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MATRICES FOR AN URBAN NETWORK USING  
TRAFFIC COUNTS

presented by

Mahkame Megan Khoshyaran

has been accepted towards fulfillment  
of the requirements for


Ph.D. degree in Civil Engineering

William C. Taylor  
Major professor

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**DYNAMIC ESTIMATION OF ORIGIN-DESTINATION  
MATRICES FOR AN URBAN NETWORK USING  
TRAFFIC COUNTS**

By

**Mahkame Megan Khoshyaran**

**A DISSERTATION**

Submitted to  
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## **ABSTRACT**

### **DYNAMIC ESTIMATION OF ORIGIN-DESTINATION MATRICES FOR AN URBAN NETWORK USING TRAFFIC COUNTS**

**By**

**Mahkame Megan Khoshyaran**

**The purpose of this research is to design a model for estimating a dynamic or time dependent origin-destination trip table for a large urban area and to test the model using traffic count data obtained from inductive loops on a sample of arterial streets. So far the only reliable way to obtain an origin destination trip table has been through surveys at origins and destinations. These surveys are expensive and they do not reflect changes in traffic flow that occur over time. The hope of this research is to provide a reliable substitute for the traditional method of obtaining an O-D trip table. The network chosen to test the model is downtown Boston.**

**The method of O-D flow estimation used in this study is sequential. The algorithm used is the Generalized Least Square Method. The input data used to test the model is the on-line traffic information obtained from inductive loops on a sample of local streets. The study period is from 6:00 a.m. to 9:00 a.m. The length of this study is 5 weekdays. The 1987 O-D table provided by the Boston**

Area Regional Planning Authority is used as the apriori O-D table for the first interval of day one.

Local street count data were provided by the City of Boston Traffic Control Center. The validity of O-D estimates were checked by examining how closely the estimated link counts derived from the O-D estimates match the actual link counts. Statistics used to measure the degree of closeness are Root Mean Square Error, and Percent Root Mean Square Error.

The model was validated based on testing using synthetic data where the true O-D volumes were known. The results of the validation proved the mathematical integrity of the model and its robustness. The model, when applied to the urban traffic data, did not produce estimates of link counts which would be suitable for real time traffic control due to the following causes: 1) partial traffic count data; 2) insufficient detector coverage; 3) lack of speed and delay information on all the links in the network; 4) lack of sufficient centroid connectors.

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# Chapter 1

## Introduction

The purpose of this research is to design a model for estimating a dynamic or time dependent origin-destination trip table for a large urban area and to test the model using traffic count data obtained from inductive loops on a sample of the arterial streets in the network. So far the only reliable way to obtain an origin destination trip table has been through surveys at origins and destinations. These surveys are expensive and they do not reflect changes in traffic flow that occur over time. The hope of this research is to provide a reliable substitute to the traditional method of obtaining an O-D trip table.

One potential use for time dependent O-D data is in controlling traffic flow in freeway corridors. Ramp metering strategies need O-D data to determine the optimum on-ramp diversion rates to parallel arterials. Likewise, incident management strategies which divert traffic from freeways to parallel service roads should consider the prevailing corridor O-D pattern. The need for O-D data is especially acute in freeways with express/collector and transfer lanes, and in metropolitan-wide control systems which consider freeway to freeway diversions during peak periods. Dynamic O-D estimates are required for the analysis of time-varying dynamics of freeway congestion growth and decay.

The use of time dependent O-D data is not limited to freeway traffic analysis. An advantage of estimating dynamic O-D data for a large urban area is for use in on-line control of traffic signals. The new generation of traffic signal models,

such as the IVHS model "MOTION" for traffic dependent signal control, rely on O-D flows for the determination of flow profiles, as well as the incremental optimization to adapt signal settings to changing traffic conditions. Control systems in use today can not adapt to the near term projected traffic conditions in the network, because they have no information about the existing O-D flows. These models, currently depend on historical data, if the system includes any forward looking capability. To develop more effective strategies for better on-line signal control it is necessary to improve the existing models for estimating O-D flows.

The ultimate goal of future looking traffic adaptive control systems is to predict variations of flows in a network so that traffic signals can react accordingly. The conventional way of collecting traffic data by traffic surveys is not suitable for this purpose. There have been a considerable number of studies conducted to determine if an O-D matrix could be derived from traffic counts, because these data can be collected and processed automatically [Cremer, Keller, Okutani, Casetta] [3,4,7,1].

The need for reliable time-dependent O-D data for traffic analysis in a metropolitan area has been identified by many European researchers as well. The European research projects DRIVE and PROMETHEUS are designed to collect more and better data about real time traffic flow. MOTION [8] is an on-line signal control system of the DRIVE project. MOTION has been developed by using the experience gained from the German off-line signal control model SIGMA and existing on-line systems in Italy.

**This model determines: (1) split as a function of traffic volume and turning movements at the intersections, (2) cycle time according to the traffic volume at critical intersections, (3) number of phases, and on a future development level, phase sequence as a function of the minimum sum of intergreen times and the time-space-diagrams and (4) offsets as a multi-functional result of origin and destination flows in the network, traffic volumes and an impact analysis of traffic and environmental criteria. Different modules of this system include O-D based estimates of flow at the intersections, and a dynamic optimization plan based on network O-D flow estimates.**

**The reference to signal timing adjustment is to a traffic adaptive signal timing arrangement, not to traditional actuated operation of traffic signals. Under advanced adaptive signal control strategies, adjustments are made in response to O-D demand profiles in a network for various time slices. This allows the system to select the most appropriate signal plan from a predeveloped library. Individual cycle phases are then adjusted based on the volume of traffic on individual streets during a cycle.**

## Chapter 2

### 2. Literature Review

The subject of dynamic origin destination trip table estimation using on-line traffic count data has been investigated by many researchers. So far the main area of concentration has been in estimating time dependent O-D trips for a section of a freeway with few entering and exit ramps, or an intersection with ( $m > 1$ ) entries and ( $n > 1$ ) exits [Cremer, Keller, Cascetta] [3,1]. Until now, only limited studies have been done on estimating time dependent O-D trips for an urban area, using traffic counts received from detectors on arterial roads. A few researchers have tested dynamic O-D estimation models on very small networks; using simulated data [Keller, Ploss, Okutani][6,7]. The networks used in testing different O-D estimation models have 6 or 7 centroids, and a minimum of 13 links and maximum of 40 links [Okutani][7]. These dynamic O-D estimation models produce good O-D estimates under controlled conditions. To validate the dynamic O-D matrix, comparisons have been made with data obtained either from a sample or through simulation.

The idea in dynamic O-D trip table calculations is to estimate time dependent variations in O-D patterns. The assumption is that the O-D volumes are not constant, but rather vary between successive time periods. In estimating O-D volumes for each interval, no matter what kind of modeling approach is used, information such as departure time from the origin, route taken to go from the origin to the destination, total travel time from an origin to a destination given a particular route, and arrival time on various links of a route are needed. The following paragraphs describe some of the issues that need to be considered in

modeling O-D travel. In addition several O-D modeling approaches will be reviewed.

## 2.1 Dynamic O-D estimation models

Several dynamic O-D estimation methods have been tried by researchers. In the early stages of dynamic O-D estimation, the type of geometry considered was an intersection. The parameter estimation was limited to finding the fraction of the volume that entered through entrance  $i$ , which exits from exit  $j$ . Later, freeway segments with a few entering and exit ramps, and small networks were considered. The following are the most significant algorithms originally used in dynamic O-D estimation for an intersection [Cremer, Keller][2,3], and later extended for use in estimating time-varying O-D matrices for networks with few intersections: Constrained optimization method, Ordinary least square estimation, Recursive estimation, and estimation by Kalman filtering. The following are the most significant algorithms used in dynamic O-D estimation for complex networks such as freeways with entering and exit ramps: Linked static -dynamic estimation method [Keller, Ploss][6], estimation by Kalman filtering[Okutani][7], Generalized Least Square Estimation[Cascetta][1]. The method of Recursive Generalized Least Square Estimation is the method used in this study to estimate O-D trip for an urban area. Therefore a thorough description of this method is provided in a later section.

## 2.2 Intersection Flow Models

In many cases traffic flows through a complex intersection can not be measured directly, and therefore must be estimated from vehicle counts at the entries and

exits of the intersection. The following methods were used in estimating the magnitude of traffic flows through complex intersections by using vehicle counts at the entries and exits of the intersection . Furthermore, it was shown that by using the time variant quality of volume counts a unique solution to the problem of estimating unknown flows was obtained.

### 2.2.1 Method of cross-correlation matrices [Cremer, Keller][2,3]

This is a least square estimation method, and was used in estimating O-D flows for an intersection with ( $m > 1$ ) entries, and ( $n > 1$ ) exits. A period  $H$  was chosen. This period was then divided into  $k$  intervals of length  $T$  each. A sample of observations was taken for each period. During each sampling period, the volume  $q(i,k)$  that entered the entrance ( $i$ ) during  $(k-1)T \leq t < kT$  was observed. Volume which left exit  $j$ ,  $y(j,k)$  during  $(k-1)T + \tau \leq t < kT + \tau$ ,  $\tau > 0$  was also observed. The objective was to estimate the part of volume  $q(i,k)$ ,  $f(i,j,k)$  which left the intersection through exit  $j$  during  $(k-1)T + \tau \leq t < kT + \tau$ . where  $\tau$  is the average travel time a vehicle needs to pass from an entry to an exit.  $f(i,j,k)$  is formulated as:

$$f(i,j,k) = q(i,k) \cdot b(i,j,k) \quad (1)$$

$b(i,j,k)$  is the fraction of  $q(i,k)$  that go from  $i$  to  $j$ . The following assumptions hold for  $b(i,j,k)$

$$0 \leq b(i,j,k) \leq 1 \text{ for all } i,j,k \quad (2)$$

$$\sum_{j=1}^n b(i,j,k) = 1 \text{ for all } i,k, \quad (3)$$

$$b(i,i,k) = 0 \text{ for } i=1,\dots,\min(m,n) \quad (4)$$

Balancing the departing and entering vehicles, for each time interval yields the following relationship

$$y(j,k) = \sum_{i=1}^m f(i,j,k) \quad (5)$$

since each O-D flow is a certain portion of the  $i$ th entering flow,  $f(i,j,k)$  can be replaced by its equivalent in equation (1). The following relationship then holds

$$y(j,k) = \sum_{i=1}^m q(i,k) \cdot b(i, j, k) \quad (6)$$

The matrix form of equation (6) is written as:

$$\mathbf{y}'(k) = \mathbf{q}'(k) \cdot \mathbf{B}(k) \quad (7)$$

where  $\mathbf{y}'(k)$  is a  $(1 \times n)$  row vector with elements  $y(j,k)$ ,  $\mathbf{q}'(k)$  is a  $(1 \times m)$  row vector with elements  $q(i,k)$  and  $\mathbf{B}(k)$  is an  $(m \times n)$  matrix with elements  $b(i, j, k)$ .

Since  $\mathbf{q}'(k)$  and  $\mathbf{y}'(k)$  are obtained through sampling, the only parameter that needs to be estimated is  $\mathbf{B}(k)$ . To calculate  $\mathbf{B}(k)$  first, the mean values  $\bar{\mathbf{q}}$ ,  $\bar{\mathbf{y}}'$ , and  $\bar{\mathbf{B}}$  are calculated.

$$\begin{aligned} \bar{\mathbf{q}} &= (1/K) \sum_{k=1}^K \mathbf{q}'(k) \\ \bar{\mathbf{y}}' &= (1/K) \sum_{k=1}^K \mathbf{y}'(k) \\ \bar{\mathbf{B}} &= (1/K) \sum_{k=1}^K \mathbf{B}(k) \end{aligned}$$



$K$  is the total number of sampling intervals.  $q'(k)$ ,  $y'(k)$ , and  $B(k)$  can be expressed as the sum of their mean plus a random deviation

$$\begin{aligned} q'(k) &= \bar{q} + \Delta q'(k) \\ y'(k) &= \bar{y}' + \Delta y'(k) \\ B(k) &= \bar{B} + \Delta B(k) \end{aligned} \quad (8)$$

inserting this into equation (7) gives

$$y'(k) = q'(k) \cdot (\bar{B} + \Delta B(k)) \quad (9)$$

or

$$\bar{y}' + \Delta y'(k) = (\bar{q} + \Delta q'(k)) \cdot (\bar{B} + \Delta B(k)) \quad (10)$$

taking the mean on both sides of equation (10) over a whole observation period, average exit flows are obtained as

$$\bar{y}' = \bar{q} \cdot \bar{B} + 1/K \sum_{k=1}^K \Delta q'(k) \cdot \Delta B(k) \quad (11)$$

subtracting equation (10) from equation (11) and substituting the summation index  $k$  with  $l$  the following relation is obtained

$$\Delta y'(k) = \Delta q'(k) \cdot \bar{B} + \{q'(k) \cdot \Delta B(k) - 1/K \sum_{l=1}^K \Delta q'(l) \cdot \Delta B(l)\} \quad (12)$$

assuming  $\Delta B(k)=0$  causes the second term in equation (12) to vanish resulting in the following

$$\Delta y'(k) = \Delta q'(k) \cdot \bar{B} \quad (13)$$

introducing the following abbreviations

$$\begin{aligned} \mathbf{Q} &= [\Delta \mathbf{q}'(1), \Delta \mathbf{q}'(2), \dots, \Delta \mathbf{q}'(K)]' \\ \mathbf{Y} &= [\Delta \mathbf{y}'(1), \Delta \mathbf{y}'(2), \dots, \Delta \mathbf{y}'(K)]' \end{aligned} \quad (14)$$

prime denotes transpose of a vector. Partitioning the observation period into subsequent time intervals and using time sequence of traffic counts provides additional information which solves the problem of an underdetermined system of equations and leads to a unique solution.

The least square error solution to equation (13) is given by

$$\hat{\mathbf{B}} = (\mathbf{Q}' \mathbf{Q})^{-1} \cdot \mathbf{Q}' \cdot \mathbf{Y} \quad (15)$$

where

$$\begin{aligned} \mathbf{Q}' \mathbf{Q} &= \sum \Delta \mathbf{q}(k) \cdot \Delta \mathbf{q}'(k) = K \cdot \phi_{qq} \\ \mathbf{Q}' \cdot \mathbf{Y} &= \sum \Delta \mathbf{q}(k) \cdot \Delta \mathbf{y}'(k) = K \cdot \phi_{qy} \end{aligned}$$

where  $\phi_{qq}$  and  $\phi_{qy}$  are the cross-correlation matrices correlating the sequences of  $\Delta \mathbf{q}(k)$  with  $\Delta \mathbf{y}(k)$ , respectively over the observation period  $k=1, \dots, K$ . With this notation equation (15) can be rewritten as

$$\hat{\mathbf{B}} = \phi_{qq}^{-1} \cdot \phi_{qy}$$

$\hat{\mathbf{B}}$  is the least square solution of  $\bar{\mathbf{B}}$ .

The above estimation procedure was tested using synthetic and real data from different studies. A 5-leg intersection was considered for testing this model. The synthetic entry flows  $q(i,k)$  were generated as integer random numbers. For the deviations  $\Delta b(i,j,k)$  random numbers from a normal distribution with zero mean and a standard deviation of 0.05 were taken first. Then these deviations were updated such that conditions (3), and (4) were preserved, and gave realistic integer flows  $f(i,j,k)$ . Comparison of the O-D table of synthetic data with those from the dynamic estimation procedure showed a close match [Cremer, Keller] [3,4].

### 2.2.2 Method of constrained optimization [Cremer, Keller] [3,4]

Here the estimation problem described above is reformulated as a constrained optimization problem. Given a sample of entries and exits over  $K$  intervals, an estimate for  $\bar{B}$ ,  $\hat{B}$  is obtained such that the squared error between the observed exit flow, and estimated exit flow is minimized.

$$J = \min \left( \frac{1}{K} \sum_{k=1}^K \| y(k) - \hat{y}(k) \|^2 \right) \quad (16)$$

where

$$y(i,k) = \sum_{i=1}^m q(i,k) \cdot b(i,j,k) \quad \text{for all } j,k$$

$$0 \leq b(i,j,k) \leq 1 \quad \text{for all } i,j,k$$

$$\sum_{j=1}^n b(i,j,k) = 1 \quad \text{for all } i,k$$

$$b(i,j,k) = 0 \quad \text{for } i=1, \dots, \min(m,n)$$

$$\hat{y}'(k) = q'(k) \cdot \hat{B} \quad (17)$$

where  $y'(k)$  is a  $(1 \times n)$  vector of observed exit flows, and  $\hat{y}'(k)$  is the  $(1 \times n)$  vector of measured flows.  $q'(k)$  is a  $(1 \times m)$  vector of volumes that enter through entrance  $i$ .  $\hat{B}$  is an  $(m \times n)$  matrix of estimate for the unknown matrix  $\bar{B}$ . The optimal solution for  $\hat{B}$  can be computed within the admissible parameter space. The solution is found as follows by substituting for  $\hat{y}(k)$  in equation (16)

$$J = ((1/K) \sum_{k=1}^K y'(k) \cdot \hat{y}(k) - 2 \sum_{k=1}^K Q'(k) \cdot \hat{B} + \sum_{k=1}^K q'(k) \cdot \hat{B} \hat{B}' \cdot q(k)) \quad (18)$$

the optimal solution for  $\hat{B}$  can be obtained by setting the first derivative of  $J$  with respect to  $\hat{B}$  equal to zero.

$$J = -2/K \left( \sum_{k=1}^K q(k) \cdot y'(k) \right) + 2/K \left( \sum_{k=1}^K q(k) q'(k) \right) \cdot \hat{B}^* = 0 \quad (19)$$

Here  $\hat{B}^*$  represents the optimal solution under the assumption that the solution lies inside the admissible parameter space. The optimal solution  $\hat{B}^*$  to equation (16) is given by

$$\hat{B}^* = \phi_{qq}^{-1} \phi_{qy}$$

$\phi_{qq}$  and  $\phi_{qy}$  were defined earlier. This method when tried under the same geometry and data described for the cross-correlation method gave better O-D estimates mainly because it uses the information from all equations formulated.

### 2.2.3 Method of Recursive Estimation [Cremer,Keller] [3,4]

Unlike the two methods described before, the predicted exit flow deviations are computed from the measured entry flow deviations based on the estimation of the (k-1)th step by a model similar to equation (13) as follows

$$\Delta y'(k) = \Delta q'(k) \cdot \hat{B}(k-1) \quad (20)$$

this prediction then is inserted into the following recursive correction formula;

$$\hat{B}(k) = \hat{B}(k-1) + \gamma \cdot K(1/K \sum \Delta q(k) \cdot (\Delta y(k) - \Delta \hat{y}(k)) , \text{ summed over } k=1,...,K \quad (21)$$

where  $\gamma$  is a gain factor.

The following assumptions assure the convergence and stability of the above equation

1) The expected values of  $\Delta q(k)$  and  $\Delta B(k)$  are zero.  $\Delta B(k)$  is an  $(m \times n)$  matrix of random deviations in split parameter  $B$ .

$$\begin{aligned} E(\Delta q(k)) &= 0 \\ E(\Delta B(k)) &= 0 \end{aligned} \quad (22)$$

2) Covariances are given by an  $(m \times m)$  matrix

$$E\{\Delta q(k) \cdot \Delta q'(l)\} = (S_q, 0) \quad (23)$$

$S_q$  for  $k=l$ , and 0 for  $k \neq l$

$$E(\Delta \mathbf{b}(i, k) \cdot \Delta \mathbf{b}'(j, l)) = (\mathbf{V}_i, 0) \quad (24)$$

$\mathbf{V}_i$  for  $i=j$ , and  $k=l$ , and 0 for  $i \neq j$  or  $k \neq l$

$$E(\Delta \mathbf{q}(k) \cdot \Delta \mathbf{b}'(i, l)) = 0 \quad \text{for all } i, k, l$$

3) the gain parameter  $\gamma$  should be chosen according to

$$0 < \gamma < 2 / (m \cdot \sigma_q^2)$$

$$\sigma_q^2 = \max (E(\Delta \mathbf{q}_1^2(k))) \quad (25)$$

The exit flows are updated based on the estimated entry flows  $\Delta \mathbf{q}(k)$ . This procedure was tested under the same geometry and with synthetic data as the previous models. The results were better than the other two methods above when the split parameters  $b(i, j, k)$  varied significantly from hour to hour (i.e. when the O-D pattern was different at different hours) due to better tracking abilities.

#### 2.2.4 Estimation by Kalman Filtering [Cremer, Keller] [3,4]

The Kalman filtering technique is another recursive estimation method.

Consider the equation (6)

$$y(j, k) = \sum_{i=1}^m q(i, k) \cdot b(i, j, k) = \mathbf{q}'(k) \cdot \mathbf{b}(j, k) \quad (26)$$

$\mathbf{b}(j,k)$  is the  $j$ th column of matrix  $\mathbf{B}(k)$  containing all split parameters which are related to the  $j$ th exit flow. The dynamic nature of the split parameter  $\mathbf{b}(j,k)$  is incorporated via the state equation, wherein

$$\mathbf{b}(j,k) = \mathbf{b}(j-1, k) + \mathbf{w}(k) \quad (27)$$

$\mathbf{w}(k)$  being a random error term with mean zero and known covariance  $\mathbf{W}$ .

$$E\{\mathbf{w}(k)\} = 0$$

$$E\{\Delta\mathbf{w}(k) \cdot \Delta\mathbf{w}'(k)\} = (\mathbf{W}, 0) \quad (28)$$

$\mathbf{W}$  for  $k=l$ , and zero for  $k \neq l$ .

Information about the state variables is given by the measurement of the  $j$ th exit flows which are related to the  $\mathbf{b}(j,k)$  by equation (26), plus a random measurement error vector  $\mathbf{v}(k)$ . The relationship between  $\mathbf{y}(j,k)$  and  $\mathbf{q}(k)$  is linear and is specified by the measurement equation

$$\mathbf{y}(j,k) = \mathbf{q}'(k) \cdot \mathbf{b}(j, k) + \mathbf{v}(k) \quad (29)$$

with

$$E\{\mathbf{v}(k)\} = 0$$

$$E\{\mathbf{v}^2(k)\} = (\mathbf{R}, 0) \quad (30)$$

Kalman filter generates bias-free estimates only if the system model is completely observable. This implies that

$$\text{rank}[\mathbf{q}(k-K), \mathbf{q}(k-K+1), \mathbf{q}(k-K+2), \dots, \mathbf{q}(k)] = n \quad (31)$$

where  $n$  is the number of exits,  $K$  is the total number of intervals, and  $k$  represents each interval. This condition implies that the matrix  $\mathbf{Q}$  of

equation(14) or equivalently matrix  $\phi_{qq}$  has full rank. It is important to note that choosing the O-D flows  $f(i,j,k)$  directly as state variables would set up a system description which is not completely observable and for which the Kalman filter could not be applied.

The split parameter  $b(j, k)$  can be estimated by the following set of equations

$$d(k) = [P(k-1) + W]q(k)[q'(k)[P(k-1) + W]q(k) + R]^{-1} \quad (32)$$

$d(k)$  is called the filter gain, and  $P$  is the estimation error covariance matrix.

Updating the estimation error covariance matrix is obtained from

$$P(k) = [I - d(k) \cdot q'(k)][P(k-1) + W] \quad (33)$$

$b(j, k)$  is estimated by the following equation

$$\hat{b}(j, k) = \hat{b}(j, k-1) + d(k) (y(j,k) - q'(k) \cdot \hat{b}(j, k-1)) \quad (34)$$

For the start of this computation an initial value of the covariance matrix of estimation errors  $P(0)$  must be specified. If the assumptions of the Kalman filter are fulfilled, the filter equations compute bias-free, minimum variance estimates. Otherwise, estimates may not necessarily be optimal, but have minimum variance. The Kalman filtering process when applied on the same geometry as the above models with the same synthetic data does not produce the best results. The reason for this is that the assumption of the random variations as being Gaussian white noise is only partially fulfilled. The results showed considerable fluctuations trying to follow the real variations.



## 2.3 Complex Networks

Three major algorithms that are used in estimating O-D matrices for complex networks will be reviewed here. The first algorithm is the Linked Static-Dynamic Correlation Method developed by Keller and Ploss [6]. This algorithm is an extension of the model developed by Cremer and Keller [3,4] which was explained earlier under the section on single intersections or partial networks. The linked static-dynamic method for the estimation of O-D flows in transportation systems from traffic counts is based on the following concept.

The model finds the coefficient of correlation (or weights) between an entry volume and an exit volume profile. The coefficient of correlation is an indicator that shows to what extent the variation of one variable is determined by the variation of another variable. These coefficients are used as weights for the O-D flows, and are linked with an entropy maximization model to find a unique O-D estimate. This new approach uses all the information that can be used for partial networks or single intersections to improve the results for the total system.

The Linked Static-Dynamic Correlation Method is the algorithm used in project "MOTION" for estimating O-D flows. "MOTION" is an on-line signal control system developed using the German off-line signal control model SIGMA and existing on-line systems in Italy.

The Kalman filtering algorithm for complex networks was developed by Okutani [7]. In this paper, the ideas introduced by Keller and Cremer [3,4] are extended to a general network. Provisions are made to solve the problem of an underdetermined system.

The third model that will be reviewed is the dynamic estimation of O-D flows based on time-varying traffic counts and any other available information such as outdated O-D matrices. Two types of optimization procedures are introduced. A simultaneous GLS estimation which produces joint estimates of the whole set of O-D matrices, one for each time interval, and a sequential GLS estimation which produces a sequence of O-D estimates for successive time intervals.

### 2.3.1 Linked Static-Dynamic Correlation Method [Keller, Ploss] [6]

The relationship between link flows and O-D demand is essential for dynamic O-D estimation. This relationship is expressed by

$$V(l) = \sum_{i=1}^I \sum_{j=1}^J p(i,j,l) \cdot f(i,j) \quad (35)$$

$V(l)$  represents volume on link  $l$ ;  $p(i,j,l)$  is the fraction of O-D flows  $f(i,j)$  on a path that includes link  $l$ . The system of linear equations based on equation (35) is highly underdetermined. However, by including the estimated turning flows at intersections and cordon line counts on the periphery of the system, additional equations for the determination of the O-D flows can be developed.

This addition reduces the degree of under determination of the systems considerably. As a result, this method has two parts: part one, is estimation of the O-D flows at intersections using a linked static dynamic correlation model. This information is then added to part two to decrease the degree of under determination. Part two is the estimation of the O-D flows in complex traffic networks with the linked static-dynamic method.

**PART ONE:**

Estimation of the O-D flows at intersections in simple networks using a linked static-dynamic correlation model.

**Definitions**

**I**      Number of entries in a system

**J**      Number of exits in a system

**T**      Number of intervals with the length  $\Delta t$  in a time period  $p$

$q(i,k)$    volume passing entry  $i$  during interval  $k$ , ( $k=1,...,T$ ,  $i=1,...,I$ )

$\bar{q}(i) = 1/T \sum q(i,k)$ , summed over  $k=1,...,T$

$y(j,k)$    volume passing exit  $j$  during interval  $k$ , ( $k=1,...,T$ ,  $j=1,...,J$ )

$\bar{y}(j) = 1/T \sum y(j,k)$ , summed over  $k=1,...,T$

$t(i,j)$    travel time from  $i$  to  $j$  (integer multiple of  $\Delta t$ )

$f(i,j)$    partial volume from  $i$  to  $j$  during the observation period  $p$

$b(i,j,k) = f(i,j)/q(i)$  (split parameter),       $0 \leq b(i,j,k) \leq 1$

The following static conditions have to be satisfied

$$(1) \quad q(i) = \sum_{j=1}^J f(i,j) ,$$

$$(2) \quad y(j) = \sum_{i=1}^I f(i,j) ,$$

$$(3) \quad \sum_{j=1}^J b(i,j,k) = 1 ,$$

### 2.3.1.1 Calculation of the Coefficient of Correlation, $w(i,j)$

The coefficient of correlation  $w(i,j)$  can be obtained from the following equation which is similar to the correlation coefficient . The correlation coefficient gives a quantitative measure of how strong the linear relationship is between x and y- i.e., to what degree x and y are related.

$$w(i,j) = \frac{\sum_{k=1}^T ((q(i,k) - \bar{q}(i)) \cdot (y(j,k+t(i,j)) - \bar{y}(j)))}{\sqrt{\sum_{k=1}^T ((q(i,k) - \bar{q}(i))^2 \cdot \sum_{k=1}^T (y(j,k+t(i,j)) - \bar{y}(j))^2)}$$

large variations in  $w(i,j)$  can result if the observation period p is small. This can be avoided by exponential smoothing of  $w(i,j)$ , using a smoothing factor  $\alpha$  as follows

$$w(i,j) = \alpha \cdot w(i,j, p) + (1-\alpha) \cdot w(i,j, p-1)$$

O-D flows can be expressed by

$$f(i,j) = w(i,j) \cdot Q(i) \cdot Y(j)$$

the unknown values  $Q(i)$  and  $Y(j)$  can be obtained by

$$Q(i) = q(i) + \sum_{j=1}^J (Y(j) \cdot w(i,j)) ,$$

$$Y(j) = Y(i) + \sum_{i=1}^I (Q(i) \cdot w(i,j)) ,$$

Using an iterative solution method such as an entropy model, a unique solution to this system can be found that satisfies the static conditions (1) and (2) exactly and includes the dynamic weights  $w(i,j)$  for the most recent period. This system of equations are added to equation (34).

