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James M. Zolnierek

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THE EFFECTS OF MARKET STRUCTURE, INDUSTRY CONDITIONS AND TECHNOLOGICAL OPPORTUNITIES ON FIRM R&D BEHAVIOR AND ECONOMIC GROWTH RATES

By

James M. Zolnierek

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ABSTRACT

THE EFFECTS OF MARKET STRUCTURE, INDUSTRY CONDITIONS AND TECHNOLOGICAL OPPORTUNITIES ON FIRM R&D BEHAVIOR AND ECONOMIC GROWTH RATES

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The four essays that form my dissertation consider the microeconomic foundations underlying economic growth. In part, economic growth results from firms successful R&D efforts to improve product quality or production processes. Each essay in my dissertation examines the behavior of firms in a continuous series of patent races aimed at producing these R&D successes.

In the first essay I construct a quality ladders growth model with decreasing returns R&D technology. This allows me to analyze the effects of firm level R&D behavior on economic growth, an analysis absent from previous quality ladders models. Furthermore this construction allows me to reexamine the relationship between participation in patent races and firm level R&D behavior in a repeated patent race setting. In contrast to previous single patent race models, I find a negative relationship between patent race participation and firm R&D expenditures.

A commonly found result in quality ladder models of growth is that state-of-the-art producers in each industry do no R&D. In the second essay of my dissertation I explore the relationship between quality leaders and industry outsiders. I show that when the common R&D technology exhibits sufficiently diminishing returns and entry into patent races is sufficiently restricted, leaders will perform R&D and may

retain their market share across product generations.

The third essay examines the implication of patent policy on firm R&D behavior. In a single industry quality ladders model I find a positive but diminishing relationship between patent policy and economic growth.

In the final essay of my dissertation, I examine a quality ladders model with alternate production and R&D technologies. The R&D behavior of both quality leaders and outside R&D firms are re-examined. It is shown that with constant returns R&D technology, leaders' cumulative R&D efforts exceed followers' efforts when leaders have a significant technological advantage in R&D.

In each of the four essays the impacts of changing industry parameters are examined. Each essay also contains a welfare analysis comparing free market outcomes and social optimums and a derivation of optimal tax and subsidy schemes that correct for market inefficiencies.

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1.1 Introduction

Recently, a series of quality ladders models have been introduced into the economic growth literature. These models are used to examine industry R&D behavior in a series of repeated patent races and the implications for economic growth[1]. In these models, each industry is composed of firms competing in an infinite series of patent races aimed at improving the quality of the state-of-the-art good in the industry. Successful firms capture a degree of monopoly power through their unique ability to produce the industry's state-of-the-art good. R&D efforts are driven by the chance to gain monopoly power. While each of the models generate results on the relationship

[1]The most often cited of these models is the model developed by Grossman and Helpman[1991a]. For extensions of this model see Segerstrom[1991], Grossman and Helpman[1991b], and Barro[1995].

between industry R&D effort and economic growth, they share the common inability to determine R&D behavior at the firm level. This is a direct result of the assumption that R&D technology is characterized by constant returns to scale[2]. With constant returns to scale at the firm level, an industry's R&D market structure will have no bearing on R&D effort, as equilibrium industry R&D effort is consistent with an infinite number of R&D market structures.

Rather than constant returns, a series of empirical studies on the nature of R&D technology suggests that R&D technology is characterized by decreasing returns. Examining the relationship between patents granted and R&D spending, Kortum[1993] reports point elasticity estimates in the range 0.1 to 0.6, while Hall, Griliches, and Hausman[1986] obtain an average elasticity estimate of 0.3. Using market value data, Thompson[1993] obtains an average R&D output elasticity with respect to R&D expenditure of 0.86. Each study suggests decreasing returns to R&D expenditures at the firm level, but the authors are quick to point out that their results may be hampered by data constraints and the difficulty of both measuring R&D output and of matching R&D output with R&D inputs. On theoretical grounds Thompson and Waldo[1994] have argued against constant returns in R&D stating that, "... R&D activity does not satisfy the usual justification for constant returns to scale in manufacturing—namely

[2]A model developed by Segerstrom[1995], with decreasing returns to R&D at the industry level provides an exception to the assumption of constant returns R&D technology, but firm-level R&D behavior remains indeterminate and characterized by constant returns in the Segerstrom model. A second model by Thompson and Waldo[1994] looks at firm-level decreasing returns technology but in a single industry where products within the industry are horizontally differentiated.

that plants can be replicated." As suggested by both this theoretical argument and the empirical evidence, a decreasing returns to scale R&D technology is employed in this article. The introduction of decreasing returns technology into the quality ladders model allows an analysis of firm-level R&D behavior, an analysis absent from previous quality ladders models.

Although the relationship between patent race participation and firm-level R&D behavior has not, until now, been examined in a repeated patent race setting the relationship has been analyzed in a single patent race setting. Glenn Loury[1979] was the first to analyze this relationship in a stochastic setting. In his model, Loury assumed a memoryless R&D technology where firms invest in a supply of R&D labor at the beginning of each R&D race, a sunk cost. With the investment the firm is assumed to gain a fixed probability, in each period the race continues, of successfully inventing a new product. The instantaneous probability of success in Loury's model is a strictly increasing function of the initial investment, exhibiting initial increasing returns to scale followed by decreasing returns as the investment in R&D grows. Loury assumed the first successful inventor would capture exclusive rights to an infinite stream of future benefits.

In the Loury specification, when the number of firms participating in the R&D race increases, each firm does a smaller amount of individual R&D. This conclusion is a result of both the single patent race format and the technological assumptions Loury adopts. With all investment occurring up front, an increase in firm participation will not effect the costs a firm expects to incur during an R&D race. Although each firm's expected costs remain fixed, each firm's expected marginal benefit from R&D

falls with participation, as the likelihood that it will succeed first diminishes. Profit maximizing firms have an unambiguous incentive to decrease R&D effects. Loury finds that the increase in industry R&D effort resulting from greater R&D participation dominates the decrease in industry R&D effort due to smaller individual efforts. The net result is an earlier expected arrival date for the industry's innovation. Loury's specification did not consider the possibility of product upgrading. With product upgrading R&D efforts in future patent races will effect the profits firms expect to earn in the present patent race. This results in intertemporal effects between patent races, effects not found in the Loury model.

Lee and Wilde[1980] modified Loury's original model to include recurrent costs in the R&D race, with a lump sum up-front cost independent of R&D intensity. Lee and Wilde also consider the instantaneous probability of success at each instant to be increasing in effort and exhibiting increasing turning to decreasing returns to scale.

In contrast to Loury, Lee and Wilde predict that when patent race participation increases, the individual level of firm R&D will increase. With recurrent R&D costs an increase in competition has a smaller discounting effect on expected research profits. While the expected marginal benefits from R&D fall as more firms participate, as in the Loury model, under the Lee and Wilde specification the expected marginal costs of R&D fall as well, as firms expect a shorter race and thus a shorter period of R&D expenditures. The fall in marginal R&D costs outweighs the fall in marginal R&D benefits and, subsequently, competition increases firm-level efforts in the Lee and Wilde model. With a greater number of firms participating in the race and each choosing a greater R&D intensity, Lee and Wilde find, as did Loury, that innovations are expected to occur sooner. Like Loury, Lee and Wilde do not consider product upgrading.

Here I adopt R&D technology similar to that used by Lee and Wilde, but with no fixed start-up costs for firms beginning an R&D race. The potential for product upgrading results in a series of patent races in each industry. Modeling repeated patent races explicitly accounts for intertemporal effects absent in the single patent race models. With intertemporal effects driving the model the relationship between R&D participation and firm R&D effort proves to be negative, reversing the result found by Lee and Wilde.

The remainder of the chapter is organized as follows. Section two introduces the R&D technology within the context of the quality ladders model. In section three, existence of a firm-level unique steady-state equilibrium is shown. A steady-state analysis is performed in this section to determine the effect of changing model parameters on equilibrium values, with particular emphasis on the relationship between participation in patent races and firm R&D behavior. Section four examines the welfare implications of the model and in section five government policy effects are considered. Section six offers concluding remarks.

1.2 The Model

The quality ladders model considered here is a model with an economy comprised of a continuum of industries indexed by ω on the unit interval [0,1]. Each industry produces a good horizontally differentiated from goods in other industries. Within each industry, goods are differentiated vertically by quality level, where quality is indexed by j. There are a countably infinite number of quality levels of each good but only those which have been invented can be successfully produced. At time $t = 0$ the state-of-the-art good in each industry has a quality index of $j = 0$. In each industry a fixed number of firms, n, have the ability to compete in repeated R&D races to create higher quality state-of-the-art goods. The winner of each R&D race increases the quality of the previous state-of-the-art product by a factor $\lambda > 1$ and becomes the industry "quality leader".

1.2.1 The Consumer Sector

Consumers are identical, have preferences that extend infinitely into the future, and have intertemporal preferences over goods of the form

$$U \equiv \int_0^\infty u(t)e^{-\rho t}\,dt. \tag{1.1}$$

The representative consumer's subjective discount rate is given by ρ, where $\rho \geq 0$. Instantaneous utility at time t is represented by $u(t)$ which takes the form

$$u(t) \equiv \int_0^1 \ln\left[\sum_{j=0}^{\infty} \lambda^j d(j,\omega,t)\right] d\omega. \tag{1.2}$$

The consumer's consumption of goods of quality j from industry ω at time t is represented by $d(j, \omega, t)$. The parameter λ, where λ^j is the measure of the quality of a good which has been improved on j times, is assumed constant across industries.

The consumer chooses a level of spending at time t, given by $E(t)$. The consumer, taking market prices as given, allocates $E(t)$ to maximize $u(t)$. The consumer's budget constraint is given by

$$\int_0^\infty E(t)e^{-R(t)t}\,dt \leq A_0. \tag{1.3}$$

A_0 is the sum of the present discounted value of the flow of wage and profit income added to the value of asset holdings at time $t = 0$. The cumulative interest factor up to time t is represented by $R(t)$.

The consumer's problem can be solved by a three step backwards induction process. First, at time t, goods within an industry are perfect substitutes and the consumer will allocate all spending on goods in the industry to goods with the lowest quality-adjusted price[3]. Second, given the nature of the Cobb-Douglas instantaneous utility function, the consumer will allocate equal shares of $E(t)$ to each industry. In the steady state equilibrium, the state-of-the-art good in each industry will have the minimum quality-adjusted price and, for price p, face demand $d = \frac{E(t)}{p}$.

The remaining problem for the consumer is to determine the optimal allocation of spending over time. The solution to this consumer intertemporal optimization

[3]Below it will be shown that, in each industry, in the steady-state equilibrium, the firm with the ability to produce the highest quality product, the quality leader in the industry, will set the lowest quality-adjusted price and be the sole producer of goods in the industry.

problem dictates that spending evolve according to

$$\frac{E'(t)}{E(t)} = r(t) - \rho. \tag{1.4}$$

When the interest rate exceeds the rate at which the consumer discounts future consumption, the consumer will spend more on future consumption than on present consumption and expenditures will increase over time. When the rate at which the consumer discounts future consumption exceeds the interest rate, the consumer will spend more for present consumption than for future consumption and expenditures will decrease over time. The steady-state equilibrium examined here, with constant consumer spending over time, requires that the instantaneous interest rate, $r(t)$, equals the consumer's subjective discount rate, ρ. With identical consumers the aggregate steady-state spending at each moment is defined to be E. In equilibrium aggregate spending each period equals the sum of wage income from non-R&D labor plus profits.

1.2.2 The Production Sector

The production technology is characterized by constant returns to scale where one unit of labor produces one unit of any good independent of time, industry, or quality. The wage rate is normalized to one, giving each firm a constant marginal cost of one.

Producers within an industry compete in prices. At time t each consumer will purchase only the lowest quality-adjusted priced goods from the industry. Without loss of generality I assume whenever two goods share the same quality-adjusted price, consumers will choose to purchase the good of the highest quality. With unitary elastic demand and constant marginal cost, profits are maximized when the state-of-the-art producer with a one-step quality lead charges a price equal to λ and are given by

$$\pi = \frac{E(\lambda - 1)}{\lambda}. \tag{1.5}$$

It is assumed that firms will never attempt to imitate the current state-of-the-art good. Imitation may be prohibited directly by assuming broad patent protection or indirectly by assuming positive costs for imitation, which may result from efforts to circumvent narrow patent protection. In the latter case, when two firms produce the state-of-the-art good price competition eliminates any market power and firms are unable to recoup imitation costs.

1.2.3 The Research Sector

Initially the number of firms participating in patent races in each industry is fixed at n. This allows an analysis of firm R&D behavior as industry participation increases, up to a maximum sustainable or free entry level. Over time, n firms compete in patent races in each industry, but for each individual race the quality leader will perform no R&D[4]. Therefore, only a subset of $n-1$ of the n firms will be competing in a patent race at any given time. The $n-1$ firms participating in each patent race will hire labor which is devoted to R&D. A firm which devotes l units of labor to R&D will

[4]It is shown below that for n sufficiently large, R&D will not be profitable for the state-of-the-art producer. I restrict attention to this case.

innovate at $\tau(l)$, prior to time t, according to the probability given by

$$prob[\tau(l) \leq t] = 1 - e^{-h(l)t}. \tag{1.6}$$

The parameter $h(l)\,dt$ measures the instantaneous probability of a successful innovation when l units of labor are devoted to R&D. The expected duration until success is given by $h(l)^{-1}$. Firms pay wages to R&D workers each period until a firm in the race successfully innovates signaling the beginning of the next patent race.

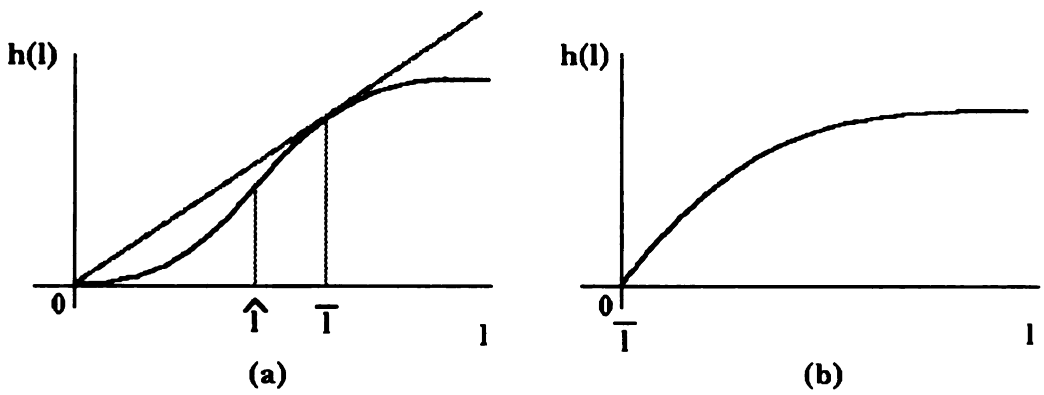

Figure 1.1: The Individual Firm R&D Technology

Each of the n firms in an industry, independent of industry or time, faces the same R&D technology. The function $h(l)$ (see Figure 1.1) is assumed to be twice continuously differentiable and strictly increasing in l. Increasing returns are assumed to prevail up to $\hat{l}$ for each firm where $\hat{l} \geq 0$. Beyond $\hat{l}$ the technology is characterized by decreasing returns. The function $h(l)$ is also assumed to satisfy $h(0) = 0 = h'(\infty)$. Picture (a) in Figure 1.1 illustrates the case with initial increasing returns ($\hat{l} > 0$) and picture (b) in Figure 1.1 illustrates the case with decreasing returns over all R&D labor choices ($\hat{l} = 0$).

The average product of labor is maximized at $\bar{l}$ (see Figure 1.1) which is defined by the R&D labor choice which satisfies $\frac{h(l)}{l} = h'(l)$ when $\dot{l} > 0$ and by $\bar{l} = 0$ when $\dot{l} = 0$. The labor choice $\bar{l}$ proves to be a critical point for firms in making their R&D decisions. Below it is shown that in any positive growth equilibrium each participant in a patent race will hire at least $\bar{l}$ units of R&D labor.

1.2.4 The Labor Sector

The labor supply is homogeneous, fixed at L, and the labor market is assumed to clear each period. In equilibrium the share of labor devoted to production is given by $\frac{E}{\lambda}$. The symmetric level of steady state labor devoted to R&D for each firm other than the leader is defined as l and the economy-wide share of labor devoted to R&D is $(n-1)l$. The labor market clearing condition is given by

$$L = \frac{E}{\lambda} + (n-1)l. \tag{1.7}$$

1.2.5 The Credit Sector

Firms finance research by borrowing from consumers at the risk free market rate $r(t)$. Through a well diversified portfolio investors can eliminate risk concerns. Investors then force firms to maximize expected returns from R&D, eliminating arbitrage possibilities.

1.3 Steady State Equilibrium

The steady-state perfect-foresight equilibrium derived here has the following properties:

1. Consumer expenditures remain constant over time, implying that the instantaneous interest rate equals the subjective discount rate.

2. Of the n firms with cutting edge R&D technology, there will be $n-1$ firms, which are non-producing followers, performing R&D. The remaining firm will be the sole producer of goods in the industry, but this firm peforms no R&D.

3. Each R&D firm will choose the same level of R&D effort independent of industry or time period.

4. Prices will be fixed across time and industry.

5. The wage rate will be constant over time.

1.3.1 Existence of the Steady State Equilibrium

Two equations determine the equilibrium for the model. The first equation is the "R&D condition" which captures the relationship during a patent race between firm R&D efforts and the benefits firms expect from winning the patent race. At the beginning of each R&D race, each firm takes the expected benefit associated with winning the patent race, V, as given. Each firm then takes the level of innovative effort by its competitors, $k = \sum_{j \neq i} h(l_j)$, as fixed and chooses R&D intensity to maximize its expected R&D profits. A non-leading firm i, investing in l_i units of R&D labor, faces expected R&D profits of

$$E\Pi(l_i, k) = \int_0^\infty V e^{-\rho t} h(l_i) e^{-h(l_i)t} e^{-kt}\, dt$$
$$- \int_0^\infty \left[\int_0^t l_i e^{-\rho s}\, ds \right] (k + h(l_i)) e^{-(k+h(l_i))t}\, dt. \qquad (1.8)$$

When making R&D choices for a patent race, each firm calculates the present value of expected benefits less costs from R&D. Each firm's expected benefits from R&D are calculated considering that it will capture the expected benefits of success, V, at time t, provided it successfully innovates at time t and no other firm has been successful in innovating prior to t. The firm's R&D labor choice determines the flow costs the firm expects. For the patent race, the expected costs from R&D for each firm are equal to the discounted value of the flow of R&D labor costs which stop accruing when the next industry success occurs. Solving equation (1.8) yields expected R&D profits for firm i of

$$E\Pi(l_i, k) = \frac{Vh(l_i) - l_i}{\rho + k + h(l_i)}. \qquad (1.9)$$

In the symmetric Nash equilibrium, $l_i = l$ for all $n - 1$ identical participants in the patent race. The R&D intensity choice which maximizes expected profits for each firm will satisfy the "R&D condition"

$$V_R = \frac{h(l) - lh'(l) + k + \rho}{h'(l)[\rho + k]} = \frac{(n-1)h(l) - lh'(l) + \rho}{h'(l)[\rho + (n-2)h(l)]}. \qquad (1.10)$$

Firms will only participate in a patent race if expected profits from R&D are non-negative. Substituting the expected benefit of winning an R&D race defined by the "R&D condition," equation (1.10), into the expected R&D profit equation, equation (1.9), yields that for each firm

$$E\Pi = \frac{h(l) - lh'(l)}{h'(l)[\rho + k]} = \frac{h(l) - lh'(l)}{h'(l)[\rho + (n-2)h(l)]}. \tag{1.11}$$

Expected R&D profits will be non-negative for each of the $n - 1$ firms in equilibrium provided the equilibrium level of labor hired by each firm i satisfies $l_i \geq \bar{l}_i$.

As a stability condition analogous to that found in Lee and Wilde, I make the following assumption about the form of R&D technology:

Assumption 1 *For all $l \geq \bar{l}$, the form of R&D technology satisfies*

$$\frac{d\frac{\partial E\Pi(l_i,k)}{\partial l_i}}{dl} \leq 0.$$

If this condition is violated then the solution to equation (1.10) will be unstable. If the solution is unstable then a multilateral increase in each firm's R&D efforts will generate the desire for each firm to increase efforts further, generating an infinitely repeated series of further increases in each firm's R&D efforts[5].

The second equation determining the steady-state equilibrium solution is the "labor market condition." This equation captures the relationship between R&D intensity in future patent races and the expected benefits from success in current patent races. In the steady state each firm is assumed to take the amount of R&D labor hired by all participants in all industries in the next R&D race as fixed at l. Given this steady state assumption of perfect foresight, the winner of the current R&D race

[5]See Appendix A for an explicit derivation of this restriction on technology. It is also demonstrated in Appendix A that a broad class of technological forms meet the requirements of this condition, including the constant returns technology adopted in previous quality ladders models.

earns expected benefits equal to expected discounted profits, or

$$V = \int_0^\infty \left[\int_0^t \frac{E(\lambda - 1)}{\lambda} e^{-\rho s} \, ds \right] (n-1)h(l) e^{-[(n-1)h(l)]t} \, dt. \tag{1.12}$$

The winner will earn leading firm profit flows until it is displaced by the innovator of the next generation product. Solving equation (1.12) yields

$$V = \frac{E(\lambda - 1)}{\lambda[\rho + (n-1)h(l)]}. \tag{1.13}$$

Equilibrium spending is defined by the labor market clearing condition, equation (1.7). Substituting this value of spending into the equation (1.13) yields the "labor market condition"

$$V_L = \frac{[L - (n-1)l][\lambda - 1]}{[\rho + (n-1)h(l)]} \tag{1.14}$$

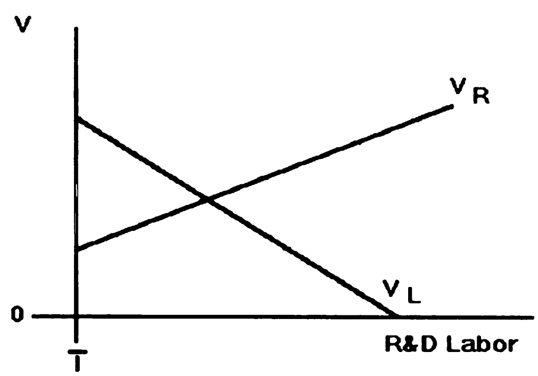

Figure 1.2: Steady State Equilibrium

The "labor market condition" is downward sloping in (l, V) space (see Figure 1.2). This relationship between firm R&D expenditure, l, and the expected benefit, V, to a successful patent race participant is defined by

$$\frac{\partial V_L}{\partial l} = -\frac{(\lambda - 1)(n-1)[\rho + Lh'(l) + (n-1)[h(l) - lh'(l)]]}{[\rho + (n-1)h(l)]^2}. \tag{1.15}$$

The relationship is negative, $\frac{\partial V_L}{\partial l} < 0$ for all values of $l \geq \bar{l}$. Increases in the level of firm R&D activity have two effects on the discounted value of winning the R&D race. First, with more resources devoted to R&D in the next patent race, fewer resources will remain for the winner to use in the production of output during the next race. This reduces the size of flow profits each current patent race participant expects if it successfully innovates. Second, with more R&D activity in the next patent race each firm in the current race expects to retain any increase in market share it gets from winning the R&D race for a shorter period of time. Both effects decrease the expected value of winning a patent race for all economically meaningful values of l and V[6].

The "R&D condition" is upward sloping in (V, l) space for all $l \geq \bar{l}$ (see Figure 1.2). This relationship between l and V is defined by

$$\frac{\partial V_R}{\partial l} = \frac{-h''(l)[\rho(2n-3)h(l) + \rho^2 + (n-1)(n-2)h(l)^2] + (n-2)h'(l)^2[lh'(l) - h(l)]}{h'(l)^2[\rho + (n-2)h(l)]^2}. \tag{1.16}$$

Given Assumption 1, greater expected benefits support greater equilibrium R&D efforts by all firms and the relationship between V_R and l is positive for all $l > \bar{l}$.

The "labor market condition," is downward sloping for all $l \geq \bar{l}$, and eventually V_L becomes negative for large l. Given Assumption 1 the "R&D equation" is upward sloping for all $l \geq \bar{l}$. Further the "R&D equation" is everywhere positive for $l \geq \bar{l}$ as firms only choose positive R&D efforts if the expected benefits from winning are

[6]For sufficiently large values of l, V_L will be negative. Any positive growth equilibrium will occur at values of V strictly greater than zero, which is shown below to occur when the economy is endowed with a sufficiently large labor supply.

positive. A unique symmetric steady-state equilibrium exists with firms choosing positive amounts of R&D labor provided $V_L(\bar{l}) \geq V_R(\bar{l})$. This equilibrium is illustrated in Figure 1.2. Assumption 2 ensures a positive growth equilibrium exists.

Assumption 2 *L is sufficiently large so that for $l \geq \bar{l}$*

$$L \geq \frac{[\rho + (n-1)h(\bar{l})]}{h'(\bar{l})(\lambda - 1)} + (n-1)\bar{l}.$$

When this assumption is met, the labor resources remaining when firms each choose the minimum profitable R&D effort, $\bar{l}$, are sufficiently large to make R&D at $\bar{l}$ profitable[7].

Proposition 1 *Given Assumptions 1 and 2, a unique steady state equilibrium exists with firms earning non-negative expected profits from R&D.*

Given existence of a unique equilibrium solution, the equilibrium value of R&D labor chosen by each patent race participant is defined implicitly by

$$\mathcal{H} = V_L - V_R = \frac{[L-(n-1)l][\lambda - 1]}{[\rho + (n-1)h(l)]} - \frac{(n-1)h(l) - lh'(l) + \rho}{h'(l)[\rho + (n-2)h(l)]} = 0. \tag{1.17}$$

[7]From the labor market clearing condition, equation (1.7), $L - (n-1)\bar{l} = \frac{E}{\lambda}$, which is market output. Assumption 2 requires resources after R&D hiring to be sufficiently large that

$$\frac{E}{\lambda} \geq \frac{\rho + (n-1)h(\bar{l})}{h'(\bar{l})(\lambda - 1)}.$$

Rearranging and substituting $\frac{h(\bar{l})}{\bar{l}}$ for $h'(\bar{l})$ yields

$$\left[\frac{\left(\frac{E(\lambda-1)}{\lambda}\right)}{\rho + (n-1)h(\bar{l})}\right] h(\bar{l}) - \bar{l} \geq 0,$$

which is precisely the condition for R&D to be profitable for each firm at $\bar{l}$.

1.3.2 Comparative Steady-State Analysis

When the economy is endowed with more labor resources, each firm will hire more R&D labor in each race. This effect works through the labor market condition. To illustrate, note that a current patent race participant knows that if the resource endowment grows and firm R&D efforts in the next patent race remain fixed then more labor will be devoted to production during the next race. With more output being produced in the next race the profit flows the winner of the current race (next period's producer) expects to earn are larger. This effect results in a shift to the right in the labor market curve, as the benefits a winner expects increase for every given R&D effort level. With an increase in the economy's resource endowment, the equilibrium value of firm R&D effort and equilibrium benefits patent race winners expect both increase.

Each firm will also hire more R&D labor in each race when the size of innovations are larger. This effect also works through the labor market condition. To illustrate, note that a current patent race participant knows that if the next innovation is relatively larger both the markup firms are able to charge for output and profit flows will be greater to the current winner (and next period producer) given fixed firm R&D efforts in the next round. This effect results in a shift to the right in the labor market curve as the benefits a winner expects increase for every given R&D effort level. With an increase in the size of innovations the equilibrium value of firm R&D effort and equilibrium benefits patent race winners expect both increase.

Firm R&D efforts will increase both when the economy's resource endowment is larger and when the size of innovations are larger. In both cases, with a constant number of firms each choosing greater R&D intensity, industry innovations are expected to arrive at a more rapid rate.

