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WITH APPLICATION TO SYNTHESIZING CORRELATION MATRICES

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A COMPARISON OF METHODS FOR CORRECTING MULTIVARIATE DATA FOR ATTENUATION WITH APPLICATION TO SYNTHESIZING CORRELATION MATRICES

BY

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ABSTRACT

A COMPARISON OF METHODS FOR CORRECTING MULTIVARIATE DATA FOR ATTENUATION WITH APPLICATION TO SYNTHESIZING CORRELATION MATRICES

By

Christine M. Schram

Corrections for attenuation have long been used to adjust sample correlations for measurement error. Current research synthesis (meta-analysis) procedures involve synthesizing correlational data. The synthesis of multivariate correlation data raises several statistical questions including correcting for measurement error. The focus of this work was to discover the most statistically sound method for correcting multivariate data for attenuation which accounts for dependence among the correlations and reliabilities.

Multivariate corrections from "errors-in-variables" regression analysis were examined, as was an existing multivariate correction from educational literature. These methods were compared to the traditional univariate correction for attenuation. Simulated and exact comparisons were made of corrected correlations and their resulting variance-covariance matrices.

All the methods examined produced similar corrected correlations. Even the simple univariate correction yielded corrected correlations that were good estimates of the population correlation. In addition, a variety of

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approximations to the population variance-covariance matrices of the corrected correlations were associated with these correction methods. A variance estimator derived in this dissertation and based on large-sample theory yielded the best estimates of variation when compared to the empirical sampling distribution. A related variance estimation method based on the correction from Fuller and Hidiroglou (1978) also gave similar results to the large-sample theory method, but relied on raw data, which are often unavailable in most research syntheses.

An example illustrated the results of a multivariate synthesis using the new procedure. The results of this example showed more variability in the average correlations and larger homogeneity test statistics when compared to previous analyses of the same studies.

Overall, if corrections are to be applied, the univariate correction, used with the large-sample variance-covariance matrix, will yield reasonable results.

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Acknowledgements

Now that this is over, I'm not sure I can produce words worthy of the gratitude I feel. I'm finding it difficult to appropriately thank those whose assistance and encouragement accompanied me on this journey. However, I will give it my best shot.

I wish to thank my committee members for helping me get through this with ease. Dr. William Schmidt and Dr. Dennis Gilliland were always cooperative and available and made this process much easier than I expected. Thanks too to Dr. Irv Lehmann, for filling in admirably, for always being encouraging, and for always having time to listen.

My undying gratitude goes out to my advisor and mentor, Dr. Betsy Becker, who not only has been supportive and essential to my progress, but is a great role model, and most of all, a great friend.

I also wish to thank those who were influential early in my academic career, especially Dr. William Mehrens and Dr. Michael Seltzer who epitomized class whenever I worked with them.

Finally, I have to thank my family and friends for their support and their attempts to understand when they knew they

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couldn't possibly. Thanks Alex, the love of my life, for keeping me sane, making me laugh, and making the sacrifices more bearable. Maybe now we can go on a vacation.

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CHAPTER I

INTRODUCTION

Meta-analysis in Educational Research

Meta-analysis is a developing statistical technique that allows for the synthesis of results from studies of the same phenomenon. Such studies generally contain data sufficient to compute an effect size, which exhibits the magnitude of the relationship studied. Meta-analytic statistical techniques are then used to combine effect sizes, and summarize the results. Further analysis allows the explanation of the variability in effect sizes by the modeling of moderating variables.

Meta-analyses have been criticized for being too simplistic and for lacking in theory (Chow, 1987). Because many meta-analyses simply summarize bivariate relationships without accounting for moderators, or other significant relationships, this criticism is justified. One potential answer to this criticism is to develop methodology which will allow the synthesis of more complicated theory-based data. Meta-analytic techniques are currently being developed which allow the summary of multivariate relationships (see, e.g., Becker & Fahrbach, 1994; Becker & Schram, 1993). These techniques allow the synthesis of interrelationships (correlations) among variables, in contrast to the current syntheses of simple bivariate relationships. Resulting synthese

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syntheses will allow for a more complex and complete picture of educational phenomena.

Multivariate syntheses examine systems of variables (models of interrelated variables), in contrast to univariate syntheses, which summarize data about one relationship. However, synthesizing models by combining correlation matrices leads to several statistical problems, one of which involves correcting for attenuation in a multivariate setting. The multivariate nature of such corrections led to several research questions. This work examined the effects of using various corrections and asked whether there is one best multivariate correction for attenuation, for the particular case of multivariate research syntheses.

Purpose of the Study

The purpose of this work was to investigate the possibilities of using multivariate corrections for attenuation. Is there a best possible correction? How do corrections proposed in the "errors-in-variables" regression literature apply? The meta-analytic context differs from the regression context in that reliabilities in educational and social-science data are often scarce or unavailable. What assumptions are necessary before these corrections can be used? Wherever multivariate situations arise when the assumptions are met, these techniques should be applicable, and in fact, the applications reach beyond meta-analysis.

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The results of this work are important because correcting for attenuation will provide better estimates of parameters. Because sample correlation coefficients underestimate population parameters, correcting for this measurement error will yield more accurate results in research syntheses. Also, the power of statistical tests is decreased if correlations are uncorrected (Williams & Zimmerman, 1982). If the methodology exists for getting better estimates, better analyses (including syntheses) will result. Certainly, the usefulness of getting the best possible estimates is obvious. We want the analyses to be as "right" as possible.

Four factors contribute to the need for further study of corrections in multivariate analyses: 1) measurement errors affect correlation coefficients, 2) dependence in multivariate data can lead to inaccurate analyses, 3) measurement errors may be correlated and 4) corrected correlations have different variances and covariances than uncorrected correlations. However, simply correcting each correlation in a matrix using traditional univariate corrections can lead to problems. Bock and Petersen (1975) noted that the resulting variance-covariance matrix associated with the correlation coefficients may not be positive definite or positive semidefinite. Preliminary simulation results showed that this possibility exists.

There are several techniques which could be used to correct correlation matrices, and ultimately their variances and cova

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and covariances, for attenuation. Applying these approaches will potentially yield different results. Contrasting them through exact work (derivations) and through simulation should provide useful information about the best way to correct correlation matrices. The use of a sample data set shows how such corrections can influence results in one case. Selecting the most appropriate and applicable procedures for meta-analytic situations is the focus of this work.

Research Questions

Several questions can be raised about multivariate attenuation corrections. The questions addressed in this study are listed below, with a brief description of each problem, a description of how each was investigated, and the anticipated results.

1. **What are the consequences of using a simple univariate correction for each of a set of correlations?** The univariate correction ignores the dependence in the data, so problems with this approach are expected. These problems may take the form of out-of-range corrected correlations (values greater than one), or correlation matrices and variance-covariance matrices for the correlations which are non-positive definite.

If the usual univariate correction-for-attenuation formulas are used, and the corrected correlations are substituted into the formula for the variance of a correlation coefficient, is the result acceptable? This first research question

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question was answered by simulating correlations and computing their variances and examining the results. It was expected that this correction would give significantly smaller variances than those found by examining the sampling distributions of disattenuated correlations.

2. **What would be the difference in variances and covariances based on the univariate correction (mentioned above), versus using a variance-covariance matrix derived from large-sample distribution theory for correlation coefficients?** If reliabilities are correlation coefficients, then corrected correlations are functions of correlations. As such, their large-sample distributions can be derived using results found in Olkin and Siotani (1976). The resulting variance-covariance matrix will take into account the covariances between the reliabilities and the sample correlations, and the covariances among the reliabilities. This method treats reliabilities as random variables rather than fixed, population quantities as is assumed by other methods mentioned below.

This method was compared to the variances and covariances associated with the univariate correction discussed in question #1, through both exact work and simulations. Exact work showed how the variances and covariances from these methods differed, and showed that the large-sample method based on Olkin and Siotani gave larger variances and covarian

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covariances than the univariate method because the large-sample correction accounts for variability in reliability coefficients.

3. **Several multivariate attenuation corrections exist for raw data, including many in the regression literature. How do these corrections compare to one another, and to the corrections mentioned above?** The multivariate corrections from Bock & Petersen (1975), Fuller and Hidiroglou (1978), and Gleser (1992) were compared to one another, and to the corrections already discussed. The assumptions necessary for the use of each method were examined, to see if the assumptions are met in multivariate syntheses. The articles by Fuller and Hidiroglou (1978) and Gleser (1992) give corrections used in regression. These formulas were examined to determine whether the corrections can be applied to multivariate synthesis, and whether they can be compared to corrections based on classical test theory, for example, the correction of Bock and Petersen (1975). This investigation determined whether any or all of these corrections are appropriate for multivariate syntheses, and whether the corrections give acceptable results.

4. **Which correction is most feasible and provides the best results?** The corrections were compared on several bases. First, the frequencies of out-of-range corrected correlations and non-positive definite variance-covariance matrices for correlations were noted. The percentage of out-of-range

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values was recorded for each method and each simulation situation. Second, the variances and covariances of corrected correlations to expected values were compared, and the degree of bias in each was assessed. Finally, the assumptions necessary for the use of each correction, and the possibility of meeting these assumptions in multivariate syntheses were discussed.

Exact work was used when possible to show how the corrections differ in theory. A simulation study showed how the corrections behaved in applications and in theory.

The results of this investigation suggest the best method of correcting multivariate data for attenuation. The best correction is the one with the best statistical properties that is feasible in terms of assumptions necessary for its use.

5. **How do these corrections affect results of multivariate syntheses?** A set of data was examined using the methods recommended by the results of this work. The applications of the corrections to this data set illustrated the different methods in a practical setting, and focused on the consequences of using the corrections described. This data was from Schmidt, Hunter and Outerbridge (1986) and consisted of correlations representing relationships among five variables, and their population and estimated reliabilities.

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Future directions for research on multivariate corrections were also addressed. Additional questions which need to be answered concern reliability distributions and correlated measurement errors. How are reliabilities distributed? Hypothetical, "assumed" reliability distributions have been used in the validity-generalization literature. Are these distributions accurate and appropriate? Should reliability in both the predictor and criterion be considered? Can correcting for both lead to further problems, especially if the reliabilities themselves are correlated? How can correlated measurement errors be estimated and what are appropriate values for such errors? These questions were not answered in this work, but seem critical for future research, thus, are discussed extensively in the final chapter of this dissertation.

Overview of the Dissertation

The dissertation contains four additional chapters. The second chapter addresses the review of the literature including the basis for corrections, their current application in multivariate syntheses, and a description of existing multivariate corrections. The third chapter details the methods used in this work. The fourth chapter summarizes the results, and the fifth chapter discusses recommendations and future research directions.

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Summary

This work investigated the statistical and practical effects of multivariate corrections for attenuation. Different methods for correcting for attenuation in multivariate situations were described, and the effects of using these different corrections and the problems arising from each were examined. Potential problems included non-positive definite covariance matrices, and out-of-range variances and correlations. Practical aspects of corrections were illustrated using a multivariate synthesis example, and the ramifications of the different corrections were discussed.

The results of this work include a justification and explanation of the most useful correction(s) for use in meta-analytic syntheses. This investigation showed that the univariate correction was a special case of each method. However, the method from Fuller and Hidiroglou (1978), and the method derived from large-sample theory yielded variances which most closely fit the sampling distributions in the simulation. Evidence is presented to show why these corrections are the best, and what problems exist with other, less useful corrections. Also, an indication of future research directions is presented.

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CHAPTER II

LITERATURE REVIEW

Synthesizing data on models of multiple interrelationships is complicated. Current meta-analysis methodology has examined the synthesis of correlation matrices between variables assumed to be measured without error (Becker, 1992). However, this assumption is not warranted, and can be problematic since measurement error can produce severely underestimated correlation coefficients. However, the best way to correct for measurement error in multivariate syntheses is unknown. Several statistical problems associated with making corrections for attenuation have arisen in such syntheses and are discussed in this chapter. The literature pertinent to this problem covers several topics, including measurement error, multivariate analysis, meta-analysis and validity generalization, and statistical methods needed for the data analysis. This chapter details literature relevant to addressing the problems outlined in the first chapter.

Measurement Error

Whenever correlation coefficients are used, issues of measurement error arise. Because of unreliability in both the predictor and criterion, observed correlations underrepresent the true (population) correlation. Formulas for correcting for unreliability date back to Spearman (1904). The basic formula

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formula for the correction for attenuation in the sample correlation $\underline{r}_{xy}$ is

$$\rho'_{xy}=\underline{r}_{xy}/(\rho_{xx}*\rho_{yy})^{1/2}, \qquad (2.1)$$

where ρ'_{xy} is the estimated population correlation between the fully reliable constructs x and y, $\underline{r}_{xy}$ is the sample correlation, and ρ_{xx} and ρ_{yy} are the known (population) reliabilities of the predictor and criterion measures respectively (Allen & Yen, 1979). The correlation ρ'_{xy} in (2.1) is the maximum sample correlation which could be obtained, if no measurement error were present.

Classical test theory forms the basis for this correction, based on the following assumptions:

1. $X = T + E$. An observed score is the sum of two parts, true score and error.

2. $E(X) = T$. The expected value of an observed score is the true score.

3. $\rho_{ET} = 0$. There is no correlation between error and true score for a population of examinees on one test.

4. $\rho_{E1E2} = 0$. The errors from two different tests are uncorrelated.

5. $\rho_{E1T2} = 0$. The error on one test is uncorrelated with the true score on another test.

6. If two tests have observed scores X and X' that satisfy Assumptions 1 through 5, and if for every population of examinees T=T' and $\sigma_E^2 = \sigma_{E'}^2$, the tests are called parallel tests.

7. If two tests have observed scores X_1 and X_2 that satisfy Assumptions 1 through 5, and if for every population of examinees, $T_1 = T_2 + c_{12}$ where c_{12} is constant, then the tests are called essentially tau-equivalent tests (Allen & Yen, 1979, p. 57).

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Given these assumptions, one can derive relationships among true, error, and observed score variances, which form the basis for one definition of reliability. Theoretically, reliability is the ratio of true score variance to total (observed) score variance. For any group of test takers, the following relationships can be derived from the assumptions above:

$$\rho_{XX} = \frac{\sigma^2_T}{\sigma^2_X} = \frac{\sigma^2_X - \sigma^2_E}{\sigma^2_X} = 1 - \frac{\sigma^2_E}{\sigma^2_X}, \qquad (2.2)$$

where ρ_{XX} is the reliability, σ^2_T is the variance of the true scores, σ^2_X is the variance of the observed scores, and σ^2_E is the variance of the errors.

Observed reliabilities can be correlations, between either scores from two administrations of the same test (test-retest reliability) or scores on two versions of a test (alternate forms). Also, reliability can be obtained through an internal consistency measure based on one administration of a test. Test-retest and alternate-forms reliabilities are likely to contain errors in measurement that are not included in the observed correlation between variables at any single time point (e.g., errors due to change over time, practice effects, fatigue, differences in forms, etc.). However, test-retest reliabilities are preferable to the others for correcting for attenuation, according to Lord and Novick (1968, p. 135). If the period before retesting is short, and fatigue factors are minimized, the other minimal errors in measurem

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measurement are likely to lead to high estimates of reliability, which will, in turn, lead to conservatively corrected correlations, according to Lord and Novick. Alternate-forms reliabilities include additional error due to differences between forms, so this type of reliability is less useful for disattenuating corrections than test-retest. Lord and Novick also claimed that internal-consistency estimates can seriously underrepresent reliabilities, especially when there is a lack of item homogeneity. Thus, the use of internal consistency reliability for attenuation corrections is discouraged. The focus of this work will be on test-retest reliabilities, and both the exact work and simulations will consider this type of reliability and its assumptions.

Multivariate Meta-analysis

Multivariate syntheses are one type of analysis where multivariate corrections for attenuation are potentially useful. Four recent multivariate syntheses, Harris and Rosenthal (1985), Schmidt, Hunter and Outerbridge (1986), Premack and Hunter (1988), and Becker and Schram (1993) illustrate how and where these corrections could be applied.

Harris and Rosenthal. Harris and Rosenthal (1985) were some of the first researchers to synthesize several relationships within one meta-analysis. They examined the literature on interpersonal expectancy effects using eight univariate meta-analyses. No attempt was made to investigate all of the paths of their model simultaneously. They did not model de

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model dependencies between paths, but did eliminate within path dependence by using the median correlation for studies which reported more than one correlation per path. No corrections for attenuation were used, nor were they mentioned in the text.

Schmidt, Hunter and Outerbridge. Schmidt et al. (1986) examined a path analysis of the impact of job experience on job knowledge, with additional paths for the effects of mental ability, work-sample performance and supervisory ratings of job performance. They had 4 studies of these 5 variables from military settings, and corrected for attenuation in all studies. Schmidt, Hunter and Outerbridge conducted a path analysis on this data, and found path coefficients along each path of their theoretical model. They averaged the disattenuated correlations across the 4 studies, then fit the path model. They did not conduct a test of overall model fit, nor did they consider the dependence in the data.

For 3 of the five variables, the reliabilities were not based on sample values. For work samples and supervisory ratings, the reliabilities for all 4 studies were set to .77 and .60, respectively. These values were determined from weighted averages of reliabilities from several studies, not including the ones used in this synthesis. The reliability for job experience was assumed to be 1.00, as the records indicated the number of months on the job.

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This study illustrates the main problem with obtaining reliability information. Often, estimates based on past research or other samples within a study are used. Schmidt et al., ignored the uncertainty in the reliability coefficients and the dependence between correlations and reliabilities. The reliabilities in their study were sample estimates, so their uncertainty should have been considered in the analysis, rather than considering them to be fixed.

Premack and Hunter. Premack and Hunter (1988) examined several univariate relationships and used them to create a multivariate (causal) model of employee decisions about unionization. In effect, they did a univariate meta-analysis on every path in the model, then combined the results and tested a causal model. However, not every study examined every pair of variables and some studies examined more than one pair. While they provided an overall test of model fit with their method, the dependence between relationships within studies was ignored. Premack and Hunter used univariate attenuation corrections for each individual correlation.

Becker and Schram. Another simple case to which this methodology might apply considers the data from Friedman (1991). In this example, data from several studies were collected to assess the relationship between math, verbal, and spatial abilities. An existing model hypothesized relationships among all three variables. Several studies were collected which contained correlations for all three

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relationships. Mathematical ability was used as an outcome with the other two variables (verbal and spatial ability) used as predictors.

Becker and Schram (1993) conducted both univariate and multivariate syntheses on a subset of this data. Their univariate analysis examined the three relationships separately (between each pair of variables) using traditional meta-analysis procedures, including conducting homogeneity tests for each path. The results from that analysis indicated whether each path was homogeneous, and if so, the magnitude of the average correlation. Although beneficial, these results did not directly test the model posited. There was no overall test of fit, and the interrelationships among the paths (and dependence in the data) were ignored.

The multivariate synthesis allowed the examination of the effects of each predictor on the outcome, and on each other simultaneously. Tests of significance and the relative importance of each predictor were also examined and prediction equations were formulated. This analysis modeled dependence in the data and gave a more complete assessment than the univariate analyses because partial relationships were considered.

All of these examples point out methodological problems in doing this type of synthesis. The dependencies in data are often ignored, or unmodeled. Other statistical problems also exist. Measurement errors have yet to be completely conside

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considered. Premack and Hunter (1988) and Schmidt, Hunter and Outerbridge (1986) corrected for attenuation on each path, but did not account for dependencies among the correlations or reliabilities. Harris and Rosenthal (1985) and Becker and Schram (1993) did not correct for measurement artifacts. All four studies attempt to address the problem in meta-analysis of synthesizing only main effects. However, much work needs to be done to solve the problem of the best way to do such a synthesis.

Validity Generalization and Corrections

Validity generalization (VG) is an approach to combining correlational study results which grew out of interest in the power of employment-selection measures to predict job success. VG has focused on correcting for so-called "artifactual variation" in studies, including measurement error. Schmidt and Hunter (1977) claimed that the variability among study results could be attributed solely to sampling error when measurement errors are eliminated. Their work has focused on the development of corrections for attenuation and range restriction in the synthesis of one outcome variable (bivariate relationships).

However, reliability information is often unavailable in published studies. Thus, Hunter and Schmidt (1990) (and others) have sampled from hypothetical distributions of reliabilities in making corrections. Distributions given by Pearlman, Schmidt, and Hunter (1980) are often used when

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reliability information is missing in studies. Others, including Raju, Burke, Normand, and Langlois (1991) have mentioned potential problems with these hypothetical reliability distributions. In particular, the accuracy of validity generalization procedures is affected by how closely the hypothetical distributions match the real population distributions (Paese & Switzer, 1988). Because the real distributions are not accessible, such a match is almost impossible to establish. Also, these hypothetical distributions were derived for the literature on personnel selection, which may not represent the distributions of reliabilities found in educational or social-science data.

Reinhardt and Mendoza (1989) also questioned the use of these hypothetical distributions. They claimed that the hypothetical distributions could be unrepresentative of the real data, and that there were no guidelines to assess the accuracy of the hypothetical distributions. As a result, they focused on using traditional VG procedures with "situational data" rather than with hypothetical data. They used reported reliabilities from samples in other studies when calculating unknown reliabilities. Their new procedures were fairly accurate when as much as 50% of the reliability data was missing, suggesting that the procedures did not require a great number of studies. A few quality studies were sufficient to produce accurate reliability estimates.

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Raju et al. (1991) approached the hypothetical reliability distribution problem from a different angle. They developed a procedure for correcting correlations that considered the sampling error arising from approximating the reliabilities. Raju et al. (1991) used averages of available reliabilities as ρ_{xx} and ρ_{yy} for making the correction in (2.1). The variance of the correlation coefficient was adjusted for the uncertainty that arises from making this substitution. When a reliability was reported, Raju et al. treated it as fixed in the derivation of the variance formulas, even though it might have been based on a sample. When the reliability was from an average, it was treated as variable. The resulting variance in the correlation coefficient was larger when a hypothetical distribution was used, because of the additional variability in the reliability. Raju et al. (1991) used simulation techniques to show that their method provided more accurate estimates of the population correlation than other procedures.

One other criticism of using hypothetical reliability distributions and simple averages to replace missing reliabilities is that, in both situations, reliability data are treated as missing at random. Hedges (1989) has argued that if low sample correlations are found in a study, the researchers may be more likely to report artifact corrections than researchers who found high correlations. Also, studies which focus on situations with greater economic and legal risk may be

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may be more likely to monitor and report reliability. Reliability information therefore seems unlikely to be missing at random.

Other Univariate Corrections

Hakstian, Schroeder, and Rogers (1988). Hakstian et al. (1988) considered the variance and covariance of univariate correlations corrected using test-retest reliabilities differently than those who used the traditional method (Equation 1). They assumed that one would have two measures of the two variables, X_1 and X_2 , and Y_1 and Y_2, and the sample correlation would be the average of the correlations between X_1 and Y_2 and X_2 and Y_1 . After this average was computed, the usual correction was used to estimate the corrected correlation.

In their study, Hakstian et al. derived, using the delta method, the variance of this corrected (separate) correlation, and the covariance between two corrected correlations estimating the same phenomenon. Using a simulation study, they found that the corrected correlations behaved fairly well, provided that the sample size was greater than 150. They concluded that correcting correlations seems to be a large-sample procedure, because three sample values with error are used to estimate the corrected value.

While the results of this study relate to the present work, the situation examined is problematic. Reliability information is seldom reported in meta-analytic studies, and finding

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finding data on two parallel measures of each variable would be even more unusual. Although the simulation used in the present study considers test-retest reliabilities, it will not consider Hakstian et al.'s formulation because of its impracticality.

Meta-analysis and corrections. More general synthesis techniques (meta-analyses) have also considered the use of corrections for attenuation and range restriction. Rosenthal (1984) recommended reporting both corrected and uncorrected results. He suggested that the majority of social-science researchers do not correct for measurement errors or report reliabilities, so uncorrected results are more typical. Hedges and Olkin (1985) gave the basic correction formulas for both mean difference effect sizes and for correlations, discussed the effect of making the correction on the variance of the correlation coefficient, and noted that their univariate methods apply to corrected or uncorrected correlations. However, Hedges and Olkin considered reliability values to be fixed and known, and therefore they did not take the variability of the reliabilities into account when adjusting the variances of corrected correlations. This assumption seems unwarranted, given that as noted above, reliabilities are often missing and estimated, and may involve much uncertainty.

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Measurement Problems in Univariate Syntheses

Winne and Belfry (1982) discussed several issues related to correcting for attenuation, including the reasons for making such corrections. The result of the correction for attenuation is an estimate of the true correlation between the variables of interest. This correlation represents a theoretical value, or, according to Winne and Belfry, a latent trait. Winne and Belfry urged caution in interpreting results from analyses which use corrected correlations, and they cited several measurement specialists who share their concern (Allen & Yen, 1979; Cronbach, 1971). The concerns stem from the claim that the resulting corrected correlation represents the true value between constructs measured without error. Factors such as sampling error of observed correlations and reliabilities and correlated measurement errors may result in poor estimates of this true correlation.

In the meta-analysis application resulting from the present work, the interest is in estimating a theoretical population value, so correcting appears appropriate. However, as important as the concerns expressed above seem, in practice, the true values are always estimated, and it is never clear how much error is affecting such estimates.

Another problem noted by Winne and Belfry (and others) was corrected correlations larger than unity. This problem was attributed to correlated measurement errors, the type of reliability, and the accuracy of the estimate of the

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reliability used (Winne & Belfry, 1982). Corrected correlations larger than unity were not acceptable, and adjusting for such potential problems in the multivariate case (e.g., by forcing the corrected correlations to be between -1 and 1) was necessary. When reliabilities are sampled from assumed distributions, the potential for out-of-range corrected correlations may be even greater.

Thomas (1989) derived, based on a classical test theory model, distributions of corrected correlations. Using his derivation, Thomas addressed the issue of out-of-range corrected correlations. He suggested a procedure which uses the distribution function (and its inverse) of a correlation coefficient so that the corrected correlation is forced to be in the interval -1 to 1. His derivations assumed that reliabilities were known and fixed, and he claimed that viewing reliabilities as random variables would complicate the picture, and would not be likely to yield practical increments of improvement better than those he derived. Thomas claimed that if the sample size is sufficiently large, the difference between estimates based on fixed versus random reliabilities should be negligible. He also stated that more work in this area was necessary, and the work here should answer some of the questions he raised. Also, this work examines the case in which reliabilities are viewed as random.

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Measurement Problems in a Multivariate Synthesis

Multivariate syntheses will have their own set of measurement and statistical problems. In univariate syntheses, covariances among variables are irrelevant. In a multivariate synthesis, covariances (i.e., between multiple study outcomes) also need to be corrected, or calculated with corrected correlations. Also, when more than one correlation arises from the same sample, and the reliabilities are sample-based, the potential for correlated measurement errors exists.

When the reliabilities and correlations of interest are calculated for the same sample, the observed reliability and the correlation are dependent. This dependence becomes more problematic when the situation is multivariate, because the reliabilities for different tests could also be interrelated when they are determined from the same sample. Accounting for covariances between (1) reliabilities for two different measures from the same sample, and (2) between a reliability estimate and the correlation it is used to correct, is a further problem that is considered in the present work.

Estimating correlated measurement errors within a set of data is another issue which needs to be addressed. The effects of such errors should be considered. Although not exactly an attenuation issue, this is a measurement-error issue with practical applications. Extensions of the work of this dissertation could consider such issues.

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Existing Multivariate Corrections

Multivariate corrections for attenuation have been studied in the regression literature (e.g., Gleser, 1992; Fuller & Hidiroglou, 1987). While these corrections are similar to ones that may apply to meta-analyses, they do not consider many of the practical problems of meta-analytic data. Also, they make assumptions about the nature of the data which are often violated in meta-analyses. For example, both techniques assume that the population reliabilities are known, and not based on sample data.

Fuller and Hidiroglou. Fuller and Hidiroglou (1978) derived regression estimators of slopes based on correcting the raw moment matrix for attenuation. Their derivations applied to situations where the error variances are not estimated from the same data used to estimate the reliabilities. They assumed that reliabilities for both the predictors and the criterion are known. They addressed cases of both correlated and uncorrelated errors.

Fuller and Hidiroglou's method corrected the moment matrix using a diagonal reliability matrix. This matrix used (1 - reliability) as the basis for the correction. The quantity (1 - reliability) is the ratio of error variance to the total variance in the predictor. By pre- and post-multiplying this reliability matrix by a diagonal matrix containing the standard deviations of the predictors, the variances were adjusted for the errors in measurement. This correct

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corrected moment matrix was then used in the traditional regression calculations for estimating the regression slopes.

The authors then examined the distributional properties of this corrected matrix, and of the corrected estimators. Their procedure guarantees that this corrected moment matrix is positive definite.

Gleser. Gleser (1992) examined measurement reliability in multivariate regression. He stated that if the goal is to assess the relationship among the true (latent) variables, then classical least squares estimation yields biased and inconsistent results. His errors in variables regression (EIVR) procedures provided alternative methods of estimation. Gleser's approach used prior information about both the reliabilities and the data to estimate a reliability matrix, which is then used in estimating the regression slopes. Unlike Fuller and Hidiroglou's approach, in Gleser's method his reliability matrix contained more than the reliabilities of the predictors. The reliability matrix also contained the correlations among the components of the reliability values (true and error variance). Gleser did not consider measurement error in the outcome variable or variables, and he assumed that the outcome and the measurement errors in the predictors were uncorrelated.

In Gleser's method the estimate of his reliability matrix, Λ, comes from previous information about the predictors, generally taken from other reliability studies.

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Gleser's preferred reliability study would consider all of the predictors to be used in the regression model. His reliability matrix, Λ, is then used in estimating the traditional regression model by multiplying the predictor matrix by the inverse of Λ in the estimate of the slope. If least squares estimation is used, this result is a generalization of the correction for attenuation used by psychometricians. Gleser uses the eigenvalues and eigenvectors of Λ to assess the influence of the measurement error on the accuracy of the estimates.

