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Learning to Teach
Mathematics for Understanding:
A Case of a Student Teacher in a Professional
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Nelli Wolf

has been accepted towards fulfillment
of the requirements for

Ph. D. degree in Teacher Education

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LEARNING TO TEACH MATHEMATICS FOR UNDERSTANDING: A CASE OF A STUDENT TEACHER IN A PROFESSIONAL DEVELOPMENT SCHOOL editions, which are often associated with PDs, may also provide a good place for novices' learning.

By

Neli Wolf

A DISSERTATION

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Michigan State University
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DOCTOR OF PHILOSOPHY

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1996

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The student teacher's personal qualities and dispositions and the relationships she built with people in the school contributed in important ways to the content of her learning. The context was also important because it offered a culture of collaboration and the chance to see and talk about teaching and learning with other novice and experienced teachers. Such conditions, which are often associated with PDSs, make any school a good place for novices' learning.

So many people have been part of my life that I don't know where to start. The list of their names is so long as not including many of them would be like forgetting mentors, family members, and teachers who have shaped who I am. I have been pushed, celebrated successes, and supported me through my struggles. I am forever grateful. I could have never achieved what I have if it were not for the help of others.

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Ron Hoz, also a professor at Ben-Gurion University, has been a tremendous support. Michigan is not that far from Israel, and I have been fortunate to have met him.

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Writing does not have to be a lonely job!

CHAPTER I

Teaching Mathematics
The Challenges
Writing this Book

Writing does not have to be a lonely job!

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CHAPTER I

LITERATURE REVIEW

Much current discourse about the desirable goals of mathematics teaching and learning focuses on the development of mathematical understanding and power, which is the capacity to make sense with and about mathematics (California State Department of Education, 1985; National Council of Teachers of Mathematics, 1989; National Research Council, 1989). Various reports identify serious deficiencies in mathematics teaching and learning and call for a kind of teaching that departs dramatically from traditional modes which promote transmission of knowledge and facts (National Commission on Excellence in Education, 1983).

In this view of teaching and learning, called teaching for understanding, the students construct their own knowledge and understandings. The teacher's main responsibility is to create worthwhile learning activities and orchestrate discourse that stimulates pupils' intellect and pushes them beyond acquisition and reproduction of facts to sense-making of the subject taught (McLaughlin & Talbert, 1993). This vision of the teaching and learning of mathematics requires teachers not only to do different things in the classrooms from what they are used to doing, but also to have different kinds of knowledge, skills, and understandings from what they typically have.

Herein lies the dilemma. The same teachers who in the past have been criticized for not doing a good job with pupils in the classrooms are now responsible for a much more challenging form of practice. And the same teacher educators who have not done a very good job of preparing teachers to teach in more traditional ways are now in charge of helping novice and experienced teachers learn a kind of practice that is much more demanding and challenging and that they might not know from the inside themselves.

How are we teacher educators going to do that? We know very little about what teaching mathematics for understanding looks like in practice, and even less about how

people might learn to teach that way. We do have some understanding about what teachers need to learn, but we are only beginning to understand what sorts of opportunities might promote that learning. Furthermore, we do not know how teachers make sense of the learning opportunities available to them and what conditions support their learning to teach academic subjects in ambitious ways. We will need to know much more about these questions in order to be able to enable teachers to learn to teach "for understanding."

My study focuses on an elementary student teacher's learning to teach mathematics in ways advocated by reformers in the company of teachers who are themselves experimenting with new approaches to mathematics teaching. It explores how a student teacher learns to teach mathematics for understanding in a professional development school. Examining her learning to teach, I focused on changes that occurred in the student teacher's knowledge and in her ways of thinking and acting as a teacher, and I analyzed how those changes relate to conceptually oriented mathematics teaching. To gain an understanding of how she learned, I examined different learning opportunities available to her and the sense she made of them. This study helps us learn about the content of a student teacher's learning, the processes through which this learning occurred, and the conditions that supported or inhibited the learning.

In order to understand how learning to teach mathematics in conceptually oriented ways occurs, we must understand what this vision of teaching mathematics looks like, what is hard about doing and learning it, what people need to learn to be able to teach that way, and how people could learn what they need to learn. What we know about these issues is the focus of the rest of this chapter.

The discussion is organized around different but related aspects of learning to teach mathematics for understanding: what kind of mathematics teaching teachers need to learn, what is hard about learning to teach that way, what teachers who have learned to do this kind of teaching have learned, and how that learning occurs. Each of these issues is the focus of one of the sections in this chapter. The first section presents a vision of teaching

mathematics for understanding, what teaching mathematics for understanding means, and what challenges it entails. The second section analyses the difficulties and complexities inherent in learning to teach in conceptual ways. The third discusses what teachers learn in the process of changing their ways of teaching mathematics and how that learning occurred. The fourth provides an account of how novices' learning to teach mathematics for understanding might occur, namely, what learning opportunities and processes contribute to this learning. I will end the chapter by presenting a rationale for choosing to study the learning of one person during student teaching in a professional development school through analyzing, synthesizing, and evaluating information and arguments, while encouraging them to communicate their reasoning to peers and teachers. The standards also recognize Teaching Mathematics for Understanding: The Vision and its Challenges as work and that they engage pupils in the The Vision activity.

Teaching for understanding as a vision of practice emphasizes pupils' conceptual understanding of subject matter, as opposed to memorization of facts and procedures (Prawat, 1989). It promotes pupils' active engagement with subject matter through explorations, problem solving, and classroom discourse. In this view, pupils construct their own knowledge and understandings, while teachers act like facilitators of this learning by asking questions, challenging pupils' thinking, and helping pupils think deeply about concepts, ideas, and relationships (McLaughlin & Talbert, 1993).

The reform proposal Professional Standards for Teaching Mathematics from the National Council of Teachers of Mathematics (1991) discusses four characteristics of teaching practice and classroom organization which feature the conceptual teaching of mathematics. These are (1) providing worthwhile mathematical tasks, (2) altering the character of classroom discourse, (3) creating a learning environment, and (4) being engaged in an ongoing analysis of teaching and learning. What follows is an elaboration of each of these characteristics in an effort to understand what they might mean for teaching practice. ils will respond to certain activities.

Worthwhile Mathematical Tasks

The National Council of Teachers of Mathematics assigns teachers the responsibility to create and engage pupils in genuinely worthwhile mathematical tasks. These are tasks that give students access to important ideas in mathematics and to ways of thinking and reasoning about mathematics. The content pupils are exposed to should be important mathematically, and pupils should be given reasons for why the content is important. Worthwhile classroom activities go beyond basic skills and the acquisition of facts. They give pupils opportunities to inquire systematically into important mathematical ideas through analyzing, synthesizing, and evaluating information and arguments, while encouraging them to communicate their reasoning to peers and teachers. The standards also recommend that teachers emphasize why certain mathematical procedures do or do not work and that they engage pupils in problem-solving activity.

To be able to create or adapt activities to meet these criteria, teachers need to know what ideas are the most central to the discipline of mathematics, how they are related, and how different pupils learn mathematics. In addition, pupils who are given these kinds of activities will come up with ideas the teacher has not anticipated, and this demands great skill in improvising responses that will move the learning forward. All this requires teachers to have a deep understanding of mathematics, as well as an understanding of diverse pupils and how they think and learn.

In addition to subject matter knowledge and knowledge about pupils and how they think, teachers will also need pedagogical content knowledge (Shulman, 1986). Teachers who possess such knowledge have an understanding of how specific ideas will be constructed by the particular pupils, what kind of previous knowledge—including misconceptions—pupils might have that they need to take into account when introducing new material, which representations are most helpful for getting certain concepts or ideas across, in which ways those ideas or concepts should be organized and sequenced, and how pupils will respond to certain activities.

the talking, modeling, and explaining (Ball, 1988). Teachers also need to make decisions

Classroom Discourse encourage students to struggle to make sense of an idea, when to ask guiding questions. Advocates of this way of teaching argue that what pupils learn is strongly related to how they learn it (Ball, 1990; Lampert, 1988a). Thus, another proposal for how mathematics teachers might move their practice toward conceptual understanding is that they alter the character of classroom discourse so as to increase pupils' active engagement with what they are learning. According to the NCTM standards, discourse involves two fundamental questions about knowledge: What makes something true or reasonable? and How can we figure out whether or not something makes sense? (NCTM, 1991, p. 34).

Learn In order for pupils to be able to define and solve problems, classroom discourse must center on reasoning and the evaluation of evidence. When confronted with a problem, pupils must be able to make conjectures about what the problem is and how it might be approached. They must be prepared to clarify and expand on ideas and to request, as well as to provide, supporting evidence for comments and opinions. Pupils also must be able to determine whether or not an argument is reasonable and the conclusions well founded. All this requires that pupils talk with and listen to one another, as well as to the teacher, and that they learn to talk about and reflect upon their own thinking, questioning, reasoning, and problem-solving strategies. This kind of dialogue also gives the teacher a way to learn about pupils' ways of reasoning, and it opens opportunities for other pupils in the class and for the teacher to challenge them and to foster the correct ones (Prawat, 1989).

encour Encouraging active involvement of pupils in sustained mathematical conversations in their classrooms requires teachers to have a very different role from the one in traditional classrooms. Teachers need to act like facilitators of pupil-pupil and teacher-pupil discussion, giving reflective feedback that will enhance the quality of the discussion. Also, teachers need to provide critical feedback related to the substance of what students are saying. To do this, they need to listen closely to their students, while students do most of

the talking, modeling, and explaining (Ball, 1988). Teachers also need to make decisions about when to let and encourage students to struggle to make sense of an idea, when to ask guiding questions, and when to tell students directly (Chazan & Ball, 1995). Such decisions depend on teachers' understanding of mathematics and on their knowledge about how pupils think and construct knowledge, because teachers need to make judgments about the things that pupils can figure out by themselves and those for which they will need help (Ball & McDiarmid, 1990; Carpenter, Fennema, Peterson, Chiang, & Loef, 1988; Lampert, 1988a, 1988b; McDiarmid, Ball, & Anderson, 1989).

Learning Environment

Still another feature of teaching mathematics for understanding is helping pupils believe in themselves as successful mathematical thinkers. This involves creating a learning environment in which serious mathematical thinking can take place, in which students are not convinced of the validity of mathematical arguments because the teacher or textbook says so, but by reasoning and justification. The teacher's role is to help students learn to expect and ask for justification and explanation from one another. An argument is not true simply because someone said so. In such an environment there is a genuine respect for pupils' ideas, a valuing of reasoning and sense-making, and a pace that allows pupils to puzzle and to think. This is also a safe place for pupils to take intellectual risks (Lampert, 1988a).

Finding ways to create and maintain a classroom environment that supports and encourages mathematical reasoning and risk taking and fosters students' competence with mathematics requires teachers to help pupils get genuinely interested in mathematics so that they can see their efforts as intrinsically worthwhile. In creating and maintaining such a classroom environment, the tasks that pupils are engaged in and the nature of the discourse that takes place in the classroom are critical. That means that in order to create this kind of learning environment, it is crucial that teachers possess deep knowledge of mathematics

(Ball, 1991; Russell, Schifter, Bastable, Yaffee, Lester, & Cohen, 1994; Simon, 1993) and knowledge of their pupils' ways of thinking and interests (Anderson, 1989; Ball & McDiarmid, 1990; Lampert 1988a, 1988b; Carpenter & Fennema, 1992).

For teaching a particular group of students, what it would mean for someone to have the Analysis of Teaching and Learning

Another attribute of teaching for understanding is its highly analytic or diagnostic nature. Researchers argue that the basis for instructional decision-making should be the analysis of what pupils are learning, which is part of the teaching itself (Prawat, 1989; Ball, 1993a; Lampert, 1986; Lampert, 1990; Lampert, 1991). Analysis of teaching involves observing and listening to the pupils in order to assess their learning. It also involves examining the influences of the tasks, discourse, and learning environment on pupils' learning of mathematics. This analysis helps teachers adapt or change activities while teaching, challenge and extend pupils' ideas, and make plans for future instruction.

Being analytical requires teachers not only to have the knowledge and the skills needed to provide worthwhile mathematical tasks, to sustain a mathematical discourse, and to create a learning environment, but also to understand the links between these and what is happening with their pupils. In addition, teachers need the dispositions to be reflective and the skills necessary to conduct such an analysis of teaching and to consider what the analysis suggests about how this environment, these tasks, and this discourse could be altered in order to help pupils learn (Ball & McDiarmid, 1990; Lampert, 1988a, 1988b).

Although the description of the vision of teaching mathematics for understanding may sound straightforward, translating it into practice is quite problematic. First, there is no clear understanding of what the rhetoric means in practice; many different interpretations exist (California State Department of Education, 1985; National Research Council, 1989; Schifter & Fosnot 1993). Second, there are many questions related to what knowledge, skills, and dispositions teachers need to be able to teach that way and how those can be used in the context of teaching. The existent research does not provide answers to

questions like what it means for a teacher to have deep knowledge of mathematics in relation to third-grade curriculum, what pedagogical content knowledge includes when it comes to fractions or place value, what kind of knowledge of pupils' thinking is necessary for teaching a particular group of students, what it would mean for someone to have the skills and dispositions to be analytical, how these kinds of knowledge, skills, and dispositions can be developed, how they can be used in practice, and what else besides knowledge, skills, and dispositions teachers need to pull it all together with a group of children. The existent research tells us, however, that teaching for understanding is inherently demanding and uncertain (Ball, 1993a) and that it requires a lot more than knowledge and skills. In the next section, I will analyze some of the complexities inherent in learning this kind of practice, as identified by studies of teachers working on changing their teaching of mathematics.

The Challenges

What Is Difficult about Learning to Teach Mathematics for Understanding

Learning to teach mathematics in conceptual ways is a difficult enterprise. This is true for at least two reasons. First, the practice itself is challenging. Representing mathematics in ways that are both authentic to the subject and appropriate for a certain group of pupils is inherently difficult and uncertain (Ball, 1993a; Lampert, 1992; Romagnano, 1994; Simon, 1995). Second, many teachers' experiences with mathematics teaching and learning, as well as their orientations toward mathematics and its pedagogy, constrain their ability to learn the practice and its enactment in the classroom. Next, I will discuss some of the challenges that are both related to the practice itself and to teachers' former experiences with mathematics teaching and learning--challenges that make learning to teach mathematics for understanding such a difficult enterprise.

In analyzing their own teaching of mathematics, Ball (1993a) and Lampert (1985) describe some of the difficulties they encountered as they attempted to represent mathematical content to their third or fifth graders. Ball, for instance, discusses the

demanding intellectual work involved in constructing authentic mathematical representations for her students, in choosing structured representational materials (such as fraction bars or colored chips), and in putting them into use in ways that would benefit her pupils' learning. She also stresses the considerable skill a teacher needs to be able to ask tactful yet incisive questions that would push pupils' thinking forward (Ball, 1992). Both Ball and Lampert talk about the need to manage complex social and moral dilemmas while orchestrating the mathematical discourse in the classroom. Issues like the girls' and minority students' participation in the mathematical discourse, pupils' feelings in relation to a particular representation, or power tensions played out in the context of pupils' participation in the discourse are at the core of any kind of practice that is more responsive to pupils (Ball, 1993a). In addition, teaching for understanding requires more than intellectual work and skills. It requires personal qualities and resources that are rarely discussed and even more rarely nurtured (Ball, 1992). Some of these are curiosity about pupils' ideas, generosity in listening to and caring about ideas that often times are very different from our own, imagination to interpret pupils' responses and to find ways to address those, and willingness to take risks (Roosevelt, 1994). Tolerance for ambiguity and patience with confusion and messiness are two other important personal qualities, for teachers who feel the need to bring closure to a lesson before its end or to have orderly structured classrooms will have a hard time letting pupils struggle with ideas and be confused (Ball 1992, Schifter & Fosnot, 1993). Self-knowledge is important as well, since the more we know about ourselves, the more we are able to learn about others, in this case pupils' experiences and ways of thinking and learning (Ball & Wilson, in press). Teaching for understanding is difficult also because of many teachers' previous experiences with traditional mathematics teaching and learning. As revealed by studies of experienced teachers trying to change their mathematics teaching, this experience may be a factor that hinders teachers' ability to learn to practice in more ambitious ways (Borko,

Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Brown & Borko, 1992; Simon, 1993). Heaton, for instance, a successful elementary teacher who learned to teach mathematics in more conceptual ways, talks about the difficulty of managing mathematical ideas in the context of complex social interaction (Heaton & Lampert, 1993). In her interactions with pupils, she needed to learn how to listen simultaneously to pupils' mathematical ideas, make sense of those, and ask questions in ways that would push pupils' thinking further. This was hard because, as a traditional teacher, she did not know how to get students to talk about their understanding of the mathematical ideas exchanged during the lesson, how to ask questions rather than provide answers. Once these ideas were on the table, she faced a second challenge: figuring out what to do about them. Deciding which ideas to pursue, which to drop, and which to postpone was hard. Her lack of experience with both pupils' mathematical thinking and discussion of mathematical ideas made learning to use classroom discourse as a means to further pupils' learning a significant challenge for her.

Teachers' previous experience with mathematics teaching and learning is not the only impediment in the process of their learning to teach mathematics in conceptual ways. Teachers' mathematical knowledge and understandings, their beliefs about learners, learning, and the teaching of mathematics, and their dispositions are also important factors that influence their ability to learn to teach in more reformed ways. These have been the focus of a growing body of research.

Teachers' Knowledge and Beliefs about Mathematics and Its Nature

Studies of what prospective and experienced teachers know and believe about mathematics and its nature paint a discouraging picture. They show that the knowledge teachers possess or gain during their participation in different classes or professional development programs poorly equips them for teaching in ways that help pupils construct knowledge and understandings.

which Ball (1988) investigated prospective teachers' mathematical knowledge and understanding of division of fractions. She found that most of the prospective elementary and secondary mathematics teachers had limited knowledge of division of fractions, since only a few were able to select correct representations of the problems given to them. Wheeler and Feghali (1983) found that elementary preservice teachers do not have an adequate concept of zero, given that 15% of the preservice teachers studied did not think it was a number and that about 75% of the teachers did not respond correctly to the question: "What is zero divided by zero?" Graeber, Tirosh, and Glover (1986) concluded that preservice elementary teachers have difficulty selecting an appropriate operation for solving arithmetic story problems and have only a partial understanding of proportion. Similar results were reported, among others, by Leinhardt and Smith (1985), Ball and Wilson (1990), and Borko, Eisenhart, Brown, Underhill, Jones, and Agard (1992). In addition, many mathematics teachers see mathematics as a cut-and-dried, rule-bound body of knowledge composed of many disparate facts and procedures with few connections among them and even fewer to the world outside the classroom (Ball, 1988; Kennedy, 1993).

Researchers also found that, with few exceptions, teachers' knowledge and beliefs about mathematics and its nature do not change as a result of their participation in different teacher education programs. A final report of the findings from the Teacher Education and Learning to Teach Study, sponsored by the National Center for Research on Teacher Education (1991), discusses elementary and secondary preservice teachers' understandings and beliefs about mathematics. The preservice students in the study came from five sites across the country. The data show that only a few prospective teachers increased or deepened their understandings of or ideas about mathematics during their undergraduate education. At the beginning of their teacher education studies, preservice teachers had weak understandings of mathematical procedures that they had, for the most part, learned to perform. They tended to see mathematics as a series of rules and procedures, much of

which was arbitrary and had to be memorized. Only 20% of the preservice elementary and 38% of the preservice secondary (who were majoring in mathematics) were able to select the correct meaning of $1\frac{3}{4} : 1\frac{1}{2}$ from among a set of alternatives on a questionnaire item. Still smaller proportions were able to generate an appropriate representation in an interview. When finishing college, many prospective teachers still had difficulties with ideas such as place value, division of fractions, and proof, and saw mathematics as a body of rules.

Teachers' Beliefs about Teaching and Learning of Mathematics

Research on teachers' beliefs about teaching and learning of mathematics does not paint a more optimistic picture. This research reveals that many beginning and experienced teachers assume that mathematics is not interesting to most students, that pupils find it boring and hard to learn. Thus, they tend to be most concerned either with engaging pupils' interest or with being direct and clear about the specific mathematical content. Some teachers perceive their role as telling and showing pupils the proper technique in the clearest way possible, thereby helping them to reach the "correct" way of thinking of mathematics (Kessler, 1985; McGalliard, 1983).

Making mathematics lessons fun is another central concern for both elementary and secondary teachers. Since they view mathematics as inherently boring and hard to learn, many teachers see it as part of their role to motivate students through the use of games or examples the pupils could "relate" to, thereby assuming that if children are having fun they will learn. Making mathematics fun is a high priority for many teachers, rather than helping students make connections to important mathematical ideas (Ball, 1988; National Center for Research on Teacher Learning, 1992; Borko & Shavelson, 1990).

Teachers' beliefs about mathematics teaching and learning are not easily changed. Teacher candidates who participated in a course sequence that promoted conceptual change in students' beliefs about mathematics teaching and learning did change their beliefs about themselves as learners of mathematics, about what it means to know mathematics, and how

mathematics is learned (Wilcox, Schram, Lappan, & Lanier, 1992). However, intensive longitudinal case studies of some of these preservice teachers showed that they were nevertheless inclined to teach mathematics in more traditional ways in the classroom. A possible explanation offered by the researchers for this outcome is that although teacher candidates' experiences in their program led them to revise their own attitudes toward mathematics and gave them images of alternative pedagogy, these experiences were not sufficient to convince them that these approaches were appropriate for elementary students or public school classrooms (Wilcox et al., 1992). Another possible explanation might be related to the change in the context of learning. In their university course, prospective teachers learned new ideas about mathematics teaching and learning. The context of student teaching, however, is different from the context of a university course. In this context, prospective teachers were not able to transfer or apply what they had learned in other contexts. Rather, they needed to relearn—or learn in different ways—what they had learned in other contexts prior to student teaching. My dissertation provides an insight into why this might be the case and how this learning occurs.

These findings pose a difficult question: How can teachers be helped to learn such a demanding form of practice when they lack knowledge, beliefs, and understandings necessary for teaching in this way, and also images of teaching to guide them (Lortie, 1975)? This is a difficult question, because we know very little about the substance and the processes of teachers' learning (Carter, 1990), and even less when it comes to learning to teach for understanding. A partial answer to this question comes from the literature that discusses teachers' learning of teaching mathematics in conceptual ways. This literature, however, is scarce and based mostly on case studies which do not offer a comprehensive picture of how people might learn to teach mathematics for understanding. Also, for the most part, the literature focuses on the learning of experienced teachers, leaving

unanswered questions about how novice teachers might be helped to learn to teach mathematics in more ambitious ways. In the next two sections I will analyze the content of the learning of teachers who changed their ways of teaching mathematics and the opportunities that contributed to that learning. I will base this analysis on four examples of interventions. Two of these examples, Rundquist and Heaton, involve close collaboration between university mathematics educators and individual teachers. The other two examples, Summer Math and Investigating Mathematics Teaching (IMT), involve groups of teachers working with mathematics educators. This analysis will be followed by a discussion of the implications of this research on our understanding of how people might learn to teach mathematics in more ambitious ways. I will begin by describing these interventions.

Four Cases

Ball, a mathematics educator, taught mathematics four days a week in Rundquist's room for three years, while Rundquist taught all other subjects to her third graders. Ball saw this opportunity mainly as an occasion for her to gain an understanding of what this kind of practice may mean and what challenges enacting such a demanding form of practice entails. She also helped Rundquist learn to teach mathematics in more reformed ways (Ball & Rundquist, 1993).

Their collaboration enabled Rundquist to observe Ball's teaching and discuss with her issues related to pupils' learning or to the mathematics involved. They also met once a week to discuss general issues related to teaching and learning. During these meetings, Rundquist often used the notes that she took while Ball was teaching to explore problems that intrigued her.

Heaton, an experienced and successful elementary teacher, read about the kind of teaching recommended by reformers as a doctoral student. Unsure how, as a prospective

teacher educator, she would be able to help novices learn to teach in ways that she herself had never taught, Heaton decided to try to learn a new kind of mathematics teaching. She arranged to teach fourth-grade mathematics next door to Lampert, a scholar and teacher educator who was teaching mathematics in the fifth grade and using her classroom as a setting for prospective and practicing teachers and teacher educators to study a new kind of teaching practice (Lampert, 1992). Once a week, Lampert observed Heaton's teaching and wrote notes about specific teaching episodes. Heaton also observed Lampert's teaching and they met regularly to talk about their practice. Sometimes they looked at samples of children's work or designed problems to use with their students. They also worked math problems and discussed connections between the elementary mathematics curriculum and the discipline of mathematics. (Schifter & Fosnot, 1993; Heaton & Lampert, 1993; Ball

& Run) Summer Math for Teachers was among the first in-service programs to introduce teachers to conceptual approaches to teaching mathematics. The program in its different versions has a history of more than 10 years. The particular intervention I am describing took place in 1989-90 and it involved elementary school teachers. The program began with a summer institute in which 36 teachers were invited to explore powerful elementary mathematics such as place value or the difference between mathematical convention and invention (Schifter & Fosnot, 1993). It continued with a full-semester mathematics course in which teachers had the opportunity to revisit the mathematics topics they taught, exploring the "big ideas" that underlie those topics. The following year, regular classroom observation and consultation were provided by the staff to teachers who requested this support while attempting to implement the ideas they learned in the institute. (Ball

et al.) Investigating Mathematics Teaching (IMT) involved a group of seven elementary teachers, two doctoral students, and a mathematics teacher educator who met bi-monthly during a period of two academic years to explore issues related to teaching mathematics in reformed ways (Featherstone, Pfeifer, & Smith, 1993). For the most part, the teachers who participated in the project were already committed to teaching mathematics in ways that

were different from those they experienced as pupils. During the beginning meetings, the group analyzed videotapes and other materials documenting teaching and learning in Ball's third-grade mathematics class. Later meetings focused on discussions of tapes of teachers' own mathematics teaching. The group also discussed issues and problems that arose in teachers' own practice.

What These Teachers Learned and What Learning Opportunities Contributed to That

Learning In learning to teach mathematics in more conceptual ways, these experienced teachers reported gaining knowledge of and about mathematics (Featherstone, Smith, Beasley, Corbin, & Shank, 1993; Schifter & Fosnot, 1993; Heaton & Lampert, 1993; Ball & Rundquist, 1993), about teaching and learning (Schifter & Fosnot, 1993; Ball & Rundquist, 1993; Heaton, 1994; Featherstone, Pfeiffer, Smith, Beasley, Corbin, Derkesen, Pasek, Shank, & Shears, 1993), about themselves and themselves as teachers (Ball & Rundquist, 1993; Featherstone, 1993). They also learned to focus on pupils and listen for their understandings (Schifter & Fosnot, 1993), developed a sense of their role and responsibilities as teachers (Heaton, 1994) and adopted ways of knowing and learning that could enable future learning of teaching mathematics for understanding (Heaton, 1994; Schifter & Fosnot, 1993).

This learning occurred mostly through observations and conversations about situated problems of practice and their possible solutions (Ball & Rundquist, 1993; Schifter & Fosnot, 1993; Heaton & Lampert, 1993), observations followed by exchange of journal entries (Heaton & Lampert, 1993), observation and conversation about competent practice with peers (Featherstone, Pfeiffer, Smith, Beasley, Corbin, Derkesen, Pasek, Shank, & Shears, 1993), and reflection on one's own practice (Featherstone, Smith, Beasley, Corbin, & Shank, 1993).

confid For example, in talking about their three-year collaboration as a context for teacher learning, Ball and Rundquist, a teacher educator and a third-grade teacher, explore three domains which were most significant in Rundquist's learning: mathematics, teaching and learning, and self. In relation to mathematics, Rundquist learned how to think about what was for her new content: How can negative numbers be subtracted? What are **role for** combinations and permutations? Is zero even or odd? Does 3×4 mean the same as 4×3 ? **used** How do we know we found all the solutions of a word problem with multiple solutions? Is 0 a multiple of 3? And is -9? She also learned to differentiate between pedagogical tools (such as minicomputers, arrow roads, beansticks, and geoboards) and the content itself (like the inverse relationship between multiplying by 3 and multiplying by $1/3$). **used**

Lamp Rundquist's learning of mathematics occurred mostly through observations of **confid** Ball's teaching and conversations with her about the content taught. Rundquist's questions about the tasks Ball used, moves she made during lessons, homework, and assessment contributed to both Ball's and Rundquist's learning because articulating ideas they held in a tacit form helped both understand their own practice. As Rundquist learned more mathematics, she also became more confident in her capabilities to learn, figure out, and enjoy mathematics, which she came to see as complicated and full of meanings and interpretations, rather than cut and dried, as she used to think about it. **used**

teach First-year conversations about mathematics and curriculum were followed by second-year experimentation with their own practice and conversations about things they were both trying and seeing happening with the children. As a result of these explorations, Rundquist learned that children come with a lot of knowledge and that they can teach each other. She also learned that part of her role is to act as a facilitator, to organize children's comments in a purposeful discussion. While listening to pupils' ideas, she began to learn more about how her pupils think and developed new insight about what it means to teach and what she wants her role as a teacher to be. The third-year conversations focused on who they were as women. These conversations helped Rundquist develop a more **teachers**

confident sense of self, which allowed her to realize that pedagogical decisions are moral and that she is both responsible and qualified to make them. Ball and Rundquist also talk about the kind of relationship they built that made this collaboration productive for both participants' learning: a relationship based on intimacy, caring, reciprocity, respect, and trust. A factor that seems to play an important role for both building this kind of relationship and creating learning opportunities for Rundquist is the significant amount of time invested in this collaboration. Three years of collaboration allows for many learning opportunities and many situations in which learning can be supported.