An increase in the subjective discount rate will decrease firm R&D efforts. In this case there are effects associated both with the labor market condition and the R&D condition. To illustrate the R&D effect note that when the subjective discount rate increases both the expected benefits from R&D and the expected costs will fall. The expected benefits from R&D fall because the benefits a firm expects to receive after winning a patent race are discounted at a greater rate. The expected costs from R&D fall because the flow of expenditures during the patent race are also discounted at a greater rate. The fall in the marginal expected costs from R&D exceeds the fall in expected marginal benefits and firms wish to increase their R&D intensity. This is reflected in a shift to the right in the R&D curve. However, the Labor market effect works in the opposite direction. Once a firm successfully innovates a greater discount rate reduces the present discounted value of the flow of profits the winner expects reducing the expected benefits of winning for any given level of R&D effort in the future. This is reflected in a shift to the left in the labor market curve. Using the implicit function theorem on equation (1.17), the effect through the labor market can be shown to dominate the R&D effect and an increase in the discount rate reduces the R&D efforts of firms in equilibrium (see Appendix B). As intuition would suggest when society cares less for the future, present consumption increases and fewer resources are devoted to R&D investments with future payoffs[8].

[8] There is an exception to this result. When there exist only two active firms in each industry

Firm R&D efforts will decrease when the subjective discount rate increases. In this case, with a constant number of firms each choosing smaller R&D intensities, industry innovations are expected to arrive at a slower rate.

Using equation (1.17) and the implicit function theorem, it also possible to show that increased patent race participation will reduce each firm's R&D efforts[9] (see Appendix C). There are three effects of increased participation on firm R&D effort. The first two effects are negative and work through the labor market condition. The third effect is positive and works through the R&D condition.

First, assuming patent race participation increases in each period and firm R&D efforts remain fixed, current patent race participants expect that in the next race fewer resources will remain for production of output. This reduces both the flow profits and total benefits winning firms expect. This effect is captured by a shift to the left in the labor market condition. Second, assuming patent race participation increases in each period and firm R&D efforts remain fixed current patent race participants expect innovations to arrive sooner. This reduces both the period of time a winner expects to receive leader profit flows and the benefit of winning an R&D race. This effect is captured by a shift to the left in the labor market condition. Both effects reduce

over time, with only one firm maintaining a high intensity R&D program and the other producing output, the R&D effect may dominate and increases in ρ may increase R&D efforts in equilibrium.

[9]There is also an exception to this result. If the number of firms active in the industry increases from two to three, then when ρ is small firm level R&D efforts may increase. When there exist only two active firms in each industry over time one maintains a high intensity R&D program and the other produces output. In this case, entry of a new firm into the patent race creates competition where it was formerly absent.

the equilibrium R&D effort chosen by each firm and the expected benefits winners receive in equilibrium.

The R&D effects resulting from increased patent race participation are positive and mirror the effects found in the Lee and Wilde model. With a constant expected benefit of winning and more participants in the patent race, the expected marginal benefit of R&D for firms falls, as it becomes less likely any one firm will be the first to innovate. At the same time the race is expected to end sooner, reducing expected marginal costs from R&D. The latter effect dominates and each firm increases its R&D effort for any given expected benefit of winning. This is captured by a shift to the right in the R&D curve increasing equilibrium R&D efforts but diminishing the expected benefit of winning in equilibrium. This R&D effect is dominated by the labor market effects and when patent race participation rises each firm reduces its R&D effort while the expected benefit to patent race winners falls in equilibrium.

Proposition 2 *Given Assumptions 1 and 2, if $n \geq 3$ the steady state solution with a greater number of firms competing in the R&D sector has each firm undertaking a strictly smaller amount of R&D.*

There are two effects of participation increases on the industry-wide instantaneous probability of success. First, each firm does less R&D, lowering the instantaneous probability of success in the industry at any moment. Second, more firms do R&D, increasing the industry-wide instantaneous probability of success. Above it was shown that increased participation unambiguously decreases the expected benefit to winners of patent races. Given that the equilibrium value of V declines, the value of V defined by the "labor market condition" will decline with the new participation level. Four possibilities occur:

(i) $(n-1)l$ is non-increasing and $(n-1)h(l)$ is non-increasing as n increases

(ii) $(n-1)l$ is non-decreasing and $(n-1)h(l)$ is non-increasing as n increases

(iii) $(n-1)l$ is non-increasing and $(n-1)h(l)$ is non-decreasing as n increases

(iv) $(n-1)l$ is non-decreasing and $(n-1)h(l)$ is non-decreasing as n increases

Case (i) is easily ruled out. To see this note that it cannot be the case that $(n-1)l$ and $(n-1)h(l)$ both are non-increasing as n increases, as in this case V_L would be non-decreasing, contrary to what was proven above.

If (ii) is true then, defining l as an implicit function of n,

$$\frac{d[(n-1)l]}{dn} = l + (n-1)\frac{dl}{dn} \geq 0, \text{ and } \frac{d[(n-1)h(l)]}{dn} = h(l) + (n-1)h'(l)\frac{dl}{dn} \leq 0,$$

where dn is strictly positive and $\frac{dl}{dn}$ is strictly negative. Then

$$1 \geq -\left(\frac{(n-1)}{l}\right)\frac{dl}{dn} \Rightarrow -\left(\frac{h(l)(n-1)}{l} - (n-1)h'(l)\right)\frac{dl}{dn} \leq 0$$
$$\Rightarrow -(n-1)\left[\frac{h(l)}{l} - h'(l)\right]\frac{dl}{dn} \leq 0.$$

In equilibrium $\frac{h(l)}{l} > h'(l)$, for $l > \bar{l}$ so case (ii) cannot occur.

The remaining two possibilities both include a non-decreasing value of the industry-wide probability of success $(n-1)h(l)$. Further, it must be that the instantaneous probability of success in the industry is strictly increasing as no change in $(n-1)h(l)$ falls under either case (i) or (ii) depending on the change in $(n-1)l$. Clearly, when participation increases the industry-wide instantaneous probability of success increases and industries expect innovations to arrive at a faster rate.

Proposition 3 *Given Assumptions 1 and 2, if $n \geq 3$ the steady state solution with a greater number of firms competing in each patent race has a strictly greater industry-wide instantaneous probability of success. Thus the steady state level of growth is strictly higher when more firms compete in R&D in each industry.*

The change in equilibrium profits from an increase in R&D participation is, using equation (1.11) and defining both l and k implicitly in n,

$$\frac{dE\pi(l,k)}{dn} = \frac{\partial E\pi(l,n)}{\partial l}\frac{dl}{dn} + \frac{\partial E\pi(l,k)}{\partial k}\frac{dk}{dn}. \tag{1.18}$$

The first term on the right hand side of equation (1.18) is zero, as $\frac{\partial E\pi(l,n)}{\partial l} = 0$ in equilibrium. When participation increases the change in k is given by

$$\frac{dk}{dn} = (n-2)h'(l)\frac{dl}{dn} + h(l) = \frac{d[(n-1)h(l)]}{dn} - h'(l)\frac{dl}{dn} \geq 0. \tag{1.19}$$

It was shown above that the industry-wide instantaneous probability of success is increasing in firm participation and individual firm R&D efforts fall with increased participation. Thus, for each firm, the instantaneous probability of a competitor succeeding increases as more firms participate in R&D. Therefore, equilibrium profits decrease as more firms participate in R&D.

Proposition 4 *Given Assumption 1, if $n \geq 3$ and equilibrium profits are positive, then as the number of firms increase in steady state, equilibrium firm profits from R&D decrease.*

It is assumed that the number of firms able to perform R&D in each industry is sufficiently large such that leaders never do R&D. This condition is met provided the number of firms with the ability to perform R&D exceeds a critical value, where the critical value is strictly less than the number of active firms in the free-entry equilibrium. Two conditions ensure both existence and uniqueness of this critical value. First, it was shown above that the profit maximizing value of R&D labor chosen by each firm is decreasing in the number of participants and that profits reach zero when equilibrium intensity falls to $\bar{l}$. Second, a current leader who wishes to join the R&D race faces the same R&D technology as a follower, but expects strictly lower benefits from a success than a follower. To illustrate, note that current leaders who successfully innovate first are able to further markup the price on their products by the factor λ, the increase in their quality-adjusted pricing advantage. The additional markup yields an increase in the flow of profits given by

$$\Delta\pi = \frac{E(\lambda - 1)}{\lambda^2}, \tag{1.20}$$

which is strictly less than the additional flow of profits to a non-leader. As both potential entrants and the leader earn the profit flows for the same expected period of time in steady state, the benefit a leader expects from a win is a fraction $\frac{1}{\lambda}$ of the benefit a follower expects from a win. Given Assumption 1 the profit maximizing choice of R&D labor is strictly decreasing in the expected value of a win, V, and since the expected benefit of a win is always lower for the current state-of-the-art producer than for a follower, the leader will always choose less R&D labor. Combining these two conditions yields that expected R&D profits for the leader will fall to zero (the profit maximizing l reaches $\bar{l}$ for the leader) before entry has reduced expected follower profits to zero. This critical number of firms is unique and strictly less than the number of participants in free entry, but the exact value is indeterminate given the general nature of the R&D technology adopted here. Focus in this article is restricted to levels at or beyond the critical value of firm participation, where profits to the leader from R&D are negative and leaders do not compete[10].

1.3.3 The Free Entry Equilibrium

The free entry equilibrium is defined as the steady state equilibrium in which expected profits from R&D are zero. The number of active firms with the ability to perform R&D in the free entry equilibrium is defined as n^*. Of these firms $(n^* - 1)$ will participate in each patent race, with the state-of-the-art producer abstaining. When the number of firms active in each patent race reaches $(n^* - 1)$, no further firms will wish to enter the patent race as R&D is not profitable.

There are two cases of free entry equilibria — with initial increasing returns in R&D, and without. It is easily seen from equation (1.11) that without initial increasing returns, there will always be an opportunity for firms to earn positive profits as long as success by another firm in the industry is not instantaneous. The free entry number of firms approaches infinity in this case. Although the number of firms approaches infinity, investment in R&D by each approaches zero. From Proposition 3 it is clear that the free entry solution involves a strictly greater amount of industry

[10]This formulation ignores the possibility of different technological capability for leaders. This is an area open for future exploration into the dynamics involved between market leaders and followers. For an example where technological opportunities differ for leaders and followers see Grossman and Helpman[1991a] and Barro[1995].

R&D than does any solution with fewer firms participating in the patent races.

If an initial range of increasing returns does occur, then as the number of firms is increased, the equilibrium value of firm R&D approaches $\bar{l}$. At $\bar{l}$ expected profits for each participating firm conducting R&D are zero. Equation (1.17), the implicit equation defining equilibrium R&D intensity, can be used to find (n^*), which is

$$n^* = \frac{Lh(\bar{l})(\lambda - 1) - \bar{l}\rho}{\lambda h(\bar{l})\bar{l}} + 1. \tag{1.21}$$

Attention below is restricted to the case where entry is finite.

1.4 Welfare Implications of the Model

By definition, any steady state equilibrium will consist of a constant number of firms participating in R&D at the same intensity, in each industry, across time. Taking the number of firms, n, in each industry as fixed and also taking the amount of R&D labor hired by each patent race participant, l, as fixed, independence of R&D effort gives a time-invariant industry R&D parameter of $(n-1)h(l)$. Using the law of large numbers and properties of the Poisson distribution, the steady state instantaneous utility at time t becomes

$$u(t) = (n-1)h(l)t \log \lambda - \log \lambda + \log E. \tag{1.22}$$

Integrating over time and discounting at the subjective discount rate ρ gives the steady state intertemporal utility discounted to time $t = 0$ of

$$U = \frac{\log E - \log \lambda}{\rho} + \frac{(n-1)h(l) \log \lambda}{\rho^2}. \tag{1.23}$$

Combined with the resource constraint, $E \leq \lambda[L - (n-1)l]$, the representative consumer's discounted utility can be used to analyze welfare implications of possible equilibria.

I examine the results of two possible social planning equilibria and compare these to the results of the model when entry is unrestricted. The first possible social planning equilibria allows the social planner to choose only the level of R&D effort of each firm, with the number of firms fixed at the free-entry level, n^*. The second possible planning equilibrium allows the social planner to choose the individual firm effort and the degree of participation in R&D in each industry.

In the first social planning equilibrium, the social planner chooses only the level of R&D done by each firm, represented by l^{**}. Taking the number of firms as fixed at n^*, the social planner maximizes intertemporal utility subject to the resource constraint by choosing l^{**} to satisfy

$$[L - (n^* - 1)l^{**}]h'(l^{**}) = \frac{\rho}{\log \lambda}. \tag{1.24}$$

This equation implicitly defines the socially optimum R&D effort for each firm. In the free market each firm chooses an effort $l = \bar{l}$ which under free entry satisfies

$$[L - (n^* - 1)\bar{l}]h'(\bar{l}) = \frac{[\rho + (n^* - 1)h(\bar{l})]}{(\lambda - 1)}. \tag{1.25}$$

A comparison of these two equations determines how the free market solution compares to the solution the planner would choose.

When $\bar{l}$ is substituted for l^{**} in equation (1.24) the left hand sides of both equations (1.24) and (1.25) are equal to $\frac{[\rho+(n^*-1)h(\bar{l})]}{(\lambda-1)}$. Equation (1.24) will be satisfied with equality at $\bar{l}$ provided $\frac{\rho}{\lambda} + \frac{Lh(\bar{l})}{\lambda l} = \frac{\rho}{\log \lambda}$. In this case the free entry equilibrium and the social planning equilibrium will be equivalent. This determines a critical value of labor supply for which the free entry solution is socially optimal. This value is given by

$$L^C = (\frac{\rho}{h'(\bar{l})})(\frac{\lambda}{\ln \lambda} - 1). \tag{1.26}$$

If the resource endowment exceeds this critical value then $\frac{\rho}{\lambda} + \frac{Lh(\bar{l})}{\lambda l} > \frac{\rho}{\log \lambda}$ and substitution of $\bar{l}$ for l^{**} into equation (1.24) will not produce equality. In this case, because the left hand side of equation (1.24) exceeds the right hand side with the substitution of $\bar{l}$ for l^{**}, and because the left hand side is decreasing in l, a value l^{**} greater than $\bar{l}$ is necessary to satisfy equation (1.24). Whenever the resource endowment exceeds the critical level defined in equation (1.26) the social planner will choose a larger R&D effort for each firm than occurs in the free entry equilibrium . It clearly follows that if the economy's resource endowment is below the critical value the social planner will choose a lower R&D effort for each firm than would occur in the free entry equilibrium.

There are three differences between private and social returns in the model. First, firms fail to appropriate all of the increases in consumer welfare that accompany an innovation because firms are unable to set perfectly discriminating prices. Second, firms do not consider that by innovating they create positive externalities for competitors in the industry by creating an increase in the knowledge base for future R&D in the industry. Both effects, the "consumer surplus" effect and the "intertemporal spillover" effect, lead to free market solutions with less individual R&D than is socially optimal. Firms also ignore the profits they will "steal" from other firms when they successfully innovate. This "business stealing" effect pushes the free market solution to a level of individual R&D greater than the socially optimal level. Equation (1.26) captures these effects and when the labor supply exceeds the critical value, L^C, the "consumer surplus" and "intertemporal spillover" effects will dominate the "business stealing" effect and welfare is raised by increasing firm efforts. When the labor supply falls short of the critical value, the "business stealing" effect dominates and welfare increases when firms reduce their R&D efforts[11].

Proposition 5 *Relative to the social planning equilibrium where the social planner chooses only the level of R&D effort by each firm, taking the number of active firms in each industry as fixed at n^*, the free entry steady state equilibrium sees each firm undertaking too little, the correct amount, or too much R&D as the labor force is greater than, the same as, or less than L^C, respectively.*

In the second social planning equilibria the planner is free to choose both the effort of each firm, l^{**}, and the number of firms participating in R&D, n^{**}. To maximize the representative consumer's discounted utility subject to the resource constraint the social planner chooses l^{**} and n^{**} to satisfy both

$$[L - (n^{**} - 1)l^{**}]h'(l^{**}) = \frac{\rho}{\log \lambda}, \text{ and} \tag{1.27}$$

[11] At the free entry equilibrium the additional societal benefit (over what the firm expects) from increasing R&D efforts associated with both the consumer surplus and intertemporal effects is $\frac{\log \lambda}{\rho}$. The loss in societal benefit associated with the business stealing effect is $\frac{\lambda}{\rho + Lh'(l)}$. Which effects will be larger depend precisely on how large the labor supply endowment is relative to the critical value L^C. This argument follows the argument for the model with constant returns technology found in Appendix A4.1 in Grossman and Helpman[1991b].

$$\frac{l^{**}}{[L-(n^{**}-1)l^{**}]}=\frac{h(l^{**})\log\lambda}{\rho}. \tag{1.28}$$

Combining the two equations yields the efficient social level of individual firm R&D effort of $l^{**}=\bar{l}$. When the social planner chooses both the number of firms doing R&D and the level of effort by each firm, the planner's choice of effort will be the same as will occur in the unrestricted entry case. The optimal number of firms is then

$$n^{**}=1+\frac{Lh(\bar{l})\log\lambda-\bar{l}\rho}{\bar{l}h(\bar{l})\log\lambda}. \tag{1.29}$$

Comparing n^{**} to n^{*} from equation (1.21) we find that $n^{**}>n^{*}$ when $L>L^{C}$, $n^{**}=n^{*}$ when $L=L^{C}$, and $n^{**}<n^{*}$ when $L<L^{C}$. As in the case of the first planning solution, equation (1.26) captures the "consumer surplus ", "intertemporal spillover", and "business stealing" effects.

Proposition 6 *Relative to the social planning equilibrium where the social planner chooses both the number of firms participating in industry R&D and the level of R&D effort by each firm, the free entry steady state solution sees each firm undertaking the socially efficient level of firm R&D effort but too few, the correct amount, or too many firms participating in the R&D race as the labor force is greater than, the same as, or less than L^{C}.*

In the free entry equilibrium, the industry R&D effort will not maximize welfare if the resource endowment does not equal L^{C}. When this occurs a social planner without control over entry will choose to change individual R&D efforts to improve welfare. A planner with control over entry will choose entry to maximize welfare but allow firms to choose R&D effort themselves since they choose the same level of effort the planner would choose for them. By controlling the number of firms the planner is able to take full advantage of returns to scale in R&D technology.

1.5 Corrective Subsidies

Consider subsidies to R&D financed by lump sum taxes on consumers. If the government pays a fraction, s, of R&D expenditures for each patent race participant then each firm's expected profits from R&D become

$$\frac{Vh(l) - (1-s)l}{\rho + k + h(l)}. \tag{1.30}$$

When the number of patent race participants is fixed, each firm maximizes profits, generating the "subsidized R&D condition"

$$V_R = (1-s)\frac{[\rho + (n-1)h(l) - lh'(l)]}{h'(l)[\rho + (n-2)h(l)]}. \tag{1.31}$$

When R&D is subsidized each firm chooses greater R&D intensity for any given expected benefit to the winner. This is because for any fixed level of firm R&D intensity R&D subsidies reduce the marginal cost of R&D but do not effect the marginal benefit of R&D. Firms then wish to increase R&D effort for any given expected benefit of winning the race. Graphically this translates into a shift to the right in the R&D curve in Figure 1.2. R&D subsidies do not enter the labor market condition. With subsidized R&D each firm increases its R&D expenditure.

Alternatively, the government could choose to subsidize production for the winner. When the government subsidizes a fraction, s, of production labor, then the limit price charged by state-of-the-art producers falls to $\lambda(1-s)$. With a lower output price the "subsidized labor market condition" becomes

$$V_L = (1-s)\frac{[L-(n-1)l][\lambda-1]}{[\rho+(n-1)h(l)]}. \tag{1.32}$$

The "subsidized labor market condition" yields that an increase in the subsidy to production will diminish the expected benefit, V_L, of winning a patent race associated with each symmetric level of individual firm R&D effort. The increased subsidy initially decreases the limit price charged by state-of-the-art producers. The lower price will increase demand for output and subsequently raise demand for production workers. With a fixed share of labor devoted to R&D, the labor supply will be insufficient to satisfy the increase in demand for production workers. To clear the labor market, aggregate consumer expenditures will fall, forcing a reduction in the demand for output and production workers. With a lower level of equilibrium consumer spending, profit flows earned by successful patent race participants fall, and the expected benefits of winning a race diminish[12]. Graphically, larger subsidies to production shift the V_L curve left in Figure 1.2. Subsidies to production do not enter the R&D condition. R&D efforts then decrease when production is subsidized.

Proposition 7 *With a fixed number of firms, increasing subsidies to R&D or decreasing subsidies to production result in unambiguous increases in the amount of steady state R&D done by each firm. As a result, industry R&D increases and the economy grows at a faster rate.*

[12]If consumer spending remains constant, profit flows to the patent race winner do not change as the decrease in profit flows from the lower limit price are exactly offset by the increase in profit flows resulting from both lower costs and increased demand.

If R&D is subsidized and firms are allowed to freely enter and exit patent races, R&D profits will diminish as the equilibrium value of R&D effort diminishes. R&D profits will reach zero when equilibrium R&D effort for each firm is $\bar{l}$. Subsequently, subsidies to R&D will not effect the level of R&D done by each firm when entry is unrestricted. The number of firms active in patent races as a result of R&D subsidies can be found by solving equation (1.17) for n when $l = \bar{l}$ and the "subsidized R&D condition" replaces the unsubsidized "R&D condition." The maximum number of firms able to do profitable R&D in each patent race is then

$$n^* = \frac{L(\lambda - 1)h'(\bar{l}) - (1 - s)\rho}{(\lambda - s)h(\bar{l})} + 1. \tag{1.33}$$

The free entry number of firms rises with subsidies to R&D.

With free entry and subsidies to production, each firm's individual R&D efforts will also remain unchanged at $\bar{l}$. Solving equation (1.17) for n when $l = \bar{l}$ and the "subsidized labor market condition" replaces the unsubsidized "labor market condition," the free-entry number of firms active in R&D is

$$n^* = \frac{L(\lambda - 1)(1 - s)h'(\bar{l}) - \rho}{[1 + (\lambda - 1)(1 - s)]h(\bar{l})} + 1. \tag{1.34}$$

Given Assumptions 1 and 2 the free entry number of firms active in R&D in each industry will decrease with subsidies to production .

Proposition 8 *In the free entry steady state, given Assumptions 1 and 2, increases in subsidies to R&D or decreases in production subsidies permit higher steady state levels of patent race participation.*

1.6 Conclusion

The introduction of R&D technology characterized by decreasing returns at the firm level into the quality ladders model generates results not previously available on the relationship between patent race participation and R&D behavior. Dynamic characteristics of firm entry into R&D races result in a negative relationship between steady state participation and R&D intensity, proving the importance of examining repeated rather than single patent race models.

While the model makes strides towards a clearer understanding of the R&D process, many areas of research remain. Relaxing the assumption of homogeneous R&D technology may give greater insight into the relationship between industry leaders and potential entrants. Imitation also plays an important part in firm R&D behavior, and the relationship between participation in patent races and firm R&D behavior when imitation occurs warrants further study. What is clear from the model is that understanding individual firm R&D behavior is important to understanding technologically driven growth, and that understanding firm R&D behavior requires understanding intertemporal aspects of repeated patent races.

Chapter 2

Market Share Retention in Quality Ladder Industries

2.1 Introduction

A common finding by authors examining endogenous growth models with quality ladder structures[1] is that industry leaders (the firms producing the state-of-the-art products in each industry) do not engage in R&D activities. All efforts to either improve the quality of the industries product line or reduce production costs are conducted by outsiders (or followers, firms without state-of-the-art production capabilities). This R&D behavior results in a continuous leap-frogging pattern of industry leadership. However, these models fail to explain the common phenomenon of market leaders creating improvements in their own products as demonstrated, for example,

[1]Examples of quality ladders growth models can be found in Grossman and Helpman[1991a, 1991b] and Segerstrom[1991].

by Intel's successful creation of successive microchip generations. The result that leaders sit out patent races to improve on their own product is a direct result of the assumption of constant returns to scale in R&D technology[2]. With constant returns to scale the economies in previous quality ladder models jump directly to steady-state equilibrium, where R&D is a break even opportunity for outsiders. Leaders who consider innovation on themselves will receive a smaller reward for R&D than outsiders, as each leader considers the business lost by replacing its own previous generation of product, a consideration not taken into account by outsiders. Outsiders who do not consider this business stealing effect will face a greater expected reward for R&D than leaders, implying leader R&D is not profitable in steady state.

When R&D technology exhibits eventual decreasing returns to scale and entry into patent races is unrestricted, the quality ladders Nash equilibrium will also find leaders sitting out patent races to improve on their own products[3]. Outsiders will enter patent races until R&D is a break even opportunity. As with constant returns to scale, because of the business stealing effect, leaders face strictly negative profits from R&D in equilibrium. However, if entry into patent races is restricted by the

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[2]An exception to this result is found in a recent quality ladders model developed by Barro and Sala-i-Martin[1995]. The result found by Barro and Sala-i-Martin, that leaders may do all R&D in equilibrium, requires an implicit assumption not found in previous quality ladders models. The assumption underlying Barro and Sala-i-Martin's result is that leaders possess a first mover advantage and can commit to a long term R&D program, and followers observe the leader's behavior and react accordingly. It is assumed here that leaders do not have a first mover advantage, or equivalently that a leader's R&D program is unobservable to outsiders.

[3]For a model with firm level decreasing returns to scale see Chapter 1.

ability of only a limited number of firms to do cutting-edge R&D, quality leaders may enter the R&D race.

This article examines the case where a limited number of firms in each industry possess cutting-edge R&D technology characterized by eventual decreasing returns. In this setting, it is shown that provided the number of firms able to do R&D is small and returns to R&D are sufficiently diminishing, leaders will participate in R&D. Thus, even in the event that leaders have no technological advantage, we may see a single firm create successive product generations. While leaders may do R&D, it is shown here they will always do strictly less R&D than outsiders when both leaders and outsiders possess the same R&D technology.

The welfare maximizing solution deviates from the free market solution in three ways. In the free market solution the monopoly power possessed by firms protected by patents on state-of-the-art products will result in a socially inefficient price of output. The price charged by each monopolist will exceed the marginal cost of production and the result will be a loss in welfare from the accumulation of deadweight losses over time. When a subsidy is levied which increases output to the socially optimal level firms will under-invest in R&D. Under-investment in R&D results from each successful firm's failure to appropriate the full benefits of innovation. Firms which obtain monopoly power will derive benefits which fall short of the gains consumers experience from successful innovation. This appropriation problem can be corrected by subsidizing R&D for each firm. The final distortion is caused by outsider's failure to consider the loss of business by previous generation producers, a distortion that the current state-of-the-art producer will consider. Although both followers and leaders under-invest in R&D, the business stealing effect will result in greater R&D under-investment by leaders. The incentive difference between leaders and followers can be corrected by granting greater subsidies to leaders for R&D with leader subsidies increasing in the size of the leaders quality advantage. This scheme of subsidies for production and for R&D corrects for inefficiencies inherent in the free market.

The remainder of the chapter is organized as follows. In Section 2 the parameters and assumptions regarding behavior by consumers and firms in the markets for labor, credit, R&D, and output are introduced. The existence and uniqueness of a steady-state equilibrium solution is presented in Section 3. In Section 4 results regarding the relative R&D efforts of leaders and followers are derived. Section 5 contains a welfare analysis and the derivation of an optimal subsidy scheme which maximizes consumer welfare. Section 6 offers concluding remarks.

2.2 The Model

The quality ladders model considered here is a model with an economy comprised of a continuum of industries indexed by ω on the unit interval [0,1]. Each industry produces a variety of goods unique to the industry. Within each industry, goods are differentiated only by quality, where quality is indexed by j. There are a countably infinite number of quality levels of each good, with j taking on integer values, but only those which have been invented can be successfully produced. At time $t = 0$ the state-of-the-art good in each industry has a quality index of $j = 0$. In each industry, a fixed number of firms, n, compete in repeated R&D races to create higher quality state-of-the-art goods. The winner of each R&D race becomes the industry "quality leader," increasing the quality of the previous state-of-the-art product by a factor $\lambda > 1$ and increasing the state-of-the-art quality index in industry ω at time t, $j(\omega, t)$, by one.

