A MULTIPORT APPROACH TO MODELING FLUID POWER SYSTEMS

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ABSTRACT

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By

Mark R. Ray

The ability to predict system dynamics is proposed as a new tool to overcome problems often encountered in the design of large scale fluid power systems. System models are constructed from component models developed by a multiport approach.

Standard techniques are used to generate the system state space equations. The component models are compiled into a catalog to be used in construction of system models. A scheme for obtaining system parameters is also introduced.

A MULTIPORT APPROACH TO MODELING FLUID POWER SYSTEMS

By

Mark R. Ray

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TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

THE PROBLEM

1.1 New Tools

The design of large scale fluid power systems has become a task of increasing difficulty in recent years due to their rapid expansion in scope and sophistication. Static analysis has been heavily employed as an aid to design but is proving less and less effective as the systems increase in complexity. Investigation of system dynamics prior to actual construction is a powerful approach to the design problem.

The ability to predict the dynamics of a fluid power system is a versatile tool and one well worth employing. The task of predicting system dynamics can be broken down into three phases. The first phase is to develop or obtain a system model, in this case a mathematical model consisting of differential equations describing the system. Next the system parameters, volumes, inertias, etc., used in the system model must be obtained. The last phase is solving the equations.

Although these operations are easy to enumerate their execution can be difficult. Trying to obtain the system equations when modeling a large scale fluid power system can be a perplexing task unless one is well armed with experience in work with fluid power systems and mathematical descriptions of physical phenomena. Often the parameters can only be obtained by testing in a laboratory. Many methods exist for solving differential equations but choosing the most effective may also require additional effort.

Prediction of system dynamics is not easy. But can it be simplified?

1.2 An Example

As an example of a design problem involving a large scale fluid power system consider the application detailed here. It may help to illustrate some of the problems previously mentioned.

A conveyor belt is to be used to move material down a line composed of individual work stations. A unique operation is performed at each station by an automated machine, so the material must be precisely positioned. The distance between each work station is identical so the belt may be indexed and all functions performed simultaneously. The limiting factor on the rate of production will be how fast the belt can be indexed while still maintaining the position accuracy required. Figure 1 shows the proposed fluid power system used to drive the system.

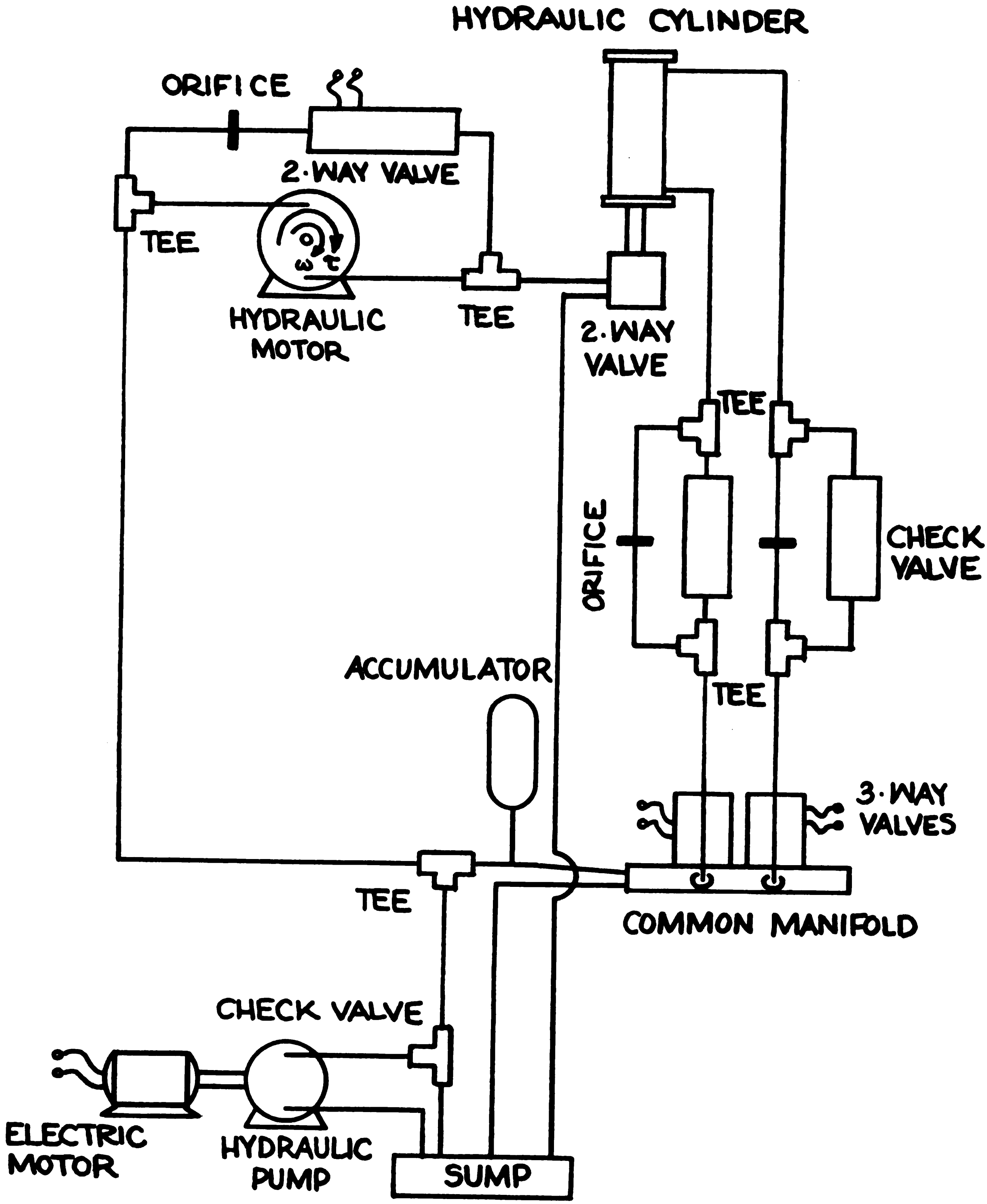

Figure 1. Hydraulic drive for conveyor system.

The hydraulic motor drives the conveyor belt through a gear arrangement but for sake of clarity they are not shown. The exhaust part of the hydraulic motor is opened and closed by means of a two way valve actuated by the hydraulic cylinder. The cylinder is in turn controlled by the two three way solenoid valves on the common manifold. The motor by-pass is also opened and closed by a solenoid valve, in this case a two way valve.

The system operates in the following manner. The solenoid valve on the left side of the manifold is normally open to the supply source. The valve on the right is normally closed to the supply source. The normally open and normally closed phases refer to the deenergized state of the valve so with no power to the valves the cylinder is up. The check valve orifice configurations between the valve and cylinder allows free flow from the valve to the cylinder but when flow is reversed the check valve is closed and the flow is metered through the orifice. By changing the orifice size the speed of the cylinder can be controlled. The solenoid valve in the motor by-pass line is normally closed and the orifice in that line is also adjustable. The solenoid valves on the manifold are energized when a signal is generated by a timing board clock. The by-pass solenoid is energized when the material being moved by the conveyor closes a switch.

Upon the signal from the timing board the manifold valves switch orientation; the left valve opens to the sump and the right valve opens to the supply source, driving the cylinder down. This action allows the two way valve closing the motor exhaust port to open and the motor begins to drive the conveyor. When the material on the conveyor closes the switch allowing the motor by-pass line to open the motor starts to decelerate. Upon another signal from the timing board the manifold valve switch orientation and begin closing the motor exhaust line. When the material reaches the desired point the conveyor is abruptly halted.

Coordination of all these activities into a smooth operation can be extremely difficult, especially when trying to operate stably at high speed.

CHAPTER 2

THE APPROACH

2.1 A Standard Format

Recognizing the need for better systems analysis, the National Fluid Power Association has formed a simulation committee "to set forth a standard format for mathematical models for fluid power systems and components [1]."

This development would be beneficial in several ways. If the format were adopted by the fluid power industry the component manufacturers could develop models for their products and supply the parameters needed in the models to describe their products. The proper format could also simplify the development of system models, making it easier to obtain the system equations. These two advances coupled with the proper method of solution could enable the designer of fluid power systems to determine the feasibility of a design before taking any action toward construction.

The format proposed here will begin by attacking the modeling problem at the component level. Standard models of common fluid power components will be developed and compiled into a catalog. A method of system model construction from component models will also be detailed. A later section on future developments will propose some interesting uses for the catalog.

The standard models for each component are developed using a multiport approach known as bond graphs. (Those unfamiliar with bond graphs are referred to Appendix A.) The bond graphs are then translated into ordinary differential equations describing the component in terms of the so-called "state variables." A set of simple algebraic equations relate the remaining component variables to the state variables.

The internal coupling structure of the component can be studied when using bond graph methods as well as the basic nature of the dynamics. The models are acausal in nature so it is unnecessary to make any decision concerning input and output at the component level. Decisions involving input and output are made when the system model is completed.

Once the problems of obtaining the system equations and parameters have been dealt with, the only remaining hurdle is the method of solution. There are a number of analog and digital techniques that can be used to solve the system equations.

A digital computer program capable of accepting a system model in bond graph form known as Enport [4] is available. The program is capable of assigning power flow directions and causality but if the system equations are nonlinear the program will not generate the state space equations. At the present time the program is only capable of manipulating small scale, continuous, linear or linearized systems. Expansion of the program capabilities is currently being studied with the ability to handle large-scale, linear systems as the next step in development, followed by nonlinear systems.

Analog computer schemes would appear to offer the best possibility for solution of large-scale fluid power system state equations. The nonlinearity and discontinuity of the equations can be nicely handled by an analog computer. The only drawback is the amount of work involved in altering the system equations when parameters are changed. This sometimes makes the use of an analog computer in an iterative manner extremely tedious. This is undesirable since iterative solution techniques are valuable in design work.

2.2 The Bond Graph Approach

The modeling of fluid power systems by employing bond graph techniques can be approached several ways in an equally valid manner. The approach taken here is by no means unique. The majority of fluid power systems of interest can be characterized as high pressure-low flow rate systems. There are several choices of power variables available to describe fluid power systems; the flow-effort pair of volume flow rate (Q) and dynamic pressure (P) shall be used here. The fluid represented in this manner has no distributed mass, compressibility, or thermal properties. However, a system may be characterized by means of lumped models whenever the significant wave lengths of all variables are large compared to the physical dimensions of the system [5]. Although this places some limitations on the models they are not too restrictive.

