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ESSAYS ON UNCERTAINTY AND SUSTAINABILITY IN THE SEMI-ARID TROPICS

By

Takeshi Sakurai

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
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DOCTOR OF PHILOSOPHY

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ABSTRACT

ESSAYS ON UNCERTAINTY AND SUSTAINABILITY IN THE SEMI-ARID TROPICS

By

Takeshi Sakurai

This study explores the effects of uncertainty on sustainability in rural areas of the semi-arid tropics characterized by mixed crop and livestock farming subject to frequent drought and lack of formal financial and insurance institutions. The fundamental hypotheses are: (i) Uncertainty due to drought induces precautionary saving; (ii) Saving takes a form of livestock holdings because livestock is a relatively safe asset and there is no formal saving institution. This combined with open access to common-pool resource results in overgrazing; (iii) Formal drought insurance will reduce overgrazing. The household's ability to smooth consumption and the village's ability to manage grazing land are the two key concepts of sustainability in this study. They are examined theoretically and empirically in four independent essays.

The first essay is a literature review. The second essay is original extensions of the models found in the literature. Effects of risks on saving are examined in the models, and it is found that precautionary demand for saving exists if saving is riskless.

In the third essay, using a static household portfolio model, conditions for overgrazing are examined by comparing an aggregation of household equilibrium livestock holdings with the social optimal livestock holding. Since households in this model are engaged in both crop production and livestock husbandry at the same time, the model predicts that overgrazing does not necessarily occur even under open-access. If crop

production risk is high enough, overgrazing will occur even if the expected return to crop production is greater than that to livestock holdings.

The fourth essay empirically explores potential demand for a hypothetical formal drought insurance scheme and the determinants of such demand using household data from Burkina Faso. Effective demand is found in all agroclimatic zones, but its size is correlated with the drought risk of the zone. The determinant analysis shows that those who have large crop stocks tend to demand less insurance, while the effects of livestock holdings and off-farm income are ambiguous. But some households keep large livestock holdings but are not sufficiently self-insured. Those households may purchase insurance to substitute for livestock holdings.

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TABLE OF CONTENTS

LIST OF TABLES xi

LIST OF FIGURES xiii

Chapter 1 Introduction

1 The Problem 1

1.1 Uncertainty and Lack of Financial Market 1

1.2 Sustainability 2

1.3 Hypotheses 5

2 Objectives of the Study 6

2.1 Objectives and Contributions of Chapters 2 and 3 7

2.2 Objectives and Contributions of Chapter 4 7

2.3 Objectives and Contributions of Chapter 5 8

References 11

Chapter 2 Household Saving in LDCs: A Critical Review

0 Summary 14

1 Introduction 16

2 Evidence of Consumption Smoothing 18

3 Household Saving Models 24

3.1 Borrowing Constraint and Uncertainty 24

3.2 Saving Under Certainty / Two-Period Model 26

3.3 Saving Under Uncertainty / Two-Period Model 27

3.3.1 Labor Income Uncertainty / Two-Period Model 27

3.3.2 Interest Rate Uncertainty / Two-Period Model....30
3.3.3 Labor Income and Interest Rate Uncertainty / Two-Period Model....31
3.4 Saving Under Certainty / Multiperiod Model....32
3.5 Saving Under Uncertainty / Multiperiod Model....33
3.5.1 Labor Income Uncertainty / Multiperiod Model....33
3.5.2 Interest Rate Uncertainty / Multiperiod Model....39
3.5.3 Labor Income and Interest Rate Uncertainty / Multiperiod Model....40
3.6 Stochastic Production / Multiperiod Model....41
3.7 Flexibility and Production/Saving....44
4 Conclusions....52
References....55
Appendix....59
A Determinants of Drought Shock in Burkina Faso....59
B Full Insurance in Burkina Faso....60

Chapter 3 Household Saving in LDCs: Extensions and New Structural Models

0 Summary....61
1 Introduction....63
2 Extensions of Saving Models....66
2.1 Correlation of Two Types of Risk....66
2.2 Stochastic Production Model....68
2.3 Flexibility and Production/Saving....71
3 Structural Models and Their Implications....73
3.1 Time Frame of Decision Making....75
3.2 A Simultaneous Decision Mode....76
3.2.1 The First Order Conditions....79
3.2.2 Capital and Labor Inputs....80

3.2.3 Saving and Borrowing....82
3.2.4 Crop Diversification....83
3.2.5 Social Insurance....84
3.2.6 Lifetime Budget Constraints....85
3.3 A Sequential Decision Model....86
3.3.1 The First Order Conditions....86
3.3.2 Effects of Increasing Risk....89
4 Conclusions....90
References....94
Appendix....103
A Labor Income and Interest Rate Uncertainty / Two-Period Model....103
B A Dynamic Household Model with Uncertain Production....104
C A Sequential Decision Model with Flexibility....106
D Effects of Increasing Risk on Decisions in a Sequential Decision Model....108

Chapter 4 Equity and Sustainable Management of Common-Pool Resources: The Case of Grazing Land in the West Africa Semi-Arid Tropics

0 Summary....110
1 Introduction....113
2 Definitions and Approach....115
2.1 Definitions....115
2.2 Approach....118
2.2.1 Relationship to Previous Work....118
2.2.2 Modeling Strategies....119
3 The Model....121
3.1 Framework....121
3.2 The Household optimum under open Access....124
3.3 Interhousehold Distribution of Optimal Investment....125

3.4 Effect of Borrowing Constraints....128
3.5 Aggregate Livestock Holdings under Open Access....129
3.6 The Social Optimum....130
3.7 Comparison between Household and Social Optima....133
4 Application of the Model to grazing Land Management....135
4.1 Physical Solutions....136
4.2 Institutional Solutions....137
4.2.1 Privatization....137
4.2.2 Quota....139
4.2.3 Transferable Permits....140
4.2.4 User Fee....142
4.3 Other Economic Solutions....142
4.3.1 Off-farm Income....142
4.3.2 Formal Drought Insurance....143
5 Conclusions....143
References....146
Appendix....154
A Proofs of Propositions 1-19....154
B Proofs for Privatization....166
C Proofs for Quota....168
D Proofs for Transferable Permits....170
E Proofs for User fee....171

Chapter 5 Crop Production under Drought Risk and Estimation of Demand for Formal Drought Insurance in the Sahel

0 Summary....177
1 Introduction....179
2 The Model....183

2.1 Time Frame....183
2.2 A Dynamic Household Model....184
2.3 The First Order Conditions....187
3 Data....191
4 Empirical Estimation....192
4.1 Production Function....192
4.2 Estimation of Demand for Drought Insurance (ϕ)....193
4.3 Determinant of Demand for Drought Insurance (ϕ)....194
5 Results and Discussion....197
5.1 Production Function....198
5.2 Estimation of Demand for Drought Insurance (ϕ)....201
5.3 Determinant of Demand for Drought Insurance (ϕ)....202
6 Conclusions....206
References....209
Appendix....221
A Change in Average Rainfall and Drought Probabilities....221
B Expected Rainfall Levels (mm)....223
C. Calculation of (30), (31), and (32)....224
D Mean and Standard Deviation of Wealth Variables....225
E Derivations for Table 3....226
F Regression Results at 35% Drought Probability....229

Chapter 6 Summary of Approach and Findings....233

LIST OF TABLES

Chapter 2 Household Saving in LDCs: A Critical Review

Table 1 Effect of Increasing Risk on Saving and Consumption.....58

Chapter 3 Household Saving in LDCs: Extensions and New Structural Models

Table 1 Relationship Between $\tilde{r}$ and $\tilde{y}_2$.....95

Table 2 Effect of Increasing Risk on Decisions (uncorrelated).....97

Table 3 Effect of Increasing Risk on Decisions (correlated).....98

Table 4 Effect of Increasing Risk on Decisions (simultaneous).....101

Table 5 Effect of Increasing Risk on Decisions (sequential).....102

Chapter 5 Crop Production under Drought Risk and Estimation of Demand for Formal Drought Insurance in the Sahel

Table 1 Comparison of Assumptions on Drought Probability.....211

Table 2 Annual Village-Level Rainfalls and Probabilities.....213

Table 3 Production Function Estimates with Fixed Effects.....214

Table 4 Summary of Signs of Marginal Productivity.....215

Table 5 Zone-level ϕ and Drought Probabilities.....216

Table 6 Determinants of ϕ (full sample).....218

Table 7 Determinants of ϕ (zone).....219

Table 8 Determinants of ϕ (stratified by livestock holdings).....220

Table A1 Segou (Sudanian zone, Mali), Drought Frequency.....221

Table A2 Koutiala (Guinean zone, Mali), Drought Frequency.....222

Table B1 Expected Rainfall (mm).....223

Table D1 Mean and Standard Deviation of Wealth variables.....225

Table E1 Signs of Marginal Productivity of Labor....227

Table E2 Signs of Marginal Productivity of Capital....227

Table E3 Signs of Marginal Productivity of Crop Diversification....228

Table E4 Signs of Marginal Productivity of Cultivated Area....228

Table F1 Determinants of ϕ (full sample)....230

Table F2 Determinants of ϕ (zone)....231

Table F3 Determinants of ϕ (stratified by livestock holdings)....232

LIST OF FIGURES

Chapter 3 Household Saving in LDCs: Extensions and New Structural Models

Figure 1 Relationship Between $\tilde{r}$ and $\tilde{y}_2$ 96

Figure 2 Time Frame of Simultaneous Decision Making 99

Figure 3 Time Frame of Sequential Decision Making 100

Chapter 4 Equity and Sustainable Management of Common-Pool Resources: The Case of Grazing Land in the West Africa Semi-Arid Tropics

Figure 1 Household's Risk Aversion Coefficient (ϕ) and Wealth 149

Figure 2 Household's Relative Risk Aversion Coefficient (ϕW) and Wealth 150

Figure 3 Optimal Investments (case 1, m-g(*)+b>0) 151

Figure 4 Optimal Investments (case 2, m-g(*)+b>0) 152

Figure 5 Optimal Investments (case 3, m-g(*)+b≤0) 153

Figure A1 Determination of Relative Size of Crop and Livestock Investments 172

Figure A2 Comparison of Household and Social Optima (case 1, m-g(*)+b<0) 173

Figure A3 Comparison of Household and Social Optima (case 2, m-g(*)+b≥0) 174

Figure A4 Comparison of Household and Social Optima (case 3, m-g(*)+b≥0) 175

Figure A5 Comparison of Household and Social Optima (case 4, m-g(*)+b≥0) 176

Chapter 5 Crop Production under Drought Risk and Estimation of Demand for Formal Drought Insurance in the Sahel

Figure 1 Time Frame of Decision Making 212

Figure 2 Interpretation of ϕ 217

Chapter 1

Introduction

1 The Problem

The main issues of this dissertation are household behavior under uncertainty in the semi-arid tropics (SAT) and its effects on common-pool resources. These two issues are linked by two major characteristics of the SAT: (i) existence of drought risk, and (ii) lack of formal credit and insurance institutions., where the agricultural system is characterized by sedentary village settlements engaged in mixed crop and livestock production such as are common in the West Africa semi-arid tropics (WASAT).

An important concept underlying the links between the two issues is sustainability. In the following sub-sections, I explain how they are linked by introducing the concept of sustainability. Then, I provide hypotheses that are theoretically explored and empirically tested in this dissertation.

1.1 Uncertainty and Lack of Formal Financial Institutions

> All human endeavors are constrained by our limited and uncertain knowledge - about external events past, present, and future; about the laws of Nature, God, and man; about our own productive and exchange opportunities; about how other people and even we ourselves are likely to behave (Hirshleifer and Riley, 1992).

On the one hand, there are many sources of uncertainty facing households: some are common over households, such as drought and volatile market prices, and some are idiosyncratic to a specific household, such as illness and accidents. But among them natural events most heavily affect people's life in rural areas of developing countries where livelihood depends on natural resources. In the SAT, drought is the major factor that determines household behavior.

On the other hand, formal insurance, and credit and capital markets are severely underdeveloped or are not accessible by the rural poor in the SAT (e.g. Aleem, 1993; Bell, 1993; Binswanger, 1986; Binswanger and Sillers, 1983; Braveman and Guasch, 1993; Christensen, 1989; Hoff and Stiglitz, 1993; Matlon, 1990; Udry, 1993a).

Faced with drought risk and the lack of those markets, rural households in the SAT engage in a variety of risk management/coping strategies to smooth interyear income and consumption. Those strategies have been empirically examined recently, e.g., earning non-agricultural income (Reardon *et al.*, 1992); investment in livestock (Christensen, 1989; Rosenzweig and Wolpin, 1993); crop storage (Udry, 1993b); transfers among households (Lucas and Stark, 1985; Rosenzweig and Stark, 1989); informal credit (Udry, 1994); crop diversification (Matlon, 1990; Norman, 1973). In addition, instead of focusing on a particular strategy, overall consumption smoothing is also examined in risk-sharing models (e.g. Deaton, 1994; Sakurai, 1995; Townsend, 1994).

However, the effects of risk management/coping strategies on common-pool resources have been rarely examined yet. To examine the links between them, sustainability is the key concept. Sustainability can be defined at different levels, such as the household level and the village level, to be explained in the next section. Sustainabilities at different levels can be complementary in some case, but can be conflicting in other case. And the links can be seen as the relationship between sustainabilities at different levels.

1.2 Sustainability

Sustainability has become an important issue in international development, but its definition differs depending on at what level and in what agricultural system the sustainability is considered (Lele, 1991; Pezzy, 1992; Ruttan, forthcoming). On the one hand, because the concept of sustainability is related to the use of natural resources, it is often defined at global, regional, or national level (e.g. Pearce and Atkinson, 1995). However, on the other hand, as sustainability is based on household behavior, it can be defined at the household level. For

example, in the Sahel, maintaining a household's agricultural productivity by controlling soil degradation is the main issue in agricultural sustainability (Reardon, forthcoming). The literature usually focuses on the "vicious cycle" between rural poverty and environmental degradation, but Reardon and Vosti (1995) argue that categorizing poverty and natural resources is necessary to analyze the links, and that the links should be understood in terms of household income and investment/saving behavior.

Combining the discussion in section 1.1 with the household-level sustainability mentioned above, sustainability must be analyzed based on a household's risk management/coping strategies in the absence of formal credit and insurance in the SAT. Because each household's objective is to sustain under risk its well-being, or more narrowly, to smooth consumption above a minimum level over a certain period, I define household-level sustainability as the ability of the household to smooth consumption over time. This comprises for example agricultural productivity by controlling soil erosion, saving, as well as other strategies in crop production, in off-farm activities, and in livestock husbandry. Consequently, a household's risk response is a function of household characteristics and other exogenous factors, although the objective is to smooth consumption. For example, some households will use off-farm income to increase agricultural productivity, but others will not; whether off-farm income is reinvested in farming will depend on nature of capital and labor market and characteristics of off-farm income flow (Reardon *et al.*, 1994). Effects of livestock income and crop diversification on agricultural productivity are unknown, too (Reardon, forthcoming)[1].

By defining sustainability at the household level as a household's ability to smooth consumption, in addition to crop production, off-farm activities and livestock husbandry become important for the household to achieve sustainability. For example, Reardon *et al.*

[1] However livestock as traction animals is known to increase household's agricultural productivity and supply response in the Sahel (Savadogo *et al.*, 1994a; Savadogo *et al.*, 1994b).

(1992) find that households that diversify income more (into off-farm and livestock sources) have more stable income and consumption.

Now I define sustainability of a given income earning activity as the maintenance of its productivity in the long-run. Thus, household-level sustainability depends on the sustainability of a mix of activities. The mix then depends on household characteristics.

The productivity, here, sustainability of off-farm activities and livestock husbandry is determined at least partially at the village level, as there is interdepedency among households. For example, off-farm activities (local and migratory) of a given household can benefit from off-farm income at the village level as the latter increases off-farm income opportunity[2]. By contrast, livestock husbandry by other villagers will have a negative externality for a particular household. That is, since livestock is kept on open-access grazing land, overgrazing will decrease availability of grass per an animal, and as a result productivity of livestock will become low. This implies that household-level sustainability is also determined by village-level factors that influence the sustainability of a given household activity.

There are many kinds of commons, such as ponds, grazing land, forest, and so on, in rural villages, and the poor tend to rely more than do the rich for their livelihood on common-pool resources in developing countries (Dasgupta and Maler, 1994). But this difference in dependence is relative, as the poor have smaller herds than do the rich, and thus put less absolute pressure on open-access grazing lands in the WASAT (Reardon and Vosti, 1995). In the Sahel, livestock is about 30% overstocked due to population growth. This has resulted in a severe degradation of grasslands (Houerou, 1989). Because livestock is a substitute for formal savings, it is used for household risk management. And as a result of each household's behavior under risk, overgrazing and degradation are occurring in the Sahel. This means that all households will finally lose the ability to smooth consumption in

---

[2] This may sound similar to the concept of agricultural growth multipliers (e.g. Haggblade and Hazell, 1989; Haggblade *et al.*, 1989; Hopkins *et al.*, 1994). But multipliers usually imply the effect of exogenous increase in tradable farm sector income on nonfarm income, and ignore each household decision under uncertainty and interdependence among households in a village.

the long-run, if they do not have a rule to control the use of grazing land. This is an important example of links between a household's risk management/coping strategies and the household's sustainability, with the links influenced by village-level factors. However, the links have not been explored based on household behavior in the literature.

Village-level factors determine household-level sustainability; sustainability at the village level can be defined as the village's ability to support household-level sustainability. It directly depends on village-level productivities of crop, livestock and off-farm activity, which are determined by village factors such as natural resource endowment, infrastructure, market development, and so on. But village-level sustainability may also depend on informal and formal village rules that govern household behavior. One example is risk-sharing within village through transfers and remittances between households. Townsend (1994) provides evidence that households in an Indian village pool risk almost fully, while Udry (1994), using data from a village in northern Nigeria, finds evidence against full risk-sharing, but in support of the existence of partial risk-sharing. Institutions that govern the local commons also affect the sustainability of local commons (Ostrom, 1990; Ostrom *et al.*, 1994), which determines household-level sustainability.

A lot of empirical evidence on informal village institutions have been found, but those studies examine performance of existing institutions based on observed behavior. But the question I raise in this dissertation is what kind of formal or informal institutions could potentially be established, how effectively they would promote sustainability at both the household level and the village level, and how sustainable the institutions themselves would be.

1.3 Hypotheses

Given discussion above, the links between household behavior under drought risk and its effects on common-pool resources are hypothesized in this dissertation as follows. (a) Households in the SAT tend to save for future uncertain income, that is, there is precautionary

demand for savings. (b) Savings take the form of livestock holdings because there is no formal saving institutions and because livestock is a relatively safe asset in times of drought in the SAT. (c) As a result of the precautionary livestock holding, overgrazing occurs because grazing land is an open-access, common-pool resource. (d) Formal financial institutions, such as drought insurance, will substitute for livestock holdings and reduce overgrazing.

These links are explored theoretically and empirically guided by the following issues. (1) How do uncertainty and borrowing constraints affect household saving? The effects break down into two categories: (i) the effects of uncertainty and borrowing constraints on household saving; (ii) the links between saving and other self-insurance strategies. However, since the latter links determine the effect of uncertainty and borrowing constraints, (i) and (ii) can be integrated. (2) What is the aggregate effect of household risk management/coping strategies (in particular, livestock holding) on the sustainable use of common-pool resources (in particular, on grazing land)? (3) Are there potential solutions for overgrazing of common-pool grazing land? Several solutions are considered, among them formal drought insurance.

2 Objectives of the Study

The objectives of this study are to address the problems presented above and to test the hypotheses presented above using household data from rural Burkina Faso in a series of essays: Chapter 2: Household Saving in LDCs: A Critical Review; Chapter 3: Household Saving in LDCs: Extensions and New Structural Models; Chapter 4: Equity and Sustainable Management of Common-Pool Resources: the Case of Grazing Land in the West Africa Semi-Arid Tropics; Chapter 5: Crop production under Drought Risk and Estimation of Demand for Formal Drought Insurance in the Sahel. Finally, in chapter 6, the four essays are synthesized to draw the conclusions.

More specific objectives and contributions of each chapter are discussed in the following sub-sections.

2.1 Objectives and Contributions of Chapters 2 and 3

Chapters 2 and 3 are closely related. Chapter 2 consists of two literature reviews on household saving in rural areas in less developed countries. In chapter 3 I extend models found in the literature reviews, and based on the reviews and the extensions, I develop new agricultural household models. The objectives are to identify gaps in the literature, to fill the gaps, and to obtain theoretical insights and empirically testable hypotheses based on the models. Thus, these chapters serve as a basis of future empirical studies on household saving in LDCs.

In chapter 2, I identify two types of gap in the literature: gaps in empirical studies and gaps in theoretical studies. The gaps in empirical studies are identified by literature review on empirical evidence of household consumption smoothing. The gaps in theoretical studies, on the other hand, are identified by review of literature on household saving models, from a static two-period model to a dynamic multiperiod model under uncertainty and flexibility.

In chapter 3, I extend models to fill the theoretical gaps identified in chapter 2. Then, I develop structural dynamic models of the agricultural household in the context of the WASAT. The models incorporate agricultural production, off-farm activities, saving, sequential decisions, and other risk management/coping strategies. These models themselves contribute to the literature because most household saving models ignore endogenous decisions for agricultural production although it is an important element of the agricultural household model (Singh, *et al.*, 1986), and because off-farm activities have seldom been treated as a risk response in a household model.

2.2 Objectives and Contributions of Chapter 4

In chapter 4, I focus on the implications of the theoretical discussions in chapter 2 and 3 of household saving behavior on overgrazing in the WASAT, where all households in the village engage in both crop production and livestock husbandry, and keep livestock on open-access grazing land. The objective of this chapter is to construct a theoretical framework

that can be used to analyze the problem of household saving in the WASAT, that is, overgrazing. The motivation behind this is to challenge the assertion since Hardin (1968) that overgrazing is inevitable in open-access grazing land, and to identify more precise conditions for overgrazing in the context of the WASAT.

First, I construct a household model that explains the household's decision to allocate its wealth between crop production and livestock husbandry. To have tractability, a static portfolio selection model is used. The model is different from those found in previous studies on resource management in two ways: (i) Each household faces two investment opportunities: risky crop production and safe livestock holdings; (ii) Households in a village are heterogeneous in wealth and as a result heterogeneous in risk preferences. These two are realistic assumptions in the WASAT, and because of them it becomes possible to examine the effects of relative returns to the two activities and the effects of wealth distribution of the village using the model. In this regard, this chapter makes a disciplinary contribution.

Second, answers to the following questions are derived from the model as theoretical insights: (i) How do WASAT households behave under uncertainty in agricultural production? (ii) Why is there overgrazing? By answering (i) and (ii), conditions for overgrazing are identified. (iii) What property regimes would solve overgrazing? Are there any solutions for overgrazing? Current property regime is assumed to be open-access, and in this chapter privatization, quota, transferable-permit, and user fee schemes are considered. But in addition to them, physical solutions and other economic solutions are considered. (iv) What would be the equity consequence of these regimes? These theoretical insights obtained from the model contribute not only to provision of testable hypotheses for empirical studies, which in turn would be useful to policy formulation in the WASAT.

2.3 Objectives and Contributions of Chapter 5

In chapter 5 hypothetical formal drought insurance is considered as a solution to overgrazing. If formal drought insurance is more cost-effective than current informal self-

insurance, it will not only benefit grazing land, but also improve household welfare and contribute to economic growth. Thus, the primary objective of this chapter is to estimate demand for (hypothetical) formal drought insurance in the Sahel.

The first hypothesis is that current informal self-insurance provides inadequate protection against drought shock for households in the Sahel. This hypothesis is empirically tested by estimating demand for hypothetical formal drought insurance given self-insurance. A hypothetical insurance scheme called "rainfall lottery" is introduced (Gautam *et al.*, 1994; Hazell, 1992). In this scheme, an insurance company can assign any rainfall cut-off point to the lottery, and if annual rainfall is lower than the predetermined cut-off point, the insurance company will pay an indemnity. This particular insurance scheme is designed to avoid moral hazard and adverse selection (that cause a supply-side problem for crop insurance, especially in LDCs). Thus, the question is whether there is demand for the drought insurance. Therefore, one objective of this chapter is to estimate potential demand for this hypothetical drought insurance.

The second hypothesis is that the a household's demand for the formal drought insurance depends on the household's current self-insurance. This hypothesis is empirically tested by explaining the demand by household asset variables and household characteristics.

To accomplish these objectives: First, a dynamic household model is constructed following chapter 3; Second, a new method to estimate demand for formal drought insurance is established. This econometric method uses observed behavior of household that informally self-insures itself through its income and asset portfolio strategies without formal insurance. The method is newly developed in this chapter, and makes a disciplinary contribution; Third, potential demand for formal drought insurance is estimated using household data from rural Burkina Faso in the WASAT. Fourth, determinants of demand for formal drought insurance are examined. These analyses reveal how households in the WASAT manage drought risk and cope with drought shock, and indirectly test the hypotheses on interlinkages of strategies derived from the models in chapters 3 and 4. In particular, the effect of livestock holding on

demand for formal drought insurance is examined to see if it will reduce overgrazing. The third and fourth results make contributions to policy formulation.

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Chapter 2

Household Saving in LDCs: A Critical Review

0 Summary

In this chapter I conduct two literature reviews: on empirical studies on household consumption smoothing and on household saving models.

In the first review, I focus on agricultural households in rural areas of LDCs. There are three findings: (i) There is empirical evidence of several strategies that smooth household consumption such as crop diversification, income diversification (particularly off-farm activities), use of risk-decreasing inputs, social insurance, and savings (livestock holdings); (ii) They are not perfect, and village insurance studies imply that households mix those strategies to smooth consumption; (iii) But those strategies are not empirically examined well in structural household models and their interactions are little known.

In the second review, models for household saving are reviewed. It starts from two-period static models, and finally deals with multiperiod dynamic models as well as flexible decision models. The main concern of this review is the effect of uncertainty and borrowing constraints. There are four findings, as follows.

First, two types of risk are often considered in literature: exogenous labor income risk and exogenous interest rate risk. The effects of increase in those risks are summarized in Table 1. The table shows that if saving is riskless, an increase in either type of risk increases precautionary demand for saving, given a convex marginal utility function ($v''' > 0$) and decreases precautionary demand for saving, given a concave marginal utility function ($v''' < 0$). On the other hand, if saving is risky, convexity of the marginal utility function ($v''' > 0$) does not necessarily induce precautionary demand for saving when interest rate risk

increases. In reality, however, those two risks coexist and must be correlated. Nevertheless there are few saving models that deal with multiple, correlated risks.

Second, in terms of precautionary demand for saving, there is little difference between two-period models and multiperiod models (as summarized in Table 1). However, the advantage of multiperiod models is dynamic simulation. Deaton (1991) shows, based on dynamic simulations, that a borrowing constraint under exogenous income risk induces precautionary demand for saving even if the marginal utility function is concave. This is an important implication that can be derived only from multiperiod models.

Third, a production function should be incorporated into a savings model. In other words, saving behavior should be considered in a structural model of agricultural household. The importance of the production function for analyzing household saving is that it includes household risk management strategies through utilizing risk-decreasing inputs, and as a result a household's input decisions may affect the saving decision. Crop diversification also may be a risk-decreasing input decision. However, I find that there are few studies that consider risk-decreasing inputs in household saving models.

Fourth, flexibility in decision making is important, especially in a model with agricultural production, because the effect of flexibility depends on whether the flexible input is risk-increasing or risk-decreasing. In addition, incorporating flexibility will allow us to deal realistically with off-farm activities. However, I find that there are few studies that deal with both off-farm activities and saving in flexible decision models.

As stated in the third and fourth conclusions, non-linearity of the marginal effect of risk and the flexibility of decision making are important concepts, but their effects cannot be predicted *a priori*. Therefore, empirical studies that treat them properly will answer how they affect household's risk responses. Such empirical studies should be based on a household structural model that incorporates not only those two but also other risk management strategies.

1 Introduction

This chapter consists of two reviews. In the first one, I review literature on empirical evidence of household consumption smoothing in rural areas of developing countries. In the second, I review literature on household saving models. Through those two reviews, gaps in the literature are identified so that this chapter can serve as a basis for future theoretical and empirical studies on household saving in LDCs.

Saving is conventionally taken to mean consumption forgone today for the purpose of greater consumption in the future. Saving is considered important for economic growth of LDCs because it provides for accumulation of capital. The view that capital formation is the key to growth, called capital fundamentalism, was reflected in the development strategies and plans of many countries (Gillis *et al.*, 1992). For example, in an influential paper, Lewis (1954) writes "The central problem in the theory of economic development is to understand the process by which a community which was previously saving and investing 4 or 5 per cent of its national income or less, converts itself into an economy where voluntary saving is running at about 12 to 15 per cent of national income or more. This is the central problem because the central fact of economic development is rapid capital accumulation (including knowledge and skills with capital)".

However, the saving behavior of households in developing countries will not necessarily lead to the accumulation of capital for several reasons. First, household saving in LDCs is to smooth consumption in the face of volatile and unpredictable income (Deaton, 1989). This is not only because agricultural production in LDCs is very uncertain, but also because households rely heavily on saving due to lack of formal consumption credit and crop insurance institutions. Second, productivity of investment is low in LDCs (Dasgupta, 1993). This is especially true in rural areas where there are few formal financial institutions. Household saving to smooth consumption takes the forms of crop stocks and livestock holdings, which are not productive. In other words, uncertainty and lack of formal financial institutions force households to save in liquid, but unproductive assets in rural areas of LDCs.

Introduction of banks is shown to shift the composition of savings toward capital, to reduce socially unnecessary capital liquidation, and as a result to promote growth (Bencivenga and Smith, 1991). On the other hand, there is evidence that household savings induced by liquidity constraints promote growth in OECD countries (Jappelli and Pagano, 1994), where productivity of investment is generally high.

Sustainability is an important issue of international development. It can be defined at different levels, but the household's sustainability can be seen as its ability to sustain its well-being or to smooth consumption above a minimum level over a certain period. In this sense, saving/dissaving is the way to achieve sustainability in the short-run. On the other hand, however, the increase in and stabilization of agricultural and non-agricultural productivity will both increase and smooth income, and as a result will smooth consumption in the long-run. To achieve long-run sustainability, use of more agricultural inputs and investment both in agricultural equipment, in maintenance and improvement of land (e.g., terracing, reforesting, digging irrigation ditches, etc.), and in human capital are critical. But the problem is, as mentioned above, that households save to smooth consumption in the short-run, and such behavior will not contribute to economic development in the long-run.

Thus, to formulate development policy, one needs to know household saving behavior: why and how they save, and what will change their behavior from unproductive to productive saving. Theoretical and empirical work at the microeconomic level has been accumulating recently (for examples, see the review by Deaton (1994)). Deaton (1994) discusses the conventional life-cycle (LC) model and the permanent-income (PI) model, and presents examples of empirical studies in LDCs based on those models. But according to Deaton there are important cases that are not covered by either one. One case is when the marginal utility of consumption is not linear under uncertainty. In this case households show a precautionary motive for saving, that is, they save more when the future becomes more uncertain. The other case is when there are borrowing constraints. To incorporate those missing features, Deaton simulates a household whose income sources are stochastic labor

income and saving, and shows that they save to smooth consumption if they know that they cannot borrow even when their income is low.

Since uncertainty and borrowing constraints are features of LDC rural economies, Deaton's model may explain saving behavior of LDC households better than LC and PI models. But the assumption of stochastic, exogenous labor income is still unrealistic because endogenous agricultural production is an important element of the agricultural household model (Singh, *et al.*, 1986). However, there are few empirical studies that consider household saving in a structural model; an exception is Rosenzweig and Wolpin (1993), who estimate a dynamic household model and show that saving/dissaving of bullock holdings as a buffer stock causes underinvestment in bullocks as productive capital. This kind of empirical study should be conducted more to understand agricultural household's behavior in LDCs. To build a basis for such empirical studies, this chapter reviews literature on household saving.

Two review sections follow this introduction: In section 2, household saving is viewed as one of the strategies to manage and to cope with risk to smooth consumption under uncertainty and market imperfection, which are common in rural areas in LDCs. Then, I review empirical evidence of consumption smoothing including not only saving but also other strategies. In section 3, household saving models are reviewed. The review starts from a two-period static model, then deals with more complicated models, such as multiperiod dynamic models and sequential decision models. The effects of uncertainty and borrowing constraints on savings are emphasized.

2 Evidence of Consumption Smoothing

Households in developing countries make use of a wide variety of strategies to smooth consumption. Since saving is one of such strategies, and since those strategies are interacted with each other, we need to know the interaction to get a complete image of household behavior under uncertainty. In this section, empirical evidence of consumption smoothing is reviewed.

The strategies can be classified as risk management and risk coping. Risk management is to reduce variability of income *ex ante* (before a risky outcome is known): crop insurance, crop/plot diversification, income diversification, and utilization of risk-decreasing technology. They reduce income risk, and as a result smooth consumption. Since formal crop insurance is rarely available in developing countries, household's strategies are mainly the latter three.

Sakurai (1993) shows that crop diversification measured by a Simpson index of planted area reduces crop income shock in the Sahelian and the Sudanian zones in Burkina Faso, but increases crop income shock in the Guinean zone (the most agro-climatically favorable zone). See Appendix A for the details. This implies that crop diversification is an *ex ante* risk management strategy in riskier areas where *ex post* risk coping is less available, but in less risky areas where *ex post* risk coping is also developed, crop diversification is rather an income-increasing strategy by diversifying to risky cash crops.

Non-farm income is a very significant part of total income in the WASAT: based on several household surveys during 1980s, Reardon *et al.* (1994) conclude that non-farm income (all income but cropping and livestock income) varies 20 to 64 percent of total incomes and non-cropping income (all income but cropping income) ranges 31 to 83 percent. In Burkina Faso it is found that income diversification indexed as the ratio of off-farm income and all income, significantly increases household income per adult equivalent, and that income diversification reduces income risk and as a result reduces food consumption risk (Reardon *et al.*, 1992). But since their non-farm income includes both income before harvest and income after harvest (sales of labor and livestock *ex post*), it is not the same as the income diversification defined above as an *ex ante* strategy. Therefore, it is not surprising that income diversification in Reardon *et al.* (1992) increases total income as well. The results give evidence, however, that income diversification, whether *ex ante* or *ex post*, smoothes consumption.

An input can be classified as either risk-increasing or risk-decreasing. If an input is risk-increasing, the marginal productivity of the input increases as use of the risky input increases, while if an input is risk-decreasing, the marginal productivity of the input decreases as use of the risky input increases. Fertilizers are often risk-increasing, while pesticides and herbicides are risk-decreasing (Ramaswani, 1993). In Burkina Faso it is found that land is a risk-decreasing input: crop production per area decreases with total cropped land, but the sensitivity of yield to rainfall variation is low when total cropped land is large (Sakurai *et al.*, 1994). Therefore, because animal traction helps area expansion (Savadogo *et al.*, 1994), animal traction may also be a risk-decreasing technology in that region.

Most studies on *ex ante* risk management except for Reardon *et al.* (1992), which includes risk coping *ex post*, do not examine whether the reduction of income risk actually reduces consumption risk. Since a household's strategy is a combination of *ex ante* risk management and *ex post* risk coping, successful risk management in one activity may not necessarily mean successful household consumption smoothing.

On the other hand, risk coping strategies can be classified as saving, or intertemporal consumption smoothing, and risk-sharing, or consumption smoothing across households. Saving includes borrowing and lending in a formal or an informal credit market, accumulating and selling assets, and storing goods for future consumption. The assets to sell may include labor, that is, households seek off-farm income sources, such as temporal migration, after a bad harvest. On the other hand, insurance is a formal institution of risk-sharing. In addition, several informal risk-sharing arrangements are found in developing countries: state-contingent transfers and remittances between relatives and neighbors, credit contracts with state-contingent repayments, share-tenancy, and so on. Informal risk-sharing as a whole is called "social insurance". There are a lot of studies on saving and social insurance in developing countries, reviewed in Gersovitz (1988) and Deaton (1994). Most studies test how well households smooth consumption, but tell little about how they achieve it. Examples of saving and social insurance follow.

Paxson (1992) tests the permanent income hypothesis (PIH) based on the following saving equation.

$$S_{irt} = \alpha + \beta \hat{Y}^P_{irt} + \gamma \hat{Y}^T_{irt} + \theta Z_{irt} + e_{irt}$$

If transitory income, $\hat{Y}^T_{irt}$, is saved and permanent income, $\hat{Y}^P_{irt}$, is consumed as PIH implies, $\beta = 0$ and $\gamma = 1$. Using time-series data from rice farmers in Thailand, she shows that γ is closed to unity but that β is positive and significantly different from zero although it is much smaller than γ. Her conclusion is that farm households save a significantly higher fraction of transitory income than nontransitory income, that is, PIH is partially supported. However, it does not necessarily mean that the saving allows them to smooth consumption. Deaton (1992a) uses two-year panel data from Cote d'Ivoire to test a permanent income model assuming that income is a stationary stochastic process. He finds that the orthogonality requirement of PIH is not supported by the data, that is, changes in consumption are affected by lagged saving and income.

An important methodological issue to test PIH is how to statistically distinguish permanent income from transitory income when only total income is available. Paxson uses a rainfall variable to estimate transitory income, because transitory income is considered to depend on the variability of rainfall. Another issue is how to define household saving, that is, to distinguish investment and saving from total income. Paxson tries three different definitions of saving and obtains different results.

Townsend (1994) uses 10-year panels of grain consumption and incomes from several sources of households in the semi-arid tropics of southern India, and tests a full insurance model for each household (133 households), based on the following equation,

$$C_t = \alpha + \beta \overline{C}_t + \gamma X_t + \theta Z_t + e_t$$

where C_t is consumption per adult equivalent of a household in year t, $\overline{C}_t$ village average consumption per adult equivalent, X_t is any other variable to test, such as household contemporaneous income in year t, and Z_t is a vector of demographic variables. Full insurance implies that $\beta = 1$ and $\gamma = 0$, that is, household consumption is not affected by its

own income. He shows that there is a tendency to reject $\beta = 0$ and accept $\beta = 1$ out of 133 households, but not all of them. However, when panel data are used, it is shown that income does matter in the determination of consumption. He concludes that full insurance is rejected, but there is partial insurance because overall the effect of income on consumption is not large. Deaton (1994) uses panel data from rural Cote d'Ivoire, and shows that the coefficients on individual income changes are significantly positive ($\gamma > 0$) in contradiction to full insurance which would require them to be zero ($\gamma = 0$). Moreover, in two of the three regions surveyed there is no effect of the village average income on individual consumption ($\beta = 0$), which is also inconsistent with full insurance which would require it to be unity ($\beta = 1$). Note that he uses village average income for $\overline{C}_t$ instead of village average consumption.

As shown above, full insurance is in general rejected, but partial insurance is still supported. This means that households smooth consumption. However, the test for full insurance does not explain how households smooth consumption. The propensity of consumption out of own income can be a measure of consumption smoothing, controlling for village consumption: Sakurai (1995) finds that within the same agroecological zone, a village with a good access to a major road has smaller propensity to consume than a village without a good access to a major road in Burkina Faso household panel data. The same pattern is observed in three different zones, the Sahelian, the Sudanian, and the Guinean. And in villages with a good access to the road, full insurance models tend to be supported, at least partially, while they are never supported in villages without good access (see Appendix B). Townsend (1994) separates households into farmers and landless, and tests full insurance for each category in each village. He finds that the landless are much less insured against income shocks than are farmers, in all three villages. Sakurai's and Townsend's findings suggest that consumption smoothing strategies other than village-level insurance may be working, such as insurance over zones including temporal migration and remittance and intertemporal smoothing through saving/dissaving. However, a full insurance model cannot distinguish

hypothetical "village insurance" from those risk coping strategies. This weakness of the full insurance model is well discussed in Alderman and Paxson (1992).

Determinant analysis and tests based on structural household models will show how agricultural households smooth consumption and why they cannot do it. However, only a few studies have been done on those issues to date. Reardon *et al.* (1992) is an example of a study of determinants. They examine determinants of income diversification (earning more off-farm income relative to farm income) and find that assets such as land and livestock holdings have positive effects. Then, they show that income diversification is an important determinant of consumption smoothing: it reduces variability of income, and as a result smoothes consumption. They compare the consumption smoothing effects of income diversification among three different agroclimatical zones, and find that the effect is the strongest in the Sahelian zone where crop production is the riskiest and income is the most diversified and the weakest in the Sudanian zone where cropping is still dominant. Livestock holdings have little effect on income smoothing as livestock income is included in off-farm income.

An example of a structural household model is Rosenzweig and Wolpin (1993). They establish a dynamic model of farm households who have precautionary demand for saving, have a floor of minimum consumption, invest in productive assets (bullocks and pumps), use productive assets (bullocks) as a buffer stock, and cannot borrow. In addition, they implicitly assume some kind of social insurance that works when household income falls below a consumption floor. Using data from the semi-arid tropics of India, they estimate structural parameters, and show in simulations that households cannot hold the optimal number of bullocks due to dissaving under borrowing constraints in the presence of weather shocks, that an actuarially-fair nonsubsidized weather insurance will not help households to hold the optimal number of bullocks because they are already self-insured, and that exogenous additional income will increase bullock holdings not only because they can now afford it but also because they become less risk averse. In their model, household profits is a function of

the number of bullocks, the number of pumps, and a weather shock; no variable inputs, such as labor, seed, or fertilizer are in the function. This may be too simplified.

In addition, in the West African semi-arid tropics, a structural household model must somehow incorporate off-farm activities: labor and capital allocation between on-farm and off-farm must be considered in a dynamic setting.

As reviewed in this section, there are several strategies that smooth household consumption in rural areas of LDCs. They are not perfect, and village insurance studies imply that households mix those strategies to smooth consumption. However, those strategies are not examined well in structural household models and their interactions are little known. Therefore, those strategies, not only saving but also crop diversification, income diversification (off-farm activities), social insurance, use of risk-decreasing inputs, should be incorporated explicitly or at least interpreted implicitly in structural models of agricultural household to analyze household saving behavior.

3 Household Saving Models

In this section, saving models are reviewed in terms of uncertainty and borrowing constraints. The review starts from two-period models, and then deals with multiperiod models. Finally, flexible decision models are introduced.

3.1 Borrowing Constraint and Uncertainty

There are two types of borrowing constraint to be considered: lifetime borrowing constraints and single-period borrowing constraints.

The lifetime borrowing constraint implies that if there is no bequest motive at the end of its lifetime, a household must consume all it has in the last period, assuming a standard utility function with positive marginal utility. In the last period, the household cannot borrow because it cannot repay the loan. In addition, if the household has a debt at the beginning of the last period, the debt must be repaid in the last period. Thus, this constraint can also be

called a lifetime budget constraint. The household's decision in the second-to-last period is affected by this constraint in the last period. If there is uncertainty in the last period, the household's decision in the second to last period is affected more by the uncertainty so that the household can avoid violating the lifetime borrowing constraint. Then, the second to last period affects the third to last period, and so on. Thus by backward induction, the lifetime borrowing constraint determines the household's lifetime optimal strategy. Even in the case of infinite life, the condition is expressed as the limit of expected discounted assets being zero, and the household's decision in each period is affected. In this respect, two-period models have an advantage over multiperiod models because it is easy to incorporate the lifetime borrowing constraint in two-period models.

However, since it is obvious that a household cannot borrow in the last period, the lifetime borrowing constraint is not usually referred to as borrowing constraint. A borrowing constraint usually means that a household cannot borrow at all or there is a ceiling above which a household cannot borrow. This type of borrowing constraint can be called a "single-period borrowing constraint". This constraint is not necessarily binding, even if it always exists. This constraint binds only when a household's optimal consumption exceeds its current wealth. In a two-period model, the single-period constraint exists only in the first period and affects the household's decision only when it is binding. On the other hand, in a multiperiod model, it can be occasionally binding depending on the household's strategies, and under the existence of a potential binding constraint the household optimizes . Therefore, the existence of a borrowing constraint affects household behavior, especially under uncertainty, even if it is not always binding. Unlike two-period models, multiperiod models can treat this problem of a "potential" binding constraint under uncertainty. Note that when one says that there is no borrowing constraint, it means that a household can borrow in each period as much as it wants. But because the borrowed money must be repaid in the future (due to lifetime borrowing constraint), the household will not borrow limitlessly.

To summarize the discussion above: (i) A lifetime borrowing constraint is always binding regardless of assumption and it rules out limitless borrowing; (ii) The existence of a single-period borrowing constraint depends on assumption; (iii) Even if a single-period borrowing constraint exists, the constraint is not always binding.

3.2 Saving Under Certainty / Two-Period Model

This review starts with the simplest saving model. This model is also the starting point of saving models in the literature, and is too basic to have a specific reference.

A household faces a two-period horizon, 1 (present) and 2 (future). Let y_1 and y_2 denote the household's exogenous labor income for period 1 and 2 respectively. It is labor income because the household has only labor to sell, and it is exogenous because its labor supply is assumed to be inelastic. Let c_1 and c_2, respectively, denote present and future consumption. In two-period models, non time-separable utility, $U(c_1, c_2)$, is sometimes used. But to make it easier to extend the model to a multiperiod model, additively time-separable utility is used here. In addition the initial wealth at the beginning of the first period, w_1, is assumed. The real interest rate, r, and the discount factor, δ, is known, fixed, and exogenous to the household.

Define saving in period 1 by $s_1 = w_1 + y_1 - c_1$. Then initial wealth at the beginning of the second period is given by $w_2 = (1+r)s_1$. Thus, the household's income for period 2 is equal to $y_2 + w_2 = y_2 + (1+r)s_1$, which must be equal to second period consumption, c_2. The problem now can be written as

$$\max_{c_1} \; u(c_1) + \delta v(y_2 + (1+r)(w_1 + y_1 - c_1)) \qquad (1)$$

Thus, from the first order condition, the famous Fisher's law is obtained:

$$\frac{u'(c_1)}{v'(c_2)} = \delta(1+r) \qquad (2)$$

In the first period, there is no single-period borrowing constraint, that is, s_1 can be negative. If the household borrows, the debt must be repaid with fixed interest, r, in the second period. In the second period, everything is already planned as assumed so that the household does not

have to violate the lifetime borrowing constraint. It is easily shown that the maximum amount that the household can borrow against future labor income is $\frac{y_2}{1+r}$. Thus, naturally when y_2 is zero, the household cannot borrow in the first period.

If there is a single-period borrowing constraint, then $s_1 \geq 0$. If it is binding, the household will choose $s_1 = 0$. Then, $y_1 = c_1$ and $y_2 = c_2$, which is "consumption tracks income perfectly". As a result of this binding constraint, c_1 is lower than the optimum, and c_2 is higher than the optimum without the borrowing constraint. Therefore, Fisher's law does not hold, that is, $\frac{u'(c_1)}{v'(c_2)} > \delta(1+r)$. In this certain world, the borrowing constraint may increases consumption variability, and consumption growth between two periods may become larger, if it is binding. Although this borrowing constraint will reduce consumption in the first period relative to the case without the binding constraint, it will never induce positive saving more than without constraint.

3.3 Saving Under Uncertainty / Two-Period Model

In this section, two-period saving models under uncertainty are reviewed. There are two kinds of uncertainty distinguished in the literature: stochastic exogenous labor income and stochastic exogenous interest rate. First, the case with labor income uncertainty, and then, the case with interest rate uncertainty are reviewed.

3.3.1 Labor Income Uncertainty / Two-Period Model

Labor income uncertainty in a two-period saving model was first considered by Leland (1968) and Sandmo (1970). They assume income risk in the second period, that is, y_2 in the certainty model is assumed to be stochastic denoted by $\tilde{y}_2$. Both use a non-separable utility function given by $U(c_1, c_2)$. They show that saving rate (the ratio of s_1 against total income in the first period) under uncertainty depends on the third derivative of the utility function: if it is quadratic, since the third derivative is zero, uncertainty has no effect on the

saving rate, but if $U_{122} - U_{222}$ is negative, uncertainty increases the saving rate higher than the optimum under certainty, that is, there is positive precautionary demand for saving. They also mention that in a special case of additively separable utility function, the condition for positive precautionary demand for saving is that the third derivative of the utility function is positive ($v''' > 0$) because the cross derivative of the separable utility function is zero. In other words, the marginal utility function is convex[1]. The difference between Leland and Sandmo is the measure of the "degree of risk". Leland uses a Taylor-series approximation for the utility function, while Sandmo introduces two parameters, additive shift and multiplicative shift for stochastic labor income and considers a mean-preserving spread.

A more general measure of increasing risk is given by Rothschild and Stiglitz (1970). Their idea is as follows: if an individual's utility is a function of a control variable α and an exogenous random variable θ, then the individual solves the utility maximization problem given by

$$\max_{\alpha} \; EU(\theta, \alpha) \tag{3}$$

The optimal α must satisfy the first order condition given by

$$E\frac{\partial U(\theta, \alpha)}{\partial \alpha} = 0 \tag{4}$$

If $\frac{\partial U}{\partial \alpha}$ is concave in θ ($\frac{\partial^3 U}{\partial \alpha \partial \theta^2} < 0$), an increase in the variability of θ will result in a decrease in the optimal choices of α, while if $\frac{\partial U}{\partial \alpha}$ is convex in θ ($\frac{\partial^3 U}{\partial \alpha \partial \theta^2} > 0$), increase in the variability of θ will result in increase in the optimal choices of α (Rothschild and Stiglitz, 1971). Since this method is often used in this paper, I will name it the RS method hereafter.

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[1] Assume a standard utility function, $u(c)$, with $u'(c) > 0$ and $u''(c) < 0$, (i) if $u(c)$ has a convex marginal utility, $u'''(c)$ is positive and the expected marginal utility increases when the variability of c increases; (ii) if $u(c)$ has a concave marginal utility, $u'''(c)$ is negative and the expected marginal utility decreases when the variability of c increases. The difference in the effect of the variability on the expected marginal utility causes the different effect of the variability on saving.

An example of the case (i) is $u(c) = \ln c$ where $c > 0$ and an example of the case (ii) is $u(c) = c - c^3$ where $0 < c < 1$. It is difficult to find an example of a utility function that globally satisfies the standard assumption, $u'(c) > 0$ and $u''(c) < 0$, as well as $u'''(c) < 0$.

Kimball (1990) applies the RS method to the two-period saving problem under income uncertainty considered by Leland and Sandmo. Following the two-period model under certainty given by (1), this problem can be written as

$$\max_{c_1} EU(\tilde{y}_2, c_1) = u(c_1) + \delta Ev(\tilde{y}_2 + w_2) = u(c_1) + \delta Ev(\tilde{y}_2 + (1+r)(w_1 + y_1 - c_1)) \qquad (5)$$

From the first order condition, the optimal condition similar to Fisher's law is obtained as follows.

$$\frac{u'(c_1)}{Ev'(c_2)} = \delta(1+r) \qquad (6)$$

The effect of increasing the variability of labor income depends on the third derivative of U. If $\frac{\partial^3 U}{\partial c_1 \partial \tilde{y}_2^2}$ is negative, that is, the marginal utility v' is convex, an increase in risk will decrease optimal consumption and will increase optimal saving in the first period (Table 1). Therefore, there is precautionary demand for saving when the third derivative of utility function is positive, which is the same conclusion as drawn by Leland and Sandmo. Kimball (1990) shows that a measure of the strength of the precautionary demand for saving is analogous to the Arrow-Pratt measure of risk aversion, and defines absolute prudence as $-\frac{v'''}{v''}$.

All the models presented above assume no single-period borrowing constraint in the first period. Consequently, if the household borrows, the debt must be repaid in the second period with interest at the same rate as the interest rate on saving. This lifetime borrowing constraint is the same as under certainty. However, the household does not know exactly how much labor income it will earn in the second period under uncertainty. Therefore the household's decision is based on the expectation of stochastic labor income, which affects the optimal consumption level depending on the nature of utility function as shown above. Consider the case of precautionary demand for saving: the optimal c_1 is lower than c_1 under certainty. Therefore, it is less likely that the single-period borrowing constraint is binding even if it exists. Thus if uncertainty increases saving in the first period, uncertainty will make the borrowing constraint being binding less likely than under certainty, while if uncertainty

decreases saving, uncertainty will make the borrowing constraint being binding more likely than under certainty. But in either case, like the case under certainty, a borrowing constraint will induce suboptimality, $\frac{u'(c_1)}{Ev'(c_2)} > \delta(1+r)$, if it is binding.

3.3.2 Interest Rate Uncertainty / Two-Period Model

In this section, interest rate uncertainty incorporated in a household portfolio selection problem is considered. In the model, a household allocates its wealth among consumption and savings with stochastic returns. Since a farm household is concerned with consumption and stochastic production, this kind of portfolio model can be considered to be a starting point of a farm household model. However, most portfolio models, including ones presented below, assume linear returns to investment and no labor input.

A portfolio model found in Sandmo (1968) can be written as follows.

$$\max_{m,\,a} \; u(c_1) + \delta E v(m + a(1+\tilde{r})) \tag{7}$$

where m is money or a riskless asset without interest, a is a risky asset with random return rate $\tilde{r}$, and consumption in the second period is $c_2 = m + a(1+\tilde{r})$. Assuming that the household has an initial asset, w_1, and has no other income source, then the budget constraint is $c_2 = m + a(1+\tilde{r}) = w_1 - c_1 + a\tilde{r}$. Because there is no exogenous income in the second period, the household cannot borrow in the first period in this model. Using a quadratic utility function, Sandmo (1968) shows that increasing the variance of stochastic returns keeping the mean constant (i) decreases consumption, c_1, (ii) decreases investment in risky assets, a, and (iii) increases investment in riskless assets, m. Precautionary demand for saving exists in this case, even though a quadratic utility function (whose third derivative is zero) is used.

More generally, Rothschild and Stiglitz (1971) use a similar model with one risky asset and no riskless asset, and show: (i) If $2v''(c_2) + v'''(c_2)c_2 < 0$, increasing risk decreases the saving rate, s, and either $v''' > 0$ or $v''' < 0$ can satisfy this condition; (ii) if $2v''(c_2) + v'''(c_2)c_2 > 0$, increasing risk increases the saving rate, s, and $v''' > 0$ is necessary

for this condition. Therefore, in this case a positive third derivative of the utility function is not sufficient for precautionary demand for saving. But note that saving in this model is risky. In this model, borrowing in the first period is impossible, too. If the quadratic utility function is used (as in Sandmo (1968)), $2v'(c_2) + v'''(c_2)c_2 < 0$ is always the case because $v''' = 0$. Sandmo's result (ii) is consistent with Rothschild and Stiglitz's result (i).

A special case is the constant relative risk aversion (CRRA) utility function, which is also found in Rothschild and Stiglitz (1971). The CRRA utility function is given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{8}$$

where σ is the coefficient of relative risk aversion. The CRRA utility function has a positive third derivative if $\sigma > 0$, which is warranted if the utility function satisfies standard assumptions: $u' > 0$ and $u'' < 0$. Therefore, because $u''' > 0$, the CRRA function has precautionary demand for saving at least in the case of stochastic labor income, and because $\lim_{c \to 0} u' = \infty$, it endogenously bounds optimal consumption away from negative or zero consumption. Moreover, risk aversion and precautionary demand for saving are controlled by the same parameter, σ, which is not necessarily true in general. With a CRRA utility function, they show that the condition $2v'(c_2) + v'''(c_2)c_2 < 0$ becomes $\sigma < 1$ and the condition $2v'(c_2) + v'''(c_2)c_2 > 0$ becomes $\sigma > 1$. Therefore, (i) if $\sigma < 1$, an increase in interest risk decreases saving; (ii) if $\sigma > 1$, an increase in interest risk increases saving. The results in this section are summarized in Table 1.

3.3.3 Labor Income and Interest Rate Uncertainty / Two-Period Model

It will be easy to extend the two-period saving model with labor income uncertainty in section 3.3.1 to a model assuming that both labor income and interest rate are uncertain. However, to my knowledge, there are few such extensions except for multiperiod models, such as, Skinner (1988) and Dasgupta (1993). Therefore, in chapter 3, I will develop a two-period model with two uncertainty sources and will examine its property, especially when they are correlated.

3.4 Saving under Certainty / Multiperiod Model

The multiperiod model is an easy extension of two-period model in section 3.2. This is also a text book problem, but here I follow Sargent (1987). The model is generally written as

$$\max_{c_t} \sum_{t=1}^{T} \delta^t u(c_t), \qquad 0 < \delta < 1 \tag{9}$$

$$\text{subject to} \qquad w_{t+1} = (1+r)(w_t + y_t - c_t)$$

y_t is exogenous labor income in period t. Borrowing is allowed in each period except in the last period, and therefore there is a lifetime budget constraint as discussed in section 3.1. The constraint can be written as

$$\sum_{t=1}^{T} (1+r)^{-t} c_t = \sum_{t=1}^{T} (1+r)^{-t} y_t + w_1 \tag{10}$$

This rules out a strategy of infinite consumption supported by unbounded borrowing. If T is infinite time, the right-hand side must converge to zero. Bellman's equation becomes

$$v(w_t, y_t) = \max_{c_t} \left[u(c_t) + \delta v(w_{t+1}, y_{t+1}) \right] \tag{11}$$

And using the first order conditions, the Euler equation is obtained as

$$\frac{\partial u}{\partial c_t} = \delta(1+r) \frac{\partial u}{\partial c_{t+1}} \tag{12}$$

Thus, the Euler equation is almost like Fisher's law obtained in the two-period saving model in section 3.2. In fact a lot of empirical studies based on multiperiod dynamic optimization are based on the Euler equation even in a stochastic setting. Since the Euler equation is one of the necessary conditions, a solution to the dynamic optimization problem above must satisfy it. However, in addition to that, the lifetime budget constraint given by (10) is also necessary, and when we solve the problem to obtain the optimal consumption path, we have to use both conditions. In the two period model, the lifetime budget constraint is already imposed on the first order conditions. But in a multiperiod model, imposing the lifetime constraint on the first order conditions is in general difficult. An example of a solvable problem is using $u(c_t) = \ln c_t$; the optimal consumption of an infinite-life household given by Sargent (1987) is

$$c_t = (1-\delta)\left[y_t + \sum_{j=1}^{\infty}(1+r)^{-j}y_{t+j} + w_t\right] \tag{13}$$

3.5 Saving Under Uncertainty / Multiperiod Model

As with the two-period models, there are three types of risk: labor income risk, interest rate risk, and both.

