This is to certify that the

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CLOSED LOOP CONTROL OF A PUMP TEST STAND

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CLOSED LOOP CONTROL OF A PUMP TEST STAND

By

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ABSTRACT

CLOSED LOOP CONTROL OF A PUMP TEST STAND

By

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The process model transfer functions for system pressure and flow rate, the corresponding closed loop controllers, and the system responses are determined for a pump test stand. The system pressure response to a step change in the control valve position is recorded, and used to determine the process model parameters for a first order transfer function. The poles of the closed loop system are placed to provide an over damped response to step changes in the pressure set point. The flow rate response to a step change in motor voltage exhibits a time delay. Frequency response analysis techniques are used to match a second order with time delay transfer function to the flow process model. Controller parameters for closed loop operation are determined to provide acceptable values for gain margin and phase margin.

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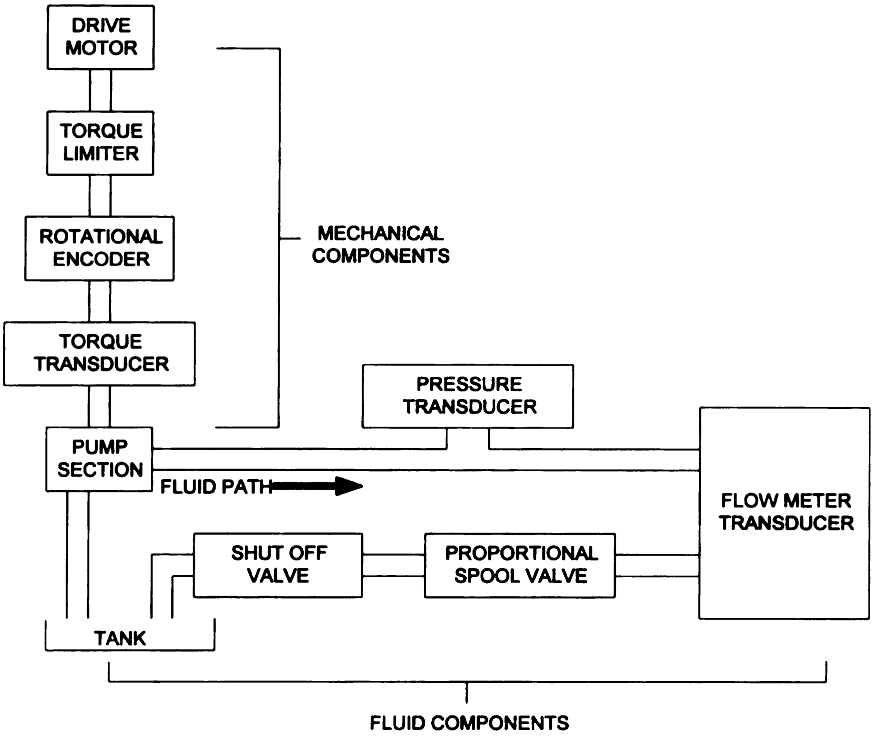
Test Stand Description

To verify the operating parameters of a new pump design, the pump must be tested in a controlled manner. Repeatable test setups and procedures are required to properly compare the results for different pump designs and design iterations. Manual testing involves the adjustment of the drive motor's voltage to reach a desired speed, and then adjusting a valve in the discharge line to reach a desired pressure. However, the movement in valve position loads the pump and motor, and this causes the speed to change. The motor voltage must be re-adjusted to correct the speed, but this changes the system pressure again. The iteration between voltage and valve position is time consuming, and it does not easily allow the exact same operating points to be recorded for every pump being tested. Using feedback control, a test stand can automatically adjust the motor voltage and valve position with a high degree of repeatability and accuracy.

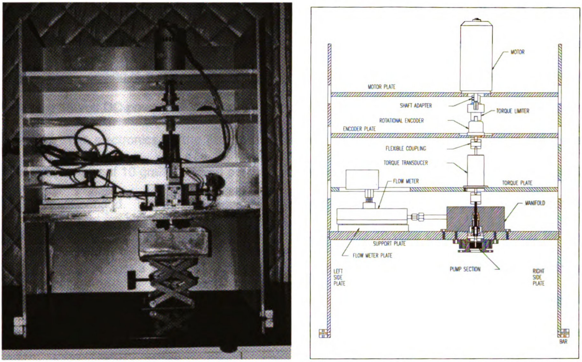
The pump test stand characterizes a pump by measuring the rotational speed and torque required to achieve a desired flow rate and pressure. A data acquisition system measures the pump parameters, and provides the control signals to regulate the motor voltage and valve position. The measured data is displayed on a computer screen, and written to the disk drives if desired. Following a brief discussion of the test stand layout and components, this paper will focus on the determination of the process model transfer functions for pressure and flow rate, the corresponding closed loop controllers, and the systems responses.

Figure 1 contains a block diagram for the test stand. The mechanical components are all connected by the rotating drive shaft. The fluid components show the flow path.



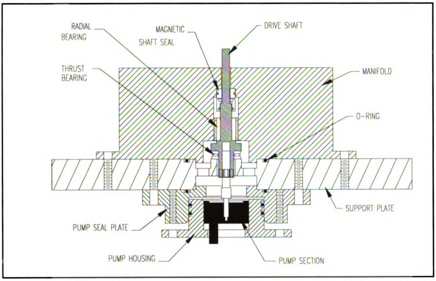
Block Diagram
Figure 1

Figure 2 shows the test stand. The drive motor sits on top of the structure. It is an one horsepower, universal motor with a DC voltage drive. The DC voltage is supplied by a 0 to 100 volts, 0 to 10 amps power supply. The power supply is controlled by a 0 to 10 volts signal. The next shelf supports the rotational encoder. The encoder provides 60 square pulses (TTL level) per revolution. Hence, the frequency output is a direct reading of the revolutions per minute (1000 Hz = 1000 rpm). Power for the encoder is from a 5 volt DC power supply. Below the encoder is the torque transducer. This transducer measures the difference in torque between its input and output shafts. Dedicated electronics for the torque transducer convert the measured torque to a 0 to 10 volts signal. The value for the full scale reading (10 volts) is selected with thumb wheels on the electronics front panel, and is set for 100 inch-ounces. The output shaft of the torque transducer is coupled to the pump drive shaft. This shaft is housed in a manifold which contains the fluid passages.



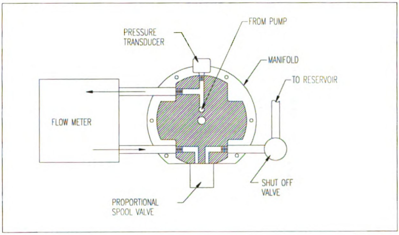
Test Stand Layout
Figure 2

Figure 3 shows a cross section of the manifold. A magnetic shaft seal prevents the fluid from leaking past the pump shaft. The housing for the pump under test is bolted to the underside of the support plate. A small fluid reservoir on a scissors jack can be raised to submerge the pump.



Manifold Cross Section
Figure 3

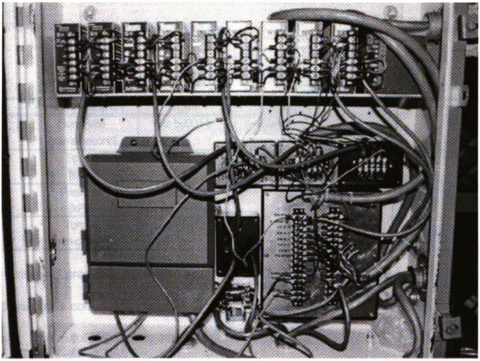
The fluid follows the path from the reservoir through the pump and into the manifold. See Figure 4. A side passage connects the pressure transducer. This transducer outputs a 1 to 6 volts signal corresponding to a gage pressure range of 0 to 200 psi. Power for the pressure transducer is from a 15 volt DC power supply. The fluid is next transferred to the flow meter and back to the manifold. The electronics for the flow meter outputs a frequency signal proportional to the mass flow rate. A flow of 10 grams/second produces a frequency of 1000 Hz. The flow range is from 0 to 50 grams/second. The fluid then passes through the proportional spool valve that is attached to the side of the manifold. This valve is controlled by a 0 to 10 volts signal. The power for the valve solenoid is from a 24 volts DC power supply. The fluid now leaves the manifold and flows through another valve. This valve is used to completely shut off the fluid passage since the spool valve will always has some degree of leakage around the spool. This two position solenoid valve is controlled by an 110 volt AC signal through a relay. The fluid then returns to the reservoir.



Fluid Path
Figure 4

A cart contains the electrical and electronic components. On the shelves are the computer, torque transducer electronics, and drive motor power supply. The electrical panel is mounted on the rear of the cart. Inside of the panel are the signal conditioning circuits, corresponding power supplies, flow meter electronics, and shut off valve relays. See Figures 5 and 6.

Electrical Cart
Figure 5



Electrical Panel
Figure 6

Signal Conditioning And Circuitry

The data acquisition system for the test stand is PC based, and was used for all the initial response testing as well as final control of the stand. The software used was Lab Tech Control from Laboratory Technologies. This package has a quick, simple set up for the I/O channels, and built in PID blocks with on-line tuning which allows for easy operation. The I/O card are from Intelligent Instrumentation. Two boards are used. An 8 channel A/D board for input, and an 8 channel D/A board for output.

The analog input card in the computer is set to read voltages that range from 0 to 10 volts. Of the four signals to be measured, only the torque transducer has this range for its output. Hence, the torque transducer is directly connected to an analog input channel. The remaining signals must be modified to convert them to the desired range. Basic op-amps circuitry and frequency to voltage converters are used for the signal conditioning. Table 1 lists the measurement devices, their outputs signals, and the signal conditioning required. The schematics for the signal conditioning circuits are shown in appendix A.

Table 1
Measurement Devices

Device	Measured Quantity	Signal	Modifications	Formula
Flow Meter	Flow Rate [grams/second]	0 to 5000 Hz	Frequency to Voltage Converter & Op-amps	V_{out1}=(1 volt/1000 Hz) f_{in} V_{out2} = -2 (V_{in1})
Rotational Encoder	Speed [rpm]	0 to 10000 Hz	Frequency to Voltage Converter & Op-amps	V_{out1} =(1 volt/1000 Hz) f_{in} V_{out2} = -V_{in1}
Pressure Transducer	Pressure [kPa]	1 to 6 volts	Op-amps	V_{out} = 2(V_{in})-2
Torque Transducer	Torque [inch-ounces]	0 to 10 volts	None	V_{out} = V_{in}

The flow rate and pressure signals provide the feedback to the computer for the control loops. Each of these lines contains a low pass filter to prevent high frequency noise from contaminating the signals. The cutoff frequency (15 Hz) for the double pole active filters is chosen to be less than half of the 40 Hz sampling frequency. No filters are used on the speed and torque signals because these values are not used in the control loops. They are displayed as moving averages which does provide some filtering. Table 2 lists the input channels.

Table 2
Analog Input Channels

Analog Input Channels			Voltage Range:	0 to 10 Volts
Channel #	Name	Engineering Units	Scale Factor	Measurement Range
0	Flow Rate	grams/second	5	0 to 50 grams/second
1	Speed	rpm	1000	0 to 10000 rpm
2	Pressure	kPa	137.9	0 to 1379 kPa
3	Torque	inch-ounces	100	0 to 100 inch-ounces

The analog output card has its voltage range set to 0 to 5 volts. The drive motor power supply and the spool valve both require a 0 to 10 volt signal, so the two control signals must be doubled. Non-inverting op-amp circuits, each with a gain of 2, modify the signals. A third output channel is used to keep the drive motor off and the valve completely open when the data acquisition program is not running. When voltage level of this channel is less than a reference voltage (4 volts), the system is off. Another channel controls a second valve in the system. This valve completely closes the fluid path. A fifth channel allows the flow meter transducer to be calibrated. Table 3 lists the output channels.

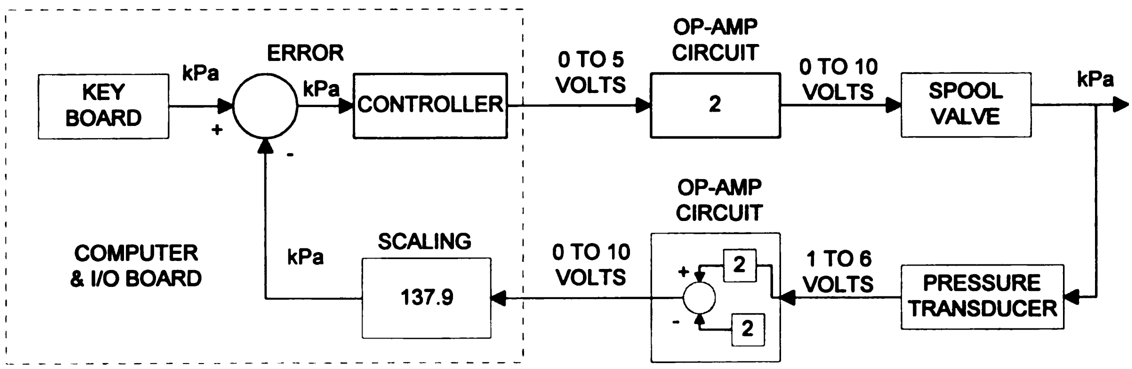
Table 3
Analog Output Channels

Analog Output Channels		Voltage Range: 0 to 5 Volts
Channel #	Name	Action
0	System Control	> 4 Volts, System On <4 Volts, System Off
1	Motor Voltage	0 to 5 Volts Motor Power Supply Control
2	Valve Voltage	0 to 5 Volts Spool Valve Control
3	Shut Off Valve Relay	> 4 Volts, Shut Off Valve Closed < 4 Volts, Shut Off Valve Open
4	Flow Meter Zeroing	> 4 Volts, Zero Flow Calibration < 4 Volts, Normal Operation

Power is supplied to the test stand by two 20 amp, 110 volt circuits. One is used for the drive motor. The other circuit powers the rest of the equipment. The solenoid valve used to shut off the fluid flow also take a 110 volt AC signal. This voltage is switched by a relay. Most of the electronic equipment contains their own internal fuses. For the power supplies that provide the signal conditioning voltages, and for the relay, an external fuse is used for protection.

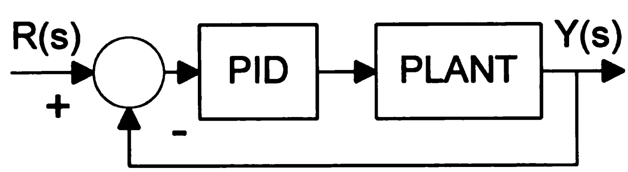
Pressure Control

The block diagram for the pressure control loop is shown in Figure 7. The input signal, entered via the keyboard, is the desired pressure in kPa. The actual pressure, also in kPa, is subtracted from the desired pressure, and the error signal is fed into the controller block. The computer outputs a 0 to 5 volt signal that is converted into a 0 to 10 volt signal with an operational amplifier circuit. This voltage controls the position of the proportional spool valve which changes the system pressure. The pressure transducer that is located between the pumping section and the valve converts the system pressure into a 1 to 6 volt signal. Another op-amp circuit scales this to a 0 to 10 volts range, and the signal is read by the computer. The computer scales this value to represent kPa, and this then becomes the actual pressure.



