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ABSTRACT

ESSAYS ON THE REAL EFFECTS OF MONETARY SHOCKS IN CLOSED AND OPEN ECONOMY

By

Scott L. Baier

This dissertation investigates the effects of unanticipated increases in the money supply on real activity, and derives the optimal monetary policy for interdependent countries. Chapters 1 and 2 derive the optimal monetary policy in a two country two sluggish cash flow model under a variety of exchange rate regimes. It is shown that the monetary authorities can achieve the Pareto optimal allocations by providing liquidity to banks when the demand for loanable funds is high. Chapter 3 adds inventories and credit goods to the basic liquidity effects model. By doing so, the dynamics of the model are vastly improved. Chapter 4 examines the econometric work of Christiano, Eichenbaum, and Evans. It is argued that only one of their models yields credible results.

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Introduction

Textbook aggregate supply and demand models suggest a simple relationship between unanticipated increases in the money supply, interest rates, and real activity: an unanticipated increase in the money supply lowers interest rates, and the lower interest rates lead to increases in output. Recently there has been a plethora of theoretical and empirical work that supports this theory. The purpose of this dissertation is threefold: First the optimal monetary policy is derived for interdependent countries in the context of a general equilibrium optimizing model. Second to improve the dynamic response of the variables in the basic Fuerst-Christiano and Eichenbaum liquidity effects models by making the real side of the economy richer by including inventories and credit goods. Third, it is my intention to quantify the liquidity effect empirically using time series econometrics.

The recent theoretical and empirical work on the negative relationship between unanticipated monetary injections and interest rates is significant since theoretical models and empirical studies from the previous decade caste doubt on the existence of the liquidity effect. In particular, from a theoretical perspective, early attempts to include money in business cycle models led to the conclusion that monetary injections that were larger than anticipated drove interest rates up causing output to fall (see Greenwood and Huffman, 1987). It is easy to see why interest rates would rise: by decomposing the nominal interest rate into the sum of expected inflation and the real rate of interest, the expected inflation component rises with the larger than anticipated monetary innovation. If the real interest rate does not fall, then the nominal rate will rise. Econometric studies, such as, Melvin (1983) and Mishkin (1981a,1981b), argue that the time period the expected inflation component of the interest rate determination process dominated the liquidity effect.

More recently the theoretical work of Lucas (1990) and Fuerst (1992) and empirical work by Christiano, Eichenbaum, and Evans (1994), Gali (1992), and Leeper and Gordon(1992) argue that the liquidity effect is theoretically plausible and that there does appear to be support for this effect in the data. Theoretically, Lucas and Fuerst show that if agents make their savings decision before the state of the world is revealed, unanticipated monetary injections can have real effects. That is, it is possible that monetary innovations that exceed agents' expectations will drive down the interest rate and lead to an increase in output. Extensions of the Fuerst-Lucas model include work by Christiano and Eichenbaum (1992b, 1995), Schlagenhauf and Wrase(1995), Fuerst (1994), and Carlstrom and Fuerst (1995).

The empirical work of Christiano and Eichenbaum (1992a) show that innovations to nonborrowed reserves decrease interest rates and cause output to increase. Leeper and Gordon (1994) also find support for the existence of the liquidity effect. Gali (1992) using a structural VAR finds that liquidity effects do exist given his identifying restrictions.

This dissertation is two parts theoretical and two parts empirical. Chapter 0 provides a more detailed review of the literature, as well as a simple two country, two currency liquidity effects model. Chapter 1 investigates the optimal monetary policy in a two country single currency economy. Similarly, Chapter 2 investigates the optimal monetary policy in a two country, two currency model with fixed and flexible exchange rates. These two theoretical chapters demonstrate that unanticipated increases in the money supply lower interest rates and increase output and show how the shocks in one country affect the other country. Once the monetary transmission mechanism is understood, the optimal monetary policy is derived. It is shown that there exists a policy rule that attains the Pareto optimal allocations.

The next two chapters are empirical investigations of the monetary transmissions mechanism. One of the problems with the liquidity effects literature is the inability of monetary shocks to have persistent effects. Chapter 3 attempts to remedy this deficiency by incorporating inventories and credit consumption goods. Including these variables make the model more consistent with the recent empirical evidence than the existing liquidity effects models. In the model with inventories and credit goods, the interest rate stays below its steady state value for several periods after the monetary innovation. This is not a feature of the other liquidity effects models.

Chapter 4 is an empirical investigation of the effects of money supply shocks using time series econometric techniques. In this chapter, a seven variable VAR investigates the model used by Christiano, Eichenbaum, and Evans(1994a,b). Christiano, Eichenbaum, and Evans claim that unanticipated increases in the money supply caused by monetary authority have real effects along the lines outlined by Friedman (1975). The responses of variables to unanticipated increases in the money supply is investigated.

Chapter 0: Literature Review and Methodology

Section I. Literature Review

All the models analyzed in Chapters 1, 2, and 3 build on a monetized, general equilibrium, production economy with an endogenous labor supply decision. Also, there are stochastic shocks that affect the productive opportunities of the economy. Fuerst (1992) was the first to show that monetary innovations in these models when they incorporate "liquidity effects" can have real effects on output. The Fuerst model begins with the notion of a representative household. Within this household there are different agents--a worker, a saver, and a shopper for the consumption good. Each of these individuals is responsible for carrying out distinct tasks. At the start of each period, the household divides its money portfolio (m) and allocates money to the agents who will need the money to conduct their daily activities. In particular, the household allots a fixed amount of money (n) to the saver. Once the savings decision is made, the household cannot alter its decision. The saver deposits these funds into the financial intermediary. The remaining m-n dollars are available for the purchase consumption goods. The shopper, therefore, is constrained by a cash-in-advance constraint given by: $m-n \geq pc$, where p is the price of the consumption good, c. In addition to the funds deposited by the saver, the financial intermediary also receives a cash injection of x dollars from the monetary authority. Thus the financial intermediary has $n+x$ dollars to loan out. Who demands these funds? Since the goods producing firms do not have any funds at the start of the period, they must borrow from the financial intermediary in order to hire inputs. That is, the goods producing firms are required to use cash to hire workers and purchase capital for next period. If the monetary injection is larger than next period. If the monetary injection is larger than anticipated, then the amount of loanable funds held by the financial intermediary will be greater than anticipated. The increase in the supply of loanable funds will cause the price of the loanable funds to fall; that is, the interest rate will fall. As long as the nominal interest rate is nonnegative, the financial intermediary will lend out all of its cash holdings to the goods producing firms. The lower interest rates entice the firms to borrow all the cash in the financial intermediaries. The firms will use this excess cash to hire more workers and purchase more capital leading to increases in contemporaneous and future output. In the Fuerst model, both consumers and firms are recipients of the additional output.

Christiano (1991) showed that the Fuerst model, when calibrated to the United States economy, interest rates do not fall when monetary innovations exceed their anticipated values. To remedy this shortcoming Christiano proposed the "sluggish capital" model. In this formulation, investment decisions are made before the state of the world is revealed and cannot be changed ex-post. With this modification, Christiano shows that, when this model is calibrated, the interest rates fall when monetary injections exceed their anticipated values. However, even though interest rates fall, the rise in expected inflation results in consumption falling precipitously in subsequent periods. This contradicts the empirical findings of Sims (1991) that monetary injections lead to a hump-shaped response of output. Also counterfactual is the contemporaneous drop in prices that Christiano's model predicts. This is easy to see by looking at the cash-in-advance constraint. The price is determined by $p=(m-n)/c$ the numerator on the right hand side is a constant, but since consumption increases as output increases prices must fall. Thus a rise in consumption must imply that contemporaneous prices fall. This fact in also inconsistent with Sims' empirical findings.

Christiano and Eichenbaum (1992b) rectify the price deflation problem by making contemporaneous wage earnings available for the purchase of consumption goods. However, as shown in Chapter 3 the dynamics of the model are not consistent with the time series data. In the Christiano and Eichenbaum model, consumption increases in the period of the shock but in the subsequent period consumption falls drastically. Again this is at odds with Sims' findings that the dynamic response of consumption to monetary innovations is hump shaped.

Schlagenhauf and Wrase (1995) construct an open economy model with complete capital mobility. They show that positive monetary innovations by one country can lead to a reduction in interest rates and a depreciation of that country's currency. While these results of their calibrated model are consistent with their reported empirical findings, their model does not account for the variability of exchange rates in the time series data. In the next sub-section, a simple open economy liquidity effects model is developed and the response of the variables in the model to an unanticipated monetary injection is discussed.

Section II. A Simple Liquidity Effects Model

The models described above are based on the premise that innovations in the money supply affect different agents in the economy differently. To demonstrate the effects of the liquidity effects models employed in Chapter 1, 2 and 3, a simple two country, open economy model is analyzed below. I assume there exist two countries populated with many infinitely lived households who have identical preferences over two consumption goods c^1 and c^2. For reasons that will be made clear below, one country is called the blue country the other country the red country. Each household within each country begins the period with the same level of wealth. Wealth is held in the form of nominal money balances. At the start of the period, households hold only the currency of their respective country. Define M^R_t (M^B_t) to be the holdings of nominal money balances held by a household of the red (blue) country at the start of the period. Each period in the red (blue) country the gross growth rate of their currency is given by $1+x^R(s_t)$ $(1+x^B(s_t))$, where s_t is the state of the world at time t. I will return shortly to describe the process that generates the state of the world in any time period, but first the endowment process for each household is described.

Each household in the blue country is endowed with a perishable "raw material" $\xi(s_t)$. Similarly, each household in the red country is endowed with a perishable "raw material" $\eta(s_t)$. Notice that the endowment depends on the state of the world at time t. The raw material can instantaneously and costlessly be transformed into consumption good one or consumption good two by a linear production process. However, during the production process the consumption good becomes discolored. In particular, when $\xi(s_t)$ is used to make consumption good one it turns blue and when it is used to make consumption good two it turns red. Similarly, when $\eta(s_t)$ is used to make consumption good one it turns red and when $\eta(s_t)$ is used to make consumption good two it turns blue. Thus the technology for the production process for both countries is $\xi(s_t)=c^{1B}(s_t)+c^{2R}(s_t)$ and $\eta(s_t)=c^{1R}(s_t)+c^{2B}(s_t)$.

Households in the blue (red) country strictly prefer blue (red) consumption goods. Also, households in both countries like consumption good one better than consumption good two. Therefore, preferences of a household in the blue country are given by

$$E\{\Sigma_{t=0}^{\infty}\beta^t(\log(c^{1B}(s_t)) + \gamma\log(c^{2B}(s_t)\ \}$$

and of a household in the red country are given by

$$E\{\Sigma_{t=0}^{\infty}\beta^t(\log(c^{1R}(s_t)) + \gamma\log(c^{2R}(s_t))\},$$

where $0<\gamma<1$, and β is the discount factor. One can interpret this set-up as implying that consumption good one is preferred because domestically produced goods are preferred to foreign produced goods, but for good two there exists snob appeal to consuming the good produced abroad whereas domestically produced good two is viewed as being cheaply made or distasteful.

I now return to describing the state of the world in any time period. The state of the world at time t, s_t, is a 4x1 vector drawn randomly from a compact set $S\subset R^4$. The distribution function is given $\Phi(ds)$ and the distribution function is constant over time. The four shocks are the two monetary innovations in each country, and the endowments in each country respectively.

Since the timing of the actions of the agents in this model is the key to understanding the model, a discussion of when the agents make their decisions and what information they have when they make their decisions is necessary. Each household consists of three agents: a shopper of consumption good one, a shopper of consumption good two, and a goods trader. At the start of each period before the state of the world is revealed, the household in the blue (red) country allocates M^B_t-N^B_t (M^R_t-N^R_t) to the consumption of good one. By assumption if a household in the blue (red) country wants to buy a good produced in the red (blue) country, they must use the currency of the red (blue) country. Since households in the blue (red) country prefer consumption good two produced in the red (blue) country, there will be a demand for and supply of foreign currency.

After the money is allocated to consumption good one, the household separates and the state of the world is revealed. At this point, the monetary shocks are revealed. Once the household decides how much it wants to spend on consumption good one, it is prohibited from altering this decision. Thus after the state of the world is revealed, the households would most likely prefer to alter its choice of n, but this is prohibited by assumption. This captures the fact that it is costly for households to adjust their portfolio with every shock to the economy without explicitly modeling the costs. The shopper for good one takes M^i_t-N^i_t ($i=R,B$) to the market place to buy good one. The goods trader goes to the marketplace where his endowment is given to him. Because he intends to buy consumption good two that is produced in the other country, the third member (the shopper for consumption good two) carrying N^i_t ($i=R$ or B) units of currency must go to the market for foreign exchange. In the market for foreign exchange the agents from the blue (red) country receive a cash injection of $X^B(s_t)$ ($X^R(s_t)$), which gives them a total of $N^B_t+X^B(s_t)$ ($N^R_t+X^R(s_t)$) to exchange for the currency of the red (blue) country. Define $D(s_t)$ to be the price of foreign exchange in terms of how many units of the red country's currency it takes to buy a unit of the blue country's currency. Market clearing in the foreign exchange market implies that $D(s_t)(N^B_t+X^B(s_t))=(N^R_t+X^R(s_t))$. Once this exchange is made, the shopper for good two travels to the goods market. At the marketplace, traders are unable to discern which agents are from their household so all transactions must be carried out using cash.

Because the technology to transform the endowment $\xi(s_t)$ into either good one or good two is assumed to be linear, the price for either consumption good $c^{1B}(s_t)$ and $c^{2R}(s_t)$ must be the same, say $P(s_t)$. Likewise, the price $(Q_t(s_t))$ for either $c^{1R}(s_t)$ and $c^{2B}(s_t)$ must be the same. The cash-in-advance constraint for a household in the blue country then implies that; $M^B_t - N^B_t \geq P_t(s_t)c^{1B}(s_t)$, and $D(s_t)(N^B_t + X^B(s_t)) \geq Q(s_t)c^{2B}(s_t)$. The cash-in-advance constraints for a household in the red country are $M^R_t - N^R_t \geq Q(s_t)c^{1R}(s_t)$, and $(1/D(s_t))(N^R_t + X^R(s_t)) \geq P(s_t)c^{2R}(s_t)$. At the end of each period, the households reunite, consume their consumption goods and pool their resources. These pooled resources form the basis of next period's wealth. For a household in the blue country, wealth next period is given by

$$M^B_{t+1} = (M^B_t - P(s_t)c^{1B}(s_t) - (1/D(s_t))Q(s_t)c^{2B}(s_t) + X^B(s_t) + P(s_t)\xi(s_t)),$$

and for a household in the red country wealth next period is

$$M^R_{t+1} = (M^R_t - Q_t(s_t)c^{1R}(s_t) - D(s_t)P(s_t)c^{2R} + X^R(s_t) + Q(s_t)\eta(s_t)).$$

Now that all the agents in the economy and their daily routines have been described, equilibria are sought. Since all agents within each country are identical, the equilibria can be analyzed by the actions of a representative household and firm in each country. For simplicity in this section, I restrict the analysis to cases in which the cash-in-advance constraints bind. Also, because I am restricting the equilibria to stationary rational expectational equilibria, all growing variables must be detrended. The only growing variables in this model are the money stocks, so all nominal variables of a particular country can be made stationary by dividing by the time t money supply of that country. In particular, the normalized variables are:

$m^i_t = 1$ where $i = R$ or B,

$p_t = (P(s_t)/M^B_t)$, $q_t = (Q(s_t)/M^R_t)$,

$n^i_t = (N^i/M^i_t)$ where $i = R$ or B, $e_t = D(s_t)(M^R_t/M^B_t)$,

$x^i(s_t) = (X^i(s_t)/M^i(s_t))$ $i = R$ or B.

The household's problem can be written in terms of its dynamic program. At this point, all time subscripts will be dropped and primes are used to denote next period's values. With a value function given by $J(m^B)$, the household in country one solves

$$J(m^B) = \max_n \int \max_{cB} \{\log(c^{1B}(s)) + \gamma \log(c^{2B}(s)) + \beta J(m^{B\prime}))\Phi(ds)\}$$

where $cB = (c^{1B}(s), c^{2B}(s))$ is the vector of choice variables for the representative household in the blue country. The household's maximization problem is subject to the cash-in-advance constraints, $m^B - n^B \geq p(s)c^{1B}(s)$ and $n^B + x^B(s) \geq (1/e(s))q(s)c^{2B}(s)$. The transition equation is given by next period's money balances are given by

$$m^{B\prime} = [m^B - p(s)c^{1B}(s) - (1/e(s))q(s)c^{2B}(s) + x^B(s) - p(s)\xi(s)]/[1 + x^B(s)].$$

The first order conditions for the representative household in the blue country are

$$\int \lambda^1{}_1(s)\Phi(ds) = \int \lambda^2{}_1(s)\Phi(ds) \qquad 0.1$$

$$(1/c^{1B}(s)) = p(s)\{\lambda^1{}_1(s) + \beta J'(m^{B\prime})/(1 + x^B(s))\} \qquad 0.2$$

$$(\gamma/c^{2B}(s)) = (1/e(s))q(s)\{\lambda^2{}_1(s) + \beta J'(m^{B\prime})/(1 + x^B(s))\} \qquad 0.3$$

$$m^B - n^B \geq p(s)c^{1B}(s) \text{ with equality if } \lambda^1{}_1(s) \geq 0 \qquad 0.4$$

$$n^B + x^B(s) \geq (1/e(s))q(s)c^{2B}(s) \text{ with equality if } \lambda^2{}_1(s)\ ) \geq 0 \qquad 0.5$$

The envelope condition is given by

$$J'(m^{B\prime}) = \int [1/(p(s)c^{1B}(s))]\Phi(ds) \qquad 0.6.$$

The representative household in the red country solves the following problem

$$J(m^R)=\max_n \int \max_{c^R}\{\log(c^{1R}(s))+\gamma\log(c^{2R}(s))+\beta J(m^{R\prime})\}$$

where $c^R=(c^{1R}(s), c^{2R}(s))$ is the vector of choice variables for the representative household in the red country, subject to the cash-in-advance constraints

$m^R-n^R\geq q(s)c^{1R}(s)$ and $n^R+x^R(s)\geq e(s)p(s)c^{2R}(s)$.

Next period's money balances are given by

$$m^{R\prime}=[m^R-q(s)c^{1R}(s)-e(s)p(s)c^{2R}(s)+x^R(s)+q(s)\eta(s)]/[1+x^R(s)].$$

The first order conditions for the representative household in the red country are given by

$$\int\lambda^1{}_2(s)\Phi(ds)=\int\lambda^2{}_2(s)\Phi(ds) \qquad 0.7$$

$$(1/c^{1R}(s))=q(s)\{\lambda^1{}_2(s)+\beta J'(m^{R\prime})/(1+x^R(s))\} \qquad 0.8$$

$$(\gamma/c^{2R}(s))=p(s)e(s)\{\lambda^2{}_2(s)+\beta J'(m^{R\prime})/(1+x^R(s))\} \qquad 0.9$$

$$m^R-n^R\geq q(s)c^{1R}(s) \text{ with equality if } \lambda^1{}_2(s)\geq 0 \qquad 0.10$$

$$n^R+x^R(s)\geq e(s)p(s)c^{2R}(s) \text{ with equality if } \lambda^2{}_2(s)\geq 0 \qquad 0.11$$

The envelope condition is given by

$$J'(m^R)=\int[1/(q(s)c^{1R}(s))]\,\Phi(ds). \qquad 0.12$$

An equilibrium for this economy is found by solving for functions $p(s)$, $q(s)$, $c^{1B}(s)$, $c^{1R}(s)$, $c^{2B}(s)$, $c^{2R}(s)$, $\lambda^1{}_1(s)$, $\lambda^1{}_2(s)$, $\lambda^2{}_1(s)$, $\lambda^2{}_2(s)$, $e(s)$ and constants $J'(m^B)$, $J'(m^R)$, n^R, and n^B that solve eq. 0.1 - eq. 0.12 and the market clearing conditions. The market clearing conditions are

$$c^{1B}(s)+c^{2R}(s)=\xi(s) \qquad 0.13$$

$$c^{1R}(s)+c^{2B}(s)=\eta(s) \qquad 0.14$$

$$e(s)(n^B+x^B(s))=(n^R+x^R(s)) \qquad 0.15$$

and $m^B=1$, $m^R=1$. I now search for an equilibrium in which the cash-in-advance constraints bind. In order for the cash-in-advance constraints to bind, restrictions are placed on n^B, n^R and the money growth process. These restrictions also imply that the equilibrium, if one exists, is unique.

Assumption 0.1: Let $0<x^i<\gamma$ for $(i=1,2)$, and

$$\int\gamma\beta(1\text{-}\max(x))\{(\gamma(1+x(s))(1+\max(x)+\beta(\max(x)\text{-}x(s))\}^{-1}>1,$$

where max(x) is the largest possible monetary shock in the red country or the blue country.

Assumption 0.1 guarantees that the lagrange multiplier on the cash-in-advance constraint is positive.

Proposition 0.1: For $x^B(s)$ and $x^R(s)$ less than γ, there exists a unique equilibrium in which all constraints bind, households maximize utility and all markets clear.

Proof: The proof proceeds as follows: First, assuming the constraints bind, I show that there exist constants n^B and n^R that are unique. Then the constants $J'(m^R)$ and $J'(m^B)$ can be recovered, as well as the equilibrium prices and quantities. Finally I solve for the lagrange multipliers. If they are found to be positive the constraints are binding and existence is proved.

Imposing the cash-in-advance constraints, eq. 0.1 - eq. 0.3 imply

$$1=\int\{\gamma(1\text{-}n^B)/(n^B+x^B(s))\}\Phi(ds) \qquad 0.16$$

and, similarly, eq. 0.7 - eq 0.9 imply

$$1=\int\{\gamma(1-n^R)/(n^R+x^R(s))\}\Phi(ds). \qquad 0.17$$

Note that eq. 0.16 and eq. 0.17 are strictly decreasing in n^B and n^R respectively. Moreover, notice if $n^B(n^R)=1$ the right hand side of eq. 0.16 (0.17) equals zero and if n^B $(n^R)=0$ then the right hand side of eq. 0.16 (0.17) exceeds unity since $x^B(s)<\gamma$ $(x^R(s)<\gamma)$. Therefore, there exist unique $n^R\varepsilon(0,1)$ and $n^B\varepsilon(0,1)$ satisfying eq. 0.16 and 0.17 respectively.

Given these values for the n's the derivatives of the value function are easily recovered and are given by

$$J'(m^B)=(1/(1-n^B)), \text{ and}$$

$$J'(m^R)=(1/(1-n^R)).$$

Prices are given by

$$p(s)=(1+x^B(s))/\xi(s),$$

$$q(s)=(1+x^R(s))/\eta(s), \text{ and}$$

$$e(s)=(n^R+x^R(s))/(n^B+x^B(s)).$$

Equilibrium quantities are given by

$$c^{1B}(s)=[(1-n^B)/(1+x^B(s))]\xi(s),$$

$$c^{1R}(s)=[(1-n^R)/(1+x^R(s))]\eta(s),$$

$$c^{2B}(s)=[(n^R+x^R(s))/(1+x^R(s))]\eta(s), \text{and}$$

$$c^{2R}(s)=[(n^B+x^B(s))/(1+x^B(s))]\xi(s).$$

The shadow prices are then given by

$$\lambda^1{}_1(s)=[1-\beta/(1+x^B(s))]/(1-n) \qquad 0.18$$

$$\lambda^1{}_2(s)=[1-\beta/(1+x^R(s))]/(1-n) \qquad 0.19$$

$$\lambda^2{}_1(s)=\{\gamma/(n^B+x^B(s)) + [(1-n^B)(1+x^B(s))]^{-1}\} \qquad 0.20$$

$$\lambda^2{}_2(s)=\{\gamma/(n^R+x^B(s)) + [(1-n^R)(1+x^R(s))]^{-1}\} \qquad 0.21$$

Clearly, $\lambda^1{}_1(s)$ and $\lambda^1{}_2(s)$ are greater than zero since $x^B(s)$ and $x^R(s)$ are constrained to be nonnegative. Also, $\lambda^2{}_1(s)$ and $\lambda^2{}_2(s)$ are positive if and only if:

$n^B<\{(\gamma(1+\max x^B(s))-\beta(\max x^B(s)))/(\gamma(1+\max x^B(s))+\beta)\}$, and

$n^R<\{(\gamma(1+\max x^R(s))-\beta(\max x^R(s)))/(\gamma(1+\max x^R(s))+\beta)\}$, where $\max x^B(s)$ and $\max x^R(s)$ are the largest magnitudes of the monetary shocks. This holds by Assumption 0.1.

As one would expect, prices are positively related to monetary injections. The exchange rate is decreasing in monetary injections in the blue country and increasing in monetary injections in the red country. In particular, a larger than average increase in the money supply in the blue country leads to a decrease in the exchange rate. The lower exchange rate makes consumption good two cheaper for people in the red country since they pay e(s)p(s), which is decreasing in the monetary innovations in the blue country. Moreover, the consumption of good one becomes more expensive--p(s) is increasing in the monetary innovations in the blue country--for members of the blue country because of the higher prices. Thus there will be an increase in the demand for consumption good two by members of the red country and a decrease in the demand for good one by members of the blue country. Consequently, production will be altered by the trader of $\xi(s)$. This trader will shift the production schedule to making more consumption good two. Unexpected cash injections leading to changes in the consumption allocation and production are precisely the liquidity effect--monetary policy has real effects. Monetary injections/withdrawals that differ from their expected values will have affects on consumers' and producers' behavior that differ from what would have occurred with full contemporaneous information. Chapters 1, 2, and 3 employ the basic concepts used above to show how monetary innovations have real effects.

Chapter 1: Optimal Monetary Policy in a Two Country, One Currency Model

As countries have become increasingly more interdependent due to the liberalization of trade policies, policy makers have been considering the benefits to be gained by coordinating policies across countries. Perhaps the most frequently proposed coordinated policy is establishing an optimal currency region. The idea here is to have only one currency in circulation for a specific group of countries. Moreover, the supply of currency would be controlled by a single central bank. The European Union is a prime example of a region that is considering the adoption of a single currency to be used by the members of the union.

The benefits of the optimal currency area include the elimination of exchange rate uncertainty and a reduction in transactions costs involved with exchanging currencies. The main argument against the adoption of an optimal currency area is that the monetary authority can not react to country specific shocks. This loss of autonomy, it is argued, may make the outcomes in optimal currency regions less efficient than the established flexible exchange rate regimes.

This chapter is a theoretical investigation of the optimal monetary policy in a two country, one currency, liquidity effects model[1]. I assume that each country specializes in the production of a single good. The monetary authority injects currency into the financial intermediaries. It is first established that monetary innovations have real effects in both countries. Then given the fact that the actions taken by the monetary authority has real effects, the optimal monetary policy is derived. It is shown that the optimal monetary policy involves contracting the money supply on average at rate equal to the household's subjective rate of time preference. However, if the

[1] These models have been popularized by Lucas (1990), Fuerst (1992), and Christaino and Eichenbaum (1992b)

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countries are "more" ("less") productive than usual, the monetary authority should increase (decrease) the growth rate of the money supply above (below) the average contraction rate. This procyclical policy accommodates money demand shocks. The ability to attain the Pareto optimal allocation hinges on the monetary authority being able to contract the money supply on average. In addition to deriving the optimal monetary policy when the growth rate of the money supply is unconstrained, I also look at the best monetary policy in the case were the money growth rate is constrained to be positive on average. In this case, a procyclical policy is still preferred. That is, the monetary authority should provide liquidity when firms are most productive. Therefore, this paper shows that the monetary authority in a single currency region can attain the Pareto optimal allocations by simply responding to the "average" level of technology.

The rest of the chapter proceeds as follows: Section I provides an overview of the model. Section II describes the behavior of the agents. In Section III, a competitive equilibrium is derived and it shown to be unique. The qualitative features of the competitive equilibrium are also investigated. Finally, in this section, the optimal monetary policy is derived. In Section IV, the "best" policy is investigated when the money supply is constrained to be positive on average. Section V concludes.

Section I. The Model: an Overview

In this chapter, a two country, liquidity effects model with a single currency is investigated. It is shown that with identically distributed shocks that a competitive equilibrium exists where all constraints bind. A property of this equilibrium is that the interest rate is decreasing in monetary innovations, and employment is a decreasing function of the interest rates.

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Therefore, monetary injections that differ from agents' expectations will cause interest rates to differ from their fundamentals and have real effects. Since monetary innovations can have real effects, the optimal monetary policy is derived. It is shown that the role of the monetary authority is non-trivial. Indeed, the monetary authority, by responding to shocks, can make the competitive equilibrium Pareto optimal.

There exist two spatially separated, infinitely lived countries populated with three types of agents: households, goods producing firms, and a financial intermediary[2]. There are many identical infinitely lived households. It is assumed that wealth is distributed uniformly across households. These households have preferences over lifetime consumption and leisure given by

$$U(c^1_j,c^2_j,L_j)=\Sigma_{t=0}^{\infty}\{\beta^t(u(c^1_{jt},c^2_{jt}) + V(T-L_{jt}))\}$$

where c^i_{jt} is consumption of good i by household j at time t, L_{jt} is the amount of labor supplied at time t, T is the fixed time endowment each period[3]. Then $(T-L_{jt})$ is the amount of leisure at time t for household j ($j=1,2$). It is assumed that u(.,.) is twice continuously differentiable, increasing and concave. V(.) is also increasing and concave. When the competitive equilibrium and the optimal monetary policy are derived, the following functional form is imposed

$$U(c^1_j,c^2_j,L_j)=\Sigma_{t=0}^{\infty}\{\beta^t(\ln(c^1_{jt}(s)) + \ln(c^2_{jt}(s)) + (T-L_{jt}))\}. \qquad 1.1$$

This specification, along with the specification of the production function, permits closed form solutions. Thus, the economic interpretations are easily understood.

Since all households are identical, I will solve for equilibria in terms of a representative household in each country. Furthermore since households in country one and households in

[2] Allowing for a finical intermediary in each country would not change the results as long as the individuals could choose where to deposit their funds after the state of the world is revealed or if the banks could make inter-bank loans after the state of the world is known.

[3] It is assumed that $T>2$, so that the Pareto optimal labor supply is feasible.

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country two are ex-ante identical, attention is restricted to the description of a representative household and firm in country one. The reader should keep in mind similar constructs hold for households and firms in country two.

Section II. Behavior of Economic Agents

The Monetary and Fiscal Authorities

There exists a single monetary authority in this model[4]. The monetary authority injects x(s) units of currency into the financial intermediary. The monetary injection in this section is assumed to be a stochastic process. In particular, $x(s) \varepsilon X$ where x(s) is iid and $X \subset R_{+}$. When the optimal monetary policy is derived, the monetary injections are a function of the state of the world. The fiscal authority in this economy collects taxes (pays transfers) in order to keep wealth constant over time. Absent this tax transfer scheme, households' wealth would change from period to period due to the technology and monetary shocks--these shocks will be discussed in greater detail below. Households prefer this tax/transfer scheme because the variability in wealth would yield lower expected lifetime utility. It is shown below that average tax/transfer is zero. Also, each period the government maintains a balanced budget. That is, it is shown below that the taxes collected from one country are equal to the transfer payments in the other country. These taxes are a form of coinsurance as suggested by Ingram (1959). That is, there are transfers that shift funds from the region which has experienced a beneficial idiosyncratic shock to the less fortunate region.

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[4] Given the equal wealth distribution and that all agents are ex-ante identical, it is possible to think of this model as a model of fixed exchange rates, as well as a single currency area.

Households

What is important in these liquidity effects models is the timing of decisions relative to the realization of the shocks, and that some decisions cannot be altered ex post. In this model, the household, which consists of a worker, and a shopper for goods one and two, begins each period with M^1_t money balances carried over from the previous period. Before the state of the world is revealed, the household chooses how much of its money balances to place in savings (N_{1t}). Once this decision is made, it cannot be altered ex-post, and the worker-shopper pair separates. With full contemporaneous information, the worker offers labor services in the labor market[5] and the shopper purchases goods in the goods market. The shopper's purchases are constrained by the amount of cash that is brought into the trading session. The cash-in-advance constraints on goods purchases are given by

$$M^1_t - N_{1t} - Q_{1t} \geq P^1_t c^1_{1t}$$

$$Q_{1t} \geq P^2_{1t} c^2_{1t},$$

where Q_{1t} is the dollar allocation for consumption of good two, and P^i is the price of good i.

At the end of the period, the household reunites consumes the consumption goods and pools its resources. The pooled resources are denominated in home currency terms and this becomes the representative household's wealth in the subsequent period. More formally

$$M^1_{t+1} = \{(M^1_t - N_{1t} - P^1_t c^1_{1t} - P^2_t c^2_{1t}) + (N_{1t}(1+R^w) + W_{1t}L_{1t}) + (\Pi^{f1} + \Pi^B) + \tau_t M^1_t\}$$

The first term in parentheses is the residual from the cash-in-advance constraints. The second term in parentheses is the return on savings deposits and labor compensation. The third set of

[5]The worker in country one (two) offers her labor services to the representative firm in country one (two). By assumption, the worker in country one (two) can not work for the representative firm in country two (one).

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terms are the dividends the household receives from owning country one's (two's) good producing firm and dividends from owning one half of the international financial intermediary. Finally τ_t is the end of the period transfer (tax). This transfer (tax) which is paid (collected) by the government maintains a constant wealth distribution across households and across countries for all time periods. Households prefer this policy to a system without these transfers (taxes) because this transfer/tax scheme yields higher expected utility.

Once the nature of the shocks is described and all nominal variables are made stationary, it is possible to solve the household's problem. The shocks are independently and identically distributed random variables drawn from a compact set S. The shock at time t is given by $s_t \varepsilon R^k$ and the distribution function is given by $\Phi(ds)$. Since monetary injections cause the money supply to vary over time, nominal variables will also vary over time. Because I am only concerned with stationary rational expectational equilibria, it is necessary to rescale all nominal variables by the contemporaneous money stock to make them stationary. Let the money growth process be given by $(M^1_{t+1}/M^1_t)=(1+x(s_t))$, where $x(s_t)$ is the time t monetary injection. The stationary variables are denoted by lowercase letters. These variables are defined as follows

$p^1(s)=P^1_t/M^1_t$	$p^2(s)=P^2_t/M^1_t$	$w^1(s)=W^1_t/M^1_t$	$w^2(s)=W^2_t/M^1_t$
$q^1(s)=Q^1_t/M^1_t$	$q^2(s)=Q^2_t/M^1_t$	$n^1=N^1_t/M^1_t$	$n^2=N^2_t/M^1_t$
$x(s)=X_t/M^1_t$.			

From this point on, unless otherwise noted, time subscripts are dropped and primes are used to denote next period's values. Given the properties of the utility function and the fact that the constraint set is compact, the household's infinite horizon maximization problem can be written as a dynamic programming problem. The household's dynamic program is given by

$$J(m)=\max_n \int_s \max_{q,c,l}\{u(c^1{}_1(s),c^2{}_1(s)) + V(T-L(s)) + \beta J(m')\}\Phi(ds)$$

subject to the two cash-in-advance constraints

$$m^1\text{-}n\text{-}q(s)_1 \geq p^1(s)c^1{}_1(s) \qquad 1.2$$

$$q_1(s) \geq p^2(s)c^2{}_1(s) \qquad 1.3$$

and the transition equation

$$m^{1\prime}=(m^1\text{-}p^1(s)c^1{}_1(s)\text{-}p^2(s)c^2{}_1(s)+n^1R^w+w^1(s)L_1(s)+\pi^{f1}+\pi^{b1}+\tau^1)/(1+x(s)).$$

The first order conditions are

$$\int_s [J'(m)R^w/(1+x(s))]\Phi(ds) = \int_s \lambda_1(s)\Phi(ds) \qquad 1.4$$

$$\lambda_1(s) = \lambda_2(s) \qquad 1.5$$

$$u_1(c^1{}_1(s),c^2{}_1(s)) = p^1(s)\{\beta J_{m'}(m')/(1+x(s)) + \lambda_1(s)\} \qquad 1.6$$

$$u_2(c^1{}_1(s),c^2{}_1(s)) = p^2(s)\{\beta J_{m'}(m')/(1+x(s)) + \lambda_2(s)\} \qquad 1.7$$

$$V_1(1\text{-}L(s)) = \beta J_{m'}(m')w_1(s)/(1+x(s)) \qquad 1.8$$

$$m^1 - n_1 - q_1(s) \geq p^1(s)c^1{}_1(s) \text{ with equality if } \lambda_1(s)>0 \qquad 1.9$$

$$q_1(s) \geq p^2(s)c(s)^2{}_1(s) \text{ with equality if } \lambda_2(s)>0 \qquad 1.10$$

where the λ_i's are the multiplier's on the cash-in-advance constraints. The envelope condition is given by

$$J_m(m)=\int_s [u_1(c^1{}_1(s),c^2{}_1(s))/\ p^1(s)]\Phi(ds). \qquad 1.11$$

Before the firms' and the financial intermediary's problem is solved, I will discuss the first order conditions. In order to aid in the discussion and avoid confusion, I renormalize by the time t money supply and reintroduce time subscripts. Combining equations 1.5-1.7 yields

$$u_1(c^1{}_{1t},c^2{}_{1t})/u_2(c^1{}_{1t},c^2{}_{1t})=P^1{}_t/P^2{}_t.$$

Thus equations 1.5-1.7 give the standard result that at any point in time the marginal rate of substitution between goods is equal to the ratio of their prices. Next by combining the envelope condition and 1.8 yields the optimal labor supply decision is obtained. That is,

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$$V'(T-L_{1t})=E_t\beta\{u_1(c^1{}_{1t+1},c^2{}_{1t+1})W^1{}_t/P^1{}_{t+1}\}$$

According to the above expression, today's labor supply is a function of the ratio of the contemporaneous nominal wage to tomorrow's price level. This implies that today's labor supply depends on the contemporaneous wage and the expected price of good one next period. The reason for this is simple: today's wages are not available for current consumption only future consumption. This equation can be rewritten so that the labor supply curve can be drawn in real wage space. To do this multiply and divide the right hand side by $P^1{}_t$. This yields

$$V'(T-L_{1t})/E_t\beta\{u_1(c^1{}_{1t+1},c^2{}_{1t+1})P^1{}_t/P^1{}_{t+1}\}= W^1{}_t/P^1{}_t.$$

Note that the labor supply equation written this way implies that increases in expected inflation will cause the labor supply curve to shift to the left.

Combining equations 1.4, 1.6, and the envelope condition yields

$$E_{t-1}u_1(c^1{}_{1t},c^2{}_{1t})/P^1{}_t=E_{t-1}\{(1+R^w)\beta\{u_1(c^1{}_{1t+1},c^2{}_{1t+1})/P^1{}_{t+1}\}.$$

This is the household's intertemporal decision rule for savings and consumption. This intertemporal condition is interpreted as follows: before the realization of the current period's shocks, the representative household allocates funds to savings so that, in expectation, the marginal cost of allocating an additional dollar to savings is equal to the marginal benefit received next period. This decision rule captures the liquidity effect. Shocks that cause the actual valuation to differ from the representative household's expected valuation will cause the interest rate to differ from Fisherian fundamentals. Therefore, cash injections into the financial intermediary that exceed the household's expectations increase the supply of loanable funds. The increase in the supply of loanable funds will cause the price of loanable funds to fall; that is, the interest rate should fall. Traditional models of this nature only capture the expected inflation effect of the monetary innovations. That is, they predict interest rates will rise with monetary i

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innovations. In the liquidity effects models, if the liquidity effect dominates the expected inflation effect then interest rates will fall. Since the firms, by assumption, must finance their expenses at least in part by loans, the lower interest rates will lower the cost of inputs and firms may employ more inputs. If this is the case, the reduction in interest rates will lead to increases in real activity.

Firms

There exists one firm in each country which is endowed at time zero a with fixed supply of capital. Production is carried out by combining capital and labor supplied by households. The capital stock endowed to each country does not depreciate nor can it be augmented. There are two shocks that affect the production function directly. The first shock is a shock to total factor productivity ($\theta_i(s)$, $i=1,2$). It is assumed that $\theta_i(s)\varepsilon R_{++}$. The other shock that affects the firms directly is a marginal productivity shock ($\alpha_i(s)$, $i=1,2$). The marginal productivity shocks falls in the range (1/2,1).

Firms must pay their workers with cash. Since households and the financial intermediary are the only ones that hold cash, firms must borrow in order to finance their wage bill. By assumption, direct borrowing from households is ruled out, so firms must obtain their funds from the financial intermediary. Therefore, firms maximize profits subject to a borrowing constraint. Given this constraint the firm's problem is given by

$$\pi^{f1} = \max_H\{p^1(s)\theta_1(s)\,H_1(s)^{\alpha_1(s)} \;\; - w_1(s)H_1(s)-b(s)R^w\},$$

where $w_1(s)$ is the wage paid to a worker in country one (rescaled by the money supply in order to make it stationary), H_1 is the amount of labor employed, b is the level of borrowing to finance the

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the wage bill, R^w is the world interest rate, and $\theta_1(s)$, and $\alpha_1(s)$ are shocks to the firms production function. The profits of the firm are paid to the representative household in country one at the end of the period[6]. The first order condition for profit maximization is given by

$$\alpha_1(s)p^1(s)\theta_1(s)\,H_1(s)^{\alpha_1(s)-1} = w_1(s)(1+R^w).$$

The firm's labor demand schedule is given by this first order condition.

The Financial Intermediary

The financial intermediary receives n_1 dollars deposited by the representative household in country one and n_2 dollars deposited by the representative household in country two. The financial intermediary is also the sole recipient of monetary injections from the monetary authority. As long as the interest rate is nonnegative, the financial intermediary will lend out all of its cash balances. Its profits are given by

$$\pi^B = (n_1 + n_2 + x(s))(1+R^w) - (n_1 + n_2)(1+R^w),$$

where the first term is the return from lending funds to the firms for production and the second term is the amount the financial intermediary owes to the household for the deposits. Clearly, profits can be simplified to: $\pi^B = (1+R^w)x(s)$. These profits are divided in half and distributed to the representative households in country one and two at the end of the period.

[6]The profits for the firm operating in country two are distributed to the representative household in country two at the end of the period.

Section III. Equilibrium

Definition: A competitive equilibrium in this economy is a sequence of prices $\{p^1(s), p^2(s), (1+R^w), w^1(s), w^2(s)\}$, allocations $\{c^1{}_1(s), c^2{}_1(s), c^1{}_2(s), c^2{}_2(s), L_2(s), L_1(s)\}$ and constants $J_{m'}(m)$, n_1, n_2, such that

i) the prices and allocations solve the households' problem,

ii) the prices and allocations solve the firms' problem,

iii) the prices and allocations solve the financial intermediary's problem,

iv) the governments' budget constraints are balanced, and

v) all markets clear.

The market clearing conditions are:

$$c^1{}_1(s) + c^1{}_2(s) = \theta_1(s) H_1(s)^{\alpha_1(s)} ,$$

$$c^2{}_1(s) + c^2{}_2(s) = \theta_2(s) H_2(s)^{\alpha_2(s)} ,$$

$$L_1(s) = H_1(s),$$

$$L_2(s) = H_2(s),$$

$$n_1 + n_2 + x(s) \geq b^1(s) + b^2(s), \text{ and}$$

$$m = m' = 1.$$

In order to guarantee an interior solution, a restriction must be placed on the monetary shocks.

Assumption 1.1:

Assume that $x(s)\geq 0$, $\int\beta[(\alpha_2(s)(.5-q(s)) - \alpha_1(s)q(s))][(x(s)(1+(x(s)/2))]^{-1}\Phi(ds) > 1$

This assumption ensures that the households will deposit funds into the financial intermediary. If not, the interest rate offered by the intermediary will exceed the discount rate. Since there is no growth in the real factors, zero deposits cannot be optimal.

Given Assumption 1.1, it is easy to show that there exists a competitive equilibrium and it is unique. In a competitive equilibrium, all markets must clear. Given the initial wealth distribution the representative household in each country will have exactly one half the total money supply. Existence of the a competitive equilibrium is stated and proven below.

Proposition 1.1: Given Assumption 1.1, the functional form of the utility function defined in 1.1, then in the class of competitive equilibria in which all constraints bind, the competitive equilibrium exists and is unique.

Proof: The solution strategy is as follows: first show that there exist constants n_1 and n_2 satisfying the households' first order condition. Then given the solutions for the n_i's the prices and wages can be derived from the first-order conditions and the market clearing conditions.

First solutions to the constants n_1 and n_2 are found. Recall that a household in country one is isomorphic to a household in country two so that $n_1 = n_2$ and $q_1(s) = q_2(s)$. Also, by combining equations 1.5, 1.6, 1.7 by the symmetry of the model, and using the binding cash-in-advance constraints yield $q^1(s)=q^2(s)=(1/2\text{-}n)/2$. Also, exploiting the symmetry and given that the shocks are iid equation 1.4 can be rewritten as:

$$1 = \int_s \{\beta(\alpha_1(s)+\alpha_2(s))((1-2n)/4)\}\{(1+x(s))(n+x(s)/2)\}^{-1}\Phi(ds). \qquad 1.4'$$

Notice that the right-hand-side of this equation is strictly decreasing in n, and when n=.5 the right hand side must equal zero. Furthermore when n=0, the right hand sides exceeds unity (from Assumption 1.1). By continuity, there exists an n^*, such that $n^* \in (0, .5)$, that is, a fixed point of this mapping in the region of zero and one-half that solves eq. 1.4'. These values for the n's imply that $J_{m'}(m)=(4/(1\text{-}2n))$ for logarithmic preferences.

The equilibrium interest rate and prices are given by

$$p^1(s)=(1-2n)/2\theta_1(s)\,L_1(s)^{\alpha_1(s)}$$

$$p^2(s)=(1-2n)/2\theta_2(s)\,L_2(s)^{\alpha_2(s)}$$

$$(1+R^w)=\{(\alpha_1(s)+\alpha_2(s))(1\text{-}2n)\}/\{2(2n+x(s))\}. \qquad 1.15$$

Consumption allocations are given by

$$c^i_j(s)=(1/2)\theta_1(s)\,L_i(s)^{\alpha_i(s)} \quad i,j=1,2$$

The equilibrium wage rate and labor supply are given by

$$w_i(s)=(1+x(s))(1\text{-}2n)/4\beta \text{ for } i=1,2 \qquad 1.16\text{ a,b}$$

and letting $\Delta(s)=\{\alpha_1(s)/(\alpha_1(s)+\alpha_2(s))\}$ then

$$L_1(s)=\Delta(s)\{(n_1+n_2+x(s))\beta 4/(1\text{-}2n)(1+x(s))\}$$

$$L_2(s)=(1\text{-}\Delta(s))\{(n_1+n_2+x(s))\beta 4/(1\text{-}2n)(1+x(s))\}. \qquad 1.16\text{ c,d}$$

The transfers are given by

$$\tau^1=\{[((\alpha_1(s)\text{-}\alpha_2(s))((1\text{-}n)/2))] - ((1\text{-}2n)/4+(\Delta(s)(2n+x(s)) + (1+x(s))))\}$$

$$\tau^2=\{((\alpha_2(s)-\alpha_1(s)))((1-n)/2)) - ((1-2n)/4+(1-\Delta(s))(2n+x(s)) + (1+x(s))))\}$$

This completes the proof.

Proposition 1.2: Given these equilibrium values, monetary injections that exceed their expected values lead to a reduction in interest rates, an increase in equilibrium employment, and an increase in output.

Proof: The reduction in interest rates follows from differentiating equation 1.15 and the fact that n is constant. Similarly, the increase in equilibrium employment follows from differentiating equation 1.16c,d. Given the production function and the fact that output is monotonically increasing in the labor, output will increase with larger than anticipated monetary innovations. This completes the proof.

As shown above, the interest rate is decreasing in x(s) and the employment level in both countries is increasing in x(s). Therefore, a monetary innovation that exceeds the households' expectations will lead to a reduction in the interest rate, an increase in employment, and thus higher output. This implies that the monetary authority can potentially alter output. The next natural question to pose is: Is this competitive outcome Pareto optimal? If not, then can the monetary authority attain a Pareto improving allocation? It seems plausible that if actions of the monetary authority can have real effects, then the monetary authority can play an important role in this economy. There are two frictions which the monetary authority may attempt to mitigate. First and most obvious, since it is common to most cash-in-advance models, is that there is an inflation tax "levied" on consumption purchases and on the firm's labor bill. Common sense would lead one to believe that the way to eliminate this distortion is through a Friedman type deflation rule. In fact, the optimal monetary policy derived below is to contract, on average, the money supply at a rate equal to the household's subjective rate of time preference. The second distortion, and particular to the liquidity effects models, is the "sluggish" cash flow that prohibits the representative household from readjusting its portfolio after the state of the world is revealed. A monetary policy that can respond to the shocks and mitigate this second distortion is welfare improving.

It is shown below that the monetary authority can attain the Pareto optimal allocations in this stochastic environment by responding to the realizations of the stochastic variables. Notice from equations 1.15 and 1.16c,d that the marginal productivity shocks, the $\alpha_i(s)$'s, affect the interest rates and labor supply but shocks to total factor productivity do not. Accordingly the monetary authority will respond to the marginal productivity shocks, the $\alpha_i(s)$'s, but not the shocks to total factor productivity.

To illustrate the potential role of the monetary authority, first consider the case where the monetary authority does not respond to the productivity shocks, and follows a constant growth rate rule. (Figures 1.1-1.5 depict this analysis graphically.) That is, the monetary authority increases the money supply by x percent each period. Also, assume that country one has a larger than anticipated marginal productivity shock while the marginal productivity shock in country two is equal to the representative household's expectation. The marginal productivity shock in country one will shift country one's demand for loanable funds to the right (See Figure 1.1). Country two's demand for loanable funds will be unaffected (Figure 1.2). The increase in country one's demand for loanable funds causes the market demand for loanable funds to shift to the right (Figure 1.3). Since the supply of loanable funds is fixed the rightward shift in the demand for

The Demand for Loanable Funds:

Country One:

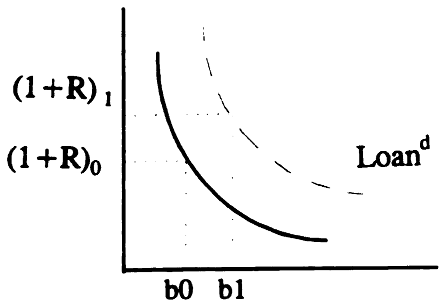

Figure 1.1

Country Two:

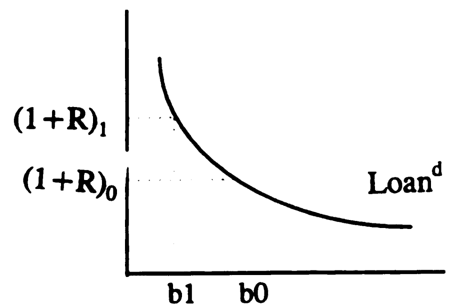

Figure 1.2

Market Demand for Loanable Funds:

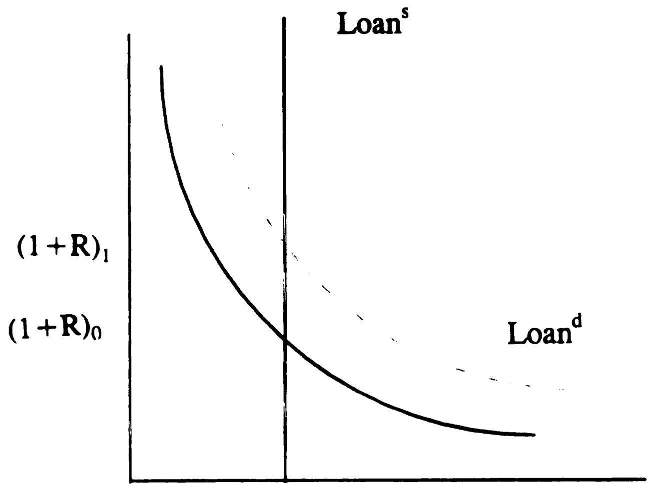

Figure 1.3

Loanable Funds

The Labor Market:

Country One:

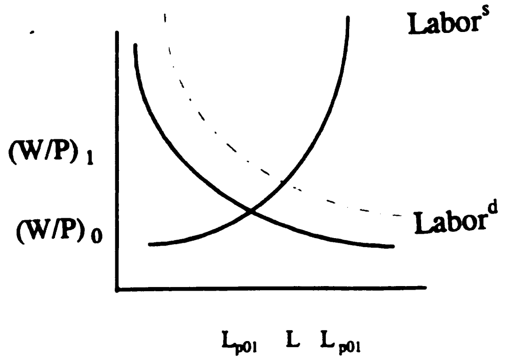

Figure 1.4

Country Two:

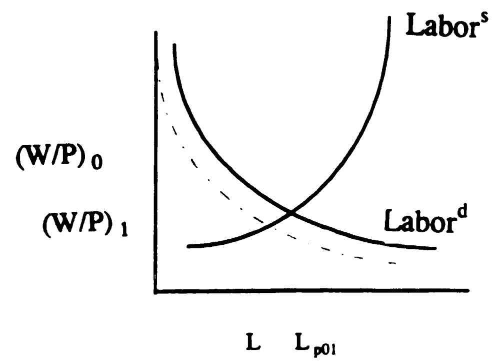

Figure 1.5

loanable funds will result in an increase in the interest rate. The higher interest rate implies that country one will get a larger portion of the loanable funds. To determine what happens to the equilibrium level of employment, recall that the firm's demand for employment is a decreasing function of the interest rate. (Figures 1.4 and 1.5 depict the labor supply and labor demand curves for countries one and two respectively. Also depicted are the Pareto optimal levels of employment when shocks are equal to their expected value (L_{po1}) and when country one's marginal productivity shock is larger than anticipated (L_{po2})).

The representative firm in country one's labor demand curve shifts to the right due to the larger than anticipated marginal productivity shock, while the increase in the interest rate shifts the labor demand curve to the left. The leftward shift arising from the increase in the interest rate is not enough to offset the rightward shift due to the marginal productivity shock. Therefore, is not enough to offset the rightward shift due to the marginal productivity shock. Therefore, equilibrium employment in country one will be greater than if the shock were equal to the households' expectation.

For the representative firm in country two, the higher interest rate shifts the labor demand curve to the left. Thus equilibrium employment in country two decreases. However, note that if the monetary authority would take an active role and increase the money supply in response to the shocks, the equilibrium interest rate would be lower than if the monetary authority followed a constant growth rate rule. The lower interest rate would allow for equilibrium employment to increase in both countries. If the monetary authority increases the money supply "just enough" the Pareto optimal allocations can be attained. Thus the role of the monetary authority is to find the Pareto optimal employment level and a money supply rule that ensures that the competitive equilibrium employment level is the Pareto optimal level of employment. To this end, the role of a fictitious social planner is postulated.

The social planner maximizes the welfare of both representative households. The only constraint the social planner faces is that consumption of goods cannot exceed production of the goods. More formally, the social planner places equal weight on each household's utility and chooses $c^1{}_1(s)$,$c^2{}_1(s)$, $c^1{}_2(s)$,and $c^2{}_2(s)$ to maximize the per period utility function given by

$$W(c_1,c_2,L_1,L_2)= u(c^1{}_1(s),c^2{}_1(s)) + V(T-L_1(s)) + u(c^1{}_2(s),c^2{}_2(s)) + V(1-L_2(s))$$

subject to the resource constraints;

$$c^1{}_1(s) + c^1{}_2(s)= \theta_1(s)\ L_1(s)^{\alpha_1(s)}\ ,\text{ and } c^2{}_1(s) + c^2{}_2(s)= \theta_2(s)\ L_2(s)^{\alpha_2(s)}$$

Carrying out this maximization, yields the following efficiency conditions,

$$V'(T-L_1(s))/u_1(c^1{}_i(s),c^2{}_i(s))=\alpha_1(s)\theta_1(s)\,L_1(s)^{\alpha_1(s)-1}\quad\text{for } i=1,2.$$

$$V'(T-L_2(s))/u_2(c^1{}_i(s),c^2{}_i(s))=\alpha_2(s)\theta_2(s)\ L_2(s)^{\alpha_2(s)-1}\quad \text{for } i=1,2.$$

Referring back to the consumer's problem, combining 1.6 and 1.8 yields

$$V'(T-L_i(s))/u_i(c^1{}_i(s),c^2{}_i(s))$$

$$=(w_1(s)/p^1(s))\{\beta J_{m'}(m')/(1+x(s))\}[\beta J_{m'}(m')/(1+x(s))+\lambda_2(s)]^{-1}\} \qquad 1.19.$$

Substituting for the $w_1(s)/p^1(s)$ from the firm's problem yields:

$$V'(T-L_1)/U_1(c^1{}_i(s),c^2{}_i(s))=$$

$$[\alpha_1(s)\theta_1(s)\,L_1(s)^{\alpha_1(s)-1}\ /(1+R^w)]\{(\beta J_{m'}(m')/(1+x(s)))[(\beta J_{m'}(m')/(1+x(s))+\lambda_2(s)]^{-1}\} \quad 1.19'.$$

It is obvious that equation 1.19' equals equation 1.18 if and only if $\lambda_2(s)=0$ and $R^w=0$ for all t. This implies that the cash-in-advance constraint cannot bind and the gross nominal interest rate should equal unity. Therefore, if the monetary authority can satiate households with cash balances and drive the nominal interest rate to unity, the monetary authority can improve upon the competitive allocation. Note also since the firm's output is constrained by how much money is held by the financial intermediary $(n_1+n_2+x(s))$, the monetary authority may be able to accommodate shocks to the production function. For example, suppose the production function experiences a positive technology shock, the firm's output is constrained by the amount it can borrow, it seems that the monetary authority could improve welfare by injecting cash into the financial intermediary. With a positive technology shock, the firm would like to borrow more than normal, but would be unable to do so unless the monetary authority responded to the shock. In order to accommodate firms, the monetary authority should increase its cash injection x(s).