## PART TWO:

### 2.3.1.2 O-D Flow Estimation For a Complex Network

The first step in this procedure is to find the shortest path between each O-D pair that includes link  $l$ . Given information such as entry  $i$ , counting site  $a$ , and exit  $j$ , and using deterministic or stochastic successive capacity restrained algorithms, these set of paths can be found. The fraction  $p(i,j,l)$  of the total number of vehicles  $f(i,j)$  that take a certain route that includes link  $l$  can be determined given a set of paths.  $p(i,j,l)$  is 1 if link  $l$  falls on a path going from  $i$  to  $j$ , and 0 otherwise. Travel time from  $i$  to  $j$ ,  $t(i,j)$  is the travel time on the shortest path between  $i$  and  $j$ . Based on the travel times, the weights  $w(i,j)$  are recalculated and applied to the flows from any entry to any exit at the cordon line of the system. O-D volumes can be formulated as

$$f(i,j) = w(i,j) \cdot X(0) \cdot \prod_l X(l) \cdot p(i,j,l)$$

$p(i,j,l)$  is equal to 1 if link  $l$  is on the path between  $i,j$  and is equal to zero otherwise.

with

$$X(0) = \left\{ \sum_{i=1}^I \sum_{j=1}^J f(i,j) \right\} + \left\{ \sum_{i=1}^I \sum_{j=1}^J w(i,j) \right\},$$

A solution for  $X(l)$  can be found iteratively by applying the balancing algorithms of the entropy model of Van Zuylen [10]

$$V(l) = \sum \sum p(i,j,l) \cdot w(i,j) \cdot X(l) \cdot \prod_l (X(l) \cdot p(i,j,l))$$

By including the turning flows of intersections inside the observed network that have already been estimated according to the algorithm in part one, new equations can be set up which decrease the degree of under determination. The entropy algorithm should converge reasonably well.

The method suggested by Keller and Ploss [8] has been incorporated in the system "MOTION", but this model is still in the experimental stage. Many improvements can be made by exploring the effects of multiple paths on the estimation of dynamic weights  $w(i,j)$ , and travel times  $t(i,j)$ . The above method considers the entropy model as the only way of finding the optimal O-D flows for a network and does not consider other optimization algorithms which may be more efficient in O-D flow estimation.

## 2.3.2 Kalman Filtering Algorithm

### 2.3.2.1 Estimating Time Varying O-D Trip Matrices From Traffic Counts [Okutani][7],[12,13]

This is an extension to a larger network of the Kalman filter formulation for estimating O-D flows at an intersection introduced by Cremer and Keller [3,4]. The model is formulated for a network with  $n$  O-D pairs and  $M$  links. Let  $f(i,t)$  denote the  $i$ th O-D flow during a time period  $t$  and  $y(j,t)$  denote the observed traffic counts on link  $j$  during the same period  $t$ . Let  $f(t)$  be an  $n$  dimensional vector with elements  $f(i,t)$ , then the next period O-D flows can be expressed by the following relation known as the state equation:

$$f(t+1) = \sum H_m(t)f(t-m) + w(t), \quad \text{summed over } m=0,\dots,q \quad (36)$$

where the input, or plant, noise  $w(t)$  is a zero-mean white-noise process, with covariance

$$E\{w(t)\} = 0$$

$$\text{cov}\{w(t), w(s)\} = R\delta_{ts} \quad \begin{array}{l} \delta_{ts} = 1 \text{ for } t=s \\ \delta_{ts} = 0 \text{ for } t \neq s \end{array}$$

Setting  $q$  (the delay time) equal to zero and  $H_0(t)$  equal to an identity matrix, equation (36) can be rewritten as

$$f(t+1) = f(t) + w(t)$$

The link volume is expressed as a fraction of O-D flows  $f(t)$  given by the linear algebraic relation, known as the measurement equation

$$y(t) = \sum P(m,t) \cdot f(t-m) + v(t), \quad \text{summed over } m=0,\dots,p \quad (37)$$

where the measurement noise  $v(t)$  is a zero-mean white noise process, with

$$E\{v(t)\} = 0$$

$$\text{cov}\{v(t), v(s)\} = R_1 \delta_{ts} \quad \begin{array}{l} \delta_{ts} = 1 \text{ for } t=s \\ \delta_{ts} = 0 \text{ for } t \neq s \end{array}$$

where

$$y(t) = (y_1(t), y_2(t), \dots, y_M(t))$$

In equation (37)  $P(m,t)$  is the proportion of O-D  $f(t-m)$  flows which use link  $j$  during period  $t$ . This is an  $(M \times n)$  matrix.  $p$  represents delay. Some transformation of equations (36) and (37) are necessary to show delay in O-D flows. Therefore, O-D flows are redefined as

$$f(m,i,t) = f(i,(t-m)) \quad (i=1,2,\dots,n, \quad m=0,1,\dots,r)$$

$$r = \max(p,q)$$

Let  $f(m,t)$  denote a matrix with elements  $f(m,i,t)$ , i.e.

$$f(m,t) = (f_1(m,t), f_2(m,t), \dots, f_n(m,t))'$$

where prime denotes a transpose. Let  $F(t)$  be the vector of dimension  $n(r+1)$

$$F(t) = (f^0(t), f^1(t), \dots, f^r(t))'$$

equations (36) and (37) can be rewritten as

$$F(t+1) = \Phi(t) F(t) + W(t)$$

$$Y(t) = P(t)F(t) + v(t)$$

where



$$\Phi(t) = \begin{bmatrix} H(t) \\ \hline I & 0 \end{bmatrix}$$

$\Phi(t)$  is an  $n(r+1) \times n(r+1)$  matrix.

$$W(t) = (w^t(t), 0)$$

$W(t)$  is an  $n(r+1) \times 1$  matrix.

$$P(t) = ((P^0(t), P^1(t), \dots, P^r(t)))$$

$P(t)$  is an  $(M \times n(r+1))$  matrix.

$$H(t) = (H_0(t), H_1(t), \dots, H_r(t))$$

$H(t)$  is an  $(n \times n(r+1))$  matrix.

$$H(m, t) = 0 : (n \times n) \text{ matrix for } m > q$$

$$P(m, t) = 0 \quad (M \times n) \text{ matrix for } m > p$$

The estimate of  $F(t)$ ,  $\hat{F}(t)$  can be obtained from the following set of equations

$$\hat{F}(t+1) = \Phi(t)\hat{F}(t-1) + K(t)[Y(t) - P(t)\Phi(t)\hat{F}(t-1)] \quad (38)$$

$K(t)$  is called the Kalman gain matrix with dimensions  $(n(r+1) \times M)$  and is estimated by

$$(39) \quad K(t) = S(t)P^t(t)[P(t)S(t)P^t(t) + R_1]^{-1}$$

$S(t)$  is and  $(n(r+1) \times n(r+1))$  estimation error covariance matrix called Apriori variance.  $S(t)$  satisfies the following system of matrix difference equations

$$S(t) = \Phi(t-1)Q(t-1)\Phi^t(t-1) + R \quad (40)$$

$$Q(t) = [I - K(t)P(t)]S(t) \quad (41)$$

$Q(t)$  is called the A posteriori variance.

The initial conditions for  $F(0)$ , and  $S(0)$  have to be selected before using the above results for estimating O-D volumes.

The Kalman Filtering methodology was tested using a network with five and eight links. Although the method performed well for a small network, it is still in the early stage. Before definitive conclusions are drawn, extensive experimentation must be carried out. The main computational difficulty of this method is that the inversion of a large matrix is required in the algorithm. Therefore, a way of reducing the dimensionality must be sought along with an efficient algorithm to achieve this inversion when applying the method to a network of practical size [Okutani] [7].

### 2.3.3 Generalized Least Square Method [Cascetta][1]

The relationship between link flows and O-D flows is not static, but varies across time intervals. The dynamic link O-D flow equation is expressed as

$$v(l,h) = \sum_{r=1}^{n_{od}} \sum_{t=1}^h p(r,t,l,h) \cdot f(r,t),$$

where  $v(l,h)$  is flow on link  $l$  during interval  $h$ .  $f(r,t)$  represents O-D flows for an O-D pair  $r$  that left the origin during some previous interval  $t$  and is on link  $l$  during  $h$ .  $p(r,t,l,h)$  is the fraction of O-D flows contributing to the flow on link  $l$  during interval  $h$ . The link O-D flow equation can be expressed in term of path flows through the following equation

$$F(k,t) = d(r,t) \cdot q(k,t) \quad (42)$$

$$v(l,h) = \alpha(k,t,l,h) \cdot F(k,t) , \quad \begin{matrix} k=1,\dots,K_r \\ t=1,\dots,h \end{matrix} \quad (43)$$

$F(k,t)$  is the flow between O-D pair  $r$  following path  $k$  that left the origin during previous intervals  $t$ .  $q(k/t)$  is the probability of choosing path  $k$  conditional on the departure interval  $t$ .  $\alpha(k,t,l,h)$  is called the link-path incidence fraction which is the fraction of path flow  $F(k,t)$  contributing to link flow  $v(l,h)$ .

By combining the two equations (42) and (43) the following relationship is obtained

$$p(r,t,l,h) = \sum_{k=1}^{K_r} \alpha(k,t,l,h) \cdot q(k/t) ,$$

$K_r$  denotes a set of all paths connecting O-D pairs  $r$ . An estimate for  $p(r,t,l,h)$  can be found by obtaining estimates for  $\alpha(k,t,l,h)$  and  $q(k/t)$  represented by  $\hat{\alpha}(k,t,l,h)$  and  $\hat{q}(k/t)$ .

$q(k/t)$  can be found by a random utility model such as

$$q(k/t) = \text{prob}(C(k,t) + \theta(k,t) \leq C(m,t) + \theta(m,t)) \quad \forall m \in K_r$$

where

$C(k,t)$  is the cost associated with taking path  $k$ , conditional on departure time  $t$   
 $C(m,t)$  is the cost associated with taking path  $m$ ,  $m \neq k$ , conditional on departure time  $t$   
 $\theta(k,t)$  is the difference between the perceived cost and the actual cost of taking path  $k$

$\theta(m,t)$  is the difference between the perceived cost and the actual cost of taking path  $m$

A path choice model can be obtained by making different assumptions on the joint distribution of error term  $\theta(k,t)$ . The most popular specifications based on zero variance, Weibull-Gumble and a multivariate normal distribution of residuals are Logit and Probit models.

The link - incident fractions  $\alpha(k,t,l,h)$  can be found based on the assumption that path flow  $F(k,t)$  is uniformly spread over the departure interval of length  $T$  and remain equally distributed over the same time length while moving on the network. The fractions  $\alpha(k,t,l,h)$  can be estimated by

$$\begin{aligned} \alpha(k,t,l,j) &= 1 - (t(k,t,l)/T - (j-1)) && \text{if } (j-1)T < t(k,t,l) < jT \\ &= 1 - \alpha(k,t,l,(j-1)) && \text{if } (j-2)T < t(k,t,l) < (j-1)T \\ &= 0 && \text{otherwise} \end{aligned}$$

$t(k,t,l)$  is the arrival time at the head of the path flow  $F(k,t)$  on link  $l$ . Knowledge of average link speed is required to calculate average travel time on each path  $k$ , and related arrival time on a link of a path.

Since  $\hat{\alpha}(k,t,l,h)$  and  $\hat{q}(k,t)$  are approximations of  $\alpha(k,t,l,j)$  and  $q(k,t)$ , estimated link flows differ from the actual ones by a random error term

$$v(l,h) = \sum_{t=1}^h \sum_{r=1}^{n_{od}} \hat{P}(r,t,l,h) \cdot f(r,t) + \delta(l,h),$$

in matrix form the above equation can be written as

$$V(h) = \sum_{t=1}^h \hat{P}(t,h) \cdot f(t) + \delta(h), \quad (44)$$

The system of equations set up this way can be solved using either simultaneous or sequential generalized least square (GLS) estimation models.

### 2.3.3.1 Simultaneous Estimation

This method provides O-D estimates for all intervals in one step. Using GLS as the desired optimization method the system is set up as

$$\delta(h) = V(h) - \left( \sum_{t=1}^h \hat{P}(t,h) \cdot s(t) \right),$$

$s(t)$  is the unknown O-D demand vector. Optimal estimates of O-D flows for all intervals can be computed by setting the first derivative of the quadratic equation (45) with respect to O-D flow estimates  $f^*(t)$  equal to zero.

$$(f_1^*(h), f_2^*(h), \dots, f_h^*(h)) = \min_{f_1^* \geq 0, f_2^* \geq 0, \dots, f_h^* \geq 0} \sum_{t=1}^h ((s(t) \hat{f}(t)' V^{-1}(t) (s(t) - \hat{f}(t)) + \delta_h' W^{-1} \delta_h) \quad (45)$$

$\hat{f}(t)$  is an O-D matrix obtained through a survey.  $V(h)$  is the variance - covariance matrix of sampling errors.  $W(h)$  is the variance - covariance matrix of assignment errors.  $f_1^*(h), f_2^*(h), \dots, f_h^*(h)$  are optimal estimates of O-D flows obtained by solving the GLS equation (45).

### 2.3.3.2 Sequential Estimation

In this case, an O-D demand vector for O-D pair  $r$  is estimated for a single interval  $h$  at each step. Equation (45) can be stated in a new form as

$$\hat{\mathbf{v}}(h) = \sum_{t=1}^h (\mathbf{P}(h,t) \cdot \mathbf{f}^*(t)) + \mathbf{P}(h,h) \cdot \mathbf{s}(h) + \delta_h,$$

$\mathbf{f}^*(t)$  is the previous interval O-D estimate. The GLS formulation of the model is as follows:

$$\delta_h = \hat{\mathbf{v}}(h) - \sum_{t=1}^h (\mathbf{P}(h,t) \cdot \mathbf{f}^*(t)) + \mathbf{P}(h,h) \cdot \mathbf{s}(h)$$

$$\mathbf{f}^*(h) = \min_{\mathbf{f}^*(h) \geq 0} \sum_{t=1}^h ((\mathbf{s}(t) - \hat{\mathbf{f}}(t))' \mathbf{V}^{-1}(t) (\mathbf{s}(t) - \hat{\mathbf{f}}(t)) + \delta_h' \mathbf{W}^{-1}(h) \delta_h)$$

The sequential generalized least square estimation method suggested by Cascetta was tested on a motorway which is part of the motorway system located in northern Italy linking Turin and Venice. True O-D volumes were available for comparison. Consistent and significant estimates of the true O-D flows over 15 minute intervals were obtained with reasonable computational requirements. The sequential generalized least square estimation method was not applied to a practical size network [Cascetta] [1].

## 2.4 Research Objective

Cascetta [1] suggests that further research is needed to assess the possibilities of dynamic O-D estimation with dynamic assignment on a more complex network. The network used by Cascetta is a linear network (Brescia-Vicenza-Padova motorway) where there is only one possible path between an origin and destination. The path choice is always the same; and therefore there is no need for dynamic path assignment.

The assignment fractions used to test the model were taken to be identity matrices. Dynamic assignment matrices were not considered. The objective of the present research is to test the sequential GLS O-D flow estimation suggested by Cascetta for an urban network using time dependent traffic data obtained from count stations on local streets. In this study path choice varies during each time interval, and the link - incident fractions  $\alpha(k,t,l,h)$  are time dependent.

The methods discussed so far have only been tried on small networks, such as intersections, with synthetic data. None of the models have been tested on a large network with real time traffic data. So far only the approach suggested by Cascetta [1] was tested on a segment of a highway, with traffic data being collected for several time intervals.

In all the methods discussed so far, the shortest path between an O-D pair is the only path considered. The issue of multiple path choice has not been addressed or considered. It is not clear from the description of these methods how they would deal with a situation where the system is underdetermined.

The general approach for finding O-D estimates in this research is the Least Square Estimation. This is the overall approach used by all the methods described so far that were tested on networks. The method used in this study is new both in its setup and in its implementation. In the setup the model is similar to the Kalman Filtering system of equations. It consists of two sets of equations: The observation equation, and the state equation. The observation equation describes the relationship between link counts and O-D trips. The State equation describes the relationship between the present interval O-D trips and the previous interval O-D trips. Though this setup is similar to the Kalman setup in concept, the State equation is a variation of the original.

The approach to finding time dependent O-D estimates in the first stage of the implementation is to find the Ordinary Least Square Estimates (OLS) of O-D trips. These estimates are by definition the best estimates given a particular data set. The OLS, O-D estimates are used in finding both the observation error and the error associated with the State equation.

The second stage of the implementation is to find the Generalized Least Square Estimates (GLS) of the O-D trips. The O-D estimates obtained through the GLS application are the unbiased minimum variance estimates. The variance incorporated in the GLS process is a function of both the observation errors and the errors associated with the State equation. This process is repeated sequentially for all the intervals in the study period. This implementation of the model is a new approach to finding the best O-D estimates for a network.



**The contribution of this research is:**

- **Introduction of a new model for estimating time dependent O-D trips for a large network.**
- **Implementing the model in an urban network using existing traffic data. The study area chosen is the City of Boston. Traffic data used is data that is available for every 15 minute interval on selected links in the network.**

## Chapter 3

### Model Development

#### 3.1 Recursive Generalized Least Square Estimation

The objective of this study is to develop a method for estimating time dependent O-D matrices. By time dependent it is meant that if a time period is divided into smaller intervals, the O-D matrices vary for each interval. The model development relies on formulating link flow as a fraction of O-D matrices during each time interval plus an error term which reflects the deviation between the estimated link flow and the real link flow. An O-D matrix for an interval is expressed as a linear combination of the O-D matrix of the previous interval plus an error term.

The model is a linear system consisting of two equations, the link flow equation or measurement equation, and the O-D matrix equation. The objective is to find an estimated O-D matrix such that a quadratic measure is minimized. The estimate which minimizes the quadratic expression will be called the Least Square estimate. The procedure used to minimize the quadratic measure, or the objective function, by adjusting the O-D matrix based on the link flow estimate error variances and the O-D estimate error variance is the Generalized Least Square Estimation method.

In this section some basic concepts of the recursive GLS algorithm are given. Start with the assumption that volume on any link  $l$  during interval  $j$ ,  $z(l, j)$  is a fraction  $h(r, t, l, j)$ , of O - D volumes  $x(r, t)$ , leaving the origin during either the

same or previous interval and are contributing to the flow on link  $l$  during interval  $j$ . In this formulation,  $r$  represents O-D pair  $r$ , and  $t$  represents the departure time from the origin. Therefore, traffic flow on any link  $l$  during interval  $j$  can be expressed by the following equation:

$$z(l, j) = \sum_{r=1}^{n_{od}} \sum_{t=1}^j h(r, t, l, j) \cdot x(r, t) + v(l, j), \quad (46)$$

where  $n_{od}$  is the number of O-D pairs,  $t$  represents intervals from 1 to interval  $j$ .  $v(l, j)$  is a zero-mean observation error term,

$$E\{v(l, j)\} = 0$$

with covariance,

$$\text{cov}\{v(l, j), v(l, k)\} = V_v(j)\delta_{jk} \quad (47)$$

In matrix form, equation (46) can be written as:

$$Z(j) = H(t, j) X(t) + V(j), \quad t=1, \dots, j \quad (48)$$

Due to the short duration of total travel times from an origin to a destination in the proposed study area, only two analysis periods are considered in this study, the present interval ( $j$ ), and the previous interval ( $j-1$ ). Equation (46) is then modified as follows:

$$z(l, j) = \sum_{r=1}^{n_{od}} \sum_{t=j-1}^j h(r, t, l, j) \cdot x(r, t) + v(l, j), \quad (49)$$

or

$$z(l, j) = \sum_{r=1}^{n_{od}} h(r, j-1, l, j) \cdot \hat{x}(r, j-1) + \sum_{r=1}^{n_{od}} h(r, j, l, j) x(r, j) + v(l, j) , \quad (50)$$

where  $\hat{x}(r, j-1)$  is the estimate of the previous interval O-D volume. Equation (50) can be rewritten as:

$$\tilde{z}(l, j) = \sum_{r=1}^{n_{od}} h(r, j, l, j) \cdot x(r, j) + v(l, j), \quad (51)$$

where

$$\tilde{z}(l, j) = z(l, j) - \sum_{r=1}^{n_{od}} h(r, j-1, l, j) \cdot \hat{x}(r, j-1) , \quad (52)$$

the matrix form of equation (51) is written as:

$$\tilde{Z}(j) = H(j, j)X(j) + V(j)$$

$\tilde{Z}$  is an  $(n_l \times 1)$  vector,  $H$  is an  $(n_l \times n_{od})$  matrix,  $X$  is an  $(n_{od} \times 1)$  vector, and  $V$  is an  $(n_l \times 1)$  vector. For  $j \geq 2$ ,  $\tilde{Z}(j)$  can be written as

$$\tilde{Z}(j) = Z(j) - H(j-1, j) \hat{X}(j-1) \quad (53)$$

for  $j=1$   $\tilde{Z}(j)$  is equal to  $Z(j)$ .

The O - D volume for interval  $j$  can be interpolated from the O - D estimates of interval  $(j-1)$  as follows:

$$\hat{X}(j-1) = X(j) + W(j) \quad (54)$$

where  $\hat{X}(j-1)$  is a vector of estimated O-D volumes for the previous interval;  $X(j)$  is the present interval O-D vector;  $W$  is a  $(n_{od} \times 1)$  error or noise vector.

The noise  $W(j)$  is a zero-mean estimation error term,

$$E\{W(j)\} = 0$$

with covariance

$$\text{cov}\{W(j), W(k)\} = V_w(j) \delta_{jk} \quad (55)$$

we can adjoin the O-D estimates for interval  $\hat{X}(j-1)$  with  $\tilde{Z}(j)$  to obtain

$$Z^*(j) = H^*(j) \hat{X}(j) + V^*(j) \quad (56)$$

where

$$\hat{X}(j) = [\tilde{Z}(j), \hat{X}(j-1)]^t$$

$$H^*(j) = [H(j, j), I]^t$$

$$V^*(j) = [\{V(j), I\}, W(j)]^t$$

The augmented matrices have the following dimensions:  $Z^*$  is a  $((n_l + n_{od}) \times 1)$  vector,  $\hat{X}$  is a  $((n_l + n_{od}) \times 1)$  matrix,  $H^*$  is an  $((n_l + n_{od}) \times (n_l + n_{od}))$  matrix.  $V^*$  is an augmented matrix of the variance-covariance of the observation errors  $V_v$  with the variance-covariance of O-D estimate errors  $V_w$ . The partitioned form of  $V^*$  can be written as

$$V^* = (V_v, \mathbf{0}_v \mid V_w, \mathbf{0}_w)$$

$V_v$  is a  $(n_l \times n_l)$  matrix,  $\mathbf{0}_v$  is an  $(n_l \times n_{od})$  matrix with zero entries,  $V_w$  is an

$(n_{od} \times n_{od})$  matrix,  $0_w$  is an  $(n_{od} \times n_l)$  matrix of zeros. The augmented matrix,  $V^*$  is an  $((n_l + n_{od}) \times (n_l + n_{od}))$  matrix.

The objective is to select an estimate  $\hat{X}(j)$  such that the quadratic measure

$$J(\hat{X}(j)) = 1/2 (Z^*(j) - H^*(j) \hat{X}(j))^t V^{*-1} (Z^*(j) - H^*(j) \hat{X}(j)) \quad (57)$$

is minimized. Because minimization of  $J(\hat{X}(j))$  is an ordinary deterministic minimization problem, the least square estimate is obtained by setting

$$d(J(\hat{X}(j))) / d(\hat{X}(j)) = 0 \quad (58)$$

By using equation (57) for  $\hat{X}(j)$  and carrying out the indicated partial differentiation, we obtain

$$d(J(\hat{X}(j))) / d(\hat{X}(j)) = H^{*t}(j) V^{*-1} (Z^*(j) - H^*(j) \hat{X}(j)) = 0 \quad (59)$$

The minimum variance unbiased estimate of the O-D matrix for interval  $j$  is expressed as:

$$\hat{X}(j) = (H^{*t}(j) V^{*-1} H^*(j))^{-1} H^{*t}(j) V^{*-1} Z^*(j) \quad (60)$$

A description of how the fraction matrix  $H$ , and error variance-covariance matrices  $V_v$ ,  $V_w$  are derived are contained in a later section.

Two system of linear equations were defined. The first equation defines a causal relationship between link counts and O-D trips. The second equation provides a connection between the previous interval O-D trips and the present interval O-D

trips. This equation states that the previous interval O-D volumes are equal to the present interval O-D volumes plus some deviation. The causal relationship defined in the second equation is significant since it allows the analyst to compensate for the problem of rank deficiency, by converting the under-determined system into a full rank system.

### 3.2 Evaluation of the Accuracy of the Sequential Generalized Least Square Estimation Procedure

The test of the credibility of the results of the GLS model (estimated O-D volumes) is to find out how close the estimated link counts are to the observed link counts. Link volumes are estimated by the following equations:

$$\hat{z}(l, j) = \sum_{t=j-1}^j \sum_{r=1}^{nod} h(r, t, l, j) \cdot \hat{x}(r, t),$$

or

$$\hat{z}(l, j) = \sum_{r=1}^{nod} h(r, j-1, l, j) \cdot \hat{x}(r, j-1) + \sum_{r=1}^{nod} h(r, j, l, j) \cdot \hat{x}(r, j), \quad (61)$$

$\hat{z}(l, j)$  is the estimated counts on link  $l$  during interval  $j$ .  $\hat{x}(r, t)$  is the estimated volume for O-D pair  $r$  during interval  $t$ .  $h(r, t, l, j)$  is an assignment matrix, specifying the fraction of  $r$ th O-D pair volumes that left the origin during interval  $t$  and are on link  $l$  during interval  $j$ .

The statistic used to measure the degree of closeness between predicted and observed link counts is the root mean square error (rmse). The rmse measures the degree of disagreement between two series of link flow values (estimated and observed) and it is given for each interval  $j$  by:

$$rmse(j) = \sqrt{\left( \sum_{l=1}^{nl} (z(l, j) - \hat{z}(l, j))^2 \right) / nl} \quad (61)$$

$$\%rmse(j) = (rmse(j) / \bar{z}(j)) * 100 \quad (62)$$



$$\bar{z}(j) = (1/n_l) \sum_{l=1}^{nl} \hat{z}(l,j)$$

$z(l,j)$  is the observed counts on link  $l$  during interval  $j$ .

## Chapter 4

### Parameter Estimation

#### 4.1 Introduction

To apply the model it is necessary to estimate the value of the model parameters. If  $z(l, j)$  denotes traffic counts on any link  $l$  during an interval  $j$ , then  $z(l, j)$  can be defined as follows:

$$z(l, j) = \sum_{t=1}^j \sum_{r=1}^{nod} h(r, t, l, j) \cdot x(r, t) + v(l, j), \quad (63)$$

In the above equation  $h(r, t, l, j)$  is the fraction of  $r$ th O-D volume that departed the origin during interval  $t$ , and is on link  $l$  during interval  $j$ . The procedure for estimating  $h(r, t, l, j)$  is adopted from Cascetta[1].  $x(r, t)$  is the  $r$ th O-D traffic volume that left the origin during interval  $t$  and is contributing to flow on link  $l$  during interval  $j$ .  $v(l, j)$  is the observation error on link  $l$  during interval  $j$ .

To estimate the fractions  $h(r, t, l, j)$ , link traffic counts must be expressed in terms of path flows. Letting  $x(k, t)$  denote the traffic flow following path  $k$  that left the origin during period  $t$ ,  $x(k, t)$  can be expressed as follows:

$$x(k, t) = x(r, t) \cdot p(k / t) \quad k \in K_r \quad (64)$$

where  $p(k / t)$  is the probability of choosing path  $k$  going between an O-D pair  $r$  given that the trip left the origin during interval  $t$ . The observed link traffic counts can be rewritten in terms of the path flow  $x(k, t)$  as follows:

$$z(l, j) = \sum_{t=j-1}^j \sum_{k=1}^{Kr} (a(k, t, l, j) \cdot x(k, t)) + v(l, j), \quad (65)$$

where  $a(k, t, l, j)$  is the fraction of path flow  $x(k, t)$  contributing to link flow  $z(l, j)$ . The assignment fraction  $h(r, t, l, j)$  can be expressed in terms of path-link incidence fractions  $a(k, t, l, j)$ , and path choice probabilities  $p(k / t)$ . Combining equation (63), and equation (64), the assignment fraction  $h(r, t, l, j)$  is expressed by the following relationship:

$$h(r, t, l, j) = \sum_{k=1}^{Kr} a(k, t, l, j) \cdot p(k / t), \quad (66)$$

Let  $\tilde{h}(r, t, l, j)$  denote the estimate of  $h(r, t, l, j)$ , and  $\tilde{\alpha}(k, t, l, j)$ , and  $\tilde{p}(k / t)$  denote estimates of  $a(k, t, l, j)$ , and  $p(k / t)$  respectively. Equation (66), can be written in terms of these estimates as follows:

If the link arrival time  $t$  is  $t=j$ , then the fraction of traffic on link  $l$  during interval  $j$  that arrived during  $j$  is:

$$\tilde{h}(r, j, l, j) = \sum_{k=1}^{Kr} \tilde{\alpha}(k, j, l, j) \cdot \tilde{p}(k / j), \quad (67)$$

If the link arrival time  $t$  is  $t=j-1$

$$\tilde{h}(r, j-1, l, j) = \sum_{k=1}^{Kr} \tilde{\alpha}(k, j-1, l, j) \cdot \tilde{p}(k / j-1), \quad (68)$$

In the next two sections, steps taken to develop estimates of the parameters  $p(k / t)$ , and  $a(k, t, l, j)$  are described.

## 4.2 Calculating the Probability of Choosing a Path $p(k/t, r)$

$p(k/t, r)$  can be estimated by a multinomial logit model. The probability of choosing any path  $k$  during any interval  $t$  can be expressed by:

$$p(k/t, r) = \text{Prob} \{ C(k,t) + q(k,t) < C(m,t) + q(m,t) \} \quad (69)$$

where,  $C(k,t)$  is the cumulative travel time for path  $k$  leaving the origin during interval  $t$ , and  $q(k,t)$  is the error term. The use of a multinomial logit model to estimate choice probabilities requires the error terms to have the following properties:

- 1) The error terms,  $q(k,t)$  are independent, and
- 2) identically distributed.

Based on the above assumption, the multinomial logit form [12] of the choice probabilities is expressed as:

$$p(k/t, r) = \exp(-\mu * (tt(r,k,t) + \beta * d_k)) / \sum_m \exp(-\mu * (tt(r,m,t) + \beta * d_m)) \quad (70)$$

where  $\mu$  is a scale parameter, and  $tt(r,k,t)$  is the total travel time on any path  $k$  for an O/D pair  $r$ , given that departure time from the origin is during interval  $t$ . The parameter  $m$  represents any other reasonable path. A brief explanation is required before variables  $\beta$ , and  $d_k$  could be defined. A network is a collection of local streets; the only exceptions are expressways. Since expressways have no traffic signals, it is reasonable to assume that the travel time for those paths

connecting origins and destinations that include an expressway should be shorter than the alternative paths. It is not possible to calculate travel time on a path that includes an expressway based on on-line traffic data, since no routine on-line traffic data for this type of roadway is available.

