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COMPUTER SIMULATION OF TURBOJET-RAMJET
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COMPUTER SIMULATION OF TURBOJET-RAMJET COMBINATION ENGINE

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ABSTRACT

COMPUTER SIMULATION OF TURBOJET-RAMJET COMBINATION ENGINE

By

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With the modern developments in the field of science and technology and growing relations among countries, the need of high-speed air travel has become increasingly important. Many countries have taken up research work to develop air-breathing engines for high supersonic flights but its application in the civilian airliners is scarcely visible.

From the standpoint of an aircraft engine designer the possibility of combining a turbojet engine with a ramjet in a single power plant that could achieve a flight Mach number close to 5 was explored. As a first step, a computer software in FORTRAN 77 has been developed that performs the thermodynamic analysis of the airflow as it passes through each component of the engine. The software consists of two major parts: the first part fixes the engine geometry based upon 14 different inputs provided by the user, and the second part simulates the off-design operation of the engine for a variety of altitudes and Mach numbers. The software helps the engine designer to conclude a conceptual design and optimize the performance parameters as per the mission requirements.

The results produced by the software are in close conformity to the theoretical results available in the literature. The software can also be used as a teaching aid in the area of aircraft propulsion.

I dedicate this thesis to my parents,

Abdul Razzaq Khan and Sultan Jahan

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LIST OF SYMBOLS AND ABBREVIATIONS

A	cross sectional area	[m^2]
c	Speed of sound	[m/s]
c_p	specific heat at constant pressure	[kJ/kg K]
f	fuel-air ratio	
h	enthalpy	[kJ/kg]
I_s	specific thrust	[N/kg/s]
k	specific heat ratio	
M	Mach number	
M_{1n}	normal component of free stream Mach number w.r.t. oblique shock	
$\dot{m}$	mass rate of flow	[kg/s]
P	static pressure at station indicated by the subscript	[kPa]
Q_R	heat of reaction of the fuel	[kJ/kmol K]
R	gas constant	[J/kg K]
T	static temperature at station indicated by the subscript	[K]
u	flow velocity at station indicated by the subscript	[m/s]

GREEK

β	oblique shock wave angle w.r.t. semi-vertex line of the wedge
γ_c	compressor pressure ratio
η	efficiency
η_b	burner efficiency

η_c	isentropic (adiabatic) efficiency of compressor	
η_{mt}	mechanical efficiency of turbine	
η_n	isentropic (adiabatic) efficiency of nozzle	
η_o	overall efficiency	
η_p	propulsive efficiency	
η_t	isentropic (adiabatic) efficiency of turbine	
η_{th}	thermal efficiency	
θ	semi-vertex angle of the intake wedge	
ν	free stream to exit velocity ratio	
ρ	static density at station indicated by the subscript	[kg/m^3]
ρ_0	stagnation density at station indicated by the subscript	[kg/m^3]

SUBSCRIPT

0	stagnation value
a	free stream air
b	burner
c	compressor
d	diffuser
e	nozzle exit
f	fuel
I	isentropic or ideal
0	stagnation value or overall as indicated by the text
p	propulsive

t turbine

th thermal

SUPERSCRIPTS

* critical condition where the flow Mach number is unity

ABBRIVIATIONS

ASP Aerospace Plan

ATOS Airplane to Orbit System

CIAM Central Institute of Aviation Motors

HSTC High Speed Civil Transport

MITI Ministry of International Trade and Industry

NASP National Aerospace Plan

TSFC Thrust Specific Fuel Consumption

UTC United Technology Corporate

CHAPTER 1

INTRODUCTION

1.1 The Need of High Speed Aircraft

Globalization of the world continues to broaden relations among countries. Accordingly the role of air travel has become increasingly important. The total number of airline passengers per year exceeded one billion at the end of the last decade [Tskhovrebov 1992]. This demonstrates an increasing trend of long range passenger aircraft in the world airliner fleet. Passenger traffic growth at the beginning of 21st century, in conjunction with increasing needs in long range transport, demands more attention to higher flight speeds.

At present, the flight speed range of air transport is limited mainly to the subsonic area owing to technical and economic factors. An increase in flight speed leads to flight time reduction and improved travel comfort, particularly on intercontinental routes. Given the expected increase in air transportation and the established practice of comfortable flight-time of no more than 2 to 3 hours for a passenger, one may infer the prospective development of a supersonic passenger aircraft of a new generation with a flight Mach in the range of 2 to 4. Furthermore, one needs to consider the distant future where air transport with hypersonic speeds will be required. This will enable flights on the longest routes to operate with considerable reduction in flight time.

The development of an effective Aerospace Plane (ASP) for launch of commercial loads is needed to continue to open up the near-earth space for peaceful goals. The key factor in low cost launch is the use of reusable launch vehicles from an Aeroplane-To-Orbit System (ATOS) with horizontal take-off/landing. Furthermore, a fuel-efficient air breathing engine is needed to affect substantial increase in the relative mass load.

1.2 Considerations for a Supersonic Civil Aircraft Engine

Several designers of transport aircraft have taken on projects for supersonic flight at Mach numbers of 2 to 4 and in the hypersonic range, even up to Mach 25. In developing the supersonic or hypersonic power plant, the main factor in defining its configuration is a wide range of operating conditions. Engine efficiency in cruise flight is ensured by optimizing thermodynamic parameters, which change considerably depending on flight speed and altitude. On the other hand, a significant requirement for the power plant of a high-speed aircraft is to provide high thrust for acceleration, which at high flight Mach numbers is determined by the air flow.

Today's air travel is mostly restricted to the use of the turbojet or turbofan engine. A typical property of a turbojet is the lowering of the pressure increase and airflow in the turbo compressor part with an increase in flight speed. Specific thrust parameters of the augmented turbojet approach ramjet specific parameters at flight Mach numbers 3 to 3.5. At higher flight speeds the ramjet gives higher specific thrust than turbojet at equal air-to-fuel coefficients. Corrected airflow in the ramjet with variable inlet and nozzle can be maintained constant over a wide flight speed range. As a result the ramjet thrust rises with Mach number more rapidly than the turbojet thrust. When the Mach number is higher than 3 to 4, the ramjet thrust exceeds augmented turbojet thrust; and in this flight speed region, the ramjet can be more effective than the turbojet. The ramjet, however, can not be used alone because it is incapable of producing static thrust and has a poor efficiency at low Mach numbers. Thus, in engines designed for a wide operating flight speed range, it is advantageous to combine high turbojet efficiency at Mach numbers up to 3 to 3.5 with the ramjet specific parameters at Mach numbers higher than 3.5 or 4.

1.3 The Turbojet-Ramjet Combination Engine

When the turbojet and the ramjet engines are combined in a single propulsion plant, it is possible to obtain air breathing engine class that is operable and effective up to Mach 5 or higher. A combined turboramjet engine is essentially a bypass engine in which a high pressure inner part is represented by a turbojet or turbofan and a low pressure external part by the ramjet.

An ideal combination is achieved when a turbojet having high value of thrust at low Mach numbers is combined with a ramjet matched to high Mach numbers. The thrust produced by each of the two engines can thus be added up. There is a common intake to this combination. Keeping the ramjet exit nozzle variable, the efficiency of air intake increases; and the turbojet takes full advantage of this situation. Usually, in these engines, afterburner chambers represent the combustion chamber of the ramjet part too. These chambers are formed by switching out the turbojet part by means of special devices and feeding fuel only to the ramjet combustion chambers. The flow requirements of the turbojet and ramjet vary inversely with variations of Mach number. The solution to this problem is to have variable geometry both for intake and exit nozzle. This arrangement displays the air flow coefficient variation much less than that of the turbojet and the ramjet used separately.

1.4 Adoption of the Concept by Various Countries

1.4.1 France

The practical application of the turboramjet engine was first seen in the form of the Griffon II aircraft. The project was started in 1953 by the French Air Ministry and a French firm, Nord Aviation. In April 1957, the Griffon II made a successful flight with the first ever operation of the combination engine in air. In May the speed of sound was exceeded and within a month supersonic flights were made up to Mach 1.3. The Mach number was progressively increased until December, when the aircraft achieved Mach 1.85 at an altitude of 13000 meters. There was, however, still a large surplus thrust giving a rate of climb of 8500 m/min. Further advance in Mach number was very slow as the turbojet limitations were being approached. Finally, the aircraft made one of the best performances with Mach 2.1 at 18500 meters, while still accelerating and climbing slightly [Daum 1959].

Griffon was acknowledged for excellent performance of the power plant and smoothness, which was not possible with a turbojet alone with an afterburner. All the pilots who flew the aircraft, praised it for maneuverability in supersonic high "g" turns and acceleration. Griffon achieved the world record of 1640 km/hr for 100 km closed circuit [Daum 1959]. This record proved not only the aircraft's speed potential and its aerodynamic qualities but also laid a milestone for the French research and development in the area. Presently, France is collaborating with Japan in the development of a zero to Mach 5 turboramjet engine program.

1.4.2 Russia

The successful experimentation on the turboramjet combination engine attracted the Russians, and their Central Institute of Aviation Motors (CIAM) started investigations on advisable engine concepts, operation process parameters, control modes and ground testing of experimental turboramjets in beginning of 60s. In the decades of 70s and 80s the wide testing program of experimental full scale turboramjets of different types were carried out at CIAM [Sosounov, Palkin 1992]. The experimental turboramjet engines corresponding to Mach numbers 4 to 4.5 were simulated at the facility. The obtained results, experimental data, and Russian experience of liquid hydrogen use opened the possibilities for the development of turboramjet engines for aerospace plane using liquid hydrogen as fuel. During research, it was found that the flight speed for transition to the ramjet mode depends on design operating parameters and the size correlation between typical propulsion system duct areas. The transition Mach number is usually in the range 2.5 to 3.5 [Sosounov, Palkin 1992].

1.4.3 United States of America

In 1986, President Ronald Regan called for congressional support for a National Aerospace Plane (NASP) - a very high-speed passenger airplane [Korthals-Altes 1987]. Concurrently, another program of NASA was demonstrated as High Speed Civil Transport (HSCT) aircraft. Studies were conducted by Boeing and McDonnell Douglas aircraft companies with a view to reduce existing 10 hour Los Angels to Tokyo trip to four hours or less. The mission required out of HSCT was to produce a non-pollutant flight and to carry 250 passengers at Mach 2.5 [Korthals-Altes 1987]. U.S. aeronautical preeminence in hypersonics is currently based on the NASP program to provide the technological basis for future hypersonic flight vehicles. The program plans to build and test a manned experimental flight vehicle, the X-30, to validate critical or enabling technologies demonstrating sustained hypersonic cruise and single-stage-to-orbit space launch capabilities. The X-30 is being designed to take-off horizontally from a conventional runway, reach hypersonic speeds of up to Mach 25, attain low earth orbit, and return to land on a conventional runway. The program is expected to develop and demonstrate the technology for future flight vehicles that will have technical, cost, and operational advantages over existing military and commercial aircraft and space launch systems [U.S. Congress Report October 4, 1991].

1.4.4 Japan

The National Space Development Agency of Japan, Institute of Space and Aeronautical Science, and the National Aerospace Laboratory are independently conducting research and development of technologies for separate but complementary aerospace vehicle concepts or systems [Fujimura 1996]. Japanese advanced propulsion systems are in various stages of maturation ranging from concept development to being operational. In terms of air-breathing engines, the air-turboramjet experimental engine (an expander-cycle air turboramjet system that uses much of the technology from the high pressure expander-cycle engine) is also in advanced development stages. Ministry of International Trade and Industry (MITI) is supporting a Mach 0 to 5 turboramjet engine development program, announced in 1989, as a basic research and development project composed mainly of component research. Planned to run through 1998, it is the first large-scale international collaboration program [U.S. Congress Report October 4, 1991]. Besides the local Japanese engine makers, the international participants are GE Aircraft Engines, United Technology Corp. (UTC), Rolls Royce, and SNECMA. Study results indicate that four engines with 270 kN take-off thrust are required for the aircraft carrying 300 passengers over a distance of 12000 km [Fujimura 1996]. This would mean a transpacific flight of only three hours from Tokyo to New York.

1.4.5 Australia

Australia is developing competence in selected subsystems for future aerospace vehicles, and its facilities are being used to test various U.S. and European aerospace vehicle concepts and components. Australia also conducted feasibility studies for an international spaceport on its Cape York Peninsula that would accommodate future aerospace plan [U.S. Congress Report October 4, 1991].

1.5 Objective of this Thesis

In this report an endeavor has been made to bring out the possibilities raised by turboramjet combination in the formula which one advocates for manned aircraft, a formula that brings out in a larger range of flight conditions the fundamental virtues of both types of engines. In order to facilitate the designer to view the combination engine through the window of software, a computer code has been developed in FORTRAN 77. This code assumes the layout of the engine as follows:

a) A common intake with variable geometry center body (wedge)

b) Turbojet parts at the inner core with outer annular area as the ramjet

c) Separate combustion chamber each for the turbojet and the ramjet

d) A mixer unit to mix the turbojet and ramjet combustion gases before entering the exit nozzle

e) A common variable area convergent-divergent exhaust nozzle

f) Doors to change the by-pass ratio and isolate the turbojet or the ramjet as per the prevailing flight conditions

The software runs in two steps. First, it provides the user with the capability to input as many as 14 different design parameters of his choice, based upon which it fixes the geometry of the engine. The main program then initiates the calculation of the pertinent flow parameters at all the stations of the turboramjet engine both for on-design and off-design flight conditions. For a known engine geometry, user input of the flight Mach number and the altitude is required to run the main program. The software has several built-in features that help the engine simulation to achieve results compatible with the real engine operation (see Section 3.2.2). It is also capable of suggesting the engine operation mode (only turbojet, ramjet, or the combination of both), based upon the selected flight conditions. The flow conditions at different stations are reflected on screen and can be printed through output file. Depending upon the conditions, various messages are flashed on screen to enable the user understand the undergoing flow process.

The software also possesses good academic value for the students of aircraft propulsion and helps them to understand the changes involved in the air flow parameters while passing through various components of the turbojet and the ramjet engines. It also serves as a means to give an insight to the subject of compressible fluid dynamics, especially in the areas of one-dimensional steady flow, isentropic flow conditions, shock waves, stagnation state, Raleigh line, combustion process, and air flow through varying area ducts.

CHAPTER 2

TECHNICAL BACKGROUND

2.1 Introduction

For high-speed operation of the aircraft, particularly for civilian use, besides the reduction in flight time maximum comfort to the passengers is also a subject of concern. It introduces constraints on the designer that the high speed must not be associated with excessive acceleration or high aerodynamic loads; and, furthermore, provision must be made for landing. In such cases the combination of ramjet and turbojet may offer decisive advantages. It may be useful to begin by recalling the problems that are posed by high speed flight with air breathing engines; the manner in which these are being tackled; and the reasons why the composite power plant, which is the object of attention, is the solution to be preferred.

The material presented in this chapter has been collected from various books and papers (see page 169) so as to serve as a quick reference to the reader on the subject of aircraft propulsion. Only some of the standard features that are of vital importance for a high-speed power plant are briefly addressed.

2.2 Engine Performance

In describing the performance of aircraft engines, it is helpful if several efficiencies and performance parameters are defined first. In this section various definitions and, for simplicity, representative expressions are presented as they would apply to an engine with a single propellant stream i.e. a turbojet or ramjet.

2.2.1 Engine Efficiency

2.2.1.1 Propulsion Efficiency

The product of thrust and flight velocity, is some times called thrust power. One measure of the performance of a propulsion system is the ratio of this thrust power to the rate of production of propellant kinetic energy. This ratio is commonly known as the propulsion efficiency, η_p. For a single propellant stream

$$\eta_p = \frac{\tau u}{\dot{m}_a\left[(1+f)(u_e^2/2) - u^2/2\right]} \tag{2.1}$$

With the following two reasonable approximations, equation (2.1) may considerably be simplified:

a) For air-breathing engines in general, $f << 1$ and may be ignored without leading to serious error.

b) In the thrust equation

$$\tau = \dot{m}_a\left[(1+f)u_e - u\right] + (P_e - P_a)A_e \tag{2.2}$$

The pressure term, $(P_e - P_a)A_e$, is usually much smaller as compared with the other terms, so that

$$\tau \approx \dot{m}_a(u_e - u). \tag{2.3}$$

thus

$$\eta_p \approx \frac{(u_e - u)u}{\left[(u_e^2/2) - u^2/2\right]} = \frac{2u/u_e}{1 + u/u_e} \tag{2.4}$$

η_p in no sense is an overall power plant efficiency because the unused enthalpy in the jet is ignored. Equation (2.3) shows that u_e must exceed u so that the right-hand side of equation (2.4) has a maximum value of unity for $u/u_e = 1$. However, for $u/u_e \to 1$, the thrust per unit mass flow is practically zero, and for a finite thrust the engine required would be infinitely large. It is, therefore, concluded that although u_e must be greater than u the difference must not be too great, and it is not realistic to try to maximize the propulsion efficiency of a jet engine. Hence, other parameters are required to evaluate the overall performance of the engine.

2.2.1.2 Thermal Efficiency

Another important performance ratio is the thermal efficiency, η_{th}, of the engine. For ramjets, turbojets, and turbofans it is defined as the ratio of the rate of addition of kinetic energy to the propellant to the total energy consumption rate $\dot{m}_f Q_R$, where Q_R is the heat of reaction of the fuel. Thus the thermal efficiency may be written, for a single propellant stream, as

$$\eta_{th} = \frac{\dot{m}_a\left[(1+f)(u_e^2/2) - u^2/2\right]}{\dot{m}_f Q_R}$$

or

$$\eta_{th} = \frac{\left[(1+f)(u_e^2/2) - u^2/2\right]}{f Q_R} \tag{2.5}$$

2.2.1.3 Overall Efficiency

The product of $\eta_p \eta_{th}$, is called the overall efficiency, η_o, and is defined by

$$\eta_o = \eta_p \eta_{th} = \frac{\tau u}{\dot{m}_f Q_R} \tag{2.6}$$

Using equation (2.4), (for $f \ll 1$)

$$\eta_o = 2\eta_{th}\left(\frac{u/u_e}{1+u/u_e}\right) \tag{2.7}$$

Thus, the overall efficiency depends only on the velocity ratio, u/u_e, and on the thermal efficiency, η_{th}; which depends somewhat on the velocity ratio. Hence, it is concluded that the efficiency of an aircraft power plant is inextricably linked to the aircraft speed.

2.2.2 Thrust Specific Fuel Consumption

The ambiguous concept of efficiency is discarded in favor of the Thrust Specific Fuel Consumption (*TSFC*) which, for aircraft engines, is usually defined as the ratio of fuel flow rate to thrust. Hence

$$TSFC = \frac{\dot{m}_f}{\tau} \tag{2.8}$$

For a turbojet with $P_a = P_e$, equation (2.2) shows that

$$TSFC = \frac{\dot{m}_f}{\dot{m}_a[(1+f)u_e - u]} \tag{2.9}$$

Equation (2.9) indicates that the $TSFC$ of a given engine depends strongly upon flight speed. Typical values of $TSFC$ for modern engines are shown in Table 2.1

Table 2.1 Typical values of TSFC for modern engines [Hills and Peterson 1992]

Engine Type	**TSFC** [kg/N . hr]
Ramjets (Mach 2)	0.17 - 0.26
Turbojets (Static)	0.075 - 0.11

2.2.3 Specific Thrust

Another performance parameter is the specific thrust I_s, namely the thrust per unit mass of air (e.g. N/kg). This provides an indication of the relative size of the engine producing the same thrust because the dimensions of the engine are primarily determined by the air flow requirements. Size is important because of its association not only with weight but also with frontal area and the consequent drag. The $TSFC$ and specific thrust are related by

$$TSFC = \frac{f}{I_s} \tag{2.10}$$

where f is the fuel/air ratio.

2.3 Turbojet

2.3.1 Air Intake System

The function of the air intake for a turbine engine is to deliver the required air mass flow at the compressor face with the highest possible stagnation pressure and with the smoothest possible velocity distribution. Any loss in stagnation pressure represents a performance loss for the engine while variation in velocity around an intake is liable to cause compressor surging or blade breakage. These requirements are to be met over a wide range of flight conditions without introducing too much complexity or weight. It is important for the intake designed on one set of conditions that the off-design performance penalties must not become unacceptably high. Modern engine installations are incorporated with a variable geometry mechanism, which ensures intake area variation with flight conditions in order to give good off-design performance.

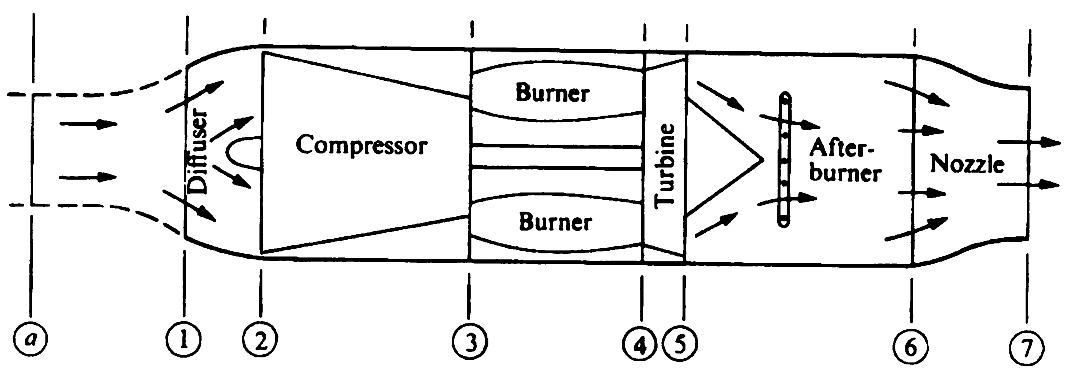

Figure 2.1 Layout of a typical turbojet engine [Courtesy Hills and Peterson 1992]

For supersonic engines there is a penalty of a high momentum drag arising from the high flight speed but there is a corresponding advantage in the high ram pressure ratio available. The ram pressure ratio of an isentropic intake is given by

$$\frac{P_0}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{k/(k-1)} \tag{2.11}$$

The formula indicates that the ram pressure ratio increases with increase in the Mach number.

In order to have a subsonic velocity at the compressor face, a supersonic intake must generate at least one shock. Deceleration through a shock is always accompanied with stagnation pressure loss, and this loss varies with the intensity of the shock. An efficient intake will use multiple shocks to keep the intensities low so as to keep the loss in stagnation pressure to a minimum.

Supersonic intakes as currently used can be classified under the headings of external compression, internal compression, and mixed external-internal compression. However, keeping in mind the requirement of the engine under study, only the external compression supersonic intake is briefly discussed.

2.3.1.1 External Compression Supersonic Intake

External compression intakes are those in which the shocks pattern is formed ahead of the outer casing of the intake. Although such intakes involve complex design, an improved performance is obtainable by their use. For Mach numbers above 1.5 the compression is achieved through shock waves by setting up a single or a double cone ahead of intake cowling.

2.3.1.2 The Single-Cone Intake

In this type of intake the cone sets up a conical shock whose intensity will be a function of the Mach number and the included angle of the cone. The flow Mach number will still be supersonic but will be lower and deflected away from the axial direction. By suitable selection of the intake geometry the shock can be made to fall on the cowl lip. For further deceleration to a subsonic Mach number, a normal shock is needed which can also be made to rest at the lip. Further subsonic diffusion takes place inside the cowl.

2.3.1.3 The Double-Cone Intake

In a double-cone intake two discontinuities of the flow are set up, first by the tip of the cone and second by the change in cone angle. With good design the two conical shocks may meet at the cowl lip, and the final normal shock may also settle here. Since the combined intensity of the two conical shocks is less than that of corresponding single one, the intake performance is improved.

In principle each intake has a two-dimensional counterpart in which cones are replaced by wedges. For these the Mach number relationships across the shocks are of the same general form as are for conical shocks. Thus, the flow characteristics can be represented in the same way both for a cone or a wedge by taking the upstream Mach number and cone or wedge angle as the prime variables.

2.3.1.4 Intake Characteristics

The intake characteristics can be well explained by drawing them to the axes of pressure ratio and non-dimensional engine intake mass flow. By keeping flight Mach number as another variable constant Mach number lines can be drawn on these characteristics.

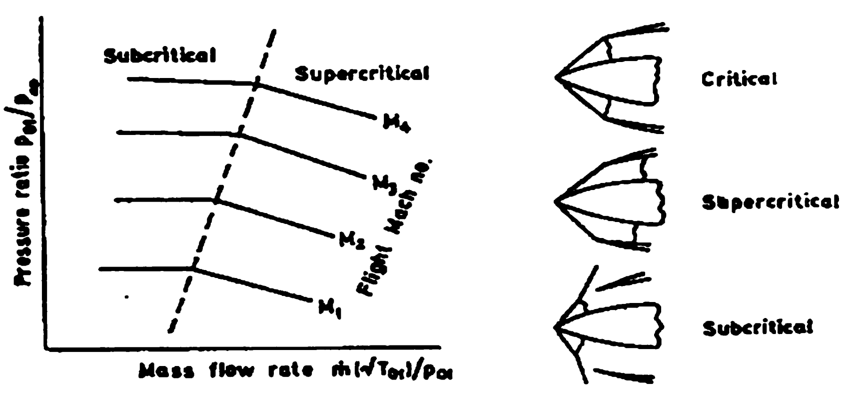

Figure 2.2 Characteristics of the external compression intake
[Courtesy McMahon 1971]

Figure 2.2 shows the typical form of intake characteristics. It is observed that the pressure ratio goes up with increasing Mach number. On the plot of these characteristics a "critical" line is drawn joining the points of maximum pressure ratio and the regions left and right to it are labeled as "subcritical" and "supercritical" respectively. By studying the shock pattern with changes in engine operating conditions it is realized that the position and intensity of the conical shock depends on the mass flow being swallowed by the engine. The critical condition represents the shock configuration for design conditions with shock resting on intake lip.

2.3.1.5 Shock Location and the Engine Air Demand

Depending upon the operating conditions if the engine air demand increases, the oblique shock will be unaffected, but the normal shock will move down into the intake as shown in the second sketch of Figure 2.2. The flow, which is still supersonic, will be accelerated in the divergent portion of the intake and the normal shock will be more intense causing greater loss in the stagnation pressure. The physical mass flow and the stagnation temperature will remain unchanged as the two depend on the flight Mach number and the intake area at the cowl lip.

If the mass flow should decrease from the critical, the normal shock will move outside the cowl lip (third sketch of Figure 2.2) but will remain about the same intensity. This concludes that the pressure ratio is virtually constant in the subcritical region. If, however, the mass flow is reduced too much the intakes are liable to become unstable.

In general the engine operation should be kept close to the critical range in order to achieve best possible pressure recovery. In the work under study the same concept is utilized by designing a variable geometry wedge, which ensures that the both oblique and normal shocks always rest on the cowl lip.

2.3.2 Compressor and Turbine

In an aircraft engine the compressor and turbine are often referred as the turbomachines. For large engines the turbomachines are preferred because they can be made much smaller in size for a given flow rate. It is because of higher internal velocities and higher fraction of internal volume allocated to the flow.

The engine performance, and thus aircraft range, is strongly dependent upon compressor pressure ratio and efficiency. Across the compressor the air is made to flow from a low pressure to a high pressure i.e. against the "natural direction" of the flow in contrast to the turbine where air flows in its natural direction.

Both compressor and turbine contain rows of stator and rotor blades. One row of a rotor and stator form a stage. In a compressor, work is done on the air and it gets accelerated as it passes through the rotor. The function of the stator row of the compressor is to accept this high velocity air and decelerate it, so as to increase the static pressure. The axial component of the air velocity almost remains constant while passing through all the stages of the compressor.

In a turbine the stators as sometimes termed as "nozzle" and the rotor as "buckets". Because of generally falling pressure in turbine flow passages, much more turning in a given blade row is possible without danger of flow separation as compared with an axial compressor blade row. This implies that more work and considerably higher pressure ratio is achievable per stage of turbine. This conclusion points at the fact that with a turbine of two to three stages it is possible to drive a multi-stage large compressor.

The airflow meets the turbine after the combustion process. Thus, the high temperature and high pressure gases entering the turbine have a large amount of available energy with respect to the exhaust conditions and must be extracted as efficiently as possible. A high percentage of this energy needs to be fed back into the cycle to drive compressor while the reminder is available for useful output, either by driving a power turbine or by forming a propelling jet stream.

As in case of an axial compressor the flow axial velocity component across turbine can also be assumed to remain constant.

2.3.3 Process of Combustion

The combustion process involves the oxidation of constituents in the fuel that are capable of being oxidized and can therefore be represented by a chemical equation. During a combustion process the mass of each element remains the same. When a hydrocarbon fuel is burned, both the carbon and the hydrogen are oxidized. Considering the combustion of methane as an example

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O \qquad (2.12)$$

In this case the products of combustion include both carbon dioxide and water. The water may be in the vapor, liquid, or solid phases, depending on the temperature and pressure of the products of combustion.

In most combustion processes the oxygen is supplied as air rather than as pure oxygen. The air is considered to be composed of 21 percent oxygen and 79 percent nitrogen by volume. This leads to the conclusion that for each mole of oxygen, 3.76 moles of nitrogen are involved. Therefore, when the oxygen for the combustion of methane is supplied as air, the reaction can be written as

$$CH_4 + 2O_2 + 2(3.76)\,N_2 \rightarrow CO_2 + 2H_2O + 7.52\,N_2 \qquad (2.13)$$

2.3.3.1 Flow Conditions During Combustion

The combustion chamber is considered to be a constant-area duct in which heat transfer takes place. The process involves changes in the stagnation enthalpy and in the stagnation temperature of a gas stream through such a duct without frictional effects. Such flows are called simple stagnation temperature-change flows.

2.3.3.2 The Raleigh Line

If the conditions upstream of the control volume are fixed (state 1), and conditions are to be found down stream (state 2), then for a particular value of V_2:

a) ρ_2 can be calculated from the continuity equation

b) P_2 may be evaluated from momentum equation

c) T_2 may be found from equation of state

Hence, the continuity equation, the momentum equation and the equation of state define a locus of states passing through state 1, which is known as the Raleigh line on the *T*-S diagram. The states on the Raleigh line are reachable from each other in the presence of heat transfer effects.

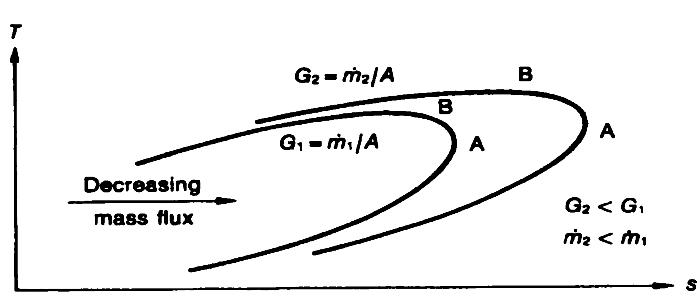

Figure 2.3 Raleigh line for different mass fluxes [Courtesy Aksel and Eralp 1994]

Raleigh lines corresponding to different mass fluxes are shown in Figure 2.2, which indicates that the entropy and temperature are maximum at points A and B respectively. At point A, where the slope of the Raleigh line is infinite, the Mach number is unity whereas, at point B where the slope is zero, $M = 1/\sqrt{k}$. Also it is to be noted that the lower branch of the Raleigh line corresponds to supersonic flow [Aksel and Eralp 1994].

Since the process of simple heating is thermodynamically reversible, heat addition must correspond to an entropy increase and the heat rejection must correspond to an entropy decrease. Therefore, the Mach number increases with heating and decreases with cooling at subsonic speeds. However, at supersonic speeds, the Mach number decreases with heating and increases with cooling. Thus, the process of heat addition tends to make the Mach number unity. The flow can not continue beyond Mach 1 with either heat addition or heat rejection alone. During the heating of a subsonic flow between points A and B of the Raleigh line, the static temperature decreases [Aksel and Eralp 1994].