Recall optimality implies that $\lambda^i{}_j(s)=0$ (for $j=1,2$) and that $(1+R^w)=1$ implies that the cash-in-advance constraints on the households and the firms "just" bind or do not bind. Thus the level of cash balances is indeterminate. This is true because the zero values on the lagrange multipliers imply that if M units of real cash balances satiates agents and an equilibrium is attained then clearly for any $\varepsilon > 0$, $M+\varepsilon$ units of real balances will also yield that equilibrium. A way around this problem is to assume that all the constraints "just" bind, however, this implies that system will be overidentified. Instead I assume, following Fuerst (1994), that the cash constraints are non-binding and the borrowing constraint just binds. This is similar to the real bills doctrine in which the monetary authority stands ready to alter the money supply to suit the needs of the borrowers.

Proposition 1.3: Given the above restrictions, and for the preferences given by equation 1.1, an equilibrium exists and is characterized by non-binding cash-in-advance constraints, a gross interest rate equal to unity, functions $p^i(s)$, $w^i(s)$, and constants n_i, and $J_m(m)$ that solve the social planner's problem.

Proof: Since the cash-in-advance constraints do not bind, prices with the logarithmic preferences are given by

$$p^1(s) = (1+x(s))/(c^1_i(s)\beta J_m(m)) \quad \text{and}$$

$$p^2(s) = (1+x(s))/(c^2_i(s)\,\beta J_m(m)).$$

By the symmetry of the households and using the goods market clearing condition, the optimal consumption allocation for each household is given by

$$c^j_i(s) = .5\theta_i(s)\,L_i(s)^{\alpha_i(s)}\ , \quad i,j=1,2$$

$$p^i(s) = 2(1+x(s))/(\beta J_{m'}(m)\theta_i(s)\,L_i(s)^{\alpha_i(s)}\ ), \quad i=1,2 \qquad 1.20\ \text{a,b}$$

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These prices can be substituted back into the envelope conditions for the representative household which yields

$$1=\int_s(\beta/(1+x(s)))\Phi(ds). \qquad 1.21$$

Equation 1.21 states that the inverse of the household's subjective rate of time preferences (β^{-1}) equals the inverse of the expected gross growth rate of money supply[7].

Wages are given by

$$w_1(s)=\{(1+x(s))/\beta J_m(m)\} \text{ and}$$

$$w_2(s)=\{(1+x(s))/\beta J_m(m)\}. \qquad 1.22\text{ a,b}$$

Recall Pareto optimal labor supply is given by

$$L_1(s)=2\alpha_1(s) \text{ and}$$

$$L_2(s)=2\alpha_2(s). \qquad 1.23\text{ a,b}$$

Substituting these values into the non-binding cash-in-advance constraint and the borrowing constraint places restrictions on the constants $J_m(m)$ and the n_i's. Thus the constraints are now given by

$$\beta J_m(m)/(1+x(s))\geq(.5\text{-}n/2)$$

$$\beta J_m(m)/(1+x(s))=2(\alpha_1(s)+\alpha_2(s)/((2n+x(s)))$$

The borrowing constraint can be rearranged and substituted back into eq. 1.21 to yield

$$J_{m'}(m)=\int\{2(\alpha_1(s)+\alpha_2(s))/(2n+x(s))\}\Phi(ds).$$

Next substitute into the non-binding cash-in-advance constraints for $\beta J_m(m)$ and rearranging yields

$$n\geq(.5(\alpha_1(s)+\alpha_2(s)\text{ - }x(s))/(2+\alpha_1(s)+\alpha_2(s)).$$

[7] This implies that agents expect the money supply to contract at a rate equal to the rate of time preference. This is just a stochastic version of the Friedman rule for the optimal quantity of money; that is, to contract the money supply equal to some real variable such as the real rate of interest or the subjective rate of time preference.

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Let α be the smallest $\alpha(s)$ in the distribution and x to be the largest x(s) in the distribution, and assuming the cash-in-advance constraint just binds at these values puts a restriction on the n's. The n's are given by

$$n^i = (\alpha - x)/(2(1+\alpha)) \quad i=1,2$$

Given these constants $J_{m'}(m)$ and n, rearranging the borrowing constraint gives the period by period money supply rule. That is, the money supply rule for any period is given by

$$x(s) = [\beta J_m(m)(2n-2(\alpha_1(s)+\alpha_2(s)))]/[2(\alpha_1(s)+\alpha_2(s))-\beta J_m(m)] \qquad \text{eq. } 1.24$$

This completes the proof.

Notice that $\partial x(s)/\partial \alpha_i(s) > 0$ which is what we would have suspected given the optimal labor supply in eq. 1.23 a,b. Thus the monetary authority will respond to shocks to the production function for high $\alpha_i(s)$'s by injecting cash into the system.

In general, the monetary authority expects to follow a policy given eq. 1.21; a monetary contraction at a rate equal to the subjective rate of time preference. The $\alpha_i(s)$'s deviating from the expected value will lead the monetary authority to deviate from this exact plan. By inspection of eq. 1.24 one may think that the monetary authority's hands are tied if both country's $\alpha_i(s)$'s change by equal magnitudes but in opposite directions. In this case the monetary authority should follow the Friedman deflation rule since the market mechanism channels the funds to the appropriate country. Furthermore, notice that the monetary authority does not respond to shocks to $\theta_i(s)$. This is not a failure of the model but rather due to the specification of preferences which yields that the Pareto optimal level of work effort is independent of the $\theta_i(s)$. With more general preferences the monetary authority does respond to $\theta_i(s)$ shocks to achieve the Pareto optimal allocations. Thus the lack of monetary independence does not inhibit the Pareto optimal allocations from being attained. That is, a single monetary authority can attain the Pareto optimal allocations.

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Section IV. Simulation Exercise

Since countries in general do not contract their money supplies, this section investigates the monetary policy when the money growth rate is constrained to be non-negative on average. Positive money growth rates may be required to generate a level of seigniorage. The idea in this section is to choose a money growth rate that corresponds to the realization of the shocks; that is, the monetary authority chooses the degree of correlation between money growth and the realization of the shock.

To keep matters simple, I restrict attention to two realizations of the α shocks. I assume α takes on the values .65 or .75 with equal probability. For this set of simulations I fix $\theta=2$ and $\beta=.95$. The experiment is to calculate how much a household would, in expectation, need to be compensated in terms of consumption goods to attain the expected Pareto optimal welfare for a given correlation between money growth rates and marginal productivity shocks. That is, I solve for Δc in the following problem:

$$E\{u(c^{1\,Po}_{i},c^{2\,Po}_{i})+(T-L_i^{Po})\}=E\{u(c^{1\,Ce}_{i}+\Delta c,c^{2\,Ce}_{i}+\Delta c)+(T-L_i^{Ce})\},$$

where Po stand for allocations in the Pareto optimal case and Ce stands for the allocations in the competitive equilibrium.

Figures 1.6 through 1.9 depict the welfare loss (Δc) for α stochastic. The welfare loss is plotted on the vertical axis and the money growth rate if the bad state ($\alpha(s)=.65$) occurs in both countries is plotted on the horizontal axis. The interpretation of this graph is as follows. Suppose the average money growth is set to Y percent, then if the bad state arises in both countries, the money

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money growth rate rule is to increase the money supply by X percent. The money growth rate rule in the good state (that is if $\alpha(s)=.75$ for both countries) is to increase the money supply by 2Y-X percent. Finally, if $\alpha(s)=.75$ for one country and $\alpha(s)=.65$ for the other country by default the monetary authority chooses to increase the money supply by its expected value (Y). This monetary rule guarantees the average growth rate of the money supply is Y percent. Thus a high x value implies a countercyclical policy. That is if the bad state occurs the growth rate of the money supply is high.

Before the simulations are analyzed, a few comments should be made about how the figures would appear in the absence of the portfolio rigidity with the only stochastic variable being the money growth process. For lack of a better label, this case is referred to as the base model. The graph for the base model would actually be symmetric and hump-shaped, which implies monetary variance is preferred. This is true because work effort in the competitive equilibrium is given by

$$V'(T-L_i(s))/u(c^1{}_i,c^2{}_i)= \alpha_1(s)\theta_i(s)\, L_i\,(s)^{\alpha_i(s)-1}\ /(1+R)^2,$$

and the interest rate is given by

$$(1+R)=[\int(\beta/(1+x(s))\Phi(ds)]^{-1}.$$

Recall in the previous section, it was shown that the Pareto optimal interest rate is unity. Therefore, in this model the closer the interest rate is to unity the closer the economy is to the Pareto optimal allocations and the lower Δc will be. The above equation for the interest rate implies that the larger the variance in the money growth process the lower interest rate will be. Therefore, the more variable the money growth process is the lower the interest rate. Higher

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money supply variance, therefore, would be preferred by households. In the simulations, I look for significant deviations from the base model.

As stated above, Figures 1.6 through 1.9 depict the welfare loss for the case where θ is fixed and α variable. Figure 1.6 shows the welfare loss when the shocks are iid and the average growth rate is 10 percent. The variable on the horizontal axis is the money growth rate if the bad state occurs in both countries. This implies that the growth rate of the money supply in case the good state occurs in both countries is .20-x, and the money growth rate will be 10 percent if a good state occurs in one country and a bad state in the other. This policy yields an expected money growth rate of 10 percent. As can be seen from the figure, the graph is hardly symmetric or hump shaped. In fact, the peak has shifted well to the right. Therefore, a procyclical policy is favored. Note the role of the monetary authority in this case is to smooth interest rates. This is easy to see. Recall the definition of interest rates:

$$(1+R)=(\alpha_1(s)+\alpha_2(s))(1-2n)[2(2n+x)]^{-1}.$$

Looking at the effect on interest rates from this procyclical policy, if the α shocks are high (low) then the above policy recommendation is to increase (decrease) the money growth rate. The large (small) α's would drive up (down) interest rates, while the increase in the money supply above (below) its expected value would counteract the upward (downward) pressure on interest rates.

Figure 1.7 depicts the welfare loss when the money growth rate is constrained to be 20 percent on average. In this figure, the graph is more "hump-shaped" but it is hardly symmetric. The implication is the same; a procyclical monetary policy is preferred.

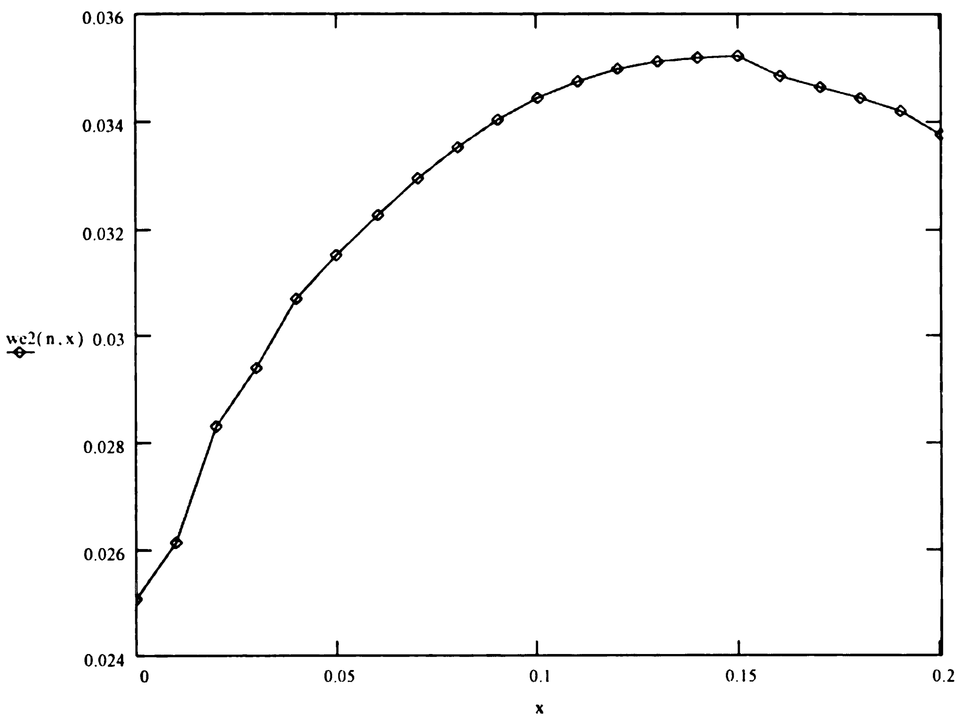
Figures 1.8 and 1.9 show the expected welfare loss when the shocks are correlated with each other but independent of all other factors. That is $\alpha_1(s)=\alpha_2(s)=.65$ or $\alpha_1(s)=\alpha_2(s)=.75$ with equal probability. Again the results are the same that a procyclical policy is preferred.

Conclusion

It was shown in this open economy liquidity effects model with a single currency used for exchange that under certain restrictions, there exists a competitive equilibrium in which all constraints bind and cash injections lead to a reduction in the interest rate and an increase in real activity. Furthermore, the competitive outcome could be improved upon by a procyclical monetary policy rule that smooths interest rates. More specifically, the role of the monetary authority is to satiate consumers with cash balances and, on average, contract the money supply at a rate equal to the subjective rate of time preference. Moreover, the monetary authority increases household's expected lifetime welfare in by responding to positive (negative) shocks to marginal productivity by decreasing (increasing) the cash contraction. These policy rules keep the interest rate at its Pareto optimal level--unity.

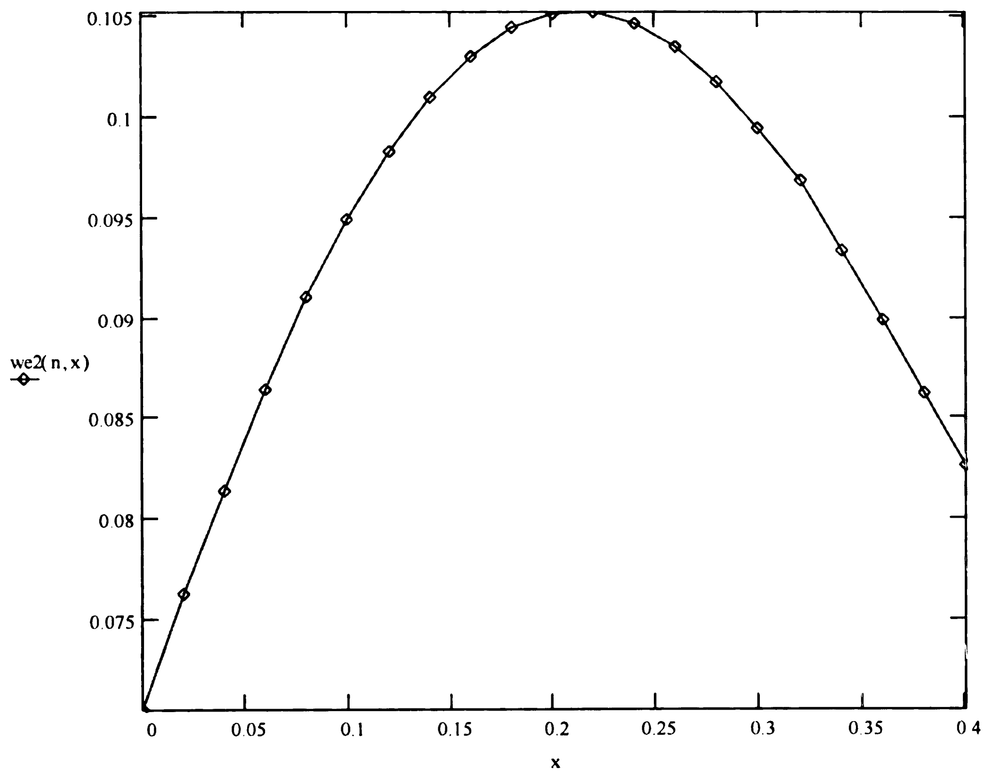
For policy advice, the monetary authority is not restricted by using only one currency. In this model, the monetary authority reacts to the average marginal productivity shock. Then the firms bid for the loanable funds. The more productive firm will receive the lion's share of the funds and thus the efficient allocation can be achieved. However, for this analysis to go through it must be the case that the monetary authority can respond quickly to shocks households can not respond to.

Figure 1.6: Welfare Loss When the Average Growth Rate of the Money Supply is 10 Percent and the Shocks are iid



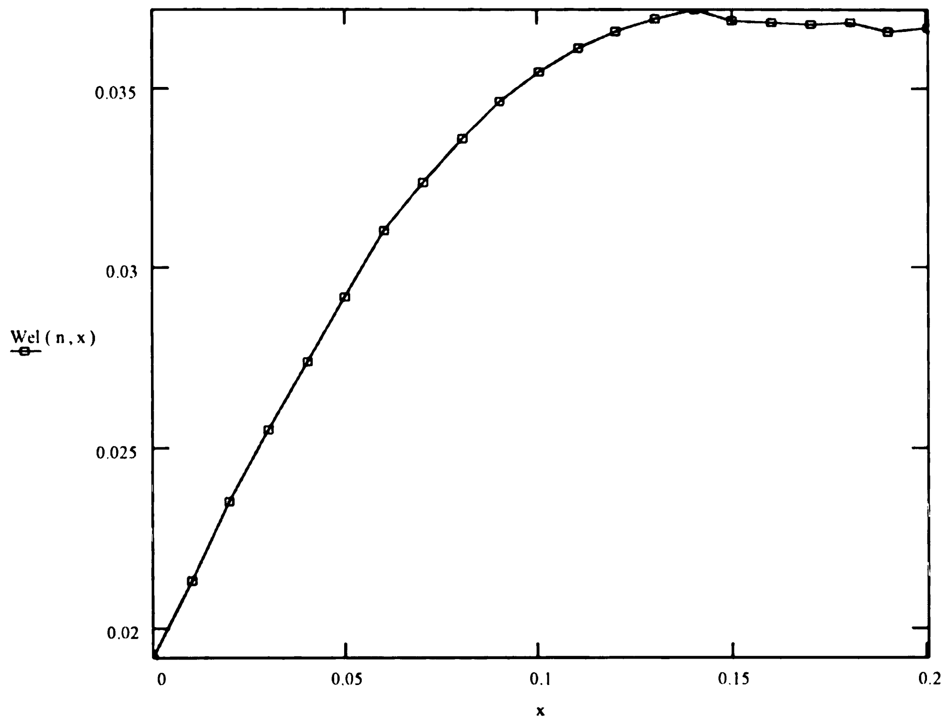
x is the growth rate of the money supply when the bad state occurs

Figure 1.7: Welfare Loss When the Average Growth Rate of the Money Supply is 20 Percent and the Shocks are iid



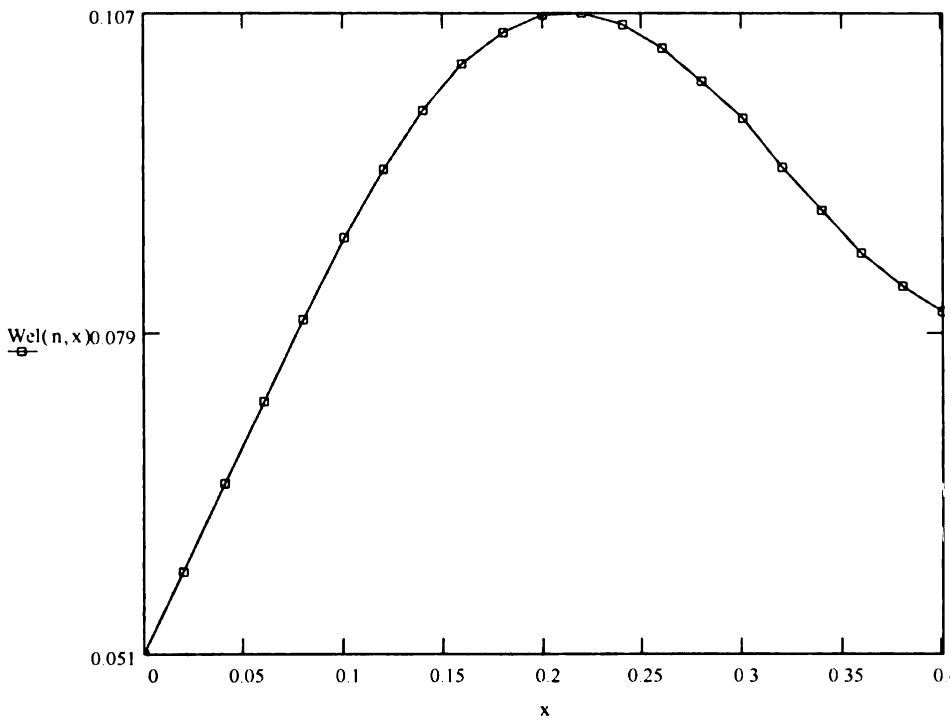
x is the growth rate of the money supply when the bad state occurs

Figure 1.8: Welfare Loss When the Average Growth Rate of the Money Supply is 10 Percent and the Shocks are Correlated



x is the growth rate of the money supply when the bad state occurs

Figure 1.9: Welfare Loss When the Average Growth Rate of the Money Supply is 20 Percent and the Shocks are Correlated



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Chapter 2: Optimal Monetary Policy in a Two Country, Two Currency Model with Fixed and Flexible Exchange Rates

In developed economies, where goods and assets are traded across borders, events that happen in one country generally affect the production possibilities and the welfare of agents in other countries. Therefore, the post World War II adage can be generalized to "when a developed economy sneezes the world is susceptible to a cold." Acknowledging the increased economic interdependence, some groups of countries have been considering the benefits of coordinating monetary and/or fiscal policies.

There are two primary decisions that need to be made in developing coordinated policies. The first is establishing optimal institutional arrangements. The second involves specifying the policy rules that the policy maker in each country will use to respond to shocks that affect the countries.

This chapter considers the optimal monetary policy arrangement among interdependent economies. The model employed is a two country, two currency sluggish cash flow model with fixed and flexible exchange rates[1]. Money is valued in this economy because households must use cash to purchase consumption goods and firms must use cash to pay their workers. I assume countries coordinate policies in order yield the highest level of welfare for all agents. I take this approach because there are several groups of countries that are attempting to coordinate their policies. Also, this model establishes a benchmark for models of non-cooperative behavior.

[1] The sluggish cash flow models have been popularized by Lucas(1990), Fuerst (1992,1994) and Christiano and Eichenbaum (1995).

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The questions this chapter provides answers to are: In an open economy setting, what are the effects of monetary innovations? What is the role of the monetary authorities? Finally, what is the best institutional arrangement for currency exchange--a flexible exchange rate regime or a fixed exchange rate regime?

To anticipate the results, brief answers to the questions posed above are: monetary innovations do have real effects. The role of the monetary authority is to smooth interest rates. Lastly, if the monetary authority is unconstrained in both the fixed and flexible exchange rate regimes, the monetary authority can attain the Pareto optimal allocations. However, if the money growth rate is constrained to be positive on average, then the fixed exchange rate regime is best.

The rest of the paper proceeds as follows. Section I sets up the model used throughout. Section II proves the existence of a competitive equilibrium with fixed and flexible exchange rates and discusses the qualitative features of the model. Section III derives the optimal monetary policies for the fixed and flexible exchange rate regimes. Section IV investigates which policy is best, a constant money growth rate rule or an interest rate target, if the monetary authority is constrained to keep the average money growth rate positive. Also in Section IV comparisons are made between the fixed exchange rate regime and the flexible exchange rate regime. Section V concludes.

Section I: The Model

There are two infinitely-lived countries populated with many infinitely-lived identical agents. There are three types of agents in this model: households, firms, and financial intermediaries. There also exists governments that collect taxes from (pay transfers to) the households in the economy, and monetary authorities that inject (withdraw) cash into (from) the financial intermediaries. Since there are numerous agents all of which behave competitively, I will restrict my attention to a representative agent of each type in each country.

Section I.1 The Representative Household

The representative household in each country consists of three members: a worker, a shopper, and a saver. Each member performs distinctive tasks. The shopper purchases the two consumption goods produced in this economy. Consumption good one is produced solely in country one and consumption good two is produced only in country two. The saver's role is to deposit the funds allocated to savings into the financial intermediaries. The worker in country i allocates time to the production of good i (i =1,2). Households have preferences defined over consumption and leisure. The representative household's lifetime utility function is given by

$$u = E_0\Sigma\{\beta^t(U(c^1_{it,}c^2_{it}) + V(T-L_{it}))\},$$

where c^j_{it} is consumption of good j by a household in country i at time t (i,j=1,2), T is the household's time endowment[2] and L_{it} is work effort by the worker in country i at time t, β is the discount factor ($\beta\varepsilon(0,1)$). It is assumed that U(.,.) is continuous, increasing and concave in both arguments, and V(.) is assumed to be continuous, increasing and concave. When the competitive equilibrium is shown to exist and when deriving the optimal monetary policies, the following utility function is employed:

[2] It is assumed that T >2 so that the Pareto optimal level of employment is feasible.

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$$u = E_0\Sigma\{\beta^t(\log(c^1_{it}) + \log(c^2_{it}) + (T-L_{it}))\},$$

where the expectations are over the realization of world wide shocks. This specification is used for two reasons. The first is that this utility function, along with the production function, allows for analytic solutions. The second is, given the closed form solutions, the general equilibrium effects of different shocks are easily understood and the optimal monetary policy rule that attains the highest welfare has a clear economic interpretation. The shocks in the model are all iid in nature and are discussed in greater detail below.

Since the timing of the decisions is important in this model, I will be very careful about describing the behavior of the agents and timing of their decisions. (The timing of the decisions and the realization of the shocks are listed chronologically on page 53.) The representative household in country i (i=1,2) begins each period holding only country i's currency and none of country j's (j≠i). At the start of each period, before the technology shocks and the money shocks are known, the household divides its currency holdings (M_{it}) between funds allocated to savings (N_{it}) and funds allocated to consumption purchases (M_{it}-N_{it}). Once made, this savings/consumption decision can not be altered.

After the savings decision is made the household separates. The worker offers her services in the labor market (L_{it}) (i =1,2). The shopper uses the funds allocated for consumption purchases to buy consumption goods one and two. It is assumed that in order for the shopper to purchase the consumption good produced in country i, he must be using the currency of country i (i=1,2). This framework, due to Lucas (1982), creates a transactions demand for foreign currency. Thus, a shopper in country one (two) takes D_{1t} (D_{2t}) units of currency to the foreign exchange market and buys foreign exchange at a price Z_t^{-1} (Z_t). The shopper for the representative household in country one (two) uses these funds to purchase consumption good two (one). The remaining funds allocated to consumption can be used to purchase consumption good one (two). The cash-in-advance constraints for the household in country one, therefore, are given by

$$M^1_t - N_{1t} - D_{1t} \geq P^1_t c^1_{1t} \text{ and } D_{1t} \geq Z_t P^2_t c^2_{1t},$$

where P^i is the price of the consumption good. Similarly for a household in country two, the cash-in-advance constraints are given by:

$$M^2_t - N_{2t} - D_{2t} \geq P^2_t c^2_{2t} \text{ and } D_{2t} \geq (1/Z_t) P^1_t c^1_{2t}.$$

The saver in country one (two) takes the N_{1t} (N_{2t}) dollars allocated to savings and goes to the foreign exchange market. There he uses Q^2_{1t} (Q^1_{2t}) units of currency to buy foreign exchange at a price Z_t^{-1} (Z_t). The $Z_t^{-1} Q^2_{1t}$ ($Z_t\, Q^1_{2t}$) units of foreign exchange are deposited in the representative financial intermediary in the foreign country. The remaining funds are then available to be deposited in the representative financial intermediary in the domestic country. This set up implies the cash constraint facing the saver in country one (two) is given by

$$N_{1t} \geq Q^1_{1t} + Q^2_{1t} \ (N_{1t} \geq Q^2_{2t} + Q^1_{2t}).$$

The gross nominal return for a dollar deposited in the financial intermediary in country one (two) is given by $1 + R^1_t$ ($1 + R^2_t$).

Note that in this model there are two reasons for a household to obtain foreign exchange: to purchase consumption goods--a transactions demand--and to deposit funds into the foreign country's financial intermediary--a financial demand. At the end of the period, the deposits by a household in country one (two) placed in the representative financial intermediary in country two (one) plus interest earned on the deposits are exchanged back into country one's (two's) currency at the end of period exchange rate given by $\hat{Z}_t$ ($1/\hat{Z}_t$). Also, any excess currency from the household one's (two's) cash-in-advance constraint for foreign

<u>Timing:</u>

1. Households decide how much to allocate to goods consumption and to savings.

STATE OF THE WORLD IS REVEALED

2. The saver allocates savings to the financial intermediaries in both countries. The shopper allocates its funds to consumption goods one and two.

3. Firms borrow funds, hire labor and produce the consumption goods.

4. The shopper purchases the consumption goods.

5. Firms repay their loans.

6. Households reunite and pool their resources. These resources include the return on savings, the residuals from the cash-in-advance constraints on consumption goods, wage income, profits from the domestic firms and intermediaries, and the tax/transfer payment.

produced goods is exchanged back into country one's (two's) currency at the end of period exchange rate $\hat{Z}_t$ $(1/\hat{Z}_t)$.

One of the salient features of this model is that even though the amount allocated to savings can not be altered, the portion of the household's savings placed in the financial intermediaries in countries one and two is at the discretion of the household and the placement decision is made with full contemporaneous information. I assume there is some degree of separability between the financial market and the goods market. That is, for a given level of risk, one can imagine that households will allocate their savings to the accounts that offer the highest return. Therefore, in this model I allow households to trade assets as new information concerning the returns on these assets becomes available. However, it is doubtful that households are willing to liquidate assets and transfer resources from the financial sector to the goods sector whenever laundry detergent goes on sale. Therefore, it is assumed that households cannot alter their portfolio decision that was made at the outset. In this model, the households allocate funds to the account that offers the highest return given the riskiness of the asset. Since the deposits are made after the state of the world is revealed, there is no risk and the households seek the highest return possible.

Thus far, there has been no need to delineate between the fixed exchange rate model and the flexible exchange rate model. The differences between models arise in how the exchange rate is determined. In the fixed exchange rate model the exchange rate is set at the beginning of the each period before the state of the world is revealed. Once determined, the monetary authorities are committed to exchange currency at the beginning of the period and at the end of the period at this predetermined rate. This implies that $Z_t = \hat{Z}_t$. Since the households begin each period with the same level of wealth and the shocks are iid across both countries, the logical choice for the fixed exchange rate is $Z_t = \hat{Z}_t = M^1_t / M^2_t$. In the flexible exchange rate model, the exchange rate is the price that equates the demand and supply for foreign currency. In this model, the foreign exchange market opens at the beginning and at the end of the period and the households' demands and supplies of foreign currency at this time determines the exchange rate.

The representative household in country i owns the goods producing firm and the financial intermediary in the country i (i=1,2). Also, the representative household pays (receives) a tax (transfer) from the government. This tax/transfer scheme is a form of a social insurance that insures the households against the idiosyncratic shocks. The tax/transfer scheme keeps wealth constant over time. In a no growth economy with iid shocks, this type of policy would be preferred by households since expected utility would be higher for constant wealth than for variable wealth. In addition the expected value of the tax/transfer is zero.

At the end of the period the representative household pools its resources, consumes it consumption goods, and enjoys its leisure. The pooled resources become the basis for next period's wealth. For the representative household in country one the pooled resources are given by

$$M^1_{t+1} = (M^1_t - P^1_t c^1_{1t} - Z_t P_t^2 c^2_{1t}) + (Q^1_{1t} R^1_t - Q^2_{1t} + Q^2_{1t}(\hat{Z}_t / Z_t)(1 + R^2_t)) +$$
$$(W^1_t L_{1t} + \Pi^{f1}_t + \Pi^{I1}_t + \tau^1_t M^1_t).$$

The first set of terms is the unspent cash from goods market transactions. The second set of terms is the return from deposits in the financial intermediaries. The final set of terms is the profits from the firm and the financial intermediary plus (minus) transfers (taxes).

Similarly, for the representative household in country two next period's wealth is given by

$$M^2_{t+1}=(M^2_t- P^2_tc^2_{2t}-(1/Z_t)P_t{}^1c^1_{2t})+ (Q^2_{2t}R^2_t-Q^1_{2t}(Z_t/\hat{Z}_t)+Q^1_{2t}(1+R_t))$$
$$+(W^2_tL_{2t}+\Pi^{f2}_t+\Pi^{I2}_t+\tau^2_tM^2_t).$$

Recall in the fixed exchange rate regime $Z_t=\hat{Z}_t=M^1_t/M^2_t$, whereas in the flexible exchange rate regime the end of the period exchange rate is not guaranteed to be equal to the beginning of the period exchange rate.

I am only concerned with stationary rational expectational equilibria. Therefore all growing variables must be detrended. The only growing variables in this model are the money stocks of each country. Lower case letters denote the detrended variables. The only exception is that the detrended beginning of the period exchange rate is denoted by e(s) and the end of the period exchange rate is denoted $\hat{e}(s)$. The detrended transition equation for the household in country one is given by

$$m^{1\prime}=(m^1-e(s)\,p^2(s)c^2{}_1(s)\,-p^1(s)c^1{}_1(s)+q^1{}_1(s)R^1(s)-q^2{}_1(s)+q^2{}_1(s)(1+R^2)(\hat{e}(s)/e(s))$$
$$+w^1(s)L_1(s)+\Pi^{f1}+\Pi^{I1}+\tau^1)/(1+x^1(s)), \qquad 2.1$$

where primes denote next period's values. The transition equation for the representative household in country two is given by

$$m^{2\prime}=(m^2-(1/e(s))p^1(s)c^1{}_2(s)-p^2(s)c^2{}_2(s))+q^2{}_2(s)R^2(s)-q^1{}_2(s)+q^1{}_2(s)(1+R^1)\,e(s)/\hat{e}(s)$$
$$+w^2(s)L_2(s)+\Pi^{f2}+\Pi^{I2}+\tau^2)/(1+x^2(s)). \qquad 2.2$$

Given the continuity and concavity of the utility function and the compactness of the constraint set, the household's problem can be cast in terms of a dynamic program. The programming problem for the household in country one is given by

$$J(m^1)=\max_{n\varepsilon[0,m]}\int \max_{c,L,q}\{U(c^1{}_1(s),c^2{}_1(s))+V(T-L_1(s))+\beta J(m^{1'})\}\Phi(ds)$$

where the maximization is subject to

$$m^1-n^1-d_1(s)\geq p^1(s)c^1{}_1(s),$$

$$d_1(s)\geq e(s)p^2(s)c^2{}_1(s),$$

$$n^1\geq q^1{}_1(s)+q^2{}_1(s).$$

The transition equation for $m^{1'}$ is given by eq 2.1. The first order conditions of the representative household in country one are given as follows

$$\int\lambda^1{}_1(s)\Phi(ds)=\int\lambda^3{}_1(s)\Phi(ds) \qquad 2.3.a$$

$$\lambda^2{}_1(s)=\lambda^1{}_1(s) \qquad 2.3.b$$

$$U_1(c^1{}_1(s),c^2{}_1(s))=p^1(s)\{\beta J'(m^{1'})/(1+x^1(s))+\lambda^1{}_1(s)\} \qquad 2.3.c$$

$$U_2(c^1{}_1(s),c^2{}_1(s))=e(s)p^2(s)\{\beta J'(m^{1'})(1+x^1(s))+\lambda^2{}_1(s)\} \qquad 2.3.d$$

$$V'(T-L_1(s))=\beta J'(m^{1'})w^1(s)/(1+x^1(s)) \qquad 2.3.e$$

$$\beta\{J'(m^{1'})/(1+x^1(s))\}R^1(s)=\lambda^3{}_1 \qquad 2.3.f$$

$$\beta\{J'(m^{1'})/(1+x^1(s))\}[(1+R^2)(\hat{e}(s)/e(s))-1]=\lambda^3{}_1(s) \qquad 2.3.g$$

$$m^1-n^1-d_1(s)\geq p^1(s)c^1{}_1(s) \text{ with equality if } \lambda^1{}_1(s)>0 \qquad 2.3.h$$

$$d_1(s)\geq e(s)p_2(s)c^2{}_1(s) \text{ with equality if } \lambda^2{}_1(s)>0 \qquad 2.3.i$$

$$n^1\geq q^1{}_1(s)+q^2{}_1(s) \text{ with equality if } \lambda^3{}_1(s)>0 \qquad 2.3.j$$

where $\lambda^1{}_1(s)$, $\lambda^2{}_1(s)$, and $\lambda^3{}_1(s)$ are the multipliers for the cash-in-advance constraint for good one, the cash-in-advance constraint for good two, and the cash-in-advance constraint on the savings allocation decisions respectively. The envelope condition is given by:

$$J'(m^{1'})=\int\{U(c^1{}_1(s),c^2{}_1(s))/p^1(s)\}\Phi(ds). \qquad 2.3.k$$

Similarly, the representative household in country two's maximization problem is given by

$$J(m^2)=\max_{n\varepsilon[0,m]}\int \max_{c,L,q}\{U(c^2{}_2(s),c^2{}_2(s))+V(T\text{-}L_2(s))\}+\beta J(m^{1\prime})\}\Phi(ds)$$

where the maximization is subject to

$$m^2\text{-}n^2\text{-}d_2(s)\geq p^2(s)c^2{}_2(s),$$

$$d_2(s)\geq e(s)^{-1}p^1(s)c^1{}_2(s),$$

$$n^2\geq q^2{}_2(s)+q^1{}_2(s).$$

The transition equation for $m^{2\prime}$ is given by eq. 2.2. The first order conditions of the representative household in country two are given as follows

$$\int\lambda^1{}_2(s)\Phi(ds)=\int\lambda^3{}_2(s)\Phi(ds) \quad 2.4.a$$

$$\lambda^1{}_2(s)=\lambda^2{}_2(s) \quad 2.4.b$$

$$U_1(c^1{}_2(s),c^2{}_2(s))=e(s)^{-1}p^1(s)\{\beta J'(m^{2\prime})/(1+x^2(s))+\lambda^1{}_2(s)\} \quad 2.4.c$$

$$U_2(c^1{}_2(s),c^2{}_2(s))=p^2(s)\{\beta J'(m^{2\prime})/(1+x^2(s))+\lambda^2{}_2(s)\} \quad 2.4.d$$

$$V'(T\text{-}L_2)=\beta J'(m^{2\prime})w^2(s)/(1+x^2(s)) \quad 2.4.e$$

$$\beta\{J'(m^{2\prime})/(1+x^2(s))\}(1+R^2(s)e(s)/\ \hat{e}(s)\text{ - }1)=\lambda^3{}_2 \quad 2.4.f$$

$$\beta\{J'(m^{2\prime})/(1+x^2(s))\}[(1+R^1)]=\lambda^3{}_2(s) \quad 2.4.g$$

$$m^2\text{-}n^2\text{- }d_2(s)\geq p^2(s)c^2{}_2(s)\text{ with equality if }\lambda^2{}_2(s)>0 \quad 2.4.h$$

$$d_2(s)\geq e(s)^{-1}p^1(s)c^1{}_2(s)\text{ with equality if }\lambda^1{}_2(s)>0 \quad 2.4.i$$

$$n^2\geq q^2{}_2(s)+q^1{}_2(s)\text{ with equality if }\lambda^3{}_2(s)>0 \quad 2.4.j$$

where $\lambda^1{}_2(s)$, $\lambda^2{}_2(s)$, and $\lambda^3{}_2(s)$ are the multipliers for the cash-in-advance constraint for good one, the cash-in-advance constraint for good two, and the cash-in-advance constraint on savings allocation decisions respectively. The envelope condition is given by

$$J'(m^{2\prime})=\int\{U(c^1{}_2(s),c^2{}_2(s))/p^2(s)\}\Phi(ds). \quad 2.4.k$$

These equations represent the optimality conditions of the representative households.

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Section I.2 The Representative Firms and the Representative Financial Intermediaries

The Firms

The representative firm operating in country one receives $p^1(s)$ for each unit of output sold. The representative firm has the production technology given by $\theta(s)H_1(s)^{\alpha(s)}$ where $H_1(s)$ is the amount of labor employed by the firm at time t, $\theta(s)\varepsilon\Theta$ where Θ is a continuous mapping from the abstract state space into the real line. Similarly, $\alpha(s)\varepsilon A$ where A is a continuous mapping from the abstract state space into (1/2,1). The wage rate is given by $w^1(s)$. Since the representative firm starts the period without any money balances, it must finance its wage bill by borrowing funds from the domestic financial intermediary. Its borrowings are restricted by the amount of loanable funds held at the domestic financial intermediary[3]. Total deposits at the intermediary are given by: $q^1{}_1(s)+e(s)q^1{}_2(s)+x^1(s)$. The first two terms are deposits by the households in country one and two respectively. The third term is the monetary injection that occurs through the financial intermediary. Since the representative firm must borrow to hire labor, the wage bill and hence employment at the firm is restricted by

$q^1{}_1(s)+e(s)q^1{}_2(s)+x^1(s)\geq w^1(s)H_1(s)$.

Therefore, the firm's problem is given by

$$\Pi^{f1}=\max_H\{p^1(s)\theta(s)H_1(s)^{\alpha(s)}-w^1(s)H_1(s)(1+R^1)\text{ subject to}$$

$$q^1{}_1(s)+e(s)q^1{}_2(s)+x^1(s)\geq w^1(s)H_1(s).$$

The representative firm in country two operates in a similar fashion. The production technology for the representative firm is given by: $\zeta(s)H(s)_2{}^{\gamma(s)}$ where $\zeta(s)\varepsilon\Theta$, and $\gamma(s)\varepsilon A$. The

[3] Financial intermediaries could be allowed to exchange deposits. Allowing for this would not change the results.

representative firm operating in country two obtains funds from the representative financial intermediary in country two. The firm's problem in country two is given by:

$$\Pi^{f2} = \max_H\{p^2(s)\theta(s)H_2(s)^{\gamma(s)} - w^2(s)H_2(s)(1+R^2)\} \text{ subject to}$$

$$q^2{}_2(s) + (1/e(s))q^2{}_1(s) + x^2(s) \geq w^2(s)H_2(s).$$

The Financial Intermediaries

The representative financial intermediary in country one receives deposits from the households in country one and country two and promises to pay them a gross nominal return of $(1+R^1)$ for each unit of currency deposited. Also, as aforementioned, the representative financial intermediary in country one is the sole recipient of country one's monetary injection. As long as the interest rate is non-negative, the intermediary will loan out all its funds to the firm. Profits for the financial intermediary in country one are given by

$$\Pi^{I1} = (q^1{}_1(s) + e(s)q^1{}_2(s) + x^1(s))(1+R^1) - (q^1{}_1(s) + e(s)q^1{}_2(s))(1+R^1).$$

The first term is the return from lending all the funds in the intermediary to the firms and the second term is the repayment of the principal plus interest to the representative households. Clearly, this can be simplified so that profits are given by

$$\Pi^{I1} = x^1(s)(1+R^1).$$

Similar reasoning implies profits for the representative financial intermediary in country two is given by:

$$\Pi^{I2} = x^2(s)(1+R^2).$$

Section I.3 The Monetary and Fiscal Authorities

Each period country one's (two's) monetary authority injects cash into the representative financial intermediary in country one (two) of the amount $x^1(s)$ $(x^2(s))$. In addition to the injections that occur through the financial intermediary, in the fixed exchange rate model, the monetary authorities also accommodate each household's demand for foreign exchange at the beginning and at the end of the period. In the flexible exchange rate model, however, the monetary authorities do not intervene in the foreign exchange market. In the section where the competitive equilibrium is derived and the qualitative features of the model are discussed, the monetary injections are assumed to be iid and $x^i(s) \varepsilon X$ where X is a continuous mapping from the abstract state space into the positive reals. In the section of the paper where the optimal monetary policies are derived, the monetary injections are a function of the state of the world at time t. That is, the monetary authorities increase (decrease) the money supplies after the households have made their savings decision and after the technology shocks are realized. This assumption is intended to capture the ease with which the monetary authorities can react to shocks in the economy.

The fiscal authorities in this economy collect taxes from (give transfers to) the households. The tax transfer scheme keeps the households' wealth constant for all time periods. Without this transfer scheme the households' wealth would vary over time. Households prefer this policy since the variability in wealth absent this scheme would yield lower expected lifetime utility. The governments in this economy maintain a "balanced" budget each period; that is, taxes collected from households in one country are equal to the transfers paid out to the households in the other country. Thus, the tax collections by country i's government are handed over to country j's government and distributed as transfers to the households. As aforementioned, this tax transfer scheme provides a form of social insurance that insures the households against the idiosyncratic shocks. Thus the governments provide a service--a form of income maintenance.[4]

Section II. Equilibrium

Now that all the actors in the economy have been described the next step is to prove the existence of a competitive equilibrium and describe the qualitative aspects of the models. In this section, I will search for a competitive equilibrium where all constraints bind. The qualitative features of the fixed and flexible exchange rate economies are then analyzed. I begin by defining the competitive equilibrium for these models.

Definition: A competitive equilibrium is a sequence of prices and allocations and constants, n^i, and $J'(m^i)$, such that, given these prices, allocations, and constants

(i) the households' problems are solved,

(ii) the firms' problems are solved,

(iii) the financial intermediaries' problems are solved,

(iv) arbitrage conditions on deposits are satisfied; that is,

$$(1+R^1)=(1+R^2) \text{ for the fixed exchange rate model}$$

$$(1+R^1)(e(s)/\ \hat{e}(s))=(1+R^2) \text{ for the flexible exchange rate model,}$$

[4] Examples of governments giving lump sum transfers is not uncommon. This transfer scheme is similar to coinsurance proposed by Ingram (1959) and others. In addition, the European Union has in several of its plans to coordinate policies have discussed similar coordinated fiscal policies as well.

(v) the joint government budget constraint balances, and

(vi) all markets clear.

There are six markets in the fixed exchange rate economy. The market clearing conditions are given by:

$$c^1{}_2(s)+c^1{}_1(s)=\theta(s)L_1(s)^{\alpha(s)},$$

$$c^2{}_1(s)+c^2{}_2(s)=\zeta(s)L_2(s)^{\gamma(s)},$$

$$L_i(s)=H_i(s) \quad i=1,2$$

$$m^i=m^{i\prime} \quad i=1,2$$

The flexible exchange rate model imposes two additional market clearing conditions that arise because the foreign exchange market opens at the beginning of the period and again at the end of the period. The market clearing conditions for foreign exchange are given by:

$$[d_1(s)+q^2{}_1(s)]=[d_2(s)+q^1{}_2(s)]e(s), \text{ and}$$

$$[d_2(s)\text{-}p^1(s)c^1{}_2(s)+q^1{}_2(s)(1+R^1)]=[d_1(s)\text{-}p^2(s)c^2{}_1(s)+q^2{}_1(s)(1+R^2)]\ \hat{e}(s).$$

Under the assumption of initial equal wealth distribution, iid shocks and the tax/transfer scheme the equilibrium will be a symmetric equilibrium. Existence of a unique competitive equilibrium for the fixed and flexible exchange rate regimes are proved separately in the following two subsections.

II.1 Competitive Equilibrium in the Fixed Exchange Rate Regime

Proposition 2.1: Assume the exchange rate is fixed. In the class of equilibria where all the constraints bind, there exists a unique competitive equilibrium.

<u>Proof:</u> See Appendix 2.1.

Given the symmetry of the model, it is shown in the appendix that the optimal savings decisions for the households in both countries are identical. Given this result the allocations are as follows

Consumption allocations: $c^1_j(s)=[\theta(s)L_1(s)^{\alpha(s)}]/2$ $c^2_j(s)=[\zeta(s)L_2(s)^{\gamma(s)}]/2$ $j=1,2$

Labor supply: $L_1(s)=\beta(\alpha(s)/(\alpha(s)+\gamma(s)))2(2n+x^1+x^2)/((1-n)(1+x^1))$,

$L_2(s)=\beta(\gamma(s)/(\alpha(s)+\gamma(s)))2(2n+x^1+x^2)/((1-n)(1+x^2))$,

prices: $p^1(s)=(1-n)/\theta(s)L_1(s)^{\alpha(s)}$,

$p^2(s)=(1-n)/\zeta(s)L_2(s)^{\gamma(s)}$,

$w^i(s)=(1+x^i(s))(1-n)/2\beta$, $i=1,2$

$(1+R^1)=(1+R^2)=[(\alpha(s)+\gamma(s))(1-n)]/(2n+x^1(s)+x^2(s))$,

deposits: $q^1{}_1(s)=q^1{}_2(s)=[2\alpha(s)n+\alpha(s)x^1(s)-\gamma(s)x^2(s)]/[2(\alpha(s)+\gamma(s))]$

$q^2{}_1(s)=q^2{}_2(s)=[2\gamma(s)n+\gamma(s)x^2(s)-\alpha(s)x^1(s)]/[2(\alpha(s)+\gamma(s))]$

taxes: $\tau^1(s)=1+x^1(s)-(\alpha(s)/(\alpha(s)+\gamma(s)))(2n+x^1(s)+x^2(s))-$

$(n+x^1(s))((\alpha(s)+\gamma(s))(1-n)/(2n+x^1(s)+x^2(s))-(1-\alpha(s))(1-n)$

$\tau^2(s)=1+x^2(s)-(\gamma(s)/(\alpha(s)+\gamma(s)))(2n+x^1(s)+x^2(s))-$

$(n+x^2(s))((\alpha(s)+\gamma(s))(1-n)/(2n+x^1(s)+x^2(s))-(1-\gamma(s))(1-n)$.

Note first that the interest rate is primarily determined by three factors: how productive the firms are (as measured by the marginal productivity shocks $\alpha(s)$ and $\gamma(s)$), the amount of currency circulating in the goods market, $(1-n)$, and the amount of loanable funds in country one and country two $(2n+x^1(s)+x^2(s))$. The intuition behind this result is simple: the interest rate is determined by the supply and the demand for loanable funds.

In the fixed exchange rate model, monetary injections in one country that are different than what households' anticipate will have spillover effects in the other country. This result is stated more formally in the following proposition.

Proposition 2.2. Monetary injections that are larger (less) than anticipated will cause interest rates in both countries to fall (rise), and cause output and employment in both countries to rise (fall).

Proof: The interest rate results follows from differentiating the interest rate with respect to $x^i(s)$ $(i=1,2)$. Similarly differentiating the equilibrium labor supply with respect to $x^i(s)$ $(i=1,2)$ gives the result that monetary innovations increase work effort.

Monetary injections that are larger than anticipated will lower the interest rates in both countries because they increase the supply of loanable funds. For instance, consider an unanticipated increase in country one's money supply. The increase in loanable funds shifts the supply of funds rightward, lowering the interest rates in country one. The interest rate in country two is initially higher than the interest rate in country one. However, since country two's interest rate is higher depositors will want to deposit more funds into the representative financial intermediary in country two. Deposits increase (decrease) in country two's (one's) financial intermediary until the rates of return are equated across countries. As a result, the increase in loanable funds decreases the interest rates in both countries. In response to the lower interest rates, firms will be able to hire more labor and output will increase.

Note also that larger than anticipated marginal productivity shocks will increase the interest rates. If the marginal productivity shocks are above (below) their mean values, firms will demand more (fewer) loanable funds and thus exert upward (downward) pressure on interest rates. Consider a marginal productivity shock in country one that is larger than anticipated. This shock increases the representative firm's demand for loanable funds. This effect increases the interest rates in country one. Since interest rates are higher in country one, depositors will allocate more funds to country one's financial intermediary and fewer funds to country two's financial intermediary. Therefore, the supply of loanable funds will increase in country one's representative financial intermediary and fall in country two's representative financial intermediary. The net effect is interest rates are higher than normal in both countries with country one receiving a higher percentage of the available loanable funds.

II.2 Competitive Equilibrium in the Flexible Exchange Rate Regime

Proposition 2.3: Assume the exchange rate is flexible in the class of equilibria where all the constraints bind, there exists a unique competitive equilibrium.

Proof: See Appendix 2.2.

It is shown in the appendix that the optimal savings decision for the households in both countries is identical. This implies the allocations and prices are given by:

Consumption: $c^1{}_j(s)=[\theta(s)L_1(s)^{\alpha(s)}]/2 \quad c^2{}_j(s)=[\zeta(s)L_2(s)^{\gamma(s)}]/2 \quad j=1,2$

Labor supply: $L_1(s)=2\beta[n+x^1(s)]/[(1+x^1(s))(1\text{-}n)]$,

$L_2(s)=2\beta[n+x^2(s)]/[(1+x^2(s))(1\text{-}n)]$,

Prices: $p^1(s)=(1\text{-}n)/[\theta(s)L_1(s)^{\alpha(s)}]$,

$p^2(s)=(1\text{-}n)/[\zeta(s)L_2(s)^{\gamma(s)}]$,

$(1+R^1)=\alpha(s)(1\text{-}n)/(n+x^1(s))$,

$(1+R^2)=\gamma(s)(1\text{-}n)/(n+x^2(s))$,

$e(s)=1$, and

$\hat{e}(s) = [\gamma(s)(n+x^1(s))]/[\alpha(s)(n+x^2(s))]$,

Taxes: $\tau^1=0$

$\tau^2=0$

The qualitative features of this model are given in the following proposition.

Proposition 2.4: Monetary injections of country one's (two's) currency that are greater than anticipated lower the interest rate in country one (two) and increase output in country one (two), there is no impact on interest rates and output in country two (one).

Proof: Follows immediately from differentiating the equilibrium conditions.

Note the nominal interest rate in country one (two) depends only on the discount factor and the money growth rate in country one (two). That is, the interest rate in country i is independent of the money growth rate in country j ($i \neq j$). This is similar to the result in Helpman and Razin (1981). There too the monetary growth rates in country j only affect the interest rate in country j. In their model, unanticipated monetary shocks have real effects because it changes the value of real debt. In this paper unanticipated monetary shocks have real effects due to the portfolio rigidity.

Therefore, because the interest rate in one country is unaffected by the money growth rate in the other country in the flexible exchange rate model, monetary innovations in one country do not have any real effects in the other country.The intuition behind this result is as follows: Consider again an unanticipated monetary injection by the monetary authority in country one, given the households' preferences, the representative household in country one will take the same amount of funds to the foreign exchange market as the representative household in country two. This implies the beginning of the period exchange rate will be unity. Then for a larger than anticipated monetary injection in country one, interest rates will fall. This alone would cause agents to deposit fewer funds in country one's financial intermediaries and more into country two's financial intermediaries. However, in this model the end of the period exchange rate adjusts to equate the return per dollar of savings across countries. Therefore, there is no incentive to alter the household's a priori deposit decision. Because of this, the supply of loanable funds will not increase in country two and employment in country two can not expand. Only country one absorbs the monetary innovation.

In the flexible exchange rate model monetary innovations only have real effects in the country where the innovation originated, whereas in the fixed exchange rate economy monetary innovations in one country have real effects in both countries. Intuitively, it would seem that the optimal monetary policy would involve coordination of the monetary authorities' activities in the fixed exchange rate economy while in the flexible exchange rate economy no coordination is necessary. The next two sections verify this.

Section III. Optimal Monetary Policy

In this section of the chapter, I search for the optimal monetary policy under the fixed and flexible exchange rate regimes. The optimal monetary policy for each regime is derived in two steps. First the social planner's problem is solved. Second it is shown that the social planner's problem can be supported by a competitive equilibrium. It is shown that in order to attain the Pareto optimal allocations, the monetary authority must react to the realization of the technology shocks and "undo" the rigidities that prevent the Pareto optimal allocations from being attained. There are two rigidities that the monetary authority needs to eliminate. The first is due to the cash-in-advance constraints, the second rigidity is due to the portfolio decision.

In both exchange rate regimes, the optimal monetary policy derived below is an interest rate peg. That is, the monetary authority reacts to shocks to keep the gross nominal interest rate at a fixed level--unity. Also, it is verified below that the optimal monetary policy in the fixed exchange rate regime does in fact require a coordinated policy. This is not the case for the model with flexible exchange rates. With flexible exchange rates, the optimal monetary policy requires that the monetary authority in country i should respond only to shocks in country i (i=1,2).

The first step in deriving the optimal monetary policy is to solve the social planner's problem. The social planner's problem is the same in both regimes. The planner's problem is to maximize the utility of each country's representative household given the resource constraints. The only constraints facing the social planner is that consumption allocations are feasible and that the time allotted to work effort is feasible. The solution to the problem includes the following efficiency conditions:

$$V'(T-L_1(s))/U_1(c^1{}_i(s),c^2{}_i(s))=\alpha(s)\theta(s)L_1(s)^{\alpha(s)-1} \qquad i=1,2 \qquad 2.5.a$$

$$V'(T-L_2(s))/U_2(c^1{}_i(s),c^2{}_i(s))=\gamma(s)\zeta(s)L_2(s)^{\gamma(s)-1} \qquad i=1,2 \qquad 2.5.b$$

The optimal monetary policy must be consistent with efficiency conditions 2.5.a,b.

