Essays on Growth and International Trade

By

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ABSTRACT

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Chapter 1 analyzes the growth effect of common tariffs using a two country version of the model developed by Young [1993]. In one of the two equilibria in the model, it is shown that an increase in the common tariff rate initially leads to faster growth but eventually leads to slower growth as the tariff rate rises. As the tariff rate approaches infinity, the growth rate asymptotically approaches the free trade growth rate. The result in this complementarity dominant equilibrium, at which innovators are pessimistic about the future, is the exact opposite of that obtained by Romer and Rivera-Batiz [1991]. The key difference is that Romer and Rivera-Batiz assume that newly developed products always take market share away from existing products. However, by allowing new products to complement old products, I show that the complementarity dominant equilibrium behaves as explained above.

Chapter 2 presents a R&D-based growth model in which innovations are in terms of intermediate product differentiation. Like in Jones [1995b] and Segerstrom [1995], the model presented here is free of the scale effects found in many R&D-based growth models. The long-run growth rate in this model is determined by the combination of the endogenously determined human capital accumulation rate and the exogenously given population growth rate. When consumption at two points in time is highly substitutable, an improvement in the educational environment lowers the human capital growth rate and the long-run economic growth rate. A higher population growth rate

also induces slower economic growth for some median value of substitutability.

Finally in Chapter 3, using an endogenous growth model with North-South technology trade, I analyze the long-run growth effects of changes in labor supply. Innovation occurs only in the North. Increases in human capital in each region have a positive growth effect, whereas increases in northern unskilled labor negatively affect the long-run growth rate. Effects of increases in southern unskilled labor could be either positive or negative. Results in this chapter are new, compared to the imitation model of Lai [1995], as changes in northern labor supply don't have growth effects in his paper. Results here are also different from Liu [1994], where only one type of labor is considered and increases in labor always have a positive growth effect.

To my wife and my children
For their Love, Support, and Sacrifice

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Chapter 1

The Growth Effects of Tariffs When There is Inovation Complementarity

1.1 Introduction

In trade between two countries, the conventional wisdom is that less obstacles to the flow of goods and services across the border is better for economic growth in both countries. The ideas of free trade helping economic growth can be implicitly found as early as Ricardo [1815]. Formalized rather recently by Samuelson [1959] and Pasinetti [1960], Ricardo's idea could be used for advocating free trade as it allocates resources in the world economy more efficiently allowing countries to gain from the free flow of goods and services across borders. A detailed discussions of numerous theoretical eff

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efforts[1] can be found in Findlay [1984]. Those theoretical backgrounds as well as the past experiences like the Smooth-Hawley Act passed in the U.S. during the Great Depression are well incorporated into the successive tariff reductions of the General Agreement on Tariffs and Trade negotiations.

Assuming increasing return to scale and imperfect competitions, theoretical research [2] in 1980s, however, give a mixed view about free trade arguments. Even though the new trade theory has not been able to give any conclusion, in Krugman's term[3], it makes free trade irretrievably lose its innocence. That is, there now is a possibility that free trade might not be the only universally desirable trade policy. As recent models with endogenous technological progress also involve market imperfections and increasing returns to scale, new growth theory also gives inconclusive results. By allowing for productivity differences between countries, Grossman and Helpman[1990] show certain conditions under which a small increase in the tariff rate increases the long-run growth rate. In their model, a small tariff increase by a country causes opposite movements of resources in and out of the research sectors in each country and the net effect depends on the productivity difference. Extending the Romer[1990] model to accommodate international trade in a symmetric setting, Rivera-Batiz and Romer[1991] show that the increase in the common tariff rate slows the economic growth at lower tariff levels. As tariff rates keep on rising the trends of slow growth reverse and eventually go back to the original free trade growth level. In

[1] For example, Uzawa [1961] and Vanek [1971] presented two-sector neoclassical growth models that connect growth and trade.

[2] Such as Brander and Spencer [1985] and Krugman [1986].

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their model, new products only substitute old products in the sense that newly developed products always take market share away from previously introduced products.

New products, however, do not always take market share away from existing products. New inventions often complement the use of existing products. For example, the introduction of a new advanced microprocessor used in personal computer systems allows consumers to have faster computers at more affordable prices. This can boost not only the sales of computers themselves but also the usages of many other existing parts used in the production of the computers. Often ignored in Schumpeterian growth models, as Young [1993] pointed out, this rent creating feature of newly introduced products may play an important role in determining the economy's growth rate. Thus, it is interesting to use Young's model that allows for innovation complementarity and to ask the same question as Rivera-Batiz and Romer did.

By modifying Young's model for a two country setting, this paper shows that an increase in the tariff rate from initially low levels can enhance the economic growth rate for both countries at one of two possible equilibria. In fact, in this complementarity dominant equilibrium, increases in the tariff rate initially increase the growth rate when the tariff rate is low, after which any further increase in the tariff rate decreases the growth rate, which eventually approaches the free trade growth rate. This result is exactly opposite to the Rivera-Batiz and Romer model, while the other equilibrium with substitution dominance behaves as in their model.

The introduction of innovation complementarity to the standard Schumpeterian growth model gives expectations about future innovation a crucial role in determining the effect of tariffs on the long-run growth rate. When the economy is in the compler

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mentarity dominant equilibrium where people are pessimistic about the future rate of innovation, a reduction in the tariff rate at lower levels has a negative growth effect. However, if the economy is in the substitution dominant equilibrium where people are optimistic about the future rate of innovation, the reduction in the tariff rate at lower levels will have a positive growth effect, as conventional free trade theories suggest. Although the selection of a single equilibrium is a matter of how to incorporate the information structure in a model and this in turn is an empirical matter, the introduction of innovation complementarity enlarges the set of possibilities in considering the effect of the change in the tariff rate on the economy's long-run growth rate.

In Section 2 below, the structure of the model is described. The solution of the model for steady state equilibria with positive growth rates is developed in Section 3. Then in Section 4, the effect of tariffs on the economic growth rate is analyzed. Section 5 compares the differences in the intuitions between this model and the Rivera-Batiz and Romer model. Finally, in Section 6, policy implications of the findings and suggestions for future research are discussed.

1.2 The Model

The model here considers a "home" country and a "foreign" country whose economies are symmetric in every respect. In each country, there are three types of economic activity on the production side. Researchers develop new types of intermediate goods that are used in the production of final goods. If successful, they are awarded with infinite patent protection. The manufacturer of the intermediate good, who acquires i

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the patent right from the researcher,[4] can charge the monopoly price for the product to final goods manufacturers. As a by-product of the development of the new intermediate goods, it becomes known how to produce a new type of a final good which is manufactured by using all existing intermediate products.[5] Since this knowledge is known to everybody, perfect competition prevails in the final goods manufacturing sector.

With this specification, the development of new intermediate goods has two effects. The substitution effect of new product development occurs as market share is taken away from the producers of existing intermediate goods in the sense that a wider variety of intermediate goods now competes to share the existing level of expenditure. However, as a new type of final good, which uses all existing intermediate goods in its production, emerges as a by-product, we have a new market for every intermediate good. This is the complementarity effect.

A common ad valorem tariff of rate δ is imposed on intermediate products that cross the border.[6] Tariff revenue collected by each importing country's government is redistributed back to its own consumers in a lump-sum fashion. Because of the symmetry between the countries, we will have symmetric calculations for each country on many occasions. In the analysis below, we will focus on the home country and for

[4]Even though the sales of the patent right across the border is not prohibited, the symmetric assumption of this model requires an equal variety of intermediate goods produced in each country. For simplicity, however, we consider the model in which international sales of patent right does not occur in the analysis below.

[5]In Young's model, there is an upper bound for each intermediate good can be used in final goods production. That is, a production of a final good there uses only relatively new intermediate products. For simplicity, we remove the limit in our model.

[6]We assume that there exists transportation costs of final goods across the border. With perfect competitions in the final goods sector, the presence of transportation costs then prohibit the trade of final goods between countries.

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the most part omit the corresponding calculations for the foreign country.

1.2.1 Consumers

Consumers in each country live forever and try to maximize their intertemporal utility

$$U \equiv \int_t^{\infty} e^{-\rho(s-t)} v(s) ds \tag{1.1}$$

where the consumer's subjective discount rate, ρ, is greater than 0 and the instantaneous utility $v(t)$ is given by

$$v(t) \equiv \int_0^{n(t)} lnC(j,t)dj + \int_0^{n(t)} lnC(j^*,t)dj^* \tag{1.2}$$

where $C(j,t)$ is the consumption a final good, j, originated[7] from the home country and $C(j^*,t)$ is the consumption of a final good, j^*, originated from the foreign country. Note that, at time t, $n(t)$ products are originated from each country.

As the intertemporal utility function is time separable, the consumer maximization problem can be solved in two stages. That is, we can first maximize for the instantaneous utility with a given expenditure level and then solve for a equilibrium time path for consumers' expenditure. Since $v(t)$ is logarithmic utility, the consumers will evenly distribute their expenditure, $E(t)$, on each product. Thus denoting j to be a index of a home originated final product, the expenditure for the final good, $E(j,t)$, is given as

[7] Final goods produced in a county are originated either from the home or the foreign counties as intermediate goods are developed.

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$$E(j,t) = C(j,t)P(j,t) = \frac{E(t)}{2n(t)} \tag{1.3}$$

As shown in Appendix, using (1.3) into (1.2), (1.2) in (1.1) and then maximizing (1.1) with respect to an intertemporal budget constraint, we obtain that the consumers' optimal expenditure path must satisfy

$$\frac{E'(t)}{E(t)} = R'(t) - \rho + g(t) \tag{1.4}$$

where $R(t)$ is the cumulative interest rate and $g(t) \equiv n'(t)/n(t)$ is the growth rate of product variety at time t.

Since the preference represented by (1.1) and (1.2) is homothetic, the aggregate expenditure is proportional to that of a representative consumer. Therefore, if we consider E without subscripts t as the total expenditure level for a country,[8] a path satisfying (1.4) can also describe the whole economy.

If we normalize this economy-wide expenditure as $E = E^{*} = 1$, we will have

$$R' = \rho - g \tag{1.5}$$

Note that the presence of logarithmic signs in front of $C(\cdot)$s in (1.2) make the failure of consuming any variety lead to infinitely negative utility[9]. Consumers in this economy have a tendency to postpone their expenditure to the future as the growth rate, g,

[8] We will drop the time subscript t in the analysis below.

[9] As Young himself pointed out, these preferences exhibit exceedingly strong preferences for variety. However, we can show that basic features of the model presented here carried over to the more complicated model with more general form of a CES utility function.

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goes up, and we can see that in (1.4). As we normalize the expenditure level, however, the interest rate should decrease to offset this tendency as in (1.5).

Also noting $E = E^* = 1$ in (1.3), we get

$$E(j) = E^*(j) = \frac{1}{2n} \tag{1.6}$$

1.2.2 Final Goods Manufacturer

Manufacturers of a home originated final good, j, uses labor[10], $L_Y(j)$, and all home and foreign intermediate goods developed prior to the introduction of the final good,[11] $x(i,j)$ and $x(i^*,j)$, in their production

$$Y(j) = L_Y(j)^{1-\beta}\left[\int_0^j x(i,j)^\alpha di + \int_0^j x(i^*,j)^\alpha di^*\right]^{\beta/\alpha} \tag{1.7}$$

where $0 < \alpha, \beta < 1$. Note that, in each country, there are j intermediate products developed prior to the final product j and this is why we have integration of intermediate goods, i and i^*, over the range $[0, j]$ in the production function above.

Denoting w as the wage rate, perfect competition in the final goods sector gives the demand for labor from producers of the final good with the production function (1.7) as

[10]In Young's model, labor is not used in final goods manufacturing. Without labor in the final goods sector, however, tariff rates do not affect the growth rate as their effects on patent prices and wage rates cancel each other.

[11]Also in Young's model, instead of using all previously developed intermediate goods, productions of final goods use only relatively new intermediate products. As mentioned earlier, this change is only for the simplicity and does not affect the result of the model.

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$$L_Y(j) = \frac{1-\beta}{w} \cdot \frac{1}{2n} \tag{1.8}$$

Also denoting $P_m(\cdot)$ as the price for an intermediate good faced by home manufacturers of final goods,[12] the demands for intermediate products, i and i^*, by producers of a home originated final good j are

$$x(i,j) = \frac{P_m(i,j)^{1/\alpha-1}}{\int_0^j P_m(i,j)^{\alpha/\alpha-1}di + \int_0^j P_m(i^*,j)^{\alpha/\alpha-1}di^*} \cdot \beta \cdot \frac{1}{2n} \tag{1.9}$$

and

$$x(i^*,j) = \frac{P_m(i^*,j)^{1/\alpha-1}}{\int_0^j P_m(i,j)^{\alpha/\alpha-1}di + \int_0^j P_m(i^*,j)^{\alpha/\alpha-1}di^*} \cdot \beta \cdot \frac{1}{2n} \tag{1.10}$$

A detailed derivations of the equations above are found in Appendix. Even though the equations above are for the manufacturers of a home originated final good, we can have the similar equations for the manufacturers of a foreign originated final good by switching j with j^*. We can also use (1.9) and (1.10) for foreign manufacturers of a final good by changing x to x^* and P_m to P_m^*.

1.2.3 Intermediate Goods

A manufacturer of an intermediate good, who acquires the right to produce the good from a researcher, can produce one unit of the intermediate good by using one unit of labor which is paid the wage rate w. Given the demand function (1.9) from home

[12]It could be different from the price intermediate goods producers actually get because, in the case of the border crossing, it is assessed with tariffs.

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manufacturers of a home originated final good, a home producer of an intermediate good then maximizes operating profits

$$\pi(i,j) = [P_m(i,j) - w] \cdot x(i,j) \tag{1.11}$$

Using (1.9), if we solve the maximization problem of (1.11), we get the mark-up pricing by a home manufacturer of intermediate goods to home manufacturers of a final good as

$$P_m(i,j) = \frac{w}{\alpha} \tag{1.12}$$

The mark-up to foreign manufacturers of final goods by a home producer of an intermediate good can be derived by maximizing after tariff operating profits

$$\pi^*(i,j) = [\frac{1}{1+\delta} \cdot P_m^*(i,j) - w] \cdot x^*(i,j) \tag{1.13}$$

Using (1.9) with $*$, if we solve the maximizing problem of (1.13), we get the mark-up by a home manufacturer of intermediate goods to foreign manufacturers of a final good as

$$P_m^*(i,j) = \frac{w(1+\delta)}{\alpha} \tag{1.14}$$

From (1.9), we can see that the demand for a home intermediate product i from producers of a home manufactured final good depends on prices of all home and foreign intermediate goods developed by the time when the final good is introduced.

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Substituting (1.12) and (1.14)[13] in (1.9) we get the demand for a home intermediate good i from producers of a home final good as

$$x(i,j) = \frac{\alpha}{jw} \cdot \frac{1}{1 + (1+\delta)^{\alpha/\alpha-1}} \cdot \frac{\beta}{2n} \tag{1.15}$$

An increase in the tariff rate raises the overall price level of intermediate products used in the production of a home manufactured final good as prices for imported intermediate products increase. This will help the sales of a home intermediate product to producers of a home final good as its own price stays the same. In (1.15), the second fraction with δ represents this. Also substituting (1.12) and (1.14) in (1.10) with appropriate changes,[14] we get the demand for a home intermediate good i from producers of a foreign final good as

$$x^{*}(i,j) = \frac{\alpha}{jw} \cdot \frac{(1+\delta)^{1/\alpha-1}}{1 + (1+\delta)^{\alpha/\alpha-1}} \cdot \frac{\beta}{2n} \tag{1.16}$$

As the fraction with δ in (1.15) represents, the denominator of the fraction with δ in (1.16) represents the increase in the overall price level of intermediate products used in the production of a foreign final good. The increase in the tariff rate, this time, raises the prices of all exported intermediate products to the foreign market. Holding constant the price of the intermediate good under consideration, this increase in the overall price level helps the sales of the intermediate good in that market. The increase in the tariff rate, however, also raises the own price and the sales of the good

[13] We actually need $P_m(i^*, j)$ in (1.9). But (1.14) is the symmetric expression for that.

[14] We need to change $x(i^*, j)$ to $x^*(i,j)$, $P_m(i,j)$ to $P_m^*(i^*, j)$, and $P_m(i^*, j)$ to $P_m^*(i,j)$

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Substituting (1.12) and (1.14)[13] in (1.9) we get the demand for a home intermediate good i from producers of a home final good as

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$$x^*(i,j) = \frac{\alpha}{jw} \cdot \frac{(1+\delta)^{1/\alpha-1}}{1+(1+\delta)^{\alpha/\alpha-1}} \cdot \frac{\beta}{2n} \tag{1.16}$$

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[13]We actually need $P_m(i^*, j)$ in (1.9). But (1.14) is the symmetric expression for that.

[14]We need to change $x(i^*, j)$ to $x^*(i, j)$, $P_m(i, j)$ to $P_m^*(i^*, j)$, and $P_m(i^*, j)$ to $P_m^*(i, j)$

under consideration as it is exported to the foreign country. The numerator in the fraction with δ in (1.16) represents this negative direct price effect.

The ith home intermediate product would be used in the production of all home and foreign originated final products introduced later than the intermediate product. As each type of final product is produced in both countries, the overall demand for the intermediate good i can be found as[15]

$$x(i) = 2\int_i^n [x(i,j) + x^*(i,j)]dj$$

Substituting (1.15) and (1.16), we have

$$x(i) = \frac{\alpha\beta}{wn} \cdot \phi(\delta) ln(\frac{n}{i}) \tag{1.17}$$

where

$$\phi(\delta) \equiv \frac{1 + (1+\delta)^{1/\alpha - 1}}{1 + (1+\delta)^{\alpha/\alpha - 1}} \tag{1.18}$$

Because $\phi(\delta)$ is crucial to explain the movement of the equilibrium growth rates later, it is important to note that the function is initially decreasing and then increasing as it asymptotically approaches the initial level as the tariff rate, δ, increases from zero to infinity,[16]. As we can see from (1.15) and (1.16), $\phi(\delta)$ is the sum of the two terms representing sales of intermediate good to producers of final goods in both

[15]As Young's model has an upper bound of intermediate goods used in the production of a final good, it is integrated over $j \in [i/\theta,\ n]$ there.

[16]$\phi(\delta)$ here is equivalent to $f(\tau)$ in Rivera-Batiz and Romer [1991].

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countries. An increase in the tariff rate raises the overall price level and positively affects the sales of the intermediate good in consideration in both countries. However, the increase in the tariff rate also raises the own price of the intermediate good and decreases its sale in the foreign market. With the tariff rate close to zero, this negative effect dominates the positive overall price effect. This explains the initial decrease in the $\phi(\delta)$. As the tariff rate is increased further, however, the dominance of the negative direct price effect for sales in the foreign market weakens. As the tariff rate approaches infinity, sales in the foreign market approaches zero. However, the sales in the home market becomes twice as much as the free trade level. This explains why the $\phi(\delta)$ function goes back to the initial level as δ approaches infinity.

The overall profits for the producer of the intermediate good i can be found as

$$\pi(i) = 2\int_i^n [\pi(i,j) + \pi^*(i,j)]dj$$

By substituting (1.15) into (1.11), (1.16) into (1.13) and those results into the above, we have[17]

$$\pi(i) = \frac{(1-\alpha)\beta}{n} \cdot \phi(\delta) ln(\frac{n}{i}) \qquad (1.19)$$

By differentiating (1.19)with respect to n, we get

$$\frac{\partial \pi(i)}{\partial n} = \frac{(1-\alpha)\beta\phi(\delta)}{n^2} - \frac{(1-\alpha)\beta\phi(\delta)}{n^2} ln(\frac{n}{i}) \qquad (1.20)$$

[17]Even though we omitted time variable t here, it is the operating profit for the intermediate good i at time t.

Because a new final product uses all existing intermediate products, it will increase the demand for each intermediate good developed earlier than the final product. This is the complementarity effect and the first term on the right hand side of (1.20) represents this. As the new final product is introduced, however, consumers have to divide their given level of expenditures over a wider variety of final products. This will reduce their demand for each final product and thus decrease the demand for each intermediate product. This is the substitution effect and the second term on the right hand side of (1.20) represents this.

