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A CONTINUOUS MONTE CARLO METHOD FOR SIMULTANIOUS
GROWTH AND EQUILIBRATION IN GELATION

By

Rajasinghe Nimalakirithi

A THESIS

submitted to
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ABSTRACT

A CONTINUOUS MONTE CARLO METHOD FOR SIMULTANEOUS GROWTH AND EQUILIBRATION IN GELATION

By

Rajasinghe Nimalakirithi

We have proposed a new realistic model for kinetic gelation studies, and its computational feasibility was successfully studied by constant NVT-ensemble Monte Carlo simulation. A continuous simulation method for polymer network formation in the presence of cross-linking agents was developed with simultaneous equilibration.

Important and ignored aspects of earlier models, such as molecular mobility and realistic potentials were incorporated into our proposed model. Monomer and polymer mobility were allowed and the effect of polymer mobility on simultaneous equilibration and growth process was carefully studied. It was important to avoid large density changes during the growth. Such density changes can occur due to reduced mobility of polymer units and the shift of intramolecular potential coordinate away from that of intermolecular potential. Growth and equilibration did not compete in our model. Chain cyclizations were allowed, and it was shown that the cage trapping was rare during the early part of the growth. During this early growth the percentage of trapped centers was very much smaller than that predicted by previous gelation simulations. Our model gave a better insight of an internal structure as this was determined by the simultaneous equilibration process, and was governed by the employed potentials.

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1. INTRODUCTION

It is well known and practically important that, when sufficiently frequent branching or cross linking of chains occurs during the course of a polymerization the enhanced capacity for further growth of the larger molecules (due to their larger number of unreacted end groups) may lead ultimately to the formation of a spanning network structures so huge that the material changes rather abruptly from a fluid to a gel. The statistical theory of such reactions, developed by Flory [1,3] has had conspicuous success in predicting the gel point for the several polycondensation to which it has been applied [4].

At the gel point, a three dimensional network appears and the average cluster size is divergent. This is a very peculiar state of matter that is neither liquid nor solid but exists at the boundary between these states. For example, the viscosity is infinite. This incipient gel consists of a distribution of self-similar clusters of all sizes, and includes an infinitely massive cluster that is fractal on all length scales. If one were to pick a monomer at random, there would be zero probability that it belonged to the infinite cluster. One might now ask what happens to this perfectly self-similar arrangement as the system moves beyond the gel point [5].

Just beyond the gel point the very largest cluster combines with the infinite cluster to form a three-dimensional net work that is no longer a mass fractal on all length scales. Embedded in this network is a sol of fractal clusters whose average size continues to

decrease as the reaction proceeds and the largest remaining cluster accrete on to the network. The probability of a monomer belonging to this cluster (called the gel fraction) thus grows beyond the gel point until all monomers have reacted. It seems remarkable that if we are able to stop the reaction and extract the sol from the gel, the sol would have the same fractal dimension and self-similar size distribution as beneath the gel point.

Many authors have generalized an early statistical description of Flory and Stockmayer by relaxing some of the assumptions of ideal random branching. These models consider tree-like polymer structure built from monomer units according to probabilistic rules for bond formation. The recursive nature of the polymer structure is exploited to obtain equations for statistical averages. Some of the statistical computational schemes reported in the literature include the recursive method [6,7], the fragment method [8,9] and the probability generating function method [10]. Many of the computational schemes are based on the same underlying model which incorporates the assumptions of ideal random cross-linking, including conversion independent kinetics, termination by chain transfer or disproportionation, equal reactivity and no cyclization.

The statistical method using probability generating functions allows expressions for structural averages to be obtained most efficiently and systematically. Although statistical methods provide an efficient way to obtain structural averages, application of this method depends on the assumption of relatively small concentrations of the multifunctional cross-linking agent.

Kinetic descriptions of free radical cross-linking polymerizations have been reported by Mikos and collaborators [11,12], where reaction kinetics is described in terms of the concentrations of the mono, di and pendent vinyl species, and moment method is

used to calculate averages for the linear primary chains. Tobita and Hamielec [13] used a pseudo kinetic rate constant method to simplify the kinetic equations. These authors illustrated that many reaction nonidealities could be included in their analysis, including unequal reactivity, conversion dependent kinetics, and cyclization in an approximate manner. Because free radical polymerizations are kinetically controlled, the structure of the polymer chains depends upon the conversion at which they were formed. Additionally, the history of the process is important. These features are more readily incorporated into the kinetic descriptions. However, the biggest disadvantage of kinetic descriptions relative to statistical description is the inability of obtaining structural information about the gel. Both methods are based upon the mean field approach, and therefore cannot accurately account for heterogeneity or topological constraints [14].

2. PERCOLATION AND KINETIC GELATION MODELS

2.1. Percolation Models

The review article by Stauffer and collaborators [15] compares classical approach (Flory-Stockmayer type theory) with the percolation theories. They discussed a solution of monomers with functionality $f > 3$; from each monomer may emanate zero to f bonds to neighboring molecules and thus this monomer may participate in the formation of a large cluster which is called a macromolecule. Two monomers in the same cluster or macromolecule are thus connected directly or indirectly (through other monomers in the same cluster) by such bonds whereas two monomers in two different macromolecules are not connected by such bonds. They denoted the number of monomers in one macromolecule by s and then called this macromolecule also an s cluster; an isolated monomer without bonds to its neighbors is thus designated as an 1-cluster with $s=1$. Under certain conditions, an "infinite" cluster can be formed, a network which extends from one end of the sample to the other. This infinitely large macromolecule is called a gel; a collection of finite cluster is called a sol. A gel usually coexist with a sol; the finite clusters are then trapped in the interior of the gel.

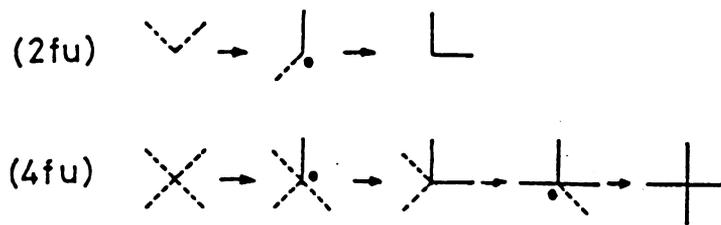
Gelation is the phase transition from a state without a gel to a state with a gel. The conversion factor p is the fraction of bonds which have been formed between the monomers of the system, i.e. the ratio of the actual number of bonds at the given moment to the maximally possible number of such bonds. Thus, for $p=0$, no bonds have been formed and all monomers remain isolated 1-clusters. In the other extreme $p=1$, all possible bonds between monomers have been formed and thus all monomers in the system have clustered into one infinite network, with no sol phase left. Thus for small

p no gel is present whereas for p close to unity one such network exists. There is in general a sharp phase transition at some intermediate critical point $p=p_c$, where an infinite cluster starts to appear; a gel for p above p_c , a sol for p below p_c . This point $p=p_c$ is the gel point and may be the analog of a liquid-gas critical point: For p below p_c only a sol is present. But for p above p_c , sol and gel coexist with each other.

In percolation model, monomers are thought to occupy the sites of a periodic lattice, and between two nearest neighbors of lattice sites a bond is formed randomly with probability p .

2.2. Kinetic Gelation Models

In one of the earlier model, by Manneville and Seze [16] gelation process of bi-functional (2fu) and tetra-functional (4fu) units were studied in the absence of any solvent. In this model monomers are placed at the nodes of a simple cubic lattice.



Different states of 2fu and 4fu.

- dotted lines indicate functions available for linking
- heavy lines indicate functions used for linking with a neighboring unit
- dots indicate that the unit is an "active center" i.e. has a free radical able to open a double bond of a neighboring unit and to link with it (active centers were formed by introduction of free radicals in the system).

Before starting the simulation, the 4fu are distributed randomly in the sample with a concentration c_{4fu} , the other nodes being occupied by 2fu. The process is initiated by activation of a given concentration c_i of randomly chosen units.

At each step:

- 1) One of the active centers is selected at random
- 2) One of its eighteen first or second neighbors having available functions is selected at random
- 3) The two units are linked;
- 4) The active center is transferred to the neighbor or annihilated with the active center of the neighbor.

In this model existence of a “infinite” cluster is characterized by the formation of a gel phase.

An important drawback of many theories, in particular of both random percolation theory on a three-dimensional lattice and of the original Flory-Stockmayer theory, is their assumption that chemical bonds are formed randomly. In reality, for irreversible gelation the bonds are formed as a result of a kinetic process which contains both deterministic and random elements. The history of the process is important since the structure of the chains depends on the conversion at which they were formed.

A model for irreversible gelation was suggested by Herrmann, Stauffer and Landau [17]. In this model bifunctional units and tetra-functional units are put into an inert solvent. Bi-functional unit has one and tetra-functional has two outer carbon double bonds which can be opened. An initiator is included in the sol which dissociates into two radicals. Each radical can saturate by breaking up a carbon double bond but leaving

the other bond unsaturate. This free bond acts as new radical opening up another double bond, and thus in a series of reactions a chain is created. Chains can cross-link at the tetra-functional molecules, thus forming branched macromolecules. The gel point is reached when for the first time an infinite macromolecule appears forming the gel which has a finite shear modulus. If two unsaturated carbon atoms encounter each other they can form a bond, thus annihilating their radical character.

Each site of the simple cubic lattice is initially occupied randomly by one of three types of molecules: tetra-functional units with probability c_t , bifunctional units with probability c_b and zero-functional solvent molecules with probability c_s .

$$c_t + c_b + c_s = 1$$

Chemical bonds can only be made between nearest neighbors on lattice. The functionality of each molecule or unit gives the maximum number of chemical bonds which it can form with its neighbors; thus the zero-functional solvent is chemically inert and the bifunctional units alone would only form chains without cross-links and without a transition to a gel. Gelation is made possible by the presence of tetra-functional units which allows for chain intersections and for the formation of larger networks of chemically bonded molecules.

The growth process is governed by the active center. Active centers are produced by the dissociation of more complex molecules. If this dissociation is relatively slow then new active centers may be formed during the gelation process; if it is relatively fast, all active centers are produced at the beginning of the reaction. In the special case of no solvent and no mobility of condensable units, all active centers in the beginning of the simulation are assumed to occur in pairs since they did not have

enough time to physically separate after their chemical dissociation. Thus initially a fraction c_i of the links connecting nearest neighbors in the lattice is assumed to be occupied randomly; and an occupied bond means that the two units connected by that link each carry an active center and are permanently bond together. Since the number of bonds is three times the number of sites in a simple cubic lattice, and since each initiator contributes two active centers, the total number of active centers is $6c_iL^3$ in a lattice with L^3 sites at an initiator concentration c_i . When the polymerizable units are mobile in a solution, active centers are highly mobile and distribution is random at the beginning, no longer pair wise, with the only resreiction that no unit carries more than one radical.

Gelation process is defined by the motion of active centers: each active center can move from its present site to a nearest neighbor site, and then the bond between its old site and the new site is called occupied. Groups of polymerizable units connected by occupied bonds are called clusters and correspond to macromolecules.

The motion of an active center to its nearest-neighbor site is random except that the new site must have at least one free functionality. That means at most three (one) of the four (two) possible bonds emanating from the tetra-functional neighbor are allowed to be occupied before the active center jumps to that neighbor. Solvent molecules never carry an active center and are never connected by occupied bonds. Should the active center jump to a site carrying another active center, the two radicals annihilate each other and the bond between the two sites become occupied. Thus the number of occupied bonds increases with time, and these bonds trace the motion of active centers, the number of which decreases in time due to annihilation. Active centers can also become trapped even if they are not annihilated, if all their neighbors are chemically saturated, that is, have

all their possible two or four bonds occupied.

Since the occupied bonds describe the path of motion of an active center they are spatially correlated, in contrast to random percolation.

Any finite mobility is allowed in the model with help of the solvent molecules. Each solvent site may exchange place with a neighbor, be it another solvent molecule or a polymerizable monomer. A movement of a polymerisable unit is allowed only if it is connected by at most one occupied bond to another unit.

In contrast to earlier studies of Manneville and de Seze, this model allows for bonds between nearest neighbors only, and the initial distribution of initiators has a constraint.

The method is similar to Monte Carlo studies of random percolation. At the beginning of each simulation they randomly distributed tetra-functional units with probability c_t on a simple cubic lattice of $L \times L \times L$ sites, with L up to 60. All other sites are occupied by bifunctional units with a concentration c_b or by solvent molecules with a concentration c_s . The positions of bi- and tetra-functional units are fixed throughout the simulation, with the exception of some runs. In the case of $c_t=1$ all sites are occupied by tetra-functional units but even then the system does not corresponds to random percolation. One reason is that in random percolation on this simple cubic lattice each site has functionality six, whereas these sites are only tetra-functional. In this model bonds are formed by a process of gradual initiation of neighboring bonds in contrast to random percolation, where bonds are formed randomly.

Initially no unit can have more than one occupied bond, in other words, the initiators are not allowed to be nearest neighbors; apart from that restriction, the spatial

distribution of initiators is taken as random. The kinetic process following the initial distribution of radicals is defined as follows: each initially occupied bond is regarded to connect a pair of free radicals. At each time unit (monte carlo step per radical), a radical randomly selects one of the six bonds emanating from this radical lattice site. This newly selected bond leads to another neighboring lattice site. Now one of two possibilities occurs. In the first case this neighboring lattice site has all its two or four bonds occupied, for bi- and tetra-functional lattice sites, respectively. In this case no additional bond can be occupied for this neighbor, and the original radical stays at its old place without creating any new occupied bond. In the other case, the number of occupied bonds emanating from this neighbor is smaller than the functionality of their neighbor. Then the bond connecting the original radical and its selected neighbor is regarded as occupied. If in this case of a successful attempt to create a new bond the neighbor happened to be another radical, the two radicals are regarded as annihilating each other.

In one Monte Carlo attempt, a radical site was selected randomly. If all its six neighbors are already saturated, then the radical is called trapped. If all radicals in the system are trapped, then no change will occur even if we make an infinite number of attempts. Initially, beside isolated monomers we have only clusters of size two, that is macromolecules containing two lattice sites connected by the initially occupied bonds. During the growth process described above the clusters can grow by the addition of one previously isolated site, or by coalescence of two clusters. Thus during the simulation it is rather easy to keep track of the number n_s of clusters containing s monomers each. s is proportional to the molecular weight if the bifunctional and tetra-functional units have the same weight; otherwise s should be called the degree of polymerization for

that particular macromolecule.

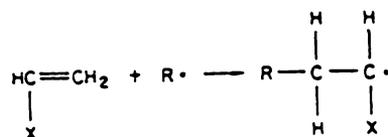
An improved model was suggested by Bansil, Herrmann and Stauffer [18] which doesn't assume that the bond formation is completely random, and it does incorporate the kinetic aspect of irreversible gelation. They considered the effects of incorporating solvent and mobility of monomers on the kinetics of free radical initiated copolymerization of a schematic vinyl and divinyl monomer.

The steps of their simulation was as follows.

- 1) Distribution of vinyl (or bifunctional) monomers, (b), divinyl (or tetra-functional) monomers (t), and solvent molecules (s), with concentration C_b , C_t and C_s respectively, on a simple cubic lattice with L^3 sites with the conservation condition

$$C_b + C_t + C_s = 1$$

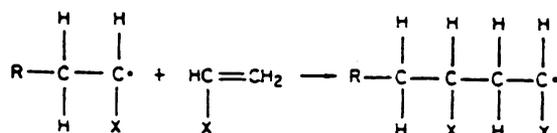
- 2) Growth process was initiated by randomly placing a free radical on a fraction C_t of the bi- and tetra-functional monomers and drawing a bond along the lattice between the monomer and the radical to represent one of the reacted functionalities of the bi or tetra-functional monomer.



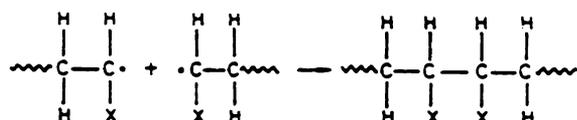
X denotes the side chain of the vinyl monomer and $\text{R}\cdot$ the free radical obtained by the dissociation of the initiator.

- 3) The polymer growth process, the reaction is simulated by transferring a randomly chosen radical from its present position to a nearest-neighbor site along one of the six

bonds emanating from each lattice site and occupying this bond, provided the neighbor chosen is not occupied by solvent molecule and has at least one free functionality left. If these conditions are not met, the radical stays, for the present time, on the initial site.



- 4) If the nearest neighbor site also has a radical on it, then the two radicals combine, thereby simulating the reaction of chain termination by combination.



Molecular mobility was taken into account by allowing diffusion of monomers. They allowed each solvent molecule to exchange position with a nearest-neighbor molecule, provided the nearest-neighbor site is occupied by an unreacted bi- or tetra-functional monomer or by another solvent molecule. The growth process is continued until a specified number of bonds has been produced. Growth process terminates due to the trapping of radicals. This trapping process occurs since a radical attempting to transfer may find all its neighbor sites occupied either by solvent molecules or by fully reacted monomers. If all radicals in the system are trapped, then no further growth is possible, and the reaction is terminated. The entire process can be simulated

many times by starting with a new initial distribution of monomers on the lattice.

During free-radical cross-linking polymerization microgel particles are formed. One reason for this is the inherent inhomogeneity of the growth process. In kinetic gelation model, conformational rearrangements are not taken into account, so that the approach to a quasi-equilibrium state by diffusion of polymer segments is not described. Diffusion control of termination at low conversion in solvents has been demonstrated experimentally by the dependence of the termination constant on the solvent viscosity. In general, we may expect that the absence of conformational rearrangements by diffusion is an important drawback of the kinetic gelation model. In particular, diffusion could lead to a stronger compaction of microgel particles. Growth model without diffusion may be applicable as a model for the cyclization of a single chain. In the kinetic gelation model free-radical cross-linking polymerization of many chains is studied. Diffusion will increase the termination rate and decrease the kinetic chain length. Thus a model without diffusion is expected to overestimate the inhomogeneity. In fact it is well-known experimentally that the kinetic chain length increases when diffusion of radical units is slowed down at increasing conversion; at high conversion the kinetic chain length decreases due to the enhanced trapping probability and the restricted diffusion of the monomer. In the kinetic gelation model, which does not contain diffusion, only the latter effect is present [19].

In one of the recent models by Bowman and Peppas [20] a face-centered cubic lattice was chosen instead of the simple-cubic lattice used by past kinetic gelation simulations. This change allows for a site to have twelve nearest neighbors instead of only six. A second basic change is the incorporation of void sites in the simulation in

order to represent free volume. Both of these changes allow for more facile mobility of the different species.

Other changes which have been made are the inclusion of monomer and initiator molecules which occupy multiple sites. By having monomers occupy multiple sites, the multiple functional groups of the monomer molecule are separated by a finite amount as opposed to being represented at the same site. The model utilizes an initiator molecule which decays into two radicals instead of generating a single radical upon decay. Finally, the simulation includes mobility of all species including the reacted polymer.

Within the simulation a single time step is the period of time required to examine each active radical and one nearest-neighbor site to determine if reaction and propagation will occur. One of the key developments of Bowman and Peppas [20] model is the ability of all species in the reaction to move.

Monomer, initiator and polymer may move and diffuse with the only constraints being that all bonds must be preserved and movement must occur from an occupied site to an unoccupied site. Mobility increases as void sites increase and as the bonding between sites decreases.

The specific mechanism of mobility is as follows. First, two nearest-neighbor sites, site no. 1 and site no.2, are selected random. Site no. 1 is designated as the site from which movement will occur while site no. 2 is designated as the site which movement will occur. Movement will occur if all the necessary conditions are met.

- 1) Site no. 1 is occupied.
- 2) Site no. 2 is a void space.
- 3) all sites to which site no. 1 is bonded are nearest neighbors of site no. 2.

During each time step a fraction of sites will be selected to be site no. 1 and a nearest-neighbor site is selected as site no. 2. If movement is possible as described by the above conditions, then it will occur. As a part of movement, bonds are also changed to ensure that bonds are between sites. As mobility increases, the heterogeneity will decrease while the tendency to trap radicals will also decrease. This will lead to a higher maximum conversion being reached as the polymer becomes more mobile. Mobility can be increased either by increasing the fraction of sites examined for mobility checks during each time step.

After the radicals have been generated from initiator decay, they are able to propagate via reaction with any nearest-neighbor functional group. In turn, if a second radical is on a nearest-neighbor site the radicals may react with each other and terminate. During each time step every radical that remains active may propagate or terminate. If the adjacent site is a void, an unreactive monomer link, or a portion of the polymer, that radical will remain inactive until at least the next time step. If, however, the adjacent site is an unreacted functional group, propagation of the radical will occur from its present site to the adjacent site. The parameter that must be set for each simulation are the initiator mole fraction, the fraction of sites which are void, the lattice size, and the fraction of sites which are inspected during each time step for mobility.

Despite the improvements in monomer size, initiation mechanism, and species mobility it is necessary to understand that this simulation still contains some very unrealistic elements. The first and the most obvious short coming of this model is that it is performed on a lattice. Polymerization do not occur on a lattice but rather the

molecules involved take on a continuous distribution of orientations. Also, the model does not account for forces and potentials at the molecular level, and the size of the monomer is still not accurately represented. The addition of a molecular size into the model by incorporating a monomer that occupies a number of sites is a great advance, but it will only approach the behavior of the actual monomer molecules.

2.3. Monte Carlo Simulations

The goal of our simulation effort describe in following chapters is to account for molecular mobility in a realistic manner. Rather than fixing reactive monomer sites in space as in most kinetic gelation simulations reported in literature, in our simulations the monomer units will be allowed to move freely in three-dimensional space as they interact according to defined intermolecular potentials. A typical simulation will include several hundred monomer units interacting in a cell with periodic boundary conditions. Spherical as well as dumbbell shaped molecules will be allowed to interact according to Lennard-Jones potentials.

Our first attempt is to calculate the potential energy in a system of Lennard-Jones atoms. The Lennard-Jones potential is given by the formula.

$$v^{LJ}(r) = 4\varepsilon((\sigma/r)^{12} - (\sigma/r)^6) \quad (1)$$

ε = Well depth

σ = Radius at which potential equal to zero

(The well depth is often quoted in units of temperature as ε/k_B , where k_B is Boltzmann's constant)

We store the coordinate vectors of our atoms in three FORTRAN arrays RX(I), RY(I) and RZ(I), with the particle index I varying from 1 to N. For the Lennard-Jones

potential it is useful to have precomputed the value of σ^2 , which is stored in the variable SIGSQ. The potential energy will be stored in a variable V, which is zeroed initially, and is then accumulated in a double loop over all distinct pairs of atoms, taking care to count each pair only once. Information about code development is given in reference 21 and here we highlight some of the important steps given there.

Periodic Boundary Conditions:

A cubic box is replicated throughout space to form an infinite lattice. In the course of the simulation, as a molecule moves in the original box, its periodic image in each of the neighboring boxes moves in exactly the same way. Thus, as a molecule leaves the central box, one of its images will enter through the opposite face. There are no walls at the boundary of the central box, and no surface molecules. This box simply forms a convenient axis system for measuring the coordinates of the N molecules. The Figure 1 (Figure 1.9 in reference 21) illustrates this in two dimension. It is not necessary to store the coordinates of all the images in a simulation, just the molecules in the central box. When a molecule leaves the box by crossing a boundary, attention may be switched to the image just entering. For a fluid of Lennard-Jones atoms, it should be possible to perform a simulation in a cubic box of side $L=6\sigma$, without a particle being able to 'sense' the symmetry of the periodic lattice. We use a cubic box in our simulation because of its geometrical simplicity.

