This is to certify that the

thesis entitled

NATURAL CONVECTION FROM A HEATED, HORIZONTAL CYLINDER WITHIN AN INCLINED RECTANGULAR DUCT

presented by

ROBERT W. VANCE

has been accepted towards fulfillment of the requirements for

MS degree in Engineering

Craig W. Somerton

Major professor

Date 8/20/96

PLACE IN RETURN BOX to remove this checkout from your record.
TO AVOID FINES return on or before date due.

DATE DUE	DATE DUE	DATE DUE

NATURAL CONVECTION FROM A HEATED, HORIZONTAL CYLINDER WITHIN AN INCLINED RECTANGULAR DUCT

BY

ROBERT W. VANCE

A THESIS

Submitted to

Michigan State University

in partial fulfillment of the requirements

for the degree of

MASTER OF SCIENCE

Department of Mechanical Engineering

1996

ABSTRACT

NATURAL CONVECTION FROM A
HEATED, HORIZONTAL CYLINDER
WITHIN AN INCLINED RECTANGULAR
DUCT

By

Robert W. Vance

An experiment was conducted to investigate the effects of confining boundaries on the convective heat transfer from a heated cylinder. This was accomplished by using a duct with a rectangular cross section and embedding a horizontal cylinder inside it. The duct was then rotated to various angles of inclination to ascertain any angular effects. The results show that there is indeed an angular dependence to the heat transfer coefficient, as well as increased natural convection heat transfer rates with the confined configuration compared to convection in an infinite medium. Additionally, Nusselt number correlations are made, which take into account not only modified Rayleigh number dependence but also angular dependence. Several attempts are made to model the problem in a simple manner, but none are successful.

In dedication to the two most important women in my life. To my mother, who always knew when to show me the road and when to let me find it on my own. She taught me the value of a good education. And to Jaima, whose constant prodding to get things done keeps me ever focused on the light at the end of the tunnel.

ACKNOWLEDGEMENTS

Their have been many people who have helped me along the way in completing this work. First, and foremost, the fine professors in this department, who not only taught me subject matter, but also how to think and organize my thoughts, without them none of this would be possible. In particular, Dr. Somerton who was never too busy to listen to my endless barrage of questions. The department's secretaries were extremely helpful in completing all the necessary forms and requirements to graduate. They did not seem to mind retyping forms when titles were changed again and again.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

Arabic

a	-	Correlation constant
b	-	Vertical axis intercept of linear plot
Bi	-	Biot number
c	-	Correlation constant
C_P	-	Specific heat at constant pressure
d	-	Diameter
D	-	Diameter
E	-	Energy
g	-	Gravitational vector
Gr*	-	Modified Grashoff number
h_c	-	Heat transfer coefficient
i	-	Electric current
k	-	Thermal Conductivity
L	-	Cylinder length
m	-	mass
m	-	Slope of linear plot

n - Mixed convection correlation constant

Nu - Nusselt number

Pr - Prandtl number

q - Heat transfer rate

Ra - Rayleigh number

Ra* - Modified Rayleigh number

Re_D - Reynolds number

T - Temperature

t_c - time constant

v - Voltage

v_b - Buoyant velocity

Y - Weighting factor

Greek

α - Thermal diffusivity

β - Thermal expansion coefficient

δ - Cylinder wall thickness

ν - Kinematic viscosity

θ - Angular inclination

Subscripts

D	-	Based on diameter
s	-	Surface
w	-	wall
∞	-	Free stream

Superscripts

"	-	Flux

INTRODUCTION

Description of Problem

Natural convection heat transfer has been a topic of study for many years. The analysis of this buoyancy driven phenomena has found its way into many text books devoted to the basics of fluid flow and heat transfer. A multitude of problems have been studied ranging from natural convection over a flat plate, to flow over a cylinder, to flow inside an enclosed cavity. The *International Journal of Heat and Mass Transfer* [6] provides a wide variety of these different problems. With so many years of research, computer simulation, and theoretical efforts one might think that the topic of natural convection has been thoroughly studied. Although many situations have been looked at, there are still subtle nuances to be examined which make way for a large variety of "new problems". Hence the present study was undertaken, a "bridging of the gap" between two well studied problems: flow over a cylinder heated by a constant heat flux, and natural convection from the walls of inclined ducts. A simple sketch of the problem appears in Figure 1.

Buoyancy driven flow from a constant heat flux cylinder in an infinite fluid is a well known and studied problem. However, if the fluid flow is confined in an unheated inclined duct, the problem is changed and the previous solutions may no longer apply. This problem was examined experimentally in hopes the results would offer some insight into how this subtle difference would effect the heat transfer.

Applications

Natural convection in a duct has direct applications in electronic component cooling. Many electronic components produce a large amount of heat which needs to be removed. The most typical method is to install a small fan and remove this unwanted heat via forced convection. However, fans are not always practical for components which need only a "small" amount of heat removed. Designing the component casings with the drafting effect encountered from this experiment may augment the heat transfer, reducing the need for the installation of fans. Aside from the cooling aspects of drafting natural convection, there may be aspects that could aid in the better design of residential fire places or industrial smoke stacks.

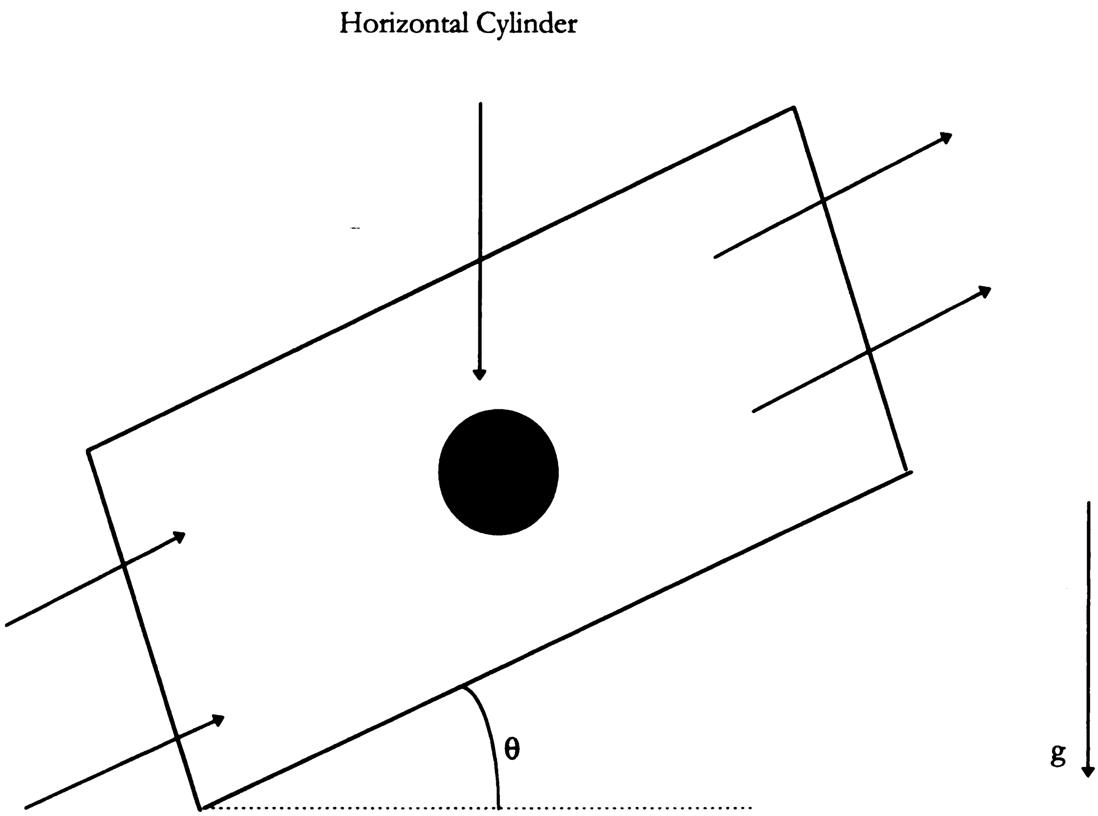

Figure 1. Schematic of the drafting problem

Literature Review

It has already been discussed that the problem of drafting natural convection has been limited to heated walls in ducts. It is for this reason that literature pertaining to this particular problem is not in abundance. In fact, no literature was found which addressed the problem. It is therefore necessary to compare the present results to similar cases, more specifically, cases dealing with natural convection in an infinite medium. Although this will not solidify or refute the results, it will indicate whether or not there is any significant increase in the average heat transfer coefficient. The following articles have been used as references for comparative data, and to offer insight to unexpected results.

