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THE ACTIVE NOISE CONTROL OF A
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Clark Radcliffe
Major professor

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THE ACTIVE NOISE CONTROL OF A ONE-DIMENSIONAL DUCT

By

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ABSTRACT

THE ACTIVE NOISE CONTROL OF A ONE-DIMENSIONAL DUCT

By

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Active noise control can reduce acoustic noise in situations where passive material sound absorption is ineffective. The specific problem studied is the active noise control of a one-dimensional, hard-walled duct with a partially dissipative boundary condition. Feedforward control technique have been used to cancel noise measured at a specific location. The control technique presented here is the modern state feedback control to globally reduce noise in a one-dimensional duct.

A state-space model of a one-dimensional acoustic duct was derived using the finite element method, which discretized the acoustic duct model continuum. A duct can be assumed to be one-dimensional model when its length is relatively long compared to cross-sectional diameter. The state-space model allows the computation of the system response and truncation of the system model.

State feedback control with a state estimator are developed from the truncated model to reduce noise level in a duct. The state estimator observes the system state through a pressure measurement microphone in the duct. The state feedback control is implemented to change the eigenstructure of the system and globally reduce noise through the control speaker. The estimator gains are determined by linear, quadratic optimum control theory. The results are discussed and the simulation is demonstrated in this paper.

To my parents

ACKNOWLEDGMENTS

I wish to express my sincere appreciation and gratitude to my major professor Dr. Clark J. Radcliffe for his valuable guidance and understanding during course of this study.

I also would like to recognize the guidance and encouragement of Dr. Larry J. Segerlind to me. His comments and advice were greatly appreciated.

Special thanks to my parents, brother and sister for their constant support and encouragement they have given me the entire time I have been in school.

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TABLE OF CONTENTS

LIST OF FIGURES

1. INTRODUCTION

Indoor noise reduction has increasingly become an important issue. The growth of noise in the workplace containing long duct assemblies is mostly due to the increase in the usage of high speed fans and complex machinery. This threatens the health of people and adversely affects the productivity. This problem can be solved by either passive or active noise control.

Passive noise control is control that uses noise absorption materials. These noise absoption materials counteract the undesirable effects of sound reflection by the hard, rigid, interior surfaces which they cover or replace. The advantage of passive noise control is simplicity and stability. Effective passive noise absorption materials need to have a thickness at least 10% of the noise wave length (Mankovsky, 1971). However, they are not suitable to reduce noise at low frequencies. The noise absorption might not be effective when the system or the operating conditions are changed because the noise absorption materials are designed for the specific systems or operating conditions.

The active noise control uses the intentional superposition of acoustic waves to reduce noise, and is able to adapt for system changes. In active noise control, low frequency noise reduction can be obtained. In feedforward control, the system inputs are measured and the cancellation sound, which contains exactly the same frequency, magnitude, and phase components as the noise to be cancelled, is generated. In this case, the noise cancellation is limited to a small region where noise is measured and cancelled. At the other locations away from the measurement location, noise is often increased because errors of frequency, magnitude, and phase are eveloped. In the feedback control, the system states are measured or estimated and the pole locations are changed to reduce but not eliminate the noise level at all positions in the duct.

The objectives of the research presented here are modeling of a one-dimensional hard-walled acoustic duct with a partially dissipative boundary condition, and the application of the state feedback control to achieve active noise control in the duct. This paper uses the finite element method to develop a discretized state space model of the duct. The state feedback control is applied to obtain noise reduction in the duct. The spillover problem is also quantified in this paper.

2. SYSTEM MODEL

2-1. Physical System Model

The system to modeled (Fig. 1) is a one-dimensional, hard-walled duct. The length of duct is longer enough that the wave propagation in the cross sectional direction can be ignored. The wall of duct is assumed to be rigid that the energy dissipation occurs only at the ends. The gas in the duct is assumed to be uniform density ρ, and temperature T.

The acoustic pressure of the system is related to the spatial gradient of the particle displacement by (Seto, 1971)

$$P(x,t) = -\rho c^2 \frac{\partial u(x,t)}{\partial x} \tag{1}$$

The duct model is governed by a linear wave equation for the particle displacement u as a function of time, and the spatial location x (Kinsler and Frey, 1962).

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2} \tag{2}$$

This model equates the spatial gradient of pressure the particle acceleration.

The boundary condition at x=0 is related to the pressure input.

$$\frac{\partial u(0,t)}{\partial x} = -\frac{1}{\rho c^2} P(0,t) \tag{3}$$

The partially reflective boundary condition at $x = L$ is the relationship between the spatial

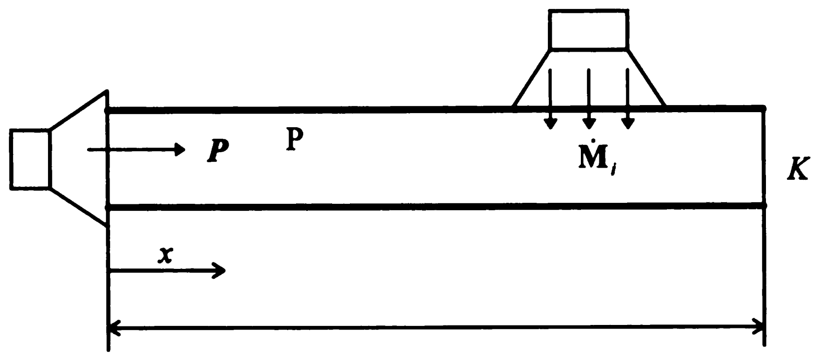

Figure 1 Physical acoustic duct model

gradient and the time gradient of the particle displacement. This is expressed as (Seto, 1971).

$$\frac{\partial u(L,t)}{\partial x} = -\frac{K}{c}\frac{\partial u(L,t)}{\partial t} \quad , K \neq \quad 0, 1, \infty \tag{4}$$

where K is dimensionless complex impedance of the termination end. The real part of K is associated with energy dissipation. The imaginary part of K is associated with conservative fluid compliance and inertial effects. When real part of K equals zero, or infinity, the termination end of the duct reflects all the acoustic energy and the response is composed of standing waves only. When K=1, the termination end of duct absorbs all the acoustic energy and the response is composed of propagating waves only. All other values of K yield some combination of propagating and standing wave response (Spiekermann and Radcliffe, 1988).