Potential Truncation:

The heart of the Monte Carlo program involves the calculation of the potential energy of a particular configuration. We must include interactions between molecule 1

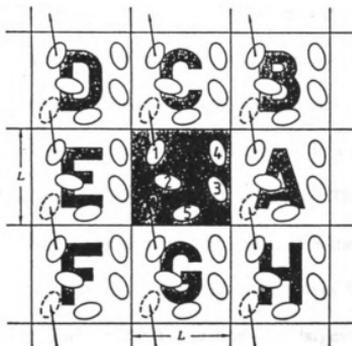


Figure 1. A two-dimensional periodic system. Molecules can enter and leave each box across each of the four edges (From Figure 1.9 in reference 21).

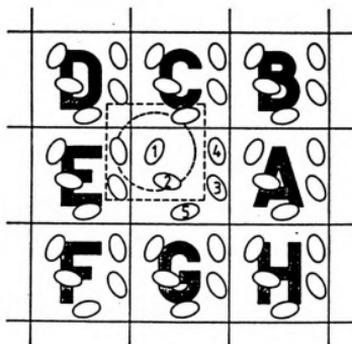


Figure 2. The minimum image convention in a two-dimensional system. The central box contains five molecules. The 'box' constructed with molecule 1 at its center also contains five molecules. The dashed circle represents a potential cutoff (From Figure 1.12 in reference 21).

and every other molecule i in the simulation box. In principle this is an $(N-1)$ number of terms. For a short-range potential energy function, we restrict this summation by making an approximation. Consider molecule 1 in Figure 2 (Figure 1.12 in reference 21) to rest at center of a region which has the same size and shape as the basic simulation box. Molecule 1 interacts with all the molecules whose centers lie within this region, that is with the closest periodic images of the other $N-1$ molecules. This is called the 'minimum image convention'. In the minimum image convention, then the calculation of the potential energy due to pairwise-additive interactions involves $N(N-1)/2$ terms. A further approximation significantly improves this situation. The largest contribution to the potential comes from neighbors close to the molecule of interest, and for short range potential we apply a spherical cutoff. This means setting the pair potential to zero for $r > r_c$, where r_c is the cutoff distance. In a cubic simulation box of side L , the number of neighbors explicitly considered is reduced by a factor of approximately $4\pi r_c^3 / 3L^3$, and this may be a substantial saving. The cutoff distance must be no greater than $L/2$ for consistency with the minimum image convention.

Initially, the N molecules in the simulation lie within a cubic box of side L , with the origin at its center, all coordinates lie in the range $(-L/2, L/2)$. As the simulation proceeds, these molecules move about the infinite periodic system. When a molecule leaves the box by crossing one of the boundaries, it is usual to switch attention to the image molecule entering the box, by simply adding L to, or subtracting L from, the approximate coordinate. For the coding information of this and the minimum image convention we refer the reader to reference 21.

We set the pair potential to zero if the pair separation lies outside some distance r_c .

It is easy to compare the square of the particle separation r_{ij} and, compare this with the square of r_c .

Metropolis Monte Carlo Method:

Classical statistics is assumed, only two-body forces are considered, and the potential field of a molecule is assumed spherically symmetric. System consists of a cube containing N particles. If we know the positions of the N particles in the cube, we can easily calculate, for example, the potential energy of the system (22)

$$V = (1/2) \sum_{i=1}^N \sum_{\substack{j=1 \\ i \neq j}}^N V(d_{ij}) \quad (2)$$

(Here V is the potential between molecules, and d_{ij} is the minimum distance between particles i and j)

In order to calculate the properties of our system we use the canonical ensemble. So, to calculate the equilibrium value of any quantity of interest F ,

$$\bar{F} = \int F \exp\left(-\frac{E}{kT}\right) d^{3N} p d^{3N} q / \int \exp\left(-\frac{E}{kT}\right) d^{3N} p d^{3N} q \quad (3)$$

Where $d^{3N} p d^{3N} q$ is a volume element in the $6N$ -dimensional phase space.

Moreover, since forces between particles are velocity-independent, the momentum integrals may be separated off, and we need perform only the integration over the $3N$ -dimensional configuration space. The Monte Carlo method for many-dimensional integrals consists simply of integrating over a random sampling of points instead of over a regular array of points.

Thus the most naive method of carrying out the integration would be to put each of the N particles at a random position in the cube, then calculate the energy of the system,

and give this configuration a weight $\exp(-E/kT)$. This method, however is not practical for close-packed configuration, since with high probability we choose a configuration where $\exp(-E/kT)$ is very small; hence a configuration of very low weight. So the method we employ is actually a modified Monte Carlo scheme, where instead of choosing configurations randomly, then weighting them with $\exp(-E/kT)$, we choose configurations with a probability $\exp(-E/kT)$ and weight them evenly.

We place the N particles in any configuration, for example, in a regular lattice.

Then we move each of the particles in succession according to the following prescription:

$$\begin{aligned} X &\rightarrow X + \alpha\xi_1 \\ Y &\rightarrow Y + \alpha\xi_2 \\ Z &\rightarrow Z + \alpha\xi_3 \end{aligned} \tag{4}$$

where α is the maximum allowed displacement, which for the sake of this argument is arbitrary, and ξ_1 , ξ_2 and ξ_3 are random numbers between (-1) and 1 . Then, after we move a particle, it is equally likely to be any where within a cube of side 2α centered about its original position.

We then calculate the change in energy of the system E , which is caused by the move. If $E < 0$, if the move would bring the system to a state of lower energy, we allow the move and put the particle in its new position. If $E > 0$, we allow the move with probability $\exp(-E/kT)$; we take a random number ξ between 0 and 1 , and if $\xi < \exp(-E/kT)$, we move the particle to its new position. If $\xi > \exp(-E/kT)$, we return it to its old position. Then, whether the move has been allowed or not, whether we are in a different configuration or in the original configuration, we consider that we are in a new configuration for the purpose of taking our averages. So

$$F = (1/M) \sum_{j=1}^M F_j \tag{5}$$

where F_j is the value of the property F of the system after the j th move is carried out according to the complete prescription above. Having attempted to move a particle we proceed similarly with the next one.

A common practice in MC simulation is to select the atoms to move sequentially rather than randomly. This cuts down on the amount of random number generation and is an equally valid method of generating the correctly weighted states. The simulation of hard spheres is particularly easy using the MC method. The same Metropolis procedure is used, except that, in this case, the overlap of two spheres results in an infinite positive energy change and $\exp(-E/kT)=0$. All trial moves involving an overlap are immediately rejected since $\exp(-E/kT)$ would be smaller than any random number generated on $(0,1)$. Equally all moves that do not involve overlap are immediately accepted. As before in the case of a rejection the old configuration is recounted in the average (21).

Hard molecule fluids have played an important role in the development of our understanding of the liquid state. They have served as a useful check of statistical theories and have often provided the underlying fluid structure in perturbation theories for realistic molecular systems. The results of those theories have been successfully applied to many problems of simple liquids, to mixtures, and to diatomic liquids.

In the MC simulation of a molecular liquid the underlying matrix of the Markov chain is altered to allow moves which usually consists of a combined translation and rotation of one molecule. Chains involving a number of purely translational and purely rotational steps are perfectly proper but are not usually exploited in the simulation of molecular liquids. The translational part of the move is carried out by randomly

displacing the center of mass of a molecule along each of the space-fixed axes. As before the maximum displacement is governed by the adjustable parameter δr_{\max} .

A method for changing the orientation of a molecule has been suggested by Jansoone [23]. It is more convenient to represent the molecular orientation by a unit vector \mathbf{e} fixed in the molecule. The new trial orientation, \mathbf{e}^m , is chosen randomly and uniformly on a region of the surface of a sphere with the constraint that,

$$1 - \mathbf{e}^n \cdot \mathbf{e}^m < d \ll 1. \quad (6)$$

d controls the size of the maximum displacement, and a sensible first guess for this parameter is 0.2 [21]. There are a number of methods for generating a random vector on the surface of a sphere. The simplest of these uses the acceptance-rejection technique of Von-Neumann [24]. The procedure is iterative.

- a) Generate three uniform random variates, ζ_1 , ζ_2 , and ζ_3 , on (0,1);
- b) Calculate $\zeta_i = 1 - 2\xi_i$ for $i=1,3$ so that the vector $\bar{\zeta} = (\zeta_1, \zeta_2, \zeta_3)$ is distributed uniformly in a cube of side 2, centered at the origin;
- c) Form the sum $\zeta^2 = \zeta_1^2 + \zeta_2^2 + \zeta_3^2$
- d) For $\zeta^2 < 1$ (inside the inscribed sphere) take $\bar{\zeta} = (\zeta_1/\zeta, \zeta_2/\zeta, \zeta_3/\zeta)$ as the vector;
- e) For $\zeta^2 > 1$ reject the vector and return to step (a).

Marsaglia [21] has suggested an interesting improvement;

- a) Generate two uniform random variates ξ_1, ξ_2 on (0,1);
- b) Calculate $\zeta_i = 1 - 2\xi_i$ for $i=1,2$
- c) For $\zeta^2 < 1$ take the vector, $\bar{\zeta} = [2 \zeta_1(1 - \zeta^2)^{1/2}, \zeta_2(1 - \zeta^2)^{1/2}, 1 - 2\zeta^2]$;
- d) For $\zeta^2 > 1$, reject and return to step (a).

A computer program developed to calculate Lennard-Jones potential of hard dumbbell system, by following Marsaglia's mathematical method is included in the appendix D.

(Note: This is a modification of the original program in reference 21)

One difficulty with MC methods for molecular fluids is that there are usually a number of parameters governing the maximum translational and orientational displacement of a molecule during a move. As usual these parameters can be adjusted automatically to give an acceptance rate of 0.5, but there is not a unique set of maximum displacement parameters which will achieve this. A sensible set of values is best obtained by trial and error for the particular simulation in hand. In our simulations maximum displacement was allowed to adjusted automatically to obtain 50% acceptance.

3. RECENT ADVANCES IN POLYMERIC GLASS STRUCTURES

Molecular dynamics simulation of bulk liquid and glass of long-chain molecules has been performed by Rigby and Roe [25] using a fairly realistic off-lattice chain model. Initial configurations of highly intertwined chains were constructed by first generating a random distribution of points representing the centers of CH₂ groups (in the manner usually employed for mono-atomic systems). Groups in close proximity were joined to form several polymer chains. Individual chains are modeled as sequences of spherical segments connected by spring like valence bonds subject to the potential energy,

$$V_{b,j} = (1/2)K_b(l_j - l_0)^2, \quad (7)$$

where l_0 is the equilibrium bond length. The valence angle between successive pairs of bonds is maintained close to the tetrahedral value θ_0 by a potential quadratic in $\text{Cos}\theta$:

$$V_{\theta,j} = (1/2)k (\text{Cos } \theta_j - \text{Cos } \theta_0)^2. \quad (8)$$

Finally, nonbonded interactions, between segments in different chains and between segments separated by more than three bonds along the chain backbone, are given according to a truncated Lennard-Jones potential. Simulation program was run with weakened force constants for all potentials (except for the LJ potential which was maintained “full strength” at all stages). Force constants were then gradually increased to their final values as the C-C bond lengths and the C-C-C valence angles decreased towards their equilibrium values.

An extension of Rigby and Roe method was recently carried out by Khare, Paulaitis and Lustig [26]. Their approach for generating glass structures was carried out in three stages. In the first stage, the monomers are initially placed in the simulation box in an order array. The system is then heated and relaxed by molecular

dynamics at an elevated temperature. The purpose at this stage of the procedure is to achieve a uniform spatial distribution and a random orientational distribution of monomers in the simulation box, and not an equilibrium liquid.

A polymerization of this configuration is then carried out in the second stage by connecting spatially adjacent monomers. Periodic boundary conditions are employed to eliminate surface effects, and the minimum image convention is used. The problem of connecting a given number of monomers to form the shortest polymer chain is similar to the classical "Traveling Salesman" problem. Traveling Salesman problem has been solved previously by simulated annealing, a multivariable optimization technique based on the Monte Carlo method. After connecting monomers in an initial polymerization sequence, simulated annealing is carried out by randomly altering this sequence and evaluating the cost of each move in terms of its effect on the total chain length of the polymer. Moves are accepted or rejected on the basis of standard Metropolis probabilities.

Many of the bond lengths and bond angles in the polymer chain backbone at the end of this stage will have values far from their equilibrium values, and hence the potential energy of this structure must be minimized primarily with respect to these degrees of freedom. Structures are relaxed using a combination of energy minimization and molecular dynamics at elevated temperature.

An non-lattice Monte Carlo method for growth and equilibration of dense liquids consisting of polymer chains with realistic intramolecular architecture was studied by Gupta, Westermann and Bitsanis [27]. The method succeeded in growing alkane chain length up to 25. They ignored the torsional potential until all the chains have grown to their full length. After selecting a chain randomly, a decision is made on

whether to further grow the chin or to attempt an equilibration move. At this stage growth and equilibration compete. Growth and equilibration moves are accepted and rejected unless they result in physical overlap between segments. Attempts to grow all the chains without simultaneous equilibration are bound to fail.

4. A NEW MODEL FOR GELATION

4.1. Description of the Model

We have developed a new model for free radical cross-linking polymerizations based upon Monte Carlo simulations. Bi-functional and tetra-functional units were considered as spheres. Monomers were initially placed in a simulation box in a cubic lattice. A random distribution and orientations were achieved by Monte Carlo simulation. Before the initiation, monomers interacted via Lennard-Jones potential. Predetermined number of monomers was randomly initiated to create active centers. The growth of linear polymer chains were achieved by bonding physically closest monomers to the active centers. These centers became active centers, and propagation occurred. The nearest neighbor status was examined before bond formation, so that the same monomer could not be bonded to more than one active center at a given instant of time (within a given Monte Carlo cycle). If at any given instant of time the same monomer was closest to more than one active center, the closest active center was bonded to that monomer and the next nearest neighbor(s) was (were) bonded to other(s). Cross-linking was made possible whenever an already bonded tetra-functional unit in one chain became the closest unit to the active center of another chain. The possibility for a given tetra-functional unit to form two consecutive bonds with the same monomer was avoided.

Within each Monte Carlo cycle all particle displacements were accepted or rejected according to their energies, and all active centers were allowed to form one bond each (provided bond length is less than reaction distance). Covalent bonds within individual chains were modeled by the harmonic oscillator potential with predetermined force constants. Along the chain backbone for non-neighboring units the

Lennard-Jones potential was not evaluated in our model because it usually falls beyond the cutoff region ($r_c=2.2\sigma$, and our monomer units consist of at least 2 c-c double bonds). For all other units, LJ potential was maintained.

In our model, mobility of both monomer and polymer were allowed in a realistic manner throughout the polymer growth. The maximum displacement allowed within the Monte Carlo simulation was reduced by a predetermined factor for all bonded units. This was done by multiplication of maximum displacement of bonded units by a factor of 0.8. In addition, bonded unit displacement was further reduced by a very small factor between each Monte Carlo cycle. This was done by multiplying the current factor by 0.995 within each MC cycle. This will account for the bulkiness of the polymer as it grows. Any displacement which moved monomers within the overlapping distance was immediately rejected. Bond formation was always carried out with the closest neighbor, provided it fell within a predetermined maximum bond length (reaction distance). As a result, the length of the growing linear chains could differ at any given instant of time, as well as at the end of the simulation. The maximum bond length was selected to be within the attractive region of the LJ potential. Bonded units were treated as harmonic oscillators with the minimum close to the LJ potential minimum. The oscillator overlapping distance was shifted to the left of the overlapping distance of LJ potential. A rough sketch of placements of potentials are shown in the Figure 3.

To facilitate homogeneous growth process, oscillator potential minima was selected to be same as LJ potential minima, for most of our simulations. Any given displacement for bonded units was accepted if the move is towards the oscillator minimum. Otherwise it was rejected. Any given displacement of a bonded unit was

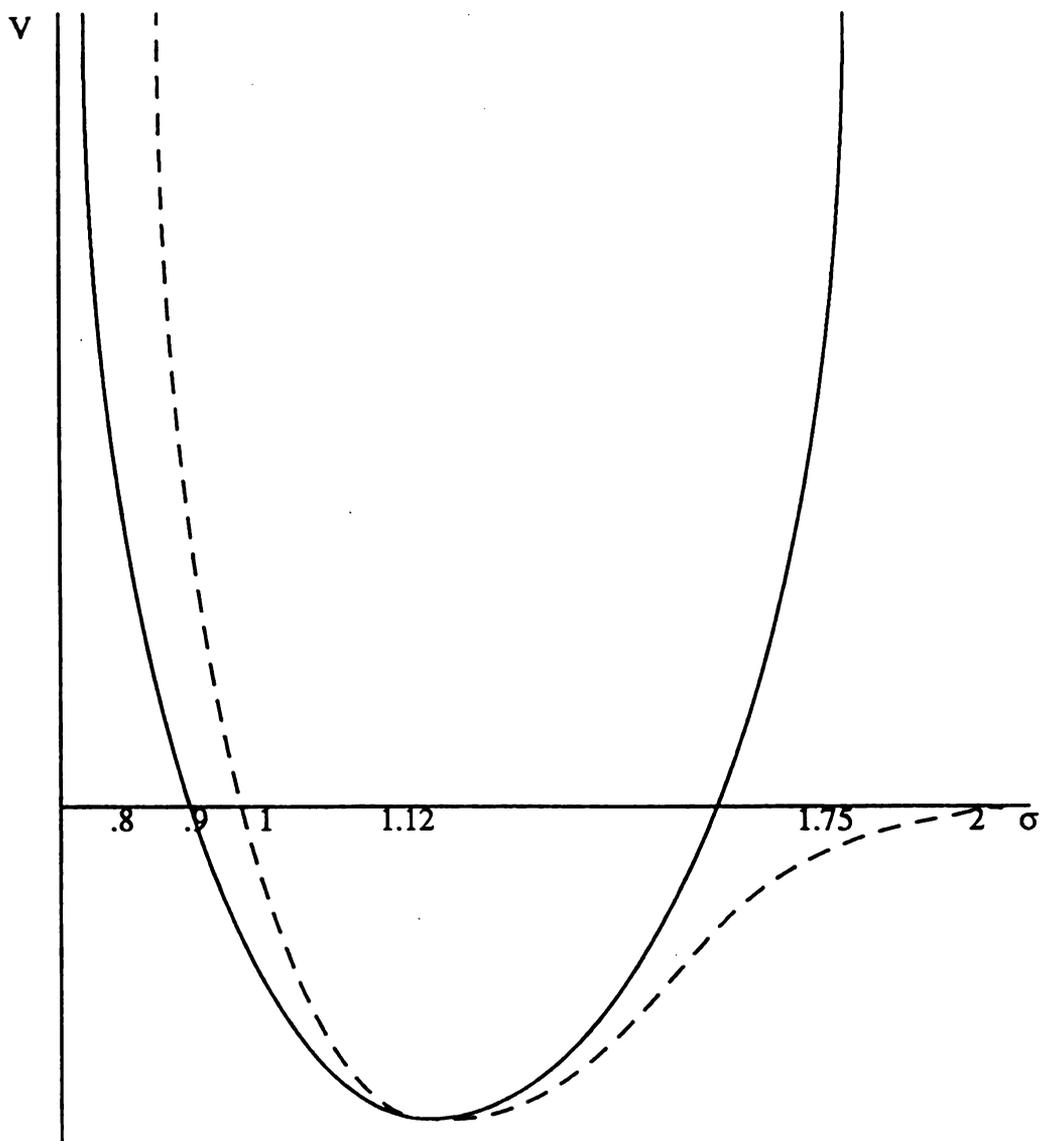


Figure 3. Rough Sketch of Oscillator and LJ Potential Placements.
 LJ potential minimum = Oscillator potential minimum = 1.12σ
 LJ potential overlap distance = 0.9σ
 Oscillator potential overlap distance = 0.8σ
 Reaction distance = Maximum bond length = 1.75σ
 LJ long-range cutoff = 2.2σ
 Reduced density = density $\cdot (\sigma^3) = 0.64$
 Reduced temperature = $k_B T / \epsilon = 1.06$

immediately rejected if it results in an overlap with its bonded neighbor or the resulting bond length exceeds the reaction distance. A large reduction of bonded unit displacement will delay the move towards the oscillator minimum. This was concluded by calculating the fraction of bonds reached to their equilibrium distance at different stages of the given simulation. At different stages of the simulation the configuration was analyzed to calculate bond lengths. Necessary computer programs for this process is included in the appendix C.

As described in earlier paragraphs, to account for the reduced mobility of the polymer as compared to unreacted monomer, bonded units were given a reduced displacement in the simulations. This feature as well as the choice of the cut off of maximum allowed bond length (a technical choice) took steric hindrance into account and led to trapped active centers, but, in comparison to most earlier models, our trappings were not permanent, since the radical center could escape or a monomer could move within the vicinity of attractive potential after some time. Such trappings could have affected the gel point, if they occurred before the gel point was reached. At the gel point, enough chains were linked together to form a large macromolecular network that spanned the entire reaction system. With higher multiple center concentration, the cross-linking was complete, and an infinite network formed, before any trapping occurred. Our results showed that chain cross-linking was complete, before any trappings of the radicals, with reasonable concentration of multiple centers.

By having multiple centers which could enter into the same linear chain at a later stage of the growth, chain cyclization was allowed in our model. Some of the earlier models did not take this into consideration. Destruction of active centers by annihilation

was not allowed in our model, since we kept the concentration of active centers at a very low percentage. The probability of such an annihilation was considered to be negligible.

The major advantage of our technique over previously published work is that the molecular mobility of non-bonded, as well as bonded units, are allowed in a realistic manner during the polymer growth. Incorporation of realistic potentials during the growth is another advantage over earlier work by many authors. At the end of our simulation the bond lengths of the cross-linked polymer backbone are closer to their equilibrium values.

It is possible to turn on a torsional potential after this stage, and the system can be left to reach to conformational equilibrium. Alternatively, one can incorporate the torsional potential along with the oscillator potential and use energy minimization.

4.2. Simulation Results and Discussion

Nine hundred and thirty four monomer units within a simulation box with periodic boundary conditions were allowed to polymerize, according to above algorithm. Four initiators and required number of multiple centers were labeled randomly at the beginning. All our simulations were carried out at reduced LJ temperature of 1.06 and an initial reduced LJ density of 0.64. All parameters in our simulations used LJ diameter (σ) as the unit of length. Our monomers were assumed to be at least two c-c bond length long. The simulation box was of length 2 holding approximately 1000 monomer units evenly spaced in a cubic lattice at the beginning. The correspondence with the physical values can be related through these. LJ reduced temperature and initial reduced density was selected to assure the literature LJ potential values at the beginning.

One of the most significant contributions of our simulation results is an enhanced understanding of radical trapping. In earlier work by many authors, the simulation was based on a random walk on a cubic lattice and an active center was considered as trapped when all its nearest neighbors (on the lattice) were occupied [17]. As a result, these authors observe early trapping at a very low conversion, a very high fraction of trapped active centers and a relatively low limiting conversion at which all active centers are trapped. For example, simulations by Herrmann, Stauffer and Landau [17] have predicted a limiting conversion of 55% for a system in which experimental observations indicates conversion over 80% are readily achieved. Their predictions are shown in the Figure 4.