1.2.4 Research

Researchers in the home country produce new designs for intermediate products according to[18]

$$n'(t) = \frac{2L_R(t) \cdot n(t)}{a} \tag{1.21}$$

With free entry into the research sector, the unit cost of a new design for this technology, $wa/2n$, equals the price of the nth patent design, $P_R[n(t)]$, which is the present value of the operating profits that will accrue to the manufacturer of the intermediate product.

$$UC \equiv \frac{wa}{2n(t)} = P_R \equiv \int_t^{\infty} e^{-[R(s)-R(t)]} \cdot \pi[n(t), s] ds \tag{1.22}$$

[18]There is no restrictions on the flow of ideas among researchers worldwide as they can use all previous knowledge in their research.

where the equality in the middle holds when there is positive growth.

By substituting (1.19) for the intermediate good $n(t)$ at time s into (1.22), we have

$$\frac{wa}{2n(t)} = \int_t^\infty \frac{(1-\alpha)\beta}{n(s)}\phi(\delta) ln[\frac{n(s)}{n(t)}]e^{-[R(s)-R(t)]}ds \tag{1.23}$$

1.2.5 Labor Market

To close the model, we need to describe the labor market. Labor demand in the research sector can be found from (1.21) as

$$L_R(t) = \frac{an'}{2n} = \frac{ag}{2} \tag{1.24}$$

Labor demand in the final goods manufacturing sector can be found by using (1.8) as

$$L_Y(t) = 2\int_0^n L_Y(j)dj = \frac{1-\beta}{w} \tag{1.25}$$

As one unit of labor is required for a unit production of an intermediate good, labor demand in the intermediate goods manufacturing sector can be found by using (1.17) as

$$L_M(t) = \int_0^n x(i)di = \frac{\alpha\beta\phi(\delta)}{wn}\int_0^n ln(\frac{n}{i})di = \frac{\alpha\beta\phi(\delta)}{w} \tag{1.26}$$

where the last equality holds as $\int_0^n ln(\frac{n}{i})di = n$. Denoting $L = L^*$ as the total labor endowment in each country, we get the labor market clearing condition by combining (1.24), (1.25) and (1.26) as

$$\frac{ag}{2} + \frac{1-\beta}{w} + \frac{\alpha\beta\phi(\delta)}{w} = L \tag{1.27}$$

By substituting (1.27) into (1.23)

$$\frac{\alpha\beta\phi(\delta) + 1 - \beta}{\frac{2L}{a} - g} = (1-\alpha)\beta\phi(\delta) \int_t^\infty \frac{n(t)}{n(s)} ln[\frac{n(s)}{n(t)}] e^{-[R(s)-R(t)]} ds \tag{1.28}$$

It is useful to note from (1.5) that along the equilibrium path, we have $e^{-[R(s)-R(t)]} = e^{-\rho(s-t)} \cdot e^{\int_t^s g(v)dv}$. It is also useful to note $ln[n(s)/n(t)] = \int_t^s g(v)dv$. Using these, we have

$$\frac{\alpha\beta\phi(\delta) + 1 - \beta}{\frac{2L}{a} - g} = (1-\alpha)\beta\phi(\delta) \int_t^\infty [\int_t^s g(v)dv] e^{-\rho(s-t)} ds \tag{1.29}$$

1.3 Steady State Equilibria

In a steady state equilibrium, we will have a constant growth rate g. To find the steady state value of g, we integrate the last part of (1.29) for a constant value of g and get

$$\int_t^\infty [\int_t^s g dv] e^{-\rho(s-t)} ds = \frac{g}{\rho^2} \tag{1.30}$$

Substituting (1.30) into (1.29), we have

$$\frac{\alpha\beta\phi(\delta)+1-\beta}{\frac{2L}{a}-g} = \frac{(1-\alpha)\beta\phi(\delta)g}{\rho^2} \tag{1.31}$$

If we multiply the left and the right hand sides of (1.31) by $1/n$ they become, respectively, the unit cost and the patent price of a new product design.

$$UC = \frac{\alpha\beta\phi(\delta)+1-\beta}{\frac{2L}{a}-g}\cdot\frac{1}{n} \tag{1.32}$$

and

$$P_R = \frac{(1-\alpha)\beta\phi(\delta)g}{\rho^2}\cdot\frac{1}{n} \tag{1.33}$$

It is useful to note that they are both increasing in g. With the fixed labor endowment in the economy, an increase in research activities puts upward pressure on the wage rate as we see in (1.27) and thus raises the unit cost of a patent design. As the growth rate increases, the number of final products that use an intermediate product grows faster and the price of a patent design increases because the patent price is the present value of a flow of operating profits.[19] Figure 1.1 below represents the unit cost and the patent price against the growth rate (g) and equilibrium growth rates are represented by the intersections of two curves. Denoting

$$K \equiv \frac{\rho^2[\alpha\beta\phi(\delta)+1-\beta]}{(1-\alpha)\beta\phi(\delta)} \tag{1.34}$$

[19]It is linearly increasing in g as the effect of g on the interest rate and on the substitution effect cancels out as we see in the transition of (1.28) to (1.29).

if $\frac{L}{a} < \sqrt{K}$, the two curves will never intersect and there will be no steady state equilibrium with positive growth. If $\frac{L}{a} = \sqrt{K}$, the two curves will touch each other once and the tangent point, $g = \frac{L}{a}$, will be the only steady state equilibrium with positive growth. However, rather than analyzing this knife edge case, our attention will be on the case where $\frac{L}{a} > \sqrt{K}$, in which the two curves intersect twice and we have two equilibria with positive growth. As we see in the Figure 1.1, the price of the patent is increasing at a constant rate while the unit cost of the new patent design is increasing at an increasing rate. Holding n constant, if we differentiate (1.32) and (1.33) with respect to g and divide those results by the originals, we get

$$\frac{UC'(g)}{UC} = \frac{1}{\frac{2L}{a} - g} \tag{1.35}$$

and

$$\frac{P_R'(g)}{P_R} = \frac{1}{g} \tag{1.36}$$

From (1.35), we can see that the rate at which the unit cost of a new design increases is increasing with the growth rate. As we can see in (1.24), an increase in the growth rate directly increases the demand for labor in the research sector. Holding demand for labor from manufacturing sectors constant, this increase in the demand for labor in the research sector raises the equilibrium wage rate. At a lower growth rate, however, employment in the research sector is relatively small compared to that in manufacturing sectors. Thus, additional demand for employment in the research sector due to increases in the growth rate will not put much pressure on the

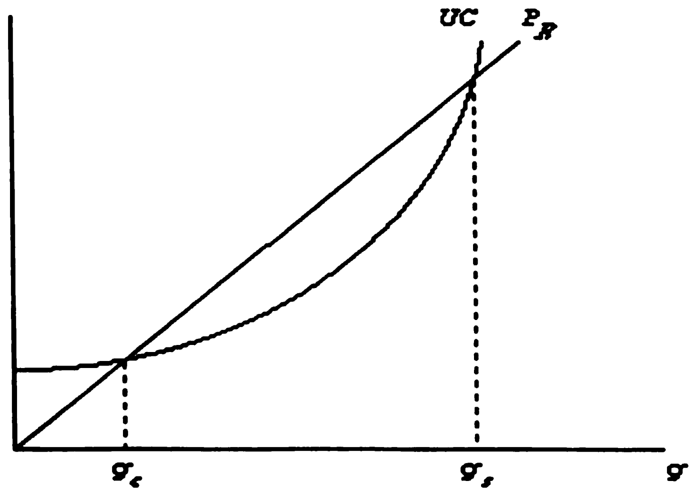

Figure 1.1: UC and P_R

wage rate and thus the unit cost to increase. As the growth rate further increases and the portion of labor employed in the research sector becomes significant, however, additional increases in the growth rate and in the demand for labor from the research sector will put greater pressure on the wage rate. This will significantly raise the unit cost of developing new products. Thus, there exists a positive relationship between the growth rate and the speed at which unit cost increases.

From (1.20), we can see that the complementarity and the substitution effects of new product development affect the instantaneous operating profits in opposite directions. As mentioned earlier, since the patent price is the present value of the flow of this operating profits, we can see that the increase in the growth rate and thus that in the number of products have two conflicting effects on the patent price. When i is relatively close to n (i.e. when the intermediate good i is a relatively new product), the complementarity effect dominates the substitution effect. As n grows further, however, the substitution effect becomes dominant. If the growth rate is slow, a new product will enjoy a long period of complementarity dominance and the effect of the growth rate on the patent price through complementarity will be relatively large. As the growth rate increases, however, the period of complementarity dominance will be shortened and the effect of the growth rate on the patent price through the complementarity will be relatively small. Thus, slower growth corresponds to faster patent price increases and faster growth corresponds to the slower patent price increases, as shown in (1.36). At lower growth rates, the patent price grows faster than the unit cost does, and later, at higher levels of growth , the speed at which these two increase is reversed. In fact, (1.35) and (1.36) indicate that the patent price is increasing at a higher rate than the unit cost when g is less than $\frac{L}{a}$, and vice versa.

In Figure 1.1, two equilibria with a positive growth rate occur at points where the unit cost and the patent price are equal. The equilibrium actually achieved by the economy depends on the expectations of researchers. If they are pessimistic about the rate of future research activities, the economy will settle down on a lower growth rate equilibrium. On the other hand, if they expect a higher rate of the research activities in the future, the economy will attain a higher growth rate equilibrium. The equilibrium with the growth rate less than L/a is the complementarity dominant equilibrium as it occurs where the complementarity effect is relatively larger. Similarly the equilibrium with the growth rate higher than L/a is the substitution dominant equilibrium as it occurs where the substitution effect is relatively larger. The two equilibria can be algebraically obtained by solving (1.31) and they are

$$g_c = \frac{L}{a} - \sqrt{(\frac{L}{a})^2 - K} \tag{1.37}$$

and

$$g_s = \frac{L}{a} + \sqrt{(\frac{L}{a})^2 - K} \tag{1.38}$$

There are issues related to the fact that only the complementarity dominant equilibrium is stable in this model. However, as Young points out, the stability problem should not necessarily lead one to conclude that the complementarity dominant equilibrium is the most likely long-run outcome. In models with learning environment,[20] different assumptions about the informational structure lead to quite different stories about stability. In fact, using a more general form of the CES utility function, we could construct a model in which both the complementarity and the substitution dominant equilibria are stable. In the analysis below, however, we will bypass these stability issues and assume that both equilibria could be achieved through appropriate processes.[21]

1.4 Growth Effects of Tariffs

For a comparative static analysis of the effects of tariffs on both equilibrium growth rates, we need to examine (1.37) and (1.38). However, as K in (1.37) and (1.38) is

[20] See Grandmont [1985] and Lucas [1986].

[21] For example, once off the substitution dominant equilibrium, we could jump back to the new substitution dominant equilibrium.

the only term that contains the tariff rate(δ), we simply need to differentiate (1.34) with respect to δ to get

$$\frac{\partial K}{\partial \delta} = -\frac{\rho^2(1-\beta)}{(1-\alpha)\beta} \cdot \frac{\phi'(\delta)}{[\phi(\delta)]^2} \tag{1.39}$$

Recall from (1.18) and the explanation after that, the $\phi(\delta)$ function is first decreasing and then increasing as it asymptotically approaches the initial level. If we take a look at the complementarity dominant equilibrium first, as there are total of three negative signs at the beginning of the right hand side of (1.39) and in front of K and the square root sign in (1.37), we can infer the movement in the equilibrium growth rate. As shown in Figure 1.2 below, an initial increase in the tariff rate from $\delta = 0$ increases the complementarity dominant equilibrium growth rate. The effect will be reversed at higher tariff rates, and, as the tariff rate goes to infinity, the growth rate will asymptotically approach the zero tariff growth level.

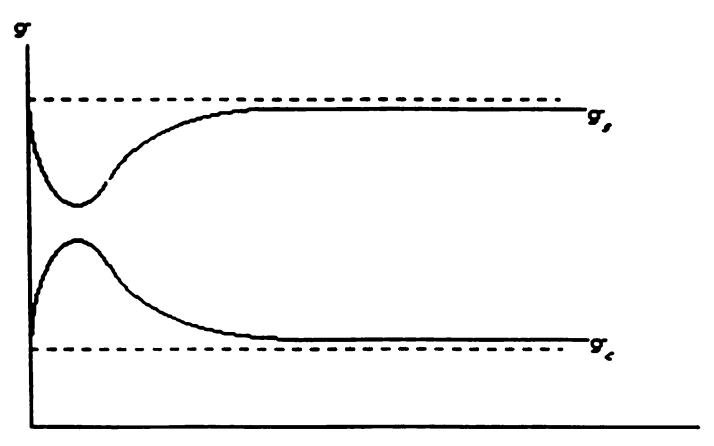

Figure 1.2: Changes in the Growth Rate

Since the signs in front of the square roots of both equilibrium growth rates in (1.37) and (1.38) are the only difference between them, tariff rates have exactly opposite effects on the substitution dominant equilibrium growth rate.

To get an insight into the movements in equilibrium growth rates, we need to examine the unit cost and the patent price in (1.32) and (1.33). Differentiating them with respect to the tariff rate (δ), we get

$$\frac{\partial UC}{\partial \delta} = \frac{\alpha\beta}{(\frac{2L}{a} - g)n} \cdot \phi'(\delta) \tag{1.40}$$

and

$$\frac{\partial P_R}{\partial \delta} = \frac{(1-\alpha)\beta g}{n\rho^2} \cdot \phi'(\delta) \tag{1.41}$$

As we note in (1.40) and (1.41), both have the term $\phi'(\delta)$ and this indicates that both UC and P_R move in the same direction as $\phi(\delta)$. That is, both the unit cost and the patent price decrease as the tariff rate increases at a lower level and then increase at a higher tariff level.

Figure 1.3 shows how the UC and P_R curves respond to the changes in the tariff rate. Starting from a tariff rate of zero, both curves shift to the right as the tariff rates increases, indicating that both the unit cost and the patent price are decreasing. Above a certain level of the tariff , however, both curves start to move back and asymptotically approach their original positions.

Recall that the patent price is the present value of the profit flows for intermediate good manufacturers and depends on sales of the product at home and abroad. The

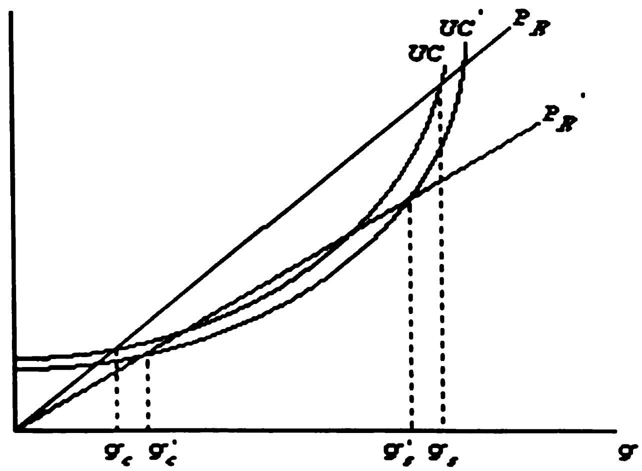

Figure 1.3: Effects of tariff on UC and P_R

sales in both markets, in turn, depend on the tariff rate. As we explained after the introduction of $\phi(\delta)$ function in (1.18), the sales and thus the patent price of a product decrease at lower levels of the tariff rate and then at higher levels of the tariff rate increase back to the level associated with zero tariff rate. This is why the P_R curve in Figure 1.3 first shifts to the right and then back to its original position.

An increase in tariff rates at lower levels decreases the demand for intermediate goods, as we noted earlier. Since one unit of labor is required for a unit of intermediate goods, this reduces the demand for labor from the intermediate goods sector. Holding labor demand in other sectors constant, the released labor from the research sector decreases the wage rate and thus lowers the unit cost of developing a patent design. At a higher tariff rate, however, the demand for intermediate goods and for the labor from the intermediate goods manufacturing sector bounce back as increasing domestic sales outweigh decreasing foreign sales. Therefore, the UC curve in Figure 1.3 also initially shifts to the right and then goes back to its original position as the tariff rates increase.

If we divide (1.40) by (1.32) and (1.41) by (1.33), we get

$$\frac{UC'(\delta)}{UC} = \frac{\alpha\beta\phi(\delta)}{1-\beta+\alpha\beta\phi(\delta)} \cdot \frac{\phi'(\delta)}{\phi(\delta)} \tag{1.42}$$

and

$$\frac{P_R'(\delta)}{P_R} = \frac{\phi'(\delta)}{\phi(\delta)} \tag{1.43}$$

As $0 < \beta < 1$, the first fraction on the right hand side of (1.42) is less than 1 and, together with (1.43), this indicates that the rate at which the patent price is changing is greater than that of the unit cost, whatever direction they are moving. The effects of a tariff on the patent price is more direct than its effects on the unit cost of a newly developed product. This is because the patent price depends on the sales and profits of the intermediate goods producer and the tariff rate affects them directly, while tariff rates directly affect the intermediate goods manufacturers' demand for labor, which is only a part of the labor market. This change in the labor demand indirectly affects the wage rate, and thus the unit cost of a product development. In whatever direction they are moving, therefore, the P_R curve shifts more than the UC curve.

At lower levels of the tariff rate, where the two curves are shifting to the right, (1.35) and (1.36) along with (1.42) and (1.43) indicate that an increase in the tariff rate moves any equilibrium growth rate below $g = L/a$ to the right and any equilibrium growth rate above $g = L/a$ moves to the left. At higher levels of the tariff rate, where both curves are shifting to the left, the movements of the equilibrium growth rate are reversed as both curves asymptotically reapproach their original positions. Thus, we will have movements of the equilibrium growth rates in response to the change in the tariff rate as in Figure 1.2.

We can find the intuition behind the movement of the equilibrium growth rates by investigating equations related to the labor market. Holding the wage rate constant, we can see the initial decrease in the demand for labor from the intermediate goods manufacturing sector by differentiating (1.26) with respect to δ

$$\left.\frac{\partial L_M}{\partial \delta}\right|_{\bar{w}} = \frac{\alpha\beta\phi'(\delta)}{w} \tag{1.44}$$

As the $\phi'(\delta)$ term, included in the numerator of (1.44), first decreases and then increases, we can see that the demand for labor in the manufacturing of intermediate goods is initially decreasing. This decrease in labor demand lowers the wage rate. This can be seen by differentiating (1.27) with respect to δ while holding L_Y and L_R constant,

$$\left.\frac{\partial w}{\partial \delta}\right|_{\bar{L_Y},\bar{L_R}} = \frac{\alpha\beta\phi'(\delta)}{L - ag/2}$$

This decrease in the wage rate increases the demand for labor from the intermediate goods sector, but not enough to overcome the initial decrease. Thus, the overall effect of the increase in the tariff rate at lower levels will be to decrease the employment of labor in the intermediate goods sector.

This decrease in the wage rate will also affect the demand for labor from both the final goods manufacturing sector and research sector. As tariff rates increase at a lower level, the value of marginal product of labor in both the research sector and the final goods manufacturing sector decreases. Holding the labor employment in each sector constant, if we differentiate the values of marginal product of labor in both the research and the final goods manufacturing sectors, we have

$$\frac{\partial \mathrm{VMPL}_R}{\partial \delta} = \frac{\alpha\beta\phi'(\delta)}{L - ag/2} \tag{1.45}$$

and

$$\frac{\partial \mathrm{VMPL}_Y}{\partial \delta} = (1-\beta)^{\beta+1} L_Y^{-\beta} [\frac{\alpha\beta\phi(\delta) + 1 - \beta}{L - ag/2}]^{-\beta} \cdot \frac{\alpha\beta\phi'(\delta)}{L - ag/2} \tag{1.46}$$

As perfect competition prevails in both the research and the final goods manufacturing sectors, decreases in the value of the marginal product of labor in both sectors tends to reduce the demand for labor in both sectors.

