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Learning to teach mathematics: Preservice teachers, their collaborating teachers and instructional contexts

presented by

Jian Wang

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Major professor

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LEARNING TO TEACH MATHEMATICS: PRESERVICE TEACHERS, THEIR COLLABORATING TEACHERS AND INSTRUCTIONAL CONTEXTS

By

Jian Wang

A DISSERTATION

Submitted to
Michigan State University
for the degree of

DOCTOR OF PHILOSOPHY

Department of Teacher Education

1998

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ABSTRACT

LEARNING TO TEACH MATHEMATICS: PRESERVICE TEACHERS, THEIR COLLABORATING TEACHERS AND INSTRUCTIONAL CONTEXTS

By

Jian Wang

This study investigates four elementary preservice teachers' learning to teach mathematics with collaborating teachers in a year-long internship. The project poses four questions. What do they learn about mathematics instruction? What is the influence of their collaborating teachers on their learning to teach? How do their collaborating teachers influence their learning? How do their instructional contexts shape what they learn?

These questions arise from a situation that many reform-oriented teacher education programs face. Mathematics education reformers have been pushing mathematics instruction toward a constructivist direction that is quite different from the prevailing mathematics teaching practice in schools. Teacher education reformers want to help their students learn to teach in new ways by helping them develop constructivist ideas and providing them longer internships that feature a gradual transition into teaching and close, supportive relationships with experienced teachers. However, many reform-oriented teacher education programs have to send their students into internships with collaborating teachers who may not necessarily teach in the way encouraged by mathematics education reformers. Research suggests that different instructional contexts have different impacts on the quality of teaching.

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My analysis leads to several findings about the preservice teachers' learning. First, although bringing many constructivist ideas into their internship, three of these preservice teachers were unable to retain all these ideas or articulate them into their practice. Instead, they developed some ideas that were contradictory to their program's expectations. Their mathematics teaching did not resemble the standards that mathematics education reformers are embracing. Second, comparing what they learned with what their collaborating teachers thought and did in mathematics instruction, I come to the conclusion that these preservice teachers actually moved closer to their collaborating teachers at both conceptual and practical levels. The ideas they shared with their collaborating teachers were able to be retained and practiced in their teaching. The ideas they failed to share with their collaborating teachers disappeared or were not enacted in their practice. The new ideas they developed in their internship were often those their collaborating teachers held and practiced. Third, the expectations both preservice and collaborating teachers had for their roles in the internship had a strong impact on what they were able to do in their collaboration. The kind of collaboration they developed, in turn, contributed to the chances for and the quality of these preservice teachers' learning. Fourth, different instructional contexts in their internship also shaped what they were able to learn. The culture of teaching in each school was different and not always supportive of their constructivist ideas. Schools did not always offer the specific images of teaching that these ideas implied. Curriculum guidelines and resources available to them were neither consistent with the constructivist vision nor specific about pedagogy. Finally, the students they taught also shaped their learning to teach.

To Weiling and Yang

ACKNOWLEDGMENTS

As I completed this dissertation work and started to enjoy this fruit of my academic career, many gardeners, who helped and nurtured its growth along the way, came to my mind. It is impossible for me to enjoy this accomplishment without giving credit to each of them.

I would first like to give my gratitude to my academic adviser, Lynn Paine. It was Lynn who saved my intellectual life from being trapped in the political crisis in China and welcomed me to the field of teacher education. Over the passed six years, she has been stimulating, advising, nurturing and challenging me through each step of my academic career, my dissertation data collection and writing. Moreover, it was Lynn who had been teaching and supporting me to learn how to live, work and research in a world I was not familiar with. I feel greatly indebted to what she has done for me.

Each of my committee members made important contributions to my dissertation work one way or the other. This work would not have been done without the academic soils and fertilizer Sharon Feiman-Nemser has been supplying. The topic of my dissertation grew out of my four years research assistant work with her in a cross-national mentoring study at the National Center for Research on Teacher Learning. It was also inspired by her pioneering studies in the field and enriched by my countless formal and informal discussions with her. She created some important opportunities for me

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to pursue my ideas and relate them to the broader professional world of teacher education.

Helen Featherstone supported me to develop my practical experience about U.S. elementary mathematics teaching, which became an important part of my dissertation. It was through a field experience course I took with her that I was able to find schools and teachers to observe and work with. Thus, I was able to put my conceptions into the real world and examine them. She was patient enough to wait for my immature ideas to grow into a final report. She gave me encouragement and constructive suggestions on how to improve it into a part of my dissertation work.

Comparative analysis within a case and across cases is the basic research method I used for my data analysis. It was through my independent study of comparative research methods in education with Jack Schwille that I started to think carefully about comparison as a research method to analyze qualitative data. It was Jack who helped me establish an important foundation for my dissertation analysis and created several opportunities for me to discuss my ideas with international teacher education audiences.

From a graduate course on educational policy I took with Brian DeLany, I learned how to look at mentoring and learning to teach from a policy perspective. Brian helped me greatly in considering the policy implications of my research findings. Moreover, without his kind sponsorship and continuous encouragement, it would have been impossible for me to keep pursuing in my professional education.

I would also like to thank my colleagues, Sharon Schwille, Neli Wolf, Martial Dembele and Liping Ma, in the cross-national mentoring project at the National Center

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for Research on Teacher Learning, This group supported me in many ways with my dissertation work. Our weekly conversations helped me stay on track and pushed my ideas further. As the last one in this group to finish their dissertation, I was able to be inspired and benefited continuously from their unique endeavors and experiences in thinking about and approaching their dissertations on teacher learning and teaching.

I would not have been able to complete this work without the love and support of my mother, Shuyan Wu. My mother has been a strong support for my pursuing an academic career and my raising a family. As I juggle my multiple roles of collecting data, writing draft, teaching undergraduate course and taking care of my family, she flew thousands of miles from China to come to my help.

Finally I would like to thank my wife, Weiling Yang, and my son, Yang, for their endless patience. Yang showed a strong interest in what I am doing and helped me in transcribing and understanding some classroom mathematics instruction with his first hand learning experience in a U. S. elementary school. Weiling's steadfast love, encouragement and support have sustained me throughout my academic career. She took many of my responsibilities as a parent in taking care of our family and educating our child, especially during the time it took to complete this dissertation. I can never thank her enough for what she has done for my endeavor in pursuing an academic career.

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Chapter 1

INTRODUCTION: RESEARCH QUESTIONS AND CONCEPTUALIZATION

Two Equivalent Fraction Units

Martha and Louis, both elementary preservice teachers from the same five-year teacher education program, were doing their internship with their collaborating teachers at the fifth grade level in two different elementary schools. Near the end of their internship, each decided to teach an equivalent fraction unit for the mathematics lead teaching unit required by their teacher preparation program. Martha taught her first lesson in her equivalent fraction unit with three instructional tasks for her students. She first asked students to define the term, equivalent, with real life examples and predict the meaning of equivalent fraction based upon their ideas of equivalent. Then Martha grouped her students into pairs and asked each pair to make three same-size hexagons with pattern blocks. Each hexagon, she required, needed to be made by more than two pattern blocks of the same color and shape, and different pattern blocks needed to be used for different hexagons. She asked her students to flip these hexagons with each other to further develop or modify their previous predictions about the definition of equivalent fraction. In the last 15 minutes of her lesson, Martha asked some groups to show how their hexagons were similar to or different from each other and how these similarities and differences helped them define the meaning of equiva

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meaning of equivalent fractions. After each group's presentation, Martha encouraged other students to support or challenge the idea presented and justify comments.

In her remaining lessons of this unit, Martha was able to retain this way of teaching. She often started her new lessons with a fraction idea her students had left with in the previous class. Then she would continuously push her students to develop a deeper understanding about equivalent fractions by getting students working in groups and in the whole class discussion and by asking them to form ideas, show their ideas to each other and then prove or disprove these ideas. During this unit, Martha gradually pushed her students to understand the concept of equivalent fraction, as she said by moving "from concrete, to pictorial and then to symbolic."

Louis developed two kinds of lessons in his equivalent fraction unit. In his first lesson, he designed the following tasks for his students. First, he required his students to use their knowledge of fractions to represent the relationship between a dollar and quarter. After his students come up with different answers, like 4/1, 1/4, 1/100, 1/5, 5/100 and 5/20, Louis began to push his students to see conflicts between some of their representations and the concrete meaning of money, such as representing a dollar and a quarter with 1 /100. He also encouraged his students to challenge each other's representations and come up with a more reasonable representation that all the students agreed upon. This kind of learning activity continued to the end of his first class.

In the rest of his lessons in the unit, Louis's way of teaching totally changed. Sometimes, he spent about the first 10 minutes of his lesson directly telling his students a concept or rule and then asking students to practice some problems related to the concept and rule. Sometimes he used these first 10 minutes to ask a few students what they thought about a

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thought about a particular concept. He would either confirm the right answers that students gave or give the class the right answer when student answers were wrong. Unlike what he did in his first lesson, Louis never required his students to explain their answers nor challenged them to question and support each other's answer in these classes. Most of his class time was spent on students individually practicing the problems and the skills that he illustrated or confirmed in these remaining lessons.

The practices Martha and Louis developed in the above units were clearly different. Their different ways of teaching raise several questions about their mathematics teaching practice and how they came to these. How do we explain what these two preservice teachers were able to do in their mathematics lessons? What kinds of thinking pushed them to teach in the ways described? What were the external factors that influenced their way of teaching. How do such influences happen? These become extremely important for us to explore when we situate their mathematics teaching practice and these questions about their learning to teach in the broad context of current mathematics education and teacher education reforms. In this chapter, I lay out the conceptualization of the study that these questions prompt, identify my research questions and consider prior research related to the inquiry.

Conceptualization of My Dissertation Study

Mathematics education reform and learning to teach

In his work, *The Child and Curriculum,* Dewey (1990) proposes three kinds of ideas of teaching children. The first is a position that considers classified subject knowledge, facts and skills as absolute and unchangeable truth and fills the child with these impartial and objective facts "without reference to their place in one's own experience" (p 184

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experience" (p.184). Literature in mathematics education defines this kind of teaching as prevailing mathematics instruction that features "telling" followed by repetitious practice (Smith III, 1996). This kind of teaching often results in the unsatisfactory mathematics performance of American students at both national (Davis & Hersh, 1981; Peterson, 1988; Schoenfield, 1985) and international levels (Husen, 1967; Stevenson & Stigler, 1992; Travers & Westbury, 1989; Schmidt, 1996). Such a teaching practice reflects an absolutist vision of mathematics, mathematics learning and teaching. Under this vision, mathematics knowledge is regarded as a collection of infallible facts, rules, formulas and skills, and mathematics learning as only retaining these facts, rules, formulas and practicing the skills (Burns, 1986; Romberg, 1992). To teach mathematics often means to tell or illustrate to students the infallible facts, rules, formulas and skills and then reinforce them through practice (Smith III, 1996). Teachers with such a teaching practice often function as judges and sources of mathematics knowledge and teach students the procedures and algorithms to manipulate numbers and symbols individually without helping them understand the meaning of symbolic representations (Goodlad, 1984; Stodolsky, 1987).

Dewey's second category of teaching reflects the vision of child self-realization. From this position, mathematics knowledge is regarded as personal meaning that an individual learner constructs (von Glaserfeld, 1985). Children's mathematics learning is considered as ideal and "it alone furnishes the standard" (Dewey, 1990, p. 187). Mathematics teaching is to focus on the children's individual construction of mathematics knowledge and the teacher's role in teaching mathematics is to facilitate and stimulate individual learners' learning (Kush & Ball, 1986). This approach to mathematics teac

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mathematics teaching has been criticized as creating "the risk of ignoring the importance of mathematics convention. Conventions that must be learned as they are used in our society if they are to serve as powerful tools for mathematical thinking and communication" (Putnam, Lampert, & Peterson, 1990, p.135).

The third model proposed by Dewey (1990) is a constructivist position which looks at knowledge, learning and teaching in a different way from both the absolutist and child self-realization models. It is also a model that mathematics education reformers have been pushing to improve the quality of mathematics education (National Commission on Excellence in Education, 1983; National Council of Teachers of Mathematics, 1989; National Research Council, 1989).

First, under this constructivist vision, mathematics knowledge is a cultural artifact of human beings that is produced, shared and transformed by individuals and groups and it is fallible, growing and changing (Leinhardt, 1992; Romberg, 1992). Its central activities are to carry out mathematical reasoning, use mathematics models to represent our physical and social realities, and apply our mathematics knowledge to solve real world problems (Putnam, Lampert, & Peterson, 1990).

Second, mathematics learning under this vision reflects three features. In learning mathematics, individual learners need to actively participate in central mathematics activities, and properly use their prior knowledge and personal experiences to construct and make sense of mathematical ideas (Cobb, 1994; Noddings, 1985; Resnick, 1983). In learning mathematics, learners need to continuously communicate, prove and examine their ideas among members of groups because this mathematics knowledge shared and developed in the collaborative work is greater than the knowledge constructed by any individual member (Leinhar

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member (Leinhardt, 1992). Mathematics learners need to be assisted and coached to the optimal level of their learning, a point learners would find hard to reach by themselves without support (Cobb, 1994; Vygotsky, 1978).

Third, according to this vision, the contents of mathematics instruction should reflect three aspects (Ball, 1989). It involves substantial mathematics knowledge and skills. It also includes mathematics knowledge about the mathematics, such as how mathematical ideas are formed, justified, validated and generalized and how this knowledge is related to other subjects and the outside world. Moreover, it encompasses disposition toward mathematics, to encourage passions about mathematics. Teachers with a constructivist vision are facilitators, challengers, assistants and organizers of students' active learning, sharing and examining of mathematical ideas as well as students' mathematics problem solving (National Council of Teachers of Mathematics, 1991).

Research (Ball, 1990; Devaney, 1983/1984; Lampert, 1986; Resnick, 1983) assumes that, to teach in such a way, teachers are not only required to have a conceptual understanding of mathematics and know why such an understanding is important. They also need to know how to help students gain that understanding. Thus, it is important for teacher educators to find ways to educate their student teachers to learn to teach in this way. Martha and Louis were preservice teachers from the same teacher education program, one that actively encouraged and supported them to develop such a constructivist conceptual understanding of mathematics and mathematics instruction.

Teacher education reform and learning to teach

In traditional teacher education programs, preservice teachers are unable to successfully develop these constructive conceptions and better understand the relationship betwe

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relationship between these beliefs of teaching and learning and real teaching situations (Kennedy, 1991). One of the important reasons is that these programs fail to focus on and find effective strategies of reducing the influence of the teachers' prior absolutist beliefs of teaching (Kennedy, 1991). These beliefs were often developed through their apprenticeship of observation (Ball & McDiarmid, 1990; Lortie, 1975). Another reason is that teacher education students are often thrown into their student teaching too soon and left without any support for learning to teach (Dewey, 1964; Feiman-Nemser, 1983). Such student teaching puts beginners in a difficult position to observe and think about the principles of teaching and its relationship to teaching practice. It implicitly encourages them to pay more attention to the technical part of their teaching rather than its goals and purposes.

Literature in teacher learning and professional development suggests that teacher educators may help preservice teachers to learn to teach in such a constructivist way by creating particular opportunities for student teachers. First, they need to focus on transforming student teachers' beliefs of instruction (Kennedy, 1991), especially their conceptions of subject matter and its learning and teaching (Ball, 1989). They need to do so because their prior concepts and beliefs often stand in their way of receiving new ideas and conceptions of teaching (Hollingsworth, 1989; Kennedy, 1997; Pajares, 1992).

Second, they also need to situate their learning in the context of teaching, from which they will have chances to understand the relationship between the ambitious ideas of teaching and specific teaching practice (Feiman-Nemser & Remillard, 1996; Resnick, 1987). Such a situated learning is important because all knowledge and theories are situated in and gr

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situated in and grow out of the contexts of their use (Brown, Collins, & Duguid, 1989; Lave & Wenger, 1991).

Third, student teachers needed to have a chance to learn how to think and act like teachers in the context of teaching by working closely with a supportive and collaborative experienced teacher. With such a relationship, preservice teachers can be assisted or coached by the experienced teacher to learn to teach at levels beyond their independent performance (Feiman-Nemser & Remillard, 1996; Tharp & Gallimore, 1988; Vygotsky, 1978). With such a relationship, preservice teachers would have chances to develop the values, standards and agreements for teaching collectively with the experienced teacher (Cochran-Smith, 1991) and "open their intentions and practices to public examination" (Little, 1990, p.521).

Fourth, reformers argue that teacher educators need to support novices' learning to teach in the context of teaching through a structure of gradual transition into practice from observation, co-teaching and planning with experienced teachers, to independent teaching (Dewey, 1964; Feiman-Nemser, 1983). This gradual transition structure would give preservice teachers time to concentrate on observing and understanding the principles of teaching in practice before stepping into real practice and "offer a more flexible framework and results in better integration of theory and practice" (Feiman-Nemser, 1989, p. 217).

Teacher education reformers have started to think about reorganizing their programs to help preservice teachers learn to teach along these directions (Holmes Group, 1986; Holmes Group, 1990). Teacher educators have begun to focus on transforming their students' connections and beliefs in their mathematics education courses (Schram

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courses (Schram, Wilcox, Lanier, & Lappan, 1988). They have extended their student teaching into a year-long, gradual induction internship and situated their student teachers in real teaching environments with contextualized support and cooperating work with an experienced teacher (Feiman-Nemser, 1989).

However, in restructuring their programs and field experiences for their students, teacher educators also face a situation of having to send their students to do their internships with different collaborating teachers who may not necessarily believe in or practice constructivist mathematics instruction. Many teacher education students have to do their internship in different school settings where constructivist mathematics instruction is neither clearly encouraged nor practiced.

Martha and Louis were learning to teach in a teacher education program and internship that reflected many aspects of this teacher education reform but with different collaborating teachers and in different school settings. The mathematics teaching practices both of them developed through their internship were also clearly different. Their experience leads me to examine the following issues: What can teacher education students in a constructivist-oriented internship learn about mathematics instruction? What are the possible influences of their collaborating teachers on their learning? How do these influences occur? How do school instructional contexts shape their learning? My dissertation is designed to explore these research questions.

Research Questions

In this dissertation, I explore these issues by studying four elementary preservice teachers' learning to teach mathematics with collaborating teachers in a year long internship in diffe

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internship in different school contexts. The following research questions and a range of subsidiary questions are designed to guide my analysis.

<u>What do these preservice teachers learn?</u>

I assume that learning to teach mathematics can happen at both conceptual and practical levels. Thus, to better answer this research question, I need to analyze what they learn at both conceptual and practical levels in relation to their mathematics instruction in their internship. Two groups of subsidiary questions are designed to tackle each part of this question.

First, I develop subsidiary questions about their conceptual development:.

- What beliefs and conceptions about mathematics and its learning and teaching do these preservice teachers bring into their internship?
- What kinds of beliefs and conceptions about mathematics and its learning and teaching do these preservice teachers end up with through their internship?
- Whether and to what extent do their beliefs of mathematics instruction move closer toward or away from a constructivist vision of mathematics instruction?

Second are the subsidiary questions about the teaching practice they have learned:

- What kinds of instructional tasks and processes do these preservice teachers develop for their mathematics teaching by the end of their internship?
- To what extent do their instructional tasks and processes reflect their beliefs of mathematics and its learning and teaching?
- To what extent did the mathematics teaching practice they learned reflect standards of constructivist mathematics instruction?

<u>What are the influences of their collaborating teachers?</u>

To answer this research question, I examine both conceptual and practical influences that collaborating teachers had on what their preservice teachers learned about mathematics instruction. Again, two groups of subsidiary questions are designed to deal with each part of their influences.

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First, I explore their conceptual influence:

- What are the beliefs and conceptions collaborating teachers have about mathematics and mathematics instruction?
- To what extent do the beliefs preservice teachers develop in their internship reflect their collaborating teachers' conceptions?

Second are subsidiary questions about their influence at a practical level:

- What kinds of instructional tasks and instructional process do collaborating teachers develop for their mathematics teaching?
- To what extent does the mathematics teaching practice that their preservice teachers develop look similar to or different from their own mathematics instruction?

How do the influences of collaborating teachers occur?

To explore this question, my analysis is carried out at three levels. First, I investigate the kinds of expectations both collaborating and preservice teachers brought into the internship. Then I study what actually happened in their collaboration. In the end I examine the relationship between what preservice teachers learned and the nature of their collaboration. For each level, I designed a group of subsidiary questions to guide my analysis.

My subsidiary questions about their expectations includes:

- What expectations do collaborating teachers develop for what their preservice teachers need to learn about mathematics instruction in the internship?
- What kind of role do they expect to play in their preservice teacher's learning to mathematics?
- What do preservice teachers think they need to learn from their collaborating teachers about mathematics teaching?
- What do preservice teachers expect their collaborating teachers to do for their learning to teach mathematics in the internship?

Subsidiary questions about their actual collaboration focus on:.

- What are the important things collaborating teachers claim they did for their preservice teachers' learning to teach mathematics?

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- What are the important things preservice teachers claim their collaborating teachers have done that benefit their learning to teach mathematics?
- What is the relationship between preservice teachers' learning and the nature of their collaboration with their collaborating teachers?

Subsidiary questions about the relationship between their actual collaborations and what preservice teachers learned ask:

- What are the important things collaborating teachers claim their preservice teachers learned about mathematics instruction?
- What are the important things preservice teachers claim they about mathematics instruction from working with their collaborating teachers?
- What is the relationship between their expectations for collaboration and their actual collaboration?

How do the instructional contexts shape their learning?

In this part, my analysis focuses on three instructional contexts and their influences. These contextual factors are the culture of teaching in the school setting, curriculum resources and support, and the kind of students preservice teachers were assigned to teach. As I will discuss later in this chapter, these three instructional contexts in different schools have different features and they are found to have a strong influence on different ways of teaching in different schools. However, our understanding about their influences on preservice teachers' learning to teach mathematics in different schools has not been properly developed.

To answer my research question, I first investigate the features of three instructional contexts in each setting where the four elementary preservice teachers worked. Then I examine the reactions these preservice teachers develop toward these instructional contexts and estimate the influences of these instructional contexts on what they learned. I designed a group of subsidiary questions for each part of my analysis.

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- What kinds of mathematics teaching are valued in each school's culture
- What kinds of teaching practice does the culture of teaching in the school expose its preservice teacher(s) to?
- What kinds of curriculum resources and supports are available to the preservice teacher(s) in each setting?
- What kinds of students do they have to teach?

Second are subsidiary questions about their reactions toward the instructional contexts in each setting:

- How do preservice teachers define the mathematics teaching and the kinds of teaching practice they are able to observe in the school?
- What curriculum resources and support do preservice teachers rely on in planning their mathematics unit and lessons?
- How do preservice teachers think about their students and their influences on their mathematics curriculum development and implementation?

Prior Relevant Research

The questions listed above reflect some of the important issues in teacher education and its reform. Although there are some prior studies that help inform me about these issues, yet these studies still leave open many questions. Our understanding about these issues is not fully developed and some of these questions have not yet received careful examination. Thus, it is necessary for us to further explore them.

Beliefs and practice in mathematics instruction

The action research done by mathematics educators in mathematics education (Ball, 1988; Lampert, 1986) suggests that without a deep understanding about subject matter and its learning and teaching, it is impossible for teachers to teach in a constructivist approach. However, studies in mathematics education have failed to provide enough evidence to support that there is a consistent relationship between teachers' beliefs about mathematics and teaching and their mathematics teaching practice (Thompson, 1992). While some researchers report a strong agreement between teachers' professed beliefs

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professed beliefs of mathematics, mathematics learning and teaching (Grant, 1984; Shirk, 1973; Steinberg, Haymore, & Marks, 1985), others reported contradictory findings and suggest the strong impact of teachers' social contexts on their practice (Cooney, 1985; McGalliard, 1983; Thompson, 1982). For example, a more recent case study further indicates that an experienced elementary teacher, who had already developed some constructivist ideas of mathematics teaching, still had problems in conducting teaching practice reflecting her vision (Cohen, 1990). In addition, most studies about the relationship between beliefs of mathematics instruction and practice have been done with school teachers and in action research situations where researchers were teachers. Few studies are on preservice teachers.

These studies in mathematics education have several implications for my dissertation study. First, we still do not know much about the relationship between preservice teachers' beliefs about mathematics instruction and their mathematics teaching practice, even when these teachers somehow develop a constructivist view of mathematics instruction. Secondly, learning to teach mathematics can happen at both conceptual and practical levels but they may not go hand in hand all the time. To understand what preservice teachers learn about mathematics instruction, we should understand their learning at both levels. Third, to understand what kinds of mathematics instruction practice preservice teachers learn, we should pay attention to both their beliefs and the instructional contexts where their learning occurs. As the above studies suggest, one's beliefs about mathematics instruction may not be the only or most important factor shaping one's mathematics teaching practice. Thus, we need to understand what contextual factors shape one's learning to teach mathematics and how these factors inte

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Collaboration and preservice teacher's learning

Research on professional development (Cochran-Smith, 1991) suggests when the relationship between preservice and collaborating teachers builds upon their shared values, standards and agreements for teaching, such relationship helps preservice teachers successfully develop an ambitious teaching practice. However, as for how to develop such collaboration among teachers, in the field of teacher education there exist two different ideas. On the one hand, using England's teaching reforms in the last ten years as an example, D. H. Hargreaves (1994) suggests that institutional initiatives, like the centralized curriculum, school-based planning, mentoring, appraisal and partnership help teachers to open their doors to their colleagues and work collaboratively in reaching the intellectual goal of schooling. On the other hand, Andy Hargreaves (1990) argues that institutionally-contrived relationships among teachers restrict the potential for teachers to learn from each other, because such contrived relationships damage the mutual trust between teachers and the autonomy teachers have in their work. He argues that such trust and autonomy are important bases upon which teachers are able to develop their collegial relationship.

These studies provide a useful framework for us to think about the relationship between collaborating and preservice teachers. However, not only do these studies fail to target the particular relationship between collaborating and preservice teachers, but also their implications for understanding the influences of collaboration on preservice teachers' learning are conflicting. Since the collaboration between collaborating and preservice teachers often cor

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teachers often combine contrived and voluntary elements, it is important for us to develop an understanding about how such a semi-contrived, semi-autonomous relationship between collaborating and preservice teachers influences the learning of preservice teachers.

Recent studies on mentoring start to show that mentor teachers may not necessarily assume a supporting role in their work in such a relationship. A study on mentors in two U.S. induction programs suggests that mentors in different programs had quite different expectations for the roles they needed to play (Feiman-Nemser & Parker, 1992). Even in the reformed teacher education program, collaborating teachers who are experienced in constructivist teaching or are embracing its philosophy still develop different expectations for what their preservice teachers need to learn and what they need to do for preservice teachers (Dembele, 1996; Feiman-Nemser, 1995; Wang, 1997).

We can speculate that the different expectations collaborating teachers have for their work and for what their preservice teachers need to learn influence preservice teachers' learning to teach. However, there are few empirical studies that actually show us: What are the influences of such diverse expectations from collaborating teachers on preservice teachers' learning to teach mathematics? How do these influences happen, if they do? Moreover, there are very few studies developed to understand the interaction between the expectations of collaborating teachers and those of preservice teachers, as well as the impact of such interaction on the results of preservice teachers' learning to teach mathematics. Thus, it is necessary for us to understand what expectations both collaborating and preservice teachers bring into their collaboration and how collaborating and preservice teachers interact with each other in shaping what preservice teachers learn in their internship.

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Gradual transition and preservice teachers' learning

The internship of gradual-transition to practice reflects an important part of current teacher education reform in many teacher education programs (Feiman-Nemser, 1989). Teacher educators believe that when teacher education students are able to gradually evolve into a practice with a sequence involving careful observing, partially participating in management and teaching, and finally teaching independently, they have a better chance to see ideas in practice (Dewey, 1964; Feiman-Nemser, 1989).

Yet in reality, many teacher education programs have to put their preservice teachers in schools where the constructivist approach of teaching is not clearly encouraged or practiced and to work with collaborating teachers who are actually not experienced in constructivist instruction. If preservice teachers are exposed to teaching practice that fails to reflect the principles and ideas they need to observe, what kinds of relationship between principles and practice do they develop? If they have to gradually transit into the kind of teaching practice with collaborating teachers who may not necessarily teach in the same direction, what kind of mathematics instruction would they gradually learn? We have little empirical evidence to answer these questions. Thus, we need to develop these understandings.

Instructional contexts and preservice teachers' learning

Research on instructional contexts and their influences on the quality of teaching provide implications for my dissertation study from another angle. First, studies in this area suggest that the kind of teaching culture in a school has important influence on the relationship among teachers and, thus, on their teaching practice. Research on the U.S. school teaching culture (Feiman-Nemser & Floden, 1986; Lieberman & Miller, 1991) shows that many

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shows that many schools organize teachers to spend most of their day in their classroom and to have few chances to talk to, observe or seek help from their colleagues. This organizational feature of teachers' work reinforces the norm of individualism and non-interference among teachers. These norms hinder the chances for teachers to foster their shared conceptions about pedagogical purpose, content, approaches and develop higher professional standards for their work (Lortie, 1975; Little, 1990).

Studies in this area imply the following things. First, different kinds of teaching cultures shape differently the ways in which teachers define what can be counted as acceptable teaching or good teaching practice. Under the individualist culture of teaching, different kinds of teaching may be acceptable, while in a contrived culture of teaching, teachers work with a shared teaching goal. Since preservice teachers are often assigned to do their internship in settings with different cultures, it is reasonable to hypothesize that their way of defining acceptable teaching practice may be differentially influenced by the school settings.

The kind of culture under which preservice teachers work can also provide or limit their chances to observe certain kinds of teaching practices. Thus, their chance to see connections between constructivist ideas of teaching and actual teaching practice can also vary. Their different chances to see principles at work can be assumed to have quite different influences on what they are able to learn about mathematics teaching.

Second, studies on instructional context also indicate that the different ways that school curriculum is structured and its resources are organized have different impacts on the quality of teaching. By analyzing the relationship between the structure of the curriculum and teaching, Cohen and Spillane (1992) make the following argument.

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When schools lack an external curriculum system that is authoritatively, specifically, and consistently prescribed, teachers do not have substantial pedagogical standards as their bases to make their teaching decisions and assess their teaching results. Stevenson and Baker (1991) show that "when control of curricular issues is at the national level, the amount of the mathematics curriculum that teachers teach is generally not related to the characteristics of the teachers or of the students, whereas in educational system with provincial or local control, it is related to teachers' and students' characteristics" (p. 11). More recent comparative study by the Third International Mathematics and Science Study (Schmidt et al., 1996) further suggests that when mathematics curriculum and textbooks are organized in a fragmented, repetitious and superficial way, it is hard for teachers to develop good mathematics teaching practice.

The implication from these curriculum studies is clear. To help preservice teachers learn to teach in a constructivist approach, one of the important steps is to support their development of curriculum and units or lesson plans that reflect constructivist ideas. However, curriculum development is impossible without the support of carefully designed, consistent and pedagogically specific curriculum resources. The kind of curriculum resources and supports can vary when preservice teachers work with different collaborating teacher and in different schools. Thus, we need to understand what kinds of curriculum resources are available to preservice teachers and how different resources and supports influence their curriculum development.

Third, the different kinds of students that teachers encounter can also have an impact on their definition of goals and their ways of teaching. A number of studies from a sociological perspective (Anyon, 1983; Metz, 1993; Powell, Farrar, and Cohen, 1985) indicate that the

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indicate that the ways in which teachers conceptualize the purpose, content, and approaches of their teaching and what they do in their classroom are greatly shaped by the social backgrounds of their students. In a historical study, Sedlak at al.(1986) also find out that increased lack of tangible rewards for students that are available to schools and teachers creates situations in which schools and teachers bargain with students to learn in their classes.

When preservice teachers are placed in different schools and classrooms, their students can be quite different from each other. These differences can occur in a complex way. For example, their students can come from cultural and social backgrounds unfamiliar to preservice teachers. These students can be also exposed to the kind of teaching practiced by collaborating teachers and the other teachers in the school which may be different from the kinds of teaching preservice teachers want to learn and engaged in. These factors may well have potential impact on preservice teachers' instruction. Thus, if preservice teachers end up with quite different learning experiences in mathematics teaching in their internship, the kind of students and their preparation for preservice teachers' teaching need to be carefully examined.

Studies in teacher learning have begun to pay attention to how the organization of teachers' work and different structures of school curriculum influence the expectations that mentor teachers have for their roles, the focuses of their mentoring practice and their behavior patterns in their mentoring. By comparing two pairs of mentor and novice teachers in a preservice teacher education program with the two pairs in an induction program, Parker et al (1994) found that the attention mentor and novice teachers paid to curriculum issues in their collaboration varied by school curriculum and program context. International comparative studies (Wang, 1997; Wang & Paine, 1994) also suggest that different curriculum structure and requirements may have a strong influence on the focus of mentoring and expectations that cooperating or mentor teachers have for their work and their mentoring practice.

However, these studies failed to touch on many aspects of the possible relationship between the instructional contexts, what preservice teachers are able to learn about mathematics instruction and how they learn. We do not know whether the different cultures of teaching will contribute differently to preservice teachers' belief development and their making connections between ideas of teaching and actual practice. Although we know that different curriculum structures would have different influences on collaborating teachers' expectations for their role and what preservice teachers need to learn, we still do not know how these curriculum differences affect preservice teachers' unit and lesson planning. We still need to know what the potential impacts from students are on preservice teachers' learning to teach mathematics and how the impact happens.

Summary of Chapter

Current mathematics education reform is pushing a model of mathematics education that reflects a constructivist vision of knowledge, learning and teaching. This model of teaching is different from the traditional mathematics teaching model and self-realization model at both conceptual and practical levels. This reform poses an important and challenging task for teacher educators in helping preservice teachers learn to teach in a constructivist way.

Many teacher education programs began restructuring their program to meet this challenge. They support their students to develop constructivist ideas of mathematics instruction throu

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instruction through transforming their prior beliefs and conceptions of mathematics teaching. They extend their students' field experience by providing an internship featuring a gradual transition to practice in the hope that their students will have a better chance to observe the teaching principles at work. They situate their students' learning in an actual classroom with a closer relationship to and assistance from an experienced teacher. Martha and Louis participated in a teacher education program with these features, yet at the opening of this chapter, we observed real differences in their teaching practices.

We know that in making these structural changes, teacher educators have to send their students to do their internship in different schools where constructivist mathematics instruction may not be practiced or even encouraged. Like Martha and Louis, student teachers may have to work with different collaborating teachers who may not necessarily believe in or practice the kind of teaching their program encourages them to learn. These challenges teacher educators face force us to ask the following questions. (1) What do the preservice teachers learn about the mathematics instruction in their internship? (2) What are the influences of their collaborating teachers on their learning? (3) How do their collaborating teachers influence their learning? (4) How do their instructional contexts shape their learning?

A literature review suggests that these are important questions for the improvement of teacher education and its reform strategies. Our knowledge about some of these questions has not been fully developed or carefully examined. My dissertation aims to develop an understanding about these issues.

Chapter 2

METHODS: SAMPLES, DATA AND ANALYSIS

Samples of My Dissertation

Subjects and sites

My dissertation examines four pairs of elementary preservice and collaborating teachers--Martha and Nick, Jaime and Bank, Louis and Ben, and Kelly and Lisa. I chose these pairs of teachers to study because they were working with different student populations in four classes of three elementary schools. I expected that these differences would allow me to observe variations of instructional contexts, collaborating teachers' teaching, collaborations between preservice and collaborating teachers and to explore their influences on preservice teachers' learning to teach mathematics.

Martha was doing an internship with her collaborating teacher, Nick, in a fifth-grade classroom in Well Elementary School. The only elementary school in a small suburban town, well served a predominantly white middle-class professional and farming community. Nick had twelve years of teaching experience, of which ten years were spent in the fourth and fifth grades in Well Elementary. Martha was his first intern.

Jaime also worked at Well Elementary but with a different collaborating teacher, Bank, in a different fifth-grade classroom. Bank had taught about twenty years. Before he began teaching

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began teaching the fifth grade class in this school sixteen years ago, he had taught mathematics and science for four years in a middle school in the same school district. Jaime was his second intern from the teacher education program.

Louis and his collaborating teacher, Ben, worked in a fifth grade class in Bell Elementary School, a new school founded several years ago in an affluent residential district. The school was racially mixed, including White, African-American and Hispanic Students. Ben had twenty-five years of teaching experience teaching the fourth and fifth grades in this area. He had moved to Bell when it was founded. Louis was his second intern from the same program.

Kelly was interning with a collaborating teacher, Lisa, in a first grade class at Mall Elementary in an urban downtown. The racially mixed school included White, African-American and Hispanic students, with most of them from working-class backgrounds. Lisa had fifteen years teaching experience, ten years of which were as a special education teacher working with visually impaired students in a different district. She moved to Mall to teach first grade five years ago. Lisa had an intern from the same program a year before she worked with Kelly.

In spite of variations in the schools, classes and students, these preservice and collaborating teachers shared several similarities. These provided a common base upon which I can compare one case with another.

All the preservice teachers came from the same five-year teacher education program developed by a large state university in the Midwest. In the program, they all took similar teacher education courses. They began and ended their internship at the same time. All the collaborating teachers volunteered to work with the teacher education program becaus

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program because of the program's restructured internship. Before the internship, each went to a week-long collaborating teacher orientation organized by the program. Thus, they received similar but limited formal training for their work.

Program requirements and structure

The program in which these teachers worked was a new program restructured in 1998 under the influence of the Holmes Report, *Tomorrow's School*, a reform document which among other things urged universities to reconnect teacher education to schools and classrooms. The program was designed with many big themes that required their students to pursue a constructivist vision and reform agenda. These themes are:

1. Deep understanding of subject matter disciplines and pedagogy that "teach for understanding."
2. A democratic commitment to the education of everybody's children--to classrooms and schools that would embrace diversity.
3. Helping TE student learn how to establish true learning communities in classrooms and schools.
4. Graduates able to participate in the process of remaking the teaching profession, renewing schools, and making a better world.
5. A better integration of theory and practice, field experience and reflection on that experience.[1]

The program's internship was designed as one year long and organized into four periods of learning that strongly suggest a gradual transition model with a focus on the relationship between theory and practice. The internship started with preservice teachers observing and assisting collaborating teachers in planning, teaching and management for about six weeks. Then they spent about another six weeks to partially participate in planning, teaching and classroom management through co-planning and co-teaching two or more subjects with their collaborating teachers. Next, they were required to conduct

[1] These visions are quoted from the program's *Elementary Collaborating Teacher Handbook* (p.7).

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ten weeks of lead teaching on two or more subjects for which they take primary responsibility for planning and teaching most days with coaching and other assistance from their collaborating teachers. In the end, they spent about eight weeks reflecting on the year's work and progress and completed all their projects and assignments while taking on a modest teaching or co-teaching load.

Along with these field experiences, the program also required its students to take two year-long graduate level seminars during their internship. One guides them to understand, think and explore issues like teachers' ethics, responsibilities, and school organization as well as their relationship with parents and community through school-based inquiry projects. The other focuses on deepening their understanding of subject matter teaching and curriculum development, and supports them to learn to adapt their curriculum for the students they teach.

In terms of preservice teachers' learning to teach mathematics in the internship, the program strongly encourages its students to learn to teach as the National Council of Teachers of Mathematics (1989, 1991) envisions. The following internship goals definitely highlight this encouragement.

> Learn to plan, teach and evaluate units that are carefully focused on important concepts, that pay serious attention to children's mathematical ideas and theories, that actively engage children in doing, writing and talking about mathematics, and that challenge and foster children's meaning making about mathematics.[2]

The role of collaborating teachers in the internship is also regarded as important in the program. Instead of only providing a classroom in which preservice teachers can

[2] These goals for learning to teach mathematics are quoted from the program's *Elementary Internship Guide*.

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apply ideas learned in their program course work, they were required to guide, support and assess preservice teachers' learning to teach across the internship year. The program defines these responsibilities as:

1. stage appropriate, classroom-based learning opportunities for intern(s) across the school year;
2. meet with intern at least once a week at regularly scheduled time to co-plan and discuss concerns;
3. assist interns in developing and implementing personal /professional learning goals;
4. help interns gain familiarity with district curriculum and grade level objectives, school policy, curriculum resources;
5. model the intellectual work of teaching by sharing goals and beliefs, co-planning, discuss dilemmas, etc.;
6. participate in appraising intern's progress at midterm, end of semester and end of the year conference;
7. participate in professional development activities for collaborating teachers.[3]

Data Sources and Collection

Goals for data collection

My four general research questions guided my data collection on these preservice and collaborating teachers and their schools. To find out what they learn about mathematics instruction in their internship, I collected two kinds of information from the preservice teachers. These data allowed me to develop a big picture of what they learned at both conceptual and practical levels:

1. Their beliefs of mathematics, its learning and teaching that they brought into their internship and those they ended up with through their internship;
2. Information about their teaching practices they learned over their internship.

To explore the influences of their cooperating teachers on what they were able to learn, I wanted to be able to compare the conceptions and practice that collaborating

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[3] These responsibilities of collaborating teachers are quoted from the program's collaborating teacher handbook.

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teachers had with the beliefs and practice that the preservice teachers developed over their internship. I gathered the following two kinds of information about collaborating teacher's mathematics instruction:

1. Their beliefs of mathematics, its learning and teaching that these collaborating teacher had when they were working during their internship;
2. Information about their mathematics teaching practice these collaborating teachers often exposed their preservice teachers to.

As I discussed in Chapter 1, collaboration is an important medium through which collaborating teachers exert their influence on preservice teachers' learning to teach. To understand how these collaborating teachers influence their preservice teachers' learning, I also collected four kinds of information from both preservice teachers and collaborating teachers on the nature of their collaboration:

1. Expectations collaborating teachers developed for their preservice teachers' learning about mathematics instruction and their roles in helping their preservice teachers' learning to teach;
2. Expectations preservice teachers had for what they needed to learn and the roles their collaborating teachers need to play for their learning;
3. Collaboration between collaborating and preservice teachers during the internship;
4. Functions of the collaboration on preservice teacher's learning from both collaborating and preservice teachers' perspectives.

To help me understand my last question about how instructional contexts shape the preservice teachers' learning, I garnered four types of information from collaborating teachers, preservice teachers and their schools. This included:

1. Kinds of mathematics teaching valued by and specific teaching practices that each preservice teacher is exposed to in each school;
2. Kinds of curriculum resources and support available to each preservice teacher in his or her setting;
3. Kinds of students each preservice teacher had to work with and their preparation for the kind of mathematics teaching the preservice teachers want to pursue;

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4. Reactions of these preservice teachers toward the culture of teaching, curriculum resources and students in the school and class.

Types of data collected

To get these kinds of information for my study, I collected documents, conducted interviews and did a lot of observation. For document collection, I gathered school mathematics curriculum guidelines, yearly reports and policies about teaching responsibilities and learning goals for students in each site. I also collected several teacher preparation program documents, such as the handbooks for elementary collaborating and preservice teachers, internship guides and schedules. The third kind of documents were artifacts that collaborating and preservice teachers used in their planning and teaching mathematics units and lessons, and in their assessing students' mathematics learning, such as the textbook, work sheets, assignments and test papers and other materials they used for their teaching. I gathered teaching plans and reflections on teaching that preservice teachers did for their mathematics units and lessons.

Another source of information I collected was observational data that included the following three categories. First, I observed four to five mathematics lessons taught by each preservice teacher in their lead teaching. The mathematics lead teaching unit was the last part of mathematics instruction these preservice teachers did in their internship for which they took primary responsibility for planning and teaching. All these lessons were videotaped.

I also observed three mathematics lessons taught by each collaborating teacher, of which two lessons were videotaped and one was observed with field notes. One videotaped lesson and one observed lesson happened in the early part of the internship and the other v

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I also spent two full days observing teaching and mentoring activities in each classroom in order to see an entire day, the flow of activities and how these related to collaborating teachers' mathematics teaching and preservice teachers' learning to teach mathematics. I did these observations in the latter part of internship. The mathematics teaching- or mentoring-connected activities in the two days were videotaped or audio-taped while the other activities were recorded with field notes.

In addition to my observations of mathematics teaching and mentoring, I also conducted observations of several lessons on the other subjects taught by both preservice teachers and collaborating teachers as well as some school activities over the year. These observations were made to see how mathematics teaching might be related to the teaching of other subjects.

The third kind of data I gathered from the these collaborating and preservice teachers was interviews. First, I conducted two one-hour interviews with each preservice teacher. The first interview was conducted in the second period of his or her internship when he or she had begun to take part in planning, teaching and managing. In this interview, I mainly asked four types of questions about their views of elementary mathematics, its learning and teaching, their mathematics planning and teaching, their views of learning to teach mathematics with collaborating teachers and the influences of instructional contexts.[4]

[4]The protocol of this interview and other interviews are included in Appendix 1.

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My second interview with each preservice teacher occurred in the last period of the internship when each preservice teacher had finished his or her mathematics lead teaching unit and had begun to summarize and reflect on what they learned. In this interview, I asked about their mathematics education and mathematics teaching preparation, views of good math teaching practices, their mathematics teaching and learning experiences in the internship, their learning to teach mathematics with collaborating teachers, and their relationship with their mentors.

I conducted two one-hour interviews with each collaborating teacher. My first interview occurred during the second phase of internship as they had begun to share teaching, planning and management responsibility with their preservice teacher. In this interview, I asked about collaborating teachers' mentoring and teaching experiences, their mentoring and teaching experiences, their views of the influence of instructional contexts and their view of mentoring and their mentoring practice.

My second interview with each collaborating teacher was at the end of the internship when each collaborating teacher had begun to take back most of his or her teaching responsibility from his or her preservice teacher. In this interview, each collaborating teacher was asked about their training in math teaching, their views of good teaching practices, their preservice teachers' learning to teach math and about their roles, ways, dilemmas and influences in collaboration.

In addition, I had a lot of informal interactions with these teachers. These interactions happened either before, during or after their teaching during my visits to their classrooms. Some of these interactions also help me in making sense of my research questions in this study.

Process and issues of data collection

My data collection occurred in the 1995--1996 school year when the four elementary preservice teachers were doing their internship. Since the goal of my data collection was to collect information about their learning to teach mathematics and the factors influencing their learning as it naturally occurred, I maintained the role of impartial observer and non-interfering interviewer over the course of the internship. I was aware, however, that it was impossible not to have any influence on what happened in each site. As an international graduate student, I clearly knew that some of them might look at me as a total outsider to their teaching and work. Thus, my presence in their classroom might exert some influence on their work.

To reduce my possible influence on the participants in this study, I structured my data collection into three periods. First, I spent about three weeks sitting interchangeably in the four classes only observing and without engaging in any formal data collection. I hoped that in doing so, teachers and students in each site would have a chance to become used to my presence in the class.

As their internship entered the second phase and each preservice teacher started to partially plan, teach and manage the class, I increased my informal interaction with them and started some of my formal data collection. During this period, I observed one and videotaped the other mathematics lessons taught by each collaborating teacher. I had the first interview with each preservice and collaborating teacher and collected some curriculum, teaching materials each teacher used for his or her mathematics classes.

The intensive data collection started when preservice teachers began their mathematics lead teaching and continued until they completed their mathematics teaching. I vid

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teaching. I videotaped all of each preservice teacher's mathematics lessons as well as his or her collaborating teachers'. I had my second interview with all these teachers and I conducted two full-day observations of each classroom and collected all the other school documents and teaching artifacts mentioned above.

As is true for all qualitative studies that involve several cases and collect data over a longer period of time, I also have had to face the dilemma of how to balance extended data collection for one case and broader data gathering for different cases. Even though I narrowed down my subjects to four pairs of teachers, in the end I still could not follow any one of the pairs as frequently as I expected. For example, each week I was only able to visit their class twice. While I was unable to develop the depth in a single case that researchers who do an individual case study, my data collection from several cases allowed me to make comparisons and arguments that would be hard to make from a single case study. Thus, the issue of depth is somewhat compensated by the values of broader and comparative data.