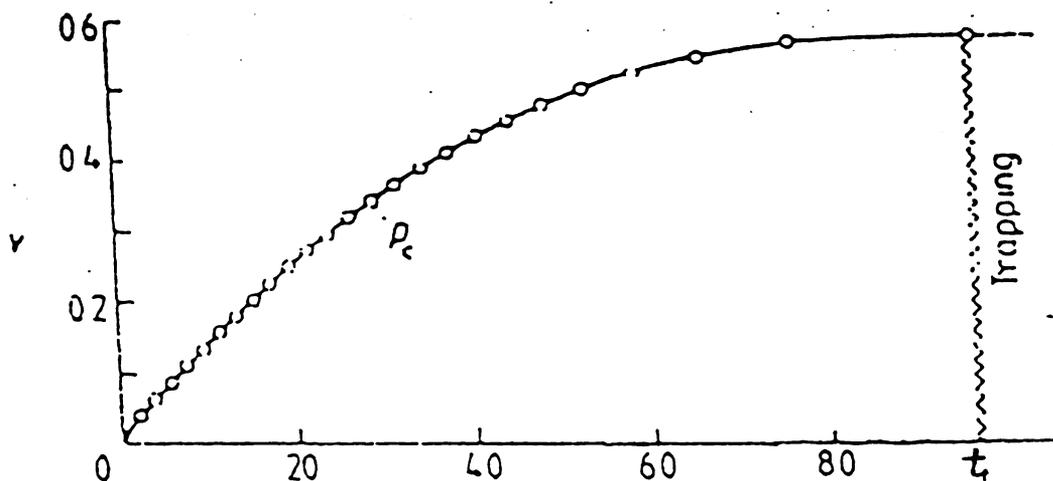


Figure 4. Conversion (v) against time for $C_1 = 0.004$, $C_1 = 0.1$ and $L=42$. The gel point is marked by p_c (From Herrmann, Stauffer and Landau [17]).

In addition, the simple kinetic gelation simulations predict that trapping increases as the fraction of tetra-functional monomer is decreased. These limitations ultimately arises from the fact that the simulations neglect molecular mobility altogether, and therefore active centers become trapped after propagating with their nearest neighbors.

In our work, we distinguished between two types of trapping mechanisms. The first kind was a temporary trapping due to the possibility that the closest neighbor to the active center was beyond the attractive region of the potential well. However, the situation might soon change due to the mobility of units. We observed that this type of early trapping did not usually occur with a reasonable choice of the reaction distance for bonding. Even if they occurred at early stage of the growth, they were only temporary and did not fall in the category of a permanent trapping. The second kind results from an active center being trapped inside a cage of bonded units. This trapped center usually did not get reactivated again due to steric hindrance. This was more permanent in nature and usually occurred at a latter part of the growth process.

Typical results from our Monte Carlo simulations are shown in Figure 5. As in all the simulations in this thesis 934 monomer units were contained in a simulation box with four propagating active centers. The specific conditions for each simulation are listed at the top of each Figure. Examination of Figure 5 reveals that the initial portion of the conversion against Monte Carlo time (cycles) profile is linear. This arises from the fact that all four active centers propagate each Monte Carlo cycle since there is no trapping. After 155 MC cycles the first active center became permanently trapped and the slope of the profile exhibits a discontinuous change from 0.428% / MC cycle to 0.3219% / MC cycle. Similarly, the second and third active centers became trapped after 174

and 200 MC cycles, respectively, with corresponding shifts in the slope of the conversion against time profile. Our simulation ended when the final active center was trapped.

In contrast to previous simulations (i.e. Figure 4), the results shown in Figure 2 correspond to a relatively high limiting conversion approaching 80%. At the limiting conversion, all active centers become trapped and no further reaction is possible. Previous simulations overestimate trapping since molecular mobility is neglected. In our simulations, the value of the limiting conversion is sensitive to the reaction distance parameter, as illustrated in Figures 5-7. These Figures illustrate that as the reaction distance is decreased, trapping occurs at a lower conversion, and the limiting conversion is decreased. As shown in Figure 7 in our work a limiting conversion about 60% can be achieved with a reaction distance of 1.6σ . With a reaction distance of 1.7σ , it was shown in our work that all active centers did not get trapped until 72% conversion. One thing to notice is that even the distance 1.75σ is well within the attractive region of the LJ potential well. With 1.75σ as the reaction distance, it was possible to reach up to 80% limiting conversion, consistent with experimental observations.

Figure 8 illustrates the effect of tetra-functional groups on cross-linking. Higher percentage of tetra-functional groups in general will lead to more cross-linking within the same number of MC cycles. After 140 MC cycles sample with 30% tetra-functional groups has formed 52 cross-links whereas sample with 20% tetra-functional groups has formed 38 cross-links. Figure 9 (30% tetra-functional groups) illustrates a typical relationship between the total number of cross-links and chain cyclizations. Within first 140 MC cycles there were 52 total cross-links with 20 cyclizations. Probabilities of cross-linking and chain cyclizations can be obtained by this technique.

R

P

L

C

C

C

V

% Conversion

Run number 1:

Parameters for potentials

LJ overlapping distance = 0.9σ
 Oscillator overlapping distance = 0.6σ
 Oscillator maximum distance = 1.75σ
 Oscillator equilibrium distance = 1.12σ

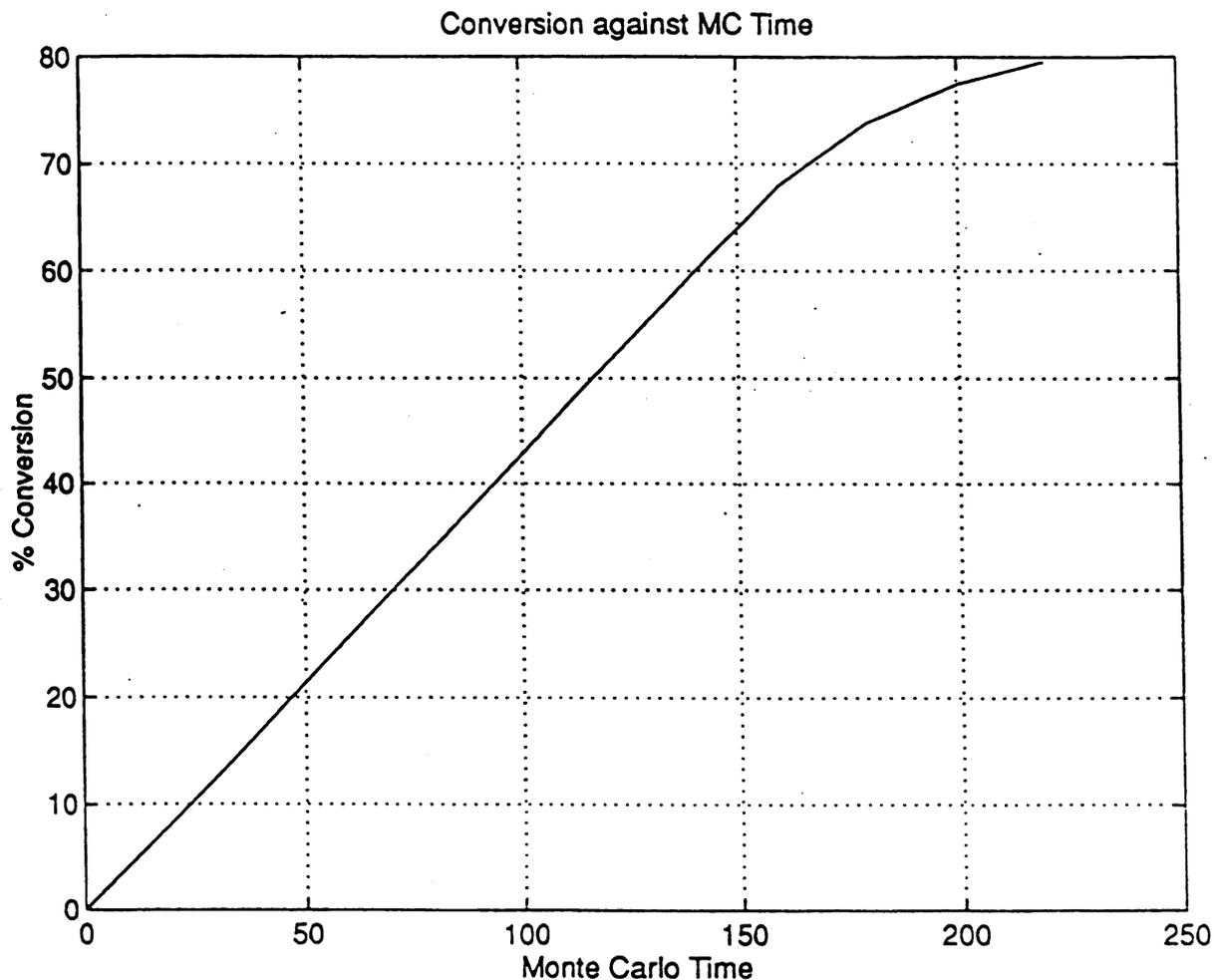


Figure 5. Conversion against MC time for a reaction distance of 1.75σ . $C_i = 0.004$, $C_t = 0.2$, 80% reduction of polymer displacement (with further reduction by a factor of 0.995 between each MC cycle).
 [C_i = initiator concentration, C_t = tetra-functional unit concentration]

Re

Pa

L

0

0

0

a

7

4

% Conversion

Run number 2:

Parameters for potentials

LJ overlapping distance = 0.9σ
 Oscillator overlapping distance = 0.6σ
 Oscillator maximum distance = 1.7σ
 Oscillator equilibrium distance = 1.12σ

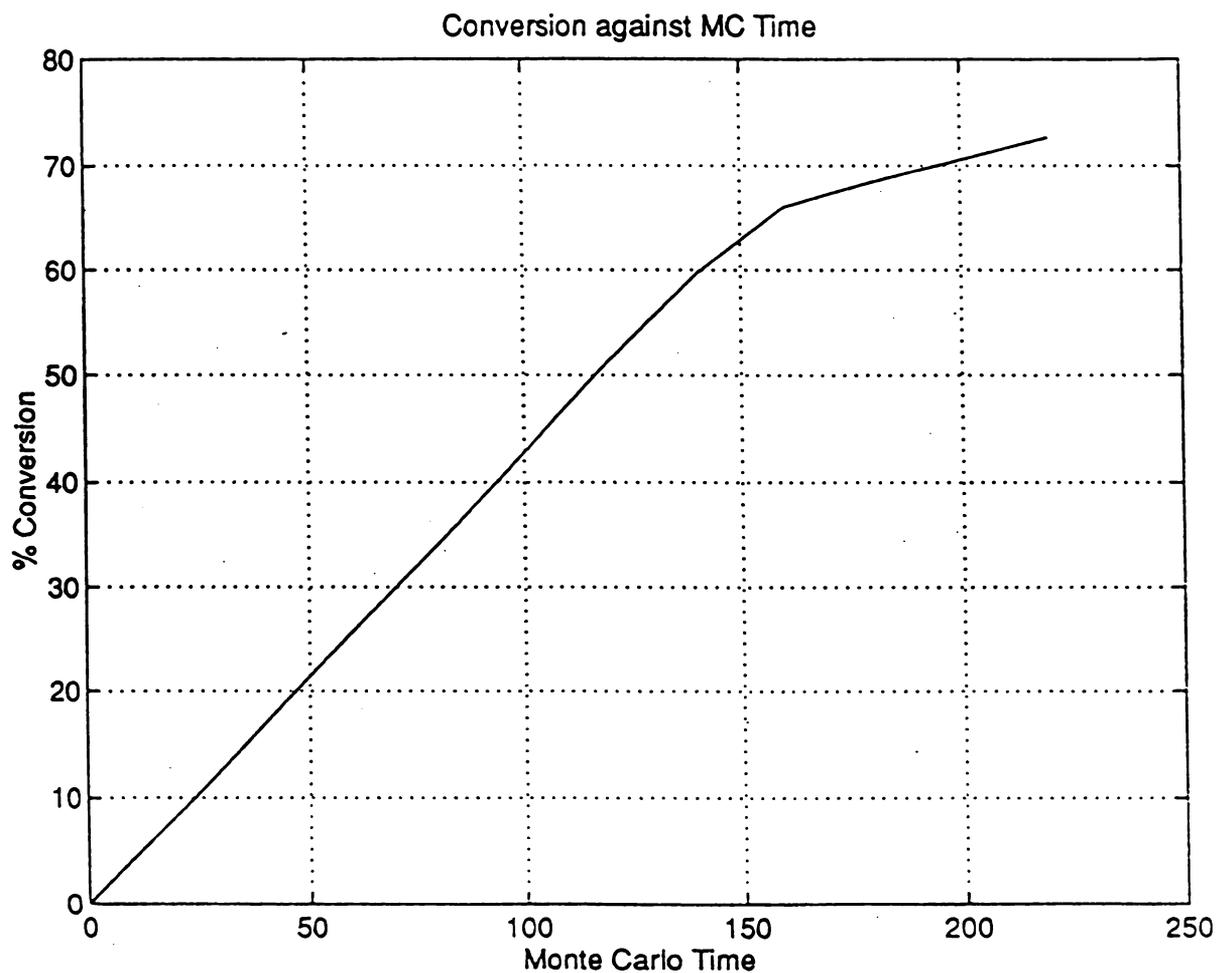


Figure 6. Conversion against MC time for a reaction distance of 1.7σ . $C_i = 0.004$, $C_t = 0.2$, 80% reduction of polymer displacement (with further reduction by a factor of 0.995 between each MC cycle).
 [C_i = initiator concentration, C_t = tetra-functional unit concentration]

Run number 3:

Parameters for potentials

LJ overlapping distance = 0.9σ
 Oscillator overlapping distance = 0.6σ
 Oscillator maximum distance = 1.6σ
 Oscillator equilibrium distance = 1.12σ

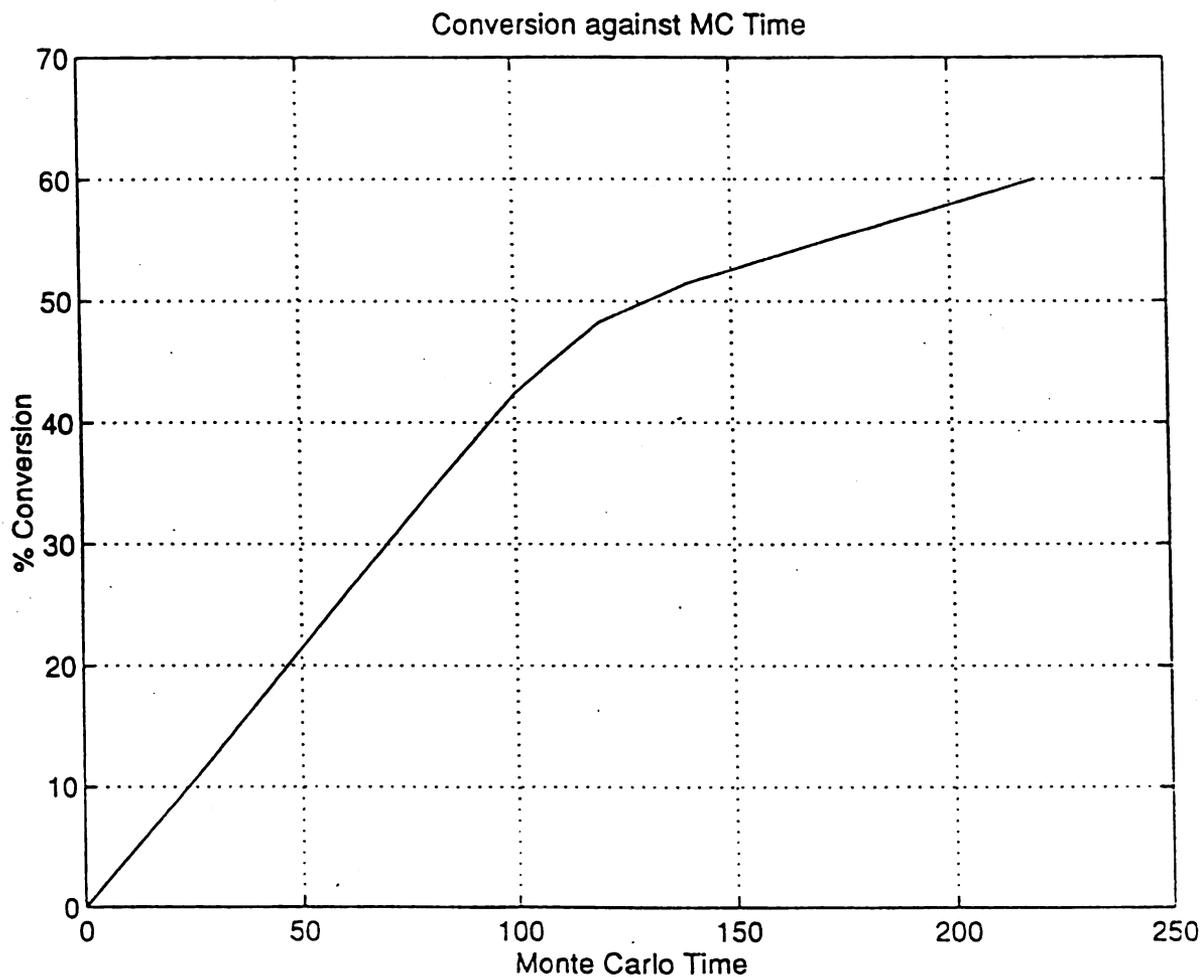


Figure 7. Conversion against MC time for a reaction distance of 1.6σ . $C_i = 0.004$, $C_t = 0.2$, 80% reduction of polymer displacement (with further reduction by a factor of 0.995 between each MC cycle).
 [C_i = initiator concentration, C_t = tetra-functional unit concentration]

At

Pa

L

0

0

0

30

50

Number of Cross-links

4

2

Analysis of cross-linking and cyclizations:

Parameters for potentials

LJ overlapping distance	= 0.9σ
Oscillator overlapping distance	= 0.6σ
Oscillator maximum distance	= 1.75σ
Oscillator equilibrium distance	= 1.12σ

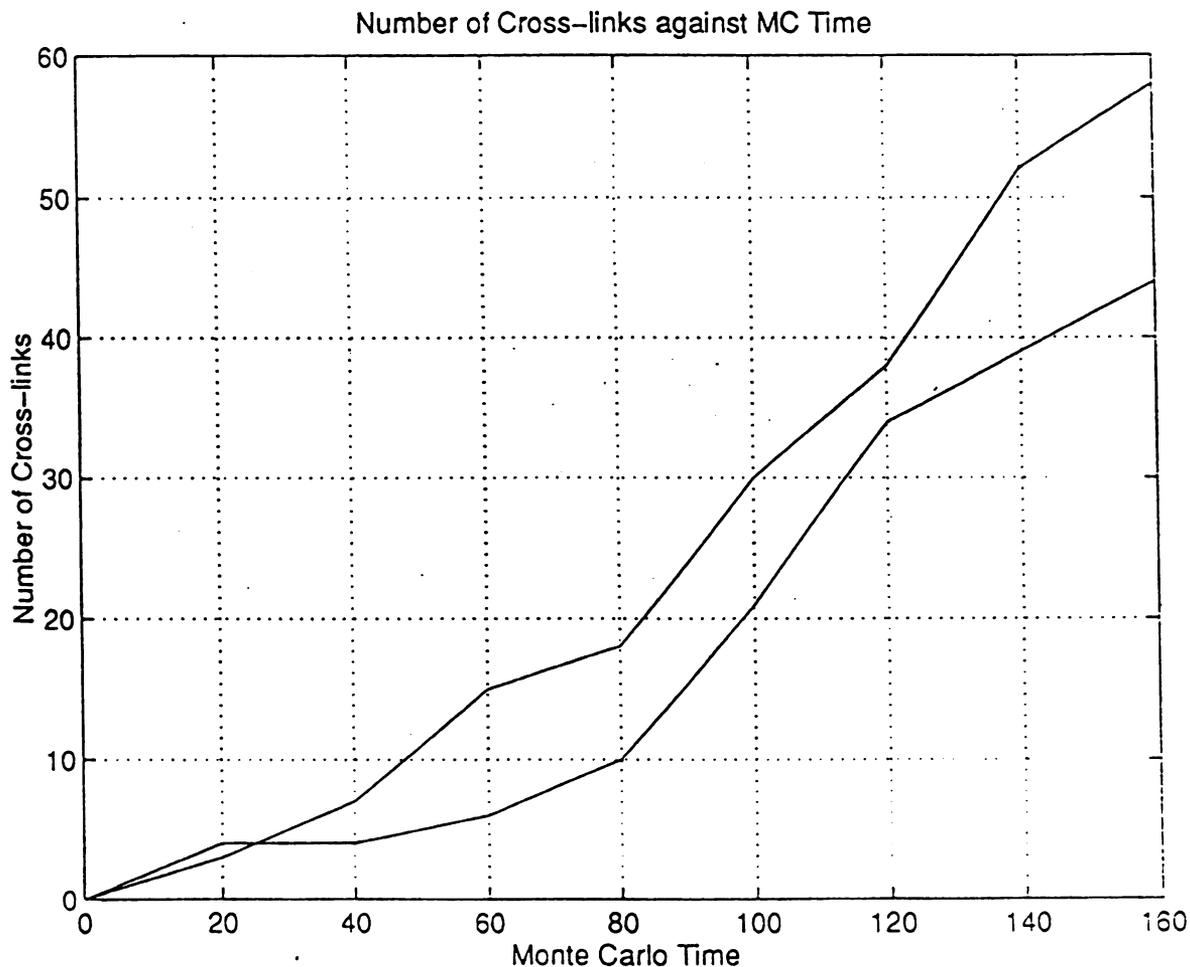


Figure 8. Total number of cross-links against MC time for $C_i = 0.004$, $C_t = 0.2$ (lower curve). $C_i = 0.004$, $C_t = 0.3$ (upper curve). 80% reduction of polymer displacement (with further reduction by a factor of 0.995 between each MC cycle). [C_i = initiator concentration, C_t = tetra-functional unit concentration]

Number of Cross-links ... Number of Cyclizations ...