Therefore, an increase in tariff rates at lower levels has a tendency to release labor from all three sectors. If we consider the tendency of releasing labor from the intermediate and the final goods manufacturing sectors together as opposing forces against the tendency of releasing labor from the research sector, the relative force releasing labor from the manufacturing sector outweighs that from the research sector at the complementarity dominant equilibrium. Thus labor in the research sector and the complementarity dominant growth rate actually increase as tariff rates increase from zero. At the substitution dominant equilibrium, the relative sizes of forces out of each sector are reversed and the equilibrium growth rate initially decreases as the tariff rate increases.[22]

The above analysis considers only the case of lower levels of the tariff rate. Above a certain level, a further increase in the tariff rate will lower the complementarity dominant growth rate and will raise the substitution dominant growth rate as each sector tends to attract labor. Furthermore, as the tariff rate goes to infinity, the employment level in each sector and the growth rate go back to the initial free trade levels. As we explained in the movement of $\phi(\delta)$ function in (1.18), this is because that the sales of an intermediate good to producers of final goods in the home country doubles while the sales in the foreign country becomes zero. This makes the patent price and the unit cost of a design for a new intermediate good go back to the original level associate with free trade.

1.5 Comparison with Rivera-Batiz and Romer

In Rivera-Batiz and Romer, tariffs alter the allocation of resources (human capital) between the manufacturing and research sectors which in turn affects growth. In their model, human capital is used in both manufacturing of final goods and research activities. As the tariff rate increases, the flow of intermediate goods between two countries is hindered. In manufacturing of final goods, producers use less imported

[22]In this model, the critical value for the growth rate at which the relative size of forces releasing labor is reversed is $g = L/a$. Substituting this into the R&D labor employment condition (1.24), we get the critical value for the employment of labor in the research sector as $L/2$. Then, the R&D labor employment in the substitution dominant equilibrium is more than 50% of the population and this is obviously unrealistic. However, as in Young, by changing the range of intermediate goods used in the production of a final good from $[i, n]$ to $[i/\theta, n]$ so that there is an upper limit for the index of intermediate goods that can be used in the production of advanced final products, we can have the substitution dominant equilibrium with labor in the research sector less than $L/2$.

intermediate inputs. This will lower the marginal product of human capital in the manufacturing sector. The fact that manufacturing of final goods uses less imported material means that the manufacturers of intermediate goods export less. This decrease in sales abroad results in a decline in patent prices, which equal the present value of operating profit flows for intermediate goods producers. This decrease in the patent price, in turn, decreases the value of the marginal product of human capital in the research sector.

As patent price is increasing linearly with the quantity of intermediate goods sold, and is not a function of anything else, the increase in the tariff rate directly affects the marginal product of human capital in the research sector. On the contrary, the effects of tariff increases in manufacturing is indirect in the sense that increases in the tariff rate only affect the imported intermediate goods which must be combined with other inputs, namely, labor, human capital, and domestically produced intermediate goods. Thus, with tariff rates close to zero, we have decreases in the value of marginal product of human capital in the research sector greater than that in the manufacturing sector. This causes human capital to move from the research sector to the manufacturing sector and, consequently, the growth rate to decrease. As tariff rates keep on growing, the marginal product of imported intermediate goods keeps rising because fewer of them are used in production. Thus at higher tariff rates, the decrease in usage of imported intermediate goods will cause a large enough reduction in the marginal product of human capital in manufacturing to outweigh that in the research sector and the trends in resource allocation is reversed. Therefore, the increase in tariff rates at higher levels increases the growth rate.

In our model, the source of growth effect of tariff changes is also the allocation of resources among sectors. The resource here is labor instead of human capital. Furthermore, effects of a tariff on the value of the marginal product of labor in both research and manufacturing of final goods sectors are the same as that on the value of marginal product of human capital in the Rivera-Batiz and Romer model. In fact, if we divide (1.45) by the value of marginal product of labor in the research sector and (1.46) by that in the final good sector, we can show that the force moving labor in and out of the research sector is always stronger than that in and out of the final goods manufacturing sector. However, contrary to human capital being used only in the research and final goods manufacturing sectors in their model, labor is used in all sectors here, including the intermediate goods sector. Therefore, we will have an additional source of resource allocation in our model, namely, the flow of labor in and out of the intermediate goods sector. At lower tariff rates, labor moves out of the intermediate goods sector. At the complementarity dominant equilibrium, the combined labor flow out of manufacturing of both intermediate and final goods overwhelms the labor flow out of the research sector while the magnitudes are reversed at the substitution dominant equilibrium. Thus, we will have increasing complementarity dominant and decreasing substitution dominant growth rates at the lower tariff rates, as explained in the previous section.

1.6 Conclusion

Effects of the change in the tariff rate on the equilibrium growth rate in this model crucially depend on people's expectations about the future. With the tariff rate low enough, if the economy is currently in the substitution dominant equilibrium where researchers are expecting a higher rate of invention, the decrease in the tariff rate raises the common growth rate. This effect of the tariff reduction on the substitution dominant equilibrium is in line with the last four decades of GATT's continuing efforts of reducing tariffs among countries. On the other hand, if researchers are expecting a lower rate of invention so that the economy is in a complementarity dominant equilibrium, a mutual reduction in the common tariff rate decreases the growth rate in both countries. This may explain the productivity slow down in late 1970s and early 1980s. As is well known, it was a period of uncertainty caused by a huge increase in oil prices. This may cause innovators become pessimistic about the future innovation and economic situations might become the one which is suitably explained by complementarity dominant equilibrium. If, in fact, the economy had been in complementarity dominant equilibrium at that time, the continual tariff reductions by GATT agreements may reduces rather than enhance the world growth rate. Even though, it must be empirically answered whether the economy was at the complementarity or the substitution dominant equilibrium, the analysis here suggests that care should be taken in implementing the trade policy with a model. We may have to make sure that a certain peculiar situation in an economy does not lead to an exceptional direction away from the conventional wisdom of the free trade advocate.

Our conclusions may depend on the structure of our model and the assumptions we are made. As mentioned earlier, for example, if we adopt different structures for learning environment, we could have different conclusions about the equilibrium selection. As the name suggests, new growth theory itself is quite new and many areas of research remain. Further exploration of the model presented here is necessary in order to determine how robust our results are.

Chapter 2

A R&D base Growth Model with Human Capital Accumulation

2.1 Introduction

One of the problems found in many first generation endogenous growth models is the scale effect. That is, all the models, developed by Romer [1990], Segerstrom, Anant and Dinopoulos [1990], Grossman and Helpman [1991], and Aghion and Howitt [1992], predict a faster economic growth with larger labor force in the R&D sector. However, this scale effect is not well reflected in data. The NSF [1989] data found dramatic increases in the number of scientists and engineers engaged in R&D in the U.S. for the last four decades. If the prediction in these models is right, we should also have dramatic increases in the growth rate for the U.S. economy. However, as Jones [1995a] argues, the average per capita growth rate has been either constant or declined since 1950.

By making the marginal size of innovations decreases as the total stock of knowledge increases, Jones [1995b] successfully remove the scale effects from earlier versions of R&D based endogenous growth models. However, as only exogenously determined parameter values like the population growth rate affect the long-run economic growth rate, Jones' model has a rather strong implication. Segerstrom [1995] also develops an endogenous growth model which is free of the scale effects. In a quality ladder setting, in which innovators are trying to improve the quality of existing products, he adds an endogenous component to economic growth. That is, with or without population growth, the endogenously determined human capital growth rate can affect the long-run economic growth rate. He also shows that education subsidies can increase the long-run economic growth rate.

This paper presents a model of horizontal product differentiation, as in Jones, while incorporating human capital accumulation as in Segerstrom.[1] The long-run equilibrium economic growth rate is determined by the combination of the endogenously determined human capital growth rate and the exogenously given population growth rate. We find that an improvement in the efficiency of human capital accumulation can affect the long-run economic growth rate. Especially, we find that long-run growth effects depend on the elasticity of substitution. In fact, in the model presented here, when consumption at two different points in time are highly substitutable, increases in the productivity of human capital accumulation lead to slower economic growth. We also find that the effect of increases in the population growth

[1]Human capital in this paper is different from that in Segerstrom, as human capital and labor are separate inputs.

rate on human capital accumulation and the long-run economic growth also depends on the elasticity of substitution.

In Section 2 below, the structure of the model is described. The solution of the model for steady state equilibria with positive growth rates is developed in Section 3. Then in Section 4, the comparative dynamic analysis is presented. Section 5 summarizes the findings of this paper and suggestions for future research are discussed. Finally, in the appendix, the solution for the optimal control problem of consumers is presented.

2.2 The Model

Researchers in this economy try to come up with new designs for intermediate goods which are different from existing intermediate goods. Once successful in developing a new good, the researcher is awarded with an infinite patent protection, and then, sells its production right to the producer who submits the highest bid. With an infinite number of producers who want to produce the product, the equilibrium price for the patent design equals the present value of profits that will accrue to the producer of the intermediate good. As the producer has the exclusive right for production, he can charge producers of the final good the monopoly price for the intermediate good. Final good producers use all the existing intermediate goods along with labor and human capital in their production. The final good sector is characterized by perfect competition. Finally, consumers maximize their utility by consuming this final good.

2.2.1 Consumers

Denoting $v(t)$ as the fraction of human capital at time t devoted to its accumulation, we assume that consumers in this model accumulate human capital according to[2]

$$\dot{H}(t) = \lambda H(t)[1 - v(t)] \tag{2.1}$$

where $\lambda > 0$ is a given technology parameter that measures how effective workers are in accumulating human capital. Human capital that are not devoted to its accumulation (vH) will be employed in either research (H_A) or manufacturing of final good (H_Y).

A representative consumer who has the subjective discount rate $\rho > 0$ and consumes $c(t)$ at time t maximizes

$$\int_0^\infty e^{-\rho t} u(c_t) dt \tag{2.2}$$

subject to a dynamic budget constraint and the differential equation for the human capital accumulation given in (2.1). Assuming that the utility function, $u(\cdot)$, in (2.2)

[2] A similar specification is used in Lucas [1988] and Segerstrom [1995] so that workers can maintain a constant rate of human capital growth over time by devoting a constant fraction of their time to studying.

is a Ramsey type preference with constant elasticity of substitution equal to $1/\sigma > 0$,[3]

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

if we solve the consumer's optimal control problem as in Appendix, we get two intertemporal optimization conditions. The first one is for consumption growth,

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(r - \rho - n) \tag{2.3}$$

where r is the interest rate and n is the population growth rate. It is important to note that consumption growth is increasing in the consumer's elasticity of substitution. Consider consumption at two different points in time. If the interest rate (r) is raised, consumption at the earlier point in time becomes more expensive relative to consumption at the later point in time. Consumers will then tend to postpone their current consumption in return for more consumption in the future. With a higher elasticity of substitution, that is, with a lower value of σ, it will be easier to postpone the current consumption and the growth rate in consumption will be higher.

The second intertemporal optimization condition is related to human capital accumulation.

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[3] In Jones, $u(c_t) = \frac{c_t^{1-1/\sigma}}{1-1/\sigma}$ is said to exhibit the constant relative risk aversion of $1/\sigma$. However, if the von Neumann-Morgenstern utility integral is additively separable, both the elasticity of substitution and the relative risk aversion depend only on the curvature of the instantaneous utility function and thus two concepts are inter-changeable. See chapters 2 and 6 of Blanchard and Fischer [1989] for detailed discussions. Since there is no uncertainty involved in consumers behavior in this model, it is more appropriate to use the term of the constant elasticity of substitution.

$$\lambda + \frac{\dot{w}_h}{w_h} = r \tag{2.4}$$

where w_h is the wage rate for human capital. As the technology parameter λ measures the marginal increase in growth of human capital for a given increase in the fraction of human capital devoted to its accumulation, (2.4) requires that the marginal rate of increase in human capital plus the growth rate in the wage for human capital must equal the market interest rate.

2.2.2 Manufacturing

Final Good

Using human capital (H_Y), labor (L), and all intermediate goods developed ($x(i)$ where $i \in [0, A]$), the final good sector produces a consumption good, Y, according to constant returns to scale technology:

$$Y = H_Y^{\alpha} L^{\beta} \int_0^A x(i)^{1-\alpha-\beta} di \qquad 0 < \alpha, \beta < 1 \tag{2.5}$$

Note that intermediate goods enter into the production of final good in the form of horizontal differentiation as specified in Ethier [1982] and Dixit and Stiglitz [1977]. Innovation in this model is represented by increasing variety of these intermediate goods (i.e. increase in A). Normalizing the price of the final good to unity, perfect competition in the final good sector yields conditional demand functions:

$$H_Y = \alpha \cdot \frac{Y}{w_h} \tag{2.6}$$

and

$$p(i) = (1-\alpha-\beta) H_Y^{\alpha} L^{\beta} x(i)^{-(\alpha+\beta)} \qquad \forall i \tag{2.7}$$

where $p(i)$ is the rental price of intermediate good i.

Intermediate Goods

As specified in Romer [1990] and Jones [1995b], we assume that a producer of an intermediate good who acquires a patent design from a researcher rents a consumption good[4] at a unit rental rate r for a period and transform it into a single unit of the producer durable. The produced intermediate good is then rented out to final good manufacturers at a rate $p(i)$. At the end of the period, the producer durable is transformed back into the original form without any depreciation. The manufacturer of an intermediate good then tries to maximize the operating profit:

$$\pi(i) = p(i)x(i) - r \cdot x(i) \tag{2.8}$$

Using (2.7) in (2.8), we get the mark-up price for all intermediate goods as

[4]This is the same as saying that there exists a separate sector producing capital goods in which they uses the same technology as in final goods sector. In either case, the capital is the forgone consumption.

$$p(i) = p = \frac{r}{1 - \alpha - \beta} \tag{2.9}$$

Noting the symmetry of the mark-up price for all i in (2.7) and (2.8), we can also note the symmetry in demands and in profits for all existing intermediate products. This symmetry in demands for each intermediate good especially gives us a convenient way of expressing the instantaneous capital stock at a point in time as

$$K = \int_0^A x di = Ax \tag{2.10}$$

Using (2.10) with (2.9) and (2.7), we can express the interest rate as

$$r = (1 - \alpha - \beta)^2 \cdot \frac{Y}{K} \tag{2.11}$$

2.2.3 Research

By using human capital, researchers try to design a new type of intermediate product based on all previously developed designs as

$$\dot{A} = \delta A^{\phi} H_A \tag{2.12}$$

where $\delta > 0$ is a technology parameter for the research sector and $0 < \phi < 1$ is an externality parameter that measures the degree of externality across time in the R&D process.[5] While researchers freely use previous knowledge in their research, they have

[5] In Jones, it just assumed that $\phi < 1$ so that his model can include the negative externality case, $\phi < 0$. However, as this decreasing return to scale feature is not essential to removing scale effects in

to pay for the human capital. As there are no restrictions for entry into the research sector, the wage rate for human capital must equal the value of the marginal product of human capital in the research sector. Thus,

$$w_h = P_A \delta A^{\phi} \tag{2.13}$$

where P_A is the price for a patent design. As there are an infinite number of manufacturers who want to produce a patented intermediate good, the patent price will be the sum of the present value of operating profits which accrue to the producer of the intermediate good.

$$P_A = \int_0^{\infty} e^{-\int_0^t r(s)ds} \pi(t) dt$$

Differentiating this and dividing the result by the original form, we get the usual no arbitrage condition as

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A} \tag{2.14}$$

2.3 Steady State Equilibrium

Noting that a constancy of per capita consumption growth in a balanced equilibrium path in (2.3), we can see the constancy of the interest rate along a balanced equilibrium path. Using this, if we differentiate both sides of (2.11) with respect to time

his model, the model presented here sticks with the increasing return to scale assumption of $\phi > 0$.

and divide the result by (2.11) itself, we get the equality between the growth rates of Y and K in a steady state. Let $C \equiv cL$ denote the total consumption by the whole population, we can write total physical capital accumulation as $\dot{K} = Y - C$. Dividing both sides of this by K, differentiating both sides with respect to time, and noting the constancy of $\dot{K}/K$ in a steady state, we get

$$\frac{Y}{Y-C}\frac{\dot{Y}}{Y} - \frac{C}{Y-C}\frac{\dot{C}}{C} = \frac{\dot{K}}{K}$$

Noting the equality between the growth rates of Y and K in a steady state, we can also establish the equality between growth rates of Y and C. Thus, denoting g_x as the steady state growth rate of the placeholder x, we can show that

$$g_Y = g_K = g_C \tag{2.15}$$

Rewriting (2.12) as $\dot{A}/A = \delta A^{\phi-1} H_A$, noting the constancy of $\dot{A}/A$ in a steady state, differentiating both sides with respect to time, and dividing the result by the original form, we get

$$g_A = \frac{1}{1-\phi} g_{H_A} = \frac{1}{1-\phi} g_H \tag{2.16}$$

where the second equality is obtained by noting the constancy of a steady state share of human capital devoted to the research sector, which we will derive at the end of this section.

Differentiating both sides of (2.10) with respect to time, dividing the result by

(2.10), and noting (2.15), we get

$$g_Y = g_A + g_x \tag{2.17}$$

Noting the constancy of the rental price for intermediate goods in (2.7), if we differentiate both sides of the equation with respect to time and divide the result by (2.7), we get

$$g_x = \frac{\alpha}{\alpha+\beta}g_H + \frac{\beta}{\alpha+\beta}n$$

Substituting this into (2.17), we get

$$g_Y = g_A + \frac{\alpha}{\alpha+\beta}g_H + \frac{\beta}{\alpha+\beta}n$$

Noting (2.16), we can rewrite the above as

$$g_Y = (\frac{1}{1-\phi} + \frac{\alpha}{\alpha+\beta})g_H + \frac{\beta}{\alpha+\beta}n \tag{2.18}$$

From (2.18), we can see that long-run growth of this economy comes from two sources. They are growth in the stock of human capital and growth in the population. Growth in the population raises the long-run economic growth as increased population goes to the production of final goods. While all of the increase in the population is absorbed in the final goods sector, the increased stock of human capital will be split into two sectors. They are the research and the final good sectors. Thus the growth in the stock of human capital contributes economic growth through two channels. The first term in the parenthesis of (2.18) represents the contribution of human capital increases to economic growth through the creation of new types of products from the research sector. The second term represents the contribution of human capital increase through the direct effect of increased human capital employment in the final good sector. From (2.6), we get

$$g_{w_h} = g_Y - g_H$$

Substituting (2.18) into the above, we get

$$g_{w_h} = (\frac{1}{1-\phi} - \frac{\beta}{\alpha+\beta})g_H + \frac{\beta}{\alpha+\beta}n$$

Substituting this into (2.4), we get the interest rate required from consumer's intertemporal optimization for human capital accumulation as

$$\lambda + (\frac{1}{1-\phi} - \frac{\beta}{\alpha+\beta})g_H + \frac{\beta}{\alpha+\beta}n = r \tag{2.19}$$

As the term inside the parenthesis is greater than zero, we can see that there is a positive relationship between the growth rate of human capital and the interest rate. Noting that the last two terms in the left hand side represent the growth rate of the wage for human capital, if we have an increase in the interest rate, we need to have an increase in the growth rate of the wage for human capital. As the final good sector is perfectly competitive, the growth rate of the wage for human capital must equal the growth rate of the marginal product of human capital in the final good sector. The growth rate of the marginal product of human capital is the difference between the output growth rate and the human capital growth rate as indicated by $g_{w_h} = g_Y - g_H$ above. It seems as though the human capital growth negatively affects the growth rate of the wage for human capital. As we noted earlier in (2.18), however, growth in final good production also comes from human capital growth, as well as population growth. As growth in human capital affects the final production not only by the increase in employment of human capital but also by the increase in the number of intermediate products through R&D, the overall effect of an increase in the growth rate of human capital on that of the wage rate for human capital will be positive. Thus, we have a positive relationship between the interest rate and the human capital growth rate. We can represent this relationship as the HH curve in the $r - g_H$ plane in Figure 2.1 below.