Also as is true for any qualitative research, my other concern of this study was how to make reasonable interpretation about what I saw and heard. Coming from China, not only did I grow up in a society with totally different cultural values and social, political and economic structures, but also I had my elementary, secondary, undergraduate and a part of my graduate education in a totally different school system. Although I was involved for about four years in the mentoring project at the National Center for Research on Teacher Learning and had many chances to read data and literature on mentoring and teacher learning in US. schools, I still had little first-hand experience about the U. S. elementary education system. This situation, sometimes, created some d

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created some difficulty for me to interpret accurately what I observed. However, this situation also put me in a position that allowed me to see many issues and aspects of preservice teachers' learning to teach mathematics that many insiders may easily take for granted. It opened many chances for me to analyze my data from a perspective that results from a continuous comparison between two different school systems and classrooms. Such an outsider perspective can also be very valuable. As the Chinese poet, Li Bai says, " when you are unable to see the real face of Lu mountain, maybe it is only because you are living in it."

In addition, my effort to get a range of data and to video-tape lessons gave me an opportunity to triangulate my interpretations by repeated viewing of key events. Thus, the kind of data collection strategies I relied on helped compensate for whatever challenges my outsider status brought.

Methods and Strategies of Data Analysis

The basic method of my dissertation study is qualitative comparative analysis within and across cases. The comparative method is used here not to find a best model for learning to teach mathematics, but to develop a deeper understanding about what preservice teachers are able to learn about mathematics under the influence of collaborating teachers and their instructional contexts. As Eggan (1965) argues, the comparative method is "a technique for establishing similarities and differences that can be applied with different degree of rigor and approximation" (p.336). My analysis here is organized clearly to parallel my four research questions.

Analysis of what preservice teachers learn

One analysis focused on my first research question: What were the four elementary preservice teachers able to learn about the kinds of mathematics instruction? To answer this question, I examined what they learned about mathematics instruction at both the conceptual and practical levels. The findings from this part of my analysis are in Chapter 3 and Chapter 4.

First, I used the two interviews I did with each preservice teacher to capture the features of his or her conceptual change and development. I coded my interview data using a conception that Thompson (1992) developed to consider teachers' beliefs about mathematics instruction in terms of three related parts: their beliefs about mathematics, their beliefs about mathematics learning and their conceptions of mathematics teaching. Using this conception, I coded each teacher's beliefs of mathematics instruction into three categories: the beliefs that each preservice teacher brought into the internship, where they got these beliefs and the beliefs he or she ended up with after teaching.

With this categorization of each preservice teacher's beliefs as a base, I then conducted three levels of analysis to develop a sense of the conceptual change and development these preservice teachers experienced in their internships.

First, I analyzed the relationship between the ideas each preservice teacher brought into his or her internship, the mathematics education course work he or she took in the program and his or her earlier mathematics learning experience. This analysis led me to see the beliefs these teachers brought into their internship, to a great extent, were shaped by their program course work rather than by their apprenticeship of observation before their teacher education program.

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Then I compared each teacher's earlier beliefs of mathematics instruction with those beliefs they ended up with as a way to capture the changes and development at a conceptual level and the influence of the internship on their conceptions of mathematics instruction. Moreover, I compared his or her beliefs before and after internship with the conceptions that mathematics educator and reformers have encouraged teachers to develop to see to what extent each preservice teacher moved closer to or away from constructivist conceptions of mathematics and its learning and teaching.

In addition to analyzing their conceptual change and development, I also used observation of each preservice teachers' lead teaching lessons and the artifacts I collected around their teaching to develop an understanding about what kind of mathematics teaching practice each preservice teacher was able to learn over his or her internship. Two reasons stood behind my decision to use their mathematics lead teaching lessons to address this issue of their learning. First, their mathematics lead teaching units, as their program arranged, were the only time when these teachers took full responsibility for planning, teaching and assessing. Thus, these lessons best represent these teachers' mathematics practices and reflected relatively little direct involvement from their collaborating teachers. Second, their mathematics lead teaching units were the last and perhaps the most important time for them to apply whatever ideas about mathematics instruction they had developed. Therefore, these lessons represent many important aspects of what these teachers actually learned about mathematics teaching over their internship.

To reach this goal of my analysis, I coded the video-taped lessons taught by each preservice teacher, his or her teaching artifacts and his or her teaching plans and reflections. I c

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reflections. I coded features of instructional tasks and instructional processes he or she developed in these lessons and associated artifact, teaching plans and reflections. Instructional tasks represent the activity and assignments that each preservice teacher wanted students to accomplish in each lesson and the ways in which students are required to accomplish these. Instruction processes indicate the effort and time each preservice teacher gave in each lesson to help students succeed in accomplishing their tasks. I assume that these themes as they occurred in his or her lessons can be used to indicate the quality of his or her mathematics instructional practice because they allowed us to see in action how each teacher established their goals of teaching and implements them in his or her mathematics lesson.

Here I did the following levels of analysis and comparison to capture the features of the mathematics instructional practice these teachers learned. I also use these analyses to identify the relationship of their mathematics teaching with their beliefs in different period of the internship.

First, I used specific examples and events collected from each observed and video-taped lesson taught by each preservice teacher to capture the important features of his or her instructional tasks and processes developed in these lessons. Then, I coded four videotaped mathematics lessons taught by each preservice teacher to show the patterns of his or her instructional tasks and processes in quantitative form. I assessed the patterns of the instructional tasks and processes against a series of standards for mathematics instructional tasks and processes. These standards are adapted from the authentic instructional standards developed by the Center on the Organization and Restructuring of

School, Wisconsin Center for Educational Research (Newmann, Secada, & Welhlage, 1995). I used these standards here for several reasons.

First, the visions of students' learning in these standards reflect the constructivist visions of mathematics learning that program and mathematics education reformers expect their preservice teachers to develop among their students. Three visions underlie these standards:

> (1) students construct meaning and produce knowledge, (2) students use disciplinary inquiry to construct meaning, and (3) students aim their work toward production of discourse, products, and performance that have value or meaning beyond success in school (Newmann & Wehlege, 1993, p.9).

Second, the specific standards derived from these three visions of student learning also lead to the learning tasks and teaching process that mathematics education reformers encourage teachers to develop. These standards (Newmann, Secada, & Welhlage, 1995) are:

<u>1. Standards for assessing instructional tasks:</u>

- Organizing information: The extent to which the instructional task in a lesson requires students to organize, synthesize, interpret, explain and evaluate complex information in addressing a mathematics idea, concept or problem.

- Considering alternatives: To what extent does the task in a lesson open chances for students to consider alternative strategies, perspectives and points of mathematics concept, problem and theory.

- Disciplinary content: The degree to which the task in a lesson promotes students' understanding of and thinking about the ideas, rules, and theories considered seminal or critical within mathematics.

- Disciplinary process: The degree to which the task leads the students to use methods of inquiry, research, or discourse of mathematics.

- Elaborated communication: The extent to which the task asks students to elaborate on their ideas and conclusions in different ways that are used in the discipline of mathematics.

- Authentic problem: To what extent does the task present students with a question, issue or problem that they have actually encountered, or are likely to have encountered, in their life beyond school or allow them to use their knowledge beyond mathematics.

2. Standards for assessing instructional process

- Higher order thinking: The degree to which students use higher order thinking--manipulating information and ideas in ways that transform their meaning and implications, such as when students combine facts and ideas to synthesize, generalize, explain, hypothesize or arrive at some conclusion or interpretation.

- Deep knowledge: The degree to which the teacher involves students in dealing with, making clear distinctions between, developing arguments about, constructing explanations for significant mathematics concepts or solving problems in systematic and related ways.

- Substantive conversation: Whether the classroom conversations are indicated by the following features: (1) The talk about mathematics includes indicators of higher order thinking. (2) Sharing ideas is evident in exchanges that are not completely scripted or controlled. (3) The dialogue builds coherently on participants' ideas and promotes collective understanding of an mathematics theme or topic.

- Connection to the world: The extent to which the class has value and meaning beyond the instructional contexts, as students address real-world problems and use their personal knowledge and experiences as contexts in applying the mathematics knowledge that they are learning.

Third, the framework involves specific measures for assessing a lesson on multiple dimensions. Specific scales under each standard are designed to distinguish to what extent each aspect of the lesson is closer to the constructivist dimension instead of considering each aspect of the lesson either good or bad. Each standard for instructional tasks is conceptualized as a continuous construct from 1 to 3 or 4 and each standard for instructional process is considered as a continuous construct from 1 to 5. Scales are assigned to each lesson based upon its quality rather than its procedural and technical features.

For example, in rating how effective a teacher is in developing an instructional task aimed at pushing students to organize information, the following three levels of scale distinguish the quality of the tasks. If the task a teacher developed in a lesson calls for students to interpret nuances of a topic that goes deeper than surface exposure or familiarity, he or she will be given 3 points in the area of organization of information. If the task only asks students to gather information for reports without interpreting, evaluating or synthesizing information, the teacher will receive 2 points in the area. When the task just requires students to retrieve or reproduce isolated fragments of knowledge or repeatedly apply previously learned algorithms and procedures, he or she will only get 1 point.

The rating for instructional process takes into account the number of students and proportion of class time to which the standard applies. For example, in rating how well a teacher is supporting his or her students to engage higher order thinking, the following five levels of scales are used. When almost all the students, almost all the time, are synthesizing, generalizing, explaining, hypothesizing, or arriving at conclusions by themselves, the teacher in this lesson will receive 5 points for higher order thinking. A teacher will be given 4 points when he or she engages many students in at least one major high-order-thinking activity that occupies a substantial portion of the lesson. A teacher will get 3 points when his or her students are simply receiving and reciting factual information, or employing rules and algorithms through repetitious routines for a good share of the le

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share of the lesson, but there is at least one activity in which some students perform some high order thinking. If students in a lesson are primarily engaged in lower order thinking but at some point they perform higher order thinking as a minor activity within the lesson, the teacher's instructional process will rate 2 points. A teacher will get only 1 point if his or her students are only involved in lower order thinking and there are no activities during the lesson that allow students to go beyond lower order thinking.[5]

To compensate for the potential subjective judgment of my individual by rating each teacher's instruction, in my analysis I also use specific examples, events and dialogues from the observed and video-taped lesson to describe the features of each teacher's instructional tasks and processes developed in these lessons. In doing so, I hope to provide a fuller and more reliable portrait of the relationship between what actually happened in a classroom and the scale assigned for a certain aspect of the lesson.

I used this quantitative analysis about the teacher's instructional tasks and processes to help me understand in which areas and to what extent their mathematics instructional practice reflects or fails to reflect what mathematics education reformers encourage them to do. Then I compared the patterns of each preservice teachers' mathematics teaching practice with the beliefs he or she had at the beginning and the end of the internship. Through this comparative analysis, I hoped to develop a sense about how the mathematics teaching practice they developed was related to their beliefs at different periods.

[5] For specific criterion of each specific scale in the standard, please look at Appendix 2.

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Analysis of collaborating teachers' influence

The second part of my analysis in this study is devoted to understanding the influence of collaborating teachers on their preservice teachers' learning to teach mathematics. To develop this understanding, first I explore the relationships between the collaborating teachers' beliefs about mathematics instruction and the beliefs their preservice teacher developed. Then I examine the connections between the collaborating teachers' mathematics teaching practice and their preservice teachers' mathematics teaching in the internship. Through these two kinds of comparative analysis, I developed an understanding about the influence of collaborating teachers on their preservice teachers' learning to teach mathematics at both the conceptual and practical levels. The findings of these analyses are presented in Chapter 5 and 6.

To understand the relationship between the beliefs of each collaborating teacher and the beliefs each preservice teacher developed, I did three levels of analysis and comparisons.

I coded and categorized the two interviews I did with each collaborating teacher and summarized his or her beliefs about mathematics, its learning and teaching. My original purpose in designing two interviews with collaborating teachers was to identify any conceptual changes that would happen to the collaborating teachers because of their involvement in the internship. However, I found nothing substantially different about each teacher's beliefs as they appeared in his or her two interviews. In addition, all the collaborating teachers reported in their interview that they had begun to form their current beliefs of mathematics instruction several years earlier. In the end, I came to use both interviews as bases for recognizing domains and patterns of his or her beliefs.

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Then, I compared my findings about the beliefs of collaborating teachers with the findings I had about their preservice teacher's beliefs at different points of the internship. Through this comparison, I developed a sense about how change in each preservice teacher's belief reflected his or her collaborating teacher's conceptions about mathematics and its learning and teaching.

I also wanted to understand the relationship between collaborating teachers' mathematics teaching and their preservice teachers' teaching practice. This required additional analysis and comparisons.

I studied specific examples and events in the three observed and videotaped lessons taught by each collaborating teacher and the assignments, work sheets and lesson plans used in these lessons to gain an insight into some important features of his or her instructional tasks and processes. It is obvious that three observed lessons from each teacher can not be used to characterize all the dimensions of a teacher's practice. However, all of these teachers agreed that the lessons I observed and videotaped reflected the approaches they used most often in their mathematics teaching during the internship. Thus, they represent the approaches that their preservice teachers were most often exposed to.

I also conducted a quantitative analysis about the two videotaped lessons of each collaborating teacher to similar to what I did for the preservice teachers. This analysis helped me to get a sense about how their mathematics instructional practice reflected or failed to reflect constructivist standards.

Using these teaching data, I compared the ratings each collaborating teacher received for his or her mathematics lessons with those ratings his or her preservice teacher got fo

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teacher got for his or her mathematics lead teaching lessons. This comparison of many dimensions of interactions allowed me to see specifically in which areas and to what extent each preservice teacher's mathematics teaching was similar to or different from the mathematics teaching practice of his or her collaborating teacher. Thus, the possible practical influences of collaborating teachers on their preservice teachers' mathematics teaching could be identified.

Analysis of the functions of collaboration

In the third part of my analysis, I examine how the influence of collaborating teachers on their preservice teachers happened. The data used for this analysis were the two one-hour interviews with each collaborating teacher and with each preservice teacher, my two full-day observations and my informal observation of their interactions.

The collaboration between collaborating and preservice teachers is an important medium through which collaborating teachers exert influence on preservice teachers. On the one hand, the kind of collaboration in each case can be influenced by the expectations collaborating teachers have for what their preservice teachers need to learn and what role they need to play in their preservice teachers' learning (Feiman-Nemser, 1995; Wang, 1997). On the other hand, it can be reasonably assumed that the kind of collaboration developed between collaborating and preservice teachers is not determined only by collaborating teacher. Preservice teachers' input into the collaboration can also be an important factor shaping the nature of collaboration. I developed analysis of this aspect at three levels and the findings of my analysis in this part are presented in Chapter 7.

First, I explored the expectations collaborating and preservice teachers had for their collaboration for the internship and how these expectations were formed in each case. I created

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case. I created coding categories and used these to categorize the following four kinds of information from all the interview data: collaborating teachers' expectations for what their preservice teacher needs to learn, their expectations for the roles they are going to play, preservice teachers' expectations for what they need to learn and the roles they wanted their collaborating teachers to play. I investigated how their expectations for their collaboration and interacted with each other and transformed their actual collaboration.

Then I analyzed the focus of and approach to collaboration in each case and the ways in which the collaborations were shaped by the expectations of both collaborating and preservice teachers. I coded all the interviews and my observations mentioned above for two kinds of information about their collaboration. First, what were the important things each collaborating teacher claimed that he or she did for his or her preservice teacher's learning to teach mathematics and what were the reasons behind their action? Second, what were the important things each preservice teacher thought that his or her collaborating teacher did for their learning and how did they think these things benefit his or her learning to teach mathematics?

I used this information to analyze the kind of collaboration each pair developed for the preservice teacher's learning to teach mathematics. Then I contrasted the collaboration in each case with the expectation developed by both parties for their collaboration to consider the influences of their expectations on their actual collaboration.

In addition, I compared the collaboration in each case with what preservice teachers learned. I contrasted the results of their learning with what both collaborating and preservice teachers claimed the preservice teacher learned. Through these comparison, I

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Analysis of the influences of instructional contexts

The last part of my analysis in my dissertation is to understand how instructional contexts shaped preservice teachers' learning results in each setting. Based upon interview data collected from each pair of collaborating and preservice teachers, some school curriculum documents and artifacts gathered from each preservice teacher, I conduced the following three levels of analysis. The findings of my analysis are discussed in Chapter 8.

My first level of analysis is to identify some general features of the instructional contexts that each school and its teachers, including the collaborating teachers, offered for preservice teachers' learning to teach mathematics. These contexts include the culture of teaching in the school, the school mathematics curriculum and the resources available and the kind of students and their preparation for the preservice teachers' teaching. I coded three categories of information in each setting from all the interviews I conducted with collaborating teachers and the curriculum materials I collected. These categories are the culture of teaching in the school, the school mathematics curriculum requirements and other curriculum resources, and the kind of students in collaborating teacher's class. I further analyzed and summarized the basic features of the three kinds of instructional contexts in each settings.

Then I coded all the interviews I conducted with each preservice teachers, the curriculum resources he or she used, and his or her teaching artifacts to identify the ways in which he or she reacted toward their instructional contexts. These reactions include how they thought about the teaching they were able to observe in the school and the school mathematics instruction, how they used the curriculum resources available to them in developing their mathematics unit and lessons and how they dealt with their students' reactions toward their curriculum implementation.

In the end, I compared and contrasted these two levels of analyses with the results of preservice teacher's learning to teach. Then I discuss and interpret how the instructional contexts shaped the outcomes of each preservice teacher's mathematics teaching practices.

Summary of Chapter

In this chapter, I discuss the data, data collection and analysis strategies for my dissertation. To properly answer my research questions raised in Chapter 1, I chose to collect data from four elementary preservice teachers, their collaborating teachers and their schools. Both similarities, in terms of their program backgrounds and the differences in light of their school, teaching and students contexts, are considered when I made this choice to maximize the chances to understand my research questions.

The data for my study are from three sources: documents, interviews and observations. The process of my data collection followed a low-interference principle. I also took many measures to reduce my influence on the work of my subjects. I discussed the two limitations of my data collection and interpretations.

The method of my data analysis is comparative. The comparison in this study occurs within and across cases. Comparative analysis is conducted in the four related aspects of my general research questions. Although most of my comparative analysis is qualitative, I also used quantitative comparison in analyzing what kind of practice preservice teachers learned and how their learning in this area reflects the kind of mathematics teaching their collaborating teachers were practicing. In the next 6 chapters, I present and interpret my findings.

Chapter 3

LEARNING AT A CONCEPTUAL LEVEL

In this chapter, I explore the conceptual development that four elementary preservice teachers, Martha, Jaime, Louis and Kelly, experienced in their internship. I discuss the nature of their conceptual change. My analysis in this chapter suggests that these teachers' conceptual developments exemplify their struggles along and among three positions defined by Dewey (1990).

Martha: Move toward Constructivism

Martha received her mathematics education in her elementary and secondary schools in a very traditional manner where teachers often "told you a rule and then asked you to do exercises about it." However, when she entered Nick's fifth grade classroom in Well Elementary for her internship, she had several ideas about mathematics instruction that were different from her experiences of learning mathematics in her grade school. Martha claimed the ideas she brought into her internship are those encouraged by the NCTM standards. She "never thought anything like that before" until she took classes in her teacher education program, especially a mathematics-focused education class she took in her senior year.

> The teacher that I had for the mathematics section of the teacher education program, she was a very big influence on me. Have you heard of the NCTM

> (National Council of Teachers of Mathematics) standards? She was very influential. She thought these things were very important and that was how she taught us about those things and how can we apply that to teaching and I had a lot of those ideas there.

Beliefs with which Martha started her internship

In the early part of her internship, Martha suggested that mathematics was an activity in which people actively make sense of mathematical ideas, form hypotheses, find a pattern and then prove it. This idea of mathematics was clearly reflected in her statement that:

> It is very important that you let your students explore the math ideas. I like them to come up with their own hypothesis, their own patterns and prove it or find an idea with this worksheet. Let's find out a rule.

Martha also thought that two processes were important in learning mathematics. First, "it is important that students discover ideas by themselves because any time that happens I will see there is a lot of ownership there and they learn better." Second, it was important for her students to learn how to communicate and test their mathematical ideas among themselves. As she said, "I really wish they will be able to talk more among themselves and trade ideas, talk and comment on each other's ideas."

Martha's conceptions of elementary mathematics instruction directly built upon her ideas of mathematics and its learning. However, these ideas were only general ideas about the roles she wanted to play. They lacked pedagogical specifications. For example, Martha believed that her role in mathematics instruction was to allow and support her students to discover mathematical ideas by themselves instead of telling them these ideas. The second role she wanted to play in her mathematics instruction was to use questioning and guidance to help students figure out their ideas.

> The teacher needs to be able to step back and let students work on the problem and try to figure out for themselves. To ask a lot of questions of students, you know, "what did you think, why did you solve the problem that way and how did you solve it? Why did you choose that way or another way?"

In addition, she felt that she should not be the judge of wrong or right mathematical ideas. Rather, she wanted to encourage students to come up with a correct mathematical idea through public examination among themselves.

> I really wish they will talk more, discuss more among themselves, like trading their ideas and commenting on each other's ideas. They focus too much on teacher and I don't like that 'cause you have to and you should be able to communicate math and be able to write the stuff out and explain it.

<u>Martha's beliefs at the end of her internship</u>

Towards the end of her internship, Martha pointed out that her internship, especially her mathematics lead teaching unit on equivalent fractions, "has reinforced what I learned in the program, in that math education class." Through teaching this unit, she got a chance to apply as well as develop several ideas she brought from her college classes.

First, Martha thought her internship gave her a chance to try her idea of mathematics as an activity of hypothesizing and proving. As she said:

> I had them make the hypothesis about the ways and themes of mathematical ideas and stuff like. Like the lesson you saw today about making equivalent fractions. We were doing that about how you figure out when you had two fractions like 2/3 and 1/4. How did you know which was bigger or you can find out that maybe we can find the common denominator and then compare them and so, you know, I like how the kids are thinking like that.

Second, Martha also claimed that conceptual understanding of mathematics should be put in the center of mathematics teaching because it helped other kinds of mathematics learning. For example, she asserted if students "can understand a math idea conceptually, they should be able to do the computation well." Thus, the instructional time, activities and materials should be organized in a way to support students' conceptual understanding. This idea was shown when she was asked what she might do differently for her fraction unit if she was going to teach it again:

> I think that I am just going back to the fraction comparison, as I understand, which fraction is bigger or smaller. That is probably one area that I would do it over again. I will put more preparation and time for it because I realized how difficult that was going for them.... To add or subtract unlike denominators you need to have equivalent fractions. Once you know equivalent fractions, you have to be able to add or subtract the fractions with like denominators. So I think they build on each other.

Third, Martha learned that there were several things she needed to pay attention to in order to help her students develop conceptual understanding. She needed to start mathematics instruction from a concrete model and gradually work toward a symbolic level of understanding. She thought that was one of the biggest ideas she got from her teaching unit on equivalent fractions:

> You need to give kids a model to think about. Something that is concrete-- whether they can move around with their hands, a picture or something else before you do the symbolic part. At least that is right with elementary mathematics. I think that is in order to understand 1/2, you need to see a picture of it and put it into a related problem. I think that is the biggest thing I learned.

Martha also learned that to help students develop conceptual understanding, a teacher also needed to pay attention to developing some norms of learning in the early part of the year. As she described

> It wouldn't be fair for them to change the rules in the middle of the year. So I feel like what I need to do before I begin to teach next year and really decide how I want my class to be run and figure out how I want to run my class.

She learned that conceptual understanding required students to actively participate in the learning activities and that group work was important to involve students in active learning. However, she also realized that not all group work was automatically functional for each group member. Thus, she thought that it needed to be carefully organized to involve all the students in learning.

> I really learned how to make and form groups and structure activities. Each person in that group has to have something to do so that they can learn. You can have a group that you have one person do all the group work. I learned how to do it so that everybody has a part in it.

Martha's conceptual development and constructivism

By comparing Martha's beliefs in her internship with the constructivist vision of mathematics instruction (see Table 1 below), I come to the following interpretation about Martha's conceptual development in her internship

Table 1 - Comparison of Martha's Beliefs with a Constructivist Vision

	Constructivist Vision	**Martha's Earlier Beliefs**	**Martha's Ending Beliefs**
Mathematics	Fallible and changing, its central activity is mathematical reasoning, modeling of physical and social realities, mathematical problem solving in real world.	Active sense making of mathematical ideas, patterns and rules through discovery and proving,	Active sense making of mathematical ideas, patterns and rules through discovery, hypothesizing, and proving.
Mathematics Learning	1. Construct and discover mathematical ideas, patterns, model and solutions by using one's prior knowledge and experience. 2. Present, communicate and prove mathematical ideas, patterns and model and solutions among group. 3. Develop mathematics knowledge and abilities to the optimal level that one can not reach without support.	1. Discover mathematical ideas, rules and patterns by students themselves. 2. Present, communicate and prove their ideas, patterns and rules among students.	1. Discover mathematical ideas, rules and patterns by students themselves and with support of proper model. 2. Present, communicate and prove their ideas, patterns and rules among students. 3. Learn with proper group work and learning habits

Table 1 (cont'd)

Mathematics Teaching	1. Facilitate and challenge students to construct, model and solve problems mathematically by themselves. 2. Assist students to present, communicate and prove their ideas to each other and to rely on themselves to determine whether an idea, model or solution is right or wrong. 3. Support them to connect mathematics learning to different disciplines, their prior knowledge and the real world.	1. Support students to discover mathematical ideas by themselves through questioning and guidance. 3. Encourage students to use public examination to judge mathematical ideas as right or wrong.	1. Guide students to discover mathematical ideas, patterns and rules with a model of working gradually from concrete, pictorial and toward symbolic understanding. 2. Focus on students' conceptual understanding of mathematical ideas, patterns and rules. 3. Encourage students to communicate and prove their ideas through proper group work and.

Martha' s conceptual development in her internship was clearly moving closer toward a constructivist vision, though she still failed to develop some of the ideas envisioned by the constructivist vision of mathematics instruction. Her development can be interpreted as follows.

First, Martha's earlier beliefs of mathematics instruction reflected the constructivist vision of mathematics instruction in many ways, though not entirely matching with it. Martha believed that mathematics was an activity of hypothesizing and proving. She suggested that learning mathematics means to learn how to discover mathematical ideas, patterns and rule and how to communicate and test these things through public examination. As a mathematics teacher, Martha saw her role as challenger and supporter for students' discovery and communication of their ideas and as supporting them to rely on themselves to judge whether something is mathematically right or wrong.

All these ideas are clearly present in reformers' constructivist vision of mathematics and its learning and teaching.

Second, it was also clear, as the Table suggests, that at the beginning of the internship, Martha failed to see some constructivist ideas as important. Furthermore, most of her beliefs were at an abstract level and did not have a clear relationship with specific pedagogical ideas and teaching methods. For example, Martha did not mention anything about the relationships between mathematics, different subjects and real life experience when she was talking about elementary mathematics instruction. When she described her ideas about mathematics teaching, she only had ideas of the roles she wanted to play and did not discuss how to specifically enact her role or how to put some of her ideas into practice.

Third, Martha's internship reinforced her constructivist beliefs of mathematics in many ways. She experimented with how to get her students to conjecture and prove their ideas of mathematics. She retained her ideas about mathematics learning as helping students learn how to discover, communicate and conduct public examination. In addition, she was able to develop some important pedagogical strategies and ideas that allowed her to put her constructivist vision into practice. For example, she began to see that conceptual understanding should be placed in the center of mathematics learning because not only was it important itself but it helped students develop other mathematics skills. She started to see that instructional time, activities and materials should be properly organized to help students develop this kind of understanding. She realized it was important to use a model of gradual transition from concrete to abstract to represent the mathematical ideas she was teaching. She suggested that groups would not be functional unless they were able incorporate all students in its activity.

Jaime: Move toward Child Self-realization

Jaime started her internship in Bank's fifth grade classroom. She claimed that her mathematics learning experiences in elementary and secondary schools were quite different from what she wanted to do in her internship. Her prior experience featured lecturing followed by repetitious drills, from which she never developed any successful learning experience, only resentment for mathematics.

> I taught differently than I learned it (mathematics). I learned drilling skills like, "this is what we have to do." It was so tedious. But you can make math a lot of fun. That was what I am trying to do. I want them to want to like math.

Jaime believed that her early horrible mathematics learning experience and her teacher education program both contributed to her beliefs of mathematics instruction that she brought into her internship.

Beliefs with which Jaime started her internship

The first interview with Jaime suggested that she regarded elementary mathematics as a subject in which the different facts, rules and formulas are related and build upon each other.

> You know that it (mathematics) is the whole building block. Like you can't and I would not want to teach about the shapes before I am talking about lines and rays because you really need to know about segment of line before you can really discuss the shape.

She also thought that mathematics was closely related to our daily life and other subjects. She claimed that school mathematics education often failed to help students realize this aspect of mathematics.

> I think when I was in school, we weren't taught this way. This is math, this is reading and this is writing. As we went to college, you realized that it became important that you learned how to apply it to the real world. As I was advised to learn math, and the only place we would think to use was in math class, and that was not true. I see math is being something that you do use everyday.

Jaime believed that many schools often gave students the wrong impression that mathematics was something fixed and unchangeable, where there was always one right solution to a problem. In her eyes, that was not true a reflection of mathematics. Mathematics was not always fixed but, rather, "you can answer it from different ways."

Jaime had three assumptions about mathematics learning. First, she thought that mathematics learning should start from the very basic concepts. Mathematics learners needed to develop a deep understanding about these concepts so that they can further develop their learning to a higher level.

> I don't think they (her students) can talk about geometry without knowing those definitions. Usually the most basic things I found out are the most important because if you are trying to teach these concepts that they do not understand, they are not going to get the highest potential they could have. I think that is the most basic and simplest thing they need to know about in my opinion.

Second, Jaime thought in developing an understanding of these basic concepts, students needed to know why and how these concepts come about through their own discovery instead of being told about and then memorizing them.

> My main question always is how I get them to understand it (a basic concept) through their own thinking about it. Not just to remember it but to understand what it is, you know, instead of having them tested and memorize it as a fact. If they understand it, they can relate to it and they wouldn't forget it. So it is more I don't want them to memorize formulas and I want them to understand how the formulas came about. I think that is always the most important thing for me.

Third, Jaime further expressed that the process of mathematics learning needed to be comfortable or enjoyable for students. Whether students liked mathematics or not was quite important for the quality of their mathematics learning. She argued that long lasting learning can not be obtained through repetitious and tedious drills.

> I think it is really important for them to see it (mathematics learning) is fun cause you can just use the textbook and say, "OK, kids, do one through eight." But I don't know how I will enjoy that. And you know, they are going to learn it for fun so that they can remember it too.

Based upon her conceptions of mathematics and its learning, Jaime developed several ideas of mathematics instruction for her internship. First, she assumed that she should spend more time in her instruction on basic concepts and help her students explain them. This assumption is clear in her comments on the geometry part of the textbook used by this school.

> Well, the theme of the textbook is that they assume the kids will get something in one day. Like talking about perimeter and area. And you know, how to find out the perimeter and area. I think they could do it but that is just telling them and that is not understanding the material. And that is why I think they don't fit in because they can't explain the perimeter and area. That is the whole new concept to the kids. And you just spend one day on it when they didn't know how to find and explain what it was.

Then she suggested that in order to help students understand the basic mathematics concepts, a teacher should help students see the relationship between mathematics and other subjects. As she described:

> I don't want them (her students) to feel math is just math and reading is just reading. They are all worked together and universally. So writing and math can be combined.

Another idea Jaime had about mathematics instruction was that she should create enjoyable and comfortable learning experiences for students. To do it, it was important for her to be flexible and respectful of students' idea of mathematics and not to let them feel "dumb" because of wrong answers.

> You should be flexible and have to be respectful, too. My teaching style aims at respectful for that. I have a lot of them come up so it is just respect their rights whether they want to share. If they got the wrong answer, I don't want to make them feel dumb on themselves. So I think it is a very important thing. It is also important to be creative to find ways to teach and the ways they wanted to learn it, through games, activities, manipulative and things like.

Jaime's beliefs at the end of her internship

By the end of her internship, Jaime still held the idea that mathematics was related with other subjects and the real world and retained the idea that mathematics learning should be comfortable for her students. She claimed that her internship further helped her see that different students had different styles of learning mathematics. She explained:

> Some kids, I think, are funny, like some kids catch up geometry terms very fast. Other kids don't. They don't know why they need to use these letters. The kids thought geometry is easy but had hard time to write about it. But the kids thought geometry is hard but had easy time to write it. So it is really interesting to see that.

Her ideas about mathematics teaching were developed along the line of child-self-realization. She began to put students' feelings about and individual ways of learning in the center of her mathematics teaching. Mathematics knowledge and ways of thinking were subject to children's own preference and ways of understanding, instead of something students had to be led toward.

First, Jaime came to feel even more strongly than before that one of the important goals for her to reach in teaching mathematics was to help all her students feel confident about and like mathematics. She argued that

> Reading is easier to teach because kids usually like it. But as for math, it is tricky. I try to reach all of them because students think it is hard and they just don't want to learn it. So it is what and how to do things to make them realize that, yeah, they

> can do this kind of math. So a lot of my teaching is to do with how to get their confidence of being able to do it.

Second, her experience in her internship led her to think that to help her students feel confident about mathematics, she should figure out what kind of mathematics each child was able to do. Taking a developmental view, a teacher should avoid the content that was difficult for children to digest and that made them uncomfortable.

> I want to further understand and to do research about what is the best grade for the children to learn certain things. Sometimes you want to teach a concept but they don't get it because their brain can't digest it.

Since different students had quite different ways of understanding and learning mathematics, Jaime also developed a strong view that she should incorporate all kinds of ways of learning students had into her mathematics teaching.

> I learned I should incorporate all of the different ways of their learning into my teaching and my teaching should grow out of their learning styles. And I want to teach it in various ways, using manipulatives, and using journals, using pictures. And kids learn in such different ways and I was glad that I get the chance to experiment and try to teach all the learning styles.

Jaime's conceptual change and constructivism

By comparing Jaime's beliefs at both points of her internship with the constructivist visions of mathematics instruction (see Table 2 below), I come to see an important change in Jaime's conception of mathematics instruction. That is, she was moving closer toward a position Dewey defined as child self-realization.

Table 2 - Comparison of Jaime's Beliefs with a Constructivist Vision

	Constructivist Vision	**Jaime's Earlier Beliefs**	**Jaime's Final Beliefs**
Mathematics	Fallible and changing, its central activity is mathematical reasoning, modeling of physical and social realities, mathematical problem solving in real world.	Its problem can be approached from different ways. Its facts, rules and formulas are related to each other and to our daily life and other subjects.	Mathematics knowledge is related with other subjects and the real world.

Table 2 (cont'd)

Mathematics Learning	1. Construct and discover mathematical ideas, patterns, model and solutions by using one's prior knowledge. 2. Present, communicate and prove mathematical ideas, patterns and model and solutions among group. 3. Develop mathematics knowledge and abilities to the optimal level that one can not reach without support.	1. Construct understandings about basic concepts through their own discovery. 2. Explain why and how they came up with such understandings. 3. Be comfortable in and enjoy their learning.	1. Learning needs to be comfortable and enjoyable for her students. 2. Different ways of learning about mathematics are all important and all should be respected
Mathematics Teaching	1. Facilitate and challenge students to construct, model and solve problems mathematically by themselves. 2. Assist students to present, communicate and prove their ideas to each other and to rely on themselves to determine whether an idea, model or solution is right or wrong. 3. Support them to connect mathematics learning to different disciplines, their prior knowledge and real world.	1. Help students see the relationship between mathematics and other subjects. 2. Focus on supporting students to understand basic concepts and explain their understanding. 3. To respect students' idea and not let them feel dumb and find ways of teaching that students want to learn.	1. Help students be confident in and like mathematics by avoiding content that did not fit children's learning ability and that made them uncomfortable. 2. To incorporate all kinds of students' learning styles into mathematics teaching.

First, Jaime brought into her internship many important constructivist views of mathematics, its learning and teaching. For example, Jaime believed mathematics rules and skills were built upon each other and related to other disciplines and the real world. She also saw mathematics as not fixed and mathematics problems as approachable from different ways. She thought that in learning mathematics, it was more important for students to understand how and why mathematics concepts came about than to remember or know how to get the right answers. Students' own discovery of ideas and explanation of them were also important. With this understanding about mathematics and its learning, Jaime hoped to help her students see the relationship of mathematics to real life and other disciplines, and develop their conceptual understanding. At the same time, she believed that she needed to help them feel comfortable about and enjoyable in their learning. These ideas, to some extent, reflect constructivist visions of mathematics instruction.

At the same time, Jaime failed to develop some important ideas of constructivist mathematics instruction. Many of her ideas were abstract and were often expressed without a clear connection to specific pedagogy that she might use in teaching a particular topic. Although Jaime was able to see that mathematics as a subject was not fixed and related to other kinds of knowledge and the real world, she failed to specify how they were related. She did not seem to see conjecturing, proving and public examining as important ways of developing conceptual understanding of mathematical ideas, even though she claimed conceptual understanding of mathematical ideas and explaining their ideas were important for students to learn mathematics.

Third, by the end of her internship, Jaime strengthened and changed some beliefs about mathematics instruction, but some of her conceptual changes clearly did not follow the constructivist orientation. Instead her ideas began to move toward a vision of child-self-realization Dewey (1990) warned teachers of. She interpreted her internship experience as reinforcing her idea that mathematics learning needed to be enjoyable for her students. She developed several pedagogical measures to put this idea in practice. For example, to make students comfortable about and confident in their learning, she wanted to reduce the difficulty of what they need to learn and did not see challenging students and public examination as important ways to help them go beyond their individual performance level. She wanted to help students feel comfortable by respecting and incorporating all different ways of their mathematics learning. In doing so, she failed to see there are some ways of thinking and learning that were more valuable and crucial to mathematics and that students should learn how to use them. For example, she ignored the importance of students' learning how to prove and communicate mathematically. Whether students felt comfortable became the final principle in making decisions about her mathematics instruction. She no longer mentioned conceptual understanding as an important goal of her mathematics instruction.

Louis: Move toward A Compromised Vision

Louis said that his conceptions of mathematics instruction were different from the ways he was taught. Although he was "always the one in class who got the idea easily," before his college education, Louis claimed that his successful mathematics learning experience often hindered his mathematics teaching in his internship because mathematics came to him easily but did not come easy for a lot of his students. He often found it hard to teach mathematics to his students at their level.

Louis also claimed that many of his ideas of mathematics instruction were developed in his mathematics methods classes that he took as a part of the teacher preparation program requirement. He found these ideas were important.

> I think this self-discovery idea come from our TE math classes I took before. These classes taught me that one of things that you need to get them to do is to discover math ideas by themselves and get ownership of ideas and then you need to make it more concrete to them.

Beliefs with which Louis started his internship

Louis expressed two views of mathematics when he was interviewed in the early period of his internship. First, he thought that mathematics was an activity of "backing up math ideas with logic and reasons and proving it to others." Second, he believed mathematics patterns, rules and concepts were not only about mathematics. They were also useful and reflected in our daily life and the real world. He used fractions as an example to explain this idea:

> The fraction are important just because it is around us every day. We have everything out there that in a way can be seen as fractions. Everything you do. You know you drive three and half miles to work. You had a half tank gas to do it. You got a donut with fifty cents, that is half of a dollar.

Louis assumed that mathematics learning in elementary school was a process in which students actively discover mathematical ideas and then apply these ideas to solve problems by themselves. This way of learning mathematics, according to Louis, " is a good incentive for students to have in class, stay on task and stay focused." With this approach to mathematics learning, students would "get a lot of ownership of the ideas."

At the same time, Louis also believed that students needed to feel comfortable when they are learning mathematics and "it is important for you to learn in a way that is comfortable with you, otherwise you will have a lot of problems in learning math." However, he believed if students were able to learn mathematics by discovering and proving, they would "feel excited about it."

It was with these ideas about mathematics and its learning that Louis developed two ideas about how to teach mathematics. These ideas were very general and optimistic, and represent the goals of his mathematics instruction without pedagogical specification.

First, Louis believed that it was his responsibility to help all his students change their perception of mathematics as a collection of mathematics skills, facts and mathematics learning as a process of memorizing followed by practice. He wanted to support his students to understand mathematics through discovering mathematical ideas by themselves.

> I just had in my head when I came to this class and face those naive conceptions of students, I am going to make a difference. This is the way (students' discovering ideas) they are all going to learn math and all of them are going to be amazing when they are finding out mathematical ideas by themselves.

Second, Louis emphasized that his mathematics teaching needed to help students feel comfortable and develop a sense of success with their mathematics learning. He expressed this idea as follows:

> I will try to make them not so discouraged. They feel like behind and they feel like they hate math. And I want to try to give them as many opportunities for success as you can.

Louis's beliefs at the end of his internship

After his internship, Louis began to see that mathematics was not only about learner's active reasoning, proving and problem solving. He even began to doubt that is the most important part of mathematics and regarded it as "one of the many ways of doing math out there." The other way of doing mathematics he realized in his internship was to demonstrate a fact and a skill and then practice it.

His ideas of mathematics learning also changed. Louis claimed that it was not realistic to change the way children looked at mathematics with the instruction that pushed kids to discover ideas and used them to solve mathematics problem. He based his conceptual changes on two reasons.

First, through his internship, he found that this way of teaching might not always lead students to feel comfortable about and have a sense of success with their mathematics learning.

> A lot of my student don't like math that way because they never had this before, it is different to them. They don't care for it. They want to go with what they have been doing and feel good at.

Second, he felt it was hard to use this approach to include all the students in learning mathematics because they were so diverse in "their abilities and levels of learning mathematics." As he said:

> I think the most difficult thing to teach in this way is the fact you have students who have such different levels. You have high students who catch mathematics really easily, and you have your low students who had a lot of problems with math.

He claimed that sometimes it was even impossible to include some of his "good" students in discovering mathematical ideas because "they feel they can't do math that way." They preferred to learn mathematics by being told ideas and then practicing it in the textbook.

With this change in his conceptions of mathematics and its learning, Louis further developed several ideas of mathematics instruction. He thought he might use these ideas in his future mathematics teaching. First, Louis thought whatever approach he was going to use in his future classroom depended finally on whether his students felt comfortable with it or not. This idea was reflected strongly in his answer to a question about what kind of approach he would like to use in his future mathematics teaching:

> I used to try to use the problem-solving idea and teach with reasoning and problem-solving. Now I would still like to try some problem-solving type of ways of doing math. But eventually if student are not getting it, I will switch to more comfortable ways to students.

Second, to teach in the way students feel comfortable, Louis believed that it was important for him to learn to teach in different ways and be able to change among them according to different situations.

> About my math teaching, I feel the most important thing is the flexibility of being able to change different types of lessons and being able to teach in different ways. And being able as I saw at this point of time, be able to have this knowledge that you can teach all these different ways.

Louis was glad that in his internship, he got the chance to try both approaches--discovering and telling followed by practice. These experiments, he said, "would greatly prepare me for different job markets".

Third, he developed an idea that it was important for him to really know every detail of mathematics content that he was teaching and never take his own mathematics knowledge for granted. Louis claimed he learned this from his lecture-formatted class on prime number when he was able to answer the question from his student about if the number one was a prime number.

> One of the things I learned from that class is that you definitely need to totally research on what you are going to teach. You can't take anything for granted. You have to go through them even if it is elementary facts that you think you know. Just brush up on them anyway.

Louis's conceptual change and constructivism

In Table 3 below, I summarize Louis's beliefs of mathematics instruction at different points of his internship and compare them with the constructive views that program and mathematics education reformers encourage. This table suggests several things about Louis' conceptual change and its nature.

Table 3 - Comparison of Louis's Beliefs with a Constructivist Vision

	Constructivist Vision	Louis's Earlier Beliefs	Louis's Final Beliefs
Mathematics	Fallible and changing, its central activity is mathematical reasoning, modeling of physical and social realities, mathematical problem solving in real world.	It reflects our daily life and its central activity is to back up mathematical ideas with logic and reasoning and prove them to each other.	An activity of learner's active reasoning, proving and problem solving and a collection of facts and skills that need to be introduced, remembered and practiced.
Mathematics Learning	1. Construct and discover mathematical ideas, patterns, model and solutions by using one's prior knowledge. 2. Present, communicate and prove mathematical ideas, patterns, models and solutions among group. 3. Develop mathematics knowledge and abilities to an optimal level that one can not reach without support.	1. Be comfortable or enjoyable for learner. 2. Process of actively discover mathematics ideas and then apply these ideas to solve problems by learners.	1. Be comfortable and enjoyable for learners. This is the fundamental principle to judge which teaching method needs to be used.
Mathematics Teaching	1. Facilitate and challenge students to construct, model and solve problem mathematically by themselves. 2. Assist students to present, communicate and prove their ideas to each other and to rely on themselves to determine whether an idea, model or solution is right or wrong. 3. Support them to connect mathematics learning to different disciplines, their prior knowledge and the real world.	1. Change students' perception that mathematics learning is a process of memorizing followed by practice. 2. Support students to understand mathematical ideas through self-discovery. 3. Help students develop a sense of comfort and success with mathematics learning.	1. Know the content that he is teaching and be able to answer every question about it. 2. Support students to develop a sense of comfort and success with mathematics learning. 3. Both reasoning and problem-solving method and an approach relying on telling followed by practice are good if students feel comfortable with them.

Over his internship, Louis experienced conceptual change that featured a compromise of his earlier constructivist view with both self-realization and absolutist visions. His conceptual change during the internship can be interpreted as follows.

First, Louis came into his internship with several strong constructivist views about mathematics instruction. He wanted to change his students' mathematics learning experience with the kind of teaching guided by these views. For example, he regarded mathematics as an activity of backing up a mathematical idea with logic and reasoning and proving it to others. He also thought mathematics was reflected in our daily life. Thus, it was important for him to help students develop a new conception of mathematics and its learning in which students' own discovery and problem solving were central. He expected that his students would develop a sense of comfort and success through this way of mathematics learning and teaching.

However, Louis's beliefs were very general and optimistic. He had not yet developed any specific ideas of how to get his students to discover and solve problems. He also failed to prepare for any conflicts and difficulties in his teaching practice. For example, he was unprepared for the conflict between his belief that mathematics learning needs to get students to discover and prove mathematical ideas by themselves and his belief that mathematics learning needs to be a comfortable and fun experience for his students. Such conflict changed his conceptions about mathematics, its learning and teaching.

Third, by the end of his internship, Louis began to develop a compromised approach to mathematics instruction in which mathematics was regarded both as reasoning and problem-solving activity and as a bunch of facts and skills that students need to be informed about and practice. He still retained his idea that learning needed to be a comfortable and fun experience for his students and put that as a final principle against which all decisions about teaching methods need to be judged. Because of this view, he began to see both a reasoning and problem solving approach and the approach of telling followed with practice as equally important and necessary in his mathematics teaching. He felt happy that he was able to use both approaches in his internship mathematics teaching practice.

Kelly: Move toward Absolutism

Kelly worked in Lisa's first grade class in Mall Elementary and like the other interns, brought many ideas into her internship. Kelly claimed that her experiences of mathematics education were totally different from what she was trying to do in her internship, though she had been very good at memorization of mathematics rules and formulas in her elementary school. She believed that some of her beliefs she brought into her internship, like the idea that mathematics learning needed to be related with different disciplines and real world, were the result of her course work in the program.

> One thing I learned from my program class is that math is related to the real world and you can make math better for the kids by connecting it to the real world and different subjects. If they see math as just sitting at their desks and doing these worksheets over and over again, and having no connection to any other subjects, to any other things out in the world, it is not doing them a whole a lot of good.

Beliefs with which Kelly started her internship

In the early part of her internship, Kelly had two views of what mathematics was. The first was that mathematics was a field in which the different skills are built upon each other from the lower level to the higher level. She described:

> When I think about elementary math, you know, the first grade teacher needs to give them foundation that they are going to build on for each grade above. You know, just give them the skill they need for high math, you know, when they eventually get up to the algebra and trilogy and calculus.