Parameters for potentials

LJ overlapping distance	= 0.9σ
Oscillator overlapping distance	= 0.6σ
Oscillator maximum distance	= 1.75σ
Oscillator equilibrium distance	= 1.12σ

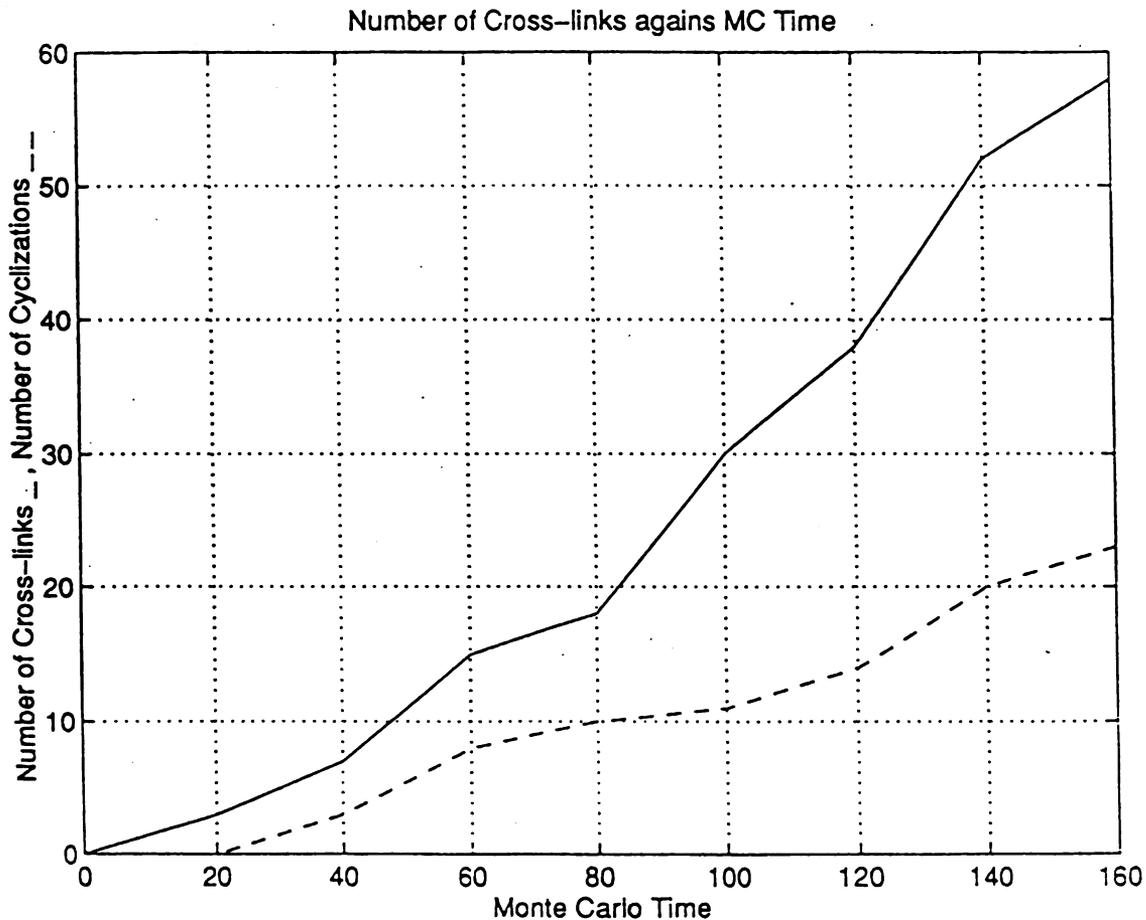


Figure 9. Total number of cross-links against MC time for $C_i = 0.004$, $C_t = 0.3$ (- curve). Number of chain cyclizations against MC time (- - curve). 80% reduction of polymer displacement (with further reduction by a factor of 0.995 between each MC cycle). [C_i = initiator concentration, C_t = tetra-functional unit concentration]

Conversion against MC time plots obtained at the end of the simulation for different variable parameters, such as maximum bond length and different degrees of polymer mobilities are included in previous pages. An illustration of a variation of cross-linking density with the concentration variation of tetra-functional units is also included. The complete computer program for this process is included in appendix B and corresponding flowcharts are included in appendix A.

If desired, one can shift the oscillator potential away from the LJ potential, but such an act could lead to density inhomogeneties within the system. We believe the above difficulty could be avoided by using a scheme which employs a reduced hard-sphere diameter at high density or using a soft (inverse ninth through inverse fourth power) repulsion [28]. In reference 28, a perturbation method was reported which divides the pair potential into a reference potential and a perturbation potential at a break point which may depend on density. A better method to avoid this problem is to work in a constant NPT-ensemble.

5. A NEW MODEL FOR POLYMERIC GLASS STRUCTURES

With some modification, the same computer code that we developed to study gelation can be used to generate long chain alkanes. In this work monomer units were considered as $-CH_2$ groups. The oscillator equilibrium length was brought towards the C-C bond length. The shift of oscillator potential minimum will change the density of the system during the growth. The corresponding change in volume can be handled in NPT-ensemble. In NVT-ensemble, replacement of stiff inverse twelfth power potential with a soft inverse ninth power repulsion [28] might be more useful here. In contrast to Gupta, Weistermann, and Bitsanis [27] model, growth and equilibrium did not compete with each other in our model, rather it was simultaneous throughout the growth (In reference 27 a decision was made on whether to further grow the chain or to attempt an equilibration move). As in their model lack of internal rotations was a disadvantage in ours too. With some modifications in our computer program, number of chains growing within the system can be increased.

There were two major advantages in our method over that of Khare, Paulaitis and Lustig [26] method. In addition to realistic mobility of particles, bond lengths reached to their equilibrium values simultaneously with the growth. A complete relaxation can be achieved by an incorporation of a torsional potential along with the harmonic oscillator potential. With some modifications, our model will provide a simplified, realistic and computationally fast way to study polymeric glass structures and long chain alkane growth.

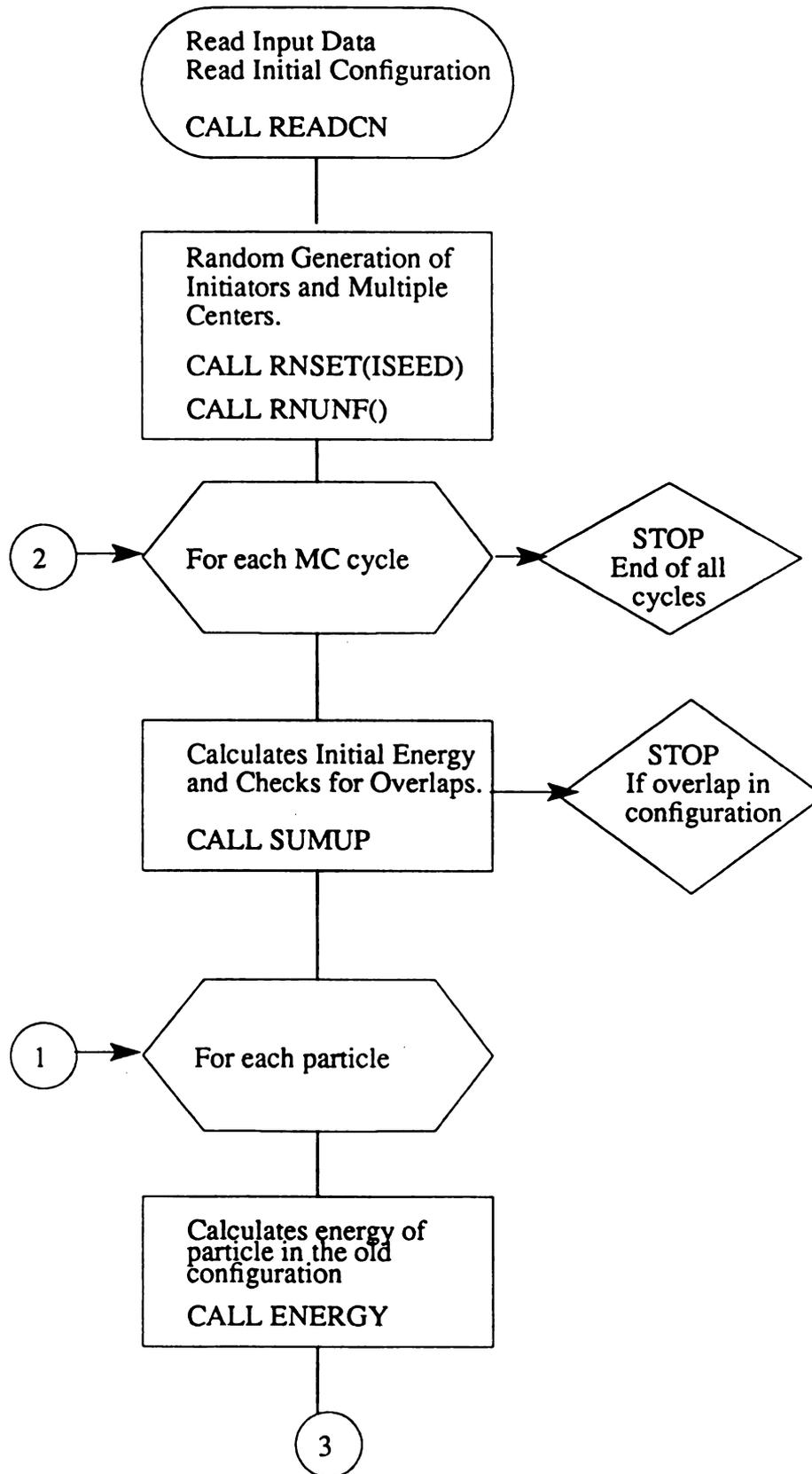
6. SUMMARY AND CONCLUSIONS

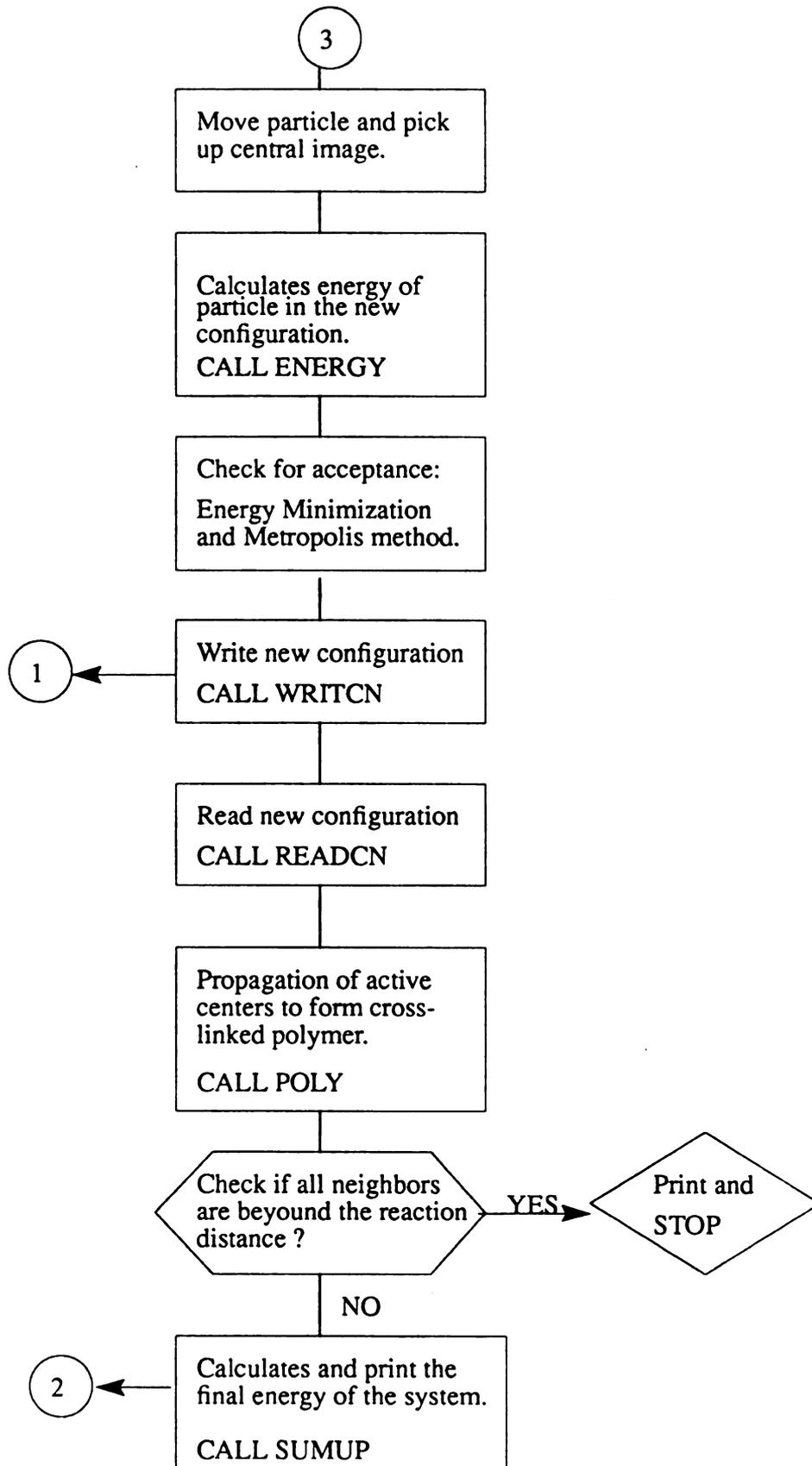
We have proposed a new realistic model for kinetic gelation studies, and its computational feasibility was studied. The importance of incorporating molecular mobility and realistic potentials in the study of cross-linking polymerization process was shown, using Monte Carlo simulations.

In contrast to most earlier simulation methods for gelation, we did not fix the monomer units on a lattice but allowed the monomer and polymer units to move in a physically realistic manner. Another important modification was to take into account, both intra- and intermolecular potentials during the growth and the simultaneous equilibration process. Polymer mobility and the potential coordinates were carefully chosen, in order to avoid possible density inhomogeneities that could build up within the growth and equilibration. The importance of this factor in gelation compared to that of long chain alkane growth was realized. An added advantage of the employment of realistic potentials during the process was to give some insight into internal architecture of the polymer backbone. Growth and the equilibration were simultaneous and did not compete with each other in our model. Chain cyclization was incorporated, and it was shown that the possibility of an early trapping was considerably reduced by taking molecular mobility into account. Chain cyclization was another ignored aspect of earlier gelation models. Cage trapping occurred at the latter part of the growth, and the ratio of percentage of trapped centers to that of active centers remained low until the system reaches a very high conversion. Though reactivation of trapped active centers was possible within our model, in most cases trapping occurred after the formation of gel phase in the presence of reasonable concentration of cross-linking agents.

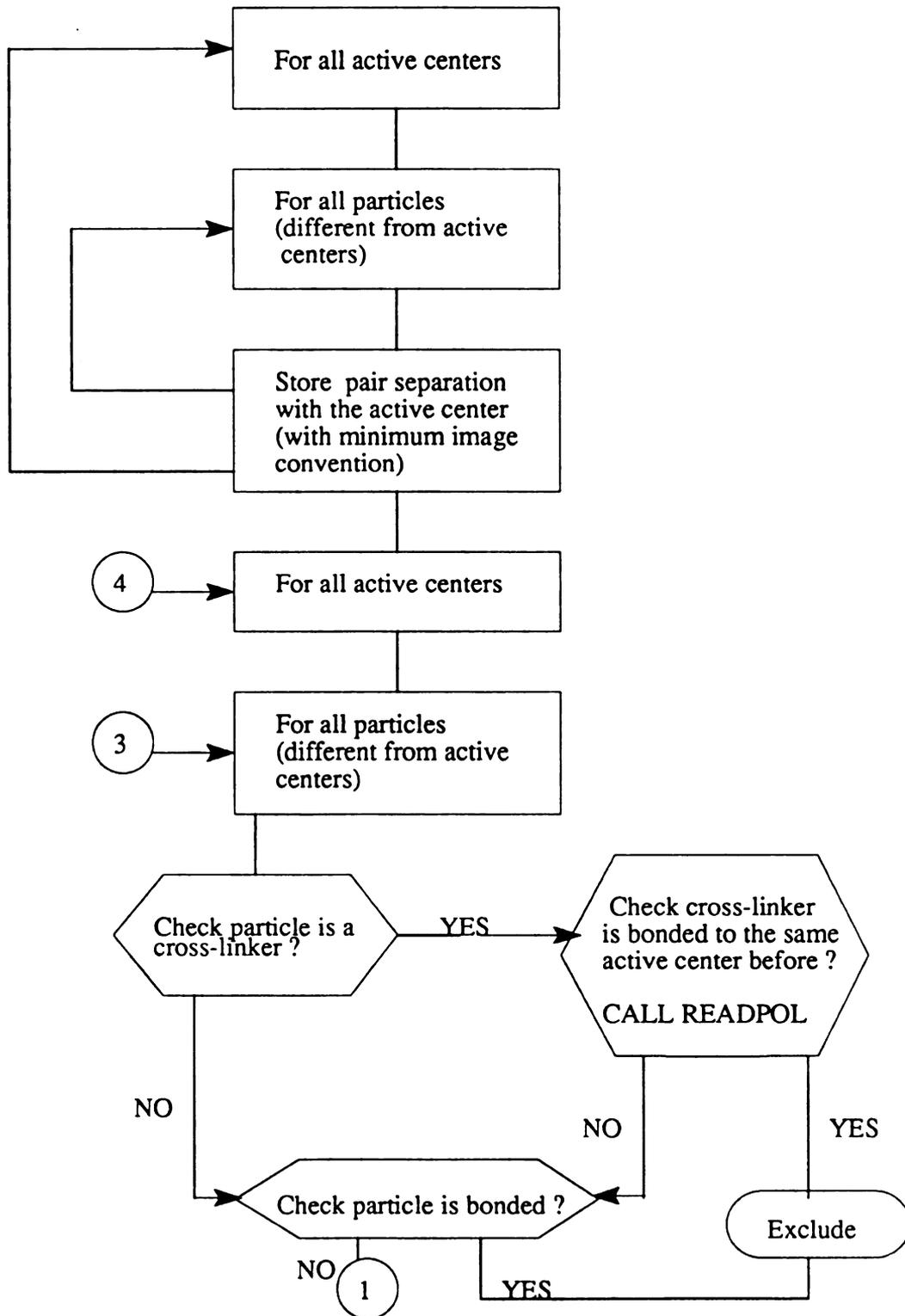
APPENDICES

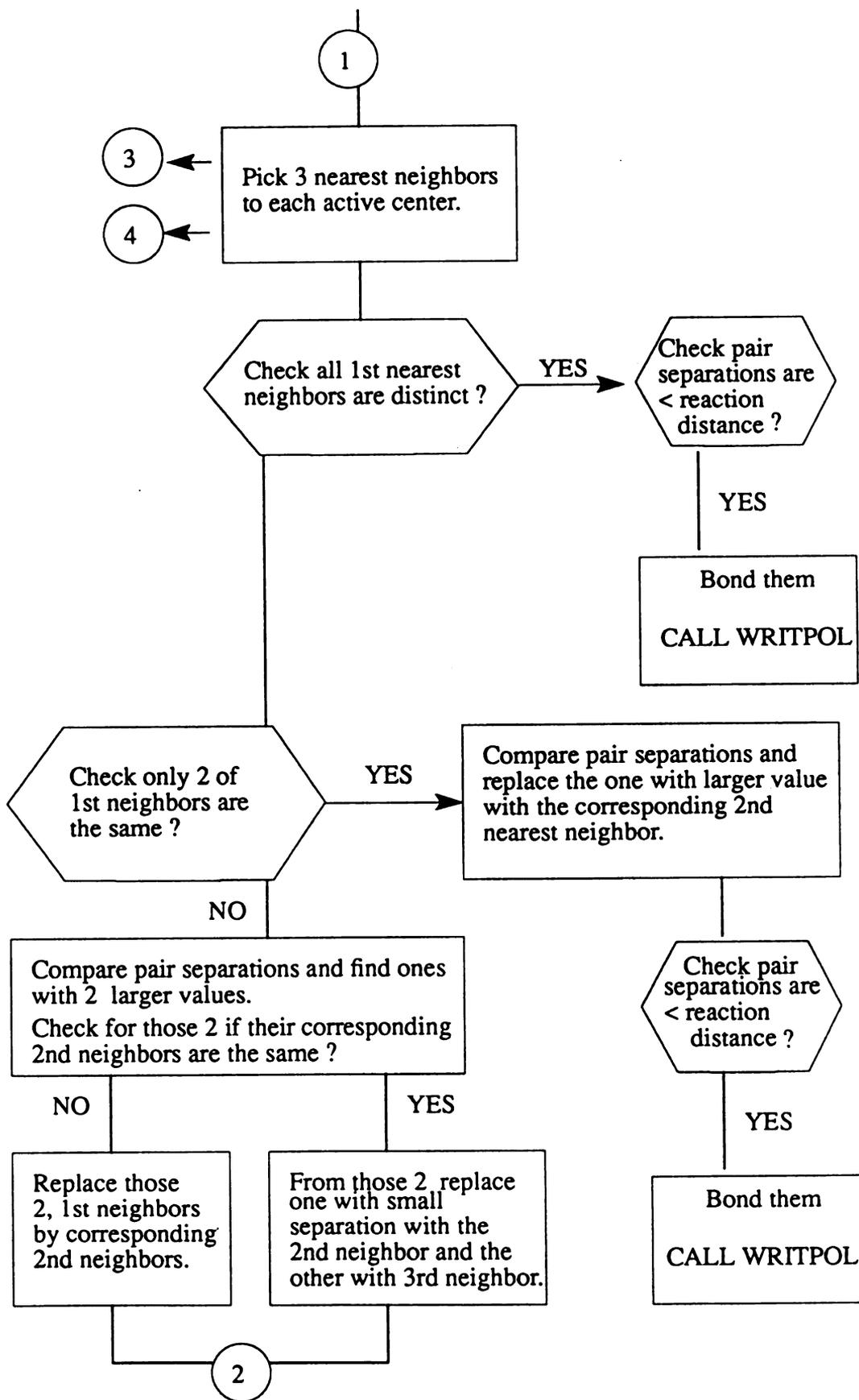
APPENDIX A
FLOW CHART FOR THE MAIN PROGRAM

