CORRELATION BETWEEN POTATO PRODUCTION PRACTICES AND YIELDS IN THE UPPER PENINSULA OF MICHIGAN

Thesis for the Degree of M. S.
MICHIGAN STATE COLLEGE

Karl Albert Vary

1945

This is to certify that the

thesis entitled

"Correlation Between Potato
Production Practice and Yields
in the Upper Peninsula of Michigan."

presented by

Karl Albert Vary

has been accepted towards fulfilment
of the requirements for

M. S. degree in Farm Management

Elton B. Hill
Major professor

Date June 9, 1945

CORRELATION BETWEEN POTATO PRODUCTION PRACTICES AND YIELDS IN THE UPPER PENINSULA OF MICHIGAN

by

KARL A. VARY

A THESIS

Submitted to the Graduate School of Michigan State College of Agriculture and Applied Science in partial fulfilment of the requirements for the degree of

MASTER OF SCIENCE

Department of Farm Management

1945

THESIS

VITA

Karl A. Vary

PERSONAL:

Age - 30 years, February 1, 1945
Birthplace - Covert, Michigan
Nationality - American
Marital status - Married
Religion - Protestant
Height - six feet
Weight - 180 pounds
Health - Excellent

EDUCATION:

High School - Creston, Grand Rapids, Michigan; graduated 1933
College - Olivet College, Olivet, Michigan; attended 1933-34
Michigan State College, East Lansing, Michigan; graduated September, 1944, B. S. Degree in Vocational Agriculture.
June, 1945, M. S. Degree in Farm Management and Economics.
Upon obtaining M. S. Degree I shall leave Michigan State College to go to Purdue University to work on my doctor's degree.

PRACTICAL TRAINING AND EXPERIENCE:

Five years had complete charge as manager of 1500-acre general purpose dairy and poultry farm.

Three years operated own 160-acre farm, renting other farms and doing extensive farm power contracting for farmers.

One year sold insurance, Auto, Fire, and Life, for State Farm Mutual Insurance Company of Bloomington, Illinois.

Six months - U. S. Government Refrigeration Engineer at Fort Custer, Michigan.

Past four years attended Michigan State College and have managed own farm at Marshall, Michigan.

Early training and experience gained from father who had charge and managed 1500-acre farm before me. (During my youth I lived on a 350-acre stock and fruit farm in Southwestern Michigan. There we maintained a show herd of Dutch Belted Cattle which we exhibited at Michigan Fairs and International Livestock Shows. We raised peaches, apples, cherries, pears, plums, and had one of the first large commercial grape vineyards in that section of Michigan.)

Particular experience:

Dairy: Registered Holstein herd of 109 head of cattle. Marketed and produced many kinds of dairy products.
Poultry: Raised broilers, turkeys, ducks, geese, guineas. Battery

and floor experience with both laying hens and broilers. Hatched own chicks for broilers and layers. Marketed - eggs, chicks, broilers, and fowl at dressed and live weights.

Sheep: 200 grade Oxford and Hampshire ewes. Marketed Easter and fancy dressed lambs.

Hogs: 15 to 20 Chester White brood sows. Marketed and dressed their litters.

Beef: Raised, bought and dressed baby beef.

Horses: Maintained and used four Belgian draft teams and two thoroughbred saddle horses.

Fish: Reared brook trout as a specialty food for hotels.

Crops: Alfalfa hay and pasture, clover, timothy, field and silage corn, wheat, barley, oats, rye, soybeans, navy beans, sudan grass, sweet clover, bromegrass, small vegetables, and as a specialty 25 acres of Idaho baking potatoes every year.

Machinery Used: Tractors - John Deere model A, B, D, G, IHC-Farmall; Caterpillar Rd-4, Rd-6; Oliver '70 Row Crop; Ford 1940 and Fordson; Allis Chalmers. (I can handle and repair these tractors with skill.)

Trucks - Ford 1½ ton; GMC 5 ton; Chevrolet 1½ ton; Ford and Chevrolet Pick-ups. (Skilled in use and repair.)

Cars - Ford, Dodge, Hudson, Chevrolet, Plymouth, Oldsmobile.

Other machinery - Pick-up hay baler, two combines, cornpicker, plows, disks, harrows, cultipackers, silo filler, numerous water pumps, irrigation equipment, electrical motors and devices; had own shop and did own welding; 2-ton and 50-ton ice machines, oil burners, feed grinders, elevators and many other types of farm machinery.

LEADERSHIP:

Directed 15 to 55 employees; planned each day and year's work. Did all the bookkeeping, including income tax reports, payroll, statements, bills, correspondence, etc.

AFFILIATION:

Phi Alpha, Exchange Club, Grand Rapids Engineers' Club, National Honor Society, Phi Kappa Phi, and Kappa Delta Pi.

ACKNOWLEDGMENTS

The writer wishes to express his appreciation to Dr. K. T. Wright, Dr. W. D. Baten, Professors B. R. Churchill, Ernest J. Wheeler, and E. B. Hill for the very helpful suggestions and counsel offered during the preparation of this manuscript.

The writer also wishes to express his gratitude to Professor D. L. Clanahan for his cooperation and the use of the Premier Potato Growers' Upper Peninsula records.

CONTENTS

CORRELATION BETWEEN POTATO PRODUCTION PRACTICES AND YIELDS IN THE UPPER PENINSULA OF MICHIGAN

Karl A. Vary

Introduction

For many years a Three Hundred Bushel Potato Club has been promoted in Michigan. From this there was developed a Premier Potato Growers' contest for certified seed growers. Still later a Premier Tablestock Growers' contest was added.

Through the efforts of the merchants and the Chamber of Commerce of the city of Escanaba a county contest for Delta County potato growers was proposed. A score card was made by D. L. Clanahan, H. C. Moore, and others giving points for recommended cultural practices. Previous to this time the contests were based only on total yield, percent of U. S. No. 1 potatoes, and placings at either county or district potato shows. Because of the interest shown in the Delta County contest, other counties have started similar contests. It has now been suggested that the results obtained from the Premier Potato Growers' contests of the Upper Peninsula be used in developing a new score card to be used for future contests.

We were asked by D. L. Clanahan to make a study of the score card and past records of each Premier Potato Grower's practices to determine the validity of the weight assigned to the various cultural practices used on the score card. The importance of

the major cultural practices was measured by correlating their influence with the potato yields of these growers.

One reason for choosing this subject for study is that we have wanted to investigate the possibility of developing a score card for use in measuring management of the entire farm business.

Factors Affecting Potato Yields

There are many factors which may affect potato yields. However, only factors over which farmers had some control were considered on the score card. The factors selected in developing new score cards are:

1. Number of sprays per acre
2. Bushels of seed planted per acre
3. Planting date - May 1st. taken as 1. (Example: June 2 = 33.)
4. Number of times worked after plowing and before cultivating
5. Number of loads of manure per acre
6. Pounds of fertilizer per acre

These factors were selected from the score card as the most important cultural practices to potato production with the help of E. J. Wheeler and B. R. Churchill of the Farm Crops Department.

Relationship of Practices to Yields

In the Upper Peninsula potato contests for the years 1942-44, there were 145 Russet Rural Premier Potato Growers. They were located principally in the counties of Menominee, Delta, Iron, and Schoolcraft. For this same period there were 109 Green Mountain Premier Potato Growers, and these men were largely in the counties of Houghton, Marquette, and Baraga. Each of the contestants produced 300 bushels or more per acre and followed one or more of the recommended cultural practices. The records of the yields and cultural practices of the above growers were made available to us for this study by D. L. Clanahan.

Correlations for the year 1944 were first computed, paying no attention to variety differences. Correlations were then calculated separately for the two varieties, Russet Rurals and Green Mountains. Premier Growers of these two varieties made up 70% of all contestants for 1942-44. The other 30% grew Chippewa, Pontiac, Menominee, and Sebago varieties. Correlations for the two varieties (Russet Rural and Green Mountain) were determined for each year of the three year period, 1942-44. Data for the three years was then combined to form a more representative grouping of factors than could be had by using any one year by itself.

Separate correlations for the tablestock and certified

potato growers were worked out for 1942 to find out if the effect of the cultural practices differed significantly. No significant differences were found; therefore this separation was not made in the other years.

Symbols were used in the study to designate factors as follows:

X_1 - yield in bushels of U. S. No. 1 potatoes

X_2 - number of sprays per acre

x_3 - number of bushels of seed planted per acre

X_4 - planting date

X_5 - number of times worked

X_6 - number of loads of manure per acre

X_7 - number of pounds of fertilizer per acre

Record of Grand Totals and Averages of Potato Growers, 1942-44

Russet Rural

No. of Growers	Year	Bu. No. 1's (+270) x_1	x_1^2	Actual Yield	No. of Sprays x_2	x_2^2	Bu. Seed Planted x_3	x_3^2	Planting Date x_4	x_4^2
23	1942	2,288	316,218	8,498	224	2,364	550	13,936	487	12,789
52	1943	5,455	819,211	19,595	471	4,537	1,161	26,709	1,431	42,081
70	1944	8,966	1,643,024	27,866	553	4,691	1,595	38,087	2,051	64,734
	Total:	16,709	2,778,453	55,959	1,248	11,592	3,306	78,732	3,969	119,604
	Average:	115.2	19161.7	386	8.6	79.9	22.8	543	27.4	824.8

Green Mountain

37	1942	3,848	492,726	13,838	257	1,953	717	14,051	1,263	45,769
19	1943	1,475	164,195	6,605	134	1,084	371	7,313	654	25,342
53	1944	5,561	692,977	19,871	341	2,369	1,037	20,450	2,097	89,247
	Total:	10,884	1,349,898	40,314	732	5,406	2,125	41,814	4,014	160,358
	Average:	99.9	12384	370	6.7	49.5	19.5	383.6	36.9	1471

Record of Grand Totals and Averages of Potato Growers, 1942-44

Russet Rural

Year	x_1x_2	x_1x_3	x_1x_4	x_2x_3	x_2x_4	x_3x_4
1942	23,677	57,777	43,518	5,627	4,636	11,386
1943	52,015	127,416	145,522	10,672	12,681	30,847
1944	74,843	219,737	260,888	12,934	15,844	47,938
Total:	150,535	404,930	449,928	29,233	33,161	90,171
Average:	1038	2793	3103	202	229	622

Green Mountain

1942	27,464	75,535	126,433	5,017	8,476	24,281
1943	12,470	29,665	45,217	2,660	4,303	12,520
1944	36,339	108,820	213,518	6,711	13,258	40,921
Total:	76,273	214,020	385,168	14,388	26,037	77,722
Average:	700	1964	3534	132	239	713

Record of Grand Totals and Averages of Potato Growers, 1942-44

Russet Rural

No. of Growers	Year	Bu. No. 1's (-270) x_1	x_1^2	Actual Yield	Times Worked x_5	x_5^2	Loads of Manure x_6	x_6^2	Pounds Fertilizer x_7	x_7^2
23	1942	2,288	316,218	8,498	275	3,735	219	2,907	15,000	11,358,750
52	1943	5,455	819,211	19,595	620	8,817	490	6,136	35,000	27,975,000
70	1944	8,966	1,643,024	27,866	935	14,659	596	8,134	48,725	37,258,125
	Total:	16,709	2,778,453	55,959	1,830	27,211	1,305	17,177	98,725	76,591,875
	Average:	115.2	19161.7	386	12.62	187.6	9	118.4	680.9	528,219

Green Mountain

37	1942	3,848	492,726	13,838	307	3,741	280	3,162	24,550	17,597,500
19	1943	1,475	164,195	6,605	145	1,477	114	1,108	14,450	11,877,500
53	1944	5,561	692,977	19,871	534	6,362	440	5,406	41,233	33,940,589
	Total:	10,884	1,349,898	40,314	986	11,580	834	9,676	80,233	63,415,589
	Average:	99.9	12384	370	9.1	106.23	7.7	88.8	736.1	581,794

Record of Grand Totals and Averages of Potato Growers, 1942-44

Russet Rural

Year	x_1x_5	x_1x_6	x_1x_7	x_5x_6	x_5x_7	x_6x_7
1942	26,849	23,784	1,562,300	2,509	181,375	139,875
1943	68,451	54,195	4,156,650	6,124	415,850	313,450
1944	125,953	86,981	6,813,350	7,998	662,650	436,725
Total:	221,253	164,960	12,532,300	16,631	1,259,875	890,050
Average:	1,525.8	1,137.6	86,429.7	114.7	8,688.7	6,138.2