To compensate for the lack of on-line data to estimate travel time; each segment of an expressway is assigned a speed. Travel time is then calculated based on the assigned speed and distance. In addition, a bonus is assigned to each segment to denote savings in time due to the absence of traffic signals. Let's denote,  $\beta < 0$  to represent a bonus; implying savings in travel time in minutes per mile and  $d_k$  as the distance of path  $k$  on an expressway in miles. If a path does not include the expressway, then  $\beta = 0$ . To use equation (70) to calculate choice probabilities, three parameters need to be estimated, one is the total travel time  $tt(r,k,t)$ , another is the value of  $\mu$ ; and once  $\mu$  is set; then for a given value of  $\mu$ ,  $\beta$  needs to be estimated .

#### 4.2.1 Procedure For Finding Multiple Paths

The first step in O-D estimation, whether it is static or dynamic estimation, is to find paths between O-D pairs. In all static O-D estimation procedures, a set of shortest paths are found. In dynamic modeling, it is assumed that there is usually more than one path choice between O-D pairs, and drivers sometimes choose a path other than the shortest path if they anticipate delay. Some models use what is called an all-or-nothing algorithm to find a set of shortest paths and ignore the possibility of multiple paths even though the modellers recognize

that in real life all drivers do not choose the shortest path to go from an origin to a destination.

As was mentioned before, much of the past research on dynamic O-D estimation has been based on freeway sections including ramps, and intersections that have simple constrained geometry. Both of these conditions have only one possible path between O-D pairs. Therefore in traffic assignment models all trips are loaded onto this one path [Cassetta][1]; and if the network represents an urban area, trips are loaded onto a set of shortest paths, one path per O-D pair. In this study the issue of multiple path choices is dealt with as follows.

Normally the most desirable approach in finding multiple paths based on variable travel time is a dynamic assignment model. In the absence of a dynamic assignment algorithm, the paths are found through a static assignment algorithm, and then the path choice probabilities are calculated. These two steps are not simultaneous, but sequential. First, all reasonable paths are found by running a static algorithm, and then the dynamic part (which is the probability of choosing a particular path during a particular interval) is found through a separate algorithm.

An equilibrium static assignment program was adopted to find a set of reasonable paths. A maximum of three most likely paths were identified. The first possible path is the shortest path obtained by running a ten iteration equilibrium assignment algorithm of the UROAD program of the UTPS [14] package. To find the second possible path, some of the major links (streets) were deleted. The choice of which links to omit is based on tracking the shortest routes for individual O-D pairs, and familiarity with the area. The equilibrium

assignment program is run again to find shortest path given the missing links. To find the third path, the same process was repeated this time with additional links missing from the second path.

#### 4.2.2 Calculation of Path Total Travel Time

Calculation of link total travel times requires information on link speed, link delay, and time of departure from the origin. In developing a dynamic trip table, the relevant data can be obtained by collecting traffic data for a number of days. This information is usually received from probe vehicles transmitting to a monitoring center, [Cremer,Putensen][5,9] . Based on the sample collected, general traffic characteristics including delay are determined. For signalized arterial links, delay is calculated implicitly within travel time estimation by looking at the variability in the link travel time experienced by individual probe vehicles on an arterial. Total link travel time within a given period is split into 3 components; the travel time prior to reaching the tail of the over saturated queue, the subsequent time required to traverse the over saturated queue, and time to reach the tail of the uniform queue and clear the stop line.

The reliability of information from probe vehicles is heavily dependent on the number of probe vehicles, their origin-destination, the path that the probe vehicle is on, and the relationship between the departure time of the probe vehicle with respect to other vehicles. Unless all vehicles are equipped and information is received continuously, the information required to estimate travel times must be derived from sample data. In this study an attempt has been made to extract the required information from data received via inductive loops

that are installed on some arterial links in an urban area. This approach is preferred over the other methods of collecting traffic information because traffic data from inductive loops is received daily and can be collected on a daily basis.

Calculation of path total travel time for each O-D pair requires two kinds of information; link average speed, and delay per vehicle. The first step is to come up with a reasonable speed for all the links in the network. Delay per vehicle is measured by dividing total delay experienced by all vehicles having to stop during the red light within time interval  $h$  by the total number of vehicles on links during the same interval  $h$ .

### 4.2.3 Calculating Link Travel Times

Total travel time on a path is calculated by adding link travel times between the origin and the destination. Link travel time is found by dividing distance by link average speed. Since both speed and delay vary from interval to interval, link travel times vary accordingly.

$$tt(r,k,t) = \sum_{l \in k} \{ (s(l,t) * d(l)) * (60) + \delta(l,t) \} \quad (71)$$

$tt(r,k,t)$  = Total travel time on path  $k$  between O-D pair  $r$ , for vehicles leaving the origin during interval  $t$ .

$s(l,t)$  = Speed on link  $l$ , during interval  $t$ .

$d(l)$  = Distance of link  $l$ .

$\delta(l,t)$  = Delay on link  $l$  during interval  $t$ .



Due to short distances between origins and destinations in the study network, almost all

O-D pairs can be reached within one 15 minute interval. Therefore, to calculate total travel time during an interval, it was assumed that drivers leave the origin within the same interval.

#### 4.2.2 Estimation of parameter $\mu$

The choice of  $\mu$  is arbitrary, but the assumption that the error terms are independent and identically distributed constrains all error terms to have the same value of  $\mu$ . The fact that each error term has the same value of  $\mu$  implies that the variances of the error terms are equal. A suitable value for  $\mu$  was chosen through an iterative process. Given that the choice probabilities must satisfy the following axioms:

$$1) 0 < p(k/t, r) < 1$$

$$2) \sum_k p(k/t, r) = 1$$

The procedure for estimating a reasonable value for  $\mu$  is as follows: given an initial value for  $\mu$ , a sequential, 12 interval OLS O-D estimation procedure was performed with observation error variances  $V = I$ , and O-D estimation error variance  $W = I$ . The estimated counts were obtained by multiplying each O-D estimate  $x(r, t)$  by the corresponding interval assignment fraction  $h(r, l, t)$  as follows:

$$\hat{z}(l,t) = \sum_{r=1}^{nod} h(r,l,t) * x(r,t)$$

in matrix form:

$$\hat{Z}(t) = H(t) \cdot X(t)$$

where  $\hat{Z}(t)$  is a  $(n_l \times 1)$  matrix of estimated link counts,  $H(t)$  is an  $(n_l \times nod)$  assignment matrix, and  $X(t)$  is a  $(nod \times 1)$  matrix of O-D estimates.  $n_l$  represents the number of links on the O-D paths for which traffic counts are available.  $nod$  is the number of O-D pairs.

The statistic used to measure the suitability of  $\mu$  was the root mean square error.

$$rmse = \sqrt{\sum_l \{(z(l,t) - \hat{z}(l,t))^2 / n_l\}}$$

where

$$\hat{z}(l,j) = \sum_{t=j-1}^j \sum_{r=1}^{nod} h(r,t,l,j) \cdot \hat{x}(r,t),$$

or

$$\hat{z}(l,j) = \sum_{r=1}^{nod} h(r,j-1,l,j) \cdot \hat{x}(r,j-1) + \sum_{r=1}^{nod} h(r,j,l,j) \cdot \hat{x}(r,j),$$

$z(l,t)$  is the observed count during interval  $t$  for link  $l$ ,  $\hat{z}(l,t)$  is the estimated count during interval  $t$  for link  $l$ . By comparing  $rmse$  values for any given value of  $\mu$  the best value for  $\mu$  can be selected.

Keeping  $\mu$  constant, different values of  $\beta$  can be tested. The statistic used to measure the suitability of  $\beta$  is the root mean square error and percent root mean square error.

$$\%rmse = (rmse / \bar{z}) * 100$$

where

$$\bar{z} = (1/nl) \sum_{l=1}^{nl} z(l)$$

By comparing  $rmse$ , and  $\%rmse$  values, suitable values for  $\beta$  can be chosen.

### 4.3 Procedure for Finding Link-Path Incidence Fraction $\alpha(k,t,l,j)$

Traffic flow on any link  $l$  during any interval  $j$  is made up of a fraction of the O-D volume leaving the origin during intervals to  $j$ , and reaching link  $l$  during interval  $j$ . To express this more formally, we define a quantity  $\alpha(l, r) \in [0,1]$ , as the fraction of the O-D volume  $x(t,r)$  contributing to the flow on link  $l$ .

$$z(l,j) = \sum_{r=1}^{n_{od}} \sum_{t=1}^h \alpha(l, r) \cdot x(t,r)$$

The fractions  $\alpha(l, r)$ s in general depend on link arrival times, which in turn depend on departure time from the origin, total travel time on the preceeding links, and identification of the origin-destination pairs whose trips use that particular link. To calculate  $\alpha(l, r)$ , first a set of paths are chosen. The path set is determined using a traffic assignment procedure, and then the arrival time on the link is found given a path and time of departure from the origin.

$\alpha(l, r)$  should be calculated based on drivers path choice, departure time from origin, and arrival time on links of a given path. To obtain this information in real time, links on a given path between origin and destination could be traversed by vehicles equipped with devices that transmit traffic related information. This option is not yet practical.

In the literature reviewed, for time-dependent O-D estimation the fraction  $\alpha(l, r)$  is taken to be  $(0,1)$ . In this study the fraction of the O-D volume that make up flow on any link  $l$ , is time dependent  $\alpha(l,j,r)$ , and is found based on the multiplicative effect of the probability of choosing a path during any period  $h$ ,

(which is a function of alternative paths total travel time, and departure time from the origin), and the fraction of O-D flows on a particular path that are on a link during period  $h$  (which is a function of link arrival time, and departure time from the origin).

The term  $\alpha(k,t,l,j)$  is the fraction of trips following path  $k$  going from origin (O) to destination (D), which left the origin during interval  $t$ , and are contributing to flow on link  $l$  during interval  $j$ . To estimate  $\alpha(k,t,l,j)$  assume that the departure flow is uniformly spread over the interval length  $T$ , and remains equally uniform while moving in the network.

Based on the above assumptions, the fraction  $\alpha(k,t,l,j)$  can be estimated as follows:

$$\begin{aligned}\alpha(k,t,l,j) &= 1 - 1/T(t(k,t,l) - T * (j-1)) && \text{if } (j-1)T < t(k,t,l) < jT \\ &= 1 - \alpha(k,t,l,(j-1)) && \text{if } (j-2)T < t(k,t,l) < (j-1)T \\ &= 0 && \text{otherwise}\end{aligned}$$

where  $t(k,t,l)$  is the arrival time on link  $l$  of the first vehicle on path  $k$  which departed the origin during interval  $t$ .  $T$  is the length of an interval and  $j$  is the present interval.

The first step in finding the link arrival time is to determine the links on any path  $k$  that have detectors. This is done by matching the path file with a data file that contains the A-node, and the B-node of the links with traffic detectors. The links are found once the A-node, and the B-node of the links on a path match the A-node, and the B-node of the links identified as having traffic detectors. The

next step is to calculate the arrival time on these links for each O-D pair  $r$  on path  $k$ . Link arrival time is calculated similar to total travel time, but the calculation stops once the link is reached. If there are several links with detectors on a path, then the algorithm calculates the cumulative arrival time on the following link until the next link with a detector is reached. The outcome is a listing of links with detectors for each O-D pair  $r$  on path  $k$ , and the corresponding link arrival times.

Due to the short link travel times in the area used for this study, only two cases are considered. Case one calculates  $\alpha(k,t,l,j)$  for link arrival time during the present interval  $(j)$ , given that traffic left the origin some time during interval  $(j)$ . Case two calculates  $\alpha(k,(j-1),l,j)$  for link arrival time of vehicles which departed the origin during the previous interval  $(j-1)$  that are still on link  $l$ . The values of  $\alpha(k,(j-2),l,j), \dots, \alpha(k,1,l,j)$ , are considered to be zero.

A check of total travel times in the study area indicates that for the majority of O-D pairs total travel times are contained within one interval. Only in a few cases does total travel time exceed one interval, and then only slightly. For example the maximum total travel time for an O-D pair is found to be approximately 17 minutes. Therefore, it is reasonable to assume that in order to be on link  $l$  during interval  $(j)$ , traffic must have left the origin during the same interval  $(j)$  or one interval earlier  $(j-1)$ . The value of  $h(r,t,l,j)$  which is the fraction of the O-D volume that departs the origin during interval  $(j)$ , and is on link  $l$  during interval  $(j)$  is calculated as follows.

$$h(r,j,l,j) = \sum_{k: k \in K_r} \alpha(k, j, l, j) \cdot \rho(k / j, r)$$

The fraction of traffic that left during interval  $(j-1)$  and is on link  $l$  during interval  $(j)$  is calculated as:

$$h(r, (j-1), l, j) = \sum_{k: k \in K_r} \alpha(k, (j-1), l, j) \cdot \rho(k/j-1, r)$$

#### 4.4 Calculation of Error Variances

The procedure used to calculate the error variances uses several steps. In the first step, the residuals from the link flow estimates are used to calculate link flow error variances for each interval by using the normal distribution to group estimated link counts into low, medium, and high volume categories. The corresponding residuals are then grouped into low, medium, and high categories based on the grouping of the estimated link counts.

The logic of grouping data is that observations with smaller variance receive a larger weight, and therefore have greater influence in the estimates obtained for the next interval. The variance estimates are measured by RMSE, and %RMSE statistics. As an initial grouping, it was decided to group data into low, medium, and high categories. The error variance matrix is then defined as:

$$\sigma^2_g = V_g V_g^t / (n_g - 1) \quad (72)$$

$V_g$  = Vector (  $1 \times nl$  ) of grouped observation errors

$V_g^t$  = Vector of size (  $nl \times 1$  ) ; Transpose of  $V_g$

$n_g$  = Number of observation errors that fall into group  $g$ .

The observation error variance  $V_v$  is a diagonal matrix of size (  $nl \times nl$  ). The diagonal entries are grouped observation error variances  $\sigma^2_g$  . The off-diagonal entries are zero.  $nl$  is the number of links in the network with detectors. A similar procedure was used to calculate the O-D estimation error variance for each interval. First, the O-D estimates for the present interval are grouped into low, medium and high categories. Then, the residuals are placed into low,



medium, high groups based on the grouping of the O-D estimates. The error variance matrix is then defined as:

$$\lambda^2_g = W_g W_g^t / (n^*_g - 1) \quad (73)$$

$W_g$  = Vector (  $1 \times nod$  ) of grouped estimated O-D volume errors

$W_g^t$  = Vector of size (  $nod \times 1$  ) ; Transpose of  $W_g$

$n^*_g$  = Number of O-D estimation errors that fall into group  $g$ .

The O-D estimates error variance  $V_w$  is a diagonal matrix of size (  $nod \times nod$  ). The diagonal entries are grouped O-D estimates of the error variances (  $\lambda^2_g$  ).  $nod$  is the number of O-D pairs.

#### 4.4.1 Formulation of the Error Variance Calculation Procedure

The first step in calculating the error variances is to find the O-D pair volume estimates and the estimated link counts. The O-D pair volume estimates for each of the 12 intervals are obtained through a sequential Ordinary Least Square estimation procedure. In OLS both the observation error variance, and the estimation error variance are identity matrices. The O-D estimates are obtained by the following equation [60]:

$$X(j) = ( H^{*t}(j) V^{*-1} H^*(j) )^{-1} H^{*t}(j) V^{*-1} Z^*(j)$$

$V^{*-1}$  is an augmented matrix of the inverse variance of the measurement noise  $V_v$  with the variance of O-D estimates errors  $V_w$ . The partitioned form of  $V^{*-1}$  can be written as

$$V^{*-1} = (V_v^{-1}, 0 \mid V_w^{-1}, 0)$$

The link volumes are then estimated as follows:

$$\hat{Z}(j) = \sum_{t=j-1}^j H(t, j) \cdot X(t),$$

$$\hat{z}(l, j) = \sum_{t=j-1}^j \sum_{r=1}^{nod} h(r, t, l, j) \cdot x(r, t),$$

or

$$\hat{z}(l, j) = \sum_{r=1}^{nod} \sum h(r, j-1, l, j) \cdot \hat{x}(r, j-1) + \sum_{r=1}^{nod} h(r, j, l, j) x(r, j),$$

In general the link counts during an interval can be expressed by the following equations

$$Z(j) = \sum_{t=j-1}^j H(t, j) X(t) + V(j),$$

$$z(l, j) = \sum_{t=j-1}^j \sum_{r=1}^{nod} h(r, t, l, j) \cdot x(r, t) + v(l, j),$$

or

$$z(l, j) = \sum_{r=1}^{nod} h(r, j-1, l, j) \cdot \hat{x}(r, j-1) + \sum_{r=1}^{nod} h(r, j, l, j) x(r, j) + v(l, j) ,$$

The observation error variance can be calculated by the following equation

$$v(l, j) = z(l, j) - \hat{z}(l, j)$$

or in matrix form as:

$$V(j) = Z(j) - \hat{Z}(j)$$

where  $V(j)$  is the observation error diagonal matrix of  $(n_l \times n_l)$ , during interval  $j$ .

The off-diagonal entries are zero.  $Z(j)$  is an  $(n_l \times n_l)$  diagonal matrix of observed link counts during interval  $j$ , and  $\hat{Z}(j)$  is an  $(n_l \times n_l)$  diagonal matrix of the predicted counts resulting from the sequential Ordinary Least Square O-D estimation procedure. The maximum likelihood estimator of the observation error variances is :

$$\sigma^2(v(j)) = \sum_{l=1}^{nl} \{ (v(l, j) * v(l, j)) / n_l \}, \quad l=1, \dots, n_l \quad (74)$$

The error variance matrix is obtained from equation (72), with diagonal entries similar to equation (74) except that ungrouped link observation error  $v(l, j)$  is replaced with grouped link observation error  $v_g(l, j)$  replacing  $n_g$  for  $n_l$ .

#### 4.4.2 Calculation of Error Variances for the Estimated Origin-Destination Volumes

The O-D volume estimates during an interval can be expressed in a matrix form as:

$$\hat{X}(j-1) = \hat{X}(j) + W(j)$$

The estimation error can be calculated by subtracting the previous interval O-D estimates from the present interval O-D estimates

$$W(j) = \hat{X}(j-1) - \hat{X}(j) \quad (75)$$

The O-D estimate  $\hat{X}$  is an  $(nod \times 1)$  matrix.  $W(j)$  is the estimation error matrix of size  $(nod \times 1)$ . The maximum likelihood estimator of the error variance is obtained through an equation similar to equation (74) as follows:

$$\lambda^2(w(j)) = \sum_{r=1}^{nod} \{ (w(r, j) * w(r, j)) / nod \}, \quad r=1, \dots, nod \quad (76)$$

The estimated errors were grouped based on the grouping of O-D estimates during interval  $(j)$ . The estimation error variances were calculated from equation (73) with diagonal entries similar to equation (76) except that the ungrouped O-D estimated error  $w(r, j)$  is substituted with grouped O-D estimated error  $w_g(r, j)$  substituting  $n_g^*$  for  $nod$ .

## Chapter 5

### Model Validation

#### 5.1 Introduction

To test the validity of the procedure adopted for the time-dependent O-D estimation, the model was tested on a synthetic network. The network is shown in Figure 1. The network consists of 7 nodes, and 7 links. The links are one directional from node 1 to node 7. Link 7 is considered to be a dummy link with distance equal to zero. There are 3 entry points (1,2,4), and 3 exit points (3,5,7). There are 7 possible O-D pairs. The O-D pairs are (1-7), (2-7), (2-3), (1-3), (1-5), (4-5), (4-7). Traffic counters are shown by a dashed line and are located on 6 of the 7 links.

Data generated for this simulation study consists of volume on 6 links, plus travel time on all the links. Data is provided for 15 intervals of 10 minutes duration each. Data that needed to be calculated were link arrival times on each link on a path between each O-D pairs, path choice probability for O-D pair (1-7) ( the rest of the O-D pairs have only one route choice). The parameter  $\alpha$ , which is the fraction of traffic that left the origin at some time  $t$  and is on link  $l$  during interval  $h$ ; the parameter  $p$  which is the probability of choosing a path that includes link  $l$  during interval  $h$ ; and subsequently the estimation of the assignment matrix. The parameter  $p$  was estimated for the O-D pair (1-7) only.  $p$  was set equal to 1 for the other O-D pairs. This was due to the unidirectional links on the network.

## 5.2 Analysis of The Results

An alternate OLS-GLS procedure was performed on the synthetic data. By an alternate OLS-GLS procedure it is meant that for each interval an OLS estimated O-D volumes and counts were found. These estimates were used to calculate the error variances as described in Chapter 4.3. Next, a GLS estimate of O-D volumes was obtained using the inverse of the error variances as weights.

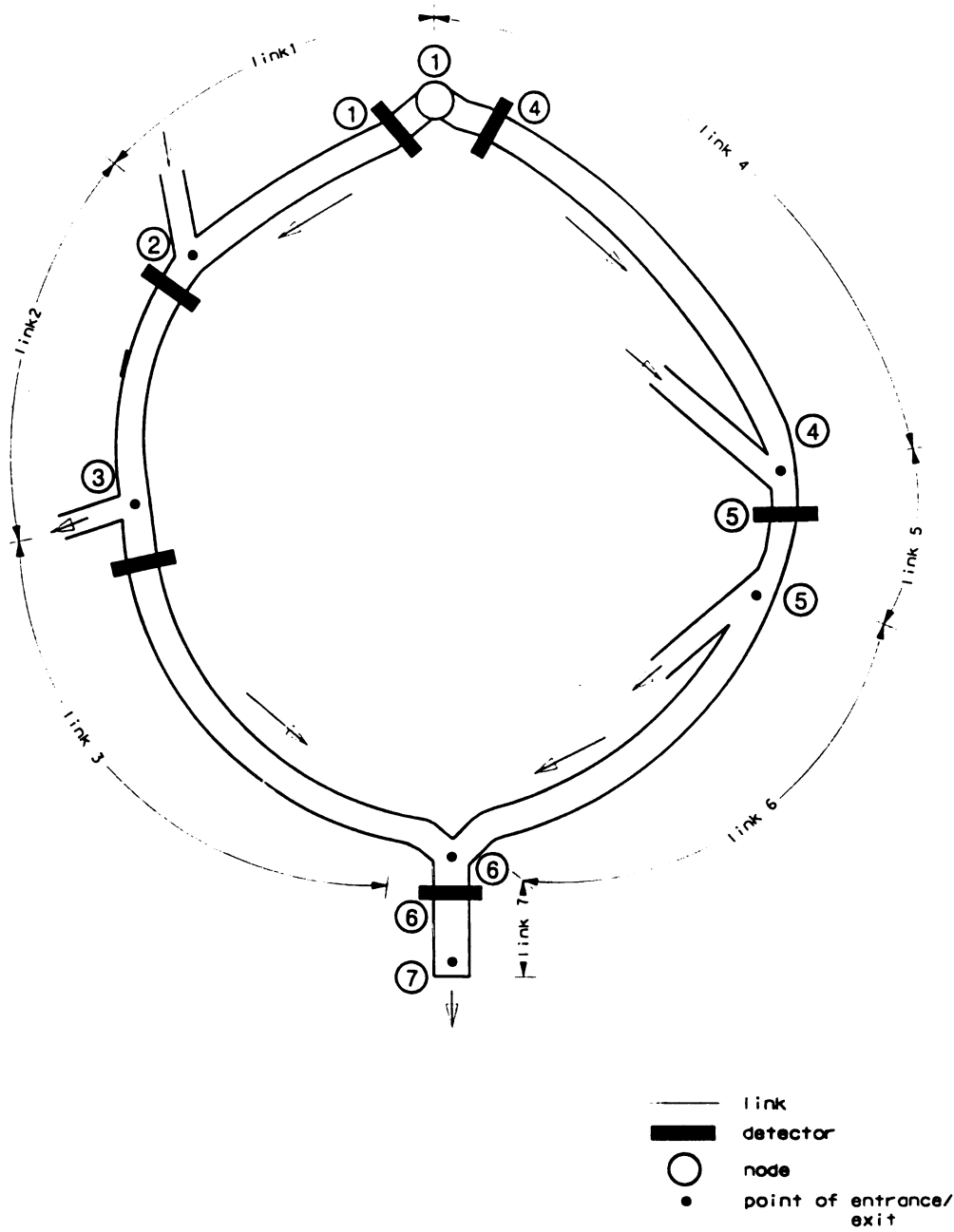
The alternate OLS-GLS procedure was performed for 2 days. Each day consists of 15 intervals of 10 minutes duration each. The estimated link counts were compared with the synthetic link counts. The measure of degree of closeness was the %RMSEs. Table 1 shows the RMSEs and %RMSEs for 15 intervals of both day 1 and day 2. Figures 2-6, show the distribution of the estimated and synthetic link counts around the mean, for day 2. For this exercise, the true O-D volumes were known. Comparison between the estimated O-D volumes, and true O-D volumes indicated a close match.

## 5.3 Sensitivity Analysis

Different values for the constant parameter  $\mu$  were tried. The purpose of doing this exercise was to find out how responsive the model is to changes in the values of the constant parameter  $\mu$ . The logit utility model for path choice was calculated for interval 1 of day 2 sample data. The %RMSE were calculated to determine the value of the constant parameter  $\mu$  that produced link estimates closest to the synthetic data. Table 2 shows the RMSE and %RMSE for different values of the constant parameter  $\mu$  for the OLS run of interval 1 of day 2. As is

apparent from the RMSEs and %RMSEs, the model does not display a significant sensitivity to changes in the values of  $\mu$ . This implies that path choice is highly invariant to different values of  $\mu$ . This is expected since only one O-D pair has a choice of separate paths.  $\mu = 1$  produces slightly lower %RMSE. The results of the sensitivity analysis are unique to this case study and are not generalizable.