Bock and Petersen. The regression formulations described above differ somewhat from that of Bock and Petersen (1975). Bock and Petersen's multivariate correction for attenuation used maximum likelihood estimation to make certain that the resulting variance-covariance matrix is positive semidefinite. They based their formulation on classical-test-theory models, and on having a known measurement-error matrix. Their formulation has been applied in studies (see, e.g., Petersen, 1976); however, the effects of using their correction have not been studied.

Bock and Peterson's method is similar to Fuller and Hidiroglou's, since both rely on adjusting eigenvalues to guarantee positive definite matrices. Bock and Petersen's derivation was based on true- and error-component covariance matrices. A restricted maximum likelihood estimate of the variance-covariance matrix was the result. Bock and Petersen

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did not make any assumptions about reliability types or values. The roots of eigenvectors of the difference between the observed and error matrices (the solution to the two matrix eigen-problem) were used to ensure that the matrix is at least positive semidefinite. In the two-variable case where the measurement errors are uncorrelated, the result was the traditional Spearman correction.

These methods were all slightly different, and may be applicable to different situations in the synthesis of correlation matrices. However, it is not clear if the corrections used in regression analyses can be applied to synthesis situations. For example, corrections in regression situations were applied to raw data (not to the correlation coefficients). Second, some of the corrections did not correct for measurement error in the relationships between the predictors and the outcome.

Summary

None of the research mentioned previously has attempted to address the role of corrections in the synthesis of multivariate correlational data, and, therefore, take into account the problems mentioned previously. No meta-analytic studies address the issue of correlations among errors for different variables within the same study.

This study focuses on methods for correcting correlation matrices from individual studies and computing the associated variance-covariance matrix of the correlations for each study.

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Once each study's correlation matrix is corrected, and its resulting variance-covariance matrix is found, then a multivariate synthesis can be completed. The next chapter details the statistical notation and theory needed to analyze the issues involving multivariate corrections for attenuation. To understand the statistical problems with correcting correlation matrices, we must examine the distributions of vectors of corrected correlation coefficients.

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CHAPTER III

METHODOLOGY

Statistical Formulations

Notation

Let X_1, ..., X_p be random variables with the multivariate normal distribution, and let the number of studies which examine correlations among these p variables be denoted as k. There are $p^{*}=p(p-1)/2$ non-redundant correlations possible in any study. Let r_{ist} and ρ_{ist} be the sample and population correlations between X_s and X_t for the ith study, where s and t = 1 to p^{*}, and i = 1 to k.

Let ρ'_{ist} represent the corrected sample correlation defined by (2.1) and assume that each study contains only one measure of each construct or variable of interest. The sample reliability for a measure of variable s will be denoted r_{iss}. The number of people in study i will be denoted n_i. In matrix notation, let $\mathbf{r}_i$ represent the vector of observed correlations $(r_{i12}, r_{i13},..,r_{i1p},...,r_{i(p-1),p})$ and let $\boldsymbol{\rho}'_i$ represent the vector of corrected correlations, and $\boldsymbol{\rho}_i$ the vector of population correlations. Let $V(\mathbf{r}_i)$ be the variance-covariance matrix of the observed correlations, $V(\boldsymbol{\rho}'_i)$ the corrected variance-covariance matrix, and $V(\boldsymbol{\rho}_i)$ the population variance-covariance matrix for the correlations. The reliability matrix or any matrix containing corrections based on reliabilities will be represented using $\boldsymbol{\Lambda}_i$.

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Multivariate Analysis

Multivariate analysis generally involves multiple outcome (dependent) variables and several predictors. Much of the notation and analysis comes from matrix algebra, and several algebraic properties of matrices are important in the statistical analyses. For example, variance-covariance and correlation matrices are known to be positive definite. This means that they are invertible and their determinants are nonzero. Knowing that these matrices must be positive definite will help determine whether the corrections attempted in this work are giving appropriate results. The existing literature on multivariate regression corrections is concerned with correcting matrices containing raw data or slopes, rather than correlation matrices, the focus of this work. The resulting changes in relevant variance-covariance matrices from using these other methods (e.g., changes in slopes) are different from the traditional corrections for attenuation.

One way to assess the consequences of the correction for attenuation is to examine the determinant of the variance-covariance matrix for the corrected correlations and that of the corrected correlation matrix itself. The determinant is a function of the elements of a matrix, and the determinant shows whether a matrix is invertible (and positive definite). Estimates of variance-covariance and correlation matrices are invalid if they are not positive definite.

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Distributions of Corrected and Uncorrected Correlations

Variances. The large-sample asymptotic variance of a correlation coefficient for a sample of size $\underline{n}_i$ is given by

$$\mathrm{Var}(\underline{r}_{ist}) = (1 - \rho_{ist}^2)^2/\underline{n}_i \ , \qquad (3.1)$$

where $\underline{r}_{ist}$ is the sample correlation, ρ_{ist} is the population correlation, and $\underline{n}_i$ is the sample size. The variance of the corrected correlation (corrected for unreliability of both the predictor and the criterion) differs from this because it considers the covariances between the reliabilities and the variances of the reliabilities. The variance of the corrected $\underline{r}$ is given in Appendix A.

Bobko and Rieck (1980), among others, have investigated the distributions of functions of correlation coefficients. They found that correlations corrected using known reliabilities are more variable than uncorrected correlations. Their results show that simply substituting a corrected correlation into the formula in (3.1) to compute the variance of corrected correlations can give misleading results.

Covariances. No research was found which showed investigations of the behavior of corrected variance-covariance matrices of correlations. The variances and covariances of univariate corrected correlations can be derived using the delta method (Rao, 1973). Although these derivations take into account the covariation between correlations, they still do not consider whether the resulting varianc

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variance-covariance matrix of the correlations is positive definite.

The covariance between two correlation coefficients which do not share a common index (the most complicated case) is as follows (Olkin & Siotani, 1976):

$$\mathrm{Cov}\ (\underline{r}_{ist}\ ,\ \underline{r}_{iuv}\ ) = [0.5\ \rho_{ist}\ \rho_{iuv}\ (\rho^2_{isu} + \rho^2_{isv} + \rho^2_{itu} + \rho^2_{itv}\ ) + \rho_{isu}\ \rho_{itv} + \rho_{isv}\ \rho_{itu} - (\rho_{ist}\ \rho_{isu}\ \rho_{isv} + \rho_{its}\ \rho_{itu}\ \rho_{itv} + \rho_{ius}\ \rho_{iut}\ \rho_{iuv} + \rho_{ivs}\ \rho_{ivt}\ \rho_{ivu}\ )]/\underline{n}_i . \quad (3.2)$$

This equation simplifies when the pair of correlations share an index. Appendix B contains the covariances between correlations for different cases, including covariances between reliabilities, and between a reliability and a correlation, based on the formulas found in Olkin and Siotani (1976). Appendix A shows the variance of a corrected correlation and the covariance of a pair of corrected correlations which were derived using the delta method for the simplest case, a 3 x 3 correlation matrix. These corrections lead to new variances and covariances. The corrected variance-covariance matrix is obtained by pre- and post-multiplying the uncorrected variance-covariance matrix by the matrix of first derivatives (the Jacobian) of the functions of the corrected correlations used in (2.1), which consider both the sample correlation and the reliabilities as random variables. The result of this multiplication is a matrix which c

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Layout of Simulation

Data Generation

A simulation was conducted to examine the frequency of problems (such as overcorrection and non-positive determinants of variance-covariance matrices) occurring because of corrections. This simulation used multivariate normal data generated with uncorrelated measurement errors. In the simulation, population correlations and reliabilities were fixed, then data were generated for each distribution of true and error scores. Observed and corrected correlations were examined, as well as sample reliabilities and determinants of resulting corrected correlation matrices and variance-covariance matrices among the corrected correlations.

This simulation examined the simplest possible multivariate case, based on three population correlations (ρ_{12}, ρ_{13}, ρ_{23}) which arise from three variables (X_1, X_2, and X_3), and the resulting three sample correlations (r_{12}, r_{13}, and r_{23}). The corresponding sample reliabilities are r_{11}, r_{22}, and r_{33}.

Two different methods for simulating the results were used. One, based on classical test theory, begins with true score and error variances, and then computes reliabilities and true correlations based on these values. A simplifying assump

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assumption of letting the variance of the observed scores equal 1 was used. This method was also used by Raju et al. (1991). When applying this method to multivariate data, in the three variable case, two correlations were fixed, and the third correlation was based on the relationship between the first two correlations. The results indicated that this third correlation varied around a fixed value (as would be expected).

In order for the correlation matrix to be positive definite, there is a distinct relationship between the three related correlations. The interval of possible values for the third correlation ($\underline{r}_{23}$), given the first two ($\underline{r}_{12}$ and $\underline{r}_{13}$), is centered around the product of the first two correlations: $\underline{r}_{12} * \underline{r}_{13}$ +/- $\sqrt{((1-\underline{r}^2_{12})*(1-\underline{r}^2_{13}))}$ (Stanley & Wang, 1969).

The other method of simulation used the multivariate normal generator in IMSL (International Mathematical and Statistical Library). A desired variance-covariance matrix for the observed scores was derived from the known true and error variances, based on the population reliabilities, and under the assumption of unit variances for the observed variables. The variance-covariance matrix for the true scores is

$$\begin{bmatrix} \rho_{11} & \rho_{12}\sqrt{\rho_{11}}\sqrt{\rho_{22}} & \rho_{13}\sqrt{\rho_{11}}\sqrt{\rho_{33}} \\ \rho_{12}\sqrt{\rho_{11}}\sqrt{\rho_{22}} & \rho_{22} & \rho_{23}\sqrt{\rho_{22}}\sqrt{\rho_{33}} \\ \rho_{13}\sqrt{\rho_{11}}\sqrt{\rho_{33}} & \rho_{23}\sqrt{\rho_{22}}\sqrt{\rho_{33}} & \rho_{33} \end{bmatrix} .$$

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The variance-covariance matrix for the errors is

$$\begin{bmatrix} 1 - \rho_{11} & 0 & 0 \\ 0 & 1 - \rho_{22} & 0 \\ 0 & 0 & 1 - \rho_{33} \end{bmatrix}.$$

With this method, the matrices of the three population correlations and the three population reliabilities were specified ahead of time. The multivariate normal generator then provided data with the given variance-covariance structure, from the Cholesky factorization of these two matrices. Once the data are generated, the true and error scores are summed, to yield observed data with the desired properties.

Both methods gave similar results in basic simulations. The method using the multivariate normal generator was chosen for further use, because it allowed the specification of the third correlation. The third correlation for the data used in the simulation was always in the range of values specified above.

Different Univariate Corrections

Differences in simulation results are found when considering which reliability definition or correction formulation to use. Raju et al. (1991) showed the derivation of corrected variances for the univariate case, in which they defined the corrected correlation to be

$$r^{c}_{xy} = \frac{r_{xy}}{r_{xxt}\ r_{yyt}}, \qquad (3.3)$$

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where $\underline{r}_{xxt} = \sqrt{\underline{r}_{xx}}$ and $\underline{r}_{yyt} = \sqrt{\underline{r}_{yy}}$ (the square root of the usual population reliabilities). This formulation led to different results, because Raju et al. took derivatives of r^c_{xy} with respect to $\underline{r}_{xxt}$ instead of $\underline{r}_{xx}$. The variances they obtained differ from those in Appendix A. The formulation in (3.3) provides much easier derivatives, but it is unclear which formulation is really more practical or accurate. For a simulation, the correlation between true scores and observed scores is readily available, but in practice, the true scores are never known. The correlations are only estimated, as are reliabilities.

The distributions of these two reliabilities ($\underline{r}_{xxt}$ and $\underline{r}_{yyt}$) and the effects of these two different formulations were examined. As expected, the distributions of $\underline{r}_{xxt}$ and $\underline{r}_{yyt}$ were much more negatively skewed than the distributions of the $\underline{r}_{xx}$ and $\underline{r}_{yy}$. Preliminary simulations showed that the formulation from Raju et al. gave variances of corrected correlations that were far smaller than those expected based on the sampling distribution. Also, one can show that in Raju et al.'s formulation, $V(\underline{r}^c)$ would always provide smaller variances than those corrections shown in Appendix A ($V(\rho')$). Therefore, further use of this formulation was not warranted, and it was not included in the final simulation study.

<u>Simulation Parameters</u>

<u>Sample sizes.</u> The size of the sample will have an influence on the magnitudes of the variances and covariances

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of the correlation matrix. Because the derivations used in this study are based on large-sample theory, and because other simulation results have shown that small sample sizes present problems with covariance-matrix estimation (Becker & Fahrbach, 1994), the sample sizes chosen for the simulation are relatively large. The sizes chosen for this study are 50, 100, 250, and 500.

Correlations. The correlation triples chosen for this study are based on practical regression situations in which one of the variables is an outcome, and the other 2 are predictors. The correlation triple (.00, .00, .00) was used so that the simplest case was represented. The rest of the correlation triples represent various population outcomes that could underlie data in regression studies. Table 1 displays the combinations used, along with the R^2 value for each combination. The first two correlations in the triples represent the population correlations between each predictor and the outcome. The third correlation in the triples represents the intercorrelation between the two predictors. The triples show varying degrees of relationship with the outcome, from weak to strong, and varying degrees of intercorrelation. The table shows that the percent of variance explained, using the second and third variables to predict the first variable ranged from .00 to .77. Many of the R^2 values are moderate, as would be expected in educational situations. All triples are possible, given the constraints mentioned previously.

Table 1

Simulation Parameters

Sample Sizes: 50, 100, 250, 500

Correlation Triples:

(0, 0, 0)	R^2 = .00
(.4, .3, .1)	R^2 = .23
(.4, .3, .7)	R^2 = .16
(.6, .4, -.2)	R^2 = .64
(.6, .4, .2)	R^2 = .44
(.7, .6, .1)	R^2 = .77
(.7, .6, .8)	R^2 = .49

Reliability Triples:

(.7, .7, .7)

(.85, .85, .85)

(.9, .8, .7)

(1, 1, 1)

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Reliabilities. The reliability triples also were based on previous research. First, the triple (1.00, 1.00, 1.00) provides the case of measurement without error. The three other triples show varying degrees of unreliability, with the lowest triple (.70, .70, .70) representing moderate measurement error. A search through several nationally administered standardized tests showed that subtest reliabilities typically ranged from .85 to .95. Therefore, when considering standardized achievement type measures, (.70, .70, .70) is low. However, Schmidt, Hunter, Pearlman, & Shane (1979) provided hypothetical criterion reliabilities which were much lower. These seemed to have a roughly normal distribution centered on .60 and ranging from .30 to .90. Reliabilities this low may be representative of employment criterion reliabilities, when the outcomes are often supervisory ratings. In education, reliabilities typically do not appear that small. In fact, hypothetical reliabilities for predictors given in Schmidt et al. (1979) ranged from .50 to .90, with 90% of them equal or above .75. This range of reliabilities seems more consistent with the educational literature. Examining reliabilities from test manuals seems more reasonable than arbitrarily using hypothetical distributions found in employment literature. For example, Bock & Vandenberg (1968) used test manuals from the Differential Aptitude Test to give the error variances (reliabilities) used in their study which used a multivariate corre

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correction for attenuation. All sample sizes, reliability and correlation triples were completely crossed so that each possibility was considered. It is expected that some overcorrection (corrected correlations greater than 1) will occur when the (.70, .70, .70) reliability triple is paired with population correlations greater than .70.

The number of replications used was 2000. Given the parameters of the simulation, there are 4 X 7 X 4 = 112 cases to be considered. Four methods (Fuller and Hidiroglou, Bock and Petersen, Gleser, and univariate) of correcting correlations were considered, and their variance-covariance matrices were computed. The limitations of the variance-covariance matrices are discussed below. The sampling distribution for each case was also examined.

Basis for Comparing Methods of Corrections

Corrected correlations. From the multivariate corrections given by Fuller and Hidiroglou (1978) and Bock and Petersen (1975), it was possible to compute a corrected correlation matrix. Appendix C contains descriptions of the various methods for finding corrected correlations. The corrected variance-covariance matrix of the raw scores was computed, and then converted to a correlation matrix. Comparing to the univariate corrected correlations in the simulation was then straightforward. The average magnitudes of the differences between the population values and these corrected correlations showed the difference between methods.

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In Gleser's method, while an estimate was determined for the matrix lambda (of errors/reliabilities), it was not clear how the corrected variance-covariance among the raw scores was adjusted. However, corrected correlations were found. All of these correction methods were computed in the simulation. Because the simulation starts with raw data, all of the needed corrections were then obtained.

Variance-covariance matrices. None of the three multivariate corrections (Bock & Petersen, 1975; Fuller & Hidiroglou, 1978; Gleser, 1992) provided estimates of a variance-covariance matrix for the corrected correlations. However, the method from Fuller and Hidiroglou (1978) did allow for a derivation of a variance-covariance matrix for the corrected correlations. This computation was a variation of the large-sample theory variances and covariances found in Appendix A.

Forms of the other two methods (Bock & Petersen; Gleser) were not amenable to such a calculation. The Gleser correction was very similar to the univariate correction, and the variance-covariance matrix for this case was identical to the large-sample theory method, therefore no computation of variance for this correction was attempted.

The correction from Bock and Petersen did not lead to any possible correction to the variance-covariance matrix of the corrected correlations. The only possibility was to insert the corrected correlation from Bock and Petersen into equatio

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equations (3.1) and (3.2) to estimate the variance-covariance matrix of the corrected correlations. This procedure yielded results similar to the univariate method variance calculation.

The fourth and fifth methods to be compared were the use of the univariate corrected correlation in the given variance and covariance formulas (Equations 3.1 and 3.2), and their use in the large-sample theory formulas shown in Appendix A. Finally, treating reliabilities as constant was also examined. This method involved using Equation (3.1) and multiplying the resulting variance by the inverse of the product of the reliability values, i.e.,

$$V(\rho'_{xy}) = \frac{1}{\rho_{xx}\rho_{yy}} * V(\underline{r}_{xy}).$$

The covariances were calculated similarly. The comparison of these six ways of calculating the variance-covariance matrices of the corrected correlations is shown in the next chapter.

Comparisons Made

Exact Comparisons

The exact comparison of formulas for the corrections and their variance-covariance matrices for the different methods was difficult. The first examination of these methods determined how they compared for the simplest case: 2 variables, 2 reliabilities, and 1 correlation. All three methods were compared to one another and to the typical univariate correction (Equation 2.1). Bock and Petersen's

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(1975) correction claims to be identical to the univariate correction for this case. Claims are not made about the equivalence of the other two corrections (Gleser, 1992; Fuller and Hidiroglou, 1978).

Next, the three variable case was examined. Differences from the above situations were expected, and the results from the three methods were not comparable. The result of these exact comparisons showed whether any of the methods provided equivalent, larger, or smaller corrected correlations and variance-covariance matrices.

Simulation Comparisons

The first results of the simulation study display the percent of out-of-range corrected correlations, the percent of invalid (non-positive definite) correlation or variance-covariance matrices, and the corrected variance-covariance matrix for each method and case. These results are compared to one another and to the sampling distribution created by the simulation. The differences between the empirical sampling distributions and the observed values from each of the methods show how the methods differ and which method(s) give results closest to the empirical sampling distribution.

Methods for the Example

After the simulation results were completed, a comparison of methods was made using data from a previous synthesis. This comparison not only examined the corrected correlations, but also showed whether the conclusions of the data analyses changed,

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changed, depending on the correction used. The analysis to be reconsidered is from Becker and Cho (1994), though the original data are from Schmidt, Hunter and Outerbridge (1986).

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CHAPTER IV

RESULTS

Exact Comparisons among the Methods

Exact work was used to compare the methods (univariate, Gleser (1992), Bock and Petersen (1975), and Fuller and Hidiroglou (1978)). The four corrections were compared algebraically. First, the 2 variable case, with 2 variables, 2 reliabilities and 1 sample correlation was examined. Bock and Petersen's (1975) claim that for this simplest case; their correction was the same as the traditional univariate correction, was verified.

Gleser (1992)

The correction from Gleser (1992) also always gives the univariate correction in the cases which were considered here. The reliability matrix ($\mathbf{\Lambda}$) in Gleser is an adjustment to the usual sum-of-squares and cross products (SSCP) matrix. The SSCP matrix is n multiplied by the variance-covariance matrix of the raw scores if the sample mean is 0 and the sample standard deviation is 1. The simulation studied here operated under those assumptions. Gleser (1992) lets $\mathbf{\Lambda} = \mathbf{\Sigma}^{-1}_{\mathbf{obs}} * \mathbf{\Sigma}_{\mathbf{true}}$ represent the reliability matrix, where $\mathbf{\Sigma}^{-1}_{\mathbf{obs}}$ is the variance-covariance matrix of the observed predictors and $\mathbf{\Sigma}_{\mathbf{true}}$ is the variance-covariance matrix of the true scores for the predictors. Then, the adjustment in the regression case is

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$\boldsymbol{\beta} = \boldsymbol{\Lambda}^{-1} * (\mathbf{X'X})^{-1}\,\mathbf{X'Y}$ (Gleser, 1992). The adjustment occurs when multiplying the inverse of $\boldsymbol{\Lambda}$ by $(\mathbf{X'X})^{-1}$. Elementwise, for the three variable case, with two predictors (assuming a mean of 0 and a variance of 1 for all variables), the matrices look like:

$$\boldsymbol{\Sigma}_{\mathbf{obs}}^{-1} = \begin{bmatrix} \dfrac{1}{1 - r^2_{23}} & \dfrac{-r_{23}}{1 - r^2_{23}} \\ \dfrac{-r_{23}}{1 - r^2_{23}} & \dfrac{1}{1 - r^2_{23}} \end{bmatrix} \quad \text{and}$$

$$\boldsymbol{\Sigma}_{\mathbf{true}} = \begin{bmatrix} \rho_{22} & r_{23} \\ r_{23} & \rho_{33} \end{bmatrix}$$

where $(\mathbf{X'X})$ =

$$\begin{bmatrix} n & r_{23}\,n \\ r_{23}\,n & n \end{bmatrix} .$$

The adjustment is occurring only to the predictor variables. The variance of the true scores is equal to the reliability when the variance of the observed scores is assumed to equal 1. The corrected correlation is then found by dividing the off-diagonal element of $(\mathbf{X'X})_{\mathbf{new}}$ by the product of the square roots of its diagonal elements where

$$(\mathbf{X'X})_{\mathbf{new}} = (\,\boldsymbol{\Lambda}^{-1} * (\mathbf{X'X})^{-1})^{-1},$$

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where $\Lambda^{-1} = (\Sigma^{-1}_{obs} * \Sigma_{true})^{-1}$. The adjustment using Λ then gives corrected correlations identical to the usual univariate correlations.

Bock and Petersen (1975)

The approach of Bock and Petersen (1975) gives the usual univariate correction if the original correlation matrix is well defined (non-negative definite), but if the corrected correlations are greater than unity, the matrix is modified further so it becomes positive definite, and a different corrected correlation is produced. This method increases the observed correlations, but not in the same way as the usual correction for attenuation.

Bock and Petersen's method manipulates the moment matrices, $\mathbf{M_e}$ and $\mathbf{M_y}$, the mean error and the mean observed sum of squares and cross products matrices, respectively. These are the matrices found by dividing the sums of squares and cross products matrices for the error and observed scores by their respective degrees of freedom. Their method involves solving the two matrix eigenproblem

$(\mathbf{M_y} - \lambda_i \mathbf{M_e})\ x_i = 0$ (Bock and Petersen, 1975, p. 674).

Once this problem is solved, the estimate of the true variance-covariance matrix of the raw scores can be made using the following formulation. Let $\mathbf{X} = (x_i, \ldots x_p)$ be the matrix of eigenvectors, let $\Lambda^* = \text{diag}(\lambda_i, \ldots, \lambda_p)$ be the matrix of eigenvalues, and let $\mathbf{I_p}$ be the p x p identity matrix, then

$\Sigma_t = \mathbf{M_y} - \mathbf{M_e} = \mathbf{B}'\ (\Lambda^* - \mathbf{I_p})\ \mathbf{B}$ where $\mathbf{B} = \mathbf{X}^{-1}$.

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If any of the elements of the $\mathbf{\Lambda}$ matrix are less than one, they are replaced by 1.0 in the calculation. This constrains $\mathbf{\Sigma}_t$ to be at least positive semidefinite. Corrected correlations are then calculated using the elements of $\mathbf{\Sigma}_t$.

Fuller and Hidiroglou (1978)

Fuller and Hidiroglou's (1978) method is similar to Bock and Petersen's (1975) in that it also uses eigenvalues to adjust the corrected correlation matrix. However, it appears that the corrected correlation from Fuller and Hidiroglou will be slightly different from that of Bock and Petersen. Fuller and Hidiroglou's method considers the already-corrected correlation matrix and forces it to be positive definite. The Fuller and Hidiroglou method only gives corrections different from the univariate correction when the corrected correlation matrix would be non-positive definite.

The Fuller and Hidiroglou method is the same as the univariate correction in the 2 x 2 case. The following shows how it compares. The Fuller and Hidiroglou method starts with the regression equation

$$\boldsymbol{\beta} = \mathbf{H}^{-1} \, (n^{-1} \, \mathbf{X}' \, \mathbf{Y}),$$

where $\mathbf{H}^{-1} = (n^{-1} \, \mathbf{X}'\mathbf{X}) - \mathbf{D} \, \mathbf{\Lambda} \, \mathbf{D}$, and where $\underline{w}$ and $\underline{x}$ are two variables in the $\mathbf{X}$ matrix and where $\mathbf{D}$ is a diagonal matrix of standard deviations of $\underline{w}$ and $\underline{x}$,

$$\mathbf{D} = \begin{bmatrix} S_w & 0 \\ 0 & S_x \end{bmatrix} .$$

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Also, $\mathbf{\Lambda}$ is a diagonal matrix containing 1 - reliability values:

$$\mathbf{\Lambda} = \begin{bmatrix} 1 - r_{ww} & 0 \\ 0 & 1 - r_{xx} \end{bmatrix} .$$

Then, for the case where the mean of each variable is assumed equal to 0 and the standard deviation is 1.0, $\mathbf{H}$ simplifies to:

$$\begin{bmatrix} S^2_w & S_{xw} \\ S_{xw} & S^2_x \end{bmatrix} - \begin{bmatrix} S^2_w\ (1 - r_{ww}) & 0 \\ 0 & S^2_x\ (1 - r_{xx}) \end{bmatrix} .$$

This in turn yields the new variance-covariance matrix for the raw scores

$$\begin{bmatrix} S^2_w\ r_{ww} & S_{xw} \\ S_{xw} & S^2_x\ r_{xx} \end{bmatrix} .$$

Solving for the correlation between $\underline{x}$ and $\underline{w}$ gives the usual correction for attenuation

$$\frac{r_{xw}}{\sqrt{r_{xx}}\ \sqrt{r_{ww}}} .$$

If the matrix of corrected correlations is not positive definite, the adjustment comes from pre-multiplying the **DAD** product by the quantity $(f - n^{-1})$, where f is the smallest root (ei

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root (eigenvalue) in the two-matrix eigenproblem $| \mathbf{M} - f\mathbf{CGC} | = 0$. Here **C** is the matrix of standard deviations of the raw scores and **G** a diagonal matrix containing reliability values. This procedure is similar to Bock and Petersen's. However, Fuller and Hidiroglou's estimate is constrained to be positive definite, while Bock and Petersen's is positive semidefinite.

<u>The Three Variable Case</u>

The formulas for the corrections in the three variable case proved to be excessively complex for all methods (except the univariate case). As such, algebraic (exact) comparisons were impossible to make. In other words, comparisons of the correction formulas did not lead to any clear conclusions. However, several small-scale examples using spreadsheets and Minitab were computed, and the following results (before the simulation part of the study was conducted) were noted.

First, the Gleser method yields corrected correlations very similar to the traditional univariate correction, in all cases. The difference between the Gleser and univariate corrections is minimal, with the difference near zero when large sample sizes are used. Second, Bock and Petersen (1975) and Fuller and Hidiroglou (1978) have similar methods, but they give different corrections; when the usual corrected correlation matrix is not positive definite or contains corrected correlations greater than unity. The simulation results show which method gives larger corrected correlations, and whic

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and which variances are closest to the expected variances in the simulation.

The results of the computation of the variance of the corrected correlations, assuming that reliabilities are fixed and known values are also of interest. In this case, the variance of a corrected correlation is simply the variance of the uncorrected correlation divided by the product of the reliability values. This variance of a correlation (assuming that the reliability is fixed) should be larger than the variance of a univariate corrected correlation computed using (3.1) and (3.2), unless the reliability values are 1.0. If the reliabilities are 1.0, the two variances will be equal.

Summary of Exact Results

This examination of the exact results from each method shows that, for legitimate corrected correlation matrices (those that are positive definite), all 4 methods produce the same values for the corrected correlations. If, however, the corrected correlation matrix would be non-positive definite, the Fuller and Hidiroglou and Bock and Petersen methods further adjust the correction. The simulation results should reflect this exact work, and additionally will show the magnitudes of differences among the corrected correlations, and the corresponding differences in the variance-covariance matrices of the corrected correlations.

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Assumptions of Each Method

Before the simulations were conducted the assumptions of the different methods were examined. The methods often violated assumptions that would need to be made in meta-analytic studies. The following details were noted, and the three multivariate methods were compared and contrasted.

Fuller and Hidiroglou's (1978) derivations apply to situations where the error variances are not estimated from the same data used to estimate the correlations. They assume that reliabilities for both the predictors and the criterion are known. This assumption was violated in the simulation since the data used to estimate the reliabilities were also the same as those used to estimate the correlations, however, the dependence was accounted for in the calculation of the variances and covariances.

Gleser's (1992) approach uses prior information about reliability values to estimate a reliability matrix which is then used in estimating the regression slopes. This reliability matrix contains more than the reliabilities of the predictors. It also takes into account the correlations among the components of the measurement error and also the correlations among the components of the true vector of predictors.