Green Mountain

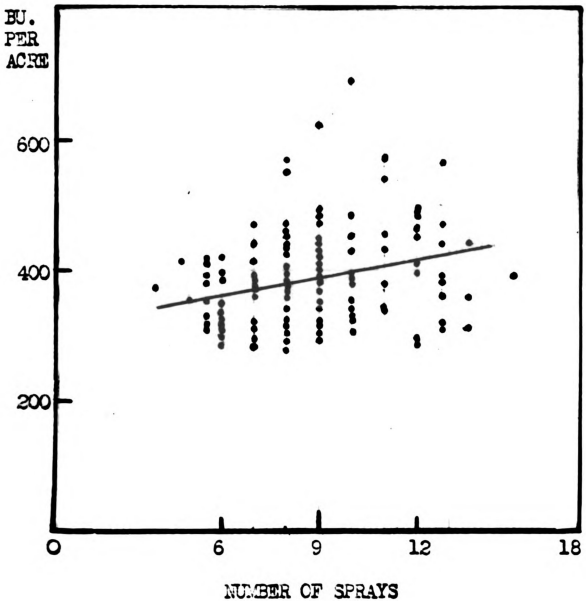
1942	31,723	31,730	2,692,950	2,383	206,350	180,000
1943	12,148	10,606	1,160,550	1,094	107,150	78,500
1944	54,463	47,147	4,372,325	4,622	408,680	317,540
Total:	98,334	89,483	8,225,825	8,099	722,180	576,040
Average:	902.1	820.9	75,466.2	74.3	6,625.5	5,284.7

Correlation Coefficients of Cultural Practices and Potato Yields in the Upper Peninsula, 1942-44

r[1]	R.R.[2]	G.Mt.[3]	r[4]	R.R.	G.Mt.	r[4]	R.R.	G.Mt.	r[4]	R.R.	G.Mt.	R[5]	R.R.	G.Mt.
12	.2535	.2910	12.3	.0327	.2488	12.4	.2410	.2069				1.23	.5044	.3371
13	.4508	.1778	13.4	.4480	.0868	13.2	.3865	.0866				1.24	.2655	.4032
14	-.0810	-.2917	14.2	-.0007	-.2080	14.3	-.0576	-.2497	14.32	-.0422	-.1399	1.34	.4566	.3377
15	.1820	-.0295	15.6	.1802	-.0623	15.7	.1681	-.0149				1.234	.4532	.3307
16	.2146	.1941	16.7	.2322	.2874	16.5	.2130	.2015				1.56	.2784	.1964
17	.4080	.1969	17.5	.4028	.1953	17.6	.4163	.2892	17.65	.3094	.2885	1.57	.4405	.1991
												1.67	.4526	.2746
												1.567	.4068	.3483

[1] Gross correlations
[2] Russet Rural variety
[3] Green Mountain variety
[4] Partial correlations
[5] Multiple correlations

NUMBER OF SPRAYS AS RELATED TO RUSSET RURAL POTATO YIELDS IN THE UPPER PENINSULA 1942-44



NUMBER OF SPRAYS

Regression Equations

1942-44 average Y = 317.21 (bushels) + 7.9 (No. of sprays) as plotted on preceding graph

1944	Y =	299.80 (bushels) + 12.4 (No. of sprays)
1943	Y =	287.86 (bushels) + 9.6 (No. of sprays)
1942	Y =	286.10 (bushels) + 6.2 (No. of sprays)

Gross and Partial Correlations 1942-44 Averages

$r_{12} = .2535$

$r_{12.3} = .0327$

$r_{12.4} = .2410$

$R_{1.234} = .4532$

Spraying is mainly done to control the following:

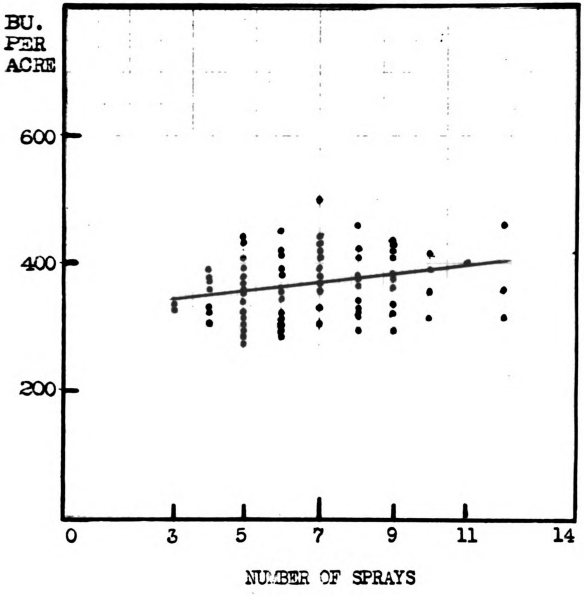
1. Insects
2. Fungus diseases
3. Virus diseases

The number of sprays is governed by the following:

1. Prevalence of the above
2. Thoroughness of spraying
3. Climatic conditions
4. Date of planting and first killing frost

Increasing the number of sprays from four or five times to fifteen increased the yield per acre of the Premier Growers of Russet Rural potatoes an average of 7.9 bushels for each spray during 1942-44, assuming that other factors remained average.

NUMBER OF SPRAYS AS RELATED TO GREEN MOUNTAIN POTATO YIELDS IN THE UPPER PENINSULA 1942-44



NUMBER OF SPRAYS

Regression Equations

1942-44 average Y = 326.27 (bushels) + 6.49 (No. of sprays) as plotted on preceding graph

1944 Y = 354.30 (bushels) + 3.19 (No. of sprays)
1943 Y = 244.70 (bushels) +14.8 (No. of sprays)
1942 Y = 286.10 (bushels) + 6.2 (No. of sprays)

Gross and Partial Correlations 1942-44 Averages

$r_{12} = .2910$

$r_{12.3} = .2488$

$r_{12.4} = .2069$

$R_{1.234} = .3307$

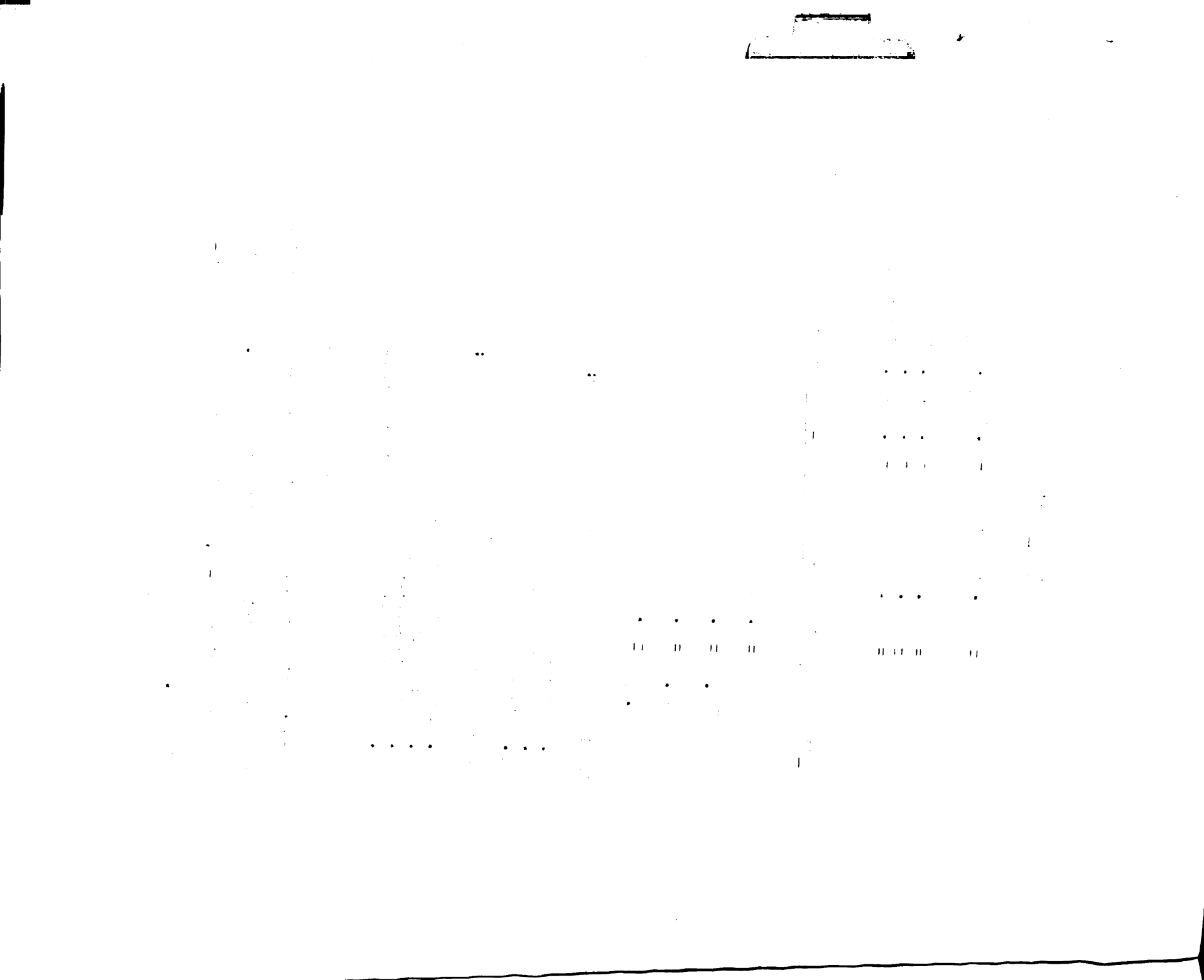
Spraying is mainly done to control the following:

1. Insects
2. Fungus diseases
3. Virus diseases

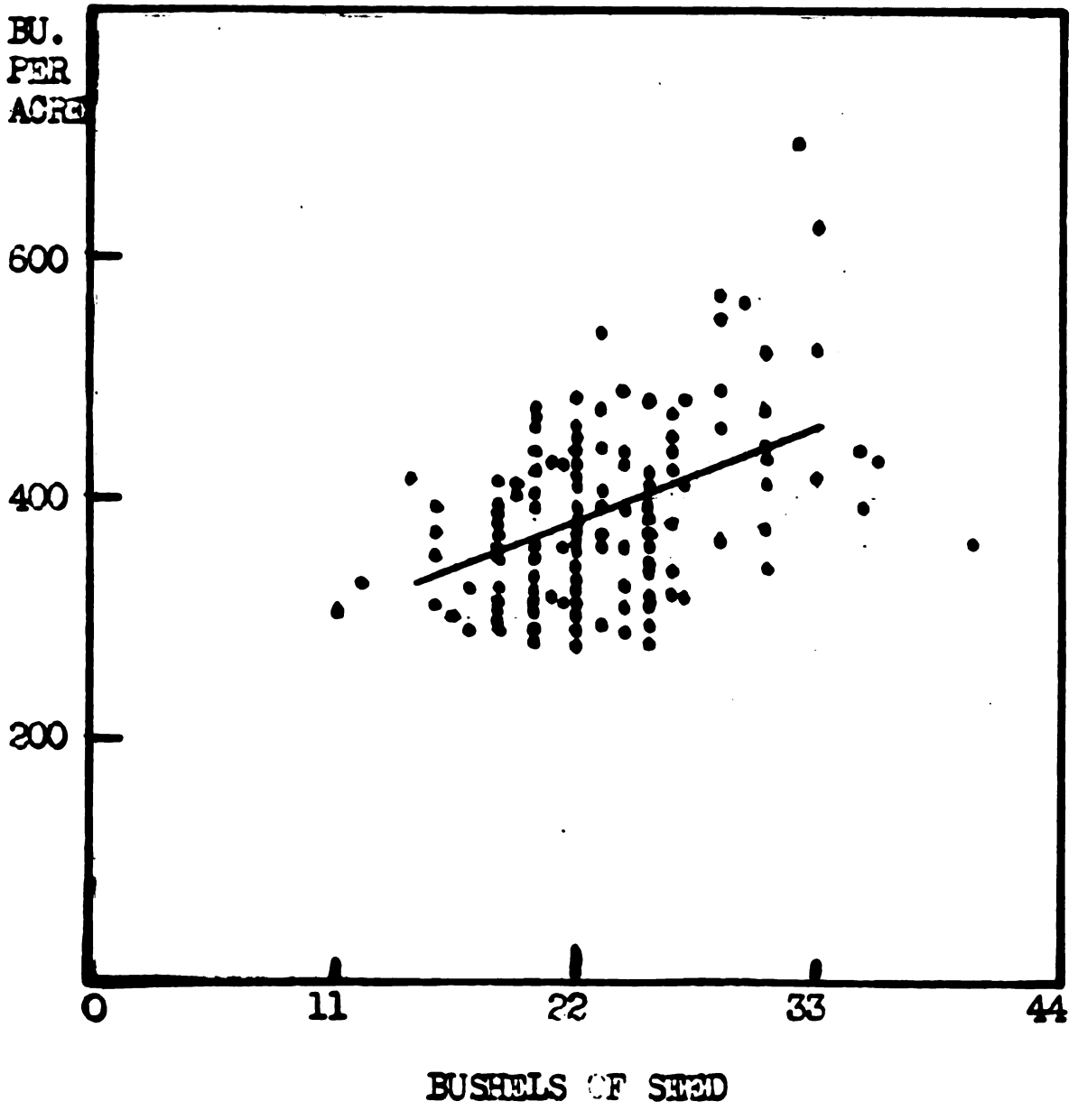
The number of sprays are governed by the following:

1. Prevalence of the above
2. Thoroughness of spraying
3. Climatic conditions
4. Date of planting and first killing frost

Increasing the number of sprays from four or five times to nine or ten times increased the yield per acre of the Premier Growers of Green Mountain potatoes an average of 6.49 bushels for each spray during 1942-44, assuming that other factors remained average.



