ADAPTATIONS TO CLIMATE CHANGE:
EXTREME EVENTS VERSUS GRADUAL CHANGES

By

Sangjun Lee

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ABSTRACT

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As extreme weather events become more prominent, there is growing interests among public and academic society in the relation between those events and climate change. While several current studies provide evidence about how climate change appears to be creating more frequent and more severe extreme events, relatively little is known about adaptation strategies compared with strategies for gradual climate change. This dissertation focuses on adaptation strategies in response to changes in the pattern of frequency and magnitude of extreme events.

This dissertation consists of three essays focused on adaptation decision making under climate change including explicit consideration of extreme events. The first essay establishes a theoretical model of adaptation, capturing the different effects of gradual climate change uncertainties versus extreme events. Employing a real options framework where underlying stochastic processes capture effects of extremes, land use decisions are examined given increased frequency and severity of extreme weather events as well as gradual climate change. Findings show that when decision makers are allowed to optimize dynamically and to learn, gradual change and extreme events can lead to different adaptation incentives as well as different likelihoods of adaptation occurring even when traditional net present value (NPV) calculations are equal; even if both exhibits same expected damage, gradual change imposes higher incentive to switch than the extreme events but the realized action may be dominated by the extreme events.

The second essay applies a real options land use conversion model to decisions in the Michigan tart cherry industry where exposure and vulnerability to extreme events has increased since 2000. Empirical yield and price processes are estimated using historical tart cherry yield and price data from 1946 to 2012. Exit decisions from farming are examined under gradual change and extreme events. Consistent with theoretical results from the first essay, the empirical assessment indicates extreme events dominate gradual change in adaption actions in the industry. Results imply that assessments of exposure and vulnerability to extremes such as spring frosts can provide valuable information to the growers.

The third essay examines the government role in designing and implementing effective policy to support individual enterprises under the effects of extreme events. Three popular forms of subsidies to support an enterprise's revenue under climate change risks are compared for effectiveness given the same cost to government: i) fixed amount subsidy; ii) fixed rate subsidy; and iii) insurance support. Empirical simulation shows that an insurance subsidy outperforms the other policy measures as it successfully transfers climate-driven risks to the government. Heavy-tail distribution of extremes, which indicates more drastic extreme events in the future, may increase government costs rapidly. Thus insurance policy needs to be designed to impose individual enterprises' own investments to protective measures in compensation of the government's risk sharing.

To my wife and daughter,
Yoonhee Choi and Olivia Jeemin Lee

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KEY TO SYMBOLS AND ABBREVIATIONS

Key to Symbols

$V(\bullet)$	Value function
r	Discount rate
$E\left[\bullet\right]$	Expected value
$E^x\left[g(X(t))\right]$	Expected value of $g(X(t))$ conditional on starting point x
$1_{\{\bullet\}}$	Indicator function
μ	Drift parameter
σ	Volatility parameter
$W(t)$	Standard Brownian motion (Wiener process)
$N(t)$	Poisson process
λ	Poisson intensity
$\sum_{i=1}^{N(t)} Y_i$	Compound Poisson process with jump distribution Y_i
η	Jump magnitude parameter
$\psi(z)$	Laplace exponent of a Lévy process
τ_h	First hitting time of h
$\overline{X}(t)$	Supremum process, i.e. $\sup_{0\le s\le t} X(s)$
$\underline{X}(t)$	Infimum process, i.e. $\inf_{0\le s\le t} X(s)$
$\mathcal{E}g(x)$	Normalized expected present value of a payoff stream $g(X(t))$, i.e. $\mathcal{E}g(x) = rE^x\left[\int_0^\infty e^{-rt} g(X(t))dt\right]$

$\mathcal{E}^+ g(x)$	Normalized expected present value of a payoff stream $g(\overline{X}(t))$, i.e. $\mathcal{E}^+ g(x) = rE^x\left[\int_0^\infty e^{-rt} g(\overline{X}(t))dt\right]$
$\mathcal{E}^- g(x)$	Normalized expected present value of a payoff stream $g(\underline{X}(t))$, i.e. $\mathcal{E}^- g(x) = rE^x\left[\int_0^\infty e^{-rt} g(\underline{X}(t))dt\right]$
$\kappa_r^+(z)$	Normalized expected present value of a payoff stream $e^{z\overline{X}(t)}$
$\kappa_r^-(z)$	Normalized expected present value of a payoff stream $e^{z\underline{X}(t)}$
h^*	Optimal switching boundary
$P\left(\tau_{h^*} \le T\right)$	Probability of switching in a given time span T
JB	Jacque-Bera statistic
SW	Shapiro-Wilk statistic
M	Insurance coverage
$\underline{M}$	Minimum effective insurance coverage
δ	Fixed rate subsidy
D	Fixed amount subsidy

Key to Abbreviations

ADF	Augmented Dickey-Fuller Test
ADP	Ad Hoc Working Group on the Durban Platform for Enhanced Action
BLS	Bureau of Labor Statistics
càdlàg	Right Continuous with Left Limits (continue à droite, limite à gauche)
CAPM	Capital Asset Pricing Model
CBA	Cost-Benefit Analysis

CPI	Consumer Price Index
DCF	Discounted Cash Flow
ENPV	Expected Net Present Value
EPV	Expected Present Value
FD	First Differencing
GCM	Global Climate Model
GHGs	Greenhouse Gases
GLS	Generalized Least Squares
IAM	Integrated assessment model
i.i.d.	Independent Identically Distributed
IPCC	Intergovernmental Panel on Climate Change
MASS	Michigan Agricultural Statistics Service
MLE	Maximum Likelihood Estimation
ODE	Ordinary Differential Equation
OIDE	Ordinary Integro-Differential Equation
PDE	Partial Differential Equation
PIDE	Partial Integro-Differential Equation
RCM	Regional Climate Model
RHS	Right Hand Side
UNFCCC	United Nations Framework Convention on Climate Change
WHF	Wiener-Hopf Factorization

CHAPTER 1: Introduction

The significance of climate change has been recognized well beyond academic debates. As anthropogenic climate change is projected to continue during this century, public concerns about adaptation strategies and potential impacts are growing. At the same time, as on-going international climate agreements such as UNFCCC (United Nations Framework Convention on Climate Change) continue to be debated, it is unlikely that international community can reduce greenhouse gases (GHGs) with deadline of the ADP (Ad Hoc Working Group on the Durban Platform for Enhanced Action). As a result, it has become more widely accepted that adaptation to climate change is inevitable (Ford, Berrang-Ford, and Paterson 2011). Especially, the general public seems to be increasingly aware of more frequent extreme weather events and the need to adapt to them.[1]

Although literature on climate change has become pervasive, formal studies about process of adaptation including decision making, implementation, and timing of adaptive actions are still relatively sparse. Assessments that do exist focus on "adaptation capacity" which refers to the ability or potential of a system to respond successfully to climate variability and change. However, the presence of adaptation capacity alone is not a sufficient condition for design and implementation of effective adaptation strategies (Adger et al. 2007). Coping with climate change impacts such as crop productivity shift and sea level rise, an agent may contemplate changing practices but that does not always result in adaptation actions. To undertake an

[1] A survey by Leiserowitz et al. (2013) reports that 59 percent of Americans respond that weather in the U.S. has been worsening over the past several years and 56 percent of Americans believe that the changing weather pattern is affected by global warming.

adaptation, individuals must consider it to be potentially profitable and demonstrate willingness to adapt as well as capacity (Winkler et al. 2010).

Benefit-cost analysis is often employed in the economic literature evaluating an individual decision maker's willingness to adapt in response to climate change. An expected cumulative sum of net cash flows, discounted back to the present using the opportunity cost of capital (expected net present value or ENPV), determines whether or not to make an investment in adaptation measures. This simplistic ENPV approach does not reflect some important characteristics of climate change and extreme events. First, individuals may be reluctant to make changes because of uncertainty surrounding future climate change and economic impacts. The challenge of evaluating potential future conditions can make individuals hesitant to make an adaptation decision, especially if the decision is not easily reversible. Furthermore, the presence of uncertainty and irreversibility stimulates a potential for learning through delaying adaptation decisions. The effects of uncertainty, irreversibility and learning can be incorporated in the decision-making process by applying a real options approach to adaptation assessments (Zhao 2012; Zilberman, Zhao, and Heiman 2012).

The shortcomings of a traditional ENPV approach are even more apparent when effects of extreme events are considered. ENPV based benefit-cost analysis treats extreme events equivalently to gradual changes since naïve expected values only capture mean tendency. Reactions to extreme events and gradual changes are the same under ENPV framework if both yield the same expected values. Yet climate extremes (extreme weather or climate event) are defined as "the occurrence of a value of weather or climate variable above (or below) a threshold value near the upper (or lower) ends of the range of observed values of the variable" (IPCC 2012: p.5). By definition, occurrences of climate extremes are rare but their effects may be drastic.

There is a fundamental difference between extreme events and gradual climate changes that suggests adaptation response to the extremes could be different from reactions to gradual climate change.

The primary research objective is to examine the role of extreme events in climate change adaptations by i) devising a formal economic model typifying adaptations under extreme events as well as gradual changes; ii) evaluating the model empirically; and iii) extending the model to policy analysis. In the following chapters, adaptations under extreme events and gradual climate change are compared using a real options approach to evaluate the decision making problem of land use switching (Dixit and Pindyck 1994; Peskir and Shiryaev 2006). The classic real options approach is extended by introducing a more general type of stochastic process which allows stochastic jumps to capture extreme events (Boyarchenko 2004; Cai and Kou 2011; Kou 2002).

Chapters are organized in the following manner. A formal economic model, which analyzes effects of extreme events as part of the decision making process and implementation timing of adaptive actions is presented in Chapter 2. A flexible probability density of extreme events magnitude is introduced to examine heavy-tailed phenomena of the extremes (Boyarchenko and Boyarchenko 2011; Cai and Kou 2011; Cai 2009). In Chapter 3 the theoretical framework is applied to decisions about exit from tart cherry production in Northwest Michigan; an industry highly susceptible to extreme weather events such as spring frost (De Melker 2012; Winkler et al. 2010; Zavalloni et al. 2006). The model is reformulated in Chapter 4 to examine alternative government policies managing climate change risks (e.g. fixed payments, fixed rate subsidies, insurance support) to evaluate cost-effectiveness in the presence of extreme events. Finally, Chapter 5 provides a summary and conclusions and suggests potential extensions for further economic research.

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CHAPTER 2: Adaptation and Extreme Events: Land Use Decisions under Uncertainty

2.1 Introduction

One of the greatest challenges posed by global climate change is the increased frequency and severity of extreme weather events, on top of the expected changes in temperature and precipitation. Although it is still extremely difficult to establish robust causal relationships between climate change and individual extreme events, there is growing evidence that human-driven climate change may induce more frequent and more severe extreme events (Rahmstorf and Coumou 2011; Stott, Stone, and Allen 2004; Trenberth 2011, 2012).[2] Thus adaptations to extreme events could be as important as adaptations to gradual changes in weather patterns. While a wide range of studies has examined adaptations to climate change over a decade, studies on adaptation to extreme events are still rather limited (Füssel 2007).

More recent studies examining adaptation process suggest that adaptations may be stimulated by extreme events than changes in average climate conditions (Berrang-Ford, Ford, and Paterson 2011; Füssel 2007). These studies argue that extreme events should be framed as propelling adaptation needs – this is known as the 'pacemaker' effect of extremes. For example, many physical investments are made only after extreme events have occurred (e.g. adoption of

[2] Climate extreme (extreme weather or climate event) is defined as 'the occurrence of a value of a weather or climate variable above (or below) a threshold value near the upper (or lower) ends of the range of observed values of the variable (IPCC 2012: p.5).' Inherently, extremes are by definition rare but drastic events, which make it notoriously challenging to assess observed changes in extremes due to quality and quantity of available data. It is not surprising that it is still under 'low confidence (in IPCC terminology)' for observed changes in a specific extreme on regional or global scale. However, more recent studies tend to show higher possibilities of changes in the extremes. Refer to IPCC (2012) for more details.

irrigation technologies after severe droughts, construction of dikes after major flooding, etc.). While these studies provide conceptual ideas framing adaptation, they do not explicitly include decision making processes that drive the pacemaker effects of extreme events.

There are few formal economic models on the *process* of decision making of physical adaptation and *timing* of adaptation actions in the presence of extreme events (Mechler et al. 2010). Most of the studies treat extreme events equally as gradual changes, both in the sense of expected adaptation costs and benefits. In the framework of traditional expected cost benefit analysis, the damages from such events are multiplied by their probabilities of occurrence, and the benefits of adaptation are calculated as the reduction in such expected damages due to reduced magnitudes of damages and/or reduced probabilities of such damages. For instance, Boyd and Ibarrarán (2008) adopt this approach to study the economic costs of severe drought in Mexico. This framework treats extreme events equally as gradual changes: both are expressed in terms of the expected net present values of their damages in cost and benefit calculations. If both cause the same expected damages, they will also lead to the same incentives of adaptation.

This framework of expected benefit cost analysis is inherently static. However, since the timing, magnitude and the impacts of climate change are uncertain, adapting to climate change is dynamic and should thus be 'proactive.' Hence, without recognizing the role of uncertainty and learning, researches focusing on adaptions to climate change that rely on static frameworks may be misleading (Zilberman, Zhao, and Heiman 2012). We argue that when decision makers are allowed to optimize dynamically and to learn, gradual changes and extreme events can lead to different adaptation incentives as well as different likelihoods of adaptation occurring even when traditional expected net present value (ENPV) calculations are equal.

A real option land conversion model is developed where the economic return from current land use follows a jump diffusion process, which is represented as a mixture of continuous variations and discrete jump variations. In this modeling strategy, gradual climate impacts are represented by a continuous Brownian motion process, while extreme events are represented by a Poisson jump process whose magnitude follows a hyper-exponential distribution. The effects on land use decisions of three different types of climate change consequences are compared: i) gradual unfavorable changes in weather patterns, ii) increased frequency of negative extreme events, and iii) increased magnitude of negative extreme events. The changes are calibrated so that they lead to the same expected losses to the current land use, so that they are equivalent in the traditional cost benefit framework.

2.2 Real option approach for land use switching under climate change

Agricultural adaptation to climate change often involves changes in land uses, such as adopting certain practices such as irrigation, growing different crop varieties, or exiting from agriculture altogether. These land use conversion decisions are characterized by uncertain payoffs and significant sunk costs. These characteristics of land use change under climate change can be well-conceptualized using real options approach (Zilberman, Zhao, and Heiman 2012).[3] The real options approach suggests that there is a value of waiting in the presence of uncertainty, learning and adjustment cost so the agents need to compare today's investment not only with no investment but with investment in the future. Under these circumstances, decision makers often balk at making changes from their status quo (Zilberman, Zhao, and Heiman 2012).

[3] See Dixit and Pindyck 1994 and references therein for more detail.

2.2.1 Theoretical land conversion model

Consider a risk-neutral agent with a parcel of land who faces the choice of continuing with the current land use or converting to an alternative use. Assume that there are two systems, $i \in \{c, a\}$ where *c* and *a* represent *current* and *alternative* land use, respectively. Switching from the current to alternative land use incurs a lump-sum establishment cost denoted by C. Let $\pi_i(t),\ i \in \{c, a\}$ denote the economic return from land use i at time t, and later the stochastic processes governing the movement of $\pi_i(t)$ will be described. The agent chooses the moment at which the agent will convert to the alternative use, and the value function $V(\bullet)$ is expressed through the agent's optimization problem as

$$V(\pi_c, \pi_a) = \sup_{\tau} E\left[\int_0^{\tau} e^{-rt}\pi_c(t)dt + \int_{\tau}^{\infty} e^{-rt}\pi_a(t)dt - e^{-r\tau}C1_{\{\tau<\infty\}}\right] \qquad (2.1)$$

where $E[\cdot]$ and $1_{\{\bullet\}}$ represent the expected value and indicator functions, and r is the discount rate. The indicator function introduces the switching cost only when switching action takes place within a finite time. Descriptively, the agent's decision problem is to find the optimal time τ which maximizes the expected return over infinite horizon. Note that the problem above allows $\tau = \infty$ for which it is optimal to stay in the *current* land use forever. Otherwise, the agent will switch to the *alternative* use since the expected profit will be greater even after accounting for switching cost C.

Since the optimal *switching* problem is naturally equivalent to an optimal *stopping* problem, equation (2.1) can be reduced to an optimal stopping problem:

$$V(\pi_c, \overline{\pi}_a) = E\left[\int_0^{\infty} e^{-rt}\pi_c(t)dt\right] + \sup_{\tau} E\left[\int_{\tau}^{\infty} e^{-rt}\left\{\overline{\pi}_a(t) - \pi_c(t)\right\}dt\right] \qquad (2.2)$$

where $\overline{\pi}_a \equiv \pi_a - rC$ which denotes adjusted return from the alternative after covering amortized switching cost. As indicated in the decision problem above, the switching problem in (2.1) is equivalent to an optimal stopping problem which finds an optimal timing when future cumulative difference in the returns between the current and alternative systems reaches a certain threshold.[4] Intuitively, if $\overline{\pi}_a(t) - \pi_c(t)$ is small, it is optimal to stay in the current use. On the other hand, if $\overline{\pi}_a(t) - \pi_c(t)$ becomes sufficiently large, it becomes optimal to convert to the alternative use.

2.2.2 Stochastic processes

We assume that the economic activity associated with current land use has a stochastic return stream (e.g. crop growing, tourism) which is dependent on climate and/or weather conditions. Although traditional real option modeling, which relies on continuous stochastic processes, provides a fertile ground for a land use switching model, it may not capture extreme events intertwined with climate change. The tail-behavior is not well-identified by standard continuous stochastic processes (e.g. geometric Brownian motion or mean-reverting process). This motivates us to extend the traditional model to incorporate jumps in the underlying stochastic process. More specifically, a mixture process, which has continuous and discrete components, is employed. Let $X(t) \equiv \ln \pi_c(t)$, then the log return stream, which is assumed to be fluctuating due to weather and/or climate conditions, can be defined using a jump diffusion process as

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[4] For each time, ENPV rule tells us that it is optimal to switch if ENPV is positive. For example, it is optimal to switch if $E\left[\int_0^\infty e^{-rt}\left\{\overline{\pi}_a(t) - \pi_c(t)\right\} dt\right] \geq 0$ in equation (2.2) under ENPV rule.

$$X(t) = x + \mu t + \sigma W(t) + \sum_{i=1}^{N(t)} Y_i \tag{2.3}$$

where x denotes a starting point of the process. The second and third terms in the right hand side (RHS) represent the Brownian motion with drift where μ and σ are drift and volatility parameters. $W(t)$ is the standard Brownian motion (Wiener process).[5] Hence, the continuous portion identifies smooth return variations. The last term in the RHS is a compound Poisson process, which captures stochastic extreme events. The number of extreme events is represented by a Poisson process $\{N(t)\}$ with arrival rate λ which captures the frequency of jump. In this setting, the probability of an extreme event is λdt in an infinitesimal time interval and the longer the time periods, the greater the chance of an extreme event occurring. $\{Y_i\}$ is a sequence of independent identically distributed (i.i.d.) random variables of jump magnitude with density $f_Y(y)$. Hence, extreme events occur according to Poisson process and once a jump happens its size is stochastic and determined by the realization of Y_i. We assume that $W(t)$, $N(t)$ and Y_i are mutually independent.[6]

As the tail behavior of the jump size distribution Y_i determines to a large extent the tail behavior of the probability density of the whole diffusion process of $X(t)$ in equation (2.3), distributional features of the impacts of extreme events can be captured by appropriate choice of

[5] Under the modeling strategy, using logarithmic specification has an advantage computationally to directly tacking the return process.

[6] Note that the stochastic process is no longer everywhere continuous since standard Brownian motion is combined with jump process. The process is càdlàg (right continuous with left limits) but increments are independent and stationary, so the process is a version of the Lévy processes. For wide class of the Lévy processes, refer to Sato (1999).

density $f_Y(y)$. Since impacts of extreme events are extensively diverse or highly uncertain across regions and sectors, there is no definite indication of what distribution should be employed in general. Yet the distribution should be able to address heavy-tailed characteristics associated with 'structural' uncertainty of climate change when representing extreme events (Weitzman 2009). Heavy-tail (or fat-tail)[7] distributions may be a natural choice in this vein.

Normal distribution has been commonly used in financial modeling since Merton (1976). Yet its relatively thin tail may not describe the fat-tail features of extreme events associated with climate change. Alternatively, exponential-type distributions such as the double-exponential distribution (Kou 2002) has been proposed more recently to study heavy tailed financial return distributions. Since the double exponential distribution can approximate a fairly flexible and general class of processes, it has been applied in many empirical approaches especially in financial market applications. Boyarchenko (2004) applies the distribution to a real option modeling to extend Gaussian stochastic processes. While exponential jump distribution supports many empirical processes with relatively fatter tail than normal distribution, it still exhibits thinner tail than power-tail distributions (e.g. Pareto or Weibull distributions).

These issues can be addressed by introducing a very flexible distribution. Specifically, in order to represent size of extreme events, spectrally negative hyper-exponential distribution is employed. The density function is given by

[7] Heavy-tail distributions refer to a class of distributions whose moment generating function does not exist such that the tail probability decays more slowly than exponentially. Power-tail (or Pareto-tail) distributions such as Pareto or Weibull distribution are standard examples. On the contrary, normal and exponential are examples of thin-tail distributions. Especially, normal distributions have a thinner tail than exponential counterparts.

$$f_Y(y) = \sum_{i=1}^{n} \alpha_i \eta_i e^{\eta_i y} 1_{\{y<0\}} \tag{2.4}$$

where $\alpha_i > 0$ and $\eta_i > 0$ for all $i = 1, 2, ..., n$, and $\sum_{i=1}^{n} \alpha_i = 1$. Here, every η_i is a jump size parameter so that the smaller η_i is, the larger is the average size of jumps. Positive jumps are ruled out in the distribution to capture only potential adverse effects due to extreme events. The most prominent advantage of using hyper-exponential distribution comes from its denseness. That is, it is possible to approximate many distributions including power-tail distributions as well as exponential-tail distributions arbitrarily closely by hyper-exponential distribution (Cai and Kou 2011; Cai 2009). For instance, it is possible to approximate Pareto or Weibull distributions by hyper-exponential distribution (See Cai and Kou 2011; Feldmann and Whitt 1998 for examples).

In addition to the denseness, employing hyper-exponential distribution has a computational advantage than directly tackling fat-tail distributions. For real options modeling such as the land use switching model discussed here, for example, researchers have to rely on numerical method to derive a solution to the problem with power-tail jump distribution. On the other hand, it is possible to obtain an analytical solution to the problem with hyper-exponential distribution. Moreover, explicit calculation of Laplace transformation to study the first passage time distribution, which will be discussed later, is possible thanks to memory-less property of the distribution.

2.3 Solution strategies

2.3.1 Economic incentive to switch: decision making process

The traditional approach for real option valuation is to employ dynamic programming or contingent claim analysis to derive the corresponding partial differential equations (PDE) or ordinary differential equations (ODE) (Dixit and Pindyck 1994). With a real option approach with jump diffusion process for an underlying stochastic process, the traditional approach yields a partial integro-differential equation (PIDE) or an ordinary integro-differential equation (OIDE), which is usually very hard or impossible to obtain analytical solutions. More recently, alternative approaches, which employ applied probability theory such as the scale function (Kyprianou 2006) or the Wiener-Hopf factorization (WHF henceforth) technique (Boyarchenko and Boyarchenko 2011; Boyarchenko and Levendorskiĭ 2007; Boyarchenko 2004) have been proposed. These methods have an advantage in that they provide clear cut approaches to obtain analytical solutions while arbitrary 'guess-and-verify' technique is necessary in the classical approaches. In addition, they provide interesting economic interpretations in the real option valuations: record-setting 'good' or 'bad' news principle (Boyarchenko 2004). This paper employs the WHF technique to derive optimal threshold indicating economic incentive to switch proposed by Boyarchenko (2004) and Boyarchenko and Levendorskiĭ (2007).