3.5.1 Labor Income Uncertainty / Multiperiod Model

Under uncertainty, the Euler equation given by (12) becomes

$$\frac{\partial u}{\partial c_t} = \delta(1+r)E_t\frac{\partial u}{\partial c_{t+1}} \tag{14}$$

This is the same as (12) except for the expectation operator. Moreover, the Euler equation above is very similar to the optimality condition for the two-period model under income uncertainty in section 3.3.1. But as discussed in section 3.4, the Euler equation in a multiperiod model does not incorporate the lifetime budget constraint, while the optimality condition in a two-period models does.

If we assume a quadratic utility function, then the Euler equation given by (14) becomes

$$c_t = \delta(1+r)E_t(c_{t+1}) \tag{15}$$

Hall (1978) tests (15) if no other variables in the information set help to predict c_{t+1} once lagged consumption, c_t, has been included, and shows that lagged income is orthogonal to current consumption. Then where is the lifetime budget constraint? The level of income at which consumption is initially set depends on lifetime resources, determined at least in part by current and future levels of labor income as shown below. A simple case of (15) is when $\delta(1+r)$ is unity, that is, $c_t = E(c_{t+1})$. Using this Euler equation with a lifetime budget constraint, the consumption level of an infinite life household is given by Deaton (1992b).

$$c_t = \frac{r}{1+r}w_t + \frac{r}{1+r}\sum_{j=0}^{\infty}(1+r)^{-j}E_t y_{t+j} \tag{16}$$

This is equivalent to (13) under uncertainty when $\delta(1+r)$ is unity. Equation (16) implies that consumption is the annuity value of expected human and non-human wealth. Deaton (1992b) takes this to be the permanent income hypothesis. Assumptions for (16) are infinite life, quadratic preferences, and a constant real interest rate equal to the rate of time-preference ($\delta(1+r)=1$). The problems of using a quadratic utility function are that consumption is allowed to be negative and that there is no precautionary motive for saving. As discussed for two-period models, precautionary saving depends on the third derivative of the utility function, but with a quadratic function, the third derivative is always zero. It can be seen from (16): even if stochastic labor income becomes more risky, if its expected mean does not change, consumption is not affected at all.

The effect of increasing risk can be easily seen using the Euler equation given by (14), which is equivalent to (6) of the two-period model. Thus, with respect to risk in stochastic labor income, if $u'''>0$, an increase in risk will increase precautionary saving, while if $u'''<0$ an increase in risk will decrease precautionary saving. However, there is a difference between the optimality condition of two-period models and the Euler equation of multiperiod models. In two-period models, risk in $\tilde{y}_2$ directly affects c_2, because $c_2=\tilde{y}_2+w_2$. In other words, if we say there is income risk in the second period, it is also consumption risk in the second period. On the other hand, in the Euler equation of multiperiod models, we do not know the relationship between income risk and consumption without making specific assumptions: even if we assume an increase in income risk in period t+1, it does not necessarily mean consumption in period t+1 becomes more risky because consumption t+1 is endogenous. Intuitively, the argument that consumption is affected by income risk is plausible if a lifetime budget constraint exists: in the last period the household has to consume all it has, like in the second period in a two-period model. Then, using backward induction, in each period consumption will be affected by the requirement in the last period under income uncertainty. It can be formally proved by going back to the Bellman's equation like (11) to show that

$$\frac{\partial^3 u(c_t)}{\partial c_t^3} \gtrless 0 \Rightarrow \frac{\partial^3 v(y_t)}{\partial y_t^3} \gtrless 0 \tag{17}$$

is true for all t (Sibley, 1975). Based on Sibley's proof, we can now examine precautionary demand for saving by examining the sign of the third derivative of utility function.

As discussed, quadratic utility functions do not show precautionary demand for saving. But if we use a utility function that has non-zero third derivative, such as the CRRA utility function given by (8), saving will depend on risk in labor income. The CRRA utility function is often used when precautionary demand for saving is concerned. But it is impossible to obtain a closed-form solution for the CRRA utility function unlike for quadratic utility function. Zeldes (1989b) uses a CRRA utility function, and calculates the optimal consumption path numerically using stochastic dynamic programming. He shows that the optimal consumption in a CRRA utility function under stochastic labor income is smaller than that in a quadratic utility function, and the optimal consumption of CRRA utility function decreases as risk in stochastic income increases: that is, there is precautionary demand for saving as expected.

Applying the CRRA utility function given by (8) to the Euler equation given by (14), equation (14) becomes

$$c_t^{-\sigma} = \delta(1+r)E_t c_{t+1}^{-\sigma} \tag{18}$$

Approximation of (18) assuming $\ln \frac{c_{t+1}}{c_t}$ is distributed normally is as follows.

$$E_t \ln \frac{c_{t+1}}{c_t} = \sigma^{-1}\left[\ln(1+r) + \ln \delta\right] + \frac{1}{2}\sigma E_t \operatorname{var}(\ln \frac{c_{t+1}}{c_t}) \tag{19}$$

If the discount rate is expressed as $\delta = \frac{1}{1+\beta}$, and if r and β are small enough to be approximated as $\ln(1+r) = r$ and $\ln(1+\beta) = \beta$, then equation (19) can be written as

$$\ln \frac{c_{t+1}}{c_t} = \sigma^{-1}\left[r - \beta\right] + \frac{1}{2}\sigma E_t \operatorname{var}(\ln \frac{c_{t+1}}{c_t}) + e_{t+1} \tag{20}$$

where an error term, nstead of an expectation operator, on the left-hand side. Equation (20) is found in Carroll (1992). As discussed above, although consumption is endogenous, a lifetime budget constraint may be able to translate labor income risk into consumption risk. Assuming

so, the variance terms in (19) and (20) are considered to come from labor income risk, and an increase in income risk increases consumption growth. Therefore, increasing risk in labor income implies larger variance, and as a result larger consumption growth, that is, higher precautionary saving. Moreover, σ determines the size of induced precautionary saving by an increase in the variance. This is consistent with the fact that σ is the measure of prudence in the case of a CRRA utility function. Carroll's interpretation is that consumers with less wealth have less ability to buffer consumption against income shocks; thus they have higher variance and faster consumption growth (Carroll, 1992). Not only wealth, but any variable that helps predict the future variability of consumption will have a role in predicting the rate of growth of consumption (Deaton, 1992b).

The multiperiod models under income uncertainty presented above assume no borrowing constraints except for the lifetime one, as discussed. Since households can borrow to smooth consumption in any period except in the last, the Euler equation given by (14) or (18) is predicted to hold. However, tests based on the Euler equation are often rejected (e.g. Flavin (1981) and Hansen and Singleton (1983)). Some authors suggest that the rejections are due to borrowing constraints (single-period constraints). As discussed in the case of two-period models, if there is a borrowing constraint and if it is binding for a household, the household's current consumption will be lower than the optimum resulting in higher marginal utility of current period. Thus, with a binding borrowing constraint, the Euler equation of the household is violated as $\frac{\partial u}{\partial c_t} > \delta(1+r)E_t\frac{\partial u}{\partial c_{t+1}}$. This may cause mixed results when we use aggregate data to test the Euler equation.

Based on this idea, Zeldes (1989a) empirically tests the existence of the single-period borrowing constraints using household panel data. In addition to the standard dynamic household model given earlier in this section, he makes an assumption that there is a single-period borrowing constraint expressed as $w_t \geq 0$, which means a household cannot be a net borrower in each period. Under this assumption, this constraint is binding for some households, while for others this constraint is not binding in a given single period. This

constraint is expressed as the Lagrange multiplier, λ_t, which will be positive if binding and zero if non-binding. Then, equation (18) can be written as

$$c_t^{-\sigma} = \delta(1+r_t)E_t c_{t+1}^{-\sigma} + \lambda_t \tag{21}$$

Using an approximation similar to (19) he finally obtains the following equation to estimate:

$$\ln\frac{c_{t+1}}{c_t} = a + HH + T_t + \sigma^{-1}\left[\ln(1+r_t) + \Theta_{t+1} - \Theta_t\right] + \ln(1+\lambda_t) + e_{t+1} \tag{22}$$

where a is a constant, HH is household fixed effect that incorporates household's discount factor, δ, $\Theta_{t+1} - \Theta_t$ is difference in household's characteristics that affect risk aversion, σ. e_{t+1} is assumed to be a mean-zero error term. Therefore, if $\ln(1+\lambda_t) + e_{t+1}$ is positive, λ_t will be positive, which means that there is a binding constraint. The variance term appearing in (19) and (20) does not appear in (22) because Zeldes assumes that the variance is constant for a household over time and includes the variance in the household fixed effect. In other words, he assumes that income risk does not change for each household. He splits the whole sample into two groups based on a wealth/income ratio, and finds that there is evidence that the borrowing constraint is binding in low wealth group.

The Euler equation tells us how a household allocates resources intertemporally and gives testable implications, but cannot show how a household reacts dynamically to the existence of single-period borrowing constraint that may bind potentially in the future. Deaton (1991) takes a different approach based on simulations. He assumes the same borrowing constraint as Zeldes (1989a) explained above, but considers the following modified Euler equation:

$$\frac{\partial u(c_t)}{\partial c_t} = \max\left[\frac{\partial u(w_t + y_t)}{\partial c_t},\ \delta(1+r)E_t\frac{\partial u(c_{t+1})}{\partial c_{t+1}}\right] \tag{23}$$

where $\delta(1+r)<1$ is assumed, which is the condition when the household will run down initial assets under certainty, or the household is "impatient" (Deaton, 1991). Equation (23) means that, under the single-period borrowing constraint ($w_t \geq 0$ for all t), if the constraint is binding, the household will consume all it has and the household's current marginal utility is equal to $\frac{\partial u(w_t + y_t)}{\partial c_t}$, but if the constraint is not binding, the household's current marginal

utility is equal to the expected marginal utility of the next period, $\delta(1+r)E_t\frac{\partial u(c_{t+1})}{\partial c_{t+1}}$, as usual. Deaton and Laroque (1992) prove that there is a unique time-invariant consumption function as $c = f(w + y)$. This function is found by backward induction on equation (23) assuming stochastic i.i.d. income and a CRRA utility function as in Zeldes (1989b) but without a borrowing constraint. Deaton shows that consumption decreases as income risk (standard deviation of stochastic income) increases and the degree of absolute prudence (σ in CRRA utility functions) increases as expected, and using the obtained consumption function, he shows in a simulation that consumption is much smoother than income where assets work as a buffer stock.

The results above imply that a household saves more with than without a borrowing constraint, because the household thinks the borrowing constraint may bind in the future, that is, the precautionary motive for saving is strengthened by the existence of borrowing constraint. This will happen even if the third derivative of the utility function is negative, and in this case the borrowing constraint will cancel the precautionary motive of consumption (uncertainty increases current consumption in this case as explained). Deaton points out that because of the borrowing restrictions, the consumption pattern changes in a fundamental way from the case when borrowing is allowed. As a results, households usually hold assets and satisfy the unrestricted Euler equation in most periods. Consequently, tests for borrowing constraints that look for zero assets and violation of the Euler equations may have few observations to work with, even when households are never allowed to borrow (Deaton, 1994).

In Deaton's model a household receives only stochastic labor income and has only choice between consumption and save. This model, even if it can explain a household behavior in LDCs, may be too simplified to understand its behavior and to obtain some policy implications.

3.5.2 Interest Rate Uncertainty / Multiperiod Model

A multiperiod model with interest rate uncertainty is reviewed in this section.

Levhari and Srinivasan (1969) consider an optimal portfolio selection problem in a multiperiod model under uncertainty, in which a household allocates its initial wealth among consumption, investment in asset A, investment in asset B. Based on their model, Rothschild and Stiglitz (1971) examine the effect of increasing risk in interest rate. The problem is given by

$$\max \; E_t \sum_{t=0}^{\infty} \delta^t u(c_t) \tag{24}$$

$$\text{subject to } w_{t+1} = (1+\tilde{r})(w_t - c_t); \qquad w_t \geq c_t \geq 0$$

where $w_t - c_t$ is saving, which must be positive because in the next period there is no other income than this saving, and $\tilde{r}$ is stochastic interest rate to the saving. The saving is to be allocated between assets A with a rate of return $\tilde{r}_A$ and asset B with a rate of return of $\tilde{r}_B$. If α is the fraction invested in the asset A, then $\tilde{r} = \alpha\tilde{r}_A + (1-\alpha)\tilde{r}_B$. Using the CRRA utility function given by (8), they show that the consumption decision is independent of the portfolio decision (the choice of α), and that the savings rate ($\frac{w_t - c_t}{w_t}$) is constant, independent of the initial wealth, w_t. Then, applying the RS method, they show the following: (i) When $\sigma < 1$, an increase in variability of $\tilde{r}_i$ reduces demand for asset i; (ii) when $\sigma > 1$, an increase in variability of $\tilde{r}_i$ has an ambiguous effect on demand for asset i; (iii) When $\sigma < 1$, an increase in the variability of $\tilde{r}$ decreases the savings rate; (iv) when $\sigma > 1$, an increase in variability of $\tilde{r}$ increases the savings rate. These results are consistent with the two-period model in section 3.3.2.

Since there is no exogenous labor income, this model excludes borrowing. This is why there is no difference in the effect of an increase in risk between two-period models and multiperiod models.

3.5.3 Labor Income and Interest Rate Uncertainty / Multiperiod Model

Skinner (1988) combines stochastic returns to saving and stochastic labor income, and using a CRRA utility function and Taylor series approximation, obtains a closed-form solution for consumption as follows:

$$\ln\frac{c_{t+1}}{c_t} = \sigma^{-1}\left[\bar{r} - \beta + v\right] + \ln\frac{L_t}{\bar{L}_t} \tag{25}$$

where v is risk premium composed of variance of income, variance of interest rate, and their covariance. $\bar{r}$ is the expected interest rate. This model allows the household to borrow (there is no single-period borrowing constraint), but takes the lifetime borrowing constraint into account by adding $\ln\frac{L_t}{\bar{L}_t}$, where L_t is lifetime resources and $\bar{L}_t$ is its anticipated present value. He shows in simulations that the life-cycle consumption pattern is smoother in the case of uncertainty than in the case of certainty and that the closer is the earning process to a random walk, the greater will be precautionary savings. In his model given in (25), positive covariance between income risk and interest rate risk (included in v) increases savings, but in his simulations he assumes the correlation is zero.

Another example is from Dasgupta (1993), where a household's asset level moves according to the equation

$$w_{t+1} = (1+r)(w_t + \tilde{y}_t - c_t - a_t - b_t) + (1+\tilde{\alpha})a_t + (1+\tilde{\beta})b_t \tag{26}$$

under the borrowing constraints expressed as $a_t \geq 0$, $b_t \geq 0$, and $w_t \geq 0$ for all t. $w_t \geq 0$ implies that the household cannot borrow at all. Equation (26) means that the household allocates its wealth $w_t + \tilde{y}_t$ among consumption, c_t, investment in risky asset A, a_t, and investment in risky asset B, b_t. Saving, given by $s_t = w_t + \tilde{y}_t - c_t - a_t - b_t$, also produces a positive, fixed return. This is very similar to the portfolio selection problem given in section 3.5.2, but the difference is the stochastic, exogenous labor income, $\tilde{y}_t$, which is a tradition in savings models and allows the household to borrow against future labor income. Note in contrast that the household cannot borrow in the problem in section 3.5.2.

A standard way gives the following three Euler equations with the Lagrange multipliers ($\phi_t \geq 0$, $\theta_t \geq 0$, and $\mu_t \geq 0$) associated with the constraints $a_t \geq 0$, $b_t \geq 0$, and $w_t \geq 0$ respectively:

$$\delta E_t(-r + \tilde{\alpha})u'(c_{t+1}) + \phi_t = 0 \tag{27}$$

$$\delta E_t(-r + \tilde{\beta})u'(c_{t+1}) + \theta_t = 0 \tag{28}$$

$$u'(c_t) = \delta(1+r)E_t u'(c_{t+1}) + \mu_t \tag{29}$$

To have a positive investment in risky assets, $\phi_t = 0$ and $\theta_t = 0$ are necessary. From the Euler equations above, $E_t\tilde{\alpha}$, $E_t\tilde{\beta} > r$ is a necessary condition given $\mathrm{cov}(\tilde{\alpha}, u'(c_{t+1})) < 0$ and $\mathrm{cov}(\tilde{\beta}, u'(c_{t+1})) < 0$.

A possible simple conclusion is that an increase in riskiness in consumption in t+1 increases the right-hand side of (29) if $u'''(c_{t+1}) > 0$, and therefore the left-hand side of the equation c_t will decline. But Dasgupta rejects such argument, because c_{t+1} is endogenous, and stops further discussion. However, because convexity of the marginal utility function implies convexity of the marginal value function (Sibley, 1975), the RS method can be applied in this case, too.

3.6 Stochastic Production / Multiperiod Model

In Dasgupta's model in section 3.5.3, two risky assets are considered as agricultural inputs, but the model assumes linear return to assets. However, the input-output relationship is often non-linear (unlike in Dasgupta's model), and consequently the effect of increasing risk is also non-linear. This non-linearity determines whether an input is risk-increasing or risk-decreasing, and therefore the input decision is a part of household risk management that may affect household saving. Therefore, the production function is important for household saving models. In this section I review models that incorporate stochastic agricultural production.

Roe and Graham-Tomasi (1986) consider a quite general dynamic model of agricultural household under uncertainty. The model explicitly incorporates stochastic

agricultural production and applies the full income concept of Singh *et al.* (1986), and assumes a perfect labor market. Their model assumes a finite-life household and a bequest motive, and they solve the problem using a constant absolute risk aversion (CARA) utility function for only two periods, the last and the second-to-last. This means that their model is reduced to a two-period model. Here, for consistency with previous multiperiod models, I assume no bequest motive in the following presentation.

The problem can be written as

$$\max_{z_t} \; E_t \sum_{t=0}^{T} \delta^t u(X_{qt}, X_{mt}, l_t); \; Z_t = (X_{qt}, X_{mt}, l_t, L_t, A_t) \tag{30}$$

subject to

$$Q_{t+1} = Q(L_t, A_t; \tilde{\varepsilon}_{t+1}) \tag{31}$$

$$c_t = p_{qt}X_{qt} + p_{mt}X_{mt} + b_t l_t \tag{32}$$

$$\begin{aligned} s_{t+1} &= w_t - a_t A_t - b_t L_t - c_t \\ &= \left[a_t \bar{A}_t + b_t \bar{L}_t + p_{qt} Q(L_{t-1}, A_{t-1}; \tilde{\varepsilon}_{t-1}) + (1+r)s_t\right] - a_t A_t - b_t L_t - c_t \end{aligned} \tag{33}$$

where X_{qt}: consumption of an agricultural staple, p_{qt}: its price, X_{mt}: a good purchased in a market, p_{mt}: its price, l_t: leisure, L_t: on-farm labor input, $\bar{L}_t$, endowment of labor, b_t: wage, A_t: land use for own production, $\bar{A}_t$, endowment of land, a_t: land rental price, s_{t+1}: saving from period t to period t+1, r: interest rate of saving, c_t: consumption that consists of staple foods, purchased goods and opportunity cost of leisure, and $Q_{t+1} = Q(L_t, A_t; \tilde{\varepsilon}_t)$: household agricultural production function with stochastic term. The output appears in the next period, t+1. The difference between $\bar{L}_t$ and L_t represents hired labor if negative or off-farm labor supply if positive, and the difference between $\bar{A}_t$ and A_t represents rented land if negative or lent land if positive. Also this model allows the household to borrow at the rate of r, which results in a negative saving in that period. In chapter 3, I will examine the effects of uncertainty on decision variables applying the RS method to this model, because Roe and Graham-Tomasi do not do it in the article.

Another example is Rosenzweig and Wolpin (1993). They consider an empirically-estimable structural model in the context of the Indian semi-arid tropics. Their model assumes that a household's profit depends on its bullock holding level, number of pumps for irrigation, and a household shock, and constructs a similar model to that of Dasgupta's explained in section 3.5.3 except for exogenous labor income. That is, although they consider a very specific input-output relationship, they assume a linear function.

In their model, a household maximizes a CRRA utility function with a consumption floor, given by

$$\max E_t \sum_{t=1}^{T} \delta^t \frac{1}{1-\sigma}(c_t - c_{\min})^{1-\sigma}$$

$$\text{subject to } c_t = \Pi(B_t, M_t, \varepsilon_t) - p^b b_{t+1} - p^m m_{t+1} - c n_{t+1} > c_{\min} + \xi \qquad (34)$$

where $\Pi(B_t, M_t, \varepsilon_t)$ is profit function, B_t is number of bullocks, M_t is number of pumps, b_{t+1} is net purchase of bullocks, m_{t+1} is purchase of pumps, n_{t+1}, is breeding of a calf, and p^b, p^m, c are their unit costs respectively. The source of the shock is unpredictable weather, but the model does not tell how the weather shock affects profitability. Since there is no interaction terms between weather and inputs, the shock just shifts total profit linearly. Since pumps are not assumed to be sold, bullocks are the only savings in this model. On the other hand, bullocks are important for production. Moreover, since no negative holdings of bullocks and pumps are allowed, the household cannot borrow. Because the model is constructed to have a consumption floor as seen in (34), it must be assumed that the household has a form of disaster insurance so that its consumption never reaches the floor, although such mechanism is not explicitly modeled.

They estimate the profit function first, then using the estimated parameters they estimate remaining structural parameters. By doing this they assume that the production decision is separable from the consumption decision. Using those parameters, they simulate life-cycle consumption patterns for an average household. Their interpretation of the model is mainly based on the simulations. Since their findings are explained in section 2 of this chapter, they are not repeated here.

They do not discuss the properties of their model, but doing so will be important to understanding the results. Their estimation of σ (the coefficient of relative risk aversion) is close to unity (0.96), but significantly different from unity. If the weather shock works like interest rate risk, $\sigma < 1$ could imply precautionary demand for consumption (according to discussion in previous sections). However, the shock variable can be seen as exogenous income risk, because there are no interaction terms between the weather shock and productive capital in the profit function, that is, the input decision does not change the size of risk. In this case, therefore, precautionary demand depends only on the sign of v'''. Since they use the CRRA utility function, $\sigma > 0$ is enough to have positive v'''. Therefore, an increasing in weather risk (that is, profit risk) is predicted to increase investment in bullocks and breeding. However, their assumption that the profit function has no interaction between weather and productive capital (bullocks and especially pumps) may not be true, that is, the effect of weather on profit could depend on the level of the productive capital. If such interactions were incorporated, the household's precautionary behavior would be different.

In this section, I review two agricultural household saving models in which agricultural production is explicitly incorporated. However, neither model assumes non-linearity of production and non-linearity of marginal effect of risk. In fact, I find no household saving model that deals with agricultural production and its non-linearity, especially of risk effect.

3.7 Flexibility and Production/Saving

In this section, models with flexibility of decision process or sequential decisions under uncertainty are reviewed. When we analyze production under uncertainty, flexibility is an important concept. In addition, the saving decision under uncertainty is also affected by information. The concept is initially introduced in firm investment analysis where capital investment is relatively less flexible than the labor input decision. Recently, a similar idea has

been applied to farm household models, where labor is also a flexible input in crop production to adjust rainfall shock.

Hartman (1972) considers the effect of price uncertainty on investment of a price-taking, risk-neutral firm that maximizes the sum of discounted profit. He introduces flexibility by assuming that labor is a flexible input which the firm can decide to maximize short-run profit in each period after price is observed. The problem is written as

$$\max_{I_t} E\sum_{t=0}^{\infty}\delta^t\left[\tilde{p}_t F(K_t, L_t^*(K_t, \tilde{p}_t, \tilde{w}_t)) - \tilde{w}_t L_t^*(K_t, \tilde{p}_t, \tilde{w}_t) - C(I_t, \tilde{q}_t)\right] \qquad (35)$$

where $F(K_t, L_t^*)$ is the production function which is assumed to be homogeneous of degree one[2]. K_t is the capital input level determined by depreciated previous capital level and investment in the current period, as $K_t = (1-d)K_{t-1} + I_t$, L_t^* is labor input optimized in the short-run under certainty and is a function of output price, wage and capital input level, $C(I_t, \tilde{q}_t)$ is investment cost function, where $\tilde{q}_t$ is a stochastic factor, $\tilde{p}_t$ is stochastic output price, and $\tilde{w}_t$ is the stochastic wage rate. By applying the RS method to the first order condition, he shows that an increase in uncertainty either in output price or wage increases investment.

Hartman (1976) generalizes the results above using a single-period model given by

$$\max_K E\left[\tilde{p}F(K, L^*(K, \tilde{p}, w)) - w L^*(K, \tilde{p}, w) - cK\right] \qquad (36)$$

or using the profit function $g(K, \tilde{p}, w)$, which is maximized in the short-run, it is written as

$$\max_K E\left[g(K, \tilde{p}, w) - cK\right] \qquad (37)$$

where uncertainty comes from only the output price and the firm is risk-neutral. He shows: (i) if $\frac{\partial^3 g}{\partial K \partial^2 \tilde{p}} > 0$, optimal K increases as output price uncertainty increases, and (ii) if $\frac{\partial^3 g}{\partial K \partial^2 \tilde{p}} < 0$, optimal K decreases as output price uncertainty increases. The result of Hartman (1972) is the same as the first case, because he assumes that the production function

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[2] A production function is homogeneous of degree one when $f(t\underline{x}) = tf(\underline{x})$ for all $t > 0$, where t is a scalar and $\underline{x}$ is a vector of inputs. Because of this assumption, the short-run profit function can be written as $h(K_t, p_t, w_t) = K_t g(p_t, w_t)$.

is homogeneous of degree one. If a risk-neutral firm decides all input before the price is observed, price uncertainty has no effect on output or on inputs. But as shown above, if flexible a decision is allowed, uncertainty in the output price has a positive or a negative effect on output or inputs as in the case of a risk-averse firm.

Hartman (1976) considers a case of risk-averse firm and shows: (i) if $\frac{\partial^2 g}{\partial K \partial \tilde{p}} > 0$ and $\frac{\partial^3 g}{\partial K \partial^2 \tilde{p}} < 0$, then uncertainty in output price reduces the optimal capital input; (ii) if $\frac{\partial^2 g}{\partial K \partial \tilde{p}} < 0$ and $\frac{\partial^3 g}{\partial K \partial^2 \tilde{p}} > 0$, then uncertainty in output price increases the optimal capital input; (iii) other combinations of those derivatives give ambiguous results. The results of Hartman's flexible decision model are different from Sandmo's earlier results that a risk-averse firm reduces output if output price is uncertain under the assumption that the input decision is made before the price is observed and the marginal profit is linear in the stochastic output price (Sandmo, 1971).

Hartman's results for a risk-averse firm presented above imply that the effects depend on whether capital is risk-increasing or risk-decreasing. But since Hartman uses a Taylor approximation and ignores the third derivative of the utility function, his results do not depend on this third derivative. His result may not hold generally considering that this third derivative determines prudence.

Although Hartman deals with a single-period model, it can be seen as a two-period model because of the flexibility: in the first period, the firm decides capital input under uncertainty, and in the second period, the firm decides labor input under certainty. This means that the firm gets perfect information at the beginning of the second period. However, the information is not necessarily perfect, although more information is available in the second period than in the first period. Thus, we can introduce a degree of information, or "informativeness". Epstein (1980) explores this idea in a three period model: in the second period there is a temporal resolution of uncertainty. Epstein's problem can be written in general as follows:

$$\max_{x_1} E_{\tilde{Y}} \max_{x_2} E_{\tilde{Z}/\tilde{Y}} U(x_1, x_2, \tilde{Z}) \tag{38}$$

In period 1 the decision about x_1 is made subject to prior expectations about the state of period 3. The uncertain future environment is represented by a random variable $\tilde{Z}$. Before the start of period 2 new information about $\tilde{Z}$ becomes available through the observation of another random variable $\tilde{Y}$ which is correlated with $\tilde{Z}$. The agent is a Bayesian decision maker and revises his prior probability distribution about $\tilde{Z}$ after observing $\tilde{Y}$ to make a decision about x_2. The amount of additional information about $\tilde{Z}$ provided by $\tilde{Y}$ is a parameter in the model. $\tilde{Y}$ is said to be more informative than $\tilde{Y}'$ if every user of information about $\tilde{Z}$ is at least as well off observing $\tilde{Y}$ before making a decision as he would be if he based his decision on an observation of $\tilde{Y}'$. This condition can be written as

$$\max_{x_1} E_{\tilde{Y}} \max_{x_2} E_{\tilde{Z}/\tilde{Y}} U(x_1, x_2, \tilde{Z}) \geq \max_{x_1} E_{\tilde{Y}'} \max_{x_2} E_{\tilde{Z}/\tilde{Y}'} U(x_1, x_2, \tilde{Z}) \tag{39}$$

This can be rewritten as

$$\max_{x_1} E_{\tilde{Y}} J(x_1, \tilde{\Pi}) \geq \max_{x_1} E_{\tilde{Y}'} J(x_1, \tilde{\Pi}') \tag{40}$$

where J is the maximum expected utility given x_1 and $\tilde{\Pi}$, and $\tilde{\Pi}$ is random variable $\tilde{Z}$ conditional on $\tilde{Y}$. Based on the definitions above, Epstein's Theorem 1 is: Let x_1^* and x_1^{**} be the solution of left-hand side and right-hand side respectively. If $\frac{\partial J}{\partial x_1}$ is concave in $\tilde{\Pi}$, then $x_1^* \leq x_1^{**}$, and if $\frac{\partial J}{\partial x_1}$ is convex in $\tilde{\Pi}$, then $x_1^* \geq x_1^{**}$. The proof of this theorem is similar to Rothschild and Stiglitz's proof of the effect of increasing risk (Rothschild and Stiglitz, 1971). This theorem implies that it depends on the model whether the prospect of greater information in period 2 discourages the adoption of an irreversible decision in period 1 to maintain some flexibility in period 2 to take advantage of the information or not.

In the three-period framework, Hartman's problem (1976) can be written as

$$\max_{0 \leq K} E - cK + E \max_{0 \leq L} \left[\tilde{p}F(K, L) - wL\right] \tag{41}$$

Thus, $J(K, w, \tilde{p})$ as defined in (40) is equivalent to profit function $g(K, \tilde{p}, w)$ defined in (37). According to Theorem 1, as $\tilde{Y}$ becomes more informative, K^* increases (decreases) if $\frac{\partial g}{\partial K}$ is convex (concave) in $\tilde{p}$. Hartman's problem is the case when $\tilde{p}$ is known in the

second period, that is, $\tilde{Y}$ is perfectly informative. As shown by Hartman, in this case an increase in uncertainty in $\tilde{p}$ increases (decreases) K^*, if $\frac{\partial g}{\partial K}$ is convex (concave) in $\tilde{p}$. Epstein concludes that, in this case, more information and more uncertainty have qualitatively similar effects, which means that prospect of more information in the second period is the same as relatively more uncertainty in the first period. On the other hand, if $\tilde{Y}$ has no information about $\tilde{p}$, the problem is reduced to a simultaneous decision problem, and since the firm is risk-neutral, uncertainty in the output price $\tilde{p}$ has no effect on its input decision.

Epstein (1980) also considers a consumption/saving decision problem in the three-period framework, which is an extension of the two-period model of Sandmo (1968) and Rothschild and Stiglitz (1971) (discussed in section 3.3.2). The problem is written as

$$\max_{0 \le s_1 \le w_1} u(w_1 - s_1) + \delta E \max_{0 \le s_2 \le rs_1} \left[u(rs_1 - s_2) + \delta E v(\tilde{z} s_2) \right] \qquad (42)$$

where w_1 denotes initial wealth, s_1 denotes saving in the first period, r denotes a known, fixed interest rate on the first-period saving, s_2 denotes saving in the second period, $\tilde{z}$ denotes stochastic interest rate on the second-period saving, and δ denotes discount rate. Like previous examples, $J(s_1, \tilde{z}) = \max_{0 \le s_2 \le rs_1} \left[u(rs_1 - s_2) + \delta E v(\tilde{z} s_2) \right]$ is defined as the maximum expected utility in the second period, and by Theorem 1, the prospect of more information in the future increases (decreases) savings in the first period if $\frac{\partial J}{\partial s_1}$ is convex (concave) in $\tilde{z}$. Epstein shows that with the CRRA utility function given by (8), $\frac{\partial J}{\partial s_1}$ is convex in $\tilde{z}$ if $\sigma < 1$ and $\frac{\partial J}{\partial s_1}$ is concave in $\tilde{z}$ if $\sigma > 1$. The two-period problem of Rothschild and Stiglitz discussed in section 3.3.2 is considered to be the case when $\tilde{Y}$ has no information about $\tilde{z}$ because the stochastic interest rate is assumed to be identically, independently distributed. As shown in section 3.3.2, their results are that an increase in uncertainty in the interest rate decreases savings in the first period if $\sigma < 1$ and increases savings in the first period if $\sigma > 1$. Epstein concludes that in this case less information and more uncertainty have qualitatively similar effects.

Epstein's general conclusion is that the relationship between the qualitative effects of earlier resolution (more information) and reduced prior uncertainty for a given information structure are model specific.

Although it is theoretically ambiguous, information or sequential decision making is empirically important. Rose (1992) develops a two-period model for her empirical study of the effect of weather risk on the labor market in rural India. She assumes that in the first period an agricultural household plants prior to the realization of weather and, in the second period after rainfall is known, the household decides how much labor to use for harvesting when the market wage has also adjusted to rainfall. In a single-period model, labor allocation between on-farm and off-farm activities must be decided prior to knowing rainfall, and no flexibility is allowed after rainfall is known. On the other hand, in the two-period model, the household can adjust harvest labor supply depending on the harvest: in a drought year the household will shift labor from on-farm to off-farm, but the wage will be lower in a drought year because labor supply to off-farm activities increases.

The model is constructed as follows: in the second period there is no uncertainty, and the household solves

$$\max_{L_2} \; w_1\hat{L}_1 + h(\varepsilon)w_2L_2 + g(\varepsilon)F(N-\hat{L}_1,\; N-L_2,\; X) + y \qquad (43)$$

where $\hat{L}_1$ denotes off-farm labor supply in the first period, which is predetermined in the second period, L_2 denotes off-farm labor supply in the second period, $N-\hat{L}_1$ denotes on-farm labor supply in the first period, $N-L_2$ denotes on-farm labor supply in the second period, X denotes a vector of fixed household characteristics, and y denotes exogenous income. The household profit from crop production is given by a profit function, $g(\varepsilon)F(N-\hat{L}_1,\; N-L_2, X)$, which depends on on-farm labor supply in the first and the second period, household characteristics, and profit rate as a function of rainfall shock ε which is known in the second period. The profit rate is given by $g(\varepsilon) = \gamma + \varepsilon(1-a)$, that is, mean rainfall γ plus random deviation from the mean rainfall, or rainfall shock ε. The size of the shock depends on the household's ability to reduce shock, such as by irrigation,

denoted by a $(0 < a < 1)$. In the first period, market wage rate is known and fixed to w_1. In the second period, market wage w_2 is affected by rainfall shock ε through $h(\varepsilon)$, which is given by $h(\varepsilon) = \gamma + \varepsilon(1-b)$ where b represents some factor that determines the response of wage rate against rainfall shock, such as availability of off-farm employment. From the first order condition of (43), the optimal L_2 is determined as $L_2^* = L_2^*(\hat{L}_1, \varepsilon)$.

In the first period, the household's problem is to maximize expected utility of income given by

$$\max_{L_1} E\,U\left[w_1 L_1 + h(\tilde{\varepsilon}) w_2 L_2^*(L_1, \tilde{\varepsilon}) + g(\tilde{\varepsilon}) F(N - L_1, N - L_2^*(L_1, \tilde{\varepsilon}), X) + y\right] \quad (44)$$

Rose uses a CRRA utility function, and assumes that on-farm labors in the first period and in the second period are complements. However, since this model is still so complicated, she cannot obtain unambiguous results unlike in a single-period model without flexibility. Even the effect of rainfall risk on off-farm labor supply in the first period is ambiguous although it is positive in her single-period model. She finally uses a linear statistical model for the empirical analysis, and shows the empirical results are largely consistent with the prediction based on her single-period model and two-period model. Empirically, she finds that rainfall risk increases off-farm labor supply in the first period, which confirms the prediction of the single-period model.

The above model is similar to that of Hartman's risk-averse firm (1976) discussed above, and the case of perfect information in firm's input decision in Epstein (1980), if we see labor supply in the first period as capital input and labor supply in the second period as labor input that maximizes short-run profit after price is known. As shown previously, even in simple models like those of Hartman and of Epstein, the effect of uncertainty on the long-run input decision is ambiguous, but the effects are systematically related to the type of technology the household uses. Following them and my previous review of various models, I will extend Rose's model including saving, and will discuss its properties in chapter 3.

Rose does not estimate structural parameters of her flexible labor input model, but uses it to obtain hypotheses for her empirical analysis. Fafchamps (1993) estimates structural

parameters of a sequential labor decision model under uncertainty using household data from rural Burkina Faso, where agriculture is rain-fed and labor is the only variable factor of crop production after the rains have started in cropping season.

Fafchamps's model is concerned with sequential decision regarding labor input to crop production, and for simplicity ignores labor allocation between on-farm and off-farm activities. His model ignores year-to-year investment and saving, too. Consequently the cost of labor is the marginal utility of leisure (unlike in Rose's model). There are two labor decisions, L_1, planting labor, and L_2, weeding labor. Cultivated acreage is assumed to be predetermined, denoted by A. Therefore, his model is essentially reduced to Epstein's three-period model defined as (38).

Using nested constant elasticity of substitution (CES) functional forms, Fafchamps writes his model as

$$\max_{L_1} E_{R_1, R_2} \max_{L_2} E_{R_2} \frac{1}{1-\phi}\left\{\delta\left[\gamma(1-L_1)^{\sigma}+(1-\gamma)(1-L_2)\right]^{\frac{\rho}{\sigma}}+(1-\delta)Y_3^{\rho}\right\}^{\frac{(1-\phi)}{\rho}} \quad (45)$$

subject to

$$Y_1 = Ae^{\tilde{R}_0} \quad (46)$$

$$Y_2 = \left[\alpha Y_1^{r_1}+(1-\alpha)L_1^{r_1}\right]^{\frac{1}{r_1}} e^{\tilde{R}_1} \quad (47)$$

$$Y_3 = b\left[\beta Y_2^{r_2}+(1-\beta)L_2^{r_2}\right]^{\frac{1}{r_2}} e^{\tilde{R}_2} \quad (48)$$

Given land, A, and initial shock, $\tilde{R}_0$, the household obtains the first information of crop growing process, Y_1, and makes planting and replanting decisions, L_1. Then, the second shock $\tilde{R}_1$ takes place after planting, which gives the second information of crop growing process, Y_2. Having Y_2, the household decides how much weeding to perform, L_2. Finally, the third shock, $\tilde{R}_2$, determines final crop output, Y_3. The household is assumed to know all the information: $\tilde{R}_0$, $\tilde{R}_1$, $\tilde{R}_2$, Y_1, Y_2, Y_3, A, L_1, and L_2. However, the econometrician knows only Y_3, A, L_1, and L_2.

Fafchamps estimates the model above using an iterative nested algorithm, full information maximum likelihood, which I will not discuss here. He compares the results with

the results of deterministic model, and concludes that the stochastic control model is superior in terms of data fitting based on Vuong's non-nested model specification test (1989). His conclusions are (i) estimation results agree with farmer' perceptions about the dependency of planting labor on early rainfall and about the occurrence of labor shortages, and (ii) they confirm the importance of flexibility in production and provide a possible explanation of why weeding is often "insufficient" on large farms or in good years.

Although Fafchamps (1993) does not explain how the flexibility affects household risk management, his results suggest that flexibility should be incorporated in agricultural household model explicitly. In the context of the semi-arid tropics, an important implication of flexibility is in labor allocation between on-farm and off-farm activities depending on rainfall, as Rose shows. However, neither Fafchamps nor Rose consider household saving behavior under flexible decision making. The effect of flexibility on saving should be examined in models as well as empirical studies.

4 Conclusions

There is empirical evidence that there exist several strategies that smooth household consumption, such as crop diversification, income diversification (particularly off-farm activities), use of risk-decreasing inputs, social insurance, and savings (livestock holdings). However, those strategies are not examined well in structural household models and their interactions are little known. Because saving is one of those strategies, household saving must be examined in the interlinkages of those risk management strategies. To analyze household saving in this way, there are four gaps in the literature.

(1) Multiple Risks and Their Correlation

Most saving models deal with two types of risk: exogenous labor income risk and exogenous interest rate risk. The effects of increase in those risks are summarized in Table 1. It shows that if saving is riskless, an increase in either type of risk increases precautionary demand for saving given convexity of the marginal utility function ($v''' > 0$) and decreases

precautionary demand for saving given concavity of the marginal utility function ($v''' < 0$). On the other hand, if saving is risky, convexity of the marginal utility function ($v''' > 0$) does not necessarily induce precautionary demand for saving when interest rate risk increases. In reality, however, those two risks coexist and must be correlated in some way. Moreover, there will be more than two risks, although drought risk dominates in the semi-arid tropics. Therefore, multiple risks and their correlation should somehow be incorporated into the model.

(2) Dynamic Models and Lifetime Budget Constraint

In terms of precautionary demand for saving, there is little difference between the two-period model and the multiperiod model. This is because in most multiperiod models, the Euler equations derived from the first order conditions are used to examine the effect of uncertainty. The Euler equations are necessary conditions for the optimal path, but as discussed in section 3.5.1 they ignore the lifetime borrowing constraint. Because it is very difficult to obtain a closed form solution reflecting the lifetime borrowing constraint, unless a very simple model is used as shown in section 3.5.1, most studies are based only on the Euler equations. To incorporate the lifetime borrowing constraint in a stochastic dynamic model, a dynamic simulation starting from the end period must be conducted. Zeldes (1989b) and Deaton (1991) are examples of this kind of simulation. Deaton shows that the borrowing constraint under exogenous income risk induces precautionary demand for saving even if marginal utility function is concave. This is an important implication that can be derived only from multiperiod models, and with this respect multiperiod models have an advantage over two-period model.

However, it will be difficult to simulate complicated household models. Therefore, it is a challenging problem how to deal with a lifetime borrowing constraint in modeling as well as in empirical studies of household saving.

(3) Production Function

The production function should be incorporated into a saving model. In other words, saving behavior should be considered in a structural household model. In Dasgupta's model in section 3.5.3, two risky assets are considered as agricultural inputs, but the model assumes linear return to the assets. However, the input-output relationship is often non-linear unlike in Dasgupta, and consequently the effect of increasing risk is also non-linear. This non-linearity determines whether an input is risk-increasing or risk-decreasing, and therefore the input decision is a part of household risk management that may affect household saving decision.

In this chapter, I reviewed two agricultural household saving models in which agricultural production is explicitly incorporated. However, neither model assumes non-linearity of production and non-linearity of marginal effect of risk. In fact, I find no household saving model that deals with agricultural production and its non-linearity of risk effect.

Crop diversification and risk-decreasing inputs, a part of household risk management strategies as reviewed, will be treated in production function in a structural household model.

(4) Flexible Decision

Flexibility in decision making is important. Fafchamps' example (1993) shows that a flexible model fits household's behavior better. Although it does not explain how the flexibility affects a household's risk management, the results suggest that flexibility should be incorporated in agricultural household model explicitly. As Epstein (1980) concludes, the effect of flexibility is model dependent.

In the context the semi-arid tropics, an important implication of flexibility is in labor allocation between on-farm and off-farm activities depending on rainfall, as Rose (1992) shows. However, neither Fafchamps nor Rose consider household saving behavior under flexible decision making. The effect of flexibility on saving should be examined in models as well as in empirical studies.

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Table 1 Effect of Increasing Risk on Saving and Consumption

Single Risk Model

	$v''' > 0$		$v''' < 0$
Source of Risk	$2v' + a(1+\tilde{r})v''' > 0$ or $\sigma > 1$	$2v' + a(1+\tilde{r})v''' < 0$ or $\sigma < 1$	$2v' + a(1+\tilde{r})v''' < 0$ or $\sigma < 1$
Income ($\tilde{y}_2$) w/o risky asset	$s \uparrow$ $c_1 \downarrow$		$s \downarrow$ $c_1 \uparrow$
Interest ($\tilde{r}$) w/o riskless asset	$a \uparrow$ $c_1 \downarrow$	$a \downarrow$ $c_1 \uparrow$	$a \downarrow$ $c_1 \uparrow$
Interest ($\tilde{r}$) with riskless asset	$m \uparrow$ $a \uparrow$ $c_1 \downarrow$	$m \uparrow$ $a \downarrow$ $c_1 \updownarrow ?$	$m \downarrow$ $a \downarrow$ $c_1 \uparrow$

Note : s is riskless saving, m is riskless asset, a is risky asset, and c_1 is consumption in the first period.

Appendix A

Determinants of Drought Shock in Burkina Faso

Table A1 Determinants of Drought Shock

Zone	ALL	Sahelian Sudanian	Guinean	
Variables	**coeff. (SigT)**	**coeff. (SigT)**	**coeff. (SigT)**	
Endogenous				
lnDIV	-0.34 (0.00)	-0.37 (0.00)	5.81 (0.12)	log of crop diversification.
NagY	2.2 (0.44)	0.89 (0.87)	-37 (0.13)	normal non-ag income (10^{-6})
Asset				
DCRSTAE	-3.8 (0.07)	-4.4 (0.06)	2.1 (0.79)	crop stock (10^{-6} per/AE)
DLVSTAE	-0.37 (0.29)	-0.38 (0.30)	33 (0.15)	livestock holding (10^{-6} per/AE)
AREAAE	0.20 (0.00)	0.21 (0.00)	-0.11 (0.73)	planted area per adult equiv.
Household				
AGEHHH	0.003 (0.04)	0.003 (0.02)	-0.03 (0.06)	age of household head
AT	0.06 (0.29)	0.06 (0.49)	0.21 (0.44)	animal traction dummy
HHAE	0.01 (0.36)	0.004 (0.58)	0.09 (0.04)	adult equivalent size of household
LEADER	0.03 (0.55)	0.06 (0.43)	0.33 (0.19)	1 if HH head=head of compound
NUMWIV	-0.08 (0.00)	-0.06 (0.05)	-0.06 (0.51)	# of wives of HH head
SIMPLE	0.04 (0.53)	0.03 (0.70)	0.78 (0.06)	1 if single conjugal unit
Plot				
AVFALL	0.003 (0.16)	0.002 (0.33)	-0.07 (0.12)	avg. years since last fallow
DISTIDX	0.035 (0.13)	0.018 (0.57)	-0.42 (0.11)	weighted avg. of plot distance(km)
LTOP	0.11 (0.24)	0.18 (0.19)	0.27 (0.48)	log of toposequence index
Region				
ACCESS	0.33 (0.00)	0.36 (0.00)	N/A	1 if road exists near village
CONST.	-0.46 (0.02)	-0.60 (0.01)	3.85 (0.09)	
# of cases	247	207	40	
R square	0.411	0.387	0.355	

Source: Sakurai, Takeshi, "Effect of Crop Diversification on Drought Shock in Burkina Faso," Draft, Michigan State University, 1993.

Note: Endogenous variables are instrumented (two-stage least squared estimation).
HH=household, AE=adult equivalent size of household.
Drought shock in drought year t (u_t) is defined as follows,

$$u_t = \frac{av.\ production\ in\ good\ years - production\ in\ drught\ year\ t}{av.\ production\ in\ good\ years}$$

Appendix B

Full Insurance in Burkina Faso

1 The model

$$c_{it} = HH_i + \alpha Y_{it} + \beta \bar{c}_t + \gamma X_{it} + \varepsilon_{it}$$

This is estimated at village level. c_{it}: household consumption per adult equivalent size of household in year t (CFA), Y_{it}: household income per AE in year t (CFA), $\bar{c}_t$: village average consumption per AE in year t (CFA), HH_i: household fixed effect, X_{it}: a vector of households characteristics, age of household head, single conjugal unit, dependency ratio and number of wives. $\alpha = 0$ and $\beta = 1$ imply full insurance.

2 Results

Table B1 Full Insurance in Burkina Faso

Village	α (own income)	β (av. consumption)	specifications
Sahelian Zone			
Access	0.036 (0.518)*	0.947 (6.920)**	village $\bar{c}_t$, without X_{it}
	0.035 (0.458)*	0.946 (5.518)**	village $\bar{c}_t$, with X_{it}
No Access	0.722 (2.933)	0.365 (1.230)	village $\bar{c}_t$, without X_{it}
	0.692 (2.083)	0.288 (0.682)	village $\bar{c}_t$, with X_{it}
Sudanian Zone			
Access	0.073 (0.494)*	0.941 (2.834)**	village $\bar{c}_t$, without X_{it}
	0.032 (0.161)*	2.256 (0.531)	village $\bar{c}_t$, with X_{it}
No Access	0.259 (2.715)	0.605 (1.879)*	village $\bar{c}_t$, without X_{it}
	0.270 (2.357)	0.833 (1.691)*	village $\bar{c}_t$, with X_{it}
Guinean Zone			
Access	0.063 (3.656)	1.021 (7.171)**	village $\bar{c}_t$, without X_{it}
	0.066 (3.856)	0.824 (4.583)**	village $\bar{c}_t$, with X_{it}
No Access	0.204 (4.075)	0.878 (2.474)**	village $\bar{c}_t$, without X_{it}
	0.198 (2.816)	0.557 (0.719)	village $\bar{c}_t$, with X_{it}

Source: Sakurai, Takeshi, "Consumption Smoothing in Burkina Faso," Draft, Michigan State University, 1995.

Note: Number in parenthesis is t-statistics of the regressions. * indicates that test result supports full insurance: * for α means that the coefficient is not significantly different from zero. * or ** for β means that the coefficient is not significantly different from unity.

Chapter 3

Household Saving in LDCs: Extensions and New Structural Models

0 Summary

There are gaps in the literature on household saving in LDCs. The gaps are both in empirical studies and in theoretical studies. The objectives of this chapter are to fill certain gaps in the theoretical literature, and to construct household models that will be used for empirical studies to fill the gaps.

There are three gaps in the theoretical literature on saving. These are drawn from the literature review in chapter 2.

First, we have little knowledge about the case when there are more than two risks and they are correlated. The findings in this chapter are: When two risks are correlated, the results under uncorrelated risks do not hold. Most cases are ambiguous. Even the most robust assertion (that when the marginal utility function is convex, there is precautionary demand for riskless saving) holds no more. The results depend on how the two risks are correlated. An interesting finding is that in the case of multiple, correlated risks, higher order correlations rather than the first order correlations (just whether positive or negative) are important determinants of the effects of increasing risk.

Second, there are few studies that examine the role of production technology in household risk management/coping strategies. Inputs can be classified as either risk-increasing or risk-decreasing based on the non-linear relationship between marginal productivity and risk. However, little is known about how risk affects the use of those inputs in an agricultural household model including saving. In other words, input has seldom been examined as a part of a household risk management/coping strategies. The findings in this

chapter are: (i) The effects depend on the non-linear response of output to risk. If the risk effect is diminishing, a convex marginal utility is sufficient for precautionary demand for saving. But if the marginal utility is concave, the effect of increasing risk is ambiguous; (ii) When the marginal utility function is convex and an input is risk-decreasing, the effect of increasing risk on the input use is positive, but otherwise, the effect is ambiguous; (iii) A risk-increasing input is overused, while a risk-decreasing input is underused compared with risk-neutral inputs.

Third, little is known about how flexibility of decision making affects household's risk management, such as saving. The findings in this chapter are: (i) Flexibility changes the standard results obtained in an inflexible model such as one presented above. The effects of increasing risk on riskless saving and input use are ambiguous; (ii) The changes are model-specific. In the model presented in this chapter, if a household increases on-farm harvest labor in a good year, the standard results will hold. But, since the off-farm wage increases in a good year, we cannot tell which labor, on-farm or off-farm, a household will increase in a good year *a priori*; (iii) However, when flexibility is allowed, the household's production risk is lowered (in other words, the household is better off under flexibility), and consequently the household's precautionary demand for saving will be lower under flexibility than under inflexibility.

On the other hand, the greatest gap found in empirical studies is that the interlinkages of household risk management/coping strategies are little known. This gap must be filled by empirical studies based on structural agricultural household models that incorporate the risk management/coping strategies. For this purpose, I develop new structural household models that satisfy the requirements. Then, I examine the properties of the models, especially the effects of increasing risks on: (i) crop diversification; (ii) use of risk-decreasing and risk-increasing inputs; (iii) on and off-farm input (labor and capital) supplies; (iv) saving. Moreover, the effect of social insurance on those decision variables, and the effect of flexible

decision making on those variables are analyzed based on the models. The followings are the main results.

First, social insurance is incorporated in the models. There is no qualitative difference in responses to increasing risk between the case of partial social insurance and the case of no social insurance, while the case of perfect social insurance is obviously different from the others. We see only a substitution effect in the case of perfect social insurance, while in the case of partial social insurance, there are both substitution and income effects.

Second, crop diversification is incorporated in the models explicitly. An increase in rainfall risk has an ambiguous effect on crop diversification. This means that it is ambiguous which effect dominates, income effect or substitute effect.

Third, I compare a simultaneous decision model with a sequential decision model. There are a few, but important differences summarized in four points as follows.

First, one important finding is that in a sequential decision model, current period risk in off-farm activities and future period risk in off-farm activities have different effects on off-farm input decisions. The effects of the current risk depend on whether the input is risk-increasing or risk-decreasing, such as the case of risk in crop production. On the other hand, the effects of the future risk is like an exogenous income risk.

Second, in sequential decision model, the sign of the marginal effect of the initial wealth of season 2 on the end wealth of season 2, expressed as, $\frac{\partial w_{1t+1}}{\partial w_{2t}}$, as well as its marginal response to risks, are important. These must be empirically estimated.

Three, an advantage of the sequential decision model is that we can treat exogenous variables known at the beginning of season 2 as risky variables at the beginning of season 1 when decisions for season 1 are made. The interest rate and the wage rate of hired labor in season 2 are treated as risky variables to examine the effect of increasing risk of those variables on the decisions in season 1.

Four, I conclude that the sequential model is better although its empirical estimation requires more data.

1 Introduction

There are gaps in the literature on household saving in LDCs identified in chapter 2. The gaps are both in empirical studies and in theoretical studies. The objectives of this chapter are to fill the gaps in theoretical literature, and to construct household models that will be used for empirical studies to fill the gaps.

The greatest gap in empirical studies is that the interlinkages of household risk management/coping strategies are little known. There is a lot of empirical evidence that there exist several strategies that smooth household consumption in rural areas of LDCs, such as crop diversification, income diversification (particularly off-farm activities), use of risk-decreasing inputs, social insurance, and savings (such as livestock holdings). However, most studies examine one particular strategy, or test consumption smoothing at the household level without specifying specific strategies used. This gap must be filled by empirical studies based on structural agricultural household models that incorporate those risk management/coping strategies listed above, and in those studies household saving in LDCs will be understood better. For this purpose, new structural models are needed.

Three gaps among the gaps in theoretical studies are particularly important to construct the structural models proposed above.

First, we have little knowledge about the case when there are more than two risks and they are correlated. Most saving models deal with either exogenous labor income risk or exogenous interest rate risk, but rarely deal with them at the same time, especially if they are correlated. However, the case of multiple risk will be important empirically, for example, crop production risk and off-farm activity risk may be different but correlated.

Second, there are few studies that examine the role of production technology in household risk management/coping strategies. Only a few household saving models include crop production explicitly. The gap is not just with respect to crop production, but the non-linear relationship between the marginal productivity of input and risk. Inputs are sometimes classified as either risk-increasing or risk-decreasing based on the non-linear relationship.

However, little is known about how risks affect those inputs decisions and how those inputs affect household saving. Empirically, this model will answer which a household prefers, saving or purchasing fertilizer, for example. Crop diversification will also be treated in this framework.

The third gap is related to flexibility in decision making. Labor is a flexible input for agricultural household: Fafchamps (1993) considers labor input adjustment based on rainfall during the cropping season; Rose (1992) considers labor allocation between on-farm and off-farm activities after harvest is known. Both empirically show that flexible models fit household behavior. However, it is little known how flexibility affects household's risk management, such as saving. There is no agricultural household model that deals with saving in a flexible decision framework, to my knowledge.

In this chapter I fill the three gaps in theoretical studies.

For the first gap, my analysis is based on a two-period model under both income risk and interest rate risk, which is a direct extension from two-period models that deal with a single risk, exogenous labor income risk or exogenous interest rate risk (e.g. Rothschild and Stiglitz, 1971; Sandmo, 1968; Sandmo, 1970). Applying a standard method given by Rothschild and Stiglitz (1971), I examine the effect of increasing those risks assuming either independence or correlation. Moreover, I try different assumptions for the correlation to see if that matters.

For the second gap, I start from a dynamic agricultural household model by Roe and Graham-Tomasi (1986). Since they do not examine the effect of increasing risk, I do it in this chapter. In this case, there is only yield risk in the model. My particular interest related with the gap is to know what effect increasing risk will have on risk-increasing and risk-decreasing inputs.

An extension from Rose (1992) is used for the third gap. I modify her model and add saving to the model. The objective of this modeling is to examine the effect of increasing risk on saving as well as other decision variables in a flexible decision model.

Then, based on the results of three extensions, I develop structural dynamic models of agricultural household in the context of the West Africa semi-arid tropics (WASAT). A specific context is needed to develop a dynamic model that has to specify timing of decisions and availability of information. Because the objective of this modeling is to construct models that can be used to fill the gap in empirical studies, the models should incorporate household risk management/coping strategies. I develop two kinds of structural model. The first model is a simultaneous decision model, in which a household decides labor and capital allocation, and crop diversification at the beginning of each harvest year, immediately after harvesting. The second model is a sequential decision model, in which a household makes a decision twice in a harvest year: at the beginning of the dry season and at the beginning of the rainy season. By doing so, the household can use information on off-farm income after the dry season, and based on the information, the household makes decisions for crop production in the rainy season.

Finally, I examine the properties of the two models, especially effects of increasing risks on: (i) crop diversification; (ii) use of risk-decreasing and risk-increasing inputs; (iii) on- and off-farm input (labor and capital) supplies; (iv) saving. Moreover, the effect of social insurance on those decision variables, and effect of flexible decision making on those variables are analyzed based on the models.

2 Extensions of Saving Models

In this section, I extend saving models to fill the remaining gaps in theoretical literature as explained in section 1.

