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DYNAMIC EFFECT OF TRAFFIC LOADING ON HIGHWAY BRIDGES

By

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ABSTRACT

DYNAMIC EFFECT OF TRAFFIC LOADING
ON
HIGHWAY BRIDGES

By

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Bridges like other structures are designed to sustain a variety of loads and load combinations including those due to their own weight, traffic, snow, wind, earthquake and more. It is the purpose of the present study, however, to focus on highway bridge traffic loads in a very fundamental manner. When a highway bridge is subjected to a moving load, the deflection and the stresses caused may be significantly higher than when the bridge is subjected to stationary loads. The mass and damping properties of crossing vehicle(s) in addition, become intrinsic to the vibrating system changing the fundamental frequencies from that determined for the structure alone. In the design of bridges, the effect of moving loads has been and continues to be represented by a 'dynamic load factor'(DLF), historically termed as "impact" factor. A review of theoretical and analytical methods used to determine the DLF is herein presented and the results compared with values obtained from current bridge design codes of the USA, Ontario (Canada) and Pakistan. Correlation of the theoretical approach with code specified values is also discussed.

Dedicated

to

my parents who continue praying for my success

and

late Brigadier Muhammad Yousaf

I have no words to express the respect and love I have for him in my heart.

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Muhammad Uzair Khan

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INTRODUCTION

1.1 General

Bridges like other structures are designed to sustain a variety of loads and load combinations including those due to their own weight, traffic, snow, wind, earthquake and more. It is the purpose of the present study, however, to focus on highway bridge traffic loads and to do that in a very fundamental manner.

1.2 Historical Review of Bridge Loads

1. Prior to the Industrial Revolution, say 1800, bridges were built of stone and timber. Traffic loads were relatively small in comparison with the mass of the bridge and moved slowly. The designer's primary concern was to design the bridge to support its own weight.

2. Over the 19th century and until after World War I several events took place successively to dramatically change bridge design:

   a. New Materials: The discovery of new construction materials such as cast iron, wrought iron, reinforced concrete, and steel lead to lighter bridges.

   b. Railroad: Introduced significantly heavier and faster traffic loads.

   c. Bridge Failures: Large number of bridge failures took place, indicating a lack of sufficient bridge design standards and codes.

   d. Analysis methods: Analysis for static loads developed rapidly. Analysis for dynamic loads lagged very badly. Designers attempted to overcome their lack of knowledge of dynamic effects with empirical methods; they assumed loading due to dynamic effects could be handled as if they were static and as a function of the span length only.

e. Bridges were mostly property of the railroad companies.

3. 20th century - post World War I:

a. Development of prestressing (lighter weight concrete bridges).

b. Development of highways, auto and truck loads.

c. In the latter part of period - development of dynamic loading analytical methods

- adoption is slow.

d. Development and adoption of standards and codes.

e. Highways and bridges are publicly owned (state-owned), control of loads becomes a policy matter.

f. Design methods, especially codes and standards, continue to be strongly affected by the 19th century empiricism.

4. Trends for the 21st century:

a. Higher speed traffic.

b. Higher loads - possibly.

c. Use of lighter materials of construction.

d. Computer design tools that fully utilize dynamic loading and corresponding response of bridge.

1.3 Dynamic Response of Highway Bridges

The primary function of a bridge is to bear traffic, and since traffic moves, it is a time-variant or dynamic form of loading. The displacement of a bridge subjected to dynamic loading is quite different from its behavior subjected to static loading even due to the same force, unless the duration over which the time-variant force is applied is quite long in comparison with the fundamental period of the structure. The dynamic effects of traffic loads can be significant. Their one special feature is that they are variant both in time and space [Fryba 1972]. The behavior of a bridge structure under the action of a moving load is referred to as its 'dynamic response' and specifically signifies its displacement or another stress resultant as it varies with time. These are important considerations in the design of bridges because of the following:

a. The dynamic interaction between the vehicle and the bridge can result in significant deflections and stresses in the bridge structure. Accordingly, the bridge has to be designed for these increased or amplified values. Resonance or near resonance conditions, that is when the frequency of the loading function approaches a fundamental frequency of the structure, can result in significant increases in these values.
b. Certain combinations of frequency of vibration and amplitude may cause discomfort and adverse psychological effects to pedestrians as well as other road users. [Kashif 1992]
c. The trend for faster and heavier vehicles and the use of ultimate-strength design criteria by most of the bridge design codes permitting possibility of lighter weight

more slender structures, which are more susceptible to the dynamic effects of moving loads. [Saadeghvaziri 1993; Humar and Kashif 1993]

The dynamic response of bridges to moving loads depends upon the interaction of the vehicle and the bridge which is a complex dynamic phenomenon [Humar and Kashif 1993] because of the large number of different parameters which influence the dynamic response. The major parameters affecting the dynamic response of the bridges to vehicular loads are [Wright and Green 1959; Nowak and Hong 1991; Humar and Kashif 1993, 1995; Huang et al. 1993, 1995; Saadeghvaziri 1993; Chang and Lee 1994; Yang et al. 1995]:

1. Bridge structure characteristics:

    a) the bridge dimensions

    b) the support conditions

    c) mass distribution

    d) stiffness distribution

    e) structural damping

In the analysis, parameters 'a' to 'd' are generally represented in terms of the mode shapes and the frequencies of the bridge structure.

2. Vehicle characteristics:

    a) the number of axles,

    b) the axle spacing and its relation to the span length,

    c) the mass and springing of the vehicle and their distribution to axles

    d) the natural frequency of the vehicle

e) the degrees of freedom assigned in the model, and

e) the damping (provided by the shock absorbers as well as interleaf friction in the springs).

3. Speed (velocity) of the vehicle,
4. Ratio of the weight(s) of the vehicle(s) to that of the bridge
5. Surface roughness of the bridge as well as the approach
6. The path followed by the vehicle, while passing over the bridge
7. Frequencies of the application of the vehicle load due to multiple axles,
8. The number of vehicles on the bridge and their relative positions
9. Forces generated by the braking (deceleration) and acceleration of the vehicle,
10. The motion induced in the bridge before application of the load under consideration, particularly in the case of continuous bridges.
11. The speed parameter i.e., the ratio of driving frequency of the vehicle to the fundamental frequency of the bridge.

1.4 Importance of Dynamic Analysis

The dynamic analysis of highway bridges subjected to vehicular traffic is essential for:

1. Accurate understanding of bridge behavior subjected to traffic loads
2. Providing criteria for the accurate design of bridges
3. Research to develop more accurate and simplified techniques for analysis and design

4. Preparing for future developments i.e.,

    a. higher speeds

    b. possible heavier loads

    c. lighter bridges

    d. higher strength conventional materials

        1) concrete; reinforced, plain or prestressed

        2) steel

        3) timber

        4) combination of the above

    e. new materials

        1) metals (e.g., aluminum)

        2) composites (fiberglass, fiber-reinforced concrete, fiber-reinforced-plastics)

In order to have a better and more accurate understanding of bridge behavior subjected to traffic loads and to provide criteria for the accurate design of bridges subjected to moving loads, research has been underway from the beginning of the railway era (cc 1820) searching for reasonable techniques to predict the dynamic response of bridges under the action of moving loads. Accordingly, numerous (analytical, model test and field) studies have been carried out to develop a relationship between the static and the dynamic responses of a bridge, and consequently to propose a simple and reliable procedure to incorporate the dynamic effect of moving loads in the design of bridges.

This approach ignores many of the dynamic features listed above such as frequency relationships.

Historically, a relationship misnamed in literature generally as an 'Impact Factor', and more exactly as the "Dynamic Load Factor" (DLF) has been used to simplify, or over-simplify, the design. It is one purpose of this study to ascertain the limits of this procedure and to suggest alternatives for greater accuracy. This factor is numerically equal to the ratio of the maximum dynamic response due to a vehicular load to the maximum static response produced by the same load acting statically minus one. In general this factor is represented as a percentage of the maximum static response.

Attempts were made to relate the "impact factor" with only one of the influencing parameters (most probably either due to lack of knowledge during the initial period of investigation about other influencing parameters, or for the ease of application in design offices) most notably, the span length and the natural frequency of the bridge. These factors were adopted by some highway bridge design code authorities as well, for example, some codes specified 'impact factors' in relation to the natural frequency of the bridge [OHBDC 1983], and others with reference to the span length [AASHTO 1992; CPHB 1967] of the bridge. Later investigations [Boehning 1953; Eichmann 1954; Wright and Green 1959,1964; Walker and Veletsos 1966; Walker 1968; Tan and Shore 1968a, b; Nieto-Ramirez and Veletsos 1966; Veletsos and Huang 1970] revealed that the dynamic response depends upon a number of other factors, given above, as well. Some researchers investigated the 'impact factors' incorporated into codes [AASHTO], and showed that the code specified values underestimate the dynamic response in certain cases (for example, for the short span bridges), and overestimate in others [Fleming and Romualdi 1961; O'Connor and Pritchard 1985; Ibanathan and Wieland 1987; Galdos et al. 1993; Huang et al. 1993; Yang et al. 1995]. The relevant code specifications regarding the 'impact factor' that is, the dynamic response of bridges to traffic loads were changed later on [OHBDC 1991; AASHTO LRFD 1994].

1.5 Measurement of Dynamic Load Effect

The dynamic effect of moving loads on a bridge, as represented by a dimensionless factor, may be mathematically defined as the increase in the maximum response (deflection, moment or shear) of a bridge structure due to a moving load, over the maximum response (deflection, moment or shear) of the structure caused by the same load when acting statically. This is usually represented as a percentage of the maximum static response, may be termed as dynamic increment, DI. Symbolically the dynamic increment, DI, is given by the percentage

$$DI = \frac{R_{dm}(x_i) - R_{sm}(x_i)}{R_{sm}(x_i)} X\,100$$

where $R_{dm}(x_i) = \max[R_d(x_i, t_j)]$ is the maximum amplitude of the dynamic response,

and $R_{sm}(x_i) = \max[R_s(x_i)]$ is the maximum static response of the bridge at section x_i under the action of moving vehicles. Bridge design codes generally express the dynamic effects of moving vehicles in terms of DI. On the other hand, theoretically the dynamic load effects are computed in terms of 'Dynamic Load Factor' (DLF), defined as the the ratio of maximum dynamic deflection to the maximum static deflection under the same load.

1.6 Objective

The objective of this study is to review the most basic approach for the analysis of the dynamic response of highway bridges under the action of vehicular traffic and to compare it with current practice; this includes:

a. A review of theoretical basis to obtain the DLF

b. Current code design specifications for dynamic effect of vehicular loads, and

c. To look for the correlation between (a) and (b) above.

To reach the objective, basic concepts pertaining to the representation and measurement of the dynamic effect of moving loads have been described in chapter 2. These concepts include the description of different methods (empirical, theoretical, model tests, and field tests), as well as the specifications of the bridge design codes currently in use in the USA, Ontario (Canada), and Pakistan. This is followed by a review of some important analytical and field test studies carried out on the problem of the bridge dynamics, in chapter 3. Theoretical basis of measuring dynamic response for some simple load models has been discussed in chapter 4. A summary of the important results obtained in chapters 3 and 4 ,and discussion thereof has been made in chapter 5 that also contains a discussion on correlation of code specified values with the results of research studies reviewed in chapter 3. Chapter 5 also includes a comparison of the dynamic increment values for a typical simple span composite bridge, obtained by using the four different bridge design codes, as well as theoretical load models. The report concludes with chapter 6, containing the findings as well as certain recommendations regarding the dynamic effect of moving loads on highway bridges.

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CHAPTER 2

CONCEPT OF DYNAMIC LOAD FACTOR

2.1 Definition

Dynamic response is defined as the history of the variation of a stress resultant such as deflection, bending moment or a stress plotted as function of time. Applied to a bridge, it is the response of the bridge when acted upon by a moving load or vehicle. One effect is to cause vibration of the bridge, a response that is significantly different from the static response since it is multi-valued and changes with time, involves frequency, and has an interactive relationship between the load and the structure. The practical outcome of this response is generally to cause an increase in the stress resultant over the static magnitude of response, that is the response under a stationary load. This increase is generally different for different bridges, depending upon a number of factors such as the mass, stiffness, and damping of the bridge; the mass, suspension, damping, and velocity of the vehicle; as well as the condition of the approach and the bridge surface.

The ratio of the magnitude of a specific parameter such as deflection y(t) compared to the deflection by the same load applied statically $y_{static} = y_{st}$ has various names such as the Dynamic Load Function. The quantity represented by the ratio of the maximum magnitude of the response caused by dynamic loading to that due to the statically applied load minus one also has many names such as Dynamic Load Factor (DLF), Dynamic Amplification Factor (DAF) and others. Through misunderstanding of the dynamic phenomenon this ratio has been called an "impact factor" in bridge design. This error is being rectified but continues to persist. (Hitting a nail with a hammer is impact - the true duration of the event is very short-lived.). The term 'impact' or 'impact factor' , has been and is still being applied incorrectly to represent this dynamic effect. The reason being that 'impact' is a phenomenon related to impulse - an effect involving action of a force acting over a very small period of time causing change in momentum of the body on which it is acting. Thus when an object having a mass and moving at some velocity 'impacts' upon another body the change in momentum in the two bodies is equivalent to an 'impulse'; a dramatic change in force over a very small time interval.

This fact is now being recognized in the literature and different bridge design codes as well as by various researchers who are now using more appropriate terms like dynamic load allowance [AASHTO LRFD 1994, OHBDC 1991], dynamic increment (DI) [Heywood 1995], dynamic increment factor (DIF) [Galdos et al. 1993]), dynamic amplification factor [Humar and Kashif 1993, 1995; Kashif 1992] and dynamic load factor (DLF) [Hwang and Nowak 1991] to represent the dynamic effect of moving loads quantitatively.

In the present study this effect will be generally referred to as the 'dynamic load factor' (DLF). The dynamic load factor, may be defined as the ratio the maximum dynamic response to the maximum static response. Symbolically DLF is given by the relation

$$DLF = \frac{R_{dm}(x_i)}{R_{sm}(x_i)}$$

where $R_{dm}(x_i) = \max[R_d(x_i, t_j)]$ is the maximum amplitude of the dynamic response, and $R_{sm}(x_i) = \max[R_s(x_i)]$ is the maximum static response of the bridge at section x_i under the action of moving vehicles. Representing the maximum static response by $y_{st\ max}$ and the maximum dynamic deflection response by $y_{dyn\ max}$, the dynamic load factor (DLF) is given by the expression

$$DLF = \frac{y_{dyn\ max}}{y_{st\ max}}$$

2.2 Code Specifications on Dynamic Load Allowance

2.2.1 AASHTO LRFD Bridge Design Specifications - 1994

The American Association of State Highway and Transportation Officials (AASHTO) Load and Resistance Factor Design (LRFD) Bridge Design Specifications - 1994 (AASHTO LRFD 1994) defines the “impact factor” as “Dynamic Load Allowance”, denoted by IM. The dynamic load allowance (DLA) is the percentage by which the static effects of the design truck or tandem are to be increased to account for the dynamic effects of traffic loads. The percentage specified for the DLA is not applicable to pedestrian loads or the design lane load (Lane load is a uniformly distributed force, usually attributable to automobiles.). The factor to be applied to the static load has been specified as: (1 + IM/100). The DLA percentages are given in Table 1 on the next page.

Relevant AASHTO LRFD code provisions regarding the dynamic load allowance are available in Appendix.

Table 2.1 : Dynamic Load Allowance, IM

Component		IM
Deck joints - All Limit States		75%
All Other Components		
* Fatigue and Fracture Limit State		15%
* All Other Limit States		33%

2.2.2 AASHTO Highway Bridge Design Specifications - 1992

In the AASHTO Highway Bridge Design Specifications - 1992, the dynamic effect of vehicle loads was accounted for through an "impact allowance" calculated using the expression:

$$I = \frac{50}{L + 125}$$

in which

I = impact fraction (maximum 30 percent);

L = length in feet of the portion of the span that is loaded to produce the maximum stress in the member.

To calculate the dynamic effect, live load was required to be multiplied by the 'impact' fraction calculated from the above expression. The dynamic effect thus obtained when added to the live load gave the total effect of the live load on the structure. Further details on AASHTO 1992 provisions on the "impact factor" are given in the Appendix.

2.2.3 Ontario Highway Bridge Design Code - 1991

The Ontario Highway Bridge Design Code-1991 (OHBDC 1991) specifies the dynamic load allowance (DLA) with reference to the number of axles of the vehicle, the maximum value being 0.40 for a single axle load and the minimum 0.25 for vehicles with three or more axles. For a 2-axle vehicle, the dynamic load allowance specified is 0.30. These values are for normal loading condition with a smooth approach road. In case of unusual approach surface or bump at expansion joint and single axle loads, the DLA is increased to 0.5 from 0.4 within a length of 3 m or one-tenth of the span, whichever is greater from the joint.

For further details about the DLA refer to the Appendix.

2.2.4 Code of Practice Highway Bridges (CPHB) 1967 (Pakistan)

The existing Code of Practice Highway Bridges (CPHB) 1967 of Pakistan provides the following formula to account for the dynamic effect of traffic loads in the design of highway bridges:

$$I = \frac{15}{L + 20}$$

where

I = impact fraction (maximum 30 percent)

L = length of span in feet

In this case, the dynamic effect of live load is obtained by multiplying the standard Truck-train loading by the "impact" fraction *I* obtained from the above formula

More details about CPHB 1967 code requirements on the calculation of dynamic effect of vehicle loads are in the Appendix.

2.3 Derivation of Dynamic Load Factor (DLF)

Numerically, the dynamic load factor (DLF) is obtained by subtracting one from the ratio of the response (deflection, moment etc.) of a bridge structure under the action of a moving load or vehicle to that obtained under the same load when applied statically to the same structure. In the actual design of a bridge, the static live load is multiplied by this fraction to obtain the dynamic effect of the live load, which is then added to other loads (dead, static live, snow, earthquake etc.) to check the resistance of various components of the bridge being designed.

The dynamic load factor, is generally derived in following ways:

2.3.1 Empirical: Using "Rule of Thumb" Based on Experience

The performance of existing bridges provides data which can and is used for "rule-of-thumb" or empirical rules for design without any detailed calculations or as a result of intuitive thought perhaps with some logical justification. In this manner a constant value or a formula may be adopted to calculate the dynamic effect of moving loads for application to particular types of bridges. These relationships may apply in limited cases or within narrow bounds only, they nevertheless constitute rules for design such as the DLF. Waddell (1918) reports one example of this type of DLF.

2.3.2 Theoretical Analysis - Computer Simulation

For a theoretical analysis the moving load and the bridge are generally modeled by making some reasonable assumptions and the bridge response under the moving load is calculated by employing well recognized mathematical and analytical techniques. For example, the load, moving at a constant speed across the bridge, may be modeled as:

1. a single force
2. a single unsprung mass

3. a multi-axle unsprung load

4. a single sprung load

5. a combination of sprung and unsprung loads,

6. a multi-axle sprung load

A few load models are shown below:

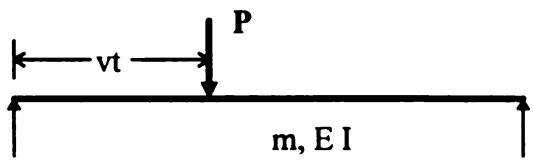

FIG. 2.1: Load without Mass

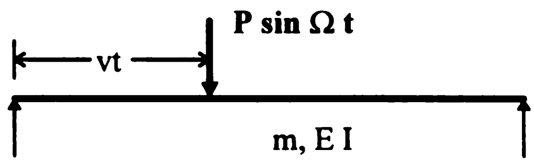

FIG. 2.2: Oscillating Load without Mass

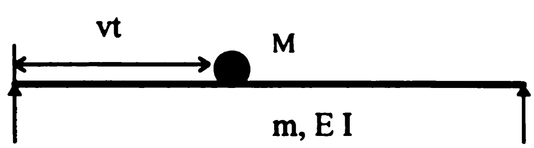

FIG. 2.3: Load with Mass

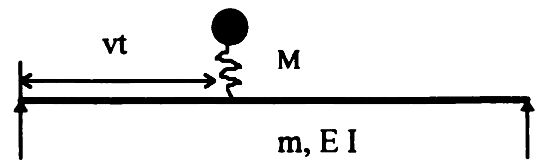

FIG. 2.4: Sprung Load with Mass

Whereas the bridge may be idealized as:

1. an elastic beam (simple, continuous or cantilever),

2. a plate (isotropic or orthotropic) structure, or

3. a combination of beam and plate structures.

The analytical expression for a force or mass moving with constant velocity over a simply supported span is an exercise designed to obtain a "complete" or "exact" solution for this unique problem.

It is necessary to clarify here that "complete" or "exact" are relative terms meaning that the solution completely or exactly satisfies the differential equation. However, the differential equation is based on certain assumptions such as elastic behavior , a simply supported span, negligible deformation due to shear and rotary inertia, and other constraints placed on the behavior of the load. The solution of the continuum equation that is, the partial differential equation, thus is a specialized format, an idealized situation. The solution of this idealized problem provides a basis for understanding the effects of traffic loads. A study of this basic situation and a review of its variation should provide significant information regarding the parameters, their interaction, and details of the dynamic response due to moving loads. However, although this idealized model and its solutions provide knowledge, in all but a few cases it does not exactly model the specifics of most bridges. Real bridges conform to a large array of types, configurations, materials , and loads.

There is an array of methods for determining the dynamic properties of structures, including any shape of bridge. Most involve modeling the structure as a system of discrete multiple degree of freedom set of masses and structural elements. These methods allow for computation of natural frequencies, numerical integration for determination of response, mode superposition, discrete models, etc. Determining the dynamic properties of a bridge is a well understood process. Knowledge gained from the solutions of the continuum problem described above can provide help for modeling loads for discrete analysis and for interpretation of the expected response details.

This is a more reliable method of finding the DLF, especially if supplemented with model tests. A number of researchers have used this method. [Inglis 1934; Scheffy 1956; Fleming and Romualdi 1961; Wen and Veletsos 1962; Sundara Raja Iyengar and Jagadish 1968, 1970; Veletsos and Huang 1970; Ibanathan and Wieland 1987; Chang and Lee 1994; Humar and Kashif 1993, 1995; Huang et al. 1995; Yang and Lin 1995]

2.3.3 Model Tests

A small sample bridge, the model, similar in dynamic characteristics to an actual one, the prototype, may be subjected to simulated representative models of actual vehicles in the laboratory, to study its dynamic response. The results obtained from such a laboratory test may also be used to arrive at suitable values or an empirical relation to reflect the dynamic effect of moving loads in actual bridge design [Boehning 1953; Eichmann 1954; Tung et al. 1956; Walker 1968; Tiedeman et al. 1993].

This is a more accurate method, especially if used in conjunction with theoretical analysis, and the results of both are in good agreement with each other. However, it is expensive as well as time consuming, and since late 1950's has declined in use in favor of computerized simulation methods.

2.3.4 Field Tests

In some cases, tests are carried out on existing bridge structures, instrumented suitably to measure dynamic response under real traffic loads [Biggs and Suer 1956; Edgerton and Beecroft 1956; Hayes and Sbarounis 1956; Foster and Oehler 1956; Fenves et al. 1962; Shepherd and Aves 1973; Billing 1984; Cantieni 1984; O'Connor and Pritchard 1985; Ibanathan and Wieland 1987; Chan and O'Connor 1990; Huang et al. 1993; Chang and Lee 1994; Heywood 1995]. Based on statistical results of these field tests, the dynamic load factor may be calculated from the relation

$$DLF = \frac{y_{dyn\ max}}{y_{st\ max}}$$

where $y_{dyn\ max}$ is the maximum dynamic deflection, and

$y_{st\ max}$ is the maximum static deflection for the same load.

Since these tests are carried out on existing full-sized bridges using actual vehicles, the results and deductions made therefrom are generally more reliable and realistic, especially if these tests are sufficient in number and cover the effects of all the factors and practical conditions that influence the dynamic response of a bridge structure the most adversely. The drawback of this method , in addition to its cost, is that the data obtained is for that particular bridge only. extrapolation or interpolation of results must rely heavily on an understanding of dynamic response characteristics obtained analytically. Further, dynamic analysis of the subject bridge, the one instrumented for testing , is a necessity if the results of the test are to be fully understood and correctly interpolated.

2.4 Modeling of the Vehicle and the Bridge

2.4.1 Vehicle Models

In the analytical studies of the dynamic response of bridges under the action of moving loads or vehicles, the mass of the vehicle has been modeled in the following ways:

2.4.1.1 As a Moving Load Having No Inertia (Mass)

This assumption holds good for the cases where the inertia (mass) of the vehicle is negligible in comparison with that of the bridge. Although less realistic than a sprung load model, a moving force model [Inglis 1934; Sundara Raja Iyengar and Jagadish 1968]:

1. helps in understanding the effect of the speed parameter (ratio of the load velocity to one-half of the bridge period of vibration).
2. indicates the relative influence of various modes of vibration to the dynamic response of the structure.

Several researchers have carried out studies based on this vehicle model [Inglis 1934; Tan and Shore 1968a, b; Timoshenko et al. 1974; Warburton 1976; Sridharan and Mallik 1979; Yang and Lin 1995].

2.4.1.2 As a Moving Mass

In cases where the inertia of the vehicle is comparable with that of the bridge, it cannot be neglected. In such cases, the load can be modeled as a moving mass. Several studies have been carried out using this vehicle model as well [Blejwas et al. 1979; Ibanathan and Wieland 1987; Akin and Mofid 1989; Yang and Lin 1995].

