This is to certify that the

thesis entitled

Optimization of a Variable
Core Geometry Radiator

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OPTIMIZATION OF A VARIABLE CORE GEOMETRY RADIATOR

By

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ABSTRACT

OPTIMIZATION OF A VARIABLE CORE GEOMETRY RADIATOR

By

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An investigation on the cost of a heat exchanger by using different geometries, specifically annular fins, has been conducted..The effect of different parameters on the cost was investigated, such as material cost, labor cost, fuel cost which is used for operating the pump and fan for the whole life of the radiator and also the effect of interest rate of the money used for operating cost. Analysis

showed that the cost of the radiator is a step function, which limits the available techniques of the optimization. Calculations are performed by using a spread sheet (Micro Soft Excel) for the different parameters of the geometries and results showed that there is no improvement in the cost by arranging different geometries in combination rather than using only one type of geometry. By this analysis we are able to find which geometry gives the minimum cost.

to my parents, Hafiz Muhammad Ashraf and Bano for
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NOMENCLATURE

Arabic

A	Area [m^2]
C_p	specific heat [J/kg K]
C	heat capacity [W/K]
$\dot{m}$	mass flow rate [kg/s]
$\dot{H}$	enthalpy [J]
$\hat{h}$	specific enthalpy [J/kg]
h	convective heat transfer coefficient [W/m^2K]

T	Temperature [C° of K]
q	heat [W]
U	thermal conductance [W/m^2 K]
R	thermal resistance [K/W]
NTU	number of transfer units.
K	thermal conductivity [W/m^2]
t	tube thickness [m]
H	height of the radiator [m]
W	width of the radiator [m]
m	slope of the straight line.
F	fin pitch [# fins/meter]
D	diameter [m]
b	y-intercept of straight line
Re	reynold's number
j	coulburn factor
E	constant
L	length [m]
P	perimeter [m]

V	volume [m^3]
Pr	prandtle number
Nu	nusselt number
f	friction factor
M	constant
(x,y)	Cartesian coordinates [m]
N	total number of terms
g	acceleration due to gravity
Kc	entrance loss coefficient
Ke	exit loss coefficient
CF	cost of fuel [$]
Ml	miles/year
CO	cost [$]
I	interest rate [%]
a	principle amount [$]
r	common ratio
$\dot{P}$	Power [W]

J	objective function
Cs	cost component related to heat exchanger sizes annual equivalent.
CP	cost component related to the pumping power annual equivalent.
CH	Cost component related to the supply of heating or cooling effect to the radiator. (heat exchange)

Greek

ε	effectiveness
_	change
ν	specific volume [m^3 /kg]
η	efficiency
ρ	density [kg/m^3]
μ	dynamic viscosity
σ	free flow area/frontal area
α	heat transfer area/total volume

Γ	correction factor

Subscripts

in	inlet
out	outlet
C	cold fluid wall side
H	hot fluid wall side
min	minimum
max	maximum
W	wall
tot	total
R	ratio
i	inside
o	outside
m	mean

avg	average
f	fin side
h	hydraulic
s	solid
act	actual
conv	convection
cond	conduction
surf	surface
F	fan
P	pump
fu	fouling

Chapter 1

Introduction

Heat exchangers provide for the transfer of heat between two moving fluids. These devices are used in power generation, chemical and food processing, heating, air conditioning and motor vehicles. It is the most recognizable heat transfer devices and one of the most widely used. Heat exchangers are classified based on flow arrangement and type of construction. There are many types of heat exchanger designs, each type with its own characteristics that make it suitable for a particular application. In motor vehicles the heat exchangers are called radiator. In a radiator the hot fluid is moving in the tubes and atmospheric air is the cooling fluid. It is a cross flow heat exchanger and these are the most successful type of heat exchangers.

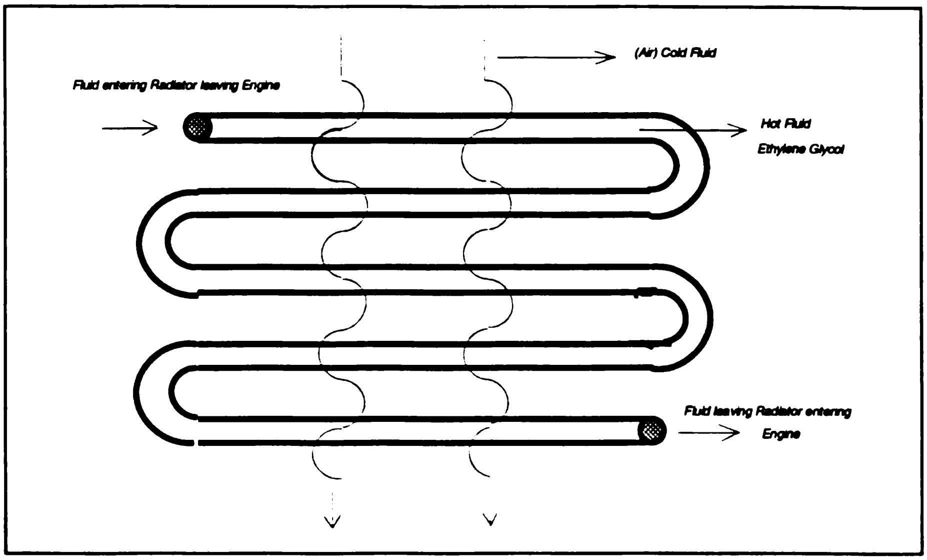


Fig .1-1: Heat flow through a Cross Flow heat exchanger

The design of heat exchangers, in general, requires consideration of the heat transfer occurring between the two fluids in addition to the mechanical energy needed to overcome the frictional forces to move the fluids through the heat exchanger. These two design criteria can be generally classified as heat transfer and pressure drop. It is typically desired to achieve large heat transfer yet maintain a small pressure drop. Large heat transfer rates can be obtained by either having a large heat transfer area or having a large heat transfer convection coefficient. Unfortunately, both of these conditions cause an increase in the pressure drop, since a larger area gives more frictional area resulting in an increase in the pressure drop and the larger flow rate to increase the convection coefficient would likewise increase the pressure drop. There is a trade-off in these two design criteria; a beneficial gain in one criteria is usually at expense of the other criteria and a compromise must be established.

Focusing on the heat transfer, consider a simplistic heat exchanger (radiator) that has two fluid streams separated by a thin wall. One fluid is moving inside the tubes and the cold air (atmospheric air) is moving across the tubes. Heat is transferred from the tube fluid to the air.

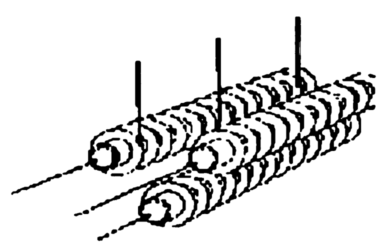


Fig 1-2: Annular fins, with liquid(hot fluid) is flowing inside the tubes and Air(gas) outside.

An effective way to increase surface area density is to make use of secondary surfaces, or fins, on one or both fluid sides of the surface. Fig. 1.2 illustrates a finned circular tube surface in which circular fins have been attached to the outside of circular tubes (annular fins). Such an arrangement is frequently used in gas-to-liquid heat exchangers where optimum design demands a maximum of surface area on the gas side. Therefore, the annular fin type heat exchanger has been chosen to study for the car radiator.

Axial conduction of heat is typically neglected in the analysis of heat transfer in a heat exchanger. This assumption may be approximate for most cases, but under certain circumstances the effect of axial conduction can become important.

Cost optimization is the most important feature of the heat exchanger. Considering the effect of cooling fluid mass flow rate and different annular fins geometries to optimize the cost of the radiator will be the focus of this thesis.

Once the type of geometry used, fluid used, inlet and outlet temperatures are selected, the problem is to get the proper type of tube geometry or combination of different tube geometries with optimum cooling fluid mass flow in order to minimize cost of the radiator. The inlet temperature of air is taken 50°C. This temperature is being achieved in some parts of asia.

The use of a variable core geometry is an attempt to make heat transfer uniform through the heat exchanger by increasing the surface area as the temperature difference decreases.

The remaining sections of this chapter will review the analysis of a heat exchanger cost optimization of heat exchangers and will conclude with a literature review. The method of solution is presented next. Heat exchanger analysis, spread sheet, optimization of cost and cost function will be addressed in chapter two, chapter three will present the results of the investigation with corresponding discussion. A summary and the resulting conclusions will be given in chapter four in addition to recommendations for future work.

1.0 Heat Exchanger Analysis:

1.0.1 Basic Analysis:

Figure 1.3 shows a heat exchanger that transfer energy between two moving fluids through a wall; the geometry and construction of this heat exchanger may be considered arbitrary. A general thermal analysis of this heat exchanger will be performed. First, taking the heat exchanger as a control volume and applying an overall energy balance, assuming no interactions (work or heat) with the surroundings and steady state, results in an enthalpy balance (H).

$$\dot{H}_{in}=\dot{H}_{out} \qquad ......(1.1)$$

This can be written in terms of the inlet, outlet and existing conditions using specific enthalpy and mass flow rate as

$$\dot{m}_H\hat{h}_H|_{in}+\dot{m}_C\hat{h}_C|_{in}=\dot{m}\hat{h}_H|_{out}+\dot{m}_C\hat{h}_C|_{out} \qquad ....(1.2)$$

and rearranged to group the fluids

$$\dot{m}_H(\hat{h}_H|_{in}-\hat{h}_H|_{out})=\dot{m}_C(\hat{h}_C|_{out}-\hat{h}_C|_{in}) \qquad ....(1.3)$$

Assuming the fluid behaves as an incompressible liquid or an ideal gas and a negligible pressure change, the change in the enthalpy can be expressed as

$$d\hat{h}=C_p dT \qquad ....(1.4)$$

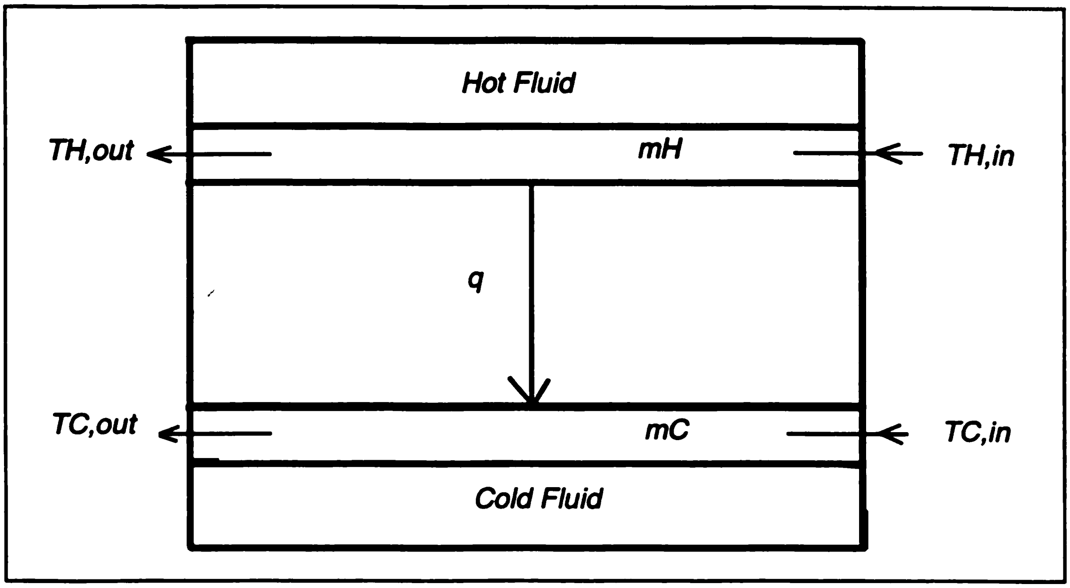


Fig.1-3: Generic heat exchanger, nonspecific design exchanging energy between the two fluids.

This can be approximated by differences ($d\hbar = \hbar 2 - \hbar 1$) using the approximate form of equation (1.4) in equation (1.3), the results for the overall energy balance are;

$$q = \dot{m}_H C_{p,H} (T_{H,in} - T_{H,out}) = \dot{m}_C C_{p,C} (T_{C,out} - T_{C,in}) \qquad \ldots\ldots(1.5)$$

Introducing the new term as the heat capacity

$$C \equiv \dot{m} C_p \qquad \ldots\ldots(1.6)$$

equation (1.5) can be rewritten as

$$q=C_H(T_{H,in}-T_{H,out})=C_C(T_{C,out}-T_{C,in}) \quad ....(1.7)$$

In general, the heat transfer between the two fluid streams is a function of the following six parameters.

$$q=f(\dot{m}_H,\dot{m}_C T_{H,in},T_{C,in},T_{C,out},T_{H,out}) \quad ......(1.8)$$

where the first four are typically given design parameters and the last two are desired results. Thus, in order to obtain a solution more information is needed. The additional information will come from the heat transfer analysis of the wall separating the two fluids.

A circuit describing the thermal communication of the two fluids is shown in Figure 1.4. This circuit shows the resistance that impedes the heat flow, neglecting any fouling on the heat exchanger wall. The total resistance of the series circuit is the sum of the individual resistances.

$$R_{tot}=R_H+R_w+R_C \quad ....(1.9)$$

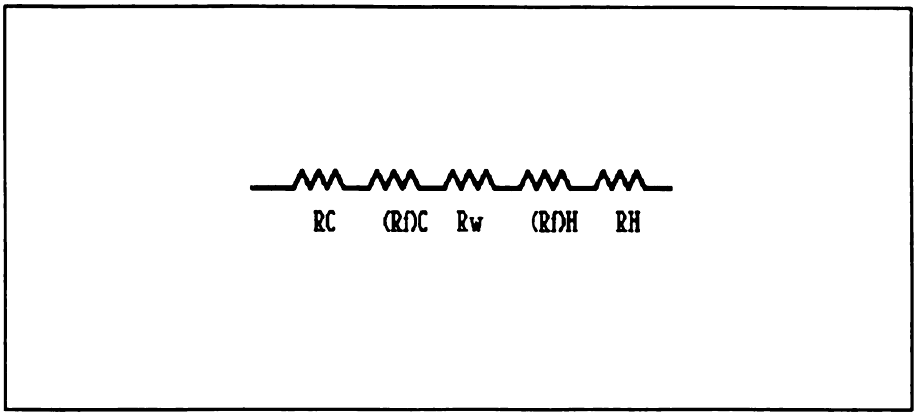


Fig.1-4: Thermal circuit for a heat exchanger wall neglecting any fouling.

Also, the overall conductance of the heat exchanger wall, that will give the total amount of heat transferred if the temperature difference is known, is

$$UA=\frac{1}{R_{tot}} \qquad \text{......(1.10)}$$

These terms are analogous to an electrical circuit, with the temperature difference as the voltage potential and the heat flow as the current. The heat transfer over a different element of length dx is

$$dq=UP[T_H(x)-T_C(x)]dx \qquad \text{......(1.11)}$$

The heat transfer over the entire length of the heat exchanger is the sum of these differentials elements over the total length

$$q=\int_0^L dq=UP\int_0^L [T_H(x)-T_C(x)]\,dx \qquad \text{......(1.12)}$$

The product UP was assumed constant and thus could be taken out of the integral, but (T_H - T_C) depends on x and cannot be removed from the integral. Because this integral is not known in general, we will define the mean temperature difference as

$$(\Delta T)_m=\frac{1}{L}\int_0^L [T_H(x)-T_C(x)]\,dx \qquad \text{.......(1.13)}$$

By substituting equation (1.13) into equation (1.12), the total heat transfer can be written as

$$q=UA(\Delta T)_m \qquad \text{......(1.14)}$$

For geometries that are more complicated, evaluating equation (1.13) is difficult if not impossible. This suggests that another approach may be necessary to remain a general analysis.

1.0.2 Dimensionless Analysis:

To introduce another approach, consider the parameters that the mean temperature difference is a function of

$$(\Delta T)_m = f(T_{H,in}, T_{C,in}, C_C, C_H, UA) \qquad \text{......(1.15)}$$

It depends on the inlet temperatures and heat capacities of the fluids and the wall conduction. The number of independent parameters can be reduced by considering a second function times the temperature difference at the inlet.

$$(\Delta T)_m = g(C_C, C_H, UA)(T_{H,in} - T_{C,in}) \qquad \text{......(1.16)}$$

Substituting equation (1.16) into equation (1.14) the functional dependence for the heat transfer is given by

$$q = UAg(C_C, C_H, UA)(T_{H,in} - T_{C,in}) \qquad \text{.......(1.17)}$$

As typical in heat transfer, scaling will be introduced to provide dimensionless parameters. Defining the maximum possible heat transfer as:

$$q_{\max}=C_{\min}(T_{H,in}-T_{C,in}) \qquad \text{......(1.18)}$$

where $C_{\min}$ = min(C_H ,C_C) and dividing equation (1.17) by equation (1.18)

$$\frac{q}{q_{\max}}=\frac{UA}{C_{\min}}g(C_C, C_H, UA) \qquad \text{......(1.19)}$$

Introducing another function that is terms of scaled independent variable gives

$$\frac{q}{q_{\max}}=\frac{UA}{C_{\min}}\acute{g}(C_R, \frac{UA}{C_{\min}}) \qquad \text{......(1.20)}$$

Where

$$C_R=\frac{C_{\min}}{C_{\max}} \qquad \text{......(1.21)}$$

Finally, noting that equation (1.20) can be written for a final function as

$$\frac{q}{q_{\max}}=h(C_R, \frac{UA}{C_{\min}}) \qquad \text{......(1.22)}$$

This demonstrates that the performance of a heat exchanger can be expressed in terms of three dimensionless variables. The first was given previously in equation (1.21) as the ratio of the heat capacities. The second is the effectiveness

$$\epsilon=\frac{q}{q_{\max}}=\frac{q}{C_{\min}(T_{H,in}-T_{C,in})} \qquad \text{......(1.23)}$$

Effectiveness (ϵ) is the ratio of actual heat transfer given by (1.7) to the maximum possible heat transfer. The number of transfer units (NTU) is the last dimensionless parameter

$$NTU=\frac{UA}{C_{\min}} \qquad \text{......(1.24)}$$

(NTU) is the ratio of the heat exchanger ability to transfer energy to the minimum fluids ability to retain energy.