Note that the jump diffusion process introduced in equations (2.3) is a special case of the Lévy process. Thus the equation allows alternative representation using moment generating function.[8] The moment generating function of a process $X(t)$ allows specific form of the process, which is given by $E\left[e^{zX(t)}\right]=\exp\{\psi(z)t\}$ where $\psi(z)$ is an exponent for the function ('Laplace'

[8] The Lévy process is a continuous time analog of random walk, which can be defined as a continuous time process with independently and identically distributed increments. When representing any Lévy processes, Lévy-Khintchine formula indicates every Lévy process can be uniquely defined with a Laplace exponent. Refer to ch.2 of Sato (1999) and ch.3 of Kyprianou (2006) for detail. For the real option modeling purpose in this paper this representation has an advantage than the representation with a stochastic differential equation since it readily provides the expected present value of a process.

exponent). A unique representation of the (log) return process exists using the Laplace exponent as

$$\psi(z) = \frac{1}{2}\sigma^2 z^2 + \mu z - \lambda \sum_{i=1}^{n} \frac{\alpha_i z}{\eta_i + z}. \tag{2.5}$$

Proof By the independence of $W(t)$, $N(t)$ and Y_i,

$$E\left[e^{zX(t)}\right] = E\left[e^{z(\sigma W(t)+\mu t)} \prod_{i=1}^{N(t)} e^{zY_i}\right].$$

Noticing that $\sigma W(t) + \mu t$ is Gaussian and $N(t)$ is Poisson process, respectively, the moment generating function is derived as

$$E\left[e^{zX(t)}\right] = \exp\left[\psi(z)t\right].$$

where

$$\begin{aligned}\psi(z) &= \frac{1}{2}\sigma^2 z^2 + \mu z + \lambda \int_{-\infty}^{0} (e^{zy} - 1) \sum\nolimits_{i=1}^{n} \alpha_i \eta_i e^{\eta_i y} dy \\ &= \frac{1}{2}\sigma^2 z^2 + \mu z - \lambda \sum_{i=1}^{n} \frac{\alpha_i z}{\eta_i + z}.\end{aligned}$$

Using the Laplace exponent above, expected value of the process (2.5) is calculated immediately as $E\left[\int_0^\infty e^{-rt} e^{x+X(t)} dt\right] = \left\{1/\left[r - \psi(1)\right]\right\} e^x$ with starting at x when $r - \psi(1) > 0$.

The optimal timing problem (2.2) is equivalent to find an optimal boundary (or threshold) separating continuation and stopping decisions such that staying or switching should be indifferent at the boundary (i.e. free-boundary problem). Denote h be a candidate of the boundary. Then define a hitting time $\tau_h \equiv \inf\left\{t \geq 0 \middle| X(t) \leq h\right\}$ where $h \in \mathbb{R}$ is a constant, which represent the first time $X(t)$ reaches or crosses h from above. Let $x \in \mathbb{R}$ be an arbitrary starting point of the process $X(t)$. Conditional on the starting point x, an expectation of a general function $g(X(t))$ is given as $E^x\left[g(X(t))\middle|X(0)=x\right]$. For notational brevity, it is useful to set $E^x\left[g(X(t))\middle|X(0)=x\right]=E^x\left[g(X(t))\right]$. Hence, $E^x\left[g(X(t))\right]$ represents the expected value of a function $g(\cdot)$ of the process at time t conditional on the process starting at value x.

In order to obtain an analytical solution of the problem, one drastic form of adaptation is assumed: abandoning current use altogether to obtain a fixed return, (e.g. through selling the land for real estate development). This adaptation measure can be considered as long-run adjustment option. Also, as this simplifying assumption enables us to obtain analytic solutions for the switching problem described in (2.2), it allow us to compare three different types of climate change consequences, which will be proposed later more concretely and systematically.

Under the constant alternative return assumption, equation (2.2) can be rewritten using the expected value (EPV) operator defined above as the EPV of switching $\hat{V}(\cdot)$ as a function of x and h:

$$\hat{V}(x;h)=E^x\left[\int_0^\infty e^{-rt}e^{X(t)}dt\right]+E^x\left[\int_{\tau_h}^\infty e^{-rt}\left\{rS-e^{X(t)}\right\}dt\right]. \qquad (2.6)$$

This $\hat{V}(\cdot)$ calculates the EPV of switching in equation given in (2.2) from when the log return $X(t)$ reaches or crosses an arbitrary threshold level h. Recall that $e^{X(t)}$ is the period t return of the current land use while rS is the (constant) per period return from the alternative use. The first term in the RHS is independent of h, which represents the EPV of staying in the current land use forever. The second the in the RHS, in turn, captures additional EPV attainable by switching. Note that $\hat{V}(\cdot)$ is not a value function, i.e. $\hat{V}(\cdot)\neq V(\cdot)$ since $\hat{V}(\cdot)$ varies by the choice of h, which may not necessarily be a maximizing argument. Thus the solution to the problem above is to find an optimal boundary h^* which maximize $\hat{V}(\cdot)$. Then it satisfies $\hat{V}^*(\cdot)\left(\equiv\hat{V}(x;h^*)\right)=V(\cdot)$.

In order to find the optimal boundary h^*, we define the normalized expected present value operator $\mathcal{E}$ which calculates the normalized EPV of a payoff stream $g(X(t))$:

$$\mathcal{E}g(x) = rE^x\left[\int_0^\infty e^{-rt} g(X(t))dt\right]. \tag{2.7}$$

We introduce the two extremum processes: the supremum process $\overline{X}(t) = \sup_{0\le s\le t} X(s)$ and the infimum process $\underline{X}(t) = \inf_{0\le s\le t} X(s)$ so that $\overline{X}(t)$ and $\underline{X}(t)$ evaluate running maxima and minima of the process $X(t)$ at time *t,* respectively. Accordingly, the (normalized) EPV-operators of a function of the supremum and infimum process can be defined:

$$\mathcal{E}^+g(x) = rE^x\left[\int_0^\infty e^{-rt} g(\overline{X}(t))dt\right] \tag{2.8}$$

$$\mathcal{E}^-g(x) = rE^x\left[\int_0^\infty e^{-rt} g(\underline{X}(t))dt\right]. \tag{2.9}$$

As equations (2.8) and (2.9) calculate expected value of $g(\cdot)$ based on running maxima and minima of the process, $\mathcal{E}^+g$ and $\mathcal{E}^-g$ may be interpreted as EPVs under best and worst scenarios if $g(\cdot)$ is non-decreasing with respect to x, respectively. Similarly, when $g(\cdot)$ is non-increasing with respect to x, the EPVs $\mathcal{E}^+g$ and $\mathcal{E}^-g$ can viewed as worst and best scenarios, respectively.

Based on these definitions in (2.7) - (2.9), a version of the WHF formula[9] states that

$$\mathcal{E}g = \mathcal{E}^+\mathcal{E}^-g = \mathcal{E}^-\mathcal{E}^+g\ . \tag{2.10}$$

This implies that the normalized EPV operator can be split into multiplication of the extremum operators. From WHF above, it is possible to deduce that the EPV in (2.6) can be rewritten in

[9] For proof and other types of representations about the WHF, refer to Boyarchenko and Levendorskiĭ (2007), ch.11 and Sato (1999), ch.9.

an equivalent form using the operators defined in (2.7)-(2.9). Specifically, the problem (2.6) is to find an optimal threshold to swap a return stream e^x to a fixed stream rS. The problem in (2.6) is represented by the WHF as following proposition.

Proposition 2.1 (EPV representation of switching problem by the WHF) *Assume that $r-\psi(1)>0$ such that the value of EPV converges where r is a discount rate and $\psi(z)$ is the Laplace exponent of the process $X(t)$ defined in* (2.5). *It can be readily checked that return stream e^x is measurable function satisfying the boundedness condition such that for some constant C*

$$\left|e^x\right| \le C\left(e^{\sigma^- x}+e^{\sigma^+ x}\right)$$

where $\sigma^- < 0 < \sigma^+$. Since $rS-e^x$ is non-increasing with respect to x which changes sign by the values of x, then the EPV in (2.6) *can be written as*

$$\hat{V}(x;h)=r^{-1}\mathcal{E}e^x + r^{-1}\mathcal{E}^- 1_{(-\infty,h]}\mathcal{E}^+(rS-e^x). \tag{2.11}$$

Proof Boyarchenko and Levendorskiĭ 2007: p.221.

As noted previously, the first term in the RHS of (2.11) is fixed regardless of h so is redundant to derive the optimal switching threshold h^*. In the second term of the RHS, since $rS-e^x$ is non-increasing and changes sign with respect to x and $\mathcal{E}^+$ is an expectation operator, $\mathcal{E}^+(rS-e^x)$ is also non-increasing and changes sign by the values of x. When starting point x [10] is sufficiently large such that $\mathcal{E}^+(rS-e^x)<0$, switching will add up some negative values to the

[10] The starting point is also a realized value which is observable to the agent. That is, whenever the agent observes a realized value, it becomes a new starting point to calculate the EPVs.

overall EPV in (2.11). If the agent switched at this situation, it should have been too early to attain enough gain from switching by giving up profit from current use. On the contrary, when x is sufficiently small such that $\mathcal{E}^{+}(rS-e^{x})>0$, it means the agent has already lost some positive gains from switching by operating under too low returns from current use. It should have been too late to switch if the agent switched at this position. Therefore, the optimality condition which maximizes the EPV in (2.11) should be $\mathcal{E}^{+}(rS-e^{h^{*}})=0$ where h^{*} denotes the optimal switching boundary.

These intuitions show some critical difference between real option and ENPV approach. Equation (2.11) can be rewritten to show the ENPV decision rule as

$$\hat{V}(x;h)=r^{-1}\mathcal{E}e^{x}+r^{-1}1_{(-\infty,h]}\mathcal{E}(rS-e^{x}). \tag{2.12}$$

Let h' be an optimal threshold chosen by the ENPV rule. The ENPV rule is to select the optimal threshold h' when $\mathcal{E}(rS-e^{x})$, which is a standard expected present value, becomes nonzero. Simply NPV approach is to evaluate a sign of $\mathcal{E}(rS-e^{x})$ in a given time, which is the standard expected present value. By the ENPV decision rule, it is optimal to switch whenever $\mathcal{E}(rS-e^{x})\geq 0$. Hence, the optimality condition under the NPV rule is given by $\mathcal{E}(rS-e^{h'})=0$. Naturally, it only considers mean of the switching benefits. On the contrary, the real option modeling takes more conservative approach as it puts more weights on the polar movements of the switching benefits by evaluating the expected present value evaluated under the supremum operator $\mathcal{E}^{+}$.

Since $rS-e^{x}$ is non-increasing with respect to x, the $\mathcal{E}^{+}(rS-e^{x})$ considers worst case than central tendency and thus disregards all temporary drops in x which seems to be profitable in terms of ENPV approach. The real option approach indicates optimal timing to switch to be the first time when $\mathcal{E}^{+}(rS - e^{x}) \geq 0$. The real option rule indicates that the agent needs to disregard positions even with positive ENPVs. That is, even if $\mathcal{E}(rS - e^{x}) \geq 0$, the agent needs to wait more as long as $\mathcal{E}^{+}(rS - e^{x}) < 0$.[11] The intuition behind this real option decision rule is that even if the realized value x becomes small which seems to be optimal under the ENPV rule, this realization could be just a temporary drop. Yet the ENPV rule, which focuses on the mean, ignores possibility of going back up.

This provides some interesting economic interpretations about the real option approach. In presence of *uncertainty*, positive ENPV values may reflect temporary increases in gain from switching but the *irreversibility* will not allow returning to the previous land use when circumstances around the use take a favorable turn. Furthermore, there might be *learning* effects as the agent can collect more information from future outcomes by waiting. Under these circumstances, it may be too optimistic to rely only on central tendency by the ENPV rule. Rather, it is optimal to employ more conservative approach based on extreme positions.

Finally, optimal decision rule under the real options framework can be explored further. At the optimal switching boundary h^{*}, the optimality condition can be evaluated as

$$\begin{aligned} \mathcal{E}^{+}(rS - e^{h^{*}}) &= rE^{h^{*}}\left[\int_{0}^{\infty} e^{-rt}(rS - e^{\bar{X}(t)})dt\right] \\ &= rS - rE\left[\int_{0}^{\infty} e^{-rt}e^{\bar{X}(t)}dt\right]e^{h^{*}} = 0. \end{aligned} \tag{2.13}$$

[11] The optimal switching threshold is derived under the supremum operator. There are other cases in which we need to derive the threshold under the infimum operator. If the switching problem is examined as a stopping problem which gives up the current system, optimal switching threshold may be derived under the infimum operator.

For the problem, an analytical solution for the optimal switching threshold can be derived as noted previously. Define a version of EPV as

$$\kappa_r^+(z) = rE\left[\int_0^\infty e^{-rt} e^{z\bar{X}(t)} dt\right]. \tag{2.14}$$

Then it is easy to find that $\mathcal{E}^+ e^{zx} = \kappa_r^+(z) e^{zx}$. Then the optimality condition in (2.13) can be expressed as

$$\mathcal{E}^+(rS - e^{h^*}) = rS - \kappa_r^+(1) e^{h^*} = 0. \tag{2.15}$$

In order to find an analytical solution of the condition given by (2.15), the explicit form of $\kappa_r^+(1)$ is needed. For the (log) return process defined in (2.5), it is well-known that there are explicit expressions for $\kappa_r^+(z)$ (e.g. See Boyarchenko and Boyarchenko 2011). Define the characteristic equation for the process as $r - \psi(z) = 0$ where r is a discounting factor and $\psi(z)$ is a Laplace exponent given by (2.5), and then the equation has only one positive root. Denoting the root as β^+, $\kappa_r^+(z)$ has an explicit expression as simple fractions of β^+:

$$\kappa_r^+(z) = \frac{\beta^+}{\beta^+ - z}. \tag{2.16}$$

This explicit expression allows us to find explicit form of the optimal switching boundary, which can be summarized as following.

Optimal switching threshold: *From the optimality condition in* (2.15), *the optimal switching boundary is given by*

$$e^{h^*} = \frac{rS}{\kappa_r^+(1)} \Leftrightarrow h^* = \ln\left(\frac{\beta^+ - 1}{\beta^+} rS\right). \tag{2.17}$$

2.3.2 Probability of switching in a given period: realized actions

While the optimal threshold to switch, which was derived above, gives an insight about dynamically optimal decision rule in which a rational agent would take, it does not informative on the realized actions under the thresholds. In order to examine the realized actions, the distribution of the first passage time $\tau_{h^*} \equiv \inf \left\{ t \geq 0 \middle| X(t) \leq h^* \right\}$ is studied since it provides information to study the probability of switching in a given period T:

$$P\left(\tau_{h^*} \leq T\right) = P\left(\min_{0 \leq t \leq T} X(t) \leq h^*\right) \tag{2.18}$$

where T is a predetermined time period and h^* is an optimal switching threshold derived above, respectively. When a process is simply a variant of the Brownian motion without a jump component, an explicit expression for the probability can be obtained using a change of measure (Girsanov theorem) and the reflection principle (e.g. Sarkar 2000; Peskir and Shiryaev 2006: ch.2). However, if a process has a jump component, it is hard to study the distribution of the first passage time due to 'undershoot'[12] problem. Specifically, in order to apply the reflection principle to $X(t)$, information about correlation between the undershoot and the terminal value $X(T)$ is required yet this is not available for jump diffusions in general (Kou and Wang 2003). Alternatively, explicit form of the Laplace transformation of the first passage time can be obtained for the jump diffusion $X(t)$ due to the memory-less property in hyper-exponential distribution. The following proposition summarizes the Laplace transformation of the first passage time.

[12] When there is a jump component, the jump diffusion may crosses a boundary by either 'hitting' the boundary exactly ($X(\tau_{h^*}) = h^*$) or 'undershoot' below the boundary ($h^* - X(\tau_{h^*}) > 0$). For detailed discussion about the undershoot (or overshoot) problem, refer to Cai (2009) and Kou and Wang (2003).

Proposition 2.2 (Laplace transformation of first passage time) *Let* x *be the starting point of the diffusion* $X(t)$. *Without loss of generality, it is possible to rearrange* η_i *'s such that* $-\eta_n < -\eta_{n-1} < \cdots < \eta_1 < 0$. *For any* $\alpha \in (0,\infty)$, *the equation* $\psi(x) = \alpha$ *where* $\psi(\cdot)$ *is the Laplace exponent of the process* $X(t)$ *has only* $n+1$ *negative roots* γ_i *'s such that*

$$\gamma_{n+1} < -\eta_n < \gamma_n < \cdots < -\eta_1 < \gamma_1 < 0 .$$

Then the Laplace transformation of the first passage time τ_{h^*} *is obtained as*

$$E\left[e^{-\alpha\tau_{h^*}}\right] = \begin{cases} 1 & \text{if } x \le h^* \\ \sum_{i=1}^{n+1} w_i e^{\gamma_i x} & \text{if } x > h^* \end{cases}. \tag{2.19}$$

Define $W \equiv (w_1, w_2, \ldots, w_{n+1})'$. *Then weighting values* w_i *'s are uniquely determined by solving the linear system* $AHW = J$ *where* A *is a* $(n+1)\times(n+1)$ *nonsingular matrix*

$$A = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \frac{\eta_1}{\eta_1+\gamma_1} & \frac{\eta_1}{\eta_1+\gamma_2} & \cdots & \frac{\eta_1}{\eta_1+\gamma_{n+1}} \\ \frac{\eta_2}{\eta_2+\gamma_1} & \frac{\eta_2}{\eta_2+\gamma_2} & \cdots & \frac{\eta_2}{\eta_2+\gamma_{n+1}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\eta_n}{\eta_n+\gamma_1} & \frac{\eta_n}{\eta_n+\gamma_2} & \cdots & \frac{\eta_n}{\eta_n+\gamma_{n+1}} \end{pmatrix},$$

$H = \mathrm{Diag}[\mathrm{e}^{\gamma_1 h^*}, \mathrm{e}^{\gamma_2 h^*}, \ldots, \mathrm{e}^{\gamma_{n+1} h^*}]$ *is a* $(n+1)\times(n+1)$ *diagonal matrix, and* $J = (1,1,\ldots,1)'$ *is a* $(n+1)\times 1$ *vector of ones.*

Proof Appendix 2.A

As noted above, while explicit formula for the Laplace transformation is derived, it is impossible to proceed further to derive explicit form of the probability of switching since the dependence structure between undershoots and the terminal condition is not available explicitly. Hence, numerical Laplace inversion technique is required to tackle the problem. It is obvious that there is a following baseline relationship for the inversion:

$$\int_0^\infty e^{-\alpha T} P\left(\tau_{h^*} \le T\right) dT = \frac{1}{\alpha}\int_0^\infty e^{-\alpha T} dP\left(\tau_{h^*} \le T\right) = \frac{1}{\alpha} E\left[e^{-\alpha \tau_{h^*}}\right]. \tag{2.20}$$

Let $\hat{P}(\alpha) = \int_0^\infty e^{-\alpha T} P\left(\tau_{h^*} \le T\right) dT$, the function $P(\cdot)$ is approximated via the numerical Laplace inversion technique given explicit form of the Laplace transformation in (2.19). For the numerical inversion, Gaver-Stehfest algorithm is employed. Since this method requires sampling of the Laplace function only on the real line, the approach to find the Laplace transformation in (2.19), especially roots finding procedure, is readily translated to the algorithm. Besides the convenience, the algorithm is known to be simple but stable with high precision computation (Kou and Wang 2003; Abate and Whitt 2006). Especially, this method turns out to be very accurate for functions of a negative exponential type, i.e. $e^{-\alpha t}$ (Hassanzadeh and Pooladi-Darvish 2007), which perfectly matches with the Laplace transformation in (2.19).

Implementation of the algorithm can be summarized as follows. Let $\tilde{f}$ be a Laplace transform of f. Then for a function $f(\cdot)$, which is bounded and real-valued on $[0,\infty)$, it is approximated as

$$f(t) \approx \frac{\ln(2)}{t}\sum_{k=1}^{2M} \omega_k \tilde{f}\left(\frac{k\ln(2)}{t}\right) \tag{2.21}$$

where

$$\omega_k = (-1)^{M+k} \sum_{j=\lfloor (k+1)/2 \rfloor}^{\min(k,M)} \frac{j^{M+1}}{M!}\binom{M}{j}\binom{2j}{j}\binom{j}{k-j}$$

with $\lfloor x \rfloor$ being the greatest integer less than or equal to x.

2.3.3 Comparing climate change consequences

As noted in the introduction section, three different types of climate change consequences are considered: gradual unfavorable changes in weather patterns, increased frequency of extreme events, and increased magnitude of extreme changes. Although these changes may not occur independently in real world, this classification enables us to extract insights concretely by comparing them in terms of an adaptation perspective.

Assume that the three climate change consequences affect return process defined in (2.5). The gradual unfavorable change indicates that the current land use becomes less favorable smoothly over time due to changing climate conditions so that it is realized with a change in the drift term. Let μ_m be a changed drift term considering gradual climate change effect. The gradual change is captured by $\Delta\mu = \mu_m - \mu$ where $\mu > \mu_m$. The extreme frequency change indicates that extreme events such as heat waves, drought or flooding occur more frequently. This change can be identified through change in the Poisson parameter by $\Delta\lambda = \lambda_f - \lambda$ where λ_f is changed frequency parameter and $\lambda_f > \lambda$.

Finally, change in the magnitude of the extreme indicates that the effect becomes severer on average once an extreme event happens. The change is captured in the change of jump size parameters. Note that although changes in average jump size can be obtained by adjusting any one or some of η_i's, that would distort original distribution arbitrarily. Hence, when adjusting jump parameters to capture magnitude change of extremes, proportions between each parameter must be maintained. Specifically, following relationship must hold. Let $\eta_{i,s}, i = 1, 2, ..., n$ be the changed magnitude parameters. For all i where $i \neq k$,

$$\frac{\Delta\eta_i}{\eta_{i,s}}=\frac{\Delta\eta_k}{\eta_{k,s}}\Leftrightarrow\frac{\Delta\eta_i}{\eta_i}=\frac{\Delta\eta_k}{\eta_k}\Leftrightarrow\frac{\eta_i}{\eta_{i,s}}=\frac{\eta_k}{\eta_{k,s}}. \tag{2.22}$$

Under the condition (2.22), define change in average jump size change such that for all i, $\Delta\eta_i=\eta_{i,s}-\eta_i$ where $\eta_i>\eta_{i,s}$.

As these consequences are not directly comparable, their effects are not readily compared in terms of economic decision making process and timing of adaptive actions. Hence, a comparable basis should be established for these changes. Let $i\in\{m,f,s\}$ to denote gradual, frequency and magnitude (size) change, respectively and these changes are translated into changes in the (log) return process $X_i(t)$ $i\in\{m,f,s\}$ for all i as

$$\psi_m(z)=\frac{1}{2}\sigma^2z^2+\mu_m z-\lambda\sum_{i=1}^{n}\frac{\alpha_i z}{\eta_i+z}, \tag{2.23}$$

$$\psi_f(z)=\frac{1}{2}\sigma^2z^2+\mu z-\lambda_f\sum_{i=1}^{n}\frac{\alpha_i z}{\eta_i+z}, \tag{2.24}$$

$$\psi_s(z)=\frac{1}{2}\sigma^2z^2+\mu z-\lambda\sum_{i=1}^{n}\frac{\alpha_i z}{\eta_{i,s}+z}. \tag{2.25}$$

where $\psi_i(z)$, $i\in\{m,f,s\}$ are corresponding Laplace exponents.

These thee consequences are calibrated to lead to same amount of expected losses each other. Let $\Delta EPV_i, i\in\{m,f,s\}$ to be corresponding expected loss from each consequence. The expected present value the process $X_i(t)$ with starting point x can be evaluated easily using the Laplace exponents as $EPV=E^x\left[\int_0^\infty e^{-rt}e^{X_i(t)}dt\right]=\frac{1}{r-\psi_i(1)}e^x$. Then the expected losses due to each change can be written as:

i) gradual unfavorable change: $\Delta EPV_m = \left[\frac{1}{r-\psi(1)} - \frac{1}{r-\psi_m(1)} \right] e^x$

ii) extreme frequency change: $\Delta EPV_f = \left[\frac{1}{r-\psi(1)} - \frac{1}{r-\psi_f(1)} \right] e^x$

iii) extreme magnitude change: $\Delta EPV_s = \left[\frac{1}{r-\psi(1)} - \frac{1}{r-\psi_s(1)} \right] e^x$.