2.4.1.3 As a Dynamic Vehicle Model

For more accurate vehicle models, the dynamic characteristics of the various vehicle components like body (mass), suspension system and tires etc. are considered separately as lumped masses, to obtain more realistic response of the bridge, as a combined effect due to interaction of the various components. This sort of vehicle models have also been used by researchers. [Hwang and Nowak 1991; Wang and Huang 1992 ; Yang and Lin 1995]

2.4.2 Bridge Models

For dynamic analyses bridges may be modeled as a continuum requiring the solution partial differential equations or as a discrete solution with a finite number of kinematic degrees of freedom which results in a set of ordinary differential equations. In the analytical studies of vehicle-bridge interaction, a bridge may further be modeled in the following ways:

2.4.2.1 As a Beam or Linear Model

The simplest way in which a bridge can be modeled is as a beam (simply supported, continuous or cantilever) of length equal to the span or spans of the bridge. A number of studies have been carried out using this model.[Biggs et al. 1959; Fleming and Romualdi 1961; Yang and Lin 1995]

2.4.2.2 As a Plate or Two-Dimensional Model

In some cases, research has been carried out by modeling the bridge as a two-dimensional or plate structure. In this case, the bridge deflection is considered along the length as well as width of the bridge [Yoshida and Weaver 1971; Sundara Raja Iyengar and Jagadish 1968, 1970; Humar and Kashif 1995].

2.4.2.3 Slab-as-a-Plate and Girder-as-a-Beam Model

In some studies, research on dynamic behavior of bridges has been carried by modeling the deck slab as a plate, and the girder as a beam element. [Yamada and Veletsos 1958; Oran and Veletsos 1961]

2.4.2.4 Studies Using Finite-Element and Finite-Strip Methods

Some researchers have carried out studies on the dynamic response of bridges, using finite-strip methods [Cheung and Cheung 1972] or finite element methods

[Rabizadeh and Shore 1975; Saadeghvaziri 1993]. These inherently result in a discrete system for mathematical analyses.

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CHAPTER 3

LITERATURE REVIEW

3.1 General

Determination of the response of bridges to moving loads i.e., vehicles, is very important for bridge users and bridge design engineers because dynamic response creates important considerations. Vibration may cause unpleasant physiological and psychological effects on bridge users, and stress resultants, e.g., deflections and bending moments in the bridge may increase considerably from that obtained under the action of static loads. The magnitude of these increases will vary in amount depending upon a number of characteristics of the bridge as well as the moving load. Thus the determination of reasonable estimates of structural loads and the response to these loads becomes increasingly more complex. One possible approach for estimating the maximum effect due to traffic loads is to increase the static load by a certain percentage or ratio based on assumptions or statistical data to account for the dynamic effect of the load, and use this increased load to design the bridge. A similar approach is used in many of the bridge design codes in the world. This approach may be sufficient for many short span stiff bridges; however, it yields insufficient information concerning bridges that because of span length or flexibility are more sensitive to vibration. Dynamic response is a complex phenomenon, affected by a number of factors which require greater computational effort. The maximum values of key stress resultants, such as bending moment at a critical section or mid-span deflection, due to dynamic loading is usually expressed in terms of the ratio of the maximum value due to dynamic loading to the value if the structure or structural element were loaded with the same force applied statically. This ratio in some form may be called an amplitude factor, a dynamic load factor, a dynamic increment, or in bridge design, incorrectly labeled as an impact factor. Accordingly, it is necessary to investigate the response of bridges to traveling loads, and to arrive at suitable methodologies for use in the design and analysis process. Highway bridges that are very stiff, usually short span - such as a masonry arch, that is, bridges with a relatively short period, may be designed using static or quasi-static methods. Bridges with a longer natural period, usually longer span and more flexible are more sensitive to dynamics and may have to be analyzed for its dynamic behavior either in part or requiring a full dynamic simulation. The large majority of highway bridges are short to medium span and built of masonry, concrete, steel, or a concrete steel composite for which static traffic loads are dominant and design utilizes the quasi-static approach. For this purpose the static load is increased by the dynamic load factor referred to earlier, labeled an "impact factor" in some codes (striking a nail with a hammer is an example of impact - the duration over which the force is applied is quite short! hardly what occurs for normal traffic loads). The study and investigation of the response of bridges to moving loads or vehicles can be traced back to 1849. Since then a number of investigations have been carried out to understand the behavior of bridges subjected to moving loads, and to explore the factors or parameters which are major contributors to this phenomenon. An overview of important research on the dynamic response of bridges subjected to moving loads and relevant to this study, is given below:

For the sake of clarity, these studies are discussed in the following two main categories:

a. Analytical and Model Studies

b. Field or Test Studies

3.2 Analytical and Model Studies

3.2.1 The history of the study of the behavior of bridges under moving loads can be traced back to 1847 when a British Royal Commission[1] was appointed to investigate the possibility of application of iron for use in construction of railway structures. The Commission was the first to investigate the effect of moving loads on bridges. After carrying out experiments on the effect of a rolling load, the Commission reached the conclusion that in the case of a light and flexible simple beam, the increase in deflection was dependent on the velocity of the load. It was also found that in certain cases, the deflection caused by the moving load was two or three times the static deflection. Professor R. Willis, a member of the Commission, tried to formulate the problem through analytical techniques and consequently developed a differential equation to calculate the deflection of the beam under the point of the load. In developing the deflection equation, Willis assumed that:

a. the mass of the beam is negligible,

b. the beam is undamped, and

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[1] The original reference was not available. The information quoted in this section has been mainly taken from the report of Wright and Green (1959).

c. the load is a concentrated mass

According to Willis[2], the deflection of the beam y_P under the point of application of load P(t) is proportional to the contact pressure of the moving load expressed as a mass m_u and can be computed from the equation for the static deflection of the beam i.e.,

$$y_P = \frac{P(t)x_P^2(L - x_P)^2}{3LEI} \qquad \text{---------(1)}$$

where x_P is the distance from left support, the origin, to the point of application of the load. For finding P(t), the inertia force, using the relationship

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = v\frac{dy}{dx}$$

is added to the rolling load of the unsprung mass m_u

$$m_u g + m_u \frac{d^2y}{dt^2} = m_u g + m_u v^2 \frac{d^2y}{dx^2}$$

assuming the velocity v, of the load to be constant. Eq(1) changes to

$$y_P = m_u g\left(1 - \frac{v^2}{g}\frac{d^2y}{dx^2}\right)\frac{x_P^2(L - x_P)^2}{3LEI} \qquad \text{---------(2)}$$

This equation was solved by G. G. Stokes[3] and the solution was in tabular form. This solution, however, holds good only for the cases where the ratio of the weight of the moving load to that of the bridge is large. In the case of present day medium span bridges, this ratio is generally not more than 0.2. Accordingly, Stoke's solution is not applicable [Wright and Green 1959].

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[2] See footnote on page 26.

[3] See footnote on page 26.

3.2.2 In 1905, A. N. Krylov[4] attempted a theoretical solution of the problem of dynamic behavior of a bridge, by assuming that:

a- the mass of the load is negligible,

b- the moving load P(x, t) can be replaced by a moving force $m_u\, g$.

Krylov represents the state of vibration of a simple beam under the action of a moving force with the differential equation

$$EI\frac{\partial^4 y}{\partial x^4}+\frac{M_g}{L}\frac{\partial^2 y}{\partial t^2}=P(x,t)=\frac{2m_u g}{L}\sum_{i=0}^{\infty}\sin i\pi x \sin 2i\pi f_b\, t \qquad \text{---------(3)}$$

Based on Krylov's solution, for most load cases, the approximate amplitude of the vibration can be found from the expression:

$$y_P=-\frac{2m_u L^3 \bar{n}}{M_g\, EI f_b}\sin 2\pi f_b\, t \sin\frac{\pi x}{L} \qquad \text{---------(4)}$$

where $\bar{n}$ is equal to $\frac{V}{2L}$

M_g is the mass of the beam or girder

f_b is the natural frequency of unloaded beam or bridge

3.2.3 H. H. Jeffcott[5] (1929) was the first researcher who included the mass of both the moving load and the bridge in his attempt to get an analytical solution to the bridge vibration problem. He neglected the effect of damping and used an iterative procedure to

[4] See footnote on page 26.
[5] See footnote on page 26.

obtain two solutions of the problem; one after neglecting the inertia term, and the other including the inertia terms. The method he used does not converge in certain cases.

3.2.4 For the analytical and experimental research carried out in Great Britain by the Bridge Stress Committee, C. E. Inglis (1934) formulated the expressions for the natural frequency of loaded and unloaded simply supported spans, respectively, given below:

$$2\pi f_{bl} = \frac{\pi^2}{L^2}\sqrt{\frac{EIL}{M_g + 2m\sin^2\frac{x_p\pi}{L}}} \qquad \text{---------(5)}$$

$$2\pi f_b = \frac{\pi^2}{L^2}\sqrt{\frac{EIL}{M_g}} \qquad \text{---------(6)}$$

where f_{bl} is natural frequency of the loaded beam or bridge.

Inglis showed that for beams with non-uniform rigidity, the natural frequency can be obtained quite accurately, using the value of the rigidity at mid-span. Using a harmonic analysis, he also developed the expressions for the following cases of loading:

a. A massless oscillating force ($P \sin \pi\,\Omega\, t$, where Ω is the frequency of the force) with damping of the bridge

b. An oscillating force with damping of the bridge:

The magnitude of vibration was obtained using a series solution. The approximate maximum amplitude is given by the relation

$$\frac{P}{K_b}\sqrt{\frac{N^2}{(w/2\pi)^2 + (C_{bl})^2}} \qquad \text{---------(7)}$$

where K_b is the stiffness of the span, and C_{bl} is the damping coefficient of the loaded span.

c. Vibrations considering locomotive springing: According to Inglis, to determine the vibration of medium and long span railway bridges accurately, the damping factors (of these bridges) must be known.

3.2.5 The research work carried out by C. F. Scheffey (1951)[6] was most probably the first study especially related to the dynamic behavior of highway bridges (the earlier studies concentrated on railroad bridges). The main objective of the study was to predict a rational impact allowance for the design of highway bridges. Scheffey developed the solution of the differential equation of motion of a simple prismatic beam traversed by a rolling mass with a constant velocity. His solution is similar to that obtained by Krylov (1905). Scheffey showed that in the case of a single load rolling smoothly over a simple span with a velocity of 60 mph, the maximum dynamic increment (i.e., addition to the static deflection due to dynamic effect of rolling load) is nearly 8 percent for bridge spans in the range of 20 to 100 feet, and nearly 4 percent in the case of a span of 300 feet [Wright and Green 1959].

Scheffey also showed that in the case of a sprung vehicle, with a spring friction about 10 percent of the vehicle weight, and entering a span with its springs in an equilibrium position, no spring action (that is, the oscillation) would take place in bridge spans longer than 40 feet. His results also lead to the conclusion that the phenomenon of resonance may take place for simply supported loaded spans of about 300 feet or more.

[6] See footnote on page 26.

He commented that while considering maximum dynamic effects, representative vehicles should be used rather than the equivalent design loads.

3.2.6 The analytical research carried out by Dr. A. Hillerborg[7] (1951) is important in that it was one of the most complete analyses performed on the general problem of dynamics of bridges considering the effects of the following parameters [Wright and Green 1959]:

"a. the mass of the load,

b. the mass of the beam,

c. the velocity of the load,

d. the spring mounting of the load,

e. viscous damping of the beam (internal and external), and

f. spring action of the load springing" (must be the spring action of the vehicle's suspension)

The analysis is an improved and modified version of the theory presented by Inglis (1934). Hillerborg also obtained the expressions for the dynamic bending moment and the shearing force as well as a general equation for the motion of a simply supported girder and several moving loads. In addition, he provided numerical solutions for the following cases [Wright and Green 1959]:

"a. a non-sprung load without damping of the beam

b. a non-sprung load with damping of the beam, and

[7] See footnote on page 26.

c. a sprung load."

The results of the analyses were compared with model tests, and both the numerical and the test results were in a good agreement. In the case of the bending moment, however, the analyses gave the dynamic increment only for a range of variables that are generally not applicable to the dynamic behavior of highway bridges.

3.2.7 The main objective of the research reported under the title "Highway Bridge Impact Investigations" and others [Boehning 1953; Eichmann 1954] carried out from 19951 to 1955 at the Engineering Experiment Station, University of Illinois[8] [Wright and Green 1959], was the development of a rational method for the prediction of the dynamic increments of the bending moment and the deflection, for a simple span under the action of normal commercial vehicles. Many of the important factors capable of affecting the response (deflections and stresses) of loaded highway bridges were considered in the analysis. Model tests, corresponding to the conditions adopted in the analysis were carried out to check the validity of the results of the analysis. Equations of motion, in this case, were derived using the energy-principles and considering the energies stored in the bridge and the vehicle as a result of the vehicle-bridge interaction.

The test models were designed to be dynamically similar to a simple span I-beam highway bridge of span length 60 ft., weight 3000 lb./ft., stiffness $20x10^9$ lb.-ft^2 and fundamental frequency 6.4 Hz, and traversed by a 30,000 lb. single axle load. The suspension system of the loading carriage was designed in accordance with

[8] Some of these references were not available. Accordingly, the report of Wright and Green (1959) is the major source of information provided in this section.

manufacturer's data about loaded trucks i.e., the natural frequency of the suspension system in the range of 1.5 to 1.84 Hz and the ratio between the sprung and unsprung weights from 7.0 to 8.0.

The following parameters represent the range of an average vehicle-bridge-system:

$$R = 0.167, \qquad R_1/R_2 = 7.0, \ \alpha = 0.04 \text{ to } 0.15, \qquad F = 3.85$$

where $R_1 = \dfrac{\text{Weight of sprung part of vehicle}}{\text{Weight of bridge}} = \dfrac{m_s}{m_g}$,

$$R_2 = \frac{\text{Weight of unsprung part of vehicle}}{\text{Weight of bridge}} = \frac{m_u}{m_g},$$

$$R = \frac{\text{Weight of vehicle}}{\text{Weight of bridge}} = R_1 + R_2 = \frac{m}{m_g},$$

$$\alpha = \frac{\text{One – half the period of bridge}}{\text{Time required for vehicle to cross span}} = \frac{V}{2FL}, \text{ and}$$

$$F = \frac{\text{Natural frequency of bridge}}{\text{Natural frequency of vehicle}}$$

The following are the parameter ranges used for both the model tests and the analytical solutions:

$$R = 0.20 \text{ to } 0.60, \qquad R_1/R_2 = 7.0, \ \alpha = 0.06 \text{ to } 0.20, \text{ and } \ F = 2.0 \text{ to } 5.0$$

The model was handled carefully to have control over the state of motion of the carriage, as it entered the beam,

The following are among the assumptions that were made in the analysis [Wright and Green 1959]:

"a. that simple slab or beam and slab bridges behave as simple flexible beams,

b. that the structures are linearly elastic,

c. that the vehicle is considered to be a single, or tandem, axle load, consisting of a sprung part (chassis and payload) and an unsprung part (wheels and frame) connected by a linear spring,

d. that the flexibility of the vehicle tires can be neglected,

e. that the vehicle speed is constant over the span,

f. that the vehicle wheels are in contact with the span throughout the crossing,

g. that the span is stationary before loading,

h. that the damping of the vehicle-bridge system, camber and deck roughness can be neglected, and

j. that the vehicle path is along the center-line of the span.

The model tests carried out were for the cases of:

a. an unsprung mass with both single and tandem axles,

b. a sprung mass with no initial vertical velocity or displacement, and

c. a sprung mass with initial velocity and displacement."

For the case of high weight ratios (i.e., the ratio of the vehicle weight to that of the bridge), the results of analytical and experimental studies for the mid-span deflection amplification factors were in good agreement. When plotted against α, the speed parameter, the amplification factors for both the mid-span deflections and the bending moment, showed a similar trend. The studies also reported an increase in the dynamic response of the loaded beam as a result of an increase in the speed parameter α.

From the test results, it was observed that unsprung single axles invariably produced larger though similar, dynamic effects than the tandem axles.

In the case of weight ratios greater than 0.20, the maximum dynamic effects for the completely unsprung vehicle were almost independent of the weight ratio. For the higher weight ratios the maximum amplification factors were observed at lower speeds, there was no change in the magnitude of the peak values.

In the case of a sprung vehicle with zero vertical displacement and velocity, the dynamic effects were similar to those produced by an 'equivalent' unsprung vehicle. In this case, the equivalent weight was the sum of the weight of the unsprung portion of the sprung vehicle plus some fraction of the sprung weight.

For normal heavy vehicles, the equivalent weight is nearly the same as the sprung portion of the vehicle carriage. The sprung vehicle with vertical motion or displacement may be responsible for greater dynamic effects in the bridge than those produced by the 'equivalent' unsprung vehicle because of the energy stored in the vehicle springs when the vehicle enters the span.

3.2.8 The studies carried out at Massachusetts Institute of Technology[9] (M.I.T) in early 1950s were aimed at developing an analytical procedure for the prediction of the motion

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[9] See footnote on page 32.

of simply supported bridges under the action of a moving sprung axle load. [Biggs and Suer 1956; Wright and Green 1959]:

Results of the field tests lead to the main conclusion that the vertical oscillation of the vehicle itself was the main influence on the motion of the bridge and that this was caused by the roughness of the approach .

The assumptions made in the analysis, were:

"a. that only first mode vibration need be considered,

b. that the vehicle system has one degree of freedom, and that its entire weight is located at the center of gravity of the vehicle mass,

c. that only viscous damping is present in the bridge vehicle system, and

d. that the deflected shape of the structure is at all times sinusoidal."

The sprung-vehicle-bridge system was represented by two single-degree of freedom systems.

The results of the laboratory studies, lead to the conclusion that a simple span beam could be represented as a single degree of freedom system , and that a single concentrated sprung load could reasonably approximate a model two axle truck.

The following further assumptions were made in the study [Wright and Green 1959]:

"a. that approach roughness induces an initial oscillation of the vehicle, the bridge deck being perfectly level,

b. that damping may be neglected,

c. that the load travels along the center-line of the structure,

d. the static and dynamic rigidities of the structure have the same characteristics,

e. that the vehicle is completely sprung, and

f. that the bridge is at rest when a vehicle enters"

The parameters considered in the study were:

ω - the frequency of the vehicle crossing,

$R = \frac{m}{m_g}$ i.e., the ratio of the total mass of the load to that of the total mass of the beam or girder,

$\frac{1}{f_v}$ where f_v is the natural frequency of the vehicle system in cps (Hz),

and $F = {}^{f_b}\!/_{f_v}$ i.e., the frequency ratio.

The results of this study, obtained for the case of a mass ratio of 0.10 and an irregularity parameter I of 0.30, (given by the relation $I = K_v\, Z_m = F\, R\, \frac{Z_m}{\delta_{st}}$ where K_v is the spring constant of the vehicle system, Z_m is the amplitude of the sprung mass motion of the vehicle, and δ_{st} is the maximum static mid-span deflection.), are as follows:

a. Phase Angle:

The maximum dynamic deflection takes place for values of the phase angle of the vehicle motion ϕ between π and 2π. Using an approximation the maximum dynamic deflection in terms of frequency is given by

$$\frac{y_m}{\delta_{st}} = 0.50F + 1.15 \qquad \text{---------(8)}$$

b. Frequency Ratio:

In the case of frequency ratios less than 2.5, the dynamic deflection increases to 1.74 for a ratio just greater than unity. The dynamic effect decreases rapidly as the frequency ratio approaches zero. For frequency ratios *F* greater than 2.5, variation in the dynamic deflection values was small. It was observed that the dynamic deflection was dependent only upon the ratio of the frequencies rather than the actual values of vehicle and bridge frequency. It was further noted, from the model tests, that a beating effect occurred for low frequency ratios when most of the energy transferred from the bridge system to the vehicle system and vice-versa. The latter was responsible for large dynamic deflections of the structure over a short time interval.

The frequency ratios range selected was based on bridge and vehicle frequencies of 2 to 8 Hz and 1 to 3 Hz respectively. The natural frequency of the bridge spans, f_b, was taken as $\frac{3.50}{L}$ Hz where L is the span length in feet.

c. Roughness:

The effect of initial vehicle amplitude upon the dynamic deflection was observed to be linear for a given frequency ratio and increased with increasing irregularity parameter I. It was further reported that the dynamic deflection was also a function of the frequency ratio, and that for frequency ratios near unity, the effect of the roughness was greater.

d. Mass Ratio:

For mass ratios greater than 0.15, the dynamic deflection was almost independent of frequency ratio. The dynamic deflection increased for mass ratios less than 0.15 and the rate of the increase was dependent upon the frequency ratio.

The results were presented as design curves [Biggs et al. 1956][10], which might be used to predict the maximum dynamic deflection of the mid-span of a medium span girder or stringer bridge, resulting from a two axle vehicle with a heavily loaded rear axle, traveling down the center line of the bridge. It was commented that it was necessary to know the dynamic characteristics of both the bridge and vehicle to apply the curves to design [Wright and Green 1959].

3.2.9 Wright and Green (1959) presented a collection and review of the analytical and field studies carried out up to that time on the problem of vibration in bridges. Their report discussed the merits and demerits of these studies besides mentioning the assumptions made and the results obtained, as mainly reported herein up to this point.

3.2.10 Fleming and Romualdi (1961) provided analytical study of single span and three span continuous highway bridges modeled as discrete multiple degree of freedom systems subjected to moving loads. The regular differential equations of motion were derived for both cases, and numerically solved with a digital computer using integration over finite intervals of time. To study the effects of vehicle characteristics (mass,

[10] As reported by Wright and Green (1959).

springing, velocity, damping) and the bridge geometry, the dynamic response of the bridge was related to the following dimension-less parameters:

$$\alpha = \frac{\text{time of crossing}}{\text{fundamental period of bridge}}$$

$$R = \frac{\text{mass of load}}{\text{mass of bridge}}$$

$$\rho = \frac{\text{Fundamental period of load}}{\text{Fundamental period of bridge}}$$

More than one thousand results were obtained by varying values of the various variables affecting the dynamic response and analyzed in terms of above mentioned dimension-less parameters. The spans used in the model study ranged from 20 to 140 ft for the case of single span, and in the case of continuous spans, the outer spans ranged from 40 to 80 ft and the central spans from 50 to 100 ft.

On the basis of their results, it was concluded that for unsprung loads, the most important factor was the velocity of the load. It was commented that the main effect of a change in the mass of the load was to change the velocity at which the maximum dynamic response would occur. It was observed that due to change in mass of the load, the change in the maximum produced dynamic effect was not large, however, it was expected that the change in dynamic load effect could be more pronounced in the case of continuous bridges. It was reported that the springing of the load, and the surface condition of the approach of the bridge were the two most important factors for the dynamic response. It was also concluded that initial vibration of the load, caused by settlement or depression in the bridge approach could result in very large DLFs.

3.2.11 Another notable and detailed analytical study on the problem of dynamic response of highway bridges was due to Wen and Veletsos(1962). The study was carried out for simple span highway bridges. The bridges analyzed were I-beam type steel-girder concrete deck slab bridges, and the span lengths analyzed were 20, 45 and 70 feet. The bridge parameters considered were span length, total weight, fundamental natural frequency of vibration, deviation of bridge surface from a straight line, and the initial dynamic condition of the bridge. The parameters considered in the case of the vehicle were its speed, total weight, distribution of weight among sprung and unsprung components, axle spacing, effective spring constant for the axle, and the initial dynamic condition of the vehicle. The analysis was carried out using a computer program in terms of the following dimension-less parameters:

1. Speed parameter, given by

$$\alpha = \frac{V T_b}{2 L} \quad \text{---------(9)}$$

where α is the speed parameter,

V is the speed of the vehicle,

T_b is the fundamental natural period of the bridge, and

L is the span length of the bridge

In the case of short span bridges, for all practical situations, T_b is proportional to the span length L. Thus, the speed parameter α is in fact a function of the vehicle speed only.

2. Weight and Weight-distribution parameters:

a. The weight parameter was defined as the ratio of the total weight of the vehicle to that of the bridge .

b. Weight distribution parameters consisted of :

I. the ratio of the static reactions on two axles, and

II. the ratios of the unsprung weight for each of two axles to total weight of the vehicle .

3. Frequency parameters:

For each axle 'j' (j = 1 or 2), a separate frequency ratio was defined as

$$\frac{(f_v)_j}{f_b} = \frac{\text{Natural Frequency of } j^{th} \text{ axle}}{\text{Fundamental Natural Frequency of Bridge}} \qquad \text{---------(10)}$$

where the axle frequency $(f_v)_j$ was defined as

$$f_v = \frac{1}{2\pi}\sqrt{\frac{K_j}{M_j}} \qquad \text{---------(11)}$$

where K_j is the estimated stiffness of 'j'th axle, and

M_j is the sprung mass of 'j'th axle.

4. Rotary Inertia Parameter was stated to be a measure of the resistance of the sprung mass of the vehicle against pitching motion, and was defined by the ratio

$$\frac{J}{\sum_{j=1}^{2} M_j a_j^2} \qquad \text{---------(12)}$$

where J is the polar moment of inertia of sprung mass about its centroidal axis,

M_j is the sprung mass of 'j'th axle, and

a_j is the horizontal distance between the 'j'th axle and the centroid of the sprung mass.

General conclusions drawn from the study, were:

a. The magnitude of the maximum dynamic effects were proportional to the speed of the vehicle.

b. In the case of axle spacing greater than the bridge length, dynamic effects under 2-axle vehicle loading were greater than that for the single axle loading.

c. For the variable axle spacing, keeping all other factors constant, a quasi-resonance condition was observed when the time interval of application of two successive axles over a point was equal to the fundamental natural period of the bridge.

d. When the bridge was considered to be initially under vibratory motion, the magnitude of the resulting dynamic effect was observed to almost linearly increase with the amplitude of the initial oscillation considered.

e. The results of the analysis lead to the conclusion that the riding surface roughness might be a source of large dynamic effects in bridges,

f. Since damping in both the vehicle and the bridge was ignored in the analysis, the actual dynamic response would be less than that obtained in the analysis.