II.1 An Optimal Monetary Policy for the Fixed Exchange Rate Regime

Comparing equation 2.5.a to the household's optimal decision rules, it is easy to verify that by combining equations 2.3.c, 2.3.e, 2.4.c, and 2.4.e and the first order conditions of the firm in country one that the combination of these equations is identical to the social planner's problem if and only if $\lambda^1{}_1(s)=0$ ($\lambda^1{}_2(s)=0$) and the gross nominal interest rate is equal to unity. Similarly, equation 2.5.b is identical to the decentralized competitive outcome if $\lambda^2{}_1(s)=0$ ($\lambda^2{}_2(s)=0$) and the nominal interest rate in country two is equal to unity. The $\lambda^i{}_j(s)$'s equaling zero imply that the cash-in-advance constraints are non-binding or are "just" binding for all states of the world; that is, households are satiated with cash balances. At this interest rate, the representative financial intermediary would still be willing to loan out all of their funds.

Before an optimal policy is derived, there are two technical details that must be addressed. First the restriction on the marginal productivity shocks ensures that the Pareto optimal work effort is feasible. Recall that $\alpha(s)$ and $\gamma(s)$ are restricted to be in the range (1/2,1). Given the logarithmic specification of the utility function and the social planner's efficiency conditions 2.5.a,b, the Pareto optimal work effort for country one (two) is given by $2\alpha(s)$ ($2\gamma(s)$). Therefore, the Pareto optimal work effort will always be feasible.

The second issue that needs to be addressed is the existence of multiple equilibria. This is easy to discern since the agents are satiated with cash balances in equilibrium. If M units of cash balances satiate the household then for any $\varepsilon > 0$ $M+\varepsilon$ units of cash balances also satiates the representative household. Also, because the nominal interest rate is unity the borrowing constraint does not bind or just binds. It is not possible to assume the cash-in-advance constraints and the borrowing constraints "just" bind since this would lead to an overidentified system. Thus the following solution strategy is pursued: I assume that the cash-in-advance constraints are non binding and the borrowing constraint "just" binds. Given this solution strategy the competitive equilibria are derived below. I show that the monetary authorities will inject (withdraw) funds into (from) the financial intermediary if the marginal productivity shocks are larger (less) than anticipated. This leads to the following proposition.

Proposition 2.5: In the fixed exchange rate economy, there exists a competitive equilibrium in which the monetary authorities can attain the Pareto optimal allocations.

Proof: The proof is by construction. First recall from the social planner's problem that the optimal labor supply for the household in country one (two) is $2\alpha(s)$ ($2\gamma(s)$). The rest of the proof proceeds as follows. The prices and allocations of the consumption goods are derived for both representative households. Then it is shown that the households expect the money supply to be contracted at a rate equal to the inverse of household's discount rate. The optimal portfolio decisions are then obtained; that is, the optimal $q^i_j(s)$'s $(i,j=1,2)$ are solved for. Then the deposits for the representative households can be obtained. Finally, the monetary policy rule is derived that ensures these Pareto optimal allocations are achieved.

Recall that if the Pareto optimal allocations are to be achieved, a necessary condition is that the cash-in-advance constraints do not bind. Rearranging 2.3.c and 2.4.c and using the market clearing condition for good one, the price for good one is (for now) given by

$$p^1(s)=[J'_2(1)(1+x^1(s))+J'_1(1)(1+x^2(s))1/e(s)][\beta J'_1(1)J'_2(1)\theta(s)L_1(s)^{\alpha(s)}]^{-1}.$$

Similarly, by combining equations 2.3.d, 2.4.d, and the market clearing conditions for good two, the price for good two is given by

$$p^2(s)=[J'_2(1)(1+x^1(s))+J'_1(1)(1+x^2(s))1/e(s)][1/e(s)\beta J'_1(1)J'_2(1)\zeta(s)L_2^{\gamma(s)}]^{-1}.$$

These prices imply that consumption allocations are given by

$$c^1{}_1(s)=\Lambda(s)\theta(s)L_1(s)^{\alpha(s)},$$

$$c^1{}_2(s)=(1-\Lambda(s))\theta(s)L_1(s)^{\alpha(s)},$$

$$c^2{}_1(s)=\Lambda(s)\zeta(s)L_2(s)^{\gamma(s)},$$

$$c^2{}_2(s)=(1-\Lambda(s))\zeta(s)L_2(s)^{\gamma(s)},$$

where $\Lambda(s)=[J'_1(1)(1+x^2(s))][J'_2(1)(1+x^1(s))+J'_1(1)(1+x^2(s))1/e(s)]^{-1}$.

These consumption allocations and prices can be substituted into the envelope condition to yield

$$1=\int(\beta/(1+x^i(s)))\Phi(ds) \quad i=1,2. \qquad 2.6.a,b$$

These conditions can be interpreted as stochastic versions of the optimal monetary policy rule given by Friedman (1969)[5].

Next, to find the optimal $q^i_j(s)$'s, recall that the representative firms in country one and country two first order conditions for profit maximization are given by

[5] Roughly defined, these equations say that the expected value of the money supply growth should equal the inverse of the household's discount factor.

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$$\alpha(s)p^1(s)\theta(s)L_1(s)^{\alpha(s)}=w^1(s)L_1(s) \quad \text{and} \quad \gamma(s)p^2(s)\zeta(s)L_2(s)^{\gamma(s)}=w^2(s)L_2(s)$$

respectively. Substitute the "just" binding borrowing constraint, the price of goods prices, the fixed exchange rate $e(s)=1$, and the fact that $n=q^1_j(s)+q^2_j(s)$ $(j=1,2)$, into the first order conditions. Then

$$q^1_j(s)=[2\alpha(s)n+\alpha(s)x^1(s)-\gamma(s)x^2(s)][2(\alpha(s)+\gamma(s))]^{-1},\ j=1,2$$

$$q^2_j(s)=[2\gamma(s)n-\alpha(s)x^1(s)+\gamma(s)x^2(s)][2(\alpha(s)+\gamma(s))]^{-1},\ j=1,2.$$

The nominal wage rate is given by

$$w^i(s)=(1+x^i(s))/[\beta J'_j(1)],\ \ j=1,2.$$

Next $J'(1)$ is solved for. To do so I use the wages, the $q^i_j(s)$'s, the equilibrium labor supply decision and substitute these into the just binding borrowing constraints to yield

$$2q^1_1(s)+x^1(s)=2\alpha(s)(1+x^1(s))/\beta J'_1(1),\ \text{and}$$

$$2q^2_2(s)+x^2(s)=2\gamma(s)(1+x^2(s))/\beta J'_2(1).$$

Solving for $\beta/(1+x^i(s))$ in the above equations and substituting into 2.6.a,b yields

$$J'_i(1)=\int[2(\gamma(s)+\alpha(s))/(2n+x^1(s)+x^2(s))]\Phi(ds).$$

Next to solve for the optimal n^i's. Recall 1/2 is the lower bound on the distribution of the $\alpha(s)$'s and $\gamma(s)$'s. Next define x to be the smallest monetary growth rate. These values are used to pin down the deposit decision. Substitute for $\beta J'(m)/(1+x^1(s))$ $(i=1,2)$ in the cash-in-advance constraint assuming the constraint just binds for the extreme values of the shocks. The n^i's are then given by

$$n^i=(1-x)/(1+x).$$

Rearranging the borrowing constraint and solving for the $x^i(s)$'s yields the optimal monetary policy rule given by

$$x^1(s)=x^2(s)=[\beta J'(1)n-\gamma(s)-\alpha(s)]/[\gamma(s)+\alpha(s)-\beta J'(1)]. \qquad 2.7$$

Since the $x^i(s)$'s and the $J'(1)$'s are the same prices and consumption allocations can be simplified to

$$c^1_j(s)=\theta(s)L_1(s)^{\alpha(s)}/2 \qquad c^2_j(s)=\zeta(s)L_2(s)^{\gamma(s)}/2 \quad j=1,2.$$

$$p^1(s)=2(1+x^1(s))/[\beta J'(1)\theta(s)L_1(s)^{\alpha(s)}],$$

$$p^2(s)=2(1+x^2(s))/[\beta J'(1)\zeta(s)L_2(s)^{\gamma(s)}].$$

This completes the proof.

Note the money supply rule given by 2.7 is increasing in the $\alpha(s)$ and $\gamma(s)$ shocks. This should not be too surprising since Proposition 2.2 showed that employment is an increasing function of positive monetary innovations and from the social planner's problem it was shown that the Pareto optimal work effort is increasing in the marginal productivity shocks. Note also that the monetary rules are the same for both countries. This should not be too surprising either, since Proposition 2.2 also showed that unanticipated monetary injections have the same effect on interest rates irrespective of the country from which they originated. Finally, note that the monetary policy rule is independent of the $\theta(s)$ and $\zeta(s)$ shocks. This is because the optimal labor supply is independent of these shocks. With a more general specification of the utility function the monetary rule would likely depend on these shocks. However, closed form solutions could not be obtained if the utility function was of the more general form of the constant relative risk aversion class. In the next section the optimal monetary policy for the flexible exchange rate regime is derived.

Section III.2: Optimal Monetary Policy in the Flexible Exchange Rate Regime

The solution strategy in this subsection is similar to the solution strategy in the previous subsection. The solution to the social planner's problem is given by 2.5.a,b. Again comparing these conditions to the representative households' conditions for optimality and the first order conditions for the firms, it is easy to verify that the representative household's efficiency condition is the same as the social planner's first order conditions if and only if $\lambda^i_j(s)=0$ (i,j=1,2) and $(1+R^i)=1$ (i=1,2). Therefore, the optimal monetary policy is one that keeps the interest rates at unity in both countries. Again, optimality implies the cash-in-advance constraints should be non-binding or "just" binding. This implies the same type of multiplicity problem as in the previous subsection. In order to deal with the multiplicity of equilibria in this case, it is assumed that the cash-in-advance constraint for consumption good one (two) purchased by the representative household in country one (two) does not bind. However, the cash-in-advance constraint for consumption good one (two) purchased by the representative household in country two (one) "just" binds. That is, households will take just enough money to the foreign exchange market so that the marginal value of a dollar used for the purchase of the foreign produced good is zero. In addition, after the foreign produced good is purchased the household has no foreign currency left over. Also, as in the previous section, it is assumed that the borrowing constraint just binds. This solution strategy leads to the following proposition.

<u>Proposition 2.6:</u> In the flexible exchange rate economy, there exists a competitive equilibrium in which the monetary authority can attain the Pareto optimal allocations.

<u>Proof:</u> The proof is again by construction. Recall from the social planner's problem that the optimal labor supply for the representative household in country one (two) is $2\alpha(s)$ ($2\gamma(s)$). Given these values the rest of the proof proceeds as follows. The prices of the consumption goods are derived. Then the amount allocated to the purchase of foreign exchange to purchase foreign produced consumption goods is derived. Then the beginning and the end of period exchange rates are solved for. Finally the optimal money supply rules are derived.

Recall from eq. 2.5.a,b that the multipliers on the cash-in-advance constraints are equal to zero. Using this fact and the market clearing conditions on consumption goods implies prices and allocations are given by

$$p^1(s)=[J'_2(1)(1+x^1(s))+J'_1(1)(1+x^2(s))(1/e(s))][\beta J'_1(1)J'_2(1)\theta(s)L_1(s)^{\alpha(s)}]^{-1}.$$

$$p^2(s)=[J'_2(1)(1+x^1(s))+J'_1(1)(1+x^2(s))(1/e(s))][1/e(s)\beta J'_1(1)J'_2(1)\zeta(s)L_2(s)^{\gamma(s)}]^{-1}$$

$$c^1{}_1(s)=\Lambda(s)\theta(s)L_1(s)^{\alpha(s)},$$

$$c^1{}_2(s)=(1-\Lambda(s))\theta(s)L_1(s)^{\alpha(s)},$$

$$c^2{}_1(s)=\Lambda(s)\zeta(s)L_2(s)^{\gamma(s)},$$

$$c^2{}_2(s)=(1-\Lambda(s))\zeta(s)L_2(s)^{\gamma(s)},$$

where $\Lambda(s)=[J'_1(1)(1+x^2(s))][J'_2(1)(1+x^1(s))+J'_1(1)(1+x^2(s))1/e(s)]^{-1}$.

These values imply

$$d_1(s)=(1+x^1(s))/[\beta J'_1(1)(1/e(s))],$$

$$d_2(s)=(1+x^2(s))(1/e(s))/[\beta J'_2(1)].$$

Using the arbitrage condition $e(s)/\hat{e}(s)\,(1+R^1)=(1+R^2)$ and the market clearing condition for end of the period exchange rate, implies that $q^1{}_2(s)=e(s)q^2{}_1(s)$. Then the beginning and the end of period exchange rates are given by

$$e(s)=[(1+x^1(s))J'_1(1)]/[J'^2(1)(1+x^2(s)], \text{ and}$$

$$\hat{e}(s)=e(s)^{-1}.$$

The consumption allocations and goods prices, therefore, can be simplified to

$$c^1{}_i(s)=[\theta(s)L_1{}^{\alpha(s)}]/2, \qquad i=1,2$$

$$c^2{}_i(s)=[\zeta(s)L_2{}^{\gamma(s)}]/2 \qquad i=1,2$$

$$p^1(s)=2(1+x^1(s))/(\beta J'_1(1)\theta(s)L_1(s)^{\alpha(s)}), \text{ and}$$

$$p^2(s)=2(1+x^2(s))/(\beta J'_2(1)\zeta(s)L_2(s)^{\gamma(s)}).$$

Wages are given by

$$w^1(s)=(1+x^1(s))/(\beta J'_1(1)), \text{ and}$$

$$w^2(s)=(1+x^2(s))/(\beta J'_2(1)).$$

Substituting the values for the $p^i(s)$'s and the $c^i{}_j(s)$'s into the envelope condition again yield the stochastic version of the Friedman rule, that is

$$1=\int(\beta/(1+x^i(s)))\Phi(ds) \quad i=1,2.$$

Given these wages and prices and the relationship of the $q^i{}_j(s)$'s, the just binding borrowing constraint can be rearranged and substituted into the above equations to yield:

$$J'_1(1)=\int[2\alpha(s)/(n+x^1(s))]\Phi(ds), \text{ and}$$

$$J'_2(1)=\int[2\gamma(s)/(n+x^2(s))]\Phi(ds).$$

Recalling that unity is the upper bound on the distribution of the $\alpha(s)$'s and $\gamma(s)$'s, and x be the smallest monetary growth rate. The n^i's are then given by

$$n^i=(1-x)/(1+x).$$

The optimal monetary policy rule is obtained from rearranging the just binding borrowing constraint from the firms' first order condition to yield

$x^1(s)=[2\alpha(s)-n\beta J'_1(1)]/[\beta J'_1(1)-2\alpha(s)]$, and 2.8.a

$x^2(s)=[2\gamma(s)-n\beta J'_2(1)]/[\beta J'_2(1)-2\gamma(s)]$. 2.8.b

This completes the proof.

Unlike the fixed exchange rate rule where the policies need to be coordinated to attain the Pareto optimal allocations, the monetary policies in this model are independent of each other. In particular, the money supply rule is simple: the monetary authority in country one (two) should alter country one's (two's) money supply in response to the marginal productivity shocks in country one (two). The money supply is increasing in the marginal productivity shocks. This is so because the Pareto optimal labor supply is increasing in the marginal productivity shocks, and recall from Proposition 2.4 that the equilibrium labor supply is increasing in larger than anticipated monetary injections. Thus for larger than anticipated marginal productivity shocks the monetary authority will increase the money supply in order to increase employment to the Pareto optimal level. The results in the previous sub-section and this sub-section are similar in some respects to the Mundell-Flemming model in that coordination of policies is needed for fixed exchange rates regimes whereas for flexible exchange rates the monetary authority in country i can tend to shocks in country i (i=1,2) alone.

Section IV: Policies in a Constrained Environment

It was shown that in both the fixed and the flexible exchange rate regimes, the monetary authority can attain the Pareto optimal allocations. This requires that the money supplies should on average be contracted at a rate along the lines suggested by Friedman (1969). In the last section both exchange rate regimes--the fixed and the flexible exchange rate--are equally good since the monetary authority can in both regimes attain the Pareto optimal allocations.

This section considers what is the best monetary policy, an interest rate peg or a constant growth rate rule, when the money growth rate is constrained to be positive on average. This may be the case if a certain amount of revenue must be raised by seigniorage to finance a portion of government spending. The choice of monetary policies is investigated under the fixed and flexible exchange rate regimes. Also, when the money growth rate is constrained to be non-negative on average the Pareto optimal allocations cannot be attained and the allocations in the fixed exchange rate and flexible exchange rate regimes will likely have different welfare properties. Because of this, comparisons can be made between the two regimes.

Two welfare measures are used to compare the alternative policies. The first measure is calculated by finding the expected per-period utility of the representative household under the growth rate rule and the interest rate rule. Whichever policy yields the higher expected utility will be preferred. The second method used is defined as the percent increase in consumption required so that the expected level of utility level in the competitive equilibrium where the money growth is constrained to be positive on average is equal to the expected utility level in the Pareto optimal environment. That is in the following equation Δc is solved for

$$E\{u(c^1_i(s)+\Delta c, c^2_i(s)+\Delta c)+V(T-L_i)\}=E\{u(c^{1PO}_i, c^{2PO}_i)+V(T-L^{PO})\}.$$

The welfare loss is given by $\Delta c/c$.

With these two welfare measures not only can comparisons be made between the two policies but also across regimes. That is, given the choice of a fixed exchange rate regime or a flexible exchange rate regime, which regime would agents prefer? It is shown below that in both regimes the interest rate rule is preferred to the constant money growth rate rule. Given that the interest rate peg is preferred, it is easy to verify that the fixed exchange rate economy is preferred.

Considered below are six different monetary growth rates and interest rate targets that corresponds to each money growth rate rule. The interest rate target is an interest rate that would prevail in a non-stochastic environment. That is, if the monetary growth rate rule is to increase the money supply by x percent, then the interest rate target keeps the interest rate pegged at $(1+R)=(1+\bar{x})/\beta$. This interest rate rule would keep the average growth rate of the money supply at x percent. The constant growth rates looked at are growth rates of 3, 5, 10, 20, 30, and 40 percent.

To keep things simple, I assume the marginal productivity shocks can take on two values. That is, $\alpha(s)$ and $\gamma(s)$ can take on the value of .65 or .75 with equal probability. If the shocks are independent, then there is a 25 percent chance that $\alpha(s)$ and $\gamma(s)$ equal .75, a 25 percent chance that $\alpha(s)$ and $\gamma(s)$ equal .65, and a 50 percent chance that one of the marginal productivity shocks equals .65 and the other marginal productivity shock equals .75.

Table 2.1 contains the results of the simulations for the fixed exchange rate model. The first line in each row labeled "x" percent refers to the constant money growth rate of "x"% , and interest rate is the interest rate that corresponds to that money growth rule. Note that for a given growth rate, the expected utility is higher and the welfare loss is lower for the interest rate target. The fourth column is the loss ratio, which is the ratio of the two welfare losses for a particular growth rate. The closer the numbers in the column are to zero the more preferred the interest rate rule is, if the ratio is equal to one the rules yield the same welfare loss, and if this ratio is greater than one the growth rate rule is preferred. Notice in all simulations the loss ratio is less than one. This implies the interest rate peg is preferred. The intuition to why the interest rate peg is the preferred policy is simple: with larger than anticipated marginal productivity shocks the representative firm's demand for loanable funds will be higher. This will put upward pressure on interest rates. The interest rate target accommodates these shocks by increasing the money supply in response to the marginal productivity shocks. That is, the interest rate peg is a procyclical policy that accommodates the demand for loanable funds. Because of the portfolio rigidity, this increase in the money supply increases employment, bringing the economy closer to the Pareto optimal level of employment. In particular, the money supply rule is given by

$$x^1(s)=x^2(s)=[\beta((\alpha(s)+\gamma(s))/2)(1\text{-}n)\text{-}n(1+\bar{x})]/(1+\bar{x}).$$

This policy is clearly procyclical. The monetary authority, by pursuing an interest rate peg, responds to the average marginal productivity shocks. For larger than anticipated marginal productivity shocks, the monetary authority increases the money supply and increases employment. Because of the portfolio rigidity, the increase in loanable funds allows for employment to increase. The findings here are similar to that of Poole (1970) that if shocks mainly affect money demand then the best policy is to target interest rates.

Table 2.2 shows the simulations for the flexible exchange rate economy. Since the money growth rate in country i affects only the interest rate and employment in country i, there will be a separate policy rule for each country. The results are similar but more dramatic than those in Table 2.1. That is, the interest rate target is much better than the constant money growth rate rule. Again the interest rate target is procyclical. The monetary growth rules that attain the interest rate target are given by

$$x^1(s)=\beta\alpha(s)[(1\text{-}n)\text{-}n(1+\bar{x})][(1+\bar{x})]^{-1},$$

$$x^2(s)=\beta\gamma(s)[(1\text{-}n)\text{-}n(1+\bar{x})][(1+\bar{x})]^{-1}$$

Similar to the fixed exchange rate model, the procyclical policy in country i keeps the economy closer to the Pareto optimal level of work effort than the money growth rate rule would.

Given that the interest rate rule is preferred to the constant money growth rate rule, the results of the model are compared across regimes. That is, which regime leads to the highest level of expected utility? In this model the answer is clear. Since the Pareto optimal allocations are the same for both regimes and the welfare loss is lower in the fixed exchange rate regime, it is preferred. To make the flexible exchange rate regime equivalent to the fixed exchange rate regime expected consumption of both goods would need to increase by .08 percent if the money growth rate was to be 3 percent on average to as low as .05 percent if the average money growth rate is 40 percent. The intuition why the fixed exchange rate regime is preferred is as follows. Recall that the equilibrium employment level is below the Pareto optimal level when the money growth is constrained to be positive. Consider the case where $\alpha(s)=.75$ and $\gamma(s)=.65$ and the average money growth rate is 3 percent. In the fixed exchange rate regime, the monetary authorities respond to the "average" marginal productivity shock. In this case with $\alpha(s)=.75$ and $\gamma(s)=.65$ and the average money growth rate being 3 percent, the interest rate rule implies the money growth rate is 3 percent in response to these shocks. The $\alpha(s)$, $\gamma(s)$, and money shocks affect the labor supply and labor demand curve. I will restrict my attention to analyzing the outcome in country one in both regimes[6]. First the high $\alpha(s)$ shock shifts the labor demand curve rightward. The money growth rate being equal to its expected value implies the labor supply curve does not shift. The shift in labor demand curve causes the real wage and employment to increase. Now consider the same shocks in the flexible exchange rate model. The demand curve for labor shifts to the same position. However, in country one the money growth rate is 5.88 percent. This causes the labor supply curve to shift back more than in the fixed exchange rate model. Therefore, employment is higher in the fixed exchange rate model. Therefore, equilibrium employment is closer to the Pareto optimal level. Because of this, the fixed exchange rate economy yields a lower welfare loss and higher expected utility than the flexible exchange rate economy.

Section V: Conclusion

This paper investigated the optimal monetary policy in a two country liquidity effects model. It was shown that the optimal monetary policy was an interest rate peg that keeps the nominal interest rate at unity. Moreover, it was shown that the monetary authority is capable

[6] It is easy to verify that the level of employment in country two will not be closer to the Pareto optimal level. However, the expected gain in utility from the increase in output exceeds the expected loss in utility from the lower output in the other country.

of attaining the Pareto optimal allocations irrespective of the monetary regime--a fixed exchange rate regime or a flexible exchange rate regime. Then for the case where monetary growth rates are constrained to be positive on average, it was shown that an interest rate target is preferred to a constant money growth rate rule. Also, in this case it was shown that the fixed exchange rate regime is better than the flexible exchange rate regime.

For policy advice this paper gives clear guidelines for countries attempting to coordinate their policies: A fixed exchange rate regime where the monetary authority targets interest rates is preferred. There are, however, a few caveats. First to arrive at closed form solutions a very restrictive utility function had to be used. Second since the monetary authority must respond to the portfolio rigidity, it must be true that the monetary authority is able to quickly gather information about the shocks. Further work should address these issues as well as investigate the optimal policies in a non-cooperative atmosphere as well as the optimal policies for countries that do not have identical wealth distributions.

TABLE 2.1: WELFARE UNDER FIXED EXCHANGE RATE REGIME

MONEY GROWTH	EXPECTED UTILITY	WELFARE LOSS	LOSS RATIO
3 PERCENT	3.2555	1.448%	.9223
INTEREST RATE	3.2567	1.346%	
5 PERCENT	3.2466	2.105%	.947
INTEREST RATE	3.2480	1.993%	
10 PERCENT	3.2181	4.141%	.973
INTEREST RATE	3.2195	4.054%	
20 PERCENT	3.1413	9.60%	.988
INTEREST RATE	3.1427	9.49%	
30 PERCENT	3.0470	16.16%	.993
INTEREST RATE	3.0484	16.05%	
40 PERCENT	2.9425	23.45%	.995
INTEREST RATE	2.9438	23.33%	

$\beta=.95, \Theta=\zeta=2$

TABLE 2.2 WELFARE UNDER FLEXIBLE EXCHANGE RATE REGIME

MONEY GROWTH	EXPECTED UTILITY	WELFARE LOSS	LOSS RATIO
3 PERCENT INTEREST RATE	3.2537 3.2566	1.584% 1.362%	.860
5 PERCENT INTEREST RATE	3.2447 3.2474	2.240% 2.070%	.90215
10 PERCENT INTEREST RATE	3.2156 3.2193	4.299% 4.083%	.950
20 PERCENT INTEREST RATE	3.1395 3.1424	9.278% 9.139%	.978
30 PERCENT INTEREST RATE	3.0452 3.0480	16.299% 16.095%	.988
40 PERCENT INTEREST RATE	2.9407 2.9434	23.585% 23.385%	.991

$\beta=.95, \Theta=\zeta=2$

Appendix 2.1: Proof of Proposition 1:

Proof: The proof will be by construction. By assuming the cash-in-advance constraints bind it will be shown that there exist unique n^i for the representative household in country i, i=1,2. Given the identical wealth distribution, I will make a guess that the optimal saving's decision is the same for households in countries one and two. Then the rest of the variables will be solved for. Then I will go back and confirm the initial guess.

First using the equations 2.3.b,2.3.c,2.3.d,e=1, and the binding cash-in advance constraint yields $d^1(s)=(1-n)/2$. Similarly, for the household in country two combining eq. 2.4.b,2.4.c,2.4.d, e=1, and the binding cash-in advance constraint yields

$$d^2(s)=(1-n)/2.$$

This implies the consumption allocations are

$$c^1{}_1(s)=c^1{}_2(s)=\theta(s)L_1(s)^{\alpha(s)}/2 \text{ and}$$

$$c^2{}_1(s)=c^2{}_2(s)=\zeta(s)L_2(s)^{\gamma(s)}/2.$$

Prices then are given by

$$p^1(s)=(1-n)/(\theta(s)L_1(s)^{\alpha(s)}), \text{ and}$$

$$p^2(s)=(1-n)/(\zeta(s)L_2(s)^{\gamma(s)}).$$

Wages are $\quad w^1(s)=(1+x^1(s))(1-n)/(2\beta)$

$$w^2(s)=(1+x^2(s))(1-n)/(2\beta).$$

Interest rates are given by $(1+R^1)=\alpha(s)(1-n)/(q^1{}_1(s)+q^1{}_2(s)+x^1(s))$

$$(1+R^2)=\gamma(s)(1-n)/((1/q^2{}_1(s)+q^2{}_2(s)+x^2(s)).$$

Using e=1 and the no arbitrage condition on deposits the optimal $q^i{}_j(s)$'s are given by

$$q^1{}_1(s)=q^1{}_2(s)=[2\alpha(s)n+\alpha(s)x^1(s)-\gamma(s)x^2(s)]/[2(\alpha(s)+\gamma(s))], \text{ and}$$

$$q^2{}_1(s)=q^2{}_2(s)=[2\gamma(s)n-\alpha(s)x^1(s)+\gamma(s)x^2(s)]/[2(\alpha(s)+\gamma(s))].$$

Interest rates are given by

$$(1+R^1)=(1+R^2)=[(\alpha(s)+\gamma(s))(1-n)]/[2n+x^1(s)+x^2(s)].$$

Then equilibrium labor supply can be gotten from the firm's first order conditions.

For a household in country one the equilibrium labor supply is given by

$$L_1(s)=[\alpha(s)/(\alpha(s)+\gamma(s)]\beta(2n+x^1(s)+x^2(s))2[(1-n)(1+x^1(s))]^{-1}$$

For a household in country two the equilibrium labor supply is given by

$$L_2(s)=[\gamma(s)/(\alpha(s)+\gamma(s)]\beta(2n+x^1(s)+x^2(s))2[(1-n)(1+x^2(s))]$$

From eq. 2.3.a, eq. 2.3.c, eq. 2.3.f, and the envelope condition we have

$$1=\int_s(\beta/(1+x^1(s)))[\alpha(s)(1-n^1-d^1(s)+ed^2(s)/(q^1{}_1(s)+eq^1{}_2(s)+x^1(s))]\Phi(ds)$$

Substituting for the optimal $d^1(s)$ and symmetry implies the equation can be written as

$$1=\int_s(\beta/(1+x^1(s)))[\alpha(s)(1-n)/(2q^1{}_1(s)+x^1(s))]\Phi(ds)$$

Then substitute for $q^1{}_1(s)$. This yields

$$1=\int_s(\beta/(1+x^1(s)))[(\alpha(s)+\gamma(s))(1-n)]/[2n+x^1(s)+x^2(s)]\Phi(ds) \qquad 2.9$$

Clearly eq. 2.9 is decreasing in n. If $n=1$ then the right hand side of eq. 2.9 is exactly zero. Also, if $n=0$ then the right hand side of eq. 2.9 exceeds unity by assumption one. By continuity of n there exists a unique fixed point $n^1\in(0,1)$ satisfying eq. 2.9. Similarly, for a household in country two eq 2.1,2.4.c,2.4.f, and the symmetry of the deposits and substituting for $d^2(s)$ yields

$$1=\int_s(\beta/(1+x^1(s)))[(\alpha(s)+\gamma(s))(1-n)]/[2n+x^1(s)+x^2(s)]\Phi(ds) \qquad 2.10$$

The right-hand-side of eq. 2.10 is strictly decreasing in n, and if $n=1$ then the right-hand-side equals zero. If $n=0$ then the right-hand-side exceeds one by assumption one. By continuity of n there exists a fixed point $n\in(0,1)$ satisfying eq. 2.10. Note that eq. 2.10 is identical to eq. 2.9. Next substititute these equibrium values back into household's reource constraint and solve for and solve for the τ 's so that wealth remains unchanged. This yields

$$\tau^1(s)=1+x^1(s)-(\alpha(s)/(\alpha(s)+\gamma(s)))(2n+x^1(s)+x^2(s))-$$
$$(n+x^1(s))((\alpha(s)+\gamma(s))(1-n)/(2n+x^1(s)+x^2(s))-(1-\alpha(s))(1-n)$$
$$\tau^2(s)=1+x^2(s)-(\gamma(s)/(\alpha(s)+\gamma(s)))(2n+x^1(s)+x^2(s))-$$
$$(n+x^2(s))((\alpha(s)+\gamma(s))(1-n)/(2n+x^1(s)+x^2(s))-(1-\gamma(s))(1-n).$$

This completes the proof.

Appendix 2.2: Proof of Proposition 2.3:

Proof: The proof will be by construction. I will assume that the constraints hold with equality. I will solve for all the equilibrium prices and allocations and show that these values are consistent with positive multipliers on all constraints. Since each household starts each period with the same level of wealth and they have identical preferences, the savings decision n^i will be the same for both countries. The optimal savings decision will be derived at the final stages of the proof. For now, I will take the n^i's as given and solve for the remaining variables.

To derive the consumption allocations 3b-d and the binding cash-in-advance constraints can be combined to yield

$$d^1(s)=[(1\text{-}n)/2e(s)].$$

Similarly, equations 4.b-d and the binding cash-in-advance constraints for the household in country two can be combined to yield

$$d^2(s)=[(1\text{-}n)e(s)/2].$$

The optimal value for $d^2(s)$, the cash-in-advance constraints, and the market clearing condition for good one can be combined to find the equilibrium price for good one. This price is given by

$$p^1(s)=[(1\text{-}n)(1+e(s))/\theta(s)L^{\alpha(s)}].$$

The consumption allocations can be obtained be rearranging the cash-in-advance equations from the household in country one and the household in country two. This leads to

$$c^1{}_1(s)=[1/(1+e(s))]\theta(s)L_1{}^{\alpha(s)},$$

$$c^1{}_2(s)=[e(s)/(1+e(s))]\theta(s)L_1{}^{\alpha(s)}.$$

Similarly, the prices and the consumption allocation for good two can be gotten by the same process. Combining eq. 2.3.c-d, 2.4.c-d, the cash-in-advance constraints for good two, and the market clearing conditions for good two yields the following price for good two and consumption allocations given by

$$p^2(s)=[(1\text{-}n)(1+e(s)^{-1})]/2\zeta(s)L_2^{\gamma(s)},$$

$$c^2{}_2(s)=[e(s)^{-1}/(1+e(s)^{-1})]\zeta(s)L_2^{\gamma(s)}, \text{ and}$$

$$c^2{}_1(s)=[(1/(1+e(s)^{-1})]\zeta(s)L_2^{\gamma(s)}.$$

Next substitute these consumption values for good one (two) for household one (two) along with the corresponding prices into the envelope condition eq. 2.3.k (2.4.k). to yield

$$J_1'(1)=2/(1\text{-}n) \quad (J_2'(1)=2/(1\text{-}n))).$$

Thus the derivative of the value function is the same for both countries.

Equations 2.3.e and 2.4.e and substituting for $w^i(s)L_i(s)$ $(i=1,2)$ from the firms borrowing constraint can be rearranged to obtain the household's optimal labor supply decision. The labor supplies for countries one and two are given by

$$L_1(s)=2\beta[q^1{}_1(s)+q^1{}_2(s)+x^1(s)]/[(1+x^1(s))(1\text{-}n)], \text{ and}$$

$$L_2(s)=2\beta[q^2{}_1(s)+q^2{}_2(s)+x^2(s)]/[(1+x^2(s))(1\text{-}n)].$$

Now in order to derive expressions for the interest rates in country one and country two, substitute for prices and for $w^i(s)L_i(s)$ from the borrowing constraint into the firm's problem and rearranging. Doing this for both countries yields

$$(1+R^1)=[\alpha(s)(1\text{-}n)(1+e(s))/(q^1{}_1(s)+q^1{}_2(s)+x^1(s))], \text{ and}$$

$$(1+R^2)=[\gamma(s)(1\text{-}n)(1+e(s)^{-1})/(q^2{}_1(s)+q^2{}_2(s)+x_2(s))].$$

The nominal wages are then given by substituting the optimal labor supply decision and the interest rate into the firms' first order conditions to yield

$w^1(s)=(1+x^1(s))(1-n)/2\beta$, and

$w^2(s)=(1+x^2(s))(1-n)/2\beta$.

Next use eq. 2.3.f and 2.4.g no-arbitrage conditions on bank deposits to solve for the exchange rates and interest rates. That is in equilibrium the exchange rate adjusted interest rate are equated. Rearranging eq. 2.3.g and eq. 2.4.f yields

$$(1+R^1)(e(s)/\ \hat{e}(s))=(1+R^2). \qquad 2.11$$

To solve for the exchange rates e(s) and $\hat{e}(s)$. I use the market clearing conditions for the exchange rates which are

$e(s)[d^1(s)+q^2{}_1(s)]=[d^2(s)+q^1{}_2(s)]$, and

$$\hat{e}(s)\,[q^1{}_2(s)(1+R^1)]=[q^2{}_1(s)(1+R^2)]. \qquad 2.12.a,b$$

Next substitute $(1+R^1)$ from eq. 2.11 into eq. 2.12.b which yields

$$e(s)q^2{}_1(s)=q^1{}_2(s). \qquad 2.13$$

Then substitute eq. 2.8 and the equilibrium values for the $d^i(s)$'s into eq. 2.12.a and rearranging yields

$e(s)=1$.

Given this value for e(s), $q^1{}_2(s)=q^2{}_1(s)\varepsilon(0,n)$ and eq. 2.11 implies that $\hat{e}(s)$ is defined to be

$\hat{e}(s)=\gamma(s)(n+x^1(s))/[\alpha(s)(n+x^2(s))]$.

Consumption allocations are given by

$c^1{}_j(s)=\theta(s)L_1(s)^{\alpha(s)}/2, \quad j=1,2$

$c^2{}_j(s)=\zeta(s)L_2(s)^{\gamma(s)}/2 \quad j=1,2$

Equilibrium labor supplies are given by

$L_1(s)=2\beta(n+x^1)/(1+x^1(s))(1-n)$, and

$L_2(s)=2\beta(n+x^2)/(1+x^2(s))(1-n)$.

Prices are given by

$$p^1(s)=(1\text{-}n)/\theta(s)L_1(s)^{\alpha(s)},$$

$$p^2(s)=(1\text{-}n)/\zeta s)L_2(s)^{\gamma(s)},$$

$$w^1(s)=(1+x^1(s))(1\text{-}n))/2\beta,$$

$$w^2(s)=(1+x^2(s))(1\text{-}n))/2\beta,$$

$$(1+R^1)=\alpha(s)(1\text{-}n)/(n+x^1(s)),$$

$$(1+R^2)=\gamma(s)(1\text{-}n)/(n+x^2(s)),$$

$$e(s)=1,$$

$$\hat{e}(s)=[\gamma(s)(n+x^1(s))]/[\alpha(s)(n+x^2(s))]$$

$$\tau^1=\tau^2=0$$

To determine the optimal n^i's, substitute into eq. 2.3.a (2.4.a) and 2.3.b (2.4.b) for $\lambda^1{}_1(s)$, $\lambda^3{}_1(s)$, $\lambda^1{}_2(s)$, and $\lambda^3{}_2(s)$and rearranging yields

$$1=\int_s(\beta/(1+x^1(s)))[\alpha(s)(1\text{-}n)/(n+x^1(s))]\Phi(ds),$$

for a household in country one and,

$$1=\int_s(\beta/(1+x^2(s)))[\gamma(s)(1\text{-}n)/(n+x^2(s))]\Phi(ds),$$

for a household in country two. These functional equations are strictly decreasing n. They exceed unity when n equals zero, and equal zero when n equals unity. By continuity of n there exists a unique fixed point. Since $x^1(s)$ and $x^2(s)$ are drawn from the same distribution as are $\alpha(s)$ and $\gamma(s)$, the n's that solve these functional equations are identical.

This completes the proof.

Chapter 3: Real Effects from Unanticipated Changes in the Money Supply in a Model with Inventories and Credit Goods: Friedman (1968) Revisited

In his 1968 American Economic Association Presidential Address, Milton Friedman argued that only unanticipated, not anticipated, increases in the supply of money could lead to increases in output and employment. More specifically, the unanticipated increase in the supply of money would cause the labor supply curve to shift rightward due to misperceptions of the real wage by workers. This would lead to a reduction in real wages and increases in employment and output. Friedman argued these effects would last anywhere from two to 10 years due to some type of market frictions. Rigorous models establishing these "Friedman effects" were constructed by Lucas (1972, 1975), and Barro (1977). Though these models could capture the initial effects of an unanticipated increase in the money supply process, the were less successful in accounting for the dynamic behavior outlined by Friedman. In addition, these models relied on agent's misperceptions of variables that were readily available to them. Because of their inability to capture business cycle dynamics and because they rely on misperceptions of available data, these models are no longer considered as appropriate models of capturing the effects of monetary innovations.

More recently, there has been renewed interest in the effects of monetary innovations on interest rates and real activity. Indeed, this resurgence has come about due to recent developments in theoretical models and to the advances made in time series econometrics. Empirical papers by Christiano and Eichenbaum (1992a), Bernanke and Blinder (1990), and Sims (1991) find empirical evidence consistent with the conventional wisdom concerning unanticipated increases in the money supply and real variables. That is, larger than anticipated increases in the money supply lower the interest rate and increase real activity.

Theoretically, Fuerst (1992) showed that, by incorporating Lucas' (1990) liquidity effects framework into a production economy, monetary innovations could have real effects that are consistent with conventional wisdom. Christiano (1991) showed that when the Fuerst model was calibrated to be consistent the United States economy, the expected inflation effect dominates the liquidity effect and interest rates rise rather than fall in response to larger than anticipated monetary injections.

Christiano and Eichenbaum (1992b) are able to modify the model so that larger than anticipated monetary injections cause interest rates to fall and output to rise. They accomplish this by assuming investment decisions are made before the state of the world is revealed and by allowing contemporaneous wage income to be used for consumption purchases. Though the contemporaneous response of interest rates and output are consistent with the empirical evidence cited above, the dynamic properties of the model are not.

This paper modifies the work of Christiano and Eichenbaum (1992b) and Fuerst (1992). In doing so, the economy reacts to monetary shocks in a manner similar to what Friedman described. Before describing that model, I will discuss the Christiano and Eichenbaum (1992b) model and a similar model that includes inventories alone. The problem in Christiano and Eichenbaum's model is that output and consumption fall precipitously in the periods following the unanticipated monetary injection before gradually returning to their steady state levels. This property of the model is clearly not consistent with the empirical evidence cited above. The basic liquidity effects model of Christiano and Eichenbaum is described in Section I. Also, the dynamic response of the variables in the model to a one standard deviation shock to the money supply is discussed. In Section II, inventories are added to the model. The reason inventories are added is simple: realism. On average, the total stock of inventories is roughly 90 percent of output (see Christiano (1988), Kydland and Prescott (1994), or Cooley and Hansen (1995)[1]). In addition, inventory investment is the most volatile component of GDP[2]. Because of these facts alone, inventory accumulation should be considered a potential factor in making these stylized models more consistent with the empirical evidence of business cycle fluctuations. Another reason to model inventories is that the volatility of inventory investment may aid in propagating idiosyncratic shocks, such as monetary shocks. That is, suppose that inventories assist in producing output, and suppose output increases in response to a monetary shock. If inventories accumulate in response to this shock, it is possible that this will cause output to remain above its steady state value for several periods. Inventories, therefore, may make the real effects of a monetary innovation persist longer.

It is shown that including inventories does not improve the model. In fact the contemporaneous behavior of consumption and the interest rate, as well as the dynamic behavior of these variables to a one standard deviation shock to the money supply process are not consistent with the empirical evidence

Section III attempts to rectify this problem by including credit goods. This modification vastly improves the model. In fact, the response of the variables seems to be consistent with Friedman's (1968) view of the effects of monetary innovations. However, there are two potential problems that arise with this model. The first problem is the

[1] Christiano's (1988) figure for inventories comes from the Survey of Current Business Table 1.2.
[2] Romer (1995) states that inventory accumulation's average share of the fall in GDP in recessions relative to normal growth is 30.7 percent

contemporaneous response of the interest rate and consumption may be in the wrong direction. The second problem is that this model implies that consumption lags the cycle, whereas, the data tell us consumption leads the cycle in United States. This second problem may not be too problematic. The data indicate that consumption leads the business cycle and is positively correlated with money growth. Most economists are in agreement on these facts. However, the response of consumption to unanticipated increases in the money supply process is less understood. In order to quantify how consumption responds to unanticipated increases in the money supply process one needs to take a stand on the money supply process. The profession is not in agreement about how this can be done empirically. Different identifying assumptions about the money supply process may lead to different results (Gordon and Leeper 1992). Section IV concludes.

Section I: The Basic Christiano and Eichenbaum Model

In the basic Christiano and Eichenbaum model there are three types of agents: households, firms, and financial intermediaries. Each agent of each type is assumed to be identical. Therefore, the actions of a representative agent of each type can be studied. The household consists of three members. Each member carries out distinct tasks. The first member, the saver, is responsible for depositing the current period's savings allocation (N_t) into the financial intermediary. The second member, the shopper uses the remaining funds (M_t-N_t) to purchase consumption goods. The third member, the worker, offers services in the labor market to the goods producing firm in exchange for wage compensation (W_tL_t). When the agents perform these tasks is discussed more fully below. (The timing of agent's decisions is listed on page 98.)

<u>Timing:</u>

1. Households decide how much to allocate to goods consumption and to savings.

STATE OF THE WORLD IS REVEALED

2. Savings are deposited in the financial intermediary.

3. Firms borrow funds, hire labor and produce the consumption goods.

4. The shopper purchases the consumption goods. Firms purchase capital.

5. Firms repay their loans.

6. Households reunite and pool their resources. These resources include the return on savings, the residuals from the cash-in-advance constraints on consumption goods, wage income, and profits from the firms and intermediaries.

The main difference in these liquidity effects models from the standard real business cycle models is the timing of the agents' decisions. The household starts the period holding M_t units of currency. Before the state of the world is known, the household allocates N_t units of currency to the saver. The remaining M_t - N_t units of currency plus contemporaneous labor receipts (W_tL_t) can be used to purchase the consumption good[3]. Once the household chooses N_t it is prohibited from realigning its portfolio.

The household receives utility from consuming the consumption good and from leisure. In particular, the household's per period utility is given by

$$U(c_t,1-L_t)=\gamma\ln(c_t)+(1-\gamma)\ln(1-L_t)\,, \qquad 3.1$$

where c_t is consumption at time t, L_t is labor at time t. The time endowment is normalized to unity so that $1-L_t$ is leisure at time t. The household's lifetime expected utility is given by

$$u=E_0\sum_{t=0}^{\infty}\beta^t U(c_t,1-L_t)\,, \qquad 3.1'$$

where is the β discount factor and $\beta\varepsilon(0,1)$.

After the savings decision is made, the household separates and the state of the world is revealed. Recall that decisions made before the state of the world is revealed can not be changed. Thus the amount allocated to savings can not be realigned. This portfolio rigidity is what allows for the realizations of the money supply process that differ from what was expected to have real effects.

[3] It is assumed that the current period's wage receipts are given directly to the shopper so that the shopper can use these receipts for goods purchases.

As stated above, the saver deposits funds into the financial intermediary. The shopper uses the remaining funds plus labor income to finance consumption purchases. The worker offers his services in the labor market.

The representative financial intermediary accepts deposits from the household (N_t) and receives cash injections from the monetary authority (X_t). By assumption, the representative goods producing firm needs to obtain funds in order to finance its wage bill. As long as the gross nominal interest rate exceeds unity, the representative intermediary will loan out all of its funds to the firm. At the end of the period, the profits of the representative financial intermediary are paid to the representative household as dividends.

The representative firm hires labor each period and purchases capital in order to maximize profits. As stated above in order to hire workers, firms must borrow from the financial intermediary. Firms own their own capital stock. The additions to the capital stock, thus investment, are decided on before the state of the world is known. Production is carried out using the neoclassical production function

$$F(K_t,H_t) = K_t^{\alpha}(\exp(\mu t)H_t)^{1-\alpha} \qquad 3.2$$

Profits for the firm are given by

$$\Pi_t = P_t\{F(K_t,H_t) - K_{t+1} + (1-\delta)K_t\} - W_tH_t(1+R_t) \ni W_tH_t \leq N_t + X_t . \qquad 3.3$$

The goods producing firm chooses labor and capital so to maximize the discounted value of its dividend payments to the representative household. Since the representative household owns the firm, dividend payments are paid to the household at the end of each period.

The only stochastic variable in this framework is the money supply process. Following Christiano (1991) and Christiano and Eichenbaum (1992b), I assume the money supply process follows a first order autoregressive process given by

$$X_t = (1-\rho)\bar{X} + \rho X_{t-1} + \varepsilon_t,$$

where $\varepsilon_t \sim N(0,.014)$[4].

Given the above set-up, the household's problem can be written in terms of its dynamic program. Since I am only concerned with stationary rational expectational equilibria, all growing variables must be detrended. Since technology is labor augmenting, all real variables will be growing at the rate $\exp(\mu)$. To make the time t real variables stationary they must be detrended by the $\exp(\mu t)$. Similarly, all nominal variables will be growing along with the changes in the money supply. The nominal variables dated at time t must be detrended by the time t money supply. In the dynamic programming problem, the usual convention of denoting detrended variables with lower case letters and dropping time subscripts, and using primes to denote next period's values is employed below. The dynamic program for the household is given by

$$J(m,k,\mathbf{k}) = \max_{n\in[0,m],k'} \int \max_{c,L,H,k',b} \{U(c,1-L) + \beta J(m',k',\mathbf{k}')F(s,ds)\},$$

subject to $m - n + wL \geq pc$,

$n + x \geq b$, and

$b \geq wH$,

and the transition equation is given by

$$m' = [m + nR - pc + wL + (1+R)x + p(\exp(-\mu\alpha)k^{\alpha}H^{1-\alpha} - k' + (1-\delta)k) - wH - bR]/(1+x),$$

and $\mathbf{k}$ is the aggregate capital stock, and $F(s,ds)$ is the distribution function.

After renormalizing by the time t money supply, the first order conditions from this programming problem can be rearranged to yield the following efficiency conditions

[4] This is the empirical estimate in Christiano and Eichenbaum (1992b).

$$\frac{U_c(c_t, 1-L_t)}{U_L(c_t, 1-L_t)} = \frac{W_t}{P_t} \tag{3.5.a}$$

$$E_{t-1}\{\frac{U_c(c_t, 1-L_t)}{P_t}\} = E_{t-1}\{\frac{\beta}{(1+X_t)}\frac{U_c(c_{t+1}, 1-L_{t+1})}{P_{t+1}}(1+R_t)\} \tag{3.5.b}$$

$$\begin{aligned} &E_{t-1}\{\frac{\beta U_c(c_{t+1}, 1-L_{t+1})}{P_{t+1}(1+X_t)}\} = \\ &E_{t-1}\{\frac{\beta^2}{(1+X_{t+1})}\frac{U_c(c_{t+2}, 1-L_{t+2})}{P_{t+2}} P_t(\exp(-\mu\alpha)\alpha k_{t+1}{}^{\alpha} L_{t+1}{}^{1-\alpha} + (1-\delta)) \end{aligned} \tag{3.5.c}$$

$$(1+R_t) = \left(\frac{W_t}{P_t}\right)^{-1} (1-\alpha)\exp(-\mu\alpha) k_t{}^{\alpha} L_t{}^{1-\alpha} \tag{3.5.d}$$

Equation 3.5.a is the household's leisure-labor decision. The household equates the marginal rates of substitution between leisure and consumption to the real wage. Equation 3.5.b is the household's portfolio choice decision. The household chooses how much to allot to savings and to consumption at time t based on the information at the beginning of time t.

The optimal capital accumulation equation is given by eq. 3.5.c. This decision is also based on information available at the beginning of time t. This assumption is intended to capture the fact that investment is a long range plan that is not likely to be reversed for small deviations from the expected state of the world. This efficiency condition can be interpreted as follows: if the firm decides to increase the capital stock by one unit, this will have the effect of lowering the end of period profits by P_t dollars. Because of this, the household will have fewer cash balances available for consumption in the next period. That is, they will, in expectations, buy $1/P_{t+1}$ fewer consumption goods . This causes the household to suffer discounted disutility given by $\beta U_c(c_{t+1}, 1-L_{t+1})$. Therefore, the time t cost of increasing the capital stock by one unit is given by the left-hand side of eq. 3.5.c.

The benefit is given by the right-hand side of eq. 3.5.c and is interpreted as follows. The addition to the capital stock at time t increases profits at time t+1 $(P_{t+1}(\exp(-\mu\alpha)k_{t+1}^{\alpha}L_{t+1}^{1-\alpha} - (1-\delta))$. These profits will be paid back to the household at the end of t+1, and therefore be available for consumption at time t+2 yielding discounted marginal utility $\beta^2 U_c(c_{t+2},1-L_{t+2}/P_{t+2}$. Along the optimal path, the household equates these costs and benefits. Equation 3.5.d is the firm's optimal labor demand decision. Note, labor demand is a decreasing function of the nominal interest rate.

These four equations, the cash-in-advance constraint for the consumption good, the borrowing constraint, and the market clearing conditions yield a system of seven equations and seven unknowns. The market clearing conditions are given by

$$\exp(-\mu\alpha)k_t^{\alpha}L_t^{1-\alpha} = c_t + k_{t+1} - (1-\delta)k_t, \qquad 3.6.a$$

$$H_t = L_t, \qquad 3.6.b$$

$$m_t = m_{t+1}, \qquad 3.6.c$$

where eq. 3.6.a is the aggregate resource constraint, eq. 3.6.b is the market clearing condition for the labor market and 3.6.c is money market equilibrium. The consistency condition on the capital stock implies k=k. Therefore, given these conditions, solutions to the seven unknowns (P_t, $(1+R_t)$ W_t, N_t, C_t, L_t, K_{t+1}) can be sought and their dynamic paths can be traced out.

Since this system of equations is non-linear, an approximation technique must be employed to trace out the dynamic path of the variables. The method used here is the same employed by Christiano (1991). The solution strategy involves four steps. These steps are: (1) choose values for the free parameters, (2) find the steady state values for the non-stochastic system, (3) linearize the system around the steady state, and (4) use the method of undetermined coefficients to solve for the unknowns.

The parameter values used here are as follows: $\beta=(1.03)^{-25}$, $\mu=.0041$, $\rho=.32$, and $x=.012$ (these numbers are all taken from Christiano and Eichenbaum (1992b)). In addition, γ is chosen so that the time allocated to work is consistent with the average time spent at work for the average individual; that is, percent of time at work is .219[5]. This implies $\gamma=.24$. Next, α is chosen so that the labor share of output in this model is consistent average labor share of output for the United States. This implies $\alpha=.3$. Finally, Christiano (1991) finds that the average consumption-output ratio and capital-output ratio is .73 and 10.59 respectively. Using these values, the depreciation rate is obtained from the resource constraint. This yields $\delta=.023$.

The dynamic responses of the variables are shown in Figures 3.1-3.5. The figures show the percent deviations from the steady state of the nominal interest rate, output, consumption, employment and the real wage, and work effort to a one standard deviation shock to the money supply process. Figure 3.1 depicts the percent change in the interest rate to the monetary shock. Note the interest rate initially falls then begins to rise in subsequent periods. (The intuition for the response of these variables to the shock is discussed in detail below.) Figure 3.2 shows how output responds to an unanticipated increase in the money supply. In response to the shock, output rises above its steady state level. In fact it remains above its steady state level for several periods after the shock. Consumption's deviation from its steady state level is depicted in Figure 3.3. Consumption increases in the period of the shock but then falls drastically in subsequent periods. The percent change in employment is depicted in Figure 3.4. Employment rises with the shock and remains slightly above its steady state value for six periods.

[5] This number is also taken from Christiano and Eichenbaum (1992b).

Note that the contemporaneous movements of the variables seem to be consistent with conventional wisdom. The dynamics of the model, however, are not. In particular, consumption (shown in Figure 3.3) falls after the shock and then gradually returns to its steady state value. Since prices rise, consumers buy fewer consumption goods. The additional output then must go into investment. The reason for this is simple: The rise in prices implies consumption (a cash good) will be more expensive than investment (a credit good)[6]. Therefore, consumption falls and investment increases. The data reported by Cooley and Hansen (1995) imply that consumption is positively correlated with monetary aggregates. So given a persistent innovation in the money supply process that causes the money supply growth rate to be above its steady state level of growth for several periods, then it should also be the case that consumption should be above its steady state level too. It clearly is not.

Returning to Figure 3.1, the nominal interest rate remains below the steady state value in the period following the shock then rises as the expected inflation rises. Empirically, Christiano and Eichenbaum (1992a) present evidence that the nominal interest rate remains below the steady state value for as many as 16 quarters following the shock.

In this model, the real wage rises initially, falls drastically in the period after the shock, and then gradually returns to the steady state. The reason for this initial rise in real wages is as follows. Consider the demand and supply for labor drawn in real wage space. Assume that the model is in steady state when hit by a monetary shock. The monetary innovation reduces interest rates. This implies the labor demand curve shifts to the right. The labor supply curve shifts to the left because consumption increases above its steady

[6] I am using the term credit good in the same manner as Lucas and Stokey (1983), Cooley and Hansen (1995), and Carlstrom and Fuerst (1995) use it. Credit goods are goods that are not subject to a cash-in-advance constraints.

state value (this follows from equation 3.5.a). The direction of the shifts of both of these curves implies the real wage must increase. Since the rightward shift in labor demand exceeds the leftward shift in labor supply, employment increases in the period of the shock. In the following period, the nominal interest rate increases but still remains below its steady state value. This implies the labor demand curve in the period immediately following the shock is to the right of the steady state labor demand curve. Since consumption is still above its steady state value, the labor supply schedule in this period must lie to the left of the steady state labor supply schedule. Taken together, this implies that the real wage must still be above its steady state level. In the second period after the shock, the nominal interest rate exceeds its steady state level. This in turn implies that the labor demand schedule in this period lies everywhere to the left of the steady state labor demand schedule. Likewise since consumption falls, the labor supply schedule must lie to the right of the steady state labor supply curve. As a result, it can be inferred from these shifts that not only does the real wage fall in the third period after the shock, it is also true that the real wage in this period is below the steady state real wage. In subsequent periods the nominal interest rate gradually falls back to the steady state level. As the interest rate falls, the labor demand schedule gradually shifts back to its steady state position. The real wage in this model is procyclical. However, it is procyclical for entirely different reasons than in the standard RBC model. In the standard RBC model the real wage is procyclical because technology shocks make the firm more productive increasing its demand for labor. In this model the real wage is procyclical due to the money supply's effect on interest rates and prices.