Heaton, a fifth-grade teacher and a doctoral student, describes how she used Lampert's, a teacher educator's, observations and notes on her lessons to gain "strategic knowledge" of teaching. Through their work together, Heaton, like Rundquist, learned that doing mathematics involves meaning and interpretation, not only finding right answers. She came to see that helping pupils interpret the directions of a problem is a central and important part of helping them work on the problem.

In her dissertation, Heaton describes other things about mathematics teaching and learning she learned during the year she collaborated with Lampert: that she needed to develop a purpose of her own (rather than the one given by the textbook) for what she was teaching, and how to go about constructing such a purpose; how mathematical knowledge is constructed; how to use her knowledge of mathematics to better "hear" what her students were saying and help them make connections from what they knew to what she wanted them to learn; and to differentiate between pedagogical tools and the mathematics itself. She learned all this by careful exploration of mathematics herself, by thinking hard about why different topics would be important to teach and how specific lessons would help pupils learn important mathematical ideas.

Working closely with another person for extensive periods of time in their own classroom led to similar kinds of learning for other teachers as well. For instance, teachers

who participated in the Summer Math Program and in the classroom follow-up talked about their learning of mathematics (Linda Sarage, Lisa Yaffee), learning to teach mathematical ideas as opposed to procedures and facts, to listen to pupils' ideas (Ana Maleve, Betsy Howlet, Pat Collins), learning about the learning process itself (Linda Sarage, Jill Lester), about teaching (Sherry Sajdak), and about themselves as mathematics learners (Sherry Sajdak, Lisa Yaffee).

Not all teachers who participated in the Summer Math moved their practice toward a more conceptual one. Some teachers continued to believe that their approach was fundamentally sound. Others, although involved and interested in conceptual approaches to learning, were not able to cope with unfamiliar classroom organizations and ultimately settled for smoothly running classrooms. Still others learned new techniques, but once in their classrooms, used them in the spirit of traditional practice. Some of these teachers realized they themselves learned more working cooperatively and using manipulatives. However, these experiences did not challenge their basic assumptions about mathematics learning and teaching (Schifter & Fosnot, 1993).

What Can We Learn from These Studies?

These studies show that learning to teach mathematics for understanding is an extremely difficult enterprise which requires a considerable amount of time and support offered in the context of practice. The learning involved has to do with mathematics and its nature, learning, teaching, and the self. The support that is needed takes the form of observations of competent teaching, co-experimentation with different ways of teaching mathematics, conversations with peers and with more experienced teachers about questions and problems that grow out of practice and their alternative solutions (Duckworth, 1987). The context of practice seems to be important because it can offer teachers a model of what competent teaching might look like and a vision they may be striving to reach (Lave, 1992), as well as

opportunities to engage in "authentic" activity with the assistance of others (Brown, 1987; Collins, & Duguid, 1989), and concrete, shared problems to discuss. By participating in authentic activity with the support of more experienced educators, teachers have opportunities not only to gain new knowledge, skills, and dispositions, but also to learn ways of knowing, thinking, and acting compatible with this kind of teaching. Being part of the conversation about learning to teach mathematics for understanding gives teachers who are new at this kind of practice a chance to be exposed to and maybe internalize patterns of knowing and learning which might eventually become habitual (Cochran-Smith, 1991).

If we want to design learning opportunities for teachers that will help them learn this kind of mathematics teaching, we need first to understand how and why teachers learn what they do from any given opportunity. To do so, we need to investigate both what an experience is like and what sense teachers make of it (Feiman-Nemser & Remillard, in press). In addition, we know relatively little about the kind of learning opportunities that help novice teachers learn to teach mathematics for understanding. We do not know how what novice teachers learn is similar or different from what experienced teachers learn and how they make sense of the learning opportunities available to them. My study will provide insights into these issues.

The research on experienced teachers' learning, together with theories of situated cognition, suggests some hypotheses about how novices' learning in the context of practice might happen. In the last section of this chapter, I will discuss a model of learning based on these theories that might help us understand novices' learning to teach mathematics in the context of practice. This model stands at the basis of my study.

A Model for Learning to Teach Mathematics for Understanding

If we want teachers to be able to assist children in their learning, researchers argue, the teachers themselves need opportunities to have their own learning assisted by more

experienced teachers or teacher educators (Tharp & Gallimore, 1988; Duckworth, 1987). This assistance can best occur in the context of practice where novices may have a chance to participate in "authentic" activity (Brown, Collins & Duguid, 1989) with the support of other, more skillful, practitioners. An example of an authentic activity might be planning, because teachers plan as a means to prepare for instruction and not for the purpose of helping a novice learn to teach. Practitioners who support novices in this endeavor act as teachers for the novices by making their knowledge and thinking visible to the learners. The social interactions between the novices and their teachers are critical for the former's learning, because it is through those interactions that the novices get access to the experienced teachers' thinking and ways of knowing and see how to accomplish the tasks. In this model of learning, the tasks that novices participate in are not simplified. Rather, novices' participation in the tasks is simplified through the assistance of more expert practitioners. Thus, assistance from more capable others enables novices to perform at levels at which they could not perform independently (Vygotsky, 1978; Lave, 1992). Eventually, knowledge and skills and ways of thinking and acting that initially exist only in the interactions between the novice and the more experienced practitioner will get internalized by the novice.

Although "assisted performance" (Vygotsky, 1978) need not occur only in practice, an obvious application of this theory is learning to teach in student teaching where the novice's learning is "situated" in the context of practice. To the extent that other teachers are more experienced at teaching mathematics for understanding and that they are assisting the novice's learning, the novice has a better chance to become more skillful at this kind of teaching. In such a situation, the novice would learn how to think and act like a teacher of this kind of teaching by observing competent mathematics teaching and by being engaged in activities of teaching together with the cooperating teacher and other more experienced teachers in the school. The cooperating teacher would model ways of thinking and acting

and "scaffold" the novice in her attempts to carry out tasks of teaching (Feiman-Nemser & Remillard, in press).

Being in a situation of practice does not ensure learning the practice of teaching for understanding. Many researchers warn us about the limited range of activities in which novices are engaged during student teaching (Tabachnick, Popkewitz, & Zeichner, 1979/80), the conservative role played by cooperating teachers (Goodman, 1985; Calderhead, 1987; and Bolin, 1988), the socializing pressures of the field on student teachers (Guyton & McIntyre, 1990; Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993). These conditions lead, in many cases, to student teachers' discounting the influence of the university soon after they began to student teach (Richardson-Koehler, 1988) and becoming more articulate and skillful about implementing more traditional approaches to teaching (Tabachnick & Zeichner, 1984; Bunting, 1988; Bolin, 1988).

In addition to external factors, there are also factors that have to do with who the novices are as learners that inhibit their ability to learn to teach in conceptual ways. For instance, Borko, Eisenhart, Brown, Underhill, Jones, and Agard (1992) found that, in relation to mathematics, a student teacher's conceptual understanding of the subject taught and beliefs and knowledge about good mathematics teaching and about learning to teach were among the obstacles that prevented her from teaching in ways she said she wanted to teach.

Field experiences do not have only negative consequences. They comprise a subtle and complicated set of both positive and negative characteristics which, in some cases, do lead to educative experiences (Zeichner, 1980). Researchers found that when cooperating teachers were prepared for reflective supervision, student teachers moved less in conservative directions and that cooperating teachers' level of thinking, as exhibited during their interactions with the student teachers, had an impact on student teachers' thinking about teaching (Zeichner & Liston, 1987).

learning. These results support my conjecture that learning to teach for understanding may not best happen in the context of teaching, in the company of thoughtful practitioners who can model for the novices ways of thinking and acting and assist their learning as they attempt teaching activities themselves (Cochran-Smith, 1991; Feiman-Nemser & Beasley, 1993; Feiman-Nemser & Remillard, in press).

Beginning teachers need to be coached by more experienced practitioners (Schon, 1987) who can model for them ways of thinking and acting.

Introduction to My Study
My study focuses on an elementary student teacher's learning to teach mathematics for understanding in the company of teachers who are themselves experimenting with new visions and approaches to teaching mathematics. It explores how a student teacher learns to teach mathematics for understanding in a professional development school. Examining her learning to teach, I focused on changes that occurred in the student teacher's knowledge and ways of thinking and acting as a teacher, and I analyzed how those were compatible with teaching mathematics for understanding. To gain an understanding of how she learned, I examined different learning opportunities available to her, the sense she made of them, and the cognitive processes through which she interpreted these opportunities. This study helps us learn about the content of a student teacher's learning, the processes through which this learning occurred, and the conditions that supported or inhibited her learning. Next, I will describe the rationale for choosing to study only one person's learning to teach mathematics for understanding, in a student teaching situation, in a professional development school.

Why Study Learning to Teach in Student Teaching?

While novice teachers learn subject matter, generic principles, skills of teaching, attitudes, and dispositions in a variety of settings, they must ultimately learn to use them with the particular pupils they are teaching. Learning the practice of teaching involves learning to adapt worthwhile learning activities to particular students through appropriate

learning experiences. It also involves learning to reason for and about one's actions. Since this learning depends on the context of the specific teaching situations, it occurs best in situ (Feiman-Nemser & Parker, 1990), while engaging a specific group of students in meaningful learning activities.

Novices may not learn to teach in ambitious ways without the support of others. Beginning teachers need to be coached by more experienced practitioners (Schon, 1987) who can model for them ways of thinking and acting (Duckworth, 1987) and engage them in conversations about their own and others' practice (Cochran-Smith, 1991). However, coaching cannot happen outside practice: Novices need the context of action to be able to make sense of the coaches' talk, and coaches need instances of practice to be able to decide what to do next (Schon, 1987). Thus, the context of practice is crucial in understanding both what beginning teachers learn about teaching and what they need to learn, as well as to make sense of how they learn.

Why Study Learning to Teach Mathematics for Understanding in a Professional

Development School?

Since learning to teach in conceptual ways is a difficult and demanding process, it does not happen very often. To study the process, I needed a situation where there was a high likelihood this would happen, a situation where a novice would have the support needed to be able to learn to teach in such ambitious ways. One way to provide novices with the support they need is to connect them with experienced teachers who are themselves actively engaged in the process of studying and changing their own practice (Cochran-Smith 1991). The potential of such a situation in helping student teachers learn to teach mathematics in more conceptual ways consists in the observations of this kind of teaching and social interactions with the cooperating teacher and other teachers in the school, in the opportunities to jointly construct knowledge about teaching mathematics for understanding. In this context, student teachers have the opportunity to see how others

teach mathematics for understanding and how they improve over time, to talk with more experienced teachers about teaching and pupils' learning, to get the support of others when attempting to teach themselves, and to participate with others in a process of inquiry about teaching and learning.

Two of the main goals of a PDS make this kind of setting appropriate for my study: The first is to engage teachers in ongoing conversations about teaching practice; the second is to help student teachers learn to teach for understanding, that is, in ways that attend seriously to subject matter and to pupils' thinking (The Holmes Group, 1990). Thus, in such an institution there is a good chance that the teachers who are responsible for helping student teachers learn how to teach are at the same time engaged in the process of examining their own practice, which, according to Cochran-Smith, might set the conditions for a productive collaboration between experienced and student teachers which might lead, in turn, to novices' learning to teach in more reformed ways.

Why a Case Study?

We know very little about what teaching mathematics for understanding looks like in practice, and even less about how people might learn to teach that way. By looking carefully at one person's learning we can generate hypotheses about what can be learned in practice, how this learning might occur, and what the conditions that support this learning are. A close-up study will help us understand the possible connections between different learning opportunities and their contribution to the student teacher's learning, which is missing in the existent research about teacher learning (Zeichner, 1980; Johnston, 1994; Feiman-Nemser & Remillard, in press).

Maria, the student teacher I have chosen to study, participated in a non-traditional methods course that focused on learning and teaching mathematics in ways advocated by reformers. Through her participation in this course she was exposed to new ways of teaching and learning mathematics, and she became committed to teaching in ways that

CHAPTER II Her resourcefulness continuously impressed me.

METHODS

As a learner, Maria was thoughtful, serious, and committed. She enjoyed discussion and engaged her students. She welcomed learning opportunities and fully cherished what she learned. This helped me study her learning, since her enthusiasm made it so visible to me. Maria's enthusiasm for learning was compelling. It fueled both my interest to study her learning and my desire to create conditions to enable it. I was particularly interested in changes that occurred in her knowledge, beliefs, dispositions, and ways of thinking and acting related to learning to teach mathematics for understanding. To account for how her learning occurred, I analyzed the learning opportunities available to her and the sense she made of them.

The Participant

The main participant in my study was Maria Hubbard (a pseudonym), an elementary prospective teacher from a large Midwestern university, student teaching in a fourth-grade classroom in a professional development school. As an elementary student teacher Maria had to learn to teach all school subjects. Still, she felt particularly committed to the teaching of mathematics for understanding, to which she was exposed in her math methods course. Since understanding learning requires us to understand not only the nature of the learning experiences encountered by learners, but also the learners themselves, I will introduce Maria by describing both who she was as a person and what she brought to student teaching in terms of prior experience with mathematics teaching and learning, beliefs about teaching and learning of mathematics, and dispositions.

Maria as a Person

Although successful as a high school student, Maria is white, from a middle-class background, and was 22 years old at the time of the study. When I met her, she struck me as enthusiastic, articulate, bright, and very pleasant. Throughout our work together, I also came to appreciate her sense of humor,

sensitivity to people, and deep concern for children. Her resourcefulness continuously impressed me. The pattern, I could do it, they were all the same and I could see the pattern. But sometimes I would get them wrong, if I missed a step—that's As a learner, Maria was thoughtful, serious, and committed. She enjoyed discussions and engagement with ideas. She welcomed learning opportunities and fully cherished what she learned. This helped me study her learning, since her enthusiasm made it so visible to me. Maria's orientation toward learning was compelling. It fueled both my interest to study her learning and my desire to create conditions to enable it.

In spite of the fact that she took advanced mathematics courses in high school, she scored very low in the SAT. Maria's Experience with Schooling Maria's experience with elementary schooling was marked by a lot of drill and memorization of facts in all subject areas. Although memorization of facts and repetition were still prominent, in high school she also experienced different kinds of teaching and learning. For instance, learning history and geography, Maria was required "to think, to argue, and to write." She enjoyed the intellectual challenge she faced in these classes and developed a passion for these school subjects.

Her experience with mathematics teaching and learning was pretty traditional. Maria describes her experiences with mathematics as routine, individualistic, and characterized by memorization:

Math always was the same. We sat down and reviewed the homework that we had. We might go up and do a couple of problems on the board. Our teacher would call up two or three [pupils] to do problems 1-2-3 on the board, and then he would go up and say if it was right or what was wrong, and then talk about what we were to learn for that day or what else we were going to do, and we worked on it quietly, and when we were finished we took our homework and did our homework. And that was every day--the same pattern. (Int Sept)

Although successful as a high school student, in the sense that she always got good grades, she said she did not remember much of what she learned because she "never understood anything at all":

For my homework I would copy the odd numbers from the back of the book, and the rest I could do because they were all the same pattern. Once I got into the pattern, I could do it, they were all the same and I could see the pattern. But sometimes I would get them wrong, if I missed a step--that's how I worked--doing the exact same thing. But I never understood anything at all.

Through her experience with mathematics teaching and learning, Maria came to believe that the nature of mathematics required this kind of pedagogy: "[I thought that] this is math and this is how math should be taught."

In spite of the fact that she took advanced mathematics courses in high school, she scored very low in the SAT test, and as a result, she had to take remedial algebra classes in college. However, she did not feel these classes improved in any way her mathematical knowledge and understanding. As a college student, Maria wanted to major in psychology, however, soon she became disappointed because of the "dry nature of the introductory courses" and decided to change her major. Her roommate, then a secondary school prospective teacher, convinced her that teaching is something that would suit her.

Maria's Teacher-Education Experiences

When Maria applied to the College of Education, she was still not sure that she indeed wanted to be a teacher. Participating in the introductory courses, however, convinced her that was what she wanted to become. "That was the first step that made me say, 'Yes, I want to be a teacher!'", Maria proudly declared. What attracted her to teaching was the intellectual challenge she experienced in these classes, the "interesting discussion" she was engaged in:

I liked the discussions especially when there was no right or wrong answer, or when it wasn't any answer at all...and I liked that everything was up for discussion and that was good.

Maria said that first she liked these courses because of the small number of students enrolled in, which made them personal and intimate, then she liked that discussion

was the main pedagogical tool in these courses, which made them intellectually stimulating for her, and only later she came also to like the topics discussed in class. A class that was particularly important for Maria's development as a prospective teacher was the mathematics methods course. In this class, she experienced for the first time learning mathematics for understanding. She liked this class because there, for the first time, she saw a math teacher "who was not interested whether the answer was right or wrong, but in the way we thought about the problem." This course contributed in a significant way to Maria's commitment to learning to teach mathematics for understanding.

The purpose of this course was to help prospective teachers reexamine some of their past experiences with mathematics, gain a new understanding of what mathematics is and how it might be taught and learned, as well as develop some tools for helping pupils understand mathematics. In this course, the students were also introduced to the NCTM Standards. They discussed what it may mean and take to teach mathematics in ways aligned with the Standards and analyzed how their past and present experiences with mathematics teaching and learning compared with the kind of teaching recommended by reformers.

To help students gain an insight into what it may mean to learn mathematics for understanding as well as to provide them with an image of what this kind of teaching might look like, the instructor devoted a significant part of the course to learning mathematics. Thus, during this course, the students examined in depth one topic of the mathematics curriculum--fractions. By studying one mathematical topic in depth, the instructor was hoping students would improve their own knowledge and understanding of fractions, as well as develop a feeling for what it may mean to understand something in depth. The instructor modeled teaching mathematics for understanding by teaching the course in ways consistent with the kind of teaching recommended by reformers, ways she was hoping students would become committed to learn.

Maria indeed appreciated what she learned in this class. She fully enjoyed both learning mathematics and discussing her and others' learning of mathematics and of teaching and learning. She found these discussions intellectually stimulating. The idea of teaching mathematics to elementary pupils by the means of "discourse" appealed to her. She became committed to teach her future pupils "the same way we were taught" in the math methods course.

Maria requested to student teach in a school where she would have opportunities to work with other people interested in teaching mathematics in more conceptual ways. She was hoping her collaboration with teachers interested in learning to teach mathematics for understanding would enhance her own learning. She also volunteered to participate in my study because she saw it as an opportunity for her to reflect on what she was learning and engage in discussions that could further her learning of teaching mathematics. Maria student taught in the school for one term (ten weeks).

The Setting and the Context

The setting in which Maria student taught, Springhill School, is a professional development school. Most of the teachers in the school have a history of being engaged in reforming their own teaching. University staff also support various efforts of reforming teaching in the school. Among the different projects in which Springhill teachers were involved during the year of my study, two focused on helping teachers learn to reform their teaching of mathematics. These were the math project and the math study group. A third project, Teacher Education Circle, was aimed at supporting cooperating teachers in their work with student teachers.

The math project involved university mathematics educators working with Springhill teachers on teaching mathematics in conceptual ways. This project started in 1990, two years before the beginning of this study, when a math educator from the university came almost daily and co-taught, co-planned and reflected with individual

teachers about the teaching and learning of mathematics for understanding. During the following year, another mathematics educator became involved in the project by doing weekly observations of the teachers while they taught mathematics and intervening either by co-teaching or by working with individual or small groups of pupils. After the lesson, the math educator and the teacher met to discuss the teaching, students' learning and the content taught. Maria's cooperating teacher was involved in this project since its beginning.

The math study group included teachers from Springhill and several other schools. The study group was part of a federally funded research center at the university. The teachers who participated in this project met bi-monthly at the university to think about ways to make mathematics teaching and learning more conceptually focused. During the meetings, the teachers have the opportunity to watch and discuss videotapes of models of teaching mathematics for understanding and engage with other teachers and math educators in conversations about specific concerns related to teaching mathematics for understanding. Maria's cooperating teacher joined this study group at the beginning of the year in which I conducted my study.

Springhill also had a history of involvement in teacher education. For ten years, the faculty had worked closely with teacher candidates in a small thematic program at the university. During the year I did my study, the school faculty in collaboration with one teacher educator were experimenting with a new student teaching program. The program had two main goals: (1) to help student teachers learn to teach in conceptual ways and (2) to engage the student teachers and their cooperating teachers in an inquiry about teaching, learning, and subject matter.

The student teaching program gave special attention to learning to teach mathematics for understanding. Student teachers had an opportunity to observe a week's worth of mathematics lessons in one teacher's classroom and to discuss with the teacher her actions

and decisions. This teacher, also a member of the math study group, was studying new ways of using classroom discourse to help pupils learn mathematics.

Support for the cooperating teachers working with the student teachers was provided through the Teacher Education Circle, a forum where a university teacher educator helped the cooperating teachers think through problems of practice that arose in their work with student teachers. Bi-monthly meetings between the cooperating teachers and the university faculty member served as an opportunity to talk through issues and problems and develop strategies for promoting student teachers' learning.

However, even if most of the teachers in the school were committed to reforming their own teaching and to helping novice teachers learn to teach mathematics conceptually, the ways in which their practice reflected the kind of teaching advocated by reformers varied. The teacher who offered the mathematics demonstration lessons was the most advanced in her teaching of mathematics. Her practice most resembled the kind of teaching recommended by the NCTM Standards. Maria's cooperating teacher, Ms. Elaine Barnes, a fourth-grade teacher, had also made some important changes in her mathematics teaching; however, she saw herself (and I agreed) much at the beginning of the process of changing her practice.

The school context in which Maria, the student teacher, began to learn to teach mathematics for understanding seemed promising. First, the cooperating teacher in whose classroom she student taught was interested in changing her ways of teaching mathematics and was working toward this goal. Her participation in the math project and the math study group suggested her commitment. Second, the Teacher Education Circle was supposed to help her and other cooperating teachers in the school to better assist and support student teachers' efforts to learn how to teach. Third, the math strand within the student teaching program offered the student teachers an opportunity to observe how another teacher taught mathematics, to talk to her and to other teachers and student teachers about the teaching and learning of mathematics, and to participate in planning and reflecting with more experienced

teachers. Finally, the fact that the school was a PDS and that some of the teachers were involved in different math-related projects suggested that they were committed to reforming their own teaching and that collaboration between teachers existed. All these conditions promised a school context in which student teachers and teachers would have opportunities to jointly construct knowledge about teaching and learning of mathematics. From this point of view, Maria could have the opportunity to learn to teach mathematics in reformed ways in the company of teachers who were themselves working toward that goal.

Data-Collection Strategies

I collected most of the data about the participant's learning during the ten weeks of her student teaching, Fall Term 1992. The data consisted of observations of her teaching and interactions with others, formal and informal interviews about her perceptions and thinking, and lesson plans, videotapes of her teaching, and journal entries she wrote to reflect on her own learning to teach mathematics "for understanding." In addition, I collected copies of the journal entries she wrote during her math methods course which she took in the spring term, 1992. These entries reflect her thinking about teaching mathematics for understanding prior to the beginning of the student teaching. I also did a follow-up observation of and interview about her teaching and learning from teaching in May 1994, while she was teaching in her own classroom.

The observations and formal and informal interviews were designed to help focus data collection on the participant's learning of what I thought were important features of teaching mathematics for understanding: choice of mathematical tasks, discourse patterns, grouping of students, analysis of teaching and pupils' learning. The formal interviews and observation guidelines I used were adaptations of instruments used in the Learning from Mentors Study conducted by National Center of Research on Teacher Learning at Michigan State University. I modified these instruments to focus them more on mathematics and the particular situation. Next, I will describe how these instruments helped me collect data

about the student teacher's learning of both acting and thinking in ways appropriate for conceptual mathematics teaching.

Interview Data

At the beginning of student teaching, I conducted an autobiographical interview probing Maria's experience as a student and learner, with a special focus on her learning of mathematics. I asked Maria to walk me through her schooling experiences from elementary school to student teaching with a view of establishing connections between these experiences and her ideas and beliefs about mathematical knowledge, teaching and learning of mathematics, and learning to teach.

In November, a month before the end of the student teaching, I conducted an interview for the purpose of understanding Maria's views in relation to learning to teach mathematics for understanding. In this interview, I elicited Maria's ideas about what needs to be learned, how that can best be learned, and what other people in the setting (including me) can do to assist her in that learning. Some of the questions also asked about the school context as a setting for learning to teach mathematics for understanding.

In December, at the end of the student teaching, I conducted the same interview for the purpose of not only understanding what her ideas were then, but also of learning about the changes that occurred in her views during the time she took more of an active role in the cooperating teacher's classroom.

At the end of the student teaching I conducted a version of this interview with the cooperating teacher for the purpose of understanding what she thought Maria needed to learn in order to teach mathematics in conceptual ways, what was hard about that learning, and what she could have done or had done to assist the student teacher in her learning.

Three times during the term, when Maria taught mathematics lessons on her own, I conducted pre-observation and post-observation interviews. The interviews were aimed at eliciting the student teacher's thinking about aspects of teaching mathematics for

understanding, such as the nature of the learning activities used in the lesson, classroom discourse, learning environment, and, in the case of the post-observation interview, analysis of the teaching and learning that occurred. These interviews and observations about teaching taken together provided access to the student teacher's thinking about teaching, as well as actual examples of that teaching.

Once, at the beginning of student teaching, I also conducted pre- and post-observation interviews with the cooperating teacher focused on an observation of her teaching. The interviews and observations helped me understand what kind of teaching Maria had a chance to observe and the thinking about mathematics teaching and learning that the cooperating teacher could share with her. In addition to the interviews about her teaching, I also interviewed Maria about particular interactions she had with the cooperating teacher, other teachers, teacher educators, and student teachers in the school, to gain an understanding about how she sees these interactions as contributing to her learning. These interactions included conversations she had with different people, participation in demonstration lessons, co-teaching, co-planning, and other activities aimed at helping her learn to teach mathematics.

In addition to the formal interviews, I had conversations with the student teacher during non-teaching times of the day, on the playground during recess duty, during lunch time, or during class time when she was not teaching. I noted these conversations in my field notes and incorporated them into my notes of the day.

Although I took notes during the interviews, I taped all interviews and transcribed the tapes as soon as I could.

Observational Data

To understand how Maria was learning to teach mathematics for understanding in the company of other people who experimented with their own ways of teaching mathematics, I needed to learn about her teaching and thinking about teaching, the models

of teaching she was observing and guided by, and the formal and informal interactions she had with different people in the setting, and how these contributed to her learning. The interviews and the student teacher's own reflections helped me understand her own and her cooperating teacher's perceptions, views, and ideas about what they did to help Maria learn to teach mathematics for understanding. However, I still needed to observe the novice and her interactions with others directly in order to see for myself what their teaching and their interactions were like. In this section, I will describe the observations I conducted in the setting.

In order to understand the activities Maria was engaging in and how these contributed to her learning, I observed her daily as she interacted with the cooperating teacher, teacher educators, other teachers in the school, peers, and pupils in the classroom. Most observations were followed by interviews with her which helped me learn about how she perceived these interactions and what sense she was making of them.

In observing conversations the student teacher had with other people (the cooperating teacher, other teachers, or student teachers), I paid attention to the substance of the conversation, meaning what topics were discussed and for how long and the dynamics, meaning who set the agenda, what kind of questions were asked and by whom, what kind of responses were given, and what roles the participants played. These interactions were taped most of the time and the tapes later transcribed. I also taped most of the interactions in which I was one of the participants, or, when that was impossible, as in the case of phone conversations, I took extensive notes about the substance during the conversation itself and reconstructed the dynamics immediately after.

To gain an understanding of how she was learning to act like a teacher, I observed Maria's interactions with pupils during lessons she taught or co-taught in collaboration with the cooperating teacher. During these observations, I paid attention to the kind of activities she was using with the pupils, the discourse she orchestrated, and the learning environment she was trying to establish. I paid special attention to the role she was playing in these

lessons. Twice during the term, in the middle and the end of the student teaching, I videotaped her teaching of mathematics.

In order to learn about the kind of teaching Maria had a chance to observe and in what ways it was conceptually oriented, I also observed the cooperating teacher's teaching of mathematics and the mathematics demonstration lessons offered as part of the student teaching program. I focused my attention in these lessons on aspects that characterize teaching mathematics for understanding: worthwhile learning activities, mathematical discourse between pupils and between teacher and pupils, and the learning environment created in the classroom.

Other documents, such as the student teacher's own reflections on what she saw or learned, math lesson plans she wrote either alone or in collaboration with others, and journal entries she wrote in the math methods course complement the data I collected through interviews and observations and give a fuller picture on the substance and the sources of the student teacher's learning.

Data Analysis

The first step in analyzing the data was taken during the time I observed and interviewed the student teacher as I transcribed interviews and wrote up notes of my observations. Writing up notes, reading transcribed interviews, and talking to the student teacher and other people in the setting helped refine my next observations and interviews.

Analysis of Interview Data

The interview data included my notes of the interviews and conversations I had with the student teacher and with other people in the setting, and transcriptions of taped interviews. As I read these notes and transcriptions of interviews, I tagged the data around categories that were pertinent to my question. Examples of these categories are Maria's talk about her experience with mathematics learning, Maria's beliefs about good mathematics

teaching, Maria's beliefs about planning and teaching mathematics, and Maria's talk about her own mathematics knowledge. I then organized the interview data into these categories and formulated possible conjectures in relation to these categories. These conjectures helped me refine the way I was looking at the data, which in turn helped me develop new categories around which to reconstruct my data.