2.3.3.3 Effects of Heat Transfer on Flow Properties

For the combustion process the flow enters of the chamber with a low subsonic Mach number such that it always remains less than 1/k. The effects of heat transfer on the flow properties across the combustion chamber may be summarized as follows:

a) The flow Mach number, static temperature, velocity, stagnation temperature, and entropy undergo an increase.

b) The static pressure, density, and stagnation pressure decrease.

2.3.4 Effect of Flight Mach Number on Turbojet

The turbojet with after-burner comprises the same elements as the ramjet, plus a rotating assembly (compressor and turbine) which represents an additional cycle. The essential function of this additional cycle is to increase the pressure, which must exist at the entry of the after-burner chamber. From the stand point of performance, the effect of increasing Mach number results from the following three considerations:

a) The air temperature at the compressor entry increases rapidly with Mach number.

b) The gas temperature at the turbine entry is limited to a given value, which is the function of the state of art at the time.

c) The speed of rotation of the machine can not exceed a certain figure.

These effects mean that as Mach number increases, the turbojet pressure ratio deteriorates and may even reduce to a value less than unity. The additional cycle thus becomes superfluous at first and then harmful. The specific consumption becomes higher than it would be if the rotating parts were discarded.

At the Mach number where the total delivery head to the after-burner combustion chamber is equal to the compressor entry pressure, the turbojet will stop its function of pressure amplifier. Assuming practical values of compressor and turbine efficiencies and the turbine limiting temperature as 1000^{o} C, this limiting Mach number is found to be approximately 3.3 [Daum, 1959]. The problem, however, is not to design an engine working at a given Mach number only, but to cover a complete flight envelope. For the turbojet the flight envelope may be defined by the extreme values taken in flight by the parameter $N/\sqrt{T_{0c}}$, where N is the engine RPM and T_{0c} is the air stagnation temperature at the compressor entry. For majority of the high speed aircraft taking 100 percent as the reference value at Mach 0.9, $N/\sqrt{T_{0c}}$ reduces to 65 percent for Mach 3 and to 52 percent for Mach 4, provided the RPM are kept constant [Daum 1959].

It is extremely difficult to achieve correct function of compressor over such a wide range. An adequate margin against surging (flow separation caused by pulsating flow due to back pressure at turbine face) must be preserved when the air is cold, i.e. at low Mach numbers, while maintaining still acceptable values of efficiency when the air is hot, i.e. at high Mach numbers. Various independent studies relating to fixed geometry compressor with compressor ratios ranging between 3.5 and 8 at sea level show that at turbine entry temperature of the order of 1000° C the turbojets ceases to act as pressure amplifiers when the Mach number exceeds 2.7 or thereabouts [Daum 1959]. Beyond this value, the drop in total pressure is higher the higher the compression ratio at sea level. Raising the turbine temperature by 100° C only increases the Mach number by about 0.25. If the variable stator vanes are incorporated, which can adjust as a function of flight parameters; the limiting Mach number can then reach or even slightly exceed 3 [Daum 1959].

The dimensions and, hence, the weight of the turbojet is determined by the rate of airflow through the engine. It is seen that the flow coefficient decreases as the flight Mach number increases. This is essentially due to the fact that the Mach number at compressor entry blades decreases because the blade tip speed remains constant and the air warms up. The Mach number is highest when cold and can not exceed a certain transonic value. It concludes that as the Mach number increases, a given turbojet is less and less able to absorb a large rate of air flow. If it is to deliver a high thrust at a high Mach number, it will have to be larger and larger.

Exhaust Nozzle

For a high performance engine it is highly desirable that a convergent-divergent propelling nozzle is used, but its use entails a number of design problems. The most immediate problem is that the ratio of exit to throat area for the divergent portion can only be at its correct value for only one pressure ratio of the nozzle. If the overall pressure ratio is less than the design value, a shock system will form inside the divergent portion of the nozzle and will lead to appreciable losses. There is also a risk that the shocks will be unstable and may cause vibration of the jet pipe. The next main problem with the design of the convergent-divergent nozzle is associated with the reheat system. When reheat is brought into operation there is a large increase in the volume flow through the nozzle and the area must be increased if the jet pipe pressure and the turbine entry temperature are to be maintained at a suitable value.

The two problems can be overcome if the nozzle with variable geometry is used. The study assumes a nozzle with variable geometry both at the nozzle throat and exit.

2.4 The Ramjet

2.4.1 Basic Features

The ramjet is the simplest of the air breathing engines. The engine has no moving parts in its operating cycle and can not operate until it is moving with fairly high forward speed. Having no rotating parts (excluding accessory drives), the ramjet can be made light and compact and can avoid material limitations encountered at high flying speeds by the turbine engines.

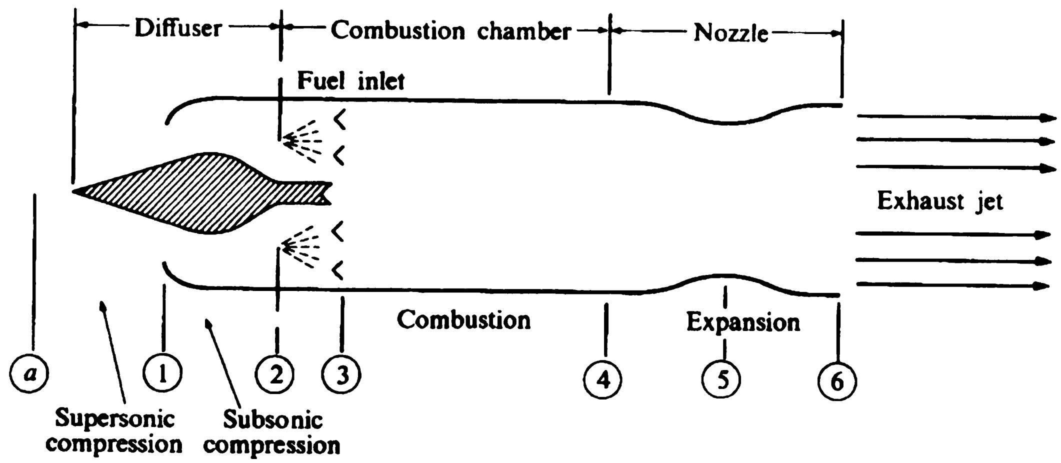

Figure 2.4 Typical layout of a ramjet engine [Courtesy Hills and Peterson 1992]

2.4.2 Effect of Intake and Nozzle Area Variation

In the case of a ramjet that incorporates an adjustable exit nozzle throat, there is no relative low temperature limitations imposed by a turbine; neither is there any compressor whose limiting speed prescribes a law to the intake flow. In this case the nozzle throat variation directly affects air flow at the entry and allows the intake to work permanently under matched conditions. Hence, the qualities of a ramjet are determined by the air intake. If an intake is selected, which has a high pressure recovery at high Mach numbers; the engine will be incomparable at high Mach numbers but poor at lower ones when its external drag will be high. On the other hand, if good characteristics are desired at low Mach numbers, poor performance will be resulted in higher Mach numbers. The net thrust will differ noticeably in different cases.

If the exit nozzle throat is kept constant, i.e. fixed gas temperature at the ramjet exit is deliberately postulated, then beyond the Mach number chosen for air flow matching, the ramjet air intake works under supercritical condition. Below that Mach number,

subcritical conditions are obtained with risk of intake buzz. Surging of the air intake is essentially linked to the intensity of the normal shock, which decreases together with the Mach number, resulting in reduced performance.

For an intake with lower efficiency the mismatching effect will be more pronounced, whereas if the intake is of progressive external compression type, matching remains correct within the range of Mach numbers for which the compression is truly isentropic.

2.5 The Turbojet-Ramjet Combination

2.5.1 Intake

For the turbojet-ramjet combination, one might be tempted to add the thrust coefficients of a ramjet and a turbojet with the aim of achieving any desired variation with Mach number. For instance, one might choose a turbojet having a high compression ratio and a ramjet matched to high Mach numbers and having a high efficiency intake and add up the thrusts. However, a better solution as advocated by Nord Aviation of France is to have a common intake. If the ramjet exit nozzle is variable, the compressor no longer imposes its law on the air intake, and the turbojet can take full advantage of the high air intake efficiency that suits the ramjet.

In the turbojet the air flow requirement is characterized by the Mach number at compressor entry, whereas in the ramjet it is governed by the Mach number at combustion chamber entry. The variation of flow with change in the Mach number in the turbojet is opposite to that in the ramjet. Because of this, it is always possible to design an air intake with a good efficiency for which matching to the total airflow will remain correct within a large range. This matching can further be augmented by using an exhaust nozzle with variable throat and exit section.

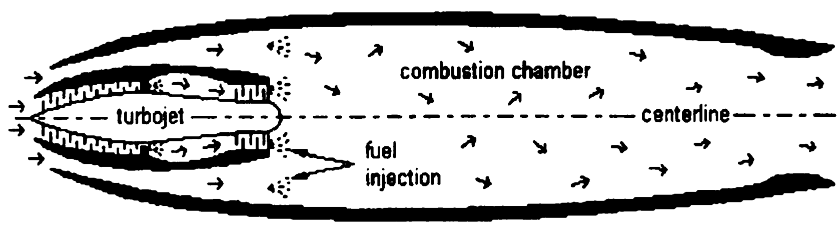

Figure 2.5 Layout of a turboramjet engine with separate combustion chambers
[Courtesy Turboramjet profile (Online Image)]

This combination will provide the sum of the thrust of a turbojet without intake losses and that of a well-matched ramjet. At the same time it will avoid the complication of having two variable geometry intake systems.

2.5.2 Engine Characteristics

For an engine intended for a very high speed, the desirable characteristics are considered one by one.

2.5.2.1 Cruise

The cruising flight at maximum Mach number is assumed to take place in accordance with the Breguet law for calculating range

$$R = \frac{V}{TSFC} \frac{L}{D} \log \frac{W_1}{W_2} \tag{2.14}$$

Here: R = range i.e. the distance covered

V = Velocity

TSFC = Thrust Specific Fuel Consumption

L = Lift

D = Drag

W_1 = Initial weight of the aircraft

W_2 = Final weight

It is observed that specific fuel consumption is one of the fundamental characteristics of the engine. It has also been observed that as soon as the Mach number reaches 2.7 to 3.0, the specific consumption of the ramjet becomes lower than that of turbojet [Daum 1959]. It means that around Mach number 2.7, the specific consumption is essentially a function of the intake characteristics. This fact is obvious as regards the ramjet, but the turbojet also behaves in the same manner because by the time Mach number reaches this value the turbojet loses its ability to amplify pressure. Thus, at Mach numbers above 2.7, the specific consumption of the turbojet would progressively increase while that of the ramjet would decrease slightly or stay constant. It should be noted here that for the same weight of fuel using a lighter engine allows a better value of W_1 / W_2 to be achieved resulting in a longer range for the same specific fuel consumption.

2.5.2.2 Climb

For that part of the flight that corresponds to acceleration and climb, it is possible to imagine many different flight patterns. However, due to the structural strength limitations most high speed aircraft projects are based on climb program. This may be decomposed as follows [Daum 1959]:

a) Acceleration at very low altitude up to Mach 0.9

b) Climb at a constant Mach number of 0.9 up to relatively high altitude, but always less than 11000 meters

c) Acceleration at constant altitude or in slight climb up to the point when maximum aerodynamic load allowed by structural strength is reached. Loading may be expressed in terms of equivalent airspeed

d) Climb at constant aerodynamic loading up to the maximum Mach number

e) Climb at maximum Mach number up to the cruise altitude

If the aircraft is to climb and accelerate, the net thrust of the engine at all points must be higher than the drag. If the thrust is chosen at too high a value, the engine may be too large and hence very heavy. If, however, the excess of thrust is too small, the fuel consumption may suffer owing to long duration of climb. This situation calls for an optimization study if the size of the engine is determined by this part of flight.

2.5.2.3 Turbojet - Ramjet Proportion

In case of the turbojet alone, the excess of thrust coefficient over drag coefficient is roughly proportional to the drag coefficient itself within a large range. Thus, the size of the turbojet may be determined either by conditions at the end of climb at cruising Mach number or by optimization of the climb and cruise portions taken together. With the turboramjet, however, an additional variable is identified which allows to achieve an almost arbitrary law of variation of thrust coefficient. This variable is the proportion of the ramjet added to the turbojet. The type of ramjet can be varied, and it is possible to achieve a law such that the excess of thrust should be large in the high supersonic range and small in the transonic range. This would mean that the turbojet should be small in size.

2.5.2.4 Specific Fuel Consumption

The high specific fuel consumption of the ramjet below Mach 2.5 will be largely compensated by the small duration of acceleration in the supersonic range. The total weight of the power plant and fuel during the climb will be less than the large turbojet designed for a high Mach. In any case there will be a large thrust margin under cruise conditions corresponding to minimum specific consumption. If the ratio of maximum drag coefficient to cruising drag coefficient is not too high, one may be tempted to decrease the proportion of the turbojet still further. However, it must then be ensured that the sea level thrust is sufficient for take-off.

2.5.3 Features of Turboramjet

The lightness of the ramjet fully justifies its use, alone or in combination with the turbojet, even under conditions in which it is very much inferior thermodynamically to the turbojet. This lightness is due to the total absence of moving parts and low value of loads due to internal pressure. Moreover, hot regions are cooled by air at a temperature equal to the stagnation temperature of ambient air, which is the lowest value obtainable. If the range is not too important, simplification may be made in the turboramjet engine with a view to reduce its weight.

The turboramjet has other virtues that are not obvious through thermodynamic analysis:

a) It is flexible so that its characteristics can be modified during manufacture or even after the first few flights. The modification may be incorporated either in variation in the drag of the aircraft or with changes in the specifications.

b) There is no need to have a turbojet capable of working at the maximum Mach number. It can be switched-off on reaching its design maximum Mach, and then kept in slow wind-milling in order to maintain flow of liquids in pipes and at bearings.

This kind of operation is made easy by the coaxial arrangement, which results in the turbojet remaining in the subsonic environments, the Mach number in the plane of the inlet being of the order of 0.4 to 0.5 on the average. In the exit plane around the turbojet, the Mach number is of the same order of magnitude. This arrangement greatly eases the problem of re-lighting the turbojet when the aircraft comes down to lower Mach number where operation of turbojet is possible.

Finally, the period required for the development of a ramjet around a turbojet is quite short as soon as some experience has been acquired through previous tests. Similar arrangements were investigated in Nord Aviation of France and CIAM of Russia.

CHAPTER 3

SOFTWARE COSIDERATIONS FOR THE TURBORAMJET

3.1 Engine Layout Criterion

The performance of the turboramjet engines largely depends on the layout used to integrate different parts of the two engines involved. The criterion advocating the layout is the principle of the use or non-use of a part of the gasturbine engine's free power to compress air flow in the ramjet part (gasturbine energy transfer to the ramjet part). An advantage for using the layout that transfers energy to the ramjet parts is that an increase in the engine thrust at take-off and low flight speeds is achievable. This layout, however, is advantageous when a turbofan engine is used in place of a turbojet in combination with the ramjet [Sosounov, Tskhovrebov, Solonin and Palkin 1992].

Turboramjet without energy transfer to the ramjet part is just a typical combination of a gasturbine and ramjet engine used on flight vehicles for that the trajectory consists of climb, acceleration, cruise and decent. In this group it is possible to have two types of engines:

a) One that has a common combustion chamber for both turbojet and the ramjet

b) Another that has separate combustion chambers for each of the two engines

Since this project is confined to the use of a combination of turbojet and the ramjet for a high-speed commercial aircraft, the study is restricted to the turboramjet engine without energy transfer to the ramjet part. Moreover, separate combustion chambers are used for each of the two engines involved, which are restricted to the use of kerosene fuel only.

3.2 Software Development for the Selected Layout

3.2.1 Capabilities of the Software

The software for the turboramjet under study is developed with a view to:

a) Allow the user to furnish the selective design parameters of his choice.

b) Fix engine geometry in the light of these parameters.

c) Observe flow conditions at different locations of the engine based upon the selected design conditions. Both on-design and off-design operations are covered.

d) Select automatically the appropriate mode of engine operation, turbojet alone, combination or only the ramjet.

e) Suggest measures for some "out of flight envelop" operations.

f) Display on screen and store in an output file the flow properties at different stations (Section 3.4) of the engine.

3.2.2 Salient Features of the Software

The software has many built in features that enhance the conformity level to the real aircraft engine design. Some of these are:

a) The calculation of free stream conditions for the selected altitude.

b) Automatic adjustment of the wedge to ensure maximum pressure recovery through shock waves at supersonic flight speeds.

c) Considerations of maximum thermal limit both for turbojet and the ramjet that are made user-defined for enhancing the flexibility for the designer.

d) Inclusion of diffusers before combustion chambers to ensure existence of flow conditions suitable for a practical combustion process.

e) Automatic checking of the choking conditions both at the intake and combustion chamber under the prevailing flight conditions and adjustment of wedge to alter the flow conditions to avoid choking problems.

f) The exhaust nozzle adjustment to remain choked at the throat and fully expanded under all flight conditions.

g) Inclusion of component efficiencies to simulate an engine as close to reality as possible.

h) Use of logic for the mode of operation of the engine, i.e., to decide whether to run turbojet, combination or the ramjet alone under the prevailing conditions.

i) Upon reaching the maximum user defined ram burner thermal limit, providing the user with the following two options:

   i. To stop the program

   ii. To continue. In this case the program limits the burner temperature to the defined limit and calculates the flow properties at further higher Mach numbers. In this case the program stops when the engine thrust drops to zero.

3.2.3 Assumptions for the Software Development

a) The flow is assumed to be one-dimensional and steady throughout.

b) The diffuser exhibits 4% stagnation pressure loss along its length under all flight conditions.

c) The diffusers installed prior to turbojet and the ramjet combustion chambers to reduce the flow velocity to the user-defined valve are isentropic.

d) Axial velocity component across the compressor and the turbine remains constant.

e) The combustion process follows the Raleigh line i.e. the combustion chambers of both the engines are constant area ducts.

3.3 Salient Features of the Selected Engine Layout

a) The turbojet is incorporated at the inner core whereas; the outer annular area is used for the ramjet.

b) A common intake and the nozzle for the two types of the engines.

c) At design point the inlet area is one square meter.

d) Maximum pressure recovery at the intake under all flight conditions that is ensured by a variable geometry moveable wedge.

e) The diffuser exit area is equally divided for flow to pass through each of the turbojet and the ramjet.

f) Bypass doors are kept fixed at 50% under all operating conditions.

g) The turbojet always operates at 100% RPM.

h) The volume flow rate (area times velocity) entering the turbojet remains fixed for all operating conditions.

i) The turbojet is incorporated with a convergent nozzle, which operates choked under all flight conditions.

j) The exhaust area of the ramjet remains equal to that of ram burner at all times.

k) The engine nozzle is convergent-divergent type, and always operates choked at the throat and fully expanded at the exit.

3.4 The Engine Layout

For the purpose of studying the flow conditions across the turboramjet, different stations are marked along the longitudinal axis of the engine. Figure 3.1 shows the schematic diagram of the engine with marking of these stations.

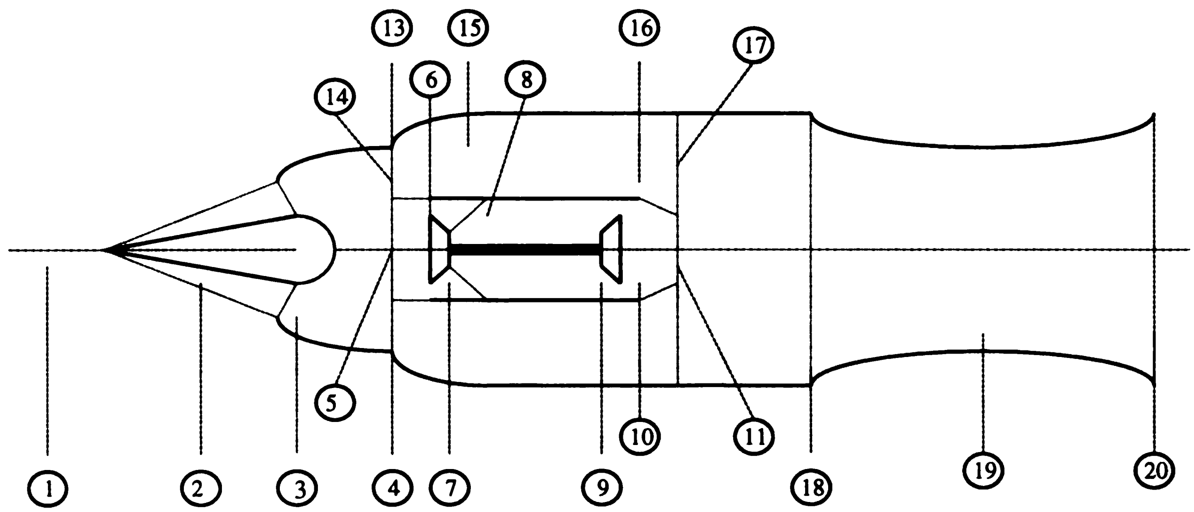

Figure 3.1 Schematic diagram of the turboramjet engine

Glossary of the stations as marked on Figure 3.1 is given in Table 3.1

Table 3.1 Explanation of the stations allocated to the turboramjet engine

Station Number	**Flow Condition/Component**
Station 1	Free stream conditions
Between 1 and 2	Oblique shock
Between 2 and 3	Normal shock
Between 3 and 4	Main diffuser
Station 5	Plane of turbojet entry at bypass doors
Between 5 and 6	Bypass doors
Between 6 and 7	Compressor
Between 7 and 8	Diffuser before turbojet combustion chamber
Between 8 and 9	Turbojet combustion
Between 9 and 10	Turbine
Between 10 and 11	Turbojet convergent nozzle
Station 12	Not presently assigned for possible future use
Station 13	Plane of ramjet entry at bypass doors
Between 13 and 14	Bypass doors (ramjet flow)
Between 14 and 15	Diffuser before ramjet combustion
Between 15 and 16	Ramjet combustion
Between 16 and 17	Ramjet exhaust
Station 18	Exit of the turbojet and the ramjet flow mixer unit
Station 19	Throat of main convergent-divergent nozzle
Station 20	Main nozzle exit

3.5 Frequently Used Equations

3.5.1 General Considerations

For the purpose of this study, the flow is assumed to be one dimensional, steady and observing ideal gas law throughout. Furthermore:

a) The gas constant R has a value of 287 [J/kg K] that remains unchanged

b) The ratio of the specific heats k varies as follows:

   i. All conditions without heat or energy transfer, $k = 1.4$

   ii. Turbojet combustion and mixer calculation, $k = 1.33$

   iii. Ramjet combustion, $k = 1.30$

c) Specific heat with constant pressure c_p is calculated from the relation

$$c_p = \frac{kR}{(k-1)} \tag{3.1}$$

d) Within one process under consideration k, R and c_p remain constant

3.5.2 Stagnation State

The Stagnation State is defined as that state which would be reached by a fluid if it were brought to rest reversibly, adiabatically, and without work. The energy equation for such a deceleration reduces to

$$\mathrm{d}h + u\,du = 0$$

which may be integrated to

$$h_0 = h + \frac{u^2}{2} \tag{3.2}$$

Here, u is the flow velocity, h is specific enthalpy of the flow and the constant of integration h_0 is called the stagnation enthalpy.

The stagnation temperature T_0 is the temperature that would be reached upon adiabatic, zero-work deceleration. For a perfect gas with constant specific heats

$$h = c_p T$$

similarly

$$h_0 = c_p\, T_0 = c_p T + \frac{u^2}{2} \tag{3.3}$$

or

$$T_0 = T + \frac{u^2}{2c_p} \tag{3.4}$$

It follows from the energy equation that if there is no heat or work transferred, T_0 will remain constant.

The stagnation (or total) pressure P_0 is defined in a similar way to T_0 but with the added restriction that the gas is imagined to be brought to rest not only adiabatically but also reversibly i.e. *isentropically*. The stagnation pressure is thus defined by:

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{k/(k-1)} \tag{3.5}$$

3.5.3 Ideal Gas Equation

$$P = \rho RT \tag{3.6}$$

3.5.4 Energy Equation

From the steady flow energy equation is represented as

$$h_{01} = h_{02} = h_1 + \frac{u_1^2}{2} = \text{Constant} \tag{3.7}$$

or

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \tag{3.8}$$

3.5.5 Conservation of Mass

The conservation of mass for steady one-dimensional flow becomes

$$\frac{\dot{m}}{A} = \text{Constant} = \rho_1 u_1 = \rho_2 u_2 \tag{3.9}$$

$$\dot{m} = \rho\, uA \tag{3.10}$$

3.5.6 Momentum equation

The momentum equation assuming negligible friction at duct walls and no body forces, is written as

$$P_1 - P_2 = \frac{\dot{m}}{A}(u_2 - u_1) \tag{3.11}$$

3.5.7 Mach Number

In compressible flow problems a frequently used variable is the Mach number which is defined as

$$M = \frac{u}{c} \tag{3.12}$$

where c is the local speed of sound in the fluid. The speed of sound is the speed of propagation of very small pressure disturbances. For a perfect gas it is given by

$$c = \sqrt{kRT} \tag{3.13}$$

hence

$$u = M\sqrt{kRT} \tag{3.14}$$

3.5.8 Isentropic Flow

Many actual processes, such as flows in nozzle and diffusers, are ideally isentropic. The simple results are presented for an isentropic flow of a perfect gas in the absence of work and the body forces.

In terms of Mach number the ratio of stagnation to static properties are

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \tag{3.15}$$

$$\frac{P_0}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{k/(k-1)} \tag{3.16}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{1/(k-1)} \tag{3.17}$$

Using continuity equation the mass flow per unit area in term of k and M can be expressed as

$$\frac{\dot{m}}{A} = \frac{P_0\sqrt{k}}{\sqrt{RT_0}} M \left(\frac{1}{1 + \frac{k-1}{2} M^2}\right)^{(k+1)/2(k-1)} \tag{3.18}$$

For a given fluid (k,R) and inlet state (P_0, T_0) it may readily be shown that the mass flow per unit area is maximum at $M=1$. If those properties of flow at $M=1$ are indicated with an asterisk, the maximum flow per unit areas becomes

$$\frac{\dot{m}}{A^*} = \frac{P_0\sqrt{k}}{\sqrt{RT_0}} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \tag{3.19}$$

The area ratio may thus be obtained by

$$\frac{A}{A^*}=\frac{1}{M}\left(\frac{2}{k+1}\left\{1+\frac{k-1}{2}M^2\right\}\right)^{(k+1)/2(k-1)} \tag{3.20}$$

It may be noted that for a given isentropic flow (i.e. known $k, R, P_0, T_0, \dot{m}$), A^* is a constant so that it may be used to normalize the actual flow area A.

3.5.9 Frictionless Constant Area Flow with Stagnation Temperature Change

Combustion and evaporation or condensation processes may be analyzed through frictionless flow in a constant-area duct in which a stagnation enthalpy change occurs. The stagnation-enthalpy change in this process can be determined from

$$\Delta h_0 = q - w$$

If subscript 1 signifies an initial condition and 2 as the final state then the continuity, momentum and energy equations with introduction of M and k, can be expressed as

$$\frac{P_2}{P_1}=\frac{1+kM_1^2}{1+kM_2^2} \tag{3.21}$$

$$\frac{P_{02}}{P_{01}}=\left(\frac{1+kM_1^2}{1+kM_2^2}\right)\left(\frac{1+\frac{k-1}{2}M_2^2}{1+\frac{k-1}{2}M_1^2}\right)^{k/(k-1)} \tag{3.22}$$

Equation (3.22) shows that the change in stagnation pressure is directly related to the Mach number. The dependence of Mach number on stagnation enthalpy (or stagnation temperature) can in turn be obtained by using the perfect-gas law, the continuity condition, and the Mach number relation

$$\frac{T_2}{T_1}=\left(\frac{P_2 M_2}{P_1 M_1}\right)^2$$

or

$$\frac{T_2}{T_1} = \left[\frac{1 + kM_1^2}{1 + kM_2^2} \left(\frac{M_2}{M_1} \right) \right]^2 \tag{3.23}$$

It can be shown that the stagnation temperature ratio is governed by

$$\frac{T_{02}}{T_{01}} = \left[\frac{1 + kM_1^2}{1 + kM_2^2} \left(\frac{M_2}{M_1} \right) \right]^2 \left(\frac{1 + \frac{k-1}{2} M_2^2}{1 + \frac{k-1}{2} M_1^2} \right) \tag{3.24}$$

Equation (3.24) shows that the Mach number depends uniquely on the ratio of stagnation temperatures and the initial Mach number.

In order to simplify these relationships the state corresponding to a unit Mach number is selected as the reference state [Hills and Peterson 1992]. Using an asterisk to signify properties at $M = 1$, it can be shown from the above equations that

$$\frac{T}{T^*} = \left(\frac{1+k}{1+kM^2} \right)^2 M^2 \tag{3.25}$$

$$\frac{T_0}{T_0^*} = \frac{2(k+1)M^2 \left(1 + \frac{k-1}{2} M^2 \right)}{(1+kM^2)^2} \tag{3.26}$$

$$\frac{P}{P^*} = \frac{1+k}{1+kM^2} \tag{3.27}$$

$$\frac{P_0}{P_0^*} = \left(\frac{2}{k+1} \right)^{k/(k-1)} \left(\frac{1+k}{1+kM^2} \right) \left(1 + \frac{k-1}{2} M^2 \right)^{k/(k-1)} \tag{3.28}$$

It may be observed that increasing the stagnation temperature drives the Mach number towards unity whether the flow is supersonic or subsonic. After the Mach number has approached unity in a given duct, further increase in stagnation enthalpy is possible only if the initial conditions change. Under such condition the flow is termed as "thermally choked".

When the stream is subjected to energy transfer, it is noted that the stagnation pressure always drops when energy is added to the stream and raises when energy is transferred from the stream. Thus, whether the flow is subsonic or supersonic, there may be significant loss of stagnation pressure due to combustion in a moving stream. Conversely, cooling tends to increase the stagnation pressure.

3.5.10 Shock Waves

3.5.10.1 Normal Shock

The conditions on the upstream side of the normal shock are denoted by subscript 1 and that on the downstream side by subscript 2. The equations below assume steady, one dimensional and adiabatic flow with no change in area across the shock because of its extremely small thickness.

In terms of k and M these relations are

$$\frac{T_2}{T_1} = \left(1 + \frac{k-1}{2} M_1^2\right)\left(\frac{2k}{k-1} M_1^2 - 1\right) \Big/ \left(\frac{(k+1)^2}{2(k-1)} M_1^2\right) \tag{3.29}$$

$$\frac{P_2}{P_1} = \left(\frac{2k}{k+1} M_1^2 - \frac{k-1}{k+1}\right) \tag{3.30}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{(k+1) M_1^2}{2 + (k-1) M_1^2}\right) \tag{3.31}$$

$$M_2 = \sqrt{\frac{(k-1) M_1^2 + 2}{2kM_1^2 - (k-1)}} \tag{3.32}$$

3.5.10.2 Oblique Shock

The normal shocks are a special case of flow discontinuities. The general situation observed in practice is for the shock to be inclined to the direction of flow as shown in the Figure 3.2. These are called oblique shocks.

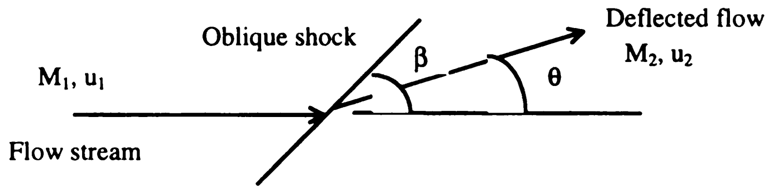

Figure 3.2 A flow stream through oblique shock

The flow illustrated as a streamline in Figure 3.2, approaches the shock wave with a velocity u_1, Mach number M_1, and at an angle β with respect to the shock. It is turned through an angle θ as it passes through the shock, leaving with a velocity u_2 and a Mach number M_2 at an angle $(\beta - \theta)$ with respect to the shock. One of the streamlines could be replaced by a solid boundary, which would represent flow around a wedge or concave corner with an angle β.