One or more control inputs associated with mass flow are allocated in the domain to control noise in the duct. The system equation with these inputs is nonhomogeneous. The linear second order wave equation modeling particle displacement in a hard-walled, one-dimensional duct is (Seto, 1971)

$$\frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = -\sum_{i=1}^{k} [\delta(x - x_i)] \frac{\partial}{\partial t} [\frac{M_i(t)}{\rho S}] \tag{5}$$

where $u(x,t)$ = particle displacement (m), c = wave speed (m/s), x = spatial location (m), t = time (s), ρ = density of the medium (kg/m^3), $M(t)$ = mass flow input in the domain (kg/s), x_i = location of mass flow input (m), S = speaker area driving the mass flow input (m^2), $P(t)$ = pressure excitation at $x = 0$ (N/m^2), and $\delta(x)$ = the Dirac delta function.

2-2. Finite Element Model

The finite element method can change this partial differential equation for the continuous medium in the duct to a system of ordinary differential equation for a discretized medium. Finite element analysis based on Galerkin's method uses weighting functions to approximate equation solutions. This method uses piecewise smooth functions, and the Galerkin residual integral formulation to generate a system of algebraic equations (Segerlind, 1984). The Galerkin's residual integral with respect to the space coordinates for fixed instant of time is

$$-\int_{x_a}^{x_b} \mathbf{W}^T (c^2 \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial^2 u(x,t)}{\partial t^2} - \sum_{i=1}^{k} [\delta(x - x_i)] \frac{\partial}{\partial t} [\frac{M_i(t)}{\rho S}]) dx = \mathbf{0}$$

(6)

where $\mathbf{W}^T$ is the Galerkin weighting functions which consists one dimensional shape functions associated with specific nodes in the finite element mesh. This residual integral can be solved for the system of differential equations for the particle displacement vector, $\mathbf{u}$.

The first term on the right hand side yields the element stiffness term and interelement term $\mathbf{I}_x^{(e)}$. The stiffness element is generated by conventional integration. The interelement term is a row vector which contains the element contributions. This vector is deleted except when derivative boundary conditions are specified (Segerlind, 1984). In the case of duct, the interelement term contains only the boundary conditions for the left and right end. The boundary condition on the right end can be separated from interelement vector, because it can be expressed as damping element. The remaining interelement vector only the left boundary condition. The second term on the right hand side of the Galerkin's residual integral turns out to be a mass element, and the third term yields point source vector which contains the mass flows at system input location. This point source vector and the interelement vector which contains the left boundary condition consist the force vector.

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{D}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}_p(t) + \mathbf{f}_M(t) \tag{7}$$

where $\mathbf{M} = \int_{x_a}^{x_b} \mathbf{N}^T \mathbf{N} dx$

$$\mathbf{K} = \int_{x_a}^{x_b} \frac{\partial \mathbf{N}^T}{\partial x} \frac{\partial \mathbf{N}}{\partial x} dx$$

$$\mathbf{D} = \mathbf{I}_{x=L}^{(e)}$$

$$\mathbf{f}_p = \mathbf{I}_{x=0}^{(e)}$$

$$\mathbf{f}_M = \sum_{i=1}^{k} \dot{M}_i \int_{x=x_i} \mathbf{N}^T \delta(x - x_i) dx$$

Dimension of each matrix, n is determined by number of elements. Each matrix row represents a specific location in the duct. The boundary conditions are incorporated into $\mathbf{D}$ and $\mathbf{f}_p(t)$. $\mathbf{D}$ is derived from $\mathbf{I}_{x=L}^{(e)}$ which represents the boundary condition of the right end. $\mathbf{f}_p(t)$ is consist of $\mathbf{I}_{x=0}^{(e)}$ term that contains the boundary condition of the left end, and $\mathbf{f}_M(t)$ is consist of the summation term which represents the mass flow inputs, $M_i(t)$ at $x = x_i$.

The frequency response of 3.66m duct (Fig. 2) show comparison between finite element solution (solid line) and analytical solution (dashed line). The response is measured at x=0 m. The impedance at the termination end is K=0.3 and 50 elements are used for finite element solution. The finite element solution is quite similar to the analytical solution.

