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This is to certify that the

thesis entitled

AN ANALYSIS OF SILAGE PRESSURES IN
CYLINDRICAL TOWER SILOS

presented by

Wei Wen Yu

has been accepted towards fulfillment
of the requirements for

Ph D. degree in Agr. Eng.

A handwritten signature in cursive script, reading "James S. Boyd".

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Date Aug. 26, 1963

117

ABSTRACT

AN ANALYSIS OF SILAGE PRESSURES IN CYLINDRICAL TOWER SILOS

by Wei Wen Yu

Constant reports of silo structural failure called for an investigation into conditions within filled silos. Horizontal, vertical and frictional silage pressures were measured from a 30 ft. silo 60 ft. high, for immediate application in silo design.

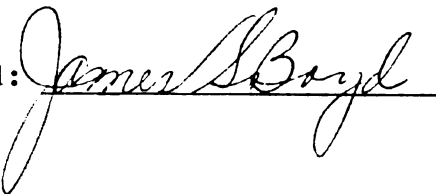
In the past four years, strain-gage pressure cells, pressure panels, air pressure gages and two-way gages were used to measure the pressures in a 30 x 60 ft. silo. These pressures are affected by unit weight of silage, coefficient of friction between silage and silo walls, the ratio of horizontal to vertical pressures, moisture contents and method of handling. The variation of unit weight of silage, coefficient of friction and the ratio of horizontal to vertical pressure were measured and formulated in terms of depth of silage.

Maximum horizontal pressure was observed at an elevation of 5 to 10 ft. from the bottom of the silo, each year. The frictional pressure on the silo wall remained almost constant when the silage depth exceeded 40 ft. Vertical pressure on the floor had several high

pressure zones. Empirical formulas were developed by least square curve fitting and may be used as a reference for the design of 30 ft. diameter silos.

An analytic investigation was made to find satisfactory formulas upon which to base silage pressure calculations. Horizontal, vertical and frictional pressures were analyzed with and without considering arch action in silage. The occurrence of maximum horizontal pressure before reaching the bottom of the silo is characteristic of the pressure distribution along the depth when considering arch action. No such occurrence is found when there is no arch action.

More experimental data are needed on density, coefficient of friction between silage and silo walls, pressures and the ratio of horizontal to vertical pressure to verify the validity of the assumptions used in this study.

Approved: 

AN ANALYSIS OF SILAGE PRESSURES IN
CYLINDRICAL TOWER SILOS

By

Wei Wen Yu

A THESIS

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CHAPTER I

INTRODUCTION

For over 80 years, silos have played an important and necessary part in livestock farming. In earlier days horizontal pits were used for feed storage. Gradually, cylindrical tower silos became more common. The number of silos made of wood and blocks decreased with the increase in the size of silos required on farms. In addition to monolithic concrete silos, stave silos have become widely accepted. During the period 1940-1945, a study of silage pressure was made by the United States Department of Agriculture. The pressure was measured in full scale farm silos of sizes up to 18 ft. diameter, 45 ft. high. Empirical formulas for silage pressure were the results of this investigation. These formulas have been used commonly since then for calculating silo pressures.

Recently, following the advance and development of field machines for harvesting, transportation of silage, and mechanization of feed handling, large silos have become more and more desirable. However, the knowledge of silage pressures in large silos, unfortunately, still was very limited. Builders designed silos according to their own experiences or by the extrapolation of existing

information on silage pressure. Numerous structural failures of large silos have demonstrated the need for information on which to base designs.

Objective

The objective of this study was to develop formulas for computing silage pressure in cylindrical tower silos.

CHAPTER II

REVIEW OF LITERATURE

Silage

Silage has been defined in the dictionary as fodder, either green or mature, converted into succulent winter feed for livestock through the process of fermentation.

Silage which has been most common in livestock farming is as follows:

1. Corn silage: The corn crop is harvested and fed in a wide variety of ways. The three most common methods of harvesting corn silage are (a) the whole-plant, (b) high moisture shelled corn, (c) high moisture ground ear corn. Sometimes the stalk is ensiled after the ears have been picked. A farm operator can use all three types of corn silage and adjust the rations as necessary because of changing needs in his livestock program.

2. Grass silage: Alfalfa, legume, and clover are very common. It is defined as silage made of uncured hay or forage crops. Usually moisture contents of grass silage are very high, at 75% or more. Excessive leakage of soluble nutrient in the juice is found as a result of high moisture contents of silage ensiled in the silo.

3. Grass haylage: To reduce the loss in nutrients, forage crops are wilted before ensiling. This process results in higher storage capacity and less loss in nutrient.

Density

Silage pressure was indirectly obtained from density by Otis and Pomroy (1957). They measured densities of corn silage by the layer method, the surface sampling method and the horizontal core sampling method at various elevations. The samples used for the density measurement were placed in airtight plastic bags and reused in the laboratory in pressure measurement. A portion of each sample was placed in a cylinder and compressed to the density in the silo at the time the core sample was taken. Comparison of the pressure obtained in the laboratory and density measured at sampling showed good agreement with high density material located in high pressure area.

Perkins et al. (1953) found that both the stage of growth of the ensiled material and the depth of silage affect the silage density. They thought fineness of cut was an important factor affecting the density of silage, for both corn and alfalfa.

James et al. (1962) used a technique of passing gamma rays through the silage and measured densities by a gamma spectrometer, in a silo 12 ft. in diameter

and 25 ft. high. They found that alfalfa silage density varied from 25 to 40 lbs. per cu. ft. and the density is not directly proportional to the depth of silage.

Aldrich (1962) compiled apparent unit weight of corn or grass silage, with moisture contents of from 68 to 72% in tower silos. Data were obtained from silos up to 20 ft. in diameter and up to 40 ft. high. The curves were extrapolated beyond 40 ft.

Friction Coefficient

Kleis and Okamura (1962), using laboratory scale storage silos, measured friction of haylage on silos made of concrete, vitrified clay, Permaglass and wood staves. All test silos were 15 in. in inside diameter and 42 in. high, except the Permaglass units which were 16 in. in diameter. They were similar to conventional silos in effective cross section. Vertical forces were measured with predetermined silage weights and densities. Frictional forces were determined by extruding the total haylage mass from the silo unit. Statical and dynamical friction were measured. The results provide information on the magnitude of coefficient and forces as related to silo materials, surface treatment, densities, moisture content temperature and crop type.

As a by-product of panel measurements by Boyd and Yu (1963), frictional coefficient was obtained in conjunction with the measured horizontal and frictional forces. The frictional coefficient for corn silage was formulated in polynomial equation of silage depth.

Pressure

Silo deterioration and failures brought demands from farmers for assistance and to meet this demand an investigation was instigated at the New Jersey Agricultural Experiment Station in 1937 with the cooperation of the Bureau of Agricultural Chemistry and Engineering, USDA, to measure the pressures exerted on silo walls by grass silage. Pressure was found to be influenced by moisture content of silage, silo size and fineness of cut. Maximum values for grass silage varied from 159 lbs. per sq. ft. in a 12 ft. diameter silo with 64% moisture silage, wet basis, at 25 ft. head to 1189 lbs. per sq. ft. in an 18 ft. silo with 77% moisture silage at 40 ft. head. This was a range of from approximately one-half to two and one-half times the pressure commonly considered in silo design which is 12 lbs. per sq. ft. per ft. of depth. The panel used for pressure measurement averaged about 4 sq. ft. in area. (Besley, 1941)

Panels were developed by McCalmont (1946) to measure silage pressure. The pressure panel was built up of two layers of 1 by 6 in. lumber as in an ordinary silo door. To this were attached two vertical pieces of 3 in. T-section iron with 12 in. between the outside faces to transfer the load to the calibrated steel bars. The panel was supported on two 5/8 in. round alloy steel bars that passed through the holes in the T-section and rested in corresponding holes in the channels along each side of the doorway. The whole door panel was used as the sensing element to measure pressure. Test work began in 1936. During the following 6 years a total of 8 separate tests were made with corn silage and 17 with grass silage. The size of silos ranged from 14 to 20 ft. in diameter and 25 to 40 ft. in height. He found that corn silage exerted less horizontal pressure than grass silage under comparable moisture conditions, but high moisture corn silage caused much higher pressures than those for which silos had been designed. The increase in pressure due to the increase in moisture contents is greater than that due to increase in diameter. The frictional pressure, measured by vertical deflection of the rod in the pressure panel, was found not significant compared with the horizontal pressures. As the silage was fed out, the frictional pressure gradually decreased

until they changed direction and actually exerted uplifting pressure on the silo walls.

One of the important developments associated with the mechanization of cattle feeding has been the use of larger diameter and higher silos. This increase in size presented a problem for the silo industry. Boyd and Yu (1960) developed a group of pressure cells, made of four A-18 SR-4 strain gages attached to a stainless steel diaphragm. The cells were 2 in. in diameter and embedded in the concrete staves so the surface of the cell was flush with the wall. The staves were placed in 4 quadrants at 6 elevations, 0, 2.5, 5, 10, 20, and 30 ft. from the ground in 60 ft. silo. Thirteen cells were placed on the floor in 4 radial directions. The highest horizontal pressure was found some distance above the bottom of the silo. During the summer of 1960, with the cooperation of the USDA, a 30 x 60 ft. silo was equipped with a column of pressure panels. These panels were similar to those used earlier in New Jersey and had an area of 5 sq. ft. per panel. Each panel was supported by two 1/2 in. heat treated steel rods mounted so that the rods were floating in a rigid support on the silo. The horizontal and frictional pressure was found from panel pressure measurement.

Pressure Formulas

In 1946, based on the panel pressure measurements by McCalmont (1946) in the previous 6 years, formulas for horizontal pressures were formulated by him.

For silos 14 ft. and less in diameter

$$H = \frac{D z^{1.195}}{2.65} \quad (1)$$

and for silos 16 ft. and more in diameter

$$H = \frac{D z^{1.45}}{5} \quad (2)$$

where

H = horizontal design pressure in psf

z = depth of silage in ft.

D = diameter of silo in ft.

The formulas were recommended for the design of silos up to 20 ft. x 45 ft. for both corn and grass silage with moisture contents below 74% wet basis.

Gurrey, et al. (1946) compiled the ACI standard for the construction of concrete farm silos in 1946. The horizontal pressure adopted for the standard was obtained in a research study as described previously, by USDA, ACI, NSA, etc. Because silos are built with heights greater than 40 ft. extrapolation became a

practical necessity. Knowing the danger of extrapolation, Gurrey obtained the curve of best fit by least squares, disregarding moisture content. The ACI horizontal pressure formula then became,

$$H = 3.3 z^{1.44} \quad (3)$$

in which

H = horizontal pressure in lb./ft.²

z = depth of silage in ft.

The maximum moisture was specified as 75%. The vertical component, i. e. frictional load, was also evaluated. The curve of best fit for the data from the panel measurement was found to be,

$$F = 5.5 z^{1.08} \quad (4)$$

in which

F = frictional pressure in lb./ft.²

z = depth of silage in ft.

Newbauer (1960), using available data from ACI, formulated the pressure function in terms of depth of silage, diameter of silo, and moisture contents of stored material. Since the available data are limited to practical existing conditions the suggested formula applies

only within certain ranges. He recommended that the following limits be used:

$$z = 5 \text{ to } 75 \text{ ft.}$$

$$d = 10 \text{ to } 20 \text{ ft.}$$

$$m = 60\% \text{ to } 90\%$$

The proposed equation for the horizontal pressure was:

$$H = 0.0133 z (d - 6) (m - 50) \quad (5)$$

in which

$$z = \text{depth of silage in ft.}$$

$$H = \text{horizontal pressure in lb./ft.}^2$$

$$d = \text{diameter of silo in ft.}$$

$$m = \text{moisture contents in per cent}$$

The most popular formula being used by silo industries is the formula by Janssen (1895). The formula was derived under the assumption that the stored material is uniform along the depth of deep bins under consideration. Coefficient of friction, moisture content of material, and the ratio of horizontal to vertical pressure are independent of the depth. The first order differential equation was derived from the equilibrium of a thin layer of material in the silo. The solution was obtained as follows.

$$V = \frac{w R}{k \mu} (1 - \exp (- k \mu h / R)) \quad (6)$$

$$L = kV \quad (7)$$

in which

V = vertical pressure of grain in lb./ft.²

L = horizontal pressure of grain in lb./ft.²

w = unit weight of grain lbs. per ft.³

U = circumference of bin in ft.

$R = A/U$ = hydraulic radius of bin in ft.

μ = coefficient of friction between grain and bin wall

k = ratio of L to V

h = depth of grain in ft.

Janssen's formula is applicable when the material stored in the silo is uniform. He did not consider the variation of w , μ and k along the depth and possible arch action in bulk material. In silage pressure calculation, as the density, coefficient of friction between silage and silo wall and the ratio k , vary with the change in depth of silage, Janssen's formula is no longer valid. A new formula is desired to include the variation of factors w , μ and k along the depth of silage and possible arch action.