BUSHELS OF SEED AS RELATED TO RUSSET RURAL POTATO YIELDS IN THE UPPER PENINSULA 1942-44



BUSHELS OF SEED

Regression Equations

1942-44 average Y = 222.44 (bushels) + 7.14(bushels of seed) as plotted on preceding graph

1944 Y = 196.40 (bushels) + 8.85 (bushels of seed)
1943 Y = 215.50 (bushels) + 7.14 (bushels of seed)
1942 Y = 275.95 (bushels) + 3.91 (bushels of seed)

Gross and Partial Correlations 1942-44 Averages

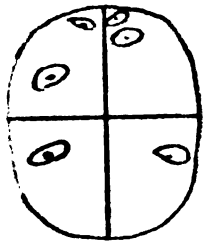
$r_{13} = .4508$

$r_{13.4} = .4480$

$r_{13.2} = .3865$

$R_{1.234} = .4532$

The number of eyes on Russet Rural seed potatoes is fewer than on the Green Mountain seed potatoes; therefore, they tend to make fewer seed pieces per bushel.

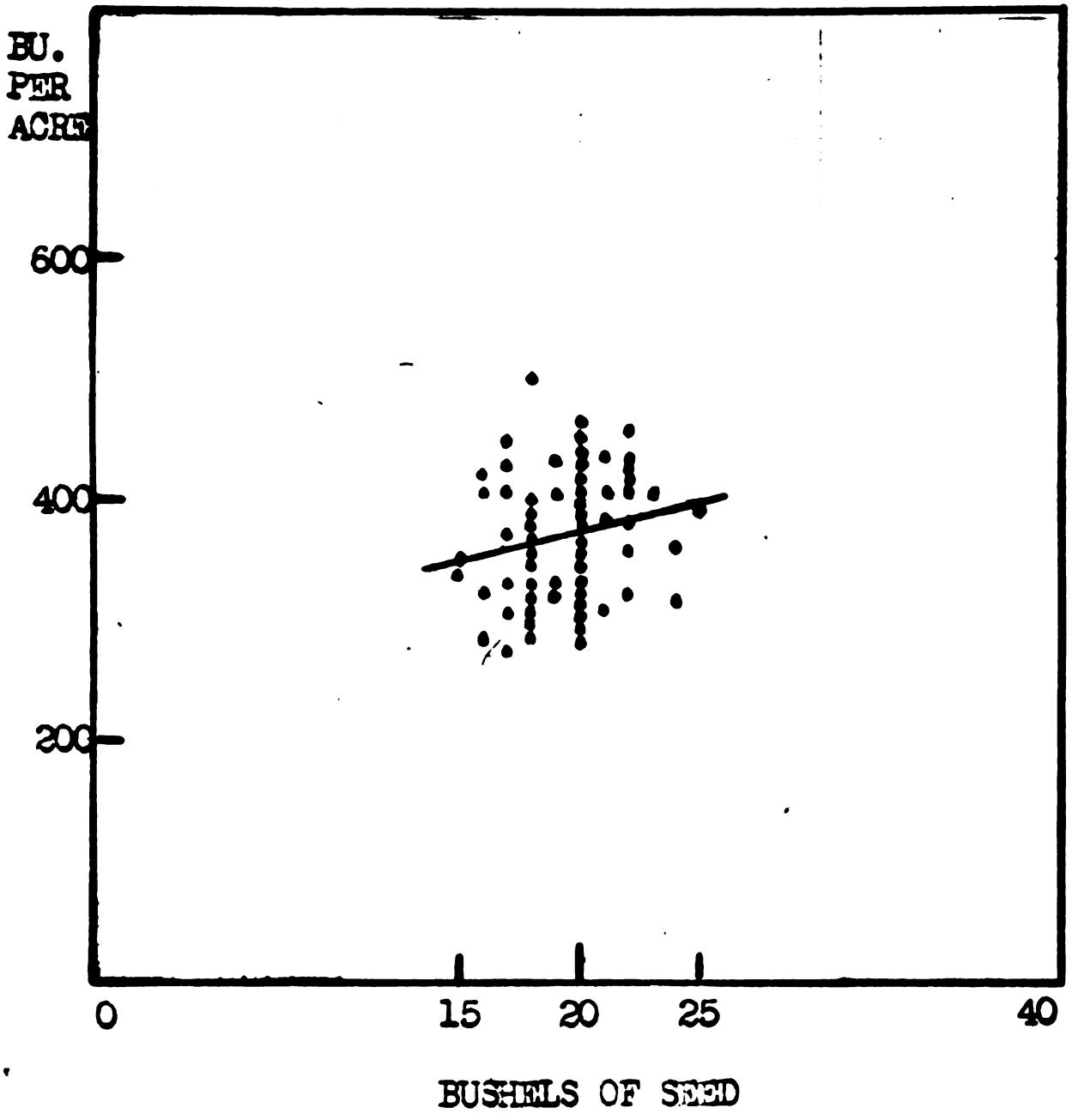


(2 to 4 seed pieces when potato is inclined to be round.)

Certified seed growers often plant whole tubers; this requires more bushels of seed per acre and makes for a smaller average sized potato.

Increasing the number of bushels of seed from twenty bushels to thirty bushels increased the yield per acre of the Premier Growers of Russet Rural potatoes an average of 7.14 bushels for each bushel of seed planted during 1942-44, assuming that other factors remained average.

BUSHELS OF SEED AS RELATED TO GREEN MOUNTAIN POTATO YIELDS IN THE UPPER PENINSULA 1942-44



BUSHELS OF SEED

Regression Equations

1942-44 average Y = 277.44 (bushels) + 4.74 (bushels of seed) as plotted on preceding graph

1944	Y =	383.2 (bushels) + .08 (bushels of seed)
1943	Y =	102.0 (bushels) + 12.6 (bushels of seed)
1942	Y =	286.1 (bushels) + 6.20 (bushels of seed)

Gross and Partial Correlations 1942-44 Averages

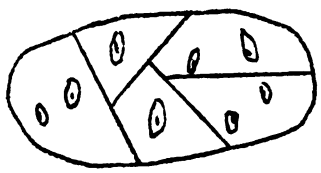
$r_{13} = .1778$

$r_{13.4} = .0868$

$r_{13.2} = .0866$

$R_{1.234} = .3307$

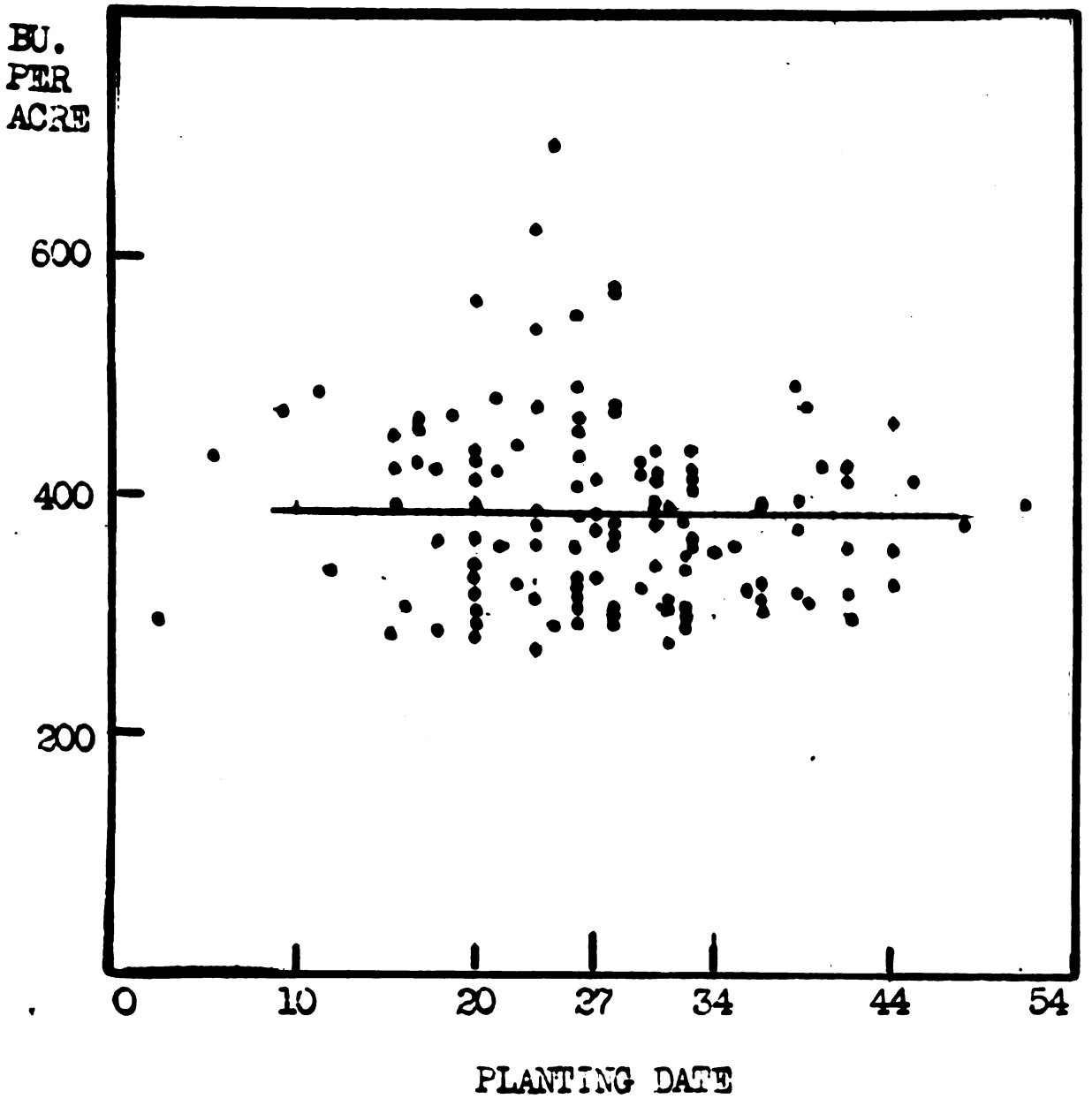
The number of eyes on Green Mountain seed potatoes is more than on the Russet Rural seed potatoes; therefore, they tend to make more seed pieces per bushel.



(4 pieces or more when the length is greater than the width.)

Increasing the number of bushels of seed from fifteen bushels to twenty-five bushels increased the yield per acre of the Premier Growers of Green Mountain potatoes an average of 4.74 bushels for each bushel of seed planted during 1942-44, assuming that the other factors remained average.