From the definition $C \equiv cL$, we get $\dot{C}/C = \dot{c}/c + n$. Substituting from (2.3), we get

$$g_C = \frac{1}{\sigma}(r - \rho - n) + n \tag{2.20}$$

Substituting (2.20) into (2.18) and noting (2.15), we get the interest rate required from consumers balanced expenditure growth as

$$\sigma[(\frac{1}{1-\phi} + \frac{\alpha}{\alpha+\beta})g_H - \frac{\alpha}{\alpha+\beta}n] + \rho + n = r \tag{2.21}$$

Note that there also is a positive relationship between the growth rate of human capital and the interest rate. Recall that we get (2.21) by equating the growth rate of total output production in (2.18) and the growth rate of consumption in (2.20). If we have an increase in the interest rate, the consumption in the early period will be relatively more expensive than the consumption in the later period. Consumers then tend to reduce their consumption in the earlier period of time and raise it later. Thus the growth rate in consumption increases with the increase in the interest rate. To keep up with the increase in consumption growth, we must have an increase in total output growth and, with the fixed rate of population growth, this could be done only by increasing human capital accumulation. Therefore, we have a positive relationship between the interest rate and the human capital growth rate and this is represented by the upward sloping CC curve in Figure 2.1 below. Another thing to note is that an increase in the interest rate induces a larger increase in the human capital growth rate with larger values of the elasticity of substitution. As the interest rate goes up, we know that consumers would postpone their consumption. This would be much easier with a higher value of the elasticity of substitution. Thus, an increase in the value of $1/\sigma$ can be represented by a decrease in the slope of the CC curve.

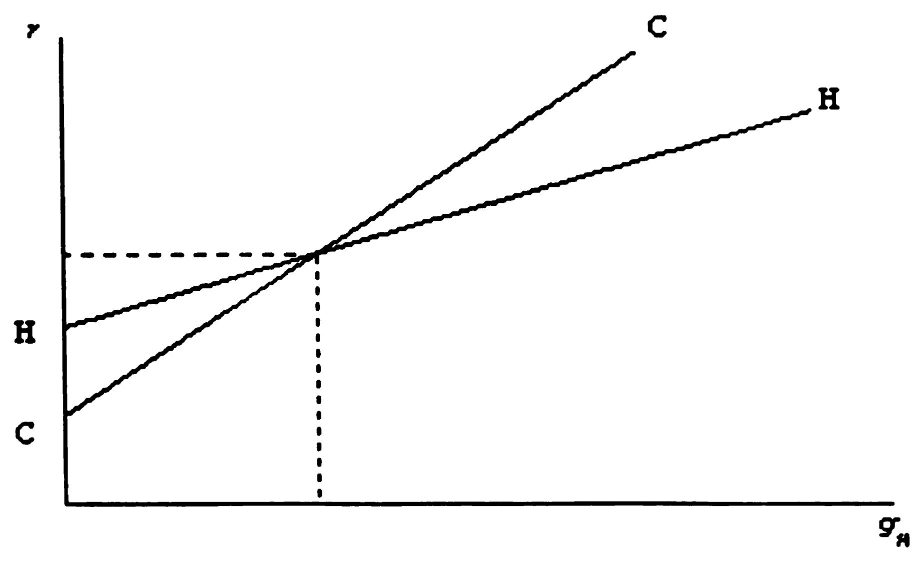

Figure 2.1: Equilibrium Human Capital Growth Rate

The equilibrium interest rate and the human capital growth rate will be determined at the intersection of the two curves. Even though Figure 2.1 above shows that the CC curve cuts through the HH curve from below, it is drawn for a higher value of σ. A decrease in the value of σ decreases the slope of the CC curve. Thus, for a sufficiently high value of the elasticity of substitution (a low σ), we would have the CC curve cutting through the HH curve from above. In fact, this would happen when $\sigma < \sigma_0 \equiv (\frac{\phi}{1-\phi} + \frac{\alpha}{\alpha+\beta})/(\frac{1}{1-\phi} + \frac{\alpha}{\alpha+\beta}) < 1$.

The equilibrium human capital growth rate can be algebraically derived by equating (2.19) and (2.21)

$$g_H = \frac{\lambda - \rho + (\sigma - 1)\frac{\alpha}{\alpha+\beta}n}{1 + (\sigma - 1)(\frac{1}{1-\phi} + \frac{\alpha}{\alpha+\beta})} \tag{2.22}$$

For this equilibrium human capital growth rate to be a meaningful interior solution, we need to consider the following. From (2.1), we get $g_H/\lambda = 1 - v$. As $1 - v$ is the fraction of human capital devoted to its accumulation, we need to assume that $0 < g_H/\lambda < 1$. For the case of $\sigma = 1$, the equilibrium growth rate for human capital is simplified as $g_H = \lambda - \rho$, we need to assume that $0 < \rho < \lambda$. For other values of $\sigma > 0$, we need to assume that the population growth rate is in between two critical values that are combinations of exogenously given parameter values. Those critical values will differ depending on the values of the elasticity of substitution (σ). In each case, we need $0 < g_H/\lambda < 1$ to be satisfied. Otherwise, the fraction of human capital devoted to its accumulation will be either 0 or 1. If $v = 0$, all human capital is devoted to its accumulation. Then there will be no human capital used in either manufacturing or research activities. In this case, we will have no production, no research, and no economic growth. On the other hand, if $v = 1$, there will be zero human capital accumulation. As we note in (2.16), without human capital accumulation, we have a zero growth rate in the number of new products. However, as we see in (2.18), we still have economic growth as long as the population grows. In this case, we can no longer call our model a R&D based endogenous growth model. In the analysis below, we assume that the condition, $0 < v < 1$, is always satisfied.

To get a steady state share of human capital employed in the research sector, we need to go back to (2.13), which states the equality between the wage rate for human capital and the value of marginal product of human capital in the research sector. Multiplying both sides of (2.13) by H_A and noting $\delta A^{\phi} H_A = \dot{A}$ from (2.12), we can rewrite (2.13) as $w_h \cdot H_A = P_A \cdot \dot{A}$. By substituting for P_A from the no arbitrage condition in (2.14) and rearranging, we can rewrite (2.13) as

$$w_h \cdot H_A = \frac{\dot{A}/A}{r - \dot{P}_A/P_A} \cdot \pi A \tag{2.23}$$

Substituting (2.9) into (2.8) and using (2.6) and (2.7), however, we can show that

$$\pi A = \frac{(\alpha+\beta)(1-\alpha-\beta)}{\alpha} w_h \cdot H_Y \tag{2.24}$$

Substituting (2.24) into (2.23) and rearranging it, we get

$$\frac{s}{1-s} = \frac{H_A}{H_Y} = \frac{\dot{A}/A}{r - \dot{P}_A/P_A} \cdot \frac{(\alpha+\beta)(1-\alpha-\beta)}{\alpha} \tag{2.25}$$

where s is the steady state share of human capital employed in the research sector.[6]

From $w_h \cdot H_A = P_A \cdot \dot{A}$ above, we can derive

$$\frac{\dot{P}_A}{P_A} = g_{w_h} + g_H - g_A \tag{2.26}$$

From (2.6), we can also derive

$$g_{w_h} = g_Y - g_H \tag{2.27}$$

Substituting (2.27) into (2.25), we get

$$\frac{\dot{P}_A}{P_A} = g_Y - g_A$$

Substituting this together with (2.15), (2.16), (2.18), (2.20), and (2.22) into (2.25), if we solve for s, we get

$$s = \frac{1}{1+\Omega}$$

where, under the assumption $0 < g_H/\lambda < 1$, we can show that

$$\Omega = \frac{\alpha}{(\alpha+\beta)(1-\alpha-\beta)}\{\frac{(\sigma-1)\lambda + (\sigma-1)(1-\phi)\frac{\alpha}{\alpha+\beta}(\lambda - n) + (1-\phi)\rho}{\lambda - \rho + (\sigma-1)\frac{\alpha}{\alpha+\beta}n} + 1\}$$

is always positive.

[6]Note that the total human capital available for the employment in either the research sector or the manufacturing sector is vH, that is not devoted to human capital accumulation. Thus, s represents the fraction of human capital out of vH.

2.4 Comparative Dynamics

In this section, we would like to study the long-run growth effects from changes in the technology parameter of human capital accumulation (λ) and from changes in the population growth rate (n). Noting that the denominator in (2.22) can be either positive or negative depending on the value of σ, we can see that the growth effect of λ and n can also be either positive or negative.

Let's consider the effect of the change in the technology parameter first. An increase in the value of λ means that people become more productive in accumulating human capital, as growth in human capital becomes faster even with the same proportion of human capital $(1 - v)$ is devoted to its accumulation. This could mean many things. For example, the increase in λ could come from an improvement in the education environment such as better libraries, computer equipment, and internet services.

We find that such an improvement in the educational environment would increase the long-run growth rate only when the consumption in two different periods of time are not highly substitutable. In fact, only when $\sigma > \sigma_0$, would the increase in λ increase the long-run growth rate of the economy. Referring to (2.18), with a given value of population growth, the long-run growth rate would increase when there is an increase in the rate of human capital accumulation. We can see from (2.22) that the growth rate of human capital is increasing in λ when the denominator is greater than zero. This requires that $\sigma > \sigma_0$. Otherwise, we would have decreases in the human capital growth rate and the long-run economic growth rate as we have a better environment for the education. Figure 2.2 below shows the effect of an

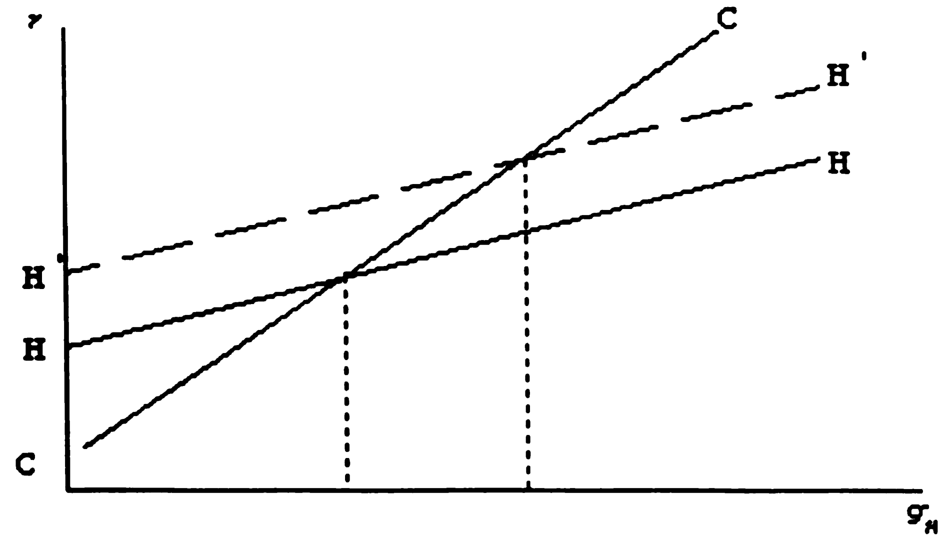

Figure 2.2: Effect of Increase in λ when $\sigma > \sigma_0$

increase in the value λ on the human capital growth rate in the case of $\sigma > \sigma_0$. As we mentioned earlier, the CC curve cuts through the HH curve from the below when $\sigma > \sigma_0$. Recalling that the HH curve is from (2.19), we can see that the HH curve will shift up as λ increases. This increases the equilibrium growth rate for human capital. Recalling that the equilibrium human capital growth rate in (2.22) is derived by equating (2.19) and (2.21), if we rewrite (2.22) as

$$\lambda + (\frac{1}{1-\phi} - \frac{\beta}{\alpha+\beta})g_H + \frac{\beta}{\alpha+\beta}n = \sigma[(\frac{1}{1-\phi} + \frac{\alpha}{\alpha+\beta})g_H - \frac{\alpha}{\alpha+\beta}n] + \rho + n \quad (2.28)$$

The left hand size represents the interest rate required for balanced human capital growth and the right hand side represents the interest rate required for balanced expenditure growth. As λ increases, we have an increase in the interest rate. This increase in the interest rate would mean an increase in the price of intermediate goods used in the production of final good, as we see from (2.9). This increase in the price of intermediate goods reduces the final good production as manufacturers of final good use a smaller amount of each intermediate good. This reduces the demand for human capital in the final good sector. The decrease in demand for intermediate goods decreases operating profits for each intermediate good producer and decreases the patent price for each design. This decrease in the patent price is reflected in the decrease in the value of the marginal product of human capital. This also reduces the demand for human capital in the research sector. Thus the increase in the interest rate decreases the demand for human capital from both manufacturers and researchers.

However, there also is a force that requires an increase in human capital. As the right hand side of (2.28) represents, the increase in the interest rate induces higher expenditure growth as consumers are trying to substitute their current consumption for consumption in the future. The overall effect of the increase in the interest rate on human capital depends on the relative size of these two opposing forces. As the increase in the interest rate induces a smaller increases in human capital with a lower value of consumer's elasticity of substitution, we have an overall decrease in the demand for human capital with higher values of σ. In fact, if $\sigma > \sigma_0$, we will have a decrease in demand for human capital. Unlike models in which no human capital accumulation is presented,[7] consumers in this model can decide the human capital growth rate. As the overall demand for human capital decreases, more human capital

[7] For example, the growth rate in Romer's model is determined by competing forces for a given human capital by the research sector and the intermediate goods sector

is devoted to its accumulation. Thus, we have increases in the human capital growth rate as shown in Figure 2.2 above.

On the other hand, if $\sigma < \sigma_0$, we will have a decrease in the equilibrium human capital growth rate as the force that increases the demand for human capital outweighs the force that decreases the demand for human capital. This can be shown as in Figure 2.3 below where the CC curve passes through the HH curve from above.

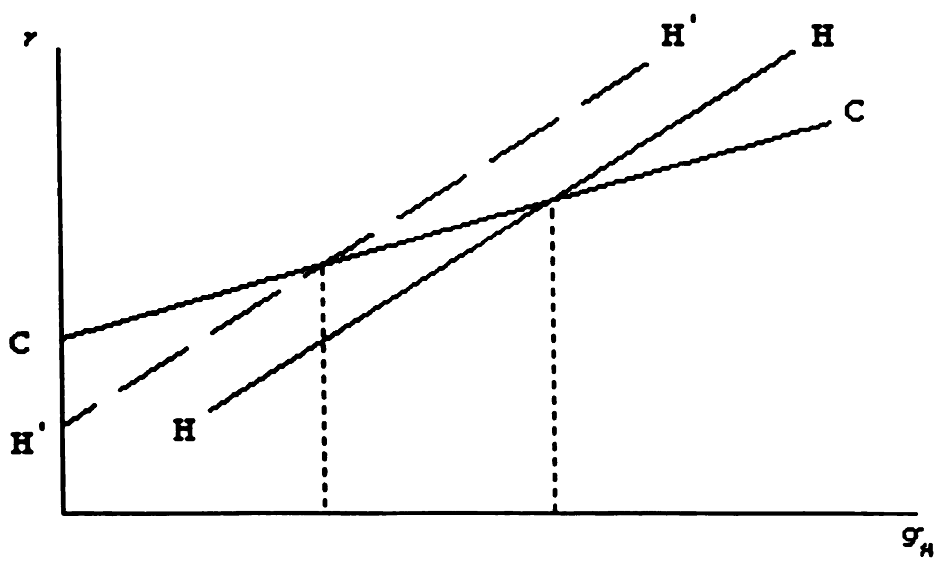

Figure 2.3: Effect of Increase in λ when $\sigma < \sigma_0$

Effects of the increase in the population growth rate on economic growth would be somewhat different from that of the technology parameter λ. If we differentiate both sides of (2.18) with respect to n, we get

$$\frac{\partial g_Y}{\partial n} = (\frac{1}{1-\phi} + \frac{\alpha}{\alpha+\beta})\frac{\partial g_H}{\partial n} + \frac{\beta}{\alpha+\beta} \tag{2.29}$$

As the part of the increased population directly goes to final good production, the increase in the population growth rate would directly increase economic growth The second term on the right hand side of the equation above represents this. However, the increase in the population growth rate also indirectly affects economic growth through change in the human capital growth rate, as is represented by the first term on the right hand side of the equation above. As in the case of the change in λ, the effect of the change in the population growth rate on the human capital growth rate could be either positive or negative depending on the value of the elasticity of substitution. Let's deal with this before we investigate the overall effect of the increase in the population growth rate on the long-run economic growth.

As we see in the left hand side of (2.28), the increase in the population raises the interest rate for balanced human capital growth. This shifts the HH curve upward as in the case of the increase in the value of λ. However, the analysis this time is more complicated because we also have a shift in the CC curve as the right hand side of (2.28) also contains n. The direction of the shift in the CC curve depends on the value of σ. Figure 2.4 below is drawn for the case of $\sigma > \sigma_1 \equiv (\alpha + \beta)/\alpha$ where the CC curve shifts down as n grows.[8] The downward shift in the CC curve can be understood by noting that the CC curve is derived by equating (2.18) and (2.20). From (2.18), we can see that the increase in the population growth rate raises the output growth rate. The increase in n also raises the consumption growth rate, as we note that $\sigma > \sigma_1 > 1$ in (2.20).[9] As $\sigma > \sigma_1$, however, the increase in the consumption growth rate is larger than the increase in the output growth rate.[10] To balance these two growth rates, we need to have a decrease in the interest rate and this is reflected

[8] As $\sigma_0 < 1 < \sigma_1$, we have the CC curve passes through the HH curve from below.

[9] For $\partial g_C/\partial n = 1 - 1/\sigma > 0$, we need that $\sigma > 1$.

[10] From (2.18) and (2.20), $\partial g_Y/\partial n < \partial g_C/\partial n$ requires that $\sigma > \sigma_1$.

in the downward shift of the CC curve.

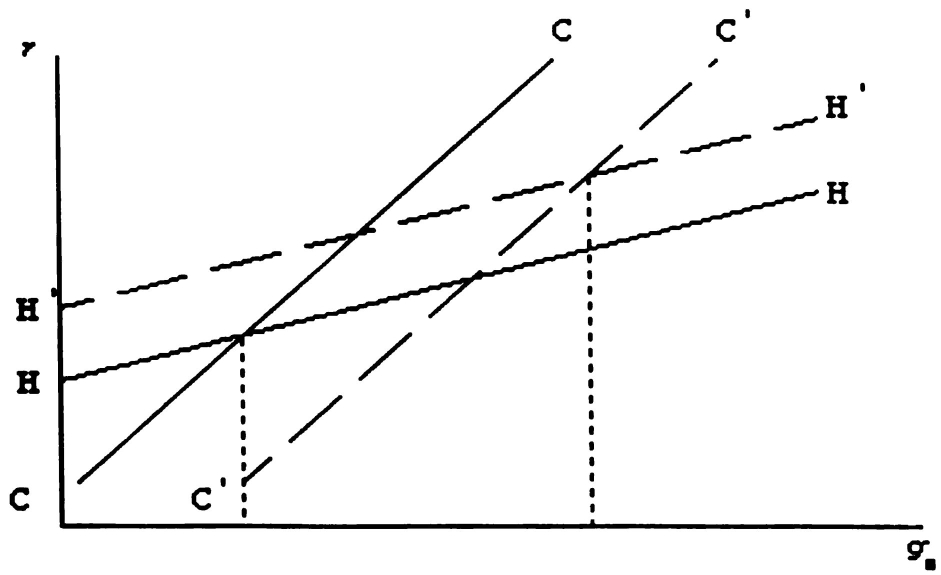

Figure 2.4: Effect of Increase in n when $\sigma > \sigma_1$

The decrease in the interest rate means that current consumption becomes less expensive relative to future consumption. Consumers then spend more and save less. The growth rates of consumption and output decrease and thus the demand for human capital decreases. This raises the human capital growth rate, as more human capital is devoted to its accumulation. This is represented by the increase in g_H up to the point where the new C'C' curve intersects with the HH curve in Figure 2.4. This increase in the growth rate for human capital is further enhanced by the upward shift in the HH curve. As the left hand side of (2.28) also contains n, the increase in the population growth rate calls for an increase in the interest rate. This is why the HH curve shifts up. The explanation for the increase in the human capital growth rate due to this shift is analogous to the case of increase in λ, which we discussed earlier. That is, noting that the left hand side of (2.28) is the equilibrium interest rate for balanced human capital growth, the increase in the population growth rate raises the interest rate and this decreases the demand for human capital from both the final good sector and the research sector. As the demand for human capital is decreased, we have an increase in the human capital growth rate.