Figure 1 - Drawing, Depicting the Synthetic Network



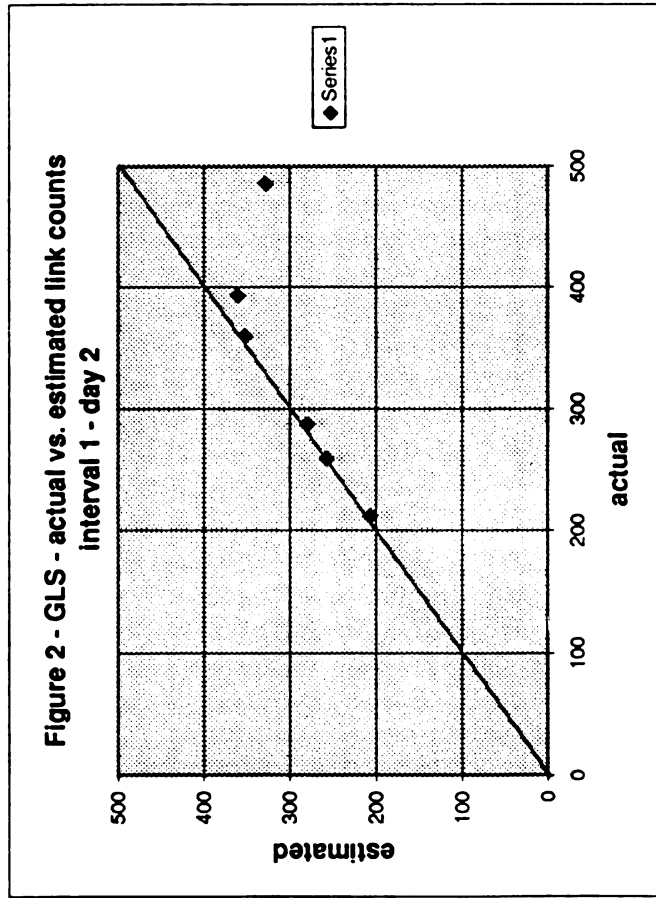


**Table 1**

Comparison of Predicted Versus Observed Link Flows -  
GLS runs - day 1 and day 2 - Sample Data

		Day 1	Day2	
Interval	rmse	%rmse	rmse	%rmse
1	58	18	66	20
2	22	7	36	11
3	43	11	36	9
4	55	12	50	11
5	60	11	64	11
6	66	12	57	10
7	83	13	71	11
8	93	14	87	13
9	101	16	95	14
10	99	16	83	13
11	80	13	72	11
12	78	13	68	11
13	62	12	53	10
14	47	9	40	8
15	44	9	32	6

Actual	Estimated
287	280
360	352
212	206
259	257
393	361
486	328

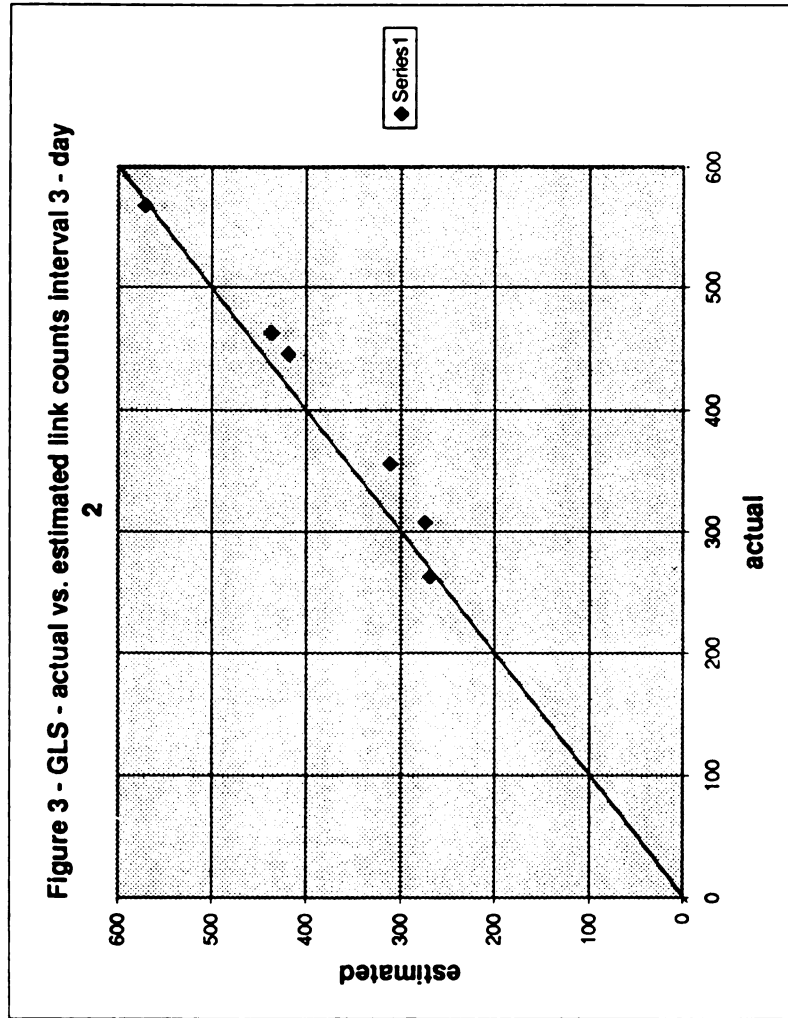


Actual

356  
446  
263  
308  
463  
568

Estimated

311  
418  
269  
274  
437  
571

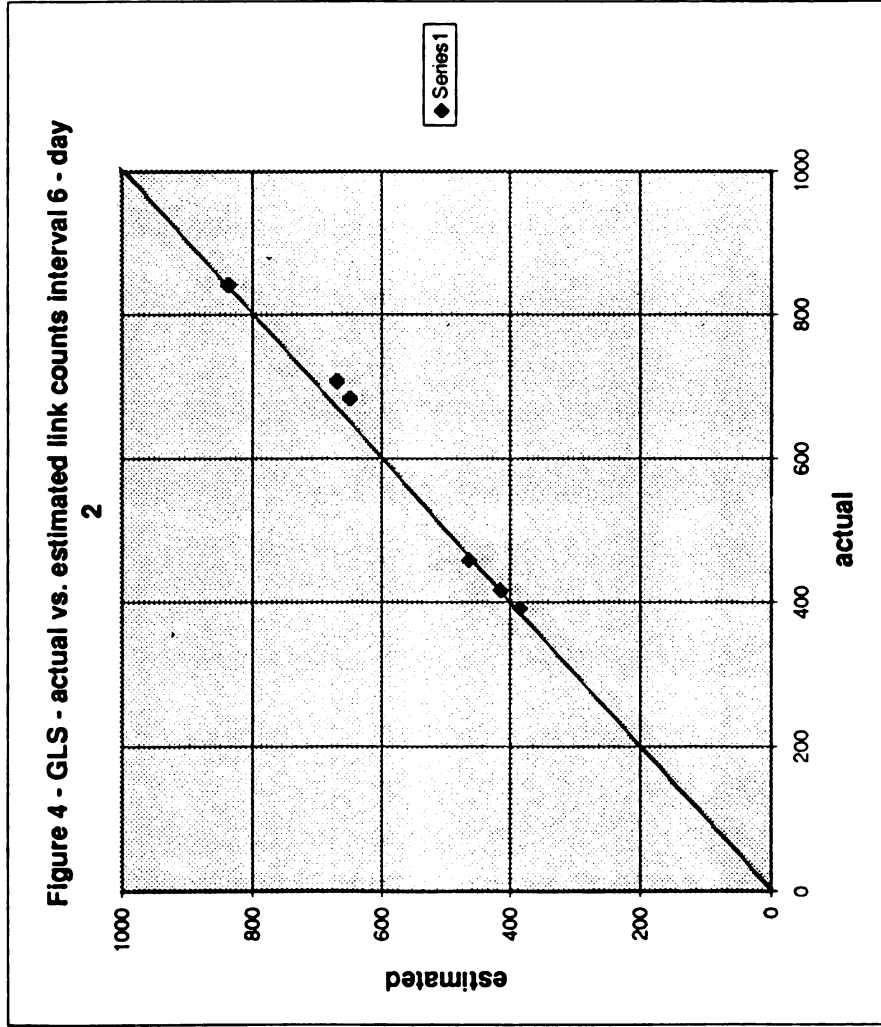


Actual

458  
683  
391  
416  
707  
841

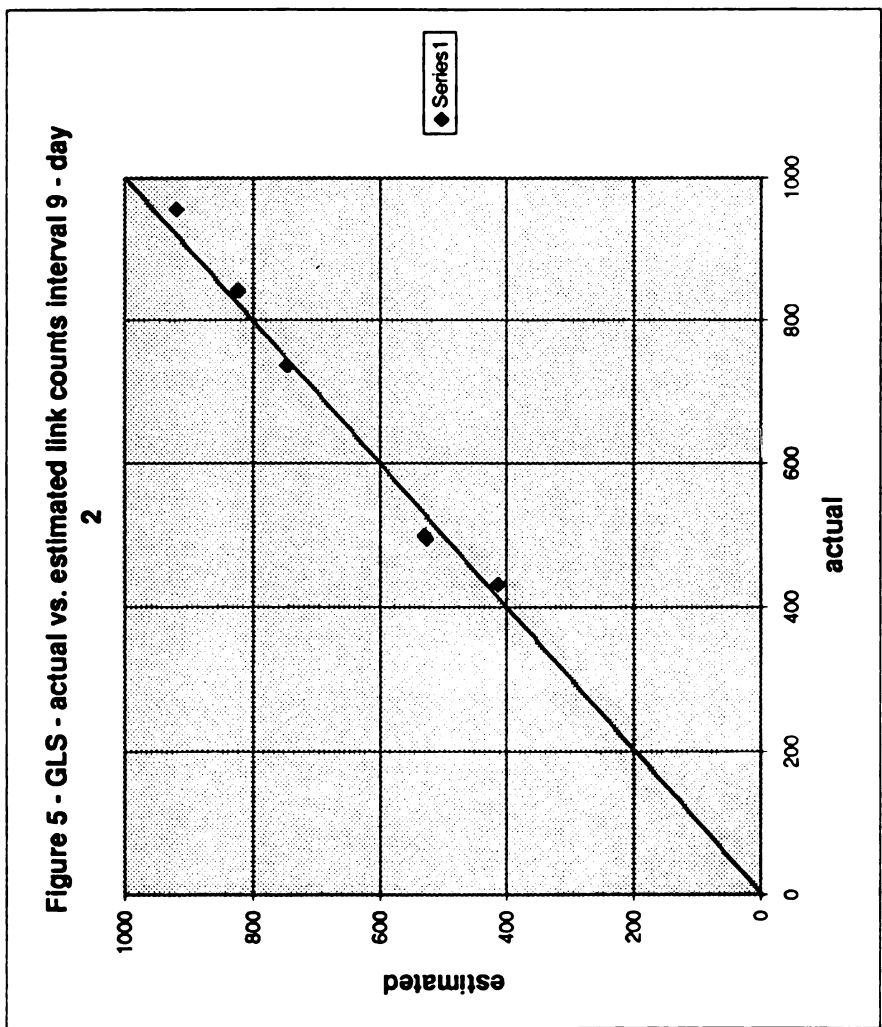
Estimated

463  
649  
384  
415  
669  
836

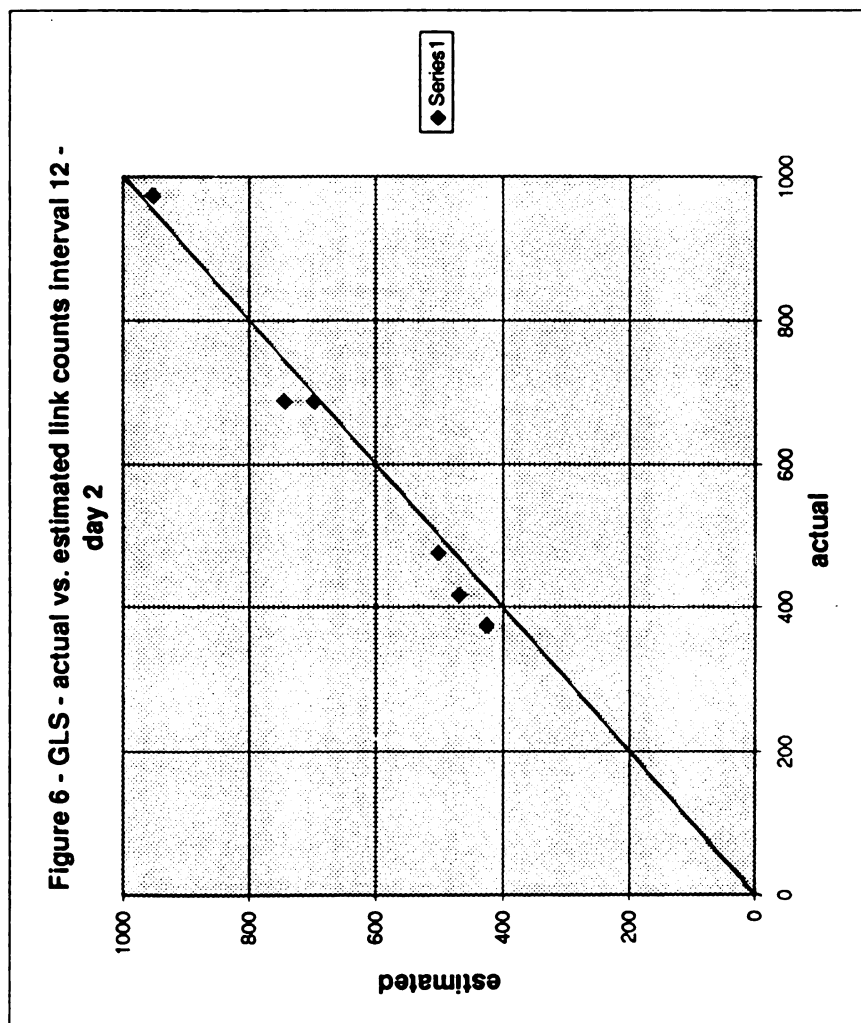


Actual  
500  
737  
432  
496  
842  
957

Estimated  
529  
746  
413  
526  
823  
920



Actual	Estimated
417	468
688	744
475	501
375	426
687	697
974	953



**Table 2**

**RMSE and %RMSE for Different Values of the Constant Parameter**  
 **$\mu$  – interval 1 of day 2 - Sample Data**

$\mu$	RMSE	%RMSE
0.3	42	13
0.8	41	13
1	41	12
1.5	43	13
2.0	43	13
3.0	44	13
4.0	45	13

## Chapter 6

### 6.1 Representation of an Urban Area as a Network

Having determined that the model gives good results when all factors are known, the next phase of the study is to see if it can be applied to an underdetermined system with several estimated parameters. In order to test the model, the first step was to define an area of interest and to represent the area as a network which is a graph  $(N,A)$  of a set of  $N$  points which are known as nodes, and a set  $A$  of ordered pairs of these nodes known as directed arcs. Arcs represent the streets. Each arc has properties such as distance, and direction.

The next step was to identify the points that traffic emanates from or tends towards such as parking lots, residences, and places of work (in other words traffic collectors/generators). Traffic collectors/generators are represented by a set of nodes that are not part of the roadway network. These nodes are called centroids. They are points on the network that represent the approximate location of traffic collectors/generators in the area. These centroids are either origins or destinations. Traffic collectors/generators are connected to the network by links with special attributes.

Once the network was coded it was necessary to match the location of traffic counters with the corresponding links on the network. This was done using special schematic maps that show the location of detectors on streets, their proximity from intersections, the direction of traffic covered by loops, and the number of lanes covered by these detectors. Coding the location of traffic counters on the network was done by identifying the arcs that represent the



corresponding streets on the schematic maps.

The inner study area consists of parts of the Beacon Hill, Back Bay, and Kenmore neighborhoods, and Commonwealth Avenue to the Boston University Bridge. The external areas include Cambridge, West end, Financial district, Downtown crossing, Chinatown, Theater district , Prudential /Copley area, Fenway, the edge of Brookline; and Brighton. The area is divided into 34 zones. The zoning scheme was adopted from the 1975 DPW (Department of Public Works) zoning systems. The zones are chosen where there is either a concentration of business activities or residents. Parking lots are chosen as traffic collectors or centroids for each zone. (See Map of Boston).

The network representation of the area has 1124 links and 34 centroids. twelve centroids represent the inner study areas, and 22 centroids represent outside areas or externals.

There are a total of 77 traffic detectors in this area. The majority of traffic detectors cover only one lane, and are located in one side of a street only. In a few cases, such as Cambridge Street, loops cover both directions, and two lanes on each side. The 77 links that had detectors were identified and the links A-node, and B-node were stored in a data file. Path files for all O-D pairs are listings of A-nodes, and B-nodes of the links on paths that join origins to destinations. The path file was matched with the 77 links file to find out which links on each of the possible paths had detectors. Of the 77 links, a total of 58 links fall on at least one paths that connects O-D pairs.

## 6.2 Data Collection

After coding the network and identifying the location of the loops, the next task was to collect data for the analysis. Local street traffic count data were provided by the City of Boston Traffic Control Center. In the Traffic Control Center, traffic controllers monitor traffic by watching link data displayed on computer screens. The location of the link on the network is displayed on a large screen where a series of flashing lights display traffic signals. A flashing arrow on a link signals either uninterrupted flow or congestion. A green arrow signals normal flow, and red congested flow. If congestion occurs traffic controllers attempt to correct the problem by adjusting the traffic signal timing. The data is not ordinarily saved since no further analyses are done; but if requested, data can be saved for any particular period, in 15 minute increments. A hard copy of the traffic data was obtained from the Traffic Control Center.

Each traffic detector is identified by a number. A set of detectors clustered in one area are identified by their area number. Each report provides two blocks of information; the first block gives traffic statistics per detector such as volume, percent occupancy, average speed, number of stops, delay, historical volume (which is an hourly average based on the previous 5 weeks), and historical occupancy (which is an average based on the previous 5 weeks). The second block of information is a summary statistic for the section. Data for this block is obtained by averaging the detector data in a section.

For this study, data were obtained for every 15 minute interval for the period from 6:00 a.m. to 9:00 a.m. for six consecutive days from a Monday to the

Monday of the following week not including the weekend. Data were collected between August 3 and August 14, 1992. The data explicitly used in parameter estimation of the recursive Generalized Least Square algorithm are; volume, average speed, and delay.

### 6.3 Data Description

A brief description of each of the items on the report is given below:

- **Sensor number** This is a reference number used for locating traffic detectors on maps used by the traffic controllers. These maps have detailed intersection schematics.
- **Traffic volume (VEH)** per 15 minute interval. Counts are received and stored during each traffic signal cycle, and then added together to get 15 minute counts.
- **Average speed (MPH)** An average speed for a particular link is calculated as follows: An algorithm sets an average vehicle length to be equal to 20 ft. A detector is assumed to have 5 ft x 5 ft dimensions. As the front of a vehicle passes over a loop, 5 ft. is added to the length of a vehicle, and as the back of a vehicle crosses the loop another 5 ft. is added to the length of a vehicle, making a vehicle 30 ft long. While a loop is occupied by a vehicle, it sends a signal to the computer. The duration of the signal is taken to be the time over the loop.

The algorithm then calculates speed by dividing the vehicles length (which is assumed to be a constant 30 ft.) by the duration of the signal. During the green interval of each cycle vehicle speeds are calculated and then divided by the number of vehicles that passed over a loop to obtain an average speed for a link during that cycle.

The average speeds for each cycle are calculated and summed, and then divided by the number of cycles in an interval to obtain an average speed for a link for that interval. The section average speed is the arithmetic average of the calculated average speeds of the sensors in a section.

- **Stops (VEH)** This is a derived parameter, assuming that some fraction of vehicles that pass over a sensor during the yellow signal are assumed to stop at the intersection for the red light.

- **Average stopped time delay (min)** This is the number of stops per cycle, multiplied by delay per vehicle stopped. This is the average delay experienced by vehicles at the stop line due to the red light. Delay is calculated per cycle summed over the number of cycles in a 15 minute interval, and then averaged.

## 6.4 Estimating Speed and Delay For All The Links in The Network

The network for the study area consists of 1124 links. A total of 77 links equipped with detectors fall within the study area. Of these 77 links, 58 links fell on one or more of the minimum paths that connect origins to destinations.

Therefore, assumptions had to be made to estimate speed and delay for the remainder of links in the network. To do this, the network was divided into 5 sections. The division followed the segmentation scheme used by the City of Boston. Using these section boundaries, the following assumptions were made:

- For each interval, all the links within the boundaries of a section are assigned the average section speed.
- For each interval, all the links within the boundaries of a section are assigned a constant section delay. This delay is calculated as follows:

$$\delta = \frac{\sum_{l=1}^m \delta(l,h)}{\sum_{l=1}^m v(l,h)} \quad (77)$$

$\delta$  = Section delay

$\delta(l,h)$  = Delay on link  $l$  during interval  $h$ . Only links that have traffic detectors are considered .

$v(l,h)$  = Volume on link  $l$  during interval  $h$ .

## 6.5 Adjustment of the Apriori (1987) O-D Matrix

The historical trip matrix was derived from the 1987 Census survey data. The O-D volumes were factored to update this table to be more consistent with the observed link counts.

The following steps were taken to adjust the apriori O-D table:

- 1) A screen line was picked. In this case due to the particular geometry of the area, and the availability of on-line count data from the streets that cross the screen line Massachusetts avenue was chosen. The direction of traffic chosen was west to east.
- 2) The apriori (1987) O-D, 3 hour volumes for those O-D pairs whose paths cross the screen line were summed.
- 3) The on-line traffic counts on the links that crossed the screen line were summed for the 12 intervals of day 1.
- 4) This volume was then divided by the total observed volumes from the links over the 3 hour period to get the adjustment factor.

The apriori O-D matrix was adjusted by the following ratio:

$$\text{ratio} = \left( \sum_{i=1}^n x(i) \right) / \left( \sum_{h=1}^{12} \sum_{j=1}^m v(j,h) \right)$$

where

$x(i)$  = trips going from origin (O) to destination (D) that cross the screen line

$n$  = Number of  $x(i)$ s.

$v(j,h)$  = Observed counts on link  $j$  during interval  $h$ , on those links on the O-D paths that cross the screen line.

$m$  = Number of  $v(j,h)$ s.

Using the above calculation, the adjustment factor was found to be equal to 1.36.

## 6.6 Choosing the Parameter $\mu$

The RMSE values of the OLS run of interval one of day one were calculated for values of  $\mu$  between 0.3 and 4.0. Table 3 lists the value of  $\mu$  vs. RMSE. The

results show that RMSE is not very sensitive to the value of  $\mu$ . Choosing  $\mu = 1.5$ , causes the shortest path to be the prominent path choice; but still assigns some trips to the second and third paths defined for each origin and destination. A value of  $\mu = 1$  means that the probability of trips assigned to the minimum path is not significantly higher than an alternate path. As  $\mu$  increases, the probability of trips being assigned to the minimum path as opposed to an alternate path increases. The minimum path update algorithm developed by the Bureau of Public Roads (BPR) uses an update value of 1.5 for the exponent of  $v/c$ , which is somewhat analogous, and supports the choice of  $\mu$  for this study.

## 6.7 Choosing the Bonus factor $\beta$

Using a  $\mu$  value of 1.5, different values of  $\beta$  from -0.2 to -1.0 were used to obtain the RMSE values of the OLS run of interval one of day one. Table 4 shows the values of  $\beta$  vs. RMSE. Since  $\beta$  is the travel time bonus used to modify the travel time on Storrow Drive (an expressway), the value is constrained by reasonableness. Due to the link lengths, the maximum travel time on any segment of "Storrow Drive" is 0.5 minutes, therefore a bonus factor  $\beta = -0.3$  minutes was used.

## 6.8 Test of Model Responsiveness When Different Weights are Assigned to the Count Data versus Prior O-D Estimates Data

To test how sensitive the model is to input data (Count data), a weighting factor  $\alpha$  was assigned to the counts and  $(1 - \alpha)$  was assigned to the prior O-D estimates. The purpose of this exercise was to find out if the link estimates improve by assigning more weight to the counts and less weight to the prior O-D matrix.

The test was performed as follows: The inverse counts error variance were multiplied by  $\alpha$ , and the inverse O-D estimates error variances were multiplied by  $(1-\alpha)$ . The model was tested for several values of  $\alpha = 0.05, 0.2, 0.6$  for one day only, without a prior day O-D estimate. The prior interval O-D estimate used came from the 'historical' O-D matrix. The measure of how close the estimated counts were to the actual counts was the RMSE values. Table 5 shows the RMSEs for different values of  $\alpha$ .

Compare this Table with table 6 which contains the RMSE values of a 12 interval GLS run when equal weights are assigned to the inverse error variance for the counts and the O-D estimates. The model produced better estimates of link counts when almost all the weight was assigned to the counts. The implication is that more count data would produce better link count estimates which in turn implied better O-D estimates. Basically, the less underdetermined the system, the better the estimating ability of the model.

These results indicate that the model acts rationally, since it produces better estimates of link counts when more weight is assigned to the counts and less to the prior O-D estimates. This weight assigning process though reasonable, could not be adopted to the test network in the study because count data from all links are not available. The obvious deduction from this exercise is that to get better O-D estimates additional traffic count data is needed. However, since these data are not available, and it can be assumed that successive O-D flows are not randomly distributed, but in fact are related to the previous interval flows, a value of  $\alpha = 1$  was selected. Therefore, the weights assigned to the counts and the O-D estimates are as described earlier derived from observation error



variances and O-D estimates error variances; and are not artificially augmented by any factor  $\alpha$ .

Table 3

RMSE for different values of the constant parameter  
 $\mu$  – interval 1 of day 1

$\mu$	RMSE
0.3	46
0.8	45
1	44
1.5	43
2	42
3	41
4	40

Table 4

RMSE and %RMSE for different values of the bonus factor  
 $(\beta)$  – interval 1 of day 1

$\beta$	RMSE	%RMSE
-0.2	63	240
-0.3	64	242
-0.5	63	243

Table 5

RMSEs For Different Values of the Weighting Factor  
 $\alpha$       12 intervals - GLS Run

Interval	$\alpha = 0.05$	$\alpha = 0.2$	$\alpha = 0.6$
	RMSE		
1	20	23	31
2	17	21	29
3	20	23	29
4	27	30	36
5	37	42	50
6	39	44	51
7	43	50	61
8	52	61	74
9	64	75	93
10	69	80	98
11	74	86	105
12	77	86	105

**Table 6**  
**Comparison of Actual Versus Estimated Counts -**  
**12 intervals - GLS Run**

Interval	RMSE
1	36
2	32
3	31
4	36
5	50
6	50
7	59
8	72
9	90
10	95
11	102
12	102

## Chapter 7

### Analysis of the Results

#### 7.1 Introduction

The intent of this chapter is to report the results of the application of the model described in chapters 3 and 4 to estimate O-D trips for the study network using the data described in chapter 6. The approach used in the analyses is depicted in the flow chart shown Figure 7. The results of the initial model application were used to identify problems with the data. The data were then adjusted, and the analyses were repeated.

#### 7.2 The OLS Procedure

As shown in Figure 7, the model was initially run using the OLS procedure. The OLS procedure will result in the O-D estimate that produces the lowest %RMSE given a particular data set. The OLS procedure is identical to the GLS; except that the error variance of the link counts and the prior O-D estimates are not weighted. The OLS O-D estimation equation is similar to equation (60) of chapter 3 with an the inverse variance matrix  $V^{*-1} = I$ , an identity matrix.

$$\hat{X}(j) = (H^{*t}(j) V^{*-1} H^{*}(j))^{-1} H^{*t}(j) V^{*-1} Z^{*}(j)$$

The OLS procedure was performed in a sequential manner for 12 consecutive intervals for each of 5 days. To check the validity of the O-D estimates, these estimates were used to calculate link count estimates for each interval using

equation 61 of chapter 3. The RMSE and %RMSE statistics measure how close the link estimates were to the actual counts. The RMSEs and %RMSEs are exhibited in Tables 7 through 11. A comparison of RMSE's and %RMSE's of the 12 intervals shows little change in RMSE values between day 4 and day 5. The predicted and observed link flows for intervals (1,3,6,9,12) of day 5 are shown in Figures 9-13. A review of these charts shows:

- 1) A significant over-estimation of almost all links in interval 1 (54 of 58).
- 2) The identification of 3 outlier links (overestimated).
- 3) Most of the links being underestimated by interval 12 (39 of 58 links).

### 7.3 The GLS Procedure

The GLS procedure attempts to minimize the deviations of the estimated counts from the observed counts while also weighting the magnitude of deviations between the prior O-D estimates and the present O-D estimates. The O-D estimates and the link count estimates of the OLS procedure for the same interval of the same day are used in calculating errors and estimating the variance as was described in section 4.3.

The GLS procedure was also run sequentially for 12 intervals of each day for 5 days. The O-D estimates were obtained using equation (60) of section 3.1 (shown here as a reminder)

$$\hat{X}(j) = (H^{*t}(j) V^{*-1} H^{*}(j))^{-1} H^{*t}(j) V^{*-1} Z^{*}(j)$$

The link counts were then estimated from the O-D estimates for each interval.

The link counts are estimated using equation (50) of chapter 3

$$z(l, j) = \sum h(r, j-1, l, j) \cdot \hat{x}(r, j-1) + \sum h(r, j, l, j) x(r, j) + v(l, j) \quad , \quad r=1, \dots, n_{od}$$

in matrix form the above equation is stated as

$$Z(j) = H(j-1) \hat{X}(j) + H(j) X(j)$$

The measure of the goodness- of -fit of the estimated link counts to the actual link counts was the RMSE and %RMSE statistics . A comparison of RMSE's and %RMSE's of the 12 intervals for 5 days are shown in Tables 12 through 16 show little change in the RMSEs and %RMSEs between day 4 and day 5. The predicted and observed link flows for intervals (1,3,6,9,12) of day 5 are shown in Figures 14-18. A review of these charts show:

- 1) Most of the links are overestimated in interval 1, with 3 links identified as outliers.
- 2) In interval 3 and 6, the majority of the links are underestimated, with both high and low outliers.
- 3) In interval 9, there is nearly an equal number of links that are over and under estimated. The low outliers are the most significant.
- 4) In interval 12, there is an equal number of links overestimated and underestimated, with no evidence of convergence between the actual and estimated link counts.



## 7.4 A Revision of the Assumptions

At the end of day 5 there is still a considerable deviation between the actual and estimated counts for some of the links. This means that either the number of trips assigned to the links by the assignment matrix is not correct, or there are errors in the actual traffic counts. Since calculation of the assignment matrix is dependent on travel times, which are a function of speed and delay, errors in estimating these parameters could lead to assignment errors..

In an attempt to improve the assignment matrix, the assumptions made in deriving average speed and delay were reviewed. The under-estimation of many of the interval 12 day 5 link counts was alarming. One explanation for under-estimation could be a low average speed on some links or in some sections of the network which would lead to the link being assigned low volumes. In checking the raw data obtained from the Traffic Control Center, some links were being recorded with an average speed as low as 5 mph. Since this does not appear reasonable, the following procedure was adopted to increase average speed and reduce average stopped delay.

In each section those detectors that showed speeds that were not within the range of the level of service A, B, and C were removed from the calculation of an average speed for the section.

The link average stopped delay was also recalculated considering only those links used to calculate the section average speed.

The section average delay was then calculated from equation (77) of chapter 6

$$\delta = \frac{\sum_{l=1}^m \delta(l, h)}{\sum_{l=1}^m v(l, h)}$$

where the numerator is the sum of delay from detectors that show level of service A, B, or C and the denominator is the sum of the volume of those detectors that fall within these levels of service.

A review of the detector data used to represent "actual" counts on the links indicated that in some cases the detectors only covered one lane of a multi-lane street. In these locations, the detector counts were multiplied by the ratio of total lanes on the links to the number of lanes covered by the counter to obtain new link volumes.

## 7.5 Adjustment Procedure

The counts were adjusted and the speed and delay were revised as described above and the OLS procedure was run again for 12 intervals of day 5. Table 17 shows the values of the RMSEs and %RMSEs for day 5. The %RMSE was reduced from 64 to 45 by interval 12, and the distribution around the mean appears to be better with the adjusted data, (Figures 19-23). Observations from these charts are:

- 1) Almost all links are overestimated in interval 1 (similar to the original OLS results).

- 2) The trend of most links being overestimated in interval 1 and most links being underestimated in interval 12 is retained.
- 3) The same three links that were overestimated in the initial OLS runs remain outliers with the adjusted data.

The location of these three links and their proximity to a centroid is shown in Figure 8. The over-estimation of these three links constitutes 15 % of the RMSE values of the adjusted OLS estimates for interval 12.

Another indicator of how close the link estimates are to the actual counts is the frequency with which links fall within a specified percent absolute difference between the estimated and the actual counts for the final interval of day 5:

- 1) 16 Percent of the links fall in interval { 0, 25]
- 2) 31 Percent of the links fall in interval {25, 50]
- 3) 27 Percent of the links fall in interval {50,100]
- 4) 26 Percent of the links fall in interval { >100 }

Comparison of the RMSEs and %RMSEs ( Table 18 and Table 16) of the GLS runs before and after data adjustment show lower RMSEs and %RMSEs for the unadjusted GLS runs. This is a result of the weighting factors used to calculate successive estimates. In the GLS estimation procedure, the inverse of the error variance ( this includes link count error variance and O-D error variance) act as weighting factors. These weights adjust for the differences between the estimated link counts and the observed counts, and smooth out large variations that might occur between the present interval and the previous interval O-D

estimates. Therefore, high weights are assigned to data with a low error variance, and low weights are assigned to data with a high error variance.

In the adjusted GLS estimation procedure the weights assigned to the link counts for intervals 4-12 are very small compared to the weights assigned to the link counts in the unadjusted GLS runs. The weights assigned to prior O-D estimates in the adjusted GLS estimation procedure are also smaller than the weights assigned to prior O-D estimates in the unadjusted GLS estimation.

The effect of the high volume links (which are generally overestimated) being assigned extremely low weights in the GLS process is an over-estimation of the O-D trips which use these links. This results in an increase in the volume assigned to these links and consequently to even larger estimated link flows in the next interval. Comparison between the estimated and actual link counts for the GLS runs using the adjusted data show some improvement in the distribution around the mean (Figures 24-28). Reviewing these figures leads to the following observations:

- 1) There is approximately an equal number of links overestimated and underestimated in interval 1.
- 2) There are a number of outliers on both the high and low side in interval 1.
- 3) By interval 12, most of the links are overestimated, but outliers remain on both the high and low side.
- 4) The three outliers from the previous runs are no longer identifiable as a special case.