A departure from conventional bond graph notation will be made in that the effort variables will be represented by P (pressure) and F (force) rather than the generalized effort symbol e. The flow variables will be represented by Q (volume flow rate), V (linear velocity) and W (rotational velocity), rather than the generalized flow symbol f. This is done in an effort to make the presentation in terms more familiar to the average engineer. A consistent set of units will be maintained throughout the development which will conflict somewhat with general practice in the fluid power industry. The volume flow rate, for example, is given in cubic inches per second rather than gallons per minute. Other quantities will be defined as required.

Several models will now be developed to illustrate the methods used to obtain the standard models for common fluid power system components in the catalog (Appendix B).

2.3 Typical Models

The common hydraulic cylinder shown in Figure 2a has many applications in fluid power systems and can be used to demonstrate the bond graph modeling technique.

If the fluid is considered incompressible and the piston and shaft massless, the device will exhibit no dynamics. If the motion of the piston and shaft occurs without dissipation and no leakage occurs the device may be considered ideal. After making these assumptions the next step is to examine the external ports of the device. Figure 2b is a word bond graph of the hydraulic cylinder showing the device as a 3 port element. The power convention on the fluid ports is chosen only for convenience and causes no loss of generality to the model. By means of the free body diagram in Figure 2c the basic coupling structure may be examined. The sum of forces on the free body diagram is given as

$$F_1 - F_2 = F_3 \qquad 2.3.1$$

Substituting for F_1 and F_2

$$P_1A_1 - P_2A_2 = F_3 \qquad 2.3.2$$

Equation 2.3.2 shows the relationship between the chamber pressures and the force on the shaft. By

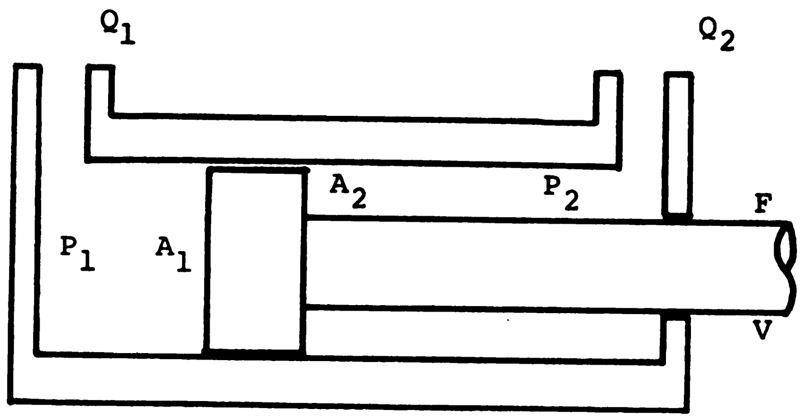

Figure 2a

Figure 2b

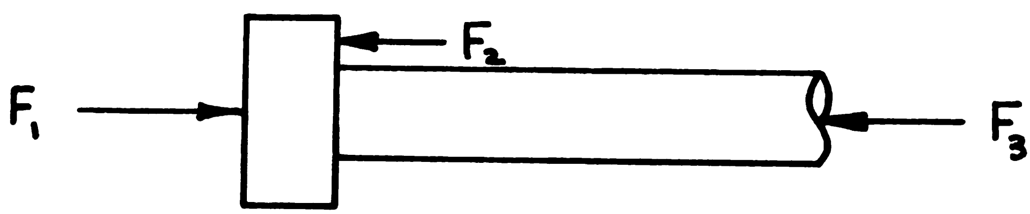

Figure 2c

Figure 2. (a) Hydraulic cylinder; (b) word bond graph; (c) free body diagram.

examining the flow equations the relationship between the piston velocity and the inlet and outlet flow is revealed.

$$V = \frac{Q_1}{\bar{A}_1} = \frac{Q_2}{\bar{A}_2} \qquad 2.3.3$$

The ideal device is represented by the bond graph model in Figure 3a. The relationships are modeled by two transformers and a 1-junction. Now that the ideal model has been developed, the static and dynamic models require only some additions to the basic model.

First consider the static model, which includes losses but no dynamics. Two mechanical losses occur that are relatively simple to model. A force is needed to overcome sealing friction around the shaft. This force is a function of the velocity of the shaft and is added to the 1-junction as a resistance or dissipating element. The other mechanical loss is due to the force needed to shear the fluid in the narrow clearance between the cylinder and piston. This is also a function of the velocity of the piston and is added as a resistance element on the 1-junction. Two fluid power losses occur due to leakage. One is internal leakage past the piston, which is dependent on the chamber pressures and the piston velocity. The component of leakage flow due to the piston velocity will be neglected in the model developed here. The leakage flow will be considered to be dependent on chamber pressure only. This effect can be modeled by establishing a 0-junction to represent the chamber pressure and inserting a resistance element on a 1-junction between them. The flow is shown from P_1 to P_2. This is done to establish a convention and causes no loss of generality. The other leakage loss occurs where the shaft passes out of the cylinder body. This leakage is dependent on the pressure in the chamber, P_2, the velocity of the shaft and the pressure outside the cylinder. By establishing a 0-junction for that pressure the leakage past the shaft can be represented by inserting a resistance element on a 1-junction between the 0-junctions representing the two pressures. The effect of shaft velocity is ignored. Figure 3b shows the static bond graph model.

The last model to be developed is the dynamic model. If the inertia of the piston and shaft are lumped together it can be simply added to the 1-junction representing their velocity. The compliance of the chamber is dependent on the pressure in the respective chambers and can be modeled by the simple addition of two capacitance elements to the 0-junctions referring to these pressures. The inertia of the fluid is assumed negligible when compared to the inertia of the piston and shaft and so is not considered to affect the dynamics enough to warrant inclusion. The shaft and cylinder body are

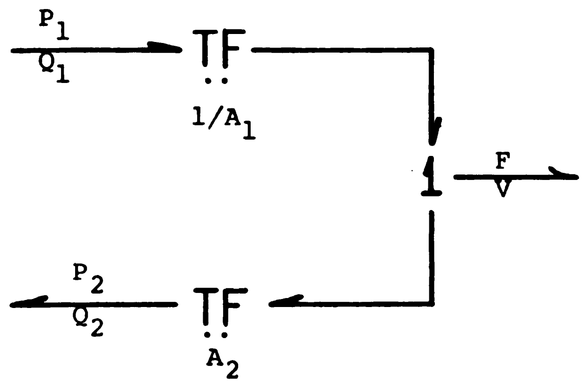

Figure 3a

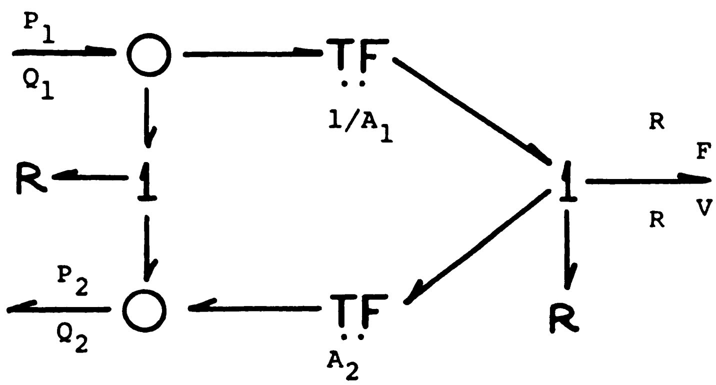

Figure 3b

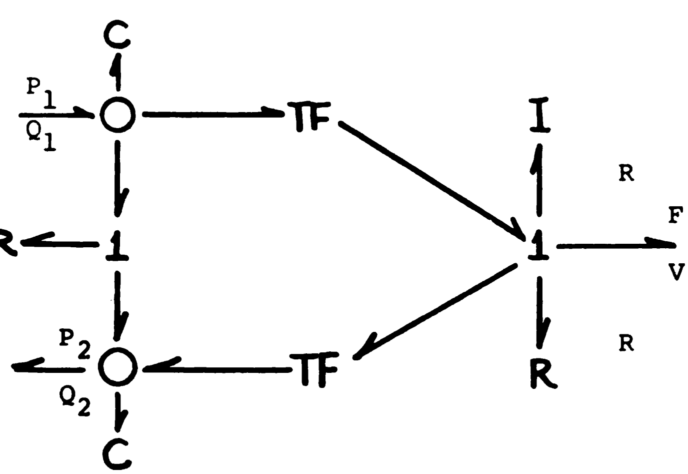

Figure 3c

Figure 3. (a) Ideal bond graph model; (b) static bond graph model; (c) dynamic model.

modeled with no compliance though this need not be the case. The dynamic model developed above is only one of many which may include some of the effects which may be important in predicting the response of the component. Figure 3c shows the dynamic bond graph model.

The hydraulic cylinder was a relatively simple device to illustrate the bond graph modeling technique. Figure 4a is an axial piston positive displacement pump which will be modeled using a slightly different approach.

Initially the pump will be considered as an ideal device having no mass or compliance. The fluid will also be considered ideal. No fluid leakage will occur nor any mechanical losses of any kind. The shaft shall be rotated causing the piston to both move in the cylinder and rotate. When the piston is at bottom dead center the volume of the cylinder shall be considered minimum. When the piston is at top dead center the volume shall be maximum. Both these positions shall be centered between the high and low pressure ports. The volume of the piston cylinder is given given by equation 2.3.4.

$$V = A_p R_p (1-\cos \Theta_s) \text{SIN } \Theta_Y \qquad 2.3.4$$

These terms are defined in Figure 4b by taking the derivative of this equation with respect to time and expression for the volume flow rate is obtained

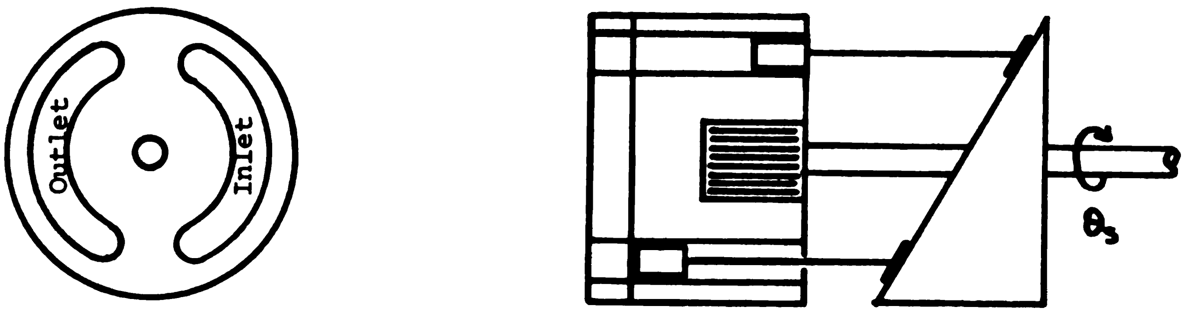

Figure 4a

Figure 4b

Θ_y = Yoke Angle (Radians)