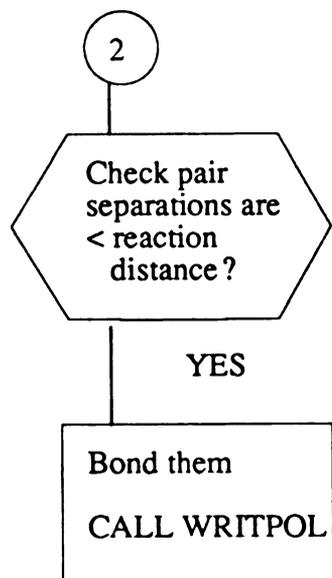




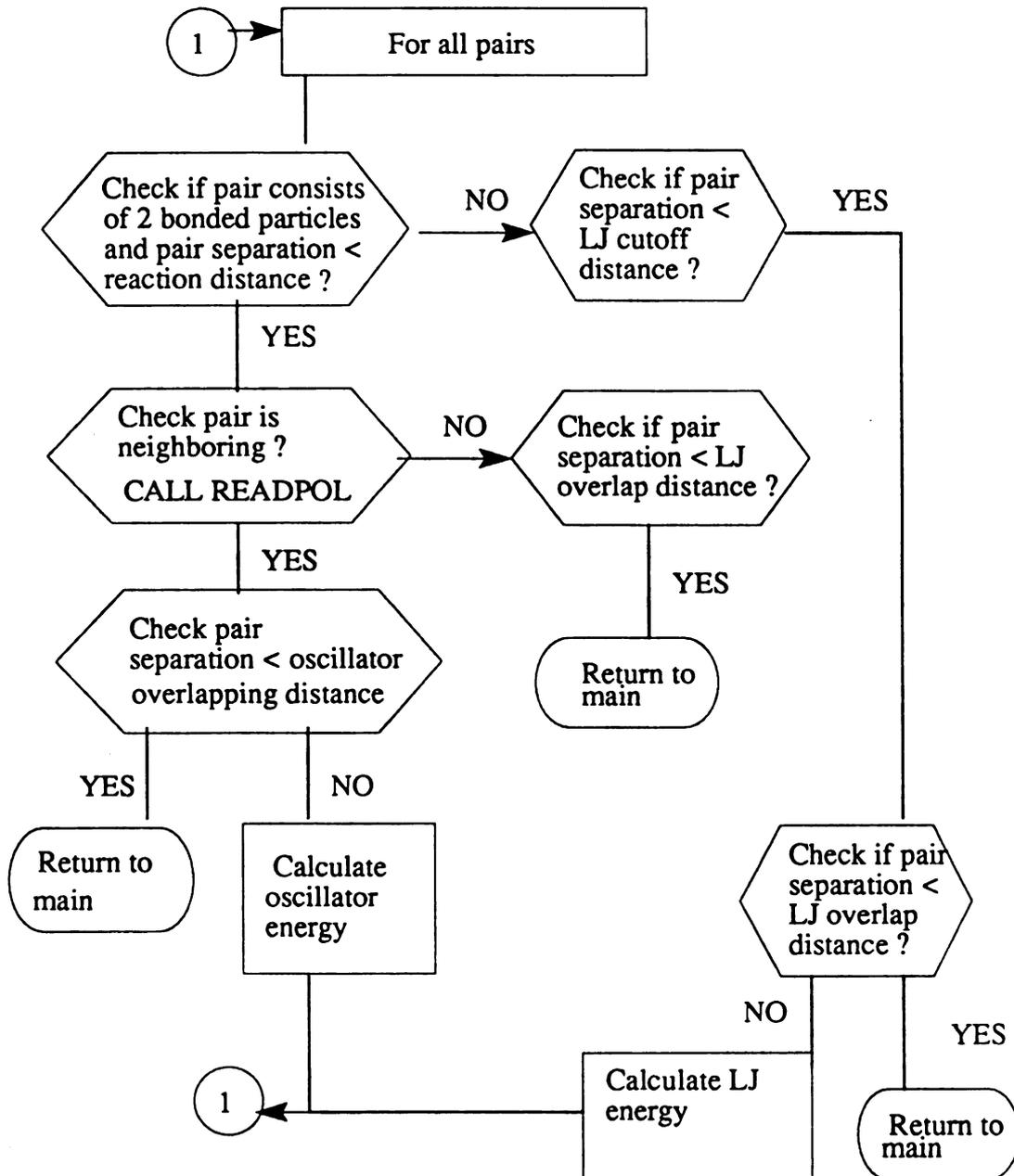
FLOW CHART FOR THE POLYMER SUBROUTINE



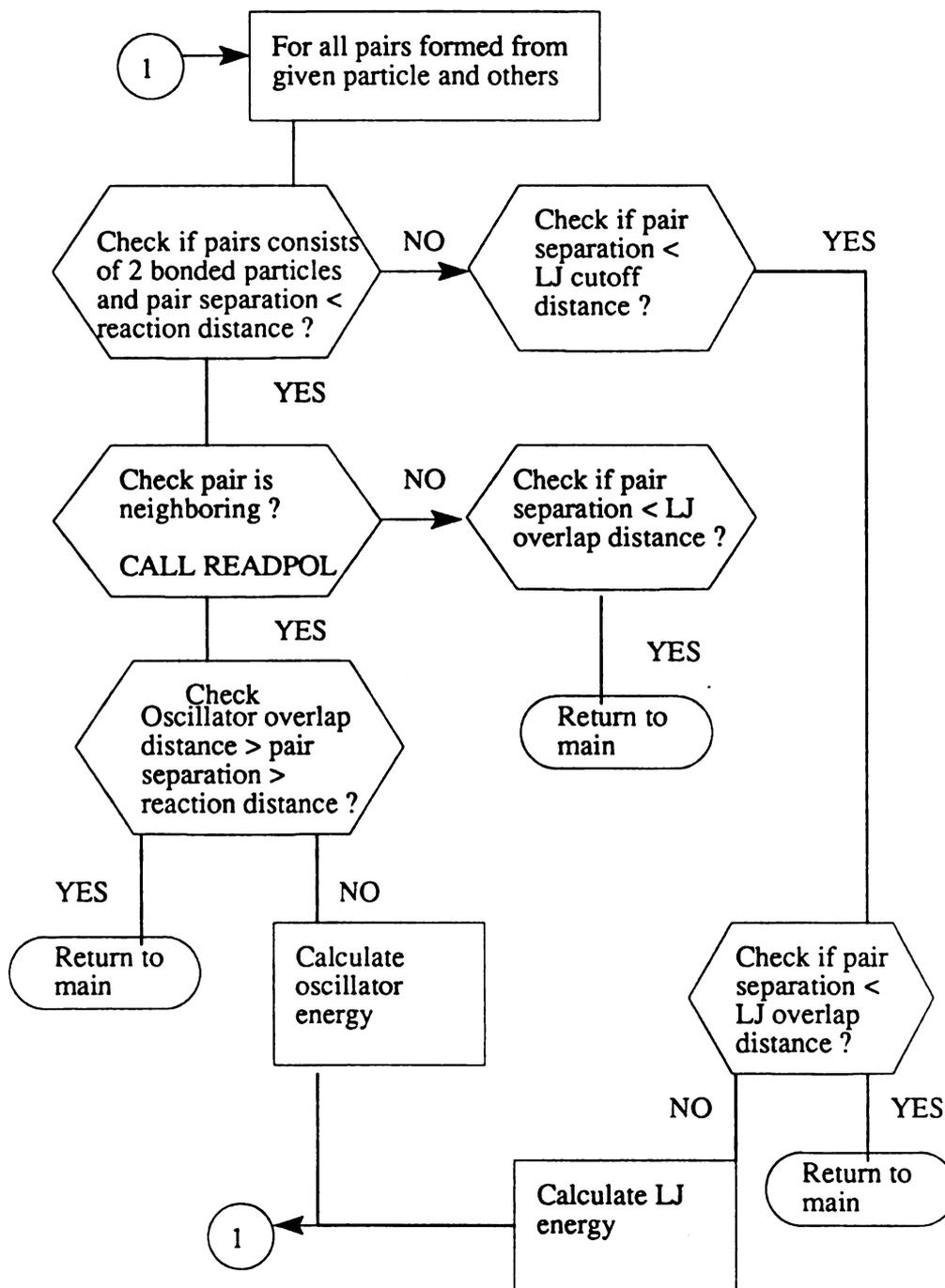




FLOW CHART FOR OVERLAP CHECK SUBROUTINE



FLOW CHART FOR SUBROUTINE TO CALCULATE ENERGY



APPENDIX B

```

c*****
c ** Constant-nvt monte carlo program for hard spheres **
c ** with a routine to form a crosslinked polymer. **
c ** ** **
c ** Program can be use for gel point calculations and **
c ** molecular weight distribution evaluations. **
c ** ** **
c ** Molecular mobility is allowed towards the energy **
c ** minimum throughout the polymerization. Mobility of **
c ** bonded units were reduced with the growth. This **
c ** is limited by the approach to oscillator equilibrium. **
c *****
c
c *****
c ** Program can be used for any given number of monomers **
c ** with an appropriate coordinate file and minor changes. **
c ** ** **
c ** Number of initiaters are resticted to four in polymer **
c ** routine, but can be change to any number with an **
c ** appropriate modifications. **
c *****

program ljgel
common /block1/ rx,ry,rz

c *****
c ** The box is of unit length, -0.5 to +0.5 **
c ** Box length can be changed with an appropriate change in **
c ** periodic boundary conditions. **
c ** ** **
c ** Principal variables: **
c ** ** **
c ** integer n number of monomers **
c ** integer nstep maximum number of cycles **
c ** integer iprint print interval **
c ** integer isave save interval **
c ** integer iratio max displacement update **
c ** interval **
c ** real rx(n),ry(n),rz(n) positions **
c ** real dens reduced density **
c ** real sigma hard sphere (LJ) diameter **
c ** real drmax reduced maximum displacement **
c ** real temp reduced temperature **
c ** real rmin minimum reduced pair **
c ** separation **
c ** real rcut reduced cutoff distance **
c ** real nbond maximum number of bonds in a **
c ** given linear chain **
c ** real bonle maximum bondlength **
c ** real eql equilibrium bondlength **
c ** real foc force constant **

```

```

c ** real      tvbmax          maximum Boltzman factor      **
c ** real      control        energy difference control      **
c ** real      factr          max displacement reduction      **
c ** real      v              the potential energy            **
c ** real      voscil         the oscillator energy          **
c ** real      red            multiplicative factor to      **
c **                                adjust rmin for the oscillator **
c ** logical  overlap        true if 2 units overlap      **
c ** logical  ovrrpol        true if 2 bonded units overlap**
c **                                after the displacement          **
c ** logical  touch          true if others overlap          **
c ** logical  neig           true if 2 bonded units are      **
c **                                neighbors                        **
c ** logical  flip           true if all possible bonds      **
c **                                were formed before the end of **
c **                                all cycles                      **
c **                                **
c ** Units:                  **
c **                                **
c ** The program uses Lennard-Jones units for user input      **
c ** and output but conducts the simulation in a box of      **
c ** unit length.                                               **
c ** For example, for a boxlength L, and Lennard-Jones      **
c ** parameters epsilon and sigma, the units are:            **
c **                                **
c ** Property      LJ Units      Program Units          **
c **                                **
c ** temp          epsilon/K      epsilon/K          **
c ** pres          epsilon/sigma**3  epsilon/L**3      **
c ** v             epsilon        epsilon             **
c ** dens          1/sigma**3      1/L**3           **
c **                                **
c ** Routines referenced:  **
c **                                **
c ** Subroutine sumup      **
c **   Calculates the potential energy for a configuration    **
c ** Subroutine energy     **
c **   Calculates the potential energy of atom i with all    **
c **   the other atoms in the liquid. For bonded neighbors   **
c **   the oscillator energy is evaluated                     **
c ** Subroutine poly       **
c **   Forms the crosslinked polymer as the simulation      **
c **   proceeds                                               **
c ** Subroutine readcn     **
c **   Reads in a configuration                               **
c ** Subroutine writcn    **
c **   Write out a configuration                             **
c ** Subroutine readpol   **
c **   Read the polymer file                                **
c ** Subroutine writpol   **
c **   Write out polymer file                               **
c *****

```

```

integer n,i2(1,4),jcoun,ij(10)
parameter (n=100)
real rx(n),ry(n),rz(n)
real drmax,dens,temp,sigma,rcut,beta,eql,foc
real acm,accept,pi,ratio,rmin,tvbmax,vos,red
real v,vnew,vold,vlast,vn,deltv,deltvb,vs,voscil
real vlrc,vlrc6,vlrc12,control,factr,bonle
real rxibold,ryibold,rzibold,rxinew,ryinew,rzinew
real voso,vosn,delvso
real vosi,vosf
integer step,i,nstep,iprint,isave,iratio
integer iflag(100),nbond
integer iseed
external rnset,rnunf
logical overlap,flip,ovrpol,neig
character cnfile*10,polymer*10
parameter (pi=3.1415927)
c *****read input data*****
open (11,file='inputdata',status='old',form='formatted')
c *****write input data*****
read(11,'(i10)')nstep
read(11,'(i10)')nbond
read(11,'(f10.4)')red
read(11,'(f10.4)')bonle
read(11,'(f10.4)')eql
read(11,'(f10.4)')foc
read(11,'(i10)')iprint
read(11,'(i10)')isave
read(11,'(i10)')iratio
read(11,'(f10.4)')temp
read(11,'(f10.4)')dens
read(11,'(f10.4)')rmin
read(11,'(f10.4)')drmax
read(11,'(f10.4)')tvbmax
read(11,'(f10.4)')rcut
read(11,'(e12.4)')control
read(11,'(a)')cnfile
read(11,'(a)')polymer
write(*,('' end of inputdata ''))
c *****read initial configuration*****
call readcn (cnfile)
c *****convert input data to program units*****
beta =1.0/temp
sigma =(dens/real(n))**(1.0/3.0)
rmin =rmin*sigma
rcut=rcut*sigma
bonle=bonle*sigma
eql=eql*sigma
drmax =drmax*sigma
denslj=dens
dens=dens/(sigma**3)
if(rcut.gt.0.5)stop' cutoff too largr '
acv=0.0
acvsq=0.0

```

```

flv=0.0
acm=0.0
accept=0.0
factr=0.35
do 1 jcoun=1,100
  iflag(jcoun)=1
1 continue
c ****calculate long range corrections*****
  sr3 = (sigma/rcut)**3
  sr9 = sr3**3
  vlrc12 = 8.0*pi*denslj*real(n)*sr9/9.0
  vlrc6 = 8.0*pi*denslj*real(n)*sr3/3.0
  vlrc = vlrc12-vlrc6
c ****write out information*****
  write(*,('sigma/box = ',f10.4))sigma
  write(*,('rmin/box = ',f10.4))rmin
  write(*,('rcut/box = ',f10.4))rcut
  write(*,('bonle/box = ',f10.4))bonle
  write(*,('eql/box = ',f10.4))eql
  write(*,('drmax/box = ',f10.4))drmax
  write(*,('tvbmax/box = ',f10.4))tvbmax
  write(*,('control = ',e12.4))control
  write(*,('lrc for V = ',f10.4))vlrc
c ****select initioators and multiple centers*****
  izeed=123457
  call rnset(izeed)
  i2(1,1)=1+int(n*rnunf())
  i2(1,2)=1+int(n*rnunf())
  i2(1,3)=1+int(n*rnunf())
  i2(1,4)=1+int(n*rnunf())
  do 2 l=1,4
    iflag(i2(1,l))=0
2 continue
  do 3 l=1,10
    ij(l)=1+int(n*rnunf())
    iflag(ij(l))=3
3 continue
  vos=0.0
c ****loops over requied bond formation per linear chain*****
  do 102 k=1,25
c ****calculates initial energy and checks for overlap*****
  call sumup(rcut,rmin,sigma,ovrlap,v,iflag,eql,foc,vosi,
+          polymer,neig,bonle,red)
  if(ovrlap)stop 'ovrlap in initial con '
  vs=(v+vlrc)/real(n)
  write(*,('initial v = ',e20.6)) vs
  write(*,('/' start of markov chain '/'))
  write(*,(' nmove          ratio          v/n          '/))
c ****loops over all cycles and all molecules*****
  do 100 step=1,nstep
    do 99 i=1,n
      rxiold=rx(i)
      ryiold=ry(i)
      rziold=rz(i)

```

```

c **** calculates the energy of i in the old configuration****
  call energy(rxiold,ryiold,rziold,i,rcut,sigma,vold,
+   iflag,eql,foc,voso,ovrpol,rmin,polymer,neig,bonle,red)
c *****move i and pickup the central image*****
  if (iflag(i).eq.0.or.iflag(i).eq.2) then
    drmaxl=drmax*factr
    rxinew=rxiold+(2.0*rnunf()-1.0)*drmaxl
    ryinew=ryiold+(2.0*rnunf()-1.0)*drmaxl
    rzinew=rziold+(2.0*rnunf()-1.0)*drmaxl
  else
    rxinew=rxiold+(2.0*rnunf()-1.0)*drmax
    ryinew=ryiold+(2.0*rnunf()-1.0)*drmax
    rzinew=rziold+(2.0*rnunf()-1.0)*drmax
  end if
  if (rxinew.gt.0.5) then
    rxinew=rxinew-1.0
  else if (rxinew.lt.-0.5) then
    rxinew=rxinew+1.0
  end if
  if (ryinew.gt.0.5) then
    ryinew=ryinew-1.0
  else if (ryinew.lt.-0.5) then
    ryinew=ryinew+1.0
  end if
  if (rzinew.gt.0.5) then
    rzinew=rzinew-1.0
  else if (rzinew.lt.-0.5) then
    rzinew=rzinew+1.0
  end if
c *****calculate the energy of i in the new config*****
  call energy(rxinew,ryinew,rzinew,i,rcut,sigma,vnew,
+   iflag,eql,foc,vosn,ovrpol,rmin,polymer,neig,bonle,red)
c *****check for acceptance*****
  deltv=vnew-vold
  deltvb=beta*deltv
  delvos=vosn-voso
  if(.not.ovrpol.and.delvos.le.0.0)then
    if(.not.touch) then
      if(deltv.le.0.0) then
        v=v+deltv
        rx(i)=rxinew
        ry(i)=ryinew
        rz(i)=rzinew
        accept=accept+1.0
      else if (exp(-deltvb).gt.rnunf()) then
        v=v+deltv
        rx(i)=rxinew
        ry(i)=ryinew
        rz(i)=rzinew
        accept=accept+1.0
      end if
    end if
  end if
end if
99 continue

```

```

    acm=acm+1.0
    vn=(v+v1rc)/real(n)
    acv=acv+vn
    acvsq=acvsq+vn*vn
c *****perform periodic operations*****
    if (mod(step,iratio).eq.0) then
        ratio=accept/real(n*iratio)
        if (ratio.gt.0.5) then
            drmax=drmax*1.05
        else
            drmax=drmax*0.95
        end if
        accept=0.0
        end if
        if (mod(step,iprint).eq.0) then
            write (*,'(i8,e14.4,e20.6)') int(acm),ratio,vn
        end if
        if (mod(step,isave).eq.0) then
            call writcn (cnfile)
        end if
        call readcn (cnfile)
100 continue
        write(*,'(//' end of markov chain '//)')
        call readcn (cnfile)
c **calculates energy of the configuration and check for overlaps**
    call sumup (rcut,rmin,sigma,ovrlap,vlast,iflag,eql,
+             foc,vosi,polymer,neig,bonle,red)
        if (abs(vlast-v).gt.control) then
            write(*,'(' problem with energy ,')')
            write(*,'('vlast = ',e20.6)') vlast
            write(*,'(' v = ',e20.6)') v
            end if
            call writcn (cnfile)
            avv=acv/acm
            write(*,'(//' average '//)')
            write(*,'('<v/n> = ',e20.6)') avv
            write(*,'('polymer oscil potential= ',e20.6)')vosi
            call readcn (cnfile)
            print *,'i2=   k= ',i2,k
            call poly(iflag,i2,k,bonle,flip,factr,polymer,neig)
            if(flip)go to 16
c **calculates final energy and check for overlaps*****
            call sumup(rcut,rmin,sigma,ovrlap,vlast,iflag,eql,
+             foc,vosf,polymer,neig)
            write(*,'('polymer oscil potentia= ',e20.6)')vosf
102 continue
            16 voscil=4*vosf/k
            write(*,'('total oscilat potential= ',e20.6)')vosf
            write(*,'('average oscil potential= ',e20.6)')voscil
500    format (t5,e12.4,t19,e12.4,t48,e12.4)
            stop
            end

```

```

subroutine poly(iflag,i2,k,bonle,flip,factr,polymer,neig)

c *****
c **  Forms the cross-linked polymer, and write out polymer **
c **  file. **
c ** ** **
c **  Principal variables: **
c ** ** **
c **  integer jj(3,4)      up to 3 nearest neighbors for a **
c **                       given active center **
c **  integer ipoly(n,4)   linear polymer chains **
c **  integer iflag(n)     bonded or nonbonded status **
c **  integer i2(1,4)     propagating active centers **
c **  integer near(n-1)   neighboring status **
c **  integer m1(1,3),etc  centers for various combinations**
c **  integer m10,m11,etc  of neighboring status **
c **  real   path(99,4)    distance from active centers to **
c **                       all other points **
c **  real   path1(3,40)   distance from active centers to **
c **                       up to 3 nearest neighbors **
c **  charact polymer     name of polymer file **
c *****

common /block1/rx,ry,rz
common /block2/ipoly
integer n,jj(3,4),l,m1(1,3),m2(1,2),m3
integer m10,m11,m12,m13,m14,m20,m21,m22,m23
parameter (n=100)
real rx(n),ry(n),rz(n)
integer ipoly(100,4),iflag(100),i2(1,4),near(99)
real path(99,4),bonle,rxij,ryij,rzij,path1(3,4)
logical flip,neig
character polymer*(*)
flip=.false.
do 1 j=1,3
m1(1,j)=0
1 continue
m2(1,1)=0
m2(1,2)=0
m10=0
m11=0
m12=0
m13=0
m14=0
m20=0
m21=0
m22=0
m23=0
m31=0
m32=0
m33=0
m34=0
do 2 j=1,99
near(j)=0

```

```

2 continue
  do 3 l=1,4
    path1(1,l)=1.2
3 continue
  if(k.eq.1)then
    do 4 l=1,4
      ipoly(k,l)=i2(1,l)
4 continue
  call writpol(polymer)
  return
  else
    do 6 l=1,4
      do 5 i=1,n
        if(i.ne.i2(1,l))then
          rxij=rx(i)-rx(i2(1,l))
          ryij=ry(i)-ry(i2(1,l))
          rzij=rz(i)-rz(i2(1,l))
          if( rxij.gt.0.5) then
            rxij=rxij-1.0
          else if(rxij.lt.-0.5) then
            rxij=rxij+1.0
          end if
          if( ryij.gt.0.5) then
            ryij=ryij-1.0
          else if(ryij.lt.-0.5) then
            ryij=ryij+1.0
          end if
          if( rzij.gt.0.5) then
            rzij=rzij-1.0
          else if(rzij.lt.-0.5) then
            rzij=rzij+1.0
          end if
          path(i,l)=sqrt((rxij)**2+(ryij)**2+(rzij)**2)
        end if
      end do
5 continue
6 continue
  do 8 l=1,4
    do 7 i=1,99
      if(iflag(i).eq.2) then
        j=i2(1,l)
        call readpol(polymer,i,j,neig)
        if(neig)then
          near(i)=1
        end if
      end if
      if(i.ne.i2(1,l)) then
        if((iflag(i).ne.0).and.(near(i).eq.0).and.
+      (path(i,l).lt.path1(1,l)))then
          path1(1,l)=path(i,l)
          jj(1,l)=i
        end if
      end if
    end do
7 continue
8 continue

```

```

do 9 l=1,4
near(jj(1,l))=1
pathl(2,l)=1.2
9 continue
do 11 l=1,4
do 10 i=1,99
if((iflag(i).ne.0).and.(near(i).eq.0).and.
+ (path(i,l).lt.pathl(2,l)))then
pathl(2,l)=path(i,l)
jj(2,l)=i
end if
10 continue
11 continue
do 12 l=1,4
near(jj(2,l))=1
pathl(3,l)=1.2
12 continue
do 14 l=1,4
do 13 i=1,99
if((iflag(i).ne.0).and.(near(i).eq.0).and.
+ (path(i,l).lt.pathl(3,l)))then
pathl(3,l)=path(i,l)
jj(3,l)=i
end if
13 continue
14 continue
end if
do 18 j=1,3
if(jj(1,j).eq.jj(1,j+1)) then
m1(1,j)=j
end if
18 continue
do 19 j=1,2
if (jj(1,j).eq.jj(1,j+2)) then
m2(1,j)=j
end if
19 continue
if (jj(1,1).eq.jj(1,4)) then
m3=1
end if
if((m1(1,1).eq.0).and.(m1(1,2).eq.0).and.
+ (m1(1,3).eq.0))then
m10=1
end if
if((m1(1,1).ne.0).and.(m1(1,2).eq.0).and.
+ (m1(1,3).eq.0))then
m11=1
else if((m1(1,1).eq.0).and.(m1(1,2).ne.0).and.
+ (m1(1,3).eq.0))then
m12=1
else if((m1(1,1).eq.0).and.(m1(1,2).eq.0).and.
+ (m1(1,3).ne.0))then
m13=1
else if((m1(1,1).ne.0).and.(m1(1,2).eq.0).and.