## 7.6 An alternate OLS-GLS approach

In an attempt to improve the distribution of link estimates around the mean, a variation of the OLS - GLS procedure was tried. The procedure is as follows: Interval one OLS O-D estimates are calculated. Subsequently interval one GLS O-D estimates are calculated. For the second and each subsequent interval OLS O-D estimation, the prior interval GLS O-D estimates were used as an apriori O-D matrix.

A comparison of the RMSEs and %RMSEs of 12 intervals of 5 days are shown in Tables 19-23, both the RMSE and %RMSE values increase towards the later intervals. However, they are much lower than the GLS results shown in Table 18 when the sequential OLS - GLS was employed. The predicted and observed link flows for intervals (1, 3, 6, 9, 12) of day 5 are shown in Figures 29-33. Comparison between the estimated and actual link counts for the alternate GLS runs using the adjusted data show some improvement in the distribution around the mean. Reviewing these figures leads to the following observations:

- 1) There is approximately an equal number of links overestimated and underestimated in interval 1.
- 2) There are a number of outliers on both the high and low side in interval 1.
- 3) By interval 12, most of the links are overestimated, but outliers remain on both the high and low side.

## **7.7 Possible Sources of Differences Between the Estimated and Actual Link Counts**

There are three outliers that appear in many of the figures showing link counts versus estimated link volumes. In the alternating OLS-GLS procedure they represent 32% of the RMSE and 54% of the %RMSE terms in interval 12 day 5 results. Two of these links are on the one way street which provides the only access to and from the centroid shown in Figure 8. Therefore, in the estimating process, all paths going from any origin to this particular centroid (destination) must include link 1. Likewise, all paths departing this origin must include link 2. These links are also used by trips from other origins to other destinations. Because path assignment must be continuous, trips on a link must continue on the adjacent links on the same paths. Thus overestimating link 2 results in overestimating link 3.

The concentration of a significant percent of the error on these three links is thought to be a result of the zone structure. Using centroids with multiple access links to the network should result in fewer outliers, and a lower RMSE.

Certain other assumptions or simplifications had to be made in the application of the model structure to the data available. These items are perceived to contribute to the error terms when applying the model to the study area network. First, the selection of links to be eliminated in order to find the second and third alternative path for the trip assignment process was heuristic, and constrained to be on paths that included one of the 58 links where the detectors were located.

The use of a probabilistic assignment algorithm that would identify the three alternate paths in one run might result in a different set of paths.

If the constraint that all paths must include one of the 58 links were relaxed (by virtue of having more detectors), the vehicle trip distribution would be more uniformly distributed over the network, thus reducing the probability of over assignment on the links with detectors.

A second parameter that would change the number of trips assigned to various paths in the network is the parameter  $\beta$ .

This parameter (as was described in chapter 4) represents savings in travel time (min) for trips using expressway links. Thus  $\beta$  represents the drivers perception of time saved by choosing a route that includes a freeway segment. In this research, the value of  $\beta$  was selected as a maximum reasonable value for the shortest freeway link in the network.

In fact, different values of  $\beta$  might apply to links other than freeway links. For example, motorists may perceive that a one way street with good progression would provide a higher average speed than a two way arterial with a high density of traffic signals. The use of  $\beta$  for freeway links,  $\beta_2$  for one-way links and  $\beta_3$  for other arterial links may provide a better representation of driver perception of the disutility of using links on various paths.

In addition,  $\beta$  is considered to be time independent in the model formulation. Thus, the motorist is assigned the same perceived bonus at 6:00 in the morning

and in the height of rush hour. A feature that should be considered in future applications of the model would be to use a  $\beta(t)$  formulation, with  $\beta$  varying according to the time of day.

Other assumptions or procedural steps used in the application of the model may contribute to the high RMSE and %RMSE values include:

- 1) Grouping of the link counts and the estimated O-D volumes.

Link counts and O-D volumes were categorized into 3 groups of low, medium, and high. Other grouping schemes such as clustering data into groups of 2 or 4 were not considered.

- 2) Adjustment of the prior O-D estimates.

During interval (j), the prior O-D estimate  $X(d,j-1)$ , ( $d$ =day,  $j$ =interval) is multiplied by a correction factor. The correction factor is:

$$\begin{array}{lll} \text{if} & X(d-1,j-1) > 0 & \text{then} \quad X(d-1,j) / X(d-1,j-1) \\ \text{if} & X(d-1,j-1) = 0 & \text{then} \quad X(d-1,j) / X(d-1,j-1) = 1.0 \end{array}$$

- 3) Problems with the On-line data. Observed link counts.

Volume data collected by the detectors in the streets often provides volume data for only one or two lanes out of a multi - lane street. Therefore, the volume data that is a major input data in the model is incomplete. To adjust for lack of data, a correction factor based on the number of lanes was used.

- 4) Observed link speeds

The speed information obtained from traffic counters is an average speed of vehicles going over detectors during the green phase of traffic signals. This speed does not represents the distribution of a vehicle's speed while traveling across a link. It only reflects the speed of the vehicles passing over the detectors,



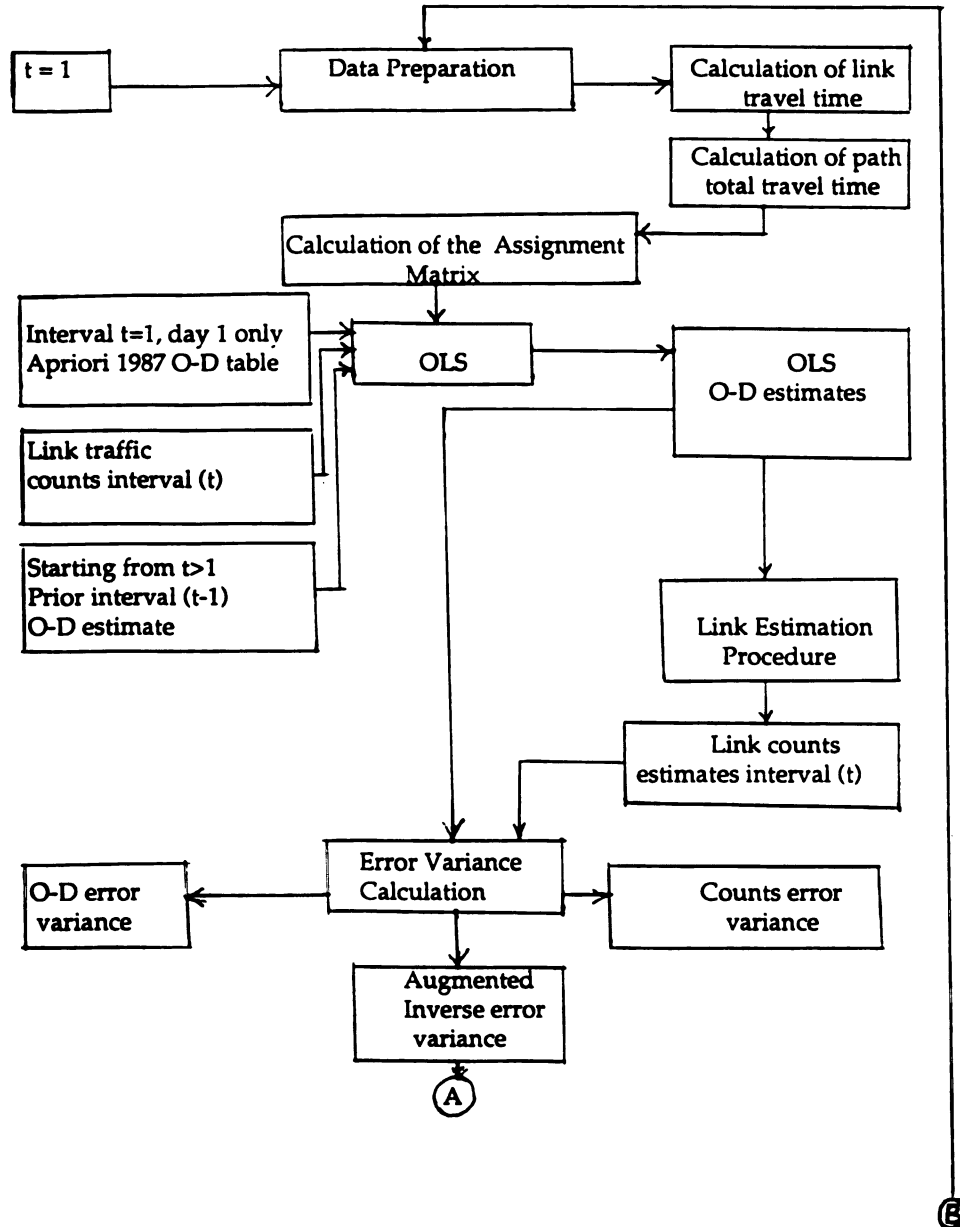
and this may vary according to the distance from the counter location to the intersection.

5) Observed link delay

Delay values obtained from the detectors represent delay due to stopping at an intersection due to a red traffic signal. No on-line delay information is available for random midblock delay on streets.

Figure 7

Flow Chart Depicting the overall OLS - GLS  
Procedure



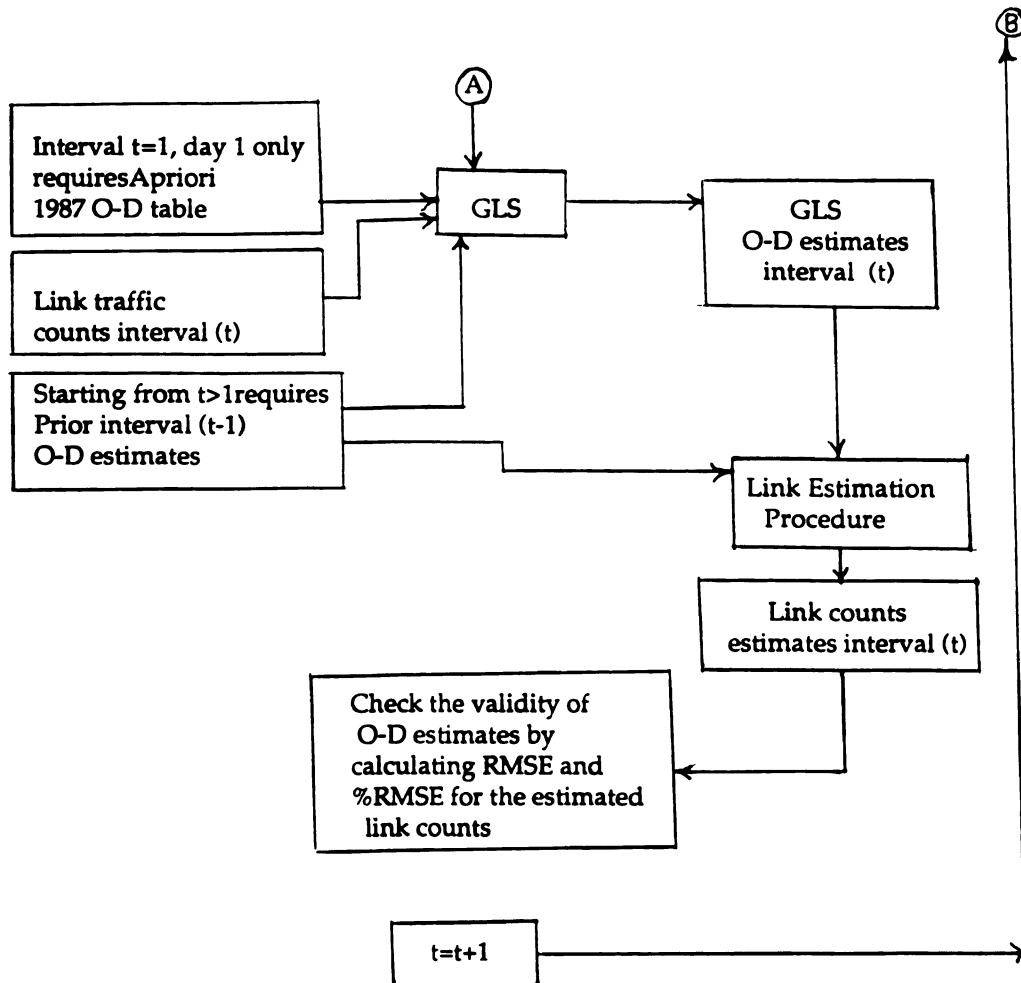


Figure 8

Drawing, Depicting the Location of the Over-Estimated Access/Egress Links to a Centroid



**Table 7****Comparison of Predicted Versus Observed Link Flows - OLS run - Day 1**

<b>Intervals</b>	<b>Xobs</b>	<b>Xest</b>	<b>RMSE</b>	<b>%RMSE</b>
1	1536	4184	64	242
2	1485	2995	43	167
3	2205	3592	46	121
4	3055	4660	57	108
5	3613	4985	61	98
6	3904	4990	57	85
7	4532	5658	65	83
8	5648	6391	70	72
9	6495	6958	81	72
10	6878	6903	83	70
11	6920	6930	80	67
12	7070	6688	80	65

Table 8

Comparison of Predicted Versus Observed Link Fows - OLS run - Day 2

Intervals	Xobs	Xest	RMSE	%RMSE
1	1509	3823	55	212
2	1638	2950	42	148
3	2569	3765	46	103
4	2928	4109	49	95
5	3757	4877	55	86
6	4348	5418	63	83
7	5071	5842	62	71
8	5928	6630	66	65
9	6760	7129	78	67
10	7076	7153	81	66
11	7172	7040	82	66
12	7384	7098	83	65

Table 9

Comparison of Predicted Versus Observed Link Flows - OLS run - Day 3

Intervals	Xobs	Xest	RMSE	%RMSE
1	1553	3935	58	217
2	1648	3021	45	157
3	2428	3602	44	106
4	3010	4204	50	96
5	3809	4841	60	91
6	4468	5576	63	82
7	5011	6026	65	75
8	5922	6762	73	72
9	6594	7215	80	70
10	7133	7616	84	69
11	7570	7816	88	67
12	7421	7436	85	66

**Table 10**

**Comparison of Predicted Versus Observed Link Flows - OLS run - Day 4**

<b>Intervals</b>	<b>Xobs</b>	<b>Xest</b>	<b>RMSE</b>	<b>%RMSE</b>
1	1532	3780	57	215
2	1622	2820	42	150
3	2431	3598	46	109
4	3058	4355	51	98
5	4060	5421	61	86
6	4490	5631	65	84
7	5134	6284	70	79
8	6150	7303	79	74
9	6754	7652	87	75
10	6661	6668	82	71
11	7524	6507	84	65
12	7419	6302	87	68



Table 11

Comparison of Predicted Versus Observed Link Flows - OLS run - Day 5

Intervals	Xobs	Xest	RMSE	%RMSE
1	1521	3788	54	205
2	1616	3055	43	153
3	2378	3485	42	102
4	2872	4128	50	102
5	3805	4890	54	83
6	4157	5132	57	79
7	4882	5986	65	77
8	5616	6248	67	69
9	6374	6973	78	71
10	6654	6705	73	64
11	7245	7422	83	66
12	7331	7249	81	64

