THE EFFECT OF A MATH TRADING GAME ON ACHIEVEMENT AND ATTITUDE IN FIFTH GRADE DIVISION

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ABSTRACT

THE EFFECT OF A MATH TRADING GAME ON ACHIEVEMENT AND ATTITUDE IN FIFTH GRADE DIVISION

By

Frank E. Fishell

The purpose of this research study was to investigate the effects of a math trading game on achievement in division and attitude towards mathematics of fifth grade elementary students.

Eight intact fifth grade classes were selected from the Montcalm Area Intermediate School District. Four of the classes were randomly assigned to the treatment group (T_1) and four were randomly assigned to the control group (T_2). Each class studied a division unit prepared by the investigator for forty five to fifty five minutes per day for fifteen class days. The treatment group played a math trading game during this period and were shown a division process which used the principles of the trading game. The control group did only the division unit, using no manipulatives or other laboratory devices.

Each group was pretested with a division achievement test constructed by the researcher and an adapted version of

Dutton and Blum's attitude inventory. Univariate analysis of variance completed on the pretest and preattitude inventory revealed that T_1 and T_2 were not significantly different on the dependent variables of achievement and attitude toward mathematics at the beginning of the study. The author constructed parallel achievement posttest and delayed achievement posttest with reliability estimates of .90 and .92, respectively.

The achievement posttest and attitude inventory were administered the fifteenth day of the division unit. The delayed achievement posttest was administered six weeks later.

Univariate analysis of variance was used to assess the effect of the treatment, on achievement and attitude towards mathematics, by sex. Univariate analysis of variance was used with the repeated measures, split plot design to assess the treatment effect, over time, on achievement by sex. The achievement posttest and delayed posttest were the repeated measures, split into male and female plots. The two levels were T_1 and T_2.

The results of the study indicated:

1. The use of the math trading game did not significantly improve achievement in division at the fifth grade level.

2. The use of the math trading game did not significantly improve attitude toward mathematics of fifth grade students.
3. There were no significant sex differences related to achievement.
4. There were no significant sex differences related to attitude toward mathematics.
5. There was no significant effect, over time, on achievement associated with the playing of the math trading game.
6. To be termed successful in the division operation students must know the subtraction and multiplication facts.
7. Students can understand the division process and the place value system of numeration and still not be termed successful in division, as measured by achievement tests, because of slowness in completing problems.

This thesis is Dedicated

to

Ipha, my Wife,

to

Lyn and Valerie

Daughters,

and

to

Jay and Bret

Sons.

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CHAPTER I

THE PROBLEM

NEED AND PURPOSE

Games in mathematics are becoming widespread in both mathematics literature and in classroom usage. Some issues of the Mathematics Teacher and most issues of the Arithmetic Teacher, both journals of the National Council of Teachers of Mathematics, have articles about mathematical games and their use in the classroom. Some of these articles imply that games are the long sought after "fountain of easy learning."

Hirsch [1975] in his article "Graphs and Games," lists as his objectives:

> Objectives: The student will (1) discover and apply Euler's formulas regarding connected planar graphs and (2) play and analyze the game of Sprouts [p. 125].

Schlinsog [1968] talks about expectations of kindergarten children and says:

> The mathematics taught in kindergarten should not concentrate on the mastery of specific facts but, rather, on concepts and ideas. Games, manipulative devices, and group experiences can be used to expose children to mathematical concepts [p. 701].

Statements like this one by Rode [1971] appear in articles about games. "The game called 'Make a Whole' helps develop the concept of fractional numbers by using concrete examples" [p. 116].

Browne [1974] said, "the game of 'Tic-Tac-Toe' has long been used as a motivational device in mathematics instruction" [p. 128].

The above statements are not isolated cases, but are examples of statements made in many articles about mathematical games. There is, however, a very definite lack of conclusive research on games to justify these statements.

None of the above articles included any research evidence that games helped either achievement or attitude in mathematics. The eight issues of Volume 19 [1972] of The Arithmetic Teacher was selected by the author and a search of the volume revealed the following:

1. A total of twenty-one games were explained in the volume.
2. The January and April issues contained complete articles about using games in the mathematics classroom.
3. Six additional articles mentioned how useful games were for various purposes in the classroom.
4. Not a single mention was made in any of the above articles about any research which had been done with games in the mathematics classroom.

Will games help improve mathematical attitude? Will games improve mathematical achievement? These questions are difficult to answer. Many teachers include games of some kind in the mathematics education of elementary school children. Although the wide use of games probably is an indication that teachers believe games are helpful in teaching mathematics, the author is of the opinion that few teachers realize how games affect the achievement of students in mathematics or how games affect the attitude of students toward mathematics.

A review of the research literature does indicate mixed results on the use of games in helping students learn elementary mathematics. Some researchers have reported significant results while many more have reported non-significant results but do feel the need for more definitive research on the question of "How helpful are games in teaching mathematics?"

The author has asked many elementary teachers, "What is the most difficult topic in elementary mathematics to teach?" Many feel as did DeSpigno [1971] when he said, "Learning to work division problems is probably the most difficult arithmetic task for elementary school children to handle" [p. 373]. He then went on to state that he felt even when students had mastered the mechanics of division they had little or no understanding of the division process.

Coburn [1975] in the Michigan Educational Assessment Program Mathematics Interpretive Report, when reporting on the division objective of a one-digit divisor with dividend less than one hundred, for seventh grade, states "The objective is minimal and the results are disappointing" [p. 24]. He further suggests that curriculum changes be made in the grade placement of two or three digit divisors because of the "unsatisfactory results, overall, on division" [p. 24].

Copeland [1972] in his chapter on division of whole numbers states; "Division . . . is more difficult for children to understand than is addition, subtraction, or multiplication" [p. 168].

This is similar to what Cacha [1972] said about "Division of large numbers" being a difficult process for children to learn [p. 349].

The author feels that a trading game may help teach the division operation. Since the division operation requires application of several different concepts, (mainly place value, subtraction and multiplication), it was felt that a math trading game might help students learn the division concept because this game emphasizes place value and addition or subtraction. A complete description of the math trading game is given in Appendix A.

It is the purpose of this research study to investigate the effects of a math trading game on achievement in division and attitude towards mathematics of fifth grade elementary students. If mathematical games can be used to improve achievement and attitude in elementary mathematics, then games may indeed by a valuable contribution to the curriculum. Learning and teaching may both be more enjoyable.

DEFINITION OF TERMS

Cooperating classes: Eight intact classrooms in the Montcalm Area Intermediate School District which consented to participate in this study.

Division Unit: The division unit was an author constructed, three week unit in elementary division suitable for the fifth grade. Since the cooperating schools used different textbooks it was necessary to write material for use by all eight of the classes. The complete division unit is included in Appendix B.

T_1--Treatment Group: Four of the eight cooperating classes were randomly assigned to the treatment group. The treatment group played the math trading game at the same time they were doing the division unit.

T_2--Control Group: Four of the eight cooperating classes were randomly assigned to the control group. The control group did the three week division unit without the trading game or any other manipulatives.

Trading Game: A game that uses a system of barter and different colors for the trading principles. Complete instructions are given in Appendix A.

HYPOTHESES

The following research hypotheses were investigated.

H_1: The mean score of T_1 will be significantly higher than the mean score of T_2 on the post achievement test.

H_2: The mean scores of T_1 will be significantly higher than the mean scores of T_2 on the post attitude test.

H_3: The mean scores of T_1 on the post attitude test will be significantly higher than their mean scores on the pre-attitude test.

H_4: The mean score of the treatment group will be significantly higher on the post six week achievement test than the mean scores of the control group on the post six week achievement.

H_5: The mean score of girls will be significantly higher than the mean score of boys on the post achievement test.

H_6: The attitude of the girls in the treatment group will improve significantly more than the attitude of the boys in the treatment group.

To help control for confounding variables and increase external validity, analysis of variance will be used to test the following hypotheses.

H_7: The eight cooperating classes are all from the same population on the achievement pretest

H_8: The eight cooperating classes are all from the same population on the attitude pretest

All hypotheses will be tested at the $\alpha = .05$ level of significance.

INSTRUMENTATION

The data for this study will be obtained by testing T_1 and T_2 with the following instruments:

1. Initial Aptitude in Division (IAD-test)
2. Attitude Inventory Pretest
3. Achievement Posttest
4. Attitude Inventory Posttest
5. Delayed Achievement Posttest

The achievement tests were constructed by the author and the attitude inventory is adapted from the one developed by Dutton and Blum [1968]. These instruments appear in Appendix C.

METHOD AND PROCEDURE

The sample for this study was eight cooperating fifth grade division classes from the Montcalm Intermediate

School District. Four classes were randomly assigned to the control group. Both T_1 and T_2 were instructed in the division operation using the division unit. T_1 played the math trading game while T_2 did not.

The basic design for the study was the pretest, posttest, control group design of Campbell and Stanley [1963, p. 13].

Table 1. Pretest, Posttest, Control Group Design.

	Division Unit	Trading Game	Pre-Ach Test	Pre-Att Test	Post-Ach Test	Post-Att Test	Delayed Post-Ach Test
T_1	X	X	0	0	0	0	0
T_2	X		0	0	0	0	0

Of additional interest to this study was the investigation of division strategies used by students. Several students were interviewed individually to see what strategies they used in solving division problems.

THEORY

Educational theory is embedded in educational and learning psychology. Therefore these fields were examined for a theoretical look at attitudes and achievement. Bruner [1963] when writing about the Woods Hole Conference states:

> The fourth theme related to the desire to learn and how it may be stimulated. Ideally, interest in the material to be learned is the best stimulus to learning. . .[p. 14].

A desirable learning environment is one where learning takes place as the result of self motivation and the learner continues to be self-motivated. This usually is the result of successful experiences or as the saying goes, "Success breeds success."

Proctor [1965] found success experiences were a good technique for giving self confidence to slow learners and also for changing their attitudes toward mathematics. Earlier Hartung [1953] and Fehr [1967] found that to produce a favorable attitude toward mathematics the student must have repeated successful experiences. Successful experiences are pleasant experiences but there are also other kinds of pleasant experiences. An interesting game can be a pleasant experience even though the person is not successful in terms of winning the game. The contestant may feel that he is improving and will want to play the game again. There are also strategies to use where the level of competition seeks its own level and students are then usually more successful in terms of winning. Ashlock [1971] describes it this way:

> During normal instruction, the right answer is "expected;" but when playing a game, losing is acceptable. It is not possible for everyone to win all the time. Further, the competitive aspect of a game often encourages a quick response [p. 363].

It may be as Aiken [1972] states:

> Perhaps the soundest principle that the teacher can apply in trying to improve students' attitudes is to associate mathematics with things that the learner views as pleasant, interesting, or of potential value to him [p. 232].

Our interests, actions and feelings are greatly determined by our attitude towards something. An individual's attitude toward mathematics is determined by how he perceives himself in relation to mathematics. Johnson [1957] says that attitude is an emotional set or "predisposition to react in a characteristic way toward a given person, object, idea, or situation" [p. 114]. He also observes later that:

> If our students are to learn to like mathematics they must find pleasure in performing the learning activities in and out of the mathematics classroom. And our students will find pleasure in doing that which they can do successfully, that which seems significant in meeting their needs, that which gives them the status they esteem [p. 116].

What are a student's feelings toward mathematics? Does he have an interest of any kind in mathematics? The answers to these kinds of questions determine, in a large part, a person's attitude toward mathematics.

Biggs and MacLean [1969] emphasize "That it is an attitude toward learning" [p. 6] that must be developed if students are to be successful in mathematics. Willoughby [1970] writes:

> Attitudes of other people affect a student's desire to learn. In some cases, friends and family do not think learning is important, or they are actively opposed to it [p. 278].

Attitudes are formed or learned in the same way other things are learned. Aiken [1972] found that a student's attitude toward mathematics was related to the student's perception of the attitude and ability of teachers and parents. Tocco [1971] found student attitude toward mathematics was positively correlated with student achievement in mathematics.

These studies seem to indicate that attitudes are learned at both home and school. Learning takes place inside the individual, not externally, although external stimuli can affect the individual's attitude and hence affect learning.

Achievement and Evaluation of Achievement in Mathematics

Much has been written on the subject of student achievement in mathematics. The period of the late 1950s and 1960s was a curriculum revolution in an effort to improve student achievement in mathematics. When the discussion is about achievement one of the important aspects is the measurement or evaluation of achievement. The following comments are intended to show the complexity of evaluating achievement in mathematics.

One of the most monumental evaluation programs of recent times was the National Longitudinal Study of Mathematics Abilities (NLSMA) conducted by SMSG to evaluate the SMSG curriculum. Two people who were closely associated with this project were Edward G. Begle and James W. Wilson.

Begle and Wilson [1970] writing on student achievement in mathematics said there should be a whole range of pupil-performance criteria. Before presenting their model for mathematics achievement the authors said:

> The model described below assumes that mathematics achievement is a many component phenomenon. That is, mathematics achievement is not a unitary trait, and therefore a strategy needs to be available to insure sampling of a whole range of measures of mathematics achievement. One strategy is to classify mathematics achievement outcomes in two ways--first by categories of content, and second by levels of cognitive behavior assumed to be associated with the outcome or its measures [p. 372].