2.1 Correlation of Two Types of Risk

I develop a two-period model with two kinds of risk: labor income risk and interest rate risk.

The problem can be written as

$$\max_{m,a} EU(\tilde{r},\ \tilde{y}_2,\ m,\ a) = u(c_1) + \delta Ev(c_2) = u(w_1 + y_1 - m - a) + \delta Ev(\tilde{y}_2 + m + a(1+\tilde{r})) \tag{1}$$

In the first period, the household has initial wealth, w_1, and labor income, y_1, the sum of which is allocated among consumption, c_1, riskless asset, m, and risky asset, a, with stochastic interest rate, $\tilde{r}$. In the second period, the household has initial wealth given by $\tilde{w}_2 = m + a(1+\tilde{r})$ and stochastic labor income, $\tilde{y}_2$, the sum of which is consumed in the second period. The first order conditions with respect to m and a respectively are

$$E\frac{\partial U}{\partial m} = -u' + \delta Ev' = 0 \tag{2}$$

$$E\frac{\partial U}{\partial a} = -u' + \delta E(1+\tilde{r})v' = 0 \tag{3}$$

Differentiating twice (2) and (3) with respect to $\tilde{r}$ and $\tilde{y}_2$ respectively, the effects of increasing risk are examined (following Rothschild and Stiglitz (1971), or the RS method hereafter). The third derivatives used for the RS method are shown in Appendix A. There are two cases: one assumes that income risk and interest rate risk are independent, and the other assumes that they are correlated.

When the two risks are uncorrelated, the results are as follows: (i) Increasing risk in either the interest rate or labor income increases holdings of riskless assets when $v''' > 0$ and decreases holdings of riskless assets when $v''' < 0$. This is a standard result of saving under income uncertainty; (ii) The effect of increasing risk in the interest rate on holdings of risky assets is ambiguous when $v''' > 0$, and negative when $v''' < 0$; (iii) Increasing labor income risk increases investment in risky assets when $v''' > 0$ and decreases investment in risky asset when $v''' < 0$; (iv) Those results are not different from those when only one risk is considered (in sections 3.3.1 and 3.3.2 of chapter 2). These results are summarized in Table 2.

Then, assume that two risks are correlated. As shown in Table 1, four cases are considered. The relation of the two risks are non-linear as graphically presented in Figure 1. The four cases simplify the reality, but it is much more realistic than assuming a linear relationship. In the negatively correlated cases, exogenous income is considered like ordinary

gifts within a village or a local area; when a drought occurs, such gifts are discouraged and the interest rate becomes high. On the other hand, in the positively correlated cases, exogenous income may be remittances from non-household members and/or public food aid; they are high when a drought damages local economy and interest rate becomes high.

The results are shown in Table 3; when the two risks are correlated, the results under uncorrelated risks do not hold any more. Even the most robust assertion that when $v''' > 0$, there is precautionary demand for riskless saving, depends on how the two risks are correlated. As seen from Table 3, most results are ambiguous. This suggests that the effects should be empirically determined. But one interesting point obtained from Table 3 is that the effect of increasing risk depends more on the second-order relationship between the two risks than on the first-order relationship (that is, just whether positively or negatively correlated). For example, when $v''' > 0$, cases 1 and 3 are similar, while cases 2 and 4 are similar. A lesson from this analysis is that when we consider multiple, correlated risks, we need to pay attention to higher order correlations.

2.2 Stochastic Production Model

In this section, I examine the effect of increasing risk on production input decision. This analysis is an extension of Roe and Graham-Tomasi (1986) who consider a quite general dynamic model of agricultural household under uncertainty. For simplicity I assume no bequest motive, unlike Roe and Graham-Tomasi.

The problem can be written as

$$\max_{z_t} \; E_t \sum_{t=0}^{T} \delta^t u(X_{qt}, X_{mt}, l_t); \quad Z_t = (X_{qt}, X_{mt}, l_t, L_t, A_t) \tag{4}$$

subject to

$$Q_{t+1} = Q(L_t, A_t; \tilde{\varepsilon}_{t+1}) \tag{5}$$

$$c_t = p_{qt}X_{qt} + p_{mt}X_{mt} + b_t l_t \tag{6}$$

$$\begin{aligned} w_t &= a_t\bar{A}_t + b_t\bar{L}_t + p_{qt}Q(L_{t-1}, A_{t-1}; \tilde{\varepsilon}_{t-1}) + (1+r)s_{t-1} \\ &= s_t + a_tA_t + b_tL_t + c_t \end{aligned} \tag{7}$$

where X_{qt}: consumption of an agricultural staple, p_{qt}: its price, X_{mt}: a good purchased in a market, p_{mt}: its price, l_t: leisure, L_t: on-farm labor input, $\bar{L}_t$, endowment of labor, b_t: wage, A_t: land use for own production, $\bar{A}_t$, endowment of land, a_t: land rental price, s_{t+1}: saving from period t to period t+1, r: interest rate of saving, c_t: consumption that consists of staple foods, purchased goods and opportunity cost of leisure, and $Q_{t+1} = Q(L_t, A_t; \tilde{\varepsilon}_t)$: household agricultural production function with stochastic term. The output appears in the next period, t+1. The difference between $\bar{L}_t$ and L_t represents hired labor if negative or off-farm labor supply if positive, and the difference between $\bar{A}_t$ and A_t represents rented land if negative or lent land if positive. Also this model allows the household to borrow at the rate of r, which results in a negative saving in that period.

Bellman's equation is:

$$v(w_t) = \max_{s} \; u(c_t) + \delta E_t v(w_{t+1}) \tag{8}$$

This equation is different from what is found in Roe and Graham-Tomasi (1986). Here I assume that utility is a function of total consumption given by (6) and that the value function is a function of initial wealth given by w_t. From Bellman's equation, the following first order conditions are obtained. The first order condition with respect to s_t, is:

$$-u'(c_t) + \delta(1+r)E_t v'(w_{t+1}) = 0 \tag{9}$$

The first order conditions with respect to L_t and A_t respectively are

$$-b_t u'(c_t) + \delta p_{qt+1} E_t v'(w_{t+1}) \frac{\partial Q}{\partial L_t} = 0 \tag{10}$$

$$-a_t u'(c_t) + \delta p_{qt+1} E_t v'(w_{t+1}) \frac{\partial Q}{\partial A_t} = 0 \tag{11}$$

Equations (9)-(11) are necessary conditions for the optimal path. Thus, consumption decisions and production decisions are not separable.

Now, unlike Roe and Graham-Tomasi (1986), I examine effects of increasing risk by applying the RS method to the first order conditions (9)-(11). The second derivatives of the first order conditions (9)-(11) with respect to $\tilde{\varepsilon}_t$ are in Appendix B.

Some assumptions are made before analyzing the effects. First, with respect to the effect of the risk on output: $\frac{\partial Q}{\partial \tilde{\varepsilon}_t} > 0$ and $\frac{\partial^2 Q}{\partial \tilde{\varepsilon}_t^2} < 0$ are assumed. This means that a production shock increases output, but the effect is diminishing. They are plausible assumptions considering $\tilde{\varepsilon}_t$ like a rainfall shock. Second, with respect to the effect of the risk on the marginal productivity of inputs: There is no *a priori* assumption for $\frac{\partial^2 Q}{\partial L_t \partial \tilde{\varepsilon}_t}$, $\frac{\partial^2 Q}{\partial A_t \partial \tilde{\varepsilon}_t}$, $\frac{\partial^3 Q}{\partial L_t \partial \tilde{\varepsilon}_t^2}$ and $\frac{\partial^3 Q}{\partial A_t \partial \tilde{\varepsilon}_t^2}$. But here I assume that labor is risk-increasing and the effect is diminishing ($\frac{\partial^2 Q}{\partial L_t \partial \tilde{\varepsilon}_t} > 0$ and $\frac{\partial^3 Q}{\partial L_t \partial \tilde{\varepsilon}_t^2} < 0$) and that land is risk-decreasing and the effect is diminishing ($\frac{\partial^2 Q}{\partial A_t \partial \tilde{\varepsilon}_t} < 0$ and $\frac{\partial^3 Q}{\partial A_t \partial \tilde{\varepsilon}_t^2} > 0$).

The results of the effect of increasing risk are summarized as follows: (i) When $v''' \geq 0$, saving increase when production risk increases; (ii) When $v''' < 0$, the effect of increasing risk on saving is ambiguous; (iii) The result (i) is consistent with a standard precautionary demand for saving, while the result (ii) is not consistent. These results critically depend on the non-linear response of output against risk ($\frac{\partial^2 Q}{\partial \tilde{\varepsilon}_t^2} < 0$); (iv) When $v''' \geq 0$, the effect of increasing risk on the use of risk-decreasing inputs is positive, but that on the use of risk-increasing inputs is ambiguous; (v) When $v''' < 0$, the effect of increasing risk on input use is ambiguous; (vi) If inputs are risk-neutral, the effect of increasing risk on input use is positive when $v''' \geq 0$, but is ambiguous when $v''' < 0$. Proofs are in Appendix B.

From (10) and (11) with the assumptions above, it can be shown that risk-increasing labor is underused at the optimum, that is, $p_{qt+1} E_t \frac{\partial Q}{\partial L_t} - (1+r) b_t > 0$, and that risk-decreasing land is overused at the optimum, that is, $p_{qt+1} E_t \frac{\partial Q}{\partial A_t} - (1+r) a_t < 0$, relative to the optimum when those inputs are risk-neutral (proofs are in Appendix B). Note that since input is one-year lagged, there is an interest for input prices.

As seen from the first order conditions (9)-(11), input decisions are not separable from consumption decisions, and the results above show that even if saving increases due to

increasing production risk, whether consumption increases or decreases is unknown. Therefore, the type of the input, that is, whether it is risk-increasing or risk-decreasing, is important to analyze an agricultural household's behavior. To do so, we need to make more specific assumptions in the context of the agricultural system with which we are concerned.

2.3 Flexibility and Production/Saving

To examine the effect of flexibility, I develop a model based on Rose (1992). The model is given by

$$\max_{L_1,\, s_1} \; u(y + w_1 L_1 - s_1) + \delta E v(c_2) \tag{12}$$

$$\text{where} \qquad c_2 = (1+r)s_1 + F(N - L_1,\, N - L_2^*,\, \tilde{R},\, X) + w_2(\tilde{R})L_2^*$$

y, exogenous income; L_1, off-farm labor supply in the first period; L_2, off-farm labor supply in the second period; $N - L_1$, on-farm labor supply in the first period; $N - L_2$, on-farm labor supply in the second period; $\tilde{R}$, stochastic rainfall, unknown in the first period and known in the second period; w_1, fixed wage in the first period; $w_2(\tilde{R})$, wage depending on rainfall in the second period; X, a vector of fixed household characteristics; s_1, saving in the first period; r, known and fixed interest rate for saving; c_2, consumption in the second period. Note that this model follows Rose's assumption that on-farm labor and off-farm labor are perfect substitute setting leisure constant, and consequently when one increases, the other always decreases.

Unlike Rose, I use additively separate utility here and add saving s_1 in the first period. The saving produces a fixed, known interest rs_1. Following Rose, off-farm labor supply in the second period L_2 is optimized in the short-run, denoted by L_2^*. This model looks very much like a two-period saving model, but the differences are that the model includes endogenous and flexible labor allocation decisions, that there is a production in the second period, and that stochastic labor income in the second period is endogenous.

The first order conditions with respect to s_1 and L_1 respectively are

$$-u' + \delta(1+r)Ev' = 0 \tag{13}$$

$$w_1 u' - \delta E v' F_1 = 0 \qquad (14)$$

where $F_2 = w_2$ is used because L_2 is optimized in the short-run. From those equations, $Ev'(EF_1 - (1+r)w_1) + \mathrm{cov}(v', F_1) = 0$ is obtained. Based on this, the following results are obtained: (i) If first-period on-farm labor is risk-increasing, labor is underused (lower than when labor is risk-neutral). This is predicted in the single-period model, but not by the two-period flexible model given by Rose (1992); (ii) If first-period on-farm labor is risk-decreasing, labor is overused (higher than when labor is risk-neutral). The proofs are in Appendix C. The results are the same result as in section 2.2. This implies that the results depend on whether first-period labor, e.g. ploughing, planting, and weeding, is risk-increasing or risk-decreasing. However, Rose assumes only the case of risk-increasing without explanation.

Then, the effect of increasing risk is examined using the RS method: (iii) An increase in rainfall risk has an ambiguous effect on riskless saving in the first period, regardless of the sign of v'''; (iv) An increase in rainfall risk has an ambiguous effect on off-farm labor supply in the first period, regardless of the sign of v'''. Results (iii) and (iv) are different from those obtained in section 2.2 using inflexible model. Proofs are in Appendix C.

The ambiguity comes from the ambiguous sign of $\frac{\partial L_2^*}{\partial \tilde{R}}$; the off-farm labor supply in the second period is optimized after knowing rainfall, that is, it depends on the rainfall. If a household allocates more labor to on-farm activity (harvest) in a good year, the sign of $\frac{\partial L_2^*}{\partial \tilde{R}}$ should be negative. However, since the off-farm wage in the second period also increases in a good year due to high demand for harvest labor, a household may allocate more labor to off-farm activity to in a good year. In this case, $\frac{\partial L_2^*}{\partial \tilde{R}}$ should be positive. We cannot tell whether $\frac{\partial L_2^*}{\partial \tilde{R}}$ is positive or negative *a priori*; rather it should be empirically determined. But if we assume that $\frac{\partial L_2^*}{\partial \tilde{R}}$ is negative, the results become as follows: (v) An increase in rainfall risk has a positive effect on riskless saving in the first period when $v''' \geq 0$, but has an ambiguous

effect when $v''' < 0$; (vi) An increase in rainfall risk has a positive effect on off-farm labor supply in the first period when $v''' \geq 0$ and off-farm labor is risk-decreasing. Otherwise, the effect of increasing rainfall risk is ambiguous. Proofs are in Appendix C. The results (v) and (vi) are the same as those in section 2.2.

Although the effects are ambiguous *a priori*, the household's production risk is lowered (in other words, the household is better-off under the flexibility) when flexibility is allowed, and consequently the household's precautionary demand for saving will be lower under flexibility than under inflexibility.

3 Structural Models and Their Implications

In the following sections, dynamic household models are developed in the context of the West Africa Semi-Arid Tropics (WASAT). The objective is to construct household models that will be used for empirical studies to fill the gaps in the literature. The greatest gaps in empirical studies is that the interlinkages of household risk management/coping strategies are little known. This gap must be filled by empirical studies based on structural agricultural household models that incorporate risk management/coping strategies.

Therefore, the models I develop here must have (i) use of risk-decreasing and risk-increasing inputs; (ii) on- and off-farm input (labor and capital) supplies; (iv) saving; (iv) crop diversification; (v) social insurance. Moreover, following the discussion in sections 2.1-2.3, (a) multiple, correlated risk, (b) non-linear production function, and (c) flexible decision making, are incorporated into the models as follows.

(a) Multiple, Correlated Risks

I assume two risks: stochastic rainfall and stochastic exogenous income. Rainfall affects crop production. Exogenous income can be independent of rainfall, but may be correlated with it. Unlike a simple saving model without production, exogenous income is not labor income from inelastic labor supply. I assume that exogenous income is from remittances, gifts, and/or public food aid. Therefore, we can consider that the stochastic

exogenous income is "social insurance". Thus, the degree of the correlation between rainfall and exogenous income can be considered as the degree of social insurance.

(b) Non-linear Production Function

I assume a crop/off-farm production function, which is a function of labor and capital inputs for on/off-farm activities, crop diversification, rainfall, and household characteristics. By defining signs of second and third order derivatives, I make assumptions about responses of the marginal productivities of those inputs and crop diversification against increasing risk. In the WASAT, off-farm activities are not limited to agricultural employment. Therefore, I assume that off-farm income is a function of off-farm labor and capital inputs, not just wage times labor supply (unlike Rose (1992)).

(c) Flexible Decision Making

I develop two kinds of structural models. The first model is a simultaneous decision model, in which a household decides labor and capital allocation, and crop diversification at the beginning of each harvest year, immediately after harvesting. The second model is a sequential decision model, in which a household makes a decision twice in a harvest year: at the beginning of the dry season and at the beginning of the rainy season. By doing so, the household can use information on off-farm income after the dry season, and based on the information the household make decisions for crop production in the rainy season.

Note that the simultaneous decision model and the sequential decision model are similar to Rose's (1992) one-period model and two-period model respectively, but my models deal not only with labor allocation but also with capital allocation and saving in a dynamic framework. And as a result, information about wealth is the key in decision making in my sequential decision model unlike Rose's.

Both models are based on the concept of "harvest year", which is explained in section 3.1. Then in section 3.2, I develop a simultaneous decision model, and finally it is modified to incorporate sequential decisions in section 3.3.

3.1 Time Frame of Decision Making

There are two seasons in the WASAT: the rainy season and the dry season. Only in the rainy season, crop production takes place. Crop production is rain-fed and unstable as rainfall is very variable in the WASAT. Households are engaged in a variety of off-farm activities to compensate shortfalls of crop income due to drought and to stabilize total household income over years as explained in chapter 2. Off-farm activities mainly occur in dry season after the harvest is known, but some of them are conducted in rainy season as well.

That crop output is a key variable leads us to the concept of "harvest year", which begins with the harvest at the beginning of the dry season and ends at the end of the rainy season, with the simplifying assumption that all harvesting is done instantly at the beginning of each harvest year, as shown in Figures 2 and 3. The two figures differ in assumptions about information the household uses in decision making. Figure 2 corresponds to a simultaneous decision model, in which the household makes all the decisions at the beginning of the dry season of year t. Figure 3 corresponds to a sequential decision model, in which there are two sequential decisions in year t: at the beginning of the dry season and at the beginning of the rainy season.

Both models start from the dry season of year t. At the beginning of the dry season, the household knows its initial wealth level which consists of the harvest of the previous year t-1, carry-over wealth from year t-1, and stochastic exogenous income. Carry-over wealth consists of crop stocks and livestock holdings. Stochastic exogenous income may depend on the harvest of year t-1 and household characteristics. It includes public food aid, gifts, and remittances from non-household members, and acts as if social insurance. I assume that such income is uncertain in the previous year, but at the beginning of year t, it becomes certain. Off-farm income includes income from local employment, self-employment, and migration. In both the simultaneous decision model and the sequential decision model, idiosyncratic risk in off-farm income is assumed.

In the sequential decision model, decisions are made as follows. At the beginning of the dry season, given knowledge about initial wealth as well as its characteristics and being under idiosyncratic risk in the dry season and uncertainty about the future rainy season, the household decides: (i) how to allocate family labor between leisure and off-farm activities in the dry season; (ii) how to allocate its initial wealth among consuming, precautionary saving, and off-farm activities in the dry season. Then, at the beginning of the rainy season, knowing its new wealth level given by the sum of net off-farm earnings and saving and being under rainfall risk and idiosyncratic risk, the household decides: (i) how to allocate family labor among leisure, on-farm, and off-farm activities in the rainy season; (ii) how to allocate its wealth among consuming, precautionary saving, off-farm activities, and purchasing variable inputs for crop production in the rainy season; (iii) how to diversify crops.

In the simultaneous decision model, on the other hand, the household can make all decisions including on-farm and off-farm activities at the beginning of the dry season.

3.2 A Simultaneous Decision Model

I apply a standard intertemporal utility maximization problem given by a time-separable lifetime utility function for household as follows:

$$\max \; E_t \sum_{k=0}^{T} \delta^k u(c_{t+k}, l_{t+k}) \tag{15}$$

where t refers to harvest year, $u(\,)$ is the one-period utility function, c_t is real consumption in year t, l_t is leisure in year t, δ is the discount factor, E_t is the expectation operator (conditional on information available at year t), and T is the end of the household's planning horizon. Standard assumptions about the utility function apply: $u(\,)$ is monotonically increasing and concave in c_t and l_t.

As is shown in Figure 3, year t starts with the harvest from rainy season in year t-1, and therefore the household's initial wealth is given by

$$w_t = (1+r)s_{t-1} + Q_{t-1} + y_t \tag{16}$$

where s_{t-1} is saving or carry-over wealth from year t-1 to year t and Q_{t-1} is total income (on-farm and off-farm) in year t-1, y_t is stochastic, exogenous income that is realized and known at the beginning of year t with certainty.

Since there is no formal saving institutions, the saving takes the form of crop stocks and livestock holdings that produce no interest. But considering livestock, reproduction may yield a positive interest rate, while death is a negative interest rate. This interest rate is stochastic by nature, and is known only at the end of year t-1. However, here for simplicity interest rate, r, is assumed to be known, fixed, and positive.

Total income, Q_{t-1}, includes both crop income and off-farm income. Crop income cannot be consumed in year t-1 due to the time frame of this model, but part of off-farm income can be consumed in year t-1. However, here Q_{t-1} is assumed to be the sum of total crop income and total off-farm income. Thus, to allow off-farm income earned in year t-1 to be consumed in year t-1, the consumed off-farm income is considered to be borrowing from year t, denoted by b_{t-1}. Since this borrowing is internal, naturally it is limited by the amount of off-farm income in year t-1. Now, equation (16) will become

$$w_t = (1+r)s_{t-1} + Q_{t-1} + y_t - b_{t-1} \tag{17}$$

Equation (17) means that part of total off-farm income, denoted by b_{t-1}, is already consumed in year t-1 and is not available as the initial wealth of year t.

If the household is allowed to borrow from external sources, borrowing is handled as a subset of saving: if the saving decision in year t-1 is negative ($s_{t-1} < 0$), the household will be a net borrower in year t-1. The interest rate of this borrowing is the same as that of saving. If borrowing is not allowed, there will be a constraint as $s_t \geq 0$ for all t. In addition, a lifetime budget constraint must be imposed regardless of the single-period borrowing constraint. Since the household's finite life is determined to end in year T and no bequest motive is assumed, all assets must be consumed in year T in this model. This constraint will be discussed later, but for now, only intertemporal optimal allocations are considered as if the household's life is infinite.

y_t includes gifts, public food aid, and remittances from non-household members to be received in year t. It depends on household characteristics and is conditioned by the previous year's harvest, and therefore is predetermined at the beginning of year t. However, at the beginning of year t, y_{t+1} is unknown because it depends on rainfall in year t.

In a simultaneous decision model, the initial wealth given by (17) is allocated at the beginning of harvest year t among consumption, variable capital inputs for crop production including hired labor, off-farm activities, and savings as follows:

$$w_t + \frac{b_t}{1+r} \equiv c_t + K_t^a + a_t L_t^h + K_t^o + s_t \tag{18}$$

where K_t^a is variable inputs for crop production including seeds and fertilizer, K_t^o is variable input for off-farm activities, $a_t L_t^h$ is cost for hired labor, that is, wage rate a_t times amount of hired labor L_t^h, and b_t is off-farm income consumed in year t as explained above.

Labor constraints are as follows:

$$N_t + L_t^h \equiv l_t + L_t^a + L_t^o \tag{19}$$

where N_t is the exogenous, total time available to the household in year t, L_t^h is hired labor and l_t is leisure that appears in the utility function. Equation (19) means that leisure is determined by on-farm labor supply, L_t^a, hired labor, L_t^h, and off-farm labor supply including local on-farm employment, L_t^o.

Total income generated in year t, Q_t, is a function of endogenous capital and labor inputs for on-farm and off-farm activities and crop diversification, D_t, in year t, and exogenous or predetermined variables including stochastic yearly rainfall, $\tilde{R}_t$, idiosyncratic risk in off-farm earnings, $\tilde{\varepsilon}_t$, and a vector of household characteristics, X_t:

$$Q_t = Q(K_t^a, K_t^o, L_t^a, L_t^h, L_t^o, D_t, \tilde{R}_t, \tilde{\varepsilon}_t, X_t) \tag{20}$$

Crop diversification, D_t, is a Simpson index[1]. D_t is greater than or equal to zero and less than unity: D_t is exactly equal to zero when a household produces only one crop, and is unity when a household produces infinite kinds of crops, which is impossible.

This dynamic optimization problem for the household can be written as

$$V_t(w_t) = \max_{\Pi_t} u(c_t, l_t) + \delta E_t V_{t+1}(w_{t+1})$$

$$= \max_{\Pi_t} u(w_t + \frac{b_t}{1+r} - K_t^a - a_t L_t^h - K_t^o - s_t,\ N_t + L_t^h - L_t^a - L_t^o) \qquad (21)$$

$$+ \delta E_t V_{t+1}((1+r)s_t + \tilde{Q}_t + \tilde{y}_{t+1} - b_t)$$

where $\Pi_t = (K_t^a,\ K_t^o,\ L_t^a,\ L_t^h,\ L_t^o,\ D_t,\ S_t,\ b_t)$ is the vector of decision variables of harvest year t, and $V_t(\ )$ is the maximum value of future utility discounted back to t. Given the standard assumptions for the utility function, the value function, $V(\ )$, is also monotonically increasing and concave in w.

3.2.1 The First Order Conditions

The first order conditions for each decision variable given by Π_t is as follows:

K_t^a: $$-\frac{\partial u}{\partial c_t} + \delta E_t \frac{\partial V}{\partial w_{t+1}} \frac{\partial Q}{\partial K_t^a} = 0 \qquad (22)$$

K_t^o: $$-\frac{\partial u}{\partial c_t} + \delta E_t \frac{\partial Q}{\partial K_t^o} \frac{\partial V}{\partial w_{t+1}} = 0 \qquad (23)$$

L_t^a: $$-\frac{\partial u}{\partial l_t} + \delta E_t \frac{\partial V}{\partial w_{t+1}} \frac{\partial Q}{\partial L_t^a} = 0 \qquad (24)$$

L_t^h: $$-a_t \frac{\partial u}{\partial c_t} + \frac{\partial u}{\partial l_t} + \delta E_t \frac{\partial V}{\partial w_{t+1}} \frac{\partial Q}{\partial L_t^h} = 0 \qquad (25)$$

L_t^o: $$-\frac{\partial u}{\partial l_t} + \delta E_t \frac{\partial Q}{\partial L_t^o} \frac{\partial V}{\partial w_{t+1}} = 0 \qquad (26)$$

D_t: $$\delta E_t \frac{\partial V}{\partial w_{t+1}} \frac{\partial Q}{\partial D_t} = 0 \qquad (27)$$

s_t: $$-\frac{\partial u}{\partial c_t} + \delta(1+r) E_t \frac{\partial V}{\partial w_{t+1}} = 0 \qquad (28)$$

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[1] $$D_t = 1 - \sum_k \left[\frac{\textit{planted area for crop k (ha)}}{\textit{household's total planted area (ha)}} \right]^2$$

b_t: $$\frac{1}{1+r}\frac{\partial u}{\partial c_t} - \delta E_t \frac{\partial V}{\partial w_{t+1}} = 0 \qquad (29)$$

The effect of increasing risk is examined following the RS method in the following sub-sections.

3.2.2 Capital and Labor Inputs

Effects of increasing risk on on-farm inputs, K_t^a, L_t^a, and L_t^h are examined first using the RS method. From equations (22), (24), and (25), differentiating twice with respect to stochastic variables, $\tilde{R}_t$ $\tilde{\varepsilon}_t$, and $\tilde{y}_{t+1}$, respectively yields:

$\tilde{R}_t$:

$$V'''(\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t})^2 \frac{\partial Q}{\partial K_t^a} + V''(\frac{\partial^2 Q}{\partial \tilde{R}_t^2} + \frac{\partial^2 \tilde{y}_{t+1}}{\partial \tilde{R}_t^2})\frac{\partial Q}{\partial K_t^a} + 2V''(\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t})\frac{\partial^2 Q}{\partial K_t^a \partial \tilde{R}_t} + V'\frac{\partial^3 Q}{\partial K_t^a \partial \tilde{R}_t^2} \qquad (30)$$

$\tilde{\varepsilon}_t$: $$V'''(\frac{\partial Q}{\partial \tilde{\varepsilon}_t})^2 \frac{\partial Q}{\partial K_t^a} + V''\frac{\partial^2 Q}{\partial \tilde{\varepsilon}_t^2}\frac{\partial Q}{\partial K_t^a} \qquad (31)$$

$\tilde{y}_{t+1}$: $$V'''\frac{\partial Q}{\partial K_t^a} \qquad (32)$$

Before signing them, I have to make some more assumptions, (a) and (b).

(a) On $\tilde{y}_{t+1}$ or social insurance:

$\tilde{y}_{t+1}$ is stochastic exogenous income conditioned by the harvest of year t, and $\tilde{R}_t$. There are three cases about the relationship of the two: (i) If $\tilde{y}_{t+1}$ acts like social insurance, they will be negatively correlated ($\frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} < 0$). And if it is a perfect insurance, $\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} = 0$ will hold, while if it is a partial insurance, $\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} > 0$ will hold. Note that this does not necessarily smooth consumption. (ii) In isolated villages with few economic links with external world, this relation will be the opposite because the drought shock affects all the village economy. That is, $\frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} > 0$ will hold. In this case, however, $\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} > 0$ is still true. (iii) They can be independent, that is, $\frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} = 0$. In any case except for the perfect

insurance, marginal productivity of rainfall should be diminishing, that is, $\frac{\partial^2 Q}{\partial \tilde{R}_t^2} + \frac{\partial^2 \tilde{y}_{t+1}}{\partial \tilde{R}_t^2} < 0$ will hold although the sign of $\frac{\partial^2 \tilde{y}_{t+1}}{\partial \tilde{R}_t^2}$ is ambiguous.

However, in signing (30), because the sign of $\frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t}$ does not directly affect the signs of (30), there are only two cases: one is the case of imperfect social insurance, that is, when $\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} > 0$ holds, and the other is the case of perfect insurance, that is, when $\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} = 0$. In the latter case, only the effect of rainfall uncertainty is examined.

(b) On risk-increasing and risk-decreasing inputs

(i) If K_t^a is risk-increasing, $\frac{\partial^2 Q}{\partial K_t^a \partial \tilde{R}_t} > 0$ will hold and its effect is diminishing with risk, that is, $\frac{\partial^3 Q}{\partial K_t^a \partial \tilde{R}_t^2} < 0$. (ii) If K_t^a is risk-decreasing, $\frac{\partial^2 Q}{\partial K_t^a \partial \tilde{R}_t} < 0$ will hold and its effect is increasing with risk, that is, $\frac{\partial^3 Q}{\partial K_t^a \partial \tilde{R}_t^2} > 0$.

Based on the assumptions above, (30), (31), and (32) are signed. The results are summarized in Table 4.

Then, effects of increasing risk on off-farm inputs, K_t^o, and L_t^o are examined. From equations (23) and (26), differentiating twice with respect to stochastic variables, $\tilde{R}_t$, $\tilde{\varepsilon}_t$, and $\tilde{y}_{t+1}$, respectively yields:

$\tilde{R}_t$: $$V'''\left(\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t}\right)^2 \frac{\partial Q}{\partial K_t^o} + V''\left(\frac{\partial^2 Q}{\partial \tilde{R}_t^2} + \frac{\partial^2 \tilde{y}_{t+1}}{\partial \tilde{R}_t^2}\right)\frac{\partial Q}{\partial K_t^o} \quad (33)$$

$\tilde{\varepsilon}_t$: $$V'''\left(\frac{\partial Q}{\partial \tilde{\varepsilon}_t}\right)^2 \frac{\partial Q}{\partial K_t^o} + V''\frac{\partial^2 Q}{\partial \tilde{\varepsilon}_t^2}\frac{\partial Q}{\partial K_t^o} + 2V''\frac{\partial Q}{\partial \tilde{\varepsilon}_t}\frac{\partial^2 Q}{\partial K_t^o \partial \tilde{\varepsilon}_t} + V'\frac{\partial^3 Q}{\partial K_t^o \partial \tilde{\varepsilon}_t^2} \quad (34)$$

$\tilde{y}_{t+1}$: $$V'''\frac{\partial Q}{\partial K_t^o} \quad (35)$$

(33)-(35) are signed as done for (30)-(32).

The results are summarized in Table 4. The following is found under imperfect social insurance: (i) When $V''' > 0$, an increase in rainfall variability increases off-farm inputs and risk-decreasing on-farm inputs , but has an ambiguous effect on risk-increasing on-farm

inputs; (ii) When $V''' > 0$, an increase in idiosyncratic risk in off-farm earning increases on-farm inputs and risk-decreasing off-farm inputs, but has an ambiguous effect on risk-increasing off-farm inputs; (iii) When $V''' > 0$, an increase in social insurance risk increases both on-farm and off-farm inputs; (iv) When $V''' < 0$, an increase in rainfall variability has an ambiguous effect on both on-farm and off-farm inputs; (v) When $V''' < 0$, an increase in idiosyncratic risk in off-farm earning has an ambiguous effect on both on-farm and off-farm inputs; (vi) When $V''' < 0$, an increase in social insurance risk decreases both on-farm and off-farm inputs.

From (22) and (28), the following equation is obtained:

$$\frac{\partial u}{\partial c_t} = \delta E_t \frac{\partial V}{\partial w_{t+1}} E_t \frac{\partial Q}{\partial K_t^a} + \delta \operatorname{cov}\left(\frac{\partial V}{\partial w_{t+1}}, \frac{\partial Q}{\partial K_t^a}\right) = \delta(1+r) E_t \frac{\partial V}{\partial w_{t+1}} \qquad (35)$$

The covariance will be positive if $\frac{\partial^2 Q}{\partial K_t^a \partial \tilde{R}_t} < 0$, that is, K_t^a is risk-decreasing, and it will be negative if $\frac{\partial^2 Q}{\partial K_t^a \partial \tilde{R}_t} > 0$, that is, K_t^a is risk-increasing. If the covariance is positive, $E_t \frac{\partial Q}{\partial K_t^a} < 1 + r$ must hold, and if the covariance is negative, $E_t \frac{\partial Q}{\partial K_t^a} > 1 + r$ must hold. These imply that at the equilibrium, a risk-decreasing input for crop production is overused in a sense its marginal productivity is less than the interest rate while a risk-increasing input for crop production is underused. Similar results for labor input for crop production can be obtained from (24) and (28). Moreover, capital input and labor input for off-farm activities have the same properties, which can be obtained from (23) and (28), and (26) and (28) respectively. These results are consistent with findings in section 2.3.

3.2.3 Saving and Borrowing

Similarly to section 3.2.2, the effect of increasing risk on s_t and b_t is examined from (28) and (29). From (28), differentiating twice with respect to stochastic variables, $\tilde{R}_t$, $\tilde{\varepsilon}_t$, and $\tilde{y}_{t+1}$, respectively yields:

$$\tilde{R}_t: \qquad (1+r)V'''\left(\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t}\right) + (1+r)V''\left(\frac{\partial^2 Q}{\partial \tilde{R}_t^2} + \frac{\partial^2 \tilde{y}_{t+1}}{\partial \tilde{R}_t^2}\right) \qquad (36)$$

$$\tilde{\varepsilon}_t: \qquad (1+r)V'''(\frac{\partial Q}{\partial \tilde{\varepsilon}_t})^2 + (1+r)V''\frac{\partial^2 Q}{\partial \tilde{\varepsilon}_t^2} \tag{37}$$

$$\tilde{\varepsilon}_t: \qquad (1+r)V''' \tag{38}$$

From (29), the same equations as above but with the opposite sign are obtained. The signs of those equations are determined using the assumptions given in section 3.2.2.

Under imperfect social insurance (i) if $V''' > 0$, an increase in any risk will increase saving and decrease borrowing; (ii) if $V''' < 0$, an increase in rainfall risk and idiosyncratic risk have an ambiguous effect on saving and borrowing, but an increase in social insurance risk increases saving and decreasing borrowing. The results are summarized in Table 4.

Equations (28) and (29) are the same. Thus, at the equilibrium, the marginal gain from saving is equated to the marginal cost of internal borrowing from off-farm income. This is because the interest rate for internal borrowing from off-farm income is the same as the interest rate of saving. But b_t is bounded by the amount of off-farm income, and consequently equation (29) will possibly become $\frac{\partial u}{\partial c_t} < \delta(1+r)E_t\frac{\partial V}{\partial w_{t+1}}$ due to the binding constraint. This may affect the optimal level of other decision variables, but the first order conditions (22)-(28) will still hold.

If the optimal internal borrowing from off-farm income exceeds off-farm income, the household will have to borrow from external sources at the same interest rate as saving, and consequently negative saving will be the optimum. However, if such borrowing is not allowed, saving cannot be negative. Thus, if the borrowing constraint is binding, the household's saving in year t will be zero and equation (28) will become $\delta(1+r)E_t\frac{\partial V}{\partial w_{t+1}} < \frac{\partial u}{\partial c_t}$.

3.2.4 Crop Diversification

Equation (27) can be written as

$$\delta E_t\frac{\partial V}{\partial w_{t+1}}E_t\frac{\partial Q}{\partial D_t} + \delta\,\mathrm{cov}(\frac{\partial V}{\partial w_{t+1}}, \frac{\partial Q}{\partial D_t}) = 0 \tag{39}$$

where $\frac{\partial V}{\partial w_{t+1}} > 0$. Thus, based on equation (39), if $\frac{\partial Q}{\partial D_t} > 0$, the covariance must be negative and if $\frac{\partial Q}{\partial D_t} < 0$, the covariance must be positive. In addition, $\frac{\partial^2 V}{\partial w_{t+1} \partial \tilde{R}_t} < 0$ is assumed to hold. Consequently, negative covariance requires $\frac{\partial^2 Q}{\partial D_t \partial \tilde{R}_t} > 0$ and positive covariance requires $\frac{\partial^2 Q}{\partial D_t \partial \tilde{R}_t} < 0$. Therefore, there are two cases that satisfy equation (39): one is $\frac{\partial Q}{\partial D_t} > 0$ and $\frac{\partial^2 Q}{\partial D_t \partial \tilde{R}_t} > 0$ (crop diversification is output-enhancing and risk-increasing); the other is $\frac{\partial Q}{\partial D_t} < 0$ and $\frac{\partial^2 Q}{\partial D_t \partial \tilde{R}_t} < 0$ (crop diversification is output-reducing and risk-decreasing). If crop diversification is used to reduce crop production risk, $\frac{\partial Q}{\partial D_t} < 0$ will hold. But it is not necessarily the case, as shown Sakurai (1993).

The effect of increasing risk is examined using (30), (31), and (32) by replacing K_t^a with D_t. Given the assumptions above, under imperfect insurance: (i) When $V''' > 0$, an increase in risk in rainfall has an ambiguous effect on crop diversification; (ii) When $V''' > 0$, an increase in idiosyncratic risk in off-farm activities increases crop diversification if crop diversification is risk-increasing, and decreases crop diversification if crop diversification is risk-decreasing; (iii) When $V''' > 0$, an increase in social insurance risk increases crop diversification if crop diversification is risk-increasing and decreases crop diversification if crop diversification is risk-decreasing; (v) When $V''' < 0$, (i) is the same, (ii) becomes ambiguous, and (iii) becomes the opposite. The results are summarized in Tables 4.

3.2.5 Social Insurance

Social insurance is defined here as informal institutions that make the household total income uncorrelated with rainfall, or reduce the correlation. There are several institutions for social insurance observed in LDCs, but in this model the institutions are not specified; it is assumed to reduce income risk due to rainfall costlessly. If this insurance requires some *ex ante* cost, this model can be seen as a model for formal insurance.

The following is found under perfect social insurance: (i) An increase in rainfall risk has no effect on saving, borrowing, and off-farm inputs; (ii) An increase in rainfall risk decreases risk-increasing on-farm inputs and risk-increasing crop diversification ($\frac{\partial Q}{\partial D_t} > 0$); (iii) An increase in rainfall risk increases risk-decreasing on-farm inputs and risk-decreasing crop diversification ($\frac{\partial Q}{\partial D_t} < 0$); (iv) Those effects do not depend on the sign of V'''. Although there is perfect insurance, the household responds to an increase in rainfall risk. This is because the marginal productivities of on-farm inputs and crop diversification are not linearly related to rainfall; the response is seen as a pure substitution effect between inputs keeping expected income constant.

3.2.6 Lifetime Budget Constraints

The household's wealth at the end of its lifetime must be zero if no bequest motive is assumed, and this constraint affect household's behavior backward from the last period. Intertemporal optimal conditions like (22)-(29) are necessary for a dynamic optimum, but cannot give a solution without the lifetime constraint. If a lifetime constraint is imposed, the household will be less likely to borrow even if borrowing is allowed in each period. It does not affect the intertemporal optimal conditions, but the consumption level will become smaller than without the constraint. Therefore, if we are concerned with the level, we need to incorporate the lifetime constraint.

But here, the effects of risk are the main concern, and so we can ignore the lifetime constraint. In addition, since households in rural LDCs are extended family with several generations, it will be difficult to apply the concept of "lifetime" used for nuclear family in developed countries. Deaton (1989) calls it a "dynastic" household, and assumes that the household lives forever.

3.3 A Sequential Decision Model

In this section sequential decisions shown in Figure 3 are incorporated in the dynamic model. Now the dry season is called "season 1" and the rainy season is called "season 2". At the beginning of season 1, the household's decision is based on its initial wealth consisting of harvest of year t-1, saving and exogenous income, and at the beginning of season 2, its decision is based on the outcome of off-farm activities in season 1. Since there is uncertainty in off-farm income of season 1, the household will be better off if it makes decisions for season 2 with the information on season 1. The sequential decision model is set up as follows: First, the optimal decision rule for season 2 is found in short-run utility maximization. This is the two-period production/saving problem. Second, given the optimal decision rule for season 2, a standard dynamic optimization is solved for off-farm activities in season 1.

3.3.1 The First Order Conditions

A standard intertemporal utility maximization problem given by a time-separable lifetime utility function (the same as (15)) is applied. But in addition, utility for each season is assumed to be additively separable, that is, $u(c_t, l_t) = u(c_{1t}, l_{1t}) + \delta u(c_{2t}, l_{2t})$, where c_{1t} and l_{1t} are consumption and leisure in season 1 of year t, and c_{2t} and l_{2t} are consumption and leisure in season 2 year t.

In season 2, the household is assumed to solve two-period production/saving problem given the outcome of off-farm activities in season 1.

$$\max_{\Theta_2} \; u(c_{2t}, l_{2t}) + \delta E_{2t} v(w_{1t+1}) \tag{40}$$

The initial wealth of season 2 is given by $w_{2t} = (1 + r_{1t})s_{1t}$, which is the sum of saving and its interest in season 1. Initial wealth and the time endowment are allocated respectively as follows:

$$w_{2t} = c_{2t} + K^a_{2t} + K^o_{2t} + a_{2t}L^h_{2t} + s_{2t} - \frac{b_{2t}}{1+b}; \quad N_{2t} + L^h_{2t} = l_{2t} + L^a_{2t} + L^o_{2t} \tag{41}$$

where K^a_{2t} and K^o_{2t} are purchased variable input for crop production and off-farm activities respectively in season 2 of year t, L^a_{2t} and L^o_{2t} are labor input for crop production and off-

farm activities respectively in season 2 of year t, L^h_{2t} is hired labor for crop production in season 2 of year t, s_{2t} is saving and b_{2t} is internal borrowing as discussed in section 3.2. Hired labor is valued at exogenous market wage, a_{2t}, known to the household when the decision is made. Then, at the beginning of the season 1 of the next year, the household wealth becomes

$$w_{1t+1} = Q(K^a_{2t}, K^o_{2t}, L^a_{2t}, L^h_{2t}, L^o_{2t}, D_{2t}, \tilde{R}_{2t}, X_t, \tilde{\varepsilon}_{2t}) + (1+r_{2t})s_{2t} - b_{2t} + \tilde{y}_{1t+1} \quad (42)$$

where Q is crop/off-farm income function in season 2. Crop income is assumed to depend on stochastic rainfall, $\tilde{R}_{2t}$, and off-farm income is assumed to be independent of rainfall but has an idiosyncratic risk, $\tilde{\varepsilon}_{2t}$. D_{2t} is crop diversification index as defined in section 3.2, which affects only crop production. $\tilde{y}_{1t+1}$ is the stochastic, exogenous income known to the household at the beginning of season 1 in year t+1. Its properties are as discussed in section 3.2.

The first order conditions are obtained by differentiating (37) with respect to Θ_{2t}, a vector of the decision variables of this problem: $\Theta_{2t} = (K^a_{2t}, K^o_{2t}, L^a_{2t}, L^h_{2t}, L^o_{2t}, D_{2t}, s_{2t}, b_{2t})$.

K^a_{2t}: $$-\frac{\partial u}{\partial c_{2t}} + \delta E_{2t}\frac{\partial v}{\partial w_{1t+1}}\frac{\partial Q}{\partial K^a_{2t}} = 0 \quad (43)$$

K^o_{2t}: $$-\frac{\partial u}{\partial c_{2t}} + \delta E_{2t}\frac{\partial v}{\partial w_{1t+1}}\frac{\partial Q}{\partial K^o_{2t}} = 0 \quad (44)$$

L^a_{2t}: $$-\frac{\partial u}{\partial l_{2t}} + \delta E_{2t}\frac{\partial v}{\partial w_{1t+1}}\frac{\partial Q}{\partial L^a_{2t}} = 0 \quad (45)$$

L^h_{2t}: $$-a_{2t}\frac{\partial u}{\partial c_{2t}} + \frac{\partial u}{\partial l_{2t}} + \delta E_{2t}\frac{\partial v}{\partial w_{1t+1}}\frac{\partial Q}{\partial L^h_{2t}} = 0 \quad (46)$$

L^o_{2t}: $$-\frac{\partial u}{\partial l_{2t}} + \delta E_{2t}\frac{\partial v}{\partial w_{1t+1}}\frac{\partial Q}{\partial L^o_{2t}} = 0 \quad (47)$$

D_{2t}: $$\delta E_{2t}\frac{\partial v}{\partial w_{1t+1}}\frac{\partial Q}{\partial D_{2t}} = 0 \quad (48)$$

s_{2t}: $$-\frac{\partial u}{\partial c_{2t}} + (1+r_{2t})\delta E_{2t}\frac{\partial v}{\partial w_{1t+1}} = 0 \quad (49)$$

b_{2t}: $$\frac{1}{1+b}\frac{\partial u}{\partial c_{2t}} - \delta E_{2t}\frac{\partial v}{\partial w_{1t+1}} = 0 \quad (50)$$

Those first order conditions (43)-(50) are very similar to the first order conditions (22)-(29), and the discussion on the first order conditions derived from the simultaneous decision model in section 3.2 is valid (I will not repeat it here).

From (43) to (50), the optimal level of the decision variables can be written as a function of wealth at the beginning of season 2, w_{2t}, the interest rate, r_{2t}, the market wage, a_{2t}, household characteristics, X_t, and distribution of stochastic variables, $\tilde{R}_{2t}$, $\tilde{\varepsilon}_{2t}$, and $\tilde{y}_{1t+1}$. Those variables, except for w_{2t}, are exogenously given. The household's expectation about the levels of the exogenous stochastic variables at the beginning of season 2 of year t is the same as that of the beginning of season 1 because no new information about those stochastic variables is given at the beginning of season 2. Thus, the optimal Θ_{2t} will depend on the household's decision in season 1 through w_{2t}.

The multiperiod utility maximization problem can be written as follows:

$$V(w_{1t}) = \max_{\Theta_{1t}} u(c_{1t}, l_{1t}) + \delta E_{1t}\left[u(c^*_{2t}, l^*_{2t}) + \delta v((1+r_{2t})s^*_{2t} + \tilde{Q}^* - b^*_{2t} + \tilde{y}_{1t+1})\right]$$

$$\text{where} \quad c_{1t} = w_{1t} - K^o_{1t} - s_{1t} \qquad (51)$$

$$l_{1t} = N_{1t} - L^o_{1t}$$

$\Theta_{1t} = (s_{1t}, K^o_{1t}, L^o_{1t})$ is a vector of decision variables in season 1, and superscript * means that those variables are determined by w_{2t} according to the optimal decision rule implied by the first order conditions (43)-(50). w_{2t} is determined by decision variables in season 1, but it is uncertain due to idiosyncratic risk:

$$w_{2t} = (1+r_{1t})s_{1t} + F(K^o_{1t}, L^o_{1t}, X_t, \tilde{\varepsilon}_{1t}) \qquad (52)$$

where F is off-farm income function in season 1. Thus, the household's problem is to allocate its initial wealth, w_{1t}, among consumption in season 1, c_{1t}, purchased variable input for off-farm activities in season 1, K^o_{1t}, and saving in season 1, s_{1t}, and to allocate its labor endowment, N_{1t}, between leisure in season 1, l_{1t}, and labor input for off-farm activities in season 1, L^o_{1t}, to maximize equation (51).

The first order conditions with respect to Θ_{1t} can be simplified using the short-run first order conditions (43)-(50) as follows:

$$s_{1t}: \quad -\frac{\partial u}{\partial c_{1t}} + \delta^2(1+r_{1t})E_{1t}v'\frac{\partial w_{1t+1}}{\partial w_{2t}} = 0 \tag{53}$$

$$K^o_{1t}: \quad -\frac{\partial u}{\partial c_{1t}} + \delta^2(1+r_{1t})E_{1t}v'\frac{\partial F}{\partial K^o_{1t}}\frac{\partial w_{1t+1}}{\partial w_{2t}} = 0 \tag{54}$$

$$L^o_{1t}: \quad -\frac{\partial u}{\partial l_{1t}} + \delta^2(1+r_{1t})E_{1t}v'\frac{\partial F}{\partial L^o_{1t}}\frac{\partial w_{1t+1}}{\partial w_{2t}} = 0 \tag{55}$$

The household does not know the interest rate in season 2, r_{2t}, and the hired labor wage rate in season 2, a_{2t}, at the beginning of season 1: they are assumed to be known at the beginning of season 2. Therefore, in addition to rainfall risk and idiosyncratic risk in season 2 and idiosyncratic risk in season 1, there are interest risk and hired wage risk for the household who makes decisions at the beginning of season 1 in the optimization problem above.

3.3.2 Effects of Increasing Risk

From the first order conditions (53)-(55), the effect of increasing risk is examined following the RS method. The second derivatives from the first order conditions are shown in Appendix D.

The results depend on $\frac{\partial w_{1t+1}}{\partial w_{2t}}$, which is given by

$$\frac{\partial w_{1t+1}}{\partial w_{2t}} = \frac{\partial Q}{\partial K^a_{2t}}\frac{\partial K^a_{2t}}{\partial w_{2t}} + \frac{\partial Q}{\partial L^a_{2t}}\frac{\partial L^a_{2t}}{\partial w_{2t}} + \frac{\partial Q}{\partial K^o_{2t}}\frac{\partial K^o_{2t}}{\partial w_{2t}} + \frac{\partial Q}{\partial L^o_{2t}}\frac{\partial L^o_{2t}}{\partial w_{2t}} + \frac{\partial Q}{\partial D_{2t}}\frac{\partial D_{2t}}{\partial w_{2t}} + (1+r_{2t})\frac{\partial s_{2t}}{\partial w_{2t}} - \frac{\partial b_{2t}}{\partial w_{2t}} \tag{56}$$

whose sign is likely positive, but can be negative: the sign depends on the household's marginal propensity to save, to consume, and to invest out of w_{2t}, as well as on the marginal productivities of the investment. It can be negative, for example, when a household has very low w_{2t} due to drought in previous year and/or a failure in off-farm activity in dry season of current year. Here, however, $\frac{\partial w_{1t+1}}{\partial w_{2t}}$ is assumed to be positive for simplicity. Moreover, we need to specify whether this positive marginal effect is risk-increasing or risk-decreasing as done before. Here, its is assumed to be risk-decreasing against all risk.

Given the assumptions above, effect of an increase in the following risks are examined: (1) rainfall risk in crop production in season 2 under imperfect social insurance, $\tilde{R}_{2t}$; (2) $\tilde{R}_{2t}$ under perfect insurance; (3) idiosyncratic risk in off-farm activities in season 1, $\tilde{\varepsilon}_{1t}$; (4) exogenous income risk, or risk in social insurance, $\tilde{y}_{1t+1}$; (5) interest rate risk in season 2, r_{2t}; (6) wage rate risk in season 2, a_{2t}; (7) idiosyncratic risk in off-farm activities season 2, $\tilde{\varepsilon}_{2t}$.

The results, except for (2) rainfall risk under perfect social insurance, (3) idiosyncratic risk in off-farm activities in season 1, and (6) wage rate risk are: (i) saving in season 1 increases when risks increase; (ii) off-farm inputs (capital and labor) in season 1 increase when risks increase and when inputs are less the optimum. The results (i)-(ii) mean that those risks have the same effect as the risk of future exogenous income. As for (3) idiosyncratic risk in off-farm activities in season 1: (iii) saving in season 1 increases; (iv) off-farm inputs increase if they are risk-decreasing. Otherwise, the effect on off-farm inputs are ambiguous. The results (ii) and (iii) are different. As for (2) rainfall risk under perfect insurance: (vi) an increase in risk has no effect on the household's behavior in season 1. Finally as for (6) wage rate risk in season 2: (vii) an increase in risk has an ambiguous effect on the household's behavior season 1 under the assumptions given above. The results are summarized in Table 5.

4 Conclusions

From sections 2.1-2.3, three points are concluded.

First, when there are more than two risks, how they are correlated is an important determinant of the effects of increasing risk. As shown in Table 3, most results are ambiguous, but the second order relations determine the effects rather than the first order relations. This finding is reflected in modeling in section 3.

Second, the effects of increasing risk depend on how the marginal productivity of inputs responds to change in risk, and whether the input is risk-increasing or risk-decreasing.

However, as shown in section 2.2, this makes results ambiguous. Risk-increasing inputs often show an ambiguous effect because benefit from increasing output and cost from increasing risk are canceling each other. The conclusion is that the non-linearity of risk response is important.

Third, introduction of flexibility changes the results. Most results obtained in an inflexible model in section 2.2 do not hold in a flexible model in section 2.3. The changes are model-specific, but in general in flexibility models, production risk in the second period becomes similar to exogenous income risk.

Based on the findings listed above, I develop two structural household models in section 3 and examine their properties to compare a simultaneous decision model and a sequential decision model.

First, social insurance is incorporated in the models. As shown in Table 4, there is no qualitative difference in responses to increasing risk between the case of partial social insurance ($\frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} < 0$ and $\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} > 0$) and the case of no social insurance ($\frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} > 0$ and $\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} > 0$), while the case of perfect social insurance ($\frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} < 0$ and $\frac{\partial Q}{\partial \tilde{R}_t} + \frac{\partial \tilde{y}_{t+1}}{\partial \tilde{R}_t} = 0$) is obviously different from others. As discussed in section 3.2.5, we see only a substitution effect in the case of perfect social insurance. Thus, in the case of partial insurance, we may see an income effect in addition to a substitution effect. Since perfect insurance is an extreme case, this result predicts that the difference among villages in terms of risk response will not be qualitative, but quantitatively distributed between perfect and nothing, that is, the income effect differs.

Second, crop diversification is incorporated in the models explicitly. Crop diversification is not necessarily a strategy that reduces risk by having a smaller expected output. Rather, it can be an output-enhancing, risky strategy. This point is treated by assuming that crop diversification can be either risk-increasing or risk-decreasing. Either case can be optimal in the dynamic models considered. As shown in Table 4, an increase in

rainfall risk has an ambiguous effect on crop diversification. This means that it is ambiguous which effect dominates, the income effect or the substitution effect. If there is perfect social insurance, only the substitution effect will exist. In this case, as shown in Table 4, an increase in rainfall risk decreases risk-increasing crop diversification and increase risk-decreasing crop diversification, as expected. This finding implies that crop diversification must be treated carefully in empirical studies. Otherwise, empirical studies will show mixed, ambiguous results.

Third, I compare a simultaneous decision model with a sequential decision model. There are a few, but important differences summarized in three points as follows.

(1) One important finding is that in the sequential decision model current period risk in off-farm activities, $\tilde{\varepsilon}_{1t}$, and future period risk in off-farm activities, $\tilde{\varepsilon}_{2t}$, have different effects on off-farm input decisions. The effects of the current risk depend on whether the input is risk-increasing or risk-decreasing like the case of risk in crop production. On the other hand, the effects of future risk do not depend on the technology because risk affects current decisions just like exogenous income risk does. Therefore, the effects are the same as exogenous income risk effects. Since off-farm activities are conducted mainly in season 1, the ambiguity of the effects of idiosyncratic risk in season 1 is important. This risk may discourage off-farm activities, even if there is income risk in the future. This point can be applied to saving as well, if saving is assumed to be risky. In fact, that saving is riskless may be a strong assumption, especially if we assume crop stocks and livestock holdings as a substitute for saving in the context of the WASAT, even though it may be true that such savings are relatively riskless compared with other risky investment. Then, if saving is risky, the non-linear relationship between risk and return will become important. In the WASAT, livestock holdings may be risk-increasing or risk-decreasing, which determines the effect of increasing risk as shown previous sections.

(2) In the sequential decision model, the sign of $\frac{\partial w_{1t+1}}{\partial w_{2t}}$ and its marginal response against risks are important to examine the effect of uncertainty, as discussed in section 3.3.2.

In Table 5, $\frac{\partial w_{1t+1}}{\partial w_{2t}}$ is assumed to be positive for simplicity, but it can be negative as argued, and therefore in empirical studies it must be estimated. Moreover, the magnitude of $\frac{\partial w_{1t+1}}{\partial w_{2t}}$ will also crucial to understand the household's behavior under uncertainty.

(3) An advantage of the sequential decision model is that we can treat exogenous variables known at the beginning season 2 as risky variables at the beginning of season 1 when decisions for season 1 are made. In section 3.2.5, the interest rate and the wage rate of hired labor in season 2 are treated as risky variables to examine the effect of increasing risk of those variables on the decisions in season 1. Although the effect of wage rate risk is ambiguous, the effect of interest rate risk is found to be the same as exogenous income risk.

(4) I conclude that the sequential model is better although its empirical estimation requires more data.