Both Fuller and Hidiroglou's (1978) and Gleser's (1992) corrections occur in regression models, and both make assumptions that would not necessarily be reasonable in meta-ana

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meta-analysis. In most primary studies included in a meta-analysis, reliability information (if given at all) is from the sample, thus is not a population parameter. Also, both of these methods correct a raw data matrix rather than a correlation matrix, and both assume reliabilities are nonrandom.

Bock and Petersen (1975) considered the whole correlation matrix, rather than the individual elements. Their multivariate correction for attenuation uses restricted maximum likelihood estimation to make certain that the resulting variance-covariance matrix is positive definite. It is not clear what assumptions they make about the reliabilities. However, in the example given in their paper, they use a known value for estimating measurement error of human characteristics. In another study, Bock and Vandenberg (1968) have used known reliabilities in estimating error variances. It is unclear whether using reliabilities based on sample data would violate any assumptions for the Bock and Petersen procedure.

Although the methods discussed violate some assumptions of multivariate meta-analyses, all were used in the simulation part of this study. Putting all of the corrected correlations into the derived formulas for the variances and covariances of the correlation matrices produced variances and covariances which were compared to the sampling distribution. The effect of violating these assumptions (if any) was then determined.

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Simulation Results

Caveats

The simulation program was written in FORTRAN, and a version of this program is given in Appendix D. The data were summarized using a SAS program. Some assumptions were made during the data generation and analysis. First, as noted above, the Gleser (1992) correction was virtually identical to the traditional univariate correction. However, in the program, the univariate correction was calculated using the sample correlation and reliability statistics in Equation (2.1). The Gleser correction was estimated based on the raw data matrices, as shown in Chapter 3. Therefore, slightly different results were expected for these two methods.

Second, although there were five different calculations for the variance-covariance matrices, only 3 unique methods existed. These methods are: (1) the traditional variance-covariance corrections using Equations (3.1) and (3.2), which were used to get the univariate and the Bock and Petersen variance-covariance matrices, (2) the large-sample theory variance-covariance matrices (shown in Appendix A), which were adjusted for use with the Fuller and Hidiroglou method, and also with a traditional univariate correction, and (3) the fixed reliability calculation. These methods were then compared to the empirical variances computed from the sampling distribution of the corrected correlations across replications in the simulation.

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In each of the 112 simulation runs, counts of corrected correlations greater than unity, non-positive definite corrected correlation matrices and non-positive definite variance-covariance matrices were recorded. One problem encountered in this process was that the determinants of the variance-covariance matrices of the corrected correlations were small. Because the order of these matrices is proportional to $1/\underline{n}^3$, and because variances and covariances of correlations are small as well, the determinants of the variance-covariance matrices of the corrected correlations were often extremely tiny ($<10^{-10}$), especially for the cases when $\underline{n} = 500$. For this reason, after these determinants were calculated, they were multiplied by $\underline{n}^3$ before counts and comparisons among the methods were made. Without this convention, the majority of the cases would have shown all 2000 replications to have "improper" variance-covariance matrices.

Results from Count Data

Corrected correlations greater than unity. Here, a "case" refers to one of the seven different population correlation sets from Table 1. The first count examined was the percent of corrected correlations greater than unity, for each case and method of correcting the correlations. In general, the frequency of corrected correlations greater than unity, as shown in Table 2, was small (no more than three percent). In two of the cases ($\rho.$ = (.00, .00, .00) and

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ρ = (.40, .30, .10)), no corrected correlations were greater than unity for any sample size or reliability combination. In the cases where ρ = (.60, .40, .20) and ρ = (.60, .40, -.20), the number of corrected correlations greater than unity was tiny (less than .11%). In these cases, the only methods to produce corrected correlations greater than unity were the univariate and the Gleser corrections, when the sample size was 50, and the reliability triple was (.70, .70, .70). The overcorrected correlations occurred for the first correlation (and largest) in the triple (.60). The Bock and Petersen (1975) and Fuller and Hidiroglou (1978) corrections never gave corrected correlations greater than unity in any case.

As the population correlations increased, the percent of improper corrected correlations increased. In the case where ρ = (.40, .30, .70) and $\underline{n}$ = 50, for the reliability triple (.70, .70, .70) both the univariate and Gleser corrections had 0.75% (15 out of 2000) invalid corrected correlations. In the same case, but where the reliability triple (.90, .80, .70) was used, the percents were 0.20% and 0.15% for the univariate and Gleser methods, respectively. All of the invalid corrected correlations occurred when the largest (third) population correlation (ρ = .70) was corrected. In fact, in all cases, no sample estimate of a population correlation less than .60 gave a corrected correlation greater than unity.

Table 2

Percent of Co

Case 1. ρ_{22} =

Pop. Corr.	0.0
ρ_{11} = .7 ρ_{22} =	
Univar	0.0
Bock	0.0
Fuller	0.0
Gleser	0.0
ρ_{11} = .85 ρ_{22} =	
Univar	0.0
Bock	0.0
Fuller	0.0
Gleser	0.0
ρ_{11} = .9 ρ_{22} =	
Univar	0.00
Bock	0.00
Fuller	0.00
Gleser	0.00
ρ_{11} = 1.00 ρ_{22}	
Univar	0.00
Bock	0.00
Fuller	0.00
Gleser	0.00

Table 2

Percent of Corrected Correlations Greater than Unity

Case 1. ρ_{12} = .0 ρ_{13} = .0 ρ_{23} = .0

		n=50			n=100			n=250			n=500	
Pop. Corr.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2 (Cont'd)

Case 2. ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .1

	n=50			n=100			n=250			n=500		
Pop. Corr.	0.40	0.30	0.10	0.40	0.30	0.10	0.40	0.30	0.10	0.40	0.30	0.10
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2 (Cont'd)

Case 3. ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .7

	n=50			n=100			n=250			n=500		
Pop. Corr.	0.40	0.30	0.70	0.40	0.30	0.70	0.40	0.30	0.70	0.40	0.30	0.70
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7												
Univar	0.00	0.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2 (Cont'd)

Case 4. ρ_{12} = .6 ρ_{13} = .4 ρ_{23} = .2

	n=50			n=100			n=250			n=500		
Pop. Corr.	0.60	0.40	0.20	0.60	0.40	0.20	0.60	0.40	0.20	0.60	0.40	0.20
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7												
Univar	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2 (Cont'd)

Case 5. ρ_{12} = .6 ρ_{13} = .4 ρ_{23} =-.2

	n=50			n=100			n=250			n=500		
Pop. Corr.	0.60	0.40	-0.20	0.60	0.40	-0.20	0.60	0.40	-0.20	0.60	0.40	-0.20
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7												
Univar	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2 (Cont'd)

Case 6. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .1

	n=50			n=100			n=250			n=500		
Pop. Corr.	0.70	0.60	0.10	0.70	0.60	0.10	0.70	0.60	0.10	0.70	0.60	0.10
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7												
Univar	0.90	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.75	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2 (Cont'd)

Case 7. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .8

	n=50			n=100			n=250			n=500		
Pop. Corr.	0.70	0.60	0.80	0.70	0.60	0.80	0.70	0.60	0.80	0.70	0.60	0.80
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7												
Univar	1.15	0.15	3.00	0.00	0.00	0.40	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	1.15	0.30	2.95	0.00	0.00	0.40	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univar	0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univar	0.00	0.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univar	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

In the case ρ = (.70, .60, .10), the results were similar to the case mentioned previously. For n = 50, 0.90% of the corrected correlations from the univariate and 0.75% for the Gleser methods gave improper corrected correlations, for the first correlation in the triple (.70). The second correlation in the triple (.60) yielded .15% corrected correlations greater than unity for both the univariate and Gleser cases with reliabilities equal to .70 and sample size of 50. No other reliability and sample size combinations produced invalid corrections for this case.

The case which showed the highest percent of invalid corrected correlations was ρ = (.70, .60, .80), as expected. The univariate method produced 1.15% improper corrected correlations for the first element in the triple (.70), 0.15% for the second element (.60), and 3.00% for the third element (.80) for n = 50 and the reliability triple (.70, .70, .70). These numbers were 0.95%, 0.30%, and 2.95% respectively for the Gleser corrected correlations for the same combination. The third correlation in the triple (.80) also yielded non-zero percents for the cases where reliability triples were (.85, .85, .85) and (.90, .80, .70) with n = 50 reliabilities (.70, .70, .70) with n = 100. For the univariate correction these percents ranged from 0.05% to 1.45% and for the Gleser correction, from 0.05% to 1.60%.

Overall, in no case where the sample size was 250 or 500, or the reliabilities were 1.0, did any invalid corrected correlations occur. These results show that where univariate corrections are applied, for moderate to large correlations with small sample sizes and somewhat lower reliabilities, the chance of corrected correlations greater than one is non-zero. Because the Bock and Petersen (1975) and Fuller and Hidiroglou (1978) corrections adjust for these problems, their use may be warranted when such problems are anticipated.

Determinants of the corrected correlation matrices. The vast majority of the matrices of the corrected correlations were positive definite. Table 3 displays, for each method, the percentages of the corrected correlation matrices that were less than or equal to 0. The cases, ρ =(.40, .30, .10) and ρ = (.00, .00, .00), did not produce any invalid corrected correlation matrices and the case ρ = (.60, .40, .20) produced only a few invalid matrices, as shown in the table.

For the case ρ = (.60, .40, -.20) with reliability triple (.70, .70, .70) the univariate, Bock and Petersen (1975), and Gleser (1992) methods produced 7.2%, 2.15%, and 7.35% improper corrected correlation matrices, respectively. These percentages declined to 1.65%, 0.05%, and 1.55% when, for the same reliability triple, n = 100. With the (.85, .85, .85) reliability triple and n = 50, the univariate and Gleser methods gave 0.15% invalid matrices, while the Bock and Petersen method gave 0.05%. Finally, for the reliability triple (.90, .80, .70) and n = 50, the percentages for the univariate, Bock and Petersen, and Gleser methods were 0.55%, 0.25%, and 0.60%, respectively.

Table 3

Percent of Determinants of Corrected Correlation Matrices Less than or Equal to Zero (2000 replications)

Case 1. ρ_{12} = .0 ρ_{13} = .0 ρ_{23} = .0

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univar	0.00	0.00	0.00	0.00
Bock	0.05	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00

Table 3 (Cont'd)

Case 2. ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .1

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00

Table 3 (Cont'd)

<u>Case 3.</u> ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .7

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univar	1.60	0.00	0.00	0.00
Bock	0.80	0.05	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	1.55	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univar	0.30	0.00	0.00	0.00
Bock	0.30	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.30	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00

Table 3 (Cont'd)

Case 4. ρ_{12} = .6 ρ_{13} = .4 ρ_{23} = .2

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univar	0.40	0.00	0.00	0.00
Bock	0.50	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.45	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univar	0.00	0.00	0.00	0.00
Bock	0.05	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00

Table 3 (Cont'd)

Case 5. ρ_{12} = .6 ρ_{13} = .4 ρ_{23} =-.2

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univar	7.20	1.65	0.00	0.00
Bock	2.15	0.05	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	7.35	1.55	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univar	0.15	0.00	0.00	0.00
Bock	0.05	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.15	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univar	0.55	0.00	0.00	0.00
Bock	0.25	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.60	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00

Table 3 (Cont'd)

Case 6. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .1

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univar	14.65	3.85	0.20	0.00
Bock	5.70	0.45	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	14.85	3.80	0.20	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univar	1.75	0.00	0.00	0.00
Bock	0.45	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	1.80	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univar	1.95	0.10	0.00	0.00
Bock	0.85	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	2.25	0.05	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00

Table 3 (Cont'd)

Case 7. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .8

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univar	8.20	1.00	0.00	0.00
Bock	4.60	0.25	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	8.25	1.10	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univar	0.15	0.00	0.00	0.00
Bock	0.10	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.10	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univar	2.40	0.05	0.00	0.00
Bock	0.95	0.05	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	2.65	0.05	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univar	0.00	0.00	0.00	0.00
Bock	0.00	0.00	0.00	0.00
Fuller	0.00	0.00	0.00	0.00
Gleser	0.00	0.00	0.00	0.00

For the case ρ = (.40, .30, .70) a smaller number of invalid corrected correlation matrices were found. Again, the reliability triple (.70, .70, .70) in combination with $\underline{n}$ = 50 produced the most problems, with 1.60%, 0.80%, and 1.55% invalid matrices for the univariate, Bock and Petersen, and Gleser methods respectively. The Bock and Petersen method also produced one (0.05%) invalid matrix for the same reliability combination with a sample size $\underline{n}$ = 100. For the reliability triple (.90, .80, .70) with n = 50, each of the three methods mentioned above produce 0.30% improper corrected correlation matrices.

The case with the greatest number of invalid corrected correlation matrices was ρ = (.70, .60, .10). Nearly 15% of the matrices were invalid for the reliability triple (.70, .70, .70) and n = 50 case for the univariate and Gleser methods. The Bock and Petersen method in this same combination produced 5.70% invalid matrices. Within the same reliability triple, but with n = 100, the univariate, Bock and Petersen and Gleser method produced 3.85%, 0.45%, and 3.80% invalid corrected correlation matrices respectively. This particular correlation triple was the only one to produce invalid results when the sample size was 250. With this sample size and the reliability triple (.70, .70, .70), the univariate and Gleser methods yielded 0.20% invalid matrices. When the (.85, .85, .85) reliability triple was used, somewhat fewer problems were found. With this triple and n = 50, the univariate, Bock and Petersen, and Gleser methods produced invalid matrices 1.75%, 0.45%, and 1.80% of the time, respectively. When the reliability values were changed to (.90, .80, .70), the three methods gave 1.95%, 0.85%, and 2.25% invalid matrices for n = 50. With this same reliability triple, but with n = 100, the univariate and Gleser methods gave 0.10% and 0.05% improper matrices, respectively.

In the last case, ρ =(.70, .60, .80), with reliability triple (.70, .70, .70) and n = 50, the percentages of improper corrected correlation matrices were 8.20%, 4.60% and 8.25% for the univariate, Bock and Petersen, and Gleser methods, respectively. With n = 100 and the same reliability values, these percentages changed to 1.00%, 0.25%, and 1.10%, respectively. This case also produced a small number of invalid matrices for the (.85, .85, .85) reliability triple with n = 50. These numbers were 0.15% (univariate), 0.10% (Bock and Petersen), and 0.10% (Gleser). The reliability triple (.90, .80, .70) yielded 2.40% invalid matrices for the univariate method, 0.95% for the Bock and Petersen method, and 2.65% for the Gleser method when the sample size was 50. With the same reliability triple and n = 100, all three of the above methods gave 0.05% improper matrices, or one matrix out of 2000.

These results indicate that the Fuller and Hidiroglou method does prevent improper corrected correlation matrices. None of the combinations, when the Fuller and Hidiroglou method was used, produced improper results. Problems with the determinants of the corrected correlation matrices appeared to be related in part to the presence of corrected correlations larger than unity. However, invalid matrices occur more frequently than correlations larger than one. This result indicates that other problems result from these corrections, which would imply that the Stanley and Wang (1969) inequality is being violated in situations other than when correlations greater than unity occur. In no case did the original sample correlation matrix have a determinant less than 0. Therefore, the problems occurred after the correction had been made.

Determinants of variance-covariance matrices of the corrected correlations. Also recorded were the percentages of invalid variance-covariance matrices of the corrected correlations for each method. Table 4 displays these results. The case ρ =(.40, .30, .10) did not produce any invalid matrices, while the case ρ= (.00, .00, .00) produced 1 problematic matrix, for the Bock and Petersen correction with reliability triple (.70, .70, .70) and n = 50. As with the results for the corrected correlations greater than unity and the number of improper corrected correlation matrices, the majority of the combinations that caused problems had small sample sizes and reliability values.

Table 4

Percent of Determinants of Variance/Covariance Matrices
Less than or Equal to Zero (2000 replications)

Case 1. ρ_{12} = .0 ρ_{13} = .0 ρ_{23} = .0

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00

Table 4 (Cont'd)

<u>Case 2.</u> $\rho_{12} = .4$ $\rho_{13} = .3$ $\rho_{23} = .1$

	n=50	n=100	n=250	n=500
$\rho_{11} = .7$ $\rho_{22} = .7$ $\rho_{33} = .7$				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
$\rho_{11} = .85$ $\rho_{22} = .85$ $\rho_{33} = .85$				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
$\rho_{11} = .9$ $\rho_{22} = .8$ $\rho_{33} = .7$				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
$\rho_{11} = 1.00$ $\rho_{22} = 1.00$ $\rho_{33} = 1.00$				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00

Table 4 (Cont'd)

Case 3. ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .7

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univariate	0.95	0.00	0.00	0.00
Bock & Petersen	1.00	0.05	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univariate	0.20	0.00	0.00	0.00
Bock & Petersen	0.30	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00

Table 4 (Cont'd)

<u>Case 4.</u> ρ_{12} = .6 ρ_{13} = .4 ρ_{23} = .2

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univariate	0.10	0.00	0.00	0.00
Bock & Petersen	0.55	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.05	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00

Table 4 (Cont'd)

Case 5. ρ_{12} = .6 ρ_{13} = .4 ρ_{23} = -.2

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univariate	0.25	0.05	0.00	0.00
Bock & Petersen	3.25	0.15	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.10	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.35	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00

Table 4 (Cont'd)

Case 6. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .1

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univariate	2.40	0.05	0.05	0.00
Bock & Petersen	8.05	0.90	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univariate	0.15	0.00	0.00	0.00
Bock & Petersen	0.65	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univariate	0.05	0.00	0.00	0.00
Bock & Petersen	0.95	0.05	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00

Table 4 (Cont'd)

Case 7. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .8

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univariate	7.00	1.10	0.00	0.00
Bock & Petersen	5.25	0.25	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univariate	0.10	0.00	0.00	0.00
Bock & Petersen	0.20	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univariate	2.55	0.05	0.00	0.00
Bock & Petersen	1.55	0.05	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univariate	0.00	0.00	0.00	0.00
Bock & Petersen	0.00	0.00	0.00	0.00
Fuller & Hidiroglou	0.00	0.00	0.00	0.00
Large Sample	0.00	0.00	0.00	0.00
Fixed Reliability	0.00	0.00	0.00	0.00

For the case where ρ = (.60, .40, .20) the Bock and Petersen method produced 0.55% and 0.05% invalid matrices when n = 50 and the reliability triples (.70, .70, .70) and (.85, .85, .85) were used, respectively. For the first reliability triple, the univariate correction yielded 0.10% invalid variance-covariance matrices. The rest of the combinations for this case did not provide any problematic matrices.

When the ρ = (.60, .40, -.20) case was examined, again the Bock and Petersen and univariate methods were the only ones to produce invalid matrices. For the reliability triple (.70, .70, .70) and n = 50, the univariate and Bock and Petersen methods produced 0.25% and 3.25% improper matrices respectively. These percentages were reduced when n = 100 and the percentages were also non-zero when other reliability triples were used.

For the case where ρ = (.40, .30, .70) with reliability triple (.70, .70, .70) the only combinations with notable results were the univariate method, which gave 0.95% invalid variance-covariance matrices, and the Bock and Petersen method which showed 1.00% invalid matrices for n = 50.

When ρ = (.70, .60, .10) the univariate and Bock and Petersen methods again produced invalid variance-covariance matrices, particularly in the case where the reliability triple (.70, .70, .70) and the sample size n = 50 was used. In that case, the univariate method produced 2.40% improper matrices, while the Bock and Petersen gave 8.05%. Other reliability and sample size combinations produced non-zero percentages smaller than this case, as shown in the table.

Finally, the last case, ρ = (.70, .60, .80) produced improper matrices for the reliability triples (.70, .70, .70), (.85, .85, .85) and (.90, .80, .70) for n = 50. The rates for these three cases for the univariate method were 7.00%, 0.10% and 2.55% respectively. For the Bock and Petersen method, these rates were 5.25%, 0.20%, and 1.55%, respectively. When n =100, the reliability triples (.70, .70, .70) and (.90, .80, .70) also yielded non-zero percents of invalid matrices. For the former triple, the rates were 1.10% for the univariate method, and 0.25% for the Bock and Petersen method, while for the latter triple, the rates for these methods were both 0.05%.

The size of the determinants of the variance-covariance matrices. Besides counting the variance-covariance matrices of the corrected correlations that were non-positive definite, the size of the determinant of each matrix was also examined. Table 5 displays the mean determinants for each case and combination, for the 2000 replications. The means displayed in the tables are actually the mean determinants multiplied by n^3. As shown in the tables, the univariate and Bock and Petersen methods produced the smallest determinants. These were considerably smaller for several of the cases. For all of the cases, when the reliability triple (1.00, 1.00, 1.00) was used, the determinants were virtually identical for each method. These results fit with the number of improper variance-covariance matrices found above. The methods and combinations which yielded the smallest determinants also produced the highest numbers of invalid variance-covariance matrices.

Table 5

Mean Determinants of Variance-Covariance Matrices of the Corrected Correlations (2000 replications)

Case 1. ρ_{12} = .0 ρ_{13} = .0 ρ_{23} = .0

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univar	0.6909	0.8336	0.9280	0.9629
Bock	0.5827	0.7541	0.8911	0.9428
Fuller	7.4797	8.0457	8.2551	8.3875
Large Sample	7.4797	8.0457	8.2551	8.3875
Fixed Rel.	7.7899	8.2097	8.3239	8.4231
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univar	0.7777	0.8829	0.9512	0.9757
Bock	0.6365	0.7885	0.9072	0.9536
Fuller	2.2706	2.4440	2.5716	2.6130
Large Sample	2.2706	2.4440	2.5716	2.6130
Fixed Rel.	2.3159	2.4678	2.5816	2.6180
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univar	0.7462	0.8649	0.9438	0.9730
Bock	0.4650	0.6388	0.8123	0.8980
Fuller	3.4131	3.6518	3.8194	3.8720
Large Sample	3.4131	3.6518	3.8194	3.8720
Fixed Rel.	3.5079	3.7015	3.8401	3.8819
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univar	0.8352	0.9131	0.9635	0.9824
Bock	0.6821	0.8164	0.9218	0.9593
Fuller	0.8357	0.9137	0.9641	0.9830
Large Sample	0.8357	0.9137	0.9641	0.9830
Fixed Rel.	0.8358	0.9137	0.9641	0.9830

Table 5 (Cont'd)

Case 2. $\rho_{12} = .4$ $\rho_{13} = .3$ $\rho_{23} = .1$

	n=50	n=100	n=250	n=500
$\rho_{11} = .7$ $\rho_{22} = .7$ $\rho_{33} = .7$				
Univar	0.3587	0.3956	0.4253	0.4368
Bock	0.2843	0.3437	0.3920	0.4188
Fuller	5.2764	5.1876	5.2849	5.3285
Large Sample	5.2764	5.1876	5.2849	5.3285
Fixed Rel.	5.8371	5.7004	5.7761	5.8119
$\rho_{11} = .85$ $\rho_{22} = .85$ $\rho_{33} = .85$				
Univar	0.3843	0.4156	0.4336	0.4389
Bock	0.2961	0.3597	0.4080	0.4198
Fuller	1.3273	1.3654	1.3877	1.3969
Large Sample	1.3273	1.3654	1.3877	1.3969
Fixed Rel.	1.4243	1.4588	1.4793	1.4883
$\rho_{11} = .9$ $\rho_{22} = .8$ $\rho_{33} = .7$				
Univar	0.3853	0.4128	0.4334	0.4386
Bock	0.1509	0.1518	0.1473	0.1461
Fuller	2.1002	2.0806	2.1233	2.1252
Large Sample	2.1002	2.0806	2.1233	2.1252
Fixed Rel.	2.2710	2.2371	2.2762	2.2769
$\rho_{11} = 1.00$ $\rho_{22} = 1.00$ $\rho_{33} = 1.00$				
Univar	0.4054	0.4246	0.4373	0.4380
Bock	0.3074	0.3636	0.4110	0.4241
Fuller	0.4057	0.4249	0.4376	0.4383
Large Sample	0.4057	0.4249	0.4376	0.4383
Fixed Rel.	0.4058	0.4249	0.4376	0.4383

Table 5 (Cont'd)

Case 3. ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .7

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univar	0.0843	0.0816	0.0776	0.0770
Bock	0.0633	0.0716	0.0741	0.0750
Fuller	2.5411	2.4373	2.2704	2.2407
Large Sample	2.5447	2.4373	2.2704	2.2407
Fixed Rel.	3.1538	3.0535	2.8715	2.8425
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univar	0.0833	0.0797	0.0786	0.0770
Bock	0.0655	0.0694	0.0778	0.0761
Fuller	0.4492	0.4158	0.4044	0.3964
Large Sample	0.4492	0.4158	0.4044	0.3964
Fixed Rel.	0.5401	0.5038	0.4918	0.4833
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univar	0.0907	0.0832	0.0791	0.0773
Bock	0.0261	0.0186	0.0142	0.0128
Fuller	0.9488	0.8754	0.8290	0.8123
Large Sample	0.9489	0.8754	0.8290	0.8123
Fixed Rel.	1.1539	1.0799	1.0313	1.0139
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univar	0.0828	0.0807	0.0771	0.0771
Bock	0.0666	0.0734	0.0745	0.0761
Fuller	0.0829	0.0808	0.0772	0.0772
Large Sample	0.0829	0.0808	0.0772	0.0772
Fixed Rel.	0.0829	0.0808	0.0772	0.0772

Table 5 (Cont'd)

<u>Case 4.</u> $\rho_{12} = .6$ $\rho_{13} = .4$ $\rho_{23} = .2$

	n=50	n=100	n=250	n=500
$\rho_{11} = .7$ $\rho_{22} = .7$ $\rho_{33} = .7$				
Univar	0.1489	0.1566	0.1521	0.1529
Bock	0.1171	0.1358	0.1447	0.1492
Fuller	3.4035	3.2531	3.1028	3.0834
Large Sample	3.4056	3.2531	3.1028	3.0834
Fixed Rel.	4.0520	3.8803	3.7250	3.7056
$\rho_{11} = .85$ $\rho_{22} = .85$ $\rho_{33} = .85$				
Univar	0.1555	0.1531	0.1542	0.1538
Bock	0.1222	0.1382	0.1457	0.1501
Fuller	0.6819	0.6528	0.6449	0.6413
Large Sample	0.6819	0.6528	0.6449	0.6413
Fixed Rel.	0.7831	0.7529	0.7451	0.7415
$\rho_{11} = .9$ $\rho_{22} = .8$ $\rho_{33} = .7$				
Univar	0.1464	0.1535	0.1500	0.1529
Bock	0.0445	0.0396	0.0359	0.0347
Fuller	1.0419	1.0434	0.9910	0.9960
Large Sample	1.0419	1.0434	0.9910	0.9960
Fixed Rel.	1.2147	1.2159	1.1602	1.1652
$\rho_{11} = 1.00$ $\rho_{22} = 1.00$ $\rho_{33} = 1.00$				
Univar	0.1572	0.1522	0.1551	0.1550
Bock	0.1214	0.1352	0.1471	0.1515
Fuller	0.1573	0.1523	0.1553	0.1551
Large Sample	0.1573	0.1523	0.1553	0.1551
Fixed Rel.	0.1573	0.1524	0.1553	0.1552

Table 5 (Cont'd)

<u>Case 5.</u> ρ_{12} = .6 ρ_{13} = .4 ρ_{23} = -.2

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univar	0.0652	0.0614	0.0572	0.0570
Bock	0.0451	0.0474	0.0520	0.0528
Fuller	2.5025	2.3815	2.3277	2.3139
Large Sample	2.5268	2.3840	2.3277	2.3139
Fixed Rel.	3.2051	3.0605	3.0124	2.9989
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univar	0.0541	0.0553	0.0547	0.0553
Bock	0.0433	0.0495	0.0526	0.0532
Fuller	0.3682	0.3680	0.3576	0.3620
Large Sample	0.3682	0.3680	0.3576	0.3620
Fixed Rel.	0.4685	0.4699	0.4583	0.4641
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univar	0.0559	0.0533	0.0550	0.0550
Bock	0.0158	0.0157	0.0157	0.0158
Fuller	0.6635	0.6238	0.6200	0.6127
Large Sample	0.6637	0.6238	0.6200	0.6127
Fixed Rel.	0.8487	0.8075	0.8032	0.7950
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univar	0.0532	0.0546	0.0547	0.0549
Bock	0.0455	0.0491	0.0525	0.0535
Fuller	0.0532	0.0547	0.0547	0.0550
Large Sample	0.0532	0.0547	0.0547	0.0550
Fixed Rel.	0.0532	0.0547	0.0548	0.0550

Table 5 (Cont'd)

Case 6. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .1

	n=50	n=100	n=250	n=500
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7				
Univar	0.0250	0.0207	0.0174	0.0165
Bock	0.0153	0.0156	0.0152	0.0150
Fuller	1.6838	1.5858	1.4702	1.4259
Large Sample	1.7260	1.5911	1.4703	1.4259
Fixed Rel.	2.2958	2.1665	2.0319	1.9802
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85				
Univar	0.0185	0.0163	0.0160	0.0157
Bock	0.0150	0.0144	0.0152	0.0153
Fuller	0.1937	0.1745	0.1704	0.1688
Large Sample	0.1938	0.1745	0.1704	0.1688
Fixed Rel.	0.2638	0.2413	0.2365	0.2350
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7				
Univar	0.0197	0.0164	0.0161	0.0155
Bock	0.0045	0.0035	0.0030	0.0028
Fuller	0.3482	0.3051	0.2937	0.2861
Large Sample	0.3487	0.3052	0.2937	0.2861
Fixed Rel.	0.4815	0.4308	0.4179	0.4089
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00				
Univar	0.0173	0.0160	0.0154	0.0155
Bock	0.0146	0.0149	0.0150	0.0153
Fuller	0.0174	0.0160	0.0155	0.0155
Large Sample	0.0174	0.0160	0.0155	0.0155
Fixed Rel.	0.0174	0.0160	0.0155	0.0156

Table 5 (Cont'd)

<u>Case 7.</u> $\rho_{12} = .7$ $\rho_{13} = .6$ $\rho_{23} = .8$

$\rho_{11} = .7$ $\rho_{22} = .7$ $\rho_{33} = .7$

	n=50	n=100	n=250	n=500
Univar	0.0118	0.0078	0.0065	0.0060
Bock	0.0085	0.0069	0.0061	0.0057
Fuller	1.1104	0.8342	0.7459	0.6990
Large Sample	1.1240	0.8351	0.7459	0.6990
Fixed Rel.	1.5552	1.2132	1.1087	1.0486

$\rho_{11} = .85$ $\rho_{22} = .85$ $\rho_{33} = .85$

	n=50	n=100	n=250	n=500
Univar	0.0085	0.0068	0.0059	0.0057
Bock	0.0075	0.0068	0.0059	0.0057
Fuller	0.0952	0.0778	0.0700	0.0669
Large Sample	0.0952	0.0778	0.0700	0.0669
Fixed Rel.	0.1320	0.1101	0.1002	0.0961

$\rho_{11} = .9$ $\rho_{22} = .8$ $\rho_{33} = .7$

	n=50	n=100	n=250	n=500
Univar	0.0100	0.0072	0.0062	0.0057
Bock	0.0017	0.0010	0.0008	0.0007
Fuller	0.2532	0.1960	0.1763	0.1683
Large Sample	0.2537	0.1960	0.9910	0.9960
Fixed Rel.	0.3494	0.2795	0.2549	0.2447

$\rho_{11} = 1.00$ $\rho_{22} = 1.00$ $\rho_{33} = 1.00$

	n=50	n=100	n=250	n=500
Univar	0.0078	0.0068	0.0059	0.0057
Bock	0.0076	0.0069	0.0062	0.0059
Fuller	0.0078	0.0068	0.0060	0.0057
Large Sample	0.0078	0.0068	0.0060	0.0057
Fixed Rel.	0.0078	0.0068	0.0060	0.0057

<u>Summary of counts.</u> The overcorrected correlation counts, the invalid correlation matrices counts, and the counts of the improper variance-covariance matrices of the corrected correlations all show similar patterns. Problems tend to occur when the sample sizes are small, and the reliability triples contain moderate reliability values (e.g., .70). In no case did problems occur when the reliability values were unity. Also, the cases with high percentages for one of the three counts, tended to have high percentages on all three indices. Particularly problematic were the ρ = (.70, .60, .10) and the ρ = (.70, .60, .80) cases. When the higher correlations were matched with lower reliabilities, problems were expected, and they did occur.

