{2})(\eta - \mathbf{y} \cdot \mathbf{r}_{2}^{2})(\xi - \mathbf{x} \cdot \mathbf{r}_{2}^{2}) + 8(\mathbf{x} - \xi)(\eta - \mathbf{y} \cdot \mathbf{r}_{2}^{2})^{2} \cdot [(1 - \mathbf{r}_{1}^{2})(1 - \mathbf{r}_{2}^{2}) + \mathbf{r}_{12}^{2}]^{3}}{[(1 - \mathbf{r}_{1}^{2})(1 - \mathbf{r}_{2}^{2}) + \mathbf{r}_{12}^{2}]^{3}}$$

•

$$- 8(x-\xi) (x-\xi r_1^2) (\eta-y \cdot r_2^2)^2 - 4r_{12}^2 (1-2x\xi) (\eta-y \cdot r_2^2)^2$$

$$-\frac{24r_{12}^{2}(x-\xi r_{1}^{2})(\eta-y\cdot r_{2}^{2})^{2}(\xi-x\cdot r_{2}^{2})}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{4}}\}$$

•

$$\frac{\partial^{4}G}{\partial x \partial y^{2} \partial \eta} = -\frac{a^{2}}{4\pi D} \left\{ -\frac{6(x-\xi)(y-\eta)}{r_{12}^{4}} + \frac{8(y-\eta)^{3}(x-\xi)}{r_{12}^{6}} + \frac{4x\eta-2x\eta r_{2}^{2}}{(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}} + \frac{-2y(1-2y\eta)(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{1}^{2})(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{1}^{2})(\xi-x\cdot r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-2\eta(1-r_{2}^{2})(\xi-x\cdot r_{2})-2\eta(1-r_{2}^{2})(\xi-x\cdot r_{2})-2\eta(1-r_{2}^{2})(\xi-x-r_{2})-2\eta(1-r$$

$$\frac{-2(y-\eta r_{1}^{2})(\xi-x\cdot r_{2}^{2})-2(y-\eta)(1-2y\eta)(\xi-x\cdot r_{2}^{2})+4(\eta-y\cdot r_{2}^{2})(\xi-x\cdot r_{2}^{2})-1}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{2}}$$

$$\frac{-2r_{2}^{2}(y-(\xi-x\cdot r_{2}^{2})-2x(1-2y\eta)(\eta-y\cdot r_{2}^{2})(1-r_{2}^{2})+8(y-\eta)x\eta(\eta-y\cdot r_{2}^{2})+1}{(1-r_{2}^{2})(1-r_{2}^{2})+8(y-\eta)x\eta(\eta-y\cdot r_{2}^{2})+1}$$

$$\frac{+2r_{2}^{2}(x-\xi)(y-\eta r_{1}^{2})-2x\eta r_{12}^{2}r_{2}^{2}-2(x-\xi)(1-2y\eta)(\eta-y\cdot r_{2}^{2})}{(1-r_{2}^{2})(\xi-x\cdot r_{2}^{2})-16(y-\eta)(y-\eta r_{1}^{2})\cdot}$$

$$\frac{+(\eta-y\cdot r_{2}^{2})(\xi-x\cdot r_{2}^{2})+8(y-\eta)(\eta-y\cdot r_{2}^{2})(\xi-x\cdot r_{2}^{2})-16(y-\eta)(y-\eta r_{1}^{2})\cdot}{[(1-r_{1}^{2})(1-r_{2}^{2})+r_{12}^{2}]^{3}}$$

$$\frac{\cdot(\xi-x\cdot r_{2}^{2})-4r_{12}^{2}(1-2y\eta)(\eta-y\cdot r_{2}^{2})(\xi-x\cdot r_{2}^{2})-8(x-\xi)(y-\eta r_{1}^{2})\cdot}{[(1-r_{1}^{2})(\eta-y\cdot r_{2}^{2})(\xi-x\cdot r_{2}^{2})-8(x-\xi)(y-\eta r_{1}^{2})\cdot}]^{3}}$$

$$\frac{\cdot (\eta - y \cdot r_2^{2})^2 + 8r_{12}^2 x_{\eta} (\eta - y \cdot r_2^{2})}{[(1 - r_1^2) (\eta - y \cdot r_2^{2}) + r_{12}^{2}]^4} + \frac{24r_{12}^2 (y - \eta r_1^2) (\eta - y \cdot r_2^{2})^2 (\xi - x \cdot r_2^{2})}{[(1 - r_1^2) (1 - r_2^2) + r_{12}^2]^4}$$

-

APPENDIX B

COMPUTER PROGRAM FOR THE BOUNDARY INTEGRAL METHOD

	CO	MF	יטי	T	EI	R	F	۶F	۱C)G	R	A	M		F	01	R	5	ΓI	H	Ξ	ł	30	JC	JN	IE)A	١F	۲Y		I	N	Т	E	G	R/	J	Ĺ	Μ	E	T	Η	0]	D				
-		P	R O	61	R A	Ħ	P	L	C۱	. 0	L	D (1	NF	۶U	T	, 0)U	T	Ρι	JT	,	T/	٨P	E	5 =	•1	N	PU	T		T /	P	E	5=	0ι	T	₽l	JT)								
ĊĊĊ	***	••	**	•	• •	•	* 1	•	• 1		•	• •	• •	•	• •	•	• •	•	•	* 1	• •		•	• •		• •	•		• •	• •		• 1		•	• •	• •	• •	•	• •	•	• •	•	• •	• •	• •			
č	BOU	ND	AR	Y	1	N	T E	6	R I			EG	IJ	A 1	11	0	N	M	E	TH	10	D									-									-		-						*
ç			••	•	_	-			_																																							*
Ç	LIN	EA	R	PI		T	Ē	Ŧ	R. Hi	ÊÔ	R	Y																																				÷
č	ARB	IŦ	R A R A	R	1	P	L A R A		S		R	M Se		L (D																																*
Č	ARB	ĪŤ	RĂ	R	r 	Ē	?U	İN	Ďi	ŇŘ	Y	(: 	Ñ I	DI	Ť	10) N	S		_	_			_		_				-							-		_								*
ç	THE	ł	CL) u 1 d	11	NG	•	1,	• P	'U	1 m c	V		ט. ר	E	5 1	כ הו	A' T	N E e	B N	E		EN	n.	L 7 A 7	ς Ε ,	D	6 0 1	/ 1 h)	TI T	M C	F	RI	: Ł		FU	RI	1 A	1	• •	•	• •	• •				*
č				NI	IL P				1		Ŭ		Ë	R	ò	F	Ê	Õ	Ļ			Ř	ž	ių IN	Ĕ	ŝŀ	ł	E	ËŇ	Ġ	t	H E	S															*
Č				X E B /	34	¥I X	Р , 8		N 1	r P Y S	Õ	Ŭ,	IT PE	S Q	V E	N	T S]	Â	Ť F	U	H	I	ΪH	N	Ď	i C M	Å	Ļ	R	Ē	200	5 A	T I	S	FJ Xe		D Y	3	-					•			*
ç				X		ç	Y Y F T	8	1	= L = F = T	I	D E L T F	D	U. F		I	ŇŢ	S	•	-	9 E	5 T	H 1 N 1	t 5 T 6		A L	. U	IN (6	T	н	Ł	P	A 1	1 14	C	16	1		T	F 6	R	A 1	1	0*	•		
č				Ĺ	,	•	•		1	Ē	ŝ	; i = 0	ÌÌ] (R) N N E	R		2	Ĭ	NÍ		R	N /	ĂĽ I	N	•)A E R	DN	A L	ŝ	I	N 1	S															*
Ĉ				0 DI	? <u>s</u>	1			1	E V	Î	LL EL	JE D		SU SU	P	T A D I	A V	Ñ	S \ S I	Ē	Ř	S	Ē	Ļ		D	Ģ	AT	[X	N 1 - [Ē	R R		L 1	Į	2	D	١	PC	1(N 1	T				*
ç				E		Ê	UE		1	= } = Y = T	C	EL	D IG	S	5 U 1	D C			U	SJ SJ	Ó	F		SF TH	Ē			6 T	EF	I N	Ă	-	1	RI	: C	11	10	N.)									*
č					(D	I	US		1	R			ŝ	ŝ	- 2 1 1	FR	Ť	H	Ē	'(I	R	ci	ונ	Å	R	C	L	A۴	P	E	D	P	L	\ T	E												*
Č				٩ł	T	Y	PE				Ú. T	ŪN Y F	Ë	Ă A = 1	i Y I	-)Ñ C	Ď		II P	0 E	N D	T	¥1	P E		F	CF	2	E	A (H	1	1E	Sł	1	L	EN	6	Tŀ	1						*
ç											T	Y F Y F	Ē	-	5	-	-	Ş	R	ĒĒ	Ľ	Y		5U	P	PC	JR	T	EC)																		*
č	***	**	**	#1	• •	•	* 4	•	*	• •	•	* 1	•	* 1	•	•	* *	•	*	• 1	•	*	• 1	* *	•	* 1	•	•	* 1	*	•	* 1	*	• •	•	* 1	*	*1	*	•	* 1	*	•	•	* 1	•	* *	•
-		DD	IM	E	NS NS	ł	C'N C'N		X		32	6) 5)	2	YE		32	6) 5)) , e	X	X E) (3	7))]	ž	Y E 5 J	30	3	7)) ,	N	P 1	Y 1	PI	E (37	7) 	•		•								
		D	12	EP	15	ļ			RUXI		1	źź	;) ; {	ť			// 7)	;) ; ;	ų	R P (1	7	5	,(Śŕ	X	2	17	5	,8		Y	1	17	5	ΥE	A ((r	22	•	K	L (. 7	21	,				
č		U	4 (*		• •	•		•				``	,0		, 0				5		,																											
Ċ	FDR	MA r	1 00	51	1 A -	T	EM	E	N1	r s 	-						_				• •	_	-	-								-	-	• •				_	-					•				
	200	1,	4 X	E		B	ŢŸ	P	Ę	, 1 • /		40		2	Ĩ	5	, e		į	Ş,	4	4	ļ	ξŚ	2	/	- #		9 - P			× , • ,	,	A 1	17) 7	" 4 ¥	, 0 , ,	×.	, " 	ж. н,	Χt Λ'	• /	• '	/ *	•	. 1	18	
	300	1		Ê	j² Jr	6	3×	-	È	ŝŠ		ð		X	Ę	Ž	ĵ.	1	ĥ	5) L (Ś	i	3	٦ ډ x	0	ž		Ş	ģ,	; ê	S	2]	/ (, . 14	د . م ا	8)	, (_	E	, 2 0		8.	.1	6)	Χ,				
	400	1		Ē	R	m)	C	• 1	L	00	T	•	, 1	9	x,	, 	P	SF	, H		34	6 X	,	• F	°S	M	• /	,	0	• /	' (14	۰,	8)	(,	E	20	•	ع	, 1	6)	κ,				
	200	٦ ء	0 0	E i F () R	2			ç	5)L		I	-	£ 1	9	X		X	51		3	52	X		¥ (4	•	22	::	q	14	6 ¹	B X	1	ĘZ	<u>C</u>	. 8	ç	10	5 X	,	E Z	0	. 8)))
	700	11	6 X 0 R	F	ė	È.	Ý 1 M	Ż	ţ	ÎĤ	6	<mark>آ</mark> ۴۱	ĬĬ	ŏ	2	F		s.	1	Ĵ,	3	E	Ş		1	2) 71)) IX		~ ' ~ h	1 M		, 1		•	, ' . 1	3.			N	F	P	' =	•	. 1	3.			
		17	1H	2		Y	e.	N N	6		4		<u>></u> U	Ļ	ÞR		8		6	, j	1	3		»' !,		ĎF	ÞS -	1	R/	D	Ĩ	Ç.	5	0	},	# . T I	I E	D	ĔŤ	Â	T		=	, 1	2	3	•	
	800	٦F	7 Oř	Ē	(ī	ç	ļ 9	T is	H) 4		K		Į	Š.	.4 .4	F	Į	52	E	; c	ינ י	A.	TI TI	E E o	=		•		F (5. 	.3) -			: 7	•	•	2										
	103	F	ŬR NR	M	Ì	Ę	Í	Ś	-	Į	2	ģ.	ļ	Ž)	0	r 11	r N		۳ ۱	15		•		1	L S 1	r	0	F) 3 D	, 00	– Uf	8 L	e E	ء د ۲	NI	 F C	K)	, I N	6	1)F	1	•	с.	. S'	"	
	•••	1)	71 E-	Ş	0	Ň	ÓÏ	Ē	s	5,	•	- Ř E	İ S	Ĭ	ĺ	L		I	Ă	Ĕ	Ň	Ē	Ë	UĒ In	Š	٥Į	50	Š	Ť	E Ď Ş H	A	Ř P (Ĺ	δi	۱` ,	ÄÌ	ŔĔ		ÍŇ		Tİ	İÈ	ĺ	ĎŔ	Ďİ	R	C	F
ç		1/	Ι,	41		N	۴L -	•	1	ג ר	•	" E	?.	С.	•		1 -	,	1	3)	۱,		B	. C	•		Z		•	, ,)						•											
č	TAN	U I •	V E A	D	LU (5	E	د •)	•		••		••• 15	••	•	Fp	• _	PP	2 -	Þ	P	57	-	DI	E T		. •	EV		LI	JF		R /	۱D	11	JS		11	A 1	LU	F								
		Ü	ŘÍ EA	Ĭ	Ĕ((5	8	\$	70 (0 X	5 A 8 (Ĭ	Ī,	, G , I	Fi		ĥ	ļ	Ş	P	R	, Č	Þ	Ś	Ī,	Ď	E1	ľÅ		ĒÌ	/Ā	Ľ	Ü		ŔÌ	ĬĎ	Ĩi	ĴŠ	,	ĪV	Ā	Ll	JE						
		R	E A E A F A	DDC			*) *)			8 (91	Ĭ	è	Ķ	I		N			N	MI	.)																											
					• •				•		-	• •		•	• -		•			•																												

APPENDIX B

READ(5,*)(BANY(I),I=1,NML) READ(5,*)(XXB(I),I=1,NML) WEITE(6,100)(I,XB(I),YB(I),BANX(I),BANY(I),XXB(I),YYB(I),NBTYPE(I) II=1,NML) XXB(MML+1)=XXB(1) YYB(NML+1)=YYB(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(NML+1)=NBTYPE(1) NBTYPE(1),I=1,NFP) READ(5,*)(YF(I),I=1,NIP) READ(5,*)(Q(I),I=1,NIP) WRITE(6,200)(I,XI(I),YI(I),L(I),Q(I),I=1,NIP) **** THE PLATE PROBLEM IS TO SOLVE THE FOLLOWING SET OF EQUATIONS . • ٠ ٠ -٠ ٠ • ٠ • ٠ ٠ • * ۰ . . ٠ * • ٠ . ٠ . RM(I,J) RM(I,J+NML) PL(I) ٠ ٠ ٠ ٠ F# . ٠ . ٠ . . ٠ 4 . . • ٠ ٠ ٠ *** * . . ***** -. • ٠ ٠ ٠ 1 . ÷ RM(I+NML,J+NML) * RM(I+NML,J) * . * 8. ۰ RL(I+NML) ٠ • ٠ . * ٠ ٠ ٠ ********** THE FIRST STEP IS TO DETERMINE THE RL VECTOR. THESE ARE THE VALUES OF AREA INTEGRALS OF BOUNDARY CONDI-TION EQUATIONS. ٠ ٠ THE FOLLOWING IS FOR TRANSFERING TO NON-DIMENSIONAL FULLUWING IS FUR TRANSPERING TO NUM-DIMER NBP=NML+1 NML2=2*KML DFLATE=EVALUE+HVALUE+*3/(12.*(1.-PR+*2)) FI=4.*ATAN(1.) CDEFf=-1./(16.*PI) CDEFf=-1./(16.*PI) CDEF2=2.0*CDEF1 DD PO I=1.NML XB(I)=XP(I)/RADIUS YB(I)=YP(I)/RADIUS YB(I)=YYP(I)/RADIUS XXB(NMLP1)=XXP(1) YPB(N=LF1)=YYB(1) DD P1 I=1.NFP XF(I)=XF(I)/RADIUS YF(I)=XI(I)/RADIUS YI(I)=XI(I)/RADIUS VI(I)=YI(I)/RADIUS VI(I)=YI(I)/RADIUS 80 81 82 CCCCCCCCCCC NCHECK IS AN ARGUMENT NUMPER FOR DOUBLE CHECKING B.C. VALUES. AFTEP THE FICTITIOUS LOADS P* AND M* HAVE BEEN COMPUTED, THESE VALUES ARE SUBSTITUTED BACK TO THE ORIGINAL BOUNDARY CONDITION EQUATIONS TO DOUBLE CHECK WHETHER F(W)=ZEPO ARE SATISFIED. ACCORDING TO EXPERIMENTS, ERRORS LE. E-1^ INDICATE* THE RM MATRIX IS NOT POORLY ILLCONDITIONED, AND POTH TRUNCATION AND ROUND-OFF ERRORS ARE NOT GREAT. ...

```
C
         NCHECK='
                THE FOLLOWING EQUATIONS ARE FOR ALL THE THREE B.C. TYPES
                                                         FULLOWING EQUATIONS ARE FOR ALL THE THREE B.C. TYP

D0 5 I=1,NML

RL(I)=0.0

R1(I+NML)=C.0

R1x=XP(I)

R1y=YE(I)

                                                        29=K1Y-R2Y

24=R125/(23+R125)

1F(24.LE.1.E-12)GCTD 17

25=ALDG(24)

GC TC 15

25=C.0

21%=(21+22+R125+25)+Q(J)
                             17
 C
                                                             IF (NBTYFE(I).EQ.1)60TC 30
 C
                                                           ANX 2= ANX ++2
ANY 2= ANY ++2
2 TR12= Z3 + R12 S
2 SR12 S= Z3 R12 + 2
Z SR12 C= Z3 R12 + 2 3 R12 S
Z SF12 G= Z3 R12 S++2
C
                                                             IF (NBTYPE(I).EQ.3)GOTO 34
                       If (NBITPE(I).EQ.5)60T0 34
ZZ3=A\X2+(R2S+2.+Z7++2/R12S-(4.+Z7+Z6+R12S+R2S)/Z3R12
1+2.+R12S+Z6+2/Z3R12S+Z5)
ZZ4=4.+ANX+ANY+(Z7+Z9/R12S-(Z9+Z6+Z7+Z8)/Z3R12+R12S+Z6+ZP/Z3R12S)
ZZ5=A\Y2+(R2S+2.+Z7+2)P12S-(4.+Z9+Z8+R12S+R2S)/Z3R12
1+2.+P12S+Z++2/Z3R12S+Z5)
Z11=(ZZ3+ZZ4+ZZ5)+Q(J)
6CT0 34
30 CDNTINUE
Z11=(ANX+(Z6-Z6+Z4+Z7+Z5)+ANY+(Z8-Z8+Z4+Z9+Z5))+Q(J)
34 CDNTINUE
IF (L(J).6T.1) 60 T0 1
Z12=C0EF1+DPS1+DETA/4.0
Z13=C0EF2+DPS1+DETA/4.0
Z13=C0EF2+DPS1+DETA/4.0
Z13=C0EF2+DPS1+DETA/2.0
Z13=C0EF2+DPS1+DETA/2.0
Z13=C0EF2+DPS1+DETA/2.0
Z13=C0EF2+DPS1+DETA/2.0
Z12=C0EF1+DPS1+DETA/2.0
Z12=C0EF1+DPS1+DETA/2.0
Z12=C0EF1+DPS1+DETA/2.0
Z12=C0EF1+DPS1+DETA/2.0
Z12=C0EF1+DPS1+DETA/2.0
Z13=C0EF2+DPS1+DETA/2.0
Z13=C0EF2+DPS1+DETA/2.0
Z13=C0EF2+DPS1+DETA/2.0
Z13=C0EF1+DPS1+DETA/2.0
Z14=C0EF1+DPS1+DETA/2.0
Z14=C0EF1+DPS1+DETA/2.0
Z14=C0EF1+DF1+DETA/2.0
Z14=C0EF1+DF1+DETA/2.0
Z14=
C
                                      2 Z12=COEF1+DPSI+DETA
Z13=COEF2+DPSI+DETA
3 CONTINUE
C
                                                              IF (NETYPE(I).E0.3)GDTD 50
                                                           RL(1)=RL(1)+Z12+Z1?
RL(1+NML)=RL(1+NML)+Z13+Z11
GDT0 4
C
 CCC
                THE FOLLOWING IS FOR FREE EDGES
                           5: CONTINUE

CONSTI=ANX+(ANX2+(2,-PR)+ANY2)

CONSTI=ANY+((2.+PR-1.)+ANX2+(2.-PR)+ANY2)

CONSTJ=ANX+((2.+PR-1.)+ANY2+(2.-PR)+ANX2)

CONSTJ=ANX+(ANY2+(2.-PR)+ANX2)

CONSTS=ANX2+PR+ANY2
```

.

CONST6=2.*(1.-PR)*ANX*ANY CONST7=ANY2+PR*ANX2 ZF1=1.-2.*R1X*R2X ZF2=1.-2.*R1Y*R2Y ZF3=R1X-R2X*R1S ZF4=R1Y-R2Y*R1S AA3=6.*Z7/R12S-4.*Z7**3/R12S**2-(6.*Z6+6.*R2S*Z7)/Z3R12 1*(12.*Z6*2*Z7+6.*R12S*R2S*Z6)/Z3R12S-8.*R12S*Z6+3/Z3F12C DD3=6.*T0/P12S-4.*Z0**2*Z9/P12S**2+(-2.*R2S*Z0+6.*Z7)/Z3P12 1*(12.*Z0*Z**2*6.*R12S*R2S*Z8)/Z3R12S* AA2=2.*Z0/R12S-4.*Z7**2*Z9/P12S**2+(-2.*R2S*Z0+2.*Z8)/Z3P12 1*(6.*Z7*Z6*2*Z*Z6/Z5*R2S*Z8)/Z3R12S* DD2=2.*Z7/R12S-4.*Z0**2*Z7/R12S**2-(2.*R2S*Z7+2.*Z6)/Z3R12 1*(6.*Z7*Z6**2*Z6/Z5*R12C DD2=2.*Z7/R12S-4.*Z0**2*Z7/R12S**2-(2.*R2S*Z7+2.*Z6)/Z3R12 1*(8.*Z0*Z6**2*Z6/Z5*R12C AA4=R2S*2.*Z7**2/R12S-(4.*Z7*Z6*R12S*R2S)/Z3R12 1*2.*R12S*Z6**2*Z7/R12S+Z5 DDD=R2S*2.*Z7**2/R12S+Z5 GG6=2.*Z7*Z0/R12S-2.*(Z0*Z6*Z7*Z8)/Z3R12 1*2.*R12S*Z6**2/Z3P12S+Z5 T0TAL1=(CONST1*AA3*CONST2*AA2*CONST3*DD2*CONST4*DD3)*Z13*Q(J) RL(1)*RL(1)*RL(1*NML)*T0TAL2 CONTINUE CONTINUE WRITE (6,300) (I,RL(I),RL(I+NML),I=1,NML) 45 CCCCCCC ***** DETERMINATION OF COEFFICIENT MATRIX Rm(I,J), Rm(I,J+NML), Rm(I+NML,J), AND Rm(I+NML,J+NML) COEF1=-COFF1 COEF2=-COEF2 DO R I=1, MML R1x=xB(I) R ZS=ALDG(ZA) C Z6=R1X-R2X+R1S Z7=R1X-R2X Z7=R1X-R2X Z7=R1X-R2X Z9=P1Y-R2Y Z10=R1X+R2S-R2X Z11=P1Y+P2S-R2Y Z12=1, ^-2.0+P1Y+R2X Z13=-2.^+R1X+R2Y Z14=-2.0+R1Y+R2X Z15=-2.^+R1Y+R2X Z15=-2.0+R1Y+R2X Z15=-2.0+R1Y+R2X RM(I,J)=C0EF1+(Z3+P12S+Z5)+S2 RM(I,J+NML)=C0EF2+(ANX2+(Z6-Z6+Z4+Z7+Z5)+ANY2+(Z6-Z8+Z4+Z9+Z5))+S2 IF(NBTYPE(I).EQ.3)60T0 51 C 1 C IF (NBTYPE(I).EQ.2)60T0 40 C

