SOURCES OF BUSINESS CYCLES IN AN ECONOMY WITH MONEY, REAL SHOCKS, AND NOMINAL RIGIDITY —A STUDY OF THE UNITED STATES: 1954 - 1991

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ABSTRACT

SOURCES OF BUSINESS CYCLES IN AN ECONOMY
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This dissertation examines the sources of postwar US economic fluctuations in a VAR framework with cointegration constraints. A theoretical macroeconomic model consisting of equations that describe the labor and goods market behavior and featuring an ex ante nominal wage contract is used to guide the empirical setup in this study. The theory prescribes five variables for the economic system, real output, real balances, real wages, nominal interest rates and inflation and postulates two cointegrating relationships, the velocity of money and the ex post real interest rate. Overidentifying restriction tests for the structural restrictions derived from the theory indicate that the steady-state structure of the system is consistent with the postwar US data but the dynamic structure is not. As a result only the permanent shocks specified by the theory, the nominal shock, the technology shock and the labor-market shock, are identified whereas transitory shocks are unidentified. The long-run effects of the permanent shocks on the five variables are constrained by their long-run multipliers derived from the theory. Their contributions to the shorter-run economic fluctuations are documented in this study and the results are consistent with several prominent studies in the literature.

Dedicated to my parents, Tsui Chih-An and Li Chiu-Chi.

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Chapter 1
INTRODUCTION

In this dissertation the sources of the post-war macroeconomic fluctuations in the United States are investigated in a vector autoregression framework with cointegration constraints. The common-trends model is employed to identify permanent innovations in the economic system before their individual contributions to business cycles can be chronicled. Five nonstationary I(1) macroeconomic aggregates with significant business-cycle characteristics are examined: the private-sector real output (y_t), real money balances (mr_t), real wages (wr_t), short-term nominal interest rates (R_t) and inflation rate (π_t). They are included based on an expanded theoretical model adapted from Blanchard and Fischer's (1989) business-cycle model. The model provides equations descriptive of labor and goods market behavior and a nominal wage contract to feature nominal rigidity in an economy. From the solutions of the theoretical model we find two cointegrating relations, the velocity of money ($y_t - mr_t$) and the ex post real interest rate ($R_t - \pi_t$). It also provides a well defined set of simultaneous relationships among the five variables which are later rejected by the post-war U.S. data. The model nevertheless has long-run information that is useful to identify the permanent shocks that constitute the common stochastic trends.

For almost two decades vector autoregression (VAR) has been a very popular tool for macroeconomic studies. Prior to its introduction, researchers typically had been criticized for estimating large systems of equations with strong over-identifying restrictions. VAR in contrast is a more explorative or descriptive approach to empirical analysis. To illustrate, for a vector of I(0) variables x_t, write its VAR form compactly as

$$A(L)\, x_t = \varepsilon_t \tag{1.1}$$

where ε_t is the error vector. Estimable parameters of the VAR contained in the lag-operator polynomial, $A(L)$, are not restricted except for $A(0) = I$, an identity matrix. Rather, an understanding about the system is gained primarily by analyzing the impulse-response functions and the forecast-error variance decomposition. The analyses are done on the structural vector-moving-average (VMA) representation of the VAR,

$$\Delta x_t = R(L)\, \nu_t. \tag{1.2}$$

Vector ν_t consists of mutually-independent structural innovations that are propagated through the system to cause fluctuations. $R(L)$ is an infinite-order polynomial matrix that contains the structural impulse-response functions. The reduced-form VMA for the first-difference of x_t can be obtained from (1.1) as

$$\Delta x_t = C(L)\, \varepsilon_t \tag{1.3}$$

where $C(L)$ is also an infinite-order polynomial matrix. The task of identifying structural impulses from reduced-form residuals amounts to finding an unique matrix F such that

$$\nu_t = F \varepsilon_t. \tag{1.4}$$

Then the impulse-response functions are available as

$$R(L) = C(L)\, F^{-1} \tag{1.5}$$

which comes from equating (1.2) and (1.3) and then substituting into (1.4).

Traditionally there existed a dichotomy in studying economic growth and business cycles (King, Plosser and Rebelo 1988a). Stochastic innovations were thought to be responsible for business cycles whereas growth was considered driven by deterministic trends. Later, the notion that many economic time-series contain stochastic growth trends, advocated by Beveridge and Nelson (1981) and Nelson and Plosser (1982), gained prominence. This suggests that stochastic shocks that form the trends may in fact also cause short-run fluctuations (King, Plosser and Rebelo 1988b). The subsequent progress in cointegration research further recognized that different non-stationary variables may share common stochastic trends. Stock and Watson (1988) formally outline a common-trends model of the form

$$x_t = A\tau_t + B(L)\varepsilon_t. \tag{1.6}$$

where x_t is $n \times 1$ and τ_t is the $k \times 1$ common stochastic trends

$$\tau_t = \tau_{t-1} + \nu_t^P. \tag{1.7}$$

A is a $n \times k$ common-trends loading matrix which brings the impact of τ_t onto x_t. Thus $A\tau_t$ represents the permanent or nonstationary component of x_t. The $n \times n$ lag-polynomial $B(L)$ is stable so that $B(L)\varepsilon_t$ is a stationary component of x_t. Equation (1.7) shows the common trends are driven by the k-dimensional structural innovations, ν_t^P, that exert permanent effects. In this dissertation, the permanent innovations are specified as a technology, a permanent nominal shock and a labor-market shock. The other r $(\equiv n - k)$ innovations in this equation system produce only transitory effects and are denoted ν_t^T such that $\nu_t' = \left(\nu_t^{P\prime}\ \nu_t^{T\prime}\right)$. It is interesting to note that in (1.7) the innovations to the permanent trends, i.e., ν_t^P, are not independent of the disturbances, ε_t, in light of $\nu_t = F\varepsilon_t$ in (1.4). Clearly, the dichotomy between growth and cycles mentioned above no longer stands.

One major contribution of the common-trends model to VAR analyses is that it provides special identification schemes that are lacking in VAR models containing only transitory shocks. The fact that only permanent shocks deliver impacts in the long run means long-run restrictions are available to identify permanent shocks. To show this, we first note that over the long run the transitory component in (1.6) effectively drops out so that the first difference of (1.6) is

$$\Delta x_t = A\nu_t^P. \tag{1.8}$$

Thus the loading matrix A also represents the long-run impacts of permanent shocks on the first-difference of the variables. Comparing (1.8) with the structural long-run VMA , $\Delta x_t = R(1)\,\nu_t$, reveals that the long-run multiplier matrix is

$$R(1) = \left[A \vdots \mathbf{0}\right] \tag{1.9}$$

Then notice the long-run version of (1.5) is

$$R(1)\,F = C(1) \tag{1.10}$$

where $C(1)$ is estimable from a reduced-from VMA. Define $F \equiv \left[F_k' \; F_{n-k}'\right]'$ and substitute (1.9) into (1.10) to get

$$AF_k = C(1)\,. \tag{1.11}$$

Here we see in (1.11) that knowledge about the common-trends loading matrix A provides very useful linear restrictions to identify the permanent shocks since $\nu_t^P = F_k\varepsilon_t$.

In this dissertation the conjecture about the form of A is provided by the mentioned theoretical model. It proposes a form of the long-run equation $\Delta x_t = A\nu_t^P$ as

$$\begin{bmatrix} \Delta y_t \\ \Delta mr_t \\ \Delta wr_t \\ \Delta R_t \\ \Delta \pi_t \end{bmatrix} = \begin{bmatrix} a & c & 0 \\ a & c & 0 \\ b & e & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_t^{\text{tech}} \\ \nu_t^{\text{labor}} \\ \nu_t^{\text{money}} \end{bmatrix}, \tag{1.12}$$

where a, b, c and e are functions of behavioral parameters in the economic model. Equation (1.12) says that the technology shock and the labor-market shock have zero long-run impact on the nominal variables, R and π. On the other hand, the nominal shock has zero long-run effect on the real variables, y, mr and wr. Thus, as is in King et al. (1991), a long-run nominal neutrality is featured in the model.

The theoretical model adapted from Blanchard and Fischer (1989) is used to guide most empirical setup in this study. The model has the advantage of including both the real-business-cycle and the Keynesian assumptions about the economy. Two permanent real shocks are treated as important sources of economic fluctuations. A forward-looking wage-setting rule serves to account for the prevalent wage rigidity phenomenon in the economy. In Chapter 3, the wage-contract model is presented and solved by the rational expectation techniques. The solutions are consistent with both the Keynesian and the RBC predictions of price and real-wage movements over business cycles. The vector error-correction model (VECM) and the VMA representation of the wage-contract model are also derived in Chapter 3. In the process we obtain the simultaneous structure of F as well as the long-run multiplier of permanent shocks, or A.

Chapter 2 provides a review of VARs with cointegration constraints and of the VECM representation. The common-trends representation and how it provides extra information for identification are covered in detail. Methods to identify permanent shocks and transitory shocks are provided. An application of the common-trends methodology is demonstrated with an example from Rasche (1992). In Chapter 4 an analysis is presented of the stationarity of the time series using unit-root tests and graphs. Then various specifications of dummy variables and lag lengths are tested to estimate an optimal VAR model. Cointegration-rank tests are also done to ensure that two conintegrating vectors can be imposed in estimation. As a result, a VECM with three lag terms and five dummy variables is estimated. Lastly in Chapter 4 the identification of the structural-form VMA with restrictions on A as stated above is discussed . Chapter 5 covers the analysis of impulse-response functions and forecast-error variance decompositions. Chapter 6 presents a summary of findings, remarks on potential contributions and shortcomings of this study and concludes the dissertation.

Chapter 2

VECTOR AUTOREGRESSION AND THE COMMON-TRENDS MODEL

2.0 Introduction

This chapter provides a review of the econometric methodology required for the empirical analysis in this dissertation. A brief discussion on VAR modeling techniques is provided in Section 2.1 with attention focused on various identification approaches. The theory of a multivariate system characterized by cointegration and the vector error-correction representation is introduced in Section 2.2. The common-trends model approach which is useful in identifying permanent economic impulses is covered in Section 2.3. A detailed review of the methods of permanent and transitory shock identification is covered in Section 2.4. The presentation of Section 2.4 closely follows the approach in Chapters 3 and 4 of Hoffman and Rasche (1996). An implementation of the identification methods is illustrated in Section 2.5 by an example from Rasche (1992).

2.1 Vector Autoregression Model

2.1.1 Structural VAR and Reduced-Form VAR

Vector autoregression (VAR), first advocated by Sims (1980), has become one of the most widely applied time-series techniques by macroeconomists. The VAR approach is in spirit compatible to Frisch's (1933) view that macroeconomic time series are the result of the interaction of stochastic economic impulses and an implicit propagation mechanism in the economy. With VARs, economists can identify the role of individual disturbance in generating the business cycles and discern their dynamic effects on the economy.

To illustrate the strategy of VARs, let y_t be a $n \times 1$ vector of I(1) variables that has a finite p order autoregressive representation

$$A(L)\,y_t = \mu + \varepsilon_t. \tag{2.1}$$

where $A(L) \equiv I - A_1L - A_2L^2 - \cdots - A_pL^p$ is a matrix-polynomial in the lag operator. The usual assumption about $A(L)$ is that all roots of the polynomial equation $|A(L)| = 0$ lie outside the unit circle in the complex domain. The mean of y_t is denoted μ and the error term ε_t is assumed independently and identically normally distributed.

$$\begin{aligned}
&\varepsilon_t \sim \text{iid } N(0, \Sigma) \\
&E(\varepsilon_t) = 0, \quad E(\varepsilon_t\varepsilon_t') = \Sigma \\
&E(\varepsilon_t\varepsilon_s') = 0, \quad t \neq s.
\end{aligned}$$

Equation (2.1) is actually estimated with data and is a reduced-form model. Our ultimate interest is to uncover the structural relationships in the economy that determine the dynamics of the variables.

The structural-form VAR is as

$$B(L)\,y_t = \theta + \nu_t \tag{2.2}$$

or

$$B_0y_t = \theta + B^*(L)\,y_{t-1} + \nu_t \tag{2.3}$$

where θ is the vector of means, $B(L) \equiv B_0 - B_1L - B_2L^2 - \cdots - B_pL^p$ is a $n \times n$ polynomial in the lag operator, and $B^*(L)$ is defined by the equation

$$B^*(L) \equiv B_1 + B_2L + \cdots + B_pL^{p-1}.$$

Contained in $B(L)$ are the structural economic relations that represent the propagation mechanism mentioned above. The disturbance term ν_t represents the exogenous impulses that shock the economy and has the distribution assumption,

$$\nu_t \sim \text{iid } N(0, D)$$

$$E(\nu_t) = 0, \quad E(\nu_t \nu_t') = D, \text{ where } D \text{ is diagonal}$$

$$E(\nu_t \nu_s') = 0, \quad t \neq s.$$

The zero-covariance assumption for ν_t, implied by a diagonal D, is essential to isolating the individual influence of innovations on the variables.

2.1.2 Moving-Average Representation

If the stability condition of the polynomial matrix $A(L)$ is satisfied, i.e., the polynomial equation $|A(L)| = 0$ has all roots outside unit circle, then $A(L)$ has a inverse as

$$A(L)^{-1} = C(L) \equiv \sum_{j=0}^{\infty} C_j L^j$$

where $C(L)$ is an infinite-order matrix-polynomial. Then y_t has a vector moving-average (VMA) representation or Wold representation,

$$y_t = \delta + C(L)\,\varepsilon_t, \tag{2.4}$$

where $\delta = A^{-1}(1)\,\mu$. Here C_j for j from 0 to ∞ are the impulse-response matrix because each of the matrix elements measures the impact on y_t over different time-horizons of a unit change in the error term ε_t,

$$C_{ij\,s} = \frac{\partial y_{i\,t+s}}{\partial \varepsilon_{j\,t}} \qquad s = 0, 1, \cdots, \infty$$

where $C_{ij\,s}$ is the (i,j)th element of C_s.

However, we are interested in the impulse responses of the structural innovations. Unlike an unit change in ε_t that does not have intuitive meanings, the effect of one unit or one standard-deviation economic shock is interpretable. Similar to the analysis above, if the stability condition of $B(L)$ is satisfied, its inverse exists as

$$B^{-1}(L) = R(L) \equiv \sum_{j=0}^{\infty} R_j L^j \tag{2.5}$$

where $R(L)$ is an infinite-order polynomial matrix. The set of matrices R_j in (2.5) is the impulse-response function and can be defined by

$$R_{ij\,s} = \frac{\partial y_{i\,t+s}}{\partial \nu_{j\,t}} \quad s = 0, 1, \cdots, \infty.$$

The vector moving-average representation of the structural-form VAR is then

$$y_t = \delta + R(L)\,\nu_t. \tag{2.6}$$

2.1.3 Structural VAR Identification

To find out the relation between the reduced-form VAR and the structural-form VAR, premultiply (2.3) by B_0^{-1} to obtain its reduced form as

$$y_t = B_0^{-1}\theta + B_0^{-1}B^*(L)\,y_{t-1} + B_0^{-1}\nu_t. \tag{2.7}$$

By comparing (2.1) and (2.7), we see their parameters can be related by

$$B(L) = B_0 A(L) \tag{2.8}$$

$$\nu_t = B_0 \varepsilon_t \tag{2.9}$$

$$D = B_0 \Sigma B_0'. \tag{2.10}$$

Thus, the issue of identifying the structural-form from the reduced-form VAR amounts to locating the unique matrix B_0 (to be referred to as the identification matrix) so that the left-hand-side in equations (2.8) to (2.10) are obtained.

Here we experience the same identification problem as is encountered in the simultaneous-equation model[1]. To exactly identify the structural form as that in

[1]To show this, we pre-multiply (2.1) through by any $n \times n$ nonsingular matrix H to get

$$HA(L)\,y_t = H\mu + H\varepsilon_t.$$

equation (2.3), we have to limit the choice of the identification matrix to B_0. This initially requires n^2 restrictions to solve for the n^2 elements in B_0. Normalizing the diagonal elements of B_0 to unity reduces the required restrictions to $n(n-1)$. Equation (2.10) also provides $\frac{n(n-1)}{2}$ zero restrictions because D is diagonal. Thus this standard VAR model will require $\frac{n(n-1)}{2}$ additional restrictions on B_0 based on theory or sound economic intuition in order to achieve exact identification.

The first generation of VAR practitioners often applied Cholesky decomposition of the reduced-form covariance matrix Σ to achieve identification. That is, the upper off-diagonal elements of B_0, in (2.10), are assumed zero. This method provides the additional $\frac{n(n-1)}{2}$ restrictions required for exact identification. This practice, however, implies a particular recursive ordering in the contemporaneous relations of the variables. This set of identification restrictions was originally proposed by Wold (1954). The choice in most cases cannot be justified on theoretical grounds and therefore is often arbitrary. Improved identification schemes were devised later based on specific structural assumptions consistent with economic analysis. Bernanke (1986) and Sims (1986) provide two examples in which the contemporaneous relations of variables are set up this way.

To illustrate this later approach, in equations (2.5) and (2.6) we set L to zero to get the contemporaneous relations between the reduced-form and the structural-form VAR as

$$C(0)\,\varepsilon_t = R(0)\,\nu_t \tag{2.11}$$

or

$$\varepsilon_t = R(0)\,\nu_t \tag{2.12}$$

since $C(0) = A(0)^{-1} = I_n$ by the form of $A(L)$ in (2.1)[2]. As long as economic

Then it is easy to verify the above equation has exactly the same reduced form as that in (2.7).

[2]By comparing (2.12) to (2.9), we note that $B_0 = R(0)^{-1}$.

reasonings provide at least $\frac{n(n-1)}{2}$ such contemporaneous restrictions in equation (2.12), we can achieve just- or over-identification of the structural VAR model.

With increasing emphasis on the nonstationary nature of many economic time-series, it is natural to add long-run restrictions for identification that are unavailable to VARs with purely stationary variables. One long-run restriction implied by nominal neutrality is frequently used for identification (King et al. 1991). It requires that a permanent inflation shock has zero long-run effect on real variables. Another example is Blanchard and Quah (1989) in which the demand shock has no long-run impact on output. In each case, the assumption provides one zero-restriction on the cumulative multiplier matrix

$$R(1) = \sum_{j=0}^{\infty} R_j$$

which measures the long-run impacts of various shocks occurring in the distant past. This technique, first credited to Blanchard and Quah (1989), is later used by King et al. (1991) and Gali (1992). Compared to the practice of using Cholesky decomposition or even the contemporaneous restrictions as in (2.11), this technique is often more justified by economic theories about the long-run effects.

2.2 Cointegration and the Vector Error-Correction Model

In Section 2.1, we discussed the standard VAR methodology where y_t is assumed to be multivariate covariance-stationary. In this section we will deal with the change in the formulation of VARs when y_t represents a vector of nonstationary variables. More specifically, elements of y_t here are integrated of order one, or I(1). We first provide a formal definition on the order of integration adapted from Engle and Granger (1986).

Definition 1 *A series with no deterministic component which has a stationary, invertible, ARMA representation after differencing d times, is said to be integrated* *of order d, denoted* $x_t \sim I(d)$*.*

Among economic time-series that are of the same $I(1)$ order, there can be certain linear combinations of the series that are $I(0)$. In this case, the variables are said to be cointegrated. The following definition of cointegration is also from Engle and Granger (1986).

Definition 2 *The components of the vector* x_t *are said to be cointegrated of order* d, b*, denoted* $x_t \sim CI(d, b)$*, if all components of* x_t *are* $I(d)$ *and there exists a non-zero vector* β *so that* $\beta' x_t \sim I(d-b)$*,* $b > 0$*. The vector* β *is called the cointegrating vector.*

For the case where $d = b = 1$, cointegration means if the components of y_t are all $I(1)$, then $\beta' y_t \sim I(0)$ is stationary. The relation $\beta' y_t$ is often interpreted as an economic equilibrium relationship that is true only in the long run. When $\beta' y_t \neq 0$, it is interpreted as a deviation from the long-run equilibrium or an equilibrium error. The equilibrium error is stationary and reverts to its mean of zero over time. In that case the equilibrium relationship among the economic variables is restored. When the dimension of y_t is greater than two, there may be multiple independent cointegrating vectors since it is reasonable for the joint behavior of the variables to be governed by several equilibrium relations. Gather all the r (> 1) existing cointegrating vectors to form a $n \times r$ matrix β. The rank of β, i.e., r, is called the cointegrating rank of y_t.

We now note a standard VAR(p) model in the level of y_t,

$$A(L)\, y_t = \mu + \varepsilon_t,$$

as in equation (2.1) can be written as a VAR($p-1$) model in the first-difference of y_t plus a lag level term. It is called the vector error-correction model (VECM)

$$\Gamma(L)\,\Delta y_t = -\Pi y_{t-1} + \mu + \varepsilon_t \tag{2.13}$$

or

$$\Delta y_t = \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \cdots + \Gamma_{p-1} \Delta y_{t-p+1} - \Pi\, y_{t-1} + \mu + \varepsilon_t \tag{2.14}$$

where

$$\begin{aligned} \Pi &\equiv A(1) = I - A_1 - A_2 - \cdots - A_p \\ \Gamma(L) &= I - \Gamma_1 L - \Gamma_2 L^2 - \cdots - \Gamma_{p-1} L^{p-1} \\ \Gamma_i &= -\textstyle\sum_{j=i+1}^{p} A_j; \quad (i = 1, \cdots, p-1). \end{aligned} \tag{2.15}$$

It is clear that in a VECM the response to the long-run equilibrium error $(-\Pi y_{t-1})$ is expressed separately from terms that represent the short-run movements. This distinction is an important part of what has come to be known as the Engle-Granger two-step procedure (Engle and Granger 1986) for VECM estimation. This representation is also used by Johansen (1988) to develop his maximum-likelihood procedure for estimating the cointegrating rank and the cointegrating space.

We note that the rank of Π should be equal to the number of unit roots in the polynomial equation $|A(\lambda)| = 0$ since $\Pi = A(1)$[3]. The following discussion about the rank of Π is broken down to three cases concerning the times-series nature of y_t:

A. If y_t is a vector of stationary series, because all roots of $|A(\lambda)| = 0$ are outside the unit circle, Π is of full rank n.

B. If all elements of y_t are $I(1)$ and no cointegration exists, then y_t does not have a VECM representation but a pure VAR$(p-1)$ in the first-difference of y_t.

[3]This is by Corollary 4.3 in Johansen (1995).

This amounts to that $\Pi = 0$ and the rank of Π is equal to zero[4]. Indeed for the original VAR in levels in (2.1), the polynomial equation $|A(\lambda)| = 0$ has n unit roots and therefore $A(1) = \Pi$ is a zero-rank matrix.

C. When all the n elements of y_t are $I(1)$ and r cointegrating relations exist among the individual variables, the rank of the n-dimensional Π is reduced by r to equal $n - r \equiv k$. There are k unit roots for the polynomial equation $|A(\lambda)| = 0$ and $n - k$ roots outside the unit circle if the equilibrium relationships are stable.

For case *C*, let β be the $n \times r$ matrix consisting of the r cointegrating vectors then there exists an $n \times r$ matrix α such that $\Pi = \alpha\beta'$ is the coefficient matrix of y_{t-1} in the VECM representation in (2.14). The error-correction matrix α is often regarded as a speed of adjustment coefficient. It determines how much change there will be in the y_t in (2.14) in proportion to the size of the equilibrium error, $\beta' y_t$, in each period. The size of the total adjustment is equal to $\alpha\beta' y_t$ every period[5]. These results are formally established in the influential Granger Representation Theorem (Engle and Granger 1986) or GRT,

Theorem (GRT) Suppose the $n \times 1$ $I(1)$ vector y_t can be expressed as (2.1). Then the model can be written in VECM form as (2.14). Assume the $n \times n$ matrix Π has reduced rank $r < n$ and therefore can be expressed as the product of two full-column-rank $n \times r$ matrices α and β, i.e., $\Pi = \alpha\beta'$. Furthermore, let the $n \times k$ matrices $\alpha_\perp$ and $\beta_\perp$ be the orthogonal complements of α and β so that $\alpha'_\perp \alpha = 0$ and $\beta'_\perp \beta = 0$. Then:

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[4]This is because no linear combination of y_t (including Πy_{t-1}) is stationary. Therefore, Πy_{t-1} should not appear on the right-hand-side of (2.14) because Δy_t and all its lag terms on the RHS of (2.14) are $I(0)$ processes.

[5]But since VECM is a reduced-form represenatation it is more appropriate to treat α as loading matrix than the speed of adjustment matrix from a structural point of view.

(1) Δy_t and $\beta' y_t$ are stationary

(2) Δy_t has a moving average representation $\Delta y_t = C(L)(\mu + \varepsilon_t)$

(3) $C(1) = \beta_\perp (\alpha'_\perp \Gamma \beta_\perp)^{-1} \alpha'_\perp$ has rank k[6]

(4) y_t has the representation $y_t = y_0 + C(1)\left(\mu + \sum_{s=1}^{t} \varepsilon_s\right) + C^*(L)\varepsilon_t$ where $C^*(L)$ is defined by $C(L) = C(1) + C^*(L)(1-L)$[7].

There are two versions of proof for GRT. The original proof by Engle and Granger (1986) deals mainly with a reduced-rank VMA representation of vector y_t. In contrast, Johansen (1991) works on a VAR representation and expresses the theorem in terms of conditions on parameters for cointegration. The theorem presented above is adapted from Johansen's version because in this dissertation a VAR (VECM) is fitted to the data. Regardless of the approach, the theorem establishes that a cointegrated system of variables can be represented in three equivalent forms: a vector autoregression with cointegration constraints, a vector error-correction and a reduced-rank vector moving-average representation.

2.3 The Common-Trends Model

A univariate time series that contains a unit root in its autoregressive representation is said to be driven by one stochastic trend. Let x_t be such a process expressed as

$$(1-L)A(L)\,x_t = \mu + \varepsilon_t \tag{2.16}$$

where μ is the mean of x_t and ε_t is a white noise disturbance term. For ease of discussion, define $z_t \equiv A(L)\,x_t$ and then (2.16) becomes

$$z_t = z_{t-1} + \mu + \varepsilon_t. \tag{2.17}$$

[6] Γ is defined by $\Gamma \equiv \Gamma(1)$ and $\Gamma(L)$ is given in equation (2.15). It is also immediately evident that $\beta' C(1) = C(1)\alpha = 0$.

[7] Result (4) will be discussed in greater details in Section 2.3

Successive substitution of the above results in

$$z_t = z_0 + \mu t + \sum_{s=1}^{t} \varepsilon_s. \tag{2.18}$$

Aside from z_0, z_t is characterized by cumulative trends, μt being the deterministic trend and $\sum_{s=1}^{t} \varepsilon_s$ the stochastic trend. In fact, the disturbance term ε_t that adds up to the stochastic trend could itself be a linear combination of several random shocks. In that case the unit-root process z_t is actually driven not by one but by several underlying stochastic trends.