Θ_s = Shaft Angle (Radians)

R_p = Radius of Piston Circle (Inches)

A_p = Surface Area of Piston (Inches^2)

Figure 4. (a) Positive displacement axial piston pump; (b) definition of variables.

$$\dot{V} = \frac{\partial V}{\partial \Theta_s}\dot{\Theta}_s + \frac{\partial V}{\partial \Theta_y}\dot{\Theta}_y \qquad 2.3.5$$

$$\dot{V} = (A_p R_p \mathrm{SIN}\Theta_y \mathrm{SIN}\Theta_s)\dot{\Theta}_s + (A_p R_p[1-\cos\Theta_s]\mathrm{COS}\Theta_y)\dot{\Theta}_y \qquad 2.3.6$$

Since in the case of this particular motor $\dot{\Theta}_y = 0$

$$\dot{V} = (A_p R_p \mathrm{SIN}\Theta_y \mathrm{SIN}\Theta_s)\dot{\Theta}_s \qquad 2.3.7$$

Neglecting the discontinuity at $\Theta_s = 0°$ and 180° the piston and cylinder charge when $0°<\Theta_s<180°$ and discharge when $180°<\Theta_s<360°$. The pressure acting on the face of the piston creates a force which acts through a moment arm to resist motion when $0°<\Theta_s<180°$ and assist motion when $180°<\Theta_s<360°$ as shown by the equation below

$$T_s = P(A_p R_p \mathrm{SIN}\Theta_y \mathrm{SIN}\Theta_s) \qquad 2.3.8$$

With one piston this action is highly discontinuous. When the shaft turns two pistons 180° apart if we disregard the discontinuity at top and bottom dead center the action can be considered continuous. One cylinder always charging, one discharging, one torque opposing, one assisting. If four pistons 90° apart are considered the action is even more continuous. The relationship between T and P and Q and $\dot{\Theta}_s$ is governed by the same modulus for each piston. The quantity $R_p A_p \mathrm{SIN}\Theta_y$ is a constant, actually the displacement of the cylinder, D_m . $\mathrm{SIN}\Theta_s$ simply indicates the direction of the action and the percentage of completion. If the action can be split into two distinct regimes and averaged we can model the ideal pump in the following way. For each rotation of 360° the cylinder charges and discharges once, likewise the torque acts in two opposite directions. Using two transformers and grouping the action together the bond graph of Figure 5a results.

Now that the ideal model has been developed, again, by including the dissipating elements the static model can be obtained. The leakage flow from the high pressure port to the low pressure port is a laminar leakage flow dependent on the pressure difference between the ports. It can be added by establishing a 0-junction for the inlet and outlet pressures and inserting a resistance element on a 1-junction between them. The leakage past the piston in both the high and low pressure sides is a laminar flow dependent on the piston speed and the respective pressure difference between the port and the pressure in the case. Again the leakage flow component owing to piston velocity is neglected. By establishing the 0-junction for this pressure these leakage flows may be inserted as resistance elements in 1-junctions. A resistive torque proportional to the speed of the pump required to shear the fluid in the small pump passages can be included in each model. The models can be simplified or expanded on each level. Effects not considered important can be removed,

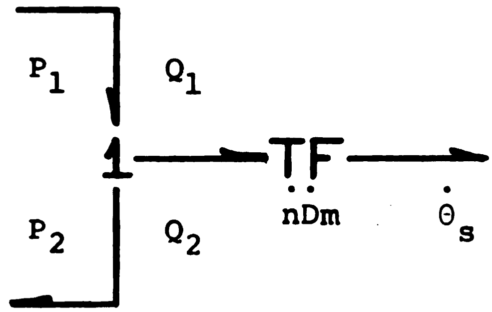

Figure 5a

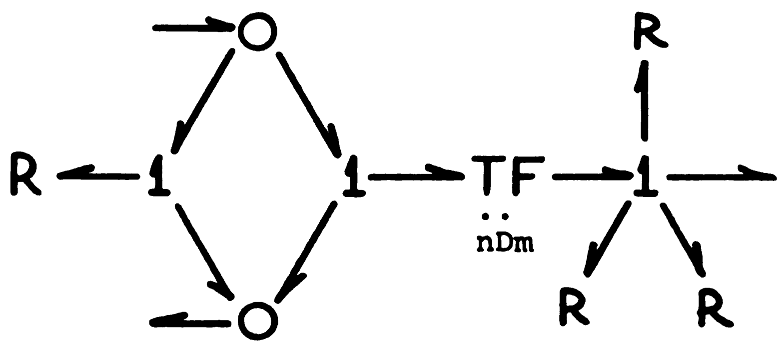

Figure 5b

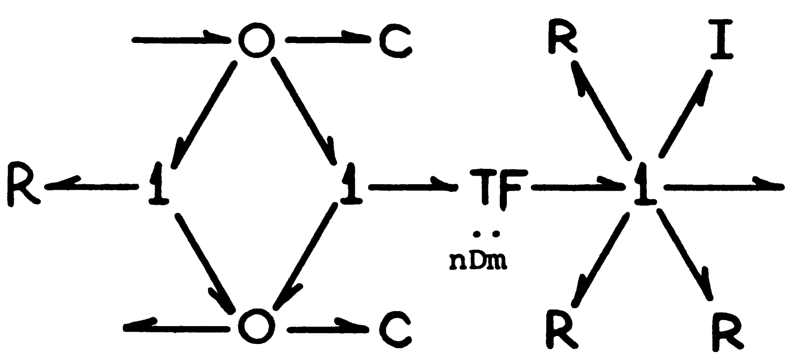

Figure 5c

n = Number of Pistons

Figure 5. Stages of development for the bond graph.

and others added between mechanical elements in relative motion can modeled as a resistance added to the 1-junction representing the speed of the pump. A small torque is also required to overcome seal friction. This is also dependent on pump speed and can be modeled as a resistance element added to the 1-junction representing the pump speed. There is also a torque required to overcome windage loss which is also a function of pump speed and can be added in the same manner as the others. Figure 5b shows the static bond graph model.

The dynamic model, for the sake of simplicity, will only include fluid compressibility in the high and low pressure ports and a lumped inertia to represent both the fluid trapped in the cylinders and the mechanical parts. The dynamic bond graph is shown in Figure 5c.

The models developed to represent the hydraulic cylinder and hydraulic pump are only general models and should not be considered unique. Three levels of model were presented to give an idea.

2.4 System Synthesis

Models of fluid power system components developed in the same manner as the examples in the last section have been compiled in Appendix B as the beginning of an extensive collection of component models. A "building block" construction method for system synthesis is presented below. Not all the steps listed are important and as the approach becomes more familiar they can be omitted, but initially they may help to clarify the procedure.

1. Examine the overall system and its components.
2. Determine the function of each component.
3. Isolate the component from the system by drawing a circle around it.
4. Determine the configuration of ports.
5. Construct a word bond graph of the system.
6. Determine the level of completeness desired in the system model.
7. Go to the catalog and obtain the component model.
8. Substitute into the word bond graph.
9. Attach bonds.

The conveyor belt example from Chapter 1 will be used to demonstrate the system synthesis procedure. For convenience the schematic is reproduced in Figure 6. Lines have been drawn around each component to isolate it as suggested. Figure 7 shows the word bond graph constructed using Figure 6 as a guide. In Figure 8 the components have been replaced by their bond graph models and the system has been reduced in Figure 9.

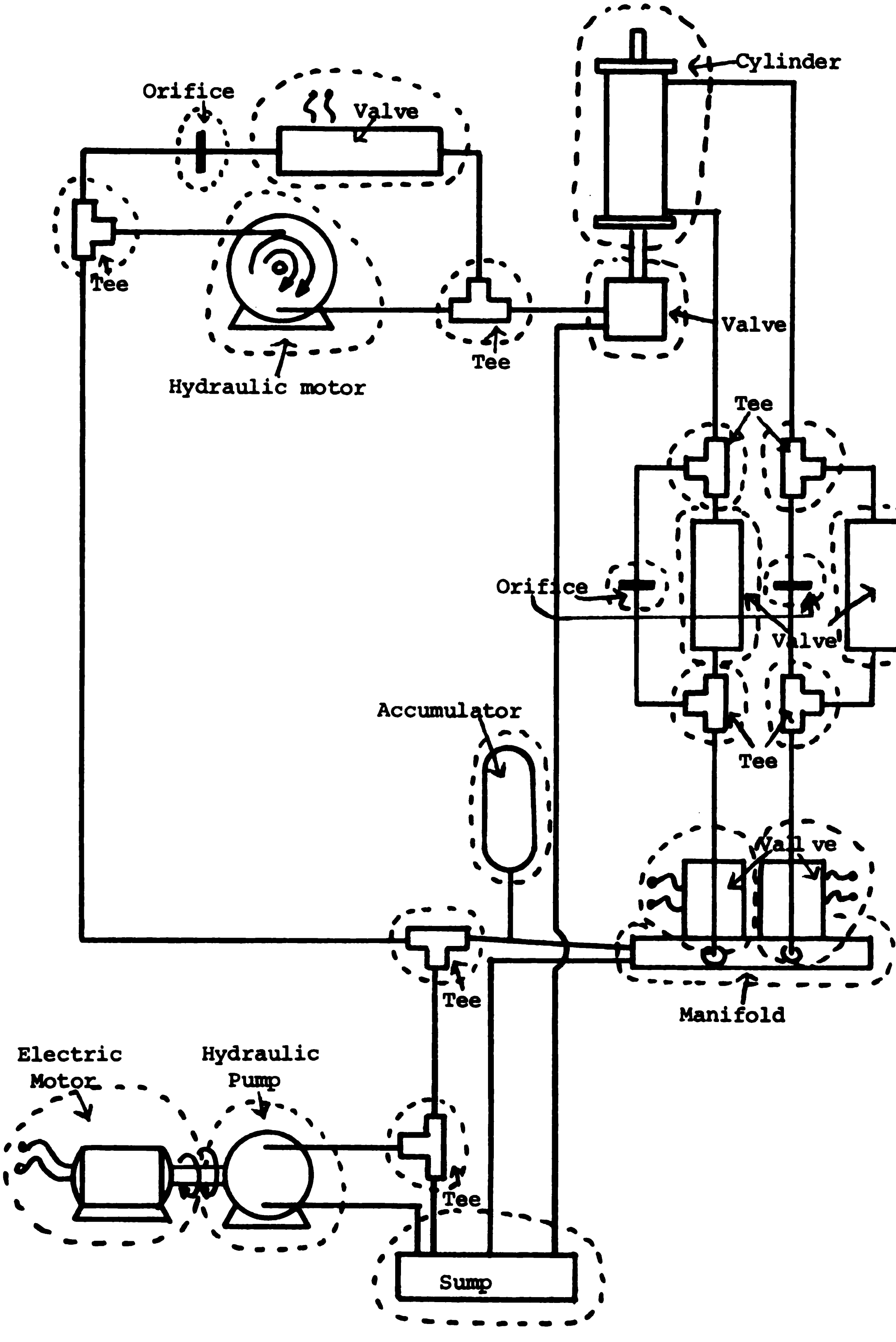

Figure 6. Schematic with components isolated.