The other view was that mathematics was not just narrowly bounded skills. It was also was closely related with our daily life and different school subjects. She pointed out:

> I like the kids to see math in the other area of their life, too. That they use math with science and that they use math with social studies. You know, you use your math skills in every other area too. So it is not just math class. That is how I look at math.

To learn mathematics, according to Kelly, was to develop a conceptual understanding of the meaning of mathematical ideas rather than to "memorize one right way to solve a problem." She used addition and subtraction as an example to illustrate her view of understanding the meaning of mathematics.

> You can memorize all the facts of addition and subtraction. But I want them to understand when they are adding, they are actually combining addends and that the sum is getting higher, when they are subtracting, they are actually taking the amount away and you always are getting smaller. So I want them to understand what they are doing and understand concepts.

To learn mathematics also meant to explore one's own ways of doing mathematics. Since there are always different ways to solve a mathematics problem, it is important and possible to develop one's own way of doing mathematics.

> They (her students) need to know that in math there are so many problem-solving situations, there are a lot of ways to approach a problem and that is one thing they need to be aware of. They have chances to explore their own ways of solving math problems.

Kelly's conceptions of elementary mathematics instruction reflected how she thought about mathematics and its learning. She believed that one of the important things she needed to do in her mathematics class was to help students understand the connection between mathematics concepts and real world or other subject areas.

> I would like to find more ways of integrating other areas into math teaching. It was like what I was saying about connecting what you were doing in math to the real world. For example, bring in actual objects they see every day in the

> classroom. you know, somehow to apply that...Bring in more science or tie the math to the other areas. Tie math to language arts and math into science and social studies.

She also thought that she needed to find different methods to involve students in learning mathematics so that her students would be able to develop their ways of doing mathematics. As she said:

> I want to introduce as many methods that kids can use because every child learned differently. So when adding, I want them to count their fingers, want them to use counting cubes or want them to use match boxes. So I try to think of as many different ways to do the problem. So that each child can find a way that is best for him or her.

Kelly's beliefs at the end of her internship

By the end of her interview, Kelly still stuck to the idea that mathematics was a collection of skills that were built upon each other and closely related with different disciplines and the real world. She claimed that her internship provided her a chance to experiment with this idea by connecting mathematics with music and other subjects in her teaching practice.

> I loved using music and art that ties in with, you know, math lessons. Any kind of art project and music. A lot of music has been written in patterns and rhythm. When we talked about math, we also talked about patterns and rhythm. I liked that I had a chance to use music to my math lesson.

By the end of the her internship, like Louis, Kelly began to see mathematics learning through self-discovery as simply one of the ways for students to learn mathematics and not even the most important one. She contributed this conceptual change through her internship to her realization that students had quite different ways of learning mathematics.

> You are going to have, even in a classroom, wide varieties of learners. Some kids are going to pick up just doing worksheet and some are able to memorize math

> facts and equations very quickly. Others are going to need the manipulatives and a lot of example problems they can think about and work out.

Kelly assumed that mathematics learning needed to be situated in a class environment where every student felt comfortable doing mathematics in their own way. She believed that such an environment was important because “we all like to do things that way that is most comfortable to us.” She said:

> You always need to make the classroom comfortable for every student. So they don’t feel trapped or don’t feel like being forced to do something in someone else’s way. That is one thing really important.

As for mathematics instruction, Kelly claimed that she learned three things. First, she learned that it was important for a teacher to know what mathematics skills is required by the school curriculum and where your students were in relation to the requirements.

> What a teacher needs to know is what math skills are required by the district. It would be in your curriculum. And a teacher also needed to know his or her students in a classroom ’cause I need to know where my kids are and how my kids learn. So you are going to get to know your class.

Second, she thought the approaches to mathematics instruction that she learned from her program were limited and even biased because it the program failed to teach her how to teach mathematics by closely following the curriculum and textbook. She claimed that her mathematics-based courses in the program “really encourage us not using worksheet and not strictly following textbook.” However, from her internship, she learned that it was “not always bad to follow the textbook and kids might need it.”

> One thing I would not mind doing at all is going to older 4th and 5th classroom. In most schools, 4th and 5th grade usually have textbook. I think that will be very different format of room because we don’t follow textbook exactly. We pull from different textbooks like I talked about. So I think it will be very different. I hope my collaborating teacher I would like to work with can use textbooks. You know,

> who might still teach as whole group before sending their kids to do their textbook. I think kids need that.

Third, Kelly thought that she needed to provide different options for her students to practice mathematics skills because students were different. When students can choose their own option to practice, they feel comfortable. She described this idea with an example from her subtraction unit lead teaching:

> You do have a classroom of wide range of learners. And so the subtraction is a perfect example that you need to give the kids as many options to practice math skills. You know, you don't tell them, "You have to do it this way, your way is wrong." Let them do it whatever way that is the easiest for them. Everyone is a different individual.

Kelly's conceptual change and constructivism

From Table 4 below, we can get a sense of Kelly's conceptual change in her internship compared with the constructivist vision of mathematics instruction. It suggests that over her internship, Kelly experienced a conceptual change from a more constructivist vision toward an absolutist position.

Table 4 - Comparison of Kelly's Beliefs with a Constructivist Vision

	Constructivist Vision	**Kelly's Earlier Beliefs**	**Kelly's Final Beliefs**
Mathematics	Fallible and changing, its central activity is mathematical reasoning, modeling of physical and social realities, mathematical problem solving in real world.	A collection of skills that are related to each other, to different disciplines and the real world. Its problems can be approached from different ways.	A collection of skills that are related to each other, to different disciplines and the real world.
Mathematics Learning	1. Construct and discover mathematical ideas, patterns, models and solutions by using one's prior knowledge. 2. Present, communicate and prove mathematical ideas,	1. Develop a conceptual understanding of mathematical idea. 2. Find their own way to do mathematics.	1. Practice mathematics skill in the way one feels comfortable and enjoys. 2. All the ways of mathematics learning are important because

Table 4 (cont'd)

	patterns, models and solutions among group. 3. Develop mathematics knowledge and abilities to the optimal level that one can not reach without support.		learners are different.
Mathematics Teaching	1. Facilitate and challenge students to construct, model and solve problems mathematically by themselves. 2. Assist students to present, communicate and prove their ideas to each other and to rely on themselves to determine whether an idea, model or solution is right or wrong. 3. Support them to connect mathematics learning to different disciplines, their prior knowledge and real world.	1. Help students understand this connection between mathematics concepts and the real world or other subject areas. 2. Support students to develop their own ways of doing mathematics.	1. Need to understand curriculum and textbook and where students are in relation to the requirements of the curriculum. 2. Know how to lecture and follow curriculum and textbook. 3. To find practice options for students to practice their skills so that they are comfortable with their learning.

First, Kelly came into her internship with some ideas that reflected a constructivist vision of mathematics and its learning and teaching, though her ideas lacked some crucial elements found in the reformers' constructivist vision. Kelly thought that mathematics skills were closely related with each other and there were different ways to approach a mathematics problem. She also believed that mathematics learning needed to focus on conceptual understanding and students' own exploration of mathematics solutions. Her ideas of mathematics instruction was similar. However, she was unable to see mathematics as an activity of carrying out mathematical argumentation and proof and failed to identify communication and public examination of mathematical ideas, as important mathematics discourse students have to learn.

Second, in addition to these constructivist elements, her beliefs also included some elements that reflected absolutist and self-realization positions. It was also clear that Kelly was unaware of her conceptual inconsistency. For example, on the one hand, Kelly tended to see mathematics only as a collection of skills her students needed to acquire. On the other hand, she assumed that conceptual understanding of mathematical ideas was more important than memorization of facts. Moreover, she claimed that she needed to develop all the teaching methods to cater to all the different ways of mathematics learning that her students brought into the classroom. She failed to see the possible conflicts between her conceptions, such as, the possibility that not all her students would feel comfortable with their own exploration and instead might want to memorize the facts and rules and then apply them.

Third, by the end of her internship, many of Kelly's constructivist ideas were weakened but she had developed and strengthened several absolutist ideas. For example, not only did Kelly stick to the idea that mathematics was a collection of skills hierarchically structured and related, but also she began to see that it was very important for her to understand the requirements of mathematics skills and build her instruction around skill practice. Kelly believed that she needed to learn to teach how to strictly follow textbook and get students to practice mathematics skills required by the curriculum. By the end of her internship, she no longer regarded students' own discovery of mathematical ideas as necessary and conceptual understanding of ideas as important.

Summary of Chapter

What beliefs and conceptions about mathematics and its learning and teaching do these preservice teachers bring into their internship? What kinds of beliefs and conceptions about mathematics and its learning and teaching do these preservice teachers end up with through their internship? To what extent did their beliefs of mathematics instruction move closer toward or away from the constructivist vision of mathematics instruction? This chapter considered these questions and found several things.

First, all the preservice teachers agreed that the conceptions of mathematics, learning and teaching they brought into their internship were contributed to greatly by their mathematics-based program course work. They all claimed that these conceptions were totally different from their mathematics education and learning experience in their elementary and secondary years.

Second, through their program, all four preservice teachers were able to develop some beliefs reflecting constructivist vision of mathematics, its learning and teaching, though many of these ideas were often general and abstract in some cases and integrated with some non-constructivist, even contradictory ideas.

Third, all of these preservice teachers experienced some conceptual development and change through their internship. However, the result of their learning at the conceptual level was quite varied. They did not all follow in the direction of a constructivist vision. Martha was able to strengthen many of her constructivist ideas and somehow pushed these ideas toward constructivist pedagogical understanding. Although starting her internship with some similar ideas and working at the same grade level of the same school with Martha, Jaime had a very different experience conceptually. Several constructivist ideas she brought into her internship were weakened. Over time, she developed some ideas that reflected a child self-realization position. Louis had wanted to use many of his constructivist ideas to transform his students' mathematics learning experiences but ended up with a compromised stance toward mathematics, its learning and teaching. Kelly clearly strengthened or developed many beliefs that reflected an absolutist position. By the end of her internship, her constructivist ideas were either weakened or disappeared.

Such findings imply that even under similar reformed internship arrangement and requirements, preservice teachers may not necessarily develop constructivist approaches to teaching. The conceptions developed and formed in their internship can even be contradictory to the constructivist vision that they had been set to pursue. The puzzle of their change leads us to examine the practice these preservice teachers developed and ask several questions. What were they were able to learn at a practical level in their internship? What was the nature of mathematics teaching practice they learned? How was their learning at a practical level related with their conceptual development? In the next chapter, I explore these questions.

Chapter 4

LEARNING TEACHING PRACTICE

In this chapter, I explore what Martha, Jaime, Louis and Kelly learned about mathematics teaching in their internship. I focus my analysis on the practice these preservice teachers developed by considering three questions. What kinds of mathematics instructional tasks and processes did each preservice teacher develop over his or her internship? To what extent did his or her teaching practice reflect constructivist standards? What was the relationship between his or her teaching practice and his or her beliefs of mathematics instruction?

My analysis in this chapter leads me to two findings. First, these preservice teachers developed quite different kinds of mathematics teaching practice that did not necessarily reflect constructivist standards. Second, the mathematics teaching practice they developed over their internship may not necessarily be related in a causal fashion with the conceptions they brought into their internship. Instead, the relationship between their conceptions and practice was mutual and interactive.

Martha: Practice toward the Constructivist Direction

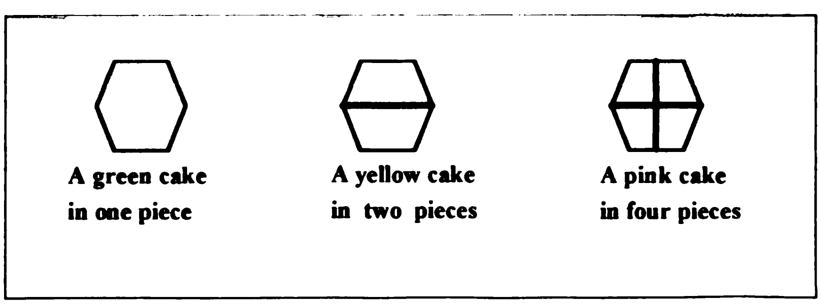
Martha taught an equivalent fraction unit in her lead teaching period that constituted her major and last mathematics instructional practice she conducted in her internship. The mathematics instructional tasks and processes Martha developed in this unit showed her great efforts in developing a constructivist approach to mathematics instruction.

Martha's instructional tasks

The instructional task Martha developed in her fraction unit was characterized by three features. First, Martha was able to design her instructional tasks by requiring her students to form, develop and interpret their own views about equivalent fractions. This feature was reflected in the first lesson videotaped.

In this lesson, to develop their primary understanding about equivalent fraction, Martha started the class by requiring students to define the term, "equivalent," with real life examples. Then she asked them to predict the meaning of equivalent fractions based upon their definitions. She then grouped her students into pairs and asked them to make three hexagon cakes of the same size with pattern blocks as shown in Figure 1 below. She asked each pair to cover these hexagons with each other and further develop or modify their previous predictions about equivalent fractions. In the end, Martha requested some groups present to the whole class the ways these cakes were similar to and different from each other. She asked them how these cakes helped them understand the meaning of equivalent fractions.

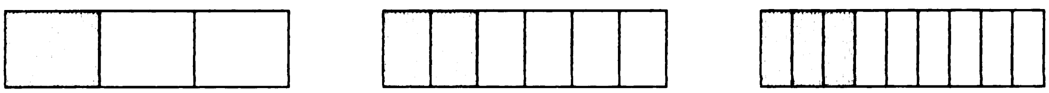
Figure 1 - Three Cakes Martha Required Students to Make



Second, Martha's instructional tasks in these lessons also strongly emphasized students' proving and disproving each other's ideas through public examination to develop a shared understanding about fraction concepts. For example, in the third lesson, Martha designed particular tasks with this in mind for her students to deepen their understanding about the concept of equivalent fractions.

First, she requested students to work in pairs to split the 1/3, 6/7, 5/12, 2/2 fraction bars evenly into two, three or four equal parts. For example, she asked students to make a 1/3 Bar into two and three equal parts as shown in Figure 2 below.

Figure 2 - Fraction Bar Martha Used in Her Third Lesson



In this activity, each pair was required to have one student split and explain why the splitting he or she did was equivalent. The other was asked to check or question his or her peer's work. If they were unable to agree with each other, they should consult with another pair to resolve the difference.

In the end, Martha demanded some groups prove to the class how their group got equivalent fractions and what their definitions of equivalent fraction were with the support of their fraction bars. At the same time, she questioned or encouraged other groups to challenge the presenters with counter examples. She summarized the kind of fraction ideas that students reached for the lesson.

Although Martha carefully considered how to support her students to discover and prove or disprove each other's equivalent fractions, her instructional tasks in her lead teaching unit were relatively weak in incorporating real life examples and events. In all her observed mathematics classes, the instructional tasks she designed were neither carefully connected to the real world problems that students might experience in their daily life nor closely related with other subjects that students learned or were learning.

Martha's instructional processes

Martha' instructional process reflected similar features to her instructional tasks. First, Martha was able to devote most of her instructional time in these observed lessons to developing students' conceptual understanding through their own exploration. In her fourth videotaped lesson, for instance, Martha spent almost all of her class time to support her students to find out and prove to each other whether the fractions in a sheet shown in Figure 3 were equivalent or not.

Figure 3 - Fraction Pair Worksheet Martha Used in Her Fourth Lesson

A. 1/2 and 3/6	**B. 7/8 and 14/16**	**C. 2/5 and 6/16**	**D. 4/6 and 5/24**

During this period, she asked her students to work in groups and come up with their answers to these fraction pairs and the principles they used to get their answers. Then in the whole class time, she asked her students to present their principles and encouraged them to challenge each other's principles to come up with a definition of equivalent fraction all would accept.

Second, Martha's interaction with students in her lessons consistently focused on clarifying their understanding of a concept and encouraging alternative ways of thinking. In her conversation with students, she questioned or insisted students show the principles and reasons they used to get answers and encouraged students to find the alternative answers to the questions. The following dialogue in her fourth lesson is an example of the kind instructional conversation she developed in her mathematics teaching. She started this conversation by asking students whether 1/2 and 3/6 were equal[6]:

P: Who would like to give me the message about number one? [1/2 and 3/6], S1.
S1: Times 2 by 3.
P: OK, do you say it is true or false?
S1: True.
P: You said 1/2 is equal to 3/6. How did you find it out? [She ask students to explain their reasoning]
S1: Times 3.
P: Times what?
S1: Times 1 by 3.
P: 1 times 3 equals 3? Do you times the numerator by 3? What did you do to the denominator? [She requires S1 to clarify his method]
S1: Times by 3
P: So if we time 1 by 3 and 2 time 3 we got 3/6. So we say 1/2 equals 3/6. Why does this support the hypothesis S2 said before? [She asks students to give reason]
S1: ...
P: Speak up so that we all can hear you.
S1: Because....
P: What does the hypothesis say, keep that in mind.
S1: You multiply the same number by numerator and d.... I can't say that
S2: Denominator.
S1: Denominator
P: Everyone say that.
SS: Denominator [laugh].
P: Do you mean you time denominator and numerator by the same number and so they are the same? Is there anybody disagree with that? Does anybody do it differently? [She pursues an alternative way of thinking]
S3: I don't disagree, but I bet there's something in math that's called terminator in fraction. It will destroy the whole problem and it will be false or whatever.
P: Did anybody figure it out differently, use different strategies?[She wants
an alternative way of thinking]
S4: You can divide it by 3.

---

[6] In this dialogue and the dialogues followed, P is the preservice teacher. C represents the collaborating teacher. Students who speaks individually are labeled by S1, S2, and S3, and SS represents the whole class. Underlining is emphasis I add to highlight my analysis of the interaction.

P: You say 3/6 and divide each by 3. Division! Is that what you did or you did it differently?[She requires S4 to clarify his method]....

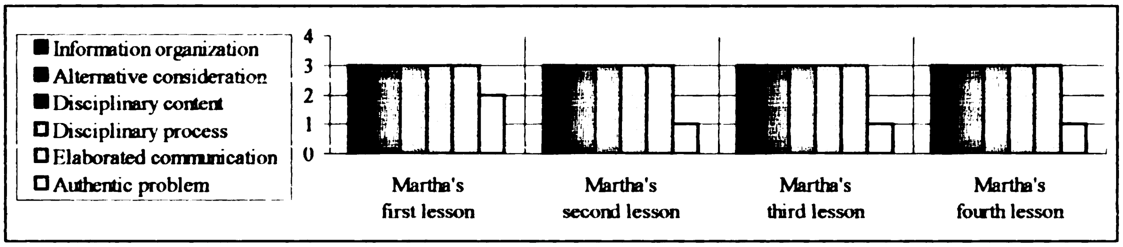
In this short conversation, we can see that Martha, first, asked S1 to identify whether 1/2 and 3/6 were equal. When S1 gave his seemingly "correct" answer, Martha was not satisfied with it. She demanded S1 describe his whole process and explain why his answer supported the definition of equivalent fractions they had reached so far. The class finally connected S1's answer to the definition they had come up with (to multiply both denominator and nominator by the same number to see whether a pair of fractions were equivalent). But Martha kept pushing her class to use an alternative way to find equivalent fractions until another student, S4, came up with the division method. Then she began demanding S4 give a clear explanation about his method.

While even this short excerpt demonstrates Martha pushing her students in several ways, in the process of her teaching, Martha paid little attention to helping her students make sense of the relationship between what they were learning, students' real life experiences and the mathematics knowledge they had learned before. Of the classes I observed, the only time she tried to build this relationship was in her first lesson, when she asked her students to find examples from their daily life experiences to show their understanding about the word, "equivalent." In all her other lessons, she rarely did anything to help her students make this connection.

Martha's practice and constructivist standards

The mathematics teaching Martha developed in her internship was consistent and strongly reflected all the dimensions of constructivist standards in almost all the aspects except for building connection between what students learn and their daily life experience. Comparing her instructional tasks developed in her four video-taped lessons with the constructivist standards for instructional tasks, I came to the following findings, summarized in Table 5.

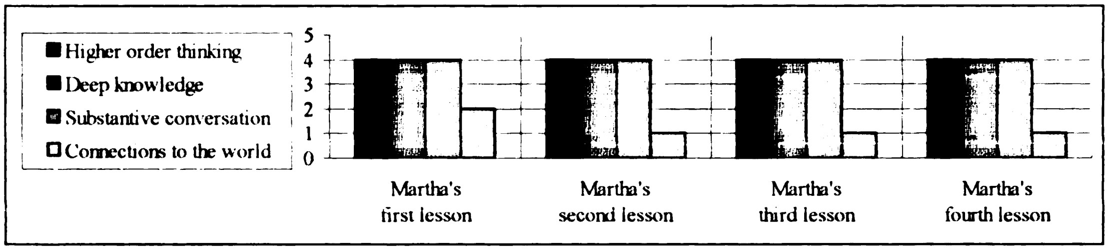
Table 5 - Martha's Instructional Tasks and Constructivist Standards



This table allows us to see how well Martha was able to develop constructivist instructional tasks. For all her four video-taped lessons, she was rated 3 on a 3 point scale in terms of requiring students to organize information, consider alternative ways of solving problem and thinking, pay attention to both disciplinary content and disciplinary inquiry. She was only slightly less successful (3 on a 4 point scale) in her efforts to design extended and elaborated communication. However, she was less effective in designing tasks that incorporate issues and problems from students' daily life. She was only rated 2 on a 3 point scale in her first lesson and 1 in the other three lessons for authentic problem.

Martha's instructional process ratings, like those for her instructional tasks, are also consistent throughout all four observed lessons and strongly reflect the constructivist standards in all the aspects except for the category of connection to the world. Table 6 summarizes these ratings.

Table 6 - Martha's Instructional Processes and Constructivist Standards



Martha's instructional processes in all the four lessons were strong in support of her students' higher order thinking, developing deep knowledge and substantive conversation. For all of the areas she was rated 4 on a 5 point scale. Such ratings suggest that Martha was able to devote a large chunk of her class time to helping students understand the important concept of fraction. She made a great effort to encourage and challenge students to form, explain, prove and disprove each other's ideas. Moreover she was able to develop focused and extensive discussions to support students to clarify their understanding about the concepts.

However, Martha had more difficulty in carrying out a practice that helps students make connections to the real world. Compared with the other aspects of her teaching process, she was only rated 2 in her first lesson and 1 in the other lessons on a 5 point scale. Such rating suggests that she failed to do as good a job in helping her students see the implications of what they learned for the real world in her teaching process as she did in helping other aspects of their learning.

Martha's beliefs and her practice

Coming into her internship, Martha brought with her several ideas of mathematics instruction that reflected a constructivist vision and that she was able to implement in her teaching practice. As I discussed in Chapter 3, Martha believed that mathematics is an activity of forming, proving and disproving each other's hypotheses. She thought that students' own discovering, proving, communicating and sharing each other's ideas were important ways of learning mathematics. She wanted to support her students to learn to form, prove and examine ideas and develop conceptual understanding about mathematical ideas.

Both my quantitative and qualitative analyses of her instructional practice clearly suggest that Martha was able to implement these ideas in designing her instructional tasks and conducting her instructional processes. In her teaching, she was able to give a central place to students' conceptual understanding of mathematical ideas through their own discovery. She was able to support them to interpret and prove or disprove each other's ideas and develop a shared understanding of mathematical ideas through public examination while demanding students develop alternative ways of thinking. Most of her class time and activities were devoted to supporting these learning tasks.

Martha's teaching practice helped her strengthen many of these ideas and somehow push these ideas into pedagogical thinking. After her internship, Martha agreed that she was able to learn how to use a concrete model to present mathematical ideas and push students to develop abstract thinking by moving from the concrete and pictorial presentation of the ideas. She realized that group work was important but needed to be carefully structured. These conceptual developments contributed to her teaching practice in her internship, especially to her lead teaching practice in which she experimented with the use of concrete models and group or pair work to help students learn to understand equivalent fractions.

While certain ideas she developed enriched her teaching, Martha failed to develop teaching practice in the areas that she failed to believe and stress. In her teaching practice, Martha did very little in integrating students' life experiences and real life examples into her instructional tasks and also did a relatively weak job in getting students to see the relationship between mathematics learning and students' real life experiences during her teaching. However, Martha never discussed that she needed to build this connection in her teaching for her students and failed to stress this as an idea she needed to follow in her teaching.

Jaime: Practice with a Non-mediating Stance

Jaime taught a long geometry unit in her mathematics lead teaching, for which she took full responsibility of planning and teaching. In the five lessons observed, she helped her students understand and differentiate some geometrical concepts, such as line, ray, and segments. She chose to teach these concepts because she saw these as basic building blocks for geometry.

Jaime's instructional tasks

Two features stood out in the instructional tasks that Jaime developed in these lessons. First, Jaime paid substantial attention to integrating real life examples that students might experience into her instructional tasks and required students to find information and ideas from their daily life to form an idea about a geometry concept she was teaching. One of the examples came from her first lesson as Jaime helped students develop some primary understanding of what count as lines and shape.

Jaime started her lesson by assigning students to work in groups of four or five and requiring them to generate some ideas about what geometry was, based upon their own experience. She gave each group an envelope containing stuff from her apartment, a picture of the Tower of London, dice and maps and so on. She asked each group to identify geometric lines and shapes from these objects. Then she asked her students to report their findings to the whole class.

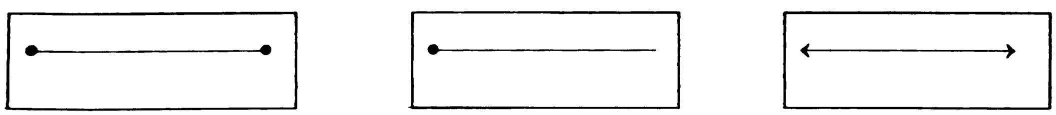
A second feature of her teaching task was that Jaime was not able to create instructional tasks that clearly engaged students in interpreting, elaborating, supporting and challenging each other's ideas, though she was able to get them to come up with ideas. For example, in her second lesson, Jaime planned to help her students "learn the definitions of line, segments, ray." Jaime assigned her students to work in groups to find out lines with dots from a map of France and identify the difference between the line with a dot on one end and the line with dots on both ends. Her students were required to report their findings after their group work. However, in this process, Jaime did not clearly demand students elaborate their reasons and justify their answers, nor did she do anything to clarify all the different answers and unanswered questions her students brought up.

Jaime's instructional processes

The instructional processes Jaime developed in her lessons observed suggest similar features to those characterizing her instructional tasks. First, during her teaching, Jaime made an effort to use real life objects to help her students make sense of what they were learning. In her first videotaped class, she used a picture, dice and a map to help her students see how lines, segments and rays are used or displayed in these real life examples. In her second lesson, Jaime spent a substantial chunk of time in her class to get her students to work in groups to find the lines and shapes around their classroom and then she emphasized how geometry can be reflected in every aspect of our life.

At the same time, Jaime made little effort to assist her students to clarify the principles that they used to form their ideas even when students came up with a right answer. Although sometimes she asked students to form an idea and even report their idea to the class, she spent little time to push students to develop a shared idea through public examination. For instance, in the third videotaped lesson--on the definitions of line, ray and segment--she spent about 20 minutes assigning her students to work in groups to find out the differences between the following figures, shown in Figure 4 below.

Figure 4 - The Graphics Jaime Used in her Third Lesson



Jaime spent the rest of the lesson asking students to report their findings to the whole class. During students' presentation, Jaime did not ask students to support their findings with evidence and reasoning nor did she push her students to challenge each other's findings with counter examples. One student came up with the idea that a line with arrows in both directions would represent something continuous, an idea, as she wrote in her teaching notes, that exactly implied the definition of line. Yet she neither drew attention to the issue nor did she help her students elaborate and make an argument about it. Instead, she kept letting students say whatever ideas they had until the class ended.

Third, Jaime's interaction with students as a whole group rarely took a substantial portion of her teaching time. When she did have conversation with the class, it often represented students reporting their ideas. Neither did she facilitate their discussion and help them elaborate their ideas, nor did she push her students to find alternative answers or think at a higher level. The following conversation that happened in her fourth lesson observed is a typical example of such conversations. Here she discussed with her students the differences between two graphics (see Figure 5 below).

Figure 5 - The Graphics Jaime Used in Her Fourth Lesson

Graphic 1 Graphic 2

P: We are looking at this one [she points to Graphic 1]. Do you think it is going on forever? S1.
S1: No.
P: Why?
S1: Because the dots means stop.
P: S2.
S2: If it covers a ball, it will go around and it will not stop.
P: OK, S8.
S3: It does not mean it does not stop. It could mean down the road after two places.
S2: That is why I say it does not stop.
P: So you are saying it keeps going. Then if it keeps going, it becomes a line. So OK, S3 was saying it might keep going on the either side. If that is the case, then we will put arrows on it [She points to Graphic 2 and tells the answer to her students].

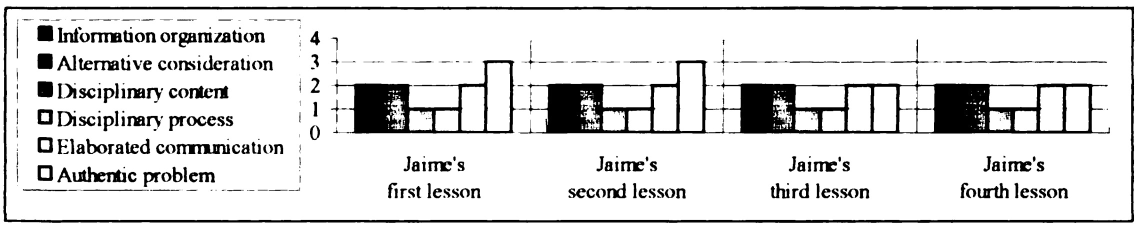
It was obvious that in this conversation, Jaime did not help students to elaborate their answer and nor did she drew attention to the different definitions in students' minds. S1 suggested that the graphic 1 meant stop, while S2 and S3 argued it also implied continuity. Jaime simply wrapped up this discussion by giving her answer. She continued her conversation with students in this way for a few more minutes before she assigned her students to work in groups to find a name for segment.

Jaime's practice and constructivist standards

The mathematics instructional practice Jaime developed in her internship, to a large extent, failed to reflect the constructivist visions in almost all the aspects of the criteria except for the way she was able to build a connection between what students learn, their real life experience and the real world.

As shown in Table 7 below, Jaime's instructional task ratings in her four videotaped lessons were consistent but average or low in all the areas of criteria except for the category of authentic problems.

Table 7 - Jaime's Instructional Tasks and Constructivist Standards



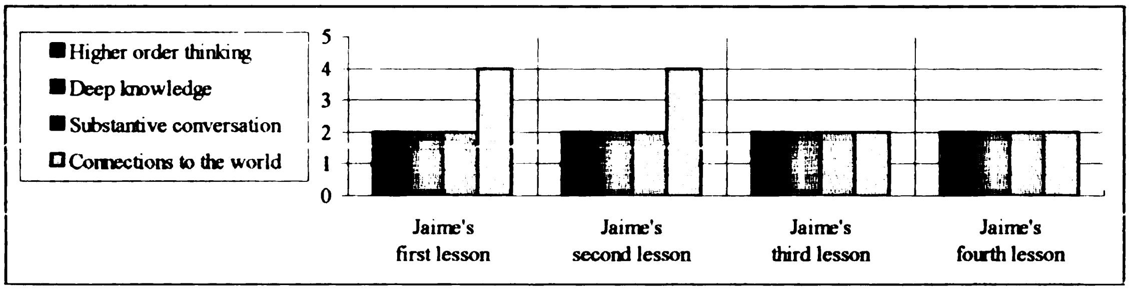
Jaime received her highest score (3 points) for her instructional tasks in the first two videotaped lessons in the area of authentic problem and 2 points in her last two classes. Such ratings indicate that Jaime did a good job in integrating students' daily life experience and real life examples into her instructional tasks in her first two lessons. Although she failed to do as good a job in her last two lesson, she was still working in that direction.

In terms of her effort to encourage organization of information, consideration of alternatives and support for elaborated communication, however, her ratings were less able to reflect constructivist standards. Although her students had opportunities to develop their ideas about geometrical concepts and report their ideas to the other classmates, she did not consider how to get students to interpret information and explain their answers to each other in designing her teaching activities.

Jaime appeared even weaker, getting 1 point, in disciplinary content and process. Her instructional tasks in these lessons were weak in terms of developing students' conceptual understanding of geometrical ideas and pushing them to publicly prove and disprove each other's idea.

The instructional processes Jaime developed in these mathematics lessons were similar to her instructional tasks. Across different lessons, her processes were weak in comparison with the constructivist standards except for the category of connection to the world.

Table 8 - Jaime's Instructional Processes and Constructivist Standards



Jaime appeared especially weak, receiving 2 out of a 5-point rating, in higher order thinking, deep knowledge and substantive conversation. Jaime was unable to devote a large chunk of her class time to helping students develop conceptual understanding of the geometrical definitions. Although she gave her students time and opportunities to develop their own ideas, she failed to encourage and support students to explain, prove and challenge each other's ideas in the process of her teaching. She was unable to develop focused and extensive discussions to clarify or get her students to clarify by themselves their misconceptions and confusions about the concepts and definitions. She rarely challenged or got students to challenge each other to reach a higher level of understanding of a concept beyond what they arrived at individually.

Her instructional process was strongest in the area of connection to world. For her first two classes, scoring 4 out of 5 points indicates that she was able to spend time in supporting her students to see connections between what they were learning and its real world implications. However, such efforts were less successful in her last two classes, when she only received 2 points.

Jaime's beliefs and her practice

Jaime brought several constructivist ideas of mathematics instruction from her program into her internship and wanted to try them out in her teaching practice. She believed mathematical ideas, rules and skills were related with other disciplines and the real world. She felt that mathematics instruction needed to be organized in a way to help students see this relationship. She also thought that in her mathematics instruction, she needed to help students develop conceptual understanding of basic concepts through their own discovery and to help them explain their understandings instead of telling them the idea and asking them to memorize.

Although she was able to design her instructional tasks and conduct her instructional processes by connecting what students were learning with their daily life experience and real life examples, she failed to implement many of her other ideas or only superficially implemented these ideas in her teaching practice. In her teaching practice, she took simple concepts for the basic concepts. Throughout her lead teaching unit, she only focused on line, ray and segment and presented them without challenging her students to connect them to other concepts they had learned and were going to learn. Although students were encouraged to come up with ideas and report them to the class, these activities became the substitute for students' conceptual understanding through public examination. That is, her students were never encouraged to interpret and prove or disprove each other's ideas. Students' thoughts were left unexamined.

By the end of the semester, Jaime no longer stressed students' conceptual understanding of basic ideas through their own discovery and explanation. Instead, she reinforced some of her ideas and developed them into a self-realization stance. For example, Jaime's idea that mathematics learning needed to be enjoyable and fun for students was clearly strengthened by the end of her internship. After her teaching practice, not only did she believe that mathematics learning needed to be enjoyable, but also she thought that she needed to respect all the different ways of learning that students had developed. She claimed that she would use different teaching methods to make them happy. The quality of their understanding of mathematical ideas was no longer an important goal for her to pursue.

Louis: Move from Constructivism to Absolutism

Like Martha, Louis taught an equivalent fraction unit during his lead teaching. However, unlike Martha, Louis was also able to teach independently a geometry unit before his formal lead teaching started and, thus, Louis had more diverse experiences in teaching than Martha.

Louis's instructional tasks

The instructional tasks Louis developed in the five observed lessons can be divided into two totally different types. In his first lesson, Louis was able to incorporate examples from the real world into his design. He emphasized his students' conceptual understanding through their own discovery, elaboration and examination. In this lesson, he designed several learning tasks for his students to develop a primary understanding of equivalent fractions. At the beginning of his class, Louis showed his students a dollar and a quarter on an overhead and required his students to use their own experiences about money and fractions to represent the relationship between a dollar and quarter. After his students came up with different answers, like 4/1, 1/4, 1/100, 1/5, 5/100 and 5/20, Louis began to challenge students to see the contradiction between their symbolic representations, 1/100, and the concrete meaning of a quarter and a dollar. He also encouraged his students to find similar conflicts in their other representations. Through their own discovery and public examination of this money ratio case, he pushed students to come up with a definition for equivalent fraction that would be consistent across different cases, like a dollar with a nickel, and two dollars with a quarter.

The second type of instructional task Louis developed is represented by the rest of his classes. In these lessons, Louis' instructional tasks changed greatly from his first class in the following ways. Louis began to stress telling and demonstrating a rule or an idea, followed by students individually practicing it. Although he still asked his students questions in his demonstration, he seldom required them to elaborate or explain their ideas and answers. Finally, he no longer incorporated students' experiences or real world examples into his instructional tasks.

For example, in the second lesson, Louis designed the following activities to teach further equivalent fractions. At the start, Louis wrote on the blackboard the following pairs of fractions and required his students to find a way to define whether each pair were equivalent: (1) 2/5 = 4/10, (2) 3/6=4/7 and (3)1/5=1/15. A girl said 3/6 would be equal to 4/7 because she could add 1 to both denominator and nominator to get them. Some students clearly agreed with her ideas. However, a boy disagreed with it and he went on to claim that 2/5 was equal to 4/10 because he could multiply both denominator and nominator of 2/5 with 2 to get 4/10. Instead of helping students compare and explore their hypotheses of equivalent fraction, as he did in the first lesson, Louis simply confirmed the boy's answer and asked the whole class to write it down. Then he required his students to use this idea to accomplish a similar exercise in the textbook and the incorrect answer was never fully refuted but merely abandoned.

Louis's instructional processes

Louis's instructional processes in these observed lessons also developed along two lines. In his first observed fraction lesson, Louis was able to devote almost all of his instructional time to developing students' conceptual understanding of equivalent fraction. Not only were his students constantly pushed to give their fraction representation of a money ratio, but also they were required to explain their reasons and prove their thinking to the other students.

Whenever students had problem in resolving differences, Louis was able to draw their attention back to the real world example—dollars, quarters, dimes and nickels. He challenged them to compare their fraction representation with the value of money it represented and think about it in the context of the real world.

Louis developed a long and sustained conversation on the concept of equivalent fractions in which he not only helped his students explain and share their ideas of equivalent fractions. He also challenged his students to make distinctions and raise questions about each other's ideas to reach a shared understanding. Here was a typical part of his conversation with his students in this lesson:

P:	...What if I put a nickel out of 2 dollars be?
S1:	That will be 1 out of 40.
P:	1 out of 40.
S2:	1 out of 40 and 5 out of 200.
P:	<u>Are these equal? How can we check? Any ideas? Is there any way to check if they are equal?</u> [He requires students to explain the reasons]
S3:	Divide 1 out of 40 into 5 out of 200.
P:	OK, what would it be? 1 out of 40 and 5 out of 200 are equal. OK, <u>how can we check if 1 out of 40 equals 5 out 200.</u> [He requires students to explain the reasons again.]
S4:	You time..
S6:	It is from your head.
S7:	You know it when you see it.
P:	<u>What would you time? S4.</u>[He asks for a clarified answer from S4.]
S4:	Times 1 out of 40 by 5
P:	5?
S4:	Each times by 5.
P:	You times each by 5. So what would I find if times 1 by 5 here.
S4:	ER, 5
P:	So what would be if I times 40 by?
S4:	200.
P:	<u>Is that reasonable</u>? [He pushes students to challenge S4's answer]
S8:	1 times 5 equals 5
P:	1 times 5 equals 5. [He write on overhead]
S8:	5 times 40 equal 200.
P:	5 times 40 equal 200 [write on overhead]. <u>Is that possible? How many people can do this?</u> [He requires students to explain the reasons]. <u>Here down the bottom. S4 said he started with 1 out of 40 and he times 1 by 5, equals 5. And he times 40 by 5 equals 200.</u>[He clarifies his question and challenge]
S8:	Yes, that is right.
SS:	Yes....
S9:	Why do you times it by 5?
S5:	But yes, in this case it is. You can times by...
S9:	I bet you can time by any number...
SS:	Yes.
SS:	No, you can't do it.
S9:	Why I have to time by 5?
SS:	...[They are debating with each other for a moment]
P:	<u>OK, so S9 is asking why we have to time by 5?</u> [He draws students' attention to the question S9 raises]

S9: Why not 50?
P: Yes, why not 50?
S11: That is the wrong number...
P: That is a legitimate question, isn't it? [He challenge students to give a reason for his answer.]

In this conversation, Louis was clearly encouraging his students to use proper fractions to represent a nickel compared with two dollars. When students came up with two different fraction representations, 1/40 and 5/200, he further challenged his students to explain and prove why both fractions can be used. When a student tried to question why a fraction had to be multiply by five to get another equivalent fraction, Louis helped clarify his question to the class and encouraged them to resolve this issue by themselves. This class continued like this until the end.

However, Louis structured his remaining lessons in a totally different way. In these classes, he usually started his lesson with students individually practicing something on mathematics facts. Then he would demonstrate or model a new rule or procedure for a short period and spend the rest of his class to get students to do individual practice on the rule or procedures he modeled with textbook. Louis spent little of his class time developing his students' conceptual understanding of fractions. He neither challenged or engaged students to challenge each other's ideas, nor did he connect what students were learning with real life examples. Instead, his instructional processes were oriented towards lower order operations.

For example, in his last videotaped lesson, Louis spent the first ten minutes asking students to practice twenty-five simple division questions and check each other's answers. Then he used about ten minutes in asking students to give all the factors for the number 78. When they gave their answers, they were not asked to give any clarification how and why they got these ideas and no real life examples were used to help students get a clear sense of what factor meant. The rest of the class, students worked individually on some questions chosen from their textbooks. Louis walked around and helped when a student had a question about his exercises.

During the demonstration period, Louis would interact with students but his conversations with students were usually short and often strictly controlled by Louis. Louis sometimes asked his students to give an answer to the question he raised. If students failed to come up with the right answer, he would provide one or ask another student to get the right answer. Once the right answer was brought up, he would confirm it and then require his students to practice individually on some similar questions from their textbook. Here is a typical example of his conversations with his students in his second lesson:

P: OK, factoring. Anybody would know or has any ideas about this? S1.
S1: OK, a number with common...
P: Like what?
S1: One of the factor can be used as an even number.
P: OK, 2 is a factor for 4 [He gives the answer directly]. Any ideas? S2.
S2: I have no idea.
P: OK, I spare you to say this for the first time. Mr. Smith [Ben] especially have trouble with this because of a small attention span. Any way, A factor is the number that you can use to get another number. In the other words, for the number 24. 1 and 24 are factors because 1 times 24 is 24 [He gives students a rule]. Any other factors?
S3: 12.
P: 12 what? 12 times what is 24? [He implies the answer]
S3: 2.
P: 2 is a factor.
S4: 6.
P: What would come with 6?
S4: 4.
P: S5, any ideas, any factors?
S5: 3
P: Any other factors?

S6: 8.
P: Any others. I think they are pretty much all. How do you see it? Are these easy or they are pretty hard?
SS: No.
P: No what? Not complicated? It is easy. Number 10. Factoring? S7

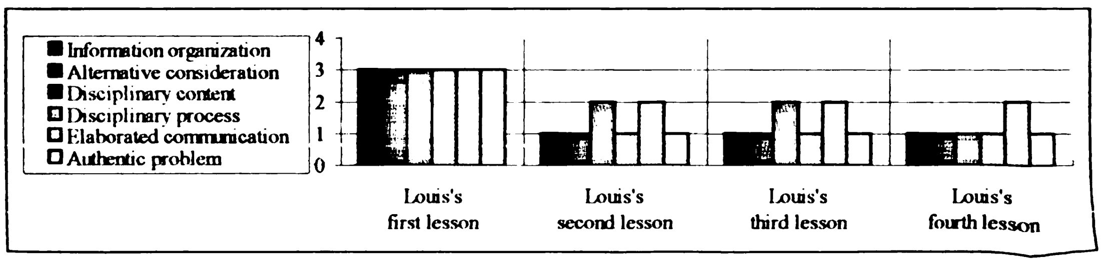
From this short conversation, we can see that Louis first posed a question about the definition of factoring and asked two of his students, S1 and S2, to answer it. When his students failed to do so, he immediately gave his own explanation and then got his students to practice factoring with the idea he provided. The conversation kept going like this for another five minutes before he assigned his students to practice some problems from the textbook for the rest of his class.

Louis's practice and constructivist standards

By assessing his mathematics instruction against constructivist standards, I find that the teaching practice Louis developed was not consistent. It reflected two different kinds of values. While his first lesson strongly reflected constructivist standards, the rest of his lessons were totally traditional and reflected an absolutist vision.

Table 9 below summarizes the ratings that Louis got for his instructional tasks in all his video-taped lessons. It demonstrates that Louis's instructional tasks changed dramatically from a constructivist approach in his first lesson to a traditional approach in his last three lessons.

Table 9 - Louis's Instructional Tasks and Constructivist Standards

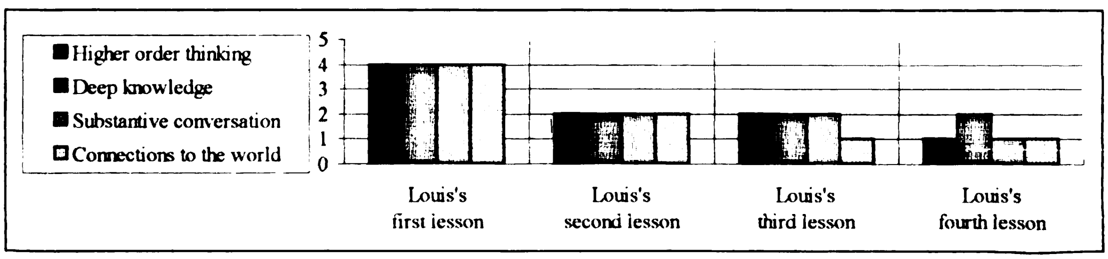


In his first lesson, Louis's 3 point rating in all the areas of instructional tasks suggests that Louis made efforts to help students organize information and consider alternative ways of thinking. He was also able to design instructional tasks that allowed students' to develop conceptual understanding through their own discovery, elaboration and examination. Moreover, he was able to integrate students' real life experiences in his instructional tasks.

However, in his remaining lessons, Louis's instructional tasks were less successful in all the areas. His rating for disciplinary inquiry was 2 points for his second and third lessons and 1 point for his last lesson. He received only 1 point in information organization, consideration of alternatives and disciplinary content. His scale for authentic problem was reduced from 2 points in his second and third lesson to 1 point in his last lesson.

The instructional processes Louis developed in these mathematics lessons were also similar to his instructional tasks. In his first lesson, his processes compared favorably with the constructivist standards. However, in his remaining lessons, his processes became increasingly weaker in all the areas. These features of his instructional process are shown in Table 10 below.

Table 10 - Louis's Instructional Processes and Constructivist Standards



Louis appeared especially strong, getting 4 out of 5 points in higher order thinking, deep knowledge, substantive conversation and connection to the world in his first lesson. In this lesson, not only was Louis able to organize a long and sustained discussion to help his students develop conceptual understanding of equivalent fractions, but also he was able to constantly encourage and support students to develop this understanding through their own discovery, interpretation and public examination. He paid enough attention to assisting his students to see the relationship between the concept of equivalent fraction and its real life representations. He continuously drew their attention to the real life context when his students had conflicting ideas.

In the rest of his lessons, Louis's processes dramatically moved away from the constructivist approach in all the aspects. His ratings in higher order thinking, deep knowledge, substantial conversation and connection to the world started to be reduced from 2 points in his second lesson and third lesson gradually to only 1 point in all the areas except for deep knowledge (2 points). Such ratings demonstrates that Louis spent less and less time to supporting his students to develop conceptual understanding of fractions. He devoted fewer and fewer efforts to connect mathematical ideas to his students' daily life experience. Moreover, he became more and more unwilling to encourage and support students to form, prove and challenge each other's ideas. Most of his instructional time in these classes were used for students to individually practice mathematics rules and procedures that he lectured and illustrated. His conversation with students was only used to get simple and correct answers from students with easy questions.