Previously mentioned demensionless parameters at least for the moderately simple heat exchanger geometry, can result during the analytical analysis of the heat transfer occurring in a heat exchanger. For example, consider a cross flow one fluid mixed and one unmixed heat exchanger. The correlation describing the performance of this heat exchanger.

(i) for C_{min} (unmixed)

C_{max} (mixed)

$$\epsilon=\left(\frac{1}{C_R}\right)(1-\exp(-C_R[1-\exp(-NTU)])$$

.....(1.25)

(ii) for C_{min} (mixed)

C_{max} (unmixed)

$$\epsilon=1-\exp(-C_R^{-1}[1-\exp(-NTU)]) \qquad .....(1.26)$$

1.1 Literature Review:

Previously the optimization of cost is done to optimize the heat transfer in radiative heat exchangers. The idea is that if we optimize the heat transfer then the cost also optimize.

Kuprys[1] used the concentric heating element with an additional central heat exchanger in the form of field's tube is analyzed. Experiment showed that effective thermal power developed by such heat exchangers depends basically on the flowrates, distribution and initial pressure of the cooling gas. A number of design and operating parameters (heating-element emissivity, cross-sectional area of the field tube, etc) affect the effective thermal power developed by the heat exchanger under the specified constraints on the temperature of its elements. He used a computer package to find the optimum heat flux in a radiator.

Another way adopted by Kovarik [2] was to get a maximum of the objective function. Basically this is the aim of optimization. An objective function, J, is defined as the ratio of performance to cost.

$$J = \frac{H}{C_s + C_p + C_h} \qquad \text{......(1.27)}$$

From equation (1.27)

$$J = \frac{1}{(C_S + C_P)/H + a4} \qquad \text{.....(1.28)}$$

Clearly, the position of the maximum of J coincides with the minimum of the first term in the denominator of the right hand side of equation (1.28), and is independent of the value of the energy cost factor a4 . Therefore an optimal heat exchanger is optimal for any cost of energy. Hence by applying the optimization techniques we can found the optimum of J.

Evans [3] used the fundamental principles of differential second law analysis to find the optimum number of transfer units.

$$NTU = (1/b)\ \ln(1+bnH) \qquad \text{---------(1.29)}$$

Equation (1.29) gives the optimum number of transfer units for all elementary flow arrangements (i.e, counterflow, parallel flow and elementary cross flow) and for all possible values of the heat capacity rate ratios.

The optimization done in this thesis differ from previous work in the formulation and solution of the problem. Previous techniques of optimization involves the optimization of the heat flux [1], optimization of objective (performance to cost) function by getting the maxima [2], and the optimization of transfer units [3]. Although the previous techniques of optimization are better to some extent, these are not the direct approaches to develop the appropriate cost function and the use the optimization. Therefore, the tasks are to develop an appropriate cost function, modify the existing spread sheet program, and perform the iterative calculation to minimize cost function.

Chapter 2

Method of Solution

The method employed to get the optimum design is the Microsoft Excel Ver-4. Two different spread sheets, one for sizing and one for rating problems, were employed.

Operating data used was for of General Motor Cavelier 3.1L. The radiator is of cross flow, annular fin type. Air is cooling fluid and ethylene glycol is the working(hot) fluid.

2.1 Sizing Process:

In first step, a sizing analysis was done for uniform cores using the 6 (six) different geometries A,B,C,D,E, and F one by one.For one type of geometry sizing is done by iterating with the mass flow of air to get the optimum mass flow where the cost is minimum. This is done by changing the outlet temperature of air with a particular mass flow of air such that we achieved the required outlet temperature of hot fluid (ethylene hlycol).

By getting the cost for different mass flow rates we get the minimum cost required for geometries A,B,C,D,E, & F at a particular mass flow rate. From these observations we can then get the geometry and air mass flow rate for which the cost is minimized.

S.No	Geometry	Fin Pitch	Ctr-Ctr Dis	Ctr-Ctr Dis.
1.	A	289/m	0.0248 m	0.0203 m
2.	B	343/m	0.0247 m	0.0203 m
3.	C	343/m	0.0247 m	0.0203 m
4.	D	276/m	0.0313 m	0.0343 m
5.	E	343/m	0.0313 m	0.0343 m
6.	F	343/m	0.0469 m	0.0343 m

Table 2.1: Showing the important features of the six geometries used

2.2 Rating Process:

For the variable geometry core, that is a core consisting of two or more geometries,it is useful to perform a rating problem.By adding one layer of a particular type of geometry and using the mass flow rate that gives the minimum cost in the sizing problem for that particular geometry the outlet temperatures are calculated. If outlet temperature of hot fluid is equal of slightly less than the required one then the process stops, otherwise, another layer of same type of geometry or of other type is added. This process is continued until we get the required temperature. This process is iterated with all possible combination to get the minimum possible cost.

2.3 Heat Exchanger Analysis:

In heat exchanger analysis there are two types of problems:

1- Sizing Problem

2- Rating Problem

2.3.1 Sizing Problem:

In the sizing problem we know the inlet and outlet temperatures of the fluids, mass flow rates, frontal area of the exchanger the geometry used and its parameters. Hence all the operating conditions are known or can be calculated from the first law of thermodynamics. We can calculate effectiveness (ϵ) from the given informations but we have to know the ϵ-Ntu relationship according to the geometry specification (annular fin one fluid mixed (air) and one unmixed (ethylene glycol)). Once the Ntu is calculated, the heat transfer surface area can be determined.

2.4 Concerning Equations:

After deciding the type of heat exchanger, working fluids (cooling and hot fluid) the problem is set up in the following manner.

Cold Fluid (Air)		Hot Fluid (Ethylene Glycol)	
1- Inlet temp. of air $T_{C,in}$	=?	Inlet temp. $T_{H,in}$	=?
2- Outlet temp. of air $T_{C,out}$	=?	Outlet temp. $T_{H,out}$	=?
3- Average temp. TH,avg	=?	Average temp. $T_{H,avg}$	=?
4- $C_{p,C}$	=?	$C_{p,H}$	=?
5- $\dot{m}_C$ (Air)	=?	$\dot{m}_H$(Ethylene Glycol)	=?
6- $C_C = C_{max}$ or C_{min}	=?	$C_H = C_{max}$ or C_{min}	=?

Other inputs include the material of the tubes and the fins.

Tube Material	Copper(Cu)
Fin Material	Aluminum(Al)

2.4.1 Frontal Area:

We know that height (H) and width (W) of the radiator, which

is fixed for a particular design then we can calculate the frontal area

$$\text{Frontal Area} = A = W \times H \qquad \text{--------} \ (2.1)$$

2.4.2 Fixed Parameters for a geometry:

I used the informations about he geometry (annular fin) form [1]. The following parameter are fixed parameters for a particular geometry and these are read out directly from the geometry specifications (Appendix "D")

(a) Free flow area/frontal area, σ

(b) Heat transfer area/Total area, α

(c) Flow passage hydraulic diameter, 4r =(4xtube dia)

(d) Tube outer diameter = D_o

(e) Inside tube diameter = D_i

(f) Fin diameter = D_f

(g) Fin Pitch (Fins/meter) = P

(h) Fin thickness = t

(i) Fin Area/Total Area = A_f/A_t

(j) Exch. Height: Ctr. to Ctr. Tube Distance = h

(k) Exch. Depth: Ctr. to Ctr. Tube Distance = d

The spread sheet program requires specification of the constant and exponent associated with the colburn factor $j_H=CRe^m$. The six types of annular fin geometries used have, correlating relations that are both straight lines curves. All have been approximated as a straight line, and it appear accuracy is not very much affected.

The standard equation for the straight line is

$$y = mx + b \qquad \text{-------(2.1)}$$

Where, m = slope of the line

b = y intercept

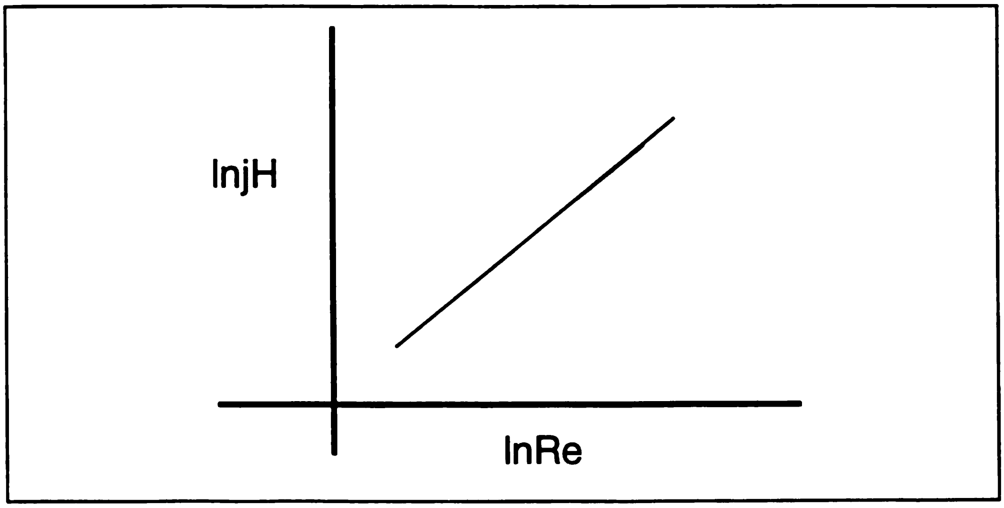


Fig.2-1: Graphic relationship between Colburn factor and Reynold's number.

$$\ln j_H = m \ln Re + b \qquad .....(2.2)$$

Slope m is as follow:

$$m=\frac{(y_2-y_1)}{(x_2-x_1)}=\frac{(\ln j_{H,2}-\ln j_{H,1})}{\ln(Re_2)-\ln(Re_1)} \qquad .....(2.3)$$

and the y-intercept

$$b=\ln j_{H,2}-m\ln Re_2 \qquad ......(2.4)$$

$$\ln j_H = \ln Re^m + b$$

$$\ln j_H = \ln Re^m + \ln e^b$$

$$\ln j_H = \ln Re^m e^b$$

$$j_H = e^b Re^m \qquad ......(2.5)$$

$$m=\frac{\ln j_{H,2}-\ln j_{H,1}}{\ln(Re_2)-\ln(Re_1)} \qquad ......(2.6)$$

Hence, $$C = e^b \qquad ......(2.6a)$$

$$j_H = C\ Re^m \qquad ......(2.7)$$

Equation (2.5) gives the relation for the Colburn Factor.

2.4.4 Friction Factor Equation:

Similar to Colburn's factor, we approximate the friction factor relation to be a power law and find the equation in a similar way.

$$f = C\ Re^{m} \qquad \text{......(2.8)}$$

$$C = e^{b} \qquad \text{......(2.9)}$$

$$b = \ln f_2 - m \ln Re_2 \qquad \text{.....(2.10)}$$

$$m = \frac{\ln f_2 - \ln f_1}{\ln(Re_2) - \ln(Re_1)} \qquad \text{.....(2.11)}$$

2.5 Calculated Values For a Geometry:

2.5.1 Fin Length:

Since the focus has been on annular fin type heat exchangers the fins may be modeled as shown bel

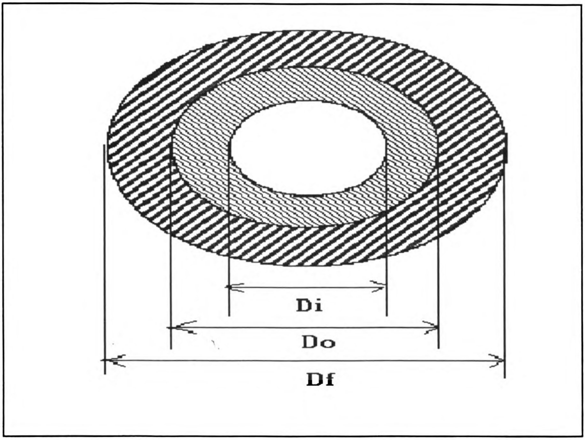


Fig.2-2: ***Showing the thickness of tubes and fin diameter.***

$$Finlength = L_f = \frac{(D_f - D_0)}{2} = (R_f - R_0) \qquad ......(2.12)$$

2.5.2 Fin Perimeter:

For annular fin type heat exchangers

$$Fin\ Perimeter = \frac{\pi}{2}\,\frac{(D_f^2 - D_9^2)}{Lf}$$

$$P_f=\Pi(D_f+D_o)$$

......(2.13)

2.5.3 Fin Cross-Sectional Area:

$$A_c=t\Pi\frac{(D_f-D_o)}{4}2 \qquad \text{......(2.14)}$$

$$A_c=t\frac{\Pi}{2}(D_f-D_o)$$

2.5.4 Inside Surf. Area/Outside Surf. Area(U.C):

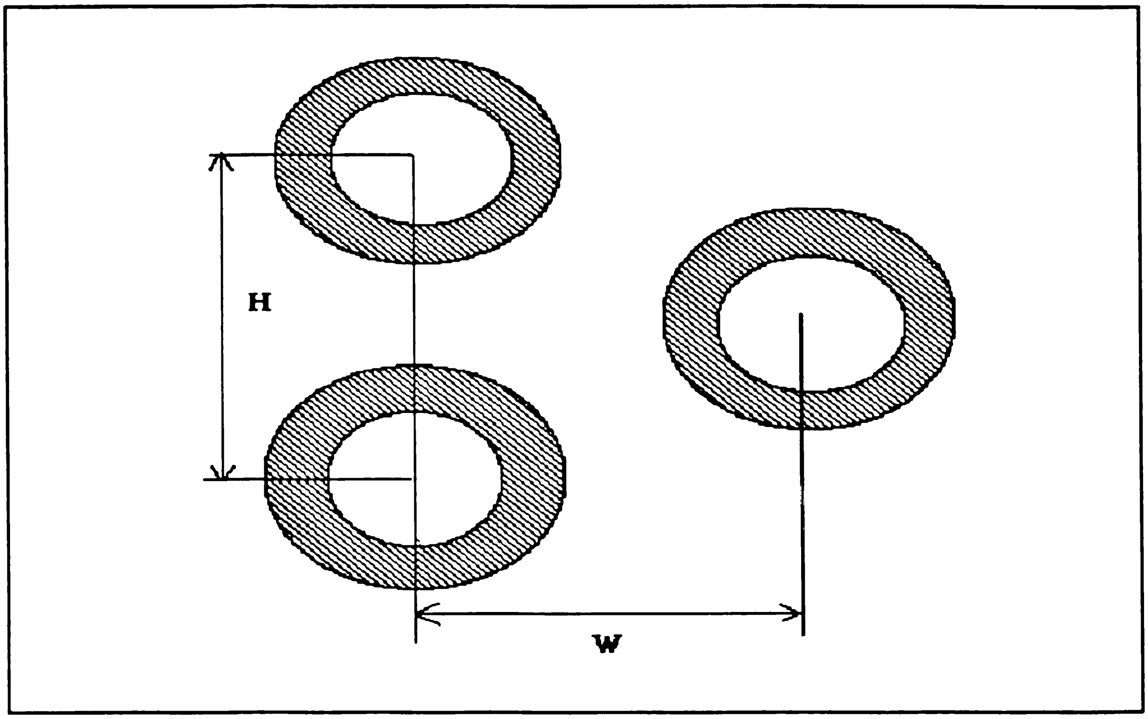


Fig.2-3: Showing the arrangement of tubes in radiator to get width and height.

$$\frac{A_i}{A_o} = \frac{Inside\ Surface\ Area\ of\ Tubes}{(Surf.\ Area\ of\ Fins)x2 + Nacked\ surf.\ area\ of\ tubes}$$

$$\frac{A_c}{A_o} = \frac{\pi D_i L}{LP(\frac{\pi}{4}D_f^2 - \frac{\pi}{4}D_o^2)2 + [\pi D_o L - LP\pi D_o t]}$$

$$\frac{A_c}{A_o} = \frac{D_i}{P\frac{(D_f^2 - D_o^2)}{2} + D_o - PD_o t} \qquad \ldots\ldots(2.15)$$

$$\frac{A_c}{A_o}=\frac{D_i}{0.5P(D_f^2-D_o^2)+D_o-PD_o t} \qquad \ldots\ldots(2.16)$$

2.5.5 Volume of cell Solid/Volume of Cell:

$$\left[\frac{V_s}{V_{tot}}\right]_{cell}=\frac{Volume\ of\ Solid\ material}{Total\ Volume}$$

$$\left(\frac{V_s}{V_{tot}}\right)_{cell}=\frac{\frac{\pi}{4}(D_o^2-D_i^2)L+PL\frac{\pi}{4}(D_f^2-D_o^2)t}{HWL}$$

$$\left(\frac{V_s}{V_{tot}}\right)_{cell}=\frac{\pi(D_o^2-D_i^2)+P\pi(D_f^2-D_o^2)t}{4HW}$$

$$\ldots\ldots(2.17)$$

2.6 Method to find the outlet Temperature of Coolant (Ethylene Glycol):

Outlet temperature of working fluid can be found by the use of first law of thermodynamics.