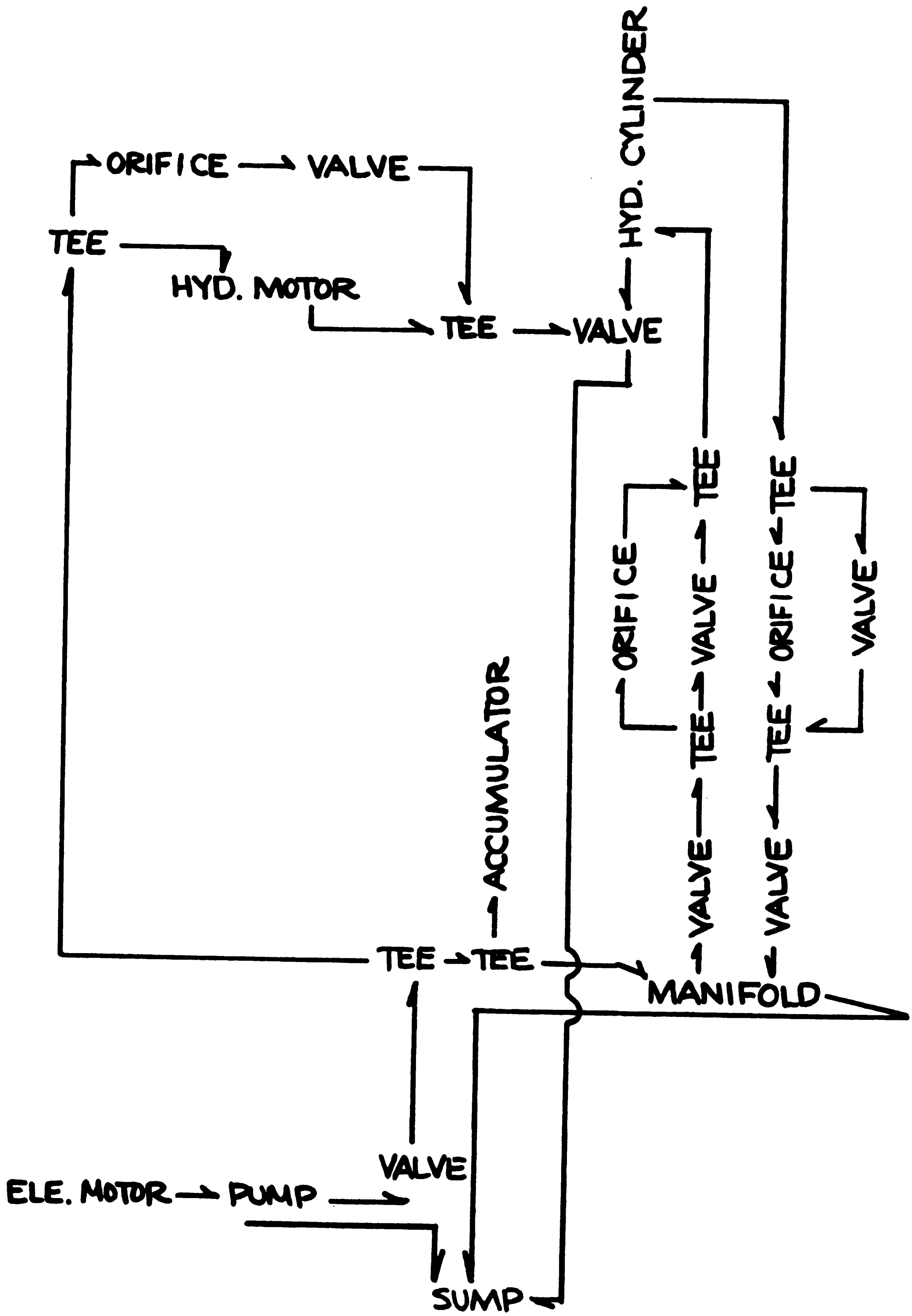

Figure 7. Word bond graph of conveyor belt model.

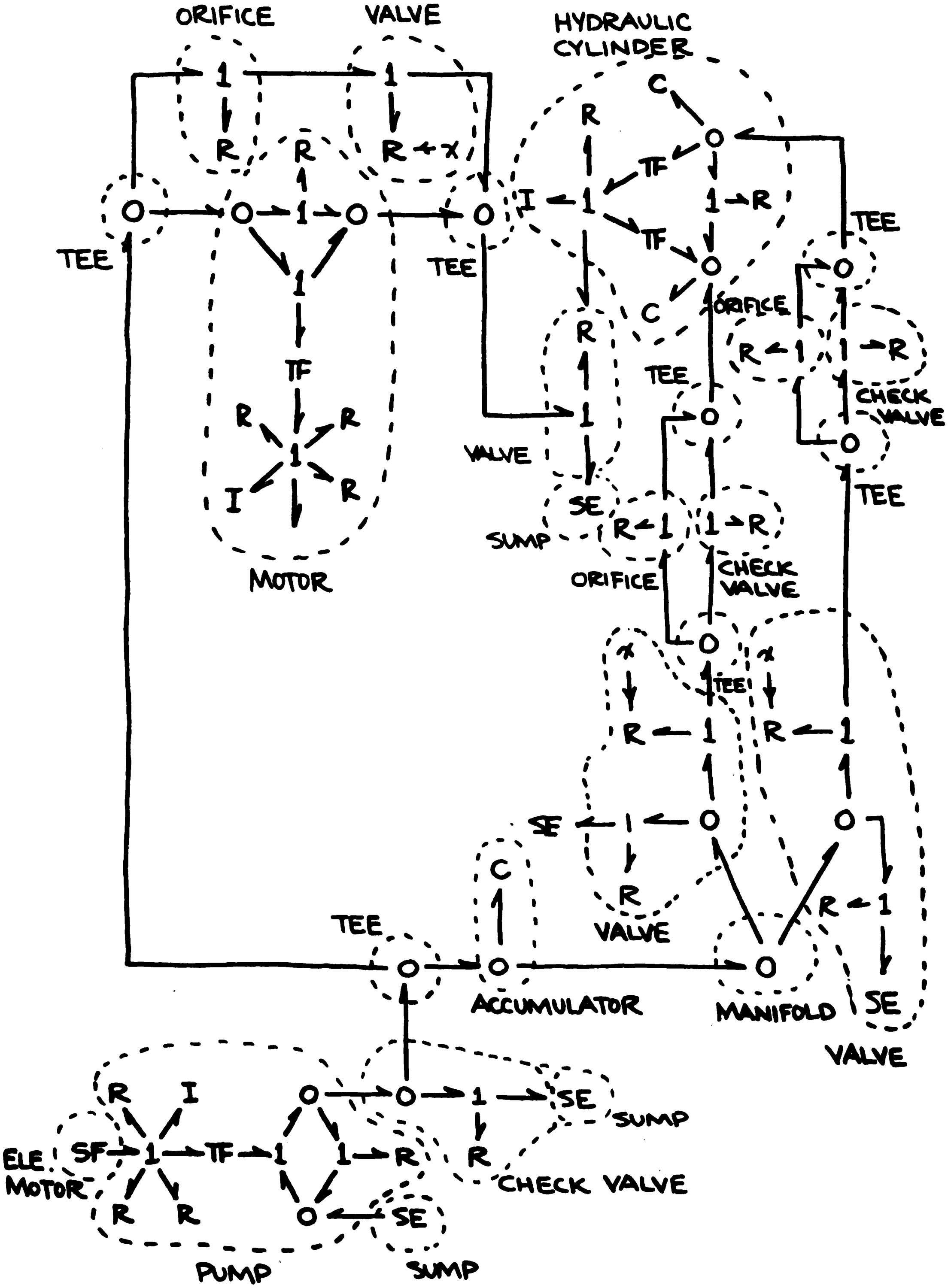

Figure 8. Preliminary bond graph of conveyor belt model.