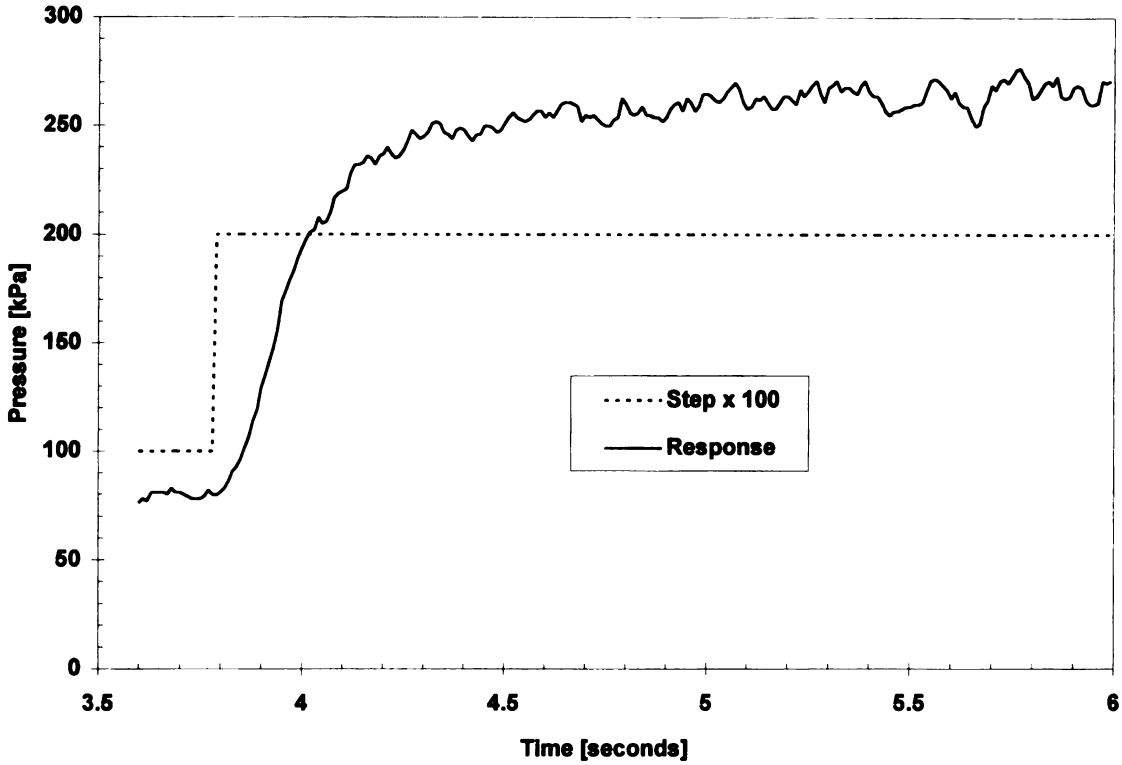
Pressure Control Loop
Figure 7

To model the complete plant for pressure control, and to determine the appropriate form for the controller, the simplified block diagram of Figure 8 is used. All external circuitry, the valve, the transducer, and computer scaling have been lumped into the plant expression. The input signal and the actual signal are still in units of kPa. The controller consists of only the PID block of the data acquisition software. The signal between the controller and the plant is 0 to 5 volts.



Control System
Figure 8

To determine the plant transfer function, the response of the system pressure to a step change in valve control voltage was measured. The procedure used was to first stabilize the system with a constant voltage applied to the drive motor circuitry to ensure sufficient fluid flow, and 1 volt was applied to the valve circuit. The valve voltage was then changed to 2 volts, and the pressure response recorded. This data is shown in Figure 9.



Pressure Response To A Step In Valve Voltage
Step From 1 To 2 Volts
Figure 9

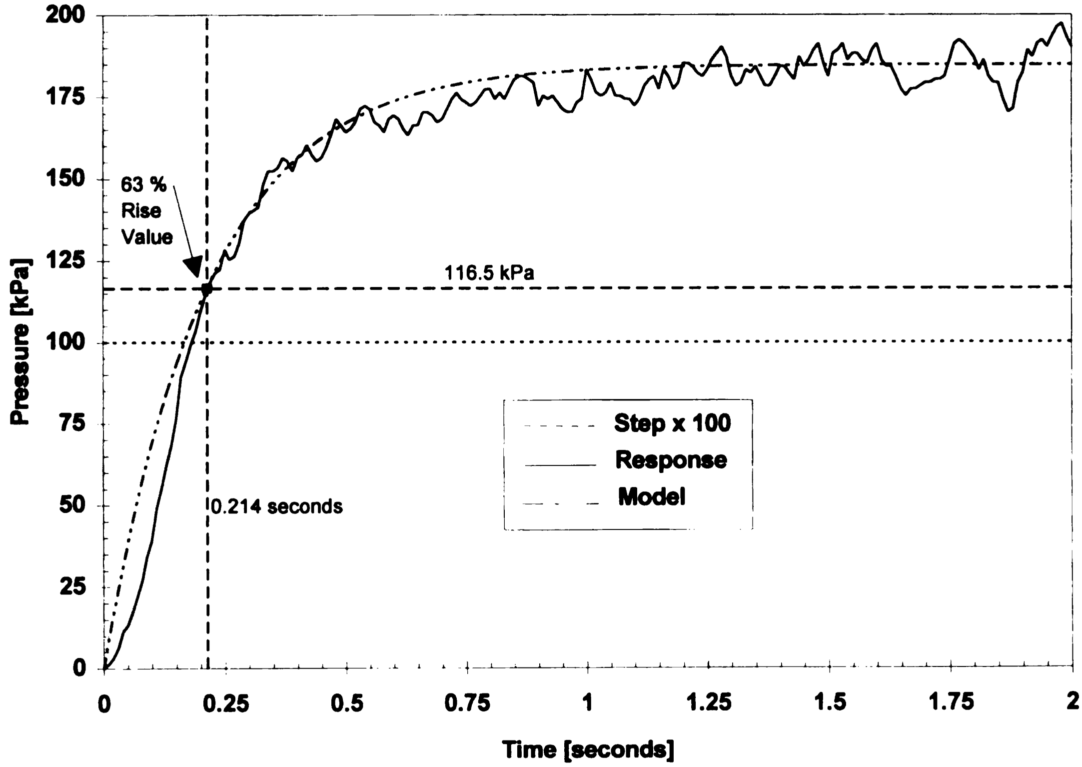
For the pressure control plant, a first order system of the form

$$\frac{Y(s)}{R(s)} = \frac{k}{\tau s + 1}$$

is assumed with R(s) and Y(s) the input and the output signals respectively. Since the step input did not start from zero, and since the final value of the step is 2 rather than 1, the signals R(s) and Y(s) must be modified to map the above step response to a system which has as its input an unit step with zero steady state conditions. The deviation variables are found by subtracting the steady state value of the signals from the physical values for all time after the step has occurred. Let $Y^*(s) = Y(s) - Y_{SS}(s)$, and $R^*(s) = R(s) - R_{SS}(s)$, where the starred values are the deviation variables, Y(s) and R(s) are the physical variables, and the initial values are $Y_{SS}(s) = 79.7$ kPa and $R_{SS}(s) = 1$ volt. The transfer function in deviation variables is then

$$\frac{Y^*(s)}{R^*(s)} = \frac{k}{\tau s + 1}$$

Figure 10 shows the step response for the deviation variables. The values for k and τ are determined from the graph. The time constant τ is the amount of time required for the pressure to rise to 63 % of the final value. The final value averages to be 184.9 kPa. 63% of 184.9 is 116.5 kPa. The time when the pressure equals 116.5 kPa is 0.214 seconds. Hence, the time constant τ = 0.214 seconds. The gain k is determined from the ratio of the final value of the pressure deviation variable over the final value of the input deviation variable. For the input, r^* = 1 volt. The final value of the output is y^* = 184.9 kPa. The gain k = 184.9 kPa / 1 volt = 184.9 kPa / volt.



Pressure Response To A Unit Step In Valve Voltage
[Deviation Variables]
Figure 10

The transfer function is then

$$\frac{Y^*(s)}{R^*(s)} = \frac{184.9}{0.214s + 1}$$

The time domain equation for this transfer function is

$$y^* = kr^*\left(1 - e^{-t^*/\tau}\right) = 184.9r^*\left(1 - e^{-t^*/0.214}\right)$$

Note that time t* is also a deviation variable since the step change in the input for the physical variables did not occur at zero seconds, but at 3.79 seconds. The above equation is plotted in figure 10 as the model.

Substituting for the deviation variables yields

$$y - y_{SS} = k(r - r_{SS})\left(1 - e^{-t^*/\tau}\right)$$

$$y - 79.7 = 184.9(r - 1)\left(1 - e^{-(t-3.79)/0.214}\right)$$

As a check, the physical value for pressure at t = 3.79 seconds (the steady state condition), r = 1 volt is

$$y - 79.7 = 184.9(1 - 1)\left(1 - e^{-(3.79-3.79)/0.214}\right) = 0$$

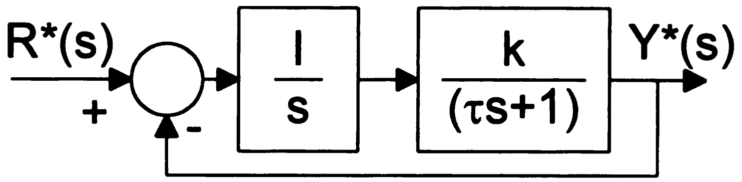
or, $y = 0 + 79.7 = 79.7$ kPa.

The final physical value for the pressure at $t = \infty$, r = 2 volts is

$$y - 79.7 = 184.9(2 - 1)\left(1 - e^{-\infty/0.214}\right) = 184.9$$

$$y = 184.9 + 79.7 = 264.6 \text{ kPa}$$

With an estimate for the for plant transfer function determined, the controller can be specified. For this case, only the integrator portion of the PID block is used. Figure 11 shows the block diagram.



Pressure Closed Loop System
Figure 11

The Laplace transfer function of the integrator is I/s where I is the gain. The closed loop transfer function is then

$$\frac{Y^*}{R^*} = \frac{\frac{k}{\tau}I}{s^2 + \frac{1}{\tau}s + \frac{k}{\tau}I}$$

To determine the value for I, the root locus of the characteristic equation

$$s^2 + \frac{1}{\tau}s + \frac{k}{\tau}I = 0; \quad k = 184.9, \quad \tau = 0.214$$

is plotted in figure 12. For values of I between 0 and 0.0063181, the roots are negative and real. This will produce an over damped response. For values greater than 0.0063181, the roots are complex, and the response will be under damped. For the closed loop response to be non-oscillatory, and to account for the uncertainty in the plant model, a conservative value for the integrator gain is chosen to provide over damped response. The value used for the integrator gain is 0.00515. The closed loop transfer function is then

$$\frac{Y^*}{R^*} = \frac{4.44967}{s^2 + 4.67290s + 4.44967}$$

The poles are located at s = -1.33208 and s = -3.34109.

Figure 13 shows the actual response of the closed loop system to a step from 200 to 300 kPa. Also plotted is the response of the above closed loop transfer function to the same input. Note, however, that the deviation conditions were again considered in determining the time domain function for this model. The time domain deviation function is

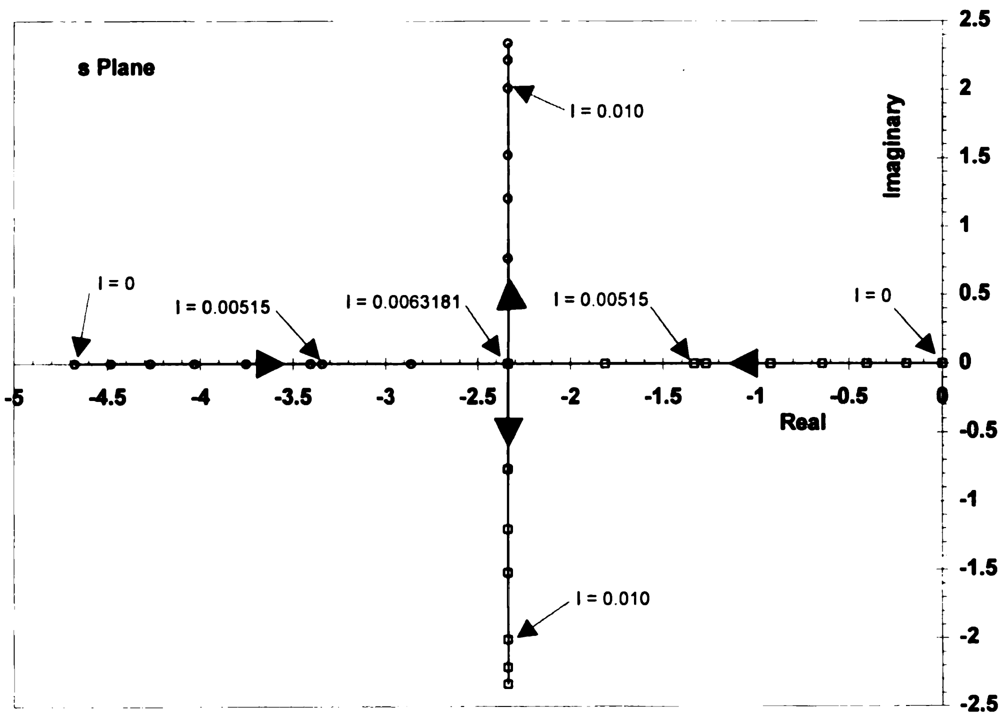
$$y^* = r^*\left[1 + 0.663e^{-3.34109t^*} - 1.663e^{-1.33208t^*}\right]$$

Substituting r* = r - r_{SS}, with r = 300 kPa and r_{SS} = 200 kPa, y* = y - y_{SS}, with y_{SS} = 200 kPa, and t* = t - 19.18 yields

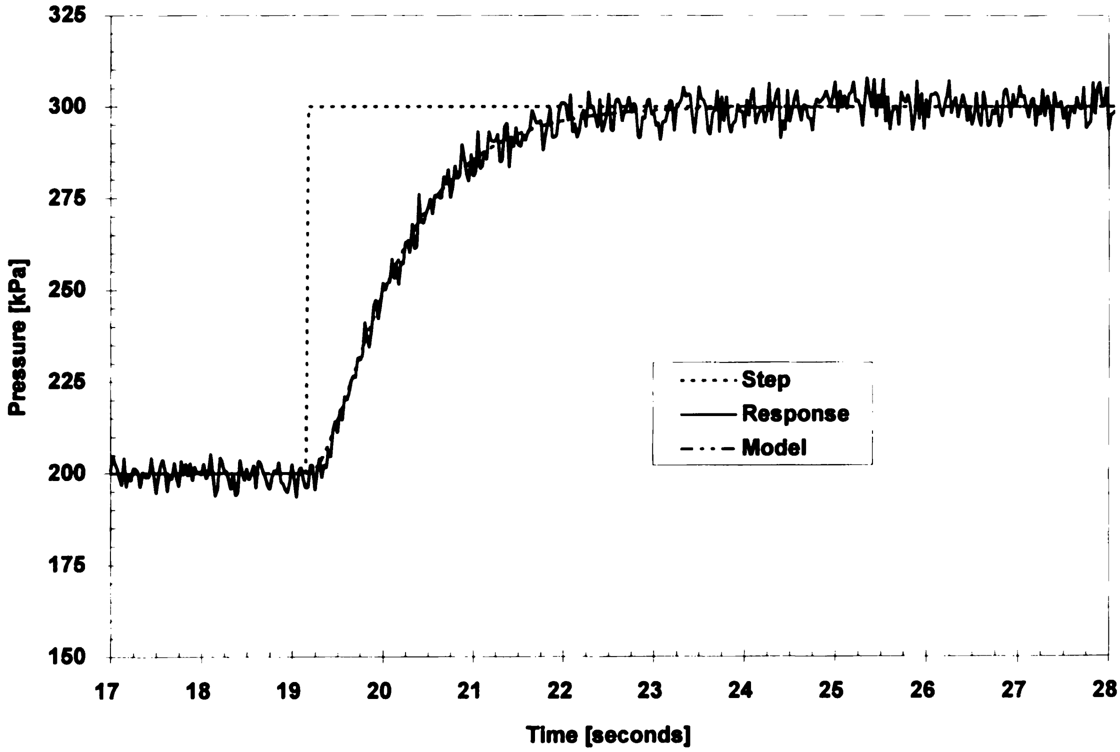
$$y - 200 = (300 - 200)\left[1 + 0.663e^{-3.34109(t-19.18)} - 1.663e^{-1.33208(t-19.18)}\right]$$

At t = ∞, r = 300 kPa; y = r = 300 kPa.