2.2.1 The Consumer Sector

Consumers are identical, have preferences that extend infinitely into the future, and have intertemporal preferences over goods of the form

$$U \equiv \int_0^\infty u(t)e^{-\rho t}\,dt. \tag{2.1}$$

The representative consumer's subjective discount rate is given by ρ. Instantaneous utility at time t is represented by $u(t)$, which takes the form

$$u(t) \equiv \int_0^1 \ln\left[\sum_0^\infty \lambda^j d(j, \omega, t)\right] d\omega. \tag{2.2}$$

The consumer's consumption of goods of quality j from industry ω at time t is represented by $d(j, \omega, t)$. The parameter λ, where λ^j is the measure of quality of a good which has been improved on j times, is assumed constant across industries.

Each consumer chooses a level of spending at time t, given by $E(t)$. The consumer, taking market prices as given, allocates $E(t)$ to maximize $u(t)$. The budget constraint on the consumer is given by

$$\int_0^\infty E(t)e^{-R(t)t}\,dt \le A_0. \tag{2.3}$$

A_0 is the sum of the present discounted value of the flow of wage and profit income added to the value of asset holdings at time $t = 0$. The cumulative interest factor up to time t is represented by $R(t)$.

The representative consumer's problem can be solved by a three step backwards induction process. First, at time t, goods within an industry are perfect substitutes and the consumer will allocate all spending on goods in the industry to goods with the lowest quality adjusted price[4]. Second, given the nature of the Cobb-Douglas instantaneous utility function, the consumer will allocate equal shares of $E(t)$ to each industry. In the steady-state equilibrium, the state-of-the-art good in each industry will have the lowest quality adjusted price at each moment and, for price $p(\omega, t)$, face demand $d(\omega, t) = \frac{E(t)}{p(\omega,t)}$.

The solution to the consumer's intertemporal optimization problem dictates that spending evolve according to

$$\frac{E'(t)}{E(t)} = r(t) - \rho. \tag{2.4}$$

In any steady-state equilibrium where consumer spending is constant over time, the instantaneous interest rate, $r(t)$, must equal the consumer's subjective discount rate, ρ. Spending each period is given by the sum of wage income from non-R&D labor plus profits. The aggregate steady-state flow of spending is defined to be E.

2.2.2 The Production Sector

The production technology is characterized by constant returns to scale where one unit of labor produces one unit of any good independent of time, industry, or quality.

[4]Below it will be shown that, in each industry, in the steady-state equilibrium, the firm with the ability to produce the highest quality product, the quality leader in the industry, will set the lowest quality adjusted price and be the sole producer of goods in the industry.

The wage rate is normalized to one, giving each firm a constant marginal cost of one.

Producers within an industry compete in prices. In the steady-state consumers allocate a portion of their income E to spending on each industry's goods, with all of E allocated to the good with the lowest quality adjusted price. It is assumed, without loss of generality, that when faced with multiple goods from an industry with differing qualities but the same quality adjusted price, consumers will prefer the good of the highest quality. A firm with the ability to produce the state-of-the-art good k quality steps ahead of its nearest rival can, by charging a price less than or equal to λ^k, capture the entire market for goods within its industry. With unitary elastic demand and constant marginal cost, profits are maximized when the firm charges a price equal to λ^k and are given by

$$\pi^k = \frac{E(\lambda^k - 1)}{\lambda^k}. \tag{2.5}$$

For an outsider with no current market share in an industry, successful creation of a new state-of-the-art product results in an increase in profit flows for the firm of

$$\pi^1 = \frac{E(\lambda - 1)}{\lambda}. \tag{2.6}$$

A quality leading firm which is successful in increasing its quality lead from k steps ahead to $k + 1$ steps ahead of its nearest rival will increase its profit flows by

$$\pi^{k+1} - \pi^k = \frac{E(\lambda^{k+1} - 1)}{\lambda^{k+1}} - \frac{E(\lambda^k - 1)}{\lambda^k} = \frac{E(\lambda - 1)}{\lambda^{k+1}}. \tag{2.7}$$

The increase in profit flows experienced by a leader successful in gaining an additional quality step lead is strictly smaller than the increase in profit flows expected by an outsider and strictly decreases as the leader's quality advantage grows.

2.2.3 The Research Sector

In each industry there is assumed to be only a limited number of firms with the ability to do cutting-edge research. Of the n firms able to do viable R&D, a subset of $(n-1)$ of the n firms will be followers or outsider firms competing for entry into the market. The remaining firm, the single leader and sole producer, will compete to obtain a greater lead when one step ahead and do no R&D when it has a two-step advantage[5]. The n firms participating in each patent race will hire labor which is devoted to R&D. A firm which devotes x units of labor to R&D will innovate at time τ which arrives before the length of time t has expired according to the probability given by

$$prob[\tau(x) \leq t] = 1 - e^{-h(x)t}. \tag{2.8}$$

The R&D technology is assumed the same for both leaders and followers[6]. The parameter $h(x)\,dt$ measures the instantaneous probability of a successful innovation when x units of labor are devoted to R&D. The expected duration until success is given by $h(x)^{-1}$. Firms pay wages to R&D workers each period until a firm in the

[5]It is assumed leaders will not attempt to gain a three step advantage. Leaders will behave in this manner whenever the number of firms able to do cutting-edge R&D is sufficiently close to the number which would occur under free entry, as the expected discounted profits from R&D are decreasing in the size of the quality lead obtained. When the number of firms on the cutting-edge of R&D is sufficiently close to the free entry level, but far enough away that one-step leader R&D is profitable, expected profits from obtaining a three-step lead will be negative. Interest is restricted to this case.

[6]For a quality ladders model in an international setting, where leaders do R&D better than followers see Grossman and Helpman[1991c].

race successfully innovates.

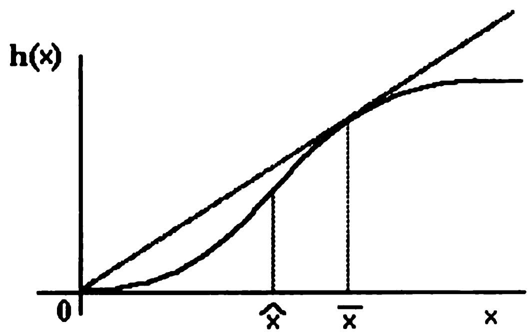

Figure 2.1: The Individual Firm R&D Technology

The function $h(x)$ (see Figure 2.1) is assumed to be twice continuously differentiable and strictly increasing in x. Increasing returns ($h''(x) > 0$) are assumed to prevail up to $\hat{x}$ for each firm where $\hat{x} > 0$. Beyond $\hat{x}$ the technology is characterized by decreasing returns[7] ($h''(x) < 0$). The function $h(x)$ is also assumed to satisfy $h(0) = 0 = h'(\infty)$. The average product of labor is maximized at $\bar{x}$, which is defined by the R&D labor choice which satisfies $\frac{h(x)}{x} = h'(x)$ when $\hat{x} > 0$.

2.2.4 The Labor Sector

The labor supply is homogeneous, fixed at L, and the labor market is assumed to clear each period. In equilibrium the share of labor devoted to production in industries with a one-step-ahead leader is given by $\frac{E}{\lambda}$. The labor devoted to production in

[7]Examining the relationship between patents granted and R&D spending, Kortum[1993] reports point elasticity estimates in the range 0.1 to 0.6, while Hall, Griliches, and Hausman[1986] obtain an average elasticity estimate of 0.3. Using market value data, Thompson[1993] obtains an R&D output elasticity with respect to R&D expenditure of 0.86.

industries with a two-step-ahead leader is given by $\frac{E}{\lambda^2}$. The level of steady-state labor devoted to R&D by the leader is defined as l and the symmetric level of steady-state labor devoted to R&D by each follower is defined as f. The share of labor devoted to R&D in industries with a one-step-ahead leader is $l + (n-1)f$, while the labor devoted to R&D in industries with a two-step-ahead leader is $(n-1)f$. When Θ is the fraction of industries with a one-step-ahead leader and $(1-\Theta)$ is the fraction of industries with a two-step-ahead leader the labor market clearing condition is given by

$$L = \Theta\left[\frac{E}{\lambda} + l + (n-1)f\right] + (1-\Theta)\left[\frac{E}{\lambda^2} + (n-1)f\right]. \tag{2.9}$$

2.2.5 The Credit Market

Firms finance research by borrowing at the risk-free market rate $r(t)$. Through a well diversified portfolio investors can eliminate risk concerns. Investors will then force firms to maximize expected returns from R&D.

2.3 Steady-State Equilibrium

The steady-state-perfect-foresight equilibrium examined here has the following properties:

1. Consumer expenditures do not vary over time implying that the instantaneous interest rate equals the discount rate

2. The proportion of industries each period with a one-step-ahead quality leader remains constant with all technologically able firms in these industries engaging

in R&D activities

3. The proportion of industries each period with a two-step-ahead quality leader remains constant with only outsiders in these industries engaging in R&D activities

4. The portion of labor allocated to R&D is constant over time as is the portion allocated to production

5. Both wages and the price index for the economy are time invariant

2.3.1 Steady-State Spending

In the steady state innovation efforts by followers and by one-step-ahead quality leaders are fixed across time and industries. For each industry with a one-step-ahead leader, the probability at each instant that the leader will increase its quality lead to two steps is $h(l)$. With a continuum of industries with a one-step-ahead leader, a fraction $h(l)$ of the industries will become industries with a two-step quality leader at each instant. For each industry with a two-step quality leader, the instantaneous probability that the two-step quality leader is replaced by an outsider is $(n-1)h(f)$. With a continuum of industries with a two-step quality leader, a fraction $(n-1)h(f)$ of the industries will become industries with a one-step leader at each instant. For the fraction of industries with a one-step leader and the fraction of industries with a two-step leader to remain constant, Θ must solve

$$\Theta h(l) = (1-\Theta)(n-1)h(f). \tag{2.10}$$

This condition ensures that at each instant the number of industries in which a leader moves from a one-step quality lead to a two-step quality lead is equivalent to the number of industries in which a two-step quality leader is replaced by a new entrant.

Substituting the industry divisions, defined by equation (2.10), into the labor market clearing condition, defined by equation (2.9), and simplifying gives consumer spending in steady state, defined in terms of R&D efforts and the model parameters as

$$E = \lambda \left\{ \frac{(n-1)h(f)[L-l-(n-1)f] + h(l)[L-(n-1)f]}{[(n-1)h(f) + \frac{h(l)}{\lambda}]} \right\}. \tag{2.11}$$

2.3.2 The No-Arbitrage Conditions

In the steady state the value of each firm can be determined from the no-arbitrage conditions for each firm type: one-step quality leader, two-step quality leader, and potential entrant. In each case, given the assumptions of a continuum of industries and independence of returns to R&D across firms and industries, arbitrage possibilities for well-diversified investors will be eliminated when the returns on each asset are consistent with those available in the risk free market.

A potential entrant which spends f each instant on R&D generates a probability $h(f)\,dt$ of creating a new state-of-the-art product during the interval of length dt, in which case the firm's market value increases to that of a one-step-ahead quality leader. No firm will experience nominal capital gains during dt, as all nominal variables are constant in the steady state. Taking the value of a one-step-ahead quality leader as fixed at V^1, the firm value for a potential entrant, defined as V^0, when the potential entrant spends f at each instant to finance R&D during an interval of time dt, is given by

$$h(f)[V^1 - V^0]\,dt - f\,dt = rV^0\,dt. \tag{2.12}$$

During the interval of length dt the one-step-ahead quality leader receives profit flows of π^1 and by spending l on R&D at each instant generates a probability $h(l)\,dt$ of creating a new state-of-the-art product. In this case the firm's market value increases to that of a two-step-ahead leader. With $(n-1)$ followers each using f units of R&D at each instant, the one-step-ahead leader will, with probability $(n-1)h(f)\,dt$, be replaced as quality leader by an outsider during the interval. Taking the market value of a two-step-ahead quality leader as fixed at V^2, and effort by each potential entrant as fixed at f, the market value for a one-step-ahead quality leader, defined as V^1, when the leader spends l each instant to finance R&D during an interval of time dt and receives one-step-ahead leader profits of π^1, is given by

$$\pi^1\,dt + h(l)[V^2 - V^1]\,dt - l\,dt - (n-1)h(f)[V^1 - V^0]\,dt = rV^1\,dt. \tag{2.13}$$

In each interval of length dt a two-step-ahead leader receives profit flows of π^2, and with $(n-1)$ followers each using f units of R&D at each instant, the two-step-ahead leader will have its state-of-the-art product replaced by an outsider's product with probability $(n-1)h(f)\,dt$. With flow profits of π^2 at each instant and R&D assumed unprofitable for leaders with a two-step quality lead, the market value for a two-step-ahead quality leader, defined as V^2, when each follower's R&D efforts are fixed at f, is

$$\pi^2\,dt - (n-1)h(f)[V^2 - V^0]\,dt = rV^2\,dt. \tag{2.14}$$

2.3.3 The Rewards for Innovation

The no-arbitrage conditions for one-step-ahead leaders and two-step-ahead leaders, equation (2.13) and equation (2.14), can be combined to solve for the increase in market value associated with a leader that is successful in gaining a two-step quality advantage. The gain in firm value from increasing a quality lead to two steps is

$$[V^2 - V^1] = \frac{\pi^2 - \pi^1 - \{h(l)[V^2 - V^1] - l\}}{r + (n-1)h(f)} = \frac{\pi^2 - \pi^1 + l}{r + (n-1)h(f) + h(l)}. \tag{2.15}$$

The firm which is successful in becoming a two-step-ahead leader increases its instantaneous returns by the profit flows of a two-step-ahead leader but loses the instantaneous returns from profit flows of a one-step-ahead leader and the expected profits from research and development. The successful innovator receives this increase in instantaneous returns until an outsider is successful in creating the industry's next generation product. The increase in firm value the leader obtains by increasing its quality lead an extra step will then equal the expected present discounted value of the increase in instantaneous returns.

The no-arbitrage conditions for followers and one-step-ahead leaders, equation (2.12) and equation (2.13), can be combined to solve for the increase in market value associated with a follower that is successful in gaining a one-step quality advantage. Solving gives

$$\begin{aligned}[V^1 - V^0] &= \frac{\pi^1 - \{h(f)[V^1 - V^0] - f\} + h(l)[V^2 - V^1] - l}{[r + (n-1)h(f)]} \\ &= \frac{\pi^1 + f + h(l)[V^2 - V^1] - l}{[r + nh(f)]}. \end{aligned} \tag{2.16}$$

The potential entrant that is successful in becoming a one-step-ahead leader increases its instantaneous returns by the profit flows of a one-step-ahead leader plus the expected profits from one-step-ahead leader R&D but loses the instantaneous expected returns from potential entrant R&D. The successful innovator receives this increase in instantaneous returns until an outsider is successful in creating the industry's next generation product. The increase in firm value the leader obtains by moving from an outsider to a one-step quality leader will then equal the expected present discounted value of the increase in instantaneous returns.

2.3.4 Firm R&D Choices

When the no-arbitrage condition for the one-step-ahead leader is solved, the one-step-ahead firm has a market value of

$$V^1 = \frac{\pi^1 + h(l)V^2 - l + (n-1)h(f)V^0}{r + h(l) + (n-1)h(f)}. \tag{2.17}$$

Each one-step-ahead leader chooses R&D to maximize its market value. The profit maximizing choice of R&D labor for each leader solves

$$\begin{aligned} \frac{dV^1}{dl} &= \frac{[r + h(l) + (n-1)h(f)][V^2 h'(l) - 1] - [\pi^1 + h(l)V^2 - l + (n-1)h(f)V^0]h'(l)}{[r + h(l) + (n-1)h(f)]^2} \\ &= \frac{h'(l)V^2 - 1 - h'(l)V^1}{[r + h(l) + (n-1)h(f)]^2} = 0. \end{aligned} \tag{2.18}$$

Reduction of equation (2.18) gives the implicit equation defining each leader's profit maximizing R&D labor choice as

$$h'(l)[V^2 - V^1] = 1. \tag{2.19}$$

One-step-ahead leaders will hire R&D labor until the marginal expected benefits from R&D equal marginal R&D costs.

For a one-step-ahead leader with no current R&D program, market value as defined in equation (2.17) will equal

$$V^1(l=0) = \frac{\pi^1 + (n-1)h(f)V^0}{r+(n-1)h(f)} \tag{2.20}$$

Solving the equation defining the leader's profit maximizing R&D labor choice, equation (2.18), for two-step-ahead firm market value and substituting into the equation defining the one-step-ahead leader's market value, equation (2.17), gives a one-step-ahead leader a market value of

$$\begin{aligned} V^1(l=l^*) &= \frac{\pi^1 + (n-1)h(f)V^0 + h(l)[V^1 + \frac{1}{h'(l)}] - l}{r+(n-1)h(f)} \\ &= V^1(l=0) + \frac{[\frac{h(l)}{h'(l)} - l]}{r+(n-1)h(f)}. \end{aligned} \tag{2.21}$$

Leaders will choose to do positive R&D only if their profit maximizing labor choice defined by equation (2.18) exceeds $\bar{x}$, in which case the market value of a one-step-ahead leader doing R&D at the profit maximizing level exceeds the value of a one-step-ahead leader doing no R&D.

Substituting into the leader's profit maximizing condition, defined by equation (2.19), for one-step-ahead and two-step-ahead leader profit flows, defined by equation (2.6) and equation (2.7), and for the increase in firm value for a leader that gains a two-step advantage, defined by equation (2.15), gives the leader's R&D condition in terms of the leader R&D effort, l, follower R&D effort, f, and equilibrium spending, E. This equation is

$$\frac{\left[\frac{E(\lambda-1)}{\lambda^2}\right] - \left[\frac{h(l)}{h'(l)} - l\right]}{r+(n-1)h(f)} = \frac{1}{h'(l)}. \tag{2.22}$$

The change in firm value a leader expects as a result of becoming a two-step-ahead quality leader equals the present discounted value of the change in profit flows to the leader less the loss in expected R&D profits at each instant, where one-step-ahead leader R&D profits are $\left(\frac{h(l)}{h'(l)} - l\right)$. The discount factor accounts for both the rate of return the firm must earn at each instant and the expected duration for which the firm expects to retain its gains considering outside R&D efforts.

The no-arbitrage condition for the potential entrant gives the potential entrant a market value of

$$V^0 = \frac{h(f)V^1 - f}{r + h(f)}. \tag{2.23}$$

Each potential entrant chooses R&D to maximize its market value. The profit maximizing choice of R&D labor for each potential entrant solves

$$\frac{[r + h(f)][h'(f)V^1 - 1] - h'(f)[h(f)V^1 - f]}{[r + h(f)]^2} = \frac{h'(f)V^1 - 1 - h'(f)V^0}{[r + h(f)]^2} = 0 \tag{2.24}$$

Reduction of equation (2.24) gives the implicit equation defining each potential entrant's profit maximizing R&D labor choice as

$$h'(f)[V^1 - V^0] = 1. \tag{2.25}$$

Each potential entrant will hire R&D labor until the marginal expected benefit of R&D equals the marginal cost of R&D.

For a potential entrant, market value as defined in equation (2.23) will equal zero when the potential entrant conducts no R&D. Solving the equation which defines the potential entrant's profit maximizing R&D labor choice, equation (2.24), for a one-step-ahead leader's market value and substituting into the equation defining the potential entrant's market value, equation (2.23), gives a potential entrant a market value of

$$V^0 = \left(\frac{1}{r}\right)\left[\frac{h(f)}{h'(f)} - f\right] \tag{2.26}$$

when they choose R&D in accordance with equation (2.24). Followers will choose to do positive R&D only if the solution defined by equation (2.24) exceeds $\bar{x}$, in which case the profit maximizing R&D labor choice gives the potential entrant a market value greater than or equal to zero.

Substituting into the follower's profit maximizing condition, defined by equation (2.25), for one-step-ahead and two-step-ahead leader profit flows, defined by equation (2.6) and equation (2.7), and for the increase in firm value for a follower that gains a one-step advantage, defined by equation (2.16), gives the follower R&D condition in terms of the leader R&D effort, l, follower R&D effort, f, and equilibrium spending, E. This equation is

$$\frac{\left[\frac{E(\lambda-1)}{\lambda}\right] - \left\{\left[\frac{h(f)}{h'(f)} - f\right] - \left[\frac{h(l)}{h'(l)} - l\right]\right\}}{r + (n-1)h(f)} = \frac{1}{h'(f)}. \tag{2.27}$$

A follower successful in gaining a one-step quality lead increases its market value by one-step leader flow profits less the difference between the expected returns from R&D to a follower, given by $\left(\frac{h(f)}{h'(f)} - f\right)$, and the expected returns to a one-step quality leader at each instant discounted to the present. The discount factor for potential entrants also accounts for both the rate of return the firm must earn at each instant and the expected duration for which the firm expects to retain its gains considering outside R&D efforts.

2.3.5 Existence of the Steady-State Solution

There are two equations which define the model, the "R&D condition" equation and the "labor market condition" equation. Solving the follower R&D condition, defined by equation (2.27), for equilibrium spending, E, and plugging this into the leader's R&D condition, defined by equation (2.22), yields the "R&D condition" in (l, f) space, given by

$$C_R(l,f) = \frac{h'(l)}{h'(f)}[r + nh(f)] - \lambda[r + (n-1)h(f) + h(l) - lh'(l)] - h'(l)[f - l] - h(l) = 0. \tag{2.28}$$

Substituting the value of equilibrium spending defined by the labor market clearing condition, defined by equation (2.11), into the leader's R&D condition, defined by equation (2.22), yields the "labor market condition" in (l, f) space, given by

$$C_L(l,f) = \left(\frac{\lambda - 1}{\lambda}\right)\left(\frac{(n-1)h(f)[L - l - (n-1)f] + h(l)[L - (n-1)f]}{[(n-1)h(f) + h(l)/\lambda]}\right)$$
$$- \left(\frac{r + (n-1)h(f) + h(l) - lh'(l)}{h'(l)}\right) = 0. \tag{2.29}$$

Together the "R&D condition" and "labor market condition" define the steady-state equilibrium R&D choices for both potential entrants and one-step leaders in each industry.

A unique solution with both one-step-ahead leaders and potential entrants hiring R&D labor will exist (as pictured in Figure 2.2), provided the R&D condition and the labor market condition uniquely intersect in the region of (l, f) space in which equilibrium R&D profits are positive for both one-step-ahead leaders and potential entrants. It was shown that for non-negative R&D profits, the R&D efforts of

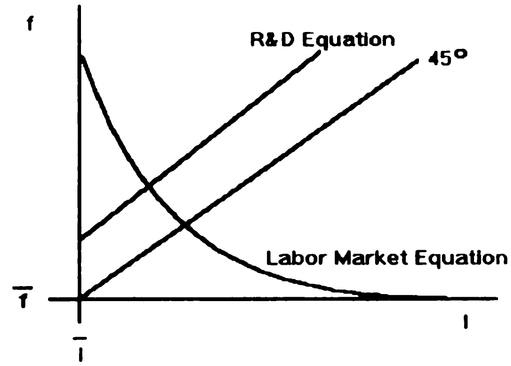

Figure 2.2: Equilibrium

potential entrants and one-step-ahead quality leaders must exceed $\bar{x}$. The region of positive R&D profits is defined by the quadrant in (l, f) space for which $l \geq \bar{l}$ and $f \geq \bar{f}$.

The labor market condition is downward sloping in the positive R&D quadrant as shown by the following analysis of the effects of changes in R&D intensities on the labor market condition. Differentiating the labor market condition with respect to changes in follower R&D intensity yields

$$\frac{\partial C_L}{\partial f} = -\left(\frac{\lambda-1}{\lambda}\right)\left\{\frac{[(n-1)^2h(f)+(n-1)h(l)][(n-1)h(f)+h(l)/\lambda]}{[(n-1)h(f)+h(l)/\lambda]^2}\right\}$$

$$-\left(\frac{\lambda-1}{\lambda}\right)\left\{\frac{(n-1)h'(f)[h(l)/\lambda]l}{[(n-1)h(f)+h(l)/\lambda]^2}\right\}$$

$$-\left(\frac{\lambda-1}{\lambda}\right)^2\left\{\frac{(n-1)h'(f)h(l)[L-(n-1)f]}{[(n-1)h(f)+h(l)/\lambda]^2}\right\}-\frac{(n-1)h'(f)}{h'(l)}, \tag{2.30}$$

which is negative inside the positive R&D quadrant[8].

[8]When the value of f is such that $(n-1)f$ exceeds L, then returns from R&D are strictly negative and the labor market condition will lie outside the positive R&D quadrant. An equilibrium with positive R&D effort requires that any equilibrium solution must have $(n-1)f < L$.

To understand the negative slope of the labor market condition it helps to think of the labor market condition in terms of the leader R&D condition. The labor market condition is the difference between the marginal expected benefit of R&D and the marginal expected cost of R&D for one-step-ahead leaders, where equilibrium spending is defined by labor market clearing. Changes in follower R&D efforts affect the one-step-ahead leader's marginal expected benefit from R&D directly and through the labor market. The direct impact of increased follower R&D is to diminish the marginal expected benefit one-step-ahead quality leaders receive from R&D. Follower R&D effort effects the marginal expected benefit one-step-leaders receive from R&D by lowering the expected increase in firm value a leader experiences by increasing its lead. A larger R&D effort by followers lowers the expected period of time a firm successful in gaining a two-step advantage expects to retain its lead. With a new innovation expected sooner, a one-step-leader expects increased profit flows from gaining a two-step lead to last for a shorter period of time. The result is a smaller increase in firm value for a firm successful in becoming a two-step quality leader.

Increased follower R&D effort also reduces the expected increase in firm value a leader gains by becoming a two-step leader through the labor market. Increased follower R&D increases the labor devoted to R&D in both industries with one-step-ahead and two-step-ahead leaders. The increase reduces the labor available in the economy for production. An increase in follower R&D effort also increases the fraction of industries in the economy with a one-step-ahead leader and decreases the fraction with a two-step leader. With more R&D done in industries with a one-step-leader, the increase in follower R&D creates an increase in labor used for R&D and a decrease in the labor available for production. Both labor market effects raise the labor devoted to R&D and reduce the labor devoted to production. With less production expected increases in profit flows from successfully innovating will diminish. The net impact of the sum of the direct effect and labor market effects of increased follower R&D effort is to diminish the marginal expected benefit one-step-ahead quality leaders receive from R&D.

Differentiating the labor market condition with respect to changes in leader R&D intensity yields

$$\frac{\partial C_L}{\partial l} = \left(\frac{\lambda-1}{\lambda}\right)\left\{\frac{[(n-1)h(f)][-(n-1)h(f)+h'(l)[L-(n-1)f]]}{[(n-1)h(f)+h(l)/\lambda]^2}\right\}$$
$$-\left(\frac{\lambda-1}{\lambda}\right)\left\{\frac{(n-1)h(f)h(l)/\lambda}{[(n-1)h(f)+h(l)/\lambda]^2}\right\}$$
$$-\left(\frac{\lambda-1}{\lambda}\right)\left\{\frac{[h'(l)/\lambda][(n-1)h(f)][L-l-(n-1)f]}{[(n-1)h(f)+h(l)/\lambda]^2}\right\}+\frac{h''(l)[r+(n-1)h(f)+h(l)]}{h'(l)^2}. \quad (2.31)$$

Substituting the labor market equation into the derivative gives

$$\frac{\partial C_L}{\partial l} = +\left(\frac{\lambda-1}{\lambda}\right)\left(\frac{h''(l)}{h'(l)}\right)\left[\frac{(n-1)h(f)[L-l-(n-1)f]}{[(n-1)h(f)+h(l)/\lambda]}\right]+\left(\frac{h''(l)l}{h'(l)}\right)$$
$$+\left(\frac{\lambda-1}{\lambda}\right)\left\{\frac{-[(n-1)h(f)]^2-(n-1)h(f)h(l)/\lambda-[h'(l)/\lambda][(n-1)h(f)][L-l-(n-1)f]}{[(n-1)h(f)+h(l)/\lambda]^2}\right\}$$
$$+\left(\frac{\lambda-1}{\lambda}\right)\left(\frac{[L-(n-1)f]\quad\cdot}{[(n-1)h(f)+h(l)/\lambda]^2}\right)\left((n-1)h(f)\left[\frac{h''(l)h(l)}{h'(l)}+h'(l)\right]+\frac{h''(l)h(l)^2}{\lambda h'(l)}\right). \quad (2.32)$$

The change in C_L with respect to changes in leader R&D will be negative for all values of $l, f \geq \bar{x}$ given the following assumption.