Perhaps the most useful piece of literature was the 1987 article by Ahmad and Qureshi [2] dealing with a numerical solution for laminar natural convection around a horizontal cylinder subject to an uniform heat flux boundary condition. This article allowed the comparison of both computational and experimental results for the infinite fluid case to that of inclined duct flow. The 1992 paper by Ahmad and Qureshi [1] examined mixed convection around a horizontal cylinder subject to an uniform heat flux. This also was a computational study that looked at the relationship between Reynolds numbers, Grashoff numbers, drag coefficients, and Nusselt numbers. The following three articles examined the infinite fluid problem and provided comparative results: a 1975 article by Morgan [8], and the 1974 and 1975 papers by Churchill [3,4].

Papers dealing with duct flow natural convection are fairly common, but offer little insight to this problem as they deal with heated walls, not cylinders. However, the 1993 article by Yan [9] looked at inclining the duct at various angles of inclination. Although the walls were still heated, the trends encountered were useful in interpreting the results of the present investigation. Deschamps [5] modeled a horizontal heat flux cylinder as a line source for numerical work. This paper is more concerned with fluid mechanics than heat and mass transfer, but does provide interesting results for enclosed natural convection. Finally, the text by Mills [7] was useful in examining mixed convection, in particular modeling a buoyant velocity for the natural convection.

Road Map

In the following three chapters, the experimental apparatus and the procedure which was used to derive a heat transfer coefficient will be discussed. The results will be presented and discussed, as well as compared with existing data for similar cases. Finally, conclusions will be drawn and recommendations for future work will be made.

METHOD OF SOLUTION

Apparatus

The experimental apparatus used in examining this problem is a simple one. The configuration consists of a horizontal, hollow aluminum cylinder with a copper-constantan thermocouple embedded in it, a Hewlett Packard 6011A DC power supply which is used to run a variable current through the cylinder, and a Hewlett Packard 3468A digital multi-meter to record the voltage drop across the cylinder and the thermocouple. The cylinder is confined in a Plexiglas duct eleven inches in length. The duct is attached to a stationary frame which allows it to be rotated to the desired angle of inclination. It is assumed that the cylinder is thermally lumped, thus leading to the neglecting of any angular variation in temperature. This allows any internal natural convection and radiation to be ignored. These assumption will be examined latter in this chapter. A schematic of the setup appears in Fig. 2.

Procedure

The evaluation of the natural convection heat transfer coefficient was done under steady conditions so an energy balance of the cylinder would produce an easily evaluated expression for h_c. Performing the energy balance (energy in equals energy out) yields the relation in Equation (1).

$$h_c = \frac{vi}{(T_s - T_\infty)\pi d L} \qquad (1)$$

Now that a relation for the heat transfer coefficient exists, it is necessary to evaluate it for various cases. This was done by starting with the duct in a horizontal position and recording the needed values for eight different current settings of the power supply. The duct was then rotated to various angles of inclination, from horizontal to vertical, and the data was recorded for these cases. The fluid properties needed to map the heat transfer coefficient to a Nusselt number are evaluated at the film temperature of the air. This is done by using a curve fit for the various properties which is valid over the temperature range encountered in this experiment. These curve fits can be found in appendix A.

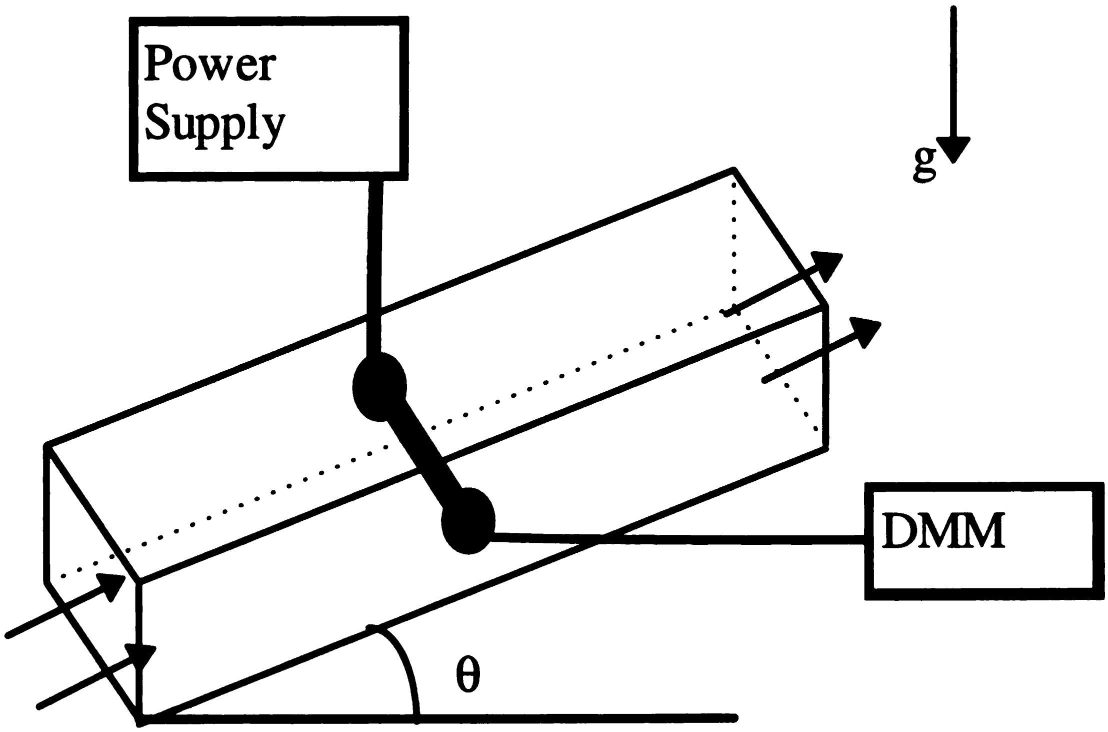

Figure 2. Schematic of experimental setup

Recording the surface temperature, and hence voltage drop, was not as straight forward as it might seem. As it turns out, natural convection with a constant heat flux boundary condition has an oscillating steady state. That is, in order to keep the heat flux constant the temperature and voltage drop are constantly adjusting. It was, therefore, necessary to either use a data acquisition system to record the data every few seconds, or make a judgment on the temperature and voltage. Both these methods were employed for several cases, and the results are shown in the next chapter. The data acquisition used was a National Instruments Lab View virtual instrument template. However, the data acquisition system proved to be too time consuming, as it was necessary to get enough full oscillations to justify the use of this method, Fig. 3 shows a one hour profile for the surface temperature. Thus, a means to consistently record the data was needed.

The data used to evaluate the heat transfer coefficient was recorded when the surface temperature of the cylinder was at a maximum. As can be seen from Eq. (1) this will underpredict the heat transfer coefficient. Two different size cylinders were used in order to get a wide variety of Rayleigh numbers. Diameters of 2.83 and 6.33 millimeters were used with both cylinders having the same length of 85 millimeters. Both temperature and constant heat flux based Rayleigh numbers were recorded to aid in the modeling of the problem and in correlating Nusselt number expressions. The expressions for both Rayleigh numbers are shown in equations 2 and 3.

$$Ra = \frac{g\beta \Delta T D^3}{\alpha v} \tag{2}$$

$$Ra^* = \frac{g\beta q D^4}{\alpha v k} \tag{3}$$

Note the following relationship between the Nusselt number and Ra* and Ra.