```

```

+          (m1(1,3).ne.0)) then
  m14=1
  end if
  if ((m2(1,1).eq.0).and.(m2(1,2).eq.0)) then
    m20=1
  end if
  if ((m2(1,1).ne.0).and.(m2(1,2).eq.0)) then
    m21=1
  else if ((m2(1,1).eq.0).and.(m2(1,2).ne.0)) then
    m22=1
  else if ((m2(1,1).ne.0).and.(m2(1,2).ne.0)) then
    m23=1
  end if
  if ((m1(1,1).ne.0).and.(m1(1,2).ne.0).and.(m21.ne.0)) then
    m31=1
  else if ((m1(1,3).ne.0).and.(m21.ne.0).and.(m3.ne.0)) then
    m32=1
  else if ((m1(1,1).ne.0).and.(m22.ne.0).and.(m3.ne.0)) then
    m33=1
  else if ((m1(1,2).ne.0).and.(m22.ne.0).and.(m3.eq.0)) then
    m34=1
  end if
  if ((m10.eq.1).and.(m20.eq.1).and.(m3.eq.0)) then
    do 20 l=1,4
      if (path1(1,l).lt.bonle) then
        ipoly(k,l)=jj(1,l)
        i2(1,l)=jj(1,l)
          if (iflag(jj(1,l)).eq.3) then
            iflag(jj(1,l))=2
          else if (iflag(jj(1,l)).eq.1) then
            iflag(jj(1,l))=0
          else if (iflag(jj(1,l)).eq.2) then
            iflag(jj(1,l))=0
          end if
        end if
      end if
20 continue
      factr=factr*.95
      call writpol(polymer)
      short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
        if (short.gt.bonle) then
          flip=.true.
        end if
      return
    else
      do 21 l=1,4
        if (iflag(jj(1,l)).eq.3) then
          iflag(jj(1,l))=2
        else if (iflag(jj(1,l)).eq.1) then
          iflag(jj(1,l))=0
        else if (iflag(jj(1,l)).eq.2) then
          iflag(jj(1,l))=0
        end if
21 continue
      end if

```

```

if ((m3.eq.0).and.(m20.eq.1).and.(m11.eq.1))then
  if (path1(1,1).le.path1(1,2)) then
    jj(1,2)=jj(2,2)
    path1(1,2)=path1(2,2)
    if (iflag(jj(1,2)).eq.3)then
      iflag(jj(1,2))=2
    else if (iflag(jj(1,2)).eq.1)then
      iflag(jj(1,2))=0
    else if (iflag(jj(1,2)).eq.2)then
      iflag(jj(1,2))=0
    end if
  else
    jj(1,1)=jj(2,1)
    path1(1,1)=path1(2,1)
    if (iflag(jj(1,1)).eq.3)then
      iflag(jj(1,1))=2
    else if (iflag(jj(1,1)).eq.1)then
      iflag(jj(1,1))=0
    else if (iflag(jj(1,1)).eq.2)then
      iflag(jj(1,1))=0
    end if
  end if
do 22 l=1,4
  if(path1(1,l).lt.bonle) then
    ipoly(k,l)=jj(1,l)
    i2(1,l)=jj(1,l)
  end if
22 continue
  factr=factr*.95
  call writpol(polymer)
  short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
  if(short.gt.bonle) then
    flip=.true.
  end if
return
else if((m3.eq.0).and.(m21.eq.1).and.(m10.eq.1)) then
  if(path1(1,1).le.path1(1,3)) then
    jj(1,3)=jj(2,3)
    path1(1,3)=path1(2,3)
    if (iflag(jj(1,3)).eq.3)then
      iflag(jj(1,3))=2
    else if (iflag(jj(1,3)).eq.1)then
      iflag(jj(1,3))=0
    else if (iflag(jj(1,3)).eq.2)then
      iflag(jj(1,3))=0
    end if
  else
    jj(1,1)=jj(2,1)
    path1(1,1)=path1(2,1)
    if (iflag(jj(1,1)).eq.3)then
      iflag(jj(1,1))=2
    else if (iflag(jj(1,1)).eq.1)then
      iflag(jj(1,1))=0
    else if (iflag(jj(1,1)).eq.2)then

```

```

        iflag(jj(1,1))=0
        end if
    end if
do 23 l=1,4
if (path1(1,l).lt.bonle)then
ipoly(k,l)=jj(1,l)
i2(1,l)=jj(1,l)
end if
23 continue
factr=factr*.95
call writpol(polymer)
short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
    if(short.gt.bonle) then
        flip=.true.
    end if
return
else if ((m3.eq.1).and.(m20.eq.1).and.(m10.eq.1))then
    if (path1(1,1).le.path1(1,4)) then
        jj(1,4)=jj(2,4)
        path1(1,4)=path1(2,4)
        if (iflag(jj(1,4)).eq.3)then
            iflag(jj(1,4))=2
        else if (iflag(jj(1,4)).eq.1)then
            iflag(jj(1,4))=0
        else if (iflag(jj(1,4)).eq.2)then
            iflag(jj(1,4))=0
        end if
    else
        jj(1,1)=jj(2,1)
        path1(1,1)=path1(2,1)
        if (iflag(jj(1,1)).eq.3)then
            iflag(jj(1,1))=2
        else if (iflag(jj(1,1)).eq.1)then
            iflag(jj(1,1))=0
        else if (iflag(jj(1,1)).eq.2)then
            iflag(jj(1,1))=0
        end if
    end if
do 24 l=1,4
if (path1(1,l).lt.bonle)then
ipoly(k,l)=jj(1,l)
i2(1,l)=jj(1,l)
end if
24 continue
factr=factr*.95
call writpol(polymer)
short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
    if (short.gt.bonle)then
        flip=.true.
    end if
return
else if ((m3.eq.0).and.(m20.eq.1).and.(m12.eq.1))then
    if (path1(1,2).le.path1(1,3)) then
        jj(1,3)=jj(2,3)

```

```

path1(1,3)=path1(2,3)
  if(iflag(jj(1,3)).eq.3)then
    iflag(jj(1,3))=2
  else if(iflag(jj(1,3)).eq.1)then
    iflag(jj(1,3))=0
  else if(iflag(jj(1,3)).eq.2)then
    iflag(jj(1,3))=0
  end if
else
  jj(1,2)=jj(2,2)
  path1(1,2)=path1(2,2)
  if(iflag(jj(1,2)).eq.3)then
    iflag(jj(1,2))=2
  else if(iflag(jj(1,2)).eq.1)then
    iflag(jj(1,2))=0
  else if(iflag(jj(1,2)).eq.2)then
    iflag(jj(1,2))=0
  end if
end if
do 25 l=1,4
  if(path1(1,l).lt.bonle)then
    ipoly(k,l)=jj(1,l)
    i2(1,l)=jj(1,l)
  end if
25 continue
  factr=factr*.95
  call writpol(polymer)
  short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
  if(short.gt.bonle)then
    flip=.true.
  end if
return
else if((m3.eq.0).and.(m22.eq.1).and.(m10.eq.1))then
  if(path1(1,2).le.path1(1,4)) then
    jj(1,4)=jj(2,4)
    path1(1,4)=path1(2,4)
    if(iflag(jj(1,4)).eq.3)then
      iflag(jj(1,4))=2
    else if(iflag(jj(1,4)).eq.1)then
      iflag(jj(1,4))=0
    else if(iflag(jj(1,4)).eq.2)then
      iflag(jj(1,4))=0
    end if
  else
    jj(1,2)=jj(2,2)
    path1(1,2)=path1(2,2)
    if(iflag(jj(1,2)).eq.3)then
      iflag(jj(1,2))=2
    else if(iflag(jj(1,2)).eq.1)then
      iflag(jj(1,2))=0
    else if(iflag(jj(1,2)).eq.2)then
      iflag(jj(1,2))=0
    end if
  end if
end if

```

```

do 26 l=1,4
  if(path1(1,l).lt.bonle)then
    ipoly(k,l)=jj(1,l)
    i2(1,l)=jj(1,l)
  end if
26 continue
  factr=factr*.95
  call writpol(polymer)
  short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
    if(short.gt.bonle)then
      flip=.true.
    end if
  return
else if((m3.eq.0).and.(m20.eq.1).and.(m13.eq.1))then
  if(path1(1,3).le.path1(1,4))then
    jj(1,4)=jj(2,4)
    path1(1,4)=path1(2,4)
    if(iflag(jj(1,4)).eq.3)then
      iflag(jj(1,4))=2
    else if(iflag(jj(1,4)).eq.1)then
      iflag(jj(1,4))=0
    else if(iflag(jj(1,4)).eq.2)then
      iflag(jj(1,4))=0
    end if
  else
    jj(1,3)=jj(2,3)
    path1(1,3)=path1(2,3)
    if(iflag(jj(1,3)).eq.3)then
      iflag(jj(1,3))=2
    else if(iflag(jj(1,3)).eq.1)then
      iflag(jj(1,3))=0
    else if(iflag(jj(1,3)).eq.2)then
      iflag(jj(1,3))=0
    end if
  end if
do 27 l=1,4
  if(path1(1,l).lt.bonle)then
    ipoly(k,l)=jj(1,l)
    i2(1,l)=jj(1,l)
  end if
27 continue
  factr=factr*.95
  call writpol(polymer)
  short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
    if(short.gt.bonle)then
      flip=.true.
    end if
  return
else if((m3.eq.0).and.(m20.eq.1).and.(m14.eq.1))then
  if(path1(1,1).le.path1(1,2)) then
    jj(1,2)=jj(2,2)
    path1(1,2)=path1(2,2)
    if(iflag(jj(1,2)).eq.3)then
      iflag(jj(1,2))=2

```

```

        else if (iflag(jj(1,2)).eq.1) then
            iflag(jj(1,2))=0
        else if (iflag(jj(1,2)).eq.2) then
            iflag(jj(1,2))=0
        end if
    else
        jj(1,1)=jj(2,1)
        path1(1,1)=path1(2,1)
        if (iflag(jj(1,1)).eq.3) then
            iflag(jj(1,1))=2
        else if (iflag(jj(1,1)).eq.1) then
            iflag(jj(1,1))=0
        else if (iflag(jj(1,1)).eq.2) then
            iflag(jj(1,1))=0
        end if
    end if
    if (path1(1,3).lt.path1(1,4)) then
        jj(1,4)=jj(2,4)
        path1(1,4)=path1(2,4)
        if (iflag(jj(1,4)).eq.3) then
            iflag(jj(1,4))=2
        else if (iflag(jj(1,4)).eq.1) then
            iflag(jj(1,4))=0
        else if (iflag(jj(1,4)).eq.2) then
            iflag(jj(1,4))=0
        end if
    else
        jj(1,3)=jj(2,3)
        path1(1,3)=path1(2,3)
        if (iflag(jj(1,3)).eq.3) then
            iflag(jj(1,3))=2
        else if (iflag(jj(1,3)).eq.1) then
            iflag(jj(1,3))=0
        else if (iflag(jj(1,3)).eq.2) then
            iflag(jj(1,3))=0
        end if
    end if
    do 28 l=1,4
        if (path1(1,l).lt.bonle) then
            ipoly(k,l)=jj(1,l)
            i2(1,l)=jj(1,l)
        end if
28 continue
        factr=factr*.95
        call writpol(polymer)
        short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
        if (short.gt.bonle) then
            flip=.true.
        end if
    return
    else if ((m3.eq.0).and.(m23.eq.1).and.(m10.eq.1)) then
        if (path1(1,1).le.path1(1,3)) then
            jj(1,3)=jj(2,3)
            path1(1,3)=path1(2,3)

```

```

    if (iflag(jj(1,3)).eq.3) then
      iflag(jj(1,3))=2
    else if (iflag(jj(1,3)).eq.1) then
      iflag(jj(1,3))=0
    else if (iflag(jj(1,3)).eq.2) then
      iflag(jj(1,3))=0
    end if
  else
    jj(1,1)=jj(2,1)
    path1(1,1)=path1(2,1)
    if (iflag(jj(1,1)).eq.3) then
      iflag(jj(1,1))=2
    else if (iflag(jj(1,1)).eq.1) then
      iflag(jj(1,1))=0
    else if (iflag(jj(1,1)).eq.2) then
      iflag(jj(1,1))=0
    end if
  end if
  if (path1(1,2).le.path1(1,4)) then
    jj(1,4)=jj(2,4)
    path1(1,4)=path1(2,4)
    if (iflag(jj(1,4)).eq.3) then
      iflag(jj(1,4))=2
    else if (iflag(jj(1,4)).eq.1) then
      iflag(jj(1,4))=0
    else if (iflag(jj(1,4)).eq.2) then
      iflag(jj(1,4))=0
    end if
  else
    jj(1,2)=jj(2,2)
    path1(1,2)=path1(2,2)
    if (iflag(jj(1,2)).eq.3) then
      iflag(jj(1,2))=2
    else if (iflag(jj(1,2)).eq.1) then
      iflag(jj(1,2))=0
    else if (iflag(jj(1,2)).eq.2) then
      iflag(jj(1,2))=0
    end if
  end if
do 29 l=1,4
  if (path1(1,l).lt.bonle) then
    ipoly(k,l)=jj(1,l)
    i2(1,l)=jj(1,l)
  end if
29 continue
  factr=factr*.95
  call writpol(polymer)
  short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
  if (short.gt.bonle) then
    flip=.true.
  end if
return
else if ((m3.eq.1).and.(m20.eq.1).and.(m12.eq.1)) then
  if (path1(1,1).le.path1(1,4)) then

```

```

jj(1,4)=jj(2,4)
path1(1,4)=path1(2,4)
  if (iflag(jj(1,4)).eq.3) then
    iflag(jj(1,4))=2
  else if (iflag(jj(1,4)).eq.1) then
    iflag(jj(1,4))=0
  else if (iflag(jj(1,4)).eq.2) then
    iflag(jj(1,4))=0
  end if
else
jj(1,1)=jj(2,1)
path1(1,1)=path1(2,1)
  if (iflag(jj(1,1)).eq.3) then
    iflag(jj(1,1))=2
  else if (iflag(jj(1,1)).eq.1) then
    iflag(jj(1,1))=0
  else if (iflag(jj(1,1)).eq.2) then
    iflag(jj(1,1))=0
  end if
end if
if (path1(1,2) .le. path1(1,3)) then
jj(1,3)=jj(2,3)
path1(1,3)=path1(2,3)
  if (iflag(jj(1,3)).eq.3) then
    iflag(jj(1,3))=2
  else if (iflag(jj(1,3)).eq.1) then
    iflag(jj(1,3))=0
  else if (iflag(jj(1,3)).eq.2) then
    iflag(jj(1,3))=0
  end if
else
jj(1,2)=jj(2,2)
path1(1,2)=path1(2,2)
  if (iflag(jj(1,2)).eq.3) then
    iflag(jj(1,2))=2
  else if (iflag(jj(1,2)).eq.1) then
    iflag(jj(1,2))=0
  else if (iflag(jj(1,2)).eq.2) then
    iflag(jj(1,2))=0
  end if
end if
do 30 l=1,4
if (path1(1,l) .lt. bonle) then
ipoly(k,l)=jj(1,l)
i2(1,l)=jj(1,l)
end if
30 continue
factr=factr*.95
call writpol(polymer)
short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
  if (short.gt.bonle) then
    flip=.true.
  end if
return

```

```

else if(m31.ne.0)then
  if((path1(1,1).le.path1(1,2)).and.(path1(1,1).
+   le.path1(1,3)).and.(jj(2,2).ne.jj(2,3)))then
    jj(1,2)=jj(2,2)
    path1(1,2)=path1(2,2)
    jj(1,3)=jj(2,3)
    path1(1,3)=path1(2,3)
    if(iflag(jj(1,2)).eq.3)then
      iflag(jj(1,2))=2
    else if(iflag(jj(1,2)).eq.1)then
      iflag(jj(1,2))=0
    else if(iflag(jj(1,2)).eq.2)then
      iflag(jj(1,2))=0
    end if
    if(iflag(jj(1,3)).eq.3)then
      iflag(jj(1,3))=2
    else if(iflag(jj(1,3)).eq.1)then
      iflag(jj(1,3))=0
    else if(iflag(jj(1,3)).eq.2)then
      iflag(jj(1,3))=0
    end if
  end if
  if((path1(1,1).le.path1(1,3)).and.(path1(1,1).le.
+   path1(1,2)).and.(jj(2,2).eq.jj(2,3)))then
    jj(1,3)=jj(2,3)
    path1(1,3)=path1(2,3)
    jj(1,2)=jj(3,2)
    path1(1,2)=path1(3,2)
    if(iflag(jj(1,2)).eq.3)then
      iflag(jj(1,2))=2
    else if(iflag(jj(1,2)).eq.1)then
      iflag(jj(1,2))=0
    else if(iflag(jj(1,2)).eq.2)then
      iflag(jj(1,2))=0
    end if
    if(iflag(jj(1,3)).eq.3)then
      iflag(jj(1,3))=2
    else if(iflag(jj(1,3)).eq.1)then
      iflag(jj(1,3))=0
    else if(iflag(jj(1,3)).eq.2)then
      iflag(jj(1,3))=0
    end if
  end if
  if((path1(1,2).le.path1(1,1)).and.(path1(1,2).le.
+   path1(1,3)).and.(jj(2,1).ne.jj(2,3)))then
    jj(1,1)=jj(2,1)
    path1(1,1)=path1(2,1)
    jj(1,3)=jj(2,3)
    path1(1,3)=path1(2,3)
    if(iflag(jj(1,1)).eq.3)then
      iflag(jj(1,1))=2
    else if(iflag(jj(1,1)).eq.1)then
      iflag(jj(1,1))=0
    else if(iflag(jj(1,1)).eq.2)then

```

```

        iflag(jj(1,1))=0
        end if
        if (iflag(jj(1,3)).eq.3) then
            iflag(jj(1,3))=2
        else if (iflag(jj(1,3)).eq.1) then
            iflag(jj(1,3))=0
        else if (iflag(jj(1,3)).eq.2) then
            iflag(jj(1,3))=0
        end if
    end if
+   if ((path1(1,2).le.path1(1,1)).and.(path1(1,2).le.
        path1(1,3)).and.(jj(2,1).eq.jj(2,3))) then
        jj(1,3)=jj(2,3)
        path1(1,3)=path1(2,3)
        jj(1,1)=jj(3,1)
        path1(1,1)=path1(3,1)
        if (iflag(jj(1,1)).eq.3) then
            iflag(jj(1,1))=2
        else if (iflag(jj(1,1)).eq.1) then
            iflag(jj(1,1))=0
        else if (iflag(jj(1,1)).eq.2) then
            iflag(jj(1,1))=0
        end if
        if (iflag(jj(1,3)).eq.3) then
            iflag(jj(1,3))=2
        else if (iflag(jj(1,3)).eq.1) then
            iflag(jj(1,3))=0
        else if (iflag(jj(1,3)).eq.2) then
            iflag(jj(1,3))=0
        end if
    end if
+   if ((path1(1,3).le.path1(1,1)).and.(path1(1,3).le.
        path1(1,2)).and.(jj(2,1).ne.jj(2,2))) then
        jj(1,1)=jj(2,1)
        path1(1,1)=path1(2,1)
        jj(1,2)=jj(2,2)
        path1(1,2)=path1(2,2)
        if (iflag(jj(1,1)).eq.3) then
            iflag(jj(1,1))=2
        else if (iflag(jj(1,1)).eq.1) then
            iflag(jj(1,1))=0
        else if (iflag(jj(1,1)).eq.2) then
            iflag(jj(1,1))=0
        end if
        if (iflag(jj(1,2)).eq.3) then
            iflag(jj(1,2))=2
        else if (iflag(jj(1,2)).eq.1) then
            iflag(jj(1,2))=0
        else if (iflag(jj(1,2)).eq.2) then
            iflag(jj(1,2))=0
        end if
    end if
+   if ((path1(1,3).le.path1(1,2)).and.(path1(1,3).le.
        path1(1,2)).and.(jj(2,1).eq.jj(2,2))) then

```

```

jj(1,2)=jj(2,2)
path1(1,2)=path1(2,2)
jj(1,1)=jj(3,1)
path1(1,1)=path1(3,1)
  if (iflag(jj(1,1)).eq.3) then
    iflag(jj(1,1))=2
  else if (iflag(jj(1,1)).eq.1) then
    iflag(jj(1,1))=0
  else if (iflag(jj(1,1)).eq.2) then
    iflag(jj(1,1))=0
  end if
  if (iflag(jj(1,2)).eq.3) then
    iflag(jj(1,2))=2
  else if (iflag(jj(1,2)).eq.1) then
    iflag(jj(1,2))=0
  else if (iflag(jj(1,2)).eq.2) then
    iflag(jj(1,2))=0
  end if
  end if
do 31 l=1,4
  if (path1(1,l).lt.bonle) then
    ipoly(k,l)=jj(1,l)
    i2(1,l)=jj(1,l)
  end if
31 continue
  factr=factr*.95
  call writpol(polymer)
  short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
  if (short.gt.bonle) then
    flip=.true.
  end if
  return
else if (m32.ne.0) then
  if ((path1(1,1).le.path1(1,3)).and.(path1(1,1).
+   le.path1(1,4)).and.(jj(2,3).ne.path1(2,4))) then
    jj(1,3)=jj(2,3)
    path1(1,3)=path1(2,3)
    jj(1,4)=jj(2,4)
    path1(1,4)=path1(2,4)
    if (iflag(jj(1,3)).eq.3) then
      iflag(jj(1,3))=2
    else if (iflag(jj(1,3)).eq.1) then
      iflag(jj(1,3))=0
    else if (iflag(jj(1,3)).eq.2) then
      iflag(jj(1,3))=0
    end if
    if (iflag(jj(1,4)).eq.3) then
      iflag(jj(1,4))=2
    else if (iflag(jj(1,4)).eq.1) then
      iflag(jj(1,4))=0
    else if (iflag(jj(1,4)).eq.2) then
      iflag(jj(1,4))=0
    end if
  end if
end if

```

```

if((path1(1,1).le.path1(1,4)).and.(path1(1,1).le.
+   path1(1,3)).and.(jj(2,3).eq.jj(2,4)))then
jj(1,4)=jj(2,4)
path1(1,4)=path1(2,4)
jj(1,3)=jj(3,3)
path1(1,3)=path1(3,3)
  if(iflag(jj(1,4)).eq.3)then
    iflag(jj(1,4))=2
  else if(iflag(jj(1,4)).eq.1)then
    iflag(jj(1,4))=0
  else if(iflag(jj(1,4)).eq.2)then
    iflag(jj(1,4))=0
  end if
  if(iflag(jj(1,3)).eq.3)then
    iflag(jj(1,3))=2
  else if(iflag(jj(1,3)).eq.1)then
    iflag(jj(1,3))=0
  else if(iflag(jj(1,3)).eq.2)then
    iflag(jj(1,3))=0
  end if
end if
if((path1(1,3).le.path1(1,1)).and.(path1(1,3).le.
+   path1(1,4)).and.(jj(2,1).ne.jj(2,4)))then
jj(1,1)=jj(2,1)
path1(1,1)=path1(2,1)
jj(1,4)=jj(2,4)
path1(1,4)=path1(2,4)
  if(iflag(jj(1,1)).eq.3)then
    iflag(jj(1,1))=2
  else if(iflag(jj(1,1)).eq.1)then
    iflag(jj(1,1))=0
  else if(iflag(jj(1,1)).eq.2)then
    iflag(jj(1,1))=0
  end if
  if(iflag(jj(1,4)).eq.3)then
    iflag(jj(1,4))=2
  else if(iflag(jj(1,4)).eq.1)then
    iflag(jj(1,4))=0
  else if(iflag(jj(1,4)).eq.2)then
    iflag(jj(1,4))=0
  end if
end if
if((path1(1,3).le.path1(1,4)).and.(path1(1,3).le.
+   path1(1,1)).and.(jj(2,1).eq.jj(2,4)))then
jj(1,4)=jj(2,4)
path1(1,4)=path1(2,4)
jj(1,1)=jj(3,1)
path1(1,1)=path1(3,1)
  if(iflag(jj(1,1)).eq.3)then
    iflag(jj(1,1))=2
  else if(iflag(jj(1,1)).eq.1)then
    iflag(jj(1,1))=0
  else if(iflag(jj(1,1)).eq.2)then
    iflag(jj(1,1))=0
  end if