3.2.12 Sundara Raja Iyengar and Jagadish (1968,1970) performed numerical studies on dynamic response characteristics of two bridges, consisting of one each of slab- and slab-and-beam type bridges. The bridges were modeled as two-dimensional orthotropic plate structures, and the vehicle was modeled as a moving force in the first study, and as a moving mass in the second study. The general conclusions were as follows:

1. The mid-span response of the highway bridge was influenced mostly by the first three modes of vibration. Although this conclusion had been drawn with reference to two bridges, it might be extended to other bridges where the frequencies of the first three modes were clustered together. It has been quoted with reference to Sundara Raja Iyengar et al. (1967) that the frequencies of most of the highway bridges follow the above mentioned pattern.

2. When the bridge was subjected to a concentric (acting along center line of the bridge) moving force, the amplification factors for the points below the moving force followed the trend found using beam theory. The values of the amplifications found by the orthotropic plate theory were smaller than the values found by the beam theory. When the bridge, modeled as a plate structure, was under an eccentric moving force (acting along a line parallel to the center line of the loading lane), the amplifications for points below the moving force did not closely follow the trend observed in the case of beam theory.

3. In the case of a moving force model, the maximum amplification on the transverse section was higher at points away from the line of loading, than that on the line of loading. The largest amplifications were realized at the unloaded edge of a bridge

under an eccentric loading. This effect was more pronounced in the beam-and-slab bridge than in the slab bridge. If the first two frequencies of a bridge were very close, the unloaded edge executed a beating type motion, when the bridge was subjected to an eccentric moving force.

When the vehicle was modeled as a moving mass, the following conclusions were drawn:

a. The maximum amplification factors on the transverse section were larger for points on the bridge away from the line of loading, than on the line of loading. It was observed that in the case of an eccentrically loaded bridge, the effect was more pronounced, and the unloaded edge experienced quite large amplifications irrespective of the frequency and mass ratios. The amplifications in the beam and slab bridge were reported to be much larger than in the slab bridge.

b. The amplifications given by the beam theory were slightly on the conservative side when compared to the point of maximum amplification in the orthotropic plate theory.

c. The moment and deflection amplifications were about equal for points away from the line of loading . Whereas for points on the line of loading, the maximum moment amplification factors were observed generally smaller than the deflection amplification factors. This effect was more pronounced for larger values of the speed parameter.

It was observed that the maximum value of the moment at the mid-span point on the line of loading always occurred when the load was at or very close to mid-span even if the interaction force was not large at this instant. It was also concluded that for the larger values of the speed parameter, the load would act near the third quarter point and the maximum deflection amplification occurred when the load had traveled beyond the midspan. Thus, the variation of interaction force between the bridge and the moving vehicle had a stronger influence on the maximum deflection rather than on the maximum moment, for points on the line of loading. The above might be responsible for the considerable differences between maximum amplification factors for the deflection and the moment for such points.

The frequency and mass ratios had definite, although secondary, influences on the response characteristics. When the frequency ratio was between 0.6 and 1.0, large dynamic effects were observed. The effect of mass ratio was found to be less pronounced, although the amplifications generally increased with the mass ratio.

It was concluded that the initial oscillation generally lead to higher amplitude oscillations all over the bridge. It was remarked that the maximum moment, at the mid-span point on the line of loading, would be increased significantly by initial oscillation only if the load would be acting at or near the mid-span. It was commented that for a flexible bridge, an eccentric loading with initial oscillation might produce pronounced oscillations at the unloaded edge.

3.2.13 The report of Veletsos and Huang (1970) provides a summary and a list of references regarding studies conducted for predicting the dynamic response of highway bridges at the University of Illinois since 1950. These studies were of following types:

a. Theoretical and numerical studies,

b. Laboratory studies of highway bridge models, and

c. Field tests on a number of specially designed bridges.

In the analytical studies, the bridge was modeled in two different ways. In the first case, it was assumed to behave like a single beam (used in the analysis of simple-span, three span continuous, and three span cantilever bridges), and in the second (used to analyze simple-span bridges only), it was treated as a plate continuous over a series of flexible beams. The main objective of the study was to present the most general method of analysis developed in University of Illinois studies, using single-beam modeling of bridges. The secondary objective of the study was to analyze other types of bridges specifically with respect to susceptibility of the bridges to different kinds to vibrations.

In the method of analysis presented for the prediction of dynamic response of bridges to moving loads, the bridge was modeled as a single linearly elastic beam with distributed flexibility and concentrated point masses . The vehicle was idealized as a three-axle sprung load, taking into account the effect of friction present in the suspension system.

On the basis of the results of the analysis and other relevant data[Nieto-Ramirez and Veletsos 1966], it was concluded that cantilever type bridges are most susceptible to vibrations caused by moving loads, simple-span and continuous-type bridges being next in the same order.

3.2.14 Ibanathan and Wieland (1987) conducted a stochastic analysis to predict the dynamic response of a simply supported box girder bridge subjected to a vehicle moving at a constant speed. The bridge was modeled as a one-dimensional elastic beam, and the vehicle as a single-point mass moving at constant speed.

From the results of analysis, it was concluded that:

a. at high speeds and at high ratios of the vehicle mass to the bridge mass, the the mass of the vehicle had significant effect on the response of the bridge,

b. the maximum response was almost unaffected by the presence of damping (in either the bridge or the vehicle), and

c. mainly trucks traveling at high speeds were responsible for the maximum dynamic response, and the dynamic effect due to passenger cars could be ignored.

3.2.15 The objective of the detailed analytical study performed by Hwang and Nowak (1991) were:

a. to develop a procedure for the calculation of the dynamic load, and

b. to determine the statistical parameters of the dynamic load to be used in the development of a reliability-based bridge design code.

The analysis was carried out for cases of :

a. composite steel girder bridges, and

b. prestressed concrete girder bridges

with simply supported spans of 12 to 30 m. Strains were considered to be in the elastic range only.

For the dynamic analysis, the vehicle was considered to be composed of:

a. the vehicle body, represented by a distributed mass, and

b. tires, represented by linear elastic springs.

A three-axle single truck and a five-axle tractor-trailer were used in the analysis. The bridge was modeled as a prismatic beam. The damping of the bridge was assumed to be 2% of the critical damping for each mode. A computer program was used to solve differential equations of the motion for the vehicle-bridge system, using Newmark β method. For the three-axle single single truck, 20% of the total weight was assumed to be acting on the front axle and the rest 80% on the rear axle. In the case of five-axle tractor-trailor 60% of the total weight was assumed to be acting on the front axle and 40% on the rear one.

A parametric study was carried out by calculating the dynamic load factors (as the ratio of the maximum dynamic deflection to the maximum static deflection), as a function of the gross weight, the speed, and the axle distance of the vehicle. Dynamic load effects caused by one and two vehicles were also compared.

3.2.16 In the study carried out by Chang and Lee (1994), the vehicle was modeled as a 2 degree-of-freedom vibrating system, and the bridge was represented by a simply supported beam. The study was carried out to find the effects of the surface roughness, vehicle speed, and the bridge span on the 'impact factor'. The results were compared with 'impact factor' given by the Korean code(1982) i.e.,

$$i = \frac{20}{50 + L} \qquad \text{---------(13)}$$

where L is the span length in meters.

Using 4 different vehicle models it was shown that the response was different when the road roughness was taken into account than when the effect of road roughness was neglected. It was also observed that 'impact factors' increased with surface roughness, and vehicle speed. It was pointed out that the Korean code underestimated the dynamic load effect, especially for vehicles traveling with high speeds over a long rough bridge.

3.2.17 Humar and Kashif (1995) also treated the problem of bridge vibration analytically. The bridge deck was idealized by rectangular thin plate elements. The plate bending element was based on a triangular element derived by Hsiah (Clough and Tocher 1966) for ensuring monotonic convergence. The vehicle was modeled as a single sprung mass and a dashpot (representing viscous damping). Damping was ignored in the bridge. The vehicle configuration and the road roughness were also ignored.

From the study of dynamic response of a slab type structure under the action of moving vehicles, the following conclusions were made:

a. The characterizing parameters of a bridge modeled as an isotropic plate structure were the aspect ratio, the speed parameter α, the frequency ratio φ, and the mass ratio κ.

b. Two different bridges with the same four parameters had identical responses.

c. For an orthotropic plate model of the bridge, the governing parameters were α, ϕ, κ , and factors (χ) and (θ) related to the geometry and stiffness of the plate.

d. For an isotropic plate model of the bridge, the maximum response of two bridges with different aspect ratio were not the same but still quite close.

e. In general, dynamic amplification factors for the presence of multiple vehicles were lower than those for the single vehicle.

f. The response of two vehicles each of mass m, traveling along parallel paths but one lagging behind the other was less than that produced by the two when traveling parallel to each other.

g. For the isotropic plate case, the moment amplification factors were generally less than the deflection amplification factors.

3.2.18 Huang et al.(1995) analyzed a thin walled box-girder bridge, using beam elements. The vehicle used in the analysis was an AASHTO HS20-44 truck, modeled as an 11 degrees-of-freedom system. Four classes of road roughness (very good, good, average, and poor) were considered in the analysis.

The results of the analysis were stated to be in good agreement with that obtained by using a folded plate model [Jones[11] 1972].

From the results of the study, it was concluded that:

a. A decrease in torsional rigidity of the end diaphragm, resulted in a decrease of the 'impact factors' for simply supported box-girder bridges.

b. 'Impact factors' varied with change in transverse positions and number of vehicles.

c. For 'very good' and 'good' road surfaces, the 'impact factors' were less than 0.10 and varied slightly with increased speed of the vehicle.

d. For the 'average' and 'poor' road surfaces, the 'impact factors' increased significantly.

3.2.19 Using finite element method approach for modeling the bridge and a modified dynamic condensation method, Yang and Lin (1995) presented an analytical model for the dynamic analysis of a vehicle-bridge interaction system. In the model, the vehicle was represented by sprung lumped masses and dashpots, and the bridge was idealized with beam elements and a rough pavement.

3.2.20 In the analytical study of Yang et al.(1995), the vehicle was modeled with suspended lumped masses i.e., lumped masses supported by springs and dashpots (viscous damping), whereas the bridge was modeled as a series of beam elements. The

[11] The original reference not available.

analysis was carried out only for "beam like" bridges consisting of simple and continuous span bridges for all (short, medium and long) spans. Separate formulas were proposed for determining 'impact factors' in case of deflections, bending moments, and shear forces for the simple as well as continuous bridges..

From the results of the analysis, it was concluded that:

a. AASHTO (1989) 'impact formula' underestimates the dynamic load effect of moving loads in certain cases, especially in case of mid-span deflection, and mid-span moment of simple span beams.

b. The suspension stiffness of the vehicle had negligible effect on the DLF.

c. The pavement roughness had little effect on the dynamic response, such that it could be neglected while using the proposed formulas. (Although the effect of pavement is generally significantly pronounced, especially if the approach road or the riding surface of the bridge contains a settlement or a small bump [Fleming and Romualdi 1961; Heywood 1995].)

d. When heavier bridge cross-sections were used, the 'impact factor' might be slightly less than that predicted by the proposed formulas.

3.3 Field or Test Studies

In case of field or test studies the majority of the types of the highway bridges considered consisted of the simple and continuous span slab-and-beam or slab-and-girder bridges. Although in these tests generally the bridges were well-instrumented, nevertheless, the test vehicles were not always true representatives of normal commercial vehicles.

3.3.1 Some of the early test studies carried out in various parts of world up to about 1918, to determine the dynamic effect of moving loads on bridges, have been reported by Waddell (1918). Although the original references of most of these studies are not available, nevertheless these studies are important in that these highlight general trends followed in the early studies on determination of dynamic load effect of moving loads on bridges. Accordingly, the following information is mainly based on the report of Waddell (1918):

Recognition of the existence of the dynamic effect caused by moving vehicles had taken place at least in case of railway bridges as long ago as the middle of the 19th century. Experimental determination of dynamic effects due to moving loads had commenced as early as 1849 when an English engineer named Willis[12] performed some very generalized experiments on a pair of 9 feet long bars, using a carriage running at different speeds with variable loads. It was concluded that the deflections increased with the speed of the carriage up to a certain point. The deflections recorded were in some cases two or three times those obtained under static load. However, the bars used in the tests were too flexible to be compared with real bridges. Accordingly reasonable prediction of load factors for real bridges was not possible [Waddell 1918].

[12] The original reference is not available. The information presented has been chiefly taken from Waddell (1918).

In 1881, the deflections measured at the center and the quarter points of a 60-feet continuous Howe truss span, by H. Sabine[13] and Professor S.W. Robinson resulted in DLF values ranging from 6 to 12 percent for velocities at or under 30 mph. In a paper, based on the results of the deflection tests conducted between 1881 and 1887 on Ohio railroad bridges [Robinson 1887], Professor Robinson showed that the dynamic load effects may be as high as 50 percent with an average of about 26.5 percent.

In France, a notable experimental work for determination of dynamic load effect was started by Col. Charles Rabut[14]. The investigation yielded the DLF values from 10 to 50 percent. A formula for 'impact factors' of short spans was proposed as follows [Waddell 1918]:

$$I = \frac{1}{1 + \left(\frac{L}{4}\right)^2} \qquad \text{---------(14)}$$

where I is the impact factor, and

L is the span in meters

In 1893, J. Melan[15] of Austria carried out a mathematical study of dynamic effects for different span lengths. His results indicated impact values of 0.77 to 0.15 for the span lengths from 5 to 120 m respectively [Waddell 1918].

In 1887, C.C. Schneider[16], proposed the following formula:

$$I = \frac{300}{L + 300} \qquad \text{---------(15)}$$

[13] Same as footnote No. 12.
[14] See the footnote on page 54.
[15] See the footnote on page 54.
[16] See the footnote on page 54.

where I is the coefficient of 'impact' and

L is the length of that portion of the span which is covered by the moving load when the member under consideration receives its greatest live load stress.

This formula remained in use until about 1912-13 [Waddell 1918].

Another important step towards the experimental determination of the dynamic effect of moving loads on bridges was the formation of an American Railway Engineering Association (AREA) committee for a thorough investigation of the impact effect. The report of the committee[17] issued in 1911, based on the experimental work carried out on :

a. 21 plate-girder spans of lengths up to 100 feet, and

b. 24 truss spans of length ranging from 100 to 250 feet

for loaded cars with speeds ranging from 10 to greater than 60 miles an hour, proposed following two formulas for calculation of impact [Waddell 1918]:

$$I = \frac{1}{1 + \frac{L^2}{20{,}000}} \qquad \text{---------(16)}$$

and

$$I = \frac{60}{L} \qquad \text{---------(17)}$$

where I and L are defined similar to that for the Schneider formula given above.

[17] See the footnote on page 54.

In a later report of the committee[18] [AREA *Proceedings* 1916], a revised formula has been given as [Waddell 1918]:

$$I = \frac{1}{1 + \frac{L^2}{30,000}} \qquad \text{---------(18)}$$

In a paper (published in Volume CC of the *Proceedings* of the Institution of Civil Engineers, Charles William Anderson)[19], based on some experiments conducted in India by the government bridge engineer H. S. Sales, proposed the following 'impact' formula [Waddell 1918]:

$$I = \frac{50}{L + 50} \qquad \text{---------(19)}$$

The author of the paper admitted that the formula had no direct scientific basis [Waddell 1918].

After indicating the inadequacy of various impact formulas, Waddell (1918), based on the analysis of these formulas, recommended the following formulas, for various kinds of bridges:

Structure Type	Proposed Impact Formula
1. Steam Railway Structures	$I = \frac{165A}{nL + 150}$ ---------(20)
2. Highway and Electric-Railway Structures	$I = \frac{100A}{nL + 200}$ --------(21)

[18] See the footnote on page 54.
[19] See the footnote on page 54.

where n is the number of tracks on the bridge,

L is the span length in feet, and

A = 1.00 for steel bridges with open or light floors,

= 0.75 for steel bridges with heavy, solid floors,

= 0.50 for reinforced concrete structures, except spandrel-filled arch bridges

= 0.25 for spandrel-filled arch bridges

3.3.2 The real effort on the problem of the highway bridge dynamic response started in the early 1930s when a special committee of the American Society of Civil Engineers published an important report on the subject of the 'impact' in highway bridges [Committee 1931]. The recommendations made in this report, based on data obtained from a series of field tests carried out using 3-1/2 ton Liberty trucks traveling at a maximum speed of 15 mph, became the basis of most of the specifications (on the computation of dynamic effects of moving loads on highway bridges) being used in the 1960s [AASHTO] and some of which are still being used [CPHB 1967]. In this report the committee recommended, in the case of girders and trusses of highway bridges that:

a. in the case of span lengths less than 40 ft, the 'impact increment' (that is, the dynamic load factor) for the stress should be assumed as 25 percent of the live load stress, and

b. in the case of span lengths of 40 feet or more, the impact increment of the stress should be assumed as a fraction of the live load stress determined by the formula:

$$I = \frac{50}{L + 160} \qquad \text{---------(22)}$$

where I is the impact fraction, and

L is the span length in feet.

3.3.3 In 1944, Vandegrift[20] performed tests on several types of bridges in Ohio, to determine their dynamic properties and to check if some sort of damper could be fixed to the bridge to reduce vibration. The major conclusions drawn from the study were:

1. Rigidity: Composite action was present in simply supported and continuous deck and slab bridges although not considered in the design. Accordingly, the stiffness and natural frequency of the spans were increased. However, the stiffening provided by the deck system did not increase the rigidity of through plate-girder bridges and trusses.

2. Frequency of Vibration: "First mode natural frequencies were usual for both loaded and unloaded spans." [Wright and Green 1959] (Might be similar in both loaded and unloaded cases).

3. Dampers and Amplitude: In the case of 3-span continuous bridge, the dampers used were effective in reducing the amplitude of vibration.

3.3.4 Dynamic load effect was the main concern of the bridge tests reported by G. R. Mitchell[21] (1948-1952 D.S.I.R. - Great Britain) [Wright and Green 1959]. The tested bridges were stated to have relatively good approaches and the surfaces free of pot holes

[20] See footnote on page 26.
[21] See footnote on page 26.

or irregularities that were thought to be responsible for the initiation of vertical motion of the vehicle. The spans of the bridges were in the range of 28 to 35 feet. The test vehicle consisted of a tractor-trailer combination and a sprung suspension system with solid tires. The vehicle axle spacing was about 24 feet and axle loads were 14 kips for the front and 50 kips for the rear axle. The results and conclusions of the study were as follows:

a. In one of the tests, the observed dynamic response was similar regardless of whether the vehicle traveled along the center line, or in a traffic lane of the bridge."

b. It was noticed that the test vehicles had a vertical motion as they crossed the bridges and, moreover, their wheels did not leave the deck of the bridge. In the case of loaded bridges, the frequency of vibration was different than the natural frequency of the unloaded bridge, and thus it was the case of a forced vibration.

c. For each axle of the trailer, values of the forced frequency of vibration were found to be constant and did not change significantly even from bridge to bridge.

d. In the small range (Original reference is not available. So the speed range is not known) of vehicle speeds considered during the tests the amplitude of the bridge vibration increased considerably with the speed of the test vehicle.

3.3.5 The tests reported by Norman[22] (1949) [Wright and Green 1959] were carried out to investigate the causes of significant vibration of a bridge under normal loading conditions. The bridge was a two lane, two span composite structure of total length 160

[22] See footnote on page 26.

feet, with a 20 degree. skew angle and a slight grade (1/60). The bridge was designed for H-20 S16 loading. The bridge approaches were reported to be good. The results and the conclusions drawn were:

a. The maximum amplitude of motion occurred when the test vehicle had left one span and had reached the middle of the adjacent span. Vibrations caused by the two-axle test vehicle continued for one and a half minutes in some cases. The structure was so flexible that it started vibrating due to pedestrians walking over it.

b. For calculating the natural frequency the values of stiffness were computed from measured static deflections. The bridge had a first vibration mode equivalent to the second mode of a simple span, with a 2% difference between the measured and computed frequencies. The test vehicle did not affect the frequency of the bridge vibration.

c. The magnitude of the vibration of a bridge was dependent upon the vehicle speed; the motion of the bridge was not much significant at a speed of 37.2 mph whereas at 41.2 mph it became very pronounced. The latter speed did not correspond to the resonance condition given by the product of vehicle axle spacing and natural frequency of the bridge (32 mph).

d. The maximum recorded value of the amplitude was 1.7×10^{-3} inches, and the natural frequency of the bridge 3.58 Hz. "Assuming the motion of the bridge to be sinusoidal over this maximum cycle, the resulting velocity and acceleration were 38,000 micro-inches per second, and 0.002g. feet per $(\text{second})^2$ respectively. This motion was very significant for an observer on the bridge." [Wright and Green 1959]

3.3.6 In the field tests carried out at M.I.T.[23] (Massachusetts Institute of Technology) [Wright and Green 1959], five simply supported bridges with spans from 69 to 114 feet were subjected to a two axle moving vehicle, to observe the mid span deflections. Two of these bridges were later on retested using an instrumented two axle vehicle as well.

The study lead to the following major conclusions:

1. Rigidity: The values of simple static deflections and natural frequencies of vibration revealed that the bridges generally had a greater rigidity than that computed from the design data.

2. Frequency of Vibration: The loaded bridges exhibited the forcing frequency of the test vehicles instead of the natural frequency of the bridge. Originally the bridge vibrated at its natural frequency, later on the forcing frequency dominated the bridge motion. The irregularity of the deck of one bridge modified the vehicle motion, and correspondingly, resulting in a change in the frequency of the bridge vibration from a forced to a natural frequency. In the unloaded condition the bridges vibrated with their first mode frequency.

The natural frequency of a simple beam bridge was found to be approximated by the empirical equation:

$$f_b = \frac{1.2\pi}{2L^2}\sqrt{\frac{EIL}{M_g}} \qquad \text{---------(23)}$$

where f_b is the natural frequency of the bridge, Hz

L is the span length,

EI is the flexural rigidity, and

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[23] See footnote on page 26.

M_g is the mass of the beam or girder

3. Amplitude: No apparent relationship could be found between the amplitude of vibration under load and vehicle speed. In case of a smoothly rolling load, the observed amplitudes were larger than that obtained by theory. The largest amplitudes were observed either in the case of an irregular bridge approach or when the natural frequency of the span was low compared to the other spans. The surface roughness of the bridge deck was not thought to be a major cause in producing large amplitudes[Wright and Green 1959]. Assuming viscous damping, the mean values of damping coefficient were found to be 0.007 for girder bridges and 0.022 for stringer bridges.

4. Vehicle Effects: The rear axle load was varied up to 82 per cent of the total vehicle load. The results lead to the conclusion that variation of the forcing frequency was proportional to the frequency of load variation. The magnitude of this variation was a function of pavement irregularity and thought to be a primary factor affecting the amplitude of vibration. The expansion joints were found to have a significant effect on the axle load variation. The observed frequency of (dynamic) load variation was in agreement with the computed natural frequency of the loaded tires. It was observed that in the case of the test vehicle, the actual vehicle springing was due to the tires, rather than the suspension system. It was pointed out that a vehicle with a blocked suspension system (totally unsprung vehicle load) might result in much larger bridge deflections than from a vehicle with a free system.

5. Analysis: All the results obtained through analytical studies, mentioned earlier (section 3.2.8 above), were in good agreement with the corresponding field test results.

6. Design: It was warned that the present tendency toward long span slender bridges should be regarded with caution because combination of low natural frequency (approaching that of heavy vehicles) and approach road surface irregularities might result in excessive vibration for these structures.

3.3.7 Hayes and Sbarounis (1954) reported observations made on the dynamic and static characteristics of a three-span continuous beam and slab bridge. The approaches and deck of the test structure were described as smooth.

Composite action was found to be present in various amounts between the deck and the beams, and was responsible for increased structural rigidity. However, a comparison of static and dynamic strain measurements showed that the composite action was reduced under dynamic loading.

During and as a result of the motion of the test vehicle, the natural frequency of the structure was changing continuously and was found to be a function of the percentage of the composite action present between beam and slab for varying positions of the test vehicle.

Whenever the period of load application coincided with the natural period of the structure, the amplitudes were a maximum.. At low speeds, the tandem axles of the test vehicle lead to a resonant condition; whereas at higher speeds the tandem axles acted dynamically as a single axle, the axle spacing of the test vehicle being a major factor. Reportedly, the natural frequency of the test vehicle had no coincidence with the frequencies of the bridge motion resulting from dynamic load tests.

3.3.8 Tests carried out by the Michigan State Highway Department[24] (1952-1957) to determine the static and dynamic characteristics of several types of highway bridges,[Foster 1952; Foster and Oehler 1956; Wright and Green 1959] were of two types:

a. extensive tests on two bridges, using a wide variety of test vehicles, and

b. tests on a variety of bridge types, using only one standard instrumented test vehicle, to compare the vibration susceptibility of the bridge types.:

a. Extensive Tests on Two Bridges:

These tests were carried out on the following two bridges:

I. An eight span bridge, consisting of a three-span continuous (steel) plate girder and slab structure and five simple spans, and

II. a bridge consisting of six simple spans.

Following were the major findings from these tests:

1. Rigidity: The ratio of the actual to computed deflections of the spans under load, was observed to be less than 40 per cent, the deflections being computed in accordance with AASHO specifications[25] (1953)[Wright and Green 1959]. It was concluded that the increase in rigidity was due to composite action of the deck and girders.

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[24] See footnote on page 32.
[25] See footnote on page 32.

2. Frequency of Vibration: In general, the bridge vibrations exhibited a first mode natural frequency. The natural frequencies of the spans, not designed for composite action, were calculated considering the presence of the composite action. The difference between the computed and the field values of natural frequency was found to be less than 4 per cent. The decay of the residual vibrations was observed to have a recurring pattern, although it was not regular.