Assumption 3 *The R&D technology exhibits sufficient diminishing returns such that*

$$|h''(x)h(x)| \geq |h'(x)^2|.$$

Assumption 3 ensures that one-step-ahead firm R&D labor choices lead to maximum one-step-ahead firm market values in the steady state.

The impact of an increase in one-step-ahead leader R&D is to lower the marginal expected benefit of one-step-ahead leader R&D through the changes in the expected benefits leaders receive by gaining a two-step advantage and through diminishing returns in R&D. One result of moving from a one-step-ahead quality lead to a two-step-ahead quality lead is the loss of one-step-ahead leader expected R&D profits. Larger R&D efforts by one-step-ahead leaders increase both one-step-ahead instantaneous expected R&D profits and the loss of these expected profits suffered by firms successful in becoming two-step-ahead quality leaders. The increased loss of expected R&D profits will lower the increase in market value one-step-ahead firms expect from R&D and the marginal expected benefit of R&D for these firms.

The expected change in market value is also affected by increased leader efforts through the labor market. As one-step-ahead leader effort increases more resources are devoted to R&D. This lowers the resources left for production and the increase in flow profits a firm receives by increasing its lead to two steps diminishes. Also, with an increase in one-step-ahead leader efforts the fraction of industries with a one-step-ahead quality leader must decrease for the industry divisions to remain constant in the steady-state equilibrium. This also implies that the fraction of industries with a two-step leader will rise. In the steady state more R&D labor is consumed by industries with a one-step leader than in industries with a two-step leader and when the economy contains a relatively larger fraction of industries with a two-step leader the fraction of labor devoted to R&D will diminish. This effect increases the amount of labor left for production and the associated increase in flow profits for a firm successful in gaining a two-step advantage. The net effect of increased labor use by one-step-ahead leaders through the labor market, and the subsequent effect on the expected gains to a one-step-ahead leader from innovation will depend on the model parameters.

The remaining effect on the marginal expected benefit of R&D caused by an increase in one-step-ahead leader R&D efforts occurs through R&D technology. With a greater R&D effort the marginal product of R&D falls, a direct result of diminishing returns in R&D. Examining the above derivative shows that, given Assumption 3, the sum of the direct effects, labor market effects, and the technology effect is to lower the marginal expected benefit of R&D when one-step-ahead leaders hire more R&D labor.

Given Assumption 3, the implicit function theorem shows the labor market condition to be downward sloping in the positive R&D quadrant as

$$\frac{df}{dl} = -\frac{\frac{\partial C_L}{\partial l}}{\frac{\partial C_L}{\partial f}} \leq 0 \text{ in the positive profit quadrant.} \tag{2.33}$$

Given the above derivatives we see that when followers do more R&D, the marginal benefit of R&D for a one-step-ahead leader defined by the labor market equation falls, requiring one-step-ahead leaders to reduce R&D efforts in order to maximize profits.

The R&D equation slopes up in the positive profit quadrant. Differentiating the R&D condition with respect to changes in leader R&D intensity yields

$$\frac{\partial C_R}{\partial l} = \frac{h''(l)}{h'(f)}[r + (n-1)h(f) + h(f) - fh'(f)] + (\lambda + 1)lh''(l), \tag{2.34}$$

which is negative in the positive profit quadrant.

The R&D condition can also be defined in terms of the difference between expected marginal benefits and expected marginal costs of R&D for the leader with equilibrium spending in this instance defined by the follower R&D equation. Increases in one-step leader R&D can be broken down into direct effects, effects through the follower R&D equation, and a technology effect. As for the labor market condition, increased one-step-ahead leader R&D effort will increase the loss in expected instantaneous R&D profits for a firm successful in capturing a two-step quality lead. This reduces the increase in market value the successful firm obtains and the marginal benefit of one-step-ahead leader R&D.

The expected marginal benefits are also affected through the follower R&D condition. All else equal, changes in leader R&D will require changes in consumer spending flows in order to satisfy follower profit maximizing conditions. The resulting change in consumer spending flows will then effect one-step leader's marginal expected benefit from R&D. Increased R&D effort by one-step leaders increases the expected R&D profits one-step-ahead leaders receive at each instant. A follower which successfully innovates will then receive a smaller loss in expected R&D flow profits for greater one-step-ahead leader R&D choices. This will increase the marginal benefit of R&D for followers. The follower profit maximization condition holds for the same follower R&D intensity provided the consumer spending level falls, reducing the marginal benefit of follower R&D. A lower level of consumer spending decreases the marginal expected benefit of R&D for leaders.

The final effect on the marginal expected benefit of R&D for a one-step leader is through R&D technology. With diminishing returns to R&D an increase in one-step-ahead leader R&D reduces the marginal expected benefit of R&D. The net result of the sum of the direct effects, the effects through the follower R&D condition, and the technology effect is that increases in one-step-ahead leader R&D will reduce the marginal expected benefit of R&D for the one-step-ahead leader.

Differentiating the R&D condition with respect to changes in follower R&D intensity yields

$$\frac{\partial C_R}{\partial f} = (n-1)h'(l) - \frac{h''(f)H'(l)}{h'(f)^2}\{[r + nh(f)]\} - \lambda(n-1)h'(f). \tag{2.35}$$

Substituting in the R&D equation gives

$$\frac{\partial C_R}{\partial f} = -\left(\frac{h''(l)}{h'(f)}\right)\{\lambda[r + h(l) - h'(l)l] + f + h(l) - h'(l)l\}$$
$$+(n-1)h'(l) + \lambda(n-1)h'(f)\left[-\frac{h''(f)h(f)}{h'(f)^2} - 1\right]. \tag{2.36}$$

C_R is increasing in f in the positive profit quadrant given Assumption 3.

Increased follower R&D increases the marginal benefit of R&D for a one-step leader defined by the R&D condition. The direct impact on the marginal benefit is to reduce the period a leader expects to remain a two-step leader by increasing outsider efforts to unseat the leader. This effect on the marginal benefit of R&D is negative as the expected gain in market value a leader expects by increasing its lead falls.

The effect of increased follower R&D on the one-step leader's marginal benefit through the follower R&D equation is to increase consumer spending and profit flows and subsequently increase the marginal benefit of one-step leader R&D. This occurs as increased follower effort is unambiguously associated with greater equilibrium spending by consumers through the follower R&D equation. Given Assumption 3, the net effect of the sum of the direct effect and the effects through the follower R&D equation of increased follower R&D efforts is an increase in the marginal expected benefit of R&D for the one-step ahead quality leader.

The implicit function theorem dictates that given Assumption 3,

$$\frac{df}{dl} = -\frac{\frac{\partial C_R}{\partial l}}{\frac{\partial C_R}{\partial f}} \geq 0 \text{ in the positive profit quadrant.} \tag{2.37}$$

Given the R&D equation, increasing follower R&D will result in an increase in the marginal expected benefit from R&D for a one-step-ahead leader. Profit maximization implies a greater one-step-ahead leader R&D effort associated with the higher level of follower R&D. This is illustrated by the positive slope of the R&D equation in the positive quadrant.

The R&D curve in (l, f) space intersects the $l = \bar{l}$ axis at some positive finite value of $f \geq \bar{f}$ and remains increasing and positive[9] in the positive profit quadrant.

The labor market curve in (l, f) space will intersect both the $l = \bar{l}$ and $f = \bar{f}$ axis for values of $f \geq \bar{f}$ and values of $l \geq \bar{l}$ respectively given a sufficiently large labor supply. Both intercepts are increasing in the labor supply and approach infinity as the labor supply approaches infinity. As the intercept to the R&D equation is independent of the labor supply it must be that for all values of L beyond a critical value there exists a unique intersection of the R&D and labor market curves in the positive profit quadrant. This is the result pictured in Figure 2.2.

Proposition 9 *Given Assumption 3, for all values of the labor supply beyond a critical value, a unique positive profit equilibrium will exist with both leaders and followers doing research in industries with a one-step quality leader.*

[9]See the following section for a proof of the result that the solution to the R&D equation will lie everywhere above the curve $f = l$ in the quadrant defined by $l \geq \bar{l}$ and $f \geq \bar{f}$ in (l, f) space.

2.4 R&D Intensities

For large labor supplies and sufficiently diminishing returns, a unique solution with both leaders and followers doing R&D was shown to exist in the previous section. When a solution with leaders and followers both doing R&D exists it is possible to determine whether the leader or each of the followers will invest more in R&D by examining the R&D equation. From the R&D equation we see that when $l = f$,

$$C_R = -(\lambda - 1)[r + (n-1)h(f)] - \lambda[h(f) - fh'(f)]. \tag{2.38}$$

If the common R&D choice falls in the positive profit quadrant, $C_R < 0$. As both $\frac{\partial C_R}{\partial f} \geq 0$ and $\frac{\partial C_R}{\partial l} \leq 0$ the solution to the R&D equation will lie everywhere above the $f = l$ curve in the quadrant defined by $f \geq \bar{f}$ and $l \geq \bar{l}$ in (l, f) space and followers will always do more research than the one-step-ahead leader. This result follows directly from the fact that the increase in profit flows to a leader are strictly smaller than the increase in profit flows to a successful follower. Combined with the fact that the duration benefits are received from a successful innovation are equivalent for both successful leaders and followers, follower expected R&D profits will always exceed the expected R&D profits of leaders.

Proposition 10 *For equilibrium solutions with both leaders and followers performing R&D in industries with a one-step-ahead leader, each follower will always do strictly more R&D than will the leader.*

In equilibrium the expected returns to R&D for a firm hiring x units of R&D labor are given by $\left[\frac{h(x)}{h'(x)} - x\right]$ and are strictly increasing in R&D labor. Proposition 10 implies that with followers hiring strictly greater supplies of R&D labor the expected returns from R&D for followers will exceed the expected returns from R&D for leaders.

2.5 Welfare

2.5.1 The Maximum Social Welfare Solution

The socially optimal solution is taken to be the solution which provides the representative consumer with maximum discounted utility. The optimum is found by reducing the welfare maximization problem to its component parts. Given an allotment of production labor of D(t), to achieve maximum utility at each instant, production resources must be allocated to solve

$$\max_{d(\omega,t)} \int_0^1 \ln\left[\lambda^{j(\omega,t)} d(\omega,t)\right] d\omega. \tag{2.39}$$

The solution is constrained by the state equation,

$$\frac{dY(\omega)}{d\omega} = d(\omega,t), \tag{2.40}$$

where $Y(\omega) = \int_0^\omega d(s,t)\, ds$ and the resource constraint

$$D(t) = Y(1) = \int_0^1 d(s,t)\, ds. \tag{2.41}$$

Given the independence of marginal production costs with respect to quality, the social cost of each unit of output is constant. With constant societal prices, the state-of-the art good will bear the minimum quality adjusted price and production of resources in each industry are best allocated to the state-of-the-art good in each industry with a quality $\lambda^{j(\omega,t)}$. Production of the state-of-the-art good in each industry, which coincides with consumption, at time t is given by $d(\omega,t)$.

The Hamiltonian for the inter-industry production allocation problem is

$$\mathcal{H} = \ln\left[\lambda^{j(\omega,t)} d(\omega,t)\right] + \mu(\omega) d(\omega,t). \tag{2.42}$$

The solution of this optimal control problem will solve the resource constraint, given in equation (2.41), the first order condition, given by

$$\frac{\partial \mathcal{H}}{\partial d(\omega,t)} = \frac{1}{d(\omega,t)} + \mu(\omega) = 0, \tag{2.43}$$

and the costate equation, given by

$$-\frac{\partial \mu(\omega)}{\partial \omega} = \frac{\partial \mathcal{H}}{\partial Y(\omega)} = 0. \tag{2.44}$$

For maximum social welfare the optimal control solution dictates the distribution of production must be equally divided across industries. Consumers will consume equal portions of goods from each industry as dictated by the Cobb-Douglas utility function. Output of goods in each industry at time t will satisfy $D(t) = d(\omega,t)$ for all ω.

The problem of allocation of R&D resources can be solved by first finding the optimal allocation within each industry and then finding the optimal allocation across industries. Given an allotment of labor for R&D at time t, welfare maximization requires that in each industry R&D labor be allocated to maximize the instantaneous probability of success in the industry. Given $l(\omega,t)$ units of R&D labor in industry ω, the welfare maximization problem is

$$\max \sum_{i=1}^{n} h[l_i(\omega,t)],$$

$$\text{subject to } l(\omega,t) = \sum_{i=1}^{n} l_i(\omega,t). \tag{2.45}$$

The Lagrangian for the problem is

$$\mathcal{L} = h[l_i(\omega,t)] - \mu[\sum_{i=1}^{n} l_i(\omega,t) - l(\omega,t)]. \tag{2.46}$$

The solution to this static problem will satisfy the resource constraint defined in equation (2.45) and the first order conditions given by

$$\frac{\partial \mathcal{L}}{\partial l_i(\omega, t)} = h'[l_i(\omega, t)] - \mu = 0 \text{ for all } i. \tag{2.47}$$

The optimal distribution scheme requires allocating equal R&D labor shares to each firm including the leader. For each R&D firm $l_i(\omega, t) = l(\omega, t)/n$. This allocation results from returns to scale in the R&D technology. With constant returns to scale, as modeled in previous quality ladders models, R&D allocation within an industry is trivial with any allocation yielding the same industry growth rate. When R&D technology exhibits decreasing returns to scale the problem becomes non-trivial, with welfare maximization requiring equal R&D labor allotments and subsequently equal marginal R&D benefits across firms.

With an allotment of R&D labor of $l(t)$ to distribute across industries at time t, the social planner will maximize the rate of change in instantaneous utility at each instant by allocating labor to solve

$$\max_{l(\omega,t)} \int_0^1 nh\left[\frac{l(\omega, t)}{n}\right](\lambda - 1)\, d\omega. \tag{2.48}$$

subject to the state equation given by

$$\frac{dR(\omega)}{d\omega} = l(\omega, t), \tag{2.49}$$

where $R(\omega) = \int_0^\omega l(s, t)\, ds$ and the resource constraint given by

$$l(t) = R(1) = \int_0^1 l(s, t)\, ds. \tag{2.50}$$

The Hamiltonian for the welfare maximization problem is

$$\mathcal{H} = nh\left[\frac{l(\omega, t)}{n}\right](\lambda - 1) + \mu(\omega)l(\omega, t). \tag{2.51}$$

The solution to the optimal control problem will solve the resource constraint defined in equation (2.50), the first order equation, given by

$$\frac{\partial \mathcal{H}}{\partial l(\omega,t)} = h' \left[\frac{l(\omega,t)}{n}\right] (\lambda - 1) + \mu(\omega) = 0, \tag{2.52}$$

and the costate equation defined by

$$-\frac{\partial \mu(\omega)}{\partial \omega} = \frac{\partial \mathcal{H}}{\partial R(\omega)} = 0. \tag{2.53}$$

The maximum rate of change in instantaneous utility is achieved by allocating equal shares of R&D labor to each industry, a result obtained from combining the first order equations. Equal allocations result from returns to scale in R&D. Unequal allocations of R&D will result in unequal marginal benefits to each industry from R&D. Whenever marginal R&D benefits are unequal the rate of change in instantaneous utility can be increased by reallocating R&D labor from industries with lower marginal R&D benefits to industries with higher marginal R&D benefits. Taking advantage of returns to scale implies eliminating the differences and allocating equal shares of R&D labor to each industry resulting in equal marginal benefits of R&D in each industry.

The optimum allocation of R&D labor is to allocate each firm with cutting-edge research abilities the same amount of R&D labor. Defining the optimum R&D labor for each firm as l^* and concentrating on the steady-state solution where growth is constant across time the economy's budget constraint at each instant becomes

$$L = D + nl^*. \tag{2.54}$$

With equal production and R&D intensities across industries[10], instantaneous utility at each moment is given by

$$u(t) = \ln D + tI \ln \lambda. \tag{2.55}$$

Substitution of instantaneous utility and the R&D technology into the representative consumer's intertemporal utility function defined by equation (2.1) and integrating with respect to time yields

$$U = \frac{\ln D}{\rho} + \frac{nh(l^*) \ln \lambda}{\rho^2}. \tag{2.56}$$

Consumer welfare is chosen to maximize the present discounted value of intertemporal utility with respect to the economy's budget constraint given in equation (2.54). Substitution of the budget constraint into the discounted value of utility gives

$$U = \frac{\ln [L - nl^*]}{\rho} + \frac{nh(l^*) \ln \lambda}{\rho^2}. \tag{2.57}$$

The welfare maximizing choice of R&D labor allocated to each R&D firm solves

$$\frac{[L - nl^*] \ln \lambda}{\rho} = \frac{1}{h'(l^*)}. \tag{2.58}$$

Welfare is maximized when R&D is conducted to the point where the marginal benefit from future R&D success equals the marginal cost resulting from reduced production.

[10]With equal R&D intensities across industries, the probability in each industry that exactly m improvements have occurred in an interval of length τ is given by

$$f(m, \tau) = \frac{(I\tau)^m e^{-I\tau}}{m!}.$$

Using properties of the Poisson distribution gives

$$\int_0^1 \ln \lambda^{j(\omega,t)} = \sum_0^{\infty} f(m,t) \ln \lambda^m = tI \ln \lambda.$$

This argument can be found in both Grossman and Helpman[1991a] and Segerstrom[1991].

2.5.2 Optimal Tax and Subsidy Policy

To achieve maximum social welfare a two tiered tax and subsidy scheme is necessary to correct for inefficiencies in the free market. A subsidy is necessary to correct for under-production and a subsidy is necessary to equate R&D incentives and achieve optimal R&D output. Given that a successful innovator is able to exercise monopoly pricing power, limited only by lower quality goods, the price of the state-of-the-art good in each industry will exceed its marginal cost. This deadweight loss can be corrected by a production subsidy for output. When the state-of-the-art producer has a k-step quality lead, a subsidy of S_p^k on all output will lead to a limit price for a leader with an k step quality lead of

$$p^k = \lambda^k[1 - S_p^k]. \tag{2.59}$$

To reduce the price of output to its unitary marginal cost the production subsidy for a firm with a k step quality lead is

$$S_p^k = \frac{\lambda^k - 1}{\lambda^k}. \tag{2.60}$$

Given a unitary price of output, production will stop at the point where the social marginal cost of consumption equals the social marginal benefit.

For maximum welfare to be achieved two corrections must be made in the R&D market. Firms must each be given the incentives through R&D subsidies to hire the same amount of labor and the amount of R&D labor hired by each firm must equal l^*, the welfare maximizing R&D output. The first correction is necessary to eliminate the difference in expected R&D returns between leaders and followers, a result of the differences in the flow profits each receives from success. The second correction is necessary to equate the increase in consumer surplus resulting from innovation with the expected benefits to the innovating firm.

To equate R&D efforts for followers and leaders with any size quality lead a subsidy which covers a fraction of S_R^k of labor costs for the k-step ahead leader must be employed. In the social optimum R&D must be done by leaders independent of the leader's quality advantage. To find the subsidy scheme which achieves this goal, firm R&D incentives must be analyzed. With the appropriate R&D subsidies, the return on firm value each period of length dt must equal the profit flows a k-step leader receives during the period plus the expected profits from R&D when hiring l_k units of R&D at each instant less the expected loss due to outside innovation during the period where, with optimal subsidies, outsiders will hire the same amount of R&D labor as the leader defined as l. The no-arbitrage condition defining the market value for k-step ahead leader, V^k, is

$$rV^k\,dt = \pi^k\,dt + [h(l_k)[V^{k+1} - V^k] - l_k(1 - S_R^k)]\,dt - (n-1)h(l)[V^k - V^0]\,dt. \quad (2.61)$$

The k-step quality leader's firm value is

$$V^k = \frac{\pi^k + h(l_k)V^{k+1} - l_k(1 - S_R^k) - (n-1)h(l)[V^{k-1} - V^0]}{r + h(l_k) + (n-1)h(l)}. \quad (2.62)$$

A leader choosing R&D labor to maximize firm value will choose a level of R&D effort to equate the marginal benefits of R&D to the marginal costs. This occurs when the R&D choice satisfies

$$h'(l_k)[V^{k+1} - V^k] - [1 - S_R^k] = 0. \quad (2.63)$$

In each industry followers and leaders of any quality lead will hire equal amounts of R&D labor provided

$$\left[\frac{V^{k+1} - V^{k}}{1 - S_R^k}\right] = \left[\frac{V^{k+2} - V^{k+1}}{1 - S_R^{k+1}}\right] \text{ for all } k \geq 0. \tag{2.64}$$

As the difference in follower and leader incentives emanate from differences in expected profit flows changes the optimal subsidy will be of the form

$$S_R^{k+1} - S_R^k = \phi[\pi^{k+1} - \pi^k], \tag{2.65}$$

where ϕ is a constant independent of k to be defined below.

Finding the increase in market value between a k-step leader and a $(k+1)$-step leader requires first noting that from equation (2.64) the optimal subsidy must solve

$$[V^{k+2} - V^{k+1}] = \frac{[V^{k+1} - V^k][1 - S_R^{k+1}]}{[1 - S_R^k]}. \tag{2.66}$$

Using this fact, when each firm chooses l units of labor, combining the no-arbitrage conditions for a k-step-ahead leader and a $(k+1)$-step-ahead leader, each defined by equation (2.61), the change in firm value associated with moving from a k-step lead to a $(k+1)$-step lead is given by

$$[V^{k+1} - V^k] = \frac{\pi^{k+1} - \pi^k - \left[\frac{h(l)}{h'(l)} - l\right][S_R^{k+1} - S_R^k]}{r + (n-1)h(l)}. \tag{2.67}$$

The increase in value a firm receives from successful innovation is the increase in profit flows it receives less the change in the expectations of instantaneous R&D profits which are now affected by R&D subsidies. With the appropriate subsidy the expected R&D profits from R&D for a k-step ahead firm are given by $[1 - S_R^k]\left[\frac{h(l)}{h'(l)} - l\right]$.

Firm value defined in equation (2.67) can be substituted into the optimal subsidy condition, defined by equation (2.64), to give the implicit equation defining the optimal subsidy as

$$\frac{\pi^{k+1} - \pi^k - \left[\frac{h(l)}{h'(l)} - l\right][S_R^{k+1} - S_R^k]}{[r + (n-1)h(l)][1 - S_R^k]} = \frac{\pi^{k+2} - \pi^{k+1} - \left[\frac{h(l)}{h'(l)} - l\right][S_R^{k+2} - S_R^{k+1}]}{[r + (n-1)h(l)][1 - S_R^{k+1}]} \quad (2.68)$$

With the assumed form of the subsidy defined by equation (2.65), the optimal subsidy necessary to equate R&D efforts across firms in an industry is

$$S_R^k = 1 - \frac{\phi}{\lambda^k}. \quad (2.69)$$

This form of the subsidy where ϕ is fixed across firms implies optimal subsidies for leaders get larger as the leader's quality advantage increases. This form also implies leaders should receive strictly greater subsidies than followers[11].

To determine the explicit subsidy, ϕ must be chosen so that when the labor choice by each firm in each industry is l^* the following condition is satisfied:

$$\frac{\left[\frac{E(\lambda-1)}{\lambda}\right]\left[\frac{1}{\phi} - \left(\frac{h(l^*)}{h'(l^*)} - l^*\right)\right]}{\rho + (n-1)h(l^*)} = \frac{1}{h'(l^*)} = \frac{E \ln \lambda}{\rho}. \quad (2.70)$$

Solving for ϕ yields

$$\phi = \left[\frac{[\rho + (n-1)h(l^*)][\lambda \ln \lambda]}{\rho[\lambda - 1]} + \frac{h(l^*) - l^* h'(l^*)}{h'(l^*)}\right]^{-1}. \quad (2.71)$$

As pointed out above, the positive ϕ ensures that subsidies are larger for leaders, reducing the discrepancies between profit incentives of leaders and followers. The R&D subsidy necessary to achieve maximum social welfare is then

$$S_R^k = \frac{\frac{[\rho+(n-1)h(l^*)][\lambda \ln \lambda]}{\rho[\lambda-1]} - \frac{1}{\lambda^k} + \frac{h(l^*) - l^* h'(l^*)}{h'(l^*)}}{\left[\frac{[\rho+(n-1)h(l^*)][\lambda \ln \lambda]}{\rho[\lambda-1]} + \frac{h(l^*) - l^* h'(l^*)}{h'(l^*)}\right]}. \quad (2.72)$$

[11]This result assumes ϕ positive, a result proven below.

R&D subsidies are positive as a result of the successful firm's failure to appropriate all gains from R&D. Gains from firms fall short of gains to consumers, as the increase in profit flows firms receive by gaining a k-step lead, given by $\frac{E(\lambda-1)}{\lambda^{k+1}}$, are strictly less than gains in consumer surplus to consumers, given by $E \ln \lambda$ for all $\lambda > 1$. This shortfall is worsened by both the loss in value firms experience due to diminished expected R&D profits at each instant and the fact that, given positive outsider R&D efforts, firms expect to keep the gains from success for a limited period of time. Subsidies are positive[12] and increase for firms with a larger quality lead as the shortfall between the change in profit flows and gains to consumers from a success increase.

2.6 Conclusion

Prior quality ladders models of growth have found all research to be conducted by outsiders in an industry, contrary to what one observes in the economy. Models where leaders perform R&D rest on the assumption that leaders are better at conducting R&D than their potential successors. This model gives an alternative explanation

[12]With proper subsidies for production of output firms will under-invest in R&D and corrective R&D subsidies will be necessary. Without production subsidies firms may over-invest in R&D if the economy's resource endowment is small and R&D taxes may be necessary. The cause of over-investment in R&D is the under-use of labor resources in production. In the absence of production subsidies firms will set monopoly prices and output will be under-produced. Under-production of output creates excess labor supply reducing the costs of R&D and creating a tendency for firms to over-invest in R&D. Without production subsidies this tendency to over-invest may dominate the tendencies to under-invest in R&D that were outlined above.

for the ability of leaders to retain their market share. When a finite number of firms have the ability to do cutting-edge R&D, market share leaders may obtain a portion of the profits resulting from this market imperfection. We would then expect to see leaders holding market share more often in industries with a limited number of firms conducting R&D.

With only a limited number of firms able to do R&D, maximum welfare is achieved by taking advantage of returns to scale in the common R&D technology. To take full advantage of returns to scale in R&D, all firms, including leaders of any quality advantage, must participate at the same intensity level. Achieving equal R&D effort across firms requires a graduated system of R&D subsidies with larger subsidies to firms with greater quality advantages. Combined with production subsidies, which eliminate under-production by market leaders, R&D subsidies will equate firm gains and consumer gains from R&D, resulting in the social optimum.

Chapter 3

Patent Length and Economic Growth

3.1 Introduction

Traditionally, optimal patent length analysis has been conducted within the framework of a single patent race. An article by Nordhaus[1969] serves as a cornerstone for the early patent policy literature. The model adopted by Nordhaus, and later graphically reinterpreted in an article by Scherer[1989], is of a single innovative episode. R&D technology is assumed deterministic, with each R&D labor choice purchasing a fixed date of successful innovation. Once an innovation occurs, the innovating firm, by right of patent, controls all production of output of the good until its patent expires. Nordhaus found increasing the patent length decreases welfare by increasing the length of time the innovating firm is able to exert monopoly power, but increases welfare by increasing innovative effort and shorting the duration until a new product is introduced.

The single episode-deterministic R&D technology format of Nordhaus's model fails to capture important intertemporal and strategic effects on firm R&D behavior resulting from patent length policy. The adoption of a single episode model is of particular importance when considering industries in which goods are continually created to improve on and replace existing products and are eventually improved on and replaced themselves in a process of continuous product upgrading. By ignoring product upgrading, future R&D effort has little effect on current R&D effort as products never become obsolete. Strategic effects of Nordhaus's model are also limited by the deterministic nature of R&D as firms' R&D labor choices determine a new products introduction date with certainty. At each instant only one firm will do R&D directed towards creating a particular product, a result of the deterministic nature of R&D. The partial equilibrium nature of the model also limits the model's ability to adequately capture resource costs associated with R&D.