The major shortcoming of this model is its inability to replicate the dynamic behavior of output, consumption, interest rates, and real wages. The model, however, is consistent with the contemporaneous response of variables to a money shock. Christiano and Eichenbaum (1992b) rectify this problem by making it costly for households to adjust their portfolios for several periods after the shock. This costly portfolio adjustment assumption helps make the model more consistent with the empirical evidence. Instead of taking this approach, I investigate the implications of the dynamic behavior of the variables if the real side of the economy is modeled more explicitly. In particular, in the next section, inventories are added to the model.

Section II. The Christiano and Eichenbaum Model with Inventories

This section incorporates inventories into the model investigated in Section I. Recall the two reasons for adding inventories to the model: the first is that the stock of inventories at a point in time is non-negligible--roughly 90 percent of output. Also, recall that if inventories accumulate in response to a monetary innovation, then it may be possible that the real effects of a monetary innovation will persist for several periods. (The timing of the model is discussed on page 108.)

In this section, I assume inventory decisions are made after the state of the world is revealed. Moreover, I assume the inventory purchases are not subject to a cash-in-advance constraint. I assume, as in Christiano (1988) and Kydland and Prescott (1994) that inventories assist in the production of output. The production function specification is

Timing:

1. Households decide how much to allocate to goods consumption and to savings.

STATE OF THE WORLD IS REVEALED

2. Savings are deposited in the financial intermediary.

3. Firms borrow funds, hire labor and produce the consumption goods.

4. The shopper purchases the consumption goods. Firms acquire capital and inventories.

5. Firms repay their loans.

6. Households reunite and pool their resources. These resources include the return on savings, the residuals from the cash-in-advance constraints on consumption goods, wage income, and profits from the firms and intermediaries.

as in Kydland and Prescott (1994) given by

$$F(\hat{K}_t, H_t, S_t) = [(1-\psi)(\hat{K}_t^{\alpha}(\exp(\mu t)H_t)^{1-\alpha})^{\frac{-1}{v}} + \psi S_t^{\frac{-1}{v}}]^{-v},$$

where S_t are inventories at time t, and the capital stock in this section ($\hat{K}_t$) is net of inventories.

Removing the growth trend from the production function yields

$$f(\hat{k}_t, H_t, s_t) = [(1-\psi)(\exp(-\alpha\mu)\hat{k}_t^{\alpha} H_t^{1-\alpha})^{\frac{-1}{v}} + \exp(\mu / v)\psi s_t^{\frac{-1}{v}}]^{-v}.$$

Also as in Kydland and Prescott, I assume that in the growth economy inventories do not depreciate and the ratio of inventories to output is .90[7]. The parameters α, v and ψ are free parameters. The parameter v is chosen to be 3[8]. This value implies that inventories are complements to capital and labor in producing output. The parameters α and ψ are chosen so that the capital-output ratio and the inventory-output ratio in this model is the same as the average capital-output ratio and the inventory-output ratio over the cycle for the United States economy. The values for α and ψ that achieve this are .363 and .0062, respectively. Except for the expression for goods market clearing and an additional efficiency condition for inventory accumulation, the model is same as in Section I. The efficiency conditions are

$$\frac{U_c(c_t, 1-L_t)}{U_L(c_t, 1-L_t)} = \frac{W_t}{P_t} \qquad 3.6.a$$

$$E_{t-1}\{\frac{U_c(c_t, 1-L_t)}{P_t}\} = E_{t-1}\{\frac{\beta}{(1+X_t)} \frac{U_c(c_{t+1}, 1-L_{t+1})}{P_{t+1}}(1+R_t)\} \qquad 3.6.b$$

[7] The aggregate capital stock is then adjusted so that the sum of inventories and capital is equal to 10.59 as in Section I. Experiments with inventory to output ratios of .25 ranging to 1.5 had little affect on the qualitative aspects of the model.

[8] Sensitivity analysis was done by letting v=1.5 and v=4.5. These changes do not qualitatively change the results. In fact, they hardly affect the quantitative results.

$$E_{t-1}\{\frac{\beta U_c(c_{t+1},1-L_{t+1})}{P_{t+1}(1+X_t)}\} =$$

$$E_{t-1}\{\frac{\beta^2}{(1+X_{t+1})}\frac{U_c(c_{t+2},1-L_{t+2})}{P_{t+2}}P_t(f_1(\hat{k}_t,L_t,s_t)+(1-\delta))\tag{3.6.c}$$

$$E_t\{\frac{\beta U_c(c_{t+1},1-L_{t+1})}{P_{t+1}(1+X_t)}\} =$$

$$E_t\{\frac{\beta^2}{(1+X_{t+1})}\frac{U_c(c_{t+2},1-L_{t+2})}{P_{t+2}}P_t((f_3(\hat{k}_t,L_t,s_t)+(1-\mu))\tag{3.6.d}$$

$$(1+R_t)=\left(\frac{W_t}{P_t}\right)^{-1}(f_2(\hat{k}_t,L_t,s_t)) \tag{3.6.e.}$$

Equation 3.6.d is the efficiency condition for optimal inventory accumulation.

The resource constraint is given by

$$f(\hat{k}_t,H_t,s_t)=[(1-\psi)(\exp(-\alpha\mu)\hat{k}_t^{\alpha}H_t^{1-\alpha})^{\frac{-1}{v}}+\exp(\mu/v)\psi s_t^{\frac{-1}{v}}]^{-v} =$$

$$c_t + \hat{k}_{t+1}-(1-\delta)\hat{k}_t + s_{t+1} + \mu s_t$$

The cash-in-advance constraint on consumption goods and the borrowing constraints are as in Section I.

To examine the dynamics of the model the same solution strategy is employed as in Section I. The responses of the interest rate, consumption, the real wage, and work effort to a one standard deviation shock to the money supply are reported shown in Figures 3.6-3.10. Note that the interest rate rises in response to a larger than anticipated monetary injection. The reason for this is simple: the real rate of interest increases. The real rate of interest is the amount of consumption goods the household is willing to give up this period in exchange for an expected increase in consumption next period; that is,

$\beta U_c(c_{t+1}, 1-L_{t+1})/ U_c(c_t, 1-L_t)$. The increase in the real rate of interest occurs due to the drop in consumption this period and the expected increase in consumption next period. Given that the combination of this effect and the rise in expected inflation dominate the non-Fisherian effect, the nominal interest rate increases. Thus, not even the contemporaneous response of the interest rate is consistent with the time series evidence. Before I try to amend this model, I will first investigate how the other variables behave in response to the monetary shock.

The real wage in this model falls in response to the monetary shock. This occurs because the increase in the interest rate shifts the labor demand curve to the left. Labor supply shifts out because of the contemporaneous drop in consumption. Both of these shifts cause the real wage to fall. Since equilibrium work effort goes up, it must be the case the rightward shift of the labor supply curve exceeds the shift in labor demand. Since work effort increases in the period of the shock, output must rise. Notice also that consumption falls, and investment in capital goods does not change. This implies inventory investment must rise by a higher percent than the increase in output. In fact, inventory investment increases by over 40 percent. In the period after the shock, the nominal interest rate rises again due to the real interest rate being above its steady state level and due to the expected inflation effect. The increase in the nominal rate causes the labor demand curve to shift even further back than in the initial period. Labor supply shifts further to the left because consumption again falls. Because the shift in labor demand exceeds the shift in labor supply, employment falls. In subsequent periods, the interest rate falls shifting the labor demand curve back toward its steady state position. Also, the fact that consumption is slowly rising causes the labor supply curve to shift gradually back to it steady state position. The shiftings of these curves cause the real wage to gradually return to its steady state level, and for employment to increase until it returns to its steady state level.

Clearly, the addition of inventories does not improve the dynamic behavior of the model. In fact, it makes the model worse. In the next section, I will add a credit good to the model to try to remedy the deficiencies cited above.

Section III. Inventories and Credit Goods: Friedman (1968)

In this section of the paper, credit goods are added to the model[9]. I add credit goods for two reasons. First credit goods account for significant percent of consumption purchases (see Cooley and Hansen (1995)). Second it is possible that in response to a monetary shock consumption of the credit good and inventory investment both increase. This could cause total consumption to increase and the addition to inventories may make the effects of the monetary shock more persistent.

Adding credit goods changes the model in four ways. First, the utility function needs to be modified to incorporate credit goods. Second, an additional efficiency condition is added. Third the aggregate resource constraint needs to be modified. The budget constraint gets modified to incorporate credit goods. Everything else in the model is the same as in Section II.

The timing of the model is slightly different than the models in Sections I and II. In this case after the savings decision is made and goods are produced, both consumption

[9] Again, these are credit goods as defined in Lucas and Stokey (1983). The best way to think of these credit goods is that they are like purchasing goods on a credit card where no interest is owed if you pay-in-full at the end of the period.

goods are purchased. The cash goods are paid for at the time of purchase. The credit goods, on the other hand, are paid for at the end of the period when the household pools its resources. The timing of the agents' actions are given on page 115.

The new utility function is given by

$$U(c_{1t},c_{2t},1-L_t)=\gamma_1\ln(c_{1t})+\gamma_2\ln(c_{2t})+(1-\gamma_1-\gamma_2)\ln(1-L_t)\,, \qquad 3.7$$

where c_{it} is consumption of good i at time t (i=1,2 where c_{1t} is the cash good and c_{2t} is the credit good[10]), L_t is labor supplied at time t, the time endowment is normalized to unity so that 1-L_t is leisure at time t. The household's lifetime expected utility is given by

$$u=E_0\sum_{t=0}^{\infty}\beta^t U(c_{1t},c_{2t},1-L_t)\,, \qquad 3.7'$$

where $\beta\varepsilon(0,1)$.

The efficiency conditions are given by

$$\frac{U_{c_1}(c_{1t},c_{2t},1-L_t)}{U_L(c_{1t},c_{2t},1-L_t)}=\frac{W_t}{P_t} \qquad 3.8.a$$

$$E_{t-1}\{\frac{U_{c_1}(c_{1t},c_{2t},1-L_t)}{P_t}\}=E_{t-1}\{\frac{\beta}{(1+X_t)}\frac{U_{c_1}(c_{1t+1},c_{2t+1},1-L_{t+1})}{P_{t+1}}(1+R_t) \qquad 3.8.b$$

$$E_{t-1}\{\frac{\beta U_{c_1}(c_{1t+1},c_{2t+1},1-L_{t+1})}{P_{t+1}(1+X_t)}\}=$$

$$E_{t-1}\{\frac{\beta^2}{(1+X_{t+1})}\frac{U_{c_1}(c_{1t+2},c_{2t+2},1-L_{t+2})}{P_{t+2}}P_t(f_1(\hat{k}_t,L_t,s_t)+(1-\delta)) \qquad 3.8.c$$

[10] I assume that credit goods account for 7.5 percent of total expenditures on consumption goods. This value implies that in steady state the average propensity to save is roughly .05 which is consistent with time series average in Baier 1990.

$$E_t\{\frac{\beta U_{c_1}(c_{1t+1},c_{2t+1},1-L_{t+1})}{P_{t+1}(1+X_t)}\} =$$

$$E_t\{\frac{\beta^2}{(1+X_{t+1})}\frac{U_{c_1}(c_{1t+2},c_{2t+2},1-L_{t+2})}{P_{t+2}}P_t((f_3(\hat{k}_t,L_t,s_t)+(1-\mu))\qquad 3.8.d$$

$$\{\frac{U_{c_2}(c_{1t},c_{2t},1-L_t)}{P_t}\} = E_t\{\frac{\beta}{(1+X_t)}\frac{U_{c_1}(c_{1t+1},c_{2t+1},1-L_{t+1})}{P_{t+1}}(1+R_t)\}\qquad 3.8.e$$

$$(1+R_t)=\left(\frac{W_t}{P_t}\right)^{-1}(f_2(\hat{k}_t,L_t,s_t)\qquad 3.8.f.$$

The dynamics of this model are depicted in Figures 3.10-3.15. Not much has changed from Section II in regard to the initial response of the variables. Notice, however, that the nominal interest rate falls in the period after the shock. The nominal interest rate falls because the of the expected price deflation[11]. Because the nominal rate falls, the labor demand curve shifts to the right. The labor supply curve shifts to the left due to the increase in consumption[12]. Since equilibrium work effort increases, the shift in labor demand is greater than the shift in labor supply. In the following period, the nominal interest rate remains below its steady state level due to prices again falling (consumption in this period barely changes). Since the nominal interest is below the steady state level, the labor demand curve will be everywhere to the right of the steady state level, but to the left of the labor demand curve in the previous period. The labor supply curve does not move much from the previous period because consumption is essentially unchanged. The shift in labor demand exceeds the shift in labor supply since employment is above its steady state

[11] Prices will still be above their steady state level, but the drop in prices is enough to lower the nominal interest rates.

[12] Because consumption in this period is above the steady state level of consumption the labor supply curve will lie everywhere to the right of the steady state labor supply curve.

Timing:

1. Households decide how much to allocate to purchase cash goods and to savings.

STATE OF THE WORLD IS REVEALED

2. Savings are deposited in the financial intermediary.

3. Firms borrow funds, hire labor and produce the consumption goods.

4. The shopper purchases the consumption goods. The cash goods are purchased using cash. The credit goods obtained and the household pays for these goods at the end of the period when the household reunites and pools its resources.

5. Firms repay their loans.

6. Households reunite and pool their resources. These resources include the return on savings, the residuals from the cash-in-advance constraints on consumption goods, wage income, andprofits from the firms and intermediaries.

level. In subsequent periods, the nominal interest rate rises and consumption falls, the former falling moves the labor demand curve back to it original position, and the latter decreasing causes the labor supply to shift back to it original position.

An interpretation of this model is that a monetary injection that exceeds household's anticipated values affects the economy in a way that was described by Friedman (1968). That is, an unanticipated increase in the money supply initially affects goods prices more than nominal wages. In response, the real wage falls. Employment increases because the higher prices reduce consumption which causes the labor supply curve to shift to the right. After the initial impact of the shock, the effects are propagated over time due to the increase in inventories and the price deflation. In addition in this model, the interest rate falls below its steady state level for several periods following the shock. This persistence effect on interest rates in response to an unanticipated increase in the money supply process is also unique to this model[13]. Also, this model predicts the real wage is procyclical after the initial monetary shock.

Section IV: Conclusion

In a general equilibrium model, this paper produces a response of output and employment to a monetary innovation that is consistent with the story outlined by Friedman (1968). What is more, the effects are persistent. That is, output and employment remain above their steady state levels for several period's following the shock. In the period following the shock, the nominal interest falls below its steady state level and remains below its steady state level for four quarters. In addition to getting persistent effects on output, employment and the interest rate, the model also predicts the real wage

[13] Christiano and Eichenbaum are able to get persistence by making it costly for agents to adjust their portfolio. There is no such adjustment cost incorporated in this model.

will be procyclical and inventory investment will be the most volatile of all variables examined. The main shortcoming of the model is that consumption lags the business cycle rather than leading the cycle as the data suggests it does.

Fig 3.1: Change in Interest Rate

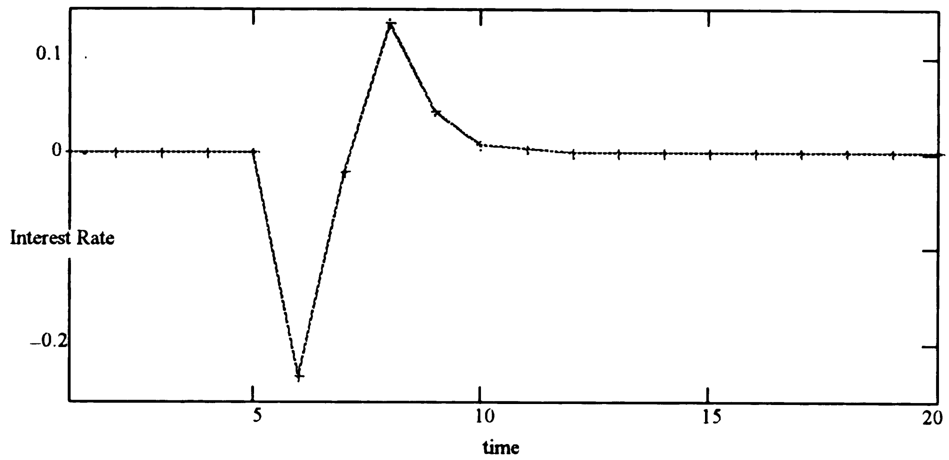


Fig 3.2: Change in Output

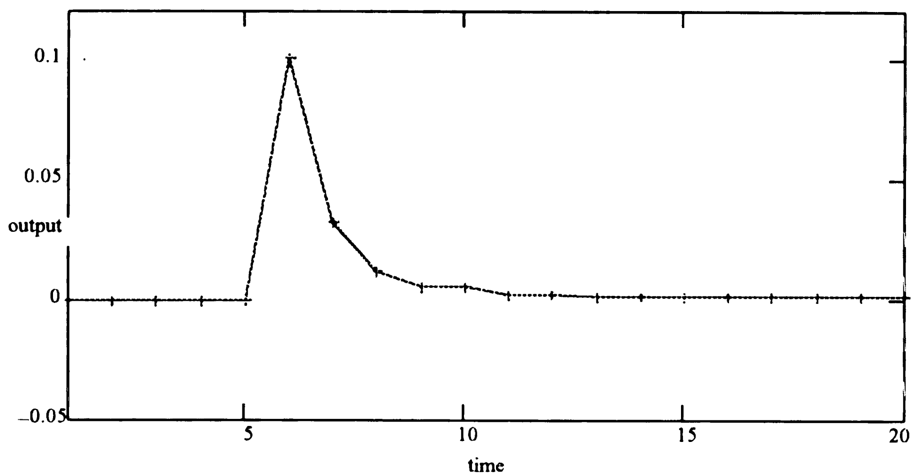


Fig 3.3:Change in Employment

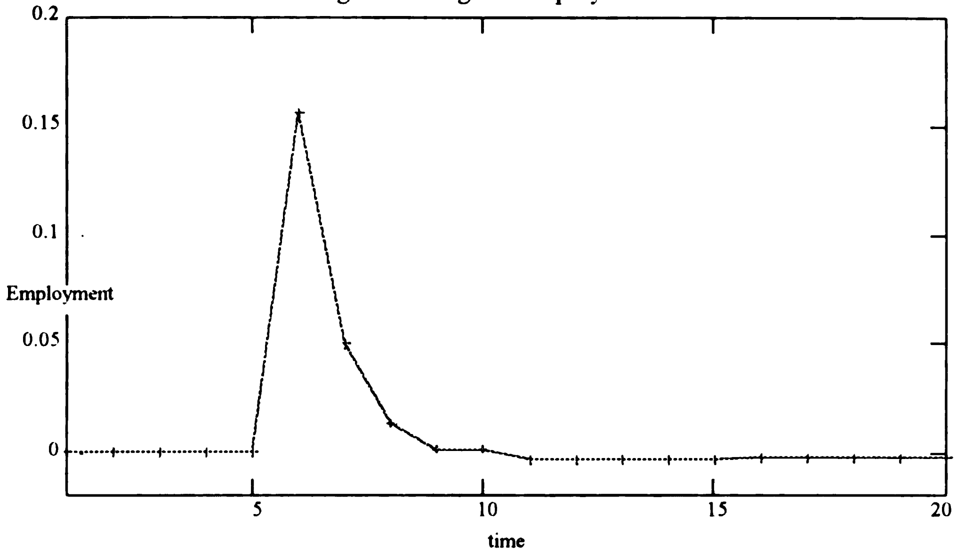


Fig 3.4: Change in Consumption

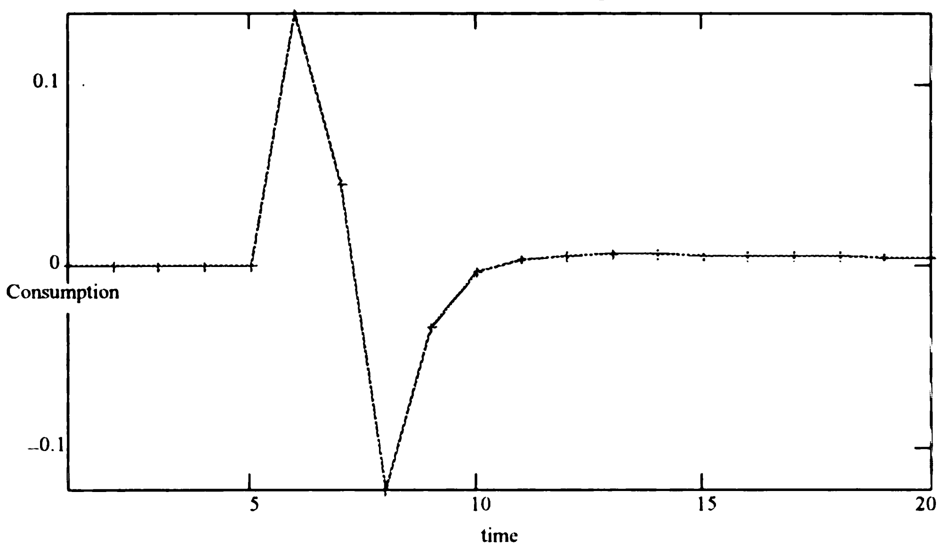


Fig 3.5: Change in Output

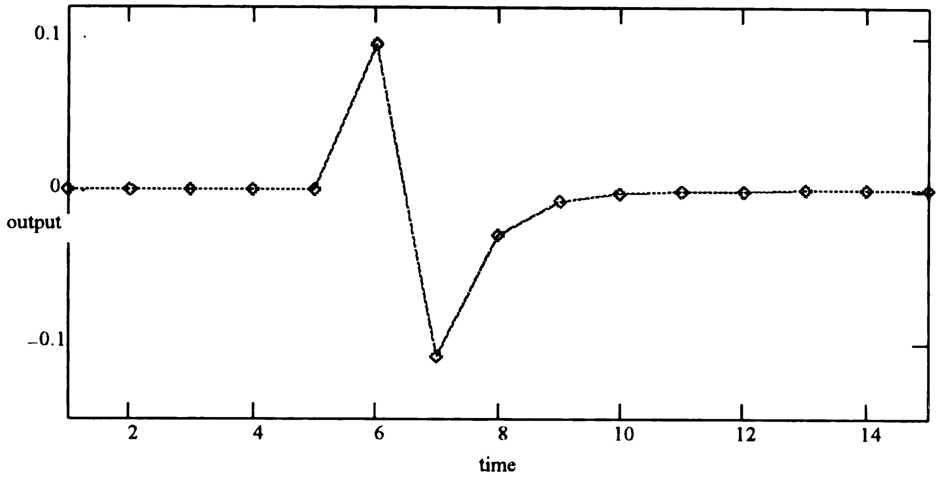


Fig 3.6:Change in Interest Rate

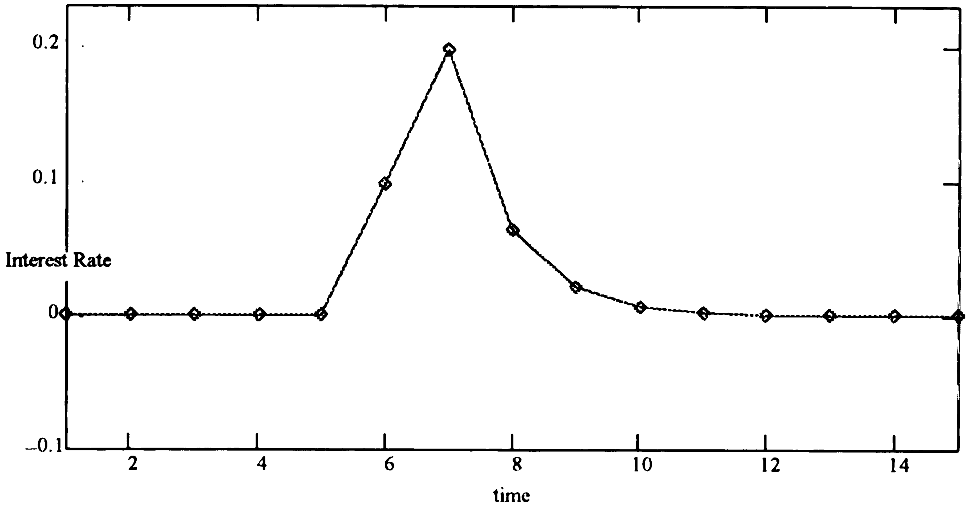


Fig 3.7: Change in Employment

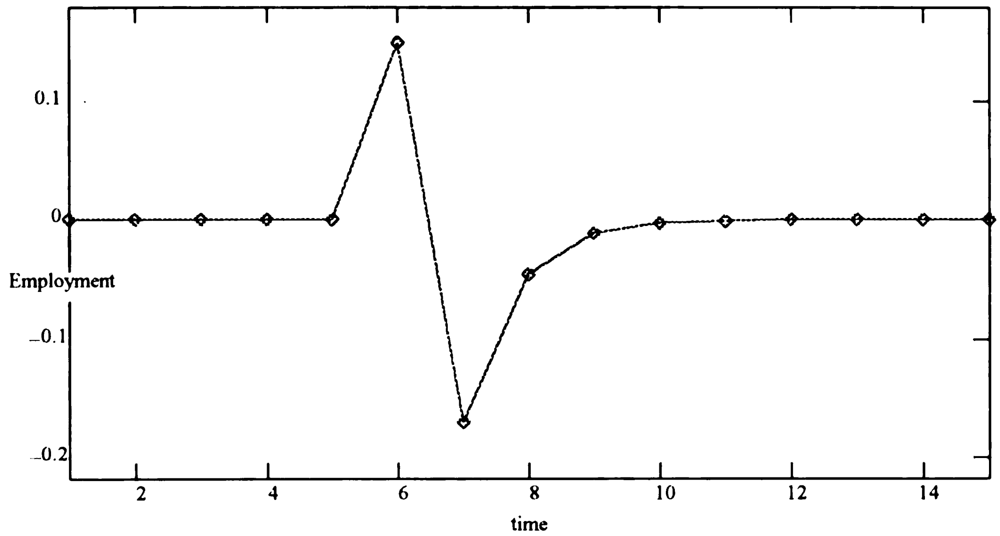


Fig 3.8: Change in Consumption

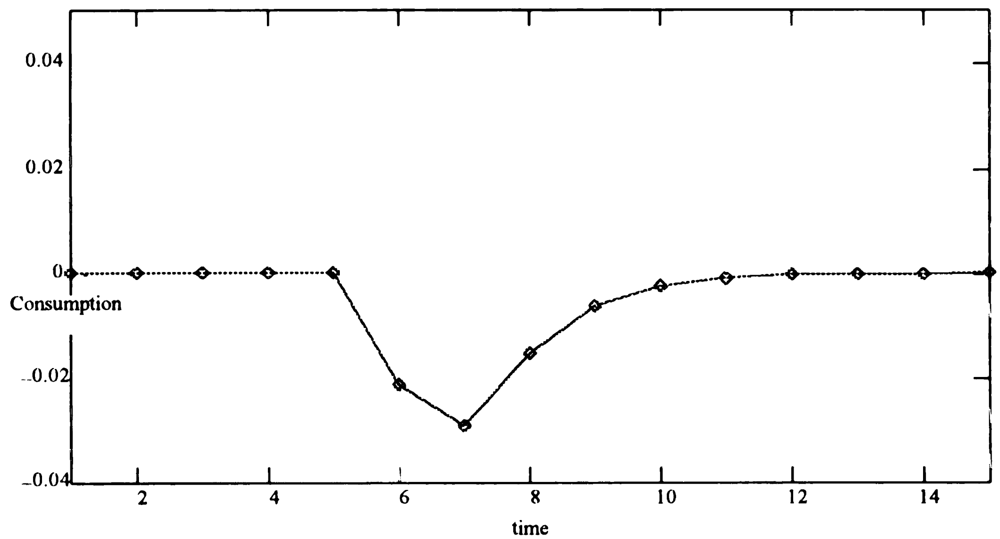


Fig 3.9: Change in Inventories

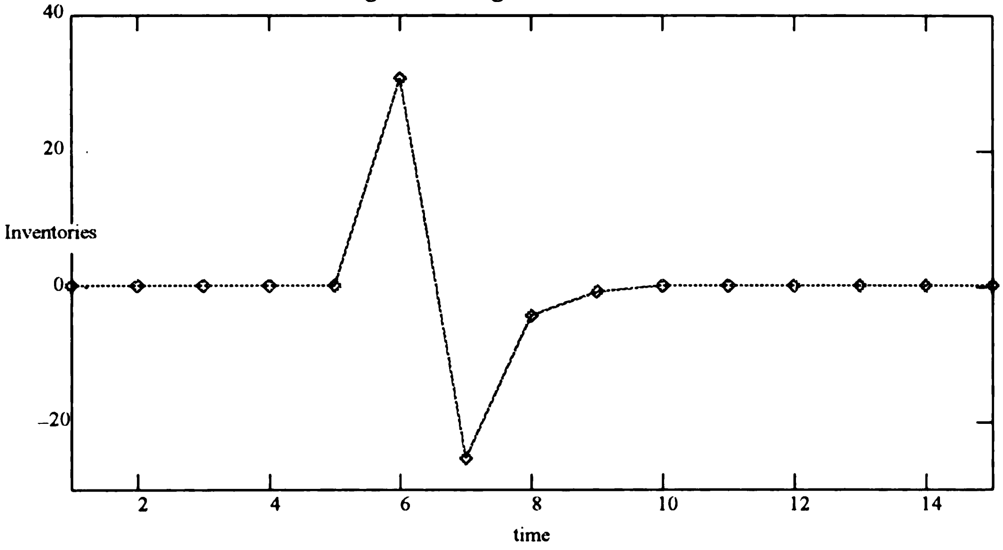


Fig 3.10: Change in Output

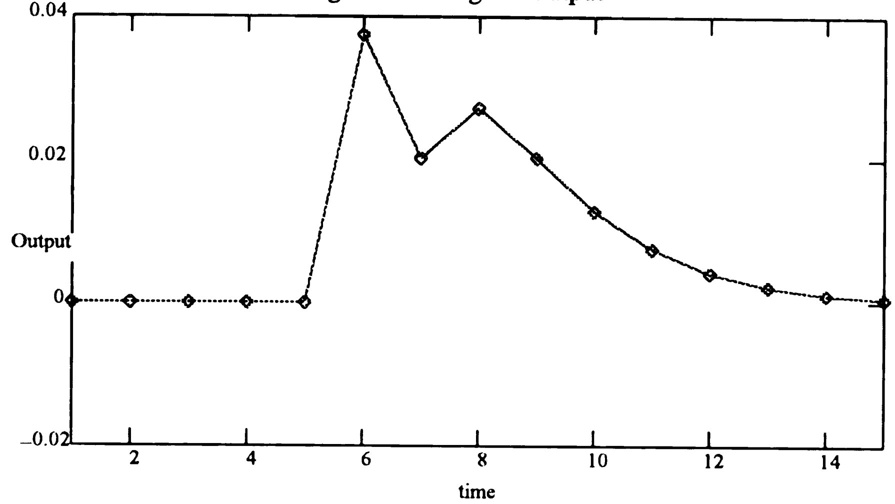


Fig 3.11: Change in Interest Rate

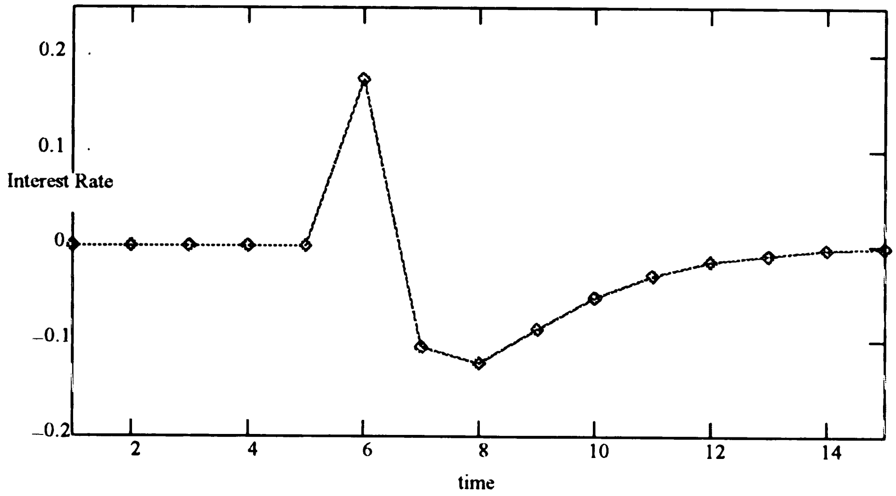


Fig 3.12: Change in Investment

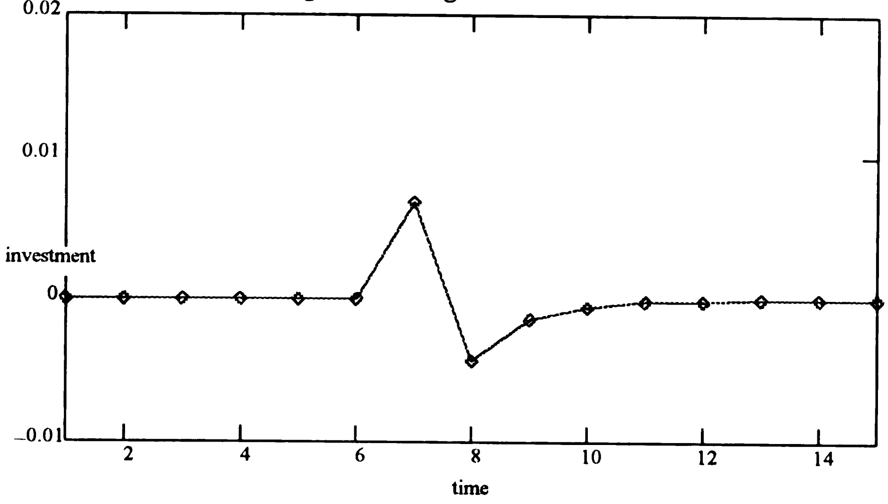


Fig 3.13: Change in Employment

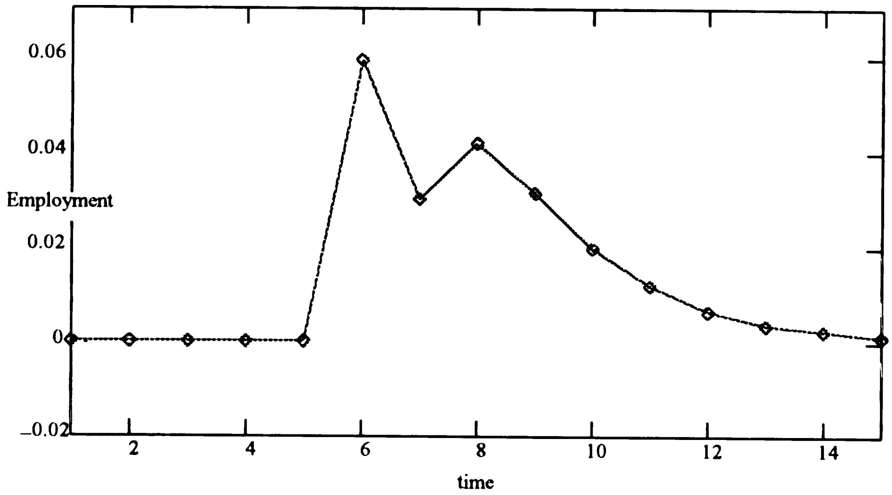


Fig 3.14: Change in Consumption

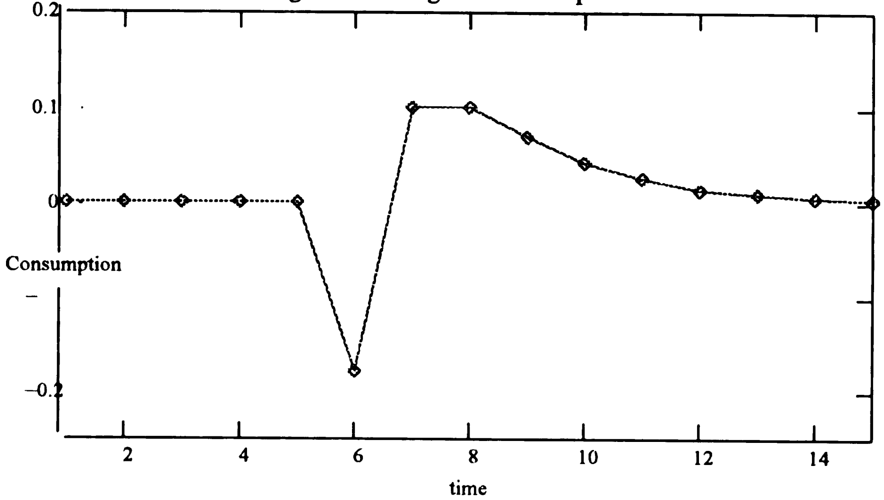
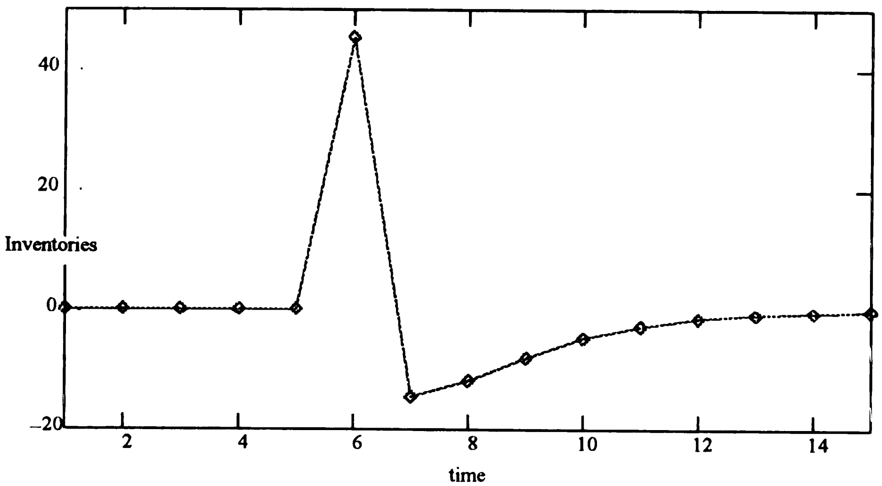


Fig 3.15: Change in Inventories



Chapter 4: Time Series Evidence of Liquidity Effects

In a series of papers, Christiano and Eichenbaum (1992a,b) and Christiano, Eichenbaum and Evans (1994a,b) (hereafter CEE) use VAR models to search empirically for the existence of liquidity effects. Briefly summarizing their results, under their identification scheme, they find liquidity effects do exist and can last for several periods. In this chapter, the identification scheme employed by CEE is discussed and the VAR models are reestimated using quarterly data from 1959.1-1993.4. In order to choose among the competing models, impulse response functions are constructed and the models are judged on the criterion outlined in Eichenbaum (1995). In addition, an alternative detrending procedure is employed and the models are reestimated. The impulse response functions from these models are also evaluated according to Eichenbaum's criterion. If the impulse response functions of these models are qualitatively similar using different detrending techniques and the models do not have "incredible" predictions, then more faith can be placed in CEE's empirical results.

In Section I.1, Eichenbaum's method for evaluating just identified VAR models is reviewed. The models of CEE are also reestimated and evaluated according to Eichenbaum's criterion. In Section I.2, the variables are made stationary by differencing the data rather than passing the data through the Hodrick-Prescott filter as CEE did. In addition to these models being evaluated according to Eichenbaum's criterion, the impulse response functions of these models are compared and contrasted with the impulse response functions of the HP filtered models.

Section II examines other shocks in the CEE specification. The impulse response functions are compared to what theory would predict about these different types of shocks. In addition, the impulse response functions are compared and contrasted with related empirical papers.

Section III concludes.

Section I.1: CEE Model

In order to verify or refute an empirical model or the claims of a theoretical model, the models must pass a battery of tests. It is not unusual to require that a theoretical model be consistent with the data on all dimensions, also it is typically required that the empirical results come from a system of economic or reduced form equations that arise from some "well-specified" theoretical model. The problem with the former is that a theoretical model is a simplified version of reality and therefore is likely to be misspecified in some way or another. The problem with the latter is that it is possible to work backward from any empirical results to some theoretical model. More to the point, there is an identification problem associated with most empirical models and the identifying assumptions that are required to "disentangle" the different shocks are seldom innocuous. The problem of blending theory and econometrics is not particular to macroeconomics. However, it seems to be more evident in this branch of the science. Given this, Eichenbaum (1995) addresses this problem and suggests guidelines for theorists and macroeconometricians to follow. For the macrotheorist, Eichenbaum cites Lucas (1980) as the basis for evaluating models. That is, suppose it is known how certain economic variables react to a certain type of shock, then, at a minimum, the responses of the variables to a shock in the theorists' abstract economy should look like the responses of the variables in the real economy to the same type of shock. The more a model can answer simple questions correctly the more faith that can be placed in the model's ability to answer more difficult questions. To apply this type of test to an artificially constructed economy, theorists need to know how the economy responds to certain types of shocks. Additionally , the theorists require some type of metric to compare the results of their models to how the actual economy responds to the same type of stimulus. To quantify how the economy responds to "simple" shocks is one of the jobs of the econometrician.

In attempting to gauge qualitatively and quantitatively the effects of different types of shocks, Eichenbaum states the econometrician must first decide which variables to include in her model. That is, using Eichenbaum's example, if the researcher wants to investigate how the economy responds to increases in the money supply caused by actions taken by the monetary authority, higher order monetary aggregates (say M1 or M2) may be inappropriate. These higher order aggregates, Eichenbaum argues, may be inappropriate because they also capture changes in the money supply due to actions taken by financial institutions and demand side factors. Narrower monetary aggregates, such as the monetary base or nonborrowed reserves, are likely to be less contaminated.

In addition to the choosing the "right" variables to include, Eichenbaum states that if different models, employing different identifying restrictions, yield similar answers to a particular question then some guidance is given for the construction of theoretical models. The theorist's model should be consistent with this "robust" empirical finding. Therefore, one way to aid the macrotheorist is for the macroeconometricain to conduct sensitivity analysis by employing different identifying restrictions and see how sensitive the results are to different econometric models.

Another check of the model Eichenbaum suggests is for the econometrician to ask: are the responses of the variables to innovations "incredible"? For example, suppose a shock causes output to rise for several periods and the model predicts that the unemployment rate also increases. This model is discarded because of its incredible prediction--since in reality unemployment is countercyclical not procyclical.

Given Eichenbaum's guidelines for setting up econometric models and determining the plausibility of the model's predictions, I reexamine the models studied in CEE using the guidelines given above. Before the CEE models are estimated and examined, I will briefly summarize the results of similar work by Christiano and Eichenbaum (1992a,b). The purpose of their study is to quantify the effects on interest rates of an unanticipated increase in the money supply. Christiano and Eichenbaum use a four variable VAR model. The data are quarterly observations from 1960.1 to 1993.4. The variables used are the federal funds rate, the natural logarithm of gross domestic product (GDP), the natural logarithm of the GDP deflator (price), and the natural logarithm of a monetary aggregate (M). Among the monetary aggregates experimented with were the natural logarithm of the monetary base (Mbase) and the natural logarithm of nonborrowed reserves (NBRC). They argue that these monetary aggregates seem to be the most appropriate in attempting to quantify the effects of unanticipated increases in the money supply since these aggregates are not as "contaminated" by other supply and demand disturbances as higher order monetary aggregates. Christiano and Eichenbaum's identification scheme employs the assumption that unanticipated changes in the monetary policy rule are some orthogonalized component of the of the monetary aggregate. That is, they use the Wold causal ordering to identify unanticipated money supply shocks. For example, the following Wold causal {M, Fedfun, GDP, Prices} implies the monetary authority's feedback rule does not include contemporaneous values of the federal funds rate, the price level, or GDP, whereas an ordering {Fedfun, Price, M, GDP} implies that the monetary authority's feedback rule includes the contemporaneous values of the federal funds rate and the price level but it does not respond to the contemporaneous value of GDP. Christiano and Eichenbaum use four lags of the variables to remove the serial correlation from the residuals.

In order to make the system stationary, Christiano and Eichenbaum pass the data through the HP filter before estimating their model. In addition to this stationary inducing transformation, they also report the results of their model when the variables are made stationary by removing a piecewise-linear trend, and when the variables are made stationary by diffferencing the data until they are stationary. Christiano and Eichenbaum find that their results are qualitatively similar for the different stationary inducing transformations. Using the ordering {Mbase, Fedfun, GDP, Price} they find that an unanticipated increase in the money supply process leads to a contemporaneous rise in the federal funds rate. The federal funds rate remains "high" for several periods following the shock. Therefore, when unanticipated changes in the monetary base are used as the unexpected component of the money supply process the liquidity effect is not present in the data. That is, unanticipated increases in the money supply process increase the federal funds rate rather than lowering it.

Christiano and Eichenbaum argue that orthogonalized components of the monetary base may not be the appropriate variable to use as the unexpected component of monetary policy. Their reason for this is as follows: liquidity effects are believed to arise through the purchase of securities by the Federal Open Market Committee (FOMC). The decisions of the FOMC are made largely independent of the amount of borrowed reserves. Therefore, the appropriate measure of unanticipated policy should not include the transactions carried out at the discount window. Because of this, Christiano and Eichenbaum argue that the unanticipated component of nonborrowed reserves is a better measure of unexpected changes in monetary policy.

Using the ordering {NBRC, Fedfun, GDP, Price}, they find that the interest rate remains below its pre-shock level for 16 quarters. Thus, Christiano and Eichenbaum show that by choosing a narrower definition of money (NBRC) unanticipated increases in the money supply process lower interest rates. In addition to these findings, Christiano and Eichenbaum investigate a different type of unexpected change in the monetary authority's policy. Following Sims (1991) and Bernanke and Blinder (1990), they argue that innovations in the federal funds rate may be an appropriate measure of unexpected monetary policy actions. This is true if the monetary authority has been pursuing an interest rate target. Empirically, they find that an unexpected increase in the federal funds rate is accompanied by a reduction in nonborrowed reserves and a drop in output. A reasonable interpretation is as follows. In order to raise interest rates, the FOMC sells bonds in the open market, thereby reducing nonborrowed reserves and the supply of loanable funds. In response to the reduction in loanable funds, interest rates rise. Output falls in response to the higher interest rates.

CEE (1994 a,b) build on the work of Christiano and Eichenbaum. They expand the list of variables in the VAR model to include the log of commodity pieces (PCOM), the unemployment rate (UNEM) and the log of total reserves (TR). CEE work exclusively with data that has been detrended by passing the data through the HP filter. Figures 4.1 - 4.7 show the actual series plotted against the HP-trend. To denote a HP detrended series X a "C" will be placed in front of that series so it will read CX and a T will be placed in front of the variable to denote the HP-trend. The data used are quarterly data from 1959.1-1993.4. Figures 4.8 and 4.9 show the detrended federal funds rate plotted against detrended nonborrowed reserves (Figure 4.8) and plotted against the detrended monetary base (Figure 4.9) along with the contemporaneous correlations. Note that the federal funds rate is negatively correlated with nonborrowed reserves and nearly uncorrelated with the monetary base. Also, the cyclical component of nonborrowed reserves is much more volatile than the cyclical component of the monetary base.

Eichenbaum's guidelines suggest that the variables the researcher chooses should generate the shocks that the researcher wants to investigate. Here interest lies in unexpected monetary policy shocks. Given the narrow definitions of money and the interest rate used, CEE's choice of variables seems reasonable. Another point made by Eichenbaum is that the responses of variables to shocks should not be incredible. A few potentially incredible responses are worth discussing. First, unanticipated increases in nonborrowed reserves should not lead to a reduction in total reserves. It seems unlikely that as a result of financial institutions receiving more funds than expected from the monetary authority, they would end up holding fewer funds. Also, suppose in response to an unanticipated increase in the money supply the unemployment rate and output increases. Given Okun's Law this prediction seems "incredible". Finally, suppose in response to an unanticipated increase in the money supply, interest rates fall, output increases and prices fall. This price deflation is not inconsistent with some theoretical models. For this response to be plausible the interest rate must affect the supply side more than the demand side. Thus, a temporary price deflation would not be "incredible". However, most economists believe money is neutral in the long run. Therefore, it should be the case that after a period of time prices would rise. A model that did not predict this would also be discarded.

CEE achieve identification by the Wold causal ordering. This identification scheme leaves open the question: Which ordering is correct[1]? Before proceeding, I will describe why commodity prices were included in the model. CEE's reasoning for the inclusion of the commodity prices is as follows. CEE assert the Federal Reserve has likely reacted to movements in commodity prices. In particular, CEE claim the Federal Reserve has responded to changes in the price of crude oil. As a result of including commodity prices in the Federal

[1] CEE only report results when unanticipated increases in the money supply lower the federal funds rate. It should be noted that this result is not robust to all specifications. Typically, if the federal funds rate is placed before the monetary aggregate an unanticipated increase in the money supply process causes interest rate to increase. CEE discard these models based on the "incredible" responses of other variables in the system.

Reserve's reaction function, the "price puzzle" (as labeled by Sims (1991)) vanishes. That is, Sims (1991) found that in response to monetary policy innovations prices responded in a manner that was counterintuitive. Sims also found that in response to unanticipated increases in the federal funds rate, output fell for several periods and prices rose for several periods. One would expect prices to fall in response to a shock that causes a sustained reduction in output. This finding was peculiar enough that Sims labeled this fining as the "price puzzle".

To illustrate how the price puzzle disappears with the inclusion of commodity prices, consider the a six variable VAR without commodity prices. One of CEE's orderings excluded commodity prices. The ordering of the variables CEE used is as follows {CGDP, CPRICE, CNBRC, CFEDFUN, CTR, CUNEM}. The unanticipated component of the money supply process is the orthogonalized component of nonborrowed reserves. The response of the variables to a one standard deviation shock to the money supply process is depicted in Figure 4.10 row 3. First, notice that in response to the monetary innovation the federal funds rate falls by 33 basis points and total reserves increase. Both of these responses are significantly different from zero. In response to the increase in nonborrowed reserves, there is a price deflation that lasts two and a half years. This response, however, is not significantly different from zero. It seems that prices should rise in response to an increase in the money supply. The point estimates, however, indicate prices will fall for several periods. The response of prices in this model to a monetary innovation is not consistent with conventional wisdom.

Next consider the dynamic response of variables in a seven variable VAR model employed by CEE that includes commodity prices. The ordering of the variables is {CGDP, CPRICE, CPCOM, CNBRC, CFEDFUN, CTR, CUNEM}. Figure 4.11 row 4 depicts the response of the variables to an unanticipated increase in the money supply process. The response of the variables given this ordering seems more plausible. In particular, an unanticipated increase in the money supply still lowers the federal funds rate (by 18 basis points). The point estimates of output show that output increases in response to the monetary innovation. However, these point estimates are not significantly different from zero. Prices rise in the period after the shock, but these responses are not significant. The unemployment rate, however, increases for two periods before falling. This seems peculiar especially since the point estimates of output indicate output is rising. I think this prediction of output and unemployment moving in the same direction is peculiar enough to discard the model on the grounds that it gives incredible predictions. The rest of this sub-section experiments with different orderings to seek plausible models of the effects of unanticipated increases in the money supply.

For brevity, I investigate three other orderings used by CEE. I first place CNBRC first so that the ordering is given by {CNBRC, CGDP, CPRICE, CPCOM, CFEDFUN, CTR, CUNEM}. This ordering implies that the contemporaneous values of all variables are excluded from the monetary authority's feedback rule and the unanticipated increases in the money supply affect all variables in the current period. Figure 4.12 row 3 depicts the response of the variables in the system to an innovation in the money supply process. In this case, the federal funds rate falls by nearly 27 basis points in the period of the innovation. The response shows that interest rates will remain significantly below their pre-shock level for three quarters. Output falls while the unemployment rate increases. The response of prices is not significant. Is this plausible? Consider an IS-LM model with a Lucas AS curve. Suppose there was an unanticipated increase in the money supply process. If prices do not immediately adjust to keep real money balances unchanged, interest rates will fall, shifting out the LM curve. The aggregate demand shifts to the right because of the shift in the LM curve. Thus, output increases and, in time, prices rise. This version of the IS-LM model is not consistent with the impulse response functions discussed above. Consider the prototype monetized RBC model. An unanticipated increase in the money supply acts as a tax on labor income so workers supply fewer hours and employment and output fall. In the monetized business cycle model, prices increase for two reasons. The first is because the increase in the money supply puts upward pressure on prices. The second is due to the reduction in output. Interest rates, in this model, also increase. Now consider the limited participation models of Lucas (1990) and Fuerst(1992). In these models, unanticipated increases in the money supply lower interest rates. Output and employment increase. Therefore, the empirical results of this model are not consistent with any of the standard models that are "well equipped" to discuss the affects of monetary policy shocks.

The next ordering experimented with places unemployment first. That is, the ordering {CUNEM, CGDP, CPCOM, CPRICE, CNBRC, CFEDFUN, CTR}. This implies that the money rule responds to the contemporaneous values of the unemployment rate, output, and prices. Also, innovations in the money supply process only affect the contemporaneous values of total reserves and interest rates. The results of this experiment are depicted in Figure 4.13 row 5. Using this ordering, an unanticipated increase in the money supply significantly lowers the interest rate for two quarters. Initially interest rates fall by 15 basis points and remain below their pre-shock level in the subsequent period. Thus the liquidity effect lasts roughly two quarters. In periods following the innovation, the point estimates show that the unemployment rate falls and output increases. More importantly, this ordering yields results that are consistent with the limited participation models and with IS-LM/AS-AD models. However, the point estimates of the unemployment rate, output and prices are not significant.

The last ordering investigated puts the federal funds rate before nonborrowed reserves. The ordering used with this specification is {CGDP, CPRICE, CCOM, CFEDFUN, CNBRC, CTR, CUNEM}. The impulse response functions using this ordering are depicted in Figure 4.14 row 5. With this identification scheme, unanticipated increases in the money supply cause output and prices to rise in the period after the shock. Again it is the case that these responses are not significant. Interest rates fall by almost seven basis points. However, in the period of the shock the unemployment rate increases. This increase is significantly different from zero. Also the point estimates show that the unemployment rate will increase for three quarters after the shock. The fact that output and the unemployment rate move in the same direction for several periods casts doubt on the plausibility of this model.

In sum, when orthogonalized components of nonborrowed reserves are considered as the unanticipated component of monetary policy, the ordering that is "most" consistent with existing theoretical models of unanticipated increases in the money supply is {CUNEM, CGDP, CCOM, CPRICE, CNBRC, CFEDFUN, CTR}. The point estimates of this model are consistent with a version of the IS-LM/AS-AD model and with limited participation models. Other specifications investigated were discarded due to implausible implications of the models.

The next set of experiments conducted by CEE investigates the effects of unanticipated changes in the federal funds rate. They pursue this strategy because Bernanke (1990), McCallum (1983), and Sims (1991) have argued that for the majority of the post World War II period the Federal Reserve has been targeting the federal funds rate. Therefore, CEE claim that unanticipated changes in the federal funds rate are good candidates for innovations in monetary policy. CEE use the federal funds rate as the interest rate in their model because the Federal Reserve has more control over this interest rate than most other interest rates where other actors affect the demand and supply curves.

I consider three different orderings of the seven variable VAR. In all cases the orthogonalized component of federal funds rate is the unexpected component of monetary policy. Most treatments in intermediate macroeconomics textbooks state that an unanticipated increases in interest rates due to contractionary monetary policy cause output and prices to fall and the unemployment rate to increase. The limited participation models also predict that output would fall and unemployment would rise in response to an unanticipated monetary contraction that leads to higher interest rates. Monetized RBC models predict that an unanticipated increase in the interest rate is due to increases in the money supply. Employment and output fall in response to the increase in the money supply that drives up interest rates.

The first ordering considered is {CGDP, CPRICE, CCOM, CFEDFUN, CNBRC, CTR, CUNEM}. The dynamic responses of the variables to an unanticipated increase in the federal funds rate are reported in Figures 4.15 row 4. Note that the unemployment rate falls initially before rising in subsequent periods. Output falls and the reduction in output is not significant. In fact, none of the point estimates, with the exception of the federal funds rate, are significant. Notice that prices rise in response to a monetary contraction. This is not consistent with either the IS-LM/AD-AS model or the limited participation models. Because of this, I discard this model because of its incredible predictions.

The next ordering places the monetary policy rule first. The dynamic response of the variables to an innovation in the federal funds rate is depicted in Figures 4.16 row 1. Using the ordering {CFEDFUN, CGDP, CPRICE, CCOM, CNBRC, CTR, CUNEM}, it is easy to see that the impulse response functions are implausible. That is, the model predicts that nonborrowed reserves fall and total reserves increase. If the unanticipated increase in interest rates is caused by the FOMC selling bonds to financial institutions then it seems reasonable to expect to see both nonborrowed reserves and total reserves falling. Note, that this model predicts total reserves and nonborrowed reserves move in opposite directions. This could conceivably occur if the Fed held discount rate constant the federal funds rate increased. If this occurred then the level of borrowings may increase so that total reserves increased and nonborrowed reserves fell. However, the unemployment rate and output move in the same direction. This finding is not consistent with any of the theoretical models of monetary innovations discussed above.