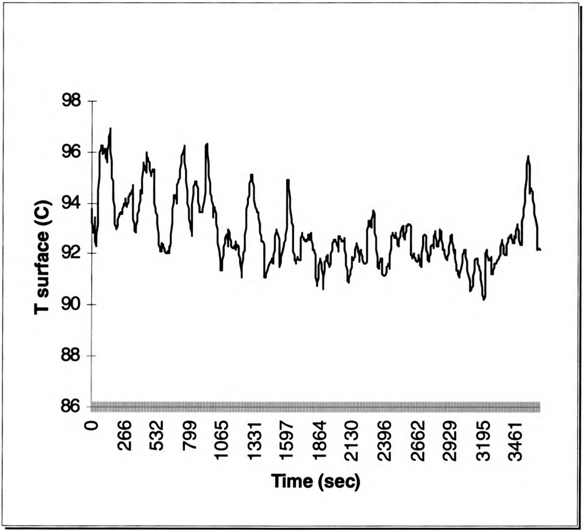

Figure 3. Periodic behavior of the surface temperature.

$$\frac{Ra^*}{Ra} = \frac{q''D}{\Delta Tk} = \frac{h_c D}{k} = Nu_D \qquad (4)$$

Now that a means of evaluating the driving force for the fluid mechanics, and the resulting heat transfer coefficient exists, it is necessary to use these results to glean some insight into this problem. In the following chapter the efforts to model this problem are examined, and useful Nusselt number correlations are obtained.

Measurement uncertainties

When doing any experimental work it is necessary to evaluate how well a certain measurement is being recorded. Since it is impossible to measure anything exactly, it is important that the researcher know how much uncertainty there is in the evaluation of a particular parameter. In regards to the present study these parameters are the heat transfer coefficient, the Nusselt number, and the modified Rayleigh number. The method used to evaluate the uncertainties comes from the use of multi-variant calculus, and produce the uncertainty equations shown below.

$$dRa^* = \frac{g\beta D^4 q''}{\alpha\nu k}\left(\frac{d\beta}{\beta} + 4\frac{dD}{D} + \frac{dq''}{q''} + \frac{d\alpha}{\alpha} + \frac{d\nu}{\nu} + \frac{dk}{k}\right) \qquad (5)$$

$$dh_c = \frac{vi}{\pi Dl(T_w - T_\infty)}\left(\frac{dv}{v} + \frac{di}{i} + \frac{dD}{D} + \frac{dl}{L}\right) + \frac{vi}{\pi Dl}\left[\frac{dT_w + dT_\infty}{(T_w - T_\infty)^2}\right] \qquad (6)$$

$$dNu_D = \frac{h_c D}{k}\left(\frac{dh_c}{h_c} + \frac{dD}{D} + \frac{dk}{k}\right) \qquad (7)$$

The calculations and uncertainty values can be found in appendix B. It should be noted that all the uncertainty values were obtained from the accuracy of the measurement equipment used, except for the temperature. Since there was the judgment call on the maximum temperature, the uncertainty in the surface temperature was taken as the difference between the maximum and the minimum temperatures from the time history plots. Furthermore, it was assumed that the fluid properties were known exactly, this is not a bad assumption since the correlations are very accurate and their effect would be quite small.

Using the above method there was found to be an average uncertainty of seven percent for the heat transfer coefficient, of ten percent for the Nusselt number and of eleven percent for the modified Rayleigh number.

It is necessary to justify the assumptions made at the beginning of this chapter. To verify the lumped capacitance model the Biot number is examined. The Biot number represents a comparison between the energy transported by convection and the energy transported via conduction within the solid. The Biot number is defined in Eq.(8),

$$Bi = \frac{h_c \delta}{k} \tag{8}$$

where δ is the hollow cylinder wall thickness. This number is found to be on the order of 10^{-5} which is much smaller than the standard value of 0.1 which is accepted as the maximum value for the Biot number to validate the use of a lumped capacitance model.

Having verified the model it is necessary to evaluate the system time constant and compare it to the transient response of the periodic steady state. Using the lumped capacitance model the system time constant is given by the relation,

$$t_c = \frac{mC_P}{h_c A} \tag{9}$$

where m is the cylinder mass, and A is the surface area. Evaluating this time constant a value of roughly 20 seconds is obtained. Comparing this to the period of oscillation of approximately 120 seconds it is found to be significantly less. Since the system time constant is smaller than the oscillation time the temperature results are indicative of the actual process.

The final justification for using a steady state approximation is the comparison of terms from the lumped capacitance model. These terms are the transient energy storage term, the energy output term, and the energy input term. These appear in Eqs.(10-12), respectively.

$$E_{stored} = mC_P \frac{dT}{dt} \tag{10}$$

$$E_{out} = h_c A(T_w - T_\infty) \tag{11}$$

$$E_{in} = vi \tag{12}$$

Comparing the transient storage term to the energy output term, it can be seen that $E_{stored}/E_{in} \approx 0.95$, and $E_{out}/E_{in} \approx 0.02$. From these results it can be seen that the neglecting of the transient term is not a poor assumption since it is much smaller than the other two terms.

RESULTS AND DISCUSSION

Introduction

The analysis of the data is primarily concerned with developing useful Nusselt number correlations. Additionally, comparisons to parallel problems are studied to help support conclusions drawn about the "drafting" effect of the inclined duct. Along with these comparisons are several lab tests to further enhance the notion of "drafting" flow. Modeling efforts are also undertaken in this chapter.

Results

Convective heat transfer analysis is generally concerned with examining the Nusselt numbers dependence on a driving force parameter. For forced convection this parameter is the Reynolds number, for natural convection the driving force is characterized by the Rayleigh number. It is for this reason that this studies data is presented in Nusselt number versus Rayleigh number plots. These plots appear below in two figures, the first illustrates low Rayleigh number dependence, and the overall Ra* dependence.

These graphs not only provide a concise way to present the data, they provide an indication that there may be a drafting effect present. This conclusion is drawn by comparing the lower angle Nusselt numbers to those for the larger inclination angles. It is clear that the larger angles induce a larger convective heat transfer rate. Furthermore, the functional relationship between the Nusselt number and the modified Rayleigh number changes as Ra* increases. This will greatly effect the correlations made at the end of the chapter. In the following section, basic trends encountered are discussed, as well as the air flow characteristics.

Trends

Any trends that exist in the heat transfer coefficient are due to trends in the fluid flow. For pure natural convection the fluid flow is described as plume flow. This type of flow is a