Uncertainty in R&D technology, introduced in a model by Loury[1979], allowed strategic behavior to be modeled with more accuracy. Loury looked at a patent race where firms obtained an uncertain date of introduction through R&D spending. With uncertain innovation a multitude of firms will take part in the race to introduce a new product with the first uncertain successor capturing patent protection on the good. Although expanding on the strategic analysis in Nordhaus's model, the strategic considerations analyzed in the Loury model remained limited by the single patent race format.

The recent introduction of endogenous growth models into the economic literature

has resulted in a reexamination of industry R&D behavior. In particular, the quality ladders growth models focus on a series of patent races to upgrade product quality. New higher quality products replace older products rendering them obsolete. The quality ladders format introduces intertemporal effects into the patent race analysis, effects missed in the previous single episode models. Adopting R&D technology similar to that found in the Loury model, the quality ladders models consider R&D to be uncertain. In the context of these models, firms compete against one another to be the first to create a new generation of products. The first successful firm receives a patent on the new good, enabling it to exercise monopoly power until either the good's patent expires or another firm is successful in upgrading the good's quality. The threat of future quality upgrading forces firms to consider future R&D behavior when making their R&D choices in the present.

Although the quality ladders growth model provides an ideal framework in which to reexamine the optimal patent length problem, patent lengths have received only limited treatment in the literature. In these models[1] patents enter either directly, where they are assumed to be of infinite length, or indirectly, where patent length is implicitly captured in imitation costs. An exception is a recent model developed by Horowitz and Lai[1994]. Although, in their model, goods within an industry are assumed to climb the quality ladder, their analysis of strategic and intertemporal R&D behavior is limited by the assumptions that R&D is deterministic and that a single firm does all R&D in each industry. In the Horowitz and Lai model the

[1]For examples of quality ladders models see Grossman and Helpman[1991b], Segerstrom[1991], and Barro and Sala-i-Martin[1995].

single firm capable of R&D will do R&D only on the date for which its patent is to expire. The more frequent are innovations the smaller are the quality increases. Horowitz and Lai's optimal patent length analysis focuses on the trade-off between the frequency and size of product innovations a trade-off that is a direct result of the lack of competition in R&D.

The model considered here adopts the framework of the quality ladders model. Products in the single industry modeled climb the quality ladder, with new generations replacing older generations. Multiple firms have the ability to do R&D towards creating new products with R&D technology that produces an uncertain date of innovation. With the uncertainty comes a series of patent races where multiple firms do parallel research aimed at creating a new product generation. When determining R&D effort each firm must consider both the duration for which it receives patent protection on its generation of product and the expected duration until future R&D renders its generation of product obsolete. R&D technology is assumed to exhibit initial increasing returns followed by decreasing returns. If firms are free to enter and exit patent races, this R&D technology will result in a finite number of firms performing R&D in each race.

A comparative steady-state analysis of the model generates the result that increasing patent length will unambiguously increase effort towards innovation. When entry is unrestricted the steady-state level of R&D conducted by each firm in the industry will remain constant. Longer patent lengths induce more firms to join each race, increasing the expected rate of growth in the industry. While increasing the patent length will stimulate R&D, the effect diminishes as the patent length increases.

Increases in patent length when the patent length is already long have little effect on R&D. This occurs because both larger patents and greater R&D efforts reduce the probability that firms reach patent expiration.

Welfare analysis suggest that both the direct effects of a longer patent length and the indirect effects created by increased R&D effort must be considered when determining the optimal length of industry patents. The optimal patent length is the patent length which maximizes the combination of the base utility consumers receive from resources devoted to production, the expected gains to consumers from more rapid growth, and the expected gains to consumers from future patent expirations.

Computational simulations with two general decreasing returns to scale R&D technologies suggest that infinite patent protection is optimal whenever labor resources are sufficiently large to support competition in R&D markets. The simulations suggest that welfare is everywhere increasing in patent length but will converge as patent length increases. When the economy's resource endowment is relatively large convergence occurs quickly with social welfare approaching its limiting value for even short patent length duration. This rapid convergence suggests that in quality ladders where the economy's resources are sufficiently large to support competition in R&D the current finite patent system may suffer only minimal inefficiencies. In light of these results, previous criticisms of the patent systems for its arbitrary and inefficient structure may be without basis.

The remainder of the chapter is organized as follows. Section two introduces the model, specifying the assumptions governing consumer behavior, firm behavior in both production and R&D, and the mechanism by which the labor market clears. In section three a unique steady state equilibrium is shown to exist. The R&D efforts of firms operating when a patent is active are compared with the R&D efforts of firms operating without an active patent. The section concludes with a description of the free entry steady state. The fourth section contains a comparative steady state analysis of the effects of patent policy on firm and industry R&D behavior with the welfare implications of patent policy examined in section five. Section six offers concluding remarks.

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3.2 The Model

The economy modeled here consists of two sectors, a production sector, and a R&D sector. In the production sector the economy's single class of consumption goods are produced. Goods within the class are differentiated with respect to quality. In the R&D sector, firms compete in an infinite series of patent races aimed toward creating new blueprints for new generations of products within the economy's class of consumption goods. The first firm to succeed in any given patent race creates a blueprint for producing a consumption good with quality exceeding the previous state-of-the-art good by a factor λ. The quality improvement is assumed constant across patent races and exceeds unity. The successful firm receives a patent on the innovation which is enforced perfectly while in effect. At the expiration of the patent the blueprint becomes public domain. Below it is shown that in the steady state the firm with a patent on the state-of-the-art good will be the sole producer of consumption goods in the economy. In the event the patent on the state-of-the-art good expires prior to the introduction of a new generation of the product, the industry reverts to a perfectly competitive structure with marginal cost pricing and an indeterminate number of break-even producers. In this manner the economy will experience a succession of innovations, which create better quality products, reducing previous product generations to obsolescence. At the same time the economy will experience intermittent patent expirations when patent races proceed beyond the statutory patent length.

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3.2.1 The Consumer Sector

The model consists of a fixed number of infinitely lived consumers with identical preferences represented by the intertemporal utility function

$$U \equiv \int_0^\infty u(t)e^{-\rho t}\,dt. \tag{3.1}$$

Time is indexed by t and each consumer's subjective discount parameter is given by ρ. Normalizing the quality of the state-of-the-art good at time 0 to unity, the quality of the state-of-the-art good after j successes is given by λ^j. At each instant the representative consumer's instantaneous utility function is

$$u(t) \equiv \ln \sum_{j=0}^{\infty} \lambda^j x(j,t). \tag{3.2}$$

Consumer demand for goods of quality λ^j at time t is given by $x(j,t)$. The intertemporal budget constraint for each consumer is given by

$$A_0 = \int_0^\infty \left[\sum_{j=0}^{\infty} p(j,t)x(j,t)\right] e^{-R(t)}\,dt. \tag{3.3}$$

The minimum cost to the consumer for goods of quality λ^j at time t is given by $p(j,t)$. The cumulative interest factor up to time t is represented by $R(t)$ and A_0 represents the sum of the present discounted value of all future income and the value of asset holdings at time $t = 0$. .

At any instant t the consumer faces a choice between goods which substitute perfectly for one another when adjusted for quality. The consumer will then exclusively consume from the sub-class of goods with the lowest quality-adjusted price. Below it will be shown that this sub-class will consist of only the state-of-the-art good which

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will be produced by the inventor of its blueprint while the patent for the good is in effect and by any firm when the patent has expired. The intertemporal problem for the consumer has the solution

$$\frac{\dot{E}}{E} = r(t) - \rho. \tag{3.4}$$

In the steady state equilibrium consumer spending will be constant over time with the instantaneous interest rate $r(t)$ equal to the representative consumer's subjective discount rate ρ. The aggregate constant level of consumer spending is defined as E.

3.2.2 The Production Sector

Labor units are chosen such that one unit of homogeneous labor, the sole factor of production, produces one unit of output of any quality good independent of time. The wage rate while a patent is in effect is chosen as the numeraire and set to unity. A firm which possesses the patent on the state-of-the-art good one quality step ahead of its nearest rival[2] can, by setting a price of p, earn flow profits of

$$\pi_p = \frac{E(p-1)}{p}. \tag{3.5}$$

The firm earns these flow profits provided the price it chooses is sufficiently low such that its quality-adjusted price is less than or equal to the quality-adjusted price of its nearest follower when the follower prices at marginal cost. It is assumed without

[2] In the free entry steady-state it is shown below that the firm which holds the patent on the state-of-the-art good will not undertake R&D while patent protected. This ensures that each current patent protected leader has monopoly power which is limited by a firm with the ability to produce a product one quality step below that of the state-of-the-art producer.

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loss of generality that consumers, when faced with two goods with equal quality-adjusted prices but differing qualities, will purchase only the good of higher quality. The patent protected state-of-the-art producer maximizes flow profits by pricing at the limit price λ. The flow of profits to the patent protected quality leader are then given by

$$\pi_p = E\left(\frac{\lambda - 1}{\lambda}\right). \tag{3.6}$$

When the patent on the state-of-the-art good expires an infinite number of firms have access to the blueprint for the good and competition will force price to marginal cost[3]. The wage rate when the patent on the state-of-the-art good has expired is given by w_{np} which is equivalent to the price of output. The blueprint for the state-of-the-art product becomes public domain with the expiration of its patent and profits for each firm are driven to zero when the holder of the of the blueprint loses its monopoly power. Given the assumption of constant returns to scale in production each firm will be indifferent as whether to produce or not. An indeterminate number of break-even producers will then satisfy consumer demand for the industry's good.

3.2.3 The R&D Sector

Initially n firms are assumed to possess R&D resources sufficiently advanced to allow them to compete profitably in a series of R&D races. During each patent race, while

[3]The assumption that imitation is costless when no patent is in effect restricts attention to industries such as pharmaceuticals where the cost of imitation of new drugs is negligible when the new drugs lose patent protection. See Mansfield, Schwartz, and Wagner[1981] for evidence on imitation costs in four industrial sectors.

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the quality leader is protected by patent, the leader will not participate in R&D, leaving a total of $(n-1)$ firms performing R&D[4]. When a patent expires without the introduction of a new product generation, the previous quality leader will reenter the race, making the total number of firms competing in a patent race when no patent is in effect equal to n. Each firm possesses the same R&D technology where the time of success, given l units labor devoted to R&D, $\tau(l)$, arrives prior to time t with probability given by

$$\text{prob}[\tau(l) \leq t] = 1 - e^{-h(l)t}. \tag{3.7}$$

The expected time until success for each firm devoting l units of labor to R&D is then given by $\frac{1}{h(l)}$. The R&D parameter $h(l)$ is assumed increasing over all positive l. The rate of increase in $h(l)$ is assumed positive when R&D labor choices are less than $\hat{l}$ and negative beyond $\hat{l}$, where $\hat{l}$ is constant over time and across firms and greater than zero. This form implies initial increasing returns to scale in R&D turning to decreasing returns at greater R&D efforts[5]. The maximum average R&D output is

[4]This paper focuses attention on the free entry case where the number of firms with competitive R&D resources is sufficiently large such that R&D profits are driven to zero. In this case, or when the number of firms capable of performing R&D competitively is in the neighborhood of the free entry number, the state-of-the-art producer will not perform R&D while benefiting from patent protection as shown below.

[5]Examining the relationship between patents granted and R&D spending, Kortum[1993] reports point elasticity estimates in the range 0.1 to 0.6, while Hall, Griliches, and Hausman[1986] obtain an average elasticity estimate of 0.3. Using market value data, Thompson[1993] obtains an R&D output elasticity with respect to R&D expenditure of 0.86. Each study suggests decreasing returns to R&D expenditure at the firm level, although to differing degrees.

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defined as $\bar{l}$ and occurs at the point where the average R&D output, $\frac{h(l)}{l}$, equals the marginal R&D output $h'(l)$, a value of R&D effort strictly greater than $\dot{l}$.

3.2.4 The Labor Sector

When the state-of-the-art good is protected by patent demand for labor in the production sector is given by $\frac{E}{\lambda}$ while demand in the R&D sector is given by $(n-1)l_p$, where l_p is defined as the symmetric steady state choice of R&D effort by each firm participating in a patent race when the state-of-the-art good is patent protected. The condition for labor market clearing with an active patent is then

$$L = \frac{E}{\lambda} + (n-1)l_p. \tag{3.8}$$

During periods when the patent on the state-of-the-art good has expired, demand for labor in the production sector is given by $\frac{E}{w_{np}}$ and demand for labor by the R&D sector is given by nl_{np}, where l_{np} is defined as the symmetric steady state choice of R&D effort by each firm participating in a patent race when the state-of-the-art good is no longer patent protected. The condition for labor market clearing with an expired patent is then

$$L = \frac{E}{w_{np}} + nl_{np}. \tag{3.9}$$

Combining equation (3.8) and equation (3.9) generates the wage rate during periods when no patent is in effect in terms of the model parameters and R&D efforts. This wage rate is given by

$$w_{np} = \frac{\lambda[L-(n-1)l_p]}{[L-nl_{np}]}. \tag{3.10}$$

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3.3 Steady State Equilibrium

3.3.1 The Expected Reward For Successful R&D

R&D is undertaken by each firm with the goal of capturing the stream of benefits resulting from the monopoly power bestowed by patent protection on the producer of a new state-of-the-art good. In calculating the expected benefit to winning a patent race, each firm takes the level of R&D efforts of each other R&D firm during its term as quality leader as fixed at the steady state value l_p. Given independently distributed returns to R&D across firms, the date at which a firm expects to be replaced by a new quality leader, $\mu(l_p)$ occurs prior to the length of time t with probability

$$\text{prob}[\mu(l_p) \leq t] = 1 - e^{-I_p t}, \tag{3.11}$$

where $I_p = (n-1)h(l_p)$ is the aggregate R&D parameter while a patent is in effect. The expected benefit from an R&D success discounted to the date of success, when the patent length is T, is then given by

$$V = \int_0^T \left[\int_0^t \pi_p e^{-\rho s}\, ds\right] I_p e^{-I_p t}\, dt + e^{I_p T}\left[\int_0^T \pi_p e^{-\rho s}\, ds\right]. \tag{3.12}$$

The successful firm earns flow profits of π_p until either another firm succeeds in creating a higher quality product or until the patent on its product expires, whichever comes first. Solving equation (3.12) yields

$$V = \pi_p \left[\frac{1 - e^{-(\rho + I_p)T}}{I_p + \rho}\right]. \tag{3.13}$$

Substituting equilibrium spending as given in the labor market equation, equation (3.8), into the flow of profits a leader experiences, given by equation (3.6), and placing

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this value into each firm's discounted benefit calculation gives the expected benefit of winning a patent race in steady state,

$$V_\pi = [\lambda - 1][L - (n-1)l_p]\left[\frac{1 - e^{-(\rho + I_p)T}}{I_p + \rho}\right]. \tag{3.14}$$

This equation is termed the "profit equation."

In the steady state the industry R&D effort during active patent periods remains constant over time. The expected benefit of winning an R&D race will depend only on the industry R&D effort while the firm's patent is in effect, as at patent expiration the successful firm's pricing power is eliminated. Because only active patent R&D efforts in the period following a success enter each firm's expected benefit calculations, both firms performing R&D while a patent is in effect and firms performing R&D while no patent is in effect will expect the same benefit from winning an R&D race, given by equation (3.14).

3.3.2 The No-Arbitrage Conditions

Taking the expected benefit of an R&D success as given at V, each firm attempting to innovate on an active patent will have a firm value V_p given by the no-arbitrage condition

$$rV_p = h(l_p)[V - V_p] - l_p. \tag{3.15}$$

The no-arbitrage condition specifies that, provided managers or firm owners act in a manner consistent with risk neutrality, the return on firm value must be equal to that available by loaning funds in the risk free market. The value of a firm attempting to innovate on an active patent is then

$$V_p = \frac{h(l_p)V - l_p}{r + h(l_p)}. \tag{3.16}$$

To maximize firm value, a firm attempting to innovate on a current patent chooses R&D labor to solve

$$r[h'(l_p)V - 1] - h(l_p) + h'(l_p)l_p = 0. \tag{3.17}$$

The steady state value of R&D labor chosen by each firm participating in a patent race with a current patent protected state-of-the-art good and when the expected benefit of winning the race is given by V_π is implicitly defined by

$$V_{Rp} = \frac{r + h(l_p) - l_p h'(l_p)}{rh'(l_p)}. \tag{3.18}$$

This equation is termed the "active patent R&D equation."

Substituting equation (3.18), the equation implicitly defining the profit maximizing active patent R&D choice, into equation (3.16), the active patent firm value equation, gives steady state equilibrium firm value for firms participating in an active patent race of

$$V_p = \frac{h(l_p) - l_p h'(l_p)}{rh'(l_p)}. \tag{3.19}$$

Firm value will be positive, and firms will choose to enter the active patent R&D race provided firm value is non-negative. This occurs for all parameter values such that the equilibrium choice of labor exceeds $\bar{l}$.

Taking the expected benefit of winning an R&D race as fixed at V, firms which are attempting to innovate on an expired patent will have a firm value V_{np} given by the no-arbitrage condition

$$rV_{np} = h(l_{np})[V - V_{np}] - l_{np}w_{np}. \tag{3.20}$$

As in the active patent case the no-arbitrage condition specifies that the return on value for an R&D firm participating in a patent race with no active patent must equal the rate of return available in the risk free market. The value of a firm attempting to innovate on an expired patent is then

$$V_{np} = \frac{h(l_{np})V - l_{np}w_{np}}{r + h(l_{np})}. \tag{3.21}$$

To maximize firm value, a firm attempting to innovate on an expired patent chooses R&D labor to solve

$$r[h'(l_{np})V - w_{np}] - h(l_{np})w_{np} + h'(l_{np})l_{np}w_{np} = 0. \tag{3.22}$$

The steady state value of R&D labor chosen by each firm participating in a patent race without a patent protected state-of-the-art good and when the expected benefit of winning the race is given by V_{π} is implicitly defined by

$$V_{Rnp} = \frac{w_{np}[r + h(l_{np}) - l_{np}h'(l_{np})]}{rh'(l_{np})}. \tag{3.23}$$

This equation is termed the "non-active patent R&D equation."

Substituting equation (3.23), the equation implicitly defining the profit maximizing non-active patent R&D choice, into equation (3.21), the non-active patent firm value equation, gives steady state equilibrium firm value for firms participating in a non-active patent race of

$$V_{np} = \frac{w_{np}[h(l_{np}) - l_{np}h'(l_{np})]}{rh'(l_{np})}. \tag{3.24}$$

Firm value will be positive, and firms will choose to enter the non-active patent R&D race provided firm value is non-negative. This occurs for all parameter values such that the equilibrium choice of labor exceeds $\bar{l}$.

3.3.3 Relative R&D Efforts

Given the implicit equations defining R&D labor choices with and without an active patent, equation (3.17) and equation (3.22) respectively, the non-active patent wage rate defined in equation (3.10) can be shown with a proof by contradiction to exceed unity for any steady state equilibrium. Assume that the wage rate with a non-active patent is less than or equal to unity. If w_{np} is equal to unity then the profit maximizing equations for both firms in active patent and non-active patent races, equation (3.17) and equation (3.22), are identical and any steady state equilibrium has $l_p = l_{np}$, the R&D labor choices for firms in both the active and non-active patent state will be the same.

If the wage rate w_{np} decreases to a value below unity the level of effort chosen by each firm when no patent exists will increase. To see this, note that, given a fixed benefit of winning an R&D race of V_π, when no patent is active the R&D labor choice of each firm participating in the patent race can be defined implicitly by

$$\mathcal{N} = r[h'(l_{np})V_\pi - w_{np}] - h(l_{np})w_{np} + h'(l_{np})l_{np}w_{np} = 0. \tag{3.25}$$

Increases in l_{np} will decrease $\mathcal{N}$ for all economically significant parameter values as shown by

$$\frac{\partial \mathcal{N}}{\partial l_{np}} = (rV_\pi + l_{np}w_{np})h''(l_{np}). \tag{3.26}$$

Increases in w_{np} will decrease $\mathcal{N}$ for all economically significant parameter values as shown by

$$\frac{\partial \mathcal{N}}{\partial w_{np}} = -r - [h(l_{np}) - l_{np}h'(l_{np})]. \tag{3.27}$$

Applying the implicit function theorem gives that for any value of V_π

$$\frac{dl_{np}}{dw_{np}} = -\frac{\frac{\partial \mathcal{N}}{\partial w_{np}}}{\frac{\partial \mathcal{N}}{\partial l_{np}}}, \tag{3.28}$$

which is negative for any economically significant parameter values. For a fixed expected benefit of winning an R&D race, firms will do more R&D when the cost of R&D labor is smaller. Given that the non-active patent R&D labor choice for each firm, l_{np}, is decreasing in the wage rate, w_{np}, if the wage rate falls below unity then in the steady state each firm in a non-active patent race will do strictly more R&D than each firm in an active patent race, l_{np} will rise above l_p.

If the wage rate in the non-active patent state is less than or equal to unity, the amount of R&D labor hired by each firm in the non-active patent state will be equal to or exceed the amount of labor hired in the active patent state, $l_{np} \geq l_p$. Given that more firms participate in the non-active patent state, if each firm hires more labor in the non-active patent state the total labor hired by the industry in the non-active state must exceed that hired in the active patent state, $nl_{np} \geq (n-1)l_p$. From equation (3.10) the wage rate w_{np} must then exceed λ which strictly exceeds unity. But this contradicts the original assumption that the wage rate w_{np} was less than or equal to unity. Thus, the wage rate w_{np} must exceed unity in any steady state solution. Given that the wage rate exceeds unity, it must be that in the steady state equilibrium each firm in an active patent race does more R&D than each firm in a non-active patent race, $l_p > l_{np}$.

In equilibrium, when a patent expires, the price of output will initially fall as the holder of the state-of-the-art blueprint loses monopoly power. With falling prices the demand for labor in the production sector increases because of increased consumer demand for the good. Further, the former quality leader will now wish to enter the R&D race, increasing the demand for R&D labor. The increased demand for labor will necessarily increase the wage rate, diminishing individual firm R&D efforts and reducing the size of the increase in production.

Proposition 11 *In any steady state equilibrium each firm participating in a patent race during a period in which the state-of-the-art good is protected by patent will do strictly more R&D than will each firm in a patent race during a period in which the patent on the state-of-the-art good has expired.*

3.3.4 Uniqueness of the Steady State Equilibrium

The steady state equilibrium value of labor chosen by each firm in an active patent race is defined implicitly by setting the expected benefit of winning the race from the "profit equation" equal to the expected benefit of winning defined by the "active patent R&D equation." The non-active patent R&D choice does not enter this defining equation as non-active R&D will only occur after a successful firm has lost the gains from innovation granted by patent protection and has no bearing on firms' expected benefits from R&D.

The equilibrium value of labor chosen by each firm in an active patent race will determine the expected benefit of winning an R&D race in both the active and non-active patent states. The equilibrium non-active patent R&D effort is then implicitly defined by equating the expected benefit of winning determined by the "profit equation" to the expected benefit of winning determined by the "non-active patent R&D equation."

In (l, V) space the function defining V_π is downward sloping in l_p as shown by

$$\frac{\partial V_\pi}{\partial l_p} = -\frac{(\lambda-1)[L-(n-1)l_p](n-1)h'(l_p)[1-\{1+T[\rho+(n-1)h(l_p)]\}e^{-[\rho+(n-1)h(l_p)]T}]}{[\rho+(n-1)h(l_p)]^2}$$
$$-(n-1)(\lambda-1)\left[\frac{1-e^{-[\rho+(n-1)h(l_p)]T}}{\rho+(n-1)h(l_p)}\right], \tag{3.29}$$

which is negative for all significant parameter values[6]. For active patent labor choices exceeding the value for which $l \geq \frac{L}{(n-1)}$ the expected benefit of winning an R&D race is strictly negative.

As R&D efforts increase three separate effects occur on the expected benefit of a victory. Increased R&D efforts decrease the resources remaining for production. This reduces the flow of profits winners receive and reduces the expected benefit of a victory. With increased effort, the expected length of time a firm expects to remain in the quality lead is reduced and the expected benefit of a win falls. With the next innovation expected at a sooner date, the expectation that the firm will lose its lead to patent expiration is diminished, increasing the expected benefit of a win. The last effect is strictly dominated by first two effects, resulting in a net decrease in the

[6]The negative relationship occurs provided $l \geq \bar{l}$ and l sufficiently small such that $(n-1)l \leq L$. For firms to participate in R&D in the active patent race, the equilibrium labor choice must satisfy these two requirement. In Appendix D a proof is given that for equilibrium R&D labor choices satisfying these conditions,$[1-\{1+T[\rho+(n-1)h(l_p)]\}e^{-[\rho+(n-1)h(l_p)]T}] \geq 0$.

expected benefit of a win from increased R&D effort.

In (l, V) space the function defining V_p is upward sloping in l_p for all economically significant parameters, capturing the fact that for firms to employ greater R&D labor, they must obtain a higher reward if a success occurs. The upward slope is shown by

$$\frac{\partial V_p}{\partial l_p} = -\frac{\rho h''(l_p)[\rho + h(l_p)]}{[\rho h'(l_p)]^2}. \tag{3.30}$$

which is positive for economically significant values of l_p. The expected benefit of a win defined by the "active patent R&D equation" is strictly positive at $\bar{l}$ and increases to infinity in the limit as l_p increases.

A unique positive R&D solution exists in the active patent state provided $V_\pi \geq V_p$ at $\bar{l}$. In this case the expected benefit of a win defined by the "profit condition" exceeds the expected value of a win defined by the "active patent R&D equation" at $\bar{l}$ and decreases continuously until becoming negative. A unique positive active patent R&D effort occurs provided

$$L \geq \frac{[\rho + (n-1)h(\bar{l})]}{h'(\bar{l})(\lambda - 1)(1 - e^{-[\rho+(n-1)h(\bar{l})]T})} + (n-1)\bar{l}. \tag{3.31}$$

If the condition in equation (3.31) does not hold then $l_p = 0$ and R&D will not be profitable in steady-state.

The intersection of the "profit equation" and the "active patent R&D equation" determines the equilibrium expected benefit of winning a patent race. It was shown above that firms will do strictly less R&D when no patent is active. The V_{Rnp} curve lies everywhere to the left of the V_{Rp} curve and will be upward sloping as shown by

$$\frac{\partial V_{np}}{\partial l_{np}} = -w_{np}\frac{\rho h''(l_{np})[\rho + h(l_{np})]}{[\rho h'(l_{np})]^2}. \tag{3.32}$$

Firms in the non-active patent state also require greater expected rewards to R&D in order to increase efforts. As the expected value of winning an R&D race is independent of non-active patent R&D effort, a positive solution will exist in the non-active state provided $V_\pi \geq V_{Rnp}$ at $\bar{l}$. This occurs provided

$$L \geq \left[\frac{\lambda[L-(n-1)l_{np}]}{[L-n\bar{l}]}\right]\left[\frac{[\rho+(n-1)h(\bar{l})]}{h'(\bar{l})(\lambda-1)(1-e^{-[\rho+(n-1)h(\bar{l})]T})}\right]+(n-1)\bar{l}, \qquad (3.33)$$

where l_p is the steady state active patent R&D choice determined independently as described above. If the condition in equation (3.33) does not hold then $l_{np}=0$ and R&D will not be profitable in the non-active patent state.

Three solutions are possible conditional on the parameters of the model. If the labor supply is sufficiently small then no R&D will be conducted in either the patent or non-patent state. If the labor supply is large and the number of firms is small then R&D will be conducted in both the patent and non-patent states. If the labor supply is large and the number of firms is close to the free-entry level then R&D will be conducted in the patent state but not in the non-patent state.