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Table 1 Relationship Between $\tilde{r}$ and $\tilde{y}_2$

Correlation	Case	
Negative	1	$\frac{\partial \tilde{y}_2}{\partial \tilde{r}} < 0,\ \frac{\partial \tilde{r}}{\partial \tilde{y}_2} < 0,\ \frac{\partial^2 \tilde{y}_2}{\partial \tilde{r}^2} < 0,\ \frac{\partial^2 \tilde{r}}{\partial \tilde{y}_2^2} < 0$
Negative	2	$\frac{\partial \tilde{y}_2}{\partial \tilde{r}} < 0,\ \frac{\partial \tilde{r}}{\partial \tilde{y}_2} < 0,\ \frac{\partial^2 \tilde{y}_2}{\partial \tilde{r}^2} > 0,\ \frac{\partial^2 \tilde{r}}{\partial \tilde{y}_2^2} > 0$
Positive	3	$\frac{\partial \tilde{y}_2}{\partial \tilde{r}} > 0,\ \frac{\partial \tilde{r}}{\partial \tilde{y}_2} > 0,\ \frac{\partial^2 \tilde{y}_2}{\partial \tilde{r}^2} > 0,\ \frac{\partial^2 \tilde{r}}{\partial \tilde{y}_2^2} < 0$
Positive	4	$\frac{\partial \tilde{y}_2}{\partial \tilde{r}} > 0,\ \frac{\partial \tilde{r}}{\partial \tilde{y}_2} > 0,\ \frac{\partial^2 \tilde{y}_2}{\partial \tilde{r}^2} < 0,\ \frac{\partial^2 \tilde{r}}{\partial \tilde{y}_2^2} > 0$

Graphs for each case are presented in Figure 1.

Figure 1 Relationship Between $\tilde{r}$ and $\tilde{y}_2$

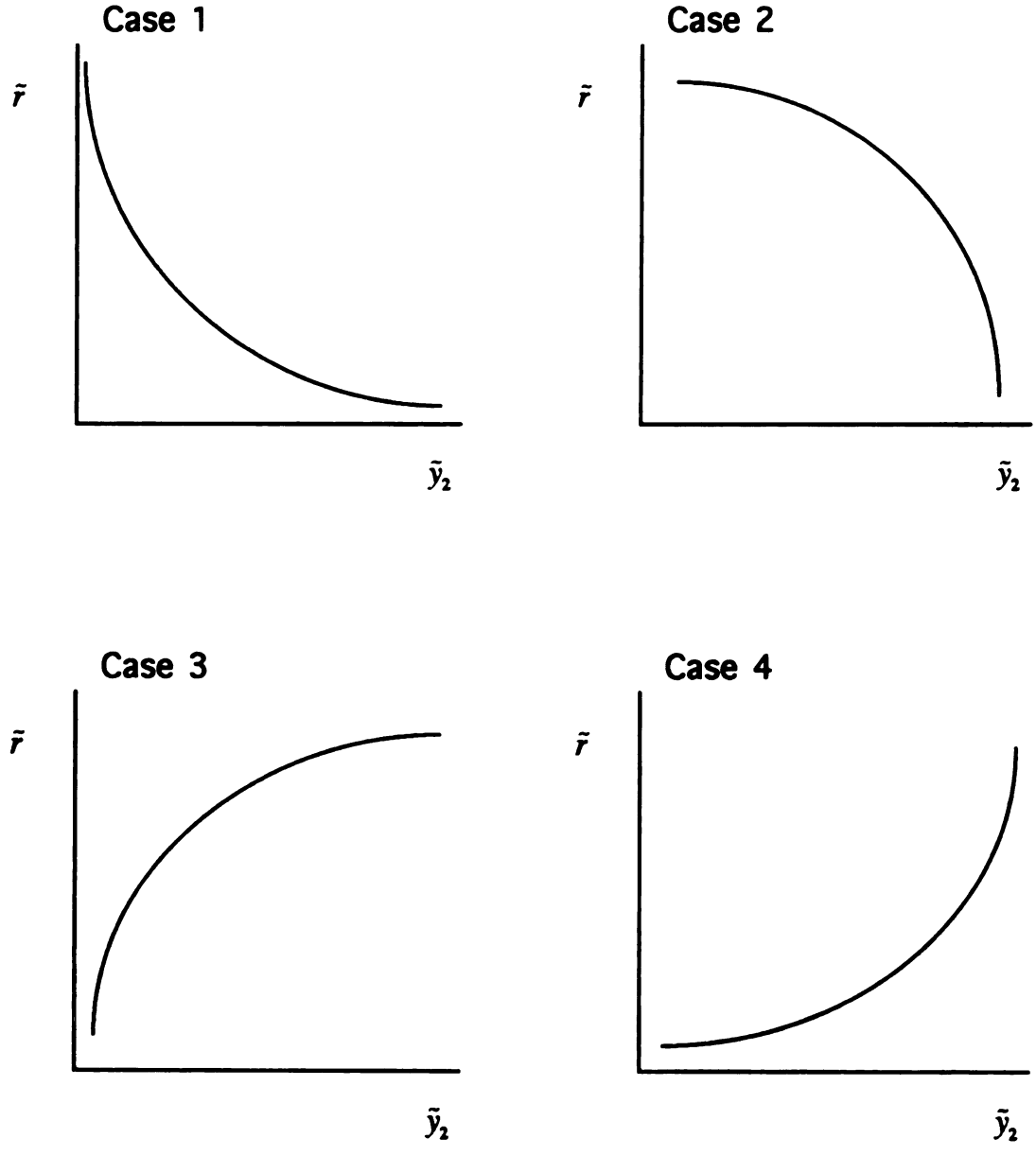

Table 2 Effect of Increasing Risk on Decisions (uncorrelated)

	$v''' > 0$		$v''' < 0$
Source of Risk	$2v' + a(1+\tilde{r})v''' > 0$	$2v' + a(1+\tilde{r})v''' < 0$	$2v' + a(1+\tilde{r})v''' < 0$
Income ($\tilde{y}_2$)	$m\uparrow$ $a\uparrow$ $c_1\downarrow$		$m\downarrow$ $a\downarrow$ $c_1\uparrow$
Interest ($\tilde{r}$)	$m\uparrow$ $a\uparrow$ $c_1\downarrow$	$m\uparrow$ $a\downarrow$ $c_1 \updownarrow ?$	$m\downarrow$ $a\downarrow$ $c_1\uparrow$

Note : m is riskless asset, a is risky asset, and c_1 is consumption in the first period.

Table 3 Effect of Increasing Risk on Decisions (correlated)

	$v''' > 0$			
Source of Risk	CASE 1 (negative)	CASE 2 (negative)	CASE 3 (positive)	CASE 4 (positive)
Income ($\tilde{y}_2$)	$m\uparrow$ $a\updownarrow?$ $c_1\updownarrow?$	$m\updownarrow?$ $a\updownarrow?$ $c_1\updownarrow?$	$m\uparrow$ $a\updownarrow?$ $c_1\updownarrow?$	$m\updownarrow?$ $a\updownarrow?$ $c_1\updownarrow?$
Interest ($\tilde{r}$)	$m\uparrow$ $a\updownarrow?^1$ $c_1\updownarrow?^1$	$m\updownarrow?$ $a\updownarrow?$ $c_1\updownarrow?$	$m\updownarrow?$ $a\updownarrow?$ $c_1\updownarrow?$	$m\uparrow$ $a\updownarrow?^1$ $c_1\updownarrow?^1$

	$v''' < 0$			
Source of Risk	CASE 1 (negative)	CASE 2 (negative)	CASE 3 (positive)	CASE 4 (positive)
Income ($\tilde{y}_2$)	$m\updownarrow?$ $a\updownarrow?$ $c_1\updownarrow?$	$m\downarrow$ $a\updownarrow?$ $c_1\updownarrow?$	$m\updownarrow?$ $a\updownarrow?$ $c_1\updownarrow?$	$m\downarrow$ $a\updownarrow?$ $c_1\updownarrow?$
Interest ($\tilde{r}$)	$m\updownarrow?$ $a\updownarrow?$ $c_1\updownarrow?$	$m\downarrow$ $a\updownarrow?^2$ $c_1\updownarrow?^2$	$m\downarrow$ $a\downarrow$ $c_1\uparrow$	$m\updownarrow?$ $a\updownarrow?$ $c_1\updownarrow?$

Note : m is riskless asset, a is risky asset, and c_1 is consumption in the first period.

Note 1: When $\frac{\partial\tilde{y}}{\partial\tilde{r}} + a < 0$ holds, $a\uparrow$ and as a result $c_1\downarrow$. Otherwise, they are ambiguous.

Note 2: When $\frac{\partial\tilde{y}}{\partial\tilde{r}} + a > 0$ holds, $a\downarrow$ and as a result $c_1\uparrow$. Otherwise, they are ambiguous.

Figure 2 Time Frame of Simultaneous Decision Making

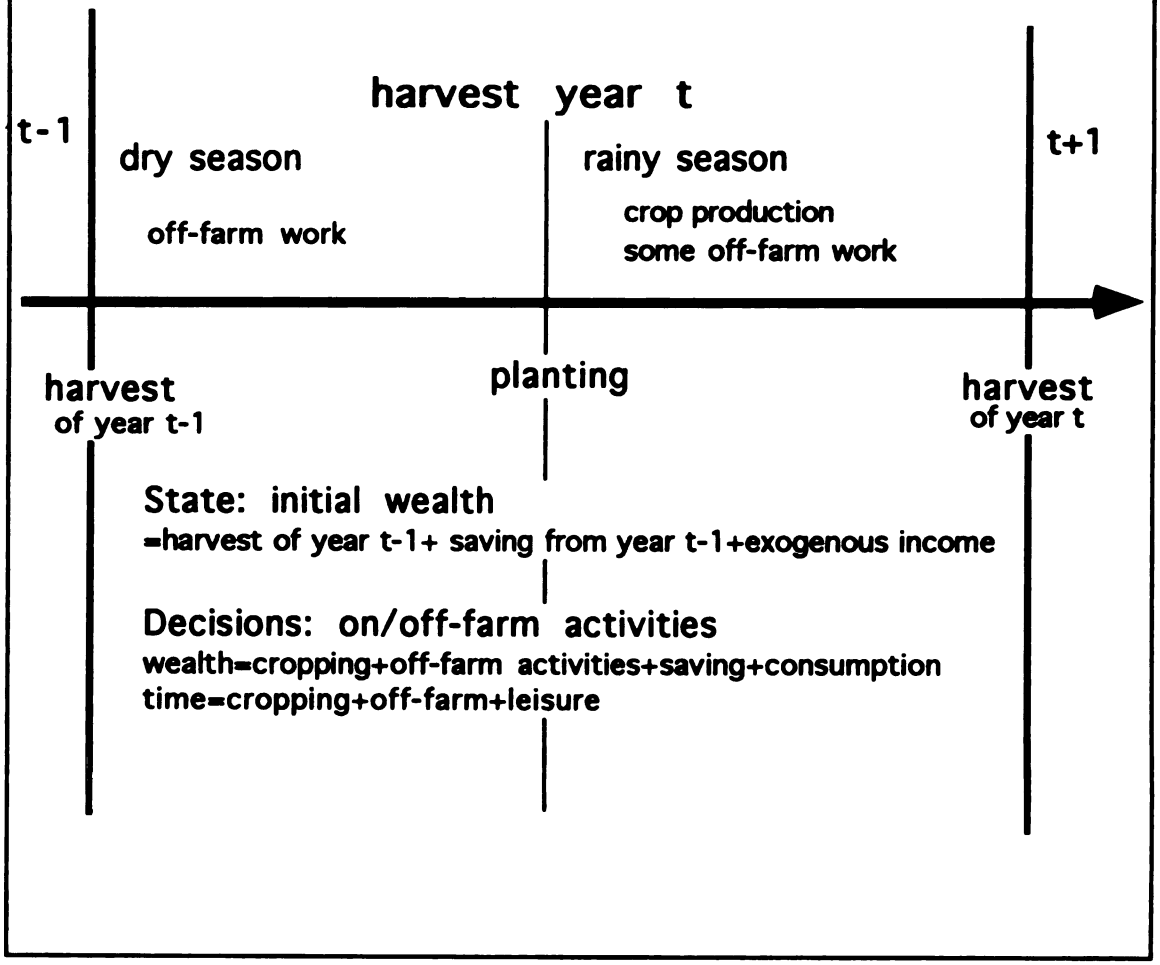

Figure 3 Time Frame of Sequential Decision Making

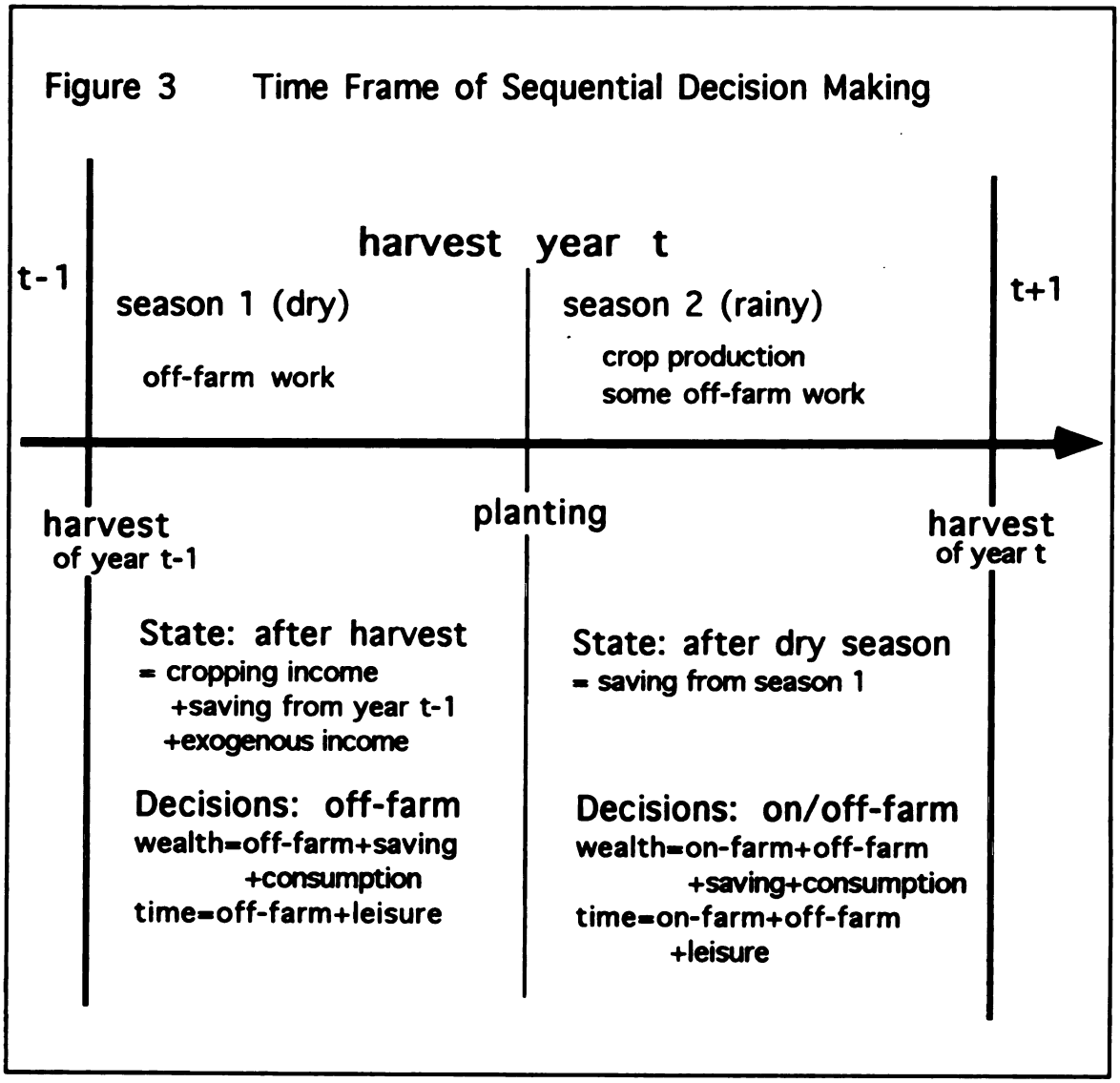

Table 4 Effect of Increasing Risk on Decisions (simultaneous)

Utility Type	$V''' > 0$				$V''' < 0$			
Risk Source	$\tilde{R}_t$ (imperfect insurance)	$\tilde{\varepsilon}_t$	$\tilde{y}_{t+1}$	$\tilde{R}_t$ (perfect insurance)	$\tilde{R}_t$ (imperfect insurance)	$\tilde{\varepsilon}_t$	$\tilde{y}_{t+1}$	$\tilde{R}_t$ (perfect insurance)
On-Farm Inputs (L_t^a, K_t^a, L_t^h)								
Risk-Inc.	ambiguous	↑	↑	↓	ambiguous	ambiguous	↓	↑
Risk-Dec.	↑	↑	↑	↑	ambiguous	ambiguous	↓	↑
Off-Farm Inputs (L_t^o, K_t^o)								
Risk-Inc.	↑	ambiguous	↑	no effect	ambiguous	ambiguous	↓	no effect
Risk-Dec.	↑	↑	↑	no effect	ambiguous	ambiguous	↓	no effect
Savings	↑	↑	↑	no effect	ambiguous	ambiguous	↓	no effect
Borrowing	↓	↓	↓	no effect	ambiguous	ambiguous	↑	no effect
Crop Diversification								
Risk-Inc.	ambiguous	↑	↑	↓	ambiguous	ambiguous	↓	↓
Risk-Dec.	ambiguous	↓	↓	↑	ambiguous	ambiguous	↑	↑

Note: $\tilde{R}_t$, rainfall risk; $\tilde{\varepsilon}_t$, idiosyncratic risk in off-farm activities; $\tilde{y}_{t+1}$, social insurance risk.

Table 5 Effect of Increasing Risk on Decisions (sequential)

Source of Risk	1 $\tilde{R}_{2t}$	2 $\tilde{R}_{2t}$ (perfect insurance)	3 $\tilde{\varepsilon}_{2t}$	4 $\tilde{y}_{t+1}$	5 r_{2t}	6 a_{2t}	7 $\tilde{\varepsilon}_{1t}$
Savings in season 1	↑	no effect	↑	↑	↑	ambiguous	↑
Cap. for Off-Farm							
Risk-Increasing	↑	no effect	↑	↑	↑	ambiguous	ambiguous
Risk-Decreasing	↑	no effect	↑	↑	↑	ambiguous	↑
Labor for Off-farm							
Risk-Increasing	↑	no effect	↑	↑	↑	ambiguous	ambiguous
Risk-Decreasing	↑	no effect	↑	↑	↑	ambiguous	↑

($V''' > 0$ only)

Note : $\frac{\partial w_{1t+1}}{\partial \tilde{\varepsilon}_{1t}} > 0$ is assumed.

Column 1 assumes $\frac{\partial^2 w_{1t+1}}{\partial \tilde{R}_{2t} \partial w_{2t}} < 0$ and $\frac{\partial^3 w_{1t+1}}{\partial \tilde{R}_{2t}^2 \partial w_{2t}} > 0$.

Column 3 assumes $\frac{\partial^2 w_{1t+1}}{\partial \tilde{\varepsilon}_{2t} \partial w_{2t}} < 0$ and $\frac{\partial^3 w_{1t+1}}{\partial \tilde{\varepsilon}_{2t}^2 \partial w_{2t}} > 0$.

Column 5 assumes $\frac{\partial^2 w_{1t+1}}{\partial r_{2t} \partial w_{2t}} < 0$ and $\frac{\partial^3 w_{1t+1}}{\partial r_{2t}^2 \partial w_{2t}} > 0$.

Column 7 assumes $\frac{\partial^2 Q_1}{\partial \tilde{\varepsilon}_{1t} \partial K_{1t}^o} > 0$, $\frac{\partial^3 Q_1}{\partial \tilde{\varepsilon}_{1t}^2 \partial K_{1t}^o} < 0$, $\frac{\partial^2 Q_1}{\partial \tilde{\varepsilon}_{1t} \partial L_{1t}^o} > 0$, and $\frac{\partial^3 Q_1}{\partial \tilde{\varepsilon}_{1t}^2 \partial L_{1t}^o} < 0$ for risk-increasing inputs, and $\frac{\partial^2 Q_1}{\partial \tilde{\varepsilon}_{1t} \partial K_{1t}^o} < 0$, $\frac{\partial^3 Q_1}{\partial \tilde{\varepsilon}_{1t}^2 \partial K_{1t}^o} > 0$, $\frac{\partial^2 Q_1}{\partial \tilde{\varepsilon}_{1t} \partial L_{1t}^o} < 0$, and $\frac{\partial^3 Q_1}{\partial \tilde{\varepsilon}_{1t}^2 \partial L_{1t}^o} > 0$ for risk-decreasing inputs.

Appendix A

Labor Income and Interest Rate Uncertainty / Two-Period Model

This is used in section 2.1 to derive the results in Tables 2 and 3.

From (2),

$$\frac{\partial^3 U}{\partial m \partial \tilde{r}^2} = \delta\left(\frac{\partial \tilde{y}_2}{\partial \tilde{r}} + a\right)^2 v''' + \delta \frac{\partial^2 \tilde{y}_2}{\partial \tilde{r}^2} v'' \tag{A1}$$

$$\frac{\partial^3 U}{\partial m \partial \tilde{y}_2^2} = \delta\left(1 + a\frac{\partial \tilde{r}}{\partial \tilde{y}_2}\right)^2 v''' + \delta a \frac{\partial^2 \tilde{r}}{\partial \tilde{y}_2^2} v'' \tag{A2}$$

From (3),

$$\frac{\partial^3 U}{\partial a \partial \tilde{r}^2} = \delta(1+\tilde{r})\left(\frac{\partial \tilde{y}_2}{\partial \tilde{r}} + a\right)^2 v''' + 2\delta\left(\frac{\partial \tilde{y}_2}{\partial \tilde{r}} + a\right) v'' + \delta(1+\tilde{r}) \frac{\partial^2 \tilde{y}_2}{\partial \tilde{r}^2} v'' \tag{A3}$$

$$\frac{\partial^3 U}{\partial a \partial \tilde{y}_2^2} = \delta(1+\tilde{r})\left(1 + a\frac{\partial \tilde{r}}{\partial \tilde{y}_2}\right)^2 v'' + 2\delta \frac{\partial \tilde{r}}{\partial \tilde{y}_2}\left(1 + a\frac{\partial \tilde{r}}{\partial \tilde{y}_2}\right) v'' + \delta(1+\tilde{r}) a \frac{\partial^2 \tilde{r}}{\partial \tilde{y}_2^2} v'' + \delta \frac{\partial^2 \tilde{r}}{\partial \tilde{y}_2^2} v' \tag{A4}$$

When we assume that income risk and interest rate risk are independent, $\frac{\partial \tilde{y}_2}{\partial \tilde{r}} = 0$ and $\frac{\partial^2 \tilde{y}_2}{\partial \tilde{r}^2} = 0$ are assumed in (A1) and (A3), and $\frac{\partial \tilde{r}}{\partial \tilde{y}_2} = 0$ and $\frac{\partial^2 \tilde{r}}{\partial \tilde{y}_2^2} = 0$ are assumed in (A2) and (A4). On the other hand, when we assume that they are correlated, there are four cases as shown in Table 1 and Figure 1.

Appendix B

A Dynamic Household Model with Uncertain Production

In this appendix, effects of increasing risk in production are examined in the dynamic model in section 2.2. Analysis starts from the first order conditions (9), (10), and (11).

(1) Examination of Effects of Risk

From (9)

$$v'''\left(\frac{\partial Q}{\partial \tilde{\varepsilon}_t}\right)^2 + v''\frac{\partial^2 Q}{\partial \tilde{\varepsilon}_t^2} \tag{B1}$$

From (10)

$$v'''p_{qt+1}\left(\frac{\partial Q}{\partial \tilde{\varepsilon}_t}\right)^2 \frac{\partial Q}{\partial L_t} + v''\frac{\partial^2 Q}{\partial \tilde{\varepsilon}_t^2}\frac{\partial Q}{\partial L_t} + 2v''p_{qt+1}\frac{\partial Q}{\partial \tilde{\varepsilon}_t}\frac{\partial^2 Q}{\partial L_t \partial \tilde{\varepsilon}_t} + v'\frac{\partial^3 Q}{\partial L_t \partial \tilde{\varepsilon}_t^2} \tag{B2}$$

And from (11)

$$v'''p_{qt+1}\left(\frac{\partial Q}{\partial \tilde{\varepsilon}_t}\right)^2 \frac{\partial Q}{\partial A_t} + v''\frac{\partial^2 Q}{\partial \tilde{\varepsilon}_t^2}\frac{\partial Q}{\partial A_t} + 2v''p_{qt+1}\frac{\partial Q}{\partial \tilde{\varepsilon}_t}\frac{\partial^2 Q}{\partial A_t \partial \tilde{\varepsilon}_t} + v'\frac{\partial^3 Q}{\partial A_t \partial \tilde{\varepsilon}_t^2} \tag{B3}$$

To evaluate the sign of (B1)-(B3), we need to make specific assumptions about the production function. As mentioned in the text, the assumptions are $\frac{\partial Q}{\partial \tilde{\varepsilon}_t} > 0$, $\frac{\partial^2 Q}{\partial \tilde{\varepsilon}_t^2} < 0$, $\frac{\partial^2 Q}{\partial \tilde{\varepsilon}_t \partial L_t} > 0$, $\frac{\partial^3 Q}{\partial L_t \partial \tilde{\varepsilon}_t^2} < 0$, $\frac{\partial^2 Q}{\partial A_t \partial \tilde{\varepsilon}_t} < 0$, and $\frac{\partial^3 Q}{\partial A_t \partial \tilde{\varepsilon}_t^2} > 0$.

The sign of (B1) is positive when $v''' \geq 0$, and ambiguous when $v''' < 0$. The sign of (B2) is ambiguous regardless of the sign of v'''. The sign of (B3) is positive when $v''' \geq 0$, and ambiguous when $v''' < 0$.

If both inputs are risk-neutral, (B2) and (B3) will become the same as (B1), and consequently the results will be that the sign of (B2) and (B3) is positive when $v''' \geq 0$, and ambiguous when $v''' < 0$.

(2) Proof of "labor is underused"

From (9) and (10), the following is obtained:

$$p_{qt+1}E_t v' E_t \frac{\partial Q}{\partial L_t} + p_{qt+1}\mathrm{cov}(v', \frac{\partial Q}{\partial L_t}) - (1+r)b_t E_t v' = 0 \qquad \text{(B4)}$$

In (B4), the covariance is negative when labor is risk-increasing input. Therefore, form (B4)

$$p_{qt+1}E_t \frac{\partial Q}{\partial L_t} - (1+r)b_t > 0 \qquad \text{(B5)}$$

is obtained. This means that labor is underused at the optimum relative to the optimum when labor is risk-neutral (that is, the covariance is zero).

(3) Proof of "land is overused"

From (9) and (11), the following is obtained:

$$p_{qt+1}E_t v' E_t \frac{\partial Q}{\partial A_t} + p_{qt+1}\mathrm{cov}(v', \frac{\partial Q}{\partial A_t}) - (1+r)b_t E_t v' = 0 \qquad \text{(B6)}$$

In (B6), the covariance is positive when land is risk-decreasing input. Therefore, from (B6)

$$p_{qt+1}E_t \frac{\partial Q}{\partial A_t} - (1+r)a_t < 0 \qquad \text{(B7)}$$

is obtained, This means that land is overused at the optimum relative to the optimum when land is risk-neutral (that is, the covariance is zero).

Appendix C

A Sequential Decision Model with Flexibility

Proofs for the flexibility model in section 2.3 are below.

(1) Proofs of (i) and (ii)

From equations (13) and (14),

$$Ev'(EF_1 - (1+r)w_1) + \mathrm{cov}(v',\ F_1) = 0 \qquad \text{(C1)}$$

is obtained. In (C1), (i) when the covariance is negative, $EF_1 - (1+r)w_1 > 0$ holds. That is, on-farm labor in the first period is underused; (ii) when the covariance is positive, $EF_1 - (1+r)w_1 < 0$ holds. That is, on-farm labor in the first period is overused. Thus, we need to determine the sign of the covariance, which depends on the signs of $\frac{\partial v'}{\partial \tilde{R}}$ and $\frac{\partial F_1}{\partial \tilde{R}}$.

From $\frac{\partial v'}{\partial \tilde{R}}$ with $F_2 = w_2$, the following is obtained:

$$\frac{\partial v'}{\partial \tilde{R}} = v''\left(\frac{\partial F}{\partial \tilde{R}} + \frac{\partial w_2}{\partial \tilde{R}} L_2^* - F_2 \frac{\partial L_2^*}{\partial \tilde{R}} + w_2 \frac{\partial L_2^*}{\partial \tilde{R}}\right) = v''\left(\frac{\partial F}{\partial \tilde{R}} + \frac{\partial w_2}{\partial \tilde{R}} L_2^*\right) \qquad \text{(C2)}$$

Therefore, the sign of $\frac{\partial v'}{\partial \tilde{R}}$ negative. This means that total income is high when rainfall is high. Thus, (i) when $\frac{\partial F_1}{\partial \tilde{R}} > 0$, the covariance is negative, and consequently on-farm labor in the first period is underused; (ii) when $\frac{\partial F_1}{\partial \tilde{R}} < 0$, the covariance is positive, and consequently on-farm labor in the first period is overused. Moreover, $\frac{\partial F_1}{\partial \tilde{R}} > 0$ implies that on-farm labor in the first period is risk-increasing, while $\frac{\partial F_1}{\partial \tilde{R}} < 0$ implies that on-farm labor in the first period is risk-decreasing.

(2) Proofs of (iii)

Differentiating the first order condition (13) with respect to $\tilde{R}$ twice yields

$$v'''\left(\frac{\partial c_2}{\partial \tilde{R}}\right)^2 + v''\frac{\partial^2 c_2}{\partial \tilde{R}^2} \qquad \text{(C3)}$$

where $\frac{\partial c_2}{\partial \tilde{R}} = \frac{\partial F}{\partial \tilde{R}} + \frac{\partial w_2}{\partial \tilde{R}} - F_2 \frac{\partial L_2^*}{\partial \tilde{R}} + w_2 \frac{\partial L_2^*}{\partial \tilde{R}} = \frac{\partial F}{\partial \tilde{R}} + \frac{\partial w_2}{\partial \tilde{R}} > 0$, and from $\frac{\partial c_2}{\partial \tilde{R}}$

$$\frac{\partial^2 c_2}{\partial \tilde{R}^2} = \frac{\partial^2 F}{\partial \tilde{R}^2} + \frac{\partial^2 w_2}{\partial \tilde{R}^2} L_2^* + \frac{\partial w_2}{\partial \tilde{R}} \frac{\partial L_2^*}{\partial \tilde{R}} \tag{C4}$$

is obtained. In (C4), the sign of $\frac{\partial L_2^*}{\partial \tilde{R}}$ is ambiguous *a priori*. Therefore, the sign of $\frac{\partial^2 c_2}{\partial \tilde{R}^2}$ is ambiguous, and consequently the sign of (C3) is also ambiguous. However, if $\frac{\partial L_2^*}{\partial \tilde{R}}$ is negative, the sign of $\frac{\partial^2 c_2}{\partial \tilde{R}^2}$ becomes negative, and consequently: if $v''' \geq 0$, the sign of (C3) is positive; if $v''' < 0$, the sign of (C3) is ambiguous.

(3) Proofs of (iv)

Differentiating the first order condition (14) with respect to $\tilde{R}$ twice yields

$$-v''(\frac{\partial c_2}{\partial \tilde{R}})^2 F_1 - 2v' \frac{\partial c_2}{\partial \tilde{R}} \frac{\partial F_1}{\partial \tilde{R}} - v' \frac{\partial^2 c_2}{\partial \tilde{R}^2} F_1 - v' \frac{\partial^2 F_1}{\partial \tilde{R}^2} \tag{C5}$$

The sign of (C5) is ambiguous because the sign of $\frac{\partial^2 c_2}{\partial \tilde{R}^2}$ is ambiguous as discussed above. However, if $\frac{\partial L_2^*}{\partial \tilde{R}}$ is negative, the sign of $\frac{\partial^2 c_2}{\partial \tilde{R}^2}$ becomes negative, and consequently: when $v''' > 0$, and $\frac{\partial F_1}{\partial \tilde{R}} < 0$ and $\frac{\partial^2 F_1}{\partial \tilde{R}^2} > 0$ (on-farm labor in the first period is risk-decreasing and its effect is diminishing), the sign of (C3) is negative; but other combinations of the signs give ambiguous results.

Appendix D

Effects of Increasing Risk on Decisions in Sequential Decision Model

From the first order conditions, (53)-(55) effect of increasing risk can be examined by differentiating twice with respect to stochastic variables following the RS method. The derivatives are as follows.

From (53):

$$\tilde{R}_{2t}: \quad V'''\left(\frac{\partial w_{1t+1}}{\partial \tilde{R}_{2t}}\right)^2 \frac{\partial w_{1t+1}}{\partial w_{2t}} + V''\frac{\partial^2 w_{1t+1}}{\partial \tilde{R}_{2t}^2}\frac{\partial w_{1t+1}}{\partial w_{2t}} + 2V''\frac{\partial w_{1t+1}}{\partial \tilde{R}_{2t}}\frac{\partial^2 w_{1t+1}}{\partial \tilde{R}_{2t}\partial w_{2t}} + V'\frac{\partial^3 w_{1t+1}}{\partial \tilde{R}_{2t}^2 \partial w_{2t}} \tag{D1}$$

$$\tilde{\varepsilon}_{2t}: \quad V'''\left(\frac{\partial w_{1t+1}}{\partial \tilde{\varepsilon}_{2t}}\right)^2 \frac{\partial w_{1t+1}}{\partial w_{2t}} + V''\frac{\partial^2 w_{1t+1}}{\partial \tilde{\varepsilon}_{2t}^2}\frac{\partial w_{1t+1}}{\partial w_{2t}} + 2V''\frac{\partial w_{1t+1}}{\partial \tilde{\varepsilon}_{2t}}\frac{\partial^2 w_{1t+1}}{\partial \tilde{\varepsilon}_{2t}\partial w_{2t}} + V'\frac{\partial^3 w_{1t+1}}{\partial \tilde{\varepsilon}_{2t}^2 \partial w_{2t}} \tag{D2}$$

$$\tilde{y}_{1t+1}: \quad V'''\frac{\partial w_{1t+1}}{\partial w_{2t}} \tag{D3}$$

$$r_{2t}: \quad V'''\left(\frac{\partial w_{1t+1}}{\partial r_{2t}}\right)^2 \frac{\partial w_{1t+1}}{\partial w_{2t}} + 2V''\left(\frac{\partial w_{1t+1}}{\partial r_{2t}}\right)\frac{\partial^2 w_{1t+1}}{\partial r_{2t}\partial w_{2t}} \tag{D4}$$

$$a_{2t}: \quad V'''\left(\frac{\partial w_{1t+1}}{\partial a_{2t}}\right)^2 \frac{\partial w_{1t+1}}{\partial w_{2t}} + V''\frac{\partial^2 w_{1t+1}}{\partial a_{2t}^2}\frac{\partial w_{1t+1}}{\partial w_{2t}} + 2V''\frac{\partial w_{1t+1}}{\partial a_{2t}}\frac{\partial^2 w_{1t+1}}{\partial a_{2t}\partial w_{2t}} + V'\frac{\partial^3 w_{1t+1}}{\partial a_{2t}^2 \partial w_{2t}} \tag{D5}$$

$$\tilde{\varepsilon}_{1t}: \quad V'''\left(\frac{\partial w_{1t+1}}{\partial \tilde{\varepsilon}_{1t}}\right)^2 \frac{\partial w_{1t+1}}{\partial w_{2t}} + V''\frac{\partial^2 w_{1t+1}}{\partial \tilde{\varepsilon}_{1t}^2}\frac{\partial w_{1t+1}}{\partial w_{2t}} + 2V''\frac{\partial w_{1t+1}}{\partial \tilde{\varepsilon}_{2t}}\frac{\partial^2 w_{1t+1}}{\partial \tilde{\varepsilon}_{1t}\partial w_{2t}} + V'\frac{\partial^3 w_{1t+1}}{\partial \tilde{\varepsilon}_{1t}^2 \partial w_{2t}} \tag{D6}$$

Similarly from (54):

$$\tilde{R}_{2t}: \quad (D1) * \frac{\partial F}{\partial K_{1t}^o} \tag{D7}$$

$$\tilde{\varepsilon}_{2t}: \quad (D2) * \frac{\partial F}{\partial K_{1t}^o} \tag{D8}$$

$$\tilde{y}_{1t+1}: \quad (D3) * \frac{\partial F}{\partial K_{1t}^o} \tag{D9}$$

$$r_{2t}: \quad (D4) * \frac{\partial F}{\partial K_{1t}^o} \tag{D10}$$

$$a_{2t}: \quad (D5) * \frac{\partial F}{\partial K_{1t}^o} \tag{D11}$$

$$\tilde{\varepsilon}_{1t}: \qquad (D6)^* \frac{\partial F}{\partial K^o_{1t}} + 2(V'' \frac{\partial w_{1t+1}}{\partial \tilde{\varepsilon}_{1t}} \frac{\partial w_{1t+1}}{\partial w_{2t}} + V' \frac{\partial^2 w_{1t+1}}{\partial w_{2t} \partial \tilde{\varepsilon}_{1t}}) + V' \frac{\partial w_{1t+1}}{\partial w_{2t}} \frac{\partial^3 F}{\partial K^o_{1t} \partial \tilde{\varepsilon}^2_{1t}} \quad (D12)$$

Finally from (55):

$$\tilde{R}_{2t}: \qquad (D1)^* \frac{\partial F}{\partial L^o_{1t}} \quad (D13)$$

$$\tilde{\varepsilon}_{2t}: \qquad (D2)^* \frac{\partial F}{\partial L^o_{1t}} \quad (D14)$$

$$\tilde{y}_{1t+1}: \qquad (D3)^* \frac{\partial F}{\partial L^o_{1t}} \quad (D15)$$

$$r_{2t}: \qquad (D4)^* \frac{\partial F}{\partial L^o_{1t}} \quad (D16)$$

$$a_{2t}: \qquad (D5)^* \frac{\partial F}{\partial L^o_{1t}} \quad (D17)$$

$$\tilde{\varepsilon}_{1t}: \qquad (D6)^* \frac{\partial F}{\partial L^o_{1t}} + 2(V'' \frac{\partial w_{1t+1}}{\partial \tilde{\varepsilon}_{1t}} \frac{\partial w_{1t+1}}{\partial w_{2t}} + V' \frac{\partial^2 w_{1t+1}}{\partial w_{2t} \partial \tilde{\varepsilon}_{1t}}) + V' \frac{\partial w_{1t+1}}{\partial w_{2t}} \frac{\partial^3 F}{\partial L^o_{1t} \partial \tilde{\varepsilon}^2_{1t}} \quad (D18)$$

Chapter 4

Equity and Sustainable Management of Common-Pool Resources: The Case of Grazing Land in the West Africa Semi-arid Tropics

0 Summary

In the West Africa Semi-arid Tropics (WASAT), it is common to find sedentary settlement mixed crop and livestock system of production. Livestock is used as a major form of saving because there are no formal saving or credit institutions. Since crop production is uncertain due to frequent drought, households tend to keep livestock to insure income. Livestock is kept on open-access grazing land, and overgrazing occurs due to lack of governing rules. It is predicted that degradation of grazing land due to overgrazing will decrease productivity of livestock, and consequently households will lose both income and their ability to smooth income, which is the basis of households' sustainability. However, few studies have so far tried to analyze grazing land management in terms of households' behavior under uncertainty. In this chapter, the decline of livestock productivity is assumed to occur by a simplified mechanism: given fixed grazing land, the more livestock, the less grass available per animal, and as a result the slower animal growth (lower livestock productivity). Degradation of grazing land will further decrease the amount of grass per an animal, while an introduction of feed-efficient animals will increase livestock productivity.

This chapter addresses four main questions: (1) How do WASAT households behave under uncertainty, especially under uncertainty in agricultural production? (2) Why is there overgrazing? (3) What property regimes would solve overgrazing? (4) What would be the equity consequence of these regimes?

To answer these questions, a portfolio selection model that explains household's behavior under uncertainty is developed. The model is different from previous studies on resource management in two points: (1) Each household faces two investment opportunities: risky crop production and safe livestock holdings. (2) Households in a village are heterogeneous in wealth and as a result heterogeneous in risk preference. Prices are not allowed to change in this analysis. Optimal investments are derived from the model under an open-access regime as a Nash-Cournot equilibrium.

The optimal livestock holding under an open-access regime is compared with the social optimal livestock holding, where economic overgrazing is defined as a case when the open-access optimum is greater than the social optimum. Then, the relationship between wealth distribution in a village and the degree of overgrazing is examined by comparing two villages with the same aggregated wealth, the same number of households, but a different distribution of household wealth.

The following are the key findings from these analyses: (1) A household's optimal livestock holding is high when crop production risk is high and/or when the household's initial wealth is high; (2) Overgrazing is inevitable when the expected return to livestock holdings is greater than the expected return to crop production; (3) Overgrazing does not necessarily occur when the expected return to crop production is greater than the expected return to livestock holdings. Crop production risk must reach a threshold level for overgrazing to occur; (4) The degree of overgrazing increases when household wealth in the village approaches equality keeping the average wealth constant. In other words, the social gain by moving from the household optimum to the social optimum is large when household wealth is distributed equally. In this analysis wealth distribution, not wealth level determines the effect. But this is true when household wealth is above a certain level; (5) Moving from the household optimum to the social optimum is not necessarily a Pareto improvement, that is, very rich and very poor households are likely to be worse off absolutely at the socially optimal equilibrium; (6) Although overgrazing can be severe in a village with relatively equal

wealth distribution, a sustainable common property management rule will be established more easily than in a heterogeneous village, according to conclusion (4) and (5).

Finally, using the model, several institutions for grazing land are examined. In these analyses, households are assumed to be in the household optimum under an open access regime, and welfare effects of institutions are considered. The findings are : (1) Privatization will reduce overgrazing, but wealthier households are more likely to be better off absolutely by privatization; (2) A quota will reduce overgrazing, but its welfare effect is ambiguous although a Pareto improvement may be possible; (3) Transferable permits and user fees will also reduce overgrazing, but their welfare effect for individual households is ambiguous; (4) The feasibility of these institutions depends on transaction costs, monitoring costs, and enforcing costs, which is because the grazing land is a common-pool resource. Together with conclusion (5) in the previous paragraph, if village wealth is more equally distributed, enforcement costs will be lower and consequently institutions, such as quotas, will be more feasible; (5) Increase in livestock productivity (by conservation of grazing land and/or improvement of animals) will be difficult due to free-riding and will not reduce overgrazing,. But increase in crop and off-farm productivity (the expected return to them) and/or decrease in their risk could reduce overgrazing by decreasing demand for livestock.

The last result above predicts that off-farm income will play an important role in determining demand for livestock holdings, considering the difficulty in increasing productivity of crop production. However, the result is true only if a household's wealth does not change by an increase in productivity of off-farm income and/or an decrease in its risk. If a household's wealth increases by them, their effect on livestock holding is ambiguous because increasing off-farm income decreases demand for precautionary demand for livestock holdings on the one hand, but increasing household wealth increases the optimal livestock holding on the other hand. Therefore, risk in on- and off-farm activities needs to be so small that households will invest in them and will reduce livestock holding in spite of increasing total wealth.

1 Introduction

Livestock has an important role in many agricultural systems in rural areas of developing countries. In the West Africa Semi-arid Tropics (WASAT) especially, it is used not only for animal traction but also as a major form of saving because there are no formal saving or credit institutions. Since crop production is uncertain due to frequent drought in the WASAT, households tend to keep livestock to insure income (Reardon *et al.*, 1988). Although livestock is important for income smoothing, not all households can accumulate enough livestock to cope with income shortfall due to drought. Consequently, livestock income makes income distribution more unequal (Reardon and Taylor, 1994).

The WASAT livestock population has increased over the last three decades, mainly because of increasing human population[1]. Due to the sharp increase in stock numbers, in conjunction with prolonged drought, the Sahel grasslands have undergone extremely severe degradation. In northern Burkina Faso, the long-term carrying capacity of livestock of rangelands is estimated at 9 hectares/TLU (tropical livestock unit), or 28 kg live weight per hectare, while the actual stocking rate averages 6 hectares/TLU, that is, 33% overstocking. In the Sahel as a whole, although carrying capacity varies, about 30% overstocking is estimated (Houerou, 1989).

That grazing lands are "open access" has been an economic explanation of overgrazing since the publication of Hardin's "Tragedy of the Commons": livestock is private property, but cattle graze on the commons (Hardin, 1968)[2]. "Open access" or "lack of a communal management scheme for grazing land" was not a problem and even could be

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1 Growth rates in stock numbers between 1950 and 1983 in this region are: cattle (2.52), sheep (2.67), goats (2.24) and camels (1.90). The rate of demographic growth is 1.5 to 2.5% per annum among pastoralists and 3.0 to 3.5% per annum among settled farmers. These human and livestock growth rates are very close; and therefore, the human to livestock ratio has remained unchanged from 1950 to 1983, 0.9 TLU (tropical livestock unit) per person in the overall population and about one TLU per inhabitant in the rural population. The growth rate of cultivated land is also about 2.5% per annum, which is very close to the growth rates of both human and livestock population (Houerou, 1989).

2 I will define "open access" in section 2.

considered optimal when human population density was low in the WASAT[3]. Now, although overgrazing and degradation are common, village rules have not adjusted to the new situation and allow overgrazing by permitting open access to grazing land by villagers. It is predicted that degradation of grazing land due to overgrazing will decrease available grass per animal, and as a result, will lower animal growth rate. Economically, this means that average returns to livestock holdings (or productivity of livestock) will be lower, and consequently households will lose both income and their ability to smooth income. Since households' sustainability[4] depends on their ability to insure consumption by smoothing income, maintaining productivity of livestock, that is, sustainable management of grazing land, is important for households' sustainability in the WASAT.

However, because productivity of livestock is not the only concern of households in the WASAT, performance of grazing land management cannot be judged only by livestock productivity. Even if villagers succeed in establishing new rules to achieve long-run sustainable use of grazing land, if it reduces their ability to smooth income in the short-run, such rules may not be adopted. In the short-run, overgrazing may be even better for them than other welfare loss, such as severe income shortfall in drought years. Therefore, we should not analyze the livestock husbandry independently of farm management of rural households that are risk averse, face several income opportunities, and are probably under borrowing constraints. However, few studies have so far tried to analyze grazing land management in terms of households' behavior under uncertainty.

Thus, this chapter addresses four main questions: (1) How do WASAT households behave under uncertainty, especially under uncertainty in agricultural production? (2) Why is

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[3] Population densities in this region are rather low in absolute terms, but they have sharply increased over the last four decades as the following statistics show: for example, Burkina Faso, 12.7 (1950), 19.8 (1970), 27.3 (1983), 32.8 (1990); Mali, 2.1 (1950), 2.8 (1970), 4.6 (1983), 6.9 (1990); Niger, 1.7 (1950), 3.2 (1970), 4.6 (1983), 6.1 (1990); Senegal, 2.7 (1950), 21.8 (1970), 31.3 (1983), 37.5 (1990). (in inhabitants per km^2, from several sources.)

[4] Sustainability at the household level is defined as the ability of the household to smooth consumption over time (see chapter 1).

there overgrazing? (3) What property regimes would solve overgrazing? (4) What would be the equity consequence of these regimes? To answer these questions, a model that explains household's behavior under uncertainty is developed in section 3. In the model, a household faces two investment opportunities: risky crop production and safe livestock holdings. The optimal investments are derived from the model under an open-access regime. In section 4, using the model, several types of institutions for grazing land supported by other property right regimes than open access, such as privatization, quotas, user fees, and transferable permits are compared in terms of efficiency and equity.

2 Definitions and Approach

2.1 Definitions

When we analyze resource management, it is important to distinguish physical attributes of a resource and human institutions that govern its use. Schmid (1987) uses the terms "situation" and "structure" to distinguish property right regimes from the resource itself. The "situation" is the set of attributes of the resource and the "structure" is a rights regime assigned to it. Following this definition, physical attributes of grazing land in the WASAT are described first, and then rights regimes are considered.

Grazing land in the WASAT is interspersed with other farm land distant from where villagers live. Villagers keep their privately-owned livestock on the grazing land. Exclusion of others from using a particular spot of the grazing land is inherently difficult because fencing and monitoring are costly relative to the gains from exclusion. Moreover, quality of grass is not homogeneous, especially when a drought occurs. Therefore, considering drought risk, the gain from exclusion is very low or negative. Schmid (1987) classifies this kind of situation as a high-exclusion-cost good. On the other hand, since livestock is privately owned, consumption of grass by the livestock of a villager reduces the availability of grass for others. That is, there is rivalry or subtractability in use of the grazing land among villagers. Schmid (1987) classifies this kind of situation as an incompatible-use good. In sum, grazing land in

the WASAT has two attributes: difficult exclusion and high subtractability. Ostrom *et al.* (1994) classify goods that have those two attributes as common-pool resources or CPRs, and call a case where a CPR is involved a CPR situation. In this chapter I use the term "common-pool resource" for grazing land in the WASAT according to this definition.

However, CPR situations are not necessarily tragic: when the quantity demanded of the resource is not sufficiently large, individual strategies do not produce suboptimal outcomes, and only when CPR situations produce suboptimal outcomes, is it called a CPR dilemma (Ostrom *et al.* 1994). This distinction between CPR situations and CPR dilemmas implies that although grazing land in the WASAT is a common-pool resource, it is not necessarily destructively used.

Now, given the physical attributes, we will consider human institutions that govern the use of the grazing land. Berkes and Farvar (1988) distinguishes a common-property resource (the same as a common-pool resource as defined above) from a common-property regime, and state that common-property resources may be held within: (1) open access; (2) communal property; (3) state property; (4) private property. In addition, Stevenson (1991) divides open access into two categories: open access to limited users and open access to unlimited users. Resources open to only members are not common property, but open access (the former case) by his definition because members can extract unlimitedly. On the other hand, common property by his definition limits both users and amount of use by rules. As Bromley (1991) points out, "common property" has been often used confusingly for common-pool resources that are open access. But here, following Bromley, I restrict the term common property to express a regime of common-pool resources that are communally owned and managed, distinguishing it from open-access property, state property, and private property.

To characterize the current regime of grazing land in the WASAT in general, "open access limited to villagers" will be the most appropriate. There is no state-owned or privately-owned grazing land. Grazing lands that are communally owned and managed may possibly

exist, but they are not many and they do not reduce the importance of analyzing the open-access grazing lands in the WASAT. Therefore, in this chapter, grazing land is assumed to be open access only to villagers. This means that every villager can use the grazing land as much as he/she likes, but grazing land is open only to villagers[5].

Thus, the questions can be restated: (1) Are there CPR dilemmas? (2) Are there institutionally feasible alternatives to improve the dilemma outcomes? (3) Are there any other potential solutions?

First, as discussed above, if the quantity demanded of grazing land is not large, there will be no dilemma. This may be obvious, but there has been little analysis on when the demand is low enough to avoid the dilemma. Therefore, this is one of the questions to be answered in section 3. Second, as classified by Berkes and Farvar (1988), there are three other property right regimes for common-pool resources: communal property, state property, and private property. In section 4, the performance of those regimes is predicted using the model presented in section 3. Privatization, quota, transferable permit, and user fee schemes are considered as institutional solutions assuming communal property or state property. These institutions are concerned with how to control the use of grazing land, that is, how to assign and enforce a quota to households. A user fee scheme controls a household's livestock holdings by adjusting the fee rate without assigning a quota. But if communication is allowed among households, they may be able to reach a cooperative solution to implement a quota. On the other hand, a transferable permit scheme is a non-cooperative solution that depends on the market to allocate a quota. Finally, as mentioned above, reduction of demand for grazing land will solve the CPR dilemma, even without new institutions. The model will also tell how such solutions can be obtained.

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[5] There may be conflicts between villages and conflicts between peasants and herders. However, in this paper I focus on the issue of resource management within a village where all households are engaged in both crop production and animal husbandry. They are common in the WASAT.

2.2 Approach

2.2.1 Relationship to Previous Work

This chapter follows previous work on open-access resource management in that I compare the open-access equilibrium with the socially optimal equilibrium and examine effects of institutions on the open-access equilibrium, as reviewed next. However, there are fundamental differences: (1) Households in my model face several income opportunities; (2) Households are heterogeneous in wealth and as a result differs in risk preferences. These two are main contributions of this chapter to the literature on resource management.

Resource management under open access (sometimes called common property, as discussed above) has been well examined since Gordon's static model of fishery management (1954). He compares a socially optimal monopoly solution with an uncontrolled competitive equilibrium in which the monopolist's rent is dissipated, and concludes that the uncontrolled equilibrium means a higher expenditure of effort, higher fish landings, and a lower continuing fish population than in the socially optimal equilibrium. More generally, exploiters in the uncontrolled competitive equilibrium equate the average product of the variable input to the input's real rental rate, while a single exploiter equates the marginal product of the variable input to its real rental rate (Haveman, 1973; Weitzman, 1974).

Since Gordon, dynamics and uncertainty have been incorporated into the model. Examples of dynamic models are Brown (1974), Clark and Munro (1975), and Dasgupta and Heal (1979). In dynamic models, the discount rate is one of the important factors, while in static models the discount rate is assumed to be zero.

However, here a static model under uncertainty is considered. Andersen (1982) introduces price uncertainty in a static model of competitive commercial fishery. His conclusion is that, for risk averse firms, price uncertainty independent of fishing activity reduces the number of fishing units and total fishing effort when compared with certainty, and that the total fishing effort in an open-access fishery under price uncertainty may be smaller than in a socially optimal fishery if the variance of price is high and/or fishery firms

are highly risk averse. That is, over-exploitation under open access does not always hold under uncertainty. Sandler and Sterbenz (1990) assume harvest uncertainty in a static model and demonstrate that a fixed number of risk-averse firms faced with harvest uncertainty owing to resource stock uncertainty reduce their exploitation of commons.

Although cropping/livestock husbandry households and cropping/fishing households are common in rural areas in developing countries, there are few papers that discuss the issue of common-pool resource management for those households. Wilson and Thompson (1993) is one exception. They analyze *ejidos*, communal land holdings groups in Mexican semi-arid highlands, producing both crops and livestock. They argue that communal grazing lands have an advantage over private ownership because communal ownership reduces the individual's risk due to uneven distribution of rainfall within an *ejido*. However, their analysis does not discuss the relationship between the livestock husbandry and the crop production. Unlike in Wilson and Thompson, the relationship between these two activities is central to the present chapter; it is assumed that a household's uncertainty comes from crop production and that livestock holdings which are less risky play the role of a hedge against risk.

In addition, most literature on common-pool resource management including Wilson and Thompson assumes homogeneous agents that all have the same wealth. By contrast, my model assumes that households are heterogeneous in wealth and as a result are heterogeneous in risk preference. This assumption allows me to analyze the effect of wealth distribution on overgrazing and the effect of resource management rules on wealth distribution.

2.2.2 Modeling Strategies

My model is to be fully presented in section 3, but in this sub-section modeling strategies are briefly explained. My model focuses on both common-pool resource management and micro-behavior of agricultural households, and its key elements are (1) major characteristics of WASAT households: uncertainty, risk aversion, and credit constraints, (2) a standard portfolio selection model under uncertainty, e.g., Newbery and Stiglitz (1981),

Robinson and Barry (1987), and Hirshleifer and Riley (1992), (3) Just and Zilberman's treatment of heterogeneity of households in wealth and risk preference (1983, 1988a, 1988b), and (4) Nash-Cournot solution to obtain optimal household behavior under an open-access regime.

The behavior of each household is represented as a portfolio management problem in which initial wealth (human and non-human except land) is allocated between risky crop production and riskless livestock husbandry to maximize household utility. I follow Just and Zilberman's portfolio selection model of micro-behavior of agricultural households and their treatment of heterogeneity of households in wealth and risk preference, where household's risk preference is a function of its initial wealth: e.g., the relationship between farm size and technology adoption (1983), the effects of agricultural policies on income distribution (1988a), and the effects of environmental policies on income distribution (1988b).

In addition, the subtractability of grazing land, an attribute of common-pool resources, is incorporated by a livestock productivity function, which depends on total village livestock holdings. I use a function given by Gibbons. Following Gibbons (1992), I apply a Nash-Cournot solution to my model to obtain optimal household behavior under an open-access regime in which each household behaves optimally assuming that other households make optimal decisions[6].

Overgrazing is defined as follows. First, I assume that the total grazing land available to villagers is constant, livestock is homogeneous, its amount is continuous, and its price is constant. Consequently investment in livestock (expressed in monetary terms) is proportional to the total amount of livestock (expressed in live weight), and since grazing land is constant, the population density of livestock is proportional to the total investment in livestock. Therefore, summing optimal livestock holdings over all households in the village is

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[6] Since a static one-period model is used, Nash-Cournot equilibrium is a one-shot, noncooperative solution that assumes no communication. The inefficiency of the open-access regime will be reduced if the game is infinitely repeated (Friedman, 1990) and/or communication is allowed (Ostrom *et al.*, 1994).

considered to express the stocking level of livestock. Second, the socially optimal livestock holding is obtained as a monopoly solution, or a solution of a so-called social planner. Finally, by comparing the two stocking levels, overgrazing is defined as a case when livestock holdings at the open-access equilibrium are higher than at the socially optimal equilibrium.

The relationship between wealth distribution in a village and the degree of overgrazing is examined by comparing two villages that have the same number of households and the same aggregated wealth (as a result, average wealth per household is the same)[7]. However, wealth is more widely distributed in one village (unequal village) than the other (equal village). Because this comparison is done by spreading wealth while keeping the mean constant, I call it "a mean-preserving spread of wealth" in this chapter.

3 The Model

3.1 Framework

Consider initially a single household i with fixed productive capacity, W_i, which is simply assumed to be the household's initial wealth except land to be invested in crop production and livestock holdings. The productive capacity can take any form, either human or non-human, but it is expressed in monetary terms, that is, the initial wealth is assumed to be perfectly liquefiable. The household can allocate its initial wealth in any proportion between them, that is, $W_i = A_i + L_i$, where A_i is input for crop production and L_i is livestock holdings[8]. A_i and L_i are also in monetary terms, and their prices are assumed to be fixed. Since crop production is risky, its return is given by πA_i, where π is the stochastic net return

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[7] These two conditions are enough when there are so many households that the wealth distribution can be approximated as being continuous, or when wealth is evenly distributed even if it is discrete. When wealth is discrete and is distributed unevenly, one more condition as follows is needed to compare wealth distribution; wealth distribution is spread proportionally to the distance to the mean. In either case, this analysis is concerned with the wealth distribution, not the level of wealth.