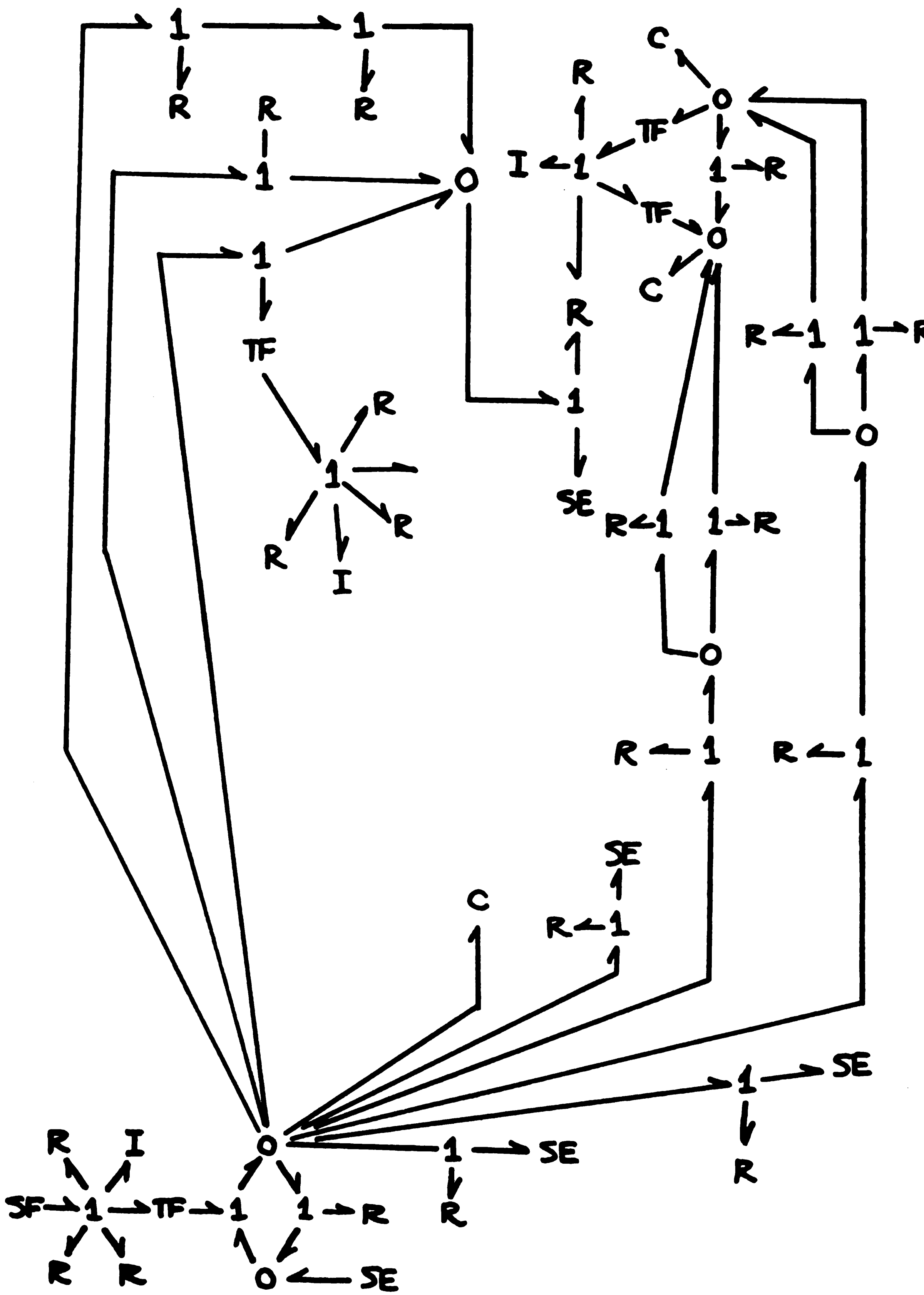

Figure 9. Bond graph of conveyor belt model.

CHAPTER 3

THE CATALOG

3.1 Format

The catalog, though not extensive, contains a representative collection of common fluid power system components. The models themselves do not represent all possible configurations but hopefully some of the most useful. Instructions for the use of the catalog are relatively simple. The components are grouped by main function only. For example, all motors are under a common listing, whether hydraulic or electric. There is no other organization other than the general groups at present; as the catalog grows more formal organization may be necessary. An attempt was made at consistency but in some instances it was difficult to maintain. A typical catalog page is shown in Figure 10. It should provide some idea of how the material in the catalog is presented. Three models are usually presented for each component representing an ideal, static and dynamic level of analysis. Occasionally more than one model is presented at certain levels to detail a special case. Appendix B contains the catalog.

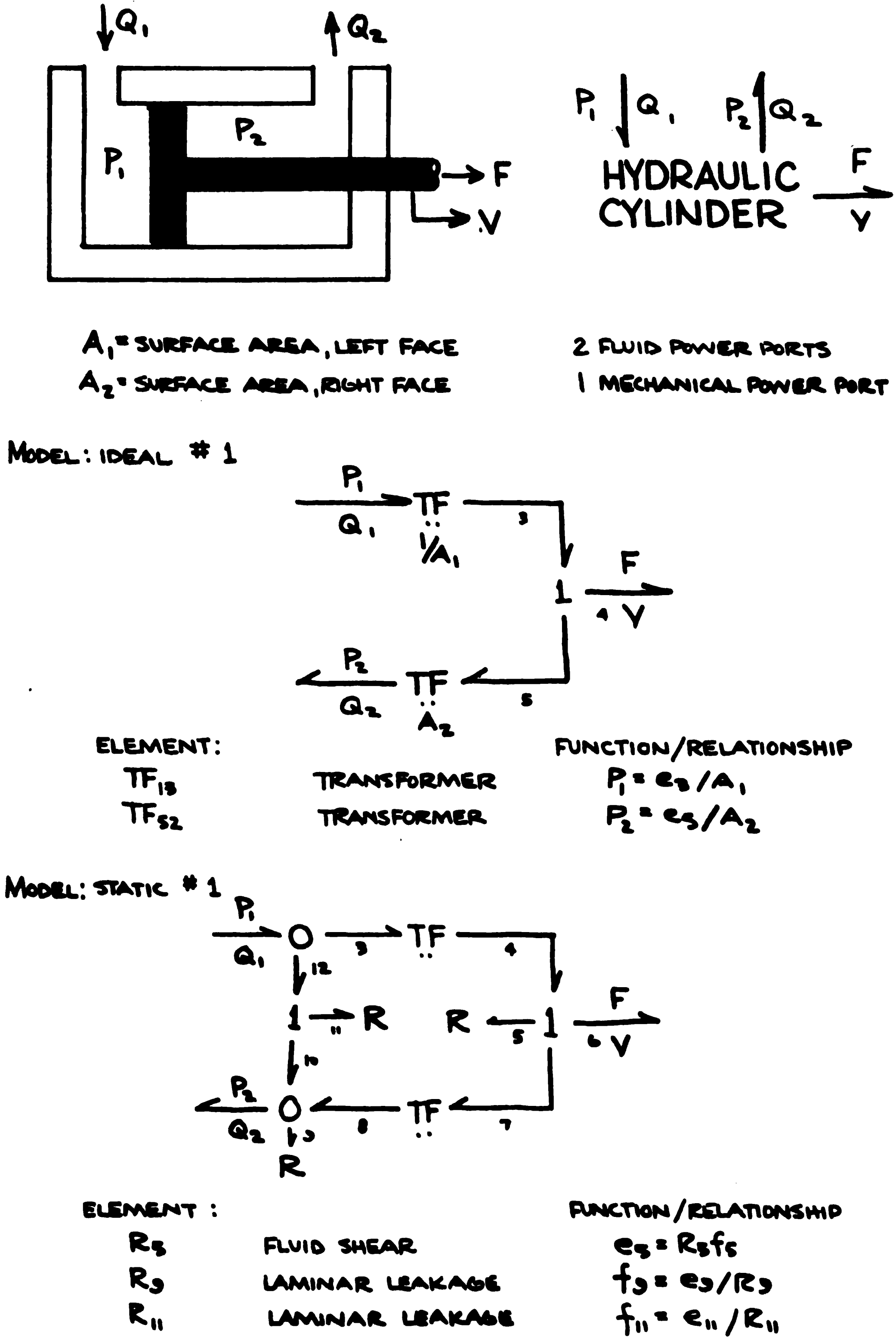

Figure 10. Example of a cataloged component.

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

4.1 Summary

In a presentation of the National Fluid Power Association Simulation Committee's proposed standard format to the National Conference on Fluid Power [1] in September of 1972 the following was stated.

The Simulation Committee avoided the enticing avenue of writing a separate general equation, with variable parameters which each manufacturer would supply, for each type of valve, pump, cylinder, etc., commercially available. Such an approach would merely temporarily side-step the more general problem of guaranteeing compatibility of mathematical equations of two components which can be physically connected in the laboratory.

It has been demonstrated that bond graph techniques can be used to effectively model fluid power system components not only in a manner that allows standard models of common components to be developed by also guarantees mathematical compatibility of the equations obtained when the component models are connected. A method was also introduced to simplify the system synthesis from the component models. A beginning was made at collecting the component models into a catalog useful to the manufacturer and designer for exchanging information. The use of bond graph techniques also simplifies the task of obtaining the system equations.

4.2 Future Work

If the models developed in this thesis using bond graph techniques prove unsatisfactory, further development must be done to develop models which can be standardized throughout the fluid power industry. Once these goals are accomplished manufacturers could provide the component parameters specified by the standard model.

A computer library of models could be developed for use with a simulation program. Ideally the component models could be called out and assembled according to designer specification. The program would generate and solve the system equations and simulate system response. The designer may be able to use the design program without any specific knowledge about the component models, other than choosing appropriate levels of complexity.

APPENDICES

APPENDIX A

A DEFINITION OF THE BOND GRAPH LANGUAGE

R. C. ROSENBERG
Associate Professor, Department of Mechanical Engineering, Michigan State University, East Lansing, Mich.

D. C. KARNOPP
Professor, Department of Mechanical Engineering, University of California, Davis, Calif.

A Definition of the Bond Graph Language

Introduction

The purpose of this paper is to present the basic definitions of the bond graph language in a compact but general form. The language presented herein is a formal mathematical system of definitions and symbolism. The descriptive names are stated in terms related to energy and power, because that is the historical basis of the multiport concept.

It is important that the fundamental definitions of the language be standardized because an increasing number of people around the world are using and developing the bond graph language as a modeling tool in relation to multiport systems. A common set of reference definitions will be an aid to all in promoting ease of communication.

Some care has been taken from the start to construct definitions and notation which are helpful in communicating with digital computers through special programs, such as ENPORT [5].[1] It is hoped that any subsequent modifications and extensions to the language will give due consideration to this goal.

Principal sources of extended descriptions of the language and physical applications and interpretations will be found in Paynter [1], Karnopp and Rosenberg [2, 3], and Takahashi, et al. [4]. This paper is the most highly codified version of language definition, drawing as it does upon all previous efforts.

Basic Definitions

Multiport Elements, Ports, and Bonds. *Multiport elements* are the nodes of the graph, and are designated by alpha-numeric characters. They are referred to as elements, for convenience. For example, in Fig. 1(*a*) two multiport elements, 1 and *R*, are shown. *Ports* of a multiport element are designated by line segments incident on the element at one end. Ports are places where the element can interact with its environment.

For example, in Fig. 1(*b*) the 1 element has three ports and the *R* element has one port. We say that the 1 element is a 3-port, and the *R* element is a 1-port.

Bonds are formed when pairs of ports are joined. Thus bonds are connections between pairs of multiport elements.

For example, in Fig. 1(*c*) two ports have been joined, forming a bond between the 1 and the *R*.

Bond Graphs. A *bond graph* is a collection of multiport elements bonded together. In the general sense it is a linear graph whose nodes are multiport elements and whose branches are bonds.

A bond graph may have one part or several parts, may have no loops or several loops, and in general has the characteristics of any linear graph.

An example of a bond graph is given in Fig. 2. In part (*a*) a bond graph with seven elements and six bonds is shown. In part (*b*) the same graph has had its powers directed and bonds labeled.

A *bond graph fragment* is a bond graph not all of whose ports have been paired as bonds.

An example of a bond graph fragment is given in Fig. 1(*c*), which has one bond and two open, or unconnected, ports.

Port Variables. Associated with a given port are three direct and three integral quantities.

Effort, $e(t)$, and *flow*, $f(t)$, are directly associated with a given port, and are called the port power variables. They are assumed to be scalar functions of an independent variable (t).