By calibrating the changes such that $\Delta EPV_m = \Delta EPV_f = \Delta EPV_s$, expected loss condition is summarized by the following proposition.

Proposition 2.3 (Equivalent expected loss condition) *The changes in return process* $\pi(t)$ *under three climate change consequences exhibit same expected loss in terms of return if and only if*

$$\begin{aligned} \Delta\mu &= -\left(\sum_{i=1}^{n} \frac{\alpha_i}{\eta_i + 1} \right) \Delta\lambda, \\ \Delta\lambda &= -\left(\sum_{i=1}^{n} \frac{\alpha_i}{\eta_i + 1} \right)^{-1} \frac{\lambda}{\eta_{k,s}} \left(\sum_{i=1}^{n} \frac{\alpha_i \eta_{i,s}}{(\eta_{i,s}+1)(\eta_i+1)} \right) \Delta\eta_k, \end{aligned} \tag{2.26}$$

for an arbitrary k.

Proof Appendix 2.B

In response to these three consequences, the agent will change optimal switching boundary given by (2.17) from status quo accordingly. The agent will have higher incentive to switch for the climate change consequences as these changes imply negative impacts on current use. Since each change should exhibit same EPV under the Proposition 2.3, it is noteworthy that three consequences are equivalent in terms of the ENPV framework under the condition. That is, an agent who obeys ENPV decision rule described in (2.12) will impose same optimal switching boundary for all three consequences. Note that the optimal switching boundary represents economic incentive to switch. Thus the ENPV decision rule suggests same economic incentive to switch will be same regardless of the climate consequences.

On the other hand, under the Proposition 2.3, real option decision making exhibits significantly different economic incentive to switch. Let h^* optimal level derived in (2.17) describing status quo threshold to switch. This level indicates that the agent will opt for switching once the realized value (starting point) becomes smaller than the boundary. For the three climate change consequences, changes in the boundary exhibit clear ranking across the three consequences. The agent will have greater incentive to switch (higher threshold) under the gradual change than the extreme event cases. Between two extreme event cases, the agent will have greater incentive to switch under the frequency change than the magnitude change. This result is summarized as the following proposition.

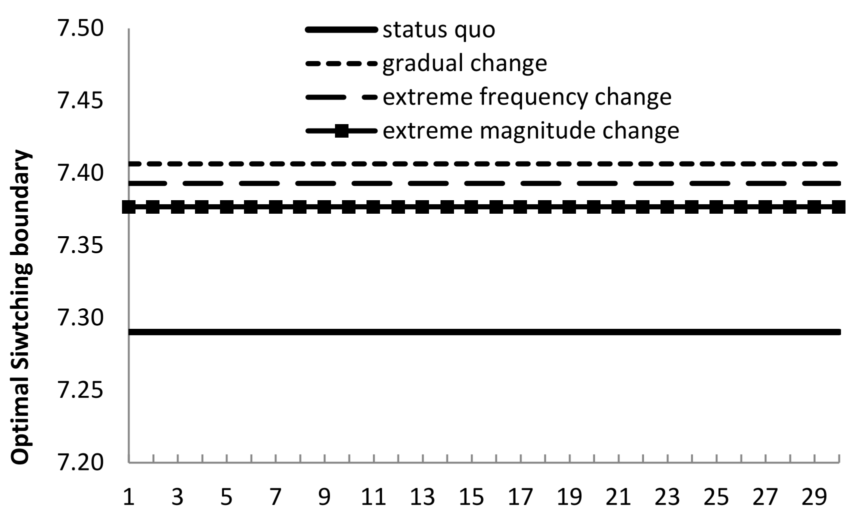
Proposition 2.4 (Economic incentive to switch under climate change) *Assume that Proposition 2.3 holds so that three climate change scenarios have same expected losses. Let $\Delta h_i^*,\ \ i \in \{m, f, s\}$ denote changes of optimal switching boundary from status quo under gradual, frequency and magnitude changes, respectively. Then changes in optimal switching boundary are ranked as*

$$\Delta h_m^* > \Delta h_f^* > \Delta h_s^* . \tag{2.27}$$

Proof Appendix 2.C

Let $h_i^*,\ \ i \in \{m, f, s\}$ denote new optimal boundaries to switch for each consequence under Proposition 2.3. Then from Proposition 2.4, $h_m^* > h_f^* > h_s^*$. Hence, the agent will impose higher level as an optimal switching boundary under the gradual change than the extreme events. Note that starting point x must be located above the boundary before switching such that higher boundary is closer to the starting point. Therefore, higher boundary indicates higher economic incentive to switch. Following figure depicts numerical examples describing changes in optimal switching boundary. In order to derive the figure, 50 percent in frequency of extreme event ($\lambda_f = 0.075$) is assumed then calibrated parameters of gradual (μ_m) and magnitude change ($\eta_{1,s}, \eta_{2,s}$) according to the Proposition 2.3. The parameter values are provided in the note of the figure. As summarized in Proposition 2.4, the changes in optimal switching boundary exhibit highest (least stringent) under gradual change and lowest (most stringent) under magnitude changes of extreme events.

Figure 2.1 Changes in optimal switching boundary



Note: Two-parameter hyper-exponential distribution is used for the numerical example. Parameter values are as follows: For status quo, $\mu = 0.02$, $\sigma = 0.3$, $\lambda = 0.05$, $\eta_1 = 2$, and $\eta_2 = 3$. For gradual change, $\mu_m = 0.0127$. For frequency change of extremes, $\lambda_f = 0.075$. For magnitude change of extremes, $\eta_{1,s} = 1.0525$, and $\eta_2 = 1.5787$. Other parameters are same as status quo and expected loss are same for all cases.

Intuitively, the gradual change, which captured in the deterministic portion, is little to do with uncertainty and learning. It is obvious that there is little value of waiting associated with the uncertainty and learning, which induce higher incentive to switch than the extreme events. This is consistent with the stylized facts in the real world, and reflects the higher incentive to wait and see under learning patterns represented by a jump process. Between jump frequency and magnitude changes, it is noteworthy that jump size is more closely related to the tail-behavior of given stochastic process. As such, this implies the jump size encompasses higher uncertainty and learning, which give higher incentive to wait and see.

While the optimal switching thresholds exhibit clear ranking according to different climate change scenarios, it does not necessarily imply that realized action may end up with the same hierarchy. In order to examine the realized action, the first passage time problem is examined as proposed in the previous section. The probability of switching in a given period captures timing actions given new boundaries under the consequences.

While next Proposition 2.5 shows systematic ranking about probability of switching under climate change, it cannot be proved analytically as it is impossible to find explicit expressions for the probabilities. The numerical method is employed as proposed in the previous section. Also, sensitivity analysis is conducted for ±10 percent range of all parameters from provided values. This ensures robustness of ranking of probability of switching depicted in Figure 2.2 and Figure 2.3. The results show that other parameters have little effects on the ranking of the probability but starting point x has an implication to the ranking of probability of switch. This implies that current exposure and vulnerability to climate change may have strong implications to determine adaptation actions. The numerical result can be found in Figure 2.2 and

Figure 2.3. Following proposition summarize the numerical results of probability of switching for every scenario.

Proposition 2.5 (Probability of switching in a given period under climate change) *From Proposition 2.4, it is possible to deduce that* $h_m^* > h_f^* > h_s^*$. *Let* $P_i(T) \equiv P(\tau_{h_i^*} \leq T)$ *, for* $i \in \{m, f, s\}$ *denote probability of switching in a given period* $T > 0$ *for each process* $X_i(t)$ *as defined in (2.23) - (2.25), respectively. Then following two results are obtained.*

i) *If starting point* x *is sufficiently distant from the optimal switching boundary,*

$$P_m(T) < P_f(T) < P_s(T) \;\; for \;\; all \;\; T > 0. \tag{2.28}$$

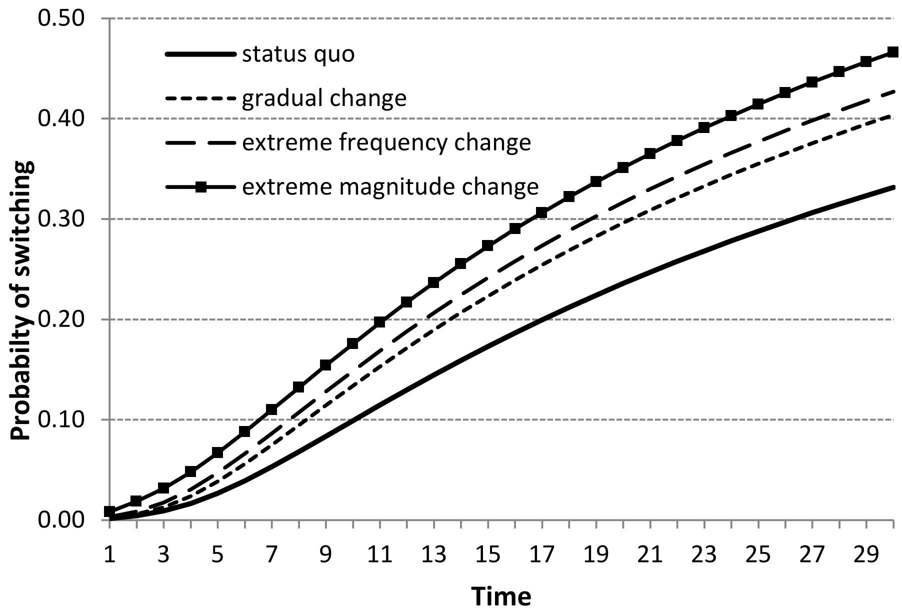
ii) *If starting point* x *is sufficiently close from the optimal switching boundary,*

$$P_m(T) > P_f(T) > P_s(T) \;\; for \;\; all \;\; T > 0. \tag{2.29}$$

Note that real option decision rule stated in Proposition 2.4 imposes more stringent decision boundary to extreme event cases than the gradual change. First, Proposition 2.5 above implies it is more likely to switch under extreme events despite the lower switching incentive when starting point is sufficiently far from the boundary (Figure 2.2).

This provides an insight about the decision making in the real world. Though the real option theory stresses to disregard temporary or sudden drops in the decision making, the realized action may be dominated by the extreme events. This finding is consistent with another stylized fact: historically most adaptation occurred in response to risks of extreme weather events than the mean changes (‘pacemaker’ effect of extreme events), e.g. in adopting irrigation technologies (Ford, Berrang-Ford, and Paterson 2011; Füssel 2007; Negri, Gollehon, and Aillery 2005).

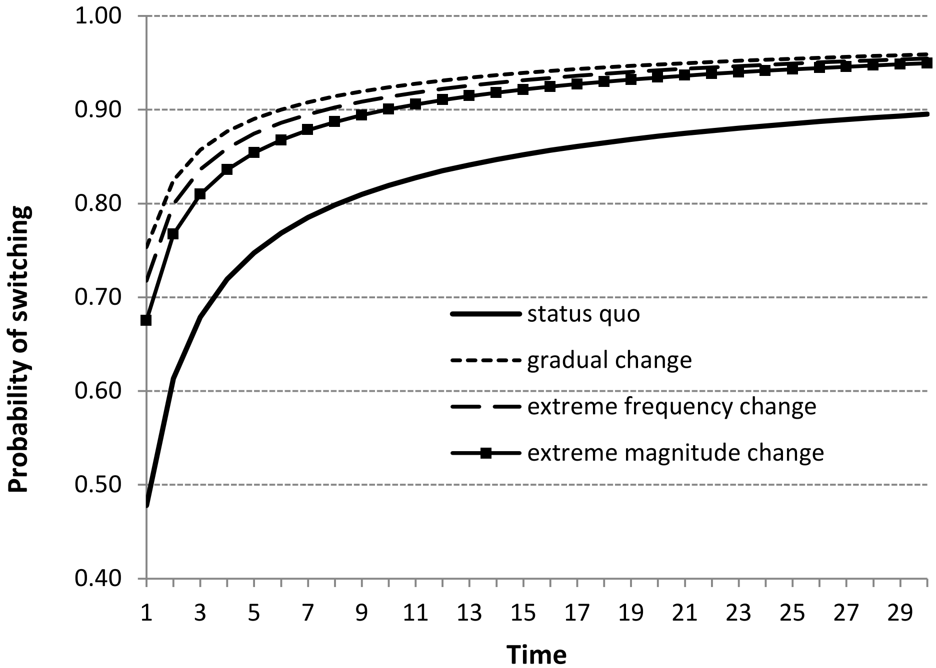
Figure 2.2 Probability of switching (distant from starting point)



Note: Starting point is set as $x = 9$. Other parameter values are same as parameters used to derive Figure 2.1.

On the contrary, the numerical results also suggest that extreme events do not dominate actions always. If starting point is close to switching boundary, gradual change will be more likely to induce actions (Figure 2.3). The starting point which is close to the switching boundary implies current economic activity is already not so favorable. In terms of climate change, this represents regions and/or sectors which have been exposed to climate change and thus are highly vulnerable. In the case, gradual change will stimulate adaptation actions as even small change may be a strong strike to the sectors or regions.

Figure 2.3 Probability of switching (far from starting point)



Note: Starting point is set as $x = 7.5$. Other parameter values are same as parameters used to derive Figure 2.1.

Furthermore, the numerical result implies that the contrast between the switching incentive and the switching probability varies with the time period considered. Extreme events become much more significant in the long run where the starting point is far from the switch boundary. Even when gradual changes dominate actions where starting point is close to boundary, the gap between the gradual and extreme events shrinks. Also, there is a difference between frequency change and magnitude change of extremes. Intuitively, the frequency increase implies that if the time horizon increases, it is highly likely to face at least one occurrence of extreme event. But the size of extreme event may not be sufficiently large to pass the predetermined boundary. In the case of the size change, in contrast, if the extreme event comes with larger size on average, one drastic extreme event may easily undershoot across the boundary even if the decision rule is set to be very stringent. This pattern signifies the importance of tail-behavior in affecting decisions in the real world. That is, the pacemaker effect of extreme events may be present even in dynamically consistent decision making processes.

The last findings have important implications for adaptation decisions in the real world: while gradual changes might play a significant role in short-run adaptation decisions, in the long run and all else equal, it is the extreme events that will play a more significant role. The dynamic decision making model proposed here shows significantly different decision rules than the traditional ENPV framework. As climate vulnerability is inherently dynamic, correctly specifying the dynamic nature in the adaptation policies would be critical. Also there might be clear disparity between economic incentives and realized actions. As extreme events may induce more actions in the long run, the finding stresses the importance of these events for the design and implementation of long term adaptation policies.

2.4 Conclusion

This chapter show that there is disparity between the *ex ante* incentive to adapt and the likelihood of taking adaptation actions in presence of extreme events. In the decision making process, under the dynamically optimal framework, a rational agent's incentive to adapt is driven more by gradual changes than by extreme events as gradual changes have little uncertainty and more predictable than rare extremes. However, in terms of the timing of actions, extreme events are more likely to stimulate adaptation actions even with more stringent switching boundary. The timing of adaptive actions depends on the distance between the starting point that captures the current state and the adaptation threshold. Gradual changes may stimulate adaptive actions in sectors and/or regions which are highly vulnerable to climate change currently.

This study suggests some areas of extensions. First, this study employs the geometric jump diffusion process as an underlying stochastic process representing the effects of climate change. Although a highly flexible jump distribution is assumed, other jump diffusion models may be worth exploring. For example, arithmetic jump diffusion model or mean reverting jump diffusion could be possible other representations of effects of climate change.

Empirical applications based on the theoretical model will be extremely useful. The application requires this model to be combined with real world data and climate scenarios (e.g. GCMs). For example, adaptations in a sector or region are examined and adaptive actions can be predicted by climate change scenarios (e.g. crop switching in agriculture in presence of extreme events). Especially, this model can contribute to integrated assessment models (IAM) by providing information about decision process and timing of actions.

APPENDICES

Appendix 2.A: Proof of Proposition 2.2

Let x be the starting point of the jump diffusion $X(t)$. For any $\alpha \in (0,\infty)$, Let $\psi(\cdot)$ be the Laplace exponent of the process $X(t)$ defined by (2.5). Then the equation $\psi(x)=\alpha$ has $n+1$ negative roots γ_i 's and one positive root γ^+ such that

$$\gamma_{n+1} < -\eta_n < \gamma_n < \cdots < -\eta_1 < \gamma_1 < 0 < \gamma^+ .$$

Let $u(x)$ be the bounded solution of $(L-\alpha)u(x)=0$ for all $x > h^*$ where L is an infinitesimal generator of jump diffusion $X(t)$ and $u(x)=1$ for all $x \le h^*$. By the Theorem 3.2 of Cai and Kou (2011), for $x > h^*$, $u(x)$ will have the form

$$u(x) = Ae^{\gamma^+ x} + \sum_{j=1}^{n+1} w_j e^{\gamma_j x} ,$$

where A and $w_i, i=1,2...n+1$ are undetermined coefficients. For $u(x)$ to be bounded near ∞, it is obvious that $A=0$. Using the continuity condition $u(h^*+)=1$, we have

$$\sum_{j=1}^{n+1} w_j e^{\gamma_j h^*} = 1. \tag{2.30}$$

Expanding $(L-\alpha)u(x)=0$ yields

$$\frac{1}{2}\sigma^2 u''(x) + \mu u'(x) - (\lambda+\alpha)u(x) + \int_{-\infty}^{0} u(x+y) f_y(y)dy = 0. \tag{2.31}$$

Note that the integration in (2.31) can be evaluated as

$$\begin{aligned}&\int_{-\infty}^{0} u(x+y) f_y(y)dy \\ &= \int_{-\infty}^{h^*-x} 1\cdot\left(\sum_{i=1}^{n} \alpha_i \eta_i e^{\eta_i y}\right)dy + \int_{h^*-x}^{0} u(x+y)\left(\sum_{i=1}^{n} \alpha_i \eta_i e^{\eta_i y}\right)dy.\end{aligned}$$

Using change of variable $z = x+y$ and plugging $u(z) = \sum_{j=1}^{n+1} w_j e^{\gamma_j x}$ into above expression yields

$$
\begin{aligned}
&\int_{-\infty}^{0} u(x+y) f_y(y) dy \\
&= \int_{-\infty}^{h^*-x} 1 \cdot \left(\sum_{i=1}^{n} \alpha_i \eta_i e^{\eta_i y} \right) dy + \sum_{i=1}^{n} \left(\alpha_i \eta_i e^{-\eta_i x} \int_{h^*}^{x} u(z) e^{\eta_i z} dz \right) \\
&= \sum_{i=1}^{n} \alpha_i e^{\eta_i (h^*-x)} + \sum_{i=1}^{n} \sum_{j=1}^{n+1} \left(w_j \frac{\alpha_i \eta_i}{\eta_i + \gamma_j} e^{\gamma_j x} \right) \\
&\quad - \sum_{i=1}^{n} \sum_{j=1}^{n+1} \left(w_j \frac{\alpha_i \eta_i}{\eta_i + \gamma_j} e^{\eta_i (h^*-x) + \gamma_j h^*} \right).
\end{aligned}
$$

By plugging the above result into (2.31), for any $x > h^*$,

$$
\begin{aligned}
&\frac{1}{2}\sigma^2 u''(x) + \mu u'(x) - (\lambda + \alpha) u(x) + \int_{-\infty}^{0} u(x+y) f_y(y) dy \\
&= \sum_{j=1}^{n+1} \left[w_j e^{\gamma_j x} \left(\psi(\gamma_j) - \alpha \right) \right] - \lambda \sum_{i=1}^{n} \left\{ \alpha_i e^{\eta_i (h^*-x)} \left[\sum_{j=1}^{n+1} \left(w_j \frac{\alpha_i \eta_i}{\eta_i + \gamma_j} e^{\gamma_j h^*} \right) - 1 \right] \right\} \\
&= 0.
\end{aligned}
$$

Since $\gamma_j, i = 1, 2 \ldots n+1$ are roots of equation $\psi(x) = \alpha$, the expression above becomes

$$
\sum_{i=1}^{n} \left\{ \alpha_i e^{\eta_i (h^*-x)} \left[\sum_{j=1}^{n+1} \left(w_j \frac{\alpha_i \eta_i}{\eta_i + \gamma_j} e^{\gamma_j h^*} \right) - 1 \right] \right\} = 0.
$$

We know that $\eta_1, \eta_2, \ldots, \eta_n$ are distinct, thus it is possible to deduce that

$$
\sum_{j=1}^{n+1} \left(w_j \frac{\alpha_i \eta_i}{\eta_i + \gamma_j} e^{\gamma_j h^*} \right) = 1 \qquad \text{for all } i = 1, 2, \ldots, n. \tag{2.32}
$$

From (2.30) and (2.32), a system of equations with $n+1$ equations and $n+1$ unknowns is derived. This can readily be expressed as $AHW = J$ where A, H, W and J are same as defined in Proposition 2.2. Hence, $u(x)$ can be expressed as

$$
u(x) = \begin{cases} 1 & x \le h^* \\ \sum_{i=1}^{n+1} w_i e^{\gamma_i x} & x > h^* \end{cases}.
$$

In order to prove $u(x) = E[e^{-\alpha\tau_{h^*}}]$, a martingale approach can be employed. Refer to Cai and Kou (2011) for detail.