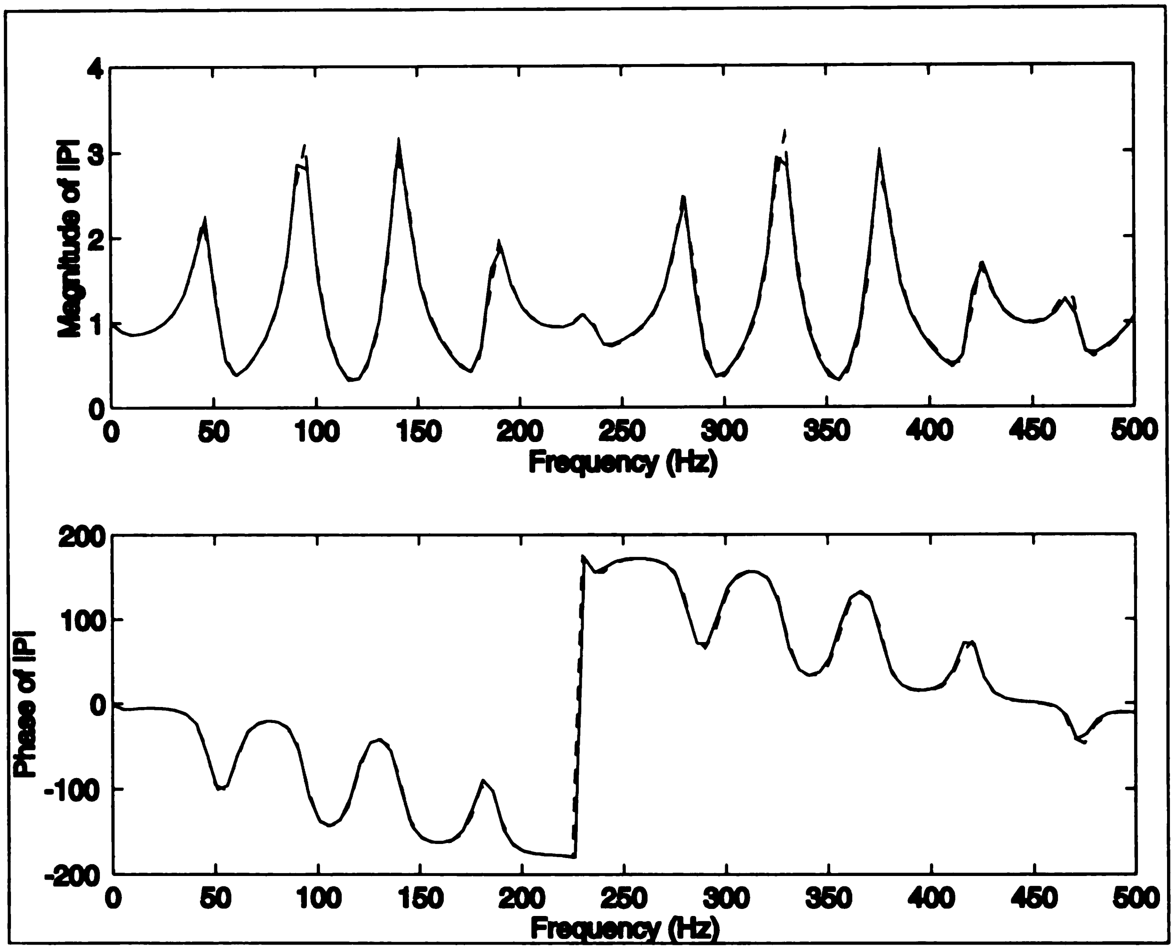

Figure 2 The frequency response of 3.66*m* duct by finite element solution (solid line) and analytical solution (dashed line). Impedance constant K=0.3, input location x=0*m*, and output location x=0.762*m*. Number of element for finite solution is 50.

3. STATE SPACE REPRESENTATION

The state space representation used in the control system observer and simulation can be derived from the finite element form of governing equation. Defining that the state vector, $\mathbf{x}_1$ represents the vector of particle displacements, $\mathbf{u}$ in equation (7) and $\mathbf{x}_2$ represents the vector of particle velocities, $\dot{\mathbf{u}}$, the second order differential equation of the system can be changed to the first order differential equation. Letting $\mathbf{x}_1 = \mathbf{u}$, $\mathbf{x}_2 = \dot{\mathbf{u}}$, and $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]^T$ gives

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_p\mathbf{f}_p(t) + \mathbf{B}_M\mathbf{f}_M(t) \tag{8-1}$$

$$\mathbf{P} = \mathbf{c}^T\mathbf{x}(t) \tag{8-2}$$

where $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix}$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}$$

$\mathbf{A}$ is $2n \times 2n$ matrix which contains the characteristic of physical duct system. $\mathbf{B}$ has dimension $2n \times n$ and plays role of gain of force vectors $\mathbf{f}_p$ and $\mathbf{f}_M$ in state space. Force vectors $\mathbf{f}_p$ and $\mathbf{f}_M$ are the disturbance input and control input, respectively. $2n \times 1$ matrix, $\mathbf{c}$ contains the spatial gradient of particle displacement which is expressed in equation (1).

A can be diagonalized using modal transformation, $\mathbf{x} = \mathbf{\Phi z}$. $\mathbf{\Phi}$ is modal matrix and **z** is the state vector in modal coordinate. Substituting $\mathbf{x} = \mathbf{\Phi z}$ and premultipling inverse of modal matrix gives

$$\dot{\mathbf{z}} = \mathbf{A}_D\mathbf{z} + \mathbf{B}_{PD}\mathbf{f}_p + \mathbf{B}_{MD}\mathbf{f}_M \tag{9-1}$$

$$\mathbf{P} = \mathbf{c}_D{}^T\mathbf{z} \tag{9-2}$$

where $\mathbf{A}_D = \mathbf{\Phi}^{-1}\mathbf{A\Phi}$

$$\mathbf{B}_{PD} = \mathbf{\Phi}^{-1}\mathbf{B}_P$$

$$\mathbf{B}_{MD} = \mathbf{\Phi}^{-1}\mathbf{B}_M$$

$$\mathbf{c}_D{}^T = \mathbf{c}^T\mathbf{\Phi}$$

$\mathbf{A}_D$ is diagonal matrix and has the same characteristic equation and eigenvalues as what **A** has, because $\mathbf{A}_D$ is mathematically similar to **A**. This state space representation allows the individual modes to be decoupled from each other.

4. TRUNCATION

A truncated finite element model is needed because the exact solution to the acoustic duct problem requires an infinite number of elements which is not available. A truncated model can be obtained by simply truncating high frequency modes of the state space model shown in equation (9), because the state space model has decoupled individual modes. This truncated model allows only dominant modes to be observed and controlled.