The following is the author's adaptation of Begle and Wilson's model for use with the basic operations in the elementary school. This study is interested, mainly, in the area of achievement in division and in evaluating that achievement.

Table 2. A Model for Mathematics Achievement.

	Addition	Subtraction	Multiplication	Division
Computation				
Comprehension				
Application				
Analysis				

The essential idea of the model is that measures of achievement, test items in this case, can be classified in two ways; (a) by categories of mathematical content and (b) by levels of behavior. Here the levels of behavior reflect the complexity of a task and not simply the difficulty of a task. In the model the categories of content are subject matter while the levels of behavior are computation, comprehension, application and analysis.

The levels of behavior are both hierarchical and ordered. The levels are ordered in the sense that analysis is more cognitively complex than application, which is in turn more cognitively complex than comprehension and so on. The levels are hierarchical in that an item at the application level may require both comprehension and computation skills for its solution.

This model is by no means unique in either of its dimensions but serves as an illustration of the complexity of evaluating achievement in mathematics.

Begle and Wilson [1970] conclude with this statement:

> In emphasizing measure of mathematics achievement, all cognitive outcomes, there is no intent to disregard affective outcomes such as attitude, appreciation, interest, and anxiety . . . The first concern of mathematics program evaluation, however, has been, and will continue to be, with cognitive outcomes, or achievement. The affective outcomes are supportive and important but secondary to, or at most, of equal importance to, achievement [p. 375].

Much could be written on the evaluation procedure, but the author feels the statement by Gagne' [1970] outlines the essential qualities of cognitive test items. Gagne' writes that when testing for immediate outcomes, the test items should have the following characteristics:

1. Pose questions that reflect directly the defined objective of the learning.
2. Conform to the class of performances that the

> learner has been told represent the achievement to be reached at the end of the learning session, and
>
> 3. Represent this class of performances without being specifically recallable as verbal chains from within the learning session itself (except in those instances in which specific verbal recall is itself the objective) [p. 342].

In summary, test items for measures of achievement are cognitive and should be of the appropriate level for the students being evaluated.

OVERVIEW

Chapter two is a review of the pertinent literature on games' effect on attitude toward mathematics and achievement in mathematics.

In chapter three the design of the study is explained. This includes a description of how the sample was selected, what measures were used to gather the data, the design used to set up the test of the hypotheses, a statement of the hypotheses in null form, and the appropriate analysis for testing the hypotheses and the procedures for conducting the case studies.

Chapter four contains an analysis of the results including an interpretation of the results and statements of significance of the results.

Chapter five contains a summary of the study and recommendations for future studies.

CHAPTER II

REVIEW OF LITERATURE

Extensive research has been conducted on various aspects of the modern mathematics curriculum. This research has been reported in newspapers, journals, periodicals, monograms and in books. However, games and their effect on achievement and attitude have not enjoyed the research interest shown some of the other topics. Although there has been interest in math games, relatively little research has been done on their effect upon the elementary school pupil and his achievement in mathematics. The review of research presented in this study has been divided into the following sections, (1) games and achievement and (2) games and attitude.

GAMES AND ACHIEVEMENT

Many games, expensive and inexpensive, are used for various reasons in elementary school today. Willoughby [1970], in the Sixty Ninth Yearbook of the National Society for the Study of Education, while writing on the subject of motivation, states, "If he (the teacher) cannot make the subject intrinsically interesting to all the children, he should try to add excitement by means of games" [p. 278].

Willoughby thinks of games as a motivational aid which will improve achievement.

Rosenbloom [1965] suggests that activities with immediate recreational value are good reinforcers for cognitive learning. He suggests mathematical games in which are embedded relevant mathematical principles.

Professor Zolton P. Dienes [1972] writing in Lamon's book, Learning and the Nature of Mathematics, says:

> Apart from such skilled questioning and suggestions there would, of course, have to be a large number of standard games in the classroom that could be played with the materials. These games would be ordered in a certain way, because some games would be too difficult to play without some previous games having been played. Most mathematical structures can be learned by playing skillfully contrived and excitingly motivating games of a mathematical nature [p. 64].

Biggs and MacLean [1969] make a strong plea for games in the teaching of basic arithmetic skills. They say "Practice in computational skills is just as effective and much more palatable when disguised in a game context" [p. 50]. These authors also feel that the weak student will benefit most and that "games can be a potent influence on changing children's attitudes toward mathematics" [p. 52].

Underhill [1972] in his book Teaching Elementary School Mathematics, talks about meaning, attention and child involvement as factors enhancing memory and the learning of mathematics. He later says that "games . . . will enrich their mathematical background, stimulate interest and curiosity and help maintain a high level of motivation" [p. 64].

Stanford [1970] set out to determine the effects, as measured by standardized achievement tests and teacher evaluation, of certain supplementary activities on mathematics achievement. The treatment group played academic games on a daily basis for fifteen minutes for eighteen weeks. The treatment group did significantly better than the control group at the .05 level of significance with girls making the greatest gain. He conducted a follow-up study five weeks later and found that during a five day period 141 students in the control group, not all different, returned to their room during the lunch period to play games. This study was conducted at Tupelo, Mississippi using seventh grade boys and girls.

Wynroth [1970] tested the hypothesis that "young school children can learn the natural numbers faster and better than by traditional teaching methods, if they initially learn its basic form (natural numbers) by playing a series of competitive games." His subjects were a first grade class and a kindergarten class in the Ithaca, New York school system. Two comparable first grade and a kindergarten class were used as controls.

Written work was introduced only after several months of previous verbal learning of concepts playing games. The written work consisted mainly of specially designed looseleaf workbooks, which were completed at the student's individual pace. The experiment was carried on for most of the 1968-1969 school year.

Wynroth's reported results were significant in favor of the experimental groups at the .05 level.

Part of a study of Addleman [1972] was to determine if mathematical games influenced achievement in mathematics. The treatment groups were teachers-in-training at East Texas State University. The treatment was carried out for eight weeks. She found a significant gain in numerical achievement in favor of the treatment group. The researcher concluded that games were likely to affect numerical achievement for in-training-teachers.

Crist [1968] claims he had long known about the educative powers of active game situations and felt educators had neglected the capabilities of active play as a means of fostering intellectual growth.

The purpose of Crist's study was to determine how well certain time-telling concepts could be developed in an active game situation at the third-grade level as compared with more traditional methods of teaching this material. The treatment was carried out for ten class days.

His results were not significant and he concluded that the active game method of teaching time-telling was as good as the more traditional methods. He failed to determine if attitude was significantly improved.

Henry [1974] investigated to determine if either the 'EQUATIONS GAME' a mathematics game or 'TAC-TICKLE', a strategy game could significantly improve students' cognitive abilities in mathematics.

Three experienced teachers assisted in the experiment. Each teacher instructed one control class, one 'EQUATIONS' experimental class and one 'TAC-TICKLE' experimental class. All nine classes were pretested and posttested with the Dutton Attitude scale and two batteries of the Cognitive Abilities Test. The researcher reported no significant difference in cognitive abilities scores for the treatment groups.

The experimental groups played the games for approximately half of each class period, every other day, for six weeks. The subjects were seventh grade mathematics classes.

The effects of a nonsimulation game and student teams on achievement in mathematics was investigated by Edwards [1972]. Two low ability and two average ability seventh grade classes participated in the study. One class in each group received the treatment while the other class was used as a control. The treatment classes played the game 'EQUATIONS' twice a week for nine weeks in addition to the regular instruction. The 'EQUATIONS' tournament was based upon team competition using four member teams.

Students were pretested and posttested with the computations subtest of the Stanford Achievement Test in mathematics and a divergent solutions test designed by the experimentor. The experimental classes had significantly greater gains on both the computations subtest and the divergent solutions test. The low ability treatment group obtained a posttest score on the divergent solution tests that was almost double their pretest score, while the low ability control group showed no gain.

These studies seem to indicate that games could be the solution for many of the problems associated with achievement in elementary mathematics. Achievement, however, is a multi-dimensional phenomenon and there are no easy answers as shown in the following studies where games failed to produce significant changes in achievement.

Bowen [1969] tested the hypothesis "that students who use the 'WFF'N PROOF' game as an instructional aid will demonstrate a significantly higher degree of proficiency in obtaining logical principles than peer models who are instructed through a structured approach." His subjects were three classes of fourth grade honor students with I.Q. range of 131-159.

One class was used as a control, another class received treatment A (textbook) while the third class received treatment B (WFF'N PROOF). The hypothesis was rejected at the .05 level. The girls receiving treatment A had the highest score, followed by the boys in the same group. Next were the girls in treatment B followed by the boys in treatment B. The correlation between raw scores and intelligence was $r = .79$ for treatment A and $r = .65$ for treatment B.

This study can be criticized because of the use of single classes for controls and treatments. Significance would be almost impossible to obtain under these circumstances.

Burgess [1969] conducted a study to determine whether a strategy involving regular use of mathematical games would prove effective for teaching mathematics to low achieving secondary students.

He found significant differences in achievement measures favoring the control group occurred with females on multiplication and division tests and younger subjects on addition and subtraction tests. No substantial relationship existed between ability, achievement, attitude or socioeconomic levels. The treatment was carried out for eight weeks.

Burgess concluded more definitive research is needed concerning the relationship between type of game and the learning of specific concepts. Since results indicated younger students benefited from the games strategy, for addition and subtraction, he suggests research be done with elementary students.

Although the literature is inconclusive regarding the benefits of teaching games, there are strong indications that games can help achieve specific objectives in elementary school mathematics.

A search of the expository and research literature reveals nothing has been reported about experiments with math trading games. The writer feels the trading games are important in elementary mathematics because they illustrate the basic principles of a place value number system.

In summary, the research regarding games and achievement is inconclusive. The research does suggest working with younger students and aiming the games at specific mathematical problem areas.

GAMES AND ATTITUDE

Jones [1968] used mathematical games as an integral part of his classroom instruction during a summer program for ninth-grade low students. The games were introduced to demonstrate to the students some practical applications of mathematics and ways of having fun with mathematical principles. He found significant positive attitude change toward mathematics at the end of the summer program. Programmed lectures were also a part of the instructional program and the researcher failed to do an analysis to determine whether programmed lectures or mathematical games were responsible for the attitude shift.

This early success of changing attitudes with the use of games, however, was short lived. The studies of the next several years did not support the findings of Jones.

Knaupp [1971] found that although he varied the instruction from the usual arithmetic lessons (he used manipulative models to illustrate the decimal numeration system) no changes were found in attitudes towards learning arithmetic. He did report a significant increase in understanding for all the groups however. His subjects were second grade students and he developed his own attitude scale.

In a seven week study with low-achieving, ninth grade students, Cech [1972] found that the use of calculators in the instructional program did not improve student attitude toward mathematics.

Similar results were found in the following study by Fink et al. [1971]. The researchers in a study at Indiana University, Bloomington, Indiana, examined whether the systematic use of motivational games by teachers of the culturally disadvantaged and educable mentally retarded can improve students' attending behaviors. Results showed that the use of games in either regular or educable mentally retarded classes did not significantly affect overall behavior. They did report, however, that individual teachers produced considerably different results. Noted were differences in deviancy patterns between two types of classes and games' effects on specific forms of deviant behavior.

Part of the study by Addleman [1972] was to determine if mathematical games influenced attitudes toward mathematics. The treatment was carried out for eight weeks with teachers-in-training. The researcher reported no change in attitude for the period although there was a significant increase in achievement.

This result is the same as Henry [1974] found when he used a mathematics game and a strategy game to see if they would improve students' attitude toward mathematics. He reported no significant difference in attitude for the treatment group. These subjects were seventh grade mathematics classes.

The literature of games' effects on attitude change is indeed scanty. The following studies are of some interest however in that they relate to attitude and achievement in elementary school.

Dutton and Blum [1968] found no significant difference in the attitude toward mathematics of boys and girls. They did report however, that younger children have a more positive attitude towards mathematics than older children. The pupils tested were sixth, seventh and eighth graders.

Keane [1969] found inconclusive data on the relationship between teachers' attitude toward mathematics and their pupils' attitudes. He reported that teacher attitude had no effect on pupil achievement and there was no relationship between pupil attitude and achievement. His attitude measure was the Dutton scale.

Neale [1969] in a review of the literature, found correlations between attitude and achievement in mathematics to be low, between .2 and .4, and that unfavorable attitudes increases as students continued through school.

Suydam and Weaver [1970] report that:

> first of all, there is no consistent body of research evidence to support the popular belief that there is a <u>significant</u> positive relationship between pupil attitudes toward mathematics and pupil achievement in mathematics. We have little research basis for believing that these two things are causally related [p. 2].

One should be very careful in interpreting correlation as cause and effect but correlation can be interpreted as the strength of relationship between two variables.

Although early studies seem to show no correlation between mathematical attitude and mathematical achievement, later studies are showing a correlation.

Burbank [1970] found significant correlations between students' mathematical attitude and mathematical achievement. This study involved 411 seventh grade students who were given the Dutton Mathematics Attitude Scale and SRA's Achievement Test Battery from which the mathematics score was used. The research also reported a significant correlation between parents' attitude and student attitude.