Appendix 2.B: Proof of Proposition 2.3

For process $X(t)$, we have $EPV = \dfrac{1}{r-\psi(1)}e^{x}$. To make each change has same expected loss, we set $\Delta EPV_m = \Delta EPV_f = \Delta EPV_s$.

$$\Delta EPV_m = \Delta EPV_f \Leftrightarrow \psi_m(1) = \psi_f(1),$$

$$\Delta EPV_f = \Delta EPV_s \Leftrightarrow \psi_f(1) = \psi_s(1).$$

By solving equations above, the two equalities are obtained: By the condition (2.22) for an arbitrary η_k,

$$\begin{aligned}
\Delta\mu &= -\left(\sum_{i=1}^{n}\frac{\alpha_i}{\eta_i+1}\right)\Delta\lambda,\\
\Delta\lambda &= -\left(\sum_{i=1}^{n}\frac{\alpha_i}{\eta_i+1}\right)^{-1}\frac{\lambda}{\eta_{k,s}}\left(\sum_{i=1}^{n}\frac{\alpha_i\eta_{i,s}}{(\eta_{i,s}+1)(\eta_i+1)}\right)\Delta\eta_k.
\end{aligned}$$

Appendix 2.C: Proof of Proposition 2.4

Define an implicit function $G(\beta^+;\mu_Y,\sigma,\lambda,\eta)\equiv r-\psi(\beta^+)$. Since the characteristic equation $r-\psi(z)$ is a concave function on $(-\eta,\infty)$ and β^+ is a positive roots on the interval, G is decreasing in the neighborhood of β^+ i.e. $G_{\beta^+}=\partial F/\partial\beta^+<0$. Define ΔG^i, $i=\{m,f,s\}$ as

$$\begin{aligned}\Delta G^m &= G(\mu_m)-G(\mu)\\ \Delta G^f &= G(\lambda_f)-G(\lambda)\\ \Delta G^s &= G(\eta_{i,s})-G(\eta_i).\end{aligned}$$

Using these conditions and by chain rule and implicit function theorem, we have

$$\Delta h_m^* = \frac{\partial h^*}{\partial\mu}\Delta\mu = \left\{-\frac{1}{\kappa_r^+(1)}\right\}\left\{-\frac{1}{(\beta^+-1)^2}\right\}\left\{-\frac{1}{G_{\beta^+}}\right\}\Delta G^m \equiv \Phi\cdot\Delta\mu\,,$$

where

$$\Phi = \left\{-\frac{rS}{[\kappa_r^+(1)]^2}\right\}\left\{-\frac{1}{(\beta^+-1)^2}\right\}\left\{\frac{\beta^+}{G_{\beta^+}}\right\}<0\,.$$

Similarly,

$$\Delta h_f^* = \frac{\partial h^*}{\partial\lambda}\Delta\lambda = \Phi\cdot\left\{-\sum_{i=1}^{n}\frac{\alpha_i}{\eta_i+\beta^+}\right\}\Delta\lambda\,,$$

and under the condition (2.22), for an arbitrary k,

$$\Delta h_s^* = \sum_{i=1}^{n}\frac{\partial h^*}{\partial\eta_i}\Delta\eta_i \;= \Phi\cdot\frac{\lambda}{\eta_{k,s}}\left\{\sum_{i=1}^{n}\left(\frac{\alpha_i\eta_{i,s}}{(\eta_{i,s}+\beta^+)(\eta_i+\beta^+)}\right)\right\}\Delta\eta_k.$$

Under the Proposition 2.3, we obtain

$$\Delta h_m^* - \Delta h_f^* = \Phi\cdot\left\{\sum_{i=1}^{n}\frac{\alpha_i}{\eta_i+\beta^+}-\sum_{i=1}^{n}\frac{\alpha_i}{\eta_i+1}\right\}\Delta\lambda>0. \qquad (2.33)$$

Similarly, under the Proposition 2.3, we also obtain

$$\Delta h_f^* - \Delta h_s^*$$
$$= \Phi \cdot \left\{ \left(\sum_{i=1}^{n} \frac{\alpha_i \eta_{i,s}}{(\eta_{i,s} + \beta^+)(\eta_i + \beta^+)} \right) \left(\sum_{i=1}^{n} \frac{\alpha_i \eta_{i,s}}{(\eta_{i,s} + 1)(\eta_i + 1)} \right)^{-1} \left(\sum_{i=1}^{n} \frac{\alpha_i}{\eta_i + 1} \right) - \left(\sum_{i=1}^{n} \frac{\alpha_i}{\eta_i + \beta^+} \right) \right\} \Delta\lambda. \tag{2.34}$$

Let $\left(\sum_{i=1}^{n} \frac{\alpha_i \eta_{i,s}}{(\eta_{i,s} + 1)(\eta_i + 1)} \right) \left(\sum_{i=1}^{n} \frac{\alpha_i}{\eta_i + \beta^+} \right) - \left(\sum_{i=1}^{n} \frac{\alpha_i \eta_{i,s}}{(\eta_{i,s} + \beta^+)(\eta_i + \beta^+)} \right) \left(\sum_{i=1}^{n} \frac{\alpha_i}{\eta_i + 1} \right) = \Theta.$ Define $n \times 1$ matrices

$$B = \left(\frac{\alpha_1}{(\eta_1 + 1)}, \frac{\alpha_2}{(\eta_2 + 1)}, \ldots, \frac{\alpha_2}{(\eta_n + 1)} \right)',$$

$$B_\beta = \left(\frac{\alpha_1}{(\eta_1 + \beta^+)}, \frac{\alpha_2}{(\eta_2 + \beta^+)}, \ldots, \frac{\alpha_2}{(\eta_n + \beta^+)} \right)',$$

$$C = \left(\frac{\eta_{1,s}}{(\eta_{1,s} + 1)(\eta_1 + 1)}, \frac{\eta_{2,s}}{(\eta_{2,s} + 1)(\eta_2 + 1)}, \ldots, \frac{\eta_{n,s}}{(\eta_{n,s} + 1)(\eta_n + 1)} \right)', \text{ and}$$

$$C_\beta = \left(\frac{\eta_{1,s}}{(\eta_{1,s} + \beta^+)(\eta_1 + \beta^+)}, \frac{\eta_{2,s}}{(\eta_{2,s} + \beta^+)(\eta_2 + \beta^+)}, \ldots, \frac{\eta_{n,s}}{(\eta_{n,s} + \beta^+)(\eta_n + \beta^+)} \right)'.$$

Then Θ can be written as $\Theta = C'BB'_\beta 1_n - C'_\beta B_\beta B' 1_n$ where $1_n = (1,1,\ldots,1)'$ is a $n \times 1$ vector of ones.

Without loss of generality, it is possible to set

$$0 < \eta_{1,s} < \eta_{2,s} < \cdots < \eta_{n,s}$$
$$0 < \eta_1 < \eta_2 < \cdots < \eta_n.$$

Define a $n \times n$ diagonal matrix as $\Omega = \mathrm{Diag}\left[\frac{\eta_{1,s}}{\eta_{1,s}}, \frac{\eta_{1,s}}{\eta_{2,s}}, \ldots, \frac{\eta_{1,s}}{\eta_{n,s}} \right]$. Using the diagonal matrix,

$$
\begin{aligned}
\Theta &= C'BB'_\beta 1_n - C'_\beta B_\beta B' 1_n \\
&> (\Omega C)'BB'_\beta 1_n - (\Omega C_\beta)' B_\beta B' 1_n \\
&> (\Omega C)' B_\beta B' 1_n - (\Omega C_\beta)' B_\beta B' 1_n \\
&= \left[(\Omega C)' - (\Omega C_\beta)' \right] B_\beta B' 1_n > 0.
\end{aligned} \tag{2.35}
$$

The first inequality in equation (2.35) holds because every vector has only positive elements and every element of Ω is less than unity. The second inequality can be proved as follows.

$$
\begin{aligned}
&\left[(\Omega C)'BB'_\beta 1_n - (\Omega C_\beta)'_\beta B_\beta B' 1_n \right] - \left[(\Omega C)' B_\beta B'_\beta 1_n - (\Omega C_\beta)'_\beta B_\beta B' 1_n \right] \\
&= (\Omega C)'BB'_\beta 1_n - (\Omega C)' B_\beta B'_\beta 1_n \\
&= (\Omega C)' \left[BB'_\beta - B_\beta B' \right] 1_n.
\end{aligned} \tag{2.36}
$$

Note that BB'_β and $B_\beta B'$ in (2.36) are transpose matrix each other. For $i < j$,

$$
\begin{aligned}
\left[BB'_\beta - B_\beta B' \right]_{ij} &= \frac{\alpha_i}{\eta_i + 1} \frac{\alpha_j}{\eta_j + \beta^+} - \frac{\alpha_j}{\eta_j + 1} \frac{\alpha_i}{\eta_i + \beta^+} \\
&= \frac{\alpha_i \alpha_j (\beta^+ - 1)(\eta_j - \eta_i)}{(\eta_i + 1)(\eta_j + 1)(\eta_i + \beta^+)(\eta_j + \beta^+)} \\
&\equiv \varepsilon_{ij} > 0.
\end{aligned}
$$

Thus $\left[BB'_\beta - B_\beta B' \right]$ is a $n \times n$ matrix which has a form

$$
\left[BB'_\beta - B_\beta B' \right] = \begin{pmatrix} 0 & \varepsilon_{12} & \cdots & \varepsilon_{1n} \\ -\varepsilon_{12} & 0 & \cdots & \varepsilon_{2n} \\ -\varepsilon_{13} & -\varepsilon_{23} & \cdots & \varepsilon_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ -\varepsilon_{1n} & -\varepsilon_{2n} & \cdots & 0 \end{pmatrix}
$$

where $\varepsilon_{ij} > 0$ for all i and j. It follows from equation (2.36),

$$
(\Omega C)' \left[BB'_\beta - B_\beta B' \right] 1_n = \eta_{1,s} \sum_{\substack{j=2 \\ j>i}}^{n} \sum_{i=1}^{n} \left(\frac{1}{\eta_{i,s} + 1} - \frac{1}{\eta_{j,s} + 1} \right) \varepsilon_{ij} > 0.
$$

By this result, the second inequality in (2.35) holds. Therefore,

$$\Delta h_f^* - \Delta h_s^* = -\Phi \cdot \Theta \cdot \left(\sum_{i=1}^{n} \frac{\alpha_i \eta_{i,s}}{(\eta_{i,s}+1)(\eta_i+1)} \right)^{-1} \Delta\lambda > 0. \qquad (2.37)$$

Combining (2.37) with the result in (2.33), we have

$$\Delta h_m^* > \Delta h_f^* > \Delta h_s^* .$$

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CHAPTER 3: Impact of Extreme Events and Climate Change on Land Use Decisions for the Michigan Tart Cherry Industry

3.1 Introduction

Crop production is directly exposed to weather conditions, extreme events as well as gradual climate change. Recent examples of weather-related crop loss in agriculture include a severe and extensive drought across the United States in 2012; more than 50 percent of farms and cropland were under severe drought conditions by mid-August 2012 (USDA Economic Research Service 2012). Impact is also realized at a regional or local level; for example, in California unusual rainy days in harvest season reduce both quality and yield of wine grapes due to soggy grapes (Husted 2012).

There is growing evidence of sizable rise in impacts of extreme events. For instance, premium U.S. wine production is predicted to decline drastically under extreme heat (White et al. 2006). The effects of climate change on major crop yields such as corn, soybean and cotton in the U.S. are also shown to be highly nonlinear functions where temperatures beyond certain thresholds can be very harmful to production (Schlenker and Roberts 2009). Specific types of extreme events such as hailstorm damage are not negligible as well (Botzen et al. 2010).

Tart cherry production in Northwest Michigan[13] is highly sensitive to spring frost events (Winkler et al. 2010). In Michigan 2012 spring frost almost destroyed fruit production in the region (De Melker 2012). After record-high early warm days in the spring, freeze events killed a large proportion of cherry buds which resulted in nearly zero harvest.[14]

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[13] This region includes counties around Traverse City, MI: Grand Traverse, Leelanau, Antrim, Benzie counties. Production sites are particularly concentrated in Leelanau and Grand Traverse counties.

[14] Similar spring freeze events occurred in the region in 1945, 1981 and 2002.

3.1.1 Potential climate change impact on the Northwest Michigan tart cherry industry

Gradual climate change has been recognized widely in agricultural impact assessments. In addition to the 2012 spring frost event, there are other indications that tart cherry production in Northwest Michigan may be highly vulnerable to climate change in the future (Winkler et al. 2010). Wet days during the flower stage are less favorable for pollination rate and tend to lower yields. Wet days during harvest season may increase leaf spot disease (*Coccomyces hiemalis*), which can lead to early defoliation and poor fruit quality in future seasons. Hot and windy weather before harvest can negatively impact fruit quality through wind whip and soft fruit (Zavalloni et al. 2006).

As cherry trees come out of dormancy in the spring, risk increases rapidly as cold resistance declines. Vulnerability to freeze increases dramatically when trees are at a bud stage, so a period of warm weather followed by freeze has the potential to completely destroy buds and the resulting crop. There is evidence that trees are developing earlier in the spring than they were in previous decades and thus are increasingly vulnerable to a freeze event in the spring (Andresen 2012).

Major spring frost events in 2002 and 2012 showed empirical signs of change in terms of frequency and magnitude (Zavalloni et al. 2006). Prior to 2002, spring frost events were observed in 1945 and 1981. Many growers in the region considered these years to be a once in a lifetime situation but the 2002 and 2012 frosts may indicate frequency change in the future. Warm temperature in spring 2012 pushed cherry trees to a development stage about five week ahead of normal (Andresen 2012).[15]

[15] Similarly, early dormancy releases for fruit crops have been observed in Germany (Chmielewski, Blümel, and Pálešová 2012)

The Michigan spring frost events in 2002 and 2012 have some disparate aspects which can provide new insights for future seasons. While the 2002 frost primarily impacted tart cherry production, the 2012 event was more widespread and other fruit crops in the region (e.g. apples, peaches, sweet cherries and juice grapes) also suffered losses. As early spring cold weather is normal in the region, preceding warm temperatures may not only increase the likelihood of freeze but also lengthen the exposure days once a freeze occurs.

3.1.2 Adaptation under gradual changes vs. extreme events

This chapter applies a real options adaptation model to land use decisions undertaken by tart cherry growers in Northwest Michigan in the face of climate change. The tart cherry industry, as a perennial crop production, has intrinsic investment or establishment costs which are (partly) irreversible and thus there are more limited short-term adaptation strategies than annual crops. Tart cherry growers cannot readily switch their orchards to alternative land use year-by-year in response to weather conditions or price signals. Hence, the discounted cash flow (DCF) method as a version of expected net present value (ENPV) model does not reflect the real world pace of adjustment in land use for this industry.

A real option land use model is applied where tart cherry yield follows a jump diffusion process which represents a mixture of continuous variation and discrete jump variation. Three different types of climate adjustments (gradual unfavorable changes in weather patterns, increased frequency of extreme events, and increased magnitude of extremes) are compared for their impacts on land use decisions. The adjustments are calibrated so that they result in the same expected losses to the current land use (i.e. tart cherry production). Therefore, results are equivalent in the traditional ENPV framework.[16]

Using observed historical yields from 1947 to 2012, a jump diffusion model of tart cherry yields is estimated to obtain continuous and jump parameters of the distribution. The distribution is used to compare returns from continued production versus removal of land from agricultural use to obtain a fixed return (i.e. selling the land for real estate development). Climate change consequences are compared in terms of i) the economic incentive to switch, which is represented by optimal switching thresholds and ii) the probability of switching in a given period, which captures realized actions given the thresholds.

3.2 Decision making and timing of adaptive actions

3.2.1 Yield and price processes

Consider a representative tart cherry grower in Northwest Michigan with a stochastic return stream. In order to evaluate climate and/or weather effects on land use, the return stream is broken into separate stochastic processes for yield and price. Weather conditions and extreme events are reflected in a yield process whereas a price process identifies market adjustments. Assuming operating costs are constant, the economic return per unit of land (per acre) from the crop production can be written as

$$\pi_c(t) \equiv \tilde{P}(t)\tilde{Y}(t) - K$$

[16] For real option applications to production restructuring problems in agriculture, see Odening, Mußhoff, and Balmann 2005; Hinrichs, Mußhoff, and Odening 2008; Kuminoff and Wossink 2010; Song, Zhao, and Swinton 2011.

where $\tilde{P}(t)$ and $\tilde{Y}(t)$ are unit price (per pound) and yield (pounds per acre) at time *t*, respectively. Operating cost (per acre) is denoted by K. Let $Y(t) \equiv \ln \tilde{Y}(t)$, then the log yield process, assumed to be fluctuating due to weather and/or climate conditions, can be described by the following stochastic process:

$$Y(t) = y + \mu_Y t + \sigma_Y W_Y(t) + \sum_{i=1}^{N(t)} Q_i . \tag{3.1}$$

where *y* is the starting point of the process and μ_Y and σ_Y are drift and volatility parameters, and $W_Y(t)$ is the standard Brownian motion (Wiener process). The continuous portion identifies smooth yield variations, and the last term in the right hand side (RHS) is a compound Poisson process which captures rare extreme events. $\{N(t)\}$ is the Poisson process with arrival rate λ and $\{Q_i\}$ is a sequence of independent identically distributed (i.i.d.) random variables which determines magnitude of extreme events with density $f_Q(q)$. $W_Y(t)$, $N(t)$ and Q_i are assumed to be mutually independent.

As the tail behavior of the jump magnitude distribution Q_i determines to a large extent the tail behavior of the probability density of the whole jump diffusion process of $Y(t)$ in equation (3.1), distributional features of the impacts of extreme climate events can be captured by the density $f_Q(q)$. The distribution should coincide with a class of yield distributions proposed in the theoretical literature and should capture empirical impacts of extreme events due to climate change.

As a version of exponential-type distributions, a spectrally negative exponential distribution is employed as the distribution of the extreme magnitude, which allows only negative jumps.[17] This specification is well-suited to capture the impacts of extreme events on tart cherry yields. First, by introducing negative exponential jump size into equation (3.1), the yield process transition density approximates a negatively skewed beta distribution, which is commonly used for crop yield distribution (Goodwin 2009; Hennessy 2011; Nelson and Preckel 1989; Turvey and Zhao 1999). Second, since an exponential distribution has a higher peak and fatter tail than the corresponding normal distribution, it can reflect heavy-tail phenomena of climate change impacts. As positive jumps are ruled out, the density for jump size can be written as

$$f_Q(q) = \eta e^{\eta q} 1_{\{-\infty,0\}} \tag{3.2}$$

where $\eta > 0$ is a jump size parameter so that a smaller η represents a larger average magnitude of jump.

Let $P(t) \equiv \ln \tilde{P}(t)$. Changes in logarithmic price is assumed not to include the jump component but instead follow a Brownian motion with drift that can be written as

$$P(t) = p + \mu_P t + \sigma_P W_P(t) \tag{3.3}$$

where p is the starting point of the price process and μ_P and σ_P are drift and volatility parameters and $W_P(t)$ is the Wiener process. The variations in the Gaussian portions of yield and price processes are correlated with each other: $E\left[dW_Y(t), dW_P(t)\right] = \rho_{PY} dt$ where ρ_{PY} denotes the correlation coefficient between yield and price. Similarly, the jump component of yield is assumed to be correlated with the price process $E\left[d\left(\sum_{i=1}^{N(t)} Q_i\right), dW_P(t)\right] = \rho_{PY} dt$.

[17] Examples include Mordecki (2002) to model option pricing and Egami and Xu (2008) to model job switching model.

A smooth price process for tart cherries is justified empirically. Agricultural products can be subject to certain regulations stabilizing prices and the tart cherry industry is regulated by the Federal Marketing Order. Over 95 percent of tart cherry are processed (e.g. frozen, dried, or canned) and can be stored. The marketing order restricts marketable volume when overall production exceeds a predetermined amount and releases volume in short crop years. Thus, price movements are smoothed out under the order. Storage can play a role to stabilize agricultural prices for some commodities including tart cherries. To the extent storage acts as a buffer against a short crop year, upside movement is smoothed out. In addition, tart cherry prices won't move as sharply as yield variations due to substitution effects since tart cherries are primarily used as production ingredient so buyers find it relatively easy to substitute with other fruit products. Finally, extreme events are not evenly distributed across tart cherry production regions and so only regional supplies may be impacted. National or international supplies may limit price impact due to yield change in a particular region.

Since both stochastic processes introduced in equations (3.1) and (3.3) are special cases of the Lévy process, the equations allow alternative representations using Laplace exponents (Refer Sato 1999 Ch.1 for detail). The log yield and price processes can be represented using the Laplace exponent as

$$\psi_Y(z) = \frac{1}{2}\sigma_Y^2 z^2 + \mu_Y z - \frac{\lambda z}{\eta + z} \tag{3.4}$$

$$\psi_P(z) = \frac{1}{2}\sigma_P^2 z^2 + \mu_P z. \tag{3.5}$$

3.2.2 Land use switching decision

In the tart cherry growing area of Northwest Michigan soils are mostly sandy, which are excellent for fruit growing; however, they are less conducive to annual crop production limiting grower options to switch to these crops. Furthermore, the widespread impact from the 2012 spring frost event suggests that switching to another fruit crop may not be a good adaptation option. In 2012 most fruit crops in the region were adversely affected including apples and peaches as well as cherries. Many regional cherry growing sites are located on the shore of Lake Michigan which is an attractive environment for real-estate development. With historically high land prices, moving out of agricultural use is likely a reasonable adaptation option for growers.

Therefore, abandoning agricultural use (e.g. through selling land for real estate development) can be viewed as a feasible adaptation option in the region. Let the price of land be denoted by S . Let τ_h be the hitting time when the sum of log yield and price processes first reaches an arbitrary value h as

$$\tau_h \equiv \inf \left\{ t \geq 0 \middle| Y(t) + P(t) \leq h \right\}.$$

Then given the log yield and price processes defined in (3.1) and (3.3), the grower's land use switching problem can be written as

$$\hat{V}(y, p; h) = E^{y,p}\left[\int_0^{\infty} e^{-rt}[e^{Y(t)+P(t)} - K]dt\right] + E^{y,p}\left[\int_{\tau_h}^{\infty} e^{-rt}\left\{rS - [e^{Y(t)+P(t)} - K]\right\}dt\right] \quad (3.6)$$

(Boyarchenko & Levendorskiĭ, 2007: ch. 11 for detail). The first term in the RHS captures value of current system (tart cherry production) without switching. The second term identifies value of switching when $Y(t) + P(t)$ reaches a certain threshold h . Selling the land at land price S provides the same constant return stream rS each period.

Since both price and yield are stochastic in this setting i.e. $\pi_c(t) \equiv \tilde{P}(t)\tilde{Y}(t) - K$, the economic incentive to switch will depend on relative size of both variables given constant tart cherry production costs and fixed income from sale for real estate development. Let $\tilde{X}(t) = \tilde{P}(t)\tilde{Y}(t)$ be a gross revenue process. When tart cherry price is sufficiently high to provide a certain level of $\tilde{X}(t)$, a grower will have less incentive to convert even if yield is low and *vice versa*. Hence, the solution to this problem will depend on the level of gross revenue, which is a function of price and yield. Since price and yield processes are correlated, a dimension is reduced by constructing the log gross revenue process. Define the logarithmic gross revenue process as $X(t) = P(t) + Y(t)$. Then $X(t)$ can be represented as following process

$$X(t) = x + \mu_X dt + \sigma_X dW_X(t) + (1 + \rho_{PY}) \sum_{i=1}^{N(t)} Q_i$$

where $x \equiv p + y$ is the starting point of the log revenue process $X(t)$ and $\mu_X = \mu_P + \mu_Y$, $\sigma_X = \sqrt{\sigma_P^2 + \sigma_Y^2 + 2\rho_{PY}\sigma_P\sigma_Y}$ and $W_X = \frac{1}{\sigma_X}\left[\sigma_P W_P(t) + \sigma_Y W_Y(t)\right]$. It can be shown that W_X is a standard Brownian motion. That is,

$$\begin{aligned} d\left[W_X, W_X\right]_t &= \frac{1}{\sigma_X^2}\left(\sigma_Y^2 d[W_Y, W_Y]_t + 2\sigma_Y\sigma_P d[W_Y, W_P]_t + \sigma_P^2 d[W_P, W_P]_t\right) \\ &= \frac{1}{\sigma_X^2}\left(\sigma_Y^2 + 2\rho_{PY}\sigma_Y\sigma_P + \sigma_P^2\right)dt \\ &= dt \end{aligned}$$

where $[\cdot]_t$ denotes a quadratic variation. The correlation between Gaussian components is reflected in σ_X while the yield jumps are adjusted by the factor $(1 + \rho_{PY})$ in the revenue process. That is, if ρ_{PY} is negative, which is the typical relationship between agricultural price and yield, negative yield shocks are absorbed by price adjustments. Since W_X is the standard Brownian motion, the (log) gross process above can be represented with corresponding Laplace exponent $\psi_X(z)$ as

$$\psi_X(z) = \frac{1}{2}\sigma_X^2 z^2 + \mu_X z - (1+\rho_{PY})\frac{\lambda z}{\eta + z}. \tag{3.7}$$

The expected present value (EPV) of switching land use $\hat{V}(\cdot)$ in equation (3.6) can be rewritten as a function of starting revenue x and revenue threshold h :

$$\hat{V}(x;h) = E^x\left[\int_0^\infty e^{-rt}\left\{e^{X(t)} - K\right\}dt\right] + E^x\left[\int_{\tau_h}^\infty e^{-rt}\left\{rS - (e^{X(t)} - K)\right\}dt\right]. \tag{3.8}$$

Thus $\hat{V}(\cdot)$ calculates the EPV of switching in equation (3.8) when the log of gross revenue $X(t)$ reaches or crosses an arbitrary threshold level h . In order to find the optimal threshold h^*, define the normalized expected present value operator $\mathcal{E}$ which calculates the normalized EPV of an arbitrary function $g(X(t))$:

$$\mathcal{E}g(x) = rE^x\left[\int_0^\infty e^{-rt} g(X(t))dt\right]. \tag{3.9}$$

Introduce the two extremum processes: the supremum process $\overline{X}(t) = \sup_{0\le s\le t} X(s)$ and the infimum process $\underline{X}(t) = \inf_{0\le s\le t} X(s)$ so that $\overline{X}(t)$ and $\underline{X}(t)$ evaluate running maxima and minima of a process $X(t)$ at time *t,* respectively. The (normalized) EPV-operators of a function of the supremum and infimum process are defined accordingly:

$$\mathcal{E}^+ g(x) = rE^x\left[\int_0^\infty e^{-rt} g(\overline{X}(t))dt\right] \tag{3.10}$$

$$\mathcal{E}^- g(x) = rE^x\left[\int_0^\infty e^{-rt} g(\underline{X}(t))dt\right]. \tag{3.11}$$

Hence, it is possible to write the EPV in (3.8) as

$$\hat{V}(x;h) = r^{-1}\mathcal{E}(e^x - K) + r^{-1}\mathcal{E}^{-}1_{(-\infty,h]}\mathcal{E}^{+}(rS + K - e^x).$$ [18]

Then the optimality condition becomes

$$\mathcal{E}^{+}(rS + K - e^x) = 0.$$

Define a special version of EPVs applied to the exponential functions

$$\kappa_r^{+}(z) = rE\left[\int_0^{\infty} e^{-rt} e^{z\bar{X}(t)} dt\right]. \tag{3.12}$$

Then the optimality condition can be evaluated as

$$\mathcal{E}^{+}(rS + K - e^{h^*}) = rS + K - \kappa_r^{+}(1)e^{h^*} = 0. \tag{3.13}$$

For the (log) gross revenue process defined in (3.7), explicit expressions for $\kappa_r^{+}(z)$ can be derived. Define the characteristic equation for the process as $r - \psi_X(z) = 0$, then the equation has unique positive root denoted by β^{+} and $\kappa_r^{+}(z)$ has explicit expression as

$$\kappa_r^{+}(z) = \frac{\beta^{+}}{\beta^{+} - z}. \tag{3.14}$$

Plugging this expression into the optimality condition in (3.13), the optimal switching boundary is expressed as

$$h^* = \ln\left(\frac{rS + K}{\kappa_r^{+}(1)}\right). \tag{3.15}$$

3.2.3 Timing of adaptation: probability of switching

While the timing of adaptive action will depend on both price and yield realizations, these movements are reflected in the log gross revenue process. Hence, it is required to identify

[18] Note that this switching problem in (3.8) allows same representation with extremum operators as Boyarchenko and Levendorskiĭ (2007).

when log revenue $X(t)$ reaches or falls below the optimal switching boundary h^* for the first time (i.e. first passage time). The probability of switching is considered:

$$P\left(\tau_{h^*} \le T\right) = P\left(\min_{0 \le t \le T} X(t) \le h^*\right) \tag{3.16}$$

where T is a predetermined time period, h^* is an optimal switching threshold derived above, and τ_{h^*} is the first passage time defined by $\tau_{h^*} \equiv \inf\left\{t \ge 0 \middle| X(t) \le h^*\right\}$.