Heat lost by working fluid(Ethy. Gly.) = Heat gain by air

$$(\dot{m}C_p)_C(T_{C,out}-T_{Cin})=(\dot{m}C_p)_H(T_{H,in}-T_{H,out}) \qquad \ldots\ldots(2.18)$$

which specifies all of the temperatures.

2.7 Finding of Specific heats (C_p 's):

The values of C_p 's are found at their respective mean temperature. The existing spread sheet incorporate a curve fit to data to represent C_p as a function of temperature.

2.8 Calculation of Prandtle Number:

The Prandtl number is

$$P_r = \frac{\mu C_p}{K} \qquad \text{......(2.19)}$$

The values of μ, Cp and K are evaluated at the mean temperature.

2.9 Effectiveness (ε):

The effectiveness now can be calculated by the following formula. It is the ratio of the actual heat transfer to the theoretical heat transfe

$$\epsilon = \frac{q_{act}}{q} = \frac{C_H(T_{H,in}-T_{H,out})}{C_{min}(T_{H,in}-T_{C,in})} = \frac{C_C(T_{C,out}-T_{C,in})}{C_{min}(T_{H,in}-T_{C,in})} \qquad \text{.....(2.20)}$$

The spread sheet uses an if statement to determine C_{min}.

2.10 Calculation of Ntu:

There are different relationships between ϵ and Ntu for different conditions. The relationship for the cross flow heat exchanger with one fluid mixed (air) and one unmixed (ethylene glycol) is being used in this analysis.

If air is (C_{max}) and Ethylene Glycol in (C_{min}) then relation is;

$$Ntu=-\ln\left[1+\left(\frac{1}{C_R}\right)\ln(1-\epsilon C_R)\right] \qquad .....(2.21)$$

and if air is (C_{min}) and Ethylene Glycol (C_{max})

$$Ntu=-\left(\frac{1}{C_R}\right)\ln\left[C_R\ln(1-\epsilon)+1\right] \qquad ......(2.22)$$

Where

$$C_R=\frac{C_{min}}{C_{max}}$$

2.11 Outside Reynold's Number:

We can find the Reynold's number by using the following formula.

$$Re=\frac{GD_h}{\mu} \qquad \text{......(2.23)}$$

Where,

$$G=\frac{\dot{m}_H C_H}{\sigma A_{fr}}$$

2.12 Outside Heat Transfer Coefficient:

Outside heat transfer coefficient is extracted from the colburn factor

$$h_o=j_H\frac{GC_p}{Pr^{2/3}} \qquad \text{.....(2.24)}$$

Where,

$$j_H = C\ Re^m$$

Value of m, C and b can be found by using equations (2.11),(2.9) and (2.10) respectively. Prandtl number of the outside fluid has been evaluated by the equation (2.18)

2.13 Inside Heat Transfer Coefficient:

If inside Reynolds number is less than 2300 flow is laminar and

$$h_i=\frac{4K_H}{D_h}$$

......(2.25)

and Nusselt Number = Nu = 4

But if flow is turbulent then

$$h_i=\frac{NuK_H}{D_h}$$

.....(2.26)

and

$$Nu=\frac{(\frac{f}{8})(Re_D-1000)Pr}{1+12.7(\frac{f}{8})^{\frac{1}{2}}(Pr^{\frac{2}{3}}-1)} \quad [6]$$

where f is the friction factor and it is

$$f=(0.79lnRe_D-1.64)^{-2} \quad [6]$$

.....(2.28)

This relation is valid for $0.5<Pr<2000$ and $2300<Re <5 \times 10^5$.

2.14 Overall Surface Efficiency (η_o):

The quantity η_o is termed the overall surface efficiency or temperature effectiveness of a finned surface. It is defined such that, for the hot or cold surface, the heat transfer rate is

$$q=\eta_o hA(T_b-T_\infty) \qquad \text{......(2.29)}$$

It can be introduced easily into the expression for the overall heat transfer coefficient,The overall surface fin efficiency is related to the fin effectiveness as

$$\eta_o=1-\frac{A_f}{A}(1-\eta_f) \qquad \text{.....(2.30)}$$

For simplicity , it is assumed that a straight or pin fin of length "L" can be used to model the annular fin. Assuming an adiabatic tip

$$\eta_o=\frac{\tanh(ML_f)}{ML_f} \qquad \text{.....(2.31)}$$

$$M=\sqrt{\frac{h_o P_{surf}}{A_c K_f}} \quad \ldots\ldots(2.32)$$

The derivation of can be found in Appendix (A-2)

2.15 Overall Heat Transfer Coefficient:

Now, the overall heat transfer heat transfer coefficient can be found. An essential , and often the most uncertain, part of any heat exchanger analysis is determination of the overall heat transfer coefficient. Overall coefficient is defined in terms of the total thermal resistance to heat transfer between two fluids. The coefficients can be determined by accounting for conduction and convection resistances between fluids separated by composite plane and cylindrical walls respectively.

2.15.1 Fouling:

During normal heat exchanger operation surfaces are often subject to fouling by fluid impurities, rust formation, or other reaction between the fluid and the wall material. The deposition of a film or scale on the surface can greatly increase the resistance to heat transfer between the fluids.

This effect can be treated by introducing in additional thermal resistance, termed the fouling factor, R_f . Its value depends on the operating temperatures, fluid velocity, and length of service of heat exchanger.

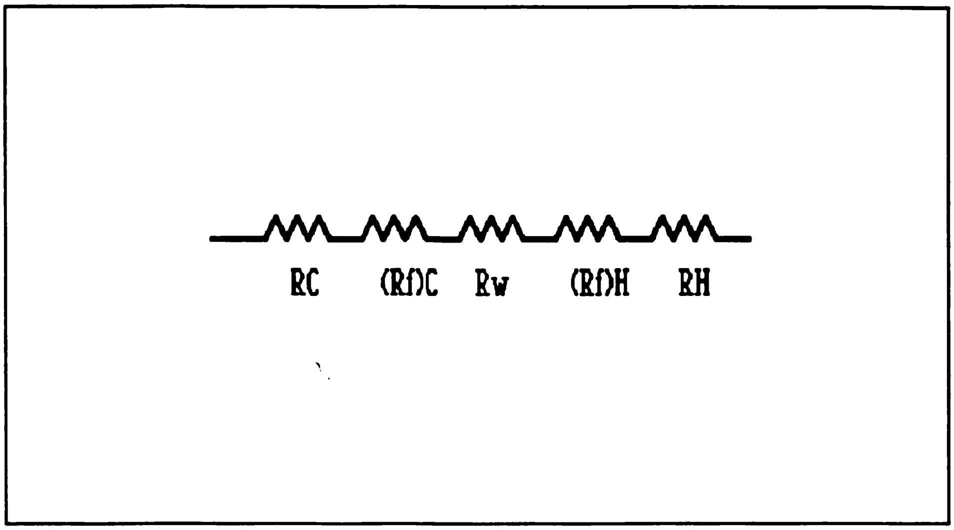


Fig.2-4: Thermal resistance circuit showing all resistances and also fouling resistances of cold side and hot side fluids.

$$\frac{1}{UA}=\frac{1}{\eta_o hA_C}+\frac{Rf,C}{(\eta_o A)_C}+R_w+\frac{(R_{f,H})}{\eta_o A_H}+\frac{1}{\eta_o hA_H}$$

.....(2.33)

Where, $R_{f,c}$ and $R_{f,H}$ are the fouling factors. Because ethylene glycol which is corrosion resistance and also anti-freeze, fouling is reduced to its minimum value. So, in further analysis fouling will be ignored. Rw depends on the geometry used. For cylindrical tube using the annular fin $\ln(D_o/D_i)/(2\pi KL/A_c)$ is the tube wall resistance.

$$U=\left[\frac{1}{h_H\frac{A_H}{A_C}}+\ln\frac{\left(\frac{D_o}{D_i}\right)}{2\pi K\frac{L}{A_c}}+\frac{1}{h_C\eta_{o,i}}\right]^{-1} \quad .....(2.34)$$

Hence we need to know the five parameter h_H , A_H/A_C , $\eta,_o$, h_o L/A_c. All of these can be calculated through equations (2.28), (2.15) ,(2.25), & (2.30).

2.16 Depth and Number of Tubes Required:

The sizing problem really comes down to determining the depth of the heat exchanger core and the number of tubes required. First of all we find the total volume

$$V_{tot}=\frac{A_C}{\alpha} \quad ......(2.35)$$

Where A_o is the required heat transfer surface area on the fin side and has been determined form the E-Ntu analysis

$$A_c=\frac{NtuC_{\min}}{U_C} \quad .....(2.36)$$

The depth of the heat exchanger can then be calculate

$$Depth = d = \frac{V_{tot}}{A_{fr}} \qquad \text{......(2.37)}$$

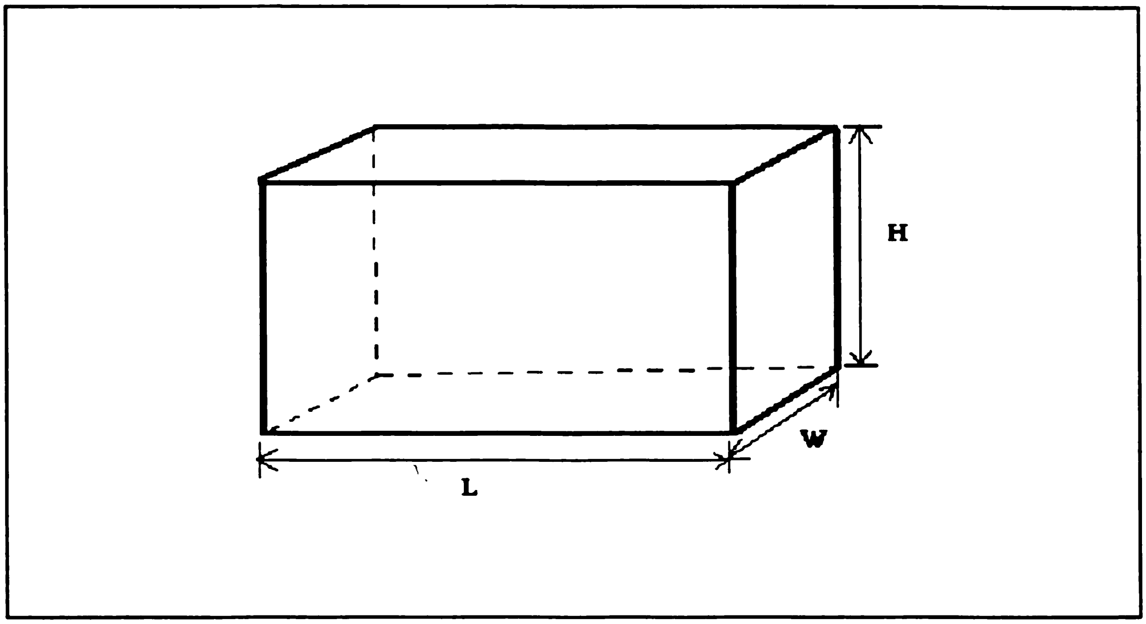


Fig.2-5: Showing the sketch of radiator for length, width and height.

The number of tubes for a particular geometry for one pass can be calculated by dividing depth with the center to center distance of tubes.

$$Number\ of\ Tubes\ in\ one\ pass \quad \frac{Frontal\ Area\ (A_{fr})}{Exch.\ Depth\ Ctr\text{-}Ctr.\ Tube\ Dist.}$$

Similarly the number of passes (number columns in the depth

$$umber\ of\ Passes\ Depth\ Columns = \frac{Depth\ of\ Exchanger(d)}{Exch.DepthCtr\text{-}Ctr.\ Tube\ Dist}$$

direction) can be calculated by Total Number of Tubes = (# of tubes in one pass) x

(# of passes in depth columns)

.....(2.37)

2.17 Power Requirements:

Power requirements for heat exchangers required

Outside pressure change

Inside pressure change

Outside power loss

Inside power loss

Total operating power

2.17.1 Outside Pressure Change:

According to equation (2.8) friction factor can be found

$$f = C\ Re_o^m$$

There two conditions for the outside pressure loss.

If,

$$(1+\sigma^2)\left(\frac{\nu_o}{\nu_i}-1\right)+\left(\frac{f_o \alpha V \nu_m}{\sigma A_f \nu i}\right)<0 \qquad ......(2.38)$$

if this condition satisfied than the change in pressure is zero

$$\Delta P = 0$$

If, the above condition do not satisfy than the pressure drop is

$$\Delta P_i=\left[(1+\sigma^2)\left(\frac{\nu_o}{\nu_i}-1\right)+\left(\frac{f_o \alpha V \nu_m}{\sigma A_f \nu i}\right]\left(\frac{\nu_i m_i^2}{2\sigma^2 A_f^2}\right) \qquad ......(2.39)$$

2.17.2 Inside Pressure Change:

For the tube side flow,

If Re <2300, then the friction factor is

$$f=\frac{64}{Re} \qquad .....(2.40)$$

But, if the Reynolds number is larger than 2300 then the equation (2.40) no longer valid. The equation for this case is as follow.

$$f=\frac{1}{(0.79\ln Re-1.64)^2} \quad .....(2.41)$$

Inside pressure loss can be found by the following relationship.

$$\Delta P_o=\frac{8m_2^2 wnf_i}{\pi^2 (D_i)^5 \rho_2 H^2} \quad .....(2.42)$$

Details of this development can be found in Appendix A-3

2.17.3 Outside Power Loss:

Power required to maintain the mass flow of air to get the required temperature. of the hot side fluid. This is actually the fan power required.

$$\text{Outside Power required} = P_o = \frac{m_1 \ \Delta P_o}{\rho_1} \quad .....(2.43)$$

2.17.4 Inside Power Loss:

Power required to maintain the mass flow of hot Fluid (ethylene glycol) to get the required outlet temperature, so this is the pumping power required.

$$\text{Inside Power required} = P_i = \frac{m_2 \Delta Pi}{\rho_2} \quad .....(2.44)$$

2.17.5 Total Operating Power required:

Total operating power is the sum of Fan Power and Pumping Power (outside power loss + inside power loss).

$$P_{tot} = P_o + P_i \quad -----(2.45)$$

2.18 Rating Problems:

For the rating problem we know the core geometry, the mass flow rates and the entering fluid temperature.If the heat transfer rate and exchanger effectiveness are predicted rate the resulting outlet temperatures can be determined. This is like using a heat exchanger given to us off the shelf, where we know everything about it except for its operating conditions.

The rating problem follows the solution methodology outlined below

(i) First of all find the $C_{p,H}$ and $C_{p,C}$ at the mean temperature,
if both of the temperatures are not known then we find the values at the inlet temperatures.
(ii) Calculate the C_{min} and C_{max} ($\dot{m}C_p$) and then calculate $C_R = C_{min}/C_{max}$.

(iii) Find the h_o, h_i, η_o, A_i/A_o , as in the sizing problem.

(iv) hi can be found out by first checking the Re number to ensure the type of flow whether it is laminar or turbulent.

$$Re=\frac{UD_h}{\nu} \qquad .....(2.46)$$

$$h_i=\frac{NuK_H}{D_h} \qquad .....(2.47)$$

If flow is laminar Nu=4
If flow is turbulent

$$Nu=\frac{(\frac{f}{8})(Re_D-1000)Pr}{1+12.7(\frac{f}{8})^{\frac{1}{2}}(Pr^{\frac{2}{3}}-1)} \qquad ....(2.48)$$

and $f = (0.79 \ln Re - 1.64)^{-2}$

.....(2.49)

This correlation is valid for $0.5<Pr<2000$ and $2300<Pr<5x10^5$

.

(v) Now we calculate the h_o (outer heat transfer coefficient) by the same equation(2.25) as in the sizing problem.

(vi) The Ntu can now be calculated.

(vii) Next we find the effectiveness by using the appropriate effectiveness Ntu equation.

Since we are using an annular fin heat exchanger with one fluid mixed(air) and one fluid unmixed(ethylene glycol), then

if C_{max} = Air, and C_{min} = Ethylene Glycol

$$\epsilon=(\frac{1}{C_R})(1-\exp(-C_R^{-1}[1-\exp(-Ntu))]$$

......(2.49)

and if

C_{max} = mixed and C_{min} = unmixed

$$\epsilon=1-\exp(C_R^{-1}(1-\exp[-C_R(Ntu))])$$

......(2.50)

(vi) Next the Ntu can be found by

$$Ntu=\frac{UA}{C_{\min}}$$ ......(2.51)

With the effectiveness known, the actual heat transfer rate is determined from

Where,

$$q_{act}=\epsilon q_{\max}=\epsilon C_{\min}(T_{H,in}-T_{C,in})$$ .....(2.52)

(vii) Now by applying the 1st law of thermodynamics we can calculate the exit temperatures of hot and cold fluid.

$$T_{H,out}=T_{H,in}-\frac{q_{act}}{C_H}$$ .....(2.53)

$$T_{C,out}=T_{C,in}-\frac{q_{act}}{C_C}$$ .....(2.54)

The operating power required will be calculated in the same manner as in the sizing problem in sectio

2.17.5,12.17.6,2.17.3 .