[8] A_i is not necessarily limited to crop production, but can be considered to be sum of all investment opportunities except livestock. In this case, A_i includes not only crop production, but off-farm activities and migration, and because it includes risky crop production and because other income source cannot cancel the risk perfectly, A_i can still be considered to be risky investment relative to L_i.

per unit of agricultural investment. π has a normal distribution with mean, m, and variance, v.

In the livestock husbandry, the subtractability of the open-access grazing land is incorporated following Gibbons (1992). The total livestock holding of the village determines the level of subtractability of grass, and consequently determines the village productivity of livestock. On the other hand, household i's total cost to keep livestock depends only on its holdings of livestock. The net return to livestock is given by

$$L_i g(\sum_{i=1}^{n} L_i) - bL_i \qquad (1)$$

Equation (1) means that if household i decides to keep the livestock level at L_i, the value of the livestock will be $L_i g(\sum_{i=1}^{n} L_i) - bL_i$ at the end of this period. This production includes both growth of each animal and net births. The return to livestock husbandry is assumed to be deterministic, for simplicity[9]. b is the cost per unit of livestock investment (variable cost of keeping animals), and $g(\ )$ is the village productivity function. This productivity function implies that the household's return to livestock husbandry depends on the total livestock holdings of the village. Since village grazing land is fixed, increasing total livestock in the village decreases livestock productivity due to congestion, that is, $g' < 0$, and the decrease in productivity is accelerated by increasing total livestock due to degradation, that is, $g'' < 0$. As mentioned above, a fixed livestock price is assumed, so L_i and $L_i g(\sum_{i=1}^{n} L_i) - bL_i$ can also be interpreted as the amount of livestock. Livestock holdings are similar to formal saving with interest. However, this interest rate is not exogenous, but is determined by the aggregation of each household's livestock holdings, that is, this interest rate is endogenous.

Thus, household i's net wealth at the end of the period is given by

$$Y_i = \pi A_i + L_i g(\sum_{i=1}^{n} L_i) - bL_i \qquad (2)$$

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[9] Even if the return to livestock husbandry is stochastic, and has some positive correlation with crop production, the following model will not change much if crop production is more risky than livestock.

Because π is normally distributed, Y_i is normally distributed. The household maximizes expected utility of Y_i. Except for the endogenous interest rate of livestock holding, the maximization problem is the same as a static portfolio problem with a riskless asset. The solution to this problem depends on the form of the utility function (Newbery and Stiglitz, 1981), but since Y_i is assumed to be normally distributed, the utility function can be treated as a function only of the mean and standard deviation of Y_i (Hirshleifer and Riley, 1992). Thus, maximizing the expected utility of Y_i is equivalent to maximizing the certainty-equivalent Y_i given by

$$\begin{aligned} CE(Y_i) &= EY_i - \frac{1}{2}\phi_i \operatorname{var}(Y_i) \\ &= mA_i + L_i g(\sum_{i=1}^{n} L_i) - bL_i - \frac{1}{2}v\phi_i A_i^2 \end{aligned} \tag{3}$$

where m and v are mean and variance of π respectively, and ϕ_i is the coefficient of absolute risk aversion at the expected income. The use of certainty-equivalent income rather than expected utility provides analytic advantages (Robinson and Barry, 1987).

The coefficient of absolute risk aversion, ϕ_i, is given by $-\frac{U''(Y_i)}{U'(Y_i)}$. Therefore, household i's ϕ_i is a function of household i's expected final wealth. However, to obtain an explicit solution, ϕ_i is assumed to be constant for each household in this chapter, that is, each household has constant absolute risk aversion. Such a utility function is given by $U(Y_i) = -ke^{-\phi Y_i}$, where k is a constant. It has the standard property of a utility function: $U' > 0$ and $U'' < 0$. For distributional analysis, the household-specific ϕ_i is assumed to be a function of the household's initial wealth following Just and Zilberman (1988a; 1988b). This means that in a cross-sectional comparison of households in the village, ϕ_i is given by

$$\phi_i = \phi(W_i) \tag{4}$$

where W_i is household i's initial wealth. Initial wealth can be used here because each household has constant absolute risk aversion during the single period of the model. (However, since the household's initial wealth differs in each period, the household's absolute risk aversion is not constant over periods, depending on initial wealth, as expressed in (4).)

In the following analysis, $\frac{\partial \phi}{\partial W} < 0$ is assumed, that is, the wealthier the household, the less risk averse (Newbery and Stiglitz, 1981). In this case though $\frac{\partial \phi}{\partial W} < 0$ is assumed, ϕ must be positive by definition. Consequently, $\lim_{W \to \infty} \phi$ must be zero or some positive constant. Therefore, $\frac{\partial^2 \phi}{\partial W^2} > 0$ is required. In addition, increasing relative risk aversion, $\frac{\partial \phi}{\partial W}\frac{W}{\phi} > -1$, is assumed, following Arrow (1971). This condition is equivalent to saying that ϕW is an increasing function of W although ϕ is a decreasing function of W. However, the sign of $\frac{\partial^2(\phi W)}{\partial W^2} = 2\frac{\partial \phi}{\partial W} + W\frac{\partial^2 \phi}{\partial W^2}$ is still ambiguous because my only assumptions concerning the relationship between ϕ and W are $\frac{\partial \phi}{\partial W} < 0$ and $\frac{\partial^2 \phi}{\partial W^2} > 0$. But now I assume $\frac{\partial^2(\phi W)}{\partial W^2} < 0$. Based on the discussion above, ϕ and ϕW are graphed in Figure 1 and Figure 2 respectively in which the following additional assumptions are reflected: $\lim_{w \to 0} \phi = \infty$, $\lim_{w \to \infty} \phi = 0$, $\lim_{w \to 0} \phi W = 0$, $\lim_{w \to \infty} \phi W = \infty$. An example that meets these assumptions is $\phi = W^{\alpha}$ where $-1 < \alpha < 0$.

3.2 The Household Optimum under Open Access

The household optimum under open access is solved by n-player Nash equilibrium. Assume that each household in the village decides investment in livestock holding and in crop production simultaneously. Then, household i's optimality condition is obtained by the first order condition of (3) with respect to L under the constraint $W_i = A_i + L_i$ assuming that other households behave optimally. The first order condition is

$$\frac{\partial CE(Y_i)}{\partial L} = -m + g(\sum_{i=1}^{n} L_i^*) - b + L^* g'(\sum_{i=1}^{n} L_i^*) + v\phi A^* = 0 \qquad (5)$$

From (5), the optimal household investment in crop production and livestock holdings are obtained as

$$A^* = \frac{-m + g(\sum_{i=1}^{n} L_i^*) - b + g'(\sum_{i=1}^{n} L_i^*)W}{-v\phi + g'(\sum_{i=1}^{n} L_i^*)} \qquad (6)$$

$$L^* = \frac{m - g(\sum_{i=1}^{n} L_i^*) + b - v\phi W}{-v\phi + g'(\sum_{i=1}^{n} L_i^*)} \tag{7}$$

Equations (6) and (7) imply that household i's optimal A_i and L_i depend on initial wealth, W_i, and the coefficient of absolute risk aversion, ϕ_i. They also depend on village level parameters, m, b, and v, and on the village livestock productivity function. Superscript * means a household optimum. A^* or L^* can be negative. A negative optimum means the household specializes in one activity and needs to borrow money. If there are borrowing or liquidity constraints, that household will have a corner solution, which will be discussed in section 3.4.

When households' optima are compared in the village, the relationship between optimal investment and initial wealth over households is revealed to be:

$$\frac{\partial A^*}{\partial W} = \frac{g'(\sum_{i=1}^{n} L_i^*) + v\frac{\partial \phi}{\partial W}A^*}{-v\phi + g'(\sum_{i=1}^{n} L_i^*)} \tag{8}$$

$$\frac{\partial L^*}{\partial W} = \frac{-v\phi - v\frac{\partial \phi}{\partial W}A^*}{-v\phi + g'(\sum_{i=1}^{n} L_i^*)} \tag{9}$$

To derive (8) and (9), it is assumed that $\sum_{i=1}^{n} L_i^*$ is already given. Consequently, $g(\sum_{i=1}^{n} L_i^*)$ and $g'(\sum_{i=1}^{n} L_i^*)$ can be considered to be constant. In the following section, they are sometimes written as $g(*)$ and $g'(*)$ respectively for simplicity.

3.3 Interhousehold Distribution of the Optimal Investment

Using (6)-(9), how the household's optimum changes with its wealth can be known. There are two distinguishable cases: when $m - g(*) + b > 0$ and when $m - g(*) + b \leq 0$. The former condition means that the expected return to crop production (m) is greater than the

expected return to livestock holding ($g(*) - b$). The latter condition means that the expected return to livestock holding is greater than or equal to the expected return to crop production.

When $m - g(*) + b > 0$ (crop returns are higher than livestock returns), the following five propositions are obtained from the model. Their proofs are in Appendix A.

Proposition 1: *All households invest in crop production.*

Proposition 2: *There is a unique wealth level under which households do not invest in livestock and do specialize in crop production.*

Proposition 3: *The smaller the difference in expected returns to crop production and to livestock holdings, the less likely is specialization in crop production.*

Proposition 4: *The riskier is crop production, the less likely is specialization in crop production.*

Proposition 5: *All households invest more in crop production than in livestock holdings when the difference in expected returns to crop production and to livestock holdings is large. But when the difference in expected returns is small, there are some households that invest more in livestock holdings than in crop production even with the initial assumption that crop returns are higher than livestock returns.*

The unique wealth level defined in proposition 2 is denoted by W_c. The propositions above, together with $\lim_{W \to 0} A^* = 0$, $\lim_{W \to \infty} A^* = \infty$, $\frac{\partial A^*}{\partial W} > 0$ for all W, $\lim_{W \to 0} L^* = 0$, $\lim_{W \to \infty} L^* = \infty$, $\frac{\partial L^*}{\partial W} < 0$ for $0 < W < W_c$, and $\frac{\partial L^*}{\partial W} > 0$ for $W_c < W$ where W_c satisfies $\frac{\partial L^*}{\partial W} = 0$, give a graphic presentation of the relationship between optimal investments and households' wealth when the expected return to crop production is greater than that to livestock holdings (shown in Figure 3 and Figure 4). There are two cases: one when A^* always exceeds L^* (Figure 3) and the other when A^* is lower than L^* in a range of initial wealth (Figure 4).

When $m - g(*) + b \leq 0$ (livestock returns are higher than crop returns), propositions similar to the case where $m - g(*) + b > 0$ can be made as follows. Their proofs are in Appendix A. Because proofs of them are very similar to those of propositions 1-5, details are omitted.

Proposition 6: *All households invest in livestock holdings.*

Proposition 7: *There is a unique wealth level under which households do not invest in crop production and do specialize in livestock husbandry.*

Proposition 8: *The smaller the difference in the expected return to crop production and to livestock holdings, the less likely is specialization in livestock husbandry.*

Proposition 9: *The greater the sensitivity of livestock productivity to changes in total village livestock holdings, the less likely is specialization in livestock husbandry*[10].

Proposition 10: *Poorer households invest more in livestock holdings than in crop production, while richer households invest more in crop production than in livestock holdings.*

The unique wealth level given by proposition 7 is denoted by W_d. The propositions above together with $\lim_{W \to 0} A^* = 0$, $\lim_{W \to \infty} A^* = \infty$, $\frac{\partial A^*}{\partial W} < 0$ for $0 < W < W_d$, $\frac{\partial A^*}{\partial W} > 0$ for $W_d < W$, where W_d satisfies $\frac{\partial A^*}{\partial W} = 0$, $\lim_{W \to 0} L^* = 0$, $\lim_{W \to \infty} L^* = \infty$, $\frac{\partial L^*}{\partial W} > 0$ for all W, give a graphic presentation of the relationship between optimal investments and households' wealth when the expected return to livestock holdings is greater than to crop production, which is shown in Figure 5.

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10 The sensitivity is expressed in $g'(*)$, that is, it is determined by village stock level and the nature of grass. Since $g''(*)$ is negative, the sensitivity is high when village stock level is high.

3.4 Effects of Borrowing Constraints

In this section, effects of borrowing constraints on optimal investments are considered based on the relationships obtained in section 3.3 (shown in Figures 3-5). As explained previously, a negative optimal investment theoretically means that the household needs to borrow to optimize its investment and that it specializes in one activity. This borrowing must be paid with interest at the end of the period in this one-period model, which is expressed as a negative return to the negative investment. Because of this cost of borrowing, limitless borrowing is excluded from the model.

On the other hand, if households cannot borrow, the model predicts that relatively poor households are forced to choose suboptimal behavior to specialize in one activity: if the expected return to crop production is greater than to livestock holdings, households whose initial wealth is equal to or less than W_A will choose $A^* = W$ and $L^* = 0$ (Figures 3 and 4), and if the expected return to livestock holdings is greater than or equal to the expected return to crop production, households whose initial wealth is equal to or less than W_L will choose $A^* = 0$ and $L^* = W$ (Figure 5).

Based on the discussion above, whether borrowing constraints exist or not, the model predicts that households in the low wealth state will specialize in one activity. The question is whether there is an important number of such households. This question cannot be answered by the model, but rather requires empirical evidence. According to observation, most Sahel households are engaged in both activities, which means that their initial wealth is high enough not to require borrowing according to the model. Therefore, I assume that zero or negative investment due to low wealth has no effect when I aggregate household investment to the village level, although the model itself does not eliminate the possibility of negative investment.

The static model has to ignore several effects of borrowing constraints that a dynamic model can capture. First, borrowing constraints that are binding occasionally induce precautionary saving as Deaton (1991) shows in a simulation. This may induce investment in

livestock more than without binding constraints. Second, even if the borrowing constraints are not binding, that is, a household's initial wealth is large enough not to borrow as discussed above, if the household believes that there is some probability that its initial wealth may occasionally be small enough to require that it borrow in the future, the household may try to avoid such a binding situation by choosing a safer portfolio (Carroll, 1992; Deaton, 1992). This behavior may also induce more investment in livestock than would be the case without constraints.

3.5 Aggregate Livestock Holdings under Open Access

Village livestock holdings are the sum of the household optimal livestock holdings obtained as equation (7). But aggregation does not make sense if there are households that have negative livestock holdings at their optimum (borrowing for crop production), which is possible as shown in Figures 3 and 4: however, as discussed in section 3.4, when village livestock holdings are considered, I assume that there is no negative number, or that the negative number effect is minimal.

The relationship between wealth distribution in a village and the village livestock holdings under open access is analyzed by applying "a mean-preserving spread of wealth" explained in section 2.2.2. If L^* is a convex function of W, that is, if $\frac{\partial^2 L^*}{\partial W^2} > 0$, a mean-preserving spread of wealth increases the village livestock holdings, and if L^* is a concave function of W, that is, if $\frac{\partial^2 L^*}{\partial W^2} < 0$, a mean-preserving spread of wealth decreases the village livestock holdings. However, based on equation (7) and the assumptions made above, it is impossible to determine the sign of $\frac{\partial^2 L^*}{\partial W^2}$ in general, as seen in Figures 3-5. But by restricting conditions, the following proposition is obtained; its proof and an example are in Appendix A.

Proposition 11: *The more unequally wealth is distributed in the village, the smaller are village livestock holdings under open access, given that household wealth is sufficiently large for all households. How large is large enough for this to hold depends on the all factors that determine optimal livestock holdings and the relationship between coefficient for absolute risk aversion and household initial wealth.*

Proposition 11 intuitively implies that, the richer a household is, the lower the marginal increase in livestock holdings per unit wealth. Therefore, if wealth is transferred from a poor household to a rich household (mean-preserving redistribution of wealth), the sum of livestock of the two households will be lower than before the redistribution. This relationship is based on only wealth distribution of the village, but it does not hold for a very poor household, because the marginal increase in livestock holdings becomes low among the very poor, that is, a very poor household will invest in crop production rather than in livestock holdings.

The relationship between wealth distribution and village livestock holdings is from the non-linear relationship between wealth and the household optimal livestock holdings, shown in Figures 3-5 (see proof of proposition 11 in Appendix A). Thus, the key to the proposition above is the concavity of the optimal livestock holding curve to household wealth.

3.6 The Social Optimum

Overgrazing is defined as a case where the sum of the household optimal livestock holdings is greater than the social optimum at which social welfare is maximized. Thus, the social optimum is introduced in this section to compare with the household optimum discussed in previous sections. The socially optimal allocation is obtained assuming that village total wealth is allocated between crop production and livestock holding to maximize the sum of household certainty-equivalent incomes. This is a problem of a "social planner" who decides the optimal livestock holdings for the society as a whole and also the optimal

crop production for each household. In other words, the social planner decides how to allocate the social optimal livestock holdings to each household.

Village total income is the sum of household incomes

$$\sum_{i=1}^{n} Y_i = \pi \sum_{i=1}^{n} A_i + \sum_{i=1}^{n} L_i g(\sum_{i=1}^{n} L_i) - b \sum_{i=1}^{n} L_i \tag{10}$$

But since crop production is stochastic, the social planner maximizes certainty-equivalent income as is done for the household's optimum. The social certainty-equivalent income is the sum of household certainty-equivalent incomes as follows:

$$CE(\sum_{i=1}^{n} Y_i) = m \sum_{i=1}^{n} A_i + \sum_{i=1}^{n} L_i g(\sum_{i=1}^{n} L_i) - b \sum_{i=1}^{n} L_i - \frac{1}{2} v \sum_{i=1}^{n} \phi_i A_i^2 \tag{11}$$

Maximizing the sum of certainty-equivalent income given by (11) implies that the planner weights each household equally and as a result social planner's risk preference is uniquely determined by this summation.

There are two kinds of budget constraint: for the village and for the individual household. At the village level, the budget constraint is $\sum_{i=1}^{n} W_i = \sum_{i=1}^{n} A_i + \sum_{i=1}^{n} L_i$ and the budget constraint for each household is $W_i = A_i + L_i$. Thus, the social optimum is obtained from the first order conditions with respect to $\sum_{i=1}^{n} L_i$ and A under the budget constraints mentioned above. The first order condition with respect to $\sum_{i=1}^{n} L_i$ is

$$\frac{\partial CE(\sum_{i=1}^{n} Y_i)}{\partial \sum_{i=1}^{n} L_i} = -m + g(\sum_{i=1}^{n} L_i^s) + \sum_{i=1}^{n} L_i^s g'(\sum_{i=1}^{n} L_i^s) - b + v \sum_{i=1}^{n} \phi_i A_i^s \frac{\partial L}{\partial \sum_{i=1}^{n} L_i} = 0 \tag{12}$$

and the first order condition with respect to A is

$$\frac{\partial CE(\sum_{i=1}^{n} Y_i)}{\partial A} = m - g(\sum_{i=1}^{n} L_i^s) - \sum_{i=1}^{n} L_i^s g'(\sum_{i=1}^{n} L_i^s) + b - v\phi A^s = 0 \tag{13}$$

Superscript s means the social optimum. From (13), ϕA^s must be the same for all households because other variables in the equation are common to the village. Thus, because ϕA^s is

constant and because $\sum_{i=1}^{n} \frac{\partial L_i}{\partial \sum_{i=1}^{n} L_i} = 1$, equation (12) is the same as equation (13). The condition for the social optimum given by (13) is different from the condition for the household optimum given by (5) because the social optimum requires ϕA^s to be constant. However, because ϕ_i is household specific, socially optimal A_i^s differs over households.

ϕA^s is not necessarily positive, but can be zero or negative. If it is not positive, all households in the village specialize in livestock at the social optimum. Although this is possible in this model, such a case is ignored for this analysis to make a sense, that is $A_i^s > 0$ is assumed here. This is supported by the following proposition. Its proof is in Appendix A.

Proposition 12: *All households invest in crop production to some extent at the social optimum when the expected return to livestock holdings is small enough even if it is greater than the expected return to crop production.*

When A^s is positive, $\sum_{i=1}^{n} A_i^s$ is also positive. However, the social optimal livestock holdings, $\sum_{i=1}^{n} L_i^s$, can be either positive, zero, or negative. It depends on the size of $\sum_{i=1}^{n} A_i^s$: if $\sum_{i=1}^{n} A_i^s < \sum_{i=1}^{n} W_i$, then $\sum_{i=1}^{n} L_i^s > 0$, if $\sum_{i=1}^{n} A_i^s = \sum_{i=1}^{n} W_i$, then $\sum_{i=1}^{n} L_i^s = 0$, and if $\sum_{i=1}^{n} A_i^s > \sum_{i=1}^{n} W_i$, then $\sum_{i=1}^{n} L_i^s < 0$. Even if $\sum_{i=1}^{n} L_i^s$ is zero or negative, there can be some households that invest in livestock. However, in the following analysis, only the case where $\sum_{i=1}^{n} A_i^s < \sum_{i=1}^{n} W_i$, and $\sum_{i=1}^{n} L_i^s > 0$ is dealt with. This assumption implies that the village as a whole will not be a net borrower.

With respect to the social optimum, the following propositions are obtained. Their proofs are in Appendix A.

Proposition 13: *There is a unique social optimum.*

Proposition 14: *The more unequal is household wealth in the village, the larger the socially optimal livestock holdings.*

Proposition 15: *The socially expected return at the social optimum is lower when household wealth is more unequally distributed in the village. This relationship is strengthened when crop production risk is large.*

Propositions 14 and 15 compare two villages that have the same number of households and the same aggregated wealth (see footnote 7). However, wealth is more widely distributed in one village (unequal village) than the other (equal village). Proposition 14 implies that the socially optimal livestock holdings are greater in the unequal village than in the equal village. However, proposition 15 implies that the expected return at the social optimum is lower in the unequal village than in the equal village.

3.7 Comparison between Household and Social Optima

Overgrazing is defined as a case where $\sum_{i=1}^{n} L_i^* > \sum_{i=1}^{n} L_i^s$, that is, the sum of the household's optimal livestock holdings exceeds the social optimum. $\sum_{i=1}^{n} L_i^*$ is obtained from the first order condition given by equation (5) by summing over households:

$$n(-m + g(*) - b) + \sum_{i=1}^{n} L_i^* g'(*) + v\sum_{i=1}^{n} \phi_i A_i^* = 0 \qquad (14)$$

Equation (14) is to be compared with equation (13), which gives the social optimum. Equation (13) is rewritten using $\phi A^s = K^s$ as

$$-m + g(s) - b + \sum_{i=1}^{n} L_i^s g'(s) + vK^s = 0 \qquad (15)$$

When $n = 1$, (14) and (15) are equal, which means the social optimum is the same as the household optimum. Thus, in the following discussion, $n \geq 2$ is assumed. The following propositions are obtained by comparing (14) and (15). Their proofs are in Appendix A.

Proposition 16: *Overgrazing always occurs when the expected return to livestock holding is greater than the expected return to crop production.*

Proposition 17: *Overgrazing does not necessarily occur when the expected return to livestock holding is smaller than or equal to the expected return to crop production.*

Proposition 18: *When the expected return to crop production is greater than the expected return to livestock holding, overgrazing will occur if crop production risk is high, households are relatively equal in wealth and/or the difference between the two returns are small.*

The final question is whether all households will be better off in the social optimum. In other words, is the change from the household optimum under open access to the social optimum a Pareto improvement? Household i's certainty-equivalent income at the household optimum is obtained from equations (4) and (5) as

$$\begin{aligned} CE(Y_i^*) &= mA_i^* + L_i^* g(\sum_{i=1}^{n} L_i^*) - bL_i^* - \frac{1}{2} v\phi_i (A_i^*)^2 \\ &= A_i^* L_i^* g'(\sum_{i=1}^{n} L_i^*) + \frac{1}{2} v\phi_i (A_i^*)^2 + W_i (g(\sum_{i=1}^{n} L_i^*) - b) \end{aligned} \tag{16}$$

And household i's certainty-equivalent income at the social optimum is obtained from equations (4) and (13) as

$$\begin{aligned} CE(Y_i^s) &= mA_i^s + L_i^s g(\sum_{i=1}^{n} L_i^s) - bL_i^s - \frac{1}{2} v\phi_i (A_i^s)^2 \\ &= A_i^s \sum_{i=1}^{n} L_i^s g'(\sum_{i=1}^{n} L_i^s) + \frac{1}{2} v\phi_i (A_i^s)^2 + W_i (g(\sum_{i=1}^{n} L_i^s) - b) \end{aligned} \tag{17}$$

By comparing $CE(Y_i^*)$ and $CE(Y_i^s)$, it can be seen that $CE(Y_i^s)$ is not necessarily greater than $CE(Y_i^*)$. Therefore, the social optimum is not always a Pareto improvement, although society as a whole maximizes total certainty-equivalent income at the social optimum. However, it is impossible to obtain explicit conditions for $CE(Y_i^s) \geq CE(Y_i^*)$ based on (16) and (17). The following proposition is obtained. The proof is in Appendix A.

Proposition 19: *When there is overgrazing, there will be a certain level of initial wealth below which no household will be better off at the social optimum, and another certain level of initial wealth above which no household will be better off at the social optimum.*

Proposition 19 implies that in a village whose wealth distribution is equal, the social optimum will be a Pareto improvement, while in a village whose wealth distribution is unequal, the social optimum will not be a Pareto improvement (the proof is not based on the wealth distribution, but on the wealth level in this case unlike previous ones). In the case of non-Pareto improvement, the poor who are more risk averse are forced to have a more risky portfolio at the social optimum, while the rich who are less risk averse are forced to have a portfolio with less expected return at the social optimum.

Section 3.7 together with pervious sections concludes: (1) Village livestock holdings under an open-access regime is greater than village livestock holdings at the socially optimal equilibrium when the expected return to livestock holdings is greater than the expected return to crop production; (2) If the expected return to crop production is greater than the expected return to livestock holdings, crop production risk must be high enough for overgrazing; (3) The degree of overgrazing is high when household wealth in the village is relatively equal. But in other words, the social gain at the social optimum is large when household wealth is relatively equal; (4) The social optimum is not necessarily a Pareto improvement, that is, very rich and very poor households are likely worse off at the socially optimal equilibrium if such households exist in the village; (5) Although overgrazing can be severe in a relatively equal village, a common property management rule will be established more easily than in a heterogeneous village, according to conclusion (3) and (4).

4 Application of the Model to Grazing Land Management

In this section, I discuss potential solutions for overgrazing based on the model presented in previous sections. They are classified into three categories: physical solutions,

institutional solutions, and other economic solutions. The other economic solutions may need new institutions, but the distinction between the institutional solutions and the other economic solutions here is that the institutional solutions are involved in a change in property rights. Although solutions for common-pool resource management have been considered in various contexts, to my knowledge there has been no model for grazing land management in the WASAT. Therefore, the following analyses are new in the theoretical literature.

4.1 Physical Solutions

Physical solutions include increase in productivity of livestock, increase in productivity of crop production, and utilization of risk-decreasing technology in crop production.

The increase in livestock productivity may be achieved by conservation investment to increase carrying capacity, and/or by introduction of highly productive grass and/or feed-efficient animals from outside.. From equation (15), the social optimum is given by

$$\sum_{i=1}^{n} L_i^s = \frac{m - g(s) + b - vK^s}{g'(s)} \qquad (18)$$

A conservation investment that lowers the negative effect of livestock population on productivity will give a smaller negative $g'(s)$, and thus will increase the social optimum, $\sum_{i=1}^{n} L_i^s$. The open-access problem is seen in equation (18): the private cost of conservation investment that is included in b will easily offset the gain from conservation, a smaller negative $g'(s)$. Conservation investment is often suboptimal due to lack of land tenure and lack of immediate benefit (Clay and Reardon, 1993). Conservation investment will be more difficult in grazing land because there is far less immediate benefit from the investment and the benefit cannot be privatized if it is open access. In addition, since the grazing land is a common-pool resource, investment in it requires coordination, which is difficult.

On the other hand, if the expected return to livestock (livestock productivity) becomes much greater than that to crop production (crop productivity), the socially optimum livestock

holding will also rise; this is the case when $g(s)-b$ is greater than m and $m-g(s)+b$ is a large negative number. However, if grazing land is open access, any improvements in livestock productivity will also increase investment in livestock. As seen in equation (7), the household optimal livestock holding will increase and, as propositions 16 and 17 state, when the expected return to livestock is higher than that to crop production, overgrazing is inevitable. Therefore, improvement of livestock productivity will not solve the problem under open access.

As shown in propositions 16, 17, and 18, increase in crop productivity and/or decrease in crop production risk will reduce overgrazing. Thus, introduction of highly productive varieties and risk-decreasing technology, such as fertilizer and small-scale irrigation, and control of soil erosion will solve the overgrazing physically. Theoretically it is true, and it is of course another important issue. But it is not easy to achieve (Reardon, forthcoming).

4.2 Institutional Solutions

As discussed above, in a situation of open access, physical solutions have limited power without appropriate institutions. Therefore, a change in property rights, such as privatization, quotas, transferable permits, and user fees should be considered.

4.2.1 Privatization

Privatization is often considered to be a solution for the tragedy of the commons because it internalizes the cost of degrading grazing land. It can be introduced into the model as follows. Assume that each household is allowed to privatize some specific area of village grazing land based on its current share of village livestock holdings, $\frac{L_i^*}{\sum_{i=1}^{n} L_i^*}$. For simplicity, all households are assumed to have some livestock ($L_i^* > 0$) and quality of grass is assumed to be uniformly distributed[11] so that productivity of a piece of the total grazing land depends only

[11] This assumption avoids the problem of heterogeneous grazing land described in section 2.1.

on livestock density. Also assume all animals have the same feed-conversion efficiency. Since the total grazing land of the village is assumed to be fixed, it can be assumed to be unity in this analysis. Consequently, each household will have $\frac{L_i^*}{\sum_{i=1}^{n} L_i^*}$ of grazing land and if household i's optimal livestock holding under privatization is L_i^p, the livestock density for household i will be $\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*$ and the livestock productivity function for household i will be $g(\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*)$.

Under these assumptions, it can be shown that $L_i^p < L_i^*$ must hold. Its proof is in Appendix B. This means that privatization of grazing land decreases optimal livestock holdings for each household, and as a result total village livestock holdings, $\sum_{i=1}^{n} L_i^p$, also becomes smaller.

At issue is whether privatization is a Pareto improvement, that is, whether $CE(Y_i^p) \geq CE(Y_i^*)$ for all i. As shown in Appendix B, $CE(Y_i^p) \geq CE(Y_i^*)$ does not always hold but depends on the relative magnitudes of L_i^* and L_i^p, household's initial wealth, and the features of the livestock productivity function. The proof suggests that wealthier households that have larger optimal livestock holdings under open access are likely to be better off absolutely by privatization.

However, the above comparison of certainty-equivalent incomes ignores the private costs associated with privatization: allocation of village grazing land to each household will cause high transaction costs, especially if quality is not uniform (unlike my assumption); Even if it is somehow allocated, there would be high exclusion costs to stop animals invading privatized grazing land. Such additional costs can be seen included in b. If b under private property is sufficiently greater than b under open access, no household will be better off by the privatization (in the previous discussion b is assumed to be the same in both regimes, that is, no exclusion cost is assumed). Therefore privatization of open-access grazing land will be difficult.

4.2.2 Quota

A quota scheme may be suitable for grazing land because it restricts the total number of livestock in the village on which livestock productivity of the village depends, but it avoids the problem of physical allocation of grazing land in the case of privatization. However, allocation of the quota is still a problem. In a quota scheme, villagers (or policy makers) have to choose a desired level of village livestock holdings[12], and decide an allocation rule in which the desired level is achieved: a quota would be used to allocate livestock holdings to each household, but it is not necessarily the least cost way.

The total amount of livestock that the village must give up, D, to achieve a desired level of livestock, L^D, is given by $\sum_{i=1}^{n} L_i^* - L^D = D$. D is allocated among households by $\sum_{i=1}^{n} d_i = D$. Different allocation rules translate into different social costs associated with the quota. If the quota is proportional to current livestock holdings under open access, the allocation rule will be $d_i = \frac{DL_i^*}{\sum_{i=1}^{n} L_i^*}$. This quota scheme is not a Pareto improvement, that is, some households will be worse off under this quota than under open access (the proof in Appendix C). It depends on household wealth level, and the relationships between household wealth and the optimal livestock holding under open access and between household wealth and household preference. Although the model predicts an ambiguous effect on welfare, it also shows that it may be possible to design a quota that will achieve a Pareto improvement, and that it will be easier if household wealth and as a result preference is less heterogeneous.

In the model above, no monitoring and enforcement costs are taken into account. If the costs are high, they will offset the benefit and will make the scheme infeasible; it will happen especially if some households are worse off as shown above. Even if every household is better off under the quota, each household will have incentive to violate the quota, and

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[12] The socially desired level is not necessarily the social optimum because the social optimum is not known to villagers or policy makers; it is something they decide based on available information.

therefore monitoring and enforcement costs will still be required. The costs partly depend on the characteristics of the village: if it tends to be cooperative, the costs will be low. As shown in proposition 19, a village whose wealth distribution is relatively equal will have more incentive to cooperate because the gain from cooperation is greater.

4.2.3 Transferable Permits

The transferable permit is theoretically a cost effective way to achieve a desired level of resource use (Baumol and Oates, 1988). Transferable permits are considered to be the way to allocate a quota in the market instead of merely assigning it administratively. In the previous section, an assigned quota is not assumed to be transferable, but here the quota can be sold and bought. Consequently, this system is flexible to change in an individual household's economic condition. Moreover, the market price of the permits could increase if demand increases. Therefore, the permits system can meet human population growth, which is considered to be one of main causes of increasing total livestock holdings.

However, an assignment problem still remains in this transferable permit system. Some government assignment of rights is antecedent to market trade so the market is not an alternative to government in this sense. There are two ways to allocate permits. One is to assume households share equal property rights, and the other is to assume the village possesses the property rights. In the former case (equal property rights), each household is given an equal number of permits initially, and they can then trade the permits. Thus, households that want to keep more livestock than initially permitted must pay households that want to keep less livestock than initially permitted. As a result, there is income transfer from the rich to the poor because the poor have fewer animals to graze as shown in Figures 3-5. Because the poor benefit from this scheme, they will have an incentive to monitor the rich if monitoring cost does not exceed the benefits from the permits sold. This may not be realistic, for example, they may not have the capacity to monitor, but it is possible at least theoretically.

Even so, the benefit will not necessarily exceed the enforcement and monitoring costs, and therefore its feasibility is not promised.

The latter case (village ownership) requires a village agency that sells permits, and any household that wants to keep livestock must pay according to the size of its livestock holdings. The permits can be traded between households to meet changing demand. Thus the income transfer effect still exists, but is smaller than in the former case (equal property rights), and similarly flexible to achieve least-cost allocation. The money collected by the agency will be spent for monitoring and some investment in the commons. Again, if monitoring costs exceeds the permits sales, this scheme is not feasible. This works like a user fee, but the difference is that the permit scheme keeps the total number of livestock constant or less, while a user fee cannot control the total number without adjusting the fee rate.

The scheme of transferable permits where households start with an equal allotment is as follows. First, every household is entitled to an equal number of permits, which is given by a socially desired level (see footnote 9) divided by the number of households, that is, $\frac{L^D}{n}$. Then, they trade them to achieve maximization of expected utility at the unit price p. It is assumed that the desired level of livestock holdings, L^D, is smaller than the level under open access, $\sum_{i=1}^{n} L_i^*$, and there is no excess supply of permits. Thus, the optimal livestock holdings for each household under the transferable permit scheme, L_i^m, should sum to the desired level, that is, $\sum_{i=1}^{n} L_i^m = L^D$ and price of the permits, p, which is also endogenously determined in the model, should be positive.

Comparing L_i^m with the household's optimum under open access, L_i^*, given by (7), it is not possible to tell which is greater in general. Thus, although the transferable permit scheme enforces a reduction in total village livestock holdings, we cannot predict who will increase and who will decrease livestock holding in this model (see Appendix D). Nor can we tell who will be better off and who will be worse off. But obviously there will be some worse

off. Although there is an income transfer to the poor, the welfare effect is not clear because they have to increase risky crop production.

4.2.4 User Fee

A user fee scheme is as follows. Let t denote the user fee rate proportional to household i's livestock holdings. t is given exogenously and is assumed to be positive. Then household i has to pay tL_i. Under this scheme, all households will reduce optimal livestock holdings (see Appendix E). However, we cannot tell who becomes better off and who becomes worse off. Again, a user fee requires monitoring and enforcement, and the collected fee must be spent on these costs. However, who owns the grazing land, state or village, will affect the costs: Communal ownership will be less costly than state ownership in monitoring and enforcement, although it depends on characteristics of the community. But even if this scheme works, the amount of livestock reduction cannot be controlled directly unlike in the case of transferable permits, but is controlled indirectly by adjusting the fee rate.

4.3 Other Economic Solutions

Because livestock is used as an asset to smooth income in the WASAT, if cheaper (or at least as cheap as livestock) alternatives are available, they will be able to reduce the number of livestock, and as a result, the CPR dilemma will be reduced. As implied by propositions 17 and 18, off-farm activities, included in crop production in this model as note in footnote 6, is one possible alternative. Another potential alternative is formal drought insurance.

4.3.1 Off-farm Income

Off-farm income is known to be a significant part of total income of households the WASAT (Reardon *et al.*, 1994),. The effect of off-farm income may be to increase the expected return to crop production (higher m) and/or to reduce crop production risk (lower v). Therefore, off-farm income will reduce the degree of overgrazing, according to

proposition 18. However, this is true only if a household's wealth does not change by an increase in productivity of off-farm income and an decrease in its risk. If a household's wealth increases by them, their effect on livestock holding is ambiguous because household wealth increases the optimal livestock holding on the other hand. Because of these two opposite effects, off-farm income will have ambiguous effects on livestock holdings in the long-run. Christensen (1989) empirically finds that the effect of off-farm income on livestock holdings is ambiguous. For off-farm income to reduce livestock holdings, risk in crop production and off-farm activities should be so small that households invest in those activities instead of livestock husbandry. The issue is riskiness, cost, and availability of other self-insurance strategies relative to livestock holdings. However, the problem in the WASAT is that there are few other strategies, either formal or informal.

4.3.2 Formal Drought Insurance

If formal financial institutions, such as drought insurance, consumption credit and savings were available in the WASAT, households will reduce number of livestock. In the case of formal drought insurance, it will reduce livestock holding if (i) it is less costly than households' self-insurance, and it is demanded by those who keep large livestock holdings; (ii) it substitutes for their livestock holdings, which is one of their self-insurance strategies. Therefore, these must be empirically examined to see if formal drought insurance will reduce the pressure on grazing land. Probably, only the rich may be able to utilize such financial institutions. But even if only the rich benefit from the financial institutions, they will work: because in general the rich keep more livestock, if they reduce the amount, the poor may be able to enjoy higher productivity of livestock and be better off.

5 Conclusions

Because I assume livestock is a riskless asset, household optimal livestock investment under an open-access regime increases as households become more risk averse or crop

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production becomes more risky, as expected. In addition, I assume households have increasing relative risk aversion, thus wealthier households tend to keep larger livestock holdings. But regardless of the household's risk attitude, if $-m + g(*) - b > 0$ holds, there is overgrazing. This is the condition when the expected return to livestock is greater than the expected return to crop production. This condition is not affected by crop production risk, but the degree of overgrazing depends on risk. An important implication is that increasing crop and/or off-farm productivity (higher returns to them, or higher m) may reduce overgrazing, even if grazing land is open access, and that decreasing their risks (smaller v) may also reduce overgrazing.

When the expected return to livestock holdings, however, is smaller than the expected return to crop production, overgrazing does not necessarily occur even if grazing land is open access. Under this condition, if crop production risk is high, overgrazing is likely. Wealth distribution in the village is one of factors that determine the degree of overgrazing. So, if wealth is more equally distributed, overgrazing is more likely under open access. Wealth distribution also determines distribution of cost and benefit of management scheme for grazing land. In a village where there are very rich and very poor households, they are likely to be worse off at the social optimum than at the household optimum. This suggests that communal rule will be difficult to enforce if village wealth is unequal, although overgrazing may be less severe in an unequal village.

A quota scheme will be an effective approach if it works with small costs relative to the benefit. The costs depend on a community's potential to cooperate. If it is not feasible, other institutions that create more incentive to monitor neighbors or to generate revenue to cover monitoring/enforcement cost will be required. Transferable permit and user fee schemes are examined in this chapter. Both reduce total livestock holdings, but have ambiguous effects on household investment and on household welfare if households are heterogeneous. If the only alternative to livestock holdings is risky crop production, both schemes will impose a high cost to households that reduce livestock holdings. Therefore other

economic solutions mentioned in section 4.3 are important to implement those institutions. I think that a mix of solutions will be a better way.

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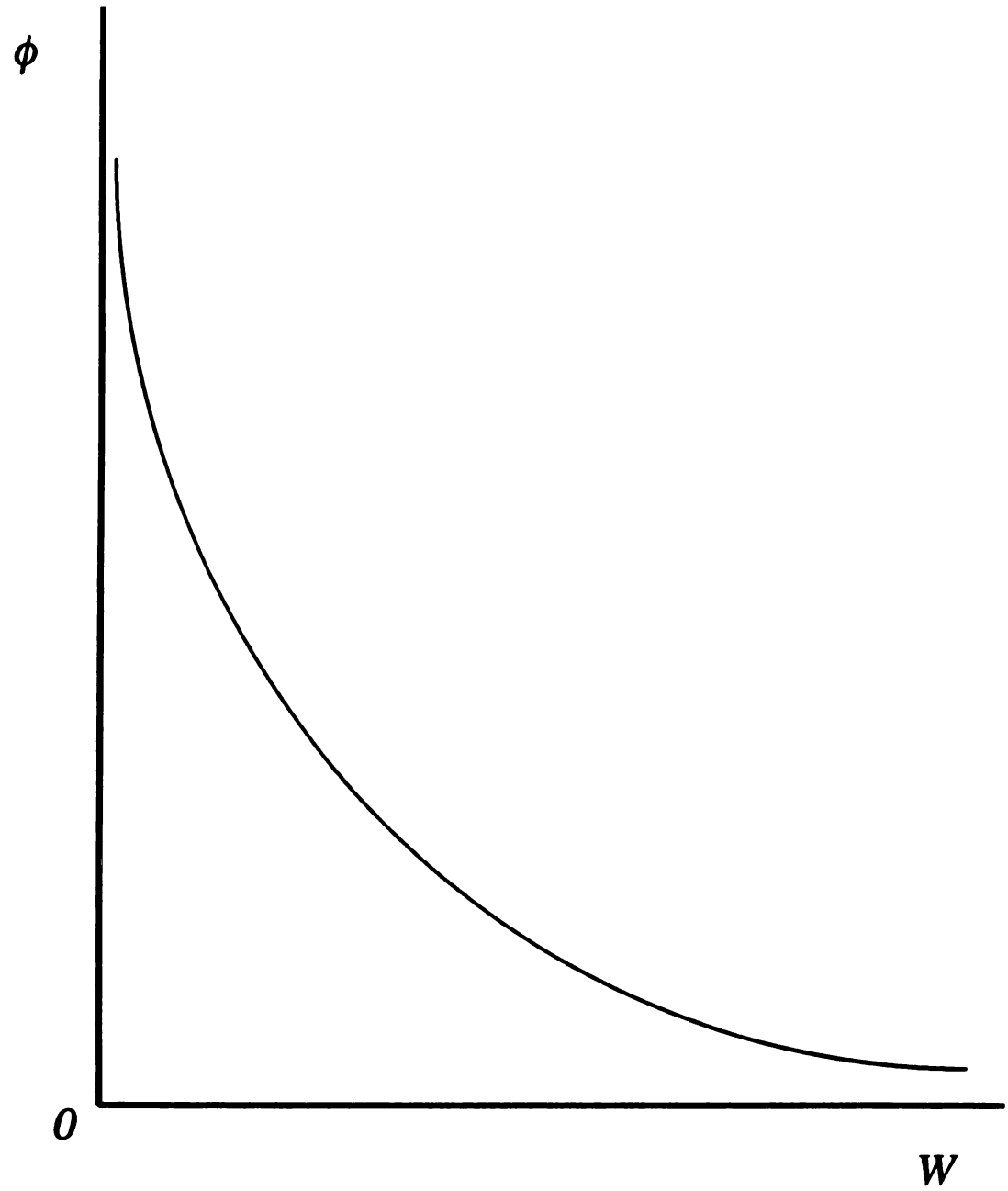
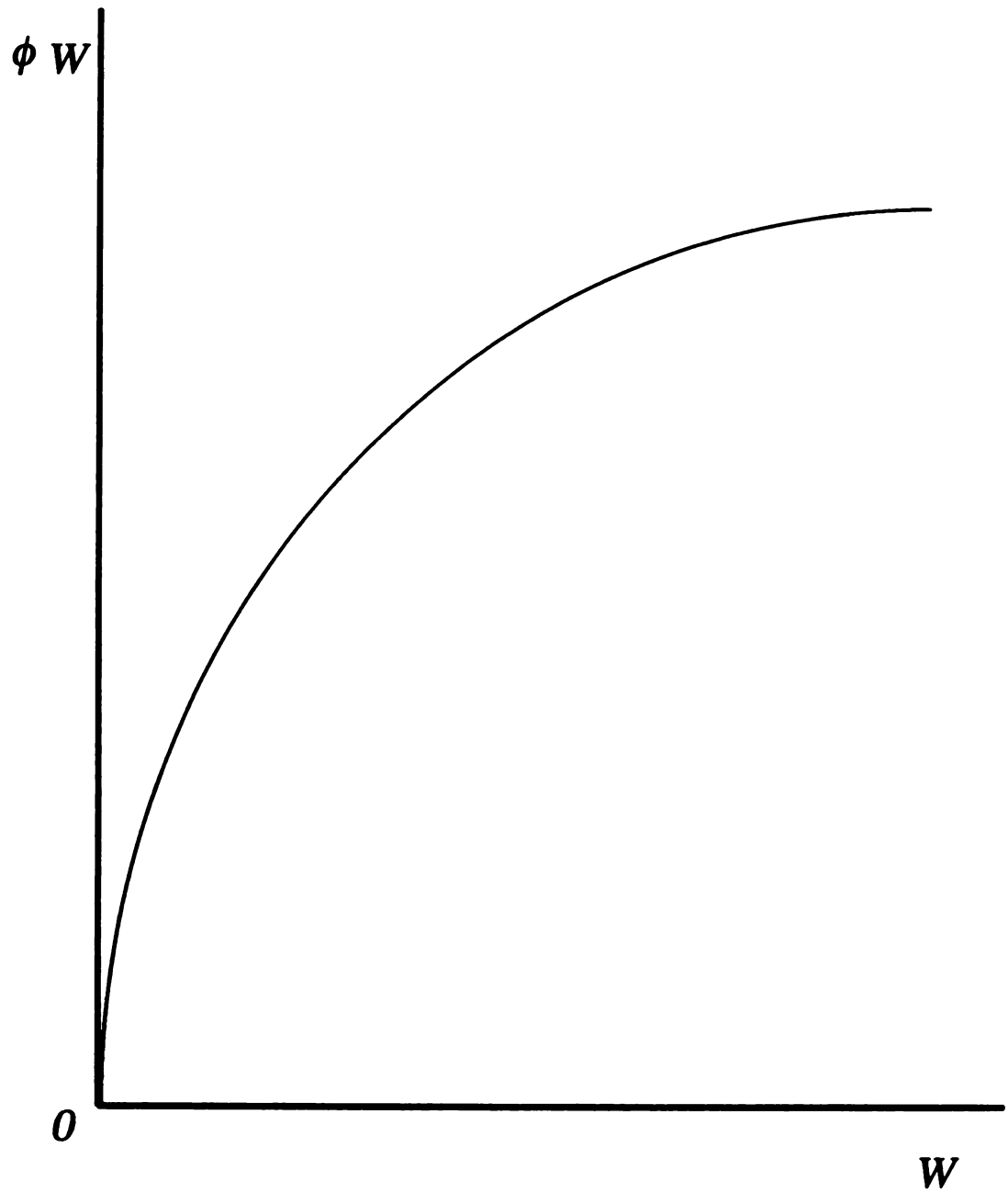
Figure 1 Household's Risk Aversion Coefficient (ϕ) and Wealth



Assumptions: $\frac{\partial \phi}{\partial W} < 0$, $\frac{\partial^2 \phi}{\partial W^2} > 0$, $\lim_{w \to 0} \phi = \infty$, and $\lim_{w \to \infty} \phi = 0$

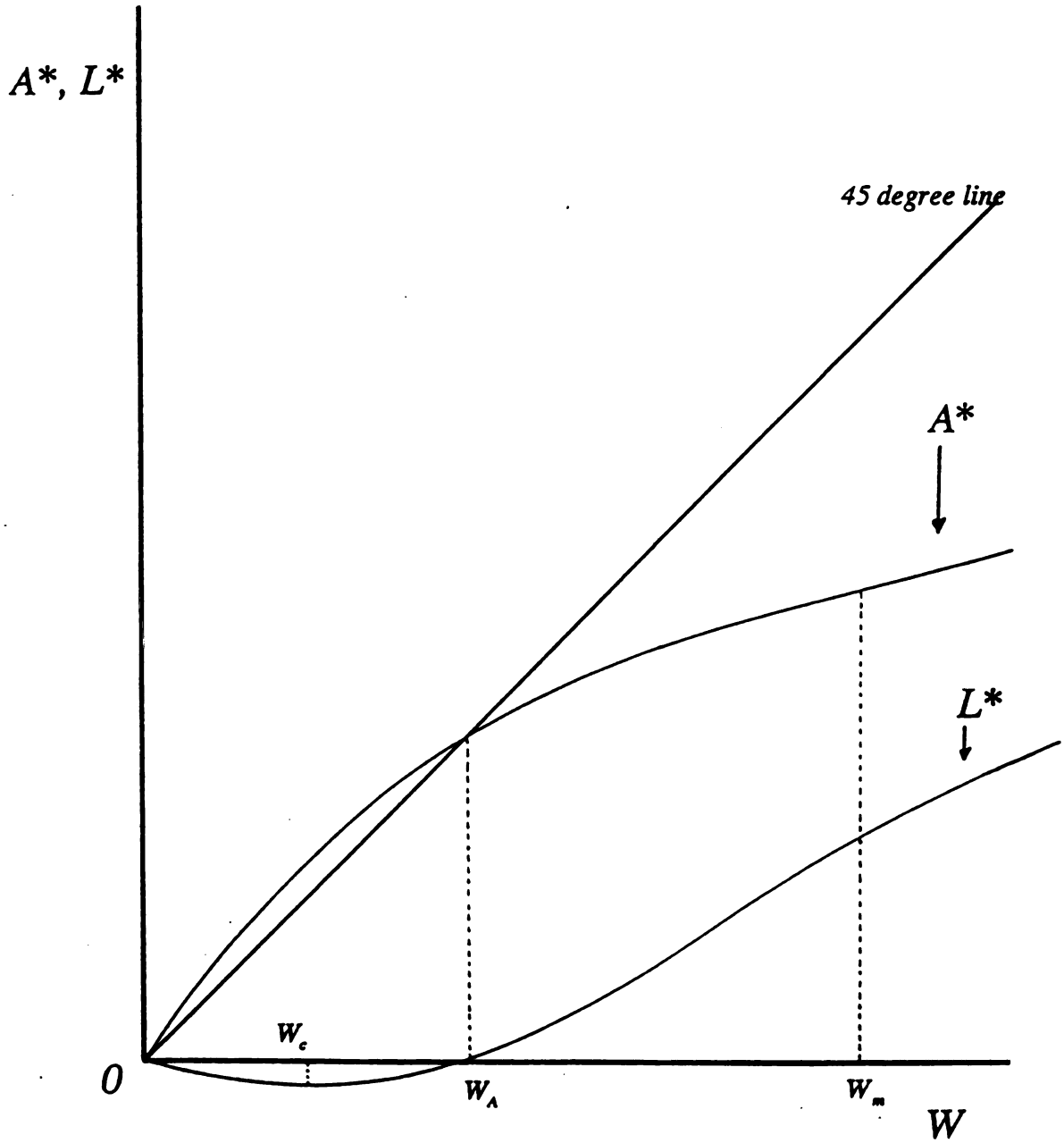
Figure 2

Household's Relative Risk Aversion Coefficient (ϕW) and Wealth



Assumptions: $\frac{\partial(\phi W)}{\partial W} > 0$, $\frac{\partial^2(\phi W)}{\partial W^2} < 0$, $\lim_{w \to 0}(\phi W) = 0$, and $\lim_{w \to \infty}(\phi W) = \infty$

Figure 3 **Optimal Investments (Case 1, $m - g(*) + b > 0$)**

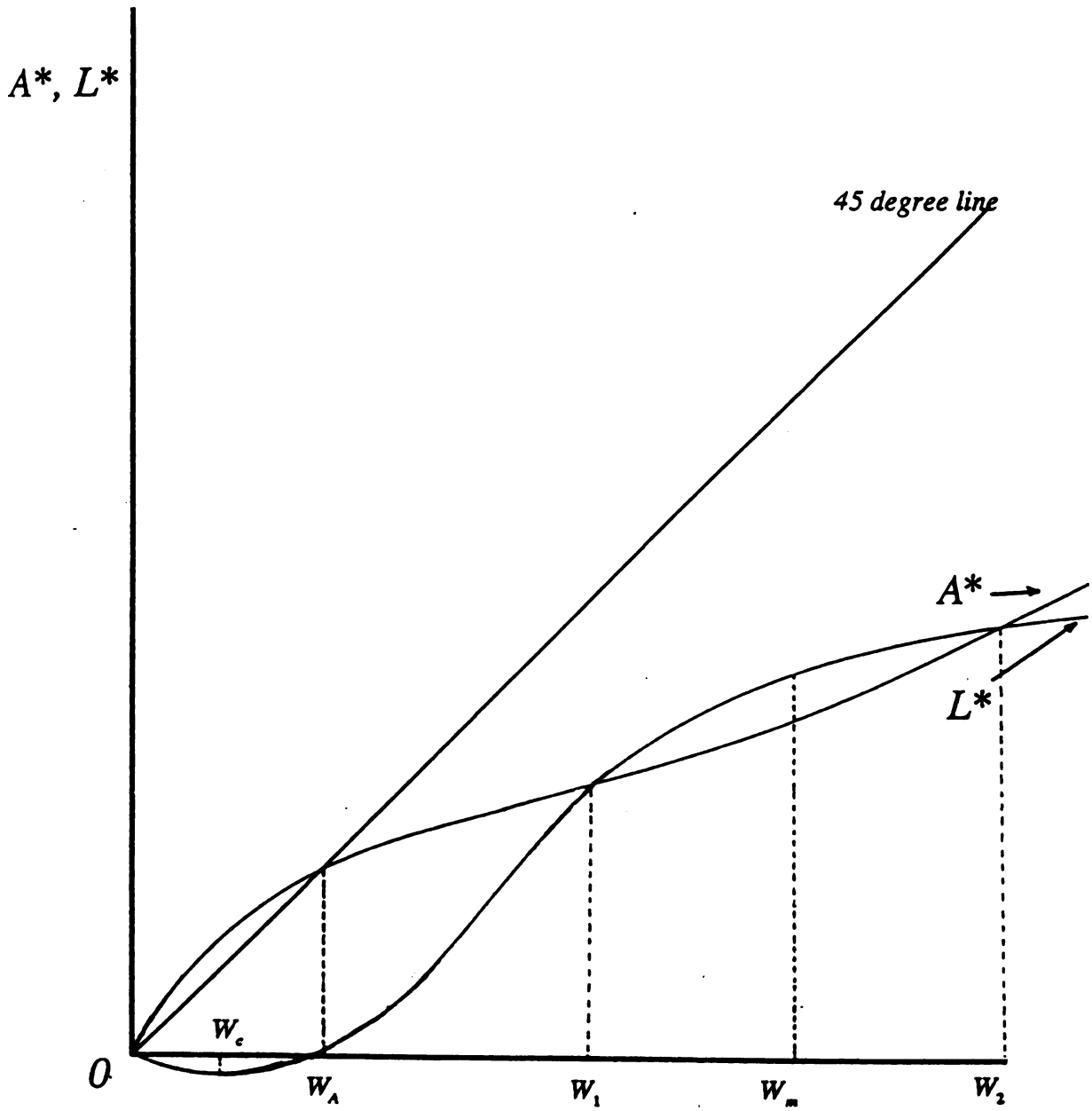


Expected return to crop production is greater than that to livestock.

At W_A, $L^* = 0$, at W_C, $\frac{\partial L^*}{\partial W} = 0$, and at W_m, $A^* - L^*$ is minimum.

Always $A^* > L^*$.

Figure 4 Optimal Investments (Case 2, $m - g(*) + b > 0$)



Expected return to crop production is greater than that to livestock.

At W_A, $L^* = 0$, at W_C, $\frac{\partial L^*}{\partial W} = 0$, and at W_m, $A^* - L^*$ is minimum.

$A^* \geq L^*$ when $W \leq W_1$ or $W_2 \leq W$, and $A^* < L^*$ when $W_1 < W < W_2$.