```

```

end if
if (iflag(jj(1,4)).eq.3) then
  iflag(jj(1,4))=2
else if (iflag(jj(1,4)).eq.1) then
  iflag(jj(1,4))=0
else if (iflag(jj(1,4)).eq.2) then
  iflag(jj(1,4))=0
end if
end if
+ if ((path1(1,4).le.path1(1,1)).and.(path1(1,4).le.
  path1(1,3)).and.(jj(2,1).ne.jj(2,3))) then
  jj(1,1)=jj(2,1)
  path1(1,1)=path1(2,1)
  jj(1,3)=jj(2,3)
  path1(1,3)=path1(2,3)
  if (iflag(jj(1,1)).eq.3) then
    iflag(jj(1,1))=2
  else if (iflag(jj(1,1)).eq.1) then
    iflag(jj(1,1))=0
  else if (iflag(jj(1,1)).eq.2) then
    iflag(jj(1,1))=0
  end if
  if (iflag(jj(1,3)).eq.3) then
    iflag(jj(1,3))=2
  else if (iflag(jj(1,3)).eq.1) then
    iflag(jj(1,3))=0
  else if (iflag(jj(1,3)).eq.2) then
    iflag(jj(1,3))=0
  end if
end if
+ if ((path1(1,4).le.path1(1,3)).and.(path1(1,4).le.
  path1(1,1)).and.(jj(2,1).eq.jj(2,3))) then
  jj(1,3)=jj(2,3)
  path1(1,3)=path1(2,3)
  jj(1,1)=jj(3,1)
  path1(1,1)=path1(3,1)
  if (iflag(jj(1,1)).eq.3) then
    iflag(jj(1,1))=2
  else if (iflag(jj(1,1)).eq.1) then
    iflag(jj(1,1))=0
  else if (iflag(jj(1,1)).eq.2) then
    iflag(jj(1,1))=0
  end if
  if (iflag(jj(1,3)).eq.3) then
    iflag(jj(1,3))=2
  else if (iflag(jj(1,3)).eq.1) then
    iflag(jj(1,3))=0
  else if (iflag(jj(1,3)).eq.2) then
    iflag(jj(1,3))=0
  end if
end if
do 32 l=1,4
if (path1(1,l).lt.bonle) then
ipoly(k,l)=jj(1,l)

```

```

i2(1,1)=jj(1,1)
end if
32 continue
factr=factr*.95
call writpol(polymer)
short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
  if(short.gt.bonle) then
    flip=.true.
  end if
return
else if(m33.ne.0) then
  if((path1(1,1).le.path1(1,2)).and.(path1(1,1).
+   le.path1(1,4)).and.(jj(2,2).ne.jj(2,4)))then
    jj(1,2)=jj(2,2)
    path1(1,2)=path1(2,2)
    jj(1,4)=jj(2,4)
    path1(1,4)=path1(2,4)
    if(iflag(jj(1,2)).eq.3)then
      iflag(jj(1,2))=2
    else if(iflag(jj(1,2)).eq.1)then
      iflag(jj(1,2))=0
    else if(iflag(jj(1,2)).eq.2)then
      iflag(jj(1,2))=0
    end if
    if(iflag(jj(1,4)).eq.3)then
      iflag(jj(1,4))=2
    else if(iflag(jj(1,4)).eq.1)then
      iflag(jj(1,4))=0
    else if(iflag(jj(1,4)).eq.2)then
      iflag(jj(1,4))=0
    end if
  end if
  if((path1(1,1).le.path1(1,4)).and.(path1(1,1).le.
+   path1(1,2)).and.(jj(2,4).eq.jj(2,2)))then
    jj(1,4)=jj(2,4)
    path1(1,4)=path1(2,4)
    jj(1,2)=jj(3,2)
    path1(1,2)=path1(3,2)
    if(iflag(jj(1,2)).eq.3)then
      iflag(jj(1,2))=2
    else if(iflag(jj(1,2)).eq.1)then
      iflag(jj(1,2))=0
    else if(iflag(jj(1,2)).eq.2)then
      iflag(jj(1,2))=0
    end if
    if(iflag(jj(1,4)).eq.3)then
      iflag(jj(1,4))=2
    else if(iflag(jj(1,4)).eq.1)then
      iflag(jj(1,4))=0
    else if(iflag(jj(1,4)).eq.2)then
      iflag(jj(1,4))=0
    end if
  end if
  if((path1(1,2).le.path1(1,1)).and.(path1(1,2).le.

```

```

+       path1(1,4) .and. (jj(2,1) .ne. jj(2,4)) then
      jj(1,1)=jj(2,1)
      path1(1,1)=path1(2,1)
      jj(1,4)=jj(2,4)
      path1(1,4)=path1(2,4)
      if (iflag(jj(1,1)) .eq. 3) then
        iflag(jj(1,1))=2
      else if (iflag(jj(1,1)) .eq. 1) then
        iflag(jj(1,1))=0
      else if (iflag(jj(1,1)) .eq. 2) then
        iflag(jj(1,1))=0
      end if
      if (iflag(jj(1,4)) .eq. 3) then
        iflag(jj(1,4))=2
      else if (iflag(jj(1,4)) .eq. 1) then
        iflag(jj(1,4))=0
      else if (iflag(jj(1,4)) .eq. 2) then
        iflag(jj(1,4))=0
      end if
    end if
    if ((path1(1,2) .le. path1(1,4)) .and. (path1(1,2) .le.
+     path1(1,1)) .and. (jj(2,1) .eq. jj(2,4))) then
      jj(1,4)=jj(2,4)
      path1(1,4)=path1(2,4)
      jj(1,1)=jj(3,1)
      path1(1,1)=path1(3,1)
      if (iflag(jj(1,1)) .eq. 3) then
        iflag(jj(1,1))=2
      else if (iflag(jj(1,1)) .eq. 1) then
        iflag(jj(1,1))=0
      else if (iflag(jj(1,1)) .eq. 2) then
        iflag(jj(1,1))=0
      end if
      if (iflag(jj(1,4)) .eq. 3) then
        iflag(jj(1,4))=2
      else if (iflag(jj(1,4)) .eq. 1) then
        iflag(jj(1,4))=0
      else if (iflag(jj(1,4)) .eq. 2) then
        iflag(jj(1,4))=0
      end if
    end if
    if ((path1(1,4) .le. path1(1,1)) .and. (path1(1,4) .le.
+     path1(1,2)) .and. (jj(2,1) .ne. jj(2,2))) then
      jj(1,1)=jj(2,1)
      path1(1,1)=path1(2,1)
      jj(1,2)=jj(2,2)
      path1(1,2)=path1(2,2)
      if (iflag(jj(1,1)) .eq. 3) then
        iflag(jj(1,1))=2
      else if (iflag(jj(1,1)) .eq. 1) then
        iflag(jj(1,1))=0
      else if (iflag(jj(1,1)) .eq. 2) then
        iflag(jj(1,1))=0
      end if

```

```

        if (iflag(jj(1,2)).eq.3) then
            iflag(jj(1,2))=2
        else if (iflag(jj(1,2)).eq.1) then
            iflag(jj(1,2))=0
        else if (iflag(jj(1,2)).eq.2) then
            iflag(jj(1,2))=0
        end if
    end if
    if ((path1(1,4).le.path1(1,2)).and.(path1(1,4).le.
+       path1(1,1)).and.(jj(2,1).eq.jj(2,2))) then
        jj(1,2)=jj(2,2)
        path1(1,2)=path1(2,2)
        jj(1,1)=jj(3,1)
        path1(1,1)=path1(3,1)
        if (iflag(jj(1,1)).eq.3) then
            iflag(jj(1,1))=2
        else if (iflag(jj(1,1)).eq.1) then
            iflag(jj(1,1))=0
        else if (iflag(jj(1,1)).eq.2) then
            iflag(jj(1,1))=0
        end if
        if (iflag(jj(1,2)).eq.3) then
            iflag(jj(1,2))=2
        else if (iflag(jj(1,2)).eq.1) then
            iflag(jj(1,2))=0
        else if (iflag(jj(1,2)).eq.2) then
            iflag(jj(1,2))=0
        end if
    end if
    do 33 l=1,4
    if (path1(1,l).lt.bonle) then
        ipoly(k,l)=jj(1,l)
        i2(1,l)=jj(1,l)
    end if
33 continue
    factr=factr*.95
    call writpol(polymer)
    short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
        if (short.gt.bonle) then
            flip=.true.
        end if
    return
    else if (m34.ne.0) then
        if ((path1(1,2).le.path1(1,3)).and.(path1(1,2).
+         le.path1(1,4)).and.(jj(2,3).eq.jj(2,4))) then
            jj(1,3)=jj(2,3)
            path1(1,3)=path1(2,3)
            jj(1,4)=jj(2,4)
            path1(1,4)=path1(2,4)
            if (iflag(jj(1,3)).eq.3) then
                iflag(jj(1,3))=2
            else if (iflag(jj(1,3)).eq.1) then
                iflag(jj(1,3))=0
            else if (iflag(jj(1,3)).eq.2) then

```

```

    iflag(jj(1,3))=0
    end if
    if(iflag(jj(1,4)).eq.3)then
    iflag(jj(1,4))=2
    else if(iflag(jj(1,4)).eq.1)then
    iflag(jj(1,4))=0
    else if(iflag(jj(1,4)).eq.2)then
    iflag(jj(1,4))=0
    end if
end if
+   if((path1(1,2).le.path1(1,4)).and.(path1(1,2).le.
    path1(1,3)).and.(jj(2,4).eq.jj(2,3)))then
    jj(1,4)=jj(2,4)
    path1(1,4)=path1(2,4)
    jj(1,3)=jj(3,3)
    path1(1,3)=path1(3,3)
    if(iflag(jj(1,3)).eq.3)then
    iflag(jj(1,3))=2
    else if(iflag(jj(1,3)).eq.1)then
    iflag(jj(1,3))=0
    else if(iflag(jj(1,3)).eq.2)then
    iflag(jj(1,3))=0
    end if
    if(iflag(jj(1,4)).eq.3)then
    iflag(jj(1,4))=2
    else if(iflag(jj(1,4)).eq.1)then
    iflag(jj(1,4))=0
    else if(iflag(jj(1,4)).eq.2)then
    iflag(jj(1,4))=0
    end if
end if
+   if((path1(1,3).le.path1(1,2)).and.(path1(1,3).le.
    path1(1,4)).and.(jj(2,2).ne.jj(2,4)))then
    jj(1,2)=jj(2,2)
    path1(1,2)=path1(2,2)
    jj(1,4)=jj(2,4)
    path1(1,4)=path1(2,4)
    if(iflag(jj(1,2)).eq.3)then
    iflag(jj(1,2))=2
    else if(iflag(jj(1,2)).eq.1)then
    iflag(jj(1,2))=0
    else if(iflag(jj(1,2)).eq.2)then
    iflag(jj(1,2))=0
    end if
    if(iflag(jj(1,4)).eq.3)then
    iflag(jj(1,4))=2
    else if(iflag(jj(1,4)).eq.1)then
    iflag(jj(1,4))=0
    else if(iflag(jj(1,4)).eq.2)then
    iflag(jj(1,4))=0
    end if
end if
+   if((path1(1,3).le.path1(1,4)).and.(path1(1,3).le.
    path1(1,2)).and.(jj(2,4).eq.jj(2,2)))then

```

```

jj(1,4)=jj(2,4)
path1(1,4)=path1(2,4)
jj(1,2)=jj(3,2)
path1(1,2)=path1(3,2)
  if(iflag(jj(1,2)).eq.3)then
    iflag(jj(1,2))=2
  else if(iflag(jj(1,2)).eq.1)then
    iflag(jj(1,2))=0
  else if(iflag(jj(1,2)).eq.2)then
    iflag(jj(1,2))=0
  end if
  if(iflag(jj(1,4)).eq.3)then
    iflag(jj(1,4))=2
  else if(iflag(jj(1,4)).eq.1)then
    iflag(jj(1,4))=0
  else if(iflag(jj(1,4)).eq.2)then
    iflag(jj(1,4))=0
  end if
end if
+ if((path1(1,4).le.path1(1,2)).and.(path1(1,4).le.
  path1(1,3)).and.(jj(2,2).ne.jj(2,3)))then
  jj(1,2)=jj(2,2)
  path1(1,2)=path1(2,2)
  jj(1,4)=jj(2,3)
  path1(1,4)=path1(2,3)
    if(iflag(jj(1,2)).eq.3)then
      iflag(jj(1,2))=2
    else if(iflag(jj(1,2)).eq.1)then
      iflag(jj(1,2))=0
    else if(iflag(jj(1,2)).eq.2)then
      iflag(jj(1,2))=0
    end if
    if(iflag(jj(1,3)).eq.3)then
      iflag(jj(1,3))=2
    else if(iflag(jj(1,3)).eq.1)then
      iflag(jj(1,3))=0
    else if(iflag(jj(1,3)).eq.2)then
      iflag(jj(1,3))=0
    end if
  end if
+ if((path1(1,4).le.path1(1,3)).and.(path1(1,4).le.
  path1(1,2)).and.(jj(2,2).eq.jj(2,3)))then
  jj(1,3)=jj(2,3)
  path1(1,3)=path1(2,3)
  jj(1,2)=jj(3,2)
  path1(1,2)=path1(3,2)
    if(iflag(jj(1,2)).eq.3)then
      iflag(jj(1,2))=2
    else if(iflag(jj(1,2)).eq.1)then
      iflag(jj(1,2))=0
    else if(iflag(jj(1,2)).eq.2)then
      iflag(jj(1,2))=0
    end if
    if(iflag(jj(1,3)).eq.3)then

```

```

        iflag(jj(1,3))=2
        else if(iflag(jj(1,3)).eq.1)then
            iflag(jj(1,3))=0
        else if(iflag(jj(1,3)).eq.2)then
            iflag(jj(1,3))=0
        end if
    end if
do 34 l=1,4
if(path1(1,l).lt.bonle)then
ipoly(k,l)=jj(1,l)
i2(1,l)=jj(1,l)
end if
34 continue
factr=factr*.95
call writpol(polymer)
short=amin1(path1(1,1),path1(1,2),path1(1,3),path1(1,4))
if(short.gt.bonle) then
flip=.true.
end if
return
end if
return
end
end
c*****

subroutine sumup(rcut,rmin,sigma,ovrlap,v,iflag,eql,foc,
+               vos,polymer,neig,red)

common /block1/rx,ry,rz
common /block2/ipoly
c*****
c **
c ** Calculates the total potential energy
c ** For bonded neighbors calculates oscilator energy and for
c ** others calculates LJ energy.
c **
c ** Principal variables:
c **
c ** integer n                the number of atoms
c ** integer iflag(n)         bonded status
c ** integer ipoly(n,4)       linear polymer chains
c ** real    rx,ry,rz         the positions of the atoms
c ** real    v                the LJ potential energy
c ** real    vos              the oscilator potential energy
c ** real    rmin             LJ overlapping distance
c ** real    rminl            oscilatoroverlapping distance
c ** logical overlap          true for substantial atom overlap**
c ** logical neig             true if i and j are neighbors in
c **                          any linear chain
c **
c ** Returns the total potential energy at the begining and
c ** at the end of the run.Check for the overlaps in oscilator**
c ** potential and the LJ potential separetely.
c *****

```

```

integer n
parameter (n=100)
real rx(n),ry(n),rz(n)
real sigma,rmin,rcut,v,foc,vos,red,rminl
logical overlap,neig
real rcutsq,rminsq,sigsq,rxij,ryij,rzij,bonle,bonll
real rxi,ryi,rzi,vij,sr2,sr6,rijsq,eql,rmisql
integer i,j,iflag(100),ipoly(100,4)
c *****
character polymer*(*)
overlap=.false.
rcutsq=rcut*rcut
rminsq=rmin*rmin
rminl=rmin*red
rmisql=rminl*rminl
bonll=bonle*1.3
sigsq=sigma*sigma
v=0.0
vos=0.0
do 100 i=1,n-1
  rxi=rx(i)
  ryi=ry(i)
  rzi=rz(i)
  do 99 j=i+1,n
    rxij=rxi-rx(j)
    ryij=ryi-ry(j)
    rzij=rzi-rz(j)
c ***** minimum image pair separations*****
    if( rxij.gt.0.5) then
      rxij=rxij-1.0
    else if(rxij.lt.-0.5) then
      rxij=rxij+1.0
    end if
    if( ryij.gt.0.5) then
      ryij=ryij-1.0
    else if(ryij.lt.-0.5) then
      ryij=ryij+1.0
    end if
    if( rzij.gt.0.5) then
      rzij=rzij-1.0
    else if(rzij.lt.-0.5) then
      rzij=rzij+1.0
    end if
    rijsq=rxij*rxij+ryij*ryij+rzij*rzij
    if (rijsq.lt.rminsq) then
      overlap=.true.
      return
    if((iflag(i).eq.0.or.iflag(i).eq.2).and.
+      (iflag(j).eq.0.or.iflag(j).eq.2))then
      if (rijsq.lt.bonll) then
        call readpol(polymer,i,j,neig)
        if(neig)then
          if (rijsq.lt.rmisql) then
            overlap=.true.

```

```

        return
      else
        rij=sqrt(rijsq)
        vos=vos+0.5*foc*(rij-eql)**2
      end if
    else if((rijsq.lt.rminsq) then
      overlap=.true.
      return
    end if
  end if
else if (rijsq.lt.rcutsq) then
  if((rijsq.lt.rminsq) then
    overlap=.true.
    return
  else
    sr2=sigsq/rijsq
    sr6=sr2*sr2*sr2
    vij=sr6*(sr6-1.0)
    v=v+vij
  end if
end if
99 continue
100 continue
    v=4.0*v
    return
  end

```

C*****

```

subroutine energy(rxi,ryi,rzi,i,rcut,sigma,v,iflag,eql,
+               foc,vos,ovrpol,rmin,polymer,neig,red)

```

```

common /block1/rx,ry,rz
common /block2/ipoly

```

```

C*****
C ** This subroutine is used to calculate the change of *
C ** energy during a trial move of atom i.it is caled before*
C ** and after the random displacement of i. For bonded **
C ** neighbors displacements are accected towrds the equli.**
C ** ** **
C ** Principal variables: **
C ** ** **
C ** integer n number of monomers **
C ** integer iflag(n) bonded status **
C ** integer ipoly(n,4) linear polymer chains **
C ** logical ovrpol true if 2 neighbors in a linear **
C ** chain overlaps or they exceed the**
C ** maximum bond length **
C ** logical touch true for all other overlaps **
C ** logical neig true if i and j are neighbors in **
C ** a linear chain **
C ** character polymer name of polymer file **
C*****

```

```

integer n,i
parameter (n=100)
real rx(n),ry(n),rz(n)
real rcut,sigma,rxi,ryi,rzi,v,rmin
real rcutsq,sigsq,sr2,sr6,eql
real rxij,ryij,rzij,rijsq,vij
integer j,iflag(100),ipoly(100,4)
character polymer*(*)
logical ovrpol,neig,touch

c *****
ovrpol=.false.
touch =.false.
rcutsq=rcut*rcut
rminsq=rmin*rmin
sigsq =sigma*sigma
v=0.0
vos=0.0
c *****loop over all molecules except i*****

do 100 j=1,n
  if(i.ne.j)then
    rxij=rxi-rx(j)
    ryij=ryi-ry(j)
    rzij=rzi-rz(j)
    if (rxij.gt.0.5) then
      rxij=rxij-1.0
    else if (rxij.lt.-0.5) then
      rxij=rxij+1.0
    end if
    if (ryij.gt.0.5) then
      ryij=ryij-1.0
    else if (ryij.lt.-0.5) then
      ryij=ryij+1.0
    end if
    if (rzij.gt.0.5) then
      rzij=rzij-1.0
    else if (rzij.lt.-0.5) then
      rzij=rzij+1.0
    end if
    rijsq=(rxij*rxij+ryij*ryij+rzij*rzij)
    if((iflag(i).eq.0.or.iflag(i).eq.2).and.
+ (iflag(j).eq.0. or.iflag(j).eq.2))then
      if (rijsq.lt.bon11) then
        call readpol(polymer,i,j,neig)
        if(neig)then
          if((rijsq.lt.rmisq1).or.rijsq.gt.bonle)) then
            ovrpol=.true.
            return
          else
            rij=sqrt(rijsq)
            vos=vos+0.5*foc*(rij-eql)**2
          end if
        else if((rijsq.lt.rminsq) then

```

```

        ovrpol=.true.
        return
    end if
end if
else if (rijsq.lt.rcutsq) then
    if(rijsq.lt.rminsq) then
        touch=.true.
        return
    else
        sr2=sigsq/rijsq
        sr6=sr2*sr2*sr2
        vij=sr6*(sr6-1.0)
        v=v+vij
    end if
end if
100 continue
    v=4.0*v
    return
end

c *****

subroutine readcn (cnfile )

    common /block1/rx,ry,rz

c *****
c ** Subroutine to read cordintes from unit 10          **
c ** Reads cordintes for new MOnTe Carlo cycle          **
c **                                                    **
c ** Principal variables:                                **
c **                                                    **
c ** integer      n          number of monomers          **
c ** real          rx(n),etc  cordintes of mpnomer units **
c *****

    integer n
    parameter (n=100)
    real rx(n),ry(n),rz(n)
    character cnfile*(*)
    integer cnunit
    parameter (cnunit=10)
    open(unit=cnunit,file='cnfile',status='old',
+     form='formatted')
    rewind(unit=cnunit)
    do 30 i=1,n
        read (unit= cnunit,fmt=500,end=31) rx(i), ry(i),rz(i)
30 continue
31 write(*,(''end of file read''))
    close (unit=cnunit)
500 format(t5,e12.4,t19,e12.4,t48,e12.4)
    return
end

```

```

C *****
C      subroutine writcn (cnfile)
C
C      common /block1/rx,ry,rz
C
C*****
C  ** Subroutine to write cordinate file after each MC cycle  **
C  **                                                         **
C  ** Principal variables:                                     **
C  **                                                         **
C  ** integer      n          number of monomer units        **
C  ** real         rx(n),etc  cordinates if monomer units   **
C  ** *****
C
C      integer n
C      parameter (n=100)
C      character cnfile*(*)
C      real rx(n),ry(n),rz(n)
C      integer cnunit
C      parameter (cnunit=10)
C      open (unit=cnunit,file='cnfile',form='formatted')
C      do 40 i=1,n
C      write (cnunit,500) rx(i),ry(i),rz(i)
40 continue
C      endfile (unit=cnunit)
C      close (unit=cnunit)
500 format(t5,e12.4,t19,e12.4,t48,e12.4)
C      return
C      end
C
C *****
C      subroutine readpol(polymer,i,j,neig)
C
C      common /block2/ipoly
C
C *****
C  ** Subrutine to read polymer file and identify neighbors  **
C  **                                                         **
C  ** Principal variables:                                     **
C  **                                                         **
C  ** integer  ipoly(n,4)  polymer chains                    **
C  ** logical  neig        true if i and j are neighbors in **
C  **                                     the same linear chain **
C  ** *****
C
C      character polymer*(*)
C      logical neig
C      integer i,j,ipoly(100,4),j3,j4,jcoun,j5,j6,j7,j8,j9,j10
C      neig=.false.
C      open(12,file='polymer',status='unknown',form='formatted')
C      rewind (unit=12)
C      read(12,fmt=502,end=25)((ipoly(jcoun,lco),lco=1,4),
+      jcoun=1,100)

```

```

do 17 jcoun=1,100
do 16 lco=1,4
if(ipoly(jcoun,lco).eq.i) go to 18
16 continue
17 continue
18 j3=(jcoun-1)
j4=lco
j7=(jcoun+1)
do 20 jcoun=1,100
do 19 loc=1,4
if(ipoly(jcoun,lco).eq.j) go to 21
19 continue
20 continue
21 j5=jcoun
j6=lco
if((j3.eq.j5.and.j4.eq.j6).or.(j7.eq.j5.and.j4.eq.j6)then
neig=.true.
go to 25
end if
do 23 jcoun=100,0,-1
do 22 loc=1,4
if(ipoly(jcoun,lco).eq.j) go to 24
22 continue
23 continue
24 j8=jcoun-1
j9=lco
j10=jcoun+1
if((j8.eq.j5.and.j9.eq.j6).or.(j10.eq.j5.and.j9.eq.j6)then
neig=.true.
end if
25 close(unit=12)
502 format(100(4i10/))
return
end

```

```

c *****

```

```

subroutine writpol(polymer)

```

```

common /block2/ipoly

```

```

c *****

```

```

c ** subroutine to write polymer file at end of each MC cycle**

```

```

c **

```

```

c ** Principal variables:

```

```

c **

```

```

c ** integer ipoly(n,4) polymer chains

```

```

c *****

```

```

integer ipoly(100,4),jcoun

```

```

character polymer*(*)

```

```

open(12,file='polymer',status='unknown',form='formatted')

```

```

write(12,fmt=501)((ipoly(jcoun,lco),lco=1,4),jcoun=1,100)

```

```

endfile(unit=12)

```

```
close(unit=12)
501 format(100(4i10/))
return
end
```

```
*****
```

APPENDIX C

This appendix contains some programs need to analyze the structure of the reaction system at different stages of the growth.