Appendix C contains the complete derivation for both the plant and closed loop transfer functions, and the time domain equations.



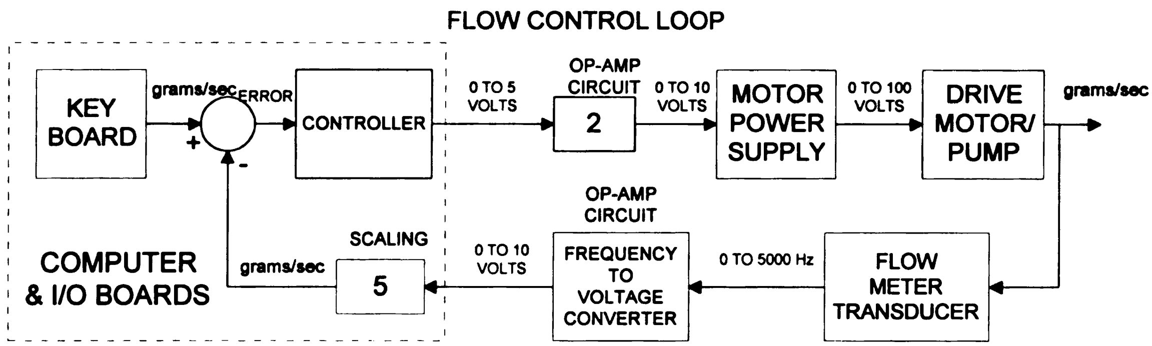
Root - Locus Diagram
Figure 12



Closed Loop Pressure Response To A Step Input
Step From 200 To 300 kPa
Figure 13

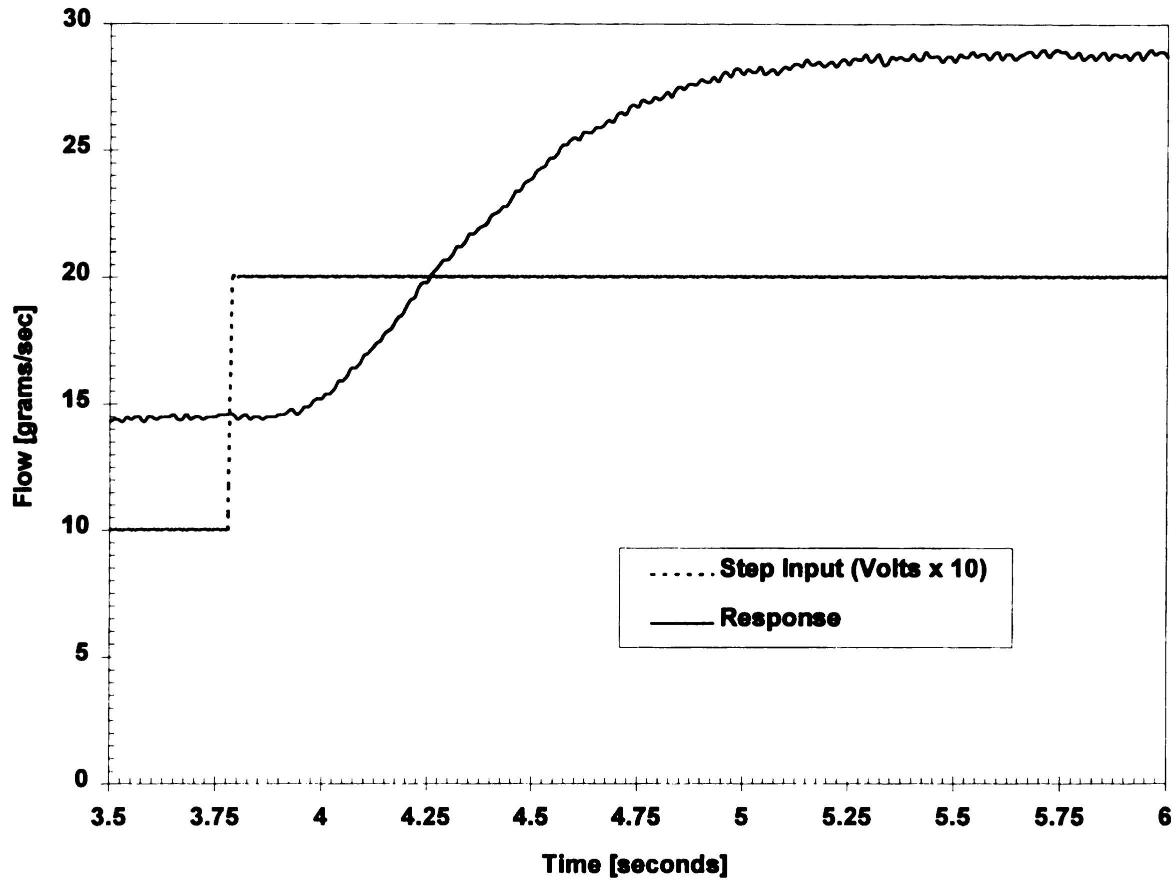
Flow Control

The desired flow rate is achieved using the feedback loop shown in Figure 14. The flow produced by the drive motor and pump is measured by the flow meter. This signal is read by the I/O cards in the computer, and is compared to the desired flow rate. The resulting error signal is processed, and the control voltage to the motor power supply is adjusted.



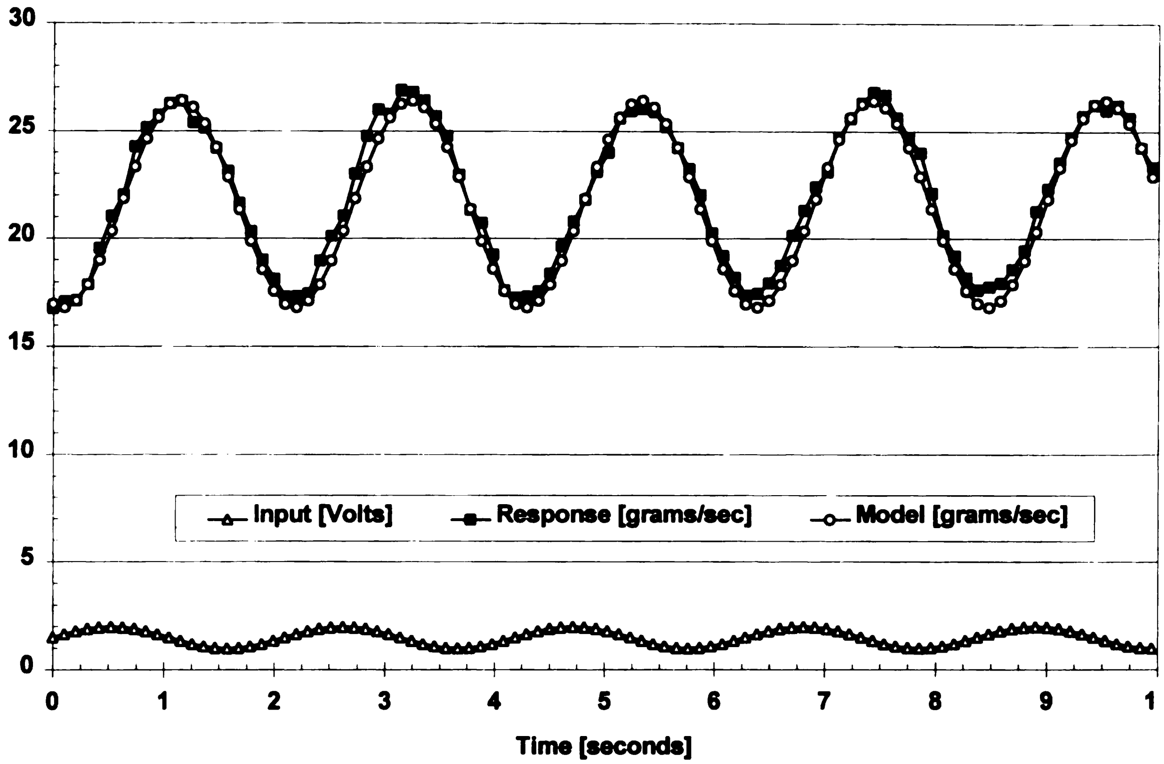
Flow Control Loop
Figure 14

The response of the system to a step input in motor control voltage is shown in Figure 15. The voltage was stepped from 1 to 2 volts, and the response recorded. From the graph, it appears that a time delay exists in the response. Since a time delay results in a non-rational transfer function, the techniques used in frequency response analysis must be used to determine the process model.



Flow Response To A Step In Motor Control Voltage
Step From 1 To 2 Volts
Figure 15

The procedure for determining the frequency response of the system involves subjecting the system to sinusoidal input signals with different frequencies. The amplitude ratios of the output to input signals, and the phase angles between the signals are determined. A typical plot of input and output signals is shown in Figure 16. The amplitude and phase angle of the curve labeled as the model in Figure 16 are determined by using a spread sheet program to fit the curve to the output data. The input signal for all cases is $1.5 + 0.5 \sin(\omega t)$ where ω is varied for each test.



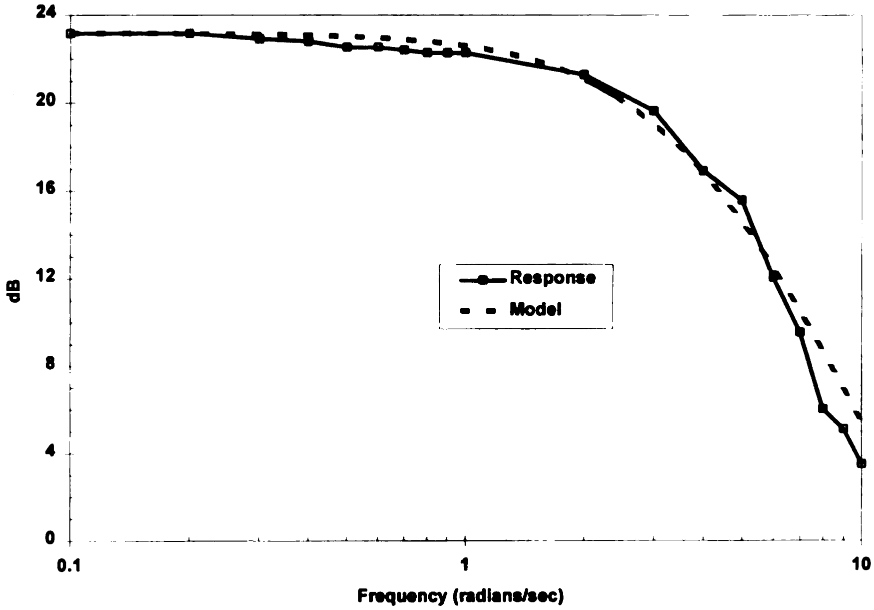
Sine Wave Response
Frequency: 3 radians/second
Figure 16

For the response in Figure 16, the model parameters are found to fit the formula 21.615 + 4.8 sin(3t-1.8326). The amplitude ratio is then 4.8/0.5 = 9.6. The phase angle is -1.8326 radians/second, or -105 degrees. The steady state gain is derived from the ratio of the DC components, 21.615/1.5 = 14.41. Table 4 contains the amplitude ratios and phase angles for all the different frequencies.

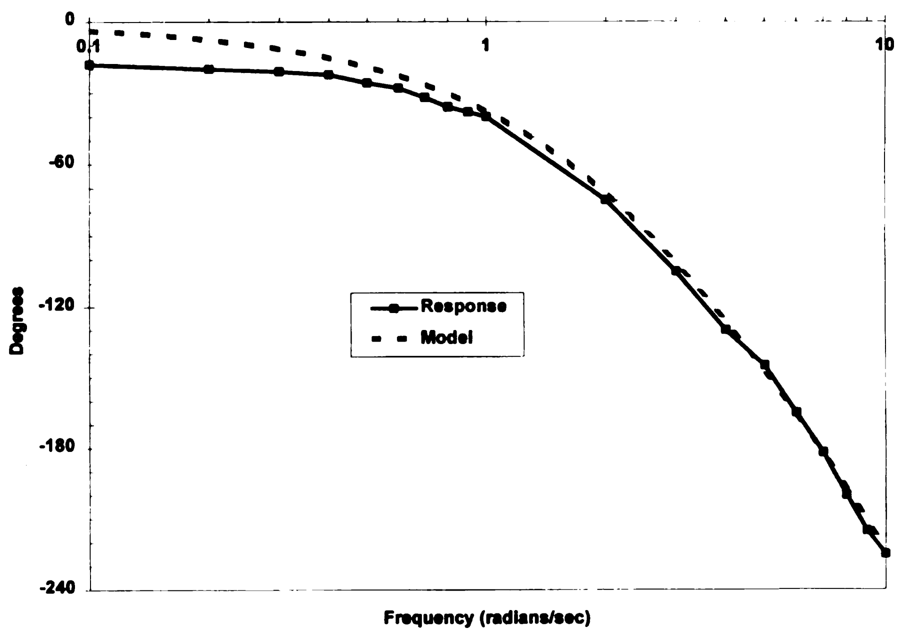
Table 4
Frequency Response Data

Frequency [rads/sec]	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2	3	4	5	6	7	8	9	10
Amplitude Ratio	14.4	14.4	14	13.8	13.4	13.4	13.2	13	13	13	11.6	9.6	7	6	4	3	2	1.8	1.5
Phase Angle [degrees]	-18	-20	-21	-22.5	-26	-28	-32	-36	-38	-40	-75	-105	-130	-145	-165	-182	-200	-215	-225

The amplitude ratios are converted to decibels, and these and the phase angles are plotted versus frequency in Figures 17 and 18 respectively. Also plotted are the equations representing the process model. A second order model with a time delay is assumed. The parameters of the model are based on trial and error fits with a spread sheet program. The values that best fit the empirical data are as follows: gain k=14.4 grams / second / volt, time constants τ_1 = 0.24 seconds, τ_2 = 0.28 seconds, and time delay θ = 0.15 seconds.