Spillover is generated by truncation. Spillover is system contamination by unmodeled modes (Balas, 1978). Spillover occurs when unmodeled modes in the system interacts with the control system action. This Spillover degrades control performance and can cause instability.

The frequency response of a 3.66m duct (Fig. 3) shows the effect of truncation. The response is measured at x=0.792m with the pressure excitation at x=0m. The impedance is K=0.3+0.2i and a truncated model with 7 term is used to approximate the solution (solid line). This truncated model with 7 term has only 3 dominant modes while a 62 term model (dashed line) includes higher frequency modes.

Time response of the finite element model (Fig. 4) shows the effect of truncation in time domain. The input into the duct is a harmonic pressure excitation with a frequency of 150 Hertz initiated at t=0 second. The input location is x=0 and the response is calculated at location x=0.792m. The impedance constant K=0.3+0.2i is used. This time response shows the predicted response with only 7 term (solid line) is quite similar to that for 62 term (dashed line) because truncated error is reduced by decoupling the states.

Another time response of the finite element model (Fig. 5) shows the response to the pulse pressure input with 7 term model (solid line) and 62 term (dashed line). The magnitude of pulse input is 1 N/m^2 and width is 0.005 second. The input location is x=0 and the response is calculated at x=0.792m. The impedance constant is K=0.3+0.2i. The 7 term finite element model shows the same response in each of the states but lower resolution due to the reduction in state number.

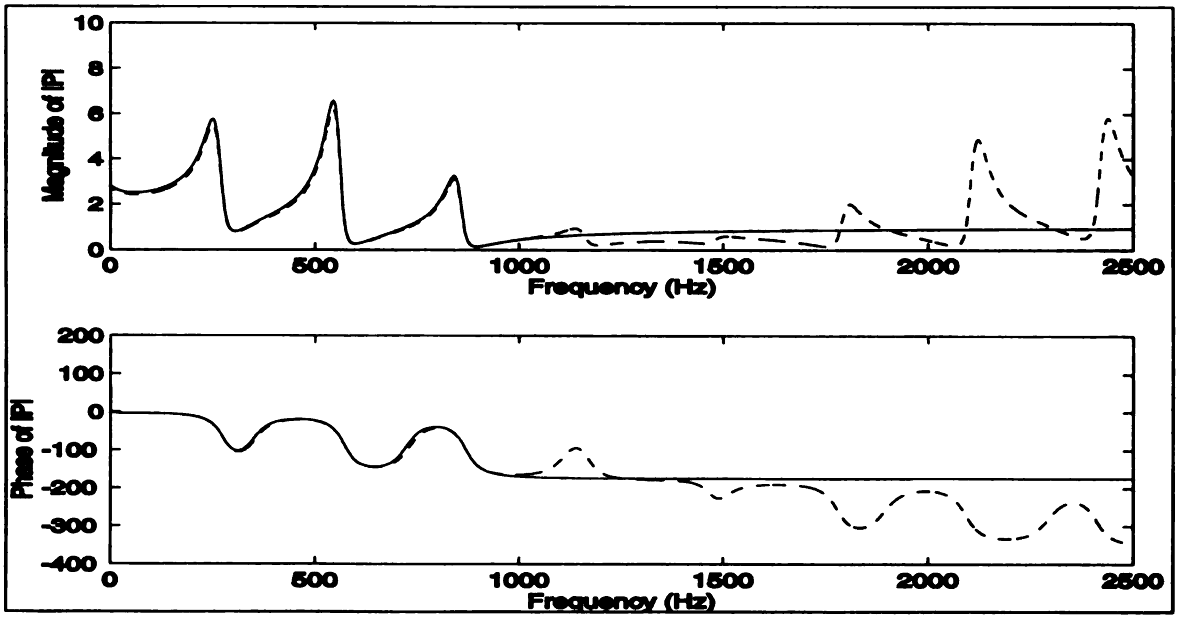

Figure 3 The frequency response of 7 term (solid line) and 62 term (dashed line) duct model with impedance constant K=0.3+0.2i. Total length of duct L=3.66m, input location x=0m, and output location x=0.792m.

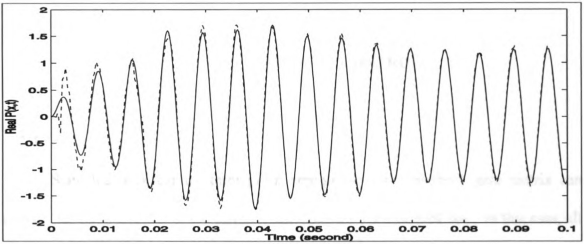

Figure 4 The time response of 7 term (solid line) and 62 term (dashed line) duct model to the harmonic pressure input with a frequency of 150 Hz initiated at t = 0 second. Total length of duct L=3.66m and impedance constant K=0.3+0.2i. Input location x=0m, and output location x=0.792m.

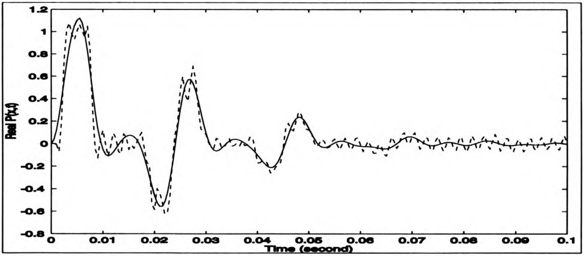

Figure 5 The time response of 7 term (solid line) and 62 term (dashed line) duct model to the pulse pressure input with a magnitude of 1 N/m^2 and width of 0.005 second initiated at t=0 second. Total length of duct L=3.66m and impedance constant K=0.3+0.2i. Input location x=0m and output location x=0.792m.