Based on this concept, we now discuss the concept of common stochastic-trends in the context of a VAR for the n-dimensional y_t. Given all elements of y_t are I(1), the n variables together contain a maximum of n distinct stochastic trends that can derive from the n-dimensional structural innovations, ν_t. The disturbance terms, ε_t, are linear combinations of the structural innovations. It is possible that the n stochastic trends that affect each element of y_t are not independent. If y_t is driven by a reduced number of independent trends then certain elements of y_t must share some common trends. Stock and Watson (1988) formalize the idea by asserting that a common-trends model (CTM) exists for a cointegrated system of nonstationary variables. Specifically, for a n-dimensional vector $I(1)$ time series with r distinct cointegrating relations, the first difference of the variables can be characterized as driven by $n - r$ common stochastic trends[8]. Because GRT also guarantees the equivalence between cointegration and VECM, it follows that a CTM can be derived for a VECM system and vice versa. Thus the VECM estimation results computed from the maximum likelihood procedure of Johansen (1988, 1991) are useful in solving for the common-trends representation.

In presenting the common-trends framework, Stock and Watson (1988) direct

[8]Conversely, if there exists $k\,(= n - r)$ common trends for the n-dimensional vector, then r independent linear combinations of the $I(1)$ variables can be found such that they are stationary.

their analysis on the vector moving-average representation which can be obtained by inverting the corresponding VAR model with cointegration constraints. The inversion method is different from the usual method for VARs without cointegration considerations and is discussed in Warne (1990). Write the reduced-from VMA as

$$\Delta y_t = \delta + C(L)\,\varepsilon_t \tag{2.19}$$

where $\delta = C(1)\,\mu$. Recursive substitution of (2.19) results in

$$\begin{aligned} y_t &= y_0 + \delta t + C(L)\left(1 + L + L^2 + \cdots + L^t\right)\varepsilon_t && (2.20)\\ &= y_0 + C(1)(\mu t + \sum_{s=1}^{t}\varepsilon_s) + C^*(L)\,\varepsilon_t && (2.21)\end{aligned}$$

Noting in the last step use is made of the relation

$$C(L) = C(1) + C^*(L)(1 - L)\,. \tag{2.22}$$

where

$$\begin{aligned} C^*(L) &= C_0^* L + C_1^* L + C_2^* L^2 + \cdots \\ C_j^* &= -\sum_{k=j+1}^{\infty} C_k \text{ for } j = 0, 1, 2, \cdots. \end{aligned}$$

Now (2.21) can be written as

$$y_t = y_0 + C(1)\,\varphi_t + C^*(L)\,\varepsilon_t \tag{2.23}$$

where φ_t is a random walk with drift process

$$\begin{aligned} \varphi_t &\equiv \varphi_{t-1} + \mu + \varepsilon_t && (2.24)\\ &= \varphi_0 + \mu t + \sum_{s=1}^{t}\epsilon_s. && (2.25)\end{aligned}$$

Recall from Granger's Representation Theorem in 2.2 that with r cointegrating vectors the rank of $C(1)$ is $n - r$ and $\beta' C(1) = 0$. Since $C(1)$ has rank $n - r$, there exists a $n \times r$ full-column-rank matrix B_r such that $C(1)\,B_r = 0$. Furthermore, a full-column-rank $n \times k$ matrix can also be found, denoted B_k, with its columns orthogonal to the columns of B_r. Define $A \equiv C(1)\,B_k$ which has rank k. Create the nonsingular $n \times n$ matrix $B = \left(B_k \vdots B_r\right)$. Then

$$C(1)\,B = \left(A \vdots 0\right) = AS_k$$

where $S_k \equiv \left(I_k \vdots 0\right)$ is a $k\times\, n$ selection matrix. Now (2.23) can be rewritten as

$$\begin{aligned} y_t &= y_0 + C(1)\,BB^{-1}\varphi_t + C^*(L)\,\varepsilon_t \\ &= y_0 + A\,S_k\,B^{-1}\varphi_t + C^*(L)\,\varepsilon_t \\ &= y_0 + A\tau_t + C^*(L)\,\varepsilon_t. \end{aligned} \tag{2.26}$$

The k-dimensional common stochastic-trends τ_t follows the random walk with drift process[9]

$$\tau_t = \gamma + \tau_{t-1} + \nu_t^P. \tag{2.27}$$

We see that $n \times 1$ y_t is driven by a reduced number (k) of stochastic trends which, in turn, arise from the innovations ν_t^P. Equation (2.26) is the common-trends representation proposed by Stock and Watson (1988). They use this representation to develop test methods regarding the number of common trends or, equivalently, the number of cointegrating vectors in a system of nonstationary variables. Note the common-trends representation can be regarded as a multivariate extension of the Beveridge and Nelson (1981) decomposition of a univariate time series into a permanent component and a transitory component. The second term on the RHS

[9]Define $B^{-1} \equiv \begin{pmatrix} B_k^* \\ B_r^* \end{pmatrix}$ where B_k^* is $k \times n$ and B_r^* is $r \times n$. Then (2.27) is derived as

$$\begin{aligned} \tau_t &= S_k B^{-1}\varphi_t = B_k^*\varphi_t \\ &= B_k^*\left(\mu + \varphi_{t-1} + \varepsilon_t\right) \\ &\equiv \gamma + \tau_{t-1} + u_t. \end{aligned}$$

of (2.26), $A\tau_t$, is the nonstationary or permanent component of y_t expressed by a linear combination of $n-r$ stochastic trends. The third term, $C^*(L)\,\epsilon_t$, represents the non-integrated transitory component.

The CTM specified in (2.26) can be called the structural-form CTM. The reason is that it involves the part of the permanent-shocks ν_t^P from the vector structural-innovations ν_t. The other part of ν_t, denoted ν_t^T, is called the transitory shocks because its impacts die out with time. Identifying the structural-form CTM from the reduced-from CTM in (2.23) is similar in spirit to the problem of the structural VAR from the reduced-form VAR. Comparing (2.23) to (2.26) provides the relation

$$A\tau_t = C(1)\,\varphi_t \tag{2.28}$$

or furthermore, by (2.24) and (2.27), the three equations

$$A\gamma = C(1)\,\mu \tag{2.29}$$

$$A\nu_t^P = C(1)\,\varepsilon_t \tag{2.30}$$

and

$$AA' = C(1)\,\Sigma C(1)'. \tag{2.31}$$

Note (2.31) holds because we assume $E(\nu_t\nu_t') = I_n$[10].

To identify A which has $n\cdot k$ elements, as many restrictions are required. First of all, there is one set of restrictions that can be derived from knowledge about cointegration. Specifically, cointegration requires $\beta' A = 0$ which supplies $k\cdot r$ zero restrictions. The result $\beta' A = 0$ obtains because it is required for the cointegration

[10]The covariance of structural innovations equal to I_n means the diagonal elements of B_0 in the structural-form VAR in 2.1,

$$B_0 y_t = \theta + B^*(L)\,y_{t-1} + \nu_t,$$

are not normalized to 1. Therefore to identify B_0 we now need n^2 restrictions instead of $n(n-1)$.

of y_t in (2.26). That is, for the equilibrium error,

$$\beta' y_t = \beta' y_0 + \beta' A \tau_t + \beta' C^*(1)\varepsilon_t, \tag{2.32}$$

to be stationary, the trend term, $\beta' A \tau_t$, cannot appear in the RHS of (2.26). Secondly, equation (2.30) provides another $\frac{k(k+1)}{2}$ restrictions. $\frac{n(n+1)}{2}$ restrictions are not supplied because the $n \times n$ symmetric matrix AA' has rank k, the same as that of $C(1)$. With the two sets of restrictions, there are still $\frac{k(k-1)}{2}$ restrictions lacking to exactly identify A. These extra restrictions could be specified for A based on knowledge about the long-run effects of the structural innovations ν_t. One example mentioned in Section 2.1.3 is the nominal neutrality condition which requires the long-run impact of nominal shocks on real variables to be zero. How this knowledge about the total impacts of ν_t (specifically ν_t^P) helps identify will be clarified in the following discussions.

2.4 Permanent and Transitory Shock Identification

In Section 2.3, we consider issues of identifying the common-trends loading matrix A. This sections covers the issue of identifying the entire cointegrated VAR system. We learn from Section 2.3 that the $n \times 1$ structural innovation vector ν_t is composed of k permanent shocks, ν_t^P, and $n-k$ transitory shocks ν_t^T so that $\nu_t = \left[\nu_t^{P\prime}\ \nu_t^{T\prime}\right]'$. The task is to uncover the vector ν_t from the reduced-form VAR residuals, ε_t. This is operationally achieved by finding a n-dimensional square matrix F such that $\nu_t = F\varepsilon_t$. Partition F so that $F = [F_k'\ F_r']'$ and F_k is $k \times n$ and F_r is $r \times n$. Then ν_t^P and ν_t^T are individually identified as

$$\begin{aligned} \nu_t^P &= F_k \varepsilon_t \\ \nu_t^T &= F_r \varepsilon_t. \end{aligned}$$

We will use a two-step procedure by which an initial candidate of F is first found before obtaining the final F. The concept of the orthogonal compliment of the cointegration space is essential for the identification.

The Granger Representation Theorem in Section 2.2 provides the reduced-form total impact matrix as

$$C(1) = \beta_{\perp} (\alpha'_{\perp} \Gamma \beta_{\perp})^{-1} \alpha'_{\perp} \tag{2.33}$$

where $a_{\perp}$ and $\beta_{\perp}$ are orthogonal compliments of α and β respectively so that $\beta'_{\perp}\beta = \alpha'_{\perp}\alpha = 0$ and $\Gamma = I - \sum_{i=1}^{p-1}\Gamma_i$ is from (2.15). The values of $C(1)$, Γ_i α and β can be obtained by estimating a VECM. It is then possible to specify the permanent shocks ν_t^P and the loading matrix A as

$$\begin{aligned} \nu_t^P &= \alpha'_{\perp}\varepsilon_t & (2.34) \\ A &= \beta_{\perp} (\alpha'_{\perp} \Gamma \beta_{\perp})^{-1}. & (2.35) \end{aligned}$$

The specification of (2.34) and (2.35) is based on $A\nu_t^P = C(1)\varepsilon_t$ in (2.30). Equation (2.34) and (2.35) implicitly defines

$$F_k \equiv \alpha'_{\perp} = (A'A)^{-1} A'C(1). \tag{2.36}$$

The uniquely derived $\alpha_{\perp} = F_k$ is what defines the stochastic trends. Now, by the structural-form long-run VMA of the first-differenced variables

$$\Delta y_t = R(1)\nu_t = A\nu_t^P, \tag{2.37}$$

and by recalling $\nu_t \equiv \begin{bmatrix} \nu_t^P \\ \nu_t^T \end{bmatrix}$, we arrive at a special condition for the long-run multiplier of cointegrated VARs as

$$R(1) = \left[A \vdots 0\right]. \tag{2.38}$$

The interpretation of (2.37) and (2.38) is that in the long-run only permanent shocks impact on the economy while effects of transitory impulses die out. A is called the common-trends loading matrix because it transmits the long-run effect of ν_t^P onto variables in the system.

From now on, for notational simplicity and without loss of generality, we will write the loading matrix in (2.35), $\beta_\perp (\alpha'_\perp \Gamma \beta_\perp)^{-1}$, as $\beta_\perp$ so now $A = \beta_\perp$. Then (2.33) is written as $C(1) = \beta_\perp \alpha'_\perp$ and permanent shocks are still

$$\begin{aligned} \nu_t^P &= (\beta'_\perp \beta_\perp)^{-1} \beta'_\perp C(1) \varepsilon_t \\ &= \alpha'_\perp \varepsilon_t \end{aligned} \tag{2.39}$$

(given $C(1)$ known, knowing either $\alpha_\perp$ or $\beta_\perp$ automatically yields the other). The identification problem arises here because even with $C(1)$ known and both α and β given, there are still infinitely many choices of $\alpha_\perp$ and $\beta_\perp$[11]. Restrictions have to be imposed on $\alpha_\perp$ and/or $\beta_\perp$ to accomplish identification. Usually researchers have better ideas as to the structure of cointegration relations contained in β.

Therefore, as in the case of identifying A in Section 2.3, we make use of the orthogonal condition $\beta' \beta_\perp = 0$. We have to come up first with an initial candidate for $\beta_\perp$, denoted $\beta_\perp$, such that $\beta' \beta_\perp^0 = 0$. This imposes $r \cdot k$ zero restrictions on $\beta_\perp^0$ toward identifying $\beta_\perp$. The tentatively identified permanent innovations are thus

$$\alpha_\perp^{0\prime} \varepsilon_t = \left(\beta_\perp^{0\prime} \beta_\perp^0\right)^{-1} \beta_\perp^{0\prime} C(1) \varepsilon_t. \tag{2.40}$$

Next, it is necessary to use the conventional assumption of independence among structural innovations. Inspect the covariance matrix of the tentative permanent innovations,

$$\Sigma_P = \alpha_\perp^{0\prime} \Sigma \alpha_\perp^0, \tag{2.41}$$

which is restricted to be a diagonal matrix. But this will rarely be the case when it is based on the initial $\alpha_\perp^0$ and $\beta_\perp^0$. To ensure a diagonal covariance matrix we find

[11]This can be easily shown by obtaining $\alpha_\perp^{*\prime} = H\alpha'_\perp$ and $\beta_\perp^* = \beta_\perp H^{-1}$ and verify $\alpha_\perp^{*\prime} \sum_{s=1}^t \varepsilon_s$ is also a common trends that satisfies

$$C(1) = \beta_\perp \alpha'_\perp = \beta_\perp^* \alpha_\perp^{*\prime}.$$

the lower triangular Cholesky decomposition of Σ_P, denoted π, such that

$$\Sigma_P = \pi D_P \pi'. \tag{2.42}$$

Then we officially identify $\alpha_\perp$ and $\beta_\perp$ by

$$\begin{aligned} \alpha_\perp &= \alpha_\perp^0 (\pi')^{-1} \\ \beta_\perp &= \beta_\perp^0 \pi. \end{aligned}$$

Note that $\beta_\perp^0 \alpha_\perp^{0\prime} = \beta_\perp \alpha_\perp' = C(1)$ is satisfied and thus $\alpha_\perp$ and $\beta_\perp$ are related by

$$\alpha_\perp' = (\beta_\perp' \beta_\perp)^{-1} \beta_\perp' C(1).$$

Now it is straightforward to show that the covariance matrix of the new permanent innovations, $\nu_t^P = \alpha_\perp' \varepsilon_t$, is diagonal as

$$\begin{aligned} E\left(\nu_t^P \nu_t^{P\prime}\right) &= \alpha_\perp' \Sigma \alpha_\perp \\ &= \pi^{-1} \alpha_\perp^{0\prime} \Sigma \alpha_\perp^0 (\pi')^{-1} \\ &= \pi^{-1} \Sigma_P (\pi')^{-1} \\ &= D_P \end{aligned}$$

where the last step is based on (2.42). Thus we have uniquely determined $F_k = \alpha_\perp'$ and identify the permanent innovations according to

$$\nu_t^P = F_k \varepsilon_t = \left(\pi^{-1} \alpha_\perp^{0\prime}\right) \varepsilon_t. \tag{2.43}$$

Whether conditions for identifying $\alpha_\perp$ and $\beta_\perp$ are met can be checked by counting the number of available restrictions. Since $\alpha_\perp$ and $\beta_\perp$ are $n \times k$ matrices, $n \cdot k$ restrictions are needed to exactly identify either. By using Cholesky decomposition for the symmetric $\alpha_\perp^{0\prime} \Sigma \alpha_\perp^0$, we get $\frac{k(k+1)}{2}$ zero restrictions useful in identifying $\beta_\perp$ and $\alpha_\perp$. The initial candidate, $\beta_\perp^0$, exhibits $r \cdot k$ restrictions. Thus, additional $\frac{k(k-1)}{2}$ (=

$nk - \frac{k(k+1)}{2} - rk)$ identifying restrictions are required on $\beta_\perp^0$ (or $\beta_\perp$). This is usually provided by the long-run neutrality condition that restricts certain stochastic trends not to affect y_t in the long run. This sets the corresponding elements of the common-trends loading matrix, $\beta_\perp$, equal to zero.

The discussion on how to identify F_r follows the work of Warne (1990) which is very similar to the strategy used to identify F_k. The assumption of independence among structural innovations is still critical for the task. First, independence between the permanent and transitory shocks requires

$$Cov\left(\nu_t^P \nu_t^T\right) = \alpha_\perp' \Sigma F_r' = 0. \tag{2.44}$$

Warne suggests specifying an initial candidate of F_r as

$$F_r^0 = \alpha^{0\prime}\Sigma^{-1} \tag{2.45}$$

where α^0 is any space spanned by columns of α so that $\alpha_\perp'\alpha^0 = 0$ and therefore the independence assumption holds as stated in (2.44)[12]. Second, independence among the elements of transitory shocks also requires the covariance matrix of ν_t^T,

$$\Sigma_T \equiv \alpha^{0\prime}\Sigma^{-1}\Sigma\Sigma^{-1}\alpha^0 = \alpha^{0\prime}\Sigma^{-1}\alpha^0, \tag{2.46}$$

to be diagonal. Similar to discussions on identifying F_k, this requires getting the lower-triangular Cholesky decomposition, denoted Q, such that

$$\Sigma_T = QD_TQ' \tag{2.47}$$

Then we can improve on the initial candidate by getting

$$\begin{aligned} F_r &= Q^{-1}F_r^0 \\ &= Q^{-1}\left(\alpha^0\right)'\Sigma^{-1} \\ &= \alpha^{*\prime}\Sigma^{-1} \end{aligned} \tag{2.48}$$

[12] About the choice of α^0, Englund et al. (1994) recommends an $r \times n$ selection matrix U, chosen so that $\alpha^0 = \alpha(U\alpha)^{-1}$ with $U\alpha$ a nonsingular matrix.

where $\alpha^* \equiv \alpha^0(Q')^{-1}$. By (2.48) we know the newly identified transitory shocks,

$$\nu_t^T = \alpha^{*\prime}\Sigma^{-1}\varepsilon_t, \tag{2.49}$$

satisfy the zero-covariance requirement of (2.44). It is also easy to verify that the covariance of ν_t^T is the diagonal D_T in (2.47). Now stacking the results derived from above for F_k and F_r, we have the unique matrix F to identify the structural shocks as

$$\begin{aligned} \nu_t &= F\varepsilon_t \\ &= \begin{bmatrix} \left(\beta_\perp'\beta_\perp\right)^{-1}\beta_\perp' C(1) \\ Q^{-1}\left(\alpha^0\right)'\Sigma^{-1} \end{bmatrix} \varepsilon_t. \end{aligned}$$

Still, we need to count the number of available restrictions to determine whether exact identification is indeed achieved. There are $n \cdot r$ elements in F_r. Zero covariance between permanent and transitory shocks entails $k \cdot r$ zero restrictions on F_r. The assumption of zero covariance for transitory shocks or, more precisely, getting the Cholesky decomposition of the symmetric Σ_T in (2.47), provides additional $\frac{r(r+1)}{2}$ restrictions on F_r. To complete the identification, $\frac{r(r-1)}{2}$ $\left(= rn - \frac{r(r+1)}{2} - rk\right)$ more restrictions need to be furnished. These extra restrictions are typically cast in a way to produce zero impact effects for certain transitory shocks on variables in the system. Englund et al. (1994) discuss how zero contemporaneous effect restrictions can be imposed by choosing the selection matrix U considered above. However, I choose a different approach (Rasche 1992) that will be presented in Section 2.5 with a numeric example. This approach, without using U matrix, also imposes zero contemporaneous effect restrictions to identify transitory innovations.

2.5 Example of a Common-Trends Model

The implementation of the identification procedure can be illustrated by a simple example of term structure of interest rate and money demand study by Rasche (1992). Four variables were studied with two cointegrating relations present among the variables, real balances $(m-p)$, real output (y), long-term interest rate (R_L) and short-term interest rate (R_S). Denote $x_t' = [m-p\ R_L\ y\ R_S]$ and the 2×4 cointegrating matrix is estimated as

$$\beta = \begin{bmatrix} 1 & 0 & -1 & \lambda \\ 0 & 1 & 0 & -1 \end{bmatrix} \tag{2.50}$$

where λ is estimated to be 0.124. The first row of β defines a typical money demand equation with unit coefficient for real output. The second equation describes a stationary term structure of interest rate relation.

To identify F such that $F\varepsilon_t = \nu_t$, an initial candidate for F, denoted F^0, is first selected before eventually achieving identification according to $F = H^{-1}F^0$ where H^{-1} is the Cholesky decomposition of $F^0 E\left(\varepsilon_t\varepsilon_t'\right) F^{0\prime}$. There are 16 $(= 4^2)$ elements in F so as many restrictions have to be specified. The Cholesky decomposition provides ten and six remain to be devised.

The 4×4 reduced-from long-run multiplier $C(1)$ is obtained by applying result *(3)* of the GRT and has the form of

$$C(1) = \begin{bmatrix} -\lambda c_1 + c_2 \\ c_1 \\ c_2 \\ c_1 \end{bmatrix} \tag{2.50}$$

where c_1 and c_2 are estimated to be

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -17.734 & .080 & 5.558 & -.103 \\ .827 & .026 & .668 & -.019 \end{bmatrix}.$$

It is obvious that $C(1)$ has a reduced rank equal to 2. Notice the requirement $\beta' C(1) = 0$ is satisfied. Next the initial candidate of F_k is set as

$$F_k^0 = \alpha_\perp^{0\prime} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{2.51}$$

and $\beta_{\perp}^{0}$ is obtained, by the conditions $\beta_{\perp}^{0}\alpha_{\perp}^{0\prime} = C(1)$, as

$$\beta_{\perp}^{0} = \begin{bmatrix} -\lambda & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{2.52}$$

With the form of $\beta_{\perp}^{0\prime}$ chosen as above, the first permanent shock can be interpreted as a nominal shock that permanently impacts on both the long and the short rate (second and fourth variables)[13]. The second permanent shock is interpreted as a real shock that impacts permanently upon real output (third variable) but not nominal interest rates. Notice the condition $\beta'\beta_{\perp}^{0} = 0$ is satisfied and this provides four identifying restrictions. There are two restrictions yet to be specified, one shall be a zero long-run impact restriction to identify F_k and the other a zero first-period-impact restriction to identify F_r.

The initial candidate for F_r is chosen to be

$$F_r^0 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{2.53}$$

This specification defines the first transitory shock to have unit first-period impact on the fourth variable, the short rate, while the second transitory shock to have unit first-period impact on the second variable, the long rate. The particular form of F_r^0 chosen above, as will be shown shortly, is essential to imposing the zero contemporaneous-effect restriction for transitory shock identification. Now stack F_k^0 and F_r^0 to form

$$F^0 = \begin{bmatrix} -17.734 & .080 & 5.558 & -.103 \\ .827 & .026 & .668 & -.019 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \end{bmatrix}. \tag{2.54}$$

[13]Recall that, in the long run, $\Delta x_t = \beta_{\perp}\nu_t^p$.

Then obtain the lower-triangular Cholesky decomposition of $F^0 E\left(\varepsilon_t \varepsilon_t'\right) F^{0\prime}$ as

$$H = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0085 & 1.0 & 0.0 & 0.0 \\ -1.9296 & -19.429 & 1.0 & 0.0 \\ 0.0782 & 9.4637 & 0.6039 & 1.0 \end{bmatrix}. \tag{2.55}$$

Then F is identified by

$$\begin{aligned} F &= H^{-1}F^0 \\ &= \begin{bmatrix} -17.154 & 0.0950 & 5.92 & -.1138 \\ 1.0426 & 0.0270 & 0.6613 & -0.0193 \\ -12.844 & 0.7080 & 24.272 & .4050 \\ -0.7691 & 0.3095 & -21.379 & -0.0528 \end{bmatrix}. \end{aligned}$$

Now we show how the zero impact-period-effect restriction is embodied in the form of F_r^0 in (2.53) in order to exactly identify the two transitory shocks. First calculate the inverse of F_0 as

$$F^{0-1} = \begin{bmatrix} -0.0406 & 0.3380 & 0.0022 & -0.0055 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0503 & 1.0785 & 0.0257 & -0.0321 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}.$$

The arithmetic of inverting a matrix ensures that as long as each row of F_r^0 has only one element equal to 1 and all other elements equal to 0 as in (2.54), then it will turn out that one row in $\left(F^0\right)^{-1}$ will have its fourth element as 1 while the other elements all equal to 0. It will also turn out that a second row in $\left(F^0\right)^{-1}$ will have its third element equal to 1 while the rest are zero. To inspect the impact-period effects we need to calculate the impact matrix $R(0)$, using (2.12) in Section 2.1,

$$\begin{aligned} R(0) &= F^{-1} = \left(F^0\right)^{-1} H \\ &= \begin{bmatrix} -0.0425 & 0.2421 & -0.00111 & -0.00554 \\ 0.0782 & 9.4637 & 0.6039 & 1.0 \\ 0.00743 & 0.1763 & 0.00631 & -0.0321 \\ -1.9296 & -19.4294 & 1.0 & 0.0 \end{bmatrix}. \end{aligned}$$

With the particular form of $(F^0)^{-1}$ along with the fact that H is lower triangular, the product of $(F^0)^{-1}$ and H is guaranteed to have a zero element in its fourth column. Thus our particular form of F_τ^0 carries the assumption that the second transitory shock has zero contemporaneous effect on the fourth variable, the short rate.

As for the long-run neutrality restriction, it is assumed that the first permanent shock, the nominal shock, should not have any long-run impact on the third variable, real output. This requirement imposes a zero restriction on $R(1)$. Specifically, the restriction is imposed on the common-trends loading matrix $\beta_\perp$ since by (2.38), $R(1) = \begin{bmatrix} \beta_\perp & 0 \end{bmatrix}$. Recall in Section 2.4 that $\beta_\perp$ is related to the initial choice of $\beta_\perp^0$ by

$$\begin{aligned} \beta_\perp &= \beta_\perp^0 \pi \\ &= \begin{bmatrix} -\lambda & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \pi_{21} & 1 \end{bmatrix} \\ &= \begin{bmatrix} -\lambda + \pi_{21} & 1 \\ 1 & 0 \\ \pi_{21} & 1 \\ 1 & 0 \end{bmatrix}. \end{aligned}$$

Then for the neutrality condition to hold, π_{21} has to be zero [14]. This supplies one overidentifying restriction toward recovery of the structural model.

[14] It will be shown in Chapter 4 that π is the same as B_{11} which is a partition of the 4×4 lower-triangular matrix $B = \begin{bmatrix} B_{11} & 0 \\ B_{21} & B_{22} \end{bmatrix}$ where every partition is a 2×2 matrix. More discussion on this is in Section 4.5 where my identification issues are addressed. Also, a zero restriction on B_{11} essentially requires a restricted Cholesky decomposition to be estimated. A RATS SVAR procedure is useful for such purposes.