Flow: Plant Model Frequency Response
Magnitude
Figure 17



Flow: Plant Model Frequency Response
Phase
Figure 18

The transfer function for the model of the plant for flow then has the form

$$G_p(s) = \frac{ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

or with the values substituted

$$G_p(s) = \frac{14.4e^{-0.15s}}{(0.24s + 1)(0.28s + 1)}$$

The equation plotted in figure f4 for the magnitude frequency response is

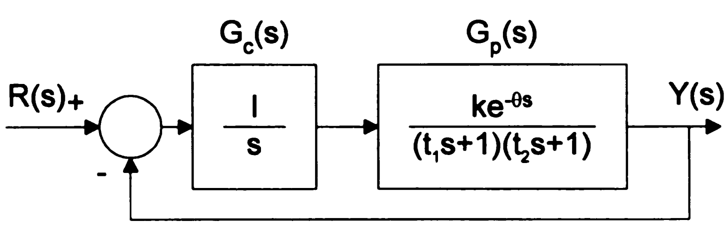
$$|G_p(j\omega)| = \frac{14.4}{\left(\sqrt{0.24^2\omega^2 + 1}\right)\left(\sqrt{0.28^2\omega^2 + 1}\right)}$$

The equation plotted in figure f5 for the phase angle frequency response is

$$\angle G_p(j\omega) = -0.15\omega - \tan^{-1}[0.24\omega] - \tan^{-1}[0.28\omega]$$

With the plant specified, the controller, $G_c(s)$, can be chosen. A simple integrator with gain, $G_c(s) = {}^{I}\!/_{s}$, is used for closed loop control. Figure 19 shows the closed loop block diagram. The integrator term eliminates any off set error between the set point and process variable. It also reduces the effect of high frequency signals by attenuating the amplitude by a factor proportional to the reciprocal of the signal frequency. The integrator acts as a low pass filter.

To ensure that the system remains stable, the open loop transfer function frequency response is analyzed. The Bode Stability Criterion requires that the amplitude ratio of the open loop transfer function, $G_{OL}(s)$, be less than unity when the phase angle equals -180 degrees. The open loop transfer function is derived from the closed loop system when the feed back leg is disconnected.



Flow Closed Loop System
Figure 19

The open loop transfer function is then

$$G_{OL}(s) = G_c(s)G_p(s) = \left(\frac{I}{s}\right)\left(\frac{14.4e^{-0.15s}}{(0.24s+1)(0.28s+1)}\right)$$

The magnitude of this transfer function when s = jω is

$$\left|G_{OL}(j\omega)\right| = \frac{14.4I}{\omega\left(\sqrt{\omega^2(0.24)^2+1}\right)\left(\sqrt{\omega^2(0.28)^2+1}\right)}$$

and the phase angle is

$$\angle G(j\omega) = -\frac{\pi}{2} - 0.15\omega - \tan^{-1}[0.24\omega] - \tan^{-1}[0.28\omega]$$

Note that the open loop magnitude equation, $|G_{OL}(j\omega)|$, is simply the process model magnitude equation, $|G_P(j\omega)|$, multiplied by I / ω.

$$\left|G_{OL}(j\omega)\right| = \left(\frac{I}{\omega}\right)\left|G_P(j\omega)\right|$$

Also, the phase angle equation for the open loop system, $\angle G_{OP}(j\omega)$, is the process model phase angle equation with an added (- π / 2) term. The π / 2 term shifts the phase of the process model by - 90° at all frequencies. Hence, the stability of the closed loop system can be determined using the process model frequency response equations with the critical frequency adjusted from -180° to - 90°, (- π / 2). Since the equation for the phase angle does not contain any terms with the gain I, the frequency, ω, where the angle equals -π / 2 radians can be calculated.

$$-\frac{\pi}{2} = -0.15\omega - \tan^{-1}[0.24\omega] - \tan^{-1}[0.28\omega]$$

This is the critical frequency, and is equal to 2.5875 radians / second. The process model magnitude at this frequency is then

$$\left|G_p(j2.5875)\right| = \frac{14.4}{\left(\sqrt{0.24^2(2.5875)^2+1}\right)\left(\sqrt{0.28^2(2.5875)^2+1}\right)} = 9.906$$

In decibels,

$$20 \log (9.906) = 19.92 \text{ dB}.$$

To satisfy the stability criterion, the integrator gain, I / ω, must reduce the above gain to less than zero. Hence,

$$(19.92) + 20 \log (I / \omega) < 0$$

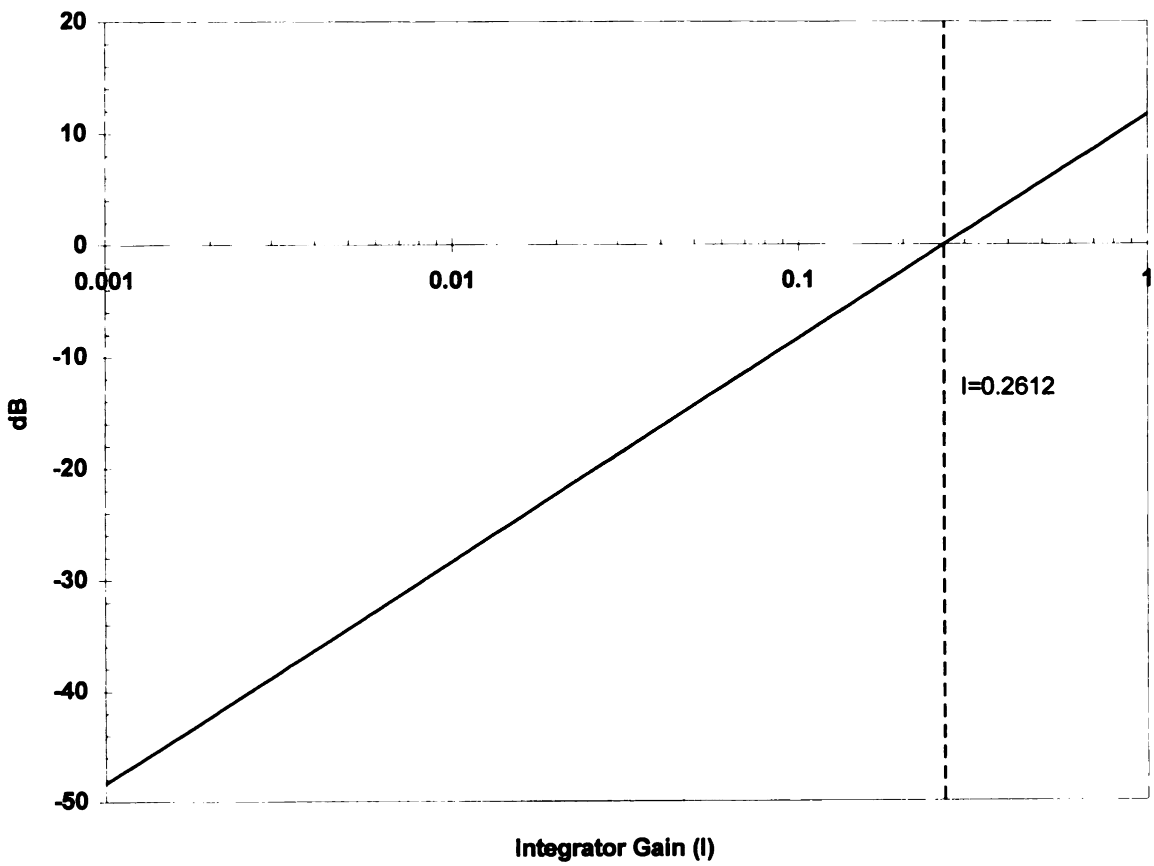
$$I / \omega < 0.10095$$

However, $\omega = 2.5875$ radians / second, and

$$I < (0.10095)(2.5875)$$

$$I < 0.2612$$

The open loop gain at this frequency is called the gain margin. Figure 20 plots the gain margin in decibels as a function of integrator gain I.



Gain Margin vs. Integrator Gain
Figure 20

Based on the above graph, the integrator gain must be less than I=0.2612 to ensure stability. A decent value for gain margin is around -10 dB, so a value of I = 0.07 is chosen. This gives a gain margin of 19.92 + 20 log (0.07 / 2.5875) = -11.4 dB.

The phase margin for the open loop transfer function is related to the phase at the frequency where the open loop gain now equals 0 in dB. Recall

$$|G_{OL}(j\omega)| = \left(\frac{I}{\omega}\right)|G_P(j\omega)|$$

$$|G_{OL}(j\omega)| = \left(\frac{I}{\omega}\right)|G_P(j\omega)| = 0$$

$$0 = \left(\frac{0.07}{\omega}\right)\frac{14.4}{\left(\sqrt{0.24^2\omega^2+1}\right)\left(\sqrt{0.28^2\omega^2+1}\right)}$$

$$\omega = 0.9497 \text{ radians / second}$$

At this frequency the process model phase is

$$\angle G_p(j0.9497) = -(0.15)(0.9497) - \tan^{-1}[(0.24)(0.9497)] - \tan^{-1}[(0.28)(0.9497)]$$

$$\angle G_p(j0.9497) = -0.6265 \text{ radians /second} = -35.9^\circ$$

The phase margin is the sum between this value and 90°, or

$$\phi = 90^\circ + (-35.9^\circ) = 54.1^\circ$$

This is an acceptable value.

With the controller now specified, the closed loop response can now be examined. This transfer function is

$$G_{cl}(s) = \frac{\left(\frac{I}{s}\right)\frac{ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}}{1+\left(\frac{I}{s}\right)\frac{ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}}$$

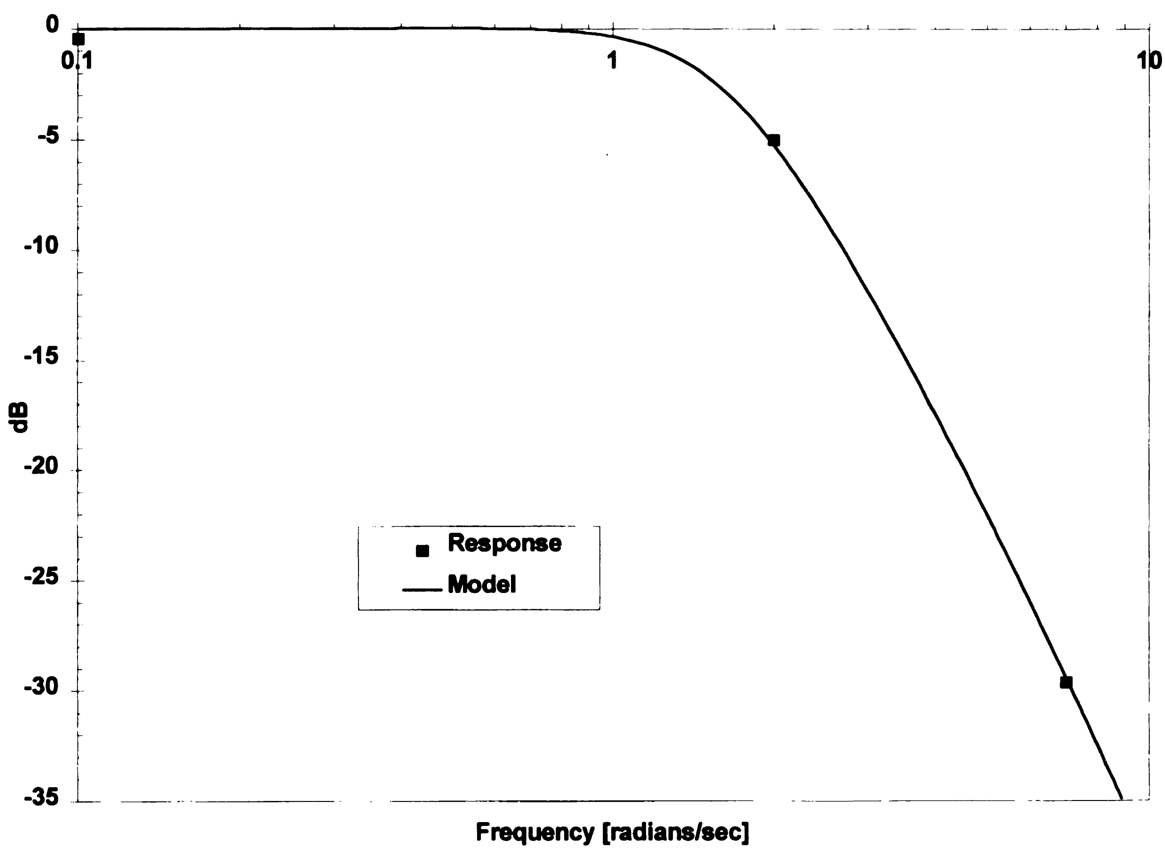
Substituting the values, the magnitude as a function of frequency is

$$|G_{cl}(j\omega)| = \frac{1008}{\sqrt{\left(1008\cos(0.15\omega) - 0.52\omega^2\right)^2 + \left(\omega - 0.0672\omega^3 - 1008\sin(0.15\omega)\right)^2}}$$

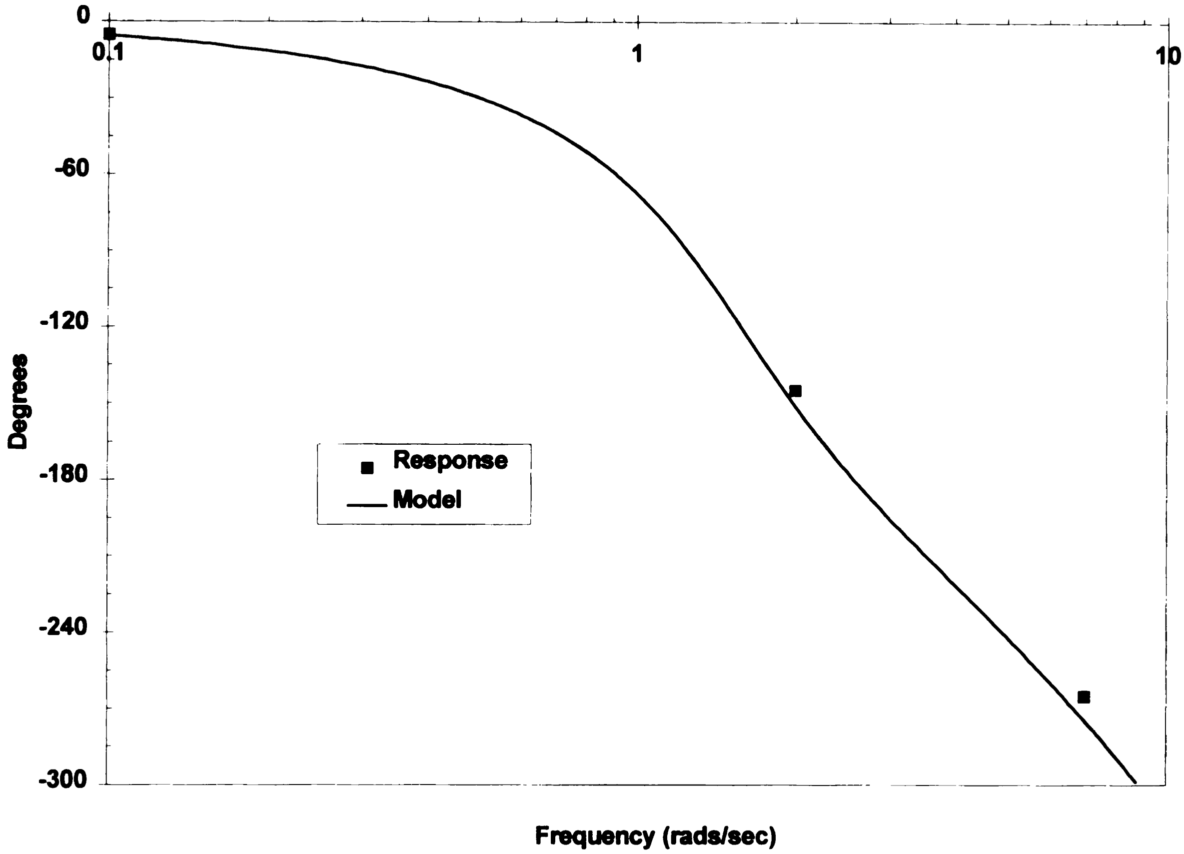
This equation is plotted in Figure 21. The phase angle equation is

$$\angle G_{cl}(j\omega) = -0.15\omega - \tan^{-1}\left[\frac{\omega - 0.0672\omega^3 - 1008\sin(0.15\omega)}{1008\cos(0.15\omega) - .052\omega^2}\right]$$

and this is plotted is Figure 22. Also shown on these two plots are the measured magnitudes and phase angles of the closed loop system driven at frequencies of 0.1, 2.0, and 7.0 radians/seconds. The model has good agreement with the actual data.