In a study of the relationship between attitude toward mathematics and selected pupil characteristics, Spickerman [1970] found favorable attitudes toward mathematics are associated with high course mark aspirations while unfavorable attitudes are associated with low course mark aspirations. This study was conducted in grades eight through twelve of a Kentucky high school. The researcher also found that students with unfavorable attitudes towards mathematics still value mathematics as a useful subject.

Tocco [1971] did a correlational study designed to examine the non-cognitive factors which might affect mathematical achievement in junior high school. The author found that student achievement in mathematics is directly related to student attitude toward mathematics.

Many studies on attitudes are of short duration but Beattie [1973] did a longitudinal study whose results may be more significant than the shorter studies. He investigated changes in attitude towards mathematics which occurred over a three year period in relation to mathematics achievement, sex, reading achievement and I.Q. Results showed attitudes stable over the period and that attitudes were less valuable for predicting achievement than were the other variables.

Aiken [1972] reviewed the latest research on the subject of attitudes toward mathematics. He feels the affective goals of mathematics instruction are represented by Objective VI of the National Assessment of Education Progress.[1]

> VI. Appreciation and use of Mathematics
>
> A. Reocgnizing the importance and relevance of mathematics to the individual and to society.
> B. Enjoyment of mathematics [Aiken, p. 229].

In attempting to find out what influences student attitude Aiken [1972] studied eighty-five girls and ninety-seven boys in the eighth grade and concluded:

> (1) there is a general variable of attitude toward mathematics that includes attitude toward routine computations, terms, symbols, and word problems;

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[1] This is from the National Assessment of Educational Progress. Mathematics Objectives. Ann Arbor, Michigan. NAEP, 1970.

> (2) there are sex differences in the direction and degree of the relationship of mathematics attitude to interests in other subjects and to personality characteristics;
>
> (3) attitude toward mathematics is positively correlated with grades in arithmetic and mathematics; and
>
> (4) attitude toward mathematics is related to students' perceptions of the attitude and abilities of their teachers and parents [p. 233].

He concedes that these results are not new but feels that when they are considered along with previous results;

> They point to the fact that since the relationships vary with educational level and sex, a whole complex of variables needs to be taken into account if more comprehensive statements about the origin, effects, and modifications of attitudes towards mathematics are to be made [p. 233].

This is in agreement with an early study by Poffenberger and Norton [1955] who set out to determine factors that were related to positive and negative attitudes towards mathematics.

Three hundred ninety entering freshmen at the University of California at Davis comprised the sample. Poffenberger and Norton found that ones attitude toward mathematics is a cumulative phenomenon, one experience building on another. Another result was that attitudes were developed in the home and in some cases before the child started school. Also, a significantly higher number of females disliked mathematics than males although they were comparable on other variables.

SUMMARY

This review resulted from a key word search of ERIC, Dissertation Abstracts, Periodical Literature and numerous bibliographies from research studies examined by the author.

The review of the literature makes the following conclusions seem appropriate. There is a definite need to find effective means of modifying and improving attitudes and achievement of elementary school children towards mathematics. One potential way of doing this is by using games to make mathematics more interesting and more satisfying for students.

Because of the complexities of achievement and attitudes, studies investigating these topics must also be complex if comprehensive statements about effects of achievement and modifications of attitudes towards mathematics are to be made. Correlation studies will not answer the important questions which remain.

CHAPTER III

DESIGN OF THE STUDY

THE SAMPLE

The population for this study was the fifth grade classes of the six smaller school districts of the Montcalm Intermediate School District. The district is comprised of seven separate districts, with Greenville being the only large city school district. The six schools which comprise the population of this study are in small rural towns with populations between 1,000 and 2,000 people in each town. The Intermediate District has a rural population of about 20,000.

A sample of eight cooperating classrooms was selected from this population by talking to individual teachers, explaining the project and asking if they would be willing to help. The investigator explained that participation would mean everyone would have to follow the same schedule, teach the same division unit and if selected, their class would be playing the math trading game. Not all of the teachers contacted were willing to participate in the project. The eight cooperating classrooms in this study were taught by five female and three male teachers.

Four of the cooperating classes were randomly assigned to T_1 and four classes were randomly assigned to T_2. The teachers in T_1 are two female and two male, while the teachers in T_2 are three female and one male.

The teachers are confounded with treatment in this study but very specific instructions given the teachers should minimize this problem. The author visited each classroom and very subjectively rated the teachers. On a seven point scale the author felt the teachers differed by less than one point, from approximately 4.5 to 5.5.

The students in the fifth grade classes are from the towns and rural areas surrounding each town. The rural areas are agricultural, producing agricultural products for shipment out of the area. The towns where the schools are located, are mostly dependent upon the rural areas for their business. There are some small factories but most of the people not engaged in agriculture drive outside the area to work in factories in the larger cities.

The number of students from each class who completed the five testing instruments are as follows: T_{11} had 21 students, T_{12} had 22 students, T_{13} had 17 students, T_{14} had 12 students, T_{21} had 20 students, T_{22} had 21 students, T_{23} had 25 students, and T_{24} had 20 students. T_{1i} and T_{2i} are the treatment and control groups respectively from Appendix D.

THE DIVISION UNIT

The division unit used in this study was constructed by the author. The unit was designed to achieve the following objectives:

1. Review previously learned material in division, mainly single digit divisors.
2. Introduce the division operation as the inverse of the multiplication operation.
3. Relate division as an operation to the idea of repeated subtraction for single digit and double digit divisors.
4. Show the impossibility of division by zero by having students remove the empty set in measurement division.
5. Introduce three digit divisors.
6. Introduce the standard division algorithm.
7. Have students work some application problems using the division operation.

The unit is mostly a composite of the various texts presently in use in the cooperating classrooms. The division process is introduced using sets and repeated subtraction. Later the dividend is written in expanded form and this is used as an alternative method of showing the division process. The unit ends with some lessons showing the standard division algorithm along with story problems which illustrate how division is used in different fields. The complete unit is included as Appendix B.

Each of the classes studied the division unit for three weeks (fifteen class days). The treatment group played the math trading game (see Appendix A) for at least thirty minutes of class time each week and were encouraged to play at other times on their own.

INSTRUMENTATION

Four instruments were used to gather data: (1) an attitude inventory developed by Dutton and Blum [1968] was adapted for use in this study; (2) a test for initial aptitude in division (IAD-test); (3) a division achievement posttest; and (4) a division achievement delayed posttest. The IAD, post-, and delayed posttest were constructed by the author. The two post achievement tests are comparable forms, while the test for initial aptitude in division estimates entering division skills. Each of these instruments may be found in Appendix C.

The original Dutton Attitude Inventory has been widely used and respected in research involving attitudes toward mathematics. Dutton [1954] developed a scale for measuring attitude toward mathematics using the method described by Thurstone and Clave in 1937. This scale was developed for use mainly with older school children and adults. It has been found to be extremely reliable for measuring attitude toward mathematics. Dutton [1954] used the test-retest method to estimate the reliability of his scale and reported a correlation of .94. In a later study [1962] he reported a reliability correlation of .84.

Litwiller [1970] used Dutton's scale and found at the .01 level that correlation coefficients of .75 and .74

were significant, thus indicating a high degree of reliability.

Wall [1972] used the Dutton Inventory and found the correlation between pretest and posttest was .785 which he concluded was extremely reliable.

For a study with elementary school children Dutton and Blum [1968] changed Dutton's original scale into a Likert-type test. A Likert-type scale is composed of third-person statements to which a subject may make one of five responses: strongly agree, agree, neutral, disagree or strongly disagree. These are scored from one to five, with five being most favorable.

Dutton and Blum prepared the scale using the strongest items from the Dutton-Thurstone type scale and reworded the statements to make them third-person statements. Each item on the scale was checked to see that it had only one main idea, that it had no ambiguities, that it was not too long and that it would discriminate between positive and negative feelings. Because these items had been given scale values when used in Dutton's previous scale, considerable information was known about their discrimination and usefulness in measuring attitudes.

The reliability of the scale, using the Spearman-Brown test and retest formula, was .84.

When the attitude inventory is administered under conditions where the subjects have no reason to lie, the scale has been shown to be reliable. The directions given the subjects during this study should have minimized this problem.

The author constructed achievement tests were content validated by examination by other mathematics educators. The tests were constructed so as to measure the objectives of the unit. Each test was also constructed so as to obtain a range of scores from zero to one hundred on a percentage basis. This was done to eliminate a floor or ceiling effect on the achievement test. The Initial Aptitude in Division (IAD) test was used to check for equality of groups at the beginning of the study. This is more thoroughly examined in a later section of this chapter and again in chapter four.

The posttest and delayed posttest are equivalent forms of the division achievement test constructed by the author. There was no appropriate commercial division tests available for this level. Content validity was assured by careful construction of the test to meet the objectives of the division unit.

The reliability of the achievement post and delayed posttest were estimated by the author using the Hoyt Reliability Coefficient [Hoyt, 1941] through an analysis of variance technique. Ten items were randomly selected from the posttest and the delayed posttest and combined into a single test using the technique suggested by Cook and Stufflebeam [1967]. This test was administered to two intact fifth grade classes in the Montcalm Intermediate School District. The data from these tests were used to compute the Hoyt Reliability Coefficients given in the following table.

Table 3. Reliability Coefficients of the Division Achievement Posttest and Delayed Posttest.

	Hoyt Coefficient	Total Test Reliability
Posttest	.82	.90
Delayed Posttest	.86	.92

The Hoyt [1941] coefficient of reliability is given as:

$$R_{tt} = \frac{\sigma^2 \text{ among individuals} - \sigma^2 \text{residual}}{\sigma^2 \text{ among individuals}}$$

Hoyt claims this formula gives the same results as those found by using the Kuder and Richardson formula 20 [p. 156].

The total test reliability was computed from the Hoyt Coefficient using the Spearman-Brown formula:

$$R_{tt} = \frac{2\ r_{st}}{1 + r_{st}} ,$$

where R_{tt} is the total test reliability and r_{st} is the reliability coefficient of the fifty percent sampled-item test.

HYPOTHESES, EXPERIMENTAL DESIGN AND DATA ANALYSIS

In this section the hypotheses are stated in testable form. Then the experimental design is presented. This is followed by an explanation of how the data was gathered. Next the appropriate analysis is presented. The section ends with statements on the assumptions for the study and limitations of the study.

Testable Hypotheses of This Study

The main purpose is to determine if the math trading game has an effect on student attitude toward mathematics and on student achievement in fifth grade division. The following hypotheses were tested at the $\alpha = .05$ level of significance. The calculations were performed by the Michigan State University Computer Center utilizing the C.D.C. 3600 and 6400.

1. There is no significant difference between T_1 and T_2 on the IAD test as measured by the class means.
2. There is no significant difference between T_1 and T_2 on the preattitude inventory as measured by the class means.
3. There is no significant difference in achievement in division between T_1 and T_2 as measured by the achievement posttest.
4. There is no significant difference in achievement in division between T_1 and T_2 as measured by the delayed achievement posttest.
5. There is no significant treatment by sex interaction on the achievement posttest.

6. There is no significant treatment by sex interaction on the delayed achievement posttest.

7. There is no significant difference between the male scores and female scores for T_1 and T_2 on the achievement posttest.

8. There is no significant difference between the male scores and female scores for T_1 and T_2 on the delayed achievement posttest.

9. There is no significant difference in the attitude mean scores of T_1 and T_2 as measured by the attitude postinventory.

10. There is no significant sex differences in attitude toward mathematics between T_1 and T_2 as measured by the preattitude inventory.

11. There is no significant sex differences in attitude toward mathematics between T_1 and T_2 as measured by the postattitude inventory.

12. There is no significant treatment main effect for T_1 vs T_2.

13. There is no significant testing main effect for posttest, delayed posttest.

14. There is no significant sex main effect for male vs female.

15. There is no significant treatment by sex interaction.

The use of the Michigan State University computing facilities were made possible through support, in part, from the National Science Foundation.

The basic experimental design used in this study was Campbell and Stanley's Design Four [1963, p. 13]. This is the pretest, posttest, control group design shown in Table 1 of this study.

In addition to the hypotheses stated above an added interest of this study was to investigate strategies

used by elementary school students doing the division operation.

The achievement IAD test and posttest scores were compared for the purpose of identifying:

(a) Students who scored poorly on both exams.
(b) Students who scored high on both exams.
(c) Students who did well on one exam and poorly on the other exam.

Thirteen students were chosen from these groups and were interviewed individually in order to identify strategies used by students doing the division operation.

Analysis: Mathematical Achievement and Attitude

In order to test the stated hypotheses, data was gathered through the administration of pre- and postattitude inventories, IAD-, post-, and delayed postachievement tests to each of the eight cooperating classes. The IAD-test for achievement and preattitude was administered during the first period of the three week unit. The posttest, for attitude and achievement, was given during the fifteenth lesson of the division unit. The delayed posttest for achievement was given six weeks after the close of the division unit. The IAD and posttests were administered in the same order both times, achievement test and then attitude inventory, so that external influence would be comparable both times. The students were told in each case that the results were for the author's purposes only but they should do as well as they could on each exam. They were also asked to answer the statements on the attitude inventory with their true feelings since these answers would in no way influence their grades in class.

To avoid violating the independence assumption, intact classes were the treatment unit, and hence the class is the unit of analysis. The class means, by sex (male and female) and total, were computed for each of the five measures and are shown for each of the eight classes in Appendix D.