Equations for calculating the change in pressure, temperature, and Mach number across a normal shock may be used to calculate these changes across an oblique shock by using the normal components. It implies that whenever the term M_1 appears in the equations for the normal shock, it is replaced by the normal inlet component M_{1n}, such that

$$M_{1n} = M_1 \sin \theta \qquad (3.33)$$

For the oblique shock these relationships are

$$\frac{T_2}{T_1} = \left(1 + \frac{k-1}{2} M_{1n}^2\right)\left(\frac{2k}{k-1} M_{1n}^2 - 1\right) \Bigg/ \left(\frac{(k+1)^2}{2(k-1)} M_{1n}^2\right) \tag{3.34}$$

$$\frac{P_2}{P_1} = \left(\frac{2k}{k+1} M_{1n}^2 - \frac{k-1}{k+1}\right) \tag{3.35}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{(k+1) M_{1n}^2}{2 + (k-1) M_{1n}^2}\right) \tag{3.36}$$

$$M_2^2 \sin^2(\beta - \theta) = \left(\frac{(k-1) M_{1n}^2 + 2}{2k M_{1n}^2 - (k-1)}\right) \tag{3.37}$$

CHAPTER 4

DESCRIPTION OF THE SOFTWARE OPERATION

4.1 Introduction

In this chapter the rationale and methodology of the software for the turboramjet engine are discussed. It explains how the evaluation of flow conditions at various stations (Table 3.1) was made possible through the use of equations referred in Chapter 3. The topics discussed in this chapter follow the same order as appeared in the software.

A general rule followed in calculating flow properties is that from the known conditions at some point an endeavor is made to first find out the flow Mach number and then various functions (Appendix A) are used to calculate the respective flow properties.

4.2 The Software Structure

The software developed for the turboramjet operates in two steps:

a) At first the design program is run based upon a number of design parameters the user wishes to select. A list of required parameters along with the respective practical ranges appears on the screen to facilitate the user. This part of the program fixes the engine geometry, and the output is stored in a file.

b) Considering the known engine geometry obtained from the design program, the main program is run. The user is required to input only the desired flight Mach number and the altitude. The program itself calculates the required parameters and the pertinent flow conditions at all the stations of the turboramjet engine both for on-design and off-design flight conditions.

4.3 The Flow Chart

In the next four pages the flow chart of the software is presented in the form of block diagram. Each block represents a subroutine for that the respective details will be discussed in the following sections. Some parameters required for decision making are also shown in rectangular blocks. The representation is done using the standard symbols for the flow charts as far as possible.

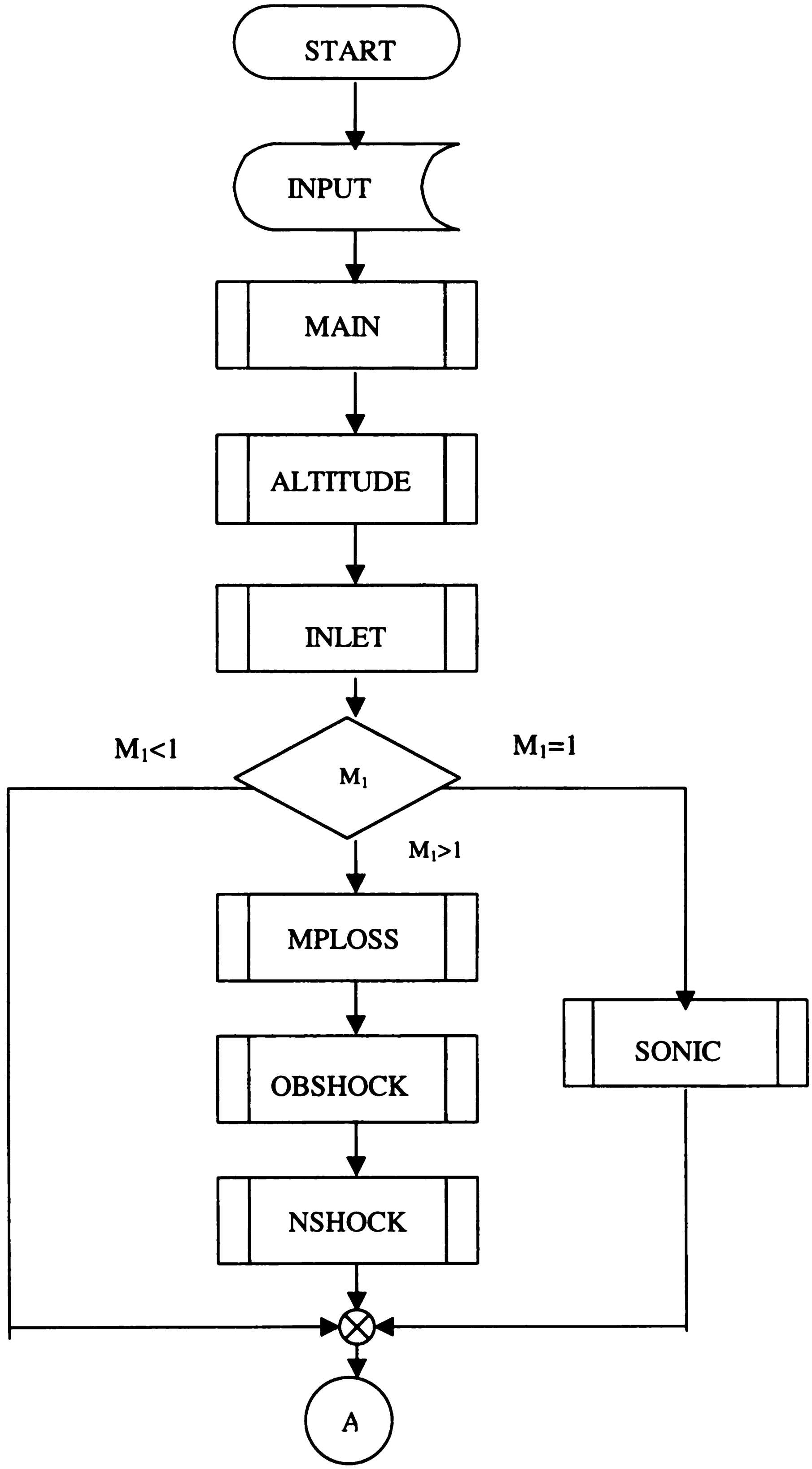

Figure 4.1 Flow chart of the software

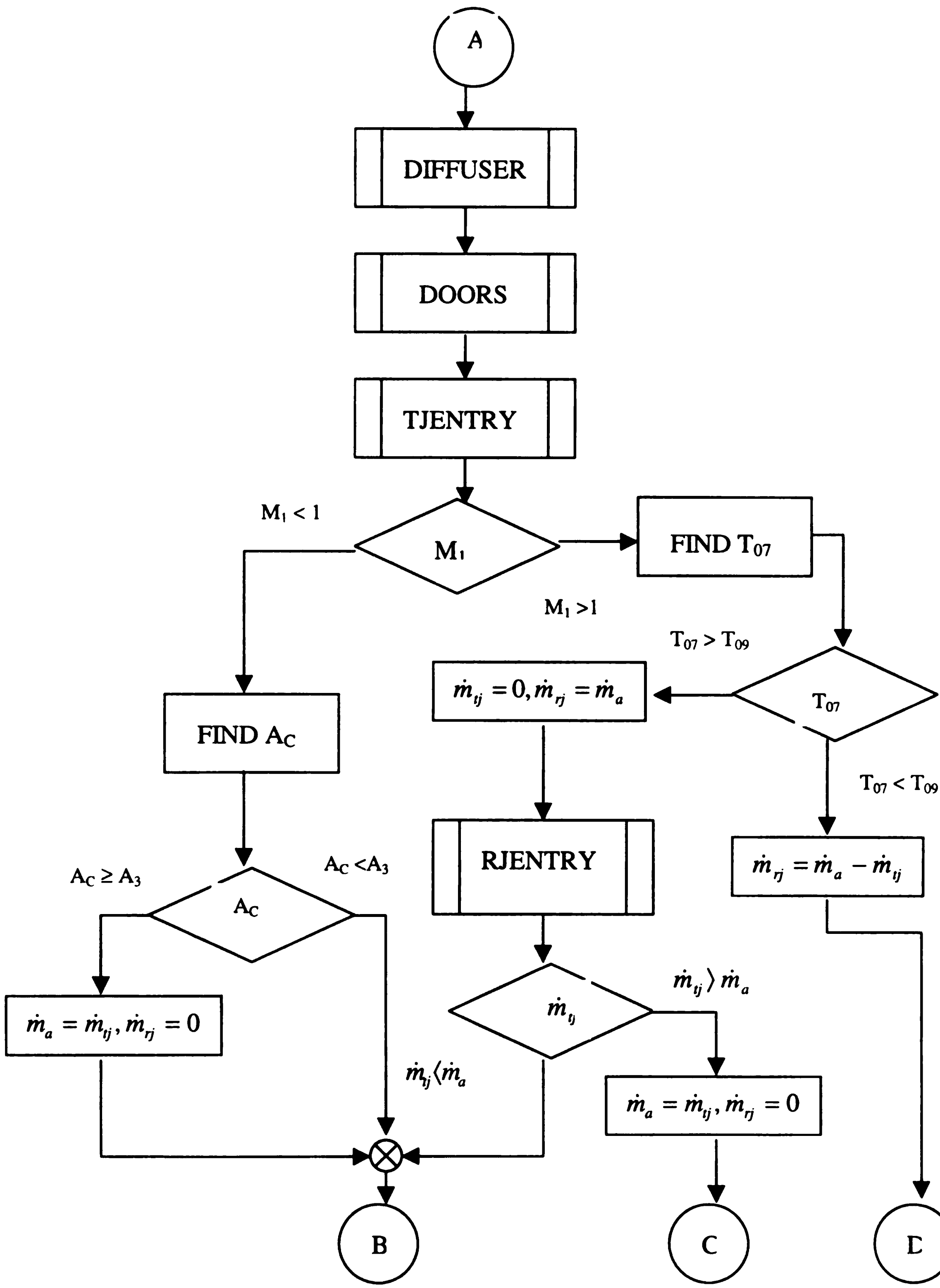

Figure 4.1 (Continued)

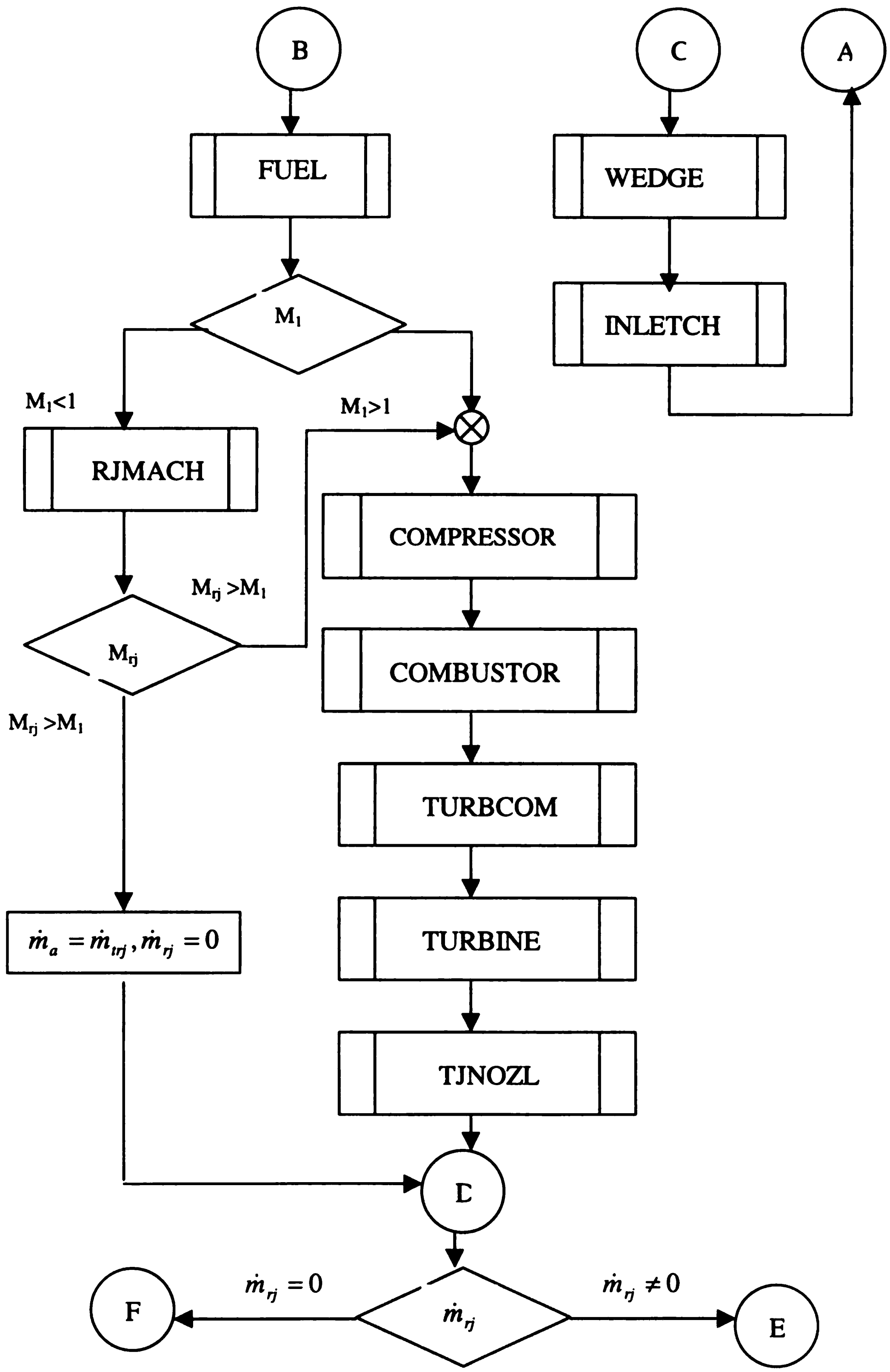

Figure 4.1 (Continued)

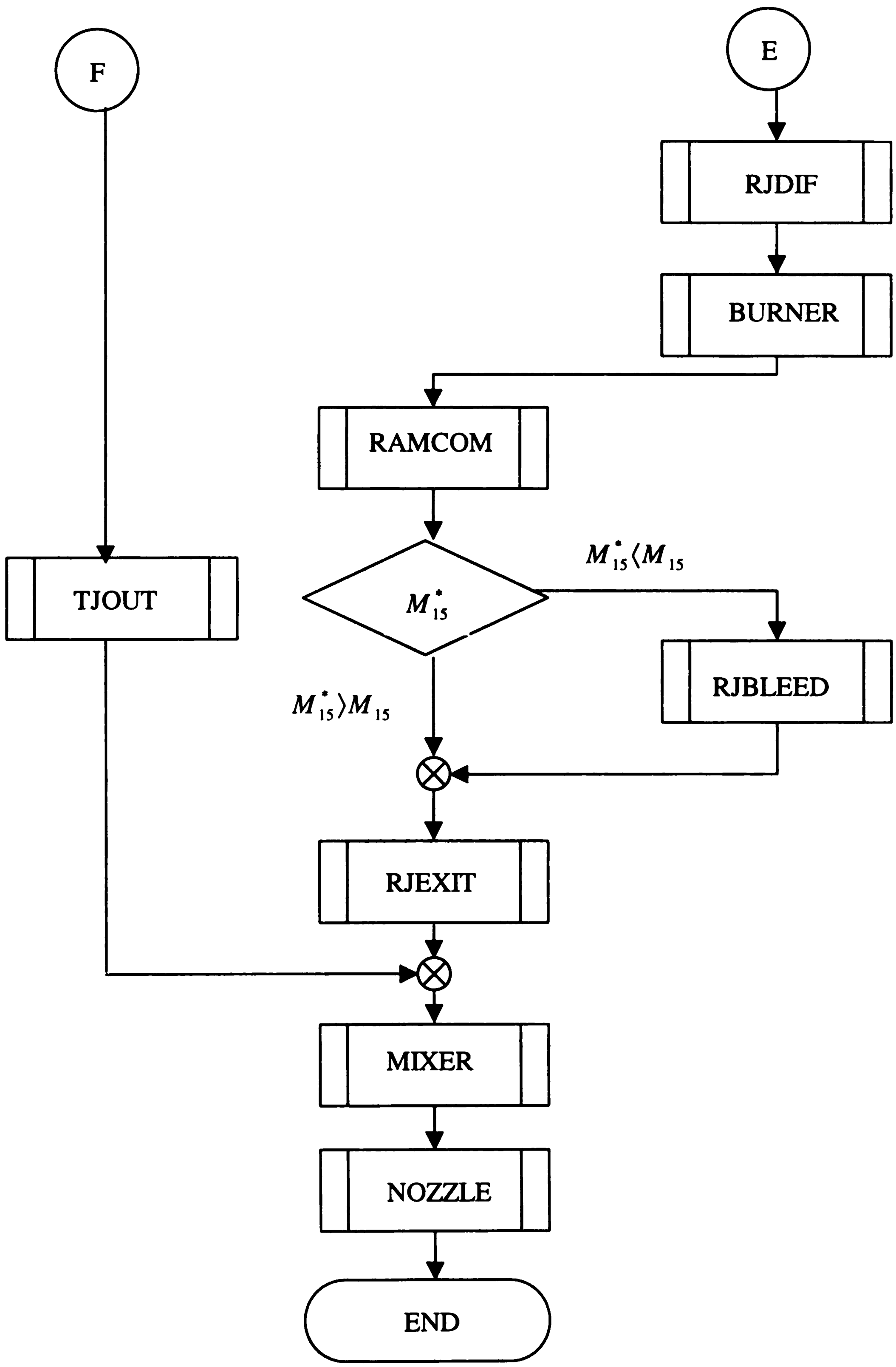

Figure 4.1 (Continued)

4.4 The Atmosphere

The domain of the air breathing engines extends up to almost 30,000 meters above sea level; however, the regularly used altitude for the supersonic transport stays within 18,000 to 21,000 meters. In line with the altitude requirement of the turboramjet the present study is restricted up to 30,000 meters and U.S. Standard Atmosphere (1962) is used as a reference.

The earth's surface is generally regarded as divided by altitude into regions. Closest to the earth surface is the troposphere that extends up to a height of about 11,000 meters and has specific temperature and pressure variation terminating into the tropopause. Above this is the stratosphere that extends from tropopause to the stratopause at about 51,000 meters. In this region up to the air breathing range, the temperature stays constant.

4.4.1 Variation of Temperature and Pressure with Altitude

In the troposphere the atmospheric variation is defined by a constant lapse rate or decrease of temperature (denoted by λ) and has a value equal to -0.0065 K/geopotential meter. With a defined standard sea-level condition the variation in temperature and pressure can be established by using hydrostatic and perfect gas equations as follows:

$$T = \left(T_1 - \lambda Z\right) \tag{4.1}$$

and

$$P = P_1\left[1 - \left(\lambda Z/T_1\right)\right]^{1/\lambda R} \tag{4.2}$$

In the above two equations:

a) T and P are the temperature and pressure respectively required to be calculated at altitude Z

b) T_1 and P_1 are the temperature and pressure respectively at standard sea-level conditions

c) λ is the lapse rate and R is the gas constant

Beyond troposphere up to the air breathing range, the temperature stays constant. The variation of pressure, however, is governed by the relationship that has been established with the use of hydrostatic and perfect gas equation

$$\frac{P}{P_1} = \exp\left[(Z_1 - Z)/RT\right] \tag{4.3}$$

Owing to the existence of an isothermal or quasi-isothermal region, there is a good reason for an altitude of about 11000 meters being a common cruising altitude for long-range transport aircraft. It is because, within the air breathing range, the lowest temperature is encountered at 11000 meters that benefits the turboengine performance [Shepherd 1972].

4.5 Free Stream Conditions

Considering the user input of the flight Mach number and the altitude, the program runs the subroutine ALTITUDE and returns the values of static temperature and pressure. Assuming constant values of R and k, the other flow parameters can be calculated as follows:

a) Velocity from equation (3.14), $u = M\sqrt{kRT}$

b) Density from ideal gas relation (equation [3.6]), $P = \rho\ RT$

c) Equation (3.1) is used to calculate c_p

d) Enthalpy is evaluated by using equation (3.3), $h = c_p \Delta T$

e) Stagnation conditions:

i. Enthalpy from equation (3.2), $h_0 = h + u^2/2$

ii. Temperature T_0 by using function TTKM

iii. Pressure P_0 from function PTKM

iv. Function DTKM is used for calculating density ρ_0

4.6 Intake System

Since the engine under study is designed for higher supersonic speeds, the design parameter selected for the intake design is the maximum pressure recovery. Another important consideration was to have a common intake system both for the turbojet and the ramjet. The efforts were aimed at designing an intake that could provide satisfactory operation over a wide range of flight Mach numbers with minimum pressure losses encountered to the airflow.

An external compression supersonic intake located at the nose section of the aircraft is selected for this purpose. The intake embodies a wedge for the shock formation during supersonic flights. The wedge designed for this intake is of variable geometry and capable of changing its shape and position in the intake according to the prevailing flight conditions. For subsonic flight the wedge moves fully inside the intake rendering it able to behave as a simple pitot intake. Under supersonic conditions the wedge adjusts its shape and position such that both the oblique and the normal shocks are always located on the intake lip. In other words the intake always operates in critical mode and hence maximum pressure recovery is ensured under all operating conditions.

4.6.1 Subsonic Flight

Keeping in mind the role of the engine under study, it is expected that the user will always opt for a supersonic Mach number for the design program operation. As such, the subsonic flight conditions will be encountered only during off-design flight conditions of the engine. Moreover, the turbojet alone covers most of the subsonic flight regime, and the ramjet gets into operation close to transonic Mach range. The ramjet operating Mach number depends upon the prevailing flight conditions based upon that the software calculates the minimum Mach number at that the ramjet assumes operation (See Appendix C).

One of the design considerations for the turboramjet is that the volume flow rate (area times velocity entering the intake) remains constant for the turbojet unless the intake is choked. In the subsonic range depending upon the flight speed and the mass flow demanded by the engine, the inlet might have to operate with a wide range of incident stream conditions. The streamline patterns for two typical subsonic conditions are shown in Figure 4.2.

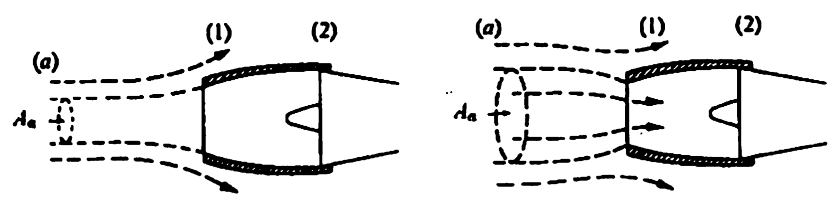

Figure 4.2 Typical stream pattern for subsonic inlets [Courtesy Hills and Peterson 1992]

During level cruise the streamline pattern may include some deceleration of the entering fluid external to the inlet plane. Under these conditions the upstream captured area A_c is less than the inlet area (Figure 4.2[a]). During low-speed high-thrust operation (take-off and climb), the engine demands more mass flow and the stream pattern resembles Figure 4.2(b). In this case the captured area will be more than the inlet area. It indicates a flight condition where the joint operation of the two engines is not possible. The mass flow rate barely meets the turbojet demand by being accelerated at the entrance of the intake to maintain constant volume flow rate.

The software makes use of this fact and does not allow flow to pass through the ramjet if the captured area is greater or equal to the intake area. In such a case only the turbojet operation is possible. Even if at some flight condition, the intake area becomes greater than the captured area, the flow is not directed to the ramjet until the time minimum Mach number for the ramjet operation is achieved.

Since there is no shock formed at the intake, the flow conditions at the plane of entry remain the same as that of free stream. The mass flow rate entering the intake is calculated with the help of continuity relation (equation [3.3]).

4.6.2 Sonic Flight

At the sonic flight Mach number the wedge remains fully in and the intake has a simple pitot configuration. A normal shock is formed at the entry of the intake and the flow properties are obtained using normal shock relationships, equations (3.29) through (3.32). The software handles these calculations through subroutine SONIC.

4.6.3 Supersonic Flight

At a supersonic Mach number the wedge assumes a shape and transverse location in the intake such that the intake operates in critical mode. The software calls the subroutine MPLOSS that determines the wedge geometry and location for critical operation of the intake. A detailed analysis follows in Section 4.7. The software calculates the flow properties after the oblique shock by calling subroutine OBSHOCK. Another subroutine NSHOCK is called for the properties after the normal shock. These subroutines make use of equations (3.29) through (3.37). At this stage, however, in order to find out other properties of the flow, the software follows two different approaches:

a) At the design point it is assumed that with the corresponding position of the wedge the intake area is one square meter [Section 3.3 (c)]. Hence the continuity equation and other relevant functions (Appendix A) are used to find out the mass flow rate and the stagnation conditions respectively.

b) For off-design operation the wedge adjusts itself for critical intake condition and the intake geometry changes. The intake area calculation will be discussed in Section 4.8. After having known the new intake area the procedure as discussed in paragraph (a) of this section is adopted for calculation of other properties of the flow.

4.7 Wedge Design Calculations

Figure 4.2 shows the schematic diagram of a wedge with physical and geometrical details.

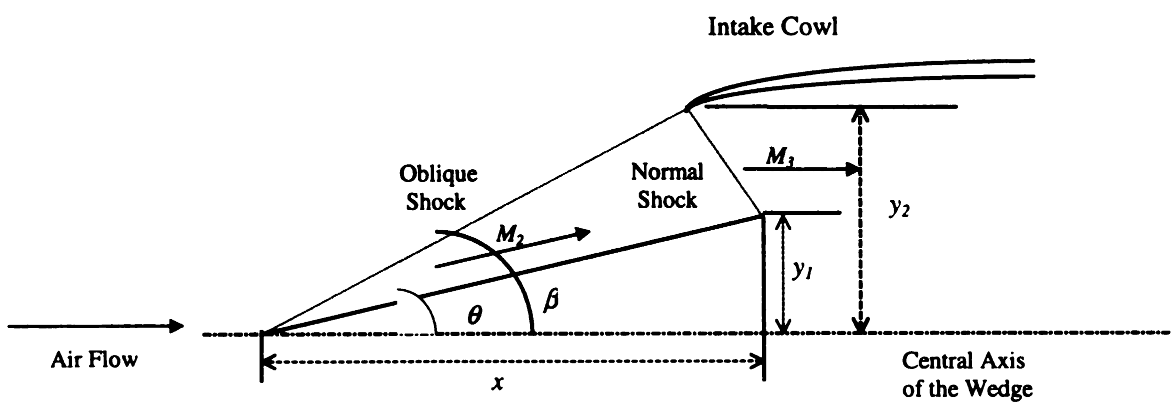

Figure 4.3 Half wedge showing shock formation and geometrical details

Since the wedge is symmetrical about its longitudinal axis, the Figure reflects only the half wedge with θ as semi-vertex angle and β as ensuing oblique shock angle under critical condition. Furthermore, for a two-dimensional analysis the half wedge may be considered a right angle triangle with base as x and height at inlet plane as y_1. Under this condition the inlet radius is represented as y_2.

The first objective is to find out the values of θ and β that correspond to the critical operation of the intake for a given flight Mach number. After having known these values a relationship between θ, β, x, y_1 and y_2 may be established.

4.7.1 Maximum Pressure Recovery

For supersonic flights, pressure recovery is the ratio of stagnation pressure of the flow after it has passes through the shock system to the free stream stagnation pressure. For a given flight condition maximum pressure recovery will be established with some values of θ and β which satisfy the following relation between the two

$$\cot\theta = \tan\beta\left[\frac{(k+1)M_1^2}{2\left(M_{1n}^2-1\right)}-1\right] \qquad (4.4)$$

where $M_{1n} = M_1 \sin\beta$ is the normal component of the free stream Mach number at the oblique shock.

The correct values of θ and β are obtained through iterative process in that for some value of β corresponding value of θ is calculated followed by the pressure recovery. β is varied in small step and new pressure recovery thus obtained is compared with the previously calculated value. The iteration stops where maximum recovery is obtained and corresponding values of θ and β are picked up for subsequent use.

The lowest value of β is determined from the Mach wave relation

$$\beta_{\min} = \sin^{-1}\left(\frac{1}{M_1}\right) \tag{4.5}$$

After obtaining on set of θ and β values, the software calculates the other related parameters in the following sequence:

a) The Mach number after the oblique shock from equation (3.37)

$$M_2^2 \sin^2(\beta - \theta) = \left[\frac{(k-1)M_{1n}^2 + 2}{2kM_{1n}^2 - (k-1)}\right]$$

b) The stagnation pressure ratio after the oblique shock is obtained from

$$\frac{P_{02}}{P_{01}} = \left[\frac{(k+1)M_{1n}^2}{2+(k-1)M_{1n}^2}\right]^{k/(k-1)} \left[\left(\frac{2k}{k+1}\right)M_{1n}^2 - \left(\frac{k-1}{k+1}\right)\right]^{1/(1-k)} \tag{4.6}$$

c) If the Mach number after the oblique shock is supersonic then:

i. Equation (3.31) is used to find Mach number after the normal shock

ii. $M_3 = \sqrt{\dfrac{(k-1)M_2^2 + 2}{2kM_2^2 - (k-1)}}$

iii. Stagnation pressure ratio across the normal shock is calculated with the help the following equation

$$\frac{P_{03}}{P_{02}} = \left[\frac{(k+1)M_2^2}{2+(k-1)M_2^2}\right]^{k/(k-1)} \left[\left(\frac{2k}{k+1}\right)M_2^2 - \left(\frac{k-1}{k+1}\right)\right]^{1/(1-k)} \tag{4.7}$$

d) If the Mach number after the oblique shock is subsonic:

i. M_3 is equal to M_2

ii. $\dfrac{P_{03}}{P_{02}} = \dfrac{P_{02}}{P_{01}}$

e) The overall stagnation pressure drop across the two shock waves (pressure recovery) thus can be obtained as

$$\frac{P_{03}}{P_{01}} = \frac{P_{03}}{P_{02}} \times \frac{P_{02}}{P_{01}} \tag{4.8}$$

f) β is increased with a step size of one-tenth of a degree and iteration continues till the time maximum value of the pressure recovery is achieved.

4.8 Intake Area Calculation

4.8.1 Subsonic Flight Conditions

At the design point the inlet radius y_2 is calculated that remains unchanged for any other flight condition for the same design. Since under subsonic flight conditions the wedge is fully receded inside and configures the intake to simple pitot type, its area can simply be obtained as

$$A_3 = \pi\ y_2^2 \tag{4.9}$$

4.8.2 Supersonic Flight Conditions

At the design point the intake area is assumed to be one square meter. For any off-design flight condition the wedge adjusts itself for maximum pressure recovery giving new values of θ and β. Accordingly, the new values of base of the wedge x and wedge height at inlet plane y_1 are obtained. The intake area under such condition is calculated from the right angle triangle rule such that

$$x = \sqrt{\frac{1}{\pi}\left(\tan^2\beta - \tan^2\theta\right)} \tag{4.10}$$

and

$$A_3 = \pi\ \left(y_2^2 - y_1^2\right) \tag{4.11}$$

The calculation of the inlet area in all the cases is done through subroutine INLET.