For instance, initially I created a category which I called "views about Maria's learning during student teaching," which included her own reports as well as others' comments (including mine) about her learning. After looking at the transcripts of the interviews that pertained to this category more closely, it became clear that Maria's learning could be better described in more refined ways. Thus, I set up more categories, such as "learning about the teachers role" or "learning about planning," rather than one general category. After setting up new categories, I went back to previous interviews and pulled out additional comments which pertained to them.

I engaged in a similar process for all the interviews and formal and informal conversations I had transcripts or notes for. These categories helped me learn not only about different people's views at certain points of time, but also how they changed during the time. These categories were subject to further refinement once I began the analysis of the observational data.

Analysis of Observational Data

After taking extensive field notes about the events I observed, I "wrote up" each of my observations using the observation guide which was developed for the "Learning from Mentors Study" as a structure for my notes. These "write ups" included narrative description of the day's observation as well as answers to interpretive questions from the guide. In the case of a conversation, for example, the narrative began by a description of the place where it happened, the length of the time it took, and the people involved in it. In the body of the narrative I recreated the chronology of the conversation, what people talked

about, and the dynamic of the conversation: who initiated what parts, what kind of topics were raised, and what roles people played. After writing a descriptive summary of the conversation, I answered questions about the substance (what could be learned from this conversation), structure (what topics got discussed and how they related to teaching mathematics for understanding), processes (what was the pattern of interaction), and roles (who decided what gets discussed, when, and how) the people involved played in the conversation.

When I observed the demonstration lessons, for example, I paid attention to both the ways in which the teaching was conceptually oriented and the roles the student teacher played during the lesson. The narrative included the chronology of the events, what pupils did and said, and how the teacher responded, as well as the dynamics of the lesson. I also described the student teacher's interventions in the lesson if there were any. The interpretive questions focused on the ways in which the lesson was conceptually oriented and had to do with the nature of classroom activities, classroom discourse, and roles the pupils played during the lesson. These are examples of interpretive questions from the observation guide which I answered in relation to demonstration lessons:

Do the learning tasks/activities embody important mathematical ideas? Do the activities give pupils a chance to reason and evaluate ideas, arguments, evidence? How are pupils organized for learning? How is knowledge construed in the class (e.g., as a fixed set of ideas or body of facts? a changing set of ideas that needs to be re-examined?)? Are students expected to question and challenge each other's ideas? What does the teacher do to encourage this?

These write ups helped me identify themes and frame assertions about the student teacher's learning. They also helped me refine the categories for the interview data. Finally, I separated the observation data into categories similar to the interview data.

After analyzing the interview and observational data separately, I combined and re-organized them into categories that would suit both kinds of data. In this process, some of the categories expanded, others became parts of new categories. For instance, a category

such as "Maria's talk about her own mathematical knowledge" became "Maria's mathematical knowledge." This category comprised my analysis of both Maria's description and assessment of her own mathematical knowledge and my interpretations of her knowledge based on observations of her teaching and conversations with others. A category such as "Maria's learning about planning" became part of two other categories: "Maria's learning about pupils' thinking" and "Maria's learning about teacher's role."

Through this organization of data I noticed that Maria's learning occurred mostly in three domains: subject matter, pupils' thinking, and teacher's role. Maria's learning in relation to each of these domains is the subject of the next three chapters. Learning in these domains is not only prominent in Maria's case; it is also crucial for teachers who change their practice or learn to teach in more conceptual ways.

In making claims about the data, I used multiple sources of evidence from the study for the purpose of triangulation. Interviews, observational field notes, documents such as lesson plans and reflections and videotapes of teaching all contributed to my construction of meaning about the events that occurred in the setting and ultimately to my understanding of the content and the process of Maria's learning.

My Role as a Researcher

My role in the study has changed and evolved during the data collection process, as Maria, the student teacher, progressed through the semester and her work with people in the setting, especially with the cooperating teacher, put new demands on her. The changes that occurred in the way I saw and enacted my role reflect my understanding of the conflicts and moral dilemmas that appear when the researcher's role and other past and present roles can not be, or we do not want them to be, separated so easily. I use this section as an opportunity to explain what my role in the study was and how it changed during the semester, as a response to the moral dilemmas that I faced.

Originally, I intended to be a field researcher in the setting. I planned to observe the student teacher as she was teaching or interacting with other people and to interview her about what she was learning on these occasions. With this role in mind I began my study. The situation I encountered in the setting proved to be rather complicated, which forced me to re-think my role in the setting.

Soon after I started my observations in the setting, Maria began to ask me questions related to her learning to teach mathematics. At first I tried to be true to my intended role, sending her to others or turning the question back to her. For instance, to a question like "What examples should I use?" I would say, "Who do you think might be able to help you with that?", or to a question like "Shall I do this or shall I do that", I would ask, "What might help you decide?" However, as I got to understand better the ways in which the student teacher and the cooperating teacher interacted, I no longer felt comfortable giving this kind of answer.

One day, as Maria was beginning to take more of an active role during the math lessons, Ms. Barnes, the cooperating teacher, asked her to work on "problem solving" with the pupils. Ms. Barnes chose the problems the pupils were supposed to work on. Maria introduced the activity and asked the pupils to work on the problems until the end of the period. At the end of the lesson, Ms. Barnes asked Maria to conduct "mathematical discourse" the next day about this problem-solving activity. Maria did not feel she could deal with the complexities of such a discussion. She asked Ms. Barnes, "How should I do that? What should I talk about? What should I do with the many ways in which the kids solved the problems?" Annoyed with the student teacher's inability to "decide and act upon her decision," and busy with her own schedule, the cooperating teacher walked away.

This situation posed a moral dilemma to me. On the one hand, I was supposed to observe how the student teacher interacted with the cooperating teacher. On the other hand, there was the moral aspect of the problem: the student teacher who realizes the difficulties inherent in teaching mathematics for understanding struggles to learn to teach that way, but

help and guidance are not always available to her and the pupils experience the results. It was very hard for me to know that I, as a mathematics teacher educator, could do something to contribute to the student teacher's and pupils' learning, and not do anything besides observe the struggle. I interpreted Maria's repetitive requests for my assistance as a sign of her seriousness and commitment to learning to teach.

In deciding whether to intervene--and if so, how--I took into consideration the cooperating teacher's feelings and the repercussions my intervention could have on the student teacher, pupils, and my own relationship with the cooperating teacher. The decision I made was to help the student teacher while working along the cooperating teacher's lines. This meant that Ms. Barnes made the curricular decisions, she decided what Maria was supposed to do, and I tried to help Maria think about what she could do to improve her teaching and thinking about teaching. I thought that by doing that I could contribute to Maria's learning without jeopardizing her and my relationship with the cooperating teacher. Acknowledging her busy schedule, Ms. Barnes welcomed the opportunity for me to help Maria in her learning to teach mathematics.

An example of my working in this way with the student teacher would be when she asked me to go over the questions she wanted to ask pupils during the math lesson, and to help her anticipate some of the pupils' answers. I suggested that she role-play this discussion. Since I played the role of the fourth-grade student, I helped her think about some of the pupils' responses, while at the same time she realized some of the weaknesses in her questioning. During the time I worked with Maria this way, she took the initiative mostly by asking me questions or asking advice for the things she wanted to do. I saw my role not as providing answers, but always addressing her concerns.

A second shift in the way I worked with Maria happened when she began to think about the place-value unit, a unit that, according to the program requirements, she had to develop and teach on her own. Before she actually started the planning and development of the unit, Maria had a conversation with other student teachers about how they needed to

approach the task. By listening to their conversation, I learned that Maria had difficulties thinking about the concepts and ideas that should be included in the unit and how those could be translated into a fourth-grade curriculum.

Following this conversation, Maria requested help on planning the unit from the cooperating teacher. But Ms. Barnes was extremely busy and did not have the time or possibility to assist Maria. Ms. Barnes suggested the general lines for constructing the unit, but this was not the kind of substantial support the student teacher needed. After a discussion between the cooperating teacher, student teacher, program director, and me, I took more of an active role in the student teacher's learning to teach mathematics.

During the time I worked with the student teacher, I helped her think about the subject matter she was teaching, planned and reflected with her about the lessons she taught, and worked with her with individual children. During this time, I kept a journal about my work with the student teacher, and I interviewed her periodically to learn about what she was making of our work together. This way I tried to combine and make peace between the two roles that could have conflicted: that of a researcher and that of a person who offers on-site support.

Changing my role in the setting presented both benefits and pitfalls for my study. On the one hand, working intensively with Maria allowed me to get a clearer understanding of her thinking and learning. Also, because of the intensive joint work, we got close to each other, which encouraged Maria to talk openly with me. On the other hand, as we became closer, I noticed I was less predisposed to be critical of her thinking and of the learning process she was going through.

Once I became aware of the fact that close relationship might interfere with the study, I started to take careful notes about our interactions, to audiotape as many of our conversations as I could, and, in the case of phone conversations, to take notes during the conversation and immediately after to summarize it. However, most of the data reported in the study was collected prior to our developing a close friendship.

"I see Neli as a friend and as a teacher," Maria explained the essence of our relationship in one of her last journals. "As a friend, because she always makes time to help me, no matter what; as a teacher, because she helps me with how to learn to teach math." For me, my collaboration with Maria represented one of the rare occasions someone can have to get close to a person's learning, understand the substance and process of it, and learn to appreciate the struggle and joy that learning entails. For the reader, I hope my work will provide an insight into the beginning of processes of learning to teach (Zeichner, 1980), leading him or her to an understanding of the subtleties often missed in more generalized kind of research.

CHAPTER III

LEARNING MATHEMATICS

Content knowledge--I thought it was important and I am still thinking that now, but now I have a better understanding of what I think I know, and what I actually know, or what I know only at surface and not in depth. And I also thought that in order to teach it was important to know it yourself well, but now I know what it means to know it well--you really have to explore it before you even dare to teach it to the kids. (Int Dec)

This chapter focuses on Maria's learning of mathematics: what mathematics she learned and how this learning came about. I will also analyze what Maria learned about her own knowledge of mathematics and what implications this learning had on her understanding of what it means to know subject matter in a way she could teach it to pupils. Analyzing the processes through which Maria learned mathematics, I will focus on co-planning and conversations with pupils, because these seem to be the occasions that most enabled her learning of mathematics.

Learning Mathematics from Planning

Co-planning a Unit

One of the requirements of the program in which Maria participated was that student teachers develop a math unit to teach in their cooperating teachers' classrooms. Student teachers were supposed to work on developing this unit together, so that they could offer each other advice and support. To facilitate collaboration among the student teachers, the program developers chose a topic common to all grades: place value. Since this concept is taught in every elementary class and around the same time of the year, student teachers could work together on developing the unit, while they would still need to make necessary adjustments for the particular pupils they were working with.

Not sure how to approach the task, but knowing she was supposed to take into consideration students' prior knowledge about place value, Maria asked the cooperating

teacher what she taught and thought pupils already knew about the subject. Ms. Barnes told her the pupils had already been taught "place value" in previous years, and they knew it because they were able to tell the names of the places, and they could read and write numbers if given the values of the digits in different places. Maria was puzzled: If the pupils knew that, what else she could teach them?

If they know the names of the different places and they know that 1 ten equals 10 ones, and 1 hundred equals 10 tens, and so on what shall I do? What else is there to know about place value? (Oct 19)

Ms. Barnes suggested that she teach rounding and estimation because that was what the district curriculum recommended to be taught as place value in fourth grade. Since Maria thought that knowing "place value" means only knowing the names of the places and the relationship between them, she could not see how rounding and estimation could be related to "place value." Instead, she read in one of the textbooks that fourth graders are supposed to know "place value up to millions." So she decided to "find one good activity and go on and on through millions" (Oct 19). She and other student teachers started to look for "good activities" that could help pupils understand "place value up to millions." When Maria could not find any activities, she requested my help.

I suggested that she think about what important ideas and concepts are embedded in the idea of "place value." I recommended that she look at different textbooks and at NCTM Curriculum and Evaluation Standards and come up with a list of important concepts that fall under the concept of place value. Maria agreed. The next day, she had a list that included trading, borrowing, comparing numbers, grouping by ten, computation, number sense, rounding, and estimation. She was still not sure how rounding and estimation relate to place value. Since from this list I could not know if Maria understood the big ideas embedded or connected to the concept of place value, as well as how the different concepts and ideas are related, I suggested that maybe we could draw a map that would help us better understand what place value is and why it might be important to teach. What follows

are excerpts from our taped conversation which I am presenting in detail because it shows not only what Maria's understanding of place value was, but also how it changed during our conversation.

I started by writing "place value" in the middle of a big sheet of paper. Maria drew an arrow and wrote "names of the places and the relationship between them." I asked why it would be important to know that. Maria thought for a couple of seconds and said, "Well, it's important because you have to know that 1 ten equals 10 ones, and 1 hundred equal 10 tens. You need that for trading and borrowing." Then she drew two more arrows from "place value" and wrote "trading" and "borrowing." I asked if knowing the relationship between different places would help me better understand what numbers are. She again thought for a while and then said, "Yeah, I think so. If you understood that in a number like 2405, 2 represents two thousands and 5 represents 5 ones, you would know that's different than a number like, let's say, 5402. So maybe place value can help children understand how big numbers are." "And maybe to compare numbers," I added. "I see that, and maybe with ordering of numbers, too?" Maria asked, at the same time drawing two more arrows: "understanding numbers" and "ordering and comparing numbers." At that point it seemed to me that Maria was beginning to see that place value includes much more than just "knowing the names of the places and the relationships between them." I wanted to help her get clearer about two more ideas. First, in response to her expressed concern, I wanted her to understand what rounding and estimation have to do with place value. Second, I was not sure what she meant by "trading" and "borrowing," how she saw them as similar and/or different, and how she saw them as related to place value. I asked her if there was some place on the map for "rounding" and "estimation." "I don't know," Maria answered, "I have to think first why we round or estimate, and how we do that." Maria started to look through the NCTM Standards in search for an answer to that question:

Instruction should emphasize the development of an estimation mind set. Children should come to know what is meant by an estimate, when it is appropriate to estimate, and how close an estimate is required in a given situation....When checking estimates, a teacher can reinforce place value ideas by having the children place the estimated items in groups of ten and then in hundreds whenever possible. It is also important for the teacher and the children to identify a range for "good estimates" (Standard 5: Estimation, p. 36-37).

Maria puzzled for a moment:

So that means that when I teach estimation, I should emphasize its relationship to place value...I should have them count and group things to check their estimates...Hmmm...But if I teach place value, how do I get to estimation?

- I: And how is this "trading" and "borrowing" related to all this?
- M: That's easy. Trading is related to subtraction. Like if I had 18-9 I would trade but not borrow. I would trade 1 ten for 10 ones. To do that you need to know place value. But...if I get into addition and subtraction...hmmm...how would I connect that to rounding and estimation?
- I: Well, when you do operations with numbers, how do you know that your result makes sense?
- M: Hmmm. You mean I estimate? Yeah, I do that, don't I? Yeah, if I add two big numbers like 289 and 599 how do I know if my answer makes sense? I would do 289 is almost 300, and 599 is almost 600, so the result should be close to 900. Yeah, I see that.
- I: Is that rounding or estimation?
- M: Yeah, I see. It can be either one. 289 can be rounded to the nearest ten and then it's 290, or to the nearest hundred and then it's 300, so I can estimate the result to be 900.
- I: So you round to estimate?
- M: Yeah, that's what I did, didn't I? I used rounding as a tool to estimate. There are many ways to estimate, and one of them can be rounding, right? And what does all this have to do with place value? Well, it's hard to round if you do not know place value: how would you know what to round to if you don't have a sense of the places?
- I: Let's go back to this "trading" and "borrowing." They seem to be a little confusing. Maybe we can use a different term instead? Maybe we can talk a little how are those different and how are they alike?

M: See, "trading" is easier for me to think about. I am not quite sure when I would use borrowing. Let's use "trading" for now and think about it later.

I: Okay.

Co-planning as an Occasion for Learning Mathematics

When Maria came to student teaching, she believed that knowing subject matter was important, but she also believed that she knew mathematics well enough to teach it to elementary students (int, Nov). She began to question this assumption only when confronted with situations in which her knowledge of mathematics proved to be problematic. One of these situations was planning a unit on place value. When she began the planning itself, she believed that knowing the names of the places and the relationships between them is all there is to know about place value. The cooperating teacher's advice, to teach rounding and estimation as part of the unit on place value, did not challenge her belief, or maybe did not even make sense to her, perhaps because when the advice was given she was not yet in the context of doing, "in the midst of a task (and perhaps stuck in it)" (Schon, 1987, p. 102). When Maria approached me, she was stuck. She wanted me to help her plan a good activity that could teach the students the names and the value of the places up to millions.

From Maria's request I learned that she needed help understanding what "place value" means and entails and how this important mathematical concept is related to other mathematical concepts and ideas. Helping her better understand what she was supposed to teach was the first step in planning a lesson or a sequence of lessons. Partly because she was a student eager to learn as much as she could about teaching and partly because she had a problem and believed I could help solving it, Maria listened and cooperated in a way that allowed her to further her understanding of place value. The map we drew is far from perfect, and it by no means exhausts the whole set of concepts that are mathematically related to "place value." In fact, it even contains ideas that are not mathematically correct. However, it represents the development of Maria's understanding of this concept, as we talked about it. My purpose in working on the map was to help her see and appreciate the

richness of ideas related to place value, the relationship between them, and the implications of these ideas on the way we think and operate with numbers in everyday life.

By working on the map and thinking about how and why, for instance, we round or estimate, she came to better understand the mathematical territory she was supposed to teach. Our discussion did not help her think about how to represent this content to the students, or how to assess what they already knew about it. But it did help her understand a little better what was there to be taught and learned. Reading the Standards helped Maria think about connections among the different ideas she encountered. Realizing that in teaching estimation, she was supposed to make connection to place value and that grouping by ten and hundreds could help students check if their estimates are reasonable made her think about how she could attend to estimation when the focus of her teaching was place value.

A problematic issue in this conversation was Maria's understanding of "trading" and "borrowing." I did not know what she meant by these words or how she saw them as similar or different. I wanted to push her to think about "regrouping" and how the idea of regrouping can help us understand the algorithm of addition and subtraction. But Maria seemed to be overwhelmed by the number of ideas she encountered in only one conversation. This is how I interpreted her request to "use trading for now and think about it later." I did not think that continuing the discussion with her would bring any more desired results. We discussed her ideas about "trading" and "borrowing" in a different conversation which I initiated a couple of days later.

What Maria learned from this conversation is, I think, that place value is an idea which has implications for the way we order, compare, and operate with numbers. She also learned that rounding can be used as a tool to estimate and that we can estimate to see if the result of an operation makes sense. From her talking about the necessity to "have a sense of the places" in order to know what to round to, I was not sure if she perceived rounding as a

way to make sense of a number. The way she later talked about rounding proves she did not.

The Conditions That Enabled Maria's Learning

The formal occasion for Maria's learning was created by the program requirement that student teachers plan a unit on place value. Faced with an assignment that she did not know how to address, Maria asked the cooperating teacher for help. However, she did not feel she could use the advice she received, since she did not know how rounding and estimation were related to place value. Maria did not choose to teach without understanding this relationship. She did not feel she could request more help from the cooperating teacher because she was "afraid since I knew [Mrs. Barnes] expected me to know much more than I did. I was afraid she would say, 'Maria!! You should know that!'" She also thought that I might be willing to help her, so she saw an alternative.

Maria brought to our conversation a lot of trust and willingness to take risks. She trusted me that I would support and help her with the best of my knowledge even when I did not answer her concerns directly. Maria approached me with a request to help her find a good activity. As a response, I asked her to think about important ideas and concepts embedded or related to the concept of place value. Although I did not seem to address her concern, she agreed to do so, even if that meant more work for her. "I saw you as a partner," she said. "I knew you would help me. And I felt it was easier for me to talk to you [than with Mrs. Barnes], and I could joke around and not feel stupid that I don't know something."

Maria's dispositions played an important role in her learning in this case. She was willing to learn, reflect ("I do not know, I have to think first why we round or estimate and how we do that"), make connections ("If I get into addition and subtraction, how would I connect that to rounding and estimation?"), work hard to make sense of ideas ("maybe place value can help children understand how big numbers are"), and make mistakes ("If I had 18-9, I would trade but not borrow").

I had a double role in this interaction. On the one hand, I felt committed to help Maria; on the other, I was trying myself to think about the same ideas and what they imply for fourth graders. I did not feel I was teaching her something that I knew. Although my knowledge of place value was stronger than hers, I did not have at my fingertips a full map of what place value involves, much less an understanding of what place value means in relation to fourth graders. In helping Maria, I approached the task the same way I would have approached it if I had to teach the subject myself. In this sense the situation we were engaged in was real and the task authentic. Although I did not have to teach the unit myself, I felt I had to help Maria plan something that she could teach real pupils in a real classroom.

By approaching the task of planning the way I did, as I would have approached it if I had to plan the unit myself, I made my own thinking in relation to planning visible to Maria and thus offered her an opportunity to see how a more experienced learner thinks about this kind of task. Our discussion might have helped her not only learn more about place value, but also realize that understanding the subject matter to be taught is more than a legitimate part of planning; it is at the heart of it.

As we co-planned this unit, I felt we were together engaged in the process of solving the problem that we had. I saw Maria the same way she saw me: as a partner, a co-experimenter, engaged in the process of solving the given task. It seems that this sense of "togetherness" (Schon, 1987) helped Maria trust me and become totally engaged in the process of figuring out what place value means and involves. Maria's efforts to learn, apparent in the questions that she asked me and herself, play an important role in pushing her learning forward. Questions like "How would I connect addition and subtraction to place value?" or "What does all this have to do with place value?" help her make connections between estimation and mathematical operations, or between rounding and place value.

Planning a Lesson

As part of the unit on place value, Maria decided to teach a lesson on "rounding." She planned the lesson by herself, but before teaching it she wanted to discuss her plans with me. She believed that since the pupils had previous experience with rounding, they knew the procedures for rounding three and four digit numbers. Her main purpose for that lesson was to remind the pupils the procedures for rounding numbers and to have them practice rounding bigger numbers. She prepared a worksheet with questions which pupils were supposed to answer during the lesson. The questions, which she took from different textbooks, asked pupils to round different numbers to the nearest ten, hundred, or thousand. I asked Maria what ideas she wanted the pupils to learn in this lesson. She said she wanted them to learn how to round. "Yes, but how to round depends on the situation for which they round." "What do you mean?!?" Maria asked. I did not know what Maria was puzzled about. I thought she knew that how to round depends on the context of the problem, and the difficulty for her was to represent this idea to her pupils. Or maybe she thought this was an obvious idea to the children since they have been already taught about rounding in previous grades? I asked her how she uses rounding. "I use the rule," she said. "Let's say I have a two-digit number. If the ones digit is 5 or greater than five, I round up; if it's less than 5, I round down." I was beginning to realize that for Maria rounding was a rule to follow, not a tool she could use to make sense of a situation. I wanted her to think about reasons for rounding, about situations in which it would make sense to round and those in which it would not, and about how the way we round depends on the situation at hand. I decided to discuss some examples with her. I started by using an example from the worksheet she prepared for the children. "The distance between the center of the earth and the center of the moon is 238857 miles." These are excerpts from our taped conversation.

I: How would you round this number, for instance?

M: You want me to round to the nearest ten? hundred? thousand?

- I: If I did not tell you how to round it, how would you round?
- M: Hmmm. That's a good question. How would I round? I guess it depends. Well, I could round it to the nearest ten--would that make sense? Probably not. I could round it to the nearest hundred. I guess it depends on my purpose. Hmmm. I do not know.
- I: If you want to remember this number, or if you want to make sense of how big this number is, how would you round it?
- M: I would probably say 240000. I see. I could also say it's almost 239000, but it's harder to remember that way.
- I: Now let's take a different example: What if I told you that there were 279 people at a party, how would you round that?
- M: I guess it does not really matter. I could say there were about 280 people or about 300 people. Ok. So I see that how I round it's a question of purpose, it depends on the problem and on the situation. Wow, I have never thought of that. That's great! And I thought I knew how to round. I want my pupils to see that! I want them to learn that how to round depends on the situation that you have.
- I: Are there any situations in which it would not be good to round, or in which we do not need to round?
- M: Oh, my god, this is so complex! Situations in which it would not be good to round? Hmmm, are there any?
- I: What about medicine? If a person is on a special diet, or needs to take a certain kind of medicine?
- M: Yeah, I see. Then you'd want to be exact. You would not want to round anything. I guess that's a situation in which every little amount counts.
- I: Right. I think that's an idea in rounding: you need to decide what counts and what not and that depends on the situation. Like if I told you about my salary, which is \$1050, I would probably say 1000 because that's what's important to me. \$50 is less important. (Laugh.)
- M: You mean you won't say 1100?!? You don't always round 5 up?!? I thought that was a rule! I thought you have to round 5 up! Even that depends on the situation? Oh, my goodness!
- I: It also depends on who is the person who is doing the rounding.
- M: What do you mean?!?
- I: Well, if I were 35, and I would talk to you about my age, I would probably round it to 30 (laugh); someone else might round it to 40.
- M: I have never thought this way about rounding! It's really great! I thought all there is to know about it is the rule. I want my children to see this. I want them to see that how you round depends on the situation in which you round, and I want them to be

able to decide when to round and how to round! I am going to change my lesson plan right now!

- I: And maybe you can also think about some situations when you do not need to round. We talked about situations when you can not round, but maybe there are also situations in which you can, but you don't need to round?

Maria indeed changed her lesson plans. Instead of teaching one lesson on rounding as a rule to follow, as she originally intended, she taught a sequence of three lessons in which she asked pupils to make decisions about how to round different numbers in context, and to think about situations in which it would necessary, appropriate, or not appropriate to round.

Planning as an Occasion for Learning Subject Matter

The purpose of this planning conversation was twofold: to help Maria better understand what rounding means and entails, and to help her think about what ideas about rounding would be important for her pupils to learn. I saw the second purpose as strongly dependent on the first. In the planning conversation that I first described, we mapped out some of the territory that was connected to place value for fourth graders. In the second, we focused on one concept, rounding, and tried to deepen our understanding of what it means, how we use it in real life situations, and what would be important to know about it.

My original purpose for this conversation was not to deepen Maria's understanding of rounding, but to help her construct an activity that would both represent the ideas embedded in the content she wanted to teach and push pupils' thinking forward. When we began our conversation, I did not know anything about what Maria's understanding of rounding was. If I had known, even a little, I probably would have not begun the conversation with a statement such as "how to round depends on the situation for which we round," because I would have thought it would not make sense to her. As it turned out, even if this statement did not make sense to her, or maybe exactly because of that, it intrigued her, and she decided to pursue its meaning. This conversation opened a window into Maria's thinking, which allowed me to learn about her understanding of rounding.

What I learned made me decide to work purposefully on enlarging her understanding of what rounding was all about.

I did not begin the conversation with a clear idea of how to help Maria learn that rounding was context-dependent. I decided to start by asking her one of the questions she prepared for her pupils. The reason I chose to do that was that I did not think that how the question was formulated on the worksheet was appropriate for pupils. The question asked pupils to round to the nearest thousand the number representing the distance between the center of the earth and the center of the moon. I thought that rounding to the nearest thousand was not appropriate in this context, and that pupils should--and are able to--decide what to round the number to. I saw making this decision as an integral part of the problem.

The question proved fruitful: Maria did not think that deciding how to round a number implies also deciding what the number should be rounded to. So she asked me to tell her what I wanted the number to be rounded to. Since I believed she could and should decide by herself, I returned the question back to her. My question, "How would you round?", pushed her to think about what would make sense in the specific situation. Even if she could not come up with an answer by herself, she began to ask herself questions that helped her figure out at least part of the answer: It depends on the purpose for which you round. I thought if I supplied her with ideas of what some of these purposes might be, she could figure out by herself what kind of rounding would make sense. And indeed, she realized that there are at least two reasonable ways of rounding the number, one of which made more sense than the other.

Deciding how to round a given number is not the only decision that needs to be made in the specific context. Whether or not we can or need to round is also an issue that stands at the heart of rounding. I wanted Maria to understand that, so I asked her to think about situations for which rounding would not be appropriate. Since Maria could not think of any such situations, I gave her an example of a person being on a special diet in which it

might be important to know the exact amounts of food the person might need. This example helped her realize that what she needed to look for in making these decisions was whether or not small amounts count in the specific situation. I thought her insight was important, but I was not sure if she also understood that she was the one to decide what can be considered a "small amount" and what not depending on the situation at hand. This was the reason why I rephrased her statement while summarizing what I thought was the main point in our discussion: "I think that's an idea in rounding: you need to decide what counts and what not and that depends on the situation."

To help her make sense of this generalization, and also to show her how I decide what counts and what does not, I brought up a new example, that of my salary, in which I would round 1050 to 1000, because in this case I do not consider \$50 to be an important amount. When I chose this example, I was not aware that Maria still held the "rule" that "5 is rounded up." My assumption was that if she understood that rounding is context-dependent, she also understood that other "rules" she held did not make sense. I could not imagine a situation in which she would hold two contradictory beliefs: that rounding is context-dependent and that any 5 is rounded up. This example, however, helped Maria become aware of her own assumption and maybe even realize, once more, its limited validity.