Louis's beliefs and his practice

Louis came into his internship with several strong constructivist views about mathematics instruction. For example, Louis wanted to help students develop conceptual understanding through students' own discovery, explanation and public examination and he hoped to connect real life examples to their learning and expected that his students would develop a sense of comfort and success through this way of mathematics teaching.

It is clear that Louis was able to implement many of these ideas in his first lesson. In this lesson, he used a money ratio as an example to design his instructional tasks and engaged his students to use their knowledge about money and fractions to construct a model to represent money problems. He required his students to prove or challenge each other's ideas to arrive at a shared understanding about equivalent fractions. He played the role of facilitator and participant in this mathematics discourse and kept drawing students' attention back to clarify the issue when discussion went off track or students missed an important point.

However, by the end of his internship, not only were some of these beliefs changed and weakened, but also his mathematics teaching practice moved toward a totally different direction in ways that reflected changed ideas. In these lessons, Louis did not require his students to construct, explain and prove their own understanding about the concepts of fraction. He paid little attention to encouraging students to find alternative ways of thinking. Conceptual understanding and its relationship with students' own experiences and real world examples were not stressed. They gave way to pure symbol manipulation. Louis began to directly offer symbolic rules and formulas followed by students' individually practicing them without his clarifying their meaning and implication for the real world.

Louis's ideas of mathematics instruction also changed as he finished his internship. He began to see mathematics as a reasoning and problem-solving activity and as a group of facts and skills that student needed to be informed of and practice. He considered the idea that learning needed to be a comfortable and fun experience as a final principle against which all teaching methods should be judged. He felt no problem with using the teaching approach that featured telling followed by practice as long as it made students happy.

Kelly: Practice with an Absolutist Approach

Instead of teaching a longer unit in her mathematics lead teaching, Kelly taught several short topics. One of the topics was time reading, one was addition and subtraction of numbers over 10, and the last was estimation of numbers. In her lead leading, Kelly claimed that she taught different topics instead of a long unit as her program required, because Lisa's curriculum were structured in this way.

Kelly's instructional tasks

One of the central instructional tasks Kelly developed in her lessons was to help students know a correct rule through direct demonstration and modeling. This feature of her instructional tasks could be seen clearly from what she did in her first videotaped lesson on addition, in which Kelly taught students the addition of two single digit numbers that lead to a result above 10, like 4+7 = ?.

In this lesson, she first showed her students that she could get results for this number sentence by keeping the bigger number 7 in her mind, and then counting four more numbers after 7 with her figures as aids. She demonstrated some more examples by asking her students to mimic her method and then she assigned each student a work sheet with similar addition questions individually. During the class, Kelly neither encouraged students to find alternative approaches to this kind of addition, nor did she explain or get her students to discuss why the approach demonstrated was correct.

Another feature of her instructional tasks was that Kelly emphasized more students' individual practice or their applying the correct procedures rather than interpretation and examination of their answers. In her second lesson on estimation, for example, she started her lesson by explaining a worksheet and assigned students to estimate different colored candy hearts in a transparent box by following the same steps she explained on the worksheet. Then she asked students to report their estimation and compared the actual candy hearts in the box with students' reports. During the whole class, she provided no chances for her students to reason, elaborate or analyze the strategies behind their estimation nor did she help them clarify better strategies of estimating.

Third, Kelly was able to incorporate real life objects or teaching aids into her instructional tasks. In both her addition and subtraction lessons, she used different kinds of manipulatives--like little fish, flowers and toys--to demonstrate her way of adding and subtracting. She provided students different kinds of manipulatives as aids to complete their work sheets. In her two estimation lessons, she used different color candy hearts as counting aids for her students to predict and check their estimation since it was close to Valentine's Day. However, in her teaching, the manipulatives and candy hearts were only used to motivate students and to support their practice of the rules and procedures she demonstrated and modeled. They were not used as tools to help students reason and explain the procedures.

Kelly's instructional processes

All the lessons Kelly taught in her lead teaching period were clearly structured into two parts. The first involved about five to ten minutes of demonstration and modeling of a correct way of doing a mathematics problem. The second part was about fifty minutes of students' individual practice on problems on a worksheet that was similar to what she had modeled and demonstrated. All of her observed classes ended up with students' individual practice except for her fourth lesson, at the end of which she asked students to report what they did on their work sheet.

During her demonstration, Kelly rarely supported and challenged students to discover mathematical ideas, though she was able to integrate some real life examples into her instructional tasks. Whenever used, these examples often functioned as a way to attract students' attention or as an aid for her to show the right strategy that students needed to follow.

In her subtraction lesson, for instance, since some students had stopped paying attention to her demonstration of how to calculate the number sentence 11-3 = 8, Kelly quickly drew a tree on the board with eleven toys in it. She told her students that eleven of her toys were caught in a tree and three fell off after she shot them. She continued to demonstrate her subtraction strategy to calculate how many toys were still in the tree by showing students that she could count the toys in the tree before she shot and keep the number, 11, in her mind. Then she could count 3 back from 11 to get the result of 8. Following this example, she asked some of her students to mimic the same strategy to solve similar problems with other objects with little flowers, fish and seashells as counting aids.

Sometimes Kelly was able to develop some interaction with her students in her demonstration. However, her conversations with students were often controlled by the teacher and focused on practicing procedures or explaining worksheets rather than on students' conceptual understanding and reasoning. The dialogue below is an example in her addition lesson in which she explained the first two questions of a worksheet.

P: On the first side of the paper, though, you have some piggy banks, and we are going to try some of these together first. And you can try them at your desk when you have them. In the piggy bank in the first No. 1. How many of you can tell me how many cents are in that piggy bank? On the first square right there? S2.

S2: 1.

P: It's a 1 in there?

S3: No,...7.

P: <u>It's a what?</u> [She ask students to read the worksheet again]

SS: 7.

P: It's a 7. Yes. 7 cents already in the piggy bank and we have 1 more penny to add to that 7. And we put that in, would someone raise their hand and tell me how many cents we will have all together if we add one more in? S4.

S4: 8.

P: We will have eight since 7 plus 1 more is 8. In this piggy bank, how many do we have? <u>Could someone raise your hand and tell me? In No. 2, this piggy bank. How many cents do we have?</u> [She starts another example]

SS: 4.

P: S5?

S5: 4.

P: We have 4 cents. How many cents are we going to add to our piggy bank here?

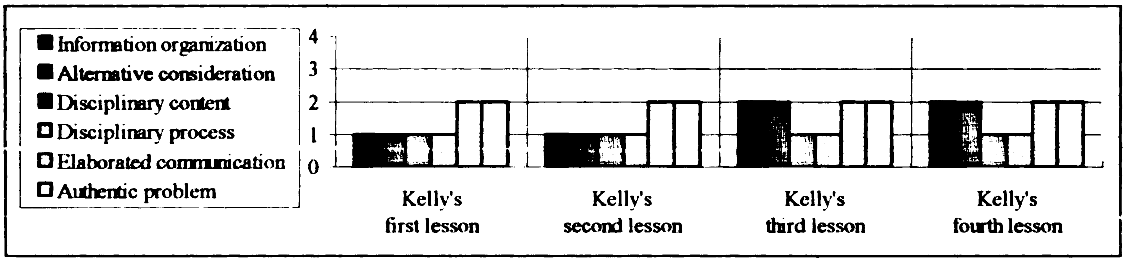
SS: 2.

In this conversation with students, Kelly was modeling how to do some addition exercise using examples from a worksheet that students were going to practice for the rest of this class. It was clear that she only asked simple questions in the service of her demonstration. She did not show her attention to students' conceptual understanding about the procedures, nor did she encourage students to find alternative ways of doing such addition. This kind of conversation continued with several other examples demonstrated and then students were assigned to work individually on the worksheet.

Kelly's practice and constructivist standards

The mathematics instructional tasks Kelly developed in her internship were consistent but strongly reflected an absolutist approach and were traditional in nature. Table 11 below summarizes the ratings Kelly got for her instructional tasks in all four lessons.

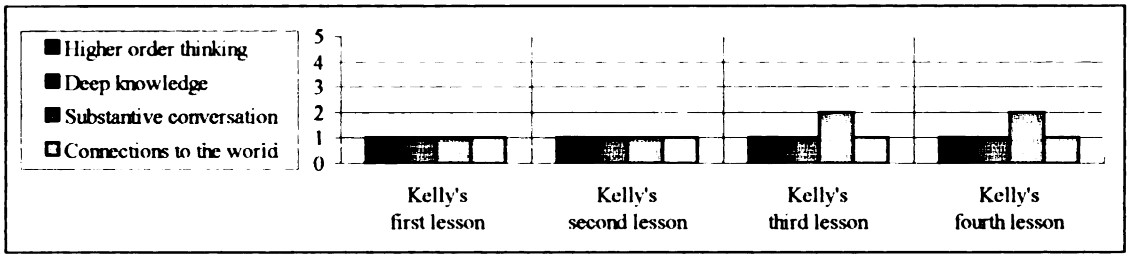
Table 11 - Kelly's Instructional Tasks and Constructivist Standards



Kelly appeared weak in developing her instructional tasks in support of students' elaborated communication and their solving authentic problem, criteria for which she only received 2 points. Her instructional tasks were even weaker in involving students to develop conceptual understanding of mathematical ideas through mathematical discourse. In these areas, she was rated only 1 point. Her ratings in information organization and consideration of alternatives was also only 1 point in her first two lessons, though she was able to do a little bit better job in her last two (2 points). Such scales indicate that Kelly was not successful in designing instructional tasks that pushed students to form or explain their ideas and find alternative ways of thinking.

Kelly's instructional processes look similar to her instructional tasks. They were again consistent but far from the constructivist standards in almost all the categories, as Table 12 shows.

Table 12 - Kelly's Instructional Processes and Constructivist Standards



Kelly was not effective in all four lessons in organizing students to carry out higher order thinking, develop deeper knowledge and make connection to the world . She only received 1 out of 5 points in all her lessons in these aspects. Her processes of engaging students to have substantive conversation were also not much better (1 point in her first two classes, although she made a little bit of progress in her last two lessons, getting 2 points). These ratings demonstrates that in her teaching, Kelly devoted little time and effort to supporting students to develop conceptual understanding and to form, prove and challenge each other's ideas. Most of her instructional time in these classes was used for students to practice mathematics rules and procedures that she had modeled. Even though sometimes she was able to get her students to report their findings after their individual practice, her conversation with her students was only used to get simple answers from students. Although she was able to create real life examples to attract her students' attention to her demonstration, the connections between what she was teaching and the real world were never clearly explained nor justified in her teaching.

Kelly's beliefs and her practice

Kelly brought into her internship several constructivist beliefs of mathematics instruction. For example, she believed that there were different ways to approach a mathematics problem and that mathematics learning needed to focus on conceptual understanding through students' own exploration. Kelly assumed that mathematics instruction needed to help students connect what they were learning with their life experience and the other subjects.

However, from her actual teaching practice, I find that most of these ideas were not activated and implemented in her teaching. In all her observed lessons, Kelly never encouraged students to form, prove and disprove each other's idea through public examination. She did not require students to come up with an alternative way of thinking about mathematics and mathematics problems. Conceptual understanding was clearly not her focus in teaching. Instead, she directly demonstrated a right way of doing mathematics and then asked students to practice similar questions on worksheets in the same way. Kelly's teaching practice in these lessons was more traditional in nature, though she did use some manipulatives and real life examples in her demonstration and students' individual practice. She seemed to give great weight to students individually practicing the rules and procedures she modeled and demonstrated or her getting correct answers from her students.

Instead of implementing some of her constructivist ideas of mathematics instruction into her teaching, Kelly's internship weakened or changed some of these ideas. After her mathematics teaching practice, Kelly came up with some new ideas of mathematics instruction which mirrored the traditional mathematics teaching practice she conducted in her lessons. For example, Kelly began to stress skills practice as an important goal of her mathematics instruction. She wanted to learn about the curriculum requirements for skill practice and how to get students to practice their skills by exactly following the textbooks. She no longer talked about conceptual understanding and students' own exploration in learning mathematics.

Summary of Chapter

In this chapter, I have analyzed the four preservice teachers' learning to teach mathematics in terms of what they were able to learn about mathematics instruction at the practical level. I have examined the nature of mathematics teaching practice they learned and how their learning at the practical level was related to their conceptual development. These analyses lead to several findings.

First, the mathematics teaching practices that the four preservice teachers developed through their internship were qualitatively different from each other. Not all of the mathematics teaching they developed in their internship reflected the constructivist standards or went as the program expected.

Martha was able to practice in the direction of the constructivist standards. Many aspects of her teaching practice reflected the constructivist standards and what the program encouraged their students to develop. Although Jaime was able to practice mathematics teaching by connecting students' learning with real life examples, she was reluctant to support, challenge and push students to learn mathematics beyond the level of their individual performance. Instead she retained a non-mediating stance in her mathematics teaching practice. Louis was able to practice mathematics teaching in a constructivist approach in his first lesson. However, he was unable to maintain this kind of mathematics teaching and quickly changed to an absolutist approach in the rest of his mathematics lessons. Kelly taught in the traditional way throughout her lead teaching practice and her teaching practice was distant from constructivist standards.

Second, the relationship between what these teachers believed and what they were able to do was neither linear nor causal. Instead, the relationship between their beliefs and practices of mathematics instruction tended to be mutual and interactive. My analysis in this chapter suggests several conclusions about the relationship between preservice teachers' beliefs and practices in their learning to teach.

If preservice teachers failed to develop certain beliefs, it would be hard for them to learn to practice what was not in their mind. Martha's teaching practice reflected the constructivist standards in most areas. However, she was not successful in integrating students' life experiences into her instructional task since she never stressed this idea in both points of her internship. Jaime did not see mathematics and mathematics learning as an activity of forming, proving and testing mathematics hypothesis. In accordance, her rating in higher order thinking and deep knowledge in instructional process and disciplinary knowledge and disciplinary inquiry in the instructional tasks was consistently low across each of her lessons.

Even if preservice teachers had a constructivist belief, they were not necessarily able to implement the idea in their practice. Sometime these beliefs stayed at a nominal level and had no impact on practice. Although Martha was able to implement many of her constructivist ideas throughout her teaching practice, Kelly was obviously unable to do so. For example, Kelly believed that mathematics instruction should focus on students' conceptual understanding through their own discovery. What she did in her teaching was totally contradictory to what she believed and she never tried to implement any of these ideas.

Even if they were able to practice what they believed, they were not necessarily able to maintain it throughout their practice. Like Martha, Louis also believed that mathematics is an activity of forming, proving and disproving each other's ideas and was able to develop his instructional task and process that reflected this conception in the first lesson. However, he was unable to maintain this idea throughout all his fraction lessons and ended up with a teaching practice that was traditional in nature.

The beliefs these teachers brought into their practice can change and develop as they practice their mathematics instruction. Martha ended up believing that a concrete model of presenting mathematical ideas and proper group work were important for contextualizing a constructivist vision of mathematics teaching. It was obvious that in her teaching practice, she did experiment with the function of concrete models and group or pair work in helping students learn fractions. Jaime's idea that mathematics learning needed to be enjoyable and fun for students was strengthened and contributed to her developing a self-realization stance through her teaching practices. Louis developed the idea that mathematics is a group of skills that needs to be practiced. He started to see the approach of telling followed by practice as an important way to teach mathematics if students felt happy about it. In his lead teaching practice, it was apparent that he had been teaching in an absolutist direction after his first try at a constructivist approach. Kelly's teaching in the traditional approach helped her realize that it was very important for her to understand the specific requirements of curriculum and teach by following a textbook.

My findings in this chapter strongly imply that even under similar reformed internship arrangement and requirements, preservice teachers can still develop mathematics teaching practice that is distant or moves away from constructivist standards. What happened during their internship that produced these diverse learning results at both conceptual and practical levels? To answer this question, my focus naturally turns to their collaborating teachers, since they are assumed to play such an important role in their preservice teachers' learning to teach. Not only did they need to provide models for their preservice teacher to observe, but also they were expected to closely work with preservice teachers and support their learning in each phase of the internship. Thus, to understand what shaped these preservice teachers' diverse learning results, we need to examine collaborating teachers' influence. In the next two chapters, I answer the question by exploring such influences at both conceptual and practical levels.

Chapter 5

BELIEFS OF COLLABORATING AND PRESERVICE TEACHERS

In this chapter, I explore the influence of the collaborating teacher on his or her preservice teacher's conceptual learning. My analysis focuses on the relationship between the conceptions of mathematics instruction each collaborating teacher had and the early and later beliefs his or her preservice teacher had in the internship. To understand such conceptual influences, I considered two questions for each pair of collaborating and preservice teachers. What were the beliefs and conceptions that the collaborating teachers had about mathematics instruction? To what extent did the beliefs that preservice teachers developed in their internship reflect their collaborating teachers' conceptions?

My analysis suggests that the four collaborating teachers--Nick, Bank, Ben and Lisa--had quite different conceptions about mathematics and its learning and teaching. The beliefs each preservice teacher developed and formed in the internship largely reflected his or her collaborating teacher's conceptions. Through their internship, all the preservice teachers, to a great extent, were moving closer conceptually to their collaborating teachers.

Relationship Between Nick's Beliefs and Martha's Conceptions

Nick had twelve years of teaching experience. About five years ago, Well Elementary started restructuring its mathematics curriculum under the influence of the NCTM standards. It initiated a series of workshops to train its teachers to teach the new curriculum. These activities and his frustration about his students' mathematics learning in his traditional classroom began to change Nick's way of looking at mathematics instruction and his teaching practice. Nick claimed that the conceptual change he experienced was not easy without any support. He wanted to help new teachers shorten their learning to teach in the new way that took him "a long time to be able to learn and to be able to do." Since the five-year program that Martha was in reflected what he and his school were experimenting with and pursuing, he made up his mind to volunteer to be a collaborating teacher.

Nick's beliefs of mathematics instruction

Nick developed the following beliefs about elementary mathematics and its learning and teaching over the past five years. Nick thought that mathematics is a sense making activity about "the relationship and balance between the amounts and numbers." To develop a sense of the relationship and balance between amounts and numbers basically is "to understand how the concepts are related with each other. And how you go from here to there."

Nick's conceptions about mathematical learning were closely related with his beliefs of mathematics and were reflected in the following three aspects. First, he believed this kind of mathematics understanding is more important for his students than memorizing facts and rules and applying them to get right answers.

> If kids don't learn to understand the relationship between the numbers and amounts and how geometric shapes are related, then they are going to break down pretty quickly in the middle and high school. They begin to memorize how to get the answer and do not work very well in algebra.

Second, to develop such conceptual understanding, students need to be encouraged to learn how to develop, communicate and prove their hypotheses. He described that "if we have a second grader who is learning adding 9+6, he should be able to learn to make a model of that and show to the other people how you get that answer."

Third, he believed that conceptual understanding about mathematics for the elementary students needed to be a process of gradual transition from a concrete model to the symbolic. He expressed:

> Kids learn much better by being able to actually hands-on the manipulatives. You worked on solving math problems in a way saying numbers but never really writing down and seeing the symbols and signs. Then you move to add along with what you build. You begin drawing pictures to represent your idea. And then you go back to the manipulatives and begin to use symbols. And then the pictures in symbols until you are just using the symbols and signs.

Nick claimed that his mathematics teaching approach changed as he changed his way of looking at mathematics and its learning. One "biggest" change was that in his mathematics teaching, he paid more attention to gradually developing students' conceptual understanding about mathematics concepts through a sequence moving from concrete understanding to pictorial understanding and then to symbolic understanding.

> It is a system. I used some manipulatives and some concrete hands-on activities to help them understand math. I asked them to draw pictures to help them understand and communicate their ideas. I also bring in the mathematics symbols at the right time, after they understand the concepts. For example, if I am teaching division, they might be doing division for two weeks before they even see the numbers. And then we begin to bring in those numbers to relate to what they have already done so they have an understanding about the process before they actually deal with symbols. So that is a big change for me.

The second approach Nick developed to teach mathematics was that mathematics teaching should be organized to support his students to discover concepts and connections by themselves and then test and prove them to each other. As he said:

> The other way is through discovery model. I try to teach mathematics in a similar way where I am teaching science. You can understand them much better if you discover these connections and are supported to discover these connections. So we may work on a math unit, to encourage the kids to come up with a hypothesis, they see a rule and begin to see something they think. It is pretty consistent in mathematics. We put that on the board or put it somewhere. As we continue we keep testing that, either changing it or seeing whether someone helps to prove that. So that is a difference. That is not the way again I was taught. It is new in a different way of having you think yourself and teaching.

The third approach Nick claimed in his interview was that he paid attention to helping students learn how to communicate their mathematical ideas to each other or to other people. In his mathematics teaching, Nick assumed that he always required students to clearly explain their ideas to each other or to "write as much in math and science as we do in a reading class."

In addition, Nick thought that to teach with this approach, a teacher would have to face and resolve two difficulties. One is how to develop classroom discourse. In his words, "the actual managing of the manipulatives and group work." The other "is to be patient enough to go through the processes" and not to skip to the results in a hurry.

Nick's conceptions and Martha's belief development

In Chapter 3, I presented a picture of the beliefs that Martha brought into her internship and conceptions she ended up with. Summarized in Table 13 below is a comparison between the beliefs Martha brought in and developed from her internship and beliefs that Nick had for his mathematics instruction. From this comparison, I come to come interesting findings about the relationship between Nick's beliefs and Martha's conceptual development.

Table 13 - Conceptual Comparison between Martha and Nick

	Martha's early beliefs	Martha's late beliefs	Nick's beliefs
Mathematics	Active sense making of mathematical ideas, patterns and rules through discovery and proving,	Active sense making of mathematical ideas, patterns and rules through discovery, hypothesizing, and proving.	Active sense making of different mathematics concepts and their relationship through forming hypotheses, and communicating and proving them.
Mathematics Learning	1. Discover mathematical ideas, rules and patterns by students themselves. 2. Present, communicate and prove their ideas, patterns and rules among students.	1. Discover mathematical ideas, rules and patterns by students themselves and with support of proper model. 2. Present, communicate and prove their ideas, patterns and rules among students.	1. Develop conceptual understanding of different parts of mathematics and how they are related with and build upon each other. 2. Develop, communicate and prove their hypotheses by themselves. 3. A process of gradual transition from a concrete model to the symbolic.
Mathematics Teaching	1. Support students to discover mathematical ideas by themselves through questioning and guidance. 3. Encourage students to use public examination to judge mathematical ideas as right or wrong.	1. Guide students to discover mathematical ideas, patterns and rules with a model of working gradually from concrete, pictorial and toward symbolic understanding. 2. Focus on students' conceptual understanding of mathematical ideas, patterns and rules. 3. Encourage students to communicate and prove their ideas through proper group work and hands on experience.	1. Support students to understand mathematics concepts and connections through a model from concrete, pictorial to symbolic. 2. Assist them to form, develop and prove their hypotheses and ideas by themselves. 3. Push them to clearly communicate their ideas to each other in different forms. 4. To slow down the pace of teaching and pay attention to management of class discourse.

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The beliefs Martha brought into her internship were consistent with the beliefs held by her collaborating teacher, Nick. However, Nick's beliefs about mathematics and its learning and teaching were clearly contextualized and grounded in mathematics elementary teaching practices, while Martha's ideas were more abstract and without specific pedagogical consideration.

Martha and Nick shared the idea that mathematics is an active sense making and conceptual understanding activity. Mathematics learning is to discover, prove and communicate mathematical ideas, and mathematics instruction is to support this kind of mathematics sense making and learning.

However, when discussing their ideas, Nick emphasized more the conceptual understanding of the relationship among different concepts. He was able to see a specific and progressive model of mathematics learning that he would use to realize his goal of mathematics instruction. He had a clear sense about how to help his students learn mathematics and what potential difficulties in enacting such mathematics teaching are. For example, he required his students to explain their ideas clearly and to write them out. He used many specific examples of mathematics, like fractions, addition, long division, to show how he would like to implement some of his philosophy of mathematics instruction. He clearly suggested that management of such teaching discourse was important. As a teacher, he needed to set up a pace of teaching that allowed his students to learn kinds of mathematics that were difficult to learn.

By the end of her internship, not only did Martha retain her early ideas of mathematics instruction, but she was also able to develop some important pedagogical strategies for how to put her ideas of mathematics instruction into practice. All her conceptual

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Martha began to realize how to use group work to involve all the students in learning mathematics by assigning different responsibilities to different students. She started to see how to design a concrete but progressive model to gradually lead students to develop a deep and abstract understanding about a mathematics concept. These ideas that Martha developed and reinforced through her internship were exactly what Nick believed about mathematics learning and instruction: a concrete but progressive model of mathematics instruction, managing classroom discourse and the pace of learning to make sure students conceptually understand what was taught.

Relationship Between Bank's Beliefs and Jaime's Conceptions

Bank also worked at the fifth grade level at Well Elementary and he had about fifteen years teaching experience. Like Nick, Bank also experienced a conceptual change about five years ago as his school restructured its mathematics curriculum under the influences of the NCTM standards.

Bank was willing to be involved in and stayed in the internship program for two reasons. First, the new ideas that the program was pursuing closely connected with the philosophy of teaching he and his school was moving toward. It allowed him to "get in touch with some of the newest things being done." Second, the teacher education program that Jaime was in was very open and listened to the input of school teachers.

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Bank's beliefs of mathematics instruction

Bank had two ideas about mathematics. First he believed that mathematics is a collection of skills to apply mathematics concepts and rules to solve problems and it was important to "understand how division process works and how fraction works, etc. The basic skills was important of course." Then he saw mathematics rules, concepts and ideas as closely related with other subjects and reflected in our daily life.

> I am also thinking about math in general is in daily life around you. How do we use math in the building we are in? How do we use math in figuring out how much food we feed students lunch, estimating and measuring?.... I feel all subjects are connected together in the real life.

As for how to learn mathematics, two ideas stood out in his description. First, he thought that students' own discovery was a more efficient way to learn different mathematics skills and concepts than telling followed by memorizing. As he described:

> I will try to ask them to think about it and look for the order and look for patterns and help guide them to discover the basic ideas of math rather than telling them: "Here is the way it works. Now do five of them." I would rather help them discover those relationships. It seems to last longer in that way.

Second, he believed that mathematics learning needed to be comfortable and enjoyable for students. Otherwise it was very hard for students to do their best job. He said in the interview when asked how he thought about mathematics learning for elementary students:

> You can't do your best work if you are not happy. You can't do your best work if you are not comfortable and happy in your environment. So we try to make every student feel like fun.... We really go out of way to make sure everybody again feeling good and feeling comfortable about what they are doing.

Bank's beliefs of mathematics teaching were closely related with his views of mathematics and his ideas that mathematics needed to be fun and enjoyable. First, Banks thought that

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thought that to help his students be more engaged in their learning and be willing to work harder and longer, it was very important for him to make mathematics teaching interesting and fun. One way to do that was to support students to make connections between the mathematics concepts he wanted to get across and other subjects. When he was asked what he would like to do if he was going to teach the same geometry unit that Jaime did for her mathematics lead teaching, he replied:

> First of all, the overall goal of our unit. What are the main concepts that I want to get across in this unit? Secondly, how can I interpret those main ideas with other subject matter like science, art or writing or literature? How can I find a connection there? I also look for projects that would be valuable experience to apply the knowledge and extend and their basic concepts that I am trying to apply.

Another way to do it was to put what students need to learn in the real life context or support them to build the connections between what they need to learn and the real world. In this way, students would see why they need to learn this. This view was reflected in his comments on how to teach geometry to his fifth grade students.

> So for our level, I will guess it will be the overall understanding that the geometry is a part of life around us. It is a base we can construct. Basically all we build is a relationship to geometry. In fact most things in nature have a relationship to geometry, symmetry and things of that nature. So again I try to make the connection to the real world and help them see why we study this.

Third, Bank wanted to use "many more projects and more manipulatives" to make student feel mathematics was "more of an interesting, fun, exacting and meaningful thing to learn."

Bank's conceptions and Jaime's belief change

Table 14 below compares Bank's beliefs of mathematics instruction with what Jaime brought into her internship and what she ended up with.

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Table 14 - Conceptual Comparison between Jaime and Bank

	Jaime's early beliefs	Jaime's later beliefs	Bank's beliefs
Mathematics	Its problem can be approached from different ways. Its facts, rules and formulas are related to each other and to our daily life and other subjects.	Mathematics knowledge is related with other subjects and the real world.	subjects and reflected in the real world.
Mathematics Learning	1. Construct understandings about basic concepts through their own discovery. 2. Explain why and how they came up with such understandings. 3. Be comfortable or enjoyable in their learning.	1. Learning needs to be comfortable and enjoyable for her students. 2. Different ways of learning about mathematics are all important and all should be respected.	1. To learn different mathematical ideas and concepts by their own discovery. 2. Learning needs to be comfortable and enjoyable for students to make them do their best job.
Mathematics Teaching	1. Help students see the relationship between mathematics and other subjects. 2. Support students to understand basic concepts and explain their understanding. 3. To respect students' ideas and not to let them feel uncomfortable and find ways of teaching that students are happy to learn.	1. Help students see the relationship between mathematics and other subjects. 2. Help students be confident in and like mathematics by avoiding content that did not fit children's learning ability and that makes them uncomfortable. 3. To incorporate all kinds of ways of learning students have into mathematics teaching.	1. Use projects and manipulatives to make students feel fun about and enjoyed mathematics. 2. Connect what student is learning with other subjects. 3. Connect what student is learning to real world examples.

It is clear that Bank had several ideas of mathematics and mathematics learning and teaching that were similar to some of the ideas Jaime brought into her internship. He also differed from Jaime in some other aspects. For example, Bank shared with Jaime that mathematics was closely related with the real world and different subjects and students' own discovery should be encouraged in their learning. Also sharing with Jaime, Bank believed that mathematics learning had to be enjoyable and involve comfortable learning expe

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learning experiences. On the other hand, Bank also thought that mathematics was fundamentally skills and the central goal of mathematics learning was to retain these skills. Jaime not only emphasized constructing understandings about basic concepts through students' own discovery but also stressed students' explanation of why and how they came up with their understandings.

Among all the ideas shared by Bank and Jaime, Bank's beliefs were often expressed in a way that was grounded in his elementary mathematics teaching practice while Jaime's ideas were more abstract and lacked specific pedagogical consideration. When discussing how to help students feel happy about mathematics learning, Bank was able to contextualize this idea in a specific teaching situation. For example, he saw that projects and manipulatives were important tools for him to realize this learning goal and at the same time, students could build their mathematics skills with them. Jaime was clearly unable to contextualize her ideas of helping students feel mathematics learning was fun.

By the end of her internship, Jaime strengthened and further developed many of her beliefs that she shared with Banks. For example, she obviously stuck to the idea that mathematics was closely related to different subjects and the real world. She clearly reinforced her idea that mathematics learning needed to be enjoyable and fun for her students. All these ideas were clearly held by her collaborating teacher, Bank. In addition, Jaime developed several pedagogical ideas about how to make this kind of happy learning happen. For example, she thought mathematics teaching was to build the confidence of her students in learning mathematics. The content of teaching should not be so difficult as to make students uncomfortable. Since students had quite different ways of learning m

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Some of Jaime's early beliefs were obviously weakened throughout her internship and these ideas were ones Bank failed to develop in his mathematics instruction. By the end of her internship, for example, she no longer stressed that mathematics learning had to support students developing conceptual understanding about basic mathematics concepts and assist them to explain how and why their understanding came about. Although these ideas were pushed by her program, Bank never took them as important or thought that he had to implement them.

Relationship Between Ben's Beliefs and Louis's Conceptions

Bell Elementary where Ben taught had actively been pursuing the kind of teaching envisioned by the NCTM standards and emphasized conceptual understanding. Ben had been in the mathematics curriculum committee and participated in restructuring the school mathematics curriculum. About two years ago, however, the school refocused its mathematics teaching back on computation because students in the school had failed to get high grades in the computation part of the state mathematics examination. As an elementary school in the affluent area, it felt a great pressure from the parents to improve the grades of its students in this aspect. To respond to the school's call of "going back to basics," at the time Louis began his internship in his class, Ben had reoriented his mathematics toward the model of telling and illustrating followed by practice.

Ben was willing to be involved in the internship program for the two reasons. First, he felt that he was almost at retirement age and it was his responsibility to bring some new

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some new people along and the new teachers needed chances to learn how to teach. Another reason was many teacher education programs only trained female elementary teachers. He wanted to see some males in the profession and the program was able to assign male preservice teachers in his classroom. It turned out that both of his preservice teachers from the program were male. Louis was his second intern.

Ben's beliefs of mathematics instruction

Ben suggested that school mathematics was a collection of facts, concepts, and skills that were related with our daily life, as in his word, "like you use math facts and concepts all the time in real life." On the other hand, he also thought that all the different facts and concepts of mathematics were built upon each other, for example, "the fifth grade math builds on what the first and second, third and fourth grade." He used fractions that Louis taught in his lead teaching unit as an example to illustrate this point:

> My big thing, I would like to see my kids understand the relationship between the fraction and decimals. He (Louis) did a lot with reducing and equivalent that is good. That is very positive. They need to know that. But the big thing, I think the big thing they need to know is the relationship between decimals and fractions.

His ideas about mathematics learning were closely related with his view of mathematics and were reflected in the following three aspects. First, he believed that his students should learn how different mathematics facts and concepts were related to each other. He described:

> I think they need to learn to make connections between different math facts and concepts, and we need to help the kids make connections. Or what is that you are going to use and when. And that is really important things. Or even to say to them, "You might not be using this now but when you get into the curriculum two years down the road, you are going to need this in order to help you do it."

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Second, he also emphasized that it was very important for his students to learn to remember and apply different mathematics facts and skills they needed to learn at their level. Ben believed that to remember facts was even more important than to understand the connections conceptually.

> While we still believe conceptual understanding, they have to learn concepts. They also have to learn facts...We are feeling right now we have done so much on concepts. Some of the kids are losing out on computation. And we are going back now and spend a little more time on computation. I don't care what it is. You might want to know what it means, but if you don't know the fact, you are not going to do anything.

In addition, Ben believed mathematics learning should be enjoyable and fun. He wanted his students to realize and experience this aspect of mathematics learning.

> I think that is really important thing--making math learning fun. You know, you need to know how to entertain the kids and help them enjoy it. I guess that is pretty much important.

Ben's views of mathematics instruction were built upon his views of mathematics and its learning. First, Ben thought there were two goals for his mathematics teaching. One was to help students feel fun in and enjoy mathematics and overcome the mathematics phobia his students had.

> Number one I think it is our responsibility as teachers not to make kids learn math but to have kids like math and not to be fearful of math... To me they have to feel comfortable with math and to enjoy it so.

Not only did he think this was important for him to make his students happy about mathematics in his teaching, but also he believed that it was the most difficult thing to do. He described this belief as follows:

> I think the most difficult thing is to help kids overcome their fear of math. Some kids have this real math phobia. To me that is the most difficult thing to have kids to say, "I can do it." And you know, "I have problem doing this. Can you help

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> me?" I never allow the kids to say, "I can't. I am having a problem. Can you help me to do this?" I think helping them overcome their feeling about math.

Second, Ben thought that he needed to help his students to "see the connection between the math facts and the real life." He should prepare his students to see the connection between different mathematics facts and concepts.

> If I will teach fraction, I would put the connection between decimals and fractions together. I will just start from the beginning and ended up that way. I am doing the way with division, I am teaching division right now. And they have to write their answers both 22 remainder of 3 and 22 and 3/4, showing the remainder is a fraction number.

Third, Ben believed that to have his students memorize facts and apply them to get right answers were important teaching methods. He claimed that he would also like to help students see it was very important to remember mathematics facts.

> It is important to help them understand that if they don't know math facts, there is nothing they can do. When we get them into fifth grade, they don't know math facts. That is just horrible. But I think it is the kids' attitude and the learners' attitude... I may have just made them memorize the rules. That may be dumb but I think they need to know it. That would be one of the important things I want to do.

Ben's conceptions and Louis's belief change

The comparison between what Ben believed and Louis's early and later conceptions of mathematics instruction (shown in Table 15 below) indicates several interesting things about the conceptual influence Ben had on Louis's understanding about mathematics instruction.

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Table 15 - Conceptual Comparison between Louis and Ben

	Louis's early beliefs	Louis's later beliefs	Ben's beliefs
Mathematics	An activity of learner's active reasoning, proving and problem solving and it is reflected in our daily life.	1. An activity of learner's active reasoning, proving and problem solving 2. A collection of facts and skills that need to be informed, remembered and practiced.	A collection of facts, concepts, and skills that are built upon each other and related to real world.
Mathematics Learning	1. Process of actively discovering mathematical ideas and then applying these ideas to solve problems by learners. 2. Be comfortable or enjoyable for learner.	1. Being comfortable and enjoyable for learners is the fundamental principle of learning mathematics.	1. See the relationship between different mathematics facts and concepts. 2. Remember and apply different mathematics facts and skills. 3. Learning should be enjoyable and fun.
Mathematics Teaching	1. Change students' perception that mathematics learning is a process of memorizing and practicing. 2. Support students to understand mathematical idea through self-discovery. 3. Help students develop a sense of comfort and success with mathematics learning.	1. Support students to develop a sense of comfort and success with mathematics learning. 2. Either support students to reason and solve problems or tell students answers followed with practice, depending upon which way students feel more comfortable with. 3. Know the content of teaching by heart and be able to answer every question about it.	1. Support students to feel fun and enjoy mathematics and overcome mathematics phobia. 2. Help students see the connection between different mathematics concepts and between mathematics and the real world. 3. To have his students memorize facts and apply them to mathematics problems.

From the above table, we can see that Louis came into his internship with several beliefs of mathematics, some of which were clearly shared with his collaborating teacher, Ben. For example, Louis shared with Ben that mathematics facts, concepts and skills were closely related to the real world and that mathematics learning needed to be comfortable and fun for students.

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Louis also started with several ideas about mathematics instruction that were clearly different from Ben. Louis assumed that to support students' own discovery of mathematical ideas would result in his students' being comfortable and having a fun learning experience. He claimed that mathematics was an active sense making activity of forming, backing up and proving mathematical ideas with logic and reasoning. Ben was clearly not thinking in that direction. Instead, Ben emphasized that mathematics is a collections of facts, concepts and skills that were built upon each other. It was important to support students to both memorize mathematics facts and understand their relationships. In his eyes, the former was even more important than the latter for his teaching at the time of Louis's internship. Thus, Ben thought in his teaching he would like to directly inform or show his students the mathematics connections and have them memorize mathematics facts rather than support them to discover the relationship by themselves.

By the end of his internship, it was obvious that those beliefs Louis shared with Ben were retained and strengthened while those he failed to share with Ben were weakened, changed or disappeared. For example, Louis began to see mathematics not only as an activity of learner's active reasoning, hypothesizing, proving and problem solving, but also as a collection of facts and skills that students should be informed about, remembered and practiced. He assumed that all the mathematics teaching methods had equal value no matter whether it was lecturing followed by practice according to a textbook or supporting students' own construction of mathematical ideas. He developed his idea that students needed to feel comfortable and experience fun in mathematics learning int

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learning into a fundamental principle against which he selected his teaching methods. All these ideas were clearly held by his collaborating teacher.

On the other hand, he no longer mentioned students' own discovery of mathematical ideas and proving them as the most important way for students to learn mathematics and he did not want to use this method to change students' conceptions of mathematics and mathematics learning. Instead, he claimed that it proved to be an ineffective way for his students to learn mathematics in his internship. These ideas were clearly not held by Ben for his mathematics teaching.

Relationship Between Lisa's Beliefs and Kelly's Conceptions

Working at the first grade level in Mall Elementary for five years, Lisa volunteered to be a collaborating teacher because she thought that would give her a chance to do some research work. She had planned to pursue a research career by doing doctoral study after she got her masters degree in teaching and curriculum but her family situation then did not allow her to do it.

Lisa's beliefs of mathematics instruction

Lisa thought that school mathematics was a subject in which different skills and facts were structured in a hierarchical way. Not only was this hierarchical and progressive structure of skills reflected in the mathematics of different grade levels, but also it was reflected within a grade level and a class:

> I think mathematics is the knowledge of skill progression. Like what things are needed from how children learned to count and how they learned to correspond? For example, somebody teaches 7th grade. I think she needs to know hierarchy, not only what is taught in each grade level but as you move along and continue, how kids move from one thing to the next.

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Lisa thought that to acquire mathematics skills, not only did students need to remember facts and rules of a mathematics problem, but also they needed to develop the conceptual understanding of mathematics concepts and rules. She used addition of number 5 to illustrate her point.

> When you add one more, what is going to happen to my number? What is going to happen to my pile of things? Taking as very elementary, "What does 5 mean?" 5 means 5 dinosaurs, 5 this and 5 that, you know whatever. It means 25 is 5 groups of five. It could be this and it could be that. This place value is the same way you continued to understand the concept of number.

However, instead of letting her students learn the concepts by their own discovery and exploration, she believed that the conceptual understanding of the rule and strategies needs to be learned through students being shown and informed. She explained:

> I can put boxes of manipulatives in their hand and I can get them over. I can get them to do all these little tasks. But if I don't turn them on by giving them strategies to figure out, they will turn their mind off because they have not had that a lot of strategies yet.

Lisa developed several ideas about how to teach mathematics in her classroom and all these ideas were closely related with her beliefs of mathematics and mathematics learning. First, as a mathematics teacher, Lisa often faced students with quite different levels of mathematics skills. Thus, she assumed that it was important for her to understand the mathematics skill progression:

> Because you will have kids all stages so you got to know your progression. To what extent the kids have problems in certain areas, maybe it is in place value. You might need to check back three or four steps back to see where the child's problem was. If you don't know the progression, you are going to have problem working with him or her...

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Second, she suggested that it was important to demonstrate to her students the meaning of a mathematics concept with manipulatives and get them to understand it. She used an addition example to show her idea:

> So I am hoping the most important part of instruction is that they get to understand what 47 is than just adding 47 by 2. That is fact. If I can get them to know more about 47, I will show them how to write 47, and then show them with 4 groups of 10 cubes, or I can show them 4 piles of tens sticks and 7 ones sticks. I can show them in different ways. That 47 come after 46, 47 come before 48. Just the whole concept.

In addition, Lisa believed that it was very important for her to give students enough chances to practice the skills that she wanted them to acquire. She thought that students needed to practice the skills in the school as well as in their daily life. For example, they could practice number skills by counting merchandise when they were going shopping with their parents or practice counting in the car when they went somewhere. Otherwise, it was impossible for students to get that skill.

> I can put addition paper in front of my kids and they look at you like "what?" And then you do the first problem together, or you do some together. They go through it, they complete the entire page. Next day I gave them another page and maybe it is subtraction. A week later, I gave them a page of addition. They are lost and they don't know what to do. So I haven't given them enough practice to internalize it and I got to get them to practice that.

<u>Lisa's conceptions and Kelly's belief change</u>

Table 16 compares Lisa's conceptions of mathematics instruction and Kelly's early and later beliefs over her internship. From it, I come to the following findings about the relationship between the beliefs held by Lisa and Kelly about mathematics instruction.

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Table 16 - Conceptual Comparison between Kelly and Lisa

	Kelly's early beliefs	Kelly's later beliefs	Lisa's beliefs
Mathematics	A collection of skills that are related to each other, to different disciplines and the real world. Its problems can be approached from different ways.	A collection of skills that are related to each other, to different disciplines and the real world.	A subject in which different skills and facts are structured in a hierarchical and progressive way.
Mathematics Learning	1. Develop a conceptual understanding of mathematical idea through their own exploration.	1. Practice mathematics skill in the way one feel comfortable and enjoyable. 2. All the ways of mathematics learning are important because learners are different.	1. Remember mathematics facts and rule and use the rules to solve problem. 2. Understand mathematics concepts and rules.
Mathematics Teaching	1. Help students understand this connection between mathematics concepts and real world or the other subject areas. 2. Support students to develop their own ways of doing mathematics.	1. Need to understand the requirements of curriculum and textbook for what students need to know. 2. Know how to lecture and follow curriculum and textbook. 3. Provide chances for students to practice their mathematics skills in way that they feel comfortable.	1. Understand the mathematics skill progression for the students you are teaching. 2. Demonstrate to her students the meaning of mathematics concepts and rules with manipulatives. 3. To provide students with enough chances to practice mathematics skills.

Kelly brought into her internship some ideas about mathematics and mathematics learning that were quite similar to what Lisa believed. For example, Kelly shared with Lisa the idea that mathematics is a collection of skills that were related with each other and conceptual understanding of mathematical ideas were very important.

However, she clearly differed from Lisa in other thoughts. Kelly believed that conceptual understanding of mathematics skills was an active sense making process and needed to be developed through students' own discovery and exploration, while Lisa

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thought that students needed to be shown the mathematics concepts, strategies and skills. Kelly wanted students to see the relationship between mathematics skills and the real world but Lisa emphasized that a teacher needed to know the skill progression for her students, support them to practice and internalize the skills.

By the end of her internship, Kelly was able to strengthen some of these beliefs she brought into her internship and develop some new conceptions of mathematics and its learning and teaching. However, the beliefs she was able to strengthen and develop were those that she shared with Lisa.

For example, not only did Kelly stick to the idea that mathematics was a collection of skills hierarchically structured and related, but also she began to see that it was very important for her to understand the requirements of mathematics skills and the skill progression. Moreover, she began to see that she needed to learn how to follow the school curriculum and how to help her students practice these mathematics skills. All these conceptions, to a great extent, reflected many beliefs that Lisa had about mathematics instruction.

In addition, by the end of her internship, the beliefs Kelly failed to share with Lisa in the beginning were apparently weakened or disappeared. For example, Kelly no longer stressed that students needed to understand concepts through their own discovery and exploration as the most important way of learning mathematics. Instead, she began to see it as one of many ways of mathematics learning. Its value, like memorization and repetitious practices, only depended upon whether students felt happy or comfortable about it or not.

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Summary of Chapter

In Chapter 3, I discussed that all the preservice teachers in this study experienced some conceptual development and change through their internship. However, except for Martha, the three other preservice teachers all strengthened and developed some beliefs that were not along the line of constructivist mathematics instruction. Many of their constructivist views in fact were weakened or disappeared.

My analysis in this chapter suggests that while three preservice teachers in this study walked away from constructivism at a conceptual level, all four preservice teachers were walking closer toward their collaborating teachers in their thinking about mathematics, its learning and teaching. Their moving toward their collaborating teachers at the conceptual level occurred in three forms.

First, they weakened or no longer stuck to those beliefs that were not shared with their collaborating teachers, even though these beliefs reflected the expectations of their program and mathematics education reformers. Second, they kept or reinforced many beliefs that they brought into their internship, if they clearly shared them with their collaborating teachers. Third, they further developed some new conceptions and beliefs of mathematics instruction that their collaborating teachers clearly held or emphasized.

The four collaborating teachers had quite different conceptions of mathematics and its learning and teaching and these were not consistent with the expectation of the program and constructivist mathematics instruction. Thus, not surprisingly, their influences on their preservice teachers' thinking were qualitatively different and were not all positively related with constructivist mathematics instruction.

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What was the influence of collaborating teachers on their preservice teachers' mathematics teaching practice? In next chapter, I explore this question. My analysis will focus on the relationship between the mathematics teaching practice each collaborating teacher exposed his or her preservice teachers to and the teaching practice that each preservice teacher developed in his or her internship.

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Chapter 6

PRACTICES OF COLLABORATING AND PRESERVICE TEACHERS

In this chapter, I explore the influence of each collaborating teacher's teaching on the mathematics instructional practice his or her preservice teacher developed in their internship. My analysis first focuses on what kind of mathematics teaching each collaborating teacher was practicing during the internship. Then I compare the practice of each collaborating teacher with the mathematics teaching his or her preservice teacher developed over his or her internship. Through this comparison, I develop an understanding about the relationship between collaborating teacher's teaching and the teaching the preservice teacher was able to practice.

My analysis suggests that the mathematics teaching the four collaborating teachers practiced and exposed their preservice teachers to in the internship differed qualitatively. The mathematics instructional practice each preservice teacher developed looked quite similar to or moved closer to what his or her collaborating teacher was practicing.