Figure 9 - OLS run - actual vs. estimated link counts interval 1 - day 5

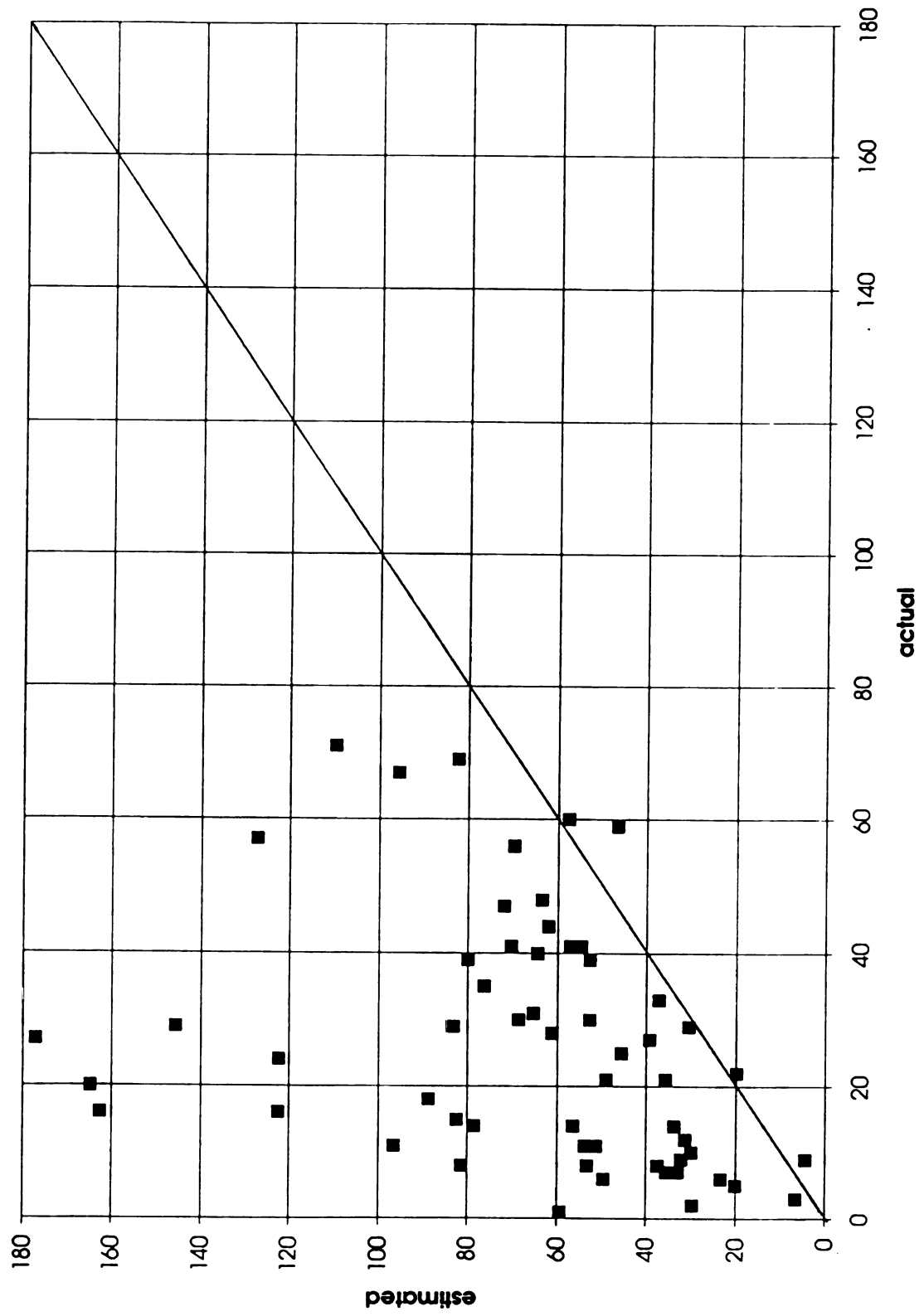


Figure 10 - OLS run - actual vs. estimated link counts interval 3 - day 5

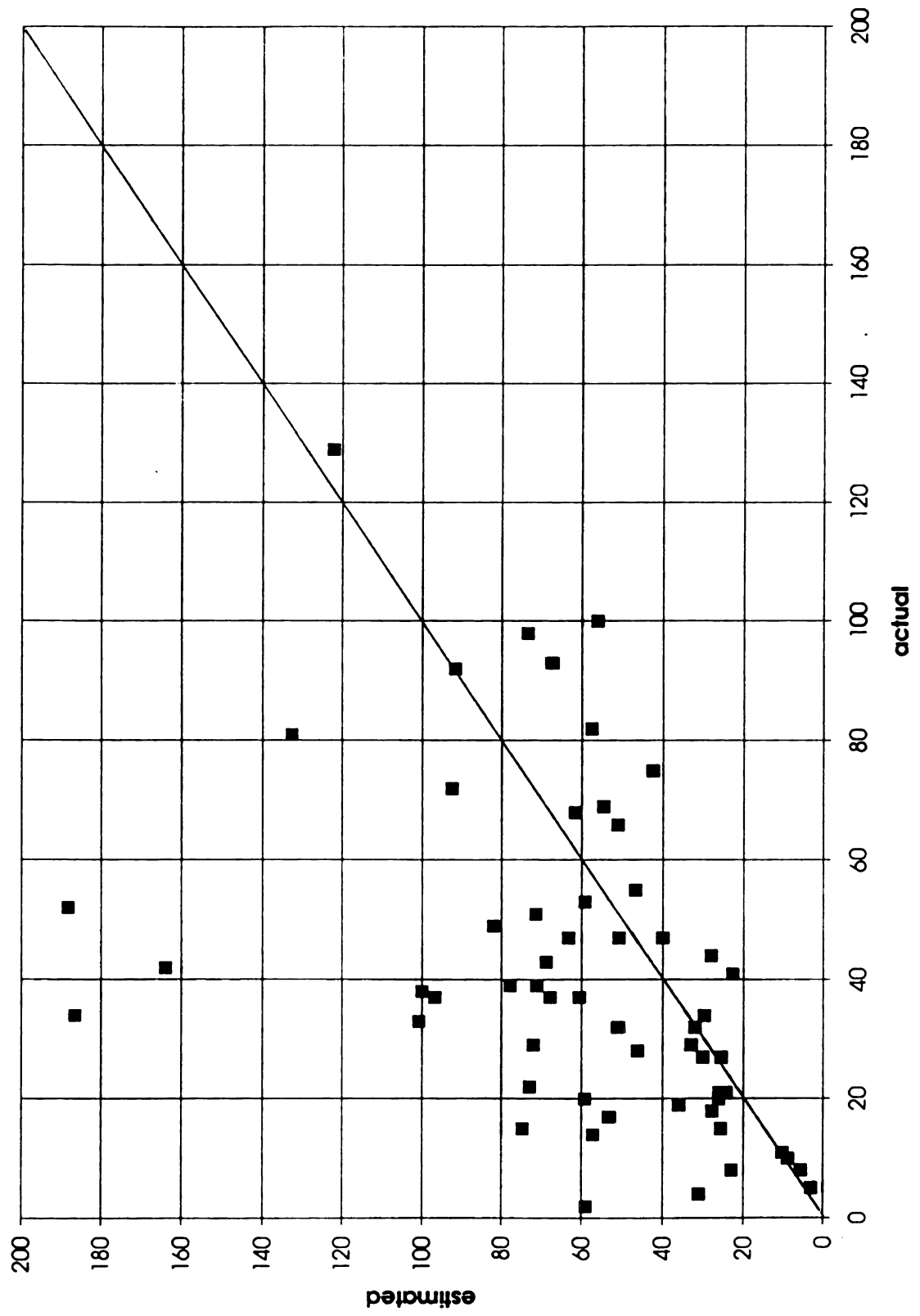


Figure 11 - actual vs. estimated link counts interval 6 - day 5

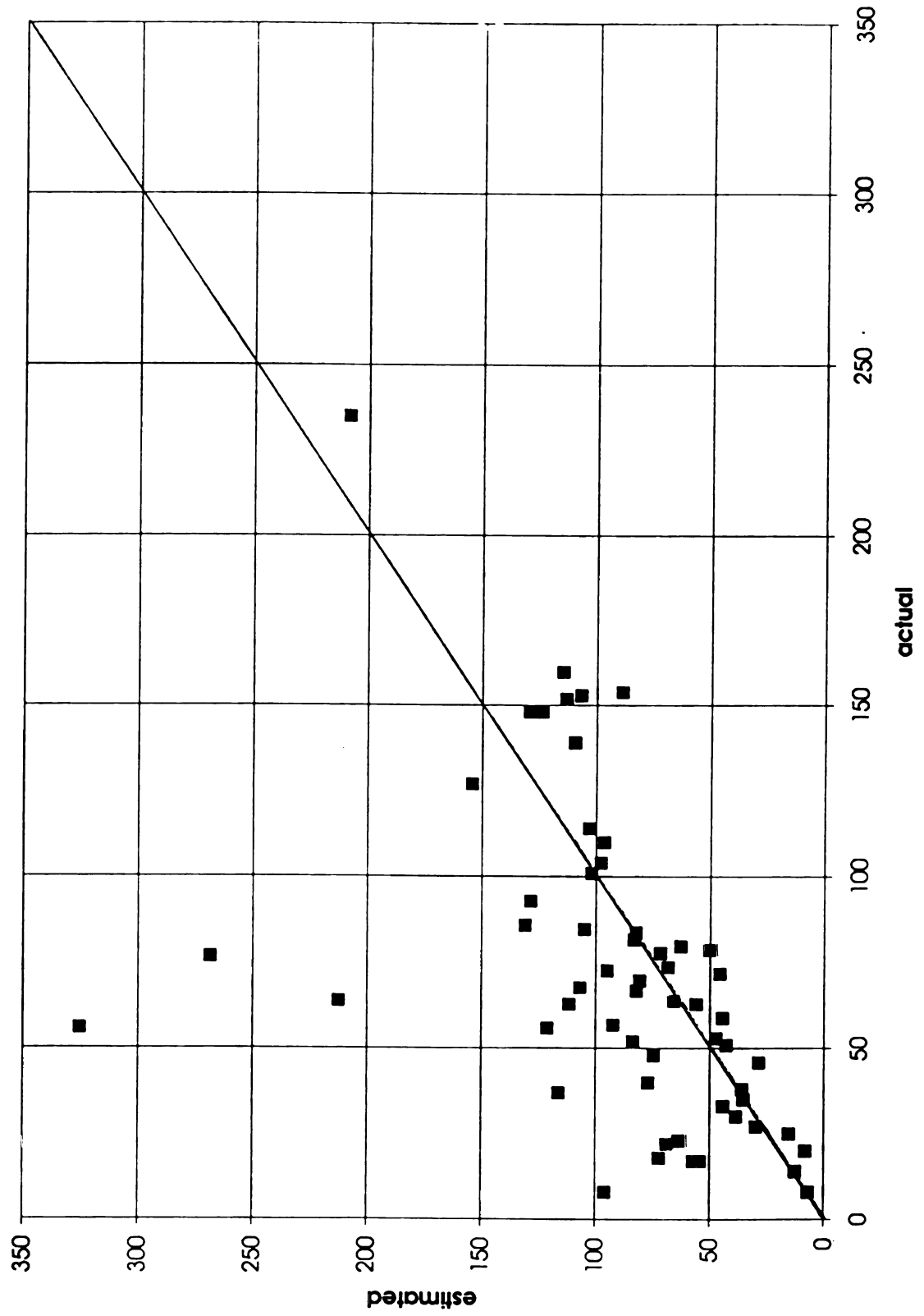


Figure 12 - OLS run - actual vs. estimated link counts interval 9 - day 5

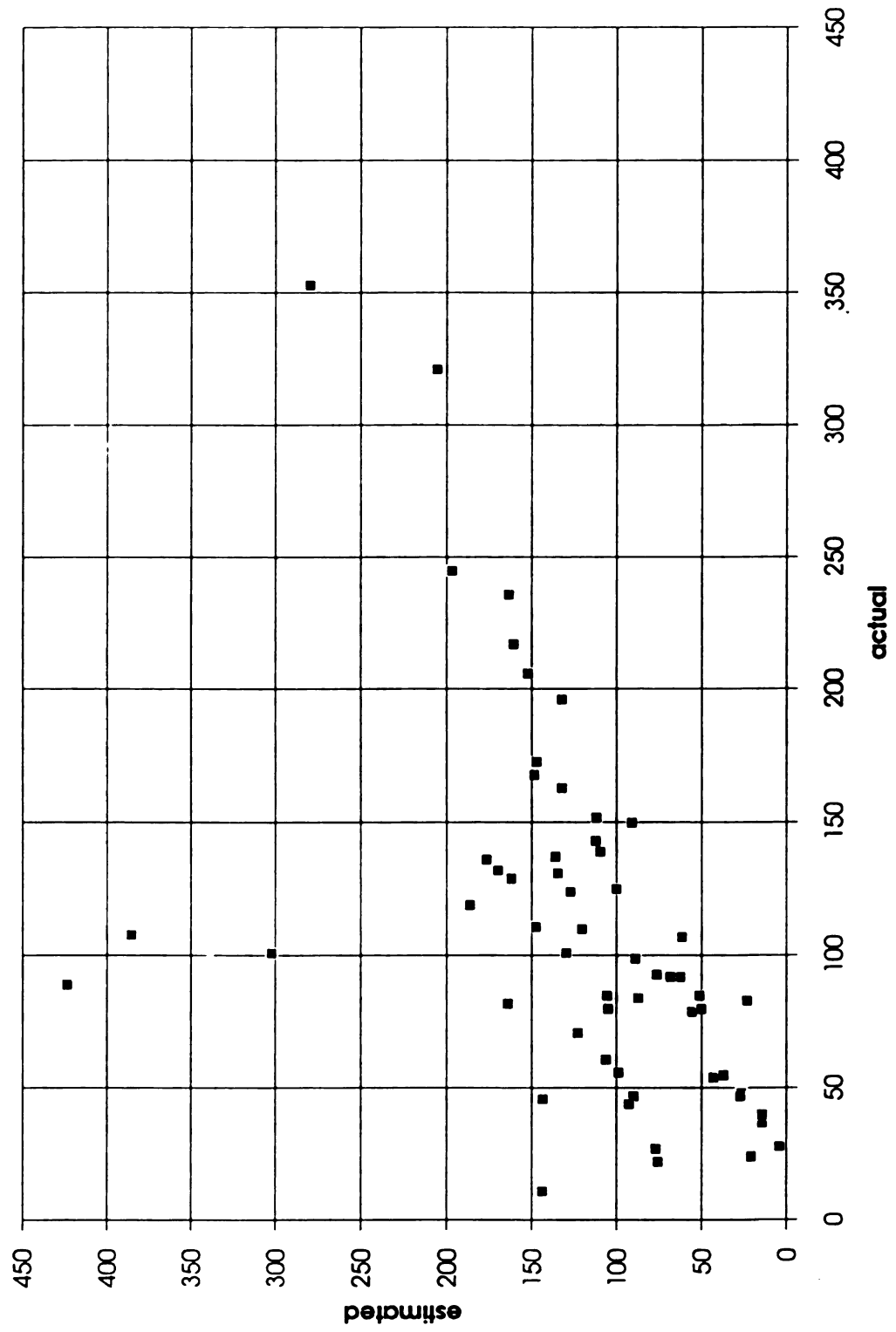


Figure 13 - OLS run - actual vs. estimated link counts interval 12 - day 5

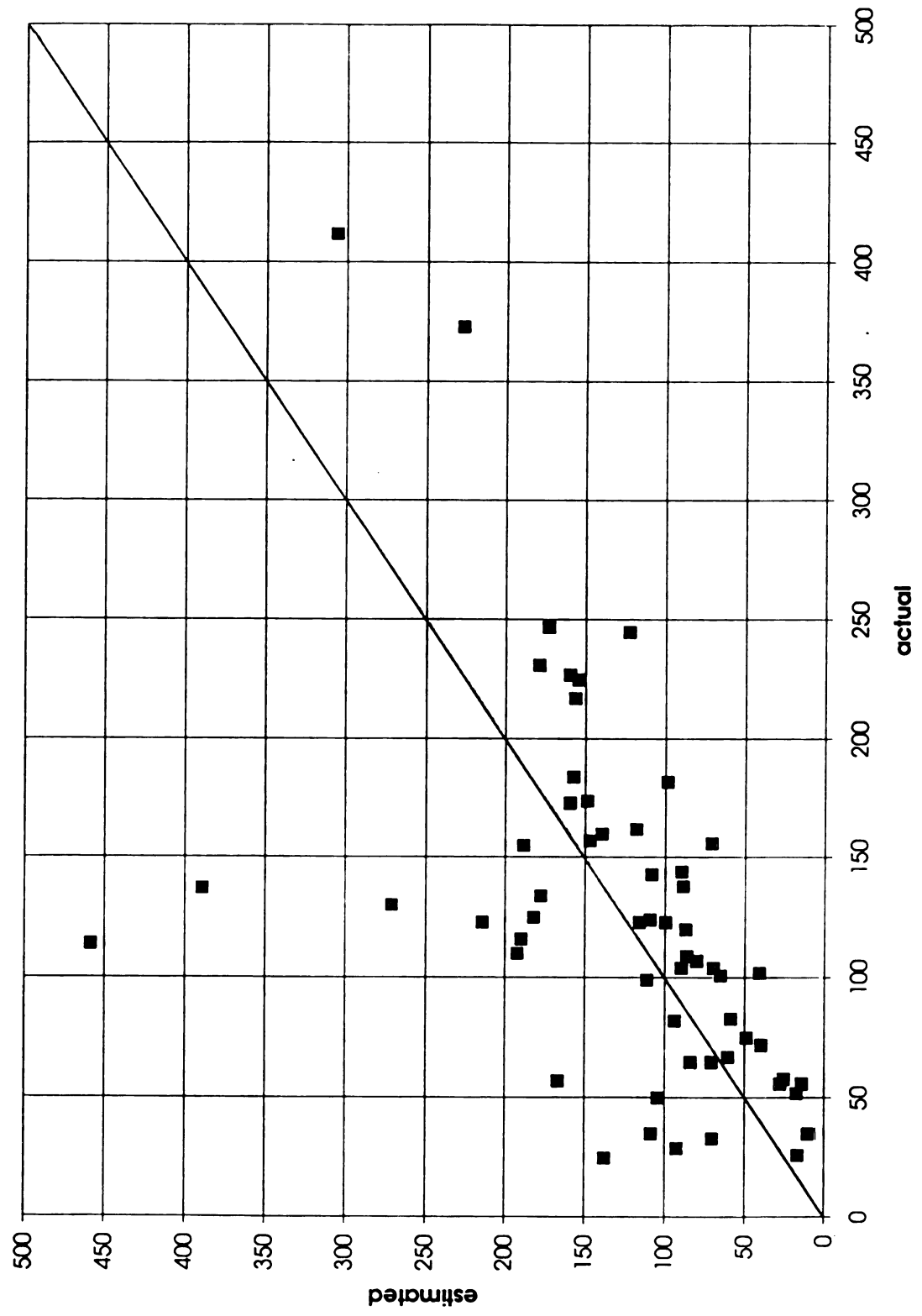


Table 12

Comparison of Predicted Versus Observed Link Flows - GLS run - Day 1

Intervals	Xobs	Xest	RMSE	%RMSE
1	1536	2787	51	194
2	1485	2269	43	166
3	2205	2335	43	113
4	3055	2439	49	93
5	3613	2445	57	91
6	3904	2475	58	85
7	4532	2623	67	86
8	5648	2746	80	82
9	6495	2935	95	85
10	6878	2945	101	85
11	6920	3057	102	85
12	7070	3131	100	82

Table 13

Comparison of Predicted Versus Observed Link Flows - GLS run - Day 2

Intervals	Xobs	Xest	RMSE	%RMSE
1	1509	2630	47	179
2	1638	2217	42	149
3	2569	2313	40	91
4	2928	2466	42	83
5	3757	2724	49	76
6	4348	2879	57	76
7	5071	3114	62	71
8	5928	3703	68	67
9	6760	4159	79	68
10	7076	4316	83	68
11	7172	4424	86	69
12	7384	4441	88	69



Table 14

Comparison of Predicted Versus Observed Link Flows - GLS run - Day 3

Intervals	Xobs	Xest	RMSE	%RMSE
1	1553	2621	47	176
2	1648	2123	39	136
3	2428	2204	34	81
4	3010	2463	38	73
5	3809	2931	46	70
6	4468	3285	51	67
7	5011	3635	56	65
8	5922	4590	59	57
9	6594	5064	67	59
10	7133	5338	74	60
11	7570	5711	80	61
12	7421	5655	79	62

Table 15

Comparison of Predicted Versus Observed Link Flows - GLS run - Day 4

Intervals	Xobs	Xest	RMSE	%RMSE
1	1532	2601	48	180
2	1622	1869	33	118
3	2431	2197	33	78
4	3058	2591	37	70
5	4060	3209	47	67
6	4490	3531	51	66
7	5134	3976	58	65
8	6150	5131	63	59
9	6754	5661	71	61
10	6661	5410	70	61
11	7524	5293	81	63
12	7419	5503	82	64

Table 16

Comparison of Predicted Versus Observed Link Flows - GLS run - Day 5

Intervals	Xobs	Xest	RMSE	%RMSE
1	1521	2769	49	186
2	1616	2020	33	119
3	2378	2195	30	73
4	2872	2583	35	71
5	3805	3659	51	77
6	4157	3933	55	77
7	4882	4423	58	69
8	5616	5512	60	62
9	6374	6387	71	64
10	6654	6967	75	65
11	7245	7558	86	69
12	7331	7662	87	69

Figure 14 - GLS run - actual vs. estimated link counts interval 1 - day 5

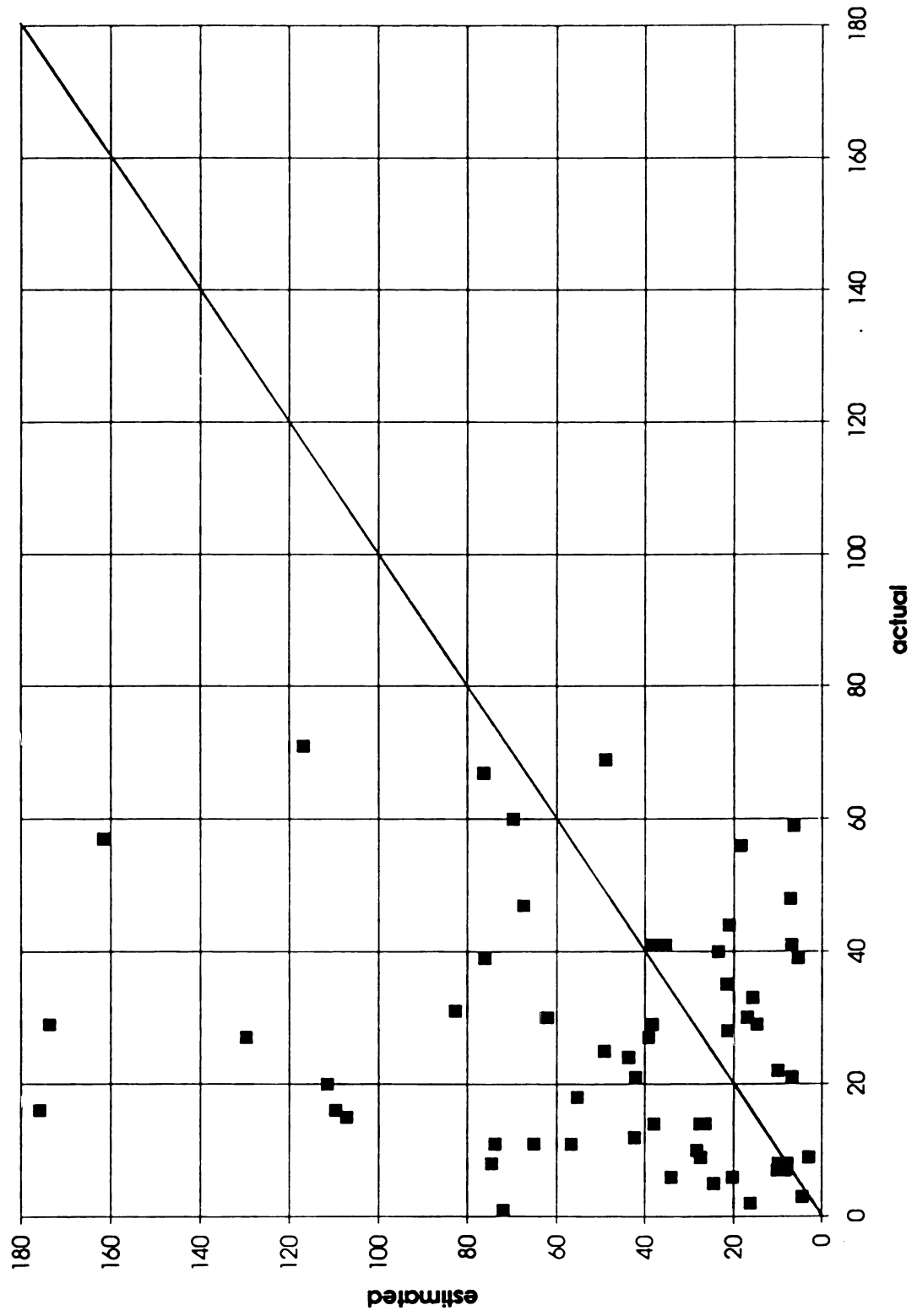


Figure 15 - GLS run - actual vs. estimated link counts interval 3 - day 5

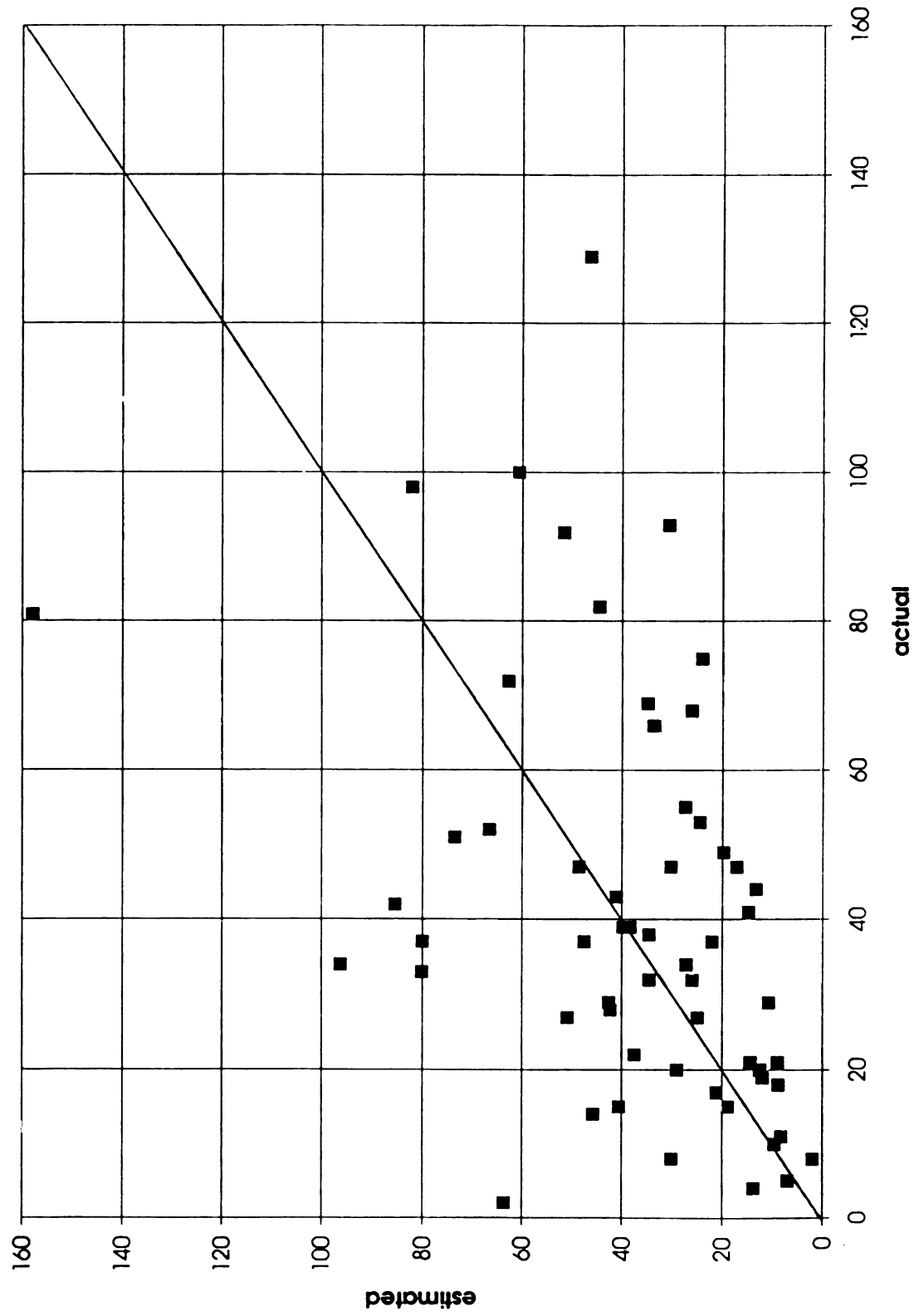


Figure 16 - GLS run - actual vs. estimated link counts interval 6 - day 5

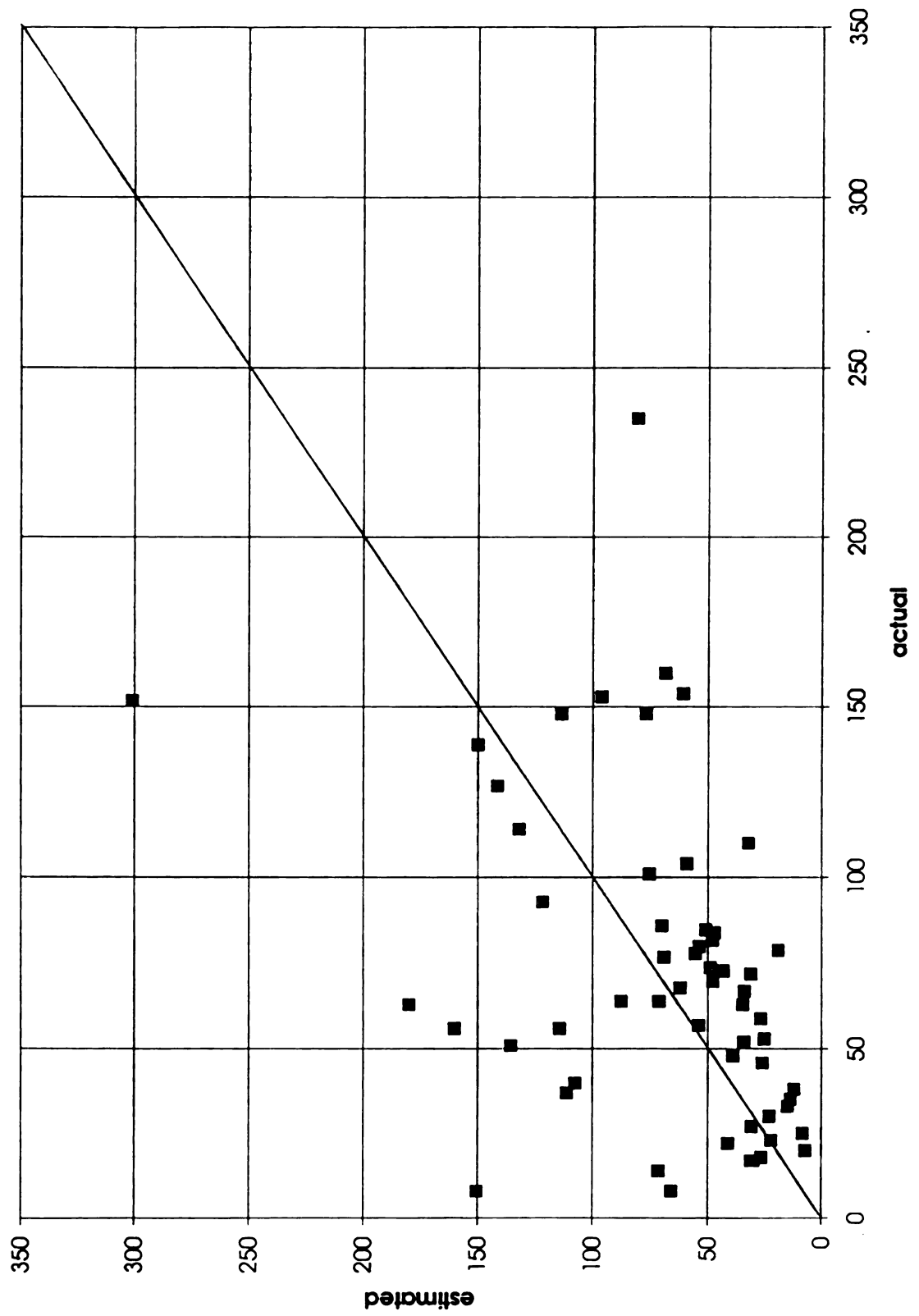


Figure 17 - GLS run - actual vs. estimated link counts interval 9 - day 5

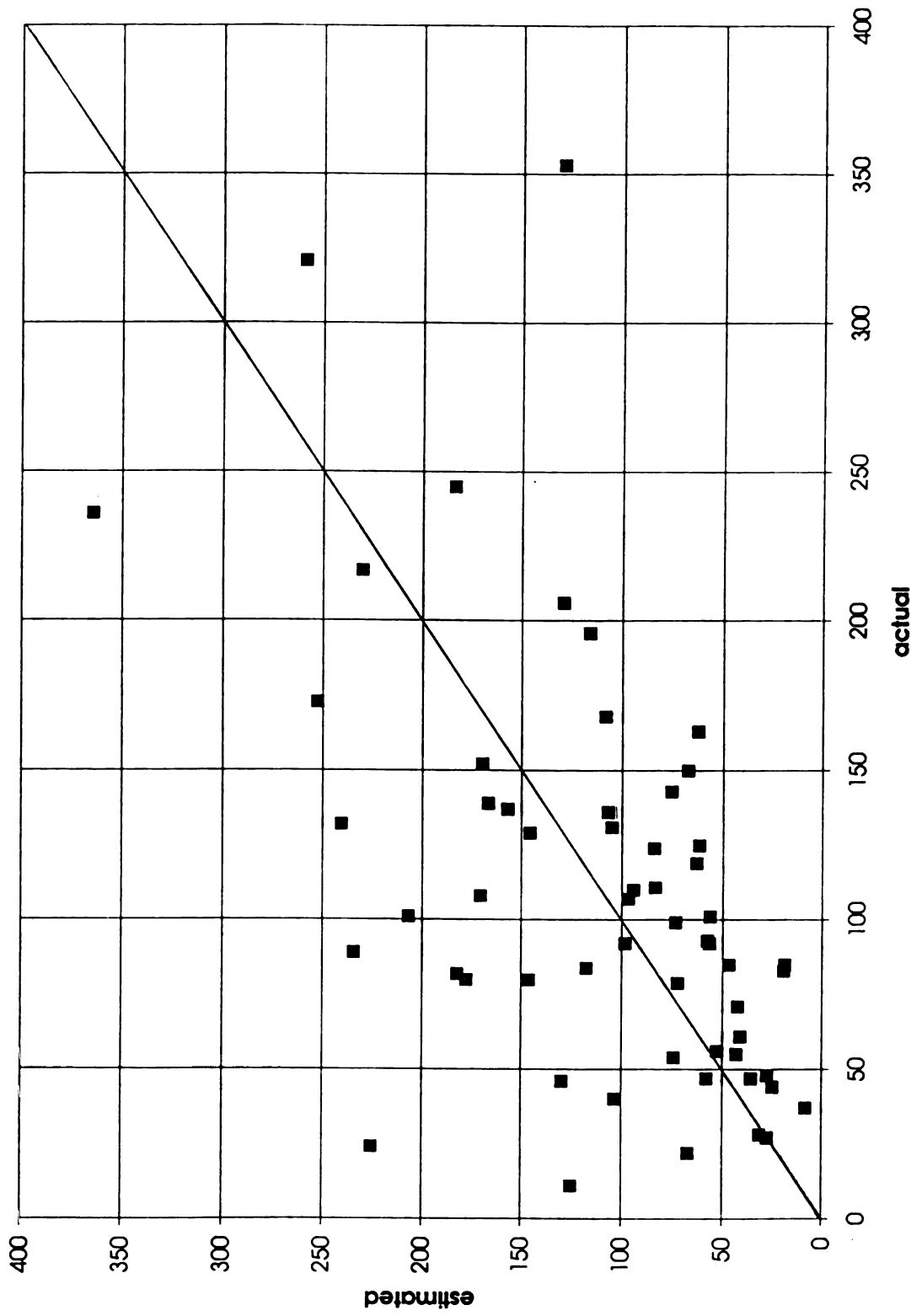


Figure 18 - GLS run -actual vs. estimated link counts interval 12 - day 5

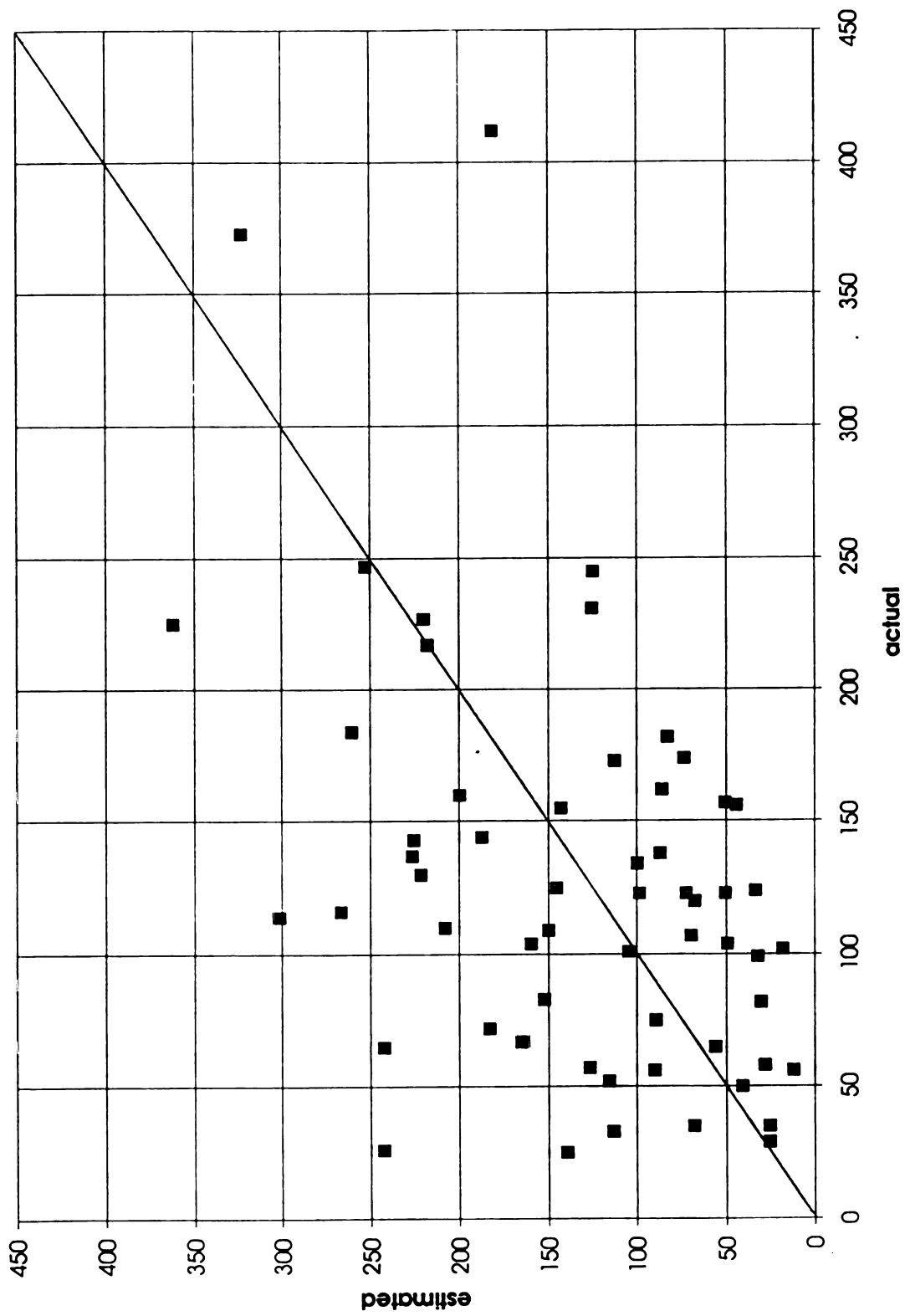




Table 17

Comparison of Predicted Versus Observed Link Flows - OLS run -  
Adjusted Data - Day 5

Intervals	Xobs	Xest	RMSE	%RMSE
1	2917	5735	69	138
2	3109	4879	55	102
3	4580	6052	60	76
4	5483	7037	72	76
5	7331	8838	84	66
6	8052	9480	91	66
7	9452	10821	99	61
8	11072	12184	107	56
9	12213	13524	121	57
10	12844	12907	105	47
11	13949	13781	115	48
12	14271	13895	112	45

**Figure 19 - OLS run -actual vs. estimated link counts interval 1 - adjusted day 5**  
data

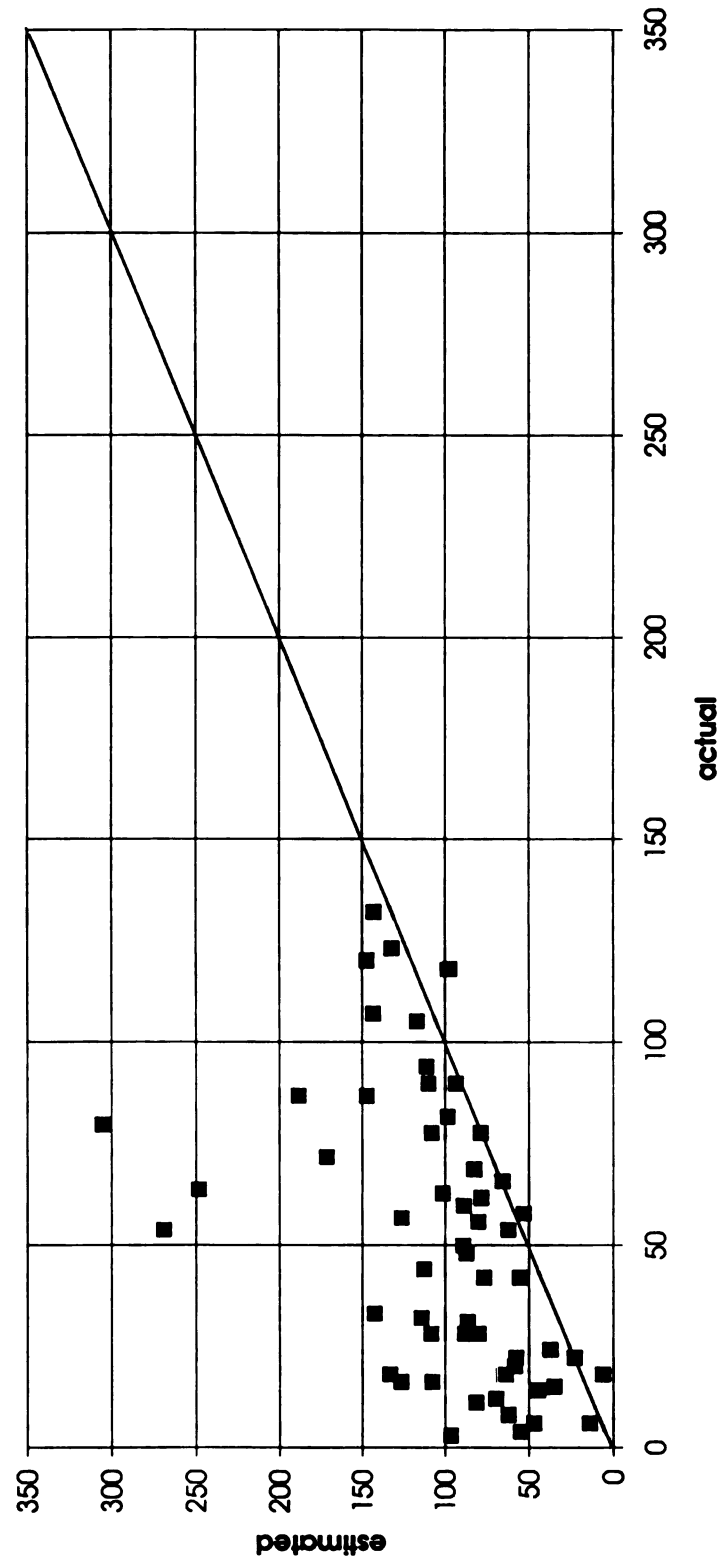
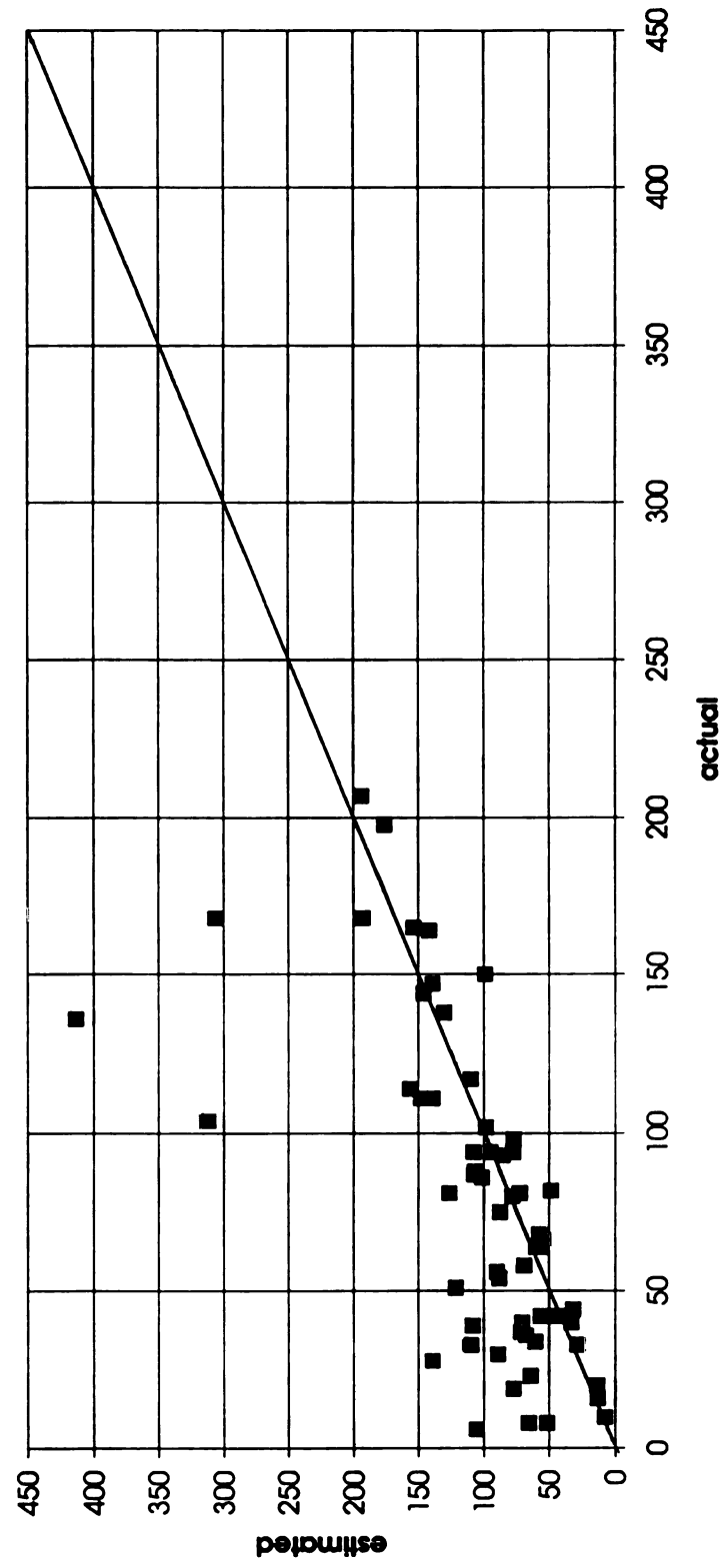