CHAPTER III

INVESTIGATION

As tools for investigation for silage pressures, direct and indirect methods have been used. The direct method includes panel, transducer, pressure strip and cell pressure measurements. The indirect method includes density measurement and the pressure is found by reproducing the equivalent density with applied force.

Most research workers found it extremely difficult to obtain the data on silage pressure for silo design. Silage pressure was affected by many factors such as density, friction coefficient between silage and silo wall, k , the ratio of horizontal to vertical pressure, maturity of silage, method of filling silos, and elapsed time after filling. Among those factors, density, coefficient of friction and ratio k are more significant and can be measured and controlled in the design of the silo.

Silage Density

An important factor causing pressure is silage density. In the process of ensiling, forage, grass and whole plant of corn are all chopped, and hauled by wagon to the blower. Silos are filled by blowing

material to be ensiled in at the top of the silo. At the bottom of the silo, silage is compressed by the filled silage. Silage is usually cut $3/4$ to $3/8$ in. in length, but comparing with the fine grain material, it is irregular in size. The interlocking of leaves and stalks causes the mass of silage to become spongy. The additional load of silage causes the consolidation of material, similar to clayey soil in nature. Liquid in silage, then, is exuded and becomes free flowing liquid.

From the observation of silo failures in recent years silos failed most often immediately after filling with excessively wet silage. When silage under the surcharge load arrives at the saturation limit, the excessive moisture will flow out from a drain or from openings in silo walls. The density of saturated silage may become very high. Even at the saturation point, the density of silage may still be greater than that of water. In general, due to the cavity and air space in the silage mass, the upper limit of silage density may be considered as the unit weight of water.

We may assume that density is directly proportional to the increase in moisture content before saturation. Choppers now available all have adjustable length of cut. For forage crops, leaves and stalks do not vary much in fineness of texture and relative structure and size. But

for corn silage, there are great variations. With a proper combination of sizes extremely high density is possible. On the contrary, extremely low density is also possible if only the light leaves accumulate at the down wind side of the blower pipe. The fineness of chopped silage thus may affect density.

During the filling operation, interruptions are very common. The filling operation may stop for several hours or several days and this unsteady rate of weight increase affects the density increase.

Research workers have found it extremely difficult to estimate density specifically from one silage to another. Several compilations of silage density were found, in Table 1. (Otis and Pomroy, 1957). For calculating pressure by formula it is desirable to express density as a function of depth, which can be specified definitely. Density can be defined as a function of depth of silage, moisture content, length of cut, method of handling, rate of filling, maturity of silage crops and weather condition during the ensiling operation. Depth of silage and moisture content can be expressed in numbers while the other factors are qualitative and not controllable. Experience and investigation have shown that to avoid spoilage and keep the juice loss at the minimum the moisture content of the silage should be held between 50 and..

85%. When the moisture content is low, due to its spongy property, the storage capacity is reduced. At the same time, silage spoilage may increase. If the moisture content of silage is kept below 85% and above 50%, density can be specified in terms of depth of silage only as shown in Figure 1.

Polynomial equations are derived by means of least square curve fitting. Sets of points from Table 1 were fitted into least square polynomials of second and third degree. Coefficients of the least squares polynomials are tabulated in Table 2.

The second degree polynomial from data A in Table 2 is most desirable in relating density and depth for actual application. The equations recommended by author for use are as follows:

$$f_{11}(z) = w = 40.519 + 0.434 z \quad (8)$$

$$f_{12}(z) = w = 34.779 + 1.121 z - 0.0104 z^2$$

$$f_{13}(z) = w = 30.051 + 2.076 z - 0.0454 z^2 + 0.000332 z^3$$

Friction Coefficient

Two types of coefficient of friction arise in tower silos, coefficient of internal friction and coefficient of friction between silage and silo walls.

Coefficient of internal friction is defined as tangent of the angle of repose of silage. From 1959 to 1962, the angle of repose for corn silage was observed by author

TABLE 1
DENSITIES OF CORN SILAGE IN LBS./CU. FT.

Depth of Silage Ft.	A	B	C	D	E	F
	ASAE Data 1963 Year Book	Univ. of Minn. 1940-1942	USDA Circ. 603 1941	Kan. Agr. Exp. Sta. 1919	USDA FB- 1820 1939	Wisc. Agr. Exp. Sta. 1897
5	40	34	40	35	26	25
10	48	39	47	38	35	33
15	53	42	51	40	42	40
20	55.5	44	53	41	43	46
25	57	46	54	42	54	52
30	58.5	47	54.5	43	58	56
35	59.5	48	55	43.5	63	60
40	61	-	-	-	-	-
45	62	-	-	-	-	-
50	63	-	-	-	-	-
55	63.5	-	-	-	-	-
60	64	-	-	-	-	-
65	64.5	-	-	-	-	-
70	65	-	-	-	-	-

TABLE 2

COEFFICIENT OF 2ND AND 3RD DEGREE POLYNOMIALS FOR DENSITIES OF CORN SILAGE
BY LEAST SQUARE CURVE FITTING, w IN LBS./CU. FT. AND z IN FT.

	$w = a_0 + a_1z + a_2z^2$			$w = a_0 + a_1z + a_2z^2 + a_3z^3$			
	a_0	a_1	a_2	a_0	a_1	a_2	a_3
A	34.770	1.129	-0.0104	30.051	2.076	-0.0454	0.000332
B	29.714	1.014	-0.014	27.714	1.469	-0.041	0.000490
C	34.215	1.441	-0.0248	29.714	2.465	-0.085	0.001000
D	32.500	0.601	-0.0083	31.000	0.943	-0.0284	0.000330
E	17.429	1.874	-0.0167	15.429	2.329	-0.0434	0.000450
F	16.390	1.831	-0.0170	16.390	1.831	-0.0170	0.000000

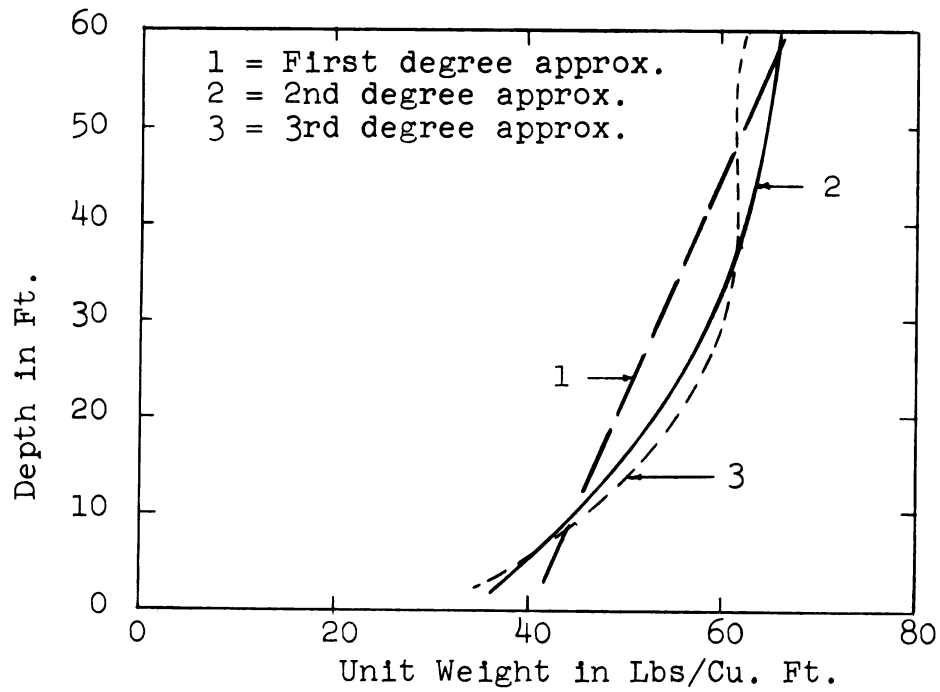


Figure 1. Unit Weight of Corn Silage from Least Square Curve Fitting

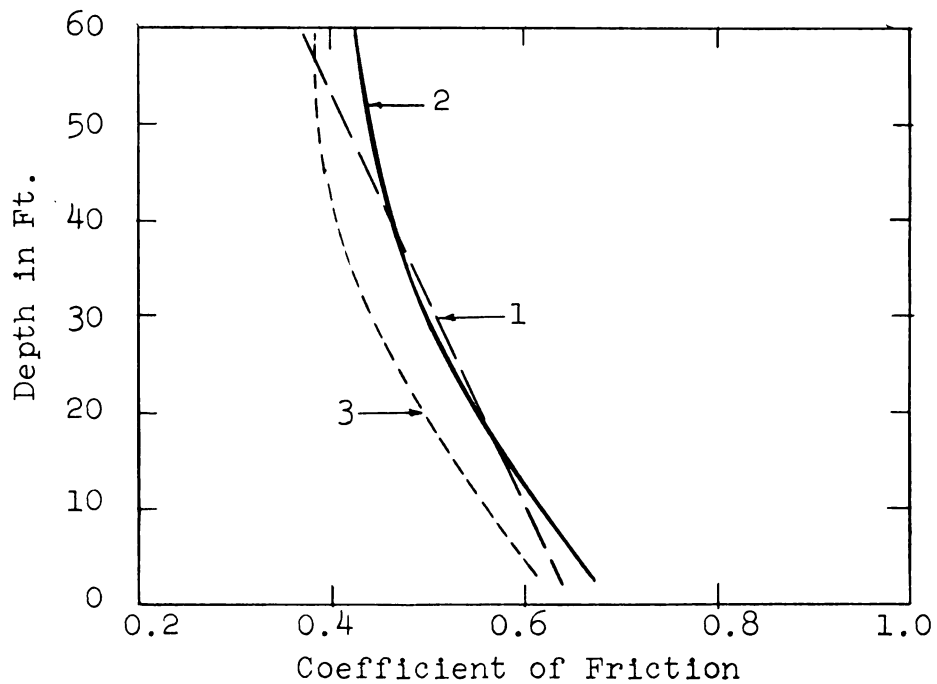


Figure 2. Coefficient of Friction Between Corn Silage and Stave Silo Walls by Curve Fitting

at two farms in Michigan. The measured angle of repose for corn silage ranged from 31 degrees to 41 degrees, which correspond to the coefficient of internal friction of 0.601 to 0.90.

Coefficient of friction between silage and silo walls was obtained as a by-product of panel pressure measurement. Panel measurements give horizontal (normal to the silo wall) pressure and frictional pressure. By directly calculating the ratio of a horizontal pressure to the corresponding frictional pressure, a coefficient of friction is obtained. Using horizontal and frictional pressures measured in 1962, the curves (Figure 2) were obtained by least square curve fitting.

The 1st, 2nd and 3rd degree polynomials are:

$$f_{21}(z) = \mu = 0.657 - 0.00482 z$$

$$f_{22}(z) = \mu = 0.698 - 0.00876 z + 0.0000706 z^2$$

$$f_{23}(z) = \mu = 0.636 - 0.00650 z - 0.0000275 z^2 + 0.00000115 z^3$$

in which

z = depth of silage in ft.

The three curves show good agreement but the 2nd degree curve is slightly higher in magnitude. Therefore, the second degree curve is recommended for pressure calculation.

Ratio $k = H/V$

The value k is defined as the ratio of horizontal to vertical pressure at any one point in the silo.

In a statical analysis of pressure exerted by bulk material, gravitational force is the only significant source of pressure. The analysis is generally made by static equilibrium along the vertical direction. Bulk material differs from a solid material, and has a tendency to settle and change its physical properties along the depth of material. The pressure exerted by silage in a horizontal direction is of prime interest here. The load on a horizontal plane in a vertical direction can be evaluated easily by weighing. The horizontal pressure needs some modification. Other than direct measurement of pressure in horizontal direction, it may be considered as a partial pressure caused by the vertical pressure. Once this assumption is accepted, a factor relating horizontal pressure to vertical pressure can be selected.

From the Rankine theory, (Taylor, 1960), soil pressure can be calculated by

$$P = wz \frac{1 - \sin \phi}{1 + \sin \phi} \quad (10)$$

in which

P = unit lateral pressure in lbs. per sq. ft.

w = unit weight of soil in lbs. per cu. ft.

z = depth in ft.

$\emptyset$ = angle of repose of the soil

By taking,

$$k = (1 - \sin \emptyset) / (1 + \sin \emptyset) \quad (11)$$

the relation between horizontal and vertical pressure can be defined. This will result in a constant k , for a constant $\emptyset$. In a non-homogeneous, semi-fluid material like silage, the ratio may be much more complicated. A transducer was used to measure the pressure in two directions at a point. For a close approximation to pressure at a point in silo, a pressure transducer was built. The transducer has two diaphragm gages, Bouffler 1.5 in. 4-ED, 45 ohm, (Saunders-Roe Foil Strain gages, 1960), cemented to the backs of thin stainless steel plates which were firmly soldered on the steel cells. The cells were embedded in two faces of a 2 in. cubic steel block, mutually perpendicular to each other and flush with the face. The transducer was calibrated with hydrostatic pressure and mounted on a guide inside the silo about 4 in. from the wall surface. Pressures in both directions were measured simultaneously. The ratio of measured horizontal to vertical pressures were calculated. Least square curve fitting was applied to arrive at the polynomial equations shown below. Figure 3 shows their variations with the depth of silage.