PLANTING DATE AS RELATED TO RUSSET RURAL POTATO YIELDS IN THE UPPER PENINSULA 1942-44



PLANTING DATE

Regression Equations

1942-44 average Y = 386.6 (bushels) - .05 (planting date after May 1.) as plotted on preceding graph

1944 Y = 409.6 (bushels) - .39 (days after May 1)
1943 Y = 422.0 (bushels) - 1.7 (days after May 1)
1942 Y = 411.3 (bushels) - 1.9 (days after May 1)

Gross and Partial Correlations 1942-44 Averages

$r_{14} = -.0810$

$r_{14.2} = -.0007$

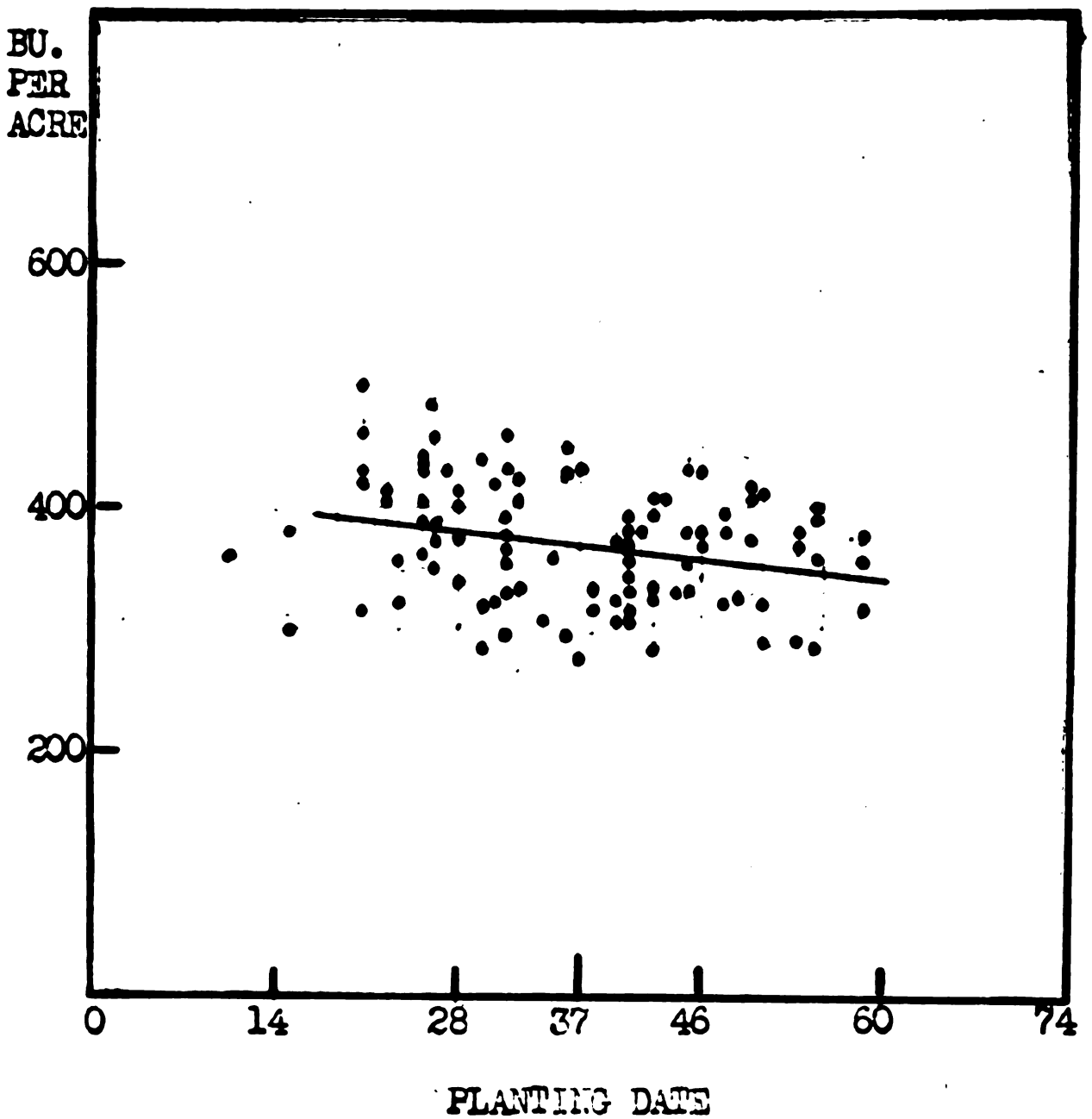
$r_{14.3} = -.0576$

$r_{14.32} = -.0422$

Russet Rural potatoes are grown in a district of the Upper Peninsula where climatic conditions generally favor an earlier planting date than that where Green Mountain potatoes are grown.

Increasing the number of days from May fifteenth to June thirteenth decreased the yield per acre of the Premier Growers of Russet Rural potatoes an average of .05 bushel for each day of delayed planting during 1942-44, assuming that the other factors remained average.

PLANTING DATE AS RELATED TO GREEN MOUNTAIN POTATO YIELDS IN THE UPPER PENINSULA 1942-44



PLANTING DATE

Regression Equations

1942-44 average Y = 415.89 (bushels) - 1.25 (planting date after May 1st) as plotted on preceding graph

1944	Y =	415.9 (bushels) - 1.04 (days after May 1)
1943	Y =	280.2 (bushels) - 1.96 (days after May 1)
1942	Y =	437.0 (bushels) - 1.85 (days after May 1)

Gross and Partial Correlations 1942-44 Averages

$r_{14} = -.2917$

$r_{14.2} = -.2080$

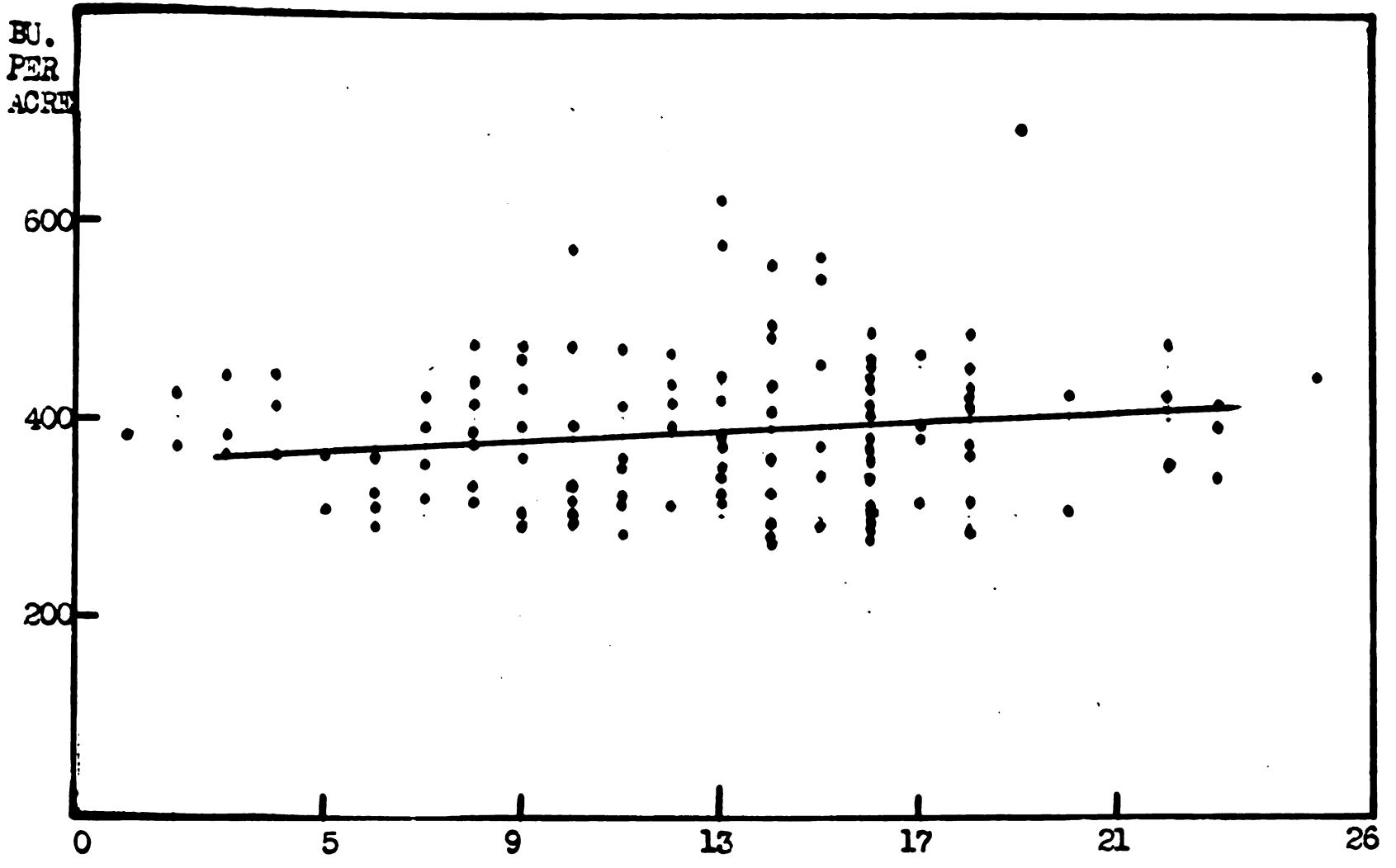
$r_{14.3} = -.2497$

$r_{14.32} = -.1399$

Potatoes in the Green Mountain area are generally planted ten days later than in the Russet Rural area. Green Mountain potatoes are more susceptible to drouthy weather conditions and earlier planting is important.

Increasing the number of days from May 15 to June 20 decreased the yield per acre of the Premier Growers of Green Mountain potatoes an average of 1.25 bushels for each day of delayed planting during 1942-44, assuming that the other factors remained average.

NUMBER OF TIMES WORKED AS RELATED TO RUSSET RURAL POTATO YIELDS
IN THE UPPER PENINSULA 1942-44



NUMBER OF TIMES WORKED

Regression Equations

1942-44 average Y = 353.43 (bushels) + 2.52 (times worked) as plotted on preceding graph

1944	Y =	369.6 (bushels) + 2.2	(times worked)
1943	Y =	346.4 (bushels) + 2.39	(times worked)
1942	Y =	382.6 (bushels) - 1.1	(times worked)

Gross and Partial Correlations 1942-44 Averages

$$r_{15} = .1820$$

$$r_{15.6} = .1802$$

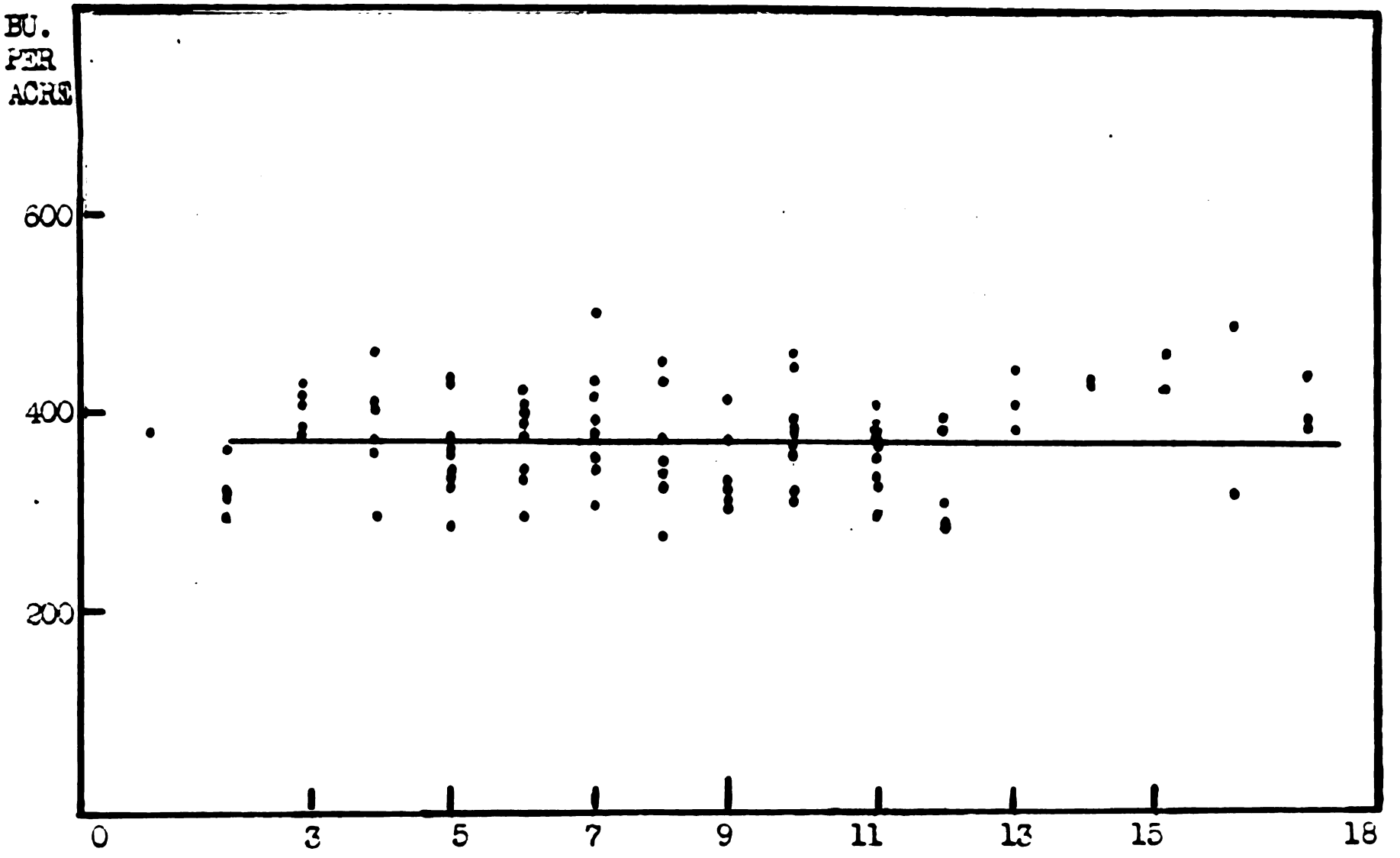
$$r_{15.7} = .1681$$

$$R_{1.567} = .4068$$

The number of times worked refers to the working of the soil after plowing and prior to the planting of potatoes. An effort is made to secure a deep mellow seed bed that is relatively free from weeds and one which will require a minimum of cultivation during the growing season.