The next ordering investigated is given by {CUNEM, CGDP, CPRICE, CCOM, CFEDFUN, CNBRC, CTR}. Figure 4.17 row 5 depicts the impulse responses of an innovation to the federal funds rate. The point estimates show that the unemployment rate increases, output falls and nonborrowed reserves fall. However, total reserves rise in response to an unanticipated increase in the interest rate. As stated above, this could conceivably happen if the discount rate does not change and the federal funds rate increased by a great deal.

Section I.1: CEE Model with Difference Stationary Data

CEE make the data in their VAR models stationarity by passing the data through the HP filter. In this section, a different stationary inducing transformation is employed. More specifically, the data is differenced until it is integrated of order zero. Using these transformed series, the VAR models in the previous section are re-estimated and the impulse response functions are evaluated using Eichenbaum's criteria. Unit root tests were done on all variables. The results are reported in Appendix 4.1. Using the Augmented Dickey Fuller tests, it is shown that the data are I(1) with the only exception being prices which are I(2).

Before the VAR models are estimated it is instructive to look at HP filtered series plotted against the difference data. Figures 4.18-4.25 depict the difference data (denoted with a D) and the HP filtered data (denoted as before with a C). Also, the correlation coefficients for each series is given. Notice in all cases that the HP filtered data and the difference data are positively correlated. The correlation coefficients range from as high as .31 for the federal funds rate and as low as .11 for prices.

As before, the first set of orderings in the VAR specification is used to examine the effects of unanticipated changes in the monetary policy when the unanticipated change is defined to be the orthogonalized component of non-borrowed reserves. Again five lags of each variable are used to remove the serial correlation from the residuals. The first ordering is {DGDP, D2PRICE, DCOM, DNBRC, DFEDFUN, DTR, DUNEM}. This ordering implies the money supply rule depends on the contemporaneous values of the change in GDP, the change in inflation, and the change in commodity prices. In addition, unanticipated changes in the money supply will not affect these variables in the current period. Figure 4.26 depicts the response of the variables to a one standard deviation shock to DNBRC. In this case, the federal funds rate falls by 23 basis points and total reserves increase. The change in output and inflation increase with a lag, but these point estimates are not significant. Thus the behavior of the variables to a shock to DNBRC is similar to the model with HP filtered data. As in the VAR model with HP filtered data, the unemployment rate increases. Again, this finding seems peculiar especially since point estimates indicate that GDP increases.

The next ordering experimented with is {DNBRC, DGDP, D2PRICE, DFEDFUN, DTR, DUNEM}. Using this ordering unanticipated increases in the money supply (innovations to DNBRC) will affect all variables contemporaneously. In this case the interest rate falls by 29 basis points (compared to 27 basis points in the model with HP filtered data), the unemployment rate increases and output falls (as in the model with HP filtered data). This finding seems implausible. Certainly the three theoretical models discussed above do not yield this prediction. Therefore, I discard this model due to its incredible predictions. The impulse response functions for this specification are depicted in Figure 4.27.

The ordering that was most "successful" concerning the effects of unanticipated increases in the money supply with HP filtered data is also the most consistent with differenced data. The dynamic response to an unanticipated monetary shock is depicted in Figure 4.28. An innovation to DNBRC with the ordering {DUNEM, DGPD, D2PRICE, DCON, DNBRC, DFEDFUN, DTR} causes the federal funds rate to fall by 20 basis points. In the period following the shock, the unemployment rate falls, output, inflation and commodity prices increase. Thus, the responses of the variables in the model with difference stationary data looks qualitatively similar to the responses with HP filtered data. However, only the point estimates of the federal funds rate are significant.

The last ordering I investigated puts the change in the Federal funds rate before the change in nonborrowed reserves. With the ordering, the federal funds rate falls by almost 12 basis points in the period following the shock. This point estimate is not significantly different from zero. Output, inflation and commodity prices all rise, as does the unemployment rate. These point estimates are also not significant. Therefore, given the point estimates of the responses of the change in output, the change in the unemployment rate behaves in a manner that is "incredible". This ordering using difference stationary data is also qualitatively similar to HP filtered data. In fact, all the results are very similar to the HP filtered results. These results are depicted in Figure 4.29.

Instead of the orthogonalized component of the change in nonborrowed reserves being interpreted as an unanticipated change is monetary policy, the orthogonalized component of the change in the federal funds rule is now considered the unanticipated change in monetary policy. The same orderings will be investigated as when orthogonalized components of the HP filtered federal funds rate were considered as unanticipated monetary policy actions.

The first ordering is {DGDP, D2PRICE, DPCOM, DFEDFUN, DNBRC, DTR, DUNEM}. The response of the variables in the system to a one standard deviation increase in the change in the federal funds rate is depicted in Figure 4.30. As in the model with the HP filtered data, the unemployment rate and nonborrowed reserves initially fall. Since output proceeds the federal funds rate it will not respond to contemporaneous shocks to the federal funds rate. In the period following the shock, output falls and continues to fall in subsequent periods. These point estimates are not significant. The point estimates also show that total reserves initially rise before falling. The findings in this model are not at all consistent with conventional wisdom. That is, if the Federal Reserve wanted to raise interest rates we would expect them to sell bonds thus decreasing total reserves. With this ordering, the exact opposite happens--total reserves increase.

If the federal funds rate is placed first, the ordering is {DFEDFUN, DGDP, D2PRICE, DPCOM, DNBRC, DTR, DUNEM}. Then the response of the variables to an unanticipated change in the federal funds rate is quantitatively similar to the case when HP filter was used. As with the model with HP filter data, output and employment respond in a manner that is "incredible". That is, output increases and unemployment falls. The impulse response functions are depicted in Figure 4.31.

Finally when DNBRC is placed before the change in the federal funds rate and the ordering is given by {DGDP, D2PRICE, DPCOM, DNBRC, DFEDFUN, DTR, DUNEM}. An innovation to the change in the federal funds rate results in responses that are not all consistent with conventional wisdom. In particular, GDP increases, unemployment falls, non-borrowed reserves and total reserves increase and inflation increases. This response is depicted in Figure 4.32.

Section II: Other Shocks in the CEE Model

CEE only report the responses of the variables in their VAR model to what they claim are monetary policy innovations. If their identification scheme is correct then their findings indicate that unanticipated increases in the money supply lower interest rates and may increase output. But how do we know if their identification scheme is correct? One way to check their model is to see how the variables respond to other types of shocks in the system. In particular for the HP detrended data, I focus on three additional shocks. They are shocks to the cyclical component of GDP, the unemployment rate, and total reserves. For the differenced data, I investigate additional shocks as well. If the responses of these variables are credible then CEE may have identified monetary policy innovations. The idea employed here is to apply Lucas' criteria for judging a model. Can the model provide adequate answers to simple questions, that is, are the responses of variables to other shocks in the model credible? If so, then more weight can be placed on the model's ability to answer more difficult questions (how does the economy respond to monetary innovations).

As stated above, I consider the effects of innovations to the cyclical component of GDP, the unemployment rate, and total reserves for the HP filtered data. Because the effects on the variables differ depending on the method used to detrend the variables, the remainder of this section proceeds as follows. In Section II.1, the HP filtered data is analyzed. In Section II.2 the difference stationary is analyzed. In these subsections, I first discuss the effects of these shocks in terms of what theory would predict, then I compare the prediction to the impulse response functions for the different shocks. Section II.3 concludes.

Section II.1

Detrending the data using the HP-filter allegedly removes the estimated long-run trend from the variables. What remains after the detrending are the estimated cyclical components of the data. Therefore, the shocks to CGDP, CUNEM, and CTR should be thought of as transitory shocks to GDP, the unemployment rate, and total reserves respectively. Given that these shocks are transitory, how would the variables in the system respond to each of these shocks? I will discuss each in turn.

Suppose there is a transitory shock that temporally raises GDP for some time, say, a positive technology shock. This positive technology shock would increase the marginal product of labor causing the labor demand curve to shift outward. This would cause an increase in employment, and most likely a reduction in the unemployment rate. If this shock were persistent then firms would increase investment since the marginal product of capital would also be higher. Since the shock is temporary, household's consumption should not change too much. The increase in the supply of output and the increase in the demand for goods due to an increase in investment will have an ambiguous effect on the interest rate. If the increase in the supply of goods is greater (less) than the increase in the demand for goods the real interest rate will fall (rise) (see Barro (1994) p. 231-234). If the real interest rate increases it is likely that banks will lend out more funds. The increase in loans implies banks are drawing down their excess reserves. In sum, a positive technology shock to GDP should decrease the unemployment rate, interest rates should rise and total reserves should fall as banks are more willing to loan funds out due to the higher real interest rate. However, this is not the only story for these responses. Another plausible story is that there was an aggregate demand shock and there is some nominal rigidity in the model so that the aggregate supply curve is upward sloping. Given the identification scheme, there is no way to delineate between a transitory supply shocks and aggregate demand shocks.

Figure 4.11 row 1 depicts the response of the variables to an innovation to CGDP when CGDP is placed first in the ordering. Note the unemployment rate, the interest rate and nonborrowed reserves all fall. The point estimates indicate that total reserves fall. However, this point estimate is not significantly different from zero. Figure 4.12 row 2 depicts the results to an innovation in the cyclical component of GDP when the ordering is given by {CNBRC, CGDP, CPRICE, CCOM, CFEDFUN, CTR, CUNEM}. In this case, the unemployment rate falls, the interest rate increases and the point estimates are significantly different from zero. Total reserves initially rise before falling. The point estimate of the cyclical component of total reserves, however, is not significantly different from zero. Neither of the point estimates for CPRICE and CCOM is significant.

When the ordering is given by {CUNEM, CGDP, CPRICE, CCOM, CNBRC, CFEDFUN, CTR}, an innovation to the cyclical component of GDP, only the response of CGPD is significantly different from zero. The point estimate of the CFEDFUN shows that the cyclical component of the federal funds rate will be above its pre-shock level for two years. These responses are depicted in Figure 4.13 row 2.

Figure 4.14 row 1 shows the responses of the variables when the ordering is given by {CGDP, CPRICE, CCOM, CFEDFUN, CNBRC, CTR, CUNEM}. These impulses indicate that CUNEM falls. In addition to the fall in CUNEM, CCOM and CFEDFUN increase. The point estimates of these variables are significantly different from zero. CCOM may increase as the increase in demand for investment goods increases the prices of these goods. Also, the interest rate will increase if firms attempt to borrow more funds to purchase investment goods.

Figure 4.15 row 2 and Figure 4.16 row 2 depicts the response of the variables to a shock to CGDP when the ordering is given by {CFEDFUN, CGDP, CPRICE, CCOM, CNBRC, CTR, CUNEM} and { CUNEM, CGDP, CPRICE, CCOM, CFEDFUN, CNBRC, CTR}, respectively. In both cases, the responses of the variables are consistent with a positive transitory technology shock or a shock to aggregate demand.

The next shock investigated is a shock to the cyclical component of the unemployment rate. Theoretically, a transitory shock to the unemployment rate should affect variables much like a negative technology shock. That is a shock that increases the unemployment rate should lower output and put downward pressure on interest rates and prices. According to row 7 in Figures 4.11, 4.12, 4.14, and 4.15, none of the responses to a shock to CUNEM are significantly different from zero. Thus, these orderings provide little insight. However, it is possible that a shock to the unemployment rate has completely different effects. Consider a shock to labor force participation where more people decide to look for work, but there is no increase in the number of people who find jobs. For example, an increase in individuals looking for work in the summer because school is no longer in session[2]. In this case the effects from this type of shock may be small. When CUNEM is placed first in the ordering as is the case in Figures 4.13 and 4.16, an innovation to CUNEM decreases output, commodity prices and interest rates. These variables are all significantly different from zero. These results seem plausible. A possible story behind these responses is in response to a shock to the unemployment rate that causes output to fall, firms will produce less and demand fewer inputs. This will lower the demand for inputs and lower prices. In addition, firms will demand fewer loanable funds and the interest rate will fall. If the Fed increases the money supply in response to the negative shock, nonborrowed reserves and total reserves will increase and interest rates will fall even farther.

[2] I thank Robert Rasche for making me aware of this point.

The last shock I investigate is a shock to total reserves. One possible story is that in the absence of any actions by the monetary authority total reserves would increase if banks decide to make fewer loans at all interest rates. That is, if the reserve to deposit ratio increases. If the reserve to deposit ratio increased, total reserves initially would increase. The money supply would eventually fall since an increase in the banks desire to lend funds would decrease the money multiplier. This would have the effect of driving up the real rate of interest. The increased interest rates would lower consumption and investment. Also as a result of the higher interest rates, output would fall and the unemployment rate would also increase. Prices would fall in response to the shocks. As time passes, output would increase and the unemployment rate would fall. Looking at Figures 4.11, 4.12, 4.14, 4.15 row 6 and Figures 4.13 and 4.17 row 7, the response of these variables to an innovation in total reserves is consistent with this story.

In sum, these orderings are consistent with the conventional wisdom about how the economy responds to transitory shocks to GDP, the unemployment rate, and total reserves. When unemployment is placed first in the ordering, a transitory shock to the unemployment rate leads to responses that are significantly different from zero. Recall in Section I, this ordering yielded the most credible results. However, even if this is the correct ordering, the liquidity effect is smallest in this case (interest rates fall by only 8 basis points) and the effects on output and the unemployment rate are not significantly different from zero.

Section II.2

In this section, I investigate the total effect on the seven variables to different shocks in the system. These responses are depicted in Figures 4.32 to 4.40. As in the last section, the responses of the variables are compared to what different theoretical models would predict. Also, given the advances in time series econometrics, estimating small scale VAR models and reporting the results of the impulse response functions to transitory and permanent shocks has been somewhat of a growth industry. Because of this, there is a growing body of literature that documents how economic variables respond to different permanent and transitory shocks. The impulse response functions of the sum of the effects in the difference stationary version of the CEE model can be compared and contrasted with this literature as well. In particular, are the responses of the variables to shocks to output and shocks to the unemployment rate in this model similar to the findings by other authors who have investigated these types of shocks? In addition to comparing the impulse response functions to other researcher's studies, I draw comparisons between the responses in the CEE model to what theoretical models predict. Finally, I reexamine the shocks CEE called monetary policy shocks to see if the long-run responses of the variables are credible.

Recently there have been several papers investigating the effects of permanent and transitory shocks on real and nominal variables. A few noteworthy contributions whose results I make comparisons to are: King, Plosser, Stock and Watson (1991) (hereafter KPSW), Gali (1992), and Crowder, Hoffman, and Rasche (1996) (hereafter CHR). In KPSW they identify the permanent shocks in a six variable VECM model that includes nominal variables. The permanent shocks are a balanced growth shock (a technology shock), an inflation shock, and a real interest rate shock. They find that a technology shock permanently increases output, whereas, a permanent shock to inflation has no permanent effect on output. Gali (1992) uses a structural VAR model and imposes a priori restrictions on the economic relationships among the four variables in the system. Gali finds that a technology shock increases output by .71 percent initially and 1.1 percent after 20 quarters. Also, the nominal money stock increases in response to the shock, however, the point estimates are not significant. Prices fall initially but then return to their pre-shock level. Also in Gali's model, nominal interest rates hardly move in response to technology shocks. CHR (1996) is a hybrid of the KPSW and Gali models. They do not impose any a priori restrictions on the model. Instead, they test for cointegrating relationships among the variables and use these relationships to help determine the economics of the model. In response to a supply shock, CHR find that output increases permanently. Real balances and the nominal money supply (M1) also end up permanently higher. Prices, however, end up permanently lower. The nominal interest remains significantly below its pre-shock level for roughly 30 periods.

Given these results, how do "technology" shocks in the difference stationary model compare to these other results. There are two candidates for technology shocks in the seven variable difference stationary model. These are shocks to DGDP and DUNEM. I will first discuss the effects of shock to DGDP. The effects of shocks to DGDP are depicted in figures 4.33 row 1, 4.34 row 2, 4.35 row 2, 4.36 row 1, 4.37 row 1, 4.38 row 2, 4.39 row 2 and 4.40 row 1. The results of these models are qualitatively similar so I will restrict my discussions to the impulse responses in Figure 4.33 row 1. An innovation to DGDP results in output being permanently higher. In response to this shock the unemployment rate falls, inflation increases, nominal interest rates rise and nonborrowed reserves fall. Are these findings consistent with the other empirical work cited above? In short, no.

With exception of output, all the variables that are common across models move in opposite directions. In particular, the interest rate in this model increases. In CHR, the interest rate falls and in Gali the interest rate falls slightly. In CHR inflation falls initially. In this model, inflation increases but the point estimates are not significant. Do these contradictions imply that the CEE identification scheme is incorrect? Not entirely. There is no way to explain away the differences in the models; however, there is a plausible theoretical explanation underlying these impulse functions. In response to a permanent technology shock, suppose that individuals increase their consumption because the shock is believed to be permanent and investment increases because the marginal product of capital is higher. As a result, the real interest rate ends up higher. Because the interest rate is higher, the opportunity cost of holding money increases banks and accordingly hold fewer reserves, so total reserves and nonborrowed reserves will fall.

Alternatively, a shock that permanently increases the unemployment rate could be considered a shock that increases the natural rate of unemployment. Such a shock is depicted in Figures 4.33 row 7, 4.34 row 7, 4.35 row 2, 4.36 row 7, 4.37 row 7, 4.38 row 7, 4.39 row 1 and 4.40 row 2. Since the impulse response functions are qualitatively similar I will restrict my attention to Figure 4.35 row 1. In this case, an innovation to the unemployment rate causes the point estimate of the unemployment rate to be higher six years after the shock. Output remains below its pre-shock level for six years after the shock.

The next shock I consider is an inflation shock. The responses of the variables in the CEE model can be roughly compared to the impulse responses in what CHR term an inflationary expectations shock. The sum of the effects of the variables to an inflationary shock is depicted in Figures 4.33-4.40, in the row that is labeled with D2PRICE. Note, in response to an inflation shock output temporarily increases, the unemployment rate falls, interest rate rises, nonborrowed reserves and total reserves fall. These findings are roughly consistent with CHR. In CHR, an inflationary expectations shock temporarily increases output and nominal interest rates. The inflationary expectations shock also lowers the money supply. Thus, CHR reason that in response to an increase in inflationary expectations the Federal Reserve contracts the money supply and as a result the nominal interest rates will rise. If however, the increase in t

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in the nominal interest rate is lower than the increase in the expected rate of inflation, then the real interest rate has fallen and output may increase. Thus, the sum of responses to an inflation shock in CEE model is consistent with the findings in CHR.

The next shock investigated is a shock to commodity prices. The corresponding long-run responses of the variables to a shock to commodity prices are depicted in the row labeled DCOM in Figures 4.33-4.40. A shock to PCOM lowers GDP and the unemployment rate for six years. These point estimates are significantly different from zero. In addition, the federal funds rate rises and total reserves and nonborrowed reserves fall. Also, inflation increases but this response is not significant at any horizon. A plausible explanation for these impulses is a negative supply shock due to an increase in oil prices. This shock could lower output. The unemployment rate would increase. The decrease in supply and the higher oil prices would increase inflation. The higher inflation would increase the federal funds rate and in response to the higher opportunity cost of holding money banks would hold fewer reserves. This explains the drop in total reserves and nonborrowed reserves. Thus, an innovation to commodity prices is consistent with an adverse supply shock[3].

Finally, I look at the sum of effects on the variables that CEE call monetary policy shocks. In Figures 4.33-4.40 the row labeled NBRC shows the long-run effects on the variables to innovation in DNBRC. That is, these figures show what happens to the levels of the variables to different innovations. Note these responses are consistent with conventional wisdom: An increase in the money supply will lower interest rates in the short run, output will increase and prices will rise. In the long run, there will be no effects on any of the real variables. However, the long-run effect on the variables due to an innovation in the Federal funds rate is quite implausible. That is, output and the unemployment rate are "permanently"

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[3] An example of this would be an increase in the price of oil.

affected by monetary policy shocks. One would expect that monetary policy shocks would only have transitory real effects. Because there are permanent real effects, I do not believe that the shocks to the federal funds rate in the difference stationary version of the CEE data can be considered a monetary policy innovation. Rather it is a shock that permanently effects output and the unemployment rate, such as a shock to the real interest rate that is discussed in KPSW.

Section III: Conclusion

This chapter investigated the empirical work of CEE (1994 a,b) and subjected their model to the criteria outlined in Eichenbaum (1995). It was shown that the model produces the most credible results when the unemployment rate is placed first in the ordering. In this instance, unanticipated increases in the money supply caused by monetary policy initially decrease the interest rate. Because CEE only report "monetary policy" shocks it is hard to gauge how credible their results are. Because of this, I investigated other shocks in the system. It was shown that the impulse response functions of the variables to shocks to the cyclical component of GDP, the unemployment rate, and total reserves are credible. Thus, using HP filtered data, an ordering that places the unemployment rate first may provide insights about the effects of unanticipated monetary policy shocks.

Instead of passing the data through the HP filter, I also consider differencing the data until the data are I(0). In this case, the initial response of the variables is qualitatively similar to the HP filtered model. However, some of the long-run responses are not credible. In particular, an innovation to the federal funds rate has an effect on output and the unemployment rate that is significant even six years after the shock. We would not expect a transitory monetary policy shock to have such long-run effects. Also, the long-run responses of the variables are not consistent with the findings reported in Crowder, Hoffman, and Rasche (1996) or Gali (1992).

Because of this, I am skeptical that the Wold causal ordering with difference stationary data can identify monetary policy shocks.

Finally, neither of these transformations schemes are innocuous. Cogley and Nason (1995) and Amsler (1996) argue that passing the data through the HP filter may add a cyclical component to the data. Therefore, the results using the HP filtered data may not tell us anything about the true cyclical component. Also, univariate "detrending" may introduce specification error if some of the data are nonstationary but cointegrated.

Differencing the data to make it stationary may also lead to improper inferences. By differencing the data to make it stationary, the researcher may be ignoring long-run relationships that are present in the data. Thus, the researcher may be throwing out information. In fact, a test for cointegrating relationships among the seven variables GDP, inflation, commodity prices, nonborrowed reserves, the federal funds rate, total reserves, and the unemployment rate, using Johansen's maximum likelihood test, finds four cointegrating relationships at the 5 percent significance level. A study that incorporates these cointegrating relationships may be more successful in identifying monetary policy shocks and if the model can identify a money supply rule, this formulation may be able to answer the more difficult question: what is the effect on other variables when the money supply rule changes?

Figure 4.1: GDP

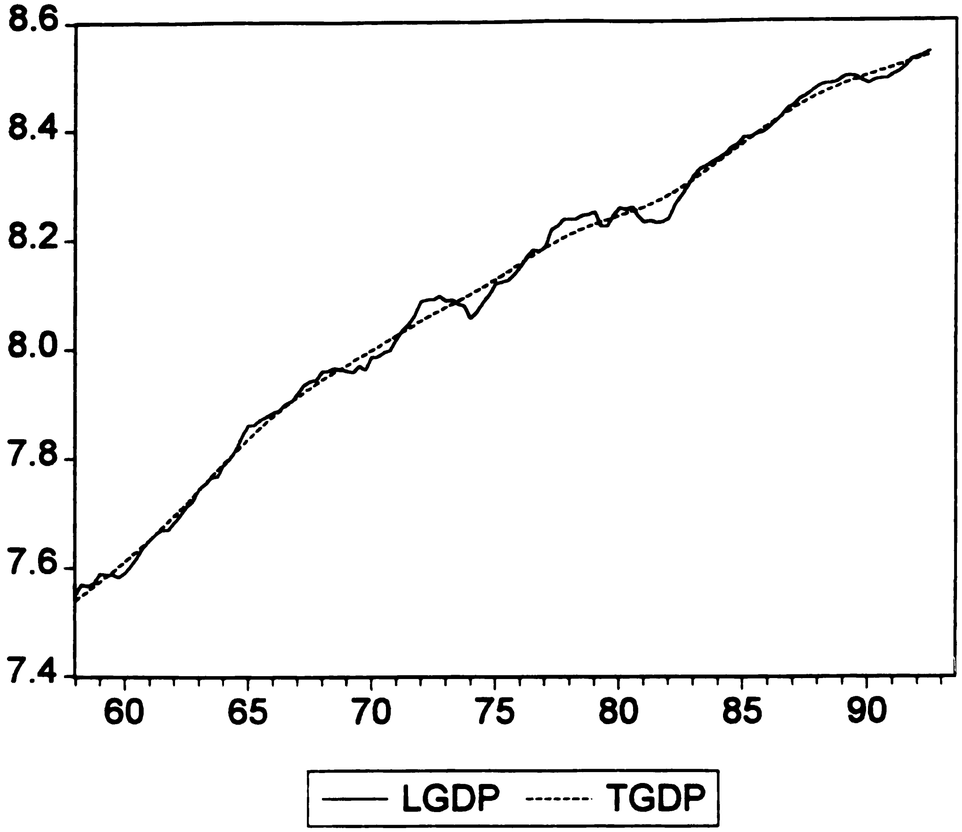


Figure 4.2: Federal Funds Rate

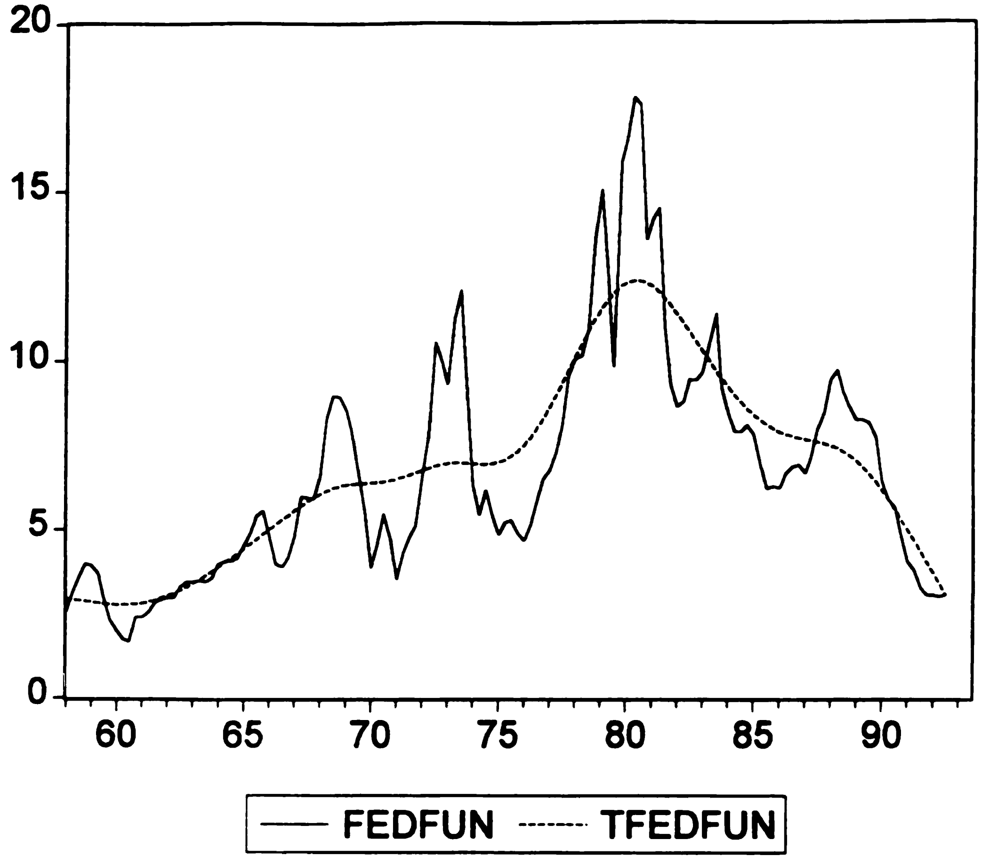


Figure 4.3: Nonborrowed Reserves

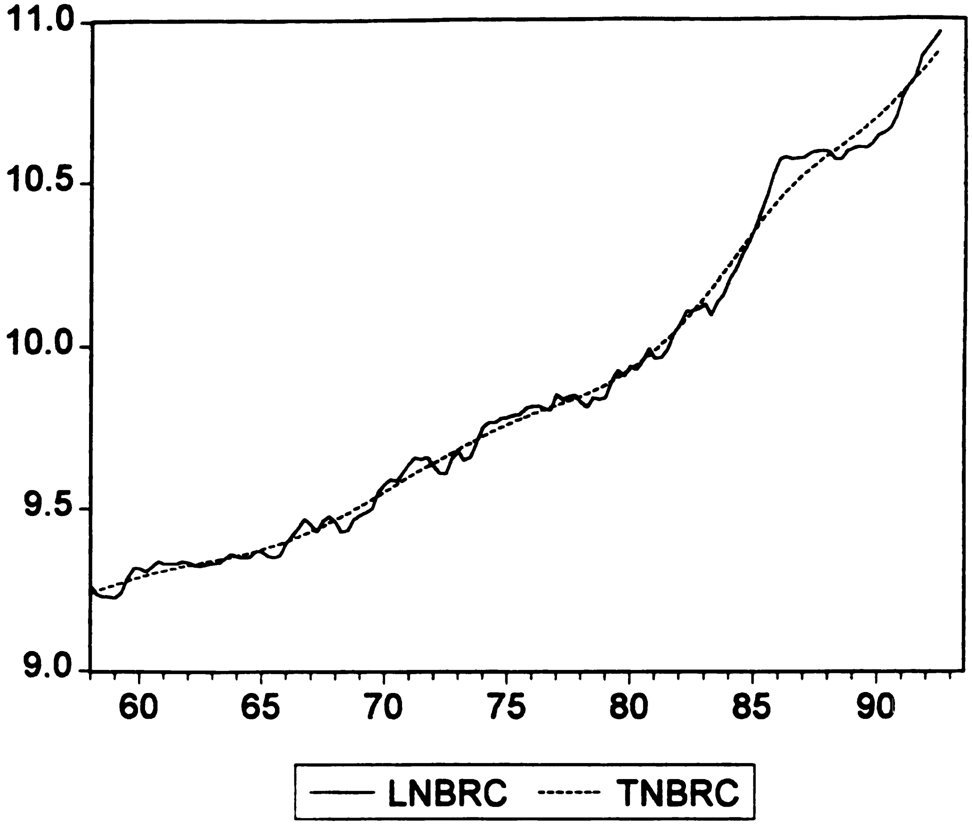


Figure 4.4: Monetary Base

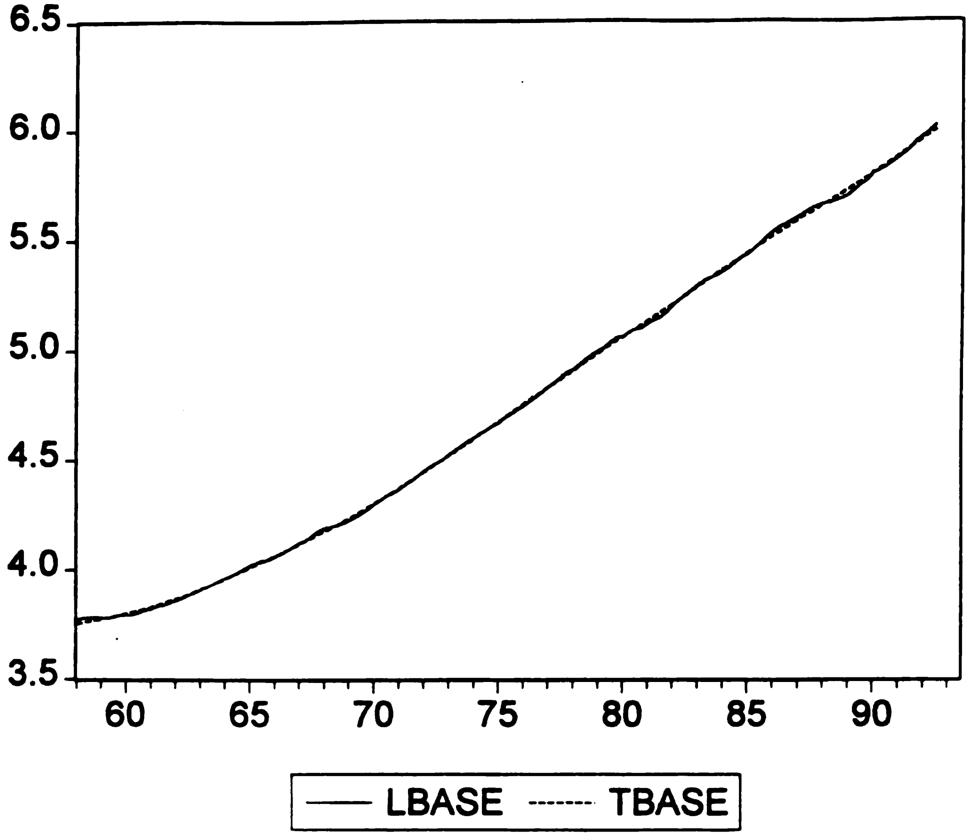


Figure 4.5: Total Reserves

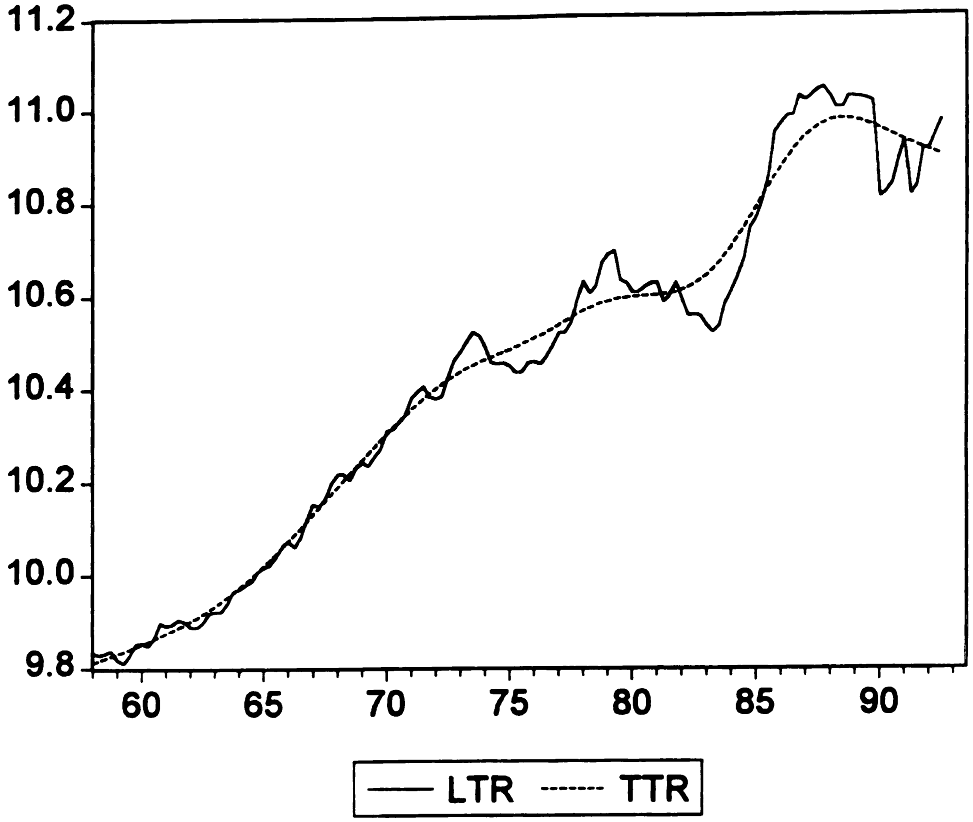


Figure 4.6: Price Level

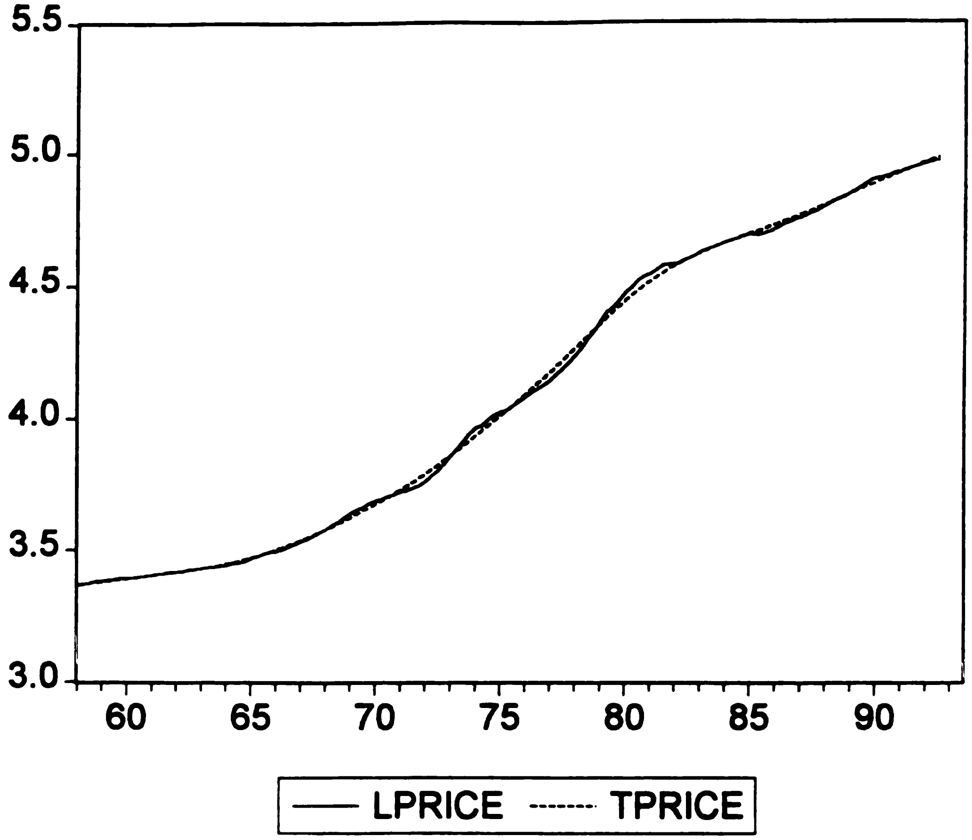


Figure 4.7: Unemployment Rate

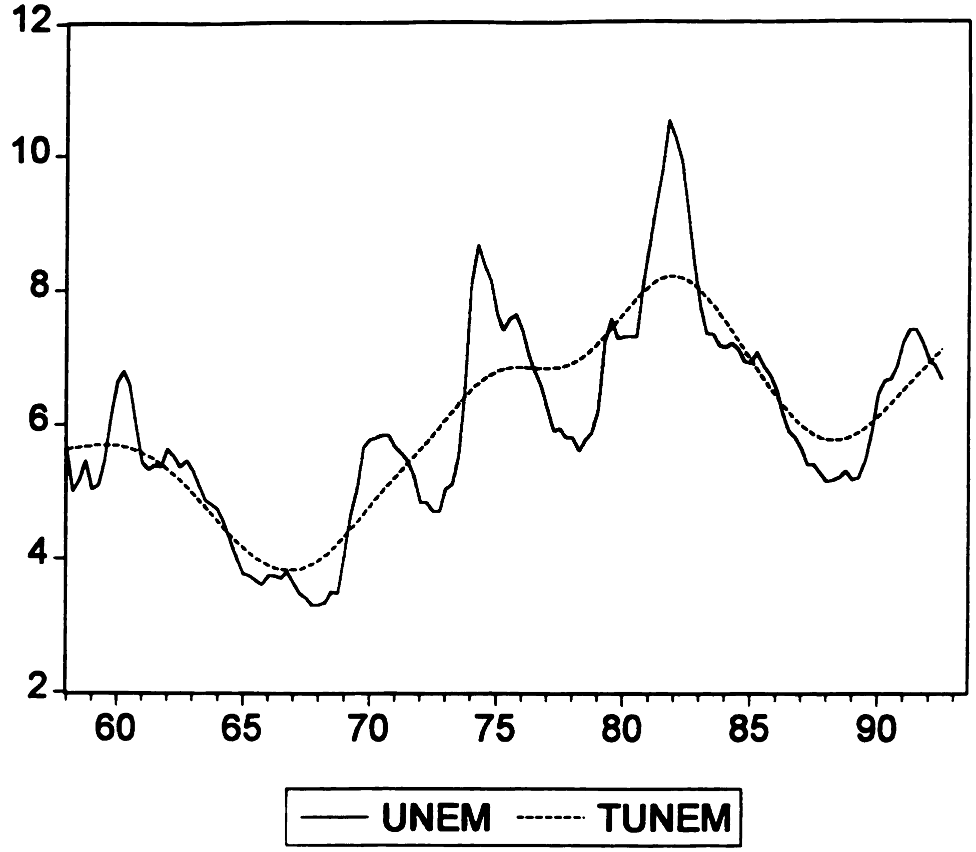
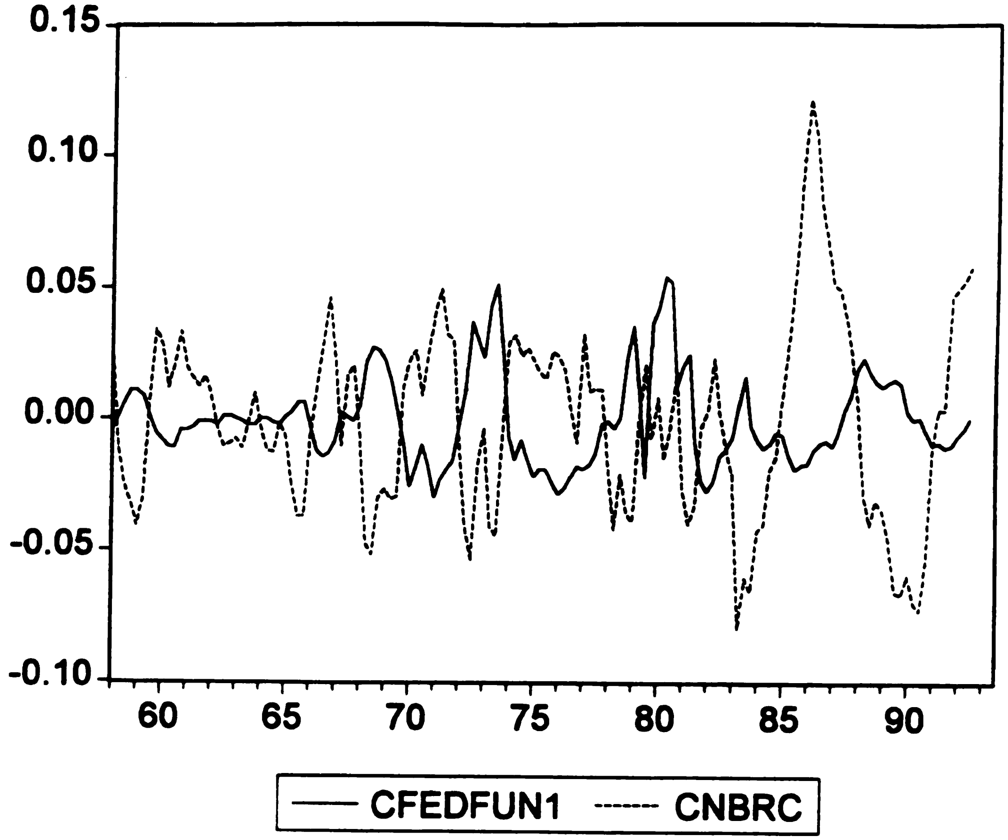
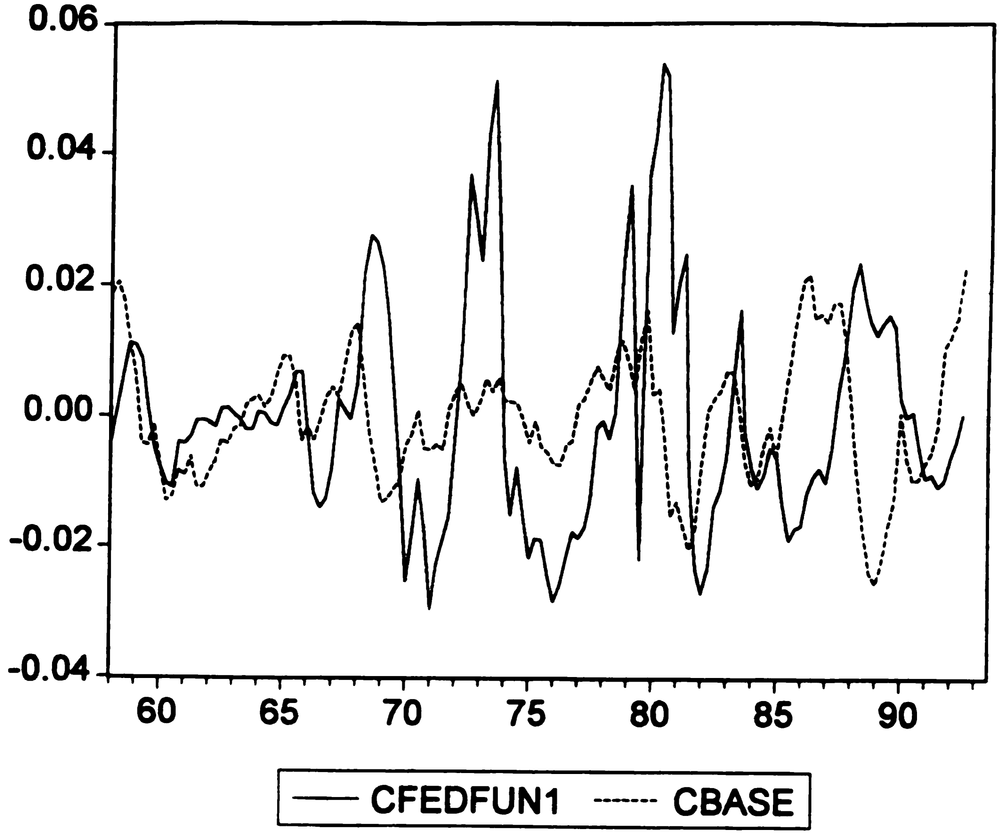


Figure 4.8: Detrended Federal Funds Rate and Nonborrowed Reserves



Correlatiom -.520809

Figure 4.9: Detrended Federal Funds Rate and Monetary Base



Correlation .004421

Figure 4.10

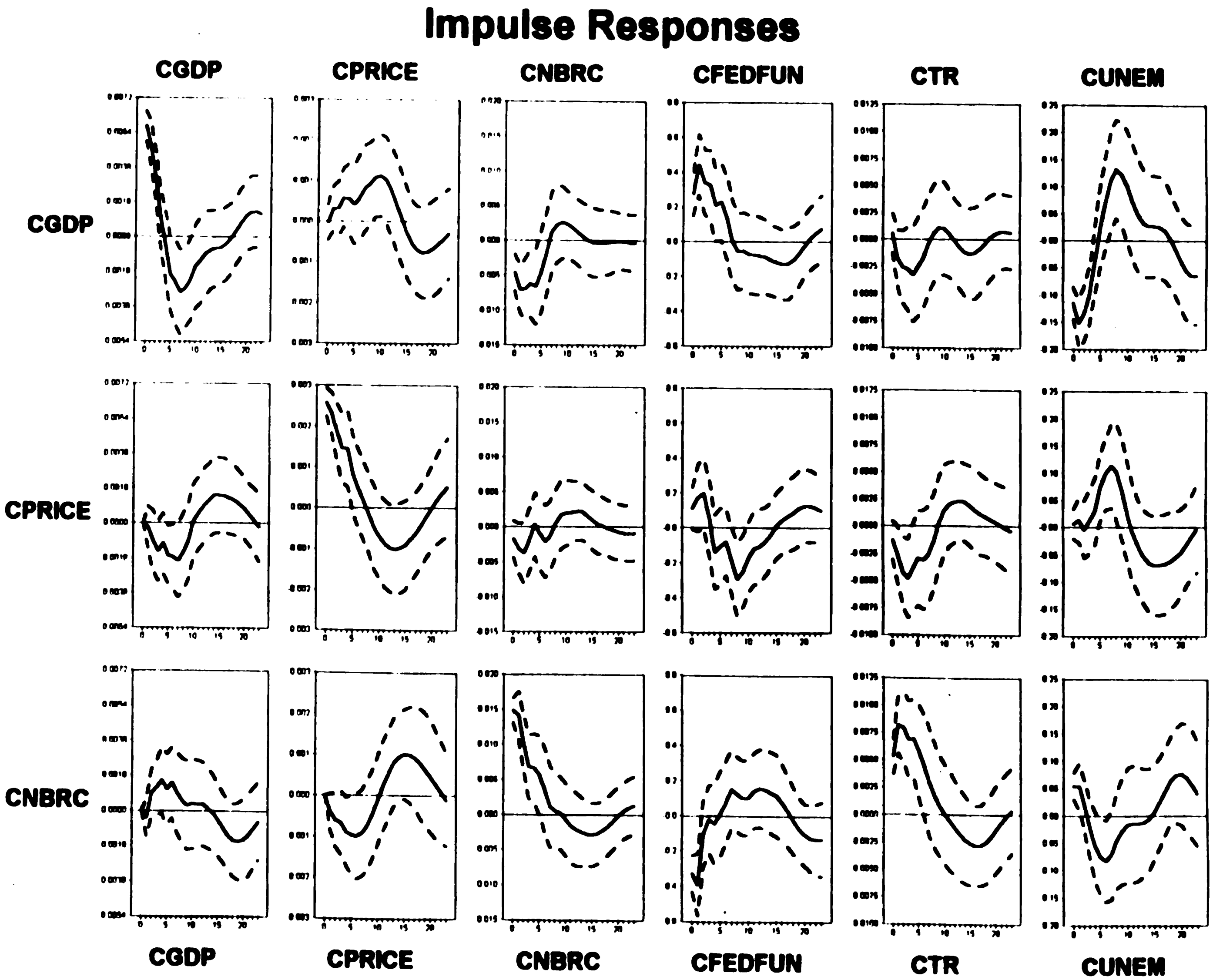

Impulse Responses
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Figure 4.10 (Cont'd)

Impulse Responses

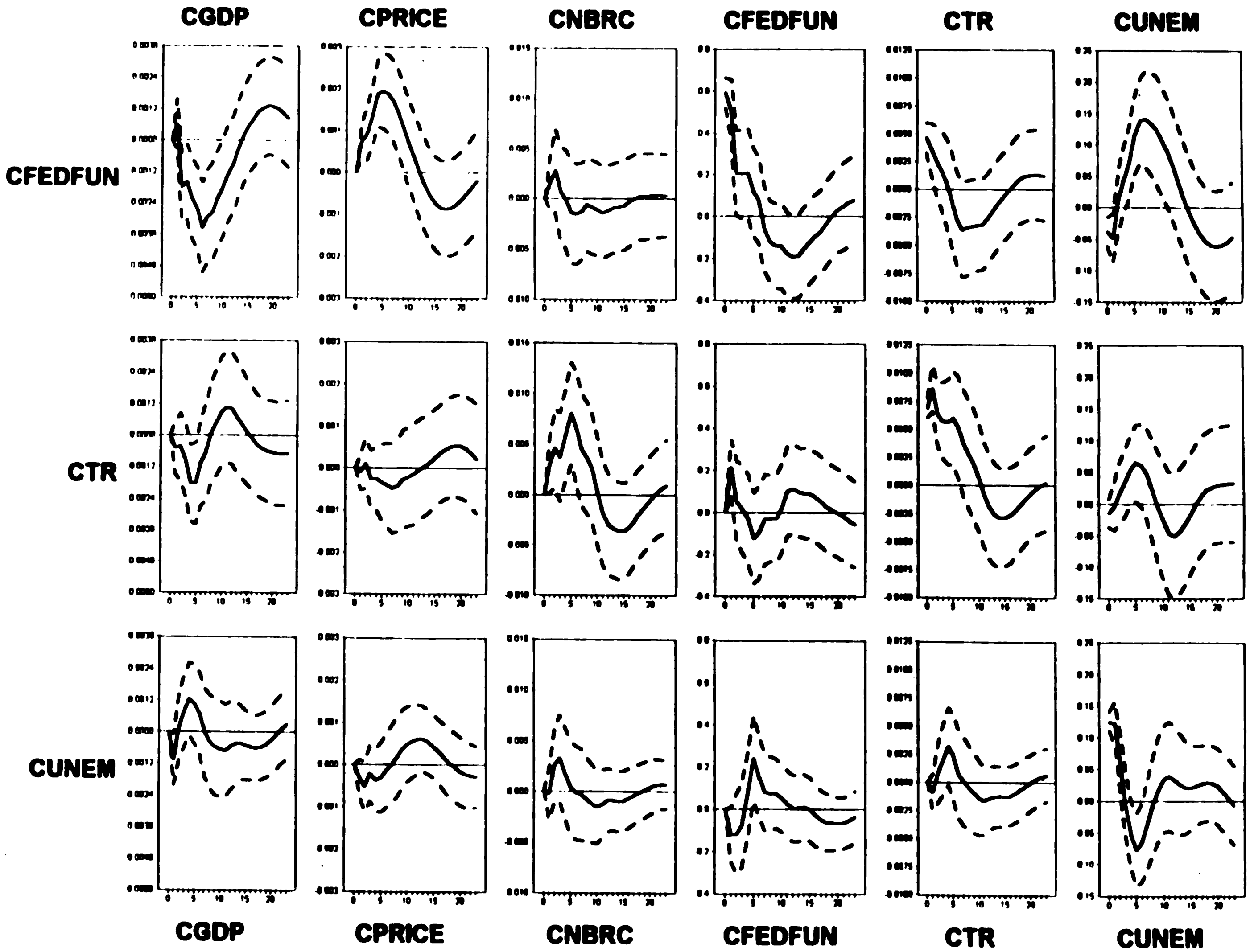

Figure 4.11

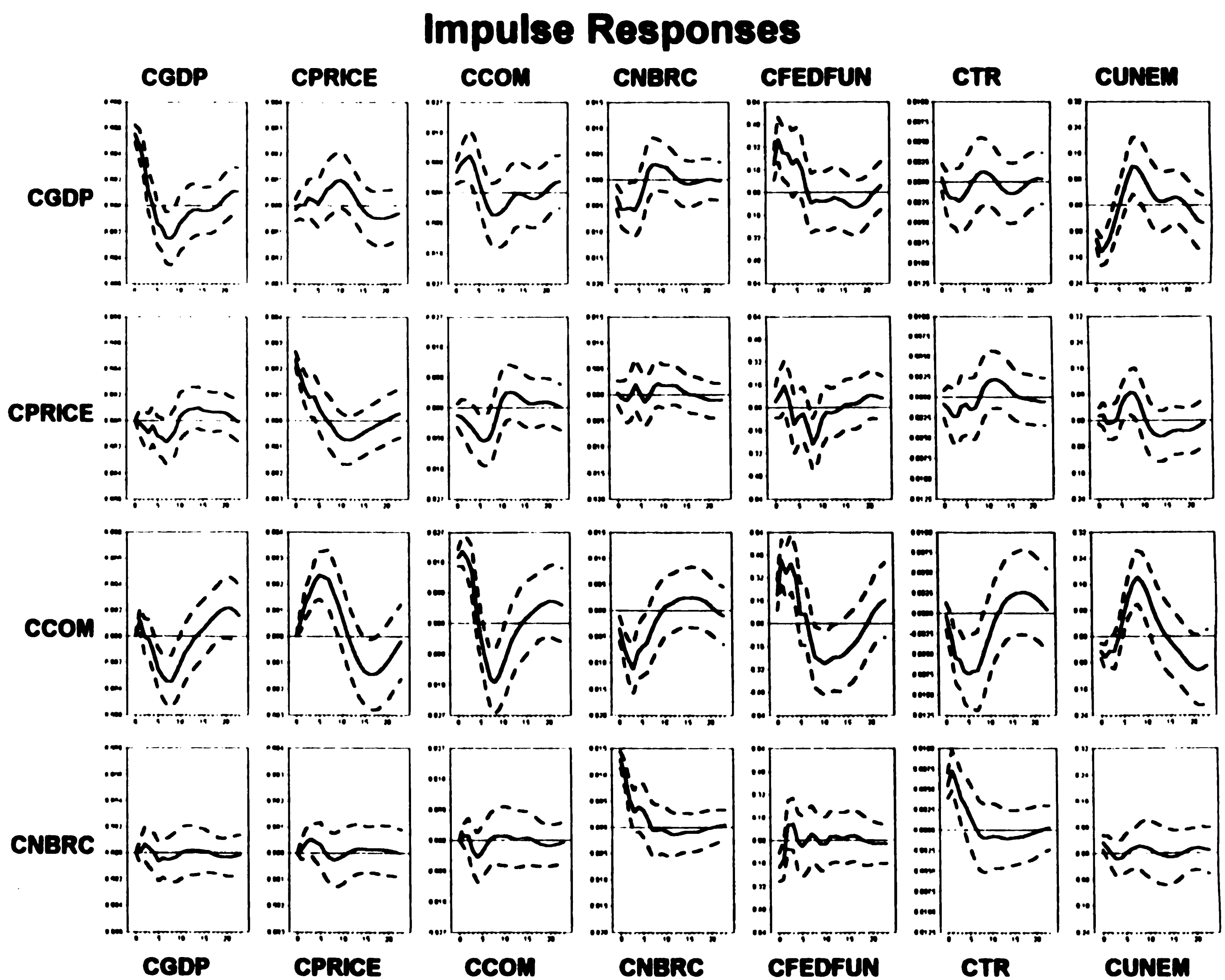
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Figure 4.11 (Cont'd)