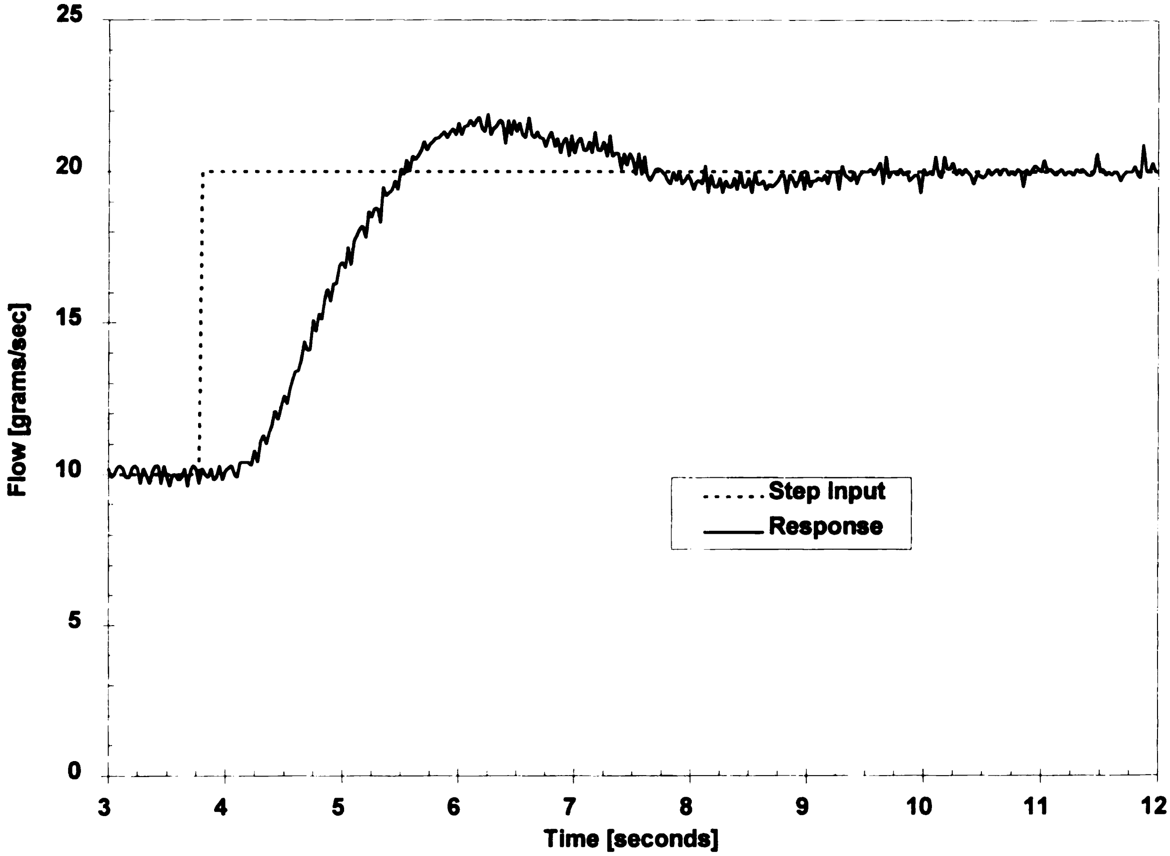


Flow: Closed Loop Frequency Response
Magnitude
Figure 21



Flow: Closed Loop Frequency Response
Phase
Figure 22

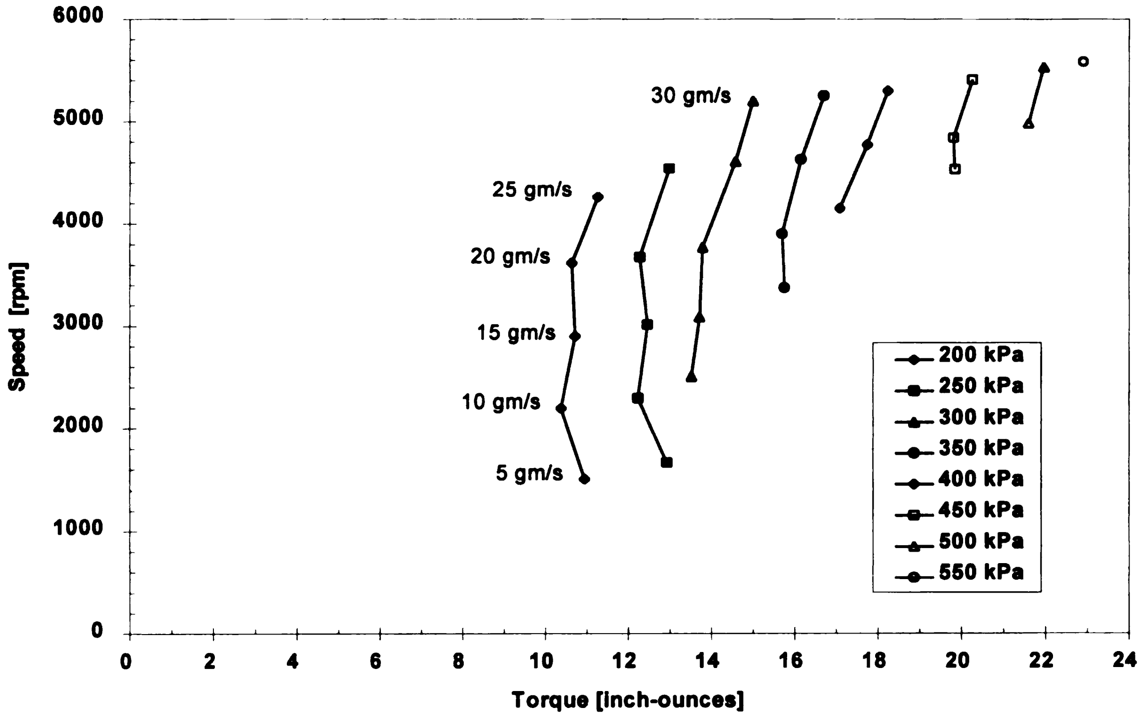
Under normal operation, the test stand responds to step changes in the set point. The response of the flow to a step from 10 to 20 grams/second is shown in Figure 23. From the graph, the time delay is still evident, but the flow does reach its set point value with only a modest amount of overshoot and oscillation. Appendix B contains the complete derivation of all of the above equations.



Flow: Closed Loop Step Response
Figure 23

Test Stand Performance

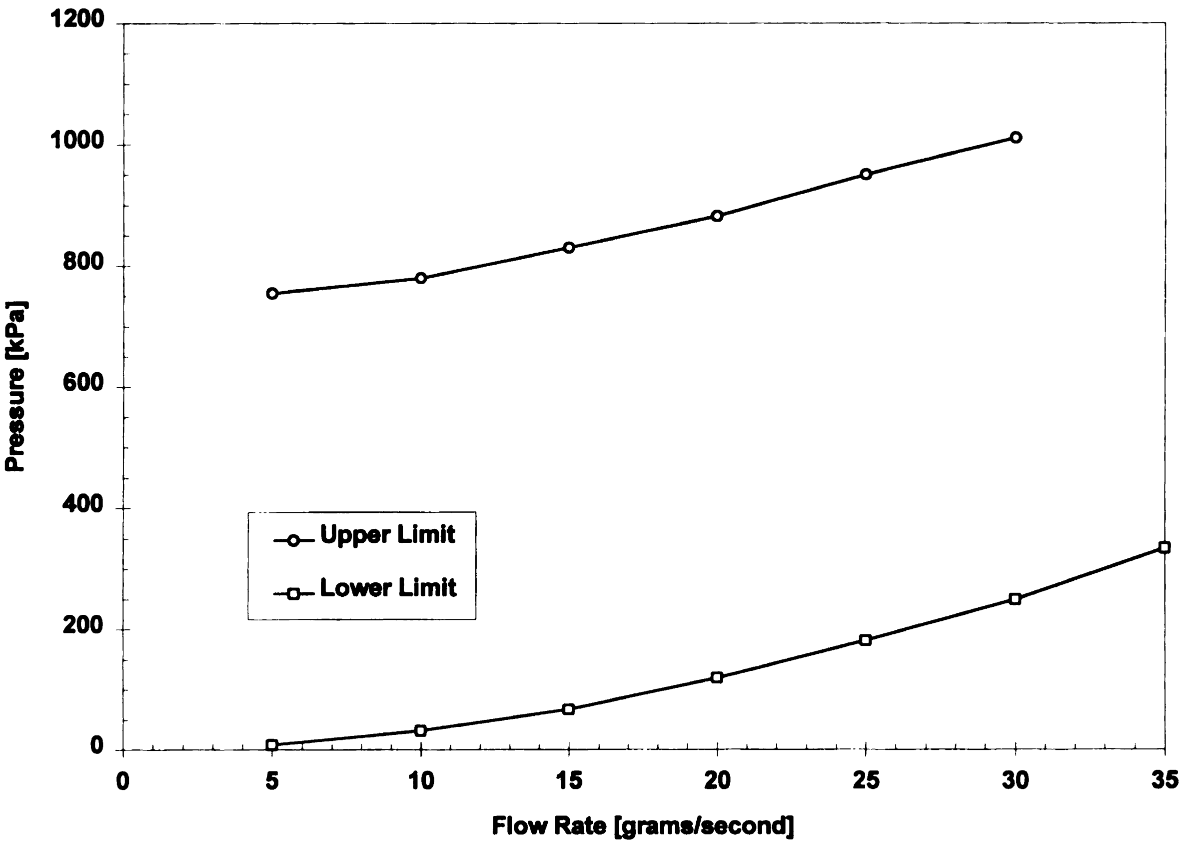
The data recorded by the acquisition system are the speed and torque required to reach the desired flow rate and pressure. These four values are written to a floppy disk for each combination of flow and pressure set points. Using a spread sheet program, the data can be graphed. Figure 24 shows a typical performance graph. The graph shows how the speed and torque vary along lines of constant pressure. Specific flow rate points are also displayed. This type of graph is used to determine the required speed and torque an electric motor should produce for this pump to generate a given flow rate and pressure.



Pump Section
Speed vs. Torque At Constant Pressures
Figure 24

The constant pressure lines in the above graph do not all start at the same flow value because of limitations in the flow stand. A lower limit on pressure for each flow rate exists. This is due to the system restriction from the size of the pipes, flow meter, and valves. An upper limit also exits. This limit is adjustable by the setting on the proportional spool valve. This value is chosen to provide enough resolution for valve control in the flow and pressure range of interest. The upper limit on the valve results from the spool valve being in its fully closed position. Since there is leakage around the valve, a further increase in flow will also increase pressure. Hence, there is a maximum pressure for each flow rate. Higher pressures can be achieved by shutting off the flow with the second valve, but the flow rate is then no longer adjustable.

A second source for an upper limit on flow is the top speed of the drive motor. This speed is limited by the maximum amount of current the power supply can deliver which is 10 amps. Non-linear performance results when the set point for flow causes the motor power supply to reach this limit. Figure 25 shows these limitations.



Test Stand Operating Range
Figure 25

For improved performance, and to eliminate the above limitations, the following modifications to the test stand should be incorporated. A second identical power supply should be slaved to, and connected in parallel to the first power supply. The non-linear motor response will be reduced with this doubling of the current capacity. The next larger sized flow meter should be used to reduce the system restriction. Going to a larger spool valve should also reduced the restriction, but a larger valve may not readily mount on the manifold. Finally, the controlling software should have an option to enter a desired speed, and have the system converge to that set point. This would allow the standard flow versus pressure curves at constant speeds to be easily generated. It would, however, require the determination of the process model for speed control, but the methods used in this paper would provide adequate results.

Two control loops were required to operate this test stand. The pressure process model was analyzed using a step function input, and was determined to have a first order response. The controller for closed loop operation was designed setting the gain of the integrator so the poles of the characteristic equation of the system transfer function would produce an over damped response. The flow control loop required frequency response analysis because the flow process model included a dead time factor. The corresponding transfer function with an exponential in the numerator is irrational, and cannot be resolved with simple algebraic math. The system was tested with sinusoidal inputs with differing frequencies, and the gain and phase margins were selected to provide stable operation.