Increasing the number of times worked from five to eighteen increased the yield per acre of the Premier Growers of Russet Rural potatoes an average of 2.52 bushels for each time worked during 1942-44, assuming that the other factors remained average.

NUMBER OF TIMES WORKED AS RELATED TO GREEN MOUNTAIN POTATO YIELDS
IN THE UPPER PENINSULA 1942-44



NUMBER OF TIMES WORKED

Regression Equations

1942-44 average Y = 300.3 (bushels) - .05 (times worked) as plotted on preceding graph

1944	Y =	396.0 (bushels) - 1.6 (times worked)
1943	Y =	329.2 (bushels) + 2.41(times worked)
1942	Y =	280.2 (bushels) - 1.96(times worked)

Gross and Partial Correlations 1942-44 Averages

$$r_{15} = -.0295$$

$$r_{15.6} = -.0623$$

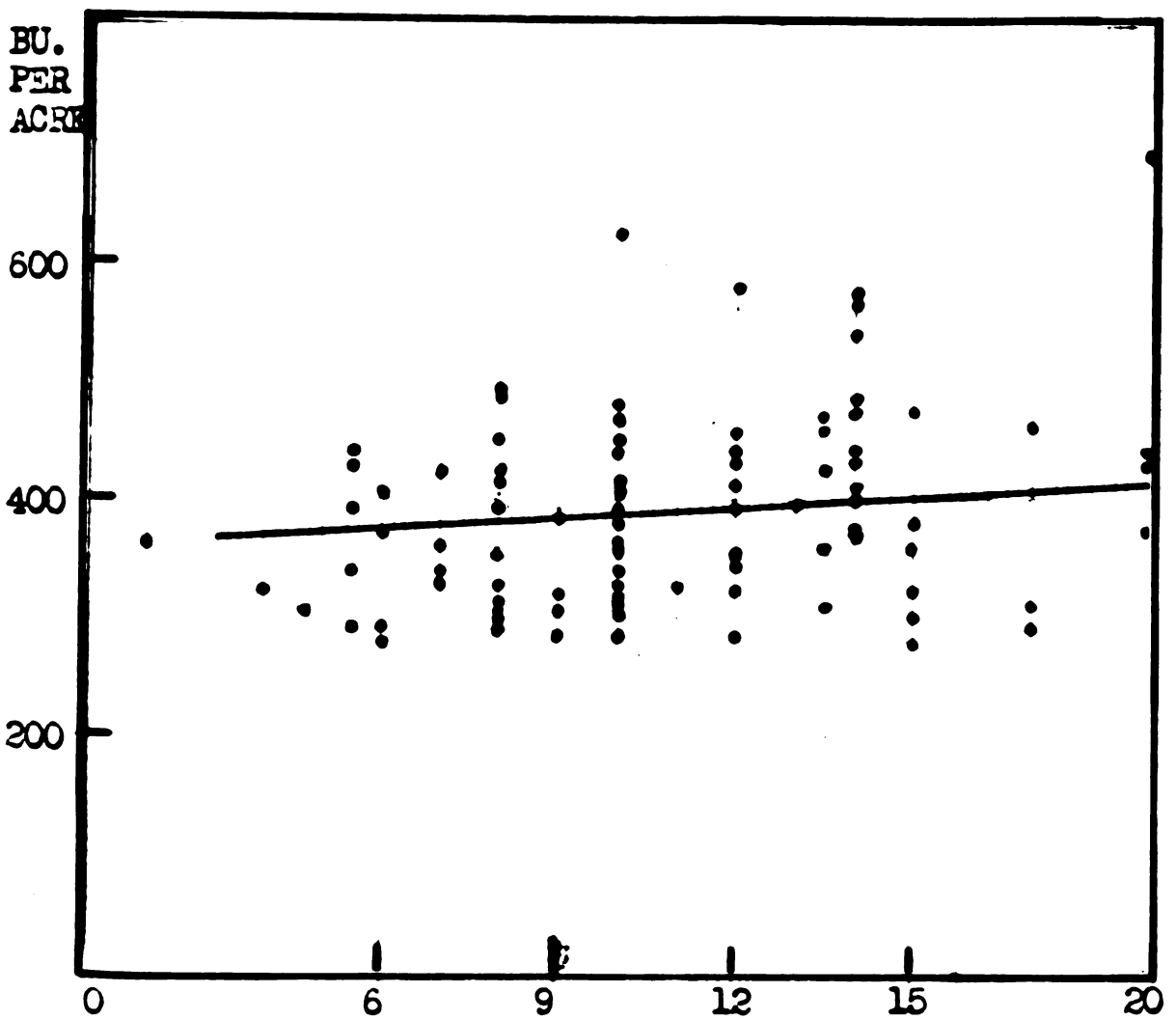
$$r_{15.7} = -.0149$$

$$R_{1.567} = .3483$$

The number of times worked refers to the working of the soil after plowing and prior to the planting of potatoes. An effort is made to secure a deep mellow seed bed that is relatively free from weeds and one which will require a minimum of cultivation during the growing season. In the Green Mountain area the above is more of a problem because of the type of soil, a shorter length of time in which to prepare the seed bed, and the prevalence of more quack grass.

Increasing the number of times worked from five to eleven times decreased the yield per acre of the Premier Growers of Green Mountain potatoes an average of .05 bushel for each time worked during 1942-44, assuming that the other factors remained average.

NUMBER OF LOADS OF MANURE AS RELATED TO RUSSET RURAL POTATO YIELDS IN THE UPPER PENINSULA 1942-44



NUMBER OF LOADS OF MANURE

Regression Equations

1942-44 average Y = 361.1 (bushels) + 2.68 (loads of manure) as plotted on preceding graph

1944 Y = 372.0 (bushels) + 3.20 (loads of manure)
1943 Y = 357.6 (bushels) + 1.84 (loads of manure)
1942 Y = 346.3 (bushels) + 2.43 (loads of manure)

Gross and Partial Correlations 1942-44 Averages

$r_{16} = .2146$

$r_{16.5} = .2130$

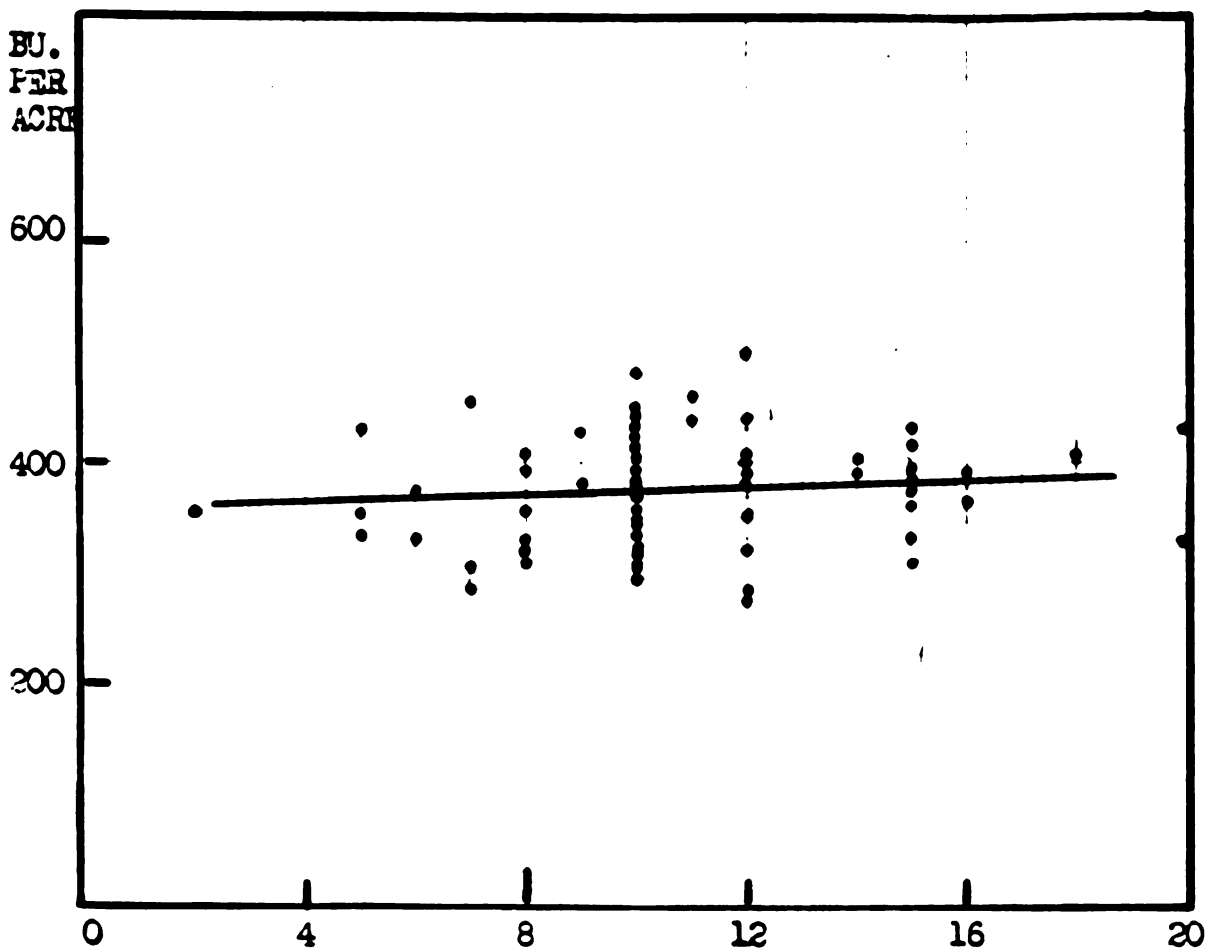
$r_{16.7} = .2322$

$R_{1.567} = .4068$

In general, loads of manure applied to potato ground increased the yield. This was true where manure alone was applied and where manure and fertilizer together was used.

Increasing the loads of manure from six to fifteen increased the yield per acre of the Premier Growers of Russet Rural potatoes an average of 2.7 bushels for each load of manure added during 1942-44, assuming that the other factors remained average.

NUMBER OF LOADS OF MANURE AS RELATED TO GREEN MOUNTAIN POTATO YIELDS IN THE UPPER PENINSULA 1942-44



NUMBER OF LOADS OF MANURE

REGRESSION EQUATIONS

1942-44 average Y = 355.5 (bushels) + 1.88 (loads of manure) as plotted on preceding graph

1944	Y =	355.5 (bushels) + .03 (loads of manure)
1943	Y =	322.8 (bushels) + 4.14 (loads of manure)
1942	Y =	355.0 (bushels) + 2.50 (loads of manure)

Gross and Partial Correlations 1942-44 Averages

$$r_{16} = .1941$$

$$r_{16.5} = .2874$$

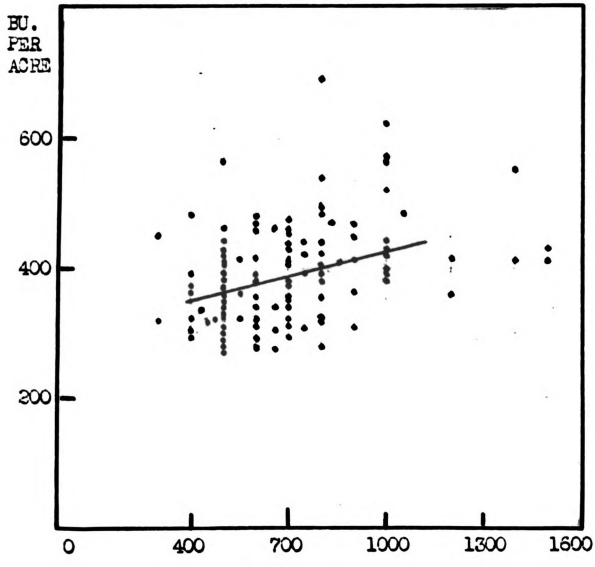
$$r_{16.7} = .2015$$

$$R_{1.567} = .3483$$

In general, loads of manure applied to potato ground increased the yield. This was true where manure alone was applied and also where manure and fertilizer together was used.