It must be noted, however, that the percentage of times that problems occurred was small for all three indices. No more than 3% of the replications gave out-of-range corrected correlations for any case and reliability combination. The percents were larger for the determinants of the variance-covariances matrices of the corrected correlations, ranging up to 8% with improper values. Finally, the most problematic of the three indices was the determinants of the corrected correlation matrices, showing that up to 15% of the replications in one case yielded improper results.

I also recorded the number of times per replication that the Bock and Petersen (1975) and Fuller and Hidiroglou (1978) did not default to the univariate correction. The number of cases seems to be associated with the number of invalid correlation matrices. Table 6 displays the cases where the Bock and Petersen and Fuller and Hidiroglou methods did not match the usual univariate correction, these cases are referred to as "adjusted" cases. In these cases, both of the methods adjusted for the problem of a non-positive definite correlation matrix, and used eigenvalues to yield new matrices. These adjustments did not occur very frequently, especially when sample sizes were large. The table displays the number of times per combination that each method "adjusted". Also in the table is the number of replications on which the Fuller and Hidiroglou method required the added adjustment and in the same replication, the univariate and Gleser-method corrected correlation matrices were non-positive definite. Most of the time, the use of the adjustment was related to the invalid nature of the univariate correlation matrix. The Fuller and Hidiroglou correction obviously worked to correct this problem because the method did not yield any problematic correlation matrices.

Table 6

Number of Times Per Case where the Fuller and Hidiroglou and Bock and Petersen adjustments were needed.*

Case	Reliability	n	Fuller Adjust	Fuller & Invalid Univ. Corr. Matrix	Fuller & Invalid Gleser Corr. Matrix	Bock Adjust	Bock & Invalid Bock Corr. Matrix
(0, 0, 0)	(.7, .7, .7))	50	0	0	0	1	1
(.4, .3. .7)	(.7, .7, .7)	50	34	29	29	19	16
		100	0	0	0	1	1
	(.9, .8, .7)	50	6	6	5	6	6
(.6, .4, .2)	(.7, .7, .7)	50	10	8	8	11	10
	(.85, .85, .85)	50	1	0	0	0	0
	(.9, .8, .7)	50	0	0	0	1	1
(.6, .4, -.2)	(.7, .7, .7)	50	170	132	132	78	43
		100	33	29	28	3	1
	(.85, .85, .85)	50	3	3	3	3	1
	(.9, .8, .7)	50	12	10	10	7	5
(.7, .6, .1)	(.7, .7, .7)	50	343	281	281	162	114
		100	93	73	73	15	9
		250	6	4	4	0	0
	(.85, .85, .85)	50	44	33	33	11	9
	(.9, .8, .7)	50	48	38	39	19	17
		100	2	1	1	0	0
(.7, .6, .8)	(.7, .7, .7)	50	178	152	149	94	92
		100	27	19	19	5	5
	(.85, .85, .85)	50	3	1	1	2	2
	(.9, .8, .7)	50	63	45	47	19	19
		100	2	1	1	1	1

* Only cases where either the Fuller & Hidiroglou or Bock & Petersen adjustments were needed are included in this table.

The Bock and Petersen method also showed similar results. However, even after the adjustment was applied, many of the resulting corrected correlation matrices were still invalid. An examination of Table 6 along with the raw numbers produced from Table 3 show that virtually any time there was a problem with the corrected correlation matrix, the Fuller and Hidiroglou and Bock and Petersen methods adjusted. These methods also adjusted at other times, but the univariate corrected correlation matrices were not necessarily non-positive definite. These results could be due to rounding error.

Results of Magnitude Data

Corrected correlations. The average corrected correlations across replications appear in Table 7. These values reflect differences in the methods, as well as how the corrections become more accurate depending on sample size and reliability values. As shown in the tables, the Bock correction is most different from the others. In fact, when the Bock correction is used in combination with the (.90, .80, .70) reliability, the corrected correlations become much larger than their corresponding population values.

Other results visible in the table show that when the reliability values are unity, the univariate, Fuller & Hidiroglou, and Gleser methods all yield identical corrected correlations. Only the Bock and Petersen correction differs.

Table 7

<u>Mean Corrected Correlations</u>

<u>Case 1</u>. ρ_{12} = .0 ρ_{13} = .0 ρ_{23} = .0

Pop. Corr.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7			n=50		n=100			n=250			n=500	
Univariate	0.003	0.001	0.006	-0.001	-0.002	0.000	0.003	0.004	-0.001	0.000	0.002	-0.001
Bock	0.002	0.002	0.006	0.000	-0.003	-0.003	0.002	0.004	-0.003	0.001	0.001	0.000
Fuller	0.002	-0.001	0.005	-0.001	-0.001	0.001	0.003	0.004	-0.001	0.000	0.002	-0.001
Gleser	0.003	-0.001	0.005	-0.001	-0.002	0.000	0.003	0.004	-0.001	0.000	0.002	-0.001
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univariate	-0.003	-0.001	0.004	-0.002	-0.001	-0.001	-0.002	0.001	0.001	-0.002	0.000	-0.002
Bock	-0.007	0.001	0.004	-0.006	0.000	0.001	-0.003	0.002	0.000	-0.002	0.001	-0.002
Fuller	-0.002	-0.001	0.005	-0.002	-0.001	-0.001	-0.002	0.001	0.001	-0.002	0.000	-0.001
Gleser	-0.002	-0.001	0.005	-0.002	-0.001	-0.001	-0.002	0.001	0.001	-0.002	0.000	-0.001
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univariate	-0.004	-0.002	-0.005	-0.002	0.001	0.002	-0.003	0.000	-0.002	0.002	0.001	-0.002
Bock	-0.007	-0.006	-0.001	-0.004	0.007	0.007	-0.004	-0.002	-0.002	0.004	0.001	-0.002
Fuller	-0.003	-0.003	-0.006	-0.002	0.002	0.002	-0.003	0.000	-0.002	0.001	0.001	-0.002
Gleser	-0.003	-0.003	-0.006	-0.002	0.002	0.002	-0.003	0.000	-0.002	0.001	0.001	-0.002
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univariate	0.002	-0.002	0.005	0.001	-0.001	-0.004	0.000	0.001	-0.002	0.000	0.001	0.000
Bock	0.005	-0.003	0.008	0.004	-0.003	-0.007	-0.002	-0.002	-0.002	0.000	0.000	0.002
Fuller	0.002	-0.002	0.005	0.001	-0.001	-0.004	0.000	0.001	-0.002	0.000	0.001	0.000
Gleser	0.002	-0.002	0.005	0.001	-0.001	-0.004	0.000	0.001	-0.002	0.000	0.001	0.000

Table 7 (Cont'd)

Case 2. ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .1

Pop. Corr.	0.40	0.30	0.10	0.40	0.30	0.10	0.40	0.30	0.10	0.40	0.30	0.10
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7		n=50			n=100			n=250			n=500	
Univariate	0.390	0.305	0.100	0.398	0.296	0.097	0.399	0.299	0.099	0.401	0.300	0.099
Bock	0.433	0.335	0.120	0.428	0.320	0.115	0.414	0.311	0.102	0.408	0.307	0.103
Fuller	0.387	0.301	0.099	0.396	0.295	0.097	0.399	0.298	0.099	0.401	0.300	0.099
Gleser	0.391	0.305	0.100	0.398	0.296	0.097	0.399	0.299	0.099	0.401	0.300	0.099
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univariate	0.396	0.296	0.097	0.402	0.299	0.106	0.399	0.300	0.100	0.401	0.300	0.101
Bock	0.441	0.334	0.120	0.433	0.324	0.122	0.413	0.311	0.105	0.409	0.307	0.105
Fuller	0.395	0.295	0.097	0.402	0.299	0.106	0.399	0.300	0.100	0.401	0.300	0.101
Gleser	0.397	0.296	0.097	0.402	0.299	0.106	0.399	0.300	0.100	0.401	0.300	0.101
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univariate	0.395	0.295	0.103	0.398	0.296	0.101	0.402	0.300	0.102	0.401	0.298	0.101
Bock	0.535	0.438	0.236	0.547	0.457	0.236	0.554	0.472	0.235	0.554	0.479	0.226
Fuller	0.394	0.293	0.103	0.397	0.295	0.100	0.401	0.300	0.102	0.401	0.297	0.101
Gleser	0.395	0.295	0.104	0.398	0.296	0.101	0.402	0.300	0.102	0.401	0.298	0.101
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univariate	0.401	0.296	0.106	0.397	0.298	0.096	0.399	0.299	0.099	0.400	0.300	0.101
Bock	0.449	0.336	0.129	0.427	0.323	0.111	0.415	0.312	0.106	0.407	0.306	0.104
Fuller	0.401	0.296	0.106	0.397	0.297	0.096	0.399	0.299	0.099	0.400	0.300	0.101
Gleser	0.401	0.296	0.106	0.397	0.297	0.096	0.399	0.299	0.099	0.400	0.300	0.101

Table 7 (Cont'd)

<u>Case 3.</u> ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .7

Pop. Corr.	0.40	0.30	0.70	0.40	0.30	0.70	0.40	0.30	0.70	0.40	0.30	0.70
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7		n=50			n=100			n=250			n=500	
Univariate	0.399	0.295	0.698	0.398	0.294	0.694	0.401	0.299	0.700	0.399	0.298	0.700
Bock	0.427	0.323	0.722	0.415	0.307	0.714	0.408	0.305	0.707	0.403	0.302	0.703
Fuller	0.397	0.292	0.691	0.396	0.292	0.691	0.400	0.299	0.699	0.399	0.298	0.699
Gleser	0.400	0.296	0.698	0.397	0.294	0.694	0.401	0.299	0.700	0.399	0.298	0.700
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univariate	0.401	0.294	0.700	0.400	0.301	0.699	0.400	0.298	0.699	0.400	0.302	0.700
Bock	0.424	0.318	0.726	0.412	0.313	0.713	0.404	0.302	0.706	0.403	0.304	0.704
Fuller	0.399	0.293	0.697	0.399	0.301	0.698	0.399	0.297	0.698	0.400	0.302	0.700
Gleser	0.401	0.294	0.700	0.400	0.301	0.699	0.400	0.298	0.699	0.400	0.302	0.700
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univariate	0.399	0.294	0.692	0.400	0.300	0.699	0.399	0.300	0.699	0.400	0.300	0.699
Bock	0.518	0.418	0.792	0.532	0.435	0.805	0.545	0.449	0.812	0.547	0.449	0.814
Fuller	0.397	0.293	0.687	0.399	0.299	0.697	0.399	0.300	0.698	0.400	0.299	0.698
Gleser	0.399	0.295	0.692	0.400	0.300	0.700	0.399	0.300	0.699	0.400	0.300	0.699
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univariate	0.393	0.294	0.694	0.399	0.300	0.699	0.399	0.298	0.698	0.400	0.301	0.700
Bock	0.418	0.320	0.723	0.409	0.309	0.711	0.405	0.304	0.705	0.403	0.303	0.703
Fuller	0.394	0.295	0.694	0.400	0.300	0.699	0.399	0.298	0.699	0.400	0.301	0.700
Gleser	0.394	0.295	0.694	0.400	0.300	0.699	0.399	0.298	0.699	0.400	0.301	0.700

Table 7 (Cont'd)

<u>Case 4.</u> $\rho_{12} = .6$ $\rho_{13} = .4$ $\rho_{23} = .2$

Pop. Corr.	0.60	0.40	0.200	0.60	0.40	0.20	0.60	0.40	0.20	0.60	0.40	0.20
$\rho_{11} = .7$ $\rho_{22} = .7$ $\rho_{33} = .7$		n=50			n=100			n=250			n=500	
Univariate	0.598	0.404	0.199	0.597	0.402	0.199	0.599	0.401	0.200	0.598	0.400	0.199
Bock	0.628	0.431	0.219	0.615	0.419	0.214	0.608	0.406	0.203	0.603	0.404	0.203
Fuller	0.592	0.400	0.197	0.594	0.400	0.198	0.598	0.401	0.200	0.598	0.400	0.199
Gleser	0.598	0.404	0.199	0.597	0.402	0.199	0.599	0.401	0.200	0.598	0.400	0.199
$\rho_{11} = .85$ $\rho_{22} = .85$ $\rho_{33} = .85$												
Univariate	0.600	0.396	0.195	0.602	0.397	0.199	0.600	0.400	0.201	0.600	0.400	0.199
Bock	0.632	0.425	0.225	0.618	0.410	0.210	0.608	0.406	0.205	0.604	0.404	0.202
Fuller	0.597	0.394	0.194	0.600	0.396	0.198	0.599	0.400	0.201	0.599	0.400	0.199
Gleser	0.600	0.396	0.195	0.601	0.397	0.199	0.600	0.401	0.201	0.600	0.400	0.199
$\rho_{11} = .9$ $\rho_{22} = .8$ $\rho_{33} = .7$												
Univariate	0.597	0.398	0.203	0.602	0.397	0.200	0.600	0.400	0.199	0.600	0.400	0.203
Bock	0.710	0.544	0.375	0.721	0.545	0.381	0.726	0.544	0.379	0.732	0.542	0.382
Fuller	0.595	0.397	0.203	0.601	0.396	0.199	0.600	0.400	0.199	0.600	0.400	0.203
Gleser	0.597	0.399	0.204	0.602	0.397	0.200	0.600	0.400	0.199	0.600	0.400	0.203
$\rho_{11} = 1.00$ $\rho_{22} = 1.00$ $\rho_{33} = 1.00$												
Univariate	0.601	0.394	0.197	0.600	0.399	0.201	0.600	0.401	0.202	0.601	0.400	0.201
Bock	0.635	0.424	0.223	0.618	0.414	0.214	0.609	0.409	0.210	0.604	0.402	0.202
Fuller	0.601	0.394	0.198	0.600	0.399	0.201	0.600	0.401	0.202	0.601	0.400	0.201
Gleser	0.601	0.394	0.198	0.600	0.399	0.201	0.600	0.401	0.202	0.601	0.400	0.201

Table 7 (Cont'd)

<u>Case 5.</u> ρ_{12} = .6 ρ_{13} = .4 ρ_{23} = -.2

Pop. Corr.	0.60	0.40	-0.20	0.60	0.40	-0.20	0.60	0.40	-0.20	0.60	0.40	-0.20
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7			n=50		n=100			n=250			n=500	
Univariate	0.597	0.393	-0.195	0.596	0.395	-0.203	0.597	0.402	-0.200	0.599	0.399	-0.201
Bock	0.617	0.400	-0.185	0.612	0.403	-0.186	0.608	0.405	-0.187	0.605	0.402	-0.192
Fuller	0.590	0.387	-0.193	0.593	0.394	-0.202	0.596	0.402	-0.200	0.598	0.399	-0.201
Gleser	0.598	0.393	-0.195	0.596	0.396	-0.203	0.597	0.402	-0.200	0.599	0.399	-0.201
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univariate	0.600	0.397	-0.194	0.598	0.403	-0.199	0.600	0.396	-0.204	0.599	0.401	-0.200
Bock	0.619	0.401	-0.174	0.611	0.410	-0.179	0.609	0.400	-0.189	0.605	0.403	-0.193
Fuller	0.598	0.395	-0.194	0.597	0.403	-0.199	0.599	0.396	-0.204	0.599	0.401	-0.200
Gleser	0.600	0.396	-0.195	0.598	0.403	-0.199	0.600	0.396	-0.204	0.599	0.401	-0.200
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univariate	0.597	0.399	-0.196	0.599	0.398	-0.199	0.598	0.396	-0.203	0.600	0.399	-0.200
Bock	0.731	0.481	0.016	0.743	0.484	0.032	0.750	0.480	0.037	0.753	0.485	0.043
Fuller	0.595	0.397	-0.194	0.597	0.397	-0.198	0.598	0.396	-0.202	0.600	0.399	-0.200
Gleser	0.597	0.399	-0.195	0.599	0.398	-0.199	0.598	0.396	-0.202	0.600	0.399	-0.200
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univariate	0.596	0.396	-0.199	0.597	0.398	-0.199	0.600	0.400	-0.198	0.599	0.400	-0.201
Bock	0.616	0.403	-0.173	0.614	0.404	-0.177	0.609	0.404	-0.184	0.605	0.404	-0.192
Fuller	0.596	0.396	-0.199	0.597	0.398	-0.199	0.600	0.400	-0.198	0.599	0.400	-0.201
Gleser	0.596	0.396	-0.199	0.597	0.398	-0.199	0.600	0.400	-0.198	0.599	0.400	-0.201

Table 7 (Cont'd)

Case 6. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .1

Pop. Corr.	0.70	0.60	0.10	0.70	0.60	0.10	0.70	0.60	0.10	0.70	0.60	0.10
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7			n=50		n=100			n=250			n=500	
Univariate	0.696	0.599	0.102	0.696	0.600	0.095	0.696	0.598	0.094	0.699	0.599	0.098
Bock	0.718	0.619	0.140	0.707	0.611	0.116	0.702	0.604	0.104	0.702	0.602	0.102
Fuller	0.684	0.589	0.101	0.692	0.597	0.095	0.695	0.597	0.094	0.699	0.598	0.098
Gleser	0.696	0.599	0.102	0.696	0.600	0.095	0.696	0.598	0.094	0.699	0.599	0.098
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univariate	0.699	0.596	0.101	0.699	0.597	0.099	0.700	0.597	0.097	0.699	0.601	0.101
Bock	0.715	0.613	0.132	0.708	0.608	0.120	0.705	0.602	0.107	0.702	0.603	0.106
Fuller	0.697	0.593	0.100	0.698	0.596	0.099	0.700	0.596	0.097	0.699	0.600	0.101
Gleser	0.700	0.595	0.101	0.699	0.597	0.099	0.700	0.597	0.097	0.699	0.601	0.101
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univariate	0.698	0.599	0.100	0.699	0.601	0.104	0.697	0.598	0.097	0.700	0.602	0.103
Bock	0.805	0.699	0.353	0.810	0.703	0.366	0.812	0.702	0.368	0.815	0.705	0.378
Fuller	0.694	0.595	0.099	0.698	0.599	0.104	0.697	0.597	0.097	0.700	0.602	0.103
Gleser	0.698	0.599	0.100	0.699	0.601	0.105	0.697	0.598	0.097	0.700	0.602	0.103
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univariate	0.695	0.599	0.099	0.697	0.597	0.096	0.700	0.600	0.102	0.699	0.599	0.099
Bock	0.712	0.619	0.140	0.706	0.608	0.120	0.704	0.604	0.111	0.702	0.602	0.105
Fuller	0.695	0.599	0.099	0.697	0.597	0.096	0.700	0.600	0.102	0.699	0.599	0.099
Gleser	0.695	0.599	0.099	0.697	0.597	0.096	0.700	0.600	0.102	0.699	0.599	0.099

Table 7 (Cont'd)

Case 7. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .8

Pop. Corr.	0.70	0.60	0.80	0.70	0.60	0.80	0.70	0.60	0.80	0.70	0.60	0.80
ρ_{11} = .7 ρ_{22} =.7 ρ_{33} = .7	n=50			n=100			n=250			n=500		
Univariate	0.700	0.595	0.799	0.700	0.598	0.798	0.698	0.597	0.799	0.700	0.601	0.800
Bock	0.710	0.607	0.817	0.706	0.603	0.808	0.701	0.598	0.803	0.702	0.602	0.803
Fuller	0.691	0.588	0.789	0.697	0.595	0.794	0.697	0.596	0.797	0.700	0.600	0.799
Gleser	0.700	0.595	0.800	0.701	0.598	0.798	0.698	0.597	0.799	0.700	0.601	0.800
ρ_{11} = .85 ρ_{22} =.85 ρ_{33} = .85												
Univariate	0.695	0.594	0.799	0.696	0.597	0.799	0.700	0.600	0.801	0.700	0.600	0.800
Bock	0.710	0.605	0.813	0.702	0.600	0.808	0.701	0.600	0.804	0.701	0.600	0.802
Fuller	0.693	0.592	0.796	0.695	0.596	0.797	0.699	0.600	0.801	0.700	0.600	0.800
Gleser	0.696	0.595	0.799	0.696	0.597	0.799	0.700	0.600	0.801	0.700	0.600	0.800
ρ_{11} = .9 ρ_{22} =.8 ρ_{33} = .7												
Univariate	0.698	0.596	0.796	0.702	0.600	0.796	0.698	0.597	0.800	0.701	0.600	0.800
Bock	0.764	0.688	0.881	0.766	0.687	0.884	0.762	0.680	0.885	0.766	0.685	0.886
Fuller	0.696	0.593	0.791	0.701	0.598	0.794	0.697	0.596	0.799	0.700	0.600	0.800
Gleser	0.699	0.596	0.797	0.702	0.600	0.797	0.698	0.597	0.800	0.701	0.600	0.800
ρ_{11} = 1.00 ρ_{22} = 1.00 ρ_{33} = 1.00												
Univariate	0.697	0.597	0.797	0.699	0.596	0.798	0.699	0.599	0.799	0.700	0.600	0.800
Bock	0.706	0.602	0.812	0.703	0.598	0.805	0.700	0.599	0.802	0.700	0.599	0.801
Fuller	0.697	0.597	0.797	0.699	0.596	0.798	0.699	0.599	0.799	0.700	0.600	0.800
Gleser	0.697	0.597	0.797	0.699	0.596	0.798	0.699	0.599	0.799	0.700	0.600	0.800

The Bock and Petersen method yields corrected correlations larger than the other methods. Trends in the table show that higher reliabilities lead to more accurate corrections. Also, the larger the sample size, the more accurate the correction. Overall, the corrected correlations are all remarkably close to the population values, with the exception of the Bock and Petersen correction when the reliabilities are (.90, .80, .70). This may have something to do with reliabilities being unequal in these cases. An investigation of why these results occurred yielded no solutions.

Variances. The estimated variances of the corrected correlations can be compared to the empirical variances based on the sampling distributions (the 2000 cases) of the corrected correlations. Table 8 displays the variances of the corrected correlations for the different cases and combinations of factors. The third line at the top of each table shows the theoretical variance if one were to substitute the population correlation into the usual variance formula (Equation 3.1). As shown in the tables, this theoretical variance is smaller than the variance of the corrected correlations from the sampling distributions. This result was also found in Becker and Fahrbach (1995). This result was expected, given the work of Bobko and Rieck (1980) (among others) who showed that corrected correlations are more variable than uncorrected correlations.

Table 8

Mean Variances of Corrected Correlations

Case 1. ρ_{12} = .0 ρ_{13} = .0 ρ_{23} = .0

	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}
Correlation	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Expect Var.	0.0200	0.0200	0.0200	0.0100	0.0100	0.0100	0.0040	0.0040	0.0040	0.0020	0.0020	0.0020
$\rho_{11}=\rho_{22}=\rho_{33}=.7$		n=50			n=100			n=250			n=500	
Emp. Sam. Var.	0.0445	0.0415	0.0425	0.0213	0.0199	0.0210	0.0083	0.0082	0.0087	0.0041	0.0043	0.0043
Univariate	0.0183	0.0184	0.0184	0.0096	0.0096	0.0096	0.0039	0.0039	0.0039	0.0020	0.0020	0.0020
Large Sample	0.0394	0.0396	0.0396	0.0202	0.0202	0.0201	0.0081	0.0081	0.0081	0.0041	0.0041	0.0041
Fuller	0.0394	0.0396	0.0396	0.0202	0.0202	0.0201	0.0081	0.0081	0.0081	0.0041	0.0041	0.0041
Bock	0.0177	0.0177	0.0176	0.0094	0.0094	0.0094	0.0039	0.0039	0.0039	0.0020	0.0020	0.0020
Fixed Reliab.	0.0400	0.0401	0.0402	0.0203	0.0203	0.0203	0.0081	0.0081	0.0081	0.0041	0.0041	0.0041
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Sam. Var.	0.0297	0.0288	0.0284	0.0143	0.0141	0.0137	0.0056	0.0055	0.0057	0.0027	0.0028	0.0027
Univariate	0.0189	0.0189	0.0189	0.0097	0.0097	0.0097	0.0040	0.0040	0.0040	0.0020	0.0020	0.0020
Large Sample	0.0267	0.0267	0.0268	0.0136	0.0136	0.0136	0.0055	0.0055	0.0055	0.0028	0.0028	0.0028
Fuller	0.0267	0.0267	0.0268	0.0136	0.0136	0.0136	0.0055	0.0055	0.0055	0.0028	0.0028	0.0028
Bock	0.0180	0.0180	0.0181	0.0095	0.0095	0.0095	0.0039	0.0039	0.0039	0.0020	0.0020	0.0020
Fixed Reliab.	0.0269	0.0269	0.0270	0.0136	0.0136	0.0136	0.0055	0.0055	0.0055	0.0028	0.0028	0.0028
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Sam. Var.	0.0283	0.0335	0.0398	0.0150	0.0163	0.0180	0.0057	0.0065	0.0072	0.0027	0.0030	0.0035
Univariate	0.0189	0.0187	0.0185	0.0097	0.0097	0.0097	0.0040	0.0039	0.0039	0.0020	0.0020	0.0020
Large Sample	0.0269	0.0307	0.0346	0.0136	0.0156	0.0176	0.0055	0.0063	0.0071	0.0028	0.0192	0.0036
Fuller	0.0269	0.0307	0.0346	0.0136	0.0156	0.0176	0.0055	0.0063	0.0071	0.0028	0.0192	0.0036
Bock	0.0170	0.0161	0.0170	0.0091	0.0088	0.0092	0.0038	0.0038	0.0038	0.0020	0.0183	0.0020
Fixed Reliab.	0.0271	0.0310	0.0350	0.0137	0.0157	0.0177	0.0055	0.0063	0.0071	0.0028	0.0192	0.0036
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Sam. Var.	0.0197	0.0210	0.0208	0.0106	0.0101	0.0101	0.0040	0.0040	0.0044	0.0019	0.0021	0.0019
Univariate	0.0192	0.0192	0.0192	0.0098	0.0098	0.0098	0.0040	0.0040	0.0040	0.0020	0.0020	0.0020
Large Sample	0.0192	0.0192	0.0192	0.0098	0.0098	0.0098	0.0040	0.0040	0.0040	0.0020	0.0020	0.0020
Fuller	0.0192	0.0192	0.0192	0.0098	0.0098	0.0098	0.0040	0.0040	0.0040	0.0020	0.0020	0.0020
Bock	0.0184	0.0183	0.0183	0.0095	0.0096	0.0095	0.0039	0.0039	0.0039	0.0020	0.0020	0.0020
Fixed Reliab.	0.0192	0.0192	0.0192	0.0098	0.0098	0.0098	0.0040	0.0040	0.0040	0.0020	0.0020	0.0020

Table 8 (Cont'd)