Appendices

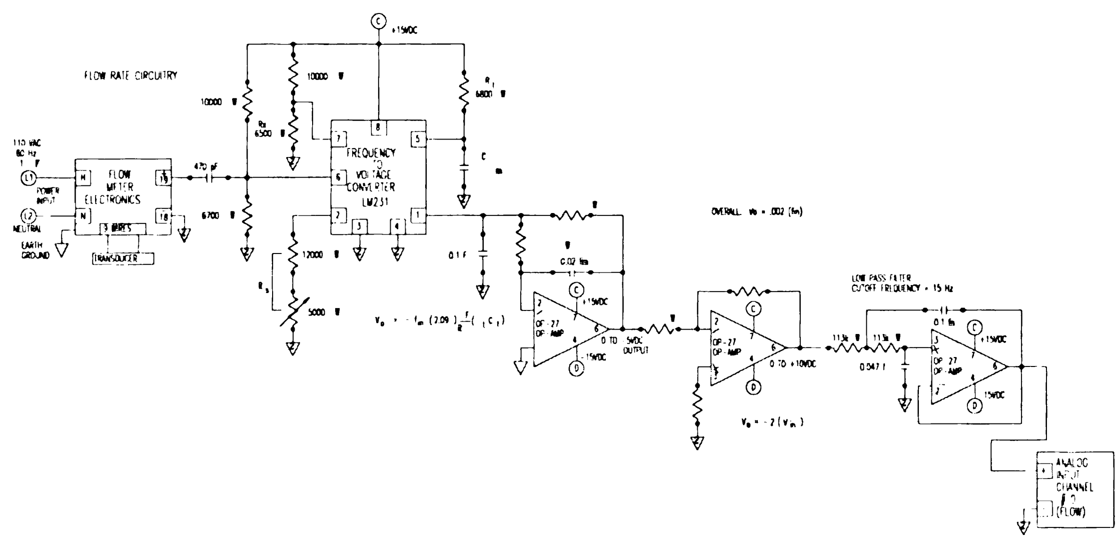
Appendix A

Signal Conditioning And Wiring Diagrams

Analog Input Channels: (0 to 10 Volts)

#0: Flow Rate.

The flow meter produces a 0 to 5000 Hertz square wave signal corresponding to a 0 to 50 grams/ second flow rate. A frequency-to-voltage (f-t-v) converter maps the signal to a 0 to -5 vdc range. Since the I/O board is set for a range of 0 to 10 volts, the f-t-v voltage is scaled by a inverting op-amp circuit with a gain of 2. Next, a second order low pass active filter with a cut off frequency of 15 Hertz eliminates any higher frequency noise from the signal. Figure A1 contains the schematic for channel # 0.

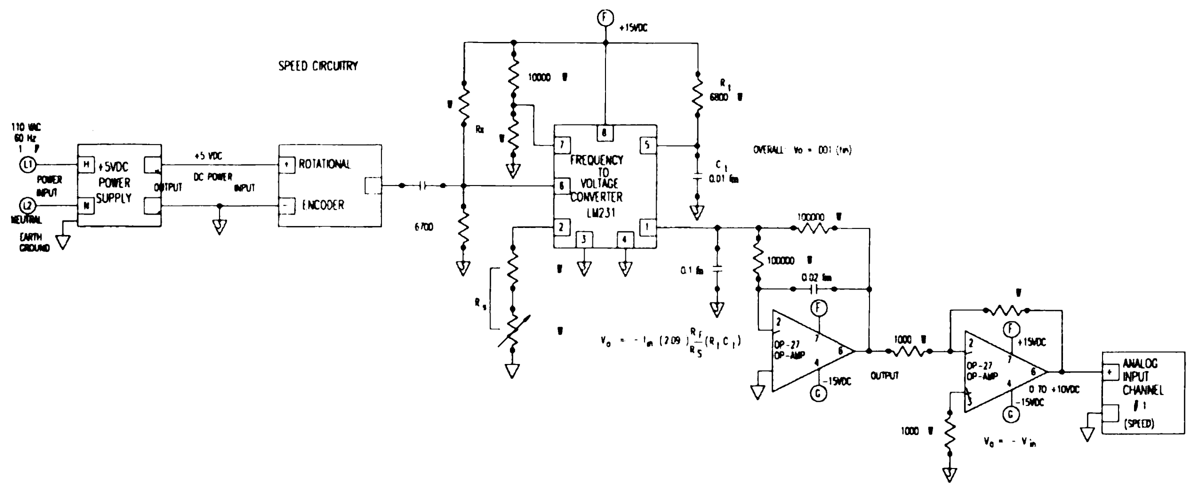


Flow Rate Circuitry
Figure A1

#1: Speed.

The rotational speed of the pump is measured by an optical rotational encoder. The encoder outputs 60 TTL pulses each revolution. The speed in rpm is the frequency of the encoder signal. The speed range is 0 to 10000 rpm, so the frequency range is 0 to 10 kHz. A f-t-v converts this frequency to a 0 to -10 vdc signal. A inverting op-amp circuit with unity gain makes this signal positive. Figure A2 contains the schematic for channel #1.

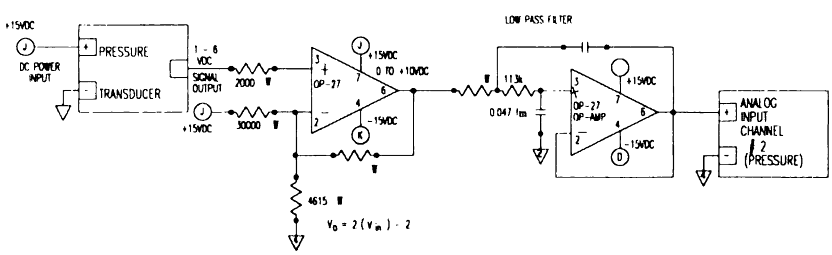
frequency to a 0 to -10 vdc signal. A inverting op-amp circuit with unity gain makes this signal positive. Figure A2 contains the schematic for channel #1.



Speed Circuitry
Figure A2

#2: Pressure.

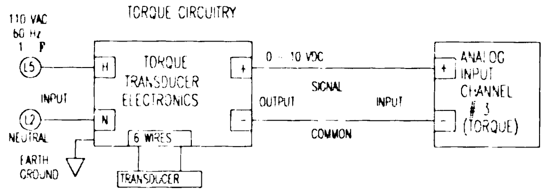
The pressure transducer voltage range is 1 to 6 volts which correspond to a 0 to 200 psi values. An op-amp circuit scales and off sets this range to be 0 to 10 vdc. A second order low pass active filter with cut off frequency of 15 Hertz eliminates higher frequency noise. Figure A3 contains the schematic for channel #2.



Pressure Scaling Circuitry
Figure A3

#3: Torque.

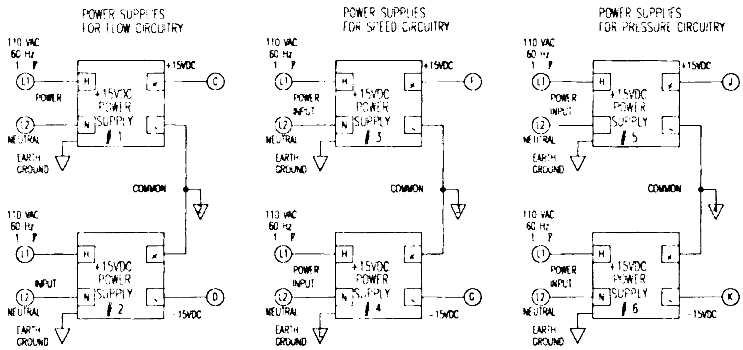
The signal from the torque transducer and its electronics is 0 to 10 vdc. This signal can be directly coupled to the I/O board. The range of torque values represented by the 0 to 10 vdc signal is set on the torque transducer electronics to be 0 to 100 inch-ounces. Figure A4 contains the schematic for channel #3.



Torque Circuitry
Figure A4

Power Supplies.

The schematics for the power supplies for the flow, speed, and pressure circuitry are shown in Figure A5. For each circuit, two +15 vdc power supplies are connected to provide +15 and -15 volts to power the op-amps. The torque signal does not require any additional power supplies.

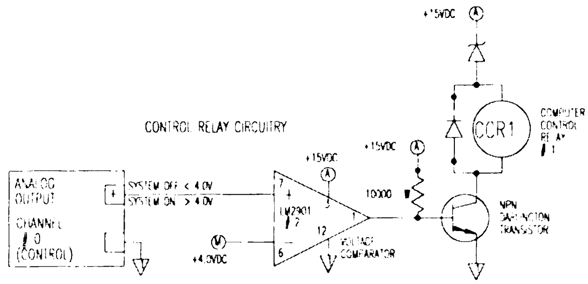


Power Supply Circuitry
Figure A5

Analog Output Channels: (0 to 5 Volts)

#0: System Control.

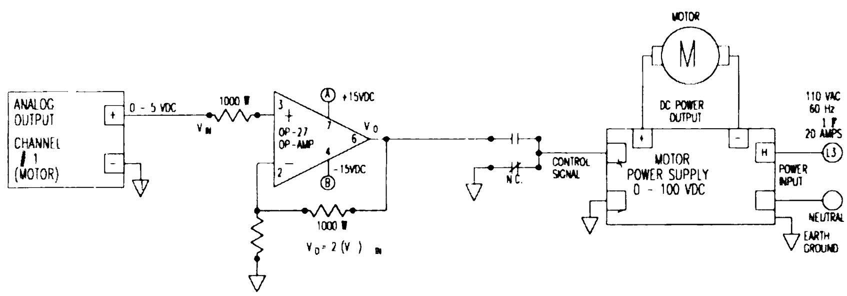
A voltage comparator is used to switch a transistor to control a relay. When the channel output voltage exceeds 4 volts, the comparator output turns on the Darlington transistor. The collector is pulled to ground, and current flows through the relay coil. Computer controlled relay #1 (ccr1) prevents the motor and spool valve from operating when the data acquisition program is not running. Figure A6 contains this schematic.



Control Relay Circuitry
Figure A6

#1: Motor Speed.

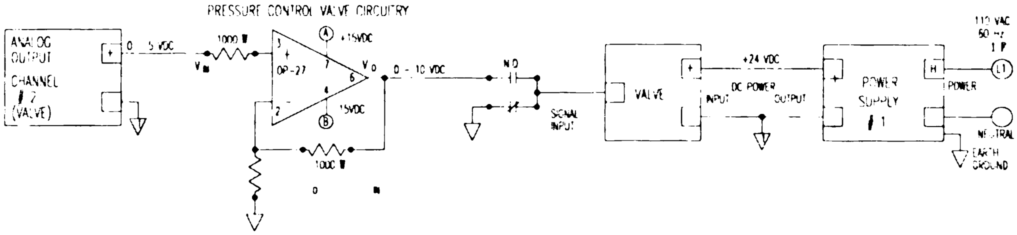
The I/O board outputs a 0 to 5 vdc signal. Since the motor power supply requires a 0 to 10 vdc range, a non-inverting op-amp circuit with gain of 2 is used. The op-amp output is switched by the first set of normally open contacts in ccr1. The motor power supply output 0 to 100 vdc to drive the universal motor. Figure A7 contains this schematic.



Drive Motor Circuitry
Figure A7

#2: Pressure.

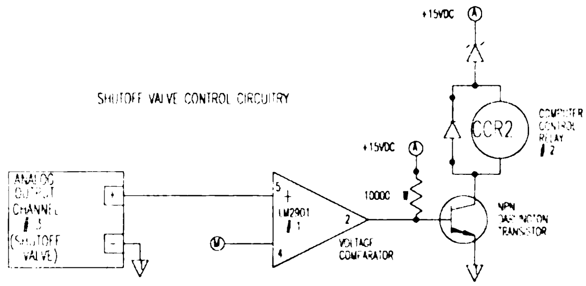
The I/O board outputs a 0 to 5 vdc signal. Since the spool valve requires a 0 to 10 vdc range, a non-inverting op-amp circuit with gain of 2 is used. The op-amp output is switched by the second set of normally open contacts in ccr1. The spool valve is powered by a +24 vdc power supply. Figure A8 contains this schematic.



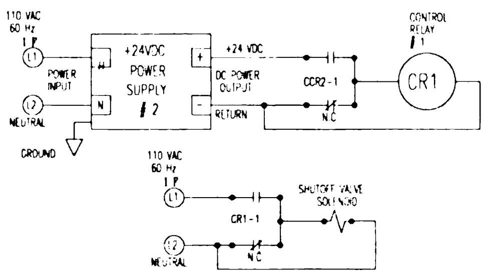
Proportional Spool Valve Circuitry
Figure A8

#3: Shut Off Valve.

A voltage comparator is used to switch a transistor to control a relay. When the channel output voltage exceeds 4 volts, the comparator output turns on the Darlington transistor. The collector is pulled to ground, and current flows through the relay coil. Computer controlled relay #2 (ccr2) switches a 24 vdc line that energizes control relay #1 (cr1). Cr1 switches a 110 vac line that powers the shut off valve. Figure A9 contains the dc voltage schematic, and Figure A10 shows the 24 vdc power supply and relay for the 110 vac line.



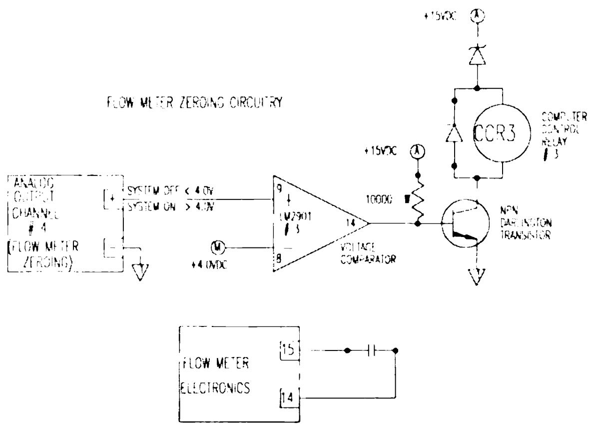
Shut Off Valve Circuitry
Figure A9



Relay Power Supply Circuitry
Figure A10

#4: Flow Meter Zeroing Circuitry.

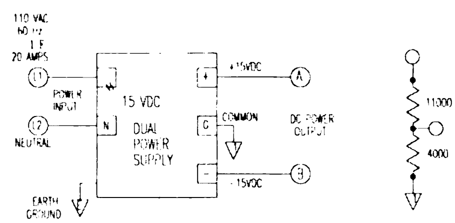
A voltage comparator is used to switch a transistor to control a relay. When the channel output voltage exceeds 4 volts, the comparator output turns on the Darlington transistor. The collector is pulled to ground, and current flows through the relay coil. Computer controlled relay #3 (ccr3) connects two terminals on the flow meter. This connection enables the calibration and zero flow level adjustment on the flow meter. Figure A11 contains this schematic.