2.19 Cost Function:

The cost is defined, for this purpose, as the annual equivalent of all the present value of expenditure necessarily incurred in acquiring and operating any equipment over its useful lifetime.

The cost function associated with the cost of heat exchanger is as follow:

Cost≡ C = Material Cost
+ Fabrication Cost
+ Fan Capital Cost
+ Pump Capital Cost
+ Operating Cost
(Pump & Fan)

.....(2.55)

2.19.1 Material Cost:

Material cost includes the cost of tubes and fins.

Material Cost = Tube Material Cost + Fin Material Cost

.....(2.56)

2.19.1.1 Tubes Cost:

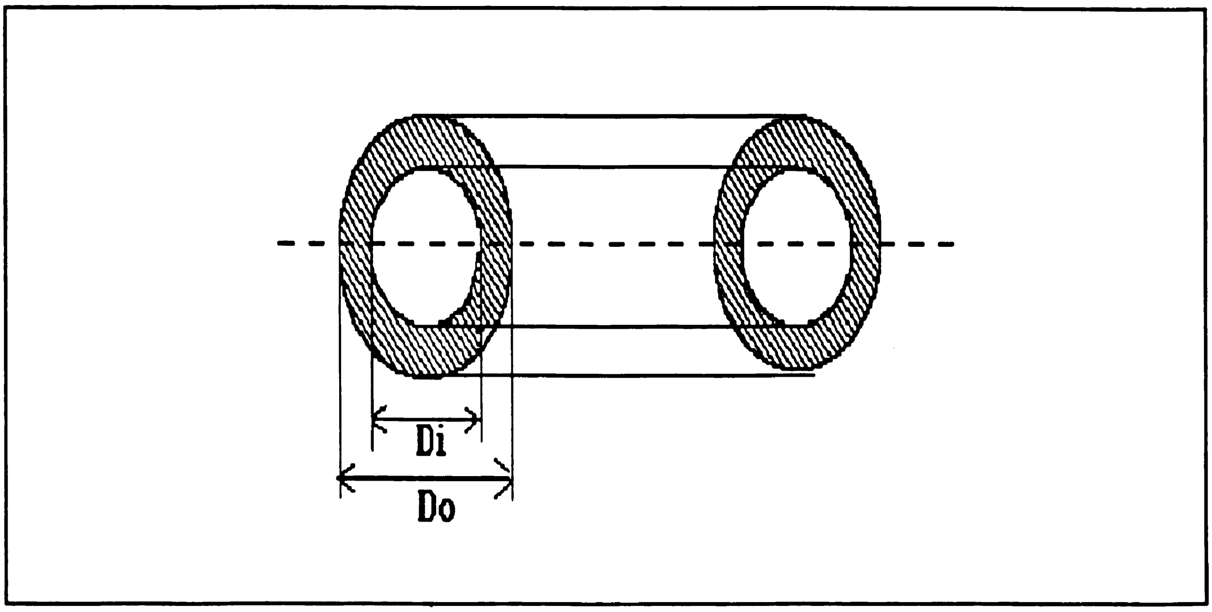


Fig.2-6: Showing the internal and outer diameter of the tubes.

$$\text{Tube Cost} = (\pi/4)(D_o^2 - D_i^2)\ (\text{width of Radiator}) \times (\text{\# of tubes in each column}) \times (\text{\# of columns of tubes}) \times (\text{Cost per kg of tube material(Cu)}) \quad .....(2.57)$$

2.19.1.2 Fins Cost:

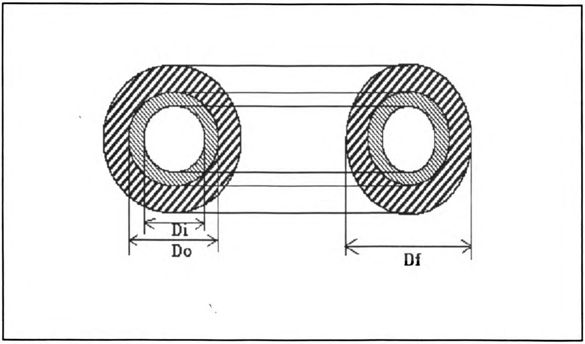


Fig.2-7: ***Showing tube and fin cross sectional areas***

$$
\begin{aligned}
\text{Fin Cost} = {} & (\pi/4)\ (D_o^2 - D_i^2)t \\
& \times \text{(Fin Pitch)} \\
& \times \text{(\# tubes)} \\
& \times \text{(width of Radiator)} \\
& \times \text{(cost of fin material per kg(Al))}
\end{aligned}
\quad \ldots.(2.58)
$$

2.19.2 Fabrication Cost:

The cost associated to manufacture the radiator is the fabrication cost. It is "n' times of material cost. The value of "n" depends on the experience and the process used. I am using the value of n=2 im this analysi

$$\text{Fabrication Cost} = n \times (\text{Material Cost}) \quad \ldots.(2.59)$$

2.19.3 Fan and Pump Cost:

The capital costs of fan and pump is fixed, and there is no need to include in the total cost of the radiator for optimization.

2.19.4 Pump and Fan Operating Cost:

Cost associated with the operating power required for fan and pump.

$$C_{PF}=[(\dot{P}_P+\dot{P}_F)\frac{C_F}{mpg}\frac{M}{\dot{P}}e] \quad \ldots\ldots(2.60)$$

By checking the units

=[(kw)x($/kw.hr)(# or hours of operating/year)

= \$/year (U.S. Dollars)

Since these operating costs are represented as annual costs and we wish to compare with capital costs, the value of money must be accounted for. We are taking the interest rate of 7% (because recently in United States of America the interest rate on cars sold on installments is this) but this rate is flexible.

Life of the radiator is also flexible. I have taken the life of the radiator 10 years.

The series for the 7% compound interest is the Geometric Progression (G.P.).

$$C_{PF} + (C_{PF} \times I + C_{PF}) + [(C_{PF} \times I + C_{PF}) \times I + C_{PF}] + ---$$

Upto 11th term.

$$r = \frac{(C_{PF} I + C_{PF})}{C_{PF}} \qquad .....(2.61)$$

Now for n=10 the N=10+1=11.

Hence after 10 years the term will be 11th term.

$$S_n = \frac{P(r^N - 1)}{(r-1)}$$

.....(2.62)

This will gives the total cost associated with the operating power of fan and pump.

Hence, the Cost Function is

Cost = Fin Cost + Tube Cost + F(fabrication Cost)
+ operating Cost associated with fan and pump
+ Interest cost associated with fan and pump operating cost.

2.20 Microsoft Excel Spread Sheet:

To solve the problem the existing spread sheet made by Edith Brown [7] was modified into two spread sheets now

(1) For Sizing Problems
(2) For Rating Problems

2.20.1 Spread Sheet For Sizing Problem:

In the spread sheet some cells are the inputs and the some cells are the calculated results. Red border cells are the user's input and the other one's are of calculated results.

Outlined - Red Border Cells are User - Inputted

ALL UNITS SI

Finned Compact Heat Exchanger Design

Part "A"

Material of Exch. Fins	Aluminum	k =	237	W/(m^2K)
Material of Exch. Tubes	Copper	k =	401	
Front view: Height of Exchanger:		0.36	meters	
Front view : Width of Exchanger :		0.6	meters	
Frontal area(m^2):		0.216	square meters	

Part "B"

Heat Exchanger Core Configuration Parameters

Part "B-1"

Enter the appropriate code for Core Configuration

Geometry	Code
Geom-A	*1*
Geom-B	*2*
Geom-C	*3*
Geom-D	*4*
Geom-E	*5*
Geom-F	*6*

Type of Geometry used: *2*

Geometry used is: *Geom-B*

	Number	Units
1. Free Flow Area/Frontal Area:	0.524	none
2. Surface Area / Volume:	535	meters
3. Colburn Factor, Re Exponent:	-0.4506	none
4. Colburn Factor, Re constant:	0.295	none
5. Friction Factor, Re Exponent:	-0.2084	none
6. Friction Factor, Re constant:	0.1856	none
7. Hydraulic Diameter:	0.003929	meters
8. Exchanger Height:Ctr.toCtr. Tube dist.:	0.024765	meters
9. Exchanger Depth:Ctr.-to-Ctr. Tube dist.::	0.02032	meters
10.Outside Tube Diameter:	0.00762	meters

	Outlined - Red Border Cells are User - Inputted					
		ALL UNITS SI				
		Finned Compact Heat Exchanger Design				
11.Inside Tube Diameter:				0.00512	meters	
12. Fin Diameter:				0.023368	meter	
13.Fin Pitch				343	meter	
14.Fin Area/Total Area:				0.91	none	
15.Fin thickness				0.00046	none	
16.Fin Length:				0.007874	meters	
17.Fin Perimeter:				0.097351673	meters	
18.Fin Cross-sectional Area:				1.1379E-05	m^2	
19.Inside Surf. Area/outside Surf. Area(U.C)				0.056819727	none	
20.Volume of Cell Solid / Volume of Cell				0.169880002	none	
Part "B-2"						
	Enter the appropriate Code for the Fluid to be used:					
		Fluid	***Code***			
		Air	***1***			
		Engine Oil	***2***			
		Ethylene Glycol	***3***			
		Superheated Water Vapor 4				
		Water	***5***			
Part "B-3"						
		Fluid Properties				
Outside (Fin Side) Fluid				**Inside (Tube side) Fluid**		
Fluid:	1			Fluid:	3	
	Air				***Ethylene Glycol***	
Properties:		Units:	Tout guess:	Properties:		**Units:**
M dot	0.8	Kg/s	360.89	M dot	0.87	Kg /s
T in	323	K	Tavg.guess	T in	370	K
T out	349.45	K	365.445	T out	360.886041	K
T ave	336.23	K	% Diff.:	T ave	365.4430205	K
Cp	1012.039	J/(Kg.K)	-0.001097017	Cp	2700.764304	J/(Kg.K)
Dyn.Visc.	2.0251E-05	N s/m^2		Dyn.Visc.	0.002478693	N s/m^2
T. Cond	0.029	W/(m K)		T. Cond	0.262638386	W/(m K)
Density	1.024	Kg/m^3		Density	1070.693697	Kg/m^3
Prandtl	0.700	none		Prandtl	25.4889092	none

	Outlined - Red Border Cells are User - Inputted					
			ALL UNITS SI			
		Finned Compact Heat Exchanger Design				
Part "B-4"						
			Calculated Parameters			
1-Effectiveness			0.562765957			
2-Cmin/Cmax			0.344573118			
3-Ntu			0.973835971			
4-Outside Re			1371.329671			
5-Colburn factor			0.011382191			
6-Outside heat trans. coef.			103.2417125			
7-Inside Re			6004.439525	Type of Tube Side Flow:		
8-Inside heat trans. coef.			3943.58572			
9-Fin efficiency primed			0.935833879		Turbulent Flow	
10-Fin efficiency factor			0.929487779			
11-Out(fin side)overall coef.			67.30547272			
12-Total Volume Needed			0.021896212	m^3		
13-Depth of Exchanger			0.10137135	meters		
14-Number of Tubes (Height)			14.53664446			
15-Numb.of passes,depth cols.			5			
16-Total Number of Tubes			73			
Part "B-5"						
Outside (Fin Side) Fluid						
Specific Volume (m^3/kg)						
At T in	At T out	At Tave				
9.39E-01	1.01E+00	0.97640248				
			Exchanger Power Requirements			
					Units	
		1-Outside Friction factor		0.041190551	none	
		2-Inside Friction factor		0.036514484	none	
		3-Outside Pressure change		106.091734	Pa	
		4-Inside pressure change		84425.1395	Pa	
		5-Outside Power Loss		82.87058533	Watts	
		6-Inside Power Loss		68.60026503	Watts	
		7-Total Operating Power		151.4708504	Watts	
Part "C"						

	Outlined - Red Border Cells are User - Inputted			
	ALL UNITS SI			
	Finned Compact Heat Exchanger Design			
	Calculation of Cost Of Exchanger			
Part "C-1"				
1-Density of Tube Material	COPPER		8933	kg/m^3
2-Density of Fin material	ALUMINIUM		2702	kg/m^3
3-Cost per kg of tube material			4	$
4-Cost per kg of fin material			5	$
5-Fabrication Cost Factor			2	none
6-Cost of Fuel in ($/gallon)			1.05	$/gallon
7-Miles/Year of car (approx. car running)			20000	
8-MPG (Miles per Gallon)			27	miles/year
9-Total horse power of the Engine			160	Horse Power
10-Life of the Radiator			10	years
11-Interest Rate/Year			0.07	%
	Calculated Values			
Part "C-2"				
1-Tube Material Used			9.787477181	kg
2-Fin Material Used			7.156822426	kg
3-Tube Material Cost			39.14990872	$
4-Fin Material Cost			35.78411213	$
5-Fabrication Cost			149.8680417	$
6-Operating Cost(Fan & Pump) per Year			0.987019616	$
7-Multiplier factor for each year			1.07	none
8-Total Operating Cost for the entire life of radiator			15.57872214	$
9-Total Cost of Radiator			240.3807847	$

Part "D"

Property Correlations

Outside (Fin Side) Fluid:

Property Code	Spec.Heat	Dyn.Visc.	T. Cond.	Density	Prandtl
1	1012.04	2.03E-05	2.91E-02	1.02E+00	7.00E-01
2	2060.87	4.44E-02	1.40E-01	8.62E+02	6.52E+02
3	2592.62	4.78E-03	2.60E-01	1.09E+03	4.76E+01
4	N.A.	N.A.	N.A.	N.A.	N.A.
5	4185.04	4.42E-04	6.53E-01	9.82E+02	2.83E+00

Spec. Volume at:	Temp.In.	Temp.Ave.	Temp.Out
1	0.94	0.976	1.005

	Outlined - Red Border Cells are User - Inputted					
			ALL UNITS SI			
		Finned Compact Heat Exchanger Design				
	2	0.00	0.001	0.001		
	3	0.00	0.001	0.001		
	4	N.A.	N.A.	N.A.		
	5	0.00	0.001	0.001		
Property Code:		Inside (Tube Side) Fluid:				
		Spec.Heat	Dyn.Visc.	T Cond.	Density	Prandtl
	1	1014.29	2.16E-05	3.13E-02	9.82E-01	6.96E-01
	2	2185.85	2.12E-02	1.38E-01	8.45E+02	3.37E+02
	3	2700.76	2.48E-03	2.63E-01	1.07E+03	2.55E+01
	4	N.A.	N.A.	N.A.	N.A.	N.A.
	5	4206.53	3.03E-04	6.76E-01	9.66E+02	1.88E+00

2.20.1.1 Part "A" Exchanger Fixed Parameter for a Design:

Red border cells shows the user input or the input by some feed-data. It has five parameter. Four are user input and the one is calculated.

1- Fins material thermal conductivity Al(237 w/m°k)
2- Tubes material thermal conductivity Cu(401 w/m°k)
3- Height of Exchanger (in m)
4- Width of exchanger (in m)
5- Frontal area in (m^2)

Frontal area is calculated by multiplying parameter 3 and 4. This is done by specifying equation at that cell which has the multiplication of the cell number of 3 and 4.

For example in the existing spread sheet.

at E11 = E9 + E10

2.20.1.2 Part "B" Heat Exchanger Core Configuration Parameter:

This part deals with the fixed and calculated parameters of heat exchanger, for a particular geometry of a heat exchanger core configuration.

Part "B-1":

I am using the six different geometries of a annular fin, and I have restrict to only these six geometries, therefore, I have made the spread sheet user friendly. For doing this I have used the code of different geometries. For example for Geometry-A configuration if we put "1" then all the fixed data relating the geometry "A" will automatically entered at the required cells.

So I have made the cell "C27" is the input cell for a geometry code. Making reference of this cell and using "IF" statement one can easily entere the 15 fixed parameters for a geometry.

These fifteen parameters are as follow;

1- Free flow Area/Frontal Area
2- Surface Area/Volume
3- Colburn Factor, Re exponent
4- Colburn Factor, Re constant
5- Friction Factor, Re exponent
6- Friction Factor, Re constant
7- Hydraulic Diameter.
8- Exch. Height ctr. to ctr. Tube dist.
9- Exch. Depth ctr. to ctr. Tube dist.
10- Outside tube diameter

11- Inside tube diameter

12- Fin diameter

13- Fin pitch

14- Fin Area/Total Area

15- Fin Thickness.

For example for parameter 1 (σ) for

Geom.-A = 0.538

Geom.-B = 0.524

Geom.-C = 0.494

Geom.-D = 0.449

Geom.-E = 0.443

Geom.-F = 0.628

By using IF statement at cell F31

```
=IF($C$27=1,0.538,IF($C$27=2,0.524,IF($C$27=3,0.494,IF($C$27=4,0.
449,IF($C$27=5,0.443,IF($C$27=6,0.628,"ERROR"))))))
```

It compares with IF statement and replace the value of parameter for the respective geometry.

In this part five parameters are calculated which are as follow;

1- Fin Length

2- Fin Perimeter

3- Fin Cross-sectional area

4- Inside surface Area/outside surf. Area (U.C.)

5- Volume of Cell solid/Volume of Cell

These values are calculated by using the equations (2.12) ,(2.13), (2.14), (2.16) and (2.17) respectively.

The only difference in inputting the formula is not to enter variable directly, insted specify the cell numbers to enter the formula

Part "B-2":

This part gives the opportunity to the user to select the tube fluid and the outside (cooling) fluid. In this part we listed only the codes for different fluids.