Chapter 3

AN ECONOMIC MODEL CHARACTERIZED BY COINTEGRATION

3.0 Introduction

A simple economic model with a wage contract is presented in Section 3.1. The rational expectation solution of the model is obtained in Section 3.2 and cyclical properties of the model are provided. A real interest rate identity is added to the economic model in Section 3.3 and a resulting econometric system of five equations is obtained. In Section 3.4, a structural-form vector error-correction representation of the system is derived, with two cointegration relations present, M2 velocity and ex post real interest rate. The structural-form moving-average representation is derived in Section 3.5 and the long-run multiplier of structural innovations is used to guide identification in Chapter 4. The presentation of this chapter closely follows that of Chapter 5 of Hoffman and Rasche (1996).

3.1 An Economy With Wage Contract

From Chapter 1 we know it is desirable to use identification restrictions that can be derived from economic theory to identify a common-trend model. For this purpose we will utilize a model of macroeconomic fluctuations adapted from Blanchard and Fischer (1989, p.518). This model contains a wage contract that provides the model economy with a nominal rigidity.

$$
\begin{aligned}
y_t^d &= m_t - p_t + v_t, && && (3.1)\\
y_t^s &= \beta(p_t - w_t + u_{1t}), && \beta > 0, && (3.2)\\
n_t^d &= \gamma(p_t - w_t + \alpha u_{1t}) + u_{3t}, && \gamma > 0,\ 0 \le \alpha \le 1, && (3.3)\\
n_t^s &= \delta(w_t - p_t), && \delta \ge 0, && (3.4)\\
w_t \mid E_{t-1} n_t^d &= E_{t-1} n_t^s, && n_t = n_t^d. && (3.5)
\end{aligned}
$$

The variables y_t, n_t, w_t, and p_t are, respectively, the logarithms of aggregate output, employment, the nominal wage, and the price level and u_{1t} and v_t are supply and demand shocks. More specifically, since the aggregate supply equation (3.2) is a variant of the Cobb-Douglas production function, u_{1t} is considered the technology or productivity shock. The aggregate demand equation (3.1) is in the form of a velocity equation so the demand shock, v_t, alternatively has the interpretation of velocity shock.

Labor demand is affected by the labor demand shock u_{3t} in addition to shocks to labor productivity, i.e., u_{1t}. Included in u_{3t} could be exogenous job creations and eliminations such as that due to input price shocks, increasing market integration and specialization, demographic changes, and shocks that affect inventory and capacity utilization. It is critical to recognize that only the portions of these forces that do not directly affect aggregate supply in (3.2) are included in u_{3t}. They influence the aggregate supply but only through their effects on the labor market. In (3.4) labor supply is assumed positively related to the real wage. The cause of nominal rigidity is revealed in (3.5) where wage contracts are set one-period in advance and are intended to equate the next period expected labor demand and supply. The actual employment is assumed to be demand-determined, so $n_t = n_t^d$. Thus when unforeseen shocks take place in the upcoming period only firms can adjust the level of employment and the pre-set wage cannot be changed. Alternatively, u_{3t} can be specified as a labor supply shock so u_{3t} is put in (3.4) instead of (3.3). This is similar to Shapiro and Watson's (1988) specification of a random walk with drift labor-supply process. Nothing other than a few mathematical signs will be affected by this change in the model. For this reason, we can label u_{3t} the labor-market shock rather than a labor demand shock.

3.2 Solution of the Rational Expectation Model

We can solve the above equation system with Rational Expectation techniques for the solution of p, w, and y. To save space, we leave the details of the solution process in Appendix A. Introduce the notation $\widehat{x}_t \equiv E_{t-1}x_t$ for any variable x_t so that $\widehat{x}_t$ is the rational expectation of x_t made at time $t-1$ subject to all information available then. The solutions for w_t, p_t and y_t are all expressed as combinations of the expectation terms, $\widehat{x}_t$, and expectation error terms, $x_t - \widehat{x}_t$, as follow:

$$\begin{aligned} p_t &= \widehat{m}_t + \widehat{v}_t + \beta(a-1)\widehat{u}_{1t} + \frac{\beta}{\delta+\gamma}\widehat{u}_{3t} \\ &\quad + \frac{1}{\beta+1}\left[(m_t - \widehat{m}_t) + (v_t - \widehat{v}_t)\right] - \frac{\beta}{\beta+1}(u_{1t} - \widehat{u}_{1t}). \end{aligned} \tag{3.6}$$

$$w_t = \widehat{m}_t + \widehat{v}_t + \left[\beta(a-1) + a\right]\widehat{u}_{1t} + \frac{\beta+1}{\delta+\gamma}\widehat{u}_{3t}. \tag{3.7}$$

$$\begin{aligned} y_t &= \beta(1-a)\widehat{u}_{1t} - \frac{\beta}{\delta+\gamma}\widehat{u}_{3t} \\ &\quad + \frac{\beta}{\beta+1}\left[(m_t - \widehat{m}_t) + (v_t - \widehat{v}_t) + (u_{1t} - \widehat{u}_{1t})\right]. \end{aligned} \tag{3.8}$$

This model has properties resembling those of Benassy's (1995) model that allows for both the traditional Keynesian and the Real Business Cycle interpretations of price and real wage movements over business cycles. To see this, first calculate the real wage according to $wr_t \equiv w_t - p_t$ to get

$$\begin{aligned} wr_t &= a\widehat{u}_{1t} + \frac{1}{\delta+\gamma}\widehat{u}_{3t} \\ &\quad - \frac{1}{\beta+1}\left[(m_t - \widehat{m}_t) + (v_t - \widehat{v}_t)\right] + \frac{\beta}{\beta+1}(u_{1t} - \widehat{u}_{1t}). \end{aligned} \tag{3.9}$$

As can be seen from (3.6), (3.8) and (3.9), unexpected technology disturbances cause price p and output y levels to be negatively related and real wages $w-p$ and output y to be positively related. This results are typical of the RBC prescription of price and real wage behaviors over the business cycle (Els 1995). On the other hand, monetary surprises produce positive correlations between p and y and negative relations between $w-p$ and y. This is in line with traditional Keynesian views of procyclical prices and counter-cyclical real wages due to nominal rigidity (Hairault and Portier 1993). The observed cyclical patterns of p and $w-p$ in a given time period are likely to depend on the relative strength of monetary and technology shocks in that period.

3.3 Complete Model Specification

Now we specify both the technology shock u_{1t} and the labor demand shock u_{3t} are random walk with drift processes,

$$u_{it} = \tau_i + u_{it-1} + \epsilon_{it} \quad i = 1, 3,$$

and consequently

$$\left.\begin{aligned} \widehat{u}_{it} &= \tau_i + u_{it-1} \\ u_{it} - \widehat{u}_{it} &= \epsilon_{it} \end{aligned}\right\} \quad i = 1, 3.$$

As for the money supply process, we use the specification of Hoffman and Rasche (1996) and have

$$\begin{aligned} m_t &= m_{t-1} + u_{2t} \\ u_{2t} &= \tau_2 + u_{2t-1} + \epsilon_{2t} \end{aligned} \tag{3.10}$$

It then follows that

$$\widehat{m}_t = m_{t-1} + u_{2t} - \epsilon_t$$

$$m_t - \widehat{m}_t = \epsilon_{2t}.$$

The aggregate demand shock in (3.1) is also considered a stationary process with a moving-average representation, $v_t = \theta(L)\epsilon_{5t}$, where $\theta(L) = \theta_0 + \theta_1 L^1 + \theta_2 L^2 + \cdots + \theta_p L^p$. Innovations ϵ_{1t}, $\epsilon_{2t}, \epsilon_{3t}$ and ϵ_{5t} are all assumed white noise processes.

With all the stochastic terms fully specified, (3.8) can be expressed as

$$\begin{aligned} u_{5t} \;\equiv\; & y_t = \beta(1-a)\left(\tau_1 + u_{1t-1}\right) - \frac{\beta}{\delta+\gamma}\left(\tau_3 + u_{3t-1}\right) \\ & + \frac{\beta}{\beta+1}\left[\epsilon_{1t} + \epsilon_{2t} + \theta_0 \epsilon_{5t}\right], \end{aligned} \tag{3.11}$$

Here we define $u_{5t} \equiv y_t$ to be used below where u_{5t} is seen as a new composite variable dominated by two random walk with drift processes. The solved wage-contract model can now be summarized by four equations, (3.11), (3.1), (3.2) and (3.10), involving four variables, y_t, m_t, p_t and w_t.

To make this theoretical model complete with more nominal variables represented, we use the real interest rate identity

$$rr_t \equiv R_t - \widehat{\pi}_{t+1} \equiv \phi\,(L)\,\epsilon_{6t} \tag{3.12}$$

where R_t is the nominal interest rate and π_{t+1} is the inflation rate defined by $\pi_{t+1} = p_{t+1} - p_t$. The real interest rate, rr_t, has been found to be a stationary long-run relationship by Mishkin (1992) and Crowder and Hoffman (1996). Here I assume it is governed by a stable moving-average process $\phi\,(L) = \phi_0 + \phi_1 L^1 + \phi_2 L^2 + \cdots + \phi_q L^q$ and ϵ_{6t} is white noise.

The real interest rate identity provides one additional equation but two extra variables to the model. We can get around this problem by eliminating p_t in (3.1) and (3.2) and replace m_t and w_t respectively by the real balance, $mr_t \equiv m_t - p_t$, and the real wage, $wr_t \equiv w_t - p_t$. Now the first equation in (3.10) can be manipulated to be

$$mr_t = mr_{t-1} - \pi_t + u_{2t}. \tag{3.13}$$

by using $mr_t \equiv m_t - p_t$ and $\pi_t \equiv p_t - p_{t-1}$. The last problem lies in the next-period inflation rate in (3.12) whose ex post form, π_{t+1}, is not available for estimation along with y_t, mr_t, wr_t and R_t at time t . The solution is to use (3.12) to acquire a new equation for the stationary process, $R_t - \pi_t - mr_t + mr_{t-1}$, which includes the current inflation, π_t. We show this equation in (3.14) below and leave the derivation details in Appendix B. We note the equation can be given the economic meaning that the ex post real interest rate, defined as $R_t - \pi_t$, is about equal to, in the long run, the growth in real balances $mr_t - mr_{t-1}$.

Collecting (3.1), (3.2), (3.11), (3.13) and (3.14), we have a five equation system

$$\begin{aligned} y_t &\equiv u_{5t} && (3.11) \\ y_t + \beta\, wr_t &= \beta u_{1t} && (3.2) \\ mr_t + \pi_t &= mr_{t-1} + u_{2t} && (3.13) \\ y_t - mr_t &= \theta(L)\,\epsilon_{5t} && (3.1) \\ R_t - \pi_t - mr_t + mr_{t-1} &= \tau_2 - \beta(1-a)\,\Delta u_{1t} + \frac{\beta}{\delta+\gamma}\Delta u_{3t} \\ &\quad + \frac{\beta}{\beta+1}(\epsilon_{1t}+\epsilon_{2t}) + \psi(L)\,\epsilon_{5t} + \phi(L)\,\epsilon_{6t}. && (3.14) \end{aligned}$$

Defining $d \equiv \beta(1-a)$, $g \equiv \frac{\beta}{\beta+1}$ and $h \equiv \frac{\beta}{\delta+\gamma}$ for (3.14), then the five-equation economic system above can be presented in matrix form as

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} y_t \\ mr_t \\ wr_t \\ R_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ mr_{t-1} \\ wr_{t-1} \\ R_{t-1} \\ \pi_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 1 \\ \beta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{5t} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -d & 0 & h & 0 \end{bmatrix} \begin{bmatrix} \Delta u_{1t} \\ \Delta u_{2t} \\ \Delta u_{3t} \\ \Delta u_{5t} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta(L) & 0 \\ g & g & 0 & \psi(L) & \phi(L) \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{5t} \\ \epsilon_{6t} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \tau_2 \end{bmatrix}. \qquad (3.15)$$

The coefficient matrix on the LHS of (3.15) contains the contemporaneous structure of the five $I(1)$ variables in the system. This information may be applied to identify a 5×5 F matrix such that $F\Sigma F' = D$ where D, a diagonal matrix, and Σ are respectively the covariance matrix of structural innovations and reduced-form errors. Exact-identification for this model requires 10 parameters in F and 5 in D to be estimated. For ease of analysis, premultiply the coefficient matrix, denoted F, by a transformation matrix W so the resulting matrix has unit diagonals as in

$$
\begin{aligned}
W \cdot F &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & \frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ \frac{1}{\beta} & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}.
\end{aligned}
$$

This matrix appears to have extra zero and unit restrictions in a Wold causal chain structure, with the exception that one nonzero element exists in the upper triangular. There is only one free parameter to estimate and thus there are 9 over-identifying restrictions. Sets of reduced number of overidentifying restrictions are available such as that provided by the common-trends loading matrix $\beta_{\perp}$. They are discussed in the following sections.

3.4 The Vector Error-Correction Representation

We first formulate a separate equation system below to solve for the VECM later. This system is constructed for u_{it} for $i = 1, 2, 3, 5$ based on specification assumptions in Section 3.2 as

$$\begin{bmatrix} 1-L & 0 & 0 & 0 \\ 0 & 1-L & 0 & 0 \\ 0 & 0 & 1-L & 0 \\ -dL & 0 & hL & 1 \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{5t} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ g & g & 0 & g\,\theta_0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{5t} \\ \epsilon_{6t} \end{bmatrix} + \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ d\tau_1 - h\tau_3 \end{bmatrix} \tag{3.16}$$

Now the inverse of the polynomial matrix on the LHS of (3.16) is

$$(1-L)^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ dL & 0 & -hL & (1-L) \end{bmatrix}. \tag{3.17}$$

By premultiplying through (3.16) by the square matrix in (3.17), we get

$$\begin{bmatrix} \Delta u_{1t} \\ \Delta u_{2t} \\ \Delta u_{3t} \\ \Delta u_{5t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ dL + g\,(1-L) & g\,(1-L) & -hL & \theta_0\, g\,(1-L) & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{5t} \\ \epsilon_{6t} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ d & 0 & -h \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}. \tag{3.18}$$

To derive the VECM, add one-period-lagged (3.11) (3.2) and (3.13) respectively to the RHS of (3.15) to obtain

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} y_t \\ mr_t \\ wr_t \\ R_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \beta & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ mr_{t-1} \\ wr_{t-1} \\ R_{t-1} \\ \pi_{t-1} \end{bmatrix}$$

$$+\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} y_{t-2} \\ mr_{t-2} \\ w_{t-2} \\ R_{t-2} \\ \pi_{t-2} \end{bmatrix}+\begin{bmatrix} 0 & 0 & 0 & 1 \\ \beta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -d & 0 & h & 0 \end{bmatrix}\begin{bmatrix} \Delta u_{1t} \\ \Delta u_{2t} \\ \Delta u_{3t} \\ \Delta u_{5t} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \tau_2 \end{bmatrix}+\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta(L) & 0 \\ g & g & 0 & \psi(L) & \phi(L) \end{bmatrix}\begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{5t} \\ \epsilon_{6t} \end{bmatrix}. \tag{3.19}$$

Subtract $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} y_{t-1} \\ mr_{t-1} \\ wr_{t-1} \\ R_{t-1} \\ \pi_{t-1} \end{bmatrix}$ from both sides of (3.19) to get

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} \Delta y_t \\ \Delta mr_t \\ \Delta wr_t \\ \Delta R_t \\ \Delta \pi_t \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} \Delta y_{t-1} \\ \Delta mr_{t-1} \\ \Delta wr_{t-1} \\ \Delta R_{t-1} \\ \Delta \pi_{t-1} \end{bmatrix}$$

$$+\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} y_{t-2} \\ mr_{t-2} \\ wr_{t-2} \\ R_{t-2} \\ \pi_{t-2} \end{bmatrix}+\begin{bmatrix} 0 & 0 & 0 & 1 \\ \beta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -d & 0 & h & 0 \end{bmatrix}\begin{bmatrix} \Delta u_{1t} \\ \Delta u_{2t} \\ \Delta u_{3t} \\ \Delta u_{5t} \end{bmatrix}$$

$$+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \tau_2 \end{bmatrix}+\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta(L) & 0 \\ g & g & 0 & \psi(L) & \phi(L) \end{bmatrix}\begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{5t} \\ \epsilon_{6t} \end{bmatrix}. \tag{3.20}$$

Then substitute (3.18) into (3.20) to obtain the structural VECM:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} \Delta y_t \\ \Delta mr_t \\ \Delta wr_t \\ \Delta R_t \\ \Delta \pi_t \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} \Delta y_{t-1} \\ \Delta mr_{t-1} \\ \Delta wr_{t-1} \\ \Delta R_{t-1} \\ \Delta \pi_{t-1} \end{bmatrix}$$

$$+\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_{t-2} \\ mr_{t-2} \\ wr_{t-2} \\ R_{t-2} \\ \pi_{t-2} \end{bmatrix} + \begin{bmatrix} d & 0 & -h \\ \beta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ -d & 1 & h \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$

$$+\begin{bmatrix} dL + g(1-L) & g(1-L) & -hL & \theta_0 g(1-L) & 0 \\ \beta & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta(L) & 0 \\ g-d & g & h & \psi(L) & \phi(L) \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{5t} \\ \epsilon_{6t} \end{bmatrix} \tag{3.21}$$

To obtain the reduced-form VECM, first take inverse of the coefficient matrix on the LHS of (3.20) as

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ -\frac{1}{\beta} & \frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

Now premultiply it through (3.20) and rearrange the error-correction term to get the reduced-form VECM:

$$\begin{bmatrix} \Delta y_t \\ \Delta mr_t \\ \Delta wr_t \\ \Delta R_t \\ \Delta \pi_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ -1 & 2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \Delta mr_{t-1} \\ \Delta wr_{t-1} \\ \Delta R_{t-1} \\ \Delta \pi_{t-1} \end{bmatrix}$$

$$+\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} y_{t-2} \\ mr_{t-2} \\ wr_{t-2} \\ R_{t-2} \\ \pi_{t-2} \end{bmatrix}$$

$$+\begin{bmatrix} dL + g(1-L) & g(1-L) & -hL & \theta_0 g(1-L) & 0 \\ dL + g(1-L) & g(1-L) & -hL & \theta_0 g(1-L) - \theta(L) & 0 \\ -\frac{-\beta + dL + g(1-L)}{\beta} & -\frac{1}{\beta} g(1-L) & \frac{1}{\beta} hL & -\frac{1}{\beta}\theta_0 g(1-L) & 0 \\ g-d & g+1 & h & \psi(L) & \phi(L) \\ -dL - g(1-L) & 1 - g(1-L) & hL & -\theta_0 g(1-L) + \theta(L) & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{5t} \\ \epsilon_{6t} \end{bmatrix} + \begin{bmatrix} d & 0 & -h \\ d & 0 & -h \\ -\frac{-\beta+d}{\beta} & 0 & \frac{1}{\beta}h \\ -d & 2 & h \\ -d & 1 & h \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}. \tag{3.22}$$

In the above derivations, a structural vector error-correction representation is obtained from a fully specified economic model characterized by two cointegration relations. With the time-series nature of the stochastic shocks specified as in Section 2.3, the existing long-run equilibrium relations between the variables are already revealed in (3.1) and (3.12). The first cointegration relation is the velocity of money, $y_t - mr_t$, and the second one is the ex post real interest rate, $R_t - \pi_t$. The knowledge about their exact forms will be useful in terms of improving statistical efficiency when we estimate a reduced-from VECM model in Chapter 4. In other words, the theory superimposes 'known' cointegrating vectors so no parameters need to be estimated in a VECM. We notice that even though no speed of adjustment parameters are specified in the theoretical model, a VECM representation is derived that specifies 'known' adjustment parameters in α.

Now we show that the top 3×5 partition of the structural matrix F on the LHS of (3.21) is a common-trends matrix $\alpha'_{\perp}$. The structural error-correction coefficient in (3.21) can be written as $F\alpha$ where α is the 5×2 reduced-from adjustment matrix. Define $F = \begin{bmatrix} F_k \\ F_r \end{bmatrix}$ where F_k and F_r are 3×5 and 2×5 respectively. Then

$$\begin{aligned} F\alpha &= \begin{bmatrix} F_k\alpha \\ F_r\alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

implies $F_k\alpha = 0$. Thus F_k is an orthogonal compliment of α and may be treated as a common-trends matrix $\alpha'_\perp$. If we only impose identification restrictions included in F_k and allow F_r to be freely estimated, the number of overidentifying restrictions is reduced for the model. Such an approach also amounts to ignoring all the coefficient restrictions in (3.1) and (3.14) which compose the dynamic structure of the economic model. This is a meaningful approach only if the entire structure of F is rejected by an overidentifying restriction test. Only in this case would we ask the question whether the long-run structure of the model alone can be successfully estimated by the data.

3.5 The Vector Moving-Average Representation

To construct a VMA for the 5-dimensional vector variables, we first write the structural VAR model in (3.19) in lag operator as

$$\begin{bmatrix} 1-L & 0 & 0 & 0 & 0 \\ 1-L & 0 & \beta(1-L) & 0 & 0 \\ 0 & (1-L)^2 & 0 & 0 & (1-L) \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -(1-L) & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} y_t \\ mr_t \\ wr_t \\ R_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \tau_2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 1 \\ \beta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -d & 0 & h & 0 \end{bmatrix} \begin{bmatrix} \Delta u_{1t} \\ \Delta u_{2t} \\ \Delta u_{3t} \\ \Delta u_{5t} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta(L) & 0 \\ g & g & 0 & \psi(L) & \phi(L) \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{5t} \\ \epsilon_{6t} \end{bmatrix}. \qquad (3.23)$$

Substitute (3.18) into (3.23) to get

$$\begin{bmatrix} 1-L & 0 & 0 & 0 & 0 \\ 1-L & 0 & \beta(1-L) & 0 & 0 \\ 0 & (1-L)^2 & 0 & 0 & (1-L) \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -(1-L) & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} y_t \\ mr_t \\ wr_t \\ R_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} d & 0 & -h \\ \beta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ -d & 1 & h \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$

$$+ \begin{bmatrix} dL + g(1-L) & g(1-L) & -hL & \theta_0 g(1-L) & 0 \\ \beta & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta(L) & 0 \\ g-d & g & h & \psi(L) & \phi(L) \end{bmatrix} \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{5t} \\ \epsilon_{6t} \end{bmatrix}. \qquad (3.24)$$

Now construct the inverse of the polynomial matrix on the LHS of (3.24),

$$(1-L)^{-1}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -(1-L) & 0 \\ -\frac{1}{\beta} & \frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & (1-L) \\ -(1-L) & 0 & 1 & (1-L)^2 & 0 \end{bmatrix}. \tag{3.25}$$

Premultiply through (3.24) by the matrix in (3.25), without $(1-L)^{-1}$, to obtain the VMA model $\Delta x_t = \delta + R(L)\nu_t$. After collecting terms we derive the VMA representation of the economic model as

$$\begin{bmatrix} \Delta y_t \\ \Delta mr_t \\ \Delta wr_t \\ \Delta r_t \\ \Delta \pi_t \end{bmatrix} = \begin{bmatrix} dL + g(1-L) \\ dL + g(1-L) \\ 1 - \frac{1}{\beta}(dL + g(1-L)) \\ (g-d)(1-L) \\ -dL(1-L) - d(1-L)^2 \end{bmatrix}\epsilon_{1t} + \begin{bmatrix} g(1-L) \\ g(1-L) \\ -\frac{1}{\beta}g(1-L) \\ 1 + g(1-L) \\ 1 - g(1-L)^2 \end{bmatrix}\epsilon_{2t}$$

$$+\begin{bmatrix} -hL \\ -hL \\ \frac{h}{\beta}L \\ h(1-L) \\ hL(1-L) \end{bmatrix}\epsilon_{3t} + \begin{bmatrix} \theta_0 g(1-L) \\ (\theta_0 g - \theta(L))(1-L) \\ \frac{-\theta_0}{\beta}g(1-L) \\ \psi(L)(1-L) \\ -[\theta_0 g - \theta(L)](1-L)^2 \end{bmatrix}\epsilon_{5t} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \phi(L)(1-L) \\ 0 \end{bmatrix}\epsilon_{6t}$$

$$+\begin{bmatrix} d & 0 & -h \\ d & 0 & -h \\ 1-\frac{1}{\beta}d & 0 & \frac{h}{\beta} \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \tag{3.26}$$

To find out the long-run multiplier $R(1) = [\beta_\perp \; 0]$ of the structural innovations ν_t, set all L equal to 1 in (3.26) to get

$$R(1)\nu_t = \begin{bmatrix} d & 0 & -h & 0 & 0 \\ d & 0 & -h & 0 & 0 \\ 1-\frac{d}{\beta} & 0 & \frac{h}{\beta} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{5t} \\ \epsilon_{6t} \end{bmatrix} \tag{3.27}$$

and the constant-term vector

$$\delta = \begin{bmatrix} d & 0 & -h \\ d & 0 & -h \\ 1-\frac{d}{\beta} & 0 & \frac{d}{\beta} \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}. \tag{3.28}$$

The 5-dimensional vector δ forms the deterministic trend component in the level of $x_t' = [y_t \, mr_t \, wr_t \, R_t \, \pi_t]$ and the 3-dimensional τ can be treated as the common deterministic trends. The particular form of $R(1)$ in (3.27) reveals that innovations ϵ_{5t} and ϵ_{6t}, with no long-term effects on the first-difference of x_t, are transitory shocks. In fact, they are, respectively, the underlying force that forms the stationary demand shock in (3.1) and the stationary real interest rate shock in (3.12). On the other hand, the technology shock ϵ_{1t} and the labor-market shock ϵ_{3t} have nonzero long-term effects on real variables Δy_t, $\Delta\, mr_t$ and Δwr_t. The monetary shock ϵ_{2t} has long-run effects on nominal variables ΔR_t and $\Delta\, \pi_t$ but not on real variables. Thus a long-run neutrality of money is a property of the economic model. The three permanent shocks constitute three common stochastic trends in the level of x_t.

As considered in Chapter 2, the particular form of $\beta_\perp$ contained in $R(1)$ is valuable to identify common stochastic trends or permanent shocks. In the preceding sections structural information included in F and its subset $\alpha_\perp'$ also are shown useful for identification. Between $\alpha_\perp$ and $\beta_\perp$ however, only one is required for identification by the relation $R(1)\,F = \beta_\perp \alpha_\perp' = C(1)$. As an reduced-form moving average model or $C(1)$ can be derived from the same theory, the knowledge of either $\alpha_\perp$ or $\beta_\perp$ yields the other immediately. In principle identification based on $\alpha_\perp$ or $\beta_\perp$ should also have equal statistical power for overidentifying restriction tests.