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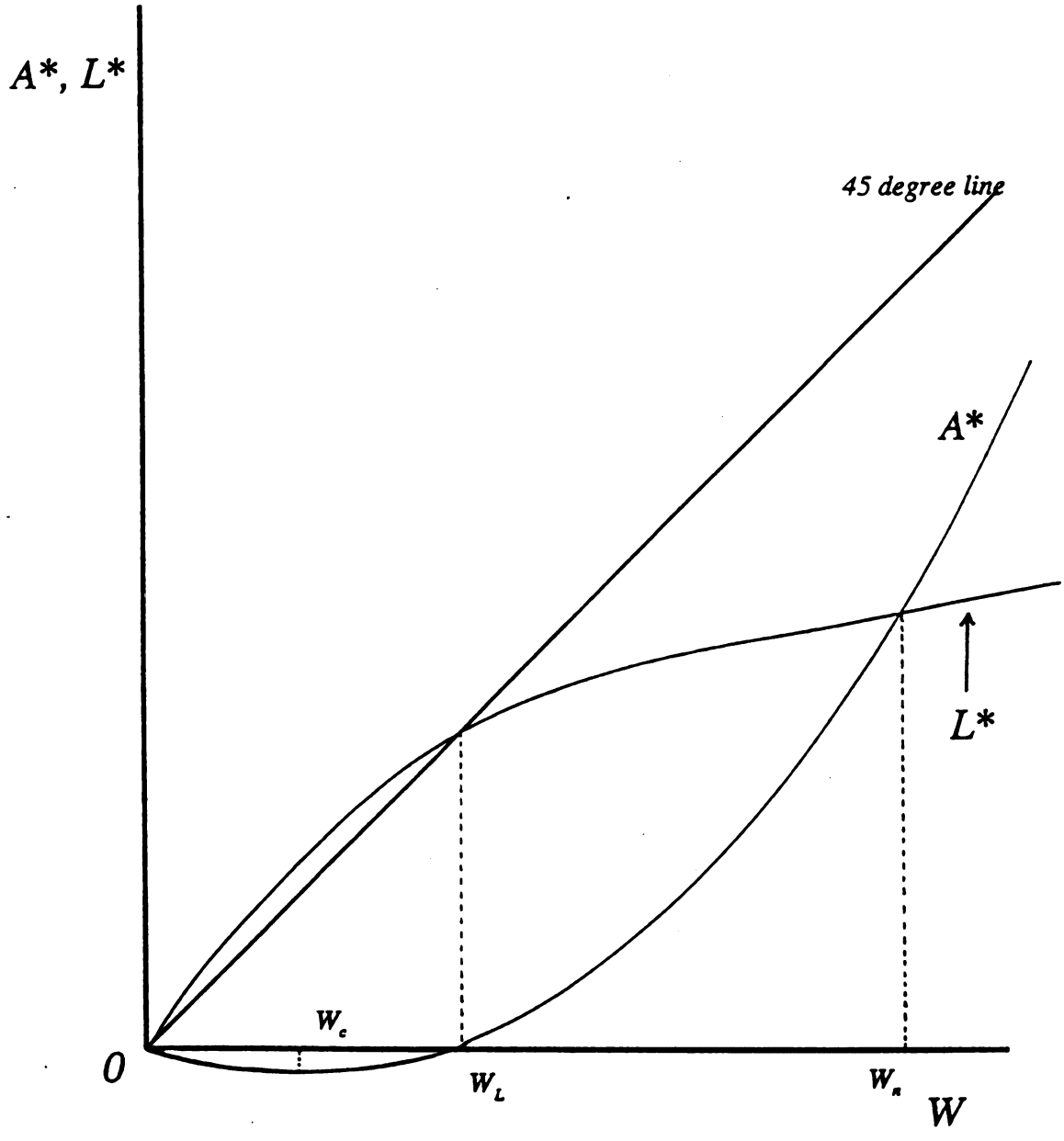
Figure 5 **Optimal Investments (Case 3, $m - g(*) + b \leq 0$)**



Expected return to crop production is smaller than that to livestock.

At W_L, $A^* = 0$, at W_C, $\frac{\partial A^*}{\partial W} = 0$,.

$A^* < L^*$ when $W < W_n$ and $A^* \geq L^*$ when $W_n \leq W$.

Appendix A

Proof of Proposition 1

In equation (6), the numerator is always negative because $m - g(*) + b > 0$ and $g'(*) < 0$, and the denominator is always negative because $v\phi > 0$ and $g'(*) < 0$. Therefore, A^* is always positive. Q.E.D.

Proof of Proposition 2

In equation (7), the denominator is always negative because $v\phi > 0$ and $g'(*) < 0$, but the numerator is negative only when $\frac{m - g(*) + b}{v} < \phi W$. The left-hand side of this inequality is a positive constant, and the right-hand side is an increasing function of wealth, W, as assumed. Therefore, there must be a unique W that satisfies $\frac{m - g(*) + b}{v} = \phi W$, which is denoted as W_A. Thus, at and below W_A, L^* is non-positive, which means that the poorer households do not invest in livestock and specialize in crop production. Q.E.D.

Proof of Proposition 3

As shown previously, the unique W_A at and below which households do not invest in livestock is determined by $\frac{m - g(*) + b}{v} = \phi W$. Therefore, the smaller is the difference in expected returns, that is, $m - g(*) + b$ is smaller, the smaller is W_A. It means that most households invest in livestock in addition to crop production when those returns are close. Q.E.D.

Proof of Proposition 4

Similarly to the proof of proposition 3, if crop production becomes riskier, that is, if v becomes higher, W_A will be smaller. It means that only very poor households specialize in crop production when crop production is very risky (because of proposition 2). Q.E.D.

Proof of Proposition 5

Difference between the optimal investment in crop production and the optimal investment in livestock holdings is given by

$$A^* - L^* = \frac{-2[m - g(*) + b] + [g'(*) + v\phi]W}{-v\phi + g'(*)} \qquad \text{(A1)}$$

In equation (A1), because the denominator is always negative, $A^* - L^*$ is positive when the numerator is negative, that is when $[g'(*) + v\phi]W < 2[m - g(*) + b]$. Since $2[m - g(*) + b]$ is a positive constant, this condition implies that when $[g'(*) + v\phi]W$ is smaller than that constant, all households regardless of initial wealth invest more in crop production than in livestock. Let $f(W) = [g'(*) + v\phi]W$, then $\lim_{W \to 0} f(W) = 0$ and $\lim_{W \to \infty} f(W) = -\infty$. Its first derivative is $f'(W) = g'(*) + v\phi + vW\frac{\partial \phi}{\partial W}$ whose limits are: $\lim_{W \to 0} f'(W) = \infty$ and $\lim_{W \to \infty} f'(W) = g'(*) < 0$. In addition, there is a unique W satisfying $f'(W) = 0$, which is easy to see if the condition is rewritten as $\phi + W\frac{\partial \phi}{\partial W} = -\frac{g'(*)}{v}$ where the right-hand side is a positive constant and the left-hand side is the first derivative of ϕW which is already assumed to decrease monotonously from infinity to zero. Therefore, the unique W, say W_m, satisfying $f'(W) = 0$ gives the maximum value of $f(W) = [g'(*) + v\phi]W$, and the value must be positive, that is, $f(W_m) = [g'(*) + v\phi]W_m > 0$. Thus, $A^* - L^*$ is always positive when $f(W_m) < 2[m - g(*) + b]$. But if $f(W_m) > 2[m - g(*) + b]$ holds, $A^* - L^*$ will be negative in some range of wealth around W_m and is positive in the lower tail and the upper tail of wealth distribution. In other words, only some households in the wealth range around W_m invest more in livestock holdings than in crop production, but only when $f(W_m) > 2[m - g(*) + b]$ is satisfied.

Using the first order condition, $f(W) = [g'(*) + v\phi]W$ can be written as $f(W_m) = -vW_m^2\frac{\partial \phi}{\partial W}$, which does not depend on m, $g'(*)$ or b. Therefore, when $2[m - g(*) + b]$ increases, or when the difference in expected returns to crop production and to livestock holdings increases, no household invests more in livestock holdings than in crop production. But when the difference decreases, some households invest more in livestock

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holdings than in crop production, and the wealth range where livestock investment exceeds crop production investment becomes wider as the difference in expected returns decreases. This proof of proposition 5 is graphically presented in Figure A1. Q.E.D.

Proof of Proposition 6

To show $L^* > 0$ for all W is very similar to the proof of proposition 1. Q.E.D.

Proof of Proposition 7

There is a unique W, say W_L, that satisfies $\frac{m - g(*) + b}{g'(*)} = W$. Thus, $A^* \leq 0$ for $W \leq W_L$.

Q.E.D.

Proof of Proposition 8

In $\frac{m - g(*) + b}{g'(*)} = W_L$, when $m - g(*) + b$ becomes closer to zero, W_L also becomes closer to zero. It means that there are less households specializing in livestock husbandry. When $m - g(*) + b = 0$, W_L becomes zero, that is, there is no household specialized in livestock husbandry. Q.E.D.

Proof of Proposition 9

The sensitivity is expressed in $g'(*)$. It is negative, which means that when total village livestock holdings decrease, livestock productivity increases. Therefore, greater sensitivity implies a smaller (or larger in absolute value) $g'(*)$ for a given $\sum_{i=1}^{n} L_i^*$. In $\frac{m - g(*) + b}{g'(*)} = W_L$, when the absolute value of $g'(*)$ increases, W_L becomes smaller. It means that there are less households specializing in livestock husbandry. Q.E.D.

Proof of Proposition 10

Similarly to the proof of proposition 5, $A^* - L^*$ is positive when $[g'(*) + v\phi]W < 2[m - g(*) + b]$. However, in this case, the right hand side of this inequality is a negative constant or zero. According to Figure A1, when $2[m - g(*) + b]$ is negative or zero, there is a unique, positive W, say W_n, above which $[g'(*) + v\phi]W < 2[m - g(*) + b]$ is satisfied. That is, $A^* < L^*$ for $W < W_n$ and $A^* \geq L^*$ for $W \geq W_n$. Q.E.D.

Proof of Proposition 11

As discussed in section 3.4, most households are considered to have positive livestock holdings ($L^* > 0$). But even if $L^* > 0$ is assumed, we cannot sign $\frac{\partial^2 L^*}{\partial W^2}$ *a priori*. However, as shown in Figures 3-5, $\frac{\partial L^*}{\partial W} > 0$ holds (for $W > W_c$ if $m - g(*) + b > 0$ and for all W if $m - g(*) + b \leq 0$) and $\lim_{W \to \infty} \frac{\partial L^*}{\partial W} = 0$ also holds. This suggests that $\frac{\partial^2 L^*}{\partial W^2} < 0$ must hold for sufficiently large W and above. As discussed in the text, if $\frac{\partial^2 L^*}{\partial W^2} < 0$ holds, a mean-preserving spread of wealth decreases the village livestock holdings.

For example, if $\phi = W^\alpha$ where $-1 < \alpha < 0$ is assumed (this function satisfies all assumptions made for ϕ as explained in section 3.1), it is possible to show that $\frac{\partial^2 L^*}{\partial W^2} < 0$ holds for all W if $m - g(*) + b \leq 0$, or the expected return to crop production is smaller than to livestock holdings, and that $\frac{\partial^2 L^*}{\partial W^2} < 0$ holds for sufficiently large W if $m - g(*) + b > 0$, or the expected return to crop production is larger than to livestock holdings. In the latter case, the exact condition for W cannot be obtained, but $W = \left(\frac{-g'(*)}{v}\right)^{-\frac{1}{\alpha}}$ is large enough for $\frac{\partial^2 L^*}{\partial W^2} < 0$ (derivations are not shown).

Proof of Proposition 12

When ϕA^s is positive, A^s will be positive because ϕ is assumed to be positive, that is, all households invest in crop production to some extent at the social optimum. In equation (13),

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when ϕA^s is positive, $m - g(s) - \sum_{i=1}^{n} L_i^s g'(s) + b$ must be positive. Because $g'(s)$ is assumed to be negative, if $m - g(s) + b$ is positive, the condition for ϕA^s to be positive is met (the expected return to crop production is greater than the expected return to livestock holdings). On the other hand, when $m - g(s) + b \leq 0$, the sign of ϕA^s is ambiguous, but if the difference in the expected returns is small enough, ϕA^s can be positive (the expected return to crop production is smaller than the expected return to livestock holdings).

Proof of Proposition 13

The proposition is equivalent to saying that equation (13) gives a unique, positive ϕA^s under assumptions of $\sum_{i=1}^{n} A_i^s > 0$ and $\sum_{i=1}^{n} L_i^s > 0$. Let $\phi A^s = K^s$, so equation (13) can be written as a function of K:

$$h(K) = m - g(\sum_{i=1}^{n} W_i - K\sum_{i=1}^{n} \frac{1}{\phi_i}) - (\sum_{i=1}^{n} W_i - K\sum_{i=1}^{n} \frac{1}{\phi_i})g'(\sum_{i=1}^{n} W_i - K\sum_{i=1}^{n} \frac{1}{\phi_i}) + b - vK \qquad \text{(A2)}$$

Differentiation with respect to K gives

$$h'(K) = 2g'(\,)\sum_{i=1}^{n} \frac{1}{\phi_i} + (\sum_{i=1}^{n} W_i - K\sum_{i=1}^{n} \frac{1}{\phi_i})\sum_{i=1}^{n} \frac{1}{\phi_i} g''(\,) - v \qquad \text{(A3)}$$

$h'(K)$ is negative for positive K, because $g'(\,)$ and $g''(\,)$ are assumed to be negative and $\sum_{i=1}^{n} W_i - K\sum_{i=1}^{n} \frac{1}{\phi_i} = \sum_{i=1}^{n} L_i$ is assumed to be positive.

When $K = 0$, function (A2) becomes $h(0) = m - g(\sum_{i=1}^{n} W_i) - (\sum_{i=1}^{n} W_i)g'(\sum_{i=1}^{n} W_i) + b$.

This is the same as the left-hand side of equation (12) with a negative sign when $\sum_{i=1}^{n} L_i = \sum_{i=1}^{n} W_i$. Therefore, when the socially optimal livestock holdings is smaller than total wealth, that is, when $\sum_{i=1}^{n} L_i^s < \sum_{i=1}^{n} W_i$, $h(0) > 0$ holds. On the other hand, when $K = \dfrac{\sum_{i=1}^{n} W_i}{\sum_{i=1}^{n} \frac{1}{\phi_i}}$,

function (A2) becomes $h(\frac{\sum_{i=1}^{n} W_i}{\sum_{i=1}^{n} \frac{1}{\phi_i}}) = m - g(0) - 0 * g'(0) + b - v\frac{\sum_{i=1}^{n} W_i}{\sum_{i=1}^{n} \frac{1}{\phi_i}}$. Because

$\frac{\sum_{i=1}^{n} W_i}{\sum_{i=1}^{n} \frac{1}{\phi_i}} \geq \frac{\min W_i}{\max \frac{1}{\phi_i}} = \min(\phi_i W_i) > 0$, the following inequality holds:

$$h(\frac{\sum_{i=1}^{n} W_i}{\sum_{i=1}^{n} \frac{1}{\phi_i}}) \leq m - g(0) - 0 * g'(0) + b - v\min(\phi_i W_i) \qquad \text{(A3)}$$

In equation (12), when $\sum_{i=1}^{n} L_i = 0$, the left-hand side is positive, because the optimal level exists where $0 < \sum_{i=1}^{n} L_i^s$. This means that the right hand side of inequality (A3) is negative. Therefore $h(\frac{\sum_{i=1}^{n} W_i}{\sum_{i=1}^{n} \frac{1}{\phi_i}}) < 0$ holds. Thus, $h'(K) < 0$, $h(0) > 0$, and $h(\frac{\sum_{i=1}^{n} W_i}{\sum_{i=1}^{n} \frac{1}{\phi_i}}) < 0$ indicate that there is a unique, positive K between 0 and $\frac{\sum_{i=1}^{n} W_i}{\sum_{i=1}^{n} \frac{1}{\phi_i}}$ that gives $h(K) = 0$. Such K is now denoted by K^s. Q.E.D.

Proof of Proposition 14

As a measure of inequality, the mean-preserving spread of wealth distribution is used here. So, if A^s is concave in W, $\sum_{i=1}^{n} A_i^s$ will be lower and $\sum_{i=1}^{n} L_i^s$ will be higher when wealth is distributed less equally.

Differentiating A^s with respect to W yields

$$\frac{\partial A^s}{\partial W} = \frac{\partial(\frac{K^s}{\phi})}{\partial W} = K^s(-\frac{1}{\phi^2}\frac{\partial \phi}{\partial W}) \qquad \text{(A4)}$$

$$\frac{\partial^2 A^s}{\partial W^2} = \frac{\partial^2 (\frac{K^s}{\phi})}{\partial W^2} = K^s (\frac{2}{\phi^3}\frac{\partial \phi}{\partial W} - \frac{1}{\phi^2}\frac{\partial^2 \phi}{\partial W^2}) \qquad \text{(A5)}$$

Because ϕ is assumed to be a convex function of W with $\phi' < 0$ and $\phi'' > 0$, (A4) is positive, and (A5) is negative. Therefore, A^s is concave in W. Q.E.D.

Proof of Proposition 15

The socially expected return is given in equation (11). When livestock holdings are allocated optimally in the society, equation (11) can be written using K^s as

$$EC(\sum_{i=1}^{n} Y_i^s) = mK^s \sum_{i=1}^{n} \frac{1}{\phi_i} + \left(\sum_{i=1}^{n} W_i - K^s \sum_{i=1}^{n} \frac{1}{\phi_i}\right) g(\sum_{i=1}^{n} W_i - K^s \sum_{i=1}^{n} \frac{1}{\phi_i})$$
$$- b\left(\sum_{i=1}^{n} W_i - K^s \sum_{i=1}^{n} \frac{1}{\phi_i}\right) - \frac{1}{2} v (K^s)^2 \sum_{i=1}^{n} \frac{1}{\phi_i} \qquad \text{(A6)}$$

Differentiating equation (A6) with respect to $\sum_{i=1}^{n} \frac{1}{\phi_i}$ results in

$$\frac{\partial EC(\sum_{i=1}^{n} Y_i^s)}{\partial \sum_{i=1}^{n} \frac{1}{\phi_i}} = \frac{1}{2} v (K^s)^2 > 0 \qquad \text{(A7)}$$

As proved for proposition 14, $\sum_{i=1}^{n} \frac{1}{\phi_i}$ is lower when wealth is more unequally distributed. Therefore, equation (A7) implies that the socially expected return is lower when the village wealth is more unequal. When v, crop production risk, is large, this effect is strengthened as seen in equation (A7). Q.E.D.

Proof of Proposition 16

The condition above is written as $-m + g(*) - b > 0$ as already discussed in section 3.2. Assume contrarily that $\sum_{i=1}^{n} L_i^* \le \sum_{i=1}^{n} L_i^s$, then $0 < g(s) \le g(*)$ and $g'(s) \le g'(*) < 0$ hold and consequently the following inequalities hold:

$$-m + g(s) - b \le [-m + g(*) - b] < n(-m + g(*) - b) \qquad \text{(A8)}$$

$$\sum_{i=1}^{n} L_i^s g'(s) \le \left[\sum_{i=1}^{n} L_i^s g'(*) \right] \le \sum_{i=1}^{n} L_i^* g'(*) \tag{A9}$$

$$v\sum_{i=1}^{n} \phi_i A_i^* = \left[v\sum_{i=1}^{n} \frac{K^s}{A_i^s} A_i^* = vK^s \sum_{i=1}^{n} \frac{A_i^*}{A_i^s} \ge vK^s \frac{\sum_{i=1}^{n} A_i^*}{\max A_i^s} \ge vK^s \frac{\sum_{i=1}^{n} A_i^*}{\sum_{i=1}^{n} A_i^s} \right] \ge vK^s \tag{A10}$$

where the terms in [ ] are used only to prove the inequalities. Inequalities (A8), (A9) and (A10) imply that (14) and (15) cannot hold simultaneously, which contradicts the proofs that both household and social optima exist. This contradiction is due to the additional assumption of $\sum_{i=1}^{n} L_i^* \le \sum_{i=1}^{n} L_i^s$. Therefore, $\sum_{i=1}^{n} L_i^* > \sum_{i=1}^{n} L_i^s$ must hold when $-m + g(*) - b > 0$. That is, overgrazing always occurs. Q.E.D.

Proof of Proposition 17

Now the condition is $-m + g(*) - b \le 0$. If $\sum_{i=1}^{n} L_i^* \le \sum_{i=1}^{n} L_i^s$ is contrarily assumed, inequalities (A9) and (A10) still hold, but inequality (A8) does not always hold. There are two possible inequalities under those assumptions above:

$$0 \ge -m + g(*) - b > n(-m + g(*) - b) > -m + g(s) - b \tag{A11}$$

and

$$0 \ge -m + g(*) - b \ge -m + g(s) - b \ge n(-m + g(*) - b) \tag{A12}$$

If inequality (A11) holds, then inequality (A8) also holds and consequently $\sum_{i=1}^{n} L_i^* > \sum_{i=1}^{n} L_i^s$ must hold, which means there is overgrazing. On the other hand, if inequality (A12) holds, inequality (A8) does not hold, which means that either $\sum_{i=1}^{n} L_i^* > \sum_{i=1}^{n} L_i^s$ or $\sum_{i=1}^{n} L_i^* \le \sum_{i=1}^{n} L_i^s$ can be correct. That is, overgrazing does not necessarily occur when $-m + g(*) - b \le 0$. Q.E.D.

Graphic Presentation of Proofs of Propositions 16 and 17

The proofs of propositions 16 and 17 are presented in graphs as follows. When $-m+g(*)-b>0$ (overgrazing always occurs according to proposition 16), the socially optimal investment in crop production for each household, $A^s = \frac{K^s}{\phi}$, and the household optimal investment in crop production under open access, $A^* = \frac{-m+g(*)-b+g'(*)W}{-v\phi+g'(*)}$, are graphed in Figure A2 as functions of household initial wealth, W. There is a unique W that equates A^s and A^*, which is given by

$$W_e = A^s + \frac{m-g(*)+b-vK^s}{g'(*)} = A^s + C_e \qquad \text{(A13)}$$

W_e is a sum of A^s and a positive constant, C_e. Below W_e, $A^s > A^*$, and above W_e, $A^s < A^*$ as shown in Figure A2. The size of W_e is associated with the degree of overgrazing; the larger C_e, the larger W_e, and consequently wider range of wealth supports $A^s > A^*$. C_e is large when (1) $-m+g(*)-b$ is large and/or absolute value of $g'(*)$ is small, which is the condition that many households specialize in livestock husbandry according to propositions 7 and 8, (2) v is large, which implies that crop production is risky, and (3) K^S is large, which implies relatively equal distribution of wealth (see the proof at the end of this section). This means that large village livestock holdings under open access according to section 3.5 and small socially optimal livestock holdings according to proposition 14. These conditions are reasonable for overgrazing. However, if all households have wealth greater than W_e, there is no overgrazing because $\sum_{i=1}^{n} L_i^s > \sum_{i=1}^{n} L_i^*$. This contradicts proposition 16. Therefore, it is considered that W_e is determined by wealth distribution so that some households have wealth below W_e.

When $-m+g(*)-b \le 0$ (overgrazing does not always occur according to proposition 17), C_e in equation (A13) can be either positive, zero, or negative depending on $m-g(*)+b-vK^s$.

When $m-g(*)+b-vK^s$ is negative, $-m+g(*)-b$ must be small negative or zero and/or vK^s must be large positive. Small negative or zero $-m+g(*)-b$ is the condition

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when specialization in crop production is unlikely according to proposition 3. Large v implies risky crop production, and is also the condition when specialization in crop production is unlikely according to proposition 4. These conditions are associated with large livestock holdings under open access when the expected return to crop production is higher than the expected return to livestock holdings. Large K^S implies that large village livestock holdings under open access and small socially optimal livestock holdings, as explained above. In this case C_e is positive and there is a unique W_e (Figure A3). Therefore, the consequence will be either overgrazing or undergrazing.

On the other hand, when $m - g(*) + b - vK^s$ is positive, which is the case of large negative $-m + g(*) - b$ and/or small positive vK^s, C_e is negative. These conditions are associated with small livestock holdings under open access, and large livestock holdings at the social optimum. In this case there are two possible patterns of A^s and A^* depending on the position of C_e and curvatures of A^s and A^* as shown in Figure A4 and Figure A5. Figure A4 is the case when overgrazing is still possible, but is less likely than the case in Figure A3. Figure A5 is the case when there is no W_e, and $A^s < A^*$ always holds. This means undergrazing always occurs, when C_e is small enough (or large negative enough), that is, $-m + g(*) - b$ is positive and very large and/or vK^s is very small though positive. Thus, proposition 17 can be conditioned as proposition 18.

The relationship between K^s and the wealth distribution of a village is explained as follows. Totally differentiating equation (13) with respect to K^s and $\sum_{i=1}^{n} \frac{1}{\phi_i}$ and rearranging yield

$$\frac{\partial K^s}{\partial \sum_{i=1}^{n} \frac{1}{\phi_i}} = \frac{\left[2g'(s) + (\sum_{i=1}^{n} W_i - K^s \sum_{i=1}^{n} \frac{1}{\phi_i}) g''(s)\right] K^s}{\left[2g'(s) + (\sum_{i=1}^{n} W_i - K^s \sum_{i=1}^{n} \frac{1}{\phi_i}) g''(s)\right] \sum_{i=1}^{n} \frac{1}{\phi_i} - v} \qquad \text{(A14)}$$

Equation (A13) is positive, because both numerator and denominator are negative; $g'(s) < 0$, $g''(s) < 0$, and $(\sum_{i=1}^{n} W_i - K^s \sum_{i=1}^{n} \frac{1}{\phi_i}) > 0$. As shown in Proof of Proposition 14, $\frac{1}{\phi}$ is a

concave function of W, and consequently $\sum_{i=1}^{n}\frac{1}{\phi_i}$ decreases with an increase in the spread of wealth. Therefore, K^s is large in a village with relatively equal wealth distribution.

Proof of Proposition 18

See the discussion about Figures A3-A5 above.

Proof of Proposition 19

When there is overgrazing, $\sum_{i=1}^{n}L_i^s < \sum_{i=1}^{n}L_i^*$ hold by definition. Thus, $g(\sum_{i=1}^{n}L_i^s) > g(\sum_{i=1}^{n}L_i^*) > 0$ and $0 > g'(\sum_{i=1}^{n}L_i^s) > g'(\sum_{i=1}^{n}L_i^*)$ hold. Consequently with respect to the second terms of (16) and (17), $W_i(g(\sum_{i=1}^{n}L_i^s) - b) > W_i(g(\sum_{i=1}^{n}L_i^*) - b)$ always holds.

When $A^s > A^*$, or $W < W_e$ in Figure A2 and A3, $\frac{1}{2}v\phi_i(A_i^s)^2 > \frac{1}{2}v\phi_i(A_i^*)^2$ always holds, too. Therefore, $CE(Y_i^s) < CE(Y_i^*)$ requires sufficiently small $A_i^s\sum_{i=1}^{n}L_i^s g'(\sum_{i=1}^{n}L_i^s)$ relative to $A_i^*L_i^*g'(\sum_{i=1}^{n}L_i^*)$ in order to offset $W_i(g(\sum_{i=1}^{n}L_i^s) - b) > W_i(g(\sum_{i=1}^{n}L_i^*) - b)$ and $\frac{1}{2}v\phi_i(A_i^s)^2 > \frac{1}{2}v\phi_i(A_i^*)^2$. For $A_i^s\sum_{i=1}^{n}L_i^s g'(\sum_{i=1}^{n}L_i^s) < A_i^*L_i^*g'(\sum_{i=1}^{n}L_i^*)$ to hold, $A_i^s\sum_{i=1}^{n}L_i^s > A_i^*L_i^*$ is necessary, because $0 > g'(\sum_{i=1}^{n}L_i^s) > g'(\sum_{i=1}^{n}L_i^*)$ is assumed. Because $\sum_{i=1}^{n}L_i^s > L_i^*$ is considered to hold in most cases, $A_i^s\sum_{i=1}^{n}L_i^s > A_i^*L_i^*$ also holds in most cases assuming $A^s > A^*$. Thus, the conditions for $CE(Y_i^s) < CE(Y_i^*)$ will be that L_i^* and/or A_i^* are very small, that is, household i's initial wealth is very low.

On the other hand, when $A^s \le A^*$, or $W \ge W_e$ in Figures A2 and A3, $\frac{1}{2}v\phi_i(A_i^s)^2 < \frac{1}{2}v\phi_i(A_i^*)^2$ always holds. If the difference between the two sides are sufficiently large, $CE(Y_i^s) < CE(Y_i^*)$ will hold regardless of the first terms in (16) and (17). That is, households whose initial wealth is very large will be worse off at the social optimum. Even if

the difference between $\frac{1}{2}v\phi_i(A_i^s)^2$ and $\frac{1}{2}v\phi_i(A_i^*)^2$ is not sufficiently large, if $A_i^s\sum_{i=1}^{n}L_i^s g'(\sum_{i=1}^{n}L_i^s)$ is sufficiently small relative to $A_i^* L_i^* g'(\sum_{i=1}^{n}L_i^*)$, $CE(Y_i^s) < CE(Y_i^*)$ will hold. Since $0 > g'(\sum_{i=1}^{n}L_i^s) > g'(\sum_{i=1}^{n}L_i^*)$ and $A^s \le A^*$ are assumed, $\sum_{i=1}^{n}L_i^s < L_i^*$ is necessary for $A_i^s\sum_{i=1}^{n}L_i^s g'(\sum_{i=1}^{n}L_i^s)$ to be sufficiently small relative to $A_i^* L_i^* g'(\sum_{i=1}^{n}L_i^*)$. $\sum_{i=1}^{n}L_i^s < L_i^*$ is the condition when household i's livestock holdings exceed the livestock holdings that the village as whole will have at the social optimum. It implies wealth distribution is extremely unequal. Q.E.D.

Appendix B

Effects of Privatization

Proof of $L_i^p \geq L_i^*$

Under this privatization scheme, household i's certainty-equivalent income when grazing land is privately owned is

$$CE(Y_i^p) = mA_i + L_i g(\frac{L_i}{L_i^*}\sum_{i=1}^{n} L_i^*) - bL_i - \frac{1}{2}v\phi A_i^2 \qquad \text{(B1)}$$

And the first order condition with respect to L_i is

$$\frac{\partial CE(Y_i^p)}{\partial L_i} = -m + g(\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*) + \frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^* g'(\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*) - b + v\phi A_i^p = 0 \qquad \text{(B2)}$$

From (5), the first order condition under open access is given by

$$-m + g(\sum_{i=1}^{n} L_i^*) + L_i^* g'(\sum_{i=1}^{n} L_i^*) - b + v\phi A_i^* = 0 \qquad \text{(B3)}$$

Comparing (B2) and (B3), it can be shown that household i's optimal livestock holdings under private ownership is smaller than under open access. Assuming contrarily that $L_i^p \geq L_i^*$, then $\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^* \geq \sum_{i=1}^{n} L_i^*$ holds. Under this condition, the following three inequalities hold: $g(\sum_{i=1}^{n} L_i^*) \leq g(\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*)$, $\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^* g'(\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*) < L_i^p g'(\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*) \leq L_i^* g'(\sum_{i=1}^{n} L_i^*)$, and $v\phi A_i^p \leq v\phi A_i^*$. If the three inequalities simultaneously hold, then either (B2) or (B3) cannot be true. This is a contradiction to the fact that both optima exist. Therefore, $L_i^p < L_i^*$ must hold.

Condition for $CE(Y_i^p) \geq CE(Y_i^*)$

Using the first order conditions under open access and private ownership respectively, the certainty-equivalent income can be expressed as

$$\begin{aligned} CE(Y_i^*) &= mA_i^* + L_i^* g(\sum_{i=1}^{n} L_i^*) - bL_i^* - \frac{1}{2}v\phi_i (A_i^*)^2 \\ &= A_i^* L_i^* g'(\sum_{i=1}^{n} L_i^*) + \frac{1}{2}v\phi_i (A_i^*)^2 + W_i (g(\sum_{i=1}^{n} L_i^*) - b) \end{aligned} \qquad \text{(B4)}$$

$$CE(Y_i^p) = mA_i^p + L_i^p g(\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*) - bL_i^p - \frac{1}{2}v\phi_i(A_i^p)^2$$
$$= A_i^p \frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^* g'(\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*) + \frac{1}{2}v\phi_i(A_i^p)^2 + W_i(g(\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*) - b) \quad \text{(B5)}$$

Comparing (B4) and (B5), it can be concluded that $CE(Y_i^p) \geq CE(Y_i^*)$ does not always hold but depends on the relative magnitudes of L_i^* and L_i^p, household's wealth and the features of the livestock productivity function. Because $A_i^* < A_i^p$, $g(\sum_{i=1}^{n} L_i^*) < g(\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*)$ and $g'(\sum_{i=1}^{n} L_i^*) < g'(\frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*)$ hold, $CE(Y_i^p) > CE(Y_i^*)$ holds for households that satisfies $L_i^* > \frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*$. Even if $L_i^* \leq \frac{L_i^p}{L_i^*}\sum_{i=1}^{n} L_i^*$, if the difference is small enough, $CE(Y_i^p) \geq CE(Y_i^*)$ will still hold. This suggests that wealthier households that have larger optimal livestock holdings, L_i^*, under open access are likely to be better off by privatization.

Appendix C

Effects of Quota

Under this quota scheme, the certainty-equivalent income for household i under the quota scheme is

$$CE(Y_i^q) = m(A_i^* + \frac{DL_i^*}{\sum_{i=1}^{n} L_i^*}) + (L_i^* - \frac{DL_i^*}{\sum_{i=1}^{n} L_i^*})g(\sum_{i=1}^{n} L_i^* - D) - b(L_i^* - \frac{DL_i^*}{\sum_{i=1}^{n} L_i^*}) - \frac{1}{2}v\phi_i(A_i^* + \frac{DL_i^*}{\sum_{i=1}^{n} L_i^*})^2 \tag{C1}$$

The difference in certainty-equivalent incomes for household i between open access and quota is

$$\begin{aligned} & CE(Y_i^q) - CE(Y_i^*) \\ &= (L_i^* - \frac{DL_i^*}{\sum_{i=1}^{n} L_i^*})g(\sum_{i=1}^{n} L_i^* - D) - L_i^* g(\sum_{i=1}^{n} L_i^*) + \frac{DL_i^*}{\sum_{i=1}^{n} L_i^*}(m + b - v\phi_i A_i^* - \frac{1}{2}v\phi_i \frac{DL_i^*}{\sum_{i=1}^{n} L_i^*}) \\ &= \frac{L_i^*}{\sum_{i=1}^{n} L_i^*}\left[(\sum_{i=1}^{n} L_i^* - D)(g(\sum_{i=1}^{n} L_i^* - D) - g(\sum_{i=1}^{n} L_i^*)) + DL_i^*(g'(\sum_{i=1}^{n} L_i^*) - \frac{1}{2}v\phi_i \frac{D}{\sum_{i=1}^{n} L_i^*})\right] \end{aligned} \tag{C2}$$

From the middle expression to the last, the first order condition under open access, $m - g(*) + b - v\phi_i A_i^* - L_i^* g'(*) = 0$, is used.

The first term in the large parenthesis is positive and common for the all households. Thus, the sign of equation (C2) depends on how large the second term is relative to the fixed first term, which is determined by household L_i^* and $\phi_i L_i^*$. It is impossible to determine specific conditions when the latter negative term is greater than the former fixed positive term (in absolute values), but (C2) implies that some households may have smaller certainty-equivalent income in this quota scheme than under open access.

Since D changes the sizes of fixed positive term and the household-specific negative term, it may be possible to design the quota through adjusting D so that all (or most) households can be better off by the quota.

Appendix D

Effects of Transferable Permits

Under this transferable permit scheme, the budget constraint for household i is

$$W_i + (\frac{L^D}{n} - L_i)p = A_i + L_i \tag{D1}$$

Using (D1), household i's certainty-equivalent income is given by

$$CE(Y_i^m) = m(W_i - L_i(p+1) + \frac{p}{n}\sum_{i=1}^{n} L_i) + L_i g(\sum_{i=1}^{n} L_i) - bL_i - \frac{1}{2}v\phi_i(W_i - L_i(p+1) + \frac{p}{n}\sum_{i=1}^{n} L_i)^2 \tag{D2}$$

The first order condition with respect to L_i is given by

$$-m(1+p-\frac{p}{n}) + g(\sum_{i=1}^{n} L_i^m) - b + L_i^m g'(\sum_{i=1}^{n} L_i^m) + v\phi_i(W_i - L_i^m(p+1) + \frac{p}{n}\sum_{i=1}^{n} L_i^m)(1+p-\frac{p}{n}) = 0 \tag{D3}$$

From (42), optimal livestock holdings for household i under the transferable permits scheme is

$$L_i^m = \frac{m(1+p-\frac{p}{n}) - g(\sum_{i=1}^{n} L_i^m) + b - v\phi_i(W_i + \frac{p}{n}\sum_{i=1}^{n} L_i^m)(1+p-\frac{p}{n})}{-v\phi_i(p+1)(1+p-\frac{p}{n}) + g'(\sum_{i=1}^{n} L_i^m)} \tag{D4}$$

Comparing L_i^m given by (D4) with the household's optimum under open access, L_i^*, given by (7), it is not possible to tell which is greater in general.

Appendix E

Effects of User Fee

The budget constraint under this user fee scheme for household i will be

$$W_i - tL_i = A_i + L_i \tag{E1}$$

where household i's certainty-equivalent income is given by

$$CE(Y_i^t) = m(W_i - (1+t)L_i) + L_i g(\sum_{i=1}^{n} L_i) - bL_i - \frac{1}{2} v\phi_i (W_i - (1+t)L_i)^2 \tag{E2}$$

And the first order condition with respect to L_i is

$$\frac{\partial CE(Y_i^t)}{\partial L_i} = -(1+t)m + g(\sum_{i=1}^{n} L_i^t) - b + L_i^t g'(\sum_{i=1}^{n} L_i^t) - v\phi_i A_i^t = 0 \tag{E3}$$

Equation (E3) is to be compared to the first order condition under open access, given by equation (5). Assume contrarily that $L_i^t \geq L_i^*$, then $g(L_i^t) \leq g(L_i^*)$, $g'(L_i^t) \leq g'(L_i^*)$, and $A_i^t \leq A_i^*$ must hold. But those conditions cannot satisfy both (5) and (E3) at the same time. Therefore, $L_i^t < L_i^*$ is required.

Figure A1 Determination of Relative Size of Crop and Livestock Investments

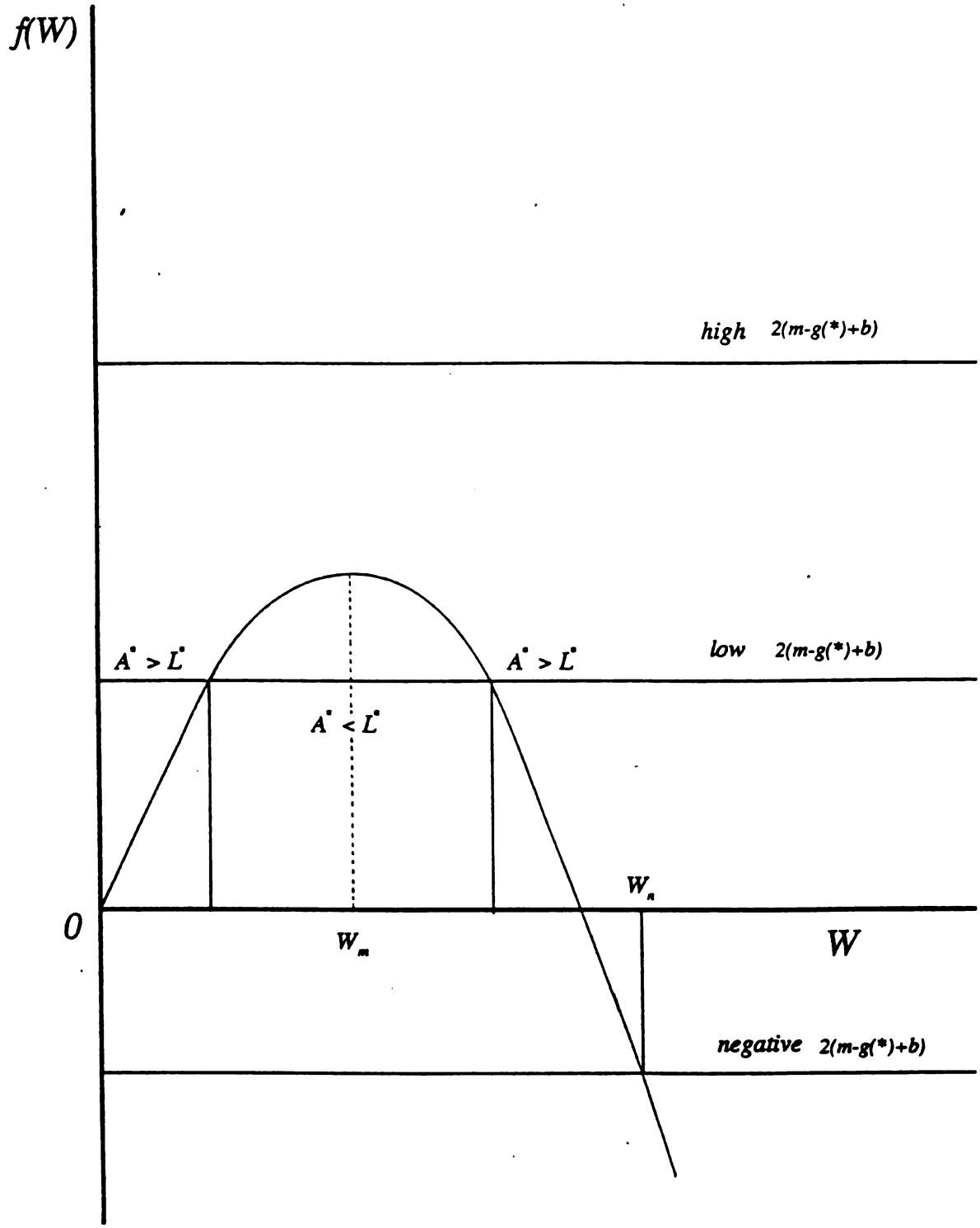
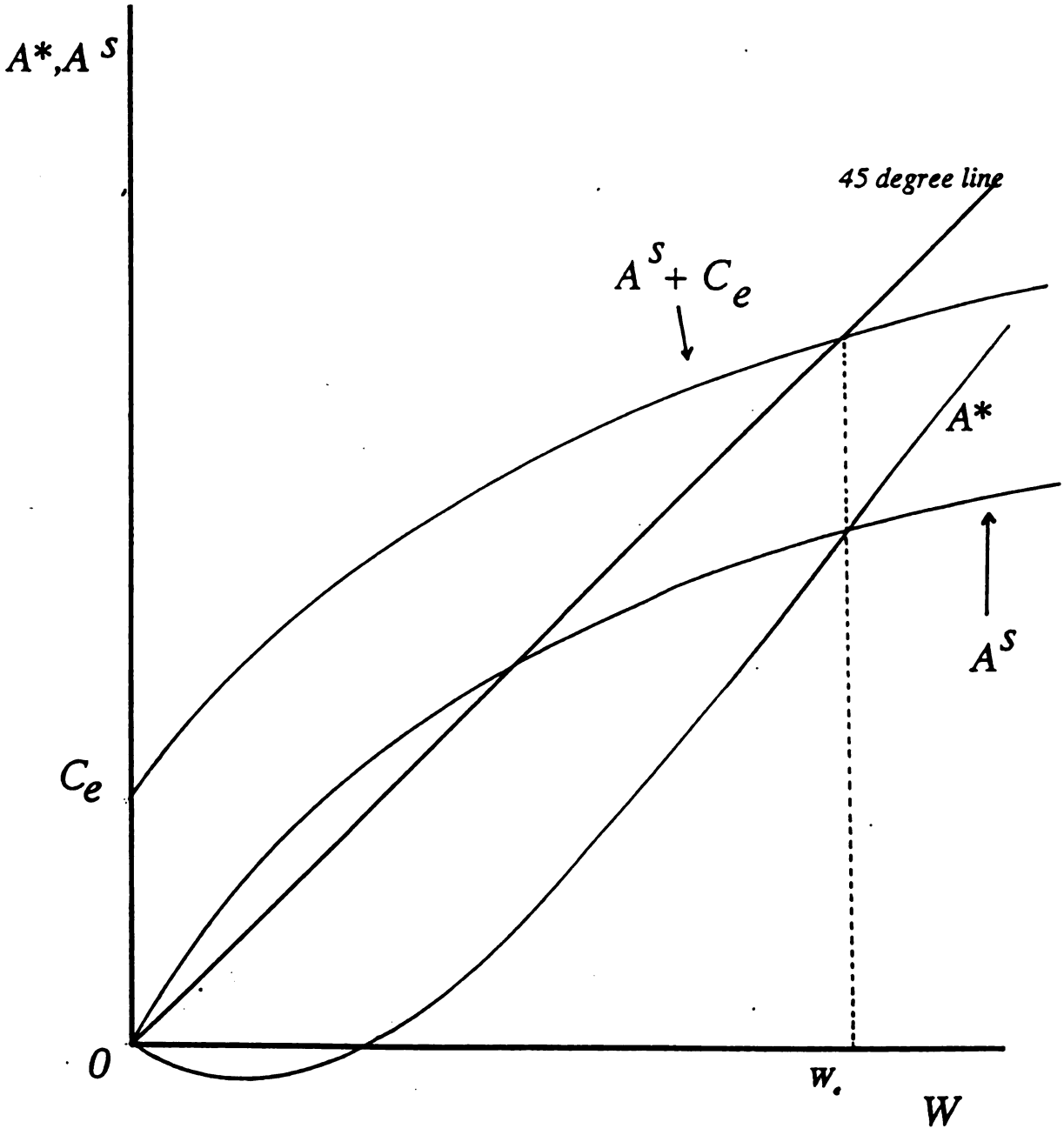


Figure A2

Comparison of Household and Social Optima (Case 1, $m - g(*) + b < 0$)



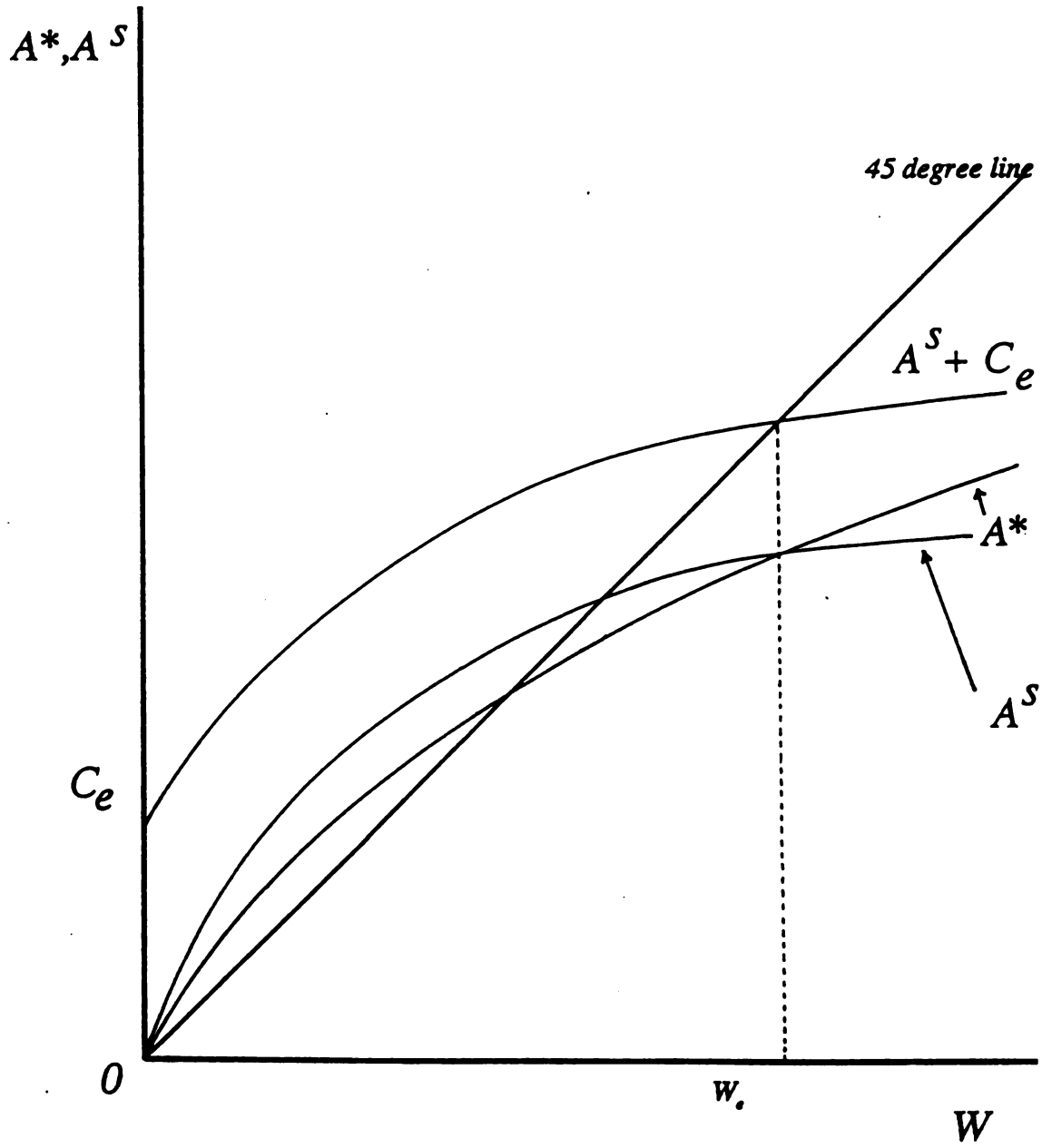
Expected return to crop production is smaller than that to livestock.

$C_e > 0$

$A^s \geq A^*$ when $W \leq W_e$, and $A^s < A^*$ when $W > W_e$

Figure A3

Comparison of Household and Social Optima (Case 2, $m - g(*) + b \geq 0$)



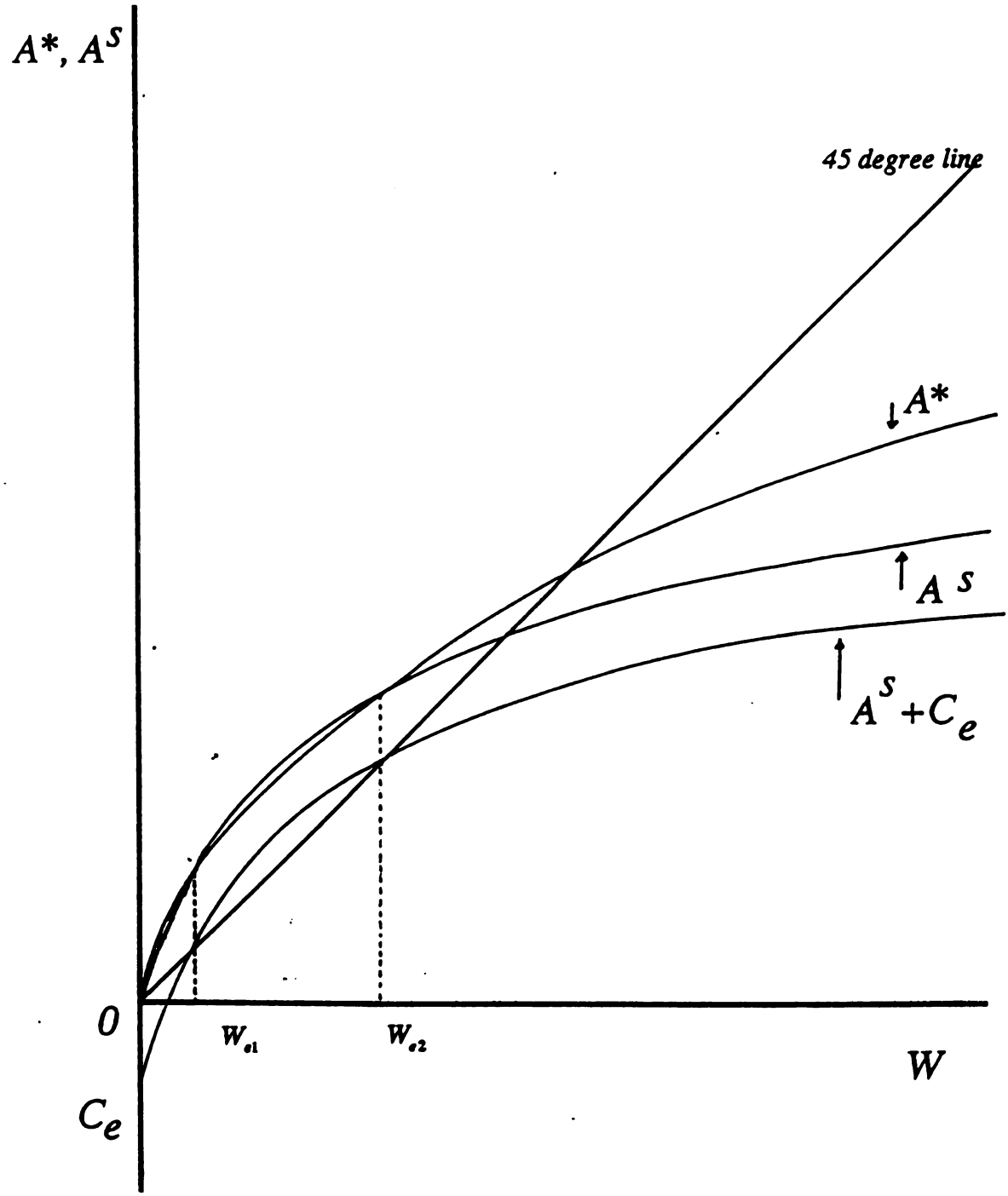
Expected return to crop production is greater than that to livestock.

$C_e > 0$

$A^s \geq A^*$ when $W \leq W_e$, and $A^s < A^*$ when $W > W_e$

Figure A4

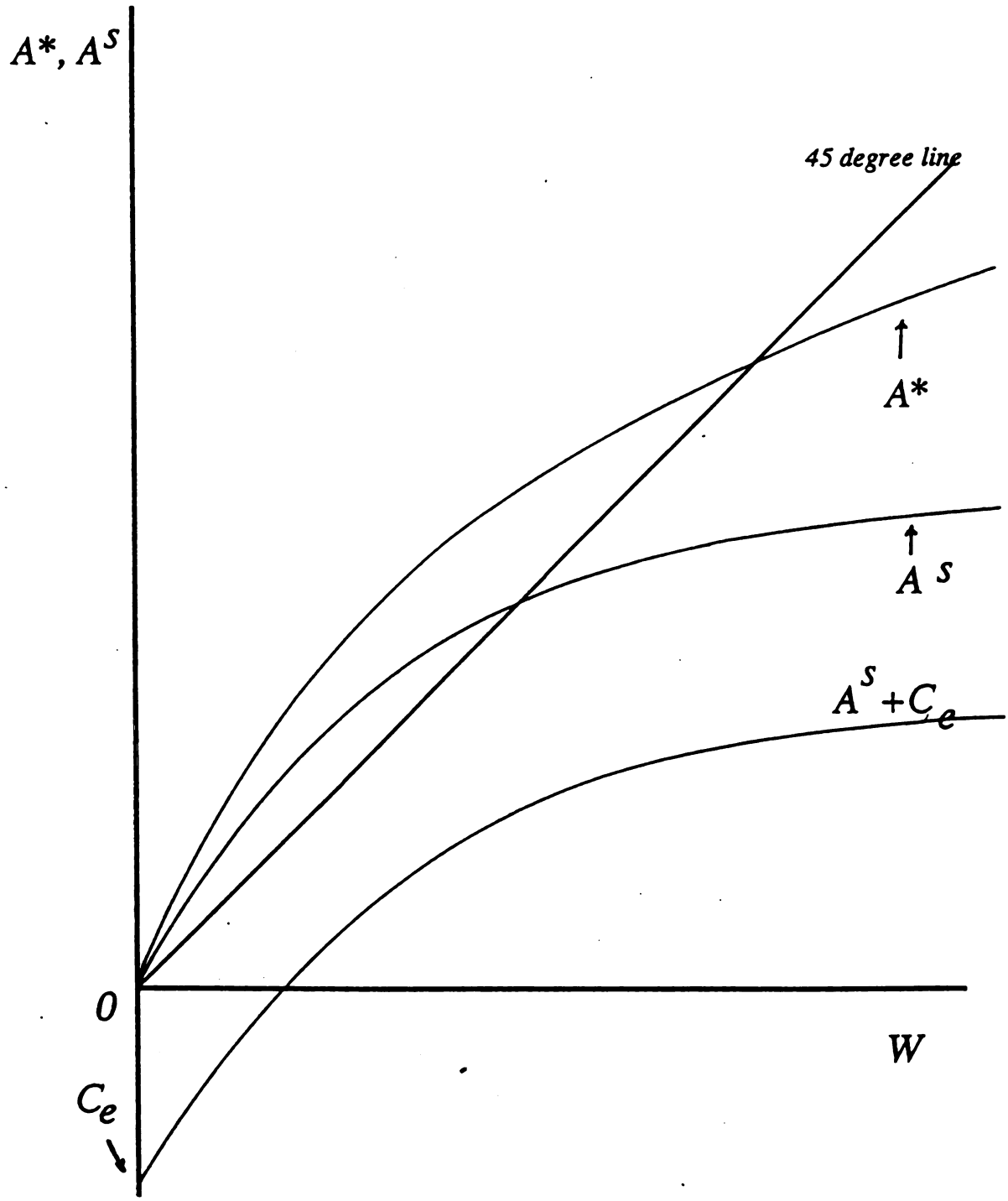
Comparison of Household and Social Optima (Case 3, $m - g(*) + b \geq 0$)



Expected return to crop production is greater than that to livestock. $C_e \leq 0$, but close to zero. $A^s < A^*$ when $W < W_{e1}$, $A^s \geq A^*$ when $W_{e1} \leq W \leq W_{e2}$, and $A^s < A^*$ when $W_{e2} < W$.

Figure A5

Comparison of Household and Social Optima (Case 4, $m - g(*) + b \geq 0$)



Expected return to crop production is greater than that to livestock.

$C_e \leq 0$, but close to zero.

Always $A^s < A^*$.

Chapter 5

Crop Production under Drought Risk and Estimation of Demand for Formal Drought Insurance in the Sahel

0 Summary

The Sahel is a drought-prone region. But there is currently no formal drought insurance; rather, households informally insure themselves through their income and asset portfolio strategies. The concern is that their self-insurance is costly and not effective when a drought affects all households over large areas. Formal drought insurance, on the other hand, can pool risks across zones, and its cost can be below self-insurance. This chapter explores the level and determinants of potential demand in the Sahel for a hypothetical formal drought insurance called rainfall lottery as follows.

First, the conceptual part consists of six steps. The paper: (i) Sets out a utility maximizing model under budget and labor constraints with self-insurance; (ii) Derives the first order conditions that represent "self-insurance equilibrium", which we are observing; (iii) Sets out a similar model, but with the hypothetical formal drought insurance, or the rainfall lottery; (iv) Derives the first order conditions that represent "formal-insurance equilibrium", which we cannot observe; (v) Defines "equilibrium premium rate". This is the premium rate at which the household is indifferent between the self-insurance equilibrium and the formal-insurance equilibrium; (vi) In the case of a rainfall lottery, the equilibrium premium rate is expressed by the ratio of expected marginal values of a good year (non-drought year) and a drought year, i.e. the utility trade-off between two types of year.