Figure 20 - OLS run - actual vs. estimated link counts interval 3 - adjusted day 5



**Figure 21 - OLS run - actual vs. estimated link counts interval 6 - adjusted day 5**

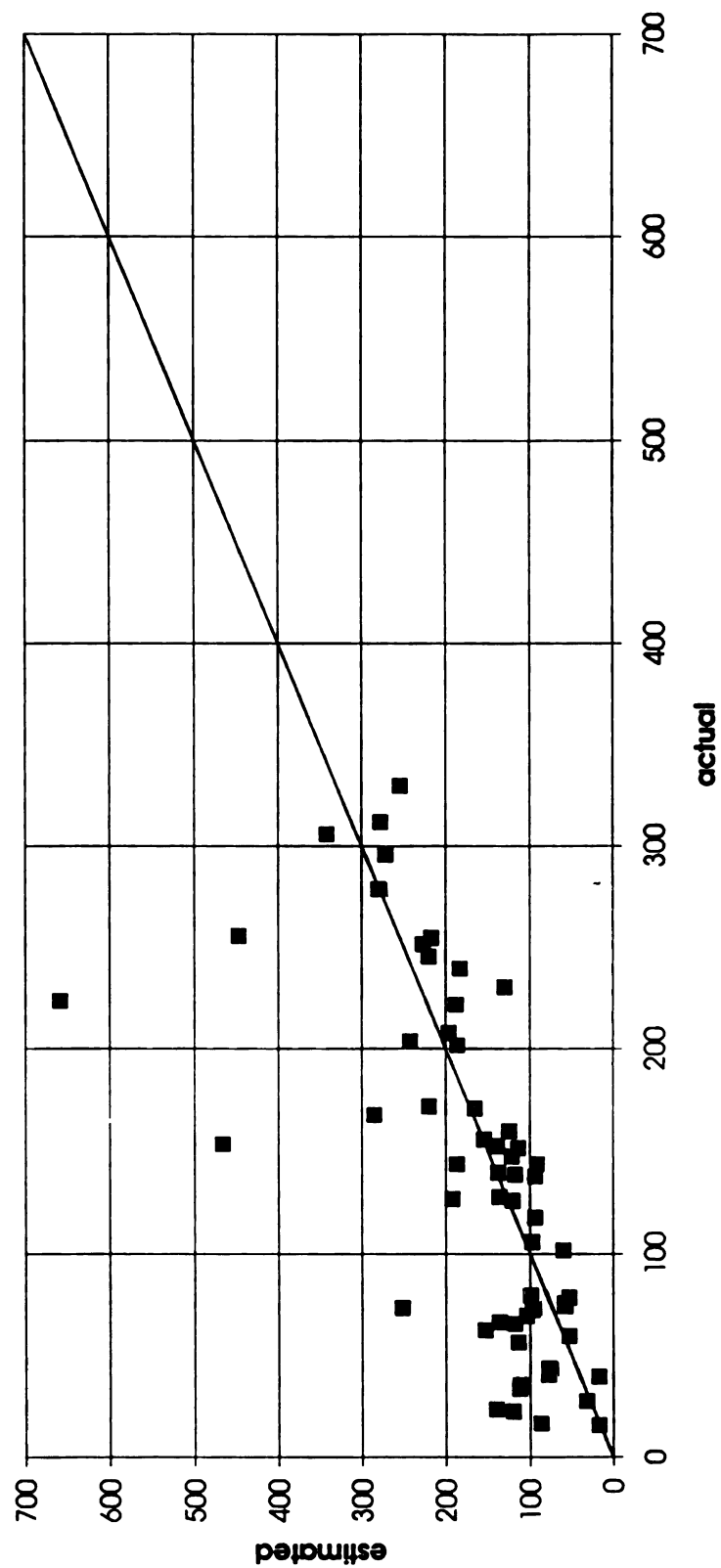


Figure 22 - OLS run actual vs. estimated link counts interval 9 - adjusted day 5 data

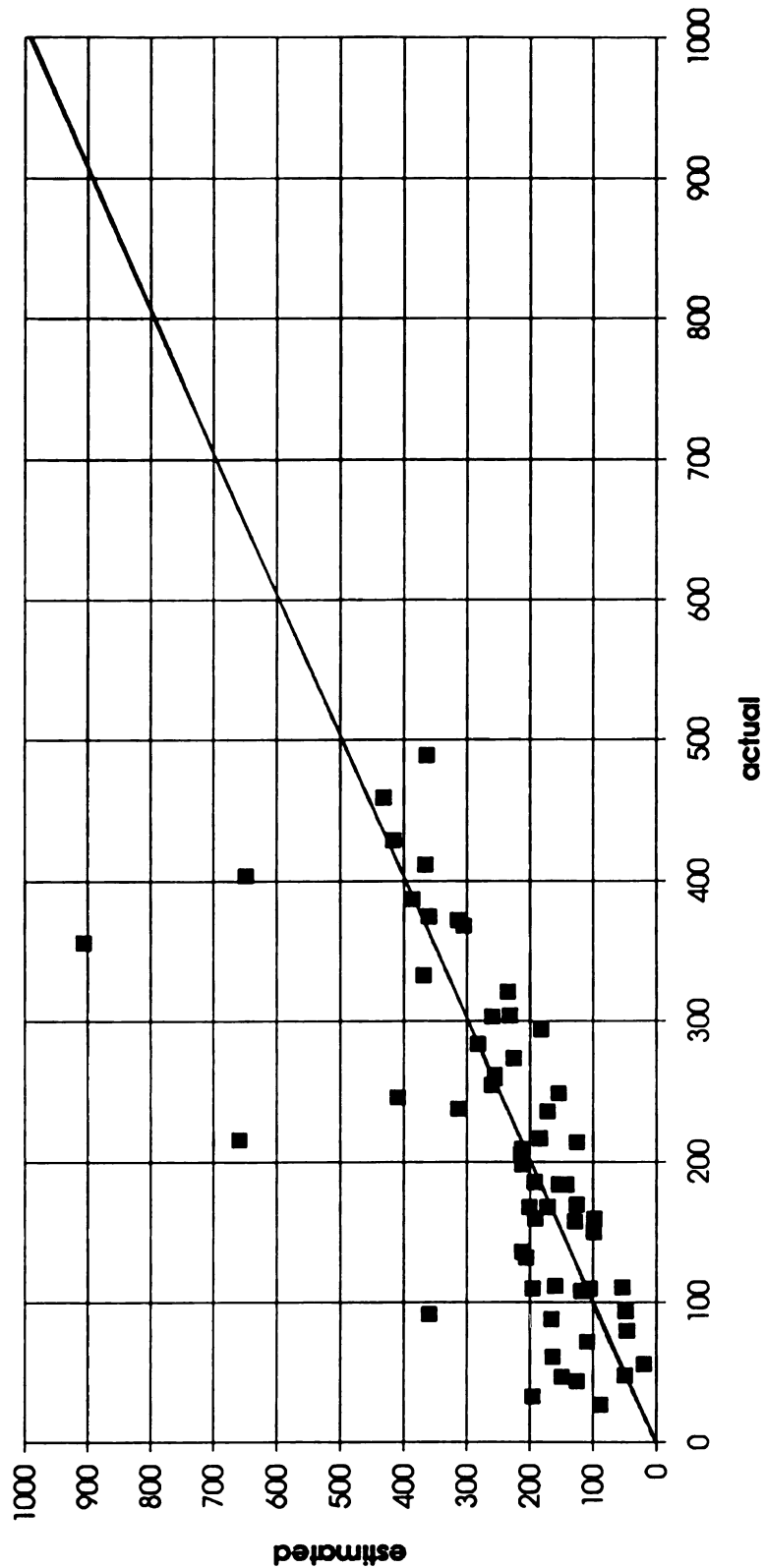


Figure 23 - OLS run - actual vs. estimated link counts interval 12 - adjusted day 5

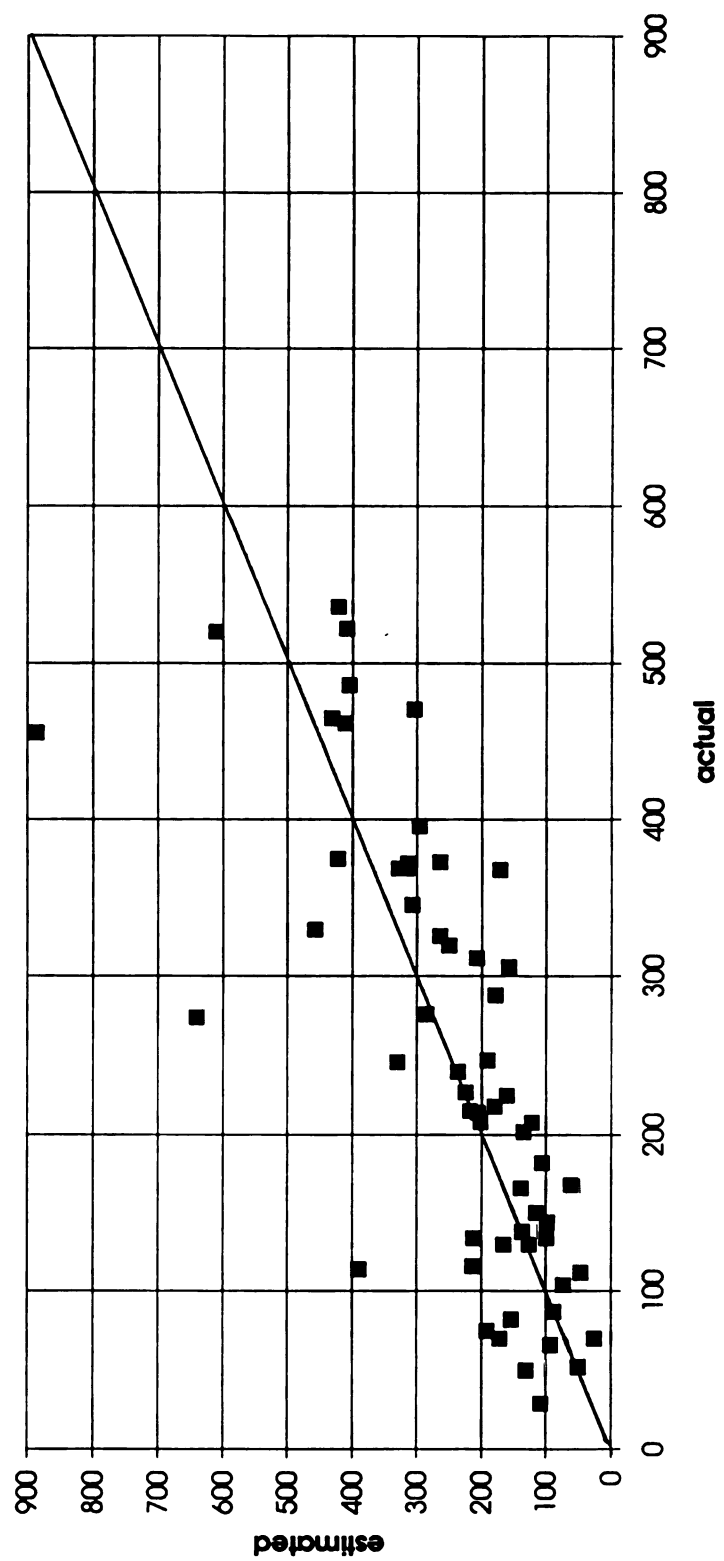


Table 18

Comparison of Predicted Versus Observed Link Flows - GLS run -  
Adjusted Data - Day 5

Intervals	Xobs	Xest	RMSE	%RMSE
1	2917	2963	49	94
2	3109	2773	38	67
3	4580	3592	55	67
4	5483	5352	75	75
5	7331	7954	123	94
6	8052	8978	141	101
7	9452	11855	165	101
8	11072	18480	284	149
9	12213	23404	396	188
10	12844	26780	479	216
11	13949	29884	543	226
12	14271	32507	630	256

Figure 24 - GLS run actual vs. estimated link counts interval 1 - adjusted day 5

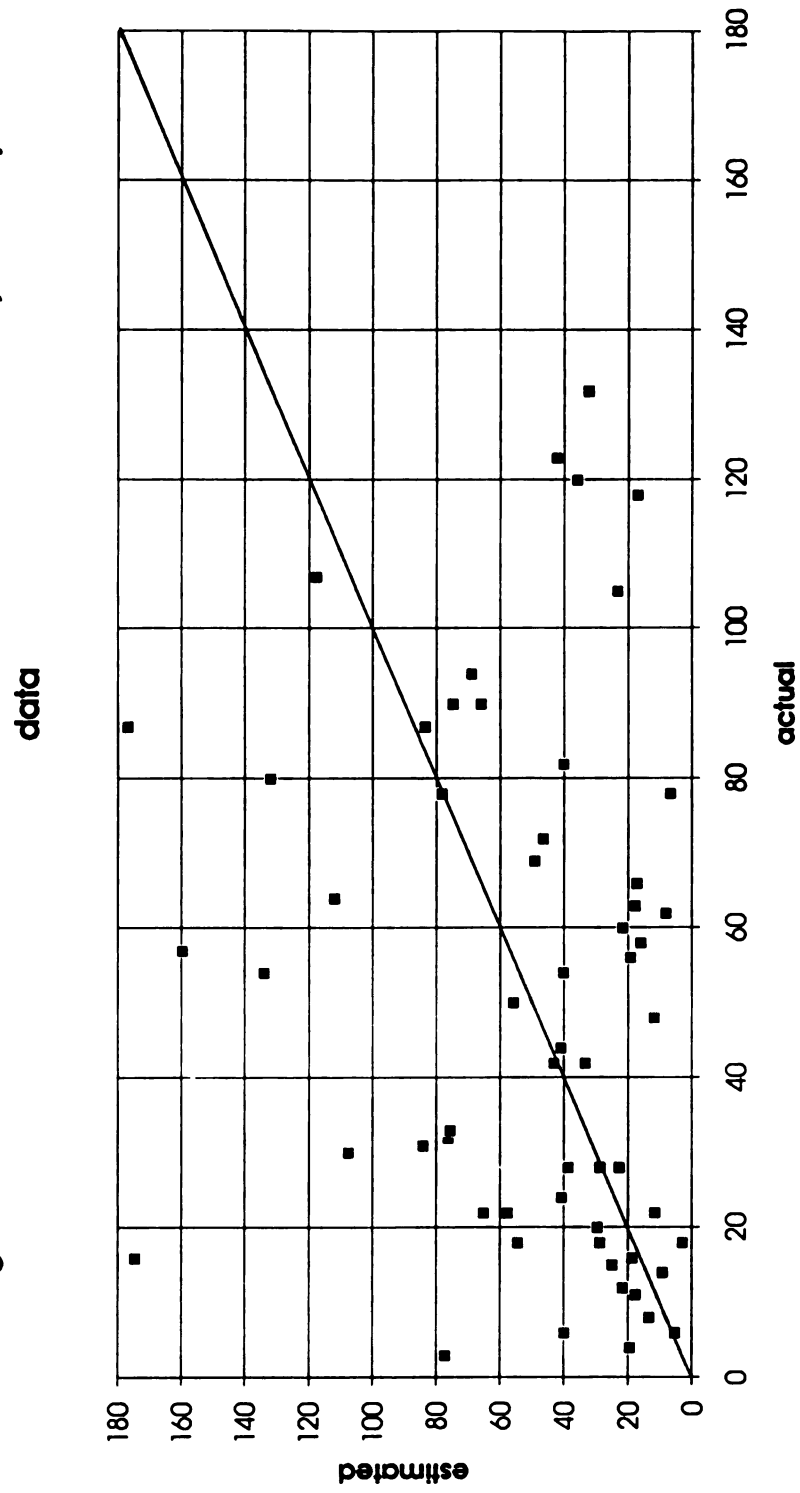




Figure 25 - GLS run - actual vs. estimated link counts interval 3 - adjusted day  
5 data

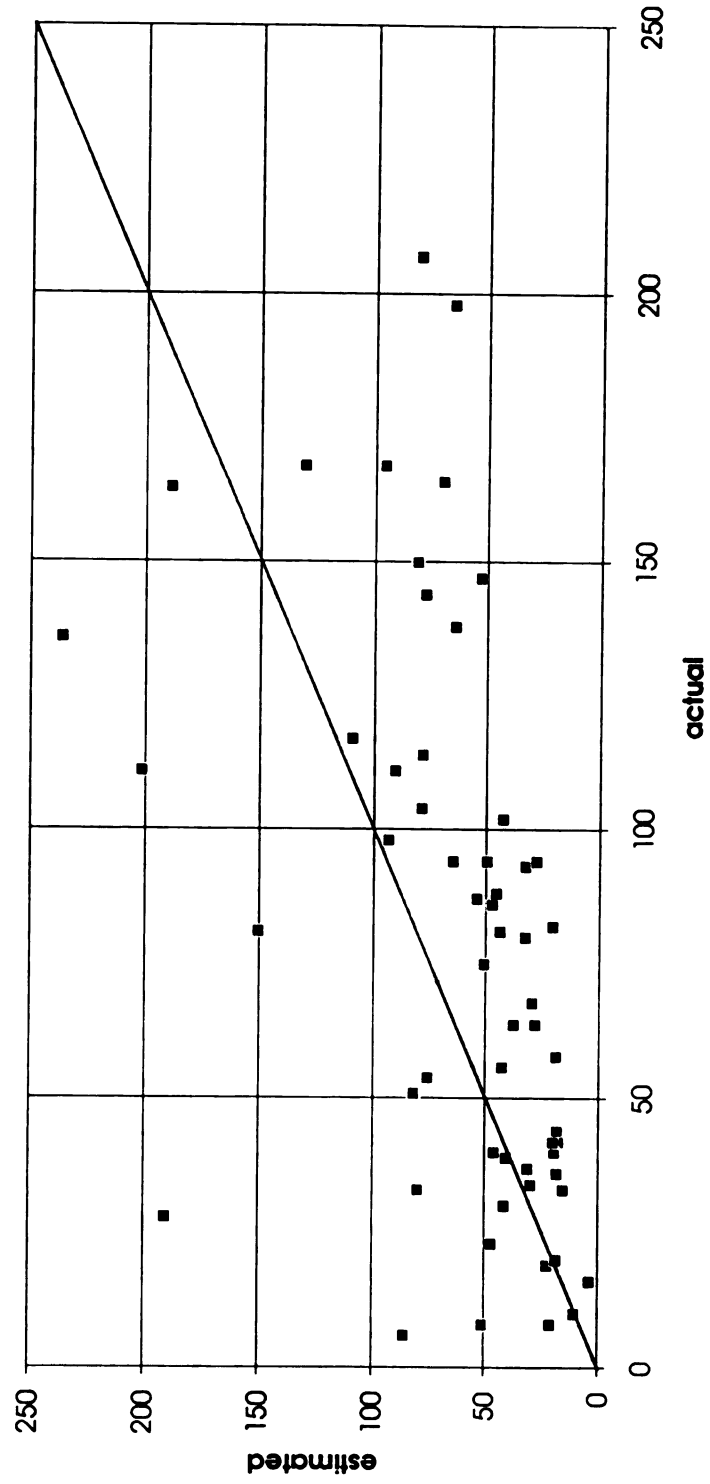


Figure 26 - GLS run - actual vs. estimated link counts interval 6 - adjusted day  
5 data

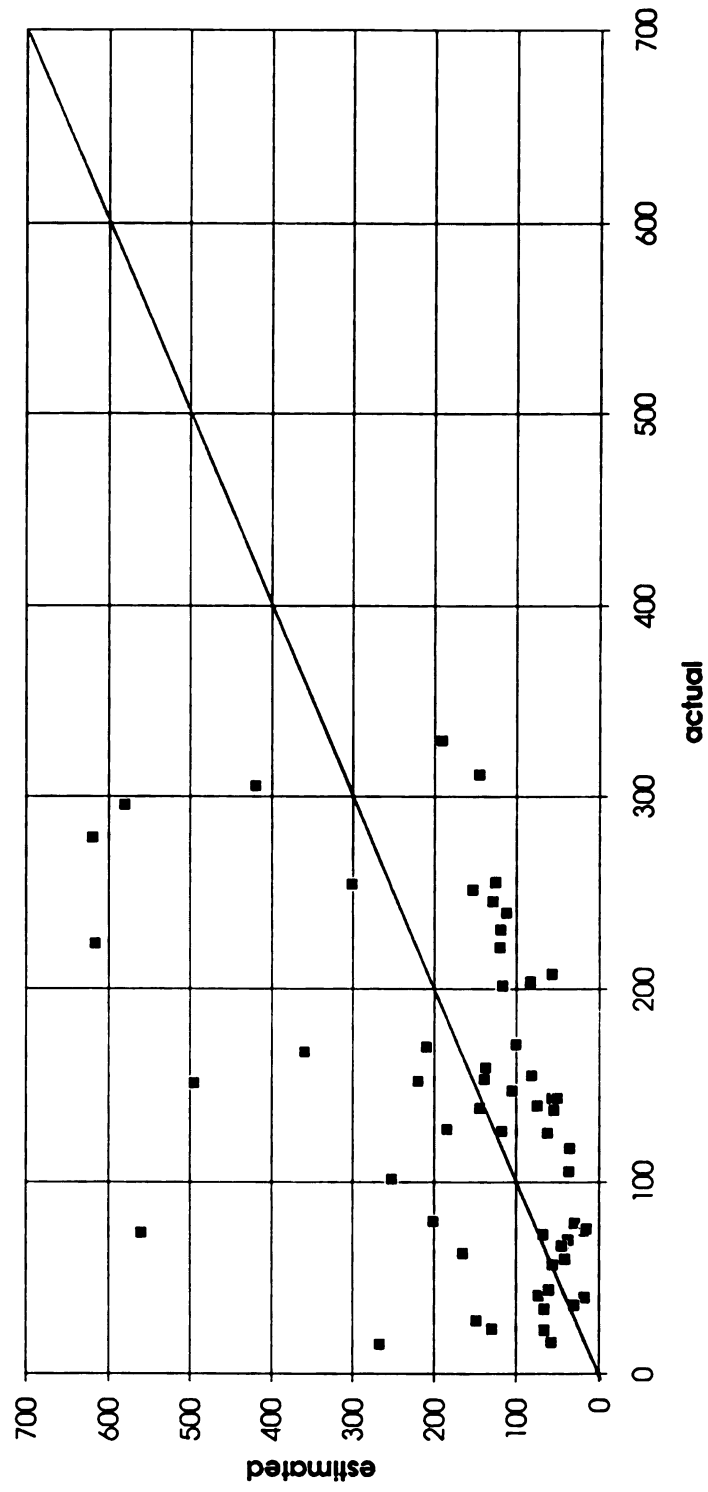


Figure 27 - OLS run actual vs. estimated link counts interval 9 - adjusted day 5  
data

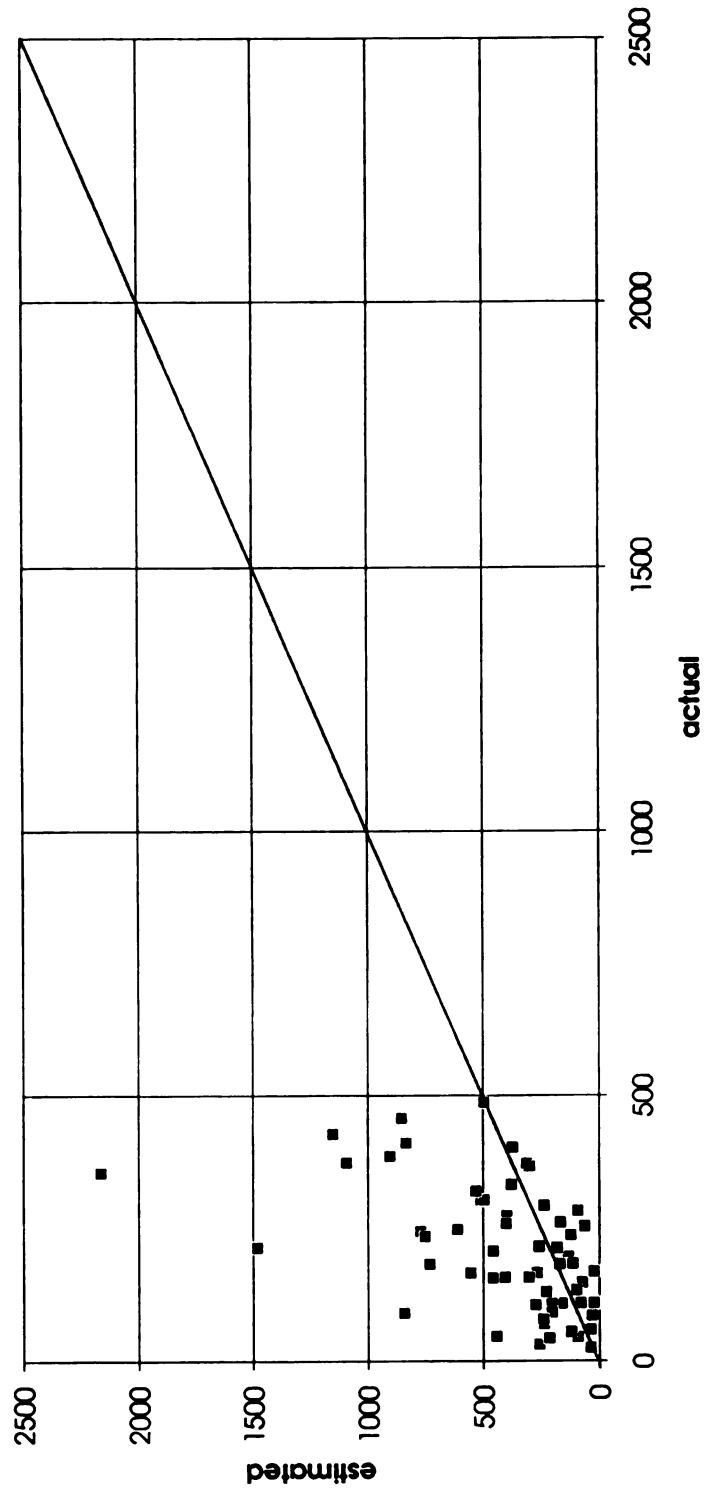


Figure 28 - GLS - actual vs. estimated link counts interval 12 - adjusted day 5  
data