The unique steady state value of R&D effort chosen by each firm in an active patent race is implicitly defined by

$$\mathcal{H}_1 = V_\pi - V_{Rp} = [L-(n-1)l_p][\lambda-1]\left[\frac{1-e^{-[\rho+(n-1)h(l_p)]T}}{\rho+(n-1)h(l_p)}\right] - \frac{1}{h'(l_p)} = 0 \qquad (3.34)$$

Proposition 12 *A unique steady state equilibrium value exists for all parameter values. If the economy is endowed with relatively few resources no R&D will be done. If the economy is endowed with relatively many resourçes and the number of firms with the ability to do R&D is small then R&D will be done both when a patent is active and* *when no patent is active. If the economy is endowed with relatively many resources and the number of firms with the ability to do R&D is large then R&D will be done only when a patent is active in the industry.*

3.3.5 The Free Entry Steady State

The remainder of this article focuses on the case where the number of firms able to do R&D is sufficiently large such that profits in the active patent state are reduced to zero. To show that entry reduces profits note that under free entry firms will enter R&D races provided R&D is profitable. The effect of additional participation on active patent R&D effort can be determined by applying the implicit function theorem to equation (3.34).

As active patent R&D effort increases, $\mathcal{H}_1$ falls as shown by

$$\frac{\partial \mathcal{H}_1}{\partial l_p} = \frac{\partial V_\pi}{\partial l_p} - \frac{\partial V_{Rp}}{\partial l_p}, \tag{3.35}$$

which is negative for $l \geq \bar{l}$ given that $\frac{\partial V_\pi}{\partial l_p} \leq 0$ and $\frac{\partial V_{Rp}}{\partial l_p} \geq 0$ as shown previously.

With more firms capable of competitive R&D the expected value of winning an R&D race V_π falls for each given level of active patent research as does $\mathcal{H}_1$. This is shown by

$$\frac{\partial \mathcal{H}_1}{\partial n} = \frac{\partial V_\pi}{\partial n} = -\frac{(\lambda-1)[L-(n-1)l]h(l_p)[1-\{1+T[\rho+(n-1)h(l_p)]\}e^{-[\rho+(n-1)h(l_p)]T}]}{[\rho+(n-1)h(l_p)]^2}$$
$$-l_p(\lambda-1)\left[\frac{1-e^{-[\rho+(n-1)h(l_p)]T}}{\rho+(n-1)h(l_p)}\right] \tag{3.36}$$

which is negative for all significant parameter values[7]. The effects of increasing the

[7] The negative relationship is conditional on $1-\{1+T[\rho+(n-1)h(l_p)]\}e^{-[\rho+(n-1)h(l_p)]T} \geq 0$ which is shown in Appendix D to hold for all positive significant parameter values.

number of firms participating in R&D are equivalent to increasing the R&D efforts of each active patent race participant. Increased R&D efforts decrease the resources remaining for production, reducing the flow of profits winners receive and the expected benefit of a victory. Increased R&D efforts reduce the expected length of time a firm expects to remain in the quality lead, but the expectation that the firm will lose its lead to patent expiration is diminished. The last effect is again dominated and the result is a net decrease in the expected benefit of a win from increased R&D effort.

The change in active patent R&D effort resulting from increased participation is then negative, as shown by

$$\frac{dl_p}{dn} = -\frac{\frac{\partial \mathcal{H}_1}{\partial n}}{\frac{\partial \mathcal{H}_1}{\partial l_p}}. \tag{3.37}$$

Subsequently as the number of firms participating in each patent race increases, both the patent and non-patent R&D efforts fall. If free entry is allowed to occur firms will continue to enter active-patent races until profits are reduced to zero. At this point the profits from non-patent R&D are strictly negative and no non-active patent R&D will not be done. This comes as a result of the increased demand for labor in the production sector. Wages are forced up and R&D, which generated expected profits of zero in the active-patent state, now becomes strictly unprofitable.

Proposition 13 *When entry into patent races is unrestricted firms enter R&D races until profits from R&D are driven to zero in active patent races. With free entry no R&D will be conducted following a patent expiration, as increased R&D costs make R&D unprofitable.*

In the free entry case a leader with a patent protected product will not perform

R&D. To illustrate, consider a firm contemplating entering a patent race while it holds the patent on the state-of-the-art good one quality step ahead of its nearest rival in the industry. If the current leader is the first successful firm to create a new state-of-the-art product it will obtain a two-step quality lead. With a two-step quality lead, the firm maximizes flow profits by choosing the limit price λ^2. The two-step ahead leader then receives flow profits of

$$\pi_p' = \frac{E(\lambda^2 - 1)}{\lambda^2}. \tag{3.38}$$

The increase in the firm's profit flow is then

$$\pi_p' - \pi_p = \frac{E(\lambda^2 - 1)}{\lambda^2} - \frac{E(\lambda - 1)}{\lambda} = \frac{\pi_p}{\lambda}. \tag{3.39}$$

Given the a steady state R&D effort of l_p by all other firms while its patent is in effect, the expected benefit to becoming a two step quality leader for a current one step quality leader is

$$V_{Rp}' = \frac{\pi_p}{\lambda}\left[\frac{1 - e^{-(\rho + I_p)T}}{I_p + \rho}\right] = \frac{V_\pi}{\lambda}. \tag{3.40}$$

In steady state the expected benefit of a win is then equal to $\frac{V_{R_p}}{\lambda}$.

Elimination of arbitrage possibilities requires that for a R&D labor choice of k_p the return on value for a one-step ahead quality leader be given as follows,

$$rV_{Rp} = \pi_p + h(k_p)[V_{Rp}'] - k_p - (n-1)h(l_p)V_{Rp}. \tag{3.41}$$

The one-step ahead quality leaders firm value is then

$$V_{Rp} = \frac{\pi_p + h(k_p)V_{Rp}' - k_p}{r + (n-1)h(l_p)}. \tag{3.42}$$

To maximize firm value the one-step ahead leader chooses R&D labor to maximize its firm value, yielding the first order equation

$$h'(k_p)V'_{Rp} - 1 = h'(k_p)\left(\frac{V_{Rp}}{\lambda}\right) - 1 = 0. \tag{3.43}$$

Combining the steady state solution $l_p = \bar{l}$, equation (3.18), and equation (3.43) gives that in the steady state the leader's R&D labor choice must solve

$$\frac{h'(k_p)}{\lambda h'(l_p)} = 1. \tag{3.44}$$

This implies that the leader will choose a smaller R&D effort than non-leader firms and $k_p < \bar{l}$.

The value of a one step ahead quality leader is given by inserting V'_{Rp} as defined in the profit maximizing condition, equation (3.43), into firm value as defined by equation (3.42), yielding

$$V_{Rp} = \frac{\pi + \left(\frac{h(k_p)}{h'(k_p)}\right) - k_p}{r + (n-1)h(l_p)}. \tag{3.45}$$

The firm value when the leader does no R&D will exceed the firm value when the leader performs R&D whenever $k_p < \bar{l}$. Above it was shown that the leader will choose a value smaller than $\bar{l}$, which implies that in the free steady state the leader will do no R&D.

Proposition 14 *With free entry into R&D races profits from R&D are strictly negative for firms attempting to further a quality lead and leaders will do no R&D while patent protected.*

3.4 Patent Effects in Free Entry

As shown above with free entry no R&D will be conducted after a patent expires, but while active the number of firms participating in each patent race at the R&D level $\bar{l}$ is defined by

$$\mathcal{H}_2 = [L-(n-1)\bar{l}][\lambda-1]\left[\frac{1-e^{-[\rho+(n-1)h(\bar{l})]T}}{\rho+(n-1)h(\bar{l})}\right]-\frac{1}{h'(\bar{l})}=0. \tag{3.46}$$

Using the implicit function theorem, it is shown below that the number of firms participating in each patent race will increase with statutory patent length. Increases in the patent length will increase $\mathcal{H}_2$ as shown by the following derivative.

$$\frac{\partial\mathcal{H}_2}{\partial T}=e^{-[\rho+(n-1)h(\bar{l})]T}. \tag{3.47}$$

When the patent length increases the expected benefit of winning a patent race will increase for each R&D effort, as profit flows are expected to be received for a longer period of time. Increases in firm participation will for reasons outlined above diminish $\mathcal{H}_2$ as shown by the following derivative.

$$\frac{\partial\mathcal{H}_2}{\partial n}=-\frac{h(\bar{l})[1-\{1+[\rho+(n-1)h(\bar{l})]T\}e^{-[\rho+(n-1)h(\bar{l})]T}]}{[\rho+(n-1)h(\bar{l})]^2}-\frac{\bar{l}}{(\lambda-1)h'(\bar{l})[L-(n-1)\bar{l}]^2}. \tag{3.48}$$

Subsequently for $\mathcal{H}_2$ to hold with equality, if the patent length rises the number of participants must also rise. The implicit function theorem dictates that for all economically relevant parameter values

$$\frac{dn}{dT}=-\frac{\frac{\partial\mathcal{H}_2}{\partial T}}{\frac{\partial\mathcal{H}_2}{\partial n}}=\left[\frac{h(\bar{l})[e^{[\rho+(n-1)h(\bar{l})]T}-\{1+[\rho+(n-1)h(\bar{l})]T\}]}{[\rho+(n-1)h(\bar{l})]^2}\right.$$
$$\left.+\frac{\bar{l}e^{[\rho+(n-1)h(\bar{l})]T}}{(\lambda-1)h'(\bar{l})[L-(n-1)\bar{l}]^2}\right]^{-1}, \tag{3.49}$$

which is greater than or equal to zero for all significant parameter values[8]. With free entry individual R&D effort will always be driven to the zero profit level or $\bar{l}$. With R&D effort fixed and the number of patent race participants increasing the aggregate industry R&D effort increases along with the expected rate of growth in the industry.

Proposition 15 *As the statutory patent length is increased, entry into each patent race will increase, as will the expected rate of growth in the industry.*

While increasing patent length increases industry R&D effort, the impact of patent length increases diminish as the patent length grows. To illustrate note that a direct increase in patent length diminishes the effect of patent length changes on R&D effort as shown by the following derivative.

$$\frac{\partial\left[\frac{dn}{dT}\right]}{\partial T} = -\frac{h(\bar{l})e^{[\rho+(n-1)h(\bar{l})]T} + \frac{[\rho+(n-1)h(\bar{l})]\bar{l}e^{[\rho+(n-1)h(\bar{l})]T}}{(\lambda-1)h'(l)[L-(n-1)l]^2}}{\left[\frac{h(\bar{l})[e^{[\rho+(n-1)h(\bar{l})]T}-\{1+[\rho+(n-1)h(\bar{l})]T\}]}{[\rho+(n-1)h(l)]^2} + \frac{\bar{l}e^{[\rho+(n-1)h(\bar{l})]T}}{(\lambda-1)h'(l)[L-(n-1)l]^2}\right]^2} \tag{3.50}$$

As the patent length increases the direct effect of a patent change on the expected benefit of a win falls, as the expected benefits are received further into the future and are discounted at a higher rate. Changes in the patent length have a smaller effect on the expected benefit of a win and on $\mathcal{H}_2$. With a longer patent length, increases in the patent length have a smaller effect on expected benefits, and greater expected benefits result in smaller increases in participation. The unambiguous direct result of increasing patent length is the diminishing of the effect of patent policy on R&D efforts.

[8] $e^{[\rho+(n-1)h(\bar{l})]T}-\{1+[\rho+(n-1)h(\bar{l})]T\} \geq 0$ whenever $1-\{1+[\rho+(n-1)h(\bar{l})]T\}e^{-[\rho+(n-1)h(\bar{l})]T} \geq 0$ which is shown to be true in Appendix D.

Increased patent length will also increase R&D efforts, indirectly influencing the effectiveness of patent length changes through R&D efforts. With more firms participating patent length increases have a smaller impact on the expected benefits of winners, as firms are less likely to last as quality leader until their patent expires. With greater R&D effort, an increase in benefits will cause a smaller increases in participation, diminishing the impact of patent length increases. Thus a smaller increase in participation is generated by patent length increases for longer patent lengths. This is illustrated by the derivative

$$\frac{\partial\left[\frac{dn}{dT}\right]}{\partial n} = -\frac{\left(\frac{h(\bar{l})^2}{[\rho+(n-1)h(l)]^3}\right)[\rho+(n-1)h(\bar{l})]T[e^{[\rho+(n-1)h(\bar{l})]T}-1+2]}{\left[\frac{h(\bar{l})[e^{[\rho+(n-1)h(\bar{l})]T}-\{1+[\rho+(n-1)h(\bar{l})]T\}]}{[\rho+(n-1)h(l)]^2}+\frac{\bar{l}e^{[\rho+(n-1)h(\bar{l})]T}}{(\lambda-1)h'(l)[L-(n-1)l]^2}\right]^2}$$
$$+\frac{\left(\frac{h(\bar{l})^2}{[\rho+(n-1)h(l)]^3}\right)2[e^{[\rho+(n-1)h(\bar{l})]T}-1]}{\left[\frac{h(\bar{l})[e^{[\rho+(n-1)h(\bar{l})]T}-\{1+[\rho+(n-1)h(\bar{l})]T\}]}{[\rho+(n-1)h(l)]^2}+\frac{\bar{l}e^{[\rho+(n-1)h(\bar{l})]T}}{(\lambda-1)h'(l)[L-(n-1)l]^2}\right]^2}$$
$$-\frac{\frac{(\lambda-1)h'(\bar{l})\bar{l}[L-(n-1)\bar{l}]e^{[\rho+(n-1)h(\bar{l})]T}[2+Th(\bar{l})[L-(n-1)\bar{l}]]}{\{(\lambda-1)h'(l)[L-(n-1)h(l)]\}^4}}{\left[\frac{h(\bar{l})[e^{[\rho+(n-1)h(\bar{l})]T}-\{1+[\rho+(n-1)h(\bar{l})]T\}]}{[\rho+(n-1)h(l)]^2}+\frac{\bar{l}e^{[\rho+(n-1)h(\bar{l})]T}}{(\lambda-1)h'(l)[L-(n-1)l]^2}\right]^2}, \tag{3.51}$$

which is negative for all significant parameter values[9]. The effect of patent length changes on R&D efforts diminishes as the patent length is extended as shown by

$$\frac{d\left[\frac{dn}{dT}\right]}{dT} = \frac{\partial\left[\frac{dn}{dT}\right]}{\partial n}\left[\frac{dn}{dT}\right] + \frac{\partial\left[\frac{dn}{dT}\right]}{\partial T} \tag{3.52}$$

which is negative for all significant parameter values.

Proposition 16 *Although patent length increases will unambiguously increase R&D effort, the increase in effort will diminish as the patent length increases.*

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[9]The partial derivative is negative provided $[\rho+(n-1)h(\bar{l})]T[e^{[\rho+(n-1)h(\bar{l})]T}-1+2] - 2[e^{[\rho+(n-1)h(\bar{l})]T}-1] \geq 0$ which is shown to hold in Appendix E.

3.5 Optimal Patent Length

Above it was shown that each choice of patent length is associated with a unique level of R&D effort in both the active patent and non-active patent states. In the case of free entry no R&D is conducted if the patent on the state-of-the-art good expires prior to the creation of the next generation of good. The nature of growth in an industry is for the industry's good to proceed up the quality ladder at random intervals until at any given quality level a patent expires, in which case the climb up the quality ladder ceases. Longer patent lengths reduce the likelihood that industry growth will cease[10].

Maximum expected welfare in the economy can be achieved by maximizing the representative consumer's expected discounted intertemporal utility function. Assuming an active patent at time $t = 0$, which is active for a length of time T, expected social welfare equals

$$
\begin{aligned}
EW(T) = &\int_0^T \left[\int_0^t \ln\left(\frac{E}{\lambda}\right) e^{-\rho s}\, ds\right] I_p e^{-I_p t}\, dt \\
&+ e^{-I_p T}\left[\int_0^T \ln\left(\frac{E}{\lambda}\right) e^{-\rho s}\, ds + \int_T^\infty \ln\left(\frac{E}{W_{np}}\right) E^{-\rho s}\, ds\right] \\
&+ \int_0^T \left[\int_0^T \left[\int_0^u \ln\left(\frac{\lambda E}{\lambda}\right) e^{-\rho s}\, ds\right] I_p e^{-I_p u}\, du\right. \\
&\left. + e^{-I_p T}\left[\int_0^T \ln\left(\frac{\lambda E}{\lambda}\right) e^{-\rho s}\, ds + \int_T^\infty \ln\left(\frac{\lambda E}{W_{np}}\right) e^{-\rho s}\, ds\right]\right] I_p e^{-(I_p+\rho)t}\, dt \\
&+ \int_0^T \left[\int_0^T \left[\int_0^T \left[\int_0^v \ln\left(\frac{\lambda^2 E}{\lambda}\right) e^{-\rho s}\, ds\right] I_p e^{-I_p v}\, dv\right.\right.
\end{aligned}
$$

[10] If entry is sufficiently restricted, such that R&D in the non-active state is positive, then a patent expiration will result in a smaller R&D effort and the industry will proceed up the quality ladder but progression will be at a slower expected rate.

$$+e^{-I_pT}\left[\int_0^T \ln\left(\frac{\lambda^2 E}{\lambda}\right)e^{-\rho s}\,ds + \int_T^\infty \ln\left(\frac{\lambda^2 E}{W_{np}}\right)e^{-\rho s}\,ds\right]\Bigg] I_p e^{-(I_p+\rho)u}\,du\Bigg] I_p e^{-(I_p+\rho)t}\,dt$$

$$+\ldots \tag{3.53}$$

Integrating and summing terms, the expected welfare function reduces to

$$EW(T) = \frac{\ln L - (n-1)\bar{l}}{\rho} + \left[\frac{\ln\lambda}{\rho}\right]\left[\frac{I_p(1-e^{-[\rho+I_p]T})}{\rho + I_p e^{-[\rho+I_p]T}}\right]$$

$$+\left[\frac{\ln L - \ln\,[L-(n-1)\bar{l}]}{\rho}\right]\left[\frac{(I_p+\rho)e^{-[\rho+I_p]T}}{\rho + I_p e^{-[\rho+I_p]T}}\right] \tag{3.54}$$

Social welfare can be separated into three component parts. The first portion of social welfare is the base welfare the consumer receives independent of new discoveries or expired patents. Initially, the instantaneous utility for a consumer is given by

$$u(0) = \ln\frac{E}{\lambda} = \ln\,[L-(n-1)\bar{l}]. \tag{3.55}$$

Both innovation success and patent expiration increase the consumer's instantaneous utility. In the steady state the consumer is guaranteed instantaneous utility of at least $u(0)$ throughout time. If innovation effort is fixed at the steady state level, discounting the minimum flow of instantaneous utility to the present yields the consumer's base welfare regardless of patent length or variability of research success.

The second term on the right hand side of the expected welfare condition is added welfare the consumer expects as a result of gains from increases in product quality. The increase in the representative consumer's instantaneous utility function from the introduction of a new generation of consumption good is given by

$$\ln\frac{\lambda^{j+1}E}{\lambda} - \ln\frac{\lambda^j E}{\lambda} = \ln\lambda. \tag{3.56}$$

The change in instantaneous utility from a new innovation is independent of the initial quality level and time in the steady state. The second portion of social welfare is the present discounted value of all expected increases in utility as a result of new innovation.

The third portion of social welfare represents potential increases in welfare due to patent expirations. The increase in utility associated with loss of monopoly power due to patent expiration is given at each instant by

$$\ln \frac{E}{w_{np}} - \ln \frac{E}{\lambda} = \ln L - \ln \left[L - (n-1)\bar{l}\right]. \tag{3.57}$$

The third term in the expected welfare function measures the expected increase in welfare as a result of the expected present discounted value of all future gains from patent rate expiration.

Examining these three terms illustrates the impact increasing patent length has on expected welfare. The effect of increasing patent length on base utility is unambiguously negative. An increase in patent length increases the expected gains from innovation. With a greater reward for success more firms enter each patent race. The steady state fraction of resources devoted to R&D increases and the fraction of resources devoted to production diminishes. With fewer resources devoted to production the base utility consumers expect independent of both innovation success and patent length falls.

Increasing patent length will have two effects on the benefits consumers expect to receive from future innovation. Increasing the patent length will increase expected benefits from R&D and directly increase the amount of resources devoted to R&D in the industry. With more resources devoted to R&D the expected gains from future innovation increase. Even with fixed innovative efforts in both the patent and non-patent states, a longer patent length increases the expected rate of innovation. Above, it was shown that each firm does strictly less R&D after a patent expires. Extending the patent length increases the period of time that R&D is conducted at the higher active patent level and reduces the period of time that R&D is conducted at the lower non-active patent level. The net effect of patent length increases on the expected benefits consumers receive from future innovation is unambiguously positive.

Three effects on the expected gains consumers receive from firms loss of monopoly power occur as a result of patent length increases. The direct of effect of increasing the patent length is to increase the duration that firms exercise monopoly power, extending the period that the society suffers dead-weight losses in the absence of further innovation. Adding to the negative impact on consumer welfare is an increase in innovative effort as a result of the patent length increase. With greater innovative effort each patent race is less likely to extend past patent expiration and consumers are less likely to experience the increase in utility associated with the loss of monopoly power. Mitigating these two effects is a third effect resulting from an increase in patent length. With a longer patent length, the difference between resources devoted to production in the non-active patent state and the amount of resources devoted to production in the active patent state increases. The deadweight losses associated with monopoly power are greater when patent length is longer. Because deadweight losses are greater, in the event a patent expires, consumers experience greater benefits by recovering these losses and therefore the expected welfare associated with patent expiration increases. The net results of patent length increases on social welfare, occurring as a result of changes in the expected benefits from patent expiration, are ambiguous depending on the model parameters.

The optimal patent length must be chosen by considering the combination of effects of patent length increases on consumer welfare as occurring through changes in the base consumer welfare, changes in the expected gains from quality improvement, and the changes in expected gains from patent expiration. Previous models of a single innovative episode failed to capture the intertemporal effects of patent length changes, among these, the effect of patent length changes on the economy's growth rate over time, the effect of future innovative effort on current firm R&D efforts, and the effect of changes in the rate of innovation on the welfare consumers receive from patent expirations. Clearly, a policy concentrating on a single innovation will be remiss in addressing societal welfare when the industry under examination is of the dynamic nature found in quality ladders industries.

Typical welfare results for some specific R&D technologies and model parameters, resulting from a computational simulation, are reported below. For the simulations two R&D technologies are considered with typical examples of each reported below. In the first case it is assumed that for R&D labor choices at or beyond $\bar{l}$, the R&D technology can be represented by the general decreasing returns to scale form

$$h(l) = (l - a)^{\beta}, \tag{3.58}$$

where β takes on values in the interval $(0, 1)$ and $a < \bar{l}$. In each simulation, for each combination of parameters examined, the outcome is the same. Expected consumer welfare is strictly non-decreasing in the patent length whenever the economy's resources are large enough to support competitive R&D. Although non-decreasing, at some point welfare gains diminish. As the patent length is increased consumer welfare gains increase by an ever smaller portion. Eventually the effect of increasing the patent length yields minimal gains to welfare. The larger the economy's resource base, the faster the gains from increasing patent length die out.

In Table F.1, in Appendix F, simulations are reported for a range of patent lengths, two sample labor supplies, and the parameter values $\rho = 0.01$, $\lambda = 2.5$, $\beta = 0.8$, and $a = 2$. The point of maximum average R&D output at which each firm conducts R&D is assumed to be at $l = 10$. The labor supply $L = 17$ is the supply for which competition is just profitable for multiple firms. For each value of the labor supply patent policy is non-decreasing over all patent lengths and diminishes in impact rapidly. Table F.1 represents a situation in which current patent policy proves to deviate minimally from the socially efficient outcome.

In Table F.2, in Appendix F, simulations are reported for a range of patent lengths, two sample labor supplies, and the parameter values $\rho = 0.008$, $\lambda = 1.56$, $\beta = 0.5$, and $a = 0.1$. The point of maximum average R&D output is assumed at $l = 0.2$. Typically, social welfare is non-decreasing in patent length with gains from increasing the patent length diminishing as the patent length increases. Table F.2 represents a situation where increasing the length of patents from the current length may cause significant increases in social welfare. The labor supply necessary for profitable competition is $L = 1$ in this case. When the labor resource endowment for the economy is greater than this minimum the gains from changing current patent policy will once again prove minimal as shown when $L = 10$ in Table 2.

Tables F.3 and F.4 in Appendix F report typical simulation results for the natural log form of R&D technology. For these cases it is assumed that for labor choices greater or equal to $\bar{l}$, R&D technology can be represented by the form

$$h(l) = \ln(l - a). \tag{3.59}$$

In Table F.3 results are reported for a range of patent lengths, two sample labor supplies, and the parameter values $\rho = 0.008$, $\lambda = 1.56$, and $a = 10$. The point of maximum average R&D output is taken to be $l = 1.96322$. In Table F.4 results are reported for a range of patent lengths, two sample labor supplies, and the parameter values $\rho = 0.1$, $\lambda = 2.5$, and $a = 0.2$. The point of maximum average R&D output in Table F.4 is $l = 11.76322$. The results with the natural log form of technology mirror the results found above for the technology form defined in equation (3.58).

The results from simulations run with the two technological forms defined above are clear. The optimal patent length whenever resources are sufficient to support competition is infinite, with gains from increasing the patent length diminishing as patent length is increased. An arbitrary patent length as practiced in many industrialized nations will cause minimal social losses when each economy is resource rich but may cause larger inefficiencies if each economy has a poor resource base. In the framework of repeated R&D races patent policy will, in industrialized nations, in general, prove to have small order welfare implications as currently enforced. The significance of patent policy is diminished by firms' expectations of further innovation prior to patent expiration a significant omission from single episode patent analysis.

3.6 Conclusion

Traditional patent race analysis has been conducted in models of single innovative episodes. The analysis conducted here demonstrates the importance of identifying intertemporal effects of patent policy when choosing the optimal patent length. Social planners must consider not only the implications of patent length on current races but on future R&D efforts and the rate of progression up the quality ladder. The above analysis also demonstrates the importance of understanding the strategic behavior of firms competing in R&D races where success occurs in a non-deterministic manner. Optimal patent policy must be set considering the entry into patent races and the resulting changes in R&D effort and growth. For optimal patent policy to be rooted in the logic of economic theory it must incorporate both the intertemporal and strategic considerations.

An interesting result of the above analysis is that while the current statutory patent length almost certainly deviates from the optimal infinite length, this may have little significance on welfare. Above it was shown that the effects generated by increases in patent length diminish as patent length increases. This implies that the current patent rate may be sufficiently large such that changes in its length result in only small order changes in R&D efforts and subsequently little change in expected welfare. Given the potential for only small welfare changes from moving to the optimal patent length, the costs of passing legislation which moves the economy to the optimum infinite length may not be warranted. Although the welfare implications of the above analysis are clear, the analysis assumes that firms freely enter each R&D

race and no barriers to entry or fixed start up costs are involved in entry. When competition is imperfect patent length may have a more significant impact on social welfare. This area of patent policy certainly deserves further examination.

Chapter 4

The R&D Incentives of Leaders and Followers[1]

3.1 Introduction

One of the striking features of many R&D-driven models of economic growth[2] is that industry leaders, the firms that produce the highest quality products or have the lowest costs of production, do not engage in R&D activities. In these models, it is not profit-maximizing for industry leaders to do R&D. Instead, all of the R&D investment that drives economic growth is undertaken by follower firms, firms that are not technologically advanced enough to compete in product markets. Industry leaders earn monopoly profits as a reward for their past research success but they do not make any effort to improve their products until after they have lost their

[1]This chapter is joint work with Paul S. Segerstrom.

[2]See Segerstrom, Anant and Dinopoulos [1990], Grossman and Helpman [1991b, chap. 4], and Aghion and Howitt [1992].

leadership positions and no longer actively produce.

This type of equilibrium R&D behavior is not consistent with casual observation. For example, in the computer sector, virtually all of the industry leaders (IBM, Microsoft, Apple, Hewlett-Packard, etc.) devote considerable resources to R&D activities. In the microprocessor industry, Intel has maintained its leadership position by aggressively doing R&D ever since it introduced the 4004 chip back in 1971.[2] Clearly, one of the challenges facing economic theorists is to explain why we see industry leaders engaging in R&D to improve their own products.