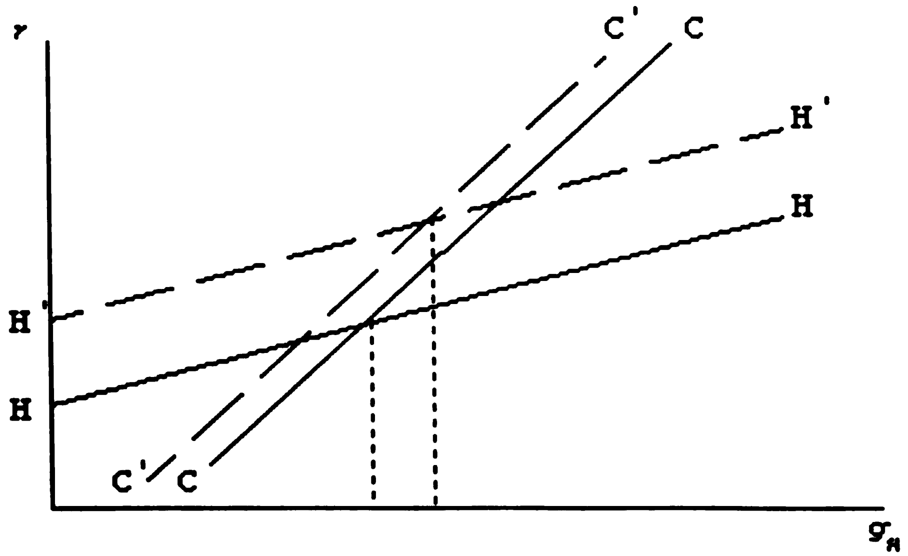

Figure 2.5: Effect of Increase in n when $1 < \sigma < \sigma_1$

If $\sigma < \sigma_1$, we will have the CC curve shifting up instead of shifting down.[11] Continuing on the case of $\sigma > \sigma_0$ so that the CC curve passes through the HH curve from the below, an upward shift in the CC curve causes a decrease in the human capital accumulation rate, with the reasoning opposite to the one given in the case of $\sigma > \sigma_1$ above. However, as the HH curve also shifts up, we have a force requiring an increasing the growth rate of human capital. The overall effect of an increase in the population growth rate on the human capital growth rate would depend on the relative magnitude of these two opposing forces. It turns out to be the that the force

[11]When $\sigma = \sigma_1$, the CC curve does not shift. However, as the HH curve still shifts up, we will have an increase in the human capital growth rate.

that increases the human capital accumulation rate outweighs the other force, when $\sigma > 1$ as the HH curve shifts more than the CC curve. Thus, in this case, we will have an increase in the human capital growth rate as the population growth rate increases as in Figure 2.5 above. If $\sigma_0 < \sigma < 1$,[12] on the other hand, we will have a decrease in the human capital growth rate since the CC curve shifts more than the HH curve.

Finally, for the case $\sigma < \sigma_0$, we have the CC curve pass through the HH curve from the above. The increase in the population growth rate shifts both the CC and the HH curves upward. Thus the shifts of the two curves affect the human capital in opposite directions. However, as we have $\sigma < 1$, the CC curve shift more than the HH curve. Thus, in this case, the increase in the population growth rate raises the human capital growth rate as shown in Figure 2.6 below.

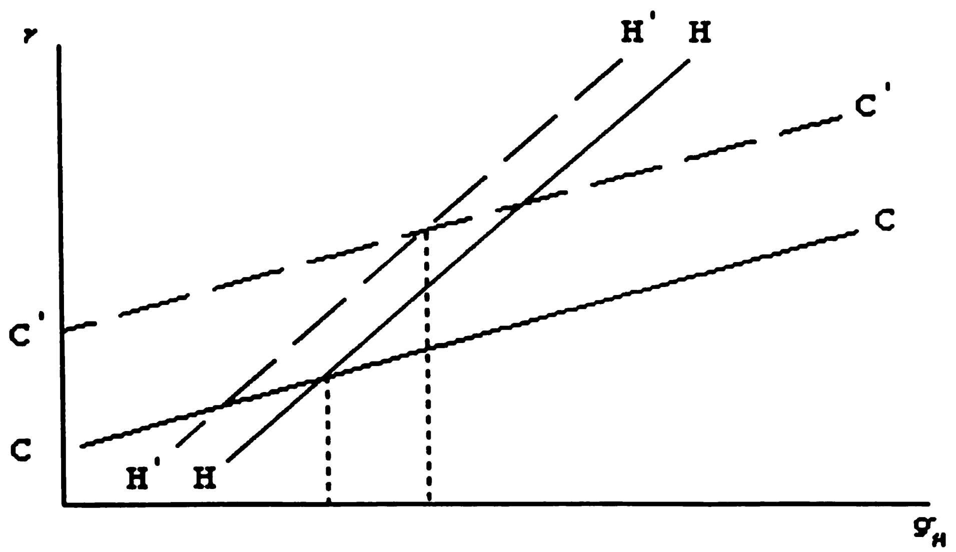

Figure 2.6: Effect of Increase in n when $\sigma < \sigma_0$

Now we can see that the increase in the population growth rate increases the

[12]When $\sigma = 1$, we have $g_H = \lambda - \rho$. Thus, the human capital growth rate does not depend on the population growth rate.

growth rate of human capital accumulation for all values of σ, except for the intermediate range $\sigma_0 < \sigma \leq 1$.

Aside from this range, the increase in the population growth rate increases the human capital growth rate and this helps the long-run economic growth in addition to the direct effect through the increase in the employment of human capital in the final good sector as indicated by the second term on the right hand side of (2.29). In the case of $\sigma_0 < \sigma \leq 1$, the direct employment effects and indirect effects through human capital accumulation work in opposite directions. By differentiating the equilibrium human capital growth rate in (2.22) with respect to n and substituting the result into (2.29), we can show that the direct employment effect is larger than the indirect effect when $\sigma_2 < \sigma < 1$ where $\sigma_2 \equiv (\frac{\phi}{1-\phi} + \frac{2\alpha}{\alpha+\beta})/(\frac{1}{1-\phi} + \frac{\alpha}{\alpha+\beta})$. Thus, we can say that the increase in the population growth rate increases the economy's long-run growth rate for all ranges of σ, except for $\sigma_0 < \sigma \leq \sigma_2$.

In this section, we show that growth effects of an improvement in the educational environment and an increase in the population growth rate critically depends on the value of the elasticity of substitution. However, Deaton [1992] reports various measures of its value between 0.265 and 0.736.[13] According to these figures, we have σ in the range of 1.36 to 3.77 and we can say that an improvement in the educational environment and an increase in population growth rate increase the long-run economic growth rate. However, it is an empirical matter and, as Deaton pointed out, we don't have good estimates of actual intertemporal consumption responses. Thus, the result in this section remains as a possibility.

[13]See *Understanding Consumption*, 1992, p.73.

2.5 Conclusion

Jones [1995b] uses the term of semi-endogenous growth in describing his model, since R&D activities are done by profit seeking firms while the long-run economic growth rate is determined by exogenously given parameter values like the population growth rate. Long-run economic growth in the model presented here is also determined partly by the population growth rate. However, long-run economic growth in this model is also affected by the endogenously determined human capital growth rate. Thus, the model presented here can be called a true-endogenous growth model, in the sense that the long-run growth rate is determined, at least by, human capital investment decisions. This feature is also presented in Segerstrom [1995]. However, innovations in his model are different as innovators are trying to improve the quality of the existing products.

Consumers' elasticity of substitution in this model play an important role in determining long-run economic growth. An increase in the technology parameter for human capital accumulation (λ) increases the interest rate, and this increase in the interest rate raises the consumption growth rate, as consumers substitute current spending for that at a later point in time. The increase in the consumers' expenditure growth rate requires more human capital to be devoted to the production of final good. On the other hand, increases in the interest rate calls for less human capital as the demand for that from both the manufacturing of the final good and the research sector decreases. Thus, there are two opposing forces for the demand for human capital and the dominance of one force over the other depends on the value of the elasticity of substitution. As the substitution of current consumption for future consumption is easier with higher values of the elasticity of substitution, the force that calls for more human capital in the final good sector is larger when we have a higher value of σ. Unlike models in which no human capital accumulation is present, consumers in this model can decide the human capital growth rate. As forces that demand more human capital in the production side[14] outweighs that demand less, it is natural that consumers devote a larger proportion of human capital to production activities and smaller proportion to human capital accumulation. This decreases the growth rate for human capital and, consequently, the long-run economic growth rate. This finding has an interesting implication that better environment for the education does not necessarily lead to faster growth in the economy.

Effects of increases in the population growth rate on the equilibrium human capital growth rate also depend on the value of the elasticity of substitution. Unlike changes in λ, in which the initial effect on the interest is only on the human capital accumulation, changes in the population growth rate affect the interest rate required for balanced growth in both human capital and expenditure. Although the analysis in this case becomes little bit more complicated due to the changes in the interest rate on both sides, it is basically the same as the above case in the sense that the relative dominance of two opposing forces depends on the value of the elasticity of substitution. It turns out to be the case that, for most of values of the elasticity of substitution, the force that calls for more human capital growth outweighs the other force except for some intermediate range of the elasticity of substitution. Therefore,

[14]The final good and the research sectors.

aside from this intermediate range, increases in the population growth rate increase the human capital growth rate.

As the increased population is absorbed into productions of the final good, changes in the population growth rate also directly affects the long-run economic growth rate. Except for the intermediate range of the elasticity of substitution mentioned above, therefore, increases in the population growth rate increase the long-run economic growth rate through both the direct and the indirect effects. In the intermediate range of the elasticity of substitution, the direct effect increases the long-run growth rate and the indirect effect decreases the long-run growth rate. The direct effect dominates the indirect effect for larger values of σ in this range. Thus, we can also say that the increase in the population growth rate increases the long-run economic growth rate for most of values of the elasticity of substitution, except for some intermediate range.

As mentioned above, the model presented here improves Jones' model in the sense that the long-run economic growth rate is determined, at least partly, by the endogenously determined human capital growth rate. However, the model presented here is still like Jones' model in the sense that no policy parameters like tax and subsidy rates appear in the determination of the equilibrium growth rate. However, the findings in this model suggest the possibility of developing endogenous growth models without scale effects but with policy parameters affecting long-run economic growth. Further exploration of models with decreasing returns to scale in R&D is needed to see whether it is actually possible to develop models in which policy parameters affect the long-run growth rate.

Chapter 3

Factor Endowments and Economic Growth in a Model With Technology Trade

3.1 Introduction

Since the publication of a seminal paper by Vernon [1966], there have been many attempts to incorporate his idea of an international product cycle into a formal dynamic model. Krugman [1979] considers an economy with two regions, innovating North and imitating South, and presented a model in which a constant fraction of goods are produced in each region. However, innovation and imitation rates in his model are given exogenously. Dollar [1986] presents a model that allows endogenous technological diffusion, while the innovation remains exogenous. With the development of new growth theory in the 1980s, that emphasizes the increasing returns to scale and imperfect competition, Segerstrom et al. [1990] present a model that allows endogenous innovation in a model of product cycle. While imitation is exogenous in their model, in the sense that patent length is exogenously given, Grossman and Helpman [1991] present models that allow both endogenous innovation and endogenous imitation.

The model presented here involves an issue of how changes in labor endowments affect the long-run growth rate of the economy. In a horizontal product differentiation framework, Grossman and Helpman show that only the expansion of the southern effective labor force has a positive growth effect.[1] Introducing two types of labor (Human Capital and Unskilled Labor) in the Grossman and Helpman's wide-gap case, Lai [1995] shows that the expansion of southern human capital has a positive growth effect, whereas the expansion of southern unskilled labor has a negative growth effect. However, changes in either labor supply in the North continue to have no growth effect.

As all the models mentioned above consider imitation as the only vehicle of technological diffusion between the North and the South, they tend to ignore the rapid growth of technology trade between countries in recent years. For example, U.S. earnings from royalties and licensing fees abroad has grown from \$5.1 billion to \$15.3 billion during the period between 1982 and 1990.[2] By making the assumption that imitation is costlier than purchasing technology, Liu [1994] shows that there exists a long-run steady state equilibrium with technology trade. In his model, expansion of

[1]This is a result for the wide-gap case in their model. In the narrow-gap case, expansion of effective labor in both regions has a positive growth effect.

[2]See *Survey of Current Business*, June 1991, p.45.

the labor forces in both regions has a positive growth effect.

Introducing two types of labor in Liu's model, as in Lai, this paper shows that the expansion of different types of labor in each region have different growth effects. Increases in human capital in each region has a positive growth effect, whereas increases in northern unskilled labor negatively affect long-run growth. Increases in southern unskilled labor could be either positive or negative. Results in this paper are new, compared to Lai, as changes in northern labor forces now have long-run growth effects. Unlike Liu's model, where only one type of labor is considered and increases in labor endowments in either region always have a positive growth effect, the model presented here has results more in depth.

In Section 2 below, the structure of the model is described. The solution of the model for a steady state equilibrium with positive growth rates is developed in Section 3. Then in Section 4, comparative dynamic analysis is done for each type of labor in each region. Finally, in Section 5, we summarize implications of the findings of this paper.

3.2 The Model

At time t, $n_N(t)$ products are produced in the North and $n_S(t)$ products are produced in the South. Goods are freely traded across the border. All the $n(t) = n_N(t) + n_S(t)$ products are developed in the North. That is, only the North is capable of designing new products which are horizontally differentiated from the existing products. The diffusion of technology from the North to the South is as specified in Liu [1994]. That is, some proportion of the products developed in the North immediately finds producers in the South. Even though the South is capable of imitating northern developed products, because of high imitation costs, it is more profitable for southern manufacturers to buy technology directly from northern innovators. Thus, imitation never occurs. Also as in many R&D based growth models[3], if two or more producers are producing the same product, each firm earns zero profit due to Bertrand competition, and cannot recover initial development costs. Thus, a single firm, located either in the North or in the South, produces each product in equilibrium.

3.2.1 Consumers

A representative consumer in the world (in the North and the South), who lives forever, maximizes the intertemporal utility

$$U(t) \equiv \int_t^\infty e^{-\rho(\tau-t)} \log u(\tau) d\tau \tag{3.1}$$

subject to intertemporal budget constraints

$$\int_t^\infty e^{-[R(\tau)-R(t)]} E(\tau) d\tau \leq \int_t^\infty e^{-[R(\tau)-R(t)]} Y(\tau) d\tau + A(\tau) \tag{3.2}$$

where $\rho > 0$ is the consumer's subjective discount rate, $R(t)$ is the cumulative interest rate, $Y(t)$ is the consumer's income, and $A(t)$ is the asset value. Also, the instantaneous utility, $u(t)$, and the instantaneous expenditure level, $E(t)$, are given,

[3] For example, Grossman and Helpman [1991]

respectively, as

$$u(\tau) \equiv [\int_0^{n(\tau)} x(j)^\alpha dj]^{1/\alpha} \tag{3.3}$$

and

$$E(\tau) = \int_0^{n(\tau)} p(j)x(j)dj \tag{3.4}$$

where $0 < \alpha < 1$ and $x(j)$ and $p(j)$[4] are, respectively, the consumption and the price of product j.

As the intertemporal utility function in (3.1) is time separable, we can solve the consumer's problem in two stages. That is, for the expenditure level given in (3.4), we can first solve for the maximization of the instantaneous utility in (3.3). In doing so, we get the instantaneous demand for product j as

$$x(j) = \frac{p(j)^{-\epsilon}}{\int_0^n p(j')^{1-\epsilon} dj'} E \tag{3.5}$$

where $\epsilon \equiv 1/(1-\alpha)$ is the elasticity of substitution between goods and E is the aggregate world expenditure.[5] Substituting (3.5) into (3.3), (3.3) into (3.1), and maximizing (3.1) subject to (3.2), we get the consumer's optimal expenditure path as

$$\frac{\dot{E}}{E} = \dot{R} - \rho$$

[4]As goods are freely traded, the price of a good is the same for consumers and producers.

[5]Consumers are homothetic, the representative consumer's behavior is proportional to the aggregate behavior of consumers in the world. Thus, we can interpret E and $x(j)$ in (3.5), respectively, as the aggregate world expenditure and the aggregate demand for good j.

where, throughout this paper, dots above variables represent the time derivatives of those variables. As in Lai [1995], imposing normalization $E(t) = n(t)$ for all t, we find that

$$\dot{R} = \rho + g \tag{3.6}$$

where $g \equiv \dot{n}/n$.

3.2.2 Innovation

Innovation in this model is in terms of final goods and it occurs only in the North. The innovation process is described as in Romer [1990]. That is, based on public knowledge $k = n$ at the time, innovators in the North develop designs for new products by using human capital according to

$$\dot{n} = \frac{H_N^R \cdot n}{a_D} \tag{3.7}$$

where H_N^R is northern human capital used in the research sector and a_D is the productivity parameter for the innovation. With this specification, denoting w_H^N to be the wage rate for human capital in the North, the unit cost of a new product design will be $w_H^N a_D/n$. Free entry into the research sector ensures that the price for a new product design equals the unit cost. Thus, denoting P_B as the price of the blueprint for a new product, we have

$$P_B = \frac{w_H^N a_D}{n} \tag{3.8}$$

3.2.3 Northern Production

The setup cost for a northern manufacturer is the purchasing cost of a product design from an innovator. Once paid P_B for a product design, a northern manufacturer tries to maximizes the discounted present value of lifetime operating profits

$$\Pi_N(t) = \int_t^{\infty} e^{-[R(\tau)-R(t)]}\pi_N(\tau)d\tau \tag{3.9}$$

where the northern instantaneous operating profits, $\pi_N(\tau)$, are

$$\pi_N(\tau) = p_N(\tau)x_N(\tau) - c_N(\tau)x_N(\tau) \tag{3.10}$$

in which p_N and c_N are, respectively, the price and the unit production cost of a northern product. Given the demand for a northern product, x_N, from (3.5), the mark-up price for a northern product can be calculated as

$$p_N = \frac{c_N}{\alpha} \tag{3.11}$$

Substituting (3.11) into (3.10), we get

$$\pi_N = \frac{1-\alpha}{\alpha}c_N x_N \tag{3.12}$$

As in Lai [1995], manufacturers in the North use both human capital and unskilled labor. Th

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labor. That is, the production function for a good i is given as

$$x(i) = \Lambda[\beta h(i)^{\gamma} + (1-\beta)l(i)^{\gamma}]^{1/\gamma} \tag{3.13}$$

where $0 < \beta < 1$, $-1 \leq \gamma \leq 0$,[6] and $h(i)$ and $l(i)$, respectively, are quantities of human capital and unskilled labor devoted to the production of good i. Λ is a productivity parameter. Note that the elasticity of substitution between inputs in this production function, $\xi \equiv 1/(1-\gamma)$, takes the highest value of 1, when $\gamma = 0$ (the Cobb-Douglas case).

3.2.4 Southern Production

The same production technology in (3.13) is also applied to southern manufacturers. However, southern manufacturers have to go through an additional process after purchasing blueprints of product designs from northern innovators and before actually producing the products. This process involves a learning cost such as those coming from the training of local labor, education for managerial level employees, or translation of blueprints into the local language. As in Liu [1994], this process requires a_L/n_S units of human capital,[7] where a_L is the productivity parameter for the learning activity. The set up cost for a southern manufacturer will then be the sum of

[6] In Lai, $\gamma \leq 0$ is assumed to ensure that both types of labor are used in manufacturing. In this model, $\gamma \leq 0$ is continued to be assumed. However, algebraic complication arising in this model forces us to impose an additional assumption $\gamma \geq -1$. As a result, the production function in (3.13) cannot include the Leontief case ($\gamma = -\infty$). However, our assumed range of γ continues to include the Cobb-Douglas case ($\gamma = 0$).