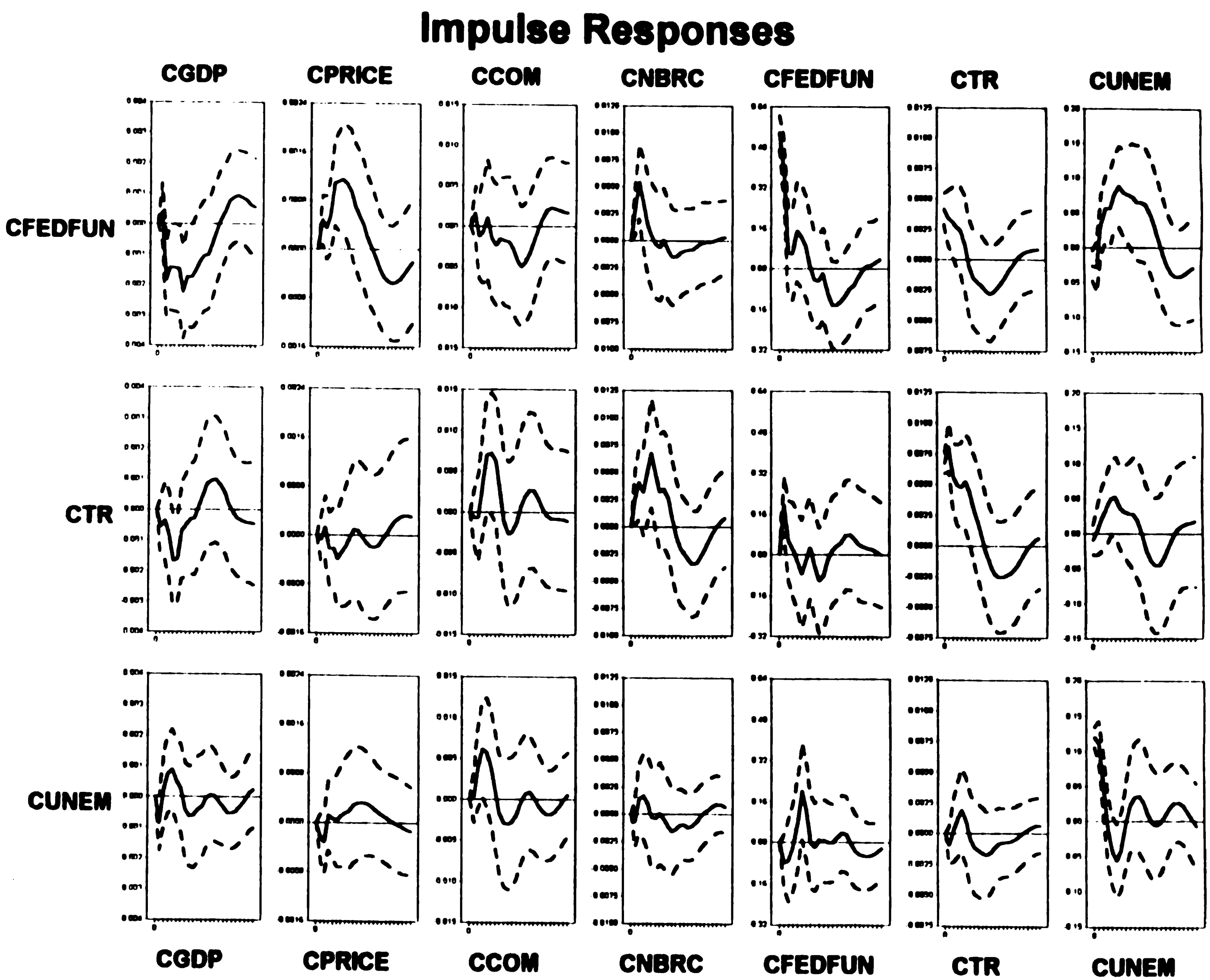

Figure 4.12

Impulse Responses

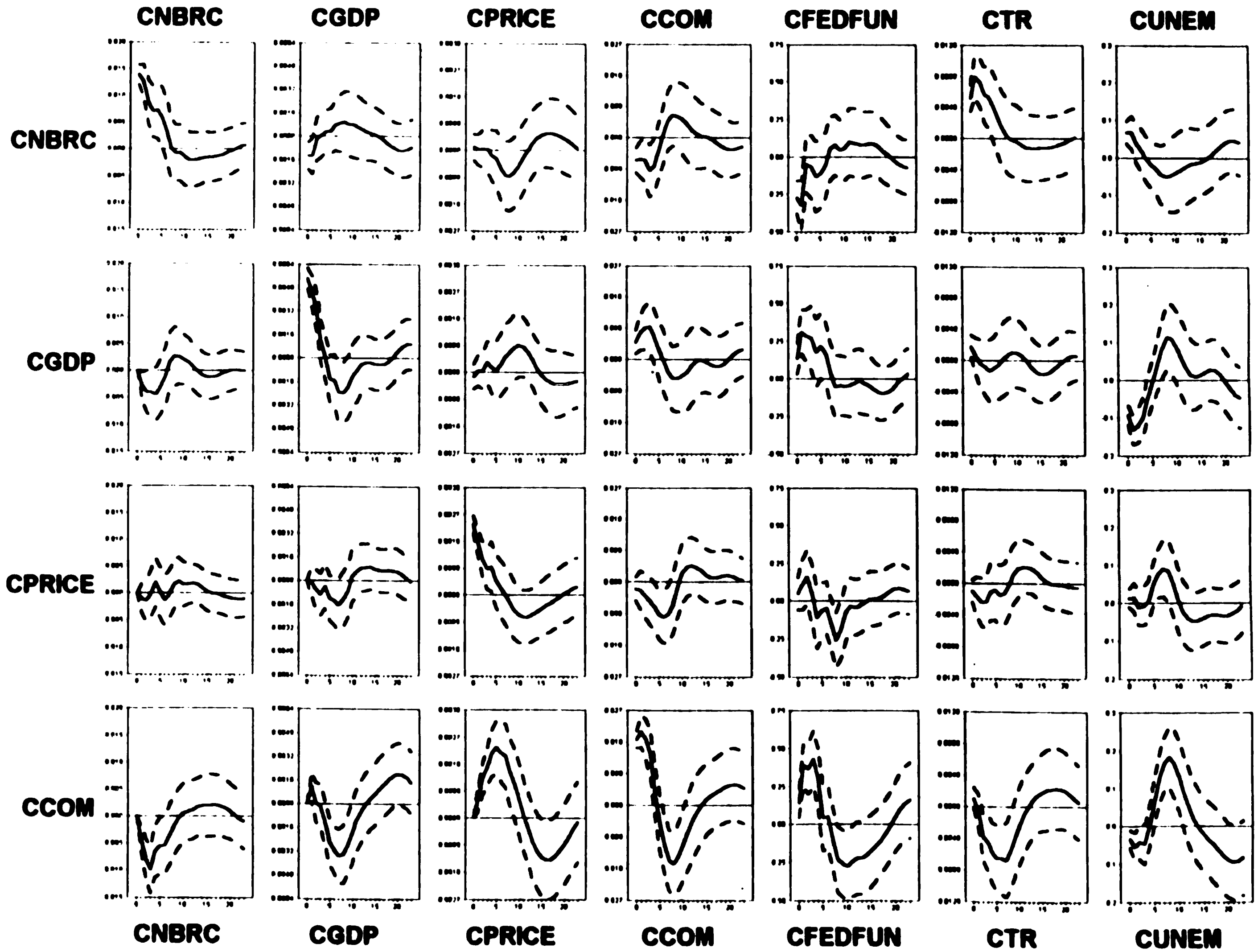

Figure 4.12 (Cont'd)

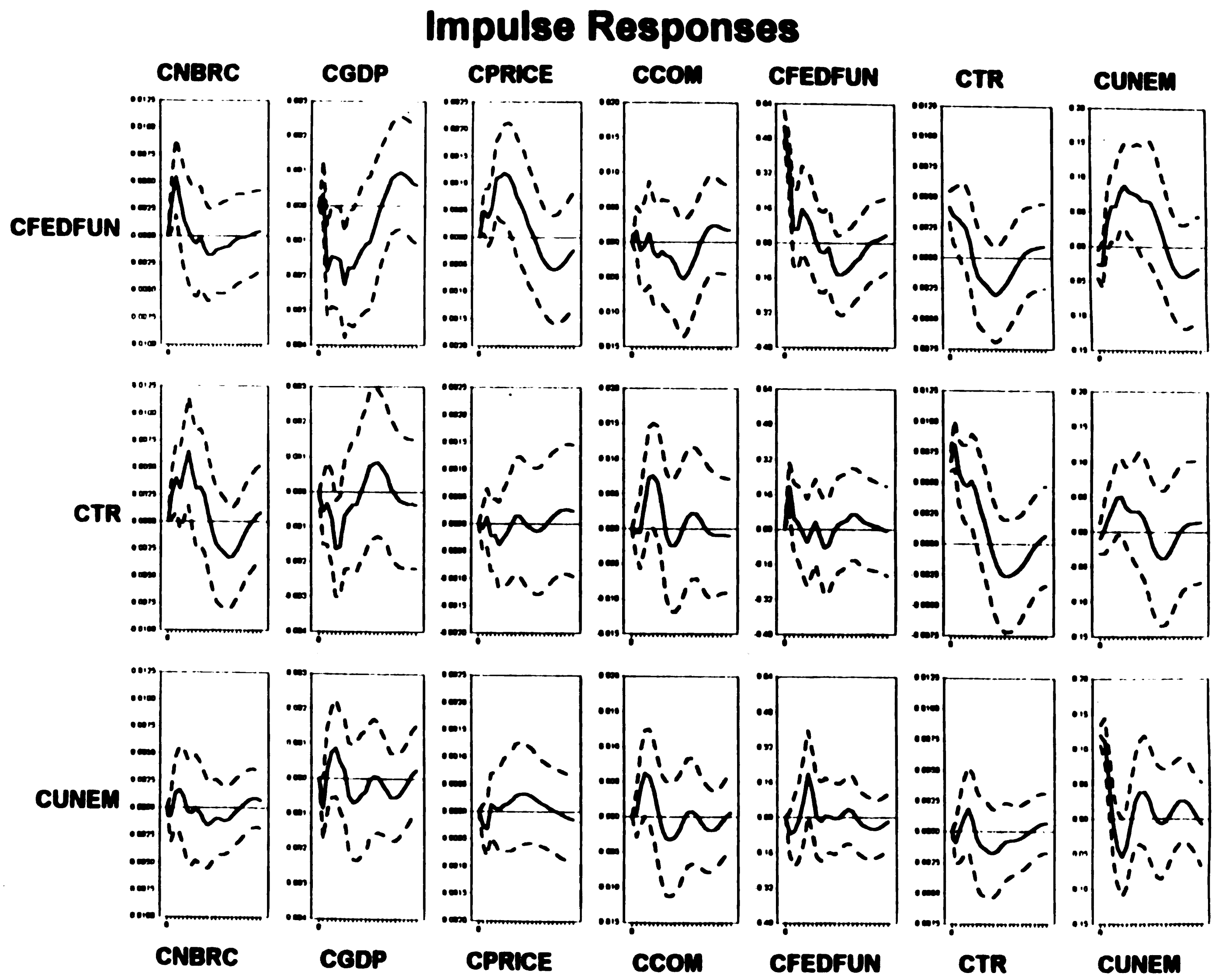

Figure 4.13

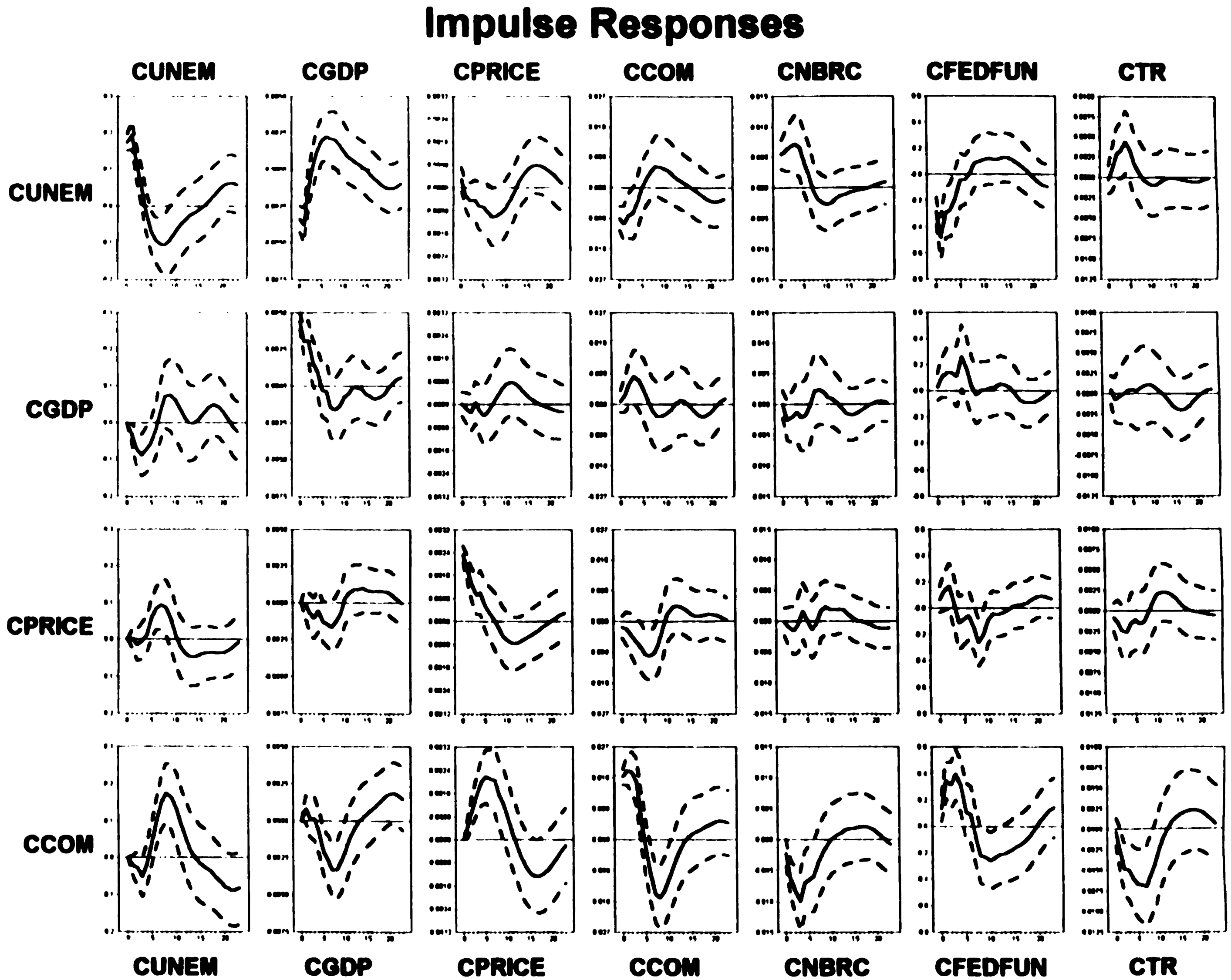
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Figure 4.13 (Cont'd)

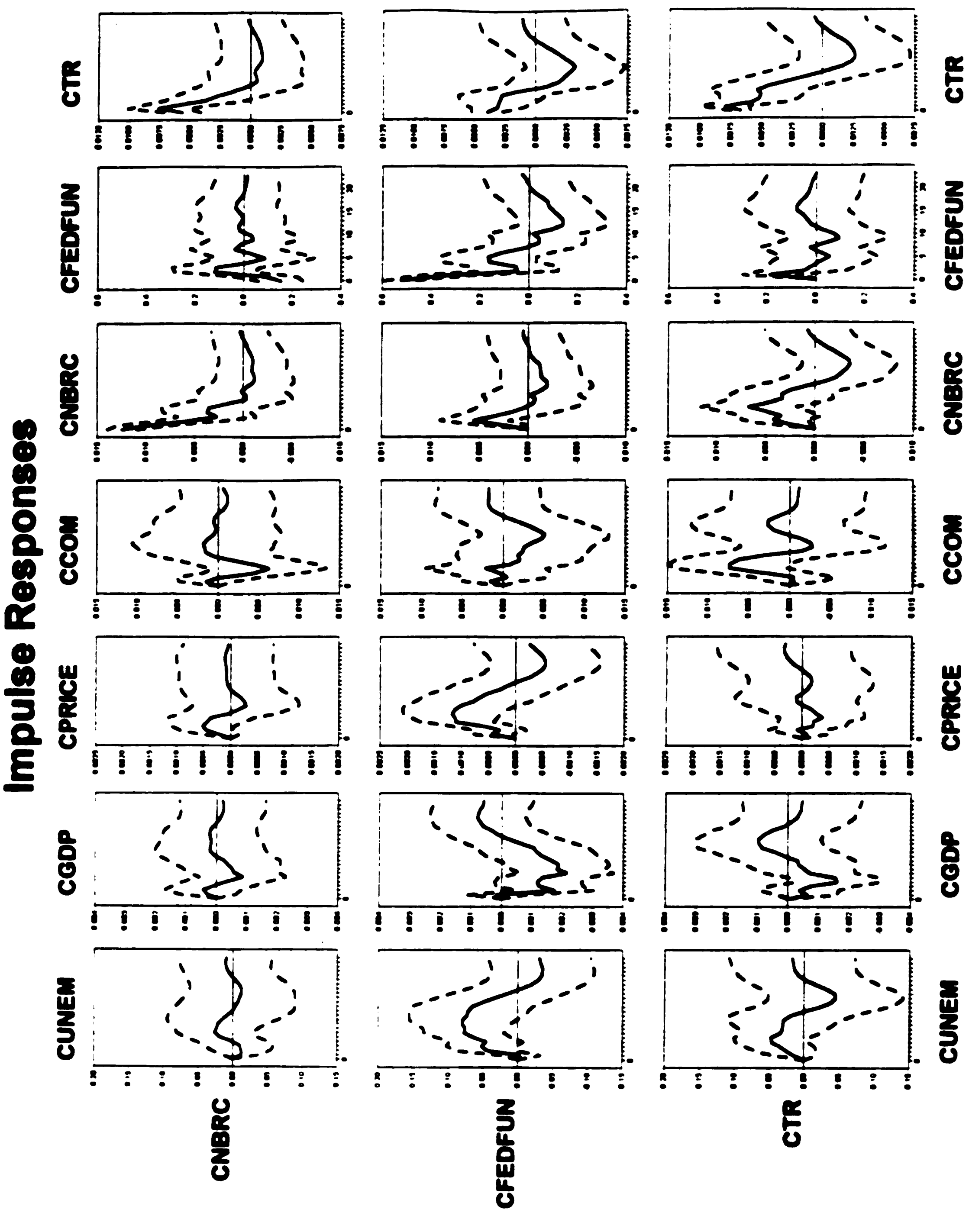

Figure 4.14

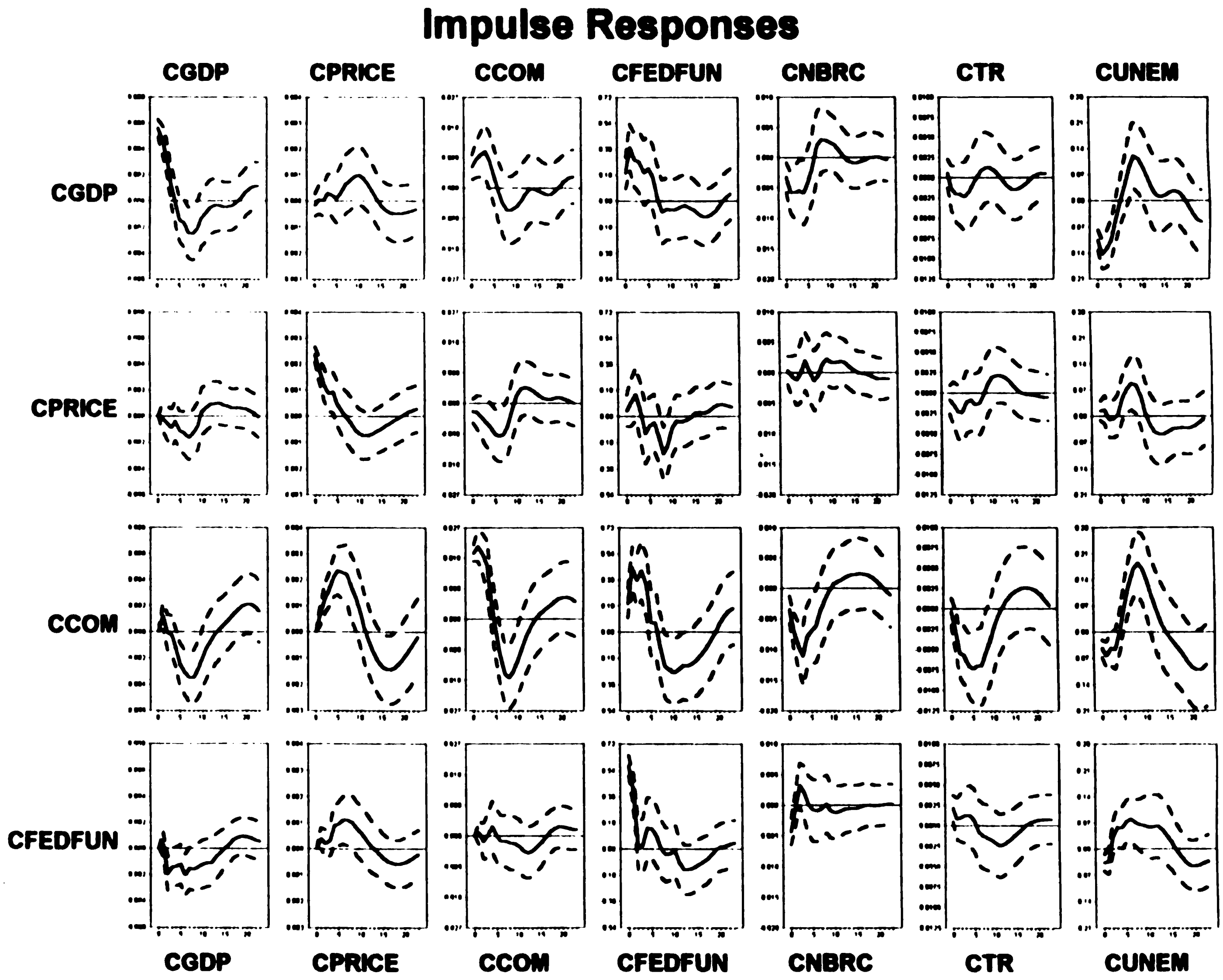
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Figure 4.14 (Cont'd)

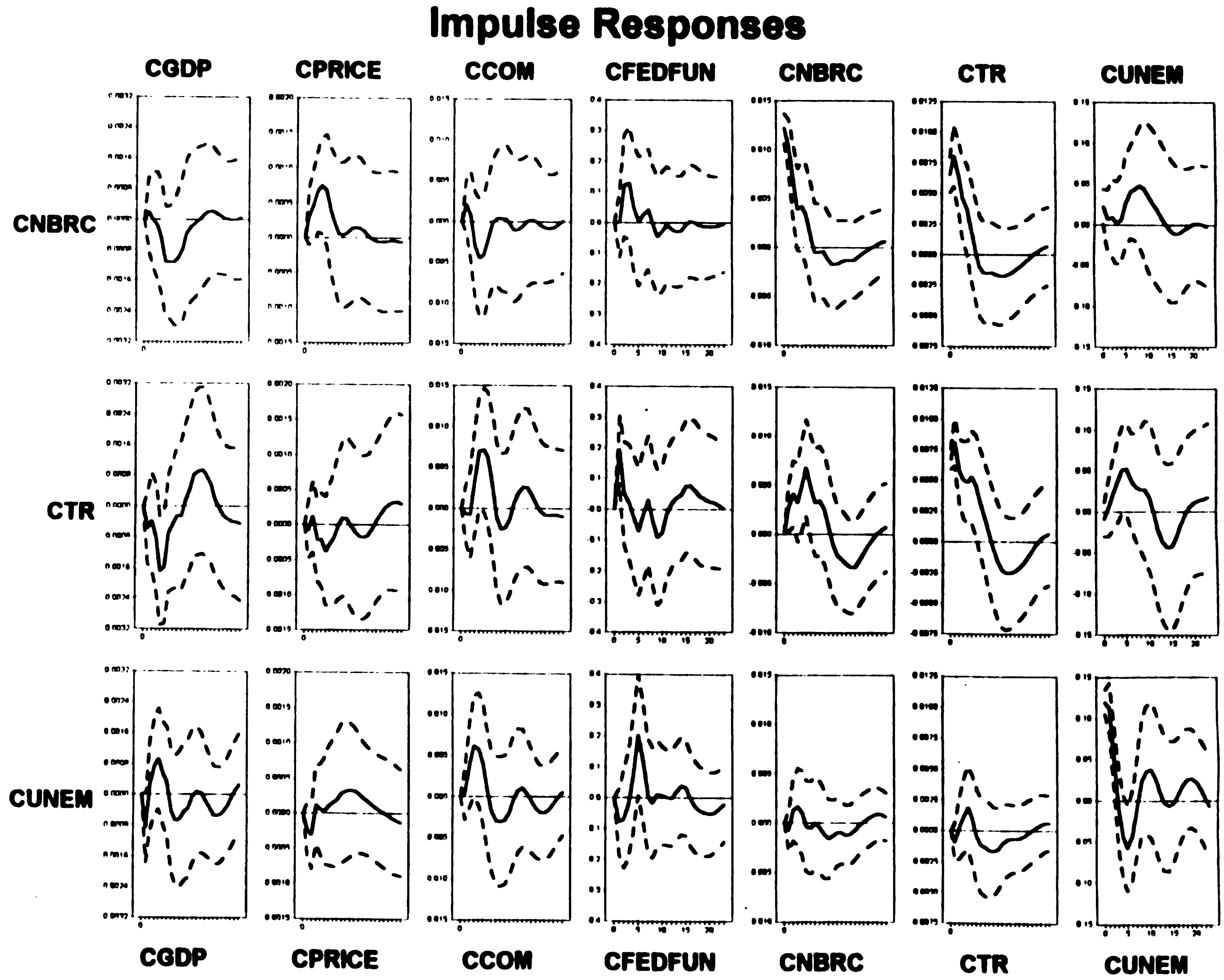

Figure 4.15

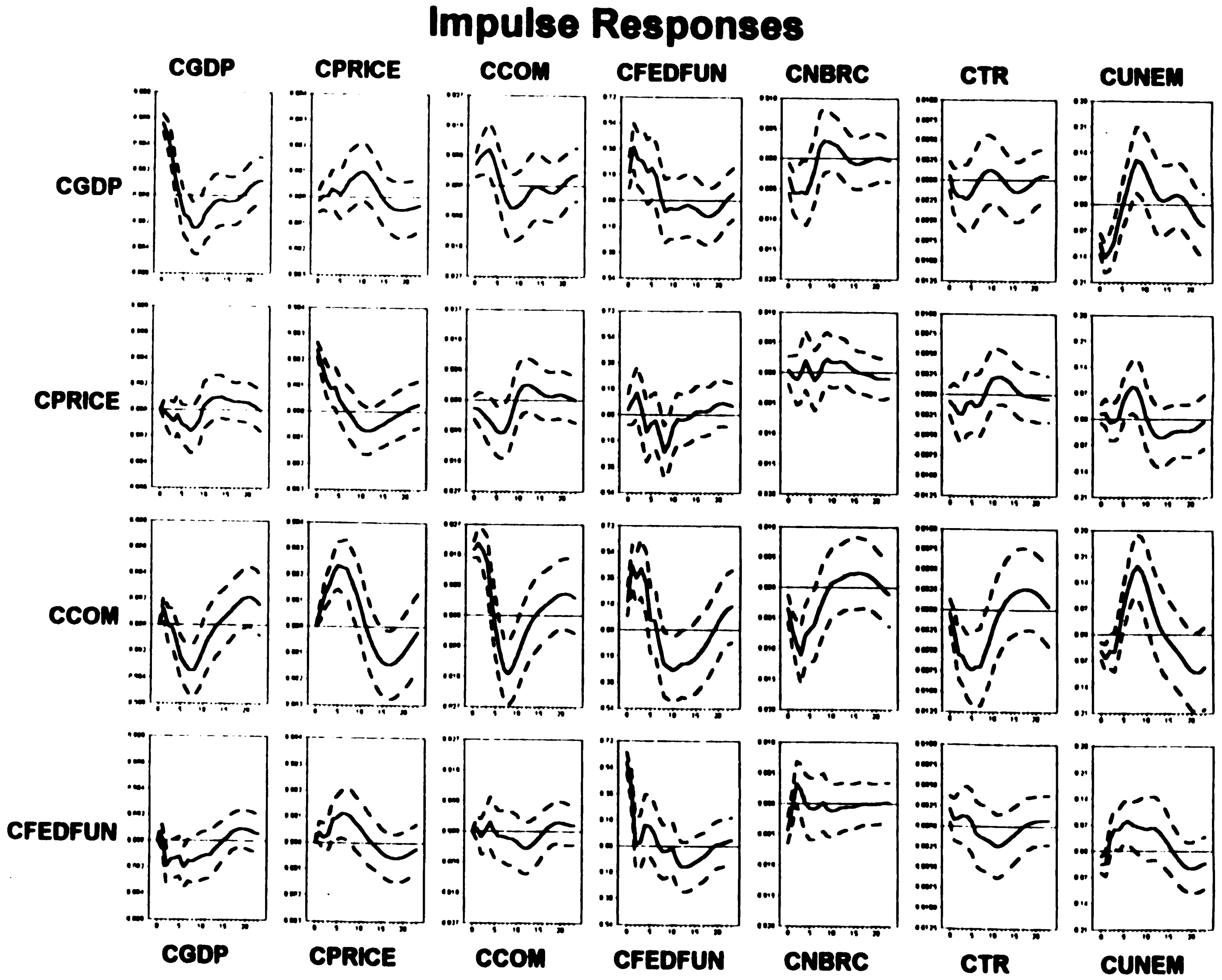
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Figure 4.15 (Cont'd)

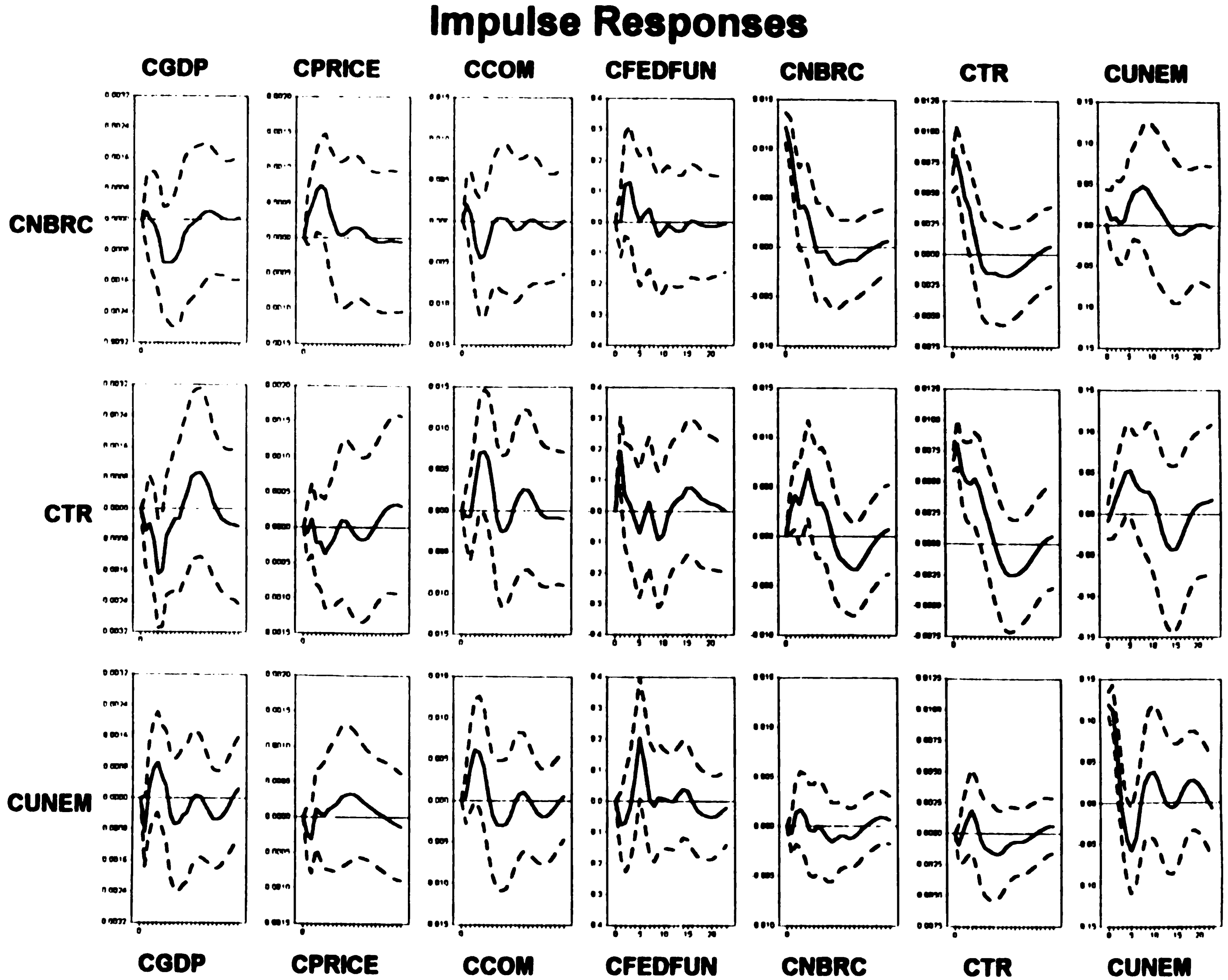

Figure 4.16

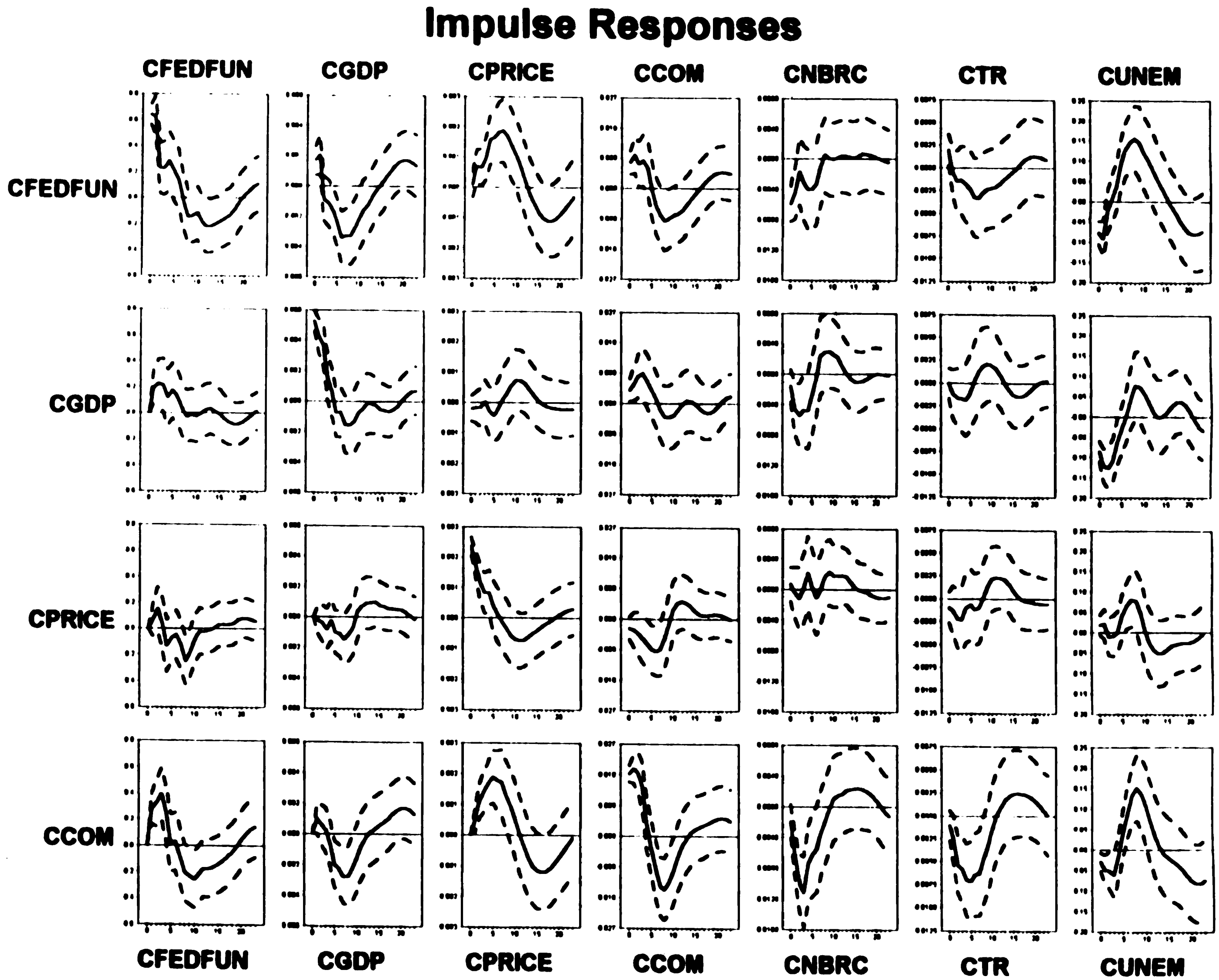
Impulse Responses
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Figure 4.16 (Cont'd)

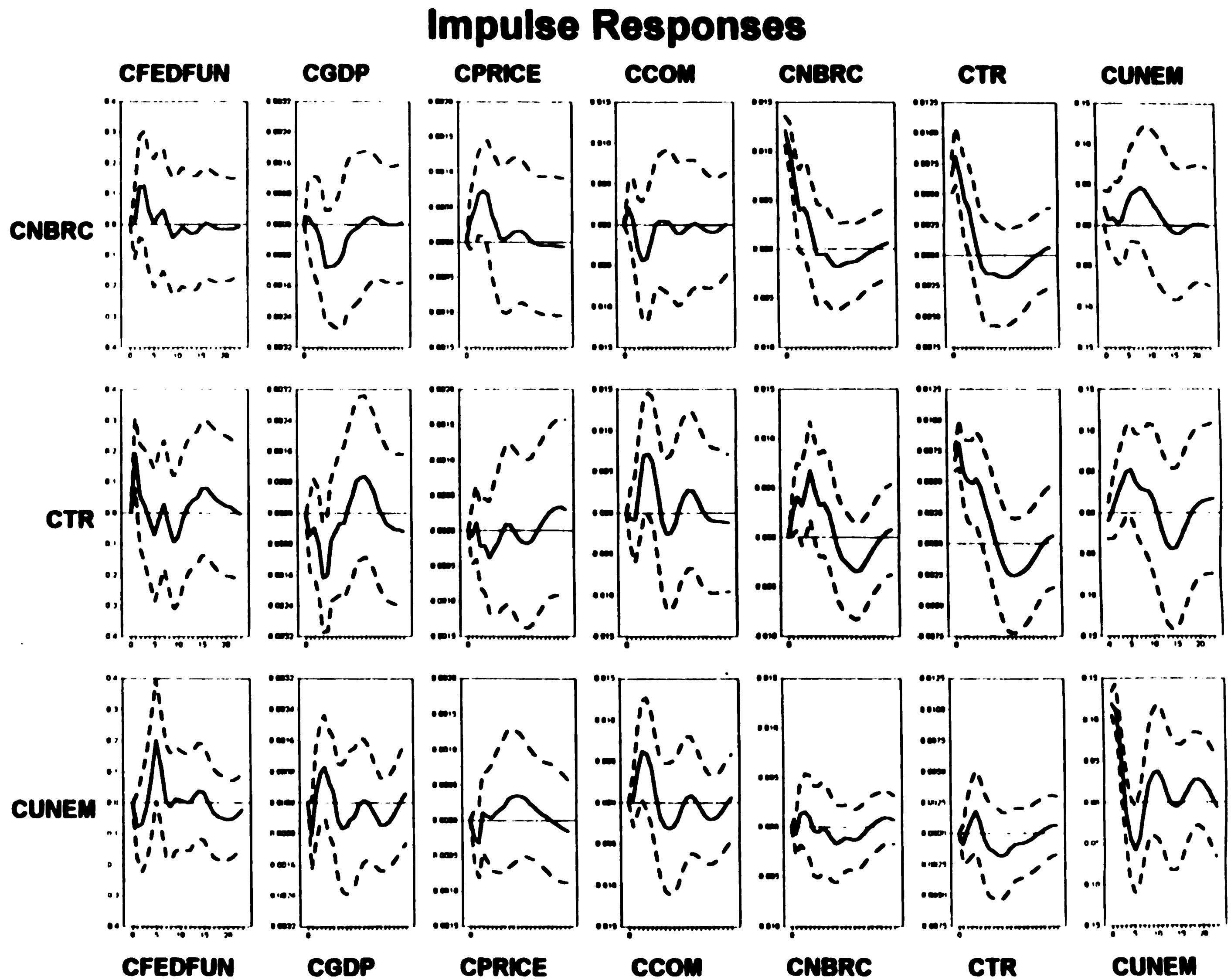


Figure 4.17

Impulse Responses

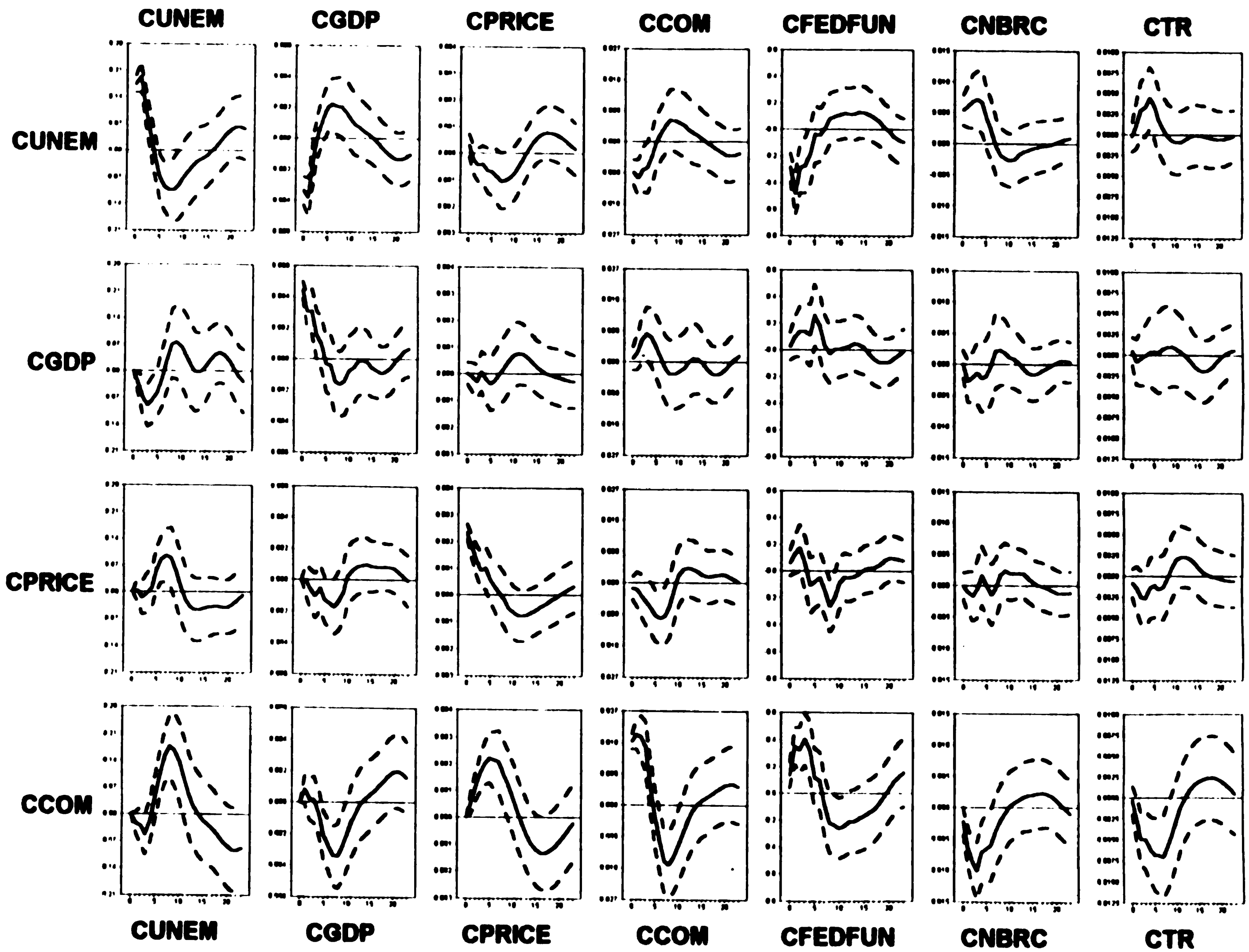


Figure 4.17 (Cont'd)

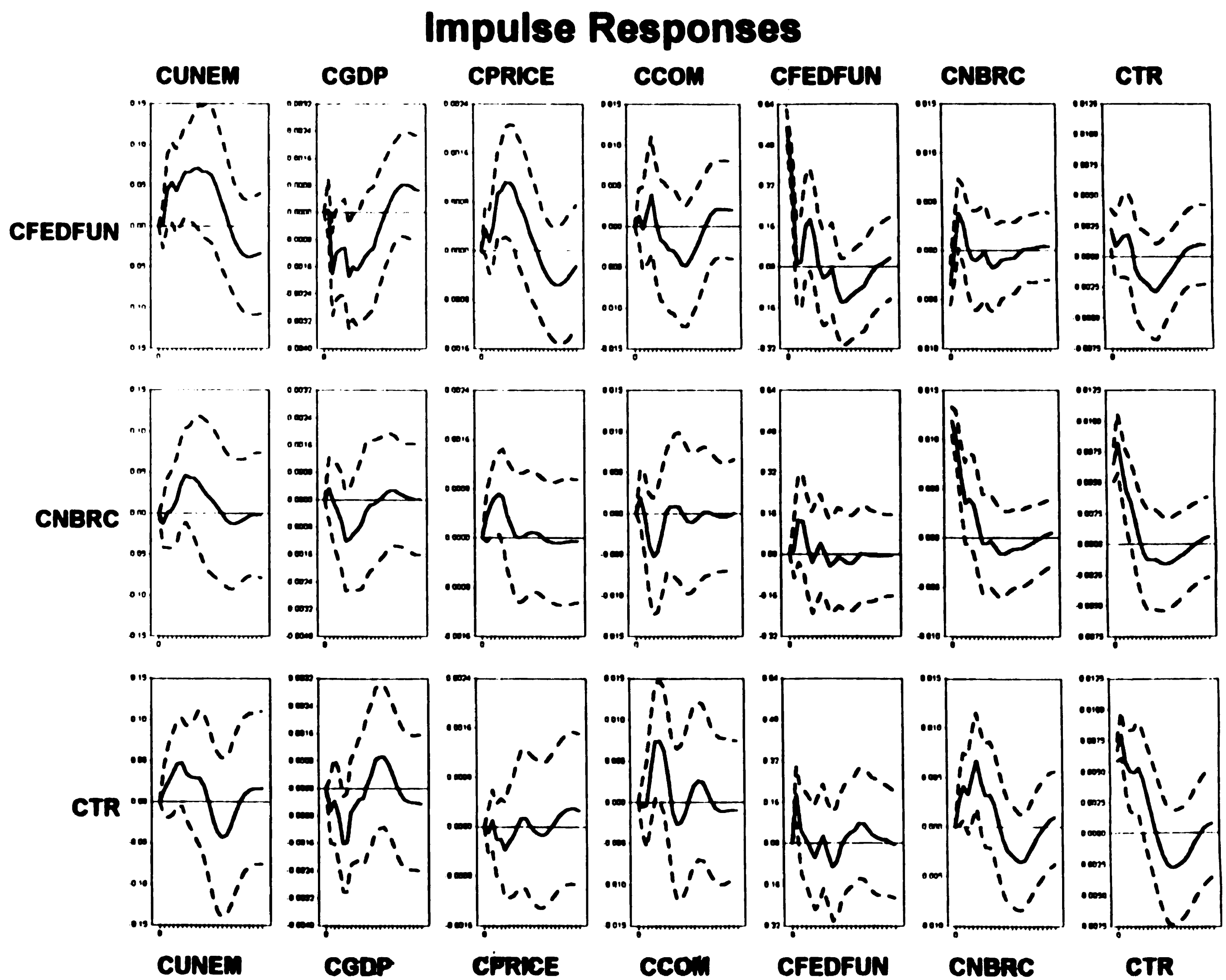
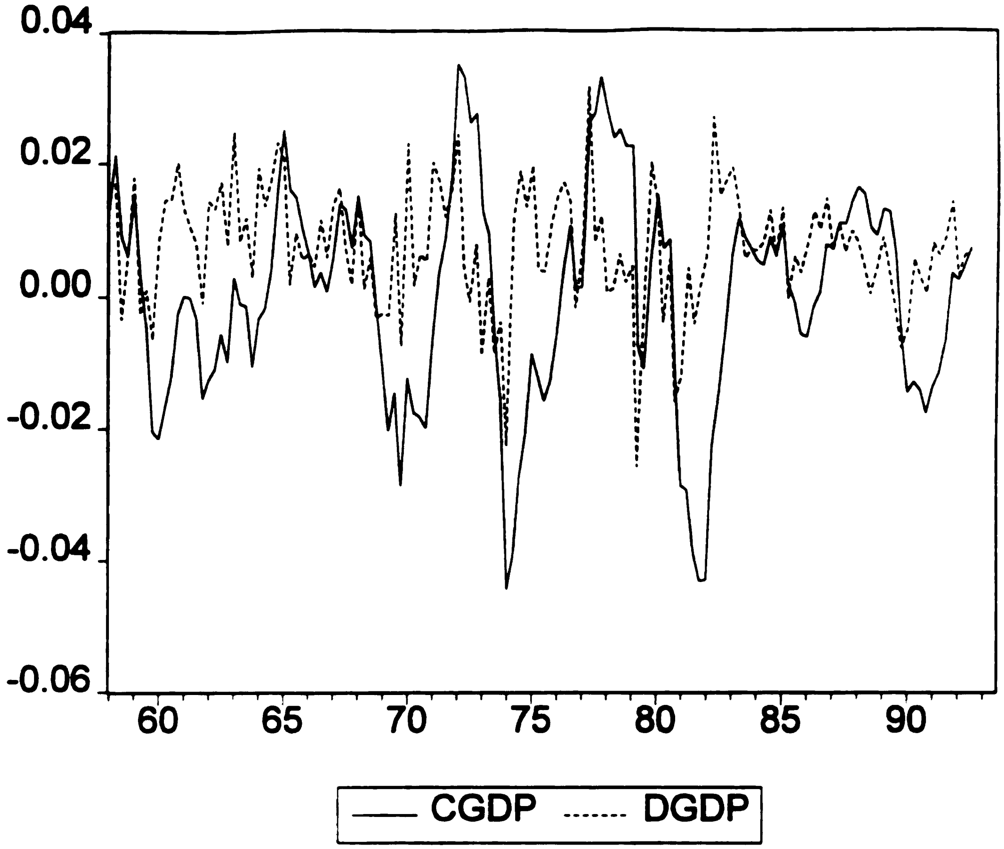
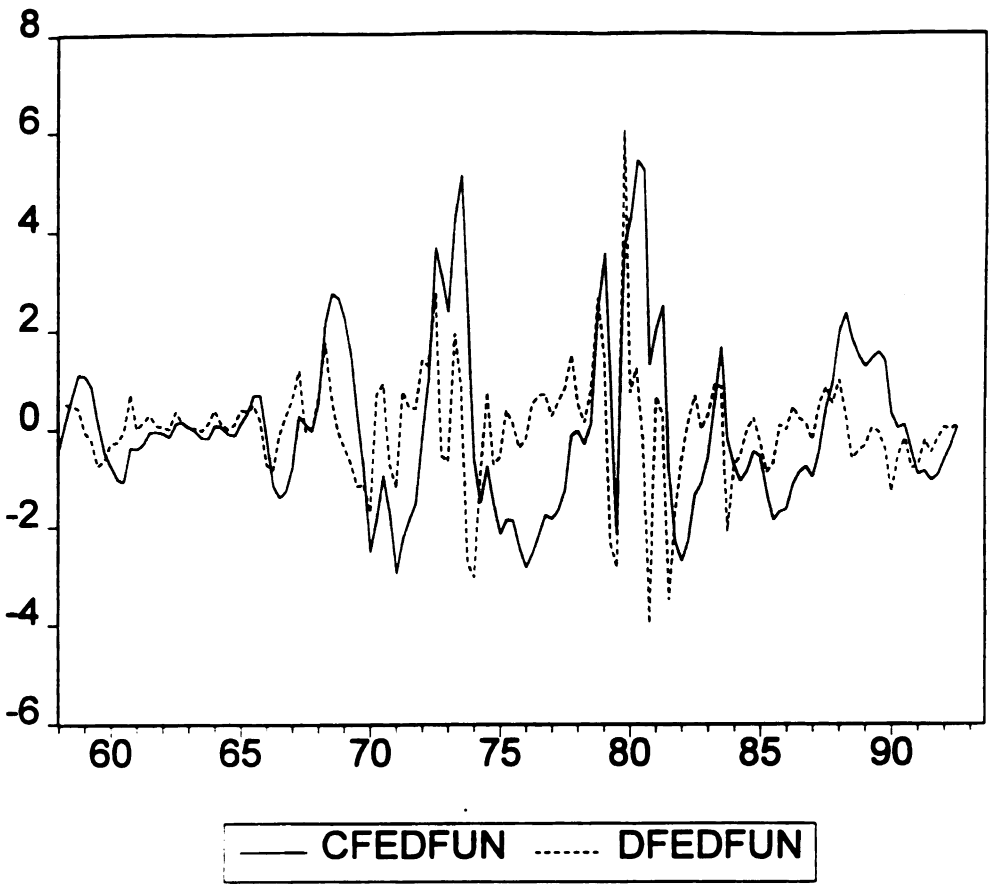