3. Amplitude and Duration of Vibration: The effect of the period of axle load application (i.e., axle spacing/vehicle speed) was very significant. It was observed that when this period was a simple fraction of the natural period of the span, the amplitude and duration of vibration of the bridge were maximum, except in the case of two axle vehicles - probably because of unequal load distribution between axles. The effect of tandem axles was found to be smaller than that of an equivalent single axle. Considering viscous damping, the damping coefficients obtained were 0.025 for beam and slab bridges, 0.011 for plate girder and slab bridges, and 0.004 for the continuous bridge. The bridges were generally of composite concrete-deck over steel-girder construction.

b. General Tests: In this part of study, thirty four spans were tested using a standard instrumented 2S-1 vehicle. The bridges tested were beam-and- slab type simple, continuous or cantilever-type structures. The data recorded consisted of the mid-span deflections and axle load variation of the test vehicle.

The main conclusions reached were [Wright and Green 1959]:

"1. Rigidity: Using AASHO specifications (1953), the ratio of observed to theoretical deflection for the bridges tested was found to be less than 0.50. The average ratio was:

a) for simple spans, with composite design, 0.32,

b) for simple spans, without composite design, 0.16,

c) for continuous spans, either steel beam and slab without composite design or reinforced concrete, less than 0.35, and

d) for cantilever-type bridges with composite design, for both anchor arm span and suspended span, 0.44 and without composite design 0.22." [Wright and Green 1959]

2. Frequency of Vibration: The spans exhibited a first mode vibration significantly. The computed frequencies were in a very good agreement with the field results. The numerical method of A. S. Veletsos and N. M. Newmark (1955)[26], was used for calculating the frequency values of the continuous beam or girder and slab bridges. Whereas, for the continuous reinforced concrete and cantilever-type spans the cumbersome influence coefficient method (Thomson 1953)[27] was used. It was observed that the suspended spans of the cantilever-type bridges did not vibrate as simple spans but with a frequency dependent upon the structure as a whole.

3. Amplitude and Duration of Vibration: It was observed that the loaded simple spans with natural frequencies of less than 6.5 Hz generally developed greater amplitudes than

[26] Reported by Wright and Green (1959).
[27] Same as footnote No. 26.

spans with frequencies of less than 6.5 Hz. The cantilever bridges were observed to be the most flexible. "The duration of vibration of the various types of spans was approximately proportional to their observed rigidity. The mean values of damping factor obtained for the various bridge types were:

"a. for simple spans, composite and non-composite design, 0.012,

b. for continuous beam or girder and slab spans, non composite design, 0.010,

c. for continuous reinforced concrete spans, 0.020, and

d. for cantilever-type spans, both composite and non-composite design, 0.010."

[Wright and Green 1959]

4. Vehicle: The test vehicle consisted of two heavily loaded axles; one with a conventional suspension system and the other connected directly to the vehicle frame of the vehicle. The DLFs for the various loaded spans were proportional to the variation of the maximum axle load except in cases of the cantilever spans and the extremely rigid non-composite simple spans. "The maximum load variation was found to be proportional to vehicle speed, and varied with the irregularity of the bridge approaches."

"5. Vibration Susceptibility: An attempt was made to compare the various type of bridges tested with respect to human perception of motion, by using Janeway's[28] perceptibility limits [Wright and Green 1959]. The results suggested simple spans and cantilever type structures give the same degree of unpleasantness under the action of the test vehicle.

[28] See footnote on page 26.

However, the field observers considered the cantilever structures the more unpleasant of the two types."

3.3.9 "Two similar three-span continuous and plate-girder bridges were tested by Edgerton and Beecroft (1953) of the Oregon State Highway Department. One Bridge was studied because of reports of perceptible vibration and floor panel damage, while the second study was made for comparison. The test vehicle approximated an H-20 S-16 design load.

The conclusions reached from the tests were:

1. Rigidity: Composite action between the girders and the floor slab, though not considered in design, increased the stiffness of the spans.

2. Frequency of Vibration: Two modes of vibration were evident (first symmetric mode and first asymmetric mode of vibration). No forcing frequencies were observed. Computed frequencies for the bridges were about 10 per cent lower than observed. (The structures had varying moment of inertia and dead weight; the frequencies for the bridges were computed in terms of an equivalent prismatic structure).

3. Amplitude: Values of amplitude for both bridges were dependent upon the speed of the test vehicle. The values of amplitude obtained could be associated with deviations in the deck profiles of the two bridges."

3.3.10 "Observations were carried out by the California Highways Department[29] [Wright and Green 1959] on a beam and slab bridge to determine a) the degree of composite action present (in design such action was not considered), and b) the load distribution characteristics of the structure."

"The bridge was composed of a concrete deck supported by three longitudinal steel beams. the general layout of the 23-span structure was such that the girders were continuous over two spans, with the third span a suspended (might be cantilever or overhanging) span. The vehicle used during the tests was similar to a standard two-axle H-truck but weighed nearly 34 tons."

"The records of deflection indicated that dynamic deflection was a function of vehicle speed." [Wright and Green 1959]

3.3.11 The report of Fenves et al. (1962) gave a brief review of the program of the AASHO Road Test bridges, and presented some of the major results of investigation. The dynamic tests were performed on 15 test bridges, using 14 test vehicles. More than 1800 test runs were carried out with different combinations of vehicles and bridges. Vehicle speeds ranged from 10 to 60 mph. The bridges consisted of composite and non-composite steel, and reinforced, and prestressed concrete. Two and three axle vehicles were used in the tests. The dynamic response was studied with respect to four dimensionless parameters, namely the speed parameter, the weight ratio, the frequency ratio and the axle spacing ratio. Spans of test bridges were in the range of 50 ft, average measuresd natural

[29] See footnote on page 26.

frequency from 3.2 to 6.9 Hz, and average damping coefficient ranged from 0.8 to 4.5% of the the critical vaue. The dynamic load factors obtained from strain readings were in the range 0.03 to 0.25, and those calculated from deflection readings were in the range 0.02 to 0.42.

3.3.12 Billing (1984) reports the results of the dynamic field tests carried out on 27 highway bridges in Ontario, Canada. The tests were carried out to support and confirm the provisions for dynamic loading and dynamic response of bridges, designed using the Ontario Highway Bridge Design Code (OHBDC) - based on the frequency of the bridge. The bridges were of various configurations and consisted of 14 steel bridges with spans from 22 to 122 m, 3 timber bridges each of about 5 m span, and 10 concrete bridges of spans from 16 to 41 m. All the tests were performed according to a standardized procedure. Data was collected for 100 or more vehicle passes over each bridge in order that statistical parameters of dynamic response could be established to determine the load factors of a limit states code.

The results of the study revealed that:

a. The timber bridges apparently did not exhibit vibration.

b. The bridges were generally considerably stiffer under the service loading of a single truck than they were assumed to be in design.

c. The data generally confirmed that the form and the values selected in the first edition of OHBDC were adequate, except requirement of a few minor changes.

3.3.13. To investigate the main parameters influencing the dynamic response of highway bridges due to traffic loads, a comprehensive field test study was carried out in Ontario, Canada [Wright and Green 1964]. A total of 52 bridges, consisting mostly of simple span bridges, were selected to observe vertical motion at or near the mid-span, when subjected to moving vehicles. The spans of the 47 bridges varied from 50 to 320 feet, and the widths from 16 to 100 feet with the number of lanes varying from 1 to 8. The fundamental natural frequencies of the unloaded bridges varied between 1.96 and 10.6 Hz, and those for loaded bridges, in the range of 2.1 to 10.6 Hz., the bridges with extreme values being different in both cases. The study was not specifically undertaken for determination of dynamic amplification due to moving loads, nevertheless results of the tests reflected the absolute maximum values of 'dynamic increments' for the deflection to be between 0.23 and 1.16.

3.3.14 Shepherd and Aves (1973) have reported the results of field tests carried out on 22 spans of 10 different concrete bridges along with comparison of the test results with that of a proposed simple mathematical model. The bridges consisted of 7 simply supported spans of lengths from 31 to 80 feet, and 3 continuous spans of lengths from 22.5 to 48 feet. The experimental and theoretical frequencies ranged from 5.72 to 12.5 Hz and 6 to 10.2 Hz, respectively for the simple spans, and from 9.5 to 11.4 Hz and 9 to 12 Hz respectively for the continuous spans. The report compared the DLFs specified by the bridge design codes (of that time) of AASHO, Australia, New Zealand, India, Hungary, Germany and the United Kingdom.

On the basis of results of the field and model studies, it was concluded that :

a. The code specified dynamic load allowances were frequently exceeded in case of relatively short span simply supported bridges when the single vehicle load represented a large proportion of the total design live load and the frequency ratio of the vehicle and the bridge was relatively high.

b. The load variation was due mainly to the roughness of the approach, and the roughness of the bridge deck had a smaller contribution. On the basis of the limited experiment results, and a simple theoretical analysis, a formula had been proposed for the determination of DLFs against those specified in the bridge codes. It was, however, admitted that more theoretical research and field tests are needed to adopt the proposed formula for all type of bridges.

3.3.15 The report of Reto Cantieni (1984) summarizes the dynamic test results carried out by Swiss federal Laboratories for Material Testing and Research (EMPA) from 1958 to 1981 on 226 beam and slab type highway bridges. The bridges tested consisted of prestressed concrete (205), reinforced concrete (5), composite steel and concrete (14), and prestressed lightweight concrete structures. The number of spans of these bridges varied between 1 and 42, with an average of 4. Of these 226 bridges, 72% were multi-span continuous beam type, and 12% were single-span simply supported beam type structures. The minimum total length was 13 meters (43 ft) with maximum span equal to 11 meters (36 ft). The maximum total length was 3147.5 meters (10,300 ft) with maximum span equal to 118.8 meters (390 ft). A total of 109 bridges were straight, 97 skewed or curved, and 20 both skewed and curved.

It was concluded that a highway bridge would exhibit a pronounced dynamic response, provided that its frequency range is the same as that for one of the two vehicle modes concerned, and provided also that the vehicle speed and the surface roughness combine in such a way that corresponding mode of vibration of the vehicle would be excited.

3.3.16 In England, a field test study was performed by Leonard[30] et al. (1974), under Transportation and Road Research Laboratory, Crowthorne. The study was carried out on 30 highway bridges using eight commercial vehicles with gross weights more than 32 tons. The dynamic load factor values from 0.09 to 0.75 were recorded. Further details not available due to, non availability of the original report.

3.3.17 The report of Green[31] (1977), regarding the study of the dynamic response of bridge superstructure in Ontario, Canada, indicated dynamic load factors of 0.10 to 0.87 in the case of highway bridges. Further details not available due to, non availability of the original report.

3.3.18 O'connor and Pritchard (1985) have reported the results of two dynamic test studies performed on Six Mile Creek Bridge; a short span , steel and concrete highway

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[30] The original report was not available. The information presented was taken from Huang et. al. (1993).
[31] See the footnote No.16.

bridge in Australia. Field strains were used to calculate the maximum mid-span bending moments for 170 trucks in normal traffic. For vthe gross vehicle weights of 27 to 44 ton, the dynamic load factors ranged from -0.08 to 1.32.

3.3.19 Chan and O'connor (1990) have mentioned the results of some of the dynamic test studies performed in various countries, indicating that the DLF values higher than those given by certain bridge design codes [AASHTO 1977], have been obtained in several of studies. A vehicle model has been proposed for the highway bridge dynamic load investigations. Among others, it has been concluded that the natural frequencies of a range of simply supported bridges, with spans from 10 to 32 m, satisfy a relationship of the type Lf = constant, where L is the span and f is the fundamental frequency of the bridge, with reasonable accuracy.

3.3.20 To study the effect of truck suspension type on the dynamic response of short-span bridges was the main concern of the field investigation carried out by R. J. Heywood (1995). The study was carried out on three bridges consisting of:

- an R.C.C bridge having 3-spans of length 8.2 m each,
- a prestressed concrete bridge with 4-spans of length 9.14 m each, and
- a 3-span timber girder bridge with span lengths 7.62, 9.14, and 7.62 m repectively.

Vehicle vibrations may generally be expressed in terms of body bounce and axle hop frequencies. The body bounce frequencies for the vehicles were from 1.5 to 2 Hz for air-suspended, and from 2.5 to 4 Hz for steel-suspension vehicles. The axle hop frequencies generally range between 8 and 15 Hz. These might couple dynamically with with bridges having similar frequencies or the span lengths from 8 to 15 m.

The results of the study lead to the following conclusion:

a. Dynamic increments (the percent increase of dynamic deflection over the static deflection) were small for speeds less than 40 km/hr.

b. Dynamic increments (DI's) greater than 50% were relatively common. DI's of 100% or more were observed. The largest DI recorded was 137% for an air suspended vehicle excited by pavement repair at the prestressed concrete bridge.

c. DI for air-suspended vehicles was generally less than that for steel suspended vehicles.

d. DI due to steel suspended vehicles tended to increase with the speed of the vehicle.

3.4 Factors Affecting Dynamic Response

From the literature review, it follows that the dynamic response of highway bridges to moving loads is a complex phenomenon, depending upon a number of variables pertaining to both the load and the bridge. The main factors affecting the dynamic response and their effects on the bridge behavior are given below:

3.4.1 Bridge Span

The dynamic response decreases, in general, with increase in span length of the bridge. Comparison with analytical results shows that the AASHTO - 1992 formula:

$$I = \frac{50}{L + 125} \qquad \text{---------(24)}$$

underestimates the dynamic effect of vehicle load for short span bridges whereas for other spans [Fleming and Romualdi 1961; Huang et al. 1993], it is fairly accurate when the vehicle is modeled as an unsprung load [Fleming and Romualdi 1961]. The above formula overestimates the bridge response in the case of spans more than 80 feet [Huang et al. 1993].

3.4.2 Weight of the Vehicle

Dynamic response decreases with an increase in weight of the vehicle. For shorter spans, the dynamic effect will increase more rapidly with the reduction in weight of the vehicle [Huang et al. 1993]. An increase in mass of the load will result in an increase in the velocity at which the maximum response will take place [Fleming and Romualdi 1961]. In the case of 12-30 m span simply supported steel girder bridges subjected to a five-axle tractor trailer, the dynamic effect represented by the ratio of the dynamic to the static deflection decreases with increase in truck weight. However the decrease is due to the fact that the static deflection increases more than dynamic deflection, thereby decreasing the ratio of the two [Hwang and Nowak 1991].

3.4.3 Speed of the Vehicle

The dynamic effect of the load does change with the velocity of the vehicle. The response however, does not follow the same trend for vehicles with different gross vehicle weights [Hwang and Nowak 1991].

3.4.4 Number of Loaded Lanes

An increase in number of loading lanes will generally result in an increase in the dynamic load factor in the case of short span bridges, but has little effect on the maximum value of dynamic load factors [Huang et al. 1993].

3.4.5 Lateral Position of the Vehicles on the bridge

In the case of multi-girder bridges and especially for short spans, the lateral static and dynamic distributions are significantly different. The dynamic load factor in this case is inversely proportional to the static lateral distribution factor [Huang et al. 1993].

3.4.6 Approach Surface Roughness and Initial Conditions

Uneven approaches cause the initial displacement and velocity in the vehicle entering the bridge span. These two conditions can significantly increase the dynamic response of the bridge [Humar and Kashif 1993]. According to the investigations carried out at Massachusetts Institute of Technology (M.I.T) in 1953 (see section 3.2.8), the vertical oscillation of the vehicle itself, caused by the bridge surface roughness, had the main influence on the dynamic response of the bridge [Wright and green 1959]. The presence of a settlement of area in the approach may lead to extremely large dynamic effects in the bridges due to initial vibration of the load or vehicle. The value of the dynamic load factor may be much higher than that caused by several other load and structure parameters generally considered in the dynamic analysis of the bridges due to moving loads [Fleming and Romualdi 1961]. The existence of initial conditions, that is, initial vertical displacement and velocity of the vehicle before entering the bridge span generally results in higher amplitudes of vibration when compared with zero initial conditions. The effect is more significant and pronounced in the case the load is acting at or close to mid-span [Sundara Raja Iyengar and Jagadish 1970]. Initial displacement significantly increases the dynamic amplification in the case of moving loads over that without initial displacement [Shepherd and Aves 1973; Humar and Kashif 1993] . Presence of a small irregularity in the shape of a small sized bump or depression in the approach road introduces initial conditions and this may drastically increase the amplitude of dynamic response. The difference between the values of the dynamic response with and without initial conditions decreases, as the value of the speed parameter increases [Humar and Kashif 1993]. It is believed [Tung et al. 1956] that the initial vibration of the bridge would further increase the dynamic effect of the moving load.

3.4.7 Number of Vehicles on the Bridge

Generally the dynamic amplification factors are lower for multiple presence of vehicles on the bridge, than for a single vehicle. Dynamic amplification due to two parallel abreast (in phase) vehicles is significantly greater than when one of them is lagging behind the other (out of phase). For isotropic plate model, the amplification factors for the moment are generally less than those for the deflection [Humar and Kashif 1995]. The ratio of the dynamic to the static deflection for two side by side trucks is less than that for a single truck [Hwang and Nowak 1991].

3.4.8 Damping

Field tests have shown that the vehicle damping ratio is fairly low e.g., for a 10-ton dump truck [Biggs et al. 1959], the average value of damping is 2.2% of the critical value. It has been noticed that although damping of vehicle does reduce the dynamic response, but not by a large amount. The damping effect reduces the dynamic response more significantly in situations where the vehicle enters the bridge with initial conditions particularly when coupled with small vehicle weights and low values of the speed parameter [Humar and Kashif 1993].

3.4.9 Springing of the vehicle

There is not much difference in the dynamic response of the bridge for the sprung and the unsprung moving loads, in absence of the initial conditions (i.e., no initial displacement and velocity). However, when the initial conditions that result from the irregularities in the approach road, are taken into account the dynamic effects are considerably larger in the case of a sprung load, for the ranges of speed parameter and the mass ratios considered [Tung et al. 1956]. Dynamic effects are higher for a single unsprung load than for a single sprung load regardless of consideration or neglecting the effects of initial conditions [Fleming and Romualdi 1961]. According to the studies carried out at the University of Illinois[1951-55] in the case of a sprung vehicle with zero vertical displacement and velocity, the dynamic effect was found to be the same as for an equivalent unsprung load [Wright and Green 1959]. From the nature of the dynamic response for the vehicles of various gross weights as a function of the speed of the vehicle, it follows that the characteristics of the suspension system depend upon the weight and the speed of the vehicle [Hwang and Nowak 1991]. It has been reported [Heywood 1995] that while investigating the effect of vehicle suspension, the dynamic load factor values in excess of 50 percent were commonly observed. Values recorded were more than 100 percent in some cases. The maximum recorded value was 137 percent for an air-suspension system vehicle that was excited by a pavement repair [Heywood 1995] .

3.4.10 Type of Span

It has been reported [Fleming and Romualdi 1961] that the change in the maximum dynamic response due to change in mass might be significantly larger for continuous bridges than for single span bridges.

3.4.11 Axle Spacing

Axle spacing is an important factor in computing the static as well as the dynamic response of a bridge subjected to moving loads. The dynamic effect represented by the ratio of the dynamic to the static deflection behaves differently from span to span with the axle spacing. For longer spans (24 and 30 meters in this case), it increases with the increase in axle distance. Both the maximum mid-span static and the dynamic deflections increase with increase in axle spacing. In the case of smallest span considered (that is, 12 m), static deflections are constant as the tractor dominates the response, and the dynamic deflections are also nearly constant. [Hwang and Nowak 1991]. The dynamic deflection response is significantly affected by the axle spacing, particularly for higher values of the speed parameter α (= V/2Lf). Generally the response produced by a single-axle vehicle is significantly larger than a two-axle vehicle, when both have the same masses [Humar and Kashif 1993].

3.4.12 Speed Parameter

Speed parameter is defined as the ratio of the vehicle speed to the half period of first mode of vibration of the bridge. The speed parameter generally ranges up to 0.30 [Humar and Kashif 1993 and others]. In the case of the points on the line of loading, the moment amplification is generally smaller with large values of speed parameter, as compared to the deflection amplification. For larger values of speed parameter, maximum deflection amplification for the deflection takes place after the load moves away from the mid-span. The maximum deflection amplification factors are generally greater than the maximum moment amplification factors [Sundara Raja Iyengar and Jagadish 1970]. The dynamic response is generally higher for higher values of the speed parameter as well as the mass ratio (i.e., the ratio of the vehicle mass to that of the bridge), although the maximum values are at different values of the two variables. The dynamic response in general increases with increase in the value of the speed parameter [Humar and Kashif 1993]. The dynamic effect for the maximum moment is small in the case of an unsprung load. Considerable variation has been observed for amplification factors, as a function of the speed parameter [Tung et al. 1956]. The magnitude of the maximum dynamic effects were proportional to the speed of the vehicle [Wen and Veletsos 1962].

3.4.13 Frequency Ratio

Assuming frequency of the vehicle to be 2.75 Hz, the general range of frequency ratio (that is, ratio of the vehicle frequency to that of the bridge) is ≤ 1.4 whereas if the vehicle frequency is taken as 5 Hz, the frequency ratio reaches to about ≤ 2.5. The maximum value of dynamic amplification does not occur at a frequency ratio of 1.0, as generally believed, but takes place for frequency ratios from 0.4 to above 1.0 for speed parameter values between 0.10 and 0.30. Contrary to the case of the speed parameter, the variation of the dynamic response with the frequency ratio does not follow a general trend [Humar and Kashif 1993]. In case of slab or beam-and-slab bridges modeled as orthotropic plates, large dynamic effects were observed for frequency ratios between 0.6 and 1.0 [Sundara Raja Iyengar and Jagadish 1970].

3.4.14 Mass Ratio

In the case of a single unsprung load, there is a small increase in the dynamic effect with variation of mass ratio from zero to 1.0 [Fleming and Romualdi 1961]. In case of slab or beam-and-slab bridges modeled as orthotropic plates, the dynamic amplification is generally directly proportional to the mass ratio (i.e., the ratio of the vehicle mass to that of the bridge), although the effect is of secondary nature and less pronounced [Sundara Raja Iyengar and Jagadish 1970]. For the commonly used heavy vehicles, the mass ratio generally ranges to about ≤ 1.4 [Humar and Kashif 1993].

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CHAPTER 4

BEAM DEFLECTIONS DUE TO DYNAMIC LOADS

4.1 Differential Equation of Motion

The differential equation of the elastic curve of a prismatic beam with x-axis coincident with the longitudinal axis, loaded in the plane of a principal axis through the shear center, the x-y plane, thus resulting in bending about the other, or perpendicular, principal axis, that is the z-axis, is given by the expression

$$EI\frac{d^2y}{dx^2} = -M(x) \qquad \text{---------(a)}$$

where EI is the stiffness of the beam,

y is the deflection of the beam, and

M(x) is the bending moment, as a function of the distance, *x*, along the length of the beam, due to the load acting on the beam

Differentiating Eq(a) successively with respect to *x*, the distance along the beam, one gets

$$EI\frac{d^3y}{dx^3} = -\frac{dM}{dx} = -V(x) \qquad \text{---------(b)}$$

$$EI\frac{d^4y}{dx^4} = -\frac{dV}{dx} = q(x) \qquad \text{---------(c)}$$

where V(x) is the shear force, and

q(x) is the load intensity per unit length of beam

In case of a beam oscillating in free vibration, the only force acting on the beam is the inertia force, given by $$-m\,a = -m\frac{d^2y}{dt^2}$$

where a is the acceleration

m is the mass per unit length of the beam = ρA = $\gamma A/g$.

ρ is the density per unit length

γ is the specific weight

g is the acceleration due to gravity = 32.2 ft. per second^2

Substituting this force into Eq(c), one obtains

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{---------(1)}$$

This partial differential equation is the general beam equation for an elastic prismatic undamped beam in free vibration. The solution of general beam equation (Eq 1) can be obtained by applying the method of separation of variables. Consequently, the solution is of the form

$$y(x, t) = \varphi(x)Y(t) \qquad \text{---------(2)}$$

Substituting Eq(2) into Eq(1), one obtains

$$EI\varphi^{IV}Y + m\varphi\ddot{Y} = 0 \qquad \text{---------(3)}$$

or,

$$\varphi^{IV}Y + \frac{m}{EI}\varphi\ddot{Y} = 0 \qquad \text{---------(4)}$$

where φ^{IV} is the 4th derivative of $\varphi(x)$ with respect to 'x' = $\dfrac{d^4\varphi}{dx^4}$

and $\ddot{Y}$ is the second derivative of Y(t) with respect to 't' = $\dfrac{d^2Y}{dt^2}$

Dividing both sides of Eq(4) by Eq(2), one gets

$$\frac{\varphi^{IV}}{\varphi} + \frac{m}{EI}\frac{\ddot{Y}}{Y} = 0 \qquad \text{---------(5)}$$

$$\frac{\varphi^{IV}}{\varphi} = -\frac{m}{EI}\frac{\ddot{Y}}{Y} = a^4 \qquad \text{---------(6)}$$

Thus, since each term is independent,

$$\varphi^{IV}(x) = a^4\varphi(x) \qquad \text{---------(7)}$$

and

$$\frac{m}{EI}\ddot{Y}(t) + a^4\,Y(t) = 0 \qquad \text{---------(8)}$$

Restating Eq(8),

$$\ddot{Y}(t) + \omega^2 Y(t) = 0 \qquad \text{---------(9)}$$

where

$$\omega^2 = a^4\frac{EI}{m} \quad \Rightarrow \quad a^4 = \omega^2\frac{m}{EI} \qquad \text{---------(10)}$$

The general solution of Eq(7) is

$$\varphi(x) = A_1\cos ax + A_2\sin ax + A_3\cosh ax + A_4\sinh ax$$

Determination of the constants of integration A_i depends upon boundary conditions (bc) but, in all cases, results in a series of roots λ_n, n = 1, 2, ..., ∞ such that Eq(7) may be restated as

$$\varphi_n^{IV}(x) = \lambda_n \varphi_n(x) \qquad \text{---------(11)}$$

where $$\lambda_n = a_n^4 \qquad \text{---------(d)}$$

For example, in case of a simply supported single span beam, the solution is in a sinusoidal form with $a_n = \frac{n\pi}{L}$

where L = the span length,

and $\varphi_n(x) = D_n \sin\left(\frac{n\pi}{L}\right)$. In this case, $a^4 = \left(\frac{n\pi}{L}\right)^4 = \omega_n^2 \frac{m}{EI}$.