4.9 Programming for Diffuser

By the time the flow enters the intake, its velocity reduces to a subsonic Mach number. In order to have a suitable Mach or the velocity of the flow at the compressor face, flow is made to pass through a diffuser. The criteria for the flow condition at the compressor is:

a) The axial flow approaching a subsonic compressor should not be much higher than 0.4 [Hills and Peterson 1992]

b) The axial velocity at compressor entry remains within a value 150 meter per second [Kuchemann 1953] and [Hills and Peterson 1970]

For the engine design under study, option (b) is selected and the software uses the maximum velocity limit entering the compressor (velocity limit at diffuser exit) as the user input. This option is selected with a viewpoint of a common user who may have more familiarity with the velocity parameter over the Mach number

While the flow passes through the diffuser, a rise in static pressure and reduction of the velocity (to a practical value) is achieved at the diffuser exit. Since the efficiency of the diffuser is a function of the flight Mach number, the ratio of the specific heats and stagnation pressure ratio across diffuser, its handling especially in off-design conditions becomes complicated. However, to include the effect of diffuser efficiency a conservative approach is followed, and it is assumed that the flow encounters 4% stagnation pressure loss across diffuser under all flight conditions.

Since the flow through the diffuser is essentially adiabatic with no work transfer, h_0, T_0, k and c_p of the flow remain unchanged across diffuser. Calculation of other flow conditions at diffuser exit is discussed in the subsequent sections.

4.9.1 On-Design Performance

At the design point the diffuser exit velocity, u_4, is user defined, T_{04} remains the same as T_{03} and c_p is assumed constant. The static conditions T_4 and P_4 are obtained using equations (3.4) and (3.5) respectively. Latter ρ_4 is obtained from ideal gas relation (equation [3.6]) followed by diffuser exit area A_4 from the continuity relation (equation [3.10]). Equation (3.14) is used to calculate the Mach number. Relevant functions are then used for evaluation of other flow properties.

4.9.2 Off-Design Performance

For off-design operation the known parameters are T_{04}, P_{04}, A_4, and the mass flow rate $\dot{m}$. For known k and gas constant R, the Mach number at the diffuser is first calculated from the function XMACH (Appendix B) followed by T_4 and P_4 from respective functions [Appendix A]. u_4 is then found out from Mach number (equation [3.14]) and ρ_4 from ideal gas relation (equation [3.6]). For h equation (3.3) is used.

After having calculated the flow properties at diffuser exit, the conditions at the entry of bypass doors are known.

4.10 By-pass Doors

At the main diffuser exit the bypass doors are installed, which allows the flow to enter either the turbojet or the ramjet or, in case of joint operation, to both. The doors are attached with the turbojet compressor casing and can be actuated to increase or decrease the incoming flow area for either of the engines. However, for the present study the bypass is kept fixed at 50% such that physically the doors remain parallel to the longitudinal axis of the engine at their position under all operating conditions.

4.10.1 On-Design Conditions

Based upon the design conditions, the calculated area of the diffuser exit is divided into two equal halves: one half each assigned to the turbojet and the ramjet. This corresponds to stations (5) and (13) respectively to the turbojet and the ramjet in Figure 3.1. The local flow conditions at these two stations will be the same as that of station (4), the diffuser exit, since the flow has not yet passed through the bypass doors. The software uses the subroutine DOORS for evaluating flow conditions at stations (5) and (13).

At this stage half of the mass flow available at the diffuser exit is assumed to be entering in each of the two engines isentropically such that

$$\text{Turbojet mass flow} = \text{Ramjet mass flow} = \frac{1}{2}(\text{Total mass flow captured by the combjet})$$

Across the bypass doors the flow reaches at the plane of entry of the two engines, station (6) for turbojet and (14) for the ramjet. Since the flow is isentropic, the stagnation conditions do not change. The other flow properties at the two engines entry are evaluated as follows:

a) For the turbojet it is assumed that the volume flow rate (area times velocity) remains constant [Section 3.3(h)]. At design point it is termed as the "engine demand". Using continuity equation at stations (4) and (6) reveals that

$$\dot{m}_4 = \rho_4 u_4 A_4 \tag{4.12}$$

and

$$\dot{m}_6 = \rho_6 u_6 A_6 \tag{4.13}$$

but $\dot{m}_6 = \dfrac{\dot{m}_4}{2}$ and $A_6 = \dfrac{A_4}{2}$

Therefore equation (4.10) becomes

$$\frac{\dot{m}_4}{2} = \frac{\rho_6 u_6 A_4}{2}$$

or
$$\dot{m}_4 = \rho_6 u_6 A_4 \tag{4.14}$$

Since at low Mach numbers such as at stations (4) and (6) the air behaves as an incompressible fluid

$$\rho_6 = \rho_4$$

and equation (4.13) may be written as

$$\dot{m}_4 = \rho_4 u_6 A_4 \tag{4.15}$$

Comparing equations (4.12) and (4.15) it can be seen that

$$u_6 = u_4 \tag{4.16}$$

If the flow velocities are same at two stations, it can be proved from the isentropic relationships (Section 3.5.8) that for the unchanged stagnation conditions, static conditions of the flow will also be the same.

b) At the entry to the ramjet the same argument applies, and it is concluded that the flow properties as calculated at the diffuser exit are valid at the plane of turbojet and the ramjet entry except for the mass flow rate and the respective areas which are reduced to half.

The flow velocity entering the turbojet at the design point has an important significance. When the bypass doors are fixed at 50 %, the area of turbojet entry calculated at the design point will remain unchanged for the same design under all operations and the engine demand (assumed always fixed) will solely be governed by this value of the flow velocity. In the software this velocity is described as the "fixed velocity" and denoted by v_f. The flow calculations at the entry of the two engines are done through subroutine ENGENTRY.

4.10.2 Off-Design Conditions

a) The flow through the turbojet is governed by the assumption of a fixed volume flow rate unless the intake is choked. The stagnation conditions at the turbojet and ramjet entries remain the same as that of the diffuser exit. The other properties, however, may be calculated keeping in mind the rationale of the fixed velocity that is also known under off-design conditions. Static temperature, enthalpy and Mach number are found using the definition of stagnation temperature (equation [3.4]) and the Mach number (equation [3.12]) respectively. Static pressure and density are then calculated from the functions PKM and DKM respectively (Appendix A) followed by mass flow rate through the turbojet from continuity (equation [3.10]). Subroutine TJENTRY is used for these calculations.

b) From the known stagnation conditions at the ramjet entry (same as the diffuser exit), the Mach number is first obtained from the function XMACH (Appendix B). Respective functions are then used to calculate static temperature, pressure, and density and equation (3.3) for enthalpy. After having known all the relevant parameters, the flow velocity is evaluated from the continuity equation. These calculations for the ramjet are performed using the subroutine RJENTRY.

4.11 Compressor

The study assumes usage of an axial flow compressor for which the compression ratio (γ_c) and the adiabatic efficiency (η_c) are user defined. Furthermore, the flow velocity across compressor is assumed constant (Section 3.2.3). Hence

$$u_7 = u_6$$

The attention, however, is focused on the calculation of flow properties across the compressor treating it as one unit. As such inter-stage flow conditions are not addressed.

From the two known parameters (γ_c and η_c) of the compressor, the stagnation pressure rise is first obtained from

$$P_{07} = (\gamma_c)(P_{06}) \tag{4.17}$$

The ideal stagnation temperature (T_{07i}), that could have been achieved through an isentropic process at the compressor exit is calculated from isentropic relation, equation (3.5), such that

$$T_{07i} = T_{06}(\gamma_c)^{\frac{k-1}{k}} \tag{4.18}$$

After having known T_{07i}, the actual stagnation temperature at compressor exit may be obtained from the definition of the adiabatic efficiency of the compressor (η_c)

$$\eta_c = \frac{(T_{07i} - T_{06})}{(T_{07} - T_{06})} \tag{4.19}$$

such that

$$T_{07} = T_{06} + \frac{(T_{07i} - T_{06})}{\eta_c} \tag{4.20}$$

The other stagnation conditions ρ_{07} and h_{07} are calculated from the equation of state and equation (3.3) respectively.

For the static conditions the assumption of constant axial velocity across compressor is used, and T_7 is obtained from definition of h_{07} (equation [3.4]) followed by Mach number from equation (3.12). The functions PKM and DKM (Appendix A) are called for obtaining P_7, and ρ_7, and for h_7 equation (3.2) is used.

Rise in stagnation enthalpy reflects the compressor work such that

$$\text{Compressor Work} = h_{07} - h_{06} \tag{4.21}$$

4.11.1 On-Design and Off-design Calculations

The procedure followed in the software to calculate design and off-design conditions is the same except that at the design point the compressor exit area is evaluated using the continuity equation whereas, in the off-design mode, flow conditions are obtained assuming the known exit area of the compressor. The software uses the subroutine COMPRESSOR for all these calculations.

4.12 Diffusers Before Combustion

The performance of the air-breathing jet engine strongly depends upon the mass flow rate per unit cross-section area of the engine. Keeping in mind the continuity relation (equation [3.10]) the requirement of a larger mass flow for a given flight condition may be met in two ways

a) By having a larger cross-sectional area of the engine, but it induces higher drags and weight penalties

b) Allowing higher velocity flow entering the engine to compensate for small frontal area

A velocity limitation is, however, imposed by the combustor since it is necessary to maintain a stationary flame within a moving air stream. A flame remains stationary in a traveling mixture of reactants if the speed of the flame relative to the reactants is just equal to the reactant-mixture velocity. For this purpose it is desirable that the average velocity of the reactants passing through the combustor is of the order of 30-60 m/s [Cohen, Rogers and Saravanamuttoo 1987].

In the software the velocity limitations in terms of Mach number entering the combustion chamber are made user defined. While giving inputs for the design of the engine, the user is expected to feed a velocity within the specified range entering the combustion chamber. The software handles the velocity limitation criteria through the subroutines COMBDIF and RJDIF for the turbojet and the ramjet respectively. In both the cases the software assumes an isentropic diffuser installed before the two combustion chambers in order to reduce the flow velocity in accordance with the input conditions.

4.12.1 On-Design Performance

For the isentropic diffuser the stagnation properties of the flow remain unchanged at the diffuser exit. With the help of known (user defined) flow velocity the static temperature is calculated first using equation (3.4) followed by Mach number from equation (3.14). The static properties of the flow may then be obtained using the respective functions (Appendix A) and the diffuser exit area from the continuity relation, equation (3.10).

4.12.2 Off-Design Performance

Stagnation flow conditions at the diffuser exit are known from its isentropic behavior. Mach number is obtained from the function XMACH and the respective static properties from the relevant functions (Appendix A). Velocity at this point is calculated from the Mach relation, equation (3.14).

4.13 The Combustion Process

The function of the combustion chamber is identical for a gas turbine or a ramjet engine. The objectives are to introduce and burn fuel in the compressed air flowing through the combustion chamber with minimum pressure loss and with as complete a utilization of fuel as possible. In the aircraft jet engines a fresh supply of air is continuously maintained and the direct method of burning fuel in the working stream is employed. For a one-dimensional steady flow, this process of combustion is analogous to a flow process in a constant area duct with stagnation temperature change and is described by the Raleigh process (Section 2.3.3.2).

For the work under study, the combustion chamber for the turbojet is termed as "combustor" and that for ramjet as "burner". In both the engines the combustion process is tackled by assuming that the flow observes the Raleigh line. The attention is focused to determine the flow properties and combustion parameters at the end of combustion process without addressing flow conditions within the combustion zone. An annular type of combustion chambers layout is considered with a stable combustion process over a wide range of operating altitude and the forward speed. The losses in the combustion process are addressed by including:

a) A user defined combustion efficiency (η_b)

b) A fixed 2% stagnation pressure loss under all operating conditions to compensate for the presence of flame holders in the burning area

4.13.1 Combustion Calculations at the Design Point

4.13.1.1 Turbojet Combustion

The known conditions at the combustor exit, station (9), are:

a) The stagnation temperature (T_{09}) that is also the turbine inlet temperature

b) The area (A_{09}) from the Raleigh process consideration that assumes a constant area heat addition process

Considering the known parameters, the software first evaluates the chamber inlet choking temperature T_0^* from the Raleigh line stagnation temperature ratio (equation [3.26]) and compares it with T_{09}. The operation continues only if the stagnation temperature at chamber exit (T_{09}) stays lower than that of corresponding choking temperature T_0^*. This condition ensures that the chamber is not choked and the combustion is possible without changing the inlet conditions. If, however, T_{09} is greater than T_0^*; the program terminates after suggesting that the user either enhance turbine inlet temperature limit or modify the other design inputs for successful operation of the design program.

In the absence of the choking condition the Mach number at the chamber exit is calculated from the critical temperature ratio (equation [3.26]). It forms a quadratic in terms of Mach number, and the root representing the subsonic Mach number is picked up as the solution. Stagnation pressure at the chamber exit is obtained from equation (3.22). The static pressure and temperature at the chamber exit are calculated from equations (3.21) and (3.23). The exit velocity is evaluated from the Mach relation and static and stagnation enthalpies from equation (3.3). The software handles these calculations through the subroutines COMBUSTOR and TURBCOMB.

4.13.1.1 Ramjet Combustion

Comparatively higher temperature limits are achievable in the ramjet burner because it contains no moving parts. The maximum thermal limit (TRC), burner efficiency (η_b), and the fuel type are user defined. The software calculates the adiabatic flame temperature (AFT) of the corresponding fuel, compares it with the maximum thermal limit, and picks the burner temperature (T_{016}) for any of the two conditions:

a) If AFT is less than TRC, AFT is assigned as the burner exit temperature (T_{016}). In this case the theoretical air-fuel ratio (AF_{th}) is obtained from the relation

$$AF_{th} = \frac{\dot{m}_{air}}{\dot{m}_{fuel}} \tag{4.22}$$

and the fuel-air ratio (f) from its reciprocal i.e.

$$f = \frac{1}{AF_{th}} \tag{4.23}$$

Stagnation enthalpy (h_{016}) is then calculated by using the relationship combining fuel-air ratio, burner efficiency, and heat of combustion of the fuel (Q_R)

$$f = \frac{\left(h_{016}/h_{015}\right) - 1}{\left(\eta_b Q_R/h_{015}\right) - \left(h_{016}/h_{015}\right)} \tag{4.24}$$

The burner temperature (T_{016}) is then obtained from stagnation enthalpy (h_{016}) using equation (3.3).

b) If AFT turns out to be greater than TRC, the program terminates suggesting that the user either increase the TRC or change other design parameters to make engine operation possible.

4.13.2 Off-Design Combustion Calculations

4.13.2.1 Turbojet Combustion

The off-design calculations for the turbojet combustion follow exactly the same procedure as that of the design point calculations. The reason for this similarity is that the design program is run to fix the engine geometry, but for the combustion process the Raleigh line approach is used that by default assumes constant area combustion chamber. As such the ramjet burner area is already known by fixing its diffuser geometry, and it remains unchanged for the same design. Therefore, for any set of design conditions the calculation procedure will remain unaltered.

4.13.2.2 Ramjet Combustion

The process adopted for the off-design calculation of the ramjet combustion is different than the design point operation in two ways:

a) When the defined temperature limit (TRC) is exceeded, i.e., AFT becomes greater than TRC, the user has two options:

   i. To limit the burner temperature equal to TRC and continue

   ii. Quit the program for exceeding burner thermal limit

   If the user adopts the first option, the TRC equals T_{016}, h_{016} is calculated from T_{016}, the actual fuel-air ratio (f) from equation (4.24) and the actual air-fuel ratio (Af_{act}) from reciprocal of f. For the other flow conditions, at first, the critical temperature is obtained based on the conditions at burner entry, station (15), by using equation (3.26). It may be noted that the critical temperature T_0^* stays the same for both stations (15) and (16) as it describes the burner choking condition. When T_0^* is used to represent station (16), the Mach number (M_{16}) can be obtained using the critical temperature ratio, equation (3.26) that returns a forth order equation in M_{16}. The subsonic positive root is picked up representing M_{16}. The stagnation pressure ratio (equation [3.28]) is utilized to calculate P_{016} and to include the flame holder effect; 2% of calculated P_{016} is subtracted from its value. The static conditions like T_{16}, P_{16} and ρ_{16} are calculated using the functions TKM, PKM and DKM respectively and h from the product of c_p and T_{16}. Velocity is obtained from the Mach equation and A_{16} is assigned equal to A_{15} because of the Raleigh process.

b) If the combustion temperature (T_{016}) exceeds the critical temperature (T_0^*), the software performs calculations to limit the burner inlet Mach (M_{15}) through an iterative process such that T_{016} becomes less or equal to T_0^*. In this case the new flow conditions at burner entry are established through subroutine RJBLEED, the new (reduced) mass flow through the ramjet is compared with that without choking and excess mass flow is bled out. The calculations for the combustion process are then repeated for the modified input conditions at the burner entry.

Excluding the above mentioned two conditions, the software makes use of the same logic and relationships as used for design point calculations for the purpose of evaluating the flow properties under off-design flight conditions. Subroutines BURNER and RAMCOMB are used for these calculations.

4.13.1 Combustion Parameters

Some parameters associated with the combustion process are calculated in the program that contributes in establishing engine performance.

a) Fuel mass flow rate $\left(\dot{m}_f\right)$ is obtained from the product of fuel-air ratio and the air mass flow rate through the engine. Hence

$$\dot{m}_f = f \times \dot{m}_a \tag{4.25}$$

b) Net mass flow rate at the combustion chamber exit is the sum of air and fuel mass flow rate

$$\dot{m}_e = \dot{m}_a + \dot{m}_f \tag{4.26}$$

c) Actual fuel-air ratio (*f*) from equation (4.24).

d) Actual air-fuel ratio (Af_{act}) by taking inverse of the actual air-fuel ratio (*f*).

e) Excess air is obtained using the relation

$$\%\text{ excess air } = \left(\frac{AF_{act} - AF_{th}}{AF_{th}}\right)(100\%) \tag{4.27}$$

f) Theoretical air (*TA*) is calculated from the relation

$$\%\text{ theoretical air } = 100\% + \%\text{ excess air} \qquad (4.\ 28)$$

4.14 The Turbine

The performance of the turbine is limited principally by two factors: compressibility and stress. Compressibility limits the mass flow rate that can pass through a given turbine whereas the stress limits the wheel speed. The performance of the engine depends very strongly on the maximum operating temperature. For a given pressure ratio and adiabatic efficiency the turbine work per unit mass is proportional to the inlet stagnation temperature.

In the software the turbine inlet temperature (T_{09}), the adiabatic efficiency (η_t) and mechanical efficiency (η_{tm}) are made user defined. As such a flexibility is offered to the user to study the changes in engine performance with varying turbine inputs. In doing so, however, the matter of concern is to calculate the flow properties after the expansion process through turbine and not within the turbine stages. The software handles the calculation of flow properties across turbine by calling subroutine TURBINE.

4.14.1 Operation at the Design Point

The turbine inlet conditions at station (9) are known from the turbojet combustion calculations. The objective is to find out flow properties at turbine exit, station (10). At first the turbine work is calculated using the compressor work and the mechanical efficiency of turbine (η_{tm}) such that

$$\text{Turbine Work} = \eta_{tm}\,(\text{Compressor Work}) \tag{4.29}$$

also in terms of stagnation enthalpies

$$\text{Turbine Work} = (h_{09} - h_{010}) \tag{4.30}$$

that helps in evaluating h_{010} and subsequently T_{010} by using equation (3.3).

The adiabatic efficiency (η_t) is user defined and in terms of stagnation temperatures can be expressed as

$$\eta_t = \frac{(T_{09} - T_{010})}{(T_{09} - T_{010i})} \tag{4.31}$$

where T_{010i} is the ideal stagnation temperature that could have been achieved through an isentropic expansion across the turbine and is calculated from equation (4.31). The value of T_{010i} is used to find out P_{010} from the isentropic relation between pressure and temperature ratios, equation (3.5) such that

$$P_{010} = P_{09}\left(\frac{T_{010i}}{T_{09}}\right)^{\frac{k}{k-1}}$$

Among the remaining flow properties, h_{010} and ρ_{010} are calculated using equations (3.3) and (3.6) respectively and using the assumption of constant axial velocity across the turbine (Section 3.2.3), T_{10} is calculated from equation (3.4), M_{10} form equation (3.12), h_{10} from the definition of enthalpy and P_{10} and ρ_{10} using functions PKM and DKM (Appendix A) respectively. Later A_{10} may be obtained by using the continuity relation (equation [3.10]).

4.14.2 Off-Design Operation

In the off-design calculations the software follows the same procedure that is used in the design program. It is because of this fact that depending upon the input conditions and the adopted procedure, the area does not contribute to the evaluation of the flow properties across turbine.

4.15 The Turbojet Nozzle

The process in the nozzle is very close to adiabatic because the heat transfer per unit mass flow is much smaller than the difference of enthalpies between inlet and exit. Therefore, the equation of one-dimensional isentropic flow can describe the nozzle process quite well.

For the purpose of analysis, the expansion process in the turbojet nozzle is assumed to be isentropic. A convergent nozzle is selected for the turbojet, and it is assumed that it operates always choked whenever the turbojet is in operation.

From the preceding conditions it is obvious that the stagnation state at nozzle exit (station 11) will be the same as that at its entry (station 10) and the Mach number (M_{11}) will be always unity. Thus making use of equation (3.15) static temperature (T_{11}) is obtained, such that

$$T_{11} = \frac{T_{011}}{\left(1 + \frac{k-1}{2} M_{11}\right)}$$

Thereafter, P_{11} and ρ_{11} are calculated using the functions PKM and DKM respectively, u_{11} from relation, h_{11} from definition of enthalpy and A_{11} with the help of continuity equation. It may, however, be noted that mass flow rate through the turbojet nozzle will be the sum of turbojet air mass flow rate and the corresponding fuel mass flow rate under that condition.

4.15.1 On-Design and the Off-Design Operation

Since the turbojet nozzle is considered to be operating always choked, its exit area (A_{11}) can not be fixed. It will keep adjusting for different conditions for the choking flow. As such, the area at the design point does not contribute to the calculation of flow properties at any other operating conditions and the procedure adopted for calculating on-design and the off-design flow conditions remains the same.

4.16 The Mixer

The mixer is the cylindrical channel having a diameter equal to the outer diameter of the engine. It is installed between the exhaust of the ramjet (station 17) and the entry of the main convergent-divergent nozzle (station 18) of the turboramjet. Combustion products from both turbojet and the ramjet enter into the mixer. The two jet streams are mixed such that fully stabilized and steady flow is achieved before entering into the exhaust nozzle.

4.16.1 On-Design Calculations

The mass flow in the mixer is the sum of the mass flows out of the two engines. The mass flow from the turbojet is

$$\dot{m}_{11} = (\dot{m}_a)_{11} + (\dot{m}_f)_{11}$$

similarly, the ramjet contribution of the mass flow is

$$\dot{m}_{17} = (\dot{m}_a)_{17} + (\dot{m}_f)_{17}$$

where $\dot{m}_a$ and $\dot{m}_f$ are the mass flow rates of air and fuel and the subscript 11 and 17 indicate the flow from turbojet and the ramjet respectively.

The total mass flow in the mixer becomes

$$\dot{m}_{18} = \dot{m}_{11} + \dot{m}_{17} \tag{4.32}$$

The molecular weights (*MW*) of the flow from the turbojet and the ramjet are obtained from the analysis of the respective combustion products. The combustion products are assumed to consist of nitrogen, oxygen, water vapor and carbon dioxide. In each case the fraction of constituent gases in the combustion product is multiplied with their respective molecular weights and added up. The subroutine IGPROPL is used to calculate the molecular weights.

The mole flow rate ($\dot{N}$) entering the mixer can now be found from

$$\dot{N} = \frac{\dot{m}}{MW} \tag{4.33}$$

such that

$$\dot{N}_{18} = \dot{N}_{11} + \dot{N}_{17} \tag{4.34}$$

The molar composition of the turbojet and the ramjet exhaust gases entering into the mixer are calculated from the subroutine IGPROPL. At the mixer exit the molar composition of the turboramjet is obtained by first multiplying the composition of the two exhausts with the respective mole flow rate and the sum of the two compositions is divided by the total mole flow rate. Thus

$$\text{Combjet Molar Composition} = \left[\frac{\dot{N}_{11}(\text{Turbojet composition}) + \dot{N}_{17}(\text{Ramjet Composition})}{\dot{N}_{18}}\right]$$

The stagnation enthalpy at the mixer exit (h_{018}) is calculated from the energy equation

$$h_{018} = h_{011} + h_{017} \tag{4.35}$$

and stagnation temperature is obtained from the enthalpy definition

$$T_{018} = \frac{h_{018}}{c_p} \tag{4.36}$$

The velocity (u_{18}) and pressure (P_{18}) are obtained by simultaneously solving the flow equations of first law of thermodynamics, conservation of mass, momentum integral and the ideal gas relation. The procedure is elaborated in appendix D.

Once u_{18} and P_{18} are known, T_{18} is obtained from temperature and velocity relationship (equation [3.4]), ρ_{18} from continuity, Mach number from equation (3.12) and functions PTKM and DTKM are used to calculate P_{018} and ρ_{018}.

4.16.2 Off-Design Calculations

During off-design conditions as long as both the turbojet and the ramjet are in operation the procedure adopted for the flow conditions in the mixer is the same as that at the design point. The procedure, however, is different for the following two conditions:

a) At low Mach number when the ramjet is not yet put into operation, the door hinged at the mixer exit operates and closes the ramjet exhaust by resting on the turbojet nozzle (see Figure 4.4). Only the turbojet exhaust flow passes through the mixer and the main nozzle.

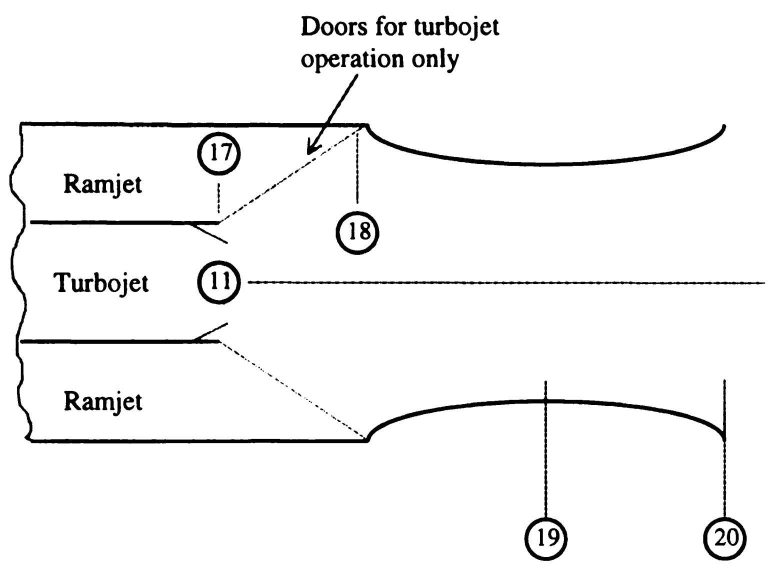

Figure 4.4 Section of the turboramjet showing the doors when only the turbojet is in operation

By doing so a diffuser is formed between stations (11) and (18) and that is assumed to be isentropic. The stagnation conditions at station (18) remains equal to that at (11), M_{18} is obtained from function XMACH, T_{18}, P_{18} and ρ_{18}

from functions TKM, PKM and DKM respectively. The static enthalpy (h_{18}) is calculated from the product of c_p and T_{18}, and u_{18} from the Mach relation.

b) When the turbojet is shut down for exceeding the turbine inlet temperature limit, it allows no flow to pass through. Under this condition $\dot{m}_{11}$ and N_{11} are set equal to zero and the molar composition of the turboramjet is calculated based upon the ramjet flow only. There on the software follows the same procedure that is used at the design point calculation.

The calculations for the mixer are handled by the software through subroutine MIXER. However, an additional subroutine TJOUT is called for the case when only the turbojet is in operation.

4.17 The Exhaust Nozzle

The flow from the mixer is exhausted to the atmosphere through the variable area convergent-divergent nozzle that generates the engine thrust. The nozzle attached with the turboramjet is designed such that it always operates choked at the throat and fully expanded to the atmosphere. In Figure 4.4 the entry to the nozzle is represented by station (18), and throat and exit by stations (19) and (20) respectively. The nozzle efficiency (η_n) is user defined.

From the given conditions it is evident that

$$M_{19} = 1$$

and

$$P_{20} = P_1$$

since the flow in the nozzle is assumed to be adiabatic

$$h_{020} = h_{019} = h_{018}$$

and for constant specific heat

$$T_{020} = T_{019} = T_{018}$$

4.17.1 Flow Conditions at the Throat

Since the nozzle is assumed to be always choked the static temperature (T_{19}) is calculated from the temperature ratio (equation [3.15]) using value of M_{19} as unity

$$T_{19} = T_{019}\left(\frac{2}{k+1}\right) \qquad (4.37)$$

and

$$u_{19} = M_{19}\sqrt{kRT_{19}} \qquad (4.38)$$

Assuming that the nozzle efficiency does not change at the throat, the ideal temperature (T_{ti}) is calculated from the definition of the efficiency (η_n)

$$T_{ti} = T_{018} - \left(\frac{T_{018} - T_{19}}{\eta_n}\right) \qquad (4.39)$$

Using the value of (T_{ti}) in the relationship between temperature and pressure ratio (equation [3.5]), P_{19} can be obtained

$$P_{19} = P_{019}\left(\frac{T_{019}}{Tti}\right)^{\frac{k-1}{k}} \qquad (4.40)$$

The local density (ρ_{19}) and the throat area (A_{19}) are evaluated making use of the equation of state and the continuity equation respectively.

4.17.2 Flow Conditions at the Exit

For the expansion process through the nozzle the ideal temperature at the exhaust T_{020i} may be obtained from the known parameters. The ideal temperature is the temperature that could have been achieved at the nozzle exit through an isentropic expansion process. Using isentropic relation between temperature and pressure ratio (equation [3.5])

$$T_{020i} = T_{018}\left(\frac{P_{20}}{P_{018}}\right)^{\frac{k-1}{k}} \tag{4.41}$$

the static temperature T_{20} can now be found out by introducing the nozzle efficiency

$$\eta_n = \frac{(T_{018} - T_{20})}{(T_{018} - T_{20i})}$$

or

$$T_{20} = T_{018} - \eta_n(T_{018} - T_{20i}) \tag{4.42}$$

By using equation of state (equation [3.6]) the static density (ρ_{20}) is calculated. Mach number (M_{20}) is obtained from the ratio of the temperature (equation [3.15]), u_{20} from Mach relation and A_{20} from the continuity equation. The functions PTKM and DTKM are used to return the values of P_{020} and ρ_{020}. The static enthalpy is calculated from the definition of stagnation enthalpy in terms of velocity (equation [3.2]).

4.17.3 On-Design and Off-Design Calculations

The nozzle used in the turboramjet is variable area both at the throat and the exit. It is so in order to accommodate the design considerations of always-choked throat and fully expanded exhaust. These considerations call for the requirement of throat and exit areas adjustment with varying operating conditions and the question of fixing the areas at design point has no practical applicability. As such the same procedure is followed for the flow parameters calculation in the nozzle both at the design point and off-design operations.

4.18 Engine Performance Parameters

During the flow expansion process in the nozzle there is no addition or expulsion of either the air or fuel in the flow stream, and the total mass flow at the nozzle exit is the same as at the mixer exit. Therefore

$$\dot{m}_{20} = \dot{m}_{18}$$

The total mass flow contains the air and fuel fractions that are received at the mixer, followed by the nozzle, each from the turbojet and the ramjet such that:

the air mass in the nozzle is

$$\dot{m}_{a20} = \dot{m}_{a11} + \dot{m}_{a17} \tag{4.43}$$

and the fuel fraction in the nozzle flow

$$\dot{m}_{f20} = \dot{m}_{f11} + \dot{m}_{f17} \tag{4.44}$$

where subscript (11) and (17) represents fractions from the turbojet and the ramjet respectively.