5. STATE FEEDBACK CONTROL

5-1. State Estimator

State feedback control through an appropriate state feedback gain matrix can change the eigenstructure of the physical duct system to the desired one. In this case, all states need to be measurable and available. However, none of states variables of the physical duct are directly measurable, therefore a state variable estimator is needed. A state observer is a device or computer program that runs in parallel with the actual system and estimates the state variables based on the measurements of input and output from physical system. An estimate of the infinite number of state variables of the physical duct is not possible. So a truncated state observer is used.

The state observer equations for single disturbance are

$$\dot{\hat{\mathbf{z}}}_N = \mathbf{A}_N\hat{\mathbf{z}}_N + \mathbf{B}_{MN}f_M + \mathbf{B}_{PN}f_P + \mathbf{g}_N(P - \hat{P}) \tag{10-1}$$

$$\hat{P} = \mathbf{c}_N^T\hat{\mathbf{z}}_N \tag{10-2}$$

where $\mathbf{A}_N$ is a truncated matrix which contains the N dominant dynamic characteristics of the duct system, $\hat{P}$ is the estimated sound pressure by observer, f_M is control input vector which is mass flow and located at $z \neq 0$, $\mathbf{B}_{MN}$ is control input matrix, f_P is disturbance input which is a pressure excitation and located at the left end, $\mathbf{B}_{PN}$ is the disturbance input matrix, $\mathbf{g}_N$ is the observer gain vector, and P is the pressure measurement output from the actual duct. $\hat{\mathbf{z}}_N$ designates the observed state vector which approximates the actual state vector, $\mathbf{z}_N$. The last term on the right side of equation (10-1) is a correction term. This correction term involves the difference between actual P and estimated $\hat{P}$, and is weighted by $\mathbf{g}_N$. This observer gain vector, $\mathbf{g}_N$ determines the error dynamics between the actual states and the estimated states.

The observer error equation can be expressed by

$$\dot{\mathbf{e}} = \mathbf{A}_e\mathbf{e} + \mathbf{B}_{PN}\mathbf{f}_P \tag{11}$$

where $\mathbf{e} = \mathbf{z}_N - \hat{\mathbf{z}}_N$ that is the error vector which designates the difference between the actual states and the estimated states, and $\mathbf{A}_e = \mathbf{A}_N - \mathbf{g}_N\mathbf{c}_N^T$. The poles of observer error equation determine the rate of convergence of the error to zero. If matrix $\mathbf{A}_e$ is a stable matrix, error vector will converge, which means the estimated states will converge to the actual states. Hence, if the eigenvalue of matrix $\mathbf{A}_e$ is chosen such that the dynamic behavior of the error vector is asymptotically stable and adequately fast, then any error will converge with adequate speed. However, the error equation is driven by any unknown input, $\mathbf{f}_P$. The desired poles of matrix $\mathbf{A}_e$ can be located by determining the observer gain matrix, $\mathbf{g}_N$. This pole placement can be done using Ackermann's formula. The Ackermann's formula for an observer is (Phillips and Harbor, 1996)

$$\mathbf{g}_N = \alpha_e(\mathbf{A}_N)\begin{bmatrix} \mathbf{c}_N \\ \mathbf{c}_N\mathbf{A}_N \\ \vdots \\ \mathbf{c}_N\mathbf{A}_N^{n-1} \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{12-1}$$

$$\alpha_e(\mathbf{A}_N) = \mathbf{A}_N^n + \alpha_{n-1}\mathbf{A}_N^{n-1} + \cdots + \alpha_1\mathbf{A}_N + \alpha_0\mathbf{I} = \mathbf{0} \tag{12-2}$$

where $\alpha_e(\mathbf{A}_N)$ is the desired characteristic polynomial whose roots are eigenvalues for error system. The observer gain is recommended to be chosen so that the dynamic speed of the observer is twice or fourth times faster than those of the system (Phillips and Harbor, 1996).

5-2. State Feedback Control with Optimized Feedback Gain

Pole location determines the speed and damping of the dynamic response. Changing pole location gives different eigenstructures of the system. Pole location is generally desired to be as far to the left of the origin as possible, because they have faster dynamic response. However, locating poles far to the left of the origin yields high control gain and signals which are too large to be produced within the system power limitations. Such high gain systems also have signal noise problems, and limiting the speed of response is necessary to avoid saturation and noise problems. The pole location for the speed limitation of response can be determined by Ackermann's formula expressed as

$$\mathbf{k}_{ac} = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{B}_{PN} & \mathbf{A}_N\mathbf{B}_{PN} & \cdots & \mathbf{A}_N{}^{n-1}\mathbf{B}_{PN} \end{bmatrix} \alpha_c(\mathbf{A}_N) \tag{13}$$

where $\mathbf{k}_{ac}$ is the state feedback gain vector and $\alpha_c(\mathbf{A}_N)$ is the desired characteristic polynomial. The state feedback gain matrix can be decided by the desired poles and can be used for control law $f_M = -\mathbf{k}_{ac}\hat{\mathbf{z}}_N$.

Another technique to determine the state feedback gain vector is linear quadratic optimal control technique. This optimal control technique generates control which minimize both system error and the input required to a device. These system error and the input can be expressed by performance index. Performance index in this paper consists the integral of a quadratic form of the truncated state vector, $\mathbf{z}_N$ plus an another quadratic form of the control input, f_M, that is

$$J = \int_0^\infty (\hat{\mathbf{z}}_N^* \mathbf{Q} \hat{\mathbf{z}}_N + f_M \mathbf{R} f_M) dt \tag{14}$$

The first term on the right side of the equation (14) represents a penalty on the deviation of the N dominant state vector , $\mathbf{z}_N$ from the origin, while the second term represents the control effort. $\mathbf{Q}$ is a state weighting matrix which determines the relative importance of the each state variable, and assume to be positive definite Hermitian or real symmetric matrix. In this paper, $\mathbf{Q}$ is selected as a diagonal matrix which has the different diagonal elements to weight each state variable differently. $\mathbf{R}$ is a matrix which contains the weighting value for control vector, and chosen 1×1 matrix with weighting value 1.