Second, the empirical part consists of six steps. The paper: (i) Estimates parameters of the behavioral equations at the self-insurance equilibrium; (ii) Calculates the expected

marginal values of a good year and a drought year using the parameters estimated in the first step, and obtains the equilibrium premium rate as defined above. To have the expected values, I use rainfall distribution to be defined in (iii) and a rainfall cut-off point that separates a drought year from a good year to be defined in (iv); (iii) Assumes that annual rainfall is normally distributed. Its mean and variance are obtained from long-run rainfall statistics and are assumed not to change over years; (iv) Defines a drought year by setting a rainfall cut-off point on the normal rainfall distribution. This point can be assigned arbitrarily, and if rainfall of a year is below the cut-off point, the insurant will win the rainfall lottery; (v) Calculates a drought probability for each cut-off point, given mean and variance of normally distributed rainfall; (vi) Defines "effective demand" for the formal drought insurance as the case when the estimated equilibrium premium rate is greater than the drought probability that the cut-off point implies. Note that the effective demand means that the household will be willing to pay more premium than in the case of actuarially fair drought insurance, and that the insurance firm will be able to make positive gross revenues.

Third, the analysis part is as follows. This paper (i) Examines the existence of effective demand for the formal drought insurance for several drought probabilities; (ii) Examines how the individual equilibrium premium rates vary with household assets and characteristics via determinant analysis; (iii) Exogenously stratifies the sample in various ways to see if that affects determinants. Panel data on households from three agroclimatic zones in Burkina Faso are used: the Sahelian (unfavorable agroclimatically), the Guinean (favorable agroclimatically), and the Sudanian (between the two). From the analysis part, the following findings are salient.

First, effective demand for the hypothetical formal drought insurance is found in all zones, which implies inadequacies in households' current self-insurance strategies. But the demand is correlated with agroclimatic zone because drought risk differs over zones: it is highest in the Sahelian zone and lowest in the Guinean zone. The effective demand is distributed in the most narrow range of drought probability in the Sahelian zone (15%-25%)

and in the most wide range of drought probability in the Guinean zone (10%-35%). This implies that households are more heterogeneous in terms of the level of self-insurance in the agroclimatically favorable zone. In the determinant analysis, a 15% drought probability is chosen as that at which most households are considered to demand the insurance.

Second, the determinant analysis shows that the effective demand depends on how households manage drought risk *ex ante* and *ex post*; there is much heterogeneity in the strategies to self-insure even within zones. The results show that this is true with off-farm income only in the middle livestock stratum and with livestock holdings only in the lower livestock stratum. On the other hand, in the upper livestock stratum, neither livestock holdings nor off-farm income has a significant effect on demand for formal drought insurance. However, since in this stratum households on average show the largest demand for the drought insurance, their self-insurance is not sufficient. This is a sign that formal drought insurance can substitute for livestock holdings at least partially among those who keep relatively large livestock holdings without changing their welfare.

1 Introduction

This chapter explores the level and determinants of effective demand for formal drought insurance in the Sahel, the drought-prone region of the West African semi-arid tropics (WASAT). There is currently no such formal insurance; rather, households informally insure themselves through their income and asset portfolio strategies. By studying how households allocate inputs to production under informal self-insurance, their willingness to pay for a hypothetical formal drought insurance is inferred.

In the WASAT, rural households engage in a variety of risk management/coping strategies to smooth interyear consumption in the face of unstable output and drought risk (Matlon, 1990); e.g. income diversification (Reardon *et al.*, 1992), crop diversification (Matlon, 1990; Norman, 1973), livestock holdings (Christensen, 1989), crop storage (Udry,

1993), and informal credit (Udry, 1994). However, this paper is concerned with formal drought insurance for the following reasons:

First, most informal strategies do not fully insure in the case of drought, which affects all households within a village or even over large areas. During severe droughts, informal transfers within a rural community, such as state-contingent transfers and remittances among friends and neighbors, will not be effective if the community as a whole does not have enough food. In addition, local interest rates increase because many households seek credit at the same time, local wages decline due to increase in labor supply, and livestock prices drop because of distress sales of livestock. Few informal strategies are immune against the covariability problem; exceptions are seasonal migration and remittance from non-household members living far away or abroad[1].

Second, except for the few informal risk-sharing across zones that do not experience drought simultaneously, a formal institution at this moment is public food aid. However, expectation of food aid may be causing a moral hazard problem to households.

Third, existing WASAT household strategies for risk management may be neither cost-effective nor shared by all households. It is natural that self-insurance costly. But here, a few important costs associated with self-insurance are to be mentioned. One cost is that livestock holdings and crop stocks as precautionary savings are not productive. If households can invest in more productive activities, their welfare will increase in the long-run. The other cost is the degradation of grazing land. Since livestock is kept on open-access grazing land, household self-insurance strategy through livestock holdings is likely cause overgrazing.

To overcome the problems above, therefore, formal insurance will be needed. Formal insurance can pool risks across agroclimatic zones and will possibly be less costly than current informal mechanisms. But formal insurance institutions are rarely developed in drought-prone regions of developing countries; in Burkina Faso there is none. Supply-side

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[1] There is an evidence that villages that have a good access to the urban area (capital city) are better insured against droughts in Burkina Faso. This results implies that informal mechanisms across zones are working (Sakurai, 1995).

causes of market failure, such as moral hazard and high transaction costs, have been relatively well studied (Binswanger, 1986). To avoid the supply-side problems, a "rainfall lottery" has been considered, in which if annual rainfall reported by the regional whether station is lower than a level predetermined by an insurance company, the company will pay an indemnity (Gautam *et al.*, 1994; Hazell, 1992). This formal drought insurance scheme can require less transaction cost because the insurance firm does not need to inspect drought damage of each insurant, and can avoid moral hazard because indemnity payment does not depend on individual's drought damage[2]. Following them, this paper applies this hypothetical drought insurance scheme.

Since the rainfall lottery is not a perfect insurance, the question is whether households are willing to buy the hypothetical drought insurance as a substitute for and/or as a supplement to the existing informal insurance strategies or not. If the answer is yes, then the next question is who will buy it. This paper addresses those demand issues, which has not been empirically examined.

Demand for the hypothetical drought insurance is inferred from household production input decisions without formal insurance. This approach was developed by Gautam *et al.* (1994), following from literature which infers risk attitudes from risk avoidance behavior in agricultural production (Antle, 1987; Antle, 1989; Moscardi and Janvry, 1977). The model has been modified in the context of the WASAT and used to estimate demand for the hypothetical drought insurance (Sakurai *et al.*, 1994). The method this chapter uses is summarized as follows: (i) Set out a dynamic household model, in which a household maximizes its utility under budget and labor constraints with self-insurance; (ii) Derive the first order conditions that become the behavioral equations for precautionary saving, crop production, and off-farm activities. They represent "self-insurance equilibrium", which we are observing; (iii) Set out a similar dynamic household model, but with the hypothetical

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[2] Exactly speaking, there will be small moral hazard because a household will choose riskier strategies with the formal drought insurance than without it.

formal drought insurance, or the rainfall lottery; (iv) Derive the first order conditions that represent "formal-insurance equilibrium", which we cannot observe; (v) Define "equilibrium premium rate". This is the premium rate at which the household is indifferent between the self-insurance equilibrium and the formal-insurance equilibrium; (vi) In the case of rainfall lottery, the equilibrium premium rate is expressed by the ratio of the expected marginal values of a good year (non-drought year) and a drought year. This is the marginal rate of substitution between a good year and a drought year, measuring the utility trade-off between the two types of year; (v) If the equilibrium premium rate estimated above is greater than the probability of drought[3], there exists "effective demand".

The approach of this chapter significantly differs from previous papers in the assumption about the household's view of the world (summarized in Table 1). In Gautam *et al.* (1994), for example, all households have the same binary view of the world: a good year and a drought year, and believe the same, fixed probability of drought based on which each household optimizes the intertemporal allocation of resources. The household's definition of a drought is shared by the insurance company which runs the rainfall lottery. That is, the definition of a drought is exogenous to both households and to the insurance company. In Sakurai *et al.* (1994) there is no *a priori* definition of a drought. Each household has a binary view, but households' subjective beliefs of the probability of drought differ although they all know the true distribution of annual rainfall. This means that households define a drought based on their welfare, and consequently their subjective probabilities change according to their wealth and experience of drought. That is, they are Bayesian decision makers: Based on (subjective) belief, each household optimizes the intertemporal allocation of resources. On the other hand, the insurance company can set any arbitrary rainfall level to define a drought. Therefore, a drought year by a household's subjective judgment is not necessarily a drought year by the insurance company's definition. In this chapter, however, households do not have a binary view of drought, but they all know the true distribution of

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3 The probability of drought is defined later in this section.

rainfall based on which each household optimizes the intertemporal allocation of resources. If an insurance company provides a rainfall lottery, then households come to have a binary assessment of drought with the probability of drought arbitrarily set by the company. The new approach of this chapter can, thus, avoid the strong assumption that a household has a binary view without formal insurance, either objective or subjective. This is important because we cannot know whether households have a binary view or not and what probability they believe, merely based on observation of households' behavior without formal drought insurance.

Based on that assumption, probability of a drought year is defined as follows: (i) Annual rainfall is assumed to be distributed normally with fixed mean and variance obtained from long-term rainfall statistics; (ii) Households and the insurance firm know this distribution, but there is no objective definition of a drought year; (iii) The insurance firm sets a rainfall cut-off point arbitrarily to define a drought year for the rainfall lottery. Households who buy this lottery now have a binary view of the world; (iv) Since a normal distribution is assumed, the probability of a drought year is easily calculated for a given cut-off point. Thus, the cut-off point can be expressed as either a rainfall level or a drought probability. Since they are equivalent, the drought probability is used instead of a rainfall cut-off point; (v) Thus, a low drought probability implies a low cut-off rainfall level and low expected rainfall in a drought year, that is, a severe drought occurring with low frequency.

2 The Model

2.1 Time Frame

Roughly speaking, a household makes two major decisions about its income strategies during a year: off-farm work (mainly done in the dry season in rural Burkina) and agricultural production (done only in the rainy season). That agricultural output is a key variable leads us to the concept of "harvest year", which begins with the harvest at the beginning of the dry season and ends at the end of the rainy season, with the simplifying

assumption that all harvesting is done instantly at the beginning of each harvest year. The harvest year in the survey is defined to begin September 1 and to end August 31. The dry season of harvest year t is named "season 1 of year t" and the rainy season of harvest year t is named "season 2 of year t", as shown in Figure 1.

At the beginning of season 1 of year t, the household knows its initial wealth which consists of the harvest of the previous year t-1, carry-over wealth from year t-1, and off-farm income. Carry-over wealth consists of crop stocks and livestock holdings. Off-farm income is assumed to be "expected with certainty" at the beginning of season 1. Off-farm income includes pensions, remittances from non-household members, and income from local employment and migration. Given knowledge about initial wealth as well as its characteristics, the household decides: (i) how to allocate family labor between on-farm and off-farm activities in season 2; (ii) how to allocate its initial wealth among consuming, precautionary saving, buying inputs for agricultural production, and buying (hypothetical) insurance if available; and (iii) how much to diversify crops. Although agricultural production takes place in season 2, it is assumed that decisions about agricultural production, such as input use and crop diversification, are made simultaneously at the beginning of the harvest year.

In season 2 the household undertakes both agricultural production and off-farm work according to the input decisions made in season 1. However, agricultural output also depends on rainfall in season 2 and on household-specific shocks, which are stochastic. I assume here that uncertainty in agricultural production comes from rainfall uncertainty, but the magnitude of the drought shock on agricultural production depends on the household's characteristics and its decisions about input use and crop diversification.

2.2 A Dynamic Household Model

First a model without formal drought insurance is considered. I apply a standard intertemporal utility maximization problem given by a time-separable lifetime utility function for household i as follows:

$$\max \; E_{it} \sum_{k=0}^{T} \delta_i^k U_i(C_{it+k}, L_{it+k}) \qquad (1)$$

where i refers to household, t refers to year, $U_i(\ )$ is the one-period utility function, C_{it} is real consumption in year t, L_{it} is leisure in year t, δ_i is the discount factor, E_{it} is the expectation operator (conditional on information available at year t), and T is the end of the household's planning horizon. Standard assumptions about the utility function apply: $U_i(\ )$ is monotonically increasing and concave in C_{it} and L_{it}. See Gautam *et al.* (1994) for theoretical considerations.

As is shown in Figure 1, year t starts with the harvest from season 2 in year t-1, and therefore the household's initial wealth is given by

$$W_{it} = S_{it-1} + Y_{it-1} + M_{it} \qquad (2)$$

where S_{it-1} is carry-over wealth from year t-1 to year t and Y_{it-1} is agricultural output in year t-1. M_{it} is wealth that is realized in year t with certainty, but is conditioned by year t-1's state (good or drought), and therefore is predetermined at the beginning of year t. M_{it} includes off-farm income to be earned (with certainty) in season 1 of year t, plus gifts and public food aid received in year t.

Labor constraints are as follows:

$$N_{it} + L_{it}^h = L_{it} + L_{it}^f + L_{it}^o \qquad (3)$$

where N_{it} is the exogenous, total time available to the household in year t, L_{it}^h is hired labor and L_{it} is leisure that appears in the utility function. Equation (3) means that leisure is determined by on-farm labor supply (L_{it}^f), hired labor (L_{it}^h) and off-farm labor supply including local on-farm employment in season 2 (L_{it}^o). Off-farm labor supply in season 1 is predetermined at the time of decision making for year t and does not appear in the labor constraints. This implies that off-farm labor supply in season 1 is already subtracted from total available labor, N_{it}. The labor variables are expressed in monetary terms.

The budget constraints are as follows:

$$W_{it} + L_{it}^o = C_{it} + K_{it} + L_{it}^h + S_{it} \qquad (4)$$

Equation (4) means that the household's initial wealth (W_{it}) and off-farm income in season 2 (L_{it}^{o}) are allocated in period t among consumption (C_{it}), purchased farm inputs for agricultural production (K_{it}), hired labor (L_{it}^{h}), and savings (S_{it}).

As defined in equation (2), initial wealth (W_{it}) includes carry-over wealth from year t-1 (S_{it-1}), but in equation (4) I assume that there is no interest accrual. In rural Burkina Faso there is no formal savings mechanism (rural bank): livestock holdings and crop stocks are used as substitutes. Livestock may provide positive interest (reproduction), but on the other hand it is subject to capital risks due to price fluctuation. To make the model simple, I assume there is no capital gain or loss on wealth. Borrowing is handled as a subset of saving: if the saving decision in period t (S_{it}) is negative, it implies that the household will be a net borrower in year t. No interest for borrowing is assumed[4].

Agricultural output in year t (Y_{it}) is a function of endogenous purchased farm inputs and labor (K_{it} and L_{it}^{f}) and crop diversification (D_{it}) in year t, and exogenous or predetermined variables including yearly rainfall (R_t) and a vector of household/plot characteristics (X_{it}):

$$Y_{it} = F(L_{it}^{f}, K_{it}, D_{it}, R_t, X_{it}) \qquad (5)$$

Crop diversification (D_{it}) is a Simpson index[5]. D_{it} is greater than or equal to zero and less than unity; D_{it} is exactly equal to zero when a household produces only one crop, and is unity when a household produces infinite kinds of crops, which is impossible.

Without formal drought insurance, the household forms its expectations based on a given rainfall distribution. By using the equations above, the dynamic optimization problem for the household can be written as

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[4] Although no interest for borrowing is assumed, the household cannot borrow limitlessly because by the end of the household's planning horizon (period T), all debt must be repaid. In other words, the household is not free from life-time budget constraint. Moreover, since there is uncertainty in future income, borrowing is riskier for the household than without uncertainty.

[5] $$D_{it} = 1 - \sum_{k} \left[\frac{\textit{planted area for crop k (ha)}}{\textit{household's total planted area (ha)}} \right]^2$$

$$V_{it}(W_{it}) = \max_{\Pi_t} U_i(C_{it}, L_{it}) + \delta_i \int_{-\infty}^{\infty} V_{it+1}(W_{it}) f(R) dR \tag{6}$$

or

$$\begin{aligned} V_{it}(W_{it}) = \max_{\Pi_t} & U_i(W_{it} + L_{it}^o - K_{it} - L_{it}^h - S_{it},\ N_{it} + L_{it}^h - L_{it}^f - L_{it}^o) \\ & + \delta_i \int_{-\infty}^{\infty} V_{it+1}(S_{it} + Y_{it} + M_{it+1}) f(R) dR \end{aligned} \tag{7}$$

where $\Pi_{it} = (D_{it}, L_{it}^f, L_{it}^o, L_{it}^h, K_{it}, S_{it})$ is the vector of decision variables of period t, and $V_{it}(\ )$ is the maximum value of future utility discounted back to t, and $f(R)$ is the probability density function of rainfall. Given the standard assumptions for the utility function, the value function, $V(\ )$, is also monotonically increasing and concave in W.

2.3 The First Order Conditions

Assuming that households behave optimally without formal drought insurance, what we observe is a "self-insurance equilibrium", given by the first order conditions obtained by differentiating equation (7) with respect to the control variables, Π_{it}. They are as follows:

D_{it}:
$$\delta_i \int_{-\infty}^{\infty} \frac{\partial V_{it+1}(W_{it+1})}{\partial W_{it+1}} \frac{\partial Y_{it}}{\partial D_{it}} f(R) dR = 0 \tag{8}$$

L_{it}^f:
$$-\frac{\partial U_i}{\partial L_{it}} + \delta_i \int_{-\infty}^{\infty} \frac{\partial V_{it+1}(W_{it+1})}{\partial W_{it+1}} \frac{\partial Y_{it}}{\partial L_{it}^f} f(R) dR = 0 \tag{9}$$

L_{it}^o:
$$\frac{\partial U_i}{\partial C_{it}} - \frac{\partial U_i}{\partial L_{it}} = 0 \tag{10}$$

L_{it}^h:
$$-\frac{\partial U_i}{\partial C_{it}} + \frac{\partial U_i}{\partial L_{it}} = 0 \tag{11}$$

K_{it}:
$$-\frac{\partial U_i}{\partial C_{it}} + \delta_i \int_{-\infty}^{\infty} \frac{\partial V_{it+1}(W_{it+1})}{\partial W_{it+1}} \frac{\partial Y_{it}}{\partial K_{it}} f(R) dR = 0 \tag{12}$$

S_{it}:
$$-\frac{\partial U_i}{\partial C_{it}} + \delta_i \int_{-\infty}^{\infty} \frac{\partial V_{it+1}(W_{it+1})}{\partial W_{it+1}} f(R) dR = 0 \tag{13}$$

Under the hypothetical formal drought insurance scheme, or the rainfall lottery, the insurance company sets a rainfall cut-off point denoted by R^*. If the accumulated rainfall for the year is below the cut-off point, the year is defined as a "drought year"; a "good year" has rainfall greater than or equal to R^*. Consequently the definition of drought year is weather-station-specific rather than household-specific.

If this insurance scheme were available, household i would have a binary view of the world, and be at a new equilibrium called "formal-insurance equilibrium". Budget constraint (4) would become

$$W_{it} + L_{it}^{o} = C_{it} + K_{it} + L_{it}^{h} + S_{it} + \rho I_{it} \tag{14}$$

where ρI_{it} is the insurance premium that the household would pay if such a scheme were available, and ρ is the known exogenous premium rate against total indemnity I_{it}. The total indemnity, I_{it}, is decided by the household with other input decisions, Π_{it}. If year t is a good year, using superscript g, the initial wealth of year t+1 is $W_{it+1}^{g} = S_{it} + Y_{it}^{g} + M_{it}^{g}$. And similarly, if year t is a drought year, using superscript d, initial wealth in year t+1 is $W_{it+1}^{d} = S_{it} + Y_{it}^{d} + M_{it}^{d} + I_{it}$. These two equations go into the dynamic optimization given by (7).

The formal-insurance equilibrium is given by values $(\tilde{W}_{it+1}^{g}, \tilde{W}_{it+1}^{d}, \tilde{C}_{it}, \tilde{L}_{it}, \tilde{\Pi}_{it}, I_{it})$. If $I_{it} > 0$, the first order conditions will be

$$D_{it}: \quad \delta_i \int_{-\infty}^{R^*} \frac{\partial V_{it+1}(\tilde{W}_{it+1}^{d})}{\partial W_{it+1}} \frac{\partial Y_{it}}{\partial D_{it}} f^{d}(R)dR + \delta_i \int_{R^*}^{\infty} \frac{\partial V_{it+1}(\tilde{W}_{it+1}^{g})}{\partial W_{it+1}} \frac{\partial Y_{it}}{\partial D_{it}} f^{g}(R)dR = 0 \tag{15}$$

$$L_{it}^{f}: -\frac{\partial U_i(\tilde{C}_{it}, \tilde{L}_{it})}{\partial L_{it}} + \delta_i \int_{-\infty}^{R^*} \frac{\partial V_{it+1}(\tilde{W}_{it+1}^{d})}{\partial W_{it+1}} \frac{\partial Y_{it}}{\partial L_{it}^{f}} f^{d}(R)dR + \delta_i \int_{R^*}^{\infty} \frac{\partial V_{it+1}(\tilde{W}_{it+1}^{g})}{\partial W_{it+1}} \frac{\partial Y_{it}}{\partial L_{it}^{f}} f^{g}(R)dR = 0 \tag{16}$$

$$L_{it}^{o}: \quad \frac{\partial U_i(\tilde{C}_{it}, \tilde{L}_{it})}{\partial C_{it}} - \frac{\partial U_i(\tilde{C}_{it}, \tilde{L}_{it})}{\partial L_{it}} = 0 \tag{17}$$

$$L_{it}^{h}: \quad -\frac{\partial U_i(\tilde{C}_{it}, \tilde{L}_{it})}{\partial C_{it}} + \frac{\partial U_i(\tilde{C}_{it}, \tilde{L}_{it})}{\partial L_{it}} = 0 \tag{18}$$

$$K_{it}: -\frac{\partial U_i(\tilde{C}_{it}, \tilde{L}_{it})}{\partial C_{it}} + \delta_i \int_{-\infty}^{R^*} \frac{\partial V_{it+1}(\tilde{W}_{it+1}^{d})}{\partial W_{it+1}} \frac{\partial Y_{it}}{\partial K_{it}} f^{d}(R)dR + \delta_i \int_{R^*}^{\infty} \frac{\partial V_{it+1}(\tilde{W}_{it+1}^{g})}{\partial W_{it+1}} \frac{\partial Y_{it}}{\partial K_{it}} f^{g}(R)dR = 0 \tag{19}$$

$$S_{it}: -\frac{\partial U_i(\tilde{C}_{it}, \tilde{L}_{it})}{\partial C_{it}} + \delta_i \int_{-\infty}^{R^*} \frac{\partial V_{it+1}(\tilde{W}_{it+1}^{d})}{\partial W_{it+1}} f^{d}(R)dR + \delta_i \int_{R^*}^{\infty} \frac{\partial V_{it+1}(\tilde{W}_{it+1}^{g})}{\partial W_{it+1}} f^{g}(R)dR = 0 \tag{20}$$

$$I_{it}: \quad -\rho \frac{\partial U_i(\tilde{C}_{it}, \tilde{L}_{it})}{\partial C_{it}} + \delta_i \int_{-\infty}^{R^*} \frac{\partial V_{it+1}(\tilde{W}_{it+1}^{d})}{\partial W_{it+1}} f^{d}(R)dR = g_i(\tilde{C}_{it}, \tilde{L}_{it}, \tilde{W}_{it+1}^{d}) = 0 \tag{21}$$

where $f^{d}(R)$ and $f^{g}(R)$ are truncated distribution density functions: that is, the rainfall distribution density function, $f(R)$, is truncated by the rainfall cut-off point, R^*.

If the first order condition (21) is evaluated at the self-insurance equilibrium, it will become

$$-\rho\frac{\partial U_i(C_{it}, L_{it})}{\partial C_{it}} + \delta_i\int_{-\infty}^{R^*}\frac{\partial V_{it+1}(W^d_{it+1})}{\partial W_{it+1}}f^d(R)dR = g_i(C_{it}, L_{it}, W^d_{it+1}) \tag{22}$$

Then, $g_i(C_{it}, L_{it}, W^d_{it+1}) \le 0$ shows no potential demand for the formal drought insurance at the self-insurance equilibrium, while $g_i(C_{it}, L_{it}, W^d_{it+1}) > 0$ shows potential demand. Similarly, all the remaining first order conditions (15)-(20) are evaluated at the self-insurance equilibrium. From (20) and (22), conditions for potential demand for the formal drought insurance at the self-insurance equilibrium is expressed as

$$\rho < \frac{\int_{-\infty}^{R^*}\frac{\partial V_{it+1}(W^d_{it+1})}{\partial W_{it+1}}f^d(R)dR}{\int_{-\infty}^{R^*}\frac{\partial V_{it+1}(W^d_{it+1})}{\partial W_{it+1}}f^d(R)dR + \int_{R^*}^{\infty}\frac{\partial V_{it+1}(W^s_{it+1})}{\partial W_{it+1}}f^s(R)dR} = \frac{\phi_{it}}{1+\phi_{it}} \tag{23}$$

where ϕ_{it} is the marginal rate of substitution at the self-insurance equilibrium between drought and good years for a given threshold R^*. ϕ_{it} is the utility weight for a drought year if the weight for a good year is set to be unity, and measures the utility trade-off between two types of years. ϕ_{it} is defined by

$$\phi_{it} = \frac{\int_{-\infty}^{R^*}\frac{\partial V_{it+1}(W^d_{it+1})}{\partial W_{it+1}}f^d(R)dR}{\int_{R^*}^{\infty}\frac{\partial V_{it+1}(W^s_{it+1})}{\partial W_{it+1}}f^s(R)dR} = \frac{V^{d'}}{V^{s'}} \tag{24}$$

In (23), if $\frac{\phi_{it}}{1+\phi_{it}}$ is equal to ρ, the household will be indifferent between purchasing and not-purchasing the formal drought insurance. Therefore, $\frac{\phi_{it}}{1+\phi_{it}}$ is called household i's "equilibrium premium rate" at the self-insurance equilibrium, and is denoted by ρ^*_{it}. The demand is "effective", when a household is willing to pay more premium than the premium of an actuarially fair drought insurance. For the demand to be effective, it is necessary that ρ^*_{it} is greater than the probability of a drought year given by $q = \Pr(R \le R^*)$. When effective demand exists, the insurance firm can set premium rate higher than the probability of a drought year so that it make enough revenue to cover administration costs and to make a

profit from the insurance scheme[6]. From (23), the condition becomes $q < \rho_{it}^{*} = \frac{\phi_{it}}{1+\phi_{it}}$, which is equivalent to

$$\phi_{it} > \frac{q}{1-q} \tag{25}$$

Using ϕ_{it} from (24), the first order conditions (15)-(20) evaluated at self-insurance equilibrium give the self-insurance equilibrium relationships that show how the household weights future cost and benefit[7].

Equation (15) can be rewritten as

$$\int_{R^*}^{\infty} \frac{\partial Y_{it}}{\partial D_{it}} f^s(R)dR = -\phi_{it} \int_{-\infty}^{R^*} \frac{\partial Y_{it}}{\partial D_{it}} f^d(R)dR \tag{26}$$

The left hand side of equation (26) is the marginal cost of crop diversification, and the right hand side is the marginal benefit of crop diversification accrued only in drought years. These derivatives should have opposite signs, but we cannot assign *a priori* a sign to those derivatives. Moreover, both sides can be zero: in this case household i chooses crop diversification to maximize both good year and drought year outputs. If such D_{it} exists, equation (26) will have no role in estimating ϕ_{it}.

From (16)-(20), the labor condition is obtained as

$$1+\phi_{it} = \int_{R^*}^{\infty} \frac{\partial Y_{it}}{\partial L_{it}^f} f^s(R)dR + \phi_{it} \int_{-\infty}^{R^*} \frac{\partial Y_{it}}{\partial L_{it}^f} f^d(R)dR \tag{27}$$

The left hand side of equation (27) is the marginal return to off-farm labor, i.e. the wage rate (=1), weighted by 1 and ϕ_{it}. The right hand side is the marginal return to labor used in agricultural production, weighted by 1 and ϕ_{it}. If $\int_{R^*}^{\infty} \frac{\partial Y_{it}}{\partial L_{it}^f} f^s(R)dR = 1$, any ϕ_{it} is not determined by this condition. To have a unique, positive ϕ_{it}, both $\int_{R^*}^{\infty} \frac{\partial Y_{it}}{\partial L_{it}^f} f^s(R)dR > 1$ and

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[6] This is a condition that the insurance firm can make a positive, gross revenues. It is a necessary condition for the firm to cover administration cost and to make a net profit, but not sufficient.

[7] A critical assumption to derive the relationships is that the covariances between marginal value of value function and the marginal productivities of inputs are zero, although they are affected by rainfall. That is, $\text{cov}\left(\frac{\partial V_{it+1}}{\partial W_{it}}, \frac{\partial Y_{it}}{\partial D_{it}}\right) = 0$, $\text{cov}\left(\frac{\partial V_{it+1}}{\partial W_{it}}, \frac{\partial Y_{it}}{\partial L_{it}^f}\right) = 0$, and $\text{cov}\left(\frac{\partial V_{it+1}}{\partial W_{it}}, \frac{\partial Y_{it}}{\partial K_{it}}\right) = 0$.

$\int_{-\infty}^{R^*} \frac{\partial Y_{it}}{\partial L_{it}^f} f^d(R)dR < 1$ are required, assuming $\int_{R^*}^{\infty} Y_{it} f^s(R)dR > \int_{-\infty}^{R^*} Y_{it} f^d(R)dR$. This condition means labor input is less than the optimum in a good year, and greater than the optimum in a drought year.

Equating (19) and (20) yields a saving equation as

$$1 + \phi_{it} = \int_{R^*}^{\infty} \frac{\partial Y_{it}}{\partial K_{it}} f^s(R)dR + \phi_{it} \int_{-\infty}^{R^*} \frac{\partial Y_{it}}{\partial K_{it}} f^d(R)dR \qquad (28)$$

The left side is the marginal return to savings, which is assumed to be unity, weighted by 1 and ϕ_{it}. The right hand side of equation (28) is the marginal agricultural product of purchased variable inputs, weighted by 1 and ϕ_{it}. This saving equation has implications for determination of ϕ_{it} similar to those discussed with respect to the labor equation.

3 Data

The household data used to estimate the models were collected in a farm household survey in Burkina Faso conducted by International Crops Research Institute for the Semi-Arid Tropics (ICRISAT). The panel data used cover three full harvest-years, 1981/82-1983/84 (see Figure 1), with a sample of 89 households in each year, spread over five villages, in three agroclimatic zones: the Sahelian, in the northwest; the Sudanian, in the Central Plateau; and the Guinean, in the southwest.

These three zones differ in rainfall level and variability. Annual rainfall data and long-term statistics are shown in Table 2. The village-level rainfall data were observed by survey enumerators in the surveyed villages, and the long-term averages and standard deviations are from weather stations that cover those villages[8]. The Sahelian zone has the lowest and most variable rainfall, so cropping is the riskiest. The Guinean zone is moderately-favored agroclimatically, with medium to high rainfall that is least variable, thus cropping is

[8] The weather stations are located in Djibo (Sahelian), Yako (Sudanian), and Boromo (Guinean). Data are through 1983 and include 29 years for Djibo, 38 years for Yako, and 58 years for Boromo. Source: Sivakumar and Gnoumou (1987).

much less risky. The Sudanian zone is between the two zones in rainfall level and variability. Cropping is only rainfed in all three zones, and undertaken only by smallholders. Note that most rainfall levels during the survey period are lower than the long-term average.

4 Empirical Estimation

The empirical analysis goes as follows: (i) Estimate parameters of the behavioral equations at the self-insurance equilibrium; (ii) Calculate expected marginal values of a good year and a drought year using the parameters estimated in the first step, and obtain the equilibrium premium rate. To have expected values, we need to know rainfall distribution and a rainfall cut-off point that separates a drought year from a good year. As noted before, the cut-off point is expressed as a probability of drought; (iii) Examine whether there is effective demand for the formal drought insurance for several drought probabilities; (iv) Examine how the household equilibrium premium rate varies with household assets and characteristics via determinant analysis; (v) Split the sample in various ways to see if that affects determinants.

4.1 Production Function

For equation (5) a modified Cobb-Douglas function with interaction terms is specified as follows:

$$\ln Y_{it} = \alpha_1 \ln L^f_{it} + \beta_1 \ln K_{it} + \eta_1 \ln D_{it} + \mu_1 \ln A_{it} + \omega R_{vt} + \alpha_2 R_{vt} \ln L^f_{it} + \beta_2 R_{vt} \ln K_{it} + \eta_2 R_{vt} \ln D_{it} + \mu_2 R_{vt} \ln A_{it} + \sum_{i=1}^{n} h_i HH_i + e_{it} \quad (29)$$

where subscript i refers to household, v refers to village and t refers to year. The variables are: Y_{it}, household i's agricultural output in 1000*Franc CFA (FCFA) per hectare; L^f_{it}, household i's farm labor input (both own and hired) per hectare (in 1000*FCFA) assuming that labor is paid the market wage; K_{it}, household i's purchased variable inputs per hectare (in 1000*FCFA), which includes purchased fertilizer, and seed (both own and purchased); D_{it}, household i's crop diversification index as defined in footnote 1; A_{it}, household i's total cultivated area (hectares); R_{vt}, rainfall in village v in year t (in 1000 mm); HH_i, a dummy for

household i, or household i's fixed effect; e_{it}, the disturbance term. This specification includes interaction terms between inputs and rainfall, which capture change in marginal productivity depending on rainfall. This production function is estimated separately in fixed-effect estimation in each agroclimatic zone (Table 3).

4.2 Estimation of Demand for Drought Insurance (ϕ)

Applying the production function to the optimality conditions (26), (27), and (28), and solving for ϕ_{it}, results in the following three equations:

$$\phi_{it}^{D} = -\frac{\int_{R^*}^{\infty}(\eta_1 + \eta_2 R_z)Y_{it}f_z^s(R)dR}{\int_{-\infty}^{R^*}(\eta_1 + \eta_2 R_z)Y_{it}f_z^d(R)dR} \tag{30}$$

$$\phi_{it}^{L} = \frac{\int_{R^*}^{\infty}(\alpha_1 + \alpha_2 R_z)Y_{it}f_z^s(R)dR - L_{it}^f}{L_{it}^f - \int_{-\infty}^{R^*}(\alpha_1 + \alpha_2 R_z)Y_{it}f_z^d(R)dR} \tag{31}$$

$$\phi_{it}^{K} = \frac{\int_{R^*}^{\infty}(\beta_1 + \beta_2 R_z)Y_{it}f_z^s(R)dR - K_{it}}{K_{it} - \int_{-\infty}^{R^*}(\beta_1 + \beta_2 R_z)Y_{it}f_z^d(R)dR} \tag{32}$$

Since ϕ_{it} is obtained in each equation above, it now has superscripts D, L, or K. α_1, α_2, β_1, β_2, η_1, and η_2 are parameters obtained from the estimation of the production function specified as (29). L_{it}^f, and K_{it} are production input decisions, the same as defined in the specification of the production function. They are endogenous and are assumed to be decided at the beginning of each year before knowing the level of rainfall. R_z^* is rainfall cut-off point for zone z, and $f_z^s(R)$ and $f_z^d(R)$ are truncated rainfall distribution density functions for zone z. To have expected values based on limited long-term rainfall statistics (mean and variance) for each zone, rainfall is assumed to be normally distributed and independent over years. Here we use long-term mean and variance because we think that they are more appropriate for estimating demand for drought insurance[9]. The relationship

[9] As noted in section 3 and shown in Table 1, average rainfall during the survey period is lower than the long-term average. Using rainfall series in this region, we examine how the change in rainfall distribution affects frequency of drought defined by this scheme (Appendix A). The results suggest that the insurance company should use the long-term distribution because the short-term distributions (in this case 10 years) vary a lot depending on which decade to choose.

between the assigned probability of a drought year, q, and the rainfall cut-off point, R_z^*, in each zone are shown in Appendix B. Using those R_z^*, ϕ_{it}^D, ϕ_{it}^L, and ϕ_{it}^K are calculated for each case in equations (30), (31), and (32) respectively. The methods are in Appendix C.

A unique ϕ attributed to the group is estimated in a system as follows:

$$\phi_{it}^D = \phi + e_{it}^D \tag{33}$$

$$\phi_{it}^L = \phi + e_{it}^L \tag{34}$$

$$\phi_{it}^K = \phi + e_{it}^K \tag{35}$$

The system (33)-(35) is estimated by iterated feasible generalized least squares and ϕ is obtained as a coefficient of unity for three equations under equality restriction across equations. Correlation in disturbances is assumed. The three zones are estimated together, and then separately to know the differences between zones for each probability of drought given in Appendix B. The results are shown in Table 5.

4.3 Determinants of Demand for Drought Insurance (ϕ)

ϕ is estimated as a measure of risk aversion for the community. However, it varies with household wealth across a community or even across years for a given household depending on shocks received. Thus I examine determinants of ϕ. Within a household, ϕ changes each year depending on initial wealth. But because ϕ is used as a measure of demand for drought insurance (reflecting a fixed preference), I use two different specifications to examine effects of household wealth. The first is a cross-sectional specification, and the second one is a fixed-effect specification. The cross-sectional system specifications as follows[10]:

$$\phi_{it}^D = \sum_j \xi_j \bar{X}_{ji} + \sum_k \zeta_k X_{kit} + v_{it}^D \tag{36}$$

$$\phi_{it}^L = \sum_j \xi_j \bar{X}_{ji} + \sum_k \zeta_k X_{kit} + v_{it}^L \tag{37}$$

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[10] This is not perfectly cross-sectional, because time-variant variables are used as in the pooled regression. However, here it is named cross-sectional regression, because the time-invariant variables used in equations are time-means to capture cross-section effects.

$$\phi_{it}^{K} = \sum_{j} \xi_j \bar{X}_{ji} + \sum_{k} \zeta_k X_{kit} + \nu_{it}^{K} \qquad (38)$$

ϕ_{it}^{D}, ϕ_{it}^{L}, and ϕ_{it}^{K} are calculated based on (30), (31), and (32) respectively, as explained above. ν_{it}^{D}, ν_{it}^{L}, and ν_{it}^{K} are disturbances. The system is estimated by iterated feasible generalized least squares. Equality restrictions across equations are imposed on ζ_k and ξ_j respectively, and correlation in disturbances is assumed.

$\bar{X}_{ji}$ are time-invariant variables so as to reflect average characteristics rather than time-varying decisions. The variable set includes a constant, the household's average off-farm income in season 1 (local self-employment, local agriculture employment and migration income), its average livestock holdings, its average crop production, its average amount of crop gifts received from other households, the average amount of public food aid the household received, its cultivated area, and overall soil quality of the household's plots (how many years a household continues to use the same plot on average weighted by plot area). All variables are expressed in 1000 FCFA per adult equivalent (AE) except for cultivated area which is in hectares per AE and overall soil quality which is in years. All of these variables are part of the wealth that the household expects to have. Means and standard deviations of those wealth variables are shown in Appendix D.

X_{kit} are time-variant variables. They include the household's characteristics and the characteristics of the household's plots. The household characteristics variables are the AE size of the household, the age of the household head, the number of wives of the household head, the dependency ratio (percentage children younger than 15), a dummy for compound leader (1 if the leader of the household is also compound leader), a dummy for single conjugal unit (1 if the household has a single conjugal unit), a dummy for animal traction (1 if the household uses animal traction). Household plot characteristics are: soil-type diversification index (using a Simpson index), a toposequence index, and the average distance between the residence and the plots weighted by plot size (Km). The soil-type diversification index measures heterogeneity of soil types among the household's plots. In general, the more

diversified the soil types on a given farm, the less risky is crop production. The toposequence index is the household's average of plots' toposequence index assigned as 1=uppermost, 2=next to 1, 3=mid-slope, 4=next to 5, 5=swamp. In general, the higher the toposequence index, the less risky is crop production.

Characteristics of a household that increase its capacity to diversify income sources, either within crop production or within overall income, will have a negative effect on ϕ, and those making the household more risk averse will have a positive effect on ϕ. But those effects are ambiguous *a priori.*

The fixed effect specification is as follows:

$$\phi_{it}^{D} = \sum_{k} \zeta_k X_{kit} + u_i + v_{it}^{D} \tag{39}$$

$$\phi_{it}^{L} = \sum_{k} \zeta_k X_{kit} + u_i + v_{it}^{L} \tag{40}$$

$$\phi_{it}^{K} = \sum_{k} \zeta_k X_{kit} + u + v_{it}^{K} \tag{41}$$

u_i is a household specific (fixed) effect, and the X_{kit} are time-variant variables. ϕ_{it}^{D}, ϕ_{it}^{L}, and ϕ_{it}^{K} are the calculated ϕ, and v_{it}^{D}, v_{it}^{L}, and v_{it}^{K} are disturbances. The system is estimated by iterated feasible generalized least squares. Equality restrictions across equations are imposed on ζ_k and u_i respectively and correlation in disturbances is assumed. There is no time-invariant variable ($\overline{X}_{ji}$) other than u_i. Therefore, u_i captures the time-invariant part of household and plot characteristics, as well as the household's average wealth as a fixed household characteristic. The time-variant variables X_{kit} are the initial assets of each year: off-farm income to be earned, livestock holdings, crop production of the previous year, public food aid to be received, crop gifts from other households to be received (in 1000 FCFA per AE), and cultivated area in hectares per AE. As explained above in the section on the dynamic household model, I assume that off-farm income in season 1, food aid, and crop gifts are expected with certainty when a household makes a decision.

In this fixed effect specification, we can know the effects of change in the household wealth level on demand for drought insurance. Because I use the agricultural output of the previous year as an initial asset variable, data from the first harvest year has to be dropped.

Using those two specifications, I estimate the effects of exogenous or predetermined variables on demand for drought insurance, using the full sample as well as using sub-groups. Sub-groups are formed by zone or by stratification based on household average livestock holdings per AE: below 25 percentile, above 75 percentile, and between the two.

5 Results and Discussion

ϕ is a function of the expected marginal productivities of drought and good years, that is, the household's input decisions to avoid drought risk directly determine demand for drought insurance. Since the expected marginal productivities depend on rainfall distribution and the drought probability assigned, ϕ depends on them too. ϕ is estimated over a range of drought probabilities, from 10% to 40%, to see how ϕ changes as assigned drought probability, q, changes.

The relationship between group average ϕ and q is interpreted as follows: When considering a household group, except in the case of an extremely low q, the lower q, the more households in the group have effective demand, and when q is above some high point, no household demands the insurance hence the group's average ϕ is close to zero. This is because a low q implies a severe drought, while a high q implies a mild drought as explained in section 1. Note that since this ϕ is a zone-level average, zero ϕ does not necessarily mean that no household demands the drought insurance.

The change of individual ϕ_{it} is interpreted based on the definition given by (24), $\phi_{it} = \frac{V^{d'}}{V^{s'}}$, where $V^{d'}$ and $V^{s'}$ are marginal values of the value function given by the dynamic optimization problem, equation (6). As intuitively seen in Figure 2, drought probability, q, affects the combination of the two slopes and as a result gives a different ϕ_{it}. This relationship implies that a higher probability of drought makes the difference in expected wealth between good and drought years smaller, and as a result, gives a smaller

$\frac{V^{d'}}{V^{s}}$. If $V^{d'}$ is large enough relative to $V^{s'}$, the household will demand the drought insurance, while if they are close, the household will not demand it.

Therefore, households that are highly risk-averse and/or have a large drought shock have high ϕ' s. Thus, the relationship between asset variables and ϕ is considered as follows:

(i) Assets can have a positive effect. In this case, there will be two different situations. One is the case when the asset allows households to make risky decisions because the asset provides ability to smooth consumption. The other is the case when the asset is safe, but its insurance effect is not sufficient. For example, a household will tend to keep a safe asset if the household is very risk averse, but if the asset does not effectively smooth income, the household will have high ϕ. This means that a more risk-averse household will allocate more labor and capital to relatively safe assets and/or risk reducing activities, such as off-farm activities and savings (livestock holdings) than to risky crop production. But if they do not reduce the size of drought shock enough, the household will have a large ϕ. These two effects can be distinguished by regression specifications: the former is observed in the fixed effect regression and the latter is observed in the cross-section regression.

(ii) Assets can be a negative effect. This is the case when accumulated assets insure household income effectively. For example, a household that is able to earn enough off-farm income and/or to keep large livestock holdings will have a small drought shock, and also have a small ϕ.

Thus, assets have two opposite effects on ϕ. I name the case (i) "risk aversion effect" and the case (ii) "income effect". Because of those two effects, impacts of wealth variables, especially off-farm income and livestock holdings, are *a priori* ambiguous.

5.1 Production Function

Table 3 presents the parameter estimates of the production function. Using estimated parameters, I evaluate signs of marginal productivity and effects of rainfall on marginal

productivity at drought probabilities of 15% and 35% to show the production function is reasonably estimated. The results are summarized in Table 4, and the derivations are in Appendix E. Since the production function has interaction terms between inputs and rainfall as shown equation (29), the marginal productivities depend on rainfall level. Therefore, it is possible that a marginal productivity has different signs at 15% and 35% drought probabilities[11]. The reason why I choose 15% and 35% drought probabilities is based on the results of ϕ estimation explained in section 5.2.

In general, when $\frac{\partial^2 Y}{\partial Q \partial R} > 0$, input Q is risk-increasing (a decrease in rainfall causes a decrease in the marginal productivity of Q, or equivalently, the marginal productivity of rainfall increases with increasing use of Q), and when $\frac{\partial^2 Y}{\partial Q \partial R} < 0$, input Q is risk-decreasing (a decrease in rainfall causes an increase in the marginal productivity of Q, or equivalently, the marginal productivity of rainfall decreases with increasing use of Q). The optimal application of a risk-increasing (decreasing) input under risk averse preferences is always less (more) than the risk-neutral level of input use (Ramaswani, 1993).

The results shown in Table 4 are interpreted as follows:

(i) Labor

With respect to labor input, positive marginal productivity in both good and drought years is expected, but whether its use increases or decreases risk is ambiguous *a priori.*

In the Sahelian zone, labor has a negative marginal productivity and is risk-decreasing in a good year, while it has a positive marginal productivity and is risk-increasing in a drought year. Both are significant. The marginal productivity turns negative at 447 mm rainfall and above, and at the mean rainfall (480 mm) it is still close to zero, although it is

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[11] The signs in Table 4 are obtained as follows; For example, to know the sign of the Sahelian $\frac{\partial Y}{\partial L^f}$ in drought years at 15% drought probability, first, from Appendix B expected level of rainfall in a drought year at 15 % drought probability is found, which is 227 mm. Second, looking at the column that covers 227mm in Table E1, positive sign for the Sahelian $\frac{\partial Y}{\partial L^f}$ is found.

negative. Those signs imply that labor input is close to the optimum in a good year and is less than optimum in a drought year at a given labor input level decided before knowing rainfall.

In the Sudanian and the Guinean zones, labor has a positive marginal productivity and is risk-increasing. These signs imply that if a household uses more labor per hectare, yield per hectare will be higher in both good and drought years, but greater labor use causes the drought shock to productivity to be larger.

(ii) Capital (purchased variable inputs)

Capital also should have a positive marginal productivity in both good and drought years, but whether its use increases or decreases risk is also ambiguous *a priori*. The estimates are not significant in the Sahelian and the Guinean zones, probably because their use of fertilizer is small and ineffective and/or seed is overused.

In the Sahelian zone, the signs imply that at a given capital input level, capital has a negative marginal productivity and reduces risk in a drought year, while it has a positive marginal productivity and increases risk in a good year. Therefore, capital acts as insurance in a drought year.

In the Sudanian zone, capital has the same property as labor (see above).

In the Guinean zone, capital decreases risk by reducing output regardless of rainfall level. This property is the same as the capital input in the Sahelian zone in a drought year.

(iii) Crop Diversification

The effect of crop diversification is ambiguous *a priori*, as explained in section 2.3, although the standard view based on portfolio theory, such as Newbery and Stiglitz (1981), predicts that diversification reduces the variance of the return by reducing the expected return.

In all the three zones, estimates are significant, and crop diversification has similar effects: in a drought year, it gives a low output and a low risk, while in a good year it gives a high output and a high risk. Crop diversification acts as if an insurance in a drought year, but acts as a gamble in a good year. In the Sahelian zone, even in a good year crop diversification

is insurance if rainfall is not so high. This result implies that crop diversification measured by a Simpson index includes both risk-decreasing crop diversification, such as choosing safer portfolio, and risk-increasing crop diversification, such as growing cash crop.

(iv) Land

Note that the signs for land input in Table 4 is the effect of total cultivated land (hectare) on output per hectare, not total output. Land is assumed to be a fixed input, and therefore the results should be interpreted as a cross-sectional comparison.

In the Sahelian zone, the estimates are significant, and land has the same property as capital and crop diversification: insurance in a drought year. This may because a large cultivated area increases the probability of survival of crops in a drought year.

In the Sudanian and the Guinean zones, the estimate is not significant. The negative signs imply that land act as insurance regardless of rainfall level.

5.2 Estimation of Demand for Drought Insurance (ϕ)

ϕ's are estimated for each zone separately and using the full sample (combined zones). The results are shown in Table 5.

In the Sahelian zone there is effective demand for the insurance at between 15% and 25% probability and at other probabilities there is no demand. At 25%, ϕ begins to drop, and at 30% and above, it is zero or negative. This implies that most households are already insured at this level of drought, which is consistent with the finding that the majority of households in the Sahelian zone are self-insured for an occasional drought (Reardon *et al.*, 1992).

On the other hand, both the Sudanian and the Guinean zones have effective demand for the insurance over all the range of drought probabilities. In the Sudanian zone, there is effective demand at the full range of drought probabilities. ϕ, with patterns similar to the Sahelian's, peaks at 15-25% drought probabilities and then drops at 30% (yet, unlike the Sahelian zone, ϕ stays positive through 40%). In the Guinean zone, ϕ increases with drought

probability, but the effectiveness of demand (the difference between ϕ and $\frac{1-q}{q}$) decreases with the drought probability, as expected. The Guinean ϕ pattern implies that expected wealth is less sensitive to rainfall and/or households are less risk averse on average in the Guinean zone compared with other zones.

Comparing the three zones, demand is highest in the Sahelian zone, smallest in the Guinean zone and between the two in the Sudanian zone. Moreover, the Sahelian zone is the most homogeneous, while the Guinean zone is the most heterogeneous in terms of assets and other characteristics that drive insurance demand, or in other word, ability to self-insure. Again, the Sudanian zone is in the middle. This pattern is consistent with the riskiness of crop production in those zones.

Finally, ϕ is estimated using the full sample (combined zones). Between 15% and 30%, ϕ' s are very close to each other, and at 35%, ϕ is the largest. That is, demand for the drought insurance is distributed over a wide range of drought probabilities. Consequently, for the subsequent analysis, two probabilities of drought, 15% and 35%, are used; 15% is the first peak of ϕ, at which most households are considered to show effective demand for the drought insurance, and 35% is the peak just before a sudden fall of ϕ at which probably some households (that are distinguishable from others in terms of risk management) still show effective demand[12].

5.3 Determinants of Demand for Drought Insurance (ϕ)

Demand being "effective" at the zone level (that is, where zone-level ϕ exceeds $\frac{q}{1-q}$) does not mean that all households in the zone have effective demand; conversely, lack of demand at the zone level can mask demand by certain groups within it. Determinant analysis will help to understand who has demand for the formal drought insurance.

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[12] Comparing between 15% and 35% drought probabilities, at 15% drought probability drought is severe, but it occurs infrequently, while at 35% drought probability drought is mild, but it occurs frequently.

As discussed in section 4.3, two different regression specifications are used: cross-section and fixed effects. The former specification explains ϕ by average household wealth variables so that we can compare across households. The latter specification explains change in ϕ over time within a household by change in household wealth so that we can know how ϕ changes each year depending on initial household wealth.

Table 6 shows regression results for the full sample using ϕ evaluated at 15% drought probability[13]. As explained in section 5.2, most households are considered to have demand for the drought insurance at 15% drought probability. Coefficients for household fixed effects are estimated in the fixed effect regressions, but the results are not shown in the table; not all of them are statistically significant.

In both the cross-section regression and the fixed effect regression, crop production has a negative significant effect and cultivated land has a positive significant effect. The former implies that initial crop stock decreases demand for the drought insurance, as expected. The latter implies that households with more land are more risk averse or land is a risky asset. In full sample regression, the positive effect of cultivated land seems to reflect that land per AE is the highest in the Sahelian zone where estimated ϕ is also the highest (see Appendix D). The positive sign means that a risk averse household tends to cultivate a large area, but risk-decreasing effect of land is not effective to smooth output and/or a household that relies on more crop production (using more land) has a larger drought shock. In both regressions, on the other hand, off-farm income and livestock holdings do not show significant effect, which may be explained by the two opposing effects (risk averse and income effects) of those variables as discussed above.

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[13] Similar analyses were done for a 35% probability of drought, but results are not discussed here (tables of the results are in Appendix F). As explained in section 5.2, at 35% some households that are well self-insured for this level drought show no demand. Therefore, at 35% only drought-sensitive households are considered to have effective demand for the drought insurance. Some regression results differ from those when 15% drought probability is assumed. The changes of signs and significance can be explained by the assumption that only drought-sensitive households still demand the drought insurance at 35% drought probability.

Public food aid significantly decreases demand for drought insurance in the fixed effect regression, as expected . But in the cross-section regression food aid increases demand, which implies that more food aid is given to households that demand drought insurance more when we compare between households cross-sectionally, because such households suffer larger drought shock. Actually in the Sahelian zone, public food aid per AE is the largest (Appendix D).

In Table 7, regression results for each agroclimatic zone are shown. In the Sahelian zone, the signs of initial crop stocks, cultivated area, and public food aid are similar to those for the full sample, but the significance level is lower. In addition to those, inter-household gifts have a significant positive effect in the cross-section specification, which means that those who receive more gifts tend to demand drought insurance if other things are equal, probably because they have larger drought shocks. This means that gifts in the Sahelian zone work like state-contingent transfer (they go to households that have large shocks), but the gifts themselves are not effective to smooth income.

In the Sudanian zone, initial crop stocks decrease demand for drought insurance in the fixed-effect regression, as expected. But cultivated area has a negative significant effect in the fixed-effect regression, too. The cultivated area result is contrary to those obtained for the full sample, and also to those of the Sahelian zone and the Guinean zone. The negative effect of cultivated area in the fixed effect regression implies that a household has a smaller shock when it cultivates more land. The extra land acts as a buffer, by supplementing their limited fertile land and stabilizing output over years. The result implies that the risk-decreasing effect of land is in effect in the Sudanian zone. Much less public food aid was distributed in the Sudanian zone (compared to the Sahelian zone), but it has a significant effect unlike the Sahelian zone.

In the Guinean zone, initial crop stocks have a positive significant effect in the fixed effect regression. Cultivated area also has a positive significant effect in the fixed effect regression, but a negative significant effect in the cross-section regression. The negative effect

of cultivated area implies that land has income effects when we compare between households. This will be because more land means more room for cotton, the main cash crop in the Guinean zone. On the other hand, the positive effects in the fixed-effect regression imply that a household tends to have a large shock when it cultivates more land and has more crop stocks, because the household can make risky decisions. This is consistent with the fact that crop production is a major source of cash in this zone. Inter-household gifts have a negative, significant effect in the cross-section regression, which means that a household that receives more gifts on average is insured better when we compare between households. This is the income effect unlike the case of the Sahelian zone. In the Guinean zone where more gifts are observed than other zones, gifts may not be state-contingent transfers, but rather reflect household wealth.

Although off-farm income and livestock holdings are important income sources in the three zones, the results show no significant effect for them except for livestock holdings in the Guinean zone. This is probably because risk aversion and income effects cancel each other.

But exogenous stratification by wealth reveals significant effects of wealth variables[14]. The full sample is divided into three: lower 25 percentile of livestock holdings, upper 25 percentile, and between them. Average ϕ is not significantly different from zero in the lower stratum, and is the highest in the upper stratum (Table 8). In the lower stratum there is no demand for drought insurance, while in other strata there is effective demand. The difference in ϕ partially reflects that livestock holdings per AE are the largest in the Sahelian zone

[14] Splitting a sample of households on *a priori* grounds by wealth variables is found in the literature e.g., on liquidity constraints (Bernanke, 1984; Hayashi, 1985; Zeldes, 1989), and on the rate of time preference (Lawrance, 1991). They split a sample to separate poor households from the rich, and examine if those two groups differ. There is a possible selectivity bias in splitting a sample by endogenous wealth variables. Hayashi (1985) uses Tobit to account for the bias. Zeldes (1989) is also aware of the bias, and tries to split the sample based on predicted wealth by a logistic regression. But due to poor prediction, the results based on predicted wealth are not satisfactory and he uses the results based on actual wealth in his paper. Using time-varying wealth variables, Hayashi and Zeldes are concerned with a liquidity constraint of temporarily poor households. On the other hand, Bernanke (1984) and Lawrence (1991) do not consider the bias. Lawrence divides the full sample into seven strata by average real labor income. In my case, I am considering permanently (in a relative sense) poor households by stratifying the full sample by average wealth, and therefore selectivity bias is less problematic like in Lawrence.

where ϕ is also the highest. However, only half of the upper stratum households are from the Sahelian zone. The results imply that a household that is highly risk averse or tends to have a large shock keeps large livestock holdings, but the income smoothing effect is not sufficient.

In Table 8, the results of regression analyses for each stratum are shown. In households with low livestock holdings, livestock holdings have a significant negative effect in both specifications. This means that livestock holdings effectively insure in this household group. But when a household has high off-farm income and/or high initial crop stocks, the household tends to have a large drought shock because those allow the household to make risky decisions. In the middle stratum, livestock holdings show only a cross-sectional effect; a household that tends to be more risk averse or to have a larger shock, keeps more livestock holdings when comparing between households. On the other hand, off-farm income has a negative significant effect in the fixed-effect specification, that is, off-farm income is effectively used for self-insurance. In the upper stratum, neither off-farm income nor livestock holdings has a significant effect. Since those households on average demand the drought insurance more than do other strata, the insignificant results do not mean that they are already self-insured enough. Rather, the results imply (i) risk averse households or households with high risk keep more livestock holdings than do those with low risk. (ii) But because the level of livestock holding per AE and off-farm income per AE do not affect the degree of insurance demand (the size of ϕ), those households may need a better insurance. Probably, their livestock holdings is larger that the minimum livestock holdings that give the same level of self-insurance. Therefore, it will be possible that formal drought insurance substitutes for some of livestock holdings making the household's welfare at least unchanged or even improved.