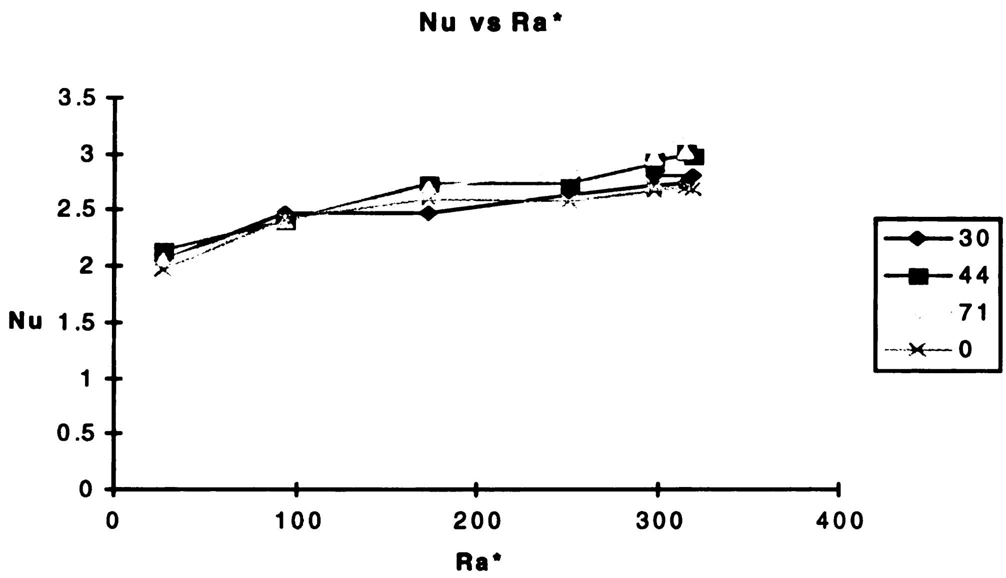

Figure 4. Nusselt number dependence for low Rayleigh

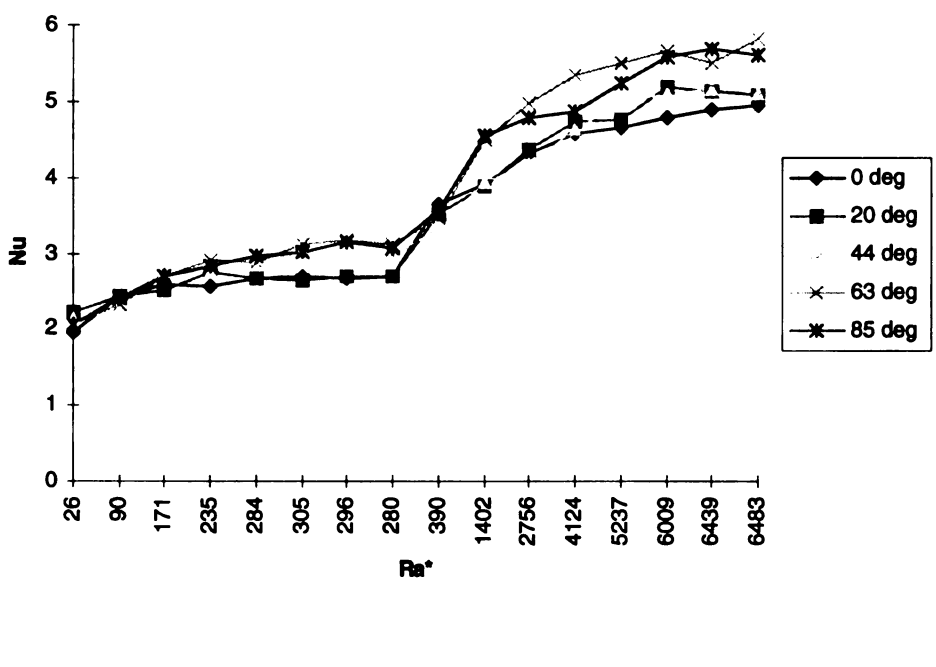

Figure 5. Overall Rayleigh number dependence

buoyancy driven flow caused by the density difference between the hot air near the cylinder surface and the colder free stream air. This flow is in the vertical direction, 180 degrees from the direction which gravity acts. It is hypothesized that there may also exist a different type of fluid motion, drafting flow. This flow type would be caused by confining walls of the duct. That is, as the air comes in from the bottom opening of the duct to replace the air taken away by the plume flow, it pushes the excess air out of the top opening of the duct(since the applied pressure gradient inside the duct is assumed zero).

In order to assess whether "drafting" was indeed occurring , two methods were employed. The first method was to place thermocouples on the top and bottom of the duct, before and after the cylinder. If the convection from the cylinder was a natural plume, one would expect the top thermocouple after the cylinder to register a higher voltage drop, i.e. be hotter, than the others. The data for several angles of inclination, for approximately the same Rayleigh number, are shown in table 1.

Table 1. Thermocouple voltages at various locations on the duct.

Angle/Position	Top/After	Top/Before	Bottom/After	Bottom/Before
0 degrees	0.48 volts	0.52	0.15	0.13
10 degrees	0.54	0.26	0.1	0.1
63 degrees	0.07	0.03	0.04	0.04
71 degrees	0.01	0.02	0.05	0.08

The data in table 1 shows that for near horizontal angles of inclination the top thermocouple temperatures are much higher than those on the bottom. This would indicate a plume effect of the hot air rising. At larger angles of inclination all the thermocouple readings are about the same, showing the plume effect not to be the dominate flow. As has been stated, the "steady state" for this case is oscillatory in nature. The non-steady behavior of the heat transfer indicates a non-steady behavior in the fluid flow. Therefore, even though the thermocouple readings at higher angles of inclination show a more streamline flow field, this is not really the case. A more accurate means of evaluating flow behavior was needed.

The flow was visualized by placing a small amount of baby oil on the surface of the heated cylinder. Baby oil produces a light gray smoke which, presumably, will illustrate the flow direction. By photographing this flow a better understanding of the flow phenomena was obtained. The flow around the cylinder can be broken up into three stages: plume flow, drafting flow, and flow reversal. The first two stages seem to interact periodically, as was illustrated by the periodic "steady state" of the heat transfer. The reversal stage only happens on the rare occasion when the transition from drafting flow to plume flow occurs quickly. This stage is characterized by much mixing about the cylinder as the flow tries to restore the plume. Photographs of these regions appear in Fig. 6 and 7.

The flow reversal region, was very difficult to photograph as its duration was very short and the mixing of light gray smoke did not show up well in the pictures. It is obvious from the two photographed cases that there is indeed some excess fluid motion beyond pure natural convection. Although finding exact solutions to the fluid motion would be difficult, and probably has to be done numerically as opposed to experimentally, it is desirable to have a simple model to describe the "drafting" problem. The attempts to use a simple model to describe this problem are the topic of the next section.

Having described the fluid motion, it is necessary to describe the trends in the heat transfer. It is expected that the heat transfer will be better at higher inclination angles than lower angles because of the drafting flow. In order to illustrate this, the experimental Nusselt numbers were plotted as a ratio of pure natural convection based Nusselt numbers versus Rayleigh number. This plot is shown below as Fig. 8.

Examination of Fig. 8 shows that there is indeed a higher heat transfer rate at higher angles of inclination than for lower. Furthermore, there appears to be little difference between 53 degrees and 85 degrees. It may be possible that there is little increase in the heat transfer coefficient after some critical inclination angle. It would take many more experiments before this could be verified, but it is apparent that the larger angles produce higher Nusselt numbers than do lower angles.

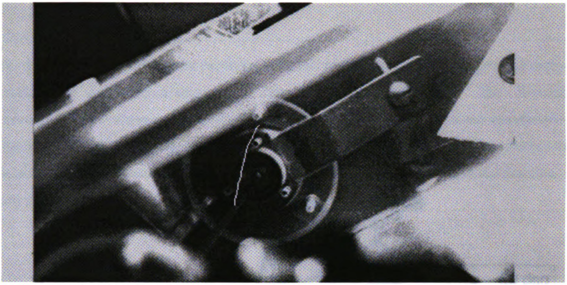

Figure 6. Plume flow visualization around the cylinder.(Graphically enhanced)

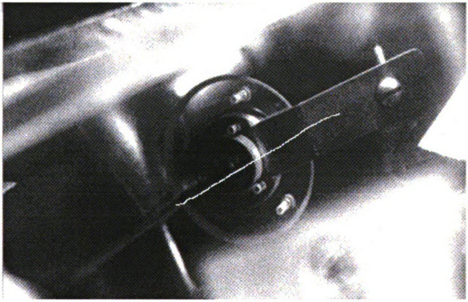

Figure 7. Drafting flow visualization around the cylinder.(Graphically enhanced)