Fluid	Code
Air	1
Engine Oil	2
Ethylene Glycol	3

Superheated water vapor	4
Water	5

Part "B-3": Fluid Properties:

In this section we evaluate the fluid properties for hot and cold fluid, once selecting the two; by entering the appropriate code in cell "B65" and "F65"

In this five (5) quantities are input quantities.

1- Mass flow rate of cooling fluid in (kg/s)
2- Mass flow rate of hot fluid in (kg/s)
3- $T_{in,C}$ of Cold fluid in (K)
4- $T_{out,C}$ of Cold fluid in (K)
5- $T_{in,H}$ of Hot fluid in (K)

All other parameters are either calculated or replaced by comparing two conditions.

Calculated values are of T_{out} of hot fluid and T_{ave} of both fluid (hot and cold)

Specific heats of hot and cold (C_p) fluid
Dynamic viscosity of hot and cold (υ) fluids
Thermal conductivity of hot and cold fluids

Density of hot and cold fluids.

Prandtl number of hot and cold fluids.

These quantities are replaced by comparing the appropriate condition through the macros in Part-D at the end of spread sheet. The value of C_p is calculated at the mean temperature and finalized value is taken when the constraint codition is satisfied. This give the correct values for the properties of the fluid.

Part "B-4" Calculated parameters for heat Exchanger Design:

This part include all the calculated parameters relating the final design of the heat exchanger. In this part we get all the desired values regarding the design of any heat exchanger. There are sixteen parameters.

1- Effectiveness ϵ by using Equation(2.20)

2- C_{min}/C_{max}

3- Ntu by using equations (2.21) & (2.22)

4- Outside Reynolds number through equations(2.23)

5- Colburn factor by using equation(2.7)

6- Outside heat transfer coefficient by equation(2.25)

7- Inside Reynold number through(

8- Inside heat transfer coefficient by equation(2.28)

9- η_o through equation(2.30)

10- η_f fin efficiency by using equation(2.31)

11- Outside(fin side) overall coefficient equation(2.34)

12- Total volume needed by using equation(2.35)

13- Depth of exchanger by using equation(2.36)

14- Number of tubes (Height)

15- Number of passes, depth columns.

16- Total number of tubes by using equation(2.37)

From these quantities we can find how many tubes are required and how many passes are there in a the heat exchanger.We also get the performance parameters such as effectiveness(ϵ), Ntu (net transfer units), and fin efficiencey (η_f), etc.

Part-B-5 Exchanger Power Requirements:

In this section we have three parameters which will be taken by comparing the statements. All of them are related to the outside fluid (air)

υ_1 = Specific volume at T_{in}

υ_2 = Specific volume at T_{out}

υ_m = Specific volume at T_{mean} (mean temperature)

There are seven calculated parameters regarding the power required to run fan pump. These are

Outside friction factor f_o by equation(2.8)
Inside friction factor f_i by equation(2.38)or(2.39)
Outside pressure change by equation(2.40)
Inside pressure change by equation(2.41)
Outside power loss (Fan) by equation(2.42)
Inside power loss (pump) by equation(2.43)
Total power loss through equation(2.44)

Hence we get the operating power for fan and pump.

Part C:

This part is concerned with the cost of the heat exchanger.

Part C-1:

This part include the parameters which are user defined and supplied the user, therefore, these are red bordered cells. These are as follow.

1- Density of tube material.
2- Density of fin material.

3- Cost per kg of tube material.

4- Cost per kg of fin material.

5- Fabrication cost factor.

6- Cost of fuel in ($/gallon).

7- Miles/year of a car(approx. car running).

8- MPG (miles per gallon).

9- Total horse power of the engine.

10- Life of the radiator.

11- Interest rate/year.

Part C-2:

This part include all the calculated parameters regarding the cost of heat exchanger. If we are able to calculate eight (8) quantities then we obtain the total cost of the heat exchanger. These are as follow:

1- Tube material used.

2- Fin material used.

3- Tube material cost.

4- Fin material cost.

5- Fabrication cost.

6- Operating cost (Fan & Pump) per year.

7- Multiplication factor for each year.

8- Total operating cost for the entire life of radiator.

9- Total cost.

This will complete the design and will calculate the cost of heat exchanger.

Part-D:

This is a small macro and gives the value of the properties of fluids used for cooling and the working fluid.
These properties are:

Specific heat

Dynamic Viscosity.

Thermal conductivity

Density and

Specific volume of outside fluid (Air). at T_{in}, T_{out} , and at the mean temperature (T_{mean}).

2.20.2 Spread sheet for Rating Problems:

There are no major differences between the rating and sizing spread sheets. The differences are in part "B-3" and "B-4".

In "B-3" now only two temperature are known.

T_{in} of cold fluid (air)

T_{in} of Hot fluid (Ethylene Glycol)

and the other two temperature are calculated.

T_{out} of cold fluid by Equation (2.51)

T_{out} of hot fluid by Equation (2.52)

2nd difference is in part "B-4". In this the depth of the exchanger is a user input and the formulae used for effectiveness and Ntu are little bit different those are equation(2.49) or (2.50) for effectiveness and (2.50.b) for Ntu. In this the only is inlet temperatures of the hot and cold fluids and the depth of exchanger and we get the required outlet temperatures of hot and cold fluids. Keep changing the depth of exchanger until we get the required outlet temperature of hot fluid.

2.21 Optimal Heat Exchanger:

An optimal heat exchanger is defined as one that, while satisfying imposed constraints, achieves the required task at the lowest possible cost.

2.21.1 Optimization with uniform geometry:

In this step the sizing problem spread sheet is use. By using the one type of geometry A,B,C,D,E, and F and iterating with the mass flow rate of air and the outlet temperature of air we get the required outlet temperature of ethylene glycol.

Then taking the mass flow rate of air then we change the outlet temperature of air (cold fluid) such that we achieve the required outlet temperature of ethylene glycol. If the temperature of ethylene glycol is more than the required, we increase the outlet temperature of the air, and if it is less then we decrease the temperature of air.

This process continues for different mass flow rates of ethylene glycol for all the geometries. By the observations we can able to decide which geometry and at what mass flow rate of air we get the minimum cost (optimal solution).

2.21.2 Optimization with variable geometries:

After determining the optimum cost using the unifrm geometry core,we now want to further reduce the cost by using different geometries together. Single type of geometry optimization will show that which geometry gives the minimum cost at what mass flow.

The spread sheet used in this case is the rating problem spread sheet, in which we calculate the outlet temperatures of air and ethylene glycol. The depth of the exchanger is now the input. By using one layer of tubes of particular geometry which gives the minimum cost and also the proper mass flow of air, we get the outlet temperature of air and ethylene glycol. A 2nd layer is then added of same geometry or of other geometry keeping mass flow constant. When we are adding the 2nd layer the outlet temperatures achieved by adding the first layer will be the inlet temperatures of air and ethylene glycol. This process continue with different combinations of geometries, until we achieve the minimum cost.

Chapter 3

RESULTS AND DISCUSSION

3.0 Source of given data:

I have taken the data of General Motor Car Cavalier-uplevel. Engine Capacity is 3.1L.

Height of Exchanger H = 0.36 meters

Width of Exchanger W = 0.6 meters

Thermal Conductivity of Tubes(Cu) = 401 w/m k

Thermal Conductivity of Fins(Al) = 237 w/m k

Working Fluid (Hot Fluid) Ethylene Glycol

Cooling Fluid is ambient air.

3.0.1 Hot Fluid (Ethylene Glycol)

1- Mass flow rate of hot fluid $(m^{\circ}_{H}) = 0.87$ kg/s

2- Temperature leaving engine entering radiator $T_{H,in} = 370$ K

3- Temperature leaving radiator entering engine $T_{H,out} = 360.89$ K

3.0.2 Cold Fluid (Air)

1- Mass flow rate (m°_{C}) (Found through iteration the optimize mass flow)

2- Temperature entering radiator $T_{C,in}$ = 323 K

3- Temperature leaving radiator (Calculated through formula)

3.0.3 Some Engine Properties:

1- Engine capacity = 3.1 L

2- Horse Power of the engine = 160 HP.

3- Miles per gallon (MPG) = 27 miles/gallon

3.0.4 Thickness of the Copper tubes:

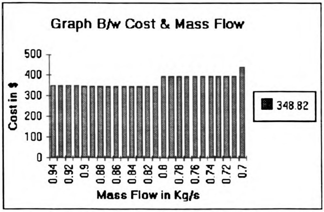
Thickness of the copper tubes is taken 0.0025m from the hand book [6].

3.1 Cost of Radiator by using one Type of Geometry by Sizing method.

In this we use geometry A,B,C,D,E and F one by one. By taking one type of geometry and iterating through mass flow rate of air to get the minimum cost of the radiator.

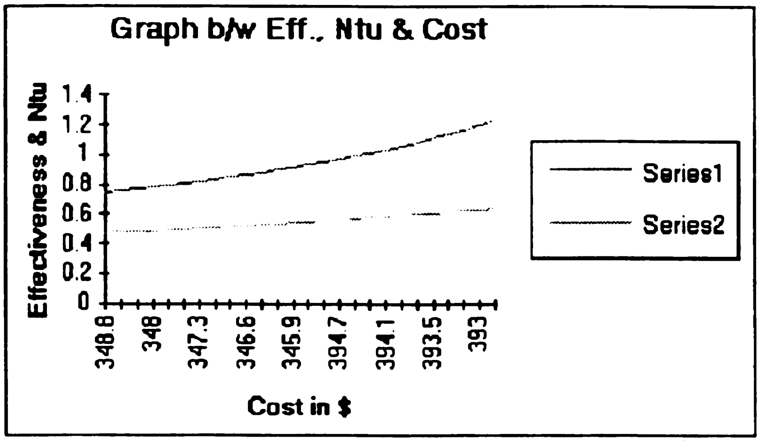
3.1.1 Cost of Radiator using Geometry 'A' only:

By iterating through the mass flow rate of air starting from 0.95 kg/s to 0.70 kg/s. Table 3.1 in appendix C-1 shows the mass flow rate of air, Cost, Ntu and Effectiveness. Figures (3.1)and (3.1a) also tells the same story. Minimum cost is $345.48 incurred at 0.81 kg/s.



Graph 3.1: Shows the cost of Geometry 'A' with different mass flow and the optimum mass flow is 0.81 kg/s where cost is $345.49.

Fig.3.1: Graph cost vs mass flow for the geometry "A"

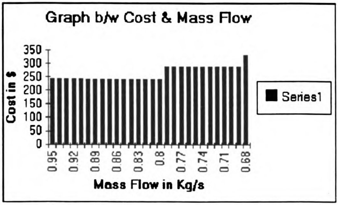


Graph 3.1.1: Showing the cost vs Ntu and Effectiveness at different costs.

Fig.3.1a: Graph effectiveness & Ntu vs cost for geometry "A"

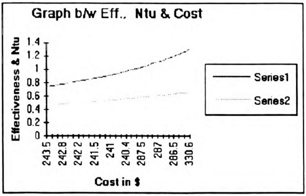
3.1.2 Cost of Radiator using Geometry 'B' only:

Table 3.2 in appendix C-2 shows the mass flow rate of air, cost, Nut and Effectiveness for the geometry 'B'. Minimum cost is $240.38 incurred at 0.81 kg/s of air. Graphs (3.2) and (3.2a) also tells the same story.



Graph 3.2: Showing the cost vs different mass flow rate. It showing that the minimum cost is at 0.8kg/s which is $240.38.

Fig.3.2: Graph cost vs mass flow for geometry "B"

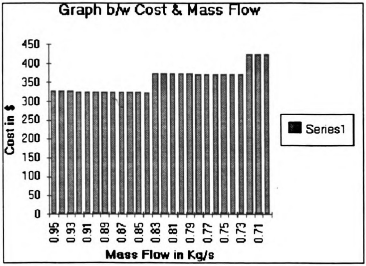


Graph 3.2.1: Series 1 showing the Ntu and Series 2 showing the effectiveness vs different costs.

Fig.3.2a: Graph effectiveness & Ntu vs cost for geometry "B"

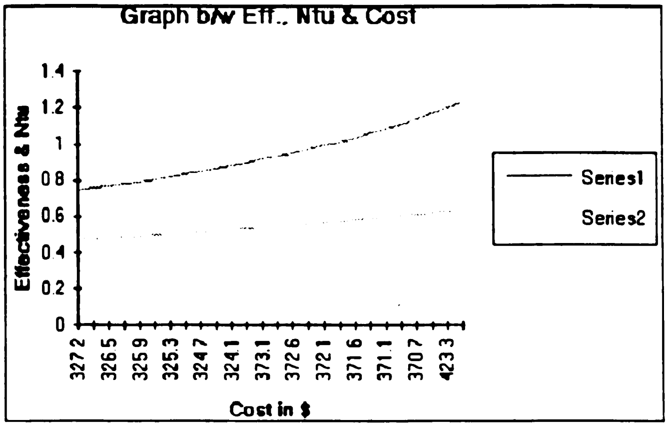
3.1.3 Cost of Radiator using Geometry 'C' only:

Table 3.3 in appendix (C-3) showing the mass flow of air, cost, Ntu and effectiveness for geometry 'C'. Minimum cost is $324.86 incurred at 0.84 kg/s. Graphs (3.3) and (3.3a) also shows the same data in the graphical form.



Graph 3.3: Showing the cost vs mass flow for geometry 'C'. At mass flow .84kg/s the cost is $323.87.

Fig.3.3: Graph cost vs mass flow for the geometry "C"

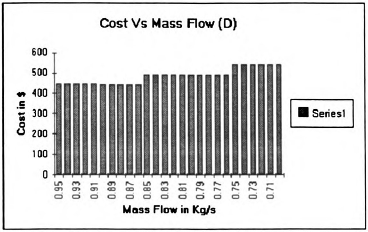


Graph 3.3.1: Series 1 is showing the Ntu and Series 2 is showing effectiveness vs different costs.

Fig.3.3a: Graph showing tahe effectiveness & Ntu vs cost.

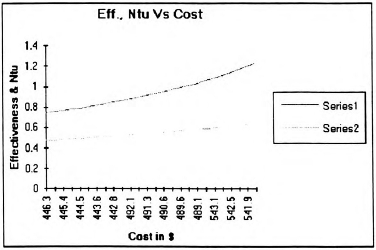
3.1.4 Cost of Radiator using Geometry 'D' only:

Table 3.4 in the appendix C-4 shows the same four quantities as in table 3.3 for geometry 'D'. Minimum cost at 0.86 kg/s is $442.38. Graphs (3.4) and (3.4a) are also the graphical representation of the data in table 3.4.



Graph-4: Showing Cost vs Mass Flow Rate for Geometry "D". Minimum Cost is $442.38 at 0.86 Kg/s

Fig.3.4: Graph cost vs mass flow for geometry "D"

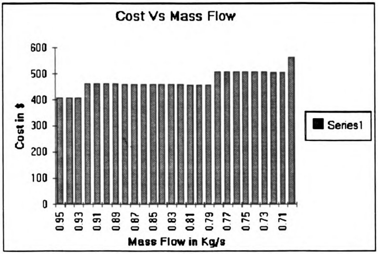


Graph 3.4.1: Series 1 is showing Ntu and Seriese 2 is showing Effectiveness for geometry 'D'

Fig.3.4a: Graph effectiveness vs Ntu for geometry "D"

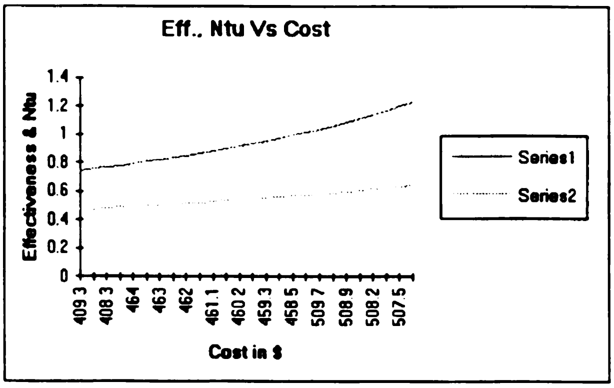
3.1.5 Cost of Radiator using Geometry 'E' only:

Table 3.5 in appendix C-5 shows the four quantities as in the other tables like 3.3 and 3.4 for geometry 'E'. Minimum cost is \$408.29 at mass flow of air 0.92 kg/s. Graphs (3.5) and (3.5a) also tells the same story of table 3.5 in graphical form.



Graph 3.5: Showing cost vs mass flow. It is showing that cost is minimum at 0.93kg/s which is 409.30

Fig.3.5: Graph cost vs mass flow for the geometry "E"

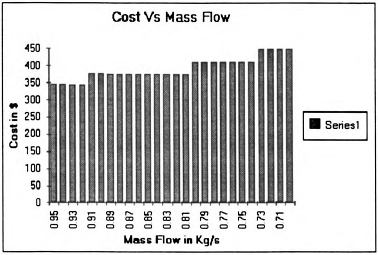


Graph-5a: Showing NTU, Effec. vs Cost for Geometry "E".