Power, $P(t)$, is found directly from the scalar product of effort and flow, as

$$P(t) = e(t)\cdot f(t).$$

The direction of positive power is indicated by a half-arrow on the bond.

Momentum, $p(t)$, and *displacement*, $q(t)$, are related to the effort and flow at a port by integral relations. That is,

[1] Numbers in brackets designate References at end of paper.

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Copies will be available until September, 1973.

Discussion on this paper will be accepted at ASME Headquarters until January 2, 1973

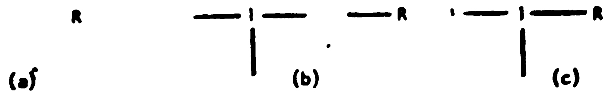


Fig. 1 **Multiport elements, ports, and bonds: (a) two multiport elements; (b) the elements and their ports; (c) formation of a bond**

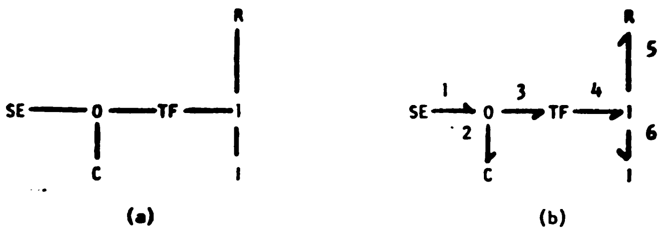


Fig. 2 **An example of a bond graph: (a) a bond graph; (b) the bond graph with powers directed and bonds labeled**

$$p(t) = p(t_0) + \int_{t_0}^{t} e(\lambda)d\lambda$$

and $q(t) = q(t_0) + \int_{t_0}^{t} f(\lambda)d\lambda$, respectively.

Momentum and displacement are sometimes referred to as energy variables.

Energy, $E(t)$, is related to the power at a port by

$$E(t) = E(t_0) + \int_{t_0}^{t} P(\lambda)d\lambda.$$

The quantity $E(t) - E(t_0)$ represents the net energy transferred through the port in the direction of the half-arrow (i.e., positive power) over the interval (t_0, t).

In common bond graph usage the effort and the flow are often shown explicitly next to the port (or bond). The power, displacement, momentum, and energy quantities are all implied.

Basic Multiport Elements. There are nine basic multiport elements, grouped into four categories according to their energy characteristics. These elements and their definitions are summarized in Fig. 3.

Sources.

Source of effort, written $SE\ \underline{e}$, is defined by $e = e(t)$.

Source of flow, written $SF\ \underline{f}$, is defined by $f = f(t)$.

Storages.

Capacitance, written $\frac{e}{f}\ C$, is defined by

$$e = \Phi(q) \text{ and } q(t) = q(t_0) + \int_{t_0}^{t} f(\lambda)d\lambda.$$

That is, the effort is a static function of the displacement and the displacement is the time integral of the flow.

Inertance, written $\frac{e}{f}\ I$, is defined by

$$f = \Phi(p) \text{ and } p(t) = p(t_0) + \int_{t_0}^{t} e(\lambda)d\lambda.$$

That is, the flow is a static function of the momentum and the momentum is the time integral of the effort.

Dissipation.

Resistance, written $\frac{e}{f}\ R$, is defined by

SYMBOL	DEFINITION	NAME
$SE \xrightarrow{e}$	$e = e(t)$	source of effort
$SF \xrightarrow{f}$	$f = f(t)$	source of flow
$C \leftarrow \frac{e}{f}$	$e = \Phi(q)$ $q(t) = q(t_0) + \int f\cdot dt$	capacitance
$I \leftarrow \frac{e}{f}$	$f = \Phi(p)$ $p(t) = p(t_0) + \int e\cdot dt$	inertance
$R \leftarrow \frac{e}{f}$	$\Phi(e,f) = 0$	resistance
$\xrightarrow{1} TF \xrightarrow{2}$ (1:m)	$e_1 = m\cdot e_2$ $m\cdot f_1 = f_2$	transformer
$\xrightarrow{1} GY \xrightarrow{2}$ (r)	$e_1 = r\cdot f_2$ $e_2 = r\cdot f_1$	gyrator
$\xrightarrow{1} 0 \xrightarrow{3}$, 2	$e_1 = e_2 = e_3$ $f_1 + f_2 - f_3 = 0$	common effort junction
$\xrightarrow{1} 1 \xrightarrow{3}$, 2	$f_1 = f_2 = f_3$ $e_1 + e_2 - e_3 = 0$	common flow junction

Fig. 3 **Definitions of the basic multiport elements**

That is, a static relation exists between the effort and flow at the port.

Junctions: 2-Port.

Transformer, written $\frac{e_1}{f_1}\ TF\ \frac{e_2}{f_2}$, is a linear 2-port element defined by

$$e_1 = m\cdot e_2$$

and

$$m\cdot f_1 = f_2,$$

where m is the modulus.

Gyrator, written $\frac{e_1}{f_1}\ GY\ \frac{e_2}{f_2}$, is a linear 2-port element defined by

$$e_1 = r\cdot f_2$$

and

$$e_2 = r\cdot f_1,$$

where r is the modulus.

Both the transformer and gyrator preserve power (i.e., $P_1 = P_2$ in each case shown), and they must each have two ports, so they are called essential 2-port junctions.

Junctions: 3-Port.

Common effort junction, written $\xrightarrow{1} 0 \xrightarrow{3}$ with bond 2 entering from below,

is a linear 3-port element defined by

$$e_1 = e_2 = e_3 \qquad \text{(common effort)}$$

and

$$f_1 + f_2 - f_3 = 0. \qquad \text{(flow summation)}$$

zero junction. *Common flow junction*, written $\overset{1}{\rightharpoonup} 1 \overset{3}{\rightharpoonup}$, with bond 2 ↑,

is a linear 3-port element defined by

$$f_1 = f_2 = f_3 \qquad \text{(common flow)}$$

and

$$e_1 + e_2 - e_3 = 0. \qquad \text{(effort summation)}$$

Other names for this element are the *effort* junction and the *one* junction.

Both the common effort junction and the common flow junction preserve power (i.e., the *net* power in is zero at all times), so they are called junctions. If the reference power directions are changed the signs on the summation relation must change accordingly.

Extended Definitions

Multiport Fields.

Storage Fields. *Multiport capacitances*, or *C-fields*, are written $\overset{1}{\rightharpoonup} C \overset{n}{\leftharpoondown}$, with bonds 2 ↑ …, and characterized by

$$e_i = \Phi_i(q_1, q_2, \dots q_n),\ i = 1 \text{ to } n,$$

$$\text{and } q_i(t) = q_i(t_0) + \int_{t_0}^{t} f_i(\lambda)d\lambda,\ i = 1 \text{ to } n.$$

Multiport inertances, or *I-fields*, are written $\overset{1}{\rightharpoonup} I \overset{n}{\leftharpoondown}$, with bonds 2 ↑ …,

and characterized by

$$f_i = \Phi_i(p_1, p_2, \dots p_n),\ i = 1 \text{ to } n,$$

$$\text{and} \quad p_i(t) = p_i(t_0) + \int_{t_0}^{t} e_i(\lambda)d\lambda,\ i = 1 \text{ to } n.$$

If a *C*-field or *I*-field is to have an associated "energy" state function then certain integrability conditions must be met by the Φ_i functions. In multiport terms the relations given in the foregoing are sufficient to define a *C*-field and *I*-field, respectively.

Mixed multiport storage fields can arise when both *C* and *I*-type storage effects are present simultaneously. The symbol for such an element consists of a set of *C*'s and *I*'s with appropriate ports indicated.

For example, $\overset{1}{\rightharpoonup} ICI \overset{3}{\leftharpoondown}$, with bond 2 ↑, indicates the existence of a set of relations

$$f_1 = \Phi_1(p_1, q_2, p_3),$$

$$e_2 = \Phi_2(p_1, q_2, p_3),$$

$$f_3 = \Phi_3(p_1, q_2, p_3),$$

and

$$p_1(t) = p_1(t_0) + \int_{t_0}^{t} e_1(\lambda)d\lambda,$$

$$q_2(t) = q_2(t_0) + \int_{t_0}^{t} f_2(\lambda)d\lambda,$$

$$p_3(t) = p_3(t_0) + \int_{t_0}^{t} e_3(\lambda)d\lambda.$$

Multiport dissipators, or *R*-fields, are written $\overset{1}{\rightharpoonup} R \overset{n}{\rightharpoonup}$, with bonds 2 ↑ …,

and are characterized by

$$\Phi_i(e_1, f_1, e_2, f_2, \dots e_n, f_n) = 0,\ i = 1 \text{ to } n.$$

If the *R*-field is to represent pure dissipation, then the power function associated with the *R*-field must be positive definite.

Multiport junctions include 0 junctions and 1 - junctions with *n* ports, $n \geq 2$. The general case for each junction is given in the following.

$\overset{1}{\rightharpoonup} 0 \overset{n}{\rightharpoonup}$, with bonds 2 ↑, *i* ↑, … $\qquad$ $\overset{1}{\rightharpoonup} 1 \overset{n}{\rightharpoonup}$, with bonds 2 ↑, *i* ↑, …

$$e_1 = e_2 = \dots = e_n \qquad f_1 = f_2 = \dots = f_n$$

$$\sum_{i=1}^{n} f_i = 0 \qquad \sum_{i=1}^{n} e_i = 0$$

Modulated 2-Port Junctions. The *modulated transformer*, or *MTF* written $\overset{1}{\rightharpoonup} \overset{\downarrow m(\mathbf{x})}{MTF} \overset{2}{\rightharpoonup}$ implies the realtions

$$e_1 = m(\mathbf{x})\cdot e_2$$

and

$$m(\mathbf{x})\cdot f_1 = f_2,$$

where $m(\mathbf{x})$ is a function of a set of variables, $\mathbf{x}$. The modulated transformer preserves power; i.e., $P_1(t) = P_2(t)$.

The *modulated gyrator*, or *MGY*, written $\overset{1}{\rightharpoonup} \overset{r(\mathbf{x})\downarrow}{MGY} \overset{2}{\rightharpoonup}$ implies the relations

$$e_1 = r(\mathbf{x})\cdot f_2$$

and

$$e_2 = r(\mathbf{x})\cdot f_1,$$

where $r(\mathbf{x})$ is a function of set of variables, $\mathbf{x}$. The modulated gyrator preserves power; i.e., $P_1(t) = P_2(t)$.