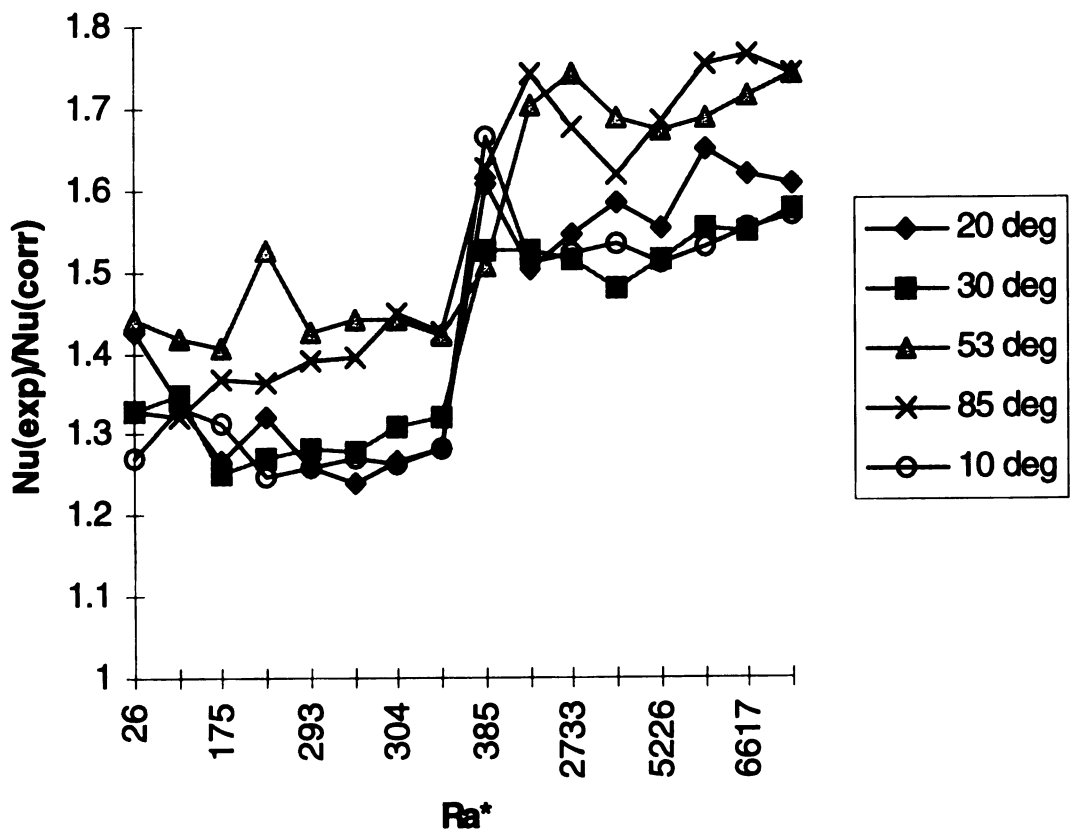

Figure 8. Nusselt ratio versus Rayleigh

Modeling

The first attempt at modeling the process was to treat the problem as a quasi-mixed convection problem. It was therefore necessary to estimate the "buoyant" velocity. The Bernoulli equation for flow along a streamline was used with the assumption that the pressure was constant. The driving force for the flow would therefore be the density difference between the flow at the inlet and around the cylinder. The result is a relation for v_b, which is a function of only density, and assuming an ideal gas, only a function of temperature.

$$v_b = \sqrt{2\left(\frac{\frac{1}{T_1}-\frac{1}{T_2}}{\frac{1}{T_2}}\right)g\Delta h} \tag{13}$$

By using Eq.(13) it was possible to obtain estimates for Reynolds numbers and compare the results from the experiment to established results for numerical and experimental mixed convection. Mill's [7] suggests that for mixed convection there is a power relationship between the mixed convection Nusselt number and those for natural and forced convection. The suggested relationship is shown in Eq.(14), where the 0.3 is a correction factor for cylindrical geometries, and n has a value of 3.

$$(Nu-0.3)^n = (Nu_{NC}-0.3)^n + (Nu_{FC}-0.3)^n \tag{14}$$

The experimental data was used in Eq.(14) to get a feel for the extent of the mixed convection. The data did not agree well with the above relation. In fact, the left hand side only agrees with the right when n goes to infinity. This was the first indication that either the process is not a mixed convection phenomena or the buoyant velocity is being over predicted. Comparison to the numerical study of Ahmad and Qureshi [2] for mixed convection around a horizontal cylinder provided further support to this assertion. By using the buoyant velocity based Reynolds number and a modified Grashoff number comparisons were made with the findings and again showed poor agreement. The estimated Reynolds numbers are of the order of 40 to 140, with Gr* of about 2. These numbers correspond to average Nusselt numbers of about 3.5, almost 70 percent higher than experimental results.

Although the buoyant velocity is being over predicted perhaps a linearly weighted combination of forced convection and natural convection can be found to produce a simple model. The proposed model is shown as equation 15.

$$Nu = YNu_{FC} + (1-Y)NU_{NC} \tag{15}$$

This model was examined over the range of inclination angles to see if the various angles affected the Rayleigh Number averaged Y. Additionally, the Y values for one angle were examined to find a correlation with Rayleigh Number. These results appear in figures 9 and 10, respectively.

As can be seen, the weighting factor Y is not well behaved over a range of inclination angles. The Y values are less than 0.5 and do tend to increase with theta. This "trend" for Y is not enough to begin to form a concrete model. If the data had behaved more linearly, then it would have been possible to examine this further, but as it stands this model was dropped after the results in Fig. 10. Although Y is for the most part linear with respect to Ra, the values are not unique. This lack of "uniqueness" cast serious doubt about whether or not this model was capable of producing useful results, and if using the mixed convection analysis was correct.

Aside from over predicting the buoyant velocity, which can be corrected, it is necessary to evaluate whether the process is truly mixed or not. The answer is rather simple. By looking at the driving force for the flow, it is obvious that the only driving force is buoyancy. Although there seems to be a higher air velocity than pure natural convection, this can most likely be attributed to the duct area and continuity of the air flowing through the duct.