Because of the limited degrees of freedom the univariate analysis of variance method was used to test the first eleven research hypotheses instead of ANCOVA or multivariate ANOVA. This design is shown in Table 4. In this design treatment is crossed with sex and the testable hypotheses are shown in Table 4a for each separate achievement test or attitude inventory.

Table 4. Design for Univariate Analysis of Variance.

	Test (P)	
	Male	Female
T_1	X	X
T_2	X	X

Table 4a. Testable Hypotheses.

T	(treatment main effect)
R : T	(error, replications nested in treatment)
S	(sex main effect)
TS	(treatment by sex interaction)
RS : T	(error)

The repeated measures, split plot design was used to test the last four research hypotheses for the effect of treatment, over time, and sex differences in achievement. The design is a 2 X 2 X 2 design with the two factors being achievement posttest and achievement delayed posttest, split into male and female plots. The two levels of the design are T_1 and T_2. This design is shown in Table 4b and the associated testable research hypotheses are given in Table 4c.

Table 4b. Repeated Measures, Split Plot Design.

		Achievement Test (P)			
		Posttest		Delayed Posttest	
		M	F	M	F
Treatment group	T_1	X	X	X	X
	T_2	X	X	X	X

This analysis allowed for the testing of interaction effects as well as the main treatment effects, over time, for the achievement posttest and delayed posttest. Again sex is crossed with treatment for analysis.

Table 4c. Repeated Measures, Split Plot Design Testable Hypotheses.

T	(treatment main effect)
R : T	(error, replications nested in treatment)
P	(test main effect)
TP	(treatment by test interaction)
RP : T	(error)
S	(sex main effect)
TS	(treatment by sex interaction)
RS : T	(error)
PS	(test by sex interaction)
TPS	(test by sex by treatment interaction)
RPS : T	(error)

R = replications.

The split plot design is described by Cox [1958] as follows:

> The essence of the method . . . is that a lower precision is accepted for one comparison, the difference between sexes, in order that the precision of the more interesting comparisons, namely of the treatment factors and their interation with sex, shall be increased [p. 146].

It is this last comparison that Aiken [1972] inferred when writing about "the whole complex of variables" that must be taken into account if meaningful answers about sex differences are to be answered.

Equality of Groups

The question "Are the treatment and control groups different, shomehow, in the beginning?" is a very important question which must be answered. Is the control group different than the treatment group? Campbell and Stanley [1963] feel the most adequate "all-purpose assurance of lack of initial biases between groups is randomization" [p. 25]. The classes in this study were randomly assigned to treatment and control groups in an effort to reduce biases concerning teacher confounding, history, maturation, testing and the other variables in Table 1 of Campbell and Stanley [1963, p. 8].

To check for equality of the groups on the dependent variables, achievement and attitude toward mathematics, the achievement IAD test and preattitude inventory were given during the first day of the division unit. An appropriate analysis to check for equality of groups on the dependent variables is the one-way analysis of variance [Glass and Stanley, 1970, p. 353]. If there is no difference then external validity will have been increased for the study. This is further discussed in Chapter IV.

Basic Assumptions

The above design requires certain basic assumptions about the sample population. Analysis of variance and repeated measures designs have the following assumptions: Normality of within cell populations, equal variances and independence. The repeated measures design has the additional assumption of "the correlations of all pairs of levels of the fixed factors across the population of random factor levels must be the same" [Glass and Stanley, 1970, pp. 13-15].

LIMITATIONS OF THE STUDY

Since the pretest, posttest, control group design of Campbell and Stanley [1963, pp. 13-15] was used, the external validity factor is a minus. Campbell and Stanley [1963, p. 18] mention the effect which an attitude pretest may have on subsequent posttest answers. They say:

> In attitude change studies, where the attitude tests themselves introduce considerable amounts of unusual content, it is quite likely that the person's attitudes and his susceptibility to persuasion are changed by the pretest [p. 18].

The directions given the students in this study should have minimized this possibility.

SUMMARY

This chapter describes the elements of the experimental design used in this research. The population for the study and the sample used in the study were described first. The division unit was described next, followed by a section on instrumentation which describes the tests used in gathering the data. The hypotheses were stated in testable form, and the experimental design and data analysis were presented in the next section of the chapter. These were followed by basic assumptions used for the study and the method of checking for equality of groups in the beginning was explained. Limitations of the study were presented last in the chapter.

CHAPTER IV

PRESENTATION AND ANALYSIS OF DATA

Presented in this chapter is a summary of the data collected during this study, an analysis of this data and the results of this analysis. The chapter consists of the following sections:

1. Analysis of the IAD achievement test and the preattitude inventory to test for initial differences of T_1 and T_2;
2. Analysis of the results of the achievement posttests;
3. Analysis of the results of the attitude inventories;
4. Further analysis of the achievement posttest and delayed posttest;
5. Cell means and standard deviations;
6. Analysis of the student interviews for strategies in division;
7. Summary of findings.

ANALYSIS OF THE IAD ACHIEVEMENT TEST AND THE PREATTITUDE INVENTORY TO TEST FOR INITIAL DIFFERENCES OF T_1 AND T_2

The question of equality of the treatment and control groups in division ability and attitude towards mathematics was investigated. The IAD-test and an attitude inventory were administered to each class on the first day of the study. The following two null hypotheses were tested using the univariate analysis of variance described in Chapter III.

H_1: There is no significant difference between T_1 and T_2 on the IAD-test as measured by the class means.

H_2: There is no significant difference between T_1 and T_2 on the preattitude inventory as measured by the class means.

Data Analysis

The IAD-test and the preattitude inventory were scored by the author and the class means were calculated for each class. These scores are given in Appendix D. An appropriate analysis for testing H_1 and H_2 is the univariate analysis of variance discussed by Armore [1973]. The results of this analysis for H_1 and H_2 are given in Tables 5 and 6, respectively.

Findings

The F-tests from the univariate analysis of variance for H_1 and H_2 were not significant at the $\alpha = .05$ level.

Table 5. Univariate ANOVA for T_1 and T_2 on IAD Achievement Test.

Source of Variation	SS	df	MS	F
Treatment	47.6	1	47.6	1.36
Error	210.4	6	35.1	
Total	258.	7		

Not significant. ($F_{1,6} = 5.99$)

Table 6. Univariate ANOVA for T_1 and T_2 on Preattitude Inventory.

Source of Variation	SS	df	MS	F
Treatment	15.9	1	15.9	.94
Error	101.3	6	16.9	
Total	117.2	7		

Not significant. ($F_{1,6} = 5.99$)

Conclusions

Since the null hypotheses were not rejected this would seem to indicate that on the dependent variables, achievement and attitude toward mathematics, T_1 and T_2 are not significantly different.

ANALYSIS OF THE RESULTS OF THE ACHIEVEMENT POSTTESTS

The results of the posttest and the delayed posttest for T_1 and T_2 were used in a univariate analysis of variance to assess the effect of the math trading game on achievement in division. The assessment was carried out in six steps by testing the following null hypotheses:

H_3: There will be no significant treatment by sex interaction on the achievement posttest.

H_4: There will be no significant differences between the male scores and female scores for T_1 and T_2 on the achievement posttest.

H_5: There will be no significant difference in achievement in division between T_1 and T_2 as measured by the achievement posttest.

H_6: There will be no significant treatment by sex interaction on the delayed achievement posttest.

H_7: There will be no significant difference between the male scores and female scores for T_1 and T_2 on the delayed achievement posttest.

H_8: There will be no significant difference in achievement in division between T_1 and T_2 as measured by the delayed achievement posttest.

Data Analysis

Data was collected through the administration of the achievement posttest and delayed posttest as described in Chapter III. The class means were determined for each group to test the above hypotheses and are included in Appendix D. The univariate analysis of variance for the above hypotheses are given in Tables 7 and 8. Table 7 contains the information for H_3, H_4, and H_5 while Table 8 contains the information for H_6, H_7, and H_8.

Table 7. Summary of ANOVA for Achievement Posttest.

Sources	df	MS	Univariate	
			F	P Less Than
(1) Treatment by Sex Interaction	1	52.92	.56	.48
(2) Sex (S)	1	16.20	.17	.69
RS : T (error)	6	94.91		
(3) Treatment (T)	1	365.76	1.11	.33
R : T (error)	6	329.20		

Table 8. Summary of ANOVA for Delayed Achievement Posttest.

Sources	df	MS	Univariate	
			F	P Less Than
(1) Treatment by Sex Interaction	1	193.2	2.65	.155
(2) Sex (S)	1	203.1	2.79	.146
RS : T (error)	6	72.9		
(3) Treatment (T)	1	665.63	1.37	.286
R : T (error)	6	486.17		

Findings

The F-test for the treatment by sex interaction on the achievement posttest was .56, with $p < .48$, which was not significant at the $\alpha = .05$ level. The F-test for the sex main effect on the achievement posttest was .17, with $p < .69$, which was not significant at the $\alpha = .05$ level. The F-test for the treatment main effect on the achievement posttest was 1.11, with $p < .33$, which was not significant at the $\alpha = .05$ level. The F-test for the treatment by sex interaction on delayed achievement posttest was 2.65, with $p < .155$, which was not significant at the $\alpha = .05$ level. The F-test for sex main effect on the delayed achievement posttest was 2.79, with $p < .146$, which was not significant at the $\alpha = .05$ level. The F-test for the treatment main effect was 1.37 for the delayed achievement posttest, with $p < .286$, which was not significant at the $\alpha = .05$ level.

Conclusions

The F-tests for hypotheses H_3 through H_8 were not significant at the $\alpha = .05$ level and the null hypotheses were not rejected. This would seem to indicate that the treatment (math trading game) had no significant effect on achievement in division as measured by the achievement posttest and the delayed achievement posttest. Also, this seems to indicate that there is little sex effect associated with learning mathematics at the fifth grade level.

ANALYSIS OF THE RESULTS OF THE ATTITUDE INVENTORIES

The univariate analysis of variance on the results of the attitude preinventory indicated no initial difference between the treatment and control groups on the attitude variable as measured by the preattitude inventory. The results of the postattitude inventory were used to assess the effect of a math trading game on student attitude toward mathematics. The univariate analysis of variance was used to test the following null hypothesis.

> H_9: There is no significant difference in the attitude mean scores of T_1 and T_2 as measured by the attitude postinventory.

The results of the pre- and postattitude inventory were used to assess the sex differences in attitude toward mathematics by fifth grade students. The class means for the males and females were figured and are included in Appendix D. These scores were used in the univariate ANOVA to test the following null hypotheses:

> H_{10}: There is no significant sex differences in attitude toward mathematics between T_1 and T_2 as measured by the preattitude inventory.
>
> H_{11}: There is no significant sex differences in attitude toward mathematics between T_1 and T_2 as measured by the post attitude inventory.

Data Analysis

The data was collected by administering the preattitude inventory on the first day of the division unit, immediately after the IAD achievement test. The postattitude inventory was administered the fifteenth day of the division unit, immediately after the achievement posttest. Class means were determined for each instrument and are included in Appendix D. This data was used to test the null hypotheses H_9, H_{10} and H_{11} given above. The data for the preattitude inventory is summarized in Table 9 while the data for the postattitude inventory is summarized in Table 10.

Table 9. Summary of the Univariate ANOVA for the Preattitude Inventory.

Sources	df	MS	Univariate	
			F	P Less Than
(1) Treatment by Sex Interaction	1	2.48	.10	.77
(2) Sex (S)	1	42.57	1.64	.25
RS : T (error)	6	25.90		
(3) Treatment (T)	1	18.71	.31	.60
R : T (error)	6	61.29		

Findings

The F-test for the treatment by sex interaction on the preattitude inventory was .10 ($p < .77$) which was not significant. The T-test for sex main effect on the preattitude inventory was 1.64 ($p < .25$) which is not significant at the $\alpha = .05$ level. The F-test for the treatment by sex interaction on the postattitude inventory was .25 which was

Table 10. Summary of the Univariate ANOVA on the Post-attitude Inventory.

Sources	df	MS	Univariate	
			F	P Less Than
(1) Treatment by Sex Interaction	1	4.41	.25	.63
(2) Sex (S)	1	53.29	3.06	.13
RS : T (error)	6	17.40		
(3) Treatment (T)	1	39.69	.96	.37
R : T (error)	6	41.42		

not significant at the $\alpha = .05$ level. The F-test for sex main effect on the postattitude inventory was 3.06 ($p < .13$) which was not significant at the $\alpha = .05$ level. The F-test for the treatment main effect on the postattitude inventory was .96 ($p < .37$) which was not significant at the $\alpha = .05$ level.

Conclusions

Since none of the F-tests for H_9, H_{10} and H_{11} were significant, the null hypotheses were not rejected and it can be concluded the math trading game had little effect on the attitude of the fifth grade students. Also, there was little sex main effect or treatment by sex interaction as measured by the attitude inventories.

FURTHER ANALYSIS OF THE ACHIEVEMENT POSTTEST AND DELAYED POSTTEST

The following secondary analysis was completed to test the effect of the treatment over time. The design used was the repeated measures, split plot design described in Chapter III. The repeated measures were achievement posttest and delayed posttest, split into male and female scores for T_1 and T_2. The number of factors over the repeated measures are two, achievement posttest and delayed posttest. The factor over the subjects has two levels T_1 and T_2.