It can now be concluded that

$$\dot{m}_{20} = \dot{m}_{a20} + \dot{m}_{f20} \tag{4.45}$$

The fuel- air ratio of the engine becomes

$$f = \frac{\dot{m}_{f20}}{\dot{m}_{a20}}$$

The engine thrust (in Newtons) can now be obtained by using the thrust equation [equation(2.2)]. Thus

$$\tau = \dot{m}_{20}\left[(1+f)u_{20} - u_1\right] + (P_{20} - P_1)A_{20}$$

for the fully expanded nozzle $P_{20} = P_1$, and the thrust equation reduces to the form

$$\tau = \dot{m}_{20}\left[(1+f)u_{20} - u_1\right] \tag{4.46}$$

The specific thrust (I_s) is the thrust per unit mass flow (in N/kg) such that

$$I_s = \frac{\tau}{\dot{m}_{20}} = \left[(1+f)u_{20} - u_1\right] \tag{4.47}$$

The thrust specific fuel consumption (TSFC) is evaluated from the thrust and the fuel mass flow rate

$$TSFC = \frac{\dot{m}_{f\,20}}{\tau}$$

Conversion of $TSFC$ in standard units (kg (fuel)/hr/kN) leads to

$$TSFC = \frac{(1000)(3600)f}{\left(\tau / \dot{m}_{a20}\right)} \tag{4.48}$$

Thrust Power (TP) is the product of the thrust and flight velocity

$$TP = \tau\, u_1$$

Propulsion Efficiency (η_p) is the ratio of this thrust power to the rate of production of propellant kinetic energy

$$\eta_p = \frac{\tau\, u_1}{\dot{m}_{a20}\left[(1+f)(u_{20}^2/2) - u_1^2/2\right]} \tag{4.49}$$

Thermal Efficiency (η_{th}) from the definition (Section 2.2.1.2) is obtained as

$$\eta_{th} = \frac{\left[(1+f)(u_{20}^2/2) - u_1^2/2\right]}{fQ_R} \tag{4.50}$$

Overall Efficiency (η_o) is the product of propulsion and the thermal efficiencies

$$\eta_o = \eta_p \eta_{th} = \frac{\tau u_1}{\dot{m}_f Q_R} \tag{4.51}$$

CHAPTER 5

BENCHMARKING OF THE COMPUTER CODE

The reliability of the computer code is established through hand calculations for a typical set of design conditions. These calculations are done using the standard relationships pertaining to the gas flow through an aircraft engine. An overview of these relationships has been presented in Chapter 3. The calculations are carried out station-by-station for the turboramjet (see Figure 3.1). After completing the calculations for each station, a comparison is made between the values of the flow parameters obtained through the hand calculations and those from the computer program.

5.1 Set of Design Conditions

The following design parameters are selected for the hand calculations:

a) Flight Mach number, $M_1 = 2.5$

b) Flight altitude = 11000 m

c) Velocity at diffuser exit, $u_4 = 120$ m/s

d) Compressor pressure ratio, $\gamma_c = 6.0$

e) Velocity through turbojet combustor, $u_8 = 30.0$ m/s

f) Turbine inlet temperature, $T_{09} = 1350\ ^0$K

g) Ramjet thermal limit, $TRC = 3000\ ^0$K

h) Velocity across the ramjet burner, $u_{15} = 30.0$ m/s

i) Compressor efficiency, $\eta_c = 0.88$

j) Turbojet combustor adiabatic efficiency, $\eta_{btj} = 0.98$

k) Adiabatic efficiency of turbine, $\eta_t = 0.93$

l) Mechanical efficiency of the turbine, $\eta_{tm} = 0.97$

m) Ramjet burner adiabatic efficiency, $\eta_{brj} = 0.98$

n) Adiabatic efficiency of the engine nozzle, $\eta_n = 0.97$

5.2 Fixed Parameters

a) The gas constant, $R = 287$ J/kg ^{0}K

b) Molecular weight of air, $MW_{air} = 28.967$ kg/kmol

5.3 The Target Flow Parameters

The flowing properties of the flow are calculated at each station of the turboramjet:

a) The stagnation temperature, T_0

b) The stagnation pressure, P_0

c) The static temperature, T

d) The static pressure, P

e) The static enthalpy, h

f) The density, ρ

g) Flow velocity, u

h) Flow Mach number, M

i) Mass flow rate, $\dot{m}$

j) The area of the engine, A

5.4 Some Important Formulae for $k = 1.4$

5.4.1 Shock Relations

$$\cot\theta = \tan\beta\left[\frac{6M_1^2}{5\left(M_1^2\sin^2\beta - 1\right)} - 1\right] \tag{5.4.1}$$

$$\frac{P_{02}}{P_{01}} = \left[\frac{6M_1^2\sin^2\beta}{M_1^2\sin^2\beta + 5}\right]^{3.5}\left[\frac{6}{7M_1^2\sin^2\beta - 1}\right]^{2.5} \tag{5.4.2}$$

$$\frac{P_{03}}{P_{02}} = \left[\frac{6M_2^2}{M_2^2 + 5}\right]^{3.5}\left[\frac{6}{7M_2^2 - 1}\right]^{2.5} \tag{5.4.3}$$

$$M_2\sin(\beta - \theta) = \sqrt{\frac{M_1^2\sin^2\beta + 5}{7M_1^2\sin^2\beta - 1}} \tag{5.4.4}$$

$$M_3 = \sqrt{\frac{M_2^2 + 5}{7M_2^2 - 1}} \tag{5.4.5}$$

5.4.2 Isentropic Relations

$$\frac{T}{T_0} = \left(1 + \frac{M^2}{5}\right)^{-1} \tag{5.4.6}$$

$$\frac{P}{P_0} = \left(1 + \frac{M^2}{5}\right)^{-3.5} \tag{5.4.7}$$

5.5 Calculation of the Flow parameters

5.5.1 Station (1) : Free Stream Conditions

From the standard altitude tables for 11000 m,

- $T_1 = 216.7$ K
- $P_1 = 22.6$ kPa
- $\rho_1 = 0.3648$ kg/m^3

- Velocity of sound, $c_1 = 295.2$ m/s

1. Obtain flow velocity from Mach number relation

   $u_1 = M_1 c_1 = 2.5(295.2) = 737.9$ m/s

2. From supersonic flow table for $M = 2.5$ and $k = 1.4$,

   $$\frac{P_1}{P_0} = 0.05853$$

   $$\frac{T_1}{T_0} = 0.4444$$

   This leads to

   $T_0 = 487.6$ K

   $P_0 = 386.1$ kPa

5.5.1.1 Evaluation of Wedge Angle (θ) and the Shock Angle (β)

The evaluation of θ and β is an iterative process for maximum pressure recovery. The algorithm to obtain these values is as follows:

a) Initialize the pressure recovery (*PR*)

b) Assume some value of the shock wave angle (β) greater than the Mach wave angle

$$\beta_{\min} = \sin^{-1}\left(\frac{1}{M_1}\right)$$

c) Find the corresponding wedge angle (θ) from equation (5.4.1)

d) Obtain pressure ratio across the oblique shock $\frac{P_{02}}{P_{01}}$ with the help of equation (5.4.2)

e) Find M_2 using equation (5.4.4)

f) Calculate M_3 from equation (5.4.5)

g) Find pressure ratio across normal shock $\frac{P_{03}}{P_{02}}$ using equation (5.4.3)

h) Calculate the overall pressure recovery

$$\frac{P_{03}}{P_{01}} = \frac{P_{03}}{P_{02}} \times \frac{P_{02}}{P_{01}}$$

i) Compare $\frac{P_{03}}{P_{01}}$ with dummy variable PR

j) If $\frac{P_{03}}{P_{01}} > PR$, put $PR = \frac{P_{03}}{P_{01}}$

k) Increase β by 1^0 and perform iteration until maximum value of $\frac{P_{03}}{P_{01}}$ is achieved

Iteration 1

Let $PR = 0.0$

and $\beta = 42^0$

this leads to $sin^2\,\beta = 0.4477$ and $M_1^2\,sin^2\,\beta = 2.7983$

1. Find θ

$$\cot\theta = \tan 42\left(\frac{37.5}{5(2.7983-1)} - 1\right)$$

$$\cot 6 = 2.853$$

$$6 = 19.31^0$$

2. Find $\frac{P_{02}}{P_{01}}$

$$\frac{P_{02}}{P_{01}} = \left(\frac{37.8\times 0.4477}{2.7983+5}\right)^{3.5}\left(\frac{6}{7(2.7983)-1}\right)^{2.5}$$

$$\frac{P_{02}}{P_{01}} = (14.6405)(0.05919) = 0.8666$$

3. Find M_2

$$M_2 \sin(\beta-\theta) = \sqrt{\frac{(2.7953+5)}{7(2.7983)-1}}$$

$$M_2 = 1.678$$

4. Find M_3

$$M_3 = \sqrt{\frac{(1.678)^2+5}{7(1.678)^2-1}} = 0.646$$

5. Find $\frac{P_{03}}{P_{02}}$

$$\frac{P_{03}}{P_{02}} = \left(\frac{6(1.678)^2}{(1.678)^2+5}\right)^{3.5}\left(\frac{6}{7(1.678)^2-1}\right)^{2.5}$$

$$\frac{P_{03}}{P_{02}} = (14.848)(0.0583)$$

$$\frac{P_{03}}{P_{02}} = 0.8647$$

6. Find $\frac{P_{03}}{P_{01}}$

$$\frac{P_{03}}{P_{01}} = \frac{P_{03}}{P_{02}}\times\frac{P_{02}}{P_{01}}$$

$$\frac{P_{03}}{P_{01}} = 0.8647\times 0.8666$$

$$\frac{P_{03}}{P_{01}} = 0.7493$$

7. Compare Pressure recovery

$\frac{P_{03}}{P_{01}} \rangle PR$, another iteration is needed.

<u>Iteration 2</u>

Let $PR = 0.7493$

and $\beta = 43^0$

this leads to $sin^2 \beta = 0.4651$ and $M_1^2 \, sin^2 \beta = 2.907$

1. Find θ

$$\cot\theta = \tan 43\left(\frac{6(2.5)^2}{5(2.907-1)} - 1\right)$$

$$\cot 6 = 2.7349$$
$$6 = 20.08^0$$

2. Find $\frac{P_{02}}{P_{01}}$

$$\frac{P_{02}}{P_{01}} = \left(\frac{6\times 2.907}{2.907+5}\right)^{3.5}\left(\frac{6}{7(2.907)-1}\right)^{2.5}$$

$$\frac{P_{02}}{P_{01}} = (15.942)(0.05355) = 0.8536$$

3. Find M_2

$$M_2 \sin(\beta - \theta) = \sqrt{\frac{(2.907+5)}{7(2.907)-1}}$$

$$M_2 = 1.641$$

4. Find M_3

$$M_3 = \sqrt{\frac{(1.641)^2 + 5}{7(1.641)^2 - 1}} = 0.656$$

5. Find $\frac{P_{03}}{P_{02}}$

$$\frac{P_{03}}{P_{02}} = \left(\frac{6(1.641)^2}{(1.641)^2 + 5}\right)^{3.5} \left(\frac{6}{7(1.641)^2 - 1}\right)^{2.5}$$

$$\frac{P_{03}}{P_{02}} = (13.440)(0.06543)$$

$$\frac{P_{03}}{P_{02}} = 0.8794$$

6. Find $\frac{P_{03}}{P_{01}}$

$$\frac{P_{03}}{P_{01}} = \frac{P_{03}}{P_{02}} \times \frac{P_{02}}{P_{01}}$$

$$\frac{P_{03}}{P_{01}} = 0.8794 \times 0.8536$$

$$\frac{P_{03}}{P_{01}} = 0.7506$$

7. Compare Pressure recovery

$\frac{P_{03}}{P_{01}} \rangle PR$, another iteration is needed.

<u>Iteration 3</u>

Let $PR = 0.75063$

and $\beta = 44^0$

this leads to $sin^2 \beta = 0.4825$ and $M_1^2 \, sin^2 \beta = 3.015$

1. **Find θ**

$$\cot\theta = \tan 44\left(\frac{6(2.5)^2}{5(3.0159-1)} - 1\right)$$

$$\cot 6 = 2.6270$$
$$6 = 20.84^0$$

2. **Find $\frac{P_{02}}{P_{01}}$**

$$\frac{P_{02}}{P_{01}} = \left(\frac{6 \times 3.0159}{3.0159+5}\right)^{3.5}\left(\frac{6}{7(3.0159)-1}\right)^{2.5}$$

$$\frac{P_{02}}{P_{01}} = (17.284)(0.0486) = 0.8403$$

3. **Find M_2**

$$M_2 \sin(\beta - \theta) = \sqrt{\frac{(3.0159+5)}{7(3.0159)-1}}$$

$$M_2 = 1.605$$

4. **Find M_3**

$$M_3 = \sqrt{\frac{(1.605)^2 + 5}{7(1.605)^2 - 1}} = 0.667$$

5. **Find $\frac{P_{03}}{P_{02}}$**

$$\frac{P_{03}}{P_{02}} = \left(\frac{6(1.605)^2}{(1.605)^2 + 5}\right)^{3.5}\left(\frac{6}{7(1.605)^2 - 1}\right)^{2.5}$$

$$\frac{P_{03}}{P_{02}} = (12.135)(0.0736)$$

$$\frac{P_{03}}{P_{02}} = 0.8932$$

6. **Find** $\frac{P_{03}}{P_{01}}$

$$\frac{P_{03}}{P_{01}} = \frac{P_{03}}{P_{02}} \times \frac{P_{02}}{P_{01}}$$

$$\frac{P_{03}}{P_{01}} = 0.8932 \times 0.8403$$

$$\frac{P_{03}}{P_{01}} = 0.7506$$

7. **Compare Pressure recovery**

$\frac{P_{03}}{P_{01}} \approx PR$, another iteration is needed to check the trend.

<u>Iteration 4</u>

Let $PR = 0.7506$

and $\beta = 45^0$

this leads to $sin^2\,\beta = 0.50$ and $M_1^{\,2}\,sin^2\,\beta = 3.125$

1. **Find** θ

$$\cot\theta = \tan 45\left(\frac{6(2.5)^2}{5(3.125-1)} - 1\right)$$

$$\cot 6 = 2.5294$$
$$6 = 21.57^0$$

2. **Find $\frac{P_{02}}{P_{01}}$**

$$\frac{P_{02}}{P_{01}} = \left(\frac{6\times 3.125}{3.125+5}\right)^{3.5}\left(\frac{6}{7(3.125)-1}\right)^{2.5}$$

$$\frac{P_{02}}{P_{01}} = (18.6690)(0.0443) = 0.8269$$

3. **Find M_2**

$$M_2 \sin(\beta-\theta) = \sqrt{\frac{(3.125+5)}{7(3.125)-1}}$$

$$M_2 = 1.569$$

4. **Find M_3**

$$M_3 = \sqrt{\frac{(1.569)^2+5}{7(1.569)^2-1}} = 0.678$$

5. **Find $\frac{P_{03}}{P_{02}}$**

$$\frac{P_{03}}{P_{02}} = \left(\frac{6(1.569)^2}{(1.569)^2+5}\right)^{3.5}\left(\frac{6}{7(1.569)^2-1}\right)^{2.5}$$

$$\frac{P_{03}}{P_{02}} = (10.913)(0.0831)$$

$$\frac{P_{03}}{P_{02}} = 0.9065$$

6. **Find $\frac{P_{03}}{P_{01}}$**

$$\frac{P_{03}}{P_{01}} = \frac{P_{03}}{P_{02}} \times \frac{P_{02}}{P_{01}}$$

$$\frac{P_{03}}{P_{01}} = 0.9065 \times 0.8268$$

$$\frac{P_{03}}{P_{01}} = 0.7495$$

7. Compare Pressure recovery

$\frac{P_{03}}{P_{01^*}} \langle PR$, the point of maxima has passed. The previous value is selected representing the maximum pressure recovery.

Thus for the given conditions the following results are obtained:

a) Maximum pressure recovery = 75.06%

b) Corresponding wedge angle (θ) = 20.84^0

c) The shock wave angle (β) = 44^0

Table 5.1 Comparison of flow Properties at Station (1)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M
By Hand	487.6	386.1	216.7	22.6	0.3648	217.4	737.9	2.5
Computer	487.6	386.9	216.7	22.6	0.3638	217.7	738.0	2.5

5.5.2 Station (2) : Oblique Shock

Known parameters:

- M_2 = 1.605 (from pressure recovery calculations)
- $\frac{P_{02}}{P_{01}} = 0.8403 \Rightarrow P_{02} = 324.4$ kPa
- $T_{02} = T_{01} = 487.6$ K
- $k = 1.4$

1. From the shock table against $M_2 = 1.605$

$$\frac{T_{02}}{T_2} = 1.515$$

$$T_2 = \frac{487.6}{1.515} = 321.8 \text{ K}$$

2. Find P_2 using isentropic relation

$$\frac{P_{02}}{P_2} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{k}{k-1}}$$

$$P_2 = \frac{324.4}{(1.515)^{3.5}} = 75.8 \text{ kPa}$$

3. Equation of state

$$\rho_2 = \frac{75.8}{(0.287)(321.8)} = 0.82073 \text{ kg / m}^3$$

4. Mach relation

$$u_2 = M_2\sqrt{kRT_2}$$

$$u_2 = 1.605\sqrt{1.4(287)(321.8)} = 577.1 \text{ m / s}$$

5. From enthalpy relation, $h = c_p T$

$$h_2 = 322.8 \text{ kJ / kg}$$

Table 5.2 Comparison of flow Properties at Station (2)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M
By Hand	487.6	324.4	321.8	75.8	0.8207	322.8	577.1	1.61
Computer	487.6	327.7	319.3	74.5	0.8119	320.7	581.7	1.62

5.5.3 Station (3) : Normal Shock

Known Parameters:

- $M_3 = 0.667$ (from pressure recovery calculations)
- $\dfrac{P_{03}}{P_{02}} = 0.8932 \Rightarrow P_{03} = 289.8$ kPa
- $T_{03} = T_{02} = 487.6$ K
- $k = 1.4$

1. Equation (5.4.6)

$$T_3 = T_{03}\left(1+\frac{(0.667)^2}{5}\right)^{-1} = 447.8 \text{ K}$$

2. Using enthalpy relation, $h = c_p T$

$$h_3 = 449.8 \text{ kJ / kg}$$

3. Equation (5.4.7)

$$P_3 = P_{03}\left(1+\frac{(0.667)^2}{5}\right)^{-3.5} = 215.0 \text{ kPa}$$

4. Ideal gas equation

$$\rho_3 = \frac{215.0}{(0.287)(447.8)} = 1.6729 \text{ kg / m}^3$$

5. Mach relation

$u_3 = M_3\sqrt{kRT_3}$ leads to

$u_3 = 282.9$ m/s

6. Find geometry of the wedge

For $\theta = 20.84^0$ and $\beta = 44^0$, obtain base X and height Y_1 of the half wedge (see Figure 4.3)

$$X = \sqrt{\frac{1}{\pi\left(\tan^2\beta - \tan^2\theta\right)}} = 0.6357 \text{ m}$$

$Y_1 = X \tan\theta = 0.2499$ m

for the critical condition (shock attached to intake lip) the intake radius Y_2 becomes

$Y_2 = X \tan\beta = 0.6139$ m

7. Obtain intake area (A_3) (see Figure 4.2)

$$A_3 = \pi\left(Y_2^2 - Y_1^2\right)$$

$A_3 = 0.988 \text{ m}^2$

8. Mass flow rate

Continuity equation at station (3), $\dot{m} = \rho uA$

$$\dot{m}_3 = (1.6729)(282.9)(0.988) = 467.6 \text{ kg/s}$$

Table 5.3 Comparison of flow Properties at Station (3)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A
By Hand	487.6	289.8	477.8	215.0	1.6729	449.8	282.9	0.667	0.998
Computer	487.6	290.5	448.3	216.6	1.6815	450.3	280.9	0.662	1.0

5.5.4 Station (4) : Diffuser

Known Parameters:

- u_4 = 120 m/s (user defined)
- $P_{04} = 0.96\ (P_{03}) = 278.2$ kPa (design consideration)
- $T_{03} = T_{02} = 487.6$ K (adiabatic flow)

- $k = 1.4$
- $\dot{m}_3 = \dot{m}_4$

1. Using isentropic relation $T_0 = T + \frac{u^2}{2c_p}$, find T_4

$$T_4 = 487.6 + \frac{(120)^2}{2(1.005)(10^3)} = 480.4 \text{ K}$$

2. Enthalpy relation $h_4 = (1.005)(480.4) = 482.8$ kJ/kg

3. Velocity of sound $c = \sqrt{kRT}$ leads to $c_4 = 439.3$ m/s

and $M = \frac{u}{c}$ gives $M_4 = 0.273$

4. Equation (5.4.7)

$$P_4 = 278.2\left(1 + \frac{(0.273)^2}{5}\right)^{-3.5} = 264.2 \text{ kPa}$$

5. Ideal gas equation

$$\rho_4 = \frac{264.2}{(0.287)(480.4)} = 1.9162 \text{ kg/m}^3$$

6. Continuity

$$A_4 = \frac{467.6}{(1.9162)(120)} = 2.034 \text{ m}^2$$

Table 5.4 Comparison of flow Properties at Station (4)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A
By Hand	487.6	278.2	480.4	264.2	1.9162	482.8	120.0	0.273	2.034
Computer	487.6	278.9	480.4	264.8	1.9187	482.6	120.0	0.273	2.051

5.5.5 Stations (5) and (13) : Bypass Doors

Design point assumptions are:

- The bypass doors are fixed at 50% for all flight conditions
- Isentropic flow over the bypass doors
- $\dot{m}_5 = \dot{m}_{13} = \dfrac{\dot{m}_4}{2} = 233.2 \text{ kg / s}$
- $A_5 = A_{13} = \dfrac{A_4}{2} = 1.017 \text{ m}^2$

With in changes in areas and mass flow rates in stations (5) and (13) as compared with station (4), it is concluded that the flow properties at all these stations will be identical to that of station (4) (see Section 4. 9).

Table 5.5 Comparison of flow Properties at Stations (5) and (13)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A
By Hand	487.6	278.2	480.4	264.2	1.9162	482.8	120.0	0.273	2.034
Computer	487.6	278.9	480.4	264.8	1.9187	482.6	120	0.273	2.051

5.5.6 Stations (6) and (14) : Turbojet and Ramjet Entry

The stations (13) and (14) have the same flow properties at the design point as those of (5) and (13) respectively. It is because the bypass doors are kept fixed at 50% throughout and flow past the doors is assumed isentropic.

Table 5.6 Comparison of flow Properties at Stations (6) and (14)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A
By Hand	487.6	278.2	480.4	264.2	1.9162	482.8	120.0	0.273	2.034
Computer	487.6	278.9	480.4	264.8	1.9187	482.6	120	0.273	2.051

5.5.7 Station (7) : Compressor

Known parameters:

- $P_{07} = (\gamma_c)\, P_{06} = 1669.2$ kPa
- $\gamma_c = 6$ (user input)
- $u_7 = u_6 = 120.0$ m/s (constant axial velocity across compressor)
- $\eta_c = 0.88$
- $k = 1.4$

1. Find ideal temperature using isentropic temperature - pressure relationship

$$T_{07i} = T_{06}(\gamma_c)^{\frac{k-1}{k}} = 813.6\ \text{K}$$

2. Obtain T_{07} from the definition of η_c

$$\eta_c = \frac{(T_{07i} - T_{06})}{(T_{07} - T_{06})} \qquad \Rightarrow \qquad T_{07} = 858.0\ \text{K}$$

3. Isentropic temperature - velocity relation

$$T_7 = 858.0 - \frac{(120)^2}{2(1.005\times 10^3)} = 850.8\ \text{K}$$

4. $h_7 = c_p T_7 = 855.1$ kJ/kg

5. Velocity of sound $c = \sqrt{kRT}$, leads to $c_7 = 584.7$ m/s

   and

$$M_7 = \frac{u_7}{c_7} = 0.205$$

6. Equation (5.4.6)

$$P_7 = 1669.2(0.971) = 1621\ \text{kPa}$$

7. Equation of state leads to $\rho_7 = 6.6386$ kg/m^3

8. Compressor Work = $c_p(T_{07} - T_{06}) = 371.5$ kJ/kg or 10759 kJ/kmol

Table 5.7 Comparison of flow Properties at Station (7)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A
By Hand	858.0	1669.2	850.8	1621.0	6.6386	855.1	120.0	0.205	1.017
Computer	858.0	1673.1	850.5	1624.8	6.6479	854.6	120.0	0.205	1.026

5.5.8 Station (8) : Diffuser for Turbojet Combustor

Known Parameters:

- Isentropic flow across diffuser (assumed), therefore:

  $T_{08} = T_{07} = 858.0$ K

  $P_{08} = P_{07} = 1669.2$ kPa

- $u_{08} = 30.0$ m/s (user input)
- $k = 1.4$
- $\dot{m}_8 = \dot{m}_6 = 233.2$ kg/s

3. Isentropic temperature - velocity relation

$$T_8 = 858.0 - \frac{(30)^2}{2(1.005 \times 10^3)} = 857.5 \text{ K}$$

4. $h_8 = c_p T_8 = 862.1$ kJ/kg

5. Velocity of sound $c = \sqrt{kRT}$, leads to $c_8 = 586.9$ m/s

and $$M_8 = \frac{u_8}{c_8} = 0.051$$

6. Using isentropic relation $\dfrac{P_{08}}{P_8} = \left(\dfrac{T_{08}}{T_8}\right)^{\frac{k}{k-1}}$

$$P_{08} = 1669.2\left(\frac{857.5}{858.0}\right)^{1.714} = 1667.5 \text{ kPa}$$

7. Equation of state leads to $\rho_8 = 6.7756$ kg/m^3

8. Continuity equation gives

$$A_8 = \frac{233.8}{(6.7756)(30)} = 1.15 \text{ m}^2$$

Table 5.8 Comparison of flow Properties at Station (8)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A
By Hand	858.0	1669.2	857.5	1667.5	6.7756	862.1	30.0	0.051	1.150
Computer	858.0	1673.1	857.5	1670.1	6.7798	861.4	30.0	0.051	1.161

5.5.9 Station (9) : Turbojet Combustion

Known Parameters:

- $T_{09} = 1350$ (user defined)
- $A_9 = A_8$ (Raleigh process approach)
- $\eta_b = 0.98$ (user defined)
- $k = 1.33$
- $c_p = 1.157$

1. Find T_0^* (choking temperature) for station (8)

$$\frac{T_{08}}{T_{08}^*} = \left[\frac{2(k+1)M_8^2\left(1+\frac{k-1}{2}M_8^2\right)}{\left(1+kM_8^2\right)^2}\right]$$

$$T_{08}^* = 858.0\left[\frac{\left(1+1.33(0.051)^2\right)^2}{2(2.33)(0.051)^2\left(1+0.2(0.051)^2\right)}\right] = 69189.0\ \text{K}$$

Since $T_{08}^* \rangle\ T_{08}$, combustion is possible.

2. Also note that $T_{08}^* = T_{09}^*$ for the same combustor, therefore, for the known T_{09} and T_{09}^*, M_9 can be obtained from the temperature ratio.

$$\frac{T_{09}}{T_{09}^*} = \left[\frac{2(1.33+1)M_9^2\left(1+\frac{1.33-1}{2}M_9^2\right)}{\left(1+1.33M_9^2\right)^2}\right]$$

$$0.0195 = \frac{4.66M_9^2 + 0.79M_9^4}{1+2.66M_9^2 + 1.77M_9^4}$$

this leads to a fourth order equation in M

$$M_9^4 + 6.099M_9^2 - 0.02581 = 0$$

Assuming $M_9^2 = X$, the equation reduces to a quadratic in X. The quadratic formula returns the roots of X as

$X_1 = 0.00423$ and $X_2 = -6.103$

Positive root is selected representing the subsonic Mach in the combustor.