Due to the undershoot problem,[19] an explicit form of the distribution of the first passage time cannot be obtained. However, the Laplace transformation of the first passage time can be derived and inverted numerically to obtain probability of switching in a given time T using the following relationship:

$$\int_0^\infty e^{-\alpha T} P\left(\tau_{h^*} \le T\right) dT = \frac{1}{\alpha}\int_0^\infty e^{-\alpha T} dP\left(\tau_{h^*} \le T\right) = \frac{1}{\alpha} E\left[e^{-\alpha \tau_{h^*}}\right].$$

An exponential distribution is employed for extreme magnitude distribution. Since the exponential distribution is a special case of exponential family of distributions, explicit form of the Laplace transformation is obtained.

Proposition 3.1 *Let* $x\ (> h^*)$ *be the starting point of the gross revenue process* $X(t)$. *For any* $\alpha \in (0,\infty)$, *the equation* $\psi_X(z) = \alpha$ *has only two negative roots* γ_1 *and* γ_2 *where* $\gamma_2 < -\eta < \gamma_1 < 0$. *Then the Laplace transform with the jump size distribution in (3.2) is given by*

$$E\left[e^{-\alpha \tau_{h^*}}\right] = \frac{\gamma_2(\eta + \gamma_1)}{\eta(\gamma_1 - \gamma_2)} e^{\gamma_1 (x - h^*)} + \frac{\gamma_1(\eta + \gamma_2)}{\eta(\gamma_2 - \gamma_1)} e^{\gamma_2 (x - h^*)} \quad . \tag{3.17}$$

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[19] With a jump component, the log revenue process may crosses a boundary by either "hitting" the boundary exactly, i.e. $X(\tau_{h^*}) = h^*$) or "undershoot" below the boundary, i.e. $h^* - X(\tau_{h^*}) > 0$ (Kou and Wang, 2003).

Proof Appendix 3.A

In order to invert the Laplace transformation, the Gaver-Stehfest algorithm is employed and the probability of switching can be calculated in predetermined time period T.[20]

3.3 Land use switching decision for the tart cherry industry in Northwest Michigan

3.3.1 Data and estimation strategy

Small scale yield information such as farm- or block-specific yield data would be ideal for this application to reflect individual decision-making. Since this level of disaggregation is not readily available, state-level data is collected and adjusted to reflect production in Northwest Michigan. Tart cherry yield and price data are taken from Michigan Agricultural Statistics (MASS) for the period of 1947-2012. CPI for fruit and vegetables from Bureau of Labor Statistics (BLS) are used to deflate the price series since it reflects overall market price changes.[21] Annual Northwest Michigan tart cherry yield is depicted in Figure 3.1

The figure below shows that the extreme events in 2002 and 2012 were devastating. The 2002 event was mainly due to wind freeze (28-29°F, northwest wind) within a short duration of time (11-12 hours). In 2012 record-high warm weather for five or six days in late March induced buddings approximately five week early, which increased exposure to freezing weather afterwards.

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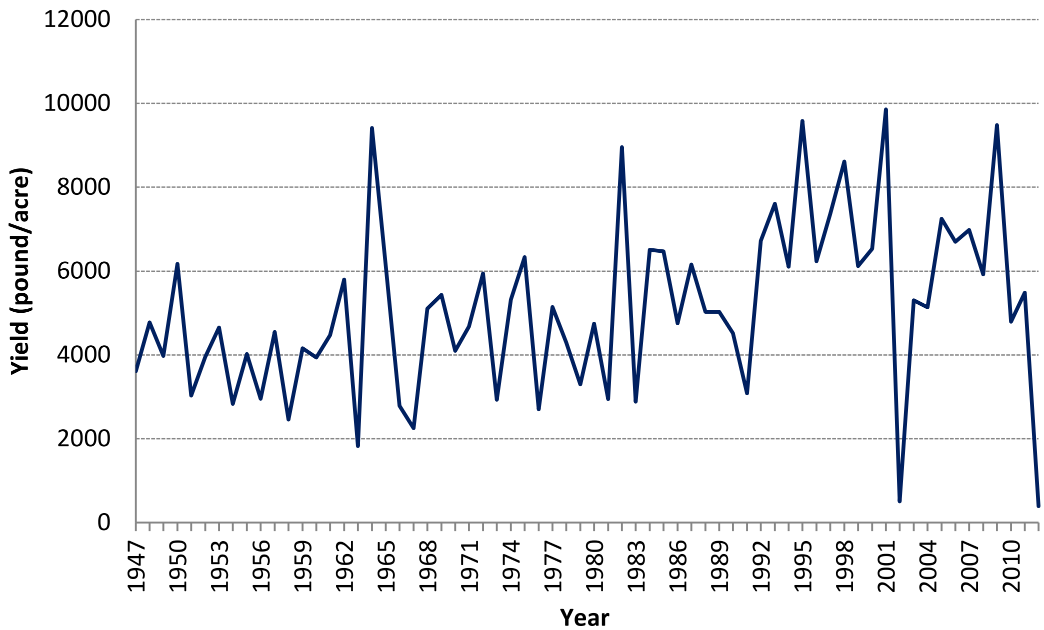
[20] For the form of the Laplace transformation given by (3.17), the algorithm is known to be fast and accurate for inversion of the Laplace transformation (Kou and Wang 2003).

[21] Base year is set to be 1982, i.e. CPI 1982=100.

Figure 3.1 Tart cherry yield/acre in Northwest Michigan



Source: Michigan Agricultural Statistics (MASS)
Note: Yield series is adjusted to reflect actual tart cherry yields in Northwest Michigan based on Black et al. (2010).

A time series of annual production costs is not available for tart cherries. While there are some surveys of Michigan production costs (e.g. Nugent 2003; Black et al. 2010), they are not consistently collected and rely on different assumptions. In this model, operating cost is assumed to be constant and taken from estimates in Black et al. (2010) for a mid-sized grower in Northwest Michigan ($688/acre). The annual operating cost consists of pruning, mowing, crop protection, herbicide and fertilizer, and others. Orchard establishment and land purchase cost are not included and assumed to have already been disbursed.

Since selling the land is the adaptation option, land value represents the price the grower can receive. The value of farmland for development purposes in the region is taken from residential land value of undeveloped land reported by Wittenberg and Wolf (2012) and assumed to be constant ($3,563/acre). All these values are deflated by the CPI.

Finally, discount rate for the switching problem should be specified. When dealing with jump diffusion process as (3.1), it is not easy to find an asset that duplicates the stochastic dynamics of the process (Dixit and Pindyck 1994). In real options, it becomes even harder to find the replicating asset thus it is impossible to specify a discount rate by the capital asset pricing model (CAPM). Alternatively, a risk-adjusted interest rate is employed for the exogenous discount rate r. As suggested in Me-Nsope (2009) and McManus (2012), a 6 percent risk premium is assumed. Also, 3.7 percent for risk-free rate is assumed based on recent three year (2010-2012) average of 30 year T-bill rate. Therefore, 9.7 percent risk-adjusted interest rate is adopted as a discount rate for the switching problem.

The logarithmic yield and price processes are considered as given in equations (3.1) and (3.3). Since the processes are special cases of the Lévy process, which is a continuous time analog of random walk, they will exhibit unit root empirically. The Lévy process for yield can better reflect events such as technological progress and extreme events. For the price process, theory suggests that commodity prices are stationary but they are often non-stationary empirically (Wang and Tomek 2007) due to multiple forces such as structural change, market structure, and macroeconomic shocks.

Based on Generalized Least Squares (GLS) based augmented Dickey-Fuller (ADF) test (Elliott, Rothenberg, and Stock 1996), the null hypothesis of the unit root is not rejected for either the log yield or price series at the 5 percent significance level (Table 3.2). Intuitively this suggest that Lévy processes (random walks) are more appropriate than other alternatives considering tart cherry market is small with historically high variations in yields and prices.

When jumps are included, additional exploratory data analysis is needed to identify the process. The change in logarithmic yield series $\Delta Y(t)$ should be leptokurtic (fat-tailed) and negatively skewed if the yield process follows the jump diffusion specified in (3.1). The skewness and kurtosis of $\Delta Y(t)$ are -0.53 and 8.50, respectively; negatively skewed and fatter-tailed than the normal distribution.[22] The change in logarithmic price series $\Delta P(t)$, on the contrary, should be close to a normal distribution since Brownian motion is assumed. The skewness and kurtosis of $\Delta P(t)$ are 0.04 and 2.74, which is reasonably close to the normal distribution.

The Jarque-Bera test of normality compares sample skewness and kurtosis with those of the standard normal distribution. The test statistic is given by

$$JB = n\left\{\frac{\hat{\alpha}_{sk}^2}{6} + \frac{(\hat{\alpha}_{ku}-3)^2}{24}\right\}$$

where n, $\hat{\alpha}_{sk}$ and $\hat{\alpha}_{ku}$ are sample size, sample skewness and kurtosis, respectively. Test results reject the null hypothesis of normality for $\Delta Y(t)$ at the 1 percent level while failing to reject the null for $\Delta P(t)$ even at the 10 percent significance level. Although the Jarque-Bara test result confirms that $\Delta Y(t)$ is non-Gaussian, the test often performs poorly with small sample and tends to over-reject the null of normality.

The Shapiro-Wilk test of normality, which is known to have a higher power with small sample, is conducted. The test statistic is given by

$$SW = \frac{\left(\sum_{i=1}^{n} a_i X_{(i)}\right)^2}{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}$$

[22] The skewness and kurtosis of the standard normal distribution are 0 and 3, respectively.

where $X_{(i)}$ is the ith order statistic and $\overline{X}$ is the sample mean. The constants a_i are defined by

$$\left(a_1, a_2, \ldots, a_n\right) = \frac{m'U^{-1}}{(m'U^{-1}U^{-1}m)^{1/2}}$$

where $m = (m_1, m_2, \ldots, m_n)'$ are the expected values of ith order statistics of i.i.d. random variables from the standard normal distribution and U is the variance-covariance matrix of the order statistics. Hence, the Shapiro-Wilk test is based on correlation between $X_{(i)}$ and corresponding ith order statistic from the standard normal distribution. Based on the Shapiro-Wilk test, results reject the null hypothesis of normality at the 1 percent level for $\Delta Y(t)$ while failing to reject the null for $\Delta P(t)$ even at the 10 percent significance level. These tests strongly support that $\Delta Y(t)$ is non-Gaussian but $\Delta P(t)$ is Gaussian.

Focusing on an estimation strategy, for a general jump diffusion process a closed-form probability density function is usually not available. However, for a jump diffusion model with exponential-type jump, the probability density function can be approximated (Kou 2002). Changes in the logarithmic yield process over time interval Δt is given by:

$$\begin{aligned} \Delta Y(t) &= Y(t + \Delta t) - Y(t) \\ &= \mu_Y \Delta t + \sigma_Y \Delta W_Y(t) + \sum_{i=N(t)+1}^{N(t+\Delta t)} Q_i \end{aligned}$$

where $\Delta W_Y(t) = W_Y(t + \Delta t) - W_Y(t)$. Since $\Delta W_Y(t)$ represent a change in Wiener process, $\Delta W_Y(t)$ has the normal distribution $N(0, \Delta t)$.

If the time interval Δt is sufficiently small,[23] Poisson process can be reduced to a Bernoulli distribution within the interval as

$$\sum_{i=N(t)+1}^{N(t+\Delta t)} Q_i \approx \begin{cases} Q_{N(t)+1} & \text{with probability } \lambda\Delta t \\ 0 & \text{with probability } (1-\lambda)\Delta t \end{cases}.$$

Consequently, the change in log yield process can be approximated in distribution as

$$\Delta Y(t) \approx \mu_Y \Delta t + \sigma_Y Z_Y \sqrt{\Delta t} + B \cdot Q \tag{3.18}$$

where Z_Y and B are standard normal and Bernoulli random variables, respectively. Q is a random variable whose density is given by (3.2), i.e. exponential distribution on the negative plane. The density w of the RHS has an explicit expression as

$$w(x) = \frac{1-\lambda\Delta t}{\sigma_Y \sqrt{\Delta t}} \varphi\left(\frac{x-\mu_Y\Delta t}{\sigma_Y\sqrt{\Delta t}}\right) + \lambda\Delta t \left\{ \eta e^{(\sigma_Y^2\eta^2\Delta t)/2} e^{(x-\mu_Y\Delta t)\eta} \Phi\left(-\frac{x-\mu_Y\Delta t+\sigma_Y^2\eta\Delta t}{\sigma_Y\sqrt{\Delta t}}\right)\right\} \tag{3.19}$$

where $\varphi(\cdot)$ and $\Phi(\cdot)$ are probability density and cumulative density functions of the standard normal distribution, respectively. By setting $\Delta t = 1$ for yearly data an estimating equation is obtained for the equation (3.18) given the density (3.19):

$$\Delta Y(t) = Y(t+1) - Y(t) = \mu_Y + \sigma_Y Z_Y + B \cdot Q\,. \tag{3.20}$$

To estimate the parameters of the logarithmic price series, the equation (3.3) is discretized to the estimating equation:

$$\Delta P(t) = P(t+1) - P(t) = \mu_P + \sigma_P Z_P\,. \tag{3.21}$$

[23] This condition is to restrict number of jumps in a time interval to be at most one. Though we are applying this approximation to relatively large time interval (yearly data), we allow at most one jump over a year as an extreme event as the crop is realized annually. Hence, the approximation is still valid in the application.

Since transitions of these processes are i.i.d., they can be viewed as a Markov process. When estimating these equations, the maximum likelihood estimation (MLE) exploiting Markov property of the processes has been known to be the best choice to estimate parameters since the MLE estimator is asymptotically efficient among all classes of estimators under weak regularity conditions. While a transition density function is rarely available explicitly for general classes of the Markov processes, the transition density is approximated explicitly for the jump diffusion model as (3.19). For a continuous diffusion model as in (3.21), the transition density is readily available. Since density functions are given explicitly, the maximum likelihood estimation (MLE) can be employed to estimate parameters of the equations (3.20) and (3.21). Finally, the correlation parameter ρ_{PY} can be estimated by the sample correlation coefficient of the price and yield processes.

Parameter estimates using historical yield and price series constitute a baseline scenario. The scenario represents current trends of yield and price without effects of climate change; the parameters from historical data remain unchanged in the future. Parameters for the baseline scenario are summarized in Table 3.1. Overall parameter estimates are not statistically significant at the 5 percent level. Since only annual yield and price data are available, this problem seems to be due to small sample size (N=66).

The jump component is assumed to be an extreme event which is a rare but drastic change, so jump frequency λ is expected to be very small. It is challenging to attain statistical significance on parameters associated with jump size and frequency. However, simulation of parameter estimate means are very close to true values (Bibby, Jacobsen, and Sørensen 2010). Volatility parameters are significant at the 1 percent level, which reflect higher yearly variations of yield and price despite the overall small sample size. Estimated correlation coefficient $\hat{\rho}_{PY}$

shows high negative correlation (-0.698) between price and yield. It seems reasonable considering that tart cherry production in the U.S. is highly concentrated in Michigan (70-75 percent of total U.S. production) so that yields in Michigan determine overall supply in the U.S.

Table 3.1 Baseline Parameters

Parameters of the stochastic processes of yield and price				
	$\Delta Y(t)$		$\Delta P(t)$	
Drift	$\hat{\mu}_Y$	0.072 (0.079)	$\hat{\mu}_P$	-0.001 (0.072)
Volatility	$\hat{\sigma}_Y$	0.580 (0.054)	$\hat{\sigma}_P$	0.582 (0.051)
Jump frequency	$\hat{\lambda}$	0.071 (0.062)		
Jump size	$\hat{\eta}$	0.668 (0.472)		
Correlation	$\hat{\rho}_{PY}$		-0.698	
Operating cost	K		\$688/acre	
Land price	S		\$3,563/acre	

Note: Standard errors are in parenthesis.

3.3.2 Gradual change and extreme events: climate change scenarios

Three different types of climate change outcomes are considered: gradual unfavorable changes in weather patterns, increased frequency of extreme events, and increased magnitude of extreme changes. Let $i \in \{m, f, s\}$ to denote gradual, frequency and magnitude change, respectively. Although in reality these changes may not occur independently, this classification enables comparison in terms of an adaptation perspective.

Assume that the three climate change outcomes affect the yield process defined in (3.1). Gradual unfavorable change indicates that the production site becomes less favorable smoothly over time due to changing climate conditions. The change is realized with a change in the drift term μ_Y. The gradual change is captured by $\Delta\mu_Y = \mu_Y^m - \mu_Y$ where $\mu_Y > \mu_Y^m$. The frequency change indicates higher frequency of extreme events such as crop failure, which is assumed to be identified through a change in the Poisson parameter by $\Delta\lambda = \lambda^f - \lambda$ where $\lambda^f > \lambda$. Finally, change in magnitude indicates that extreme effects become more severe on average once an extreme event happens. The change is captured in the jump size parameter and defined as $\Delta\eta = \eta^s - \eta$ where $\eta > \eta^s$.

Gradual change in weather patterns, increased frequency of extreme events, and increased magnitude of extreme events are calibrated to exhibit equivalent loss in terms of expected present values. These adjustments are translated into changes in the log gross revenue process $X_i(t)$, $i \in \{m, f, s\}$ as

$$\psi_X^m(z) = \frac{1}{2}\sigma_X^2 z^2 + \mu_X^m z - (1+\rho_{PY})\frac{\lambda z}{\eta + z} \tag{3.22}$$

where $\mu_X^m = \mu_Y^m + \mu_P$ and

$$\psi_X^f(z) = \frac{1}{2}\sigma_X^2 z^2 + \mu_X z - (1+\rho_{PY})\frac{\lambda^f z}{\eta + z} \tag{3.23}$$

$$\psi_X^s(z) = \frac{1}{2}\sigma_X^2 z^2 + \mu_X z - (1+\rho_{PY})\frac{\lambda z}{\eta^s + z}. \tag{3.24}$$

Let $\Delta EPV_i, i \in \{m, f, s\}$ be corresponding expected loss from each consequence. By setting $\Delta EPV_m = \Delta EPV_f = \Delta EPV_s$, the equivalent expected loss condition can be derived as

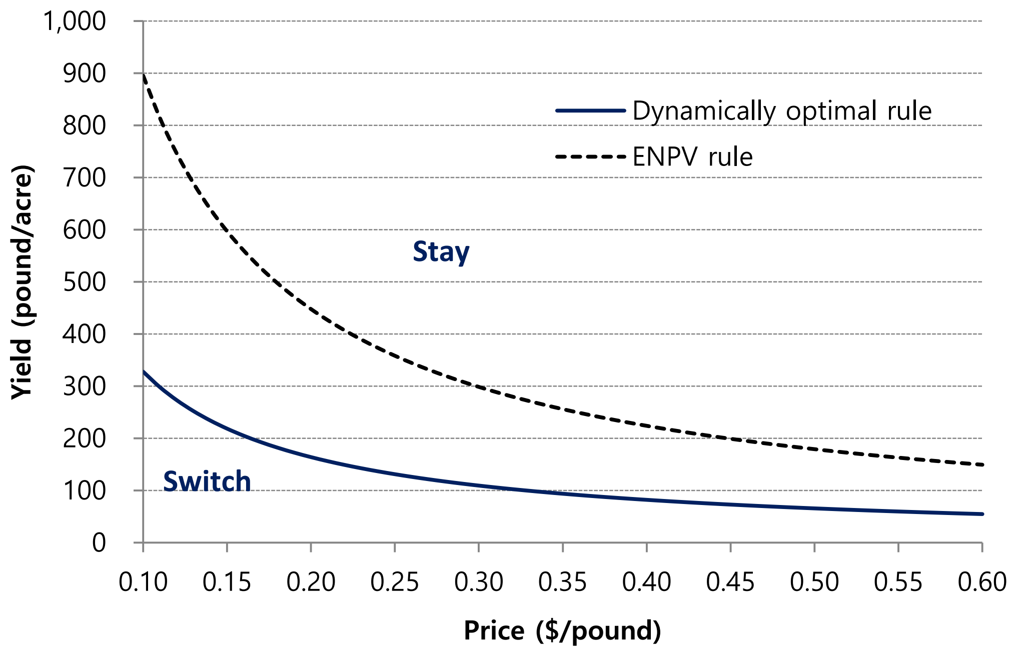
$$\begin{aligned} \Delta\mu_Y &= -\frac{1+\rho_{PY}}{\eta+1}\Delta\lambda, \\ \Delta\lambda &= -\frac{\lambda}{(\eta^s+1)}\Delta\eta. \end{aligned} \tag{3.25}$$

3.3.3 Empirical results and discussion

3.3.3.1 Baseline scenario (without climate change)

Using the estimated baseline parameters, optimal switching boundary is investigated as given in the equation (3.15).[24] Optimal switching boundaries are derived under dynamically optimal (real option) and ENPV rules (Figure 3.2). The decision rule naturally depends on the price and yield as they are stochastic variables.

Figure 3.2 Optimal switching thresholds under real option and expected net present value (ENPV) rule



As the grower realizes yields and prices every year, the grower will make a decision to stay in tart cherries or switch (i.e. remove land from production). As shown in the figure, the

[24] For convergence of the EPV operator, overall growth rate of gross revenue should be less than assumed discount rate; i.e. $r-\psi_X(1)>0$. The estimated growth rate of $X(t)$ is 0.16, which is greater than the discount rate. Based on the statistical significance, the drift parameters are set to zero in order to obtain the convergence, i.e. $\hat{\mu}_Y=\hat{\mu}_P=0$.

optimal decision rule exhibits an inverse relationship between price and yield. Specifically, the grower tends to have a higher incentive to switch facing lower yield and price. In other words, at a given annual price, the grower will have greater incentive to remove land from production if yield is significantly low.

Real option valuation requires evaluating the expected present value based on extreme processes given the stochastic variables while the ENPV rule relies on central tendency of the processes. This naturally leads to more consideration of the tail behavior of the processes using real option valuation. As traditional real option literature indicates (e.g. Dixit and Pindyck 1994), the dynamically optimal real option rule exhibits a more stringent threshold than the ENPV rule; there is less incentive to switch under the dynamically optimal rule. For example, when price is at 10 cents, the real option rule requires yield under approximately 150 pounds before land is removed from production while the ENPV rule indicates it is optimal to switch at yields less than 300 pounds at the same price. The real option rule accounts for uncertainty, learning and irreversibility in dynamic decision making so that waiting is valued more heavily.