Influence of Nick's Teaching Practice

Well Elementary began encouraging its teachers to teach as the NCTM standards envisioned five years ago. Since that time, Nick not only changed his way of looking at mathema

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mathematics instruction, he also started to explore a new way of mathematics teaching that was different from the way he had been teaching.

I examined two of Nick's lessons taught during Martha's internship. The first lesson which Nick taught before Martha's lead teaching, focused on the concept of partition as a preparation for Martha's fraction unit. In the second lesson that happened after Martha's lead teaching, Nick helped students review equivalent fractions taught by Martha in her lead teaching.

Nick's instructional tasks

The instructional tasks Nick developed in these lessons had several features. First, Nick was able to integrate students' daily life experience into his design of instructional task. In his first lesson, Nick started the class by requiring students to use real world examples that could be evenly partitioned. After students came up with things like brownies, paper, candy bars and post cards and so on, he asked them to categorize these. Then he required students to develop a story problem based upon the principles they developed for categorizing. He requested students to ask as many questions as possible about this story problem and urged them to accomplish this task with their parents' assistance after school.

Nick was also able to design the instructional tasks to push students to develop conceptual understanding through their own exploration and to elaborate their understanding. For example, Nick started his first lesson by asking his students to come up with a big list of things that can be evenly partitioned. Then he assigned his students to work in small groups to generate some principles that they could use to categorize all the things.

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the things. He challenged each group to explain the principle clearly to the class as his students reported how they categorized these things.

Moreover, Nick paid attention to designing activities that allowed students to find alternative ways to solve routine mathematics problems and to see the relationship between different mathematics concepts in these lessons. This was shown in his second lesson, which focused on how to get the lowest terms of a fraction. Nick helped students learn a routine method of finding a lowest term for a fraction, that is, first, to find all the factors for the denominator and nominator, and then identify the factors they both shared, and in the end, divide both denominator and nominator with these common factors. Then Nick started to push his students to think of alternative ways for some special case, like 8/64. He helped them to use their knowledge of even numbers to identify immediately if the fraction was in its lowest term or not without going through the routine process.

Nick's instructional processes

Nick used substantial chunks of class time to push students to see how mathematics knowledge could come from our daily life through mathematical inquiry. For instance, he used fifteen minutes of his first class to challenge his students to find how some unrelated things could be simplified into two groups by examining the ways that they were partitioned. Then he used another ten minutes to discuss in the class to help the class see the two basic methods that people could use to do even partition, by shape and by items.

Nick devoted most of his instructional time to helping students understand and apply the mathematics concepts. Even when he was teaching a review class, he still put conceptual understanding as his central focus by constantly helping and challenging his students t

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students to see the relationship between different mathematics concepts. For example, he spent half of his second class to challenge students to see that to obtain the lowest terms of a fraction was a process of using different kinds of mathematics concepts, division, multiplication and even numbers. He questioned the routine way of thinking and got students to find answers to new situations and problems. During this process, he rarely directly answered or provided explanations to his students' question and confusion.

Nick spent a lot of time to interact with his class on mathematics concepts that students were learning. Through this kind of conversation, not only did he get his students to report their ideas about the concepts but also he challenged students to explain and prove their answers. The following dialogue in his second lesson is such an example. After teaching a routine way of finding the lowest terms for a fraction, Louis continued:

C: Let's see, 8/64. <u>Would you know that is the lowest terms or not? Right off your head. Could you know it is the lowest term without figuring out if it is lowest term?</u> [He challenges students to use an alternative way]

SS: No.

SS: It is impossible.

C: Yes, you can. I guarantee that is not the lowest term. I don't even have to try what number to divide by, each of these divided by a number. I know it. Do you remember what we did in this morning? <u>Something we did for 48 and I am looking for some factors, the one that is really easy</u> [Support students to think about an alternative]. Is that lowest term? <u>How can I immediately tell without even calculating it at all</u> [He challenges students to use an alternative way]? S1.

S1. Because the nominator and denominator.... You need to go down to see how many pieces you have. So the 64, I mean it is quite a lot of them.

C: <u>How about 8/24? Is that lowest term? How can you tell about this one? Is it or is it not? How can you tell whether it is or it is not?</u> [He challenges S1 and the class to give a clear explanation with another example]. Is there any number I can divide 8/64 by?

SS: ....

C: OK, <u>you guys are thinking the nature of the number? Simple</u> [He supports students again].

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SS: You divide 64 by 8...

C: No, no, no. Do not go even that far, even easier. I can tell you within a half second whether it is lowest term or not. What I need is a half second [He challenges students again].

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S3: You can divide the numerator and denominator by the same number [S3 uses the routine way again].

C: Don't even make it too hard. As soon as you see it, yes or not.

S4: No.

C: Why not? [He pushes S4 to give a reason]

S4: You can divide both by 8. Err...

C: Even easier. Let's think about that [He pushes students again]. S5.

S5: They are both even numbers.

C: They are both even numbers. What is true about even numbers [He pushes students to elaborate his answer]?

S5: They can be divided by 2.

C: Divided by 2. You know they are both even numbers. I know they can be divided by 2. I don't know whether I can get the lowest term or not by dividing them but I know it is not the lowest term [He summarizes S5's 1 answer].

In this conversation, Nick first gave student a fraction 8/64 and required them to come up with an alternative way to find out whether it was the lowest term or not without going through the routine steps he had just taught. When his students failed to discover the idea and thought it was impossible, instead of telling them the answer, Nick challenged them and reminded them to use the concepts they learned in the morning. When his students still failed to find the method, he used another fraction, 8/24, to support their discovery by pointing out that the same method that applied to 8/64 can be used for 8/24. After S4 found out that he can divide both denominators and nominator in both fractions by 8, he kept challenging the class until S5 discovered the principle in such cases. That is, when both denominator and nominator are even numbers, it is not the lowest term.

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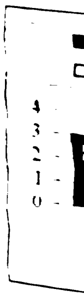

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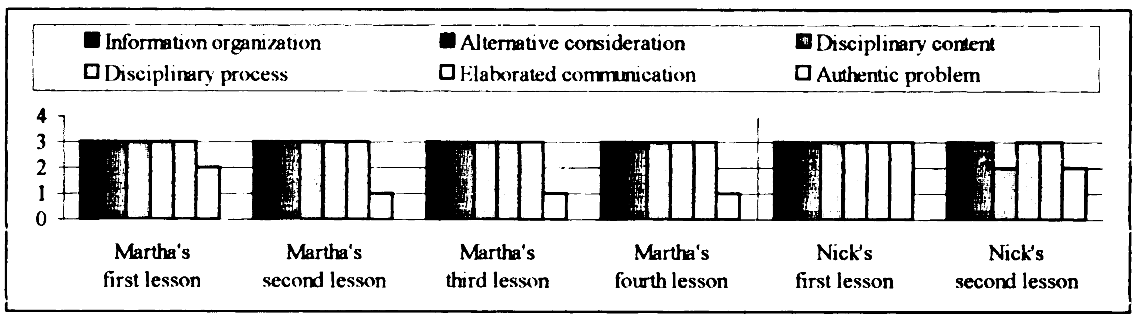
Nick's teaching and Martha's practice

Comparing Nick's teaching with the Martha's practice and constructivist standards, I find that Martha's mathematics teaching looks very similar to what Nick did in almost all the areas in both instructional tasks and process development. First, let's look at their instructional task comparison as summarized in Table 17 below. From this table, we can see the following things.

Table 17 - Instructional Task Comparison between Nick and Martha



In both of his videotaped lessons, Nick was able to develop instructional tasks that strongly reflected constructivist features. He received the highest (3-points) ratings in information organization, consideration of alternatives, disciplinary knowledge, and disciplinary inquiry. He had 3 on a 4-point scale for elaborated communication. In addition, his rating in authentic problem for his first lesson was the highest (3 points) and 2 points for his second lesson.

Such ratings suggest that Nick was not only able to pay close attention to the important concepts of mathematics and students' own discovery of mathematical ideas and procedures. He was also able to require students to interpret, explain and prove their ideas. He challenged them to put their ideas into public examination and opened chances for his students to develop alternative ways of solving mathematics problems. Moreover, he paid attention to building students' mathematics learning from their daily life experience.

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experience, though Nick did a less good job in the second. However, in that lesson, he was still able to connect what students had already learned to what they were learning. For example, he was able to help his students see whether a fraction was lowest or not by using the knowledge of even numbers that they had learned previously.

By comparing the instructional tasks Martha developed in her lead teaching unit with what Nick did in his lessons, I found out that in developing their instructional tasks they looked quite similar in many aspects. Both Nick and Martha shared the same high ratings in the categories of information organization, consideration of alternatives, disciplinary content and elaborated communication in all the mathematics lessons observed.

As I showed in Chapter 4 in detail, Martha was also able to pay close attention to the important concepts of mathematics and students' own discovery of mathematical ideas and procedures. She was also able to require students to interpret, explain and prove their ideas and challenge them to put their ideas into public examination and find alternatives. The difference here is that Martha did a relatively weak job in building instructional tasks by incorporating real life examples and events and drawing on what students already learned, especially in her last three lessons.

Table 18 compares Nick's instructional process and Martha's teaching process in terms of constructivist standards. From this table, we can also see that both were able to develop similar instructional processes that reflected constructivist standards.

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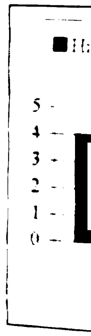

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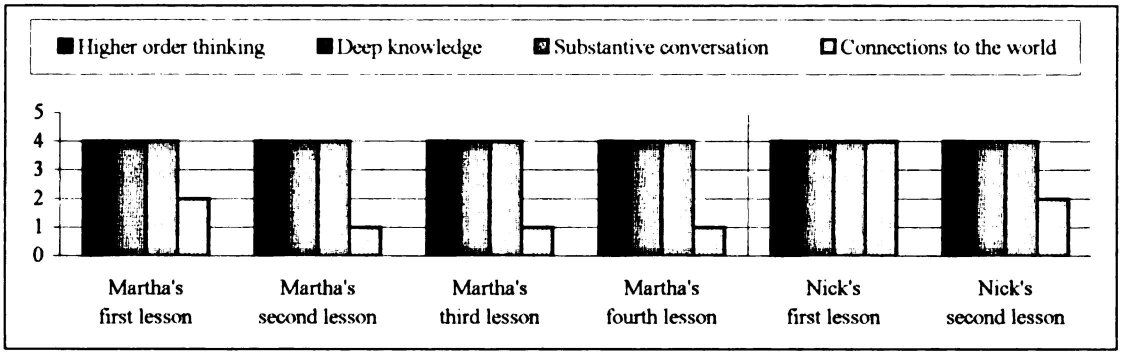
Table 18 - Instructional Process Comparison between Nick and Martha



Nick's instructional process also emulated constructivist features in all the aspects rated. He received 4 on a 5-point scale in higher order thinking, deep knowledge and substantive conversation in both of his videotaped lessons. He also got 4 in the area of making connection to the world in his first lesson and 2 in his second. These ratings suggest that large chunks of time in Nick's lessons were devoted to helping students develop conceptual understanding of mathematical ideas. In his class, students were engaged in forming, proving and challenging hypothesis and ideas. Focused and extensive discussions on the concepts were developed to get his students to further clarify their understanding of mathematical ideas and their relationships. Nick also helped his students to see their learning as related to the world beyond the classroom and justified the relationship in his first lesson.

Comparing their instructional processes in their video-taped lessons, we can see that both Martha and Nick shared the same and relatively higher ratings in most aspects of instructional process, such as, in higher order thinking, deep knowledge and substantive conversation. However, Martha did a less effective job than Nick in helping students see the relationships between what they were learning and the real world and justifying it.

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Influence of Bank's Teaching Practice

Bank began to change his way of looking at mathematics and mathematics instruction as his school started to push its mathematics instruction in the direction the NCTM standards envisioned. He claimed that as he moved conceptually closer to the standards, in his practice, he found it hard to implement his newly developed conceptions of mathematics teaching. He said he was "still more an instructor" who would like to show his students the answers instead of inspiring them to discover it by themselves.

Two of Bank's mathematics lessons are discussed here. In his first lesson, which occurred before Jaime took over, he taught long division where zero was present in the quotient. In his second lesson, that happened after Jaime' lead teaching, he taught his students how to find the lowest term for a fraction.

Bank's instructional tasks

Three features stand out in Bank's instructional tasks in his mathematics teaching. First, it was clear that Bank was able to integrate into his instructional tasks real world examples and events. For example, Bank started his second lesson by discussing a coming football game between the State University and the University of the State. He counted how many students in this class supported each team and then discussed how fractions could be used to represent the supporters for either side. For example, 15/25 of the students in the class were for the State University and 10/25 for the University of the State. Then he began discussing how 15/25 and 10/25 can be represented by simpler fractions by reducing them to their lowest term.

Second, Bank was able to be open to students' questions and give careful explanation and reasons for the mathematics concepts and rules that confused students.

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When students during his second lesson questioned why they needed to find largest common factors to reduce a fraction to the lowest term in his second lesson, Bank praised the student who raised the question and then demonstrated to the class that he could get the lowest terms of 16/24 in two ways. The first was to directly divide both nominator and denominator with its greatest common factor, 8, and the second was to divide them by 2 three times. Whatever ways were used, he explained, the result was the same and the difference was the second method took more steps and time.

However, Bank's instructional tasks did not emphasize students' own discovery of mathematics concepts and procedures, and failed to clearly require his students to learn to use mathematics inquiries and elaborate their own understanding. His instructional tasks were often designed only to convey the procedures and concepts that he wanted his students to apply and understand. To teach his student the long division with zero in its product in his first lesson, Bank first showed his students a rhythm, "Dumb Monkey Smashes Banana," to help students recall the procedure. That was divide, multiply, subtract and bring down. Then he gave some examples using this procedure and answered some questions from his students. In the end, he assigned students to practice the procedure individually with some similar problems from their textbook.

Bank's instructional processes

To help his students accomplish his instructional tasks, Bank often structured his class time into two chunks. He often had a relatively short fifteen minute demonstration followed by a long period in which students were assigned to practice individually problems selected from the textbook. During their practice, Bank would offer assistance to indivic

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to individual students when he was asked or felt that his students needed it. In the first section, Bank's teaching often reflected several features. He was able to demonstrate his understanding of a mathematics concept or model a procedure. Only when students raised questions were they shown or led to explore the complexity of mathematics concepts and the variations of mathematics procedures. Thus, mathematics concepts and procedures were treated quite unevenly in his instruction. Deep understanding of some was countered by superficial understanding of others. For example, in his first lesson, he was mainly lecturing and modeling how to do a long division with the DMSB procedure in his first chunk. It was not until a student raised a question about why in division for $724 \div 8 = 090...4$, the first zero can be omitted and the second zero had to be written out, that he began to explain the reasons for the position of zero in the product of a division.

Second, Bank often used real life examples and events to introduce a new concept and procedure to get students motivated and interested. For example, in his first lesson, he started his class with a recent football game in which the result was "nothing to nothing," and then introduced the topic of the class.

Third, teacher-led conversation was another feature of Bank's instructional process. Not only did he often show the concepts and procedures, but also he was able to open his teaching content for questions and provide answers. However, his conversation with his class was rarely used to challenge his students to discover and elaborate the concepts or procedures and prove or disprove them to the class. Rarely was it decided by students' process of inquiry. The following dialogue from his first lesson is such an example of his conversation with students. Here he was helping students understand the division problem, $8\overline{)724}$:

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C: DMSB. It stands for a little saying, Dumb monkey smash bananas. What these words really stand for? S3 (He asks students to recall the steps of division).

S1: D is divide, m is multiply, s is subtract, and b is bring down.

C: Thank you, S1. Divide, multiply, subtract and bring down. We still need those and these are important. If you want to put those in paper, it is fine to help your process. So using these steps, what thing I would think about first with this problem right here (He points to $8\overline{)724}$)? S2

S2: First divide by 8.

C: So my divisor is 8, my first candidate is 7. Can I make a pile of 8 objects?

SS: No.

C: No, do I need to put an answer up here?

SS: Yes.

C: How many piles of objects can I make?

SS: 0

C: So it is proper to put 0 on the spot. Many times we don't do that because it is an empty answer. Then there really are 0 in the place value according to this (He explains the reason putting 0 here).What should I do next? S3.

S3: Multiply.

C: What should I do next? S3 say I should multiply? What to multiply? S3.

S3: 0 multiply 8

C: And 0 multiply 8 is?

S3 0

C: 0. And next step is subtract. 7 - 0 is what? S4.

S4: 7.

C: 7. What is my next step? S5? I just subtract...

S5: Bring down.

In this conversation, Bank was using the phrase, "Dumb monkey smash bananas," to help students practice long division. He modeled and asked simple questions. Like in his other class, he did not challenge or support his student to explore mathematical ideas by themselves and then prove and explain their ideas.

Bank's teaching and Jaime's practice

Contrasting Bank's instructional practices with constructivist standards and with the mathematics teaching Jaime developed, I find that Jaime and Bank were quite similar in several ways. Neither their instructional tasks nor their processes were strong by constructivist standards. Their practices were very similar in many ways.

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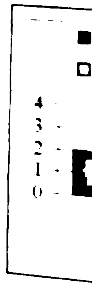

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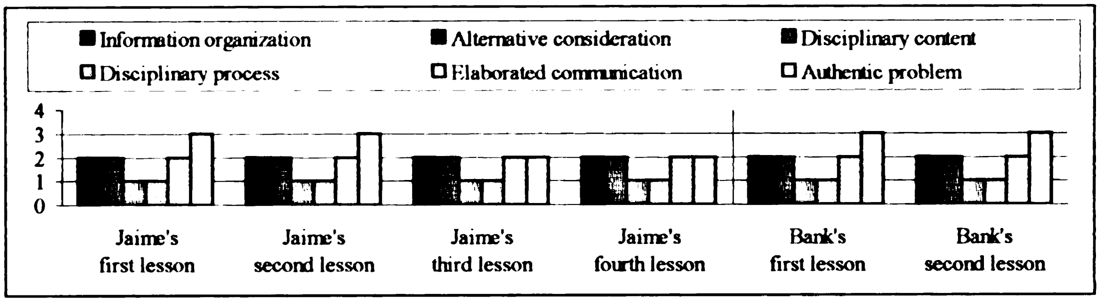
Table 19 summarizes the ratings that Bank and Jaime received in the six aspects of their instructional tasks for mathematics instruction. Comparing their ratings with each other and against the standards, I come to the following findings.

Table 19 - Instructional Task Comparison between Bank and Jaime



Bank failed to successfully develop his instructional tasks along the constructivist line. His ratings in information organization, consideration of alternative and elaborated communication averaged 2 on a 3-point scale. These ratings suggest that Bank was not very successful in designing tasks that required students to discover, interpret, elaborate their mathematical ideas. Although his students had chances to raise questions about what they were learning, they were not clearly encouraged or challenged to develop alternative ways to solve mathematics problems.

Bank had only a 1 point rating in disciplinary content and disciplinary process in both classes. This indicates that students' conceptual understanding of mathematical ideas did not get prominence in his instructional task design. Students were not clearly required to prove or disprove each other's ideas of mathematics. Instead, remembering procedures and applying rules took a central place. However, Bank received his highest 3-point rating for both classes on the dimension of authentic problem. In other words, Bank paid attention to incorporating the real world events and examples into his instructional tasks.

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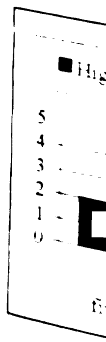

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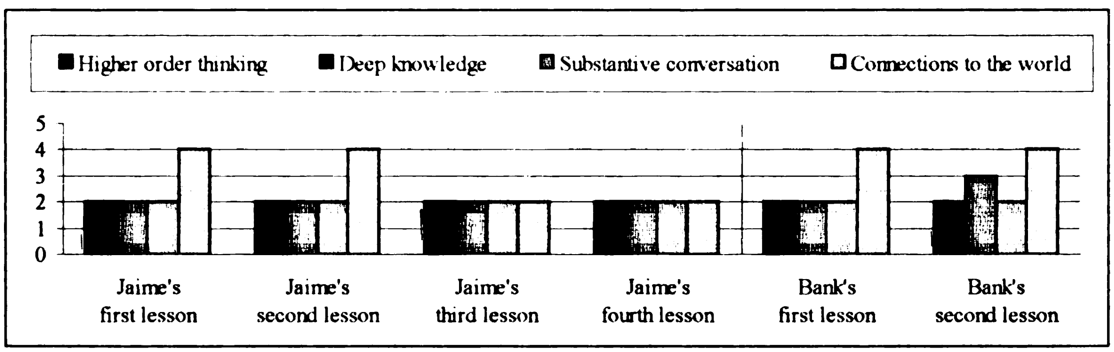
The instructional tasks developed by both Jaime and Bank were similar in several ways. From the table we can see that both teachers shared the same ratings in information organization, consideration of alternative, disciplinary process and elaborated communication. The differences between them was mainly that Bank was stronger than Jaime in her last two lessons in developing instructional tasks that incorporated students' daily life experience. In actual teaching, Bank paid more attention to students' questions and confusions and was able to turn these into further instruction. Jaime was able to get students to form their own ideas but she often stopped there without pushing students to further understand mathematical ideas.

From Table 20 below, we can see the instructional processes. Bank and Jaime each developed which also shared many features of the instructional tasks. Their ratings were not strong in most aspects assessed except for connection to the world.

Table 20 - Instructional Process Comparison between Bank and Jaime



Bank received 2 on a 5 point scale in higher order thinking and substantive conversation in both of his lessons and he was not successful in encouraging students to discover, prove and elaborate their ideas. Receiving 2 in his first lesson and 3 in his second suggests that students' conceptual understanding of mathematical ideas did not take a central place in his teaching. Although he was a little bit stronger on this criteria in his second tha

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his second than his first, in both classes, Bank failed to push students to develop their understanding through their own discovery and exploration. Modeling and demonstration was the main approach he used to help students learn mathematical ideas. However, he was able to help his students see what they were learning was related to the world beyond the classroom and justify the need to learn something as implied by his 4 point rating in connection to the world in both of his classes.

Jaime and Bank were similar in the instructional process they developed in many ways. Both were unsuccessful in pushing students to elaborate and prove their ideas and both were good at helping students connect their learning with their daily life experience. However, Bank opened more chances for students' questions and provided answers, while Jaime was able to get students to form their ideas without pushing them to reach a higher level of understanding.

Influence of Ben's Teaching Practice

Bell Elementary had actively pursued the kind of mathematics teaching that was envisioned by the NCTM standards. It moved its focus of mathematics teaching back to computation because students in the school failed to get high grades in the computation part of the state mathematics examination. As an elementary school in an affluent area, the school felt great pressure from parents to improve its students' grades. Ben was actively involved in pursuing the NCTM standards. He was a member of the school mathematics curriculum committee that was formed to restructure the school curriculum under the guidance of the NCTM standards. However, he reoriented his mathematics instruction toward a model of telling followed by practice as Bell Elementary pushed its teachers to go back to basics.

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The two videotaped mathematics lessons Ben taught are discussed and analyzed below. His first lesson dealt with number guessing games, multiplication and division facts practice, and geometry while his second lesson was filled with number guessing games, multiplication problem practice, division problem practice and fraction practice.

<u>Ben's instructional tasks</u>

Practicing mathematics facts and procedures was the main instructional task that Ben designed for his students in these lessons. It was often arranged before and after new teaching content. For example, in each of Ben's lessons, his students were required to practice one or two mathematics fact worksheets. Here is a part of such a worksheet collected from his second class.

Figure 6 - Worksheet Ben Used in his Second Lesson

<u>Division Facts</u>

$63 \div 7 =$	$2 \div 2 =$	$25 \div 5 =$	$81 \div 9 =$	$72 \div 8 =$
$32 \div 4 =$	$56 \div 8 =$	$0 \div 8 =$	$28 \div 7 =$	$9 \div 1 =$
$36 \div 6 =$	$18 \div 3 =$	$10 \div 2 =$	$20 \div 4 =$	$14 \div 2 =$
...	...	...	...	...

Efficiency was the goal of such practice. Seventy-five division facts should be done in four minutes with over ninety percent correct answers, otherwise students needed to practice more as their homework. In both classes, Ben also required students to compete with each other in practicing some mental mathematics problem, like this one collected from his first class: $\sqrt{64} \div 8 + 2 - 1$. Moreover, after teaching a new topic, he usually assigned students to work individually on some similar questions he demonstrated from their textbook.

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Second, Ben liked to incorporate real life examples and events into his instructional tasks. For instance, in his first lesson on how to get the area of a rectangle, Ben was able to create problems of carpeting his and his students' bedrooms to help his students think in context about how to estimate how much carpet they needed to buy. He insisted his student consider about units they use and emphasized the problem that might result from inconsistent use of units, like feet for the length of the room and yard for carpet they needed to buy.

Third, Ben paid less attention to students' own discovery of mathematics concepts and procedures through mathematics inquiries, like proving and disproving each other's ideas and hypotheses. His instructional tasks were often designed only to convey the procedures that he want them to apply and to illustrate his understanding of the concepts. For example, in his second lesson on long division, he first modeled a problem, $643\overline{)7261}$ by following the procedure of "Dumb Monkey Smashes Bananas," like Bank, and then asked some of his students to do two more similar problems with the same procedure. After that, Ben assigned the class to practice individually .

Ben's instructional processes

To help his students accomplish his instructional tasks, Ben's teaching in each lesson was often structured into four chunks. In the beginning, he would play some number guessing game and do mathematics facts and mental mathematics practice. He often spent a short period in teaching new content and, in the end he assigned students to work individually on some textbook questions by following his demonstration. Thus, most of Ben's class time was spent practicing what had already been learned and what was just taught.

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The time Ben devoted to new content instruction was often short. The teaching process in this part of his class often had the following features. First, Ben was able to constantly use real life examples and events to help his students see the relationship between what was being taught and the issues or problem that students might meet in their outside school life. He was able to justify the mathematics facts practice they were doing with its real life implications. For example, in teaching his students how to calculate the area of a rectangle, not only was Ben able to use the real life example, his as well as his students' bedrooms, but also he emphasized many times that multiplication could be applied to geometrical problems in our real world. Thus, he agreed that it was important for them to practice these facts and memorize them.

Lecturing and modeling were the main ways for Ben to teach the new content. However, sometimes he would do it through conversation with his students in which he asked questions that only needed a simple answer or confirmation from students. Seldom did Ben challenge his students to elaborate on or to explore their ideas of mathematics concepts or to develop a deeper conceptual understanding. This feature of his instructional process can be seen from the following dialogue in his first lesson on how to come up with a formula to calculate the area of a rectangle. After demonstrating how to calculate areas for his or his students' bedrooms, Ben continued his conversation with his students:

C: OK, these are the words that are important for us, which we are finding the areas for rectangles. We have worked with rectangle and square. See if you can come up in your mind a formula. I will do one for you. [He writes on board: S [S = A, S^2]. I will explain what I did. I will try to find out what I did for a square. I was trying to find out what if a square. A stands for?

SS: Answer.

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C: OK, A can stands for answer. What we have been working for?
S2: Area.
C: Area, OK, let's see, what is S stands for? S does not stand for square here.
SS: ....
C: I am trying to find out a square. A square has four...
SS: Sides.
C: Oh, side, side times side, or we can go S^2. Using this as an example, I want you to come up a formula like this. So we can find area for a rectangle [He gives students a model to follow].
SS: OK.
C: We will do it as a group, you have done this. You found out all the areas here today. You multiply length and width, you found area. OK, I give you a clue. Since we are going to find out area. I first wrap up a word. A stands for area. [He starts to model again and writes on blackboard: A=].
SS: ....
C: Is there anybody got this yet? You guys come up with a formula? Go and write it on the board. One of you, OK. S1. You can come up one, S2, OK, write it down. [S3 and S4 write on board: A = L[W]
S3: It is kind of easy.
C: Did you hear what S3 says it about. Please say it one more time.
S3: ...
C: S3 just said a math concept is kind of easy [the two students finishing writing on the board the formula. A= L [W]. Good job, OK. Will you have a seat. Good job. You are right, S3. It is kind of easy. What I want you to understand is A= L x W.

In this conversation, Ben first asked his students to come up with a formula for calculating areas for rectangle. Without letting his students think for themselves for a moment, he immediately gave them the formula of how to calculate areas for square as a model for them to follow, which was A = S x S. He gave students a clue that in the formula for square, the letter, A, represents area and S represents side. Then he told students in the case of rectangle, the letter, A, was still area and rectangle had width and length as their sides. In the end, he was able to have students find the formula for calculating the area of a rectangle in a way that was almost equal to telling.

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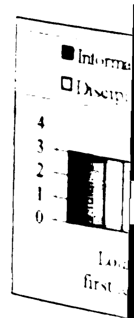

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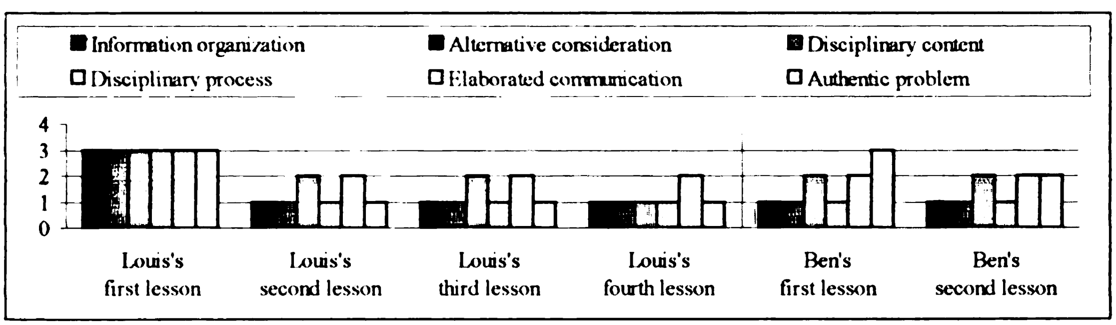
Ben's teaching and Louis's practice

Comparing the instructional practice that Louis and Ben developed with constructivist standards, we see that Louis's first lesson was different from his other lessons and Ben's classes in its strongly reflecting constructivist standards of mathematics instruction. However, his remaining lessons looked increasingly similar to the teaching practice Ben developed, a practice that was weak in developing students' conceptual thinking through their own discovery and exploration. Practice, demonstration followed by practice became the norm of their teaching.

Table 21 below gives us a summary of the ratings that Louis and Ben got for their instructional tasks in their videotaped mathematics lessons. It suggests several things.

Table 21 - Instructional Task Comparison between Ben and Louis



Ben received only 1 point in information organization, consideration of alternatives and disciplinary inquiry. This implies that Ben was weak in designing instructional tasks that required students to construct, interpret, or elaborate their ideas of mathematics. He was also less likely to challenge students to develop alternative ways to think of mathematics concepts. His 2 point rating in disciplinary inquiry and elaborated communication suggests that conceptual understanding of mathematics was not as important as practicing procedures in his tasks. Students in his class were seldom required to p

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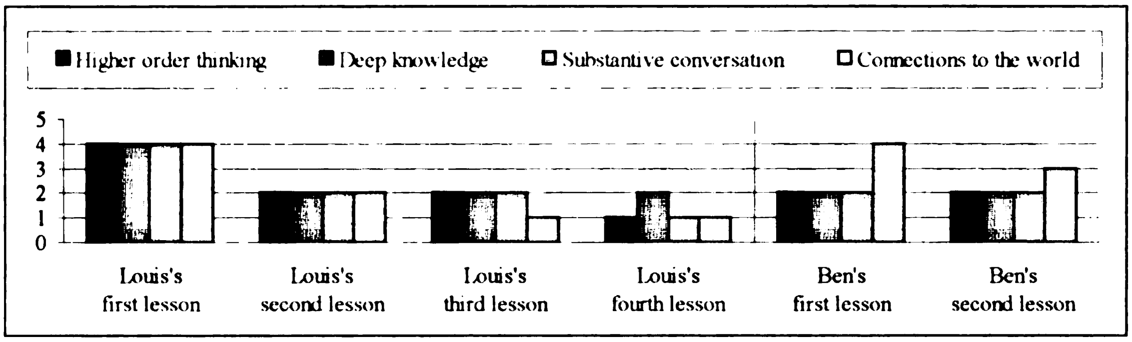
required to prove or disprove each other's ideas. However, Ben was strong in incorporating students' daily life and real world examples into his instructional tasks. He did especially well in this aspect in his first lesson, for which he was rated the highest 3 points.

Comparing Louis's and Ben's ratings, we can see that the instructional tasks developed in Louis's first lesson were much closer to constructivist standards than those in his last three lessons. For this lesson, he received 3 points in all the categories assessed. These ratings were also different from what his collaborating teacher, Ben, was able to do.

However, in his remaining lessons Louis was clearly unable to keep or further develop what he was able to do in his first lesson. Based upon the ratings they received, we can see that Louis looked increasingly similar to Ben in designing his instructional tasks. The major differences was that Ben was able to emphasize the experiences students brought into his class more than Louis was able to do in his three lessons.

The instructional processes that Ben and Louis developed also reflected similar features in their instructional tasks. Table 22 shows that the instructional process Louis developed in his first videotaped lesson was not only different from all his other lessons in all the categories assessed, but also different from those developed by Ben. To a great extent, it reflected constructivist standards. However, in his remaining lessons Louis's instructional process gradually walked away from constructivist standards and moved closer to Ben.

Table 22 - Instructional Process Comparison between Ben and Louis



Ben received 2 on a 5-point scale in higher order thinking, deep knowledge and substantive conversation. That suggests that Ben was weak in developing students' conceptual understanding and pushing their own discovery and examination of mathematical ideas. In his class, practice of procedures and rules was the dominant activity that was often modeled or illustrated by the teacher. Then students were required to practice them repeatedly. His instructional conversations were often used in way to model a correct mathematics procedure or to provide easy questions for students to answers. However, Ben did a better job in engaging students in connecting their learning to the real world, as he got 4 points in his first lesson and 3 points in his second lesson.

In his first class, Louis was really successful in engaging students in learning mathematics as reflected in constructivist standards. He received 4 out 5 points in all the areas. He was able to devote most of his class time to helping students develop conceptual understanding of mathematical ideas through their own discovery, explanation and examination. He was able to develop focused and extensive discussions with and among students. He was able to help his students see the relationship between what they were learning and its real world application.

However, Louis was unable to retain this way of teaching in his remaining lessons. His ratings in all the areas became increasingly lower one lesson after the other. He received 2 points in all areas for his third lesson and by the end of his last lesson, he was only able to get 2 points in deep knowledge and scored only 1 point in each of the other areas. His instructional processes looked increasingly similar to Ben's. The differences were that Ben did a better job in helping his students see a relationship between what they were learning and what they had already learned and experienced outside school. In addition, Louis did a worse job in conducting a sustained conversation and paid less attention to students' conceptual understanding in his last lesson than Ben.

Influence of Lisa's Teaching Practice

In recent years, Mall Elementary where Lisa worked began to stress its K-3 students' mathematics performance. On the one hand, it pushed its K-3 teachers to use different kinds of manipulatives in their mathematics instruction. On the other hand, it actively participated in a national standardized testing program for metropolitan areas sponsored by the school district "to look at how Mall Elementary students perform in certain areas [including mathematics] in comparison to their counterparts in other urban areas across nation."[7]

Lisa claimed that she actively responded to the first push from her school by using a lot of manipulatives in her teaching. At the same time, she also felt pressed to support her students to prepare for the standards examination, though she preferred not to. The two lessons Lisa taught are analyzed here. One was on calculating addition and

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[7] This is quoted from *1995 Annual Report of Mall Elementary School.*

subtraction problems with calculators that occurred before Kelly's lead-teaching period. The other, after Kelly's lead teaching, was on addition of two digit numbers without carrying.

Lisa's instructional tasks

Lisa's instructional tasks in both lessons shared several features. First, Lisa strongly emphasized conveying a correct strategy or rule of doing mathematics problem through her demonstration and modeling. For example, in her second lesson on addition of two digital numbers, Lisa used the following three problems (see Figure 7 below) to teach students how to do two-digit addition with a strategy that would be problematic when students were doing multiple-digit number addition with carrying:

Figure 7 - Three Addition Examples Lisa Used in her Second Lesson

16	15	12
+ 12	+ 13	+ 41

Lisa directly demonstrated to her students that to get the result for first problem, they needed to add the numbers in the ones column first and write their answer under the ones. Then they needed to add the number in the ten column and copy the answer under the tens. She led her students to do the next two problems by insisting they follow the strategy she used for the first question. During the whole process, she neither helped students analyze the strategy, nor opened any chances for students' questions and discussions about any alternative ways.

Practicing the procedure that Lisa modeled and demonstrated was another feature of her instructional tasks. For each of her lessons, Lisa often required her students to accomplish one or two worksheets in which all the problems were similar to those Lisa modeled and demonstrated. Here is the part of such a worksheet collected from her second lesson (see Figure 8 below) which Lisa assigned her students to do after modeling the aforementioned problems:

Figure 8 - Worksheet Lisa Used in her Second Lesson

Tens	Ones	Tens	Ones	Tens	Ones	Tens	Ones
1	2	4	2	5	0	3	1
+ 3	6	+ 1	4	+ 2	7	+ 2	8

In her mathematics teaching, Lisa was able to incorporate different kinds of manipulatives and pictorial worksheets into her instructional tasks that she used to support students' practicing rules and procedures. For example, when her students were assigned to do individual practice in her second lesson, not only did Lisa allow her students to choose four different kinds of counting aids to support calculation: the small fish, little flowers, colored sticks and small chickens. She also gave her students a worksheet in which the mathematics problems were illustrated with pictures of rabbits, monkeys, money and crayons.

<u>Lisa's instructional processes</u>

Lisa's instructional process can be divided into two clear-cut parts in these videotaped lessons: demonstrating a rule or skills for about ten minutes and students individually practicing similar questions on a worksheet for the remaining class. Those who completed their worksheet first were either given an extra worksheet to work on or were assigned to do other activities, like play computer games, draw and read, etc. Lisa usually walked around and checked or helped individual students who needed extra assistance.

Even though Lisa was able to incorporate different kinds of manipulatives and pictorial worksheets into her instructional tasks to support her students practicing procedures, Lisa rarely helped her students see connections between what students were learning and its implications for the real word. Nor did she justify the need for them to learn something. For example, even in her first lesson on using calculators, a topic which had great potential for her to make a case about the relationship between what students were learning and its real world applications, she did nothing to help her students see its daily life application and made no justification for the need to learn it. Rather Lisa simply turned this class into another modeling and practice process.

In demonstrating a procedure, Lisa often insisted her students follow her demonstration step by step and rarely allowed any alternative approach or provided any explanation and reasons. In her first lesson, for example, when seeing some students started a new calculation with calculators by pushing its "ON/OFF" key instead of using "C "--the cancel key, Lisa stopped her students' individual practice. Then she strictly insisted her students use "C" key through a step by step demonstration again without giving any explanation why the "C" key could not be used.

During her demonstration, her conversations with students were often short and conducted in a way to get students to provide affirmative response to her demonstration. Students' questions were often not clearly encouraged. The following dialogue in her second class on two digit number addition shows this feature of her teaching:

C: Now what we are going to start today is...Read this number [She writes down on the board the number 23 and points to it]
SS: 23
C: Next [She writes down the symbol + and points to it].
SS: Plus, Equals...
C: Does that say "equals," What does it say? Plus.
SS: Plus.
C: What is this? [She write 15 under 23 by +]
SS: 15...
C: What is this [She draws a line under symbol and the number 15 and then points to it] ? Equals. Let's read this once again.
SS: 23, plus...
C: What is this number?
SS: 15, equals.
C: Now, we are going to add 23, 2 tens, how many ones?
SS: 3...
C: <u>We have done a lot of that. We have done a lot of sticks, we have done unified cubes. We have been doing chicks and eggs. We have been doing a lot of tens and ones</u> [She reminds her students how to do addition they have learned]. Now we are going to add ones first. Everybody add.
SS: 3 plus 5...
C: is?
SS: 8
C: We have done ones, we are going to add tens. 2 plus 1 is?
SS: 3
C: So our answer is?
SS: 38.
C: I need to eyes and ears up here. Let's read this problem. All the problem.
SS: 23 plus 15 equals 38.

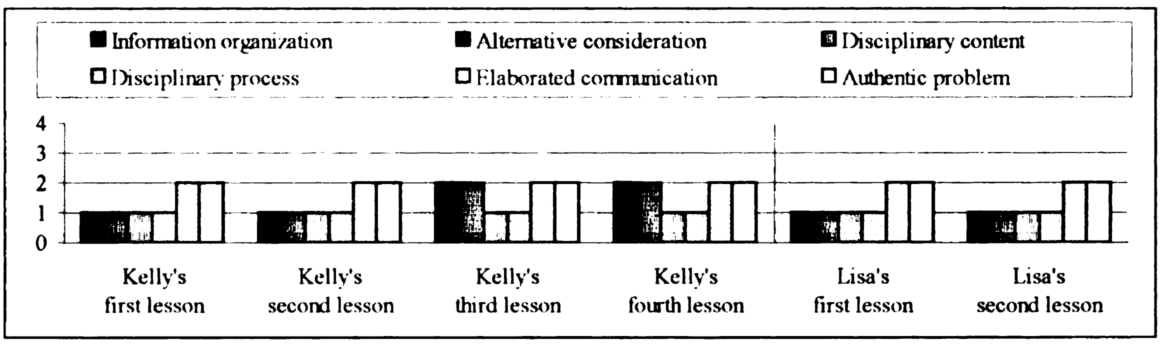
In this short conversation, Lisa, as she often did in her teaching, was demonstrating an approach to adding two digit-numbers, 23 + 15 =. She modeled her strategy step by step and asked questions that required no explanation but simple confirmation. No discussion occurred about the concepts. Students were shown directly how to do the calculation without showing why and how the procedure was used and its real world applications.

Lisa's teaching and Kelly's practice

When I compare Lisa's mathematics teaching practice with Kelly's lead teaching I discussed in Chapter 4, I find that their mathematics teaching practice was similar. Both were weak in relation to constructivist standards. Both Lisa and Kelly developed mathematics instructional practice that reflected absolutist approach.

Table 23 summarizes the rating that Kelly and Lisa received for their instructional tasks. From this table, we can see several things about their instructional tasks.

Table 23 - Instructional Task Comparison between Lisa and Kelly

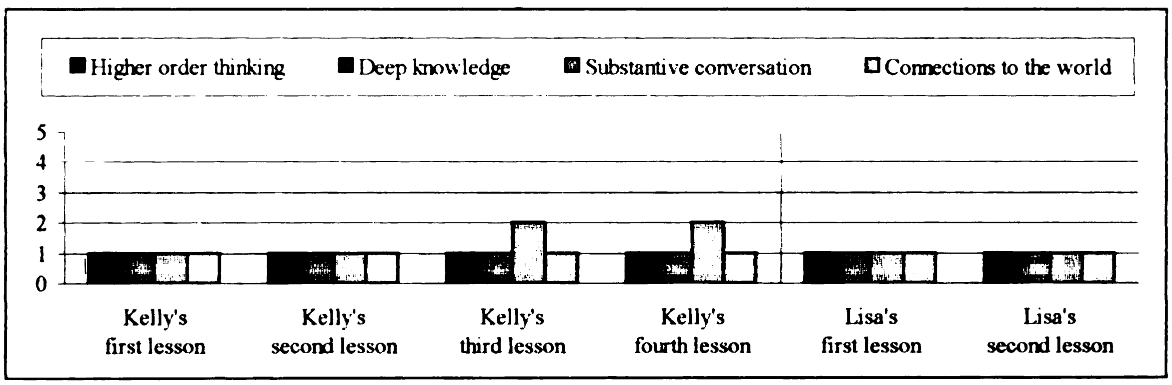


Lisa was weak in developing tasks that supported students to organize information, consider alternatives and push students' conceptual understanding through their own discovery. She only received 1 point in all these areas on a 3 point scale. Her 2 point rating in elaborated communication also suggests that she was not very successful in designing instructional tasks that clearly required her students to interpret and elaborate their ideas of mathematics. Although Lisa was able to incorporate various manipulatives into her instructional tasks (2 points for authentic problem), she only used these manipulatives as counting aids for students to practice the rules demonstrated. She rarely used them to help students construct their ideas about mathematics concepts or discover a rule.

Comparing the ratings that Lisa got with Kelly's rating in the instructional tasks, we can see that the instructional tasks Kelly and Lisa developed in their lessons were quite similar to each other in almost all the aspects assessed. They were rated low in terms of all the constructivist standards. The difference was that Kelly did a little better job in her last lesson in letting her students consider their own estimation and report their findings.

Their ratings in instructional processes reflected similar features. From Table 24 below, we see that the instructional practice they developed in these lessons were similar in many aspects and distant from constructivist standards.

Table 24 - Instructional Process Comparison between Lisa and Kelly



Lisa was weak in developing teaching processes and her rating was rated low in every aspect of constructivist standards, receiving only 1 point on a 5-point rating scale. These ratings indicate that Lisa devoted all her class time to the traditional way of teaching, which was dominated with teacher's telling followed by students' individual practice. Her conversations with students, when they occurred, served for her to model a correct mathematics procedure. The relationship between what she was teaching and its real world implications were neither identified nor justified.

Comparison of Kelly and Lisa's ratings in their instructional process suggests that in almost every aspect, their instructional processes were similar but weak from a constructivist approach. The only difference was that Kelly did a little better job in building substantive conversation in her last two lessons than Lisa, in that she was able to get students to report their answers to the class after their individual practice.

Summary of Chapter

In chapter 4, I described how the four preservice teachers in this study developed quite different mathematics teaching practices. However, except for Martha, the three others -- Jaime, Louis and Kelly -- were not able to practice or to keep practicing the constructivist ideas they brought into their internship. Their teaching practices were different from each other in light of constructivist standards or the expectation of their program.

My analysis in this chapter leads to two findings related to their learning. First, all the collaborating teachers taught mathematics in different ways. Except for Nick, all the other collaborating teachers were teaching in a way that was not what the program encouraged their preservice teachers to learn. Nick's instructional practice was highly rated in terms of constructivist standards in almost all the areas of instructional task and process. Bank's instructional practice was still distant from the standards in many areas, though he was able to relate what he was teaching to students` daily life experience and real world events and be open to students' questions. Ben was able to show to his students the relationship between what students were learning and the real world. However, he was unable to practice many other constructivist ideas, like developing conceptual understanding through students' own discovery and public examination.

Lisa's instructional practices were overwhelmingly traditional. Her dominant approach to teaching was to demonstrate procedures followed by engaging students to do individual practice.

Second, in spite of the instructional practice differences between each preservice teacher and those among their collaborating teachers, the instructional practices that each preservice teacher developed, to a great extent, was similar to or moved toward his or her collaborating teacher. Thus, it strongly implies the influence of collaborating teachers on preservice teachers' learning to teach at a practical level.

Like Nick, Martha's instructional practice was also highly rated in most aspects of constructivist standards. The only difference lies in that Martha did a relatively weak job in building instructional tasks that were closely related to real life issues and problems. Jaime's instructional practice shared similar ratings to Bank's instructional practice. Then the difference was that Jaime was less likely to provide feedback to her students' questions than Bank. Although Louis were able to develop a constructivist approach in his first class, his instructional practice dramatically changed into a traditional model that was similar to what Ben was doing. The difference lay in that Ben was stronger in connecting what he was teaching to students' daily life and what they had already learned than Louis was in his remaining lessons. Kelly's teaching exactly reflected what Lisa was doing in her mathematics classes. It strongly suggests the traditional orientation–telling followed with practice.

While my analysis in Chapter 5 and 6 show that collaborating teachers may have exerted a strong impact on what a preservice teacher was able to learn at both conceptual and practical levels, there are still questions to be answered. How did such influences happen? In the next chapter, I further examine this question by exploring the function of collaboration between collaborating and preservice teachers.