Case 2. ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .1

	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}
Correlation	0.4000	0.3000	0.1000	0.4000	0.3000	0.1000	0.4000	0.3000	0.1000	0.4000	0.3000	0.1000
Expect Var.	0.0141	0.0166	0.0196	0.0071	0.0083	0.0098	0.0028	0.0033	0.0039	0.0014	0.0017	0.0020
$\rho_{11}=\rho_{22}=\rho_{33}=.7$		n=50			n=100			n=250			n=500	
Emp. Sam. Var.	0.0351	0.0352	0.0420	0.0162	0.0187	0.0197	0.0066	0.0075	0.0076	0.0032	0.0035	0.0041
Univariate	0.0136	0.0156	0.0181	0.0069	0.0080	0.0094	0.0028	0.0033	0.0039	0.0014	0.0016	0.0019
Large Sample	0.0331	0.0358	0.0393	0.0162	0.0179	0.0198	0.0065	0.0072	0.0080	0.0033	0.0036	0.0040
Fuller	0.0331	0.0358	0.0393	0.0162	0.0179	0.0198	0.0065	0.0072	0.0080	0.0033	0.0036	0.0040
Bock	0.0127	0.0145	0.0175	0.0065	0.0078	0.0093	0.0027	0.0032	0.0038	0.0014	0.0016	0.0019
Fixed Reliab.	0.0351	0.0372	0.0399	0.0172	0.0186	0.0200	0.0069	0.0074	0.0081	0.0035	0.0037	0.0040
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Sam. Var.	0.0214	0.0251	0.0285	0.0105	0.0122	0.0146	0.0041	0.0048	0.0056	0.0021	0.0023	0.0027
Univariate	0.0137	0.0159	0.0186	0.0070	0.0081	0.0095	0.0028	0.0033	0.0039	0.0014	0.0017	0.0020
Large Sample	0.0206	0.0232	0.0264	0.0103	0.0117	0.0134	0.0041	0.0047	0.0054	0.0021	0.0024	0.0027
Fuller	0.0206	0.0232	0.0264	0.0103	0.0117	0.0134	0.0041	0.0047	0.0054	0.0021	0.0024	0.0027
Bock	0.0124	0.0149	0.0179	0.0065	0.0078	0.0094	0.0027	0.0032	0.0039	0.0014	0.0016	0.0019
Fixed Reliab.	0.0215	0.0238	0.0266	0.0108	0.0120	0.0135	0.0043	0.0048	0.0054	0.0022	0.0024	0.0027
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Sam. Var.	0.0215	0.0273	0.0363	0.0108	0.0140	0.0178	0.0042	0.0053	0.0069	0.0021	0.0027	0.0036
Univariate	0.0139	0.0160	0.0183	0.0070	0.0081	0.0095	0.0028	0.0033	0.0039	0.0014	0.0016	0.0019
Large Sample	0.0210	0.0273	0.0343	0.0104	0.0136	0.0173	0.0042	0.0055	0.0070	0.0021	0.0027	0.0035
Fuller	0.0210	0.0273	0.0343	0.0104	0.0136	0.0173	0.0042	0.0055	0.0070	0.0021	0.0027	0.0035
Bock	0.0101	0.0121	0.0159	0.0049	0.0060	0.0083	0.0019	0.0024	0.0035	0.0010	0.0012	0.0018
Fixed Reliab.	0.0219	0.0281	0.0348	0.0108	0.0140	0.0174	0.0044	0.0056	0.0070	0.0022	0.0028	0.0035
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Sam. Var.	0.0150	0.0166	0.0206	0.0076	0.0084	0.0105	0.0028	0.0032	0.0040	0.0014	0.0016	0.0020
Univariate	0.0138	0.0162	0.0188	0.0070	0.0082	0.0096	0.0028	0.0033	0.0039	0.0014	0.0016	0.0020
Large Sample	0.0138	0.0162	0.0188	0.0070	0.0082	0.0096	0.0028	0.0033	0.0039	0.0014	0.0016	0.0020
Fuller	0.0138	0.0162	0.0188	0.0070	0.0082	0.0096	0.0028	0.0033	0.0039	0.0014	0.0016	0.0020
Bock	0.0123	0.0151	0.0182	0.0066	0.0078	0.0095	0.0027	0.0032	0.0039	0.0014	0.0016	0.0019
Fixed Reliab.	0.0138	0.0162	0.0188	0.0070	0.0082	0.0096	0.0028	0.0033	0.0039	0.0014	0.0016	0.0020

Table 8 (Cont'd)

Case 3. ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .7

	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}
Correlation	0.4000	0.3000	0.7000	0.4000	0.3000	0.7000	0.4000	0.3000	0.7000	0.4000	0.3000	0.7000
Expect Var.	0.0141	0.0166	0.0052	0.0071	0.0083	0.0026	0.0028	0.0033	0.0010	0.0014	0.0017	0.0005
$\rho_{11}=\rho_{22}=\rho_{33}=.7$		n=50			n=100			n=250			n=500	
Emp. Sam. Var.	0.0330	0.0383	0.0215	0.0179	0.0189	0.0099	0.0071	0.0073	0.0039	0.0033	0.0037	0.0018
Univariate	0.0134	0.0156	0.0056	0.0069	0.0080	0.0027	0.0028	0.0033	0.0010	0.0014	0.0016	0.0005
Large Sample	0.0325	0.0356	0.0205	0.0164	0.0180	0.0100	0.0065	0.0072	0.0039	0.0033	0.0036	0.0019
Fuller	0.0325	0.0355	0.0205	0.0164	0.0180	0.0100	0.0065	0.0072	0.0039	0.0033	0.0036	0.0019
Bock	0.0128	0.0150	0.0047	0.0068	0.0079	0.0025	0.0028	0.0032	0.0010	0.0014	0.0016	0.0005
Fixed Reliab.	0.0345	0.0370	0.0244	0.0174	0.0187	0.0121	0.0069	0.0074	0.0047	0.0035	0.0037	0.0024
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Sam. Var.	0.0217	0.0244	0.0109	0.0106	0.0121	0.0049	0.0043	0.0050	0.0020	0.0020	0.0024	0.0009
Univariate	0.0138	0.0159	0.0054	0.0070	0.0081	0.0027	0.0028	0.0033	0.0011	0.0014	0.0016	0.0005
Large Sample	0.0207	0.0232	0.0103	0.0103	0.0117	0.0050	0.0042	0.0047	0.0020	0.0021	0.0024	0.0010
Fuller	0.0207	0.0232	0.0103	0.0103	0.0117	0.0050	0.0042	0.0047	0.0020	0.0021	0.0024	0.0010
Bock	0.0131	0.0153	0.0047	0.0067	0.0079	0.0024	0.0028	0.0033	0.0010	0.0014	0.0016	0.0005
Fixed Reliab.	0.0216	0.0238	0.0121	0.0108	0.0120	0.0059	0.0043	0.0048	0.0023	0.0022	0.0024	0.0012
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Sam. Var.	0.0215	0.0291	0.0166	0.0107	0.0139	0.0081	0.0041	0.0059	0.0030	0.0022	0.0028	0.0016
Univariate	0.0138	0.0159	0.0057	0.0070	0.0081	0.0027	0.0028	0.0033	0.0011	0.0014	0.0016	0.0005
Large Sample	0.0208	0.0271	0.0167	0.0104	0.0137	0.0080	0.0042	0.0055	0.0031	0.0021	0.0027	0.0015
Fuller	0.0208	0.0271	0.0167	0.0104	0.0137	0.0080	0.0042	0.0055	0.0031	0.0021	0.0027	0.0015
Bock	0.0104	0.0127	0.0031	0.0051	0.0064	0.0013	0.0020	0.0025	0.0005	0.0010	0.0013	0.0002
Fixed Reliab.	0.0217	0.0280	0.0199	0.0109	0.0141	0.0096	0.0043	0.0056	0.0038	0.0022	0.0028	0.0019
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Sam. Var.	0.0150	0.0178	0.0057	0.0072	0.0083	0.0027	0.0028	0.0033	0.0010	0.0014	0.0016	0.0005
Univariate	0.0140	0.0161	0.0054	0.0070	0.0082	0.0027	0.0028	0.0033	0.0010	0.0014	0.0017	0.0005
Large Sample	0.0140	0.0161	0.0054	0.0070	0.0082	0.0027	0.0028	0.0033	0.0010	0.0014	0.0017	0.0005
Fuller	0.0140	0.0161	0.0054	0.0070	0.0082	0.0027	0.0028	0.0033	0.0010	0.0014	0.0017	0.0005
Bock	0.0132	0.0154	0.0046	0.0068	0.0080	0.0025	0.0028	0.0033	0.0010	0.0014	0.0016	0.0005
Fixed Reliab.	0.0140	0.0161	0.0054	0.0070	0.0082	0.0027	0.0028	0.0033	0.0010	0.0014	0.0017	0.0005

Table 8 (Cont'd)

Case 4. ρ_{12} = .6 ρ_{13} = .4 ρ_{23} = .2

	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}
Correlation	0.6000	0.4000	0.2000	0.6000	0.4000	0.2000	0.6000	0.4000	0.2000	0.6000	0.4000	0.2000
Expect Var.	0.0082	0.0141	0.0184	0.0041	0.0071	0.0092	0.0016	0.0028	0.0037	0.0008	0.0014	0.0018
$\rho_{11}=\rho_{22}=\rho_{33}=.7$		n=50			n=100			n=250			n=500	
Emp. Sam. Var.	0.0242	0.0361	0.0374	0.0126	0.0174	0.0200	0.0047	0.0065	0.0082	0.0023	0.0031	0.0040
Univariate	0.0082	0.0137	0.0173	0.0042	0.0069	0.0089	0.0016	0.0028	0.0036	0.0008	0.0014	0.0018
Large Sample	0.0248	0.0331	0.0382	0.0124	0.0164	0.0191	0.0048	0.0065	0.0077	0.0024	0.0033	0.0039
Fuller	0.0248	0.0331	0.0382	0.0124	0.0164	0.0191	0.0048	0.0065	0.0077	0.0024	0.0033	0.0039
Bock	0.0073	0.0129	0.0167	0.0039	0.0067	0.0087	0.0016	0.0028	0.0036	0.0008	0.0014	0.0018
Fixed Reliab.	0.0282	0.0350	0.0391	0.0141	0.0174	0.0195	0.0056	0.0069	0.0078	0.0028	0.0035	0.0039
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Sam. Var.	0.0145	0.0215	0.0274	0.0070	0.0110	0.0133	0.0027	0.0042	0.0054	0.0013	0.0022	0.0026
Univariate	0.0085	0.0137	0.0175	0.0041	0.0070	0.0099	0.0016	0.0028	0.0037	0.0008	0.0014	0.0018
Large Sample	0.0141	0.0206	0.0251	0.0069	0.0104	0.0127	0.0027	0.0042	0.0051	0.0014	0.0021	0.0026
Fuller	0.0141	0.0206	0.0251	0.0069	0.0104	0.0127	0.0027	0.0042	0.0051	0.0014	0.0021	0.0026
Bock	0.0075	0.0128	0.0169	0.0039	0.0068	0.0088	0.0016	0.0028	0.0000	0.0008	0.0014	0.0018
Fixed Reliab.	0.0156	0.0215	0.0255	0.0077	0.0108	0.0129	0.0030	0.0043	0.0052	0.0015	0.0022	0.0026
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Sam. Var.	0.0146	0.0251	0.0341	0.0069	0.0127	0.0176	0.0027	0.0052	0.0070	0.0013	0.0023	0.0034
Univariate	0.0083	0.0137	0.0174	0.0042	0.0070	0.0089	0.0016	0.0028	0.0036	0.0008	0.0014	0.0018
Large Sample	0.0140	0.0243	0.0328	0.0070	0.0122	0.0167	0.0027	0.0049	0.0067	0.0014	0.0024	0.0034
Fuller	0.0140	0.0243	0.0328	0.0070	0.0122	0.0167	0.0027	0.0049	0.0067	0.0014	0.0024	0.0034
Bock	0.0051	0.0098	0.0137	0.0024	0.0049	0.0070	0.0009	0.0020	0.0029	0.0004	0.0010	0.0015
Fixed Reliab.	0.0155	0.0255	0.0335	0.0078	0.0129	0.0169	0.0030	0.0051	0.0068	0.0015	0.0026	0.0034
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Sam. Var.	0.0087	0.0150	0.0190	0.0041	0.0072	0.0089	0.0017	0.0027	0.0038	0.0008	0.0015	0.0019
Univariate	0.0083	0.0140	0.0178	0.0041	0.0070	0.0090	0.0016	0.0028	0.0037	0.0008	0.0014	0.0018
Large Sample	0.0083	0.0140	0.0178	0.0041	0.0070	0.0090	0.0016	0.0028	0.0037	0.0008	0.0014	0.0018
Fuller	0.0083	0.0140	0.0178	0.0041	0.0070	0.0090	0.0016	0.0028	0.0037	0.0008	0.0014	0.0018
Bock	0.0072	0.0132	0.0172	0.0038	0.0068	0.0089	0.0016	0.0028	0.0036	0.0008	0.0014	0.0018
Fixed Reliab.	0.0083	0.0140	0.0178	0.0041	0.0070	0.0090	0.0016	0.0028	0.0037	0.0008	0.0014	0.0018

Table 8 (Cont'd)

Case 5. ρ_{12} = .6 ρ_{13} = .4 ρ_{23} = -.2

	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}
Correlation	0.6000	0.4000	-0.200	0.6000	0.4000	-0.200	0.6000	0.4000	-0.200	0.6000	0.4000	-0.200
Expect Var.	0.0082	0.0141	0.0184	0.0041	0.0071	0.0092	0.0016	0.0028	0.0037	0.0008	0.0014	0.0018
$\rho_{11}=\rho_{22}=\rho_{33}=.7$		n=50			n=100			n=250			n=500	
Emp. Sam. Var.	0.0265	0.0355	0.0409	0.0125	0.0163	0.0195	0.0050	0.0065	0.0076	0.0024	0.0033	0.0039
Univariate	0.0084	0.0135	0.0171	0.0042	0.0069	0.0089	0.0017	0.0028	0.0036	0.0008	0.0014	0.0018
Large Sample	0.0252	0.0328	0.0378	0.0123	0.0163	0.0191	0.0049	0.0065	0.0077	0.0024	0.0033	0.0039
Fuller	0.0250	0.0326	0.0376	0.0123	0.0163	0.0191	0.0049	0.0065	0.0077	0.0024	0.0033	0.0039
Bock	0.0078	0.0131	0.0169	0.0039	0.0067	0.0089	0.0016	0.0028	0.0037	0.0008	0.0014	0.0018
Fixed Reliab.	0.0285	0.0348	0.0388	0.0140	0.0173	0.0194	0.0056	0.0069	0.0078	0.0018	0.0035	0.0039
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Sam. Var.	0.0147	0.0223	0.0270	0.0072	0.0110	0.0131	0.0027	0.0043	0.0054	0.0014	0.0020	0.0027
Univariate	0.0082	0.0138	0.0175	0.0041	0.0070	0.0090	0.0016	0.0028	0.0037	0.0008	0.0014	0.0018
Large Sample	0.0139	0.0206	0.0251	0.0068	0.0104	0.0128	0.0027	0.0042	0.0051	0.0014	0.0021	0.0026
Fuller	0.0139	0.0206	0.0251	0.0068	0.0104	0.0128	0.0027	0.0042	0.0051	0.0014	0.0021	0.0026
Bock	0.0077	0.0132	0.0173	0.0039	0.0069	0.0090	0.0016	0.0028	0.0037	0.0008	0.0014	0.0018
Fixed Reliab.	0.0154	0.0215	0.0255	0.0076	0.0109	0.0129	0.0030	0.0043	0.0052	0.0015	0.0022	0.0026
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Sam. Var.	0.0136	0.0258	0.0336	0.0068	0.0127	0.0173	0.0028	0.0049	0.0067	0.0014	0.0023	0.0032
Univariate	0.0082	0.0137	0.0174	0.0041	0.0069	0.0090	0.0016	0.0028	0.0036	0.0008	0.0014	0.0018
Large Sample	0.0139	0.0244	0.0332	0.0068	0.0122	0.0167	0.0027	0.0049	0.0067	0.0014	0.0024	0.0034
Fuller	0.0139	0.0244	0.0332	0.0068	0.0122	0.0167	0.0027	0.0049	0.0067	0.0014	0.0024	0.0034
Bock	0.0045	0.0112	0.0175	0.0020	0.0057	0.0093	0.0008	0.0023	0.0039	0.0004	0.0012	0.0020
Fixed Reliab.	0.0155	0.0257	0.0339	0.0076	0.0128	0.0170	0.0031	0.0051	0.0068	0.0015	0.0026	0.0034
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Sam. Var.	0.0086	0.0151	0.0185	0.0041	0.0071	0.0091	0.0017	0.0030	0.0038	0.0009	0.0015	0.0019
Univariate	0.0084	0.0139	0.0178	0.0041	0.0071	0.0091	0.0017	0.0028	0.0037	0.0008	0.0014	0.0018
Large Sample	0.0084	0.0139	0.0178	0.0041	0.0071	0.0091	0.0017	0.0028	0.0037	0.0008	0.0014	0.0018
Fuller	0.0084	0.0139	0.0178	0.0041	0.0071	0.0091	0.0017	0.0028	0.0037	0.0008	0.0014	0.0018
Bock	0.0080	0.0135	0.0176	0.0039	0.0069	0.0091	0.0016	0.0028	0.0037	0.0007	0.0014	0.0018
Fixed Reliab.	0.0084	0.0139	0.0178	0.0041	0.0071	0.0091	0.0017	0.0028	0.0037	0.0008	0.0014	0.0018

Table 8 (Cont'd)

Case 6. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .1

	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}
Correlation	0.7000	0.6000	0.1000	0.7000	0.6000	0.1000	0.7000	0.6000	0.1000	0.7000	0.6000	0.1000
Expect Var.	0.0052	0.0082	0.0196	0.0026	0.0041	0.0098	0.0010	0.0016	0.0039	0.0005	0.0008	0.0020
$\rho_{11}=\rho_{22}=\rho_{33}=.7$	n=50			n=100			n=250			n=500		
Emp. Sam. Var.	0.0204	0.0251	0.0415	0.0099	0.0123	0.0207	0.0039	0.0048	0.0081	0.0019	0.0022	0.0042
Univariate	0.0056	0.0082	0.0181	0.0027	0.0042	0.0094	0.0011	0.0016	0.0039	0.0005	0.0008	0.0019
Large Sample	0.0207	0.0247	0.0393	0.0101	0.0123	0.0199	0.0039	0.0048	0.0080	0.0019	0.0024	0.0040
Fuller	0.0204	0.0244	0.0387	0.0101	0.0123	0.0199	0.0039	0.0048	0.0080	0.0019	0.0024	0.0040
Bock	0.0051	0.0076	0.0175	0.0026	0.0040	0.0093	0.0010	0.0016	0.0038	0.0005	0.0008	0.0019
Fixed Reliab.	0.0246	0.0281	0.0399	0.0121	0.0140	0.0201	0.0048	0.0056	0.0081	0.0024	0.0028	0.0040
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Sam. Var.	0.0100	0.0134	0.0283	0.0049	0.0072	0.0132	0.0019	0.0027	0.0052	0.0009	0.0013	0.0027
Univariate	0.0054	0.0084	0.0186	0.0026	0.0041	0.0096	0.0011	0.0016	0.0039	0.0005	0.0008	0.0020
Large Sample	0.0101	0.0141	0.0264	0.0049	0.0069	0.0134	0.0020	0.0027	0.0054	0.0010	0.0014	0.0027
Fuller	0.0101	0.0141	0.0264	0.0049	0.0069	0.0134	0.0020	0.0027	0.0054	0.0010	0.0014	0.0027
Bock	0.0049	0.0079	0.0181	0.0025	0.0040	0.0093	0.0010	0.0016	0.0038	0.0005	0.0008	0.0019
Fixed Reliab.	0.0119	0.0156	0.0266	0.0058	0.0077	0.0135	0.0023	0.0030	0.0054	0.0012	0.0015	0.0027
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Sam. Var.	0.0101	0.0172	0.0368	0.0049	0.0086	0.0179	0.0018	0.0033	0.0070	0.0010	0.0016	0.0036
Univariate	0.0056	0.0084	0.0183	0.0026	0.0041	0.0095	0.0011	0.0016	0.0039	0.0005	0.0008	0.0019
Large Sample	0.0104	0.0174	0.0343	0.0050	0.0084	0.0173	0.0020	0.0034	0.0070	0.0010	0.0017	0.0035
Fuller	0.0104	0.0174	0.0342	0.0050	0.0084	0.0173	0.0020	0.0034	0.0070	0.0010	0.0017	0.0035
Bock	0.0028	0.0055	0.0143	0.0013	0.0027	0.0072	0.0005	0.0010	0.0029	0.0002	0.0005	0.0015
Fixed Reliab.	0.0122	0.0196	0.0347	0.0059	0.0095	0.0175	0.0023	0.0038	0.0070	0.0012	0.0019	0.0035
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Sam. Var.	0.0056	0.0086	0.0195	0.0029	0.0044	0.0105	0.0010	0.0017	0.0041	0.0005	0.0008	0.0019
Univariate	0.0055	0.0084	0.0189	0.0027	0.0041	0.0096	0.0010	0.0016	0.0039	0.0005	0.0008	0.0020
Large Sample	0.0055	0.0084	0.0189	0.0027	0.0041	0.0096	0.0010	0.0016	0.0039	0.0005	0.0008	0.0020
Fuller	0.0055	0.0084	0.0189	0.0027	0.0041	0.0096	0.0010	0.0016	0.0039	0.0005	0.0008	0.0020
Bock	0.0050	0.0078	0.0182	0.0025	0.0040	0.0094	0.0010	0.0016	0.0038	0.0005	0.0008	0.0019
Fixed Reliab.	0.0055	0.0084	0.0189	0.0027	0.0041	0.0096	0.0010	0.0016	0.0039	0.0005	0.0008	0.0020

Table 8 (Cont'd)

Case 7. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .8

	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}	ρ_{12}	ρ_{13}	ρ_{23}
Correlation	0.7000	0.6000	0.8000	0.7000	0.6000	0.8000	0.7000	0.6000	0.8000	0.7000	0.6000	0.8000
Expect Var.	0.0052	0.0082	0.0026	0.0026	0.0041	0.0013	0.0010	0.0016	0.0005	0.0005	0.0008	0.0003
$\rho_{11}=\rho_{22}=\rho_{33}=.7$		n=50			n=100			n=250			n=500	
Emp. Sam. Var.	0.0220	0.0258	0.0153	0.0099	0.0128	0.0069	0.0038	0.0048	0.0028	0.0018	0.0024	0.0014
Univariate	0.0057	0.0082	0.0032	0.0027	0.0041	0.0014	0.0011	0.0017	0.0005	0.0005	0.0008	0.0003
Large Sample	0.0209	0.0251	0.0164	0.0100	0.0123	0.0076	0.0039	0.0049	0.0030	0.0019	0.0024	0.0015
Fuller	0.0208	0.0249	0.0163	0.0100	0.0122	0.0076	0.0039	0.0049	0.0030	0.0019	0.0024	0.0015
Bock	0.0052	0.0079	0.0027	0.0026	0.0041	0.0013	0.0010	0.0016	0.0005	0.0005	0.0008	0.0003
Fixed Reliab.	0.0248	0.0284	0.0206	0.0120	0.0140	0.0098	0.0048	0.0056	0.0039	0.0024	0.0028	0.0019
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Sam. Var.	0.0103	0.0145	0.0067	0.0047	0.0070	0.0028	0.0019	0.0027	0.0012	0.0010	0.0014	0.0006
Univariate	0.0054	0.0082	0.0028	0.0027	0.0041	0.0013	0.0010	0.0016	0.0005	0.0005	0.0008	0.0003
Large Sample	0.0102	0.0138	0.0066	0.0050	0.0069	0.0032	0.0020	0.0027	0.0012	0.0010	0.0014	0.0006
Fuller	0.0102	0.0138	0.0066	0.0050	0.0069	0.0032	0.0020	0.0027	0.0012	0.0010	0.0014	0.0006
Bock	0.0052	0.0080	0.0025	0.0026	0.0041	0.0013	0.0010	0.0016	0.0005	0.0005	0.0008	0.0003
Fixed Reliab.	0.0120	0.0154	0.0084	0.0059	0.0077	0.0041	0.0023	0.0030	0.0016	0.0012	0.0015	0.0008
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Sam. Var.	0.0109	0.0179	0.0115	0.0047	0.0081	0.0052	0.0020	0.0034	0.0021	0.0010	0.0016	0.0010
Univariate	0.0055	0.0083	0.0030	0.0027	0.0041	0.0014	0.0011	0.0016	0.0005	0.0005	0.0008	0.0003
Large Sample	0.0105	0.0173	0.0123	0.0050	0.0085	0.0058	0.0020	0.0033	0.0023	0.0010	0.0017	0.0011
Fuller	0.0104	0.0173	0.0123	0.0050	0.0085	0.0058	0.0020	0.0033	0.0023	0.0010	0.0017	0.0011
Bock	0.0039	0.0058	0.0013	0.0019	0.0029	0.0005	0.0007	0.0011	0.0002	0.0003	0.0006	0.0001
Fixed Reliab.	0.0122	0.0194	0.0156	0.0059	0.0096	0.0075	0.0024	0.0038	0.0030	0.0012	0.0019	0.0015
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Sam. Var.	0.0054	0.0082	0.0028	0.0027	0.0042	0.0014	0.0011	0.0017	0.0005	0.0005	0.0008	0.0002
Univariate	0.0054	0.0084	0.0028	0.0027	0.0041	0.0014	0.0010	0.0016	0.0005	0.0005	0.0008	0.0003
Large Sample	0.0054	0.0084	0.0028	0.0027	0.0041	0.0014	0.0010	0.0016	0.0005	0.0005	0.0008	0.0003
Fuller	0.0054	0.0084	0.0028	0.0027	0.0041	0.0014	0.0010	0.0016	0.0005	0.0005	0.0008	0.0003
Bock	0.0051	0.0082	0.0025	0.0026	0.0041	0.0013	0.0010	0.0016	0.0005	0.0005	0.0008	0.0003
Fixed Reliab.	0.0054	0.0084	0.0028	0.0027	0.0042	0.0014	0.0011	0.0016	0.0005	0.0005	0.0008	0.0003

The other lines in the table correspond to mean variances found from various methods. These methods include the univariate method, which simply takes the corrected correlation and inserts it into Equation 3.1 and 3.2. As expected, the results using this method closely approximate the expected variance, and are much smaller than the variance of the sampling distribution.

The method from Bock and Petersen, as mentioned previously, did not yield a variance estimate of its own. Instead, the corrected correlations from this method were inserted into the univariate variance formulas for comparison purposes. The Bock and Petersen results are therefore similar to the univariate results.

The large-sample theory variances are those found using the formulas in Appendix A and Appendix B. The Fuller and Hidiroglou (1978) method also made use of this formulation, though with differing results. The Fuller and Hidiroglou correction relies on the smallest eigenvalue from the given variance matrix, and if this value is less than unity, a substitution is made. The large-sample theory method with the Fuller and Hidiroglou variation then adjusted for the use of this eigenvalue. The large-sample variance and the Fuller and Hidiroglou variation of that method should be identical, unless the original correlation matrix is non-positive definite. As shown in the tables, the large-sample variance and the Fuller and Hidiroglou variation give very close approximations to the variance of the sampling distribution of the corrected correlations. The accuracy of these estimators increases as the sample size increases. With $\underline{n}$ = 500, for example, the elements of the corrected variance-covariance matrices using the large-sample method (and the Fuller and Hidiroglou variation) are within 0.0002 of the sampling distribution, no matter what the reliability values, or the correlation case.

The final line in each grouping shows what the variance would be if one assumed that the sample reliability was a constant. This differs from the large-sample formulation in that the large-sample formulation assumes that reliabilities are variable, and that variability is accounted for in the computation. The results using this formulation are quite close to the large-sample results. However, at smaller sample sizes, they are not nearly as accurate as the other variances.

The other factor of note is the reliability value. It appears, given these data, to have no effect on the results. What is apparent, however, is that when the sample reliabilities are approximating a population reliability of 1.00, all variance corrections give similar results, and these results closely resemble the expected variance of the correlations measured without error. This result verifies that the simulation seems to be working as it should.