1. Program 'CONNECT' will read 'polymer' file and write the status of all particles (bonded or non-bonded and if bonded particle numbers) to the 'link' file.
2. Program 'IMAGE' will read 'cnfile' file and 'polymer' file and write the information (to file 'disc') of which bonded pairs are image connection.
3. Program 'CONNECT2' will read 'link' file and 'disc' file and write the information of bonded neighbors which are not image connections to the 'linkf' file.

```
program connect
common /block2/ipoly
common /block3/jpoly
integer n,m
parameter (n=934)
parameter (m=200)
integer jcoun,lco,j3,j4,ipoly(m,4),j1,k1,jpoly(n,5)
integer iflag(n),j5,j6,j7,j8,j9,j10
character polymer*10,link*6
call readpol(polymer)
do 17 lco=1,4
do 16 jcoun=1,200
i=ipoly(jcoun,lco)
if(iflag(i).eq.0)then
j3=(jcoun+1)
j4=(jcoun-1)
j1=ipoly(j3,lco)
k1=ipoly(j4,lco)
if(j1.lt.k1) then
j=j1
k=k1
else
j=k1
k=j1
end if
jpoly(i,1)=i
jpoly(i,2)=j
jpoly(i,3)=k
call writlink(link)
iflag(i)=1
else
j5=(jcoun+1)
j6=(jcoun-1)
j1=ipoly(j5,lco)
k1=ipoly(j6,lco)
if(j1.lt.k1)then
j=j1
k=k1
else
j=k1
k=j1
end if
jpoly(i,4)=j
jpoly(i,5)=k
if(jpoly(i,3).gt.jpoly(i,4))then
j7=jpoly(i,3)
jpoly(i,3)=jpoly(i,4)
jpoly(i,4)=j7
end if
if(jpoly(i,2).gt.jpoly(i,3))then
j8=jpoly(i,2)
jpoly(i,2)=jpoly(i,3)
jpoly(i,3)=j8
end if
```

```

    if(jpoly(i,4).gt.jpoly(i,5))then
    j9=jpoly(i,4)
    jpoly(i,4)=jpoly(i,5)
    jpoly(i,5)=j9
    end if
    if(jpoly(i,3).gt.jpoly(i,4))then
    j10=jpoly(i,3)
    jpoly(i,3)=jpoly(i,4)
    jpoly(i,4)=j10
    end if
    call writlink(link)
    end if
16  continue
17  continue
    stop
501  format(934(5i6/))
502  format(200(4i10/))
    end
    subroutine readpol(polymer)
    common /block2/ipoly
    integer m
    parameter (m=200)
    integer jcoun,lco,ipoly(200,4)
    open(12,file='polymer',status='unknown',form='formatted')
    rewind(unit=12)
    read(12,fmt=502,end=22)((ipoly(jcoun,lco),lco=1,4),
+                          jcoun=1,200)
22  close(unit=12)
502  format(200(4i10/))
    return
    end
    subroutine writlink(link)
    common /block3/jpoly
    integer jpoly(934,5),jco,lco
    open(14,file='link',status='unknown',form='formatted')
    write(14,fmt=501)((jpoly(jco,lco),lco=1,5),jco=1,934)
    endfile(unit=14)
    close(unit=14)
501  format(934(5i6/))
    return
    end

```

```

program image
common /block1/rx,ry,rz
common /block2/ipoly
common /block3/dis
integer n,m
parameter (n=934)
parameter (m=200)
integer ipoly(m,4),dis(m,2)
real rx(n),ry(n),rz(n)
real path(1,4),rxij,ryij,rzij
character cnfile*10,polymer*10,disc*10
call readcn(cnfile)
call readpol(polymer)
j=0
do 2 i=1,m
do 1 l=1,4
if((ipoly(i,l).ne.0).and.(ipoly(i+1,l).ne.0)) then
rxij=rx(ipoly(i+1,l))-rx(ipoly(i,l))
ryij=ry(ipoly(i+1,l))-ry(ipoly(i,l))
rzij=rz(ipoly(i+1,l))-rz(ipoly(i,l))
path(1,l)=sqrt((rxij)**2+(ryij)**2+(rzij)**2)
end if
c   print*, 'path(1,l)= ',path(1,l)
if (path(1,l).gt.1.0) then
j=j+1
dis(j,1)=ipoly(i,l)
dis(j,2)=ipoly(i+1,l)
print *, 'i,j = ',ipoly(i,l),ipoly(i+1,l)
end if
1  continue
2  continue
call wridisc(disc)
stop
end
subroutine readpol(polymer)
common /block2/ipoly
integer n
parameter (n=934)
parameter (m=200)
character polymer*(*)
integer i,l,ipoly(m,4)
open(12,file='polymer',status='unknown',form='formatted')
rewind (unit=12)
read(12,fmt=502,end=22)((ipoly(i,l),l=1,4),i=1,200)
22  close(unit=12)
502  format(200(4i10/))
return
end
subroutine readcn (cnfile )
c  ** SUBROUTINE TO READ IN THE CONFIGURATION FROM UNIT 10  **
common /block1/rx,ry,rz
integer n
parameter (n=934)
real rx(n),ry(n),rz(n)

```

```
character cnfile*(*)
integer cnunit
parameter (cnunit=10)
open(unit=cnunit,file='cnfile',status='old',form='formatted')
rewind(unit=cnunit)
do 30 i=1,n
read (unit= cnunit,fmt=500,end=31) rx(i), ry(i),rz(i)
30 continue
31 write(*,(''end of file read''))
close (unit=cnunit)
500 format(t5,e12.4,t19,e12.4,t48,e12.4)
return
end
subroutine wridisc(disc)
common /block3/dis
integer m
parameter (m=200)
integer dis(m,2)
character disc*(*)
open(16,file='disc',status='unknown',form='formatted')
write(16,fmt=503)((dis(i,1),l=1,2),i=1,200)
endfile(unit=16)
close(unit=16)
503 format(200(2i10/))
return
end
```

```

program connect2
common /block3/jpoly
common /block4/kpoly
common /block5/dis
integer i,jco,lco,jpoly(934,5),kpoly(934,5),dis(200,2)
character link*6,linkf*6,disc*6
call readlink(link)
call readisc(disc)
i=0
do 12 jco=1,934
if(jpoly(jco,1).ne.0) then
i=i+1
kpoly(i,1)=jpoly(jco,1)
do 13 j=1,200
if(jpoly(jco,1).eq.dis(j,1))then
if(dis(j,2).eq.jpoly(jco,2)) then
jpoly(jco,2)=0
else if(dis(j,2).eq.jpoly(jco,3)) then
jpoly(jco,3)=0
else if(dis(j,2).eq.jpoly(jco,4)) then
jpoly(jco,4)=0
else if(dis(j,2).eq.jpoly(jco,5)) then
jpoly(jco,5)=0
end if
else if(jpoly(jco,1).eq.dis(j,2))then
if(dis(j,1).eq.jpoly(jco,2)) then
jpoly(jco,2)=0
else if(dis(j,1).eq.jpoly(jco,3)) then
jpoly(jco,3)=0
else if(dis(j,1).eq.jpoly(jco,4)) then
jpoly(jco,4)=0
else if(dis(j,1).eq.jpoly(jco,5)) then
jpoly(jco,5)=0
end if
end if
13 continue
kpoly(i,2)=jpoly(jco,2)
kpoly(i,3)=jpoly(jco,3)
kpoly(i,4)=jpoly(jco,4)
kpoly(i,5)=jpoly(jco,5)
call writlink(linkf)
end if
12 continue
stop
501 format(934(5i6/))
502 format('CONNECT',5i6)
end
subroutine readlink(link)
common /block3/jpoly
integer jpoly(934,5),jco,lco
character link*(*)
open(14,file='link',status='unknown',form='formatted')
rewind(unit=14)
read(14,fmt=501,end=22)((jpoly(jco,lco),lco=1,5),jco=1,934)

```

```
22   close(unit=14)
501  format(934(5i6/))
      return
      end
      subroutine writlink(linkf)
      common /block4/kpoly
      integer kpoly(934,5),jco,lco
      open(15,file='linkf',status='unknown',form='formatted')
      do 23 jco=1,934
      write(15,fmt=502)kpoly(jco,1),kpoly(jco,2),kpoly(jco,3),
+          kpoly(jco,4),kpoly(jco,5)
23   continue
      endfile(unit=15)
      close(unit=15)
502  format('CONNECT',5i6)
      return
      end
      subroutine readisc(disc)
      common /block5/dis
      integer m
      parameter (m=200)
      integer dis(m,2)
      character disc*(*)
      open(16,file='disc',status='unknown',form='formatted')
      rewind(unit=16)
      read(16,fmt=503,end=33)((dis(i,1),l=1,2),i=1,200)
33   close(unit=16)
503  format(200(2i10/))
      return
      end
```

APPENDIX D

```

program dumbel
common /block1/ rx,ry,rz,ex,ey,ez
integer n,natom
parameter (n=100,natom=2)
real rx(n),ry(n),rz(n),ex(n),ey(n),ez(n)
real drmax,dens,sigma,dab(natom),dotmin,d
real acm,accept,pi,ratio,rmin,rcut,deltv,deltvb
real rxiold,ryiold,rziold,rxinew,ryinew,rzinew
real exiold,eyiold,eziold,exinew,eyinew,ezinew
real v,vnew,vold,vlast,vn,vs,temp
real vlrc,vlrc6,vlrc12,control,beta
integer step,i,nstep,iprint,isave,iratio
logical overlap
character cnfile*10
parameter (pi=3.1415927)
read input data*****
open (12,file='inputdata',status='old',form='formatted')
c*****write input data*****
read(12,'(i10)')nstep
read(12,'(i10)')iprint
read(12,'(i10)')isave
read(12,'(i10)')iratio
read(12,'(f10.4)')temp
read(12,'(f10.4)')dens
read(12,'(f10.4)')rmin
read(12,'(f10.4)')drmax
read(12,'(f10.4)')tvbmax
read(12,'(f10.4)')rcut
read(12,'(f10.4)')d
read(12,'(e12.4)')control
read(12,'(a)')cnfile
write(*,'('' end of inputdata '')')
c*****convert input data to program units*****
beta=1.0/temp
sigma =(dens/real(n))*(1.0/3.0)
dab(1)=d*sigma/2
dab(2)=-dab(1)
drmax =drmax*sigma
dotmin=0.2
rmin=rmin*sigma
rcut=rcut*sigma
denslj=dens
dens=dens/(sigma**3)
if(rcut.gt.0.5)stop' cutoff too large '
acm=0.0
accept=0.0
acv=0.0
c*****calculate long range correction
sr3=(sigma/rcut)**3
sr9=sr3**3
vlrc12=8.0*pi*denslj*real(n)*sr9/9.0

```

```

      vlrc6 =8.0*pi*denslj*real(n)*sr3/3.0
      vlrc=vlrc12-vlrc6
c*****write out information****
      write(*,('sigma/box = ',f12.6))sigma
      write(*,('dab(1)/box = ',f12.8))dab(1)
      write(*,('dab(2)/box = ',f12.8))dab(2)
      write(*,('drmax/box = ',f12.6))drmax
      write(*,('dotmin = ',f12.6))dotmin
      write(*,('rmin/box = ',f12.6))rmin
      write(*,('rcut/box = ',f12.6))rcut
      write(*,('tvbmax/box = ',f12.6))tvbmax
      write(*,('lrc for v = ',f12.6))vlrc
      write(*,('control = ',e12.4))control
      call readcn(cnfile)
c*****calculates initial energy and cheks for overlap**
      call check(rcut,rmin,sigma,dab,ovrlap,v)
      if(ovrlap)stop 'ovrlap in initial con '
      vs=(v+vlrc)/real(n)
      write(*,('initial v = ',e20.6 ))vs
      write(*,('/' start of markov chain '/'))
      write(*,(' acm      ratio      v/n  '/'))
c*****loops over all cycles and all molecules *****
      do 100 step=1,nstep
        do 99 i=1,n
          rxiold=rx(i)
          ryiold=ry(i)
          rziold=rz(i)
          exiold=ex(i)
          eyiold=ey(i)
          eziold=ez(i)
c*****calculates the energy of i in the old config
          call test(rcut,rmin,rxiold,ryiold,rziold,i,exiold,
            * eyiold,eziold,sigma,dab,vold)
c*****move i and pickup the central image*****
          iseed=12349
          rxinew=rxold+(2.0*ran(iseed)-1.0)*drmax
          ryinew=ryold+(2.0*ran(iseed)-1.0)*drmax
          rzinew=rzold+(2.0*ran(iseed)-1.0)*drmax
          if (rxinew.gt.0.5) then
            rxinew=rxinew-1.0
          else if (rxinew.lt.-0.5) then
            rxinew=rxinew+1.0
          end if
          if (ryinew.gt.0.5) then
            ryinew=ryinew-1.0
          else if (ryinew.lt.-0.5) then
            ryinew=ryinew+1.0
          end if
          if (rzinew.gt.0.5) then
            rzinew=rzinew-1.0
          else if (rzinew.lt.-0.5) then
            rzinew=rzinew+1.0
          end if
c*****calculate the energy of i in the new config*****

```

```

        call orien(exiold,eyiold,eziold,dotmin, exinew,
        *          eyinew,ezinew)
c*****check for acceptance***
        call test(rcut,rmin,rxinew,ryinew,rzinew,i,exinew,
        *          eyinew,ezinew,sigma,dab,vnew)
        deltv=vnew-vold
        deltvb=beta*deltv
        if(deltvb.lt.tvbmax) then
if(deltv.le.0.0) then
v=v+deltv
        rx(i)=rxinew
        ry(i)=ryinew
        rz(i)=rzinew
        ex(i)=exinew
        ey(i)=eyinew
        ez(i)=ezinew
        accept=accept+1.0
        else if (exp(-deltvb).gt.ran(iseed)) then
v=v+deltv
        rx(i)=rxinew
        ry(i)=ryinew
        rz(i)=rzinew
        ex(i)=exinew
        ey(i)=eyinew
        ez(i)=ezinew
        accept=accept+1.0
        endif
        endif
99  continue
        acm=acm+1.0
        vn=(v+vllrc)/real(n)
        acv=acv+vn
c*****perform periodic operations
        if (mod(step,iratio).eq.0) then
        print *, 'accept=',accept
        ratio=accept/real(n*iratio)
        if (ratio.gt.0.5) then
            drmax=drmax*1.05
dotmin=dotmin*1.025
        else
            drmax=drmax*0.95
dotmin=dotmin*0.975
        end if
        accept=0.0
        end if
        if (mod(step,iprint).eq.0) then
        write (*,'(i8,e14.4,e20.6)') int(acm),ratio,vn
        end if
        if (mod(step,isave).eq.0) then
        call writcn (cnfile)
        end if
        call readcn (cnfile)
100  continue
        write(*,'(///' end of markov chain '///)')

```

```

call check (rcut,rmin,sigma,dab,ovrlap,vlast)
if(ovrlap) stop 'overlap in final configuration'
if(abs(vlast-v).gt.control) then
write(*,('' problem with energy , ''))
write(*,(''vlast = '',e20.6)')vlast
write(*,('' v      = '',e20.6)')v
endif
call writcn (cnfile)
avv=acv/acm
write(*,('/' average ''/))
write(*,(''<v/n> = '',e20.6)')avv
500   format(e10.4,t12,e10.4,t24,e10.4,t36,e10.4,t48,e10.4,
*         t60,e10.4)
stop
end
subroutine orien(exiold,eyiold,eziold,dotmin,
*               exinew,eyinew,ezinew)
real exiold,eyiold,eziold,exinew,eyinew,ezinew,dotmin
real dot,xil,xi2,xi,xisq
dot=0.0
100   if ((1.0-dot).ge.dotmin) then
xisq=1.0
200   if(xisq.ge.1.0) then
xil=ran(iseed)*2.0-1.0
xi2=ran(iseed)*2.0-1.0
xisq=xil*xil+xi2*xi2
goto 200
endif
xi=sqrt(1.0-xisq)
exinew=2.0*xil*xi
eyinew=2.0*xi2*xi
ezinew=1.0-2.0*xisq
dot=exinew*exiold+eyinew*eyiold+ezinew*eziold
goto 100
endif
return
end

subroutine test(rcut,rmin,rx,ry,rz,i,exi,eyi,ezi,sigma,
*             dab,v)
common /block1/rx,ry,rz,ex,ey,ez
c   calculates the total potential energy
c   integer n           the number of atoms
c   real rx,ry,rz      the positions of the atoms
c   real v             the potential energy
c   real w             the virial
c   logical overlap    true for substantial atom overlap
c   returns the total potential energy at the begining and
c   at the end of the run
c   *****
integer n,natom
parameter (n=100,natom=2)
real rx(n),ry(n),rz(n)
real ex(n),ey(n),ez(n)

```

```

real rxi,ryi,rzi,exi,eyi,ezi,sr2,sr6
real sigma,dab(natom),v,rcutsq,rminsq
logical overlap
real rxij,ryij,rzij,exj,eyj,ezej,vab
real rxab,ryab,rzab,dabi,sigsq,rabsq
integer i,j,ia,jb
c      *****
overlap=.false.
rcutsq=rcut*rcut
rminsq=rmin*rmin
sigsq=sigma*sigma
v=0.0
c      ** loop over all the pairs in the liquid**
do 100 j=1,n
if(j.ne.i) then
exj=ex(j)
eyj=ey(j)
ezj=ez(j)
rxij=rxi-rx(j)
ryij=ryi-ry(j)
rzij=rzi-rz(j)
c      ** minimum image pair separations**
if( rxij.gt.0.5) then
rxij=rxij-1.0
else if(rxij.lt.-0.5) then
rxij=rxij+1.0
end if
if( ryij.gt.0.5) then
ryij=ryij-1.0
else if(ryij.lt.-0.5) then
ryij=ryij+1.0
end if
if( rzij.gt.0.5) then
rzij=rzij-1.0
else if(rzij.lt.-0.5) then
rzij=rzij+1.0
end if
do 99 ia=1,natom
dabi=dab(ia)
do 98 jb=1,natom
rxab=rxij+exi*dabi+exj*dab(jb)
ryab=ryij+eyi*dabi+eyj*dab(jb)
rzab=rzij+ezi*dabi+ezj*dab(jb)
rabsq=rxab*rxab+ryab*ryab+rzab*rzab
if (rabsq.lt.rcutsq) then
sr2=sigsq/rabsq
sr6=sr2*sr2*sr2
vab=sr6*(sr6-1.0)
v=v+vab
endif
98   continue
99   continue
endif
100  continue

```

```

v=4.0*v
return
end
subroutine check(rcut,rmin,sigma,dab,ovrlap,v)
common /block1/rx,ry,rz,ex,ey,ez
c   calculates the total potential energy
c   integer n           the number of molecules
c   real rx,ry,rz       the positions of the atoms
c   real v              the potential energy
c   real w              the virial
c   logical overlap     true for substantial atom overlap
c   returns the total potential energy at the begining and
c   at the end of the run
c   *****
integer n,natom
parameter (n=100,natom=2)
real rx(n),ry(n),rz(n)
real ex(n),ey(n),ez(n)
real sigma,dab(natom),vab,sr2,sr6
logical ovrlap
real rxi,ryi,rzi,rxij,ryij,rzij,exi,eyi,ezi
real exj,eyj,ezej,rxab,ryab,rzab,dabi,sigsq,rabsq
integer i,j,ia,jb
c   *****

overlap=.false.
rcutsq=rcut*rcut
rminsq=rmin*rmin
sigsq=sigma*sigma
v=0.0
do 100 i=1,n-1
  rxi=rx(i)
  ryi=ry(i)
  rzi=rz(i)
  exi=ex(i)
  eyi=ey(i)
  ezi=ez(i)
  do 99 j=i+1,n
    rxij=rxi-rx(j)
    ryij=ryi-ry(j)
    rzij=rzi-rz(j)
c   ** minimum image pair separations**
    if( rxij.gt.0.5) then
      rxij=rxij-1.0
    else if(rxij.lt.-0.5) then
      rxij=rxij+1.0
    end if
    if( ryij.gt.0.5) then
      ryij=ryij-1.0
    else if(ryij.lt.-0.5) then
      ryij=ryij+1.0
    end if
    if( rzij.gt.0.5) then
      rzij=rzij-1.0

```

```

else if(rzij.lt.-0.5) then
rzij=rzij+1.0
end if
exj=ex(j)
eyj=ey(j)
ezj=ez(j)
do 98 ia=1,natom
dabi=dab(ia)
do 97 jb=1,natom
  rxab=rxij+exi*dabi+exj*dab(jb)
  ryab=ryij+eyi*dabi+eyj*dab(jb)
  rzab=rzij+ezi*dabi+ezj*dab(jb)
  rxabsq=rxab*rxab
  ryabsq=ryab*ryab
  rzabsq=rzab*rzab
  rabsq=(rxabsq+ryabsq+rzabsq)
  if (rabsq.lt.rminsq) then
  overlap=.true.
  return
  else if(rabsq.lt.rcutsq) then
  sr2=sigsq/rabsq
  sr6=sr2*sr2*sr2
  vab=sr6*(sr6-1.0)
  v=v+vab
  end if
97  continue
98  continue
99  continue
100 continue
v=4.0*v
return
end
subroutine readcn (cnfile )
C  ** SUBROUTINE TO READ IN THE CONFIGURATION FROM UNIT 10      **
common /block1/rx,ry,rz,ex,ey,ez
integer n
parameter (n=100)
real rx(n),ry(n),rz(n),ex(n),ey(n),ez(n)
character cnfile*(*)
integer cnunit
parameter (cnunit=11)
open(unit=cnunit,file='cnfile',status='old',form='formatted')
rewind(unit=cnunit)
do 30 i=1,n
read (unit= cnunit,fmt=500,end=31) rx(i), ry(i),rz(i),
* ex(i),ey(i),ez(i)
write(*,fmt=500)rx(i),ry(i),rz(i),ex(i),ey(i),ez(i)
30  continue
31  write(*,(''end of file read''))
close (unit=cnunit)
500  format(e10.4,t12,e10.4,t24,e10.4,t36,e10.4,t48,e10.4,
* t60,e10.4)
return
end

```

```
subroutine writcn (cnfile)
common /block1/rx,ry,rz,ex,ey,ez
i  ninteger n
parameter (n=100)
character cnfile*(*)
real rx(n),ry(n),rz(n),ex(n),ey(n),ez(n)
integer cnunit
parameter (cnunit=11)
open (unit=cnunit,file='cnfile',form='formatted')
do 40 i=1,n
write(cnunit,fmt=500)rx(i),ry(i),rz(i),ex(i),ey(i),ez(i)
40  continue
endfile (unit=cnunit)
close (unit=cnunit)
500  format(e10.4,t12,e10.4,t24,e10.4,t36,e10.4,t48,e10.4,
*         t60,e10.4)
return
end
```

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