Chapter 7

COLLABORATION AND ITS INFLUENCES

My analysis in previous chapters leaves me with an interesting question: How did the influences of collaborating teachers on preservice teachers' learning to teach occur? In this chapter, I explore this influence by turning to the collaboration between collaborating and preservice teachers in the program. As I discussed in Chapter 1 and 2, collaboration between preservice and collaborating teachers can be considered as an important medium through which collaborating teachers exert their influence on their preservice teachers' learning to teach. In this program, collaborating teachers were organized to work closely with their preservice teachers to help these teachers learn to teach. Not only were they required to provide a model for their preservice teachers to observe, but also they were required to guide, support and assess preservice teachers' learning to teach across the internship year.

In this chapter, I explore three aspects of this collaboration. First, I examine the foundations upon which collaborating and preservice teachers developed their expectations for their collaboration. Then I analyze the focuses of and approaches to their actual collaboration. In the end, I explore the ways in which collaborating teachers and collaboration in each case influenced the results of preservice teacher's learning.

Collaboration and Its Functions in the Case of Nick and Martha

Foundations of their collaboration

The collaboration between Nick and Martha was built upon several foundations. First, the conceptions of mathematics instruction that Martha brought into the internship were similar to those ideas that Nick developed in his teaching. Martha and Nick both believed that mathematics was an activity involving sense making and conceptual understanding. Both claimed that the goal of mathematics learning is to discover, prove and communicate mathematical ideas and mathematics instruction is to support this kind of mathematics sense making and learning.

Not only did they have similar ideas about mathematics instruction, but also they were able to realize this similarity between them. Nick thought that the five-year program that Martha was in reflected what he and his school were experimenting with. Martha also claimed that when she came into Nick's class, she realized this conceptual similarity between them.

> We were similar as far as finding the hypotheses, trying to have them and help them figure out their ideas like a kind of asking them questions, guiding them to learn.

In addition, Nick was able to put many of his ideas into practice and Martha was able to identify these ideas from Nick's teaching. This fact not only allowed Martha a chance to see some of her teaching ideas in action. It also provided her a specific model to follow and think about in her learning to teach. As Martha said when she summarized why she was able to learn what she learned in her internship:

> Part of it, it is he teaches math in the same way that I was taught in my program. That is having them to think about the things and find out them by themselves

> rather than telling to them. He is teaching the way that I was taught to teach. And I hadn't actually done by myself.

Expectations for their collaboration

These foundations greatly contributed to the expectations that Nick and Martha developed for their collaboration in two ways. From Nick's side, seeing that Martha shared some basic ideas of mathematics instruction with him pushed Nick to assume what Martha needed to learn was to strengthen these ideas and connect these ideas with teaching practice by involving her in doing it .

> She is not coming with a strong philosophy, I don't think a lot of teachers do. I am not sure any of the interns really come and say this is. It is really that she has been at a point of a lot of exploration and practice that she will begin to see a little more of the ideas behind the teaching practice.

Nick developed the idea that Martha needed to learn how to support students to discover mathematical ideas by situating their learning at the particular level rather than to have them find correct answers at the symbolic level. He explained:

> Have the symbolic part sometimes, I don't want to say, being least important but not as important as students had been taught before. We used to get these numbers in front of them and they manipulate these numbers to get the right answer. Now, she needs to have the class talk about there is a sort of thing and really have kids to come up with an answer rather than you just give them the answers in math. You know that is different, because till you are really at the particular level, you are not sure exactly what they mean.

In addition, Nick claimed that to teach mathematics at the particular level was not only about what manipulative needed to be used. It was also about how to organize students to discover mathematical ideas. Thus, he expected Martha to learn both aspects of this kind of mathematics teaching.

> To teach in this way is not just how you use the manipulative to get mathematical concepts across, but also the management is a whole different thing. How do you

> distribute these, how do you have your students to use them in a way that is best use of their time? It is managing that kind of teaching. It is a lot of more difficult.

Seeing Nick able to practice many of the ideas she brought into the internship not only helped Martha recognize that she and Nick shared the same philosophy of mathematics instruction. It also pushed Martha to think that there were things in Nick's teaching that were worth her learning. For example, although Martha sensed that Nick would "kind of guide their conversation a little bit more " than she "would feel comfortable doing," she believed that Nick was a good model for her to learn from. She thought that "one of his strong points is that he is able to put his teaching on their level and that is definitely what I need to learn from him."

This foundation also helped Nick and Martha build their expectations for the roles they needed to play in their collaboration. Nick assumed three roles in working with Martha that were also reflections of what the program expected him to do. First, Nick claimed that he needed to model specific ways of this kind of mathematics teaching and provide chances for Martha to observe and think about what was happening in his teaching and students' learning in his class. He said:

> First way, which is the way they were encouraged to learn, is simply by me modeling them. Just saying here is what we are doing as I taught the class. Martha will be in the back listening as well. When it was time for the kids to work independently, she can move among the kids and learn how they will do it at the same time she was learning how to do it.

He also claimed that he needed to provide teaching materials and support for Martha to learn how to develop a mathematics curriculum that both Martha and he would agree upon and that would fit his students' learning.

> I would recommend other textbooks. I had a lot of supplemental books that will be specifically for teachers and their teaching of math and how to teach fractions.

> She relies on those a lot. What she does is to incorporate something I had and even use them. She is taking all those materials and then she is using some other resources. Once that she feels that material fits the way she understands and then can communicate it better in teaching. It still needs to fit the way I was doing things and the way kids were doing things.

The last role Nick wanted to play was to be "a bouncing board" from which Martha could get feedback and comments on what she thought and did in her mathematics teaching. In this way, Nick emphasized Martha would have better chances to "think about what students just did, where they are going and how this relates to what you have done in teaching."

Like Nick, Martha also thought that her learning to teach mathematics in the internship was not a process of trial and error. Instead she claimed that in her internship she needed supports from her collaborating teacher in the following ways. First, Martha expected that her collaborating teacher would be able to think with her about what to teach and how to teach mathematics.

> I think I like to be able to bounce ideas off another person, and have him talk with me about a different idea or a concept and what is going on in the classroom.

She hoped that her collaborating teacher would be able to carefully observe and give specific feedback to her about her mathematics teaching. As she said "to observe and think with me about my teaching. What was good and what was not and why?"

Thus, the expectations that Nick had for his collaboration with Martha seemed to match what Martha wanted Nick to do for her. Such features of their mutual expectations for their collaboration shaped what they actually did in collaboration.

Collaboration between Nick and Martha

The form of actual collaboration between Nick and Martha went in a gradual transitional model as the program arranged, but the focus of their collaboration reflected what they expected to do in their collaborative work. In the first few weeks of her internship, Martha was encouraged by Nick to observe how he was planning, organizing and teaching mathematics lessons. He encouraged her to ask any questions concerning his teaching and the reasons behind what he did in his classroom. As Martha gradually moved into partial involvement in Nick's teaching, for example, to help a group of kids or an individual student do an assignment in his mathematics class, Nick continued to discuss with her what happened, was happening and was going to happen.

As Martha began taking over more and more responsibilities of teaching, Nick began "to pull that support away little by little until you really become a safety net." However, he still supported Martha's learning to teach in the following ways. First, Nick analyzed with Martha the mathematics concepts she was going to teach and their relationship. He helped Martha develop a better sense about a sequence of how to teach these concepts. As Nick described:

> We look over what concepts and what are steps we need to introduce and start on the concepts.... Then she would e-mail over the weekend a unit and a lesson plan that she had. "Here is what I want to do but I am still confused about myself." Then we get together again so that she really made that effort to understand and to get to know some pretty difficult ways of teaching, or some concepts or pretty difficult to understand.

Second, Nick provided Martha specific materials and books that can be used for her teaching certain concepts and sent her to other teachers Nick thought were stronger in certain areas for further assistance. He said:

> I would send her to a specific person that I felt was a little stronger about that area, especially again to know some level of this curriculum. I would find resource and find specific books and materials that could be used and pointing those concepts out.

Third, Nick gave feedback to Martha and modeled the ways of teaching that he would like her to use during Martha's teaching. Nick supported Martha' teaching by asking students some questions, writing questions and comments and passing them to Martha, or taking over the class directly and modeling. Nick pointed out two things by jumping in during Martha's teaching. First, he wanted to point out important things that Martha was missing in her teaching. Second, he wanted to suggest an alternative way of doing things that "better involve students in thinking about math concepts."

Martha felt satisfied with Nick's support for her learning to teach mathematics in several ways. First, Martha claimed that Nick really worked as model for her in learning to teach mathematics. She claimed in the end of her internship that without this model, it would be hard for her to teach in the way she taught.

> I saw he taught mathematics at the beginning of the year. There were a couple of things that I really liked a lot. Most of the time, kids will trade the ideas about math. And this is what I think about hypotheses. One gave a hypothesis and others would think it differently and test that hypothesis. The kids were a little bit less inhibited and they were willing to give their ideas.

Martha also thought Nick really worked as a sounding board for her curriculum development and provided many specific ideas for her teaching plan. For example, she claimed that Nick really helped her figure out the proper sequence of instruction and resources, especially for her fraction lead teaching unit.

> He helped me a lot with how you plan a unit of math. As we sat down and we talked about, "How do you want this unit to go? What concepts do we need to talk about first? Which ones need to come after? We want to start the equivalent

> fractions before adding the similar denominators." So we talked about that and he also gave me the resources that I need to teach it.

She felt that Nick's jumping in many times in her teaching really eased difficulties that she had and supported her to see alternative ways of teaching certain mathematical ideas in context.

> A lot of time what he did was while I was teaching, he would kind of, I guess you could say, but I don't want to say interfering because it is not like that at all. He would ask the class question and it was the question that would help them think about concept in a different way that I didn't think about.

Functions of their collaboration

By the end of her internship, Martha believed that a concrete model of presenting mathematical ideas and proper group work were important for contextualizing constructivist vision of mathematics teaching. In her teaching practice, she experimented with concrete models and group work in helping students learn to teach fractions.

Nick claimed that Martha succeeded in learning three important things about mathematics instruction, each of which he was able to contribute to. First, Nick was especially satisfied with Martha's consistent efforts to support and challenge students to discover mathematical ideas and think like mathematicians and his role in making this happen.

> I think the biggest part of what she learned overall was how to get kids to reach an understanding on their own. Again that was something I was doing with her, how do you support enough and keep that challenge going and keep that kind of thinking. Sometimes the kids just want to know what the answer is. That is what I saw her really reach for and get better at through the year...That is what mathematicians do. That is mathematics exploration. It is to come up with an idea. The same thing with science and say, "I have seen this pattern and I think this works and let's test it and try it." And I think that is what she discovered and that is going to help her most in the future because again you can't do that unless you have someone to do it with.

Nick was also pleased that Martha was able to learn how to support students to develop a deeper understanding about mathematical ideas through careful explaining and organizing their thoughts. He again thought that Martha's learning in this aspect met his expectation for her to learn how to organize students to discover mathematical ideas.

> I think she achieves what I expected, the depth of understanding of the kids. One thing that she makes sure that she kept math journals along with experiences that kids would take time to think about and to put into words. The idea that until you can teach someone else about this, you really don't understand yourself. And being able to write it down on paper and organize your thoughts that way is sometimes close to the way we can get to by really teaching it. But you have to organize enough to see the result of those.

In addition, Nick thought that Martha was able to learn how to begin students' learning of mathematics at concrete level with manipulatives and other objects as their supports. He thought that Martha "got better and better a lot" in this aspect as he pushed.

Martha also realized that her collaboration with Nick made it possible for her to learn about mathematics instruction in three aspects that fit her expectations. She claimed that she learned the importance of a progressive sequence of mathematical ideas in teaching, working from the particular to the abstract level. In her word, she learned "something about the progression from concrete to pictorial, and then to symbolic," which was clearly an important expectation that Nick had for Martha's learning

Martha also thought that she was able to learn to how to relate mathematical ideas to her student's level. Again she claimed this was something she was able to learn from her collaborating.

> I also learned how to relate to them (her students) because there are times that they don't understand because I couldn't relate to them in their terms. Like let's say you had something like 3/4. What could that be? Now you wanted to add something like 1/2, you know. I think he (Nick) is really good in that aspect. Putting it at their level. I think that is really something I learned from him.

Martha felt that she began to realize that it was very important to learn how to organize group work in this kind of mathematics instruction. She regarded it as a direction for her further learning to teach mathematics after her internship. She expressed this idea in the following way:

> I think the other thing I learned and would like to see more was group work or what group work works. He (Nick) knew a lot of that. That is something I liked to do and that I want to do better at.

Collaboration and Its Functions in the Case of Bank and Jaime

Foundations of their collaboration

Although working in the same grade level in the same school with Nick and Martha, the foundations for collaboration between Bank and Jaime were not the same. Both Bank and Jaime had several similar beliefs about mathematics instruction that reflected a constructivist vision. For example, both believed that it was important for students to discover mathematical ideas. Both thought mathematics instruction needed to support students to see that what they were learning was related with their daily life experiences and other subjects. Both assumed that mathematics learning had to be enjoyable, comfortable or happy experiences for their students.

However, unlike Nick and Martha, their conceptual similarities did not function as an important base for Bank and Jaime to build a collaborative relationship. Instead, their collaboration was developed upon a different base.

Although Bank was able to help his students connect what they were learning to their daily life and to open chances for them to raise questions, Bank thought it was hard to implement his idea that mathematics needed to develop students' conceptual understanding through their own discovery. In practice, he was "still more an instructor" who liked to show his students the answers instead of inspiring them to discover it for themselves.

Because Bank felt it was hard to teach as he believed, he volunteered to be a collaborating teacher so that he would be able to learn some of the new ways to teach mathematics in a constructivist approach. To learn how to teach rather than to help a preservice teacher learn to teach was his most important motivation in entering into this relationship

Jaime's earlier observation and lack of substantial interaction with Bank on their beliefs about mathematics instruction convinced her that Bank was not teaching mathematics in the direction that she expected.

> He will give answers quicker than I would. He would give them more information than I would. A lot of times I want them to come up an answer by themselves. For example, to write the definition of a square by what we notice about the square. And I would like to make them do the work. While Bank might tell them the definition... I think that is the primary way we were dramatically different.

Expectations for their collaboration

The fact that both Bank and Jaime felt that Bank was unable to practice some ideas of mathematics instruction they agreed with pushed them to develop different expectations for their collaboration from what Nick and Martha had. Bank focused on those areas that he felt comfortable doing in his mathematics instruction when discussing what Jaime needed to learn about mathematics instruction. First, Bank was able to open chances for students to raise questions about what he was teaching. Thus, the first thing he expected Jaime to learn was to find different ways to present mathematics concepts and create different opportunities for students to express their needs in learning.

> First of all, children think in different ways. So when the particular concept expressed in certain way, some of the kids will understand that way, and some won't. And you might need to think of another way to present it or the other activity could be done to allow children to think in different way and have the same opportunities to grasp the concept equally as well. So I help my interns to see that you will need varieties of ways to present materials as well as variety of ways to allow the students to express what they know.

Bank stressed that Jaime needed to learn how to help students understand the relationship between mathematics and the real world, which again was a strong area in his mathematics teaching practice.

> Secondary, the general understanding about how mathematics works in real world is the other half of the battle she needs to fight. If kids don't see why it is relevant and why it is important, they are not thinking about it very carefully and they are not going to value it very highly. They are not going to work at it very hard.

He also expected Jaime to learn that mathematics learning is a long road for students and there are always time constraints on each teacher's teaching. Thus, as a teacher, Jaime needed to "be ready for that idea that there was going be different levels of performance among students and learn how to live with that."

> The other thing would be to help them (interns) also realize each student is not going to learn everything at mastery level this year. And we don't have time on it. We only have so much time to cover everything. The goal is try to get most of the students being able to complete that math task most of the time. But you also have to recognize they are going to do it again next year. So those they don't get complete 100 percent this year, at some point, you have to let it go and move on to the next thing because we use a spiral approach where they will get it again next year.

His lack of expertise in practicing the kind of mathematics teaching Jaime wanted to learn also pushed Bank to assume a non-mediating position in supporting Jaime's learning to teach. Bank wanted to play the role of friend in their collaboration and to provide moral support for Jaime when she had frustration in learning to teach.

> I am a friend and someone to give them (interns) some support when things are rough and things are not going in the right way. I am going to be a kind of help to pick them up a little bit and let them know tomorrow is going to be a new day. And it will be all right sometimes.

The other role Bank wanted to play was to provide alternative suggestions or resources if Jaime had difficulties in teaching mathematics and came to him for help. He said:

> If my intern gives an assignment and she finds that the forty percent of the kids didn't do acceptable work on this assignment, and she is frustrated with that. "God, I thought I taught a good lesson. I covered all of the main points. I thought I did real well. But obviously 40 % of the kids aren't getting this." When she comes to me for help, then I will try to suggest other ways to present it or "let's look at what the problem was and try to find another way to approach that problem."

Being unable to identify Bank's teaching practice as a model also changed Jaime's expectations for their collaboration and the role she expected Bank to play in her learning to teach. Jaime expected that her collaborating teacher would provide her a model of teaching that she wanted to learn. However, her early observation convinced her that Bank was not teaching in the direction she wanted to learn.

> Ideally he (Bank) would teach the way I wanted to teach but that never happened. For example, if there is anything I wanted, it would be that he had used the math journals already. For him to use the cooperative learning the way that I wanted to use. But he does not do any of these.

She also hoped that her collaborating teacher would be able to think and analyze with her about her lesson plans and her actual teaching. However, Jaime realized that it was also an unrealistic expectation.

> It would be helpful for me to go home and make my lesson plan and next morning for him to read over it and really discuss it with me. Not just glance at it but to really go over it. That is impossible because both of us had many other things to do. It would be good at the end of the day or right after the lesson to really discuss my teaching that could be better, like what makes it so good, that kind of the thing.

Thus, Jaime started to develop a realistic attitude toward the role Bank needed to play in her learning to teach mathematics. She wanted Bank to "be flexible enough" for her to try what she wanted to do in his classroom.

Collaboration between Bank and Jaime

The expectations Bank and Jaime developed for their collaboration in turn shaped what they actually did in their collaborative work. On the surface, like Martha, Jaime was able to follow the gradual-transition-to-practice structure in her internship. She started her learning with observation of Bank's teaching and other classroom work for a number of weeks. Then Jaime took partial responsibility for teaching and management. In the last 15 weeks, she began to take full responsibility for teaching.

However, the actual chances for collaboration between Bank and Jaime, different from what Nick and Martha were able to have, reflected their expectations. According to Bank, he usually kept a non-mediating position in Jaime's learning to teach mathematics unless certain situations occurred. First, when he felt that it was important for Jaime to know the school curriculum requirements for the unit she was going to teach, Bank would sit down and discuss these with Jaime.

> We will sit down and look at the textbook to see what kinds of things the book had to present and what they felt is important. Then we discuss what was missing from the book according to school curriculum, and what we felt was important. We come to an agreement on the main goals of the unit. I then showed her the materials that I used in the aspect. Bulletin boards, projects that I used in various lessons. Those types of things that I used in the past.

Another occasion was when Jaime had difficulties in making her pedagogical decision and came to ask him for support. Then he would offer assistance and suggestions.

> I don't tell her how to do it and I listen to her ideas. If she asks for help, I can be a resource and offer some to her. Yes, it is important they get to make those decisions. "Do I want to start the day with math or I only got half an hour a day for the special thing comes? Why do I leave the math for the later of the day when I have a full hour and do something different." Those are the decisions an intern needs to be able to struggle with and come to their own feeling about how to do.

Bank would intervene what Jaime was doing when he realized that she was missing something important for students' learning or failed to get the points across and students' learning was being jeopardized.

In their collaborative work, Jaime usually regarded Bank's assistance as a kind of interference. She tried to avoid Bank's influence on her teaching. Even if she had problems and difficulties in her own teaching, she felt that it was up to herself to resolve it or to consult with other teachers because she thought Bank was not teaching in the same direction she saw as her goal. Her attitude toward Bank's support can be seen in two aspects. First, unlike Martha, Jaime felt it hard to accept Bank's jumping in during her teaching

> It is very hard for me because our teaching styles were different. When I was talking, he sometimes would like to jump in the lesson and not let me finish. Because he was used to give all the information first and then let them work, and I let them work first and then come to the conclusions. He will be afraid that I was not giving them the information that I needed. I was frustrated that he would be jump in 'cause I wanted kids to come to the conclusion without his help. That was the real dilemma.

Second, Jaime also did not regard Bank's interaction with her on her teaching plan as useful because these interactions were often short. It usually ended up with Jaime teaching as she planned. As Jaime described:

> Basically, I just planned my lesson by myself and then in the morning, I arrived to ask Bank if this is OK. Usually he will say, "OK" and there are sometimes he will tease something that I have made and struggled with.

However, Jaime did think that Bank was supportive of her learning in one way. She thought Bank was very open to let her try her own ideas. She thought it really helped her develop her individual style of mathematics teaching.

> The great thing about Bank was when I wanted to teach certain ways he is always supportive and let me try. And I heard about some mentor teachers are more flexible than others. And that flexibility is very important for me. Because if I haven't the confidence to do what I wanted. If a teacher had said that I shouldn't do it, I wouldn't have to. Bank was like "Yeah, yeah, do that," which was very helpful because he never gave me a reason to not try something. So that was important.

Functions of their collaboration

By the end of her internship, Jaime was able to retain the idea that mathematics was closely related with different subjects and the real world and that mathematics learning needed to be enjoyable and fun for her students. She further developed the idea that students had quite different ways of learning mathematics and all these ways needed to be respected and teachers needed to use different methods to help students learn mathematics in their own way. Jaime no longer stressed that mathematics learning had to support students to develop conceptual understanding or that she needed to assist them to explain how and why their understanding came about.

In her teaching, Jaime was able to connect students' learning with real life examples. She tried hard not to challenge students or push them to learn beyond the level of their individual performance. She was able to get students to form their own ideas about mathematics but she failed to encourage them to explain and prove their ideas.

Bank clearly recognized these results of Jaime's learning but failed to relate Jaime's learning to the nature of his direct support. Bank believed that over her internship, Jaime was able to learn how to integrate mathematics learning with other subjects and real world experience in her mathematics teaching.

> I have been impressed with the way she did integration with arts and with writing. For example, the book she had each group do, the triangle book I showed you yesterday. I thought it was really a valuable way she tied that in. She talked with her creative art teacher and had some ideas about how the children could physically show the shapes with their bodies. She got the giant stretching rope. They are going to put around a group of the kids and then they are going to find out how that show square, parallel lines, and triangles. So the general integration approach I have been impressed with.

Looking over her internship, Jaime also pointed out that she learned two important things about mathematics instruction: how to integrate mathematics with other subjects and students' daily life in her teaching and how to respect students' different ways of learning. However, she contributed her learning exclusively to the freedom Bank gave her to experiment with her approach. Jaime claimed that she did learn something through her observation of Bank's instruction. However, the thing she learned from Bank was only limited to the technical part of classroom management.

> From Bank, I observe more management things. We perhaps talked more about discipline issues rather than about actual subject matter and teaching method.

Collaboration and Its Functions in the Case of Ben and Louis

Foundations of their collaboration

Ben and Louis started their collaboration on a different foundation. First, they did not shared their ideas about mathematics instruction. Louis brought into his internship several ideas of mathematics instruction that were different from Ben's conceptions. Louis assumed that mathematics instruction needed to support students' own discovery of ideas and he believed that discovery would result in his students' comfortable and fun learning experience. He thought that in his teaching he needed to help students form, prove and disprove each other's ideas and connect their learning with their daily life experience.

Ben thought that mathematics is a collections of facts, concepts and skills that build upon each other. It was important to support students to both memorize mathematics facts and understand their relationships. Memorizing and practicing the rules and procedures he demonstrated and modeled was very important to him.

Second, these two teachers were able to realize the conceptual difference between themselves. Louis's senior year observation in Ben's classroom allowed him to see that Ben was not teaching in the same direction as he wanted. He pointed out in the interview that "Ben is pretty much like to show the ideas to kids and help them memorize and apply them." Thus, in the beginning of his internship, Louis had made up his mind to develop his own way of teaching in his internship without Ben's support.

Ben also realized that Louis had already had some ideas about mathematics teaching he wanted to try and sensed the difference between him and Louis in thinking about mathematics instruction. Ben decided that he would like to give Louis the chance to practice his own ideas.

Expectations for their collaboration

The expectations that Ben and Louis developed for their collaboration were based upon their conceptual difference and their understanding about this difference between their thinking about and practicing mathematics instruction.

Realizing that Louis was not going to learn to teach as he was practicing, Ben expected nothing specific for Louis to learn in the internship. His expectations are reflected in the following aspects. First, Ben expected that Louis would be able to learn to "feel comfortable in what he is doing." He claimed that only if Louis was able to feel comfortable doing what he was interested in doing would he be able to succeed in his own classroom.

Ben also assumed that since Louis had already had some ideas about what kind of mathematics instruction he wanted to experiment with, he needed to have a chance to practice what he wanted to teach and develop the practical part of his ideas.

> I think what he needed to learn would be more in the line of practical things. You know, he had the ideas and theory. Now they have to write some units based upon these ideas and teach them. It would be more like the line of practical things for teaching.

Thus, Ben also expected that Louis would be able to learn different approaches of mathematics teaching and not take one approach for granted.

> In my opinion, he needs to learn there has to be a number of different ways to teach math. You have to present ideas in different ways. Somebody might understand when you do it this way, and somebody else thought to understand when you explain the other way.

In assuming his roles in helping Louis learn to teach, Ben wanted to maintain a non-mediating position in his collaboration with Louis since Ben realized the difference between what he was able to do and what Louis wanted to learned. This attitude can be seen in the three roles he planned to play. First, he identified himself as a collaborating teacher who was to provide enough freedom for Louis to experiment with his ideas and approaches.

> I think I need to provide a situation for him where he was able to feel free to try his ideas some way. If it did not work, it is not that I am going to scream and yell and kick... I think giving him the freedom to do it to me is what he needs. I think the kids need to be in non-threatening situation. I also think for him to be in an unthreatening situation where he can try things.

Another role he wanted to play was to be a companion in Louis's teaching and provide supports whenever Louis had a problem, felt frustrated and came to him for help. He said:

> I felt my role was more kind of to be there. A lot of times are just for him to ask questions. If he feels frustrated on certain things, to give suggestions. I don't know if you noticed that a couple of times I was typing on the computer and I can see something what is going on. You know I just quietly say something when he had problem. It was more just kind of discuss with him. "Do you think they got this or have you thought of doing it this way?" A support. I think mainly I need to be there for support.

Ben thought that he would like to help Louis find resource or material for the ideas Louis wanted to try. He regarded this role as the most important support he might give to Louis.

> I think my biggest help is to help him to find resources. You know, he would say he did need so and so. So I go and hunt for some of the resources and say," Hey, why don't you look at these things?"

Understanding that Ben was teaching in a different direction pushed Louis to expect Ben to be able to provide a flexible environment for him to try his ideas rather than offer direct support and modeling. Louis said:

> I hope Ben is flexible and open to multiple and different ways of teaching. And I think that will really help me because that helps me be a little bit more flexible in trying to teach in my own way. I thought he would give me all these alternatives for me to try out what I think what most important for students. And he will not think that he is threatened by the way I am teaching, meaning he will not think that I am trying to devalue the way he taught.

Second, Louis also expected Ben to be a resource person for him when he was experimenting with his ideas. He regarded this role of his collaborating teacher as most important.

> I think the most important thing is that he will give me all the different resources when I need. And give me the opportunity to succeed or fail on my own. He was there as support if I need them.

Collaboration between Ben and Louis

The collaboration between Ben and Louis, to a great extent, reflected what they expected to happen in their relationship. First, for example, seldom did Ben and Louis sit down discussing and analyzing each other's teaching, nor did they co-plan and co-teach together. They even did not observe each other's teaching often.

Louis claimed that he had already observed Ben's teaching in Louis's senior year and pretty much knew what Ben was doing. Thus, he said it was not necessary for him to observe it when he entered Ben's class for his internship.

Their collaboration around Louis's mathematics teaching also reflected their expectations. To give Louis enough freedom to develop his own ideas of teaching and help him feel comfortable about what he wanted to do, Ben generally kept an non-interfering role in Louis's learning to teach. He would not do anything to interfere with what Louis was doing or what he wanted to do unless Louis did not know what to cover and came to him for help or he was missing the point and students' learning was jeopardized.

However, whenever they interacted with each other, the interactions were often casual and informal. There were few occasions that their interactions involved their systematic analysis and deep reflection. As Ben described:

> Our talks a lot of times were not formal type talk. After school, we will probably go through and talk just generally. Well, you know, "How do you think this went? That was something that I wonder with some other students that I may not be able to do that way yet."

Ben not only had few formal and reflective interactions with Louis. He also thought that such interaction was not helpful to Louis developing his own teaching style. This attitude could be seen from the comments he made on the formal and reflective discussions between Louis and another fifth grade teacher, Mary, who helped Louis plan his fraction unit.

> Louis does a lot of formal talk with Mary, who is working in the room next door too. She was trying to show him different areas of teaching. She might say, "Hi, have you thought about this? Do they know this? What is their prior knowledge on this whole thing?" And then she might give him the backgrounds some of the kids might have, where they are coming from and some of the questions. It is a kind of going through thinking about teaching. I really found this doesn't help at all when I was teaching. I never tell him what to do.

Louis had similar views of his collaboration with Ben. He felt that Ben really offered a lot of chances to him to do whatever he wanted to do, and he never interfered with his lesson plan unless Louis asked him for support. Louis described:

> I think what often happened was that I planned for a math unit. As I had questions, I moved along with questions first. If I can't figure out, then I was able to ask him about these questions. But I never had a sit-down type of planning with him. What he said was pretty much, "This is what you need to teach in the concepts." It was more I take this stuff to do that.

Louis thought that from Ben, he got enough freedom to do whatever he planned to do in his teaching. As he said when I asked him why I had not seen him interact with Ben often before, during or after his teaching:

> For the most part, I am pretty out there on my own doing what I wanted to do. I used the teacher as resources. But I think it is important that I am actually getting to do what I want to do. And I can do what I want to do because he let me to do so. So in that sense, you know you really couldn't see anything there. I think you pretty much seen what it is and the way it is.

Louis was satisfied that Ben never challenged his authority as a teacher in front of students by taking over his teaching or raising questions when he was teaching. Even when Ben saw there were some problems and difficulties in Louis's teaching, he would use a less intrusive way to help.

> Ben knew he didn't want to give the student impression that he was so much better than I was in that. Because of that, in some way devalued my teaching. And I appreciate that pretty much anything he would say to me he did not say in front of the students. He would mention things to me like, "You might want to think these." And he did this without letting students hear or notice.

Functions of their collaboration

By the end of his internship, Louis started to see mathematics an activity of learner's active reasoning, hypothesizing, proving and problem solving. But he also came to regard mathematics as a collection of facts and skills that students should be informed of, remember and practice. He assumed that all mathematics teaching methods had equal value no matter whether it was lecturing followed by practice according to the textbook or supporting students' own construction of mathematical ideas. He stressed even more strongly that students needed to feel comfortable with and have fun in mathematics learning. This he regarded as the most important principle of teaching. His teaching practice also moved from a constructivist approach toward the traditional approach. Both his beliefs and practices come to look like those of his collaborating teacher, Ben.

Ben identified these conceptual and practical changes that Louis experienced in his internship. He pointed out that Louis was able to learn the exact idea that he expected Louis to experience through his internship. As Ben said:

> He realized he had to monitor and just re-teach a lot of things as he was going along. And he really took it very seriously and when they took the test and he saw they didn't understand. He went back and he tried to re-teach these concepts in different ways. It is also learning experience for him. I think he learned a lot in teaching it.

Ben also realized that Louis started by experimenting with his own ideas of mathematics instruction but ended up with teaching like what he was doing. He was confused why the free-to-try environment he created for Louis failed to help Louis successfully develop his own style.

> Things happened were a kind of funny because in the end, he started to do things the way I do things. And everybody has their own style... I do. I don't want everybody to do it the way I do it.

Looking over his internship, Louis claimed that he learned the following things from his collaborating teacher. He learned that forms of mathematics teaching needed to be different and as a teacher he needed to learn all the different ways. Which teaching methods to be used depended upon whether students feel comfortable about it or not.

> About math teaching I feel the most important thing is the flexibility. Being able to change different lessons and being able to teach in different ways and try different things. Being able to, as I saw at this time, be able to have this knowledge that you can teach all these different ways. But at the point of time, you teach this way, because you felt important, it is most comfortable that kids learn math in this way so. So that is way I am going to teach the math.

In relation to this, Louis believed that he learned that it is important to know all about different resources for different types of lessons. When he wanted to teach in one way, he would like to know all the reference materials for the way he wanted to teach.

> I think I learned it is important to realize that you have all those things for your teaching and know the strength and weakness of these materials. As a teacher, you need to be able to do things with this, this and this. Do what you want and plan your whole unit using these different books or using what you want out of all of them. There are a lot of strengths for teaching like this.

Collaboration and Its Functions in the Case of Lisa and Kelly

Foundations of their collaboration

Lisa and Kelly developed their expectations for collaboration based upon the following bases. First, Kelly brought several ideas of mathematics instruction into her internship. These ideas were different from what Lisa believed. Kelly believed that mathematics instruction needed to support students' conceptual understanding of mathematics and such understanding needed to be developed through the ways that matched students' different ways of learning. As a teacher, she thought one needed to be able to use different ways to help different children to get that level of understanding and to create comfortable learning experiences for students. However, she had no idea how these ideas could be put into first grade classroom teaching.

Lisa believed that students needed to be shown the mathematics concepts, strategies and skills and wanted to help students see the relationship between mathematics skills and the real world. Lisa emphasized that a teacher needed to know the skill progression for her students and support them to practice and internalize the skills. Lisa was teaching in a way that featured the model of telling followed by repetitious practice, an approach that was different from what the program expected its preservice teachers to learn. However, Lisa did not identify such difference or any difference between Kelly and her in thinking about mathematics instruction. Instead, Lisa assumed what she was teaching was what Kelly needed to learn to do.

Kelly's abstract beliefs also failed to prepare her to see and distinguish the nature of principles embedded in the mathematics instruction Lisa practiced. The different kinds of manipulatives in Lisa's teaching convinced Kelly that Lisa was teaching in a way that could satisfy different ways of students' learning. Thus, Kelly regarded Lisa as a model for her to learn from and decided to follow her approach.

Expectations for their collaboration

The expectations both Lisa and Kelly developed for their collaboration were strongly influenced by this understanding about each other's learning and teaching. What Lisa expected Kelly to learn about mathematics instruction exactly reflected her beliefs about and practice of mathematics instruction.

First, she expected Kelly to learn the specific mathematics skills that her students needed to develop in the first grade and where her students came from and where they were going in relation to these skills. She said:

> Basically what they (interns) need to learn is the hierarchy and skills of math. They need to know not just what happens in the first grade but what is the progression even if my children are in the different levels. They need to know what kids are learning and what comes after that and what comes before. If a child is able to count 50, for instance, what is the specific skills the kids need to learn.

Lisa also wanted Kelly to learn what the school curriculum required the teacher to do and how to look for the resources to support her to develop students' mathematics skills. In her words, what Kelly needed to learn was "just the objectives and what curriculum is requiring, and then look at different resources to present the curriculum."

Lisa claimed that repeated practice was very important for kids to learn mathematics skills. She expected Kelly to learn how to help students repeatedly practice their skills.

In assuming her role in their collaboration, Lisa thought she needed to play two roles for Kelly's learning. One of the things Lisa wanted to do was to provide a model of teaching for Kelly to observe and follow.

> I think my role is to provide examples and to role model the different ways that I did things. I role-modeled a lot. Sometimes, I made them (Kelly and another intern) sit more formally and watching me how to do things that I did.

Another role Lisa wanted to play was to be a support for Kelly's learning. These supports included providing material for Kelly's curriculum development, providing a place for Kelly to practice her teaching and helping her to see other teachers' teaching. She explained:

> I will also provide a place for them to teach and I provide support when they need it. I will provide materials and encourage them to seek out peers. I told them certain peers are really strong in different areas or people that will be really assertive to let them to ask for some help. So it was not just me.

Similarly, Kelly also thought that the best ways for her to learn to teach were to observe and practice. Thus, she hoped that her collaborating teacher would provide her different models of teaching to observe and give her opportunities to practice.

> Probably the two most important ways for me to learn are to observe and practice. Not only observe my mentor but also to allow me to observe all kinds of teaching and situations. I hope to go and see the wide varieties of teaching. You can get more ideas from observing. So I think that is the one way I learn a lot from. It is just observing. And the other way is just actually do it yourself. I need the opportunities to sit down and write out my lesson plans. Until I get in front of my students actually teaching it, I will not say I have learn. That is how I want learn about myself and the way I teach math from observing her and actually doing it.

<u>Collaboration between Lisa and Kelly</u>

In their actual collaboration, Lisa really pushed Kelly to learn what she wanted Kelly to learn through direct modeling and constant support. In the first few weeks, Lisa modeled her mathematics teaching for Kelly to observe. Not only did Lisa show what she actually did in her mathematics teaching, but also she took chances to explain what she did and why she did all these things.

> In the beginning, I was telling them. I talked a lot, the biggest thing mentors have trouble with. But I don't have trouble doing that. I say what I was thinking. I went through my sequence. "This is most of my goals in my head. This is what I needed to do and this is why I did it." So I would say what was in my head. I also told them what I neglect to do, when I was prepared and what I was going to do next and what I changed.

As Kelly changed her role as an observer to a participant in mathematics teaching, Lisa began to change her role and tried to do the following. First, she helped Kelly plan her lessons by shaping the content of Kelly's curriculum and providing relevant materials and resources.

> Basically, what I did was I typed out the stuff they needed to teach and they got a list of something. I let them go from there. I didn't sit down and plan lessons with them. At this point we have done talking and talking all these months now. It is time for her to spread over the wind. Depending on the skills, I showed them some materials. "Here are all the clock stuff. Here in the cupboard, you will find some stuff that might help you to teach addition over there." I told them there are some sources. All my math units are put together in the cupboard.

Then, Lisa would check Kelly's lesson plan and made sure that some of the important materials were used. She frequently had brief discussions with Kelly about the problem and missing points that occurred in Kelly's planning and teaching and helped Kelly think about the relationship between what she wanted to reach and what she did.

> We talked about her lessons the way we were not, generally speaking, in my room, When a lesson went OK, we talked over the desk. I will say we might stand there and we might sit down. We might sit at the table. But that was not planned. I might have said "How you think so and so doing that? I noticed that you really need to use this material and why did you use that instead?" If there was a problem with the kids' learning, I often told her that "I really had problem with this today or you have got to re-teach da, da, da."

Lisa also encouraged Kelly to observe mathematics teaching in other classrooms. She thought that in this way, Kelly would be able to see the relationship between different grade levels in terms of mathematics skills.

Kelly also agreed that Lisa really supported her to learn to teach in the following ways. First, Kelly assumed that Lisa showed her a model of how to cover materials and how to use manipulatives. She claimed that she learned a lot from observing her teaching in the early part of her internship.

> I think she is more an example to me. I observe her a lot. I watched how she taught a lesson. What content area she covered and what manipulatives she used while she was teaching. I learned a lot at the beginning of the year just from observing her. And she is a good example to watch and to learn from.

Kelly was also pleased that Lisa was able to directly tell her what content and material she needed to cover and how to use these materials for her mathematics teaching. She felt such information was really important because she had no idea how to build curriculum for first grade students.

> Lisa, my mentor, she is kind of giving us a list of topics that she wanted to be covered. What materials we wanted to use and what worksheet we wanted to use. She gave us the topics we need to cover for the first grade. 'Cause I don't know what should be covered for the first grade. This is my first year doing this. So she helps us out and give us the guidance.

Kelly also claimed that Lisa was able to constantly check what she planned to do and make sure her plan really reflected what she needed to cover. If there was a problem, Lisa would use examples to show how she used the material to cover the topic.

> She always looks at my lesson plans. If she sees that I am not covering certain topics, she always showed me that I haven't covered. She will say, "I have these counters you can use." She always shows me what manipulatives she has in her room. If she knows a lesson in her folder and textbooks that I hadn't seen, she will show it to me so that I want to use it. She will give me an example the way she has done it.

Kelly was also satisfied with Lisa's constant support for her teaching by offering suggestions and pointing out the places where she had problem and made mistakes. She said:

> When I was teaching a math lesson, she would observe it and she would write notes and she would talk to me immediately afterwards. She will let me see the notebook what she wrote. And so if I had any questions or need her help and suggestions, we will talk right after. Or I would talk to her after the day was over and talked about any problems I had or questions I had for her.

Functions of their collaboration

Kelly developed through her internship the ideas that mathematics was a collection of skills hierarchically structured and related and it was very important for her to understand the requirements of mathematics skills and skill progression. She started to see that she needed to learn how to follow the school curriculum and how to help her students practice these mathematics skills. All these conceptions reflected what Lisa expected her to learn. My analysis in Chapter 6 also suggests that the teaching practice Kelly developed in her internship was similar to that of her collaborating teacher, Lisa, featuring telling followed by practicing.

When asked to define what Kelly learned from her internship, both Lisa and Kelly identified some of these learning results. Lisa assumed that Kelly was able to learn how to use the first grade school curriculum and how to understand its goals, content and skills required to teach to her students.

> What she learned was more knowledge-based math at this level, the skills need to reach and the goals that curriculum has for the math. They learn to use the curriculum. They can go and look up and say I have taught this, this and this year. What do we need to teach?

Lisa also thought that Kelly was able to learn what resources could be used for her teaching and how to make some of her materials as well. She thought it was very important for Kelly to understand these materials and know how to build her own repertoire of resources.

> I think one thing she learned was she got to know that all the different manipulatives that are out there. So she can now have a repertoire of materials to get concepts across to the kids. I think that is important to know what is out there and available. She learned how to make some of these materials when you don't have a budget or in a school that does not have it.

Kelly also felt that she was able to learn several things because of Lisa's support. Kelly claimed that the most important thing she learned about mathematics teaching was to have different options for students to practice what she wanted them to learn. She claimed that she learned this through her observing Lisa's teaching.

> One just probably the biggest thing I learned from her and see her doing is that you do have a classroom of wide range of learners. And you need to give the kids many options to practice. You don't tell them you have to do this problem in this way. Your way is wrong. You let them do it whatever way is easiest for them. I learned Lisa does that a lot.

Kelly also felt that she was able to learn from Lisa how to develop a curriculum and enact it. Coming into her internship, she had no idea how to approach a topic, plan a unit on it and introduce it to her students. It was Lisa who gave her a model to follow.

> I don't think I would have been prepared as well as I did. I would not have known how to approach a topic, how do I choose a topic, how should I introduce it to the kids. I think it would be a lot of harder for me to plan a unit and a lesson. Because I watched Lisa and saw how she goes about and teaches the kids. How she has them do things, use manipulatives she wanted to use, you know. Without seeing it, I wouldn't have known it.

Summary of Chapter

My analysis in this chapter suggests several things about the relationship between the nature of collaboration and the quality of preservice teachers' learning. First, even though the program clearly specifies the roles that both collaborating and preservice teachers needed to play in their collaboration, not all of them were able to consider these requirements their responsibilities in their collaboration. They assumed quite different roles depending upon how they understood each other's learning and teaching.

Nick and Lisa assumed a teaching role in their collaboration with their preservice teachers, Martha and Kelly, because both recognized what they were practicing in their classroom was what their preservice teachers needed to learn. Martha and Kelly were also able to assume a learning role in their collaborative work with Nick and Lisa since both of them thought that their collaborating teachers were teaching in the direction they wanted to learn to teach.

However, Bank and Ben chose a non-mediating role in their preservice teachers' learning because both thought they were unable to provide a model for their preservice teachers to follow. Jaime and Louis only looked at their collaborating teachers as a resource and made up their learning to teach without specific support from Bank and Ben. They wanted to do so because they recognized that their collaborating teachers were teaching in a direction they did not want to pursue.

Second, what preservice teachers were able to do or the quality of their learning may not necessarily be guaranteed by the supportive relationship between collaborating and preservice teachers. Without a shared constructivist vision as its base, the supportive collaboration could also push preservice teachers to learned things contradictory to the expectation of the program.

Both Nick and Lisa were able to work closely with their preservice teachers in the form of mutual-observation, co-planning, and joint reflection on teaching. They were able to provide constant guidance and assistance to their preservice teachers before, during and after their teaching. Martha was able to retain a constructivist approach, but Kelly obviously was learning to teach in a traditional direction. The supportive relationship between Nick and Martha was able to be built upon a shared constructivist vision. However, Lisa's vision of teaching was more oriented toward an absolutist approach. What Lisa did for Kelly's learning to teach was to model, guide and provide suggestions around this vision of telling and practice.

However, sharing a similar constructivist vision by itself would not guarantee collaborating and preservice teachers can build a supportive relationship and give a successful learning experience to preservice teachers. It was more important that collaborating teachers are able to practice this vision and provide a model for preservice teachers to observe and identify.

Nick and Bank were both embracing a constructivist vision of mathematics instruction. Their preservice teachers, Martha and Jaime, also brought some similar beliefs about mathematics instruction into their internship. However, Nick was able to not only develop a constructivist vision but, somehow, practice it. His ability to practice his vision really helped Martha identify his vision from his teaching and made her feel that she needed to and could learn from Nick. Thus, they not only developed a supportive relationship in their collaboration but also that relationship really supported Martha to learn to teach in a constructivist direction.

However, Bank was unable to put some of his ideas into practice and his lack of confidence in helping Jaime learn a constructivist approach of teaching pushed him to develop a non-mediating role in their collaboration. The fact that Bank was unable to practice some of the constructivist vision also conveyed the message to Jaime that he was not teaching in the same direction she wanted to pursue. This understanding drove her to expect freedom to learn and closed her mind to any substantial support from Bank. Relying on trial and error in her internship, not only was Jaime was unable to make substantial progress in learning to teach in a constructivist approach but also she started to change some of her constructivist beliefs.

Third, my analysis also suggests that freedom to implement their constructivist ideas was not enough to help preservice teachers to retain and further develop their visions and learn to teach in the constructivist direction. Without constant consistent, and continuous supports from the collaborating teacher, it was difficult for preservice teachers could be able to maintain their constructivist vision and approach to teaching even if they were able to have such vision and implement it.

Martha and Louis brought into their internship some strong visions of constructivist mathematics instruction and were able to put these ideas into their teaching practice. However, Martha's collaborating teacher, Nick, assumed that it was his responsibility to support Martha to learn that kind of teaching. He exerted his influence throughout Martha's internship whenever he saw it necessary. Thus, Martha was able to not only retain and further develop some of her constructivist ideas but also keep practicing these ideas in her teaching.

Louis's collaborating teacher, Ben, maintained a non-intervening role in his collaboration with Louis and gave Louis freedom to do whatever he wanted. Louis was also able to use his freedom to implement some of his ideas in his practice as his first lesson suggests. However, the freedom he got was insufficient to support him to retain these ideas and push him to continue teaching in the constructivist direction. In the end, he used his freedom to move back toward his collaborating teacher, Ben, at both conceptual and practical levels.

My analysis in this chapter also suggests that the nature of collaboration was not the only answer to the results of each preservice teachers' learning, though it explained some of the important reasons for their learning. For example, it failed to explain why the freedom Jaime had failed to help her further develop their ideas and practice along the constructivist direction. It did not explain why Louis changed back toward his collaborating teacher's approach when Ben gave him enough freedom to develop his own style of teaching and Louis himself was able to implement constructivist ideas in his teaching. What were the other factors that shaped Jaime's and Louis's learning to teach? How did these factors contribute to Martha's and Kelly's learning? In the next chapter, I answer these questions by exploring the nature of instructional contexts in each setting and studying how these contexts influenced what the preservice teachers were able to learn about mathematics instruction.