RM(1+NML, J)=CDEF2*(ANX1*(Z1C-Z10*Z4+Z7*Z5)+ANY1*(Z11-Z11*Z4+Z9 25))*52 Z16=ANX1*ANX2*(2.0*(Z7**2)/P125+(Z12*Z3-2.0*Z7*Z6-2.0*Z7*Z10)/ Z8R12+(2.0*R125*Z6*Z10)/Z3R125+Z5) Z17=ANX1*ANY2*(2.C*Z9*Z7/R125+(Z13*Z3-2.0*Z7*Z8-2.0*Z9*Z10)/ Z18=ANY1*ANX2*(2.0*Z7*Z9/R125+(Z14*Z3-2.0*Z9*Z6-2.0*Z7*Z11)) Z18=ANY1*ANX2*(2.0*R125*Z6*Z11)/Z3R125) Z18=ANY1*ANX2*(2.0*R125*Z6*Z11)/Z3R125) Z18=ANY1*ANX2*(2.0*R125*Z6*Z11)/Z3R125) Z18=ANY1*ANX2*(2.0*R125*Z6*Z11)/Z3R125) Z18=ANY1*ANX2*(2.0*R125*Z6*Z11)/Z3R125) Z18=ANY1*ANX2*(2.0*R125*Z6*Z11)/Z3R125) Z19=ANY1*ANX2*(2.0*R125*Z6*Z11)/Z3R125*(Z15*Z3-2.0*Z9*Z8-2.0*Z9*Z11)/ Z3R12*(2.0*R125*Z6*Z11)/Z3R125*Z5) RM(1+NML,J*NML)=CDEF2*(Z16*Z17*Z18*Z19)*S2 GCTC 7 1 1 1 1 1 FOR SIMPLY SUPPORTED BOUNDARIES 4 ` Z¥1=-21 CCC FOR FREE EDGES FOR FREE EDGES 51 CONTINUE Z6=P1x+R2S-R2X 27=P1x-R2Y 2P=P1Y+R2S-R2Y CONST1=ANX1+(ANX1++2+(2.-PR)+ANY1++2) CLNST2=ANY1+((2.+PR-1.)+ANX1++2+(2.-PR)+ANY1++2) CNST3=ANX1+((2.+PR-1.)+ANY1++2+(2.-PR)+ANX1++2) CNST5=ANX1+((2.+PR-1.)+ANY1++2+(2.-PR)+ANX1++2) CONST5=ANX1+((2.+PR-1.)+ANY1++2+(2.-PR)+ANX1++2) CONST5=ANX1+((2.+PR-1.)+ANY1++2+(2.-PR)+ANX1++2) CONST5=ANX1+((2.+PR+1.)+ANY1++2+(2.-PR)+ANX1++2) CONST5=ANX1+2+PR+ANY1+2 CONST5=ANX1+2+PR+ANY1+2 CONST5=ANX1+2+PR+ANY1+2 ZF1=1.-2.+R1X+R2Y ZF3=R1X-R2X+R1S ZF4=R1Y-R2Y+R1S AA3=6.+Z7(+P12S-4.+Z7++3/R12S++2-(6.+Z6+6.+R2S+Z7)/Z3R12 +(-2.+Z2+27+6.+R12S+P2S+Z6)/Z3R12S-R.+R12S+Z6+3/Z3R12 +(-2.+Z2+27+6.+R12S+P2S+Z6)/Z3R12S-R.+R12S+Z6+3/Z3R12 +(-2.+Z2+27+6.+R12S+P2S+Z6)/Z3R12S+Z7+6.+R2S)/Z3R12 +(-2.+Z2+27+6.+R12S+Z2-4.+Z7+4.+R2Y+Z7+6.+R2S)/Z3R12 +(-2.+Z2+27+6.+R12S+Z2-4.+Z7+4.+R2Y+Z7+6.+R2S)/Z3R12 +(-2.+Z2+27+6.+R12S+Z2-4.+Z7+4.+R2Y+Z7+6.+R2S)/Z3R12 +(-2.+Z2+27+6.+R2S+Z7+Z6-2.+R2S+Z5+Z5+Z5+Z7+1+12.+R2S+Z7+Z7+2F3 +(-2.+Z6+2+12.+R2S+Z7+Z6-2.+R2S+Z5+Z5+Z5+Z5+Z7+2F3

CCCCCCCCCCC SO FAR WE HAVE FORMED BOTH RM MATRIX AND RL VECTOR. AN ISML SUPRCUTINE LEGTIF IS THEN CALLED TO SLOVE THIS SET OF SIMULTANEOUS EQUATIONS FOR VALUES OF FICTITIOUS FORCES AND BENDING MOMENTS ALONG THE THE FATH OF INTEGRATION. CALL LEGT1F(RM,1,NML2,NML2,RL,0,WKAREA,IER) D 29 I=1,NML2 29 PS(I)=RL(I) WRITE (6,400) (I,PS(I),PS(I+NML),I=1,NML) NCHECK=1 60 T0 1019 1100 CONTINUE WRITE(6,1014) D0 1101 I=1,NML2 SUM=0. DC 1101 I=1,NML2 SUM=0. DC 1102 J=1,NML2 SUM=SUM+RM(I,J)*PS(J) 1102 CONTINUE 1101 RB(I)=SUM-RLX(I) WRITE(6,1103)(I,RB(I),RB(I+NML),I=1,NML) DC 83 I=1,NML2 83 PS(I)=-FS(I)/RADIUS CCCCCCC *************** FROM MERE DOWN, DISPLACEMENTS AND BENDING MOMENTS AT ALL THE . FIELD POINTS WILL BE COMPUTED. **** RSQD=RADIUS**2/DFLATE RQD=-RSQD*RADIUS CDEF1=1./(16.*PI) CDEF2=CDEF1*2. D0 9 I=1,NFP W(I)=0.0 BMY(I)=0.0 BMY(I)=0.0 CCNTINUE PLOAD=0. 9

DC 14 I=1,NFF F1x=xF(I) R1y=yF(I) R1S=R1x++2+R1y++2 Z1=1.0-R1S DC 13 J=1,NIF R2x=xI(J) R2y=yI(J) P2S=R2y++2+F2y++2 Z2=1.C-R2S F12S=(K1x-R2x)++2+(R1y-R2y)++2 Z3=Z1+Z2 Z4=Z3+F12S Z5=F12S/Z4 Z6=R1x-F2x Z7=C1Y-R2y Z8=C2y-R1x+R2S Z9=C2y-R1x+R2S Z1^=(4.0+76+2F-C12S+R2S)/Z4 Z1=(2.0+712S+C2S)/Z4 Z1=(2.0+712S+C2S)/Z4 Z1=(2.0+712S+(29+2))/(Z4++2) IF(P12S+LE-1=F12)GDTC 19 Z14=(2.0+(Z6++2))/F12S Z15=(2.0+(Z7++2))/F12S Z16=ALCC(Z5) CD T0 Z2 Z16=-C IF SOME FIELD FOINTS ARE ASSIGF . IF SOME FIELD FOINTS ARE ASSIGNED AT THE LOAD POINTS BY MISTAKE, BOTH BMX AND BMY WILL FE AUTOMATICALLY ASSIGNED TO BE 10000000000. . ·

Z15 = (2, C* (27*+2))/R12S Z17=R1X-R2*R1S Z17=R1X-R2*R1S Z17=R1X-R2*R1S Z17=R1X-R2*R1S Z17=R1X-R2*R1S Z17=C* (27*R1S) Z17 1 1 1 1 Z44=CDEF2+(ANX2+(Z17-Z1/*Z)*Z6*Z10)*ANY2*(Z18-Z18*Z)*Z(*Z10) 1 S2 245=CDEF2*(R25+Z14+210+Z12+Z16)*S2 246=CDEF2*(R25+Z14+210+Z12+Z16)*S2 246=CDEF2*(R25+Z15+Z11+Z13+Z15)*S2 247=CDEF2*(ANX2*(Z23+Z27+Z31+Z35+Z39)*ANY2*(Z24+Z28+Z32+Z36+ 1 Z4^))*S2 248=CDEF2*(ANX2*(Z25+Z29+Z33+Z37+Z41)*ANY2*(Z26+Z30+Z34+Z38+ 1 Z42))*S2 BMX(I)=BMX(I)+(-Z45*FS(J)-Z47*PS(J*NML)-PR*Z46*FS(J)-FR*Z48* 1 PS(J+NML))*(-RADIUS) BMY(I)=BMY(I)+(-PR*Z45*FS(J)-FR*Z47*PS(J+NML)-Z46*PS(J)-FR*Z48* 1 PS(J+NML))*(-RADIUS) BMY(I)=BMY(I)+(-RADIUS) 15 CONTINUE 16 CONTINUE 16 CONTINUE 17 CONTINUE 18 YF(I)=XF(I)*RADIUS 19 A I=1 NFP XF(I)=XF(I)*RADIUS 4 RITE(C, 60())(I XF(I), W(I), BMX(I), BMY(I), I=1, NFP) 4 RITE(C, 000)PL0AD FLOAD IS THE SUM OF THE DISTRIBUTED LATERAL LOADS APPLIED. 5 FNN 1 C

 1
 5
 5
 -2
 5
 -3
 5
 -4
 5
 -2
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5
 -4
 5</

APPENDIX C

COMPUTER PROGRAM FOR THE POINT-FORCE METHOD

APPENDIX C

COMPUTER PROGRAM FOR THE POINT-FORCE METHOD

PROGRAM NEWPLCL(INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT) 000001 C 000002 C 000004 C FOINT FORCE METHOD FOR ISOTROPIC PLATE BENDING PROBLEMS ***** 000005 С 000005 C ARBITRARY PLAN FORM, TRANSVERSE LOAD, AND BOUNDARY CONDITIONS 000007 C 000008 С 000009 C REQUIRED INFUT VALUES ---000010 С 000011 C NEP =NUMBER OF BOUNDARY FOINTS 000012 =NUMBER OF INTERNAL LOAD POINTS NIP С 000013 NFP =NUMBER OF FIELD POINTS С 000014 XB,YB =POINTS ON B AT WHICH B.C. ARE SATISFIED. С 000015 С BANX, BANY = COMPONENTS OF UNIT NORMAL TO B AT XB, YB. 000016 С XXB, YYB = END POINTS OF MESHES AROUND B WHERE FICTITIOUS 000017 С FORCES ARE ASSIGNED. 000018 XF,YF С =FIELD POINTS 000019 XI,YI =INTERNAL LOAD POINTS С 000020 С PR =POISSON*S RATIO 000021 EVALUE =YOUNG*S MODULUS С 000022 С HVALUE =FLATE THICKNESS 000023 =RADIUS OF THE FICTITIOUS CIPCULAR PLATE OF WHICH RADIUS С 000024 С THE DISPLACEMENT AT THE CIRCUMFERENTIAL BOUNDARY 000025 IS SET TO ZERO. С 000026 **BOUNDARY CONDITION TYPE AT EACH BOUNDARY POINTS** С NBTYPE 000027 000028 С NBTYPE = 1 --- CLAMFED С NBTYPE = 2 --- SIMPLY SUPPORTED 000029 NBTYPE = 3 --- FREE 000030 С C 000031 C 000033 DIMENSION X8(40), Y8(40), XX8(81), YY8(81), NSTYPE(40) 000034 DIMENSION XI(100), YI(100), DEL(2) 000035 DIMENSION RB(80), RM(80,80), PS(80), WKAREA(80), RL(80) 000035 DIMENSION XF(181), YF(181), W(181), BMX(181), BMY(181) 000037 DIMENSION BANX(40), BANY(40) 000038 000039 С 100 FORMAT(#OLOCT#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X,#NBTYPE#, 000040 +/#0#/(I5,4F10.4,I5)) 000041 110 FORMAT(#0LOCT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) 000042 200 FORMAT (#0L0CT#,11X,#XI#,17X,#YI#,14X,/#0#/ 000043 +(I4,11X,F6.2,11X,F6.3)) 000044 300 FORMAT (#0LOCT#,19X,#RLD#,34X,#RLS#/#0#/(14,8X,E20.8,16X, 000045 1E20.8)) 000046 400 FORMAT (#1LOCT#,19X,#PSP#/#0#/(I4.8X,E20.8)) 000047 500 FORMAT(#0LOCT#,11X,#XF#,17X,#YF#/#0#/(14,11X,F6.3,11X,F6.3)) 000049 600 FORMAT(1H1,8X,#NODE#,12X#XF#,16X,#YF#,19X,#W#,16X,#BMX#, 000049 116X, #BMY#/(1H0, I10, 2F20.10, 3E20.12)) 000050 700 FORMAT(1H1, #INPUT VALUES #,//1X, #NBP = #,13, # NFP =#,13, 000051 +# NIP = #,13,# PR = #,F5.3, 000052 1/1H0, #YOUNGS MCDULUS = #, E14.8, # RADIUS OF THE PLATE = #, 000053 +F7.1,# THICKNESS OF THE PLATE =#,F6.3) 000054 000055 C C INPUT VALUES 000055 000057 C READ(5,*)NSP,NIP,NFP,FR,EVALUE,RADIUS,HVALUE 000058 WRITE(6,700)NBP,NFP,NIP,PR,EVALUE,RADIUS,HVALUE 000059

```
READ(5,#)(XB(I),I=1,NBP)
                                                                          000060
      READ(5,*)(YB(I),I=1,NBP)
                                                                          000061
      READ(5, #)(NBTYFE(I), I=1, NBP)
                                                                          000062
      READ(5,*)(BANX(I),I=1,NBP)
                                                                          000063
      READ(5,*)(BANY(I),I=1,NEP)
                                                                          000064
      LRITE(6,100)(I,XB(I),YB(I),BANX(I),BANY(I),NBTYPE(I)
                                                                          000065
     1,I=1,NEP)
                                                                          000066
С
                                                                          000067
C ASSIGN LOCATIONS OF FIELD POINTS, (XF,YF).
                                                                          000068
С
                                                                          000069
      X0=5.0$Y0=5.0
                                                                          000070
      XF(1)=10.-X0$YF(1)=-10.+Y0
                                                                          000071
      DO 41 I=2,11
                                                                          000072
      XF(I)=XF(I-1)-0.5
                                                                          000073
   41 YF(I)=YF(I-1)
                                                                          000074
      DO 42 J=1,10
                                                                          000075
      K=J#17
                                                                          000076
      DO 42 I=1.11
                                                                          000077
      XF(I+K)=XF(I)
                                                                          000078
   42 YF(I+K)=YF(I+K-17)+1.0
                                                                          000079
      XF(12)=10.-X0$YF(12)=-9.5+Y0
                                                                          000050
      DO 43 I=13.17
                                                                          000051
      XF(I)=XF(I-1)-1.0
                                                                          000082
   43 YF(I)=YF(I-1)
                                                                          000083
      DO 44 J=1,9
                                                                          000084
      K=J*17
                                                                          000085
      DO 44 I=12,17
                                                                          000026
      XF(I+K)=XF(I)
                                                                          000087
   44 YF(I+K)=YF(I+K-17)+1.0
                                                                          000068
      READ(5,*)(XI(I),I=1,NIP)
                                                                          000089
      READ(5,*)(YI(I),I=1,NIP)
                                                                          000090
С
                                                                          000091
C ASSIGN LOCATIONS OF FICTITIOUS FORCES, (XXB, YYB).
                                                                          000092
С
                                                                          000093
      NBP2=HBP#2$NBP2P1=NBP2+1
                                                                          000094
      NBPP1=NPP+1
                                                                          000095
      DIST1=4.0 $ DIST2=2.0
                                                                          000096
      DEL(1)=(10.+2.#DIST1)/10.
                                                                          000097
      DEL(2)=(10.+2.*DIST1+2.*DIST2)/10.
                                                                          000058
      XXB(1)=5.+DIST1-DEL(1)$YYB(1)=-5.-DIST1
                                                                          000099
      XXB(41)=5.+DIST1+DIST2-DEL(2)$YYB(41)=-5.-DIST1-DIST2
                                                                          000100
      DO 28 J=1.2
                                                                          000101
      DELT=DEL(1)$IF(J.EQ.2)DELT=DEL(2)
                                                                          000102
      DO 25 I=2,10
                                                                          000103
      K=I
                                                                          000104
      IF(J.EQ.2)K=K+40
                                                                          000105
      XXB(K)=XXB(K-1)-DELT
                                                                          000106
   25 YYB(K)=YYB(K-1)
                                                                          000107
      DO 26 I=11,20
                                                                          000108
      K=I
                                                                          000109
      IF(J.EQ.2)K=K+40
                                                                          000110
      XXB(K)=XXB(K-1)
                                                                          000111
   26 YYP(K)=YYB(K-1)+DELT
                                                                          000112
      DO 27 I=21,30
                                                                          000113
      K=I
                                                                          000114
      IF(J.EQ.2)K=K+40
                                                                          000115
      XXB(K)=XXB(K-1)+DELT
                                                                          000116
   27 YYB(K)=YYB(K-1)
                                                                          000117
      DO 28 I=31,40
                                                                          000118
                                                                          000119
      K=I
```

```
IF(J.EQ.2)K=K+40
                                                                         000120
      XXB(K)=XXB(K-1)
                                                                         000121
   28 YYB(K)=YYB(K-1)-DELT
                                                                         000122
      XXB(81)=XXB(41) $ YYB(81)=YYB(41)
                                                                         000123
      WRITE(6,110)(I,XXB(I),YYB(I),I=1,N3F2P1)
                                                                         000124
      WRITE(6,500)(I,XF(I),YF(I),I=1.NFP)
                                                                         000125
      WRITE(6,200)(I,XI(I),YI(I),I=1,NIP)
                                                                         000126
      DPLATE=EVALUE+HVALUE++3/(12.+(1.-PR++2))
                                                                         000127
      PI=4.#ATAN(1.)
                                                                         000128
      COEF1=1./(16.*PI*DPLATE)
                                                                         000129
      COEF2=COEF1*2 $ COEF4=COEF2**2 $ QLOAD=1.0 $ R2=RADIUS**2
                                                                         000130
C
                                                                         000131
C SET UP THE RL VECTOR AS SHOWN IN APPENDIX A.
                                                                         000132
С
                                                                         000133
      DO 5 I=1,HBP
                                                                         000134
                                                                         000135
      RL(I)=0.0
      RL(I+NPP)=0.0
                                                                         000135
      R1X=XB(I)
                                                                         000137
      R1Y=YB(I)
                                                                         000135
      ANX=BANX(I)
                                                                         000139
      ANY=BANY(I)
                                                                         000140
      DO 6 J=1,NIP
                                                                         000141
      R2X=XI(J)
                                                                         000142
      R2Y=YI(J)
                                                                         000143
      Z1=R1X-R2X$Z2=R1Y-R2Y$R12S=Z1##2+Z2##2
                                                                         000144
      25=ALOG(R125/R2)
                                                                         000145
      IF(NBTYPE(I).EQ.3)GOTO 18
                                                                         000146
      RL(I)=RL(I)-QLOAD*(R12S*Z5)
                                                                         000147
      IF(NBTYPE(I).EQ.1)GOTO 17
                                                                         000148
С
                                                                         000149
C FOR SIMPLY SUPPORTED EDGES ONLY ---
                                                                         000150
С
                                                                         000151
      Z15=Z1##2 $ Z25=Z2##2 $ ANXS=LNX##2 $ ANYS=ANY##2
                                                                         000152
     RL(I+NBP)=RL(I+NBP)-QLOAD*((2.*Z15/R125+1.+Z5)*ANXS
                                                                         000153
     ++4.#Z1#Z2/R125#ANX#AN1+(2.#Z25/R125+1.+Z5)#ANYS)
                                                                         000154
      GOTO 6
                                                                         000155
C
                                                                         000155
C FOR CLAMPED EDGES ONLY ---
                                                                         000157
C
                                                                         000155
   17 RL(I+NBP)=RL(I+NBP)-QLOAD*(Z1*ANX+Z2*ANY)*(1.+Z5)
                                                                         000159
      GOTO 6
                                                                         000160
                                                                         000161
C
C FOR FREE EDGES ONLY ---
                                                                         000162
C
                                                                         000163
   18 Z15=Z1##2 $ Z25=Z2##2 $ ANXS=ANX##2 $ ANYS=ANY##2
                                                                         000164
      R1255=R125*#2
                                                                         000165
      RL(I)=RL(I)-QLOAD*((ANXS+PR*ANYS)*(2.*Z1S/R12S+1.+Z5)
                                                                         000166
     ++(4.#(1.-FR)#ANX#ANY)#Z1#Z2/R125
                                                                         000167
     ++(ANYS+FR*ANXS)*(2.*Z25/P125+1.+Z5))
                                                                         000168
      C1=ANX#(1.+ANYS#(1.-PR))
                                                                         000169
      C2=((2.#PR-1.)#ANXS+(2.-PR)#ANYS)#ANY
                                                                         000170
      C3=((2.#PR-1.)#ANYS+(2.-PR)#ANXS)#ANX
                                                                         000171
      C4=ANY*(1.+ANXS*(1.-PR))
                                                                         000172
      RL(I+NBP)=RL(I+NBP)-QLOAD*(C1#Z1*(Z1S+3.#Z2S)/R12SS
                                                                         000173
     ++C2*Z2*(Z2S-Z1S)/R12SS+C3*Z1*(Z1S-Z2S)/R12SS
                                                                         000174
     ++C4*Z2*(Z2$+Z1$*3.)/R1255)
                                                                         000175
    6 CONTINUE
                                                                         000176
    5 CONTINUE
                                                                         000177
                                                                         000178
C
C SET UP THE RM MATRIX AS SHOWN IN APPENDIX B, AND NOTE THAT
                                                                         000179
```
C	FICTITIOUS FORCES ARE LOCATED AT THE CENTER OF THE ASSIGNED MESHES.	000180
С		000181
	DO 8 I=1,NSP	000182
	R1X=XB(I)	000183
	RIY=YB(I)	000184
	ANX=BANX(I)	000185
	ANY=BANY(I)	000185
	DO 7 J=1,NºF2	000187
	R2X=(XXB(J+1)+XXB(J))/2.	000183
	$\mathbf{R}\mathbf{Z}\mathbf{Y} = (\mathbf{Y}\mathbf{Y}\mathbf{B}(\mathbf{J}+1)+\mathbf{Y}\mathbf{Y}\mathbf{B}(\mathbf{J}))/2$	000189
	IF(NOPTICN.EQ.2.OR.J.NE.NBP)60T0 33	000190
	R2X=(XXB(1)+XXB(NBP))/2.	000191
	RZY=(YYB(1)+Y)B(NBP))/2.	000192
	33 CONTINUE	000193
	Z1=R1X-RZX\$Z2=R1Y-R2Y\$R125=Z1**2+Z2**2	000194
	ZJEALUGIRIZS/RZJ	000195
	IF(ND) TPE(I). 24. 5 (60) U IY	000195
	RELIJJ-RIC3763	00019/
	$\mathbf{T} \left(\mathbf{T} \left(\mathbf{T} \right) \right) = \mathbf{T} \left(\mathbf{T} \right) \left(\mathbf{T} \right) = \mathbf{T} \left(\mathbf{T} \right) \left$	000193
	EIJ-EIWE V EEJ-EEWE V ARAJ-ARIYWE V ARIJ-ARIYWE DM(TAADD 1)-(19 A718/D1061) AVENAAND	000144
	RIII 1 TNDF 7 J - LLE, */LEJ KI (23 T L, */LEJ * # NA) 14/6 - #71 #72 / Dige A Aliya Aliya (9 - #736 / Dige 1 - 1 - 75) # Aliye)	000200
	444.*21*22/RICJ*ANA*ANIT(2.*2CJ/RICJ*1.*25)*ANIS) 2010 7	000201
	90 DM(TANRD, 1)=(7)#ANYA72#ANY)#(1 475)	000202
	CO RITTOP, J /- (21 ANAV22 ANT)*(1.423) CATA 7	000203
	10 718271842 & 728272842 & ANYSZANY442 & ANYSZANY442	000204
		000205
	DM(T, 1)=((ANYSADD#ANYS)#(2 #715/D12541 475)	000207
		000208
	44(4)(4)(5)(()()()()()()()()()()()()()()	000209
	C1=ANX+(1.+ANYS+(1F7))	000210
	C2=((2, #PR-1,)#ANXS+(2, -PR)#ANYS)#ANY	000211
	C = (12 - PP - 1 - 1 + ANYS + (2 - FP) + ANYS) + ANY	000212
	C4=ANY+(1,+ANXS+(1,-FR))	000213
	RH(I+NBP,J)=(C1+Z1+(Z15+3,+Z25)/R1255	000214
	++C2*Z2*(Z25-Z15)/R1255+C3*Z1*(Z15-Z25)/R1255	000215
	++C4#Z24(Z25+Z15#3.)/R1255)	000216
	7 CONTINUE	000217
	8 CONTINUE	000218
С		000219
Ċ	SOLVE THE SET OF LINEAR EQUATIONS TO DETERMINE THE FICTITIOUS FORCES.	000220
Ĉ		000221
	CALL LEGT1F(RM,1,NBP2,NBP2,RL,0,WKAREA,IER)	000222
	DO 29 I=1,NBP2	000223
	29 PS(I)=RL(I)	000224
	WRITE (6,400) (I,PS(I),I=1,NBP2)	000225
	DO 9 I=1,15	000226
	W(I)=0.0	000227
	BMX(I)=0.0	000228
	BMY(I)=0.0	000229
	9 CONTINUE	000230
С		000231
C	COMPUTE DISPLACEMENTS AND BENDING MOMENTS AT THE PRESCRIBED FIELD POIL	N000232
C		000233
	DO 14 J=1,15	000234
	R1X=XF(I)	000235
	Rly=yf(I)	000236
	DO 13 J=1,NIP	000237
	R2X=XI(J)	000238
	R2Y=YI(J)	000239

	Z1=R1X-R2X\$Z2=R1Y-R2Y\$R12S=Z1##2+Z2##2	000240
	Z1S=Z1*+2 \$ Z2S=Z2**2	000241
	Z5=ALOG(R125/R2)	000242
	W(I)=W(I)+COEF1*(R125*Z5)*QLOAD	000243
	Z6=1.+2.#Z15/R125+Z5 \$ Z7=1.+2.#Z25/R125+Z5	000244
	Z8=DPLATE*COEF2*QLOAD	000245
	<pre>BMX(I)=BMX(I)-Z8*(Z6+PR*Z7) \$ BMY(I)=BMY(I)-Z8*(Z7+PR*Z6)</pre>	000246
	IF(I.GE.2)GOTO 13	000247
13	CONTINUE	000248
14	CONTINUE	000249
	DO 16 I=1,15	000250
	RIX=XF(I)	000251
	RlY=YF(I)	000252
	DO 15 J=1,NBP2	000253
	R2X=(XXB(J+1)+XXB(J))/2.0	000254
	R2Y=(YYB(J+1)+YYB(J))/2.0	000255
	IF(NOPTION.EQ.2.OR.J.NE.NBP)GDTO 11	000255
	R2X=(XXB(1)+XXB(NBP))/2.	000257
	R2Y=(YYB(1)+YYB(NBP))/2.	000258
11	CONTINUE	000259
	Z1=R1X-R2X\$Z2=R1Y-R2Y\$R12S=Z1##2+Z2##2	000260
	Z5=ALOG(R125/R2)	000261
	Z15=Z1++2 \$ Z25=Z2++2	000262
	W(I)=W(I)+FS(J)*COEF1*(R12S*Z5)	000263
	Z6=1.+2.*Z1S/R12S+Z5 \$ Z8=1.+2.*Z2S/R12S+Z5	000264
	Z11=PS(J)#COEF2#Z6 \$ Z12=PS(J)*COEF2#Z8	000265
	BMX(I)=BMX(I)-DPLATE*(Z11+PR*Z12)	000266
	<pre>BHY(I)=BMY(I)-DPLATE*(Z12+PR*Z11)</pre>	000267
15	CONTINUE	000268
16	CONTINUE	000269
	WRITE(6,600)(I,XF(I),YF(I),W(I),BMX(I),BMY(I),I=1,15)	000270
	END	000271