Flow Meter Zeroing Circuitry
Figure A11

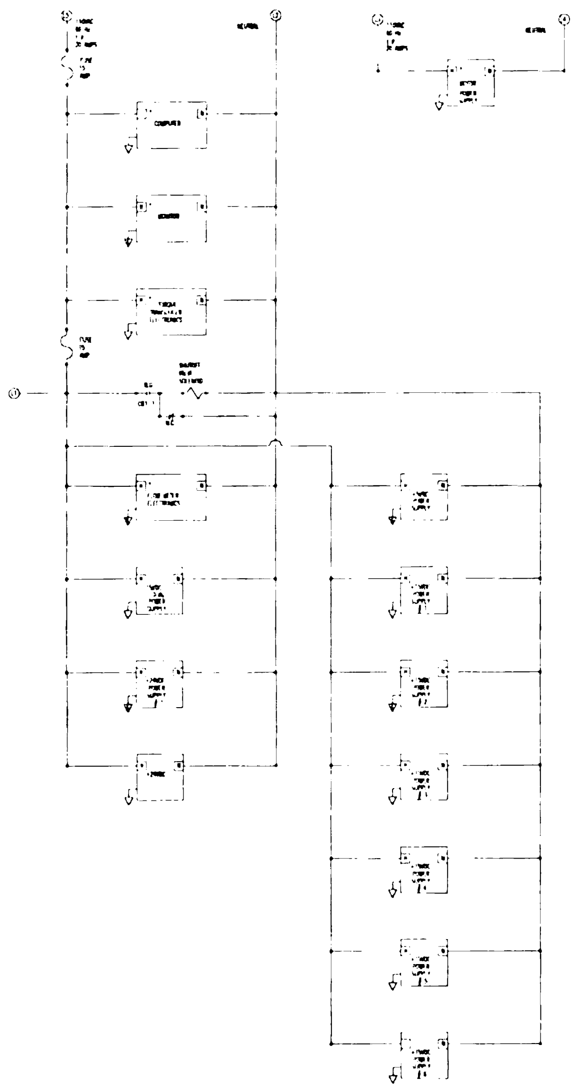
Power Supply.

Figure A12 shows the dual power supply used to power the op-amps and comparators for the analog output channels. A voltage divider is used to provide the 4 volt reference for the comparators.



Power Supply And Voltage Divider
Figure A12

Figure A13 shows the schematic for the 110 vac lines that power all the devices. A separate line is provided for the motor power supply. Each line requires a 20 amp circuit. An external fuse is located before the power supplies used for the op-amps and the shut off solenoid valve.



AC Voltage Circuitry
Figure A13

Appendix B

Flow Control

A 2nd order transfer function with time delay has the form

$$G(s) = \frac{ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

Let

$$G(s) = \frac{G_1(s)G_2(s)}{G_3(s)G_4(s)}$$

where

$$G_1(s) = k\,,\ G_2(s) = e^{-\theta s}\,,\ G_3(s) = \tau_1 s + 1\text{, and } G_4(s) = \tau_2 s + 1.$$

With $s=j\omega$, the magnitudes of the above equations are as follows:

$$|G_1(j\omega)| = k \qquad |G_2(j\omega)| = 1$$

$$|G_3(j\omega)| = \sqrt{\tau_1^2\omega^2 + 1} \qquad |G_4(j\omega)| = \sqrt{\tau_2^2\omega^2 + 1}$$

Then,

$$|G(j\omega)| = \frac{|G_1(j\omega)||G_2(j\omega)|}{|G_3(j\omega)||G_4(j\omega)|} = \frac{k}{\sqrt{\tau_1^2\omega^2 + 1}\sqrt{\tau_2^2\omega^2 + 1}}$$

or,

$$\boxed{|G(j\omega)| = \frac{14.4}{\sqrt{0.24^2\omega^2 + 1}\sqrt{0.28^2\omega^2 + 1}}}$$

The phase angle for the model is $\angle G(s) = \angle G_1(s) + \angle G_2(s) - \angle G_3(s) - \angle G_4(s)$.

With $s = j\omega$, the above angles are as follows:

$$\angle G_1(j\omega) = 0 \qquad \angle G_2(j\omega) = -\theta\omega\,,$$

$$\angle G_3(j\omega) = \tan^{-1}[\tau_1\omega] \qquad \angle G_4(j\omega) = \tan^{-1}[\tau_2\omega]$$

Then,

$$\angle G(j\omega) = -\theta\omega - \tan^{-1}[\tau_1\omega] - \tan^{-1}[\tau_2\omega] = \boxed{-0.15\omega - \tan^{-1}[0.24\omega] - \tan^{-1}[0.28\omega]}$$

The open loop transfer function is

$$G_{OL}(s) = G_C(s)G_P(s) = \left(\frac{I}{s}\right)\left(\frac{ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}\right)$$

Let $G_{OL}(s) = \dfrac{G_1(s)G_2(s)G_3(s)}{G_4(s)G_5(s)G_6(s)}$, where $G_1(s) = k$, $G_2(s) = I$, $G_3(s) = e^{-\theta s}$,

$G_4(s) = s$, $G_5(s) = \tau_1 s + 1$, and $G_6(s) = \tau_2 s + 1$. The magnitudes of these function are as follows:

$$|G_1(j\omega)| = k \qquad |G_2(j\omega)| = I$$

$$|G_3(j\omega)| = 1 \qquad |G_4(j\omega)| = \omega$$

$$|G_5(j\omega)| = \sqrt{\tau_1^2\omega^2 + 1} \qquad |G_6(j\omega)| = \sqrt{\tau_2^2\omega^2 + 1}$$

Then,

$$|G_{OL}(j\omega)| = \frac{kI}{\omega\left(\sqrt{\tau_1^2\omega^2 + 1}\right)\left(\sqrt{\tau_2^2\omega^2 + 1}\right)} = \boxed{\frac{14.4I}{\omega\left(\sqrt{0.24^2\omega^2 + 1}\right)\left(\sqrt{0.28^2\omega^2 + 1}\right)}}$$

The phase angle for the open loop transfer function is

$$\angle G(s) = \angle G_1(s) + \angle G_2(s) + \angle G_3(s) - \angle G_4(s) - \angle G_5(s) - \angle G_6(s).$$

With s = jω, the phase angles for the individual functions are as follows:

$$\angle G_1(j\omega) = 0 \qquad \angle G_2(j\omega) = 0$$

$$\angle G_3(j\omega) = -\theta\omega, \qquad \angle G_4(j\omega) = \frac{\pi}{2}$$

$$\angle G_5(j\omega) = \tan^{-1}[\tau_1\omega] \qquad \angle G_6(j\omega) = \tan^{-1}[\tau_2\omega]$$

or

$$\angle G(j\omega) = -\theta\omega - \frac{\pi}{2} - \tan^{-1}[\tau_1\omega] - \tan^{-1}[\tau_2\omega]$$

$$\boxed{\angle G(j\omega) = -0.15\omega - \frac{\pi}{2} - \tan^{-1}[0.24\omega] - \tan^{-1}[0.28\omega]}$$

The critical frequency is found by setting the phase angle equation for the open loop transfer function equal to $-\pi$.

$$-\pi = -0.15\omega - \frac{\pi}{2} - \tan^{-1}[0.24\omega] - \tan^{-1}[0.28\omega]$$

$$\therefore \omega = 2.5875 \text{ radians/second.}$$

Substituting this into the corresponding magnitude equation yields:

$$|G_{OL}(j2.5875)| = \frac{14.4I}{(2.5875)\left(\sqrt{(0.24)^2(2.5875)^2+1}\right)\left(\sqrt{(0.28)^2(2.5875)^2+1}\right)} = 3.8286\ I$$

For stability, the open loop gain must be less than unity at the critical frequency.

$$\therefore 3.8286\ I < 1$$

$$I < 0.2612$$

Let I = 0.07. The gain at the critical frequency is then

$$|G(j2.5875)| = (3.8286)(0.07) = 0.2680$$

Then gain margin is then

$$20\ \text{Log}(0.2680) = -11.4\ \text{dB}$$

To determine the phase margin, the frequency where the open loop gain equals 1 is required.

$$1 = \frac{(14.4)(0.07)}{(\omega)\left(\sqrt{0.24^2\omega^2+1}\right)\left(\sqrt{0.28^2\omega^2+1}\right)}$$

$$\omega = 0.9497 \text{ radians/second}$$

The phase angle at this frequency is

$$\angle G(j0.9497) = -(0.15)(0.9497) - \frac{\pi}{2} - \tan^{-1}[(0.24)(0.9497)] - \tan^{-1}[(0.28)(0.9497)]$$

$$\angle G(j0.9497) = -2.197 \text{ radians/second} = -125.9\ °$$

The phase margin is then 180° + (-125.9°) = 54.1 °.

Closed Loop Response.

From figure f6, the transfer function is

$$Y(s) = G_c(s)G_p(s)\left[R(s) - Y(s)\right]$$

Dropping the "(s)",

$$Y = G_cG_p(R - Y)$$

$$Y = G_cG_pR - G_cG_pY$$

$$Y + G_cG_pY = G_cG_pR$$

$$Y\left(1 + G_cG_p\right) = G_cG_pR$$

$$\frac{Y}{R} = \frac{G_cG_p}{\left(1 + G_cG_p\right)} = G_{cl}$$

With G_c = I/s, and $G_p = \dfrac{ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$,

$$G_{cl} = \frac{\dfrac{\left(\frac{I}{s}\right)ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}}{1 + \dfrac{\left(\frac{I}{s}\right)ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}}$$

Simplifying:

$$G_{cl} = \frac{\left(\frac{I}{s}\right)ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1) + \left(\frac{I}{s}\right)ke^{-\theta s}}$$

$$G_{cl} = \frac{kIe^{-\theta s}}{s(\tau_1 s + 1)(\tau_2 s + 1) + kIe^{-\theta s}}$$

For frequency response, let s=jω,

$$G_{cl} = \frac{kIe^{-j\omega\theta}}{(j\omega)(\tau_1 j\omega + 1)(\tau_2 j\omega + 1) + kIe^{-j\omega\theta}}$$

With $e^{-j\omega\theta}=\cos(-\omega\theta) + j\sin(-\omega\theta)$, with $\cos(-\omega\theta) = \cos(\omega\theta)$, and $\sin(-\omega\theta)=-\sin(\omega\theta)$, the transfer function becomes

$$G_{cl} = \frac{kl[\cos(\omega\theta) - j\sin(\omega\theta)]}{(j\omega)(\tau_1 j\omega + 1)(\tau_2 j\omega + 1) + kl[\cos(\omega\theta) - j\sin(\omega\theta)]}$$

Expanding gives

$$G_{cl} = \frac{kl\cos(\omega\theta) - jkl\sin(\omega\theta)}{-j\omega^3\tau_1\tau_2 - \omega^2(\tau_1 + \tau_2) + j\omega + kl\cos(\omega\theta) - jkl\sin(\omega\theta)}$$

Collecting terms yields

$$G_{cl} = \frac{kl\cos(\omega\theta) - jkl\sin(\omega\theta)}{\left[kl\cos(\omega\theta) - \omega^2(\tau_1 + \tau_2)\right] + j\left[\omega - \omega^3\tau_1\tau_2 - kl\sin(\omega\theta)\right]}$$

Let $G_{cl}(j\omega) = \frac{G_1(j\omega)}{G_2(j\omega)}$, then the magnitude is $|G_{cl}(j\omega)| = \frac{|G_1(j\omega)|}{|G_2(j\omega)|}$, or

$$|G_1(j\omega)| = \sqrt{k^2l^2\cos^2(\omega\theta) + k^2l^2\sin^2(\omega\theta)}$$

$$|G_1(j\omega)| = \sqrt{k^2l^2\left(\cos^2(\omega\theta) + \sin^2(\omega\theta)\right)}$$

$$|G_1(j\omega)| = \sqrt{k^2l^2(1)} = \sqrt{k^2l^2} = kl$$

and

$$|G_2(j\omega)| = \sqrt{\left[kl\cos(\omega\theta) - \omega^2(\tau_1 + \tau_2)\right]^2 + j\left[\omega - \omega^3\tau_1\tau_2 - kl\sin(\omega\theta)\right]^2}$$

$$\therefore |G_{cl}(j\omega)| = \frac{kl}{\sqrt{\left[kl\cos(\omega\theta) - \omega^2(\tau_1 + \tau_2)\right]^2 + j\left[\omega - \omega^3\tau_1\tau_2 - kl\sin(\omega\theta)\right]^2}}$$

Substituting $k=14.4$, $l=0.07$, $\tau_1=0.24$, $\tau_2=0.28$, and $\theta=0.15$ yields

$$|G_{cl}(j\omega)| = \frac{(14.4)(0.07)}{\sqrt{\left[(14.4)(0.07)\cos(0.15\omega) - \omega^2(0.24+0.28)\right]^2 + j\left[\omega - \omega^3(0.24)(0.28) - (14.4)(0.07)\sin(0.15\omega)\right]^2}}$$

$$\left|G_{cl}(j\omega)\right| = \frac{1008}{\sqrt{\left[1008\cos(0.15\omega) - 0.52\omega^2\right]^2 + j\left[\omega - 0.0672\omega^3 - 1008\sin(0.15\omega)\right]^2}}$$

The phase angle $\angle G_{cl}(j\omega) = \angle G_1(j\omega) - \angle G_2(j\omega)$.

$$\angle G_1(j\omega) = \tan^{-1}\left[\frac{-kl\sin(\omega\theta)}{kl\cos(\omega\theta)}\right] = \tan^{-1}\left[\frac{-\sin(\omega\theta)}{\cos(\omega\theta)}\right] = \tan^{-1}\left[-\tan(\omega\theta)\right] = -\omega\theta$$

$$\angle G_2(j\omega) = \tan^{-1}\left[\frac{\omega - \omega^3\tau_1\tau_2 - kl\sin(\omega\theta)}{kl\cos(\omega\theta) - \omega^2(\tau_1 + \tau_2)}\right]$$

$$\therefore \angle G_{cl}(j\omega) = -\omega\theta - \tan^{-1}\left[\frac{\omega - \omega^3\tau_1\tau_2 - kl\sin(\omega\theta)}{kl\cos(\omega\theta) - \omega^2(\tau_1 + \tau_2)}\right]$$

Substituting values yields:

$$\angle G_{cl}(j\omega) = -0.15\omega - \tan^{-1}\left[\frac{\omega - 0.672\omega^3 - 1008\sin(0.15\omega)}{1008\cos(0.15\omega) - 0.52\omega^2}\right]$$

Appendix C

Pressure Control

Plant Transfer Function:

Assume 1st order system, and step input:

$$\frac{Y^*(s)}{R^*(s)} = \frac{k}{\tau s + 1}$$

where k is the gain, t is the time constant, and Y* and R* are deviation variables defined by

$$Y^*(s) = Y(s) - Y_{SS}(s)$$

$$R^*(s) = R(s) - R_{SS}(s)$$

where Y(s) and R(s) are the physical variables, and $Y_{SS}(s)$ and $R_{SS}(s)$ are the steady state conditions. Let R*(s) = r*/s for a step input. Solving the transfer function for Y* yields

$$Y^* = \frac{kr^*}{s(\tau s + 1)}$$

Partial fraction expansion:

$$\frac{kr^*}{s(\tau s + 1)} = \frac{A}{s} + \frac{B}{\tau s + 1}$$

$$A = kr^*$$

$$B = -\tau kr^*$$

$$\therefore Y^* = \frac{kr^*}{s} + \frac{-\tau kr^*}{\tau s + 1}$$

$$Y^* = kr^* \left[\frac{1}{s} - \frac{1}{s + \frac{1}{\tau}} \right]$$

Taking the inverse Laplace transform:

$$y^*(t^*) = kr^*\left[1 - e^{\frac{-t^*}{\tau}}\right]$$

Substituting for the deviation variables yields

$$y(t) - y_{SS} = k(r - r_{SS})\left[1 - e^{\frac{-(t - t_{step})}{\tau}}\right]$$

With k=184.9 kPa / volt, t = 0.214 seconds, y_{SS} = 79.7 kPa, r_{SS} = 1 volt, and t_{step} = 3.79 seconds,

$$\boxed{y(t) = 184.9(r - 1)\left[1 - e^{\frac{-(t - 3.79)}{0.214}}\right] + 79.7}$$

Check: With r = 1 volts, t = 3.79 seconds,

y(3.79) = 184.9 (1 - 1) [1 - 1] + 79.7 = 0 + 79.7 = 79.9 kPa.