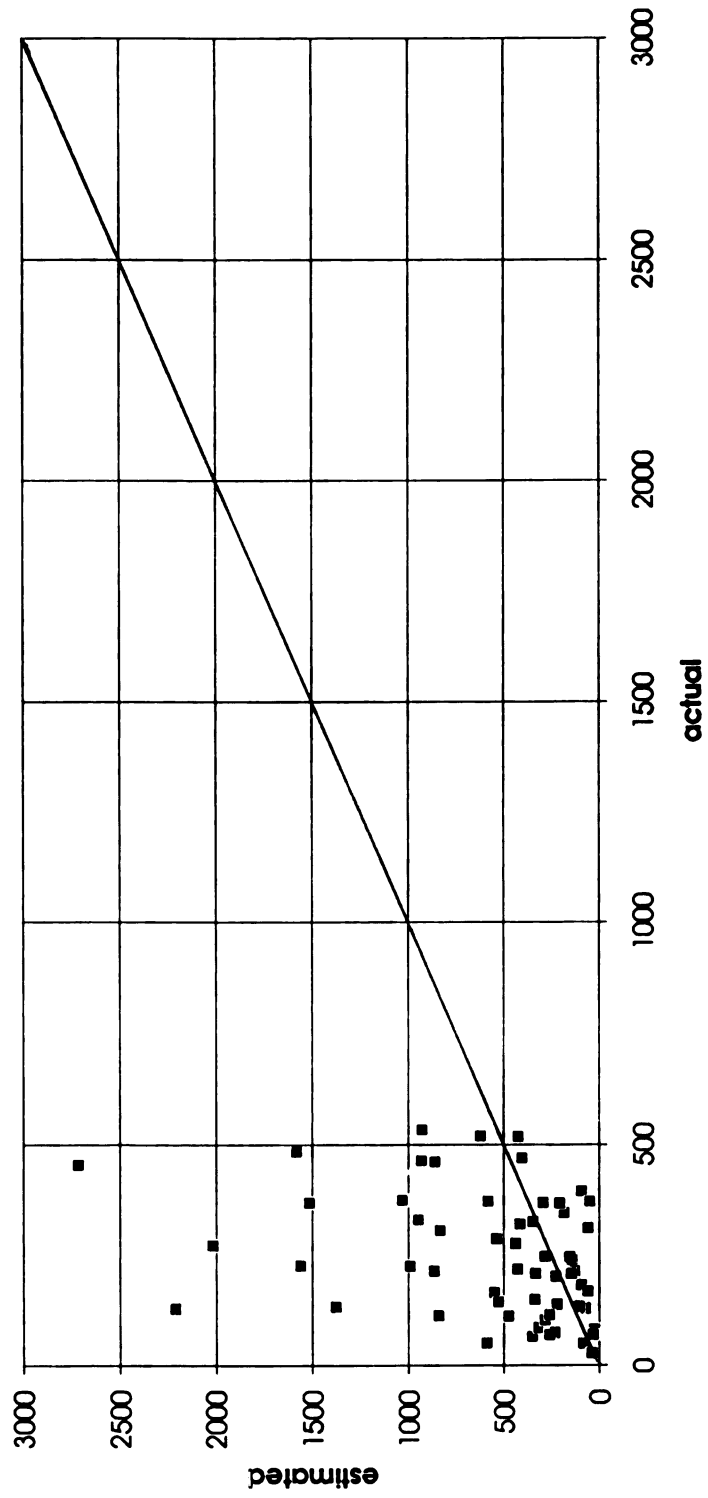


Table 19

Comparison of predicted to observed link flows -  
Alternate GLS run - day 1

Intervals	Xobs	Xest	RMSE	%RMSE
1	1536	3517	58	219
2	1485	3556	61	238
3	2205	4145	67	177
4	3055	3838	58	109
5	3613	3959	58	93
6	3904	3732	47	69
7	4532	4441	54	69
8	5648	4945	61	62
9	6495	5766	68	61
10	6878	6201	69	58
11	6920	6549	70	59
12	7070	6739	67	58

Table 20

Comparison of Predicted Versus Observed Link Flows  
Alternate GLS run - day 2

Intervals	Xobs	Xest	RMSE	%RMSE
1	1509	3278	52	200
2	1638	3367	58	207
3	2569	3928	56	126
4	2928	3653	49	97
5	3757	4109	47	73
6	4348	4495	46	62
7	5071	5221	48	55
8	5928	5908	54	53
9	6760	7169	66	57
10	7076	8127	79	65
11	7172	8427	87	70
12	7384	8693	86	67

Table 21

Comparison of Predicted Versus Observed Link Flows -  
Alternate GLS run - day 3

Intervals	Xobs	Xest	RMSE	%RMSE
1	1553	3176	51	192
2	1648	3240	56	196
3	2428	3722	51	123
4	3010	3687	47	91
5	3809	4453	50	76
6	4468	5453	75	98
7	5011	5889	78	91
8	5922	5873	94	92
9	6594	6976	68	60
10	7133	7764	80	65
11	7570	3953	99	76
12	7421	4985	80	62

Table 22

Comparison of Predicted Versus Observed Link Flows -  
Alternate GLS run - day 4

Intervals	Xobs	Xest	RMSE	%RMSE
1	1532	2601	47	179
2	1622	2724	51	181
3	2431	3092	45	107
4	3058	3143	38	73
5	4060	3895	43	61
6	4490	4888	49	63
7	5134	4699	70	79
8	6150	4285	67	64
9	6754	6162	67	57
10	6661	6690	72	62
11	7524	7293	109	84
12	7419	9776	203	160



Table 23

Comparison of Predicted Versus Observed Link Flows -  
Alternate GLS run - day 5

Intervals	Xobs	Xest	RMSE	%RMSE
1	2917	3066	48	90
2	3109	3647	45	78
3	4580	4521	47	56
4	5483	5936	57	56
5	7331	8508	86	69
6	8052	11005	121	87
7	9452	8125	93	57
8	11072	15775	159	83
9	12213	22161	287	136
10	12844	24096	382	172
11	13949	20149	246	102
12	14271	19732	231	94

Figure 29 - Alternate GLS run - actual vs. estimated link counts interval 1 - adjusted  
day 5 data

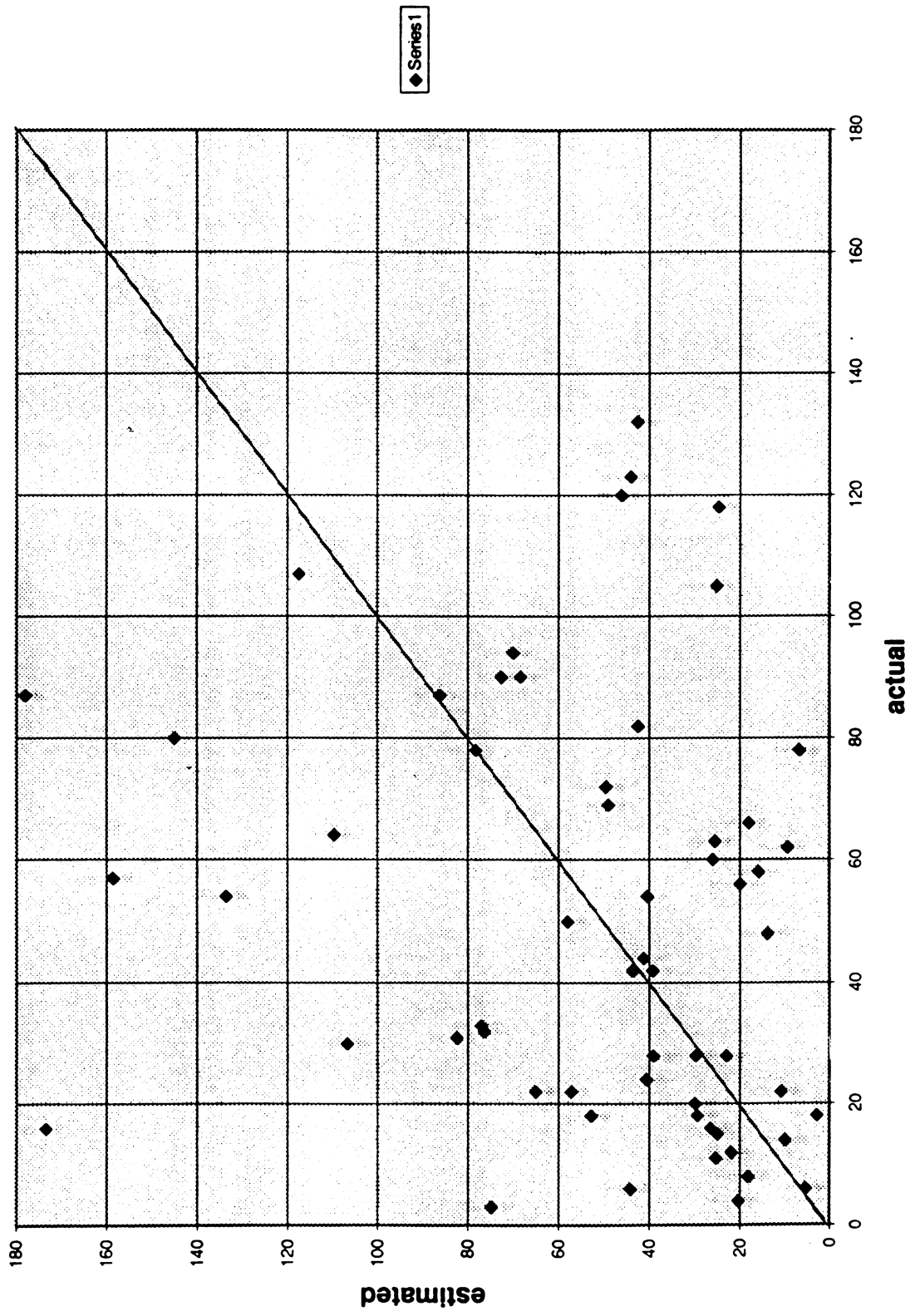


Figure 30 - Alternate GLS run - actual vs. estimated link counts Interval 3 - adjusted day 5 data

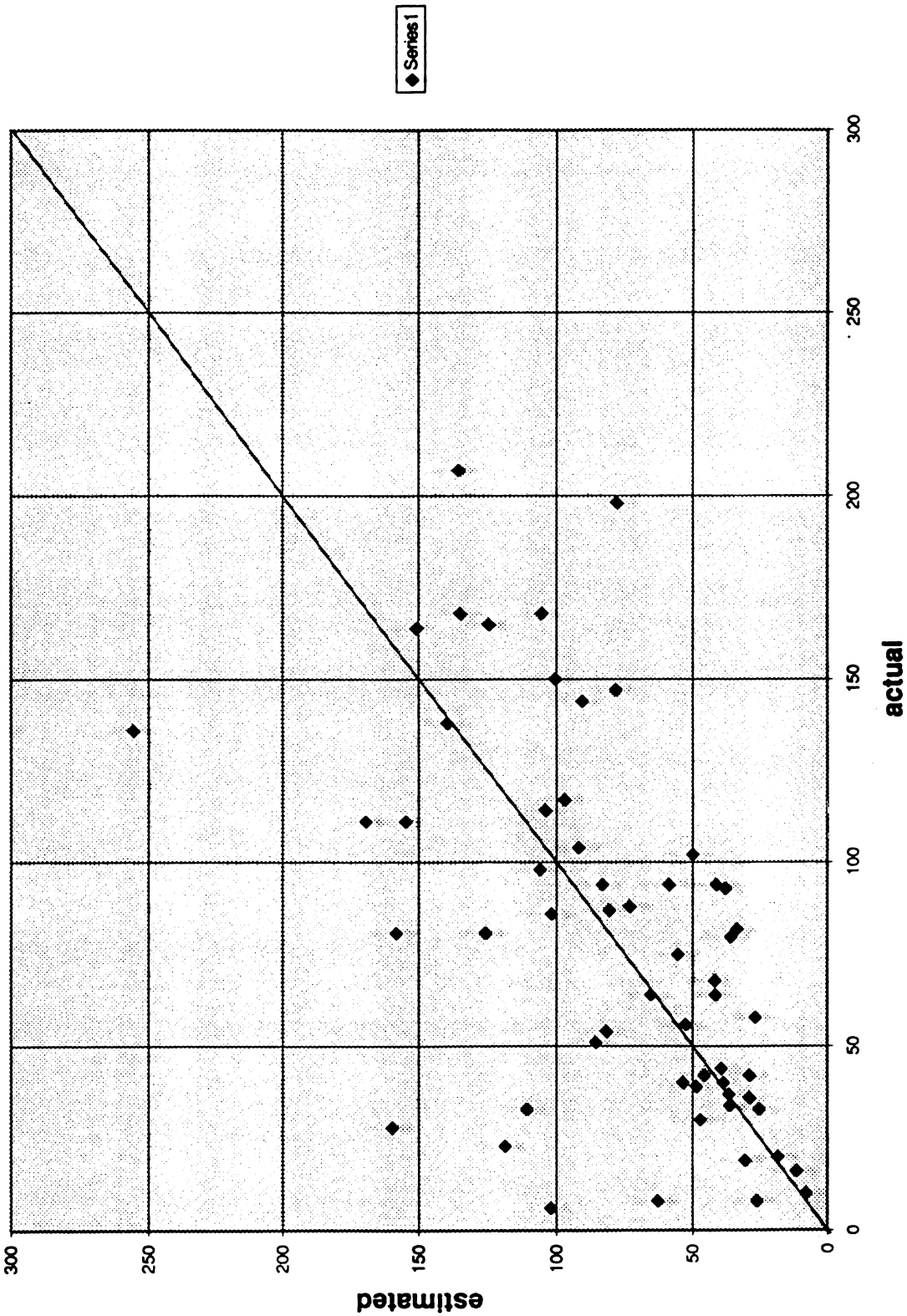


Figure 31 - Alternate GLS run - actual vs. estimated link counts Interval 6 - adjusted day 5

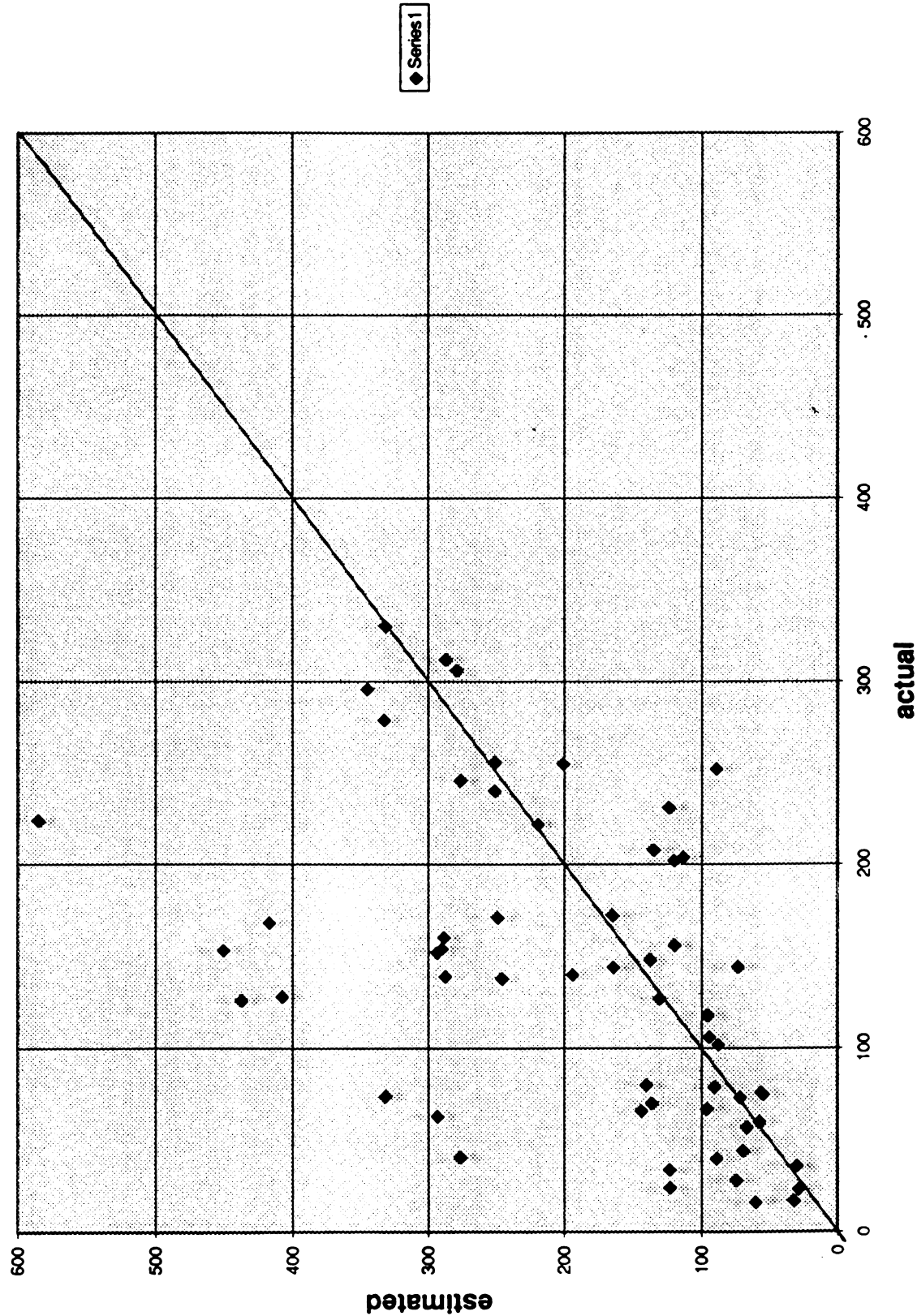


Figure 32 - Alternate GLS run - actual vs. estimated link counts Interval 9 - adjusted day 5 data

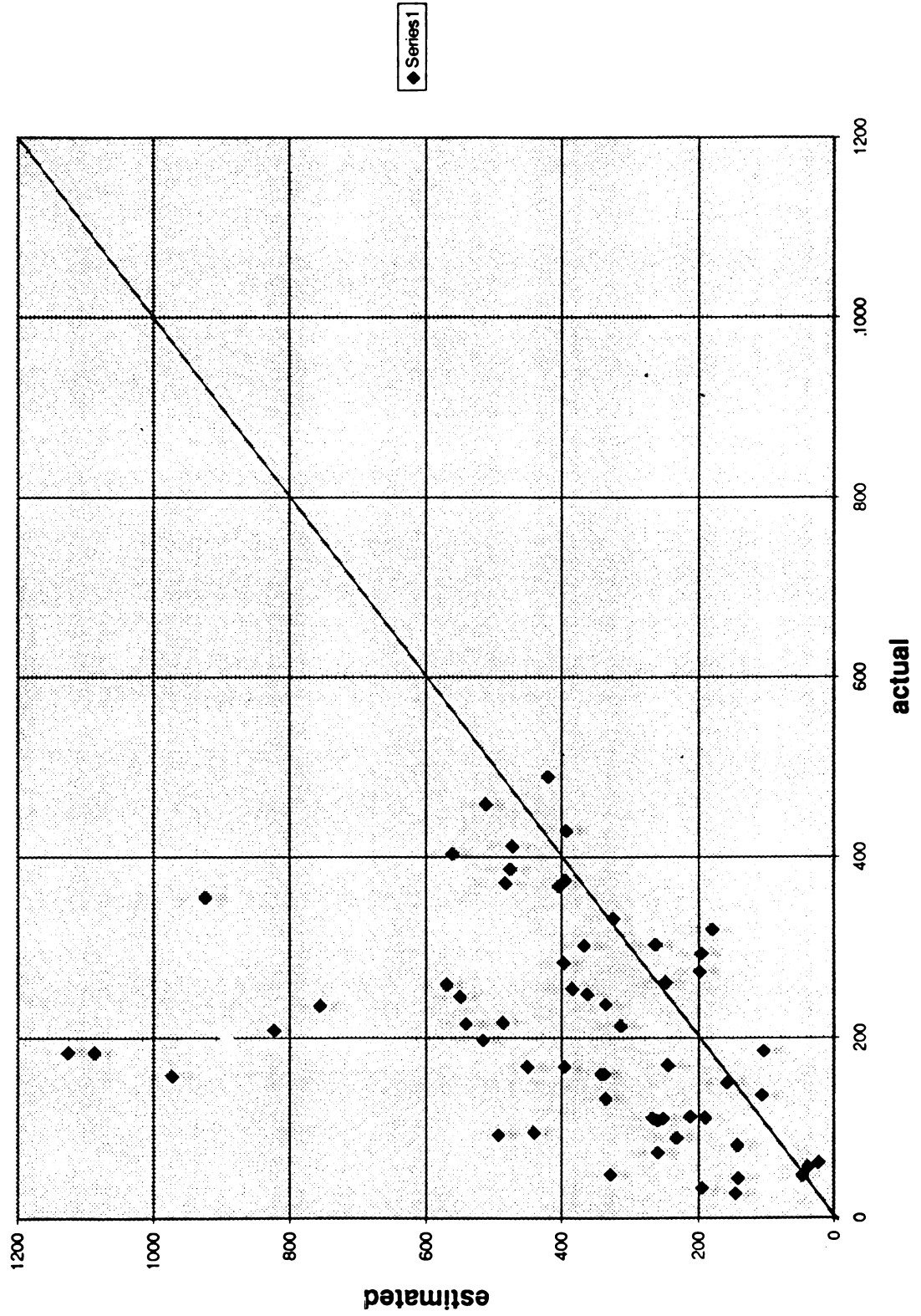
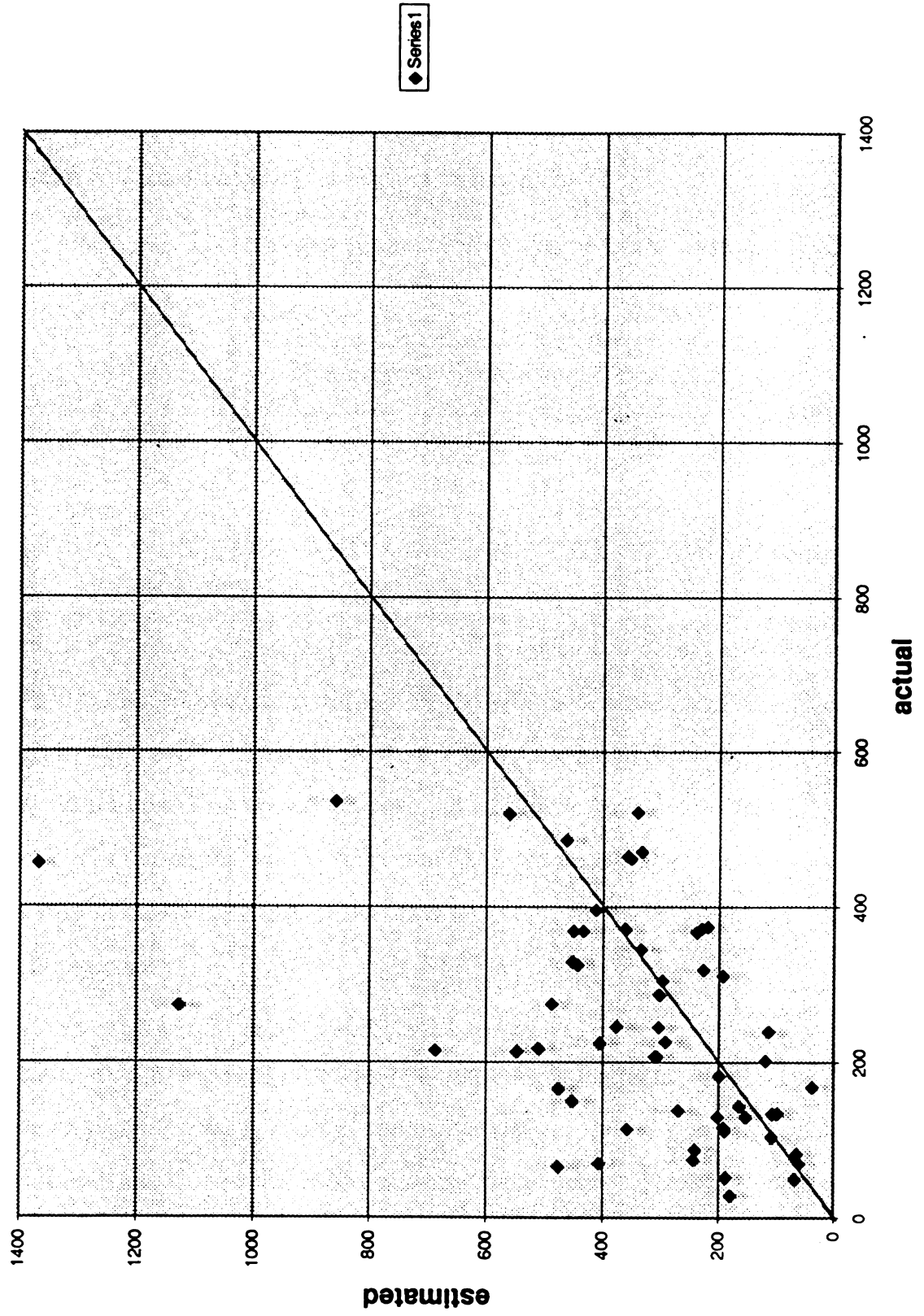


Figure 33 - Alternate GLS run - actual vs. estimates link counts interval 12 -  
adjusted day 5



## Chapter 8

### Conclusion

This research started with two objectives in mind:

- 1) Development and validation of a new dynamic O-D estimation method.
- 2) A determination of model accuracy when applied to an urban area where full data is not available.

### 8.1 Model Development

A new method for estimating origin-destination trips in an urban area based on dynamic traffic count data was proposed. The method is configured as an optimization model consisting of two linear systems of equations. The first equation treats the O-D volumes as time-variant variables, which are dependent on the time series of link flows. The second equation expresses a causal relationship between the past and present O-D volumes. This causal relationship is similar to the Kalman State equation, but varies from it in its spatial dependency.

In the Kalman filtering setup the State equation defines the future interval O-D volumes as a linear combination of the present interval O-D volumes. In the proposed method, unlike the State equation, the previous interval O-D estimate is defined in terms of the present interval O-D volumes. With this approach additional information can be obtained, which is used to identify the structure and size of the flows in urban road system.

## 8.2 Implementation

The optimization method used to derive estimates of O-D volumes is the Least Square Estimation. This approach is common in all time-dependent O-D estimation models. However, each model uses the LS method in a specific manner. For example, in the Kalman filtering approach the model starts the optimization process with some initial values for the coefficient and variance matrices, applies the LS estimation procedure, updates the coefficients and variances for the next interval and repeats the process.

No initial values for the parameters are assumed except the prior O-D trips of interval one of day one. All the parameters needed to calculate the assignment (fraction) matrix are estimated based on specified relationships. The sequential OLS application finds O-D estimates for the 12 intervals of each day of the study period. The link count estimates are then derived from the estimated O-D volumes.

The GLS estimates are based on an optimization process that finds an unbiased minimum variance estimate. This is achieved through insertion of weights, which are an inverse function of link error variances, and adjust for the deviations from the observed counts. Prior O-D estimates are adjusted by applying weights which are an inverse function of the O-D error variances to control the magnitude of variations from the previous O-D estimate. The GLS estimation process is also applied sequentially for 12 intervals. During each subsequent interval, the O-D estimates of the previous interval are used as apriori O-D trips.



### **8.3 Model Verification**

The validity of the mathematical relationships was tested by applying the process to a fully determined artificial network consisting of 7 links and 7 O-D pairs. Data such as volume on the links, and link total travel times were obtained through a random number generator process. The true O-D volumes were known. The objective was to replicate O-D volumes by applying the model. The outcome of the model implementation showed close match between estimated O-D volumes and real O-D volumes.

The conclusion drawn from implementing the model on synthetic data is that the model is mathematically sound and given enough information can produce a reliable outcome.

### **8.4 Application**

The model was applied to a network representing a portion of the city of Boston. In this network, flow on only 58 of 1124 links was known. Travel speed and delay were also only available on these same links, and no "true" O-D data exists.

#### **8.4.1 OLS Runs**

The OLS was run sequentially for 12 intervals per day for five days. The objective was to minimize the magnitude of the RMSE and %RMSE statistics. Both RMSE and %RMSE stabilized by day five. The distribution of the

difference between the estimated link counts and the actual link counts improved significantly over this analysis period.

#### **8.4.2 GLS Runs**

Similar to the OLS runs the RMSEs and the %RMSEs stabilize after day five. Both RMSEs and %RMSEs remain high.

#### **8.4.3 Alternate OLS-GLS Runs**

A comparison of the RMSEs and %RMSEs of 12 intervals of day 5, show the RMSE and %RMSE values increase towards the later intervals. Comparison between the estimated and actual link counts for the alternate GLS runs of day 5 using the adjusted data show some improvement in the distribution around the mean .

### **8.5 Revision of Assumptions**

In an attempt to improve the results, assumptions made on speed, delay, and link counts were scrutinized. All three were revised. Speed was generally raised, delay was adjusted accordingly to reflect changes in speed; and counts were raised based on the number of lanes covered by detectors.

## 8.6 Listing of the Assumptions

The following assumptions were made in this study. These assumptions will influence the results of the estimating procedure, and should be verified ( or modified) prior to using the model.

- 1) The value of the constant  $\mu$ : This constant was set equal to 1.5 based on the results of the sensitivity analysis, reasonableness of the GLS run RMSEs, and the fact that the minimum path update algorithm developed by the Bureau of Public Roads (BPR) used an update value of 1.5 for the exponent of  $v/c$ . This value effects the choice of paths between O-D pairs, and thus link flow.
- 2) The value of the constant  $\beta$ : This constant was chosen to be equal to -0.3. This choice was based on a reasonable expectation of time savings on the expressway links contained in the network. This value effects the travel time on paths which include an expressway, and thus the path choice and link flow.
- 3) Speed was calculated based on the average speed of monitored links in a section of the city. This average speed was adjusted by eliminating all the links with what was considered to be unreasonably slow speeds. This speed was then assigned to all the links that fell in that section of the city. This assumption was made because all the individual link speeds were not available. This value effects the travel time on paths passing through the section, and thus route choice and link flow.
- 4) Average delay was derived from total delay and total volume data for the monitored links within each section of the city. All the links that fell in a

particular section were assigned the same delay value. Delay has the same effect as a change in the link speed.

The OLS procedure was repeated based on the adjusted data. The distribution of the difference between the estimated link counts and the actual link counts was significantly improved.

Both the RMSEs and %RMSEs of the adjusted day 5 GLS estimates are higher than the unadjusted RMSEs and %RMSEs. The error distribution is not significantly better than the unadjusted GLS estimated link count distribution.

Overall, this dynamic optimization method is a valid mathematical model. Its applicability to an urban network requires reliable traffic data. When tested on a network with synthetic data, model gives reliable results. When the model was applied to the City of Boston network, where full data were not available, and the parameters had to be estimated from data samples, the model did not give reliable results.

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