Chapter 4

SPECIFICATION, ESTIMATION AND IDENTIFICATION OF THE ECONOMIC MODEL

4.0 Introduction

This chapter presents an econometric analysis of the five-equation cointegrated system and the identification of the structural model. The time-series properties of the variables and two assumed cointegration relations are examined in Section 4.1 using unit-root tests and graphs. Specification tests are conducted in Section 4.2 to determine an optimal vector error-correction model to estimate. Two cointegrating vectors, an M2 velocity and an ex post real interest rate, are imposed on all VECMs specifications considered. A three-lag VECM including a linear time trend on the levels and five dummy variables is selected. The validity of imposing two cointegrating vectors on a VECM is confirmed in Section 4.3 by Johansen's (1988, 1991) cointegrating rank tests and Horvath and Watson's (1995) tests for pre-specified cointegrating vectors. The VECM estimation results are presented in Section 4.4. The identification of structural VMAs using dynamic and steady-state restrictions derived in Chapter 3 is shown in Section 4.5.

4.1 Data Analysis and Unit-Root Tests

Seasonally adjusted quarterly data of the United States from 1951:1 to 1994:4 are studied in this dissertation. Particularly, real output (y_t), real balances (mr_t), real wages (wr_t), short-term nominal interest rate (R_t) and inflation rate (π_t) are of major interest. Variables are in natural logarithms, except for the nominal interest rate. The measure for real output is the private-sector GDP used by King et al. (1991) defined as GDP minus the government purchases[15]. Real balances are equal to the M2 measure divided by the price deflator. Real wages are defined as the nominal wage divided by the price deflator. The nominal-wage measure used is the average hourly earnings of workers in the manufacturing sector. The interest rate on Treasury Bills of 3-month maturity is used as the short-term nominal interest rate. Inflation is the log first difference of the price deflator. Both the nominal interest rate and the inflation rate are on per annum basis. Detailed variable definitions and data sources are presented in Appendix C.

Before formally undertaking statistical tests to determine the time-series properties of the variables, it is useful to first graph the variables and make a preliminary statement. The levels of the five variables in this study are presented in Figures 4.1, 4.3, 4.5, 4.7 and 4.9 while their first-differences are presented in Figures 4.2, 4.4, 4.6, 4.8 and 4.10. The levels of output, real money balances and real wages appear to be trending upward so they may be generated either by random walk with drift process or be trend stationary processes. The levels of the nominal interest rate and inflation, despite not showing any clear trend movement, appear nonstationary because of changing mean levels and variabilities. They may be generated by random walk processes without drift or simply be stationary series. If the first differences of variables are judged to be $I(0)$, that can lend support to the claim that their levels are $I(1)$. The first-differences of the variables appear to have constant means in the figures. However, except for that of inflation, the first differences appear to have changing variability. The inspection seems to suggest that the first differences are

[15]Professor Rasche points out that this definition of real output is not private-sector GDP but private-sector gross purchases becuase the former should only exclude government purchases of labor services from GDP rather than total government purchases.

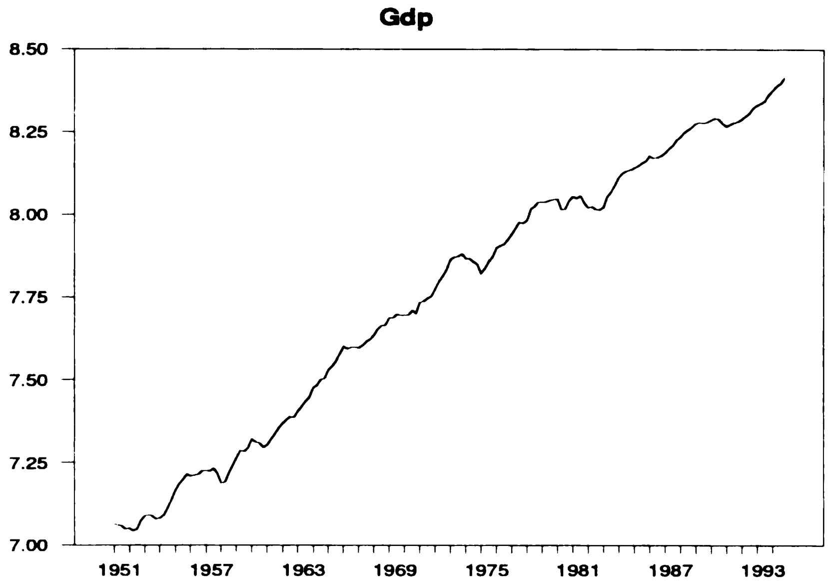

Figure 1 Real Output (Private Sector), 51:1-94:4

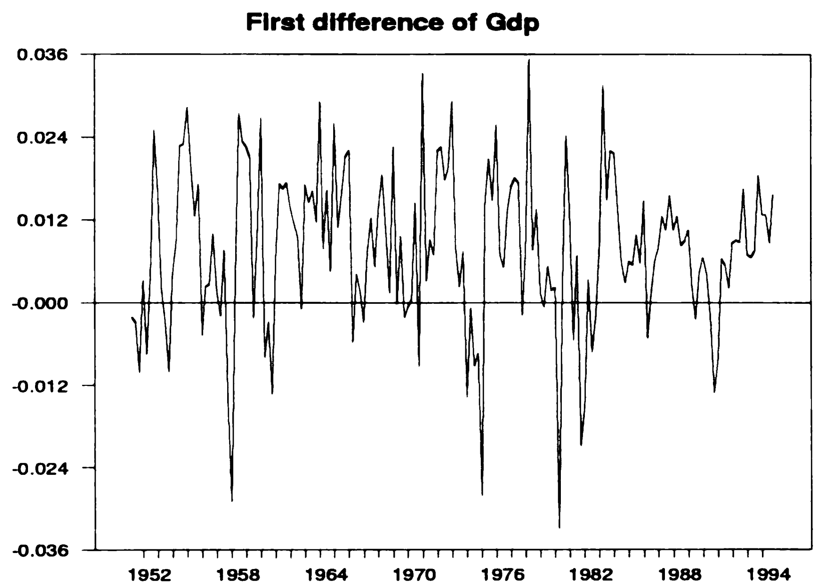

Figure 2 First-differenced Real Output, 51:1-94:4

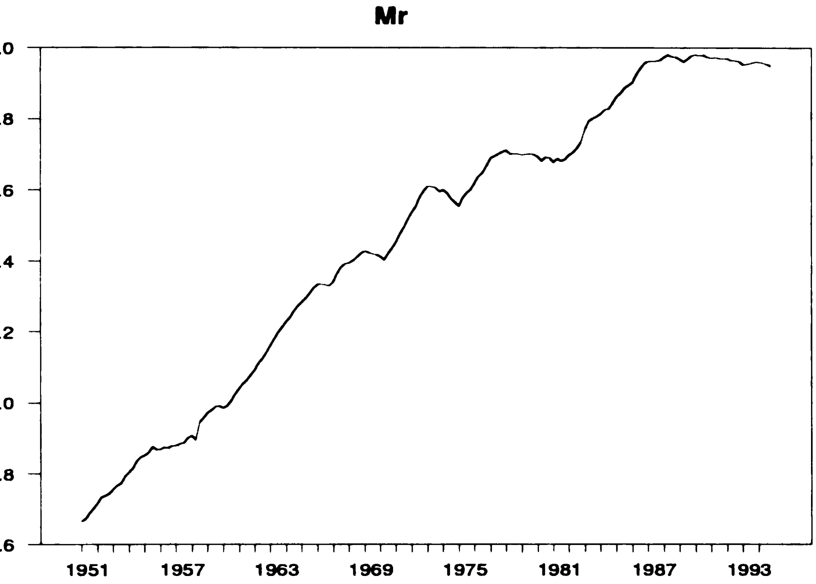

Figure 3 Real Balances, 51:1-94:4

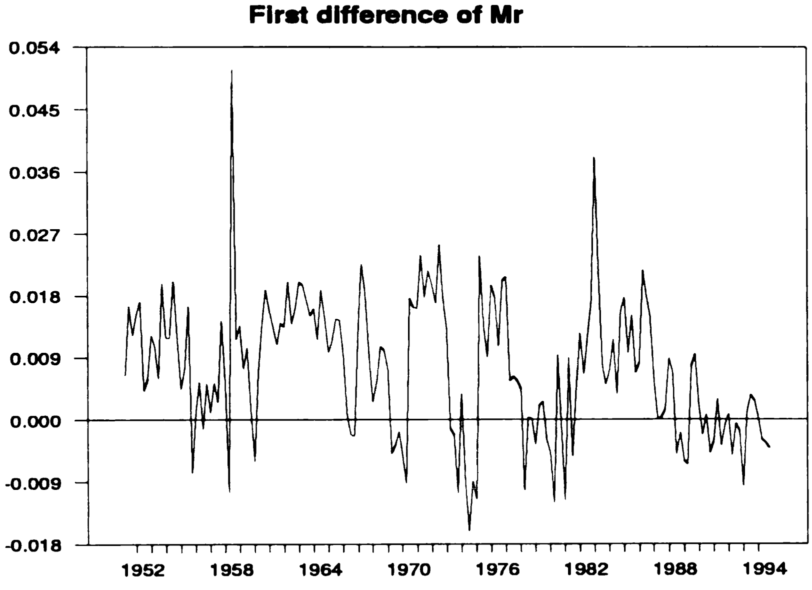

Figure 4 First-differenced Real Balances, 51:1-94:4

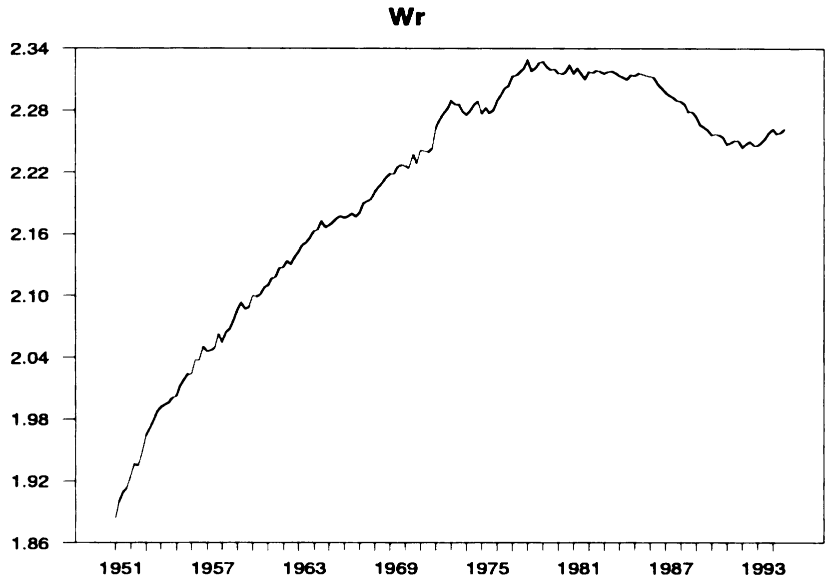

Figure 5 Real Wages, 51:1-94:4

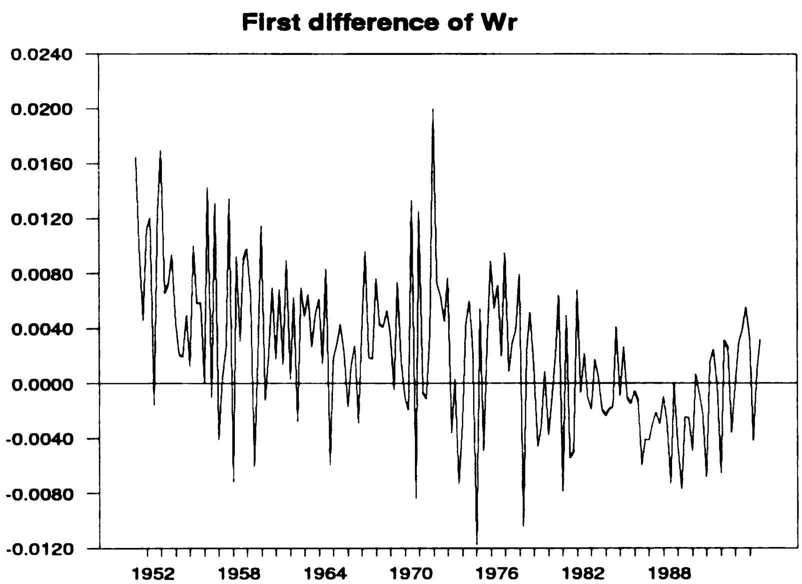

Figure 6 First-differenced Real Wages, 51:1-94:4

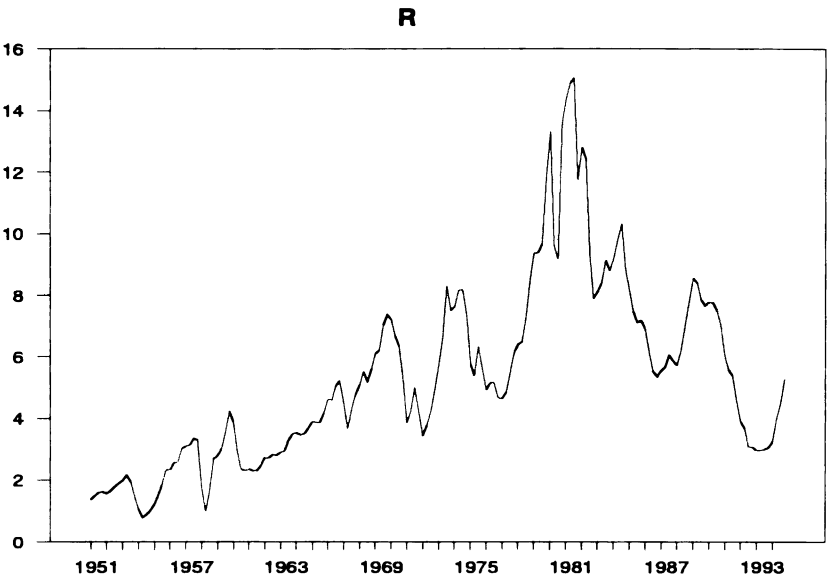

Figure 7 Nominal Interest Rate, 51:1-94:4

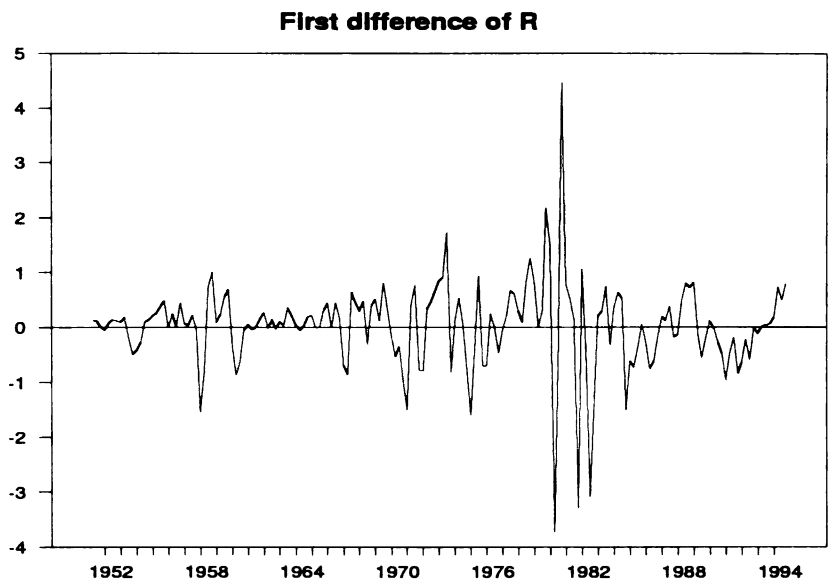

Figure 8 First-differenced Nominal Interest Rate, 51:1-94:4

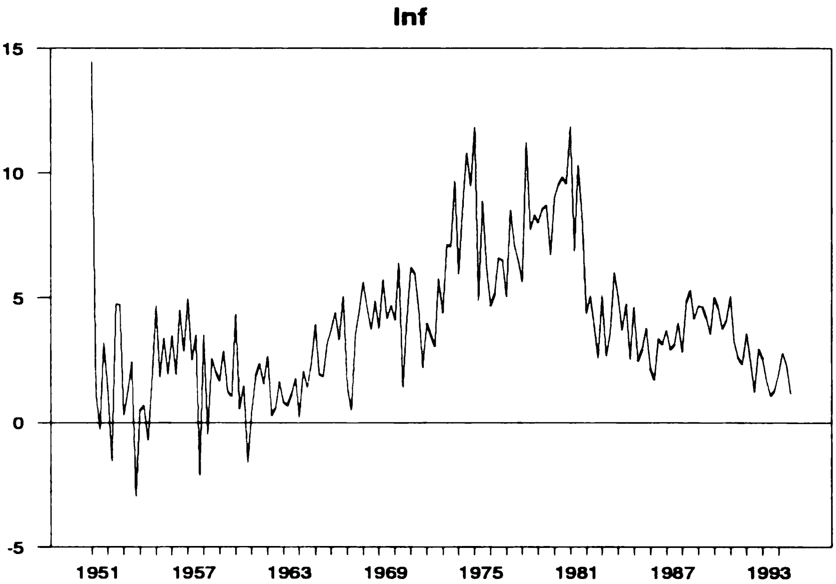

Figure 9 Inflation Rate, 51:1-94:4

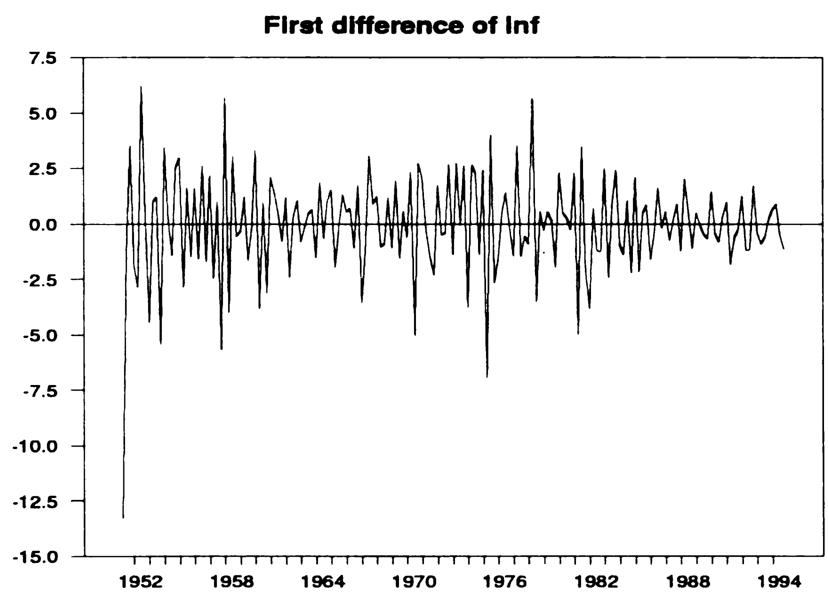

Figure 10 First-differenced Inflation Rate, 51:1-94:4

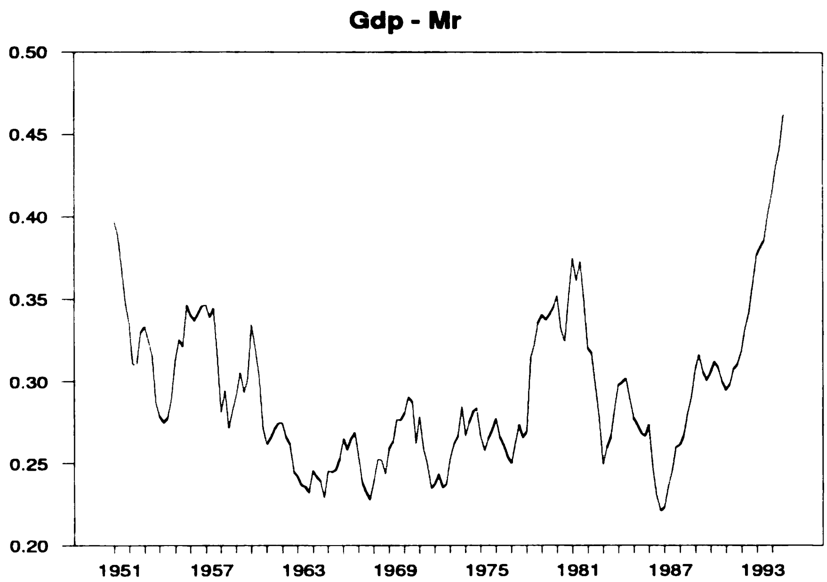

Figure 11 Velocity of Money (M2), 51:1-94:4

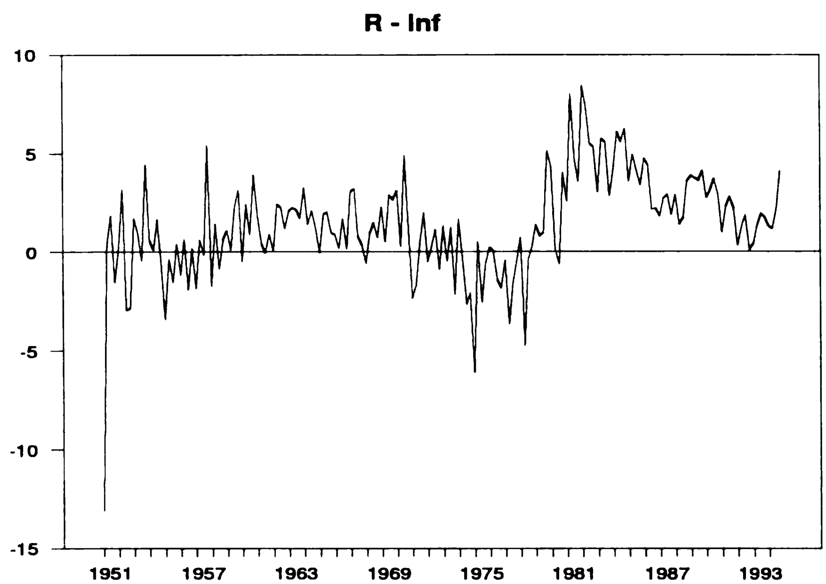

Figure 12 Ex Post Real Interest Rate, 51:1-94:4

not stationary $I(0)$ series.

Table 1 shows test results of the Augmented Dickey-Fuller unit-root t and z tests (Dickey and Fuller 1979). The tests involve running regressions of the form

$$x_t = \alpha + \delta t + \rho x_{t-1} + \sum_{j=1}^{k} b_j \Delta x_{t-j} + \epsilon_t. \tag{4.1}$$

A linear-trend term δt is included in (4.1) if graph-inspection suggests there is a trend component in the series. Conversely, a regression is run without a trend term as is the case for interest rates, inflation and all the first differences of the variables. The tests are conducted using the RATS uradf.src procedure written by Norman Morin. The t and z test statistics are calculated according to

$$\begin{aligned} t_i &= \frac{\hat{\rho}_i - 1}{\hat{\sigma}_i} \quad i = \mu, \tau \\ z_i &= T\left(\hat{\rho}_i - 1\right) \quad i = \mu, \tau \end{aligned}$$

where $\hat{\rho}_i$ and $\hat{\sigma}_i$ are respectively the estimates of ρ_i and its standard error. The subscript μ and τ indicates whether a test statistic is computed from regressions run with (τ) or without (μ) a trend term included. Both type of statistics (with subscript τ or μ) have non-standard distributions so tabulated critical values have to be consulted. The joint F tests, in cases where no trend term is included, have the null hypothesis $\rho_\mu = 1$ and $\alpha_\mu = 0$. That is, a random-walk process is hypothesized. In cases where a trend term is included, the null hypothesis is $\rho_\tau = 1$ and $\delta = 0$, i.e., a random walk with drift process. The F statistic is calculated in the usual Wald form but its asymptotic distribution is again nonstandard and appropriate tables are to be consulted (Dickey et al 1994). The number of lag differenced variables, indexed by j in equation (4.1), is determined by the Ljung-Box autocorrelation tests done on the regression residuals $\hat{\epsilon}_t$. Lags are sequentially added until the Ljung-Box test fails to reject the null of no serial correlation. This procedure is followed because ϵ_t has to be white noise for the unit-root test to be valid.

Table 1

Augmented Dickey-Fuller Tests
for a Unit Root

	Lag	t_τ (t_μ)	z_τ (z_μ)	Joint F test
y	1	-2.6023	-16.2625	3.5046
m	2	-2.3811	-5.8579	3.2850
p	2	-1.4618	-2.6930	3.1096
π	1	(-4.2423^c)	(-36.3589^c)	(9.0005^c)
w	7	-1.9781	-5.8950	2.2940
mr	1	-1.1983	-4.8765	1.2658
wr	0	-0.2084	-0.2708	22.4305^c
R	11	(-1.5204)	(-4.4261)	(1.4797)
Δy	0	(-8.8725^c)	(-110.2450^c)	(39.3661^c)
Δm	1	(-5.1760^c)	(-52.7664^c)	(13.3991^c)
$\Delta\pi$	4	(-8.0422^c)	(821.0265)	(32.3383^c)
Δw	5	(-3.6260^c)	(-30.9860^c)	(6.7682^c)
Δmr	0	(-8.0914^c)	(-93.2722^c)	(32.7395^c)
Δwr	2	(-5.1677^c)	(-66.7216^c)	(13.3692^c)
ΔR	6	(-6.1916^c)	(365.5331)	(19.1821^c)

Notes: 1. Statistics in parenthesis are t_μ and z_μ and are t_τ and z_τ otherwise. 2. a, b and c indicate statistical significance at the 10%, 5% and 1% level respectively. 3. Critical values are shown in Table 2

As shown in Table 1, the ADF test fails to reject the null of a unit-root at 10 percent level for y_t, mr_t, wr_t, and R_t and therefore supports our claim that they are I(1) variables. On the other hand, the unit-root hypothesis is strongly rejected at 1 percent level for Δy_t, π_t, $\Delta\pi_t$, Δmr_t, Δwr_t and ΔR_t suggesting they are I(0) stationary. These are almost exactly what we postulated in the theoretical model presented in Chapter 3, except that the inflation rate, π_t, is assumed an I(1) variable there. Recall inflation is cointegrated with the I(1) nominal interest rate,

R_t, to form the stationary real interest rate relation, $R_t - \hat{\pi}_{t+1}$. In the literature there is a debate about whether the order of integration for inflation is I(1) or I(0). Discussions on this can be found in Baillie, Chung and Tieslau (1996). Likewise, there are also conflicting findings regarding whether the order of p_t is I(2) or I(1) in the literature. Despite the unit-root test results, we will still treat π_t as an I(1) variable in the following analysis for two reasons. First of all, there could be a power of test problem for the unit-root test procedure. Diebold and Rudebusch (1991) found the Dickey-Fuller test has low power when the true value of ρ is near but not equal to one. Furthermore, we have just learned that the nominal interest rate is I(1) and we will also learn shortly that the ex post real interest rate, $R_t - \pi_t$, is indeed an I(0) cointegration relation. Then π_t being the difference of R_t and $R_t - \pi_t$ cannot be I(0) since I(1) +I(0) cannot be I(0).