Thus $$\omega_n^2 = \frac{EI}{m}\left(\frac{n\pi}{L}\right)^4 \quad \Rightarrow \quad \omega_n = \left(\frac{n\pi}{L}\right)^2 \sqrt{\frac{EI}{m}} \qquad \text{---------(e)}$$

Also, the frequency for the nth mode of vibration is given by the expression

$$f_n = \frac{\omega_n}{2\pi} = \frac{n^2\pi}{2L^2}\sqrt{\frac{EI}{m}} \qquad \text{---------(f)}$$

From the frequency equation, it can be concluded that in the case of beams having similar geometric properties, the frequency of transverse vibration decreases proportionally with increase in length squared, $\frac{1}{L^2}$. Or frequency is inversely proportional to length squared i.e., $f \propto \frac{1}{L^2}$.

4.2 Orthogonality Conditions

In order to establish orthogonality conditions, Eq(11) is restated for modes 'i' and 'j',

$$\varphi_i^{IV} = \lambda_i \varphi_i \qquad \text{---------(12)}$$

$$\varphi_j^{IV} = \lambda_j \varphi_j \qquad \text{---------(13)}$$

Multiplying Eq(12) by φ_j and Eq(13) by φ_i and integrating over their length yields

$$\int_0^L \varphi_i^{IV}\varphi_j\,dx = \lambda_i \int_0^L \varphi_i\,\varphi_j\,dx \qquad \text{---------(14)}$$

$$\int_0^L \varphi_j^{IV}\varphi_i\,dx = \lambda_j \int_0^L \varphi_j\,\varphi_i\,dx \qquad \text{---------(15)}$$

At this point, it is necessary to integrate Eqs(14) and (15) by parts.

For $\int_0^L \varphi_i^{IV}\varphi_j\,dx$, let $u = \varphi_j$ & $dv = \varphi_i^{IV}\,dx$

and $du = \varphi_j'\,dx$ & $v = \varphi_i'''$

Thus,

$$\int_0^L \varphi_i^{IV}\varphi_j\,dx = \varphi_j\,\varphi'''_i\Big|_0^L - \int \varphi_i'''\varphi_j'\,dx \qquad \text{---------(16)}$$

Further, for $\int_0^L \varphi_i'''\varphi_j'\,dx$, let $u = \varphi_j'$ and

$du = \varphi_j''\,dx$ & $v = \varphi_i''$

So finally,

$$\int_0^L \varphi_i^{IV}\varphi_j\,dx = \varphi_j\,\varphi_i'''\Big|_0^L - \varphi_j'\varphi_i''\Big|_0^l + \int_0^L \varphi_i''\varphi_j''\,dx$$

$$= \lambda_i \int_0^L \varphi_i\,\varphi_j\,dx \qquad \text{---------(17)}$$

Similarly, it follows from Eq(15) that,

$$\varphi_j'''\varphi_i\Big|_0^L - \varphi_j''\varphi_i'\Big|_0^L + \int_0^L \varphi_i''\varphi_j''\,dx = \lambda_j \int_0^L \varphi_i\,\varphi_j\,dx \qquad \text{---------(18)}$$

<u>Boundary or End Conditions (bc) :</u>

These are constraints or compatibility conditions imposed by the supports or end conditions at $x = \bar{x}$; $\bar{x}$ usually is equal to 0 or L (ends of span). Thus at supports, such as for the simply supported condition, displacement y at x = 0 and x = L is zero at all times t leading to the space variable φ at x = 0 and x = L becoming equal to zero.

End Conditions	Meaning	y (x, t)	φ (x)
1. Simply supported	Displacement and the moment at support are zero.	$y(x=\bar{x}, t) = 0$ $y''(x=\bar{x}, t) = 0$	$\varphi(\bar{x}) = 0$ $\varphi''(\bar{x}) = 0$
2. Fixed - end	Displacement and the slope at support are zero.	$y(x=\bar{x}, t) = 0$ $y'(x=\bar{x}, t) = 0$	$\varphi(\bar{x}) = 0$ $\varphi'(\bar{x}) = 0$
3. Free-end	Moment and the shear at support are zero.	$y''(x=\bar{x}, t) = 0$ $y'''(x=\bar{x}, t) = 0$	$\varphi''(\bar{x}) = 0$ $\varphi'''(\bar{x}) = 0$
4. Guided-end	Slope and the shear at support are zero.	$y'(x=\bar{x}, t) = 0$ $y'''(x=\bar{x}, t) = 0$	$\varphi'(\bar{x}) = 0$ $\varphi'''(\bar{x}) = 0$

Review of the boundary conditions and comparison with the first two terms in Eqs(17) & (18) shows that these are always zero. i.e.,

$$\varphi_j \varphi_i''' \Big|_0^L = 0 \quad ; \qquad \varphi_j' \varphi_i'' \Big|_0^L = 0$$

$$\varphi_j \varphi_i''' \Big|_0^L = 0 \quad ; \qquad \varphi_j' \varphi_i'' \Big|_0^L = 0$$

$$\varphi_j''' \varphi_i \Big|_0^L = 0 \quad ; \qquad \varphi_j'' \varphi_i' \Big|_0^L = 0$$

therefore, from Eqs(17) & (18)

$$\int_0^L \varphi_i^{IV} \varphi_j \, dx = \int_0^L \varphi_i'' \varphi_j'' \, dx = \lambda_i \int_0^L \varphi_i \varphi_j \, dx \qquad \text{---------(19)}$$

and

$$\int_0^L \varphi_i'' \varphi_j'' \, dx = \lambda_j \int_0^L \varphi_i \varphi_j \, dx \qquad \text{---------(20)}$$

Subtracting Eq(19) from Eq(20)

$$(\lambda_j - \lambda_i) \int_0^L \varphi_i \varphi_j \, dx = 0 \qquad \text{---------(21)}$$

It follows that if $\lambda_i \neq \lambda_j$,

$$\int_0^L \varphi_i(x) \varphi_j(x) \, dx = 0, \qquad i \neq j \qquad \text{---------(22)}$$

$$\int_0^L \varphi_i''(x) \varphi_j''(x) \, dx = 0, \qquad i \neq j \qquad \text{---------(23)}$$

and

$$\int_0^L \varphi^{IV}{}_i(x) \varphi_j(x) \, dx = 0, \qquad i \neq j \qquad \text{---------(24)}$$

Equations (22), (23), & (24) are the orthogonality conditions for elastic beams.

When i = j, Eqs(21) leads to

$$\int_0^L \varphi_i^2(x) \, dx = \alpha_i \qquad \text{---------(25)}$$

where α_i is a constant. In addition Eq(17), after substituting i = j in Eq(19) and comparing with Eq(25), transforms to

$$\int_0^L \varphi_i^{IV} \varphi_i \, dx = \int_0^L (\varphi_i'')^2 \, dx = \lambda_i \, \alpha_i \qquad \text{---------(26)}$$

These are the normalization functions.

Now, Eq(1) may be rewritten as

$$m\ddot{y} + EI\, y^{IV} = 0 \qquad \text{---------(27)}$$

and, to account for all modes of vibration, Eq(2) may be rewritten as

$$y(x,t) = \sum_{i=1}^{\infty} \varphi_i(x) Y_i(t) \qquad \text{---------(28)}$$

Consequently, substitution of Eq(28) into Eq(27), results in

$$\sum_{i=1}^{\infty} (m\varphi_i \ddot{Y}_i + EI\varphi_i^{IV} Y_i) = 0 \qquad \text{---------(29)}$$

Whereas the multiplication of Eq(29) by φ_j and subsequent integration over the entire span, yields

$$\sum_{i=1}^{\infty} (m \ddot{Y}_i \int_0^L \varphi_i \varphi_j \, dx + EI\, Y_i \int_0^L \varphi_j \varphi_i^{IV} \, dx) = 0 \qquad \text{---------(30)}$$

With the application of the orthogonality conditions i.e., Eqs(22) & (24), and the normality conditions, i.e., Eqs(25), and (26), Eq(30) transforms to

$$\ddot{Y}_i (m \int_0^L \varphi_i^2 dx) + Y_i (EI \int_0^L \varphi_i \varphi_i^{IV} \, dx) = 0 \qquad \text{---------(31)}$$

Now, defining the 'principal mass', as

$$M_n = m\int_0^L \varphi_n{}^2 dx \qquad \text{---------(32)}$$

and using Eq(26), the 'principal rigidity', as

$$R_n = EI\int_0^L \varphi_n \varphi_n^{IV} dx = EI\lambda_n \alpha_n \qquad \text{---------(33)}$$

By the substitution of Eqs(32) and (33) to Eq(31), it becomes

$$M_n \ddot{Y}_n(t) + R_n Y_n(t) = 0 \qquad \text{---------(34)}$$

However, since $\lambda_n = a_n^4$,

$$R_n = EI\lambda_n \alpha_n = EI a^4 \alpha_n = EI(\frac{\omega_n{}^2 m}{EI})\alpha_n = \omega_n{}^2 m \alpha_n \qquad \text{---------(35)}$$

So, $$R_n = \omega_n{}^2 m\int_0^L \varphi_n^2 dx = \omega_n{}^2 M_n \qquad \text{---------(36)}$$

Thus, $M_n \ddot{Y}_n(t) + \omega^2{}_n M_n Y_n(t) = 0$

that is, $$\ddot{Y}_n(t) + \omega_n^2 Y_n(t) = 0 \qquad \text{---------(37)}$$

and the equation is uncoupled into the form of a single degree of freedom equation by modes. This is the equation of a simple harmonic motion whose solution is of the form

$$Y_n(t) = A_n \cos\omega_n t + B_n \sin\omega_n t \qquad \text{---------(37a)}$$

and $$y = y(x,t) = \sum_{n=1}^{\infty} \varphi_n Y_n \qquad \text{---------(37b)}$$

Given the initial conditions(ic), i.e.,

$$y_0(x, t=0) = f_1(x) \quad \text{and} \quad \dot{y}_0(x, t=0) = f_2(x) \qquad \text{---------(38)}$$

$$y_0 = \sum_{n=1}^{\infty} \varphi_n Y_{0n} = f_1(x) \quad \text{and} \quad \dot{y}_0 = \sum_{n=1}^{\infty} \varphi_n \dot{Y}_{0n} = f_2(x) \qquad \text{---------(39)}$$

multiplying both sides of Eqs(39) stated in mode 'i' by φ_j and integrating over the span L, yields

$$\sum_{i=1}^{\infty} Y_{0i} \int_0^L \varphi_i \varphi_j \, dx = \int_0^L \varphi_j f_1(x) dx \qquad \text{---------(40)}$$

and

$$\sum_{i=1}^{\infty} \dot{Y}_{0i} \int_0^L \varphi_i \varphi_j \, dx = \int_0^L \varphi_j f_2(x) dx \qquad \text{---------(41)}$$

When *i≠j*, because of orthogonality as given by Eq(22), both the Eqs(40) and (41) equal zero; however, when i= j= n, Eqs(32) & (36) yield

$$Y_{0n} \frac{M_n}{m} = \int_0^L \varphi_n f_1(x) dx \qquad \text{---------(42)}$$

and

$$\dot{Y}_{0n} \frac{M_n}{m} = \int_0^L \varphi_n f_2(x) dx \qquad \text{--------(43)}$$

or,

$$Y_{0n} = \frac{m}{M_n} \int_0^L \varphi_n f_1(x) dx \qquad \text{--------(44)}$$

and

$$\dot{Y}_{0n} = \frac{m}{M_n} \int_0^L \varphi_n f_2(x) dx \qquad \text{---------(45)}$$

Thus,

$$y(x,t) = \sum_{n=1}^{\infty} \varphi_n (x) \left(Y_{0n} \cos \omega_n t + \frac{\dot{Y}_{0n}}{\omega_n} \sin \omega_n t \right) \qquad \text{---------(46)}$$

4.3 Forced Response

In this case, Eq(1) becomes

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + m\frac{\partial^2 y(x,t)}{\partial t^2} = p(x,t)$$

or

$$EI\, y^{IV} + m\ddot{y} = p(x,t) \qquad \text{--------(47)}$$

Using

$$y(x,t) = \sum_{i=1}^{\infty} \varphi_i(x)\, Y_i(t) \qquad \text{--------(48)}$$

Eq(47) leads to

$$\sum_{i=1}^{\infty} EI\,\varphi^{IV}{}_i(x)\, Y_i(t) + \sum_{i=1}^{\infty} m\,\varphi_i(x)\,\ddot{Y}_i(t) = p(x,t) \qquad \text{---------(49)}$$

Multiplying each term by $\varphi_j(x)$ and integrating over the span,

$$m\sum_{i=1}^{\infty} \ddot{Y}_i(t)\int_0^L \varphi_i\, \varphi_j\, dx + EI\sum_{i=1}^{\infty} Y_i(t)\int_0^L \varphi_i^{IV}\, \varphi_j\, dx = \int_0^L \varphi_j\, p(x,t)\, dx \qquad \text{---------(50)}$$

From the orthogonality conditions, Eq(22) & (24), all terms are equal to zero; therefore when i = j = n,

$$\ddot{Y}_n\,(m\int_0^L (\varphi_n(x))^2\, dx \; + \; Y_n\,(EI\int_0^L \varphi_n^{IV}(x)\, \varphi_n(x)\, dx = \int_0^L \varphi_n(x)\, p(x,t)\, dx \qquad \text{--------(51)}$$

Letting $m\int_0^L (\varphi_n(x))^2\, dx = M_n$, and

$$EI\int_0^L \varphi_n^{IV}(x)\varphi_n(x)\, dx = EI\,\lambda_i\int_0^L (\varphi_n(x))^2\, p(x,t)\, dx$$

$$= m\,\omega_n^2 \int_0^L (\varphi_n(x))^2\,dx$$

$$= \omega_n^2 M_n$$

and defining $$\int_0^L \varphi_n(x)\ p(x,t)\ dx = P_n \qquad \text{---------(52)}$$

Eq(51) transforms to the typical equation of motion for the forced vibration case in uncoupled coordinates

$$M_n \ddot{Y}_n(t) + \omega^2{}_n M_n Y_n(t) = P_n \qquad \text{---------(53)}$$

where M_n is the principal mass, lb-sec^2/ft

4.4 General Procedure

In general, the procedure for finding the response of an elastic beam is as follows:

1) Apply boundary conditions (bc) to the solution of Eq(7), i.e.,

$$\varphi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \qquad \text{---------(54)}$$

where

$a_n^4 = \lambda_n = \dfrac{\omega_n^2 m}{EI}$, are the system eigenvalues, and the corresponding functions

$\varphi_n(x)$, n = 1, 2, ..., ∞, are the corresponding system modes or mode shapes.

Note that the simply supported beam is an exception as in most of the cases, such as for a cantilever beam, the frequency equation is transcendental and the shape function, $\varphi(x)$, fairly complicated.

2) Calculate 'the principal mass', M_n , from Eq(32).

3) Calculate 'the generalized load', P_n , from Eq(52). This provides values for the uncoupled equation of motion, Eq(53).

4) Solve Eq(53) using the Method of Undetermined Coefficients, to obtain the particular solution.

5) Apply initial conditions (ic) to solve for the constants of integration. This yields the solution for $Y_n(t)$.

6) Substitute the functions $\varphi_n(x)$, found in step-1, and $Y_n(t)$, found in step-5, into Eq(28) to obtain the complete solution.

4.5 Effect of Damping

For including the effect of damping, it may be assumed to be in modes so that the expression

$$D_n \dot{Y}_n(t) = 2\,\xi_n\,\omega_n\,M_n\,\dot{Y}_n(t) \qquad \text{---------(55)}$$

may be added to Eq(53) to yield

$$M_n\,\ddot{Y}_n(t) + D_n\,\dot{Y}_n(t) + \omega_n^2\,M_n\,Y_n(t) = P_n(t) \qquad \text{---------(56)}$$

or,

$$M_n\,\ddot{Y}_n(t) + 2\,\xi_n\,\omega_n\,M_n\,\dot{Y}_n(t) + \omega_n^2\,M_n\,Y_n(t) = P_n(t)$$

or,

$$\ddot{Y}_n(t) + 2\,\xi_n\,\omega_n\,\dot{Y}_n(t) + \omega_n^2\,Y_n(t) = \frac{P_n(t)}{M_n} \qquad \text{---------(57)}$$

This is the typical under-critically damped uncoupled equation of motion with solution

$$Y_n(t) = e^{-\xi_n \omega_n t}\left(A_n \sin \omega_{nD} t + B_n \cos \omega_{nD} t\right) \qquad \text{---------(58)}$$

where $\omega_{nD} = \omega_n \sqrt{1-\xi_n^2}$, and ξ_n is the fraction of critical damping in mode 'n'.

In order to compute the bending moment, M, and the shear force, V, in an elastic beam, following relationships may be used:

$$\text{Moment: } M(x,t) = -EI\frac{\partial^2 y(x,t)}{\partial x^2} \qquad \text{---------(59)}$$

and

$$\text{Shear: } V(x,t) = \frac{\partial M}{\partial x} = -EI\frac{\partial^3 y(x,t)}{\partial x^3} \qquad \text{---------(60)}$$

Review of Integration by Parts

The following is a brief review of integration by parts:

Integration by parts is of the form

$$\int u dv = uv - \int v du \qquad \text{---------(A)}$$

Example:

$$\int xe^x dx$$

Let $u = x$ and $dv = e^x dx$, then

$du = dx$ and $v = e^x$, substituting into Eq.(A),

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C$$

4.6 Example-1: Deflection Due to a Constant Stationary Force

For a simply-supported (SS) beam, $\varphi_n(x) = \sin\frac{n\pi x}{L}$ ---------(E1a)

and, with zero damping

$$y(x,t) = \sum_{n=1}^{\infty} \varphi_n(x)(A_n \cos \omega_n t + B_n \sin \omega_n t)$$

or, $$y(x,t) = \sum_{n=1}^{\infty} \sin \frac{n\pi x}{L}(A_n \cos\omega_n t + B_n \sin\omega_n t) \quad \text{---------(E1b)}$$

To meet the orthogonality condition of Eq(32), i.e.,

$$M_n = m\int_0^L (\varphi_n(x))^2 dx \quad \text{---------(E1c)}$$

that is $$M_n = m\int_0^L \sin^2 \frac{n\pi x}{L} dx$$

Integrating, $$\int_0^L \sin^2 \frac{npx}{L} dx = \left| \frac{x}{2} - \frac{L}{4np} \sin^2 \frac{npx}{L} \right|_0^L = \frac{L}{2}$$

Therefore, $$M_n = \frac{mL}{2} \quad ; \quad \frac{m}{M_n} = \frac{2}{L} \quad \text{--------(E1d)}$$

With initial conditions Y_{0n} and $\dot{Y}_{0n}$,

$$y(x,t) = \sum_{n=1}^{\infty} \sin \frac{n\pi x}{L}(Y_{0n} \cos \omega_n t + \frac{\dot{Y}_{0n}}{\omega_n} \sin \omega_n t) \quad \text{--------(E1e)}$$

where the values for Y_{0n} and $\dot{Y}_{0n}$ are obtained from Eqs(44) & (45).

For a situation with an initial velocity, *v*, over the span except at the supports and zero displacement; from Eq(38)

$$y_0 = f_1(x) = 0, \quad \text{and} \quad \dot{y}_0 = f_2(x) = v.$$

Therefore from Eq(44), $Y_{0n} = 0$, and from Eq(45)

$$\dot{Y}_{0n} = \frac{m}{M_n}\int_0^L \varphi_n(x) f_2(x) dx$$

$$\dot{Y}_{0n} = \frac{2v}{L}\int_0^L \sin\frac{n\pi x}{L}dx$$

$$= \frac{4v}{n\pi}, \quad n = 1,3,5,\ldots,\text{(odd only)} \qquad \text{--------(E1f)}$$

Substituting into Eq(46)

$$y(x,t) = \frac{4v}{\pi}\sum_{n=1(\text{odd only})}^{\infty} \sin\frac{n\pi x}{L}(0 + \frac{1}{n\,\omega_n}\sin\omega_n t)$$

$$= \frac{4v}{\pi}\sum_{n=1(\text{odd only})}^{\infty} \frac{1}{n\,\omega_n}\sin\frac{n\pi x}{L}\sin\omega_n t$$

$$= \frac{4v}{\pi}\left[\frac{1}{\omega_1}\sin\frac{\pi x}{L}\sin\omega_1 t + \frac{1}{\omega_3}\sin\frac{\pi x}{L}\sin\omega_3 t + \ldots\right] \qquad \text{--------(E1g)}$$

4.7 Example-2: Deflection Due to a Suddenly Applied Stationary Load

For a simply supported beam with ic=0, find the response of a beam subjected to a suddenly applied mid-span load p(t).

$$p(t) = 0 \qquad t < 0$$

$$= p_0 \qquad t \geq 0$$

From Eq(52), that is, $\int_0^L \varphi_n(x)\, p(x,t)\,dx = P_n$ ---------(E2a)

$$P_n = \int_0^L \sin\frac{n\pi x}{L}p(x,t)\,dx = p_0 \sin\frac{n\pi}{2} \qquad \text{---------(E2b)}$$

Accordingly, from Eq(53),

$$\ddot{Y}_n + \omega_n^2 Y_n = \overline{P}_0 \qquad \text{---------(E2c)}$$

where $$\bar{P}_0 = \frac{2p_0}{mL} \sin\frac{n\pi}{2}$$ ---------(E2d)

Now, $$Y_n(t) = Y_{nH}(t) + Y_{nC}(t)$$ ---------(E2e)

where $Y_{nH}(t)$ is the homogeneous solution (free vibration case), and

$Y_{nC}(t)$ is the complimentary solution (forced vibration case).

with $Y_{nH}(t) = A_n \cos\omega_n t + B_n \sin\omega_n t$ ---------(E2f)

and $Y_{nC}(t) = C_1 + C_2 t$ ---------(E2g)

such that

$$Y_n(t) = A_n \cos\omega_n t + B_n \sin\omega_n t + Y_{nC}(t)$$

Thus, substituting the complimentary or particular equation,

$$Y_{nC}(t) = C_1 + C_2 t$$

$$\dot{Y}_{nC}(t) = C_2$$

into the differential equation , E2c,

one obtains $$\omega^2{}_n (C_1 + C_2 t) = \bar{P}_0$$ ---------(E2h)

Setting constants equal, Eq(E2h) yields $\omega_n{}^2 C_1 = \bar{P}_0$

which in turn yields $$C_1 = \frac{\bar{P}_0}{\omega_n{}^2}$$

and setting coefficients of 't' equal yields $C_2 = 0$

Thus, $$Y_{nC}(t) = C_1 = \frac{\bar{P}_0}{\omega_n{}^2}$$

Therefore, $$Y_n(t) = A_n \cos\omega_n t + B_n \sin\omega_n t + \frac{\overline{P}_0}{\omega_n^2}$$ ---------(E2i)

$$\dot{Y}_n(t) = B_n \omega_n \cos\omega_n t - A_n \omega_n \sin\omega_n t$$

Applying the initial conditions i.e., $Y_n(t=0)=0$, and $\dot{Y}_n(t=0)=0$, we have

$$A_n(1) + B_n(0) + \frac{\overline{P}_0}{\omega_n^2} = 0 \Rightarrow A_n = -\frac{\overline{P}_0}{\omega_n^2}$$

and, $$B_n \omega_n(1) - A_n \omega_n(0) = 0 \Rightarrow B_n = 0$$

Substituting values of the coefficients A_n and B_n , in Eq(E2i), one gets

$$Y_n(t) = -\frac{\overline{P}_0}{\omega_n^2}\cos\omega_n t + \frac{\overline{P}_0}{\omega_n^2}$$

$$= \frac{\overline{P}_0}{\omega_n^2}(1 - \cos\omega_n t)$$

$$Y_n(t) = \frac{2p_0}{mL\omega_n^2}(1 - \cos\omega_n t)\sin\frac{n\pi}{2}$$

Finally, $$y(x,t) = \sum_{n=1}^{\infty} \varphi_n(x) Y_n(t)$$

$$= \sum_{n=1}^{\infty} \frac{2p_0}{mL\omega_n^2}(1 - \cos\omega_n t)\sin\frac{n\pi}{2}\sin\frac{n\pi x}{L}$$

$$y(x,t) = \frac{2p_0}{mL}\sum_{n=1}^{\infty}\frac{1}{\omega_n^2}\sin\frac{n\pi}{2}(1 - \cos\omega_n t)\sin\frac{n\pi x}{L}$$

$$= \frac{2p_0}{mL}\left[\frac{1}{\omega_1^2}\sin\frac{\pi}{2}(1-\cos\omega_1 t)\sin\frac{\pi x}{L} + \frac{1}{\omega_2^2}\sin\pi(1-\cos\omega_2 t)\sin\frac{2\pi x}{L} + \frac{1}{\omega_3^2}\sin\frac{3\pi}{2}(1-\cos\omega_3 t)\sin\frac{3\pi x}{L} + \frac{1}{\omega_4^2}\sin 2\pi(1-\cos\omega_4 t)\sin\frac{4\pi x}{L} + \ldots\right]$$

$$y(x,t) = \frac{2p_0}{mL}\left[\frac{1}{\omega_1^2}(1-\cos\omega_1 t)\sin\frac{\pi x}{L} - \frac{1}{\omega_3^2}(1-\cos\omega_3 t)\sin\frac{3\pi x}{L} + \frac{1}{\omega_5^2}(1-\cos\omega_5 t)\sin\frac{5\pi x}{L} - \frac{1}{\omega_7^2}(1-\cos\omega_7 t)\sin\frac{7\pi x}{L} - \ldots\right]$$

or, $$y(x,t) = \frac{2p_0}{mL}\sum_{k=1(\text{odd only})}^{\infty}\left[\frac{1}{\omega_k^2}\sin\frac{k\pi}{2}(1-\cos\omega_k t)\sin\frac{k\pi x}{L}\right] \qquad \text{---------(61)}$$

This is response of an elastic beam, subjected to a quickly applied constant stationary force; a rapidly converging series because of the term $\frac{1}{\omega_k^2}$.