I also wanted Maria to realize that two different people may not necessarily round a given number in the same way, that taking the context into consideration, there is more than one legitimate way of rounding a number. One person's age was an example which quickly came to my mind as an instance in which two different people might come up with two different answers. Maria did not need to react to this specific example to let me know she understood it; her contagious joy and excitement were good proofs that she really understood what rounding was all about. Since I wanted Maria to continue to play with these ideas also later, after our conversation was over, I ended the discussion asking her to think for herself about situations in which we can, but we do not need to, round.

For Maria, thinking about rounding as context-dependent was a new and exciting idea. She also felt a sense of power coming from the fact that she could determine if and how to round in a particular situation. From her enthusiasm I understood that she really enjoyed this new sense of power she felt. Once she understood this idea, she wanted to share her discovery with both peers and pupils. In a seminar with all student teachers in the school, she responded to a student teacher who wanted to teach rounding by telling the pupils "the rule":

[About rounding] I do not want to give them [pupils] a rule. I want them to decide how to round, I want them to discover a rule. Maybe they should come up with an agreement in class, they should be able to decide whether to round up or down or maybe not to round at all. I was taught to round up, and that's what I thought the rule was, and now it occurred to me that mathematicians don't say every time we have a five we should round up! And I have never thought about that, but in fact I could use this rounding up and rounding down idea to estimate. Like if I have a list of things to round, let's say prices, I could round one up and one down to get a more accurate picture, and I never thought of that until we started this unit. (Oct 30)

In her response, Maria not only shows that she changed her way of perceiving rounding, and that she is critical of the way she learned the topic herself because it led to a limited view of what rounding was; she also takes the idea of "rounding down" further and thinks about how she could use it to estimate. The connection she made reflects her learning, because Maria came to think about rounding as a tool to estimate only a couple of days before, when we discussed how rounding and estimation might be related to place value. The pleasure and excitement she felt to discover what was for her a new mathematical idea and to make connections to other ideas made her take the risk and declare publicly: "I have never thought of that until we started this unit!" She felt the same pleasure and excitement the experienced teachers in Featherstone, Pfeiffer, and Smith's (1993) study reported to feel when they started to see connections among different mathematical topics. For her, as it was for them, being able to make connections made mathematics more meaningful and accessible.

At the end of the student teaching, Maria reflected on her learning of rounding and how that learning shaped the way she taught the concept to her pupils:

After carefully planning units and individual lessons this term, I have realized that I really took for granted that I knew the content enough to teach it. Everything was just much more complicated than I had ever imagined it would be. One of the best examples that I can give is how I learned about what rounding is. When I was in elementary school, I was told that any number from five and higher was to be rounded up. Only was it after my discussion with Neli that I realized this was not the case at all. The whole idea of my math unit was to get the students to use rounding in a context so that they could understand the idea of when, how, and why to round.

Learning Mathematics by Talking with Pupils

At the beginning of the second month of student teaching, when Maria was partially observing and partially teaching mathematics herself, she and her cooperating teacher worked together on a lesson in which they asked pupils to solve a number of word problems which involved addition and subtraction of four-digit numbers. As pupils were working on the problems, Maria told me, "I should have given an example, but I don't know what kind of example, because they are concentrating on the big numbers, they have difficulties working with these numbers and not really thinking about the problem itself."

During the time pupils worked on the problems individually or in small groups, Maria walked around trying to make sense of what they were doing. At some point she approached me saying that she could not understand what Corey was doing:

She had written 1470 and 1492 and she said the result was 22. I asked her, "Did you subtract?" But she did not really subtract. She said, "I just know it's 22." And I go, "How do you get 22 from this?" "Between 70 and 92 there is 22" she says; and I go, "OK." When you want to know how many numbers are in between those numbers, you want to know the difference." And she ended up writing 92-70."

I asked Maria what it was about Corey's solution that she did not understand. Her answer surprised me:

Well, Corey's solution is right, I know it's right because she got the right answer. But where did she take these numbers, 92 and 70, from? I was looking at what she did and I was not sure what these 70 and 92 meant. So

I asked her, "Can you write something that I can see what you did? If somebody looked at this, would they know what these numbers were?" So she said, "OK, I'll put the 14 down." But she did something I did not understand--she put a line to the numbers. It was like $14/92-14/70$. Why did she do that?

I realized Maria encountered difficulties understanding Corey's way to subtract the given numbers. She did not think that a possible strategy for subtracting 1470 from 1492 was to subtract 70 from 92. The fact that Corey's result was right intrigued Maria and kept her from telling Corey that her strategy did not work, and maybe from teaching her a strategy that "did work." What Maria did not understand was the decomposition of numbers according to the place value of their digits. As I have shown, her understanding of place value was problematic. I asked Maria what she thought Corey was doing. She answered:

I don't know. She does not quite seem to be doing subtraction. Well, she does not say two minus zero is two, nine minus seven is two, four minus four is...But wait, it doesn't really matter! Since the first two digits are the same, they don't really matter! Oh, I see! So that's what she was doing, she was subtracting 70 from 92!

To respond to Maria's expressed concern, and also to help her understand and appreciate Corey's way of explaining her thinking, I asked her what she thought Corey had in mind when "she put a line to the numbers" writing $14/92-14/70$.

Oh, yeah, I see it now. She was just trying to tell me that 14 does not matter, that's why she separated the 14. She wanted to show me that all you have to do is to subtract 70 from 92.

From talking and listening to a child and from her own reflections on what the child said, I thought, Maria learned that numbers can be decomposed according to the place value of their digits. Why did I think she learned that? A couple of days later, Maria told me that she came to the realization that she could use Corey's idea to help pupils deal with "big numbers."

Do you remember the lesson on the word problem with the four-digit numbers? Do you remember I was looking for an example to help children work with these big numbers? Not to worry about the numbers but to think about the problem instead? Well, I was thinking maybe they don't need an example. If they use Corey's strategy, they won't work with big numbers anymore. Maybe that's what I should have done: have Corey explain her idea to the class, and then have them work on these problems.

As in the previous case, when Maria's learning about what rounding is made her consider the pedagogical implications of her discovery, in this case she also thought about how she could use or could have used what she just learned in her teaching. This, of course, could not have been done, since Maria herself learned about this mathematical idea during the particular lesson by observing the child's work, discussing the idea with her, talking with me about what she noticed and not quite understood, and then reflecting some more on what she observed and talked about.

Yet it is unclear exactly what Maria did learn from talking with her pupils.

Three weeks later, Maria and other student teachers met to discuss their plans on the unit they were writing on place value. In the first meeting, the student teachers discussed what they thought the pupils in their classrooms already knew about place value. Scott, one of the student teachers in the school, presented an example of how his third graders added numbers. His example was $15+14+10+19+8$. He showed how the child added first the ones and then the tens in the following way: $5+4=9$; $9+9$ (from 19)=18; $8+8$ (from 18)=16. The sum of the ones was 16. Then the child added together the tens: 5 tens. Finally, the child added 5 tens and 16 ones together, obtaining the result of 66. Maria was confused. At first she thought what the child did was wrong. Then she added the numbers the "regular way" and was surprised to discover the result was right. But she was still not sure if the child did something wrong, and by chance got the right result, or if what he did was right. She finished the discussion by saying, "But I still don't understand it, how can you break down numbers like that?"

This incident shows that Maria did not understand that numbers can be decomposed according to the place value of their digits. She did not realize that one of the things 15

stands for is $10+5$ and that we can use that when we add. She knew the algorithm for performing addition, but she did not know why it worked. It might also be that, in a larger context, Maria, like Connie Marsh (Featherstone, Pfeiffer, & Smith, 1993), did not see that there is a "huge world of number" and perceived that world in a narrow way. What Connie Marsh described as a powerful learning experience might be the thing that Maria was missing in her learning about numbers:

What has been a real awakening for me, I think, as much as anything, is the relationship in number. I really never saw much relationship before. I mean, addition's addition and carrying was related to addition and borrowing was related to subtraction. But now the world of number is really exciting for me. [...] And I always thought 10 was $6+4$ and that there were certain facts... But it's a huge world of 10 out there, it's a whole world of all different numbers and I always looked at it as a very narrow thing (p. 14).

The contradiction between these two stories is compelling. The first story (the one that talks about Maria learning to look at the last two digits of a four digit number in order to subtract) shows that Maria learned that numbers can be decomposed; the second (how to add numbers) that she did not. How shall we understand these two apparently contradictory examples? Did Maria learn anything about mathematics? If so, what? In the next section I will analyze what Maria learned about place value from talking to pupils, and what contributed to that learning.

Talking with Pupils as an Occasion for Learning Mathematics

In the first example, Maria is intrigued by the student's mathematical idea--she tries to follow her thinking and to make sense of what the child says. The fact that the child had the "right answer" played a crucial role in the process of her learning. It made her want to understand the child's thinking, discuss what she did not understand, and reflect on what she heard the child was saying. All these led her to learn a new idea about mathematics. What she learned, however, remains a question. She did not learn, as I originally thought, that numbers can be decomposed. Maybe what she learned depended on the context in which she learned it. Maybe she just learned that in the subtraction of two four-digit

numbers, if the first two digits of the two numbers are the same, all you need is to think about the last two digits of the given numbers, and she did not see the principle behind this idea.

In the second example, Maria is also intrigued by the fact that the child had the "right answer." As in the previous case, she tries to make sense of the child's answer. However, she fails to connect this example with what happened a few weeks before. She also does not pursue the issue any further. She asks the questions, and since the questions are neither answered nor picked up by somebody else, the issue is dropped.

If we see listening to pupils' ideas as an important source for teacher's learning of subject matter (Featherstone, Pfeiffer & Smith, 1993), these two examples raise important issues about what can be learned from pupils' ideas and under which conditions this learning can occur. It seems that Maria learned that Corey's idea was right, that if you subtract 1470 from 1492, you could obtain the same result by subtracting 70 from 92. She even thought about the pedagogical implications of this idea; however, she did not think about the principle behind this idea. Therefore, she could not see a connection between this example and the way another child added a list of two-digit numbers. It also seems that in the second case, she does not push her thinking far enough to think about why the pupil's solution might be right. She is intrigued, so she asks questions about why that way of adding numbers might make sense, but she does not come up with an answer. She does not pursue the idea in a different occasion, as she did for instance, when she made a connection between rounding and estimation, or when she thought about how she could use Corey's idea in her teaching. The question is why, or what impeded Maria's learning in this case? Perhaps if she had further pursued the idea, she would have seen that the child's way of adding numbers was correct because of what the idea of place value implies, and maybe she would have connected this example to Corey's idea of subtracting numbers, which would have led her to a deeper understanding of the concept of place value.

One possible explanation is that Maria was more interested to make sense of an idea generated by the pupils in her classroom than by the pupils in a different classroom. This explanation is supported by the findings in the research conducted by Featherstone, Smith, Beasley, Corbin, & Shank (1993) with experienced teachers. "A teacher has a special relationship with ideas generated by her own students in her own classroom. This relationship includes a sense of pride and curiosity, and is different and more intense than her relationship with the ideas generated elsewhere" (p. 48). Thus, it is possible that Maria did not feel a "special relationship" with the idea presented by another student teacher, so she did not pursue it in depth, therefore missing an opportunity to get a deeper insight into the idea of place value.

Another plausible explanation of why Maria did not further pursue the example brought by Scott is the absence of an audience, or an active partner willing to pursue the idea with her until she makes sense of it. When Maria tried to make sense of Corey's subtraction, I was there to support her learning. I could ask questions, focus on the child's strategy until she understood it, help her in the process. When Maria thought about the pedagogical implications of Corey's strategy, she knew that I was a person she could share her discovery with. When Scott presented his example, and Maria struggled to make sense of it, neither the student teachers present in the room nor I picked up on her questions. I did not intervene because my help was not solicited, and also because my purpose for being there was to study the conversation the student teachers were having. Either because the other student teachers were not interested in the child's strategy, or because they totally understood why it made sense to add numbers that way, they did not support Maria in her attempts to understand. Without the support that she needed, Maria failed to make the connections that might have helped her see a new aspect of place value.

What Maria Learned in Relation to Mathematics

Student teaching was an important occasion for Maria to gain new mathematical understandings and ideas about the nature of mathematics. She learned, for instance, that estimation and place value are mathematically connected, and that place value is an idea which has implications for the way we order, compare, and operate with numbers. She also learned that rounding is context-dependent, not a rule to follow; therefore, understanding rounding entails an understanding of the context and purpose of rounding. Maria also learned to see mathematics not as a body of rules and algorithms, but as something that people, including herself and the pupils she teaches, can discover and understand.

We may ask ourselves why Maria had not possessed these understandings before coming to student teaching. Subject matter knowledge is not what we normally expect preservice teachers to acquire during student teaching. Rather, we expect them to come to student teaching with the kind of subject matter knowledge that would enable them to "hear" and make sense of pupils' mathematical ideas. However, Maria's own experience as a learner of mathematics in elementary and secondary school did not promote conceptual understanding. As described in chapter II, this is how she described her experiences:

Math always was the same. We sat down and we reviewed the homework that we had. We might go up and do a couple of problems on the board. Our teacher would call up two or three [students] to do problems 1-2-3 on the board, and then he would go up and say if it was right or wrong, and then talk about what we were to learn for that day, and we worked on it quietly. When we were finished, we took our homework and do it. And that was every day--the same pattern.

How did Maria manage to succeed as a student in this kind of environment?

I could do the problems because they were all the same pattern. Once I got into the pattern, or worked a certain set of problems, I could do it, they were all the same and I could see the pattern. But sometimes I would get them wrong, if I missed a step--that's how I worked--doing the exact same thing. But I never understood anything at all. (Int Nov)

With this kind of experience with mathematics learning, there is little wonder that her own mathematical knowledge was limited and rule-bounded. Since the courses she took in college did not directly address elementary school topics, her understanding of mathematics was left untouched, since she was an elementary student herself. Thus, she approached student teaching with a mathematical understanding which was not very different from her pupils'.

Through teaching, Maria began to develop an understanding of what kind of subject matter knowledge she needed to be able to teach mathematics conceptually, at the same time realizing she did not have this kind of understanding. However, while working on her own mathematical understanding, Maria learned how it feels to know something in depth, thus acquiring a standard of what to strive for.

Maria saw student teaching as a time when she was really engaged in "learning to teach mathematics and learning to think about mathematics" (Int Nov). By "learning to think about mathematics," she means learning that mathematics is not only rules and algorithms, but also concepts and ideas, and that knowing mathematics means understanding a mathematical concept or idea, more than memorizing rules and algorithms:

I feel that I know something raw, but when I look deep into it, I really don't?! As far as rounding, I knew how to round, at least I thought I knew, but there is a lot more to it than that.

The major thing that Maria says she learned about teaching mathematics is her realization that

I have to dig deep into something and I have to think about everything before teaching it and not to assume that I know that. I learned it, I went to school and learned it, I had it all my life and I should know that, but that's not necessarily true.

From this realization she concludes about her own experience as a student and about her teaching:

You can't, you can't just teach something and give students an algorithm. Without really thinking about it or playing around with it, they can't understand it.

As for herself, she says, "That's what I feel I am doing now--I feel I am relearning everything again!" This painful confession shows us that together with Maria's learning of and about mathematics came also the realization that she did not know enough mathematics to be able to teach in a conceptual way. When Maria talked about her realization, I felt a mixture of sadness and hope in her voice. Sadness because it was hard for her to recognize that as an almost teacher, she did not know the mathematics she was supposed to teach her fourth graders. Hope because "now I can work on these. If I had not known there are so many things I don't know, I would have not been able to work on them.

Maria's learning in relation to mathematics during student teaching raises questions about how the context of teaching specific subject matter to a particular group of pupils might contribute to one's own learning of the subject as well as to understanding what it means to know it enough to be able to teach pupils in conceptual ways. We may ask why teaching can be an appropriate context for learning subject matter, how this learning might happen, or under what circumstances this learning might best occur. These questions are the subject of the next section.

Why the Context of Teaching Enables Learning Subject Matter

Maria's case teaches us that some depth of mathematical knowledge, knowledge about one's own mathematical understanding, and insights into what it means to know mathematics for teaching, might be gained in the context of teaching a specific group of children particular mathematical concepts and ideas. The need to plan units and lessons in depth for the particular pupils in the classroom and to make sense of their mathematical ideas pushed Maria to look carefully at what there was to be learned and what pupils were actually learning or understanding. Involvement with specific subject matter for teaching through planning or conversing with pupils about their mathematical ideas contributed to Maria's realization that she did not know mathematics well enough to teach. At the same

time, these opportunities helped Maria develop a feeling for what it means to really understand mathematics in depth.

We may ask not only why student teaching is an appropriate occasion for learning subject matter, but also why this kind of knowledge and understanding is not to be gained prior to student teaching, especially in the methods courses where novices may have a chance to plan units and lessons, to watch videotapes of children working on mathematics, or observe teachers working in real classrooms. Although valuable things can be learned in methods courses, there are certain qualities that novices can begin to develop only through teaching: responsibility for the intellectual growth of the particular group of pupils and a special bond or relationship with the children in one's classroom which includes caring, curiosity, enthusiasm and pride in pupils' ideas and successes.

Feeling responsible for pupils' learning prevented Maria from trying to teach content she felt she did not really understand. A mixture of caring and curiosity for Corey's way of thinking pushed her to work hard to make sense of the child's solution, and this in turn led Maria to a new understanding of place value. Enthusiasm and pride for Corey's solution made Maria think of a mathematical way to connect this solution to other pupils' ideas. Maria did not have the same kind of relationship with pupils from other classrooms: When Scott presented an example brought up by a child from his class, Maria made an attempt to understand it, but she did not pursue it with the same tenacity that she brought to conversations she had about her own pupils' thinking.

Some of these attributes, like caring and curiosity, were already part of Maria's personal qualities before coming to student teaching; however, in the context of teaching, Maria reshaped her natural dispositions and qualities, enriching their meaning with new dimensions which were also more appropriate to teaching. For instance, a general and quite amorphous sense of caring developed into caring for pupils' intellectual growth, and interest for what pupils have to say developed into curiosity for pupils' mathematical ideas. How these qualities changed, enabling Maria to develop skills that allowed her to relate to

pupils in ways that pushed their and her learning further, will be discussed in greater detail in the next two chapters.

Even if student teaching is a somewhat sheltered situation, it still can engage the novices in authentic teaching activities. The authenticity of the situation has to do with the need to plan meaningful units and lessons and to teach those to a real group of children in real classrooms. The purpose of Maria's task, to plan a unit on place value for particular fourth graders, was different from the purpose of any university assignment she was given before. In this case, she did not have to plan for imaginary pupils, to accomplish a task only for the sake of her own learning. Rather, she was engaged in the same kind of activity in which that teachers in real classrooms are engaged: to plan and teach specific mathematical content to a particular group of pupils. While struggling to plan mathematically sound activities or listen and make sense of pupils' ideas, Maria realized her own knowledge of mathematics was not strong enough and she became motivated to work hard to enrich her understanding of the topics taught.

In the situation of listening to pupils and making sense of their ideas, Maria realized her own understanding of place value was not enabling her to make sense of their solutions, much less to push their thinking forward. However, gaining a better mathematical understanding and using it in teaching helped Maria develop a feeling for what it means to understand a mathematical idea in depth, while giving her a standard to evaluate for herself her own knowledge and understandings.

Maria's learning of and about mathematics would not be possible in any kind of student teaching situation. The cooperating teacher she was assigned to did not have the time or the ability to support Maria in her learning of mathematics. She might have assumed Maria understood place value much better than she actually did, or she might have not been aware of the depth of knowledge needed in teaching. Even if she had realized that Maria needed to strengthen her mathematical understanding, I do not think she had the time or the skill to offer Maria the intensive support she needed. Other teachers' busy schedules

in school could not allow them to offer more than sporadic support, either. Fortunately, the cooperating teacher was supportive of my contributing to Maria's learning.

Our conversations provided Maria with a rare opportunity to discuss the actual mathematics she was supposed to teach. Analyzing the mathematical territory covered by the idea of place value, looking closely at one concept or idea, or trying to understand pupils' solutions, we focused our conversations on the subject matter in question. These conversations offered Maria an opportunity to look deeply and with more mature eyes at mathematical content she had not thought about since she was an elementary student herself. Most of Maria's learning of and about mathematics was situated in our sustained conversations about the content for teaching.

Prospective teachers' learning of and about subject matter does not occur simply because they are in the context of teaching. Novices need the support of others to focus their attention on issues related to subject matter understanding and to create occasions to enable that learning. The knowledge we discussed and constructed in our conversations could not be gained elsewhere; Maria could not gain it by taking more math classes, reading textbooks or teacher's guides, or simply spending more time with pupils. She needed opportunities to challenge her mathematical knowledge and beliefs and construct for herself new understandings.

Even if my understanding of place value was richer and more flexible than Maria's, I was not teaching her things I already knew. Rather, as we talked, we both gained a sense of what each of us knew and believed, while the questions we asked of ourselves and of each other pushed us to reconceptualize these understandings.

CHAPTER IV

LEARNING ABOUT PUPILS' THINKING

I want to listen to my students. They need to be listened to to even want to learn. (Int Sept)

[I used to believe that] a lot of students have misconceptions, but if they were taught right, if their teachers had taught them well, they would not have these misconceptions. Now I think it is important to know that there are going to be a lot of misconceptions out there, no matter how perfectly you teach them, or how perfectly you think you taught something and there are going to be a lot of kids out there who are not going to understand it. And you have to find a way to figure out who does not understand what and then or later supplement it. I did not know, well, I thought if you taught it right the first time, that's it. (Int Nov)

One of the most important things in learning to teach is to have experience with kids and kids' thinking, and talking a lot with them. Given all the classes in the world, and pretend that you are a 7 year old, it doesn't mean the same, and it's going to be different every single time, because a seven year old is not going to be the same as another seven year old...I used to find it scary to probe deeper into their thoughts because I could never be sure of what they were going to say. Now, I am intrigued by their thinking process and am positive that not only myself and the child are benefiting, but the other children in the classroom as well. (Int Dec)

These three excerpts from three different interviews with Maria reflect the changes that occurred in her thinking about the role that listening to pupils plays in their and her own learning. In September, Maria talked about listening to pupils as a way to motivate them. In November, she talked about listening to pupils as a way to find out what they think in order for the teacher to be able to supplement the teaching for the pupils who have misconceptions. She also talked about her becoming aware of her own assumptions and her realization that even under the best of conditions pupils will construct knowledge in unexpected ways. In the last interview, in December, Maria talked about being intrigued by the pupils' thinking, and seeing pupils' thinking as a potential source of learning for the students in her class as well as herself. In this chapter, I will explore what Maria learned about pupils' thinking that prompted her to change her views, how she learned that, and what implications it had for the way she taught the pupils in her classroom.

The chapter is divided into three main sections, each organized around an event that contributed in some important ways to Maria's learning about pupils' thinking. In each section I first discuss Maria's ideas about pupils' thinking prior to the particular event that contributed to her learning, as represented in interviews, in conversations with me, or in her actions. Then I present the learning opportunity, analyzing what Maria could and did learn from it. In the last part of each section, I discuss how each of these learning opportunities constituted occasions for Maria's learning in relation to pupils' thinking. The first section analyzes Maria's learning about pupils' thinking from working with individual children in the classroom at the beginning of student teaching, in September. The second section focuses on Maria's learning from her first whole-class teaching experience, in October. The third and last section discusses Maria's learning about pupils' thinking from observing a demonstration lesson taught to her pupils by a mathematics teacher educator, at the end of October. The chapter ends with a discussion about what Maria learned in relation to pupils' thinking during student teaching, and how different opportunities contributed to that learning. I will begin the discussion by presenting Maria's ideas about "listening to pupils" at the beginning of student teaching, as represented in the autobiographical interview I conducted in September. This will help us understand how her ideas have changed throughout the semester.

Maria's Relation to Pupils' Thinking at the Beginning of Student Teaching

In September, at the beginning of student teaching, when Maria talked about her vision of good teaching, she mentioned her commitment to listen to pupils. She saw listening to pupils as a way to motivate them ("they need to be listened to to even want to learn"), rather than as a way for her to learn about their thinking or as a potential source for her and other students in the classroom to learn subject matter. Maria thought she could listen to her pupils because she believed she was a "good listener," a quality recognized also by her friends who often came to her to talk about their problems. She did not

differentiate between the ways she listened to her friends when they talked about their problems and the ways she would listen to her future pupils.

At the same time, Maria did make a connection between listening to students' ideas and learning mathematics. In the same interview, asked about her preparation for teaching, Maria mentioned the math methods course as being especially important to her because there she experienced a way of teaching and learning that was totally different from what she had experienced before. She liked this class because, for the first time, she saw a math teacher "who was not interested whether the answer was right or wrong, but in the way we thought about the problem." In this case it seems that what was important for Maria was the fact that the teacher was interested in the way students thought, and listened to them to find out how they thought.

I do not know why Maria thought it was important for the teacher to know how she and other students thought about the problems in question. I did not ask. Neither did I ask what she thought the teacher did with what she heard the students were saying. Now I wish I had. It might be that Maria thought it was important that the teacher listen to how students think about the problems because that motivates them. But it might also be that Maria thought about "listening to students" in two different ways depending on the situation: the situation in which she was a student, and the situation in which she was the teacher. As a learner, she came to appreciate a learning environment in which students had opportunities to talk about their thinking in relation to mathematics. As a teacher, she thought that it was important to "listen to students," but she saw "listening to students" as a way to motivate them, rather than to help them gain a conceptual understanding of mathematics.

This finding is consistent with the findings obtained by Schram, Wilcox, Lanier, and Lappan (1988), who concluded that a 10-week course can succeed in challenging preservice teachers' beliefs about how mathematics is learned, but it did not change many of the beliefs the prospective teachers held about teaching mathematics. In any case, it

seems that in September, at the beginning of student teaching, Maria did not have a clear idea in mind why in teaching mathematics it would be important to listen to students, how to listen to them, and what to do with what she hears them say. As we will see, Maria's views about "listening to pupils" changed while working with individual children in her cooperating teacher's classroom.

Learning about Pupils' Thinking by Working with Individual Children

At the beginning of the semester, Ms. Barnes taught a lesson on "problem solving." She started the lesson by writing a problem on the board: "There are 66 wheels in the parking lot. Some are on cars, some are on tricycles. How many car-tricycle combinations can you think of that equal 66?" Ms. Barnes asked the pupils to work on this problem by themselves or in small groups using beans, tiles, and popcorn.

A couple of minutes later, Gary, one of the children in the class, asked what the word "combinations" means. Ms. Barnes replied, "You can use both cars and tricycles, but the sum has to be 66." Gary looked confused, but he continued to work on the problem, as did everybody else. Half an hour later, the teacher asked the students to stop working on the problem and to write in their notebooks how they worked on that problem. "For example," she said, "you can write, 'I used beans,' 'I used tiles,' 'I asked my neighbor,' 'I did it in my head,' and so on." When the students finished their writing, the math lesson was over.

When Mrs. Barnes introduced the problem, Maria was checking papers at her desk. As kids started working on the problem, she toured the class for a couple of minutes, and then returned to her desk and continued to work on students' papers. When Gary asked his question about what the word "combinations" means, Maria became very attentive to what was going on in the classroom. She left her papers and walked around to see what kids were doing with this problem, spending a couple of minutes with each group. After a while, she reached Gary, who was working alone. Gary drew a picture to help him solve

the problem. His picture had three big circles. In each circle he drew 4 long lines, 3 short lines, and 2 dots. "I cannot do it, it will never be 66," he said to Maria. Maria asked him to explain his drawing. He did, but she did not understand his explanation. She asked Ms. Barnes to help Gary, but she was working with a different child. Maria asked me to try to understand what Gary was doing. When Gary finished his explanation, Maria pushed the manipulatives toward him and asked him to try to find a solution using them. She stayed around him most of the time he was working on the problem.

Listening to Children as an Occasion for Learning about Pupils' Thinking

When Mrs. Barnes posed the problem to the pupils, Maria did not see that necessarily as an occasion for her to learn about the ways pupils think. She toured the class at first, as she later told me, because she was more interested in checking that pupils were working on the task than how they approached the task. When she thought that more or less all were "on task," she returned to her desk and continued to do the work she had previously started. Gary's question surprised her. She did not think that some of the children might not know the meaning of the word "combinations." She sensed that children were having difficulties working on the problem. Ms. Barnes's answer that for combinations "you can use both cars and tricycles, but the sum has to be 66" did not trouble Maria. She did not ask herself what sense it made to the children. She just realized that at least one child "had problems," and since she was sensitive to children's having difficulties, she decided to walk around to see what she could do.

At the end of the lesson, I asked Maria what, if anything, was striking to her in this lesson. "Gary, definitely Gary," she said. She was surprised not so much that "Gary was confused," but that she did not understand what his confusion was. She felt at a loss, because she did not understand what Gary was trying to explain to her, and she did not know how to help him solve the problem. She asked me what I thought Gary was saying. I told her what I understood: Gary drew 3 big circles. Each circle represented one

"combination." Each "combination" was composed of 4 car's wheels (4 long lines), 3 tricycle's wheels (3 short lines), a car and a tricycle. This was how he interpreted Ms. Barnes's answer, "you can use both cars and tricycles, but the sum (of the wheels) has to be 66." In his view, each combination included 9 things, and since he realized that 66 is not divisible by 9, he said, "It will never be 66."

Even if this explanation made sense to Maria, it still surprised her. She did not expect a child to interpret in such a way what seemed to her to be a straightforward sentence of a teacher. As she said almost two months later, she believed at that time that if children were taught right, they would not have misconceptions. From her point of view, Ms. Barnes's explanation of the word "combination" was right, and she did not expect it to cause confusion. A couple of days later, when pupils' solutions for the wheels problem were discussed, Maria told me that she realized Ms. Barnes's explanation of the meaning of the word "combinations" could make sense only to those children who had already known its meaning.