Fig.3.5a: Graph effectiveness & Ntu vs cost for geomerty "E"

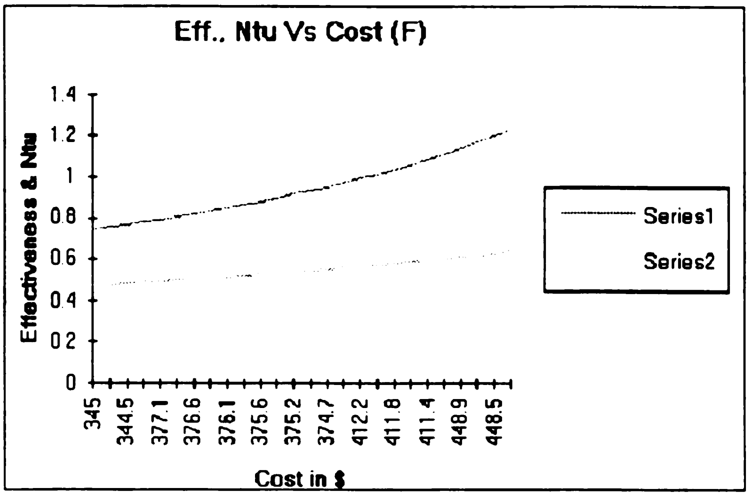
3.1.6 Cost of Radiator using Geometry 'F' only:

Table 3.6 in appendix (C-6) shows the mass flow rate of air, cost, Ntu and effectiveness for the geometry 'F'. At the optimum mass flow of 0.92 kg/s the minimum cost is $344.23. Graphs (3.6) and (3.6a) on the next two pages also tells the same story.



Graph 3.6: Showing the cost vs mass flow for Geom. 'F'.
It shows at 0.92kg/s the min. cost is $344.23

Fig.3.6: Graph cost vs mass flow for the goemtry "E"

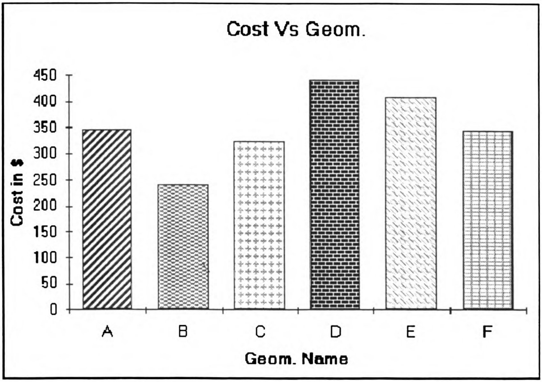


Graph 3.6.1: Series 1 is showing the Ntu and Series 2 is showing Effectiveness of Geometry 'F'.

Fig.3.6a: Graph effectiveness & Ntu vs cost for geometry "F"

3.2 Comparison of cost for Different geometries:

When we compare the cost of all the six geometries for mass flow between 0.95 to 0.70. This is better way to present the results. Table 3.7 in the appendix C-7 shows the numeric comparison and the graph (3.7) on the next page shows the story in graphical form.



Graph 3.10: Showing the comparision of minimum cost for the six geometries.

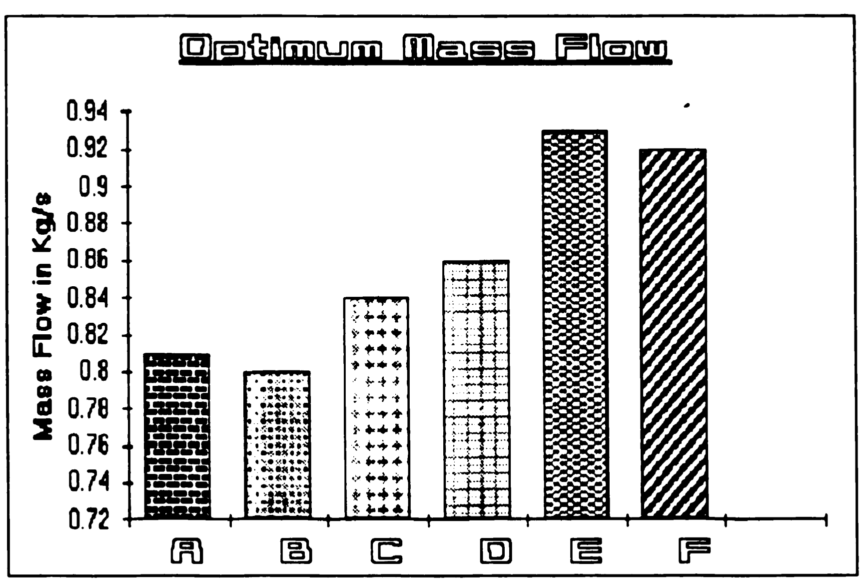
Fig.3.7: Graph showing the comparision of minimum cost vs the geometries

3.3 Comparison of minimum cost for different geometries:

Geom.	Mass Flow	Cost ($)
A	0.81	345.48
B	0.8	240.38
C	0.84	324.86
D	0.86	442.38
E	0.93	408.29
F	0.92	344.23

(Table 3.7 shows that the cost is minimum in case of geometry 'B')

Graph (3.8) shows the minimum cost Vs optimum mass flow of air for the six geometries usihg individually.

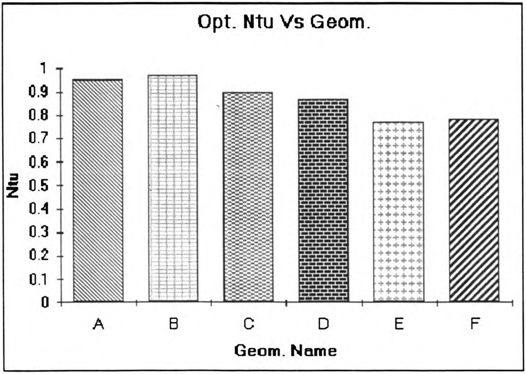


Graph 3.7: Showing the comparision of minimum mass flow for the given six geometries.

Fig.3.8: Graph showing the comparison of optimum mass flow vs the geometries.

3.4 Comparison of Ntu at minimum cost:

Similarly if we draw the graph for the Ntu of the different geometries graph (3.9) obtained.



Graph 3.9: Showing the comparision of Ntu for the six geometries at the minimum cost.

Fig.3.9: Graph showing the comparison on Ntu vs geometries at the their respctive minimum cost.

3.5 Conclusion for the one type of geometries used:

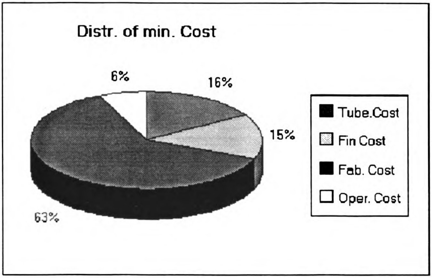
All the calculations shows that by using the geometry "B" with air mass flow rate 0.80 kg/s gives the minimum cost of all the six geometries used.

Hence the minimum cost of the radiator by using geometry 'B' with mass flow 0.80 kg/s is $240.38. And this is the optimum cost for a annular fin radiator core with one type of geometry.

3.5.1 Breakdown of minimum cost:

1- Cost of Tube Material (Cu)	=	$ 39.15
2- Cost of Fin Material (Al)	=	$ 35.78
3- Fabrication Cost	=	$149.87
4- Operating Cost	=	$ 15.58

Graph (3.10) represent the same story in pie graph on the next page.



Graph 3.11: Showing the distribution of cost of geometry 'B' which is the minimum cost of all the six.

Fig.3.10: Graph showing the distribution of cost of geometry "B"

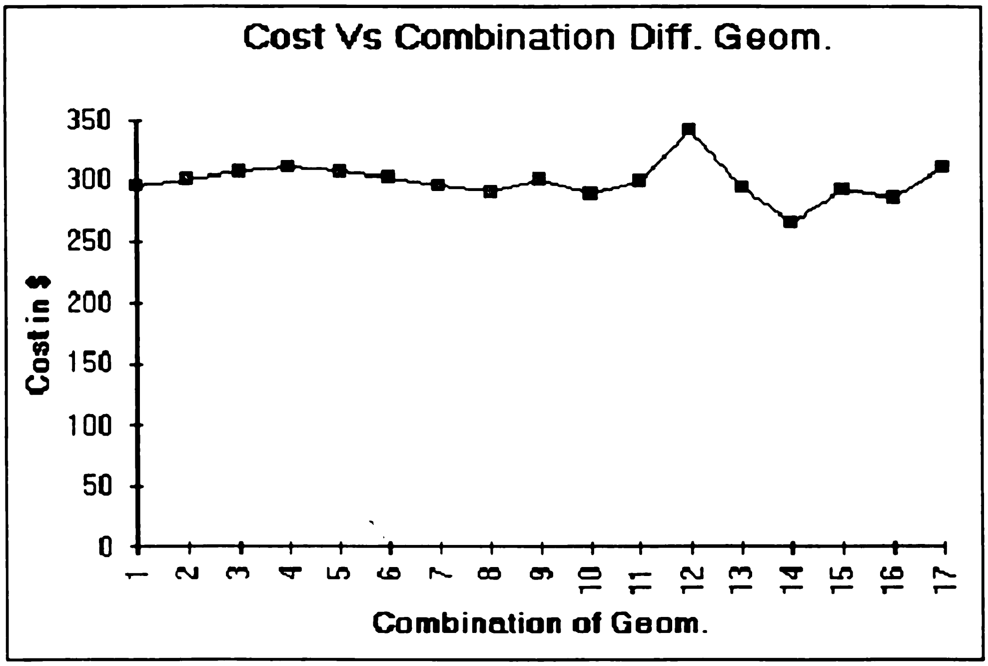
3.6 Cost of Radiator by using combination of geometries:

To further minimize the cost we used the different combinations of the given six geometries to further minimize the cost. For this purpose we use 2nd spread sheet in Appendix B-2 for rating problems of radiator. Results for using a particular geometry at a time shows that further cost can improve by combining geometries A,B,C & F.

Comb. Geom.	Mass Flow	Cost ($)
4B-2C	0.80	296.70
3B-3C	0.80	301.44
2B-4C	0.84	307.73
1B-5C	0.84	312.37
4C-2B	0.80	307.64
3C-3B	0.80	302.96
2C-4B	0.80	296.70
1C-5B	0.80	291.95
Alt B-C	0.80	301.84
4B-2A	0.80	289.91
3B-3A	0.80	300.10
2B-4A	0.81	342.19
1B-5A	0.81	294.11
4B-2F	0.80	265.89
3B-4F	0.91	292.47
2B-5F	0.91	285.73
1B-7F	0.91	312.31

(Table 3.8 Showing the cost of different combinations)

Graph (3.11) below also shows the same situation. Minimum cost is for the case using 4B-2F with mass flow rate 0.80 kg/s is $265.89.



Graph 3.12: Showing cost for the combination of different geometries (A,B,C,D,E & F)

Fig.3.11: Graph showing cost for the combinaion of different geometries (A,B,C,D,E & F}

Chapter #4

Conclusions and Recommendations for future work

By using one type of annular fin geometry and iterating with air mass flow rate the minimum cost is achieved by using Geometry "B" configuration with an air mass flow is 0.80 kg/s . This 0.80 kg/s mass flow is the optimum mass flow at which the cost is minimum. This is done using sizing spread sheet in Appendix-"B-1".

Using the combinations of different geometries A,B,C,D,E & F utilizing the use rating spread sheet (see appendix "B-2") there is no further improvement in the cost. When a combination like 4B-2C used the mass optimum mass flow of B is used and if 4C-2B is used the optimum mass flow rate of C is used and similarly for other combinations.

Analysis shows that for a uniform core, using the sizing spread sheet the cost is minimum for geometry "B" at the mass flow of 0.8 kg/s of air ($240.38). The effeciveness, Ntu of geometry "B" are also more than all the other geometries used. Variable geometry cores are not as cost effective as uniform core of geometry "B".

Cost can be reduced further with the combination of different geometries by iterating through mass flow rate of air. Hence we get the optimum mass flow where the cost is minimum.

For the future work, we have both the spread sheets for sizing (appendix A-1) and rating (appendix A-2) problems of the heat exchanger. By using these spread sheets especially rating spread sheet, taking the combinations of different geometries and iterate with air mass flow to get the minimum cost. This mass flow is the optimum mass flow for a particular combination. This is quiet long iterative process which is not inclueded in this thesis.

However, it can be accomplised by using these spread sheets given in appendix "B-1", and "B-2" then the cost of radiator can be further reduced.

Appendix-A-1

Derivation of ϵ-Ntu Relation

Cross flow arrangement and the idealized temperature conditions are pictured in the following figure.

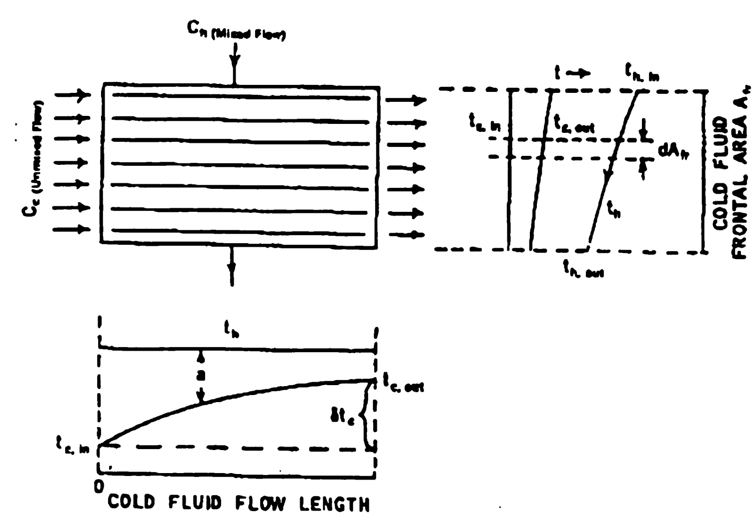


Fig.A-1.1: Showing the cross flow heat exchanger phenomenon

Uniform flow distribution for the cold stream will be specified.

So that with respect to the frontal area A_{fr} .

$$\frac{dC_c}{A_{fr}} = \frac{C_c}{A_{fr}} \qquad \text{.............(A1-1)}$$

$C_c = C_{max}$ (mixed) and $C_H = C_{min}$ (unmixed)

A uniform distribution to the heat transfer area A will be assumed so that:

$$\frac{dA}{dC_c} = \frac{A}{C_c} \qquad \text{..................(A1-2)}$$

In both cased dC_c is the differential capacity rate of the cold fluid associated with the cold fluid frontal area dA and the heat transfer surface dA.

Figure shows that for temperature conditions as a function of hot fluid flow length suggested that a condenser type of effectiveness depression is applicable.

Now we define F

$$F = \frac{dt_c}{t_H - t_{c,in}} \qquad \text{..........(A1-3)}$$

For the parallel flow [4] the ε-Ntu relation is

$$\varepsilon = \frac{[1-\exp(-Ntu(1+C_R))]}{1+C_R} \qquad \text{............(A1-4)}$$

Hence here the condenser or evaporator condition is applicable

for

$$C_R = 0$$

$$F = 1 - \exp(-Ntu) \qquad \text{...............(A1-5)}$$

From equation (A1-2)

$$F = 1 - \exp(-Ntu) = \text{Constant} \qquad \text{...............(A1-6)}$$

An energy balance for the hot fluid flowing in the entire tubes

$$dq = -C_H \, dt_H \qquad \text{...............(A1-7)}$$

dq can be written as follow

$$dq = F\,(t_H - t_{C,in})\, dC_c \qquad \text{...............(A1-8)}$$

By the combination of equation (A1-7) and equation (a1-8) we get

$$\frac{dt_h}{t_h - t_{c,in}} = -F\frac{dC_c}{C_h} = -F\frac{C_c}{C_h}\frac{dA_f}{A_{fr}} \qquad \text{................(A1-9)}$$

C_c, C_H, A_{fr} are the total magnitudes and are not variables.