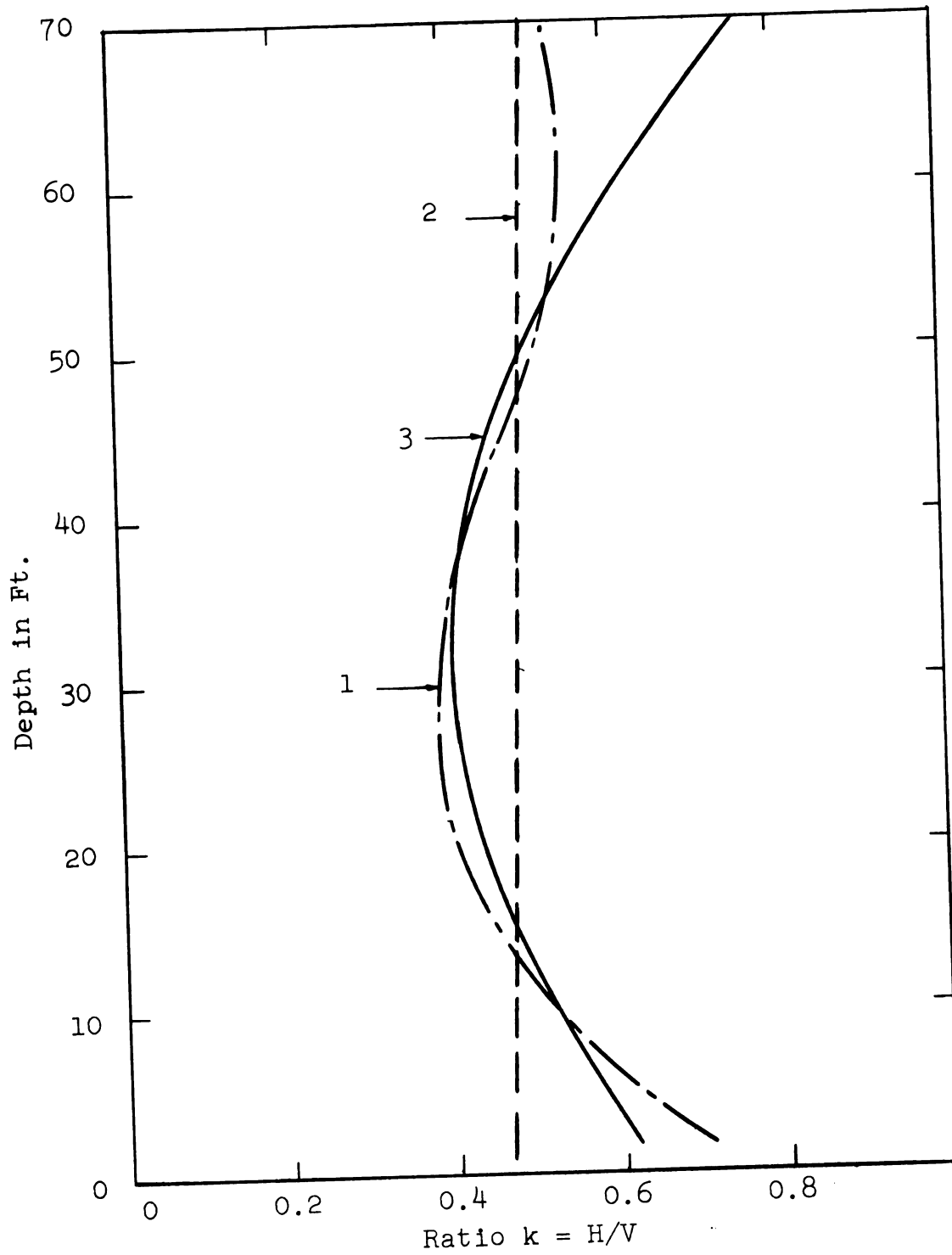


Figure 3. Ratio of Horizontal to Vertical Pressure From Two-Way Pressure Transducer

$$f_{31}(z) = k = 0.463 + 0.434 z \quad (12)$$

$$f_{32}(z) = k = 0.641 - 0.0149 z + 0.000238 z^2$$

$$f_{33}(z) = k = 0.767 - 0.0336 z + 0.000906 z^2 - 0.00000678 z^3$$

Experimental Study on Silage Pressure

Frequent structural failure in large silos prompted the study of silage pressure in large diameter silos. Early experiments resulted in the ACI Standard for farm silos. Since then, very little work has been done on silage pressure. In 1958, through the cooperation of the National Silo Association and USDA, pressure measurements on full scale silos were again initiated.

In 1959 all pressure measurements were made by Boyd and Yu (1960) with cells made of four A-18 SR-4 strain gages attached to a stainless steel diaphragm. (Figures 4 and 5) The cells were embedded flush with silo walls at elevations of 0, 2.5, 5, 10, 20, and 30 ft. from the bottom of a 30 x 60 ft. silo. The value for horizontal pressure at the 0 ft. and 5 ft. levels is the average of two cells and the value for 10 ft. is the average of three cells. This curve indicates that the highest pressures might not be at the bottom but some distance above the bottom, in this case 5 ft.

The vertical pressures in Figure 5 show variations which are caused by high density areas. A distributor was used to fill this silo, but even with uniform distribution, variations in vertical pressure are great.

Owing to the fact that the pressures are sensed by very small areas, the resulting values do not follow definite patterns or a simple curve. By checking the pressure on individual cells, areas of high pressures can be pointed out. There is a general trend indicating that the pressures at 60 ft. depth would be approximately 700 lbs./ft.².

During the summer of 1960, with the cooperation of the USDA, the 30 x 60 ft. silo was equipped with a column of pressure panels. These panels were similar to those used earlier in New Jersey and had an area of 5 sq. ft. per panel. Each panel was supported by two 1/2 in. steel rods (heat treated to 120,000 psi), mounted so that the rods were free to deflect in horizontal and vertical directions in a rigid support on the silo. The horizontal and frictional pressure were measured by the deflections of rods in horizontal and vertical directions. Deflection readings were made to the closest 0.0001 in., which represented a pressure of 1 lb./ft.² increment.

Measured horizontal pressure showed almost a straight line increase (Figure 6) and was approximately 600 psf at 60 ft. depth. The frictional pressure was slightly over 100 psf at 60 ft. depth. This frictional pressure did not change appreciably when the depth increased above 40 ft. The maximum pressure

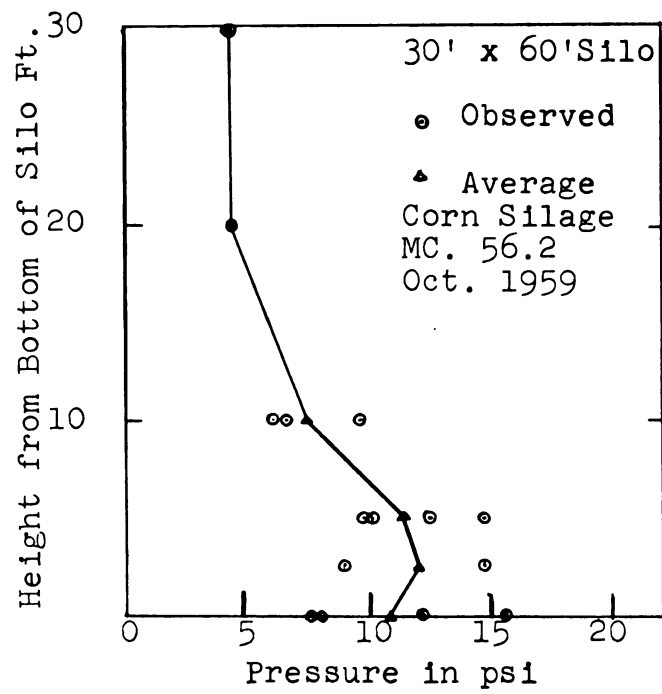


Figure 4. Horizontal Pressure Measured by Cells

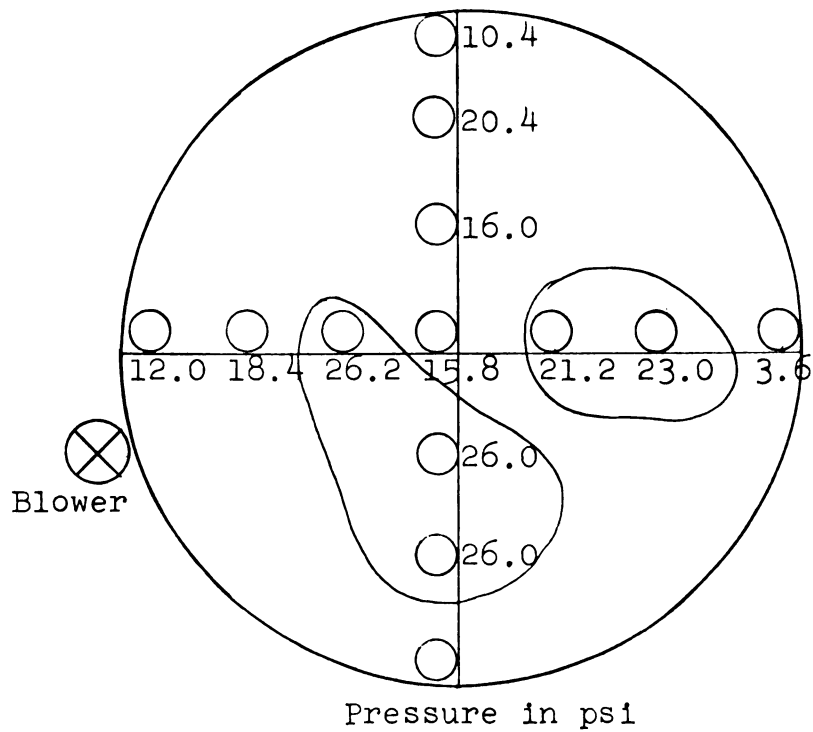


Figure 5. Vertical Pressure Measured by Cells

recorded in each panel was shown and the difference between the settled and maximum recorded pressure on each panel is nearly the same. The highest pressures recorded being on panels 2 and 4 which would be 2.5 and 7.5 ft. from the bottom of the silo. The shape of this curve was very similar to that plotted using cells, although the absolute values for horizontal pressures here are much less. The silage averaged 67.7% in moisture content. Following silo filling, pressure was continuously observed. The horizontal pressure continued to increase as the silage settled. Approximately 1 week after the completion of filling, the pressure was relieved (Figure 7) slightly. During the cold winter the annual ring of silage in contact with the silo wall was frozen about 8 in. thick. The effect on the pressure was not obtained due to the difficulties in measurement and evaluation. During the emptying of the 30 x 60 ft. silo, panel pressure measurement was continued. Figure 8 shows the pressure during filling and emptying, measured by the bottom 4 panels. Circles represent the filling pressures and dotted points represent unloading pressures. The silo was emptied at the rate of 2.5 ft. of depth per week. The horizontal pressure at any point remained constant at the maximum value until the silage surface was about 20