Chapter 8

INSTRUCTIONAL CONTEXTS AND THEIR INFLUENCES

In this chapter, I explore the influence of instructional contexts on preservice teachers' learning to teach. My analysis focuses on the interaction between the instructional contexts in each setting and preservice teachers' reaction to the contexts. First, I identify some features of three instructional contexts in each setting: the culture of teaching in the school, the school mathematics curriculum and resources available to preservice teachers, and students and their preparation for the preservice teachers' teaching. Then I examine the ways in which each preservice teacher reacted toward their contexts, the ways in which they thought about the teaching practice that he or she was able to observe, the ways in which he or she used the curriculum resources and the ways in which he or she thought about students in developing and implementing a curriculum.

Instructional Contexts and Their Influences on Martha

<u>The culture of teaching and Martha's learning</u>

The culture of teaching in Well Elementary exposed Martha to three important things. First, it encouraged the constructivist approach of mathematics instruction that Martha wanted to learn. Well Elementary strongly embraced the current mathematics education reform and a constructivist vision of teaching. About five years ago, it began to restructure its mathematics curriculum and initiated a series of workshops to educate teachers how to teach mathematics as envisioned by the NCTM standards. Teachers in this school and their mathematics instruction were strongly influenced by this reform. As Nick explained, the kind of mathematics instruction that school pushed featured supporting students to understand rather than find the right answer.

> I think it has been four years now. And some of the reasons had to do with the National Standards, NCTM standards looked at those. And not just taking them because someone says it but looking at them and saying, "Yes, it makes sense. We can understand why these things are needed. And why there is good support for it, good research to say these are kinds of the things we need to do to prepare kids." That as a big part of it. Then looking at your own professional experience and saying that was a big part of it. I have taught division for six or seven years. And I still feel that when the kids leave, they don't really understand it. A lot of them got the right answer, but they have no idea why that is the right answer or what it takes.

Being at Well also exposed Martha to a culture of collaboration among teachers. In that culture, mathematics teaching was not an individual enterprise. It was no problem for her to seek support whenever she felt she needed it. To pursue the constructivist vision of mathematics teaching, teachers in the school were encouraged to work collaboratively to explore new ways of teaching. Collaboration became the norm of their work. As Nick explained:

> Collaboration is supported in this school. If I wanted to spend an hour in other classroom to observe how they may be doing a lesson I was having trouble doing well and I may be teaching in the near future, I believed that my principal will support that. He will find some ways for my room to be covered for that hour and make some arrangement. You noticed in this building with the fourth and fifth grade teachers, we are really working together as one group in a lot of ways.

It also provided her some specific images of this kind of mathematics instruction. She could observe these and connect her ideas to actual practice, especially through her collaborating teacher, Nick. The mathematics instruction which Nick was practicing most reflected two features. The first was a "progression model" of representing mathematics concepts starting from students' concrete understanding and gradually moving toward their abstract understanding.

> For example, if I am teaching a new concept, like teaching division, they might be doing division for two weeks before they even see the symbol and number. And then we began to bring in those numbers to relate to what they are already doing so they have an understanding about the process before they actually deal with symbols. So that is been a big change in my teaching.

As discussed in Chapter 5 and 6, another feature of Nick's mathematics instruction was a discovery model. In his class, students were supported to form, prove and challenge each other's mathematical ideas.

Entering Well Elementary, Martha realized that the culture of teaching in their setting exactly reflected what she valued about mathematics instruction and what her program wanted her to learn to do. As she described:

> I think that a lot of philosophies about math teaching that Well Elementary has are the same as what we were taught. Pretty much in all the aspects, like active learning, communicate ideas among kids. Here people do not just stand in the class and look at book teaching. It is kids doing more self-discovery of concepts and writing about them.

Martha realized how her ideas of mathematics instruction were present in the actual teaching practices through her observation of Nick's instruction. This observation began to help her build a specific connection between her general ideas and specific images of teaching. Without her earlier observation, Martha said it would very hard for her to teach in the way she did.

> I saw he taught mathematics at the beginning of the year with a couple of things that I really liked a lot. Most of the time, the discussion came out of that. The kids will trade the ideas about math. And this is what I think about hypotheses. One gave a hypothesis and others would think about it differently and test that

> hypothesis. The kids were a little bit less inhibited. They were willing to give their ideas.

Martha also admitted that she benefited a lot from the school culture in which teachers supported and collaborated with each other under a shared vision of mathematics teaching. Martha thought that this collaboration among teachers gave her a role model of working together and ideas about how to seek support when necessary.

> Teachers work together really well here, which I think is a very good role model for us interns in working together. They are willing to share ideas and help each other.

<u>Curriculum resources and Martha's learning</u>

The district mathematics curriculum guidelines at Well Elementary strongly required its teachers to teach in a constructivist approach. Such a feature could be seen from the four goals defined by the school curriculum for the 5th grade mathematics instruction:

> COURSE OUTCOME A:
> Communicate mathematical ideas orally and in writing.
> COURSE OUTCOME B:
> Comprehend mathematical ideas through reading and listening.
> COURSE OUTCOME C:
> Apply conceptual understanding, estimation skills, computational proficiency, problem-solving strategies, and technological tools to all types of mathematical situations.
> COURSE OUTCOME D:
> Reason logically, recognize alternative ways to solve problem, and explore various methods to do so.[8]

The main curriculum resource that Nick used in his mathematics teaching matched these goals with a specific pedagogical model. Among several mathematics curriculum materials that Nick used for his mathematics instruction, he especially

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[8] These outcomes are quoted directly from Well Elementary School's *K-12 Mathematics Program*.

depended on the textbook called *Manipulative Experienced-Based Arithmetic* (MEBA). He claimed that this text not only was the first teaching text with which he received his training in teaching mathematics with a constructivist approach. It was also the major resource to rely on when he was planning a mathematics lesson.

> When I started changing the way I taught math, I do have a book that was produced by MEBA. That has a lot of lessons and concepts there. I will also use this math textbook for some background information and sometimes to see what is another step, and what is another way of getting from this idea to the next idea.

In this textbook, a specific model of mathematics teaching was used to help teachers represent their teaching content in a progressive way. That was to start teaching with a concrete model or hands-on experiences. Then it led students toward pictorial understanding of mathematical ideas taught and in the end, moved toward symbolic representation of mathematical ideas students needed to learn. In each step of this process, students' own exploration and communication of their ideas were encouraged. Nick described it thus:

> The one we used was really a program called the MEBA. That is a system we use the manipulatives to help students have some concrete, hands-on understanding of mathematics first. Then you help them understand then by drawing and communicating, and then bring in the mathematical symbols at the right time, after they understand the concepts.

Furthermore, Nick modeled to Martha a way of lesson planning that featured the mutual support between him and his colleagues and his continuous communication with them. He was pleased that he had an encouraging environment where his colleagues always tried to help each other out in improving their mathematics teaching practice.

> When I plan my lesson, I have to decide what I think is best for the class and what they need. I would also confer with other teachers on my plan. It is the best thing to do by sharing what we found out with each other as we do some of these things. The thing that influences me most, I think, is the other teachers. To me the

> biggest influence is your colleagues. One thing I have noticed about this school was we have a lot of people who are always looking to make things better.

In planning her mathematics lead teaching unit on fractions, Martha claimed that she was able have the following resources as her support: the school mathematics curriculum guidelines, the MEBA and Nick's suggestions. Among these supports, Martha thought the school mathematics curriculum guidelines helped her develop the general goals of her unit and gave her specific requirements for how to cover some concepts she needed to cover. For example, among the ten content areas of fractions for the fifth grade, Martha chose three to cover for her fraction unit. These areas are defined in the school curriculum as "read, write, compare and order fractions," "express equivalent fraction" and "add, subtract and multiply fraction with like or unlike denominators." [9]

Martha also relied on the MEBA program for ideas of how to cover and present these concepts and what specific approaches she needed to use to teach each specific lesson in her fraction unit.

> I used the MEBA, the Manipulative Experience-Based Arithmetic. Have you heard about it before? It stands for the manipulatives experience-based arithmetic. And it is about the curriculum that was designed for you to plan our unit. First, you do math with the concrete manipulatives. Then you move to pictures you draw. From there, you do the symbolic activity and then you go back and forth between the concrete and symbolic. So that they can understand symbols by using patterns of fraction bars or something like that.

During her unit and lesson planning, Martha had chances to work closely with Nick to further develop a sense of what fraction concepts were included in these content areas and in what sequence she need to cover these concepts. As she said:

> When I started to plan the unit, Nick and I sat down and we went throughout the sequence of what we thought it will be going. Since he had the class last year, he

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[9] These outcomes are quoted directly from Well Elementary School's *K-12 Mathematics Program*.

> knew that they hadn't learned the addition of unlike denominators or like denominators. And so we started out with like denominators, the same denominators, and then we mapped out what sequence of fractions we would going.

In addition, when she had difficulties and problems in implementing her curriculum, Martha was able to seek specific support not only from her cooperating teacher but also from the other teachers because the MEBA was widely used by many teachers in the school. Martha used the following example to describe how she was able to benefit from this culture for her mathematics lead teaching.

> One day I was starting to teach equivalent fractions. I was really confused about the activities in the MEBA. The MEBA book I really didn't understand myself. And so I came here. Nick was gone that day. He was gone and I was subbing. So I came to the lunch room to talk to Mrs. Kay. She was really open and I can get ideas from her whenever I wanted. I just sat down and asked her a little bit about equivalent fractions. She said she had this fraction bar kit that she thought really helpful. So I went to borrow that and she gave me some ideas of things that she liked and she used.

Students and their preparation for Martha's learning

The students in Nick's class were well prepared for the kind of teaching that Martha was learning. First, like the general population in Well Elementary, students in Nick's class were Caucasians from farming, professional, and state government families. Nick believed that this lack of a culturally diversified student population in his class made his students "a little easier to be able to work in groups," a central feature of his mathematics instruction.

Then, students in this class had been exposed to this way of teaching mathematics for a long time and they were used to it already. Nick assumed that not only had he worked with this class for about two years, but also his students had been exposed to this kind of teaching in their previous grades.

> Some teachers in the lower elementary level were getting the same training. It takes a period of four or five years to move kids through. Now I begin to see the benefits of that training. The kids are coming into my class now who had teachers going through this process of training a couple of years. They are better prepared for my teaching now.

Nick also realized "kids who have the support at home perform in different ways that the kids don't have" in his mathematics class. Thus, before Martha started her internship, not only did Nick pay attention to teaching his students, but also he paid attention to educating their parents about the kind of mathematics instruction he was doing.

> I had to teach parents how I was teaching the kids because again it is not the way they were taught. When they need to help their children on math concept and understanding it in a way that they had never learned, they had difficulty. I need some bases to educate the parents as well. Through materials I send home and letters I send home or personal contacts, I am sharing some of those ideas I used in my teaching because usually what kids bring back from home is how to get the right answer. Part of it is to memorize the steps.

By the end of her internship, Martha was able to identify these features of her students and their contribution toward her approach to teaching in her internship. She felt that this less diverse student population really helped her implement her mathematics as she wanted in the following ways.

First, Martha found that most of her students were coming from a cultural background similar to her own. She thought though this similarity limited her chance to develop a broad understanding about student diversity, it helped her somewhat in understanding her students in her lead teaching.

> I think they have a lot of similarities to me which is, I thought, a sort of bad because I think I need to understand different students. I don't know what will happen if I ended up with a child who is very different from me. However, I think a lot of them are like me and a lot of them came from the same background as me. It really helped me somewhat when coming to understand their learning.

Martha also realized that her students were also used to the kind of teaching she did in her class. She said "they have been working with Nick for about two years" and "the way these kids used to be taught has been consistent throughout the elementary school, not just in my classroom." Their habits of mathematics learning really supported her mathematics teaching in the way that she did not have to spend much time to develop a class learning culture to adapt to her approach to teaching. She was able to build her teaching directly upon this culture without "forcing student to change the rules of teaching and learning in the middle of the year."

Martha claimed that not only were her students used to the kind of teaching she was doing, but also their parents were supportive. They were able to really collaborate with her in helping her students do homework at home instead of challenging her ways of teaching with their own learning experiences.

> I have students whose parents were very supportive. That is not necessarily to do with their social and economic background. The more supportive parents tend to, the students tend to do a little bit better because they reinforce a lot of what we do in school at home. Like some parents make sure their students do their homework.

Instructional Contexts and Their Influences on Jaime

<u>The culture of teaching and Jaime's learning</u>

Although working in the same school at the same grade level, Jaime failed to have a chance to observe the specific images of constructivist mathematics teaching that she had hoped to develop. Thus, Jaime had fewer opportunities than Martha to connect her ideas with the specific teaching practice.

Although at the conceptual level, teachers in the building shared the general philosophy of mathematics teaching, at the practical level, the kind of teaching practice Jaime was exposed to in Bank's classroom was not consistent with the constructivist ideas shared among teachers in the school. Recall that Bank was, in his words, "still more an instructor" who would like to "show his students the answers" instead of inspiring them to discover it by themselves in his actual teaching practice, though he believed that discovery was important.

Not only was Bank's mathematics instruction inconsistent with the constructivist ideas shared among teachers in the school and with what Jaime wanted to learn, his teaching was clearly different from other teachers at the fifth grade level. There were obvious differences between Nick and Bank in teaching mathematics. These inconsistencies and differences pushed Jaime to react toward the school culture differently from Martha in two ways.

First, observing the inconsistency between her ideas of mathematics instruction and what Bank was doing, Jaime started to see the mathematics teaching conducted in Bank's class as different from what she wanted to do and decided not to follow her collaborating teacher in learning to teach mathematics. At the same time, the difference among teachers in this school and their mathematics teaching encouraged Jaime to see that mathematics instruction included a range of possible practices and teachers needed to develop individual styles of teaching. She thought such diversity was a great resource for her to learn to teach.

> By noticing teaching in this school, there were really diversities among teachers in their teaching methods, which is good. Because you can get all the different ideas and styles of teaching.

Thus, Jaime developed an idea that "it is great you can take pieces of the other teachers' teaching and put them up together to make your own style." She started to take advantage of the collaborative relationship among teachers in the school and borrow whatever she felt comfortable with from different teachers and piece together these things into her own style of mathematics teaching. She used her geometry as an example of this approach.

> Mainly it was just like when I did not know what they (students) had about geometry, I asked the fifth grade teacher what they learned. I just took that information to the other teachers and asked "Where did you start your lesson?" Because some teachers I found did not do angles until after they did shapes, that helps me think 'cause I kind of question that, because I tried to know why. They choose to do it that way and that is great, but that is not I will be comfortable doing it. So by telling you how they did in past helped me make up my mind and help me do it my way.

Curriculum resources and Jaime's learning

Although Jaime and Martha were working in the same school and at the same grade level, the curriculum resources and support for them to plan their lessons were not similar. First, the model of curriculum development that Jaime was exposed to in Bank's class was different from that in Martha's situation. In developing his curriculum, Bank only used the school curriculum as a source for him to see the specific content he needed to cover. As for the specific goals of mathematics teaching, Bank claimed that he relied more on another source -- the state standardized examination.

> In fifth grade, I know from the school curriculum guide I have to cover fractions, I have to cover multiplication, division, geometry. Another driving force is the standardized tests the state uses, the State Education Test. That gives us the goal what areas we need to be teaching, 'cause we know that is what the state deems to be valuable and we will be testing all the students on. So that kind of gives us a target to shoot for.

Second, the focuses and specific curriculum development that Jaime was exposed to in Bank's class were also different. When Bank was planning mathematics units and lessons, he did not rely on any particular texts and resources. What Bank often did was to take something from one source and use some others from another. Then he would put different parts together. In this process, Bank paid special attention to the connections between what he was teaching and the other subjects that students were learning and tried to build connections between the two.

> First of all, the overall goal of my unit. What are the main concepts that I want to get across in this unit? Secondly, how can I interpret those main ideas with other subjects like science, art, writing or literature? How can I find a connection there? I also look for projects that would be a valuable experience for my students to apply the knowledge and extend and enrich their basic concepts in the real world contexts they are familiar with.

Jaime realized that Bank was teaching in a different direction than she expected to learn to teach and decided to pursue a different way of mathematics teaching. However, she did not have the appropriate curriculum materials and support that were consistent with her ideas in developing her own units and lessons.

First, Jaime did not have sufficient support from her collaborating teacher or the other teachers to help her analyze the goals and topics of the school mathematics curriculum guidelines. For example, in deciding the specific goals and contents of her geometry unit, Jaime was totally on her own. Although the school had a strong constructivist vision of mathematics instruction, as I mentioned above, she skipped the general requirements of curriculum and directly read the fifth grade geometry content area of the school curriculum. Then she came to the conclusion that the school curriculum was requiring students to do too many things in a short period. It only focused on how to apply the formulas rather than how to help students develop deep understandings about why we need to use the concepts.

> I guess the school curriculum requires a lot of things you know, not about two dimensions but three dimensional shapes and that kind of stuff. But I think they will understand how, they know how to find out things, like the calculation. I guess they didn't understand why they are doing like the calculation that they have to find the area for a rectangle. Why do the areas of rectangles are always length times width? If they know why that make sense to them, they will never forget.

Then Jaime used the freedom her collaborating teacher gave to her and chose two geometry content areas for the fourth grade students as her lead teaching topic instead of the fifth grade level content area. She decided to help students "identify angles and use appropriate terminology (rays, vertex, obtuse and acute)" and "plot points and identify distance on a coordinate plane."[10] Jaime made all these important curriculum decision without specific supports from more experienced teachers

Second, the materials Jaime used to develop her specific teaching content and strategies in her setting were also inconsistent with her ideas of teaching. Once she decided her topic and goals, Jaime began to look at the textbook for specific material and models to help her plan her lessons and unit. However, she again felt dissatisfied with the textbook approach to the geometry content because it tried to finish many things in one day and focused on calculations rather than understanding. Thus, Jaime decided not to use the textbook for her planning.

> Well, the theme of the textbook is that they assume the kids will get something in one day. Like talking about perimeter and area. And as I told you they know how to find out the perimeter and area. I think they could but that is just telling them and that is not they are understanding the material. And that is why I think they don't fit in because they can't explain how you get the perimeter and area. That is

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[10] These outcomes are quoted from Well Elementary School's *K-12 Mathematics Program*

> a whole new concept to kids. And you just spend one day on it. You know especially when they didn't know to find or explain what it was. I chose not to use it.

Although the MEBA program that helped Martha develop her teaching unit was also available to Jaime, unfortunately it did not include geometry as part of its content. Without specific material and systematic support to rely on in planning her curriculum for her lead teaching, Jaime decided to depend upon herself to develop such a curriculum by piecing together different things from different sources.

> When I started my math unit, I wanted to teach the way I did. I went to a teacher in the downtown area. She taught that way and I wanted to teach like her. So I went to her and asked her how she set up her geometry lesson. And then I asked her about cooperative learning. I want to put that in my classroom. Then I talked to Bank and got some ideas from him. Then the teacher next door to us was great for doing that. So I use one teacher from Lansing and two teachers from here. And then I just talked to various interns and various teachers. And I just put them all together. Then I am where I want to be.

<u>Students and their preparation for Jaime's learning</u>

Students in Bank's class were similar to those in Nick's class in two ways. First, they were able to work together in groups. In fact, they were better at this than previous groups he had taught. They liked to share ideas and stayed on tasks longer. They were more friendly toward each other than the students Bank had taught before. As Bank described:

> This year, my group is little bit lower key. They are able to attend to a task a little longer. They work in groups a little bit better than last year. Last year, I had a lot of conflicts among the personalities. I worked on group skills and give and take criticism properly. This year, we are a little more friendly, lower key group. But socially a little bit better. They are able to get along together and work together in teams.

Second, in addition to a more collaborative group of students, the parents of his students were also supportive in terms of motivating his students' learning. In Bank's words:

> I have very good parent support. My students generally are cooperative, motivated and hard working partly because their family support. Family support is there and it shows we generally have very good climate for education here in the Well Elementary.

Jaime claimed that in her teaching, it was important for students to be able to form their ideas in groups and then share their ideas with the class. She recognized that her habits of learning mentioned earlier contributed to her way of teaching in the following ways. First, Jaime thought that her students being able to stay on their tasks really helped her focus more on how to make mathematics learning exciting for them rather than on how to deal with discipline and management problems.

> My kids are just behave well. I just don't worry about, you know, teaching them social skills. I love students that I have now. I guess they are not the kind of students I wanted to have. You know you will never have a bunch of kids who can't stay on their tasks here. That is just more a challenge for me to make math more fun for them.

Second, Jaime also realized that her students were quite good in working with each other and in groups. They were used to speaking about their ideas in front of the class. These habits really helped her in getting students to learn cooperatively and to discover and speak their own ideas.

> I think they were obviously used to working with each other. A lot of them had experience of cooperative learning or they can work in groups together well. They used to get in front of the class to talk about their ideas of math. So those things influence the way I teach. If they are not comfortable with talking to each other, I have to get them comfortable with that.

Instructional Contexts and Their Influences on Louis

The culture of teaching and Louis's learning

Several years ago, like Well Elementary, Bell Elementary was also actively involved in mathematics instruction reform. It reevaluated and updated its mathematics curriculum according to the NCTM standards and pushed teachers to teach in the constructivist direction. In the recent state-wide standardized mathematics examinations, students in the school did a good job in the questions about concepts but a poor job in computation questions.

When Louis began his internship in the school, this school was facing great pressure from students' parents to raise their kids' performance in the state examination. Thus, the school started to re-focus their mathematics instruction on computation and push its mathematics instruction back to the basic mathematics facts and skill practice. Louis was exposed to such school-wide change in mathematics instruction as soon as he entered his internship. As Louis said:

> Now they felt that they are in trouble. They moved back and stressed more on computation in some of the staff meetings. Such as when we come to talk about math, the school really push computation. I know the State Mathematics Test they were taking. In the reasoning and thinking logically about problems and the math story problems sections, their students are scoring well above the average than the rest of the state. But in computation part, they are well below the average. So they were happy with their reasoning scores, they will not happy with the computation scoring.

Although some teachers in the school, like Ben, began developing constructivist approach to mathematics teaching and then moved back to the traditional method, neither constructivist nor the traditional approaches actually predominated among all the teachers. The mathematics instruction in the school reflected strong individualism.

Teachers still enjoyed freedom in deciding whether or not to change. Some teachers had been teaching in a traditional way in spite of the constructivist curriculum reform several years ago. As Ben described when he was asked the influence of constructivist curriculum reform on teachers' practice:

> It is supposed to be influential. However, some teachers are still going through and using the stupid textbook all the time. We had a teacher who is excellent at social studies. Just excellent. When other teachers were using manipulatives, she still taught math in her own way. She followed the textbook a whole lot more than she should.

While the school was pushing its teaching back toward the traditional approach, there were also some teachers who were still using the constructivist approach in their mathematics instruction. For example, Mary, another fifth grade teacher who strongly supported Louis's curriculum development for his lead teaching, was still practicing a constructivist approach. Such diversity among teachers in their mathematics instruction allowed Louis a chance to observe the kind of the teaching he wanted to pursue and that was encouraged by his program.

In addition, in spite of the fact that Ben was moving back to teach basics, he had an open mind toward what Louis wanted to try and gave him the freedom to do so in his classroom. Louis realized that Ben was flexible about what he wanted to try.

> I think Ben is so flexible and open to multiple and different ways of teaching. And I think that really helps me because that helps me to try the way I want to teach. I do not want to try the two different types of teaching while I am here.

Thus, the individualism in the school instruction and the flexibility created by Ben helped Louis make up his mind to learn what he wanted to learn, even though the school was pushing its instruction back to the basics and Ben was teaching in a different direction than Louis expected to pursue in his own practice.

<u>Curriculum resources and Louis's learning</u>

Ben had little direct influence on Louis's planning and teaching. After seeing his students fail to do a good job in the area of computation, Ben changed to teach in a traditional way that was clearly not the model Louis wanted to follow. In addition, Ben's twenty-five years of teaching experiences had helped him internalize his planning processes. In his words: "I reached the stage where about four years ago, I stopped writing everything down for teaching plan." This made his process of planning often inaccessible for Louis. Thus, except for providing materials and resources, Ben did very little in directly supporting Louis to develop his curriculum.

To plan his lead teaching unit on fractions, Louis relied on three kinds of resources and support: the school mathematics curriculum and textbook available to Louis in Ben's class, some reference books Ben provided, and other teachers' suggestions and support.

First, the school curriculum gave Louis a sense of general goals and content coverage for his unit without providing him any specific pedagogical suggestions. The following three typical goals and topic coverage for fifth grade fraction instruction are specified in the school curriculum guidelines Louis used. From these two items, we can see clearly that except for teaching topic, there were no pedagogical suggestions.

It is essential that students understand

- That a fraction or decimal or percent is part of a whole divided into equal parts (including mixed numbers).
- That the same quantity can be represented in various forms (including fraction, decimal, whole, mixed number, percent) appropriate to the situation. (Students should be fluent with the renaming of common examples such as 1/4 is the same as 0.25 and 25%. Likewise, 1/2, 3/4, 1/10.)

- Equivalence of fractions.[11]

The textbook used in his class was also not very useful in helping him to develop a specific approach and pedagogy because its language was complicated for his students.

> I think that the text had a lot of good things and lot of good problems in it. And I can spend time reading problems out of the text if I wanted. But I don't I think I will use it because some of the language the textbook uses is rather complicated for the students to understand.

Unlike Jaime, Louis felt that it was hard for himself to accomplish this planning task because he had no experience in teaching fractions before. Thus, Louis began to seek some external support for his specific curriculum development. Fortunately, Louis found another fifth grade teacher, Mary, who was still practicing the constructivist approach of teaching. It was from Mary that Louis was able to receive not only specific support in planning but also get specific materials upon which he was able to build his curriculum that reflected what he wanted to do.

> When I planned this unit, I had a teacher next door, Mary, who teaches really well in the way I wanted to teach. She is a math person. She is taking a lot of math classes for her masters degree that she is getting. So I asked her what she has done. I used the book called *Mathematics way of thinking*, and that is the one she used a lot for her masters classes. She showed me some of the stuff that they (her students) were doing. So I was using that book for my unit and I want to make my unit similar to hers 'cause in this way I was not giving them answers. I was making them find them and discover them for themselves.

Finally Louis successfully developed his curriculum for his fraction lead teaching unit. This curriculum, according to Louis, aimed at getting his students to form ideas of equivalent fractions with real life examples, and then proving their ideas to each other in class without him telling students the answer. Louis was able to put this curriculum into

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[11] These standards are quoted from *Bell Public Schools' Core Curriculum: Mathematics* (p.22)

his actual teaching for the first lesson of his unit. However, he stopped carrying out his constructivist agenda afterwards and reversed to a different approach of teaching.

Students and their preparation for Louis's learning

Students in Ben's class had the following three features in terms of mathematics learning. First, many of these students were not very good at computation. This situation forced Ben to slow down and spend more time reviewing, practicing some basic mathematics facts and explaining and illustrating ideas. Thus, they were used to Ben's way of teaching, which was slow paced, involved repetitious practice and careful reviewing and illustrating. As Ben described it:

> Probably in the beginning of the year, I found that half of them did not know division and multiplication facts. At least, I felt that they should know them. Ah, I think they should have to do Bomb, Bomb, Bomb. So I have to slow down a little on some of the things I have to do with them and I have to do a whole a lot of more explaining and practicing for them.

According to Ben, there was a prevailing anxiety among these students in learning mathematics. That anxiety partly stemmed from their failure to learn mathematics well and partly because their parents conveyed the same anxiety to them. To adjust to this situation and reduce their anxiety, Ben took a more fatherly role in his teaching and reduced his content coverage and the level of difficulty in the hope that his students would feel comfortable about their learning.

> Many times we have parents coming and they will say they were never very good at math. So they pass this attitude right on to their kids and the kids will say, "I don't know how to do this." Part of the problem I think though in the past years, that I had seen that the kids are getting tired sitting down doing 50 the same problems over and over and over. You have to be nice to them, say " Oh have you found this." So I feel like I am playing a daddy role sometimes with them. Yes, it really change a lot my way of teaching. You do not cover as much and get them feel comfortable with what they are doing.

By the end of his internship, Louis claimed that his students strongly shaped his approach to mathematics instruction. This influence became apparent when the thrust of Louis's learning to teach mathematics moved from curriculum development into curriculum implementation.

Louis realized that many of his students had no problem in understanding how to get the answer but they had serious problems in getting answers fast enough. As he said:

> In the test they had that is just to see how good our kids were doing in computation, they used to be able to do 85 times 3 in three minutes. Now they had trouble, there are some kids doing it in 7 minutes. So it takes more than twice the amount of time. There are kids, we had two or three kids who can do under four minutes and none of them can do it close to 3 minutes

However, Louis believed that a constructivist approach to teaching mathematics teaching not only would change his students' perceptions of mathematics learning, but also would give them exciting experience with mathematics. As he learned from his teacher preparation program, students' own discovery of mathematical ideas and their excitement about mathematics learning could go hand in hand.

> I just had in my head when I came to this class and faced those naive conceptions of students, I am going to make a difference. This is the way they are all going to learn math and all of them are going to be amazing when they are finding out mathematical ideas by themselves.

It was with this belief that Louis decided to stick to a constructivist approach and started to seek external support to develop his own curriculum on fraction. However, his students did not feel excited about this way of mathematics learning when he began to implement his constructivist curriculum. Instead, many of them demanded he go back to the way of mathematics instruction that they were familiar with through Ben's instruction. The students' challenge created a situation in which Louis was caught in conflict between his two constructivist ideas. Mathematics instruction needed to encourage students to discover their mathematical ideas and prove them, and mathematics instruction needed to get students excited about their mathematics learning. Without any support resolving this conflict in the direction of constructivism, Louis chose to conform to his students' demands and began to teach in the way that students felt happy about.

> I thought I picked up the right way of teaching. OK, let's look at this problem, let's draw this out and let's prove it. This is the way I thought my students would really catch on. But they did not. And in the second lesson, when we geared towards equivalent fractions again, I gave them pretty much more direct answers than I did first day. And that was also from students. After you had left, some of the students said to me, "I hate that, I hate that." They said "I can't do math that way." It is like they think it is better for you to tell them the way it is. They really don't like this way of teaching, this self discovery. Before I know from our TE math classes that one of the things you need to do is to help them discover for themselves. It gives them a lot of ownership of the ideas and then it is more concrete to them and they will feel excited about it. But a lot of my students don't like it that way because it is different to them and they don't care about it. They wanted to go with what they have been doing and they are good at. And a lot of students have been really good in class and they also think "No." This is why I changed.

Instructional Contexts and Their Influences on Kelly

The culture of teaching and Kelly's learning

In recent years, Mall Elementary school began to alter its mathematics instruction. In this case, it came to emphasize its K-3 students' mathematics performance and pushed its K-3 teachers to use manipulatives in their mathematics instruction. It participated in a national standardized testing program "to look at how Mall Elementary students perform in certain areas (including mathematics) in comparison to their counterparts in other urban areas across nation."[12]

Lisa responded to the push from her school by using a lot of manipulatives in her teaching. However, the way she used the manipulatives did not reflect a constructivist approach to teaching, as I analyzed in Chapter 6. Instead of using these to help student construct their mathematics understanding and explain their ideas, she used these manipulatives as counting aids to get students to drill rules and practice repetitious questions that she illustrated. The kind of teaching Lisa practiced was not unique at Mall Elementary. Her teaching reflected prevailing practice in the school. The difference was that the lower grade teachers used different resources in their mathematics teaching while the upper level teachers tended to teach by following a textbook.

Like the three other preservice teachers, Kelly brought into her internship several constructivist ideas. She believed that mathematics learning needed to focus on students' conceptual understanding rather than memorizing. As a teacher, she wanted to use different ways to help all the students reach that level of understanding through their own discovery and make students comfortable with mathematics. However, Kelly soon realized that mathematics instruction in this school was strongly influenced by the standardized examination. She claimed that this influence could be seen even in the lower elementary classrooms and it also shaped her mathematics instruction.

> In planning a math lesson, here we sort of cover everything that would be on the standardized test that our children would be required to know. We have to make sure that we cover everything even though we don't have textbook.

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[12] This is quoted from *1995 Annual Report of Mall Elementary School*.

Kelly also found through her early observation that teachers in this school were clearly divided into two groups, the lower grades and upper grades. Teachers within a group were more likely communicate to each other about their teaching and their teaching practice than across groups.

> In this school, I would say that a lot of classes were very similar. I know that most of lower elementary classrooms did not follow the textbook page by page. They use the textbook as a reference and use it to add to their lessons or unit what ever they teach. They will pull certain pages that would help the kids. All the lower elementary use a lot of manipulatives. They followed the curriculum pretty closely so that they can cover what they are supposed to be teaching. They talked each other a lot within each grade level. And that goes for all the subjects. They liked to be around same area in their instruction and they cover a lot of similar unit. The first grade teachers will make sure that they were in the same unit every week. So there is a lot of talking and a lot of working together. As far as lower grade in this school, I would say that the math teaching was very similar.

This situation meant that Kelly was only exposed to one kind of teaching daily represented to her by collaborating teacher's practice, which featured teachers' using manipulatives to illustrate the skills and rules, followed with students using manipulatives to practice individually the skill and rule. This kind of teaching became the only practice with which she was able to connect her ideas.

Curriculum resources and Kelly's learning

Mall Elementary School's mathematics curriculum was the only source for the lower grade teachers to develop their mathematics units and lessons at the school level. However, it neither specified the topics and areas that first grade students needed to learn nor did it provide any pedagogical suggestions or recommendations on how to teach except for some general outcomes that students need to reach by the end of third grade. Here are some examples of such outcomes:

By the end of grade 3, students will

- Demonstrate the ability to compare and classify. Children will manipulate, describe, analyze, infer and invent relationships and structure
- Develop skills of sorting and organizing information and using the information to make predictions and solve new problem...[13]

Thus, to be able to teach mathematics lessons, teachers in the lower grade level had to find specific materials and develop their own units and lessons. Before Kelly entered her internship, Lisa had already gone through the following processes and constructed her specific units and lessons for all the topics she felt important and that she needed to cover for her first grade mathematics.

First, she studied the curriculum requirements and talked to teachers in the lower and upper grade levels to further develop the specific goals and content coverage for her first grade level.

> I look at the school curriculum for what are my goals and objectives for the first grade. I also look at the goals and objectives from kindergarten teachers because all my children are not right here yet, ready for what I want them to learn. So I have to check the goals and objectives for kindergarten. I do that a lot in fall to see what my kids are good at. I do check that and be aware of those and then look at the first grader's objectives. What are the second graders' objectives so where I am heading?

Then she worked very closely with three other first grade teachers in the school to further break down the content into specific skills that they needed to teach. They developed their specific units and lessons around the skills and sub-skills together.

> I will go through the skills, for instance, with three of us, first grade teachers. We look at the goals and skills. We break down that even into smaller parts. So basically what we have a report card and we put them into smaller pieces. And we break it down. For instance, our report card will say measurement. Measurement includes feet and inches. It includes measurement of time, includes volume. Or counting to 100. Well, we in first grade feel they need to count by 2s, they need to count by 5 and they need to count by 10.

[13] These outcomes are quoted from Mall Public Schools' *Mathematics Curriculum Guidelines.*

Then Lisa developed her specific units and lessons plans around these specific skills and sub-skills with different sources, the textbooks, different kinds of manipulatives and worksheets. She combined all three sources to construct her mathematics unit and lessons and organized these units and lessons in a progression.

Over the years, Lisa was able to develop a series of units and lessons, organized them in a progressive way and put them in her mathematics cabinet. She used each of these units and lessons according to what she saw as her students' needs. When Kelly came to her classroom doing their internship, these mathematics curriculum units and lessons became important resources for her teaching and planning.

In planning her lead teaching lessons, Kelly further relied upon Lisa and her well-developed units and lessons. Kelly felt that she had to do it because she had no idea about what should be covered and how to cover it for a particular unit or lesson for a first grade class, and the school curriculum failed to provide such specific information.

> I would say the math curriculum is very general. It might give you the goals but not specific content areas that need to be covered. It wouldn't tell you how to cover. I can think the science curriculum is specific right now. It talked about the concepts that children should be able to understand, how they should be able to think and what kinds of questions they should be asking themselves, and what kinds of questions you should be asking them. But math curriculum doesn't give specific topics and suggestions.

In planning her lead teaching lessons, she not only relied on Lisa for "a list of specific topics" that she needed to cover for the first grade. She also used a series of actual units and lessons designed by Lisa as her model to follow.

> Lisa has a lot of folders of all the different math units and lessons. I always go through all her files. I looked through all the manipulatives to see what Lisa has available to me to use. I had counting cubes or the math boxes to do the story problems. She had cupboards and clocks that I used for my time lesson. Those are the materials Lisa just had in her folders. She made the candy heart and work

> sheets. I just adapted the candy heart since it is getting near Valentine's to use it as my graphing and estimation project. So that was a kind how I planned the unit.

During her planning process, she could have continuous support and suggestions from Lisa whenever she asked. As I discussed in Chapter 7, Lisa would also constantly check Kelly's teaching plan for each lesson and make sure her lesson plan went as she wanted it to go.

<u>Students and their preparation for Kelly's learning</u>

The students in Lisa's class to an extent shaped what Lisa did in her mathematics teaching and influenced Kelly's learning to teach. First, most of them lacked some necessary learning experience that they needed for their first grade mathematics learning. Their interests in learning, Lisa claimed, were not properly articulated. This fact of her students pushed Lisa to focus more on direct illustration, basic skills drilling and using manipulatives to reinforce their skill development.

> My children have limited learning experience with math and reading. Just the necessary learning experiences. For example, because we live in the schools near the zoo, my kids go to the zoo a lot but kids can't classify animals. They can't tell you much about any of those animals there. Yes, they go to the zoo, but they don't know much about chick versus snake. Their learning interest has not been sparked.

Second, there were about eighteen boys in Lisa's class this year who were very active and had problems sitting down and doing their work for a longer period. Thus, these boys forced Lisa to pay more attention to the classroom management rather than thinking about how to teach mathematics.

> I got 18 boys in my class. And that matters in my teaching this year. I haven't had that before. I got 18 girls before. And that matters. These particular kids who are immature group. They are not able to sit for a longer period of time. So if I can have them sit, when I go to teach them, they are hard to control. I am dealing too much with what I think is with management issues instead of teaching math.

These features of her students directly and indirectly influenced Kelly's learning to teach in several ways. First, they shaped the kind of teaching practice Kelly was able to observe from Lisa. Kelly claimed observing was one of the important ways for her to learn to teach, and the students in Lisa's class pushed Lisa even more to concentrate on the approach of teaching as telling followed by practice.

Second, it also directly shaped what Kelly was able to do in her own planning and teaching. Kelly believed that living in the urban downtown area, her students had quite different life experiences than what she experienced when she was in elementary school in a rural community. She felt this diverse student population made it difficult for her to develop her mathematics curriculum for her lead teaching and, thus, it pushed her rely to more on her collaborating teacher for planning.

> My kids are from the inner city. They have quite different kinds of life from my experience. I came from a different type of area, not from the city. I am from a smaller town where the classroom may be a little bit more homogenous. You see in this class I am teaching now, there are all kinds of kids, all different learning levels, all different learning styles. They are from all different kinds of backgrounds. That is something I have to consider in my planning and that is also hard for me to plan my lessons.

Kelly also realized that the 18 noisy boys in her class and their behavior problems often drew her efforts away from thinking about teaching to dealing with disciplinary problems and management in her planning. She complained:

> One thing I have always keep in my mind about this class is I have 18 boys out of 23 students so our class is very active. They can be rowdy and noisy and so management is a big issue for me. I also have a couple of the students you have seen in my class that have behavior problems. I always need to take them into consideration when planning a lesson. I am still working very hard at the management because the girl (one of the kids who had behavior problems) doesn't allow you to teach many times.

Summary of Chapter

My analysis in this chapter suggests several things about the relationship between instructional contexts and the quality of preservice teachers' learning to teach. First, to successfully support preservice teachers to learn to teach mathematics in a constructivist approach, the culture of teaching in each school needed to support a vision of constructivist teaching and provide specific images of teaching that reflect the vision. My analysis suggests that both factors were important in helping these preservice teachers learn to teach mathematics in a constructivist approach. The first factor shaped a vision of good teaching accepted by the schools and the latter helped preservice teachers to build connections between their ideas and practice.

In Mall Elementary, the school's vision of mathematics teaching and images of teaching practice it provided were oriented toward traditional approach to mathematics. Thus, Kelly's general ideas never had a chance to be reinforced and connected with specific images of actual teaching. Her ideas stayed at the nominative level without finding their way to her practice.

Both Martha and Jaime were working in a school where a constructivist vision of mathematics teaching was encouraged. Martha was able to connect her ideas with the specific images of constructivist teaching that her collaborating teacher, Nick, was doing in his class. Thus, not only was she able to retain her ideas, but also she was able to continuously push these ideas into a practical level. In contrast, Jaime was unable to develop a picture of constructivist teaching because Bank was largely unable to enact this philosophy, though he shared this philosophy with Jaime at the conceptual level. Without substantial images as her support, Jaime's chances to develop and connect some of her important ideas to the practice were limited.

Under the individualist culture of teaching in Bell Elementary, good teaching practice was not clearly defined and shared among teachers. This culture gave Louis chances to connect his ideas with specific images of the constructivist teaching that was practiced by some of the teachers there. It also allowed him to validate his choice to go back to a traditional direction when he was facing conflict because both approaches were accepted in the culture.

Second, the specific curriculum resources that are consistent with a vision of constructivism are also crucial for preservice teachers' learning to teach because these resources help them develop specific unit and lesson plans that, in many cases, preservice teachers appear unable to develop by themselves. This study suggests those curriculum resources not only need to prescribe the goals and outcomes of constructivist mathematics instruction, and clearly define the content coverage, but also need to provide pedagogical suggestions that are specifically and consistently related with these goals, outcomes and contents (Cohen and Spillane, 1992).

In Kelly's situation, the school curriculum only prescribed general outcomes for mathematics instruction by the end of the third grade without any specific suggestion on what to cover and how to cover it. It gave Kelly nothing to rely on when she was planning her lead teaching units and lessons. This situation pushed her to depend only on her collaborating teacher, Lisa, for the specific curriculum materials and resources. However, these resources, though well organized and ready to use, implied a traditional approach to mathematics instruction.

In Martha and Louis's situations, though working in different schools, both had a school curriculum that prescribed both visions and specific content coverage. In addition, in planning their unit and lessons, both were able to rely on specific pedagogical resources that reflected their ideas of mathematics instruction. They were able to successfully develop a constructivist curriculum for their mathematics teaching. Martha was lucky. She was not only able to have a textbook that provided a specific model of teaching that was specific and consistent with goals and outcomes of mathematics instruction prescribed by the curriculum guide. She also had a collaborating teacher, Nick, who had successfully experimented with this model and material, as her support for her planning. Louis, though not having similar support from his cooperating teacher, was able to find Mary, another teacher who was teaching in the constructivist direction to support his curriculum development. He also able to have specific resources from this teacher to develop his unit plan. This allowed him to be able to implement his ideas in his first lesson.

Working with the same curriculum guideline as Martha, Jaime was not lucky enough to have specific and consistent curriculum material and resources to rely on in developing her unit and lesson plan. First, the same textbook Martha used for her fraction unit was also available to Jaime, but that textbook did not specifically cover geometry content. Then Jaime's collaborating teacher, Bank, was teaching in a more traditional way than Jaime expected and she decided not to follow his step in teaching mathematics. Thus, Jaime ended up taking full responsibility for curriculum development and it was clear that she was not ready for this planning task.

Last but not the least, the student population also supported or restricted preservice teachers developing a constructivist approach of teaching. My analysis suggests that the kinds of students preservice teachers are teaching will shape the focus of their curriculum development and influence the direction of their curriculum implementation.

Kelly taught at the first grade level and her students were very young and active. Many of them had not developed the learning habits for their first grade mathematics learning that their school required. Some of them even had behavior problems. These students often interrupted Kelly's teaching and distracted her attention and energy away from pedagogical issues and pushed her during both her planning and teaching to consider more how to control and manage. Such students also shaped Lisa's way of teaching by pushing Lisa to focus more on illustration and practice. Again Lisa's teaching conveyed to Kelly an even more traditional way of teaching to observe and follow.

Martha and Jaime taught fifth grade and their students were from cultural backgrounds similar to them, which gave them a better chance to understand and relate to their students. Their students were well prepared for the constructivist approach of teaching that both of them tried to develop. They could stay longer on their group work, they were not shy to speak or present ideas to their classmates and they were used to form their own ideas of mathematics if teachers encouraged these. All these factors greatly facilitated the process of their curriculum implementation. The problem for Jaime was that she was unable to have the necessary support in developing an ambitious curriculum that reflected the constructivist vision of mathematics instruction. Her collaborating teacher was not able to play the role of facilitator and supporter and Jaime felt reluctant to get support from her collaborating teacher. However, Martha was able to successfully develop her curriculum with specific and consistent supports and resources.

Louis was teaching fractions at the fifth grade in a different school from Martha. His students had anxiety about mathematics learning and were used to Ben's more traditional approach. Students felt happy about this kind of teaching because Ben was able to integrate many games and activities into their drill and reduce the difficulty of their learning tasks. Even though Louis was able to develop a curriculum that reflected many of his constructivist ideas, during his curriculum implementation, his approach was strongly challenged by his students who felt it was hard and uncomfortable to learn through this kind of teaching which led them to uncertainty. Their challenges put some of his good ideas in conflict. Without proper guidance and support for him to solve the problem in the direction of constructivism, Louis accepted the approach that students were used to and that Ben was practicing. He began to teach in a traditional way that students felt happy about.

Chapter 9

CONCLUSIONS AND IMPLICATIONS

In the beginning of this dissertation, I raised four questions for my study. What do preservice teachers learn about the mathematics instruction in their internship? What are the influences of their collaborating teachers on their learning? How do their collaborating teachers influence their learning? How do their instructional contexts shape their learning? My analysis of four elementary preservice teachers' learning in their internship and the contexts of their learning deepens my understanding about these questions. This study also helps me see some important implications for improving teacher education and preservice teachers' internships.

In this last chapter, I will summarize what I have learned about preservice teachers' learning, collaborating teachers' influences and the impacts of instructional contexts. I will also discuss implications of this study for improving teacher education and preservice teachers' learning to teach.

What did I learn from this study?

Beliefs and practice in preservice teachers' learning

Challenging preservice teachers' prior beliefs of mathematics instruction and transforming these beliefs into a constructivist vision are an important part of current teacher education reform (Kennedy, 1991; Ball & McDiarmid, 1990; Lortie, 1975). It has become an important focus of preservice teacher preparation course work, especially subject-related method classes. A tacit assumption seems to be that once preservice teachers acquire some ambitious beliefs, they will be able to gradually develop correspondent practice. Since it is so hard to change beliefs, it seems reasonable to say that once beliefs have been transformed, they can be maintained. Thus, preservice teachers' nearly developed ideas will guide them to learn relevant teaching practice when they go into internships and future teaching. Such a linear relationship between beliefs and practice is an unstated assumption of many teacher education programs.

However, this study further develops this understanding about the relationship between beliefs and practice in preservice teachers' learning to teach in three aspects. First, although preservice teachers are able to develop some constructivist beliefs of mathematics instruction, these beliefs are often not strong enough to hold constant or develop further. They are still subject to change and modification in the internship. The direction of their conceptual change and modification varies from one case to another.

Martha strengthened many of her constructivist ideas and somehow pushed these ideas toward a pedagogical thinking. Although starting her internship with some similar ideas, Jaime's constructivist ideas weakened through her internship. She formed some ideas that reflected a self-realization position. Louis wanted to use some of his constructivist ideas to transform his students' mathematics learning experiences but ended up with a compromised stance toward mathematics instruction. Kelly clearly developed several beliefs that reflected an absolutist position and the constructivist elements in her early beliefs disappeared.

Second, my study suggests that the beliefs preservice teachers brought into internship, though constructivist in nature, may not necessarily lead to correspondent practices in their internship. These entry beliefs can be advanced through practice, they can also stay at a nominative level without finding their way to practice or they can be changed by practices. Thus, the relationship between preservice teachers' beliefs and practice in the internship is not necessarily causal and linear but mutual and interactive.