Thus $M_9 = \sqrt{X_1} = 0.065$

3. Obtain P_{08} from the Raleigh line pressure ratio

$$\frac{P_{09}}{P_{08}} = \left(\frac{1+kM_8^2}{1+kM_9^2}\right)\left(\frac{1+\dfrac{k-1}{2}M_9^2}{1+\dfrac{k-1}{2}M_8^2}\right)^{k/(k-1)}$$

Putting the values and arranging

$$P_{09} = P_{08}(0.998)(1.000254)^{4.03}$$

$$P_{09} = P_{08}(0.999)$$

$$P_{09} = (1669.2)(0.999) = 1667.6 \text{ kPa}$$

4. Find T_9 from temperature - Mach relation

$$T_9 = T_{09}\left(1+0.165(0.065)^2\right)^{-1}$$

$$T_9 = 1350(0.9999) = 1349.03 \text{ K}$$

5. Velocity $u_9 = M_9\sqrt{kRT_9} = 717.6 \text{ m/s}$

6. Pressure ratio - Mach isentropic relation leads to calculation of P_9

$$P_9 = P_{09}\left(1+0.165M_9^2\right)^{-4.03}$$

$$P_9 = 1667.6(0.9971) = 1662.4 \text{ kPa}$$

7. Ideal gas relation

$$\rho_9 = \frac{1662.4}{0.287(1349)} = 4.294 \text{ kg/m}^3$$

8. Enthalpy $h_9 = 1.157(1349) = 1560.8$ kJ/kg

9. Fuel-air ratio

$$f = \frac{(T_{09}/T_{08})-1}{(Q_R/c_pT_{08})-(T_{09}/T_{08})}$$

for $C_{12}H_{26}$ the value of $Q_R = 44108.3$ kJ/kg K putting the values and solving results in,

$$f = \frac{0.573}{42.86} = 0.01338$$

10. Fuel flow rate

$$\dot{m}_f = f\dot{m}_8 = 0.01338(233.8) = 3.1282 \text{ kg/s}$$

11. Net mass flow rate

$$\dot{m}_9 = \dot{m}_8 + \dot{m}_f = 236.93 \text{ kg/s}$$

12. Air-fuel ratio (Af_{act}) = $1/f$ = 15.006

Table 5.9 Comparison of flow Properties at Station (9)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A [m^2]
By Hand	1350.0	1667.6	1349.0	1662.4	4.2940	1560.8	46.6	0.065	1.150
Computer	1350.0	1671.5	1349.0	1666.7	4.3013	1355.1	46.1	0.064	1.161

5.5.10 Station (10) : Turbine

Known Parameters:

- $u_{10} = u_9 = 46.6$ m/s (design assumption)
- $\eta_t = 0.93$ (user defined adiabatic efficiency)
- $\eta_{tm} = 0.97$ (user defined mechanical efficiency)
- $k = 1.33$
- $c_p = 1.157$ kJ/kg K

1. Turbine Work

$$W_{turb} = \frac{W_{comp}}{\eta_{tm}} = \frac{371.5}{0.97} = 383.0 \text{ kJ/kg}$$

2. Turbine work in terms of temperatures

$$W_{turb} = c_p\left(T_{09} - T_{010}\right)$$

$$383.0 = 1.157\left(1350.0 - T_{010}\right)$$

or $T_{010} = 1018.9\ \text{K}$

3. Temperature - velocity isentropic relation

$$T_{10} = T_{010} - \frac{u_{10}^2}{2c_p}$$

$$T_{10} = 1018.9 - \frac{(46.6)^2}{2\left(1.157\times10^3\right)} = 1017.9\ \text{K}$$

4. $h_{10} = (1.157)(1017.9) = 1177.7$ kJ/kg

5. $M_{10} = \dfrac{46.6}{\sqrt{1.33\times 287\times 1017.9}} = 0.075$

From the definition of adiabatic efficiency find ideal temperature (T_{010i})

$$\eta_t = \frac{T_{09} - T_{010}}{T_{09} - T_{010i}}$$

$$T_{010i} = 1350.0 - \left(\frac{1350.0 - 1018.9}{0.93}\right) = 994.0\ \text{K}$$

7. Temperature - velocity relation

$$T_{10i} = T_{010i} - \frac{u_{10}^2}{2c_p}$$

$$T_{10i} = 994.0 - 0.938 = 993.0\ \text{K}$$

8. Isentropic relation

$$P_{10} = P_9\left(\frac{T_{10i}}{T_9}\right)^{\frac{k}{k-1}} = 1662.4\left(\frac{993.0}{1349.0}\right)^{4.03} = 483.6\ \text{kPa}$$

9. Isentropic relation

$$P_{010} = P_{10}\left(\frac{T_{010}}{T_{10}}\right)^{\frac{k}{k-1}} = 483.6\left(\frac{1018.9}{1017.9}\right)^{4.03} = 485.5 \text{ kPa}$$

10. Equation of state

$$\rho_{10} = \frac{483.6}{0.287 \times 1017.9} = 1.6554 \text{ kg/m}^3$$

Table 5.10 Comparison of flow Properties at Station (10)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A [m^2]
By Hand	1018.9	485.5	1017.9	483.6	1.6554	1023.0	46.6	0.075	1.150
Computer	1018.4	485.6	1017.5	483.8	1.6554	1022.1	46.1	0.074	1.161

5.5.11 Station (11) : Turbojet Nozzle

Assumptions:

- Isentropic
- Always choked

Known parameters:

- $T_{O11} = T_{O10} = 1018.9$ K
- $P_{O11} = P_{O10} = 485.5$ kPa
- $M_{11} = 1.0$
- $k = 1.33$
- $c_p = 1.157$ kJ/kg K
- $\dot{m}_{11} = \dot{m}_{10} = 236.9$ kg/s

1. Isentropic temperature - Mach relation

$$T_{11} = T_{011}\left(1+\frac{k-1}{2}M_{11}^2\right)^{-1}$$

$$T_{11} = 1018.9\left(1+\frac{1.33-1}{2}\times 1\right)^{-1} = 874.6 \text{ K}$$

2. Velocity $u_{11} = M_{11}\sqrt{kRT_{11}} = \sqrt{1.33\times 287\times 874.6} = 577.8$ m/s

3. Isentropic relation

$$P_{11} = P_{011}\left(\frac{T_{11}}{T_{011}}\right)^{\frac{-k}{k-1}}$$

$$P_{11} = 485.5\left(\frac{874.6}{1018.9}\right)^{-4.03} = 262.4 \text{ kPa}$$

4. Equation of state

$$\rho_{11} = \frac{262.4}{0.287\times 874.6} = 1.0452 \text{ kg/m}^3$$

5. Continuity

$$A_{11} = \frac{236.9}{1.0452\times 577.8} = 0.392 \text{ m}^2$$

6. $h_{11} = c_p T_{11} = 1011.9$ kJ/kg

Table 5.11 Comparison of flow Properties at Station (11)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A [m^2]
By Hand	1018.9	485.5	874.6	262.4	1.0452	1012.0	577.8	1.0	0.392
Computer	1018.4	485.6	874.2	262.4	1.0450	1011.5	577.9	1.0	0.398

5.5.12 Station (15) : Ramjet Diffuser

Known Parameters:

- Isentropic flow across diffuser (assumed), therefore:

  $T_{015} = T_{014} = 487.6$ K

  $P_{015} = P_{014} = 278.2$ kPa

- $u_{15} = 30.0$ m/s (user input)
- $k = 1.4$
- $c_p = 1.005$ kJ/kg
- $\dot{m}_{15} = \dot{m}_{14} = 233.2$ kg/s

1. Isentropic temperature - velocity relation

$$T_{15} = 487.6 - \frac{(30)^2}{2(1.005\times 10^3)} = 487.15 \text{ K}$$

2. $h_{15} = c_p T_{15} = 489.6$ kJ/kg

3. Velocity of sound $c = \sqrt{kRT}$, leads to $c_{15} = 442.4$ m/s

   and $M_{15} = \dfrac{u_{15}}{c_{15}} = 0.068$

4. Using isentropic relation $\dfrac{P_0}{P} = \left(\dfrac{T_0}{T}\right)^{\frac{k}{k-1}}$

$$P_{15} = 287.2\left(\frac{487.6}{487.2}\right)^{-3.5} = 277.3 \text{ kPa}$$

5. Equation of state leads to $\rho_{15} = 1.9834$ kg/m^3

6. Continuity equation gives

$$A_{15} = \frac{233.8}{(1.9834)(30)} = 3.93 \text{ m}^2$$

7. $h_{015} = c_p T_{015} = 489.8$ kJ/kg

Table 5.12 Comparison of flow Properties at Station (15)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A [m^2]
By Hand	487.6	278.2	487.2	277.3	1.9834	489.6	30.0	0.068	3.93
Computer	487.6	278.9	487.1	278.0	1.9864	489.3	30.0	0.068	3.96

5.5.13 Station (16) : Ram Burner

Known Parameters:

- Burner thermal limit (TRC) = 3000 K (user defined)
- $A_{16} = A_{15} = 3.93$ m^2 (Raleigh process approach)
- $\eta_b = 0.98$ (user defined)
- $k = 1.3$
- $c_p = 1.244$ kJ/kg K

1. Establish adiabatic flame temperature and compare with known thermal limit

For the given fuel ($C_{12}H_{26}$)

$$AF_{th} = \frac{4.76 \times 18.5 \times 28.97}{170} = 15.006$$

$$f = \frac{1}{AF_{th}} = 0.06664$$

From the definition of fuel-air ratio

$$f = \frac{\left(h_{016}/h_{015}\right) - 1}{\left(\eta_b Q_R / h_{015}\right) - \left(h_{016}/h_{015}\right)}$$

(for $C_{12}H_{26}$ the value of $Q_R = 44108.3$ kJ/kg K)

putting values and arranging for h_{016}

$$h_{016} = \frac{[489.8 + (0.06664 \times 0.98 \times 44108.3)]}{1.06664} = 3159.8 \text{ kJ / kg}$$

this implies that T_{016} = 2540.0 K

T_{016} is obtained using AF_{th}, so it represents the adiabatic flame temperature (AFT). Also since AFT < TRC, the operation at this temperature is safe.

2. Mass flow through ramjet

$$\dot{m}_f = f \times \dot{m}_{RJ} = 0.06664 \times 233.8 = 15.58 \text{ kg/s}$$

$$\dot{m}_{16} = \dot{m}_{RJ} + \dot{m}_f = 249.4 \text{ kg/s}$$

3. Find T_0^* (choking temperature) for station (15)

$$\frac{T_{015}}{T_{015}^*} = \left[\frac{2(k+1)M_{15}^2\left(\frac{T_{015}}{T_{15}}\right)}{\left(1 + kM_{15}^2\right)^2}\right]$$

$$\frac{T_{015}}{T_{015}^*} = \left[\frac{4.8(0.068)^2(1.000924)}{\left(1 + (1.4)(0.068)^2\right)}\right] = 0.0218$$

$$T_{015}^* = \frac{487.6}{0.0218} = 22363 \text{ K}$$

Since $T_{015}^* \rangle T_{015}$, combustion is possible.

4. Finding M_{16}

It may be noted that $T_{015}^* = T_{016}^*$ for the same burner, therefore, for the known T_{016} and T_{016}^*, M_{16} can be obtained from the temperature ratio.

$$\frac{T_{016}}{T_{016}^*} = \left[\frac{2(1.30+1)M_{16}^2\left(1 + \frac{1.30-1}{2}M_{16}^2\right)}{\left(1 + 1.30M_{16}^2\right)^2}\right]$$

$$0.1136 = \frac{4.6M_{16}^2 + 0.69M_{16}^4}{1 + 2.6M_{16}^2 + 1.69M_{16}^4}$$

this leads to a fourth order equation in M_{16}

$$M_{16}^4 + 8.645M_{16}^2 - 0.229 = 0$$

Assuming $M_{16}^2 = Y$, the equation reduces to a quadratic in Y. The quadratic formula returns the roots of Y as

$Y_1 = 0.0264$ and $Y_2 = -8.67$

Positive root is selected representing the subsonic Mach in the ramjet burner.

Thus $M_{16} = \sqrt{Y_1} = 0.163$

5. Temperature - Mach isentropic relation

$$T_{16} = T_{016}\left(1 + \frac{k-1}{2}M_{16}^2\right)^{-1}$$

$$T_{16} = 2540.0\left(1 + \frac{1.3-1}{2}(0.163)^2\right)^{-1} = 2529.8 \text{ K}$$

6. Enthalpy

$$h_{16} = 1.244(2529.8) = 3147.1 \text{ kJ/kg}$$

7. Obtain P_{08} from the Raleigh line pressure ratio

$$\frac{P_{16}}{P_{15}} = \left(\frac{1 + kM_{15}^2}{1 + kM_{16}^2}\right)$$

Putting the values

$$\frac{P_{16}}{P_{15}} = \left(\frac{1 + 1.4(0.068)^2}{1 + 1.3(0.163)^2}\right) = 0.973$$

$$P_{16} = (0.973)P_{15} = 269.8 \text{ kPa}$$

8. Pressure - Temperature isentropic relation

$$\frac{P_{016}}{P_{16}} = \left(\frac{T_{016}}{T_{16}}\right)^{\frac{k}{k-1}}$$

$$P_{016} = P_{16}(1.0176)$$

$$P_{016} = (269.8)(1.0176) = 274.5 \text{ kPa}$$

9. Ideal gas relation

$$\rho_{16} = \frac{269.8}{0.287(2529.8)} = 0.3716 \text{ kg/m}^3$$

10. Velocity $u_{16} = M_{16}\sqrt{kRT_{16}} = 0.163(971.5) = 157.9$ m/s

Table 5.13 Comparison of flow Properties at Station (16)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A [m^2]
By Hand	2540.0	274.5	2529.8	269.8	0.3716	3147.1	157.9	0.163	3.93
Computer	2538.6	269.8	2529.0	265.4	0.3653	3146.1	154.1	0.159	3.96

5.5.14 Station (17) : Ramjet Exhaust

The passage between ramjet burner and its exhaust is assumed to be constant area duct in that the flow behaves isentropically. Therefore, the flow properties does not change between stations (16) and (17).

Table 5.14 Comparison of flow Properties at Station (17)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	u [m/s]	M	A [m^2]
By Hand	2540.0	274.5	2529.8	269.8	0.3716	3147.1	157.9	0.163	3.93
Computer	2538.6	269.8	2529.0	265.4	0.3653	3146.1	154.1	0.159	3.96

5.5.15 Station (18) : Mixer Exit

Known parameters:

- $\dot{m}_{18} = \dot{m}_{11} + \dot{m}_{17} = 486.3$ kg/s
- $A_{18} = A_8 + A_{17} = 5.08$ m^2 (maximum area of the engine)
- $k = 1.33$
- $c_p = 1.157$ kJ/kg K

1. Energy equation

$$\dot{m}_{18} c_{p18} T_{018} = \dot{m}_{11} c_{p11} T_{011} + \dot{m}_{17} c_{p17} T_{017}$$

$$\dot{m}_{18} c_{p18} T_{018} = (236.9 \times 1.157 \times 1018.9) + (249.4 \times 1.244 \times 2540) = 1067317.8 \text{ kJ/s}$$

$$T_{018} = \frac{1067317.8}{(486.3 \times 1.157)} = 1896.9 \text{ K}$$

2. Simultaneously solving first law, continuity, momentum equation and equation of state and rearranging for u_{18} (see appendix C)

$$(2c_{p18} + R)\dot{m}_{18} u_{18}^2 - 2X_1 u_{18} c_{p18} - RX_2 = 0$$

where

$$X_1 = (\dot{m}_{17} u_{17} + \dot{m}_{11} u_{11} - A_{17} P_{17} - A_{11} P_{11})$$

$$X_2 = \dot{m}_{11}(2h_{11} + u_{11}^2) + \dot{m}_{17}(2h_{17} + u_{17}^2)$$

For compatibility of the units use P in [N/m^2] and c_p in [J/kg K]

The equation is a quadratic in variable u. Putting known values to obtain coefficients and the constant term

$$(2c_{p18} + R)\dot{m}_{18} = [2(1.157 \times 10^3) + 287](486.3) = 1264866.3$$

$$2X_1c_{p18} = 2(-991651.7)(1.157\times10^3) = -2.2947\times10^9$$

$$RX_2 = 287(2.1345\times10^9) = 6.126\times10^{11}$$

Applying quadratic formula

$$u_{18(1,2)} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

where

$a = 1264866.3$

$b = -2.2947\times10^9$

$c = 6.126\times10^{11}$

the two roots are:

$u_{18(1)} = 236.2$ m/s

$u_{18(2)} = -2050.3$ m/s

Disregarding the negative root, $u_{18} = 236.2$ m/s

3. The pressure P_{18} is obtained from u_{18} (see appendix C)

$$P_{18} = \frac{\dot{m}_{18}u_{18} - X_1}{A_{18}} = \frac{(486.3\times236.2)-(-9.9165\times10^5)}{5.08} = 217823.6\ \text{N/m}^2$$

or $P_{18} = 217.8$ kPa

4. Temperature

$$T_{18} = T_{018} - \frac{u_{18}^2}{2c_{p18}} = (1896.9 - 24.1) = 1872.8\ \text{K}$$

5. Mach number

$$M_{18} = \frac{u_{18}}{\sqrt{kRT_{18}}} = \frac{236.2}{845.5} = 0.279$$

6. Enthalpy $h_{18} = (1.157 \times 1872.8) = 2166.8$ kJ/kg

7. Ideal gas equation

$$\rho_{18} = \frac{217.8}{0.287(1872.8)} = 0.4052 \text{ kg/m}^3$$

8. Pressure - Mach isentropic relation

$$P_{018} = 217.8\left[1 + 0.165(0.279)^2\right]^{4.03} = 229.3 \text{ kPa}$$

Table 5.15 Comparison of flow Properties at Station (18)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m³]	h [kJ/kg]	u [m/s]	M	A [m²]
By Hand	1896.9	229.3	1872.8	217.8	0.4052	2166.8	236.2	0.279	5.08
Computer	1894.4	225.9	1869.4	214.1	0.3987	2163.0	240.8	0.285	5.12

5.5.16 Station (20) : Turboramjet Nozzle

Assumptions:

- Fully expanded under all conditions
- Always choked at the throat
- Adiabatic

Known parameters:

- $T_{020} = T_{018} = 1896.9$ K
- $P_{20} = P_1 = 22.6$ kPa
- $M_{19} = 1.0$
- $k = 1.33$
- $c_p = 1.157$ kJ/kg K
- $\dot{m}_{20} = \dot{m}_{18} = 486.3$ kg/s

- $\eta_n = 0.97$

1. Find ideal temperature T_{20i}

$$\frac{T_{20i}}{T_{18}} = \left(\frac{P_{20}}{P_{18}}\right)^{\frac{k-1}{k}} = \left(\frac{22.6}{217.8}\right)^{0.2481} = 0.57$$

$$T_{20i} = 0.57(1872.8) = 1067.5 \text{ K}$$

2. Find T_{20} using η_n

$$\eta_n = \frac{T_{18} - T_{20}}{T_{18} - T_{20i}}$$

$$T_{20} = 1872.8 - 0.97(1872.8 - 1067.5) = 1091.6 \text{ K}$$

3. Equation of state

$$\rho_{20} = \frac{22.6}{0.287(1091.6)} = 0.0721 \text{ kg/m}^3$$

4. Find M_{20} from the temperature ratio

$$\frac{T_{020}}{T_{20}} = \left(1 + \frac{k-1}{2} M_{2.}^2\right)$$

$$M_{20} = \sqrt{\frac{(1.7378 - 1.0)}{0.165}} = 2.11$$

5. Use pressure - temperature relation for P_{020}

$$P_{020} = P_{20}\left(\frac{1896.9}{1091.6}\right)^{4.03} = 22.6(9.2725) = 209.6 \text{ kPa}$$

6. Find u_{20}

$$u_{20} = M_{20}\sqrt{kRT_{20}} = 2.11\sqrt{(1.33 \times 287 \times 1091.6)} = 1362.0 \text{ m/s}$$

7. Enthalpy $h_{20} = (1.157 \times 1091.6) = 1263.0$ kJ/kg

8. Continuity equation

$$A_{20} = \frac{\dot{m}_{20}}{\rho_{20} u_{20}} = \frac{486.3}{0.0721 \times 1362} = 4.95 \text{ m}^2$$

Table 5.16 Comparison of flow Properties at Station (20)

Flow Properties	T_0 [K]	P_0 [kPa]	T [K]	P [kPa]	ρ [kg/m^3]	h [kJ/kg]	U [m/s]	M	A [m^2]
By Hand	1896.9	209.6	1091.6	22.6	0.0721	1263.0	1362.0	2.11	4.95
Computer	1894.4	206.0	1094.8	22.6	0.0720	1266.7	1360.0	2.10	5.02

5.5.17 Engine Performance Parameters

1. Fuel flow rate

$$\dot{m}_f = \dot{m}_{ftj} + \dot{m}_{frj} = 3.13 + 15.58 = 18.71 \text{ kg/s}$$

2. Air mass flow rate

$$\dot{m}_a = \dot{m}_3 = 467.6 \text{ kg/s}$$

3. Fuel - air ratio

$$f = \frac{\dot{m}_f}{\dot{m}_a} = \frac{18.71}{467.6} = 0.040013$$

4. Thrust for fully expanded nozzle

$$\tau = \dot{m}_a \left[(1 + f) u_{20} - u_1 \right]$$

$$\tau = 467.6 \left[(1 + 0.040013)(1362) - 737.7 \right] = 317410 \text{ N}$$

or $\tau = 317.4$ kN

5. Specific thrust $= \dfrac{\tau}{\dot{m}_a} = 678.8 \ \dfrac{\text{N} \cdot \text{s}}{\text{kg}}$

6. Propulsive efficiency

$$\eta_p = \frac{\tau\, u_1}{\dot{m}_a\left[(1+f)\left(u_{20}^2/2\right)-\left(u_1^2/2\right)\right]}$$

$$\eta_p = \frac{317410 \times 737.7}{467.6\left[(1.040013)(927522)-(272101)\right]} = 0.723$$

7. Thermal efficiency

$$\eta_{th} = \frac{\left[(1+f)\left(u_{20}^2/2\right)-\left(u_1^2/2\right)\right]}{fQ_R}$$

for $C_{12}\,H_{26}$ fuel, $Q_R = 44.11\text{x}10^6$

therefore

$fQ_R = 1.765\text{x}10^6$

and

$$\eta_{th} = \frac{692534.3}{1.765 \times 10^6} = 0.392$$

8. Overall efficiency

$$\eta_o = \eta_p \eta_{th} = 0.723 \times 0.392 = 0.284$$

9. Thrust specific fuel consumption

$$TSFC = \frac{\dot{m}_f}{\tau} \text{ kg(fuel) / s / N}$$

In conventional units of [kg(fuel)/hr/kN]

$$TSFC = \frac{\dot{m}_f\,(1000)(3600)}{\tau} = 212.2 \text{ kg(fuel)/hr/kN}$$

Table 5.17 Comparison of engine performance parameters

Parameter	$\dot{m}_a$ [kg/s]	$\dot{m}_f$ [kg/s]	τ [kN]	*TSFC* [kg/hr/N]	η_p	η_{th}	η_o	f
By Hand	467.6	18.71	317.41	212.2	0.723	0.392	0.284	0.040013
Computer	472.3	19.75	320.77	221.6	0.725	0.376	0.272	0.041808

CHAPTER 6

DEMONSTRATION OF THE SOFTWARE

6.1 Design Point Performance Characteristics of the Engine

The computer software of the turboramjet is capable of handling any design considerations within the bounds discussed in Section 3.2.3. However, for the purpose of demonstration the set of design conditions as appeared in Section 5.1 are taken as reference. The goal set forth was a combination engine for a commercial aircraft use that could achieve a maximum Mach number in the range of 6 to 7. As such the design parameters, including the altitude and the Mach number, were selected to represent nearly the average values. The user defined design parameters indicate that the design point performance of the engine depends largely on a number of variables such as flight Mach number, diffuser efficiency, compressor pressure ratio, cycle temperature ratio, compressor, turbine, burner and the nozzle efficiencies.

Generally the suitability of an aircraft engine to a specific propulsion system is adjudged based upon the following parameters:

a) The thrust developed per unit of the frontal area $\left(\tau/A_F\right)$

b) The thrust developed per unit of engine weight $\left(\tau/W_e\right)$

c) The specific thrust, i.e. thrust developed per unit of mass flow rate $\left(\tau/\dot{m}_a\right)$

d) The thrust specific fuel consumption (*TSFC*)

The parameter (τ/A_F) is a measure of the thrust available for overcoming the combined drag of the airframe and the engine. To achieve high flight speeds it is essential that (τ/A_F) has a large value. This factor becomes more important as higher speeds are approached because of the rapid increase in the aerodynamic resistance of an aircraft with increase in the flight speed.

The parameter (τ/W_e) is a measure of the sum of structural weight, fuel weight and the payload that an aircraft can carry at a given flight speed. Engines having larger values of (τ/W_e) make it possible to load the aircraft with either a large quantity of fuel or more payload. Hence it is desirable that (τ/W_e) be as large as possible.

The parameters *TSFC* and (τ/W_e) are useful for predicting the range of an aircraft. It is obvious that the engine giving the lowest value for TSFC will also give the longest range for a given fuel load. Since the parameter (τ/W_e) also exercises an influence on the range, judgments regarding the range of an aircraft must be based on the considerations of both the *TSFC* and (τ/W_e).

The design parameters (τ/A_F) and (τ/W_e) are more related to the engine integration with the aircraft, as such these issues are not addressed in the software under review and the study is restricted to the performance parameters like specific thrust, *TSFC* and the engine efficiencies.

For the purpose of study, the engine operation was simulated to cover the entire operational envelope for the design point by varying Mach number and the altitude. For the selected design, the operational limits of the engine are depicted in Figure 6.1. It reveals that the turbojet operated alone between Mach 0.65 at sea level and 0.6 at 19000 meters. The joint operation of the turbojet and the ramjet was commenced upto Mach 2.89 at sea level and 3.56 at 19000 meters. Beyond this Mach number the ramjet took over alone and kept producing positive thrust upto Mach number 6.73 at sea level and upto the Mach 7.9 at 19000 meters.

At higher altitudes beyond 13000 meters, it is observed that the limiting Mach for the turbojet alone, joint operation and the combination stayed almost constant. This trend agrees with the fact that at higher Mach numbers when operational thermal limits are approached, the thrust goes down and approaches zero when free stream velocity approaches the exhaust jet velocity.

The design turbine inlet temperature (TIT) is kept as 1350 K against a value of 1700 K now achievable [Hills and Peterson 1992]. If enhanced, this limit may considerably increase the turbojet thrust and the maximum operable Mach number.

The ramjet exhaust is kept fixed equal to the burner area and exhausted in the mixer. As such, the nozzle effect for better ramjet thrust is absent. This arrangement does not take full advantage of ramjet thrust at low Mach numbers. The compressor pressure ratio required to minimize the specific fuel consumption is much less for the supersonic flight than for subsonic flight.

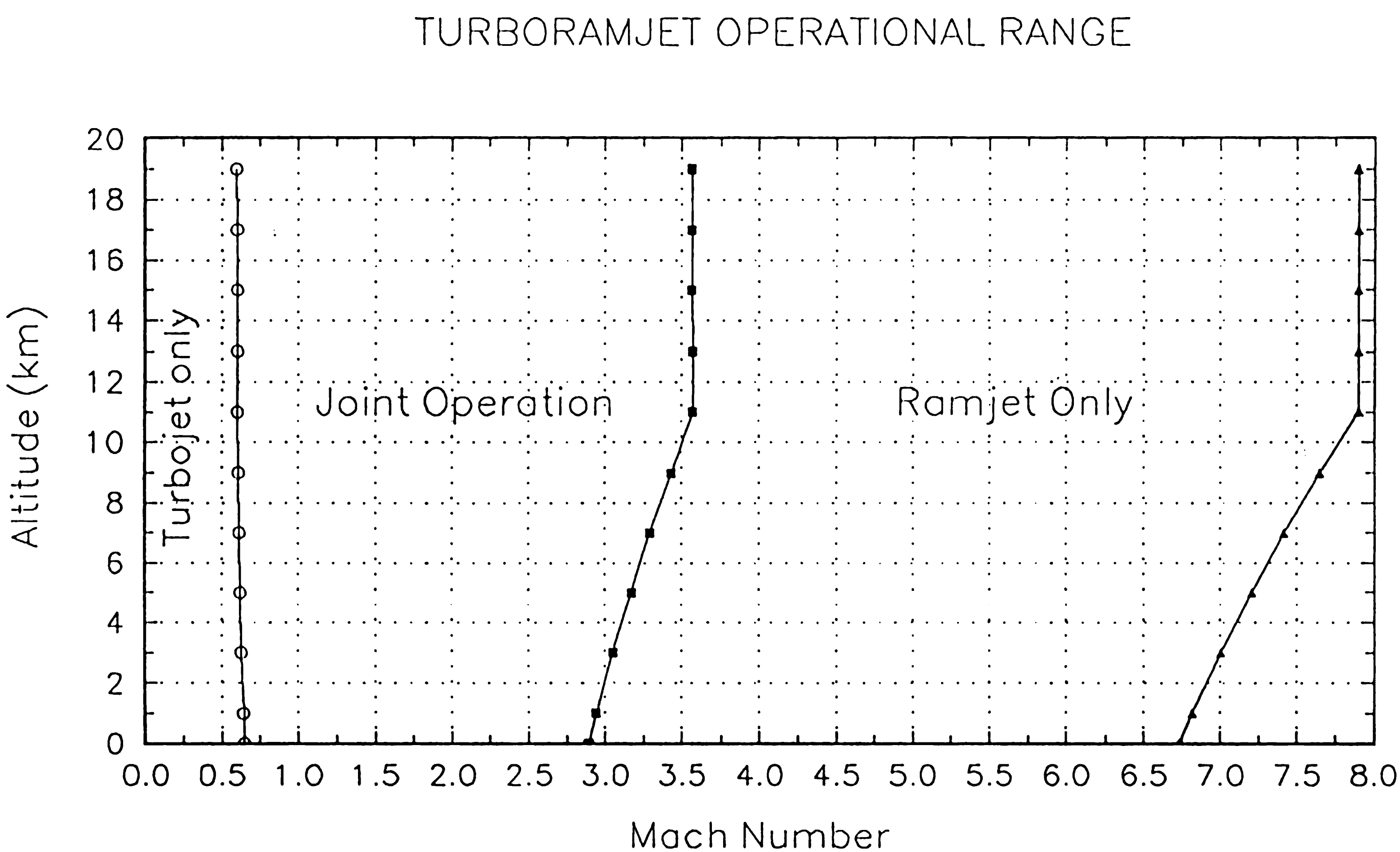

Figure 6.1 Turboramjet operational range for maximum Mach number and altitude

The lower value of compressor pressure ratio at designed point improves the turbojet thrust at supersonic Mach numbers and correspondingly lowers the specific fuel consumption. In other words the lower the compressor ratio the higher the limiting Mach number the turbojet can attain in supersonic flights.

At present the design point considerations are limited to the use of total air mass flow rate for the useful thrust and no air tapping is provided either for cooling purpose or for wind milling of the turbojet when it is shut down. Practically the air tapping is required for both of these purposes. Inclusion of these considerations may change the thrust and TSFC picture a little.

The velocities of the air entering the combustion system are of the same order of magnitude for both turbojet and the ramjet. Consequently, both engines require approximately the same combustor cross sectional area to pass the same volumetric airflow rates. For the ramjet engine, the frontal area is equal to the burner cross-sectional area, but for the turbojet the frontal area is dictated by the diameter of either the compressor or the turbine, whichever is larger. In general, the cross-sectional area of either the compressor or the turbine greatly exceeds that of the combustion chambers. Therefore, the frontal area of a turbojet is inherently larger than that of a ramjet having the same air induction capacity. The difference may become relatively small when axial flow compressor with increased air induction capacity per square meter of the frontal area, and turbine with cooled blades and disc is made available.

6.2 Off-Design Performance Characteristics of the Engine

Off-designed engine performance was checked by plotting various performance parameters by varying flight altitude and the Mach number. The results obtained from the software in respect of these parameters are discussed in the following paragraphs.

6.2.1 Specific Thrust and *TSFC*

The specific thrust (thrust per unit mass flow rate) is a criterion of the size of the engine required for producing specified total thrust. For a fully expanded nozzle the specific thrust is given by

$$\frac{\tau}{\dot{m}_a} = \left[(1+f)u_e - u_a\right]$$

since $f << 1$, it may be ignored, then

$$\frac{\tau}{\dot{m}_a} = \left(u_e - u_a\right)$$

In terms of velocity ratio $v = \frac{u_a}{u_e}$ the specific thrust becomes

$$\frac{\tau}{\dot{m}_a} = \left(1 - \frac{u_a}{u_e}\right) \qquad (6.1)$$

It is apparent from equation (6.1) that to achieve a large value of specific thrust the velocity ratio must be kept small.

From equation (4.48) the *TSFC* in standard units [kg (fuel)/hr/kN] is defined as

$$TSFC = \frac{(1000)(3600)f}{\left(\tau / \dot{m}_a\right)} \tag{6.2}$$

Clearly, for a given fuel-air ratio *TSFC* is inversely proportional to the specific thrust.

Figure 6.2 shows the comparison of plots for specific thrust and the *TSFC* verses Mach number at sea level and 11000 meter altitude. At low Mach numbers where only the turbojet operates, the engine produces a higher specific thrust value because of low velocity ratio. The plot shows a slightly decreasing trend as the Mach number increases with gradual decrease in the slope of the curve as the Mach approaches a value closer to 0.6. This refers first to an increase in the velocity ratio with increasing Mach and then stability in the velocity ratio as the Mach number increases up to 0.6. Correspondingly, the *TSFC* reflects first a gradual followed by a sharp increasing trend owing to decrease in the specific thrust in that Mach range.