This lesson was an important event in Maria's learning about pupils' thinking. First, she learned that being a good listener when a child explains his ideas in mathematics is not the same as listening to a friend talking about his or her problems. Understanding how a child makes sense of an idea requires different skills from those involved in understanding a friend's problem. She learned that by trying to listen to Gary in the same way she would listen to a friend and not understanding what he was saying. As she commented later in her journal, feeling unable to understand Gary, as well as realizing how far my interpretation was from anything she could guess, made her think that listening to pupils is something that she needs to learn and not something that she already knows how to do.

Second, I believe the discussions with and about Gary planted a seed of disbelief in her heart. Although these discussions did not make her aware of the belief she held that if children were taught right they would not have misconceptions (at least she did not talk

about it at that time), they made her think about how children interpret what they hear. They also caused her to think about how helpful Ms. Barnes's explanation was. This is the reason why, I think, she said a couple of days later that only pupils who had already known the meaning of the word "combinations" could make sense of Mrs. Barnes's explanation.

Maria's learning was facilitated by the surprise she felt during the lesson, first, when Gary asked for the meaning of the word "combination," second, when she did not understand his explanation, and third, when she realized that my explanation was far from anything she could think of. Her personal qualities played an important role in her learning from this situation. Her concern about the individual child and her desire to understand what he was saying made her focus on the child and think about alternatives that could help her make sense of the child's thinking, like asking the child to explain his thinking and asking both Mrs. Barnes's and me to help her in this endeavor. Gary's answer continued to haunt her during the next couple of days. It helped her look for connections between Ms. Barnes's response and his thinking. It made her think in depth about what Ms. Barnes's "explanation" might have meant to other children in the class. It made her realize that Ms. Barnes's answer could make sense only to those children who have already known what "combinations" meant.

It is not clear why Maria thought it was important to understand Gary's explanation. We do not know what she would have done had she understood Gary's explanation. Since she did not understand it, she pushed the manipulatives toward him, hoping that manipulatives could help him solve the problem. It would have been useful to know how she would have used the child's explanation to support his learning, if at all. It might be the case that she listened to the child because she thought that would motivate him, or that she did not think about any reasons at all. We can learn about Maria's views of reasons for listening to pupils' thinking from interviews and from observations that took place only about a month later.

It is important to note that the cooperating teacher played only a marginal role in helping Maria learn about pupils' thinking from this lesson. Of course, she was the one who gave the assignment to the pupils. By doing so, she created rich opportunities for learning not only for the pupils, but also for Maria. However, when Maria encountered difficulties understanding Gary, she could not make herself available to assist either Gary or Maria in their attempts to make sense. Moreover, at the end of the lesson, she asked the pupils to write in their notebooks about how they thought about the problem. Their writing could have been a vehicle through which both Ms. Barnes and Maria could have gotten inside students' thinking and learn about the possibly many and different ways in which pupils worked on the problem. But Mrs. Barnes immediately minimized the assignment by explaining, "For example, you can write

'I used beans,' 'I used tiles,' 'I asked my neighbor,' 'I did it in my head,' and so on." By doing so, Mrs. Barnes limited the insights she and Maria could gain about pupils' thinking.

A Month Later: Maria's Own Teaching of Mathematics

A month after she began to student teach, at the beginning of October, Maria taught her first lesson in mathematics. The purpose of the lesson was to integrate what they talked about in social studies, about Columbus, with what they were doing in math. The cooperating teacher helped her plan the lesson, which they called "Columbus Math." In preparation for that lesson, Ms. Barnes gave Maria a couple of mathematical word problems which she found in a journal for elementary teachers. The plan was to have the pupils work on these problems. When I came in, a couple of minutes before the beginning of the lesson, Maria approached me, asking me to look at her plans. She wanted to write four problems on the board, to model how to solve the first problem (by subtracting the smaller number from the bigger number), and then to ask students to solve the other problems in a similar way. "Why do you want to give them a model for solving the

problems?", I asked. "To make it easier for them to do it [to solve the problems]," Maria answered. "And why do you want to tell them to subtract the smaller number from the bigger number?" "That's how you do it, don't you?" I realized that the only way she could think about the problem was the way she would solve it herself, and she thought that was the only possible way. Since she was about to begin teaching the lesson, I could not discuss this idea with her further. I just told her, "Don't do that. Just give them the problem and let them work on it at least for a while." "Why?" Maria asked. "Let's use this problem as an occasion to learn about how kids think," I answered. As Maria told me later, she was very skeptical about there being more than one right way to think about the problem, and she did not really believe that there was something for her to learn there. She agreed, however, to write the problem on the board and give pupils time to work on it. The cooperating teacher was not part of this discussion, but she did not have any objections against Maria's taking an approach which was different from what they had originally planned.

Maria started the lesson by writing a statement on the board, "Christopher Columbus was born in 1451." She expected the students to refer to this statement every time they solved a new problem. The first problem she wrote on the board was, "Christopher's family moved to a house near Porta St. Andrea in 1455. How old was Christopher when they moved?" She asked the pupils to copy the problem from the board and to solve it. While students were working on the problem, she walked around to see what they were doing. Later, she told me that the purpose for her walking around was in fact to see who had difficulties with the problem, who needed an explanation of how to do the problem.

Maria was extremely surprised to see that only two or three pupils in the whole class thought about the problem the same way she did. She was surprised and excited to discover what was for her a new and exciting world: that of her pupils' thinking. "Look," she said to me in the middle of the lesson, "they are doing it by addition, and I never

thought you could solve it by addition!" We both walked around and took notes about the strategies pupils were using. During the lesson, Maria told me to look at what some of the pupils were doing, because their strategies did not make sense to her. I took notes about what the pupils were doing and what Maria was noticing. Part of this time the cooperating teacher observed at her desk and for part of it she walked around and looked at what pupils were doing. When the lesson was over, Ms. Barnes initiated a conversation with Maria about the lesson. The discussion focused on how to change written text of problems to make easier for the pupils to understand what they are supposed to do. Maria's concerns about pupils' strategies she did not understand were not addressed in that conversation.

Conversation about the Lesson

I started the discussion by asking Maria if there was anything about the kids' strategies that she wanted us to talk about. She said, "Yeah, Sarah. She was explaining to me how she worked the fourth problem out, and I did not understand at all. I looked back how she solved some other problems before that one and I had her explain, and I think she did it in such a different way!"

Maria was referring to the following problem: "Christopher reached America in 1492. How old was he when he arrived?" In solving the problems, Maria expected the pupils to refer back to the statement she had written first, saying that Columbus was born in 1451. To find out how old Christopher was when he reached America, she expected the pupils to subtract the year in which he was born from the year in which he reached America, namely 1451 from 1492. Sarah, a student in Ms. Barnes's class, found out Columbus's age by subtracting first 1468 from 1492, and then adding the result to 17. Maria did not know where these numbers were coming from. Her first thought was that the girl's solution was wrong, but the fact that Sarah's result was right (41) impeded her from concluding that. Maria asked Sarah to explain her solution.

In solving the problem in question, Sarah referred to one of the problems she had previously solved. That problem requested her to find the year in which Christopher's brother, Giacomo, was born, knowing that he was 17 years younger than his brother. Thus, in finding out how old was Columbus when he reached America, Sarah first found how old his brother was in that year, and then she added 17 to the result, knowing that his brother was 17 years younger than Columbus.

Maria was surprised to see that a student could think in such a way. She was very excited about Sarah's solution. Her excitement was a result of the discovery of a way of thinking she did not expect, her realizing that Sarah solved the problem correctly, and a happiness she felt as she understood the child's solution. She was proud that she created an opportunity for a student in her class, who was not even considered good at math, to find an unusual way to solve a problem and to explain the solution to her. She was enthusiastic about her learning a new way of solving a problem from a child. Maria wanted to share her discovery with me and with anybody willing to listen:

Sara, instead of referring back to when Christopher was born, she referred back to the second problem, and found out the year in which his brother was born, and went from there. So she started with 1468, and she said Christopher reached America in 1492, when his brother was 24 years old. But Columbus was older than his brother, he was 17 years older, so she added 17 to 24 and she got 41. And I didn't understand at first because I didn't understand where she was getting these numbers from. And then she had explained to me and I couldn't believe it, she did it totally different than anybody else.

But Sarah was not the only person who solved the problems in ways Maria did not expect: "They were working on the problems a lot differently than I expected them to work. Like Justin who was adding and not subtracting and I never thought you could solve these problems by addition." Maria was referring to the way Justin and others approached the first problem. To find out how old Christopher Columbus was when his family moved in 1455, they added 4 to the year he was born: $1451+4=1455$. "Or Sam,

who figured out how old Columbus was when he reached America by using the year when his family moved to Savana, which she got from another problem."

Maria did not need or want my help to make sense of pupils' strategies: She knew to ask children to explain to her their solutions and she could understand what they said. She needed me to share her pride and joy about what was for her a new and exciting world: that of her pupils' thinking:

I want the whole class to see what they did, and I want these kids to go up and explain to everybody what they did. I want Sarah to go up and explain just as she was explaining that to me...I want to have some of them come up and discuss their strategies, so that they can see there are many ways of doing this kind of problems!

At the end of the conversation, I asked Maria if she realized why I did not let her give pupils a model of "how to solve the problems." "Yes," she said, "because you can solve the problems in so many different ways, and more!"

Teaching and Talking about Teaching as an Occasion for

Learning about Pupils' Thinking

First, this lesson helped Maria learn that, if given the opportunity, pupils do think in many and unexpected ways. Although she heard this phrase before, this situation helped her see some of the ways in which real pupils in her real classroom think. Seeing so many correct strategies made a big impression on her, especially since at the beginning of the lesson she could not think of more than one correct strategy, the one she would use herself. In this case, her pupils also taught her some new ways to solve the problems herself, and gave her an insight about the potential of the task she used.

Second, this lesson shows that Maria began to learn how to listen to children. At first she did not understand Sarah's strategy, but when Sarah explained it to her, she understood and appreciated it. It seems that the fact that Sarah had the "right answer"

played an important role in Maria learning: it intrigued her and it stimulated her to listen carefully to Sarah's explanation.

Although I did not play an active role during the lesson, I believe my presence constituted a context for Maria learning. She knew that I was interested in learning about pupils' thinking, that she could always have a conversation with me about that, and that I would share her excitement about the discoveries she makes. It might be that knowing this was a stimulating factor for her to think harder and try to make sense of her students' strategies.

The trust she accorded to me was extremely significant in creating the opportunity for her learning. It made her take the risk and change the plans that she had prepared in advance just minutes before the beginning of the lesson. Although I did not give a thorough explanation of why it would not be a good idea to go with her original plans--nor did I discuss with her what she should do instead--yet Maria still trusted me and started the lesson without having a clear direction of where she wanted to go. When I asked Maria after the lesson why she decided to do so, she said that she knew that if something went wrong either I or the cooperating teacher would "jump right in and save the lesson." In reflecting about this lesson, she wrote, "I realized I can not expect to learn anything unless I try it."

The role played by the cooperating teacher both contributed to and inhibited Maria's learning on this occasion. On the one hand, Ms. Barnes helped Maria plan a lesson based on giving pupils a model to follow. Such a lesson would not have supported the pupils' learning of mathematics or Maria's efforts to learn about pupils' thinking. On the other hand, she did not object to my interfering with the plan and changing it just before the beginning of the lesson. Moreover, Maria knew that she could still count on Ms. Barnes to save the lesson if it went awry. Maria also trusted that my giving advice and her following it would not affect her future relationships with the cooperating teacher.

In the discussion that followed the lesson, Ms. Barnes did not address Maria's concern regarding pupils' strategies. The focus of the conversation she initiated was on how to change the text of the problems "to make it easier for the kids." Making the task easier for the children was also Maria's concern when she planned the lesson, which made her decide to give pupils a model to follow. Such a focus in teaching can contribute, among other things, to making the lesson go smoothly, to accomplishing more things during one lesson, but not to extending pupils' mathematical ideas. By concentrating on this goal, Mrs. Barnes failed to help Maria understand pupils' strategies and did not contribute to her thinking about why it might be important to listen to pupils.

Whenever Maria learned something new, she felt the urge to share her learning with the pupils in her class. The pride and excitement she felt realizing how many ways the problems could be solved, and that the pupils in her room could think about all these ways, made her want to share her discovery with "everybody." Again, the fact that most of the strategies pupils used were correct played a significant role in Maria's decision. More than thinking about all pupils' learning from these strategies, she was thinking about giving pupils a chance to feel excited and proud and of the many ways they could think about, especially those who solved in ways she did not expect. In the next section, I will show how Maria thought about what to do with what she learns from listening to pupils when their strategies are incorrect.

Maria's Thinking about Pupils' Misconceptions

At the end of October, the cooperating teacher gave a test to assess pupils' knowledge on addition and subtraction facts. While pupils were taking the test, Maria noticed that many students had incorrect results for the problems that involved subtraction with regrouping. At the end of the lesson, she talked about her observation with the cooperating teacher. "Yeah, I noticed that," Mrs. Barnes said. "They forgot their addition and subtraction facts. They are also doing it a lot slower than they were used to." Mrs.

Barnes thought pupils needed to be given more tests more often in order for them to regain the skills that they used to have. Maria thought differently, "If they do not understand borrowing, how is taking more tests going to help them? How is that going to teach them to subtract?" she said to me. She suggested to the cooperating teacher that they find out which kids have misconceptions about subtraction with regrouping and that she would take these kids aside and teach them "how to subtract." The cooperating teacher agreed. When I asked if she did not think these kids had been taught how to subtract before, Maria said, "Yeah, they had, but I do not think they were taught right." Maria could not think at that moment about what it would mean to teach them right or how she would do it other than using some kind of manipulatives. However, she decided to go home and think about a way to do it.

In the evening, Maria called me saying that she thought about how to teach these kids and realized that she could not teach them without understanding what misconceptions they had. She requested my help in understanding how they thought about subtraction as it was reflected in the test they took. I agreed. Analyzing their tests, Maria came up with a list of rules pupils used to solve the subtraction problems in the test. However, she did not think about using these rules in planning a lesson on subtraction. For her, the purpose of learning about pupils' thinking in this case was more to know who does not understand what. She did not think how she could use what she learned about her pupils' strategies in her teaching, other than for identifying the pupils who were having misconceptions. The way she intended to deal with the difficulties students were having was to take aside those students and to supplement the material, meaning to teach these children a lesson on subtraction with borrowing using the manipulatives, without taking into consideration what they had already known or understood about subtraction.

Maria did consider teaching a lesson on subtraction to the whole class, but she did not think that was a good idea because some kids did not need it. When I asked her about

whether she could orchestrate a whole class discussion about the rules and strategies that pupils used in solving the problems, she said,

Yeah, I guess I could. But I don't think that's a good idea. How would those kids feel about being on spot? What if other people would make fun of them? What if the other children in the room will adopt some of these wrong strategies? I'd better teach them [only the pupils who were having difficulties] during the break or maybe some time after school.

Maria did not end up teaching a supplementary lesson on subtraction. In planning the lesson she intended, she and the cooperating teacher requested the help of a math educator who was involved in a project of helping teachers teach math for understanding. The three of them planned and taught the lesson together.

This example shows that in October, although Maria began to think about the relationship between the teacher's actions and students' learning, she still perceived teaching as disconnected from pupils' thinking. For instance, she realized that giving more tests would not help her pupils learn subtraction. However, she envisioned herself teaching a lesson on subtraction without taking into consideration how her pupils already thought about it. She believed that just by supplementing the material, or using some kind of manipulatives she could help those children who were having misconceptions learn how to subtract.

Maria heard many people talking about the importance of listening to pupils, learning about pupils' thinking, and understanding what pupils are saying. Among these people were the instructor of the methods course, the cooperating teacher, other teachers in the school, and I. But it seems that none of these people discussed with her why it would be important to listen to pupils. Maria knew she was supposed to learn about pupils' thinking, she tried to do her best to understand their strategies, but she did not have a clear purpose in mind for doing that. This is why, I believe, it was important for her to understand pupils' strategies for doing subtraction, but she did not think about how she could use them in planning further instruction.

Maria also did not think it would have been good idea to orchestrate a whole-class discussion around students' conceptions about subtraction. It might be that she did not see the value of having such a discussion, since she did not know what to do once students put all their ideas on the table. It might also be that she believed what she said: that other students may learn some of the misconceptions, that it would make some students feel bad, and that it would be a waste of time for the students who had already known how to subtract. But it might also be that she opposed the idea of having such a discussion because, as she said in December, of her own fears about understanding what the children would say if given the chance. The lesson she co-planned and co-taught with the math educator helped her think differently about the reasons for having whole-class discourse organized around students' conceptions.

Observing a Lesson

To co-plan a lesson on subtraction, Mr. Miller, a math educator, took a look at the tests pupils in Maria's class had taken. Maria pointed to what appeared to be a common mistake: Pupils did not change the digit that represented the tens after "borrowing." For instance, for a problem like $35-26$, many children concluded that the result should be 19. "Yeah, I am so used to that," the cooperating teacher said. "Kids always forget that. They borrow, and then they forget that they did it." "It might be they forget it, but it might also be they do not realize that borrowing involves finding a different representation for the number they are subtracting from," Mr. Miller said. "The point is they should realize first that those are two different representations for the same number." Mr. Miller offered to begin the lesson himself, by checking first what pupils thought about representations of the numbers in subtraction.

Mr. Miller started the lesson by asking pupils to subtract 29 from 74. Stephany, a child in Ms. Barnes's class, offered to perform the subtraction and to explain

to the whole class. She began by writing the numbers vertically, . Then she explained:

$$\begin{array}{r} 74 \\ -29 \\ \hline \end{array}$$

"Cross out 7, write instead of 4, 14. Then you go 14-9 is 5, 7-2 is 5. The answer is 55."

Maria looked at Mr. Miller and smiled. From her look I thought Maria understood how the child was thinking and what Mr. Miller meant when he talked about pupils' realizing they should work with different representations of the same number. Mrs. Barnes intervened:

"Let's do this again with the manipulatives on the overhead. I'll put 7 tens and 4 ones on the overhead. Now I will change one ten with ten ones. Now what am I going to have? I am going to have 6 tens and 14 ones," Mrs. Barnes said, and crossed out the number "7" on the overhead and wrote "6" instead. "Now, what we have: 14 take away 9 is 5, 6 take away 2 is 4. So the answer is 45," Ms. Barnes continued.

When Ms. Barnes finished her explanation, Mr. Miller said, "Let's do it a little bit slower. I want you to represent 46 with your cubes, on your tables." While pupils were doing the representations, he, Maria, and Ms. Barnes walked around to see what the children were doing. Mr. Miller noticed that many pupils had the stripes representing the tens and the cubes representing the ones randomly put on the table. He asked them to organize from left to right, first the tens and then the ones. "What do I do if I want to subtract 19?", Bruce asked. "Take this one out," answered Richard, pointing to the strip representing a ten, "and put ten small pieces over here." "Let's do it right now with your blocks on the table," Mr. Miller requested. While Mr. Miller walked around, he noticed that some pupils left the ten on the table, and just added 10 more ones to the existing ones, others took away one ten and 6 ones and said, "We cannot do it. We have only 6 ones; we can not take away 9 ones!"

Mr. Miller said, "Let's back up a little. I am going to make a number. I want you to tell me what number it represents." Mr. Miller arranged on the overhead 4 tens and 3 ones. "43," a child answered. Mr. Miller changed one ten by 10 ones. "What number do I have now?", he asked. "43," some kids said. "33," others answered. Mr. Miller waited

a couple of moments without saying anything. "No, that's not true," a child said, "It's the same number; it's just done in a different way." Other pupils agreed with him. Mr. Miller put on the overhead 5 tens and 5 ones. "55," the children answered. Mr. Miller changed one ten by 10 ones. "What number do I have now?", he asked. "55," pupils answered, "It's the same number; you just changed one ten by ten ones." Ms. Barnes smiled, looked at me and at Mr. Miller and said, "Oh, that's great, I haven't thought of that!"

"Let's represent what we just did in a different way, using numbers instead of cubes," Mr. Miller continued. He drew a chart with two columns on the overhead. Above the left column he wrote "tens", above the right one, "ones." He wrote the number 55 in the chart, a "5" in each column. "How else can we represent the number?", he asked. A child went to the overhead and wrote 4 in the tens column and 15 in the ones column. "And why would I do that?", Mr. Miller asked. "So that you can take away," Matthew answered. "Which numbers I can take away?" Children answered, "0,5,2,1,3,4." A pause followed and then someone said, "6! Oh, I get it! Now you can take 6 away!" Immediately, other kids jumped in: "And 7, and 8, and 9; now you can take anything you want away!"

Mrs. Barnes picked the lesson up from this point: "Now, let's see if that helps us subtract. Let's see, how would you do 82 take away 58?" While kids were working, Mr. Miller and Maria toured the class and talked to individual children. Ms. Barnes approached me: "This is really great! I haven't thought of that! Kids really did not know that when borrowing, they are working with the same numbers!" From the back of the room, I overheard Mr. Miller telling Maria the same thing: "It is important they realize they are working with the same number in a different representation. I don't think they knew that."

Talking about the Lesson

Both Mr. Miller and Maria appreciated the lesson very much. Ms. Barnes talked about it with teachers in the staff room, emphasizing the fact that she did not understand why children were having difficulties, and explaining what Mr. Miller did to help the children comprehend the role of different representations in subtraction. Maria had a conversation with me which I initiated. Maria said she liked the lesson because kids were having fun and also because some of them learned a lot. For instance, Ahmed, with whom I worked, was getting the concept of trading. He said he only now understood that you can even subtract a number from another number which has the ones digit smaller than the second number.

"What do you think Mr. Miller did to help kids understand that?," I asked. Um... That's a good question. What did he do? First he looked at the tests the students did. He looked to see what misconceptions they had. And he did not start a brand new lesson on subtraction, as I wanted to. He worked exactly on the misconceptions he found students to have. Yeah, I see he did that...And then during the lesson, he did not go very fast. He went a lot slower than how Ms. Barnes wanted to go. He also did something I have never seen Jane doing: he made that little chart...Kids played with the manipulatives and then he did not ask them immediately to do subtractions. He asked them to represent in the table, with numbers, what they did with manipulatives. What else did he do? Well, this "representation" thing--it seemed that was very important to him. He took a lot of time to help kids understand that 4 tens and 3 ones and 3 tens and 13 ones represent in fact the same number. Yeah, it seems that cleared up a lot of confusions.

"Did anything in this lesson surprised you?," I asked.

Yeah, well not really surprised, but there were things I wouldn't think of. Like, as I told you, this representation thing. I could not guess that was a problem. But also, he decided to have a whole-class discussion about subtraction. I did not expect that. I thought he would help me take aside the children and teach them, but to openly discuss their confusions, wow, I wouldn't think of that.

When I asked Maria how she thought such an open discussion might benefit the children, she said,

Well, kids weren't put on spot, and they didn't feel bad for having all these misconceptions, as I thought they would...and Mr. Miller definitely could see what exactly kids do or do not understand, even these kids who did not seem to have misconceptions...but I think [pupils] benefited from listening

to each other, and from hearing each other's explanation, and from thinking together about Mr. Miller's questions.

Planning, Observing, and Talking as an Occasion for Maria's

Learning about Pupils' Thinking

Although this lesson was clearly an occasion for learning for both the cooperating teacher and the student teacher, I will focus only on the learning of the student teacher.

To plan the lesson, Mr. Miller first analyzed the previous work pupils did. By doing so, he could learn about pupils' knowledge, understandings, and confusions in relation to subtraction. This analysis allowed him to agree with Maria diagnosis: "Pupils do not change the digit that represents the tens after borrowing," and to take it one step further, changing it into a hypothesis that he could work on: Pupils do not realize they are working with two different representations of the same number. Rather than treating pupils' confusion as a result of pupils forgetting they borrowed, as Ms. Barnes suggested, Mr. Miller decided to start the lesson by testing his hypothesis, and if it turned out that indeed pupils changed the number while changing its representation, to work on this idea.

What seems important about this lesson is that rather than following a plan prepared in advance, Mr. Miller tried during the lesson to find where pupils' confusions were and to correct them. He marked these points in the lesson by saying, "Let's do it a little slower" and "Let's back up a little." Maria noticed these moments and realized he went more slowly than the cooperating teacher.

Mr. Miller began by asking the pupils to subtract 29 from 74. To solve this problem, pupils needed to "borrow," and therefore, to change the representation of 74. He asked a child to solve the exercise and to explain her thinking. The girl's explanation of how she got 55 as the answer for the given exercise, and the fact that nobody in the class contradicted her, confirmed Mr. Miller's hypothesis. I interpreted Maria's smile to mean that she, too, understood pupils did not realize that by changing the representation they should not change the value of the number. At this point, Mrs. Barnes, maybe because she

believed pupils "forgot" the rules of subtraction and she could remind them using manipulatives, performed the subtraction herself. Mr. Miller worked on his assertion. He asked the pupils to represent with the manipulatives the number 46. Even if Richard answered Mr. Miller's question correctly--how to subtract 19 from 46--and Mrs. Barnes did the subtraction just seconds before, some pupils still did something else. They either added 10 ones and therefore they changed the number, or did not do any "trading" and said they could not take away 9 from 6. This was an occasion for Maria to learn about how strong pupils' prior conceptions are, and how little telling or reminding can help in changing them.

Mr. Miller decided to work first on the equivalency of representations. He helped people understand that by trading one ten with 10 ones, only the representation changes and not the number they are working with. This point seemed to be, indeed, a source of confusion for pupils.

When pupils were able to recognize numbers represented in two different ways with manipulatives, Mr. Miller moved into a more abstract representation, a chart. The chart he used helped pupils make the transition between the concrete representations, with the manipulatives, to the abstract ones, with symbols. By making the chart, pupils could see how "borrowing" or "crossing out" connects to the change in representation they did with the manipulatives. Maria noticed the chart and the effect it had on students' understanding. She also realized that her cooperating teacher never took the time to build transitions between concrete and abstract representations.

Mr. Miller also helped pupils see a purpose in changing representations: to perform any kind of subtraction involving only natural numbers. It took pupils a moment to realize that by writing 4 tens and 15 ones instead of 5 tens and 5 ones, they could subtract from 55 any number less than 55. Maria appreciated this realization because of her work with Ahmed, who told her that only then did he realize that in fact two numbers can be

subtracted one from another, even if the ones digit of the second number is bigger than the ones digit of the first number.

The conversation that she had with Mr. Miller before the lesson helped Maria focus her observation during the lesson. She knew that Mr. Miller would work on pupils' misconceptions, that he would test his assumption, and if that was the case, help students realize that by changing the representation, they cannot change the number they are working with. This preparation allowed her to focus her attention on what he did during the lesson, what questions he asked, and what kind of responses his questions elicited.

The conversation Maria had with me after the lesson also, I believe, helped her be more thoughtful about the lesson. I initiated the conversation with a double goal in mind: to learn about how Maria made sense of the lesson, and also to push her thinking further, to help her think about Mr. Miller's actions and their effect on pupils' thinking. As a researcher, I asked Maria what she thought about the lesson. As a teacher educator, I wanted to push her thinking beyond liking this lesson because "kids were having fun, and some of them learned a lot." She knew she was expected to say what "a lot" might be, so she thought about her observation of Ahmed and how this lesson pushed his understanding.

I do not know if my questions served only as an opportunity for Maria to verbalize what she was already thinking about, or if they also created an opportunity for her to think about what happened in the lesson. The pauses in her responses and her hesitations made me think that she was thinking about these questions as she was speaking. It might be that without my questions she would have not thought, for instance, about what Mr. Miller did to help pupils move from the work with manipulatives to abstract representations.

Maria did not expect Mr. Miller to organize a whole-class discussion on subtraction. The fact that he did, surprised her. It might be that my question about how such a discussion benefited the children, made her re-think her original assumptions, and thus helped her realize that, in fact, the pupils as well as the teacher learned from

participating in the discourse, from listening to each other's responses and from answering Mr. Miller's questions. Indeed, Maria herself orchestrated whole-class discussions toward the end of the term. I will analyze these discussions in the next chapter, about her role.

What Maria Learned about Pupils' Thinking and How She Learned It

First, Maria learned that pupils think in many and unexpected ways. This learning enabled her, toward the end of the semester, to be critical of the university classes in which she planned generic units without taking into consideration a specific group of students. In her final self-evaluation, Maria wrote,

The knowledge that a teacher has about the students is one of the most valuable aspects of teaching. When you plan for a class that does not exist, it is impossible to know where the lesson will take you or what possible responses you might get to a given question. I have learned that for every child in the class, you need to be prepared for many responses and you can never be prepared enough.

Second, Maria learned that the way to learn about pupils' thinking is "talking and listening a lot to them." She learned that listening and understanding a child's mathematical thinking is a skill that she is just beginning to master, and not something that she already knew how to do. In talking about what she still wants to learn, Maria said,

It's hard to know what I want to learn when I do not know it. But I want to learn, to improve myself. It's my subject matter and my pupils. I don't know, I feel that I am just skimming the top sometimes with them, and then I realize I should have gone deeper. But how do I know how far I can go? How can I know when I still don't know what my kids are learning? I need to learn how to listen to them to know how they are thinking. Only then I can know how far I can go.