[7] In Liu, labor is used for this process. However, as labor in his model is used for the innovation in the North, human capital in this model is analogous to labor in his model.

the purchasing cost of a blueprint, P_B, and the learning cost, $w_H^S a_L/n_S$, in which we denote w_H^S as the wage rate for human capital in the South. Also as in Liu, for North-South technology trade to make sense, we need to assume that imitation by the South costs more than the setup cost. That is, if the South can imitate northern developed product according to $\dot{n}_S = H_S^R \cdot n_S/a_I$, it should be assumed that

$$\frac{w_H^S a_I}{n_S} > P_B + \frac{w_H^S a_L}{n_S}$$

where a_I denotes the productivity parameter for southern imitation.

Once incurring the setup cost, a southern manufacturer tries to maximize the discounted value of lifetime operating profits

$$\Pi_S(t) = \int_t^{\infty} e^{-[R(\tau)-R(t)]}\pi_S(\tau)d\tau \tag{3.14}$$

where the southern instantaneous operating profits, $\pi_S(\tau)$ is

$$\pi_S(\tau) = p_S(\tau)x_S(\tau) - c_S(\tau)x_S(\tau) \tag{3.15}$$

in which p_S and c_S are, respectively, the unit price and the unit production cost of a southern product. Given the demand for a southern product in (3.5), the mark-up price for a southern product can be calculated as

$$p_S = \frac{c_S}{\alpha} \tag{3.16}$$

Substituting (3.16) into (3.15), we get

$$\pi_S = \frac{1-\alpha}{\alpha} c_S x_S \tag{3.17}$$

3.2.5 No Arbitrage Condition

As there are many potential manufacturers for a product design in the North, innovators in the North can charge the blueprint price that equals the discounted value of a manufacturer's lifetime operating profits. Thus, we have

$$\Pi_N(t) = P_B(t) \tag{3.18}$$

Substituting (3.8) and (3.9) into (3.18) and doing logarithmic differentiation with respect to time, we get the northern no arbitrage condition as

$$\frac{\pi_N}{\Pi_N} = \dot{R} - \frac{\dot{w}_H^N}{w_H^N} + \frac{\dot{n}}{n} \tag{3.19}$$

This condition indicates that the instantaneous operating profits for a northern manufacturer must cover the interest rate and the instantaneous net capital loss.

As for the southern manufacturer, the blueprint price must equal the discounted value of a manufacturer's operating profits minus the learning cost. That is,

$$\Pi_S(t) = P_B(t) + \frac{w_H^S a_L}{n_S} \tag{3.20}$$

Substituting (3.8) and (3.14) into (3.20) and doing logarithmic differentiation to (3.20) with respect to time, we get the southern no arbitrage condition as

$$\frac{\pi_S}{\Pi_S} = \dot{R} - \frac{w_H^N a_D n_S \frac{\dot{w}_H^N}{w_H^N} + w_H^S a_L n \frac{\dot{w}_H^S}{w_H^S}}{w_H^N a_D n_S + w_H^S a_L n} + \frac{w_H^N a_D n_S \frac{\dot{n}}{n} + w_H^S a_L n \frac{\dot{n}_S}{n_S}}{w_H^N a_D n_S + w_H^S a_L n} \tag{3.21}$$

As in the no arbitrage condition for the North, the condition in (3.21) indicates that the instantaneous operating profits must cover the instantaneous operating profits and net instantaneous capital loss. Otherwise, southern manufacturers cannot recover the initial setup cost and will consequently earn negative profits.

3.2.6 Labor Market

Finally, to close the model, we need to describe the labor market. Assuming full employment for both types of labor in each region, we can write the northern human capital market clearing condition as

$$a_D \frac{\dot{n}}{n} + H_N^P = H_N \tag{3.22}$$

where the first term in the left hand side is the human capital employed in the research sector and derived from the innovation function (3.7). H_N^P and H_N, respectively, represent the human capital employed in the northern production of manufacturing goods and the total human capital endowments in the North. Denoting L_N as the total endowments for unskilled labor, the market clearing condition is simply $L_N^P = L_N$, since unskilled labor is only used in the production sector. Similarly, for the southern region, we have the market clearing condition for human capital as

$$a_L \frac{\dot{n}_S}{n_S} + H_S^P = H_S \tag{3.23}$$

where the first term in the left hand side is the human capital devoted to the learning process, H_S^P is the human capital devoted to the actual production process, and H_S is the total human capital endowment for the South. Also, denoting L_S as the total unskilled labor for the South, we can write the market clearing condition for southern unskilled labor as $L_S^P = L_S$.

3.3 Steady State Equilibrium

In this section, we solve for a steady state equilibrium where the growth rate of the number of products, $g \equiv \dot{n}/n$, and the fraction of goods produced in each region, $\sigma \equiv n_N/n_S$, are constant over time. Recalling our normalization, $E/n = 1$, then, we have $g = \dot{n}/n = \dot{n}_S/n_S = \dot{w}_H^N/w_H^N = \dot{w}_H^S/w_H^S$. Imposing these conditions in (3.19) and (3.21), we have no arbitrage conditions for the North and the South, respectively, as

$$\frac{\pi_N}{\Pi_N} = \rho + g \tag{3.24}$$

and

$$\frac{\pi_S}{\Pi_S} = \rho + g \tag{3.25}$$

As the expenditure level and the number of products grow at the same rate, with the normalization, $E/n = 1$, we have capital loss terms cancel out in (3.19) and in (3.21). Thus, the no arbitrage conditions in (3.24) and (3.25) simply imply that the instantaneous operating profits in each region need to cover the interest rate.[8] Note that the southern no arbitrage condition in (3.25) is the same as in Lai [1995]. However, the northern no arbitrage condition is different. In Lai's model, northern manufacturers face the risk of their products being imitated by southern imitators and losing the market to southern manufacturers with cost advantage. However, in this model, such imitations are not possible and we don't have the term, μ, that reflects the hazard rate.[9]

Substituting (3.12) and (3.11) along with (3.8) in (3.24), we get

$$\frac{\frac{1-\alpha}{\alpha} c_N x_N}{\frac{w_H^N a_D}{n}} = \rho + g \tag{3.26}$$

Denoting ϕ_N as the factor cost share of human capital in the northern manufacturing sector, we get

$$c_N x_N = \frac{w_H^N H_N^P}{n_N \phi_N} \tag{3.27}$$

Noting symmetry among northern manufacturers and using (3.22), from (3.13), we also get

$$\phi_N = \frac{\beta(H_N - a_D g)^\gamma}{\beta(H_N - a_D g)^\gamma + (1-\beta)L_N^\gamma} \tag{3.28}$$

[8]With $E/n = 1$, $\dot{R} = \rho + g$. See (3.6).
[9]See equation (5) in Lai.

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Substituting (3.28) into (3.27), (3.27) into (3.26), and then rearranging we get

$$\frac{n_N}{n_S} \equiv \sigma = \frac{(1-\alpha)(H_N - a_D g)^{1-\gamma}[\beta(H_N - a_D g)^\gamma + (1-\beta)L_N^\gamma]}{\alpha a_D \beta(\rho + g) - (1-\alpha)(H_N - a_D g)^{1-\gamma}[\beta(H_N - a_D g)^\gamma + (1-\beta)L_N^\gamma]} \tag{3.29}$$

This equation satisfies the equilibrium condition in the goods, capital, and labor markets in the North. Note that, if we set $\beta = 1$, (3.29) is reduced to the equation (2-23) in Liu

$$\sigma = \frac{(1-\alpha)(H_N/a_D - g)}{\alpha\rho + g - (1-\alpha)(H_N/a_D)}$$

and this indicates that his model is special case of the model presented here.[10] In Appendix, it is shown that total differentiation of (3.29) gives

$$\left.\frac{d\sigma}{dg}\right|_{NN} < 0$$

which indicates that there is a negative relationship between the growth rate ($g = \dot{n}/n$) and the fraction of goods produced in the North ($\sigma = n_N/n_S$) along the northern equilibrium condition (3.29). The negative relationship is understood as follows. Holding everything else constant, an increase in g increases the demand for human capital in the North. This increases the wage rate for human capital in the North. This increases both the unit cost of product development and, as the blue print price equals the unit cost of product development, the setup cost for manufacturers. The increase in the wage rate for human capital also increases the manufacturing cost and the price of goods in the North. Thus, the per product sale and profit decrease.

[10] His model considers the economy with only one type of labor.

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Both the increase in the setup cost and the decrease in the profit discourage entry of new manufacturers in the North and decrease the fraction of northern manufactured goods. Thus, we have a negative relationship between g and σ. Also, by considering the limit case of $\sigma \to \infty$ in (3.29), we can see that g asymptotes to the value satisfying

$$(1-\alpha)\frac{H_N}{a_D} - g = \alpha\rho - (1-\alpha)(\frac{1-\beta}{\beta})(\frac{H_N}{a_D} - g)^{1-\gamma}(\frac{L_N}{a_D})^{\gamma}$$

The NN curve in the Figure 3.1 below represents the northern equilibrium condition (3.29), based on our findings above.

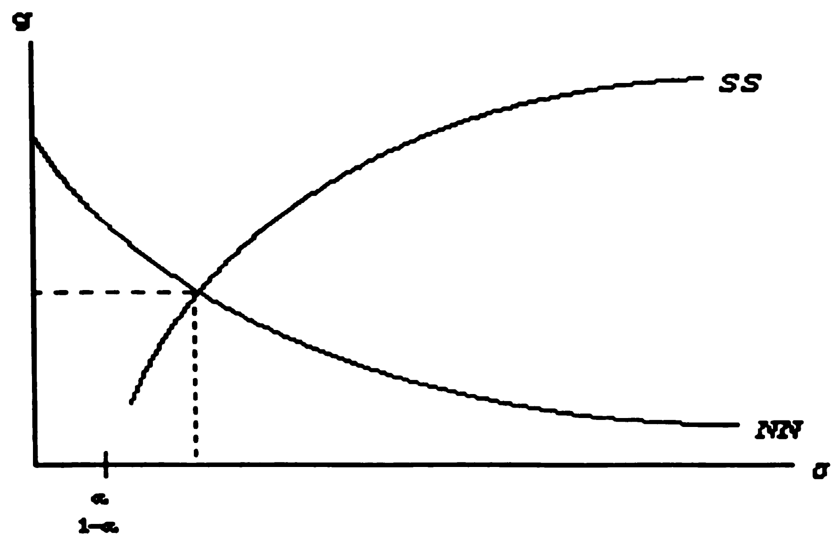

Figure 3.1: Steady State Equilibrium

Substituting (3.17) and (3.20) along with (3.8) into the southern no arbitrage condition (3.25), we get

$$\frac{\frac{1-\alpha}{\alpha}c_S x_S}{\frac{w_H^N a_D}{n} + \frac{w_H^S a_L}{n_S}} = \rho + g \tag{3.30}$$

Denoting ϕ_S to be the factor cost share of human capital in the southern manufacturing sector, we get

$$c_S x_S = \frac{w_H^S H_S^P}{n_S \phi_S} \tag{3.31}$$

Noting symmetry among southern manufacturers and using (3.23), from (3.13), we also get

$$\phi_S = \frac{\beta(H_S - a_L g)^\gamma}{\beta(H_S - a_L g)^\gamma + (1-\beta)L_S^\gamma} \tag{3.32}$$

Substituting (3.31) and (3.32) into (3.30), we get

$$\frac{\frac{1-\alpha}{\alpha}\frac{w_H^P H_S^P}{n_S}\frac{\beta(H_S-a_L g)^\gamma+(1-\beta)L_S^\gamma}{\beta(H_S-a_L g)^\gamma}}{\frac{w_H^N a_D}{n}+\frac{w_H^S a_L}{n_S}} = \rho + g$$

Substituting (3.23) for H_S^P in the above and rearranging terms, we get

$$\frac{1-\alpha}{\alpha}(H_S - a_L g)^{1-\gamma}[\beta(H_S - a_L g)^\gamma + (1-\beta)L_S^\gamma] = a_L\beta(\rho+g)[\frac{(w_H^N/w_H^S)(a_D/a_L)}{n/n_S} + 1] \tag{3.33}$$

The wage rate for human capital in each region equals the value of the marginal product of human capital in each manufacturing sector. Thus, noting symmetry among manufacturers in each region in (3.13), we get

$$w_H^N = \mathrm{VMP}_H^N = \frac{c_N}{\alpha}\beta\Lambda[\beta + (1-\beta)(\frac{L_N}{H_N - a_D g})^\gamma]^{1-\gamma/\gamma}$$

and

$$w_H^S = \mathrm{VMP}_H^S = \frac{c_S}{\alpha}\beta\Lambda[\beta + (1-\beta)(\frac{L_S}{H_S - a_L g})^\gamma]^{1-\gamma/\gamma}$$

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$$\frac{w_H^N}{w_H^S} = \frac{c_N}{c_S}\left[\frac{\beta + (1-\beta)(\frac{L_N}{H_N - a_D g})^\gamma}{\beta + (1-\beta)(\frac{L_S}{H_S - a_L g})^\gamma}\right]^{1-\gamma/\gamma} \tag{3.34}$$

Using (3.5), (3.11), and (3.16), we get

$$\frac{c_N x_N}{c_S x_S} = (\frac{c_N}{c_S})^{1-\epsilon}$$

Using $c_N x_N = w_H^N H_N^P / n_N \phi_N$ and $c_S x_S = w_H^S H_S^P / n_S \phi_S$, we can rewrite the above as

$$\frac{c_N}{c_S} = (\frac{w_H^N H_N^P / n_N \phi_N}{w_H^S H_S^P / n_S \phi_S})^{1/1-\epsilon} \tag{3.35}$$

Substituting (3.35) into (3.34) and using (3.22), (3.23), (3.28), and (3.32), we get

$$\frac{w_H^N}{w_H^S} = (\frac{n_N}{n_S})^{1-\alpha}(\frac{H_S - a_L g}{H_N - a_D g})^{1-\alpha}\left[\frac{\beta + (1-\beta)(\frac{L_S}{H_S - a_L g})^\gamma}{\beta + (1-\beta)(\frac{L_N}{H_N - a_D g})^\gamma}\right]^{\alpha(\frac{1-\alpha}{\alpha} - \frac{1-\gamma}{\gamma})} \tag{3.36}$$

Substituting (3.36) into (3.33) and noting $\sigma = n_N/n_S$, we get the southern equilibrium condition as

$$\frac{1-\alpha}{\alpha}(H_S - a_L g)[\beta + (1-\beta)(\frac{L_S}{H_S - a_L g})^\gamma] =$$

$$a_L\beta(\rho + g)\{\frac{\sigma^{1-\alpha}(\frac{H_S - a_L g}{H_N - a_D g})^{1-\alpha}\left[\frac{\beta+(1-\beta)(\frac{L_S}{H_S - a_L g})^\gamma}{\beta+(1-\beta)(\frac{L_N}{H_N - a_D g})^\gamma}\right]^{\alpha(\frac{1-\alpha}{\alpha} - \frac{1-\gamma}{\gamma})}(\frac{a_D}{a_L})}{1+\sigma} + 1\} \tag{3.37}$$

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This equation satisfies equilibrium conditions in the goods, capital, and labor markets in the South. Setting $\beta = 1$, we can also show that (3.37) is reduced to equation (2-24') in Liu.

$$\sigma = \frac{\alpha \frac{a_D}{a_L}(\rho + g)(\sigma \frac{H_S - a_L g}{H_N - a_D g})^{1-\alpha}}{(1-\alpha)\frac{H_S}{a_L} - \alpha\rho - g} - 1$$

As shown in Appendix, total differentiation of both sides of (3.37) gives a positive relationship between the growth rate and the fraction of goods produced in the North along the southern equilibrium condition,

$$\left.\frac{d\sigma}{dg}\right|_{SS} > 0$$

as long as $\sigma > \alpha/(1-\alpha)$, $L_S/a_L > L_N/a_D$, $H_S/a_L > H_N/a_D$, and $-1 \leq \gamma \leq 0$.[11] The first two conditions are also assumed in Liu, while the last two conditions are needed to accommodate newly introduced structure in this model. The first condition requires that, for a given value of α, and thus, for a given value of the elasticity of substitution between goods, $\epsilon = 1/(1-\alpha)$, a relatively larger fraction of existing goods are produced in the innovating North. The second and the third conditions require relatively large effective labor forces in the South. These conditions can be guaranteed by assuming that, for given labor endowments in each region, the southern learning activity for already developed product design is much easier than the northern innovating activity of designing new products ($a_L < a_D$). As explained in Section 2.3, the last condition restricts us in the intermediate range of the elasticity

[11]Outside the range of these assumptions, the sign of $d\sigma/dg$ is ambiguous.

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of substitution between inputs in manufacturing. The explanation for this positive relationship between g and σ is similar to that in the northern equilibrium case. That is, ceteris paribus, increases in g make the demand and the wage rate for northern human capital increase. This results in an increase in the setup cost for southern manufacturers and decreases the entry of new manufacturers in the South. Thus, we have a decrease in the fraction of goods produce in the South ($\sigma \uparrow$) and a positive relationship between g and σ. By considering the limiting case of $\sigma \to \infty$ in (3.37), we can see that g asymptotes to the value satisfying

$$(1-\alpha)\frac{H_S}{a_L} - g = \alpha\rho - (1-\alpha)(\frac{1-\beta}{\beta})(\frac{H_S}{a_L} - g)^{1-\gamma}(\frac{L_S}{a_L})^{\gamma} \tag{3.38}$$

The SS curve in Figure 3.1 represents the southern equilibrium condition, based on our findings above. The equilibrium growth rate and the fraction of products produced in each region are determined at the intersection of the SS and the NN curves as in Figure 3.1.

3.4 Comparative Dynamics

In this section, we will consider the effect of increases in human capital and unskilled labor in each region on the growth rate of the number of products. Substituting (3.5) into (3.3) and differentiating it with respect to time, we can show that

$$\frac{\dot{u}}{u} = \frac{1-\alpha}{\alpha}\frac{\dot{n}}{n}$$

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Thus, the growth rate in the instantaneous utility is proportional to the growth rate of the number of products, and the results in this section can be considered as the growth effect of consumers' welfare in the world.

3.4.1 Increases in Northern Human Capital ($H_N \uparrow$)

As shown in Appendix, holding g and L_N constant, total differentiation of (3.29) gives

$$\left.\frac{d\sigma}{dH_N}\right|_{NN} > 0$$

under our assumption that $0 < \beta < 1$ and $-1 \leq \gamma \leq 0$. This means that the NN curve shifts up[12] as H_N increases. Holding g constant, for given n and n_N, increases in northern human capital increase the human capital available for production in the North. This reduces the cost of production and the price of goods in the North. This results in increases in per product sales and operating profits for northern manufacturers. Referring back to the no arbitrage condition of the North in (3.24), this means that the per manufacturer profit rate in the left hand side exceeds the interest rate in the right hand side. This encourages the entry of new manufactures in the North. Thus, we have increases in both the fraction of goods produced in the North ($\sigma \uparrow$) and the growth rate in the number of products($g \uparrow$). This is represented by the upward shift of the NN curve as in Figure 3.2 below.

Increases in northern human capital also affects the southern equilibrium condi-

[12]This is equivalent to saying that the NN curve shifts to the right.

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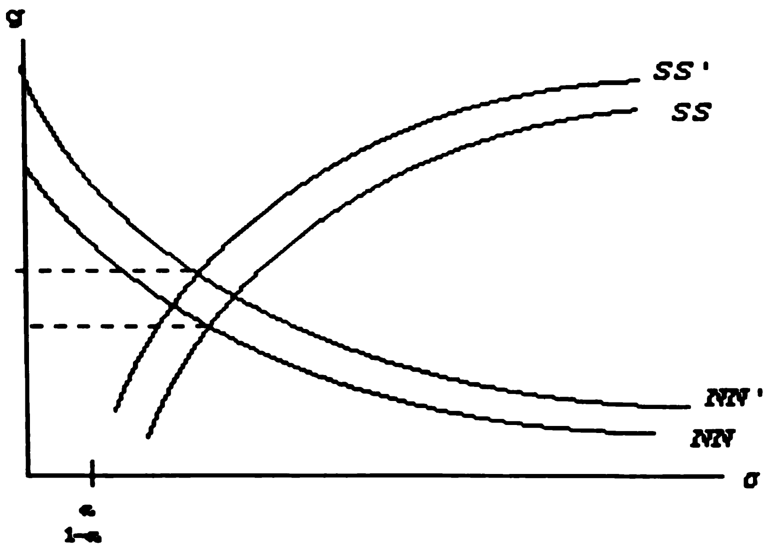

Figure 3.2: Growth Effect of $H_N \uparrow$

tion. In Appendix, we show that total differentiation of (3.37) gives

$$\left.\frac{d\sigma}{dH_N}\right|_{SS} < 0$$

under the assumption that $\sigma > (1-\alpha)/\alpha$. This means that the SS curve also shifts up as H_N increases, as in Figure 3.2 below. Increases in northern human capital affects the southern equilibrium condition through the decrease in the blueprint price. Holding g constant, for given n and n_S, increases in northern human capital lowers the unit cost, and thus the blueprint price, of a product design. As manufacturers in the South also have to buy northern product designs, the lower blueprint price means a lower setup cost for southern manufacturers. This encourages the entry of new southern manufacturers. Thus, increases in northern human capital tend to increase both the fraction of goods produced in the South ($\sigma \downarrow$) and the growth rate of the number of products ($g \uparrow$).