4.8 Example-3: Deflection Due to a Time-Variant Stationary Load

Find the response of a simply supported beam subjected to the load $p(x,t) = p_0 \sin\Omega t$ at $x = a$ with ic = 0.

$$P_n = \int_0^L \sin\frac{n\pi x}{L} p(x,t)\,dx = p_0 \sin\Omega t \sin\frac{n\pi a}{L} \qquad \text{---------(E3a)}$$

Therefore, $$\ddot{Y}_n + \omega_n^2 Y_n = \overline{P}_1 \sin\Omega t \qquad \text{---------(E3b)}$$

Where, $$\overline{P}_1 = \frac{2p_0}{mL}\sin\frac{n\pi a}{L} \qquad \text{---------(E3c)}$$

Now, $$Y_n(t) = Y_{nH}(t) + Y_{nC}(t)$$ ---------(E3d)

with, $$Y_{nH}(t) = A_n \sin \omega_n t + B_n \cos \omega_n t$$ ---------(E3e)

and for complimentary solution, assume that

$$Y_{nC}(t) = C \sin \Omega t$$ ---------(E3f)

with $$\dot{Y}_{nC}(t) = C\Omega \cos \Omega t$$ ---------(E3g)

and $$\ddot{Y}_{nC}(t) = -C\Omega^2 \sin \Omega t$$ ---------(E3h)

Substituting into the differential equation,

$$-C\Omega^2 \sin \Omega t + \omega_n^2 C \sin \Omega t = \overline{P}_1 \sin \Omega t$$

Coefficients of ' sin t ' yield $(\omega_n^2 - \Omega^2)C = \overline{P}_0$

Thus, $$C = C_n = \frac{\overline{P}_1}{\omega_n^2 - \Omega^2}$$ ---------(E3j)

and therefore,

$$Y_n(t) = A_n \sin \omega_n t + B_n \cos \omega_n t + C_n \sin \Omega t$$

and $$\dot{Y}_n(t) = A_n \omega_n \cos \omega_n t - B_n \omega_n \sin \omega_n t + C_n \Omega \cos \Omega t$$

Using zero initial conditions i.e.,

$$Y_n(t = 0) = 0 \Rightarrow B_n(1) = 0 \quad \therefore \quad B_n = 0$$

and $$\dot{Y}_n(t = 0) = 0 \Rightarrow A_n \omega_n + C_n \Omega = 0 \quad \therefore \quad A_n = -\frac{C_n \Omega}{\omega_n}$$

so, $$Y_n(t) = \frac{\overline{P}_1}{\omega_n^2 - \Omega^2}\left(\sin \Omega t - \frac{\Omega}{\omega_n} \sin \omega_n t\right)$$ ---------(E3k)

Substituting $\varphi_n(x)$ and $Y_n(t)$ into Eq(28) yields

$$y(x,t) = \sum_{n=1}^{\infty} \varphi_n(x)\, Y_n(t)$$

$$= \sum_{n=1}^{\infty} \frac{2p_0}{mL}\left(\sin\frac{n\pi a}{L}\right)\left(\frac{1}{\omega_n^2 - \Omega^2}\right)\left(\sin \Omega t - \frac{\Omega}{\omega_n}\sin \omega_n t\right)\sin\frac{n\pi x}{L}$$

$$= \frac{2p_0}{mL}\left[\sum_{n=1}^{\infty} \left(\frac{1}{\omega_n^2 - \Omega^2}\right)\left(\sin \Omega t - \frac{\Omega}{\omega_n}\sin \omega_n t\right)\sin\frac{n\pi a}{L}\sin\frac{n\pi x}{L}\right]$$

---------(62)

Concentrating on Eq(62), it follows that if the frequency, $f_P = \frac{\Omega}{2\pi}$, of the disturbing force is equal to or even close to any of the natural frequencies, $f_n = \frac{\omega_n}{2\pi}$, the corresponding term gets very large, leading to the phenomenon of resonance. Therefore, to avoid resonance, the frequency of lowest mode of vibration i.e., when n = 1, must be several times larger than the frequency of the force acting on the structure.

In case of a simply supported beam, a design engineer is generally interested in the mid-span deflection. To check the effect on deflection, consider the case of a disturbing force acting at the center of the span i.e., when $a = L / 2$. Letting $\frac{\Omega^2}{\omega_n^2} = \psi_n^2$ in Eq(62),one obtains

$$y(x,t)=\frac{2p_0}{mL}\left[\left(\frac{1}{1-\psi_1^2}\right)\frac{1}{\omega_1^2}\sin\frac{\pi x}{L}(\sin\Omega t-\psi_1\sin\omega_1 t)\right.$$

$$-\left(\frac{1}{1-\psi_3^2}\right)\frac{1}{\omega_3^2}\sin\frac{3\pi x}{L}(\sin\Omega t-\psi_3\sin\omega_3 t) \quad \text{--------(63)}$$

$$\left.+\left(\frac{1}{1-\psi_5^2}\right)\frac{1}{\omega_5^2}\sin\frac{5\pi x}{L}(\sin\Omega t-\psi_5\sin\omega_5 t)-\dots\right]$$

When the values of ψ are small i.e., when Ω is very small as compared to ω, only the first term of the above equation approximates the steady state response with good accuracy, leading to the conclusion that the ratio of the dynamic deflection, y_{dyn}, to the static deflection, y_{st}, is $\frac{1}{1-\psi^2}$. For example, if the period of the disturbing force is half of that of the natural period of the beam i.e., $\Omega=0.5\omega$, then dynamic deflection is about 33 percent greater than the static deflection.

4.9 <u>Example-4: Deflection Due to a Constant Moving Load</u>

Find the response of a simply supported beam subjected to load $P(x, t) = P_0$ that moves across the beam at a constant velocity v such that its position is given at any time t as $x = a = vt$, $0 < t < L/v$, ic=0

From Eq.(52), that is $P_n = \int_0^L \varphi_n(x)\, p(x,t)\,dx$ ---------(E4a)

For simply supported beam, $\varphi_n(x)=\sin\frac{n\pi x}{L}$

$$P_n = \int_0^L \sin\frac{n\pi\, x}{L}\, p(x,t)\, dx = p_0 \sin\frac{n\,\pi\, v}{L} t \qquad \text{---------(E4b)}$$

leads to, $$\ddot{Y}_n + \omega_n{}^2 Y_n = \frac{2\,p_0}{mL} \sin\frac{n\,\pi\, v}{L} t \qquad \text{--------(E4c)}$$

Now, $$Y_n(t) = Y_{np}(t) + Y_{nc}(t) \qquad \text{---------(E4d)}$$

where $Y_{np}(t)$ is the homogeneous solution (free vibration case), and

$Y_{nc}(t)$ is the complimentary solution (forced vibration case).

with $$Y_{nc}(t) = A_n \cos\omega_n t + B_n \sin\omega_n t \qquad \text{---------(E4e)}$$

and for particular solution assume

$$Y_{np}(t) = C\sin\frac{n\pi\, v}{L} t \qquad \text{---------(E4f)}$$

therefore,

$$\dot{Y}_{np}(t) = \frac{n\pi\, v}{L}\, C\cos\frac{n\pi\, v}{L} t \qquad \text{---------(E4g)}$$

and $$\ddot{Y}_{np}(t) = -\frac{n^2\,\pi^2\, v^2}{L^2}\, C\sin\frac{n\pi\, v}{L} t \qquad \text{---------(E4h)}$$

substitute into the differential equation (E4c)

$$-\frac{n^2\, p^2\, v^2}{L^2}\, C\ \text{in}\frac{np\, v}{L} t + w_n^2\, C\sin\frac{np\, v}{L} t = \frac{2p_0}{mL} \sin\frac{n\,p\, v}{L} t$$

Coefficients of $\sin\frac{n\pi\, v}{L} t$:

$$C(\omega_n^2 - \frac{n^2\,\pi^2\, v^2}{L^2}) = \frac{2\,p_0}{mL}$$

Therefore $$C_n = \frac{2p_0/mL}{\left(w_n^2 - \frac{n^2 p^2 v^2}{L^2}\right)} = \frac{2p_0}{mL}\frac{L^2}{w_n^2 - n^2 p^2 v^2}$$ ---------(E4i)

so that $$Y_n(t) = A_n \cos\omega_n t + B_n \sin\omega_n t + C_n \sin\frac{n\pi v}{L}t$$ ---------(E4j)

and $$\dot{Y}_n(t) = -A_n \omega_n \sin\omega_n t + B_n \omega_n \cos\omega_n t + C_n\left(\frac{n\pi v}{L}\right)\cos\frac{n\pi v}{L}t$$

---------(E4k)

so that at time t = 0,

$$Y_n(t=0) = A_n .1 + B_n .0 + C_n .0 = 0 \quad \Rightarrow \quad A_n = 0$$

and $$\dot{Y}_n(t=0) = -A_n w_n .0 + B_n w_n .1 + C_n\left(\frac{np\ v}{L}\right).1 = 0$$

or $$B_n = -\frac{C_n}{\omega_n}\left(\frac{n\pi v}{L}\right)$$ ---------(E4l)

$$Y_n(t) = C_n\left(\sin\frac{n\pi v}{L}t - \frac{1}{\omega_n}\left(\frac{n\pi v}{L}\right)\sin\omega_n t\right)$$ ---------(E4m)

Substituting (E4i) and (E4m) in Eq.(28)

$$y(x,t) = \sum_{n=1}^{\infty}\varphi_n(x)\,Y_n(t)$$

$$y(x,t) = \frac{2p_0 L}{m}\sum_{n=1}^{\infty}\frac{1}{(\omega_n^2 L^2 - n^2\pi^2 v^2)}\left(\sin\frac{n\pi v}{L}t - \left(\frac{n\pi v}{\omega_n L}\right)\sin\omega_n t\right)\sin\frac{n\pi x}{L}$$

---------(E4n)

Using definition $$\alpha_n = \frac{vT_n}{2L},$$ ---------(E4p)

where v is the velocity of the moving load, and $T_n = \dfrac{2\pi}{\omega_n}$ is the period of the beam in the mode n.

Accordingly, $\alpha_n = \dfrac{\pi v}{\omega_n L}$ yields $\pi^2 v^2 = \alpha_n{}^2 \omega_n{}^2 L^2$, $\pi v = \alpha_n \omega_n L$

Thus, $$C_n = \frac{2p_0/mL}{\left(\omega_n^2 - \dfrac{n^2\pi^2 v^2}{L^2}\right)} = \frac{2p_0/mL}{\left(\omega_n^2 - \dfrac{n^2\alpha_n^2\omega_n^2 L^2}{L^2}\right)} = \frac{2p_0}{mL\omega_n^2(1-n^2\alpha_n^2)}$$

---------(E4q)

and $$B_n = -\frac{C_n}{\omega_n}\left(\frac{n\pi v}{L}\right) = -\frac{C_n}{\omega_n}\left(\frac{n\alpha_n\omega_n L}{L}\right)$$

$$= -n\alpha_n C_n = -\frac{2n\alpha_n p_0}{mL\ \omega_n^2(1-n^2\alpha_n^2)}$$ ---------(E4r)

Therefore,

$$Y_n(t) = C_n\left(\sin\frac{n\pi v}{L}t\right) + B_n \sin\omega_n t$$

$$= C_n\left(\sin\frac{n\pi v}{L}t\right) - n\alpha_n C_n \sin\omega_n t$$

$$= \frac{2p_0}{mL\omega_n^2(1-n^2\alpha_n^2)}\left(\sin n\alpha_n\omega_n t - n\alpha_n \sin\omega_n t\right)$$ ---------(E4s)

and

$$y(x,t) = \frac{2p_0}{mL}\sum_{n=1}^{\infty}\left(\frac{\sin n\alpha_n\omega_n t - n\alpha_n\sin\omega_n t}{\omega_n^2(1-n^2\alpha_n^2)}\right)\sin\frac{n\pi x}{L}$$

may be written as:

$$y(x,t) = \frac{2p_0}{mL}\sum_{n=1}^{\infty}\left(\frac{1}{\omega_n^2(1-n^2\alpha_n^2)}\right)\sin\frac{n\pi x}{L}(\sin n\alpha_n\omega_n t - n\alpha_n\sin\omega_n t)$$

---------(E4t)

$$y(x,t) = \frac{2p_0}{mL}\left[\frac{1}{\omega_1^2(1-\alpha_1^2)}\sin\frac{\pi x}{L}(\sin\alpha_1\omega_1 t - \alpha_1\sin\omega_1 t) + \frac{1}{\omega_2^2(1-4\alpha_2^2)}\sin\frac{2\pi x}{L}(\sin 2\alpha_2\omega_2 t - 2\alpha_2\sin\omega_2 t) + \ldots\right]$$

where $\alpha_1 = \frac{\pi v}{\omega_1 L}$, $\alpha_2 = \frac{\pi v}{\omega_2 L}, \ldots$

At mid-span, i.e., $x = L/2$,

$$y_{static} = \frac{p_0 L^3}{48\,EI} \quad \text{---------(E4u)}$$

Therefore, the dynamic load factor (DLF) is

$$DLF(t) = \frac{y_{dyn}}{y_{st}} = \frac{96p_0}{mL^4}\sum_{n=1(n\,odd)}^{\infty}\left(\frac{1}{\omega_n^2(1-n^2\alpha_n^2)}\right)\sin\frac{n\pi}{2}(\sin n\alpha_n\omega_n t - n\alpha_n\sin\omega_n t)$$

---------(E4v)

Also, using

$$\omega_n = \left(\frac{n\pi}{L}\right)^2\sqrt{\frac{EI}{m}} \quad \text{---------(E4w)}$$

$$\alpha_n = \frac{\pi v}{\omega_n L} = \frac{vL}{n^2\pi\sqrt{EI/m}} \quad \text{---------(E4x)}$$

Substituting the expression for α_n and rewriting , Eq(E4v) modifies to

$$y(x,t)=\frac{2p_0}{mL}\left[\sum_{n=1}^{\infty}\left(\frac{1}{\omega_n^2-(n\pi v/L)^2}\right)\left(\sin\frac{n\pi v}{L}t-\frac{n\pi v}{\omega_n L}\sin\omega_n t\right)\sin\frac{n\pi x}{L}\right] \text{---------(64)}$$

Concentrating on the time t_1 required for the force p_0 to cross the beam span, one may get a number of solutions for the beam deflection at any point along the beam, depending upon the velocity v of the moving force. An important factor in this connection is the ratio of this time t_1 to the periods 'T_n' of modes of free vibration. For example, if the time required by the force p_0 to cross the entire length of the beam is equal to half the period T_1 of the first mode of free vibration, that is,

$$t_1=\frac{L}{v}=\frac{T_1}{2}=\frac{\pi}{\omega_1}$$

then in this case, using $\frac{L}{v}=\frac{\pi}{\omega_1}$,

$$\omega_1^2=\pi^2\frac{v^2}{L^2} \text{---------(E31)}$$

Consequently, the first term of the series solution, Eq(64), becomes infinity, and using a special technique [Timoshenko et al. (1974)], can be represented by the relation

$$y(x,t)\approx-\frac{p_0\,t}{m\pi v}\cos\frac{\pi v}{L}t\,\sin\frac{n\pi x}{L} \text{---------(65)}$$

This expression will yield the maximum value for the deflection y at the instance the time $t = L / v$, i.e., when the force will reach the far end of the beam, given by

$$y_{max}=\frac{p_0 L}{m\pi v^2}\sin\frac{n\pi x}{L} \text{---------(66)}$$

Now, using the relation

$$\omega_1^2 = \pi^4 \frac{EI}{mL^4} = \pi^2 \frac{v^2}{L^2} \qquad \text{---------(E3m)}$$

and substituting

$$v^2 = \pi^2 \frac{EI}{mL^2} \qquad \text{---------(E3n)}$$

in Eq(66), one obtains

$$y_{max} = \frac{p_0 L^3}{\pi^3 EI} \sin\frac{\pi x}{L} \qquad \text{---------(67)}$$

From the above equation, the maximum deflection of the beam under the action of moving force, p_0, is found to be about 1.5 times the static deflection, y_{st} ,

$$y_{st} = \frac{p_0 L^3}{48EI} \qquad \text{---------(68)}$$

caused by the same force while acting statically at mid-span of the beam. Considering the practical aspect i.e., the bridge response to the moving vehicles, the actual time 't_l' taken by a vehicle to cross the bridge span is generally large as compared to the period of the first natural mode of vibration, making the ratio $\alpha^2 = \frac{p^2 v^2}{\omega_1^2 L^2}$ small. Now considering only the forced vibration response , and supposing that in the most unlikely case the forced and free vibration responses may come into phase, giving rise to the phenomenon of resonance, the maximum deflection is given by the expression

$$y_{max} = \frac{2p_0}{mL}(1 + \frac{p\,v}{\omega_1 L})(\frac{1}{\omega_1^2 - (pv/L)^2})$$
$$= \frac{2p_0 L^3}{p^4 EI}(\frac{1+\alpha}{1-\alpha^2}) \qquad \text{---------(69)}$$
$$= \frac{2p_0 L^3}{p^4 EI}(\frac{1}{1-\alpha})$$

This equation leads to the conclusion that the maximum deflection of a beam, acted upon by a force moving at a slow speed, is about $\frac{1}{1-\alpha}$ times the maximum static deflection produced by the same force. It may be noted here that α is:

a. directly proportional to velocity v of the load, and

b. inversely proportional to the span L as well as the first fundamental frequency ω_1 of the beam. Thus the term $\frac{1}{1-\alpha}$ might be the origin of the DLF.

4.10 Vibration effects on Highway Bridges

It is an established fact that the load effect caused by a vehicle crossing a bridge with certain speeds is larger than when it is stationary. This is due to the dynamic effect, caused by virtue of vehicle motion. Consequently, the deformation of the bridge under the action of a moving vehicle, generally termed dynamic response, is a complex phenomenon because of a number of factors capable of affecting the vehicle-bridge interaction and consequently, the dynamic response. The dynamic characteristics of a highway bridge is a function of the mass, stiffness, damping, and load parameters of the system, including both the vehicle and the bridge. In a practical sense assuming a constant speed:

a. the stiffness is a function of the bridge only,

b. the mass and the damping are functions of both the vehicle and the bridge, and

c. the loading function depends on the vehicle velocity and the pavement surface condition of the approaches as well as the bridge.

The major causes responsible for the dynamic load effects, in the case of highway bridges, are given in section 1.3.

In the simplest case[1] of the analysis of the dynamic effect caused by the live load effect of a moving load, a design engineer may be interested in following two important situations:

I) When the mass of the moving load or vehicle is large compared to the mass of the bridge structure itself.

II) When the mass of the moving load, compared to mass of the structure, is small.

In the former situation, which is generally true in case of short-span bridges, the mass of the structure, that is, the beam can be neglected in the analysis. Accordingly, the deflection of the beam, caused by the moving load, is proportional to the pressure , P(t), caused by the unsprung mass of the moving load m_u [Timoshenko 1965] on the beam shown in Figure 4.1, shown below.

[1] The discussion in the following paragraphs has been taken mainly from Timoshenko (1965).

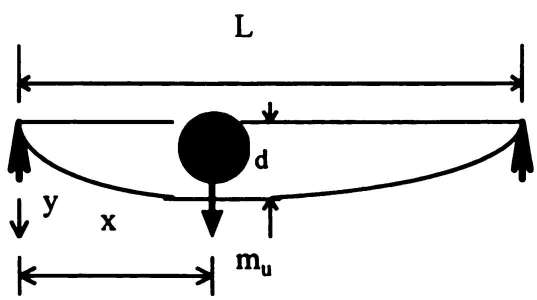

FIG. 4.1: Load as a moving mass

The deflection in this case is given by the Eq(2) of the section 3.2.1. that is the relation

$$y = m_u g\left(1 - \frac{v^2}{g}\frac{d^2y}{dx^2}\right) \frac{x^2(L-x)^2}{3LEI} \qquad \text{---------(70)}$$

It is an expression for the beam deflection due to a mass moving with constant velocity v across the beam. For a load moving along center line of the beam, an approximate solution of Eq(70) (First established by R. Willis[2] in 1849 as reported by [Timoshenko 1965]) can be obtained by replacing y with $\frac{M_u g x^2 (L-x)^2}{3LEI}$ on the right side of that equation. In this case, y will be maximum when the load is at the mid-span. Whereas, the maximum pressure is given by the expression

$$Q_{max} = M_u g\left(1 + v^2 \frac{M_u L}{3EI}\right) \qquad \text{---------(71)}$$

In this case, the maximum mid-span deflection , y_{max} , is given by a relation similar to Eq(71), which is

[2] The original reference not available. The information has been taken mainly from Timoshenko (1965).

$$y_{max} = y_{st}\left(1+\frac{v^2}{g}\frac{M_u L}{3EI}\right) \qquad \text{---------(72)}$$

From the comparison of this approximate solution with the results of an exact solution of Eq(70), obtained by G. G. Stokes, it is concluded that the approximate solution is sufficiently accurate for practical situations [Timoshenko 1965].

In the latter case, that is, when the mass of the load is small compared to the mass of the bridge, which is generally true in the case of large span bridges, the moving load can be represented by a moving force without loss of significant precision., and the results obtained in the previous discussion for the case of a moving force can be applied in this case as well.

It is pointed out here that in the foregoing discussion, the results obtained are those for a massless force or moving mass. In practical situations, the real moving loads are actually *rolling masses*, causing change in the natural frequency of the bridge with every change in position of the load moving over the bridge. This variation in the natural frequency of the vehicle-bridge system is advantageous from structural point of view, as in this situation, the oscillating moving load and the bridge will not remain in phase during all of the time taken by the load in crossing over the bridge, thereby reducing the phenomenon of resonance (it was shown in section 4.9 that resonance can occur in the case of a non-oscillating load as well, as for a given beam, it is basically a function of velocity of the load). Accordingly, both the dynamic load effect and the corresponding magnification of static deflection of the bridge, will be less than that predicted by the aforementioned theory.

In addition to above, certain other factors like unevenness caused by deterioration of the riding surface, low and high spots on the approach surface, and imperfect wheel balancing of the passing vehicles may also cause dynamic effects that may be quite high, particularly in the case of short span bridges. It therefore can be concluded that these additional dynamic effects are responsible for emphasis in the bridge design codes on the use of comparatively high dynamic load factors for the design of short-span bridges.

Theoretical solutions of the various problems to obtain the response of beams, representing highway bridges, under the different kinds of moving loads considered above readily lead to the conclusion that beam or bridge response is generally dominated by the first fundamental mode of vibration, provided the fundamental frequencies are well separated. Accordingly in such cases, the bridge response under the action of a moving load can be reasonably approximated by modeling a bridge as a single-degree-of-freedom (SDOF) system, and taking into account the first mode of vibration only. This will lead to the simplification of the real problem, which then can be represented by an ordinary differential equation along with simultaneous consideration of masses of the bridge as well as the moving load.

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CHAPTER 5

Summary and Discussion

5.1 Development of Dynamic Load Factor

Since the recognition of the existence of dynamic effects of moving loads on the railway bridges in the middle of 19th century, and the start of major research work on the problem of dynamic effects of moving loads on highway bridges in early 1930s, a number of the analytical as well as experimental studies have been carried out to identify the main factors or variables affecting the bridge response. Most of these studies were concentrated on relating the bridge response directly to the basic properties of the vehicle and the bridge, such as span and natural frequency of the bridge, and the speed and the weight of the vehicle. Initially the dynamic response was generally expressed as a function of the bridge span only [Waddell 1918; Final Report 1931]. Some of these investigations addressed the problem by using a slightly different approach. Instead of using the basic properties of the vehicle and the bridge, such as the natural frequencies and masses of both the vehicle and the bridge and the speed of the vehicle separately, the ratios of these factors were used in the investigations. The dimensionless ratios of the corresponding vehicle and bridge properties consisted of :

a. the speed parameter $= \dfrac{V}{2Lf}$

where V is the speed of vehicle,

L is the span length, and

f is the natural frequency of the bridge.

b. the frequency ratio i.e., the ratio of the vehicle frequency to that of the bridge

c. the mass or the weight ratio i.e., the ratio of the mass or weight of the vehicle to that of the bridge.

Several studies have been carried out using these dimensions parameters [University of Illinois 1951-1955; Wen and Veletsos 1962; Tan and Shore 1968a, b; Humar and Kashif 1993, 1995; Yang et al. 1995].

5. 2 Vehicle and Bridge Models

In the dynamic response studies described in chapter 3, the moving load has been essentially modeled in one or more of the following ways:

1. a single force moving at constant speed
2. a single unsprung mass moving at constant speed
3. a multi-axle unsprung load
4. a single sprung load
5. a combination of sprung and unsprung loads
6. a multi-axle sprung load

Whereas the bridge has been idealized as:

1. an elastic beam,
2. a plate (isotropic or orthotropic) structure, or
3. a combination of beam and plate structures.