Increasing the number of loads of manure from six to fifteen increased the yield per acre of the Premier Growers of Green Mountain potatoes an average of 1.88 bushels for each load of manure added during 1942-44, assuming that the other factors remained average.

POUNDS OF FERTILIZER AS RELATED TO RUSSET RURAL POTATO YIELDS IN THE UPPER PENINSULA 1942-44



NUMBER OF POUNDS OF FERTILIZER

Regression Equations

1942-44 average Y = 301.3 (bushels) + .12 (pounds of fertilizer) as plotted on preceding graph

1944 Y = 381.0 (bushels) + .03 (pounds of fertilizer)
1943 Y = 300.8 (bushels) + .12 (pounds of fertilizer)
1942 Y = 343.3 (bushels) + .04 (pounds of fertilizer)

Gross and Partial Correlations 1942-44 Averages

$r_{17} = .4080$

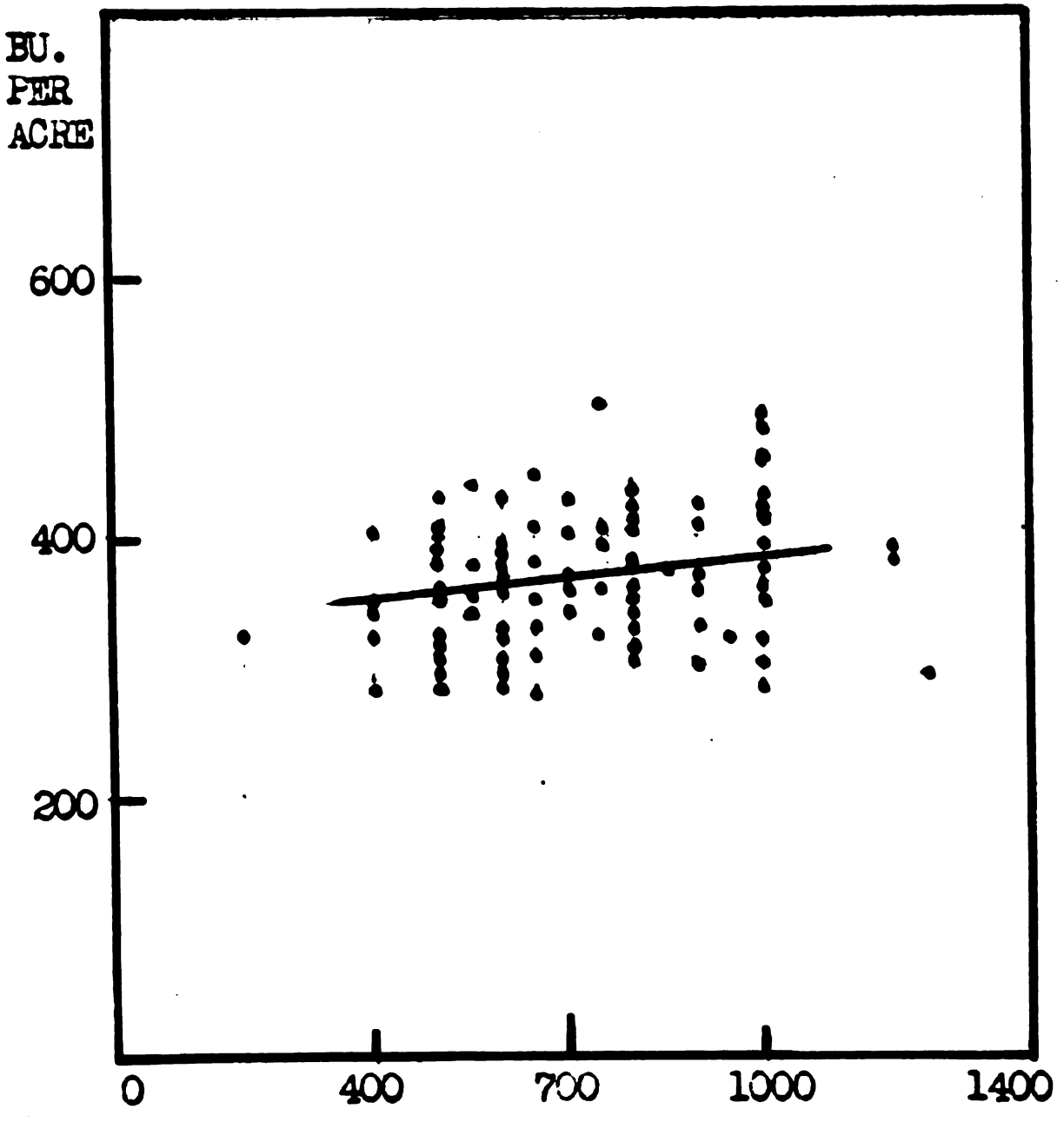
$r_{17.5} = .4028$

$r_{17.6} = .4163$

$R_{1.567} = .4068$

Increasing the amount of fertilizer from 400 pounds to 1000 pounds increased the yield per acre of the Premier Growers of Russet Rural potatoes an average of .12 bushel for each pound of fertilizer added during 1942-44, assuming that the other factors remained average.

POUNDS OF FERTILIZER AS RELATED TO GREEN MOUNTAIN POTATO YIELDS IN THE UPPER PENINSULA 1942-44



NUMBER OF POUNDS OF FERTILIZER

Regression Equations

1942-44 average Y = 333.7 (bushels) + .05 (pounds of fertilizer) as plotted on preceding graph

1944 Y = 355.5 (bushels) + .03 (pounds of fertilizer)
1943 Y = 327.1 (bushels) + .03 (pounds of fertilizer)
1942 Y = 303.0 (bushels) + .11 (pounds of fertilizer)

Gross and Partial Correlations 1942-44 Averages

$r_{17} = .1969$

$r_{17.5} = .1953$

$r_{17.6} = .2892$

$r_{17.65} = .2885$

Increasing the amount of fertilizer from 400 pounds to 1000 pounds increased the yield per acre of the Premier Growers of Green Mountain potatoes an average of .05 bushel for each pound of fertilizer added during 1942-44, assuming that the other factors remained average.

Score Cards Used in 1944

Following are examples of the actual score cards used in some counties in 1944 for determining contest winners. Score cards used in the remaining counties are similar.

ESCANABA POTATO BOOSTER ASSOCIATION

Tablestock Growers Contest

Open to any farmer growing 2 acres or more of potatoes. Points will be awarded on the basis of total acreage of all varieties grown on any one farm. Only one entry for each farm unit.

22 bu. or more per acre	Points
Certified seed	60
1 year from certification	40
2 years from certification	30
Other seed	20
Seed treatment-Semesan Bel.	65
Greensprouting (at least 10 days)	65
Planting on summer fallowed hay or sod field	60
8 loads manure per acre or plow down crop of hay	60
Commercial fertilizer 500 lbs. per acre, equivalent of 135 plant food units per acre or more	50

Planting Dates

Up to May 31	100
June 1-5 (inclusive)	75
June 6-10 (inclusive)	50

Spraying

5 times	100
6 times	150
7 or more times	200

Yield

¼ point per bushel

Any grower with 50 per cent or more of his acreage planted for certification will be automatically entered in certified contest.

Quality

For each per cent of No. 1 quality 1

Show Sample

The show sample will consist of 100 lbs. handpicked, brushed (do not wash) U.S. No. 1, nothing smaller than 2½ inches and none weighing more than 10 oz. 100

Show Placing

Best five samples, each	100
Next five samples, each	90
Next five samples, each, etc.	80

ESCANABA POTATO BOOSTER ASSOCIATION
Certified Growers' Contest

Open to any farmer growing 2 acres or more of potatoes. Points will be awarded on the basis of total acreage of all varieties grown on any one farm. Only one entry for each farm unit.

22 bu. or more per acre	Points
Certified seed	60
1 year from certification	40
2 years ffom certification	30
Other seed	20
Planting 1-10 of acreage in seed plot and ¼ of this in tuber unit plot	100
Seed treatment-Semesan Bel.	65
Greensprouting (at least 10 days)	65
Planting on summer fallowed hay or sod field	60
8 loads manure per acre or plow down crop of hay	60
Commercial fertilizer 500 lbs. per acre, equivalent of 135 plant food units per acre or more	50

Planting Dates

Up to June 5	100
June 6-10	75

(Planting dates do not apply to Seed Plots of certified growers but does apply to main field)

Spraying

5 times	100
6 times	150
7 or more times	200

Yield

¼ point per bushel

Any grower with 50 per cent or more of his acreage planted for certification will be automatically entered in certified contest.

Show Sample

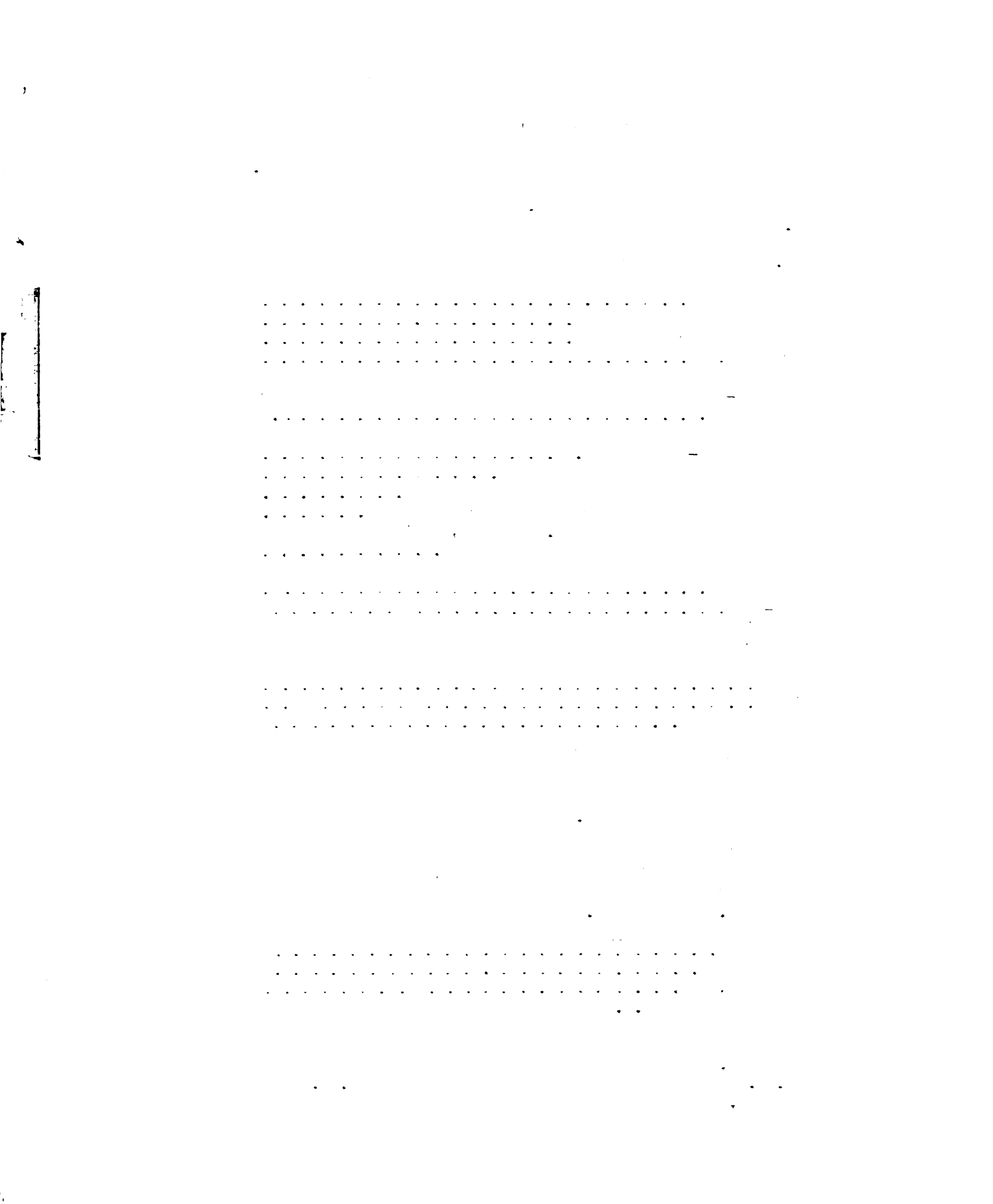
The certified show sample will consist of 128 tubers that have been hand selected for uniformity of size, shape and freedom of blemish and must be brushed and wrapped before being brought to the show. Do NOT WASH.