Martha was able to practice many of the constructivist ideas she brought into her internship and ended up with a more contextualized understanding about constructivist mathematics teaching. Jaime was only able to practice some of her ideas by integrating students' life experiences into her geometry lesson and encouraging students to express their own ideas of mathematics concepts. However, she was unable to push students' understanding of mathematics by challenging or supporting them to explain, prove and disprove their mathematical ideas. In the end, she strengthened the self-realization part of her beliefs of mathematics instruction and no longer held her idea that students' conceptual understanding needed to be the central focus of her teaching. Louis was able to practice his constructivist ideas in his first lesson. However, his experience in his teaching quickly pushed him to take an absolutist approach in the rest of his mathematics lessons and transformed his constructivist beliefs into a compromised stance toward mathematics instruction that combined constructivist, absolutist and self-realization elements. Although Kelly brought some constructivist ideas into her internship, she was never able to articulate them in her teaching practice. Her practice reflected a traditional method throughout her lead teaching. In the end, she developed several ideas that were compatible with such practice.

Third, I also learned that not all constructivist ideas can go hand in hand peacefully in their implementation. Instead, depending on the contexts in which they are implemented, they can conflict with each other in the actual teaching practice. Without a deep and clear understanding about these ideas and their relationship, these ambitious beliefs can also produce contradictory practice.

Louis's case clearly showed this feature of preservice teachers' learning to teach mathematics. He brought into his internship the idea that students' discovery of mathematics concept and their excitement about mathematics learning are related and that both were important. He expected that his students would have a new experience of learning mathematics through self-exploration and, at the same time, feel excited about this kind of learning. However, in his teaching, his students did not feel excited about their own discovery of mathematical ideas They demanded he goes back to basics--telling followed with students' individual practice--a teaching with which they were familiar and felt comfortable. Without any support to resolve this conflict in the direction of constructivism, Louis chose to satisfy his students' request and started to teach as his students wanted and felt comfortable about.

Collaborating teachers' influences on preservice teachers' learning

Literature (Brown, Collins, & Duguid, 1989; Lave & Wenger, 1991) implies that preservice teachers' learning to teach should be situated in real teaching situations since all knowledge is situated in and grows out of the contexts of their use. Teacher educators also expect that preservice teachers' learning can be assisted or coached by the experienced teacher so that novices can learn to teach at levels beyond their independent performance (Tharp & Gallimore, 1988). My study supports and expands these assumptions about preservice teachers' learning experience in the field and under the influences of collaborating teachers on their learning to teach.

First, I learned that when we put our student teachers to work closely with collaborating teachers in actual teaching situations, both their conceptions and practice can be strongly influenced by what their collaborating teachers think and practice.

All four preservice teachers in my study moved conceptually closer toward their collaborating teachers in three forms. First, they weakened and no longer stuck to those beliefs that were not shared with their collaborating teachers. Second, they kept or reinforced those beliefs that were clearly shared with their collaborating teachers. Third, they developed new conceptions of mathematics instruction that reflected ideas their collaborating teachers firmly held.

At a practical level, teaching that each preservice teacher developed was also similar to or moved toward that of his or her collaborating teacher. Martha was able to develop a constructivist approach of mathematics teaching that was similar to Nick's. The only difference is that Martha did a relatively weak job in integrating real life examples and problems into her teaching. Jaime's instructional practice was also similar to Bank's instructional practice. Both were able to integrate students' life experience into what they were teaching and neither challenged nor supported students to explain and prove their ideas. However, Jaime was less likely to provide feedback to her students' questions but more likely to get students to speak their ideas than Bank. Although Louis was able to develop a constructivist approach in his first class, his instructional practice dramatically changed into a traditional model that was similar to what Ben was practicing. Their difference is that Ben did a better job in connecting what he was teaching to what students had already learned and their daily life than Louis. Kelly's teaching exactly reflected the explaining and practice that Lisa engaged in her mathematics classes.

Second, this study also shows me that direct assistance and coaching from collaborating teachers can help preservice teachers learn to teach at levels beyond their own performance. However, without a shared constructivist vision as its base, such assisted performance can produce qualitatively different teaching performance in preservice teachers. Thus, not all the influences of collaborating teachers are necessarily positive and match teacher educators' expectations

Nick and Lisa actively coached Martha and Kelly throughout each phase of their internship using different forms of support. In the early part, they modeled their way of teaching to their preservice teachers and reflected on their reasons and goals behind their instruction. They supported their preservice teachers to understand subject matter knowledge and the sequence of representing it. They helped their preservice teachers plan their units and lessons and gave feedback about their preservice teachers' teaching. Martha and Kelly felt that they were able to learn what they might not be able to learn by themselves. However, Nick was coaching Martha toward a constructivist approach of teaching that he was practicing, while Lisa was helping Kelly learn to teach in the traditional way.

Third, the influence of collaborating teachers on preservice teachers' learning to teach may not necessarily be direct. It can happen in indirect ways. When preservice teachers only had fragile and abstract beliefs, such indirect influence can have a great impact on the quality of their learning to teach.

Bank and Ben felt reluctant to directly assist their preservice teachers, Jaime and Louis, to learn to teach mathematics, as Nick and Lisa did. Both Bank and Ben developed good personal relationships with their preservice teachers. Both clamed that they would like to provide freedom to let their preservice teachers do whatever they wanted to try. Both assumed that their support for preservice teachers' learning to teach was limited to when they were asked to do so. However, their influences on preservice teachers' learning to teach still occurred in indirect ways. They still influenced their preservice teachers' learning by limiting chances for them to connect their ideas with relevant teaching practice. They affected their peservice teachers' learning to teach by restraining their chances to get access to appropriate constructivist curriculum material with consistent but specific goals, contents and pedagogical suggestions. Finally, they exerted impact on their preservice teachers' learning by shaping their students' learning habits and dispositions.

Collaboration and preservice teachers' learning

Teacher educators encourage a collaborative relationship between collaborating and preservice teachers in which both parties share values, standards and agreements for teaching (Cochran-Smith, 1991). It is assumed that such a collaboration between collaborating and preservice teachers supports preservice teachers to learn to teach in innovative ways. However, studies in the field provide a less clear understanding of how such relationship can be developed and how the collaboration between the two parties can influence preservice teachers' learning, especially when the relationship is neither totally contrived nor totally voluntary. My study contributes to understanding these issues from three angles.

First, the kind of roles that collaborating and preservice teachers assume in their collaboration, to a great extent, depends on whether they are able to identify a discrepancy between what the preservice teacher wants to learn and what the collaborating teacher is practicing. When the collaborating teacher identifies such a discrepancy, he or she may assume a non-mediating role in preservice teacher's learning. The same understanding on a preservice teachers' part may push them to assume their learning to teach needs to be through trial and error or that they should seek external support instead of his or her collaborating teacher.

Nick and Lisa assumed a teaching role in their collaboration with their preservice teachers, Martha and Kelly, because both believed what they were practicing was what their preservice teachers needed to learn. Martha and Kelly were also able to assume a learning role in their collaborative work with Nick and Lisa, since both thought that their collaborating teachers were teaching in the direction they wanted to learn to teach. However, Bank and Ben assumed a non-mediating role in their preservice teachers' learning because both thought they were unable to provide a model for their preservice teachers to follow. Jaime and Louis only looked at their collaborating teachers as a resource. Each made up their mind to learn to teach without the specific and direct support of their collaborating teacher. They wanted to do so because they clearly recognized that their collaborating teachers were teaching in a direction they did not want to pursue.

Second, my study also suggests that to develop a supportive collaboration for preservice teachers to continuously learn to teach in a constructivist direction, certain elements need to serve as a base for their collaboration. First, both preservice and collaborating teachers need to be able to identify that they share the same constructivist vision of mathematics instruction. Second, the preservice teacher needs to be able to identify constructivist principles embedded in his or her collaborating teacher's practice and be willing to learn from the collaborating teacher. Third, the collaborating teacher needs to be able to enact a constructivist vision and be willing to play the role of teacher educator through the internship. Without any of these three elements, a supportive collaboration is hard to build.

Nick and Lisa were able to work closely with their preservice teachers in different ways. They were able to provide constant guidance and assistance to their preservice teachers throughout the internship. However, Martha was able to develop a constructivist approach, which Kelly obviously was learning in a traditional direction. The major difference between their collaborations was that Martha and Nick were able to build their collaboration upon their shared constructivist vision. In contrast, Lisa's vision of teaching was oriented toward the absolutist approach and the support Lisa provided for Kelly, thus, was based upon a traditional vision of mathematics instruction.

Nick and Bank were embracing the constructivist vision of mathematics instruction that their preservice teachers, Martha and Jaime, professed. However, Nick was able to not only develop this constructivist vision but, somehow, he was able to put it into practice. His ability to enact his vision really helped Martha identify his vision and encouraged her to learn from him. Bank was unable to practice some of his ideas. This fact not only pushed Bank to assume a non-mediating role in his collaboration with Jaime. It also conveyed the message to Jaime that Bank was not teaching in a direction she wanted to pursue and drove her to assume trial and error role in learning to teach.

Martha and Louis were able to put their constructivist ideas into their teaching practice. However, Nick assumed that it was his responsibility to support Martha to learn this kind of teaching and exert influence whenever he saw it necessary throughout Martha's internship. Louis's collaborating teacher, Ben, maintained a non-mediating role in his collaboration with Louis. What he did for Louis was mainly to let him to do whatever he wanted. However, the freedom Louis got did not support him to retain his ideas or push him to continue his effort to construct a constructivist practice. In the end, Louis used his freedom to move back toward his collaborating teacher, Ben, at both conceptual and practical levels.

Instructional contexts and preservice teachers' learning to teach

Teacher educators (Dewey, 1964; Feiman-Nemser, 1983) emphasize preservice teachers' gradual transition into practice. It is assumed that such structure will help preservice teachers learn how to teach with the guidance of ambitious principles through a progression from intensive observation, to focused curriculum development, to practicing all the aspects of their learning in teaching.

However, such gradual transition into practice often happens in different schools and classrooms where teaching practice, curriculum resources and students are different. Then how do different instructional contexts influence preservice teachers' learning to teach in a gradual transition internship? My analysis lead me to an understanding about the relationship between the instructional contexts and the quality of preservice teachers' learning to teach.

First, to successfully support preservice teachers to learn to teach mathematics in a constructivist approach, the culture of teaching in the school needs to support a vision of constructivist teaching. It also needs to provide specific images of teaching that reflect the vision. A constructivist vision of teaching accepted by the school's teachers can reinforce preservice teachers' fragile and abstract beliefs. Specific images of teaching can help preservice teachers to build the connection between their ideas and the practice--an important stage of learning to teach.

Since both vision and practice of mathematics teaching in Mall Elementary were oriented toward a traditional approach, Kelly's constructivist ideas never had a chance to be reinforced and connected with specific images of actual teaching that reflect these ideas. Thus, her ideas stayed at the nominative level without finding their way to her practice. Martha and Jaime were working in a school where constructivist visions of mathematics teaching were encouraged. Martha was able to connect her ideas with specific images of constructivist teaching that Nick practiced. Not only was she able to retain her ideas, but also she was able to continuously push these ideas into the practical level. Jaime was unable to develop specific images of a constructivist teaching through her observation of Bank, though she and Bank had many similar views of mathematics instruction that their school had been pushing. Without substantial images as her support, Jaime's chance to develop and connect some of her important ideas to the practice was limited. The individualist culture of teaching in Bell Elementary gave Louis chances to connect his ideas with the specific images of teaching practiced by some of the teachers. It also allowed him to validate his choice to go back to a traditional direction when he faced conflict.

Second, my study also suggests that the curriculum resources available for preservice teachers' curriculum development not only need to prescribe the goals and outcomes of constructivist mathematics instruction. They also need to clearly define the content coverage. Specific curriculum resources and pedagogical suggestions that are consistently and specifically related to a constructivist vision of teaching are also important. Martha and Louis were able to develop a constructivist curriculum in their internship. To a great extent, it was because both were able to access a school curriculum that prescribed both visions and specific content coverage and pedagogical resources that were consistent with their ideas of mathematics instruction. They were also able to have someone support them during their unit and lesson plan development. The school curriculum in Kelly's situation only prescribed general outcomes for mathematics instruction without any specific pedagogical suggestions. This situation pushed her to depend exclusively on Lisa for the specific curriculum materials and resources. However, Lisa's resources, though well organized and ready to use, implied a traditional approach to mathematics instruction. Using the same curriculum guideline as Martha, Jaime was not lucky enough to have specific and consistent curriculum material and resources to rely on in developing her unit and lesson plan. The same textbook Martha used, though available to Jaime, did not specifically cover the content she needed help with. In addition, Bank was teaching in a more traditional way than Jaime expected and she decided not to look to him for help. Thus, Jaime ended up taking full responsibility for curriculum development and it was clear that this planning task was not easy enough for her to successfully accomplish alone.

Third, the student population also supported or restricted preservice teachers developing a constructivist approach of teaching. My analysis suggests that the kinds of students that preservice teachers are teaching can shape the focus of their curriculum development and influence the direction of their curriculum implementation.

Kelly's students were very young and active; many of them had not developed the learning habits the school required for the first grade mathematics learning, and some even had behavior problems. These students often distracted her from thinking about pedagogical issues and forced her to focus on control and management. Students in Well Elementary were from cultural backgrounds similar to Martha and Jaime. That gave them a better chance to understand and relate to their students. Their students were also better prepared for the constructivist approach of teaching that both of them tried to develop. Louis's students had anxiety about mathematics learning and were used to Ben's traditional approach to mathematics instruction. Even though Louis was able to develop a curriculum that reflected many of his constructivist ideas, during his curriculum implementation, students challenged his approach. Without proper guidance and support for him to resolve this conflict, Louis capitulated to the students and began teaching in a traditional way.

What are the implications?

Implications for teacher education course work

Teacher education courses have been undergoing extensive reform in the wake of the Holmes Group's work (1990) and research on teacher learning. Research suggests that teacher educators need to focus on transforming student teachers' beliefs of instruction (Kennedy, 1991), especially their conceptions of subject matter and its learning and teaching (Ball, 1989) because their prior concepts and beliefs often stand in their way of receiving new ideas and conceptions of teaching (Hollingsworth, 1989;

Kennedy, 1997; Pajares, 1992). Teacher educators have begun to put transforming their students' connections and beliefs as a major focus in their mathematics education course work (Schram, Wilcox, Lanier, & Lappan, 1988).

My study suggests that even though we are able to help our preservice teachers develop some beliefs of mathematics instruction, these beliefs themselves alone are unlikely to survive an internship where collaborating teacher and school contexts are not compatible with what preservice teachers wanted to learn. Their ideas are still subject to changes if they lack skills and abilities to identify ideas from actual practice, develop their units and lessons that reflect these ideas and resolve conflicts between these ideas in their practice.

Thus, we need to better prepare our preservice teacher for their internship by enhancing their course work. Although it is important to transform preservice teachers' beliefs in their teacher education classes, goals of preservice teachers' course work can not be limited to belief transformation only. We need to pay attention to developing their ability of identifying ideas from practice, designing curriculum under the guidance of principles and resolving value conflicts while we are transforming their beliefs.

To realize these expended goals of course work, on the one hand, we need to integrate more guided field experience into our course design. On the other hand, we need to better use virtual cases of teaching in our teacher education classes, such as samples of actual school curriculum, video-, audio and written cases of teaching as well as internet resources about schools and students. As Doyle argues (1990) these virtual cases allow teacher education students to focus more on particular situations of teaching rather than only on general principles, findings and rules. They are able to draw students into situations, problems and roles that are represented in the cases and provide them chances to think in and on these situations by role playing and simulation, though not directly connecting them with teaching. Thus, we need to carefully consider how to interpret the guided field experience and cases studies in their course work, and build course work that not only pushes student teachers to more deeply understand the principles of teaching we encourage them to develop, but also give them opportunities to develop the abilities discussed.

Implications for developing collaboration

Research (Feiman-Nemser & Remillard, 1996; Tharp & Gallimore, 1988; Vygotsky, 1978) suggest that preservice teachers can better learn to teach at levels beyond their independent performance with the assistance or coaching of the experienced teachers. Teacher education reformers (Holmes Group, 1990) encourage teacher educators to build such a relationship between preservice teachers and collaborating teachers in the field. It is assumed that such a relationship would give preservice teachers chances to develop a kind of ambitious teaching practice valued by teacher educators (Cochran-Smith, 1991). However, what can be counted as expertise or the quality of experienced teachers are often vaguely defined. Studies on the difference between experienced and novice teachers (Carter, 1990) mostly focus on their differences in their technical knowledge about teaching and efficiency and accuracy in understanding teaching situations, and, thus, imply a necessary path from novice to expertise along these lines. Few studies show the qualitative differences in thinking about goals and directions of teaching. A little attention has been paid to the qualitative difference among experienced teachers in terms of their goals and values of instruction.

My study, on the one hand, suggests that collaborating teachers have an important role in shaping preservice teachers' learning experience. On the other hand, it shows that even in the same school setting, different collaborating teachers can develop qualitatively different kinds of instructional practice and their collaborative relationship with preservice teachers can also differ significantly in nature. Thus, their influences on their preservice teachers' learning can be not only different in terms of degree but in terms of quality. These findings suggests that in designing internship for preservice teachers, we not only need to redefine the expertise of teaching and carefully choose collaborating teachers for our students with new criteria. What is more important is that we need to find ways to educate experienced teachers at the same time we are thinking about how to educate our preservice teachers. Some programs have begun to pay attention to this issue. My research point to a substantive focus for that mentor education.

To support the collaborative work of preservice and collaborating teachers in this direction, we need to enhance the program influences and support for preservice teachers' learning to teach and collaborating teachers' teaching and mentoring in at least the following ways.

First, we need to enhance the roles of field instructors or liaisons need since they have chances to keep in touch with both preservice and collaborating teachers throughout the internship. Not only do they need to play a unique role in facilitating the personal and educative relationship between the two parties. They also need to support preservice teachers' learning to teach in a particular subject area, when collaborating teachers fail to exert an influence in a constructivist direction. At the same time, they need to support collaborating teachers to transform their teaching and improve their collaboration with the preservice teacher, to put the support for preservice teachers' learning at the center of their work.

Second, we need to restructure the inflexible one-to-one relationship between preservice teachers and collaborating teachers. We need to flexibly structure preservice teacher to work with different experienced teachers in a particular school or cross different schools according to the strengths of a particular teacher's teaching. Such flexible structure will liberate preservice teachers from extensive exposure to the kind of teaching practice we do not expect them to learn and at the same time, it allows collaborating teachers to better use their strengths in helping preservice teachers learn to teach. Such a flexible restructuring is especially important in the elementary level, where teachers are often required to teach all the subjects and it is very hard to find an experienced teacher who will be able to distinguish in all the areas they teach.

Third, we also need to structure more interaction among preservice teachers. My study suggests that different preservice teachers have quite different abilities in developing their curriculum and implementing their ideas along the direction the program intends for their learning to teach. Thus, peer support can be regarded as an important resource for preservice teachers learn to think about and practice teaching. As teacher educators design internships, it is important for them to consider the mutual impact among preservice teachers, especially when in their individual learning to teach, it may be hard to benefit from his or her collaborating teacher.

Implications for developing curriculum support

Curriculum studies at both the national level (Cohen & Spillane, 1992) and international level (Schmidt et al., 1996; Schmidt, McKnight & Raizen, 1996) suggest that the U.S. school curriculum often lacks a coherent and intellectual vision, its content are fragmented, low in demand and unfocused, and its pedagogical suggestions are unspecified and inconsistent, In these situations, it is hard for us to expect that teachers will do a decent job in teaching. Researchers (Feiman-Nemser & Parker, 1990) also suggest that to be able to teach academic content, beginning teachers need to learn how to deepen their own understanding of subject matter, how to think about academic content from students' perspective, how to represent subject matter and how to organize students for the purpose of teaching and learning subject matter.

My study further suggests that curriculum resources available for preservice teachers' curriculum development not only need to prescribe the goals and outcomes of constructivist mathematics instruction. They also need to clearly define content coverage. Specific curriculum resources and pedagogical suggestions that are consistently and specifically related to a constructivist vision of teaching are also important. Unfortunately, many of our student teachers are unable to get access to such curriculum in the schools where they are interning, and their collaborating teachers are often unable to help them learn to think in the above ways. Thus, to better support our students teachers to survive their internship, it is important for the teacher education programs to not only prepare them with some ambitious beliefs, but also provide them relevant curriculum resources and materials that are consistent the program's vision of teaching.

Implications for changing school contexts for teacher education

Research (Feiman-Nemser & Floden, 1986; Lieberman & Miller, 1991) on teaching culture shows that U.S. schools are often organized in a way that reinforces the norms of individualism and non-interference among teachers. These norms hinder the chances for teachers to foster their shared conceptions about pedagogical purpose, content and approaches and develop higher professional standards for their work (Lortie, 1975; Little, 1990). Studies in teacher learning further suggest that the attention mentor and novice teachers paid to curriculum issues in their collaboration varies by school curriculum and program context (Parker et al., 1994). Different curriculum structure and requirements may have a strong influence on the focus of mentoring and expectations that cooperating or mentor teachers have for their work and their mentoring behaviors (Wang, 1997; Wang & Paine, 1994). Teacher education reformers (Holmes, 1990) have been pushing school culture change by developing professional development school. They expected that in these schools "new program and technologies can be tried out and evaluated," and "faculty of the school and of the university both experience the 'whitewater' feeling of working at the edge of their knowledge." In these places, "new teachers, just forming their knowledge and technologies, taste the reality of classrooms similar to those where they are likely to get their first job" and "they also see the skill, hear the counsel and feel the support of expert teachers" (Holmes, 1990, p.2)

This study supports this calling by further suggesting that preservice teachers' learning to teach in the internship is greatly subject to the influence of different school cultures. Such culture influences can occur by defining the value of teaching and acceptable teaching practice and by providing or limiting the chances for preservice teachers to connect their ideas with specific images of teaching. This can also happen by shaping the curriculum resources preservice teachers are going to use and the students they are going to teach. Moreover, collaborating teachers often represent the school culture.

Such a reality of preservice teachers' learning to teach implies that in order to support preservice teachers learning to teach in the constructivist direction, we need a broader vision of teacher education and teacher education reform. When we design the internship for their students, we not only need to consider carefully how to structure a better internship. We also need to think about how to help teachers and their schools reform their teaching practice. Since there are not many ideal school settings for teacher educators to choose for our students' internship, we, as teacher educators, should develop an agenda to influence schools and their teaching while we are sending our students to these places to gain their field experience.

APPENDIX

APPENDIX A

INTERVIEW PROTOCOLS

Interview 1 with Preservice Teachers

Questions about their views of elementary mathematics instruction

1. What did you expect your mathematics teaching would look like before your internship?
2. Where did these ideas come from?
3. What are the most important purposes of elementary mathematics teaching?
4. How do you think about mathematics teaching compared with other subjects?
5. What are the important things about mathematics and mathematics instruction you learned from your program?

Questions about their planning and teaching

6. What are the factors influencing your mathematics planning and teaching?
7. How did they influence your planning and teaching?
8. How did you plan this unit (lesson)?
9. Did you make any change from what you planned during your teaching of this unit?
10. I noticed that your teaching follows this pattern..., why did you teach to in this way?
11. How do you know your students learn what you want to teach them in this unit?
12. What did your collaborating teacher do in helping you teach this unit (lesson)?
13. Did your collaborating teacher talk to you about your teaching after you finished a lesson in this unit (lesson)?
14. What were the important things you learned from this unit (lesson)?

Questions about their views of collaboration with collaborating teacher

15. Do you think your TE courses helped you learn to teach mathematics in your internship or not?
16. To what extent is your mathematics teaching similar to or different from your collaborating teacher's teaching?
17. If I were going to video-tape how you work with your collaborating teacher, what would you suggest I do?
18. If you could choose your collaborating teacher, what kind of collaborating teacher would you like to work with?

Questions about the influences of instructional contexts

19. Do you think this school is a good place for you to learn to teach mathematics and why?

20. Do you think the state examination has any influence on your planning and teaching mathematics in this school?
21. What kind of students would you like to teach?
22. Do you think your students in this class influenced your planning and teaching?
23. What is the influence they have on your planning and teaching mathematics?

Interview 2 with Preservice Teachers

Questions about their mathematics education and learning

1. What kinds of training and education did you receive in terms of teaching mathematics in elementary schools?
2. Which part of your training and education do you think had a more important influence on your current mathematics teaching?

Questions about their views of good mathematics instruction

3. What are the most important qualities a teacher needs to acquire in order to teach mathematics well?
4. What are the most important and difficult things in teaching elementary mathematics?

Questions about their mathematics teaching and learning experience

5. How many mathematics units did you teach and what were they?
6. Which unit do you feel most comfortable about and which you do not?

Questions about their collaboration with collaborating teacher

7. What is your collaborating teacher's role in helping you learn to teach mathematics?
8. Did your collaborating teacher help you plan a specific mathematics unit or lesson and how?
9. Did your collaborating teacher observe you teaching and talk to you about his observation?
10. In what sense did his or her assistance help you succeed in teaching this unit?
11. What did you talk about and can you give me an example?
12. If you made a mistake in teaching, what would your collaborating teacher do and why?
13. Did you observe your collaborating teacher's mathematics teaching and how did this kind of observations help your mathematics teaching?
14. Did these observations change your ways of thinking about mathematics and mathematics teaching?

Questions about what they learned in the internship

15. What are the most important things you learned from your collaborating teacher in terms of mathematics instruction?
16. Are these things you learned consistent with your beliefs before your student teaching?
17. Can you give me a general description about what you learned in teaching mathematics in this classroom?
18. What are you going to do after your internship?

Questions about their mathematics teaching in the lead teaching unit

19. What were the most important concepts in this unit your students needed to learn?
20. What were the most difficult things in this unit?
21. Do you think your collaborating teacher shared this understanding with you?
22. Can you tell me how you planned this unit?
23. To what extent do you think you succeeded in teaching this unit according to your planning?
24. What made you fail to reach these goals?
25. If you teach this unit again, what will you do differently and why?

Questions about their relationship with collaborating teacher

26. Did you have any difficulties or dilemmas when you worked with your collaborating teacher?
27. What were the most important ways that your collaborating teacher helped you learn to teach mathematics?

Interview 1 with Collaborating Teachers

Questions about their mentoring and teaching experience

1. Why did you decide to become a collaborating teacher?
2. How many interns have you worked with so far?
3. Did you receive any training for mentoring?
4. How long have you been teaching?
5. How long have you been teaching in this school?
6. What grade level have you taught so far?

Questions about their view of elementary mathematics instruction

7. What are the purposes of elementary mathematics instruction?
8. What are the specific goals of mathematics teaching at the grade level you are teaching now?
9. Did you have this understanding when you started teaching?
10. When you plan a mathematics unit, what factors do you pay more attention to?
11. What references do you usually use in your planning?
12. Do you follow certain patterns when you teach a mathematics lesson?
13. Can you describe to me the teaching method(s) you often use in your mathematics class?

Questions about their instructional contexts

14. Can you tell me a little bit about the school's mathematics curriculum?
15. What do you think about the state mathematics exam and its influences on your mathematics teaching?
16. Can you tell me something about your students in relation to your mathematics teaching?
17. Do you think they help your mathematics teaching or not, and why?

Questions about their mentoring practice

18. What do you think the important things your preservice teacher need to learn in teaching elementary mathematics?
19. What is your role in helping your preservice teacher learn these things?
20. What do you usually do in helping your preservice teacher plan a mathematics lesson?
21. What do you usually do during his or her teaching?
22. What do you usually do after his or her teaching?

Questions about their preservice teacher's teaching

23. Is there any difference between you and your preservice teacher in thinking about mathematics instruction?
24. What is the best way for your preservice teacher to learn to teach mathematics?
25. Can you tell me what your preservice teacher did in preparing, teaching and assessing this mathematics unit (lesson)?
26. Did your preservice teacher have any problems and difficulties in teaching this unit (lesson)?
27. What can he (she) do to overcome these difficulties and problems?
28. What did you usually do if your preservice teacher had a problem in planning and teaching a mathematics lesson?

Interview 2 with Collaborating Teachers

Questions about their training in mathematics teaching

1. What training and education did you receive for teaching mathematics in elementary schools?
2. Which part of your training influences your current mathematics instruction most?

Questions about their views of good mathematics teaching

3. What does an elementary teacher need to know and be able to do in order to teach mathematics well?
4. What are the most important and difficult things in teaching elementary mathematics, and why?

Questions about their preservice teachers' learning to teach

5. How many mathematics units did your preservice teacher teach in your class?
6. Which one do you think was most important for him or her to learn to teach mathematics?
7. What are the most important concepts in this unit?
8. What are the most difficult things in this unit?
9. Do you think your preservice teacher shares these understandings with you?
10. To what extent do you think your preservice teacher succeeded in teaching this unit?
11. What made him or her succeed?
12. To what extent do you think your preservice teacher failed to reach what he or she needed to reach in this unit?

13. What made her fail to reach these goals?
14. What do you think he or she needs to do in order to overcome these problems?
15. If you are teaching this unit, what will you do differently and why?
16. Can you give me a general description about what your preservice teacher learned about mathematics teaching so far?

Questions about their views of collaboration

17. What is your role in helping him or her learn to teach?
18. In what sense did you help your preservice teacher succeed in teaching mathematics and in what sense, not?
19. Did you have any difficulties or dilemmas when you worked with your preservice teacher?

Questions about their mentoring practice

20. What did you usually do in helping your preservice teacher plan a mathematics lesson?
21. What did you usually do during his or her mathematics teaching?
22. What did you usually do after your preservice teacher taught a mathematics lesson?
23. Did your preservice teacher observe your lessons and did you talk to him or her about his or her observations?
24. What did you talk about and can you give me an example?
25. If you see a mistake in his or her teaching, what would you like to do and why?
26. If you are going to have another preservice teacher, what would you change in helping him or her learn to teach mathematics?

APPENDIX B

SCALE FOR MEASURING MATHEMATICS INSTRUCTION[14]

1. Scale for Measuring Mathematics Instructional Tasks

Instructional Tasks	Scaling for Instructional Tasks
Organizing information	**3 points**: The task calls for students interpretation of nuances of a topic that goes deeper than surface exposure or familiarity. **2 points**: The task ask students to gather information for reports that indicate some selectivity or organizing but are not asked to interpret, evaluate or synthesize. **1 point**: The task requires students only to retrieve or reproduce isolated fragments of knowledge or repeatedly apply previously learned algorithms and procedures.
Considering alternatives	**3 points**: The task should clearly involve students in considering alternatives, either through explicit presentation of the alternatives or through an activity that can not be successfully completed without examination of alternatives implicit in the work. **2 points**: The task does not clearly involve students in considering alternatives, but the teacher allowed students to use alternatives when alternatives happened. **1 point**: The task does not open any chances for students to use alternatives at all. They just work and practice repeatedly in the way the teacher professed or discussed.
Disciplinary content	**3 points**: Success in the task clearly requires understanding of concepts, ideas or theories central in mathematics. **2 points**: Success in the task seems to require understanding concepts, ideas or theories central in mathematics, but task does not make these very clear. **1 point**: Success in the task can be achieved with a very superficial (or even without any) understanding of concepts, ideas or theories central to mathematics.
Disciplinary process	**3 points**: Success in the task requires the use of methods of inquiry or discourse important to mathematics, including looking for mathematical patterns and proving mathematics

[14] These scale for measuring mathematics instruction is adapted from *A guide to authentic instruction and assessment: Vision, standards and scoring (pp. 89-100)* by Fred M. Newmann, Walter G. Secada and Gary G. Wehlage, (1995). Madison, Wisconsin: Wisconsin Center for Education Research.

1. Scale for Measuring Mathematics Instructional Tasks (cont'd)

	ideas. **2 points**: Success in the task requires the use of methods of inquiry or discourse that is not central to the conduct of disciplines. **1 point**: Success in the task can be achieved without use of any specific methods of inquiry or discourse.
Elaborated communication	**4 points**: The task requires the students to show their solution path, give logical arguments, explain thinking and justify their results. **3 points**: The task requires the students to show their solution path but not required to give any mathematics arguments, explain thinking or justify their results. **2 points**: The task requires little more than giving a result. Students may be asked to show their work, but this is not emphasized and does not require much detail. **1 point:** Multiple choices exercise, fill-in-the blank exercises (answered with less than a sentence).
Authentic problem	**3 points**: The question, problem, concept or idea clearly resembles one that students encountered, or are likely to encounter in life beyond school. The resemblance is so clear that there is no need for teacher to explain. **2 points**: The question, problem, concept or idea has some resemblance to real world experiences of the students, but the connections would be not clear unless explained by the teacher **1 point:** The question, problem, concept or idea has virtually no resemblance to real world experiences of the students or what they likely encounter in their daily life.

2. Scale for Measuring Mathematics Instructional Process

Instructional Processes	**Scaling for Instructional Processes**
High order thinking	**5 points**: Almost all the students, almost all the time. are performing higher order thinking featuring synthesizing, generalizing. explaining, hypothesizing, or arriving at conclusions. **4 points**: Students are engaged in at least one major activity during the lesson in which they perform higher order thinking. This activity occupies a substantial portion of the lesson and many students are performing high order thinking **3 points**: Students are primarily engaged in routine lower order operations. During a good share of the lesson students are asked to receive or recite factual information, or simply employ rules

2. Scale for Measuring Mathematics Instructional Process (cont'd)

	and algorithms through repetitious routines. The purpose of teaching is to transmit knowledge or practice skills. There is at least one significant question or activity in which some students
	perform some higher order thinking. **2 points**: Students are primarily engaged in lower order thinking, but at some point they perform higher order thinking as a minor activity within the lesson. **1 point**: Students are only engaged in lower order thinking and there is no activities during the lesson that force students go beyond lower order thinking.
Deep knowledge	**5 points**: During the lesson, almost all the students do at least one of the following: sustain a focus on a significant topic; or demonstrate their understanding of the problematic nature of information, or demonstrate complex understanding by arriving at a reasoned, supported conclusion or explain how they solved a complex problem. **4 points**: During the lesson, many students or the teacher do at least one of the following: sustain a focus on a significant topic for a period of time; or demonstrate their understanding of the problematic nature of information, or demonstrate complex understanding by arriving at a reasoned, supported conclusion or explain how they solved a complex problem. **3 points**: Knowledge is treated unevenly during instruction. For example, deep knowledge is countered by superficial understanding of other ideas. At least one significant idea may be presented in depth, but in general the focus is not sustained. **2 points**: The knowledge remain superficial and fragmented. While some key concepts and ideas are mentioned or covered, only a superficial acquaintance or understanding of these complex ideas is evident. **1 point**: Knowledge is thin because it does not deal with significant topics or ideas. The teacher and students are involved in coverage of simple information that they are to remember.
Substantive conversation	**5 points**: All three features of substantive conversation occur with at least one example of sustained conversation, and almost all students participate. **4 points**: All three features of substantive conversation occur with at least one example of sustained conversation, and many students participate **3 points**: Feature 2 (sharing) and / or feature 3 (coherent promotion of collective understanding) occur and involve at least one example of sustained conversation. For example, the lesson includes at least 3 consecutive interchanges.

2. Scale for Measuring Mathematics Instructional Tasks (cont'd)

	2 points: Feature 2 and / or feature 3 occur briefly and involve at least one or two consecutive interchanges. **1 point**: Virtually no features of substantive conversation occur during the lesson.
Connection to the world	**5 points**: Students study or work on a topic, a problem or an issue that the teacher and students see as connected to their personal experiences or actual contemporary public situations. They recognize the connections in ways to create personal meaning and significance and to lead students to become involved in an audience beyond their classroom. **4 points**: Students study or work on a topic, a problem or an issue that the teacher and students see as connected to their personal experiences or actual contemporary public situations. They recognize the connections in ways to create personal meaning and significance for the knowledge. However, there is no effort to use the knowledge in ways that go beyond the classroom to actually influence a larger audience. **3 points**: Students study or work on a topic, a problem or an issue that the teacher and students see as connected to their personal experiences or actual contemporary public situations. They recognize the connections, but they do not explore the implications of these connections that remain abstract or hypothetical. There is no effort to use the knowledge in ways that go beyond the classroom to actually influence a larger audience. **2 points:** Students encounter a topic, a problem or an issue that the teacher tries to connect to students' personal experiences or actual contemporary public situations. The teacher informs students that there is potential value in the knowledge being studied because it relates to the world beyond the classroom. However, the connection is unspecified and there is no evidence that students make the connection. **1 point:** The lesson and activities have no clear connection to anything beyond themselves, and the teacher offers no justification.

BIBLIOGRAPHY

BIBLIOGRAPHY

Ball, D. L. (1988). Unlearning to teach mathematics. *For the learning of mathematics, 8*(1), 40-48.

Ball, D. L. (1989). *The subject matter preparation of teachers* (Issue Paper 89-4). E. Lansing: National Center of Research on Teacher Learning, Michigan State University.

Ball, D. L. (1990). *With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics* (Craft paper 90-3). East Lansing: National Center for Research on Teacher Learning.

Ball, D. L., & McDiarmid, G. W. (1990). The subject matter preparation of teachers. In W. R. Houston (Ed.), *Handbook of research on teacher education.* (pp. 437-449.). New York: MacMillan.

Brown, J. S., Collins, A., & Duguid. (1989). Situated cognition and the culture of learning. *Education researcher, 18*(1), 32-34.

Burns, M. (1986). Teaching "what to do" in arithmetic vs. teaching "what to do and why." *Educational leadership, 43*(7), 34-38.

Carter, K. (1990). Teachers' knowledge and learning to teach. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. pp. 314-338). New York: MacMillan.

Cobb, P. (1994, October). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, 13-20.

Cochran-Smith, M. (1991). Learning to teach against the grain. *Harvard Educational Review, 16*(3), 279-311.

Cohen, D. K. (1990, Fall). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis, 12*(3), 327-345.

Cohen, D. K., & Spillane. (1992). Policy and practice: the relation between governance and instruction. In G. Grant (Ed.), *Review of Research in Education* (Vol. 18,). Washington, D.C: AERA.

Cooney, T. J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education*(16), 324-336.

Davis, P. J., & Hersh, R. (1981). *The mathematical experience*. Boston: Houghton Mifflin.

Dembele, M. (1996). *Mentors and mentoring: Frames for action, ways of acting, and consequences for novice teachers' learning*. Unpublished doctoral dissertation, Michigan State University, East Lansing, MI.

Devaney, K. (1983/1984). Math beyond computation. *Educational Leadership, 41*(4), 58-60.

Dewey, J. (1964). The relation of theory to practice in education. In R. D. Archambault (Ed.), *John Dewey on education* (pp. 313-338). Chicago: The University of Chicago Press.

Dewey, J. (1990). The child and the curriculum. In J. Dewey, *The School and Society & The Child and Curriculum* (pp. 181-209). Chicago: The University of Chicago Press.

Dolye, W. (1990). Case methods in teacher education. *Teacher Education Quarterly, 17*(1), 7-15.

Eggan, F. (1965). Some reflections on comparative method in anthropology. In M. E. Spiro (Ed.), *Context and meaning in cultural anthropology* (pp. 357-372). New York: The Free Press.

Feiman-Nemser, S. (1983). Learning to teach. In L. S. Shulman & G. Sykes (Eds.), *Handbook of teaching and policy* (pp. 150-170). New York: Longman.

Feiman-Nemser, S. (1989). *Teacher Preparation: Structural and Conceptual Alternatives* (Issue Paper 89-5). East Lansing: National Center for Research on Teacher Learning, Michigan State University.

Feiman-Nemser, S. (1995). *Mentoring in the service of reform*. Paper presented at the AERA Annual Conference, San Francisco, April 21, 1994.

Feiman-Nemser, S., & Floden, R. (1986). The culture of teaching. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching* (pp. 505-526). New York: MacMillan, Feiman-Nemser, S., & Parker, M. B. (1990). *Making subject matter part of the conversation or helping beginning teachers learn to teach* (Research Report 90-3). East Lansing: National Center for Research on Teacher Learning.

Feiman-Nemser, S., & Parker, M. B. (1992, Spring). *Mentoring in context: A comparison of two U.S. programs for beginning teachers* (NCRTL special report). East Lansing, MI: National Center for Research on Teacher Learning, Michigan State University.

Feiman-Nemser, S., & Remillard, J. (1996). Perspectives on learning to teach. In F. B. Murray (Ed.), *The Teacher Educator's Handbook* (pp. 63-91). San Francisco: Jossey-Bass Publishers.

Goodlad, J. (1984). *A place called school: Prospects for the future.* New York: McGraw Hill.

Grant, C. E. (1984). *A study of the relationship between secondary mathematics teachers' beliefs about the teaching-learning process and their observed classroom behaviors.* Unpublished doctoral dissertation, University of North Dakota.

Hargreaves, D. H. (1994). The new professionalism: The synthesis of professional and institutional development. *Teaching and Teacher Education. 10(4), pp. 423-438.*

Hargreaves, A., & Dawe, R. (1990). Paths of professional development: Contrived collegiality, collaborative culture, and the case of peer coaching. *Teaching and Teacher Education, 6*(3), 227-241.

Hollingsworth, S. (1989). Prior beliefs and cognitive change in learning to teach. *American Educational Research Journal, 26*(2), 160-189.

Holmes Group. (1986). *Tomorrow's Teachers.* East Lansing, MI: Holmes Group.

Holmes Group. (1990). *Tomorrow's Schools.* East Lansing, MI: Holmes Group.

Husen, T. (1967). *International study of achievement in mathematics; A comparison of twelve countries.* Stockholm and New York: Almqvist & Wiksell Wiley.

Kennedy, M. (1991, Spring). *An agenda for research on teacher learning* (NCRTL Special Report). East Lansing, MI: National Center for Research on Teacher Learning, Michigan State University.

Kennedy, M. M. (1997). The connection between research and practice. *Educational Researcher, 26*(7), 4-12.

Kush, T. M., & Ball, D. L. (1986). *Approaches to teaching mathematics: Mapping the domains of knowledge, skills, and dispositions.* East Lansing, MI: Michigan State University, Center on Teacher Education.

Lampert, M. (1986). Teaching multiplication. *Journal of mathematical behaviors, 5*(241-280).

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambirdge: Cambridge University Press.

Leinhardt, G. (1992, April). What research on learning tells us about teaching. *Educational Leadership*, 20-25.

Lieberman, A., & Miller, L. R. (1991). Revisiting the social realities of teaching. In A. Lieberman & L. Miller (Eds.), *Staff development for education in the 90's: New realities, new perspectives* (Second ed.). New York: Teachers College Press.

Little, J. W. (1990). The persistence of privacy: autonomy and initiative in teachers' professional relationship. *Teachers College Record, 91(4), (pp. 509-536)..*

Lortie, D. C. (1975). *Schoolteacher: A sociological study.* Chicago: The University of Chicago Press.

McGalliard, W. A. J. (1982). Selected factors in the conceptual systems of geometry teachers: Four case studies (Doctoral dissertation, University of Georgia, 1982). *Dissertation Abstract International, 44*, 1364A.

National Commission on Excellence in Education. (1983). *A nation at risk.* Washington, D.C.: The U.S. Department of Education.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics* . Reston, VA: National Council of Teachers of Mathematics.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education* . Washington, D.C.: National Academy Press.

Newmann, F. M., Secada, W. G., & Welhlage, G. G. (1995). *A guide to authentic instruction and assessment: Vision, standards and scoring.* Madison, Wisconsin: Wisconsin Center for Education Research.

Newmann, N. M., & Wehlege, G. G. (1993, April). Five standards of authentic instruction. *Educational Leadership*, 9-12.

Noddings, N. (1985). Formal modes of knowing. In E. Eisner (Ed.), *Learning, teaching the ways of knowing* (pp. 116-131). Chicago: University of Chicago Press.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Clearing up a messy construct. *Review of Educational Research, 62*(3), 307-332.

Parker, M., Dembele, M., & Murdoch, J. (1994,). *Paying attention to curriculum in mentored learning to teach: A view from the U. S.* Paper presented at the 1994 Annual Conference of AERA, New Orleans, April 16, 1994.

Peterson, P. (1988). Teaching for higher order thinking in mathematics: The challenge for the next decade. In D. A. Grouws & T. J. Cooney (Eds.), *Perspectives on research on effective mathematics teaching* (pp. 2-26). Hillsdale, NJ: Erlbaum.

Putnam, R. T., Lampert, M., & Peterson, P. (1990). Alternative perspectives on knowing mathematics in elementary schools. In B. Cazden (Ed.), *Review on research in education* (Vol. 16,). Washington, DC: AERA.

Resnick, L. (1983). Mathematics and science learning: A new conception. *Science, 220,* 477-478.

Resnick, L. B. (1987, December). Learning in school and out. *Educational Researcher*, 13-20.

Romberg, T. A. (1992). Problematic features of the school mathematics curriculum. In P. N. Sackson (Ed.), *Handbook of research on curriculum* (pp. 749-788). New York: MacMillan.

Schmidt, W., & Others. (1996). *Characterizing pedagogical flow: An investigation of mathematics and science teaching in six countries*. Dordrecht, the Netherlands: Klewer Publishing.

Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (1996). *A splintered vision: An investigation of U.S. science and mathematics education*. Dordrecht, Boston, London: Klumer Academic Publishers

Schoenfield, A. (1985). *Mathematical problem solving*. San Diego, CA: Academic Press, Inc.

Schram, P., Wilcox, S. K., Lanier, P., & Lappan, G. (1988). *Changing mathematical conceptions of preservice teachers: A content and pedagogical intervention*

(Research Report 88-4). East Lansing. MI: National Center for Research on Teacher Learning.

Sedlak, M., Wheeler, C., Pullin, D., & Cusick, P. (1986). *Selling students short: Classroom bargains and academic reform in the American high school*. New York: Teachers College Press.

Shirk, G. B. (1973). *An examination of conceptual frameworks of beginning mathematics teachers*. Unpublished doctoral dissertation. University of Illinois at Urbana-Champaign

Smith III, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education, 27*(4), 387-402.

Steinberg, R., Haymore, J., & Marks, R. (1985, April). *Teachers' knowledge and structuring content in mathematics*. Paper presented at the Annual Conference of the American Educational Research Association, San Francisco, CA.

Stevenson, D. L., & Baker, D. P. (1991, January). State control curriculum and classroom instruction. *Sociology of Education, 64*, 1-10.

Stevenson, H. W., & Stigler, J. W. (1992). *Learning gap*. New York: Summit Books.

Stodolsky, S. (1987, May). *The subject matters: Instruction in math and social studies*. Paper presented at a colloquium at the Institute for Research on Teaching, College of Education, Michigan State University, East Lansing, MI.

Tharp, G., & Gallimore, R. (1988). Assisting teacher performance through the ZED: A case study. In Tharp & Gallimore (Eds.), *Rousing minds to life: Teaching, learning, and schooling in social context*. Cambridge: Cambridge University Press.

Thompson, A. (1982). *Teachers' conceptions of mathematics*, Unpublished Doctoral Dissertation. Athens. University of Georgia

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 127-146). New York: MacMillan Publishing Company.

Travers, K. J., & Westbury, I. (Eds.). (1989). *The IEA Study of Mathematics I: Analysis of Mathematics Curricula*: Pergamon Press.

von Glasersfeld, E. (1987). Learning as constructive activity. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 3-17). Hillsdale, NJ: Lawence Erlbaum

Vyg

Wa

Wa

Vygotsky, L. S. (1978). *Mind in Society: The development of higher psychological processes* . Cambridge: Harvard University Press.

Wang, J. (1997). *Institutional contexts of teaching and mentoring practices: A comparative perspective.* Paper presented at the Annual Conference of the American Educational Researchers' Association, Chicago, March 22, 1997.

Wang, J., & Paine, L. (1994). *Context as a shaping feature of mentoring in China: Curriculum and employment policies as contexts.* Paper presented at the Annual Conference of the American Educational Researchers' Association, New Orleans, April 6, 1994.