5.3 Observed Values of Dynamic Load Factor (DLF)

The ranges of the dynamic load factors observed during various studies are given in the following table [Huang et al. 1993]:

Table 5.1: Comparison of Observed DLF Values

Agency/Country	Study Reference	Range of DLF	Remarks
1. AASHTO, USA (1958-60)	Fenves et al. 1962	0.03 - 0.25 0.02 - 0.42	From strain readings From deflections
2. TRRL, England	Leonard et al. 1974	0.09 - 0.75	From 30 bridges
3. New Zealand	Shepherd and Aves 1973; Wood and Shepherd 1979	0.10 - 0.70	Mostly for a standard 2-axle truck, for 14 bridges
4. Ontario, Canada	Green 1977	0.10 - 0.87 0.07 - 0.75	Continuous bridges Other bridges
5. EMPA, Switzerland	Cantieni 1984	Maximum: 0.70 Maximum: 2.50	Normal Conditions Over a 50 mm thick Plank 300x5000mm
6.(a)Australia	O'Connor and Pritchard 1985; Chan and O'Connor 1989	-.03 - 1.32	For Six Mile Creek bridge
(b) - do -	Heywood 1995	-.10 - 1.37	Off the approach pavement repair
7. Thailand	Ibanathan and Wieland 1987	0.26 - 0.36 0.28 - 0.33	Moving force case Moving mass case
8. Florida, USA	Huang et al. 1993	-.01 - 0.41	Multiple Criteria
9. South Korea	Chang and Lee 1994	0.00 - 0.66	Surface roughness

5.4 Correlation of Field Investigations and Code Specifications

When speaking of the progress made in determining the behavior of bridges due to static loads and the sad state of affairs in understanding dynamic effects, C. E. Inglis (1934) stated:

"This stage of arrested development is largely due to the blighting influence of impact factors, which prescribe that the dynamical effect of a moving load shall be taken into account by multiplying it by a factor depending upon the span of the bridge, and usually on nothing else. The simplicity of this process, which relieves bridge designers from intellectual effort, has ensured its popularity in the past, but the imposition of these purely empirical factors, which gain their simplicity only at the cost of ignoring all the considerations of real importance, has most effectively discouraged the enterprise bridge engineers might otherwise have shown in developing their science to deal intelligently with dynamical stresses."

Before discussing the correlation of various code specifications, given in chapter 2 and the appendix, with results of theoretical and test studies described in chapters 3 and 4, it should be made clear that the purpose of design codes is not to predict the response of a structure but to ensure the structural safety. This is generally done by specifying guidelines to limit the response (deflections, moments and stresses) of a given structure under the various loading conditions within prescribed acceptable limits e.g. by specifying:

a. load factors to increase the actual loads by certain amount to account for the possible overloading of the structure while put to actual use,

b. resistance factors to decrease the calculated strength or resistance of materials of construction because of possible variation in properties of actual materials to be used in construction, and

c. span-to-depth ratios for various structural components to limit their deflections.

The purpose of this discussion or correlation is to see with reference to theoretical and test investigations discussed earlier in this study, the extent to which the codes under consideration are ensuring structural safety with respect to the dynamic effect of traffic loading in highway bridges. The discussion is as under:

1. Although under normal conditions the dynamic effect represented by the DLF should not exceed 25% [Schilling 1982], it is generally the unusual circumstances such as the existence of an area of settlement in the approach road [Fleming and Romualdi 1961] or the presence of a bump at a critical location that result in much higher dynamic effects [Schilling 1982] which may be, in some cases as high as 1.37 [Heywood 1995]. Moreover, several investigations [Fleming and Romualdi 1961; Ibanathan and Wieland 1987; Galdos et al. 1993; Huang et al. 1993; Yang et al. 1995] have shown that the AASHTO (1992) formula to account for the dynamic effect of traffic loads underestimates the dynamic effect in certain cases (e.g., in the case of short span bridges) and overestimates it in others.

5.4.1 AASHTO 1992

The formula used by the American Association of State Highway and Transport Officials (AASHTO) design specifications for highway bridges 1992, to account for the dynamic effect of moving loads is:

$$I = \frac{50}{L + 125}$$

where

I = impact fraction (maximum 30 percent);

L = length in feet of the portion of the span that is loaded to produce the maximum stress in the member.

This formula was originally adopted in the "Specifications for Steel Highway Bridges", issued by the joint Conference Committee of American Association of State Highway and Officials (AASHO) and American Railway Engineering Association (AREA) in 1927. The formula was included in the first edition of AASHO standard specifications published in 1931 [ASCE 1958]. The formula remained an integral part of AASHTO highway specifications up to and including the 15th edition published in 1992 [AASHTO 1992].

The formula is generally based on the results of dynamic load tests carried out by A. H. Fuller during 1922-1925 at the Iowa Engineering Experiment Station on railroad bridges. The general level and the upper limit of the impact value depicts the influence of A. H. Fuller's work [Fuller 1922, 1925; ASCE 1958].

A special committee of ASCE working on the dynamic effect in highway bridges, proposed a similar dynamic load formula in its final report published in ASCE Transactions in 1931 [Final Report 1931]. The Committee, in the case of girders of highway bridges recommended that:

a. for spans less than 40 ft, the impact increment i.e., the dynamic effect of the stress be taken as 25% of the live load stress, and

b. for spans equal to or than 40 ft, the fraction of the dynamic effect of the moving load be calculated from the formula:

$$I = \frac{50}{L + 160}$$

where

I = 'impact' fraction

L = span length in feet

Although under normal conditions the dynamic effect represented by the DLF, should not exceed 25% [Schilling 1982], it is generally the unusual circumstance such as the existence of an area of settlement in the approach road [Fleming and Romualdi 1961] or the presence of a bump at a critical location that result in much higher dynamic effects [Schilling 1982] which may be, in some cases as high as 1.37 [Heywood 1995] in terms of the DLF. Moreover, several investigations [Fleming and Romualdi 1961; Ibanathan and Wieland 1987; Galdos et al. 1993; Huang et al. 1993; Yang et al. 1995] have shown that the AASHTO (1992) formula to account for the dynamic effect of traffic loads underestimates the dynamic effect in certain cases (e.g., in the case of short span bridges) and overestimates it in others.

In the light of above discussion, it follows that the dynamic effect provisions of the AASHTO (1992) bridge design code are inadequate, and unable to account for the dynamic effects of real traffic loads on highway bridges, thus not always adequately ensuring structural safety properly.

5.4.2 AASHTO LRFD 1994

The dynamic load allowance (DLA) provided in AASHTO LRFD code 1994 is generally based on data and recommendations of Page[1](1976). The code commentary mentions following two sources of dynamic effects caused by the traffic loads:

1. The dynamic response due to hammering effect of wheel loads, mainly caused by surface discontinuities, crack potholes and delaminations
2. The response caused by the vehicle by virtue of its motion, caused by either long undulations in the road surface like settlement of fill, or by the resonance effects due to similarity in the bridge and vehicle frequencies of vibration.

Although as indicated in the code commentary, under normal conditions the dynamic effect represented by the DLF should not exceed 25% , it is generally the unusual circumstance such as the existence of a bump at a critical location that result in much higher dynamic effects [Schilling 1982]. Both of the above sources contribute to the possibility of unusually higher dynamic response of a bridge to moving vehicles. However, it is thought that the first one is the major contributor and more responsible for the provision of a dynamic load allowance of 75%, the likelihood of frequency match between the vehicle and the bridge being low in the majority of practical cases. Comparing the provision of a maximum value of 75% for the DLA by the code to the results of analytical and field investigations, it is observed that where several researchers [Fenves et al. 1962; Shepherd and Aves 1973; Leonard et al. 1974; Cantieni 1984; Ibanathan and Wieland 1987; Huang et al. 1993; Chang and Lee 1994] have recorded the

[1] The original reference is not available. The information presented has been taken mainly from the Commentary of AASHTO LRFD (1994).

maximum values of DLFs less than or equal to 0.75, higher values up to 1.37 have also been reported [Green 1977; O'Connor and Pritchard 1985; Chan and O'Connor 1989; Heywood 1995]. The general value of 33% DLA specified by the code, like that provided for unusual conditions (75%), also seems justified and reasonable in view of remarks made in the code commentary regarding the use of a combination of design truck and lane loading versus the design truck only [Schilling 1982]. Thus, there is a good correlation between field results and the provisions of AASHTO LRFD 1994 for the dynamic effect of vehicle loads on highway bridges.

5.4.3 Ontario Highway Bridge Design Code (OHBDC) 1991, (Canada)

OHBDC 1991 specifies the dynamic load allowance with respect to the number of axles of the vehicle, the maximum value being 0.40 for a single axle load and the minimum 0.25 for vehicles with three or more axles. For a 2-axle vehicle, the dynamic load allowance specified is 0.30.

Keeping in view the remarks given in the case of AASHTO LRFD 1994 above, and the fact that the dynamic effect of a two-axle vehicle is generally less than that of an equivalent single axle vehicle [Humar and Kashif 1993], the dynamic load allowance provisions seem reasonable for normal conditions [Schilling 1982] (of loading and smooth road surface). An unusual condition like the settlement of an area in the approach road or the presence of a bump at a critical location may result in very high dynamic effects [Schilling 1982], which may be, in some cases as high as 1.37 [Heywood 1995] in terms of DLF. However, the code commentary has some discussion on that. OHBDC

1991 DLA provisions are based in part on test results [Billing 1984], and a comparative study made of the average weight of the vehicles used in the above study (300-400 kN) and the OHBD design truck (740 kN). It was found that for the OHBD truck, using "a more-or-less inverse linear relationship" between the vehicle weight and DLF, the maximum value of DLA for the whole truck was less than 0.2, no matter how close the natural frequencies of the vehicle and the bridge are to the resonance sensitive range [OHBDC 1991]. It has been clarified in the Commentary [OHBDC 1991] that in specifying the dynamic allowance values the approach road and the bridge deck have been assumed to be paved to acceptable standards. Accordingly a conservative and minimum value of 0.25 has been specified for vehicles with 3 or more axles. In the case:

a. the approach road has not been paved for a considerable period of time or
b. the expansion joints between the approach road and the bridge super structure are not generally flush with the riding surface,

it has been recommended that that the DLA be increased from 0.4 to 0.5 within three meters or one-tenth of the span length, whichever is greater, from the location of the joint. This distance has been stated to correspond to one half cycle of the axle spring for a vehicle traveling at approximately 100 km/h (65 mph). Moreover, the design engineer has been cautioned to be careful in using design vehicles lighter than the OHBD truck, as the DLA values may not be conservative in the case of lighter vehicles.

In the case of normal load and smooth road surface, provisions of OHBDC 1991 seem to be quite reasonable for heavier design loads comparable to OHBD Truck (gross weight 740 kN ≅ 166.4 kips, the AASHTO HS20-44 design truck weighs 72 kips) in the case of normal loading and smooth riding surface conditions [Schilling 1982]. Although in the case of unusual conditions, OHBDC provisions for the DLA apparently seem inadequate on the basis of the DLF investigation results mentioned in sections 5.4.1 and 5.4.2 above, for design vehicles with gross weights comparable to that of OHBD truck these may not be inappropriate. Comparing the corresponding weights of OHBD truck and the AASHTO HS20-44 truck, and considering the fact that the DLF multiplied by the static live load gives the load caused by the dynamic effect, it follows that in the case of a 3-axle vehicle such as HS20-44, the dynamic effect indicated by OHBDC 1991 is about (166.4 / 72 * 0.25 =) 0.81 within 3 m of the expansion joint, and 0.58 for the remaining span. Whereas in the case of 2-axle vehicle, the corresponding values are 0.92 and 0.69 respectively. Values for vehicles to be treated as single-axle vehicles for the design purpose, the DLF values are still higher. Thus for the vehicles with gross weights close or higher than that of OHBD truck, provisions of OHBDC 1991 seem to have good correlation with field investigations. However, for lighter vehicles their adequacy might be questionable.

5.4.4 Code of Practice Highway Bridges (CPHB) 1967 (Pakistan)

To account for the dynamic effects caused by moving loads, the formula used by CPHB 1967 is of the same type as that used by AASHTO code 1992. The formula used by CPHB is:

$$I = \frac{15}{L + 20}$$

where I is the impact fraction (maximum 30 percent), and

L is the span length in feet.

From the great resemblance in the general layout and the fact that CPHB 1967 was prepared by an American consultant firm, it follows that it is essentially based on the same principles as that of AASHTO code in use in 1960s.

Although under normal conditions, the upper limit of 30% specified by the code seems generally acceptable [Schilling 1982], it is of concern that as was shown in the case of the AASHTO 'impact formula' used in 1992 and the earlier editions [Fleming and Romualdi 1961; Ibanathan and Wieland 1987; Huang et al. 1993; Yang et al. 1995], the CPHB formula would most probably underestimate the dynamic response in certain normal cases and especially for the unusual conditions discussed in the case of AASHTO 1992 code. In the case of Pakistan, these unusual conditions are much more likely to exist due to:

a. general deterioration of highways mostly as a result of general economic conditions prevailing in the country,

b. modest truck load limitations, and

c. lack of proper and timely repair and maintenance of highways and bridges.

Thus, from above discussion, it appears that CPHB 1967 of Pakistan does not have a desirable correlation with the results of field investigations performed in various parts of the world to determine the dynamic effects of traffic loads on highway bridges.

5.5 Review of a Bridge Design Example

5.5.1 Code-Specified Values Versus Moving Mass Model

The following is a comparison of the values of dynamic effect of traffic loading obtained using the highway bridge design codes referred to in this report as well as theoretical values using a moving mass model for the vehicle load.

The dynamic effect of traffic loading is generally represented in two different ways. Theoretically, as in chapter 4, it is expressed as a dynamic load factor (DLF) given as

$$\mathrm{DLF} = \frac{y_{dyn}}{y_{st}} = \frac{(1+\mathrm{DI})\,y_{st}}{y_{st}} = 1 + \mathrm{DI}$$

where y_{dyn} is the maximum dynamic deflection, and

y_{st} is the maximum static deflection

DI is the dynamic load effect or the increase in the static deflection due to motion of the load , expressed as a percentage of the static deflection.

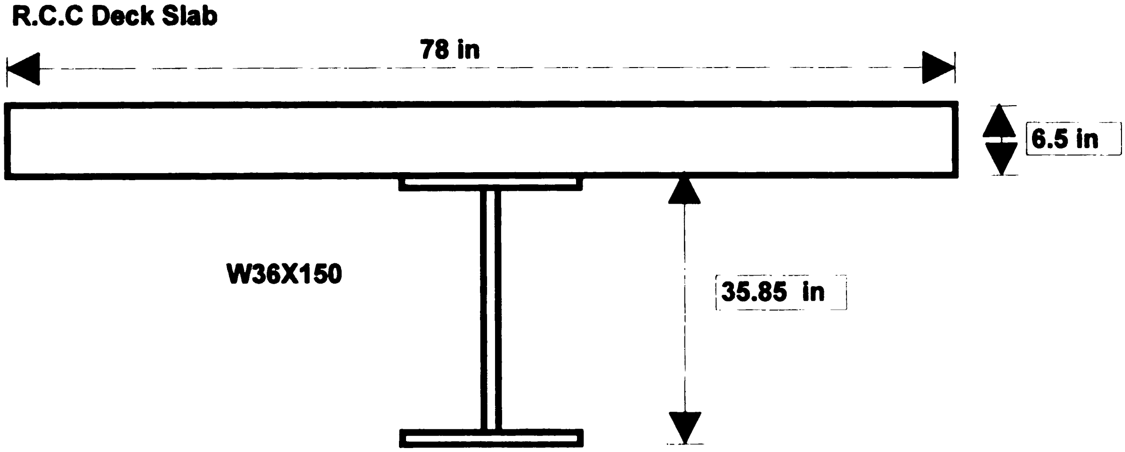
The dynamic load effect of the moving load, in this case, is equal to the simple product of the static live load and the DLF obtained using above relation. On the other hand, the bridge design codes generally give values for dynamic effect, represented by 'DI' in above expression, only. Thus, in the case of code-specified values of the dynamic effect, one (1) is added to the value of the dynamic effect such as impact [AASHTO 1992; CPHB 1967] or dynamic load allowance [OHBDC 1991; AASHTO LRFD 1994] obtained from the code directly, before multiplying with the specified static live load to determine the total effect of the moving load. In the following, only the dynamic effect due to moving load has been calculated to compare the values obtained from various sources. Thus one (1) has to be added to the indicated values before multiplying with the static load of relevant design vehicle to obtain the total load effect due to moving vehicle.

The values have been calculated using the design[2] data of a single span simply supported composite reinforced concrete-deck-steel girder bridge with following design data:

5.5.1.1 Data and Properties

- Span (center-to-center of supports) = 70 ft.
- Width = 28 ft.
- Live loading AASHTO HS20-44
- Wearing surface 30 lb./ft^2.
- Concrete strength, f'_c 3000 lb./in^2.
- A36 Steel beam (W36x150) weight = 150 lb./ft.
- Grade 40 steel reinforcement
- Modulus of Elasticity of Steel = 29×10^3 kip/in^2.
  = 4.176×10^6 kip/ft^2.
- Modulus of Elasticity of Concrete = 3.122×10^6 kip/in^2.
  = 4.496×10^5 kip/ft^2.
- Modular ratio, n = 9

[2] The design data has been taken from Example 13.1 and the composite bridge design example, given in section 19.7 of A. H. Nilson and G. Winter (1986). "Design of Concrete Structures" 10ed, McGraw-Hill Book Company, New York.



Bridge Span (center-to-center of supports) = 70 feet

FIG. 5.1: Typical Cross-section of Composite Highway Bridge

5.5.1.2 Properties of the Composite Section

In calculating the properties of the composite section, the concrete deck slab area has been transformed into equivalent steel area using the modular ratio. Accordingly, the material properties used in the calculation are essentially those of the steel, rather than of concrete. The properties of the composite section are given below:

- Self weight of the girder:

Slab weight (78x6.5/144x0.15)	= 0.528 kip/ft.
Wearing surface (78/12x0.03)	= 0.195 kip/ft.
Steel beam	= 0.15 kip/ft.
Total weight	= 0.873 kip/ft.

- Moment of Inertia (15016 in^4.) = 0.72415 ft^4.
- Mass of the girder per unit length, m

  [0.873 (kip/ft.) / 32.2 (ft./sec^2.)] = 0.027112 kip-sec^2/ft^2.

5.5.2 Code Specified Values

5.5.2.1 AASHTO - 1992

In AASHTO -1992 code, the basic load combination Group IA requires the ultimate capacity of the structure component to be not less than

$$1.3\,[D + 2.20\,(L + I)]$$

where D is the dead load,

L is the live load, and

I is the dynamic effect of live load.

The value of I is given by the expression

$$I = \frac{50}{L + 125}$$

Using the data of above design example, that is L = 70 ft, I is found to be 0.256. Thus, the dynamic load effect for the current example is 0.256 times the load of the AASHTO HS20-44 design truck weighing 72 kips.

5.5.2.2 AASHTO LRFD - 1994

The AASHTO LRFD - 1994 code, in the case of "Strength-I" load combination specifies the factored design load to be not less than

$$1.25\ DC + 1.75\ (LL + IM) + \ldots$$

where DC is the dead load of the component to be designed,

LL is the live load (design truck, tandem or lane load), and

IM is the dynamic load allowance (not applicable to the lane load).

Accordingly, using specifications given in the Appendix for AASHTO LRFD 1994, the dynamic effect of live load is found to be 0.75 for the deck joints, and 0.33 for other components in the case of Strength Limit State. The governing live load for the present example, is again HS20-44 design truck as before.

5.5.2.3 OHBDC - 1991

In the OHBDC - 1991 code, the design load in the case of Ultimate Limit State Combination-1 requires the design load to be not less than

$$1.2\,D + 1.4\,L$$

where D is the dead load of the structure, and

L is the live load inclusive of the dynamic load allowance (DLA).

Accordingly, using specifications given in the Appendix for OHBDC 1991, the dynamic effect of live load is represented by DLA . The design live load in this case is the OHBD truck weighing 740 kN (about 166.4 kips). For comparison with AASHTO codes, it is observed that the OHBD truck is about 2.31 times heavier than HS20-44 truck. Since the dynamic load effect is a percentage of the live load, it follows that total dynamic effect specified in the OHBDC - 1991 is actually 2.31xDLA when compared with AASHTO HS20-44 truck. In the case of vehicles required by the OHBDC to be considered as single axle ones, the total dynamic effect is then found to be 2.31x0.5 = 1.155 for the portions up to 3 m or 10% of the span whichever is greater; from the location of the expansion joint and 2.31x0.4 = 0.924 for the others. The corresponding values are 0.924 and 0.693 for two-axle vehicles; and 0.809 and 0.578 for vehicles with three or more axles.

5.5.2.4 <u>CPHB - 1967</u>

CPHB - 1967 specifies in the case of simple span prestressed concrete bridge structures of moderate lengths, the ultimate load capacity to be not less than

$$1.5\ D + 2.5\ (L + I)$$

where D is the dead load,

L is the design live load, and

I is the dynamic effect of live load, given by $I = \dfrac{15}{L + 20}$

in which

I = impact fraction (maximum 30 percent)

L = length of span in feet

The design truck-train load has a gross weight of 122 kips which is 1.694 times the weight of HS20-44 truck. For the present example, I = 15/(70+20) = 0.167. Thus the total dynamic load effect in this case turns out to be 1.694x0.167 = 0.283.

It is added that in case of the moving force model [Eq(E4v) of section 4.9], the values of the dynamic load effect are the same for all codes because in this case the mass of the vehicle is neglected and the calculation of the dynamic effect is dependent only on the properties of the bridge which are constant. Moreover, the dynamic effect values obtained from the 'moving force' and the 'moving mass' models cannot be compared because both models have different background. The moving force model neglects the effect of vehicle mass, and the mass model, ignores the effect of the bridge weight on the dynamic response of the bridge.

A comparison of dynamic load effect obtained using the four highway bridge design codes discussed in this report, and theoretical mass model are given in Table 5.2:

Table 5.2: Comparison of Dynamic Effect Using Design Example

Codes	Equivalent[3] Code Specified Values	Moving Mass Model[4]	Remarks
AASHTO LRFD 1994 (USA)	0.75 0.33	0.238	Deck joints Other components
AASHTO 1992 (USA)	0.256	0.238	
OHBDC 1991 (Ontario, Canada)	1.155 (0.924)[5] 0.924 (0.693) 0.809 (0.578)	0.549	Single-axle vehicle Double-axle vehicle Multi-axle vehicles
CPHB 1967 (Pakistan)	0.283	0.403	

[3] These values include the effect of the gross weight of the design vehicle load used by different codes as a ratio of the gross weight of the AASHTO HS20-44 design vehicle (72 kips).

[4] These values have been calculated from Eq(74) of section 4.10.

[5] The figures in the parentheses give dynamic effect values for the bridge portion beyond 3 meters from the expansion joint (between the approach road and the bridge); and those outside the parentheses, the values up to 3 meters from the expansion joint.

In the second column of Table 5.2 marked 'Equivalent Code Specified Values', the values of the dynamic load effect have been scaled on the basis of AASHTO HS20-44 design truck. This simply means that the dynamic load effect obtained directly from a code has been multiplied by the ratio of gross weight of the design truck of that particular code to the gross weight of AASHTO HS20-44 design truck (72 kips). In other words, if a bridge is designed, using a vehicle similar in configuration, gross weight, and axle load distribution to the AASHTO HS20-44 design truck and calculate the dynamic load effect specified by each of above mentioned design codes, then the actual dynamic effect accounted for in the design would look like that given in the second column of Table 5.2. Thus, in these terms, OHBDC - 1991 gives the highest dynamic load effects (Multi-axle vehicles; 0.809 within 3 meters of the expansion joint, 0.578 for the remaining portion), followed by AASHTO LRFD - 1994, CPHB-1967 and AASHTO-1992 in descending order.

In the case of 'Moving Mass' model, the mass of the vehicle is accounted for while calculating the dynamic load effect of moving vehicles. Accordingly, in this case, the dynamic load effect is directly proportional to the gross vehicle weight. Thus, it is seen that the dynamic load effect is the greatest in the case of OHBDC-1991, which has the heaviest design vehicle among the above codes, and the smallest in the case of AASHTO, having the lightest design vehicle of them.

5.5.3 Moving Force Model:

From the Table 5.3, it is observed that :

1. In general, the contribution of third and higher modes of vibration to overall value of DLF is very small. For example at t = 0.6 sec, DLF = 38.69 for the first mode, 0.23 for the third mode (only 0.59% of that for the first mode), and 0.05 for the 5th mode (only 0.12% of that for the first mode). Thus, it can be concluded that while calculating the response of a bridge subjected to a moving load, using moving force model as above, the effect of higher modes of vibration is negligible and the dynamic response can predicted with fairly accurately using first mode of vibration only.

2. A close look at Eq(E4v) (section 4.9), which is the basis of the Table 5.3, reveals that in the case of first mode of vibration the term in the denominator, does not become zero for the given parameters of the bridge- vehicle system, and therefore the phenomenon of resonance does not take place. A side calculation shows that at vehicle speed of about 75.44 ft/sec (51.43 miles per hour), the denominator term of the Eq(E4v) becomes zero for the system parameters considered in above calculations, thus seemingly causing an unstable condition or resonance. However, a closer inspection would reveal that this apparently resonance-type phenomenon could occur only if the system is undamped and that high amplitudes approach infinity only with infinite time, obviously not possible since the beam is of finite length defining the duration of the exciting phenomenon.

3. A further examination of Eq(E4v), Table 5.3, and the graph for the first mode only, indicates that the DLF values are increasing with each cycle, however this increase is still finite, and of course neglecting the effect of damping. Otherwise, DLF values will practically be less than indicated above, because of certain amount of damping and time required to reach the amplitude shown which has to be present in the real structures.

Table 5.3 is on the following page.