Table 9

Mean Covariances of Corrected Correlations

Case 1. ρ_{12} = .0 ρ_{13} = .0 ρ_{23} = .0

Covs.	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}
Expected	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
$\rho_{11}=\rho_{22}=\rho_{33}=.7$		n=50			n=100			n=250			n=500	
Emp. Cov.	0.00205	0.00105	0.00102	0.00006	0.00093	0.00049	-0.00015	-0.00026	-0.00011	-0.00003	-0.00010	0.00001
Univar	0.00003	0.00019	-0.00010	0.00001	0.00002	-0.00001	0.00001	0.00000	-0.00001	0.00000	0.00000	0.00000
Large-Sam.	0.00004	0.00026	-0.00016	0.00001	0.00003	-0.00002	0.00002	-0.00001	-0.00001	0.00000	0.00000	-0.00001
Fuller	0.00004	0.00026	-0.00016	0.00001	0.00003	-0.00002	0.00002	-0.00001	-0.00001	0.00000	0.00000	-0.00001
Bock	0.00004	0.00006	-0.00011	-0.00003	0.00002	-0.00001	0.00001	-0.00001	-0.00001	0.00000	0.00000	-0.00001
Fixed Rel.	0.00005	0.00027	-0.00015	0.00001	0.00003	-0.00002	0.00002	-0.00001	-0.00001	0.00000	0.00000	-0.00001
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Cov.	0.00055	-0.00035	0.00077	-0.00007	0.00027	-0.00001	-0.00013	-0.00003	0.00000	0.00002	-0.00002	-0.00001
Univar	0.00003	-0.00008	0.00004	0.00001	-0.00002	-0.00009	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000
Large-Sam.	0.00004	-0.00009	0.00037	0.00001	-0.00002	-0.00010	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000
Fuller	0.00004	-0.00009	0.00004	0.00001	-0.00002	-0.00010	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000
Bock	-0.00001	-0.00016	-0.00001	0.00004	-0.00004	-0.00009	0.00000	0.00002	0.00001	0.00000	0.00000	0.00000
Fixed Rel.	0.00004	-0.00009	0.00004	0.00001	-0.00002	-0.00010	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Cov.	0.00162	0.00091	-0.00063	0.00011	-0.00003	0.00078	-0.00014	-0.00001	0.00017	0.00008	0.00011	-0.00010
Univar	0.00009	0.00015	-0.00006	0.00004	0.00001	-0.00002	0.00002	-0.00001	-0.00002	0.00000	0.00000	0.00000
Large-Sam.	0.00009	0.00019	-0.00010	0.00005	0.00000	-0.00004	0.00002	-0.00001	-0.00003	0.00000	0.00000	-0.00001
Fuller	0.00009	0.00019	-0.00010	0.00005	0.00000	-0.00004	0.00002	-0.00001	-0.00003	0.00000	0.00000	-0.00001
Bock	0.00009	0.00027	-0.00001	0.00005	0.00002	-0.00002	0.00001	-0.00001	-0.00004	0.00000	0.00000	-0.00001
Fixed Rel.	0.00009	0.00020	-0.00010	0.00005	0.00000	-0.00004	0.00002	-0.00001	-0.00003	0.00000	0.00000	-0.00001
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Cov.	0.00030	-0.00060	-0.00012	-0.00009	-0.00029	0.00006	-0.00002	0.00019	0.00024	-0.00001	-0.00001	0.00003
Univar	-0.00013	-0.00020	0.00001	-0.00002	-0.00002	-0.00002	-0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
Large-Sam.	-0.00013	-0.00020	0.00001	-0.00002	-0.00002	-0.00002	-0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
Fuller	-0.00013	-0.00020	0.00001	-0.00002	-0.00002	-0.00002	-0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
Bock	-0.00015	-0.00028	0.00000	-0.00002	0.00001	-0.00005	-0.00001	0.00001	0.00000	-0.00001	0.00001	0.00000
Fixed Rel.	-0.00013	-0.00020	0.00001	-0.00002	-0.00002	-0.00002	-0.00001	0.00000	0.00000	0.00000	0.00000	0.00000

Table 9 (Cont'd)

Case 2. ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .1

Covs.	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}
Expected	0.00061	0.00468	0.00698	0.00031	0.00234	0.00349	0.00012	0.00094	0.00140	0.00006	0.00047	0.00070
$\rho_{11}=\rho_{22}=\rho_{33}=.7$	n=50			n=100			n=250			n=500		
Emp. Cov.	0.00008	0.00788	0.00923	0.00059	0.00328	0.00550	0.00026	0.00128	0.00217	-0.00008	0.00073	0.00092
Univar	0.00048	0.00437	0.00627	0.00028	0.00223	0.00336	0.00012	0.00092	0.00137	0.00006	0.00046	0.00069
Large-Sam.	0.00048	0.00676	0.00947	0.00029	0.00338	0.00500	0.00013	0.00139	0.00202	0.00007	0.00070	0.00101
Fuller	0.00048	0.00676	0.00947	0.00029	0.00338	0.00500	0.00013	0.00139	0.00202	0.00007	0.00070	0.00101
Bock	0.00050	0.00468	0.00639	0.00030	0.00228	0.00350	0.00012	0.00094	0.00141	0.00006	0.00047	0.00070
Fixed Rel.	0.00121	0.00736	0.01003	0.00068	0.00367	0.00525	0.00029	0.00150	0.00211	0.00015	0.00075	0.00106
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Cov.	0.00085	0.00613	0.00932	0.00027	0.00261	0.00444	0.00012	0.00110	0.00161	0.00009	0.00059	0.00090
Univar	0.00047	0.00439	0.00657	0.00027	0.00225	0.00339	0.00012	0.00092	0.00138	0.00006	0.00046	0.00070
Large-Sam.	0.00042	0.00534	0.00792	0.00025	0.00272	0.00405	0.00012	0.00111	0.00164	0.00006	0.00056	0.00083
Fuller	0.00042	0.00534	0.00792	0.00025	0.00272	0.00405	0.00012	0.00111	0.00164	0.00006	0.00056	0.00083
Bock	0.00050	0.00450	0.00691	0.00028	0.00231	0.00352	0.00012	0.00093	0.00141	0.00006	0.00047	0.00071
Fixed Rel.	0.00078	0.00562	0.00817	0.00043	0.00285	0.00416	0.00019	0.00116	0.00168	0.00009	0.00058	0.00085
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Cov.	0.00073	0.00537	0.01006	0.00058	0.00335	0.00564	-0.00009	0.00121	0.00182	0.00007	0.00054	0.00107
Univar	0.00056	0.00426	0.00640	0.00029	0.00225	0.00334	0.00012	0.00092	0.00137	0.00006	0.00047	0.00069
Large-Sam.	0.00054	0.00560	0.00963	0.00027	0.00291	0.00494	0.00012	0.00119	0.00202	0.00006	0.00060	0.00101
Fuller	0.00054	0.00560	0.00963	0.00027	0.00291	0.00494	0.00012	0.00119	0.00202	0.00006	0.00060	0.00101
Bock	0.00107	0.00479	0.00663	0.00057	0.00258	0.00349	0.00022	0.00109	0.00146	0.00009	0.00056	0.00074
Fixed Rel.	0.00080	0.00591	0.00999	0.00041	0.00305	0.00510	0.00017	0.00124	0.00207	0.00008	0.00063	0.00104
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Cov.	0.00104	0.00539	0.00704	0.00007	0.00261	0.00357	0.00007	0.00091	0.00135	0.00012	0.00047	0.00076
Univar	0.00053	0.00441	0.00668	0.00029	0.00230	0.00339	0.00012	0.00093	0.00138	0.00006	0.00047	0.00070
Large-Sam.	0.00053	0.00441	0.00668	0.00029	0.00230	0.00339	0.00012	0.00093	0.00138	0.00006	0.00047	0.00070
Fuller	0.00053	0.00441	0.00668	0.00029	0.00230	0.00339	0.00012	0.00093	0.00138	0.00006	0.00047	0.00070
Bock	0.00054	0.00453	0.00708	0.00027	0.00236	0.00354	0.00011	0.00094	0.00141	0.00006	0.00047	0.00070
Fixed Rel.	0.00053	0.00441	0.00668	0.00029	0.00230	0.00339	0.00012	0.00093	0.00138	0.00006	0.00047	0.00070

Table 9 (Cont'd)

Case 3. ρ_{12} = .4 ρ_{13} = .3 ρ_{23} = .7

Covs.	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}
Expected	0.01019	0.00137	0.00281	0.00509	0.00069	0.00141	0.00204	0.00027	0.00056	0.00102	0.00014	0.00028
$\rho_{11}=\rho_{22}=\rho_{33}=.7$	n=50				n=100			n=250			n=500	
Emp. Cov.	0.01484	0.00143	0.00512	0.00821	0.00079	0.00220	0.00331	0.00036	0.00120	0.00154	0.00010	0.00047
Univar	0.00947	0.00129	0.00274	0.00494	0.00067	0.00137	0.00201	0.00027	0.00056	0.00101	0.00013	0.00028
Large-Sam.	0.01475	0.00196	0.00487	0.00757	0.00101	0.00239	0.00305	0.00041	0.00096	0.00153	0.00020	0.00049
Fuller	0.01475	0.00196	0.00487	0.00757	0.00101	0.00239	0.00305	0.00041	0.00096	0.00153	0.00020	0.00049
Bock	0.00941	0.00116	0.00257	0.00495	0.00063	0.00131	0.00201	0.00026	0.00055	0.00101	0.00013	0.00028
Fixed Rel.	0.01630	0.00384	0.00680	0.00831	0.00197	0.00337	0.00334	0.00081	0.00136	0.00167	0.00040	0.00069
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Cov.	0.01241	0.00187	0.00364	0.00642	0.00080	0.00173	0.00266	0.00023	0.00066	0.00112	0.00013	0.00042
Univar	0.00971	0.00133	0.00269	0.00497	0.00068	0.00138	0.00203	0.00027	0.00056	0.00101	0.00014	0.00028
Large-Sam.	0.01187	0.00151	0.00345	0.00603	0.00076	0.00175	0.00245	0.00030	0.00071	0.00122	0.00015	0.00036
Fuller	0.01187	0.00151	0.00345	0.00603	0.00076	0.00175	0.00245	0.00030	0.00071	0.00122	0.00015	0.00036
Bock	0.00968	0.00119	0.00250	0.00493	0.00063	0.00132	0.00201	0.00027	0.00055	0.00101	0.00014	0.00028
Fixed Rel.	0.01258	0.00239	0.00437	0.00637	0.00121	0.00221	0.00258	0.00048	0.00089	0.00129	0.00024	0.00045
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Cov.	0.01154	0.00092	0.00524	0.00582	0.00071	0.00225	0.00227	0.00024	0.00093	0.00121	0.00020	0.00047
Univar	0.00963	0.00131	0.00278	0.00500	0.00066	0.00138	0.00201	0.00028	0.00056	0.00101	0.00014	0.00028
Large-Sam.	0.01096	0.00160	0.00487	0.00566	0.00079	0.00240	0.00227	0.00033	0.00097	0.00115	0.00017	0.00048
Fuller	0.01096	0.00160	0.00487	0.00566	0.00079	0.00240	0.00227	0.00033	0.00097	0.00115	0.00017	0.00048
Bock	0.00833	0.00102	0.00224	0.00426	0.00053	0.00110	0.00169	0.00022	0.00043	0.00085	0.00011	0.00021
Fixed Rel.	0.01175	0.00289	0.00641	0.00604	0.00146	0.00321	0.00242	0.00061	0.00130	0.00122	0.00030	0.00064
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Cov.	0.01108	0.00142	0.00285	0.00507	0.00079	0.00157	0.00200	0.00029	0.00054	0.00104	0.00017	0.00029
Univar	0.00991	0.00130	0.00269	0.00501	0.00066	0.00139	0.00202	0.00027	0.00056	0.00102	0.00014	0.00028
Large-Sam.	0.00991	0.00130	0.00269	0.00501	0.00066	0.00139	0.00202	0.00027	0.00056	0.00102	0.00014	0.00028
Fuller	0.00991	0.00130	0.00269	0.00501	0.00066	0.00139	0.00202	0.00027	0.00056	0.00102	0.00014	0.00028
Bock	0.00973	0.00119	0.00249	0.00496	0.00064	0.00134	0.00202	0.00027	0.00055	0.00101	0.00014	0.00028
Fixed Rel.	0.00991	0.00130	0.00269	0.00501	0.00066	0.00139	0.00202	0.00027	0.00056	0.00102	0.00014	0.00028

Table 9 (Cont'd)

Case 4. ρ_{12} = .6 ρ_{13} = .4 ρ_{23} = .2

Covs.	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}
Expected	0.00086	0.00427	0.00925	0.00043	0.00214	0.00462	0.00017	0.00085	0.00185	0.00009	0.00043	0.00092
$\rho_{11}=\rho_{22}=\rho_{33}=.7$		n=50			n=100			n=250			n=500	
Emp. Cov.	0.00016	0.00549	0.01454	-0.00004	0.00386	0.00721	-0.00002	0.00155	0.00274	0.00022	0.00070	0.00133
Univar	0.00079	0.00392	0.00871	0.00044	0.00209	0.00439	0.00017	0.00085	0.00182	0.00009	0.00043	0.00092
Large-Sam.	0.00084	0.00655	0.01349	0.00050	0.00340	0.00668	0.00018	0.00137	0.00274	0.00009	0.00069	0.00138
Fuller	0.00084	0.00655	0.01348	0.00050	0.00340	0.00668	0.00018	0.00137	0.00274	0.00009	0.00069	0.00138
Bock	0.00075	0.00385	0.00866	0.00041	0.00206	0.00440	0.00017	0.00084	0.00182	0.00008	0.00042	0.00092
Fixed Rel.	0.00224	0.00793	0.01465	0.00125	0.00411	0.00724	0.00049	0.00166	0.00296	0.00025	0.00083	0.00148
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Cov.	0.00016	0.00549	0.01454	-0.00004	0.00386	0.00721	-0.00002	0.00155	0.00274	0.00022	0.00070	0.00133
Univar	0.00080	0.00416	0.00863	0.00042	0.00209	0.00450	0.00017	0.00084	0.00184	0.00009	0.00042	0.00092
Large-Sam.	0.00074	0.00522	0.01052	0.00039	0.00260	0.00544	0.00015	0.00105	0.00221	0.00008	0.00053	0.00111
Fuller	0.00074	0.00522	0.01052	0.00039	0.00260	0.00544	0.00015	0.00105	0.00221	0.00008	0.00053	0.00111
Bock	0.00077	0.00401	0.00859	0.00041	0.00204	0.00450	0.00016	0.00083	0.00184	0.00008	0.00042	0.00092
Fixed Rel.	0.00141	0.00588	0.01106	0.00074	0.00294	0.00570	0.00029	0.00118	0.00231	0.00015	0.00059	0.00116
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Cov.	0.00046	0.00576	0.01358	0.00056	0.00296	0.00726	0.00017	0.00102	0.00316	0.00005	0.00051	0.00140
Univar	0.00072	0.00405	0.00872	0.00042	0.00208	0.00446	0.00017	0.00084	0.00183	0.00009	0.00042	0.00092
Large-Sam.	0.00065	0.00557	0.01337	0.00039	0.00282	0.00677	0.00016	0.00114	0.00275	0.00008	0.00057	0.00138
Fuller	0.00065	0.00557	0.01337	0.00039	0.00282	0.00677	0.00016	0.00114	0.00275	0.00008	0.00057	0.00138
Bock	0.00107	0.00349	0.00744	0.00058	0.00175	0.00382	0.00024	0.00069	0.00158	0.00012	0.00034	0.00081
Fixed Rel.	0.00118	0.00627	0.01413	0.00067	0.00317	0.00714	0.00027	0.00128	0.00290	0.00014	0.00064	0.00145
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Cov.	0.00117	0.00443	0.01000	0.00024	0.00193	0.00455	0.00017	0.00094	0.00187	0.00011	0.00044	0.00095
Univar	0.00078	0.00407	0.00894	0.00042	0.00210	0.00454	0.00017	0.00084	0.00184	0.00009	0.00043	0.00092
Large-Sam.	0.00078	0.00407	0.00894	0.00042	0.00210	0.00454	0.00017	0.00084	0.00184	0.00009	0.00043	0.00092
Fuller	0.00078	0.00407	0.00894	0.00042	0.00210	0.00454	0.00017	0.00084	0.00184	0.00009	0.00043	0.00092
Bock	0.00071	0.00389	0.00896	0.00041	0.00205	0.00453	0.00017	0.00083	0.00184	0.00009	0.00042	0.00092
Fixed Rel.	0.00078	0.00407	0.00894	0.00042	0.00210	0.00454	0.00017	0.00084	0.00184	0.00009	0.00043	0.00092

Table 9 (Cont'd)

Case 5. $\rho_{12} = .6$ $\rho_{13} = .4$ $\rho_{23} = -.2$

Covs.	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}
Expected	-0.00298	0.00533	0.00995	-0.00149	0.00266	0.00498	-0.00060	0.00107	0.00199	-0.00030	0.00053	0.00100
$\rho_{11}=\rho_{22}=\rho_{33}=.7$	n=50			n=100			n=250			n=500		
Emp. Cov.	-0.00767	0.00987	0.01579	-0.00349	0.00505	0.00759	-0.00146	0.00198	0.00299	-0.00068	0.00097	0.00154
Univar	-0.00279	0.00498	0.00908	-0.00146	0.00260	0.00474	-0.00059	0.00106	0.00195	-0.00030	0.00053	0.00099
Large-Sam.	-0.00622	0.00931	0.01487	-0.00315	0.00473	0.00760	-0.00127	0.00191	0.00309	-0.00063	0.00095	0.00156
Fuller	-0.00619	0.00926	0.01477	-0.00314	0.00473	0.00759	-0.00127	0.00191	0.00309	-0.00063	0.00095	0.00156
Bock	-0.00271	0.00487	0.00918	-0.00137	0.00256	0.00482	-0.00056	0.00105	0.00198	-0.00029	0.00052	0.00100
Fixed Rel.	-0.00604	0.00993	0.01555	-0.00301	0.00500	0.00786	-0.00121	0.00202	0.00318	-0.00060	0.00100	0.00160
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Cov.	-0.00534	0.00741	0.01289	-0.00267	0.00336	0.00636	-0.00099	0.00147	0.00258	-0.00043	0.00076	0.00126
Univar	-0.00298	0.00509	0.00947	-0.00149	0.00259	0.00487	-0.00059	0.00105	0.00198	-0.00030	0.00053	0.00099
Large-Sam.	-0.00450	0.00694	0.01189	-0.00224	0.00350	0.00609	-0.00089	0.00141	0.00246	-0.00045	0.00071	0.00123
Fuller	-0.00450	0.00694	0.01189	-0.00224	0.00350	0.00609	-0.00089	0.00141	0.00246	-0.00045	0.00071	0.00123
Bock	-0.00268	0.00498	0.00939	-0.00138	0.00253	0.00492	-0.00056	0.00101	0.00200	-0.00029	0.00053	0.00100
Fixed Rel.	-0.00444	0.00724	0.01220	-0.00220	0.00365	0.00623	-0.00087	0.00147	0.00251	-0.00044	0.00074	0.00125
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Cov.	-0.00456	0.00703	0.01646	-0.00215	0.00372	0.00851	-0.00068	0.00161	0.00314	-0.00043	0.00080	0.00143
Univar	-0.00292	0.00506	0.00935	-0.00146	0.00259	0.00484	-0.00060	0.00106	0.00196	-0.00030	0.00053	0.00099
Large-Sam.	-0.00387	0.00765	0.01516	-0.00194	0.00388	0.00774	-0.00079	0.00156	0.00311	-0.00039	0.00078	0.00156
Fuller	-0.00387	0.00765	0.01516	-0.00194	0.00388	0.00774	-0.00079	0.00156	0.00311	-0.00039	0.00078	0.00156
Bock	-0.00097	0.00414	0.00997	-0.00382	0.00205	0.00533	-0.00013	0.00082	0.00223	-0.00006	0.00041	0.00113
Fixed Rel.	-0.00426	0.00780	0.01525	-0.00214	0.00393	0.00774	-0.00087	0.00158	0.00309	-0.00043	0.00079	0.00155
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Cov.	-0.00353	0.00512	0.01024	-0.00158	0.00246	0.00499	-0.00067	0.00101	0.00212	-0.00032	0.00057	0.00100
Univar	-0.00306	0.00524	0.00955	-0.00151	0.00261	0.00491	-0.00060	0.00107	0.00197	-0.00030	0.00053	0.00099
Large-Sam.	-0.00306	0.00524	0.00955	-0.00151	0.00261	0.00491	-0.00060	0.00107	0.00197	-0.00030	0.00053	0.00099
Fuller	-0.00306	0.00524	0.00955	-0.00151	0.00261	0.00491	-0.00060	0.00107	0.00197	-0.00030	0.00053	0.00099
Bock	-0.00278	0.00504	0.00953	-0.00136	0.00256	0.00497	-0.00056	0.00106	0.00199	-0.00029	0.00053	0.00100
Fixed Rel.	-0.00306	0.00524	0.00955	-0.00151	0.00261	0.00491	-0.00060	0.00107	0.00197	-0.00030	0.00053	0.00099

Table 9 (Cont'd)

Case 6. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .1

Covs.	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}
Expected	-0.00029	0.00590	0.00874	-0.00014	0.00295	0.00437	-0.00006	0.00118	0.00175	-0.00003	0.00059	0.00087
$\rho_{11}=\rho_{22}=\rho_{33}=.7$	n=50			n=100			n=250			n=500		
Emp. Cov.	-0.00307	0.01063	0.01477	-0.00164	0.00568	0.00732	-0.00068	0.00229	0.00275	-0.00027	0.00109	0.00134
Univar	-0.00028	0.00563	0.00805	-0.00016	0.00288	0.00423	-0.00006	0.00117	0.00172	-0.00003	0.00059	0.00087
Large-Sam.	-0.00231	0.01052	0.01378	-0.00118	0.00527	0.00705	-0.00046	0.00212	0.00283	-0.00023	0.00105	0.00143
Fuller	-0.00228	0.01041	0.01361	-0.00118	0.00526	0.00704	-0.00046	0.00212	0.00283	-0.00023	0.00105	0.00143
Bock	-0.00029	0.00546	0.00784	-0.00014	0.00281	0.00417	-0.00006	0.00116	0.00170	-0.00003	0.00058	0.00087
Fixed Rel.	-0.00092	0.01252	0.01562	-0.00046	0.00625	0.00792	-0.00016	0.00251	0.00318	-0.00008	0.00125	0.00160
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Cov.	-0.00091	0.00861	0.01095	-0.00096	0.00364	0.00569	-0.00029	0.00148	0.00214	-0.00013	0.00073	0.00115
Univar	-0.00044	0.00563	0.00847	-0.00018	0.00288	0.00432	-0.00006	0.00117	0.00174	-0.00003	0.00059	0.00087
Large-Sam.	-0.00149	0.00756	0.01086	-0.00069	0.00383	0.00548	-0.00027	0.00156	0.00219	-0.00013	0.00078	0.00110
Fuller	-0.00149	0.00756	0.01086	-0.00069	0.00383	0.00548	-0.00027	0.00156	0.00219	-0.00013	0.00078	0.00110
Bock	-0.00032	0.00540	0.00821	-0.00015	0.00279	0.00422	-0.00006	0.00116	0.00172	-0.00003	0.00058	0.00087
Fixed Rel.	-0.00091	0.00852	0.01171	-0.00039	0.00431	0.00590	-0.00015	0.00175	0.00236	-0.00007	0.00087	0.00118
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Cov.	-0.00093	0.00928	0.01436	-0.00063	0.00423	0.00738	-0.00013	0.00165	0.00286	-0.00009	0.00089	0.00148
Univar	-0.00047	0.00569	0.00825	-0.00018	0.00288	0.00424	-0.00006	0.00117	0.00172	-0.00003	0.00059	0.00087
Large-Sam.	-0.00115	0.00844	0.01404	-0.00051	0.00425	0.00707	-0.00019	0.00171	0.00285	-0.00010	0.00086	0.00143
Fuller	-0.00115	0.00843	0.01402	-0.00051	0.00425	0.00707	-0.00019	0.00171	0.00285	-0.00010	0.00086	0.00143
Bock	0.00027	0.00389	0.00656	0.00021	0.00187	0.00332	0.00010	0.00074	0.00134	0.00005	0.00036	0.00067
Fixed Rel.	-0.00102	0.00923	0.01488	-0.00044	0.00464	0.00748	-0.00017	0.00187	0.00301	-0.00008	0.00093	0.00151
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Cov.	-0.00042	0.00616	0.00858	-0.00017	0.00323	0.00469	-0.00004	0.00119	0.00184	-0.00004	0.00057	0.00088
Univar	-0.00046	0.00581	0.00852	-0.00019	0.00292	0.00431	-0.00006	0.00117	0.00174	-0.00003	0.00059	0.00087
Large-Sam.	-0.00046	0.00581	0.00852	-0.00019	0.00292	0.00431	-0.00006	0.00117	0.00174	-0.00003	0.00059	0.00087
Fuller	-0.00046	0.00581	0.00852	-0.00019	0.00292	0.00431	-0.00006	0.00117	0.00174	-0.00003	0.00059	0.00087
Bock	-0.00032	0.00550	0.00814	-0.00015	0.00284	0.00420	-0.00006	0.00116	0.00172	-0.00003	0.00059	0.00087
Fixed Rel.	-0.00046	0.00581	0.00852	-0.00019	0.00292	0.00431	-0.00006	0.00117	0.00174	-0.00003	0.00059	0.00087

Table 9 (Cont'd)

Case 7. ρ_{12} = .7 ρ_{13} = .6 ρ_{23} = .8

Covs.	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}	ρ_{12},ρ_{13}	ρ_{12},ρ_{23}	ρ_{13},ρ_{23}
Expected	0.00446	0.00118	0.00235	0.00223	0.00059	0.00118	0.00089	0.00024	0.00047	0.00045	0.00012	0.00024
$\rho_{11}=\rho_{22}=\rho_{33}=.7$	n=50			n=100			n=250			n=500		
Emp. Cov.	0.00961	0.00264	0.00594	0.00463	0.00099	0.00250	0.00167	0.00045	0.00100	0.00083	0.00017	0.00045
Univar	0.00446	0.00130	0.00242	0.00226	0.00060	0.00118	0.00090	0.00024	0.00047	0.00045	0.00012	0.00024
Large-Sam.	0.00870	0.00281	0.00529	0.00431	0.00128	0.00258	0.00172	0.00051	0.00102	0.00085	0.00025	0.00051
Fuller	0.00865	0.00280	0.00528	0.00431	0.00128	0.00258	0.00172	0.00051	0.00102	0.00085	0.00025	0.00051
Bock	0.00430	0.00111	0.00227	0.00223	0.00054	0.00113	0.00089	0.00023	0.00046	0.00045	0.00012	0.00023
Fixed Rel.	0.01227	0.00624	0.00890	0.00614	0.00304	0.00444	0.00246	0.00123	0.00178	0.00122	0.00061	0.00089
$\rho_{11}=\rho_{22}=\rho_{33}=.85$												
Emp. Cov.	0.00633	0.00149	0.00360	0.00306	0.00065	0.00156	0.00107	0.00026	0.00060	0.00061	0.00016	0.00030
Univar	0.00448	0.00119	0.00236	0.00226	0.00059	0.00118	0.00089	0.00024	0.00047	0.00044	0.00012	0.00023
Large-Sam.	0.00603	0.00153	0.00333	0.00302	0.00075	0.00165	0.00119	0.00030	0.00066	0.00059	0.00015	0.00033
Fuller	0.00603	0.00153	0.00333	0.00302	0.00075	0.00165	0.00119	0.00030	0.00066	0.00059	0.00015	0.00033
Bock	0.00441	0.00105	0.00222	0.00224	0.00055	0.00114	0.00089	0.00023	0.00047	0.00045	0.00012	0.00023
Fixed Rel.	0.00764	0.00302	0.00496	0.00383	0.00150	0.00247	0.00152	0.00060	0.00099	0.00076	0.00030	0.00049
$\rho_{11}=.9$ $\rho_{22}=.8$ $\rho_{33}=.7$												
Emp. Cov.	0.00634	0.00181	0.00482	0.00247	0.00062	0.00218	0.00115	0.00022	0.00084	0.00054	0.00012	0.00044
Univar	0.00451	0.00124	0.00241	0.00224	0.00060	0.00120	0.00089	0.00024	0.00047	0.00045	0.00012	0.00024
Large-Sam.	0.00546	0.00184	0.00530	0.00270	0.00088	0.00260	0.00107	0.00036	0.00102	0.00054	0.00017	0.00051
Fuller	0.00545	0.00184	0.00530	0.00270	0.00088	0.00260	0.00107	0.00036	0.00102	0.00054	0.00017	0.00051
Bock	0.00354	0.00069	0.00135	0.00179	0.00032	0.00064	0.00070	0.00013	0.00025	0.00034	0.00006	0.00012
Fixed Rel.	0.00727	0.00402	0.00807	0.00362	0.00200	0.00402	0.00144	0.00081	0.00160	0.00072	0.00040	0.00080
$\rho_{11}=\rho_{22}=\rho_{33}=1.00$												
Emp. Cov.	0.00449	0.00117	0.00241	0.00233	0.00066	0.00130	0.00091	0.00024	0.00049	0.00045	0.00012	0.00023
Univar	0.00451	0.00118	0.00242	0.00224	0.00059	0.00120	0.00089	0.00024	0.00047	0.00045	0.00012	0.00024
Large-Sam.	0.00451	0.00118	0.00242	0.00225	0.00059	0.00120	0.00089	0.00024	0.00047	0.00045	0.00012	0.00024
Fuller	0.00451	0.00118	0.00242	0.00225	0.00059	0.00120	0.00089	0.00024	0.00047	0.00045	0.00012	0.00024
Bock	0.00446	0.00103	0.00225	0.00225	0.00055	0.00115	0.00089	0.00023	0.00047	0.00045	0.00012	0.00024
Fixed Rel.	0.00451	0.00118	0.00242	0.00225	0.00059	0.00120	0.00089	0.00024	0.00047	0.00045	0.00012	0.00024

Covariances. The covariances were derived and calculated using the same formulation as the variances, and seem to yield similar results. Table 9 displays the covariances. Again, the univariate and Bock and Petersen corrections produce covariances that are identical or nearly identical to those expected given the population values. And, these two corrections yield covariances which are far different from those found in the empirical sampling distribution of the corrected correlations.

The covariance results are not as dramatic as the variance results. Often the sampling distribution values are not close to any of the corrected results. This is especially true in the case (.60, .40, .20), and for the smaller sample sizes (n = 50 and n = 100) for many of the cases. At the larger sample sizes, again it is clear that the large-sample formulations give results closest to the sampling distribution results, and the univariate corrections seem to give covariances which are too small.

An Application of the Methods to Existing Data

The methods discussed in this study were used to reanalyze an existing meta-analysis, to see if any differences were apparent, particularly in the decision to accept or reject a homogeneity test calculated from correlation coefficients. This example is a reanalysis of the data from Becker and Cho (1994), which, in turn, was a reanalysis of the data from Schmidt, Hunter and Outerbridge (1986). A computer program was written in Fortran which allowed the synthesis of the data and the calculation of homogeneity tests. These results were then compared to the results found in Becker and Cho (1994) and Schmidt, Hunter, and Outerbridge (1986).

The original Schmidt, Hunter and Outerbridge (1986) study examined four studies, each containing 10 correlations. These ten correlations summarized the relationships among 5 variables: job knowledge, general mental ability, work sample performance, supervisory ratings of job performance, and job experience. Complete data was available for all correlations, and reliability values were given for every measure. Chapter two contains more details about this study.