Barro and Sala-i-Martin [1995, chap. 7] have recently developed a R&D-driven growth model to help explain this puzzle. Unlike in the earlier literature, they assume that industry leaders have a cost advantage in doing R&D to improve their products. By virtue of being on the technological frontier and knowing how to produce the state-of-the-art quality products in their respective industries, leaders are in a better position than other firms to improve their own products. Barro and Sala-i-Martin find that when industry leaders have the slightest R&D cost advantage over follower firms, all R&D is undertaken by industry leaders in equilibrium. Because no industry leader is ever driven out of business, they argue furthermore that their model has very different welfare properties from previous endogenous growth models which use a "quality ladders" structure. In particular, there is no need for either R&D subsidies or R&D taxes to achieve an optimal allocation of resources over time. All that the

[2]Since 1971, Intel has introduced six new generations of microprocessors (the 8080, 8086, 286, 386, 486 and Pentium chips) and in 1994, Intel spent a staggering $1.1 billion on R&D expenditures. See Malone [1995] for a fascinating account of the history of the microprocessor industry.

government has to do is appropriately subsidize production to eliminate the distortion caused by monopoly pricing in product markets.

In this paper, we also study the R&D incentives of leaders and followers when industry leaders have a R&D cost advantage over follower firms. In fact, we analyze the same R&D-driven growth model as Barro and Sala-i-Martin. Surprisingly, we reach different conclusions, indicating that Barro and Sala-i-Martin's analysis is not completely correct.

Whereas Barro and Sala-i-Martin conclude that industry leaders do all the R&D when they have the slightest R&D cost advantage, we find that industry leaders only do R&D when they have a sufficiently large R&D cost advantage over follower firms. When industry leaders have a small cost advantage, we find that all R&D is undertaken by follower firms, as was the case in the previously cited growth literature. The intuition behind our results is quite simple: since industry leaders are already earning monopoly profits, other things being equal, they have less to gain from further innovation than follower firms. Thus industry leaders will only do R&D if their cost advantage is sufficiently large to offset their smaller profit gain from innovating.

We also obtain different welfare conclusions. Whereas Barro and Sala-i-Martin conclude that only production subsidies are needed to maximize the representative consumer's equilibrium discounted utility, we find that R&D subsidies/taxes are also, in general, necessary to achieve an optimal allocation of resources. The R&D expenditures of follower firms need to be taxed heavily enough so that these firms abstain from doing R&D altogether. Otherwise resources go to firms that have higher costs of doing R&D in equilibrium. Also the R&D expenditures of leader firms need to be appropriately subsidized. Even when all R&D is undertaken by the lower cost industry leaders, they will not choose the right level of R&D effort since not all of the externalities associated with R&D investment are internalized. In particular, leaders are not able to fully appropriate the consumer surplus gains that R&D success generates.

The remainder of the chapter is organized as follows: in section 2 the model developed by Barro and Sala-i-Martin is presented and in section 3, we present our analysis of the balanced growth properties of the model. The welfare properties of the model are examined in section 4 along with an analysis of the subsidies necessary for optimal resource allocation. Section 5 contains concluding remarks.

4.2 The Model

4.2.1 Overview

Before we get into the technical details, we provide a sketch of the structure of the model first developed by Barro and Sala-i-Martin [1995, chap. 7]. There is a single competitively produced good that consumers buy. This final good is produced using labor and a variety of intermediate inputs. Each consumer is endowed with one unit of labor which is inelastically supplied to final good producers. With no population growth, the aggregate supply of labor $L(t)$ remains fixed over time that is, $(L(t) = L)$. There is a continuum $[0, 1]$ of industries which produce the horizontally differentiated intermediate inputs used in final good production.

In each industry $\omega \in [0, 1]$, firms can devote resources to R&D to improve the quality of intermediate inputs. By improving on the current best quality intermediate input produced in an industry, a successful R&D firm earns monopoly profits from selling its leading-edge quality intermediate input to final good producers. Lower quality intermediate input producers are priced out of business in equilibrium. Over time, as the quality of intermediate inputs used in final good production rises, workers become more productive and thus R&D fuels per capita consumption growth.

Each firm maximizes its expected discounted profits, taking into account both the size of the monopoly profit flow from R&D success and its likely duration. This duration is typically finite, since with other firms doing R&D, each industry leader is eventually driven out of business by further innovation. Although there is uncertainty associated with research at the industry level, since the probabilities of research success across industries are independent and there is a continuum of industries, the jumpiness in microeconomic outcomes is not transmitted to macroeconomic variables. Consumers have perfect foresight concerning the aggregate rate of technological change over time and choose their expenditure paths accordingly to maximize their discounted utilities. This is a dynamic general equilibrium model, so all markets clear throughout time.

4.2.2 Product Markets

Within each industry ω, the quality of intermediate inputs produced is indexed by j, which only takes on integer values. Higher values of j denote higher quality inputs.

At time $t = 0$, each industry's highest quality product is normalized to have quality $j = 0$. Thus at time t, the quality $j(\omega, t)$ of the highest quality product in industry ω also measures the number of successful product upgrades that have occurred in that industry since $t = 0$. The size of each product upgrade is measured by the parameter $\lambda > 1$.

For final good-producing firm i at time t, let $x_i(j, \omega, t)$ denote the amount this firm uses of the intermediate input of quality j produced by industry ω. Firm i also uses labor $L_i(t)$ to produce its final good output $Y_i(t)$ at time t. Each firm i has the same production function

$$Y_i(t) = \int_0^1 AL_i(t)^{1-\alpha} \left[\sum_j^{j(\omega,t)} \lambda^j x_i(j, \omega, t) \right]^\alpha d\omega, \tag{4.1}$$

where $0 < \alpha < 1$ and $A > 0$ are given production parameters. With $\lambda > 1$ and λ^j increasing in j, equation (4.1) implies that higher quality intermediate inputs make each worker more productive. The summation in equation (4.1) only runs up through $j(\omega, t)$ since only those intermediate inputs which have been invented by time t can be used in the production of final goods at time t. If only the highest quality intermediate inputs are used in production, which will be the case in equilibrium, then the production function reduces to

$$Y_i(t) = \int_0^1 AL_i(t)^{1-\alpha} \lambda^{j(\omega,t)\alpha} x_i(\omega, t)^\alpha \, d\omega, \tag{4.2}$$

where $x_i(\omega, t)$ is the amount of the highest quality intermediate input from industry ω used at time t.

Suppose for the moment that only the best existing quality intermediate inputs are available for use in final good production and let $p(\omega, t)$ denote the price of the leading-edge intermediate input from industry ω at time t (relative to the final good price which we treat as the numeriare). From equation (4.2), the marginal product of intermediate inputs is obtained by differentiating inside the integral with respect to $x_i(\omega, t)$. Profit maximization by firm i then implies that the marginal product of each input must equal its price:

$$A\alpha L_i(t)^{1-\alpha}\lambda^{j(\omega,t)\alpha}x_i(\omega,t)^{\alpha-1} = p(\omega,t). \tag{4.3}$$

Since all firms face the same input prices and choose the same input ratios, we can aggregate across firms to obtain the aggregate demand $X(\omega, t)$ for the highest quality intermediate input from industry ω at time t:

$$X(\omega,t) = L\left[\frac{A\lambda^{j(\omega,t)\alpha}\alpha}{p(\omega,t)}\right]^{\frac{1}{1-\alpha}}. \tag{4.4}$$

Each intermediate input is nondurable and has a unit marginal cost of production (measured in terms of final good output Y). The government subsidizes the production of all intermediate inputs by paying a fraction s_p of each firm's production costs and this subsidy policy is financed by lump-sum taxation. Thus leading-edge intermediate input producers choose their prices to solve the profit maximization problem

$$\max_{p(\omega,t)} X(\omega,t)\left[p(\omega,t) - (1-s_p)\right] \tag{4.5}$$

which yields the usual monopoly price markup

$$p(\omega,t) = \frac{1-s_p}{\alpha}. \tag{4.6}$$

Note that this monopoly price is constant over time and across industries. Furthermore, if $\frac{1}{\alpha} < \lambda$, then lower quality intermediate input producers are not able to compete even when leading-edge producers charge the unconstrained monopoly price. We assume that the size of innovations parameter λ is sufficiently large so that $\frac{1}{\alpha} < \lambda$. Then equation (4.6) holds in equilibrium and only the highest quality intermediate inputs are used in final good production. Thus, the focus in this paper is on big innovations.[3]

Given equation (4.4) and equation (4.6), a leading-edge firm with a product of quality j earns the profit flow

$$\pi(j) = \frac{L\left[\frac{1-\alpha}{\alpha}\right]\left[A\alpha^2\lambda^{j\alpha}\right]^{\frac{1}{1-\alpha}}}{(1-s_p)^{\frac{\alpha}{1-\alpha}}}. \tag{4.7}$$

The profit flows earned by a leader during the jth innovation race are independent of industry and remain constant in the interval between innovations. The total resources devoted to intermediate input production at time t are given by

$$X(t) \equiv \int_0^1 X(\omega, t)\, d\omega = \left[\frac{A\alpha^2}{1-s_p}\right]^{\frac{1}{1-\alpha}} LQ(t), \tag{4.8}$$

where

$$Q(t) \equiv \int_0^1 \lambda^{j(\omega,t)\alpha/1-\alpha}\, d\omega \tag{4.9}$$

is an intermediate input quality index. Substituting equation (4.3) and equation (4.6) into equation (4.2), and aggregating across firms gives total output of the final good at time t

$$Y(t) = \left[\frac{A^{\frac{1}{\alpha}}\alpha^2}{1-s_p}\right]^{\frac{\alpha}{1-\alpha}} LQ(t). \tag{4.10}$$

Since Q grows over time due to R&D activities (as will be shown below), output Y also grows over time. This reflects the increasing productivity of workers over time.

[3] Barro and Sala-i-Martin [1995] also solve the model when $\frac{1}{\alpha} \geq \lambda$. With small innovations, leading-edge producers practice limit-pricing and the analysis is only slightly different.

4.2.3 The Consumer Sector

All consumers have identical preferences and live forever. Each consumer born at time τ maximizes a familiar expression for utility:

$$U(\tau) \equiv \int_{\tau}^{\infty} \left(\frac{c(t)^{1-\theta} - 1}{1-\theta} \right) e^{-\rho t}\, dt, \tag{4.11}$$

where $c(t)$ is the consumer's final good consumption at time t, $\rho > 0$ is the subjective discount rate and $\theta > 0$ is the constant elasticity of marginal utility with respect to consumption. Maximizing equation (4.11) subject to the consumer's intertemporal budget constraint yields the usual intertemporal consumer optimization condition

$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho}{\theta}, \tag{4.12}$$

where $r(t)$ is the equilibrium interest rate at time t. We assume that all consumers have the same financial assets at each point in time (parents share their asset equally with their children) so that $c(t)$ also represents per capita consumption and aggregate consumption is given by $C(t) \equiv c(t)L(t)$. Note that in a *balanced growth equilibrium* where all endogenous variables grow at constant rates, equation (4.12) implies that the market interest rate r is constant over time. We restrict attention in this paper to the balanced growth properties of the model.

4.2.4 The R&D Sector

In each industry $\omega \in [0, 1]$, there are two types of firms that can do R&D; the current quality *leader* (the firm that is producing the state-of-the-art quality intermediate input) and *followers* (all other firms in the industry). There are m follower firms in each industry and each follower firm possesses the same R&D technology. The leader, however, has a cost advantage in doing R&D compared to follower firms.

Let $I_{fi}(\omega,t)$ and $I_l(\omega,t)$ denote the instantaneous probabilities of R&D success by follower firm i and the current leader in industry ω at time t. These probabilities of R&D success are independently distributed across firms, industries and over time. Thus $I(\omega,t) \equiv I_l(\omega,t) + \sum_{i=1}^{m} I_{fi}(\omega,t)$ is the instantaneous probability that some firm will innovate in industry ω at time t and $I(t) \equiv \int_0^1 I(\omega,t)\,d\omega$ is the average R&D intensity across industries in the economy.

Let $R_{fi}(\omega,t)$ and $R_l(\omega,t)$ denote the flow of resources devoted to R&D by follower firm i and the current leader in industry ω at time t (measured in units of final good output Y). The leader's instantaneous probability of R&D success is

$$I_l(\omega,t) = \frac{R_l(\omega,t)}{c_l d^{j(\omega,t)}}, \tag{4.13}$$

and follower firm i's instantaneous probability of R&D success is

$$I_{fi}(\omega,t) \equiv \frac{R_{fi}(\omega,t)}{c_f d^{j(\omega,t)}}, \tag{4.14}$$

where $1 < d$ and $0 < c_l < c_f$. With these R&D technologies for leader and follower firms, the larger are the resources that a firm devotes to R&D, the larger is the firm's instantaneous probability of R&D success.

Given $d > 1$, $d^{j(\omega,t)}$ is increasing in j. Thus equation (4.13) and equation (4.14) imply that R&D projects become more complex and challenging with each step up the quality ladder in any industry. Every time innovation occurs in an industry, the instantaneous probabilities of R&D success for both leader and follower firms decline, for any given levels of R&D effort.

We focus in this paper on the balanced growth properties of the model when R&D behavior is *symmetric* across industries (for any t, $I_{fi}(\omega, t)$ and $I_l(\omega, t)$ do not vary across industries $\omega \in [0, 1]$). In a symmetric balanced growth equilibrium, the average R&D intensity $I(t)$ also represents the R&D intensity in every industry. With all follower firms in an industry choosing the same R&D intensity and independence of returns across firms, equation (4.14) allows for convenient aggregation:

$$I_f(\omega, t) = \frac{R_f(\omega, t)}{c_f d^{j(\omega,t)}}, \tag{4.15}$$

where $R_f(\omega, t) \equiv mR_{fi}(\omega, t)$ is the total resources devoted to R&D by follower firms and $I_f(\omega, t) \equiv mI_{fi}(\omega, t)$ is the instantaneous probability of R&D success by all follower firms combined. Note that the aggregate R&D relationship equation (4.15) is independent of m. We assume that there is free entry into R&D races, that is, the number of follower firms is arbitrarily large in each industry ($m \approx +\infty$). Then individual follower firms have negligible market value in equilibrium. Comparing equation (4.15) with equation (4.13), the assumption $c_f > c_l > 0$ implies that in each industry, the current leader has a R&D cost advantage over the competitive fringe of follower firms (taken as a whole). Because leaders are already on the technology frontier, it is easier for them to advance the frontier than other firms.

4.2.5 The Resource Constraint

Differentiating equation (4.13) and equation (4.15) with respect to t , both $I_f(\omega, t)$ and $I_l(\omega, t)$ must be constants over time in a symmetric balanced growth equilibrium. Thus, we can simplify notation by letting I_f and I_l denote the R&D intensities of follower and leader firms in each industry over time and by letting I denote the aggregate intensity in each industry over time. Equation (4.13) and (4.15) also imply that the economy-wide resources devoted to R&D at time t are

$$\begin{aligned} R(t) &= \int_0^1 [R_l(\omega,t) + R_f(\omega,t)]d\omega \\ &= \{[I_l c_l] + [I_f c_f]\} \int_0^1 d^{j(\omega,t)} d\omega. \end{aligned} \tag{4.16}$$

Resources in the economy (measured in terms of final good output Y) can be either used in the production of intermediate inputs, used in the R&D sector, or consumed. Therefore, the economy-wide resource constraint at time t is

$$Y(t) = C(t) + X(t) + R(t). \tag{4.17}$$

4.3 Balanced Growth Equilibria

In order for leaders to participate in the effort to improve on their own state-of-the-art products, they must hold a significant R&D cost advantage over followers. Initially we will assume that this condition is violated and that the advantage is small enough to prohibit leaders from employing R&D resources while maintaining the industry's quality lead. With this assumption we solve for a Nash equilibrium with only followers doing R&D. We then assume leaders enjoy a large R&D cost advantage, allowing us to solve for a Nash equilibrium with only leaders doing R&D. Finally we show that in rare instances the nature of the R&D cost advantage enjoyed by leaders produces a continuum of Nash equilibria with both leaders and followers conducting R&D.

From our analysis of the model we conclude that firms do not behave the way Barro and Sala-i-Martin suggest. Barro and Sala-i-Martin conclude that leaders perform all R&D whenever they possess even the slightest R&D cost advantage over their competitors. Their conclusion is a result of the equilibrium concept they employ, a variation of the Stackelberg equilibrium concept. Implicit in their analysis, Barro and Sala-i-Martin assume that, prior to outsider's commitments to R&D, leaders are able to commit to observable long-term R&D programs. With this implied first mover advantage leaders are able to keep outsiders from entering each R&D race and the benefits leaders expect from R&D therefore equal the benefits outsiders expect from R&D. With each firm facing the same potential gains from successful R&D, Barro and Sala-i-Martin find all R&D being done by leaders whenever leaders have any cost advantage in R&D.

The equilibrium concept employed here is the Nash equilibrium where all firms choose R&D effort simultaneously and the R&D choice made by each is an optimal response to the R&D choices of all other firms performing R&D. We believe the assumptions of the Nash equilibrium more accurately reflect the actual conditions in R&D markets. Even when leaders have a first mover advantage firms will behave according to the Nash equilibrium when R&D programs are not observable by other firms.

4.3.1 A Nash Equilibrium Where Only Followers Innovate

In the balanced growth equilibrium, leaders will not employ R&D resources unless they enjoy a significant cost advantage in R&D. With an insignificant R&D cost advantage, a leader that does not participate in R&D earns a revenue flow at each instant equal to the flow of monopoly profits from intermediate production. The leader neither suffers R&D costs nor expects the benefits of R&D while in the leadership position. Following Barro and Sala-i-Martin, we also assume that the probability of successful innovation remains constant for each follower between successes. Given these assumptions, the market value for the industry's non-R&D performing quality leader during the $(j+1)$st innovation race is

$$\begin{aligned} V_l(j) &= \int_0^\infty \left[\int_0^\tau [\pi(j)]\, e^{-rs}\, ds\right] I_f e^{-I_f\tau}\, d\tau \\ &= \frac{\pi(j)}{r+I_f}. \end{aligned} \tag{4.18}$$

In the steady-state equilibrium, where the interest rate r remains constant, the market value of an industry leader will remain constant in period of the time beginning when it successfully innovates and ending when the next innovation occurs.

During each R&D race followers will each employ R&D resources which provide them with a positive probability of replacing the current industry leader as the monopoly producer of the industry's product. In the $(j+1)$st innovation race a follower that takes the cumulative R&D efforts of all other firms during the race as fixed at I_{-fi} will have an expected market value of

$$\begin{aligned} V_f(j) &= \int_0^\infty \left[\left(\int_0^\tau -R_{fi}(j)(1-s_f)e^{-rs}\, ds\right) I + V_l(j+1) I_{fi} e^{-r\tau}\right] e^{-I\tau}\, d\tau \\ &= \frac{-R_{fi}(j)(1-s_f) + I_{fi}V_l(j+1)}{r + I_{-fi} + I_{fi}}, \end{aligned} \tag{4.19}$$

where $I = I_f$ in the absence of leader R&D efforts.

Each follower in the industry will choose R&D employment to maximize its firm value at each moment. When maximizing firm value, followers will choose to employ finite non-zero resources in R&D only if the marginal expected benefit from R&D equals the marginal cost of additional R&D resources for all R&D choices. We assume firms are able to freely enter and exit R&D races. With free entry the expected profits from R&D equal zero and firms engage in finite non-zero R&D which produces steady economic growth. Given the firm value defined in equation (4.19) the free entry condition ensuring balanced growth is

$$\frac{\pi(j)}{[r + I_f][c_f d^j]} = (1 - s_f). \tag{4.20}$$

Barro and Sala-i-Martin identify two potential causes for the probability of success to vary between development races. When intermediate product quality advances so do flow profits from intermediate production. This encourages firms to commit more resources to R&D when the industry is further up the quality ladder. Offsetting this increase is the growing difficulty of R&D as product quality in the industry progresses, an effect which produces diminishing R&D efforts as the industry climbs the quality ladder. In the remainder of the analysis we restrict attention to the case where these two factors exactly offset each other. This occurs when

$$d = \lambda^{\frac{\alpha}{1-\alpha}}. \tag{4.21}$$

When this condition is met, growth in the economy proceeds at an even pace. If this condition is not met then either the profit growth effect dominates the R&D effect and the economy experiences explosive growth, or the R&D effect dominates the profit growth effect and economic growth eventually stalls.

In the balanced growth equilibrium the probability of success remaining fixed over time and, given the initial conditions of the model, across industries. With a constant probability of success the aggregate devotion of resources to R&D by all followers in all industries at any given instant equals

$$R(t) = I_f c_f Q(t). \tag{4.22}$$

In the steady state the growth of R&D resource use over time equals the growth in the economy's quality index. Because both output production and intermediate resource use grow at the same rate as the economy's quality index, the growth rate of consumption must also equal the growth rate of the quality index to satisfy the resource consraint. The growth rate of the quality index over time is given by

$$\frac{\dot{Q}(t)}{Q(t)} = \int_0^\infty I_f \lambda^{\frac{j\alpha}{1-\alpha}} \left[\lambda^{\frac{\alpha}{1-\alpha}} - 1\right] Q(t)^{-1} = I_f \left[\lambda^{\frac{\alpha}{1-\alpha}} - 1\right] \tag{4.23}$$

This is also the growth rate of per capita consumption in the economy, as a constant population ensures that economy-wide consumption growth equals per capita consumption growth.

In the steady-state the growth rate of per capita consumption defined in equation (4.23) must equal the growth rate of per capita consumption defined in each consumer's optimization problem in equation (4.12). Equating these two definitions of per capita growth generates an expression for the economy's interest rate in terms of the aggregate probability of success I_f and the model parameters. Combining this rate with equation (4.20), the free entry condition, allows us to pinpoint the aggregate probability of success in terms of the model parameters. In the equilibrium with only followers performing R&D, aggregate R&D effort produces a constant probability of success in each industry at each moment given by

$$I_f = \frac{\lambda^{\frac{\alpha}{1-\alpha}} L \left(\frac{1-\alpha}{\alpha}\right) (A\alpha^2)^{\frac{1}{1-\alpha}}}{(1-s_p)^{\frac{\alpha}{1-\alpha}}(1-s_f)c_f \left[1+\theta\left(\lambda^{\frac{\alpha}{1-\alpha}}-1\right)\right]} - \frac{\rho}{\left[1+\theta\left(\lambda^{\frac{\alpha}{1-\alpha}}-1\right)\right]}. \tag{4.24}$$

This equilibrium occurs only if the quality leader in each industry does no R&D. Considering the leader's problem, a leader choosing to employ R&D resources during the $(j+1)$st innovation race incurs positive R&D expenses which reduce its revenue flows. By employing R&D resources the leader has positive probability of retaining its quality lead and creating the next generation of intermediates. Given follower behavior the expected market value of a quality leader with a state-of-the-art product of quality j is

$$\begin{aligned} V_l(j) &= \int_0^\infty \left[\left(\int_0^\tau [\pi(j) - R_l(j)(1-s_l)]\, e^{-rs}\, ds\right) I + V_l(j+1) I_l e^{-r\tau}\right] e^{-I\tau}\, d\tau \\ &= \frac{\pi(j) - R_l(j)(1-s_l) + I_l V_l(j+1)}{r + I_l + I_f}. \end{aligned} \tag{4.25}$$

When maximizing firm value leaders will choose not to employ R&D resources only if the marginal expected benefit of R&D is less than or equal to the marginal cost of R&D resources for all resource choices. This condition is met provided

$$\frac{\left(1-\lambda^{-\frac{\alpha}{1-\alpha}}\right)\pi(j)}{(r+I_f)c_l d^j} \leq (1-s_l). \tag{4.26}$$

Combining this condition with the free entry condition in equation (4.20) we derive that a Nash equilibrium exists with only followers doing R&D whenever

$$c_l \geq \frac{c_f(1-s_f)\left(1-\lambda^{-\frac{\alpha}{1-\alpha}}\right)}{(1-s_l)}. \tag{4.27}$$

In the absence of subsidies, if leaders do not have a significant cost advantage, then followers will perform all R&D. This result contradicts that found by Barro and Sala-i-Martin.

4.3.2 A Nash Equilibrium Where Only Leaders Innovate

A significant leader R&D cost advantage creates the potential for all R&D to be done by the firm with the quality lead in each industry. If leaders assume followers will remain out of each R&D race then the expected firm value for each leader is given by equation (4.25) where $I = I_l$.

When maximizing firm value, leaders will choose a finite non-zero R&D effort only when the expected marginal benefits of R&D equal the marginal resource cost of R&D. This steady-state condition is met during the $(j+1)$st innovative episode when

$$\frac{\left(1-\lambda^{-\frac{\alpha}{1-\alpha}}\right)\pi(j)}{rc_l d^j} = (1-s_l). \tag{4.28}$$

As in the Nash equilibrium with followers performing all R&D, the parameter restriction in equation (4.21) ensures both that the probability of success remains constant as industries climb the quality ladder and that the economy experiences smooth economic growth.

Aggregating across industries, resource devotion to R&D during each time period t equals

$$R(t) = I_l c_l Q(t). \tag{4.29}$$

As in the equilibrium with only followers performing R&D, the growth of R&D resources will equal the growth in the economy's quality index. With both output of final goods and intermediate resource use growing at the same rate as the quality index, the economy-wide resource constraint will again be satisfied only if both consumption growth and per capita consumption growth proceed at the same rate as the growth in the quality index. With leaders performing all R&D the rate of growth of the quality index is given by

$$\frac{\dot{Q}(t)}{Q(t)} = \int_0^\infty I_l \lambda^{\frac{j\alpha}{1-\alpha}} \left[\lambda^{\frac{\alpha}{1-\alpha}} - 1\right] Q(t)^{-1} = I_l \left[\lambda^{\frac{\alpha}{1-\alpha}} - 1\right] \tag{4.30}$$

Per capita consumption will grow at this rate in the Nash equilibrium where leaders do all R&D.

In the steady-state the growth rate of per capita consumption defined in equation (4.30) must equal the growth rate of per capita consumption defined in each consumer's optimization problem in equation (4.12). Equating these two definitions of per capita growth generates an expression for the economy's interest rate in terms of the aggregate probability of success I_l and the model parameters. Combining this rate with equation (4.28), the steady-state condition, allows us to pinpoint the instantaneous probability of leader success in terms of the model parameters. In the equilibrium with only leaders performing R&D, the constant probability of success in each industry at each moment is given by

$$I_l = \frac{\left(\lambda^{\frac{\alpha}{1-\alpha}} - 1\right) L \left(\frac{1-\alpha}{\alpha}\right) (A\alpha^2)^{\frac{1}{1-\alpha}}}{(1-s_p)^{\frac{\alpha}{1-\alpha}}(1-s_l)c_l\left[\theta\left(\lambda^{\frac{\alpha}{1-\alpha}} - 1\right)\right]} - \frac{\rho}{\left[\theta\left(\lambda^{\frac{\alpha}{1-\alpha}} - 1\right)\right]}. \tag{4.31}$$

This equilibrium occurs only if followers in each industry do no R&D. If a follower chooses to participate in the $(j+1)$st innovation race the follower will, by employing R&D resources, obtain a positive probability of creating the next generation of product. Given leader behavior the expected market value of a follower during each R&D race where the state-of-the-art product is of quality j is defined by equation (4.19), where $I = I_l + I_f$. Followers that maximize firm value at each moment will remain out of each R&D race when the marginal expected benefit from R&D is less than the marginal cost of R&D resources. This will occur for all follower R&D resource choices only if

$$\frac{\pi(j)}{[r+I_f][c_f d^j]} \leq (1-s_f). \tag{4.32}$$

Combining this condition with the steady state condition in equation (4.28) we derive that a Nash equilibrium exists with leaders doing all R&D provided

$$c_l \leq \frac{c_f(1-s_f)\left(1-\lambda^{-\frac{\alpha}{1-\alpha}}\right)}{(1-s_l)}. \tag{4.33}$$

In the absence of subsidies, if leaders have a significant cost advantage, then followers will stay on the sidelines of R&D races.