Figure 4.18: Detrended GDP



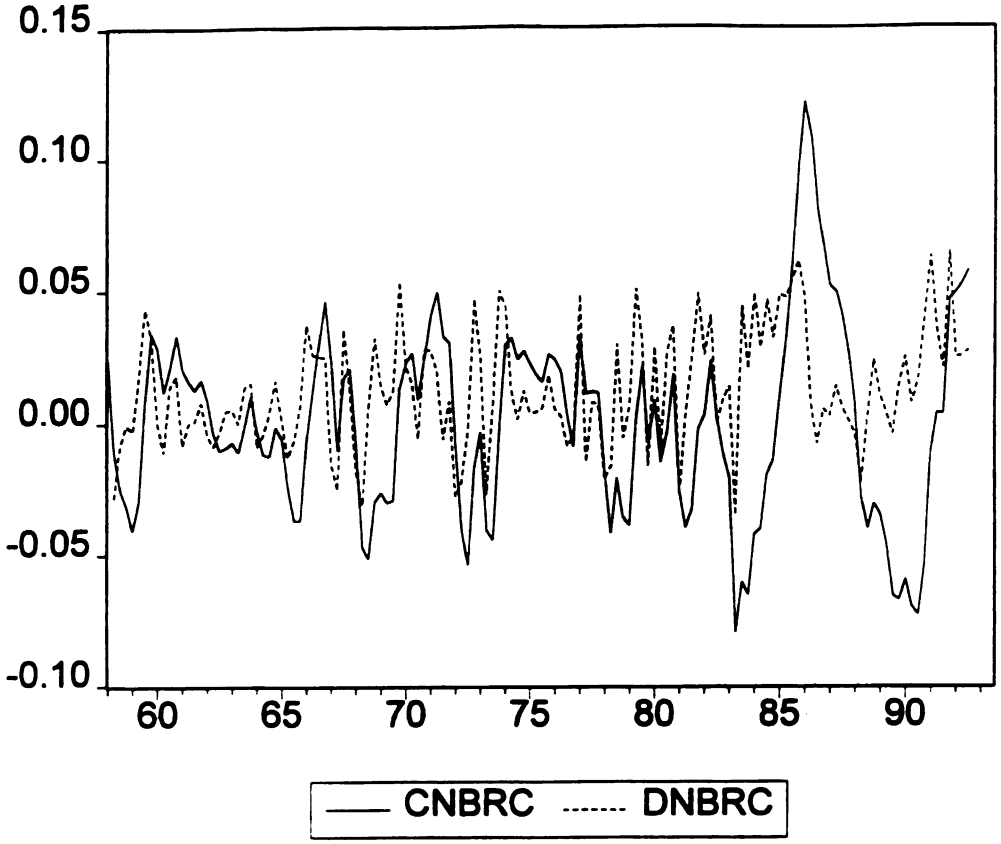
Correlation: 0.252576

Figure 4.19: Detrended Federal Funds Rate



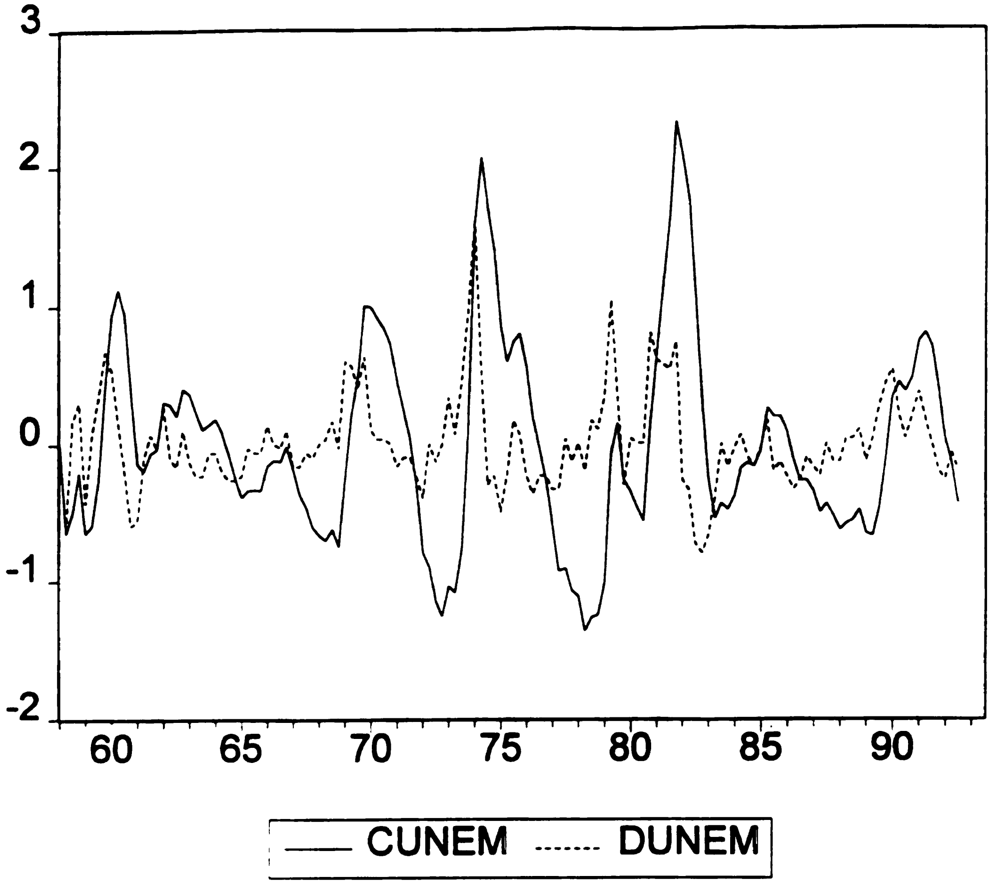
Correlation: 0.310653

Figure 4.20: Detrended Nonborrowed Reserves



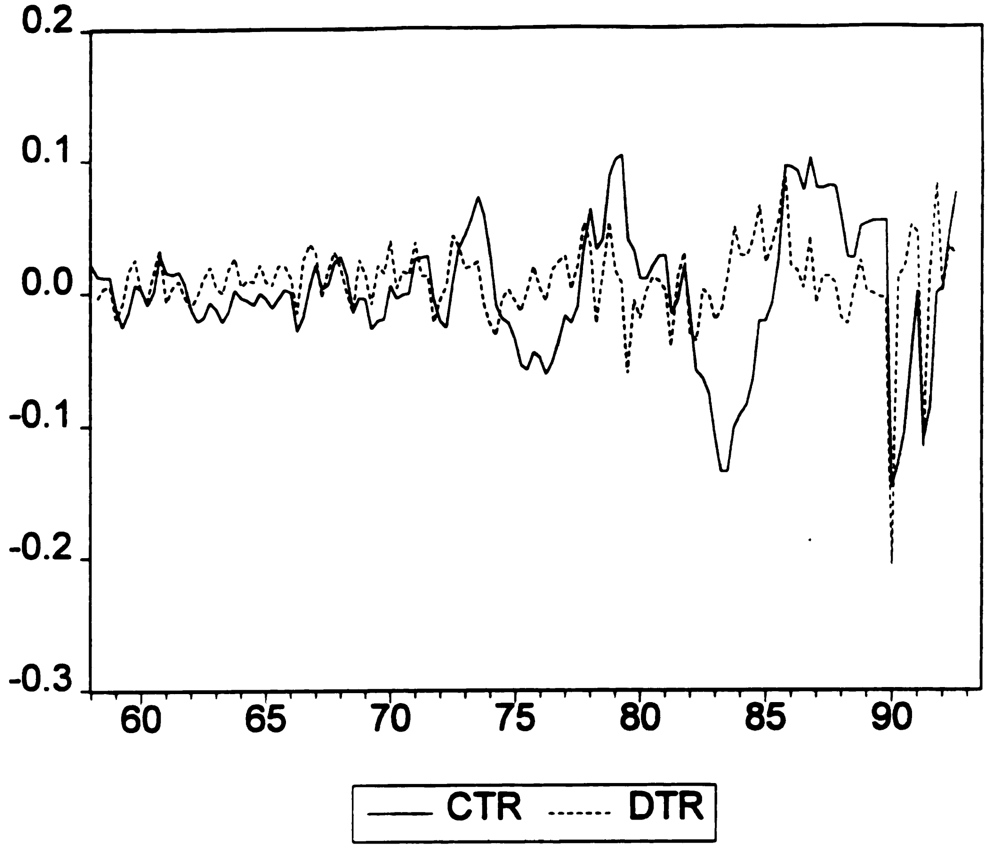
Correlation: 0.274827

Figure 4.21: Detrended Unemployment



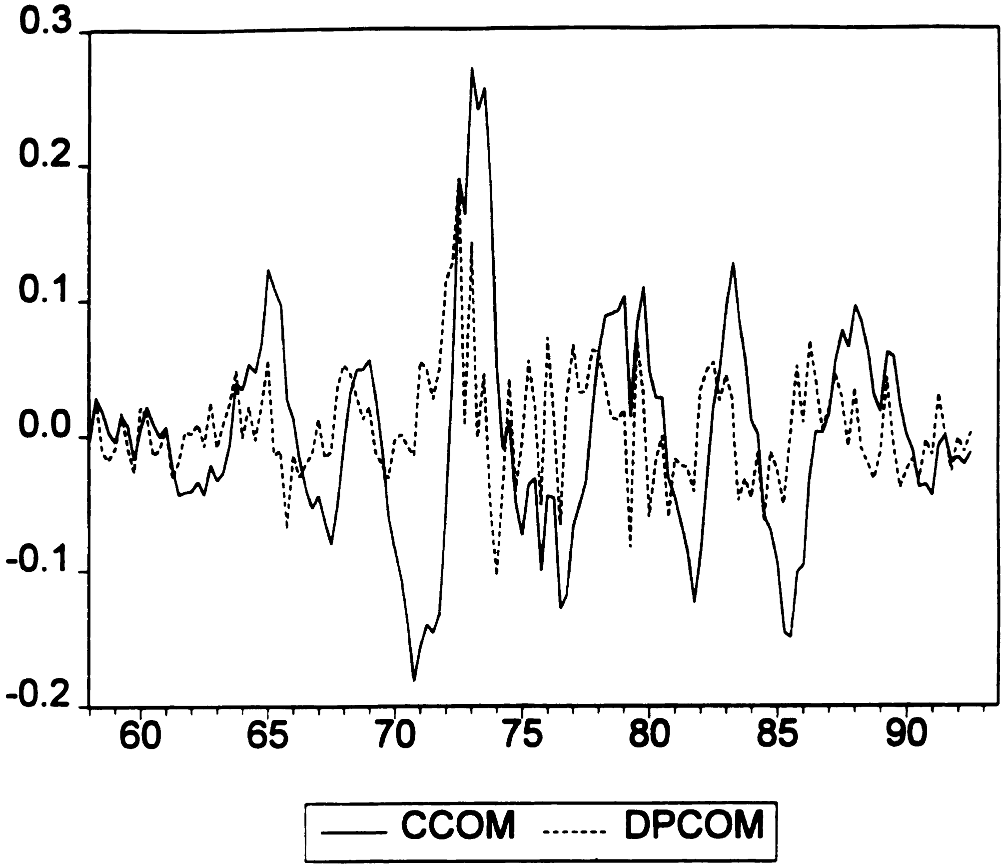
Correlation: 0.221127

Figure 4.22: Detrended Total Reserves



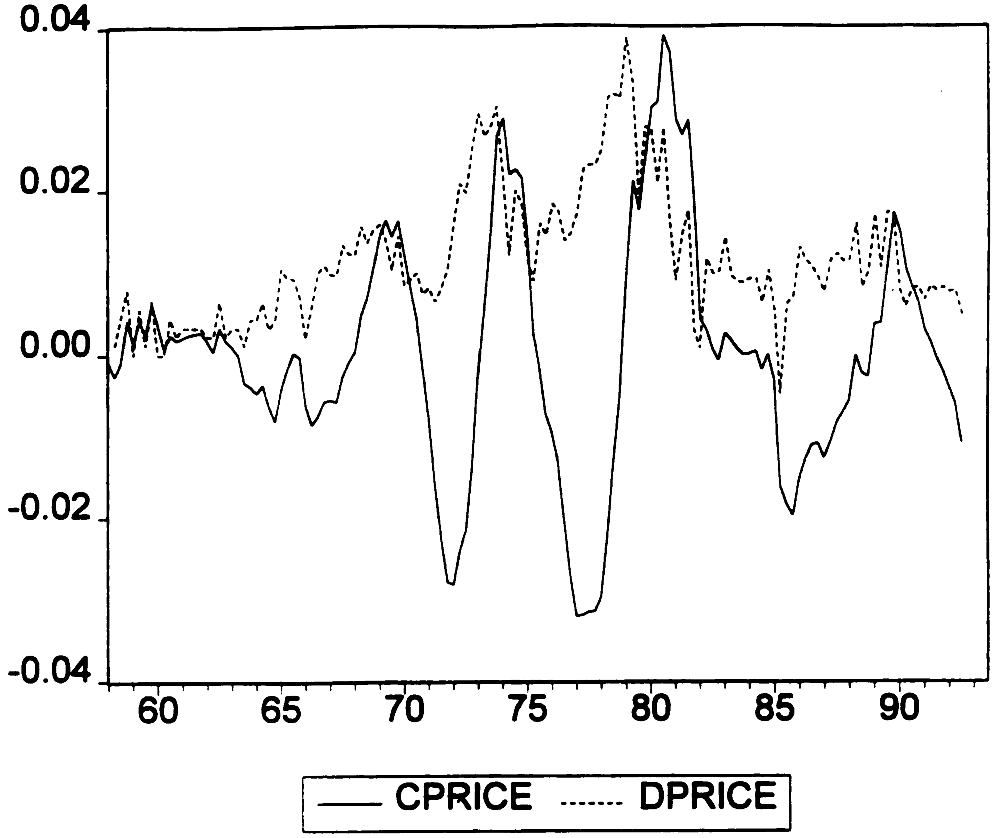
Correlation: 0.290895

Figure 4.23: Detrended Commodity Prices



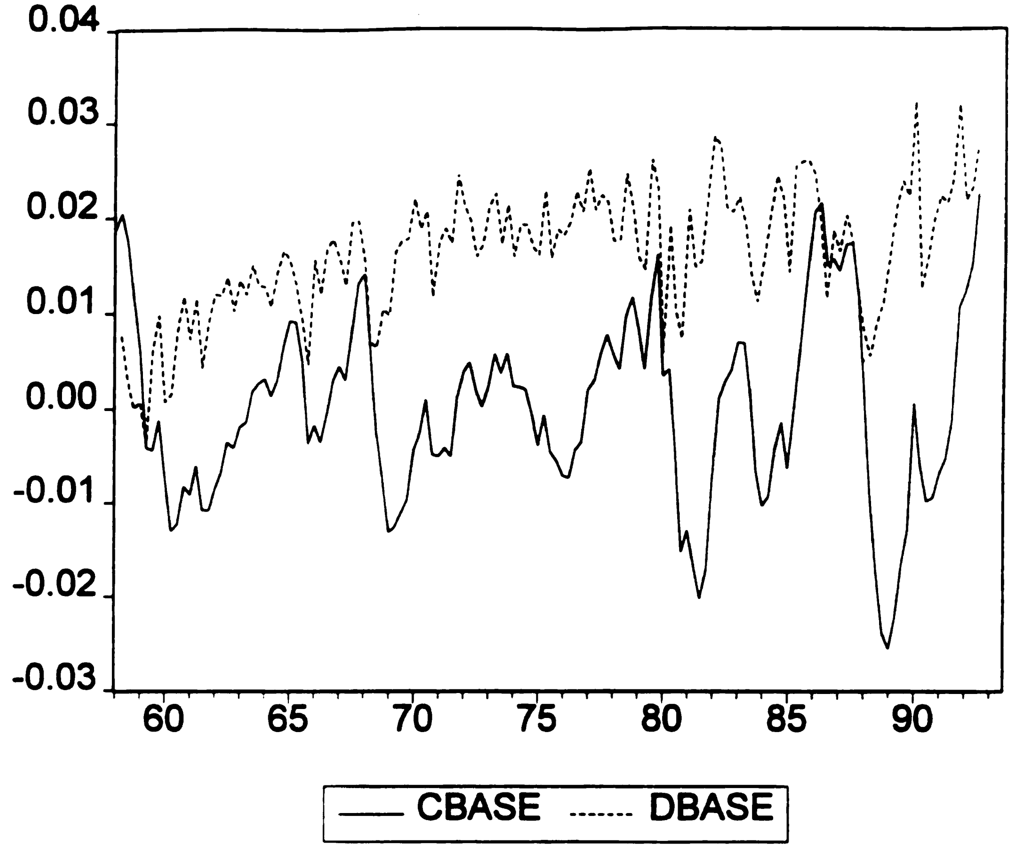
Correlation: 0.233713

Figure 4.24: Detrended Prices



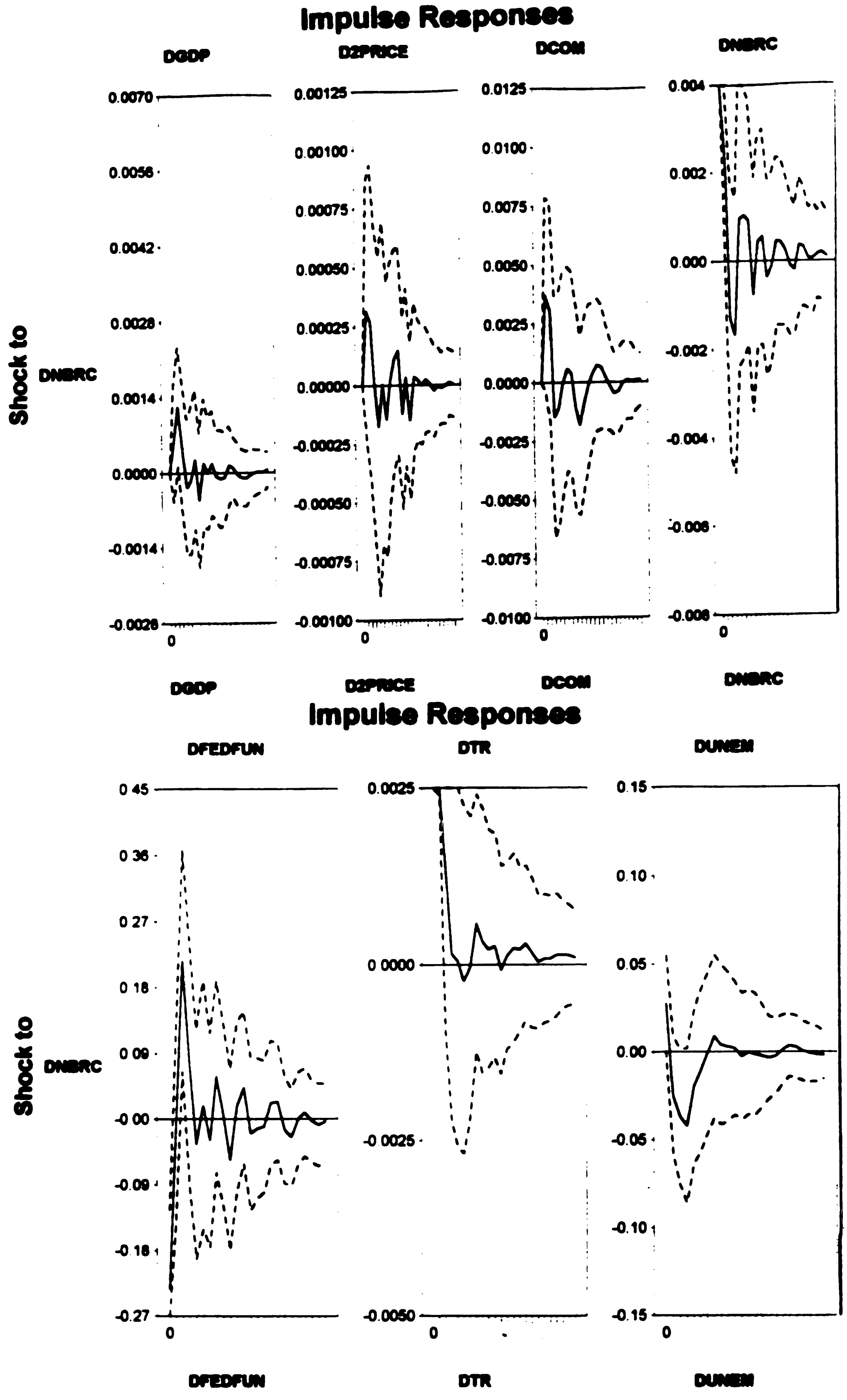
Correlation: 0.113304

Figure 4.25: Detrended Monetary Base



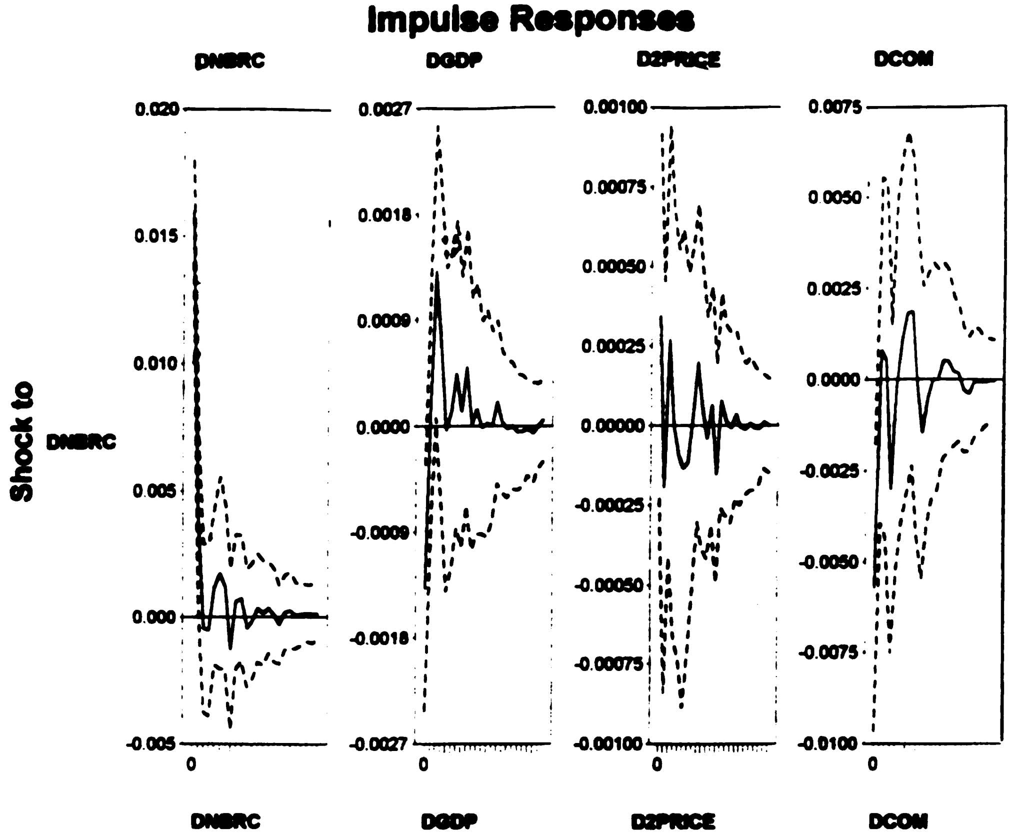
Correlation: 0.208138

Figure 4.26



Impulse Responses
DGDP
D2PRICE
DCOM
DNBRC
Shock to
DNBRC
0.0070
0.0056
0.0042
0.0028
0.0014
0.0000
-0.0014
-0.0028
0.00125
0.00100
0.00075
0.00050
0.00025
0.00000
-0.00025
-0.00050
-0.00075
-0.00100
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
-0.0025
-0.0050
-0.0075
-0.0100
0.004
0.002
0.000
-0.002
-0.004
-0.006
-0.008
0
DGDP
D2PRICE
DCOM
DNBRC
Impulse Responses
DFEDFUN
DTR
DUNEM
Shock to
DNBRC
0.45
0.36
0.27
0.18
0.09
-0.00
-0.09
-0.18
-0.27
0.0025
0.0000
-0.0025
-0.0050
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
0
DFEDFUN
DTR
DUNEM

Figure 4.27


Impulse Responses
DNBRC
DGDP
D2PRICE
DCOM
Shock to
DNBRC
DNBRC
DGDP
D2PRICE
DCOM

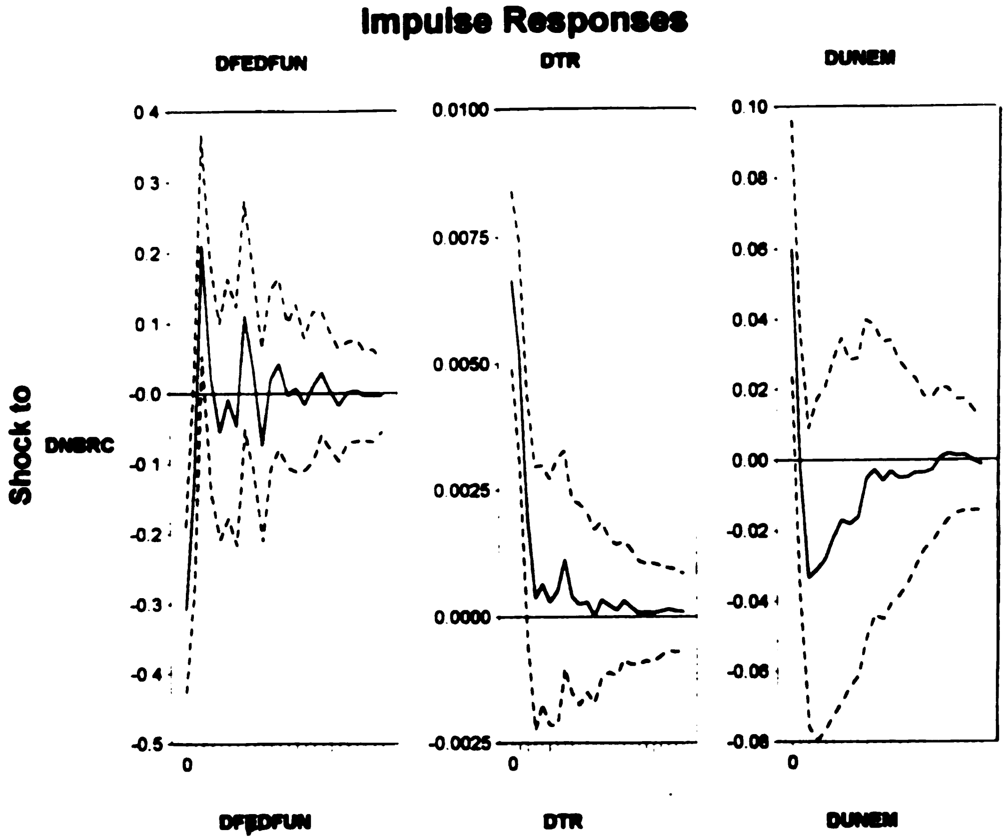
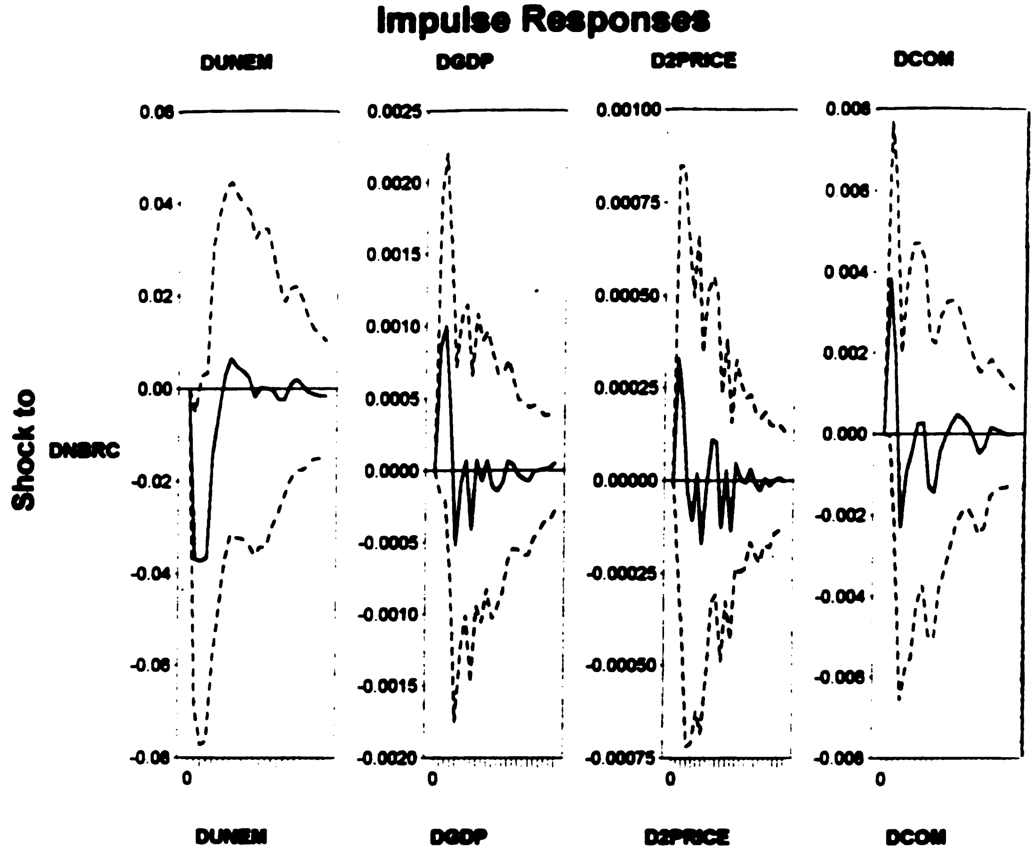

Impulse Responses
DFEDFUN
DTR
DUNEM
Shock to
DNBRC
DFEDFUN
DTR
DUNEM

Figure 4.28


Impulse Responses
DUNEM
DGDP
D2PRICE
DCOM
Shock to
DNBRC
0
DUNEM
DGDP
D2PRICE
DCOM

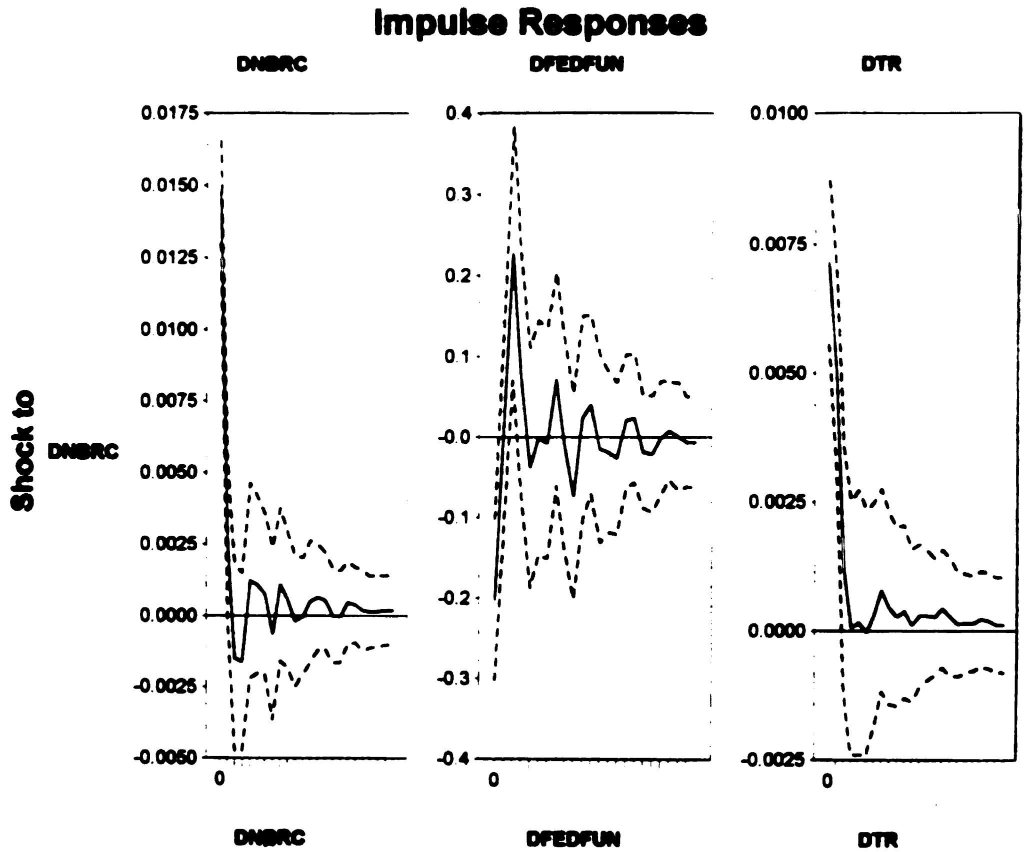
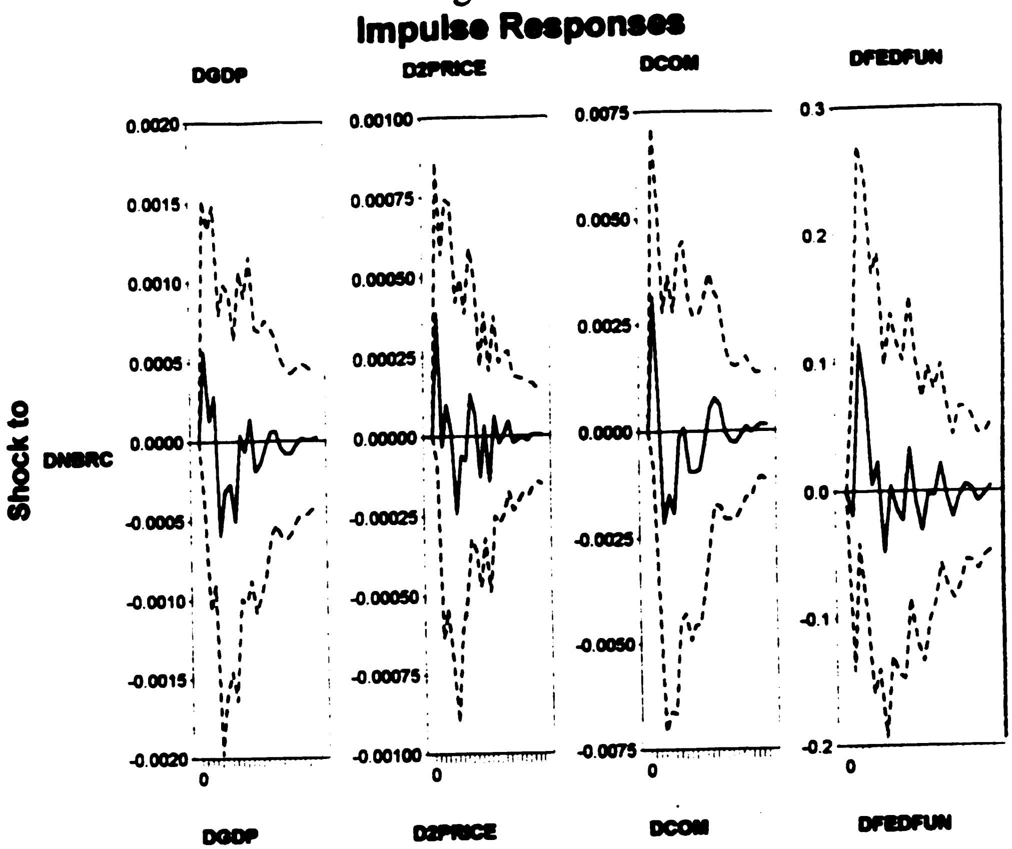

Impulse Responses
DNBRC
DFEDFUN
DTR
Shock to
DNBRC
0
DNBRC
DFEDFUN
DTR

Figure 4.29


Impulse Responses
DGDP
D2PRICE
DCOM
DFEDFUN
Shock to
DNBRC
0.0020
0.0015
0.0010
0.0005
0.0000
-0.0005
-0.0010
-0.0015
-0.0020
0.00100
0.00075
0.00050
0.00025
0.00000
-0.00025
-0.00050
-0.00075
-0.00100
0.0075
0.0050
0.0025
0.0000
-0.0025
-0.0050
-0.0075
0.3
0.2
0.1
0.0
-0.1
-0.2
0
DGDP
D2PRICE
DCOM
DFEDFUN

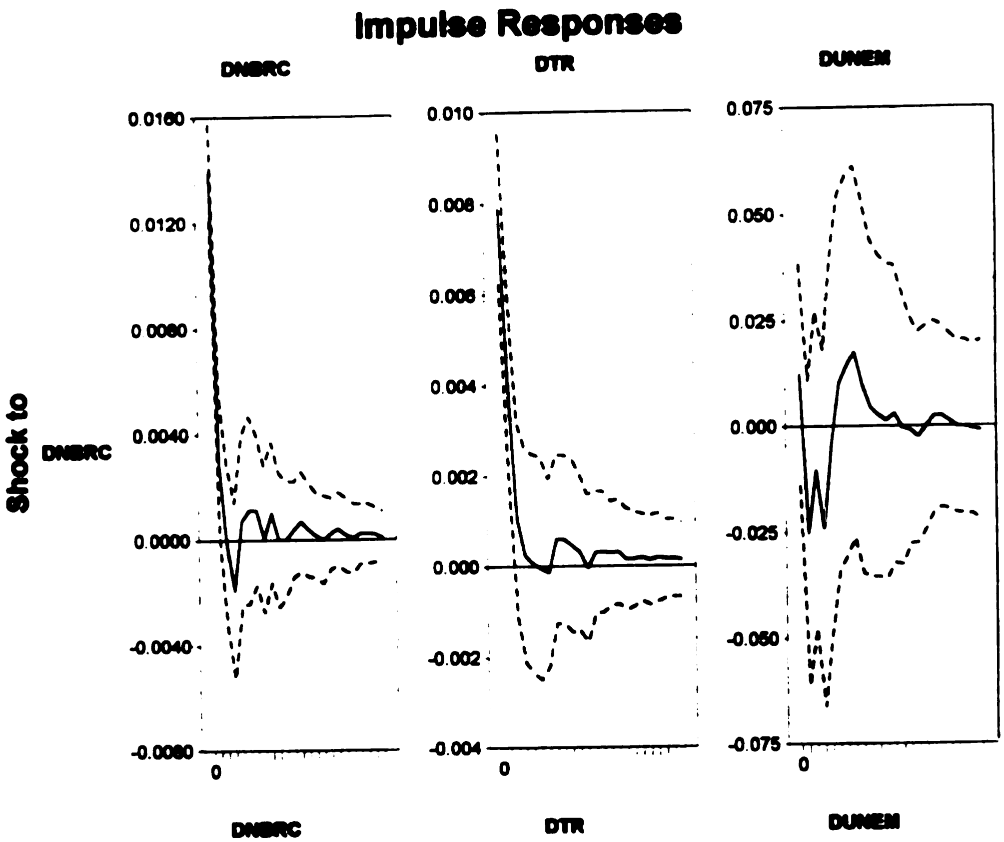
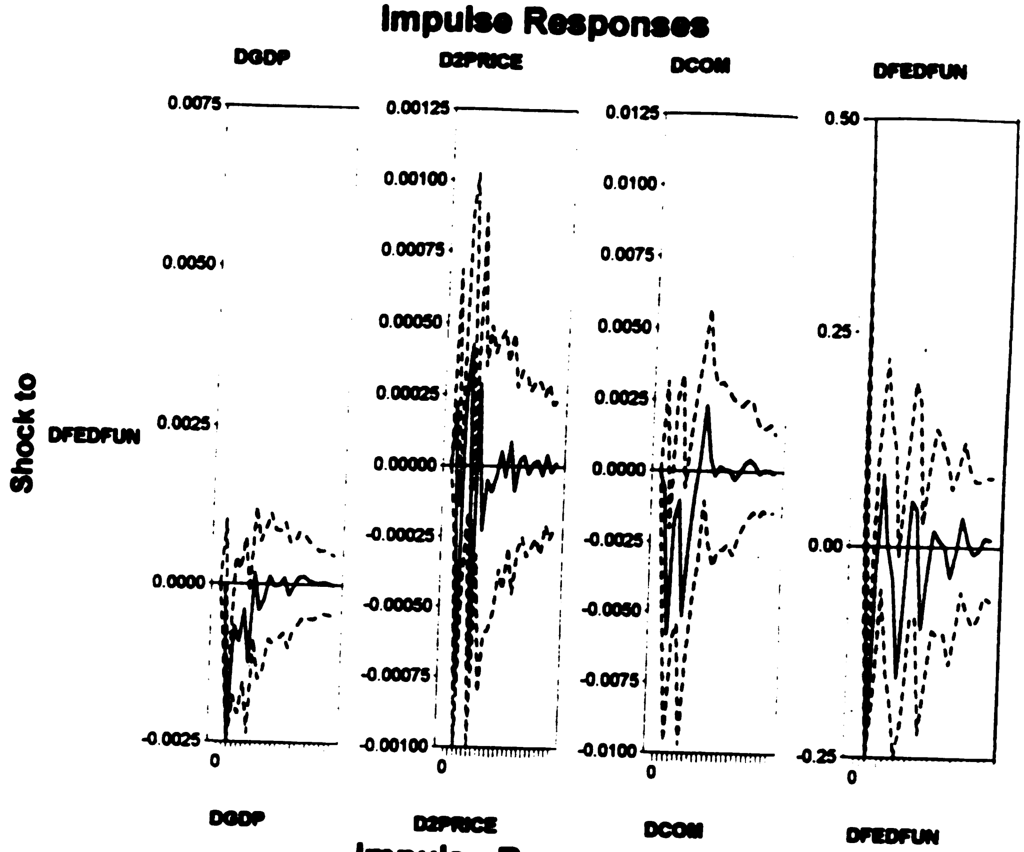

Impulse Responses
DNBRC
DTR
DUNEM
Shock to
DNBRC
0.0160
0.0120
0.0080
0.0040
0.0000
-0.0040
-0.0080
0.010
0.008
0.006
0.004
0.002
0.000
-0.002
-0.004
0.075
0.050
0.025
0.000
-0.025
-0.050
-0.075
0
DNBRC
DTR
DUNEM

Figure 4.30


Impulse Responses
DGDP
D2PRICE
DCOM
DFEDFUN
Shock to
DFEDFUN
0.0075
0.0050
0.0025
0.0000
-0.0025
0.00125
0.00100
0.00075
0.00050
0.00025
0.00000
-0.00025
-0.00050
-0.00075
-0.00100
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
-0.0025
-0.0050
-0.0075
-0.0100
0.50
0.25
0.00
-0.25
0
DGDP
D2PRICE
DCOM
DFEDFUN

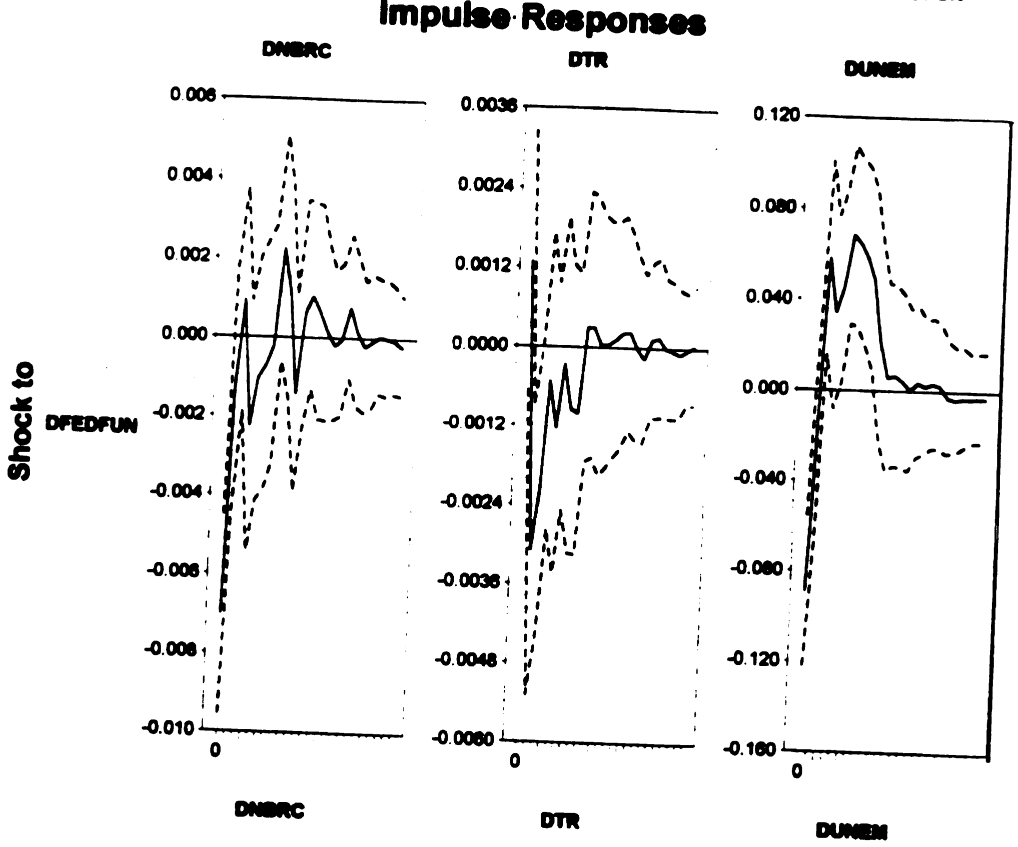

Impulse Responses
DNBRC
DTR
DUNEM
Shock to
DFEDFUN
0.006
0.004
0.002
0.000
-0.002
-0.004
-0.006
-0.008
-0.010
0.0036
0.0024
0.0012
0.0000
-0.0012
-0.0024
-0.0036
-0.0048
-0.0060
0.120
0.080
0.040
0.000
-0.040
-0.080
-0.120
-0.160
0
DNBRC
DTR
DUNEM

Figure 4.31

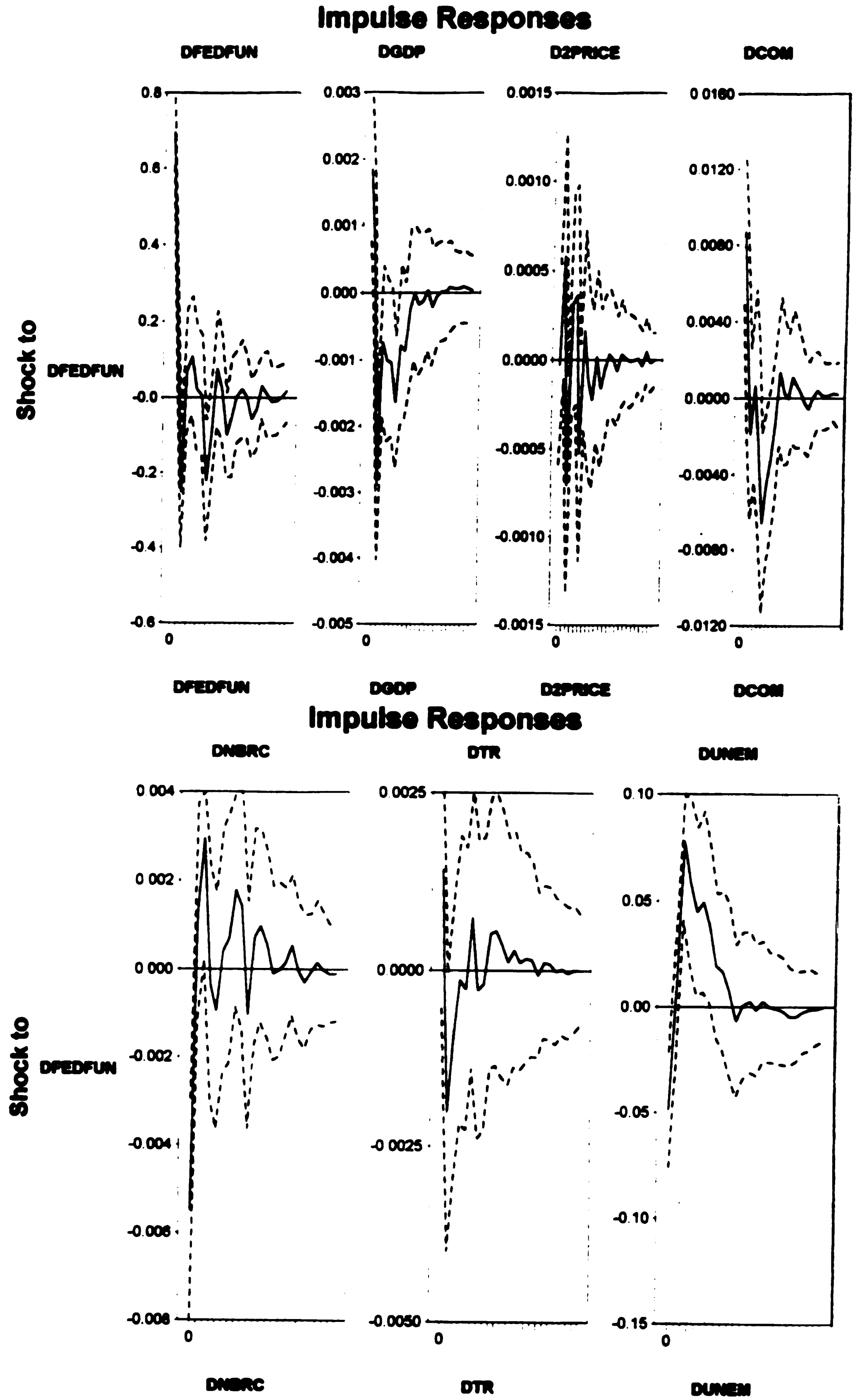
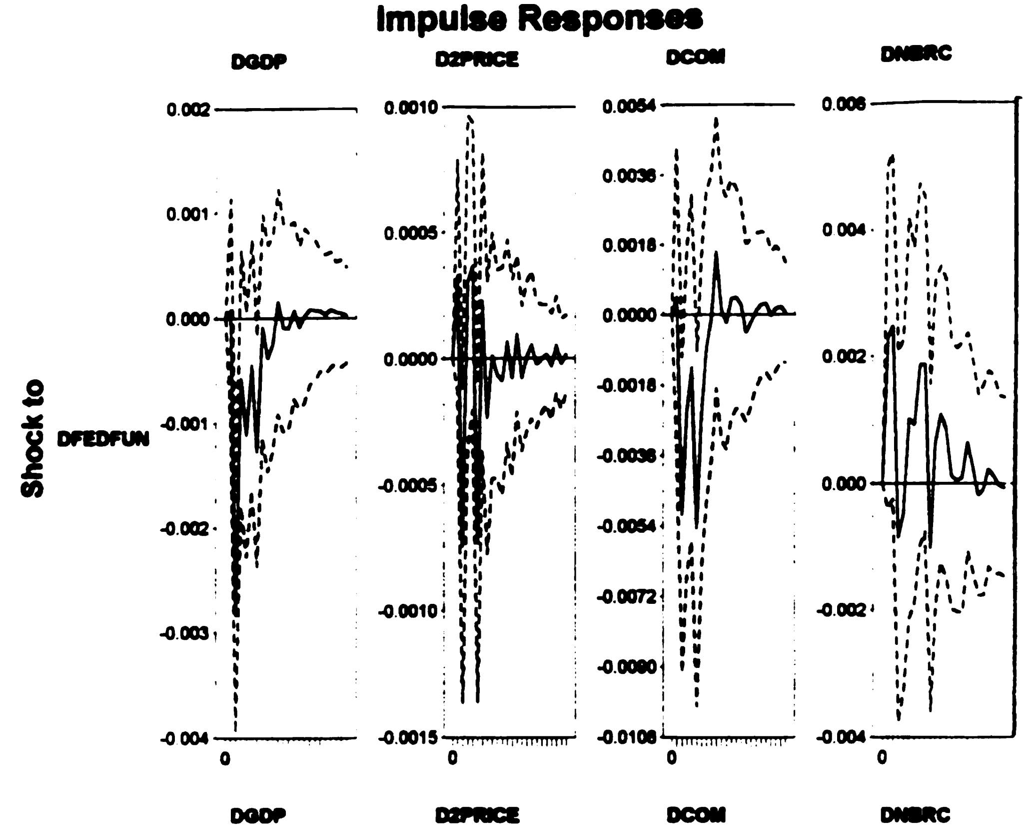

Impulse Responses
DFEDFUN
DGDP
D2PRICE
DCOM
Shock to
DFEDFUN
Impulse Responses
DNBRC
DTR
DUNEM
Shock to
DFEDFUN

Figure 4.32


Impulse Responses
DGDP
D2PRICE
DCOM
DNBRC
Shock to
DFEDFUN
0.002
0.001
0.000
-0.001
-0.002
-0.003
-0.004
0.0010
0.0005
0.0000
-0.0005
-0.0010
-0.0015
0.0054
0.0036
0.0018
0.0000
-0.0018
-0.0036
-0.0054
-0.0072
-0.0090
-0.0108
0.006
0.004
0.002
0.000
-0.002
-0.004
0
DGDP
D2PRICE
DCOM
DNBRC

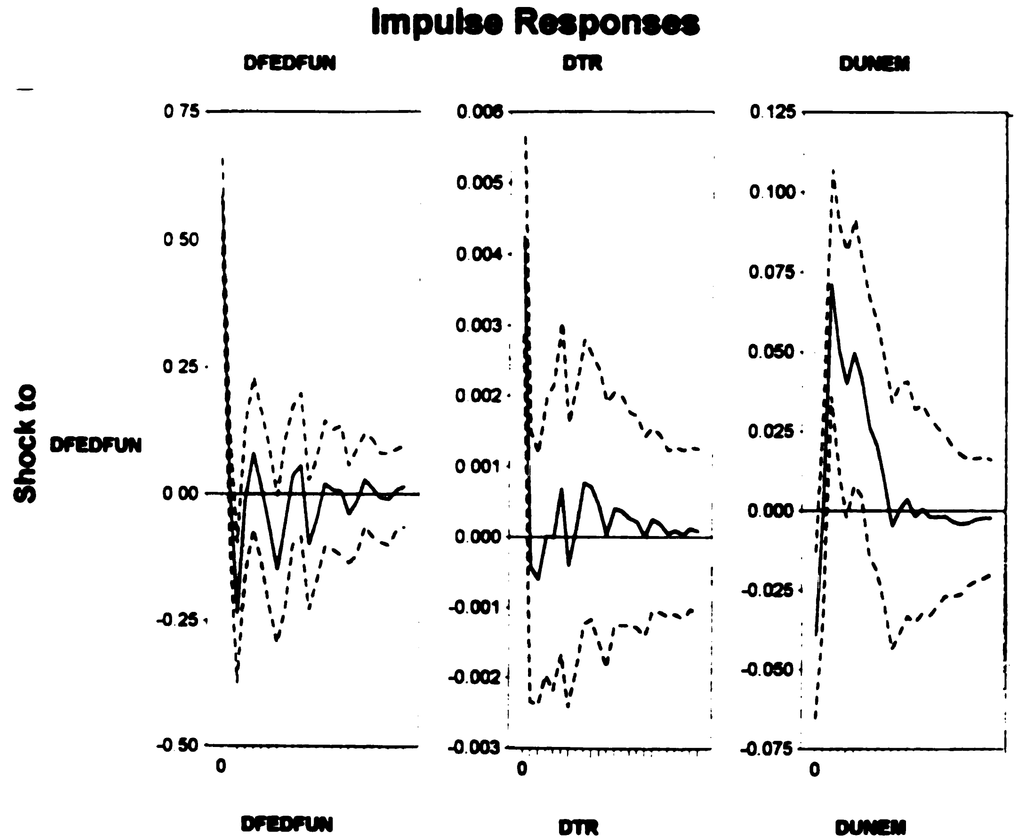

Impulse Responses
DFEDFUN
DTR
DUNEM
Shock to
DFEDFUN
0.75
0.50
0.25
0.00
-0.25
-0.50
0.006
0.005
0.004
0.003
0.002
0.001
0.000
-0.001
-0.002
-0.003
0.125
0.100
0.075
0.050
0.025
0.000
-0.025
-0.050
-0.075
0
DFEDFUN
DTR
DUNEM

Figure 4.33: Cumulative Effect

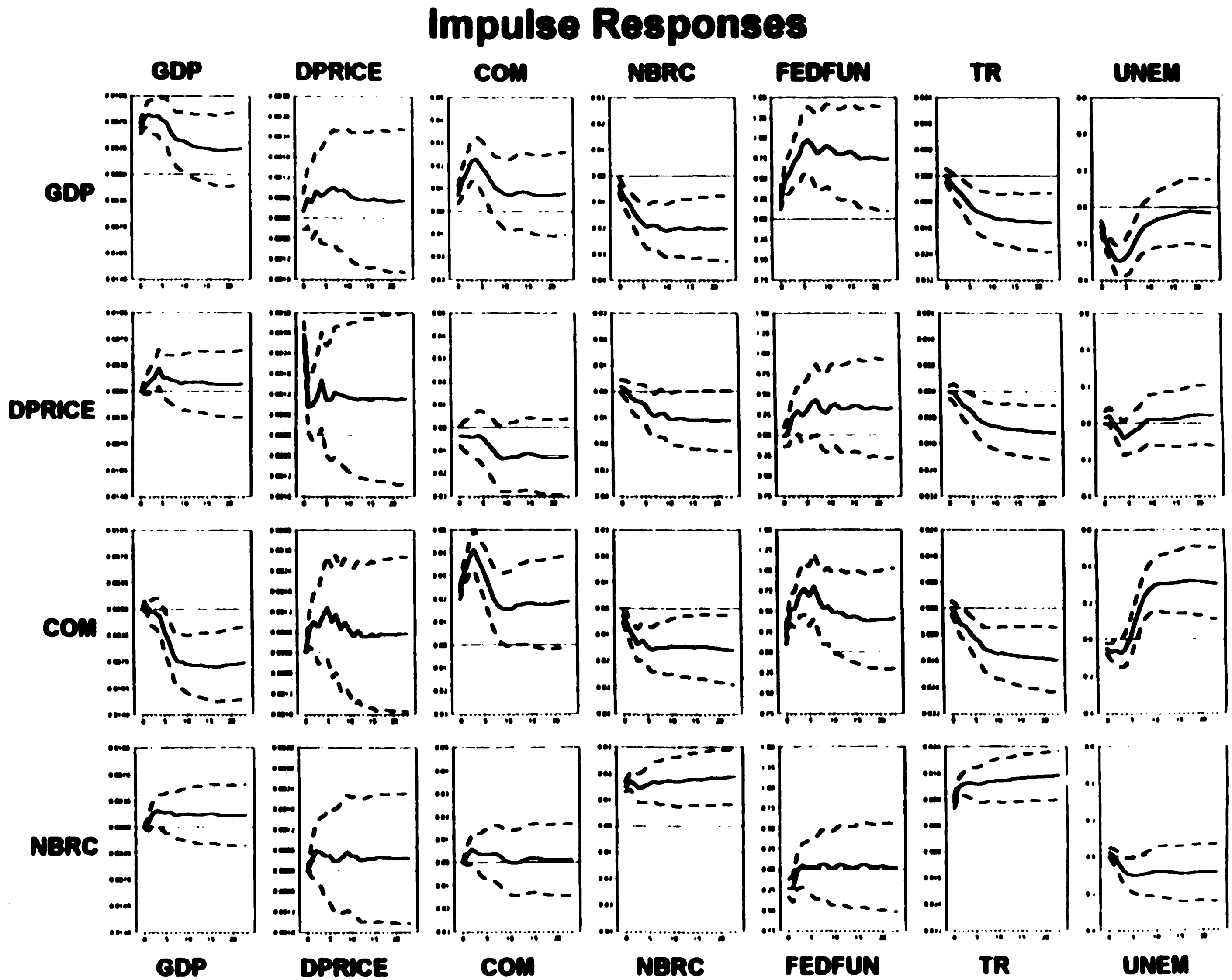


Figure 4.33 (Cont'd):