Testing the homogeneity of the correlation matrices. For this example, the generalized least squares methods used by Becker (1992) will be used. A formal hypothesis test can be used to determine whether the data obtained from several studies are consistent with the hypothesis of a common correlation matrix. Let $\underset{\sim}{\rho}_1, \ldots, \underset{\sim}{\rho}_4$ and $\mathbf{r}^c{}_1, \ldots, \mathbf{r}^c{}_4$ be the vectors of corrected correlations of length $\underline{p}(\underline{p}+1)/2 = \underline{p}^* = 10$ from each of the $\underline{k}=4$ studies, and let $\mathbf{\Sigma}_1, \ldots, \mathbf{\Sigma}_4$ be the large-sample covariance matrices of $\mathbf{r}^c{}_1, \ldots, \mathbf{r}^c{}_4$. The correlations are corrected with the univariate correction. The difference between this example and the methods from Becker and Cho (1994) will be the use of the large-sample theory variance-covariance matrices. Becker and Cho's results used the traditional univariate corrected correlations without adjusting for the variances of the reliabilities and the covariances among the correlations and reliabilities. Define the $\underline{k}$ $\underline{p}^*$ dimensional vector $\mathbf{r}$, the $\underline{k}$ $\underline{p}^*$ x $\underline{p}^*$ matrix $\mathbf{X}$, and the $\underline{k}$ $\underline{p}^*$ x $\underline{k}$ $\underline{p}^*$ matrix $\boldsymbol{\Sigma}$ by

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{r}_k \end{bmatrix}, \qquad \mathbf{X} = \begin{bmatrix} \mathbf{I}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{I}_k \end{bmatrix}, \text{and}$$

$$\boldsymbol{\Sigma} = \text{diag}(\boldsymbol{\Sigma}_1, \ \ldots, \ \boldsymbol{\Sigma}_k),$$

where $\mathbf{I}_1$, ..., $\mathbf{I}_k$ are identity matrices of order $\underline{p}^*$ (Becker 1992).

A test of the hypothesis of homogeneity of correlation matrices across studies, that is to test

$$H_0 : \underset{\sim}{\rho}_1 = \ldots = \underset{\sim}{\rho}_k,$$

uses the statistic

$$\underline{Q} = \mathbf{r}^{c\prime} \ [\boldsymbol{\Sigma}^{-1} - \boldsymbol{\Sigma}^{-1} \mathbf{X}(\mathbf{X}' \ \boldsymbol{\Sigma}^{-1} \ \mathbf{X})^{-1} \ \mathbf{X}' \ \ \boldsymbol{\Sigma}^{-1} \ ] \ \mathbf{r}^{c}$$

(Becker, 1992, p. 349). When H_0 is true then $\underline{Q}$ has approximately a chi-square distribution with ($\underline{k}$-1) $\underline{p}^*$ degrees of freedom. In this example, $\underline{k}$ = 4, and $\underline{p}^*$ =10, so the degrees of freedom are 30.

The first difference between this example and the analyses given in Schmidt, Hunter, and Outerbridge (1986) is the overall test of the hypothesis of the common correlation matrix. Schmidt et al. did not conduct any tests; they simply averaged the correlations and fit a path model using the average correlations.

Estimating a common correlation matrix. If the studies share a common population correlation matrix, then a common correlation matrix can be estimated. The estimate uses **r** as the outcome data for a generalized least squares regression analysis. To estimate a common correlation vector of length p^*, the model is

$$\mathbf{r} = \mathbf{X}\, \underset{\sim}{\rho} + \mathbf{e},$$

where $\mathbf{r}^c$ is defined as above, and $\underset{\sim}{\rho}' = (\rho_1, \rho_2, \ldots, \rho_{p*})$ is the set of common correlations to be estimated, and **X** is defined as above. The generalized least squares estimate of $\underset{\sim}{\rho}$ is given by

$$\mathbf{r}^c. = (\mathbf{X}'\, \boldsymbol{\Sigma}^{-1}\, \mathbf{X})^{-1}\, \mathbf{X}'\, \boldsymbol{\Sigma}^{-1}\, \mathbf{r}^c \quad .$$

The approximate variance-covariance matrix for this estimate is given by

$$\mathbf{V} = (\mathbf{X}'\, \boldsymbol{\Sigma}^{-1}\, \mathbf{X})^{-1}$$

(Becker, 1992, p. 348).

Results from the example. A FORTRAN program estimated the pooled correlation matrix and the homogeneity test. Three different methods were made to evaluate the effect of the new method of estimating the variance-covariance matrix of the corrected correlations. These three methods were: (1) corrected correlations were used in estimating population values, with the large-sample variance method, (2) corrected correlations were used, with the variances calculated as in

Becker and Cho (1994) by assuming reliabilities equal to 1.00 and (3) raw (uncorrected) correlations were used, and perfect reliabilities were assumed.

Table 10 shows the different values for the three methods. The Q_E statistic given in the table is the test of homogeneity (30 df). With all methods, the decision to reject this test would be the same; the correlations appear to be heterogeneous. It is clear from the Q_T statistics, that the new method of estimating the variances does suggest more heterogeneous results. This is expected given that the variability in reliabilities is now being considered.

Also shown in Table 10 are the vectors of the average correlation matrices based on the different estimates of variance. These are compared with the results found in Schmidt et al. (1986). The correlations obtained using the new method are slightly larger. Because the average correlation vector is based on a variance weighted average, and the variances using this new method are larger, there is reason to assume that these numbers will be different. The standard errors for the average correlations from the Becker and Cho and new methods are also shown. Because the new method accounts for variability in reliabilities, the standard errors from the new method are larger.

This example demonstrates the effect of using a more defensible variance estimate. Not only are the variances with this new method larger, but other statistics such as homogeneity test statistics, are impacted as well. Using this new method when correcting correlations may be warranted.

Table 10

Methods and Results from Example Data

	New Method	Becker & Cho	Raw Correlations	Schmidt, et al.
Q_T	6072.95	5993.21	4383.38	------
Q_B	5993.62	5816.34	4231.77	------
Q_E	139.33	176.87	151.60	------
Vectors of Average Correlations:				
	0.48(.025)	0.49(.022)	0.41	0.46
	0.43(.027)	0.43(.023)	0.36	0.38
	0.18(.035)	0.18(.025)	0.13	0.16
	0.06(.027)	0.07(.025)	0.06	0.00
	0.87(.015)	0.87(.014)	0.71	0.80
	0.46(.032)	0.45(.023)	0.33	0.42
	0.64(.018)	0.63(.017)	0.59	0.57
	0.40(.034)	0.39(.024)	0.27	0.37
	0.64(.020)	0.62(.018)	0.55	0.56
	0.26(.031)	0.25(.024)	0.20	0.24

CHAPTER V
SUMMARY AND CONCLUSIONS

Results

The results presented in Chapter 4 indicate success in investigating the results of using various corrections for attenuation in a multivariate setting. The purpose of this dissertation was to determine which (if any) corrections gave reasonable results, and were based on reasonable assumptions for use in multivariate syntheses. The goal was to address 5 research questions noted in Chapter 1. The results related to each question are addressed, in order, below.

1. **What are the consequences of using a simple univariate correction for each of a set of multiple correlations?** This question was addressed mainly in the simulation study. Results showed that using a univariate correction would give a good approximation to the population correlation, on average. However, some potential problems were found. The variance of the univariate corrected correlation should not be computed using the corrected value in the familiar approximate variance formula (3.1). The variance-covariance matrix of the corrected correlations depends on the correlations and on the reliabilities, and must take these values into account. Computing the corrected correlation with the univariate correction, and substituting the resulting values into Equations (3.1) and (3.2) to give the variances and covariances yielded results that were markedly different from

the empirical values. The variances and covariances using the univariate approach were too small.

Several other problems were also associated with this correction. These problems included out-of-range corrected correlations, invalid determinants of corrected correlation matrices and invalid determinants of variance-covariance matrices of the corrected correlations. The smaller the sample size, and the closer the correlation values to the reliability values, the more problems found. Having any of these problems could lead to inaccurate results in a synthesis. Therefore, the simple answer to this question is that using a univariate correction, without adjusting the variance estimates, and without considering the nature of the resulting matrices, could result in invalid intermediate results with unknown consequences if analyzed further. Other corrections and adjustments are warranted.

2. **What would be the difference in variances and covariances based on the univariate correction (mentioned above), versus using a variance-covariance matrix derived from large-sample distribution theory for correlation coefficients?** The difference between these two estimates occurs in the calculation of the variance-covariance matrices for the corrected correlations. The large-sample theory estimates were often equal to the empirical sampling-distribution values, while the univariate variances and covariances were too small (up to 50% smaller than they should be based on the

sampling distributions). This reiterates the findings reported for Question 1, which indicated that the large-sample variances and covariances should be used.

3. **Several multivariate attenuation corrections exist for raw data, including many in the regression literature. How do these corrections compare to one another and to the corrections mentioned above?** Exact comparisons showed that the existing corrections all yield the traditional univariate correction, unless the resulting corrected correlation matrix was problematic. The Bock and Petersen (1975) and Fuller and Hidiroglou (1978) corrections had contingencies for situations in which the initial corrected correlation matrix was not positive definite or had other problems. These contingencies (i.e., further adjustments) were not often used, but the Fuller and Hidiroglou correction seems to produce the best outcomes. The Bock and Petersen (1975) method produced results far from the population values when the reliability triple with unequal (and relatively large) values was used. In all other cases, the two methods produced similar results.

None of the existing methods provided estimators of the variance-covariance matrices of the corrected correlations. This was expected with the Gleser (1992) and Fuller and Hidiroglou (1978) corrections, as they were designed to correct for measurement errors in regression slopes. Similarly, the Bock and Petersen correction was designed to correct data for use in covariance-component estimation.

Because none of the three methods provided estimates of variance-covariance matrices, comparisons of these methods were made on the basis of variance-covariance estimators derived using other means. The results of those comparisons are summarized below.

4. **Which correction is most feasible and provides the best results?** The traditional univariate correction seems to give a reasonable estimate of the population correlation coefficient based on the simulation results in this work. The adjustments made in specific cases using the Bock and Petersen (1975) and Fuller and Hidiroglou (1978) methods did not improve this estimate, except when the usual estimators gave invalid results. In fact, the Bock and Petersen estimate gave poorer results in certain cases.

The Fuller and Hidiroglou correction seems best overall, because no out-of-range correlations or determinants were found, and the corrected correlations were very close to the population values. The variance-covariance matrices derived using this method along with the large-sample method also gave the best estimates. However, the Fuller and Hidiroglou method requires raw data, and therefore needs to be modified for use with summary data (sufficient statistics) such as correlations.

The results of this study indicate that if correlations are corrected, the variance-covariance matrices of these correlations must be adjusted. The traditional variance formulas do not account for dependence among the correlations and reliabilities. The method introduced in this work does provide for reasonable estimates of the variances and covariances, and should be used in synthesizing correlation matrices and in other situations where corrected correlations are to be analyzed.

Finally, the matrix of corrected correlations must be examined, to make certain that this matrix is valid. Currently there is no method for adjusting the corrected correlation matrix, unless raw data are available. If raw data exists, then the Fuller and Hidiroglou method should be used. If not, then the simple univariate correction with the large-sample variance-covariance matrix should be used.

5. **How do these corrections effect results of multivariate syntheses?** Correcting correlations without adjusting the resulting variance-covariance matrix leads to different results than not correcting or not adjusting. The results of this simulation show that modifying both the correlations and their variance-covariance matrices are necessary if the sample values are to approach the population parameters in the long run. The reanalysis of the Schmidt, Hunter and Outerbridge (1986) example showed that the homogeneity tests produced evidence for decisions similar to those in previous analyses. In other examples this may not be the case. Also, the magnitudes of the average correlations from a series of studies will most certainly change,

potentially leading to substantively different interpretations.

Other Findings

The greatest weakness of this synthesis example was the inability to transform the regression corrections into corrections that could be made directly on correlations rather than raw data. This led to the further problem of being unable to compute variances of correlations based on these methods. Further work may lead to a creative solution to these problems; the Bock and Petersen correction seems most likely to lead to a solution. However, given the results reported above for Bock and Petersen method, it is not clear whether having such a correction for summary data would be of any benefit. A conversion of the Fuller and Hidiroglou method to one which considers correlational data instead of raw data would be desirable.

One other limitation arose because of the difficulty of making exact comparisons among the methods for the three-variable case. Though the simulated data clearly showed where differences occurred, it would have helped to be able to illustrate the differences without relying on simulation data. However, the complexity of the formulas limited this process.

Applications of this Work

This work is intended to be applicable to multivariate syntheses of correlational data. The results are also applicable to any case where a correlation matrix is to be corrected and the variances and covariances of the correlations are considered important to the results. Using these corrections on raw data or for correcting slopes may be feasible. However, those methods were not part of this simulation. Certainly an extension in that direction is possible.

Further Investigations

There are several directions which can be taken to further this research. First, the Bock and Petersen correction can be investigated further, especially to determine why conflicting results were found when the reliability vector of unequal values (.90, .80, .70) was used in the simulation. More importantly, an application of the Fuller and Hidiroglou method to correlational data should be found.

Second, the effects of correcting correlations on the results of syntheses could be quantified. This would involve simulating homogeneity statistics and comparing the decisions made and the magnitudes of the average correlations under different methods of estimating variance-covariance matrices. The results of such a study would indicate the overall effect of the method chosen for practical purposes.

Another area of research could include applying these methods along with a Fisher's Z transformation. Work from Becker and Cho (1994) and Becker and Fahrbach (1994) have considered this transformation in multivariate synthesis and found it beneficial. However, no one has attempted to combine that transformation with corrected correlations, while adjusting the resulting variance-covariance matrix for the corrected correlations in a multivariate situation.

Another research possibility includes simulating correlated measurement errors to see how they function, and to see how the methods described here could be applied in such situations. Little is known about correlated errors, and the magnitudes of such errors. This work could investigate the effects of correlated errors on corrections and on the resulting variance-covariance matrices.

Finally, not enough is known about hypothetical reliability distributions and their applications. Missing data is a pernicious problem throughout all meta-analytic work, and reliability values are often missing. Combining these corrections with work on missing data in meta-analysis could be another entire dissertation.

Conclusions

The application of the findings of this work to meta-analysis is warranted. However, the use of corrections (especially the univariate method) could still lead to correlations greater than unity. As a result, data should be examined carefully to determine if correlations should be corrected, or if the reliability estimates are so uncertain that correcting may lead to more problems than it solves.

In conclusion, this dissertation outlines what is currently known about multivariate corrections for attenuation, with application to synthesis of correlation matrices. If measurement error is to be corrected, syntheses must adjust the resulting variance-covariance matrix of the corrected correlations accordingly. Unadjusted results could be misleading, since the (unadjusted) variances and covariances will be underestimated. The traditional univariate correction is appropriate in most cases. The variance-covariance matrix estimate should be the large-sample variance derived in this study.

APPENDICES

APPENDIX A

VARIANCES AND COVARIANCES OF CORRECTED CORRELATIONS

The variance of a sample correlation (r_{12}) corrected for attenuation in both variables:

$$V(\rho'_{12}) = \{V(r_{12})/(r_{11}*r_{22}) - (r_{12}*C(r_{12},r_{22}))/(r^2_{11}*r_{22}) - (r_{12}*C(r_{12},r_{22}))/(r^2_{22}*r_{11}) + (r^2_{12}*C(r_{11},r_{22}))/(2*r^2_{22}*r^2_{11}) + (r^2_{12}*V(r_{11}))/(4*r^3_{11}*r_{22}) + (r^2_{12}*V(r_{22}))/(4*r^3_{22}*r_{11})\}/n$$

$$\begin{aligned} C(\rho'_{12}, \rho'_{13}) = \{ & C(r_{12},r_{13})/(r_{11}* \sqrt{r_{22}*r_{33}}) \\ & - (r_{12}*C(r_{13},r_{11}))/(2*r^2_{11}* \sqrt{r_{22}*r_{33}}) \\ & - (r_{12}*C(r_{13},r_{22}))/(2*r_{11}*r_{22}* \sqrt{r_{22}*r_{33}}) \\ & - (r_{13}*C(r_{11},r_{12}))/(2*r^2_{11}* \sqrt{r_{22}*r_{33}}) \\ & + (r_{12}*r_{13}*V(r_{11}))/(4*r^3_{11}* \sqrt{r_{22}*r_{33}}) \\ & + (r_{12}*r_{13}*C(r_{11},r_{22}))/(4*r^2_{11}*r_{22}* \sqrt{r_{22}*r_{33}}) \\ & - (r_{13}*C(r_{12},r_{33}))/(2*r_{11}*r_{33}* \sqrt{r_{22}*r_{33}}) \\ & + (r_{12}*r_{13}*C(r_{11},r_{33}))/(4*r^2_{11}*r_{33}* \sqrt{r_{22}*r_{33}}) \\ & + (r_{12}*r_{13}*C(r_{22},r_{33}))/(4*r_{11}*r_{33}*r_{22}* \sqrt{r_{22}*r_{33}})\}/n \end{aligned}$$

Where V and C represent the usual variance and covariance functions given in Equations 3.1 and 3.2 in the text.

APPENDIX B

COVARIANCES AMONG CORRELATIONS (INCLUDING RELIABILITIES)

$$C(r_{11}, r_{12}) = \{r_{12}*(r^2{}_{11} - 1)*(0.5*r_{11} -1) + r^3{}_{12}*(r_{11} -1)\}/n$$

$$C(r_{11}, r_{13}) = \{r_{13}*(r^2{}_{11} - 1)*(0.5*r_{11} -1) + r^3{}_{13}*(r_{11} -1)\}/n$$

$$C(r_{22}, r_{12}) = \{r_{12}*(r^2{}_{22} - 1)*(0.5*r_{22} -1) + r^3{}_{12}*(r_{22} -1)\}/n$$

$$C(r_{22}, r_{23}) = \{r_{23}*(r^2{}_{22} - 1)*(0.5*r_{22} -1) + r^3{}_{23}*(r_{22} -1)\}/n$$

$$C(r_{33}, r_{13}) = \{r_{13}*(r^2{}_{33} - 1)*(0.5*r_{33} -1) + r^3{}_{13}*(r_{33} -1)\}/n$$

$$C(r_{33}, r_{23}) = \{r_{23}*(r^2{}_{33} - 1)*(0.5*r_{33} -1) + r^3{}_{23}*(r_{33} -1)\}/n$$

$$C(r_{11}, r_{23}) = \{(r_{11} - 1)*(r_{23}*r^2{}_{12} + r_{23}*r^2{}_{13} - 2*r_{12}*r_{13})\}/n$$

$$C(r_{22}, r_{13}) = \{(r_{22} - 1)*(r_{13}*r^2{}_{12} + r_{13}*r^2{}_{23} - 2*r_{12}*r_{23})\}/n$$

$$C(r_{33}, r_{12}) = \{(r_{33} - 1)*(r_{12}*r^2{}_{13} + r_{12}*r^2{}_{23} - 2*r_{13}*r_{23})\}/n$$

$$C(r_{11}, r_{22}) = \{2*r^2{}_{12} *(r_{11}*r_{22} + 1 - r_{11} - r_{22})\}/n$$

$$C(r_{11}, r_{33}) = \{2*r^2{}_{13} *(r_{11}*r_{33} + 1 - r_{11} - r_{33})\}/n$$

$$C(r_{22}, r_{33}) = \{2*r^2{}_{23} *(r_{22}*r_{33} + 1 - r_{22} - r_{33})\}/n$$

$$C(r_{12}, r_{13}) = \{0.5*(2*r_{23} - r_{12}*r_{13})*(1 - r^2{}_{12} - r^2{}_{13} - r^2{}_{23}) + r^3{}_{23}\}/n$$

$$C(r_{12}, r_{23}) = \{0.5*(2*r_{13} - r_{12}*r_{23})*(1 - r^2{}_{12} - r^2{}_{13} - r^2{}_{23}) + r^3{}_{13}\}/n$$

$$C(r_{13}, r_{23}) = \{0.5*(2*r_{12} - r_{13}*r_{23})*(1 - r^2{}_{12} - r^2{}_{13} - r^2{}_{23}) + r^3{}_{12}\}/n$$

APPENDIX C

METHODS USED FOR CORRECTING FOR ATTENUATION

Univariate Correction

The corrected correlation is approximated by

$$\rho'_{xy}=\underline{r}_{xy}/(r_{xx}*r_{yy})^{1/2},$$

where ρ'_{xy} is the estimated population $\underline{r}_{xy}$ is the sample correlation, and r_{xx} and r_{yy} are the sample reliabilities.

The Fuller and Hidiroglou Correction

The regression estimates are corrected using:

$$\boldsymbol{\beta} = \mathbf{H}^{-1}\ (n^{-1}\ \mathbf{X'}\ \mathbf{Y}),$$

where $\mathbf{H}^{-1} = (n^{-1}\ \mathbf{X'X}) - \mathbf{D}\ \boldsymbol{\Lambda}\ \mathbf{D}$, $\mathbf{D}$ is a diagonal matrix of standard deviations of the predictors and $\boldsymbol{\Lambda}$ is a diagonal matrix containing 1 - reliability values. To guarantee that $\mathbf{H}$ is positive definite, $\mathbf{DAD}$ is pre-multiplied by the quantity $(f - n^{-1})$, if $f < (1 + \underline{n}^{-1})$; where f is the smallest root (eigenvalue) in the two-matrix eigenproblem $| \mathbf{M} - f\mathbf{CGC} | = 0$. Here $\mathbf{C}$ is the matrix of standard deviations of the raw scores (including the outcome), $\mathbf{M}$ is $1/\underline{n}$ times the sum-of-squares and cross-products matrix and $\mathbf{G}$ is a diagonal matrix containing reliability values of the outcome and the predictors. This calculation also guarantees that the estimated variance-covariance of the true variables is positive definite. The corrected correlations are calculated from $\mathbf{H}^{-1}$, by dividing each off-diagonal element by the product of the square roots of the adjacent diagonal elements.

The Bock and Petersen Correction

Consider $\mathbf{M_e}$ and $\mathbf{M_y}$, the mean error and the mean observed sum of squares and cross products matrices, respectively. Solve the two matrix eigenproblem $(\mathbf{M_y} - \lambda_i \mathbf{M_e})\ x_i = 0$ (Bock and Petersen, 1975, p. 674). Once this problem is solved, the estimate of the true variance-covariance matrix of the raw scores can be made using the following formulation. Let $\mathbf{X} = (x_i, \ldots x_p)$ be the matrix of eigenvectors, let $\mathbf{\Lambda}^* = \text{diag}(\lambda_i, \ldots, \lambda_p)$ be the matrix of eigenvalues, and let $\mathbf{I_p}$ be the p x p identity matrix, then $\mathbf{\Sigma_t} = \mathbf{M_y} - \mathbf{M_e} = \mathbf{B}'\ (\mathbf{\Lambda}^* - \mathbf{I_p})\ \mathbf{B}$ where $\mathbf{B} = \mathbf{X}^{-1}$. If any of the elements of the $\mathbf{\Lambda}$ matrix are less than one, they are replaced by 1.0 in the calculation. The corrected correlations are then found by dividing each off-diagonal element of $\mathbf{\Sigma_t}$ by the product of the square roots of the adjacent diagonal elements.

The Gleser Correction

Let $\mathbf{\Lambda} = \mathbf{\Sigma^{-1}_{obs}} * \mathbf{\Sigma_{true}}$ represent the reliability matrix, where $\mathbf{\Sigma^{-1}_{obs}}$ is the variance-covariance matrix of the observed predictors and $\mathbf{\Sigma_{true}}$ is the variance-covariance matrix of the true scores for the predictors. Then, the adjustment in the regression case is $\boldsymbol{\beta} = \mathbf{\Lambda}^{-1} * (\mathbf{X}'\mathbf{X})^{-1}\ \mathbf{X}'\mathbf{Y}$. The corrected correlations are then found from taking $(\mathbf{\Lambda}^{-1} * (\mathbf{X}'\mathbf{X})^{-1})^{-1}$. Again, the correlations are a result of dividing the off-diagonal elements of the resulting matrix by the product of the square roots of the adjacent diagonal elements.