Integration yields

$$\frac{t_{H,out} - t_{c,in}}{t_{H,in} - t_{c,in}} = \exp(-F(\frac{C_c}{C_H})) \qquad \text{................(A1-10)}$$

For the case $C_R = C_{min}$, it follows from the equation of ε

$$\varepsilon = \frac{t_{H,in} - t_{H,out}}{t_{H,in} - t_{c,in}} \qquad \text{..................(A1-11)}$$

So the equation (A1-10) become,

$$\frac{t_{H,out} - t_{H,in} - t_{c,in} + t_{H,in}}{t_{H,in} - t_{c,in}} = e^{-F(C_R)} \qquad \text{.................(A1-12)}$$

$$-\frac{(t_{H,in} - t_{H,out})}{(t_{H,in} - t_{c,in})} + \frac{(t_{H,in} - t_{c,in})}{(t_{H,in} - t_{c,in})} = e^{-F(C_R)} \qquad \text{..............(A1-13)}$$

$$\varepsilon = 1 - e^{F(CR)} \qquad \text{.............(A1-14)}$$

By putting the value of F

$$\varepsilon = (\frac{1}{C_R})(1 - \exp(-C_R[1 - \exp(Ntu)])) \qquad \text{............(A1-15)}$$

For the case

$C_c = C_{min}$ (mixed)

$C_H = C_{max}$ (unmixed0

and taking the definition of effectiveness

$$\varepsilon = \frac{C_H(t_{H.in} - t_{H.out})}{C_c(t_{H.in} - t_{C.in})} \qquad \text{..............(A1-16)}$$

by using the similar approach we get

$$\varepsilon = 1 - \exp\{-CR^{-1}(1-\exp[-C_R(Ntu)]))\qquad \text{........(A1-17)}$$

Appendix-A-2

Derivation of M or Fin Efficiency

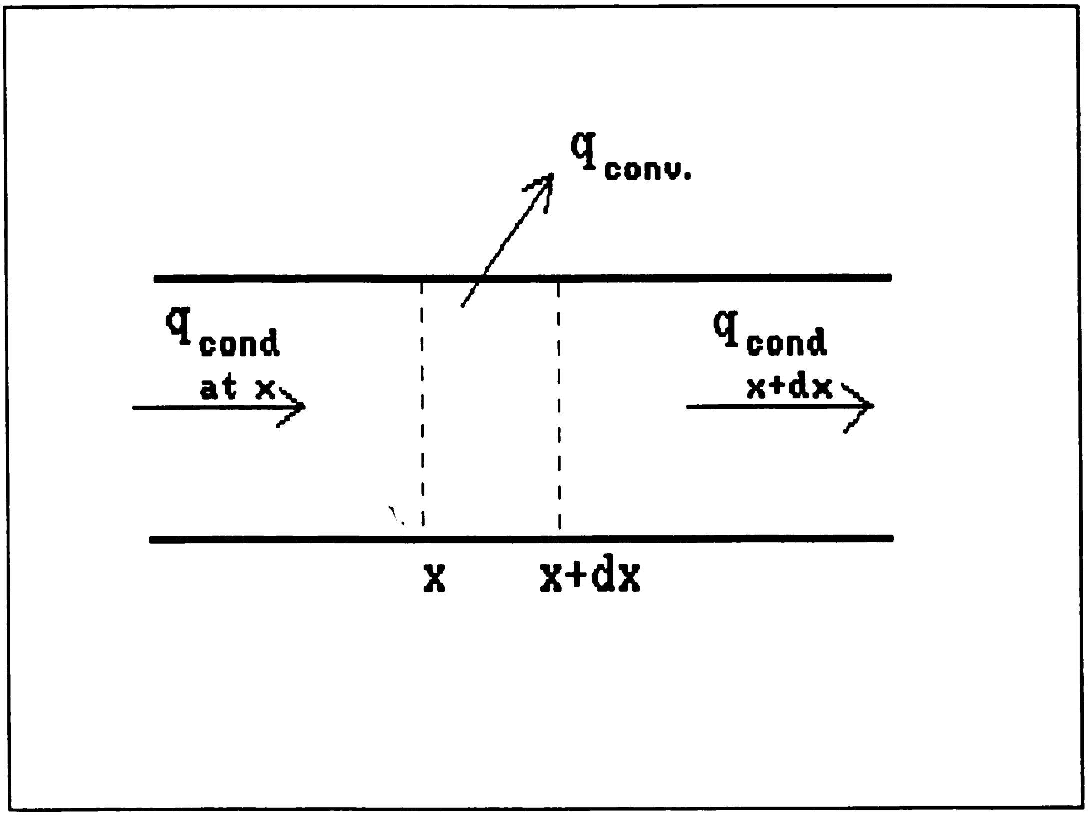


Fig.B-1: *Heat flow in small segment dx*

For "M" an eigen value from the fin equation

By using the energy balance

$$A_c[\dot{q}_{cond,x+\Delta x} - \dot{q}_{cond,x}] = -\dot{q}_{conv} P_{surf} \qquad \text{.............(A2-1)}$$

▲ A_{surf} = ▲x . P_{surf}

P_{surf} = Perimeter

Divide both sides with ▲x.

$$\frac{A_c[\dot{q}_{cond\ x+\Delta x} - \dot{q}_{cond,x}]}{\Delta x} = -\dot{q}_{conv}\, P_{surf} \qquad \text{...........(A2-2)}$$

Lim ▲x________0

$$A_c \frac{d}{dx}(q_{cond}) = -\dot{q}_{conv} P_{surf} \qquad \text{..............(A2-3)}$$

$$\dot{q}_{conv} = -k\frac{dT}{dx} \qquad \text{..................(A2-4)}$$

Equation (A2-4) is by fourier series.

According to Newton's law of cooling transforms.

$$q_{con} = h_c(T - T_\infty)$$

Put the values in equation (A2-3)

$$A_c \frac{d}{dx}(-K\frac{dT}{dx}) = -h_c(T - T_\infty).P_{surf} \qquad \text{................(A2-5)}$$

$$-KA_c \frac{d^2T}{dx^2} = -h_c(T - T_\infty).P_{surf}$$

$$KA_c \frac{d^2T}{dx^2} - h_c(T - T_\infty).P_{surf} = 0$$

$$\frac{d^2T}{dx^2} - \frac{h_c.P_{surf}}{KA_c}(T - T_\infty) = 0$$

$$\frac{d^2T}{dx^2} - M^2(T - T_\infty) = 0 \qquad \text{..............(A2-6)}$$

$$M^2 = \frac{h_c P_{surf}}{A_c K_f}$$

$$M = \sqrt{\frac{h_c P_{surf}}{A_c K_f}} \qquad \text{.......(A2-7)}$$

Calculation of P_c and A_{surs} at some mid point:

Now we calculate the P_{surf} and A_c at some mid point of the annular fin.

$$r = \frac{\frac{D_o}{2} - \frac{D_i}{2}}{2} = \frac{(D_0 - D_i}{4} \qquad \text{..................(A2-8)}$$

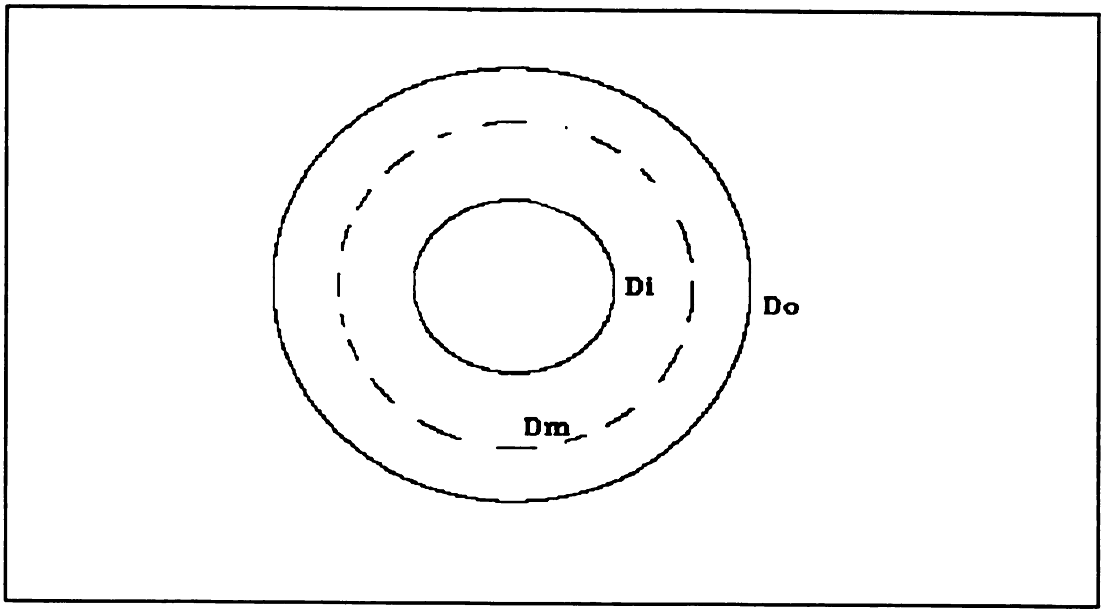


Fig.A-2.2: *Showing the dotted circle to diameter Dm.*

By putting the values of A_c and P_{surf} from equations (A2-7) and (A2-8) in equation (A2-6) we get

$$A_c = t_c 2\pi \frac{(D_o - D_i)}{4} \qquad \text{..............(A2-9)}$$

$$P_{surf} = \frac{A_{surf}}{L_f} = \frac{2\pi(\frac{D_o^2}{4} - \frac{D_i^2}{4})}{L_f}$$

$$P_{surf} = \frac{\pi}{2}\frac{(D_o^2 - D_i^2)}{L_f} \qquad \text{.............(A2-10)}$$

By putting the values of A_c and P_{surf} in equation (A2-7) we get the following value of M.

$$M = \sqrt{\frac{h_c(D_o - D_i)}{t_f\ K_f\ L_f}} \qquad \text{............(A2-11)}$$

Appendix A-3:

Pressure Loss in the Heat Exchanger

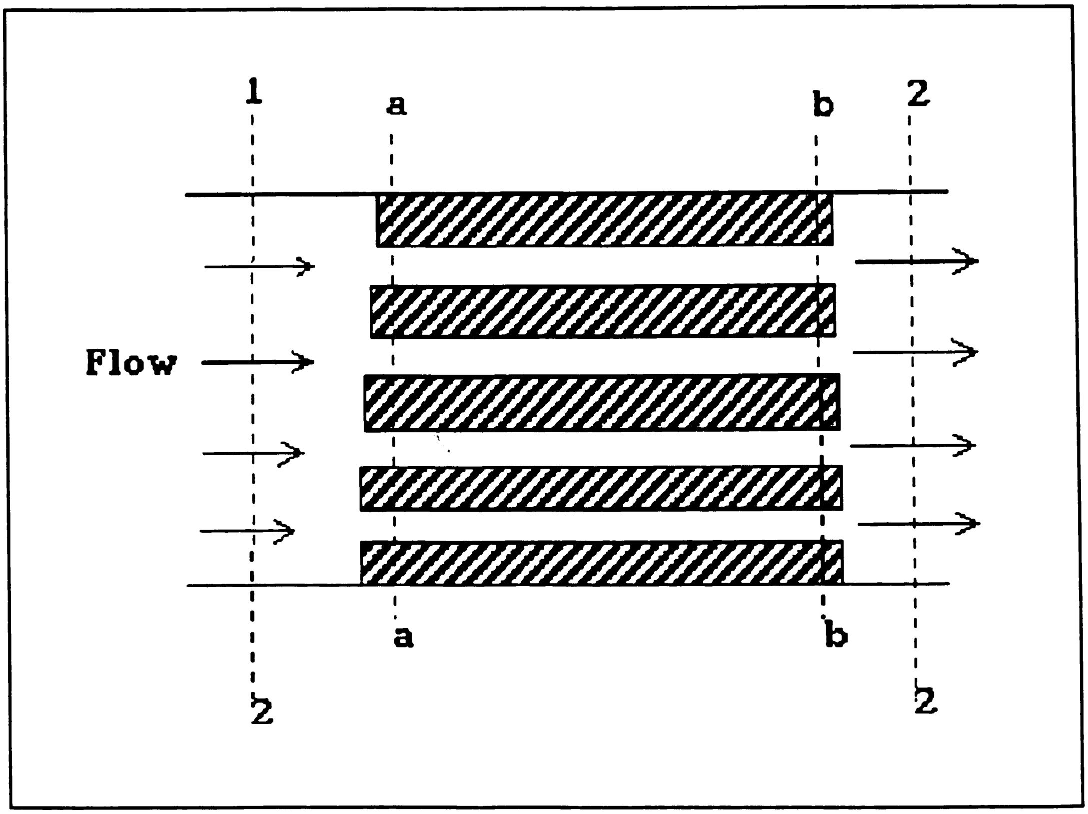


Fig A3-1: *Heat exchanger core model for pressure drop analysis*
G is based on the minimum free flow area in the core.

By the definition of the enterance and exit loss coefficients K_c K_e , and by the integration of the momentum equation (for the integration of momentum equation see [1]) through exchanger cores is:

$$\frac{\Delta P}{P_1}=\frac{G^2}{2g_c}\frac{\nu_o}{P_1}[(K_c+1-\sigma^2)+2(\frac{\nu_i}{\nu_o}-1)+f\frac{A}{A_c}\frac{\nu_m}{\nu_o}-(1-\sigma^2-K_e)\frac{\nu_i}{\nu_o}] \qquad \text{.........(A3.-1)}$$

However for flow normal to tube banks or through wire matrix surfaces, as might be employed in periodic-flow-type exchanger, enterance and exit loss effects are accounted for in the friction factor, and the equation becomes (with K_e and $k_c = 0$)

$$\frac{\Delta P}{P_1}=\frac{G^2}{2g_c}\frac{\nu_o}{P_1}[(1+\sigma^2)(\frac{\nu_i}{\nu_o}-1)+f\frac{A}{A_c}\frac{\nu_m}{\nu_o}] \qquad \text{...........(A3-2)}$$

$(1+\sigma^2)(\frac{\nu_i}{\nu_o}-1)$ Flow acceleration

$f\frac{A}{A_c}\frac{\nu_m}{\nu_o}$ Flow friction

For multipass arrangements, losses in the return headers must be accounted for separately, as must any losses in inlet and exit headers and associated ducting.

Appendix B-1

Showing the comaprison of the , mass flow, cost, Ntu, effeciveness, and number of tubes for geometry 'A'.

S.No.	Mass Flow	Cost	Ntu	Eff.	# of Tubes
1	0.95	348.82	0.7449	0.4738	102
2	0.94	348.58	0.7579	0.4794	102
3	0.93	348.30	0.7688	0.4840	102
4	0.92	348.05	0.7815	0.4894	102
5	0.91	347.80	0.7943	0.4947	102
6	0.90	347.56	0.8089	0.5006	102
7	0.89	347.31	0.8220	0.5059	102
8	0.88	347.06	0.8364	0.5117	102
9	0.87	346.82	0.8510	0.5174	102
10	0.86	346.61	0.8675	0.5238	102
11	0.85	346.36	0.8825	0.5296	102
12	0.84	346.15	0.9001	0.5362	102
13	0.83	345.92	0.9167	0.5423	102
14	0.82	345.70	0.9347	0.5489	102
15	0.81	345.49	0.9538	0.5557	102
16	0.80	394.65	0.9738	0.5627	117
17	0.79	394.46	0.9956	0.5702	117
18	0.78	394.25	1.0162	0.5772	117
19	0.77	394.05	1.0387	0.5847	117
20	0.76	393.87	1.0631	0.5926	117
21	0.75	393.67	1.0865	0.6000	117
22	0.74	393.50	1.1142	0.6085	117
23	0.73	393.32	1.1425	0.6170	117
24	0.72	393.15	1.1715	0.6255	117
25	0.71	392.97	1.2003	0.6338	117
26	0.70	438.91	1.2350	0.6434	131

Appendix B-2

Showing the comaprison of the , mass flow, cost, Ntu, effeciveness, and number of tubes for geometry 'B'.

S.No.	Mass Flow	Cost	Ntu	Eff.	# of Tubes
1	0.95	243.52	0.7449	0.4738	73
2	0.94	243.30	0.7574	0.4791	73
3	0.93	243.06	0.7688	0.4840	73
4	0.92	242.82	0.7821	0.4896	73
5	0.91	242.61	0.7943	0.4947	73
6	0.90	242.40	0.8084	0.5004	73
7	0.89	242.17	0.8215	0.5057	73
8	0.88	241.96	0.8364	0.5117	73
9	0.87	241.76	0.8520	0.5178	73
10	0.86	241.54	0.8663	0.5234	73
11	0.85	241.34	0.8831	0.5298	73
12	0.84	241.15	0.9001	0.5362	73
13	0.83	240.50	0.9173	0.5426	73
14	0.82	240.76	0.9354	0.5491	73
15	0.81	240.57	0.9545	0.5559	73
16	0.80	240.38	0.9738	0.5628	73
17	0.79	287.81	0.9956	0.5701	88
18	0.78	287.63	1.0162	0.5772	88
19	0.77	287.45	1.0387	0.5847	88
20	0.76	287.28	1.0624	0.5923	88
21	0.75	287.12	1.0887	0.6006	88
22	0.74	286.96	1.1142	0.6085	88
23	0.73	286.80	1.1425	0.6170	88
24	0.72	286.65	1.1715	0.6255	88
25	0.71	286.49	1.2011	0.6340	88
26	0.70	286.35	1.2350	0.6434	88
27	0.69	286.20	1.2690	0.6256	88

Appendix B-3

Showing the comaprison of the , mass flow, cost, Ntu, effeciveness, and number of tubes for geometry 'C'.

S.No.	Mass Flow	Cost	Ntu	Eff.	# of Tubes
1	0.95	327.16	0.7449	0.4738	88
2	0.94	326.85	0.7574	0.4791	88
3	0.93	326.54	0.7699	0.4845	88
4	0.92	326.20	0.7815	0.4894	88
5	0.91	325.89	0.7943	0.4947	88
6	0.90	325.60	0.8084	0.5004	88
7	0.89	325.28	0.8215	0.5057	88
8	0.88	325.00	0.8364	0.5117	88
9	0.87	324.70	0.8510	0.5174	88
10	0.86	324.41	0.8663	0.5234	88
11	0.85	324.14	0.8831	0.5298	88
12	0.84	323.87	0.9001	0.5362	88
13	0.83	373.10	0.9173	0.5426	102
14	0.82	372.83	0.9347	0.5489	102
15	0.81	372.57	0.9538	0.5557	102
16	0.80	372.31	0.9738	0.5628	102
17	0.79	372.08	0.9955	0.5702	102
18	0.78	371.81	1.0156	0.5770	102
19	0.77	371.57	1.0380	0.5845	102
20	0.76	371.20	1.0624	0.5923	102
21	0.75	371.10	1.0864	0.6000	102
22	0.74	370.89	1.1142	0.6085	102
23	0.73	370.68	1.1424	0.6170	102
24	0.72	423.50	1.1715	0.6255	117
25	0.71	423.28	1.2012	0.6340	117
26	0.70	423.08	1.2342	0.6432	117

Appendix B-4

Showing the comaprison of the , mass flow, cost, Ntu, effeciveness, and number of tubes for geometry 'D'.