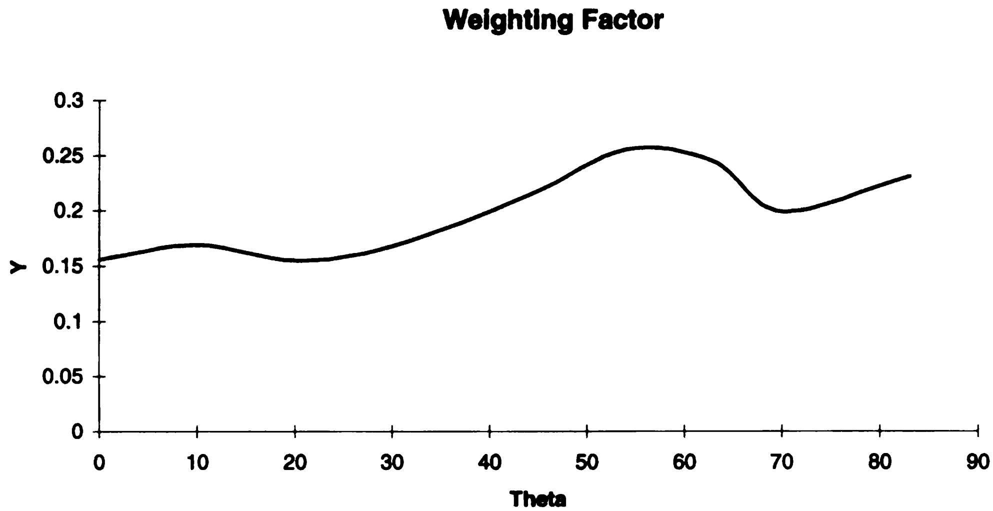

Figure 9. Weighting Factor as a function of angular inclination

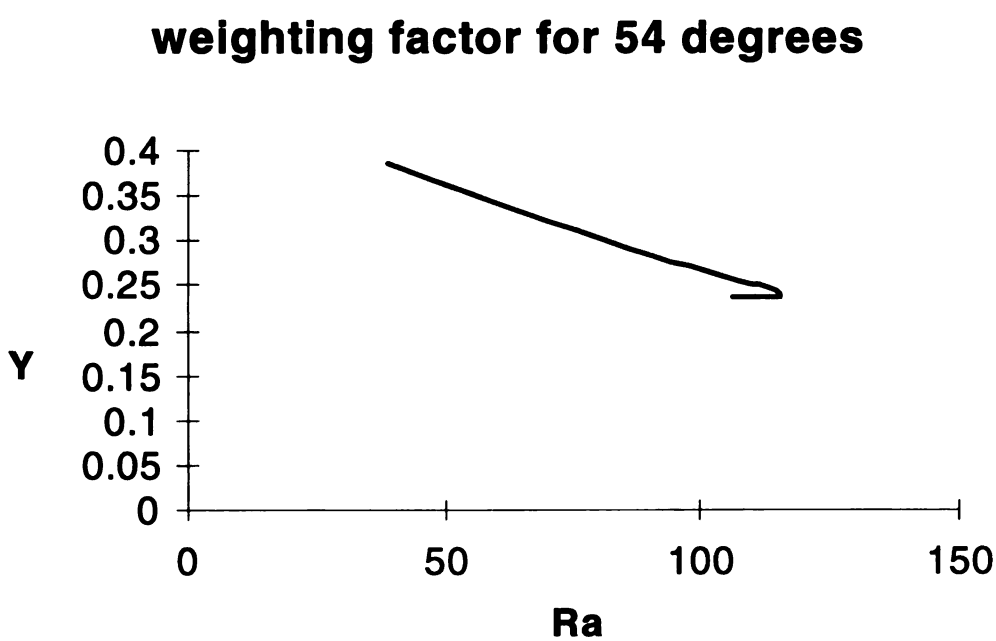

Figure 10. Rayleigh Number dependence for Y

At this point, efforts to model the problem were abandoned, hoping a better understanding of the fluid mechanics was in the near future. Several things that could be done were to find Nusselt number correlations for Rayleigh numbers and inclination angles, as well as compare the experimental results to published results . These are the topics of the next two sections.

Comparisons

As with most comparative analysis, it is useful to start with simple cases before attempting to tackle the more complex ones. Comparing the case of constant heat flux natural convection in an infinite medium to the present study provides insight. This was done by using the results of Churchill, Morgan, and Ahmad and Qureshi. Figure 11 shows the graphical comparison of their data to that obtained from the inclined duct problem.

The Nusselt correlation used can be found in the article by Morgan, and was adopted for constant heat flux conditions by using the relationship, Ra*=RaNu (see the development in chapter 2). As can be seen from Fig. 11, the non-dimensional heat transfer coefficients of the inclined duct case are larger than those for an infinite medium. This comparison involves published results of both numerical and experimental nature. Also, the angular trends encountered in this experiment agree with those encountered by Yan's numerical study of inclined mixed convection with heated walls. In this study Yan reported that increasing the inclination angle increased the heat transfer from the walls.

Nusselt Number Correlations

Finding Nusselt number correlations was the primary aim of data analysis. Although there are many aspects of the problem which are not fully understood, it is possible to provide useful equations which may help broaden the scope of "back of the envelope" calculations. Although there is a 20 percent uncertainty inherent in all Nusselt number correlations, using a correlation which is appropriate for a specific case will yield more reliable results than those for similar, but slightly different one.

The first step in finding an appropriate correlation was to plot the Nusselt number data vs modified Rayleigh to get a feel for the functional relationship between the two. These results can be found in Fig. 4 and 5 at the beginning of this chapter.

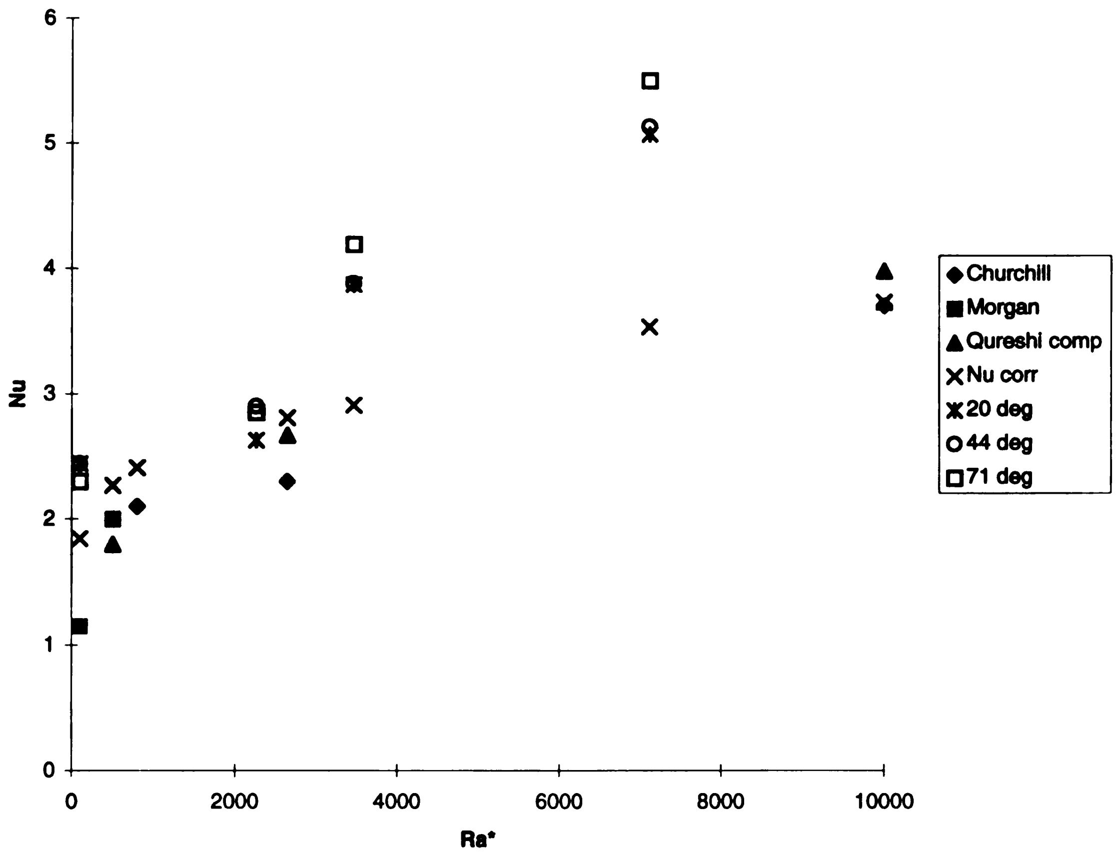

Figure 11. Comparison of average Nusselt numbers vs. Ra*

As can be seen there is an angular dependence on the heat transfer, where the more vertical the duct becomes the higher the average heat transfer coefficient becomes. Although at lower Rayleigh numbers this effect is less pronounced, it becomes noticeable as Ra* increases. This makes intuitive sense as the driving force for the problem becomes greater, the "drafting" has a more prominent influence. In addition to being supported by intuition, this angular dependence is found in the results from Yan's examination of inclined duct flow for heated wall boundary conditions. Given this dependence on inclination angle it is expected that either θ will appear in the correlation directly, or influence the correlation constants, indirectly affecting the correlation.

Since it is easier to use θ as a classification parameter for the correlation constants, this method was preferred. The power law relationship shown in Eq(16) was first tried, with the reasoning that these relations exist for natural convection in an infinite medium, perhaps only the coefficients will be effected.

$$Nu = cRa^{*m} \tag{16}$$

This was indeed the case, as it was possible to find values for c and m which would fit data for any inclination angle. However, the values for c and m did not fit well with one another, and it was not possible to create nice, neat correlations.