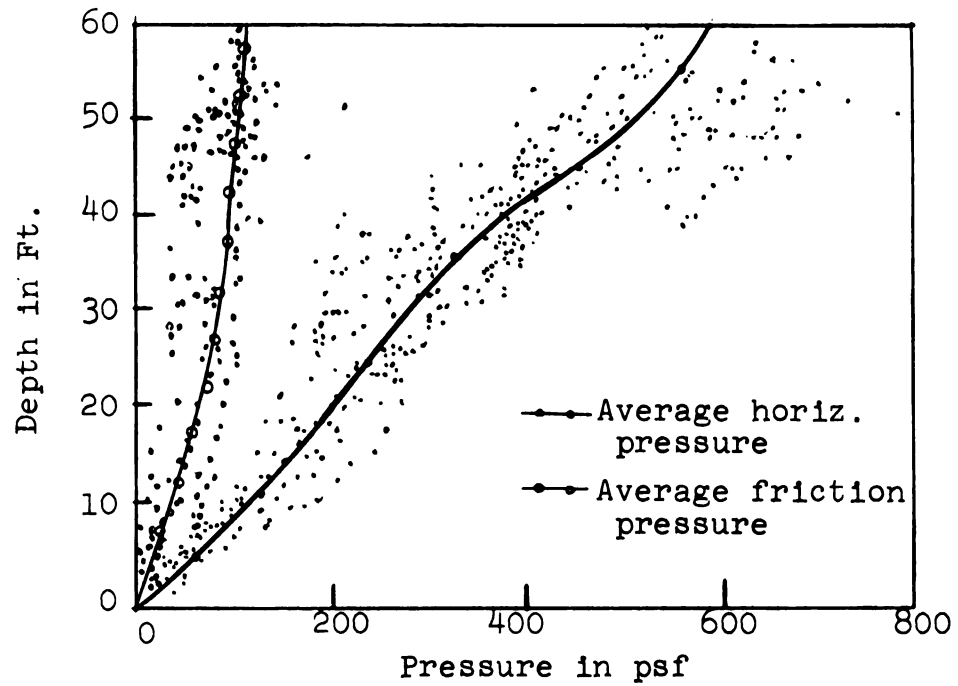


Figure 6. Silage Pressures Measured by Panels in 30 ft. Silo, 1960

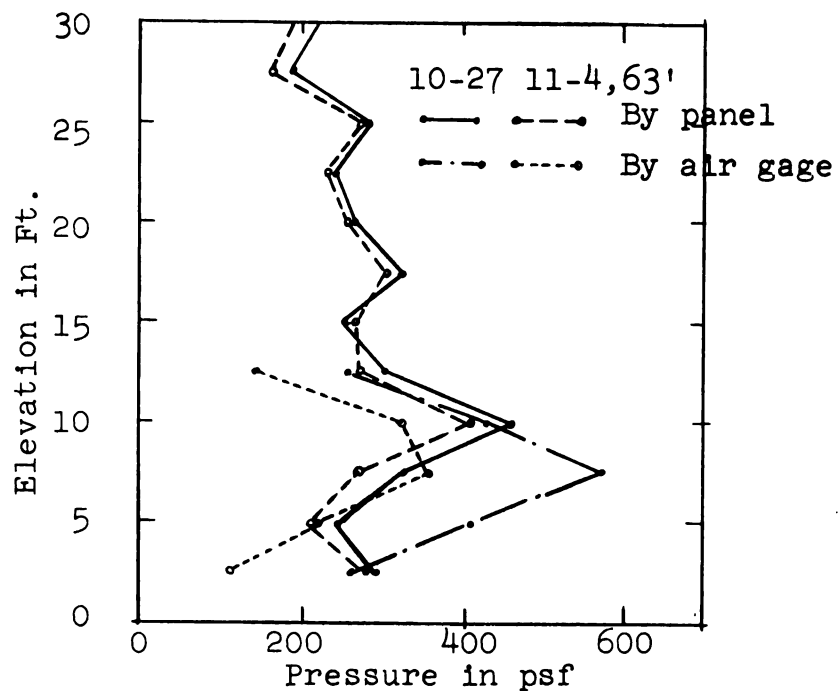


Figure 7. Horizontal Pressure Measured by Panels and Air Gages; 30 x 60 ft. Silo, Corn Silage

ft. above the measuring point, from then until the silage was removed the pressure dropped off to zero. The vertical pressure, for the same 4 panels during filling increased to 125 psf at 40 ft. depth, and did not increase appreciably from 40 ft. to 60 ft. During emptying the vertical pressure did follow very closely the loading curve. When the silage was 10 ft. deep and less, a negative pressure of 30 to 40 psf was recorded. This may be due to the spongy action of the silage.

In October, 1961, panel measurements were again made in the 30 x 60 ft. silo. Figure 9 shows the results of horizontal and frictional pressures with average M.C. of 67.7%. During filling the silo distributor did not operate which caused the silage to pile up on the silo wall opposite the panels. This caused a loose fluffy filling against the pressure panels. The distributor was readjusted to correct its distribution when the silage was 25 ft. deep. The maximum horizontal pressure measured was 400 psf at 50 ft. depth and 300 psf at 60 ft. depth at the end of filling. This pressure is much lower than that recorded in 1960, which was obtained under good distribution during filling. This is an indication that distribution had an influence on silage pressure in tower silos. The distribution of frictional pressure obtained in 1961 was similar to

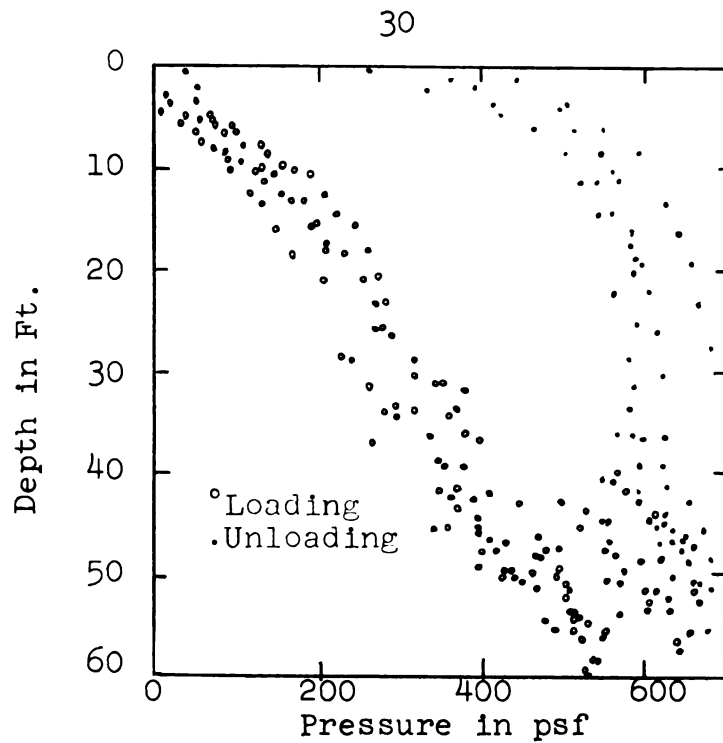


Figure 8. Horizontal Pressure during Filling and Emptying by Panels 1, 2, 3, and 4; 30' x 60' Silo, Corn Silage

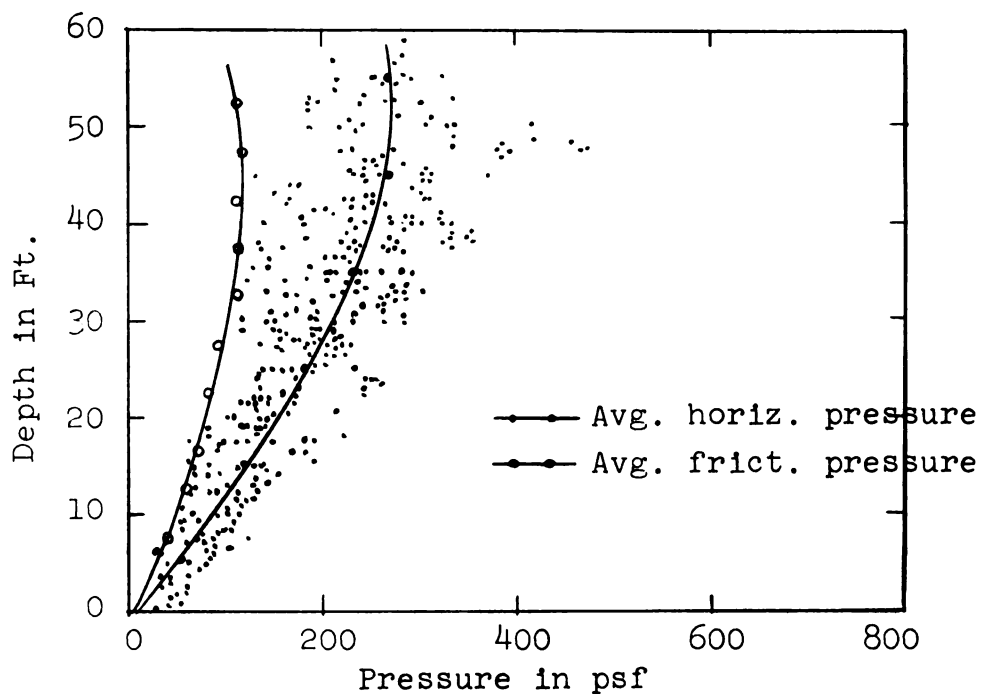


Figure 9. Silage Pressure Measured by Panels in 30 ft. Silo, 1961

that measured in 1960. The maximum frictional pressure was 150 lb./ft.²; however, a curve through those points would be close to 100 lb./ft.² from 40 ft. to 60 ft., which is slightly higher than that in 1960. This small increase in frictional pressure being associated with a large decrease in horizontal pressure.

Following the procedure of the previous years' work, pressure panels were used again in measuring pressures, in October, 1962. The average moisture contents of corn silage filled this year was 64.9% with a maximum 71.7% and minimum 53.0%. Due to the improper alignment of the distributor, the silage was piled up on the side opposite to the panels up to about 20 ft. Manual leveling was required, at the latter part of filling. The final maximum pressure distribution was very similar to the past 3 years' results. At the depth of 58 ft., maximum horizontal pressure was 640 lb./ft.². The frictional pressure also was slightly lower than the past 3 years' results, but the distribution pattern was almost identical. A summary of panel pressure measurement from 1960 to 1962 are shown in Figure 10.

Twelve air gages were built and installed during the summer of 1961. To supply air pressure, the air in the supply line was allowed to react on the inside of the shimstock diaphragm to balance the pressure exerted by the silage. As soon as the balance is correct air will by-pass the gage and cause bubbles to appear in the

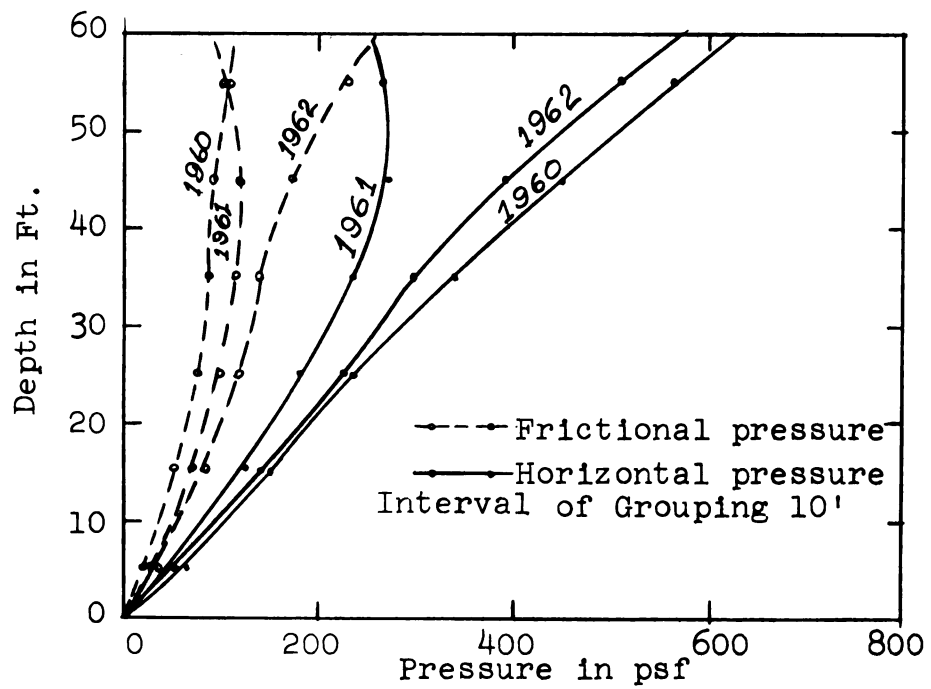


Figure 10. Average Pressure Measured by Panels from 1960 to 1962, 30 ft. Silo, Corn Silage

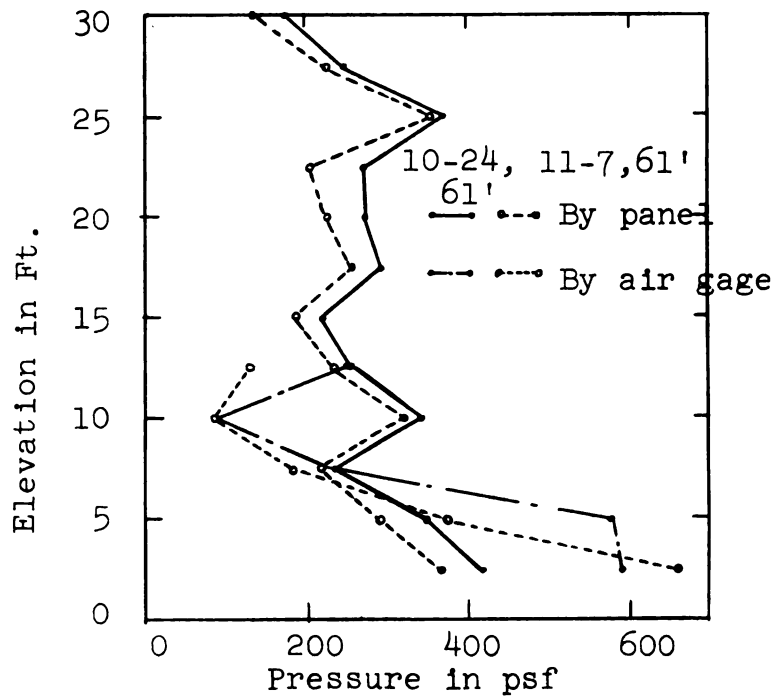


Figure 11. Horizontal Pressure Measured by Panels and Air Gages; 20 x 60 ft. Silo, Corn Silage

water bath, into which the by-pass discharges. At this time the air gage pressure will be the silage pressure. Maximum horizontal pressure measured in 20 x 60 ft. and 30 x 60 ft. silos was 700 lb./ft.² at 50 ft. and 620 lb./ft.² at 50 ft. respectively. (Figure 7 and Figure 11)

In obtaining the ratio k, both horizontal and vertical pressure were measured by a two-way pressure transducer. The measured vertical pressure here differs from frictional pressure of silage on the wall, as it is the pressure at an internal point in vertical direction. The maximum horizontal pressure at 55 ft. depth was 800 lb./ft.² and the maximum vertical pressure was 1370 lb./ft.² at 55 ft. (Figure 12)

The horizontal and frictional pressures measured from panels were grouped by a 10 ft. interval in silage depth. Groups of data were analyzed by statistical routine for Fortran 160 to test normality in a frequency distribution and to calculate mean and standard deviation. The average pressure was approximated by least square curve fitting and the 1st degree linear, 2nd and 3rd degree curvilinear polynomial equations were found by programs K-3, L7-S at the computer center, Michigan State University. Let the general equation of horizontal and frictional pressure be

$$H_N = \sum_{n=0}^N a_n z^n, \quad F_N = \sum_{n=0}^N b_n z^n \quad (13)$$

in which

$$n = 0, 1, 2, \dots$$

$$N = 1, 2, \text{ and } 3.$$

$$H_N = \text{horizontal pressure in lb./ft.}^2$$

$$F_N = \text{frictional pressure in lb./ft.}^2$$

$$z = \text{depth of silage in ft.}$$

The coefficients of the equations are tabulated in Tables 3 and 4.