Show Placing

First place	100
Second place	98
Third place, etc.	96

U.P. Show Samples

Samples from the county show will be selected for the Upper Michigan show on the basis of yield and placing at the county show. Samples shown at the county show and selected for the U. P. show must be wrapped and packed for the U. P. show by the grower.



MENOMINEE COUNTY POTATO IMPROVEMENT CLUB

Open to any Farmer in Menominee County, growing 2 acres or more of Potatoes

BASIS OF AWARDS

Points will be awarded on the basis of total acreage of all varieties grown on any one farm. Only one entry for each farm unit.

I. SEED

	Points
Planting certified seed 20 bu. or more per acre	50
Planting seed 1 yr. from certification	40
Planting seed 2 yr. from certification or 10% inspected seed plot	30
Other seed	10
Treating with corrosive sublimate, formaldehyde, or organic mercury	65
Greensprouting for 2 weeks	65

II. SEED BED PREPARATION

Plowing under a sod crop or a green manure crop	60
Fall plowing or spring plowing after summer fallowing with thorough spring fitting (6 or more harrowings or discings before planting)	60
Applying stable manure, 5 points per load - Total.	50
Applying at planting time, 400 pounds or more per acre of commercial fertilizer, such as 4-16-8; 3-12-12 or 108 plant food units	50

III. CULTURAL PRACTICES

Planting Dates - Up to May 31	100
June 1 to June 5 inc.	75
June 6 to June 10 inc.	50
Trench Planting	25
Shallow cultivation dragging field with spike-tooth harrow or weeder, 3 or more times before plants are 6" high.	50

IV. DISEASE AND INSECT CONTROL

Control of leafhopper, beetles, blights, etc., with copper lime and calcium arsenate spray or dust. At least 5 applications are necessary.

1 spray or dust	10	5 spray or dust	110
2 spray or dust	20	6 spray or dust	130
3 spray or dust	40	7 spray or dust	170
4 spray or dust	70	8 spray or dust	200

V. GRADE AND YIELD

Yield - ½ point per bushel field run yield	...
Percentage of U.S. No. 1-1 point for each per cent	...
Exhibiting 100 lb. sample of representative table stock potatoes at the Potato Show; graded as though you were selling them as a U.S. No. 1 Potato.	10 -100

Proposed Score Cards

Following are examples of the proposed score cards. The present score card is included so one may contrast it with the proposed.

SCORE CARDS

PRESENT

Tall Stock Russet Rural		Certified Green Mountain	
Bushels of Seed	**Points**		**Points**
Under 20	50		50
No. of Sprays			
1 spray	10		
2 sprays	20		
3 sprays	30		
4 sprays	40		
5 sprays	80		100
6 sprays	120		150
7 sprays	160		200
8 sprays	200		
9 sprays	240		
10 sprays	280		
Planting Date			
Up to May 31	100		100
June 1 to 6	75		75
June 7 to 12	50		50
Times Worked			
Thorough fitting	60		60

PROPOSED

Russet Rural		Green Mountain	
Bushels of Seed	**Points**		**Points**
Under 20	50	Under 20	25
20 to 24.9	100	20 & over	50
25 to 29.9	150		
30 and over	200		
No. of Sprays			
5 sprays	50	5 sprays	50
6 sprays	60	6 sprays	100
7 sprays	70	7 sprays	150
8 sprays	80	8 sprays	200
9 sprays	90	9 sprays	250
10 sprays & over	100		
Blight not controlled	0	Blight not contr.	0
Blight moderately con.	25	Blight mod. contr.	25
Blight well controlled.	50	Blight well contr.	50
Planting Date			
Thru May 20	50	Thru May 20	100
May 21 to 31	35	May 21 to 31	75
June 1 and later	20	June 1 & later	50
Times Worked			
Deep mellow seed-bed	60	Deep mellow seed-bed	60

PRESENT

Table Stock — Certified

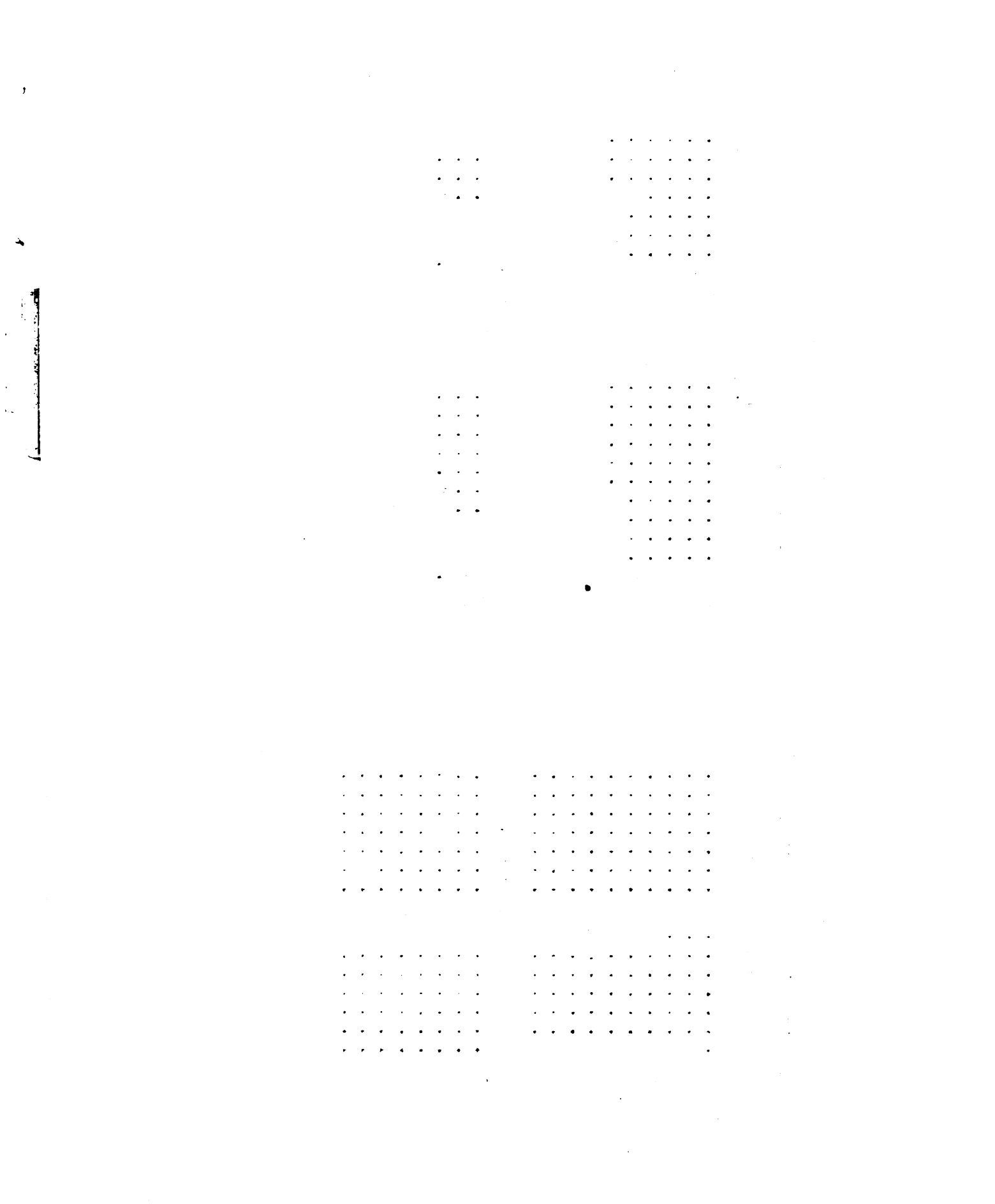
~~Russet Rural~~ — ~~Green Mountain~~

Loads of Manure	Points	Points
1 load	5	5
2 loads	10	10
3 loads	15	15
4 loads	20	20
5 loads	25	25
6 loads	30	30
7 loads	35	35
8 loads	40	40
9 loads	45	45
10 loads	50	50

Pounds of Fertilizer		
100 lbs.	10	10
200 lbs.	20	20
300 lbs.	30	30
400 lbs.	40	40
500 lbs.	50	50
600 lbs.	60	60
700 lbs.	70	70
800 lbs.	80	80

PROPOSED

Russet Rural		Green Mountain	
Loads of Manure	Points		Points
5 loads	.25	5 loads	.25
6 loads	.30	6 loads	.30
7 loads	.35	7 loads	.35
8 loads	.40	8 loads	.40
9 loads	.45	9 loads	.45
10 loads or more	.50	10 loads or more	.50
Pounds of Fertilizer			
200 to 299 lbs.	50	200 to 299 lbs.	50
300 to 399 lbs.	100	300 to 399 lbs.	75
400 lbs. and over	150	400 lbs. & over	100



Application of the Technique Used in this Study to Farm Management Studies

This study was undertaken with the idea that it might be the fore-runner to similar studies in farm management. It is felt that there is a definite need for a better way to get farmers interested in recommended practices. Considerable interest has been aroused in the Upper Peninsula area through the holding of contests and the awarding of prizes based upon the score card. Enthusiasm in the potato growing areas has prompted this question to be asked: "Why wouldn't it be possible to introduce contests in other type-of-farming areas?"

This study has shown that one can statistically relate practices to the results obtained in a specific enterprise and set up a score card. From previous study of farm management data, it is felt that a similar approach can be made through farm management practices of operating a farm.

If such a study were to be made, one would have to run multiple correlations determining the relative importance of various factors in their effect on farm earnings. The labor income would be X_1 as the dependent factor comparable to the yield in potatoes. Some of the independent factors which might be related to labor income and evaluated by a score card are as follows:

X_1 - labor income

X_2 - livestock production index (dairy, beef, hogs, poultry, sheep)

X_3 - crop yield index

X_4 - size of business (total P. M. W. U.)*

X_5 - intensity (P. M. W. U. per tillable acre)

X_6 - labor efficiency (P. M. W. U. per man)

X_7 - machinery efficiency (expense per tillable acre)

X_8 - building efficiency (expense per animal unit)

*Productive man work units

Two of these are indexes which are results of efficiency in several enterprises, namely, the livestock and crop yield indexes. Rate of production would have to be measured in dairy, beef, hogs, poultry, and sheep and each enterprise weighted according to its relative importance. This can be done in the case of livestock by multiplying the production index by the productive man work units on that enterprise. These would then be combined into a single figure indicating the average rate of livestock production. A similar procedure can be followed with crops using acres as a measure of their relative importance. Size of business could be measured by total P. M. W. U.; intensity could be measured by P. M. W. U. per tillable acre, labor efficiency by P. M. W. U. per man, machinery efficiency by expense per tillable acre, and building efficiency by expense per animal unit.

If there were a difference in soil productivity, it might be that a separate score card would have to be made up for different classes of soils.

Summary

The purpose of this study has been to make an analysis of past records of the Premier Potato Growers in the Upper Peninsula, to determine the validity of the weight assigned to various cultural practices used on the score cards, and from the results obtained to develop a new score card. It was also hoped that this investigation might lead to the possibility of developing a score card for use in farm management.

Seventy percent of the Premier Growers in the Upper Peninsula grew either Russet Rural or Green Mountain potatoes; the remaining thirty percent grew Chippewa, Pontiac, Menominee and Sebago varieties. The growers of Russet Rural potatoes were principally located in the counties of Menominee, Delta, Iron, and Schoolcraft and the Green Mountain growers in the counties of Houghton, Marquette and Baraga.