The class means for males and females of the achievement posttest and delayed posttest were used to test the following null hypostheses:

H_{12}: There is no significant treatment main effect for T_1 vs T_2.

H_{13}: There is no significant testing main effect for posttest, delayed posttest.

H_{14}: There is no significant sex main effect for male vs female.

H_{15}: There is no significant treatment by sex interaction.

Data Analysis

Data was collected through the administration of the achievement posttest and delayed posttest as described in Chapter III. The eight class means were computed by sex and are included in Appendix D. The results of the analysis of variance for the repeated measures, split plot design are shown in Table 11.

Table 11. Summary of ANOVA (Repeated Measures, Split Plot Design) on Achievement Posttest and Delayed Posttest.

Sources	df	MS	Univariate	
			F	p Less Than
(1) Treatment (T)	1	1009.13	1.59	.25
R : T (error)	6	633.65		
(2) Test (P)	1	551.95	3.04	.13
(3) Treatment by Test	1	22.28	.12	.74
RP : T (error)	6	181.79		
(4) Sex (S)	1	166.98	1.18	.32
(5) Treatment by Sex	1	244.19	1.59	.25
RS : T (error)	6	141.17		
(6) Test by Sex	1	52.28	1.96	.21
(7) Treatment by Test by Sex	1	21.95	.82	.40
RPS : T (error)	6	26.62		

R = replications.

Findings

The F-test for treatment main effect was 1.59 ($p < .25$); for test main effect was 3.04 ($p < .13$); for sex main effect was 1.18 ($p < .32$); for treatment by sex interaction was 1.59 ($p < .25$). Therefore none of the F-test for the null hypotheses are significant at the $\alpha = .05$ level.

Conclusions

Since none of the F-tests were significant at the $\alpha = .05$ level, the null hypotheses H_{12}, H_{13}, H_{14} and H_{15} were not rejected. Thus it can be concluded that there was little treatment main effect, test main effect, sex main effect and treatment by sex interaction when T_1 was compared to T_2 over time with achievement as the dependent variable.

Further Considerations

Since the repeated measures, split plot design failed to show significance, a further look at the testing main effect and treatment by testing interaction was thought advisable. Since $SS_p + SS_{TP} = SS_{P:T_1} + SS_{P:T_2}$ [Winer, 1973, p. 347] the author partitioned the sums of squares in this manner thinking this may give different results. The results are summarized in the following table.

Table 12. Summary of Repartitioned Sums of Squares.

Sources	df	MS	Univariate F
$P : T_1$	1	175.88	.97
$P : T_2$	1	398.35	2.19
RP : T (error)	6	181.79	

P = (Testing, T_1 = (Treatment), T_2 = (Control),
R = (Replications).

The SS_P and SS_{TP} above are from Table 11, sources labeled (2) and (3). These are sums of squares for test main effect and treatment by test interaction respectively.

Conclusions

The F-tests were not significant at the $\alpha = .05$ level of significance and any hypotheses about test main effect being nested within treatment would not be rejected.

CELL MEANS AND STANDARD DEVIATIONS

The following tables show the observed cell means and standard deviations for the five measures administered to the treatment and control groups.

Table 13. Observed Cell Means.

		Attitude Pretest	Attitude Posttest	IAD Test	Post Test	Delayed Posttest
T_1	Male	81.3	78.1	52.0	29.3	32.2
	Female	84.3	83.8	59.5	35.2	45.8
T_2	Male	83.1	81.6	50.7	40.4	49.3
	Female	85.2	82.6	48.2	40.6	46.9

Table 14. Observed Cell Standard Deviations.

		Attitude Pretest	Attitude Posttest	IAD Test	Post Test	Delayed Posttest
T_1	Male	13.6	14.2	11.8	30.7	26.9
	Female	13.4	12.4	18.1	28.7	26.2
T_2	Male	12.9	12.3	16.8	29.9	31.5
	Female	10.3	12.4	17.8	29.8	30.2

A study of the cell means indicate that the control group did better on the achievement posttest and delayed posttest although not significantly better. Further study of Appendix D shows that T_{14} of the treatment group had very low results on both of the above exams while T_{21} and T_{24} of the control group had very high results on the delayed achievement posttest. The author has no explanation for these scores but they undoubtedly affected the final analysis.

While the preceding analyses have been the correct ones for this study, the author thought an ANCOVA with the IAD-test and the preattitude inventory as covariates and with individual students as the unit of analysis might prove to be of some value. The results of this analysis is included in Appendix E. This analysis violates the assumption of within cell independence and must be viewed with this in mind.

STUDENT INTERVIEWS FOR STRATEGIES IN DIVISION

A further purpose of this study was to investigate, informally, division strategies used by students in this study. Thirteen students were selected and interviewed. Four students were chosen who had scored high on both the IAD test and the postachievement test; two students were chosen who had scored high on the IAD test and relatively low on the achievement posttest; three students were chosen who had scored low on the IAD test and high on the achievement posttest; and four students were chosen who had scored low on both the IAD test and the achievement posttest. Six of the interviewed students were from the treatment group and seven were from the control group.

For the interview each student was asked to work two division problems ($8\overline{)1086}$ and $252\overline{)787}$) and tell the author what they were doing as they worked the problems.

During the interview the researcher attempted to determine if the student knew the basic division operation, the place value system and how to check a division problem. Each student interviewed was asked the following questions.

1. In very basic terms, what is the division operation?
2. What does the 8 in 1086 represent?
3. How do you check a division problem?

Some students were asked the following follow-up questions for elaborations on answers to the above questions.

1. Since you seem to know the basic concepts of division, how do you start the solution of a division problem?
2. Is there another representation for the 8 in 1986?
3. Do you know the name for the division method you are using?

There were also several "Where did you learn that?" or "Why are you doing that?" type questions.

The results of the thirteen interviews are summarized in the following table.

Table 15. Results of the Student Interviews.

Group	(No.)	Number that knew the following: Subtraction facts	Multiplication facts	A division process	Place value	How to check	Standard algorithm
High	(4)	4	4	4	3	4	3
Low to High	(3)	3	3	3	2	3	3
High to Low	(2)	1	1	1	0	1	0
Low	(4)	2	1	1	1	1	0
Total	(13)	10	9	9	6	9	6

Further Analysis of Student Interviews

Two of the six students from the treatment group thought the math trading game had helped them in the division operation. One of these students was in the low group. When asked what the 8 stood for in 787, he knew it stood for

80 units but was not able to say that it was also 8 tens. This student also put an extra zero on the end of the quotient in problem two because 252 would not divide into 31, so a zero went in the quotient and then 30 became the remainder.

The other student who said the math trading game helped him in division was in the low to high group. He knew place value very well and said he liked division. Niether problem was difficult for him. This student used a close approximation to the standard division algorithm. He still put the extra zeroes in when he multiplied the number in the quotient by the divisor. For example:

$$\begin{array}{r} 1 \\ 8\ \overline{)\ 1086} \\ \underline{800} \end{array}$$

As a result of this and his answers to other questions, the author felt he knew the place value system of numeration very well. The author feels the math trading game did help this student with the numeration system.

A student in the low to high group from T_2 started the second problem using repeated subtraction and after the first step realized that the divisor could be subtracted twice more and immediately wrote the correct quotient of 3 and figured the remainder of 31. This seems to be an intermediate stage between the subtraction process and the standard algorithm.

Four of the students, who did not understand place value (based on answers to interview questions) were still able to get the correct quotient by using either the standard algorithm or the repeated subtraction method. Three of these students were in the control group and had scored high on the posttest and one was in the treatment group and had scored low on the posttest. The latter student was very slow but given time she could get the right answer. This probably helps explain her low score on the posttest which was limited to a class period for completion. She was able to work the problems during the interview session where she had more time.

It was interesting to note that none of the students used the expanded form of writing the dividend in their solutions for either problem, although this was one method explained in the division unit. The standard algorithm or repeated subtraction method was used by most of the students. One student used the "short division" method on the first problem and a close approximation of the standard algorithm for the second problem. She said she had learned the short division process last year and always used that method for single digit divisors.

One student from the "high" group worked the first problem with the standard algorithm and then without any hesitation worked the second problem with the repeated subtraction method.

The four students in the "low" group were evenly divided between T_1 and T_2. One of the students from T_1 had a fair knowledge of place value but made several mistakes in multiplication and subtraction which resulted in wrong answers. There did not seem to be any pattern to the mistakes except carelessness. The other three students were very much the same. They had trouble with multiplication and subtraction facts and lacked a knowledge of the division operation. One of these students however was fair in subtraction and after being tutored through the first problem using repeated subtraction (multiples of 10), he worked the more difficult second problem almost completely on his own.

In summary, the results of the interviews seem to indicate that mastery of the subtraction and multiplication facts are necessary for success in the division process. Students can do division successfully without knowing the standard algorithm. There are students who score low on tests simply because they are slow at working the problems even though they have a good basic understanding of the processes.

SUMMARY OF FINDINGS

Analysis of the data collected from the instruments used in this study produced the following results:

1. There was no significant treatment main effect on the achievement posttest.
2. There was no significant treatment main effect on the achievement delayed posttest.
3. There was no significant treatment by sex interaction on the achievement posttest.

4. There was no significant treatment by sex interaction on the achievement delayed posttest.
5. There was no significant difference between the male scores and female scores for T_1 and T_2 on the achievement posttest.
6. There was no significant difference between the male and female scores for T_1 and T_2 on the delayed achievement posttest.
7. There was no significant difference in attitude mean scores for T_1 and T_2 on the postattitude inventory.
8. There was no significant sex differences in attitude toward mathematics between T_1 and T_2 as measured by the preattitude inventory.
9. There was no significant sex differences in attitude toward mathematics between T_1 and T_2 as measured by the postattitude inventory.
10. There was no significant treatment main effect for T_1 vs T_2.
11. There was no significant testing main effect for posttest vs delayed posttest.
12. There was no significant sex main effect for male vs female.
13. There was no significant treatment by sex interaction.

14. A repartitioning of the sums of squares for test main effect and treatment by test interaction failed to produce significant results.
15. To be termed successful in the division operation students must know the subtraction and multiplication facts.
16. Students can know the division operation process and the place value system of numeration and still not be considered successful in division as measured by test results, because of slowness in completing problems.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

SUMMARY

The decade of the 60s saw a revolution in the elementary mathematics curriculum. Teachers returned to school in large numbers for additional study, many innovations were introduced into the classroom and, new mathematics topics were introduced into the mathematics curriculum. Teaching techniques, in general, tended to become more activity oriented. Extensive research has been done on some of these changes. There are, however, areas which need much more research before questions about the learning of mathematics can be answered.

Purpose

This study sought some answers to the questions concerning the effects of a math trading game on achievement in division and attitude toward mathematics of fifth grade students. Specifically, this study was concerned with the following questions:

1. Will a math trading game help to improve achievement in fifth grade division?
2. Will a math trading game help to improve student attitude toward mathematics?

3. Is there a sex difference associated with achievement in mathematics at the fifth grade level?
4. Is there a sex difference associated with student attitude toward mathematics at the fifth grade level?
5. What strategies are used by students in solving division problems at the fifth grade level.

The Sample

Eight intact, fifth grade classes from the Montcalm Area Intermediate School District were used in this study. Four classes were randomly assigned to T_1 and four classes were randomly assigned to T_2. A total of eighty five male and seventy three female students were involved in the study.

The students in the treatment group played a math trading game and used some of the principles of trading in the solution of division problems. The control group was limited to the division unit without the use of any manipulatives or other laboratory devices.

Literature Review

The research findings associated with achievement and games, and attitude and games was very inconclusive. Mathematics educators Willoughby, Rosenbloom, Dienes, Biggs, McLean and Underhill recommend the use of games in the teaching of mathematics. Stanford, Addleman, Edwards, and

Wynroth reported significant gains in achievement which they attributed to the effects of playing games. Crist, Henry Bowen and Burgess however reported inconclusive results after using games in the curriculum.

Research results on the effect of games on attitude toward mathematics are also contradictory. Jones reported a significant gain in attitude in his study. Knaupp, Cech, Fink, Addleman, Henry, and others reported no significant change in attitude toward mathematics as a result of using games in the curriculum.

Aiken in his review of the research feels there is a sex difference associated with attitude toward mathematics. Also, Poffenberger and Norton found a significantly higher number of females disliked mathematics than did males.

Instrumentation

The following instruments were used for data collection: (1) Initial Aptitude in Division Test (IAD test), (2) achievement posttest for division, (3) achievement delayed posttest for division, and (4) Dutton and Blum's Attitude Inventory was adapted for use in this study. The achievement posttest and delayed posttest are paralled forms.

Hypotheses

The following null hypotheses were tested to assess the effect of a math trading game on achievement in division and attitude toward mathematics of fifth grade students.

The first two null hypotheses were tested to determine if the treatment and control groups were equivalent on the dependent variables of achievement and attitude toward matheamtics.

H_1: There is no significant difference between T_1 and T_2 on the achievement IAD test as measured by the class means.

H_2: There is no significant difference between T_1 and T_2 on the preattitude inventory as measured by the class means.