40,100,181,0.3,30.E6,80.,0.4 OUC	273
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,10*-5., 000	274
-4.5,-3.5,-2.5,-1.5,-0.5,0.5,1.5,2.5,3.5,4.5,10*5. 000	275
10*-5.,-4.5,-3.5,-2.5,-1.5,-0.5,0.5,1.5,2.5,3.5,4.5, 000	276
10*5.,4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5 000	277
40*2 000	278
10+0.,10+-1.,10+0.,10+1. 000	279
10*-1.,10*0.,10*1.,10*0. 000	1280
4.5.3.5.2.5.1.5.0.50.51.52.53.54.5. 000	261
4.5.3.5.2.5.1.5.0.50.51.52.53.54.5. 000	282
4.5.3.5.2.5.1.5.0.50.51.52.53.54.5. 000	1283
4.5.3.5.2.5.1.5.0.50.51.52.53.54.5.	284
4.5.3.5.2.5.1.5.0.50.51.52.53.54.5.	285
4.5.3.5.2.5.1.5.0.50.51.52.53.54.5. 000	1286
4.5.3.5.2.5.1.5.0.50.51.52.53.54.5.	287
4.5.3.5.2.5.1.5.0.50.51.52.53.54.5.	288
A 5.3 5.2 5.1 5.0 50 51 52 53 54.5. 000	289
4.5.3.5.2.5.1.8.0.50.51.52.53.54.5.	290
104-4 5.104-3 5.104-2 5.104-1 5.104-0.5.	291
	292

DERIVATIVES OF THE GREEN'S FUNCTION OF AN INFINITE ORTHOTROPIC PLATE

APPENDIX D

APPENDIX D

`

DERIVATIVES OF THE GREEN'S FUNCTION OF AN INFINITE ORTHOTROPIC PLATE

For
$$c > 1.0$$
:

$$\frac{\partial G}{\partial \mathbf{x}} = \frac{1}{4\pi D_0 (\beta^2 - \lambda^2)} \left\{ (\mathbf{x} - \xi) \left[\beta \ln \frac{(\mathbf{x} - \xi)^2 + \lambda^2 (\mathbf{y} - r_1)^2}{\mathbf{a}^2} - \lambda \ln \frac{(\mathbf{x} - \xi)^2 + \beta^2 (\mathbf{y} - r_1)^2}{\mathbf{a}^2} \right] \right\}$$

+
$$\frac{\beta \{(x-\xi)^2 + \lambda^2 (y-\eta)^2\}}{(x-\xi)^2 + \lambda^2 (y-\eta)^2} - \frac{\lambda \{(x-\xi)^2 + \beta^2 (y-\eta)^2\}}{(x-\xi)^2 + \beta^2 (y-\eta)^2}] + 2\beta \lambda (y-\eta) [arc tg \frac{\beta (y-\eta)}{x-\xi}]$$

- arc tg
$$\frac{\lambda(y-n)}{x-\xi}$$
] - 3(x- ξ) (β - λ) }

$$\frac{\partial G}{\partial y} = \frac{\varepsilon^2}{4\pi D_0 (\beta^2 - \lambda^2)} \left\{ (y-\eta) \left[\beta \ln \frac{(x-\xi)^2 + \beta^2 (y-\eta)^2}{a^2} - \lambda \ln \frac{(x-\xi)^2 + \lambda^2 (y-\eta)^2}{a^2} \right] \right\}$$

$$-\frac{\lambda \{(x-\xi)^{2}+\lambda^{2} (y-\eta)^{2}\}}{(x-\xi)^{2}+\lambda^{2} (y-\eta)^{2}}+\frac{\beta \{(x-\xi)^{2}+\beta^{2} (y-\eta)^{2}\}}{(x-\xi)^{2}+\beta^{2} (y-\eta)}]+2(x-\xi) [arc tg\frac{\beta (y-\eta)}{x-\xi}]$$

- arc tg
$$\frac{\lambda(y-n)}{x-\xi}$$
]- 3(y-n) (β - λ) }

$$\frac{\partial^{3}G}{\partial x^{3}} = \frac{x-\xi}{2\pi D_{0}(\beta^{2}-\lambda^{2})} \left[\frac{\beta}{(x-\xi)^{2}+\lambda^{2}(y-\eta)^{2}} - \frac{\lambda}{(x-\xi)^{2}+\beta^{2}(y-\eta)^{2}}\right]$$

$$\frac{\partial^{3}G}{\partial y^{3}} = \frac{-\epsilon^{2}(y-n)}{2\pi D_{O}(\beta^{2}-\lambda^{2})} \left[\frac{\lambda^{3}}{(x-\xi)^{2}+\lambda^{2}(y-n)^{2}} - \frac{\beta^{3}}{(x-\xi)^{2}+\beta^{2}(y-n)^{2}}\right]$$

$$\frac{\partial^{3}G}{\partial x^{2} \partial y} = \frac{-\epsilon^{2} (y-\eta)}{2\pi D_{0} (\beta^{2}-\lambda^{2})} \left[\frac{\beta}{(x-\xi)^{2}+\beta^{2} (y-\eta)^{2}} - \frac{\lambda}{(x-\xi)^{2}+\lambda^{2} (y-\eta)^{2}} \right]$$

$$\frac{\partial^{3}G}{\partial x \partial y^{2}} = \frac{-\epsilon^{2} (x-\xi)}{2\pi D_{0} (\beta^{2}-\lambda^{2})} \left[\frac{\lambda}{(x-\xi)^{2}+\lambda^{2} (y-\eta)^{2}} - \frac{\beta}{(x-\eta)^{2}+\beta^{2} (y-\eta)} \right]$$

For
$$\rho < 1.0$$
:

$$\frac{\partial G}{\partial \mathbf{x}} = \frac{1}{16\pi D_0} \left[\frac{\mathbf{x} - \xi}{\mu_1} \left[\ln \frac{(\mathbf{x} - \xi)^4 + 2\rho\epsilon^2 (\mathbf{x} - \xi)^2 (\mathbf{y} - \eta)^2 + \epsilon^4 (\mathbf{y} - \eta)^4}{\mathbf{a}^4} - 6 \right] \right] \\ + \frac{2\left[(\mathbf{x} - \xi)^2 + \epsilon^2 (\mathbf{y} - \eta)^2 \right] \left[(\mathbf{x} - \xi)^3 + \rho\epsilon^2 (\mathbf{x} - \xi) (\mathbf{y} - \eta)^2 \right]}{\mu_1} - \frac{2(\mathbf{x} - \xi)}{\mu_2} \operatorname{arc} \operatorname{tg} \frac{2\mu_1 \mu_2 (\mathbf{y} - \eta)^2}{(\mathbf{x} - \xi)^2 + \rho\epsilon^2 (\mathbf{y} - \eta)^2 + \epsilon^4 (\mathbf{y} - \eta)^4} - \frac{2(\mathbf{x} - \xi)}{\mu_2} \operatorname{arc} \operatorname{tg} \frac{2\mu_1 \mu_2 (\mathbf{y} - \eta)^2}{(\mathbf{x} - \xi)^2 + \rho\epsilon^2 (\mathbf{y} - \eta)^2 + \epsilon^4 (\mathbf{y} - \eta)^4} - \frac{2(\mathbf{x} - \xi)}{\mu_2} \operatorname{arc} \operatorname{tg} \frac{2\mu_1 \mu_2 (\mathbf{y} - \eta)^2}{(\mathbf{x} - \xi)^2 + \rho\epsilon^2 (\mathbf{y} - \eta)^2} \right]$$

+
$$\frac{(\mathbf{x}-\xi)^2 - \varepsilon^2(\mathbf{y}-\eta)^2}{\mu_2} \cdot \frac{4\mu_1\mu_2(\mathbf{x}-\xi)(\mathbf{y}-\eta)^2}{[(\mathbf{x}-\xi)^2 + \rho\varepsilon^2(\mathbf{y}-\eta)^2]^2 + [2\mu_1\mu_2(\mathbf{y}-\eta)^2]^2}$$

$$-\frac{\varepsilon^{2}(y-\eta)}{\mu_{1}\mu_{2}} \ln \frac{\mu_{1}^{2}(y-\eta)^{2}+[(x-\xi)-\mu_{2}(y-\eta)]^{2}}{\mu_{1}^{2}(y-\eta)^{2}+[(x-\xi)+\mu_{2}(y-\eta)]^{2}} -$$

$$-\frac{\varepsilon^{2}(\mathbf{x}-\xi)(\mathbf{y}-\eta)}{\mu_{1}\mu_{2}}\left[\frac{2\{(\mathbf{x}-\xi)-\mu_{2}(\mathbf{y}-\eta)\}}{\mu_{1}^{2}(\mathbf{y}-\eta)^{2}+\{(\mathbf{x}-\xi)-\mu_{2}(\mathbf{y}-\eta)\}^{2}}-\frac{2\{(\mathbf{x}-\xi)+\mu_{2}(\mathbf{y}-\eta)\}}{\mu_{1}^{2}(\mathbf{y}-\eta)^{2}+\{(\mathbf{x}-\xi)+\mu_{2}(\mathbf{y}-\eta)\}^{2}}\right]$$

$$\frac{\partial G}{\partial y} = \frac{1}{16\pi D_0} \left\{ \frac{\epsilon^2 (y-n)}{\mu_1} \left[\ln \frac{(x-\xi)^4 + 2\rho\epsilon^2 (x-\xi)^2 (y-n)^2 + \epsilon^4 (y-n)^4}{a^4} - 6 \right] \right\}$$

+
$$\frac{2\{(x-\xi)^{2}+\epsilon^{2}(y-\eta)^{2}\}\{\rho\epsilon^{2}(x-\xi)^{2}(y-\eta)+\epsilon^{4}(y-\eta)^{3}\}}{\mu_{1}\{(x-\xi)^{4}+2\rho\epsilon^{2}(x-\xi)^{2}(y-\eta)^{2}+\epsilon^{4}(y-\eta)^{4}\}} + \frac{2\epsilon^{2}(y-\eta)}{\mu_{2}}$$

•arc tg
$$\frac{2\mu_{1}\mu_{2}(y-\eta)^{2}}{(x-\xi)^{2}+\rho\epsilon^{2}(y-\eta)^{2}} - \frac{(x-\xi)^{2}-\epsilon^{2}(y-\eta)^{2}}{\mu_{2}} \cdot \frac{4\mu_{1}\mu_{2}(y-\eta)(x-\xi)^{2}}{[(x-\xi)^{2}+\rho\epsilon^{2}(y-\eta)^{2}]^{2}+[2\mu_{1}\mu_{2}(y-\eta)^{2}]^{2}}$$

$$-\frac{\varepsilon^{2}(\mathbf{x}-\xi)}{\mu_{1}\mu_{2}}\ell_{n}\frac{\mu_{1}^{2}(\mathbf{y}-\eta)^{2}+[(\mathbf{x}-\xi)-\mu_{2}(\mathbf{y}-\eta)]^{2}}{\mu_{1}^{2}(\mathbf{y}-\eta)^{2}+[(\mathbf{x}-\xi)+\mu_{2}(\mathbf{y}-\eta)]^{2}}\frac{\varepsilon^{2}(\mathbf{x}-\xi)(\mathbf{y}-\eta)}{\mu_{1}\mu_{2}}$$

$$\cdot \left[\frac{2\mu_{1}^{2}(y-\eta) - 2\mu_{1}\{(x-\xi) - \mu_{2}(y-\eta)\}}{\mu_{1}^{2}(y-\eta)^{2} + \{(x-\xi) - \mu_{2}(y-\eta)\}^{2}} - \frac{2\mu_{1}^{2}(y-\eta) + 2\mu_{1}\{(x-\xi) + \mu_{2}(y-\eta)\}}{\mu_{1}^{2}(y-\eta)^{2} + \{(x-\xi) + \mu_{2}(y-\eta)\}^{2}} \right] \right\}$$

$$\frac{\partial^{3}G}{\partial x^{3}} = \frac{1}{4\pi D_{0}} \left\{ \frac{1}{\mu_{1}} \frac{(x-\xi)^{3} + \rho \varepsilon^{2} (x-\xi) (y-\eta)^{2}}{(x-\xi)^{4} + 2\rho \varepsilon^{2} (x-\xi)^{2} (y-\eta)^{2} + \varepsilon^{4} (y-\eta)^{4}} \right\}$$

+
$$\frac{2\mu_{1}(x-\xi)(y-\eta)^{2}}{[(x-\xi)^{2}+\rho\epsilon^{2}(y-\eta)^{2}]^{2}+[2\mu_{1}\mu_{2}(y-\eta)^{2}]^{2}} \}$$

$$\frac{\partial^{3}G}{\partial y^{3}} = \frac{1}{4\pi D_{0}} \left\{ \frac{1}{\mu_{1}} \frac{\rho \epsilon^{2} (x-\xi)^{2} (y-\eta) + \epsilon^{4} (y-\eta)^{3}}{(x-\xi)^{4} + 2\rho \epsilon^{2} (x-\xi)^{2} (y-\eta)^{2} + \epsilon^{4} (y-\eta)^{4}} + \frac{1}{2} \left(\frac{1}{\mu_{1}} \frac{\rho \epsilon^{2} (x-\xi)^{2} (y-\eta) + \epsilon^{4} (y-\eta)^{3}}{(x-\xi)^{4} + 2\rho \epsilon^{2} (x-\xi)^{2} (y-\eta)^{2} + \epsilon^{4} (y-\eta)^{4}} \right\}$$

+
$$\frac{2\mu_{1}(y-\eta)(x-\xi)^{2}}{[(x-\xi)^{2}+\rho\epsilon^{2}(y-\eta)^{2}]^{2}+[2\mu_{1}\mu_{2}(y-\eta)^{2}]^{2}}$$
}

$$\frac{\partial^{3}G}{\partial x^{2} \partial y} = \frac{-\epsilon^{2}}{8\pi D_{0}\mu_{1}\mu_{2}} \{ \frac{(x-\xi)-\mu_{2}(y-\eta)}{\mu_{1}^{2}(y-\eta)^{2}+[(x-\xi)-\mu_{2}(y-\eta)]^{2}}$$

$$(x-\xi)+\mu_{2}(y-\eta)$$

$$-\frac{(x-\xi)+\mu_{2}(y-\eta)}{\mu_{1}^{2}(y-\eta)^{2}+[(x-\xi)+\mu_{2}(y-\eta)]^{2}}$$

$$\frac{\partial^{3}G}{\partial x \partial y^{2}} = \frac{-\epsilon^{2}}{8\pi D_{0}\mu_{1}\mu_{2}} \{ \frac{\epsilon^{2}(y-\eta) - \mu_{2}(x-\xi)}{\mu_{1}^{2}(y-\eta)^{2} + [(x-\xi) - \mu_{2}(y-\eta)]} - \frac{\epsilon^{2}(y-\eta) + \mu_{2}(x-\xi)}{\mu_{2}^{2}(y-\eta) + \mu_{2}(x-\xi)} \}$$

$$\frac{\epsilon^{2} (y-\eta)^{2} + [(x-\xi) - \mu_{2} (y-\eta)]}{\frac{\epsilon^{2} (y-\eta) + \mu_{2} (x-\xi)}{\mu_{1}^{2} (y-\eta)^{2} + [(x-\xi) + \mu_{2} (y-\eta)]}}$$

For $\rho = 1.0$:

.

$$\frac{\partial G}{\partial \mathbf{x}} = \frac{(\mathbf{x}-\xi)}{8\pi\varepsilon D_{o}} \{ l_{n} \frac{(\mathbf{x}-\xi)^{2}+\varepsilon^{2}(\mathbf{y}-\eta)^{2}}{\mathbf{a}^{2}} - 2 \}$$

$$\frac{\partial G}{\partial y} = \frac{\varepsilon (y-\eta)}{8\pi D_0} \ln \frac{(x-\xi)^2 + \varepsilon^2 (y-\eta)^2}{a^2}$$

$$\frac{\partial^{3}G}{\partial x^{3}} = \frac{1}{4\pi\varepsilon D_{o}} \frac{(x-\xi)^{3}-\varepsilon^{2}(y-\eta)(x-\xi)}{\{(x-\xi)^{2}+\varepsilon^{2}(y-\eta)\}^{2}}$$

$$\frac{\partial^{3}G}{\partial y^{3}} = \frac{\varepsilon (y-\eta)}{4\pi D_{0}} \frac{3 (x-\xi)^{2} + \varepsilon^{2} (y-\eta)^{2}}{(x-\xi)^{2} + \varepsilon^{2} (y-\eta)^{2}}$$

$$\frac{\partial^{3}G}{\partial x^{2} \partial y} = \frac{1}{4\pi D} \frac{\left\{ \frac{\varepsilon^{2} (y-n)^{2} - (x-\xi)^{2} \right\} \varepsilon (y-n)^{2}}{\left\{ (x-\xi)^{2} + \varepsilon^{2} (y-n)^{2} \right\}^{2}}$$

$$\frac{\partial^{3}G}{\partial x \partial y^{2}} = \frac{1}{4\pi D} \frac{(x-\xi)^{2}-\varepsilon^{2}(y-\eta)^{2}\varepsilon(x-\xi)}{\{(x-\xi)^{2}+\varepsilon^{2}(y-\eta)^{2}\}^{2}}$$

s

APPENDIX E

COMPUTER PROGRAM FOR AN ORTHOTROPIC PROBLEM

APPENDIX E

COMPUTER PROGRAM FOR AN ORTHOTROPIC PROBLEM

	PROGRAM ORTPLCL(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)	000001
С		000002
С	***************************************	000003
Ċ		000004
ř	POINT FORCE METHOD FOR OPTHOTROPIC PLATE BENDING PROBLEMS.	000005
ř	ADRITIDADY DIAN FORM. TDANSVEDSE DAR. AND BOUNDADY CONTITIONS AND	0000005
ř	AREITRART FEAT FORT, TRANSVERSE LOAD, AND BOOMDART CONDITIONS ***	000000
	CUOID NERE TE AN EVANDLE FOR A ETADLY SUDDODTED COULDE DLATE	000007
L	Shuwn here is an example for a simply supported square plate.	000008
C		0000009
С	REQUIRED INPUT VALUES	000010
С		000011
С	NBP =NUMBER OF BOUTDARY POINTS	000012
С	NIP =NUMBER OF INTERMAL LOAD POINTS	000013
С	NFP _=NUMBER OF FIELD POINTS	000014
С	XB,YB =POINTS ON B AT WHICH B.C. ARE SATISFIED.	000015
Ē	BANK BANY =COMPONENTS OF UNIT NORMAL TO B AT XB. YB.	000016
ř	YYR, YYR - END DOINTS OF MESHES ADDING B LINEDE FICTITIOUS	000017
ř		000017
	FURLES ARE ASSIGNED.	000010
Ľ		000019
C	XI, TI = INTERNAL LOAD PCINTS	000020
С	VX = POISSON'S RATIO IN X DIRECTION,	000021
С	DUE TO STRESS IN Y DIRECTION	000022
С	EX,EY =YOUNG*S MODULI IN X AND Y DIRECTIONS,	000023
С	RESPECTIVELY	000024
С	HVALUE =PLATE THICKNESS	000025
С	RADIUS = RADIUS OF THE FICTITIOUS CIRCULAR PLATE OF WHICH	000026
Ċ	THE DISPLACEMENT AT THE CIRCUMFERENTIAL BOUNDARY	000027
č	IS SET TO ZERO	00002A
ř		000029
ř		000027
ž		0000030
L	RTHEFT UD/AA VB/AA VVB/AL VVB/AL LETVDF/AL	000031
	DIRENSION XE(40), TB(40), XB(CI), TB(61), ND(TPE(61)	000032
	DIMENSION X1(100), T1(100), DEL(2)	000033
	DIMENSION RLX(80),RB(60),RM(80,80),PS(80),WKAREA(80),RL(80)	000034
	DIMENSION XF(101),YF(101),W(101),BHX(101),BHY(101)	000035
	DIMENSION BANX(40),BANY(40),RMX(80,80)	000035
	REAL LUMDA, LUMDA2, MU1, MU2	000037
С		000038
	100 FORMAT(#OLOCT#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X,#NBTYPE#,	000039
	+/#0#/(15.4F10.4.15))	000040
	110 FORMAT(#0LCCT#.11X.#XXB#.14X.#YYB#/#0#/(14.8X.F9.2.8X.F9.2))	000041
	200 FORMAT (#010CT#.11X.#XT#.17X.#YT#.14X./#0#/	000042
		000042
	AAA EADMAT (ANIA-TA INA ADEDA/ANA//TA AY EAA ANI	800043
	$\begin{array}{c} \textbf{400} \textbf{Furthall} \textbf{(plucle)} $	000044
	DUE FURTIALLEULE, $11X$, $PXPP$, $1/A$, $PTPPPVV14$, $11X$, $P0$, 3 , $11X$, $P0$, 3 , $11X$, $P0$, 3 , 1 , 1 , 1 , 1 , 1 , 1 , 1 , 1	000045
	600 FORMATLINH, 8X, #NCDE#, 12X#XF#, 16X, #TF#, 19X, #W#, 16X, #BTX#,	000045
	116X,#BHY#/(1HD,110,2F20.10,3E20.12))	000047
	700 FORMAT(1H1,#INFUT VALUES#,//1X,#NML = #,I3,# NIP =#,I3,	000048
	1# NFP = #,13,# PR = #,F5.3,# DPSI = #,F5.3,# DETA = #,F5.3,	000049
	+/1H0,# EX = #,E10.4,# EY = #,E10.4,# VX = #,F5.2,# GXY = #,E10.4	000050
	+/1H0,# RADIUS OF THE PLATE = #,	000051
	+F7.1,# THICKNESS OF THE PLATE =#,F6.3,# DIST= #,F5.1)	000052
1	1103 FORMAT(15,5%,E20.12,5%,E20.12)	000053
	1014 FORMAT(1H1. THE FOLLOWING IS A LIST OF DOUBLE CHECKING OF B.C. ST.	000054
	1//1X. #NOTE IF ALL THE VALUES LISTED BELOW ADE IN THE OPDED OF	000055
	1 F-9 MP IFSS. BESHITS HTIL RE TH COOL SHADE I.	000054
	6 6-7 UN EEJDI REJULIJ MALE DE AN OUUD JNAFEIMI 1//.Avaniainy.ar p 14.137.ar p 34.//i	000055
	AFFTTATT ILTIVATEDIGI AFTAJATEDIGI GETTET 781 KODMATTINI JOV - J EIK & J OV - J KIK & J M - J KIK & J A - J	
	J = I = I = I = I = I = I = I = I = I =	0000000
	TELD.0)F VI F FIED.2)F KAU F FIED.2]	000059

702 FORMAT(1H0, #EPSLON = #, E15.7, # BETA = #, E15.7, 000060 +# LUMDA = #,E15.7) 000061 С 000062 C INPUT VALUES 000063 С 000044 READ(5,*)NHL,NIP,NFP,PR,DPSI,DETA,EX,EY,VX,GXY,PADIUS,HVALUE,DIS 000C65 WRITE(6,700 INHL, NIP, NFP, PR, DPSI, DETA, EX, EY, VX, GXY, 000056 +RADIUS, HVALUE, DIST 000057 READ(5,*)(XB(I),I=1,NML) 000063 READ(5, #)(YB(I), I=1, NHL) 000069 READ(5, #)(NBTYFE(I), I=1, NML) 000070 READ(5,#)(BANX(I),I=1,NML) 000071 READ(5.#)(BANY(I),I=1,NML) 000072 READ(5,*)(XI(I),I=1,NIP) 000073 READ(5,*)(YI(I),I=1,NIP) 000074 WRITE(6,100)(I,XB(I),YB(I),BANX(I),BANY(I),NBTYPE(I) 000075 1,I=1,NML) 000076 С 000077 С 000078 X0=5.0\$Y0=5.0 000079 XF(1)=10.-X0\$YF(1)=-10.+Y0 000030 DO 41 I=2,11 000081 XF(I)=XF(I-1)-0.5 290000 YF(I)=YF(I-1) 41 000083 DO 42 J=1,10 000094 K=J+17 000085 DO 42 I=1,11 000085 XF(I+K)=XF(I) 000087 YF(I+K)=YF(I+K-17)+1.0 42 000055 XF(12)=10.-X0\$YF(12)=-9.5+Y0 000089 DO 43 I=13.17 000090 XF(I)=XF(I-1)-1.0 000091 43 **YF(I)=YF(I-1)** 000092 DO 44 J=1,9 000093 K=J#17 000094 000095 DO 44 I=12,17 XF(I+K)=XF(I)000096 YF(I+K)=YF(I+K-17)+1.0 44 000097 NML2=NML*2\$NML2P1=NML2+1 000095 NMLP1=HML+1 000099 NOPTION=1 000100 DIST1=4.0 \$ DIST2=2.0 000101 DEL(1)=(10.+2.*DIST1)/10. 000102 DEL(2)=(10.+2.*DIST1+2.*DIST21/10. 000103 XXB(1)=5.+DIST1-DEL(1)\$YYB(1)=-5.-DIST1 000104 XXB(41)=5.+DIST1+DIST2-DEL(2)\$YYB(41)=-5.-DIST1-DIST2 000105 DO 28 J=1,2 000105 DELT=DEL(1)\$IF(J.EQ.2)DELT=DEL(2) 000107 DO 25 I=2,10 000108 K=I 000109 IF(J.EQ.2)K=K+40 000110 XXB(K)=XXB(K-1)-DELT 000111 YYB(K)=YYB(K-1) 000112 25 DO 26 I=11,20 000113 000114 K=T IF(J.EQ.2)K=K+40 000115 000116 XXB(K)=XXB(K-1)26 YYB(K)=YYB(K-1)+DELT 000117 000118 DO 27 I=21,30 K=I 000119