Junction Structure. The *junction structure* of a bond graph is the set of all 0, 1, *GY*, and *TF* elements and their bonds and ports. The junction structure is an *n*-port that preserves power (i.e., the *net* power in is zero). The junction structure may be modulated (if it contains any *MGY*'s or *MTF*'s) or unmodulated.

For example, the junction structure of the graph in Fig. 2(*b*) is a 4-port element with ports 1, 2, 5, and 6 and bonds 3 and 4. It contains the elements 0, *TF*, and 1.

Physical Interpretations

The physical interpretations given in this section are very succinctly stated. References [1], [2], and [3] contain extensive descriptions of physical applications and the interested reader is encouraged to consult them.

Mechanical Translation. To represent mechanical translational phenomena we may make the following variable associations:

1 *effort*, *e*, is interpreted as *force;*
2 *flow*, *f*, is interpreted as *velocity;*
3 momentum, *p*, is interpreted as impulse-momentum;
4 displacement, *q*, is interpreted as mechanical displacement.

Then the basic bond graph elements have the following interpretations:

1 source of effort, *SE*, is a force source;
2 source of flow, *SF*, is a velocity source (or may be thought of as a geometric constraint);

3 resistance, R, represents friction and other mechanical loss mechanisms;

4 capacitance, C, represents potential or elastic energy storage effects (or spring-like behavior);

5 inertance, I, represents kinetic energy storage (or mass effects);

6 transformer, TF, represents linear lever or linkage action (motion restricted to small angles);

7 gyrator, GY, represents gryational coupling or interaction between two ports;

8 0-junction represents a common force coupling among the several incident ports (or among the ports of the system bonded to the 0-junction); and

9 1-junction represents a common velocity constraint among the several incident ports (or among the ports of the system bonded to the 1-junction).

The extension of the interpretation to rotational mechanics is a natural one. It is based on the following associations:

1 effort, e, is associated with torque; and

2 flow, f, is associated with angular velocity.

Because the development is so similar to the one for translational mechanics it will not be repeated here.

Electrical Networks. In electrical networks the key step is to interpret a port as a terminal-pair. Then variable associations may be made as follows:

1 *effort*, e, is interpreted as *voltage*;

2 *flow*, f, is interpreted as *current*;

3 momentum, p, is interpreted as flux linkage;

4 displacement, q, is interpreted as charge.

The basic bond graph elements have the following interpretations:

1 source of effort, SE, is a voltage source;

2 source of flow, SF, is a current source;

3 resistance, R, represents electrical resistance;

4 capacitance, C, represents capacitance effect (stored electric energy);

5 inertance, I, represents inductance (stored magnetic energy);

6 transformer, TF, represents ideal transformer coupling;

7 gyrator, GY, represents gyrational coupling;

8 0-junction represents a parallel connection of ports (common voltage across the terminal pairs); and

9 1-junction represents a series connection of ports (common current through the terminal pairs).

Hydraulic Circuits. For fluid systems in which the significant fluid power is given as the product of pressure times volume flow, the following variable associations are useful:

1 *effort*, e, is interpreted as *pressure*;

2 *flow*, f, is interpreted as *volume flow*.

3 momentum, p, is interpreted as pressure-momentum;

4 displacement, q, is interpreted as volume.

The basic bond graph elements have the following interpretations:

1 source of effort, SE, is a pressure source;

2 source of flow, SF, is a volume flow source;

3 resistance, R, represents loss effects (e.g., due to leakage, valves, orifices, etc.);

4 capacitance, C, represents accumulation or tank-like effects (head storage);

5 inertance, I, represents slug-flow inertia effects;

6 0-junction represents a set of ports having a common pressure (e.g., a pipe tee);

7 1-junction represents a set of ports having a common volume flow (i.e., series).

Other Interpretations. This brief listing of physical interpretations of bond graph elements is restricted to the simplest, most direct, applications. Such applications came first by virtue of historical development, and they are a natural point of departure for most classically trained scientists and engineers. As references [1-4] and the special issue collection in the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, TRANS. ASME, Sept. 1972, indicate, bond graph elements can be used to describe an amazingly rich variety of complex dynamic systems. The limits of applicability are not bound by energy and power in the sense of physics; they include any areas in which there exist useful analogous quantities to energy.

Concluding Remarks

In this brief definition of the bond graph language two important concepts have been omitted. The first is the concept of *bond activation*, in which one of the two power variables is suppressed, producing a pure signal coupling in place of the bond. This is very useful modeling device in active systems. Further discussion of activation will be found in reference [3], section 2.4, as well as in references [1] and [2].

Another concept omitted from discussion in this definitional paper is that of *operational causality*. It is by means of causality operations applied to bond graphs that the algebraic and differential relations implied by the graph and its elements may be organized and reduced to state-space form in a systematic manner. Extensive discussion of causality will be found in reference [3], section 3.4 and chapter 5. Systematic formulation of relations is presented in reference [6].

References

1 Paynter, H. M., *Analysis and Design of Engineering Systems*, M.I.T. Press, 1961.

2 Karnopp, D. C., and Rosenberg, R. C., *Analysis and Simulation of Multiport Systems*, M.I.T. Press, 1968.

3 Karnopp, D. C., and Rosenberg, R. C., "System Dynamics: A Unified Approach," Division of Engineering Research, College of Engineering, Michigan State University, East Lansing, Mich., 1971.

4 Takahaski, Y., Rabins, M., and Auslander, D., *Control*, Addison-Wesley, Reading, Ma., 1970 (see chapter 6 in particular).

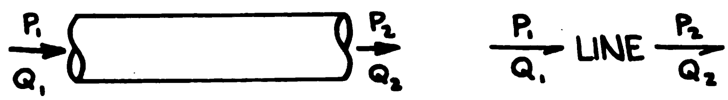
5 Rosenberg, R. C., "ENPORT User's Guide," Division of Engineering Research, College of Engineering, Michigan State University, East Lansing, Mich., 1972.

6 Rosenberg, R. C., "State-Space Formulation for Bond Graph Models of Multiport Systems," JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, TRANS. ASME, Series G, Vol. 93, No. 1, Mar. 1971, pp. 35-40.

APPENDIX B

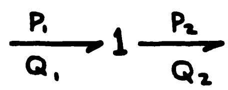
COMPONENT CATALOG

LINE - STRAIGHT

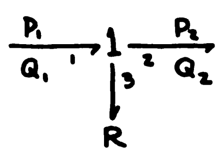


2 FLUID POWER PORTS

MODEL: IDEAL #1

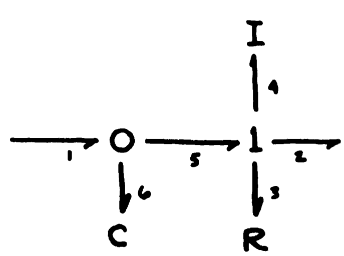


MODEL: STATIC #1



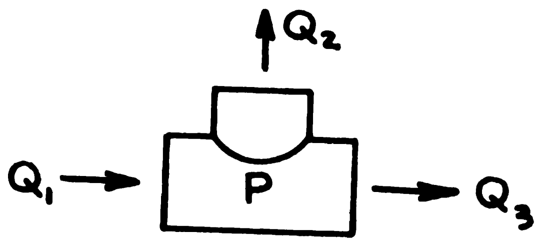
ELEMENT:		FUNCTION/RELATIONSHIP
R_3	CAPILLARY FLOW	$f_3 = e_3/R_3$

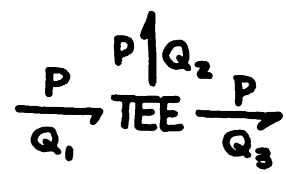
MODEL: DYNAMIC #1



ELEMENT:		FUNCTION/RELATIONSHIP
I_4	FLUID INERTIA	$f_4 = p_4/I_4$
C_6	FLUID - MECHANICAL COMBINED COMPLIANCE	$e_6 = q_6/C_6$

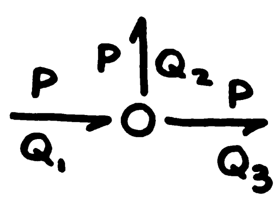
TEE



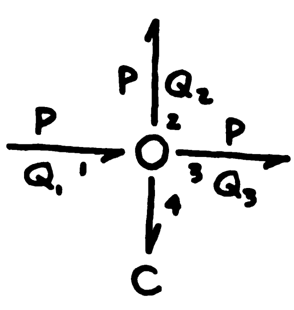


3 FLUID POWER PORTS

MODEL: IDEAL #1

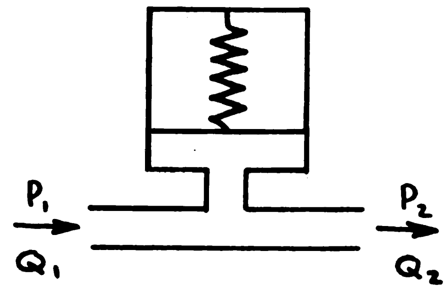


MODEL: DYNAMIC #1

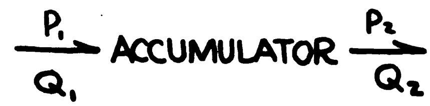


ELEMENT:		FUNCTION/RELATIONSHIP
C_4	COMBINED FLUID-MECHANICAL COMPLIANCE	$e_4 = q_4 / C_4$

ACCUMULATOR-SPRING LOADED

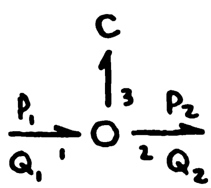


A = AREA OF PISTON



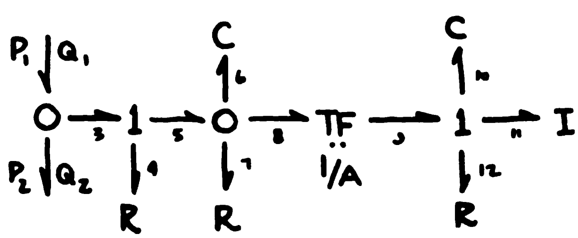
2 FLUID POWER PORTS

MODEL: DYNAMIC #1



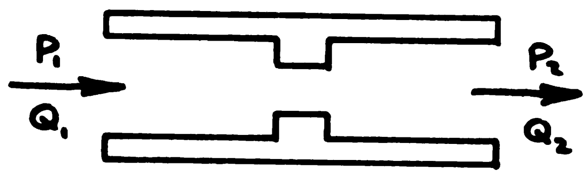
ELEMENT:		FUNCTION / RELATIONSHIP
C_3	CAPACITANCE	$e_3 = q_3/C_3$