The pressure which may occur at 90% normal distribution was calculated. Hence, empirical formulas were obtained by least square in 1st and 2nd and 3rd degree polynomials as shown in Figure 13.

The polynomial equations are as follows:

Horizontal pressures:

$$H_1 = 11.844 + 11.531 z \quad (14)$$

$$H_2 = 23.320 + 9.379 z + 0.040 z^2$$

$$H_3 = 16.115 + 13.258 z - 0.151 z^2 + 0.0023 z^3$$

Pressures measured by panels are again subdivided by 5 ft. intervals. Mean pressure and standard deviation are obtained by program L1-04. The results are shown in Figure 14.

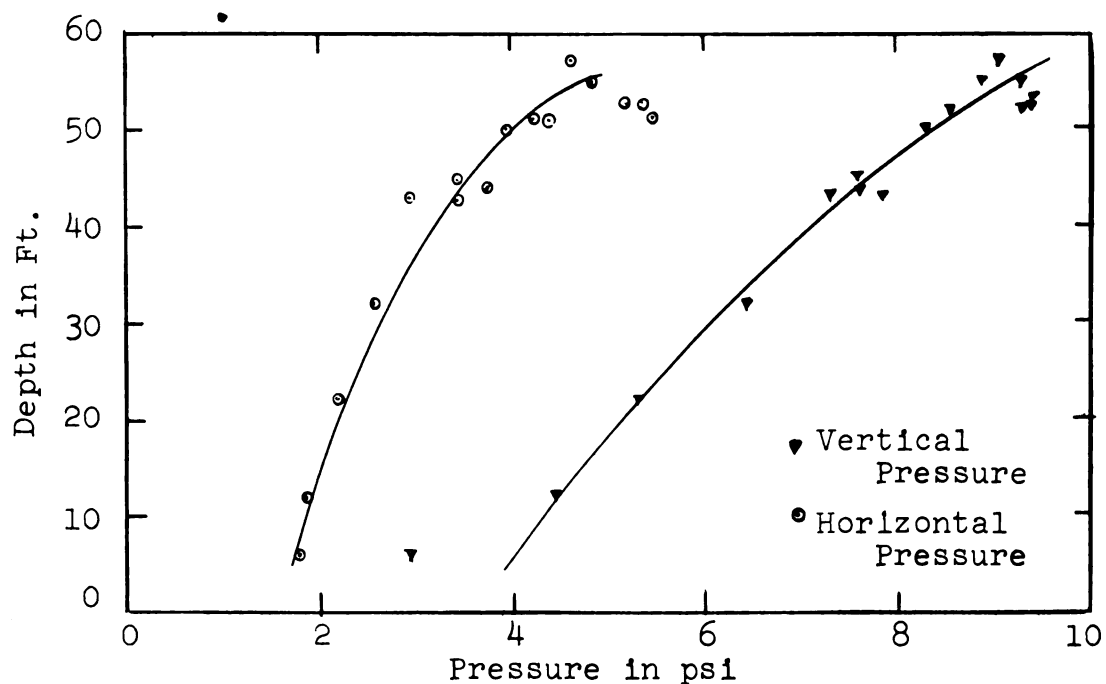


Figure 12. Horizontal Pressure and Vertical Pressure Measured from Two-Way Pressure Transducer

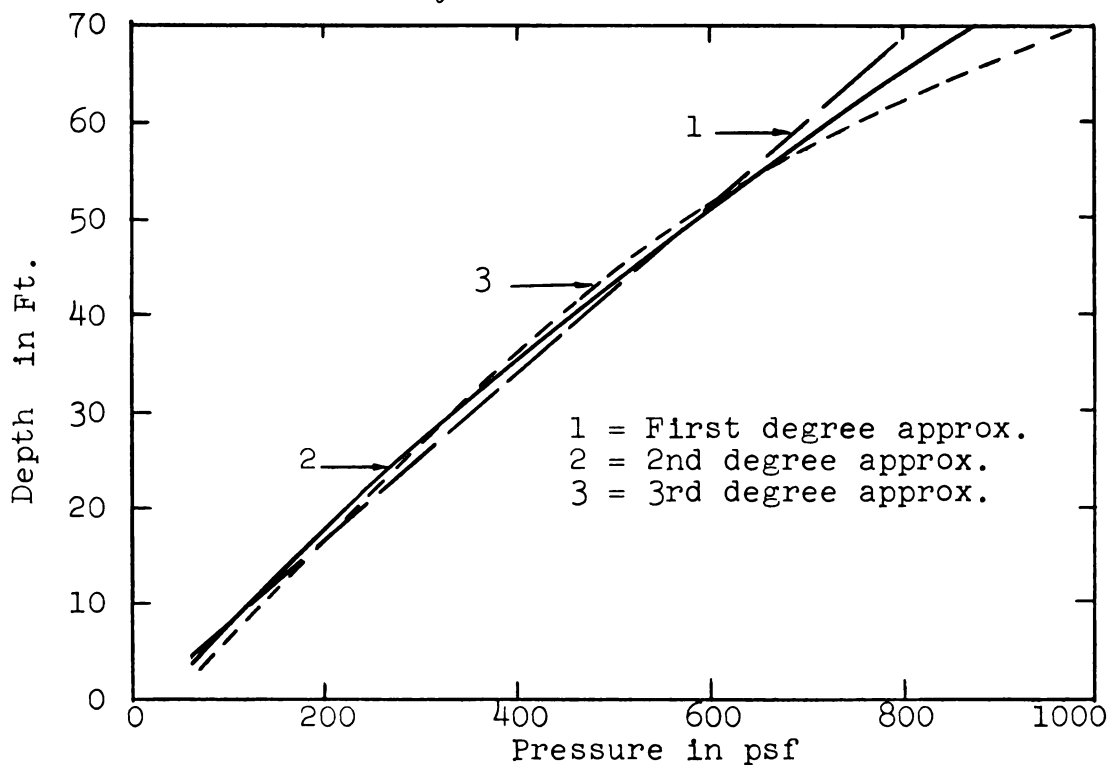


Figure 13. Estimated Horizontal Pressure at 90% Normal Distribution in 30 ft. Silo from Panel Measurements, 1960-1962.

TABLE 3
COEFFICIENTS OF POLYNOMIAL EQUATIONS FOR HORIZONTAL PRESSURE

Year	First Degree		Second Degree			Third Degree			
	a_0	a_1	a_0	a_1	a_2	a_0	a_1	a_2	a_3
1960	-2.284	10.038	12.678	7.814	0.0413	6.715	10.080	-0.0700	0.00134
1961	36.161	4.948	6.194	9.402	-0.0828	10.218	7.873	-0.0077	-0.00091
1962	3.236	8.978	8.581	8.184	0.0148	-2.254	12.302	-0.187	0.00244
1960- 1962	9.358	8.274	10.009	8.178	0.0018	5.790	9.781	-0.077	0.00095

CHAPTER IV

ANALYTICAL INVESTIGATION

The problem of calculating the pressure of silage on the silo walls is somewhat similar to the problem of a retaining wall. The theory of Rankine will apply in the case of shallow bins with smooth walls where the plane of rupture cuts the bulk material surface, but will not apply to deeper bins or bins with rough surfaces. In estimating silage pressures, more factors such as variation in unit weight along the depth, moisture content, ratio of horizontal to vertical pressures and coefficient of friction between silage and silo walls should be considered.

Two solutions to the problem of calculating pressures in tower silos are proposed:

1. Solution without considering arch action in the silage.
2. Solution considering arch action in silage.

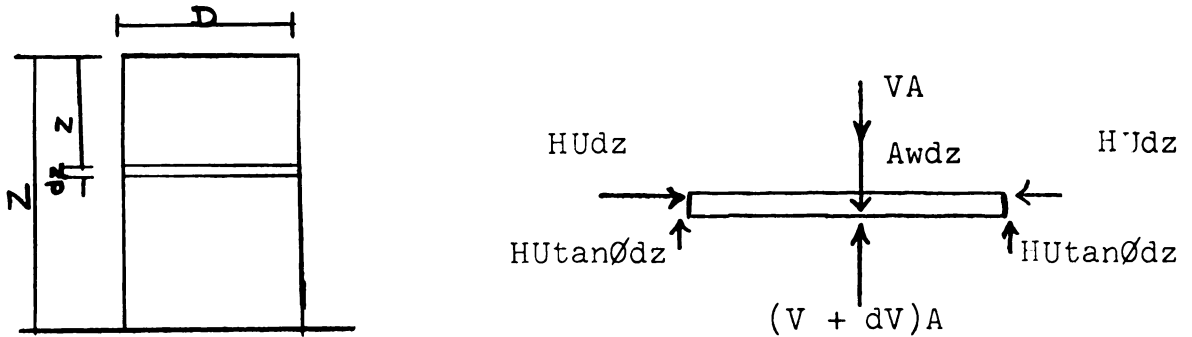
Solution Without Considering Arch Action of Silage

From the previous investigation, factors such as unit weight, coefficient of friction between silage and silo wall, and the ratio of horizontal to vertical pressure, were not constant along the depth of silage although

Janssen assumed constant values throughout the silo. When factors are assumed constant over a small increment of the depth of silage, say 5 ft., then the following derivation will be valid over a 5 ft. interval.

In cylindrical tower silos, using assumptions similar to Janssen's, the equations for the vertical and horizontal pressures were derived as follows.

Assume the equilibrium of the shown free body



$$VA - (V + dV) A + Awdz - HU \tan \emptyset dz = 0 \quad (15)$$

and

$$dV = (w - H \tan \emptyset \frac{U}{A}) dz \quad (16)$$

assuming $H = kV$, the solution is found equal to

$$V = w R (1 - \exp(-k \mu z/R)) / (k \mu) \quad (17)$$

Using the functions obtained from previous studies for

w , μ and k , let

$$f_{12}(z) = w = 34.779 + 1.121 z - 0.0104 z^2$$

$$f_{22}(z) = \mu = 0.698 - 0.00876 z + 0.0000706 z^2$$

$$f_{32}(z) = k = 0.641 - 0.0149 z + 0.000238 z^2$$

then

$$H = \frac{f_{12}(z) R}{f_{22}(z)} (1 - \exp(-f_{32}(z) f_{22}(z) z/R)) \quad (18)$$

and

$$V = H/f_{32}(z) \quad (19)$$

and

$$F = f_{22}(z) H \quad (20)$$

can be obtained.

In studying the variation in horizontal pressure along the silo wall the calculation was made in two different ways. First, assume a variable w , μ , k at each depth. The calculated pressure is approximated by least square curve fitting, and Figure 15 shows the results. Secondly by using the variable w , μ , k at each depth, calculate the difference in successive two pressures of a 5 ft. interval. Then, the difference of pressures is successively accumulated. The results are shown in Figure 16.