With the difficulty encountered in using a power law relation, another method was needed. The next step was to plot Ra*-Nu indirectly, that is plot Ra* vs Ra, and noting the relation Ra*=RaNu. The Ra*-Ra plot for Ra* less than 400 appears in Fig. 12. As can be seen the plots are nearly linear, allowing for easy curve fitting. This linear relationship between Ra* and Ra implies the following Nu number relation.

$$Nu = \left(m + \frac{b}{Ra^*}\right)^{-1} \tag{17}$$

Where m is the slope of the linear plot, and b is the y axis intercept.

Table 2. Coefficients for low Ra* correlation

Angle	m	b
0-40	0.351	5.46
41-55	0.296	11.15
56-90	0.296	12.47

This method produced results which allowed a variety of inclination angles to be represented by one set of coefficients. These results are shown in Table 2. This correlation is only valid for Ra* less than 400, it is desirable to have a more "robust" set of correlations, that is, to incorporate a larger range of Rayleigh numbers. Therefore, the same approach was repeated with data for both the large and small cylinders, resulting in a range of Rayleigh numbers form zero to 7000. This data did not produce linear plots, but rather second order curve fits, as seen in Fig. 13. However, a linear curve fit could be made between Rayleigh numbers of 400 and 7000. Thus, the same correlation form could be used over this range with variations in the coefficients, these will appear in table 3.

Table 3. Coefficients for large Ra* correlation

Angle	m	b
0-20	43.9	0.2
21-45	42.4	0.2
46-70	52.8	0.17
71-90	39.2	0.18

As a final note, these correlations were derived by using a piecemeal method. That is, the parameters were changed from their original value in hopes that a wider range of inclination angles could be incorporated. The above correlations are based on a ten percent

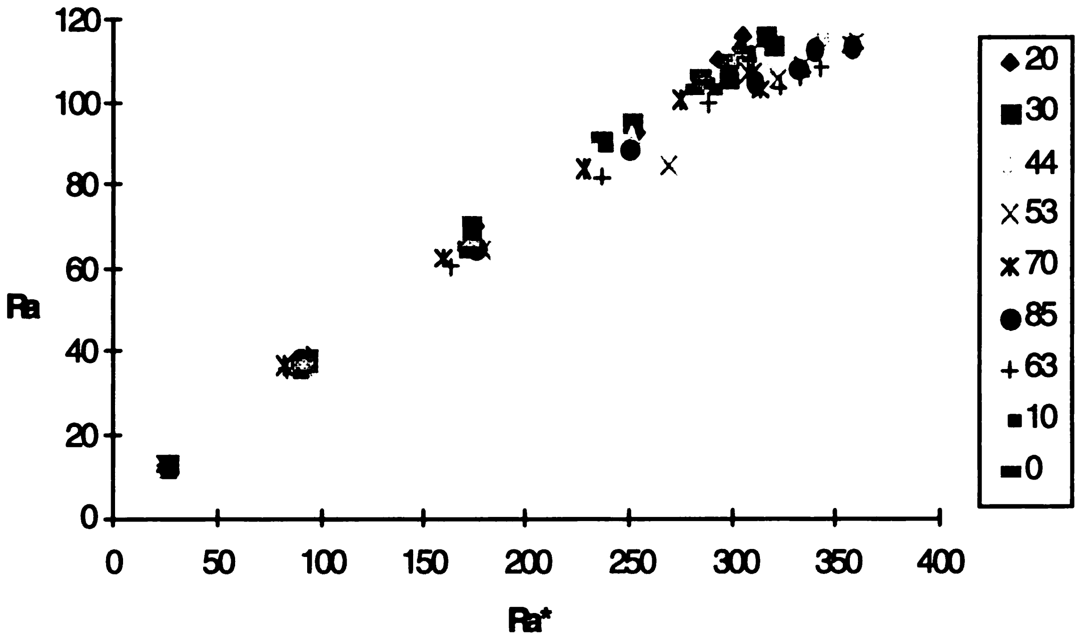

Figure 12. Rayleigh number comparison.

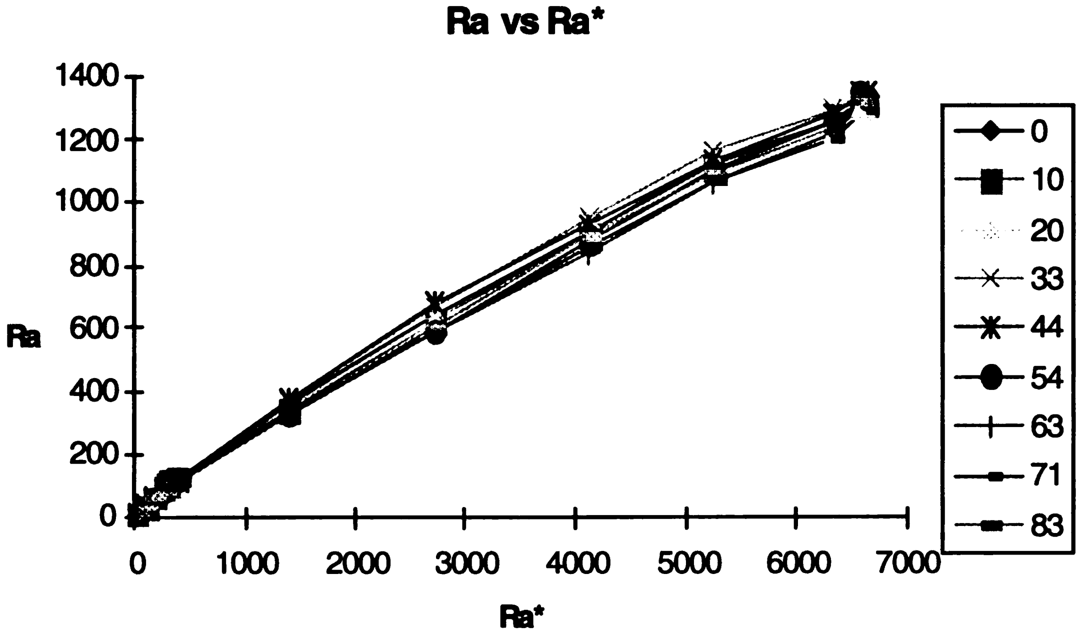

Figure 13. Large Rayleigh number comparison

maximum error between the correlation value and the experimental value. If, for any value of θ, a point had more than ten percent error, the correlation was not used for that particular angle. Combining this with the ten percent uncertainty in the experimental Nusselt number value, the correlations are roughly accurate within 20 percent. A graphical representation of the correlation accuracy is shown for a 20 degrees inclination angle in Fig. 14.