Finally, Maria changed her understanding of why she should listen to children's ideas. She began student teaching thinking the reason for listening to pupils is to motivate them. Later, she came to believe she needed to listen to pupils to find out who had misconceptions about the subject matter taught, so that she could supplement the material. At that time, Maria did not talk about teaching a lesson that would address pupils'

misconceptions. She talked about listening to pupils' ideas only as a way to identify the children who might think incorrectly, so that she could reteach them the material later.

Towards the end of student teaching, Maria began to see listening to pupils as a way of teaching and learning, from which the teacher as well as pupils may benefit.

Learning about what pupils think and know, acquiring skills to listen and understand pupils' ideas, and learning to use these ideas in classroom discourse to push pupils' thinking forward are crucial parts of learning to teach for understanding. These kinds of knowledge and skills cannot be acquired outside the context of teaching, in the absence of direct interactions with pupils in the process of learning mathematics.

Confronted with pupils' mathematical explanations which she could not understand, Maria realized that listening to pupils' mathematical ideas was a skill she was just beginning to learn and not something she had already mastered, as she used to think. Paying close attention to children's interpretations of teacher's explanations, Maria became aware of the belief she held, that if pupils are taught well they are going to know the material, understanding, at the same time, that there is no perfect way of teaching, and that no matter what teachers do, pupils are going to interpret the experience in many and unexpected ways. This realization pushed Maria to become more thoughtful about the quality of teachers' talk. It helped her discriminate between different kinds of explanations on the basis of how they would make sense to pupils.

Noticing the many different and unexpected solutions her own pupils constructed while working on a mathematical problem she perceived as straightforward, Maria gained a deeper and more grounded understanding of what she learned only at a theoretical level, that given the opportunity, pupils think in many different ways. In the situation of making sense of these solutions, Maria began to understand both what it means to give pupils this kind of opportunities, and what kind of solutions pupils might come up with.

Observing a math educator teaching her pupils a lesson and paying close attention to what she saw him doing and how pupils reacted contributed to Maria's rethinking of some

of her assumptions and beliefs about what constitutes good teaching and how pupils might learn. Confronted with the world of pupils' ideas in different occasions, Maria began to realize its richness, became curious about its wonders, and learned to see it as an important source for her and her pupils' learning.

Student teaching was not the first occasion for Maria to encounter some of these ideas. She had heard before that pupils think in many different ways and that people construct knowledge in their own special ways. In her math methods course, Maria even had a chance to explore on her own how both adults, including herself, and children in classrooms make sense of mathematical ideas. However, she learned these ideas in one context, that of a university course. Re-learning some of these ideas in a different context enabled Maria to rethink and give new and more powerful meanings to what she had already understood.

But the context of practice is not simply another context. It is the context where she can learn new skills and use what she is learning. Thus, in this context, Maria did not learn just about the practice of teaching; she learned actual teaching. In the situation of teaching, Maria learned specific instances of pupils' thinking differently, enabling pupils to think in different ways, and using pupils' ways of thinking for her own and others' benefit. Maria will encounter the next group of pupils not only knowing that she has to learn how the specific pupils think; she will also be equipped with ways to find that out and a repertoire of pupils' strategies that would enable her to hear and understand more of what they say.

CHAPTER V

MARIA'S LEARNING ABOUT THE TEACHER'S ROLE

The most important thing [in learning to teach] is to know that before you teach, first of all you need a purpose to teach. After you have the purpose, you need different plans for lessons and activities to follow up on that purpose. And you need to think about your purpose and stick with it. Once you start teaching, you need to take into consideration each child as much as you can, and then the whole class, and design the lesson in that way. (Int Dec)

This quote reflects Maria's learning about a third dimension of teaching, the teacher's role. Maria's talk about a teacher's needing a purpose to teach reflects her realization that lessons need to be directed toward a certain goal and that a teacher needs both a plan and a way of acting that would move the pupils toward that goal (Dewey, 1938). It also reflects her understanding that she was responsible for the children's intellectual growth and that it was her role to help pupils take advantage of specific learning opportunities.

When Maria came to student teaching, her images of what a teacher--and particularly a math teacher--does in the classroom were strongly influenced by her experiences during the apprenticeship of observation (Lortie, 1975) and her learning in the teacher education courses at the university. The experiences she encountered during the student teaching and the sense she made of them transformed these initial images about teacher's role and had an impact on the way she enacted that role. In this chapter, I will show what her initial perceptions or images about teacher's role were, how they changed during the time, what contributed to that change, and what implications these changes had on the way Maria taught and thought about teaching.

Maria's Initial View of the Teacher's Role

I want to have discourse in my teaching. [I believe] students should do most of the talking. I want to stop myself from talking too much before I even start. I want to be able to probe my students for the information they know and guide them to the answers without me telling them. I want

students to be able to talk about how they are thinking and learn from each other.

I am a little worried about a student of mine asking me a question that I'm not able to answer. I know that not everyone knows everything about the subjects they teach, but how do I answer a question that I don't know?

I am a little worried about [the student teaching] because I have no practice at all doing it. And I don't know how I am going to perform. And I have this one weakness, it's hard for me to speak in front of large groups, peers or children. I think when I first start teaching, I'm going to be very nervous. How am I going to lecture to a big group of kids?

These three quotes are excerpts from the same interview which Maria gave in mid-June, two and a half months before the beginning of student teaching. These excerpts reflect the conflicting perceptions she held about the teacher's role in the classroom, as influenced by her own experience as an elementary and high school student and by her experience in the math methods course.

The first quote represents mainly what she learned in the methods class: that students should do most of the talking during the lesson, they should explain their thinking, and the teacher should listen and probe pupils' answers in order to push their thinking. The second and the third quotes reflect Maria's thinking about her role in the classroom as influenced by her own experience as a student: the teacher should perform during the lessons, lecturing most of the time and knowing and answering pupils' questions. She expressed these conflicting views in the same interview, and she did not talk about a discrepancy between them.

During the first month of student teaching, when Maria did mainly observations of the cooperating teacher's teaching, she did not raise questions regarding the teacher's role in mathematics. She did, however, raise lots of questions about what Mrs. Barnes did in other subject areas. For instance, she was continuously concerned about how the cooperating teacher was making decisions in teaching literature: "How does she know when to answer [pupils' questions] and when not? How does she know whom to answer? How does she decide whom to call on?" were questions that she repeatedly asked in her

journals or in her conversations with Mrs. Barnes. As I write this, I am extremely surprised to discover that she never asked similar questions in relation to the cooperating teacher's or other's teaching of mathematics.

Since I did not notice Maria's lack of concern about the teacher's role in the teaching of mathematics at that time, I could not ask her any questions about how she saw her role in mathematics. Thinking back, I realize I was assuming that her concern about the teacher's role was general; I took the questions she was asking in one subject area as an indication of her general concern about the teacher's role in teaching.

As I look back in the journal I kept while I was studying Maria, I find my notes from a discussion I had with her two weeks after the beginning of the student teaching, after she gave pupils her first quiz. I wrote then about my astonishment to discover that she encountered difficulties talking about her reasons for choosing the particular problems or for grouping the pupils in certain ways during the quiz. She answered most of my questions with "I don't know... That's a good question... Wow, I have not thought about that...." It seemed to me that my questions were an occasion for her to think about these issues, which was surprising to me given the thoughtfulness that characterized her in relation to other subject areas.

When Maria planned her first lesson in mathematics at the beginning of October, she intended to give students a couple of word problems, to model for them how to solve the first one by subtracting the smaller number from the bigger one, and to have them solve the rest of them in a similar way. (This lesson is described in the chapter which discusses Maria's learning about pupils' thinking.)

The way she imagined her role in this lesson was quite consistent with the role her own math teachers were assuming: the teacher would explain or model a couple of problems and the students would work quietly on a few similar problems, following the pattern the teacher showed. Although very critical of this way of teaching and learning ("I

never understood anything at all"), Maria seemed to be guided in her teaching by the same image, the image that was so familiar to her.

It seems to me that the ways in which she talked about teaching and attempted to teach herself reveal three kinds of inconsistencies regarding the teacher's role. These inconsistencies raise important questions about the role and the power of one's prior experience as a student and the influence that interventions, like university courses and in particular methods courses, can have on someone learning new kinds of teaching.

First, there are the conflicting ways in which she talked about teaching before the beginning of student teaching: on the one hand she saw herself as listening to students, probing their thinking, having them do most of the talking in the classroom. On the other, she talked about her being worried about not knowing the answer to all the questions pupils would ask, or not being able to lecture to a big group of students. These views are conflicting because the first one assumes that pupils do most of the thinking, and the role of the teacher is to support them in the process of making sense of what they are learning, whereas the second assumes a more traditional view of learning, that the pupils would absorb the knowledge that the teacher presents to them.

Second, there are the discrepancies between the ways in which she approached learning about the teachers' role in mathematics and in other school subjects. In all subjects, Maria was very active in trying to make sense of the cooperating teacher's actions, raised questions about what she was doing, and tried to find reasons for her decisions. None of these were present in her learning about the teaching of mathematics. Moreover, it seemed that in mathematics Maria did not even think about the reasons behind her own choices.

Third, and last, there are the discrepancies between how Maria talked about what she appreciated in the methods course and how she herself attempted to teach. When talking about the experiences in the methods class, Maria said that the emphases the teacher put on students' thinking stood out to her. From Maria's perception, the teacher was

interested not only if the answer was right or wrong, but also how the students thought about the problems. She also had a vision of what that kind of teaching might look like: "Kids are learning by talking with each other, learning from each other, a class featured by collaboration. However, when she first attempted to teach, she fell back on the more familiar model of teaching: the teacher models a solution of a problem, and the students follow the pattern demonstrated by the teacher.

My explanations for why different images of teaching and learning existed in Maria's mind are speculations, since I was not aware of those inconsistencies while I was studying her; therefore I could not verify my conjectures with her. However, the assertions I am making now are based on my best knowledge about her and her ways of learning, and they are consistent with other findings of this study.

One possible explanation for the inconsistency I mentioned first is that in June, at the end of the academic year, Maria was under the strong impression that the math methods made on her. Therefore, when asked to talk about how she would like to teach, she talked about the ideas that she learned in that class: having discourse, listening to what pupils say, pushing their thinking forward. However, when Maria mentioned her worries, she talked from a deeper, less conscious level, and therefore her talk was influenced by the deeply rooted images that she constructed about teaching and learning during her own schooling.

Another probable explanation is that there are not actually any inconsistencies in the two ways in which she talked about teaching. It might be that, as I showed in a different chapter, she saw listening to pupils as a way to motivate them to learn, not as a way to teach them. Thus, this view did not necessarily have anything to do with the role she assumed as a teacher: to lecture and answer to pupils' questions.

How could Maria approach learning about the teacher's role in two different ways? Why did she ask so many questions about what the teacher was doing in different subject areas, and none in mathematics? It might be that her weak subject matter knowledge kept

her from paying attention to more subtle aspects of teaching, like the teacher's role, having to focus more on the mathematics itself. It might also be that the cooperating teacher's teaching of mathematics was so different from anything that Maria had previously experienced that she needed the time to make sense of it before being able to ask questions to learn anything from it. And of course it might be that a combination of these factors and more contributed to her approaching learning to teach in different ways.

Why was Maria's talk about what she liked in the math methods course so different from the way she attempted to teach in the classroom herself? It might be that the math methods course did not replace the images of teaching and learning that she has built through the years; it only added a new dimension to them. Thus, it is possible that the course helped Maria learn to appreciate certain aspects in the teaching of others, it allowed her to think what teaching for understanding was not, and planted the seeds for a view and a new commitment for a different kind of teaching of mathematics. (See previous chapters.) However, when it came to her own teaching, the course was not strong enough to overcome the images that she had developed for years about what teachers and students do in mathematics.

These are all possible explanations. I would like to offer another one, which emphasizes more Maria's construction of what mathematics is and how it should be taught.

Although Maria's experience with schooling was mostly traditional, involving a lot of drill and memorization of facts, she did have some outstanding experiences that she liked. She talks with great pleasure about her learning of history and geography in high school, where what she enjoyed most was that she was required to "think, to argue, and to write." History and geography became her favorite subjects in school, and she developed an interest in learning more about these subjects. Maria believes she owes much of what she knows in these subject areas to the way she was taught. These experiences stand in sharp contrast to her experiences in mathematics. When I asked if she saw a contradiction between the teaching and learning of these subjects and the teaching and learning of

mathematics, she said, "No, because I thought math is math and this is how math should be taught." It seems to me that her construction of what mathematics is all about and what math teachers should do to teach, had a major influence on what she learned about the teaching of mathematics, and how she learned it.

This explanation sheds some light on why Maria was preoccupied with learning about the teacher's role in some subject areas but not in mathematics: because her experiences with other subject areas gave her an image of how teaching and learning of these subjects might look, gave her a sense of possibility, whereas in mathematics she was under the impression that "this is math, and this is how math should be taught." It might also be that since she was passive as a learner of mathematics, she also assumed a passive role in learning to teach mathematics, while being an active learner of teaching in other subjects. Her belief that "this is math and this is how math should be taught" might also have determined the way she attempted to teach mathematics, in spite of what she learned on a theoretical level in the math methods course. In any case, no matter what explanation we choose, it seems that Maria's initial views about the role of the teacher were inconsistent, including both traditional and more "adventurous" aspects.

As I mentioned before, while Maria spent time in the cooperating teacher's classroom, she did not ask questions regarding the role Mrs. Barnes was assuming in teaching mathematics. Maria attempted to teach her first math lesson in a very traditional way, the same way she experienced the learning of mathematics herself.

In the middle of the second month of her student teaching, Maria and the other student teachers observed a sequence of five math lessons taught by Mrs. Kogan, a second-grade teacher who was also experimenting with new ways of teaching and learning mathematics. The purpose of this activity was to provide all student teachers with common opportunities to observe a teacher who was further along in the process of changing her practice, and to talk to her and with each other about the reasoning behind her actions and about pupils' mathematical understanding and learning. While being in Mrs. Kogan's

room, Maria observed the teacher and the pupils, worked with individual and small groups of pupils, and took notes about interesting things that she noticed.

In the following section, I will describe in detail the first lesson in the sequence that Maria observed, and I will analyze what she learned about teacher's role from these observations. I chose to focus on this particular lesson because it was the one that impressed Maria the most for the length of the mathematical discourse Mrs. Kogan orchestrated. To help the reader understand the role Mrs. Kogan played in this lesson, as well as to appreciate the difficulties Maria encountered in seeing it, I will reconstruct in great detail a large part of the lesson.

A Demonstration Lesson

In the lessons previous to the ones observed by the student teachers, the second graders wrote number sentences for different numbers using addition of integers. Mrs. Kogan's purpose was to have pupils understand the commutativity property for addition. At the same time, she wanted the pupils to practice addition in context. In the lesson prior to the one the student teachers observed, pupils in Mrs. Kogan's classroom began to struggle with the question "How do we know that we have all the number sentences written down?" Mrs. Kogan decided that was an important question to pursue, and she made it the focus of the first lesson in the sequence the student teachers observed.

Before the beginning of the lesson, Mrs. Kogan put on the board six big charts with the number sentences the pupils wrote for the numbers five, six, seven, eight, nine, and ten respectively. When the lesson started, the kids were sitting on the floor in a big circle. Mrs. Kogan asked the pupils what they figured out about the number of number sentences one could write for number nine and number ten. Andre said that "for eight you have nine number sentences." Lucy went to the board and pointed to the number sentences written on each chart: six, seven, eight, nine, ten, and eleven. Karel followed her and said, "It goes like this: six, seven, eight, nine, ten, eleven. It's one less, so probably 11

goes to twelve." Mrs. Kogan asked, "How do you know that Karel? How do you know that eleven goes to twelve?" Karel answered, "Because five goes to six, six goes to seven, seven goes to eight, eight goes to nine, nine goes to ten, and ten goes to eleven." Another child repeated what Karel said. Mrs. Kogan intervened: "A kind of pattern how the number sentences are...." The word "pattern" stimulated some of the pupils to look for different patterns within the number sentences, most of which did not appear to be related to the issue in question. Thus, one child showed that the numbers three, four, five, and six appear on each of the charts. Another student showed that the numbers seven, eight, and nine appear only on some of the charts. A girl went to the board and showed that in each number sentence you have "equal and a number." Every time Mrs. Kogan acknowledged the patterns that the pupils were finding, making all kinds of positive comments. For instance, in this last case, Mrs. Kogan said, "All number sentences seem to have that equal sign, don't they?" After a while, Deryl intervened by saying, "I want to say something about what Karel said. If you have a number like eighteen, you are going to have nineteen number sentences. Deryl's comment helped refocus the discussion. Mrs. Kogan, who seemed to let the discussion go up to this point, intervened to maintain the focus of the discussion: "For eighteen you are going to have nineteen number sentences? How did you figure that out Deryl?" "Easy," Deryl answered. "We just add one more up." "Listen to Deryl--this is an important idea," Mrs. Kogan commented to the class. Deryl continued his explanation: "For five, you have $5+1=6$ number sentences, that's one more up; for six, one more up is seven." Mrs. Kogan asked if somebody else could explain Deryl's idea. Richard showed how the original number can be obtained by subtracting one from the number of number sentences: "If you take one of these away, then you get the number you are looking for. For instance, if you have six number sentences, you know that the number you are looking for is five." Several pupils had their hands up ready to offer their suggestions about the relationship they observed between a number and the number of number sentences that can be written for that number. Mrs. Kogan asked Justin to share

his idea. Justin showed again how "you add 1 to the number to find out how many number sentences you can write for that number." At this point Mrs. Kogan suggested that she would write on the board the pupils' conjectures. She asked Deryl to tell her what to write. Deryl dictated, and Mrs. Kogan wrote on the board: "Whatever number you are finding number sentences for, just add one to that number." Then Mrs. Kogan talked again about Deryl's example: "Like you, Deryl, gave us one example: if you go all the way to eighteen, you said, there will be nineteen number sentences, and you figured that out by adding one to that number." And Mrs. Kogan asked for more examples, using different numbers. A student started to count: twelve, thirteen, fourteen, fifteen, sixteen. Mrs. Kogan asked, "When you said sixteen, do you mean 16 number sentences?" The child answered affirmatively. Mrs. Kogan asked, "When you said 16, what would be the number that you are writing number sentences for?" There was no answer for a couple of seconds. After a while, Dan, a student in the class, asked "What do you mean?" Mrs. Kogan answered, "Well, if you are trying to find the number sentences for fifteen, how many number sentences will you have?" Dan said "sixteen." Mrs. Kogan asked him, "How do you know that?" There was no response from the child, so Mrs. Kogan asked the whole class, "Does everyone agree with 16?" Most of the kids answered, "Yeah!" "Why?" Mrs. Kogan asked. Kari answered, "Because adding one more makes fifteen to be sixteen." Mrs. Kogan replied, "So you probably think there are going to be sixteen; you are not exactly sure, because [pointing to the conjecture written on the board] it says 'add one to that number'."

At this point the lesson took a different turn. Andre, who was one of the first children to notice the relationship between a number and the number of number sentences that can be written for that number, said, "But...if it is zero, it's just going to be one because...but it cannot be one because for zero there are no number sentences!" As Mrs. Kogan said later, Andre's comment made her think about the difficulties pupils encounter to understand the number zero, so she decided that the comment was important enough to

have the whole class pay attention to it. So she said, "Andre is asking another question: if you are using the number zero, how many number sentences would you have?" Some pupils said "one," some said "none." Mrs. Kogan decided that it was important for her and for the class to hear what pupils thought, so she said, "I see people have ideas. Let's hear some ideas." Three pupils, one by one, went to the board and talked about $0+0=0$. Andre wanted to explain his idea some more. So he went to the board to do some more explaining and writing, but this time he wrote: $0=0+0$. His statement generated a lot of discussion in the class: is $0=0+0$ the same as $0+0=0$? Can $0+0$ be "switched around"? Are there other number sentences that can be or cannot be "switched around"? Kids talked about the number sentence $4+4=8$ being the same as $0+0=0$ because in both cases "you can't switch the numbers around". In a conversation with the student teachers after the lesson, Mrs. Kogan said that at that moment she was not sure what kind of "switching around" pupils were talking about: "switching" around the equal sign, like in $0=0+0$ versus $0+0=0$, or the use of the commutativity property? She decided to find out: "Andre, can you think of a number sentence where you can switch?" "Sure," Andre said, and wrote on the board " $2+3=5$ "; "You can switch them around because they are not the same." "How would you switch them around?"; Mrs. Kogan asked. " $3+2=5$," Andre wrote.

Clearly, Andre was talking about the commutativity property. But what about other pupils in the class? What kind of switching around were they talking about? "Let's see...there are some other people who would like to talk about this idea. Let's give them a chance." A girl talked about finding a way to switch around $4+4$: "You write a big four and a little four and that's the same as adding a little four and a big four." Mrs. Kogan validated her idea by repeating it, and asked for other people's opinions. Another girl said that would not be the case for the number zero, "because $0+0$ you can't switch around." The first girl responded by writing on the board that a big $0+$ a small 0 is the same as a small $0+$ a big 0 . Mrs. Kogan summarized the disagreement: "These three people at the board are sort of disagreeing about something. Let me see if I understand what they are

disagreeing about. Andre is saying that $2+3=3+2$; you can switch the numbers around. Lakon is saying that $4+4$ you can switch because a big four plus a small four is the same as a small four plus a big four. Ross thinks that $4+4$ you cannot change around. What do you think?" Next, Mrs. Kogan gave them an assignment to work on until the end of the period: "In your journal, write examples of number sentences that you think you can't change around and try to explain in words why; on a different page write examples of number sentences you can switch around, and try to explain why those can be switched. Tomorrow we are going to discuss this and see why some number sentences we can change around, and some we cannot." Until the end of the hour the students worked at their desks individually or in small groups to find examples of number sentences for the two categories.

What Maria Saw in This Lesson and What She Made of It

Maria was surprised about aspects of this lesson which she liked very much. She was surprised that the pupils were so respectful of each other, that they were listening and responding to each other's comments. She was intrigued by how smoothly things went and she was curious to find out what Mrs. Kogan did at the beginning of the year to create this learning environment which she fully appreciated. Several times I asked her if she saw Mrs. Kogan doing anything to maintain this learning environment. Maria seemed to think that Mrs. Kogan did not do anything on a day-to-day basis--she did this one-time magical thing at the beginning of the year, and since then she has the perfect learning environment in her classroom. Until the end of student teaching, Maria was concerned about finding out what Mrs. Kogan did at the beginning of the year to construct the learning environment she had.

The fact that Maria did not see what Mrs. Kogan was doing on a daily basis to maintain the learning environment is consistent with another aspect of what she saw or did not see in Mrs. Kogan's lessons: she did not see Mrs. Kogan as having any role in her

teaching. Maria thought that all Mrs. Kogan did was to let pupils talk during the lesson; she just went with the flow. This view of Mrs. Kogan's role was not challenged until she started to teach herself in Mrs. Barnes's classroom.

Maria was surprised to see that the discourse took almost an hour. She was concerned about finding out reasons why would a teacher let the discourse go for so long, especially when it does not seem to go in the direction intended. Maria asked Mrs. Kogan why she let the discourse go for the whole hour. "Because that's how people learn," Mrs. Kogan answered. Maria was also curious to know why Mrs. Kogan did not stop the discussion when the pupils were finding patterns that did not seem to be related to the subject discussed (like all number sentences have an "equal" sign, the numbers three, four, five, and six appear on all the charts, etc.). Mrs. Kogan answered, "That was what students wanted to do, and that were having fun with the numbers in a way that was wonderful to them." Mrs. Kogan talked about letting her students play with the numbers, letting them have fun with them in a way that she was not allowed to when she was a student herself. She also said that what might seem meaningless to us as adults might be full of meaning for the students.

After observing Mrs. Kogan's demonstration lessons, Maria said she would like to try to teach like that. Curious to know what "like that" meant to her, I asked her to explain how she envisioned her teaching. Maria said:

I would put them [the pupils] on the floor, and have the students talk, and then one would go to the board, and talk about what he is thinking, and then somebody else would go up there and talk about his thinking, and says 'Oh, no' or 'I disagree with...' and come up with all different ideas. Her lessons seem to really drag on a long time because they were thinking really hard about what Mrs. Kogan was trying to teach.

Since at that time I was not fully aware that Maria envisioned a kind of teaching in which she would not have any role, I had not asked her, for instance, what she would do while pupils would go to the board and agree and disagree about their mathematical ideas, or what she would do to facilitate this discussion. Now I wish I had. However, from the

way she talked about her images of teaching and learning in this and other occasions, I could learn that she perceived Mrs. Kogan as having a passive role, letting pupils talk, never explaining or telling them they were right or wrong, while pupils were perceived as being actively engaged in discussing mathematics.

Maria's perception of a teacher who does not have any role in this kind of teaching is also visible in the way she planned her lessons after participating in Mrs. Kogan's demonstration lessons. For instance in planning a unit on place value, Maria intended to have a problem-solving activity, and then have the pupils "go on and on" discussing it. My question, what she would do if pupils experienced difficulties, took Maria by surprise. "What do you mean?" she said. "I would let them talk about that, and maybe they would argue about that, but other than that...." "What do you want them to learn through this problem?" I asked. "Hmm....That's a good question....Well, I want them to think about it, and I want them to talk about it, but--well, I don't know, I have not thought about that."

Maria's answer to my questions made me think that she saw having discourse in the class as a purpose in itself. To her, discourse meant letting pupils talk. She did not necessarily see that the discussion needs a focus or a purpose. I also questioned if she saw discourse as connected to or as contributing to pupils' learning. My understanding of the way Maria interpreted Mrs. Kogan's teaching and saw her own role in contributing to her pupils' learning made me decide to try to help her construct a more productive image of teaching.

I found three main issues in Maria's learning from Mrs. Kogan's lessons which are related to one theme: Maria did not see the teacher as having any role in this kind of teaching. The issues correspond to the three changes suggested by the NCTM standards: changes in the tasks, in the discourse and in the learning environment. In relation to the tasks, she did not see the assignments Mrs. Kogan gave, the problems she posed, the questions they discussed, as playing an important role in pupils' learning. In relation to the discourse, she thought that all Mrs. Kogan did was to let pupils talk for an hour about

whatever they wanted. She did not hear the questions Mrs. Kogan asked, did not notice what Mrs. Kogan did to encourage students to discuss a mathematical idea or to explain their ideas. Maria interpreted the fact that the focus of the discussion changed during the lesson as evidence that Mrs. Kogan just went with whatever the pupils said, that she did not have a purpose in mind or took an active role in directing the discussion. In relation to the learning environment, Maria admired the "perfect learning environment" that Mrs. Kogan had in her classroom, but she did not see any of the things that Mrs. Kogan was doing to maintain it. She was concerned about finding out what Mrs. Kogan did at the beginning of the year to create this learning environment. What Maria learned from Mrs. Kogan is partially a function of what is hard in learning to teach mathematics for understanding through discourse (that the role of the teacher is less visible than in the traditional teaching), and partially a function of how she made sense of what she saw Mrs. Kogan doing or saying. In the lessons she observed, Maria noticed that the pupils took an active role, were "center stage" (Ayers, 1993), were actively engaged in doing mathematics, while what the teacher was doing was not so obvious to her unexperienced eye. Since most of her experience as a student was with traditional ways of teaching mathematics, where what the teacher does is pretty obvious, it was hard for her to discern what Mrs. Kogan was doing as a teacher. It might also be that Maria's weak knowledge of mathematics kept her from appreciating the value of the questions that Mrs. Kogan asked to push pupils' thinking.

Another possible explanation for why Maria did not notice the role Mrs. Kogan was assuming during the lessons is that she experienced these lessons almost as a student in Mrs. Kogan's room (Lortie, 1975). Although Maria did participate in planning sessions and in conversations about these lessons with Mrs. Kogan and other student teachers, most of the planning was done by Mrs. Kogan alone at home in the evening. While teaching, Mrs. Kogan could not point out what she was doing and communicate the decisions she was making and the reasons for making them. Thus, the thinking involved

in Mrs. Kogan's teaching as part of what the teacher was doing in teaching mathematics for understanding remained obscure to her for a long time.

Maria's not being able to see what Mrs. Kogan was doing in the lessons reminded me of a story I read in William Ayers's (1993) book, To Teach:

When I taught preschool, much of my work was behind the scenes, quiet, unobtrusive. One year, a student teacher paid me a high compliment: "For two months, I didn't think you weren't doing anything. Your teaching was indirect, seamless, and subtle, and the kids' work was all that I could see." (p. 14)

Compliment or not, the fact is that in this kind of teaching, the role of the teacher is almost invisible to the novice.

The way Maria made sense of Mrs. Kogan's response to her questions about what she observed in this and other lessons also contributed to her perception that in teaching for understanding the teacher has no role. For instance, Maria asked Mrs. Kogan why she let the discourse go for so long. (Implicit in Maria's question is that Mrs. Kogan did not have an active role: the discourse was "going," and Mrs. Kogan "let it go" rather than stop it.) Mrs. Kogan answered, "Because that's how people learn." Then Maria asked why Mrs. Kogan did not stop the discussion when students were finding "meaningless" patterns. Mrs. Kogan answered, "Because that was what students wanted to do." Maria interpreted both answers as meaning that in order to teach for understanding it is enough to let people talk about whatever they want to talk for as long as they want to talk. This way of making sense of what she saw Mrs. Kogan doing and saying is evident in her comments in the discussions she had with other student teachers and with me, as well as in her own planning in teaching.