MODEL: DYNAMIC # 2



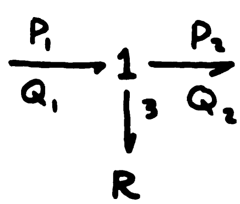
ELEMENT:		FUNCTION/RELATIONSHIP
R_4	ORIFICE FLOW	$f_4 = \sqrt{e_4}/R_4$
C_6	FLUID COMPRESSIBILITY	$e_6 = q_6/C_6$
R_7	LAMINAR LEAKAGE	$f_7 = e_7/R_7$
TF_{89}	TRANSFORMER	$e_8 = e_9/A$
C_{10}	SPRING	$e_{10} = q_{10}/C_{10}$
I_{11}	PISTON INERTIA	$f_{10} = p_{11}/I_{11}$
R_{12}	FLUID SHEAR	$e_{12} = R_{12}f_{12}$

ORIFICE-



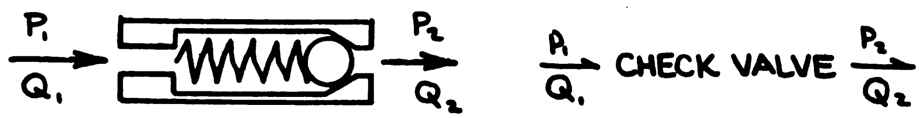
2 FLUID POWER PORTS

MODEL: STATIC #1



ELEMENT		FUNCTION/RELATIONSHIP
R_3	ORIFICE FLOW	$f_3 = \sqrt{e_3}/R_3$

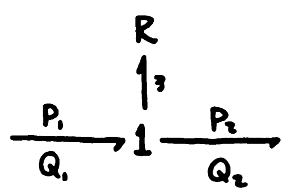
VALVE-TWO WAY, SPRING LOADED CHECK



A_E = EFFECTIVE SURFACE AREA ON BALL

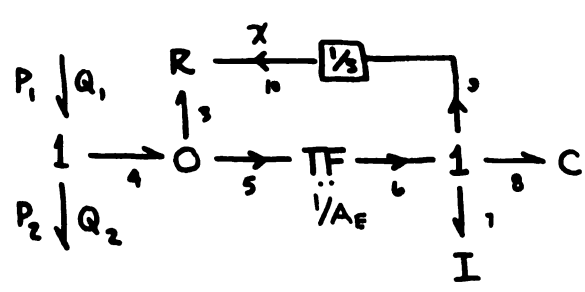
2 FLUID POWER PORTS

MODEL: STATIC #1



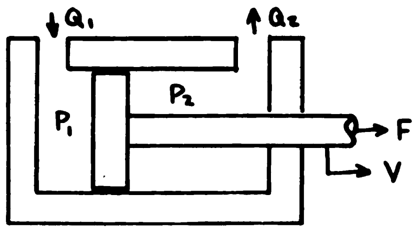
ELEMENT:		FUNCTION/RELATIONSHIP
R_3	DIODE	$P_2 > P_1 \quad Q_1 = Q_2$
		$P_2 \le P_1 \quad Q_2 = 0$

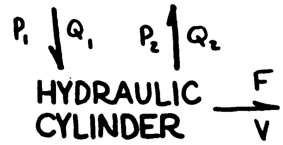
MODEL: DYNAMIC #1



ELEMENT		FUNCTION/RELATIONSHIP
R_3	MODULATED RESISTANCE	$f_3 = \phi_3(e_3, x)$
TF_{56}	TRANSFORMER	$e_5 = e_6 / A_E$
I_7	INERTIA OF BALL	$f_7 = p_7 / I_7$
C_8	SPRING	$e_8 = q_8 / C_8$

HYDRAULIC CYLINDER - DOUBLE ACTING

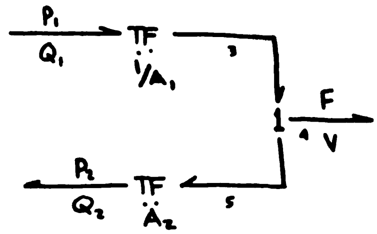




A_1 = SURFACE AREA, LEFT FACE
A_2 = SURFACE AREA, RIGHT FACE

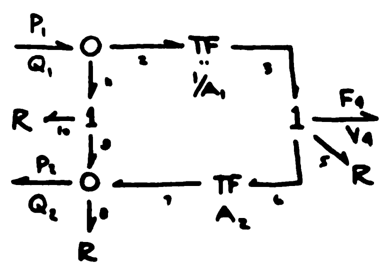
2 FLUID POWER PORTS
1 MECH. POWER PORT

MODEL: STATIC #1



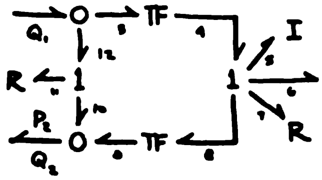
ELEMENT		FUNCTION/RELATIONSHIP
TF_{13}	TRANSFORMER	$P_1 = e_3/A_1$
TF_{25}	TRANSFORMER	$P_2 = e_5/A_2$

MODEL: STATIC #1



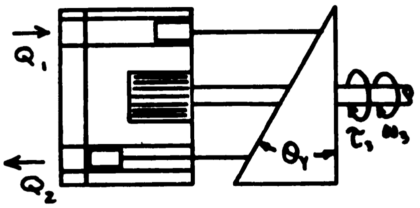
ELEMENT:		FUNCTION/RELATIONSHIP
R_5	FLUID SHEAR	$e_5 = R_5 f_5$
R_8	LAMINAR LEAKAGE	$f_8 = e_8/R_8$
R_{10}	LAMINAR LEAKAGE	$f_{10} = e_{10}/R_{10}$

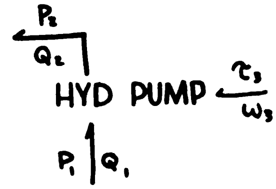
MODEL: DYNAMIC #1



ELEMENT:		FUNCTION/RELATIONSHIP
I_5	PISTON-SHAFT INERTIA	$f_5 = p_5/I_5$

PUMP-HYDRAULIC, P.D. AXIAL PISTON

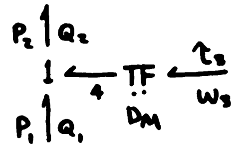




θ_Y = YOKE ANGLE
D_M = PUMP DISPLACEMENT

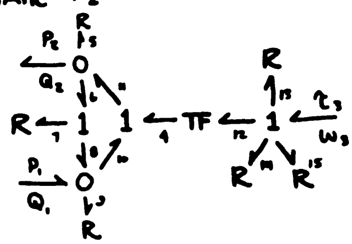
2 FLUID POWER PORTS
1 MECH. POWER PORT

MODEL: IDEAL #1



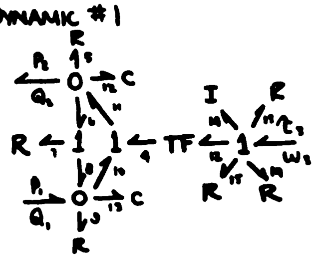
ELEMENT		FUNCTION/RELATIONSHIP
TF_{34}	TRANSFORMER	$f_3 = D_M \omega_3$

MODEL: STATIC #1



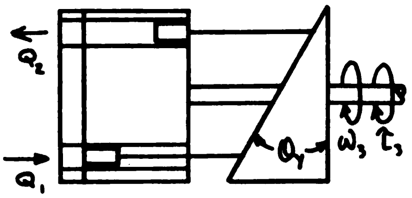
ELEMENT		FUNCTION/RELATION
R_{13}	WINDAGE LOSS	$e_{13} = R_{13} f_{13}^2$
R_{14}	FLUID SHEAR	$e_{14} = R_{14} f_{14}$
R_{15}	SEAL FRICTION	$e_{15} = R_{15} f_{15}$
R_5	LAMINAR LEAKAGE	$f_5 = e_5/R_5$
R_7	LAMINAR LEAKAGE	$f_7 = e_7/R_7$
R_9	LAMINAR LEAKAGE	$f_9 = e_9/R_9$

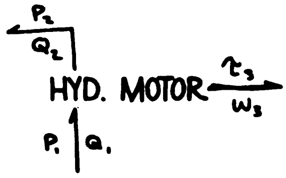
MODEL: DYNAMIC #1



ELEMENT		FUNCTION/RELATIONSHIP
C_{12}	FLUID COMPRESSIBILITY	$e_{12} = q_{12}/C_{12}$
C_{13}	FLUID COMPRESSIBILITY	$e_{13} = q_{13}/C_{13}$
I_{14}	ROTATIONAL INERTIA	$f_{14} = p_{14}/I_{14}$

MOTOR-HYDRAULIC - P.D. AXIAL PISTON

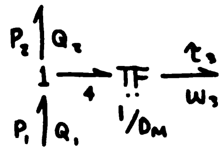




θ_Y = YOKE ANGLE
D_M = MOTOR DISPLACEMENT

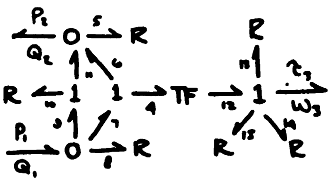
2 FLUID POWER PORTS
1 MECH. POWER PORT

MODEL: IDEAL #1



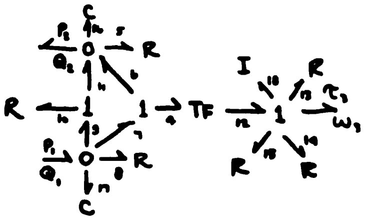
ELEMENT		FUNCTION/RELATIONSHIP
TF_{34}	TRANSFORMER	$Q_1 = D_M W_3$

MODEL: STATIC #1



ELEMENT		FUNCTION/RELATIONSHIP
R_5	LAMINAR LEAKAGE	$f_5 = e_5/R_5$
R_8	LAMINAR LEAKAGE	$f_8 = e_6/R_8$
R_{10}	LAMINAR LEAKAGE	$f_{10} = e_{10}/R_{10}$
R_{13}	WINDAGE LOSS	$e_{13} = R_{13} f_{13}^2$
R_{14}	SHEARING FLUID	$e_{14} = R_{14} f_{14}$
R_{15}	SEAL FRICTION	$e_{15} = R_{15} f_{15}$

MODEL: DYNAMIC #1



ELEMENT		FUNCTION/RELATIONSHIP
C_{16}	FLUID COMPRESSIBILITY	$e_{16} = q_{16}/C_{16}$
C_{17}	FLUID COMPRESSIBILITY	$e_{17} = q_{17}/C_{17}$
I_{18}	ROTATIONAL INERTIA	$f_{18} = p_{18}/I_{18}$

REFERENCES

REFERENCES

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