After working many statistical calculations, data for three years (1942-44) have been combined to present a more representative relationship of factors than could be had by using any one year by itself. The two cultural practices found to be of most importance in the growing of Russet Rural potatoes according to correlation analysis, were bushels of seed used in planting and pounds of fertilizer applied. In the growing of Green Mountain potatoes the correlations found with the great-

est significance were early planting and number of times sprayed.

The effect of the various cultural practices on the yield of potatoes, as shown by correlation analysis, was not as great as is frequently supposed. One possible explanation of this situation is the fact that all growers having under 300 bushels per acre were omitted, as they were not included in the contests and no data were available for them. Most of the growers having over 300 bushels per acre followed the recommended practices to a fair degree, so there were few who represent poor practices. In keeping with the above information, the facts presented in this study are valid only as they are presented within these limits of the practice.

Regression equations combining the years 1942-44 showed the effects of the cultural practices on yields to be as follows:

Effect of Cultural Practices on Yields
of Premier Potato Growers in the Upper Peninsula, 1942-44

The Addition of:	Changed the Yield	
	Russet Rural (Bushels)	Green Mountain (Bushels)
1 Spray	+ 7.90	+ 6.50
1 Bushel of seed	+ 7.10	+ 4.70
1 Day planting (after May 1st)	- .05	- 1.30
1 Time worked (after plowing & before cultivating)	+ 2.52	- .05
1 Load of manure	+ 2.70	+ 1.88
1 Pound of fertilizer	+ .12	+ .05

Proposed score cards have been developed which more nearly present a valid weight to the cultural practices based upon data from Premier Potato Growers contests, 1942-44. The points assigned to each recommended cultural practice have met with the approval of certain members of the Farm Crops Department. (Score card illustration on page 44.)

This study has shown that one may statistically relate practices followed in a specific enterprise with the results obtained, and develop a score card for use in a contest to promote better cultural practices. It also points out the possibility of statistically determining the effect of certain farm management factors on farm earnings for use in setting up a score card to encourage better farm management.

Correlation Coefficients at the 5% and 1% Levels of Significance*

Degrees of freedom	5%	1%	Degrees of freedom	5%	1%
1	.997	1.000	24	.388	.496
2	.950	.990	25	.381	.487
3	.878	.959	26	.374	.478
4	.811	.917	27	.367	.470
5	.754	.874	28	.361	.463
6	.707	.834	29	.355	.456
7	.666	.798	30	.349	.449
8	.632	.765	35	.325	.418
9	.602	.735	40	.304	.393
10	.576	.708	45	.288	.372
11	.553	.684	50	.273	.354
12	.532	.661	60	.250	.325
13	.514	.641	70	.232	.302
14	.497	.623	80	.217	.283
15	.482	.606	90	.205	.267
16	.468	.590	100	.195	.254
17	.456	.575	125	.174	.228
18	.444	.561	150	.159	.208
19	.433	.549	200	.138	.181
20	.423	.537	300	.113	.148
21	.413	.526	400	.098	.128
22	.404	.515	500	.088	.115
23	.396	.505	1,000	.062	.081

*George W. Snedecor, Statistical Methods, 1938, page 133

Formulas

Gross Correlation Coefficients:

$$r_{12} = \frac{\Delta x_1 x_2 - (\Delta x_1 \Delta x_2)}{\sqrt{\Delta x_1^2 - (\Delta x_1)^2}\ \sqrt{\Delta x_2^2 - (\Delta x_2)^2}}$$

$$r_{13} = \frac{\Delta x_1 x_3 - (\Delta x_1 \Delta x_3)}{\sqrt{\Delta x_1^2 - (\Delta x_1)^2}\ \sqrt{\Delta x_3^2 - (\Delta x_3)^2}}$$

$$r_{14} = \frac{\Delta x_1 x_4 - (\Delta x_1 \Delta x_4)}{\sqrt{\Delta x_1^2 - (\Delta x_1)^2}\ \sqrt{\Delta x_4^2 - (\Delta x_4)^2}}$$

$$r_{15} = \frac{\Delta x_1 x_5 - (\Delta x_1 \Delta x_5)}{\sqrt{\Delta x_1^2 - (\Delta x_1)^2}\ \sqrt{\Delta x_5^2 - (\Delta x_5)^2}}$$

$$r_{16} = \frac{\Delta x_1 x_6 - (\Delta x_1 \Delta x_6)}{\sqrt{\Delta x_1^2 - (\Delta x_1)^2}\ \sqrt{\Delta x_6^2 - (\Delta x_6)^2}}$$

$$r_{17} = \frac{\Delta x_1 x_7 - (\Delta x_1 \Delta x_7)}{\sqrt{\Delta x_1^2 - (\Delta x_1)^2}\ \sqrt{\Delta x_7^2 - (\Delta x_7)^2}}$$

$$r_{23} = \frac{\Delta x_2 x_3 - (\Delta x_2 \Delta x_3)}{\sqrt{\Delta x_2^2 - (\Delta x_2)^2}\ \sqrt{\Delta x_3^2 - (\Delta x_3)^2}}$$

$$r_{24} = \frac{\Delta x_2 x_4 - (\Delta x_2 \Delta x_4)}{\sqrt{\Delta x_2^2 - (\Delta x_2)^2}\ \sqrt{\Delta x_4^2 - (\Delta x_4)^2}}.$$

$$r_{34} = \frac{\Delta x_3 x_4 - (\Delta x_3 \Delta x_4)}{\sqrt{\Delta x_3^2 - (\Delta x_3)^2}\ \sqrt{\Delta x_4^2 - (\Delta x_4)^2}}$$

$$r_{56} = \frac{\Delta x_5 x_6 - (\Delta x_5 \Delta x_6)}{\sqrt{\Delta x_5^2 - (\Delta x_5)^2}\ \sqrt{\Delta x_6^2 - (\Delta x_6)^2}}$$

$$r_{57} = \frac{\Delta x_5 x_7 - (\Delta x_5 \Delta x_7)}{\sqrt{\Delta x_5^2 - (\Delta x_5)^2}\ \sqrt{\Delta x_7^2 - (\Delta x_7)^2}}$$

$$r_{67} = \frac{\Delta x_6 x_7 - (\Delta x_6 \Delta x_7)}{\sqrt{\Delta x_6^2 - (\Delta x_6)^2}\ \sqrt{\Delta x_7^2 - (\Delta x_7)^2}}$$

First Order Correlation Coefficients:

$$r_{12.3} = \frac{r_{12} - (r_{13})\ (r_{23})}{\sqrt{1 - (r_{13})^2}\ \sqrt{1 - (r_{23})^2}}$$

$$r_{12.4} = \frac{r_{12} - (r_{14})(r_{24})}{\sqrt{1 - (r_{14})^2}\ \sqrt{1 - (r_{24})^2}}$$

$$r_{13.2} = \frac{r_{13} - (r_{12})(r_{32})}{\sqrt{1 - (r_{12})^2}\sqrt{1 - (r_{32})^2}}$$

$$r_{13.4} = \frac{r_{13} - (r_{14})\ (r_{34})}{\sqrt{1 - (r_{14})^2}\ \sqrt{1 - (r_{34})^2}}$$

$$r_{14.2} = \frac{r_{14} - (r_{12})\ (r_{24})}{\sqrt{1 - (r_{12})^2}\ \sqrt{1 - (r_{24})^2}}$$

$$r_{14.3} = \frac{r_{14} - (r_{13})\ (r_{34})}{\sqrt{1 - (r_{13})^2}\ \sqrt{1 - (r_{34})^2}}$$

$$r_{15.6} = \frac{r_{15} - (r_{16})\ (r_{56})}{\sqrt{1 - (r_{16})^2}\ \sqrt{1 - (r_{56})^2}}$$

$$r_{15.7} = \frac{r_{15} - (r_{17})\ (r_{57})}{\sqrt{1 - (r_{17})^2}\ \sqrt{1 - (r_{57})^2}}$$

$$r_{16.5} = \frac{r_{16} - (r_{15})\ (r_{56})}{\sqrt{1 - (r_{15})^2}\ \sqrt{1 - (r_{56})^2}}$$

$$r_{16.7} = \frac{r_{16} - (r_{17})\ (r_{67})}{\sqrt{1 - (r_{17})^2}\ \sqrt{1 - (r_{67})^2}}$$

$$r_{17.5} = \frac{r_{17} - (r_{15})\,(r_{57})}{\sqrt{1 - (r_{15})^2}\,\sqrt{1 - (r_{57})^2}}$$

$$r_{17.6} = \frac{r_{17} - (r_{16})\,(r_{67})}{\sqrt{1 - (r_{16})^2}\,\sqrt{1 - (r_{67})^2}}$$

Second Order Correlation Coefficients:

$$r_{14.32} = \frac{r_{14.3} - (r_{12.3})\,(r_{24.3})}{\sqrt{1 - (r_{12.3})^2}\,\sqrt{1 - (r_{24.3})^2}}$$

$$r_{17.65} = \frac{r_{17.6} - (r_{15.6})\,(r_{57.6})}{\sqrt{1 - (r_{15.6})^2}\,\sqrt{1 - (r_{57.6})^2}}$$

Multiple Correlation Coefficients:

$$R^2_{1.23} = (1 - r_{12}^2)\,(1 - r_{13}^2)$$

$$R^2_{1.24} = (1 - r_{12}^2)\,(1 - r_{14}^2)$$

$$R^2_{1.34} = (1 - r_{13}^2)\,(1 - r_{14}^2)$$

$$1 - R^2_{1.234} = (1 - r_{12}^2)(1 - r_{13.2}^2)(1 - r_{14.32}^2)$$

$$R^2_{1.56} = (1 - r_{15}^2)(1 - r_{16}^2)$$

$$R^2_{1.57} = (1 - r_{15}^2)(1 - r_{17}^2)$$

$$R^2_{1.67} = (1 - r_{16}^2)(1 - r_{17}^2)$$

$$1 - R^2_{1.567} = (1 - r_{15}^2)(1 - r_{16.5}^2)(1 - r_{17.65}^2)$$

Normal Equation (to find gross predicting equation):

$$\text{I. } \Sigma(Y) = Na + b\Sigma(B)$$

$$\text{II. } \Sigma(BY) = a\Sigma(b) + b\Sigma(B)^2$$

Normal Equation for Multiple Correlation:

$$\left[\frac{\Sigma x_2^2 - (\Sigma x_2)^2}{n}\right] b + \left[\frac{\Sigma x_2 x_3 - \Sigma x_2 \Sigma x_3}{n}\right] c + \left[\frac{\Sigma x_2 x_4 - \Sigma x_2 \Sigma x_4}{n}\right] d = \frac{\Sigma x_1 x_2 - \Sigma x_1 \Sigma x_2}{n}$$

$$\left[\frac{\Sigma x_2 x_3 - \Sigma x_2 \Sigma x_3}{n}\right] b + \left[\frac{\Sigma x_3^2 - (\Sigma x_3)^2}{n}\right] c + \left[\frac{\Sigma x_3 x_4 - \Sigma x_3 \Sigma x_4}{n}\right] d = \frac{\Sigma x_1 x_3 - \Sigma x_1 \Sigma x_3}{n}$$

$$\left[\frac{\Sigma x_2 x_4 - \Sigma x_2 \Sigma x_4}{n}\right] b + \left[\frac{\Sigma x_3 x_4 - \Sigma x_3 \Sigma x_4}{n}\right] c + \left[\frac{\Sigma x_4^2 - (\Sigma x_4)^2}{n}\right] d = \frac{\Sigma x_1 x_4 - \Sigma x_1 \Sigma x_4}{n}$$

Solving for a:

$$a = My_1 + 270 - bMx_2 - cMx_3 - dMx_4$$

Substituting:

$$x_1 = a + bx_2 + cx_3 + dx_4$$

Averages Substituted in Normal Equation for Multiple Correlation, 1942-44:

$$\left[Ax_2^2 - (Ax_2)^2\right]b + \left[Ax_2x_3 - Ax_2Ax_3\right]c + \left[Ax_2x_4 - Ax_2Ax_4\right]d = Ax_1x_2 - Ax_1Ax_2$$

$$\left[Ax_2x_3 - Ax_2Ax_3\right]b + \left[Ax_3^2 - (Ax_3)^2\right]c + \left[Ax_3x_4 - Ax_3Ax_4\right]d = Ax_1x_3 - Ax_1Ax_3$$

$$\left[Ax_2x_4 - Ax_2Ax_4\right]b + \left[Ax_3x_4 - Ax_3Ax_4\right]c + \left[Ax_4^2 - (Ax_4)^2\right]d = Ax_1x_4 - Ax_1Ax_4$$