The problem with Mrs. Kogan's saying "because that's how people learn" is that Mrs. Kogan did not talk about the relationship between "people talking" and learning. Mrs. Kogan did not explain why and how talking helps people learn, what kind of talking is useful, or, specifically, how that particular "talk" contributed to pupils' learning of

mathematics. Not given these explanations, Maria took Mrs. Kogan's statements at their face value and interpreted them to mean that the key of success in teaching for understanding is to let students talk.

What Could Have Been Learned in the Demonstration Lesson

Mrs. Kogan's lesson was rich in learning opportunities in terms of the kind of changes recommended by reformers: changes in tasks, in discourse, and in learning environment.

Tasks: The problem Mrs. Kogan posed, in how many ways a number can be expressed as a sum of two positive integers (what Mrs. Kogan called number sentence), is an important problem because it gives access to pupils to some of the same issues that preoccupy mathematicians: How many solutions does a problem have? And how can we know that we found all of them? Dealing with these questions involves finding a procedure, like writing the number sentences in a systematic way, to ensure that all solutions were found. Working on this problem means for the students that they first have to think about what the problem is and how it might be approached, since the way the problem was posed does not offer hints about what pupils have to do in order to solve it. Second, pupils need to think about a method to ensure they found all the solutions. Last but not least, students are confronted with the question: Are some of the solutions equivalent (Or did we find some of the solutions more than once?) In the context of this problem, the last issue introduces pupils to an important mathematical idea: the property of commutativity. The problem also enables students to think and talk about patterns, which is an important topic in mathematics.

From the pedagogical point of view, Mrs. Kogan also talked about using this problem as an opportunity for the pupils to "practice addition in context," which meant that rather than having her pupils work on worksheets to do addition problems, they work on a mathematically meaningful problem which also enables them to practice and improve their

skills at adding two-digit numbers. Maria did not notice anything special about the task Mrs. Kogan used. She did not come to appreciate the value of the questions Mrs. Kogan asked in helping her pupils learn mathematics.

Discourse: The discourse Mrs. Kogan orchestrated during the lesson requested students to make conjectures about how this problem might be approached and to reason and justify their choices of strategies and the arguments they were making.

According to the Standards, discourse involves two fundamental questions about knowledge: What makes something true or reasonable? How can we figure out whether or not something makes sense? (NCTM, 1991, p. 34). An important part of Mrs. Kogan's lesson was focused around these questions: How do we know how many number sentences can be written for a certain number? How do we know that by adding one to a number we get the number of number sentences that can be written for that number? In dealing with these questions, the students had to reason mathematically and to provide evidence for the arguments they were making. For instance, when Karel said that there are twelve number sentences that can be written for eleven, Mrs. Kogan asked, "So how do you know that Karel?" Mrs. Kogan's question required Karel to explain her idea, "Because five goes to six, six goes to seven, seven goes to eight, eight goes to nine, nine goes to ten, and ten goes to eleven." When a couple of people observed and made conjectures about the relationship between a number and the number of number sentences that can be written for that number, Mrs. Kogan wrote the conjecture on the board in the kids' language ("Whatever number you are finding number sentences for, just add one to that number") and asked students to reason about why that pattern might work.

An important aspect of Mrs. Kogan's lesson was that the pupils were the ones who decided whether or not a conjecture or an argument makes sense. In this particular case, Mrs. Kogan put the conjecture the pupils made under the whole class's scrutiny, expecting the class as a group to decide whether or not it was true. This way, Mrs. Kogan created opportunities for pupils to think for themselves and bring examples or counterexamples for

the specific conjecture. Thus, Mrs. Kogan encouraged pupils like Andre, who was one of the first students in the class to notice the pattern, to continue to struggle to make sense of the conjecture, while at the same time she created an occasion for him to make his thinking public. When Andre exposed his thinking, saying that the conjecture might not work for zero, because "there are no number sentences for zero," Mrs. Kogan again turned his claim into a question to the whole class: "Andre is asking another question--if you are using the number zero, how many number sentences would you have?" By doing so, Mrs. Kogan showed that his question is valid and opened opportunities for other people in the class to address it. As she explained to the student teachers, Mrs. Kogan acted this way from a belief that a teacher's answer shuts off pupils' thinking. She believed that if she had answered or corrected Andre's thinking, that would have closed the discussion, would have stopped pupils from thinking further about the issue, seemingly accepting her idea but not necessarily making sense of it.

What gives quality to the discourse in Mrs. Kogan's classroom is the fact that students explain their ideas and provide arguments for the claims they are making. As a member of the community, Mrs. Kogan is also expected to expand and clarify her own ideas. For example, when Mrs. Kogan asked Dan for what number there are going to be sixteen number sentences, Dan asked her, "What do you mean?" His question reflects his expectation for Mrs. Kogan to clarify her thought. And Mrs. Kogan's clarification followed: "Well, if you are trying to find the number sentences for 15, how many number sentences will you have?"

The kind of discussions that Mrs. Kogan has in her classroom is possible because students are used to talking and listening to each other as well as to the teacher, to working in small groups, and to reflecting on their own and others' thinking. These characteristics are part of the learning environment Mrs. Kogan created in her classroom. As I said earlier, Maria did not see Mrs. Kogan as the person who orchestrates the discourse which contributes to pupils' learning. She believed that the pupils were the ones who conducted

the lesson, and Mrs. Kogan was just going with the flow. She was left with the impression that Mrs. Kogan never tells her students if something is right or wrong, and in general that she does not teach by telling.

Learning Environment: Mrs. Kogan managed to build a learning environment in her classroom in which students feel safe to take intellectual risks. Her pupils feel free to make mathematical conjectures, to agree and disagree, to expose their thinking, and to make mistakes. Although I do not know what Mrs. Kogan did to create this kind of learning environment, I can talk about what I saw her doing in this lesson to maintain it: showing respect for peoples' ideas and modeling ways for her pupils to do the same, showing that she values reasoning and sense-making, and keeping a pace that allows students to puzzle and to think.

Mrs. Kogan has a repertoire of ways to show to her pupils that she values their ideas and to help them learn how to show that to each other. Among the less conventional ways she used in this lesson to show respect for what pupils said were to use pupils' own language (for instance, she wrote on the board the conjecture that Deryl made in his own words), to encourage pupils to write their ideas on the board regardless if these ideas are right or wrong, and to accept the patterns they found, even if those patterns were not necessarily the ones she expected. At the same time, Mrs. Kogan does not accept everything: When the lesson is at a point where it might take an unwanted direction, Mrs. Kogan gently redirects the pupils to the subject discussed. For instance, when she asks for examples of how the conjecture works, and a child counts "twelve, thirteen, fourteen, fifteen, sixteen," Mrs. Kogan asks, "When you say sixteen, do you mean sixteen number sentences?" The child's answer "yeah" does not satisfy Mrs. Kogan: It does not provide evidence that the child was indeed thinking about the problem discussed or that Mrs. Kogan's question guided him in the right direction. So Mrs. Kogan asks again, "When you say sixteen, what would be the number that you are writing number sentences for?"

Deciding when and how to intervene to redirect the lesson and when to go with the flow and balancing those times is a delicate issue in Mrs. Kogan's lesson and it requires great skill. Maria did not notice anything Mrs. Kogan was doing to nurture a learning environment she worked hard to create. For Maria, things seemed to be flowing naturally, pupils were in charge of the lesson, while Mrs. Kogan had a passive role, overseeing what was going on. She was, however, concerned about finding out what Mrs. Kogan did at the beginning of the year to create a "perfect" learning environment.

Challenging Maria's Beliefs about the Teacher's Role

Maria's response to my queries about the sense she made of Mrs. Kogan's teaching and the way she translated her perceptions into practice worried me. Although I was only the researcher in the setting, I did not want her to finish student teaching thinking that in the kind of teaching recommended by reformers the teacher does not have a role--or even worse, does not have responsibilities for pupils' learning. I decided to help Maria learn more about the teacher's role and to help her reach a balance between her original images, those of a teacher's modeling steps for students, and the ones formed after watching Mrs. Kogan's teaching, of a teacher's "going with the flow." Since I knew Maria was eager to learn about teaching as much as she could, from anyone she could, I decided to propose that we have a series of conversations for the purpose of helping her integrate the conflicting images she held. For this purpose, we agreed to have conversations around a unit that we would co-plan and Maria would teach.

In deciding what unit to work on, Maria asked me if I had any preferences. I told her to think about what would make sense mathematically, taking into consideration what pupils have been taught lately, and what they had already known. Maria said:

Hmm...Let's think. We just finished a unit on place value. And they learned about base-10 numbers...and they did a unit on fractions last year, so it makes sense to teach them about decimals, which is just to extend what they learned about the whole numbers. What do you think?

I agreed. For me it was a challenge because I did not have any experience teaching decimal numbers to elementary students. The cooperating teacher agreed, too, since she was interested to learn new ways of teaching decimals. She also offered to help us in any way she could. I told Maria that as homework we should both think about what ideas, concepts, or skills we wanted to include in teaching this unit. After looking at some textbooks, Maria said she would like her pupils to learn and understand the notation, and understand the rules for performing operations with decimals. I thought that in talking about decimals, it would be important to help pupils think about the quantities represented by the notation and make connections between the written symbols and the quantities they represent. Maria wanted to know what that would mean for her fourth graders.

In thinking about how to help pupils construct meaning for decimal symbols, Maria and I decided to focus on tenths and hundredths. Maria proposed to start with tenths, and to use base-10 blocks, where the flat would be the unit, and a stick would be a tenth. I agreed, but I pointed out to her that it would be important to show pupils that the unit can change, and as it changes, the value of tenth changes. For instance, if we consider the large block as a unit, the flat will become a tenth. The idea that the value of the fraction is determined by what we consider the unit to be appealed to Maria probably because she did not think about it herself. She offered to prepare for the first day of teaching a sequence of activities that would help pupils understand the meaning of tenths in different contexts.

As she prepared to teach the first lesson on decimal numbers, Maria planned to ask pupils to construct, using base-10 blocks, representations of different decimals she read, to read representations that she constructed, and to shade in areas and fold pieces of paper in ways that different numbers would be represented. She planned for the second lesson to help pupils begin to make the connections between concrete representations and written symbols.

What Maria prepared made a lot of sense to me. However, I knew that was only the beginning. I wanted her to think about how she would know what her pupils think,

what difficulties they might encounter, and how she could help them overcome those. I said to her that those were questions we both needed to think about in preparation for teaching.

In the notes I took during and after our meetings, I wrote about Maria's astonishment when I asked her to think about what to do if people would have difficulties. "You mean, I should explain to them? I should tell them what to do?" Maria asked. In Maria's mind, "telling" was not compatible with the kind of teaching she was trying to learn--teaching by having discourse in the classroom. I was also not sure at that moment what she felt committed to: not telling, or not explaining anything to students, or helping students learn conceptual math. In other words, I did not know if she saw "not telling" as a purpose in itself, and she felt committed to that purpose, or if she wanted to teach "for understanding" and in that case, she saw "not telling" as the only means to reach her goal.

In spite of the fact that I was aware of the different ways in which she might have thought, I decided, rather than find out what she really thought, to tell her about how I saw my role as a teacher trying to teach for understanding. Although I did not tape record what I told her, I did write notes about what I said immediately after. What follows is a reconstruction of what I told her then:

I'll tell you how I think about "telling" and "explaining" in my own teaching. As I learned more and more about teaching math for understanding, I came to realize that my role is to help pupils learn mathematics in more conceptual ways, but that does not tell me too much about what kind of pedagogy I should choose. True, I figured that a lot of times that means creating a situation where pupils can discuss a problem, or can mess around with a concept or an idea without me interfering. But, as any other method, it cannot help everybody in every situation. Sometimes, depending on the situation, I would ask questions to push my pupils' thinking further, some other times I would "tell" them what there is to be seen, and some other times I would explain what I think they do not or cannot discover by themselves. I see this as part of my role: to decide what makes more sense for my students in the particular situation.

Maria did not say anything. From knowing her, I was convinced she was taking the time to think about what I just said. In the next day's lesson, I had the opportunity to

see how this conversation changed her thinking about the teacher's role. Maria started the lesson by putting a flat on the overhead. "Let's assume this is our unit today," she said. "What would you call this?" she asked, pointing to a single column in the flat. "One tenth," a child answered. "Why would you call that one tenth? Lisa?" "Because it has ten pieces in it," Lisa answered referring to the fact that there were 10 squares in each column, instead of referring to the number of columns in a flat. Maria looked puzzled. "What do other people think about that?" she asked. None of the pupils challenged Lisa's answer. Maria waited for a couple of moments. Then looked at me and smiled. "Well, what do you think about this, then?" she asked drawing on the overhead a rectangle divided into 20 parts. "What's one tenth in this case?"

From this incident I learned that Maria was beginning to think in new ways about teacher's role and also to start enacting her role in different ways. Hearing the right answer, that the column represented one tenth of the flat, did not satisfy her. She felt the need to probe Lisa's answer. Her probing proved to be fruitful: Even if Lisa offered the right answer, the reasoning behind her answer was wrong. As Maria said later, when she asked what other pupils thought about what Lisa said, she was hoping that at least some pupils in the class would challenge Lisa's answer. When that did not happen, Maria thought for a couple of moments what she should do. Thinking about the discussion we had the previous day, she decided it was her responsibility to pose a problem that would challenge Lisa's answer. Thus, she asked pupils to show the area that would represent one tenth of a rectangle divided into 20 parts. The question she posed had the potential to change pupils' minds if they thought that what makes something one tenth is the number of pieces (ten) that divide the part. In that case, pupils would shade in 10 parts out of 20 which would mean half of the total area.

Maria and I had many conversations that helped her prepare for teaching. We spent hours and hours on the phone and in meetings talking about planning, looking for interpretations of the pupils' answers, thinking about what pedagogy would make sense in

a particular situation. However, I think that the pieces of conversation I described above were the ones that constituted a breakthrough in Maria's learning about the teacher's role.

Instructional Conversation as an Occasion for Learning about the Teacher's Role

The vision of teaching mathematics for understanding includes an expanded role for teachers: facilitating student-student and teacher-student discussion, giving reflective feedback that relates to the substance of what students are saying and enhances the quality of the discussion. This requires teachers to make decisions about when to let go and encourage students to struggle to make sense of an idea, when to ask guiding questions, and when to tell students directly. Such decisions depend on teachers' understanding of mathematics and on their knowledge about how students think and construct knowledge, because they need to make judgments about the kind of things that pupils can figure out by themselves and those for which they need help (NCTM, 1991).

When Maria asked a question to challenge Lisa's answer, she enacted her role in a way that reflected a new understanding of the teacher's role, as well as a way of thinking and managing different kinds of knowledge which she was constructing through her interactions with me, the children in the class, and the mathematics in question (Zumwalt, 1989). Her new understanding, or perception, of the teacher's role included, I believe, an image of an assertive teacher who assumes responsibility for the pupils' learning, and who does what she thinks is best to facilitate that learning. Maria's assertiveness in this particular case reflected her commitment to pupils' learning, and not to what she perceived to be mathematical discourse. It signaled the beginning of Maria's perceiving herself and acting in a new role: that of the intellectual leader of a group of pupils (Dewey, 1938).

Interestingly enough, in the discussion that we had, I chose to tell her about my beliefs in relation to teacher's role in teaching mathematics for understanding, and about the place that telling has in this kind of teaching. There are two reasons why my telling her in this particular case seems to be significant. First, the consistency between what I said and

how I said it shows the ways in which I felt committed to Maria's learning. At that point my assessment of the situation was that Maria perceived teaching for understanding in a narrow, quite simplistic way. In order to enlarge her view about what teaching for understanding means and entails, I felt I needed to tell her about my views, which included aspects of teacher's role I believed she did not see. Thus, I tried to help her learn about the role of telling by telling her what my understanding is. Second, I believe, my telling played the role that modeling does in helping someone learn to teach. Maria knew I was committed to teach her for understanding about teaching mathematics in conceptual ways. When I used telling as a tool to help her with her learning, I showed her that indeed there is a place for it in teaching; I modeled for her the use of it.

In addition to helping Maria reach a new understanding of the teacher's role, our conversations also contributed to her learning how to think and manage different kinds of knowledge: knowledge about mathematics, about pupils' thinking, about pedagogy and pedagogical content knowledge. Her asking a question to challenge Lisa's answer reflected not only a way of thinking about the teacher's role, but also her understanding about decimals, about what Lisa knew and thought about decimals, and about how to represent the idea of one tenth to Lisa.

What about our conversations enabled Maria not only to construct these kinds of knowledge, but also to think and deal with them the way she did in the particular situation? It seems that the questions each of us asked, the ways in which we tried to make sense of pupils' understanding and to decide what to do next on the basis of our and pupils' understanding of decimals, helped Maria learn about the content as well as the process of teaching. From Maria's point of view, I was helping her to prepare for teaching a lesson, and at the same time I was modeling for her a way to think about teaching. In the last interview, Maria talked about her seeing me as a model of how to think about teaching:

...it's the way we worked and what questions you asked...it's all the planning that we did. Planning seems to include quite a lot. It seems that we only planned day by day or two or three days in advance. But we talked

about the kids, and what's right for the kids, and you made me think about the individual needs, about the child who was a little behind, or a little ahead of what they were being taught, and we talked about the different things we could do to help them, or find time to talk to them and work with them.

In the process of learning to think and enact her role in more sophisticated ways, Maria also came to think about her own and others' teaching in ways that reflected her learning. Her talk in December about seeing the need for a purpose as one of the most important things in learning to teach is significant in this regard:

The most important thing [in learning to teach] is to know that before you teach, first of all you need a purpose to teach. After you have the purpose, you need different plans for lessons and activities to follow up on that purpose. And you need to think about your purpose and stick with it.

I take this quote as evidence that Maria ceased to see teaching as a sequence of acts that happen by chance, just going with the flow. She began to see teaching as a purposeful act, in which she had a direction to go. In the same interview, in December, she also talked about her seeing Mrs. Kogan's teaching in a different light. Maria became aware that Mrs. Kogan was assuming an active role in her teaching, realized that questioning was an important part of that role, and came to appreciate the value of these questions in helping pupils' learn. Comparing Mrs. Kogan's teaching to her own, Maria said:

In her lessons [Mrs. Kogan's], they seemed to go with the flow, with what's going on. But that's not necessarily true, because I remember times when Mrs. Kogan had to change the direction of the talk. I did not know what to say where and to go from a certain point to another, I couldn't do that while she could. I guess Mrs. Kogan knew what questions to ask, and I never did.

CHAPTER VI

LEARNING TO TEACH IN THE CONTEXT OF TEACHING: SUMMARY AND DISCUSSION

In previous chapters I analyzed what Maria learned during student teaching and discussed specific learning opportunities that contributed to her learning. In this final chapter I will summarize what Maria learned and look across the different learning opportunities in an attempt to analyze the features that promoted her learning. I will also discuss how her personal qualities, the relationships that she built, and the resources that were available to her through the PDS contributed to her learning. I will use this discussion to draw implications for the practice of teacher education and for future research on teacher learning.

What Maria Learned

Maria's learning during student teaching corresponds to major categories of learnings that the NCTM Standards argue teachers need in order to teach mathematics in conceptual ways. These learnings have to do with mathematics, pupils' thinking, and enactment of a role that would help pupils make sense of mathematics. Maria also began to learn to be reflective about her own learning: to recognize what she knows and what she does not know and to think about ways of finding answers to questions that she learned to ask.

The data I collected do not enable me to identify all the things Maria learned. There may be many other things that she learned that I am not aware of, and there are also things that I know she did not learn. In this section I will summarize first what she learned according to the corresponding NCTM categories, and then I will discuss what she did not learn or I do not have evidence that she learned.

Learning Mathematics

Maria deepened her understanding of specific mathematical topics. For instance, she learned that rounding is context dependent and that rounding and estimation are related

to place value. She also learned that numbers can be decomposed according to the place value of their digits. More importantly, she learned what it means to know subject matter in depth, while also realizing that, for the most part, her understanding was not deep enough. Learning about a few mathematical topics in depth helped Maria develop a feeling for what it means to know mathematics in depth. This feeling helped her differentiate between situations in which she understood the mathematics in question and situations in which she did not (Schon, 1983). For example, Maria sensed when she did and did not understand the mathematics involved in pupils' strategies for subtracting four-digit numbers. When Maria did not understand, she solicited my help; when she did understand, Maria approached me simply to share her joy and enthusiasm about pupils' mathematical thinking as revealed in their strategies.

The feeling for what it means to understand mathematics in depth that Maria developed also gave her a standard of understanding to strive for and taught her that she needed to re-learn basic mathematics before attempting to teach it. Teaching a few mathematical topics to pupils helped her understand why she needed a better knowledge of mathematics herself. Maria realized she could learn subject matter through her interactions with pupils, but she also learned she needed to know subject matter to be able to hear and understand what they say.

Maria's learning about mathematics is particularly intriguing because teachers do not usually expect novices to learn subject matter while teaching it. Rather, they assume novices already possess the subject matter understanding necessary for teaching elementary pupils. Thus, subject matter knowledge is rarely the explicit focus of conversations between mentor teachers and novices (Feiman-Nemser & Parker, 1990). Maria's case not only challenges the assumption that novices already have adequate subject matter knowledge; it also shows us that novices can gain important subject matter knowledge and understandings through teaching if they have opportunities to talk about content. Planning conversations or conversations about novices' interactions with pupils can provide rich

opportunities for novices' learning of subject matter if the subject matter in question gets discussed in these conversations between novice and experienced teachers.

Learning about Pupils' Thinking

Maria also learned that, given the opportunity, pupils think in many different ways, and she learned what some of these ways are. Maria also realized that she needed to learn how to listen to her pupils. She came to see listening to pupils as an important means of enhancing both pupils' and her own learning. Listening to each other, pupils could learn mathematics. Listening to pupils, Maria could learn both mathematics and about the teaching and learning of mathematics. Through listening to pupils, Maria began to gain insights into how pupils learn mathematics as well as what she could do to help them in this endeavor. Maria also learned to value situations in which pupils talked about mathematical ideas and understandings and became interested in learning how to create such situations.

Learning to listen to pupils' ideas is important both as a means and an end. Listening to pupils helps teachers create meaningful opportunities for pupils' learning. It is also a valuable way for teachers to learn subject matter and about teaching and learning of subject matter while in the context of their own teaching (Featherstone, Smith, Beasley, Corbin, & Shank, 1993). Given that teachers do need to deepen and extend their subject matter knowledge and understandings about teaching and learning, that the opportunities to do so outside the world of the classroom are scarce, and that they take much pleasure in discovering ideas that are generated by pupils in their own classrooms (Featherstone, Smith, Beasley, Corbin, & Shank, 1993), listening to pupils' ideas can become an invaluable tool for learning for both novice and experienced teachers. Examining pupils' mathematical ideas, looking at the content through their pupils' eyes, and comparing pupils' understanding to their own can help teachers make mathematical connections they have not made before. Teachers can also gain insights into how their pupils learn mathematics and what they can do to enrich their pupils' understandings of mathematics.

Learning about the Teacher's Role

Maria came to see teaching as purposeful, with the teacher playing an important role. She understood that, in teaching, she needed both goals and means to reach these goals. She learned to develop appropriate goals for mathematics teaching and to differentiate between goals and the means to achieve these goals. For example, Maria realized that teaching mathematics for understanding is one of her goals, while telling or not telling are only pedagogical means to achieve that goal. Toward the end of student teaching, Maria learned to assume an active role as a teacher, creating thoughtful opportunities for pupils' learning.

Coming to see one's own role as promoting pupils' learning has important implications for both pupils' and teachers' learning. Teachers who see their role this way will work hard to create learning opportunities for pupils and help them make the most of these opportunities. These teachers will also have a goal for their development as teachers: to learn to create such opportunities. Just as having purposes for specific lessons gives teachers a lens through which to choose appropriate actions from an array of possibilities, having purposes for their own learning helps teachers find or construct opportunities that can help them reach their goals. Connecting their own learning with pupils' learning helps teachers learn from the very thing they are doing: learn to promote pupils' learning from creating opportunities for pupils' learning.

Learning to Reflect

I don't know how to set up the norms [for discourse], I do not know how to ask good questions, I need to work on framing questions, what should I ask first and then next, especially when the lesson takes a different turn than I expected. (Dec, 1992)

I am a lot more confident now than I was at the beginning of the year, but I am still tentative. As far as my teaching goes, I am confident, not that I know how to do it, but that I can learn how to do it. Now I know that even

if I am thrown into a totally new situation, I can figure things out by myself. It might take me a month or two to find the resources, to learn what questions to ask, to start figuring out where the kids are, but now I know that I can learn that and next year my teaching will look better, and two years from now even better, at least I hope so. (Dec, 1992)

Maria's final reflection about what she thought she learned during student teaching shows she realizes there are some important things about teaching that she learned and others that she still needs to learn. Her awareness about what remains to be learned is also a reflection of her learning--that in teaching for understanding, the kinds of questions the teacher asks are significant and asking good questions is a skill she is just beginning to learn. More importantly, Maria learned that she did not learn how to teach during student teaching. Rather, she learned how to learn to teach. In a new situation, Maria would know how to figure things out for herself, to ask appropriate questions and go about answering them, to find the necessary resources that would help her ask questions and find answers to these questions, to decide what makes sense and what does not, and to know when she has the right answer. This knowledge gives her the confidence that, in a new setting, she could learn how to teach. Maria's talk about her beginning to learn how to learn to teach is not the only evidence that she indeed began to learn that. The changes that occurred in Maria's ways of thinking and acting throughout her student teaching also speak about her learning. If in September Maria wanted to give pupils a model to follow in solving a mathematical problem and in October she was satisfied with a worksheet that reminded pupils of the rules of rounding, that was no longer the case in December. By then, Maria was interested in the reasoning behind pupils' right or wrong answers. She saw the mathematical discourse she was learning to orchestrate as a means for both pupils' and her own learning. Asking Lisa why she thought a column in the flat represents one tenth of the flat, Maria showed not only that she learned some things about teaching mathematics for understanding, but also that she found a way to learn about teaching mathematics: asking pupils questions that would open a window into their thinking.

Teacher educators cannot help novices learn everything that they need to learn in order to teach for understanding. Helping novices learn how to learn to teach seems particularly important because it enables them to continue learning on their own while teaching. Maria's case shows us what a novice can learn during student teaching and what that learning may enable in the future: Novices may learn what teaching is about and how to assess where they are in their learning of teaching. They can also learn to set goals for their own learning and how to go about reaching their goals. In addition, novices can learn how to use their own teaching as an important resource for learning about learning and teaching.

The practice of teaching involves many things that Maria did not learn. Ball (1992) discusses the considerable skill that teachers need to listen closely to a child while watching all the others, read and interpret pupils' reactions, use one's voice as a tool, and pose appropriate questions. Except for the last, which Maria knew she had not mastered, she was not even aware of the need to have these skills.

Learning to teach is a difficult and long enterprise. Novices cannot be expected to learn everything they need to learn in a limited period of time. Perhaps student teaching is a period when they can learn some important pieces about teaching, but they cannot learn to put all the pieces together in meaningful teaching acts (Schon, 1987). Perhaps they can learn only those pieces that are more obvious to them. Student teaching can be, however, the beginning of learning about teaching. Equipped with strategies to learn from teaching, novices can continue to learn and develop as long as they teach.

How Maria Learned

In previous chapters I analyzed what Maria learned from specific learning opportunities in the context of practice. In this section, I focus on how she learned, namely how particular opportunities contributed to her learning. This analysis helps us understand what different learning opportunities are good for, what can be learned from them, and

under what conditions they serve as occasions for novices' learning to teach in the context of practice.

Maria had several different opportunities to learn to teach mathematics in conceptual ways. These opportunities included (1) observations of skillful mathematics teaching and conversations around these observations; (2) co-planning conversations; (3) instructional interactions with individual pupils, followed by conversations with me or other people in the school; and (4) her own teaching and conversations around specific teaching episodes. Next, I will describe what each of these opportunities consisted of, what could be learned from them, and how they constituted occasions for Maria's learning. Drawing on Maria's learning, I will make conjectures about how each of these opportunities might help novices learn to teach in the context of practice. Since conversation and the context of her own practice are two key elements in all of these learning opportunities, I will discuss how conversations enable novices' learning in the context of practice.

Observations with Conversations

In learning to teach in conceptual ways, observations of skillful teaching may play a double role. They can offer teachers proof that it really is possible to run classrooms in ways that emphasize pupils' thinking (Duckworth, 1987). They can also help teachers construct images of what skillful performance might look like (Schon, 1987) and understand what it takes for a teacher to be able to teach that way (Ball & Rundquist, 1993; Heaton, 1994; Heaton & Lampert, 1993). Since observations are a staple in teacher education, it would be important to understand how to make them productive occasions for teacher learning.

Maria's case teaches us that observations of skillful teaching might be a powerful tool in helping novices learn new kinds of teaching, if they occur in the context of their own teaching and are accompanied by conversations that help novices understand and appreciate what they see. In the absence of these two conditions, novices might get only a

superficial image of what skillful teaching looks like, since their perceptions are shaped by their own prior experiences with teaching and learning. These experiences screen what novices are able to see and learn from observations of teaching that differs from what they have experienced as pupils themselves.