		IF(J.EQ.2)K=K+40	000120
		XXB(K)=XXB(K-1)+DELT	000121
27		YYB(K)=YYB(K-1)	000122
		DO 28 I=31,40	000123
		K=I	000124
		IF(J.EQ.2)K=K+40	000125
		XXB(K)=XXB(K-1)	000126
28		YYB(K)=YYB(K-1)-DELT	000127
		XXB(81)=XXB(41) \$ YYB(81)=YYB(41)	000128
		PI=4.#ATAN(1.) \$ 9=1.0	000120
		VY=FY#VX/FX \$ DY=FX+HVALUF##3/(12 #(1 -VY+VY))	000129
		DY=FY+HVALUF+#3/(12.#(1VY+VY)) \$ DO=SOPT(DY+DY)	000133
			000131
•	٦A	CONTRACE	000133
•			000133
			000134
		WPITE(6.110)(T.YYE(T).YE(T).T=1.8P4(201)	000135
		HRTIE(0) II0 (I) (I) (I) (I) (I) (I) (I) (I) (I) (I)	000137
		URTIE(4 200)(1)/(1)/(1) 1/(1) 1-1/((P)	000137
		#KIIC(0)CUUI(1)AI(1))I(1))I-1(NIF) #4-DV/DV & E2+CODT(E4) & EDC(0)+CODT(E2)	000135
		E4-UAZUI 4 EE-SUFILEA) 4 EPOLUN-OURILEE) EACTOD-1 E_4 & DIOAD-OXDETAXDDCT & DO+DADTHEXXO	000139
		PAULUP-1.E-D & WEUAU-WAUELAAUPSI & RE-RAUIUSPAC	000140
		RETELC, /ULJUA, ULJUA, ULJUA, VLJKUU	000141
~		IT (MBU-1. U /2,1,3	000142
L	•	FUK KNU .EW. I.U WEWERERER	000143
	1	CUEFI=1./(16.#FIREPSLON#D0) # CUEF2=2.#CUEF1	000144
		NITFE=1	000145
-			000145
C			000147
	Z	COEF1=1.7(32.*PI*DD) \$ COEF2=2.*COEF1	000148
		MUI=EPSLON*SQRT((1.+RHO)/2.)	000149
		MUZ=EPSLON*SQRT((1RHO)/2.)	000150
		NTYPE=2	000151
_		GOTO 4	000152
C	_	FOR RHO .GT. 1.0 ********	000153
	3	EETA=EPSLCH+SGRT(RHO+SGRT(RHO++2-1.)) \$ BETA2=BETA++2	000154
		LUMDA=EPSLON#SQRT(RHO-SQRT(RHO**2-1.)) \$ LUMDA2=LUMDA**2	000155
		WRITE(6,702)EPSLON, BETA, LUMDA	000155
		COEF1=1./(8.*PI*DO*(BETA2-LUMDA2)) \$ COEF2=COEF1*2.	000157
		NTYFE=3	000155
	4	CONTINUE	000159
		DO 5 I=1,NML	000160
		RL(I)=0.0	000161
		RL(I+N1L)=0.0	000162
		RIX=XB(I)	000163
		R1Y=YB(I)	000164
		ANX=BANX(I)	000165
		ANY=BANY(I)	000166
		DO 6 J=1,NIP	000167
		R2X=XI(J)	000168
		R2Y=YI(J)	000169
		Z1=R1X-R2X \$ Z2=R1Y-R2Y \$ Z1S=Z1**2 \$ Z2S=Z2**2	000170
		ANXS=ANX##2 \$ ANYS=ANY##2	000171
		AA=DX#ANXS+DY#ANYS#VX	000172
		BB=DY#ANYS+DX#ANXS*VY	000173
		CC=2.#ANX#ANY#DK	000174
C			000175
		GOTO(17,18,19),NTYFE	000176
1	17	R125=Z15+Z25#E2 \$ Z5=ALOG(R125/R2)	000177
		C1=Z5-2.#E2#Z25/R125	000178
		C2=Z5+2.#E2#Z25/R125	000179

```
C3=EPSLCN+Z1+Z2/R12S
                                                                         000180
      RL(I)=RL(I)-QLOAD*(R125*Z5-(3.*Z15+E2*Z25))
                                                                         000181
      RL(I+NML)=RL(I+NML)-QLOAD*(AA*C1+BB*E2*C2+2.*EPSLON*CC*C3)*FACTOR 000182
      GOTO 6
                                                                         000183
   18 Z7=ALOG((Z1S**2+2.*RHO*E2*Z1S*Z2S+E4*Z2S**2)/R2**2)
                                                                         000184
      Z8=ATAN(2.+MU1+MU2+Z25/(Z15+RH0+E2+Z25))
                                                                         000185
      IF(28.LT.0.)28=28+PI
                                                                         000185
      Z9=ALOG(((MU1+Z2)++2+(Z1-MU2+Z2)++2)/((MU1+Z2)++2+(Z1+MU2+Z2)++2))000187
      RL(I)=RL(I)-QLOAD*((Z1S+E2*Z2S)/MU1*Z7-2.*(Z1S-E2*Z2S)/MU2*Z8
                                                                         000185
     +-2.#E2#Z1#Z2/(MU1#MU2)#Z9-6.#(Z15+E2#Z25)/MU1)
                                                                         000189
      C1=Z7/HU1-2.#Z8/HU2 $ C2=Z7/HU1+2.#Z8/HU2 $ C3=-Z9/(HU1+HU2)
                                                                         000190
      RL(I+N'L)=RL(I+NML)-QLOAD*(AA*C1+BB*E2*C2+CC*E2*C3)*FACTOR
                                                                         000191
      GCTO 6
                                                                         000192
   19 ZLCG1=ALOG((Z1S+LUPDA2*Z2S)/R2) $ ZLOG2=ALOG((Z1S+BETA2*Z2S)/R2) 000193
      IF(APS(Z1).LE.1.E-6)GOTO 12
                                                                         000194
      Z7=ATAN(LUMDA+Z2/Z1) $ Z8=ATAN(BETA+Z2/Z1)
                                                                         000195
      GOTO 45
                                                                         000196
   12 Z7=PI/2. $ Z8=Z7
                                                                         000197
   45 CONTINUE
                                                                         000198
      IF(Z7.LT.0.)Z7=Z7+PI
                                                                         000199
      IF(Z8.LT.0.)Z8=Z8+PI
                                                                         000200
      C1=BETA#ZLOG1-LUHDA#ZLOG2 $ C2=-LUHDA#ZLOG1+BETA#ZLOG2
                                                                         000201
      C3=-77+78
                                                                         000202
      RL(I)=RL(I)-QLOAD+(BETA+(ZIS-LUMDA2+Z2S)+ZLOG1-4.+LUMDA+BETA+Z1+Z2000203
     +#(Z7-Z6)-LUMDA*(Z15-BETA2*Z25)#ZLOG2-3.#(BETA-LUMDA)
                                                                         000204
     +*(Z15+LUTDA*BETA*Z25))
                                                                         000205
     RL(I+NHL)=RL(I+NHL)-QLOAD*(AA*C1+BB*E2*C2+2.*E2*C3)*FACTOR
                                                                         000206
    6 CONTINUE
                                                                         000207
    5 CONTINUE
                                                                         000205
                                                                         000209
С
                                                                         000210
      DO 8 1=1,NML
                                                                         000211
      R1X=XB(I)
                                                                         000212
      R1Y=YB(I)
                                                                         000213
      ANX=BANX(I)
                                                                         000214
      ANY=BANY(I)
                                                                         000215
      DO 7 J=1,NML2
                                                                         000216
      R2X=(XXB(J+1)+XXB(J))/2.
                                                                         000217
      R21=(YYB(J+1)+YYB(J))/2.
                                                                         000218
      IF (NCPTICH. EQ. 2. OR. J. NE. NML)GOTO 33
                                                                         000219
      R2X=(XXB(1)+XXB(NML))/2.
                                                                         000220
      R2Y=(YYB(1)+YYE(NML))/2.
                                                                         000221
   33 CONTINUE
                                                                         000222
      Z1=R1X-R2X $ Z2=R1Y-R2Y $ Z15=Z1**2 $ Z25=Z2**2
                                                                         000223
      ANX5=ANX#42 $ ANYS=ANY##2
                                                                         000224
      AA=DX#ANXS+DY#ANYS#VX
                                                                         000225
      EB=DY#ANYS+DX#ANXS#VY
                                                                         000226
      CC=2.#ANX#ANY#DK
                                                                         000227
                                                                         000228
      GOTO (20,21,22),NTYPE
                                                                         000229
   20 R125=Z15+Z25*E2 $ Z5=ALOG(R125/R2)
                                                                         000230
      C1=Z5-2.#E2*Z25/R125
                                                                         000231
      C2=Z5+2.#E2*Z25/R125
                                                                         000232
      C3=EPSLON#Z1#Z2/R125
                                                                         000233
      RM(I,J)=R125+Z5-(3.+Z15+E2+Z25)
                                                                         000234
      RM(I+NML,J)=(AA*C1+BB*E2*C2+2.*EPSLON*CC*C3)*FACTOR
                                                                         000235
      GOTO 7
                                                                         000235
   21 Z7=ALOG((Z1S##2+2.#RHO#E2#Z1S#Z25+E4#Z25##2)/R2##2)
                                                                         000237
      Z8=ATAN(2.*MU1*MU2*Z25/(Z15+RHO*E2*Z25))
                                                                         000238
      IF(Z8.LT.0.)Z8=Z8+PI
                                                                         000239
```

С

C

```
Z9=ALOG(((MU1+Z2)++2+(Z1-MU2+Z2)++2)/((MU1+Z2)++2+(Z1+MU2+Z2)++2))000240
      RM(I, J)=((Z15+E2+Z25)/MU1+Z7-2.+(Z15-E2+Z25)/MU2+Z8
                                                                         000241
     +-2.#E2*Z1*Z2/(MU1*MU2)*Z9-6.*(Z15+E2*Z25)/HU1)
                                                                         000242
      C1=Z7/HU1-2.#Z8/HU2 $ C2=Z7/HU1+2.#Z8/HU2 $ C3=-Z9/(HU1+HU2)
                                                                         000243
      RM(I+NHL, J)=(AA*C1+BB*E2*C2+CC*E2*C3)*FACTOR
                                                                         000244
      GOTO 7
                                                                         000245
   22 ZLCG1=ALCG((ZIS+LUMDA2*Z2S)/P2) $ ZLCG2=ALCG((ZIS+BETA2*Z2S)/R2) 000245
      IF(ABS(Z1).LE.1.E-6)GOTO 23
                                                                         000247
      Z7=ATAN(LUMDA#Z2/Z1) $ Z8=ATAN(BETA#Z2/Z1)
                                                                         000245
      60T0 46
                                                                         000249
   23 Z7=PI/2. $ Z8=Z7
                                                                         000250
   46 CONTINUE
                                                                         000251
      IF(27.LT.0.)27=27+PI
                                                                         000252
      IF(Z8.LT.0.)Z9=Z8+PI
                                                                         000253
      C1=BETA*ZLC31-LUMDA*ZLCG2 $ C2=-LUMDA*ZLOG1+BETA*ZLOG2
                                                                         000254
      C3=-Z7+Z8
                                                                         000255
      RH(I,J)=(BETA*(Z1S-LUMDA2*Z2S)*ZLOG1-4.*LUMDA*BETA*Z1*Z2
                                                                         000255
     +*(Z7-Z8)-LUMDA*(Z1S-BETA2*Z2S)*ZLOG2-3.*(BETA-LUMDA)
                                                                         000257
     +#(Z1S+LUMDA+BETA+Z2S))
                                                                         000253
      RM(I+NML,J)=(AA*C1+BB*E2*C2+2.*E2*C3)*FACTOR
                                                                         000259
    7 CONTINUE
                                                                         000260
    8 CONTINUE
                                                                         000261
      DO 10 I=1,NML
                                                                         000262
      DO 10 J=1,NML2
                                                                         000263
      RHX(I,J)=RH(I,J)
                                                                         000264
   10 RMX(I+NML,J)=RH(I+NML,J)
                                                                         000265
      DO 1015 I=1,NML2
                                                                         000266
1015 RLX(I)=RL(I)
                                                                         000267
C
                                                                         000268
С
                                                                         000269
      CALL LEQTIF(FM, 1, NML2, NML2, RL, 0, WKAREA, IER)
                                                                         000270
      DO 29 I=1,101L2
                                                                         000271
   29 PS(I)=RL(I)
                                                                         000272
      WRITE (6,400) (I,PS(I),I=1,NML2)
                                                                         000273
      WRITE(6,1014)
                                                                         000274
      DO 1101 I=1,NML2
                                                                         000275
      SU11=0.
                                                                         000275
      DO 1102 J=1,NHL2
                                                                         000277
      SUM=SUM+RMX(I,J)*PS(J)
                                                                         000278
1102 CONTINUE
                                                                         000279
 1101 RB(I)=SUM-RLX(I)
                                                                         000280
      WRITE(6,1103)(I,RB(I),RB(I+NML),I=1,NML)
                                                                         000281
      DO 9 I=1,NFP
                                                                         293000
                                                                         000283
      W(I)=0.0
      BMX(I)=0.0
                                                                         000284
      BMY(I)=0.0
                                                                         000285
    9 CONTINUE
                                                                         000286
      DO 14 I=1,NFP
                                                                         000287
      R1X=XF(I)
                                                                         832000
      R1Y=YF(I)
                                                                         000289
      DO 13 J=1,NIP
                                                                         000290
      R2X=XI(J)
                                                                         000291
      R2Y=YI(J)
                                                                         000292
      Z1=R1X-R2X $ Z2=R1Y-R2Y $ Z1S=Z1**2 $ Z2S=Z2**2
                                                                         000293
C
                                                                         000294
      GOTO (30,31,32),NTYPE
                                                                         000295
   30 R125=Z15+Z25*E2 $ Z5=ALOG(R125/R2)
                                                                         000295
      Z6=2.#E2#Z25/R125
                                                                         000297
      W(I)=W(I)+QLOAD*(R125*Z5-(3.*Z15+E2*Z25))*COEF1
                                                                         000298
      BMX(I)=BMX(I)-QLOAD*COEF2*DX*((Z5-Z6)+VY*E2*(Z5+Z6))
                                                                         000299
```

		BMY(I)=BMY(I)-QLOAD*COEF2*DY* (E2*(Z5+Z6)+VX*(Z5-Z6))	000300
		6010 13	000301
	31	Z7=ALOG((Z1\$**2+2.*RHO*E2*Z1\$*Z2\$+E4*Z2\$**2)/R2**2)	000302
		Z8=ATAN(2.#MU1#MU2#Z2S/(Z15+RHO#E2#Z2S))	000303
		IF(Z8.LT.0.)Z8=Z8+PI	000304
		Z9=ALOG(((MU1+Z2)++2+(Z1-MU2+Z2)++2)/((MU1+Z2)++2+(Z1+MU2+Z2)++2)	000305
		W(I)=W(I)+QLOAD*((Z1\$+E2*Z2\$)/MU1*Z7-2.*(Z1\$-E2*Z2\$)/MU2*Z8	000306
		+-2.*E2*Z1*Z2/(HU1*HU2)*Z9-6.*(Z15+E2*Z25)/HU1)*COEF1	000307
		W2X=Z7/HU1-Z6+2./HU2 \$ W2Y=Z7/HU1+2.+Z6/HU2	000308
		BMX(I)=BMX(I)-DX#QLOAD#COEF2#(W2X+E2#VY#H2Y)	000309
		BMY(I)=BHY(I)=DY#BI0AD#COFF2*(H2Y#F2+VX#H2X)	000310
			000311
	32	2105124105((715411HDA2#725)/22) \$ 7105224105((71548FTA2#725)/22)	000312
	20		000313
		27-ATAU(1)#MAX70/711 & 78-ATAN(BETAX70/71)	000313
		Entra (1) Entra (2) (2) (2) (2) (2) (2) (2) (2) (2) (2)	000314
	94		000313
	64		000310
	47		000317
			000310
		17(26.11.0.)26=26+P1	000314
		W(I)=W(I)+QLOAD=(EETA=(ZIS-LUHDA2=Z2S)=ZLOGI-4.+LUHDAPEETA=ZI=Z2	000320
	•	+#(Z7-Z8)-LUHDA*(Z15-BETA2#Z25)#ZLOG2-3.#(BETA-LUHDA)	000321
		+#(ZIS+LUHDA#BETA#ZZS))#COEF1	000322
		W2X=BETA*ZLOG1-LUMDA*ZLCG2 \$ W2Y=BETA*ZLOG2-LUMDA*ZLOG1	000323
		BMX(I)=BMX(I)-DX#QLOAD#COEF2#(W2X+E2#VY#W2Y)	000324
		BMY(I)=BHY(I)-DY#QLOAD#COEF2#(W2Y#E2+VX#W2X)	000325
	13	CCHTINUE	000326
	14		000327
		DO 16 I=1,NFP	000328
		R1X=XF(I)	000329
		RlY=YF(I)	000330
		DO 15 J=1,NML2	000331
		R2X=(XXB(J+1)+XXB(J))/2.0	000332
		R2 1=(YYB(J+1)+YYB(J))/2.0	000333
		IF(NOPTICN.EQ.2.0R.J.NE.NML)GOTO 11	000334
		R2X=(XXB(1)+XXB(NML))/2.	000335
		R2Y=(YYB(1)+YYB(NHL))/2.	000335
	11	CONTINUE	000337
		Z1=R1X-R2X \$ Z2=R1Y-R2Y \$ Z1S=Z1##2 \$ Z2S=Z2##2	000336
С			000339
-		GOTO (34.35.36).NTYPE	000340
	34	D125=715+725#F2 & 75=A106(B125/B2)	000341
			000342
		U(T)=U(T)=U(T)=F(())=F(F)=(D)2==75-(3 = 2715=F2=725))	000343
		M(1)=M(1)++(0)=C(1)++((10-2)=(3,-(2)+(2)+(2)+(2)+(2)+(2)+(2)+(2)+(2)+(2)+	000343
		DIA(1) - DIA(1) - FS(3) = COE(1) + CO	000344
		611(1)=61((1)=F3(3)=C0Er2=01=(E2=(23=20)=(23=20))	000345
	75	0010 13 77-1107((718xx919) MBU0x59x718x795186x798xx91/D9xx91	000340
	33	2/-ALU3((213**242,*********************************	000347
		20-81AHL2,*INJIHU2*223/1213*RHU*22*223/)	000345
			000347
			0000350
		W(]]=W(]]+P3(]]*((213+E2*223)/TU]*(2/-2.*(213-E2*223)/TU2*20	000351
	•		000352
		WZX=Z//NU1-Z84Z./NU2 \$ WZT=Z//NU1+Z.#Z8/NU2	000353
		BTX(I)=BTX(I)=DX#PS(J)#COEF2#(W2X4E2#VT#W2Y)	000354
		BMY(I)=BMY(I)-DY*PS(J)*COEFZ*(WZY*EZ+VX*WZX)	000355
		60TO 15	000356
	36	ZLUG1=ALUG((Z15+LUMDA2+Z25)/R2) \$ ZLUG2=ALUG((Z15+BETA2+Z25)/R2)	000357
		IF(AB5(Z1).LE.1.E-6)GOTO 37	000358
		Z7=ATAN(LUMDA+22/Z1) \$ Z8=ATAN(BETA+Z2/Z1)	000359

	60T0 48	000360
17	77=D1/2 & 7A=77	000341
		000361
40		000362
	IF(27.L1.0.J27=27+P1	000363
	IF(28.LT.0.)28=26+PI	0 00364
	W(I)=W(I)+PS(J)*(BETA*(ZIS-LUMDA2*Z2S)*ZLOG1-4.*LUMDA*BETA*Z1*Z2	000365
	+#(Z7-Z8)-LUMDA+(Z1S-BETA2+Z2S)+ZLOG2-3.+(BETA-LUMDA)	000365
	+#(Z1S+LUTDA+BETA#Z2S))#COEF1	000367
	W2X=BETA+ZLOG1-LUMDA+ZLOG2 \$ W2Y=BETA+ZLOG2-LUMDA+ZLOG1	000368
	BMX(I)=BMX(I)-DX#PS(J)#COEF2#(W2X+E2#VY#W2Y)	000369
	BMY(I)=BMY(I)-DY*PS(J)*COEF2*(W2Y*E2+VX*W2X)	000370
15	CONTINUE	000371
16	CONTINUE	000372
	WRITE(6.600)(I.XF(I),YF(I),W(I),BMX(I),BMY(I),I=1,NFP)	000373
	GOID(53,53,51,52,53), NOUNT	000374
40		000375
47		000375
	6010 38	000376
50	RHO=2.0 \$ NCOUNT=3	000377
	GOTO 38	000378
51	RHD=0.1 \$ NCOUNT=4	000379
	6010 38	000380
52	RHO=10. \$ NCOUNT=5	000381
	6010 38	000352
67		000393
33		000353
		000384

40,100,181,0.3,1.0,1.0,30.86,2.086,0.3,7.586,80.,0.4,4.0	000386
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,10*-5.,	000387
-4.5,-3.5,-2.5,-1.5,-0.5,0.5,1.5,2.5,3.5,4.5,10*5.	000388
10*-5.,-4.5,-3.5,-2.5,-1.5,-0.5,0.5,1.5,2.5,3.5,4.5,	000389
10*5.,4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5	000390
40*2	000391
10*0.,10*-1.,10*1.,10*1.	000392
10*-1.,10*0.,10*1.,10*0.	000393
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000394
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000395
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000395
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000397
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000398
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000399
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000400
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000401
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000402
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000403
10*-4.5,10*-3.5,10*-2.5,10*-1.5,10*-0.5,	000404
10*0.5,10*1.5,10*2.5,10*3.5,10*4.5	000405

APPENDIX F

DERIVATIVES OF THE GREEN'S FUNCTION OF AN INFINITE ANISOTROPIC PLATE

APPENDIX F

DERIVATIVES OF THE GREEN'S FUNCTION OF AN INFINITE ANISOTROPIC PLATE

For simplicity, the derivatives are written in terms of the four constants ϕ_1 , ϕ_2 , ϕ_3 , and ϕ_4 , and the eight functions L_1 , L_3 , R_1 , R_3 , N_1 , N_3 , S_1 , and S_3 as shown in Eq. (38).

$$\frac{\partial G}{\partial \mathbf{x}} = \frac{1}{8\pi D_{2\,2} \phi_1 \phi_2} \left[\phi_3 \frac{\partial R_1}{\partial \mathbf{x}} + \phi_4 \frac{\partial R_3}{\partial \mathbf{x}} + 4 \left(\alpha - \gamma \right) \left(\frac{\partial S_1}{\partial \mathbf{x}} - \frac{\partial S_3}{\partial \mathbf{x}} \right) \right]$$

$$\frac{\partial G}{\partial y} = \frac{1}{8\pi D_{22} \phi_1 \phi_2} \left[\phi_3 \frac{\partial R_1}{\partial y} + \phi_4 \frac{\partial R_3}{\partial y} + 4 (\alpha - \gamma) \left(\frac{\partial S_1}{\partial y} - \frac{\partial S_3}{\partial y} \right) \right]$$

$$\frac{\partial^{3}G}{\partial x^{3}} = \frac{1}{4\pi D_{22}\phi_{1}\phi_{2}} \left[\phi_{3}\frac{\partial L_{1}}{\partial x} + \phi_{4}\frac{\partial L_{3}}{\partial x} + 4(\alpha - \gamma)\left(\frac{\partial N_{1}}{\partial x} - \frac{\partial N_{3}}{\partial x}\right)\right]$$

$$\frac{\partial^{3}G}{\partial y^{3}} = \frac{1}{4\pi D_{22}\phi_{1}\phi_{2}} \left\{ \frac{(\alpha^{2}+\beta^{2}-2\alpha\gamma)(\alpha^{2}+\beta^{2})+(\alpha^{2}-\beta^{2})(\gamma^{2}+\lambda^{2})}{\beta} \frac{\partial L_{1}}{\partial y} \right\}$$

+
$$\frac{(\gamma^2 + \lambda^2 - 2\alpha\gamma)(\gamma^2 + \lambda^2) + (\gamma^2 - \lambda^2)(\alpha^2 + \beta^2)}{\lambda} \frac{\partial L_3}{\partial y}$$

-4[
$$\alpha(\gamma^2+\lambda^2)-\gamma(\alpha^2+\beta^2)$$
]($\frac{\partial N_1}{\partial y} - \frac{\partial N_3}{\partial y}$)}

$$\frac{\partial^{3}G}{\partial x^{2} \partial y} = \frac{1}{4\pi D_{22} \phi_{1} \phi_{2}} \left[\phi_{3} \frac{\partial L_{1}}{\partial y} + \phi_{4} \frac{\partial L_{3}}{\partial y} + 4(\alpha - \gamma) \left(\frac{\partial N_{1}}{\partial y} - \frac{\partial N_{3}}{\partial y} \right) \right]$$

$$\frac{\partial^{3}G}{\partial x \partial y^{2}} = \frac{1}{4\pi D_{22} c_{1} \phi_{2}} \{ \frac{(\alpha^{2} + \beta^{2} - 2\alpha\gamma) (\alpha^{2} + \beta^{2}) + (\alpha^{2} - \beta^{2}) (\gamma^{2} + \lambda^{2})}{\beta} \frac{\partial L_{1}}{\partial x} \}$$

+
$$\frac{(\gamma^2 + \lambda^2 - 2\alpha\gamma)(\gamma^2 + \lambda^2) + (\gamma^2 - \lambda^2)(\alpha^2 + \beta^2)}{\lambda} \frac{\partial L_3}{\partial x}$$