Figures 4.11 and 4.12 graph the two cointegration relations, the M2 velocity ($y_t - mr_t$) and the ex post real interest rate ($R_t - \pi_t$). They appear to be stationary despite slightly irregular mean levels relative to the univariate time-series plots shown in Figures 4.2, 4.4, 4.6 and 4.8. Under this circumstances, we have to rely on formal tests to make inferences about their true properties. Table 2 shows the Augmented Dickey-Fuller test results for these two relations. In the top panel, regressions are run without a trend term included while one is included in the bottom panel. We will rely only on results in the top panel because neither $R_t - \pi_t$ nor $y_t - mr_t$ show any discernible trend movement in Figures 4.11 and 4.12. The bottom panel is included to provide extra reference[16]. Notice that in both panels the tests are conducted over two sample periods of different length. The shorter sample

[16]We briefly mention that the test results for regressions with a trend included. The tests still strongly reject the unit root hypothesis for the real interest rate. But all t, z and F tests fail to reject the null of a unit root for the velocity relation at 10 percent level no matter whether the long or short sample is used.

Table 2
Augmented Unit Root Tests for
$y_t - mr_t$ and $R_t - \pi_t$

	Lag	t_μ	z_μ	F test
Without linear trend in regression				
		The short sample:1953:1-1991:4		
$y_t - mr_t$	1	-3.0404^b	-17.9109^b	4.6290^b
$R_t - \pi_t$	1	-3.8976^c	-30.7874^c	7.6065^c
		The long sample:1951:1-1994:4		
$y_t - mr_t$	1	-1.5416	-8.6028	1.2536
$R_t - \pi_t$	1	-5.1136^c	-48.8346^c	13.1504^c
	Sig. level			
	10 %	-2.57	-11.2	3.81
Critical value	5 %	-2.88	-14.0	4.63
	1 %	-3.46	-20.3	6.52

	Lag	t_τ	z_τ	F test
With linear trend in regression				
		The short sample:1953:1-1991:4		
$y_t - mr_t$	1	-.3015	-17.7332	4.7997
$R_t - \pi_t$	1	-4.3712^c	-39.2619^c	9.5549^c
		The long sample:1951:1-1994:4		
$y_t - mr_t$	1	-1.7825	-9.4958	3.7957
$R_t - \pi_t$	1	-5.5728^c	-59.7335^c	15.5626^c
	Sig. level			
	10 %	-3.13	-18.0	5.39
Critical values	5 %	-3.43	-21.3	6.34
	1 %	-3.99	-28.4	8.43

Notes: 1. a, b and c indicate statistical significance at the 10%, 5% and 1% level respectively. 2. Lag indicates the lag lenth of the ADF regression. 3. Critical values are from Hamilton (1994).

covers the period from 1953:1 to 1991:4 and the longer sample covers from 1951:1 to 1994:4. Both sample periods are tested because they yield different results, suggesting there could be fundamental shifts in the long-run relationship between the two samples.

In the top panel of Table 2, and over the shorter sample, all t, z and F tests strongly reject the null of a unit root at 5 percent level for the velocity and at 1 percent level for the real interest rate. Therefore, both cointegration relations are valid for the shorter sample. The story is somewhat different when the tests are done over the longer period. While the real interest rate continues to be shown stationary at a 1 percent significance level, tests on the velocity fail to reject the unit-root hypothesis at 10 percent level for the longer sample. Thus there appears to be a regime change in the data generating process for the velocity of M2. This change causes the cointegrating velocity relationship to break down in the ADF tests. To confirm, we observe in Figure 4.11 that the velocity, before 1953:1 and after 1991:4, moves in a more drastic fashion and its mean is also higher than the mean of observations in the shorter sample. For this reason we will estimate an empirical wage-contract model using only the short-sample data that upholds both cointegrating relations.

4.2 Specification for a VAR with Cointegration

The first question to be answered in specifying a vector-autoregression model is how many lags to include. It is also customary to add seasonal or regime dummy variables to capture systematic shiftings in time-series processes. The most widely applied specification test for such decisions is the likelihood-ratio tests. Write a standard n-dimensional VAR with p lag-terms and a vector of dummy-variables D_t as

$$x_t = \sum_{i=1}^{p} A_i x_{t-i} + \mu + \Psi D_t + \varepsilon_t \tag{4.2}$$

where μ, A_i and Ψ are parameters to be estimated and the error vector ε_t is assumed distributed as i.i.d. $N(0, \Sigma)$. The likelihood-ratio test statistic Sims (1980) suggested is

$$(T-c)\left(\ln\left|\hat{\Sigma}_r\right| - \ln\left|\hat{\Sigma}_u\right|\right) \tag{4.3}$$

where $\hat{\Sigma}_r$ and $\hat{\Sigma}_u$ are respectively the maximum-likelihood estimates of the error covariance of the restricted VAR (with a shorter lag and fewer dummy variables) and the unrestricted VAR. T is the number of observations used and $c = 1 + 5p$ is the number of parameters in each equation in the unrestricted VAR. The statistic in (4.3) has an asymptotic χ^2 distribution with degree of freedom equal to the number of restrictions imposed on the system.

If cointegration is an important characteristic of the equation system, then a cointegrating-rank restriction or even explict cointegrating-vectors need to be imposed in estimating a VAR. In such cases, the VAR is usually modeled by its vector error-correction representation,

$$\Delta x_t = \mu + \sum_{i=1}^{p-1} \Gamma_i \Delta x_{t-i} + \alpha\beta' x_{t-1} + \Psi D_t + \varepsilon_t, \tag{4.4}$$

where μ and D_t are constant and dummy variables respectively and the Γ_is, α and β can solve for the A_is in (4.2). Under the assumption of r cointegrating relations, α and β are both $n \times r$ matrix and the rank of $\Pi \equiv \alpha\beta'$ is r. Typically, Johansen's (1988, 1991) maximum-likelihood procedure is used to model (4.4). In cases where specific cointegration vectors are imposed, $\beta' x_{t-1}$ become known variables and the maximum-likelihood estimates of the parameters can be obtained by running ordinary-least-squares for each of the n equations in (4.4). The OLS is a valid method because typically no cross-equation restriction is imposed on VARs or VECMs. The models in this dissertation are estimated using the CATS package.

There are three sets of dummy variables under consideration for D_t in (4.2). The first set is labelled *Dummy*0 which in fact contains no dummy variables at all so that $D_t = 0$. The second specification is labelled *Dummy*1 which includes three time dummies, $D_{67\,t}$, $D_{79\,t}$ and $D_{82\,t}$, so that D_t is a 3×1 vector. This specification has been used by Hoffman and Rasche (1996) in their studies of money-demand functions. The third set of dummy variables is *Dummy*2 which includes two time dummies, $D_{73\,t}$ and $D_{75\,t}$, in addition to the three defined in *Dummy*1. The three dummy-variable sets and the five dummy variables are defined below:

$$\begin{aligned} Dummy0 &= \{\text{No dummy variables}\} \\ Dummy1 &= \{D_{67\,t}, D_{79\,t}, D_{82\,t}\} \\ Dummy2 &= \{D_{67\,t}, D_{73\,t}, D_{75\,t}, D_{79\,t}, D_{82\,t}\} \end{aligned}$$

where

$$D_{j\,t} = \begin{cases} 0 & t < t_j \\ 1 & t \geq t_j \end{cases} \quad j = 67,\ 73,\ 75,\ 79,\ 82$$

and

$$\begin{aligned} t_{67} &= 1967:4, \quad t_{73} = 1973:4, \quad t_{75} = 1975:3, \\ t_{79} &= 1979:4, \quad t_{82} = 1982:1. \end{aligned}$$

The dummy variable $D_{73\,t}$ is zero through 1973:3 and one thereafter while $D_{75\,t}$ is zero through 1975:2 and one thereafter. They are included to account for the 1973-74 oil price shocks and the sharp increase in the price level that it caused. $D_{67\,t}$ is defined as zero from the initial time period through 1967:3 and one thereafter. It is included to capture the acceleration of inflation with the Vietnam conflict. $D_{79\,t}$ is zero through 1979:3 and one thereafter and $D_{82\,t}$ is zero through 1981:4 and one thereafter. They are included to reflect the New Operating Procedures of the Federal Reserve in place between 1979:4 and 1981:4 since means of the cointegrating vectors could shift with this policy change.

Table 3 shows values of the likelihood functions for VARs of different lag-length and dummy-variable specifications. Since the velocity and the ex post real rate coin-

Table 3
Maximum Likelihood Function of VARs

VAR lags	Dummy0	Dummy1	Dummy2
1	-28.7692	-29.1157	-29.4820
2	-30.0543	-30.2567	-30.5817
3	-30.5506	-30.7167	-30.9095
4	-30.8103	-30.9872	-31.2050
5	-31.0312	-31.1817	-31.4047
6	-31.3632	-31.5443	-31.8064
7	-31.5596	-31.7440	-31.9914
8	-31.8002	-31.9795	-32.2156

tegration relations are imposed in estimation, we in fact estimate the VECMs. The likelihood functions are then used to conduct likelihood-ratio tests to first determine the optimal dummy-variable setting as shown in Table 4. Then they are used to test for the optimal lag-length specification as shown in Table 5. Judging by the test results in Table 4, regardless of the lag-length of VARs, *Dummy*2 consistently fares better than *Dummy*0 and *Dummy*1[17]. We therefore accept the *Dummy*2 specification and include all five dummy variables in the VAR model.

The likelihood-ratio test results shown in Table 5 consistently suggest VARs with either four or six lags is the optimal specification to use. However when a likelihood-ratio test for a constrained five-lag VECM (six-lag VAR), against a non-

[17]In the case of testing for *Dummy*0 versus *Dummy*1, the degree of freedom is $15 = 3 \times 5$, the number of the reduced dummy variables (3) times the number of equations (5). Similary, the degree of freedom in the test of *Dummy*1 versus *Dummy*2 is $10 = 2 \times 5$ and in *Dummy*0 versus *Dummy*2 equal to $25 = 5 \times 5$. In the case of testing for models with sequentially shorter lags, e.g., $p-1$ versus p, the degree of freedom is equal to the number of elements in the parameter matrix A_p, or $25 = 5 \times 5$.

Table 4
Likelihood Ratio Tests for Dummy Specification

VAR Lags	D0 vs. D1	D1 vs. D2	D0 vs. D2
1	49.5595*	51.6384*	100.5048*
2	27.9271*	44.2014*	71.7237*
3	22.1020	25.2489*	47.0185*
4	22.6458*	27.4428*	49.7347*
5	18.5164	26.9842*	45.1996*
6	21.3781	30.4048*	51.4205*
7	20.8429	27.4559*	47.9298*
8	19.3687	25.0255*	44.0356*
$d.f.$	$15\,(=3\times 5)$	$10\,(=2\times 5)$	$25\,(=5\times 5)$
$\chi^2\,(\alpha=10\%)$	22.31	15.99	34.38

* indicates significant at 10%

Table 5
Likelihood Ratio Tests of Lag Length

VAR lags	Dummy0	Dummy1	Dummy2
1 vs. 2	181.2076*	157.4525*	149.5646*
2 vs. 3	67.4886*	61.1867*	42.9392*
3 vs. 4	34.0299	34.6253*	37.2418*
4 vs. 5	27.8284	23.9210	24.1625
5 vs. 6	40.1684*	42.7868*	46.5972*
6 vs. 7	22.7871	22.5684	20.5306
7 vs. 8	26.7044	25.4308	23.7663

Critical value: $\chi^2 = 34.38$ ($\alpha = 10\%$, $d.f. = 5\times 5$) for all cells. * indicates significant at 10%

Table 6
Residual Analysis for VECMs with 4 and 6 Lags

	Lags	ε_{1t}	ε_{2t}	ε_{3t}	ε_{4t}	ε_{5t}
Standard dev.	4	0.009	0.007	0.004	0.623	1.451
	6	0.008	0.007	0.004	0.569	1.401
R^2	4	0.445	0.517	0.411	0.465	0.521
	6	0.510	0.561	0.483	0.553	0.554
Normality	4	9.133^a	28.669^b	0.567	70.843^b	4.870
	6	13.299^b	18.691^b	0.223	53.182^b	7.790^a
ARCH(4)	4	5.496	7.518	15.260^b	31.292^b	2.843
ARCH(6)	6	7.279	12.210	4.817	26.604^b	3.912
Multivariate-	4	LM(4)=28.617 p-value = 0.28				
autocorr.	6	LM(4)=23.021 p-value = 0.58				

Note: The limiting distribution of the normality, ARCH(4), ARCH(6), and LM(4) test are χ^2 with $d.f.$ equal to 2, 4, 6 and 25 respectively. a and b indicates rejection of the null at 5% and 1% significance level respectively.

restricted alternative, is performed, the null is rejected at a significance level less than 1 percent. There appears to be an internal inconsistency in that VAR(6) with cointegration is selected in Table 5 to be possibly the optimal lag-length but at the same time it is rejected in favor of a VAR(6) without cointegration. On the other hand, when the VAR lag is reduced to four (three-lag VECM), the VAR with two cointegration restrictions cannot be rejected at a 10 percent significance level. It is possible the five-lag VECM is over-parameterized so as to reduce the power of the likelihood test for cointegration. We therefore will include only four lag terms, instead of six, in the cointegrated VAR model.

We also note that replacing VAR(6) with VAR(4) does not effectively change

Table 7
Cointegrating Rank Test
on Unrestricted VAR with 4 and 6 Lags

		Max-λ statistic		Criticl value	
H_0	$k=p-r$	$l=4$	$l=6$	α =10 %	α =5 %
$r=0$	$k=5$	36.54^b	37.58^b	30.77	33.18
$r=1$	$k=4$	25.02^a	33.89^a	24.71	17.17
$r=2$	$k=3$	18.11	18.66	18.70	20.78
$r=3$	$k=2$	13.09^a	13.44^a	12.10	14.04
$r=4$	$k=1$	2.36	1.28	2.82	3.96
		Trace statistic		Critical value	
H_0	$k=p-r$	$l=4$	$l=6$	α =10 %	α =5 %
$r=0$	$k=5$	95.11^b	104.84^b	65.06	68.91
$r=1$	$k=4$	58.27^b	67.26^b	43.96	47.18
$r=2$	$k=3$	33.56^b	33.38^b	26.79	29.51
$r=3$	$k=2$	15.45^b	14.72^a	13.34	15.20
$r=4$	$k=1$	2.36	1.28	2.82	3.96

Notes: a and b indicates statistical significance at 10 % and 5 % respectively.

the properties of the residuals as shown in Table 6. Listed in the table are the univariate standard deviations, unadjusted R^2, normality tests, ARCH tests and the system-wide autocorrelation LM tests. In terms of R^2 and standard deviations, VAR(6) outperforms VAR(4) simply because it includes more lagged explanatory variables. This, however, does not result in more favorable properties for VAR(6) in terms of the ARCH, the normality and the serial-correlation test statistics.

4.3 Testing for Two Cointegration Relations

An useful way of confirming the validity of the two derived cointegrating vectors is to conduct formal cointegration tests. Johansen's cointegrating rank tests

(Johansen 1988; Johansen and Juselius 1990) will first be performed. If the conjecture of the rank $r = 2$ is confirmed, even though it does not provide inference on the coefficients of cointegrating vectors, we can be more confident about the empirical results of VECM(3) with two cointegration relations imposed. A direct support for imposing the two specific cointegration relations can be gained by undertaking Horvath and Watson's (1995) tests for pre-specified cointegrating vectors. As shown below, both types of test support the use of cointegrating velocity and real interest rate relations in estimating the economic model.

4.3.1 Johansen's Cointegration Rank Tests

There are two types of cointegration-rank tests. The first one is the trace test which tests the pair of hypotheses about the rank of $\Pi \equiv \alpha\beta'$ in (4.4),

$$\begin{aligned} H_0 &: \text{rank}(\Pi) \leq r \\ H_1 &: \text{rank}\,(\Pi) = n. \end{aligned}$$

The test procedure is first set $r = 0$ and sequentially increase the value for r if the null is rejected. When the data ceases to reject a null, the particular r is then treated as the cointegrating rank. The second rank test is the maximum eigenvalue test, or Max-λ test, which has the pair of hypotheses,

$$\begin{aligned} H_0 &: \text{rank}(\Pi) = r \\ H_1 &: \text{rank}\,(\Pi) = r + 1. \end{aligned}$$

The procedure begins with zero for r and sequentially increases its value if the null is rejected. Again when a null is not rejected, the particular r value is inferred as the number of existing cointegrating vectors. The distribution of both test statistics are non-standard and simulated distributions are available (Hamilton 1994)[18].

[18]We note that the simulated distribution depends on the specification of deterministic terms

The tests are conducted on the VECM counterparts of both VAR(4) and VAR(6). As the test results in Table 7 indicate, the two types of rank test have conclusions in conflict with each other. For the Max-λ test, the $r = 2$ hypothesis is confirmed. The maximum-eigenvalue test rejects both the null of $r = 0$ and $r = 1$ but fails to reject the null of $r = 2$ against the alternative of $r = 3$ for both lag specifications. In contrast, the trace test rejects the hypothesis of $r = 2$. Specifically, the nulls of a cointegrating rank $r \leq 0$, 1, 2, and 3 are strongly rejected. What is then suggested by the trace test is a rank of 4 or 5. Since the subsequent null of $r \leq 4$ against $r = 5$ is not rejected, the trace test infers the cointegrating rank is 4.

We will rely on the $r = 2$ conclusion of the maximum-eigenvalue tests. The $r = 4$ conclusion of the trace test lacks both theoretical and intuitive justifications because it implies only one stochastic trend exists in a fairly complete economic system including five nonstationary series. It is quite common to find at least two stochastic trends in a model of more than four I(1) variables such as in Shapiro and Watson (1988) and Karras (1993). There is indeed no statistical evidence to rule out the trace test $r = 4$ result in favor of the max-λ test $r = 2$ result that is consistent with our theoretical model in Chapter 3. Nevertheless we are more interested in the question whether an empirical business-cycle model can be estimated from that theory and be consistent with the postwar U.S. macroeconomic data.

4.3.2 Horvath and Watson's Cointegrating Vector Test

Economic theories often imply parameters of 0's, 1's and -1's for cointegrating relations. In response to this phenomenon, Horvath and Watson (1995) developed the testing procedures for such explicitly specified cointegrating vectors in the context of a finite-order Gaussian VECM as in (4.4). Interests are focused on the size

in the VECM. We use the critical values as reference instead of correct values for our model that includes five dummy variables.

Table 8. Horvath/Watson Tests
of Pre-specified Cointegrating Vectors

$r_{0u} = r_{au} = 0$				Critical values		Computed
r_{0k}	r_{ak}	β_{0k}	β_{ak}	10%	1%	Wald statistic
0	1	0	MV	12.49	19.00	20.5312[b]
0	1	0	RR	12.49	19.00	31.5164[b]
0	2	0	MV, RR	23.51	31.26	56.8986[b]
1	1	MV	RR	12.49	19.00	36.2707[b]
1	1	RR	MV	12.49	19.00	25.0009[b]

Note: b indicates significance at a 1% level. MV is the cointegrating M2 velocity and RR stands for the ex post real rate.

of the cointegrating rank $r = \text{rank}(\Pi)$ and the pair of hypotheses

$$H_0 \ : \ \text{rank}\,(\Pi) = r_0$$

$$H_a \ : \ \text{rank}\,(\Pi) = r_0 + r_a \text{ with } r_0 > 0.$$

Under the null, r_0 is further defined by $r_0 = r_{0u} + r_{0k}$ where r_{0k} is the number of known cointegrating vectors while r_{0u} represents the unknown or unrestricted cointegrating vectors under the null. The number of additional cointegrating vectors present under the alternative is r_a. Similarly, the extra rank is further divided according to $r_a = r_{au} + r_{ak}$ where the subscripts u and k denote unknown and known, respectively. The Horvath and Watson tests generalize the procedures of Johansen's rank tests where no known cointegrating vectors are present. That is, his rank tests consider only cases with $r_{0k} = r_{ak} = 0$ and the hypotheses of interest for $r_{au} = 1$ is

$$H_0 \ : \ \text{rank}\,(\Pi) = r_{0u}$$

$$H_a \ : \ \text{rank}\,(\Pi) = r_{0u} + 1$$

which is shown above as the maximum-eigenvalue test.

Table 8 presents the test results on the validity of M2 velocity (MV) and ex post real rates (RR) cointegration relations. The first test specifies a null of no cointegration and the alternative of a cointegrating M2 velocity. The computed Wald statistic, 20.53, is greater than the simulated 1 percent critical value and thus strongly rejects the no-cointegration hypothesis in favor of a cointegrating velocity relation. The second test also strongly rejects a null of no cointegration in favor of a cointegrating ex post real interest rate relation. A comparison of the Wald statistic, 31.52, to that in the first test, 20.53, indicates statistical evidence is stronger for the cointegrating ex post real rate than that for the M2 velocity. This is consistent with the nonstationary test results for the two cointegration relations presented in Table 2. The third test specifies a null of no cointegration and an alternative of two cointegrating vectors. The test again strongly rejects no-cointegration at less than 1 percent significance level in favor of the two pre-specified cointegration vectors. In the last two tests shown in Table 8, one of the two cointegrating vectors is specified under the null while the other one is added under the alternative. Again both test strongly rejects the nulls of a single cointegration in favor of both cointegration relations being admitted in a VECM. Thus, the evidence obtained from the Horvath-Watson tests highly supports the practice of imposing cointegrating velocity and real interest rate relations as done in the next section.

4.4 Estimation of VECM and VMA

From the analysis in Sections 4.2 and 4.3, we have determined a specific VAR model to estimate. It has five time dummies and four lag terms. The lag is shortened to three in the VECM representation of $x_t' = [y_t \; mr_t \; wr_t \; R_t \; \pi_t]$ as

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \Gamma_3 \Delta x_{t-3} - \alpha\beta' x_{t-1} + \mu + \Psi D_t + \varepsilon_t. \qquad (4.5)$$

The cointegrating vectors derived in Chapter 3,

$$\beta' = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \tag{4.6}$$

is imposed while α is not restricted. The CATS procedures, based on Johansen's (1988, 1991 and 1995) MLE principle, estimates the other parameters in (4.5). Also see Johansen and Juselius (1990, 1992) for empirical applications of the methodology. The imposed β and the estimated α together provide a summary about the long-run dynamics of the system whereas the other parameter estimates represent the short-run dynamics of the system. The likelihood-ratio test statistic of this specification is χ^2 distributed with 6 degree of freedom. The p-value of the test statistic is 0.13 and therefore the restricted model is not rejected at the 10 percent significance level.

The estimated speed of adjustment matrix, in transpose, is

$$\alpha' = \begin{bmatrix} -0.128 & 0.003 & 0.031 & -4.189 & -9.326 \\ (-3.637) & (0.104) & (1.903) & (-1.695) & (-1.618) \\ -0.003 & 0.000 & -0.001 & -0.266 & 0.249 \\ (-3.632) & (0.647) & (-1.668) & (-4.620) & (1.850) \end{bmatrix} \tag{4.7}$$

where numbers in parenthesis are t ratios. We notice that values in the second column of α' are very small. Their t ratios also suggest that the two elements are not statistically different from zero. Thus weak exogeneity exists for real balances which is the second element of the x_t vector. The estimated α' is very different from the one obtained in the theoretical model,

$$\alpha' = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}. \tag{4.8}$$

This may indicate that both α and β derived in the theory are not concurrently consistent with the data. By imposing the theoretical β and taking the estimated α' as the correct adjustment matrix we are stating that portions of the theoretical model are misspecified so as to yield the incorrect form of α' in (4.8). We can reestimate the VECM model by, in addition to the above β, further restricting the second row of α to zero. The parameter estimates are presented in Table 8 and their t ratios in parenthesis. The likelihood-ratio test statistic for this specification against the alternative of a nonrestricted VAR has a χ^2 distribution with 8 degree of freedom. Its p-value $= 0.25$ is a substantial improvement over the p-value of 0.13 above without weak exogeneity imposed. For this reason we will take this new restricted version to be the correct specification for subsequent analysis.

Next, we convert the VECM estimation results to obtain its vector moving-average (VMA) representation as[19]

$$\Delta x_t = \delta + C(L)\varepsilon_t. \tag{4.9}$$

More importantly, for the purpose of identifying the common trend space, $\alpha_\perp$, and the factor loading matrix, $\beta_\perp$, we want to calculate the long-run multiplier matrix of the reduced-form errors ε_t as[20]

$$\begin{aligned} C(1) &= \sum_{i=0}^{\infty} C_i \\ &= \beta_\perp \alpha'_\perp. \end{aligned}$$

With $C(1)$ calculated, the long-run covariance matrix of Δx_t can be calculated as $C(1)\Sigma C(1)'$ where Σ is the covariance of the error ε_t. The estimated long-run multiplier matrix has reduced rank and has the form

[19]This involves first converting the VECM to its VAR representation and then invert $A(L)$, the lag-polynomial matrix of VAR, to get $C(L)$. The inversion techniques for a reduced-rank $A(L)$ requires special consideration and is treated in Warne (1990).

[20]The original formula for $C(1)$ as in (2.33) is $\beta_\perp (\alpha'_\perp \Gamma \beta_\perp)^{-1} \alpha'_\perp$ before we simplify $\beta_\perp (\alpha'_\perp \Gamma \beta_\perp)^{-1}$ as $\beta_\perp$ on page 23.