The optimal control law which minimize the performance index, J in equation (14) can be expressed as

$$f_M = -\mathbf{k}_{op} \hat{\mathbf{z}}_N \tag{15}$$

where $\mathbf{k}_{op}$ is a optimal feedback gain vector. Therefore, performance index can be minimized by this optimal control law, and such an optimal control law can be obtained by a proper vector $\mathbf{k}_{op}$. The equation that gives the optimal $\mathbf{k}_{op}$ is given by

$$\mathbf{k}_{op} = \mathbf{R}^{-1} \mathbf{B}_{PN}^* \mathbf{P} \tag{16}$$

where matrix $\mathbf{P}$ is the symmetric, positive definite solution of the Riccati equation expressed by (Ogata, 1970)

$$\mathbf{A}_N^{*}\mathbf{P}+\mathbf{P}\mathbf{A}_N-\mathbf{P}\mathbf{B}_{PN}\mathbf{R}^{-1}\mathbf{B}_{PN}^{*}\mathbf{P}+\mathbf{Q}=\mathbf{0} \tag{17}$$

5-3. Frequency Domain Simulation

The schematic and block diagram of the control system for duct is shown in Fig. 6 and Fig. 7. The state estimator measures pressure output from the physical system and the feedback input. The disturbance input is unknown because, in general, there is no way to determine the disturbance excitation of the duct.

A frequency response simulation of the controlled system is shown in Fig. 7. The length of the duct is 3.66*m*, the response is measured at x=0.792*m*, and the control speaker is at x=3.56*m*. The acoustic impedance at the end is *K*=0.3+0.2*i*. The number of terms used to form the model is 7. The state feedback gain is chosen to minimize the performance index **J** with

$$\mathbf{Q}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 300 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6000 \end{bmatrix}$$

$$\mathbf{R}=[1]$$

The frequency response (Fig. 7) shows that the fourth closed loop resonance tends to increase in amplitude as the closed loop resonances in the lower frequency range (solid line). The more magnitudes of closed loop resonances in frequency range of modeled dominant poles are decreased, the more likely magnitudes of closed loop resonances in frequency range of unmodeled poles are increased.

The pole locations of these system in complex plane (Fig. 8) also shows control spillover. Placing the dominant poles to the left from their original place causes some unmodeled poles to move to the right from their original place. In extreme case, those unmodeled poles move to the positive real plane, and cause the instability of the system.