APPENDIX D

FORTRAN PROGRAM USED IN SIMULATION

```
C       MAIN PROGRAM
        IMPLICIT REAL (L, M)
        COMMON IT, K, RXY(2000), RXW(2000), RWY(2000),
     + RXX(2000), RYY(2000),RWW(2000),CRXY(2000), CRXW(2000),
     + CRWY(2000), COVT(3,3), COVE(3,3), RXTXE(2000),
     + RYTYE(2000),
     + RWTWE(2000), RXTX(2000), RYTY(2000),RWTW(2000),
     + RXTX2(2000), RYTY2(2000), RWTW2(2000), SS, ITR,
     + IS, GCRXY(2000), GCRXW(2000), GCRYW(2000), HCRXY(2000),
     + HCRXW(2000), HCRYW(2000), BCRXY(2000),
     + BCRXW(2000),BCRYW(2000), FACT(2000), PXTYT, PXTWT,
     + PWTYT, PXX, PYY, PWW, DTSR(2000), DTUR(2000),
     + DTBR(2000), DTFR(2000), DTGR(2000), IMRK(2000)
        PXTYT= 0.70
        PXTWT= 0.60
        PWTYT= 0.80
        PXX= .70
        PYY= .70
        PWW= .70
        IS=  100
        SS=  100.0
        PXEYE=0.0
        PXEWE=0.0
        PWEYE=0.0
        STY=SQRT(PYY)
        STX=SQRT(PXX)
        STW=SQRT(PWW)
        SEY=SQRT(1-STY**2)
        SEX=SQRT(1-STX**2)
        SEW=SQRT(1-STW**2)
        DO 16 J=1,3
        DO 17 I=1,3
        COVE(I,J)=0.0
   17   CONTINUE
   16   CONTINUE
        COVT(1,1)=STX**2
        COVT(1,2)=PXTYT*STX*STY
        COVT(2,1)=PXTYT*STX*STY
        COVT(2,2)=STY**2
        COVT(1,3)=PXTWT*STX*STW
        COVT(3,1)=PXTWT*STX*STW
        COVT(3,3)=STW**2
        COVT(2,3)=PWTYT*STY*STW
        COVT(3,2)=PWTYT*STY*STW
        COVE(1,1)=SEX**2
        COVE(2,2)=SEY**2
        COVE(3,3)=SEW**2
```

```
      COVE(1,2)=PXEYE*SEX*SEY
      COVE(1,3)=PXEWE*SEX*SEW
      COVE(2,3)=PWEYE*SEW*SEY
      COVE(2,1)=COVE(1,2)
      COVE(3,1)=COVE(1,3)
      COVE(3,2)=COVE(2,3)
      ITR=2000
      DO 70 IT=1,ITR
      CALL GETNM
 70   CONTINUE
      CALL VARS
      STOP
      END
         --------------------------
            SUBROUTINE GETNM
      IMPLICIT REAL (L, M)
      DIMENSION X(100), Y(100), W(100), LAM1 (3,3),
     + TCOV(3,3), ECOV(3,3), TRXYW(100,3), ERXYW(100,3),
     + LAM2(3,3), LAM3(3,3), HOLD1(3,3), HOLD2(3,3),
     + HOLD3(3,3), H(3,3),
     + XTX(3,3), M(3,3), ME(3,3), MT(3,3), BEIG(3),
     + SYMINV(3,3), C(3,3),
     + BEINV(3,3), BEINVTR(3,3), FEIG(3), XTXINV(3,3),
     + SYM(3,3), BOCKM(3,3), LAM2INV(3,3), SSCPINV(3,3),
     + TSCP(3,3), ESCP(3,3),TRXYWTR(3,100),
     + ERXYWTR(3,100),
     + CLAM1(3,3), CGC(3,3), HOLD4(3,3),
     + OBS(100,3), OBSTR(3, 100), SSCP(3,3), FVEC(3,3),
     + BVEC(3,3), SSCP2(3,3), LAM2A(3,3), LIDENT(3,3),
     + OBS2INV(3,3), TRU2(3,3),OBS2(3,3)
      COMMON IT, K, RXY(2000), RXW(2000), RWY(2000),
     + RXX(2000), RYY(2000),RWW(2000),CRXY(2000),
     + CRXW(2000),
     + CRWY(2000), COVT(3,3),
     + COVE(3,3), RXTXE(2000), RYTYE(2000), RWTWE(2000),
     + RXTX(2000), RYTY(2000), RWTW(2000), RXTX2(2000),
     + RYTY2(2000), RWTW2(2000), SS,
     + ITR, IS, GCRXY(2000), GCRXW(2000), GCRYW(2000),
     + HCRXY(2000), HCRXW(2000), HCRYW(2000), BCRXY(2000),
     + BCRXW(2000), BCRYW(2000),
     + FACT(2000), PXTYT, PXTWT, PWTYT, PXX, PYY, PWW,
     + DTSR(2000), DTUR(2000), DTBR(2000), DTFR(2000),
     + DTGR(2000), IMRK(2000)
       XS=0.0
       YS=0.0
       WS=0.0
       XS2=0.0
       YS2=0.0
       WS2=0.0
       XTS=0.0
       YTS=0.0
       WTS=0.0
```

```
XTS2=0.0
YTS2=0.0
WTS2=0.0
XES=0.0
YES=0.0
WES=0.0
XES2=0.0
YES2=0.0
WES2=0.0
XYS=0.0
XWS=0.0
WYS=0.0
XTXES=0.0
YTYES=0.0
WTWES=0.0
XTXS=0.0
YTYS=0.0
WTWS=0.0
N=3
IDA=3
IRANK=3
IDR=3
CALL CHFAC(N, COVT, IDA, 0.0001, IRANK, TCOV, IDR)
CALL CHFAC(N, COVE, IDA, 0.0001, IRANK, ECOV, IDR)
CALL RNMVN(IS, 3, ECOV, 3, ERXYW, IS)
CALL RNMVN(IS, 3, TCOV, 3, TRXYW, IS)
DO 30 I=1,IS
THE X'S ARE CREATED AND SUMMED
X(I)=TRXYW(I,1)+ERXYW(I,1)
XS=X(I)+XS
XS2=XS2+X(I)**2
XTS=TRXYW(I,1)+XTS
XTS2=XTS2+TRXYW(I,1)**2
XES=ERXYW(I,1)+XES
XES2=XES2+ERXYW(I,1)**2
THE Y'S ARE CREATED AND SUMMED
Y(I)=TRXYW(I,2)+ERXYW(I,2)
YS=Y(I)+YS
YS2=YS2+Y(I)**2
YTS=TRXYW(I,2)+YTS
YTS2=YTS2+TRXYW(I,2)**2
YES=ERXYW(I,2)+YES
YES2=YES2+ERXYW(I,2)**2
THE W'S ARE CREATED AND SUMMED
W(I)=TRXYW(I,3)+ERXYW(I,3)
WS=W(I)+WS
WS2=WS2+W(I)**2
WTS=TRXYW(I,3)+WTS
WTS2=WTS2+TRXYW(I,3)**2
WES=ERXYW(I,3)+WES
WES2=WES2+ERXYW(I,3)**2
XYS=XYS+X(I)*Y(I)
```

```
      XWS=XWS+X(I)*W(I)
      WYS=WYS+W(I)*Y(I)
      XTXES=XTXES+TRXYW(I,1)*ERXYW(I,1)
      YTYES=YTYES+TRXYW(I,2)*ERXYW(I,2)
      WTWES=WTWES+TRXYW(I,3)*ERXYW(I,3)
      XTXS=XTXS+TRXYW(I,1)*X(I)
      YTYS=YTYS+TRXYW(I,2)*Y(I)
      WTWS=WTWS+TRXYW(I,3)*W(I)
   30 CONTINUE
      XM=XS/SS
      YM=YS/SS
      WM=WS/SS
      XTM=XTS/SS
      YTM=YTS/SS
      WTM=WTS/SS
      XEM=XES/SS
      YEM=YES/SS
      WEM=WES/SS
      VX=((XS2-(SS*XM*XM))/(SS-1.))
      VY=((YS2-(SS*YM*YM))/(SS-1.))
      VW=((WS2-(SS*WM*WM))/(SS-1.))
      VXT=((XTS2-(SS*XTM*XTM))/(SS-1.))
      VYT=((YTS2-(SS*YTM*YTM))/(SS-1.))
      VWT=((WTS2-(SS*WTM*WTM))/(SS-1.))
      VXE=((XES2-(SS*XEM*XEM))/(SS-1.))
      VYE=((YES2-(SS*YEM*YEM))/(SS-1.))
      VWE=((WES2-(SS*WEM*WEM))/(SS-1.))
      RXY(IT)=(XYS-(SS*XM*YM))/((SS-1.)*SQRT(VX*VY))
      RXW(IT)=(XWS-(SS*XM*WM))/((SS-1.)*SQRT(VX*VW))
      RWY(IT)=(WYS-(SS*WM*YM))/((SS-1.)*SQRT(VW*VY))
      RXX(IT)=VXT/(VXT+VXE)
      RYY(IT)=VYT/(VYT+VYE)
      RWW(IT)=VWT/(VWT+VWE)
      CRXY(IT)=RXY(IT)/SQRT(RXX(IT)*RYY(IT))
      CRXW(IT)=RXW(IT)/SQRT(RXX(IT)*RWW(IT))
      CRWY(IT)=RWY(IT)/SQRT(RWW(IT)*RYY(IT))
      OBS2(1,1)=VX
      OBS2(2,2)=VY
      OBS2(3,3)=VW
      OBS2(1,2)=RXY(IT)*SQRT(VX*VY)
      OBS2(1,3)=RXW(IT)*SQRT(VX*VW)
      OBS2(2,3)=RWY(IT)*SQRT(VY*VW)
      OBS2(2,1)=RXY(IT)*SQRT(VX*VY)
      OBS2(3,1)=RXW(IT)*SQRT(VX*VW)
      OBS2(3,2)=RWY(IT)*SQRT(VY*VW)
      TRU2(1,1)=VXT
      TRU2(2,2)=VYT
      TRU2(3,3)=VWT
     TRU2(1,2)=RXY(IT)*SQRT(VXT*VYT)/(SQRT(RXX(IT)*RYY(IT)))
     TRU2(1,3)=RXW(IT)*SQRT(VXT*VWT)/(SQRT(RXX(IT)*RWW(IT)))
     TRU2(2,3)=RWY(IT)*SQRT(VYT*VWT)/(SQRT(RWW(IT)*RYY(IT)))
```

```
      TRU2(2,1)=RXY(IT)*SQRT(VXT*VYT)/(SQRT(RXX(IT)*RYY(IT)))
      TRU2(3,1)=RXW(IT)*SQRT(VXT*VWT)/(SQRT(RXX(IT)*RWW(IT)))
      TRU2(3,2)=RWY(IT)*SQRT(VYT*VWT)/(SQRT(RWW(IT)*RYY(IT)))
       DO 39 I=1,3
           WRITE(6, 101) TRU2(I,1), TRU2(I,2), TRU2(I,3),
     + OBS2(I,1), OBS2(I,2), OBS2(I,3)
 101   FORMAT(1H ,3(F10.7,2X), 4X, 3(F10.7,2X))
  39   CONTINUE
C     THIS NEXT PART FINDS THE CORRECTED CORRELATIONS USING
C      THE METHODS OF FULLER, GLESER, AND BOCK
C      FIRST, THE SET UP, THEN THE FULLER METHOD
       DO 22 I=1, IS
       OBS(I,1)=X(I)
       OBS(I,2)=Y(I)
       OBS(I,3)=W(I)
  22   CONTINUE
       DO 23 I=1, IS
       DO 24 J=1, 3
       OBSTR(J,I)=OBS(I,J)
       TRXYWTR(J,I)=TRXYW(I,J)
       ERXYWTR(J,I)=ERXYW(I,J)
  24   CONTINUE
  23   CONTINUE
       CALL MRRRR(3,IS,OBSTR, 3, IS, 3, OBS, IS,3,3, SSCP, 3)
       DO 34 I=1, 3
       DO 35 J=1, 3
       SSCP2(I,J)=SSCP(I,J)
  35   CONTINUE
  34   CONTINUE
       CALL MRRRR(3,IS,ERXYWTR,3,IS,3,ERXYW,IS, 3,3, ESCP, 3)
       DO 25 I=1,3
       DO 29 J=1,3
       M(I,J)=SSCP(I,J)/(SS)
       ME(I,J)=ESCP(I,J)/(SS)
       LAM1(I,J)=0.0
       LAM3(I,J)=0.0
       LIDENT(I,J)=0.0
       C(I,J)=0.0
       SYM(I,J)=0.0
       SYMINV(I,J)=0.0
  29   CONTINUE
  25   CONTINUE
       LAM1(1,1) = 1.- RXX(IT)
       LAM1(2,2) = 1.- RYY(IT)
       LAM1(3,3) = 1.- RWW(IT)
       LIDENT(1,1)=1.0
       LIDENT(2,2)=1.0
       LIDENT(3,3)=1.0
       C(1,1)=SQRT(VX)
       C(2,2)=SQRT(VY)
       C(3,3)=SQRT(VW)
       CALL MRRRR(3, 3, C, 3, 3, 3, LAM1, 3, 3, 3, CLAM1, 3)
```

```
      CALL MRRRR(3, 3, CLAM1, 3, 3, 3, C, 3, 3, 3, CGC, 3)
      SYMINV(1,1)=1./SQRT(CGC(1,1))
      SYMINV(2,2)=1./SQRT(CGC(2,2))
      SYMINV(3,3)=1./SQRT(CGC(3,3))
      CALL MRRRR(3, 3,SYMINV, 3, 3, 3, M, 3, 3, 3, HOLD1, 3)
      CALL MRRRR(3, 3,HOLD1,3, 3, 3, SYMINV, 3,3,3, HOLD4,3)
C     FIND EIGENS OF CGC WITH 2 MATRIX EIGEN PROBB WITH M
      CALL EVCRG(3, HOLD4, 3, FEIG, FVEC, 3)
      CALL GVCSP(3, HOLD4, 3, LIDENT, 3, FEIG, FVEC, 3)
      IF ((FEIG(1) .LT. FEIG(2)) .AND. (FEIG(1) .LT.
     + FEIG(3))) THEN LOW=FEIG(1)
      ELSE IF (FEIG(2) .LT. FEIG(3)) THEN
        LOW=FEIG(2)
      ELSE
        LOW=FEIG(3)
      END IF
      IF (LOW .LT. (1.+1./SS)) THEN
         FACT(IT)=LOW - (1./SS)
      ELSE
         FACT(IT)=1.0
      END IF
      DO 26 I=1,3
      DO 26 J=1,3
       H(I,J)=M(I,J)-FACT(IT)*CGC(I,J)
 26   CONTINUE
      HCRXY(IT)=H(1,2)/SQRT(H(1,1)*H(2,2))
      HCRXW(IT)=H(1,3)/SQRT(H(1,1)*H(3,3))
      HCRYW(IT)=H(2,3)/SQRT(H(2,2)*H(3,3))
C     NEXT, GLESER
      CALL LINRG(3, SSCP2, 3, SSCPINV, 3)
      CALL LINRG(3, OBS2, 3, OBS2INV, 3)
      CALL MRRRR(3, 3,OBS2INV, 3, 3, 3, TRU2, 3,3,3, LAM2,3)
      CALL LINRG(3, LAM2, 3, LAM2INV, 3)
      CALL MRRRR(3, 3, LAM2INV, 3, 3, 3, SSCPINV, 3, 3, 3,
     + XTXINV,3)
C     NOW FIND THE INV OF XTXINV
      CALL LINRG(3, XTXINV, 3, XTX, 3)
      GCRXY(IT)=XTX(1,2)/SQRT(XTX(1,1)*XTX(2,2))
      GCRXW(IT)=XTX(1,3)/SQRT(XTX(1,1)*XTX(3,3))
      GCRYW(IT)=XTX(2,3)/SQRT(XTX(2,2)*XTX(3,3))
C     NOW FOR BOCK AND PETERSEN
C     FIND EIGENS OF BEIG (SOLVE 2 MATRIX PROBLEM), ALSO
C     EIGENVECS
      CALL GVCSP(3, M, 3, ME, 3, BEIG, BVEC, 3)
      IF ((BEIG(1) .LT. 1.0) .OR. (BEIG(2) .LT. 1.0) .OR.
     + (BEIG(3) .LT. 1.0)) THEN
        IMRK(IT)=1
      ELSE
         IMRK(IT)=0
      END IF
      IF (BEIG(1) .LT. 1.0) THEN
        LAM3(1,1)=0.0
```

```
      ELSE
        LAM3(1,1)=BEIG(1) - 1.0
      END IF
      IF (BEIG(2) .LT. 1.0) THEN
         LAM3(2,2)=0.0
      ELSE
         LAM3(2,2)=BEIG(2) - 1.0
      END IF
      IF (BEIG(3) .LT. 1.0) THEN
         LAM3(3,3)=0.0
      ELSE
         LAM3(3,3)=BEIG(3) - 1.0
      END IF
C     NAME BVEC THE MATRIX OF EIGENVECS, FIND BEINV
      CALL LINRG(3, BVEC, 3, BEINV, 3)
      DO 28 I=1,3
      DO 28 J=1,3
       BEINVTR(J,I)=BEINV(I,J)
28    CONTINUE
      CALL MRRRR(3, 3,BEINVTR,3, 3, 3, LAM3, 3,3,3,HOLD3, 3)
      CALL MRRRR(3, 3,HOLD3,3, 3, 3, BEINV, 3,3,3, BOCKM, 3)
      BCRXY(IT)=BOCKM(1,2)/SQRT(BOCKM(1,1)*BOCKM(2,2))
      BCRXW(IT)=BOCKM(1,3)/SQRT(BOCKM(1,1)*BOCKM(3,3))
      BCRYW(IT)=BOCKM(2,3)/SQRT(BOCKM(2,2)*BOCKM(3,3))
      DTSR(IT)=DETMN1(RXY(IT), RXW(IT), RWY(IT))
      DTUR(IT)=DETMN1(CRXY(IT), CRXW(IT), CRWY(IT))
      DTBR(IT)=DETMN1(BCRXY(IT), BCRXW(IT), BCRYW(IT))
      DTFR(IT)=DETMN1(HCRXY(IT), HCRXW(IT), HCRYW(IT))
      DTGR(IT)=DETMN1(GCRXY(IT), GCRXW(IT), GCRYW(IT))
      RETURN
      END
      FUNCTION DETMN1 (A, B, C)
      DETMN1=1.0+(2.0*A*B*C)-(A**2)-(B**2)-(C**2)
      RETURN
      END
```

```
C      ------------------------------
       SUBROUTINE VARS
       IMPLICIT REAL (L, M)
       DIMENSION V(3,3), A(3,6), PSI(6,6), AT(6,3), TEMP1(3,6),
      +Q5(3,3), DETM(2000), VARM1(2000), VARM2(2000),
      +VARM3(2000), COVM12(2000), COVM13(2000), COVM23(2000),
      +VARUV1(2000), VARUV2(2000), VARUV3(2000), CVUV1(2000),
      + CVUV2(2000), CVUV3(2000), FA(3,6), FAT(6,3),
      + TEMP2(3,6), FV(3,3), FV1(2000), FV2(2000), FV3(2000),
      + FC12(2000), FC13(2000), FC23(2000), Q1(3,3),
      + DETF(2000),  Q2(3,3), Q3(3,3), VU(3,3), VB(3,3),
      +VARB1(2000), VARB2(2000), VARB3(2000), CVB1(2000),
      + CVB2(2000), CVB3(2000), VARF1(2000), VARF2(2000),
      + VARF3(2000), COVF1(2000), COVF2(2000), COVF3(2000),
      + DETU(2000), DETB(2000), DETFIX(2000), VFX(3,3), Q4(3,3)
       COMMON IT, K, RXY(2000), RXW(2000), RWY(2000),
      + RXX(2000), RYY(2000),RWW(2000),CRXY(2000), CRXW(2000),
      + CRWY(2000), COVT(3,3),
      + COVE(3,3), RXTXE(2000), RYTYE(2000), RWTWE(2000),
      + RXTX(2000), RYTY(2000), RWTW(2000), RXTX2(2000),
      + RYTY2(2000), RWTW2(2000), SS,
      + ITR, IS, GCRXY(2000), GCRXW(2000), GCRYW(2000),
      + HCRXY(2000), HCRXW(2000), HCRYW(2000), BCRXY(2000),
      + BCRXW(2000), BCRYW(2000),
      + FACT(2000), PXTYT, PXTWT, PWTYT, PXX, PYY, PWW,
      + DTSR(2000), DTUR(2000), DTBR(2000), DTFR(2000),
      + DTGR(2000), IMRK(2000)
        DO 10 IT=1, ITR
        IMT1=0
        IMT2=0
        ICB1=0
        ICB2=0
        ICB3=0
        ICG1=0
        ICG2=0
        ICG3=0
        ICF1=0
        ICF2=0
        ICF3=0
        ICU1=0
        ICU2=0
        ICU3=0
        IDB1=0
        IDF1=0
        IDU1=0
        IDM1=0
        IDX1=0
        IDRB1=0
        IDRF1=0
        IDRU1=0
        IDRS1=0
```

```
      IDRG1=0
      IFCNT=0
      IMT3=0
      DO 15 I=1,3
      DO 16 J=1,6
      A(I,J)=0.0
      FA(I,J)=0.0
16    CONTINUE
15    CONTINUE
      IF ((IMRK (IT) .EQ. 1 ) .AND. (DTBR(IT) .LT. .0000001))
     + THEN
       IMT3=1
      ENDIF
      IF (FACT(IT) .LT. 1.0) THEN
       IFCNT=1
      ENDIF
      IF ((FACT(IT) .LT. 1.0) .AND. (DTUR(IT) .LT. .0000001))
     + THEN
         IMT1= 1
      ENDIF
      IF ((FACT(IT) .LT. 1.0) .AND. (DTGR(IT) .LT. .0000001))
     + THEN
        IMT2= 1
      ENDIF
      IF (CRXY(IT) .GT. 1.0) THEN
         ICU1=1
      ENDIF
      IF (CRXW(IT) .GT. 1.0) THEN
           ICU2=1
      ENDIF
      IF (CRWY(IT) .GT. 1.0) THEN
         ICU3=1
      ENDIF
      IF (BCRXY(IT) .GT. 1.0) THEN
         ICB1=1
      ENDIF
      IF (BCRXW(IT) .GT. 1.0) THEN
         ICB2=1
      ENDIF
      IF (BCRYW(IT) .GT. 1.0) THEN
         ICB3=1
      ENDIF
      IF (HCRXY(IT) .GT. 1.0) THEN
         ICF1=1
      ENDIF
      IF (HCRXW(IT) .GT. 1.0) THEN
         ICF2=1
      ENDIF
      IF (HCRYW(IT) .GT. 1.0) THEN
         ICF3=1
      ENDIF
      IF (GCRXY(IT) .GT. 1.0) THEN
```

```
         ICG1=1
      ENDIF
      IF (GCRXW(IT) .GT. 1.0) THEN
         ICG2=1
      ENDIF
      IF (GCRYW(IT) .GT. 1.0) THEN
         ICG3=1
      ENDIF
      IF (DTSR(IT) .LE. 0.0000001) THEN
         IDRS1=1
      ENDIF
      IF (DTFR(IT) .LE. 0.0000001) THEN
         IDRF1=1
      ENDIF
      IF (DTUR(IT) .LE. 0.0000001) THEN
         IDRU1=1
      ENDIF
      IF (DTBR(IT) .LE. 0.0000001) THEN
         IDRB1=1
      ENDIF
      IF (DTGR(IT) .LE. 0.0000001) THEN
         IDRG1=1
      ENDIF
      A(1,1)=1./SQRT(RXX(IT)*RYY(IT))
      A(2,2)=1./SQRT(RXX(IT)*RWW(IT))
      A(3,3)=1./SQRT(RWW(IT)*RYY(IT))
      A(1,4)=-RXY(IT)/(2*RXX(IT)*SQRT(RYY(IT)*RXX(IT)))
      A(1,5)=-RXY(IT)/(2*RYY(IT)*SQRT(RYY(IT)*RXX(IT)))
      A(2,4)=-RXW(IT)/(2*RXX(IT)*SQRT(RWW(IT)*RXX(IT)))
      A(2,6)=-RXW(IT)/(2*RWW(IT)*SQRT(RWW(IT)*RXX(IT)))
      A(3,5)=-RWY(IT)/(2*RYY(IT)*SQRT(RWW(IT)*RYY(IT)))
      A(3,6)=-RWY(IT)/(2*RWW(IT)*SQRT(RWW(IT)*RYY(IT)))
      FP4=(1.-FACT(IT)+FACT(IT)*RXX(IT))
      FP5=(1.-FACT(IT)+FACT(IT)*RYY(IT))
      FP6=(1.-FACT(IT)+FACT(IT)*RWW(IT))
      FA(1,1)=1./SQRT(FP4*FP5)
      FA(2,2)=1./SQRT(FP4*FP6)
      FA(3,3)=1./SQRT(FP5*FP6)
      FA(1,4)=-RXY(IT)*FACT(IT)/(2*FP4*SQRT(FP4*FP5))
      FA(1,5)=-RXY(IT)*FACT(IT)/(2*FP5*SQRT(FP4*FP5))
      FA(2,4)=-RXW(IT)*FACT(IT)/(2*FP4*SQRT(FP4*FP6))
      FA(2,6)=-RXW(IT)*FACT(IT)/(2*FP6*SQRT(FP4*FP6))
      FA(3,5)=-RWY(IT)*FACT(IT)/(2*FP5*SQRT(FP5*FP6))
      FA(3,6)=-RWY(IT)*FACT(IT)/(2*FP6*SQRT(FP5*FP6))
      DO 18 I=1,3
      DO 19 J=1,6
      AT(J,I)=A(I,J)
      FAT(J,I)=FA(I,J)
19    CONTINUE
18    CONTINUE
      PSI(1,1)=(1.-RXY(IT)*RXY(IT))**2/SS
      PSI(2,2)=(1.-RXW(IT)*RXW(IT))**2/SS
```

```
      PSI(3,3)=(1.-RWY(IT)*RWY(IT))**2/SS
      PSI(4,4)=(1.-RXX(IT)*RXX(IT))**2/SS
      PSI(5,5)=(1.-RYY(IT)*RYY(IT))**2/SS
      PSI(6,6)=(1.-RWW(IT)*RWW(IT))**2/SS
      PSI(1,2)=COVAR4(RWY(IT), RXY(IT), RXW(IT), SS)
      PSI(1,3)=COVAR4(RXW(IT), RXY(IT), RWY(IT), SS)
      PSI(1,4)=COVAR1(RXY(IT), RXX(IT), SS)
      PSI(1,5)=COVAR1(RXY(IT), RYY(IT), SS)
      PSI(1,6)=COVAR2(RWW(IT), RXY(IT), RXW(IT), RWY(IT), SS)
      PSI(2,3)=COVAR4(RXY(IT), RXW(IT), RWY(IT), SS)
      PSI(2,4)=COVAR1(RXW(IT), RXX(IT), SS)
      PSI(2,5)=COVAR2(RYY(IT), RXW(IT), RXY(IT), RWY(IT), SS)
      PSI(2,6)=COVAR1(RXW(IT), RWW(IT), SS)
      PSI(3,4)=COVAR2(RXX(IT), RWY(IT), RXY(IT), RXW(IT), SS)
      PSI(3,5)=COVAR1(RWY(IT), RYY(IT), SS)
      PSI(3,6)=COVAR1(RWY(IT), RWW(IT), SS)
      PSI(4,5)=COVAR3(RXY(IT), RXX(IT), RYY(IT), SS)
      PSI(4,6)=COVAR3(RXW(IT), RXX(IT), RWW(IT), SS)
      PSI(5,6)=COVAR3(RWY(IT), RYY(IT), RWW(IT), SS)
      DO 21 I=1,6
      DO 22 J=1,6
      PSI(J,I)=PSI(I,J)
22    CONTINUE
21    CONTINUE
      CALL MRRRR(3, 6, A, 3, 6, 6, PSI, 6, 3, 6, TEMP1, 3)
      CALL MRRRR(3, 6, TEMP1, 3, 6, 3, AT, 6, 3, 3, V, 3)
      CALL MRRRR(3, 6, FA, 3, 6, 6, PSI, 6, 3, 6, TEMP2, 3)
      CALL MRRRR(3, 6, TEMP2, 3, 6, 3, FAT, 6, 3, 3, FV, 3)
      VARM1(IT)=V(1,1)
      VARM2(IT)=V(2,2)
      VARM3(IT)=V(3,3)
      COVM12(IT)=V(1,2)
      COVM13(IT)=V(1,3)
      COVM23(IT)=V(2,3)
      FV1(IT)=FV(1,1)
      FV2(IT)=FV(2,2)
      FV3(IT)=FV(3,3)
      FC12(IT)=FV(1,2)
      FC13(IT)=FV(1,3)
      FC23(IT)=FV(2,3)
      VARUV1(IT)= ((1 - CRXY(IT)*CRXY(IT))**2)/SS
      VARUV2(IT)= ((1 - CRXW(IT)*CRXW(IT))**2)/SS
      VARUV3(IT)= ((1 - CRWY(IT)*CRWY(IT))**2)/SS
      CVUV1(IT)=COVAR4(CRWY(IT),CRXY(IT),CRXW(IT),SS)
      CVUV2(IT)=COVAR4(CRXW(IT),CRXY(IT),CRWY(IT),SS)
      CVUV3(IT)=COVAR4(CRXY(IT),CRXW(IT),CRWY(IT),SS)
      VARB1(IT)= ((1 - BCRXY(IT)*BCRXY(IT))**2)/SS
      VARB2(IT)= ((1 - BCRXW(IT)*BCRXW(IT))**2)/SS
      VARB3(IT)= ((1 - BCRYW(IT)*BCRYW(IT))**2)/SS
      CVB1(IT)=COVAR4(BCRYW(IT),BCRXY(IT),BCRXW(IT),SS)
      CVB2(IT)=COVAR4(BCRXW(IT),BCRXY(IT),BCRYW(IT),SS)
      CVB3(IT)=COVAR4(BCRXY(IT),BCRXW(IT),BCRYW(IT),SS)
```

```
      VU(1,1)=VARUV1(IT)
      VU(2,2)=VARUV2(IT)
      VU(3,3)=VARUV3(IT)
      VU(1,2)=CVUV1(IT)
      VU(1,3)=CVUV2(IT)
      VU(2,3)=CVUV3(IT)
      VU(2,1)=CVUV1(IT)
      VU(3,1)=CVUV2(IT)
      VU(3,2)=CVUV3(IT)
      VB(1,1)=VARB1(IT)
      VB(2,2)=VARB2(IT)
      VB(3,3)=VARB3(IT)
      VB(1,2)=CVB1(IT)
      VB(1,3)=CVB2(IT)
      VB(2,3)=CVB3(IT)
      VB(2,1)=CVB1(IT)
      VB(3,1)=CVB2(IT)
      VB(3,2)=CVB3(IT)
      DO 202 I=1,3
      DO 202 J=1,3
      Q1(I,J)=SS*V(I,J)
      Q2(I,J)=SS*FV(I,J)
      Q3(I,J)=SS*VU(I,J)
      Q4(I,J)=SS*VB(I,J)
202   CONTINUE
      DETM(IT)=DETMN2(Q1(1,1), Q1(2,2), Q1(3,3), Q1(1,2),
     + Q1(1,3), Q1(2,3))
      DETF(IT)=DETMN2(Q2(1,1), Q2(2,2), Q2(3,3), Q2(1,2),
     + Q2(1,3), Q2(2,3))
      DETU(IT)=DETMN2(Q3(1,1), Q3(2,2), Q3(3,3), Q3(1,2),
     + Q3(1,3),  Q3(2,3))
      DETB(IT)=DETMN2(Q4(1,1), Q4(2,2), Q4(3,3), Q4(1,2),
     + Q4(1,3), Q4(2,3))
      IF (DETM(IT) .LE. 0.0000001) THEN
        IDM1=1
      ENDIF
      IF (DETF(IT) .LE. 0.0000001) THEN
       IDF1=1
      ENDIF
      IF (DETU(IT) .LE. 0.0000001) THEN
        IDU1=1
      ENDIF
      IF (DETB(IT) .LE. 0.0000001) THEN
          IDB1=1
      ENDIF
C     HERE I DEAL WITH THE FIXED RELIABILITY ESTIMATES
      VARF1(IT)= PSI(1,1)/(RXX(IT)*RYY(IT))
      VARF2(IT)= PSI(2,2)/(RXX(IT)*RWW(IT))
      VARF3(IT)= PSI(3,3)/(RYY(IT)*RWW(IT))
      COVF1(IT)= PSI(1,2)/(RXX(IT)*SQRT(RYY(IT)*RWW(IT)))
      COVF2(IT)= PSI(1,3)/(RYY(IT)*SQRT(RXX(IT)*RWW(IT)))
      COVF3(IT)= PSI(2,3)/(RWW(IT)*SQRT(RXX(IT)*RYY(IT)))
```

```
      VFX(1,1)=VARF1(IT)
      VFX(2,2)=VARF2(IT)
      VFX(3,3)=VARF3(IT)
      VFX(1,2)=COVF1(IT)
      VFX(1,3)=COVF2(IT)
      VFX(2,3)=COVF3(IT)
      DO 179 I=1,3
      DO 179 J=1,3
      VFX(J,I)=VFX(I,J)
 179  CONTINUE
      DO 203 I=1,3
      DO 203 J=1,3
      Q5(I,J)=SS*VFX(I,J)
 203  CONTINUE
      DETFIX(IT)=DETMN2(Q5(1,1), Q5(2,2), Q5(3,3), Q5(1,2),
     + Q5(1,3), Q5(2,3))
      IF (DETFIX(IT) .LE. 0.0000001) THEN
         IDX1=1
      ENDIF
      I=IT
      WRITE(6,88) RXY(I), RXW(I), RWY(I), RXX(I), RYY(I),
     + RWW(I), CRXY(I), CRXW(I),CRWY(I), DETM(I), DETF(I),
     + DETB(I), DETU(I), DETFIX(I), PXTYT, PXTWT, PWTYT, PXX
      WRITE(6,89) VARM1(I), VARM2(I), VARM3(I), VARUV1(I),
     + VARUV2(I), VARUV3(I), COVM12(I), COVM13(I),
     + COVM23(I), CVUV1(I), CVUV2(I), CVUV3(I)
      WRITE(6,90) HCRXY(I), HCRXW(I), HCRYW(I), GCRXY(I),
     + GCRXW(I), GCRYW(I), BCRXY(I), BCRXW(I), BCRYW(I),
     + FV1(I), FV2(I), FV3(I), FC12(I), FC13(I), FC23(I), IS
      WRITE(6,91) VARB1(I), VARB2(I), VARB3(I), CVB1(I),
     + CVB2(I), CVB3(I), VARF1(I), VARF2(I), VARF3(I),
     + COVF1(I), COVF2(I), COVF3(I), PYY, PWW
 88   FORMAT(1H ,9(F6.3,1X),5(F9.4,1X), 4(F4.2,1X))
 89   FORMAT(1H ,3X, 12(F9.6,1X))
 90   FORMAT(1H ,3X,9(F6.3,1X), 6(F9.6,1X), I3)
 91   FORMAT(1H ,3X, 12(F9.6,1X), 2(F4.2,1X))
      WRITE(6,92) ICU1, ICU2, ICU3, ICB1, ICB2, ICB3, ICF1,
     + ICF2, ICG1, ICG2, ICG3, IFCNT, IMT1, IMT2, IMRK(IT),
     + IMT3, IDU1, IDB1, IDF1, IDM1, IDX1, IDRS1, IDRU1,
     + IDRB1, IDRF1, ID
 92   FORMAT(1H , 27(I1,1X))
 10   CONTINUE
      RETURN
      END
      FUNCTION DETMN2 (A, B, C, D, E, F)
      DETMN2=A*B*C+(2.0*D*E*F)-(C*D**2)-(B*E**2)-(A*F**2)
      RETURN
      END
      FUNCTION COVAR1 (A, B, SN)
      COVAR1=(A*((B**2)-1.)*((B/2.)-1.)+(A**3)*(B-1.))/SN
      RETURN
      END
```

```
      FUNCTION COVAR2 (A, B, C, D, SN)
      COVAR2=((A-1.)*(B*(C**2)+B*(D**2)-2.*C*D))/SN
      RETURN
      END
      FUNCTION COVAR3 (A, B, C, SN)
      COVAR3=(2.*A**2)*(B*C+1.-B-C)/SN
      RETURN
      END
      FUNCTION COVAR4 (A, B, C, SN)
      COVAR4=(0.5*((2.*A)-(B*C))*(1.-(B**2)-(C**2)-(A**2))+
     + A**3)/S
      RETURN
      END
```

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REFERENCES

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