Above the minimum Mach of the ramjet operation (0.604 at 11000 meters), the joint operation of the two engines is realized. At that Mach number, a sudden fall in specific thrust and sharp rise in the *TSFC* is observed. It is because the code calculates the minimum Mach number of the ramjet operation taking reference of the Mach where ramjet just starts to produce positive thrust. At that point, the ramjet gets comparatively less airflow that is surplus after meeting the turbojet engine demand and the velocity ratio is close to unity that results in a marginally positive specific thrust produced by the ramjet. Although at this point a higher proportion of specific thrust is realized by the turbojet, an overall decrease in the specific thrust is observed due to poor contribution of

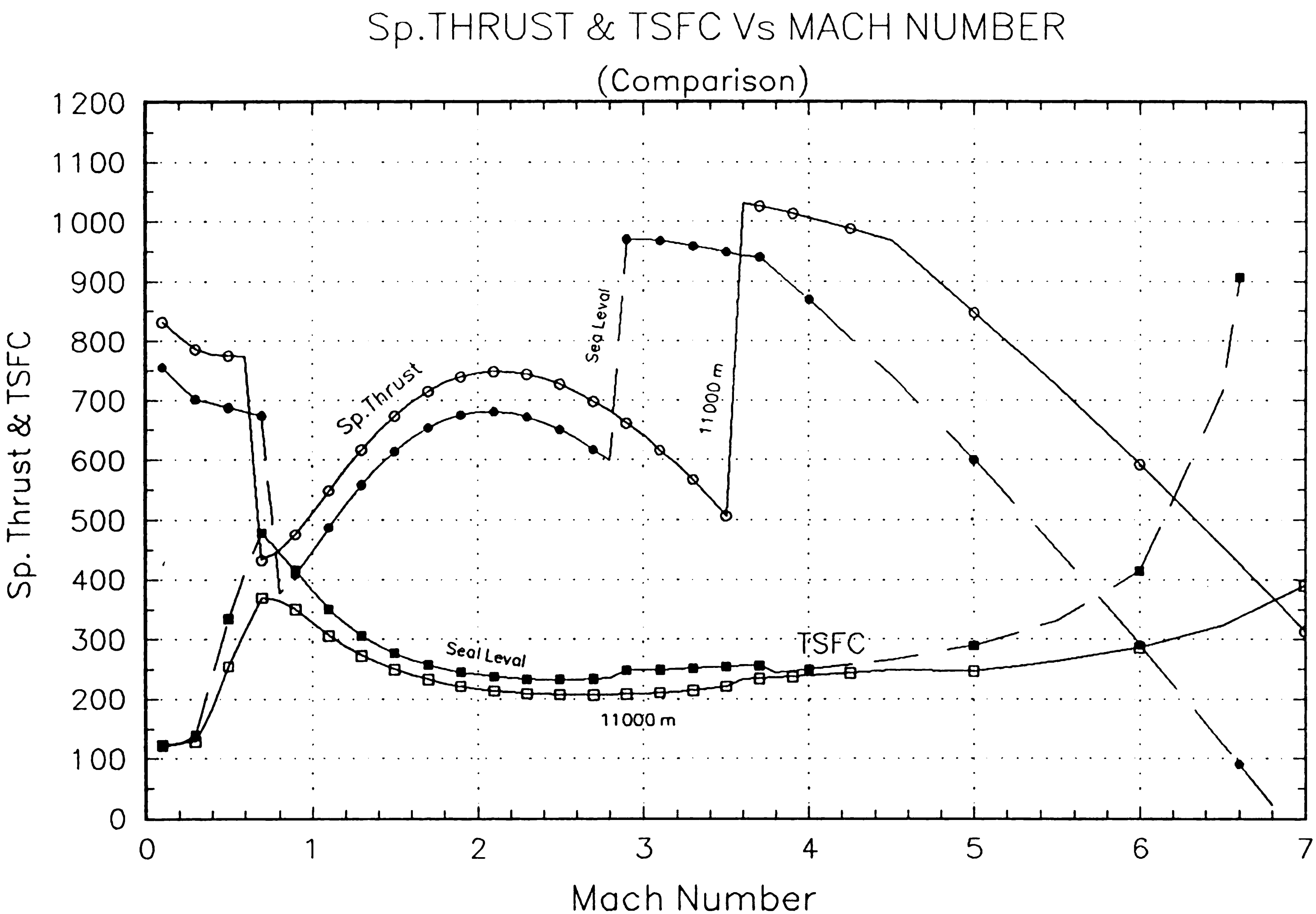

Figure 6.2 Comparison of specific thrust and TSFC with altitude variation

by the ramjet. Consequently, a sudden rise in the *TSFC* takes place at this Mach number.

Figure 6.3 shows a comparison between the specific thrust and *TSFC* for the minimum Mach for ramjet operation of 0.6 and 1.0. It can be seen that when the ramjet operation was suppressed up to Mach 1.0 a substantial increase in the engine specific thrust was observed between Mach 0.6 and 1.0. The effect on the *TSFC* can also be noted.

In the range of the Mach numbers where the two engines operated jointly it is observed that the engine produces the maximum specific thrust at Mach 2.0. The thrust curve starts to fall beyond this Mach and reaches to its minimum at Mach 3.57 where the turbojet is shut down for temperature reaching to the defined turbine inlet temperature. The reason for this behavior is that between Mach 0.6 and 2.0 both the engines contribute positively towards thrust. After Mach 2.0 the temperature difference across the turbojet combustor starts decreasing for the fixed turbine inlet temperature and less and less fuel is needed to be burned in the combustor. This reduces the turbojet thrust more rapidly than the increase in the thrust proportion by the ramjet with increasing Mach numbers. The overall effect is reflected as a marked decrease in the specific thrust. On the contrary, in this Mach range due to less fuel burning in the combustor an increase in the flight Mach has only a small effect on *TSFC* and its slope stays almost zero.

The rise in engine specific thrust with increase in the turbine inlet temperature is depicted in Figure 6.4. It can also be noted that for a given compressor pressure ratio, raising the turbine inlet temperature may or may not rise the specific fuel consumption.

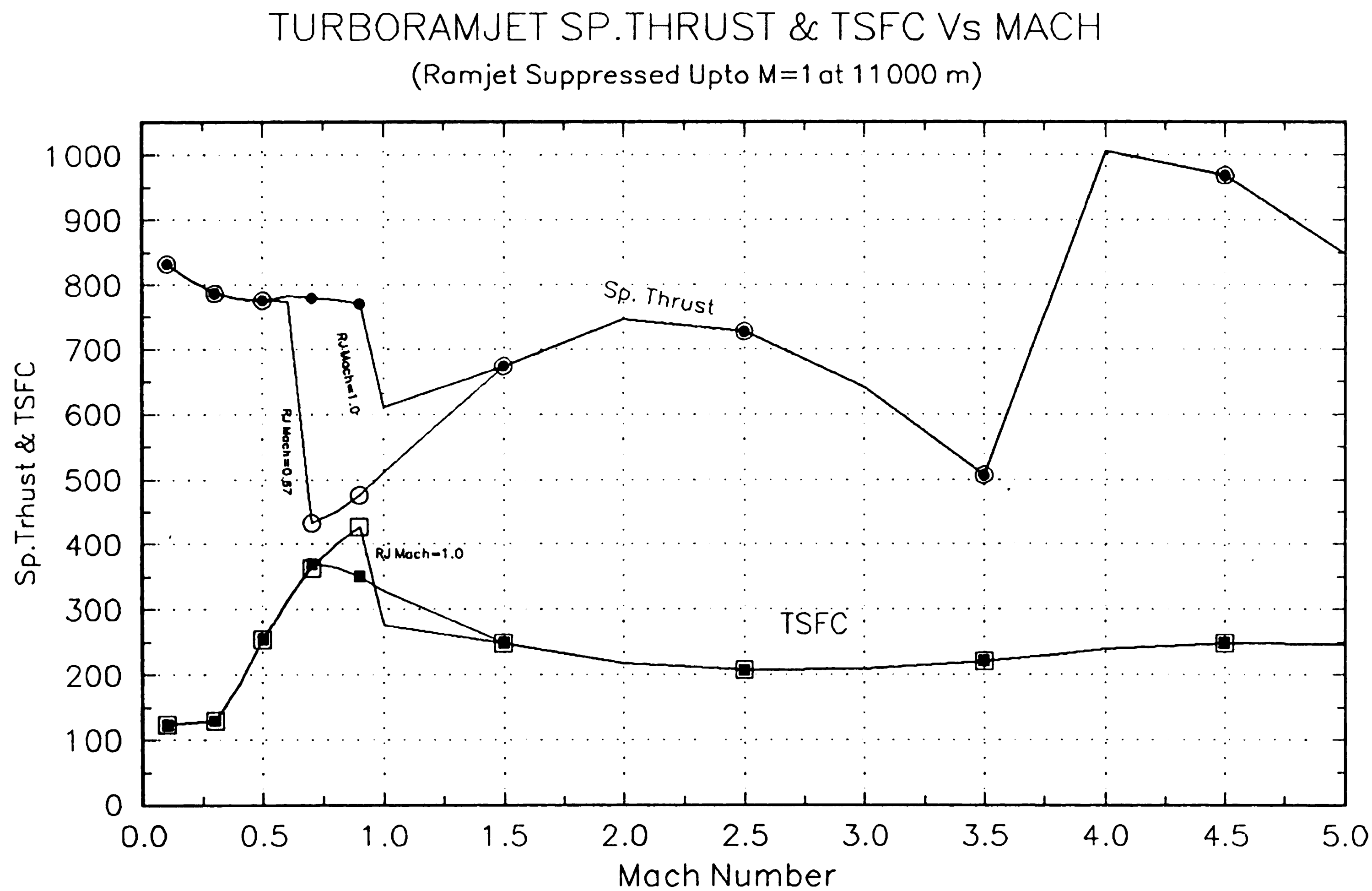

Figure 6.3 Comparison of specific thrust and TSFC with change in ramjet starting Mach at 11000 meters

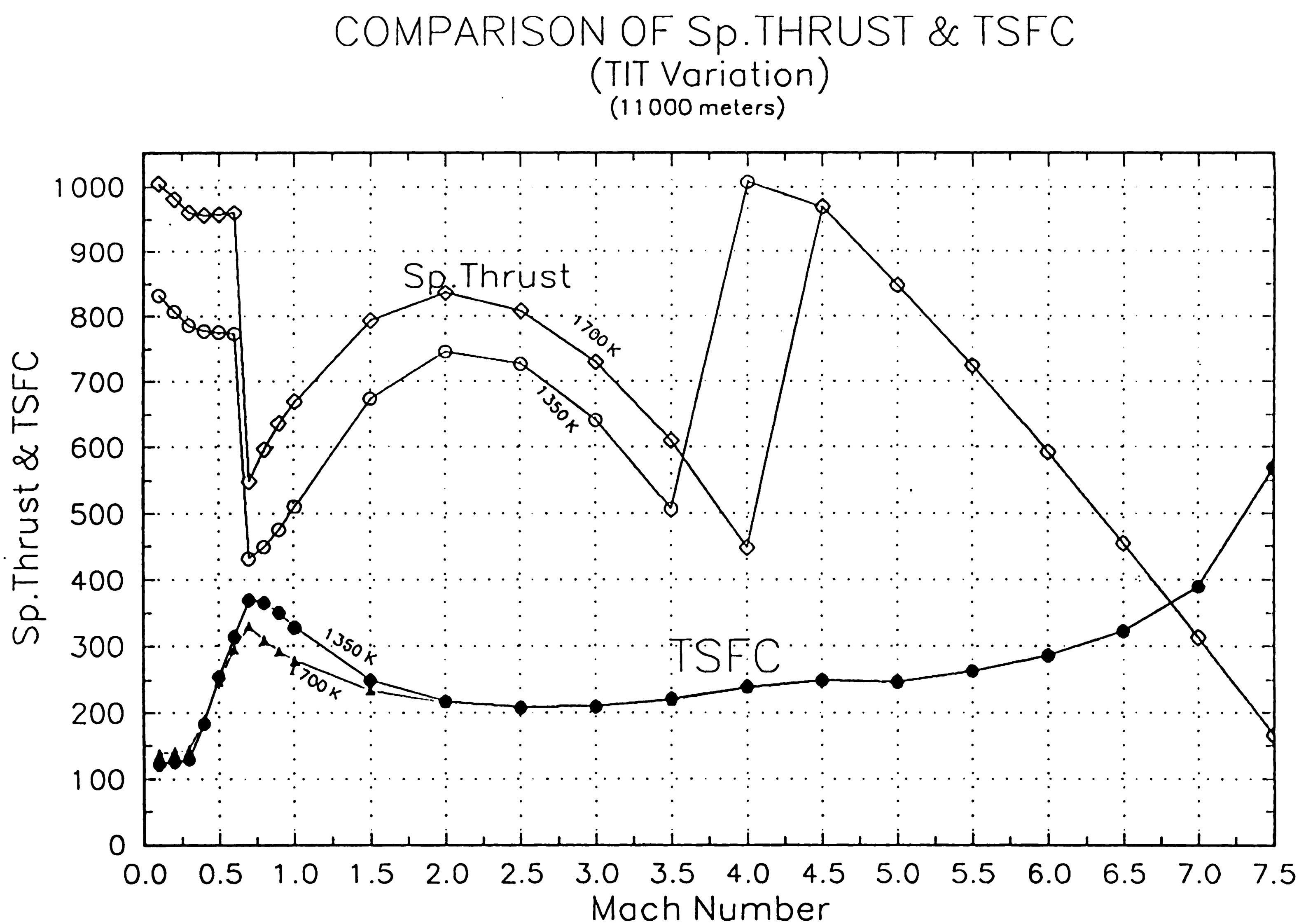

Figure 6.4 Comparison of specific thrust and TSFC for TIT variation at 11000 meters

For pressure ratios associated with minimum fuel consumption, increasing the turbine inlet temperature can reduce specific fuel consumption somewhat.

At the flight Mach numbers above 3.57, the turbojet is shut down since the temperature reaches its maximum thermal limit and the ramjet alone produces the engine thrust. Because of higher thermal limits associated with the ramjet, comparatively higher thrust values are obtainable than the turbojet for the same Mach number. A sudden rise in the engine specific thrust at Mach 3.57 is the result of sole thrust contribution of the ramjet in the absence of rather inefficient turbojet at such a high Mach number. The engine achieves the highest value of specific thrust close to Mach 3.52 because of lowest speed ratio in the sole ramjet operational Mach range.

At flight Mach of 4.52, the ramjet designed thermal limit (3000 K) was reached and the stoichiometric burning of the fuel was no longer possible. The specific thrust starts decreasing sharply and the TSFC shows an increasing trend beyond that Mach and continues to increase further as the ramjet approaches the limiting Mach number of engine operation.

6.2.2 Thrust Ratio Verses Mach Ratio

The engine performance was also studied by plotting different ratio curves. These ratios were obtained from dividing the actual parameter under the prevailing flight conditions by its respective value at design point. Figure 6.5 shows the plot of thrust ratio verses Mach ratio for different altitudes. It is observed that:

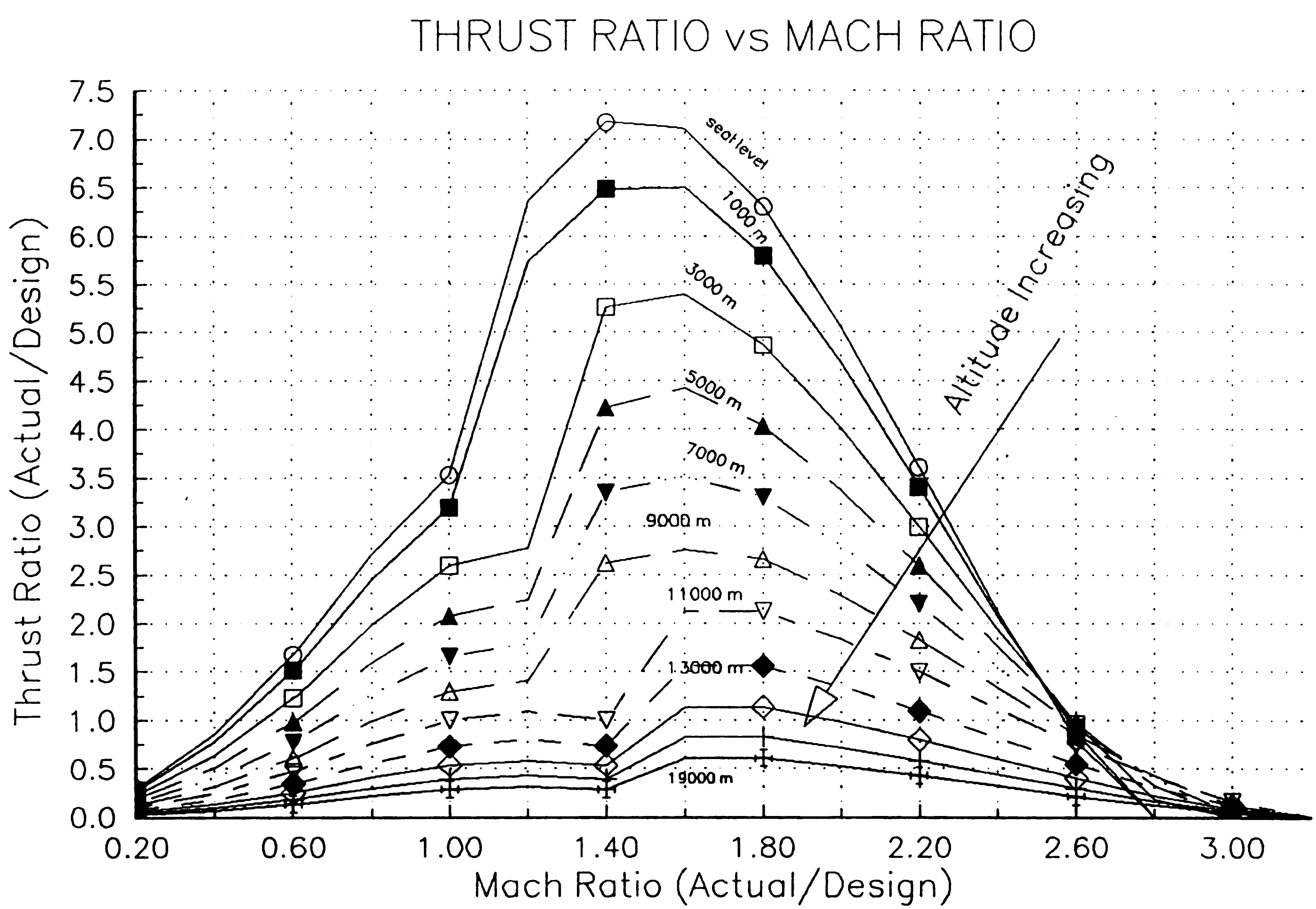

Figure 6.5 Comparison of thrust ratio verses Mach ratio with altitude variation

a) The trust ratio is minimum both at extreme Mach ratios and attains its maximum value close to the median Mach ratio. The trend is in line with the natural behavior of the thrust generation of an aircraft engine that initially increases with the Mach number, and then starts falling beyond a certain Mach when the thermal limits are approached and the mass flow through the engine decreases (also see Figure 6.6).

b) For a given Mach number, the thrust ratio decreases with increase in the altitude. It is because of decrease in density with increase in altitude that in turn reduces the mass flow rate for a given velocity and intake area. Since the thrust is directly proportional to the mass flow rate (Continuity equation), it falls as the altitude is increased.

c) At Mach ratio of 1 (flight Mach 2.5), the thrust ratio curves at lower altitudes are discontinuous and tend to shift at higher Mach ratios as the altitudes is increased. For altitudes beyond 11000 meters, the discontinuity stays at Mach ratio 1.4. This behavior refers to the thrust jump (Section 6.2.1) when turbojet is shut down and the ramjet takes over the operation alone (Figure 6.6). At lower altitudes the turbojet turbine thermal limit is reached at comparatively lower Mach numbers than that at higher altitudes (density effect). Moreover, at higher altitudes beyond 11000 meters the maximum operable Mach for the turbojet almost stays constant at 3.57 (Figure 6.1). This fact is depicted in Figure 6.5 by a noticeable reduction in the thrust ratio between altitude curves for 9000 and 11000 meters at Mach ratio 1.4.

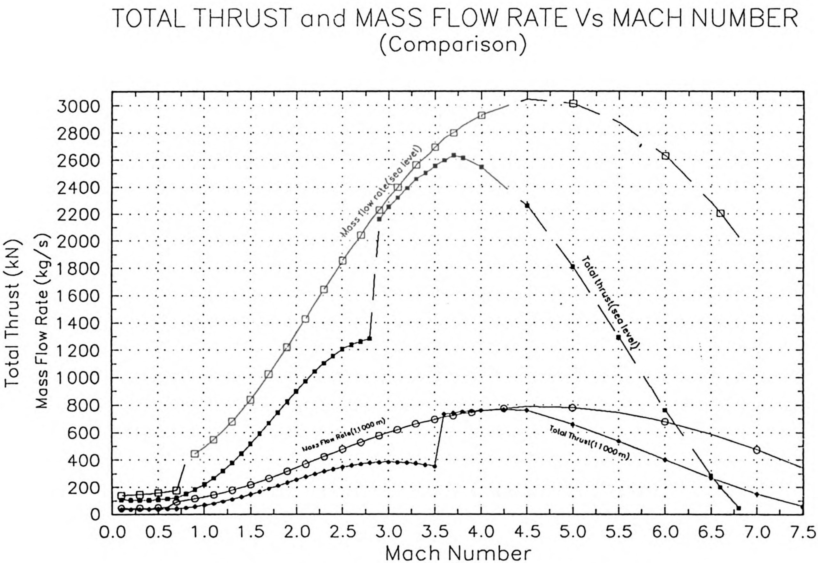

Figure 6.6 Comparison of total thrust and mass flow rate with altitude variation

d) The smooth negative slope of the thrust ratio curves beyond Mach ratio 1.8 represent the gradual decrease in the engine thrust at higher Mach numbers when only the ramjet is in operation. In this region the slope of the curves become less and less negative with increasing altitude. It refers to the maximum achievable Mach by the engine as the altitude is increased.

e) The engine maximum achievable Mach above 11000 meters stays almost constant at 7.9 (Figure 6.1) and the engine produces no positive thrust. It explains the reason for most of the thrust ratio curves representing higher altitudes to converge around Mach ratio 3.

6.2.3 Thrust Ratio Verses Altitude Ratio

Figure 6.7 shows the plots of thrust ratio verses altitude ratio for different Mach numbers. It represents the following:

a) At lower altitude ratios, the thrust ratio is higher particularly in supersonic flight below Mach 6.5. It corresponds to the fact that for the same flight Mach the mass flow rate increases more rapidly at lower altitudes than that at higher altitudes (Figure 6.6). This results in higher thrust ratios for the engine operation at low altitudes. At higher Mach numbers (4.5 at sea level and 5.5 at 11000 meters), the mass flow rate starts decreasing and the thermal limits of the engine are approached that cause a decrease in the thrust of the engine. Hence, the thrust ratio exhibits lesser and lesser change with change in altitude ratio for a given Mach number and the slope of the curve becomes almost zero

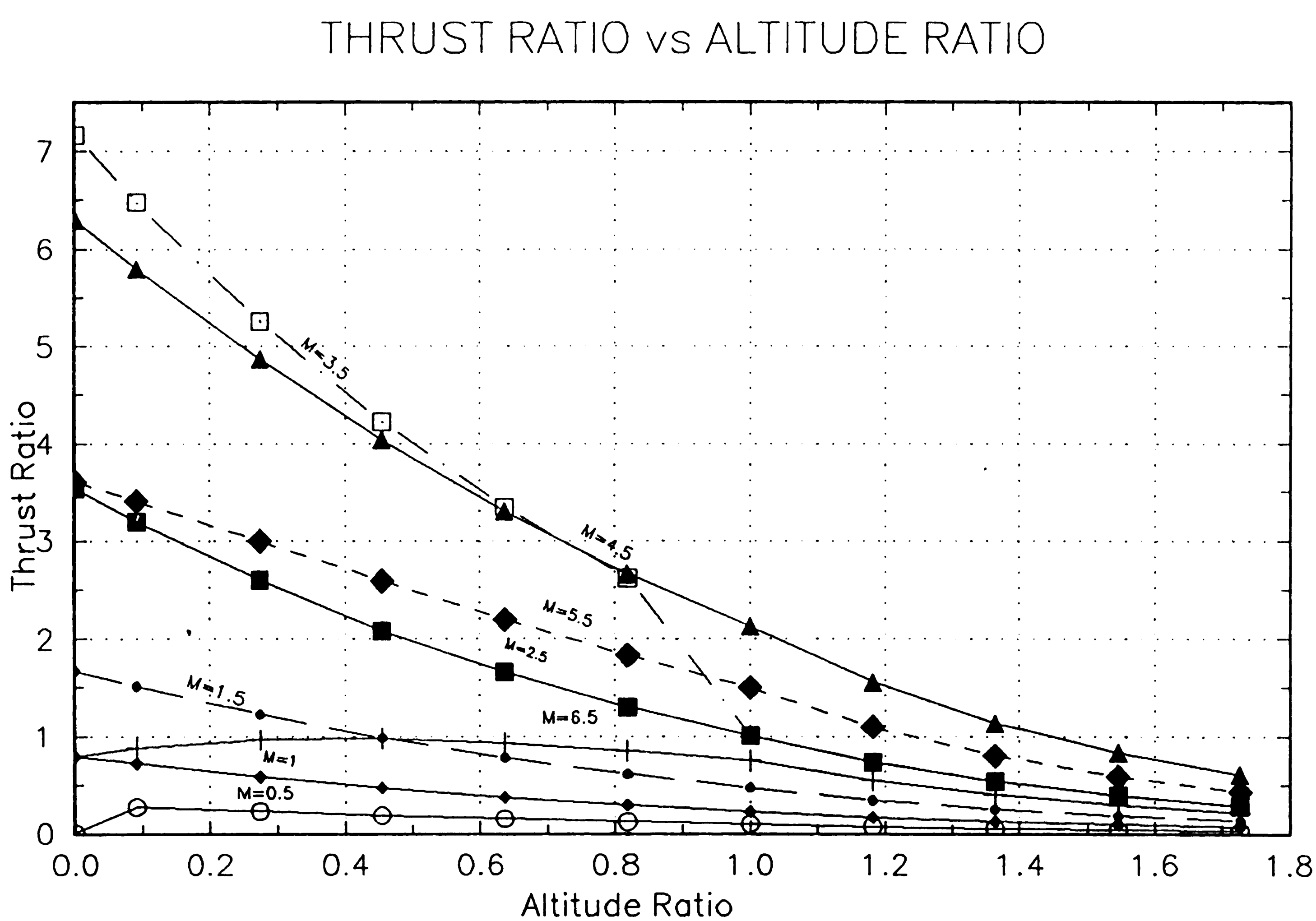

Figure 6.7 Comparison of thrust ratio verses altitude ratio with increase in Mach number

over a substantial range of altitude ratios. The curve for Mach 6.5 in Figure 6.7 represents this situation. Figure 6.6 also qualifies the conclusion where it may be noted that the thrust at Mach 6.5 has almost the same value for both sea level and that at 11000 meters.

b) For a given altitude ratio, the thrust ratio is directly proportional to the Mach number up to Mach 3. At Mach 3.5 between altitude ratios of 0.82 to 1.0, a sudden rise in the thrust ratio corresponds to the altitude range where the turbojet shuts down and ramjet alone takes over the engine operation (also see Figure 6.6).

c) The engine achieves the highest thrust ratio of 7.2 while operating at Mach 3.5 at sea level. At this Mach, higher thrust ratios are obtained up to the thrust ratio of 0.8. For altitudes ratios above 0.8 the engine produces maximum thrust ratios over the entire range of its operation while operating at Mach 4.5.

d) The convergence of all the curves close to altitude ratio 1.8 represents the engine operation at Mach ratio 3 as explained in Section 6.2.2.

6.2.4 Propulsive Efficiency

It can be seen from equation (4.49) that neglecting the fuel-air ratio for a fully expanded nozzle the propulsive efficiency may be defined as

$$\eta_p \approx \frac{2u_a/u_e}{1+u_a/u_e} \qquad (6.3)$$

Equation (6.3) indicates that large values of η_p are obtained as the ratio u_a/u_e approaches unity: i.e. when specific thrust is small.

Figure 6.8 shows the plots of specific thrust in [kN/kg/s], the velocity ratio ν and propulsive, thermal and overall efficiencies verses Mach number. The propulsive efficiency follows the trend as described by equation (6.3). In the low Mach region when only the turbojet operates η_p is lowest at Mach 0.1 where the velocity ratio is minimum. As the velocity ratio improves with the increasing Mach, η_p also rises until Mach 3.52 is reached where turbojet is shut down and sudden rise in specific thrust is observed for ramjet operation alone. Correspondingly η_p encounters a sudden drop and then gradual increase as the engine specific thrust deteriorates on higher Mach numbers.

6.3.5 Thermal Efficiency

The thermal efficiency was defined in equation (4.50) as follows

$$\eta_{th} = \frac{\left[(1+f)(u_e^2/2) - u_a^2/2\right]}{fQ_R}$$

with the assumption $(1+f) \approx 1$ it becomes

$$\eta_{th} = \frac{u_e^2 - u_a^2}{2fQ_R}$$

By introducing the velocity ratio $\nu = u_a/u_e$ the equation can be written as

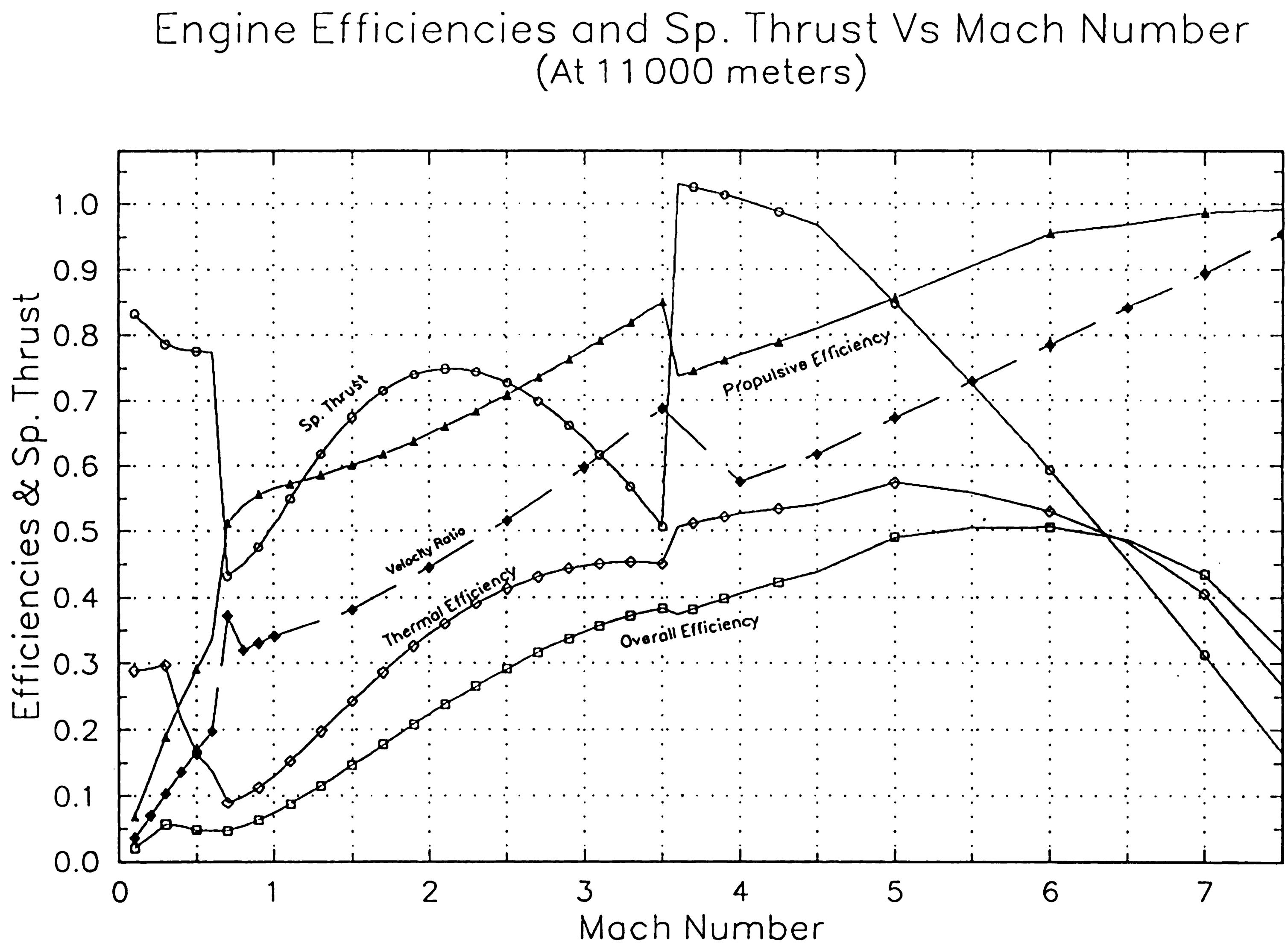

Figure 6.8 Plots of engine specific thrust, efficiencies and velocity ratio at 11000 meters

$$\eta_{th} = \frac{u_e^2\left(1-\nu^2\right)}{2fQ_R} \tag{6.4}$$

It can be seen from equation (6.4) that thermal efficiency η_{th} reduces to zero when $\nu = 1$: i.e. when $u_a = u_e$. That conclusion is logical because when $\nu = 1$, the engine develops no thrust and no energy is added to the fluid flowing through the engine. On the other hand when $\nu = 0$, the thermal efficiency of the engine depends entirely on the exhaust jet velocity and increases with u_e.