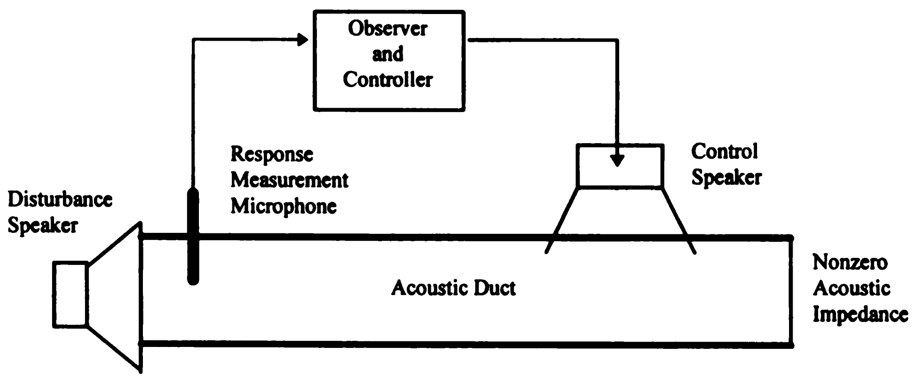

Figure 6 The schematic diagram of active noise control system

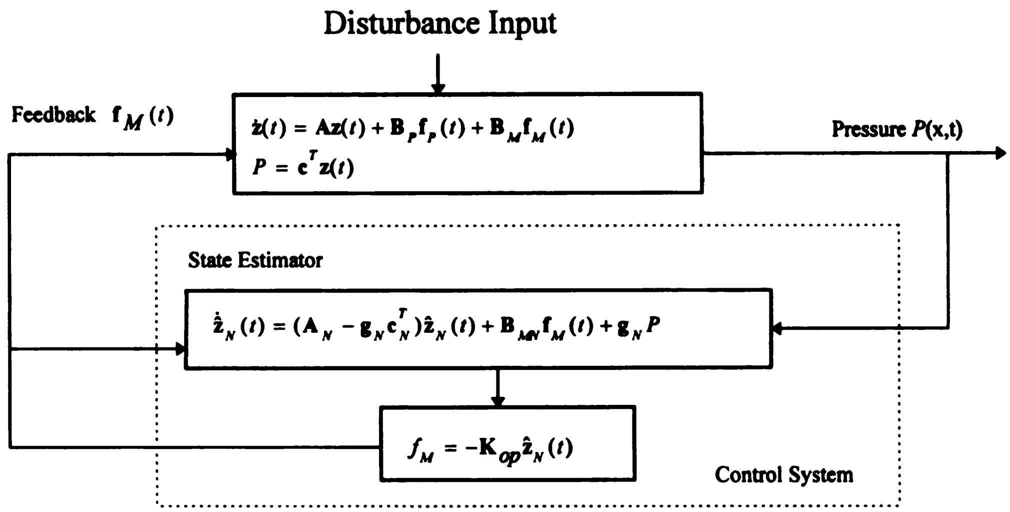

Figure 7 The block diagram of active noise control system

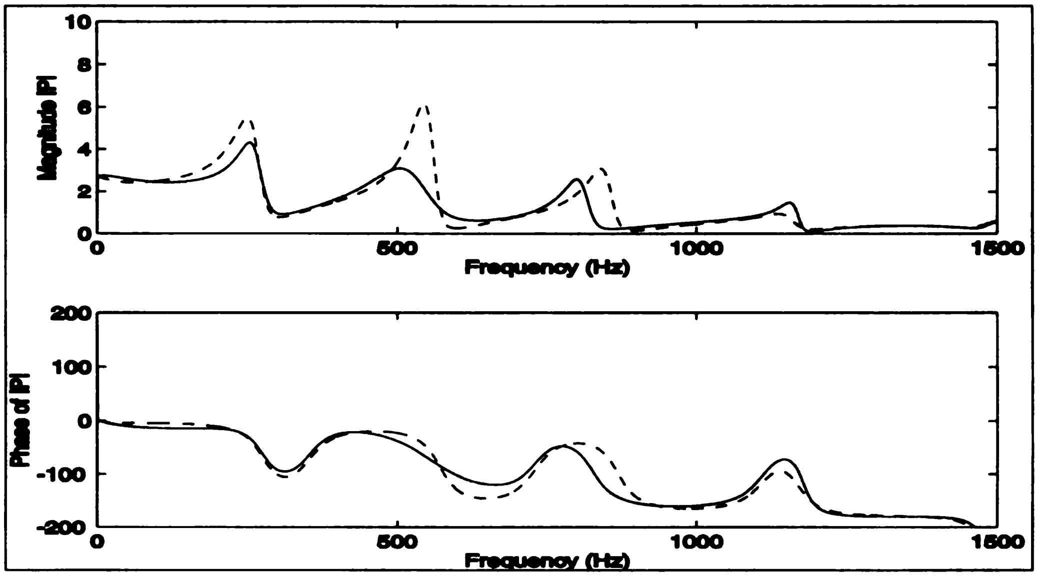

Figure 8 The frequency response of closed (solid line) and open (dashed line) loop system of 3.66m duct. Disturbance input location x=0m, states measurement location x=0.792m, K=0.3+0.2i. Number of truncation term for observer is 6.

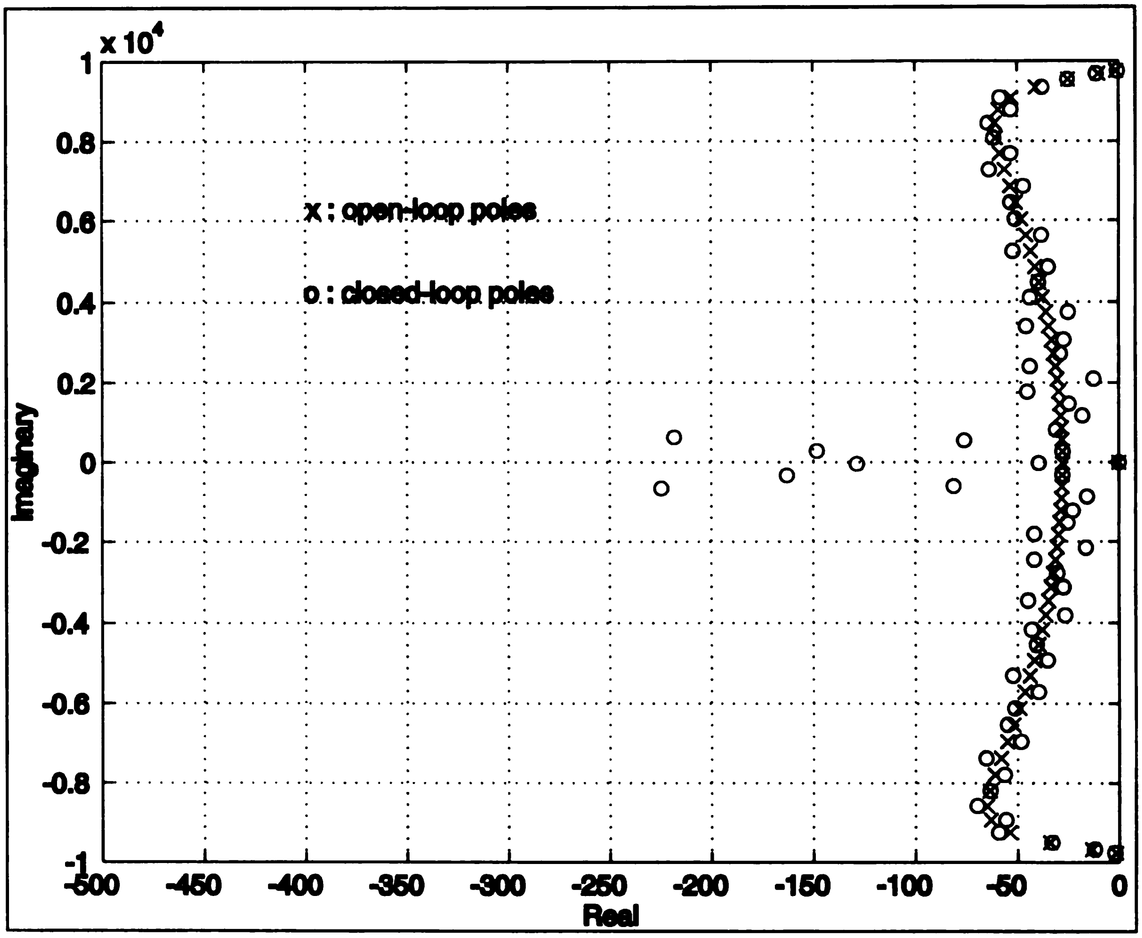

Figure 9 Pole locations of closed (o) and open (x) loop systems of a 3.66m duct. Disturbance input location x=0m, states measurement location x=0.792m, and control input location x=3.56m. Impedance constant at the termination end K=0.3+0.2i. Number of truncation term for observer is 6.