6 Conclusions

There is effective demand for the hypothetical formal drought insurance in all zones, that is, although there are a lot of informal risk management mechanisms in the Sahel, but

they are not inadequate. This may not be surprising because we observe catastrophic drought shocks during the early 1980's. But such demand is very correlated with agroclimatic zone because drought risk differs over zones. This was expected, and raised a question of the feasibility of drought insurance due to a covariability problem. However, the determinant analysis shows that the demand also depends strongly on how households manage risk *ex ante* and *ex post*, and as a result there is a lot of heterogeneity even within an agroclimatic zone. In addition to these findings, as shown in Table 2, drought does not necessarily occur simultaneously over zones. Therefore, it can be concluded that it will be possible to avoid covariability and the drought insurance scheme will be feasible.

With respect to the effects of initial assets on ϕ, in general, the results support the hypothesis that those who are wealthier and/or are more self-insured demand formal drought insurance less, because they tend to suffer smaller income shocks from drought. But in some cases the risk aversion effect dominates the income effect. In these cases, households that suffer larger shocks keep more assets, but still demand a better insurance because their self-insurance is insufficient.

Most households decrease demand for the formal drought insurance when they have more initial crop stocks. However, if households tend to invest in off-farm activities and/or livestock holdings when they have more crop output, the relation between demand for the formal drought insurance and initial crop stocks will be positive. Such exceptions are observed in the Guinean households and by households with fewer livestock (some households belong to both strata).

Public food aid has an effect similar to that of initial crop stocks, but is significant only in the full sample, in the Sudanian zone, and in the lower livestock stratum. Most households decrease demand for drought insurance when food aid is available. Those households have a larger drought shock and have a high ϕ compared to others in the cross-section.

Effects of cultivated area differ between zones although land is a risk-decreasing input in all zones. In the Sahelian zone, land does not effectively smooth income, while in the Sudanian zone land is effectively used for self-insurance. In the Guinean zone, land appear to be seen as room for cash cropping.

The negative significant effects of off-farm income and livestock holdings support the hypothesis that those who do not have self-insurance mechanisms demand the drought insurance more, because they tend to suffer a large income shock from drought. The results show that this is true with off-farm income in the middle livestock stratum and with livestock holdings in the lower livestock stratum. On the other hand, in the upper livestock stratum, neither livestock holdings nor off-farm income has no significant effect on demand for the formal drought insurance. However, since in this stratum households on average show the largest demand for the drought insurance, their self-insurance level is not sufficient. This is a sign that the formal drought insurance can substitute for livestock holdings at least partially among those who keep relatively large livestock holdings while not changing their welfare.

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Table 1 Comparison of Assumptions on Drought Probability

	household's view w/o formal insurance	household probability of drought	distribution of rainfall
Gautam *et al.* (1994)	binary, fixed	objective[1], fixed	not specified[3]
Sakurai *et al.* (1994)	binary, changing	subjective[2], changing	fixed (normal)[4]
This Chapter	continuous, fixed	not specified	fixed (normal)[4]

Table 1 (continued)

	insurance scheme	insurance firm's probability of drought	household's view under formal insurance
Gautam *et al.* (1994)	binary	same as household's	binary (unchanged)
Sakurai *et al.* (1994)	binary	arbitrary	binary (same as insurance firm)
This Chapter	binary	arbitrary	binary (same as insurance firm)

Note 1: Based on physical state, such as annual rainfall.
Note 2: Based on individual household's welfare status
Note 3: Instead of specifying rainfall distribution, a fixed, binary probability is used.
Note 4: Rainfall distribution need not be normal, but must be known to every one. In empirical analysis, normal distributions are used for simplicity.

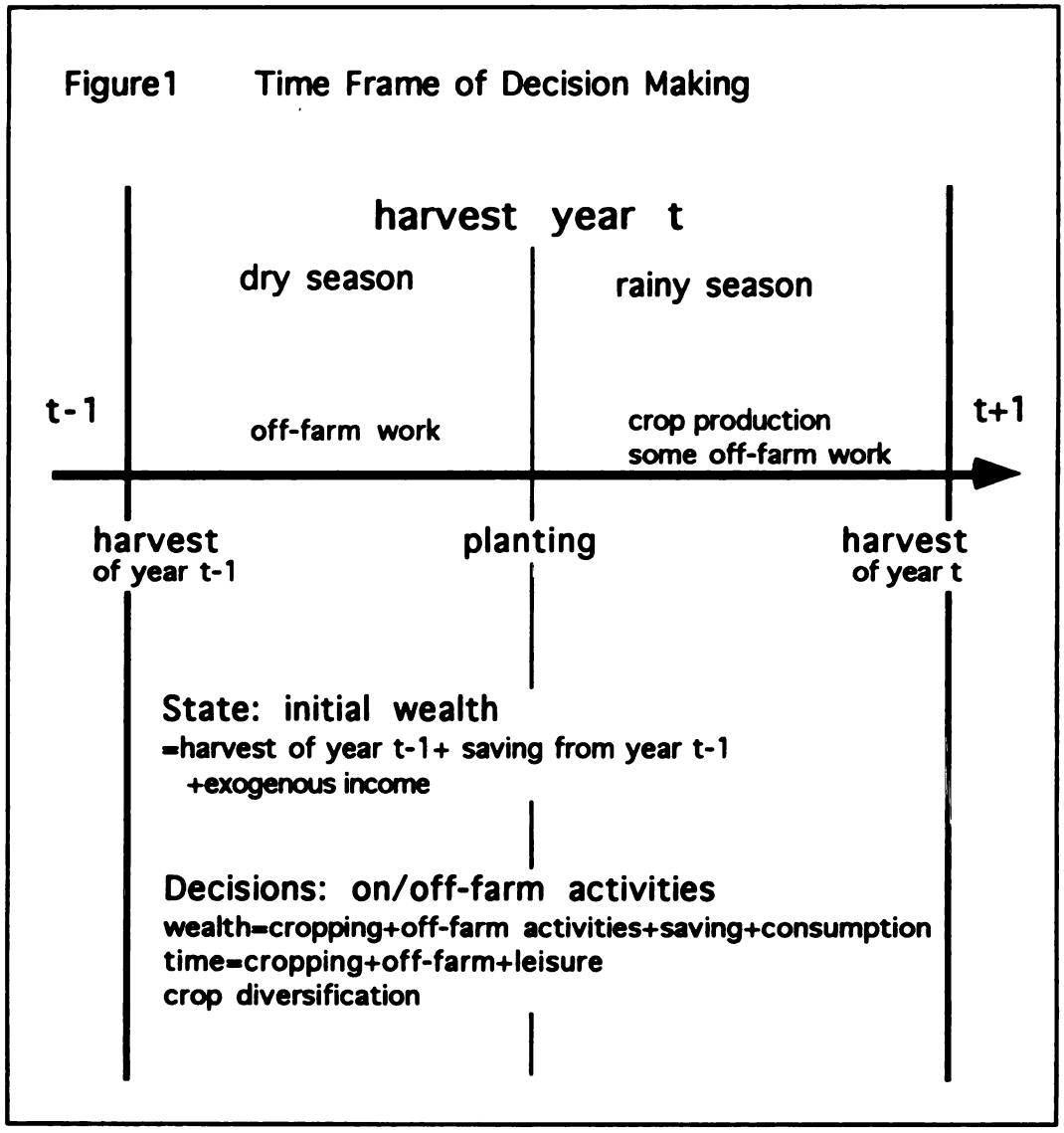


Figure1 Time Frame of Decision Making
harvest year t
dry season
rainy season
t-1
off-farm work
crop production
some off-farm work
t+1
harvest
of year t-1
planting
harvest
of year t
State: initial wealth
=harvest of year t-1+ saving from year t-1
+exogenous income
Decisions: on/off-farm activities
wealth=cropping+off-farm activities+saving+consumption
time=cropping+off-farm+leisure
crop diversification

Table 2 Annual Village-Level Rainfalls and the Probabilities

Zone	m_z	σ_z	σ_z/m_z	village	1981	1982	1983	short mean
Sahelian	480	163	0.34	village 1	454	382	476	468
				(probability)	(43.6)	(28.5)	(49.2)	
				village 2	513	347	454	438
				(probability)	(57.9)	(21.6)	(43.6)	
Sudanian	724	181	0.25	village 3	709	586	654	650
				(probability)	(47.8)	(22.5)	(34.6)	
				village 4	541	552	442	512
				(probability)	(15.7)	(17.2)	(6.0)	
Guinean	952	200	0.21	village 5	691	849	779	773
				(probability)	(0.95)	(30.2)	(19.2)	
				village 6	908	605	701	738
				(probability)	(41.3)	(4.1)	(10.4)	

Rainfall (mm), Probability (%)

Note: The means (m_z) and the standard deviations (σ_z) are based on long term rainfall data. Village-level rainfall and its probability (probability that rainfall is below it) are shown in each cell. The short mean is the average of rainfall over three years given in the table. Village 5 is not used in the analysis due to incomplete production data.

Table 3 Production Function Estimates with Fixed Effects

Variable	Sahelian Zone	Sudanian Zone	Guinean Zone
Labor (α_1)	4.09 (3.64)**	0.82 (1.89)*	1.27 (2.69)**
Labor*Rainfall (α_2)	-9.15 (3.41)**	-0.89 (1.21)	-0.80 (1.33)
Capital (β_1)	-0.18 (0.30)	-0.17 (1.02)	0.27 (0.63)
Capital*Rainfall (β_2)	0.55 (0.40)	0.39 (1.52)	-0.70 (1.17)
Crop Diversification (η_1)	-0.49 (1.13)	-4.95 (2.15)**	-6.57 (1.98)*
Diversification*Rainfall (η_2)	0.93 (0.89)	9.10 (2.34)**	8.81 (1.86)*
Total Area (μ_1)	0.88 (1.36)	0.26 (0.74)	0.79 (2.62)**
Total Area*Rainfall (μ_2)	-2.38 (1.71)*	-0.86 (1.29)	-1.19 (2.86)**
Rainfall (ω)	35.1 (4.44)**	10.8 (5.46)**	7.17 (4.02)**
R squared	0.79	0.80	0.85
Number of Cases	32*3	37*3	20*3

Note: The dependent variable is household crop output per hectare. Labor and capital input are also divided by total cultivated area. Absolute values of t-statistics are in parentheses; * means 10 % level of significance and ** means 5 % level of significance. Coefficients for household dummies are not shown, but are estimated significantly at 10 % level or better.

Table 4 Summary of Signs of Marginal Productivity

		Sahelian Zone				Sudanian Zone				Guinean Zone			
		Drought		Good		Drought		Good		Drought		Good	
Drought Prob (%)		15	35	15	35	15	35	15	35	15	35	15	35
Expected Rainfall		227	308	525	573	443	533	774	827	641	740	1007	1066
Labor	$\frac{\partial Y}{\partial L^f}$	$+^{**}$	$+^{**}$	$-^{**}$	$-^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$
	$\frac{\partial^2 Y}{\partial L^f \partial R}$	$+^{**}$	$+^{**}$	$-^{**}$	$-^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$
Capital	$\frac{\partial Y}{\partial K}$	-	-	+	+	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	-	-	-	-
	$\frac{\partial^2 Y}{\partial K \partial R}$	-	-	+	+	$+^{**}$	$+^{**}$	$+^{**}$	$+^{**}$	-	-	-	-
Crop Diverse.	$\frac{\partial Y}{\partial D}$	$-^{*}$	$-^{*}$	$-^{*}$	$+^{*}$	$-^{*}$	$-^{*}$	$+^{*}$	$+^{*}$	$-^{**}$	$-^{**}$	$+^{**}$	$+^{**}$
	$\frac{\partial^2 Y}{\partial D \partial R}$	$-^{*}$	$-^{*}$	$-^{*}$	$+^{*}$	$-^{*}$	$-^{*}$	$+^{*}$	$+^{*}$	$-^{**}$	$-^{**}$	$+^{**}$	$+^{**}$
Land	$\frac{\partial Y}{\partial A}$	$-^{**}$	$-^{**}$	$+^{**}$	$+^{**}$	-	-	-	-	-	-	-	-
	$\frac{\partial^2 Y}{\partial A \partial R}$	$-^{**}$	$-^{**}$	$+^{**}$	$+^{**}$	-	-	-	-	-	-	-	-

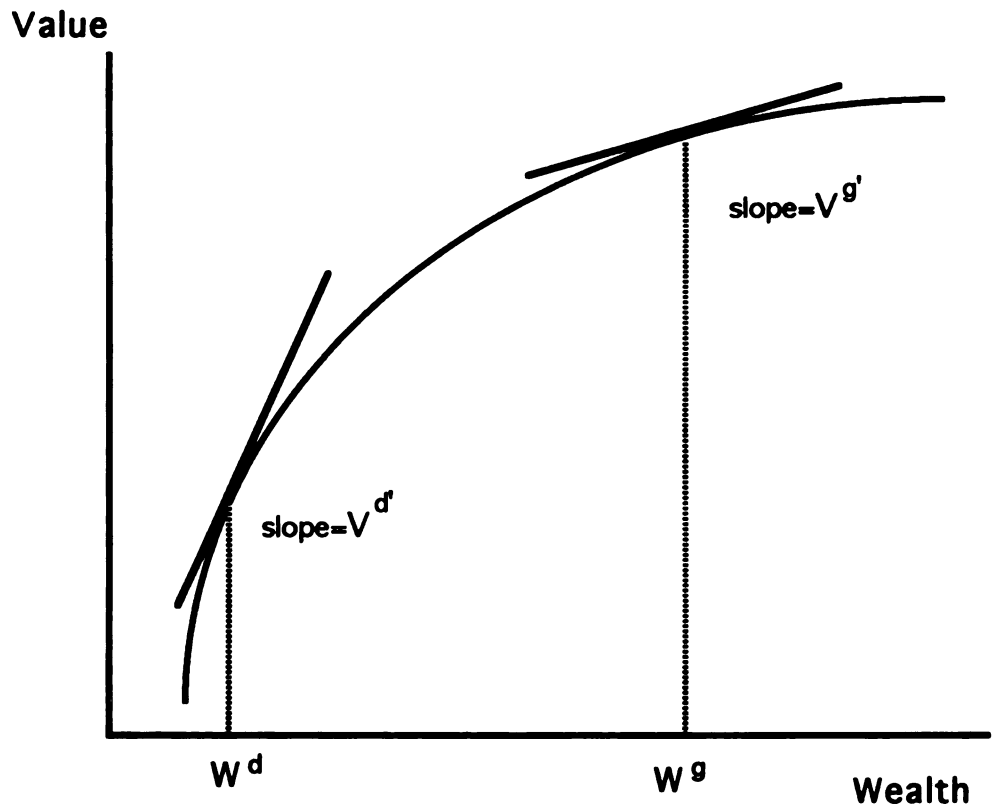
Note: The probability of a drought year is set at 15 % and 35 % respectively. Those signs are based on the estimated parameters shown in Table 3, and the derivations are in Appendix E. If the estimation of the sign is significant at 5 % level, ** is marked, and at least 10 % level, * is marked.

Table 5 Zone-Level ϕ and Drought Probabilities

Drought Probability (q)	10%	15%	20%	25%	30%	35%	40%
$\frac{q}{1-q}$	0.111	0.176	0.250	0.333	0.429	0.538	0.667
Sahelian ϕ	2.47	112**	104**	49.1**	-12.5	-0.50	-16.8*
	(6.28)	(17.3)	(21.3)	(18.9)	(15.8)	(9.37)	(9.38)
Sudanian ϕ	8.07**	14.5**	20.2**	19.9**	4.89**	4.11**	2.95**
	(0.48)	(0.75)	(1.06)	(1.16)	(0.72)	(0.66)	(0.62)
Guinean ϕ	0.38**	0.44**	0.51**	0.58**	0.66**	0.73**	0.80**
	(0.017)	(0.015)	(0.014)	(0.015)	(0.015)	(0.016)	(0.018)
Full Sample ϕ	6.63*	38.3**	35.7**	31.7**	31.6**	73.4**	-48.2**
	(3.69)	(7.07)	(8.17)	(6.91)	(6.17)	(4.98)	(9.03)

Note : According to condition (25), when ϕ is greater than $\frac{q}{1-q}$, there is effective demand. Standard errors are in parenthesis; * means 10% level of significance and ** means 1% level of significance.

Figure 2 Interpretation of ϕ



Note : ϕ is defined as $\phi_{it} = \frac{V^{d'}}{V^{g}}$. Expected wealth in a good year (W^g) and in a drought year (W^d) depend on input decisions and the definition of drought given by the threshold level of rainfall. Two marginal values depend on the expected wealth levels and the curvature of the value function, that is, the degree of risk aversion.

Table 6 Determinants of ϕ (full sample)

Probability of Drought	15%	
Specification	Cross-Section	Panel Fixed Effect
• Wealth Variables		
off-farm income (FCFA/AE)	-0.14 (0.42)	0.06 (0.20)
livestock holdings (FCFA/AE)	-0.05 (0.66)	-0.03 (0.47)
initial crop stock (FCFA/AE)	-1.87 (2.34)**	-1.13 (4.72)**
cultivated area (Ha/AE)	36.6 (3.00)**	22.9 (2.40)**
public aid (FCFA/AE)	57.8 (4.19)**	-148 (1.78)*
private gifts (FCFA/AE)	2.13 (0.68)	8.14 (1.10)
• Household Character.		
adult equivalent size (AE)	-2.63 (1.05)	-
age of household head	-0.23 (0.44)	-
number of wives	1.22 (0.14)	-
dependency ratio	4.35 (0.09)	-
compound leader	34.1 (2.19)**	-
single conjugal unit	-40.1 (2.17)**	-
animal traction	-6.11 (0.37)	-
• Plot Characteristics		
years since last fallow	-1.56 (2.10)**	-
soil type diversification	-103 (3.48)**	-
toposequence index	-11.8 (1.19)	-
distance to plot (Km)	-1.00 (0.19)	-
• Constant	119 (2.19)**	-
number of cases	267	178

Note: Absolute value of t-statistics is in parenthesis. * means 10 % level of significance and ** means 5 % level of significance.

Table 6 Determinants of ϕ (full sample)

Probability of Drought	15%	
Specification	Cross-Section	Panel Fixed Effect
• Wealth Variables		
off-farm income (FCFA/AE)	-0.14 (0.42)	0.06 (0.20)
livestock holdings (FCFA/AE)	-0.05 (0.66)	-0.03 (0.47)
initial crop stock (FCFA/AE)	-1.87 (2.34)**	-1.13 (4.72)**
cultivated area (Ha/AE)	36.6 (3.00)**	22.9 (2.40)**
public aid (FCFA/AE)	57.8 (4.19)**	-148 (1.78)*
private gifts (FCFA/AE)	2.13 (0.68)	8.14 (1.10)
• Household Character.		
adult equivalent size (AE)	-2.63 (1.05)	-
age of household head	-0.23 (0.44)	-
number of wives	1.22 (0.14)	-
dependency ratio	4.35 (0.09)	-
compound leader	34.1 (2.19)**	-
single conjugal unit	-40.1 (2.17)**	-
animal traction	-6.11 (0.37)	-
• Plot Characteristics		
years since last fallow	-1.56 (2.10)**	-
soil type diversification	-103 (3.48)**	-
toposequence index	-11.8 (1.19)	-
distance to plot (Km)	-1.00 (0.19)	-
• Constant	119 (2.19)**	-
number of cases	267	178

Note: Absolute value of t-statistics is in parenthesis. * means 10 % level of significance and ** means 5 % level of significance.

Table 7 Determinants of ϕ (zone)

Probability of Drought	15%	
Specification	Cross-Section	Panel Fixed Effect
• Sahelian Zone, ø=112 (6.47)**	n=96	n=64
off-farm income (FCFA/AE)	-0.54 (0.59)	-0.23 (0.26)
livestock holdings (FCFA/AE)	-0.13 (0.78)	0.05 (0.54)
initial crop stock (FCFA/AE)	-5.68 (1.21)	-2.38 (4.87)**
cultivated area (Ha/AE)	61.2 (1.54)	39.2 (2.09)**
public aid (FCFA/AE)	54.7 (1.28)	-186 (1.40)
private gifts (FCFA/AE)	56.5 (2.90)**	16.4 (1.12)
• Sudanian Zone, ø=14.5 (19.3)**	n=111	n=74
off-farm income (FCFA/AE)	-0.04 (0.92)	0.01 (0.10)
livestock holdings (FCFA/AE)	0.00 (0.02)	-0.08 (1.18)
initial crop stock (FCFA/AE)	0.21 (1.05)	-0.11 (1.93)*
cultivated area (Ha/AE)	-8.26 (3.01)**	-7.61 (3.18)**
public aid (FCFA/AE)	11.5 (1.75)*	-104 (3.21)**
private gifts (FCFA/AE)	0.63 (0.84)	2.06 (1.07)
• Guinean Zone, ø=0.44 (29.3)**	n=60	n=40
off-farm income (FCFA/AE)	-0.001 (1.11)	-0.001 (1.64)
livestock holdings (FCFA/AE)	0.004 (1.41)	-0.005 (1.92)*
initial crop stock (FCFA/AE)	-0.001 (0.54)	0.003 (3.53)**
cultivated area (Ha/AE)	-0.183 (1.93)*	0.239 (6.35)**
public aid (FCFA/AE)	0.118 (0.32)	0.745 (0.84)
private gifts (FCFA/AE)	-0.011 (1.86)*	0.016 (0.63)

Note: Only wealth variables are shown for average wealth specifications. T-statistics are in parenthesis.
* means 10 % level of significance and ** means 5 % level of significance.

Table 8 Determinants of ϕ (stratification by livestock holdings)

Probability of Drought	15%	
Specification	Cross-Section	Panel Fixed Effect
• Lower Stratum, ø=-3.39 (0.70)	n=66	n=44
off-farm income (FCFA/AE)	0.27 (0.53)	0.44 (2.35)**
livestock holdings (FCFA/AE)	-10.4 (2.07)**	-3.28 (6.54)**
initial crop stock (FCFA/AE)	-2.11 (3.00)**	0.47 (5.24)**
cultivated area (Ha/AE)	44.4 (3.06)**	-0.41 (0.10)
public aid (FCFA/AE)	-12.9 (0.95)	-221 (2.47)**
private gifts (FCFA/AE)	2.82 (1.32)	11.4 (5.57)**
• Middle Stratum, ø=5.26 (2.69)**	n=135	n=90
off-farm income (FCFA/AE)	-0.06 (0.52)	-0.32 (4.39)**
livestock holdings (FCFA/AE)	0.57 (1.99)*	-0.18 (1.36)
initial crop stock (FCFA/AE)	0.54 (1.66)*	-0.09 (0.87)
cultivated area (Ha/AE)	-15.6 (2.12)**	9.22 (2.49)**
public aid (FCFA/AE)	3.12 (0.61)	21.4 (1.14)
private gifts (FCFA/AE)	-0.51 (0.35)	2.34 (0.83)
• Upper Stratum, ø=24.8 (4.15)**	n=66	n=44
off-farm income (FCFA/AE)	-0.50 (1.49)	-0.33 (0.40)
livestock holdings (FCFA/AE)	0.07 (1.00)	-0.00 (0.11)
initial crop stock (FCFA/AE)	-2.30 (2.54)**	-1.20 (3.27)**
cultivated area (Ha/AE)	28.0 (3.42)**	39.9 (2.86)**
public aid (FCFA/AE)	-27.8 (1.23)	n/a
private gifts (FCFA/AE)	10.3 (1.12)	-13.7 (1.11)

Note: Only wealth variables are shown for average wealth specifications. T-statistics are in parenthesis.
* means 10 % level of significance and ** means 5 % level of significance.

Appendix A

Change in Average Rainfall and Drought Probabilities

Table A1 Segou (Sudanian zone, Mali), Drought Frequency

		long-term				short-term (1980's)			
Drought Prob.		10%	20%	30%	40%	10%	20%	30%	40%
Threshold Rainfall		505	564	607	644	384	439	479	512
Decade	Mean Rainfall								
1940	630	0.1	0.2	0.3	0.6	0	0	0.1	0.1
1950	801	0	0	0	0	0	0	0	0
1960	730	0	0.1	0.1	0.1	0	0	0	0
1970	638	0.1	0.3	0.3	0.4	0	0	0	0.2
1980	544	0.3	0.8	0.9	0.9	0	0.2	0.3	0.4
Total Frequency		0.1	0.28	0.3	0.38	0	0.04	0.08	0.14

Note : Long-term statistics are: average=678 and standard deviation=135 based on 64 years of data. Short-term statistics are: average=544 and standard deviation=125 based on 10 years of data. Each cell shows observed relative frequency of drought in the decade (number of drought years/10), when drought is defined by the threshold rainfall level given in the table.

Mean rainfall was higher during the 1950's and 1960's, and lowest during the 1980's. Even if we use long-term statistics, there is only one drought at 40% probability of drought during 1950's and 1960's. Using the same threshold level of rainfall, during the 1980's there are more drought years than predicted; at 40% probability of drought 9 out of 10 years are drought by the definition. However, in the long run, the observed frequencies of drought are very close to what is predicted. If we use short-term statistics based on data from the 1980's, it obviously gives a good prediction of frequency of drought in 1980's, but in other decades it does not give a good prediction.

Table A2 Koutiala (Guinean zone, Mali), Drought Frequency

		long-term				short-term (1980's)			
Drought Prob.		10%	20%	30%	40%	10%	20%	30%	40%
Threshold Rainfall		724	802	859	907	610	676	723	763
Decade	Mean Rainfall								
1930	921	0.1	0.3	0.4	0.5	0	0	0.1	0.2
1940*	1053	0	0.11	0.11	0.11	0	0	0	0
1950	1099	0	0	0	0.1	0	0	0	0
1960	1018	0.1	0.1	0.1	0.2	0	0	0.1	0.1
1970	864	0.1	0.2	0.6	0.7	0	0.1	0.1	0.2
1980	801	0.3	0.4	0.7	0.9	0.1	0.2	0.3	0.4
Total Frequency		0.10	0.19	0.32	0.42	0.02	0.05	0.08	0.15

Note 1: * means that there is a missing year in the 1940's.

Long-term statistics are: average=952 and standard deviation=178 based on 70 years of data. Short-term statistics are: average=801 and standard deviation=149 based on 10 years data. Each cell shows observed relative frequency of drought (number of drought years/10), when drought is defined by the threshold rainfall level given in the table.

Mean rainfall was higher during the 1940's, 1950's, and 1960's. The table shows that long-term statistics are better than short-term (1980's), as discussed on the Sudanian table.

Appendix B

Table B1 Expected Rainfall (mm)

Prob. (q)	10%	15%	20%	25%	30%	35%	40%
Truncation Z	-1.645	-1.282	-1.036	-0.842	-0.675	-0.385	-0.253
Sahelian mean 480 SD 163							
Cut-off Point	271	311	343	370	395	417	439
R_z^s	512	525	537	549	561	573	585
R_z^d	194	227	252	273	291	308	323
Sudanian mean 724 SD 181							
Cut-off Point	492	536	572	602	629	654	678
R_z^s	759	774	787	801	814	827	841
R_z^d	407	443	471	494	514	533	549
Guinean mean 952 SD 200							
Cut-off Point	696	745	784	817	847	875	901
R_z^s	991	1007	1022	1037	1051	1066	1081
R_z^d	601	641	672	698	720	740	759

R_z^s (expected good rainfall) and R_z^d (expected drought rainfall) are calculated when the probability of a drought year is q. In a standard normal distribution, truncation Z gives q probability of lower tail. Therefore, Z^s (expected z score for a good year) is given by

$$Z^s = \int_B^\infty xf(x|x > B)dx = \int_B^\infty x\frac{f(x)}{1-\Phi(B)}dx$$

where $f(x)$ is the standard normal p.d.f., $\Phi(x)$ is the standard normal c.d.f. and B=truncation Z. Z^d (expected z score for a drought year) is obtained in a similar way. Using these two Z scores and long-term, zone-level rainfall mean and standard deviation, and assuming normal distribution of rainfall, R_z^s and R_z^d are calculated for each zone.

Appendix C

Calculation of (30), (31), and (32)

Using the production function given by (29), the expectations in (30)-(32) are specified as follows.

$$\int_{R^*}^{\infty} Y_{it} f_z^g(R)dR = \exp(\alpha_1 \ln L_{it}^f + \beta_1 \ln K_{it} + \eta_1 \ln D_{it} + \mu_1 \ln A_{it} + h_i HH_i) \cdot \int_{R^*}^{\infty} \exp\left[(\alpha_2 \ln L_{it}^f + \beta_2 \ln K_{it} + \eta_2 \ln D_{it} + \mu_2 \ln A_{it} + \omega)R\right] f_z^g(R)dR \tag{C1}$$

and

$$\int_{-\infty}^{R^*} Y_{it} f_z^d(R)dR = \exp(\alpha_1 \ln L_{it}^f + \beta_1 \ln K_{it} + \eta_1 \ln D_{it} + \mu_1 \ln A_{it} + h_i HH_i) \cdot \int_{-\infty}^{R^*} \exp\left[(\alpha_2 \ln L_{it}^f + \beta_2 \ln K_{it} + \eta_2 \ln D_{it} + \mu_2 \ln A_{it} + \omega)R\right] f_z^d(R)dR \tag{C2}$$

where $f_z^g(R)$ and $f_z^d(R)$ are the probability density functions of rainfall for each zone. They have a truncated normal distribution, and are given by

$$f_z^g(R) = \frac{\exp\left[-\frac{(R-m_z)^2}{2\sigma_z^2}\right]}{(1-\Phi(R_z^*))\sigma_z\sqrt{2\pi}} - \frac{\exp\left[-\frac{(R-m_z)^2}{2\sigma_z^2}\right]}{q\sigma_z\sqrt{2\pi}} \tag{C3}$$

and

$$f_z^d(R) = \frac{\exp\left[-\frac{(R-m_z)^2}{2\sigma_z^2}\right]}{\Phi(R_z^*)\sigma_z\sqrt{2\pi}} - \frac{\exp\left[-\frac{(R-m_z)^2}{2\sigma_z^2}\right]}{(1-q)\sigma_z\sqrt{2\pi}} \tag{C4}$$

where m_z and σ_z are long-term mean and variance of rainfall for each zone, shown in Table 1, and $\Phi(R_z^*)$ is the normal cumulative distribution function.

Appendix D

Table D1 Mean and Standard Deviation of Wealth Variables

	Full Sample	Sahelian	Sudanian	Guinean
number of cases	267	96	111	40
Off-farm income (FCFA/AE)	15568	15930	11783	21990
	(18604)	(20191)	(15090)	(20258)
livestock (FCFA/AE)	39262	71500	28322	7927
	(86141)	(131810)	(36900)	(5162)
Crop Production (FCFA/AE)	17254	14904	13611	27756
	(9786)	(6959)	(5452)	(12339)
Cultivated area (Ha/AE)	1.219	1.579	0.952	1.108
	(0.612)	(0.751)	(0.416)	(0.322)
Public Food Aid (FCFA/AE)	92	231	20	5
	(35)	(39)	(7.7)	(3.5)
Private Food Gifts (FCFA/AE)	263	183	171	561
	(135)	(64.5)	(74.5)	(242)

Note: Mean and standard deviation (in parenthesis) are in each cell. Means are significantly different between zones at 0.1% level.

Appendix E

Derivations for Table 3

Q is a general input variable, that is, Q can be L^f, K, or D Differentiating the production function given in (29) with respect to Q yields

$$\frac{\partial Y}{\partial Q} = (\kappa_1 + \kappa_2 R)\frac{Y}{Q} \tag{E1}$$

where κ_1 and κ_2 are parameters corresponding to α_1, α_2, β_1, β_2, η_1, and η_2, respectively. Then, differentiating the first derivative with respect to rainfall, R, yields

$$\frac{\partial^2 Y}{\partial Q \partial R} = \{\kappa_2 + (\kappa_1 + \kappa_2 R)C\}\frac{Y}{Q} \tag{E2}$$

where $C = \omega + \alpha_2 \ln L^f + \beta_2 \ln K + \eta_2 \ln D + \mu_2 \ln A$. C depends on the household's input decisions and parameters, but independent of which variable to use for differentiation.

Based on (E1), if $\kappa_2 > 0$, then $\frac{\partial Y}{\partial Q} > 0$ when $R > -\frac{\kappa_1}{\kappa_2}$ and if $\kappa_2 < 0$, then $\frac{\partial Y}{\partial Q} > 0$ when $R < -\frac{\kappa_1}{\kappa_2}$. Based on (E2), if $\kappa_2 C > 0$, then $\frac{\partial^2 Y}{\partial Q \partial R} > 0$ when $R > -\frac{\kappa_1}{\kappa_2} - \frac{1}{C}$ and if $\kappa_2 C < 0$, then $\frac{\partial^2 Y}{\partial Q \partial R} > 0$ when $R < -\frac{\kappa_1}{\kappa_2} - \frac{1}{C}$. C for each household is calculated using actual input level and the estimated parameters shown in Table 2. $\frac{1}{C}$ is positive in most cases, and very small relative to the rainfall level given by $-\frac{\kappa_1}{\kappa_2}$, and therefore $\frac{1}{C}$ can be ignored to evaluate the sign of $\frac{\partial^2 Y}{\partial Q \partial R}$.

Based on the general derivation with respect to input Q above, signs and the significance levels of first and cross derivatives with respect to L^f, K, and D are determined based on the estimations of $-\frac{\kappa_1}{\kappa_2}$. The results are shown in Tables E1 through E3.

In the case of land input, A, signs and significance levels of the first and cross derivatives are determined based on E1 and E2, but $\kappa_1 = -\alpha_1 - \beta_1 + \mu_1$ and $\kappa_2 = -\alpha_2 - \beta_2 + \mu_2$ are substituted. Note that in this case, Y is output per hectare. The results are in Table E4.

Table E1 Signs of Marginal Productivity of Labor

Rainfall (mm)		<447	447	447-921	921	921-1596	1596	1596<
Sahelian	$\frac{\partial Y}{\partial L^f}$	+	0	-	-	-	-	-
	$\frac{\partial^2 Y}{\partial L^f \partial R}$	+/-	-	-	-	-	-	-
Sudanian	$\frac{\partial Y}{\partial L^f}$	+	+	+	0	-	-	-
	$\frac{\partial^2 Y}{\partial L^f \partial R}$	+	+	+/-	-	-	-	-
Guinean	$\frac{\partial Y}{\partial L^f}$	+	+	+	+	+	0	-
	$\frac{\partial^2 Y}{\partial L^f \partial R}$	+	+	+	+	+/-	-	-

Table E2 Signs of Marginal Productivity of Capital

Rainfall (mm)		<330	330	330-392	392	392-430	430	430<
Sahelian	$\frac{\partial Y}{\partial K}$	-	0	+	+	+	+	+
	$\frac{\partial^2 Y}{\partial K \partial R}$	-/+	+	+	+	+	+	+
Sudanian	$\frac{\partial Y}{\partial K}$	-	-	-	-	-	0	+
	$\frac{\partial^2 Y}{\partial K \partial R}$	-	-	-	-	-/+	+	+
Guinean	$\frac{\partial Y}{\partial K}$	+	+	+	0	-	-	-
	$\frac{\partial^2 Y}{\partial K \partial R}$	+	+	+/-	-	-	-	-

Table E3 Signs of Marginal Productivity of Crop Diversification

Rainfall (mm)		<530	530	530-543	543	543-746	746	746<
Sahelian	$\frac{\partial Y}{\partial D}$	-	0	+	+	+	+	+
	$\frac{\partial^2 Y}{\partial D \partial R}$	-/+	+	+	+	+	+	+
Sudanian	$\frac{\partial Y}{\partial D}$	-	-	-	0	+	+	+
	$\frac{\partial^2 Y}{\partial D \partial R}$	-	-	-/+	+	+	+	+
Guinean	$\frac{\partial Y}{\partial D}$	-	-	-	-	-	0	+
	$\frac{\partial^2 Y}{\partial D \partial R}$	-	-	-	-	-/+	+	+

Table E4 Signs of Marginal Productivity of Cultivated Area

Rainfall (mm)		<-1067	-1067	-1067-486	486	486-2419	2419	2419<
Sahelian	$\frac{\partial Y}{\partial A}$	-	-	-	0	+	+	+
	$\frac{\partial^2 Y}{\partial A \partial R}$	-	-	-/+	+	+	+	+
Sudanian	$\frac{\partial Y}{\partial A}$	+	0	-	-	-	-	-
	$\frac{\partial^2 Y}{\partial A \partial R}$	+/-	-	-	-	-	-	-
Guinean	$\frac{\partial Y}{\partial A}$	-	-	-	-	-	0	+
	$\frac{\partial^2 Y}{\partial A \partial R}$	-	-	-	-	-/+	+	+

Appendix F

Regression Results at 35% Drought Probability

As explained in section 5.2, two drought probabilities, 15% and 35%, are picked up to compare. However, in the text, only results at 15% drought probability are used. In this appendix, the results for 35% drought probability is presented. There are three tables that are corresponding to Table 7, Table 8, and Table 9 respectively.

The detailed comparison between two result sets with different drought probabilities are not conducted in this paper.

Table F1 Determinants of ϕ (full sample)

Probability of Drought	35%	
Specification	Cross-Section	Panel Fixed Effect
• Wealth Variables		
off-farm income (FCFA/AE)	-0.31 (1.29)	-0.46 (1.30)
livestock holdings (FCFA/AE)	-0.14 (2.35)**	-0.06 (0.86)
initial crop stock (FCFA/AE)	1.07 (1.92)*	-0.20 (0.70)
cultivated area (Ha/AE)	-49.3 (5.80)**	25.4 (2.20)**
public aid (FCFA/AE)	6.50 (0.68)	603 (6.01)**
private gifts (FCFA/AE)	-2.08 (0.95)	7.07 (0.79)
• Household Character.		
adult equivalent size (AE)	-2.15 (1.22)	-
age of household head	1.28 (3.49)**	-
number of wives	1.29 (0.21)	-
dependency ratio	26.2 (0.77)	-
compound leader	12.4 (1.15)	-
single conjugal unit	39.4 (3.06)**	-
animal traction	9.85 (0.87)	-
• Plot Characteristics		
years since last fallow	-1.26 (2.44)**	-
soil type diversification	55.5 (2.68)**	-
toposequence index	-7.80 (1.16)	-
distance to plot (Km)	-5.13 (1.37)	-
• Constant	60.7 (1.61)	-
number of cases	267	178

Note: Only wealth variables are shown for average wealth specifications. T-statistics are in parenthesis.
* means 10 % level of significance and ** means 5 % level of significance.

Table F2 Determinants of ϕ (zone)

Probability of Drought	35%	
Specification	Cross-Section	Panel Fixed Effect
• Sahelian Zone, ø=-0.50 (0.05)**	n=96	n=64
off-farm income (FCFA/AE)	0.10 (0.21)	0.05 (0.09)
livestock holdings (FCFA/AE)	-0.14 (1.56)	0.02 (0.35)
initial crop stock (FCFA/AE)	4.83 (1.89)*	-1.26 (4.03)**
cultivated area (Ha/AE)	-49.6 (2.28)**	11.2 (0.94)
public aid (FCFA/AE)	-5.86 (0.25)	72.1 (0.85)
private gifts (FCFA/AE)	11.2 (1.05)	-3.66 (0.39)
• Sudanian Zone, ø=4.11 (6.23)**	n=111	n=74
off-farm income (FCFA/AE)	-0.01 (0.23)	-0.002 (0.04)
livestock holdings (FCFA/AE)	-0.02 (1.00)	-0.11 (1.49)
initial crop stock (FCFA/AE)	0.12 (0.65)	-0.27 (4.15)**
cultivated area (Ha/AE)	3.24 (1.25)	-5.15 (1.95)*
public aid (FCFA/AE)	10.8 (1.73)*	-77.6 (2.17)**
private gifts (FCFA/AE)	2.35 (3.31)**	-0.43 (0.20)
• Guinean Zone, ø=0.73 (45.6)**	n=60	n=40
off-farm income (FCFA/AE)	0.001 (1.00)	-0.001 (1.20)
livestock holdings (FCFA/AE)	0.002 (0.76)	-0.001 (0.17)
initial crop stock (FCFA/AE)	-0.007 (2.99)**	-0.004 (2.70)**
cultivated area (Ha/AE)	-0.004 (0.04)	0.195 (3.33)**
public aid (FCFA/AE)	-0.365 (0.89)	1.771 (1.29)
private gifts (FCFA/AE)	-0.007 (0.98)	0.018 (0.46)

Note: Only wealth variables are shown for average wealth specifications. T-statistics are in parenthesis.
* means 10 % level of significance and ** means 5 % level of significance.

Table F3 Determinants of ϕ (stratification by livestock holdings)

Probability of Drought	35%	
Specification	Cross-Section	Panel Fixed Effect
• Lower Stratum, ø=63.2 (6.01)**	n=66	n=44
off-farm income (FCFA/AE)	-0.50 (0.49)	1.24 (2.13)**
livestock holdings (FCFA/AE)	-2.47 (0.24)	-9.63 (6.12)**
initial crop stock (FCFA/AE)	2.32 (1.64)	1.07 (3.81)**
cultivated area (Ha/AE)	-49.3 (1.69)*	-34.6 (2.64)*
public aid (FCFA/AE)	13.8 (0.50)	-96.1 (0.34)
private gifts (FCFA/AE)	-2.26 (0.53)	-3.79 (0.59)
• Middle Stratum, ø=94.9 (16.4)**	n=135	n=90
off-farm income (FCFA/AE)	-0.46 (1.62)	-0.61 (1.90)*
livestock holdings (FCFA/AE)	0.39 (0.54)	-3.95 (6.82)**
initial crop stock (FCFA/AE)	2.46 (3.03)**	-0.67 (1.50)
cultivated area (Ha/AE)	-107 (5.81)**	-4.47 (0.27)
public aid (FCFA/AE)	44.3 (3.46)**	1031 (12.4)**
private gifts (FCFA/AE)	-4.47 (1.22)	19.3 (1.54)
• Upper Stratum, ø=60.6 (7.33)**	n=66	n=44
off-farm income (FCFA/AE)	-0.52 (1.33)	0.72 (0.70)
livestock holdings (FCFA/AE)	0.07 (0.84)	-0.03 (0.43)
initial crop stock (FCFA/AE)	1.17 (1.10)	0.60 (1.32)
cultivated area (Ha/AE)	-26.9 (2.79)**	5.07 (0.29)
public aid (FCFA/AE)	-83.2 (3.13)**	n/a
private gifts (FCFA/AE)	18.5 (1.71)*	5.21 (0.34)

Note: Only wealth variables are shown for average wealth specifications. T-statistics are in parenthesis.
* means 10 % level of significance and ** means 5 % level of significance.

Chapter 6

Summary of Approach and Findings

In this dissertation, I maintain the following hypotheses: (a) Households in the Semi-arid Tropics (SAT) tend to save for future uncertain income, that is, there is precautionary demand for savings; (b) Such savings take the form of livestock holdings because there is no formal saving institutions and because livestock is a relatively safe asset in times of drought in the SAT; (c) As a result of the precautionary livestock holding, overgrazing occurs because grazing land is an open-access, common-pool resource; (d) Formal financial institutions, such as drought insurance, will substitute for livestock holdings and reduce overgrazing.

Properties and conditions of these hypotheses are examined theoretically in chapters 2-4. Then, they are empirically tested in chapter 5. However, the tests are indirect and partial because the objectives of chapter 5 are not primarily to test these hypotheses. In this chapter, I summarize how these hypotheses are tested indirectly and draw conclusions of this dissertation.

Hypothesis a: *Households in the SAT tend to save for future uncertain income, that is, there is precautionary demand for savings.*

a. 1 Conditions for Precautionary Savings

Saving is considered to be a riskless asset with a fixed interest rate in most models reviewed in chapter 2. In this case, stochastic exogenous labor income is usually the only source of risk. But sometimes risky savings are also considered. In this case, risk comes from the stochastic interest rate, but the model can include risky exogenous income as well.

The findings in the literature review in chapter 2 are summarized as follows. When saving is riskless, an increase in exogenous income risk will increase optimal saving if the marginal utility function is convex, while it will decrease optimal saving if the marginal utility

function is concave[1]. This is the most standard assertion about precautionary demand for saving. Even if risk is from a stochastic interest rate, the effects of increasing risk on riskless saving are the same as those of income risk. However, when saving is risky, the effect of increasing interest rate risk is not the same as in the case of riskless saving: if the marginal utility function is convex, the effects are ambiguous, while if the marginal utility function is concave, an increase in interest rate risk will decrease optimal saving. In the case of concave marginal utility, the effects on risky saving are the same as those on riskless saving. The ambiguity in the case of convex marginal utility is due to the mixture of prudence and risk-aversion: a positive third derivative of the utility function (i.e. convex marginal utility function) represents the prudence that increases optimal saving when risk increases; on the other hand, a negative second derivative (i.e. concave utility) represents the risk-aversion that decreases optimal holding of risky asset when risk increases. Therefore, if the household is prudent enough, an increase in interest rate risk will increase even risky saving.

In short, convex marginal utility is a necessary condition for precautionary demand for savings. It is a necessary and sufficient condition when saving is riskless, while it is just a necessary condition when saving is risky.

a. 2 Robustness of Precautionary Savings

In chapter 3, I examine the robustness of the above conclusions in several extensions of the basic models presented in chapter 2.

First, I consider the case when the two risks (exogenous income risk and exogenous interest rate risk) are correlated. In this case, when one of the risks increases it is possible that optimal riskless saving decreases even when the marginal utility function is convex, and that optimal riskless saving increases even when the marginal utility function is concave. These

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1 The convexity and concavity of the marginal utility determine how the expected marginal utility changes when risk changes. See footnote 1 in chapter 2.

results are not consistent with the results which are obtained when the risks are independent. In addition, when the risks are correlated, the effects on risky saving are ambiguous.

Second, in agricultural household models, risky agricultural production is equivalent to the risky saving in models presented in chapter 2. But the differences are (i) risky agricultural production is non-linearly related to inputs and risks, while risky saving usually assumes a linear return to investment; (ii) risky agricultural production requires multiple inputs, such as labor, capital, and land, while risky saving usually assumes a single input, money; (iii) labor income is not exogenous in an agricultural production, and therefore exogenous income may not exist or we must assume something other than labor income.

In a dynamic agricultural household model, I consider both riskless saving and capital and labor inputs for risky agricultural production, and examine the effects of increasing risk in agricultural production. The results are: (i) optimal riskless saving increases when marginal utility function is convex, but it is ambiguous when the marginal utility function is concave; (ii) optimal use of inputs increase when the inputs are risk-decreasing and when the marginal utility function is convex, but otherwise it is ambiguous. The ambiguity of the effect on optimal use of risk-increasing inputs in the case of convex marginal utility is due to mixture of prudence and risk-aversion; a risk-averse household reduces risk-increasing inputs when risk increases.

Third, in the dynamic agricultural household model, I consider the case when two correlated risks exist. In this model, the two risks are stochastic rainfall which affects agricultural production and social insurance (or transfer) risk, which are equivalent to interest rate risk and exogenous income risk respectively in the model presented first. These two risks are correlated because rainfall shock determines social insurance. However, unlike the case presented first, the correlation is one-way; rainfall affects social insurance, but there is no effect of social insurance on rainfall. By introducing this correlation of risks, the results are qualitatively the same as those in the single-risk case presented second if social insurance is imperfect. However, when social insurance is perfect, risk-neutral decisions such as riskless

saving become insensitive to change in risk, while risk-decreasing (increasing) inputs increase (decreases) when risk increases regardless of the sign of the third derivative of utility function. These are substitution effects due to change in risk, or pure risk-aversion effects.

Fourth, the effects of increasing risk are examined in a sequential decision model. If saving is riskless, the results are not different from those obtained in the simultaneous decision model presented second; that is, when marginal utility is convex, optimal riskless saving increases as risks increase. However, with respect to input decisions, the results depend on the model, that is, they depend on the assumption of availability of information and timing of decisions.

These four extensions above lead to the conclusion that precautionary demand for savings and precautionary demand for risk-decreasing inputs exist when marginal utility is convex. Even when risks are correlated, if the correlation is specific as defined above, the results will be robust.

a. 3 Endogenous Risk

So far I have considered only exogenous risks. In two period models, this is acceptable because exogenous risks can be immediately translated into consumption risk in the second period. In multiperiod models, however, precautionary demand for savings depends not directly on exogenous risks but on endogenous income and/or consumption risks (or risk in utility derived from consumption and leisure) induced by the exogenous risks. From this viewpoint, we can say that precautionary demand for savings depends on a household's ability to smooth income and/or consumption under exogenous risks. The theoretical results presented in a. 1 and a. 2 are still true, but are just qualitative, and the degree of precautionary demand for savings depends on household characteristics. Therefore, wealth or anything that helps predict the future variability of consumption will have a role in predicting the precautionary demand for savings.

This point is reflected in the agropastoral model in chapter 4. The model assumes that crop production risk is constant over the households, and predicts that the more risky is crop production, the more livestock a household will keep. But this risk can be regarded as income risk that depends on household characteristics. By assuming so, this model is consistent with the view that precautionary demand for savings depends on household's ability to reduce exogenous risk.

a. 4 Evidence of Precautionary Demand for Savings

Here, I present empirical evidence of precautionary demand for savings by showing that households in the Sahel have a convex marginal utility function. With a standard utility function, if we can show that the decline of marginal utility is diminishing as wealth increases, we can conclude that the marginal utility function is convex.

The convexity of the marginal utility function can be proven empirically based on empirically estimated ϕ, or the marginal rate of substitution of expected utility between a good year and a drought year. As seen from Figure 2 in chapter 5, the ratio between two slopes can be easily calculated using the estimated ϕ. Because this ratio decreases as drought probability increases in the Sahelian and the Sudanian zones, I conclude that the marginal value function is convex there. This suggests the existence of precautionary demand for saving. But in the Guinean zone, the result suggests no precautionary demand for saving.

a. 5 Demand for Drought Insurance and Precautionary Demand for Savings

ϕ (the marginal rate of substitution of utility between a good year and a drought year) measures demand for formal drought insurance, but it is a mixed measure of a household's degree of risk-aversion and a household's ability to reduce crop production risk. Thus, a high ϕ can be due to both high risk-aversion and high expected risk in crop production. These two effects are not consistent because a risk-averse household (high ϕ) tends to choose less risky crop production (low ϕ). Therefore if a household has a high ϕ, it

may imply that the household cannot reduce exogenous risks effectively, even if it is risk-averse. On the other hand, a low ϕ implies that a risk-averse household successfully reduces exogenous risks.

There may be three cases of the relation between a household's initial wealth and ϕ as follows. (i) Wealth will give a low ϕ if it allows a household to reduce exogenous risks. This may include the case when a poor household is forced to choose risky crop production due to binding constraints. (ii) Wealth will produce a high ϕ if it induces risky decisions because wealth provides the ability to smooth consumption. (iii) Wealth will produce a high ϕ if a household facing great exogenous risk accumulates wealth, but the wealth does not effectively reduce risk. (ii) and (iii) are distinguished by regression specifications: in fixed effect regressions we can observe (ii), while in cross-section regressions we can observe (iii). In addition to these three effects, change in wealth is likely to affect a household's risk-aversion, which also changes ϕ. Taking this effect into account, wealth will make the household less risk-averse, and as a result will reduce ϕ. Therefore, (i) will be strengthened and (ii) and (iii) will be weakened by this effect.

On the other hand, wealth will also have effects on precautionary demand for savings because wealth provides the ability to reduce exogenous risk and to smooth consumption. In case (i) above, wealth may reduce precautionary demand for savings because it reduces exogenous risk, and therefore, a high ϕ implies high precautionary demand for savings. In the case (ii) and (iii), wealth may induce or at least not reduce precautionary demand for savings because it increases or at least does not reduce income risk, and therefore, a high ϕ implies high precautionary demand for savings. Thus, in either case, ϕ measures both demand for formal drought insurance and precautionary demand for savings.

Following the discussion above, ϕ in Table 5 of chapter 5 now can be regarded as an index of precautionary demand for savings. Consequently, Table 5 can be interpreted as follows: in the Sahelian and the Sudanian zones, precautionary demand for savings decreases as rainfall risk decreases. This suggests that households in the Sahelian and the Sudanian

zones have little ability to smooth consumption and as a result rainfall risk induces precautionary demand for savings. On the other hand, in the Guinean zone, as discussed above ϕ may not imply precautionary demand for savings, but rather it can be interpreted that households in the zone have precautionary demand for dissaving, which increases as rainfall risk decreases. This suggests that households in the Guinean zone tend to gamble when they are in a less risky situation because they have enough ability to smooth consumption. Moreover, comparison of precautionary demand for savings between the Sahelian and the Sudanian zones is consistent with differences in riskiness in rainfall of the two zones.

It can be concluded from the empirical study that precautionary demand for savings exists in the two relatively risky zones, and that it can be measured by ϕ.

Hypothesis b: *Savings take the form of livestock holdings because there is no formal saving institutions and because livestock is a relatively safe asset in times of drought in the SAT.*

The following facts support the hypothesis that livestock holdings are precautionary in the Sahel.

First, as shown in Appendix D of chapter 5, livestock holdings per adult equivalent are highest in the Sahelian zone, and lowest in the Guinean zone. This implies that the more rainfall risk, the more are livestock holdings relative to crop production. This indirectly suggests that livestock is at least one form of precautionary savings.

Second, in cross-section regressions the effect of livestock holdings on ϕ is positive in the middle livestock holding stratum in Table 8 of chapter 5, which implies that when we compare households, a household that has more livestock takes more risk. In addition, when average ϕs are compared among three strata in Table 8, it is found that the larger is the average ϕ, the larger is the average livestock holding. This implies that livestock holding is precautionary saving, but it does not reduce exogenous risks effectively. These findings are consistent with the case (iii) in a. 5.

Third, the fixed effect regressions in Tables 7 and 8 of chapter 5 show that livestock holdings have a negative or an insignificant effect on ϕ: negative in the Guinean zone and the lower livestock holding stratum and insignificant in other zones and strata. The negative effect means that livestock holdings reduce exogenous risks, which is consistent with effective precautionary saving, or the case (i) in a. 5. The case of an insignificant effect may occur in one of three situations as follows: (1) although livestock holdings are precautionary, the reduction of exogenous risks is not effective; (2) household's livestock holdings are excessively so large that there is no marginal effect of livestock on risk reduction; or (3) livestock holdings make households less risk-averse (negative effect on ϕ) but allows them to gamble (positive effect on ϕ), and these two effects are canceling each other. But case (3) is not precautionary saving.

Combining the cross-section and fixed effect results, (1) and (2) will be the case in the Sahelian and the Sudanian zones. Therefore, the empirical results support the hypothesis that livestock holdings are precautionary in those zones.

However, the evidence does not necessarily imply that livestock holdings are the only precautionary savings. For example, off-farm income has a negative significant effect on ϕ in the middle livestock holding stratum. This implies that off-farm activities are also precautionary for the households in this stratum.

Hypothesis c: *As a result of the precautionary livestock holding, overgrazing occurs because grazing land is an open-access, common-pool resource.*

Empirical evidence presented in (a) and (b) supports that livestock holdings are precautionary in the Sahel. This finding justifies the agropastoral model in chapter 4, in which livestock holdings are assumed to be riskless saving to hedge against crop production risk. One important finding from the model is that overgrazing does not necessarily occur even if grazing land is open-access, and that overgrazing is likely to occur when crop production risk is high and expected return to crop production is low.

However, I have no data on quality of grazing land and governing rules of common-pool resources. And therefore, no empirical evidence is presented for this hypothesis. Empirical examination of this hypothesis is left to future research. Thus, this is the weakest link of the four hypotheses.

Hypothesis d: *Formal financial institutions, such as drought insurance, will substitute for livestock holdings and reduce overgrazing.*

This is predicted by the agropastoral model in chapter 4; if agricultural production risk decreases, livestock holdings will be lower. If drought insurance reduces crop production risk, a household that buys formal drought insurance will reduce agricultural production risk and as a result will reduce livestock holdings. Therefore, for formal drought insurance to reduce overgrazing, a necessary condition is that most households demand the insurance. This question is empirically answered in chapter 5 by examining demand for hypothetical formal drought insurance.

Since there is no demand for formal drought insurance in the lower livestock holding stratum, insurance will not reduce livestock holdings in this household group. But because total livestock holdings in this stratum are small relative to those in other strata, those households do not harm grazing land very much. On the other hand, in other strata, there is effective demand for the formal drought insurance, and consequently if households buy the insurance, livestock holdings may be reduced.

But in those strata, livestock holdings have insignificant effect on ϕ. As discussed in (c), there will be three cases: (1) Livestock holdings do not effectively reduce exogenous risk; (2) There is an excess of livestock holdings (relative to the optimal self-insurance), probably in the upper stratum; and (3) Livestock holding induces gambling behavior. In the case of (1) and (2), households still demand formal drought insurance. This implies that livestock holdings are not as effective insurance as is formal drought insurance. Therefore, households will buy the insurance to be more effectively insured, if it is available, and they are likely to reduce livestock holdings to finance it, especially if households have excess livestock

holdings. In case (3) when households demand formal drought insurance, it implies that they will substitute the insurance for current self-insurance including livestock holdings. Therefore, in either case, effective demand for the formal drought insurance implies that households will reduce livestock holdings.

However, if there are other precautionary assets, livestock is not necessarily reduced by the insurance. For example, in the middle livestock holding stratum, off-farm income has a negative effect on ϕ. This means that households may reduce off-farm activities if drought insurance reduces crop production risk effectively, and may not reduce livestock holdings very much.

Because households that have large livestock holdings are responsible for overgrazing, to avoid overgrazing they have to reduce livestock holdings significantly. As shown in chapter 5, households in the upper livestock holding stratum show strong demand for formal drought insurance, but livestock holdings have no effect on the demand. It may imply that the households in this stratum tend to keep excess livestock holdings though they do not reduce risks enough. Therefore, they are most likely to substitute formal drought insurance for livestock holdings. This means that formal drought insurance will decrease overgrazing significantly.

Conclusion

Empirical evidence supports the hypothetical story presented at the beginning of this chapter, except for hypothesis c. It is regrettable that I cannot test hypothesis c in this dissertation due to lack of data. However, all the theoretical and empirical explorations in this dissertation provide a big picture of household behavior under the drought risk in the Sahel, and will contribute to future empirical research.

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