With r = 2 volts, t = ∞,

y(∞) = 184.9 (2 - 1) [1 - 0] + 79.7 = 184.9 + 79.9 = 264.6 kPa.

Closed Loop Transfer Function:

Loop Gain from figure p4:

$$E = R^* - Y^*$$

$$Y^* = \left(\frac{I}{s}\right)\left(\frac{k}{\tau s + 1}\right)E$$

$$Y^* = \frac{kI}{s(\tau s + 1)}(R^* - Y^*)$$

$$Y^* = \frac{kI}{s(\tau s + 1)}R^* - \frac{kI}{s(\tau s + 1)}Y^*$$

$$Y^* + \frac{kI}{s(\tau s + 1)}Y^* = \frac{kI}{s(\tau s + 1)}R^*$$

$$s(\tau s + 1)Y^* + kIY^* = kIR^*$$

$$Y^*\left(s(\tau s + 1) + kI\right) = kIR^*$$

$$Y^*\left(\tau s^2 + s + kI\right) = kIR^*$$

$$\frac{Y^*}{R^*} = \frac{kI}{\tau s^2 + s + kI}$$

$$\therefore \frac{Y^*}{R^*} = \frac{\frac{kI}{\tau}}{s^2 + \frac{1}{\tau}s + \frac{kI}{\tau}}$$

Characteristic equation:

$$s^2 + \frac{1}{\tau}s + \frac{kI}{\tau} = 0$$

The poles are located at

$$s = -\frac{1}{2\tau} \pm \frac{1}{2}\sqrt{\frac{1}{\tau^2} - 4\left(\frac{kI}{\tau}\right)}$$

For under damped, oscillatory response (complex roots):

$$\frac{1}{\tau^2} - 4\left(\frac{kI}{\tau}\right) < 0$$

For critically damped response (equal, real, negative roots):

$$\frac{1}{\tau^2} - 4\left(\frac{kI}{\tau}\right) = 0$$

For over damped, non-oscillatory response (unequal, real, negative roots):

$$\frac{1}{\tau^2} - 4\left(\frac{kl}{\tau}\right) > 0$$

Choose over damped response, and let p_1 and p_2 be the poles,

with $p_1, p_2 < 0$:

Note:

$$s^2 + \frac{1}{\tau}s + \frac{kl}{\tau} = (s - p_1)(s - p_2)$$

$$s^2 + \frac{1}{\tau}s + \frac{kl}{\tau} = s^2 - (p_1 + p_2)s + p_1p_2$$

$$\therefore \frac{1}{\tau} = -(p_1 + p_2)$$

$$\therefore \frac{kl}{\tau} = p_1p_2$$

Partial fraction expansion:

$$\frac{\frac{kl}{\tau}r^*}{s\left(s^2 + \frac{1}{\tau}s + \frac{kl}{\tau}\right)} = \frac{\frac{kl}{\tau}r^*}{s(s - p_1)(s - p_2)} = \frac{A}{s} + \frac{B}{s - p_1} + \frac{C}{s - p_2}$$

$$A = \frac{\frac{kl}{\tau}r^*}{p_1p_2} = \frac{\frac{kl}{\tau}r^*}{\frac{kl}{\tau}} = r^*$$

$$B = \frac{\frac{kl}{\tau}r^*}{p_1(p_1 - p_2)}$$

$$C = \frac{\frac{kl}{\tau}r^*}{p_2(p_2 - p_1)}$$

$$\therefore Y^* = \frac{r^*}{s} + \left[\frac{\frac{kl}{\tau}r^*}{p_1(p_1 - p_2)}\right]\left(\frac{1}{s - p_1}\right) + \left[\frac{\frac{kl}{\tau}r^*}{p_2(p_2 - p_1)}\right]\left(\frac{1}{s - p_2}\right)$$

Inverse Laplace transform:

$$y^*(t^*) = r^* + \left[\frac{\frac{kl}{\tau} r^* \left(e^{p_1 t^*}\right)}{p_1(p_1 - p_2)}\right] + \left[\frac{\frac{kl}{\tau} r^* \left(e^{p_2 t^*}\right)}{p_2(p_2 - p_1)}\right]$$

Multiply second term on the right side by p_2 / p_2, and the third term by p_1 / p_1:

$$y^*(t^*) = r^* + \left[\frac{\frac{kl}{\tau} r^* p_2 \left(e^{p_1 t^*}\right)}{p_1 p_2(p_1 - p_2)}\right] + \left[\frac{\frac{kl}{\tau} r^* p_1 \left(e^{p_2 t^*}\right)}{p_1 p_2(p_2 - p_1)}\right]$$

Recall: $\frac{kl}{\tau} = p_1 p_2$

$$y^*(t^*) = r^* \left[1 + \frac{\frac{kl}{\tau} p_2 \left(e^{p_1 t^*}\right)}{\frac{kl}{\tau}(p_1 - p_2)} + \frac{\frac{kl}{\tau} p_1 \left(e^{p_2 t^*}\right)}{\frac{kl}{\tau}(p_2 - p_1)}\right]$$

$$y^*(t^*) = r^* \left[1 + \frac{p_2 \left(e^{p_1 t^*}\right)}{(p_1 - p_2)} + \frac{p_1 \left(e^{p_2 t^*}\right)}{(p_2 - p_1)}\right]$$

Rearranging yields

$$y^*(t^*) = r^* \left[1 + \frac{p_2 \left(e^{p_1 t^*}\right) - p_1 \left(e^{p_2 t^*}\right)}{(p_1 - p_2)}\right]$$

Check:

$$y^*(0) = r^* \left[1 + \frac{p_2(1) - p_1(1)}{p_1 - p_2}\right] = r^* \left[1 + \frac{p_2 - p_1}{p_1 - p_2}\right] = r^*[1 - 1] = 0$$

Check:

$$y^*(\infty) = r^* \left[1 + \frac{p_2(0) - p_1(0)}{p_1 - p_2}\right] = r^*[1 + 0] = r^*[1] = r^*$$

Converting back to the physical variables yields

$$y(t) - y_{SS} = (r - r_{SS})\left[1 + \frac{p_2\left(e^{p_1(t-t_{step})}\right) - p_1\left(e^{p_2(t-t_{step})}\right)}{(p_1 - p_2)}\right]$$

With p_1 = -3.34109, p_2 = -1.33208, y_{SS} =200 kPa, r_{SS} = 200 kPa, t_{step} = 19.18 seconds,

$$y(t) - 200 = (r - 200)\left(1 + 0.663e^{-3.34109(t-19.18)} - 1663e^{-133208(t-19.18)}\right)$$

Check: With r = 200 kPa, t = 19.18 seconds,

$$y(19.18) = (200 - 200)(1 + 0.663(1) - 1663(1)) + 200 = (0)(1 - 1) + 200 = 200$$

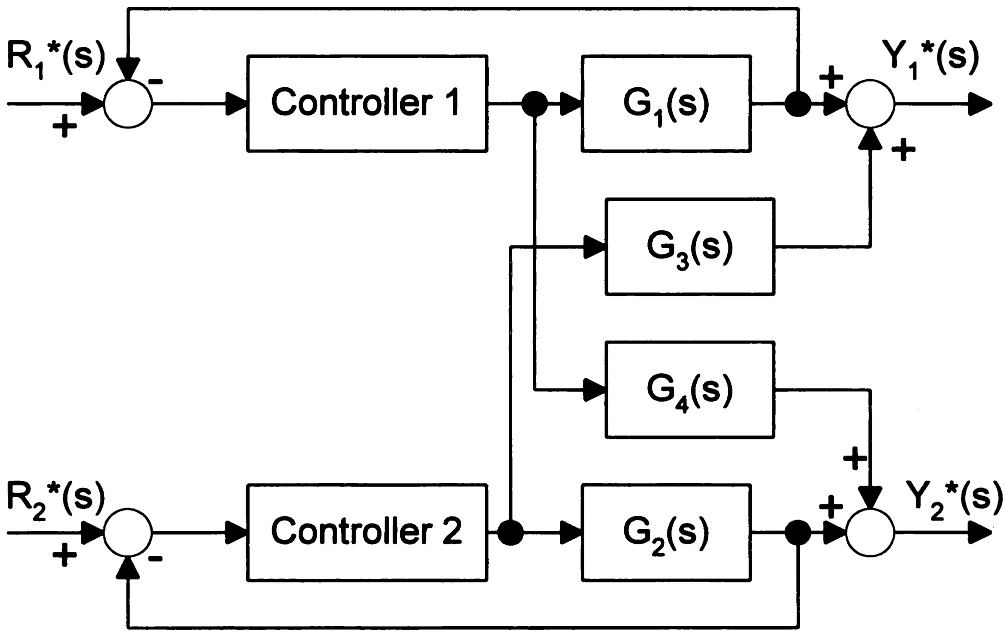
With r =300 kPa, t = ∞,

$$y(\infty) = (300 - 200)(1 + 0.663(0) - 1663(0)) + 200 = (100)(1 - 0) + 200 = 300$$

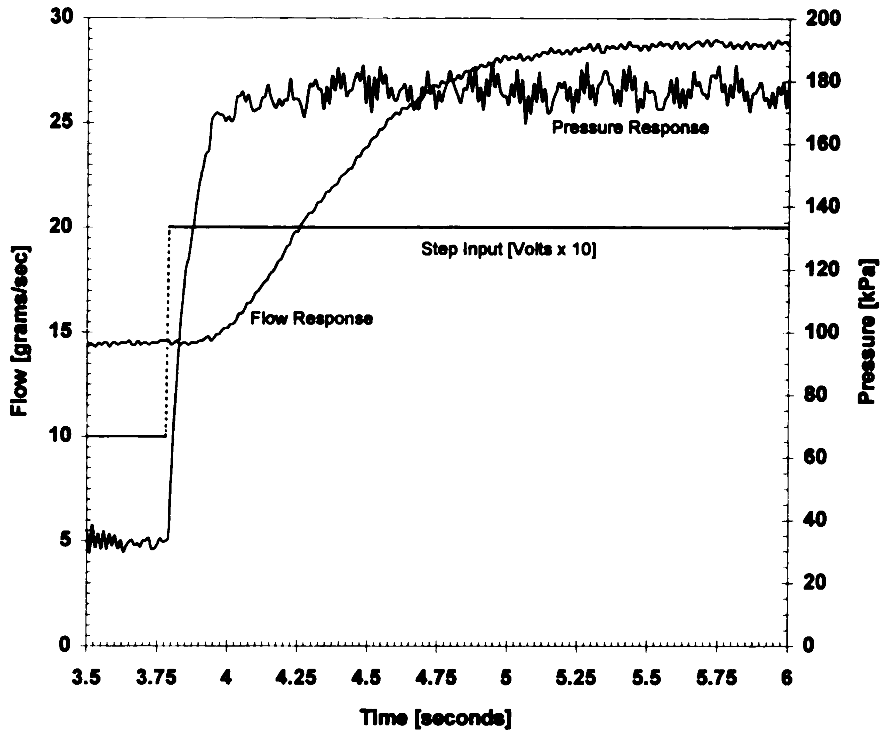
Appendix D

Flow And Pressure Responses To Step Inputs

The response of the flow to a step change in motor voltage, and the response of the pressure to a step change in valve voltage was used to determine the process models for the control loops. Note that the flow rate also changes when the valve position is altered, and the pressure changes when the motor speed is modified. Hence, The two control loops used on the test stand are not completely independent. A multiple input - multiple output (MIMO) system could be developed for the test stand. Figure D1 show a block diagram for such a system. The determination of all the process models and the corresponding controllers for the MIMO system is beyond the scope of this project; however, the flow and pressure responses to step changes in motor and valve voltages are presented. Figure D2 shows the responses to a step change in motor voltage, and Figure D3 shows the responses to a step change in valve voltage.

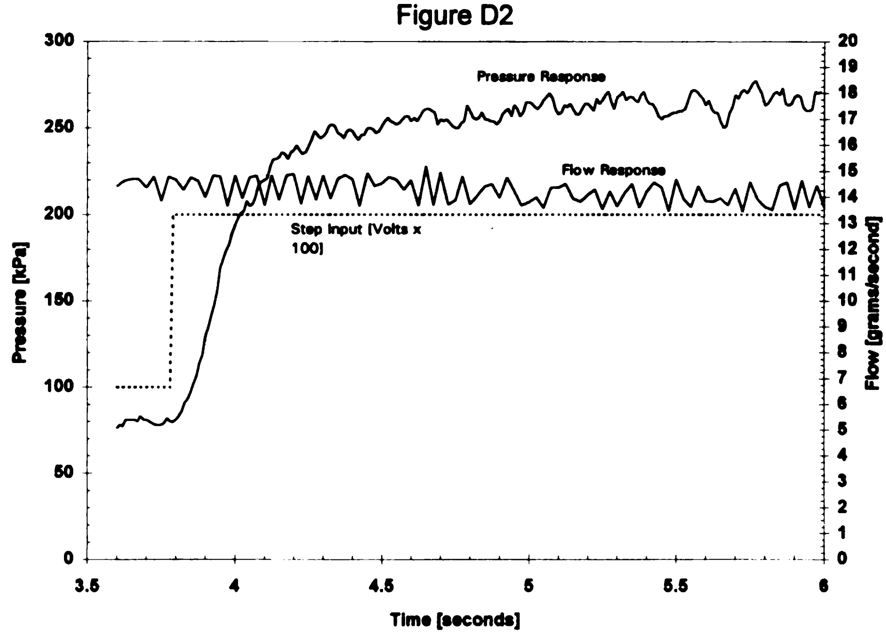


Multiple Input - Multiple Output Block Diagram
Figure D1



Flow And Pressure Responses To A Step In Motor Control Voltage
Step From 1 To 2 Volts

Figure D2



Flow And Pressure Response To A Step In Valve Voltage
Step From 1 To 2 Volts

Figure D3

Bibliography

Bibliography

Balchen, Jens G., Kenneth I. Mumme', **Process Control**, c 1988, Van Nostrand Reinhold Co., Inc., New York, New York.

Matley, Jay (editor), **Practical Process Instrumentation & Control**, c 1986, Chemical Engineering McGraw-Hill Publishing Co., New York, New York.

Phillips, Charles L., Royce D. Harbor, **Feedback Control Systems**, c 1988, Prentise-Hall, Inc., Englewood Cliffs, New Jersey.

Seborg, Dale E., Thomas F. Edgar, Duncan A. Mellichamp, **Process Dynamics And Control**, c 1989, John Wiley & Sons, Inc., New York, New York.

Takahashi, Yasundo, Michael J. Rabins, David M. Auslander, **Control And Dynamic Systems**, c 1970, Addison-Wesley Publishing Co., Reading, Massachusetts.