Solution Considering Arch Action in Silage

Wet silage tends to settle faster than drier silage. When the silo is relatively tall compared to its diameter, the silage may not settle at a certain depth. At this

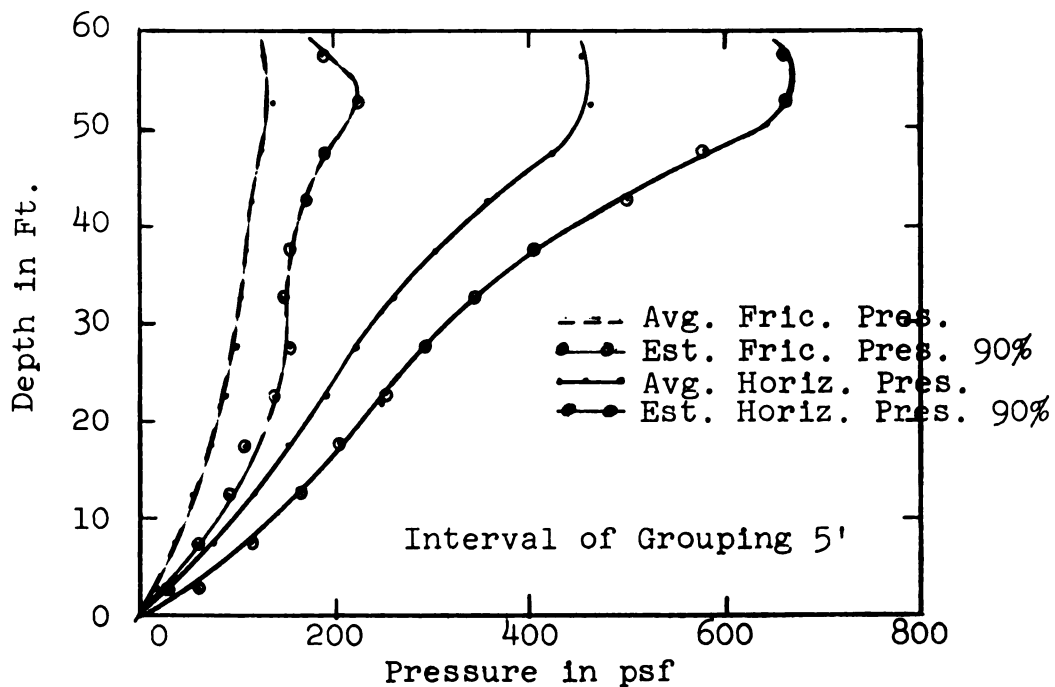


Figure 14. Silage Pressures Measured by Panels in 30 ft. Silo Corn Silage, from 1960 to 1962

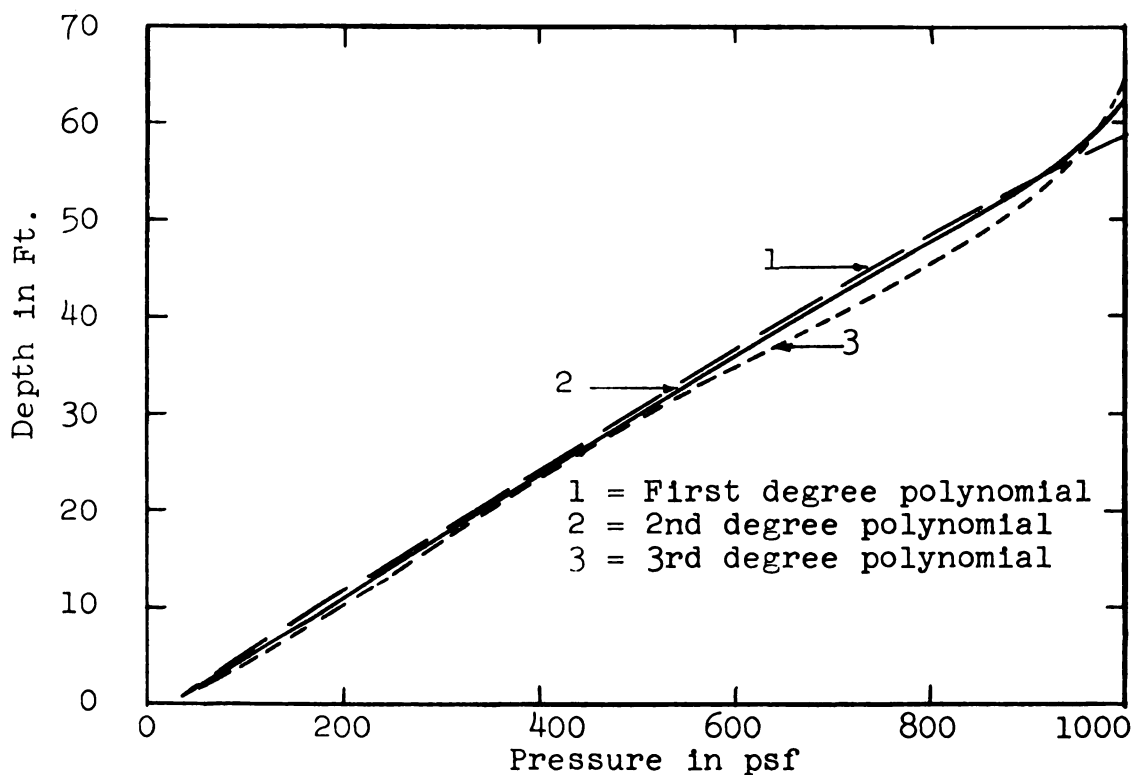


Figure 15. Horizontal Pressure Calculated from Parameters w , μ , and k Varying with Depth of Silage, 30 ft. Silo

point the weight of silage is not directly transmitted to adjacent silage but to silo walls. The silage tends to form an arch which will hold the total or partial weight of the surcharge of silage. The detailed variation of pressure in the silo is yet unknown. The ratio of horizontal to vertical pressure, k , is no longer a constant. Modification should be made in the k -function which will satisfy the boundary condition. When the arch action occurs, the vertical pressure decreases to zero at the lower boundary of the arch. The load on the arch will be taken by the resisting pressure of silo walls. Consequently, the ratio of horizontal to vertical pressure becomes very great near the bottom level of arch action. At the top surface of the silage, neither vertical nor horizontal pressure exists. Therefore, the value of k at the upper boundary where depth equals zero may be assumed equal to zero. Let the ratio of depth z to the hydraulic radius of silo R equal n then,

$$n = z/R \quad \text{or} \quad z = nR \quad (21)$$

Assume $n = 10$. Knowing that the ratio k varies from zero at zero depth to infinity at the depth of arch action, a function of n can be assigned to the ratio k then,

$$k(n) = 10 n / (100 - n^2) \quad (22)$$

From the differential relation derived in the previous section, we have

$$dV = \left(w - \frac{k(n) V \mu}{R} \right) R dn \quad (23)$$

By the change of variable the equation becomes

$$dV = \left(w - \frac{k(n) V \mu}{R} \right) dz$$

$$\frac{dV}{dn} + \mu k(n) V = w R \quad (24)$$

Solving for a particular case assuming

$$dV/V = -k(n) \mu dn \quad (25)$$

Assume $F(n) = \int_0^n k(n) dn$ has been solved, then

$$\ln V = -\mu F(n) + C_1$$

$$V = C_2 \exp(-\mu F(n)) \quad (26)$$

In order to find a proper value for the constant C_2 to satisfy the boundary conditions, differentiate eq. (26) with respect to n , by considering C_2 as a function of n .

$$\frac{dV}{dn} = \frac{dC_2}{dn} \exp(-\mu F(n)) - C_2 \mu k(n) \exp(-\mu F(n)) \quad (27)$$

Using equation (24), (26), (27)

$$dC_2/dn = w R \exp(\mu F(n)) \quad (28)$$

By solving equation (28)

$$C_2 = w R G(n) + C_3 \quad (29)$$

$$\text{where } G(n) = \int_0^n \exp(\mu F(n)) \, dn \quad (30)$$

The solution of the vertical pressure becomes

$$V = (w R G(n) + C_3) \exp(-\mu F(n)) \quad (31)$$

Apply the boundary condition $n = 0, \quad w = 0$

$$C_3 = 0 \quad (32)$$

Hence, the solution of differential equation becomes

$$V = w R G(n) \exp(-\mu F(n))$$

$$H = k(n) V$$

$$F = \mu H$$

where

$$k(n) = 10 n / (100 - n^2) \quad (33)$$

$$F(n) = 5 \ln(100 / (100 - n^2))$$

$$G(n) = G(n-1) + \exp(\mu F(n-1)) + 1/2[\exp(-\mu F(n)) - \exp(\mu F(n-1))]$$

This gives the pressure under the arch action.

In this derivation the unit weight of material and coefficient of friction are considered constant. Superimposing the effect of variations of unit weight and coefficient of friction is done by using the functions as previously defined. The results on a 30 ft. silo are shown in Figure 17.

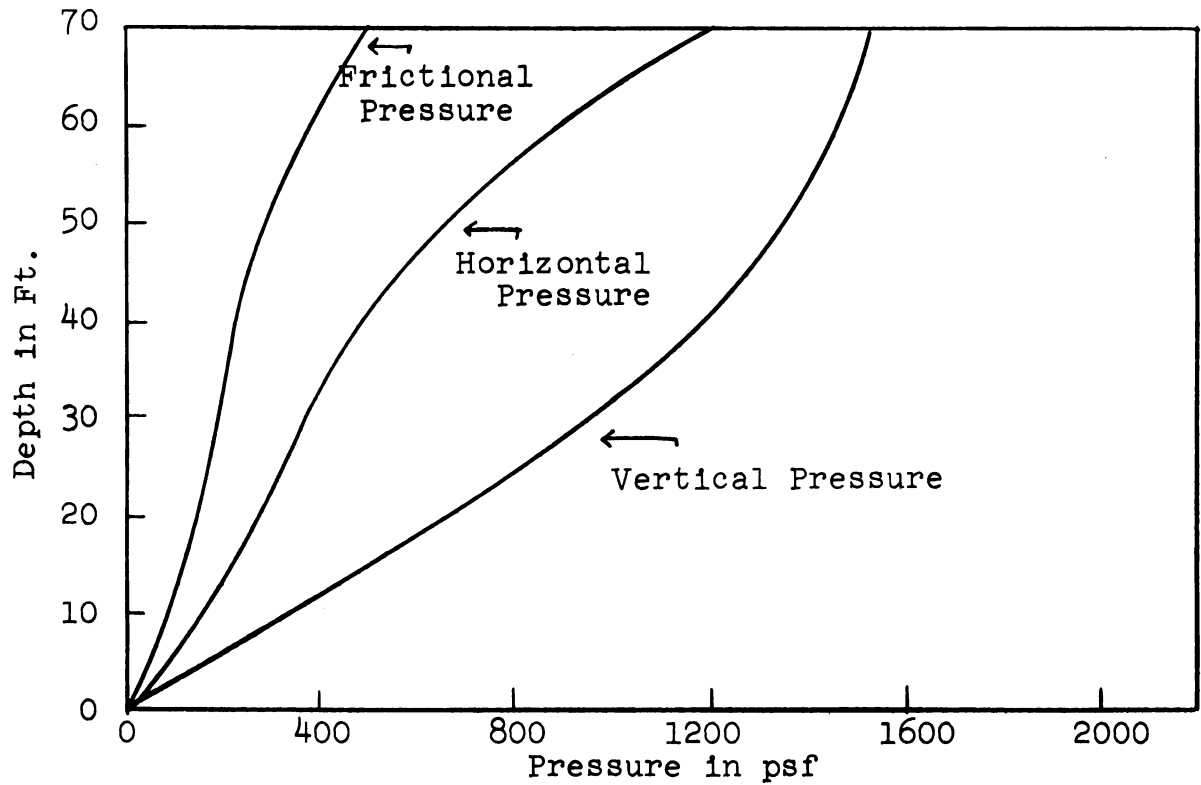


Figure 16. Pressures Calculated from Successive Superposition, 30 ft. Silo; without Considering Arch Action of Silage