Table 5.3: Composite Bridge Design Example (Nilson and Winter - 1986)

Serial No.	Time (Sec)	Dynamic Load Factor (DLF) Considering				
		Mode 1 only	Mode 3 only	Mode 5 only	Modes 1 & 3	Modes 1, 3 & 5
1	0.00	0	0	0	0	0
2	0.05	0.048682	-0.2897588	-0.07343278	-0.2410722	-0.3145050
3	0.10	0.3837934	-0.3880283	0.03120893	-0.0042349	0.02697398
4	0.15	1.2637151	0.76993809	-0.02547132	2.03365326	2.00818194
5	0.20	2.8929640	0.08072533	-0.10490460	2.97368937	2.86878477
6	0.25	5.4004934	-0.7344771	0.00665270	4.66601630	4.67266899
7	0.30	8.8239133	0.19004214	-0.02526652	9.01395548	8.98868896
8	0.35	13.100881	0.23708615	-0.10455311	13.3379677	13.2334146
9	0.40	18.068437	0.23074736	0.01418826	18.2991852	18.3133734
10	0.45	23.470466	-0.2708628	0.00684677	23.1996040	23.2064508
11	0.50	28.972885	-0.5750710	-0.07447843	28.3978142	28.3233357
12	0.55	34.185573	0.72887394	0.04345093	34.9144474	34.9578983
13	0.60	38.689576	0.22840561	0.05285684	38.9179824	38.9708393
14	0.65	42.067686	-0.6241151	-0.03833978	41.4435717	41.4052320
15	0.70	43.936250	0.00896188	0.06717071	43.9452120	44.0123827
16	0.75	43.975922	0.11708325	0.08396744	44.0930057	44.1769731
17	0.80	41.959127	0.43360824	-0.02352261	42.3927360	42.3692134
18	0.85	37.772171	-0.1866278	0.06179512	37.5855432	37.6473388
19	0.90	31.430299	-0.7243580	0.08218368	30.7059412	30.7881248
20	0.95	23.084464	0.62010027	-0.04051391	23.7045642	23.6640503
21	1.00	13.019098	0.33674797	0.02532023	13.3558463	13.3811665
22	1.05	1.6408255	-0.450725	0.05377137	1.19010032	1.24387169
23	1.10	-10.54135	-0.1302589	-0.07501657	-10.671618	-10.746634
24	1.15	-22.94330	-0.0561050	-0.02109868	-22.999407	-23.020505
25	1.20	-34.938650	0.58458202	0.02445598	-34.354068	-34.329612
26	1.25	-45.895489	-0.0528300	-0.09874059	-45.948319	-46.047060
27	1.30	-55.214333	-0.8139086	-0.04897148	-56.028242	-56.077213
28	1.35	-62.364985	0.46159146	0.01997821	-61.903393	-61.883415
29	1.40	-66.919881	0.38582549	-0.09074483	-66.534056	-66.624800
30	1.45	-68.581692	-0.2347953	-0.04381830	-68.816487	-68.860306
31	1.50	-67.203342	-0.2090817	0.04664906	-67.412424	-67.365775
32	1.55	-62.799076	-0.2603936	-0.05325778	-63.059470	-63.112728
33	1.60	-55.545786	0.66689132	-0.01596383	-54.878895	-54.894858
34	1.65	-45.774435	0.10748211	0.08654823	-45.666957	-45.580409
35	1.70	-33.952099	-0.8296888	-0.01006314	-34.781788	-34.791851
36	1.75	-20.655632	0.27791068	0.00734560	-20.377721	-20.370376
37	1.80	-6.5387519	0.36422114	0.11098211	-6.1745308	-6.0635486
38	1.85	7.7055282	-0.0025791	0.01133512	7.70294906	7.71428418
39	1.90	21.384655	-0.2179893	0.0025643	21.1666657	21.169230
40	1.95	33.846001	-0.4688507	0.10206793	33.3771511	33.4792190
41	2.00	44.513554	0.67329996	0.00033948	45.1868545	45.1871940

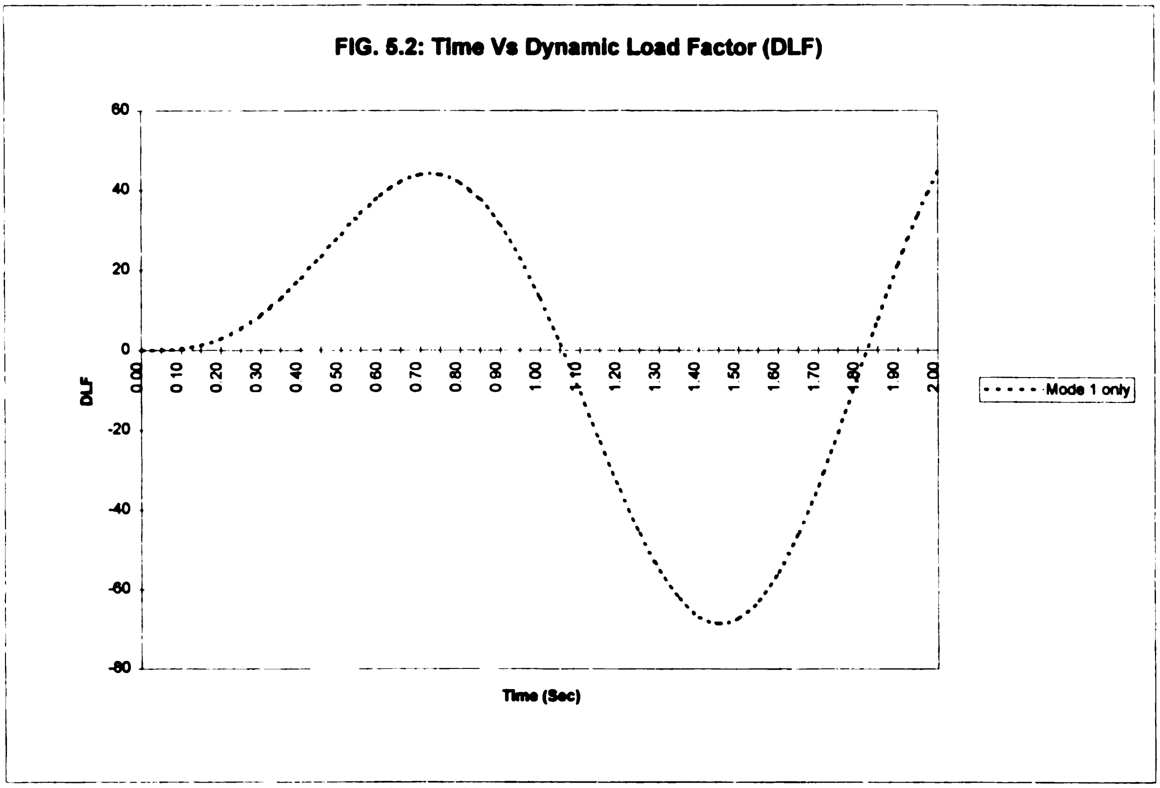


FIG. 5.2: Time Vs Dynamic Load Factor (DLF)
60
40
20
0
-20
-40
-60
-80
DLF
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
1.30
1.40
1.50
1.60
1.70
1.80
1.90
2.00
Time (Sec)
Mode 1 only

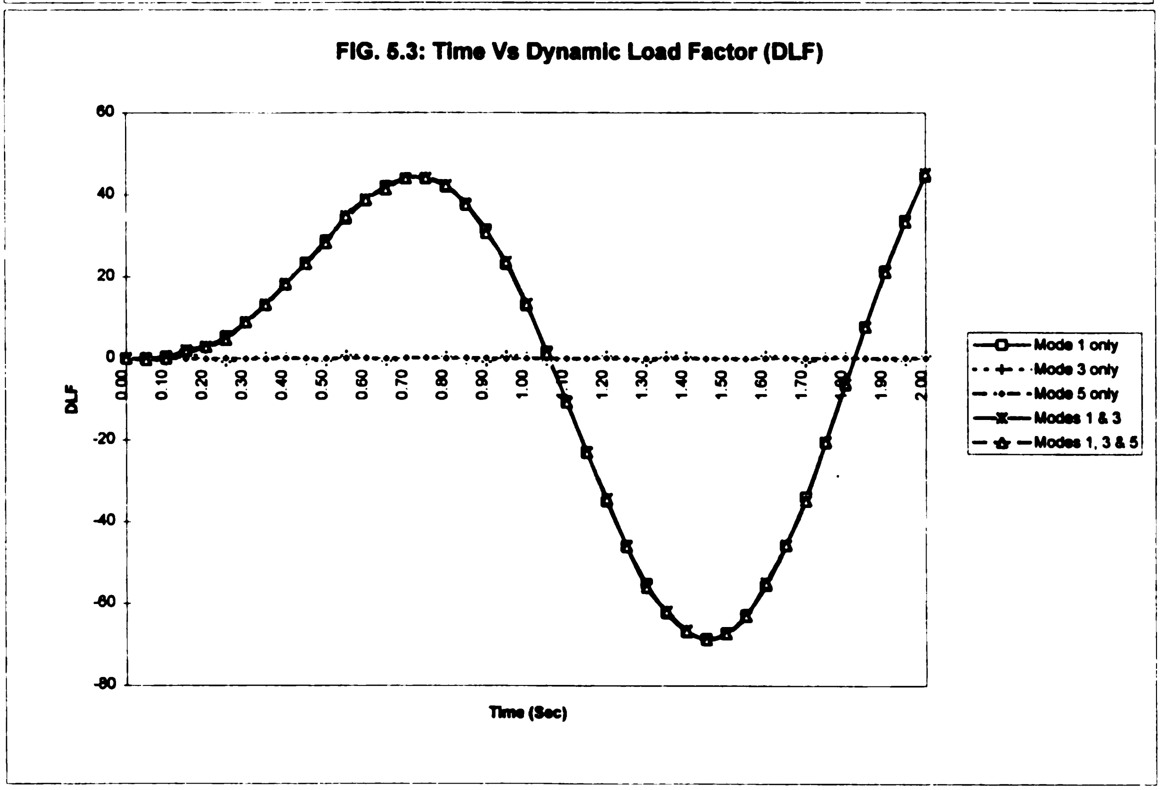


FIG. 5.3: Time Vs Dynamic Load Factor (DLF)
60
40
20
0
-20
-40
-60
-80
DLF
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
1.30
1.40
1.50
1.60
1.70
1.80
1.90
2.00
Time (Sec)
Mode 1 only
Mode 3 only
Mode 5 only
Modes 1 & 3
Modes 1, 3 & 5

CHAPTER 6

CONCLUSIONS

6.1 Findings

1. A review of analytical, model and field test studies performed to determine dynamic response of bridges subjected to traffic loads has been made. The investigations review goes back to 1849 when the first study to determine the response of railway bridges was performed.

2. Theoretical basis of derivation of dynamic deflection due to the action of simple load models such as a constant moving force, a suddenly applied mid-span load, and a time variant force have been derived in the form of examples to illustrate the theoretical concepts to determine the dynamic effects of moving loads.

3. AASHTO 1992, and CPHB 1967 highway bridge design codes, in general, do not have good agreement with the results of field investigations conducted to determine the dynamic response of highway bridges subjected to traffic loads in the case of unusual loading and approach road conditions. Thus, the corresponding provisions of both codes may be insufficient in some circumstances. The maximum values for the dynamic load factors specified by these two codes are correct only to the extent that loading conditions are normal and the approach as well as the riding surface are fairly smooth.

4. The provisions of OHBDC 1991 seem to be reasonable. This code uses heavier design truck loads (Their design truck has a gross weight of 740 kN ≅ 166.4 kips as compared to the AASHTO HS20-44 design truck weighing 72 kips) and a unique design philosophy. For lighter design vehicles and vehicles with 3- or more axles the indicated DLF may be less conservative.

5. AASHTO LRFD 1994 is based on a probabilistic approach as far as load and resistance factors are concerned. The dynamic load allowance provided in this code seems generally justified in view of most of the results of field test studies carried out to date. (The data primarily belongs to the United States.)

6. It has been remarked [Humar and Kashif 1993] that two bridges with the same spans or fundamental natural frequencies may have different values for the speed parameter, the frequency ratio and the mass ratio. Their dynamic responses will accordingly be different too. To provide for this situation, design curves have been proposed [Humar and Kashif 1993] to compute a more rational dynamic load factor (DLF). The procedure is systematic if not very simple. From a preliminary design of the bridge, the weight and the frequency of the bridge can be determined. Using the weight of the design vehicle, mass ratio can be calculated. The DLF generally increases with the speed parameter, so only maximum value of the latter needs to be considered. The response however does not vary systematically with the frequency ratio. In this case, the

response can be calculated for several possible values of frequency ratios and the one leading to the maximum DLF be used in the design [Humar and Kashif 1993].

7. Simple formulas have been proposed for calculating dynamic load factors, based on analytical research, in terms of the speed parameter only [Yang et al. 1995]. These formulas may be helpful in predicting the response of various type of bridges.

8. From the review of investigations made in chapter 3, and results of theoretical derivations made in chapter 5, and test studies given in chapter 5, it follows that the determination of dynamic response of highway bridges subjected to moving loads is a very complex problem. Use of high strength materials (both conventional and newly developed ones) , and the adoption of ultimate strength design criteria permitting bridge designers to use more slender sections in the design of bridges may lead to the bridge structure being more susceptible to vibrations. The situation becomes more aggravated keeping in view the trend and possible use of heavier and faster vehicles on the highways. This leads to the real need of development of a rational and more reliable procedure to determine the dynamic response due to traffic loads. Use of dynamic load factors can certainly make the designing process simple but does not help the designer understand the real effect of traffic loads, which is dynamic rather than static as it is generally treated in design.

6.2 Recommendations

Keeping in view the knowledge gained through the techniques used by various investigators in the study of the dynamic response of the highway bridges subjected to moving loads and other engineering knowledge, the following options are suggested for the design of a reasonable and reliable bridge:

a. Detailed dynamic analysis for vibration sensitive systems

b. A "quasi-static" approach more simplified than 'a' but rational based on partial dynamic analysis results and recommendations of theoretical, experimental, or field investigations carried out by various researchers [Humar and Kashif 1993; Yang et al. 1995] for most bridges that are neither dynamically sensitive nor insensitive.

c. Use of the static method typified by AASHTO LRFD 1994 or OHBDC 1991 for bridges which may be dynamically insensitive.

Further work is required to define the parameters representing dynamic sensitivity. The above would be enhanced or influenced by:

d. Probabilistic Approach : A detailed analysis could possibly be carried out to determine the probabilities of simultaneous occurrence of various factors responsible for the amplification of dynamic effects in highway bridges caused by the traffic loading. Based on these probabilities and the maximum effects caused by each individual factor, a suitable value would then be fixed for the dynamic load factor for use in the design of highway bridges.

In view of the complex nature of the dynamic response, and highly variable and non-deterministic nature of traffic loading to which highway bridges are subjected, probabilistic approach, as now would be in the OHBDC 1991 or the AASHTO LRFD 1994 codes, is recommended to be more widely adopted in the years to come for the reliable and cost-effective design of highway bridges. Although with modern high-speed digital computers and quality software even a very detailed design might be reasonable, nevertheless the design process should not be overly cumbersome, at least in the case of dynamically insensitive structures. Accordingly, it is suggested that the code provisions should be simple, and concise for the bridges of most common types of span lengths, for example, simply supported short and medium span bridges. For other type of structures, a comparatively detailed dynamic analysis procedure is recommended. Until, they are improved upon, the AASHTO LRFD 1994 or OHBDC 1991 may be adopted for general design. Use may also be made of the dynamic response curves developed [Humar and Kashif 1993] or the formulas proposed [Yang et al. 1995] by researchers for comparison.

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APPENDIX

APPENDIX

Code Specifications on Dynamic Load Allowance

A.1 AASHTO LRFD Bridge Design Specifications - 1994

A.1.1 American Association of State Highway and Transport Officials (AASHTO) LRFD Bridge Design Specifications - 1994 defines the dynamic load effect of the moving loads as 'Dynamic Load Allowance", denoted by IM. The relevant code provision i.e., the Article 3.6.2.1 states:

"Unless otherwise permitted in Articles 3.6.2.2 (Buried Components) and 3.6.2.3 (Wood Components), the static effects of the design truck or tandem shall be increased by the percentage specified in Table 1 for dynamic load allowance."

"The factor to be applied to the static load shall be taken as: (1 + IM/100)."

"The dynamic load allowance shall not be applied to pedestrian loads or to the design lane load."

Table 3.6.2.1-1 - Dynamic Load Allowance, IM

Component		IM
Deck joints - All Limit States		75%
All Other Components		
* Fatigue and Fracture Limit State		15%
* All Other Limit States		33%

"Dynamic load allowance need not be applied to:

- retaining walls not subject to vertical reactions from the superstructure, and
- foundation components which are entirely below ground level."

"The dynamic load allowance may be reduced for components, other than joints, if justified by sufficient evidence, in accordance with the provisions of Article 4.7.2.1."

A.1.2 According to the code Commentary, some of the above provisions were based on the report of Page (1976). The corresponding Commentary article C3.6.2.1 states:

"The dynamic load allowance (IM) in table 1 is an increment to be applied to the static wheel load to account for wheel load impact from moving vehicles."

"Dynamic effects due to moving vehicles may be attributed to two sources:

- hammering effect is the dynamic response of the wheel assembly to riding surface discontinuities, such as deck joints, cracks potholes and delaminations, and
- dynamic response of the bridge as a whole to passing vehicles which may be due to long undulations in the roadway pavement, such as those caused by settlement of fill, or to resonant excitation as a result of similar frequencies of vibration between bridge and vehicle."

"Field tests indicate that in the majority of highway bridges, the dynamic component of the response does not exceed 25% of the static response to vehicles. This is the basic for dynamic load allowance with the exception of deck joints. However, the specified live load combination of the design truck and lane load, represents a group of exclusion vehicles which are at least 4/3 of those caused by the design truck alone on short-and medium-span bridges. The specified value of 33% in Table 1 is the product of 4/3 and the basic 25%."

"This article recognizes the damping effect of soil when in contact with some buried structural components, such as footings. To qualify for relief from impact, the entire component must be buried. For the purpose of this article, a retaining type component is considered to be buried to the top of the fill"

A.2 AASHTO Highway Bridge Design Specifications - 1992

The provisions of the AASHTO Highway Bridge Design Specifications - 1992, for impact (dynamic load) allowance are as follows:

"3.8 IMPACT

3.8.1 Application

Highway Live Loads shall be increased for those structural elements in group A, below, to allow for dynamic, vibratory and impact effects. Impact allowances shall not be applied to items in group B. It is intended that impact be included as part of the loads transferred from the superstructure to substructure, but shall not be included in loads transferred to footings nor to those parts of piles or columns that are below ground."

"3.8.1.1 Group A - Impact shall be included.

(1) Superstructure, including legs of rigid frames.

(2) Piers, (with or without bearings regardless of type) excluding footings and those portions below the groundline.

(3) The portions above the groundline of concrete or steel piles that support the superstructure."

“3.8.2 Impact Formula

3.8.2.1 The amount of the impact allowance or increment is expressed as a fraction of the live load stress, and shall be determined by the formula:

$$I = \frac{50}{L + 125} \qquad (3 - 1)$$

in which

I = impact fraction (maximum 30 percent);

L = length in feet of the portion of the span that is loaded to produce the maximum stress in the member.”

“3.8.2.2 For uniformity of application, in this formula, the loaded length, L, shall be as follows:

(a) For roadway floors: the design span length.

(b) For transverse members, such as floor beams: the span length of member center to center of supports.

(c) For computing truck load moments: the span length, or for cantilever arms the length from the moment center to the furthermost axle.

(d) For shear due to truck loads: the length of the loaded portion of span from the point under consideration to the far reaction; except, for cantilever arms, use a 30 percent impact factor.

(e) For continuous spans: the length of span under consideration for positive moment, and the average of two adjacent loaded spans for negative moment.”

“ **3.8.2.3** For culverts with cover

0’ to 1’ - 0” inc. I = 30%

1' - 1" to 2' - 0" inc. I = 20%

2' - 1" to 2 - 11" inc. I = 10%"

A.3 Ontario Highway Bridge Design Code (OHBDC) 1991

A.3.1 Ontario Highway Bridge Design Code-1991 (OHBDC 1991) allows for the impact of moving vehicles in the following way:

"2-4.3.2 Dynamic Load Allowance

2-4.3.2.1 Highway live loads shall be increased by the dynamic load allowance unless otherwise specified.

2-4.3.2.2 The values specified for the dynamic load allowance shall apply unless alternative values based on tests or dynamic analysis are approved.

2-4.3.2.3 The dynamic load allowance applied to a Truck shall be as given in Table 2-4.3.2.3 for the number of axles considered in the design lane."

Table 2-4.3.2.3 Dynamic load allowance

Number of axles	DLA
1	0.40
2	0.30
3 or more	0.25

"2-4.3.2.4 The dynamic load allowance for the uniformly-distributed portion of the Lane Load shall 0.10.

2-4.3.2.5 For soil-steel structures, the dynamic load allowance shall be 0.40, reduced according to Clause 2-4.3.2.6

2-4.3.2.6 When there is a depth of earth cover, D_E, between the riding surface and the top of the structure, the dynamic load allowance shall be multiplied by the reduction factor:

$$(1-0.5D_E)$$

After this reduction, the dynamic load allowance shall not be less than 0.10. For arch and soil-steel structures, the fill depth shall be measured at the crown."

"**2-4.3.2.7** For wood components, the dynamic load allowance resulting from Clauses 2-4.3.2.3, 2-4.3.2.4, and 2-4.3.2.6 shall be multiplied by 0.70.

2-4.3.2.8 The dynamic load allowance shall be included as part of loads transferred from the superstructure to the substructure, but shall not be included in loads transferred to footings that are surrounded with earth nor to those parts of piles that are below ground."

A.3.2 The relevant sections of the OHBDC code Commentary state:

"**C2-4.3.2.1** The dynamic load allowance is an equivalent static load, expressed as a function of the OHBD Load, which is considered for design purposes to be equivalent to the dynamic and vibratory effects of the interaction of the moving vehicle and the bridge. Dynamic load allowance is not required for centrifugal, braking, collision or pedestrian loads. The maintenance vehicle load... includes an allowance for dynamic effects."

"**C2-4.3.2.2** The use of advanced dynamic analyses , or tests, either on structural models or existing structures, is subject to approval."

"**C2-4.3.2.3** Dynamic load is caused by a combination of :

- bumps in the riding surface or expansion joints which result in direct impact to the bridge deck;

- dynamic variation in axle loads due to undulations and roughness in the riding surface; and

- dynamic response of the main longitudinal bridge components to the moving vehicle loads.

The effects of these factors are generally not independent, but the relative contribution from each may vary significantly depending upon the component loaded. The dynamic load allowance for a truck, or part thereof, is specified according to the number of axles involved in generating the load effect, as follows:

i) loading by a single axle load;

ii) loading by two axle loads;

iii) loading by three or more axles, including loading by the entire Truck.

a) Components governed by a single axle unit, or part thereof, may include slabs, such as concrete deck slabs, pan fill floors, steel orthotropic decks and short-span supporting elements. Dynamic variation in axle load due to roughness in the riding surface, and impact at bumps, is directly transferred to the component, which results in a high dynamic load. The dynamic interaction of such components with the moving load is generally very small, because vibration of the component has a short period as compared to the duration of loading. the specified values of dynamic load allowance are based on test results..., assuming that the approach riding surface and bridge deck have been paved to acceptable standards. If the approach is unlikely to be paved for an extended period of time, or if expansion joints between the superstructure and approach pavement are not generally flush with the roadway riding surface, vehicles will enter the bridge in a state of excitation. To allow for this, it is recommended that the dynamic load allowance be increased from 0.4 to 0.5 within 3 m or one-tenth of the span length, whichever is greater, from the location of the joint. This distance corresponds approximately to one half cycle of the axle spring for a vehicle travelling at approximately 100 km/h (62.15 mph).

b) For longitudinal components whose design is governed by two axle loads, the axles are not generally in phase so that the impact effect is moderated. the dynamic response of the component to moving loads results in some dynamic load amplification, but spans of such components are substantially less than 20 m and the frequencies are usually sufficiently high that a frequency match between component and vehicle is unlikely. a consideration of either effect however, would load to about the same value of DLA.

c) For longer spans, for which the critical loading is due to three or more axles, a frequency match with a vehicle suspension is possible and the largest dynamic load is likely to involve dynamic interaction between the structure and the vehicle. The effect of axle load impact is not likely to be critical. the duration of loading is longer, and the bridge and vehicle may interact for more than one cycle of vibration if the bridge frequency. ... There is increased dynamic response of bridge superstructure having natural frequencies in the range 2 to 5 Hz, a range typical of the bounce frequencies of vehicles. The increase is because of interaction between the vehicle and the bridge, and has many of the characteristics of resonance in a simple oscillator."

A.4 Code of Practice Highway Bridges 1967

The existing Code of Practice Highway Bridges (CPHB) 1967 of Pakistan provides the following specifications to account for the impact effect in the design of highway bridges:

"2.9- IMPACT

Live Loads stresses produced by the standard Truck-train loading shall be increased for items Group A, by allowance as stated therein, for dynamic, vibratory and impact effects. Impact shall not be applied to items in group B."

"(A) Group A

(1) Superstructure, including steel or concrete supporting columns, steel towers, legs of rigid frames and generally those portions of the structure which extend down to the main foundation.

(2) The portions above the groundline of concrete or steel piles which are rigidly connected to the superstructure as in rigid frame or continuous designs."

"(B) Group B

(1) Abutments, retaining walls, piers, piles except Group A(2)

(2) Foundation pressures and footings

(3) Timber structure

(4) Sidewalk loads

(5) Culverts and structures having cover of 3 feet or more"

"(C) Impact Formula

The amount of this allowance or increment is expressed as a fraction of the live load stress, and shall be determined by the formula:

$$I = \frac{15}{L + 20}$$

in which

I = impact fraction (maximum 30 percent)

L = length of span in feet"

"For uniformity of application, the span length "L" shall be especially considered as follows:

For roadway floors, use the design span length.

For transverse members, such as floor beams, use the span length of member centre to centre of supports.

For computing truck load moments use the span length, except for cantilever arms use the length from the moment centre to the farthermost axle.

For continuous spans use the length of span under consideration for positive moment, and use the average of two adjacent loaded spans for negative moment.

For bridges having cantilever arms with suspended spans, use the span (length) of the cantilever arm plus half the length of the suspended span when designing the cantilever and the span length of the suspended span (when) designing the latter.

For shear due to truck loads use the length of the loaded portion of span from the point under consideration to the far reaction."

"For culverts with cover	0'	to	1' - 0"	inc. I = 30%
	1' - 1"	to	2' - 0"	inc. I = 20%
	2' - 1"	to	2 - 11"	inc. I = 10%"

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