As increases in northern human capital shift both the NN and the SS curves upward in Figure 3.2 above, their growth enhancing effects reinforce each other and we have an unambiguous increase in the growth rate.

The result here is new compared to that in Lai [1995]. In his model, the setup cost for southern manufacturers is independent of the cost of product development in the North. That is, the denominator of the left hand side of (3.30) changes to $w_H^s a_I / n_S$.[13] Then, for a given g and n from the North, the number of products manufactured in the South, n_S, and thus, the fraction of goods produced in each region, $\sigma = n_N/n_S$, are solely determined by the southern composition of labor.[14] Thus, the SS curve in Lai's model becomes horizontal. Furthermore, as the set up cost for southern manufacturers is independent of the wage rate for human capital in the North, changes in northern human capital, and the resulting changes in the wage rate, do not shift the SS curve. Without shifts in the horizontal SS curve, shifts in the NN curve alone cannot lead to changes in g.

With the collapse of the old Soviet Union, we see that lots of skilled labor from eastern European countries migrate to the western world and work for the market economies. We can consider this as an increase in human capital in the innovating North. Unlike Lai's model, the model presented here can project a positive growth effect of such an increase in northern human capital.

[13] See equation (7) in Lai.

[14] w_H^s, w_L^s, and c_S are determined by the southern composition of labor.

3.4.2 Increases in Northern Unskilled Labor ($L_N \uparrow$)

In Appendix, holding g and H_N constant, we show that the total differentiation of (3.29) gives

$$\left.\frac{d\sigma}{dL_N}\right|_{NN} \leq 0$$

which means that the NN curve shifts down as L_N increases.[15] As northern unskilled labor increases, the manufacturing cost and the price of northern products decrease. This increases the per product profit rate and encourages entry of manufacturers in the North. This tends to increase both the fraction of goods ($\sigma \uparrow$) and product development ($g \uparrow$) in the North. However, at the same time, increases in northern unskilled labor increase the marginal product of human capital in northern manufacturing. This increases the wage rate for human capital in the North. This tends to increase the manufacturing cost and offsets the above effect of encouraging entry of new manufacturers. Furthermore, since the research sector only uses human capital, the unit cost of developing new products increases and the price for a new blue print goes up. This increases the setup cost and further discourages the entry of new manufacturers in the North. As the second effect, through the increase in the marginal product of human capital, outweighs the first effect, increases in northern unskilled labor decrease both the fraction of goods produced in the North ($\sigma \downarrow$) and the rate of product development ($g \downarrow$). This is represented by the downward shift in the NN curve in Figure 3.3.

[15]When $\gamma = 0$, increases in L_N do not affect the northern equilibrium condition. The same is also true in Lai.

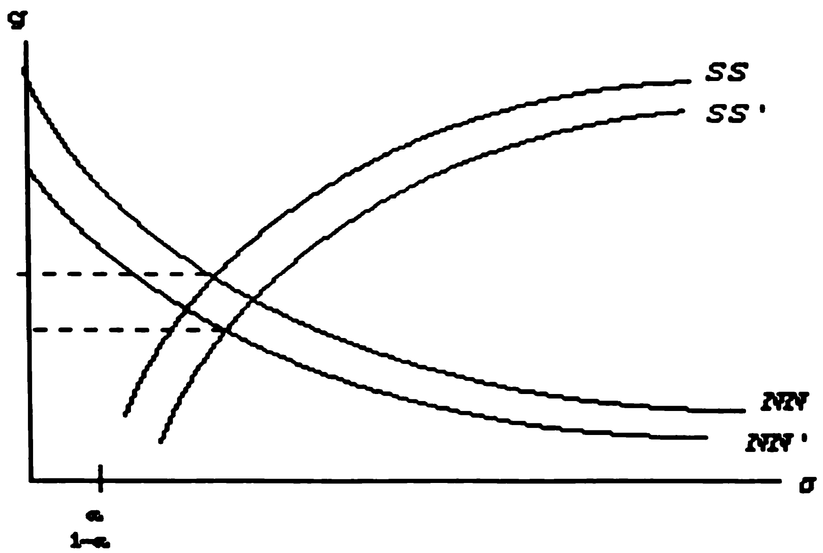

Figure 3.3: Growth Effect of $L_N \uparrow$

Total differentiation of (3.37), which is also given in Appendix, gives

$$\left.\frac{d\sigma}{dL_N}\right|_{SS} \geq 0$$

which means that the SS curve shifts down as L_N increases.[16] As mentioned above, increases in northern unskilled labor increase the marginal product of human capital and the blue print price of a new product in the North. As southern manufactures in this model also need to buy blue prints from the North, this also increases the setup cost in the South. Then, the entry of new manufacturers in the South is discouraged. Thus, the fraction of goods produced in the South decreases and the rate of product development also decreases. This is represented by the downward shift in the SS curve in Figure 3.3. As both the NN and SS curves shift down, we have an unambiguous

[16]As in the northern equilibrium condition, when $\gamma = 0$, the increases in L_N do not affect the southern equilibrium condition.

decrease in the growth rate.

This result is also new compared to Lai. As explained in the case of increasing northern human capital, the set up cost for southern manufacturers is independent of the cost of product development in the North. Thus, increases in northern unskilled labor and the resulting changes in the cost of product development in the North do not affect the southern equilibrium condition. Increases in northern unskilled labor do not shift the SS curve in Lai's model and, since the SS curve in his model is horizontal, the shift in the NN curve alone cannot result in changes in the equilibrium growth rate. The result here are also different from Liu [1994]. In his model, increases in northern labor have a positive growth effect. However, as he considers only one type of labor, the effect through changes in the relative marginal product between two different inputs cannot be addressed. Especially, as one type of labor is used in both the manufacturing and the research sectors, increases in northern labor in his model are equivalent to increases in northern human capital, which are used in both the research and the manufacturing sector in this model.

The finding in this section has an implication related to the productivity slow down in the 1980s. It is reported that there has been a sharp increase in the fraction of high school graduates relative to college graduates in the U.S. labor force in the 1980s.[17] Although we do not have population growth in this model, if we consider this increase in the fraction of high school graduates as an increase in the unskilled labor force in the U.S., which obviously is a part of the innovating North, we can

[17]Between 1979 and 1987, the number of 25-34 year old male high school graduates in the U.S. labor force grew by 40 percent, compare to an only 32 percent increase in the number of 25-34 year old male college graduates. See Levy and Murnane [1992].

expect a decrease in the world growth rate. This implication cannot be addressed in either Lai or Liu.

3.4.3 Increases in Southern Human Capital ($H_S \uparrow$)

The northern equilibrium condition in (3.29) does not contain the southern human capital term, (H_S), so increases in southern human capital do not affect the northern equilibrium condition. Thus, when there is a change in southern human capital, the NN curve stays the same. Holding g, H_N, L_N, and L_S constant, total differentiation of (3.37), which is shown in Appendix, gives

$$\left.\frac{d\sigma}{dH_S}\right|_{SS} < 0$$

which means that the SS curve shifts up as H_S increases. As southern human capital

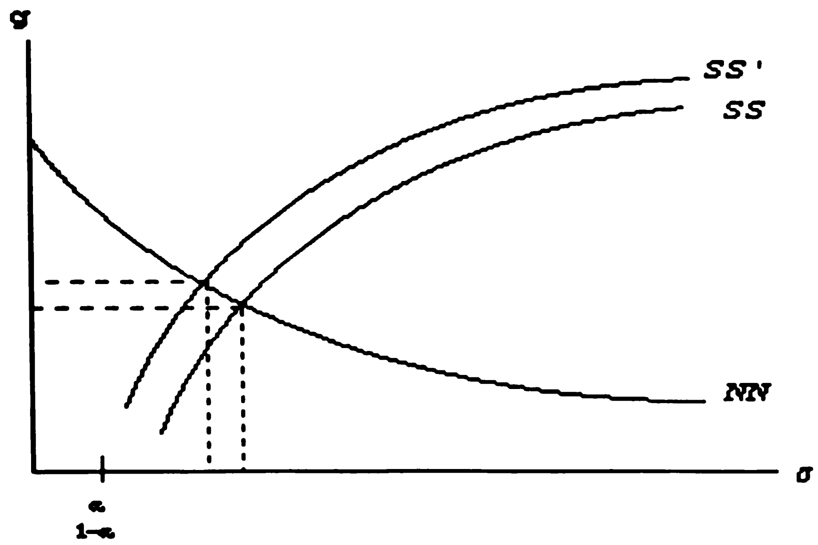

Figure 3.4: Growth Effect of $H_S \uparrow$

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manufacturers decrease. This encourages the entry of new manufacturers in the South and increases the fraction of goods produced in the South. As shown in Figure 3.4, we have an increase in g and a decrease in σ. Similar results can be obtained in Lai. In Liu's model, as labor is used in both the actual manufacturing and the learning activities, we can consider the increase in southern labor as the increase in southern human capital. Thus, the positive growth effect of southern labor in his model is qualitatively the same as the positive growth effect of the increase in southern human capital in this section.

3.4.4 Increases in Southern Unskilled Labor ($L_S \uparrow$)

Like in the southern human capital case, changes in southern unskilled labor do not affect the northern equilibrium condition. Thus, the NN curve stays the same as L_S increases. In Appendix, it is shown that total differentiation of (3.37) gives

$$\left.\frac{d\sigma}{dL_S}\right|_{SS} < 0 \qquad \text{if} \quad \theta > Q \tag{3.39}$$

where

$$\theta \equiv 1 - \frac{\alpha}{\gamma} > 1$$

and

$$Q \equiv \frac{1}{1 - \frac{a_L \beta(\rho + g)}{l}} > 1$$

in which

$$l \equiv \frac{1-\alpha}{\alpha}(H_S - a_L g)[\beta + (1-\beta)(\frac{L_S}{H_S - a_L g})^{\gamma}]$$

As explained below, the larger the elasticity of substitution between goods in consumption and the larger the elasticity of substitution between inputs in production, the larger the value of θ is. As $0 < \alpha < 1$ enters the elasticity of substitution between goods in consumption, $\epsilon \equiv 1/(1-\alpha)$, larger values of α[18] imply larger values of the elasticity of substitution between goods. Also, as $-1 \leq \gamma \leq 0$ enters the elasticity of substitution between inputs in production, $\xi \equiv 1/(1-\gamma)$, smaller values of γ[19] imply larger values of the elasticity of substitution between inputs. However, θ is larger as α is larger and γ is smaller.

We can also say that, the more abundant southern human capital is, the closer the value of Q is to 1. Note that, for a given value of g, Q becomes smaller, as l becomes larger. However, manipulating l above and using (3.32), we can rewrite l as

$$l = \frac{1-\alpha}{\alpha}\beta\frac{H_S - a_L g}{\phi_S}$$

The numerator in the last fraction becomes large, for a given g, as H_S becomes large. The denominator ϕ_S measures the factor cost share of human capital in manufacturing in the South. For a given g, relatively abundant southern human capital implies a lower wage rate for human capital in the South. This makes the factor cost share of human capital in manufacturing smaller. Thus, we can say that relatively abundant southern human capital implies larger values of l and smaller values of Q. Therefore, we can conclude that, for a given value of θ, sufficiently large amounts of southern

[18] Closer to 1.
[19] Closer to -1.

human capital ensure $\theta > Q$. Then, using (3.39), we can see that in this case the SS curve will shift up. That is, for a sufficiently large enough stock of southern human capital, increases in southern unskilled labor increase the world growth rate and the fraction of goods produced in the South. Increases in southern unskilled labor lower the wage rate for southern unskilled labor. This tends to decrease the cost of production for southern manufacturers. However, at the same time, increases in southern unskilled labor increase the marginal product of human capital in manufacturing. This raises the demand and the wage rate for human capital. This tends to offset the initial decrease in the manufacturing cost. Which of these two effects dominates depends on the relative abundance of southern human capital. That is, if human capital is relatively abundant in the South, the increase in the marginal product of human capital would not be that high, and the decrease in manufacturing cost through the decrease in the wage rate for unskilled labor dominates. Decreases in southern manufacturing cost lower the price of the goods produced in the South. This raises the demand, and thus the operating profits, for southern goods. This encourages entry of new manufacturers in the South and increases both the fraction of goods produced in the South ($\sigma \downarrow$) and the rate of product developments ($g \uparrow$).

This result is opposite to that in Lai's model of imitation. In his model, increases in southern unskilled labor also increase the marginal product of human capital in southern manufacturing. Then, demand for human capital from southern manufacturers increases. This raises the wage rate for human capital, induces human capital out of the southern research sector, and lowers the imitation rate. Decreases in the imitation rate imply decreases in the hazard rate for northern manufacturers. This tends to increase the innovation rate in the North. However, for a given number of products, as less products are imitated by the South, more manufacturers are competing for resources in the North. This lowers the per manufacturer operating profits. This tends to decrease the innovation rate in the North. As the second effect dominates the first effect, in his model, increases in southern unskilled labor result in decreases in g. Using Lai's word, the integration of labor abundant China into the world trading economy unambiguously decreases the world growth rate. However, findings in this model suggest that, if the pre-existing world trading economy is endowed with enough human capital in the non-innovating South, the integration of China into the southern economy would still have a positive growth effect.

Findings in this section can be summarized as the following

Theorem *In an economy with North-South technology trade, an increase in the supply of either northern or southern human capital increases the world growth rate. An increase in the supply of southern unskilled labor also increases the world growth rate, provided that South is endowed with relatively abundant human capital. However, an increase in northern unskilled labor has a negative growth effect.*

3.5 Conclusion

Findings in this paper have many policy implications. The positive growth effect of expanding human capital in each region has the implication related to the integration of many eastern European economies into the western trading economy. During the cold war, many communist countries tried to accumulate human capital to demonstrate the superiority of their ideology. With the collapse of the old communists bloc, a relatively rich stock of human capital is available to be integrated into the western economy. Whether this human capital is distributed in the North or in the South, we have an unambiguously positive growth effect for the world economy. The sharp increase in the fraction of high school graduates in the U.S. labor force, in 1980s, can be considered as an increase in unskilled labor force in the innovating North. Then the negative growth effect of northern unskilled labor in this model can be an explanation for the recent productivity slow down of the world economy. Integration of relatively unskilled labor abundant China into the trading economy could have positive growth effects for the world economy. However, for this result, we need to have a relative abundance of human capital in pre-existing southern economy.

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Appendix A

Chapter 1

A.1 Consumers' Expenditure Path

By substituting (1.3) into (1.2) and (1.2) in (1.1), we get

$$U = \int_t^{\infty} e^{-\rho(s-t)}\{2n(s)ln[E(s)/2n(s)] - [\int_0^{n(s)} lnP(j,s)dj + \int_0^{n^*(s)} lnP(j^*,s)dj^*]\}ds \tag{A.1}$$

If we maximize this subject to the intertemporal budget constraint

$$\int_0^{\infty} e^{-[R(s)-R(t)]} \cdot E(s)ds \leq Z(t) \tag{A.2}$$

where $Z(t)$ includes assets and tariff revenue distributed back to consumers along with the present value of consumers' wage incomes. These are either market or government determined values and they are just like given from the consumers' point of view.

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Since the prices in the second bracket of the intertemporal utility function in (A.1) is also the given values to consumers, the Hamiltonian Function for the maximization problem is given as

$$H_C \equiv e^{-\rho(s-t)} 2n(s) ln[E(s)/2n(s)] - \mu \cdot e^{-[R(s)-R(t)]} \cdot E(s)$$

where μ is the usual costate variable. The maximum principle gives

$$e^{-\rho(s-t)} \frac{2n(s)}{E(s)} = \mu \cdot e^{-[R(s)-R(t)]} \tag{A.3}$$

Differentiating (A.3) with respect to s, we get

$$-\rho \cdot e^{-\rho(s-t)} \frac{2n(s)}{E(s)} + e^{-\rho(s-t)} \frac{N'(s)E(s) - N(s)E'(s)}{E(s)^2} = -R'(s)\mu e^{-[R(s)-R(t)]} \tag{A.4}$$

By dividing (A.4) by (A.3), we get

$$\frac{E'(s)}{E(s)} = R'(s) - \rho + \frac{n'(s)}{n(s)} \tag{A.5}$$

By setting $s = t$ and using $g \equiv n'/n$, we have the expression in (1.4).