5-4. Time Domain Simulation

Time domain simulations predict the response of the system over a finite time. They are useful in determining the transient response of the controller. A time domain simulation is performed with the plant and the control system shown in Fig. 7.

A time domain simulation with 3.66*m* duct is shown in Fig. 10. The response is measured at x=0.792m and the control speaker is at x=3.56*m*. The impedance at the termination end is K=0.3+0.2i. 6 truncated term was used for state estimator. The simulation step size is 0.0002 seconds. The simulation excitation is a sine wave with unit magnitude at 150 Hertz. The state estimator and controller are both activated at t=0 second. The steady state amplitude of closed loop system (solid line) is 0.9 N/m^2 while that of open loop system (dashed line) is 1.4 N/m^2.

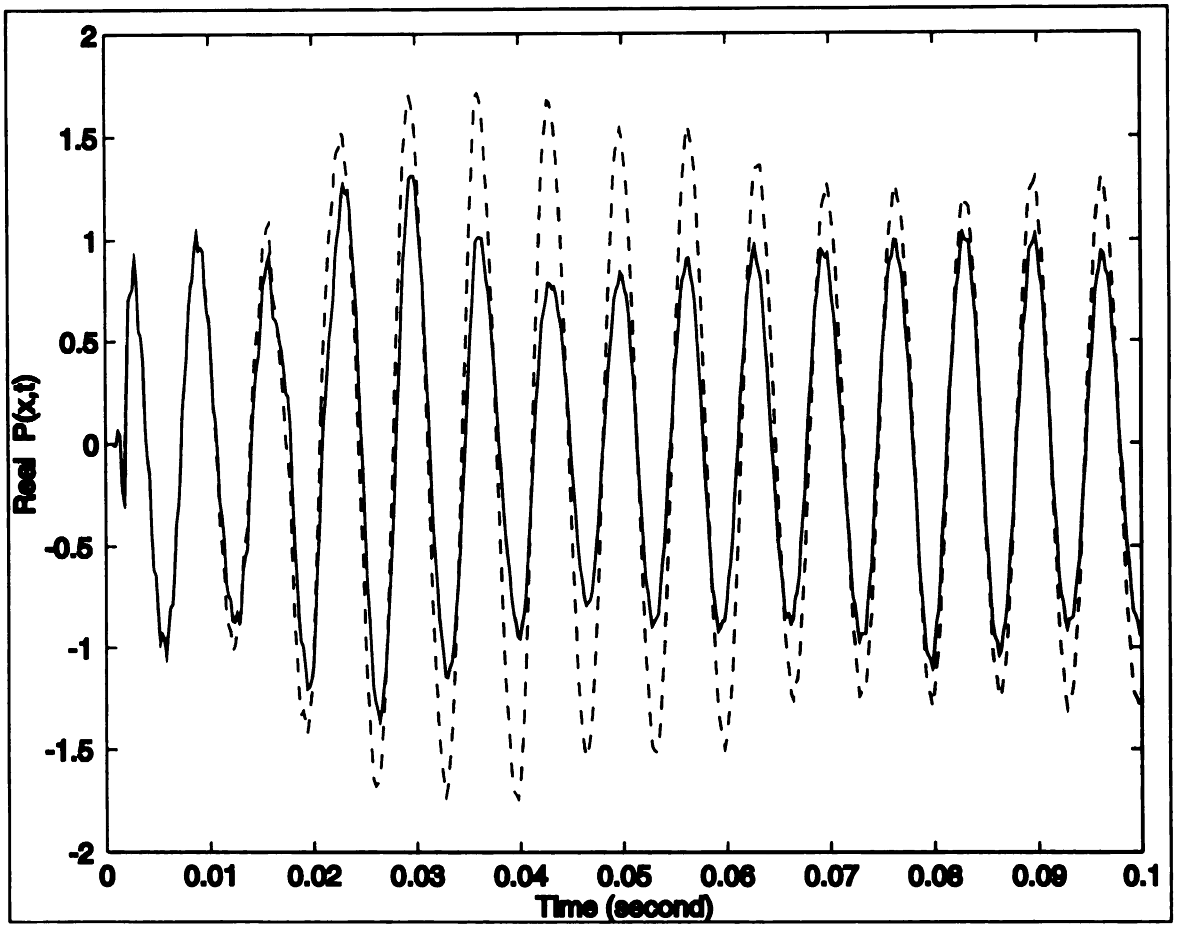

Figure 10 The time response of closed (solid line) and open (dashed line) loop systems of a 3.66m duct to the harmonic pressure input with a magnitude of $1\ N/m^2$ and a frequency of 150 Hz initiated t=0 second. Harmonic pressure input location x=0m, state measurement location x=0.792m, control input location x=3.56m. Impedance constant of the termination end K=0.3+0.2i and number of truncation terms used for observer is 6.

6. CONCLUSIONS

A one-dimensional hard-walled acoustic duct with a partially dissipative boundary condition was modeled using the finite element method. This paper showed the finite element method discretize the continuum system and approximate the exact dynamic response of the system with a convenient numerical computation.

Active noise control technique was developed to reduce noise levels. Frequency and time domain simulations are presented. These simulations showed the active feedback control techniques globally reduced noise in the duct instead of canceling the noise in particular region. Active feedback control generated spillover which degrades control performance and caused instability.

There are several areas for future work or improvement in the method. One obvious extension is to move this work into three dimensional enclosures, since most acoustic system are three-dimensional rather than one-dimensional. Three-dimensional noise reduction would be extremely useful in aircraft and factories.

A topic of improvement in the one-dimensional system is in the area of filters. Noise reduction by feedback control has limitation because of spillover problem. However implementing the filters in order to minimize the spillover problem will allow more noise reduction by feedback control.

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