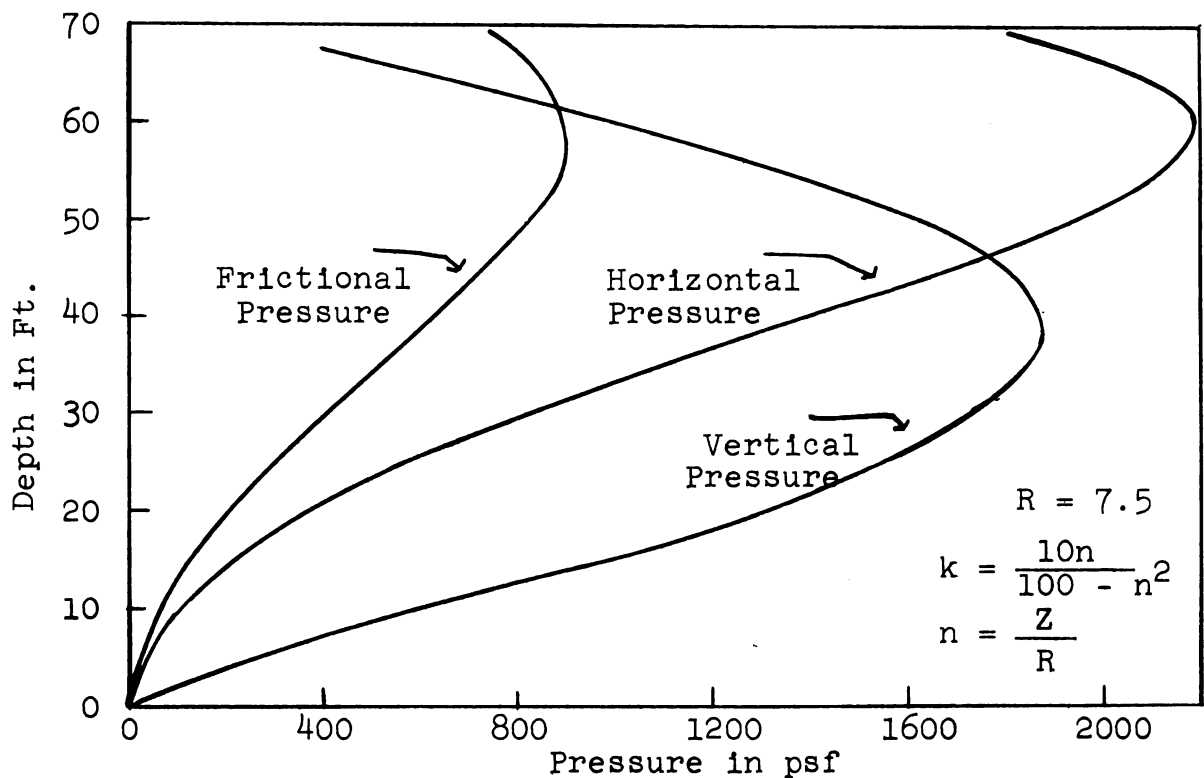


Figure 17. Pressures Calculated with Consideration of Arch Action of Silage, 30 ft. Silo

CHAPTER V

RESULTS AND DISCUSSION

The experimental study of silage pressure in full-scale silos shows that the nature of the pressures is much more complicated than has been anticipated. The least square approximation of the various factors was obtained from data collected over a period of 4 years. Care, however, should be exercised in the general application of these results. Under normal conditions, unit weight of silage, frictional coefficient between silage and concrete silo wall are readily applicable. Equation (8) may be used both for grass and corn silage. The coefficient of friction given by equation (9) is only applicable to corn silage. The ratio of horizontal to vertical pressure was obtained from one year's experimental results. The mode of variation may be correct but further experimental evidence is needed to qualify these results.

At this stage, the exact quantitative function for factors w , μ and k have not been formed, this experimental study offers additional information in an attempt to modify the existing formulas for the calculation of silage pressures in tower silos.

Figure 10 summarized the results of average horizontal and frictional pressure measured from 1960 to 1962. Grouping the data in 10 ft. intervals the horizontal pressure appears to be a linear increase with the increase in depth. The bulging of horizontal pressure was observed at the lower part of the silo when grouped in 5 ft. intervals. This bulging phenomenon has been observed each year in the panel pressure measurements. This bulging phenomenon may explain many past silo failures.

In an analytic study of silage pressures, two hypothesis were made: first, the factors w , μ , k vary according to the observed results without arch action of silage in the silo, and second, arch action does exist which affects these factors. Figure 16 shows horizontal, vertical and frictional pressure vs depth of silage when arch action is excluded in pressure calculation. The distribution of horizontal pressure is very similar to the pressure measured by panels but the magnitude is nearly 200 lbs. greater, at the depth of 60 ft. it will be inadequate and dangerous to conclude that the empirical formulas from experimental data are the final result. Observations and experiments should be carried out over much longer periods of time to include possible extreme cases of pressure developed in tower silos under different ensiling conditions.

Figure 18 shows the horizontal pressure calculated when disregarding the arch action of silage in different diameters, and may be recommended as the pressure on which farm silos should be designed.

When the arch action of silage is considered, the horizontal and vertical pressure were derived and calculated by assuming the ratio k as a function of n , where n equals the ratio of depth of silage to hydraulic radius of the silo. For function of $k = 10n/(100 - n^2)$, the pressure distribution is shown in Figure 17. The horizontal pressure, calculated considering arch action in silage, is almost three times as large in magnitude as that obtained from the panel pressure measurement but is identical in distribution pattern. The maximum pressure is found about 1.0 R to 2.0 R from the bottom of the silo.

In studying the pressure distribution corresponding to the change in k -function, let $k = \alpha n/(100 - n^2)$ and vary the parameter α . The horizontal pressure calculated by varying α from 6 to 20 was shown in Figure 19. The increase in parameter α causes a decrease in maximum horizontal pressure and a rise in the position of the action of maximum pressure. When α is less than 6, horizontal pressure increases with the depth having no bulging effect. When α equals 20 the bulging effect of horizontal pressure is only half what it is when $\alpha = 6$.

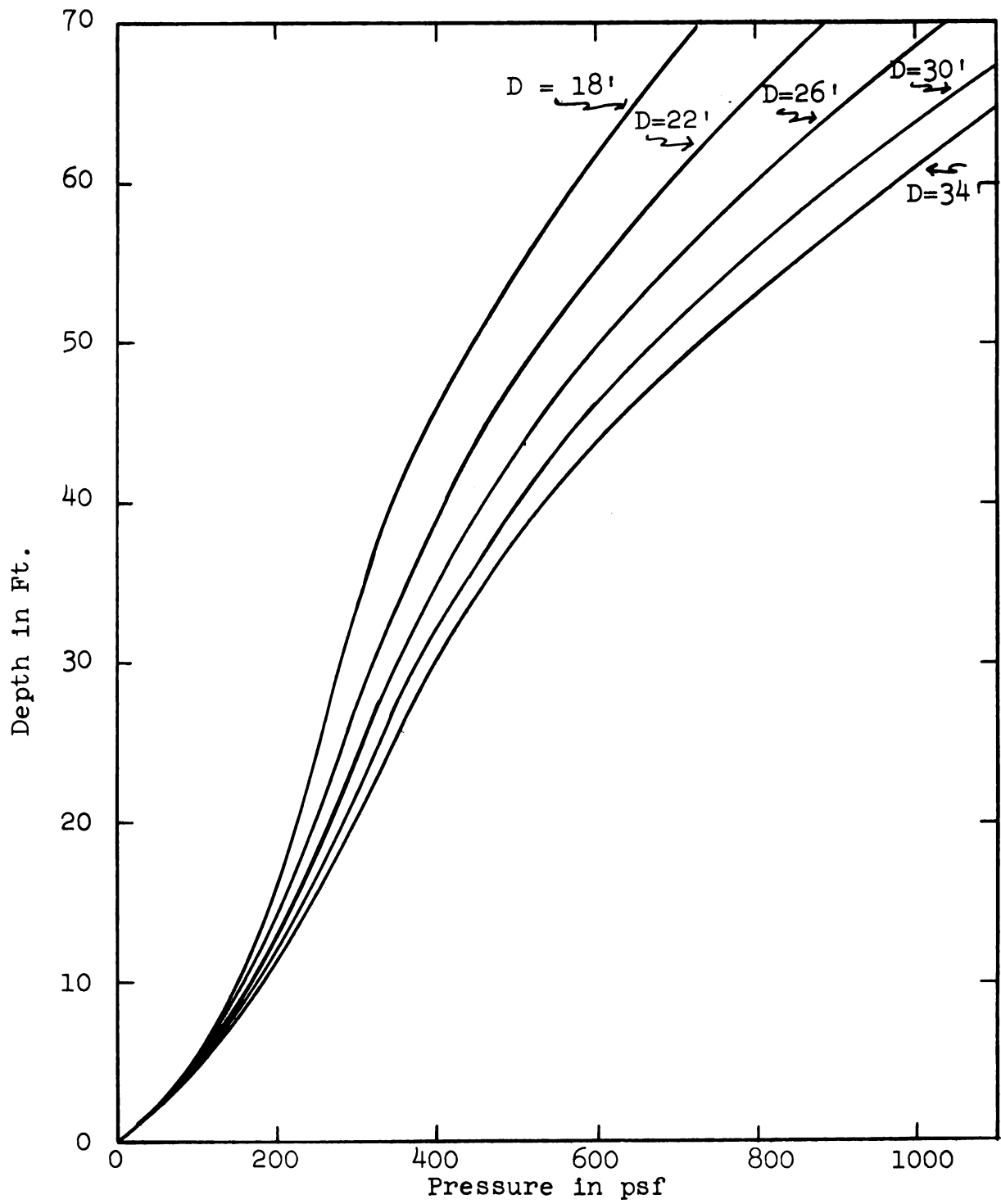


Figure 18. Horizontal Pressure by Successive Superposition for Silos of Different Diameters

Since the variation of arch action of silage in silos has not been well established, no criterion is given at present as to which ϕ to use, but for $\phi = 10$, the results seem reasonably reliable and are similar to the results actually measured. The horizontal pressures were calculated for silos of diameter 14, 18, 22, 26 and 30 ft. as shown in Figure 20.

The vertical and frictional pressure were calculated together with the horizontal pressure and are shown in Figure 21 and Figure 22.

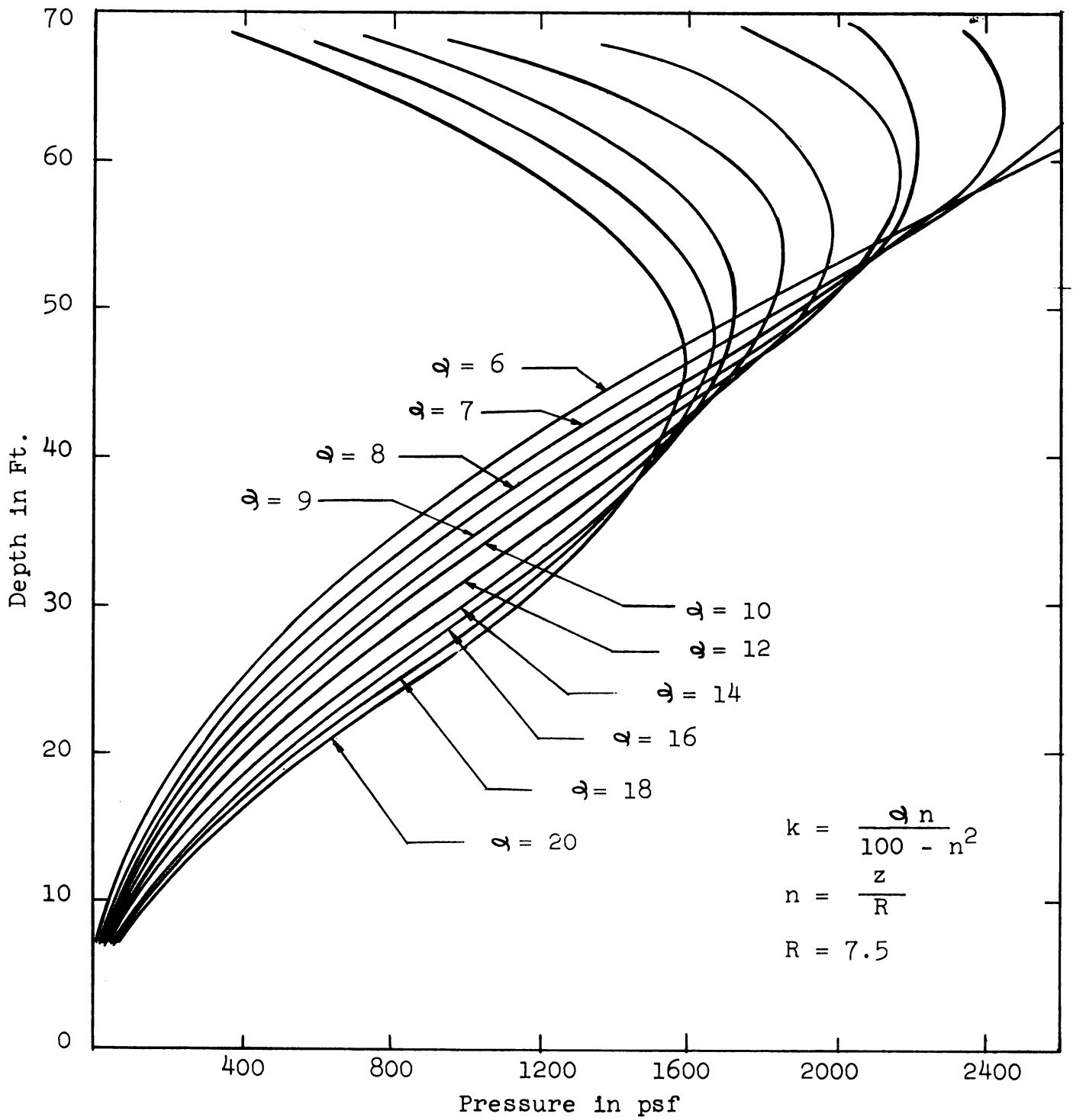


Figure 19. Horizontal Pressure Considering Arch Action of Silage, Varying Parameter α in k-Function