- 4[
$$\alpha(\gamma^2 + \lambda^2) - \gamma(\alpha^2 + \beta^2)$$
] ($\frac{\partial N_1}{\partial x} - \frac{\partial N_3}{\partial x}$) }

where,

$$\phi_{1} = (\alpha - \gamma)^{2} + (\beta - \lambda)^{2} ; \phi_{2} = (\alpha - \gamma)^{2} + (\beta + \lambda)^{2}$$

$$\phi_{3} = \frac{(\alpha - \gamma)^{2} - (\beta^{2} - \lambda^{2})}{\beta}; \phi_{4} = \frac{(\alpha - \gamma)^{2} + (\beta^{2} - \lambda^{2})}{\lambda}$$

$$L_{1} = \ln \frac{\left[(x - \xi) + \alpha (y - \eta) \right]^{2} + \beta^{2} (y - \eta)^{2}}{a^{2}}$$

$$L_{3} = \ln \frac{\left[(x - \xi) + \gamma (y - \eta) \right]^{2} + \lambda^{2} (y - \eta)^{2}}{a^{2}}$$

$$R_{1} = \left\{ \left[(x - \xi) + \alpha (y - \eta) \right]^{2} - \beta^{2} (y - \eta)^{2} \right\} \cdot (L_{1} - 3)$$

$$\mathbf{R}_{3} = \{ [(\mathbf{x}-\xi)+\gamma(\mathbf{y}-\eta)]^{2} - \lambda^{2}(\mathbf{y}-\eta)^{2} \} \cdot (\mathbf{L}_{3}-3)$$

$$N_1 = \operatorname{arc} tg \frac{\beta(y-\eta)}{(x-\xi)+\alpha(y-\eta)}$$
; $N_3 = \operatorname{arc} tg \frac{\lambda(y-\eta)}{(x-\xi)+\gamma(y-\eta)}$

$$S_{1} = \beta(y-n) [(x-\xi)+\alpha(y-n)] (L_{1}-3) + \{ [(x-\xi)+\alpha(y-n)]^{2}-\beta^{2}(y-n)^{2} \} N_{1}$$

$$S_{3} = \lambda (y-\eta) [(x-\xi)+\gamma (y-\eta)] (L_{3}-3) + \{ [(x-\xi)+\gamma (y-\eta)]^{2} - \lambda^{2} (y-\eta)^{2} \} N_{3}$$

$$\frac{\partial L_1}{\partial \mathbf{x}} = \frac{2\{(\mathbf{x}-\xi)+\alpha(\mathbf{y}-\eta)\}}{\{(\mathbf{x}-\xi)+\alpha(\mathbf{y}-\eta)\}^2+\beta^2(\mathbf{y}-\eta)^2}$$

$$\frac{\partial L_{3}}{\partial \mathbf{x}} = \frac{2\{(\mathbf{x}-\xi)+\gamma(\mathbf{y}-\eta)\}}{\{(\mathbf{x}-\xi)+\gamma(\mathbf{y}-\eta)\}^{2}+\lambda^{2}(\mathbf{y}-\eta)^{2}}$$

$$\frac{\partial L_1}{\partial y} = \frac{2\alpha (\mathbf{x}-\xi) + 2(\alpha^2 + \beta^2) (\mathbf{y}-\eta)}{\{ (\mathbf{x}-\xi) + \alpha (\mathbf{y}-\eta) \}^2 + \beta^2 (\mathbf{y}-\eta)^2 }$$

$$\frac{\partial L_{3}}{\partial y} = \frac{2\gamma (\mathbf{x}-\xi) + 2(\gamma^{2}+\lambda^{2}) (y-\eta)}{\{ (\mathbf{x}-\xi) + \gamma (y-\eta) \}^{2} + \lambda^{2} (y-\eta)^{2} }$$

$$\frac{\partial N_1}{\partial \mathbf{x}} = \frac{-\beta (\mathbf{y} - \mathbf{n})}{\{(\mathbf{x} - \xi) + \alpha (\mathbf{y} - \mathbf{n})\}^2 + \beta^2 (\mathbf{y} - \mathbf{n})^2}$$

$$\frac{\partial \mathbf{N}_{3}}{\partial \mathbf{x}} = \frac{-\gamma (\mathbf{y} - \eta)}{\{(\mathbf{x} - \xi) + \gamma (\mathbf{y} - \eta)\}^{2} + \lambda^{2} (\mathbf{y} - \eta)^{2}}$$

$$\frac{\partial N_1}{\partial y} = \frac{\beta (x-\xi)}{\{(x-\xi)+\alpha (y-\eta)\}^2+\beta^2 (y-\eta)^2}$$

$$\begin{split} \frac{\partial N_{3}}{\partial Y} &= \frac{\lambda (\mathbf{x} - \xi)}{\{(\mathbf{x} - \xi) + \gamma (\mathbf{y} - \eta)\}^{2} + \lambda^{2} (\mathbf{y} - \eta)^{2}} \\ \frac{\partial R_{1}}{\partial \mathbf{x}} &= 2[(\mathbf{x} - \xi) + \alpha (\mathbf{y} - \eta)] (\mathbf{L}_{1} - 3) + \frac{\partial \mathbf{L}_{1}}{\partial \mathbf{x}} \{[(\mathbf{x} - \xi) + \alpha (\mathbf{y} - \eta)]^{2} - \beta^{2} (\mathbf{y} - \eta)^{2}\} \\ &- 4\beta (\mathbf{y} - \eta) N_{1} - 4\beta (\mathbf{y} - \eta) [(\mathbf{x} - \xi) + \alpha (\mathbf{y} - \eta)] \frac{\partial N_{1}}{\partial \mathbf{x}} \\ \frac{\partial R_{3}}{\partial \mathbf{x}} &= 2[(\mathbf{x} - \xi) + \gamma (\mathbf{y} - \eta)] (\mathbf{L}_{3} - 3) + \frac{\partial \mathbf{L}_{3}}{\partial \mathbf{x}} \{[(\mathbf{x} - \xi) + \gamma (\mathbf{y} - \eta)]^{2} - \lambda^{2} (\mathbf{y} - \eta)^{2}\} \\ &- 4\lambda (\mathbf{y} - \eta) N_{3} - 4\lambda (\mathbf{y} - \eta) [(\mathbf{x} - \xi) + \gamma (\mathbf{y} - \eta)] \frac{\partial N_{3}}{\partial \mathbf{x}} \\ \frac{\partial R_{1}}{\partial \mathbf{y}} &= 2[\alpha (\mathbf{x} - \xi) + (\alpha^{2} - \beta^{2}) (\mathbf{y} - \eta)] (\mathbf{L}_{1} - 3) + \{[(\mathbf{x} - \xi) + \alpha (\mathbf{y} - \eta)]^{2} - \beta^{2} (\mathbf{y} - \eta)^{2}] \frac{\partial \mathbf{L}_{1}}{\partial \mathbf{y}} \\ &- 4\beta [(\mathbf{x} - \xi) + 2\alpha (\mathbf{y} - \eta)] N_{1} - 4\beta (\mathbf{y} - \eta) [(\mathbf{x} - \xi) + \alpha (\mathbf{y} - \eta)] \frac{\partial N_{1}}{\partial \mathbf{y}} \\ \frac{\partial R_{3}}{\partial \mathbf{y}} &= 2[\gamma (\mathbf{x} - \xi) + (\gamma^{2} - \lambda^{2}) (\mathbf{y} - \eta)] (\mathbf{L}_{3} - 3) + \{[(\mathbf{x} - \xi) + \gamma (\mathbf{y} - \eta)]^{2} - \lambda^{2} (\mathbf{y} - \eta)^{2}] \frac{\mathbf{L}}{\mathbf{y}} \\ &- 4\lambda [(\mathbf{x} - \xi) + 2\gamma (\mathbf{y} - \eta)] N_{3} - 4\lambda (\mathbf{y} - \eta) [(\mathbf{x} - \xi) + \gamma (\mathbf{y} - \eta)] \frac{\partial N_{3}}{\partial \mathbf{y}} \\ \frac{\partial S_{1}}{\partial \mathbf{x}} &= \beta (\mathbf{y} - \eta) (\mathbf{L}_{1} - 3) + \beta (\mathbf{y} - \eta) [(\mathbf{x} - \xi) + \alpha (\mathbf{y} - \eta)] \frac{\partial \mathbf{L}_{1}}{\partial \mathbf{x}} + 2[(\mathbf{x} - \xi) + \alpha (\mathbf{y} - \eta)]^{2} - N_{1} \\ &+ ([(\mathbf{x} - \xi) + \alpha (\mathbf{y} - \eta)]^{2} - \beta^{2} (\mathbf{y} - \eta)^{2} \frac{\partial N_{1}}{\partial \mathbf{x}} \\ \end{split}$$

$$\begin{aligned} \frac{\partial S_3}{\partial \mathbf{x}} &= \lambda \left(\mathbf{y} - \eta \right) \left(\mathbf{L}_3 - 3 \right) + \lambda \left(\mathbf{y} - \eta \right) \left[\left(\mathbf{x} - \xi \right) + \gamma \left(\mathbf{y} - \eta \right) \right] \frac{\partial \mathbf{L}_3}{\partial \mathbf{x}} + 2 \left[\left(\mathbf{x} - \xi \right) + \gamma \left(\mathbf{y} - \eta \right) \right] \cdot \mathbf{N}_3 \\ &+ \left\{ \left[\left(\mathbf{x} - \xi \right) + \gamma \left(\mathbf{y} - \eta \right) \right]^2 - \lambda^2 \left(\mathbf{y} - \eta \right)^2 \right] \frac{\partial \mathbf{N}_3}{\partial \mathbf{x}} \end{aligned}$$
$$\begin{aligned} \frac{\partial S_1}{\partial \mathbf{y}} &= \beta \left[\left(\mathbf{x} - \xi \right) + 2\alpha \left(\mathbf{y} - \eta \right) \right] \left(\mathbf{L}_1 - 3 \right) + \beta \left(\mathbf{y} - \eta \right) \left[\left(\mathbf{x} - \xi \right) + \alpha \left(\mathbf{y} - \eta \right) \right] \frac{\partial \mathbf{L}_1}{\partial \mathbf{y}} \\ &+ 2 \left[\alpha \left(\mathbf{x} - \xi \right) + \left(\alpha^2 - \beta^2 \right) \left(\mathbf{y} - \eta \right) \right] \cdot \mathbf{N}_1 + \left\{ \left[\left(\mathbf{x} - \xi \right) + \alpha \left(\mathbf{y} - \eta \right) \right]^2 - \beta^2 \left(\mathbf{y} - \eta \right)^2 \right] \frac{\partial \mathbf{N}_1}{\partial \mathbf{y}} \\ \frac{\partial S_3}{\partial \mathbf{y}} &= \lambda \left[\left(\mathbf{x} - \xi \right) + 2\gamma \left(\mathbf{y} - \eta \right) \right] \left(\mathbf{L}_3 - 3 \right) + \lambda \left(\mathbf{y} - \eta \right) \left[\left(\mathbf{x} - \xi \right) + \gamma \left(\mathbf{y} - \eta \right) \right] \frac{\partial \mathbf{L}_3}{\partial \mathbf{y}} \\ &+ 2 \left[\gamma \left(\mathbf{x} - \xi \right) + \left(\gamma^2 - \lambda^2 \right) \left(\mathbf{y} - \eta \right) \right] \cdot \mathbf{N}_3 + \left\{ \left[\left(\mathbf{x} - \xi \right) + \gamma \left(\mathbf{y} - \eta \right) \right]^2 - \lambda^2 \left(\mathbf{y} - \eta \right)^2 \right] \frac{\partial \mathbf{N}_3}{\partial \mathbf{y}} \end{aligned}$$