S.No.	Mass Flow	Cost	Ntu	Eff.	# of Tubes
1	0.95	446.30	0.7449	0.4738	93
2	0.94	445.84	0.7568	0.4789	93
3	0.93	445.38	0.7688	0.4840	93
4	0.92	444.94	0.7815	0.4894	93
5	0.91	444.40	0.7943	0.4947	93
6	0.90	444.06	0.8078	0.5002	93
7	0.89	443.64	0.8220	0.5059	93
8	0.88	443.23	0.8370	0.5119	93
9	0.87	442.80	0.8516	0.5176	93
10	0.86	442.38	0.8663	0.5234	93
11	0.85	492.07	0.8835	0.5296	104
12	0.84	491.70	0.9001	0.5362	104
13	0.83	491.31	0.9173	0.5425	104
14	0.82	490.92	0.9347	0.5489	104
15	0.81	490.56	0.9545	0.5559	104
16	0.80	490.19	0.9738	0.5628	104
17	0.79	489.55	0.9956	0.5702	104
18	0.78	489.49	1.0162	0.5772	104
19	0.77	489.13	1.0380	0.5845	104
20	0.76	488.82	1.0631	0.5926	104
21	0.75	543.11	1.0865	0.6000	116
22	0.74	542.81	1.1141	0.6085	116
23	0.73	542.51	1.1424	0.6170	116
24	0.72	542.20	1.1715	0.6255	116
25	0.71	541.88	1.2003	0.6338	116
26	0.70	541.62	1.2350	0.6434	116

Appendix B-5

Showing the comaprison of the , mass flow, cost, Ntu, effeciveness, and number of tubes for geometry 'E'.

S.No.	Mass Flow	Cost	Ntu	Eff.	# of Tubes
1	0.95	409.33	0.7449	0.4738	81
2	0.94	408.82	0.7574	0.4791	81
3	0.93	408.30	0.7699	0.4845	81
4	0.92	464.50	0.7815	0.4894	93
5	0.91	463.98	0.7943	0.4947	93
6	0.90	463.50	0.8084	0.5004	93
7	0.89	462.97	0.8215	0.5057	93
8	0.88	462.50	0.8364	0.5117	93
9	0.87	462.01	0.8510	0.5174	93
10	0.86	461.56	0.8676	0.5238	93
11	0.85	461.08	0.8831	0.5298	93
12	0.84	460.63	0.9001	0.5362	93
13	0.83	460.18	0.9173	0.5426	93
14	0.82	459.72	0.9348	0.5489	93
15	0.81	459.32	0.9552	0.5562	93
16	0.80	458.87	0.9738	0.5628	93
17	0.79	458.48	0.9956	0.5702	93
18	0.78	510.08	1.0162	0.5772	104
19	0.77	509.67	1.0380	0.5845	104
20	0.76	509.28	1.0616	0.5921	104
21	0.75	508.90	1.0864	0.6000	104
22	0.74	508.56	1.1142	0.6085	104
23	0.73	508.21	1.1425	0.6170	104
24	0.72	507.85	1.1715	0.6255	104
25	0.71	507.50	1.2012	0.6340	104
26	0.70	563.92	1.2342	0.6432	116

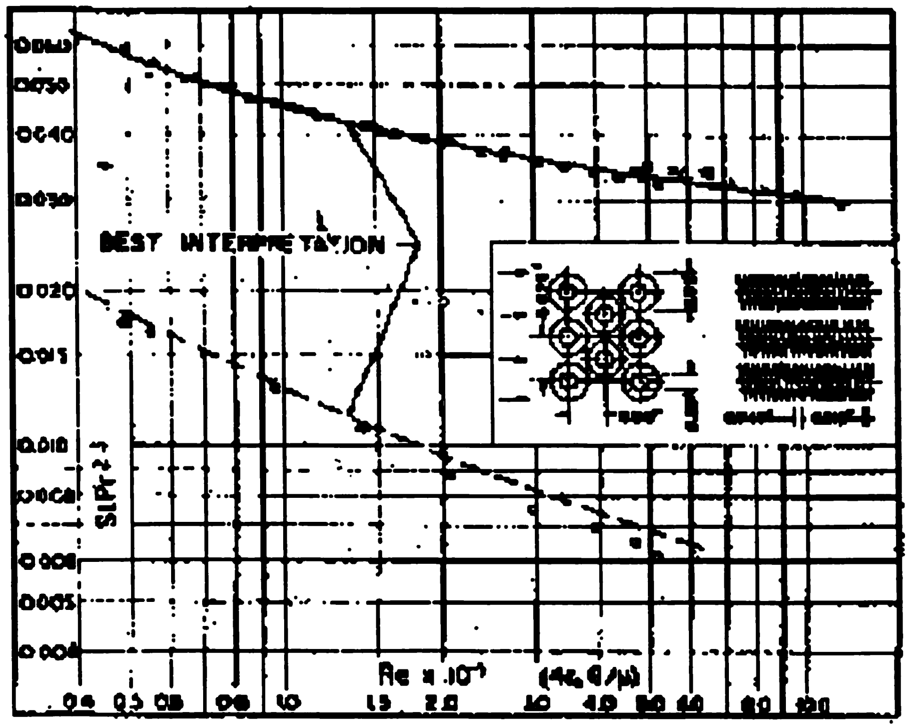
Appendix B-6

Showing the comaprison of the , mass flow, cost, Ntu, effeciveness, and number of tubes for geometry 'F'.

S.No.	Mass Flow	Cost	Ntu	Eff.	# of Tubes
1	0.95	345.03	0.7449	0.4738	70
2	0.94	344.77	0.7574	0.4791	70
3	0.93	344.49	0.7688	0.4840	70
4	0.92	344.23	0.7815	0.4894	70
5	0.91	377.10	0.7943	0.4947	77
6	0.90	376.86	0.8083	0.5004	77
7	0.89	376.61	0.8220	0.5057	77
8	0.88	376.36	0.8364	0.5117	77
9	0.87	376.11	0.8510	0.5174	77
10	0.86	375.87	0.8663	0.5234	77
11	0.85	375.65	0.8831	0.5298	77
12	0.84	375.41	0.9001	0.5362	77
13	0.83	375.19	0.9173	0.5475	77
14	0.82	374.95	0.9348	0.5489	77
15	0.81	374.74	0.9545	0.5559	77
16	0.80	412.39	0.9738	0.5627	85
17	0.79	412.19	0.9956	0.5702	85
18	0.78	411.97	1.0162	0.5772	85
19	0.77	411.77	1.0387	0.5847	85
20	0.76	411.57	1.0616	0.5921	85
21	0.75	411.37	1.0865	0.6000	85
22	0.74	411.18	1.1142	0.6085	85
23	0.73	448.88	1.1424	0.6170	93
24	0.72	448.70	1.1715	0.6255	93
25	0.71	448.52	1.2012	0.6340	93
26	0.70	448.56	1.2350	0.6434	93

Appendix C-1

For Geometry "A"

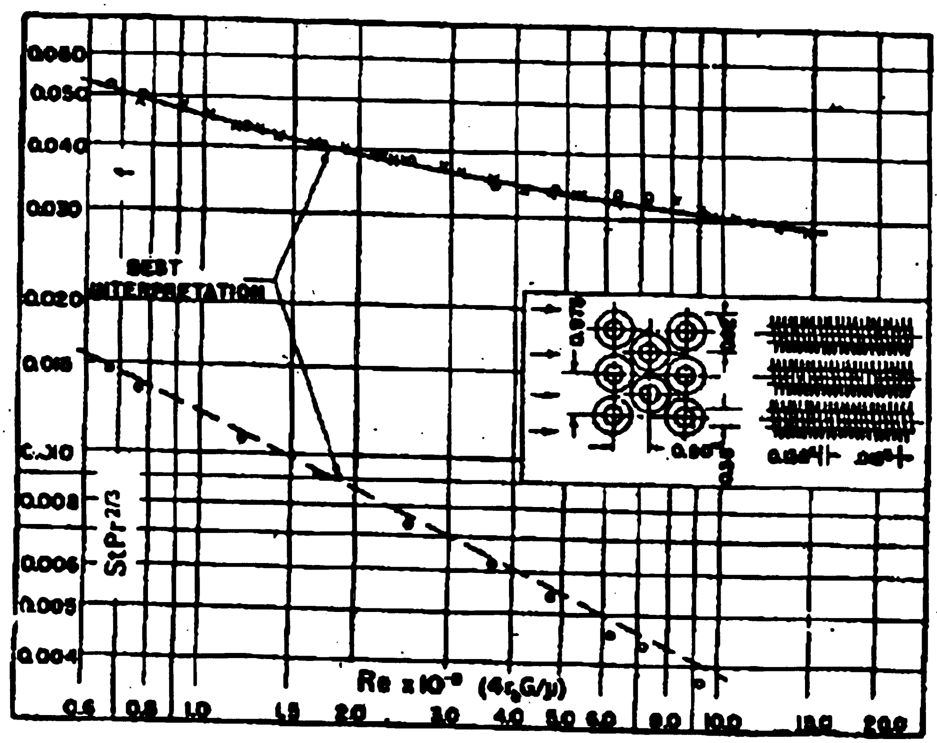


Tube outside diameter = 0.38in = 7.62x10e-3m
Fin pitch = 343 per m
Flow passage hydraulic dia = 3.929x10e-3m
Fin thickness(average) = 4.6x10e-4m
Free flow area/frontal area = 0.524
Heat transfer area/total area =635 s.m/c.m
Fin area/total area=0.91

Ref: London[5]

Appendix C-2

For geometry "B"

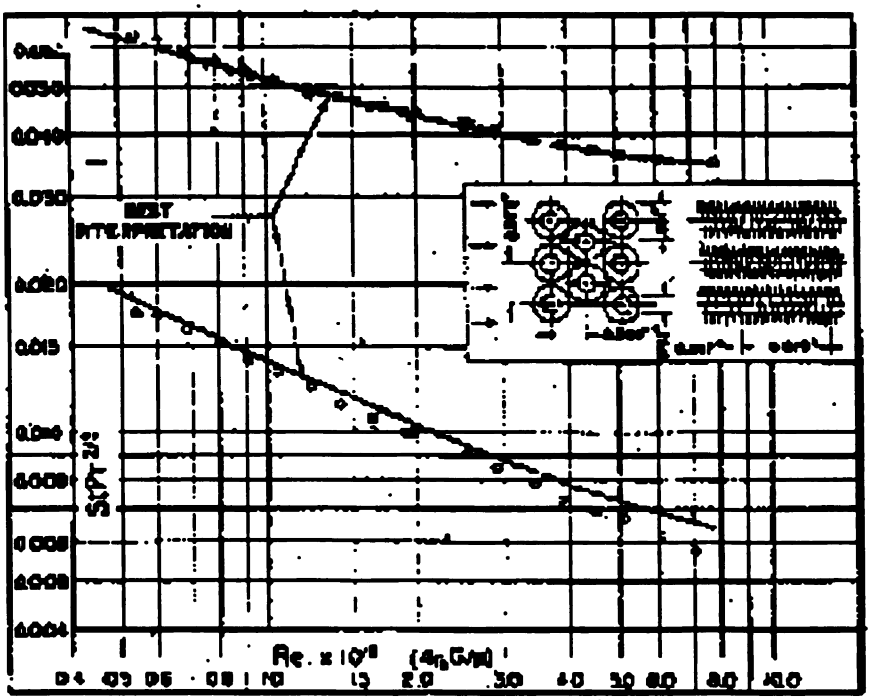


Tube outside diameter = 0.38in = 9.65 x 10e-3m
Fin pitch = 7.34 per in = 286 per m
Flow passage hydraulic diameter, = 0.0154 ft = 4.75 x 10e-3 m
Fin thickness (average) t = 0.018 in. aluminum = 0.46 x 10e-3 m
Free flow area/frontal area = 0.5380
Heat transfer area/total volume, =140 s.ft/c.ft =459 s.m/c.m
Fin area/tatal area = 0.892

Note: Experimental uncertainty for heat transfer results possibly somewhat greater than the nominal +/- 5% quoted for the other surfaces because of the necessity of estimating a contanct resistance in the bimetal tubes.

Appendix C-3

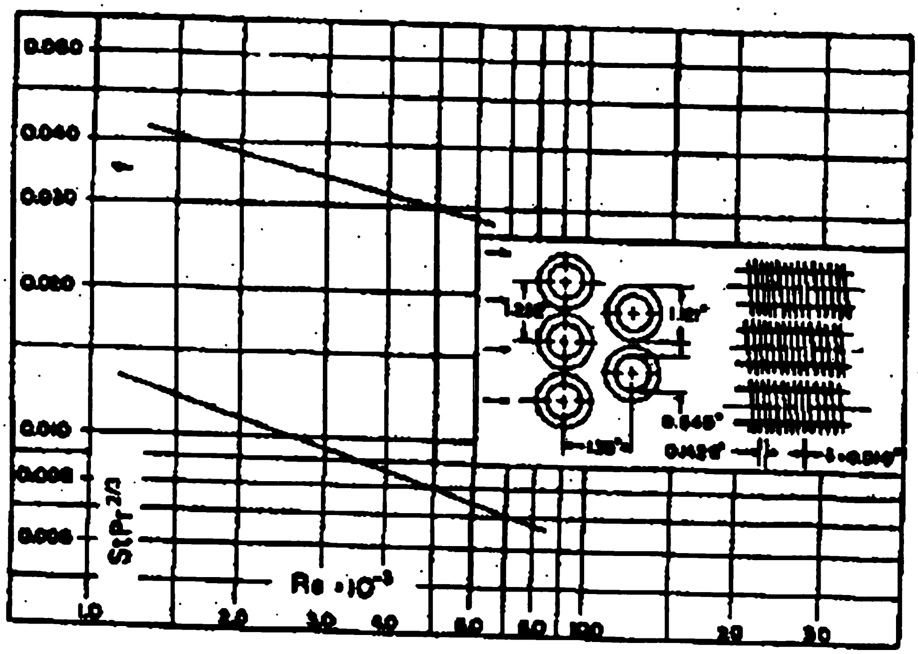
For geometry "C"



Tube outside diameter=0.42in=1.066x10e-2m
Fin pitch = 343 per m
Flow passage hydraulic diameter = 4.425x10e-3m
Fin thickness(average) t= 4.8x10e-4m
Free flow area/frontal area = 0.494
Heat transfer ara/total volume 446 s.m/c.m
Fin area/total ara=0.876

Appendix C-4

For geometry "D"

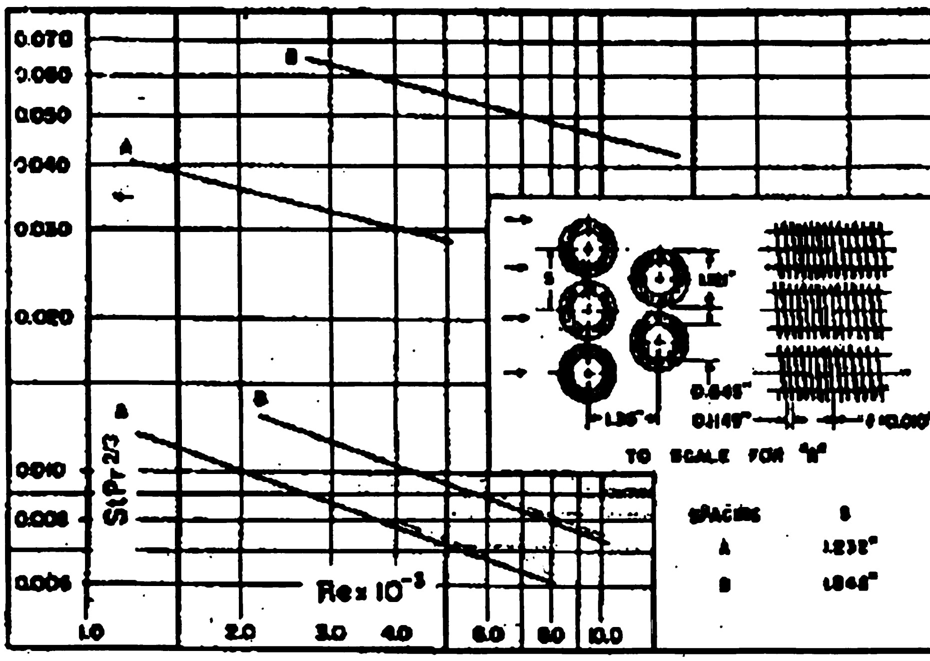


Tube outside diameter = 0.645in=16.38x10e-3m
Fin pitch=7.0per in=276 per m
Flow passage hydraulic diameter = 0.0219 ft = 6.68x10e-3 m
Fin thickness =0.010 in = 0.25x10e-3 m
Free-flow area/frontal area =0.449
Heat transfer area/tatal volume = 269x10e-3 m
Fin area/total area = 0.830

Note: Minimum free-flow aea is in spaces transverse to flow.

Appendix C-5

For geometries "E" & "F"



Tube outside diameter=0.646 in=16.38x10e-3 m
Fin pitch=6.7 per in =343 per m
Fin thickness =0.010 in =0.25 x10e-3 m
Fin area/total area=0.862

	A	B
Flow passage hydraulic diameter	0.01797	0.0383
Free flow area/frontal area	0.443	0.628
Heat transfer area/totla volume	88.7	65.7 s.ft/c.ft

Note: Minimum free-flow area is in spaces transverse to flow.

Ref: London[5]

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