A.2 Final Goods Manufacturer

Demand for labor and intermediate goods from manufacturers of a final good, j, can be found by minimizing the cost

$$C(j) = wL_Y(j) + \int_0^j P_m(i)x(i)di + \int_0^j P_m(i^*)x(i^*)di^* \tag{A.6}$$

subject to the production function given in (1.7). As the production function is Cobb-Douglas type, we know that

$$wL_Y(j) = (1-\beta)C(j) \tag{A.7}$$

and

$$\int_0^j P_m(i)x(i)di + \int_0^j P_m(i^*)x(i^*)di^* = \beta C(j) \tag{A.8}$$

Combining (A.7) and (A.8), we get

$$wL_Y(j) = \frac{1-\beta}{\beta}[\int_0^j P_m(i)x(i)di + \int_0^j P_m(i^*)x(i^*)di^*] \tag{A.9}$$

Substituting (A.9) into the cost function in (A.6), we get

$$C(j) = \frac{1}{\beta}[\int_0^j P_m(i)x(i)di + \int_0^j P_m(i^*)x(i^*)di^*] \tag{A.10}$$

As we combine two separate continuums $i \in [0, j]$ and $i^* \in [0, j]$ into one continuum $v \in [0, \ldots, j, j+0, \ldots, j+j]$, we can rewrite the cost function as

$$C(j) = \frac{1}{\beta}[\int_0^{2j} P_m(v)x(v)dv] \tag{A.11}$$

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$$v = i \qquad \text{if } v \in [0, j]$$

$$v = i^* \qquad \text{if } v \in (j, 2j]$$

We can also rewrite the production function in (1.7) under one integral sign as

$$Y(j) = L_Y(j)^{1-\beta}[\int_0^{2j} x(v)^\alpha dv]^{\beta/\alpha}$$

Rewriting this, we get

$$\int_0^{2j} x(v)^\alpha dv = [\frac{Y(j)}{L_y(j)^{1-\beta}}]^{\alpha/\beta} \tag{A.12}$$

Let $\Gamma(v) = \int_0^v x(z)^\alpha dz$ so that $\Gamma(0) = 0$ and $\Gamma(2j) = [\frac{Y(j)}{L_Y(j)^{1-\beta}}]^{\alpha/\beta}$. Then, we have

$$\dot{\Gamma} = x(v)dv$$

We can use this in place of the production function (A.12) as the constraint for the minimization of cost function in (A.11). Noting $0 < \beta < 1$ there, the Hamiltonian function for this problem can be written as

$$H_F \equiv P_m(v)x(v) - \lambda x(v)^\alpha$$

As $-\partial H/\partial\Gamma = \dot{\lambda} = 0$, we can treat λ as a constant. We also get

$$\frac{\partial H}{\partial x(v)} = P_m(v) - \lambda\alpha x(v)^{\alpha-1} = 0$$

Rearranging this, we get

$$x(v) = [\frac{P_m(v)}{\lambda\alpha}]^{1/\alpha-1} \tag{A.13}$$

or

$$\frac{1}{\lambda\alpha} = \frac{x(v)^{\alpha-1}}{P_m(v)} \tag{A.14}$$

By substituting (A.13) in (A.12), we get

$$(\frac{1}{\lambda\alpha})^{\alpha/\alpha-1}\int_0^{2j} x(v)^\alpha dv = [\frac{Y(j)}{L_y(j)^{1-\beta}}]^{\alpha/\beta}$$

Substituting from (A.14) rearranging the above, we get

$$x(v) = \frac{P_m(v)^{1/\alpha-1}}{[\int_0^{2j} P_m(v)^{\alpha/\alpha-1}dv]^{1/\alpha}} \cdot [\frac{Y(j)}{L_y(j)^{1-\beta}}]^{1/\beta}$$

If we rewrite this with separate integral signs for home and foreign intermediate goods as in the original form, we get

$$x(i) = \frac{P_m(i)^{1/\alpha-1}}{[\int_0^j P_m(i)^{\alpha/\alpha-1}di + \int_0^j P_m(i^*)^{\alpha/\alpha-1}di^*]^{1/\alpha}} \cdot [\frac{Y(j)}{L_y(j)^{1-\beta}}]^{1/\beta} \tag{A.15}$$

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$$x(i^*) = \frac{P_m(i^*)^{1/\alpha-1}}{[\int_0^j P_m(i)^{\alpha/\alpha-1}di + \int_0^j P_m(i^*)^{\alpha/\alpha-1}di^*]^{1/\alpha}} \cdot [\frac{Y(j)}{L_y(j)^{1-\beta}}]^{1/\beta} \quad (A.16)$$

Substituting (A.15) and (A.16) into (A.9) and rearranging it, we get

$$wL_Y(j) = w^{1-\beta}(\frac{1-\beta}{\beta})^{\beta}[\int_0^j P_m(i)^{\alpha/\alpha-1}di + \int_0^j P_m(i^*)^{\alpha/\alpha-1}di^*]^{\frac{\alpha-1}{\alpha}\beta}Y(j) \quad (A.17)$$

Substituting (A.17) into (A.7), we get

$$C(j) = (1-\beta)^{-(1-\beta)}\beta^{-\beta}w^{1-\beta}[\int_0^j P_m(i)^{\alpha/\alpha-1}di + \int_0^j P_m(i^*)^{\alpha/\alpha-1}di^*]^{\frac{\alpha-1}{\alpha}\beta}Y(j)$$

Differentiating this with respect to $Y(j)$, we get the marginal cost function for the final goods manufacturing and perfect competition in this sector ensures the marginal cost pricing for products. Thus,

$$P(j) = \frac{\partial C(j)}{\partial Y(j)} = (1-\beta)^{-(1-\beta)}\beta^{-\beta}w^{1-\beta}[\int_0^j P_m(i)^{\alpha/\alpha-1}di + \int_0^j P_m(i^*)^{\alpha/\alpha-1}di^*]^{\frac{\alpha-1}{\alpha}\beta} \quad (A.18)$$

Substituting (A.17) into (A.15) and (A.16) and using (A.18), we get the demand for intermediate goods from the manufacturers of a final good j as

$$x(i) = \frac{P_m(i)^{1/\alpha-1}}{[\int_0^j P_m(i)^{\alpha/\alpha-1}di + \int_0^j P_m(i^*)^{\alpha/\alpha-1}di^*]} \cdot \beta P(j)Y(j) \tag{A.19}$$

and

$$x(i^*) = \frac{P_m(i^*)^{1/\alpha-1}}{[\int_0^j P_m(i)^{\alpha/\alpha-1}di + \int_0^j P_m(i^*)^{\alpha/\alpha-1}di^*]} \cdot \beta P(j)Y(j) \tag{A.20}$$

Substituting (A.19) and (A.20) in (A.9), we get the demand for labor from the manufacturers of a final good j as

$$L_Y(j) = \frac{1-\beta}{w} P(j)Y(j) \tag{A.21}$$

As the final good j is demanded both from the home and the foreign consumers, we have $Y(j) = C(j) + C^*(j)$ and $P(j)Y(j) = E(j) + E^*(j)$. As we normalize each country's expenditure level as $E = E^* = 1$, we have expenditure for each final product as $1/2n$ as in (1.6). Noting this in (A.19), (A.20), and (A.21), we get corresponding expressions in (1.9), (1.10), and (1.8) in the main text.

Appendix B

Chapter 2

A representative consumer in this economy maximizes the discounted utility in (2.2) subject to the dynamic budget constraint:

$$\dot{K} = rK + w_l L + w_h v H + P_A \dot{A} + A\pi - cL$$

and the differential equation in (2.1) that governs human capital accumulation. The Hamiltonian function for this problem is

$$\mathcal{H} \equiv e^{-\rho t}\frac{c^{1-\sigma}}{1-\sigma} + \mu_1[rK + w_l L + w_h v H + P_A \dot{A} + A\pi - cL] + \mu_2[\lambda H(1-v)]$$

The equation of motion for the first costate variable μ_1 is given by

$$-\dot{\mu}_1 = \frac{\partial \mathcal{H}}{\partial K} = \mu_1 r$$

Rearranging this, we get

$$\frac{\dot{\mu}_1}{\mu_1} = -r \tag{B.1}$$

The first order condition for the interior consumption path is

$$\frac{\partial \mathcal{H}}{\partial c} = e^{-\rho t} c^{-\sigma} - \mu_1 L = 0$$

Rearranging this, we get

$$e^{-\rho t} c^{-\sigma} = \mu_1 L \tag{B.2}$$

Differentiating (B.2) with respect to time, we get

$$-\rho e^{-\rho t} c^{-\sigma} - \sigma e^{-\rho t} c^{-\sigma-1} \dot{c} = \dot{\mu}_1 L + \mu_1 \dot{L} \tag{B.3}$$

Dividing (B.3) by (B.2), we get

$$-\rho - \sigma \frac{\dot{c}}{c} = \frac{\dot{\mu}_1}{\mu_1} + \frac{\dot{L}}{L}$$

Denoting $\frac{\dot{L}}{L} \equiv n$ as the population growth rate and using (B.1), we can write the above as in (2.3).

The equation of motion for the second costate variable, μ_2 is given as

$$-\dot{\mu}_2 = \frac{\partial \mathcal{H}}{\partial H} = \mu_1 w_h v + \mu_2 \lambda (1 - v) \tag{B.4}$$

The first order condition for an interior v is given as

$$\frac{\partial \mathcal{H}}{\partial v} = \mu_1 w_h H - \mu_2 \lambda H = 0$$

Rearranging, we get

$$\mu_1 = \frac{\lambda}{w_h} \mu_2 \tag{B.5}$$

Differentiating (B.5) with respect to time and dividing the result with (B.5) itself, we get

$$\frac{\dot{\mu}_1}{\mu_1} = \frac{\dot{\mu}_2}{\mu_2} - \frac{\dot{w}_h}{w_h} \tag{B.6}$$

Substituting (B.1) into (B.5), we get

$$-\frac{\dot{\mu}_2}{\mu_2} = r - \frac{\dot{w}_h}{w_h} \tag{B.7}$$

Substituting (B.5) into (B.4) and rearranging, we get

$$\lambda = -\frac{\dot{\mu}_2}{\mu_2}$$

Substituting (B.7) into above, we get the expression in (2.4).

Appendix C

Chapter 3

Let's denote

$$h \equiv (H_N - a_D g)^{1-\gamma}[\beta(H_N - a_D g)^\gamma + (1-\beta)L_N^\gamma] \tag{C.1}$$

$$l \equiv \frac{1-\alpha}{\alpha}(H_S - a_L g)[\beta + (1-\beta)(\frac{L_S}{H_S - a_L g})^\gamma] \tag{C.2}$$

and

$$f \equiv A^{1-\alpha}(\frac{B}{C})^\theta \tag{C.3}$$

where

$$A \equiv (\frac{H_S - a_L g}{H_N - a_D g}) \tag{C.4}$$

$$B \equiv \beta + (1-\beta)(\frac{L_S}{H_S - a_L g})^\gamma \tag{C.5}$$

$$C \equiv \beta + (1-\beta)(\frac{L_N}{H_N - a_D g})^\gamma \tag{C.6}$$

and

$$\theta \equiv \alpha(\frac{1-\alpha}{\alpha} - \frac{1-\gamma}{\gamma}) = 1 - \frac{\alpha}{\gamma} > 1$$

Then, we can rewrite the northern and the southern equilibrium conditions, (3.29) and (3.37), respectively, as

$$\sigma = \frac{(1-\alpha)h}{\alpha\beta a_D(\rho+g)-(1-\alpha)h} \tag{C.7}$$

and

$$l - a_L\beta(\rho+g) = a_L\beta(\rho+g)\frac{(a_D/a_L)\sigma^{1-\alpha}f}{1+\sigma} \tag{C.8}$$

C.1 Slopes of the NN and the SS Curves

Total differentiation of the northern equilibrium condition (C.7) gives

$$d\sigma = \sigma\left[\frac{\partial h/\partial g}{h} - \frac{\alpha\beta a_D - (1-\alpha)\partial h/\partial g}{\alpha\beta a_D(\rho+g)-(1-\alpha)h}\right]dg \tag{C.9}$$

Partial differentiation of h in (C.1) with respect to g gives

$$\frac{\partial h}{\partial g} = -a_D\beta - a_D(1-\gamma)(1-\beta)(H_N - a_D g)^{-\gamma}L_N^{\gamma} < 0$$

as $0 < \alpha$, $\beta < 1$ and $-1 \leq \gamma \leq 0$. Noting this in (C.9), we have

$$\left.\frac{d\sigma}{dg}\right|_{NN} < 0$$

as in the main text, and this means that the NN curve is downward sloping.

Total differentiation of the southern equilibrium condition (C.8) gives

$$\frac{\partial l/\partial g}{l}dg = \frac{1}{\rho+g}dg + \frac{\frac{a_D}{a_L}\sigma^{1-\alpha}\frac{\partial f}{\partial g}}{\frac{a_D}{a_L}\sigma^{1-\alpha}f+(1+\sigma)}dg$$

$$+\frac{(1-\alpha)\frac{a_D}{a_L}\sigma^{-\alpha}f}{\frac{a_D}{a_L}\sigma^{1-\alpha}f+(1+\sigma)}d\sigma - \frac{\frac{a_D}{a_L}\sigma^{1-\alpha}f}{(1+\sigma)[\frac{a_D}{a_L}\sigma^{1-\alpha}f+(1+\sigma)]}d\sigma$$

Rearranging terms, we have

$$\frac{\frac{a_D}{a_L}f[\sigma^{1-\alpha}-(1+\sigma)(1-\alpha)\sigma^{-\alpha}]}{(1+\sigma)[\frac{a_D}{a_L}\sigma^{1-\alpha}f+(1+\sigma)]}d\sigma =$$

$$[\frac{1}{\rho+g} + \frac{\frac{a_D}{a_L}\sigma^{1-\alpha}f}{\frac{a_D}{a_L}\sigma^{1-\alpha}f+(1+\sigma)}\frac{\partial f/\partial g}{f} - \frac{\partial l/\partial g}{l}]dg \qquad \text{(C.10)}$$

However, using (C.2)-(C.6), we have

$$\frac{\partial l/\partial g}{l} < 0 \qquad \text{(C.11)}$$

and

$$\frac{\partial f/\partial g}{f} = (1-\alpha)\frac{\partial A/\partial g}{A} + \theta[\frac{\partial B/\partial g}{B} - \frac{\partial C/\partial g}{C}] > 0 \qquad \text{(C.12)}$$

as

$$\frac{\partial A/\partial g}{A} > 0$$

and

$$\frac{\partial B/\partial g}{B} - \frac{\partial C/\partial g}{C} > 0$$

as long as $\frac{H_S}{a_L} > \frac{H_N}{a_D}$, $\frac{L_S}{a_L} > \frac{L_N}{a_D}$, and $-1 \leq \gamma \leq 0$. Noting (C.11) and (C.12) in the right hand side of (C.10), we can see that it is positive. Manipulating the term inside the bracket of the numerator in the left hand side of (C.10), we can see that it is positive when $\sigma > (1-\alpha)/\alpha$. Thus, assuming $\sigma > (1-\alpha)/\alpha$, $\frac{H_S}{a_L} > \frac{H_N}{a_D}$, $\frac{L_S}{a_L} > \frac{L_N}{a_D}$, and $-1 \leq \gamma \leq 0$, we get

$$\left.\frac{d\sigma}{dg}\right|_{SS} > 0$$

as in the main text, and this means that the SS curve is upward sloping.

C.2 Increases in Northern Human Capital ($H_N \uparrow$)

Holding g and L_N constant, total differentiation of both sides of (C.7) gives

$$d\sigma = \frac{\alpha(1-\alpha)\beta a_D(\rho+g)\frac{\partial h}{\partial H_N}}{[\alpha\beta a_D(\rho+g)-(1-\alpha)h]^2} dH_N$$

As partial differentiation of h in (C.1) with respect to H_N gives

$$\frac{\partial h}{\partial H_N} = \beta + (1-\beta)(1-\gamma)(H_N - a_D g)^{-\gamma} L_N^{\gamma} > 0$$

where the last inequality is assured by our assumptions $0 < \beta < 1$ and $-1 \leq \gamma \leq 0$, we get

$$\left.\frac{d\sigma}{dH_N}\right|_{NN} > 0$$

as in the main text.

Holding g, H_S, L_N, and L_S constant, total differentiation of (C.8) gives

$$\frac{\partial l}{\partial H_N}dH_N = a_L\beta(\rho+g)[\frac{\frac{a_D}{a_L}(1-\alpha)\sigma^{-\alpha}fd\sigma + \frac{a_D}{a_L}\sigma^{1-\alpha}\frac{\partial f}{\partial H_N}dH_N}{1+\sigma} - \frac{\frac{a_D}{a_L}\sigma^{1-\alpha}fd\sigma}{(1+\sigma)^2}]$$

Noting $\partial l/\partial H_N = 0$ from (C.2), dividing the above equation by (C.8), and rearranging terms, we get

$$[\frac{1}{1+\sigma} - \frac{1-\alpha}{\sigma}]d\sigma = \frac{\partial f/\partial H_N}{f}dH_N \qquad \text{(C.13)}$$

Using (C.3)-(C.6), we can also show that

$$\frac{\partial f/\partial H_N}{f} = (1-\alpha)\frac{\partial A/\partial H_N}{A} + \theta[\frac{\partial B/\partial H_N}{B} - \frac{\partial C/\partial H_N}{C}] < 0 \qquad \text{(C.14)}$$

as

$$\frac{\partial A/\partial H_N}{A} = -\frac{1}{H_N - a_D g} < 0$$

$$\frac{\partial B/\partial H_N}{B} = 0$$

and

$$\frac{\partial C/\partial H_N}{C} = -\frac{(1-\beta)\gamma(\frac{L_N}{H_N - a_D g})^\gamma \frac{1}{H_N - a_D g}}{\beta + (1-\beta)\gamma(\frac{L_N}{H_N - a_D g})^\gamma} > 0$$

Also the term in the bracket of the left hand side of (C.13) is positive under our earlier assumption, $\sigma > (1-\alpha)/\alpha$. Noting this along with (C.14) in (C.13), we have

$$\left.\frac{d\sigma}{dH_N}\right|_{SS} < 0$$

as in the main text.

C.3 Increases in Northern Unskilled Labor ($L_N \uparrow$)

Holding g and H_N constant, total differentiation of (C.7) gives

$$\frac{d\sigma}{dL_N} = \frac{(1-\alpha)\alpha\beta a_D(\rho+g)}{[\alpha\beta a_D(\rho+g)-(1-\alpha)h]^2}\frac{\partial h}{\partial L_N} \tag{C.15}$$

From (C.1), we also get

$$\frac{\partial h}{\partial L_N} = (1-\beta)\gamma(H_N - a_D g)^{1-\gamma}L_N^{\gamma-1} \leq 0$$

Noting this in (C.15), we get

$$\left.\frac{d\sigma}{dL_N}\right|_{NN} \leq 0$$

as in the main text.

Totally differentiating (C.8), dividing the result by (C.8), and noting $\partial l/\partial L_N = 0$ from (C.2), we get

$$[\frac{1}{1+\sigma} - \frac{1-\alpha}{\sigma}]d\sigma = \frac{\partial f/\partial L_N}{f}dL_N \tag{C.16}$$

As in the previous case, the term in the bracket of the left hand side is positive as long as $\sigma > \alpha/(1-\alpha)$. Also from (C.3)-(C.6),

$$\frac{\partial f/\partial L_N}{f} = (1-\alpha)\frac{\partial A/\partial L_N}{A} + \theta[\frac{\partial B/\partial L_N}{B} - \frac{\partial C/\partial L_N}{C}] \geq 0$$

as

$$\frac{\partial A}{\partial L_N} = 0$$

$$\frac{\partial B}{\partial L_N} = 0$$

and

$$\frac{\partial C}{\partial L_N} = (1-\beta)\gamma[\frac{L_N}{H_N - a_D g}]^{\gamma-1}\frac{1}{H_N - a_D g} \leq 0$$

Thus, from (C.16), we have

$$\left.\frac{d\sigma}{dL_N}\right|_{SS} \geq 0$$

as in the main text.

C.4 Increases in Southern Human Capital ($H_S \uparrow$)

Holding g, H_N, L_N, and L_S constant, totally differentiating (C.8), and dividing the result by (C.8), we get

$$[\frac{1}{1+\sigma} - \frac{1-\alpha}{\sigma}]d\sigma = [\frac{\partial f/\partial H_S}{f} - \frac{\partial l/\partial H_S}{l}\frac{l}{l - a_L\beta(\rho+g)}]dH_S \qquad (C.17)$$

As usual, terms in the left hand side bracket is positive. Using (C.3)-(C.6), we can show that terms in the right hand side bracket becomes

$$\frac{1}{H_S - a_L g}\{[(1-Q)-\alpha] - [(1-Q)\gamma - \alpha]\frac{(1-\beta)(\frac{L_S}{H_S - a_L g})^{\gamma}}{\beta + (1-\beta)(\frac{L_S}{H_S - a_L g})^{\gamma}}\} \tag{C.18}$$

where

$$Q \equiv \frac{l}{l - a_L\beta(\rho+g)} > 1$$

With $-1 < \gamma \leq 0$[1], the absolute value of the first bracket in (C.18) is larger than that of the second bracket. As the term in the first bracket is negative, we can say that (C.18) is negative. Noting this in (C.17), we can conclude that

$$\left.\frac{d\sigma}{dH_S}\right|_{SS} < 0$$

as in the main text.

C.5 Increases in Southern Unskilled Labor ($L_S \uparrow$)

Totally differentiating (C.8) and dividing the result by (C.8), we get

$$[\frac{1}{1+\sigma} - \frac{1-\alpha}{\sigma}]d\sigma = [\frac{\partial f/\partial L_S}{f} - \frac{\partial l/\partial L_S}{l}\frac{l}{l - a_L\beta(\rho+g)}]dL_S \tag{C.19}$$

[1]When $\gamma = -1$, absolute values of bracketed terms are equal. However, the fraction terms attached to the second bracket still guarantees that terms in (C.18) is negative.

As usual, the term in the left hand side bracket is positive. Using (C.3)-(C.6), we can show that terms in the right hand side bracket become

$$[\frac{\partial f/\partial L_S}{f} - \frac{\partial l/\partial L_S}{l}\frac{l}{l - a_L\beta(\rho+g)}] = (\theta - Q)\gamma\frac{(1-\beta)(\frac{L_S}{H_S - a_L g})^\gamma}{\beta + (1-\beta)(\frac{L_S}{H_S - a_L g})^\gamma}\frac{1}{L_S} \qquad \text{(C.20)}$$

Thus, the sign of (C.20) depends on the relative size of θ and Q. That is, if $\theta > Q$, we can conclude that (C.20) is negative and, noting this in (C.19), we can say that

$$\left.\frac{d\sigma}{dL_S}\right|_{SS} < 0 \qquad \text{if} \quad \theta > Q$$

as in the main text.

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