APPENDIX G

COMPUTER PROGRAM FOR AN ANISOTROPIC PROBLEM

APPENDIX G

COMPUTER PROGRAM FOR AN ANISOTROPIC PROBLEM

	FRUGRAM ANIPI	LCL(INFUT, OUTPUT, TAPES=INPUT, TAPE6=OUTPUT)	003100
С			000110
С	******	***************************************	000120
ř			000130
ž	DOTHE FORCE METHO		000130
5	FUINT FURCE HEING	DU FUR ANISUIRUPIL PLAIE DENUING PRUBLENS,	000140
C	ARBITRARY PLAN FO	ORM, TRANSVERSE LOAD, AND BOUNDARY CONDITIONS ###	000150
С			000160
С	SHOUN HERE IS AN	EXAMPLE FOR A SIMPLY SUPPORTED SQUARE PLATE.	000170
С			000180
С	REQUIRED INPUT VA	ALUES	000190
č			000200
č		WIMBED OF BOUNDARY COINTS	000210
2	NDF	-NONDER OF BOUNDART FOINTS	000210
L A	NIP	-NUMBER OF INTERNAL LUAU PUINTS	000220
C	NFP	TNURBER OF FIELD PUINIS	000230
С	XB,YB	=FOINTS ON B AT WHICH B.C. ARE SATISFIED.	000240
С	BANX, BANY	=COMFONENTS OF UNIT NOPHAL TO B AT XB, YB.	000250
С	XXB,YYB	FOINTS OF MESHES AROUND B WHERE FICTITIOUS	000260
С		FORCES ARE ASSIGNED.	000270
C	XF.YF	=FIELD POINTS	000280
ř	XT.YT	TINTERNAL LOAD FOINTS	000290
ř		POTECONE BATIO IN V DIDECTION	000300
č	••	PUISSONS RAILU IN A DIRECTION	000300
5		DUE TU STRESS IN T DIRECTION	000310
C	EX,ET	TOUNG S MODULI IN X AND T DIRECTIONS,	000320
С		RESPECTIVELY	000330
С	GXY	SHEAR MODULUS	000340
С	HVALUE	=PLATE THICKNESS	000350
С	RADIUS	=RADIUS OF THE FICTITIOUS CIRCULAR PLATE OF WHICH	000360
č		THE DISPLACEMENT AT THE CIRCUMFERENTIAL BOUNDARY	000370
ř		TS SET TO ZEDO	000380
U		15 SET TO EERO.	
~			000300
C			000390
С С		***********	000390 #000400
C C C	*******	***********	000390 000400 000410
С С С	DIMENSION XB	**************************************	000390 000400 000410 000420
С С С	DIMENSION XB	**************************************	000390 000400 000410 000420 000420
C C C	OIMENSION XB DIMENSION XB DIMENSION XI DIMENSION RL	(40),YB(40),X×B(21),YYB(61) (100),YB(100),DEL(2) X(80),RB(80),RM(80,80),PS(80),WKAREA(80),RL(80)	000390 *000400 000410 000420 000430 000430
C C C	DIMENSION XB(DIMENSION XB(DIMENSION XI(DIMENSION RL) DIMENSION XF((40),YB(40),XXB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RM(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181)	000390 000400 000410 000420 000430 000430 000440 000450
C C C	DIMENSION XB DIMENSION XB DIMENSION XI DIMENSION RL DIMENSION FA	(40),YB(40),XXB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80)	000390 000400 000410 000420 000430 000430 000440 000450 000450
CCC	DIMENSION XB DIMENSION XI DIMENSION XI DIMENSION RL DIMENSION SA DIMENSION BA DEAL AVECTOR	(40),YB(40),XXB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RM(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX.MY.HXY	000390 000400 000410 000420 000430 000440 000450 000450 000450 000450
CCC	DIMENSION XB DIMENSION XI DIMENSION RL DIMENSION RL DIMENSION BA REAL AVECTOR COMPLEX EDDO	(40),YB(40),XXB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RM(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY	000390 000410 000410 000420 000430 000430 000450 000450 000450 000450
с с с с с	DIMENSION XB DIMENSION XI DIMENSION XI DIMENSION RL DIMENSION SA REAL AVECTOR COMPLEX ERROR	(40),YB(40),XXB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RM(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,ROOT(4)	000390 000410 000420 000420 000430 000440 000450 000450 000450 000460 000480
с с с с	DIMENSION XB(DIMENSION XI(DIMENSION XI(DIMENSION RL) DIMENSION BAN REAL AVECTOR(COMPLEX ERROR	(40),YB(40),XXB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RM(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,RODT(4)	000390 000410 000420 000430 000440 000450 000450 000450 000450 000450 000450 000450
ссс с	DIMENSION XB DIMENSION XB DIMENSION XI DIMENSION RL DIMENSION XF DIMENSION BA REAL AVECTOR COMPLEX ERROF	(40),YB(40),XYB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RH(80,80),P5(80),WKAREA(80),RL(80) (181),YF(181),W(181),BHX(181),BHY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,RODT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X,	000390 000410 000420 000420 000430 000450 000450 000450 000450 000450 000450
ссс с	DIMENSION XB(DIMENSION XI(DIMENSION XI(DIMENSION RL) DIMENSION BAN REAL AVECTOR COMPLEX ERROM	(40),YB(40),XYB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)]	000390 000410 000420 000430 000430 000450 000450 000450 000450 000470 000470 000480 000470 000510
ссс с	DIMENSION XB DIMENSION XI DIMENSION RL DIMENSION RL DIMENSION SF DIMENSION BA REAL AVECTOR COMPLEX ERROF 100 FORMAT(\$0LOCT +/\$0\$/(15,4F1) 110 FORMAT(\$0LOCT	(40),YB(40),X×B(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RM(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2))	000390 000410 000410 000420 000420 000450 000450 000450 000450 000450 000450 000450 000520
ссс с	DIMENSION XB(DIMENSION XI(DIMENSION XI(DIMENSION RL) DIMENSION BAN REAL AVECTOR(COMPLEX ERROF 100 FORMAT(#0LOCT +/#0#/(15,4F1(110 FORMAT(#0LOCT 200 FORMAT(#0LOCT	(40),YB(40),XXB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,RCOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2))	000390 000410 000420 000420 000450 000450 000450 000450 000450 000450 000450 000510 000520 000530
ссс с	DIMENSION XB(DIMENSION XI(DIMENSION XI(DIMENSION RL) DIMENSION BAN REAL AVECTOR(COMPLEX ERROF 100 FORMAT(#0L0CC +/#0#/(I5,4F1(110 FORMAT(#0L0CC 200 FORMAT(#0L0CC) +(I4,11X,F6.3)	(40),YB(40),XXB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,ROOT(4) T\$,6X,\$XB\$,8X,\$YB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 0.4)) T\$,11X,\$XXB\$,14X,\$YYB\$/\$0\$/(I4,8X,F9.2,8X,F9.2)) C\$,11X,\$XXB\$,14X,\$YYB\$/\$0\$/(I4,8X,F9.2,8X,F9.2)) C\$,11X,\$XXB\$,14X,\$YYB\$/\$0\$/(I4,8X,F9.2,8X,F9.2)) C\$,11X,\$XXB\$,14X,\$YYB\$/\$0\$/(I4,8X,F9.2,8X,F9.2))	000390 000410 000420 000420 000450 000450 000450 000450 000450 000450 000450 000510 000520 000530 000540
ссс с	DIMENSION XB(DIMENSION XI(DIMENSION XI(DIMENSION RL) DIMENSION AF(DIMENSION BAN REAL AVECTOR(COMPLEX ERROF 100 FORMAT(\$0LOC) +/\$0\$/(I5,4F1(110 FORMAT(\$0LOC) 200 FORMAT(\$0LOC) +(I4,11X,F6.3)	(40),YB(40),XYB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BHX(181),BHY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,RODT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XI#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,19X,#PSP#/#0#/(I4,8X,E20.8))	000390 000410 000420 000420 000430 000450 000450 000450 000450 000450 000450 000510 000520 000550
ссс с	DIMENSION XB(DIMENSION XI(DIMENSION XI(DIMENSION RL) DIMENSION BAN REAL AVECTOR COMPLEX ERROF 100 FORMAT(#0LOCT +/#0#/(I5,4F1(110 FORMAT(#0LOCT 200 FORMAT(#0LOCT +(I4,11X,F6.3 400 FORMAT(#0LOCT	<pre>(40),YB(40),XXB(21),YYB(61) (100),YI(100),DEL(2) X(80),RB(80),RM(80,80),PS(80),WKAREA(80),RL(80) (1R1),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,6X,#XB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XE#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,11X,#XF#,17X,#YF#/#0#/(I4,11X,F6.3,11X,F6.3))</pre>	000390 000410 000410 000420 000440 000450 000450 000450 000470 000470 000470 000470 000470 000500 000510 000550 000550
ccc c	DIMENSION XB DIMENSION XB DIMENSION XI DIMENSION RL DIMENSION RL DIMENSION BA REAL AVECTOR COMPLEX ERROS 100 FORMAT(#0LOCT +/#0#/(I5,4F1 110 FORMAT(#0LOCT 200 FORMAT(#0LOCT +(I4,11X,F6.3) 400 FORMAT(#1LOCT) 500 FORMAT(#1LOCT)	<pre>(40),YB(40),XXB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RHX(80,80) (5),MX,HY,HXY R,RODT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XI#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,19X,#PSP#/#0#/(I4,8X,E20.8)) T#,11X,#XF#,17X,#YF#/#0#/(I4,11X,F6.3,11X,F6.3)) Y, #NODTE,12Y#F#,16Y,#YE#/16Y,#YE#,16Y,#EMY#.</pre>	000390 000410 000420 000420 000440 000450 000450 000450 000450 000470 000470 000470 000470 000470 000510 000510 000550 000550 000550
c c c	DIMENSION XB DIMENSION XI DIMENSION XI DIMENSION RL DIMENSION RL DIMENSION BA REAL AVECTOR COMPLEX ERROF 100 FORMAT(#0LOC +/#0#/(I5,4F1 110 FORMAT(#0LOC +(I4,11X,F6.3 400 FORMAT(#1LOC 500 FORMAT(#1LOC	<pre>(40),YB(40),XXB(21),YYB(61) (100),YI(100),DEL(2) X(60),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XI#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,11X,#XF#,17X,#YF#/#0#/(I4,11X,F6.3,11X,F6.3)) X,#NODE#,12X#XF#,16X,#YF#,19X,#W#,16X,#BMX#, W0 710 0F20 10 7520 100 1000 1000 1000 1000 1000 1000 10</pre>	000390 000410 000420 000420 000450 000450 000450 000450 000450 000510 000510 000520 000550 000550 000550 000550
ccc c	DIMENSION XB(DIMENSION XI(DIMENSION XI(DIMENSION RL) DIMENSION RL) DIMENSION BAN REAL AVECTOR(COMPLEX ERROF 100 FORMAT(\$0L0C1 +/\$0\$/(15,4F1(110 FORMAT(\$0L0C1 +(14,11X,F6.3) 400 FORMAT(\$0L0C1 600 FORMAT(\$1L00 500 FORMAT(\$0L0C1 600 FORMAT(\$0L0C1 600 FORMAT(\$0L0C1 600 FORMAT(\$0L0C1) 500 FORMAT(\$0L0C1) 600 FORMAT(\$0L0C1) 500 FORMAT(\$0L0C1) 600 FORMAT(\$0L0C1) 500 FORMAT(\$0L0C1) 600 FORMAT(\$0L0C1) 500 FORMAT(\$	<pre>(40),YB(40),XXB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (121),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXF#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,11X,#XF#,17X,#YF#/#0#/(I4,11X,F6.3,11X,F6.3)) X,#NODE#,12X#XF#,16X,#YF#,19X,#W#,16X,#BMX#, H0,I10,2F20.10,3E20.12))</pre>	000390 000410 000420 000420 000450 000450 000450 000450 000450 000450 000510 000520 000550 000550 000550 000550
ccc c	DIMENSION XB DIMENSION XB DIMENSION XI DIMENSION RL DIMENSION RL DIMENSION BA REAL AVECTOR COMPLEX ERROF 100 FORMAT(#0LOCT +/#0#/(I5,4FIC) 200 FORMAT(#0LOCT 200 FORMAT(#0LOCT +(I4,11X,F6.3 400 FORMAT(#1LOC 500 FORMAT(#1LOCT 600 FORMAT(#1LOCT 600 FORMAT(#1LOCT 600 FORMAT(#1LOCT 16X,#BMY#/(11) 700 FCRMAT(111,#)	<pre>(40),YB(40),XYB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),B*X(181),BHY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,RODT(4) T\$,6X,\$XB\$,8X,\$YB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 0.4)) T\$,11X,\$XXB\$,14X,\$YYB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 0.4)) T\$,11X,\$XXB\$,14X,\$YYB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 0.4)) T\$,11X,\$XXB\$,14X,\$YYB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 0.4)] T\$,11X,\$XXB\$,14X,\$YYB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 0.4)] T\$,11X,\$XXB\$,14X,\$YYB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 0.4]] T\$,11X,\$XXB\$,14X,\$YYB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 11X,\$AXB\$,14X,\$YYB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 11X,\$AXB\$,14X,\$YYB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 11X,\$AXB\$,14X,\$YYB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 11X,\$AXB\$,14X,\$YYB\$,14X,\$AX\$,\$ANY\$,4X, 11X,\$AXB\$,14X,\$YYB\$,14X,\$AX\$,\$AX,\$AX,\$AX,\$AX,\$AX,\$AX,\$AX,\$AX,\$</pre>	000390 000410 000420 000430 000450 000450 000450 000450 000450 000450 000510 000520 000550 000550 000550 000550 000550 000550
ccc c	DIMENSION XB DIMENSION XB DIMENSION XI DIMENSION RL DIMENSION RL DIMENSION RA REAL AVECTOR COMPLEX ERROF 100 FORMAT(#0LOCT +/#0#/(I5,4F1) 110 FORMAT(#0LOCT 200 FORMAT(#0LOCT 200 FORMAT(#0LOCT 500 FORMAT(#1L),63 400 FORMAT(#1L),63 116X,#BMY#/(1L),63 116X,#BMY#/(1L),63 +# NFP = #,13	<pre>(40),YB(40),XXB(21),YYB(61) (100),YI(100),DEL(2) X(60),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,HY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,6X,#XB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XF#,17X,#YF#/#0#/(I4,11X,F6.3,11X,F6.3)) X,#NODE#,12X#XF#,16X,#YF#,19X,#W#,16X,#BMX#, H0,I10,2F20.10,3E20.12)) INFUT VALUES#,//1X,#NBP = #,I3,# NIP =#,I3, 3,</pre>	000390 000410 000420 000420 000450 000450 000450 000450 000470 000470 000470 000470 000470 000470 000500 000510 000550 000550 000550 000550 000550 000550 000550 000550
ccc c	DIMENSION XB DIMENSION XB DIMENSION XI DIMENSION RL DIMENSION RL DIMENSION BA REAL AVECTOR COMPLEX ERROF 100 FORMAT(#0L0CT +/#0#/(I5,4F1 110 FORMAT(#0L0CT 200 FORMAT(#0L0CT 400 FORMAT(#0L0CT 600 FORMAT(#11,8) 116X,#BMY#/(11 700 FCRMAT(111,8) +# NFP = #,II +/100,# EX = 1	<pre>(40),YB(40),XXB(21),YYB(61) (100),YI(100),DEL(2) X(60),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RHX(80,80) (5),MX,MY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XI#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,19X,#PSP#/#0#/(I4,8X,E20.8)) T#,11X,#XF#,17X,#YF#/#0#/(I4,11X,F6.3,11X,F6.3)) X,#NODE#,12X#XF#,16X,#YF#,19X,#W#,16X,#BMX#, H0,110,2F20.10,3E20.12)) INFUT VALUES#,//1X,#NBP = #,I3,# NIP =#,I3, 3, #,E10.4,# EY = #,E10.4,# VX = #,F5.2,# GXY = #,E10.4,#</pre>	000390 000410 000420 000420 000450 000450 000450 000450 000470 000470 000470 000470 000470 000470 000500 000550 00050 00000000
ccc c	DIMENSION XB DIMENSION XB DIMENSION XI DIMENSION RL DIMENSION RL DIMENSION RA REAL AVECTOR COMPLEX ERROF 100 FORMAT(\$0L0C +/\$0\$/(15,4F1 100 FORMAT(\$0L0C +(14,11X,F6.3 400 FORMAT(\$0L0C +(14,11X,F6.3 400 FORMAT(\$0L0C 600 FORMAT(\$	<pre>(40),YB(40),XXB(21),YYB(61) (100),YI(100),DEL(2) X(60),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RHX(80,80) (5),MX,MY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XF#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,11X,#XF#,17X,#YF#/#0#/(I4,11X,F6.3,11X,F6.3)) X,#NODE#,12X#XF#,16X,#YF#,19X,#W#,16X,#BHX#, H0,I10,2F20.10,3E20.12)) INFUT VALUES#,//IX,#NBP = #,I3,# NIP =#,I3, 3, #,E10.4,# EY = #,E10.4,# VX = #,F5.2,# GXY = #,E10.4, S OF THE PLATE = #,</pre>	000390 *000410 000420 000420 000450 000450 000450 000450 000450 000510 000510 000550 000500 000500 000500 000500 000500 000500 000500 000500 000500 000500 000500 000500 000500 000500 000500 000500 000000
ccc c	DIMENSION XB(DIMENSION XB(DIMENSION XI(DIMENSION RL) DIMENSION RL) DIMENSION BAN REAL AVECTOR(COMPLEX ERROF 100 FORMAT(\$0LOCT +/\$0\$/(15,4F1(110 FORMAT(\$0LOCT 200 FORMAT(\$0LOCT 200 FORMAT(\$0LOCT 200 FORMAT(\$1LOCT 200 FORMAT(<pre>(40),YB(40),XYB(21),YYB(81) (100),YI(100),DEL(2) X(80),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,RODT(4) T\$,6X,\$XB\$,8X,\$YB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 0.4)) T\$,11X,\$XXB\$,14X,\$YYB\$,7X,\$ANX\$,8X,\$ANY\$,4X, 0.4)) T\$,11X,\$XXB\$,14X,\$YYB\$,70\$/(14,8X,F9.2,8X,F9.2)) CT\$,11X,\$XXB\$,14X,\$YYB\$,\$0\$/(14,8X,F9.2,8X,F9.2)) CT\$,11X,\$XXB\$,14X,\$YYB\$,\$0\$/(14,8X,F9.2,8X,F9.2)) CT\$,11X,\$XXB\$,14X,\$YYB\$,\$0\$/(14,8X,F9.2,8X,F9.2)) CT\$,11X,\$XXB\$,14X,\$YYB\$,\$16X,\$\$0\$/(14,11X,F6.3,11X,F6.3)) X,\$NODE\$,12X\$XF\$,16X,\$YF\$,19X,\$\$W\$,16X,\$\$BMX\$, H0,110,2F20.10,3E20.12)) INFUT VALUES\$,//1X,\$\$NBP = \$,13,\$\$ NIP =\$,13,\$ \$,\$ \$,E10.4,\$\$ EY = \$,E10.4,\$\$ VX = \$,F5.2,\$\$ GXY = \$,E10.4,\$\$ \$ OF THE PLATE = \$,\$ KNESS OF THE PLATE =\$,\$ KNESS OF THE PLATE \$,\$ KNESS OF THE PLATE \$,\$ KNESS OF THE PLATE \$,\$ KNESS OF THE \$,\$ KNESS OF THE \$,\$ KNESS OF THE \$,\$ KNESS OF THE \$,\$ KNESS OF THE \$,\$ KNESS OF \$,\$ KNESS OF \$,\$ KNESS OF \$,\$ KNESS OF \$,\$ KNESS OF \$,\$ KNESS OF \$,\$ KNESS OF \$,\$ KNESS OF \$,\$ KNESS OF \$,\$ KNESS OF \$,\$ KNESS OF \$,\$ KNESS OF \$</pre>	000390 *000410 000410 000420 000430 000450 000450 000450 000450 000450 000510 000520 000550 00050 000500 000000
ccc c	DIMENSION XB(DIMENSION XB(DIMENSION XI(DIMENSION RL) DIMENSION RL) DIMENSION BAN REAL AVECTOR(COMPLEX ERROF 100 FORMAT(\$0LOC' +/\$0\$/(I5,4FI(110 FORMAT(\$0LOC' 200 FORMAT(\$0LOC' 200 FORMAT(\$1LOC' 200 FORMAT(\$1LOC' 200 FORMAT(\$1LOC' 200 FORMAT(\$1LOC' 500 FORMAT(\$1LOC' 500 FORMAT(\$1LOC' 500 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 500 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 500 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 600 FORMAT(\$1LOC' 700 FORMAT(\$1LOC' 703 FORMAT(\$1LOC' 703 FORMAT(\$1.00)	<pre>(40),YB(40),XXB(21),YYB(61) (100),YI(100),DEL(2) X(60),RB(80),RM(80,80),PS(80),WKAREA(80),RL(80) (121),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,11X,F6.3,11X,F6.3)) X,#VDDE#,12X#XF#,16X,#YF#/10X,#W#,16X,#BMX#, H0,110,2F20.10,3E20.12)) INFUT VALUES#,//IX,#NBP = #,I3,# NIP =#,I3, 3, #,EI0.4,# EY = #,EI0.4,# VX = #,F5.2,# GXY = #,EI0.4, S OF THE PLATE = #, KNESS OF THE PLATE = #,F6.3) X,#THE COEFFICIENTS OF THE CHARACTERISTIC #.</pre>	000390 000410 000420 000420 000450 000450 000450 000450 000470 000470 000470 000470 000470 000500 000510 000550
ссс с	DIMENSION XB DIMENSION XB DIMENSION XI DIMENSION RL DIMENSION RL DIMENSION RA REAL AVECTOR COMPLEX ERROF 100 FORMAT(#0LOCT +/#0#/(I5,4F1) 110 FORMAT(#0LOCT +/#0#/(I5,4F1) 110 FORMAT(#0LOCT 200 FORMAT(#0LOCT 200 FORMAT(#0LOCT 400 FORMAT(#0LOCT 500 FORMAT(#0LOCT 500 FORMAT(#0LOCT 600 FORMAT(#11,8) 116X,#BMY#/(11 700 FCRMAT(111,8) +# NFP = #,II +/1H0,# RADIU +F7.1,# TMICT 703 FORMAT(///,13) +#POLYNOMTAL	<pre>(40),YB(40),XXB(21),YYB(61) (100),YI(100),DEL(2) X(60),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RHX(80,80) (5),HX,HY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XF#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,19X,#PSP#/#0#/(I4,8X,E20.8)) T#,11X,#XF#,17X,#YF#/#0#/(I4,11X,F6.3,11X,F6.3)) X,#NODE#,12X#XF#,16X,#YF#,19X,#W#,16X,#BMX#, H0,110,2F20.10,3E20.12)) INFUT VALUES#,//1X,#NBP = #,I3,# NIP =#,I3, 3, #,E10.4,# EY = #,E10.4,# VX = #,F5.2,# GXY = #,E10.4, S OF THE PLATE = #, KNESS OF THE PLATE = #,F6.3) X,#THE COEFFICIENTS OF THE CHARACTERISTIC #, ARE#,/1X,5E12.5)</pre>	000390 000410 000420 000420 000440 000450 000450 000450 000470 000470 000470 000470 000470 000470 000500 000510 000550
ссс с	DIMENSION XB DIMENSION XB DIMENSION XI DIMENSION XI DIMENSION RL DIMENSION RL DIMENSION BA REAL AVECTOR COMPLEX ERROF 100 FORMAT(#0LOCT +/#0#/(I5,4F1(110 FORMAT(#0LOCT 200 FORMAT(#0LOCT 200 FORMAT(#0LOCT 200 FORMAT(#0LOCT 200 FORMAT(#111,# 400 FORMAT(#111,# 116X,#BMY#/(11 700 FORMAT(111,# +# NFP = #,II +/10,# RADIUS +/10,# RADIUS +#POLYNOMIAL/	<pre>(40),YB(40),XXB(21),YYB(61) (100),YI(100),DEL(2) X(60),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (121),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XI#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,19X,#PSP#/#0#/(I4,8X,E20.8)) T#,11X,#XF#,17X,#YF#/#0#/(I4,11X,F6.3,11X,F6.3)) X,#NODE#,12X#XF#,16X,#YF#,19X,#W#,16X,#BMX#, H0,I10,2F20.10,3E20.12)) INFUT VALUES#,//IX,#NBP = #,I3,# NIP =#,I3, 3, #,E10.4,# EY = #,E10.4,# VX = #,F5.2,# GXY = #,E10.4, S OF THE PLATE = #, KNESS OF THE PLATE = #,F6.3) X,#THE COEFFICIENTS OF THE CHARACTERISTIC #, ARE#,/IX,5E12.5) THE FOUR POOTS OF THE CHARACTERISTIC #, ARE#,/IX,5E12.5)</pre>	000390 000410 000420 000420 000440 000450 000450 000450 000450 000500 000550 000560 000500 000500 000500 000500 00000000
сос с	DIMENSION XB(DIMENSION XB(DIMENSION XI(DIMENSION RL) DIMENSION RL) DIMENSION BAN REAL AVECTOR(COMPLEX ERROF 100 FORMAT(\$0L0C' +/\$0\$/(I5,4F1(110 FORMAT(\$0L0C' +(I4,11X,F6.3) 400 FORMAT(\$0L0C' +(I4,11X,F6.3) 400 FORMAT(\$0L0C' 600 FORMAT(\$0L0C'	<pre>(40),YB(40),XXB(21),YYB(61) (100),YI(100),DEL(2) X(60),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (181),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RHX(80,80) (5),MX,MY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XF#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,11X,#XF#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,11X,#XF#,17X,#YF#/#0#/(I4,11X,F6.3,11X,F6.3)) X,#NODE#,12X#XF#,16X,#YF#,19X,#W#,16X,#BHX#, H0,I10,2F20.10,3E20.12)) INFUT VALUES#,//IX,#NBP = #,I3,# NIP =#,I3, 3, #,E10.4,# EY = #,E10.4,# VX = #,F5.2,# GXY = #,E10.4, S OF THE PLATE = #, KNESS OF THE PLATE =#,F6.3) X,#THE COEFFICIENTS OF THE CHARACTERISTIC #, ARE#,/IX,5E12.5) THE FOUR ROOTS OF THE CHARACTERISTIC EQUATION:#, 1</pre>	000390 *000410 000410 000420 000450 000450 000450 000450 000450 000510 000520 000550 000650 000640 000650 0000600 0000600 000600 00000000
сос с	DIMENSION XB(DIMENSION XB(DIMENSION XI(DIMENSION RL) DIMENSION RL) DIMENSION BAN REAL AVECTOR(COMPLEX ERROF 100 FORMAT(\$0LOC' +/\$0\$/(15,4F1(110 FORMAT(\$0LOC' 200 FORMAT(\$0LOC' 200 FORMAT(\$0LOC' 200 FORMAT(\$0LOC' 200 FORMAT(\$0LOC' 600 FORMAT(<pre>(40),YB(40),XYB(2),YYB(8)) (100),YI(100),DEL(2) X(80),RB(80),RH(80,80),PS(80),WKAREA(80),RL(80) (121),YF(181),W(181),BMX(181),BMY(181) NX(40),BANY(40),RMX(80,80) (5),MX,MY,HXY R,ROOT(4) T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X, 0.4)) T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XXB#,14X,#YB#/#0#/(I4,8X,F9.2,8X,F9.2)) CT#,11X,#XF#,17X,#YI#,14X,/#0#/ ,11X,F6.3)) CT#,12X,#XF#,17X,#YI#,14X,/#0#/ ,11X,#XF#,17X,#YF#/#0#/(I4,11X,F6.3,11X,F6.3)) X,#NODE#,12X#XF#,16X,#YF#,19X,#W#,16X,#BMX#, H0,110,2F20.10,3E20.12)) INFUT VALUES#,//1X,#NBP = #,I3,# NIP =#,I3, 3, #,E10.4,# EY = #,E10.4,# VX = #,F5.2,# GXY = #,E10.4, S OF THE PLATE = #, KNESS OF THE PLATE = #, KNESS OF THE PLATE = #,F6.3) X,#THE COEFFICIENTS OF THE CHARACTERISTIC #, ARE#,/1X,5E12.5) THE FOUR ROOTS OF THE CHARACTERISTIC EQUATION:#, } </pre>	000390 000410 000420 000420 000450 000450 000450 000450 000450 000520 000520 000520 000520 000550

```
707 FORMAT(1H0, #THE FOUR CONSTANTS ARE : #,
                                                                            000690
     +/1X.#ALPHA = #,E10.4.#
                                  BETA = #.E10.4.
                                                                            000700
     +# GAMMA = #,E10.4,#
                                   LUMBDA = #,E10.4)
                                                                            000710
 1103 FORMAT(15,5X,E20.12,5X,E20.12)
                                                                            000720
 1014 FORMAT(1H1, THE FOLLOWING IS A LIST OF DOUBLE CHECKING OF B.C.ST, 000730
  1//,4X#NBP#10X,#B.C. 1#,13X,#B.C. 2#,//)
708 FORMAT(1H0,#RADIUS = #,E10.3,# DIST1 = #,F6.1,
                                                                            000740
                                                                           000750
     +#
           DIST2 = #,F6.1)
                                                                            0007é0
  801 FORMAT(1H1, #THE BENDING RIGIDITIES ARE -----#.
                                                                            000770
     +/1X,#D11 = #,E10.4,# D12 = #,E10.4,# D22 = #,E10.4,
+/1X,#D66 = #,E10.4,# D16 = #,E10.4,# D26 = #,E10.4)
                                                                            000780
                                                                           000790
  802 FORMAT(1X, #SOMETHING IS WRONG WITH THE INPUT MATERIAL CONSTANTS#, 000791
     +#; THE DETERMINANT IS EITHER NEGATIVE OR ZERO#,/1X,
                                                                           000792
     +#COMPUTATION IS TERMINATED, DET = #,E15.7)
                                                                            000793
  805 FORMAT(1H1,#THE AIJ ARE ---#,/1X,#A11= #,E15.7,# A12= #,
                                                                            003030
     +E15.7,# A22= #,E15.7,# A66= #,E15.7,# A16= #,E15.7,
                                                                            000310
     +4 A26= #,E15.7)
                                                                            000820
                                                                            000830
C
C INFUT VALUES .....
                                                                            000840
C
                                                                            000850
      READ(5,*)NBP,NIP,NFP,EX,EY,VX,GXY,RADIUS,HVALUE
                                                                            000350
      READ(5,#)A16,A26
                                                                            000870
      READ(5,*)(XB(I),I=1,NBP)
                                                                            000230
      READ(5,#)(YB(I),I=1,NEP)
                                                                            000390
      READ(5,*)(BANX(1),1=1,HBP)
                                                                            000900
      READ(5,*)(BANY(I),I=1,N5P)
                                                                            000910
      READ(5,*)(XI(I),I=1,NIP)
                                                                            000920
      READ(5,*)(YI(I),I=1,NIP)
                                                                            000930
      NBP2=NBP#2$NBP2P1=NBP2+1
                                                                            000940
      NOPTION=1
                                                                            000950
      PI=4.*ATAN(1.)
                                                                            000950
С
                                                                            000970
С
                                                                            000980
      All=1./EX $ Al2=-VX/EX $ A22=1./EY $ A66=1./GXY
                                                                            000990
      WRITE(6,805)A11,A12,A22,A66,A16,A26
                                                                            001000
      DET=(A11#A22-A12##2)#A66+2.#A12#A16#A26-A11#A26##2-A22#A16##2
                                                                            001010
      IF(DET.GT.0.)GOTO 40
                                                                            001011
      WRITE(6,802)DET
                                                                            001012
      STOP
                                                                            001013
   40 CONTINUE
                                                                            001014
      ZZ=HVALUE##3/(12.#DET)
                                                                            001020
      D11=(A22*A66-A26**2)*ZZ $ D22=(A11*A66-A16**2)*ZZ
                                                                            001030
      D12=(A16*A26-A12*A66)*ZZ $ D66=(A11*A22-A12**2)*ZZ
                                                                            001040
      D16=(A12*A26-A22*A16)*ZZ $ D26=(A12*A16-A11*A26)*ZZ
                                                                            001050
      RADIUS=80. $ DIST1=2.0 $ DIST2=2.
                                                                            001060
      WRITE(6,700)NBP,NIP,NFP,EX,EY,VX,GXY,
                                                                            001070
     +RADIUS, HVALUE
                                                                            001080
      WRITE(6,708)RADIUS,DIST1,DIST2
                                                                            001090
      X0=5.0$Y0=5.0
                                                                            001100
      XF(1)=10.-X0$YF(1)=-10.+Y0
                                                                            001110
      DO 41 I=2,11
                                                                            001120
      XF(I)=XF(I-1)-0.5
                                                                            001130
   41 YF(I)=YF(I-1)
                                                                            001140
      00 42 J=1,10
                                                                            001150
      K=J+17
                                                                            001160
      DO 42 I=1,11
                                                                            001170
      XF(I+K)=XF(I)
                                                                            001180
   42 YF(I+K)=YF(I+K-17)+1.0
                                                                            001190
      XF(12)=10.-X0$YF(12)=-9.5+Y0
                                                                            001200
      00 43 I=13.17
                                                                            001210
```

		VETTI-VETTIN A	
			001220
	43	TF(I)=TF(I-I)	001230
		DO 44 J=1,9	001240
		K=.!#17	001250
		DO 44 I=12,17	001250
		XF(I+K)=XF(I)	001270
	44	YE(I+K)=>E(I+K-)7)+) 0	001290
			001235
			001290
			001300
		xxB(1)=5.+DIS(1-DEL(1)+++B(1)=-5DIS+1	001310
		XXB(41)=5.+DIST1+DIST2-DEL(2)\$YYB(41)=-5DIST1-DIST2	0013 20
		DO 28 J=1,2	001330
		DELT=DEL(1)\$IF(J.EQ.2)DELT=DEL(2)	001340
		DO 25 I=2,10	001350
		K=I	001360
		IF(J.EQ.2)K=K+40	001370
		XXB(K) = XXB(K-1) - DF(T)	001380
	25	YYB(K) = YYB(K-1)	001360
			001370
			001400
			001410
		IF(J.EQ.2)K=K+40	001420
		xxB(K)=xxB(K-1)	001430
	26	YYB(K)=YYB(K-1)+DELT	001440
		DO 27 I=21,30	001450
		K=I	001460
		IF(J.EQ.2)K=K+40	001470
		XXB(K)=XXB(K-1)+DELT	001480
	27	YYB(K) = YYB(K-1)	001490
	•••		001500
			001500
		N-4 TE(+ EQ 9)V-V-60	001910
		17 J.54.5JN-K+44U	001520
	••		001530
	28	YYB(K)=YYB(K-1)-DELT	001540
		XXB(81)=XXB(41) \$ YYB(81)=YYB(41)	001550
		WRITE(6,100)(I,XB(I),YB(I),BANX(I),BANY(I),I=1,NBP)	00 1560
		WRITE(6,110)(I,XXB(I),YYB(I),I=1,NBP2P1)	001570
		WRITE(6,500)(I,XF(I),YF(I),I=1,NFP)	001580
		WRITE(6,200)(I,XI(I),YI(I),I=1,NIP)	001590
		WRITE(6.801)D11.D12.D22.D66.D16.D26	001600
С			001610
ř	SFT	THE AND SOLVE THE FOIRTH DESCEE CHADACTERISTED DOLYNOMTAL	001420
ř	321	of and source the fourth begree characteristic formulae.	001020
L			001030
			001640
		AVECTOR(3)=2.*(D12+2.*D66)/D22	001650
		AVECTOR(4)=4.#D16/D22 \$ AVECTOR(5)=D11/D22	001650
		WRITE(6,703)(AVECTOR(I),I=1,5)	001670
		CALL ZFOLR(AVECTOR,4,RCOT,IER)	001680
		HRITE(6,704)(ROOT(I),I=1,4)	001690
		DO 706 I=1,4	001700
		ERROR=AVECTOR(1)#RODT(I)##4+AVECTCR(2)#RODT(I)##3+AVECTOR(3)#	001710
		POOT (] HH2+AVECTOP(4) HPOOT (] +AVECTOP(5)	001720
	704	WPITE(6.705)1. POOT(T). FEPOP	001730
	, 50		001740
		RA-RENEWRUILA// # RE-RAINOURUUILA//	001/40
		RJ-REAL(RUJI(J)) 7 R4-ALMAU(RUUI(J)) Imatell 707)di do di da	001/50
		WWIILLO, /U/ WI, WZ, R3, R4	001760
_		R13=R1**Z \$ R23=R2**Z \$ R35=R3**Z \$ R4 3=R4 **Z	001770
C			001780
		CONSTG=(R1-R3)*#2+(R2+R4)##2 \$ CONSTH=(R1-R3)##2+(R2-R4)##2	001790
		COEF1=1./(8.*PI*D22*CONSTG*CONSTH) \$ COEF2=2.*COEF1	801800
		CONST1=((R]-R3/##2-(R2S-R4S))/R2	001810

	CONST2=((R1-R3)*#2+(R2S-R4\$))/R4	001820
	CONST3=4.#(R1-R3)	001830
	CONST5=((R1-2.#R3)#(R1S+R2S)+R1#(R3S+R4S))/R2	001840
	CONST6=((R3-2.#R1)#(R3S+R4S)+R3#(R1S+R2S))/R4	001850
	CONST7=2.#(R1S+R2S-R3S-R4S)	001860
	CONST8=((R1S+R2S-2.#R1#R3)#(R1S+R2S)+(R1S-R2S)#(R3S+R4S))/R2	001870
	CONST9=((R3S+R4S-2,#R1#R3)#(R3S+R4S)+(R3S-R4S)#(R1S+R2S))/R4	001880
	CONST10=4.*(R1*(R3S+R4S)-R3*(R1S+R2S))	001890
	QLCAD=1.0 \$ A2=RADIUS**2	001900
	DO 5 I=1,NBP	001910
	RL(I)=0.0	001920
	RL(I+N9P)=0.0	001930
	R1X=XB(I)	001940
	R1Y=YB(I)	001950
	ANX=BANX(I)	001960
	ANY=BANY(I)	001970
	DO 6 J=1,NIP	001980
	R2X=XI(J)	001990
	R2Y=YI(J)	002000
	Z1=R1X-R2X \$ Z2=R1Y-R2Y \$ Z1S=Z1**2 \$ Z2S=Z2**2	002010
	ANXS=ANX##2 \$ ANYS=ANY##2 \$ ANXY=ANX#ANY	002020
	FUNCL1=ALOG(((Z1+R1+Z2)++2+R2S+Z2S)/A2)	002030
	FUNCL3=ALC3(((Z1+R3*Z2)**2+R4S*Z2S)/A2)	002040
	Z3=Z1+R1*Z2 \$ Z4=Z1+R3*Z2	002050
	IF(ABS(Z3).GT.1.E-6)GOTO 60	002060
	FUNCN1=PI/2.	002070
	GOTO 61	002050
60	FUNCH1=ATAN(R2+Z2/Z3)	002090
61	IF(ABS(Z4).GT.1.E-6)GOTO 62	002100
	FUNCN3=PI/2.	002110
	SOTO 63	002120
62	FURICN3=ATAN(R4+Z2/Z4)	002130
63	CONTINUE	002140
	IF(FUNCH1.LT.O.)FUNCH1=FUNCH1+PI	002150
	IF(FUNCN3.LT.O.)FUNCN3=FUNCN3+PI	002160
	AA=D11#ANXS+D12#ANYS+2.#D16#ANXY	002170
	BB=ANXS#D12+D22#ANYS+2.#ANXY#D26	002180
	CC=2.#D16#ANXS+2.#D26#ANYS+4.#D66#ANXY	002190
	FUNCR1=((Z1+R1*Z2)**2-R25*Z25)*(FUNCL1-3.)-4.*R2*Z2*(Z1+R1*Z2)	002200
•	HEUNICH1	002210
	FUNCR3=((Z1+R3#Z2)##2-R45#Z25)#(FUNCL3-3.)-4.#R4#Z2#(Z1+R3#Z2)	002220
•	▶≠FUNCN3	002230
	FUNC51=R2#Z2#(Z1+R1#Z2)#(FUNCL1-3.)+((Z1+R1#Z2)##2-R25#Z25)#FUNCH1	002240
	FUNC53=R4#Z2#(Z1+R3#Z2)#(FUNCL3-3.)+((Z1+R3#Z2)##2-R45#Z25)#FUNCN3	002250
	MXX=CONST1#FUNCL1+CONST2#FUNCL3+CONST3#(FUNCN1-FUNCN3)	002260
	MXY=CONST5*FUNCL1+CONST6*FUNCL3+CONST7*(FUNCN1-FUNCN3)	002270
	WYY=CONST8*FUNCL1+CONST9*FUNCL3-CONST10*(FUNCN1-FUNCN3)	002280
	RL(I)=RL(I)-QLDAD#(CONST1#FUNCR1+CONST2#FUNCR3+CONST3#	002290
•	(FUNCS1-FUNCS3))	002300
	RL(I+NBP)=RL(I+NBP)-QLOAD*(AA*WXX+BB*WYY+CC*WXY)	002310
6	CONTINUE	002320
5	CONTINUE	002330
		002340
		002350
	DO 8 I=1,NBP	002350
	R1X=XB(I)	002370
	R1Y=YB(I)	002350
	ANX=BANX(I)	002390
	ANY=BANY(I)	002400
	DO 7 J=1,NBP2	002410