Given the baseline thresholds, the probability of switching is derived in which a parcel of land in tart cherry production will be sold for residential development during a 20 year period (Figure 3.2).[25] The sample average price and yield for the period 1947-2012 are used for initial price and yield for tart cherries. The initial price and yield are calculated as 24 cents/pound and 5,488 pounds/acre, which approximately come to $1,100 of gross revenue per acre (note: all values are normalized using the CPI).[26] Since the explicit expression of the Laplace

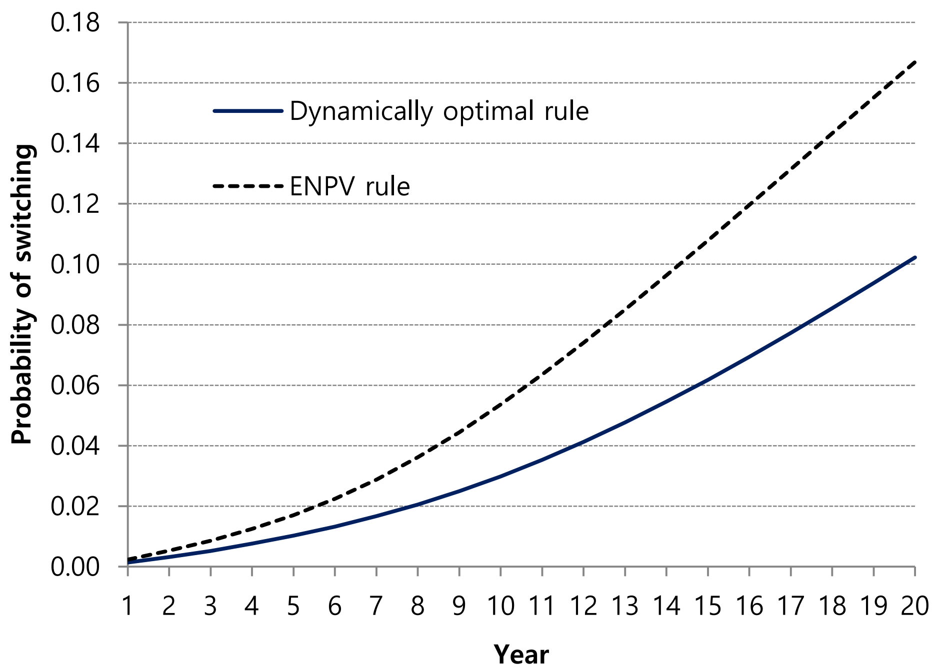
[25] 20 year period is a standard tart cherry orchard lifecycle (Black et al. 2010).

[26] The initial values are located in the 'stay' area in Figure 3.2, which is consistent with the grower's current system.

transformation is derived for the gross revenue process, the Gaver-Stehfest algorithm is directly applied to derive the probability of switching.

Since only negative jumps are allowed for the yield process and subsequently for the gross revenue process, it seems natural the probability is increasing since it is more likely to reach the threshold across a longer time horizon. As the ENPV rule sets a less stringent boundary for switching, it projects that a grower is more likely to convert the land in the longer time horizon. The ENPV ignores characteristics associated with the dynamically optimal decision rules.

Figure 3.3 Probability of switching under real option and expected net present value (ENPV) rule



3.3.3.2 Climate change scenarios

Potential yield changes are discussed in Northwest tart cherry growing regions which may be in effects in the future due to climate change.[27] In order to assess climate change effects on grower decisions, it is assumed that extreme frequency is increased by 20 percent to compare with results from the baseline scenario which maintains estimated parameters. The estimated frequency parameter (0.071) is set to increase to 20 percent to 0.085. While the baseline case indicates approximately one occurrence of an extreme event every 15 years, the change means that frequency would increase to approximately one occurrence every 12 years. Considering two spring frost events in Michigan in 10 years, this assumption is realistic and may be optimistic. Equivalent gradual change and extreme magnitude change are calculated by equation (3.25). The frequency change above is equivalent to approximately 0.3 percent yearly decrease in the drift and 71 percent increase in average size of an extreme event once it occurs, respectively.

In order to investigate robustness of the results, a sensitivity analysis is conducted. For extreme frequency change ranging from 10-40 percent with a 5 percent interval, equivalent gradual and extreme size changes are derived and resulting changes in optimal switching boundary and probability of switching are examined. The patterns for optimal thresholds and probability of switching are robust under the various set of parameters.[28]

Changes in optimal threshold to exit tart cherry production are first evaluated (Figure 3.4). Since each change is calibrated to have same expected loss as in (3.25), it is not surprising the

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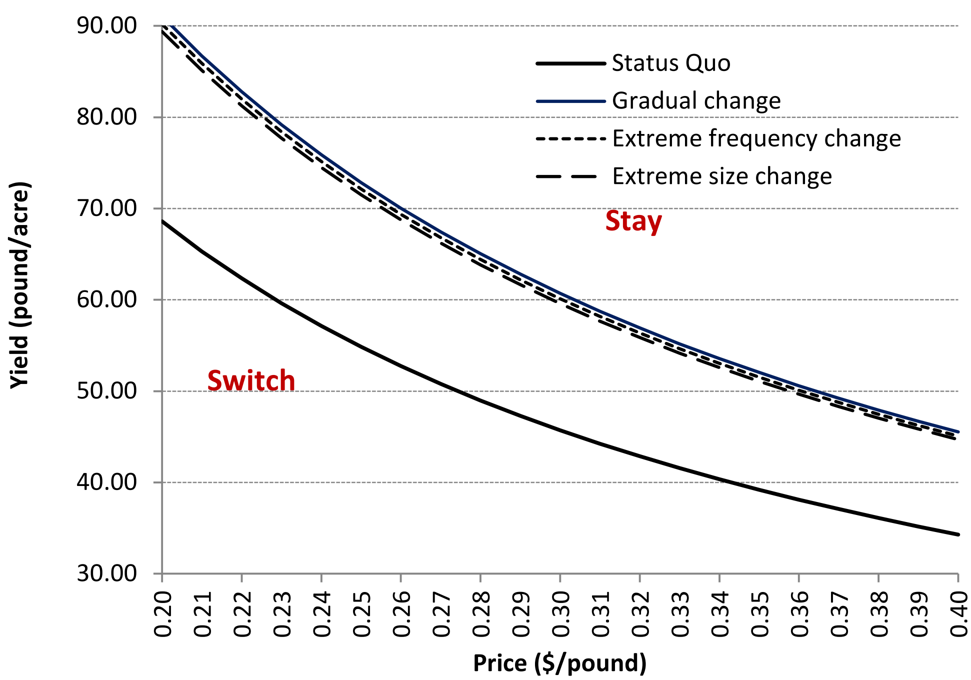
[27] For future yields under climate change, it is ideal to employ phenologically simulated data under various climate scenarios. While an international research project team for climate assessments is currently evaluating future yields of tart cherries for the Northwest Michigan, the data are not available for use at this time (Winkler et al. 2010).

[28] Detailed sensitivity analysis results are available upon request.

optimal switching threshold are close each other. Under the equivalent loss condition, the real option rule exhibits systematically different economic incentives to switch. For the three climate change scenarios (gradual, extreme frequency and extreme magnitude changes) changes in the threshold show clear ranking. A grower will have greater incentive to exit tart cherry production when climate changes gradually compared with either when an extreme frequency or magnitude increase (i.e. higher threshold). Between two extreme event cases, the grower will have greater incentive to switch under the frequency change than the magnitude change.

Figure 3.4 Changes in optimal switching thresholds



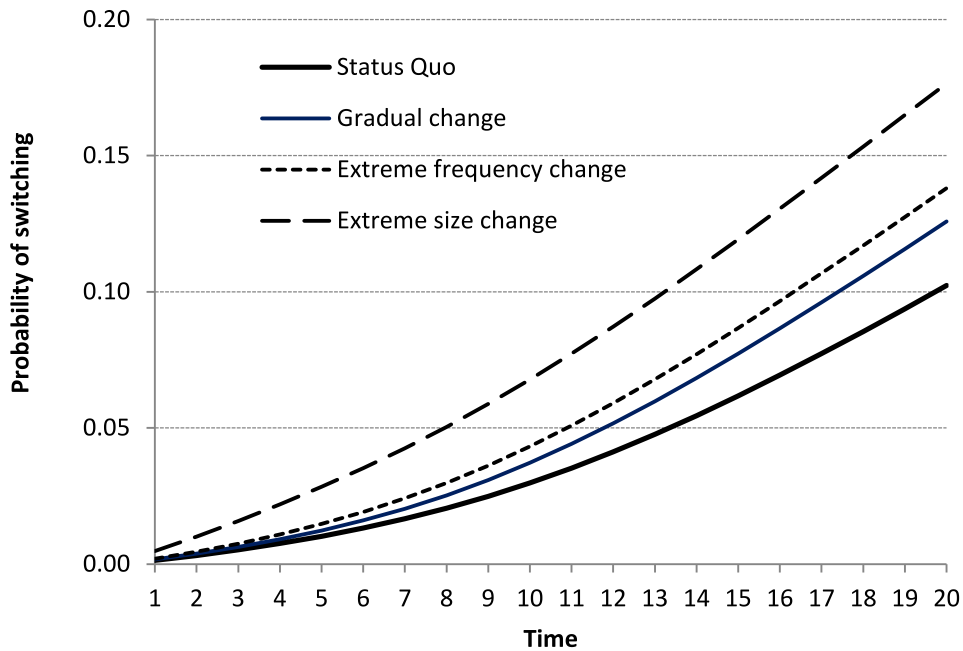
Note that gradual climate change, which is captured in the drift term of the model, has little to do with uncertainty and learning. Hence, there is little value of waiting associated with the uncertainty and learning, which induce higher incentive to switch than the extreme events. This is consistent with the stylized facts in the real world, and reflects the higher incentive to wait and see under learning patterns represented by a jump process. Between jump frequency and size changes, it is noteworthy that jump size is more closely related to the tail-behavior of given stochastic process. As such, this implies the jump size encompasses higher uncertainty and learning, which give higher incentive to wait and see.

Realized actions under changes are captured using the probability of switching (Figure 3.5). Like the baseline case, sample average price and yield for the period 1947-2012 are used for initial price and yield for tart cherries.

Recall that the real option decision rule imposes more stringent decision thresholds to extreme event cases than the gradual change. A rational grower is more reluctant to exit from tart cherry production under increasing number of extreme events or more severe extreme events than corresponding gradual change (Figure 3.4). However, the probability of switching implies it is more likely to switch under extreme event changes despite the lower exit incentive. This provides an insight about the decision making in reality. Though the real option theory stresses to disregard temporary vagaries in the decision making, the realized action may be dominated by the extreme events. This finding is consistent with another stylized fact: historically most adaptation occurred in response to risks of extreme weather events than the mean changes, e.g. in adopting irrigation technologies (Negri, Gollehon, and Aillery 2005).

Furthermore, the numerical result illustrates that the contrast between the exit incentive and probability of exit varies with the time period considered, being more significant in the long run. The differences in the incentives and probabilities of exit under the three climate change consequences also increase as the time horizon lengthens. The frequency increase implies that as the time horizon increases, it is more likely a grower will face at least one occurrence of extreme event even though that the size of that extreme event may not be sufficiently large to pass the predetermined threshold. In contrast, if the extreme event magnitude is large, one drastic extreme event may easily overshoot the threshold even if the decision rule is set to be very stringent. This pattern signifies the importance of tail-behavior in the decision making in the real world.

Figure 3.5 Changes in probability of switching



The last findings have important implications for decisions in reality: while gradual changes might play a significant role in short-run adaptation decisions, in the long run and all else equal, it is extreme events that will play a more significant role. For example, considering the high concentration of U.S. tart cherries in Northwest Michigan, extreme events may induce a significant decline of the industry in the region. This could have strong impacts on the local economy. Accordingly, the model has implications for designing adaptation policies. The dynamic decision making model has significantly different decision rules than the traditional ENPV framework. As climate vulnerability is an inherently dynamic process, correctly specifying the dynamic nature in the adaptation policies would be critical. There may be clear disparity between economic incentives and realized actions. As extreme events can induce more actions in the long run, findings here stress the importance of these events for design and implementation of long term adaptation policies.

3.4 Conclusion

As effects of climate change have been recognized more widely, there are increasing concerns about the impacts of extreme events on adaptation. A real option framework is applied to analyze the land use switching decision for tart cherry grower in Northwest Michigan. The real option decision rule implies that a rational agent may be more reluctant to switch under extreme events than under gradual changes. Decisions made in the presence of extreme events may have strong uncertainty and learning potential than under gradual climate changes, which exhibits greater incentive to stay in status quo. On the contrary, the realized actions, which are represented in terms of the probability of exit in a given time period, exhibit opposite direction compared to the economic incentives. Simulation results using Michigan tart cherry industry assure that adaptation actions actually triggered by extremes than gradual changes even with less incentive to react. Especially, the probability tends to be higher under extreme size change than other changes. This stresses the importance of tail-behavior for the adaptation to climate change in the long run.

Results in this chapter provide the following key messages to the industry. First, real options decision rule indicates that growers should be aware of uncertainty, irreversibility and learning associated with their land use decision. As shown in Figure 3.2, a dynamically optimal real option rule results in more resistance to switching than a traditional ENPV rule. In presence of uncertainty, irreversibility and learning, which is generally true for any perennial crop production, impetuous decisions can lead to sup-optimal outcomes. A real options decision rule could be particularly important in the presence of extreme events since uncertainty around them is typically large.

Second, growers must be aware of potential climate change impacts for the industry. As noted here, climate change impacts, especially potential increase in frequency and magnitude of extreme events, can no longer be considered “once in a lifetime event.” A changing pattern of dormancy release has been observed worldwide as well as in Northwest Michigan (Chmielewski, Blümel, and Pálešová 2012; Chmielewski and Rötzer 2001; Winkler et al. 2002; Zavalloni et al. 2006), which indicate greater exposure and higher vulnerability to spring frost. In addition to spring frost damage, other potential climate related impacts, which are not thoroughly explored yet, will possibly be in place in the future (Chmielewski, Müller, and Bruns 2004). Assessments of alternatives and readiness to adapt are essential to individual growers and to the industry.

Despite these broad implications, some important characteristics about tart cherry production grower decisions are not masked by employing state level data in the analysis. Tart cherry yield varies by tree age distribution per acre. Tart cherries as a perennial crop have a roughly inverse-U shaped life cycle in Northwest Michigan (Black et al. 2010; McManus 2012). Starting from establishment stage (0-5 years), which has no marketable yield, yield increases during ramping up stage (5-12 years) to maturity stage. At maturity stage (12-22 years) yield stays in peak production and then eventually starts to decline (22-30 years). Considering this yield cycle, growers typically mix trees in each age cycle block by block in their orchard to maximize profit. Thus a grower’s decision to exit may depend on tree age. Everything else equal, it is rational to abandon a block of trees at declining stage ahead of other age blocks.

Climate change impact may vary by production sites. For example, damage by spring frost depends on air movement. As cold air settles to lower level, orchards at higher elevations are less likely to be injured by frost than lower sites. Similarly, orchards adjacent to Lake Michigan are less vulnerable to frost damage due to lake effect. Since water takes longer to heat up or cool down than land, a large body of water will delay bud development during spring and thus reduce the period exposed to frost (cooling effect). This heterogeneity of sites also affects land use decision in a way that more vulnerable sites to climate change will be switched in advance.

Some extensions are worth mentioning for further empirical applications. First, as some implications are presented to adaptations to climate change in the industry, especially under presence of extreme events, policy measures need to be investigated. For the tart cherry industry revenue-based insurance scheme is under consideration with two spring freeze extremes in Michigan in 2002 and 2012. Effective insurance scheme can be examined under proposed framework in this chapter. Furthermore, other sensible policy measures can be compared with the insurance scheme under the framework. For example, the federal marketing order, which is currently in place in the industry to smooth supplies, may be evaluated with insurance scheme in terms of efficacy.

Second, though some interesting implications have been found about the effects of extreme events, the empirical application to tart cherry industry do not allow further quantitative interpretations as the model rely on assumed scenarios of future climate consequences. In order to infer quantitative implication of climate change based on the framework, a data set describing future yield consequences should be introduced to the model. The phenological data tested under various climate scenarios (e.g. GCMs or RCMs) would be extremely useful.

APPENDICES

Appendix 3.A: Proof of Proposition 3.1

Let x be the starting point of the diffusion $X(t)$. Then for any $\alpha \in (0,\infty)$, the equation $\psi_X(z)=\alpha$ has only two negative roots γ_1, γ_2 and one positive root γ^+ such that $\gamma_2 < -\eta < \gamma_1 < 0 < \gamma^+$. Let $u(x)$ be the bounded solution of $(L-\alpha)u=0$ for all $x > h^*$ where L is an infinitesimal generator of jump diffusion $X(t)$ and $u(x)=1$ for all $x \le h^*$. Then $u(x)$ can be written as $u(x) = A_1 e^{\gamma_1} + A_2 e^{\gamma_2 x} + A_3 e^{\gamma^+ x}$. For $u(x)$ to be bounded, we need to set $A_3 = 0$. Expanding $(L-\alpha)u(x)=0$ yields

$$\frac{\eta}{\eta+\gamma_1} A_1 e^{\gamma_1 h^*} + \frac{\eta}{\eta+\gamma_2} A_2 e^{\gamma_2 h^*} = 1 . \tag{3.26}$$

Also, using the continuity condition, $u(h^*+)=1$,

$$A_1 e^{\gamma_1 h^*} + A_2 e^{\gamma_2 h^*} = 1 . \tag{3.27}$$

By solving conditions above, we have $A_1 = \dfrac{\gamma_2(\eta+\gamma_1)}{\eta(\gamma_1-\gamma_2)} e^{-\gamma_1 h^*}$ and $A_2 = \dfrac{\gamma_1(\eta+\gamma_2)}{\eta(\gamma_2-\gamma_1)} e^{-\gamma_2 h^*}$. Hence, we obtain

$$u(x) = \begin{cases} 1 & x \le h^* \\ \dfrac{\gamma_2(\eta+\gamma_1)}{\eta(\gamma_1-\gamma_2)} e^{\gamma_1(x-h^*)} + \dfrac{\gamma_1(\eta+\gamma_2)}{\eta(\gamma_2-\beta^-_{1,\alpha})} e^{\gamma_2(x-h^*)} & x > h^* \end{cases} . \tag{3.28}$$

In order to prove $u(x) = E[e^{-\alpha \tau_{h^*}}]$ **,** a martingale approach can be applied. Refer to (Kou and Wang 2003) for detail.

Appendix 3.B: Testing for unit root - GLS based augmented Dickey-Fuller (ADF) test

ADF test requires fitting a regression of the form

$$\Delta y_t = \alpha + \mu t + \beta y_{t-1} + \gamma_1 \Delta y_{t-1} + \gamma_2 \Delta y_{t-2} + \cdots + \gamma_k \Delta y_{t-k} + \varepsilon_t \qquad (3.29)$$

where t is a time trend (Elliott, Rothenberg, and Stock 1996). Based on the regression ADF test is testing null hypothesis $H_0 : \beta = 0$. GLS based ADF test can be performed similarly but each variable should be GLS-detrended before run the regression above. Optimal lag length k in the equation above can be selected by removing insignificant lagged first differenced (FD) variables starting from fairly large number of lags. For log tart cherry yield and price series, there's no sign for additional serial correlation after first differencing. Thus no lagged FD variables are included in the unit root tests of log tart cherry yield and price series. The test result is given by Table 3.2.

Table 3.2 Unit roots test

	DF-GLS test statistic	1% critical value	5% critical value
$Y(t)$	-0.52	-3.71	-2.76
$P(t)$	-1.80	-3.71	-3.03

Appendix 3.C: Testing for serial correlation

The optimal switching problem in this chapter requires transition of log yield and price process to satisfy Markov property. That is, FD variables should exhibit no serial correlation. Otherwise, further transformations to these variables are needed to obtain the Markov property. In order to test for serial correlation, several kinds of tests or approaches can be employed. Serial correlation testing procedure described in Wooldridge (2012: ch.12) is applied since this approach is simple and robust. The procedure is described as following:

1) Estimate equations (3.20), save residuals $\hat{u}_t$ for all t.

2) Regress $\hat{u}_t$ on $\hat{u}_{t-1}$ and other explanatory variables in (3.20).

3) Conduct a test whether the coefficient of $\hat{u}_{t-1}$ is statistically different from zero.

4) If the coefficient of $\hat{u}_{t-1}$ is statistically different from zero, there is sign of AR(1) serial correlation.

Same procedure can be applied to log price process using the equation (3.21). Test results show that there is no sign of AR(1) serial correlation (Table 3.3).

Table 3.3 Serial correlation AR(1) test

	coefficient	p-value
$\Delta Y(t)$	-0.20	0.17
$\Delta P(t)$	-0.09	0.49

Appendix 3.D: Normality tests

As discussed in the main text, two normality tests are employed for exploratory data analysis: Jacque-Bera test and Shapiro-Wilk test of normality. Detailed results from each test can be summarized as following tables.

Table 3.4(a) Jacque-Bera test of normality

	JB test statistic	1% critical value	5% critical value
$\Delta Y(t)$	70.68	13.00	9.38
$\Delta P(t)$	0.18	5.02	4.95

Table 3.4(b) Shapiro-Wilk test of normality

	SW test statistic	p-value
$\Delta Y(t)$	0.88	0.00
$\Delta P(t)$	0.98	0.44

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CHAPTER 4: Alternative Policies to Manage Climate Change Risks in the Presence of Extreme Events

4.1 Introduction

Most recent scientific findings clearly indicate that climate change impact is inevitable unless the international community can reduce greenhouse gases (GHGs) significantly during this century (Adger et al. 2007; IPCC 2012). In the series of current international climate change negotiations (e.g. Ad Hoc Working Group on the Durban Platform for Enhanced Action: ADP), as much attention has been paid to adaptations as mitigation of GHG emissions. As interest in adaptation has grown, policy instruments which can effectively enhance adaptation practices have become more important.

Economic assessment of adaptation to major economic or environmental changes have a rich history (e.g. Hornbeck, 2012; Zilberman, Zhao, and Heiman, 2012 and references therein). In the presence of extremes it is noted that adaptation policy intervention can support to adaptive actions by individuals (See IPCC 2012 and reference therein). However, relatively little has been addressed on policy alternatives managing the climate change risks, especially in the presence of extreme events.

There are numerous policy tools which a government can use to manage climate change risks. A common form of policy intervention is to subsidize economic returns or income to the industries or individuals facing climate change risks (e.g. fixed amount subsidy, fixed rate subsidy and insurance subsidy to current enterprise). A policy measure can be employed to prevent land abandonment by encouraging continuation of a particular enterprise of other adaptive actions (Lubowski, Plantinga, and Stavins 2006; Song, Zhao, and Swinton 2011).

While abandoning a particular enterprise is a plausible adjustment for an individual decision maker, especially in the long-run (Lee and Zhao 2013), it may not be optimal from government's perspectives since the abandonment may have spillover effects for the associated industry and/or broader society. Policies can discourage rapid transitions such as mass migration between enterprises limiting spillover effects to other regions or sectors and allowing time for smoother economic adjustments. In these regards, designing and implementing a cost-effective policy is always an important objective of policymakers.

This study compare compares three types of policy measures to prevent mass land use transitions due to climate change with an explicit consideration of extremes: fixed amount subsidy, fixed rate subsidy and insurance subsidy for the current enterprise. In order to capture extreme events, return stream from current use is assumed to be a version of jump diffusion such that compound Poisson process captures extreme events. Given same government cost, the three policy tools are compared to evaluate most effective way to discourage exit from current land use.