Maria observed mathematics lessons in her cooperating teacher's room and in Mrs. Kogan's room. Both observations occurred before Maria was responsible for planning and teaching her own mathematics lessons. She had opportunities to discuss with others only what she saw in Mrs. Kogan's room. Maria did not see connections between what the demonstration teacher was doing and what pupils were learning until she attempted to teach in this way herself. Observing the cooperating teacher's mathematics teaching did not make an impact on Maria's knowledge and understandings, since at that time she was neither teaching her own lessons nor being helped to understand what she saw other teachers doing. Although the demonstration lessons were preceded and followed by conversations with the teacher and other student teachers in the school, Maria still could not see connections between teacher's and pupils' actions, since at that time she was not yet in charge of planning and teaching her own mathematics lessons. Planning and teaching specific lessons pushed Maria to think about what pupils needed to learn and how she could contribute to their learning, which allowed her to begin to look for and see connections between pupils' learning and teacher's actions.

From all the demonstration lessons that she saw, Maria made the most sense out of a lesson on subtraction taught by a university mathematics educator in the cooperating teacher's room. When he offered the demonstration lesson, Maria was already in charge of planning and teaching her own mathematics lessons. Moreover, she planned a lesson on subtraction herself. The university mathematics educator also engaged Maria in a conversation before he taught the lesson, explaining to her what he was planning to do and why, and Maria and I discussed her learning after the lesson. Conversations before or after demonstration lessons did help Maria to see what the teacher was doing to help pupils

learn. However, only after she began teaching herself, was Maria able to see how the two are really related.

Of course, other factors also contributed to what Maria did or did not see in these observations. For instance, observing pupils' learning in someone else's classroom is not the same as observing pupils in one's own classroom where one has a special kind of relationship with the pupils (Featherstone, Smith, Beasley, Corbin & Shank, 1993). This special relationship with the pupils makes teachers try harder to understand and nurture pupils' ideas. Thus, observing a demonstration lesson in Mrs. Kogan's room was not an opportunity as powerful as a lesson taught by the mathematics educator in her cooperating teacher's room. In her cooperating teacher's room, Maria might have tried harder to understand what pupils were learning and how the teacher contributed to that learning.

Experienced teachers who learned to teach mathematics in reformed ways also report a lack of ability to see connections between teacher's and pupils' roles until they have first-hand teaching experience. For instance, Heaton, who observed Lampert's teaching many times, needed the context of her own teaching to realize that Lampert did more than "merely asking repeatedly what students thought" (Heaton, 1994, p. 340), a naive conception she held. Toward the end of the semester, after having some-first hand experience with mathematics teaching, Maria, too, came to see the role that the teachers she observed played in pupils' learning. She even began to think differently about the teaching she had previously observed. Thus, she began to see the kinds of questions Mrs. Kogan asked in her lessons and how these questions contributed to pupils' learning of mathematics. When comparing Mrs. Kogan's teaching with her own, she noticed that "Mrs. Kogan knew how to ask good questions, and I never did."

Co-planning

Co-planning is a special kind of learning opportunity, since it has conversation built into it. Because novices and experienced teachers need to talk to co-plan, co-planning

gives novices an opportunity to get insights into teachers' thoughts in regard to planning. These conversations may help novices understand not only what more experienced teachers do when they plan for teaching, but also how they think about planning, what they take into consideration and why (Feiman-Nemser & Beasley, 1993).

Co-planning conversations open a window not only into the more experienced teacher's thinking, but also into the novice's sense making, revealing what he or she thinks and knows. Thus, these conversations may help experienced teachers diagnose novices' needs (Schon, 1987; Feiman-Nemser & Beasley, 1993), while also opening possibilities for "scaffolding" (Vygotsky, 1978; Lave, 1992) to occur. As long as these co-planning conversations focus on designing or adapting curriculum to fit the particulars of a given situation (Clark & Peterson, 1986), novices' learning might be scaffolded in relation to major aspects of learning to teach: learning about content and its representations, about pupils and how they think, and about the connection between them.

While the data I collected do not allow me to make a claim about what Maria learned in relation to planning, I suspect that our co-planning was an occasion for her to learn some things about how to think about planning, what is important to consider, and maybe what kinds of knowledge she needed to draw on. I do know that our co-planning conversations constituted an important occasion for Maria's learning of subject matter.

Maria's request to help her find a good activity through which pupils could learn place value up to millions made me think that she might not have had knowledge about the important ideas and concepts related to the idea of place value. Since I did not have at my fingertips that knowledge either, I recommended that she use different resources and come up with a list of important concepts that fall under the idea of place value. Finding relationships among the different concepts that Maria came up with and thinking about why they might be important helped Maria realize that place value contains much more than knowing the names of the places and the relationship between them, a conception that she held.

Looking at the content from the pupils' point of view was usually the focus of co-planning of smaller units of teaching, like specific lessons. In these conversations, we usually moved back and forth between discussing our own understanding of the content to be taught and how it could be represented for the children in the classroom. For instance, we began our conversation on rounding by discussing an activity Maria prepared for pupils to help them learn how to round. The conversation around this activity revealed that Maria did not know an important piece of subject matter, namely that rounding is context dependent. Thus, we temporarily left our conversation on how to represent the content in order to understand what the content was. Maria's new understanding about rounding had an immediate influence on her pedagogical decisions. She began to think about activities that would help pupils understand what she just learned: that when and how to round depends on the context in which we round.

Instructional Conversations with Pupils

Through interactions with individual pupils in the context of instruction, novices can begin to understand what is involved in listening to pupils and making sense of what they say. Novices can then begin to think about responding in ways that enhance both their own and pupils' understanding. These interactions may also contribute to novices' learning of subject matter as well.

Maria's conversations about subject matter with individual pupils in the classroom were occasions for her, first of all, to confront her own assumption that she already knew how to listen to pupils. Listening to pupils while they worked on mathematical problems and experiencing the frustration of not being able to make sense of what they say made her realize that she needed to learn how to listen, something she had not considered before. Through conversations with specific pupils, which she had mainly to help them work on mathematical tasks, Maria learned how to elicit more information from the pupils to help

her make sense of what they say, as well as how to respond to them in a way that would push their thinking further.

Listening to pupils' mathematical ideas and making sense of them is not only a vital skill in learning to teach for understanding. It is also an invaluable and rewarding way for teachers to enhance their own mathematical knowledge and understanding. Using pupils' ideas as a means of learning mathematics provides an opportunity for teachers to learn content while teaching it (Featherstone, Smith, Beasley, Corbin, & Shank, 1993). By analyzing closely Corey's strategy of subtracting four-digit numbers, Maria learned something about the decomposition of numbers. Looking at how Justin solved a problem by addition instead of subtraction, as she thought he would, made Maria think about possible connections between addition and subtraction. Thus, through her conversations with pupils about their own mathematical ideas, Maria learned mathematics she did not learn prior to student teaching but that she needed to know.

Experienced teachers who learned to teach mathematics in new ways began to listen to and celebrate pupils' ideas. Their motivation to change their mathematics teaching and to learn more mathematics was fueled by pupils' newly visible ideas (Featherstone, Pfeiffer, Smith, Beasley, Corbin, Derkensen, Pasek, Shank & Shears, 1993; Schifter & Fosnot, 1993). Similarly, Maria's learning was also propelled by pupils' mathematical ideas. She felt wonder and joy at the mathematical richness of pupils' ways of thinking, and she felt compelled to understand their ideas and to create conditions for pupils to generate them. The excitement and pride Maria felt when she created occasions for pupils to have these "wonderful ideas" (Duckworth, 1987) was one of the main rewards for all the hard work Maria invested to learn to teach in adventurous ways.

Maria's conversations with me around her interactions with specific pupils in the classroom contributed to her learning in an interesting way. By pushing her to think a little harder about the mathematics involved or telling her what I thought children meant, I helped Maria make sense of what she heard pupils say. I discuss how these conversations

contributed to Maria's learning in the next section when I focus specifically on conversations. But simply the presence of an interested audience helped Maria learn in an important way. Knowing that I was interested in children's thinking and that she could always share with me her discoveries about pupils' ideas stimulated Maria to work hard to see and hear more.

Teaching and Conversations Around Specific Teaching Episodes

First-hand teaching experience can provide occasions for novices to learn to create opportunities for pupils' learning. In learning how to do this, novices learn to use "in action" (Schon, 1987) different kinds of knowledge they acquired in theory, among those, knowledge of subject matter, pupils' thinking, and content related pedagogy. Through their own teaching, novices make transparent some of the images and beliefs they hold. When an interested other is there to discuss and, if necessary, challenge these beliefs, novices can enrich their views and develop their practice.

Maria's first attempts to teach mathematics helped me learn that, in spite of her description of how she saw the teacher's role, she held strong images of a mathematics teacher who gives students a model to follow. Her attempts to teach for understanding after observing the demonstration lesson made me think that she saw teaching mathematics for understanding as a form of teaching in which the teacher has no role. These two conflicting images of teaching mathematics made visible mostly through Maria's actions concerned me and led me to start thinking about ways in which I could help her reconcile the images she held, as well as gain a deeper understanding of this kind of mathematics teaching. This is how I came up with the idea of co-planning and helping Maria teach the next unit.

My talking about how I see my role in teaching mathematics helped Maria begin to think about her role; however, in the context of her own teaching, she had opportunities to refine and learn to enact that role. When a child expressed a mathematically incorrect idea,

which was left unchallenged by her classmates, Maria gave herself permission to address the particular conception. By doing so, however, she had to pull together all kinds of knowledge that she had gained. When asking a question to challenge the child's conception, Maria had to consider what the conception was about, what pupils might be thinking, how she wanted them to think, and what representation would best help them move toward that goal. Among other kinds of knowledge that she used, Maria pulled together knowledge about content, pupils' thinking, mathematical representations, and appropriateness of these representations to the situation at hand. Thus, while learning to define and enact her role as a teacher, Maria also began to learn to use in action the different kinds of knowledge she was beginning to acquire.

How Did Maria Learn?

The conversations that contributed the most to Maria's learning focused on specific issues that grew out of her teaching practice. Some of these conversations occurred as a result of Maria's questions about issues that intrigued her, like a child's mathematical thinking or the relationship between estimation and place value. Other conversations, initiated by other people in the setting, also emerged from issues in Maria's practice. For example, I initiated conversations when I thought Maria may be missing something important in a particular situation. The conversation about the teacher's role is one such example.

The most productive conversations in terms of Maria's learning were those in which another person attended to Maria's thinking in relation to the issues discussed. For instance, in the conversations that Maria and I had about rounding, I learned that Maria thought rounding was a rule to follow. This realization made me want to enlarge her understanding to include the idea of rounding as context dependent. It also helped me choose examples that could push her thinking in that direction. I chose examples in which Maria had to decide how to round or if she needed to round at all, considering the situation

in question. By contrast, in the conversation where a group of student teachers discussed pupils' understanding of place value, no one attended to Maria's thinking about place value. Maria repeatedly said she did not understand a child's strategy of adding numbers that consisted of breaking down numbers according to the place value of their digits. Other student teachers tried to explain the strategy without finding out what Maria thought about decomposition of numbers. Thus, she left the conversation confused: "But I still don't understand it! How can you break down numbers like that?"

In many of these conversations, other people made their thinking visible to Maria. For instance, the math educator who taught a lesson on subtraction with regrouping in the cooperating teacher's classroom let Maria know his assumption about pupils' understanding: "It might also be they do not realize that borrowing involves finding a different representation for the same number." I also made my thinking visible to Maria, sometimes in direct ways, sometimes in not so direct ways. Telling her how I thought about the teacher's role in teaching for understanding is an example of letting Maria know my thinking in a direct way; accepting her new mathematical understanding in relation to a specific topic that we discussed was a less direct way to let her know how I thought about the issue involved.

In the conversation in which Maria and I began to map out the territory covered by place value, I constantly asked questions to uncover her understanding. Questions like where rounding and estimation would fit on the map that we were drawing or how borrowing and trading fit together with rounding and estimation allowed me to help her think about these issues while also gaining an understanding of where she was in her thinking as our conversation was progressing. Sometimes Maria also took a leading role in checking where her understanding was in relation to others'. For instance, when she and I discussed rounding, she repeatedly asked me what I meant by my questions. Although she did not say so, it might also be the case that her contributions to the discussion were motivated by her desire to compare her understandings with mine.

Perhaps the most important characteristic of the conversations was that she found them intellectually stimulating and quite enjoyable. She learned that many ideas are more complex than they originally seemed. Seeing some of the complexities and thinking about them herself gave her a sense of joy. When she exclaimed, "Wow, this is great! I have never thought about it this way!" I sensed that indeed she was seeing an old idea in a new light. Maria used her new understandings to further generate other ideas, particularly the pedagogical implications of what she had just learned. For instance, understanding a child's strategy in solving a mathematical problem caused Maria to think about how she could use that strategy to help other children work with big numbers. Maria's new ideas further revealed her new understandings.

Why the Context of Her Own Practice

In the context of her own teaching, Maria learned important things about teaching mathematics for understanding. Some, like enacting the teacher's role, could be learned only in situ. Others, like subject matter, could be learned in other settings. Still others were relearned or refined in the context of teaching. Learning about pupils' thinking is one such example.

Why was the context of her own practice so critical for learning? First, it created certain learning opportunities by presenting specific challenges or puzzling situations to be unpacked. For instance, Maria began to understand and appreciate the complexities of addressing a child's mathematical conception while attending to the other children in the classroom only after she was in that situation herself.

Second, the fact that Maria was teaching made her more receptive to learning opportunities outside of her practice by providing lenses through which she could interpret these opportunities. For instance, observations of mathematics teaching through discourse did not contribute much to Maria's learning until she attempted to teach mathematics that way herself. Only then could she see what there was to be learned. Through her own

attempts to teach, Maria learned what she needed to pay attention to and how to make sense of what she sees.

Third, in the context of her own practice, others could act as "coaches" (Schon, 1987), helping Maria learn how to teach. Through her actions and talk, Maria opened a window into her ideas, beliefs, knowledge, and thoughts, thus enabling the people working with her to see what she did or did not understand and know how to do. These people could act then in ways that would challenge or support her understandings and ways of acting. Maria could also further act or talk, thus revealing to us her new understanding of what was discussed. For example, Maria's apparently random choice of items on a quiz she gave to pupils made me inquire about the reasons she had in making these decisions. My question helped Maria realize that she needed reasons for her actions. The next homework she gave to students reflected this new understanding.

Last but not least, teaching provided Maria with both the responsibility and the motivation to do her best to learn. Feeling responsible for pupils' learning, Maria wanted to create the best opportunities she could for them. She did not feel, for example, that she could teach content that did not make sense to her. Maria had to find resources that would help her understand the content a little better, even if that meant more time and effort on her part. Discovering the exciting world of pupils' mathematical ideas motivated Maria to try hard to create opportunities for pupils to have these ideas. Outside the context of her own practice, Maria could not feel the same kind of responsibility for pupils' learning and motivation to help them learn.

Learning in the context of teaching offers specific practice-related situations to think and talk about and with other practitioners. Through these practice-centered conversations, novices can learn a lot about teaching. What they actually learn, however, also depends on who they are as people (e.g., the knowledge, skills, and dispositions they bring to the experience), the kinds of relationships they build with other practitioners, and the resources they can mobilize on behalf of their learning. Having discussed the learning opportunities

that contributed to Maria's learning, I now consider Maria as a person, the relationships she formed with other practitioners in the setting, and the resources for learning available to her in the PDS setting.

What Influenced Maria's Learning

Personal Traits

The data I collected do not enable me to make conjectures about Maria's knowledge and skills and how they enabled or inhibited her learning of teaching. My knowledge about Maria's mathematical knowledge and understandings is also limited. Except for what was revealed in our conversations and her reflections on her own mathematical experience, I do not have a full picture of what her mathematical understanding entailed. The data that I do have lead me to conject that, in general, her mathematical understanding was limited, rule-bounded, and procedurally oriented.

Maria's mathematical understanding had an impact on both the kind of learning opportunities that were available to her and what she made of these opportunities. Since I tailored learning opportunities to fit where she was in her understanding, a significant part of these opportunities focused on helping Maria gain a better understanding of the mathematical topics involved. Thus, very important issues at the heart of teaching and learning did not get discussed. For instance, issues related to teaching as a political act or the moral and ethical dilemmas of teaching never came up in the discussions that I or other people in the setting had with Maria. Therefore, Maria missed learning about them at this stage in her professional development.

Maria's understanding of subject matter also limited what she made of some learning opportunities. For example, talking to pupils, she could only hear what her mathematical understanding allowed her to hear. The lack of a flexible repertoire of mathematical representations kept her from being able to help pupils learn and from understanding how pupils responded to the tasks she constructed for them.

If limited subject matter knowledge and understandings inhibited Maria's learning, her dispositions and personal qualities allowed her to make the most out of the learning opportunities that she did have. Her dispositions toward learning enabled her to search and create learning opportunities for herself. Maria's personal qualities made her seek the best for her pupils, which included her becoming a better teacher.

Maria's dispositions toward learning contributed enormously to what and how she learned. Her eagerness, willingness to take risks and not to know, and the seriousness with which she treated her own learning helped her take full advantage of the learning opportunities that she or others created for her.

Maria's eagerness to learn made her seek and create learning opportunities for herself. Maria welcomed any kind of learning opportunity available to her and searched for resources that could afford her new opportunities. Maria's calls to me late at night to discuss lesson plans that she was not sure about or the long conversations about teaching and learning that she initiated with me after school attest to her eagerness to learn.

Maria's willingness to take risks was remarkable. She was ready to talk about her ideas with anyone who would listen. She was also willing to experiment with new ways and ideas even if they involved a certain amount of risk. For instance, she agreed to orchestrate mathematical discourse in her lessons though she felt unsure about the mathematical content and how pupils would respond. Maria also took risks in conversations with other student teachers or with more experienced practitioners, admitting that she did not understand the topic discussed and requesting additional explanations.

Maria definitely took herself seriously as a learner. Aware of what she knew and what she did not know, she searched for ways to learn what she thought she needed. She was also very active in recruiting resources for learning and knew how to make the best use of them.

The pleasure that Maria took in learning not only made the experience very enjoyable for her, but also had an impact on others, thus influencing the learning

opportunities available to her. Maria's enthusiasm for learning fueled my desire to do the best I could to help her learn. Maria's joy was contagious: It made me spend countless hours in conversations with her or planning activities that would help her learn.

Maria's many personal qualities also played an important role in her learning to teach. Care and love for children, curiosity and generosity about how they think, as well as seriousness and commitment to children's intellectual development were only a few of the qualities that fueled Maria's learning. For instance, curiosity about children's thinking made Maria listen carefully to what children had to say. Generosity in listening to pupils led her to appreciate their ideas and take them seriously even if she did not always understand what they meant.

Most of these qualities were already part of Maria's attributes before student teaching; however, in the context of teaching, she nurtured and transformed them. For example, Maria extended a general sense of care for children into care for the development of their minds. Listening to pupils' ideas simply because she cared about them led her to discover a new and wonderful world of pupils' thinking. The more she listened, the more curious she became about these ideas, and the more interested in learning to create conditions for pupils to have and express their thinking.

Maria's commitment to children made her want to create the best opportunities for them. Thus, she did not choose to teach something that did not make sense to her. Instead, she recruited me to help her with tasks she did not know how to approach. The fact that she insisted that I help her, in spite of the fact that she knew that I was there to study her learning, speaks not only of her capacity to mobilize resources on behalf of her learning, but also of her seriousness and commitment to children. Her seriousness was further reflected in our work: Maria continued to request my help even if our collaboration put more demands on her in terms of time and amount of work she needed to do to in preparation for teaching.

Maria's case is not unique. Like Maria, many beginning teachers lack basic subject matter knowledge and skills and also other kinds of knowledge crucial to teaching and learning teaching. Like Maria, other novices also possess important personal qualities and have dispositions that can promote their learning. Teachers and teacher educators need to find ways to discover these qualities and foster them when helping novices learn to teach. They can nurture these qualities if they already exist or enhance them if weak. Ultimately, these kinds of qualities help novices learn to create learning opportunities for themselves and make the best use of these opportunities.

Professional Relationships

Though many people contributed to Maria's learning in some way or another, she did not build strong and sustained working relationships with them all. Mr. Miller, for instance, the university math educator, came to school only once a week, and even then he worked mostly with Maria's cooperating teacher and only occasionally with Maria. Mrs. Kogan, the third-grade teacher who offered the mathematics demonstration lessons, did not interact with Maria beyond the week of the demonstrations. The people who worked more intensely and over time with Maria in mathematics were the cooperating teacher, Mrs. Barnes, and I. In the next section, I will analyze the relationship that Maria built with me and the cooperating teacher and how they contributed to or inhibited her learning.

This is how Maria described our relationship in an interview taken at the end of the semester:

I see you as a friend and as a teacher. As a friend because you always make time for my concerns regardless of how busy you are. As a teacher because that's how it feels to me, you are teaching me how to teach math....And I felt you always respected me. Right after I taught a lesson, you asked me how I thought it went, or what I thought. And that made me feel that my opinion counts. And I felt it was easy for me to talk to you, and I could joke around and not feel stupid that I do not know something. I am not expected to really know something, unless I try it.

Maria appreciated the time we spent together and felt respected and her opinions validated. She felt she could talk freely about what she did not know, since I would use it only to help her learn. Also, since no formal evaluation was involved, it was probably safe for her to make mistakes and not to know certain things.

Maria brought to the relationships a lot of commitment and trust. She trusted me that I would use the best of my knowledge and skill to support her learning. She also trusted that our collaboration would not jeopardize her relationship with the cooperating teacher, who would continue to offer support. Thus, when I changed Maria's plans right before the beginning of the lesson, advising her not to give pupils a model to follow, she agreed to do so because she thought if something went wrong, either I or the cooperating teacher would intervene and save the lesson.

Maria and I were engaged in a joint inquiry about teaching and learning. Neither Maria nor I had ready-made answers in relation to problems and issues that arose in the context of practice. Together, we learned to ask questions and construct solutions for the specific situations we encountered. Although I knew more about mathematics and about teaching and learning of mathematics than Maria did and I was a more experienced inquirer, I still needed the context of Maria's learning to teach to push my thinking further. For instance, Maria's questions in relation to rounding helped me learn what it means to understand rounding, as well as what might be difficult or worth learning about it. Discussing with Maria mathematical concepts and ideas offered me an opportunity to gain insights into what it means to understand the issues discussed (Simon, 1995).

The issues that arose as she worked with subject matter and pupils, as well as the questions that she asked of me, made me think about teaching in ways I have never encountered. As much as Maria needed the context of her own teaching to learn how to teach, I needed the context of Maria's learning to teach to learn both about teaching and teacher education.

Maria finished student teaching feeling that her relationship with the cooperating teacher regarding math teaching could have been much more productive for her learning than they were. She felt she did not have the same opportunities to talk and struggle with Mrs. Barnes that she had in other subject areas. She felt separated, having to find resources on her own to help her learn how to teach. Maria saw her lack of knowledge and skill in relation to teaching mathematics as separating her from Mrs. Barnes and causing Mrs. Barnes to leave Maria to her own devices.

I felt [in math] we did not really work together. [Mrs. Barnes] was somehow separated from me. Sometimes I was really confused and I did not know even how to ask questions and that frustrated her a lot...I would have liked if [Mrs. Barnes] had been struggling with me more and talking with me more instead of being so separated from me. [Dec, 1992]

In part, Maria's assessment was right; Mrs. Barnes told me several times that she felt frustrated and disappointed about how little Maria knew in relation to the teaching and learning of mathematics. She felt much more comfortable in relation to Maria's capabilities in other subject areas. Mrs. Barnes also felt that since I could help Maria, she could concentrate on other subject areas. She welcomed my helping Maria and tried to create conditions for Maria and me to work together.

My being in the setting and helping Maria learn to teach might also have contributed to their separation. If I had not helped Maria, the cooperating teacher might have felt more responsibility toward Maria's learning. This is hard to say, especially since I began to intervene only after I realized that the cooperating teacher was not helping Maria learn to teach mathematics. It might also be the case that since Mrs. Barnes felt less sure about her mathematics teaching than about teaching other subject areas, she also felt less comfortable sharing some of her struggles with Maria.

Maria's relationship with Mrs. Barnes both contributed to and inhibited her learning to teach mathematics. The lack of collaboration, the separateness that Maria felt, did not help Maria's learning. The cooperating teacher's feelings of disappointment and frustration

did not contribute to her spending more time helping Maria learn to teach mathematics. However, their relationship was based on enough trust and respect to allow me and other people to help Maria. The cooperating teacher created conditions for Maria to spend time with others, allowed and even supported her experimentations with ideas in the classroom, and was willing to save the lesson if things went wrong.

The Professional Context

As a professional development school, the setting where Maria did her student teaching contributed to Maria's learning in two main ways: making available important resources that Maria could and did use in her learning to teach mathematics for understanding, and enabling her to participate in a culture of collaboration and conversation. Although the teachers in the school were in the process of learning to teach mathematics for understanding, they had already made significant changes in their practice. This enabled Maria to observe and discuss a kind of mathematics teaching that was moving toward the teaching she was trying to learn. In addition, Maria benefited from the help of a mathematics teacher educator who came to school regularly to assist teachers, including her cooperating teacher, in their efforts to learn to teach in more conceptual ways.

As a result of the culture of collaboration in the school, Maria and other student teachers could observe and discuss demonstration lessons offered by a teacher who was further along in the process of changing her teaching of mathematics. This activity, that required all student teachers to spend time in another teacher's room who was also recognized as more advanced at this kind of practice, was supported by all teachers in the school. The cooperating teacher's openness to my helping Maria, and even her willingness to support our work in any way she could, is also a reflection of the culture of collaboration prevalent in the school.

While participation in a PDS may mean that teachers are engaged in learning reformed kinds of practice, they may not be able to use what they have learned to help

novices learn such a demanding form of practice. Maybe because the cooperating teacher was not aware of the process she underwent in changing her own practice and how hard it was, or did not know how to use it to help Maria learn, she felt somehow disappointed about what Maria knew and did not see herself as teaching the novice what she did not know. Also, partly because this kind of teaching is so difficult, and partly because Maria, like many other novice teachers, lacked important kinds of knowledge when she came to student teaching, supporting her learning required a great amount of time. The cooperating teacher could not fit this time into her already busy schedule.

Professional development schools can indeed support novices' learning to teach in important ways, but that does not mean that novices' learning to teach is guaranteed in such schools or that other schools cannot provide conditions that support novices' learning. School schedules that allow experienced teachers to spend a considerable amount of time with novices, experienced teachers who have already moved toward more conceptual forms of practice and who have ways of helping novices learn to teach conceptually oriented forms of practice, and a school culture that encourages teacher collaboration are among the factors that contribute the most to novices' learning.

Implications for Practice and Research

What novices can or cannot learn during student teaching and how they may learn that has serious implications for the kind of student teaching that teachers and teacher educators construct to support novices' learning. Maria's experience helps us realize that learning to teach is a long and difficult process. Though this process does not start or end with student teaching, student teaching plays an important role in novices' learning to teach, since in this context novices may best learn certain things as well as relearn some of the things that they have learned only at a theoretical level in university courses. Given the many things that novices need and could learn during student teaching, student teaching

should be an experience that extends over a significant period of time. A student teaching of ten weeks is not nearly enough for novices to learn to teach.

Time alone is not enough. In addition to time, novices need many and varied opportunities to learn. These opportunities consist of observation of skillful practice, participation, with support, in teaching related activities such as planning of instruction and instruction itself, and instructional conversations with individuals or small groups of pupils in the classroom. Novices need to talk with more experienced practitioners in order to make sense of the learning opportunities available to them. Cooperating teachers or other teachers in the school need to help novices learn from what they experience. This includes helping novices see and hear what there is to be seen and heard as well as assist them with teaching activities.

To be able to help novices learn, experienced teachers need to know where the novices are in their thinking and how they make sense of what they encounter. Thus, joint activities which involve talking, such as co-planning, are especially important for novices' learning because through these activities participants have access to each other's thinking. Such activities do not only allow novices to learn to do tasks that are important in themselves and learn how experienced teachers think about doing these tasks, but also make their own thinking visible to the others. Understanding how novices think helps experienced teachers create learning opportunities that would support novices' learning better.

Assisting novices to learn to teach for understanding is not an easy task. Cooperating teachers need to devote a large amount of time to support novices' learning. In an already busy schedule, it is very hard for practitioners to find time to talk to their novices outside the lessons, to help them prepare for classes, or to discuss events that may be important for novices' learning.

Though building more time into the teachers' schedule to allow them to support novices' learning is helpful, it does not tell them how to use the time. Experienced

teachers who teach in more conceptual ways do not necessarily know how to help others learn to teach in the same way. Practitioners in charge of helping novices learn to teach need time and opportunities themselves to learn how to help novices learn to teach. As teachers need to learn to listen to their pupils and create opportunities to push pupils' thinking forward, cooperating teachers need to learn where their novices are in relation to teaching and what they can do to move them forward.

This study helped us understand what a novice teacher learned about teaching and learning of mathematics in the context of practice and what learning opportunities and conditions supported this learning. Further research is needed to help us understand better what novices who learn to teach mathematics for understanding need and can learn in the context of practice as well as how their learning might be promoted. Researchers need to pay attention not only to the learning opportunities and the conditions that support or inhibit novices' learning, but also to the processes through which novices' learning occurs. Research that focuses on how novices make sense of specific learning opportunities can help teachers and teacher educators create learning opportunities that support novices' learning.

Research is also needed to understand and improve the practice of cooperating teachers in charge of helping novice teachers learn to teach in ways advocated by reformers. First, we need to understand what thoughtful cooperating teachers do to support novices' learning, and what novices learn as a result. Then, we need to understand how cooperating teachers could learn how to support novices' learning. As we need to gain insights into how novice teachers learn to teach in ways advocated by reformers, we also need to understand how cooperating teachers learn to support novices' learning.

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