Hypotheses H_1 and H_2 were not rejected.

The following two null hypotheses were tested to determine if there was any treatment main effect on the achievement posttest and delayed posttest.

H_3: There is no significant difference in achievement in division between T_1 and T_2 as measured by the achievement posttest.

H_4: There is no significant difference in achievement in division between T_1 and T_2 as measured by the delayed achievement posttest.

Hypotheses H_3 and H_4 were not rejected.

The following four null hypotheses were tested to determine the sex differences associated with achievement in division for fifth grade students.

H_5: There will be no significant treatment by sex interaction on the achievement posttest.

H_6: There will be no significant treatment by sex interaction on the delayed achievement posttest.

H_7: There will be no significant difference between the male scores and female scores for T_1 and T_2 on the achievement posttest.

H_8: There will be no significant difference between the male scores and female scores for T_1 and T_2 on the delayed achievement posttest.

Hypotheses H_5, H_6, H_7, and H_8 were not rejected.

The following null hypothesis was tested to assess the treatment main effect on attitude toward mathematics for fifth grade students.

H_9: There is no significant difference in attitude mean scores of T_1 and T_2 as measured by the attitude post inventory.

Hypothesis H_9 was not rejected.

The following two null hypotheses were tested to assess the sex differences in attitude toward mathematics of fifth grade students.

H_{10}: There is no significant sex differences in attitude toward mathematics between T_1 and T_2 as measured by the preattitude inventory.

H_{11}: There is no significant sex differences in attitude toward mathematics between T_1 and T_2 as measured by the postattitude inventory.

Hypotheses H_{10} and H_{11} were not rejected.

The following four hypotheses were tested to assess the effect of the treatment over time. The design used was the repeated measures, split plot design using univariate analysis of variance.

H_{12}: There is no significant treatment main effect for T_1 vs T_2.

H_{13}: There is no significant testing main effect for posttest vs delayed posttest.

H_{14}: There is no significant sex main effect.

H_{15}: There is no significant treatment by sex interaction.

Hypotheses H_{12}, H_{13}, H_{14}, and H_{15} were not rejected.

Further Considerations

A repartitioning of the sums of squares for test main effect and treatment by test interaction, failed to produce any significant results.

Statistical Analysis

Univariate analysis of variance was used to test the null hypotheses H_1 through H_{11}. The design was 2 X 2, with the two factors being sex and the two levels being T_1 and T_2. For the null hypotheses H_{12} through H_{15} the repeated measures, split plot design was used with univariate analysis of variance. The repeated measures were achievement posttest and delayed posttest, which were split into male and female plots. The levels for this analysis were T_1 and T_2.

A five percent level of significance was used in accepting or rejecting each of the null hypotheses.

CONCLUSIONS

The following conclusions seem appropriate based on the findings of this study.

1. Univariate analysis of variance indicated the treatment and control groups were not significantly different on the dependent variables, achievement and attitude toward mathematics, at the beginning of the study.

2. The use of the math trading game did not significantly improve achievement in division at the fifth grade level.
3. The use of the math trading game did not significantly improve attitude toward mathematics of fifth grade students.
4. There was no significant sex difference related to achievement.
5. There was no significant sex differences related to attitude toward mathematics.
6. There was no significant effect, over time, on achievement associated with the playing of the math trading game.
7. To be termed successful in the division operation students must know the subtraction and multiplication facts.
8. Students can understand the division process and the place value system of numeration and still not be termed successful in division, as measured by test results, because of slowness in completing problems.

Discussion

While the null hypotheses were not rejected there does seem to be some data in the table of means, included as Appendix D that warrants additional comments. A neutral score on the attitude inventory was seventy-two.

The grand means on the preattitude and postattitude inventory were 84.4 and 82.1, respectively. This indicated a somewhat positive attitude toward mathematics was held by the fifth grade students in this study. There was a drop of 2.3 points in the grand mean during the three week period of the study.

Even though the results of this study were not significant, the author feels there is a place for games in the mathematics curriculum. A series of closely related games played over a longer period of time might produce significant results.

While the math trading game did not appear to help students with the division operation, the author does feel that some students had a better understanding of the number system after playing the math trading game. A plausible reason for this is that the strength of the math trading game lies primarily in concept building and not in computational skill development. The interviews indicated that some students could do a division problem and get the correct quotient without understanding much about the number system. It is quite possible that the reason the control group achieved more was because they had more time for drill work than the treatment group. This could indicate that in this study the time spent playing the math trading game was detrimental to some students in terms of their performance, as measured by the post and delayed postachievement test.

RECOMMENDATIONS

The following recommendations are based on this researcher's interpretations of the findings of this study and his personal observations made while completing this study.

For Future Research

It is recommended that future studies involving games in the mathematics curriculum be for a period of at least one semester.

It is recommended that future studies involving games in the mathematics curriculum use several games which have imbedded in them a single or closely related, mathematical concept.

It is recommended that more studies be done to investigate the strategies used in division by elementary school pupils.

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APPENDICES

APPENDIX A

THE MATH TRADING GAME

USING THE 'TRADING GAME' IN DIVISION

DIRECTIONS TO TEACHERS

THE MATH TRADING GAME

For this game sticks of yellow, blue, green and red and used. Yellow sticks have a value of one, blue sticks have a value of so many yellow, green are so many blue and red sticks are so many green. In one version of the game the winner is the first one who gets a red stick. In a different version of the game the winner is the first one to get rid of all his sticks after starting with a red one. Dice are thrown to determine the number of yellow sticks given a player.

The students are divided into groups of four or five, a banker is selected for each group. The banker exchanges sticks and pays each player in turn his value on the dice. After each game a new banker should be selected so each student has this experience. The teacher rolls the dice and calls out the number appearing (or the sum) on the face of the cube. The student whose turn it is to play then receives from the banker the number of yellow sticks called by the teacher.

The banker also trades, according to the pre-assigned value of the colors, with each student so the student can work toward the single red stick or if the other version is being played, so the player can get rid of all his sticks.

A quick game to play is for the yellow to be one, one blue equals three yellow, one green equals three blue, and one red equals three green. Two variations of this game are given below.

<u>Math Trading Game</u>: Variation one.

Try the trading game again but this time use

1 blue = 5 yellow
1 green = 5 blue
1 red = 5 green

Use two dice and add the sum appearing on top for the number of yellow chips to be traded.

Play the game both ways. Start with nothing and trade until you get a red; or you may start with a red and get rid of all your chips.

<u>Math Trading Game</u>: Variation Two

This time use three dice and the sum is the number of yellow chips to be traded.

Let 1 blue = 10 yellow
1 green = 10 blue
1 red = 10 green

Again start with nothing and obtain a red or start with a red and get rid of all your chips. This may be a long game when trading for 10 and you may want to start with a green in order to shorten the game.

USING THE 'TRADING GAME' IN DIVISION

The idea of trading may be used to help students see what is being done in the division algorithm. This is illustrated by the following example.

	G	B	Y
8)	4	3	2

Y is for yellow trades.
B is for blue trades.
G is for green trades.

Since 8 will not "divide into" 4 the four is traded for 40 and this is added to the 3, making 43.

	G	B	Y
		5	
8)	4	3	2
		43	
		40	
		3	

Now 8 will divide into the 43, five times. The five is put in the "blue" column and the 40 is subtracted from 43 leaving 3 in the blue column. Since 8 will not divide into 3, the 3 is "traded" for 30 yellow and these are added to the 2 already in the "yellow" column, making a total of 32 yellow.

	G	B	Y
		5	4
8)	4	3	2
		43	
		40	
		3	
			32
			30
			0

After the 8 is divided into the 32, the 4 is written in the quotient and the product of 4 X 8 = 32 is subtracted, leaving 0. After some practice students should be encouraged to shorten the process.

To Teachers of the Treatment Groups

On the first day of the unit, pupils should be given the preachievement test and the preattitude inventory. Differences in scores on the preachievement test and the postachievement test will be used to determine changes in achievement. Likewise, differences in scores on the pre- and postattitude inventory will be used to determine changes in attitude. Answers on the pre, post and delayed post achievement test should be considered either right or wrong, no partial credit is to be given.

During the second day introduce the math trading game along with lesson two. The children may play the game anytime it is convenient for you, (recess, before or after school). The second version of the trading game should be introduced during lesson five and the third version during lesson nine. Students should get a minimum of thirty minutes each week playing the game.

With lesson eight show how the trading game relates to division. Continue to use this method through the end of the unit.

Lesson fifteen is the posttest and the postattitude inventory and are to be given in this order. Six weeks later the delayed posttest is given. No review is to be given before the delayed test and no division, as such, is to be taught during the six week period. Do whatever else you want but no division.

To Teachers of the Control Groups

On the first day of the unit, pupils should be given the preachievement test and the preattitude inventory. Differences in scores on the preachievement test and the postachievement test will be used to determine changes in achievement. Likewise, differences in scores on the pre and post attitude inventory will be used to determine changes in attitude. Answers on the pre, post and delayed post achievement test should be considered either right or wrong, no partial credit is to be given.

The other thirteen lessons of the unit are to be taught as they are written. You are not to use any concrete materials for demonstration purposes of any kind. You may make illustrations similar to the ones in the unit, but this is all.

The posttest and the postattitude inventory are to be given as lesson fifteen. These are given in the above order, posttest first and then the attitude inventory Six weeks later the delayed postachievement test is given. No review is to be given before the delayed test and no division, as such, is to be taught during this six week period. Do whatever else you wish but no division.

APPENDIX B

DIVISION UNIT

Lesson One

Pretest for achievement.

Preattitude inventory.

Lesson Two

The standard division algorithm ($8 \overline{)432}$) is more difficult for children to understand than is addition, subtraction, or multiplication. This algorithm works from left to right and the answer is put at the top instead of below the problem. There are other problems which are not encountered in the other algorithms. For example there is generally a certain amount of guessing in getting the trial quotient in what is referred to as long division.

Subtraction is often referred to as the inverse of addition and in the same context division is the inverse of multiplication. Basically there are two types of division problems. One type is referred to as the <u>measurement</u> concept of division and the other type if referred to as the <u>partition</u> or <u>partitive</u> concept of division.

<u>MEASUREMENT</u>. Some division questions can be answered by removing subsets in a repeated subtraction sequence. Example: How many tables will be needed if I have twelve people to seat and seat four at each table? In this problem you can imagine objects being placed into equivalent sets and then counting the total number of sets. This is measurement division.

<u>PARTITION</u>. Other division questions begin with the number of sets known and require the size of each set be found. Example: How many people will be at each table if twelve people are to sit at four tables. Here the size of

each set is to be determined. This is partitive division.

In each of these concepts you know the total number of objects and one other thing. In the measurement concept you know the total number of objects and the number of objects in each subset and answer the question, "How many objects in each subset?" Note that in measurement division the divisor represents the number (size) of each set, while in partition division the divisor represents the number of sets.

The relation between these two concepts can be shown in the following table:

	Partitive	Measurement
Total to be divided	known	known
Number of sets	known	unknown
Size of sets	unknown	known

If you are to learn to interpret division in the physical world it is important that you make this distinction. An expression like "twelve divided by four" does not help to visualize the operation of division. But "twelve divided or separated into sets of four" or "twelve divided or separated into four sets" does convey a meaningful idea.

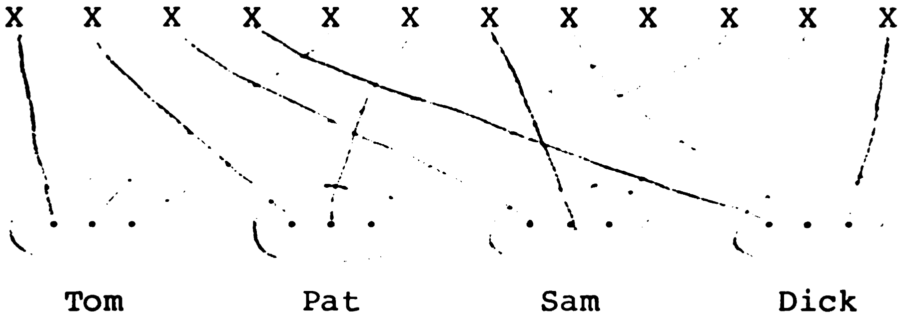
Take twelve marbles and determine the number of sets of four marbles each.

XXXXXXXXXXXX = XXXX XXXX XXXX ;

12 ÷ 4 = 3 equal sets

How many equal subsets? There are three equal subsets of four marbles each. This is measurement division.

Take twelve marbles and share these between four children. Answer the question, "How many marbles in each set?"



or 12 ÷ 4 = 3 in each set.

How large is each subset? Each person gets three marbles. This is partitive division.

In a sentence such as 12 ÷ 4 = 3, the idea should not be verbalized as measurement division if it is actually partitive division, and vice versa.

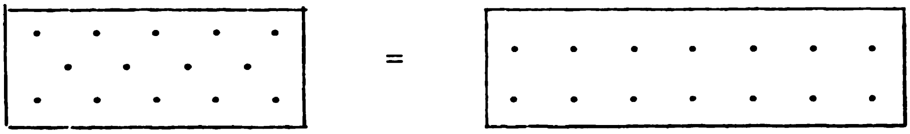
Problems:

1. Illustrate with X's and verbalize the following facts using the measurement interpretation of division.

(a) 9 ÷ 3 = ☐ (b) 6 ÷ 2 = ☐ (c) 12 ÷ 3 = ☐

2. Illustrate and verbalize each part of problem 1, using the partition interpretation of division.

3. Write a division equation for



and verbalize using both measurement and partitive forms.