4.2 Individual decision model

4.2.1 Enterprise exit model

Consider a representative agent (an enterprise operator) with a parcel of land where current land use faces climate change risk. Let $\pi(t)$ denote economic return from the current enterprise at time t. Letting $X(t) \equiv \ln \pi(t)$, assume the logarithmic return stream fluctuates due to weather and/or climate conditions. The (log) return process is given by

$$X(t) = x + \mu t + \sigma W(t) + \sum_{i=1}^{N(t)} Y_i \qquad (4.1)$$

where x denotes a starting point of the process. Normal variations of the log return process are captured by linear Brownian motion with drift μ and volatility parameter σ. In addition, returns are subject to extreme events which are captured by a compound Poisson process and represented by the last term on the right hand side of equation (4.1). Extreme events occur with Poisson process $N(t)$ such that

$$\mathrm{P}(N(t)=n)=\frac{(\lambda t)^n}{n!}e^{-\lambda t},\ n=1,2,...$$

where λ is a Poisson parameter capturing the probability of occurrence in an infinitesimal time interval. Since the expected value of the Poisson process depends on time (i.e. $E[N(t)]=\lambda t$), extreme events are more likely as time cummulates. Once an extreme event occurs, magnitude of the event Y_i is a random variable with a hyper-exponential distribution. The density of Y_i is given by

$$f_Y(y)=\sum_{i=1}^{n}\alpha_i\eta_i e^{\eta_i y}1_{\{y<0\}} \tag{4.2}$$

where $\alpha_i>0$ and $\eta_i>0$ for all $i=1,2,...,n$, and $\sum_{i=1}^{n}\alpha_i=1$. The distribution is highly flexible to encompass a rich set of probability distributions including heavy-tail distributions such as Pareto or Weibull through choice of parameter values α_i and η_i (Cai and Kou 2011; Cai 2009).

Since the jump diffusion process $X(t)$ is a version of the Lévy process, by Levy-Khintchine formula (Sato 1999: pp.37-40), $X(t)$ can be expressed using Laplace exponent $\psi(z)$ as

$$E\left[e^{zX(t)}\right]=\exp\left[\psi(z)t\right]$$

where

$$\psi(z)=\frac{1}{2}\sigma^2 z^2+\mu z-\lambda\sum_{i=1}^{n}\frac{\alpha_i z}{\eta_i+z}. \tag{4.3}$$

This expression will turn out to be convenient to find explicit solutions for the decision making problem that follows. In addition, by the Laplace exponent above, expected value of the return process $\pi(t)$ is readily calculated. Since $\pi(0)=e^x$ and $\pi(t)=e^{X(t)}$,

$$\begin{aligned}
&E\left[\int_0^\infty e^{-rt}(\pi(0)+\pi(t))dt\right]\\
&=E\left[\int_0^\infty e^{-rt}e^{X(t)}dt\right]e^x=E\left[\int_0^\infty e^{-(r-\psi(1))t}dt\right]e^x\\
&=\frac{1}{r-\psi(1)}e^x.
\end{aligned}$$

Suppose that an agent is contemplating exit from the current enterprise by selling land at the price denoted by S. For an arbitrary return function $g(\cdot)$ of the process $X(t)$, the expected value of $g(\cdot)$ given starting point x is $E^x\left[g(X(t))\right]\equiv E\left[g(X(t))\middle|X(0)=x\right]$. Thus the dynamic decision making problem can be written as optimal timing problem of exiting from the enterprise:

$$\sup_\tau E^x\left[\int_0^\tau e^{-rt}g(X(t))dt+e^{-r\tau}S1_{\{\tau<\infty\}}\right] \tag{4.4}$$

where $1_{\{\bullet\}}$ represents indicator function and discount rate is denoted by r. The decision problem above indicates that the agent wants to maximize total expected return over infinite time horizon by choosing optimal time τ to sell the land. Let $h\in\mathbb{R}$ be an arbitrary point for which $X(t)$ can reach and define a hitting time of h as

$$\tau_h\equiv\inf\left\{t\geq 0\middle|X(t)\leq h\right\},$$

which identifies the first time $X(t)$ reaches or crosses h from above. The optimal timing problem in (4.4) can be rewritten as optimal boundary (or threshold) problem

$$\sup_h E^x\left[\int_0^{\tau_h} e^{-rt} g(X(t))dt + e^{-r\tau_h} S1_{\{\tau_h<\infty\}}\right]. \tag{4.5}$$

Then the expected present value (EPV) in (4.5) can be evaluated as

$$\begin{aligned}\hat{V}(x;h) &\equiv E^x\left[\int_0^{\tau_h} e^{-rt} g(X(t))dt + e^{-r\tau_h} S1_{\{\tau<\infty\}}\right] \\ &= E^x\left[\int_0^{\infty} e^{-rt} g(X(t))dt\right] + E^x\left[\int_{\tau_h}^{\infty} e^{-rt}\left\{rS - g(X(t))\right\}dt\right].\end{aligned}$$

This $\hat{V}(\cdot)$ calculates EPV of land use given x and h. That is, this $\hat{V}(\cdot)$ represents the land value given that the agent decides to exit when $X(t)$ reaches h. Thus the solution to the problem (4.5) is to find an optimal boundary h^* which maximize $\hat{V}(\cdot)$.

In order to solve the optimization problem in (4.5), define normalized EPV operator $\mathcal{E}$ which calculates the normalized EPV of a return stream $g(X(t))$ as

$$\mathcal{E}g(x) = rE^x\left[\int_0^{\infty} e^{-rt} g(X(t))dt\right]. \tag{4.6}$$

Now introduce running maxima and minima of the process $X(t)$ as two extremum processes of $X(t)$: the supremum process $\overline{X}(t) = \sup_{0\le s\le t} X(s)$ and the infimum process $\underline{X}(t) = \inf_{0\le s\le t} X(s)$. These $\overline{X}(t)$ and $\underline{X}(t)$ evaluate running maxima and minima of the process $X(t)$ at time *t,* respectively. Using these processes, the (normalized) EPV-operators of a function of the supremum and infimum process can be defined accordingly:

$$\mathcal{E}^+ g(x) = rE^x\left[\int_0^{\infty} e^{-rt} g(\overline{X}(t))dt\right] \tag{4.7}$$

$$\mathcal{E}^- g(x) = rE^x\left[\int_0^{\infty} e^{-rt} g(\underline{X}(t))dt\right]. \tag{4.8}$$

The Wiener-Hopf factorization (WHF) allows $\hat{V}(x;h)$ to be represented using the normalized EPV operators in such that:[29]

$$\hat{V}(x;h) = r^{-1}\mathcal{E}g(x) + r^{-1}\mathcal{E}^{-}1_{(-\infty,h]}\mathcal{E}^{+}(rS - g(x))\,. \tag{4.9}$$

Now the optimality condition at the optimal boundary h^* is given by

$$\mathcal{E}^{+}(rS - g(x)) = 0\,.^{30} \tag{4.10}$$

In above expression, $\mathcal{E}^{+}(rS - g(x))$ identifies maximum possible 'regret' from selling the land. If an agent exits from the enterprise when realized value (and starting value) x is high, there will be 'regret' about exiting too early. Thus the agent should have waited until there is no room to 'regret.' The optimality condition reflects implications using a real options framework. If there is no *uncertainty,* the agent will not worry about regretting the decision since return fluctuations are known with certainty. The presence of uncertainty provides *learning* possibility as further information about the return process can be collected by delaying the decision. Similar logic applies if the decision is reversible so the agent can revert back to the previous state without any costs when the exit decision turns out to be unfavorable (See Chapter 2 for more detail)

4.2.2 Exit decisions with and without policy

Without any policy interventions, the return becomes $g(X(t)) = e^{X(t)}$ for each time since it is assumed that $X(t) \equiv \ln \pi(t)$. Suppose that a government introduces policy measures in order to stabilize the returns from the agent's current land use: i) fixed amount subsidy (D); ii) fixed rate subsidy (δ); iii) insurance support (M) where M denotes minimum level of return

[29] See Boyarchenko and Levendorskiĭ 2007: p.221 for proof.

[30] See Chapter 2 of this dissertation for derivation of the optimality condition.

predetermined by the government i.e. insurance coverage. If the government uses a fixed amount subsidy D regardless of the return level, the agent's return function will become $g(X(t)) = e^{X(t)} + D$ for each time t and shift up the return process uniformly. If the government establishes a fixed rate subsidy, payment will depend on returns each period so the return function for the time t is given by $g(X(t)) = (1+\delta)e^{X(t)}$. Thus the larger return level, the larger the government subsidy amount. Finally, insurance support indicates that the government subsidizes any deficiency below predetermined coverage level M and the return function can be written as $g(X(t)) = \max\left[e^{X(t)}, M\right]$. Let h^* be the optimal exit boundary without support and $h_i^*, i \in \{D, \delta, M\}$ be the corresponding optimal exit boundary with subsidies. For these return functions, the optimality condition in (4.10) yields

$$
\begin{aligned}
&\text{i) Without subsidy:} && \mathcal{E}^+(rS - e^{h^*}) = 0, \\
&\text{ii) Fixed amount:} && \mathcal{E}^+(rS - e^{h_D^*} - D) = 0, \\
&\text{iii) Fixed rate:} && \mathcal{E}^+(rS - (1+\delta)e^{h_\delta^*}) = 0 \text{ and} \\
&\text{iv) Insurance:} && \mathcal{E}^+(rS - \max(e^{h_M^*}, M)) = 0.
\end{aligned} \tag{4.11}
$$

Analytical solutions capturing optimal exit boundaries can be determined by the optimality conditions given in (4.11). Define a version of EPV as

$$\kappa_r^+(z) = rE\left[\int_0^\infty e^{-rt} e^{z\bar{X}(t)} dt\right]. \tag{4.12}$$

Then by the definition of normalized EPV in (4.7), we have

$$
\begin{aligned}
\mathcal{E}^+ e^x &= rE^x\left[\int_0^\infty e^{-rt} e^{\bar{X}(t)} dt\right] \\
&= rE\left[\int_0^\infty e^{-rt} e^{\bar{X}(t)} dt\right] e^x \\
&= \kappa_r^+(1) e^x.
\end{aligned}
$$

Analytical solutions for the optimality conditions in (4.11) can be derived if an explicit expression for $\kappa_r^+(z)$ exists. For the process defined in (4.1), it is known that there exists an explicit form for $\kappa_r^+(z)$ (See Boyarchenko and Boyarchenko 2011). Using the Laplace exponent in (4.3), define characteristic equation $r-\psi(z)=0$ where r is a discount rate. Then the characteristic equation has unique positive root. Denoting the root by β^+, $\kappa_r^+(z)$ is given by

$$\kappa_r^+(z)=\int_0^\infty \beta^+ e^{-\beta^+ y} e^{zy}dy=\frac{\beta^+}{\beta^+-z}. \tag{4.13}$$

For the first inequality above holds due to fluctuation theory of the Lévy process based on the WHF (See Boyarchenko and Levendorskiĭ 2007: Ch. 11 for more information). By the explicit expression of $\kappa_r^+(z)$, optimal exit boundary without subsidy can be evaluated as

$$\begin{aligned}\mathcal{E}^+(rS-e^{h^*}) &= rE^{h^*}\left[\int_0^\infty e^{-rt}(rS-e^{\bar{X}(t)})dt\right]\\ &= rS-\kappa_r^+(1)e^{h^*}=0.\end{aligned}$$

Hence, we can represent optimal exit boundary without subsidy as

$$h^*=\ln\left(\frac{rS}{\kappa_r^+(1)}\right)\Leftrightarrow \pi^*=\frac{rS}{\kappa_r^+(1)}. \tag{4.14}$$

Similarly optimal exit boundaries with fixed amount and with fixed rate subsidy can be derived by straightforward calculations as

$$h_D^*=\ln\left(\frac{rS-D}{\kappa_r^+(1)}\right)\Leftrightarrow \pi_D^*=\frac{rS-D}{\kappa_r^+(1)}, \tag{4.15}$$

$$h_\delta^*=\ln\left(\frac{rS}{(1+\delta)\kappa_r^+(1)}\right)\Leftrightarrow \pi_\delta^*=\frac{rS}{(1+\delta)\kappa_r^+(1)}. \tag{4.16}$$

For insurance subsidy, $\max(e^x, M) = M$ if $x < \ln M$ and $\max(e^x, M) = e^x$ otherwise. Thus $\mathcal{E}^+(rS - \max(e^x, M))$ can be evaluated as

$$\begin{aligned}
&\mathcal{E}^+(rS - \max(e^x, M)) \\
&= \int_0^{\ln M - x} \beta^+ e^{-\beta^+ y}(rS - M)dy + \int_{\ln M - x}^{\infty} \beta^+ e^{-\beta^+ y}(rS - e^{x+y})dy \\
&= rS + M\left[e^{\beta^+(x - \ln M)} - 1\right] + \kappa_r^+(1)e^{x + (\beta^+ - 1)(x - \ln M)},
\end{aligned}$$

where β^+ is the unique positive root of the characteristic equation $r - \psi(z) = 0$. Using the calculation above and the optimality condition in (4.11),

$$h_M^* = \frac{1}{\beta^+}\ln\left[\frac{rS - M}{M^{1-\beta^+}\left(\kappa_r^+(1) - 1\right)}\right] \Leftrightarrow \pi_M^* = \left[\frac{rS - M}{M^{1-\beta^+}\left(\kappa_r^+(1) - 1\right)}\right]^{\frac{1}{\beta^+}}. \qquad (4.17)$$

The presence of subsidy will lower the exit boundary than without subsidy by increasing overall return level from current enterprise. Intuitively, these policies make the agent less willing to exit. As increase in return level provide a buffer, the agent has incentive to wait more before exit. It is noteworthy that while fixed amount or fixed rate subsidies will be in effect with any positive values of returns, insurance subsidy may not function with positive coverage level. Specifically, it is possible that $M < \pi_M^*$ for some positive M. In the case, even if the insurance policy exists, government will not pay any subsidy because the agent will exit from the enterprise before the return level reaches the coverage level.

From the points discussed above, minimum effective coverage level can be derived for which the insurance policy can actually work. Let $\underline{M}$ denote minimum effective coverage then the insurance policy could work if $M \geq \pi_M^*$ thus $\underline{M} = \pi_M^*$. Straightforward calculation yield following proposition.

Proposition 4.1 *Insurance subsidy could take effect only if*

$$M \geq \underline{M} = \pi^* = \frac{rS}{\kappa_r^+(1)} \tag{4.18}$$

Proof Using $\underline{M} = \pi_M^*$, plug in the explicit expression for π_M^* in equation (4.17) and solve for $\underline{M}$. Then the minimum effective coverage is expressed as

$$\underline{M} = \left[\frac{rS - \underline{M}}{\underline{M}^{1-\beta^+} \left(\kappa_r^+(1) - 1 \right)} \right]^{\frac{1}{\beta^+}} \Leftrightarrow \underline{M} = \frac{rS}{\kappa_r^+(1)}.$$

Note that the minimum effective coverage exhibits same level as the exit boundary without subsidy. This indicates that if a government wants insurance policy to be functional, coverage level should be set above the agent's exit boundary without subsidy. Otherwise, the agent will not actually use the policy even if the insurance exists.

4.3 Cost-effective policies

Given optimal exit boundaries derived in the previous section, which show individual exit decision rules with and without policy measures, the government's perspective for designing and implementing the policy measures can be addressed. Assume the objective is to select a cost-effective policy from among these measures introduced above. Cost-effective policy indicates a policy intervention which can discourage the agent abandoning land more effectively than other policies with same level of government spending. In order to derive government spending to each policy, first passage time distribution need to be studied. While analytical method based on Laplace transformation provides rich information for the first passage time distribution (refer Chapter 2 for details), it is not sufficient to trace the government cost.

These problems can be resolved by employing simulation method. The parameter values used for simulation analysis are summarized in Table 4.1. In order to investigate policies under climate change, a set of parameter values used Chapter 2 as a baseline set. The second set of parameters indicates heavy-tail jump size distribution than the baseline. Note that both baseline and heavy-tail parameter set are calibrated to exhibit same total expected return over the infinite horizon. The enables to examine the effect of heavy-tail jump size distribution without noise by other factors.

Table 4.1 Simulation Parameters

		Baseline	Heavy-tail
Drift	μ	0.020	0.020
Volatility	σ	0.300	0.300
Jump frequency	λ	0.075	0.075
Jump size	α_1	0.5	0.5
	η_1	**2.0**	**1.0**
	α_2	0.5	0.5
	η_2	**3.0**	**11.0**
Selling Price (S)		\$50,000	
Discount Rate (r)		0.10	

The simulation procedure can be summarized as the following. First, simulate $N(=10,000)$ sample paths of log return processes in (4.1) with a set of parameters in Table 4.1 for 30 year span. Starting point of the log return process is set $x=9$ which is equivalent initial level of return to be approximately \$8103.

Second, let $G_i^k, i \in \{D, \delta, M\}$ denote government spending for each policy where k denotes a simulation number. For each policy measure, the government needs to spend $D, \delta e^{X(t)}$ and $\max(0, M - e^{X(t)})$ for each time, respectively. Insurance subsidy is selected as a pivot[31] and set a coverage level M. With the coverage level and parameter values, an exit boundary h_M^* can be derived. Using the simulated path, expected present value (EPV) of total government payment under insurance policy can be derived as

$$E(G_M^k) = \frac{1}{N}\sum_{k=1}^{N}\left[\sum_{t=1}^{30} e^{-rt}\max(0, M - e^{X^k(t)})1_{\{X^k(t)>h_M^*\}}\right]. \tag{4.19}$$

The indicator function identifies that the government will pay the insurance subsidy only when the agent stays in the current land use. Furthermore, insurance payment will be made only when return level is in between coverage level and exit boundary, i.e. $\ln M > X^k(t) > h_M^*$. Otherwise, if a simulated return level at time t is greater than coverage level, i.e. $X^k(t) > \ln M$, government will not make any payment even if the agent stays in the current use.

Third, the bisection algorithm is employed to find fixed amount and rate subsidy level which yield same EPV of government cost as the insurance subsidy. Pick an arbitrary level of fixed amount subsidy level D^1 and derive corresponding exit threshold $h_{D^1}^*$ where the superscript denotes iteration number. Then EPV of government cost under fixed amount subsidy can be calculated as

$$E(G_D^k) = \frac{1}{N}\sum_{k=1}^{N}\left[\sum_{t=1}^{30} e^{-rt} D 1_{\{X^k(t)>h_D^*\}}\right].$$

[31] Pivot policy is selected to calibrate so that each policy measure has same government cost. It doesn't matter which policy becomes a pivot for the calibration.

Let $f(D) = E(G_D^k) - E(G_M^k)$. If $f(D^1) < 0$, then pick a sufficiently large D^2 such that $f(D^2) > 0$. Then we can set $D^3 = (D^1 + D^2)/2$. If $f(D^3) > 0$, set $D^4 = (D^1 + D^3)/2$ and $D^4 = (D^2 + D^3)/2$ otherwise. Finally, iterate until $|f(D)| < \varepsilon$ where ε is predetermined tolerance level. The EPV of government cost under fixed rate subsidy is given by

$$E(G_\delta^k) = \frac{1}{N}\sum_{k=1}^{N}\left[\sum_{t=1}^{30} e^{-rt}\delta e^{X^k(t)} 1_{\{X^k(t) > h_\delta^*\}}\right].$$

Then same procedure can be applied as fixed amount subsidy case.

Fourth, EPV of government costs for three subsidies are equalized by the third step. Probability of exit for each policy for a given time T can be calculated as

$$P\left(\tau_{h_i^*} \le T\right) = \frac{1}{N}\sum_{k=1}^{N}\left[\sum_{t=1}^{T} 1_{\{X^k(t) \le h_i^*\}}\right],\ i \in \{D, \delta, M\},$$

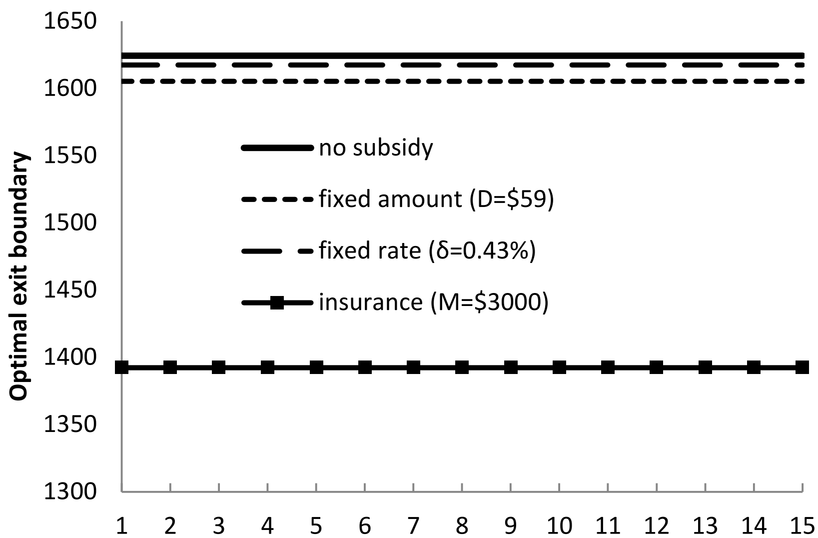
where $\tau_{h_i^*} \equiv \inf\left\{t \ge 0 \middle| X(t) \le h_i^*\right\}, i \in \{D, \delta, M\}$. The probability of exit is calculated at $T = 1, 2, ..., 30$ for each policy.

4.4 Simulation results

4.4.1 Baseline scenario

With the baseline parameters in Table 4.1, insurance coverage is set as $M = 3000$. With the coverage level, EPV of government's cost is \$482 and after costs are equalized corresponding fixed amount and rate subsidy levels are \$59 and 0.43 percent, respectively. Figure 4.1 shows changes in optimal exit boundaries comparing to no subsidy case. Under the subsidies, it is obvious that the agent lowers optimal exit boundaries, which means that the agent is less willing to exit. Since subsidies increases return stream from land, the agent has more incentive to wait before selling the land. As shown in the figure, fixed amount and rate subsidies do not exhibit prominent difference in the exit boundaries than no subsidy case. This indicates that the agent does not change economic incentive to exit significantly under these policies. In contrast, the agent noticeably lowers her willingness to exit under insurance subsidy than others.

Figure 4.1 Changes in optimal exit boundary: baseline parameters (1)



Some implication can be found from this result by looking at what derives the agent's incentive to exit. In the decision making to exit, the agent's biggest concern comes from downward movement of return process. Drops in returns could tell the coming of bad days in the future to the agent. While other subsidies also provide compensations for the drops, insurance policy removes the downward movements significantly as the risks from these drops are transferred to the government. More specifically, fixed amount subsidy induces parallel shift-up of overall return process, which does not distinguish upward and downward movements. Fixed rate subsidy, furthermore, weighs more on upward movements as higher amount is paid in the case than drops in returns. This determines the slight difference between the two policies. Insurance policy, which is targeted for downward movements, works opposite way to the fixed rate subsidy since it does not make any payment to the ups. Therefore, by eliminating considerable amount of downward risks of returns, insurance policy provides more incentive to stay in current business to the agent.