Table 9

Parameter Estimates for VECM

$$\alpha' = \begin{bmatrix} -0.128 & 0.0 & 0.031 & -4.189 & -9.326 \\ (-3.637) & (0.0) & (1.903) & (-1.695) & (-1.618) \\ -0.003 & 0.0 & -0.001 & -0.266 & 0.249 \\ (-3.632) & (0.0) & (-1.668) & (-4.620) & (1.850) \end{bmatrix}.$$

$$\mu' = \begin{bmatrix} 0.043 & 0.005 & -0.005 & 1.246 & 1.908 \\ (4.036) & (0.640) & (-0.957) & (1.660) & (1.091) \end{bmatrix}$$

$$\Psi = \begin{bmatrix} -0.007 & 0.006 & 0.010 & 0.003 & -0.008 \\ (-2.562) & (1.659) & (1.608) & (0.645) & (-2.019) \\ -0.001 & -0.002 & 0.001 & -0.001 & -0.003 \\ (-0.602) & (-0.651) & (0.301) & (-0.271) & (-1.001) \\ -0.001 & -0.002 & -0.001 & -0.001 & -0.002 \\ (-0.899) & (-0.990) & (-0.513) & (-0.319) & (-1.147) \\ -0.539 & 0.534 & 1.120 & -0.415 & -0.231 \\ (-2.818) & (1.987) & (2.588) & (-1.429) & (-0.828) \\ 0.420 & 0.486 & -1.204 & 0.330 & 0.596 \\ (0.942) & (0.774) & (-1.193) & (0.488) & (0.916) \end{bmatrix}$$

$$\Sigma = 10^{-3} \times \begin{bmatrix} 0.08 & 0.01 & 0.01 & 1.35 & 1.76 \\ 0.01 & 0.05 & 0.01 & 0.47 & -4.27 \\ 0.01 & 0.01 & 0.02 & 0.34 & -2.02 \\ 1.35 & -0.47 & 0.34 & 387.40 & 42.50 \\ 1.76 & -4.27 & -2.02 & 42.50 & 2107.00 \end{bmatrix}$$

Table 9 (cont'd)

Parameter Estimates for VECM

$$\Gamma_1 = \begin{bmatrix} 0.082 & 0.230 & -0.084 & 0.003 & -0.002 \\ (0.924) & (1.895) & (-0.420) & (2.701) & (-1.755) \\ 0.173 & 0.123 & -0.224 & -0.006 & -0.001 \\ (2.432) & (1.270) & (-1.409) & (-6.218) & (-1.374) \\ -0.062 & 0.040 & -0.182 & -0.001 & -0.000 \\ (-1.512) & (0.712) & (-1.987) & (-1.907) & (-0.459) \\ 8.439 & 8.056 & -5.187 & 0.327 & -0.111 \\ (1.356) & (0.949) & (-0.372) & (3.851) & (-1.755) \\ -1.963 & 4.334 & 96.101 & 0.583 & -0.422 \\ (-0.135) & (0.219) & (2.951) & (2.945) & (-2.865) \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} 0.067 & 0.165 & -0.280 & -0.001 & -0.001 \\ (0.742) & (1.373) & (-1.419) & (-1.184) & (1.737) \\ 0.076 & 0.105 & -0.012 & -0.002 & -0.000 \\ (1.049) & (1.095) & (-0.078) & (-2.146) & (-0.629) \\ 0.052 & 0.101 & -0.231 & 0.000 & -0.001 \\ (1.269) & (1.836) & (-2.545) & (0.014) & (-1.409) \\ 5.963 & -2.565 & -13.681 & -0.307 & 0.011 \\ (0.944) & (-0.345) & (-0.988) & (-3.604) & (0.191) \\ -22.491 & 11.816 & 48.183 & -0.004 & -0.100 \\ (-1.526) & (0.602) & (1.493) & (-0.019) & (-0.725) \end{bmatrix}$$

$$\Gamma_3 = \begin{bmatrix} -0.116 & 0.116 & 0.250 & 0.002 & -0.001 \\ (-1.374) & (1.083) & (1.281) & (1.900) & (-1.212) \\ -0.002 & 0.082 & 0.135 & -0.02 & -0.000 \\ (-0.026) & (0.962) & (0.863) & (-2.337) & (-0.894) \\ -0.038 & 0.139 & -0.079 & -0.000 & 0.000 \\ (-0.969) & (2.824) & (-0.885) & (-0.710) & (0.685) \\ 0.177 & 6.576 & 14.007 & 0.245 & 0.031 \\ (0.030) & (0.877) & (1.024) & (2.813) & (0.757) \\ -11.005 & -3.485 & 45.774 & 0.465 & -0.006 \\ (-0.797) & (-0.199) & (1.435) & (2.288) & (-0.061) \end{bmatrix}$$

$$C\,(1) = \begin{bmatrix} 0.536 & 1.008 & -0.819 & -0.010 & -0.006 \\ (1.514) & (3.587) & (-1.189) & (-2.445) & (-1.443) \\ 0.536 & 1.008 & -0.819 & -0.010 & -0.006 \\ (1.514) & (3.587) & (-1.189) & (-2.445) & (-1.443) \\ 0.194 & 0.001 & 0.844 & -0.003 & 0.001 \\ (1.815) & (0.011) & (4.601) & (-2.439) & (1.155) \\ -21.026 & 45.150 & 22.625 & 0.368 & 0.199 \\ (-2.343) & (6.340) & (1.296) & (3.736) & (1.947) \\ -21.026 & 45.150 & 22.625 & 0.368 & 0.199 \\ (-2.343) & (6.340) & (1.296) & (3.736) & (1.947) \end{bmatrix} \tag{4.10}$$

where numbers in parenthesis are t ratios. The linear trends in the level of variables x_t are calculated as

$$C(1)\mu = \begin{bmatrix} 0.0096 & 0.0096 & 0.0035 & 0.0695 & 0.0695 \end{bmatrix}'.$$

We notice the first row and the second row of $C\,(1)$ have the same values. This indicates the total impact of the errors on the first difference of output is the same as that of real balances, a condition for the stationary velocity relation. Similarly, a stationary ex post real interest rate relation requires the total effects of the errors and the same for both the nominal interest rate and the inflation rate. This is indicated by the identical fourth and fifth rows in (4.10). In contrast, the third row of $C\,(1)$ is not linearly dependent on any other row because real wages are not cointegrated with other variables.

4.5 Estimation of the Overidentifying Theoretical Model

4.5.1 The Complete Model Structure

Identification of a structural-from VMA

$$\Delta x_t = \delta + R\,(L)\,\nu_t \tag{4.11}$$

from its reduced-form counterpart in (4.9) involves finding a F matrix such that

$$\nu_t = F\varepsilon_t \tag{4.12}$$

$$R(L) = C(L)F^{-1}. \tag{4.13}$$

The n-dimensional structural innovations ν_t is composed of k-dimensional permanent shocks ν_t^P and r-dimensional transitory shocks ν_t^T. The covariance of the errors is $E(\varepsilon_t\varepsilon_t') = \Sigma$ and the innovation covariance $E(\nu_t\nu_t') = D$ is a diagonal matrix. Form the partition $F' = [F_k' \; F_r']$ where F_k is $k \times n$ and F_r is $r \times n$. As discussed in Chapter 2, the long-run multiplier of (4.8) and (4.11) are

$$\begin{aligned}\Delta x_t &= C(1)\varepsilon_t = \beta_\perp\alpha_\perp'\varepsilon_t \\ &= R(1)\nu_t = \beta_\perp\nu_t^P.\end{aligned} \tag{4.14}$$

Therefore permanent shocks are identified according to $\nu_t^P = F_k'\varepsilon_t = \alpha_\perp'\varepsilon_t$ and $\alpha_\perp' = (\beta_\perp'\beta_\perp)^{-1}\beta_\perp'C(1)$.

We begin identification with structural information available in F in Chapter 3. The explicit form of F is

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \end{bmatrix}. \tag{4.15}$$

The top 3×5 partition of F is F_k which describes the contemporaneous relations among the variables in the long-run structure of the model. The bottom 2×5 partition is F_r and represents the contemporaneous relations in the dynamic structure of the model. Estimation is done by solving an optimization problem for F and D such that

$$F\Sigma F' = D$$

subject to the pattern of F in (4.15). First normalize F to make its diagonals equal to 1 as in

$$
\begin{aligned}
F^* &= W \cdot F \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & \frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ \frac{1}{\beta} & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}.
\end{aligned} \tag{4.16}
$$

In this form we notice the three rows that form $\alpha'_\perp$ are now respectively rows 1, 3 and 5. F is subject to nine overidentifying restrictions and there is only a single free parameter to estimate in $\alpha_\perp$ (F_k).

Calculations are done by a RATS SVAR procedure written by Lansarotti and Seghelini. The result is

$$
F^* = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -0.1487 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{4.17}
$$

$$
D^{\frac{1}{2}} = \begin{bmatrix} 0.0089 & 0 & 0 & 0 & 0 \\ 0 & 0.0105 & 0 & 0 & 0 \\ 0 & 0 & 0.0039 & 0 & 0 \\ 0 & 0 & 0 & 1.5497 & 0 \\ 0 & 0 & 0 & 0 & 1.4484 \end{bmatrix}. \tag{4.18}
$$

Note that if the estimated model is not rejected by an overidentifying restriction test, we then need to premultiply the estimated F^* by W^{-1} so it returns to its theoretical form. However, the model is rejected by the overidentifying restriction test at effectively zero significance level. The test statistic is 477.99 for χ^2 distribution with 9 degrees of freedom. Thus the theoretical structure of F or the complete model in Chapter 3 as a whole is rejected by the data.

We now want to answer the question whether the data is consistent with parts of the model. The first three equations in (3.15) represent the steady-state structure of the model. They apply in equilibrium and hence are considerably less restricting than the dynamic parts of the model or the last two equations in (3.15). Therefore, we now inquire whether permanent shocks can be identified out of the long-run structure of the model, in particular, by using the long-run multiplier.

4.5.2 The Long-Run Model Structure

The common-trends loading matrix $\beta_\perp$ derived in Chapter 3, in the form of the long-run multiplier of permanent shocks $\Delta x_t = \beta_\perp \nu_t^P$, is

$$\begin{bmatrix} \Delta y_t \\ \Delta mr_t \\ \Delta wr_t \\ \Delta R_t \\ \Delta \pi_t \end{bmatrix} = \begin{bmatrix} a & c & 0 \\ a & c & 0 \\ b & e & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_t^{\text{tech}} \\ \nu_t^{\text{labor}} \\ \nu_t^{\text{nominal}} \end{bmatrix}. \tag{4.19}$$

where a, b, c, and e are constants[21]. This equation indicates that the technology shock and the labor-market shock have zero long-run impact on the nominal variables, R and π. In contrast, the nominal shock has zero long-run effect on the real variables, y, mr and wr. Estimation of the long-run model based on $\beta_\perp$ is accomplished by specifying an initial $\beta_\perp^0$ and then find a $k \times k$ lower-triangular matrix T with unit principal diagonal such that $\beta_\perp = \beta_\perp^0 T$ is of the form in (4.19). Select the initial choice as $\beta_\perp^0 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. To find out what restrictions are necessary for

[21] According to (3.28), $a = d$, $b = 1 - \frac{d}{\beta}$, $c = -h$ and $e = \frac{h}{\beta}$.

$$T = \begin{bmatrix} 1 & 0 & 0 \\ t_{21} & 1 & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}, \text{ obtain}$$

$$\begin{aligned} \beta_\perp &= \beta_\perp^0 T \\ &= \begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ t_{21} & 1 & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix} \\ &= \begin{bmatrix} -t_{21} & -1 & 0 \\ -t_{21} & -1 & 0 \\ 1+t_{21} & 1 & 0 \\ t_{31} & t_{32} & 1 \\ t_{31} & t_{32} & 1 \end{bmatrix}. \end{aligned} \tag{4.20}$$

From (4.20) it is obvious that to derive a $\beta_\perp$ of the form as in (4.19), we have to impose two zero restrictions on T, i.e., $t_{31} = t_{32} = 0$.

Note that $C(1)$ in (4.10) can be expressed as $C(1)' = \begin{bmatrix} c_1' & c_1' & c_2' & c_3' & c_3' \end{bmatrix}$ where c_i for $i = 1, 2, 3$ are all 1×5 row vectors. The initial common-trends matrix $\alpha_\perp'$ is obtained as

$$\begin{aligned} \alpha_\perp^{0\prime} &= \left(\beta_\perp^0 \beta_\perp^0\right)^{-1} \beta_\perp^{0\prime} C(1) \\ &= \begin{bmatrix} 0.5 & 0.5 & 1 & 0 & 0 \\ -0.5 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \end{bmatrix} C(1) = \begin{bmatrix} c_1 + c_2 \\ -c_1 \\ c_3 \end{bmatrix} \\ &= \begin{bmatrix} 0.7313 & 1.0090 & 0.0244 & -0.0125 & -0.0043 \\ -0.5370 & -1.0080 & 0.8190 & 0.0097 & 0.0057 \\ -21.0261 & 45.1500 & 22.6276 & 0.3734 & 0.1933 \end{bmatrix}. \end{aligned} \tag{4.21}$$

The eventually identified permanent shocks are subject to the same restrictions in T according to

$$\nu_t^P \equiv \alpha_\perp' \varepsilon_t = T^{-1} \alpha_\perp^{0\prime} \varepsilon_t \tag{4.22}$$

in order for $C(1) = \beta_\perp^0 \alpha_\perp^{0\prime} = \beta_\perp \alpha_\perp'$ to hold.

We will demonstrate the technique of identification in the setting of a transformed VECM. Premultiply the VECM in (4.5), omitting constant and dummy variable terms to save space, by a full rank $n \times n$ permutation matrix

$$W = \begin{bmatrix} W_1 C(1) \\ W_2 \end{bmatrix} \tag{4.23}$$

where W_1 is $k \times n$ and W_2 is $r \times n$. Then we have

$$\begin{bmatrix} W_1 C(1) \\ W_2 \end{bmatrix} \Delta x_t = \begin{bmatrix} W_1 C(1) \\ W_2 \end{bmatrix} \left[\sum_{i=1}^{3} \Gamma_i \Delta x_{t-i} + \alpha\beta' x_{t-1} + \varepsilon_t \right]$$

$$= \begin{bmatrix} W_1 C(1) \\ W_2 \end{bmatrix} \sum_{i=1}^{3} \Gamma_i \Delta x_{t-i} + \begin{bmatrix} 0 \\ W_2 \alpha\beta' x_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t}^* \\ \varepsilon_{2t}^* \end{bmatrix} \tag{4.24}$$

where $\varepsilon_t^* \equiv \begin{bmatrix} \varepsilon_{1t}^* \\ \varepsilon_{2t}^* \end{bmatrix} \equiv \begin{bmatrix} W_1 C(1) \\ W_2 \end{bmatrix} \varepsilon_t$. Notice the $k \times k$ covariance $E\left(\varepsilon_{1t}^* \varepsilon_{1t}^{*\prime}\right) \equiv \Sigma_1^* = W_1 C(1) \Sigma C(1)' W_1$.

Define $\varepsilon_t^* \equiv B\nu_t$ where $B = \begin{bmatrix} B_{11} & 0 \\ B_{21} & B_{22} \end{bmatrix}$ and $E\left[\nu_t \nu_t'\right] = D = \begin{bmatrix} D_p & 0 \\ 0 & D_T \end{bmatrix}$. B_{11} and B_{22} are lower-triangular matrices with unit principal diagonals. Also both D_P and D_T are diagonal matrices. Then we have the relation $B_{11}\nu_t^P = \varepsilon_{1t}^*$ and the first k equations in (4.24),

$$W_1 C(1) \Delta x_t = W_1 C(1) \sum_{i=1}^{3} \Gamma_i \Delta x_{t-i} + B_{11} \nu_t^P,$$

can be expressed as

$$B_{11}^{-1} W_1 C(1) \Delta x_t = B_{11}^{-1} W_1 C(1) \sum_{i=1}^{3} \Gamma_i \Delta x_{t-i} + \nu_t^P. \tag{4.25}$$

Working on only the k transformed long-run equations we can identify permanent shocks by

$$\nu_t^P = B_{11}^{-1} W_1 C(1) \varepsilon_t \tag{4.26}$$

and have $E\left(\nu_t^P \nu_t^{P\prime}\right) = B_{11}^{-1}\Sigma_1^* B_{11}^{-1\prime} = D_P$ that is diagonal.

To begin estimation first specify

$$W_1 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.27}$$

and so

$$\begin{aligned} W_1 C(1) &= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_1 \\ c_2 \\ c_3 \\ c_3 \end{bmatrix} \\ &= \begin{bmatrix} c_1 + c_2 \\ -c_1 \\ c_3 \end{bmatrix} = \alpha_\perp^{0\prime}. \end{aligned} \tag{4.28}$$

With $W_1 C(1) = \alpha_\perp^{0\prime}$ established in (4.28), then (4.26) can be written as

$$\nu_t^P = B_{11}^{-1}\alpha_\perp^{0\prime}\varepsilon_t. \tag{4.29}$$

Comparing (4.29) to the permanent shocks equation $\nu_t^P = T^{-1}\alpha_\perp^{0\prime}\varepsilon_t$ in (4.22), we find that $T = B_{11}$. Thus the loading matrix $\beta_\perp$ in (4.20) is also identified as $\beta_\perp = \beta_\perp^0 B_{11}$. In estimating $B_{11} = \begin{bmatrix} 1 & 0 & 0 \\ b_{21} & 1 & 0 \\ b_{31} & b_{32} & 1 \end{bmatrix}$ we also need to impose the restrictions $b_{31} = b_{32} = 0$ as is done to T in (4.20) in order for $\beta_\perp$ to match its theoretical form.

We now demonstrate how the permanent technology, the labor-market and the nominal shocks are identified by the long-run multipliers which are implicitly defined in the moving-average representation of $W_1\Delta x_t$ (Rasche 1997) or

$$\begin{aligned} W_1\Delta x_t &= W_1 C(L)\varepsilon_t \\ &= W_1 C(1)\varepsilon_t + W_1(1-L)C^*(L)\varepsilon_t \\ &= B_{11}\nu_t^P + W_1(1-L)C^*(L)\varepsilon_t. \end{aligned} \tag{4.30}$$

In the long run (4.30) becomes $W_1 \Delta x_t = B_{11} \nu_t^P$ or

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta y_t \\ \Delta mr_t \\ \Delta wr_t \\ \Delta R_t \\ \Delta \pi_t \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta y_t \\ \Delta wr_t \\ \Delta \pi_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ b_{21} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_t^{\text{tech}} \\ \nu_t^{\text{labor}} \\ \nu_t^{\text{nominal}} \end{bmatrix}. \tag{4.31}$$

Premultiply through the second equation in (4.31) by $\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$ to get

$$\begin{bmatrix} \Delta y_t \\ \Delta wr_t \\ \Delta \pi_t \end{bmatrix} = \begin{bmatrix} -b_{21} & -1 & 0 \\ 1 + b_{21} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_t^{\text{tech}} \\ \nu_t^{\text{labor}} \\ \nu_t^{\text{nominal}} \end{bmatrix}.$$

Thus the permanent technology shock is identified as having a long-run multiplier on real output equal to $-b_{21}$ (it turns out $-b_{21} = 0.906$), the labor-market shock having a unitary long-run multiplier for real wages and the permanent nominal shock also a unitary long-run multiplier for inflation.

The covariance matrix of the first k equation in (4.24) is calculated as

$$\begin{aligned} \Sigma_1^* &= B_{11} D_P B_{11}' = W_1 C(1) \Sigma C(1)' W_1' \\ &= 10^{-4} \times \begin{bmatrix} 2.258 & -2.046 & -8.825 \\ -2.046 & 2.019 & 8.810 \\ -8.825 & 8.810 & 1313 \end{bmatrix}. \end{aligned} \tag{4.32}$$

The decomposition of Σ_1^* into B_{11} and D_P is computed with SVAR. The results are

$$B_{11} = \begin{bmatrix} 1.0 & 0 & 0 \\ -0.906 & 1.0 & 0 \\ 0 & 0 & 1.0 \end{bmatrix} \tag{4.33}$$

and

$$D_P = 10^{-4} \times \begin{bmatrix} 2.58 & 0 & 0 \\ 0 & 0.1654 & 0 \\ 0 & 0 & 1313 \end{bmatrix}.$$

The overidentifying restriction test for the estimation is χ^2 distributed with two degrees of freedom. The test statistic is 4.523 with a p-value of 0.1042 and thus the overidentifying restrictions of the common-trends model is not rejected by the data at a 10 percent significance level.

The permanent shocks are then identified as

$$\begin{aligned} \nu_t^P &= B_{11}^{-1} W_1 C(1) \varepsilon_t = B_{11}^{-1} \alpha_{\perp}^{0\prime} \varepsilon_t = \begin{bmatrix} 1.0 & 0 & 0 \\ 0.906 & 1.0 & 0 \\ 0 & 0 & 1.0 \end{bmatrix} \\ &\quad \times \begin{bmatrix} 0.7313 & 1.0090 & 0.0244 & -0.0125 & -0.0043 \\ -0.5370 & -1.0080 & 0.8190 & 0.0097 & 0.0057 \\ -21.0261 & 45.1500 & 22.6276 & 0.3734 & 0.1933 \end{bmatrix} \varepsilon_t \\ &= 10^{-2} \times \begin{bmatrix} 73.13 & 100.9 & 2.44 & -1.25 & -0.43 \\ 12.57 & -9.3644 & 84.111 & -0.1628 & 0.1803 \\ -2102.6 & 4515 & 2262.8 & 37.34 & 19.33 \end{bmatrix} \varepsilon_t. \end{aligned} \tag{4.34}$$

The common-trends loading or the long-run multiplier of permanent shocks is calculated as

$$\begin{aligned} \beta_{\perp} &= \beta_{\perp} B_{11} \\ &= \begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.0 & 0 & 0 \\ -0.906 & 1.0 & 0 \\ 0 & 0 & 1.0 \end{bmatrix} \\ &= \begin{bmatrix} 0.906 & -1 & 0 \\ 0.906 & -1 & 0 \\ 0.094 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

The steady state equilibrium of the long-run economic model is expressed as

$$\begin{bmatrix} \Delta y_t \\ \Delta mr_t \\ \Delta wr_t \\ \Delta R_t \\ \Delta \pi_t \end{bmatrix} = \begin{bmatrix} 0.906 & -1 & 0 \\ 0.906 & -1 & 0 \\ 0.094 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_t^{\text{tech}} \\ \nu_t^{\text{labor}} \\ \nu_t^{\text{nominal}} \end{bmatrix}. \tag{4.35}$$

We find that the numeric values of the elements in $\beta_\perp$ are all within the range for parameters stated in the theoretical model in (3.28) and (3.2)-(3.4).

Chapter 5
MACROECONOMIC IMPULSE ANALYSIS

5.0 Introduction

This chapter presents the impulse response function and forecast-error variance decomposition analysis with respect to the identified permanent shocks or common stochastic trends (Ahmed and Park 1994). Innovation accounting is also extended to nominal balances and nominal wages which are not explicitly modeled in the previous Vector Error-Correction Models but are nonetheless integral elements of the economic analysis to be conducted.

The long-run responses of variables to the permanent shocks are restricted by the common-trends loading matrix stated in (4.35). As a result, real variable movements are strongly influenced by real permanent shocks in the long run in the sense that a high percentage of the forecast-error variance is explained by the technology and the labor-market shock. Specifically, the technology shock has long-run positive effects on both output, real balances and real wages. The labor-market shock also impacts positively on real wages but negatively on output and real balances. On the other hand, nominal variables in the long run are exclusively dominated by the permanent nominal shock also in a variance-decomposition sense. The nominal interest rate, inflation, nominal balances and nominal wages all respond positively to the permanent nominal shock. Such distinct dichotomy between the real and the nominal shock effects is only reasonable while the economy is in its steady state equilibrium.

The permanent shocks, as also found in other studies, are important sources of fluctuations in the short term. Specifically, the technology shock accounts for about 40 to 60 percent of the output variance on one- to two-year horizons. King et al. (1991) and Mellander et al. (1992) arrived at similar percentages. More significantly, the technology shock accounts for about 80 percent of the real balance forecast variance in the very short run, comparable to about 70 percent found by King et al. (1991). The labor-market shock explains about 20 percent of the output variance within the first year. This compares to the finding of Shapiro and Watson (1988) that at least 40 percent of the output variance is accounted for by a labor-supply shock. Labor-market shocks explain 40 percent of the real wage variance in the impact quarter and its influence increases thereafter. As for the permanent nominal shock, it accounts for a high proportion of the nominal interest rate variability. This is similar to the finding of Englund et al. (1994) but different from the extremely low percentages found by King et al. (1991). The nominal trend accounts for at most 30 percent of the inflation variance within the first two years. It is at least 30 percent in King et al. (1991) on the same horizon.

One seemingly unsatisfactory aspect of the impulse analysis in this chapter is that the parameter estimates are measured very imprecisely. The standard errors of the impulse response parameters and variance decompositions are calculated with the asymptotic distribution approach used by Giannini (1992) and Warne (1990). Their large size renders most of the parameter estimates insignificant. This is, however, not a phenomenon new to studies employing the VAR methodology and has been discussed in Runkle (1987). We can still gain valuable knowledge from the general patterns revealed regarding the dynamic effects of stochastic impulses on the economy. More importantly the impulse-response patterns and the variance decompositions are amenable to economic interpretations consistent with the theoretical model in Chapter 3, as is presented in the following sections.

5.1 Effects of the Permanent Technology Shock

The impacts of the technology shock on five variables in the VECM model, nominal balances, nominal wages and two cointegration relations, are illustrated by their impulse response functions (IRFs) plotted in Figures 13.1 to 13.9. As is discussed in the identification setup of Chapter 4, in equation (4.29), the technology shock is normalized to produce equal long-run effects, 0.9, on real output and real balances. We observe that the responses of output, real and nominal balances are all positive across the horizon. Output and real balances respond to technology shocks in a very timely fashion, reaching major fractions of their long-run responses only in six quarters. The short-run impacts on the velocity are negative, indicating output increases at a rate slower than real balances whose increase is aided by falling prices.

The long-run effect of the technology shock on real wages is normalized to 0.09 as shown in (4.29). In the first three years, the response of real wages overshoots its long-run steady-state level as shown in Figure 13.5. This is most likely due to the mechanism of delayed wage adjustment to price changes even though nominal wages steadily decrease. This in turn indicates prices fall at a much faster rate than nominal wages decrease in the beginning. This conjecture about the response of prices is borne out by the IRF of inflation with respect to technology shocks shown in Figure 13.8 which shows a steep drop of the inflation rate in the first year. It is reasonable that rising productivity tends to produce a disinflationary effect. It in turn could lower the inflation premium charged by the nominal rate. The temporary effect on the nominal interest rate is indeed negative, as shown in Figure 13.7, but smaller than the negative effect on inflation. As a result, there is a positive response of the ex post real interest rate as shown in Figure 13.9.

Percentages of the forecast-error variance of variables attributable to the technology shock are shown in Table 10. In the long run, output and real balance error-

Table 10
Percentage of Forecast-Error Variance Attributed to the Permanent Technology Shock

Forecast Horizon	Output	Real Balances	Nominal Balances	Real Wages	Nominal Wages	Interest Rate	Inflation
0	10.25	63.70	35.50	12.10	1.80	17.98	31.76
1	14.54	77.43	48.69	10.31	7.30	20.20	29.17
2	23.53	82.42	49.92	19.53	6.75	21.36	28.65
3	36.46	85.94	52.98	25.31	6.97	20.00	28.10
4	44.23	87.55	53.89	29.90	7.19	18.20	26.40
8	68.31	89.20	49.35	35.02	7.65	12.79	21.59
12	78.48	89.16	37.23	31.84	7.67	10.44	19.01
16	83.32	89.32	26.71	27.24	7.37	9.02	17.20
24	87.62	89.40	15.51	20.14	6.14	7.06	14.50
36	89.28	89.33	8.24	14.07	4.43	5.35	11.81
48	89.53	89.26	5.03	10.91	3.22	4.32	9.98

variances are predominantly accounted for by the technology shock. The variability of output over the near term is already strongly affected by technology shocks. About 40 to 60 percent of the error variance is explained during the second year. King et al. (1991) and Mellander et al. (1992) arrived at similar percentages for their technology shocks. The technology shock accounts for as high as over 60 percent of the forecast variance of real balances even in the impact quarter. The percentage increases on longer horizons. This is a more dramatic result compared to that of King et al. (1991) where about 70 percent of real balance variance is explained by a permanent technology shock. In comparison, its role in affecting nominal balances is not nearly as huge although still very critical for up to four years. We observe that even though in the long run the technology shock plays no role in affecting the nominal interest rate and inflation, it accounts for 20 to 30 percent of the forecast variance in both during the first year. Real wage variability is explained at most about 30 percent by the technology shock on any horizon. A bulk of the source of the real wage variance comes from the labor-market shock. On the whole nominal wages are only slightly affected by the technology shock.