Lesson Three

As mentioned earlier multiplication and division are inverse operations. Since multiplication can be thought of as repeated addition, division can be thought of as repeated subtraction.

Example: $27 \div 9 = 3$

$$\begin{array}{r} 27 \\ -\ 9 \\ \hline 18 \\ -\ 9 \\ \hline 9 \\ -\ 9 \\ \hline 0 \end{array}$$

Nine was subtracted 3 times.

Notice also that 3 X 9 = 27 and division can be thought of as renaming a product and factor as the missing factor.

$27 \div 9 = 3$

product factors

The conventional algorithm for division is written as follows:

$$\begin{array}{r} 3 \\ 9\ \overline{)27} \end{array}$$

This is read "27 divided by 9 is 3." (Notice 3 X 9 = 27). The three is the answer of the division problem or the quotient. The nine is the divisor and twenty-seven is the dividend.

$$\begin{array}{rl} & \text{quotient} \\ \text{divisor} & \overline{)\text{dividend}} \end{array}$$

Problems:

1. 28 ÷ 7 = _____ 3. 36 ÷ 9 = _____ 5. 45 ÷ 9 = _____

2. 28 ÷ 6 = _____ 4. 36 ÷ 6 = _____ 6. 64 ÷ 8 = _____

7. $7\overline{)28}$ = 9. $9\overline{)36}$ = 11. $9\overline{)45}$ =

8. $6\overline{)48}$ = 10. $6\overline{)36}$ = 12. $8\overline{)64}$ =

13. Do problems 1-6 as succesive subtraction problems.

14. A bag of marbles has one hundred marbles to be civided among the twenty children in the class. How many marbles will each child receive? Is this partitive or measurement division? Show this problem using ÷ and $\overline{)}$.

Lesson Four (Review Concepts of Division)

Work Problems: (Skill builders)

Complete the sentence.

1. 4 sets of 5 form one set of ____________.
2. 3 sets of 8 form one set of ____________.
3. A set of 15 forms __________ sets of 3.
4. A set of 45 forms __________ sets of 5.
5. 6 sets of 7 form one set of ____________.
6. A set of 40 forms __________ sets of 8.
7. A set of 48 forms __________ sets of 6.
8. A set of 64 forms __________ sets of 8.
9. 8 sets of 7 form one set of ____________.

*10. 10 sets of 5 form one set of __________.

*11. A set of 50 forms _________ sets of 5.

*12. A set of 72 forms _________ sets of 6.

Lesson Five

Division with Remainders

Division problems do not always come out exact. When this happens the amount left over (always less than the divisor) is called the remainder.

Example: How many tables will be needed to seat twenty-six people if six people are seated at each table? Is this the measurement or partitive concept of division.

$$\begin{array}{r} 4 \text{ remainder } 2 \\ 6\overline{)\,26} \\ -24 \\ \hline 2 \end{array}$$

There will be four full tables with two people at the fifth table. In the example name the divisor, dividend, quotient and remainder. Notice in the example that four 6's were subtracted from the twenty-six and that the remainder is less than the divisor. This must always be the case. Why?

Problems

Name the quotient and remainder.

1. $2\overline{)17}$
2. $3\overline{)14}$
3. $5\overline{)16}$
4. $6\overline{)37}$
5. $4\overline{)39}$
6. $8\overline{)52}$
7. $6\overline{)44}$
8. $8\overline{)29}$
9. $7\overline{)68}$
10. $9\overline{)78}$
11. $8\overline{)27}$
12. $7\overline{)37}$
13. Henry wants to plant three dozen tomato plants in rows of 8 plants each. How many full rows will he have? How many plants left over?

Division problems with remainders can be checked by multiplying quotient X divisor and then adding the remainder to get the dividend.

(quotient X divisor) + remainder = dividend

14. Check the first six problems in this exercise.

Lesson Six

Multi-Stage Division

As has been mentioned, division can be treated as repeated subtraction. We shall use this method to solve more difficult problems and then look at a shortened form of the solution.

Our quotients, up until now, have been single digit numbers. Now we shall work problems where the quotients are not single digits. We shall use repeated subtractions.

Example: We have 64 people to seat at tables with four people to each table. How many tables are needed?

```
4 ) 64
   -40    10
    24
   -24     6
     0    16 tables
```

You know what (4 X 10) is, and this subtracted from 64 leaving you with 24. Now the problem is like the ones you have been working. How many 4's in 24? Put this 6 under the 10 and you now have 16 fours or 16 tables are needed to seat the people. You may write the answer in the familiar place if you wish.

Example: Seat 96 people at tables with four people at each table. How many tables are needed?

```
4 )96
  -40    10
   56
  -40    10
   16
  -16     4
    0    24 tables
```

Why did we subtract forty twice?

Problems.

1. $2\overline{)24}$
2. $5\overline{)75}$
3. $3\overline{)54}$
4. $8\overline{)136}$
5. $7\overline{)154}$
6. $4\overline{)116}$
7. $4\overline{)68}$
8. $9\overline{)117}$
9. $6\overline{)138}$
10. $7\overline{)98}$
11. $5\overline{)135}$
12. $7\overline{)126}$
13. Check the first six problems.
14. If a bag of sugar costs $8.00, how much can be bought with $96.00?

Lessons Seven and Eight (Two days)

Another Step

Building upon the previous lesson, will lead us one step closer to the standard division algorithm.

Example: $$4\overline{)68} = 4\overset{10 \quad + \quad 7}{\overline{)40 \quad + \quad 28}} = 17$$

Here 68 is written as 40 + 28 and each part is then divided by 4 and the two parts of the quotient are added together for the final quotient. The first part of the expanded number is divisible by the divisor in multiples of ten and the last part of the dividend is what is left. Of course there are other ways to write 68 in expanded form. Can you name some other ways?

Example: $$4\overline{)96} = 4\overset{20 \quad + \quad 4}{\overline{)80 \quad + \quad 16}} = 24$$

Notice here that 80 is 2(40).

Example: $$7\overline{)259} = 7\overset{10 \quad + \quad 10 \quad + \quad 10 \quad + \quad 7}{\overline{)70 \quad + \quad 70 \quad + \quad 70 \quad + \quad 49}} = 37$$

or

$$7\overset{30 \quad + \quad 7}{\overline{)210 \quad + \quad 49}} = 37$$

In more difficult problems it is alright to write them so you understand what you are doing. Either method is acceptable in the above example. The important thing is that you understand how you write the expanded form. Relating the last example to the repeated subtraction idea from the last section we have the following example.

Example:

$$\begin{array}{rrr}
7 & \overline{)259} & \\
 & \underline{-\ 70} & 10 \\
 & 189 & \\
 & \underline{-\ 70} & 10 \\
 & 119 & \\
 & \underline{-\ 70} & 10 \\
 & 49 & \\
 & \underline{-\ 49} & \underline{7} \\
 & 0 & 37
\end{array}
\qquad \text{or} \qquad
\begin{array}{rrr}
7 & \overline{)259} & \\
 & \underline{-210} & 30 \\
 & 49 & \\
 & \underline{-\ 49} & \underline{7} \\
 & 0 & 37
\end{array}$$

Example:

$$9\overline{)198} \quad = \quad \begin{array}{rl} & \ \ 20 \ \ + \ \ 2 \\ 9 & \overline{)180 \ + \ 18} \end{array} \begin{array}{l} = \ 22 \\ \ \end{array}$$

Division problems may be written in the expanded form because division is distributive with respect to addition. Remember multiplication and division are inverse operations and multiplication is distributive over addition.

Problems

Do the following using the expanded form.

1. $3\overline{)48}$
2. $7\overline{)84}$
3. $6\overline{)84}$
4. $5\overline{)80}$
5. $4\overline{)56}$
6. $8\overline{)104}$
7. $6\overline{)96}$
8. $9\overline{)162}$
9. $3\overline{)57}$
10. $8\overline{)112}$
11. $6\overline{)108}$
12. $9\overline{)117}$
13. $9\overline{)135}$
14. $8\overline{)128}$
15. $7\overline{)119}$

With remainders.

Example:

$$4\overline{)69} \qquad \begin{array}{rlcr} & \ \ 10 & + & 7 \\ 4 & \overline{)\,40} & + & 29 \\ & & & \underline{-28} \\ & & & 1 \end{array} \begin{array}{l} = \ 17 \\ \text{remainder } 1 \\ \ \\ \ \end{array}$$

1. $3\overline{)35}$
2. $5\overline{)72}$
3. $6\overline{)112}$
4. $4\overline{)83}$
5. $9\overline{)118}$
6. $7\overline{)121}$
7. $6\overline{)93}$
8. $8\overline{)119}$
9. $9\overline{)119}$
10. $7\overline{)87}$

Try these.

<u>Example</u>:

$$14\overline{)\;294} \quad = \quad 14\begin{array}{lllll} \;10 & + & 10 & + & 1 \\ \hline)140 & + & 140 & + & 14 \end{array} = \; 21$$

or

$$14\begin{array}{lll} \;20 & + & 1 \\ \hline)280 & + & 14 \end{array} = \; 21$$

1. $11\overline{)231}$
2. $16\overline{)352}$
3. $15\overline{)540}$
4. $18\overline{)540}$
5. $12\overline{)192}$
6. $14\overline{)210}$

Lesson Nine

Zero Divisors

Do (8 ÷ 2 = ☐) by the repeated subtraction method. Also 12 ÷ 4 = ☐. Now do 7 ÷ 0 = ☐ by repeated subtraction. What is your answer? What did your neighbor get for an answer? Are they the same? Is it possible to get a single answer? If you *understand* that the last answer is no, because removal of the empty set from a set will never exhaust the given set then you should have no trouble understanding that division by zero is not allowed. Mathematicians say "division by zero is undefined."

Example: 8 ÷ 0 = undefined (no unique answer)

Zero Dividends

While 8 ÷ 0 has no single answer, what do you think would be the correct answer for 0 ÷ 8 = ☐. Solve this problem using repeated subtraction. What is your answer? If your answer is 0, and you *understand* this is because you cannot remove a set of 8 from the empty set, you should not have problems with *zero dividends*.

Problems

1. 6 ÷ 0 =
2. 8 ÷ 0 =
3. 15 ÷ 0 =
4. 36 ÷ 0 =
5. 28 ÷ 0 =
6. 53 ÷ 0 =

7. 0 ÷ 5 =

8. 0 ÷ 23 =

9. 0 ÷ 87 =

10. 0 ÷ 17 =

11. 0 ÷ 62 =

12. 0 ÷ 48 =

13. 0 ÷ 21 =

14. 0 ÷ 68 =

15. 68 ÷ 0 =

16. 23 ÷ 0 =

Lesson 10

The Standard Division Algorithm: One digit divisors.

The standard division algoritm for one digit divisors,

$$\begin{array}{rr} & 142 \\ 6 & \overline{)852} \\ & \underline{-600} \\ & 252 \\ & \underline{-240} \\ & 12 \\ & \underline{-\ 12} \\ & 0 \end{array}$$

can be derived from our previous work. If this is written as,

$$\begin{array}{rl} & 100 \quad + \quad 40 \quad + \quad 2 \;=\; 142 \\ 6 & \overline{)\,600 \quad + \quad 240 \quad + \quad 12} \end{array}$$

we can see where the various parts of the quotient, 142, come from . The one in the quotient is actually 100 (it is in the hundreds column), the 4 is really 40 and the 2 is in the units column. The 1 in the quotient is placed above the 8 in the dividend; both are in the hundreds column. The 4 in the quotient is placed above the 5 in the dividend, both are in the ten's column. The 2 in the quotient is placed above the 2 in the dividend, both are in the units column. This is very apparent in the expanded form.

Example:

$$\begin{array}{rr} & 158 \\ 4 & \overline{)635} \\ & \underline{-400} \\ & 235 \\ & \underline{-200} \\ & 35 \\ & \underline{-\ 32} \\ & 3 \end{array} \quad \text{remainder } 3, \quad 4\,\overline{)635} = \begin{array}{rl} & 100 \quad + \quad 50 \quad 8{=}158 \\ 4 & \overline{)\,400 \quad + \quad 200 + 35} \end{array}$$

Problems

Work the following using the expanded form first and the standard division algorithm second. Mentally compare your work.