The EPV of government subsidy can be decomposed into two parts as in equation (4.20) - (4.22): (i) intrinsic cost to the policy; (ii) costs due to changes in optimal exit boundary.

$$E(G_M^k) = \underbrace{\frac{1}{N}\sum_{k=1}^{N}\left[\sum_{t=1}^{30} e^{-rt}\max(0, M - e^{X^k(t)})1_{\{X^k(t)>h^*\}}\right]}_{\text{(i)}} + \underbrace{\frac{1}{N}\sum_{k=1}^{N}\left[\sum_{t=1}^{30} e^{-rt}\max(0, M - e^{X^k(t)})1_{\{h^*>X^k(t)>h_M^*\}}\right]}_{\text{(ii)}} \tag{4.20}$$

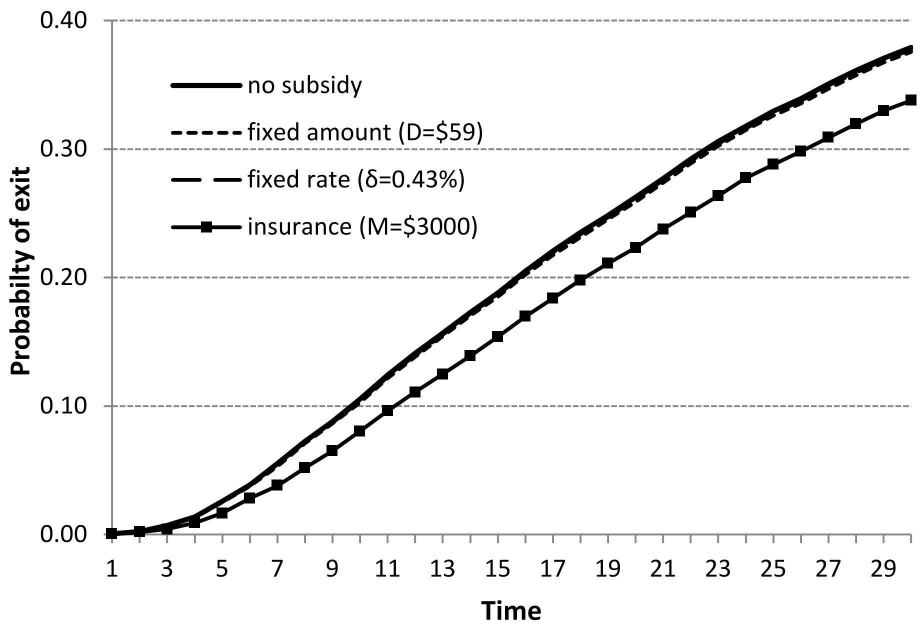
$$E(G_D^k) = \frac{1}{N}\sum_{k=1}^{N}\left[\sum_{t=1}^{30} e^{-rt} D1_{\{X^k(t)>h^*\}}\right] + \frac{1}{N}\sum_{k=1}^{N}\left[\sum_{t=1}^{30} e^{-rt} D1_{\{h^*>X^k(t)>h_D^*\}}\right] \tag{4.21}$$

$$E(G_\delta^k) = \frac{1}{N}\sum_{k=1}^{N}\left[\sum_{t=1}^{30} e^{-rt}\delta e^{X^k(t)}1_{\{X^k(t)>h^*\}}\right] + \frac{1}{N}\sum_{k=1}^{N}\left[\sum_{t=1}^{30} e^{-rt}\delta e^{X^k(t)}1_{\{h^*>X^k(t)>h_\delta^*\}}\right] \tag{4.22}$$

If a policy measure is to discourage exit from the enterprise by removing risks from current land use, the effectiveness of the policy depends on the relative size of the second portion to the first in the RHS of the equation. Of the total EPV of \$482 in this case, second part takes 30.1, 0.17 and 0.02 percent for insurance, fixed amount and fixed rate subsidies, respectively. Since more portion of total EPV is dedicated to changes in optimal exit boundary, insurance policy looks much more cost-effective than others.

These changes in optimal exit boundaries by each policy measure are translated to corresponding probability of exit. Figure 4.2 shows change in probability of exit under different policies. As expected, while fixed amount and rate subsidies barely change the probability than without policy, insurance subsidy could change exit probability significantly. Naturally this difference is attributable to the changes in the exit boundaries as discussed previously.

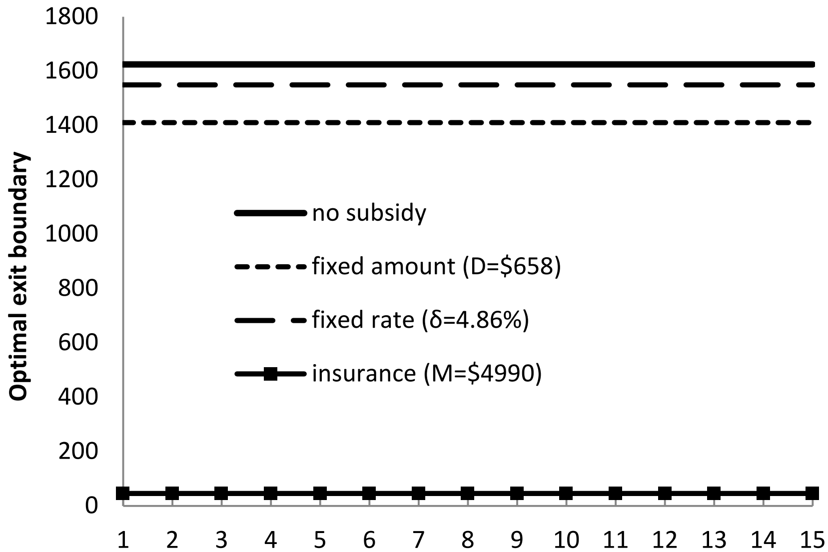
Figure 4.2 Changes in probability of exit: baseline parameters (1)



In order to examine the effectiveness of insurance policy than others more clearly, it is possible to set insurance coverage very close to fixed annual income for which the agent can attain by exiting from the enterprise. Note that if insurance coverage is set at the level, the agent's incentive to exit will be completely removed. That is, if $M > 5000 = rS$, the agent has no incentive to exit since the insurance guarantees higher income than the annual income from exiting. Setting insurance coverage level to be \$4,990, associated EPV of government cost is calculated to be \$5,483. Corresponding fixed amount and fixed rate subsidies are \$658 and 4.86 percent, respectively. The changes in optimal exit boundaries are summarized in Figure 4.3.

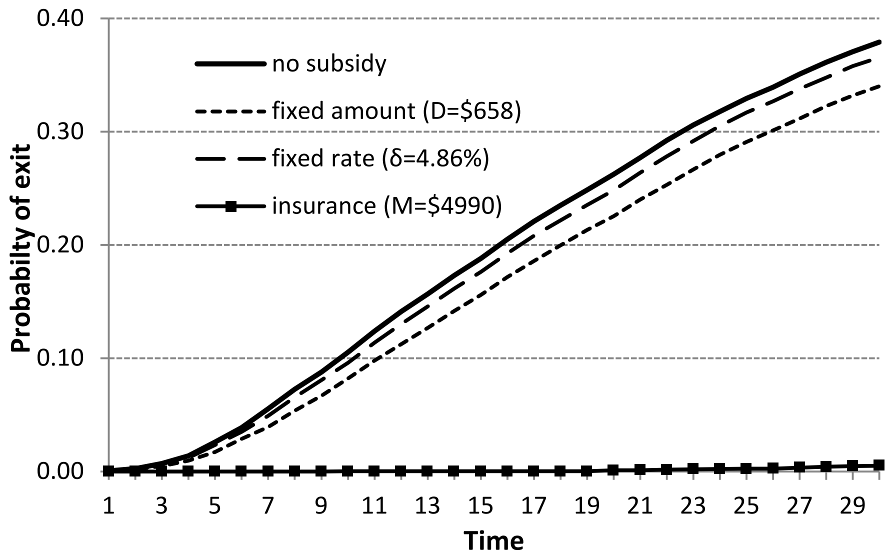
Since overall subsidy level is set to be high, it is not surprising that the agent set systematically lower exit boundaries for all policies than the previous case (Figure 4.1).

Figure 4.3 Changes in optimal exit boundary: baseline parameters (2)



However, this case also shows the significant difference between insurance policy and others. It turns out that the coverage level drives the agent's incentive to exit nearly from zero, which indicates almost no incentive to exit under the coverage level. If the proportions of total EPV attributable to changes in exit boundary are calculated according to (4.20) - (4.22), they are 53.22, 1.85 and 0.15 percent for insurance, fixed amount and fixed rate subsidies. These values indicate that the insurance support is clearly more effective than others since the insurance support actually makes the agent less willing to exit from the enterprise. As a result, following Figure 4.4 shows that probability of exit is nearly zero under insurance policy. Although other policies reduce the probability, they are not as effective as the insurance support which drives the probability of exit to zero.

Figure 4.4 Changes in probability of exit: baseline parameters (2)

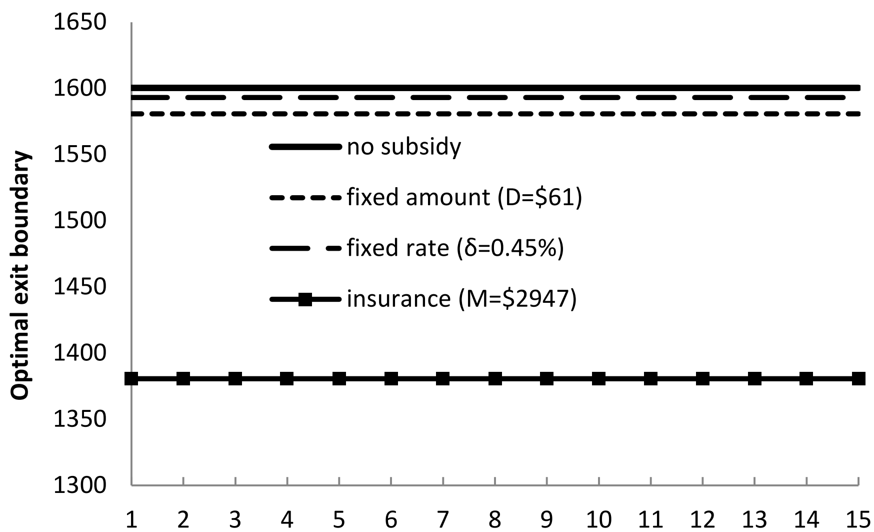


4.4.2 Heavy-tail extreme events

In this section, heavy-tail extreme events are considered. The average jumps are 0.42 and 0.55 and variances are 0.18 and 0.50 under baseline and heavy-tail scenarios, respectively. That is, once an extreme event occurs it is more likely to have a deadly drop under the heavy-tail scenario. The heavy-tail extreme events are considered in two ways. First, same government cost as the baseline case is assumed then effects of each policy is explored. Second, by maintaining same insurance coverage level and effects of each policy is examined.

Figure 4.5 summarize changes in optimal exit boundaries when same government cost is assumed as baseline scenario. Here, each policy is calibrated to reach same government cost as baseline ($482). After the calibration, associated fixed amount and fixed rate subsidy levels are calculated to be $61 and 0.45 percent which are slightly greater than the baseline ($59 and 0.43 percent), respectively. On the other hand, insurance coverage level ($2,947) is becomes lower than the baseline ($3,000). This means that government can only guarantee lower return by insurance than the baseline case.

Figure 4.5 Changes in optimal exit boundary: heavy-tail parameters (1)

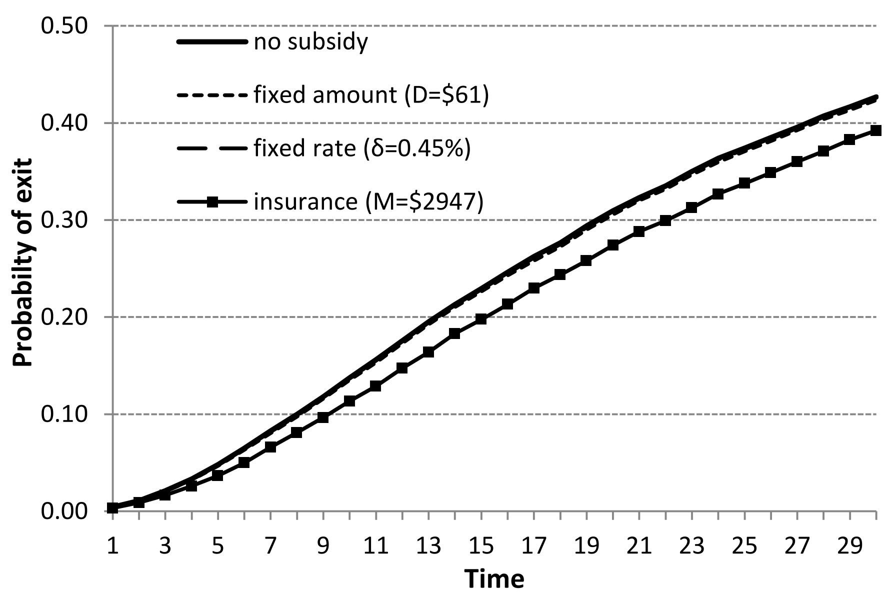


As shown in Chapter 2, probability of exit under heavy-tail extreme events is greater than the baseline scenario for every time period T without policy. Hence, government would pay fewer periods than the baseline under each policy measures. This shows that government costs may decrease due to shorter payment periods than the baseline. This is the case under fixed amount and rate subsidies. Note that in case of fixed amount and rate subsidy, a payment amount in each time is not affected considerably by heavy-tail extreme events. Hence, given same EPV of government costs, corresponding fixed amount and rate payment are greater than the baseline case due to fewer payment periods.

On the other hand, insurance payment amount is significantly affected by heavy-tail extreme events. Once insurance payment occurs, it is more likely to pay greater amount than the baseline case. While heavy-tail extreme events may shorten payment periods, overall increase in onetime payment amount dominates decreases in payment periods. Hence, for the insurance subsidy, a lower coverage level is obtained than the baseline case given same EPV of the government.

If the fractions of total costs due to changes in optimal exit boundary are calculated by (4.20) - (4.22), they are 28.01, 0.21 and 0.02 percent for insurance, fixed amount and fixed rate subsidy, respectively. Comparing these values with corresponding baseline case (30.1, 0.17 and 0.02 percent, respectively), it is obvious that smaller fraction of government spending is dedicated to prevent the agent from exiting in case insurance support. While insurance subsidy is still more effective than others, it loses its effectiveness seriously than the baseline scenario. This indicates that given same government cost insurance support become less effective under heavy-tail extreme events. Figure 4.6 summarizes changes in probability of exit under heavy-tail extreme events. The figure shows that the gap between insurance support and others shrinks under this situation than the baseline case (Figure 4.2).

Figure 4.6 Changes in probability of exit: heavy-tail parameters (1)



It is possible to ask by how much government will spend if the government set same coverage level used in the baseline (Figure 4.1). If same coverage level ($3,000) is maintained as the baseline scenario, two opposites effects may happen for insurance payments each time as discussed before. First, government costs decrease since overall probability of exit is greater under heavy-tail extreme events than the baseline so that government may pay fewer periods than the baseline case. Second, government cost increase since average payments may be greater each time due to drastic extreme events. Under the same coverage level, the latter still dominate the former so that EPV of government costs under insurance policy increase. That is, coverage level is set to be $3000 and simulation results shows that EPV of government cost increase to $534 from $482 of the baseline case. After calibrating other policies to have same EPV of government costs, corresponding fixed amount and rate subsidy levels are $67 and 0.50 percent, respectively. While coverage level is same as baseline case, fixed amount and rate subsidies increase than the baseline due to fewer payment periods.

Figure 4.7 Changes in optimal exit boundary: heavy-tail parameters (2)

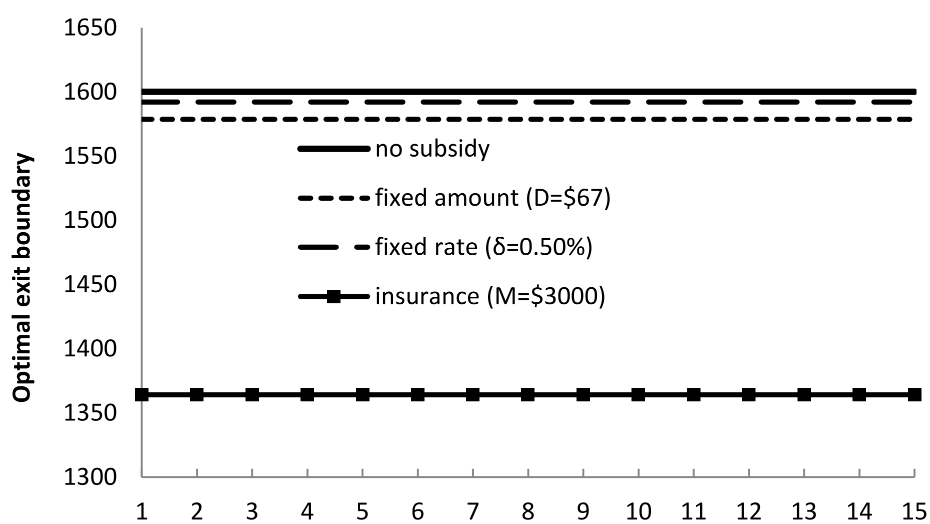
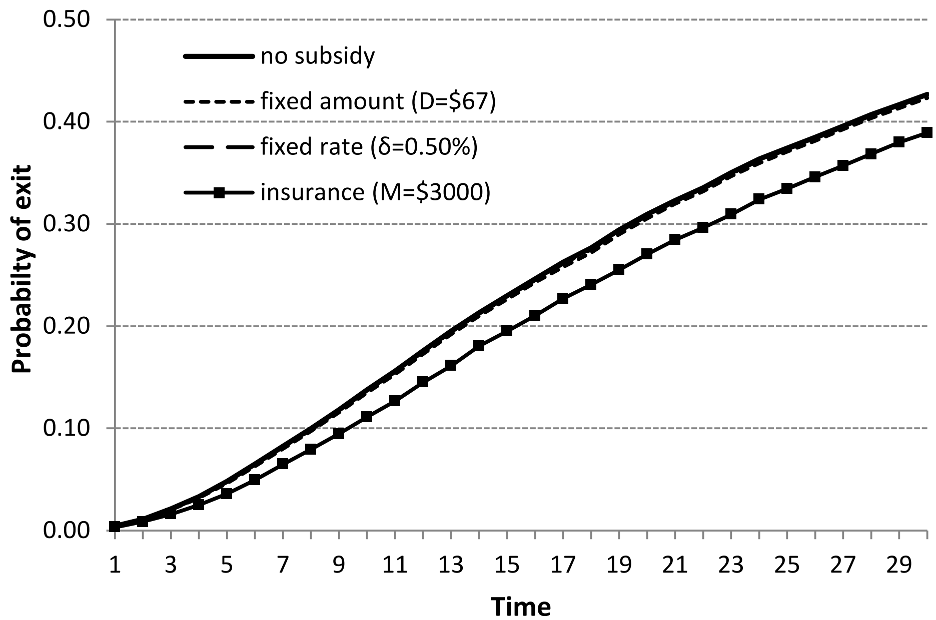


Figure 4.8 depicts probability of exit. If these results are compared to those in Figure 4.1 and Figure 4.2, insurance policy still remains less effective with same coverage as the baseline. Specifically, if proportions of EPV attributable to changes in optimal exit boundary are calculated, it comes to 29.18 percent, which is still less than the 30.10 percent of the baseline scenario.

Figure 4.8 Changes in probability of exit: heavy-tail parameters (2)



The overall discussions in this section in general indicate that insurance policy is more effective than others given same government's costs. It is mainly because insurance policy is designed to reduce downward risks of stochastic return stream in which the agent faces. In contrast, as other policies work neutrally (fixed amount subsidy) or opposite direction (fixed rate subsidy) to return levels, they do not reduce risks underlying the stochastic returns.

Although the results show that the risk transfer by insurance policy prevents exit from current land use, this should not be interpreted as discouraging overall adaptations to climate change. As stressed in IPCC (2012: ch. 9), insurance policy can promote adaptation activities by providing fund to recovery and protective activities to extreme events, which will ultimately reduce vulnerability and exposure to extreme events.

The results assuming heavy-tail extreme events, which indicate deadly extreme events occurring, show that government cost may increase if government wants to maintain same cost-effectiveness than light tail extreme events. This implies that insurance policy cannot handle such risks with same costs if extreme events become severer due to climate change.

4.5 Conclusion

This chapter compares three common policy schemes that might be used to delay land use changes due to climate change events including a focus on extreme events: fixed amount subsidy, fixed rate subsidy and insurance subsidy. Results show that given same government cost, insurance subsidy as a risk transfer mechanism appears to be most cost-effective way to prevent exit from current land use.

However, empirical simulation shows that government cost increases significantly under heavy-tail extreme events which represent severe extreme events due to climate change. This finding implies that design and implementation of insurance policy may be more important despite its cost-effectiveness among examined policy options.

Therefore, exploring well-designed insurance scheme could be an important extension from this study. The use of insurance policy combined with premium discount or other restrictions reducing climate risks can be an important topic in the future research. It is well known that insurance could be subject to adverse selection and moral hazard problem inherently so that insurance support by government may actually increase vulnerability and exposure to the climate change and extreme events as individuals do not invest in mitigating the risks. Due to these concerns, an insurance policy needs to be designed to accompany individual decision maker’s investments to reduce risk from extremes in compensation of government’s risk sharing.

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CHAPTER 5: Concluding Remarks

In climate change discussions extreme events have received increasing attention from the public and policymakers as well as from researchers. A fundamental question is how adaptive responses to extreme events are different from those to gradual climate change. This dissertation examines the effects of extreme events on adaptation by comparing them with the effects of gradual climate change, focusing on decision making and timing of adaptive actions within a real options framework.

Our results contribute to the adaptation literature by proposing a formal model to consider extreme events explicitly under the real options framework. Throughout this dissertation, major findings suggest that extreme events may not be treated equally as gradual climate change in terms of adaptation incentives and adaptation action. The theoretical model in Chapter 2 shows that adaptation decisions under extreme events and gradual change are unequivocally different from each other in both decision incentives and implementation timing. These characteristics are further validated through the empirical application and policy analysis in the following chapters.

Chapter 2 presents a formal economic framework distinguishing extreme events and gradual change in terms of willingness and likelihood of taking adaptive actions. A traditional real option model, which relies on continuous stochastic processes, is extended to incorporate jump diffusion model reflecting extreme events and gradual change altogether. This chapter highlights several key insights on the adaptation process under extreme events and gradual changes. First, gradual changes lead to higher *ex ante* willingness to adapt even if both gradual changes and extreme events give rise to the same expected damages. Second, the *ex post* likelihood of adaptive actions exhibits the opposite pattern as extreme events induce actions faster than gradual change if both has equal expected losses. This opposite rankings of willingness and likelihood of adaptations explain stylized patterns demonstrated in adoption literature where decision makers do not take extremes seriously *ex ante* but respond to them *ex post* once extreme events occur.

In Chapter 3 the theoretical framework is applied to land use decisions of exiting from a tart cherry enterprise in Northwest Michigan, which is highly susceptible to extreme weather events such as spring frost. Empirical results in this chapter reaffirm the theoretical insights in Chapter 2. The likelihood of taking adaptive action, which is estimated by the probability of exit in each time span, is higher under extreme events than gradual change. This implies that even with more stringent exit thresholds, cherry growers are more likely to act in response to the extreme cases than gradual change. Projections about extreme events in the growing region could be a critical indicator about dynamic adjustment of the industry.

The model is reformulated in Chapter 4 to examine potential government policies managing climate change risks. Three common policies (fixed payment, fixed rate subsidy and insurance support) to subsidize a decision maker's economic return stream, which is at risk due to climate change impacts, are evaluated for cost-effectiveness of government spending. Empirical simulation in the chapter indicates that insurance support outperforms other policy measures given equivalent government costs by successfully removing climate-driven risks from individuals. In further analysis, however, more severe extreme events represented by heavy-tail jump distribution may dampen the effectiveness of insurance support by increasing government spending rapidly. This insight asserts that insurance support as a risk transfer mechanism needs to be accompanied by proper risk protection measures imposed to individual beneficiaries.

Several important implications can be delivered from this dissertation. First, the role of uncertainty, learning potential over time, and non-negligible sunk costs should be addressed adequately to understand adaptation processes under extreme events. The real option modeling proposed in this dissertation is proven to successfully explain stylized patterns of real world adaptations and thus could be one of excellent tools modeling adaptations to extreme events as well as gradual change. Second, since adaptation actions are often induced by extreme events than gradual changes, decision makers should carefully assess their vulnerability to extreme events in planning and implementing adaptive actions. Third, vulnerability assessment to extreme events is also important to policy makers since it provide a basis for developing cost-effective policy tools directed at reducing risks from the extremes. Finally, detailed and accurate information about future changes in the pattern of extreme events and their impacts could be valuable to enhance effective adaptations. Hence, adaptations under extreme events require multidisciplinary efforts of researchers, decision makers, policy makers and other stakeholders.

The modeling strategy proposed in this dissertation could provide a fertile ground for modeling adaptations in presence of extreme events. First, future work could test other jump diffusions as alternative representations of climate change effects. Although the conceptual model employed here is highly flexible to capture extreme effects, other types of jump diffusion models could expand both theoretical and empirical perspectives.

Second, more empirical analyses should be done. The applications could be progressed by introducing detailed climate scenarios (e.g. GCMs or RCMs), various adaptation measures and related polices. The modeling proposed here can be employed in a broader class of adaptation assessments which investigate adaptation capacity, strategies and policies of a specific region or industry. Especially, the model fits well for a region or an industry which is susceptible to intense weather events with long-term capital investment commitment and limited adaptation options.