Impulse Responses

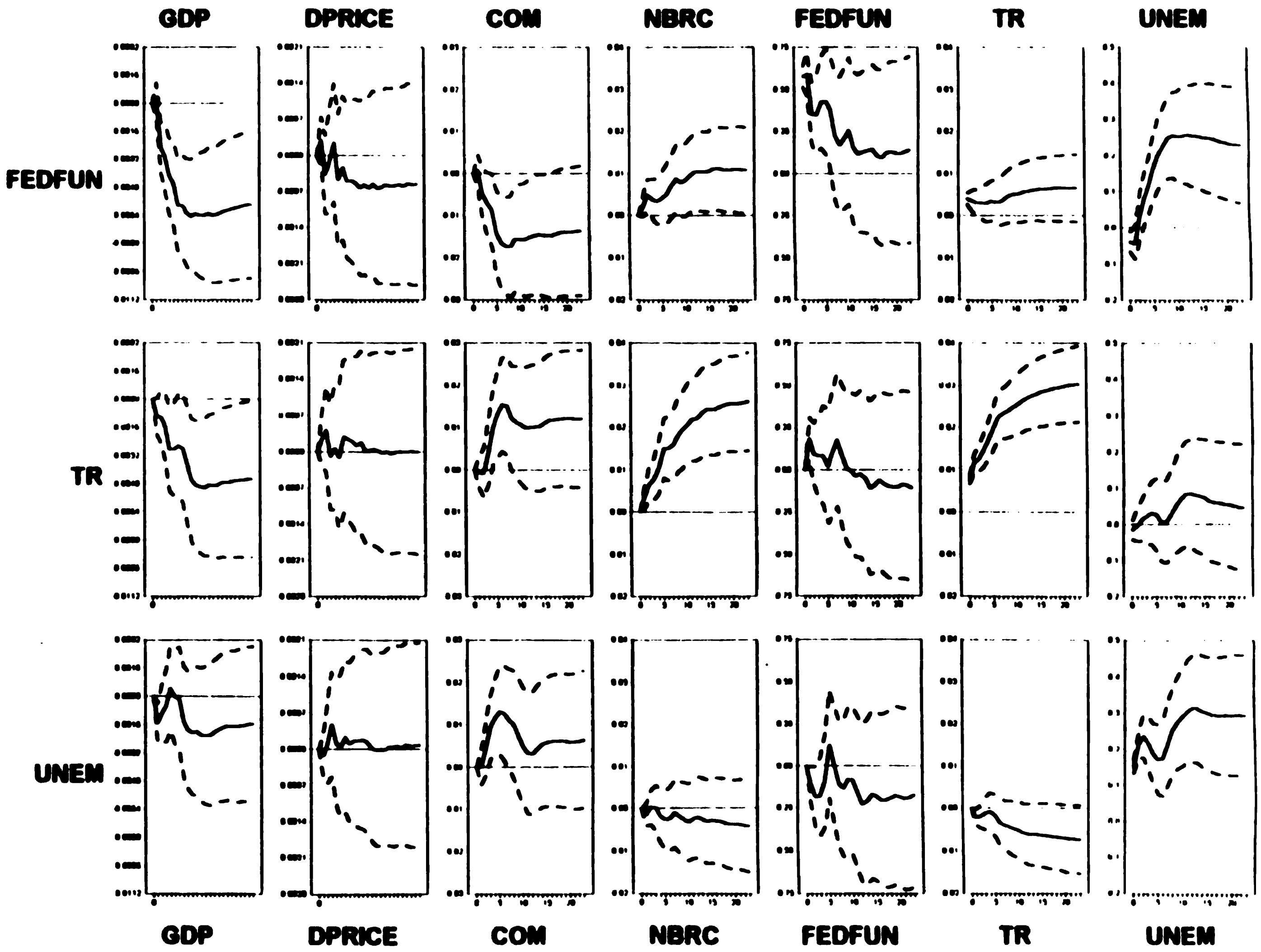


Figure 4.34: Cumulative Effect

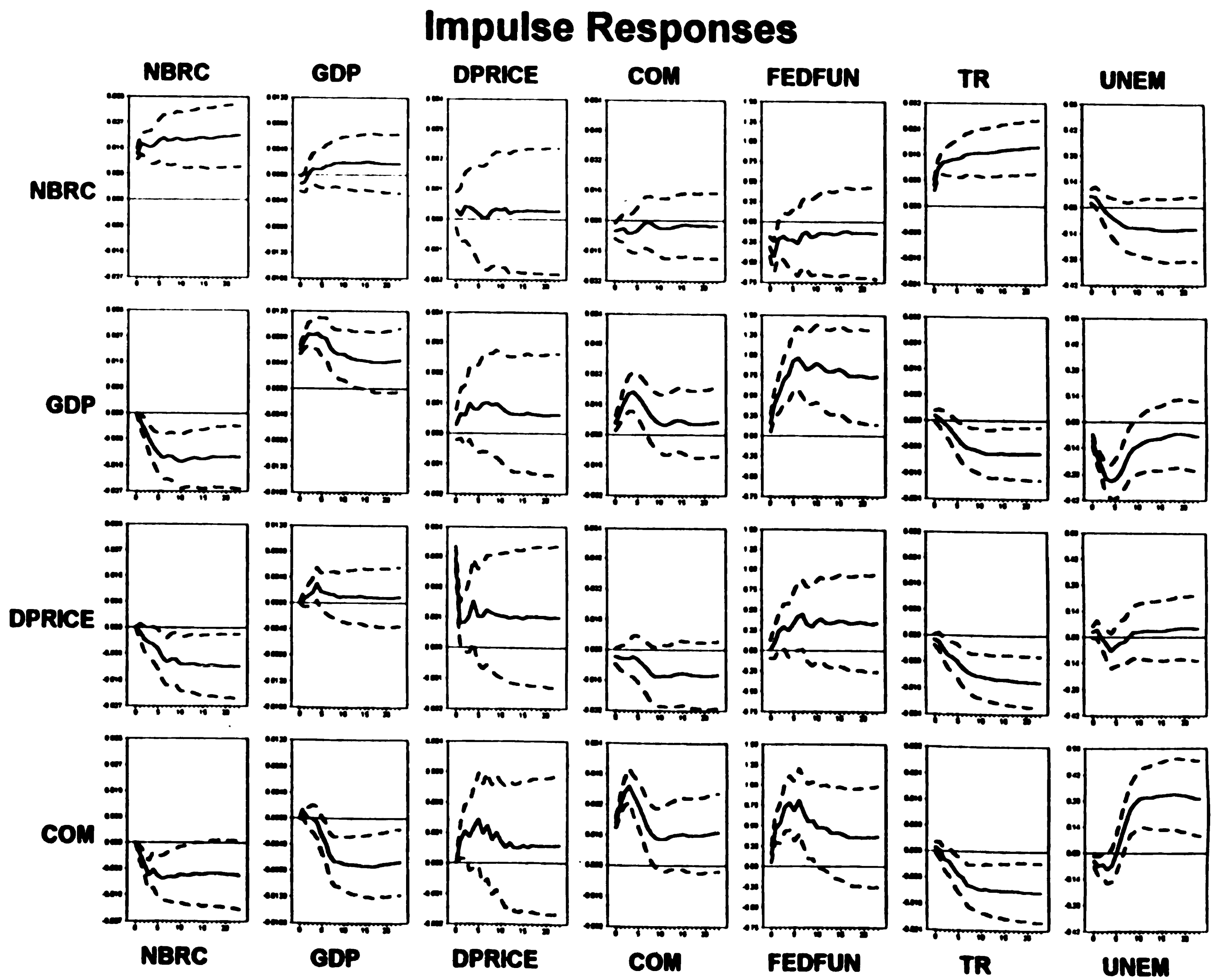


Figure 4.34 (Cont'd):

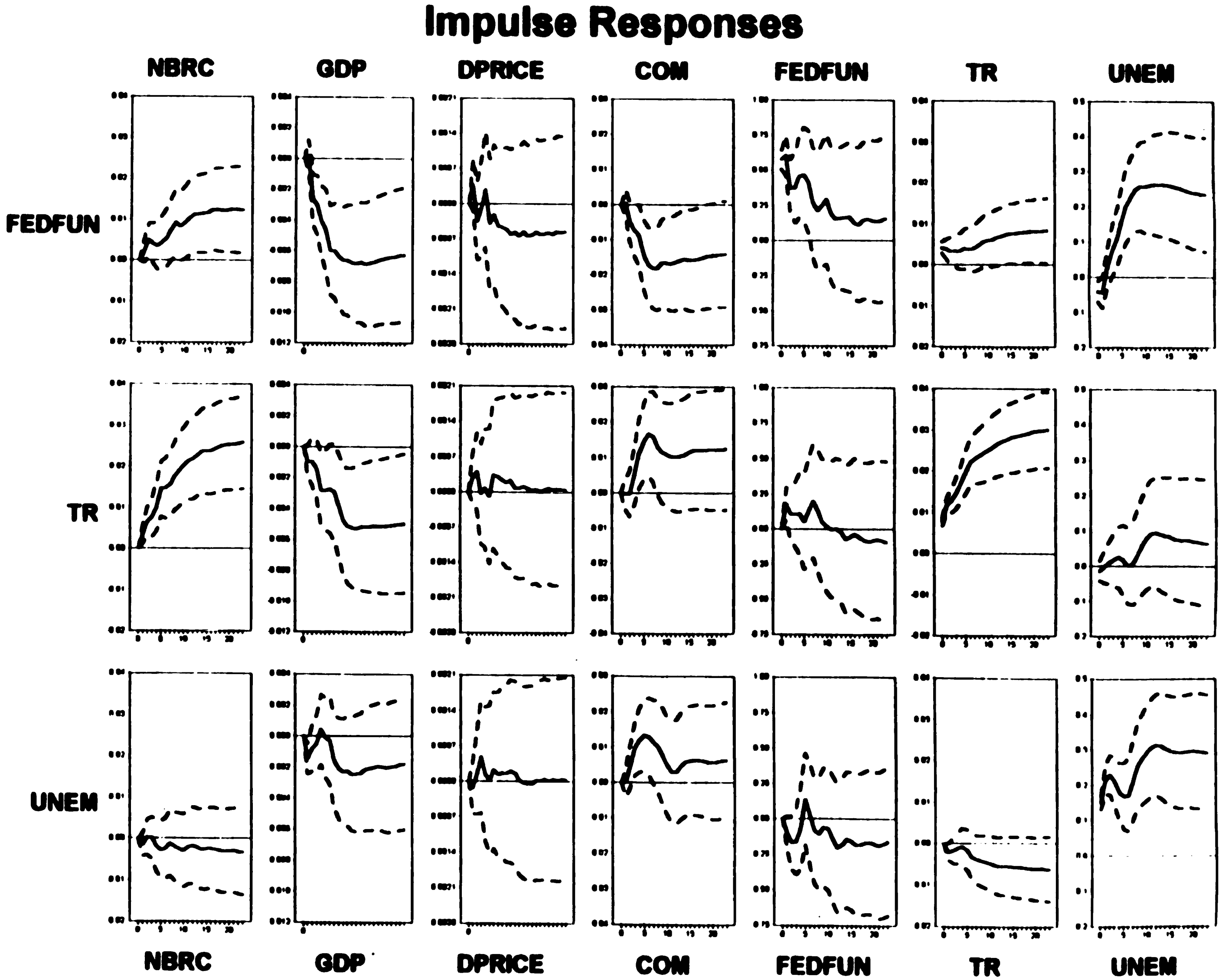


Figure 4.35: Cumulative Effect

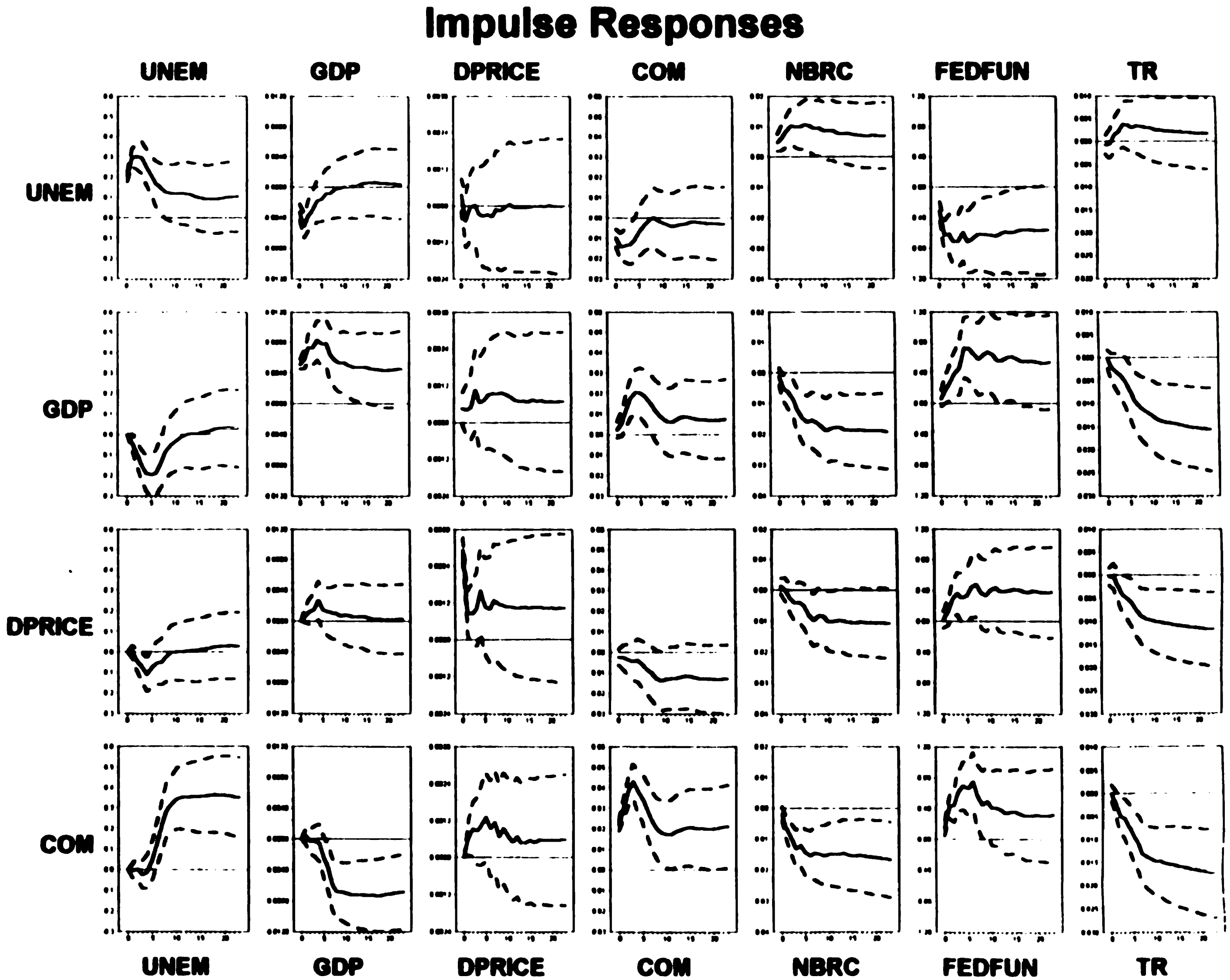


Figure 4.35(Cont'd):

Impulse Responses

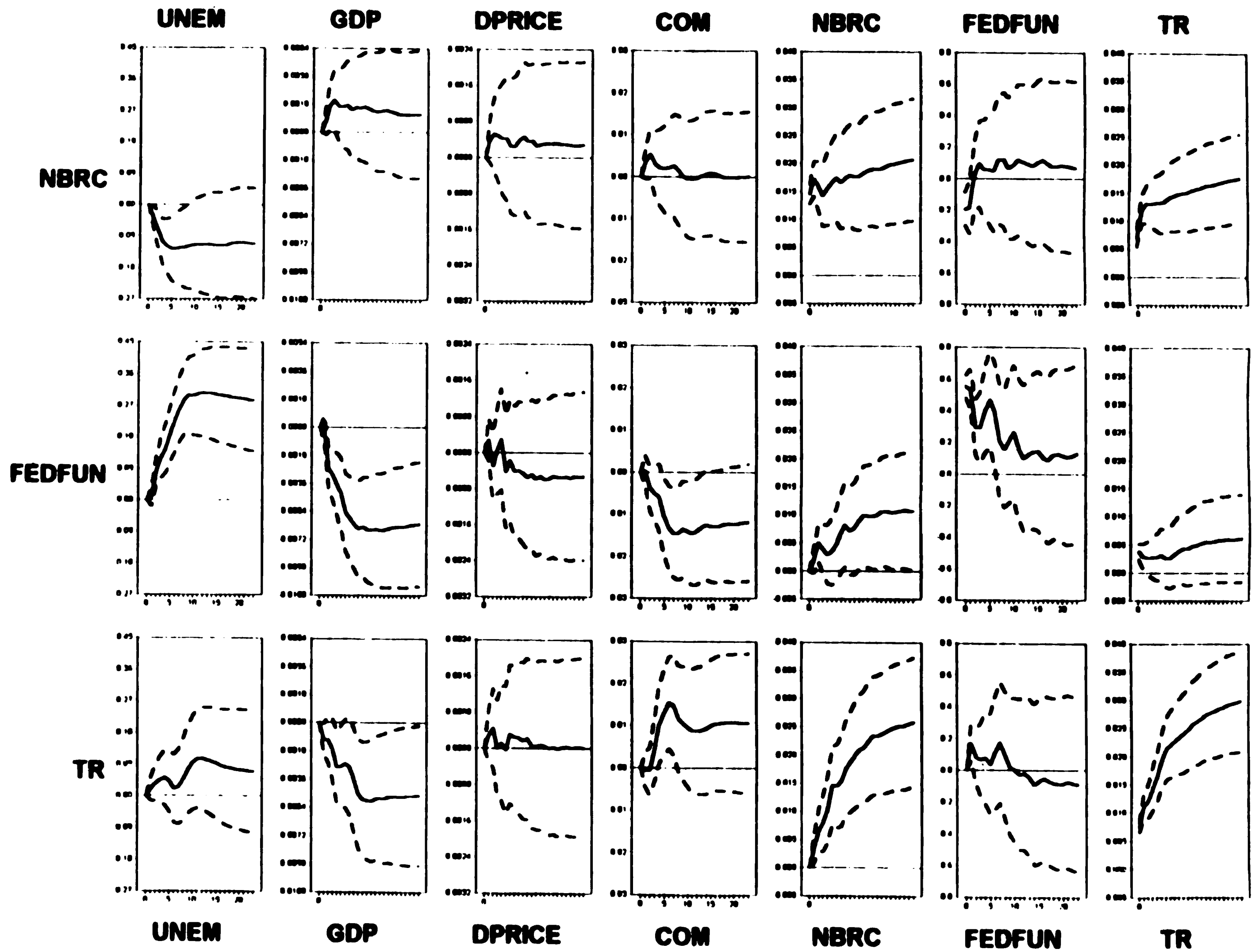


Figure 4.36: Cumulative Effect

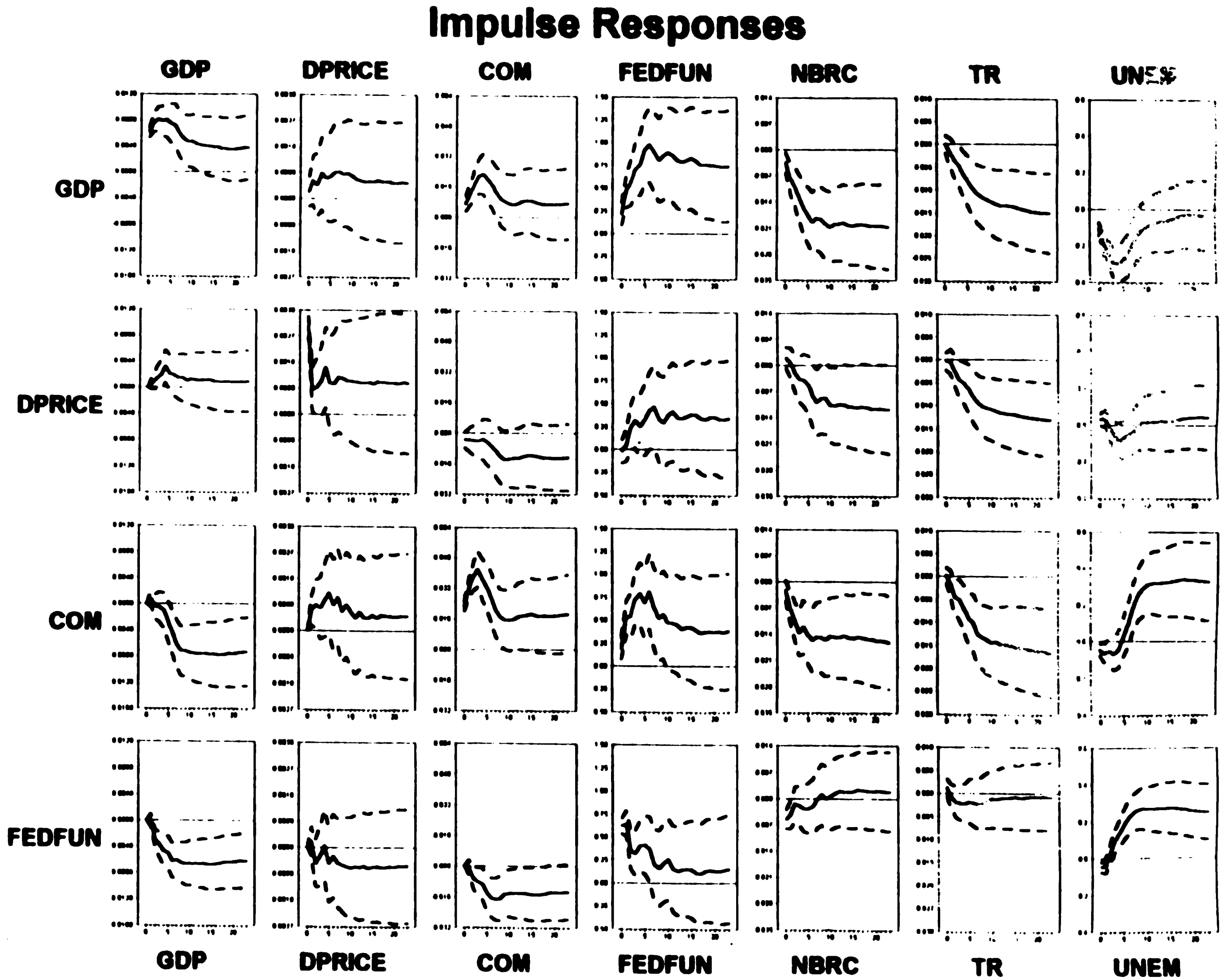


Figure 4.36 (Cont'd):

Impulse Responses

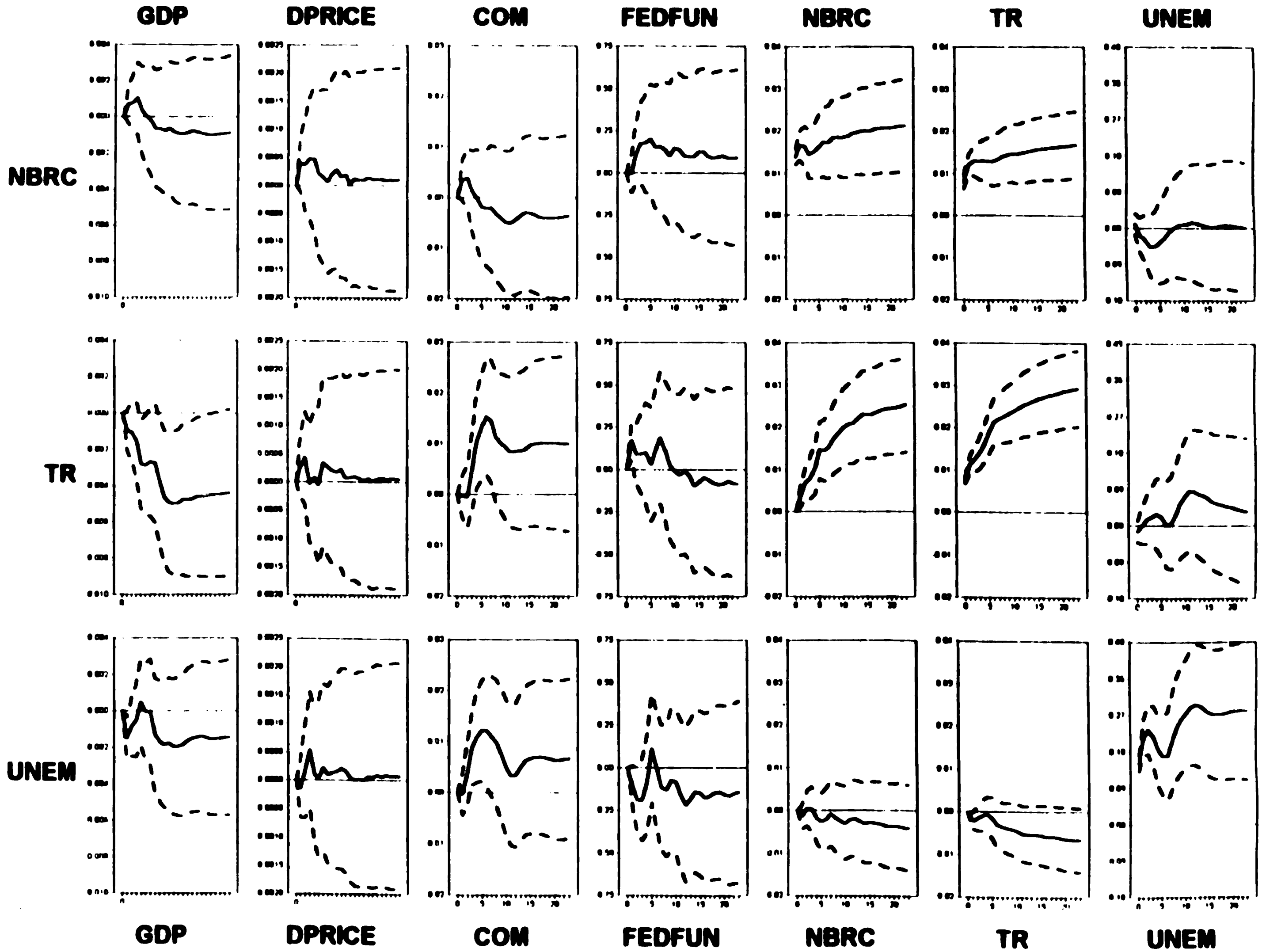


Figure 4.37: Cumulative Effect

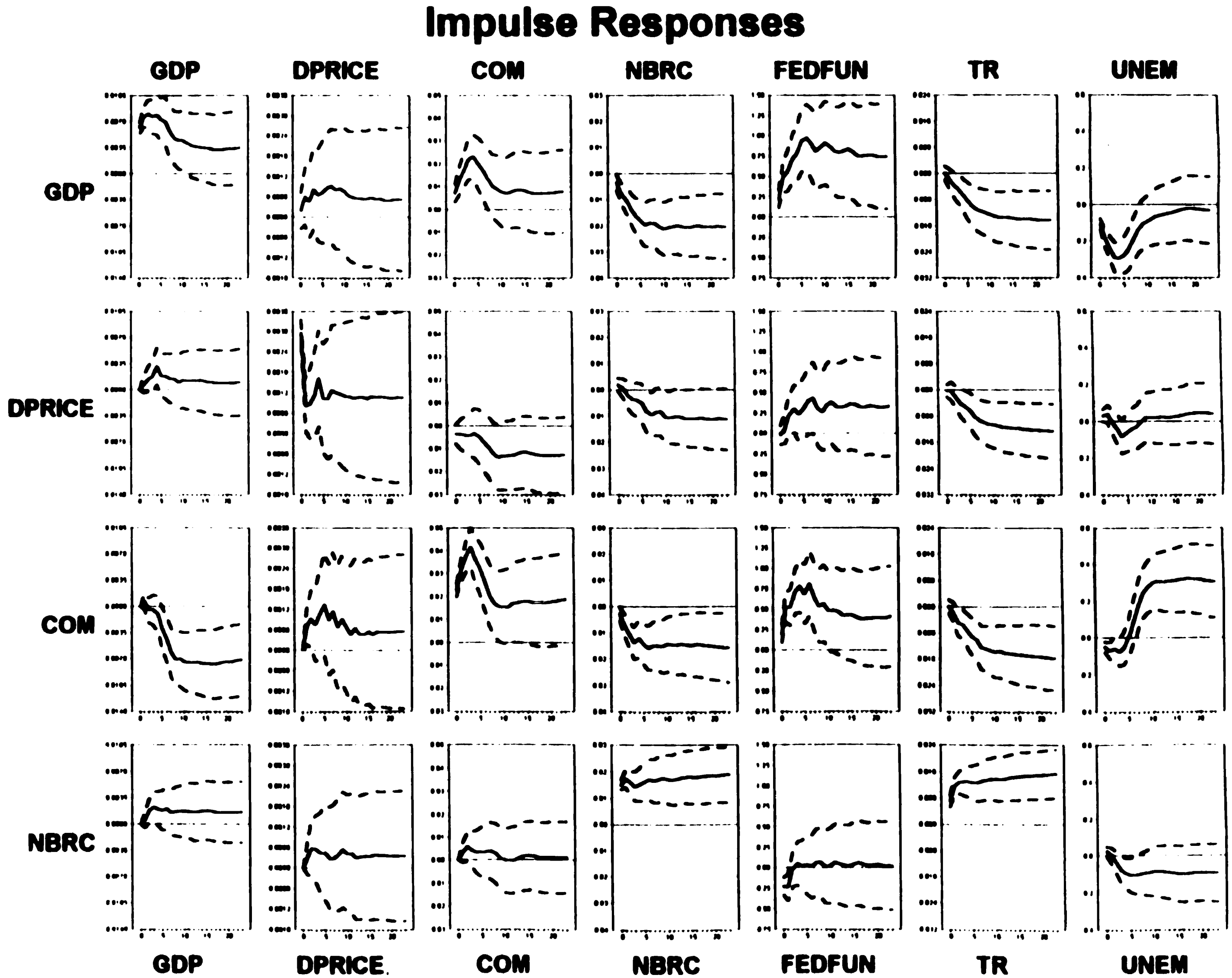


Figure 4.37 (Cont'd):
Impulse Responses

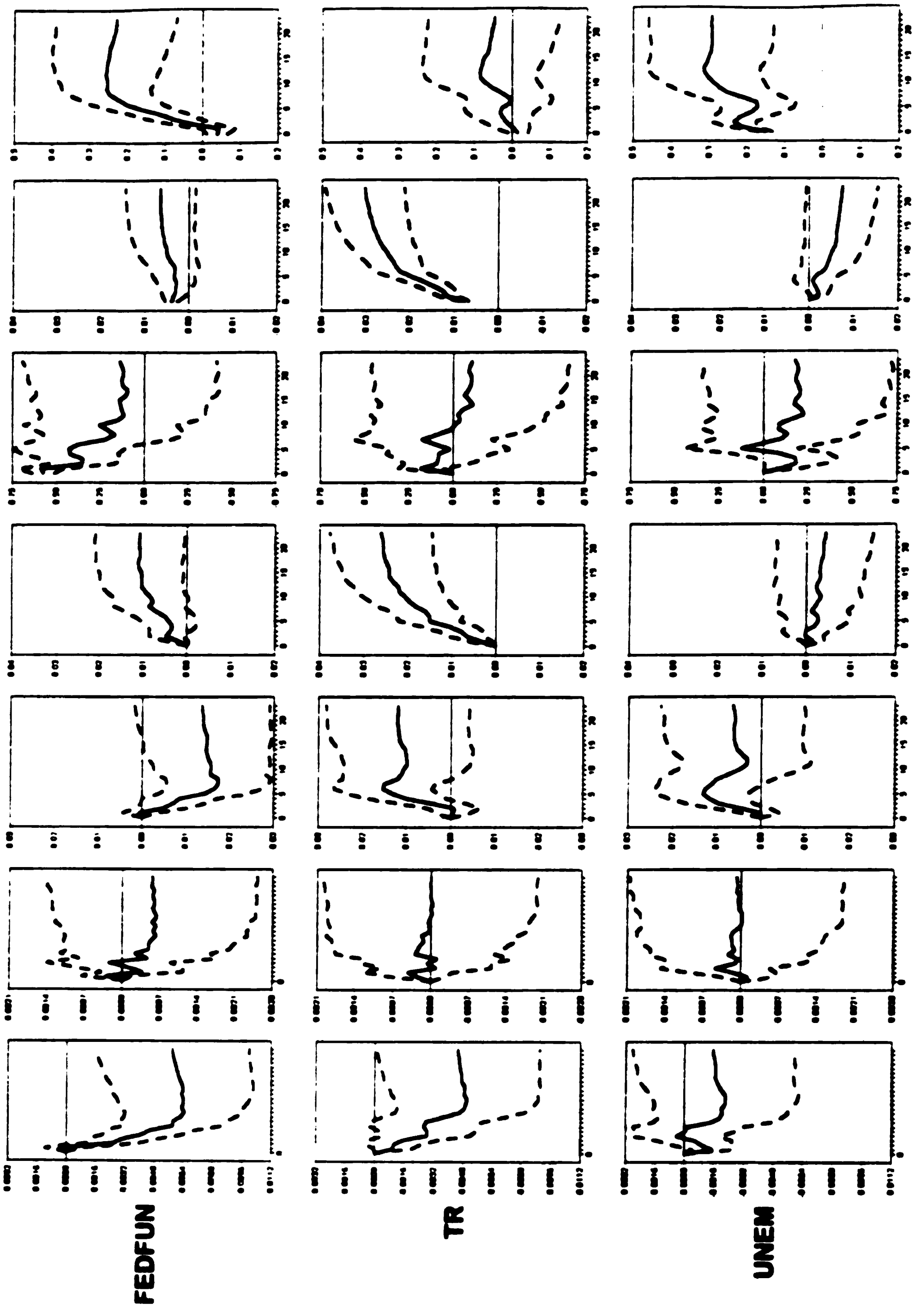


Figure 4.38: Cumulative Effect

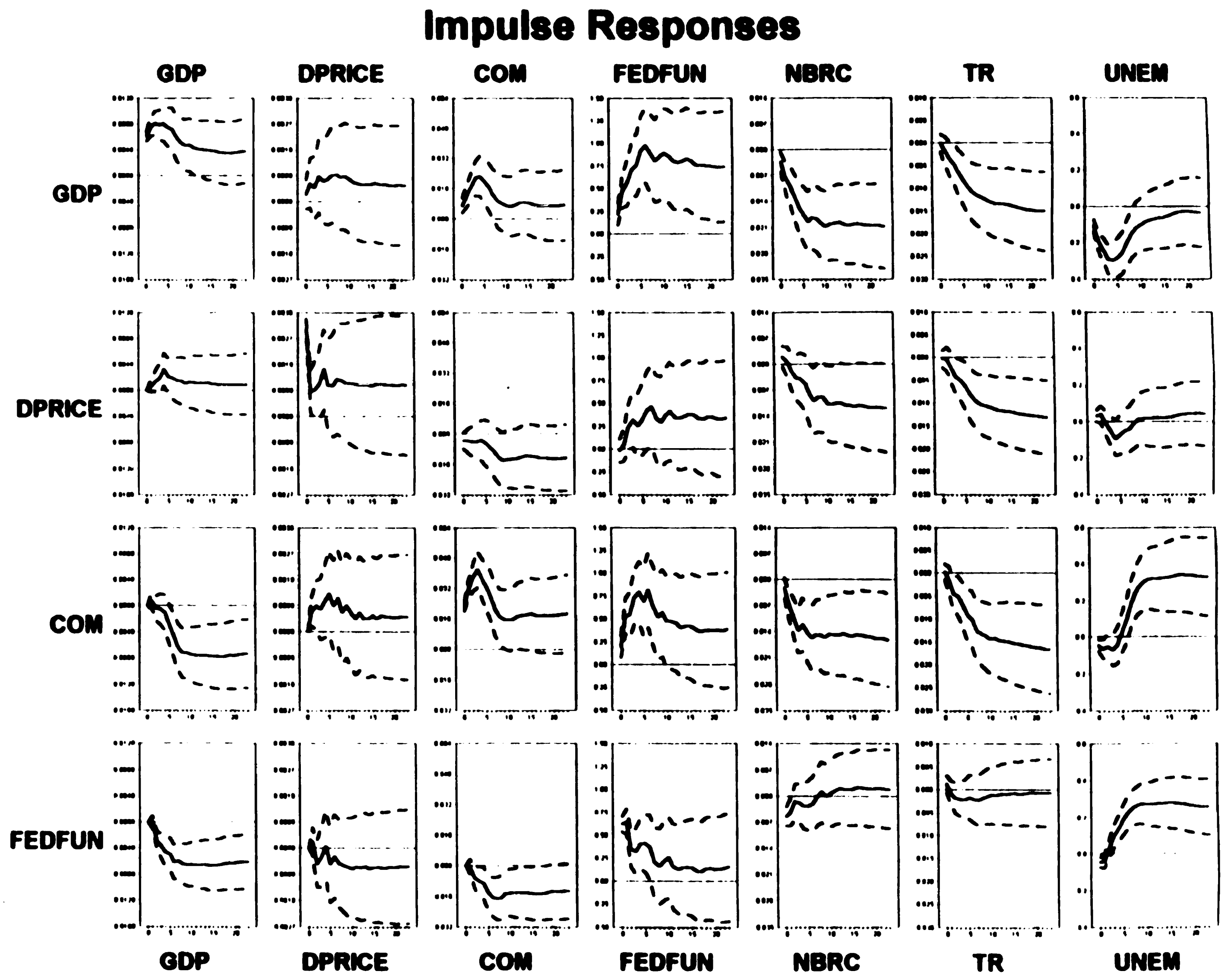


Figure 4.38 (Cont'd):

Impulse Responses

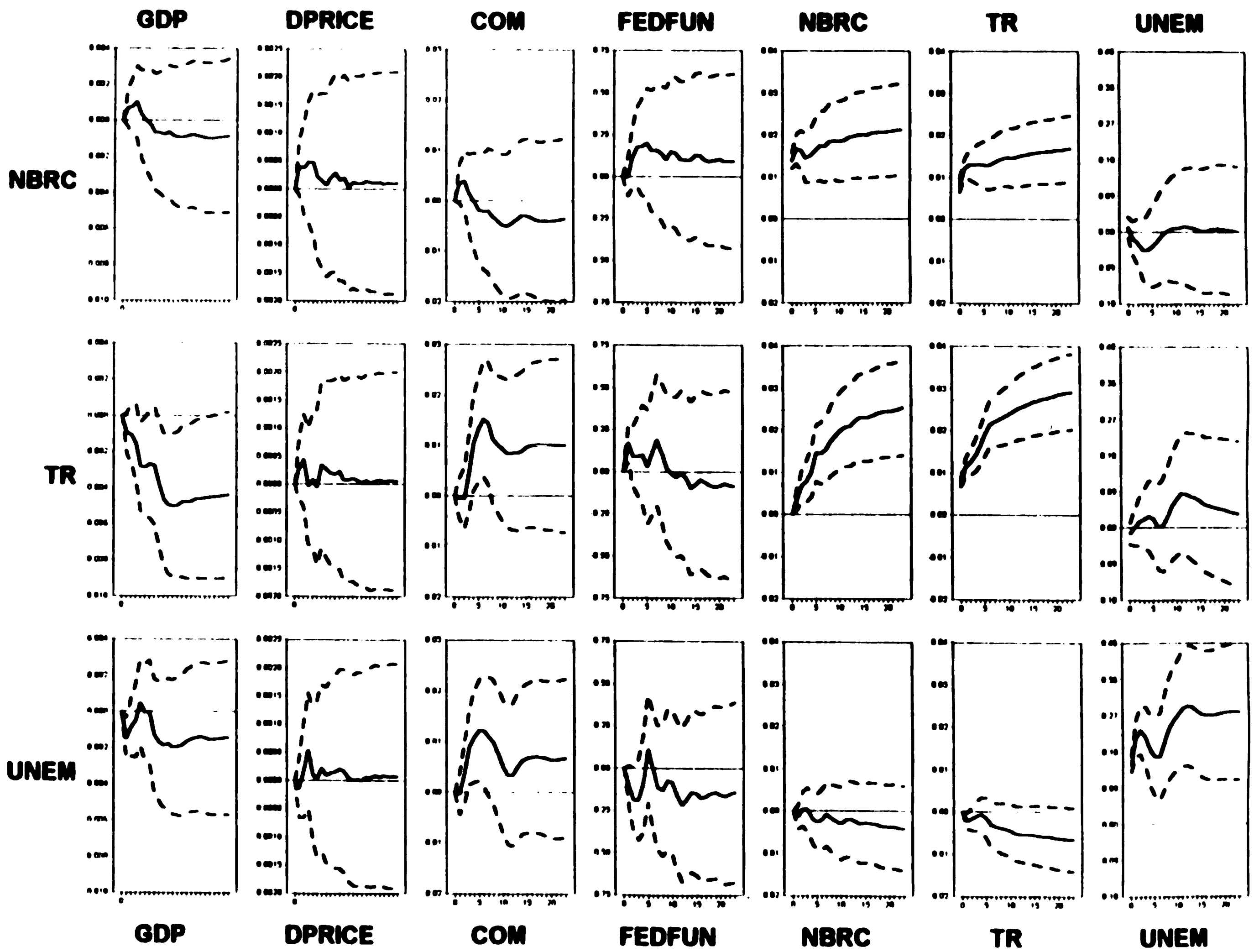


Figure 4.39: Cumulative Effect

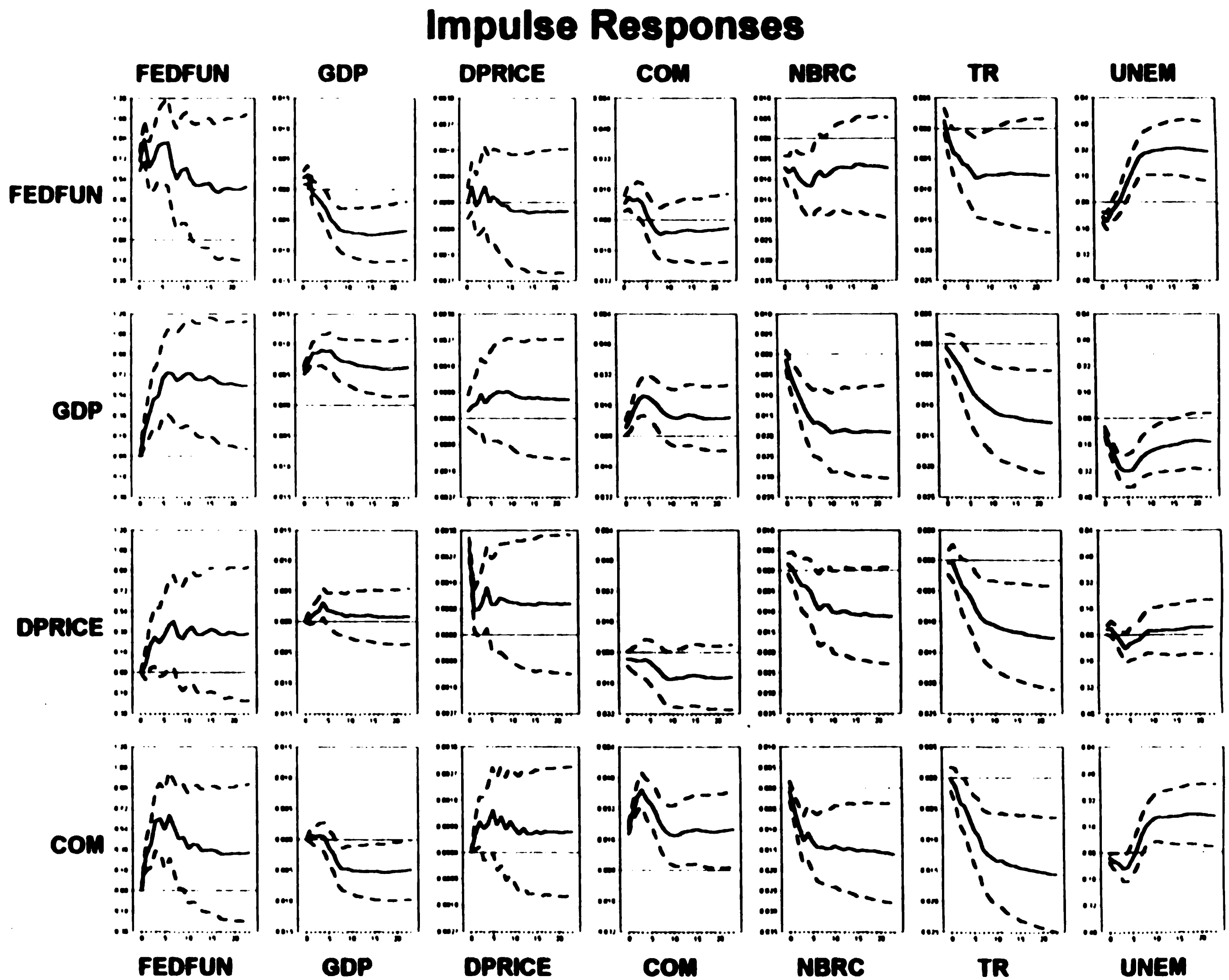


Figure 4.39 (Cont'd):

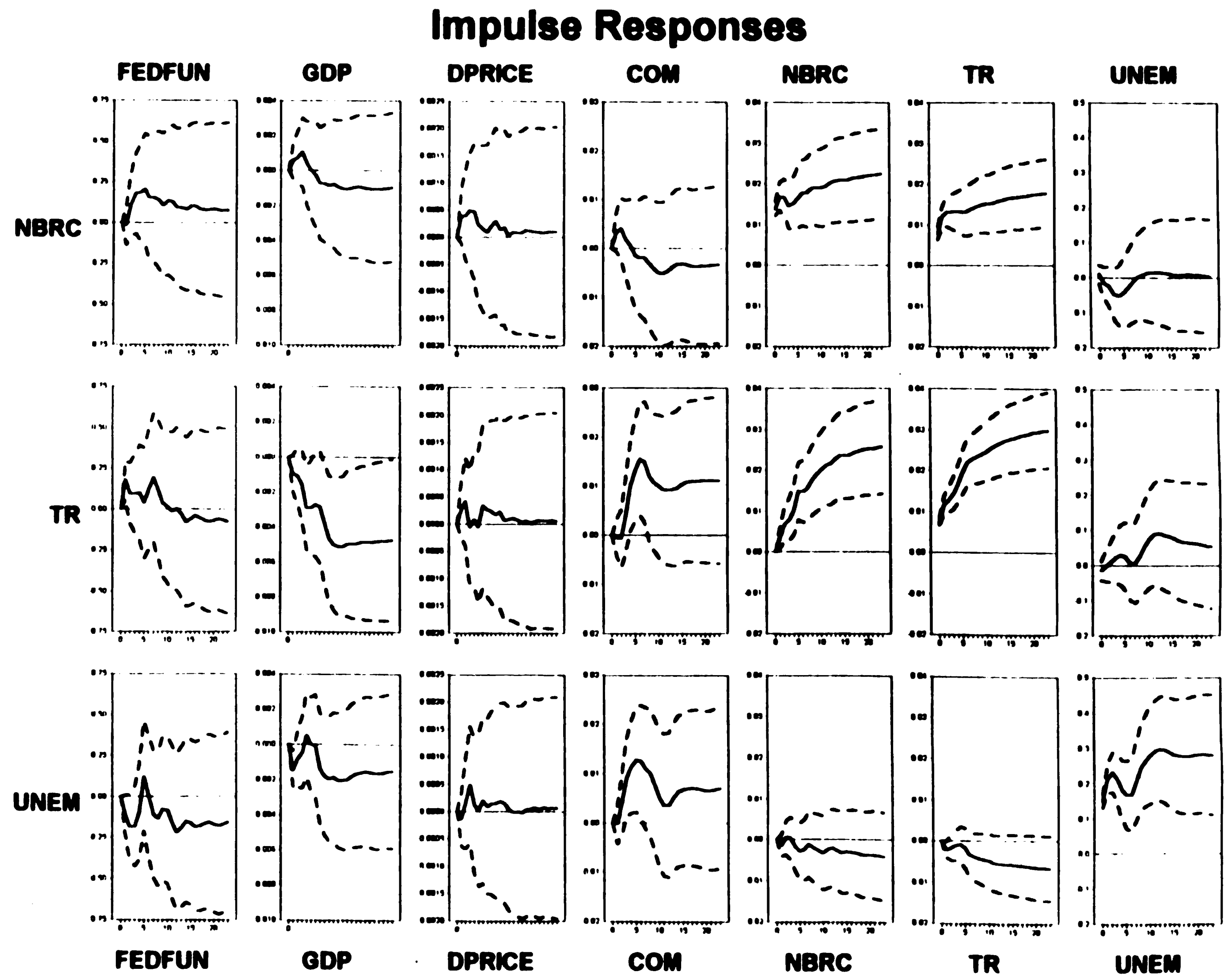


Figure 4.40: Cumulative Effect

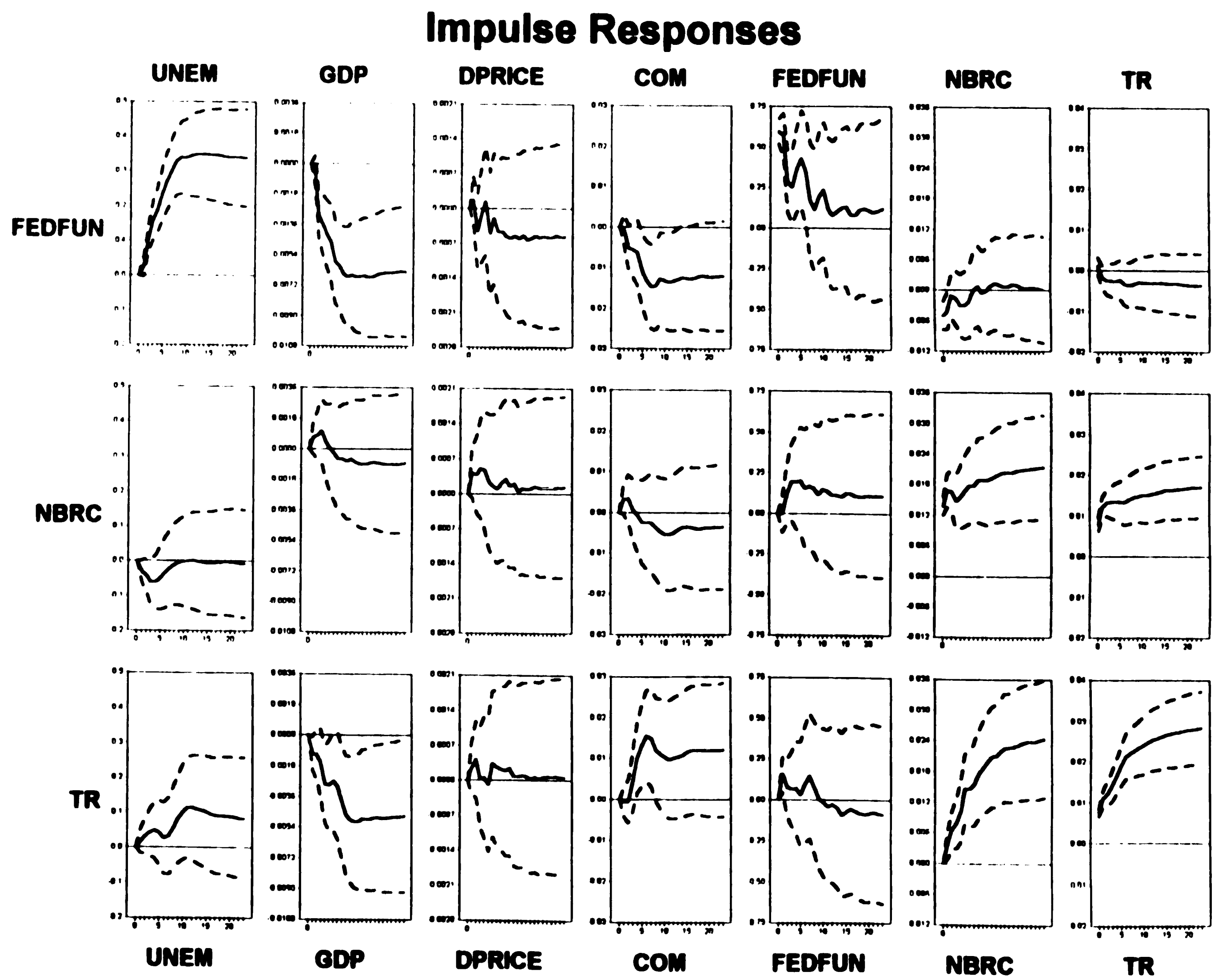
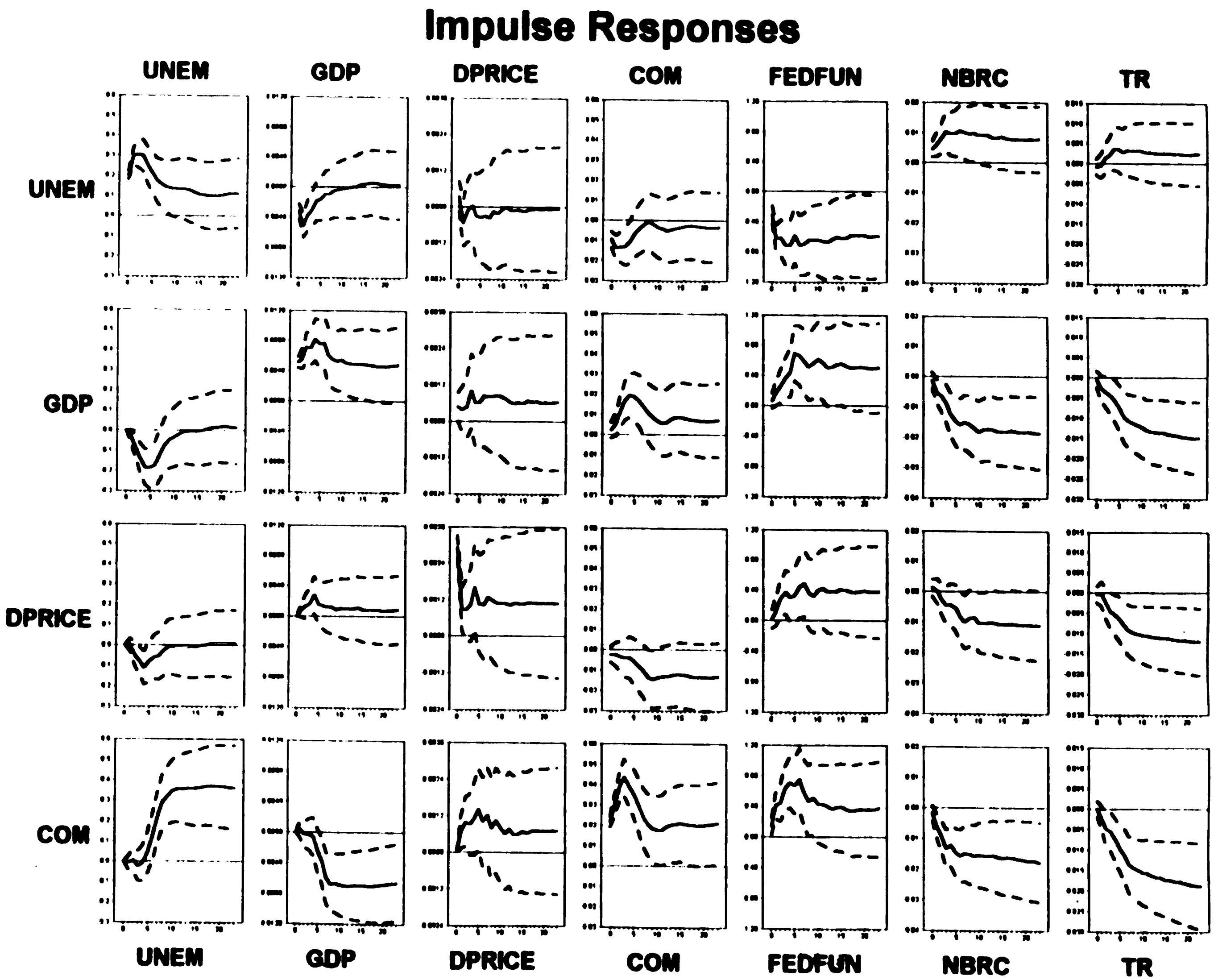


Figure 4.40 (Cont'd):



Appendix 4.1

Augmented Dickey Fuller Unit Root Tests in Levels

Variables in Logs unless otherwise noted	AR=4 No Trend	AR=4 Trend
GDP	-1.307	-2.307
Inflation (DPRICE)	-1.918	-1.667
Nonborrowed Reserves	1.737	-0.678
Total Reserves	-2.198	-2.060
Federal Funds Rate *	1.984	-0.662
Unemployment Rate*	-2.203	-2.337
Commodity Prices	-0.132	-2.196

* The Variable is in levels
**Significant at the 5 percent level.

Augmented Dickey Fuller Unit Root Tests in Differences

Variables in Logs unless otherwise noted	AR=4 No Trend	AR=4 Trend
GDP	-4.697*	-4.904*
Inflation (DPRICE)	-5.421*	-5.568*
Nonborrowed Reserves	-3.767*	-4.337*
Total Reserves	-4.068*	-4.116*
Federal Funds Rate *	-3.032*	-3.629*
Unemployment Rate*	-4.908*	-4.887*
Commodity Prices	-5.783*	-5.781*

* The Variable is in levels
**Significant at the 5 percent level.

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