1. $5\,\overline{)820}$
2. $4\,\overline{)652}$
3. $9\,\overline{)1944}$
4. $7\,\overline{)819}$
5. $6\,\overline{)1367}$
6. $5\,\overline{)1858}$
7. $7\,\overline{)1768}$
8. $8\,\overline{)958}$
9. $4\,\overline{)0}$
10. $8\,\overline{)914}$
11. $6\,\overline{)1757}$
12. $0\,\overline{)1493}$

Lessons Eleven and Twelve (Two days)

The Standard Division Algorithm: Two digit divisors,

The standard division algorithm,

$$\begin{array}{r} 14 \\ 18\,\overline{)\,252} \\ \underline{-180} \\ 72 \\ \underline{-\ 72} \\ 0 \end{array}$$

can now be arrived at with variations of our previous work. The problem above is to divide 18 into 252. From previous experience we recognize this as

$$\begin{array}{lcccl} & 10 & + & 4 & = \ 14 \\ 18\,\overline{)} & \overline{180} & \overline{+} & \overline{72} & \end{array}$$

but instead of rewriting the dividend we think about the parts and put the 1 in the quotient (which is really 10) above the $\overline{)\qquad}$ symbol, directly above the 5 in the dividend because the 5 is in the tens place in 252. Then the 4 in the quotient which is in the units place in the quotient goes directly above the last 2 in 252 because the last 2 is in the units place. Notice all the numerals which appear in the right portion of the algorithm,

$$\begin{array}{r} 14 \\ \overline{)\,252} \\ \underline{180} \\ 72 \\ \underline{72} \\ 0 \end{array}$$

are placed in the correct units, tens or hundreds column.

<u>Example</u>: By repeated subtraction: Expanded notation:

$$\begin{array}{rrr} 34 & \overline{)884} & \\ & \underline{-340} & 10 \\ & 544 & \\ & \underline{-340} & 10 \\ & 204 & \\ & \underline{-170} & 5 \\ & 34 & \\ & \underline{-\ 34} & \underline{1} \\ & 0 & 26 \end{array}$$

$$34\ \overline{)884} = \begin{array}{l} \qquad 20 \quad + \qquad 6 = 26 \\ 34\ \overline{)680 \quad + \quad 204} \end{array}$$

Standard algorithm

$$\begin{array}{rr} & 26 \\ 34 & \overline{)884} \\ & \underline{-680} \\ & 204 \\ & \underline{-204} \\ & 0 \end{array}$$

In the standard algorithm there is an estimation problem not encountered in the other methods at such a difficult level. In the standard algorithm students have a tendency to look at the 884 as 884 instead of (340 + 340 + 170 + 34) or (680 + 170 + 34) or (680 + 204). You must look at the pieces and estimate how many 34's are in 880 or 884. In arriving at the 2 in the quotient it is alright to think of the dividend as 880. Since there are 20 of the 34's in 680 the 2 goes in the tens position and after subtracting the 680 then the 6 goes in the units position.

<u>Example</u>: $22\ \overline{)1254}$

Since 10 X 22 = 220, you want the number of 220's in 1254 or 1250. How many 220's in 1250? (About 5 or 50 of the 22's.)

1st Step

$$\begin{array}{r} 5 \\ 22\ \overline{)\,1254} \\ -1100 \\ \hline 154 \end{array}$$

Put the 5 in the tens column. How many 22's in 154? (7)

2nd Step

$$\begin{array}{r} 22\ \overline{)\,1254} \\ -1100 \\ \hline 154 \\ -\ 154 \\ \hline 0 \end{array}$$

Then the 7 goes in the units column. The quotient is found to be 57. Again this can be checked by (quotient X divisor) = dividend. Do these first by repeated subtraction and then with the standard division algorithm.

Problems

1. $31\ \overline{)870}$
2. $52\ \overline{)733}$
3. $38\ \overline{)874}$
4. $22\ \overline{)792}$
5. $19\ \overline{)855}$
6. $63\ \overline{)949}$
7. $73\ \overline{)950}$
8. $42\ \overline{)966}$
9. $39\ \overline{)899}$
10. $27\ \overline{)0}$
11. $0\ \overline{)89}$
12. $34\ \overline{)1564}$
13. How many suits costing $75.00 each can be bought for $1050?

Lesson Thirteen. (Business applications)

1. If you have $1000.00 for sports coats, how many can you buy at $34 each?

2. If 14 dryers cost $2366, find the cost of one dryer.

3. What is the cost of one range if 15 ranges cost $4335?

4. Find how many 16 gallon barrels are needed to hold 512 gallons of oil?

5. How many $16 tool kits can be bought for $384?

6. A bakery sells sweet rolls, 16 to a box. How many boxes are needed for 656 sweet rolls?

Lesson Fourteen. Review. (Even professional ball players practice every day.)

Name the quotient and remainder.

1.	$4\overline{)95}$	$6\overline{)81}$	$8\overline{)1086}$	$7\overline{)85}$
2.	$2\overline{)91}$	$4\overline{)895}$	$3\overline{)870}$	$9\overline{)90}$
3.	$5\overline{)587}$	$7\overline{)2100}$	$8\overline{)32}$	$6\overline{)1220}$
4.	$7\overline{)3515}$	$4\overline{)1407}$	$6\overline{)2844}$	$7\overline{)1326}$
5.	$3\overline{)1174}$	$5\overline{)2412}$	$9\overline{)2945}$	$6\overline{)2707}$
6.	$9\overline{)2619}$	$7\overline{)2968}$	$4\overline{)1407}$	$8\overline{)5473}$
7.	$29\overline{)6259}$	$36\overline{)2975}$	$64\overline{)8053}$	$71\overline{)10634}$
8.	$64\overline{)72689}$	$49\overline{)87926}$	$73\overline{)90634}$	
9.	$83\overline{)45876}$	$58\overline{)73829}$	$62\overline{)86872}$	
10.	$0\overline{)867}$	$0\overline{)362}$	$36\overline{)0}$	$22\overline{)0}$

Lesson Fifteen.

Posttest for achievement.

Postattitude inventory.

APPENDIX C

IAD-, POST-, AND DELAYED POSTACHIEVEMENT TEST ATTITUDE INVENTORY

Lesson One.

Pretest

Complete the sentence.

1. A set of 15 equals _________ sets of 5.

2. 4 sets of 7 equals 1 set of ________.

3. 24 ÷ 6 = ________ 4. 42 ÷ 7 = _________

5. 15 ÷ 3 = ________ 6. 21 ÷ 3 = _________

7. $3\overline{)18}$ 8. $7\overline{)35}$ 9. $16\overline{)48}$

10. $41\overline{)328}$ 11. $81\overline{)972}$ 12. $62\overline{)868}$

13. $72\overline{)864}$ 14. $48\overline{)5724}$ 15. $37\overline{)8216}$

16. $63\overline{)57340}$ 17. $125\overline{)57575}$ 18. $342\overline{)168189}$

19. If 28 pieces of candy are shared equally among seven students, how many pieces does each student get? ___________

20. Is problem number 19 an example of measurement division or partition division? _________________________________

Posttest.

	a.	b.	c.
1.	$5\overline{)87}$	$8\overline{)1086}$	$4\overline{)1612}$
2.	$3\overline{)1174}$	$16\overline{)2812}$	$9\overline{)4623}$
3.	$0\overline{)1928}$	$43\overline{)1174}$	$6\overline{)54908}$
4.	$40\overline{)720}$	$81\overline{)972}$	$723\overline{)5784}$
5.	$162\overline{)868}$	$252\overline{)787}$	$333\overline{)4738}$
6.	$683\overline{)7206}$	$741\overline{)6384}$	$858\overline{)73829}$

7. How many $7 bags of seed can a farmer purchase for $100? __________ How much money, if any, is left over?_____

8. A candy company has 495 candy bars. How many boxes of 15 bars each can the company sell? __________
This is an example of (partition division) (measurement division). Underline correct one).

Posttest (Delayed).

	a.	b.	c.
1.	$6\overline{)81}$	$5\overline{)2412}$	$0\overline{)3514}$
2.	$4\overline{)2110}$	$18\overline{)2656}$	$9\overline{)3795}$
3.	$3\overline{)2532}$	$34\overline{)1419}$	$6\overline{)460174}$
4.	$60\overline{)840}$	$41\overline{)738}$	$824\overline{)6592}$
5.	$182\overline{)984}$	$254\overline{)817}$	$334\overline{)7479}$
6.	$671\overline{)8384}$	$732\overline{)4181}$	$864\overline{)72689}$

7. How many $7 shirts can a store owner purchase for $120?

_____________ How much money, if any, is left over?_______

8. A farmer has 384 eggs. How many dozen eggs does he have? ________ This is an example of (partition division) (measurement division). Underline correct line).

ATTITUDE INVENTORY

RATING

		SD	D	N	A	SA
1. Working with numbers is fun.	1.					
2. Arithmetic should be avoided whenever possible.	2.					
3. Discovering the solutions to new math problems is exciting.	3.					
4. Arithmatic is good because it makes you think.	4.					
5. Word problems are frustrating.	5.					
6. It is fun to think about problems outside class	6.					
7. Doing arithmetic problems is boring.	7.					
8. One cannot use mathematics in daily life.	8.					
9. Discovering solutions to math problems is frustrating.	9.					
10. Arithmetic is very interesting.	10.					
11. Arithmetic is too complicated.	11.					
12. Arithmetic is a stimulating activity.	12.					
13. Arithmetic is logical.	13.					
14. Arithmetic is necessary in daily life.	14.					
15. There are too many steps needed in getting the answer to a problem.	15.					
16. Arithmetic is practical.	16.					
17. There are too many chances to make mistakes in arithmetic.	17.					
18. Arithmetic takes too long.	18.					
19. Working with numbers presents a challenge.	19.					
20. Most word problems are not practical.	20.					
21. Arithmetic is a waste of time.	21.					
22. It is fun to play with numbers.	22.					
23. There are too many rules to learn in arithmetic.	23.					
24. Mathematics is frightening.	24.					

SD = Strongly Disagree
D = Disagree
N = Neutral
A = Agree
SA = Strongly Agree

APPENDIX D

CLASS MEANS

MALE, FEMALE, TOTAL

Table 16. Table of Means.

School	Attitude						Achievement								
	Preinventory			Postinventory			Pretest			Posttest			Delayed Posttest		
	M	F	T	M	F	T	M	F	T	M	F	T	M	F	T
T_{11}	78.8	81.2	80.2	79.0	85.6	82.8	49.4	70.0	61.2	15.0	30.2	23.7	19.4	40.4	31.4
T_{12}	91.4	89.6	90.6	83.0	87.1	84.7	51.5	48.9	50.5	35.8	48.9	41.1	48.8	61.1	53.9
T_{13}	74.2	83.5	77.5	76.4	81.7	78.2	57.3	59.2	57.9	46.6	40.7	44.5	36.8	46.7	40.3
T_{14}	80.0	86.3	82.1	72.2	75.0	73.2	48.1	52.5	49.6	11.0	11.2	11.1	13.1	26.2	17.5
T_{21}	92.2	89.6	90.8	89.0	86.9	87.8	51.1	46.4	48.5	34.8	38.4	36.8	62.0	55.0	58.0
T_{22}	76.2	90.9	84.6	71.6	84.8	79.1	38.3	45.4	42.4	44.6	60.8	53.8	37.8	62.5	51.9
T_{23}	91.6	86.0	88.9	87.3	82.0	84.8	53.1	50.0	51.6	36.3	33.6	35.0	31.5	22.1	27.0
T_{24}	76.2	79.6	77.4	79.5	84.1	81.1	56.5	58.6	57.2	45.5	21.9	37.2	66.2	58.6	63.5
Grand Means	82.9	86.2	84.4	80.2	84.2	82.1	53.8	53.6	53.7	35.0	38.4	36.6	41.0	47.3	43.9

M = Male subjects (85)
F = Female subjects (73)
T = Total subjects (158)

T_{1i} are classes in treatment group.
T_{2i} are classes in control group.

APPENDIX E

ANCOVA WITH PRETESTS AS COVARIATES AND STUDENTS AS THE UNIT OF ANALYSIS

ANCOVA WITH PRETESTS AS COVARIATES AND STUDENTS AS THE UNIT OF ANALYSIS

The following analysis was completed to determine the results of using the pretests as covariates and the student as the unit of analysis. The reader should keep in mind that using the student as the unit of analysis violates the assumption of within cell independence, especially since the students in the treatment group played the trading game in groups of four or five.

Data

Individual student scores on each of the five measures described in Chapter III were used for the analysis of covariance. The analysis for the post attitude inventory is summarized in the following table while the analysis for the achievement posttest and the delayed achievement posttest are summarized separately.

Table 17. Postattitude Inventory with Preattitude Inventory as Covariate (ANCOVA).

Source	df	MS	Univariate	
			F	P Less Than
(1) Sex by Treatment Interaction	1	168.34	1.56	.21
(2) Sex (S)	1	100.97	.94	.34
(3) Treatment (T)	1	11.30	.10	.75
error	153	107.93		

Findings

None of the F-tests for the ANCOVA of the postattitude inventory with the preattitude inventory as the covariate were significant at the $\alpha = .05$ level.

Conclusions

Even though the regression analysis for the test of the model was significant with $p < .0001$, there was little sex by treatment interaction associated with attitude of fifth grade students in division. There was no sex or treatment effect associated with attitude of fifth grade students in division.

ANCOVA of Achievement

The multivariate analysis of the achievement posttest and delayed posttest with the IAD test as a covariate was not significant for sex main effect with $F_{2,152} = .50$ and $p < .50$. However, the multivariate F-test for treatment main effect was significant with $F_{2,152} = 5.64$ and $p < .004$. This significance is in favor of the control group as evidenced by the table of means, Appendix D. The multivariate F-test for the sex by treatment interactions was not significant with $F_{2,152} = 1.17$ and $p < .31$.