C C

```
R2X=(X>B(J+1)+XXB(J))/2.
                                                                         002423
      R2Y=(YY6(J+1)+YY8(J))/2.
                                                                          002430
      IF(NOPTION.EQ.2.OR.J.NE.NBP)GOTO 33
                                                                          002440
      R2X=(XXB(1)+XXB(NBP))/2.
                                                                          002450
      R2Y=(YYB(1)+YYB(NBP))/2.
                                                                          002450
   33 CONTINUE
                                                                          002470
      Z1=R1X-R2X $ Z2=R1Y-R2Y $ Z15=Z1**2 $ Z25=Z2**2
                                                                          002480
      ANXS=ANX++2 $ ANYS=ANY++2 $ ANXY=ANX+ANY
                                                                         002490
      FUNCL1=ALOG(((Z1+R1*Z2)**2+R25*Z25)/A2)
                                                                          002500
      FUNCL3=ALOG(((Z1+R3*Z2)**2+R45*Z25)/A2)
                                                                          002510
      Z3=Z1+P1*Z2 $ Z4=Z1+R3*Z2
                                                                          002520
      IF(ABS(Z3).GT.1.E-6)GOTO 70
                                                                          002530
      FUNCH1=PI/2.
                                                                          002540
      GOTO 71
                                                                          002550
   70 FUNCH1=ATAN(R2#Z2/Z3)
                                                                          002560
   71 IF(ABS(Z4).GT.1.E-6)GOTO 72
                                                                         002570
      FUNCN3=PI/2.
                                                                          002580
      GOTO 73
                                                                          002590
   72 FUNCH3=ATAN(R4+Z2/Z4)
                                                                          002600
   73 CONTINUE
                                                                         002610
      IF(FUNCH1.LT.0.)FUNCH1=FUNCH1+PI
                                                                          002620
      IF(FUNCN3.LT:0.)FUNCN3=FUNCN3+PI
                                                                          002630
      AA=D11*ANX5+D12*ANY5+2.*D16*ANXY
                                                                         002640
      BB=ANXS+D12+D22+ANYS+2.+ANXY+D26
                                                                         002650
      CC=2.*D16*ANXS+2.#D26*ANYS+4.*D66*ANXY
                                                                          002660
      FUNCR1=((Z1+R1+Z2)++2-R25+Z25)+(FUNCL1-3.)-4.+R2+Z2+(Z1+R1+Z2)
                                                                          002670
     +*FUNCN1
                                                                          002680
     FUNCR3=((Z1+R3+Z2)++2-R4S+Z2S)+(FUNCL3-3.)-4.+R4+Z2+(Z1+R3+Z2)
                                                                         002690
     +#FUNCN3
                                                                          002700
      FUNCS1=R2#Z2#(Z1+R1#Z2)#(FUNCL1-3.)+((Z1+R1#Z2)##2-R25#Z25)#FUNCN1002710
      FUNCS3=R4*Z2*(Z1+R3*Z2)*(FUNCL3-3.)+((Z1+R3*Z2)**2-R4S*Z2S)*FUNCN3002720
      WXX=CONST1*FUNCL1+CONST2*FUNCL3+CONST3*(FUNCN1-FUNCN3)
                                                                         002730
     WXY=CONST5*FUNCL1+CONST6*FUNCL3+CONST7*(FUNCH1-FUNCH3)
                                                                          002740
      WYY=CONST8*FUNCL1+CONST9*FUNCL3-CONST10*(FUNCN1-FUNCN3)
                                                                          002750
      RM(I,J)=(CONST1*FUNCR1+CONST2*FUNCR3+CONST3*(FUNCS1-FUNCS3))
                                                                         002760
      RM(I+NBP,J)=(AA*WXX+BB*WYY+CC*WXY)
                                                                          002770
    7 CONTINUE
                                                                          002780
    8 CONTINUE
                                                                          002790
      DO 10 I=1,MBP
                                                                          002800
      DO 10 J=1,HEP2
                                                                          002810
      RM\times(I,J)=RM(I,J)
                                                                          002820
   10 RMX(I+NEP,J)=RM(I+NBP,J)
                                                                          002830
     DO 1015 I=1,NBP2
                                                                         002840
1015 RLX(I)=RL(I)
                                                                          002850
С
                                                                         002850
С
                                                                          002870
      CALL LEGTIF(RM,1,NBP2,NBP2,RL,0,WKAREA,IER)
                                                                         002880
     DO 29 I=1,NBP2
                                                                         002890
   29 PS(I)=RL(I)
                                                                         002900
      WRITE (6,400) (I,PS(I),I=1,NBP2)
                                                                          002910
      WRITE(6,1014)
                                                                         002920
                                                                          002930
      DO 1101 I=1,NBP2
      SU11=0.
                                                                          002940
      DO 1102 J=1,NBP2
                                                                          002950
      SUM=SUM+RMX(I,J)*PS(J)
                                                                         002950
1102 CONTINUE
                                                                          002970
1101 RB(I)=SUM-RLX(I)
                                                                          002980
     WRITE(6,1103)(I,RB(I),RB(I+NBP),I=1,NBP)
                                                                          002990
      DO 9 I=1,NFP
                                                                          003000
     H(I)=0.0
                                                                          003010
```

	BMX(I)=0.0	003020
	BMY(I)=0.0	003030
•	CONTTRUE	003040
		003040
	DO 14 1=1,N/P	003050
	R1X=XF(I)	0030 60
	R1Y=YF(I)	003070
	DO 13 1=1-NTP	003060
		003000
	K5X=X1()	003090
	R2Y=YI(J)	003100
	Z1=R1X-R2X \$ Z2=R1Y-R2Y \$ Z15=Z1##2 \$ Z25=Z2##2	003110
	E(M(1) = A) OC((7) AD(M72) = A2AD(25 + 725) / A2)	003120
		003120
	PURCES=ALUG(((ZI+R3=Z2)=#2+R4S=Z2S)/A2)	003130
	Z3=Z1+P1#Z2 \$ Z4=Z1+R3*Z2	003140
	IF(ABS(Z3).GT.1.E-6)GOTO 80	003150
		003160
		003100
	6010 81	003170
80	FUNCN1=ATAN(R2+Z2/Z3)	003 180
81	IF(ABS(Z4).GT.1.E-6)GOTO 82	003190
	FUNCN3=PI/2.	003200
	6010 83	003210
••		003210
8Z	FUNCN3=ATAN(R44Z2/Z4)	003220
83	CONTINUE	003230
	IF(FUNCH1, LT.O.)FUNCH1=FUNCH1+PI	003240
	TELEVINGNA LT A JEUNCHA-EUNCHAADT	003250
	PURCPI=((ZI+RI*Z2)**2-R25*ZL5)*(FUNCLI-3.)-4.*R2*Z2*(ZI+RI*Z2)	003260
•	++FURICN1	003270
	FUNCR3=((Z1+R3*Z2)**2-R4S*Z2S)*(FUNCL3-3.)-4.*R4*Z2*(Z1+R3*Z2)	003280
	MELINCN 3	003290
	EINICE1 = D9#79#(71+D1#79)#(EINICI1-1-1-1+((71+D1#79)##9-09##79C)#EINIC	1003300
		1003300
	FUNC53=R4+22+(21+R3+22)+(FUNCL3-3:)+((21+R3+22)++2-R45+225)+FUNCh	3003310
	WXX=CONST1#FUNICL1+CONST2#FUNICL3+CONST3#(FUNICN1-FUNCN3)	003 320
	WXY=CONST5#FUNCL1+CONST6#FUNCL3+CONST7#(FUNCN1-FUNCN3)	003330
	HYY=CONSTAFFUNC(1+CONSTG#FUNC(3-CONST10#(FUNCN1-FUNCN3)	003340
	$\mathbf{H}(\mathbf{T}) = \mathbf{H}(\mathbf{T}) \perp (\mathbf{C}) = \mathbf{C} + $	003350
	W(1)=W(1)+(CON3)1*(CON3)2*(CON3)3*(CON3)3*(CONC3))	003355
•	+#COEF1#QLOAD	003350
	MX=-COEF2#(D11#WXX+D12#WYY+2.#D16#WXY)#QLOAD	0033 70
	MY=-COEF2*(D12*WXX+D22*WYY+2,*D26*WXY)#9LOAD	003350
		003390
		003370
	BRX(I)=BRX(I)+RX	003400
	BMY(I)=BMY(I)+MY	003410
13	CONTINUE	003420
14	CONTINUE	003430
• •		003450
		003440
	R1X=XF(I)	003450
	R1Y=YF(I)	003 460
	00 15 JEL.NPP2	003470
		003480
		003480
	R2T=(TTB(J+1)+TTB(J))/2.0	003490
	IF(NOPTION.EQ.2.OR.J.NE.NBP)GOTO 11	0035 00
	R2X=(XXB(1)+XXB(NBP))/2.	003510
	P2Y=(YYB())+YYB(NBP))/2	003520
••		003530
**		003530
	Z1=R1X-R2X \$ Z2=R1Y-R2Y \$ Z15=Z14#2 \$ Z25=Z2#42	003540
	FUNCL1=ALOG(((Z1+R1#Z2)##2+R25#Z25)/A2)	0035 50
	FUNCL3=AL03(((Z1+R3#Z2)##2+R4S#Z2S)/A2)	003550
	73=71+01+72 4 74=71+03+72	003570
	BJ-BATTATEC V EVTERTRJEC VELANATATE V EVTERTRJEC	
	11(4D3(23).61.1.2-0 /6UIU 40	003580
	FUNCN1=PI/2.	003 590
	GOTO 91	003600
90	FUNCH1=ATAN(R24Z2/Z3)	003610

91	IF(ABS(Z4).GT.1.E-6)GOTO 92	003620
	FUNCN3=PI/2.	003630
	GOTO 93	003640
92	FUNCH3=ATAN(R4#Z2/Z4)	003650
93	CONTINUE	003660
	TE(EUNCH1.LT.D.)EUNCH1=EUNCH1+PT	003670
	IF (FUNCH3.LT.O.)FUNCH3=FUNCH3+PI	003680
	FINCD1=((714D1872)842-D254725)8(FINC11-3)-6 8026728(714D1872)	003690
	ANFINONI	003700
	FINCD3=((7)+D3+72)++2_D46+726)+(FINC)3_3_3_}A +D4+79+(7)+D3+72)	003710
	TONCEJ-(E2*KJ*E2/K*E*K*J*EE)/*(TONCEJ-J.)-*.*K**EE*(E2*KJ*E)////////////////////////////////////	603720
	- EIBICC1 + D24794/71xD1472)#/EIBIC11_3_3_)x/(71xD1472)##9_D9C479C \#EIBIC	11003720
	FUNC31-RE*22*(21*RI*22)*(FUNC1-3,)*((21*RI*2)**2*R23*23)*FUNC	12003730
	FU 1.33-R4*22F(21+R3*22)*(FUNCL3-3, 14(121+R3*22)**********************************	003750
		003/50
		003/50
	WTT=CONST&+FUNCLI+CONST9+FUNCL3-CUNSTIC+(FUNCNI-FUNCN3)	003//0
	W(I)=W(I)+(CONST1#FUNCR1+CONST2#FUNCR3+CONST3#(FUNCS1-FUNCS3))	003780
	+#COEF1#PS(J)	003790
	Mx=-C0EF2#(D11*Wxx+D12*Wyy+2.#D16*Wxy)#PS(J)	003800
	MY=-COEF2#(D12#WXX+D22#WYY+2.#D26#WXY)# PS (J)	003810
	HXY=-COEF2#(D16*WXX+D26*WYY+2.*D66*WXY)*FS(J)	003 820
	BMX(I)=BMX(I)+MX	003830
	BNY(I)=EMY(I)+HY	003640
15	CONTINUE	003850
16	CONTINUE	003860
	WRITE(6,600)(I,XF(I),YF(I),W(I),BMX(I),BMY(I),I=1,NFP)	003570
	END	003880

40,100,181,30.E6,2.0E6,0.3,8.8E4,80.,0.4	003900
0.,0.	003910
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,10*-5.,	003920
-4.5,-3.5,-2.5,-1.5,-0.5,0.5,1.5,2.5,3.5,4.5,10*5.	003 930
10*-5.,-4.5,-3.5,-2.5,-1.5,-0.5,0.5,1.5,2.5,3.5,4.5,	003940
10*5.,4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5	0039 50
10*0.,10*-1.,10*0.,10*1.	003950
10#-1.,10*0.,10*1.,10*0.	003970
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	003 980
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	003990
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	004000
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	004010
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	004020
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	004030
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	004040
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	004050
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	004060
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	004070
10*-4.5,10*-3.5,10*-2.5,10*-1.5,10*-0.5,	004080
10+0.5,10+1.5,10+2.5,10+3.5,10+4.5	004090
IF(DET.GT.0.)60T0 40	011011

APPENDIX H

COMPUTER PROGRAM FOR THE VERIFICATION OF EQUATIONS USED FOR ANISOTROPIC PROBLEMS

APPENDIX H

COMPUTER PROGRAM FOR THE VERIFICATION OF EQUATIONS USED FOR ANISOTROPIC PROBLEMS

	PROGRAM PLCU	CHK(INPUT,OUTFUT,TAPE5=INPUT,TAPE6=CUTPUT)	000001
C			000002
С	********	*****	000003
Ċ			000004
Ē	THIS IS TO DOUBLE	E CHECK THE ANISOTROPIC PLATE PROGRAM.	000005
č	USING & SIMPLY SI	UPPORTED SQUARE PLATE.	000006
ř			000007
ř	REQUIRED THRUT V		000007
ř	REGCIRED INFOR W		000000
ř			000007
L C		-NUMBER OF EUGIDARI FUINIS	000010
L A	NIP	FNUMBER OF INTERNAL LUAU PUINTS	000011
	NFP	ENUMBER OF FIELD PUINTS	000012
C	XB+TB	=FOINTS UN B AT WHICH B.C. APE SATISFIED.	000013
	BANX, BANY	=COMFORENTS OF UNIT NORMAL TO B AT XB, YB.	000014
C	XXB, YYB	ZEND POINTS OF MESHES AROUND B WHERE FICTITIOUS	000015
С		FORCES ARE ASSIGNED.	000016
С	XF,YF	=FIELD POINTS	000017
С	XI,YI	=INTERNAL LOAD POINTS	000018
С	VX	=POISSONAS RATIO IN X DIRECTION	000019
С		DUE TO STRESS IN Y DIRECTION	000020
С	EX,EY	=YOUNG*S MODULI IN X AND Y DIRECTIONS,	000021
С		RESPECTIVELY	000022
Ċ	GXY	SHEAR MODULUS	000023
Ĉ	HVALUE	PLATE THICKNESS	000024
č	RADTUS	FRADIUS OF THE FICTITIOUS CIPCULAR PLATE OF WHICH	000025
ř		THE DISDLACEMENT AT THE CIDCUMEEDENTIAL BOUNDADY	000026
ř		TE SET TO ZEDO	000027
ř			0000027
			000020
		***************************************	000027
		(AA) VB(AA) VVB(A)) VVB(A))	000030
	DINENSION AB	(40),1D(40),**D(01),11D(01)	000031
	DIRENSION XI	(100), 11(100), UEL(2)	000032
	DIMENSION RL	X(80), R8(80), R8(80, 80), P3(80), KAREA(80), RL(80)	000033
	DIMENSION XF	(161), TF(161), W(161), BHX(161), BHT(181)	000034
	DIMENSION BAN	NX(40), EANY(40), RMX(80,80)	000035
	DIMENSION BH	XX(151), EMYY(181), BXY(181)	000035
	DIMENSION X8	1(40),YE1(40),XXB1(81),YYB1(81),XI1(100),YI1(100)	000037
	DIMENSION XF	1(181),YF1(181),BANX1(40),BANY1(40)	000035
	REAL AVECTOR	(5),MX,MY,HXY	000039
	COMPLEX ERROR	R,RODT(4)	000040
С			000041
	100 FORMAT(#0LOC	T#,6X,#XB#,8X,#YB#,7X,#ANX#,8X,#ANY#,4X,#NBTYPE#,	000042
	+/#0#/(I5,4F1)	0.4,15))	000043
	110 FORMAT(#OLOC	T#,11X,#XXB#,14X,#YYB#/#0#/(I4,8X,F9.2,8X,F9.2))	000044
	200 FORMAT (#0LO	CT#,11X,#XI#,17X,#YI#,14X,/#0#/	000045
	+(14.11X.F6.3	,11X,F6,3))	000046
	300 FORMAT (#0LO	CT#.19X.#RLD#.34X.#RLS#/#0#/(14.8X.E20.8.16X.	000047
	1F20.A))		000048
	400 FORMAT (#110	CT#.19X.#PSP#/#0#/(T4.8X.F20.8))	000049
	500 FORMATI ADI OC'	T#.11%.#XF#.17%.#YF#/#0#/(14.11%.FA.3.11%.FA.3))	800050
	AND FORMATINAL A	x, dNODFd, 19XdXFd, 16X, dYFd, 16Y, dWd, 16X, dRNYd,	000051
	114, JENNY 11111	N, TIA, 989A 1A, 889A 191)	000051
	11041401114114	TVIJAVIST SV.AVIJSSV.ASIJ TVIDIT VALLER	000032
	JUU PURMATTINIJE	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	000053
	▼# TYP# # #j↓,]] 	000054
	+/1HU,F EX = 1	FICLU.41F ET = FICLU.41F VX = FIPD.21F GXT = FIELU.41	000055
	+/1HU,F RADIU	S UP INE PLATE = F,	000055
	+F7.1,# THIC	KNESS OF THE PLATE #, F6.31	000057
	703 FORMAT(///,1	X, #THE COEFFICIENTS OF THE CHARACTERISTIC #,	000058
	+#FOLYNOMIAL /	ARE\$,/1X,5E12.5)	000059

```
704 FCRMAT(1H0,#THE FOUR ROOTS OF THE CHARACTERISTIC EQUATION:#.
                                                                           000060
     +(/1X,2E12.5))
                                                                           000061
  705 FORMAT(1H0, # ERROR FOR ROOT NO. #,12,2X,2E13.4,# IS #,2E10.4)
                                                                           000062
  707 FORMAT(1H0, #THE FOUR CONSTANTS ARE :#,
                                                                           000063
     +/1X,#ALPHA = #,E10.4,#
                                 BETA = #,E10.4,
                                                                           000064
            GAMMA = #,E10.4,#
     42
                                   LUMEDA = #,E10.4)
                                                                           000065
 1103 FORMAT(J5,5X,E20.12,5X,E20.12)
                                                                           000066
 1014 FCPMAT(1H1,#THE FOLLOHING IS A LIST OF DOUBLE CHECKING OF B.C.S#, 000067
     1//,4X#NBP#10X,#B.C. 1#,13X,#B.C. 2#,//)
                                                                           000068
  708 FOPHAT(1H0, #RADIUS = #, E10.3,#
                                         DIST1 = #,#6.1,
                                                                           000069
            DIST2 = #.F6.1)
                                                                           000070
    +2
  801 FORMAT(1H1, #THE BENDING RIGIDITIES APE -----#,
                                                                           000071
     +/1X,#D11 = #,E10.4,# D12 = #,E10.4,# D22 = #,E10.4,
+/1X,#D65 = #,E10.4,# D16 = #,E10.4,# D26 = #,E10.4)
                                                                           000072
                                                                           000073
  805 FCRMAT(1H1, #THE AIJ ARE ---#,/1X,#A11= #,E15.7,# A12= #,
                                                                           000074
     +E15.7,# A22= #,E15.7,# A66= #,E15.7,# A16= #,E15.7,
                                                                           000075
     +# A26= #,E15.7)
                                                                           000076
                                                                           000077
C
C INFUT VALUES .....
                                                                           000078
C
                                                                           000079
      READ(5, +)NBP, NIP, NFP, EX, EY, VX, GXY, RADIUS, HVALUE
                                                                           000080
      READ(5,4)(XB(I),I=1,NBP)
                                                                           000081
      READ(5,*)(YB(I),I=1,NBP)
                                                                           000082
      READ(5,*)(BANX(I),I=1,NBP)
                                                                           000083
      READ(5,#)(PANY(I),I=1,NEP)
                                                                           000084
      READ(5,#)(XI(I),I=1,NIP)
                                                                           000085
      READ(5,*)(YI(I),I=1,NIP)
                                                                           000086
      NBP2=NBP#2$NBP2P1=NBF2+1
                                                                           000087
      NOFTION=1
                                                                           000085
      PI=4.#ATAN(1.)
                                                                           000089
С
                                                                           000090
C
                                                                           000091
      NCOUNT=1
                                                                           000092
      A16=C. $ A26=0.
                                                                           000093
      Al1=1./EX $ A12=-VX/EX $ A22=1./EY $ A66=1./GXY
                                                                           000094
      WRITE(6,805)A11,A12,A22,A66,A16,A26
                                                                           000095
      DET=(A11#A22-A12##2)#A66+2.#A12#A16#A26-A11#A26##2-A22#A16##2
                                                                           000095
      ZZ=HVALUE##3/(12.#DET)
                                                                           000097
      D11=(A22#A66-A26##2)#ZZ $ D22=(A11#A66-A16##2)#ZZ
                                                                           000098
      D12=(A16+A26-A12+A66)+ZZ $ D66=(A11+A22-A12++2)+ZZ
                                                                           000099
      D16=(A12*A26-A22*A16)*ZZ $ D26=(A12*A16-A11*A26)*ZZ
                                                                           000100
      RADIUS=80. $ DIST1=2.0 $ DIST2=2.
                                                                           000101
      DK=GXY*HVALUE*#3/12. $ VY=EY*VX/EX
                                                                           000102
      DX=D11 $ DY=D22 $ D3=DX#VX+2.#DK
                                                                           000103
      H=D3 $ RHO=H/SQRT(DX+DY)
                                                                           000104
      WRITE(6,700)NBP,NIP,NFP,EX,EY,VX,GXY,
                                                                           000105
     +RADIUS, HVALUE
                                                                           000105
      WRITE(6,708)RADIUS,DIST1,DIST2
                                                                           000107
      WRITE(6,709)DX,DY,FHO
                                                                           000108
  709 FOFMAT(///1X, #DX = #, E15.7, 5X, #DY =#, E15.7, 5X, #RHO =#, F10.5)
                                                                           000109
      X0=5.0$Y0=5.0
                                                                           000110
      XF(1)=10.-X0$YF(1)=-10.+Y0
                                                                           000111
                                                                           000112
      DO 41 I=2,11
      XF(I)=XF(I-1)-0.5
                                                                           000113
   41 YF(I)=YF(I-1)
                                                                           000114
      DO 42 J=1,10
                                                                           000115
      K=J+17
                                                                           000116
      DO 42 I=1,11
                                                                           000117
      XF(I+K)=XF(I)
                                                                           000118
   42 YF(I+K)=YF(I+K-17)+1.0
                                                                           000119
```

	XF(12)=10X0\$YF(12)=-9.5+Y0	000120
	DO 43 I=13.17	000121
	XF(I)=XF(I-1)-1.0	000122
43	YF(])=YF(]-])	000123
	DO 44 J=1,9	000124
	K=J#17	000125
	DO 44 I=12,17	000126
	XF(I+K)=XF(I)	000127
- 44	YF(I+K)=YF(I+K-17)+1.0	000128
	DEL(1)=(10.+2.#DIST1)/10.	000129
	DEL(2)=(10.+2.#DIST1+2.#DIST2)/10.	000130
	XXB(1)=5.+DIST1-DEL(1)\$YYB(1)=-5DIST1	000131
	XXB(41)=5.+DIST1+DIST2-DEL(2)\$YYB(41)=-5DIST1-DIST2	000132
	DD 28 J=1,2	000133
	DELT=DEL(1)\$IF(J.EQ.2)DELT=DEL(2)	000134
	DO 25 I=2,10	000135
	K=I	000135
	IF(J.EQ.2)K=K+40	000137
	XXB(K)=XXB(K-1)-DELT	000138
25	YYB(K)=YYB(K-1)	000139
	DO 26 I=11,20	000140
	K=I	000141
	IF(J.EQ.2)K=K+40	000142
• •	XXB(K)=XXB(K-1)	000143
26	YYB(K)=YYE(K-1)+DELT	000144
	DD 27 I=21,30	000145
		000146
	IF(J.EQ.Z)K=K+40	000147
		000148
21	$\frac{110(K) = 110(K-1)}{100}$	000149
		000150
		000151
	17(J.E4.2) K=K+40	000152
	XXD(K)=XXD(K-1)	000153
60	TID(K)-TID(K-1)-UCLI	000154
	242(21)-YVD(41) A LID(01)-LID(41)	000155
	FT1-V. UDTTE(4.807)CUT	000155
807	ENCHATING WARA LIVEN CHT TO V ET I EV VOECDEES	600157
007	ENTEDNIADIZIAN	000150
		000157
	DO 301 T=1.181	000160
	YE1(T)=YE(T)=COSDAYE(T)=STND	000101
101	YE1(1)=_YE(1)=CODEVIT(1)=J1(F	000163
301	nn 302 T=1.40	000164
	XB)(T)=XB(T)*COSP+YB(T)*STNP	000165
	YR1(T)=-XR(T)#STNP+YR(T)#COSP	000165
	BANX1(I)=BANX(I)#COSP+BANY(I)#SINP	000167
302	BANY1(I)=-BANX(I)#SINP+BANY(I)#COSP	000168
	DO 303 I=1.81	000169
	XXB1(I)=XXB(I)+COSP+YYB(I)+SINP	000170
303	YYB1(I)=-XXB(I)#SINP+YYB(I)#COSP	000171
	DO 304 I=1,100	000172
	XI1(I)=XI(I)#COSP+YI(I)#SINP	000173
304	YI1(I)=-XI(I)#SINP+YI(I)#COSP	000174
	WRITE(6,100)(I,XB1(I),YB1(I),BANX1(I),BANY1(I),I=1.NBP)	000175
	WRITE(6,110)(I,XXB1(I),YYE1(I),I=1,NSP2P1)	000176
	WRITE(6,500)(I,XF1(I),YF1(I),I=1,NFP)	000177
	WRITE(6,200)(I,XI1(I),YI1(I),I=1,NIP)	000178
		000179

.