5.2 Effects of the Permanent Labor-Market Shock

Figures 14.1 to 14.9 plot the impulse response patterns of variables to the labor-market shock. The shock is normalized to yield in the long-run an unit impact on real wages and a negative unit impact on both output and real balances. The effects on the growth of nominal balances, in Figure 14.4, are smaller than that of nominal wages, in Figure 14.6, although both are steadily increasing before reaching an almost constant rate. Due to a labor-market shock, real balances, in Figure 14.2, decrease while real wages, in Figure 14.5, increase. This implies that the growth rate of prices (the inflation level) increases at a pace faster than that of nominal balances and slower than the growth of rate nominal wages.

According to the theoretical wage-contract model, an upward adjustment of nominal wages is made slower by a wage contract mechanism. When a positive labor-market shock strikes, real wages are rising gradually. During the adjustment process, the wage contract allows firms to hire extra workers at their discretion when real wages are still sufficiently low relative to a new expected equilibrium real wage rates. For this reason there is a short-run boost in output growth as is apparent in Figure 14.1. As the nominal wage level catches up and as the short-run aggregate supply curve, whose position depends negatively on nominal wages, shifts up the positive output effect diminishes. Eventually the growth rate of output becomes negative to reflect a new tighter labor market condition caused by the labor-market shock. It is interesting to observe that while the long-run negative unitary response of real balances comes into place in about two years, it takes more than seven years for real wages and output to reach their respective equilibrium response of 1 and -1.

Table 11
Percentage of Forecast-Error Variance Attributed to
the Permanent Labor-Market Shock

Forecast Horizon	Output	Real Balances	Nominal Balances	Real Wages	Nominal Wages	Interest Rate	Inflation
0	29.95	3.23	0.04	40.75	90.46	0.31	18.58
1	26.69	2.96	0.39	52.27	80.14	0.43	20.04
2	22.32	2.60	1.02	46.44	75.84	2.83	19.73
3	18.80	2.68	1.40	44.19	66.96	5.95	20.89
4	16.45	3.42	1.39	43.79	62.57	7.67	20.63
8	8.44	6.43	1.86	47.11	51.71	9.39	20.73
12	5.77	7.78	3.45	55.38	45.09	8.64	19.65
16	4.87	8.29	4.86	63.28	40.19	7.87	18.43
24	4.89	8.91	4.62	73.70	31.68	6.58	16.11
36	6.05	9.42	3.16	81.56	22.19	5.19	13.37
48	7.07	9.71	2.16	85.34	15.95	4.27	11.39

The long-run impacts on the nominal interest rate and inflation are restricted to zero. In the short term they respond positively to the labor-market shock as shown in Figures 14.7 and 14.8. Upon impact, the inflation effect is the highest and it then gradually diminishes as the output level adjusts. The peak response of the nominal rate occurs one year after an impact and then the response tapers off. There is a huge negative initial effect in the ex post real rate. It rapidly disappears as an inflation premium is factored into the nominal rate.

The role of the labor-market shock in accounting for the forecast-error variances is shown in Table 11. Three major points are worth noting. First, as expected by its design, the labor market shock is a dominant source of variability in real wages, accounting from an initial 40 percent to 85 percent in the 48th quarter. Moreover, the proportions of the nominal wage variance explained within two years are even higher than that of the real wage, with a high of 90 percent in the impact quarter.

Second, the proportion of the output forecast variance is 30 percent initially and goes down as the horizon lengthens. Third, the labor market shock consistently explains a non-trivial portion of the inflation forecast variance, between 10 and 20 percent. There are no significant explanatory power from the labor-market shock for the variances of real balances, nominal balances and the nominal interest rate.

5.3 Effects of the Permanent Nominal Shock

The impulse responses to the permanent nominal shock are plotted in Figures 15.1 to 15.9. The nominal shock is designed to produce an unitary effect in the long run on both the nominal interest rate and the inflation rate. Since nominal neutrality is dictated by the wage-contract model, the total impacts on output, real balances and real wages are zero in the long run. Even temporary impacts on the three real quantities are very short-lived and disappear completely in less than three years. The nominal shock temporarily increases both production in the economy and the buying power of money as shown in Figures 15.1 and 15.2. There is an obvious delay in real output growth relative to the real balance increase because the peak response of output is about one year later than that of the real balance. Consequently there is an initial negative impact on the velocity from the permanent nominal shock as in Figure 15.3.

The IRFs of nominal balances and nominal wages, shown in Figures 15.4 and 15.6 respectively, are monotonically increasing at a constant rate. The reason for this originates from the identities by which the responses of nominal balances and nominal wages are obtained,

$$\Delta m_t \equiv \Delta mr_t + \pi_t \tag{5.1}$$

$$\Delta w_t \equiv \Delta wr_t + \pi_t. \tag{5.2}$$

The long-run effects of the nominal shock on both Δmr_t and Δwr_t are restricted

Table 12

Percentage of Forecast-Error Variance Attributed to the Permanent Nominal Shock

Forecast Horizon	Output	Real Balances	Nominal Balances	Real Wages	Nominal Wages	Interest Rate	Inflation
0	0.02	33.06	52.34	1.73	6.95	19.46	2.95
1	1.67	17.50	36.80	1.23	8.86	25.53	9.36
2	2.26	10.96	29.71	0.87	13.48	27.10	13.45
3	3.25	7.85	28.54	1.52	19.66	32.78	16.47
4	3.54	5.75	28.29	1.22	22.72	39.86	21.33
8	2.23	2.56	34.52	0.70	31.06	52.32	30.67
12	1.48	1.69	46.04	0.52	37.24	59.45	37.08
16	1.12	1.28	55.78	0.38	42.58	64.70	42.35
24	0.75	0.86	69.42	0.23	54.10	71.95	50.69
36	0.49	0.59	81.73	0.14	67.77	78.50	59.47
48	0.37	0.45	88.19	0.09	76.85	82.53	65.59

to be zero by the requirement of nominal neutrality. On the other hand, the long-run shock effect on the first-difference of inflation, $\Delta\pi_t$, is restricted to 1.0, making the nominal shock effects on the level of inflation, π_t, infinitely cumulating. The responses of nominal balances and nominal wages are thus dominated by this cumulative impact on the inflation level. In the long run the cumulative trend in nominal balances and nominal wages cancels out that in inflation. This allows nominal neutrality to hold with respect to real balances and real wages in the model.

The short-term effects on real wages, in Figure 15.5, are initially positive and then are erased in about three years. At first glance, the nominal interest rate and inflation have very similar response patterns. The effects are fairly moderate in the first year, overshoot the long-run unitary levels in the second year and reach the long-run near the beginning of the third year. However the fact is that their exact short-run response paths are very different. This is evidenced by the volatile response pattern of the ex post real rate during the first three years shown in Figure 15.9.

The forecast-error variance decompositions for the permanent nominal shock are shown in Table 12. Four observations are particularly worth noting. First, the nominal shock is a very important source of variability for the nominal interest rate and inflation on horizons longer than twelve quarters. This is not surprising since the nominal shock is identified by having long-run impacts only on nominal rates and inflation. Second, nominal balances and nominal wages are also dominated by the nominal shock over the longer term. This is consistent with the above discussion that shows the IRFs of both variables with respect to the nominal shock are dominated by the cumulative inflation level responses to the nominal shock. Third, over the very short run the nominal shock is less important for explaining the inflation error variance. Lastly, the nominal shock accounts for essentially none of the error variance for output and real wages. It also explains very little of the real-balance variance beyond an one-year horizon. These results seem to suggest the permanent shock which is identified by a long-run neutrality condition is also quite neutral in the short run.