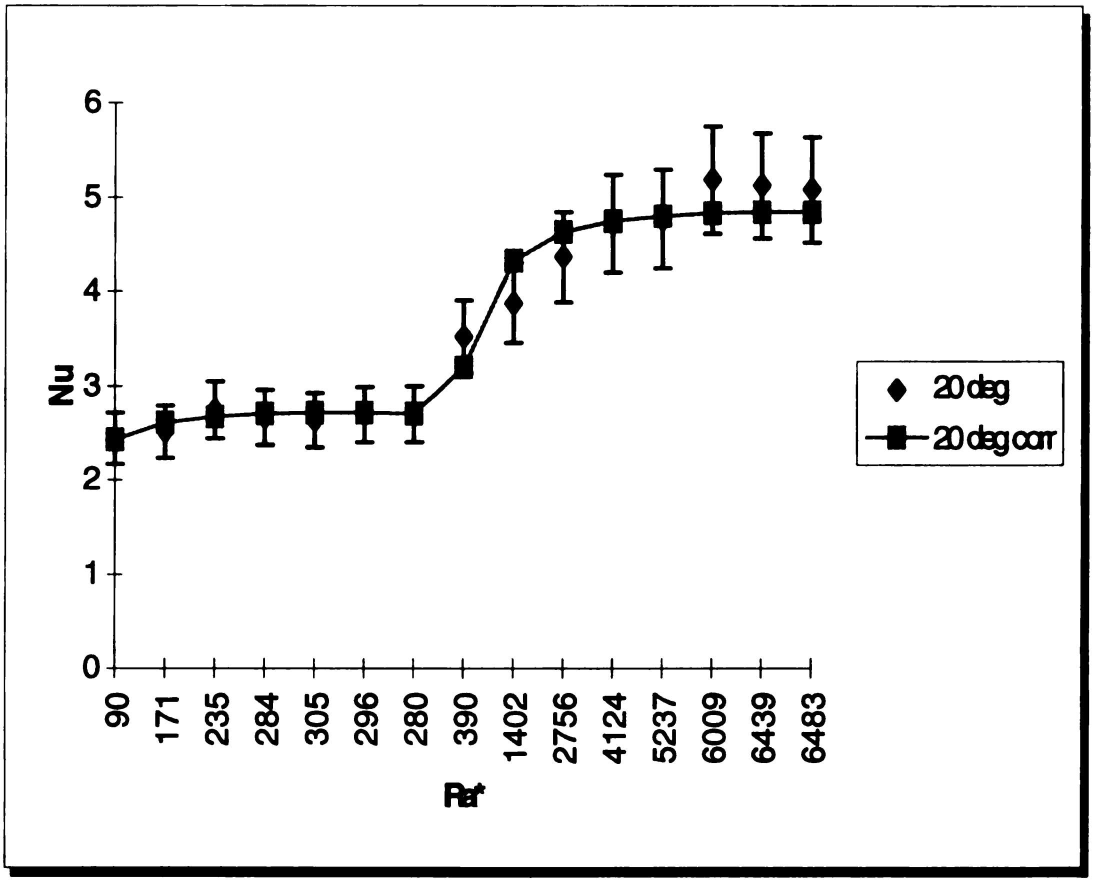

Figure 14. Graphical representation of correlation accuracy

CONCLUSIONS AND RECOMENDATIONS

Several conclusions can be drawn from the work of this thesis. Primarily, it was shown that natural convection can be enhanced by the confining of the flow. This increase in the amount of transferred thermal energy via natural convection will not have earth shattering consequences, but may broaden the scope of natural cooling. Perhaps in the near future this technique could be used to cool low power electronic components.

From an academic stand point the fluid mechanics of this research provide an interesting problem. As has been stated, the fluid motion of the drafting is quite complex. It would be very difficult to do any analytical work on a flow that is at best two dimensional and has regions of both laminar and turbulent flow. For this reason a computational solution to the two dimensional Navier-Stokes equations for constant heat flux natural convection may provide insight, not only into this problem , but also into other situations where oscillatory steady state conditions apply.

This problem has only been cursorially tested in this study, and many other aspects need to be examined. There are several parameters which would be easily examined from an experimental perspective. The inlet and exit length of the duct, the cylinder diameter to duct hydraulic diameter ratio, high Rayleigh number effects, fluid property effects, and wall boundary conditions are a few of the things which need to be addressed before this problem can be adequately understood.

Although the basic trends of the data from this examination correlate well with those of similar numerical solutions, there is still a lack of experimental data from other sources concerning this problem. For this reason, the data from this thesis is not supported by any other source, save for the trends encountered, and the values of Nusselt numbers obtained from the correlations cannot be taken as absolute.

APPENDICES

APPENDIX A

The following property curve fits are evaluated at the film temperature for the cylinder, where $T_{film} = \frac{T_s + T_\infty}{2}$, with Kelvins as the temperature units.

$$\Pr = 0.86715 - 1.0209\times10^{-3}T + 2.3688\times10^{-6}T^2 - 2.9019\times10^{-9}T^3 + 1.6783\times10^{-12}T^4$$

$$k_{air} = (-6.9091\times10^{-2} + 9.721\times10^{-2}T - 3.1469\times10^{-5}T^2)\times10^{-3} \text{ W/m}^2\text{K}$$

$$\nu = (-0.57024 + 8.9575\times10^{-3}T + 1.727\times10^{-4}T^2 - 6.622\times10^{-8}T^3)\times10^{-6} \text{ m}^2\text{/s}$$

$$C_p = \frac{8.314}{28.97}(3.653 - 1.334\times10^{-3}T + 3.291\times10^{-6}T^2 - 1.91\times10^{-9}T^3 + 2.75\times10^{-12}T^4)$$

Kj/KgK

$$\alpha = \frac{\nu}{\Pr} \text{ m}^2\text{/s}$$

$$\beta = \frac{1}{T_{film}} \text{ 1/K}$$

$g=9.81$ m/s^2

APPENDIX B

The following is a list of uncertainties used in calculating the error for the derived parameters.

di=0.05 A

dV=0.2 mV

dD=0.005 mm

dL=2.0 mm

dT_∞ =0.05 K

dT$_w$ ≈ $(T_w - T_\infty) \times 0.1$

The approximation for the cylinder wall temperature uncertainty is based on the data obtained from the Lab View data acquisition system which exhibited the above characteristics. This was necessary because the data acquisition system was not used for each case. It is believed that this uncertainty is larger than what really occurred. Furthermore, the fluid properties are assumed to be known exactly, that is, their uncertainty is zero. This is not a bad assumption as their contribution to the overall error would be small. Using these values in the following equations and taking the maximum uncertainty provides a safe measure for the error in the experiment.

$$dRa^* = \frac{g\beta D^4 q''}{\alpha\nu k}\left(\frac{d\beta}{\beta} + 4\frac{dD}{D} + \frac{dq''}{q''} + \frac{d\alpha}{\alpha} + \frac{d\nu}{\nu} + \frac{dk}{k}\right)$$

$$dh_c = \frac{vi}{\pi Dl(T_w - T_\infty)}\left(\frac{dv}{v} + \frac{di}{i} + \frac{dD}{D} + \frac{dl}{l}\right) + \frac{vi}{\pi Dl}\left[\frac{dT_w + dT_\infty}{(T_w - T_\infty)^2}\right]$$

$$dNu_D = \frac{h_c D}{k}\left(\frac{dh_c}{h_c} + \frac{dD}{D} + \frac{dk}{k}\right)$$

The maximum percent uncertainties for each parameter are as follows:

$dh_c \approx 7\%$

$dNu_D \approx 10\%$

$dRa^* \approx 11\%$

LIST OF REFERENCES

REFERENCES

1. Ahmad, R.A. and Qureshi, Z.H. , Laminar Mixed Convection from a Uniform Heat Flux Cylinder in a Crossflow, *Journal of Thermophysics and Heat Transfer* vol. 6, No. 2, April-June 1992.

2. Ahmad, R.A. and Qureshi, Z.H., Natural Convection From a Horizontal Cylinder at Moderate Rayleigh Numbers, *Numerical Heat Transfer* vol. 11 pp. 199-212, 1987.

3. Churchill, S.W. Laminar Free Convection From a Horizontal Cylinder With a Uniform Heat Flux Density, *Lett. Heat Mass Transfer* vol 1 pp. 109-112, 1974.

4. Churchill, S.W., and Chu, H.H.S., Correlating Equations For Laminar and Turbulent Free Convection From a Horizontal Cylinder, *Int. Journal of Heat Mass Transfer* vol 18 pp. 1049-1053, 1975.

5. Deschamps, Valerie, and Desrayaud, Gilles, Modeling a Horizontal Heat-Flux Cylinder as a Line Source, *Journal of Thermodynamics and Heat Transfer* vol. 8 Jan-March, 1994.

6. *International Journal of Heat and Mass Transfer*, vol. 34, Pergamon Press plc, Oxford, England, 1991.

7. Mills, A..F., *Heat Transfer,* Irwin, Boston, MA, 1992, Chap. 4.

8. Morgan, V. T. , The Overall Convection Heat Transfer from smooth circular cylinders, *Adv. Heat Transfer* vol 11 pp.199-264, 1975.

9. Yan, Wei-Mon, Mixed Convection Heat and Mass Transfer in Inclined Rectangular Ducts, *Int. Journal Heat Mass Transfer* vol. 37 pp. 1857-1866, 1994.