	SINP2=SINF##2 \$ SINP4=SINP2##2	000150
	CCS2P=CCS(2.*PHI) \$ SIN2P=SIN(2.*PHI)	000181
	D11=DX*COSF4+2.#D3*SINF2*COSF2+DY*SINP4	000182
	D22=DX=SINF4+2.#D3#SINF2#COSP2+DY#COSP4	000183
	D66=DK+(DX+DY-2.#D3)#SINP2#COSF2	000184
	D12=DY#VX+(DX+DY-2.#D3)#SINP2#COSP2	000185
	D16=0.5*(DY*SINP2-DX*COSP2+D3*COS2P)*SIN2P	000185
	D24=0.5*(DY*CCSP2-DX*SINP2-D3*CCS2P)*SIN2P	000187
	WRITE(6,801)D11,D12,D22,D66,D16,D26	000188
	AVECTOR(1)=1.0 \$ AVECTOR(2)=4.#D26/D22	000169
	AVECTOR(3)=2.#(D12+2.#D66)/D22	000190
	AVECTCR(4)=4.#D16/D22 \$ AVECTCR(5)=D11/D22	000191
	<pre>krite(6,703)(Avector(1),1=1,5)</pre>	000192
	CALL ZFOLR(AVECTOP,4,ROOT,IER)	000193
	WRITE(6,704)(RODT(I),I=1,4)	000194
	DO 705 I=1,4	000195
	ERROP=AVECTOR(1)*RCOT(I)**4+AVECTOR(2)*ROOT(I)**3+AVECTOR(3)*	000196
•	<pre>+RODT(I)**2+AVECTOR(4)*POOT(I)+AVECTOR(5)</pre>	000197
706	HRITE(6,705)I,ROOT(I),ERROR	000193
	R1=REAL(ROOT(1)) \$ R2=AIMAG(ROOT(1))	000197
	R3=PEAL(ROOT(3)) \$ R4=AIMAG(ROOT(3))	000200
	WRITE(6,7C7)R1,R2,R3,R4	000201
	R15=R1**2 \$ R25=R2**2 \$ R35=R3**2 \$ R45=R4**2	000202
	CONSTG=(R1-R3)**2+(R2+R4)**2 \$ CONSTH=(R1-R3)**2+(R2-R4)**2	000203
	COEF1=1./(8.#PI#D22*CONSTG*CONSTH) \$ COEF2=2.#COEF1	000204
	CONST1=((R1-R3)##2-(R25-R45))/R2	000205
	CUNST2=((R1-R3)##Z+(R2S-R4S))/R4	000206
	CONST3=4.#(R1-R3)	000207
	CUNS15=((R1-2,#R3)#(R15+R23)+R1#(R35+R45))/R2	000208
	CUNS16=((R3-2.#R1)*(R35+R45)+R3*(R15+R25))/R4	000209
		000210
		000211
	CONCIN-4 #(B)#(D78+D4C) D3#(B)0+D40))	000212
	GU(3)10-4.*(K1*(K33*K43)*K33*(K13*K23))	000213
	NO E Tel NBD	000214
	DU 5 1-1105- DI (T)=0 0	000215
	RL(1)-0.0 R((TAN30)=A A	000218
	B1Y=YB1(T)	000217
	B1Y=YB1(T)	000219
		000217
		000220
		000222
	P2X=X11(J)	000223
	R2Y=YI1(J)	000224
	Z1=R1X-R2X \$ 72=R1Y-R2Y \$ 215=71+#2 \$ 725=72##2	000225
	ANXSEANX#42 & ANYSEANY#42 & ANXYEANX#ANY	000225
	FUNCL1=ALOG(((Z1+R1+Z2)+++2+R25+Z25)/A2)	000227
	FUNCL3=ALDG(((Z1+R3*Z2)**2+R45*Z25)/A2)	000228
	Z3=Z1+R1+Z2 \$ Z4=Z1+R3+Z2	000229
	IF(ABS(Z3).GT.1.E-6)GOTO 60	000230
	FUNCH1=PI/2.	000231
	60TO 61	000232
60	FUNCN1=ATAH(R2*Z2/Z3)	000233
61	IF(AB5(Z4).GT.1.E-6)GOTO 62	000234
	FUNCN3=PI/2.	000235
	60T0 63	000236
62	FUNCN3=ATAN(R4#22/24)	000237
63	CONTINUE	000238
	IF(FUNCH1.LT.O.)FUNCH1=FUNCH1+PI	000239

.

•

```
IF(FUNCN3.LT.0.)FUNCN3=FUNCN3+PI
                                                                      000240
   AA=D11#ANXS+D12#ANYS+2.#D16#ANXY
                                                                      000241
   BB=ANXS+D12+D22+ANYS+2.+ANXY+D26
                                                                      000242
   CC=2.#D16#ANX5+2.#D26#ANY5+4.#D66#ANXY
                                                                      000243
  FUNCR1=((Z1+R1*Z2)**2-R2S*Z2S)*(FUNCL1-3.)-4.*R2*Z2*(Z1+R1*Z2)
                                                                      000244
  +#FUNCN1
                                                                      000245
  FUNCR3=((Z1+R3+Z2)++2-R45+Z25)+(FUNCL3-3.)-4.+R4+Z2+(Z1+R3+Z2)
                                                                      000246
  +#FUNCN3
                                                                      000247
   FUNC51=R2#Z2#(Z1+R1#Z2)#(FUNCL1-3.)+((Z1+R1#Z2)##2-R25#Z25)#FUNCN1000248
   FUNCS3=R4*Z2*(Z1+R3*Z2)*(FUNCL3-3.)+((Z1+R3*Z2)**2-R4S*Z2S)*FUNCN3000249
   MXX=CONST1+FUNCL1+CONST2+FUNCL3+CONST3+(FUNCN1-FUNCN3)
                                                                      000250
  WXY=CONST5+FUNCL1+CONST6+FUNCL3+CONST7+(FUNCN1-FUNCN3)
                                                                      000251
  WYY=CONST8*FUNCL1+CONST9*FUNCL3-CONST10*(FUNCN1-FUNCN3)
                                                                      000252
  RL(I)=RL(I)-QLOAD*(CONST1*FUNCR1+CONST2*FUNCR3+CONST3*
                                                                      000253
  +(FUNCS1-FUNCS3))
                                                                      000254
  RL(I+NBP)=RL(I+NBP)-QLOAD*(AA*WXX+BB*WYY+CC*WXY)
                                                                      000255
 6 CONTINUE
                                                                      000256
 5 CONTINUE
                                                                      000257
                                                                      000258
                                                                      000259
  DO 8 I=1,NBP
                                                                      000260
  R1X=XB1(I)
                                                                      000261
  R1Y=YB1(I)
                                                                      000262
   ANX=BANX1(I)
                                                                      000263
   ANY=BANY1(I)
                                                                      000264
  00 7 J=1,NBF2
                                                                      000265
  R2X=(XXB1(J+1)+XXB1(J))/2.
                                                                      000266
  R2Y=(YYB1(J+1)+YYB1(J))/2.
                                                                      000267
  IF(NOPTION.EQ.2.OR.J.NE.NBP)GOTO 33
                                                                      000268
  R2X=(XXB(1)+XXB(NBP))/2.
                                                                      000269
  R2Y=(YYB(1)+YYB(NBP))/2.
                                                                      000270
33 CONTINUE
                                                                      000271
   Z1=R1X-R2X $ Z2=R1Y-R2Y $ Z15=Z1**2 $ Z25=Z2**2
                                                                      000272
   ANXS=ANX##2 $ ANYS=ANY##2 $ ANXY=ANX#ANY
                                                                      000273
  FUNCL1=ALOG(((Z1+R1+Z2)++2+R25+Z25)/A2)
                                                                      000274
  FUNCL3=ALOG(((Z1+R3*Z2)**2+R45*Z25)/A2)
                                                                      000275
   Z3=Z1+R1#Z2 $ Z4=Z1+R3#Z2
                                                                      000276
   IF(ABS(Z3).GT.1.E-6)GOTO 70
                                                                      000277
   FUNCH1=PI/2.
                                                                      000278
  GOTO 71
                                                                      000279
70 FUNCH1=ATAN(R2+Z2/Z3)
                                                                      000280
71 IF(ABS(Z4).GT.1.E-6)GOTO 72
                                                                      000281
   FUNCH3=PI/2.
                                                                      000282
   60T0 73
                                                                      000283
72 FUNCH3=ATAN(R4+Z2/Z4)
                                                                      000284
73 CONTINUE
                                                                      000285
   IF(FUNCH1.LT.0.)FUNCH1=FUNCH1+PI
                                                                      000285
   IF(FUNCN3.LT.0.)FUNCN3=FUNCN3+PI
                                                                      000287
   AA=D11#ANXS+D12#ANYS+2.#D16#ANXY
                                                                      000288
  BB=AHXS*D12+D22*ANYS+2.*ANXY*D26
                                                                      000289
  CC=2.#D16#ANX5+2.#D26#ANY5+4.#D66#ANXY
                                                                      000290
  FUNCR1=((Z1+R1+Z2)++2-R25+Z25)+(FUNCL1-3.)-4.+R2+Z2+(Z1+R1+Z2)
                                                                      000291
  +#FUNCN1
                                                                      000292
  FUNCR3=((Z1+R3#Z2)##2-R45*Z25)#(FUNCL3-3.)-4.#R4#Z2#(Z1+R3#Z2)
                                                                      000293
  +#FUNCN3
                                                                      000294
  FUNCS1=R2#Z2#(Z1+R1#Z2)#(FUNCL1-3.)+((Z1+R1#Z2)##2-R25#Z25)#FUNCN1000295
  FUNCS3=R4#Z2#(Z1+R3#Z2)#(FUNCL3-3.)+((Z1+R3#Z2)##2-R4$#Z2$)#FUNCN3000296
  HOX=CONST1*FUNCL1+CONST2*FUNCL3+CONST3*(FUNCN1-FUNCN3)
                                                                      000297
  HXY=CONST5#FUNCL1+CONST6#FUNCL3+CONST7#(FUNCN1-FUNCN3)
                                                                      000298
  WYY=CONST8*****CL1+CONST9#FUNCL3-CONST10#(FUNCH1-FUNCH3)
                                                                      000299
```

С

С
	RM(I,J)=(CONST1#FUNCR1+CONST2#FUNCR3+CONST3*(FUNCS1-FUNCS3))	000300
	RM(I+NBP,J)=(AA*WXX+BB*WYY+CC*WXY)	000301
7	CONTINUE	000302
8	CONTINUE	000303
	DO 10 I=1,NEP	000 304
	DO 10 J=1,NEF2	000305
	RMX(I,J)=RH(I,J)	000306
10	RHX(I+NSP,J)=RH(I+NBP,J)	000307
	DO 1015 I=1,NEP2	000308
1015	RLX(I)=RL(I)	000309
C		000310
Č		000311
-	CALL LEGTIF(RM,1,NBP2,NBP2,RL,0,WKAREA,IER)	000312
	D0 29 I=1,NBP2	000313
29	PS(I)=PL(I)	000314
	WRITE (6,400) (I,PS(I),I=1,NBP2)	000315
	WRJTE(6.1014)	000316
	DO 1101 I=1,N3P2	000317
	SU*1=0.	000318
	DO 1102 J=1,N3P2	000319
	SUM=SUM+RHX(I,J)#PS(J)	000320
1102	CONTINUE	000321
1101	RB(I)=SUM-RLX(I)	000322
	WRITE(6,1103)(I,RB(I),RB(I+NBP),I=1,NBP)	000323
	DO 9 I=1.NFP	000324
	W(I)=0.0	000325
	BHX(I)=0.0	000326
	BHY(I)=0.0	000327
	BXY(I)=0.	000328
	BMXX(I)=0.	000329
	EMYY(I)=0.	000330
9	CONTINUE	000331
	DO 14 I=1,NFP	000332
	R1X=XF1(I)	000333
	RlY=YFl(I)	000334
	DO 13 J=1,NIP	00 0335
	R2X=XI1(J)	000335
	R2Y=YI1(J)	000337
	Z1=R1X-R2X \$ Z2=R1Y-R2Y \$ Z15=Z1**2 \$ Z25=Z2**2	000338
	FURICL1=ALOG(((Z1+R1*Z2)**2+R2S*Z2S)/A2)	000339
	FUNCL3=ALOG(((Z1+R3*Z2)**2+R4S*Z2S)/A2)	000 340
	Z3=Z1+R1#Z2 \$ Z4=Z1+R3#Z2	000341
	IF(ABS(Z3).GT.1.E-6)GOTO 80	000342
	FUNCH1=PI/2.	000343
	6010 81	000344
80	FUNCN1=ATAN(R2+Z2/Z3)	000345
81	IF(ABS(Z4).GT.1.E-6)GOTO 82	000346
	FUNCN3=PI/2.	000347
	GOTO 83	000345
82	FUNCN3=ATAN(R4*Z2/Z4)	000349
83	CONTINUE	000350
	IF(FUNCN1.LT.O.)FUNCN1=FUNCN1+PI	000351
	IF(FUNCN3.LT.O.)FUNCN3=FUNCN3+PI	000352
	FUNCE1=((Z1+R1+Z2)++2-R25+Z25)+(FUNCL1-3.)-4.+R2+Z2+(Z1+R1+Z2)	000353
		000354
	PUNCR3=((Z1+R3#Z2)##2-R43#Z25)#(FUNCL3-3.)-4.#R4#Z2#(Z1+R3*Z2)	000355
		000356
	PUNU51=RZ#ZZ#(Z1+R1#ZZ)#(FUNUL1-3.)+((Z1+R1#ZZ)##Z-RZ5#Z25)#FUNU	N1000357
	FUNU35=K4#22#121+K5#221#1FUNUL3-3. J41121+K5#22J##2-K45#225J#FUNU	846000258
	MYY- COUPIET & COUPEET & COUPIES & COUPIES & COUPEET & COUPLES)	000359

.

.

MXY=CONST5+FUNCL1+CONST6+FUNCL3+CONST7+(FUNCN1-FUNCN3) 000360
WYY=CONST8#FUNCL1+CONST9#FUNCL3-CONST10#(FUNCN1-FUNCN)	3) 000361
W(I)=W(I)+(CONST1*FUNCR1+CONST2*FUNCR3+CONST3*(FUNCS1-	-FUNCS3)) 000362
+#COEF1#QLOAD	000363
MX=-COEF2*(D11*WXX+D12*WYY+2.#D16*WXY)*QLOAD	000364
MY=-COEF2#(D12#WXX+D22#WYY+2.#D26#WXY)#QLOAD	000365
HXY=-COEF2#(D16#WXX+D26#WYY+2.#D66#WXY)#QLOAD	000366
BMX(I)=BMX(I)+MX	000357
RMY(T)=RMY(T)+HY	000369
RYY(T)=RYY(T)+HYY	000369
RMYY(T)=RMY(T)#COCD2+RMY(T)#CTND2_2 #RYY(T)#COCD#STND	000370
BMYY(T)=BMY(T)=COUPETDIN(T)=CARD2_2 =BYY(T)=COUPETDINC	000370
THE NECATIVE STON TE END & DEVEDEED ANDLE	000371
THE RECATIVE SIGN IS FOR A REVERSED ANGLE.	000372
	000373
	000374
	000373
MIX=XF1(I)	000378
NT1=1-1(T)	000377
	000376
R2X=(X)B1(J+1)+XXB1(J))/2.0	000379
R2Y=(YYE1(J+1)+YYE1(J))/2.0	000380
IF (NOPTION, EQ. 2. OR. J. NE. NBP) GOTO 11	000381
R2X=(XXE1(1)+XXB1(NBP))/2.	000382
R2Y=(YYB1(1)+YYB1(NBP))/2.	000353
11 CONTINUE	000384
Z1=R1X-R2X \$ Z2=R1Y-R2Y \$ Z1S=Z1**2 \$ Z2S=Z2**2	000385
FUNCL1=ALOG(((Z1+R1#Z2)##2+R2S#Z2S)/A2)	000385
FUNCL3=ALOG(((Z1+R3*Z2)**2+R45*Z2S)/A2)	000387
Z3=Z1+R1*Z2 \$ Z4=Z1+R3*Z2	000388
IF(ABS(Z3).GT.1.E-6)GOTO 90	000389
FUNCH1=PI/2.	000390
6 0T0 91	000391
90 FUNCH1=ATAH(R2+Z2/Z3)	000392
91 IF(ABS(Z4).GT.1.E-6)GOTO 92	000393
FUNCH3=PI/2.	000394
60T0 93	000395
92 FURICN3=ATAN(R4422/24)	000396
93 CONTINUE	000397
TELENCHI LT.O. IEUNCHIEFUNCHIAPT	000398
TELEUNCNE LT A JEUNCNEEFUNCNEADT	000399
EINICO1 = ((71 + D) + 72) + +2 = D26+726) + (FINCI 1 - 8) - 4 + D2+72+()	714P1#721 000400
**************************************	71483872) 800402
FUNCK3-1161*K3*663*663*6633*6633*1600663*3.7~4.*K4*66*17	
TTEUNUNG EINEEN-DALTAHITI -DINTAINEENEII B. 1.//71.01×701×40.04	
	5*225 J#FUNCN1000404
PUNCSS=R4+Z2+(Z1+R3+Z2)+(FUNCLS-3.)+((Z1+R3+Z2)+*Z-R4)	5+225 J+FUNLN3000405
NXX=CONST1#FUNCL1+CONST2#FUNCL3+CONST3#(FUNCN1-FUNCN3) 000406
NXY=CONST5*FUNCL1+CONST6*FUNCL3+CONST7*(FUNCN1-FUNCN3	J 000407
WYY=CONST84FUNCL1+CONST94FUNCL3-CONST10#(FUNCN1-FUNCN)	3) 000408
W(I)=W(I)+(CONST1*FUNCR1+CONST2*FUNCR3+CONST3*(FUNCS1)	-FUNC53)) 000409
+#CDEF1*P5(J)	000410
MX=-COEF2*(D11*WXX+D12*WYY+2.*D16*WXY)*PS(J)	000411
MY=-COEF2*(D12*WXX+D22*WYY+2.*D26*WXY)*PS(J)	000412
HXY=-COEF2#(D16#WXX+D26#WYY+2.#D66#WXY)#PS(J)	000413
BMX(I)=BMX(I)+MX	000414
BMY(I)=BHY(I)+HY	000415
BXY(I)=BXY(I)+HXY	000415
BMXX(I)=BMX(I)#COSP2+BMY(I)#SINP2-2.#BXY(I)#COSP#SINP	000417
BMYY(I)=BMX(I)*SINP2+BMY(I)*COSP2-2.*BXY(I)*SINP*COSP	000418
15 CONTINUE	000419

.

C

16	CONTINUE	000420
	WRITE(6,600)(I,XF(I),YF(I),W(I),BMXX(I),BMYY(I),I=1,NFP)	000421
	60T0(49,50,51,52,53)NCOUNT	000422
49	PHI=15. \$ NCOUNT=2	000423
	GOTO 804	000424
50	PHI=30. \$ NCOUNT=3	000425
	GOTO 804	000426
51	PHI=45. \$ NCOUNT=4	000427
	GOTO 804	000428
52	PHI=60. \$ NCOUNT=5	000429
	GOTO 804	000430
53	STOP	000431
	END	000432

40.100.1A1.30 F6.2 0F6.0 3.8 AF4.80 .0 4	800434
	000434
4.3,3.3,2.3,1.3,0.3,-0.3,-1.3,-2.3,-3.3,-4.3,10*-3.,	000435
-4.5,-3.5,-2.5,-1.5,-0.5,0.5,1.5,2.5,3.5,4.5,10*5.	000435
10*-5.,-4.5,-3.5,-2.5,-1.5,-0.5,0.5,1.5,2.5,3.5,4.5,	000437
10#5.,4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5	000438
10*0.,10*-1.,10*0.,10*1.	000439
10#-1.,10#0.,10#1.,10#0.	000440
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000441
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000442
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000443
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000444
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000445
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000446
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000447
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000448
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000449
4.5,3.5,2.5,1.5,0.5,-0.5,-1.5,-2.5,-3.5,-4.5,	000450
10*-4.5,10*-3.5,10*-2.5,10*-1.5,10*-0.5,	000451
10*0.5,10*1.5,10*2.5,10*3.5,10*4.5	000452

_

.

BIBLIOGRAPHY

.

BIBLIOGRAPHY

- 1. Timoshenko, S. and Woinowski-Krieger, S., <u>Theory of</u> <u>Plates and Shells</u>, McGraw-Hill Book Co., New York, (1959).
- Muskhelishvili, N. I., <u>Some Basic Problems of the Mathe-</u> matical Theory of Elasticity, Noordhoff, Holland, (1963).
- 3. Lourye, A., Application of Complex Variables Method to Plate Problems, Journal of Applied Mechanics, USSR Academy of Science, 4, (1940).
- 4. Green A. E. and Zerna, W., <u>Theoretical Elasticity</u>, Oxford University Press, New York, (1954).
- Stevenson, A. C., Complex Potentials in Two-Dimensional Elasticity, Proceedings, Royal Society of London, Series A, <u>184</u>, (1954).
- 6. Morkovin, M. V., <u>On the Deflection of Anisotropic Thin</u> <u>Plates</u>, Ph.D. Dissertation, University of Wisconsin, (1942).
- 7. Szilard, R., <u>Theory and Analysis of Plates</u>, Prentice-Hall, Inc., (1974).
- 8. Zienkiewicz, O. C., <u>The Finite Element Method</u>, McGraw-Hill Book Co., New York, (1979).
- 9. Altiero, N. J. and Sikarskie, D. L., A Boundary Integral Method Applied to Plates of Arbitrary Plan Form, <u>Computers</u> <u>and Structures</u>, (1979).
- 10. Michell, J. H., The Flexure of Circular Plates, Proceedings of the Mathematical Society of London, (1902).
- 11. Suchar, M., On Singular Solutions in the Theory of Anisotropic Plates, <u>Bulletin de l'Academie</u>, <u>Polonaise des</u> <u>Sciences</u>, <u>Series des Sciences Techniques</u>, <u>12</u>, (1964).
- 12. Mossakowski, J., Singular Solutions in the Theory of Orthotropic Plates, <u>Archives of Mechanics</u>, (1954).
- 13. Mossakowski, J., Singular Solutions in the Theory of Anisotropic Plates, Archives of Mechanics, (1955).

- 14. Lekhnitskii, S. G., <u>Anisotropic Plates</u>, Gordon and Breach Science Publishers, (1968).
- 15. Lekhnitskii, S. G., <u>Theory of Elasticity of an Anisotropic</u> Elasticity Body, Holden-Day, Inc., (1963).
- 16. Ambartsumyan, S. A., <u>Theory of Anisotropic Plates</u>, Technomic Publishing Co., Inc., (1970).
- 17. Jaswon, M. A. and Maiti, M., An Integral Equation Formulation of Plate Bending Problems, <u>Journal of Engineering</u> <u>Mechanics</u>, II, (1968).
- 18. Maiti, M. and Chakrabarty, S. K., Integral Equations for Simply Supported Polygonal Plates, <u>International Journal</u> of Engineering Science, <u>12</u>, (1974).
- 19. Jaswon, M. A., Maiti, M., and Symm, G. T., Numerical Biharmonic Analysis and some Applications, International Journal of Solids and Structures, 3, (1967).
- 20. Benjumea, R. and Sikarskie, D. L., On the Solution of Plane Orthotropic Elastic Problems by an Integral Method, <u>Journal of Applied Mechanics</u>, <u>39</u>, <u>Transaction of ASME</u>, <u>94</u>, (1972).
- 21. Altiero, N. J. and Sikarskie, D. L., An Integral Equation Method Applied to Penetration Problems in Rock Mechanics, Boundary Integral Equation Method: Computational Applications in Applied Mechanics, Edited by T. A. Cruse and F. J. Rizzo, ASME, AMD-11, (1975).
- 22. Bares, R. and Massonet C., <u>Analysis of Beam Grids and</u> <u>Orthotropic Plates</u>, Frederick Ungar Publishing Co., Inc., New York, (1968).
- 23. Vinson, J. R. and Brull, M. A., New Techniques of Solution for Problems in the Theory of Orthotropic Plates, <u>Pro-</u> <u>ceedings 4th U.S. National Congress of Applied Mechanics</u>, <u>University of California, Berkeley, June 18-21, (1962).</u>
- 24. Rajappa, N. R. and Reddy, D. V., Analysis of an Orthotropic Plate by Maclaurin's Series, <u>Journal of Royal Aeronautical</u> <u>Society</u>, <u>67</u>, (1963).
- 25. Mazumdar, J., A Method for Solving Problems of Elastic Plates of Arbitrary Shape, <u>University of Adelaide</u>, <u>Adelaide</u>, <u>Australia</u>, (1968).
- 26. Huber, M. T., Theory of Plates, L'vow, (1921).
- 27. Lekhnitzskii, S. G., Plane Statical Problem of the Theory of Elasticity of Anisotropic Body, <u>Prikladnaya Matematika</u> i Mekhanika, I, 1, (1939).

- 28. Ashton, J. E., Halpin, J. C., and Petit, P. H., Primer on Composite Materials: Analysis, Technomic Publishing Co., Inc., (1969).
- 29. Edited by Weeton, J. W. and Scala, E., Composites: State of Art, <u>Proceedings and Sessions of 1971 fall meeting</u>, <u>The Metallurgy Society of the American Institute of Mining</u>, <u>Metallurgical and Petrolium Engineers</u>, <u>Inc.</u>, <u>New York</u>, (1974).
- 30. Design with Composite Materials, The Institute of Mechanical Engineers, Great Britain, (1973).
- 31. Impact of Composite Materials on Aerospace Vehicles and Propulsion Systems, NATO Advisory Group for Aerospace Research and Development, Conference Proceedings, 112, (1972).