4.3.3 A Nash Equilibrium with all Firms Innovating

When both leaders and followers perform R&D during each innovative race, firm value for followers will be defined by equation (4.19) and firm value for leaders will be defined by equation (4.25), where in both cases $I = I_l + I_f$. Follower R&D choices will be finite and non-zero only if their expected marginal benefits from R&D equal their respective marginal resource costs. This situation will occur only if the free entry condition equation (4.20) is satisfied. For leaders, marginal expected R&D benefits equal marginal expected R&D costs when

$$\frac{\left(1-\lambda^{-\frac{\alpha}{1-\alpha}}\right)\pi(j)}{(r+I_f)c_l d^j} = (1-s_l). \tag{4.34}$$

Combining these two conditions gives us the R&D cost relationship necessary for a Nash equilibriumin which both leaders and followers conducting R&D. The cost relationship must satisfy

$$c_l = \frac{c_f(1-s_f)\left(1-\lambda^{-\frac{\alpha}{1-\alpha}}\right)}{(1-s_l)}. \tag{4.35}$$

As in the previous Nash equilibria, when the model parameters satisfy equation (4.21) the probability of a success occurring for a firm at each instant will remain constant over time. In this equilibrium the aggregate resource use by all firms at time t will equal

$$R(t) = c_l I_l Q(t) + c_f I_f Q(t). \tag{4.36}$$

Like the previous equilibria the production of output and resource use for intermediates, R&D, and consumption all grow at the same rate as the economy's quality index. When both leaders and followers perform R&D the growth rate of the quality index is given by

$$\frac{\dot{Q}(t)}{Q(t)} = \int_0^\infty (I_l + I_f)\lambda^{\frac{j\alpha}{1-\alpha}}\left[\lambda^{\frac{\alpha}{1-\alpha}} - 1\right] Q(t)^{-1} = (I_l + I_f)\left[\lambda^{\frac{\alpha}{1-\alpha}} - 1\right]. \tag{4.37}$$

This rate of growth will equal the per capita growth rate of consumption when both leaders and followers perform R&D.

Equating the growth rates defined in equation (4.37) and equation (4.12) generates the economy-wide interest rate in terms of the model parameters. Combining this expression with the free market and steady-state conditions defined in equation (4.20) and equation (4.28) generates an implicit function defining the individual firm's probability of successful innovation. This expression is

$$I_f\left[1+\theta\left(\lambda^{\frac{\alpha}{1-\alpha}}-1\right)\right] + I_l\theta\left(\lambda^{\frac{\alpha}{1-\alpha}}-1\right) = \left[\frac{\left(1-\lambda^{-\frac{\alpha}{1-\alpha}}\right)\left(\frac{1-\alpha}{\alpha}\right)L\left[A\alpha^2\lambda^\alpha\right]^{\frac{1}{1-\alpha}}}{(1-s_l)c_l(1-s_p)^{\frac{\alpha}{1-\alpha}}}\right] - \rho \tag{4.38}$$

In this case a continuum of equilibria exist with I_f taking on values between zero and the value determined in equation (4.24). For each value of I_f in this interval, a unique value of I_l exists which satisfies equation (4.38). Even though the results of the Barro and Sala-i-Martin model suggest equilibria with both leaders and followers performing R&D, the probability that the R&D cost relationship meets the requirement for such existence is extremely small. In general we would expect to observe either all R&D being performed by leaders or all R&D being performed by followers[4].

4.4 Optimal Growth and Subsidies

4.4.1 An Equilibrium with Optimal Resource Allocation

Determining the optimal distribution of resources in the economy requires solving the optimal resource allocation problem that a benevolent social planner interested in maximizing the representative consumer's welfare would solve. To achieve the

[4]One final Nash equilibrium possibility exists. If the parameter values of the model are such that the value of I_f defined in equation (4.24) and the value of I_l defined in equation (4.31) are both less than zero then no positive growth steady-state equilibria exists. For any given parameter values the determining factor as to whether a positive growth equilibrium exists is the supply of labor resources in the economy. For any set of parameter values a critical value of the labor supply exists below which no growth occurs and above which growth is positive. A positive growth equilibrium exists for the economy provided

$$L \geq \min\left[\frac{\rho c_f(1-s_f)(1-s_p)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1-\alpha}{\alpha}\right)\left[A\alpha^2\lambda^\alpha\right]^{\frac{1}{1-\alpha}}}, \frac{\rho c_l(1-s_l)(1-s_p)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1-\alpha}{\alpha}\right)\left(1-\lambda^{-\frac{\alpha}{1-\alpha}}\right)\left[A\alpha^2\lambda^\alpha\right]^{\frac{1}{1-\alpha}}}\right].$$

social optimum a social planner first chooses the level of intermediate use at each moment in time, a static allocation problem. Within each industry the marginal product of intermediates is increasing in quality. With equal production costs for goods the social planner will always exclusively use state-of-the-art intermediates from each industry. Given a unit marginal cost of production for inputs, the social planner will choose to employ intermediates from each industry up to the point where the marginal product of each intermediate equals unity. Demand for state-of-the-art intermediates from industry ω at time t will then equal

$$X(\omega,t) = L\left[A\alpha\lambda^{j(\omega,t)\alpha}\right]^{\frac{1}{1-\alpha}}. \tag{4.39}$$

Once industry specific production of intermediates has been determined, aggregating across industries reveals the economy-wide resource use in the production of intermediates to be

$$X(t) = A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}Q(t)L. \tag{4.40}$$

With this level of intermediate production, aggregate output of final goods at each instant will equal

$$Y(t) = A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}Q(t)L. \tag{4.41}$$

With constant returns to scale R&D technology, the marginal product of R&D resources for the industry leader will always be greater than the marginal product of R&D resources for an outside firm given that $c^l < c^f$ by prior assumption. Given the relative cost advantage of leader production technology, a social planner does best to apportion all of an industry's allocated R&D resources to the leader in each industry.

When the planner allocates $R(\omega, t)$ resources to industry ω at time t the expected rate of growth in the quality index equals

$$\dot{Q}(t) = \int_0^1 R(\omega, t) \frac{\lambda^{\frac{(j(\omega,t)+1)\alpha}{1-\alpha}} - \lambda^{\frac{j(\omega,t)\alpha}{1-\alpha}}}{c_l d^{j(\omega,t)}} d\omega = \frac{R(t)\left(\lambda^{\frac{\alpha}{1-\alpha}} - 1\right)}{c_l}. \tag{4.42}$$

With constant returns to scale the allocation of resources across industries has no bearing on economic growth. It is the aggregate choice of spending which affects the economy's growth over time.

The resource constraint for the social planner, subject to the optimal static allocation rules derived above, equals

$$A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right) L(t)Q(t) - R(t) - c(t)L = 0. \tag{4.43}$$

The optimal allocation of these resources will maximize expected discounted per capita utility subject to the resource constraint defined in equation (4.43) and the growth constraint defined in equation (4.42). The Hamiltonian for this optimal control problem is

$$\begin{aligned} J = \left(\frac{c^{1-\theta} - 1}{1-\theta} e^{-\rho t}\right) + v(t)\left[A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} Q(t) L \left(\frac{1-\alpha}{\alpha}\right) - R(t) - c(t)L\right] \\ + \mu(t) \frac{R(t)\left[\lambda^{\frac{\alpha}{1-\alpha}} - 1\right]}{c^l}. \end{aligned} \tag{4.44}$$

The Lagrange multipliers $v(t)$ apply to the resource constraints at each moment and the shadow price $\mu(t)$ attaches to the expression for the growth in the quality index over time.

Solving the dynamic optimal control problem yields that the optimal growth rate of consumption spending in terms of the model parameters equals

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left[\frac{\left[A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} L \left(\frac{1-\alpha}{\alpha}\right)\left[\lambda^{\frac{\alpha}{1-\alpha}} - 1\right]\right]}{c_l} - \rho\right]. \tag{4.45}$$

The per capita growth rate of consumption spending will be constant over time in the social optimum.

For the resource constraint defined in equation (4.17) to hold over time, if the rate of per capita consumption growth over time is constant then it must equal both the rate of growth in the economy's quality index and the rate of growth in R&D resource use. With a constant probability of innovative success at each instant the time invariant growth rate in the quality index equals

$$\frac{\dot{Q}(t)}{Q(t)} = I\left[\lambda^{\frac{\alpha}{1-\alpha}} - 1\right]. \tag{4.46}$$

Equating the growth rate in the quality index defined in equation (4.46) and the growth rate of consumption defined in equation (4.45) pinpoints the economy's optimal time invariant probability of success in terms of the model parameters. The probability of success at each moment in the optimum is

$$I = \frac{1}{\left[\lambda^{\frac{\alpha}{1-\alpha}} - 1\right]\theta}\left[\frac{\left[A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}L\left(\frac{1-\alpha}{\alpha}\right)\left[\lambda^{\frac{\alpha}{1-\alpha}} - 1\right]\right]}{c_l} - \rho\right]. \tag{4.47}$$

4.4.2 The Optimal Subsidy Plan

The social planner can achieve the socially optimal growth rate in the economy by correcting for three market inefficiencies inherent in the free market. The first inefficiency in the market is a direct result of the monopoly power state-of-the-art producers earn through innovative success. With monopoly pricing society suffers from traditional static dead-weight losses associated with under-production of intermediates. To correct for under-production, the social planner will choose a subsidy which reduces intermediate prices to unity, thereby producing an output level where the social marginal costs of intermediate production equal the social marginal benefits. The optimal subsidy necessary to achieve a unit price of intermediates equals

$$s_p = (1 - \alpha). \tag{4.48}$$

The second inefficiency results when a leader's R&D advantage is insufficient to prohibit followers from participation in R&D. The social planner can achieve maximum allocative efficiency by apportioning all R&D resources to the leader in each industry[5]. As shown above, the leader will perform R&D only if its cost advantage is significantly large. To correct for this inefficiency the social planner can set a prohibitive R&D tax on outsiders.

Barro and Sala-i-Martin argue that proper implementation of the two-part tax and subsidy policy outlined above produces the social optimum. By subsidizing intermediate production the social planner eliminates distortions from monopoly pricing. Prohibitive R&D taxes on outsiders, they argue, will create a situation with only leaders performing R&D. With only leaders performing R&D increased profit flows from innovation captured by the leader last forever, as do the benefits consumers receive from an innovation. Furthermore, when choosing R&D effort leaders only consider the additional profit flows from increasing the state-of-the-art quality, just as the planner only considers the extra utility created by a successful innovation. Barro and Sala-i-Martin argue this two-part tax and subsidy plan will move the economy to the social optimum.

[5]The allocation of resources between leaders and followers has no economic significance for social welfare in the case where both share the same R&D technology, $c_l = c_f$.

In their argument, Barro and Sala-i-Martin omit a further inefficiency in the R&D market which creates under-investment in R&D by leaders even when followers do not participate. Even with production subsidies the increase in profit flows achieved by a successful leader fall short of the increase in welfare gains each period to consumers. The need for a R&D subsidy comes from the successful firm's failure to capture all of consumer surplus. Although exercising monopoly power, the firm which cannot perfectly price discriminate cannot appropriate all of consumer surplus. A subsidy for intermediate production will eliminate static dead weight losses and increase the portion of consumer surplus appropriated by the firm but the difference between the static gain in consumer surplus and firm flow profits remains positive. This result occurs even with proper intermediate production subsidies. With the proper intermediate subsidy defined in equation (4.48) the instantaneous probability of success at each moment in the economy will equal the socially optimal probability only if

$$s_l = (1 - \alpha). \tag{4.49}$$

4.5 Conclusion

Our reexamination of Barro and Sala-i-Martin's model has led us to different conclusions than the previous authors regarding the nature of R&D subsidies. We find that R&D subsidies are necessary to achieve optimal resource allocations and remedy under-investment in R&D a consequence of what previous authors[6] have termed

[6]Grossman and Helpman [1991b, chapter 4] offer an analysis of "consumer surplus" effects in a growth model with the quality ladders structure.

the "consumer surplus" effect. This effect is overlooked by Barro and Sala-i-Martin, resulting in an incorrect assessment of the optimal R&D subsidies.

The most significant result of our reexamination is our conclusion that the model does not answer the question it was initially designed to address. When an appropriate equilibrium concept is employed we find that leaders only employ R&D resources when enjoying a large R&D cost advantage, a result that supports the conclusions of previous authors who have analyzed growth models with quality ladders structures. This result fails to explain why firms currently producing the state-of-the-art product in an industry will conduct R&D in the absence of a R&D cost advantage.

Appendices

Appendix A

Stability of the R&D Equation

Assumption 1 is an assumption regarding the form of technology when R&D labor exceeds $\bar{l}$. To derive the explicit restrictions imposed by Assumption 1, I solve for the inequality in the assumption. Taking expected benefits of success as fixed, a given firm chooses R&D labor to maximize expected R&D profits, generating the R&D profit maximizing condition

$$\frac{\partial E\Pi_i}{\partial l_i} = \frac{[Vh'(l_i) - 1][\rho + k] - [h(l_i) - l_i h'(l_i)]}{[\rho + k + h(l_i)]^2} = 0. \tag{A.1}$$

I restrict attention to the symmetric equilibrium, where $l = l_i$ for all participants in each patent race. If firm i increases its R&D effort about the equilibrium value of l_i defined by equation (A.1), then the resulting change in the firm's R&D profit maximizing condition is given by

$$\frac{\partial \frac{\partial E\Pi_i}{\partial l_i}}{\partial l_i} = \frac{Vh''(l_i)[\rho + k] + l_i h''(l_i)}{[\rho + k + h(l_i)]^2} \leq 0. \tag{A.2}$$

As expected, if a firm is using R&D labor in excess of its profit maximizing level, it will wish to reduce R&D labor use. If each of the other $(n - 2)$ participants in the patent race also increase their R&D efforts by the same amount, the resulting change in the LHS of equation (A.1) is given by

$$\frac{\partial \frac{\partial E\Pi_i}{\partial l_i}}{\partial k} \sum_{j \neq i} \left(\frac{\partial k}{\partial l_j} \right) = \frac{(n-2)h'(l_i)[Vh'(l_i)-1]}{[\rho + k + h(l_i)]^2} \geq 0. \tag{A.3}$$

When all other firms in the race increase their R&D efforts, firm i will wish to increase its own R&D effort. The symmetric solution to equation (1.10) is stable only if

$$\frac{d \frac{\partial E\Pi_i}{\partial l_i}}{dl_i} = \frac{\partial \frac{\partial E\Pi_i}{\partial l_i}}{\partial l_i} + \frac{\partial \frac{\partial E\Pi_i}{\partial l_i}}{\partial k} \sum_{j \neq i} \left(\frac{\partial k}{\partial l_j} \right) \leq 0. \tag{A.4}$$

When all firms raise their R&D efforts equally, and the stability condition is met, each firm's desire to reduce its own R&D, generated by its own increase in R&D efforts, outweighs the firm's desire to increase its own R&D, generated by its competitors increase in R&D efforts. The equilibrium defined by equation (1.10) is then stable. Combining equations (A.2), (A.3), and (A.4) reveals the stability condition to be met if, for $l \geq \bar{l}$, the following condition holds

$$\frac{-h''(l)[\rho(2n-3)h(l) + \rho^2 + (n-1)(n-2)h(l)^2] + (n-2)h'(l)^2[lh'(l) - h(l)]}{h'(l)^2[\rho + (n-2)h(l)]^2} \geq 0. \tag{A.5}$$

This condition holds for all economically significant parameter values, including $\rho = 0$ provided the R&D technology satisfies the restriction

$$-h''(l)h(l)^2 - h'(l)^2[h(l) - lh'(l)] \geq 0. \tag{A.6}$$

A wide variety of technological specifications will satisfy this restriction. For example, the condition is satisfied if for $l \geq \bar{l}$ the R&D technology can be represented by the functional form $h(l) = B(l-a)^b$, for any parameter values such that $B > 0$, $a \geq 0$, and $0 < b < 1$. If the R&D technology can be represented for $l \geq \bar{l}$ by the natural log specification $h(l) = ln(l - a)$, the condition will be satisfied provided $a \geq 0$. Dropping the restriction that returns to scale must eventually be decreasing, the constant returns technology adopted by Grossman and Helpman of the form $h(l) = \frac{l}{a}$ will also satisfy the stability condition, holding with strict equality.

Appendix B

Discount Rate Effects

Above it was show that given Assumptions 1 and 2, $\frac{\partial V_L}{\partial l} \leq 0$ and $\frac{\partial V_R}{\partial l} \geq 0$, which implies $\frac{\partial \mathcal{H}}{\partial l} \leq 0$. Given this result and the implicit function theorem, determining the effect of a larger discount rate on firm R&D choices only requires finding the effect of a larger discount rate on the implicit equation defining R&D hiring, l. The implicit equation is given by equation (1.17) and the resulting impact of participation changes on this function is

$$\frac{\partial \mathcal{H}}{\partial \rho} = \frac{-[L-(n-1)l][\lambda-1]}{[\rho+(n-1)h(l)]^2} - \left\{ \frac{h'(l)[\rho+(n-2)h(l)] - h'(l)[(n-1)h(l) - lh'(l) + \rho]}{[h'(l)[\rho+(n-2)h(l)]]^2} \right\}$$

$$= -\frac{V}{[\rho+(n-1)h(l)]} + \left\{ \frac{[h(l) - lh'(l)]}{[h'(l)[\rho+(n-2)h(l)]]^2} \right\}. \tag{B.1}$$

Using $V = \frac{(n-1)h(l) - lh'(l) + \rho}{h'(l)[\rho+(n-2)h(l)]}$ yields

$$\frac{\partial \mathcal{H}}{\partial \rho} = \frac{h(l)[h(l) - lh'(l)] - [(n-2)+\rho][\rho+(n-2)h(l)]}{h'(l)[\rho+(n-2)h(l)]^2[\rho+(n-1)h(l)]}. \tag{B.2}$$

Then $\frac{\partial \mathcal{H}}{\partial \rho} \leq 0$ as long as $n \geq 3$. An exception to the negative relationship between $\mathcal{H}$ and ρ occurs when $n = 2$ and ρ is small. At $n = 2$ $\frac{\partial \mathcal{H}}{\partial \rho} \geq 0$ if $\rho^2 \leq h(l)[h(l) - lh'(l)]$.

Appendix C

Participation Effects

Above it was show that given Assumptions 1 and 2, $\frac{\partial V_L}{\partial l} \leq 0$ and $\frac{\partial V_R}{\partial l} \geq 0$, which implies $\frac{\partial \mathcal{H}}{\partial l} \leq 0$. Given this result and the implicit function theorem, determining the effect of increased patent race participation on firm R&D choices only requires finding the effect of patent race participation on the implicit equation defining R&D hiring, l. The implicit equation is given by equation (1.17) and the resulting impact of participation changes on this function is

$$\frac{\partial \mathcal{H}}{\partial n} = \frac{[\rho + (n-1)h(l)][-l(\lambda - 1)] - [L - (n-1)l][\lambda - 1]h(l)}{[\rho + (n-1)h(l)]^2}$$

$$= -\left\{ \frac{h'(l)[\rho + (n-2)h(l)]h(l) - h(l)h'(l)[(n-1)h(l) - lh'(l) + \rho]}{[h'(l)[\rho + (n-2)h(l)]]^2} \right\}$$

$$= -\frac{l(\lambda - 1)}{[\rho + (n-1)h(l)]} - \frac{Vh(l)}{[\rho + (n-1)h(l)]} - \frac{h(l)}{h'(l)[\rho + (n-2)h(l)]} + \frac{Vh(l)}{[\rho + (n-2)h(l)]}$$

$$= \frac{-l(\lambda - 1)[\rho + (n-2)h(l)]h'(l) - h(l)[\rho + (n-1)h(l)] + Vh(l)^2 h'(l)}{h'(l)[\rho + (n-1)h(l)][\rho + (n-2)h(l)]}. \tag{C.1}$$

Using $V = \frac{(n-1)h(l) - lh'(l) + \rho}{h'(l)[\rho + (n-2)h(l)]}$ yields

$$\frac{\partial\mathcal{H}}{\partial n} = \frac{-l(\lambda-1)h'(l)^2[\rho+(n-2)h(l)]^2 - h(l)h'(l)[\rho+(n-1)h(l)][\rho+(n-2)h(l)]}{h'(l)^2[\rho+(n-2)h(l)]^2[\rho+(n-1)h(l)]}$$
$$+\frac{h(l)^2h'(l)[(n-1)h(l) - lh'(l)+\rho]}{h'(l)^2[\rho+(n-2)h(l)]^2[\rho+(n-1)h(l)]}$$
$$= \frac{-l(\lambda-1)h'(l)^2[\rho+(n-2)h(l)]^2 - h(l)h'(l)[\rho+(n-1)h(l)][\rho+(n-2)h(l)]}{h'(l)^2[\rho+(n-2)h(l)]^2[\rho+(n-1)h(l)]}$$
$$+\frac{-lh(l)^2h'(l)^2 + h(l)^2h'(l)[(n-1)h(l)+\rho]}{h'(l)^2[\rho+(n-2)h(l)]^2[\rho+(n-1)h(l)]}. \tag{C.2}$$

Then $\frac{\partial\mathcal{H}}{\partial n} \leq 0$ as $|-h(l)h'(l)[\rho+(n-2)h(l)]| \geq h(l)^2h'(l)$. which occurs for $n \geq 3$.

An exception to the negative relationship between $\mathcal{H}$ and n occurs when $n=2$, $l \geq \bar{l}$ and ρ is small. At $n=2$

$$\frac{\partial\mathcal{H}}{\partial n} = \frac{-l(\lambda-1)h'(l)^2\rho^2 - \rho h(l)h'(l)[\rho+h(l)] - lh(l)^2h'(l)^2 + h(l)^2h'(l)[\rho+h(l)]}{h'(l)^2\rho^2[\rho+h(l)]}$$
$$= \frac{-l(\lambda-1)h'(l)^2\rho^2 - \rho^2h(l)h'(l) - lh(l)^2h'(l)^2 + h(l)^3h'(l)}{h'(l)^2\rho^2[\rho+h(l)]}$$
$$= \frac{-l(\lambda-1)h'(l)^2\rho^2 - \rho^2h(l)h'(l) + h(l)^2h'(l)[h(l) - lh'(l)]}{h'(l)^2\rho^2[\rho+h(l)]}, \tag{C.3}$$

which has a limit of $\lim_{\rho\to 0} \left.\frac{\partial\mathcal{H}}{\partial n}\right|_{n=2} = +\infty$.

Appendix D

The Negative Slope of the Profit Equation

Demonstrating the negative slope of the profit equation requires proving that the condition

$$1 - [(I + \rho)T + 1]e^{-(I+\rho)T} \geq 0 \tag{D.1}$$

holds for all $I \geq 0$, where $I = (n - 1)h(l)$. This condition will hold as long as

$$e^{(I+\rho)T} \geq (I + \rho)T + 1. \tag{D.2}$$

Replacing $(I + \rho)T$ by X we have the condition holds for all X such that

$$e^{X} \geq X + 1. \tag{D.3}$$

At $X = 0$,

$$e^{X} = X + 1 = 1. \tag{D.4}$$

The derivative of the left hand side of equation (D.3) is

$$\frac{dLHS(x)}{dX} = e^{X} \geq 1 \text{ for all } X. \tag{D.5}$$

The derivative of the right hand side of equation (D.2) is unity.

As equation (D.2) holds with equality at $X = 0$ and the left hand side is increasing at a faster rate than the right hand side everywhere to the right of $X = 0$, the inequality in equation (D.2) holds for all values of $X \geq 0$. Subsequently the inequality in equation (D.1) holds for all values of $(I + \rho)T \geq 0$.

Appendix E

Diminishing Patent Policy Effectiveness

Demonstrating diminishing patent policy effectiveness requires proving that the following condition holds for all non-negative equilibrium I where $I = (n-1)h(l)$.

$$(\rho + I)T\left[e^{(\rho+I)T} - 1 + 2\right] - 2\left[e^{(\rho+I)T} - 1\right] \geq 0. \tag{E.1}$$

This condition will hold when $X = (\rho + I)T$ if

$$2X + (X-2)\left[e^{X} - 1\right] \geq 0. \tag{E.2}$$

The condition holds for $X \geq 2$ and will hold for $X < 2$ provided that

$$2X \geq (e^{X} - 1)(2 - x). \tag{E.3}$$

At $X = 0$ the left hand side of equation (E.3) is equal to zero as is the right hand side of equation (E.3). The derivative of the left hand side of equation (E.3) is

$$\frac{dLHS(x)}{dX} = 2. \tag{E.4}$$

The derivative of the right hand side of equation (E.3) is

$$\frac{dRHS(x)}{dX} = (1 - X)e^X + 1. \tag{E.5}$$

The derivatives are both equal to 2 at $X = 0$. The slope of the LHS is constant to the right of $X = 0$, while a check of the second derivative of the RHS reveals

$$\frac{d^2RHS(x)}{dX^2} = -Xe^X. \tag{E.6}$$

The slope of the RHS is diminishing to the right of $X = 0$.

As equation (E.3) holds with equality at $X = 0$ and the slope of the left hand side of equation (E.3) is greater than or equal to the slope of the right hand side for all values of $X \geq 0$, the inequality in equation (E.3) must hold as will the inequality in equation (E.1).

Appendix F

Simulation Tables

Table F.1: Expected Welfare Case 1

	Expected Welfare	
T	L=17	L=30
0	28.33213	34.01197
1	415.75593	885.99957
2	508.41620	891.76452
3	508.90634	891.76493
4	508.90846	891.76493
5	508.90847	891.76493
6	508.90847	891.76493
7	508.90847	891.76493
8	508.90847	891.76493
9	508.90847	891.76493
10	508.90847	891.76493
11	508.90847	891.76493
12	508.90847	891.76493
13	508.90847	891.76493
14	508.90847	891.76493
15	508.90847	891.76493
16	508.90847	891.76493
17	508.90847	891.76493
18	508.90847	891.76493
19	508.90847	891.76493
20	508.90847	891.76493

Table F.2: Expected Welfare Case 2

	Expected Welfare	
T	L=1	L=10
0	0	287.82314
1	0	11634.33789
2	52.88359	39310.03336
3	160.91667	39632.81339
4	346.19760	39633.92564
5	639.89522	39633.92944
6	1060.79573	39633.92945
7	1590.48172	39633.92945
8	2161.54430	39633.92945
9	2684.94756	39633.92945
10	3098.72839	39633.92945
11	3389.55602	39633.92945
12	3577.59704	39633.92945
13	3692.74042	39633.92945
14	3760.92542	39633.92945
15	3800.50949	39633.92945
16	3823.22675	39633.92945
17	3836.17862	39633.92945
18	3843.53537	39633.92945
19	3847.70524	39633.92945
20	3850.06594	39633.92945

Table F.3: Expected Welfare Case 3

	Expected Welfare	
T	L=15	L=30
0	338.50628	425.14967
1	375.14417	547.02661
2	455.77655	982.81837
3	601.11651	2225.52582
4	850.18146	4610.82462
5	1243.43845	7002.19575
6	1792.34779	8255.97713
7	2441.20864	8700.96305
8	3073.87052	8836.04599
9	3585.22954	8875.05207
10	3939.58145	8886.15113
11	4159.78500	8889.29611
12	4287.50155	8890.18620
13	4358.63268	8890.43803
14	4397.35734	8890.50927
15	4418.17886	8890.52942
16	4429.29941	8890.53512
17	4435.21756	8890.53673
18	4438.36109	8890.53719
19	4440.02914	8890.53732
20	4440.91379	8890.53736

Table F.4: Expected Welfare Case 4

	Expected Welfare	
T	L=2	L=10
0	6.93147	23.02585
1	8.30502	69.21215
2	12.09725	166.61510
3	16.25791	199.9593
4	20.34084	204.37557
5	23.87174	204.86146
6	26.60214	204.91378
7	28.53974	204.91940
8	29.83382	204.92000
9	30.66436	204.92007
10	31.18421	204.92007
11	31.50467	204.92007
12	31.70041	204.92007
13	31.81933	204.92007
14	31.89134	204.92007
15	31.93487	204.92007
16	31.96116	204.92007
17	31.97702	204.92007
18	31.98658	204.92007
19	31.99236	204.92007
20	31.99583	39633.92945

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