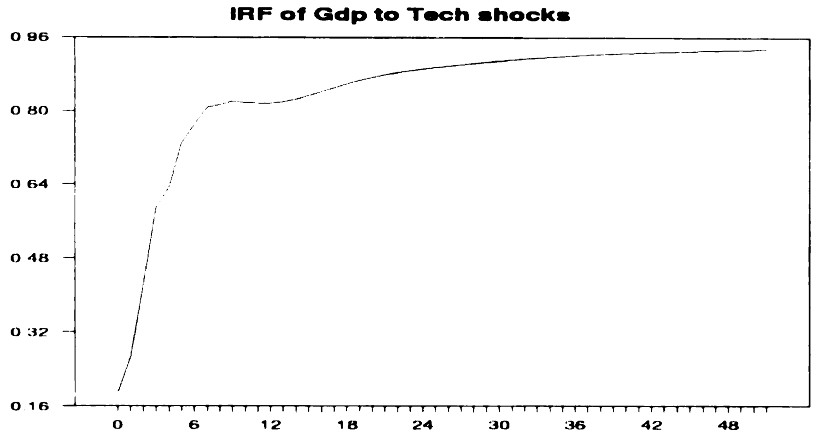

Figure 13.1 The Response of Output to Technology Shocks

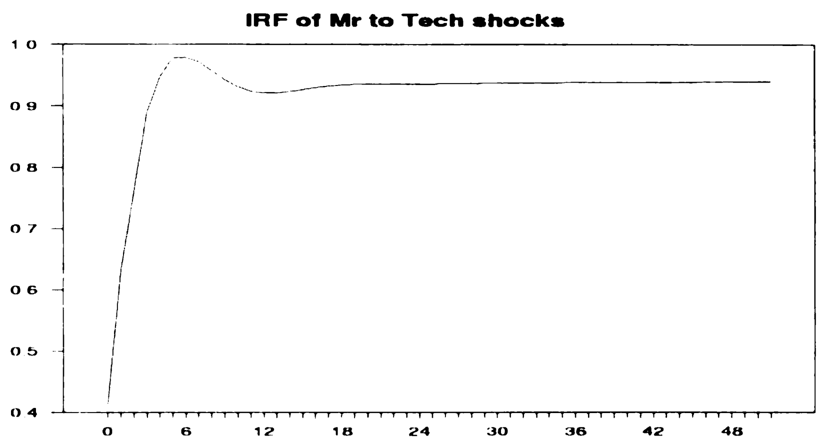

Figure 13.2 The Response of Real Balances to Technology Shocks

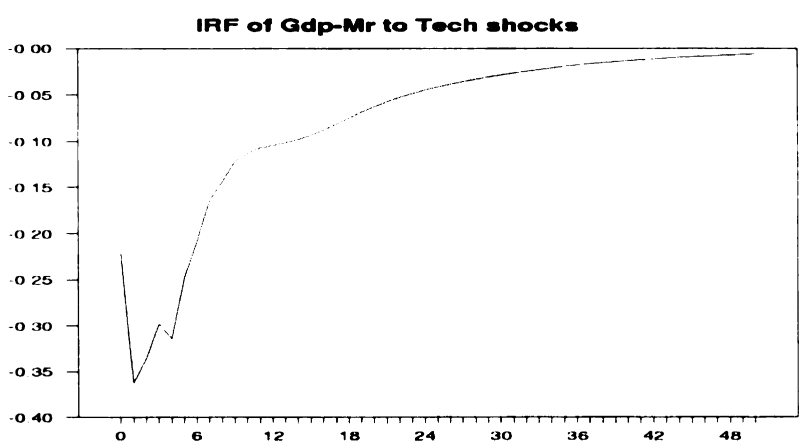

Figure 13.3 The Response of Velocities to Technology Shocks

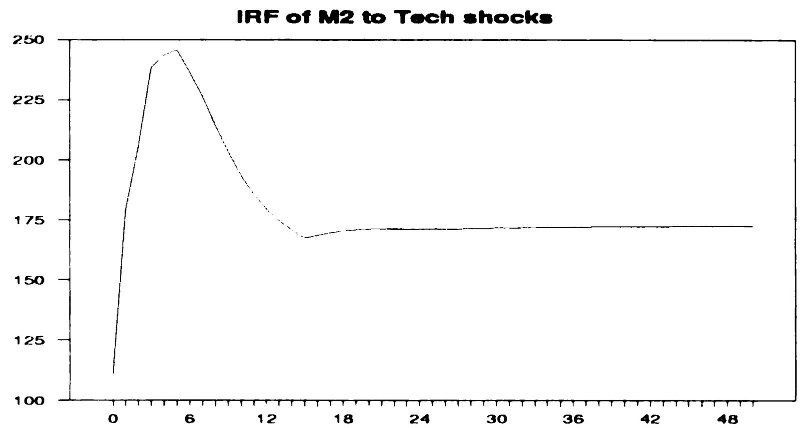

Figure 13.4 The Response of Nominal Balances to Technology Shocks

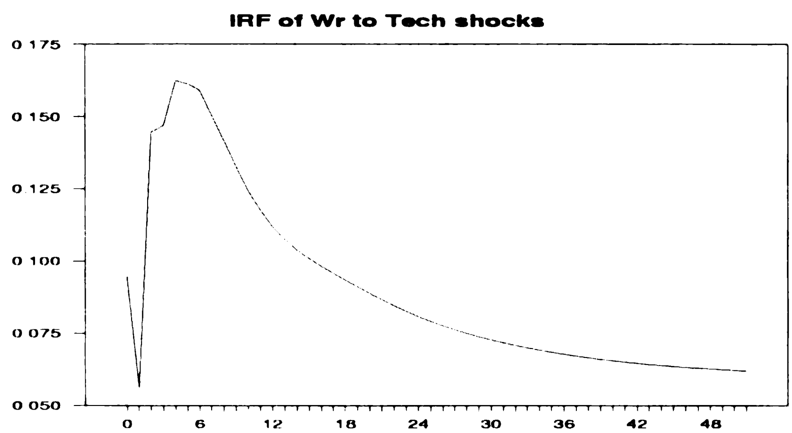

Figure 13.5 The Response of Real Wages to Technology Shocks

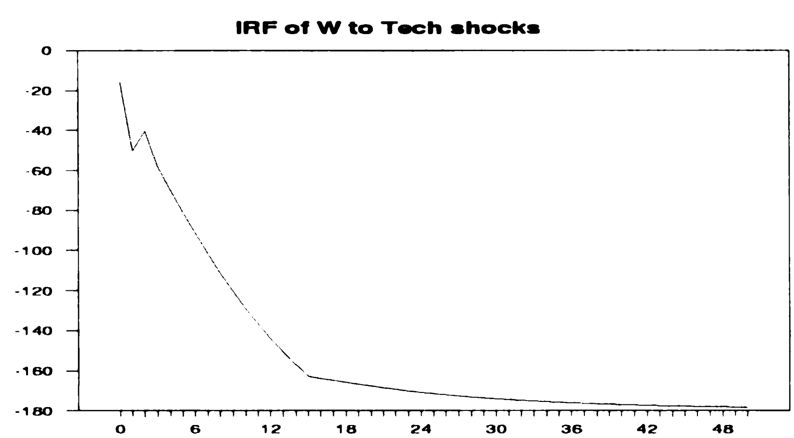

Figure 13.6 The Response of Nominal Wages to Technology Shocks

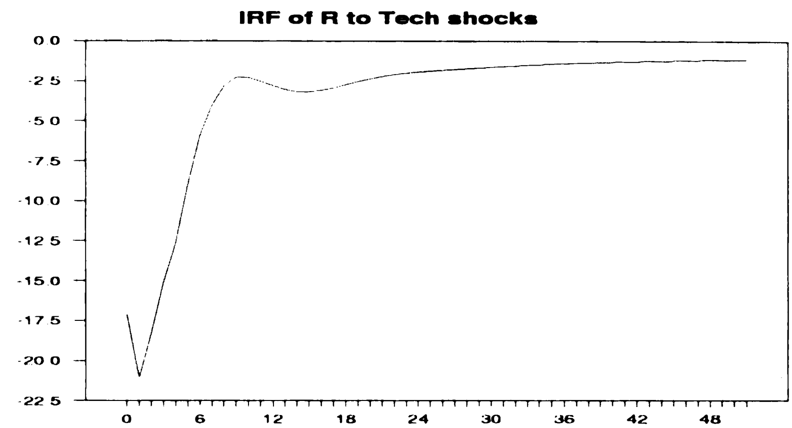

Figure 13.7 The Response of Interest Rates to Technology Shocks

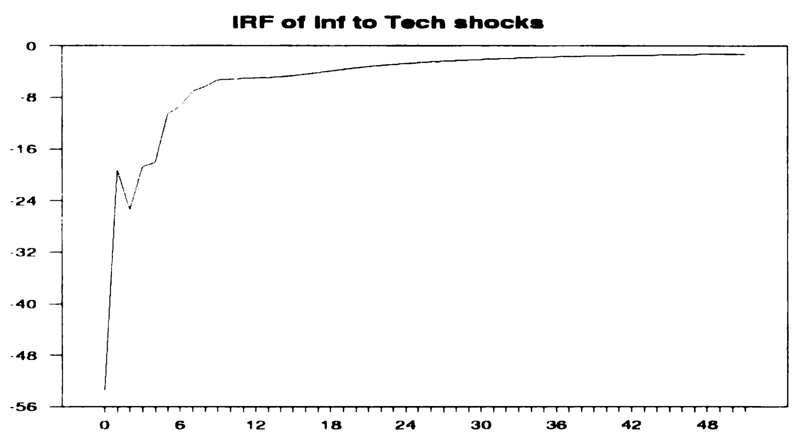

Figure 13.8 The Response of Inflation to Technology Shocks

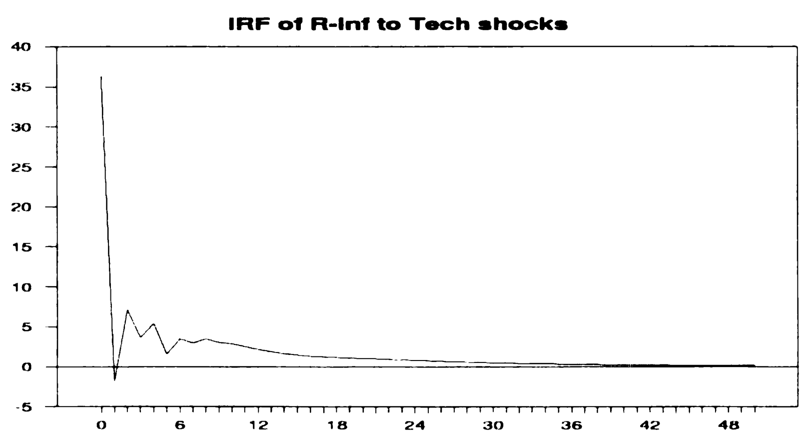

Figure 13.9 The Response of Ex Post Real Rates to Technology Shocks

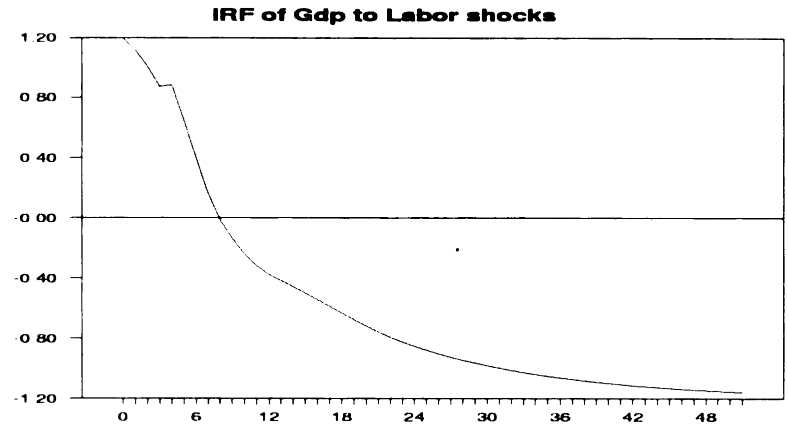

Figure 14.1 The Response of Output to Labor-Market Shocks

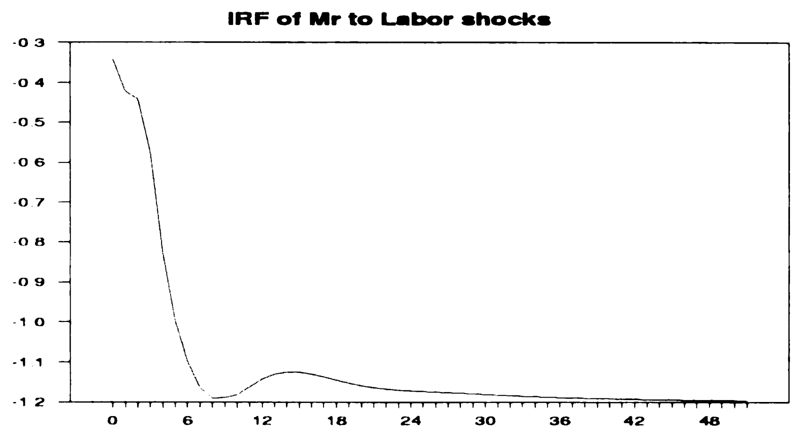

Figure 14.2 The Response of Real Balances to Labor-Market Shocks

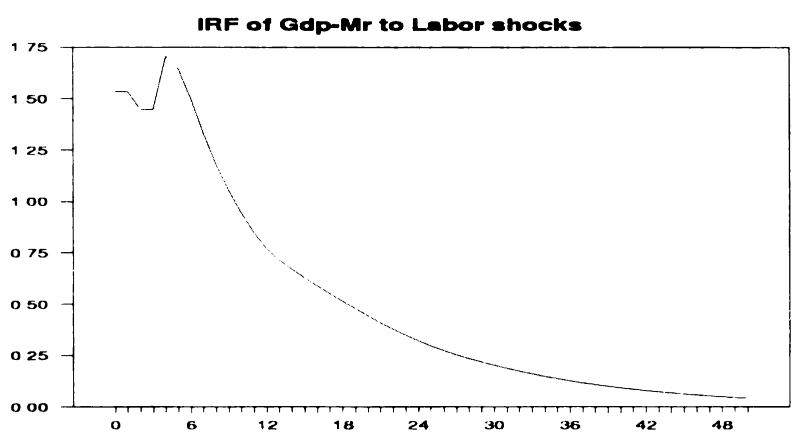

Figure 14.3 The Response of Velocities to Labor-Market Shocks

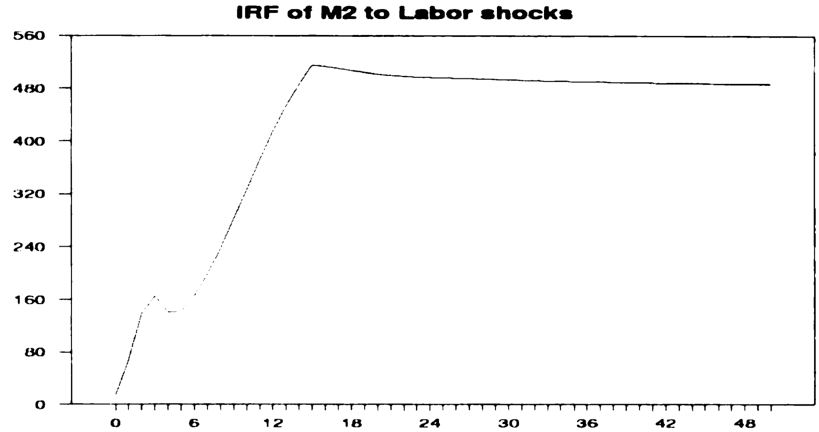

Figure 14.4 The Response of Nominal Balances to Labor-Market Shocks

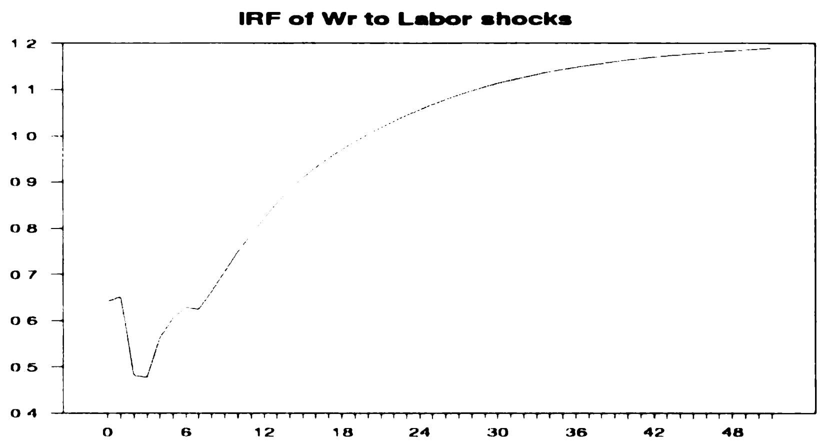

Figure 14.5 The Response of Real Wages to Labor-Market Shocks

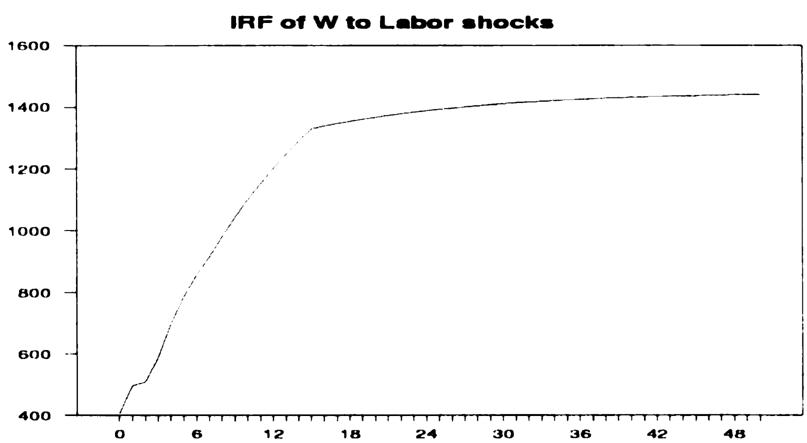

Figure 14.6 The Response of Nominal Wages to Labor-Market Shocks

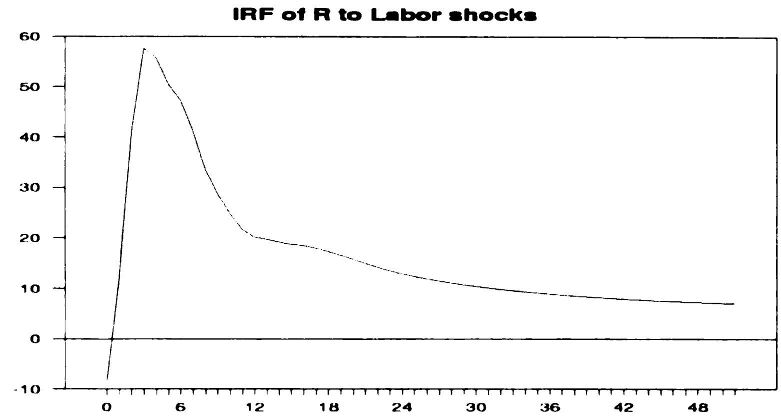

Figure 14.7 The Response of Interest Rates to Labor-Market Shocks

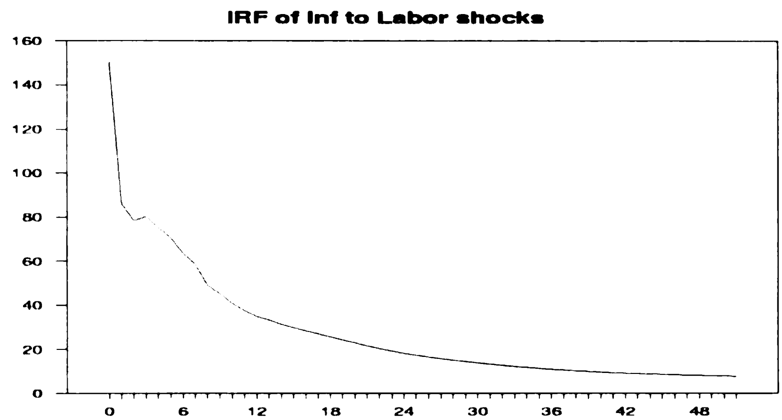

Figure 14.8 The Response of Inflation to Labor-Market Shocks

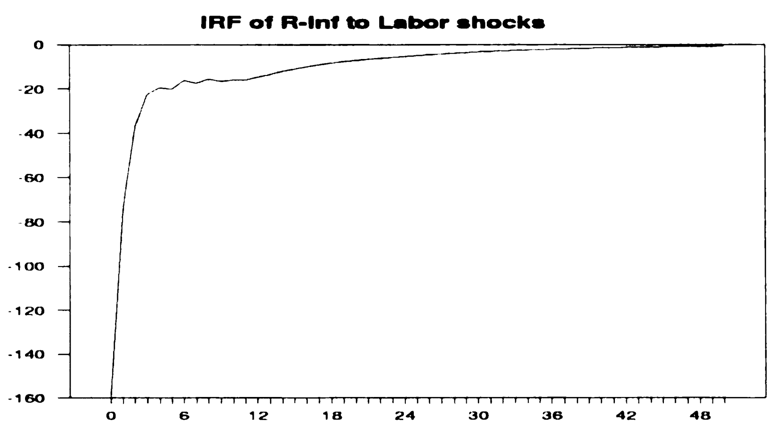

Figure 14.9 The Response of Ex Post Real Rates to Labor-Market Shocks

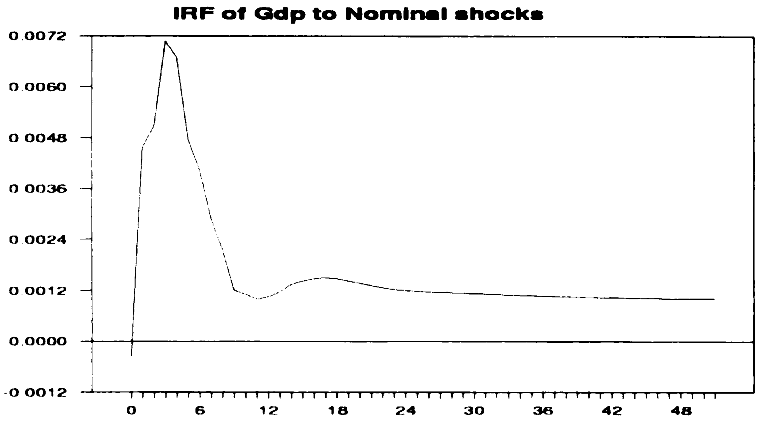

Figure 15.1 The Response of Output to Nominal Shocks

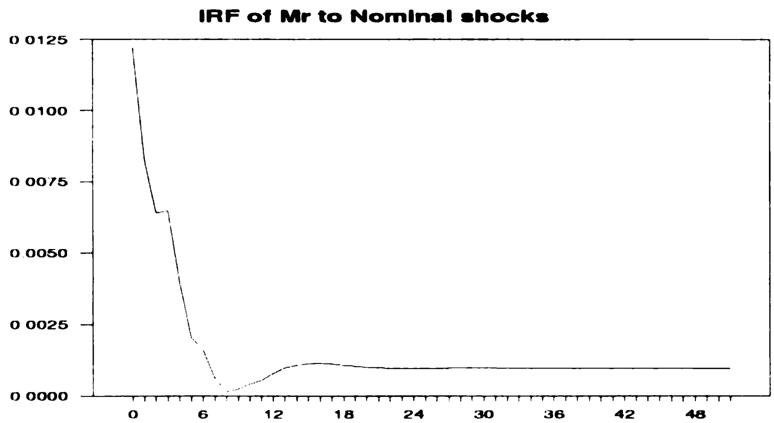

Figure 15.2 The Response of Real Balances to Nominal Shocks

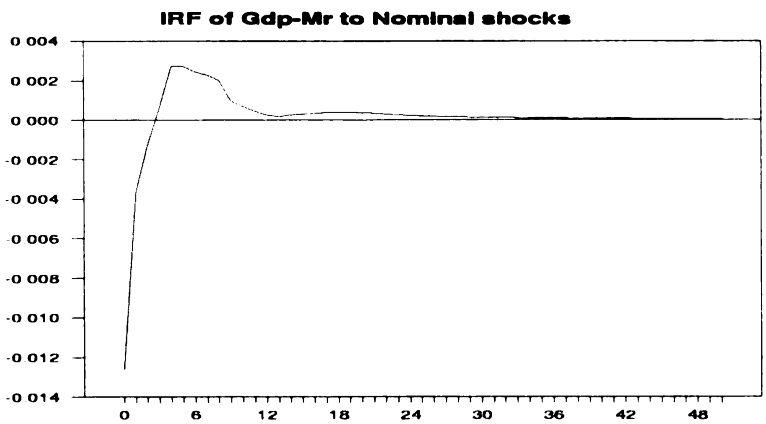

Figure 15.3 The Response of Velocities to Nominal Shocks

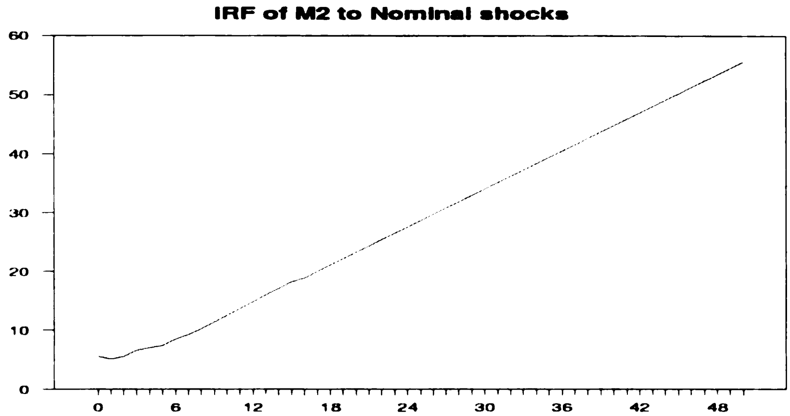

Figure 15.4 The Response of Nominal Balances to Nominal Shocks

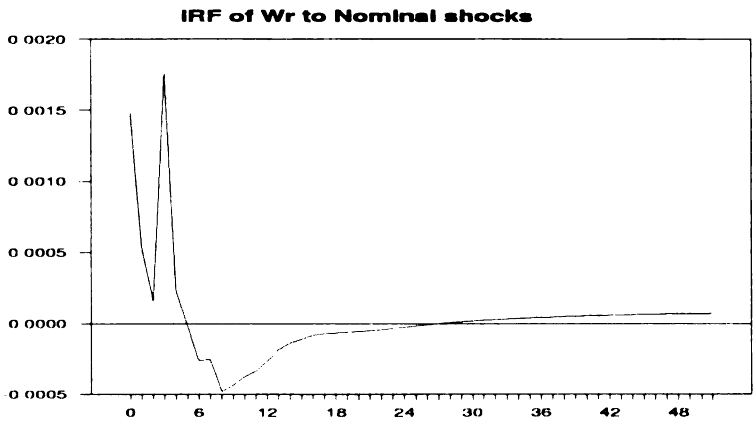

Figure 15.5 The Response of Real Wages to Nominal Shocks

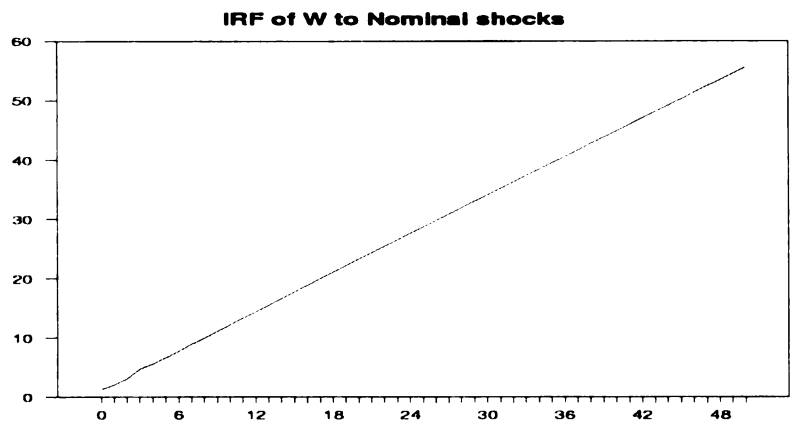

Figure 15.6 The Response of Nominal Wages to Nominal Shocks

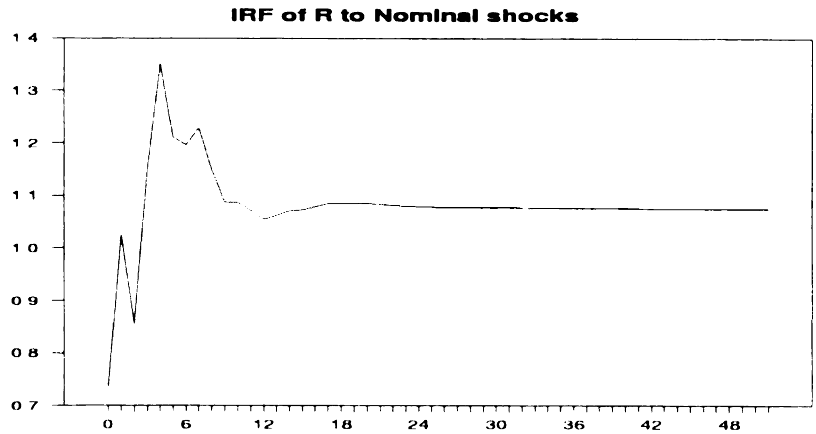

Figure 15.7 The Response of Interest Rates to Nominal Shocks

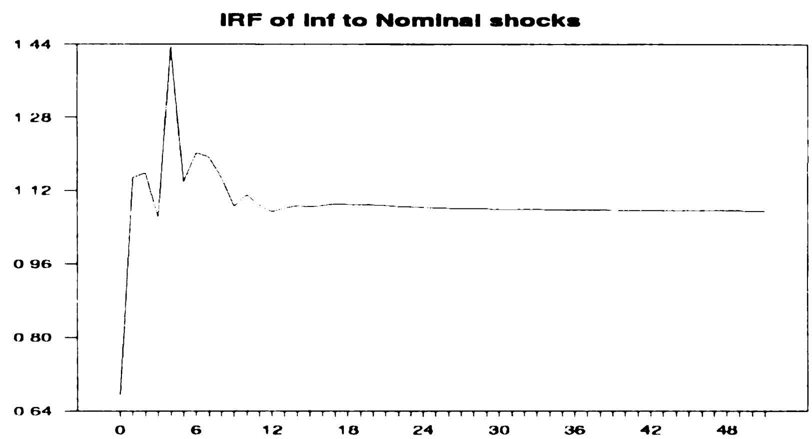

Figure 15.8 The Response of Inflation to Nominal Shocks

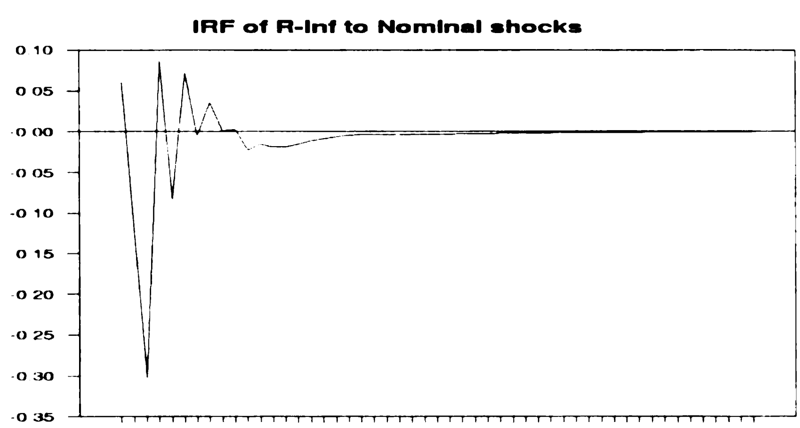

Figure 15.9 The Response of Ex Post Real Rates to Nominal Shocks

Chapter 6
SUMMARY AND CONCLUSION

This study of postwar U.S. economic fluctuations using the common-trends methodology is guided by a simple theoretical model that has properties suitable for business cycle studies. It includes a wage-contract equation to partly account for the wage rigidity characteristics in the economy. In addition, the model allows for cyclical real wage and price behaviors that are consistent with predictions from both the Keynesian and the real business cycle theories. In this model we are able to identify the roles of the permanent shocks as sources of economic fluctuations. Particularly, the permanent labor-market shock is featured to capture a separate effect on the aggregate supply that is independent of that from the technology shock. All variables used have stochastic-trend components and are widely considered to have significant cyclical properties. Among them, real wages, to my knowledge, has not been modeled previously in the common-trends model framework.

Two cointegration relations implied by the wage contract model are confirmed by the unit-root tests, cointegration rank tests and Horvath and Watson's (1995) cointegrating-vector tests. They are then imposed in estimating a VAR model that includes five time dummies. The acquired empirical model cannot be rejected by a likelihood-ratio test at a 10 percent significance level. A first attempt to identify both permanent and transitory structural shocks based on the contemporaneous relations from the theoretical model is not successful. In the next attempt we are able to identify the permanent shocks using the long-run structure available in the theoretical model though transitory shocks are not identified. Identification of the permanent shocks depends critically on their long-run multipliers predicted by the theoretical model. Findings on the significance of the permanent shocks as sources of fluctuations for output, real balances, inflation and the nominal interest rate are generally consistent with earlier studies employing the same cointegrated VAR methodology.

The common drawback suffered by VAR studies is also present in this study. Possibly due to over-parametrization (as discussed by Runkle 1987), I have found the standard errors of the structural impulse-response functions and the forecast error variance decompositions to be relatively large. The dynamic patterns revealed in such analyses can nevertheless improve our knowledge about the dynamic impacts of the permanent shocks on the economy.

In the long run, variations in output and real balances, which form the velocity relation, are dominated by the technology shock. Also for the long run, the labor-market shock dominates the variability in real wages. Given that long-run nominal neutrality is prescribed by the theory, long-run variations in the nominal interest rate and inflation are explained almost exclusively by the permanent nominal shock.

Similar to findings by others, I find permanent shocks to be important sources of fluctuations even in the short run. Specifically, the nominal trend accounts for a high proportion of the nominal interest rate variability in the short run. This is similar to the finding of Englund et al. (1994) but different from the extremely low percentages found by King et al. (1991). The impact of the nominal shock on inflation is at least 30 percent within the first two years in King et al. (1991). I find the nominal trend accounts for at most 30 percent of the inflation variance within the same horizons. The technology shock accounts for about 40 to 60 percent of the output variance on one- to two-year horizons. King et al. (1991) and Mellander et al. (1992) arrived at similar percentages. More significantly, the technology shock accounts for about 80 percent of the real balance forecast variance in the very short run, comparable to about 70 percent found by King et al. (1991). The labor-market shock already explains 40 percent of the variance in the impact quarter and only increases its influence on real wages thereafter. I found the labor-market shocks explain about 20 percent of the output variance within the first year. This compares to at least 40 percent of the output variance found by Shapiro and Watson (1988).

APPENDICES

APPENDIX A

RATIONAL EXPECTATION SOLUTIONS FOR A MODEL ECONOMY

First define $\widehat{x}_t \equiv E_{t-1}x_t$ for any variable x_t, i.e., $\widehat{x}_t$ is the rational expectation of x_t made at time t subject to all information available at time $t-1$. First apply rational expectation on the labor demand and labor supply equation and set them equal to obtain the contract wage.

$$\begin{aligned}
E_{t-1}\{\delta\,(w_t - p_t)\} &= E_{t-1}\{\gamma\,(p_t - w_t + \alpha u_{1t}) + u_{3t}\} \\
(\delta + \gamma)\,(w_t - \widehat{p}_t) &= \alpha\gamma\,\widehat{u}_{1t} + \widehat{u}_{3t} \\
w_t &= \widehat{p}_t + a\,\widehat{u}_{1t} + \frac{1}{\delta+\gamma}\widehat{u}_{3t} \qquad \text{(a.1)}
\end{aligned}$$

Note in the above $a \equiv \frac{\alpha\gamma}{\delta+\gamma}$ and $E_{t-1}w_t \equiv \widehat{w}_t = w_t$ since current wages are set in the last period. Then set aggregate supply and aggregate demand equal,

$$\begin{aligned}
m_t - p_t + v_t &= \beta\,(p_t - w_t) + \beta u_{1t} \\
p_t &= \frac{1}{\beta+1}\,(m_t + \beta w_t - \beta u_{1t} + v_t) \qquad \text{(a.2)}
\end{aligned}$$

Now substitute equation (a.1) into equation (a.2) and apply E_{t-1} on both side to get

$$\begin{aligned}
\widehat{p}_t &= \frac{1}{\beta+1}\left\{\widehat{m}_t + \beta\left(\widehat{p}_t + a\,\widehat{u}_{1t} + \frac{1}{\delta+\gamma}\widehat{u}_{3t}\right) - \beta\widehat{u}_{1t} + \widehat{v}_t\right\} \\
\widehat{p}_t &= \widehat{m}_t + \widehat{v}_t + \beta\,(a-1)\,\widehat{u}_{1t} + \frac{\beta}{\delta+\gamma}\widehat{u}_{3t} \qquad \text{(a.3)}
\end{aligned}$$

Substitute (a.3) back into (a.1) to solve for w_t in terms of predetermined terms as

$$w_t = \widehat{m}_t + \widehat{v}_t + [\beta\,(a-1) + a]\;\widehat{u}_{1t} + \frac{\beta+1}{\delta+\gamma}\widehat{u}_{3t}. \qquad \text{(a.4)}$$

The solution for p_t is obtained by substituting equation (a.4) into equation (a.2) as

$$\begin{aligned}
p_t &= \widehat{m}_t + \widehat{v}_t + \beta(a-1)\widehat{u}_{1t} + \frac{\beta}{\delta+\gamma}\widehat{u}_{3t} \\
&\quad + \frac{1}{\beta+1}\,[(m_t - \widehat{m}_t) + (v_t - \widehat{v}_t)] - \frac{\beta}{\beta+1}(u_{1t} - \widehat{u}_{1t}). \qquad \text{(3.6)}
\end{aligned}$$

The solution for real wages, $wr_r \equiv w_t - p_t$, is then

$$\begin{aligned} wr_t &\equiv w_t - p_t = a\widehat{u}_{1t} + \frac{1}{\delta+\gamma}\widehat{u}_{3t} \\ &\quad -\frac{1}{\beta+1}\left[(m_t - \widehat{m}_t) + (v_t - \widehat{v}_t)\right] + \frac{\beta}{\beta+1}(u_{1t} - \widehat{u}_{1t}). \end{aligned} \tag{3.9}$$

Finally substitute (3.6) into the aggregate supply equation (3.2) to solve for the output level as

$$\begin{aligned} y_t &= \beta(1-a)\widehat{u}_{1t} - \frac{\beta}{\delta+\gamma}\widehat{u}_{3t} \\ &\quad +\frac{\beta}{\beta+1}\left[(m_t - \widehat{m}_t) + (v_t - \widehat{v}_t) + (u_{1t} - \widehat{u}_{1t})\right]. \end{aligned} \tag{3.8}$$

APPENDIX B

THE EX POST REAL INTEREST RATE EQUATION

The real interest rate identity includes the next period ex ante inflation rate which is to be solved in terms of currently available variables. Before doing that we first define real balances by $mr_t \equiv m_t - p_t$ and inflation by $\pi_t \equiv p_t - p_{t-1}$, then the real balance process can be written as

$$mr_t = mr_{t-1} - \pi_t + u_{2t}. \tag{3.13}$$

Move forward one period on (3.13) and take expectation to get

$$\widehat{\pi}_{t+1} = \widehat{u}_{2t+1} - \widehat{mr}_{t+1} + mr_t. \tag{b.1}$$

Substitute (b.1) into (3.12) to get

$$\begin{aligned} r_t &= \widehat{\pi}_{t+1} + \phi(L)\,\epsilon_{6t} \\ &= \widehat{u}_{2t+1} - \widehat{mr}_{t+1} + mr_t + \phi(L)\,\epsilon_{6t}. \end{aligned} \tag{b.2}$$

We also need to solve for $\widehat{mr}_{t+1}$. Before doing that we subtract π_t, in (3.13), from both sides of (b.2) because we then can obtain an operational contemporaneous or ex post real-interest-rate equation as

$$r_t - \pi_t = \widehat{u}_{2t+1} - \widehat{mr}_{t+1} + mr_t + \phi(L)\,\epsilon_{6t} + mr_t - mr_{t-1} - u_{2t}$$

or

$$r_t - \pi_t - mr_t + mr_{t-1} = \widehat{u}_{2t+1} - u_{2t} + \left[mr_t - \widehat{mr}_{t+1}\right] + \phi(L)\,\epsilon_{6t} \tag{b.3}$$

The last equation will be used as one of the solution equations to form the equation system of Chapter 3.

Now we want to further solve for $mr_t - \widehat{mr}_{t+1}$ in equation (b.3). First obtain mr_t from the aggregate demand equation, (3.1), and (3.11) as

$$mr_t \;=\; \beta(1-a)(\tau_1 + u_{1t-1}) - \frac{\beta}{\delta+\gamma}(\tau_3 + u_{3t-1})$$

$$+\frac{\beta}{\beta+1}\left[\epsilon_{1t}+\epsilon_{2t}+\theta_0\epsilon_{5t}\right]-\theta(L)\epsilon_{5t}.$$

and then move one-period forward to get

$$\begin{aligned} mr_{t+1} &= \beta\,(1-a)\,(\tau_1+u_{1t})-\frac{\beta}{\delta+\gamma}\,(\tau_3+u_{3t}) \\ &+\frac{\beta}{\beta+1}\left[\epsilon_{1t+1}+\epsilon_{2t+1}+\theta_0\epsilon_{5t+1}\right]-\theta(L)\epsilon_{5t+1}. \end{aligned}$$

Take expectation conditional on information available at time t to get

$$\widehat{mr}_{t+1}=\beta\,(1-a)\,(\tau_1+u_{1t})-\frac{\beta}{\delta+\gamma}\,(\tau_3+u_{3t})-\zeta\,(L)\,\epsilon_{5t}$$

where $\zeta\,(L)=\left[\theta\,(L)-\theta_0\right]L^{-1}$. Subtracting (b.5) from (b.4) we have

$$\begin{aligned} mr_t-\widehat{mr}_{t+1} &= -\beta\,(1-a)\,\Delta u_{1t}+\frac{\beta}{\delta+\gamma}\Delta u_{3t}+\frac{\beta}{\beta+1}\,(\epsilon_{1t}+\epsilon_{2t}) \\ &+\psi\,(L)\,\epsilon_{5t} \end{aligned}$$

where $\psi\,(L)\equiv\zeta\,(L)-\theta\,(L)+\frac{\beta}{\beta+1}\theta_0$. Now (b.3) follows immediately as

$$\begin{aligned} r_t-\pi_t-mr_t+mr_{t-1} &= \tau_2-\beta\,(1-a)\,\Delta u_{1t}+\frac{\beta}{\delta+\gamma}\Delta u_{3t} \\ &+\frac{\beta}{\beta+1}\,(\epsilon_{1t}+\epsilon_{2t})+\psi\,(L)\,\epsilon_{5t}+\phi\,(L)\,\epsilon_{6t}. \quad (3.14) \end{aligned}$$

APPENDIX C

DATA SOURCES AND DEFINITIONS

All data are obtained from Citibase database except for M2 nominal balances data between 1951:1 and 1958:4 and nominal wages data. Certain data may be monthly from the data source and are averaged to form their quarterly counterparts. Data spans from 1951:1 to 1994:4 and are seasonally adjusted.

The three real measures examined in this study, real output, real balances and real wages can be derived from subtracting the price deflator form their respective nominal measures. The price deflator is in turn derived by subtracting the real output from the nominal output measure, as described in detail below.

Real output (y) is defined as the real domestic product minus the real government purchase. Using the Citibase symbols, it is

$$y = \ln(\mathrm{GDPQ} - \mathrm{GGEQ}).$$

Both GDPQ and GGEQ are measured in 1987 dollar value.

To calculate the price deflator we need the measure of the nominal output which is defined as the logarithm of nominal GDP minus the nominal government purchase, or ln(GDP-GGE) in symbols. The price deflator (p) is calculated according to

$$p = \ln\left(\frac{\mathrm{GDP} - \mathrm{GGE}}{\mathrm{GDPQ} - \mathrm{GGEQ}}\right)$$

and note $p = 0$ in 1987.

All series have been converted to their natural logarithm except for the nominal interest rate data.

The source of the nominal money supply data is the monthly Citibase M2 series which is available for 1959:1-1994:12. The M2 series for the period 1951:1-1958:12 is provided by Professor Robert Rasche. He estimated the series based on data reported in Banking and Monetary Statistics: 1941-1970 published by the Board of Governors of the Federal Reserve System in 1976. The complete monthly series are then averaged to obtain the quarterly observations. The real money balances (mr) is defined as

$$mr = \ln(\mathrm{M2}) - p.$$

The source of the nominal wage data is the monthly average hourly earnings of production workers in the manufacturing sector (Series ID EES0000006) available from the Bureau of Labor Statistics. The reason of this particular series being chosen is because it has complete observations of the sample period studied. The monthly data is then averaged to derive the quarterly nominal wages (W). The quarterly real wages is defined as

$$wr = \ln(W) - p.$$

The source of the nominal interest rate is the monthly observations of the 3-month Treasury Bill rate (FYGM3) in the secondary market measured in annual percentage. It is not seasonally adjusted. The quarterly nominal interest rate (R) is just the quarterly average of the monthly series.

Finally, following King et al. (1991), price inflation (π) is measured in annual percentage rate according to

$$\pi = 400 \times (p_t - p_{t-1}).$$

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