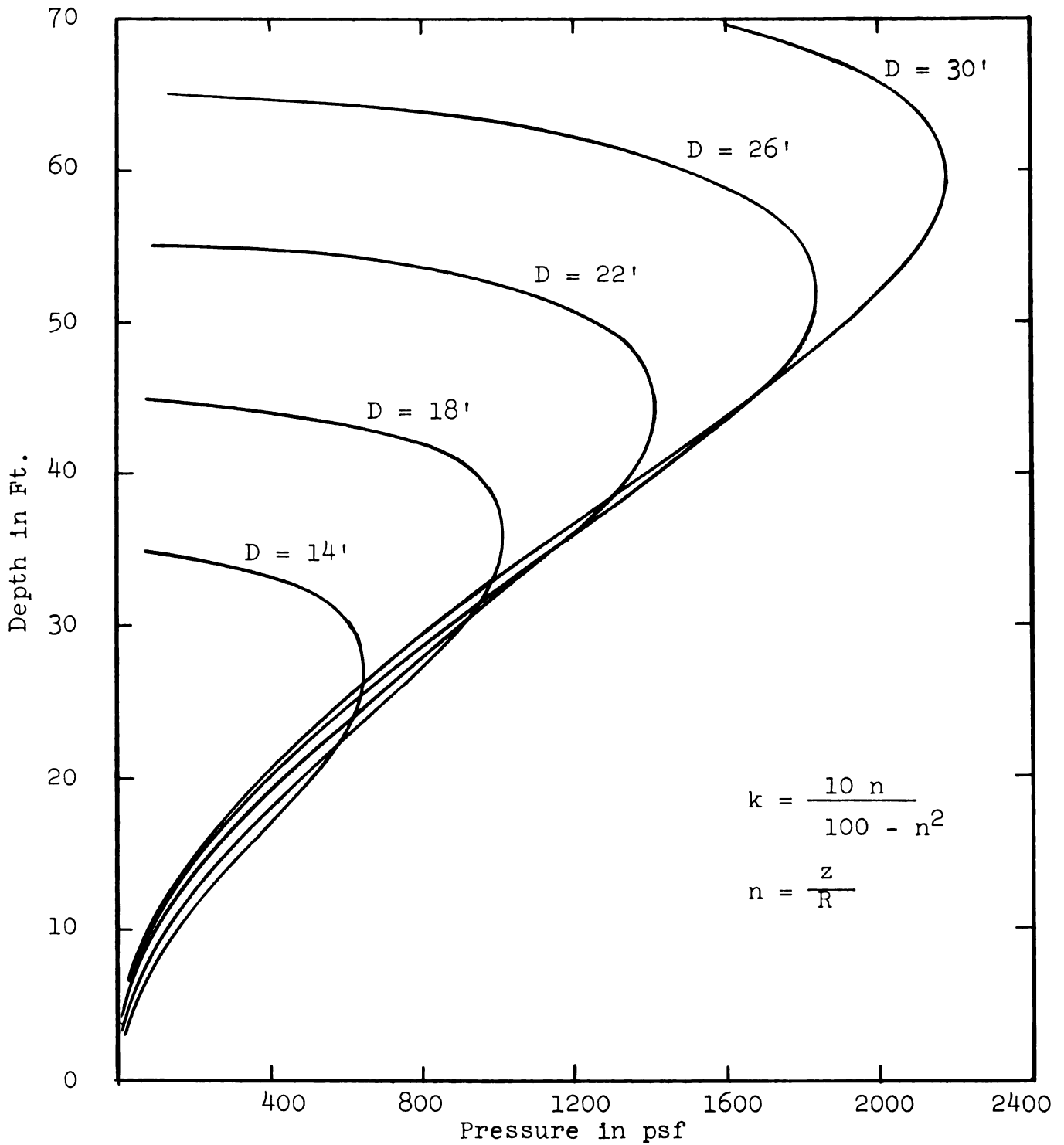


Figure 20. Horizontal Pressure Considering Arch Action of Silage for Silos of Different Diameters

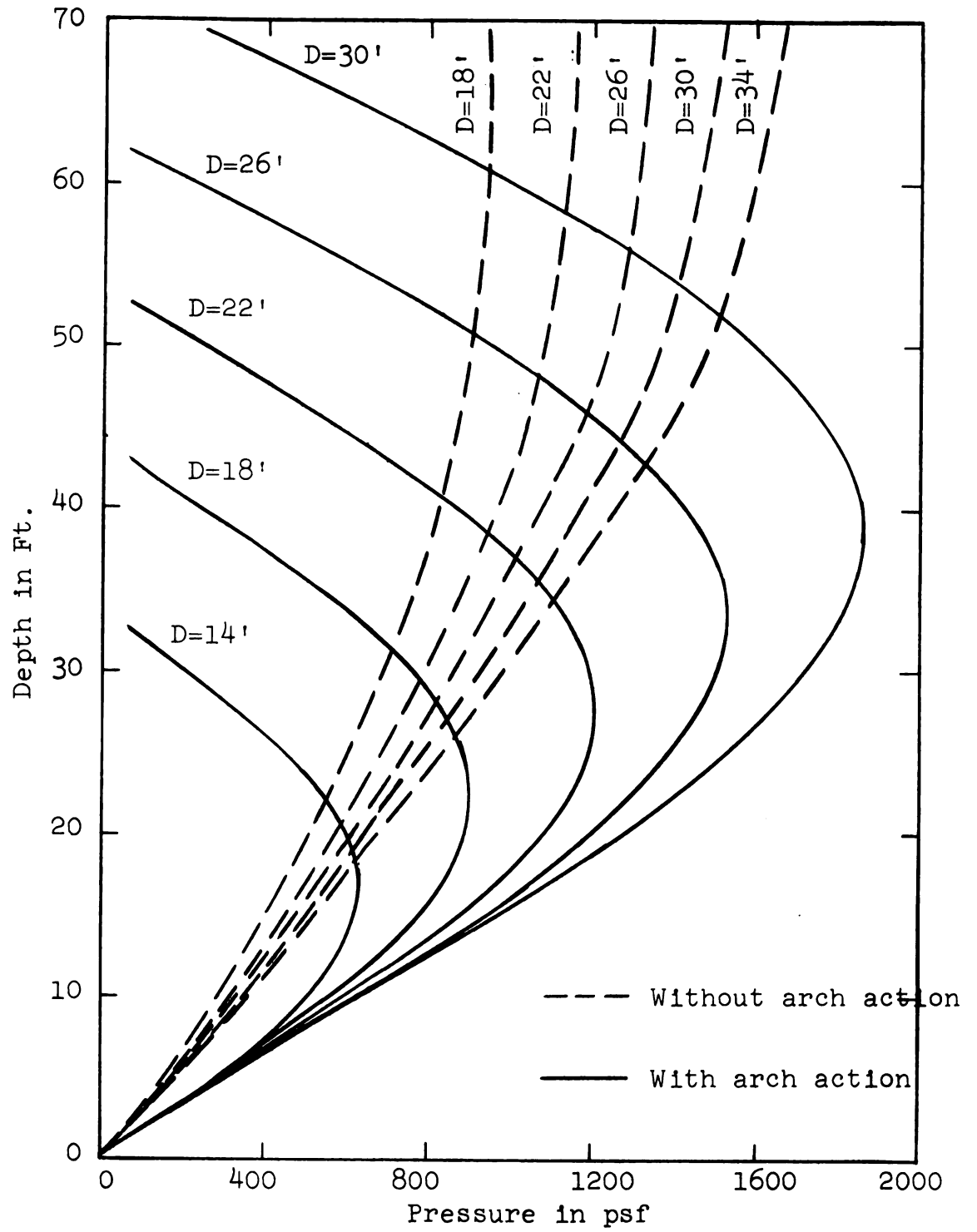


Figure 21. Vertical Pressures, With and Without Consideration of Arch Action in Silage

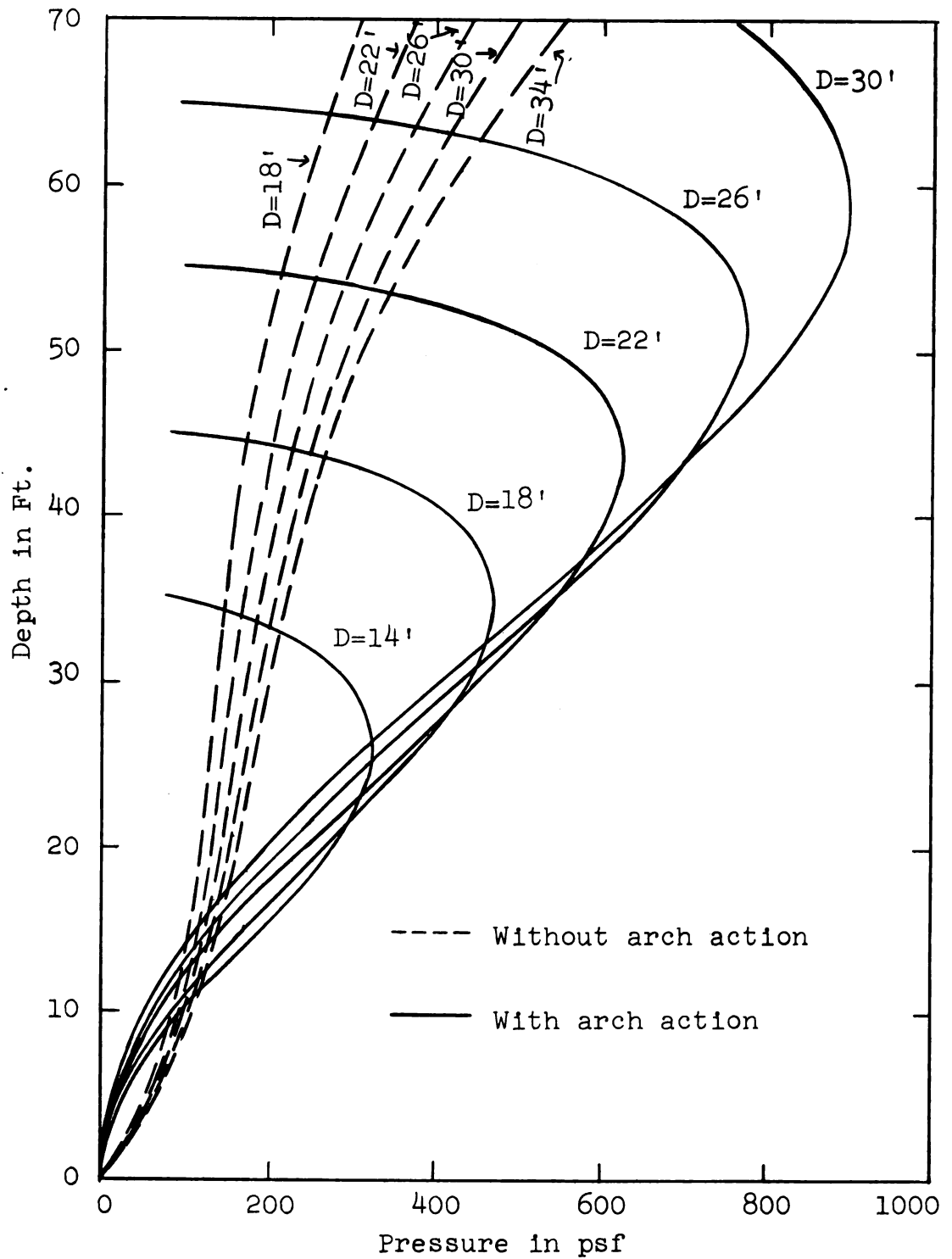


Figure 22. Frictional Pressures on the Silo Walls, With and Without Consideration of Arch Action in Silage

CHAPTER VI

SUMMARY AND CONCLUSIONS

1. The factors affecting silage pressure were evaluated. Variation of the density of silage, coefficient of friction and the ratio of horizontal to vertical pressure affect the pressure in silos.

2. The density, coefficient of friction, and ratio k were formulated as a function of a single variable under the assumption that the moisture contents of the silage lies between 50 and 80%.

3. Pressure measured in a 30 x 60 ft. silo for corn silage over 4 years was analyzed. The bulging of pressure near the bottom of the silo was observed. The accumulated pressure data from 1960 to 1962 were grouped. The average and standard deviation was calculated and empirical formula for the 30 x 60 ft. silo for corn silage was formulated by estimating the pressure at 90% normal distribution. Simple polynomial equation of a single variable was obtained by least square curve fitting.

4. Horizontal, vertical and frictional pressures were analyzed with and without considering arch action in silage. Assuming the arch action exists, the pressure

distribution is shown in Figure 17. The bulging of maximum horizontal pressure characterizes the pressure distribution with arch action. If there is no arch action of silage, pressure distribution is shown in Figure 16. No bulging of maximum horizontal pressure is found.

5. The formulas in equations (18, 19, 20) can be used when the arch action of ensiled material may be neglected. The formulas in equations (33) should be used when possible arch action may occur.

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