In Figure 6.8, the effect of ν is obvious on the thermal efficiency, and the curve follows an increasing and decreasing pattern of the specific curve as the velocity ratio affects the latter. The effect of ν on specific thrust has been discussed in Section 6.2.1.

6.2.6 Overall efficiency

From equation (4.51) it is seen that the overall Efficiency (η_o) is the product of the propulsive and the thermal efficiencies

$$\eta_o = \eta_p \eta_{th} = \frac{\tau\, u_1}{\dot{m}_f Q_R}$$

from equation (6.3) and for $f \ll 1$, it is reduced to

$$\eta_o = 2\eta_{th}\left(\frac{u_a/u_e}{1+u_a/u_e}\right)$$

$$\eta_o = 2\eta_{th}\left(\frac{\nu}{1+\nu}\right) \tag{6.5}$$

Equation (6.5) shows that the overall efficiency depends only on velocity ratio ν and on the thermal efficiency which depends somewhat on the velocity ratio.

From the Figure 6.8, the dependence of propulsive and overall efficiencies on the velocity ratio is evident. The propulsive efficiency follows almost the same trend with increase in Mach number as that of velocity ratio. Moreover, the propulsive and overall efficiencies fall to zero for $\nu = 0$ whereas, under this condition the thermal efficiency entirely depends on u_e. For $\nu = 1$, however, all the three efficiencies go down to zero.

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Summary

The concept of combining a turbojet engine with a ramjet engine in a single power plant for a high-speed aircraft is a subject of research and development in many countries today. Although in some cases the application of a turboramjet engine has been materialized in the form of prototype small aircraft, its large-scale commercial usage has not yet been witnessed.

The software for the turboramjet engine under study is developed to join the fundamental virtues of the turbojet and the ramjet in a single power plant for a supersonic aircraft that could achieve a flight Mach number over 5 and comfortably be used in civil aviation. This software is useful for an aircraft engine designer in the conceptual design phase for enabling the designer to simulate the flow conditions through the engine for the selected design parameters. As such, it may be regarded as a tool to reduce the decision time in reaching the final design of the engine. The built-in features of the software such as variable area inlet and nozzle, logic for mode of operation as per prevailing flight conditions, inclusion of maximum thermal limits criteria and minimum Mach number calculation at which to put ramjet in operation adds affinity to the actual aircraft engines. The design program of the software has a flexibility to select, as per designer's requirement, any combination of 14 different parameters important to the engine design. Through the main program, it is possible to simulate the engine operation for any Mach number and altitude of the designer's desire within air breathing range.

Simulation of the engine is possible over a large range of on-design and off-design flight conditions. However, for the purpose of demonstration a typical set of flight conditions at an altitude of 11000 meters and Mach number 2.5 was simulated. In the absence of any real turboramjet operational data, the results are compared with the turbojet and the ramjet individually and conclusions are drawn accordingly.

7.2 Conclusions

Some of the conclusions are as follows:

- For the selected design, the engine could achieve the maximum Mach number of 7.9; the limits may vary according to the design parameters selection.
- The engine could generate maximum total thrust of 800 kN at Mach 4.25, maximum specific thrust of 1.04 kN/kg at Mach 3.57, and the lowest TSFC at Mach 2.5.
- The joint operation of the turbojet and the ramjet was realized between Mach 0.64 and 3.57. The specific thrust, TSFC and the engine efficiencies in this range are comparable with a known example of an aircraft engine (Figure 7.1).
- As expected, a sudden rise in the engine thrust is observed when the turbojet shuts down and the ramjet alone assumes the operation at Mach 3.57. It is because of better operational characteristics of the ramjet at higher Mach numbers.
- The overall engine performance parameters follow the trends of a real aircraft engine.
- The changes in flow properties across the engine follow the standard one-dimensional flow rules and show trends in line with the actual aircraft engine.

Turboramjet Joint Operational Range
(11000 meters)

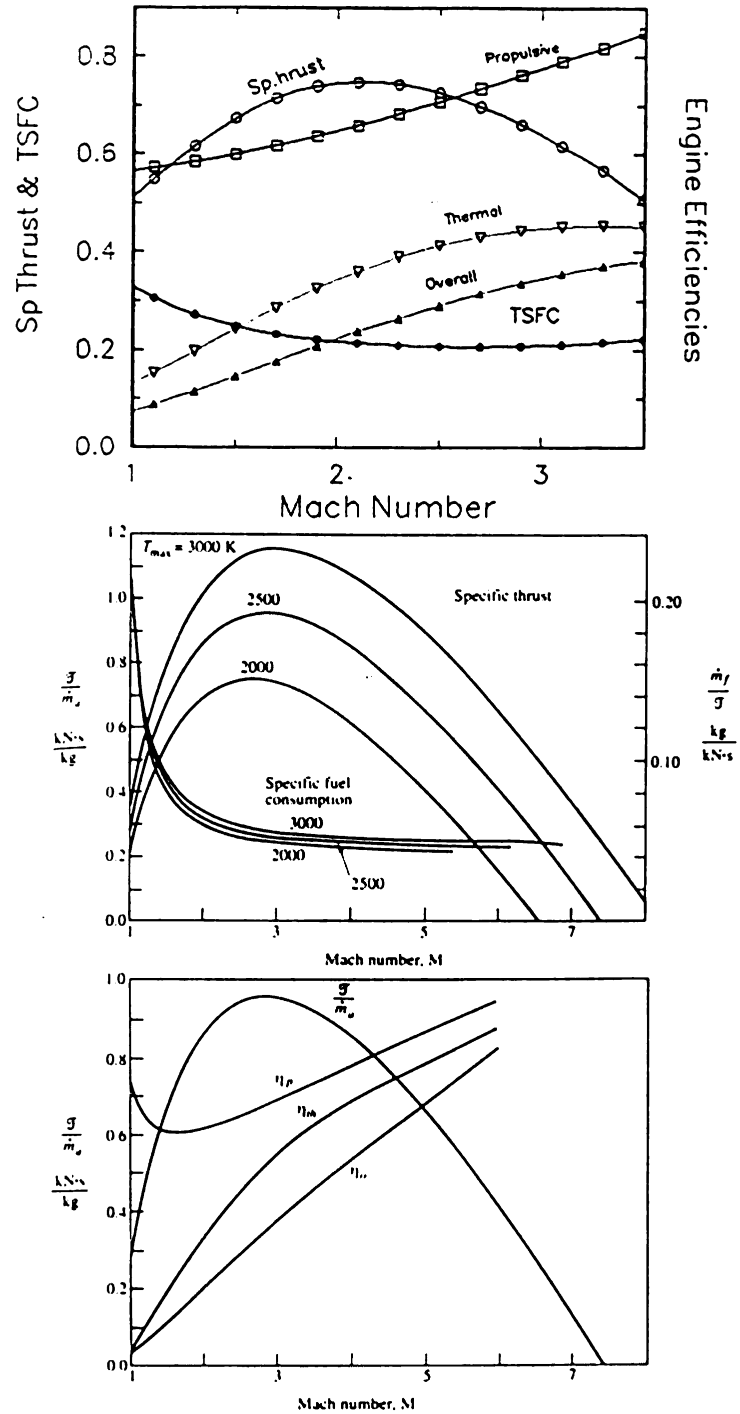

Figure 7.1 Comparison of a known ramjet performance parameters with the turboramjet in joint operation range [Ramjet polts taken from Hills and Peterson 1992]

- The results show that the software returns the logical results, is useful in understanding flow conditions across the engine, and helps in the conceptual design process of a turboramjet engine.
- The flexibility in both on-design and off-design operation in the software makes it possible to optimize the engine performance for any criteria of optimality; specific thrust, TSFC or the engine efficiencies.

7.3 Future Work

During the present development of the software, few simplifications were made following a rather conservative approach. Improvements in these areas can further enhance the versatility of the software. Some of these are identified as the areas of the future work.

- Instead of calculating the diffuser efficiency, a 4% stagnation pressure loss was introduced across the diffuser under all operating conditions. Since the diffuser efficiency depends strongly on the mass flow rate and the flight Mach number, it can not have a fixed value for all operating conditions. The criterion for obtaining diffuser efficiency may be evolved using its relationship between the stagnation pressure ratio and the Mach number.
- At present, the area allocation of the turbojet and the ramjet has been done by fixing 50% area at the diffuser exit to each of the two engines. Optimization for area allocation is required because the overall performance of the turboramjet engine is improved if a smaller but efficient turbojet at lower Mach numbers is coupled with the ramjet suitable for higher Mach numbers.

More area available to the ramjet can improve the thrust and limiting Mach of the engine.

- The bypass doors are kept fixed at 50% bypass ratio. This precludes the possibility of changing the mass flow to obtain the optimum advantage from the engine for a given flight condition. Presently the flow through the two engines entirely depends on the turbojet demand and the flight Mach number. If the bypass doors are capable of changing the bypass ratio mass flow may be changed through any of the two engines and performance can be improved for a given flight condition.
- The flow conditions within the compressor and the turbine are not presently addressed. Future work may include the effects on flow within the turbomachines that may affect the overall performance.
- Some study is required to include taping of the airflow for cooling and wind milling of the turbojet once it is shut down. The wind milling is required to keep positive pressure in the oil and fuel system, and restart the engine at low flight speeds or for coming in for landing.
- Study may be conducted to use some alternate fuel like liquid hydrogen instead of kerosene type. It can improves the thrust to weight ratio and in turn, the range of the aircraft.

APPENDICES

APPENDIX A

FUNCTIONS USED IN THE SOFTWARE

The following functions were use in the software developed for the turboramjet engine. Various relationships as discussed in Chapter 3 were used to build these functions.

A.1 FUNCTION TTKM (T, k, XMACH)

For the given value of *T*, *k*, and *M* this function returns the value of T_0 of the flow at a given station. Equation (3.12.1) is used as the basis.

A.2 FUNCTION PTKM (P, k, XMACH)

This function calculates the value of P_0 at a station for known *P*, *k* and *M* respectively. Equation (3.12.2) is used for this purpose.

A.3 FUNCTION DTKM (D, k, XMACH)

With the help of equation (3.12.3) this function returns the value of ρ_0 using ρ, *k*, and *M* as the input for a given station.

A.4 FUNCTION TKM (T0, k, XMACH)

By using equation (3.12.1) this function calculates for a given station the value of *T*. Inputs are T_0, *k*, and *M*.

A.5 FUNCTION PKM (P0, k, XMACH)

If the values of P_0, *k*, and *M* are given at some point, *P* is calculated by this function with the help of equation (3.12.2).

A.6 FUNCTION DKM (D0, k, XMACH)

For the known values of ρ_0, k, and M at some point, this function returns the corresponding value of ρ. Equation (3.12.3) is used for the calculation.

A.7 FUNCTION XMACH (TM, TT, PT, k, R, A)

For the flow condition where mass flow rate, T_0,P_0, k, R and duct area are known, the flow Mach number is found with the help of this function. The continuity equation is used for its calculation. Derivation of the function appears in appendix A.

A.8 FUNCTION CPK (k, R)

Equation (3.5.1) is used in this function in order to calculate value of c_p for known value of k and R. The unit returned for c_p is [kJ/kg ^{0}K].

A.9 FUNCTION AR (XMACH, k)

This function returns the value of Critical Area Ratio for the flow for given values of Mach number and k. Equation (3.12.6) is used for the calculation.

A.10 The IGPROPL software

A function-based software named IGPROPL was provided by the advisor. The software has the capability to calculate any of the thermodynamic properties for a variety of ideal gases provided any two properties at the state in question are known. However, in the code for the combination engine its use is restricted to the minimum and direct use of basic equations (section 3.5 through 3.15) is preferred solely for the purpose of the understanding of the process involved. The use of software IGPROPL in the code for calculation of the flow properties may reduce the length of the program and it can easily be incorporated at any time.

APPENDIX B

DERIVATION OF THE FUNCTION 'XMACH'

This Appendix explains the procedure adopted in formulating the function XMACH (Appendix A). The function evaluates the exit Mach number of a subsonic incompressible flow as it passes through a varying area duct under given conditions.

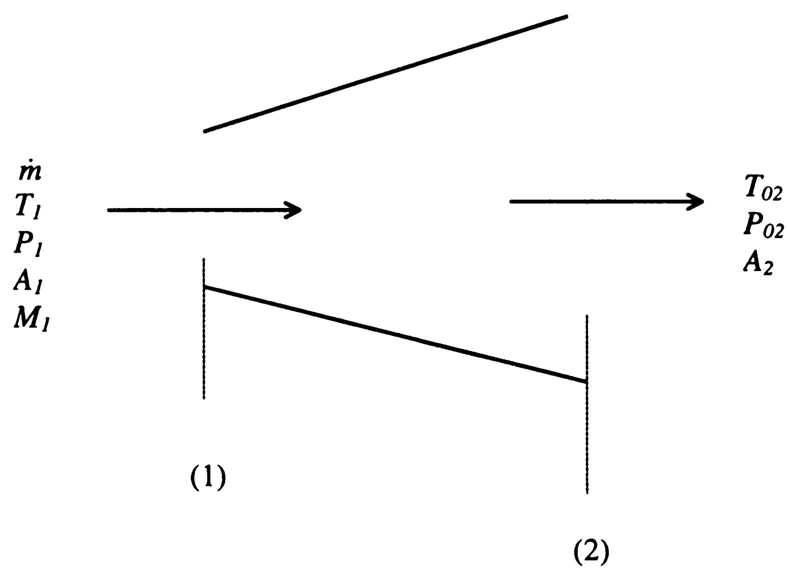

Figure B.1 Mach number calculations for a flow through a divergent passage

The known parameters are $\dot{m}$, T_1, P_1, A_1, M_1, T_{02}, P_{02}, A_2, and k, parameter M_2 is required to be calculated.

Procedure

From continuity equation

$$\dot{m} = \rho_1 u_1 A_1 = \rho_2 u_2 A_2 \qquad \text{(B-1)}$$

Using Ideal gas equation it becomes

$$\frac{P_1}{RT_1} u_1 A_1 = \frac{P_2}{RT_2} u_2 A_2 \tag{B-2}$$

it may be represented as

$$\frac{kP_1}{\sqrt{kRT_1}} \frac{u_1}{\sqrt{kRT_1}} A_1 = \frac{kP_2}{\sqrt{kRT_2}} \frac{u_2}{\sqrt{kRT_2}} A_2$$

From the definition of the Mach number, it can be written as:

$$\frac{kP_1}{\sqrt{kRT_1}} M_1 A_1 = \frac{kP_2}{\sqrt{kRT_2}} M_2 A_2 \tag{B-3}$$

$$\frac{kP_1}{\sqrt{kRT_1}} M_1 A_1 = \frac{kP_{02}}{\sqrt{kR\frac{T_2}{T_{02}}T_{02}}} \frac{P_2}{P_{02}} M_2 A_2$$

$$\frac{kP_1}{\sqrt{kRT_1}} M_1 A_1 = \frac{kP_{02}}{\sqrt{kRT_{02}}} \frac{\sqrt{\frac{T_{02}}{T_2}}}{\left(\frac{P_{02}}{P_2}\right)} M_2 A_2$$

$$\frac{kP_1}{\sqrt{kRT_1}} M_1 A_1 = \frac{kP_{02}}{\sqrt{kRT_{02}}} \frac{\left(1+\frac{k-1}{2}M_2^2\right)^{1/2}}{\left(1+\frac{k-1}{2}M_2^2\right)^{k/k-1}} (M_2 A_2)$$

$$\frac{kP_1}{\sqrt{kRT_1}} M_1 A_1 = \frac{kP_{02} M_2 A_2}{\sqrt{kRT_{02}}} \left[1+\frac{k-1}{2}M_2^2\right]^{\left(\frac{1}{2}-\frac{k}{k-1}\right)}$$

Rearranging and simplifying

$$\frac{kP_1}{\sqrt{kRT_1}} \cdot M_1 A_1 \times \frac{\sqrt{kRT_{02}}}{kP_{02}A_2} = M_2\left(1+\frac{k-1}{2}M_2^2\right)^{\frac{-(k+1)}{2(k-1)}}$$

$$\frac{P_1}{\sqrt{T_1}} \cdot M_1 A_1 \times \frac{\sqrt{T_{02}}}{P_{02}A_2} = M_2\left(1+\frac{k-1}{2}M_2^2\right)^{\frac{-(k+1)}{2(k-1)}}$$

$$\left[\frac{P_1}{P_{02}}\sqrt{\frac{T_{02}}{T_{01}}}\frac{A_1 M_1}{A_2 M_2}\right]^{\frac{-2(k-1)}{k+1}} = \left(1+\frac{k-1}{2}M_2^2\right)$$

$$M_2^2 = \left(\frac{2}{k-1}\right)\left[\left\{\frac{P_1}{P_{02}}\sqrt{\frac{T_{02}}{T_{01}}}\frac{A_1 M_1}{A_2 M_2}\right\}^{\frac{-2(k-1)}{k+1}} - 1\right]$$

$$M_2 = \sqrt{\left(\frac{2}{k-1}\right)\left[\left\{\frac{P_1}{P_{02}}\sqrt{\frac{T_{02}}{T_{01}}}\frac{A_1 M_1}{A_2 M_2}\right\}^{\frac{-2(k-1)}{k+1}} - 1\right]} \qquad \text{(B-4)}$$

The parameter M_2 appears on both sides of the equation (B-4) so the solution is obtained through an iterative process.

Let the Mach (M_2) on the Right Hand side be M_b and that on the Left Hand Side be M_a. At first M_b is put equal to zero and value of M_a is calculated. Till the time the absolute difference of (M_a-M_b) appears greater than a value (say ε), Mb is replace by Ma. When the difference converges within ε, the exit Mach (M_2) is obtained by taking the average of M_a and M_b.

APPENDIX C

CALCULATION OF MINIMUM MACH NUMBER FOR RAMJET OPERATION

The schematic diagram of a typical ramjet is shown in Figure 2.4. The flow in a ramjet engine suffers stagnation pressure losses as it flows through the engine. The Mach number at which minimum TSFC occurs and that at which TSFC becomes infinite ($\tau \to 0$), are dependent upon the severity of the component stagnation pressure losses. The compression process (a) to (02) is not isentropic hence the stagnation pressure at the end of the process is lower than it would be if the compression were isentropic.

The performance of diffusers may be characterized by a stagnation pressure ratio r_d defined by

$$r_d = \frac{P_{02}}{P_{0a}} \tag{C-1}$$

Similarly, stagnation pressure ratios can be defined for combustors (r_c) and nozzles (r_n) as follows:

$$r_c = \frac{P_{04}}{P_{02}} \tag{C-2}$$

$$r_n = \frac{P_{06}}{P_{04}} \tag{C-3}$$

The overall stagnation pressure ratio is

$$\frac{P_{06}}{P_{0a}} = r_d \, r_c \, r_n \tag{C-4}$$

The actual exhaust pressure P_e and P_6 may not equal the ambient pressure P_a. However, for constant k, the exhaust Mach number may be written as

$$M_e^2 = \frac{2}{k-1}\left[\left(1+\frac{k-1}{2}M^2\right)\left(\frac{P_{06}}{P_{0a}}\frac{P_a}{P_e}\right)^{(k-1)/k} - 1\right]$$

In terms of component pressure ratios

$$M_e^2 = \frac{2}{k-1}\left[\left(1+\frac{k-1}{2}M^2\right)\left(r_d\, r_c\, r_n \frac{P_a}{P_e}\right)^{(k-1)/k} - 1\right] \qquad \text{(C-5)}$$

If heat transfer from the engine per unit mass of fluid is assumed negligible, then the exhaust velocity in terms of exhaust stagnation temperature is given by

$$u_e = M_e \sqrt{kRT_{04} \Big/ \left(1+\frac{k-1}{2}M_e^2\right)} \qquad \text{(C-6)}$$

Since the irreversibilities have no effect on stagnation temperature throughout the engine, the fuel-air ratio necessary to produce the desired T_{04} is given by:

$$f = \frac{(T_{04}/T_{0a})-1}{(\eta_b\, Q_R / c_p\, T_{0a}) - (T_{04}/T_{0a})}$$

where η_b is the combustion efficiency and $\eta_b\, Q_R$ is the actual heat release per unit mass of fuel. The thrust per unit airflow rate then becomes

$$\frac{\tau}{\dot{m}_a} = [(1+f)u_e - u] + \frac{1}{\dot{m}_a}(P_e - P_a)A_e$$

by using equations (C-5) and (C-6) the above equation can be reduced into the form

$$\frac{\tau}{\dot{m}_a} = (1 = f)\sqrt{\frac{2kRT_{04}\,(m-1)}{(k-1)m}} - M\sqrt{kRT_a} + \frac{P_e A_e}{\dot{m}_a}\left(1-\frac{P_a}{P_e}\right) \qquad \text{(C-7)}$$

in that

$$m = \left(1 + \frac{k-1}{2} M^2\right)\left(r_d \, r_c \, r_n \, \frac{P_a}{P_e}\right)^{(k-1)/k} \qquad \text{(C-8)}$$

For a typical ramjet engine Figure B-1 shows the plots of the specific thrust and the specific fuel consumption as a function of flight Mach number and peak temperatures. The Figure B-1 indicates that the slope of the specific thrust curve is positive below Mach 2.6 and if extended, will have a point of intersection on the Mach number axis some where in the subsonic range. The study in this appendix aims at determining the point where thrust curve intersects the Mach number axis in the subsonic range. It corresponds to the minimum Mach below which the ramjet will not produce any positive thrust and should not be put in to operation below that Mach.

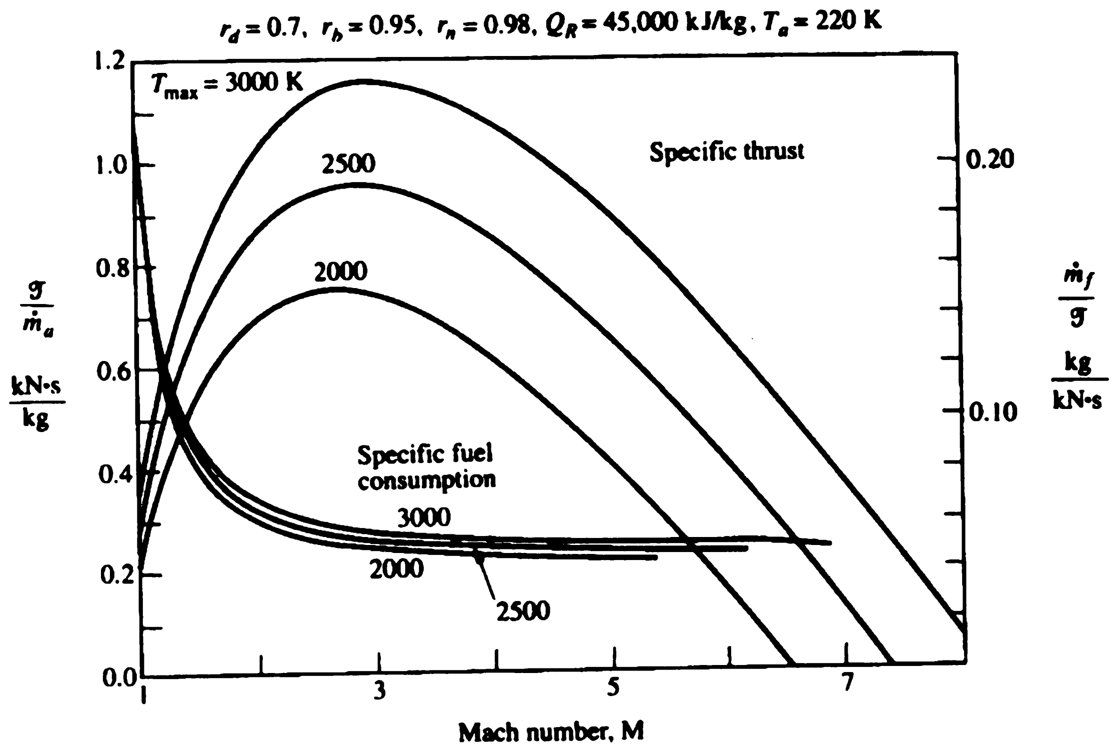

Figure C.1 Plots of a typical ramjet specific thrust and fuel consumption [Curtsey Hills and Peterson 1992]

After having obtained equation (C-7) it is now possible to find out the limiting Mach number below which the ramjet must not be operated in the combination engine. For this analysis the following assumptions are made:

a) The ramjet nozzle is fully expanded i.e. $P_e = P_a$

b) The mass of fuel consumed is negligible as compared with the mass of air

Considering the Left Hand Side of equation (C-7), it can be observed that in order to have positive thrust the following condition must be satisfied

$$(1 = f)\sqrt{\frac{2kRT_{04}\ (m-1)}{(k-1)m}} \ \rangle \ M\sqrt{kRT_a} + \frac{P_e A_e}{\dot{m}_a}\left(1 - \frac{P_a}{P_e}\right) \qquad \text{(C-9)}$$

if the above assumptions are incorporated in equation (C-9), it becomes

$$\sqrt{\frac{2kRT_{04}\ (m-1)}{(k-1)m}} \ \rangle \ M\sqrt{kRT_a} \qquad \text{(C-10)}$$

or

$$\frac{2kRT_{04}\ (m-1)}{(k-1)m} \ \rangle \ M^2\left(kRT_a\right) \qquad \text{(C-11)}$$

Let

$$\frac{2kRT_{04}}{(k-1)} = X_1$$

$$kRT_a = X_2$$

$$\left(r_d\ r_c\ r_n\right)^{(k-1)/k} = X_3$$

then equation (C-11) can be written in the form

$$X_1\left(\frac{m-1}{m}\right) \ \rangle \ M^2 X_2$$

$$X_1(m-1) \ \rangle \ mM^2 X_2 \qquad \text{(C-12)}$$

and m becomes

$$m = X_3\left(1 + \frac{k-1}{2}M^2\right) \tag{C-13}$$

putting equation (C-12) in (C-13) it is evident that

$$X_1\left[X_3\left(1 + \frac{k-1}{2}M^2\right) - 1\right] \;\rangle\; M^2\left[X_3\left(1 + \frac{k-1}{2}M^2\right)X_2\right]$$

or

$$\left[X_1X_3 + X_1X_3\left(\frac{k-1}{2}\right)M^2 - X_1\right] \;\rangle\; \left[X_2X_3M^2 + X_2X_3\left(\frac{k-1}{2}\right)M^4\right]$$

for limiting case (zero thrust) the two sides should be equal

thus $$X_1X_3 - X_1 + X_1X_3\left(\frac{k-1}{2}\right)M^2 - X_2X_3M^2 - X_2X_3\left(\frac{k-1}{2}\right)M^4 = 0$$

or $$X_2X_3\left(\frac{k-1}{2}\right)M^4 - \left[X_1X_3\left(\frac{k-1}{2}\right) - X_2X_3\right]M^2 - X_1(X_3 - 1) = 0$$

taking X_3 common

$$X_2\left(\frac{k-1}{2}\right)M^4 - \left[X_1\left(\frac{k-1}{2}\right) - X_2\right]M^2 - X_1\left(1 - \frac{1}{X_3}\right) = 0 \tag{C-14}$$

Equation (C-14) is a quadratic of the form

$$aY^2 + bY + c = 0$$

where

$$Y = M^2$$

$$a = X_2\left(\frac{k-1}{2}\right)$$

$$b = -\left[X_1\left(\frac{k-1}{2}\right) - X_2\right]$$

and

$$c = -X_1\left(1 - \frac{1}{X_3}\right)$$

The roots of Y can be obtained from quadratic formula

$$Y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and M is found out from

$$M = \pm\sqrt{Y} \qquad \text{(C-15)}$$

The value of Y and M used should be:

(a) A positive number

(b) Smaller in magnitude out of the two values

The Mach number so obtained represents the minimum Mach at which the ramjet operation is possible under the prevailing flight conditions.

APPENDIX D

EVALUATION OF FLOW VELOCITY AND PRESSURE AT MIXER EXIT

The procedure of calculating flow velocity and the static pressure at the exit of the combination engine mixer unit is elaborated in this Appendix. The mixer has a diameter equal to the outer diameter of the engine and receives flow from the both turbojet and the ramjet. Its mixes the two flows before passing through the exhaust nozzle. For the purpose of calculating the flow conditions at mixer exit the flow properties from the turbojet are represented by subscript "a", that from the ramjet by subscript "b" and at the mixer exits by subscript "c".

For the given conditions various laws pertinent to steady flow may be represented as follows:

First Law of Thermodynamics

$$\dot{m}_a\left(h_a+\frac{u_a^2}{2}\right)+\dot{m}_b\left(h_b+\frac{u_b^2}{2}\right)=\dot{m}_c\left(h_c+\frac{u_c^2}{2}\right) \tag{D-1}$$

Unknowns are h_c and u_c

Conservation of Mass

$$\begin{aligned}\dot{m}_c &= \dot{m}_a+\dot{m}_b\\ \rho\, uA)_c &= \rho\, uA)_a+\rho\, uA)_b\end{aligned} \tag{D-2}$$

Unknowns are u_c and ρ_c

Momentum Integral

$$\dot{m}_a u_a+\dot{m}_b u_b-\dot{m}_c u_c = A_a P_a + A_b P_b - A_c P_c \tag{D-3}$$

Unknowns are u_c and P_c

Ideal Gas Relation

$$\rho = \frac{P}{RT}$$

and

$$h = c_p\,T$$

By converting ρ and h in terms of P and T only three unknowns (P_c, T_c, *and* u_c) are left against three equations (1 thru 3), which can be solved simultaneously.

Substituting for ρ and h the first two equations take the form:

$$\dot{m}_a\left(2h_a + u_a^2\right) + \dot{m}_a\left(2h_b + u_b^2\right) = \dot{m}_a\left(2C_pT_c + u_c^2\right) \quad \text{(D-4)}$$

and

$$RT_c\rho_a u_a A_a + RT_c\rho_b u_b A_b - P_c\rho_c u_c A_c = 0 \quad \text{(D-5)}$$

from equations (D-2) and (D-5)

$$T_c = \frac{P_c u_c A_c}{R\left(\dot{m}_a + \dot{m}_b\right)} = \frac{P_c u_c A_c}{R\left(\dot{m}_c\right)} \quad \text{(D-6)}$$

putting equation (D-6) in (D-4) gives

$$R\left[\dot{m}_a\left(2h_a + u_a^2\right) + \dot{m}_a\left(2h_b + u_b^2\right)\right] = \left(2C_p P_c u_c A_c + R\dot{m}_c u_c^2\right) \quad \text{(D-7)}$$

arranging equation (D-3) for P_c

$$P_c = \frac{\dot{m}_c u_c - \left(\dot{m}_a u_a + \dot{m}_b u_b - A_b P_b - A_a P_a\right)}{A_c} \quad \text{(D-8)}$$

By putting equation (D-8) in (D-7) and rearranging a quadratic in u_c is obtained

$$\begin{aligned}\left(2C_p + R\right)\dot{m}_c u_c^2 - 2C_p\left(\dot{m}_a u_a + \dot{m}_b u_b - A_b P_b - A_a P_a\right)u_c \\ - R\left[\dot{m}_a\left(2h_a + u_a^2\right) + \dot{m}_a\left(2h_b + u_b^2\right)\right] = 0\end{aligned} \quad \text{(D-9)}$$

Solution of equation (D-9) returns two roots, the smaller positive root represents the mixer exit velocity (u_c) in [*m/sec*].

The mixer exit static pressure (P_c) is obtained by putting the value of u_c in equation (D-8). The static pressure P_c is calculated in [*Pascal*].

It may be noted that subscripts *a,b* and *c* correspond to stations (11), (17), and (18) respectively as appeared in Figure 3.1.

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