THE PERFORMANCE OF A PHYSICAL DISTRIBUTION CHANNEL SYSTEM UNDER VARIOUS CONDITIONS OF DEMAND UNCERTAINTY: A SIMULATION EXPERIMENT

Dissertation for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY

THOMAS W. SPEH

1974

This is to certify that the

thesis entitled

THE PERFORMANCE OF A PHYSICAL DISTRIBUTION CHANNEL
SYSTEM UNDER VARIOUS CONDITIONS OF DEMAND
UNCERTAINTY: A SIMULATION EXPERIMENT

presented by

Thomas W. Speh

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Marketing

Donald J. Bowersox
Major professor

Date May 10, 1974

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ABSTRACT

THE PERFORMANCE OF A PHYSICAL DISTRIBUTION CHANNEL
SYSTEM UNDER VARIOUS CONDITIONS OF DEMAND
UNCERTAINTY: A SIMULATION EXPERIMENT

By

Thomas W. Speh

To effectively and efficiently administer physical distribution channel systems the various types of uncertainty which effect the channel must be recognized and their impacts evaluated. One form of uncertainty which potentially has significant impact on channel performance is demand uncertainty. Variations in the number of units demanded per unit of time have the potential to influence the performance of all physical distributions activity centers, and thus the entire system in terms of cost and service capability. Therefore, the objective of this research was to measure the impact of demand uncertainties on the cost and service performance of a three echelon physical channel system.

Demand uncertainties may be described by three measures: the probability distribution of daily demand; the average demand per day; the variance of daily demand. These three characteristics of demand uncertainty were the experimental factors of this research. A number of levels of each factor were evaluated, including six probability distributions of daily demand, three levels of demand variance and two levels of average demand per day.

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Two general hypotheses were tested. They were:

1. Uncertain demand leads to higher costs and lower service performance than would occur if demand was constant per day.

2. Different levels of each type of demand uncertainty have different impacts on physical channel system cost and service performance.

The LREPS physical channel simulation model was used to test the research hypotheses. Thirty experimental simulation runs were made for a test period of ninety days each. Two runs were made with fixed demand, i.e., the same number of units were demanded each day. The demand for each of the remaining runs was generated from a specific demand distribution, with a given average demand and demand variance. The measure of each simulation run included total demand stocked out, total cost, transport cost, facility cost, thruput cost and inventory cost. These results were used to test the research hypotheses using analysis of variance techniques.

The major conclusions of the research are:

1. The comparison of the simulation runs made with each type of demand uncertainty to the runs with fixed demand per day revealed that overall channel total cost was not measurably affected by various demand uncertainties. However, the amount of demand stocked out was significantly higher as a result of demand uncertainties.

2. The comparison of cost and service performance of the channel <u>among</u> the types of uncertainty and <u>among</u> the different levels of each type revealed significant differences in selected cases. However, certain levels of each type demand uncertainty consistely

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affected channel performance. The exponential and normal were the demand distributions which created the greatest impact on channel performance. High levels of demand variability repeatedly led to high stockout percentages, and in some cases, high activity center costs. The costs and stockouts resulting from low average level of demand were always different than those resulting from the high average level of demand.

3. The three characteristics of demand uncertainty differed in terms of the response variables they affected. The impact of distributions was primarily on inventory and transport costs; the level of variance affected transport costs; the impact of the average demand level was observed on all response variables. All three types of demand uncertainty affected the amount of demand stocked out.

4. In general, the amount of demand stocked out is more sensitive to demand uncertainty than are total costs. Total cost varied only as the average demand level varied; it was not sensitive to variances and distributions.

5. The effects of different characteristics of demand uncertainty were felt at different echelons in the channel. The more symmetrical distributions, the lower demand variances, and the high average demand level appeared to create cost and service impacts at the wholesaler and manufacturer level in the channel. The less symmetrical distributions, the high variances and low average demand level led to a higher incidence of stockouts at the retail level. However, the most extreme variability of demand resulted in large amounts of stockouts at all levels in the channel.

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6. A number of implications for physical channel system administration follow from the results of this research. The research suggests the need for empirically estimating the nature of the demand pattern faced by the channel. Then, policies may be formulated to plan for, or mitigate the effects of the type of demand uncertainty that prevails. For some types of demand uncertainty retail performance was most affected, while for other types the impacts were felt at the wholesale and manufacturer level. Not only must these impacts be considered in determining individual firm policies, but total channel-wide planning is called for. Thus, because uncertainty had different effects within the entire channel, the systems approach to integrated channel operation is reaffirmed. Because some types of demand uncertainties have more favorable effects on channel performance, efforts to alter the demand pattern may lead to improved channel performance. Adjustments to the marketing mix, in the area of advertising and special promotions, might be employed in an effort to affect demand patterns. Conversely, adjustments in the marketing and/or the physical distribution mix may produce demand patterns which negatively influence the performance of the channel. Thus, such adjustments must be evaluated in light of their impact on the pattern of demand.

7. In an effort to provide a tentative indication as to the nature and scope of future research in this area, three additional simulation runs were completed where both demand and lead time in the channel were allowed to vary in a controlled experiment. The results of these runs indicated that lead time variations may have a much

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greater impact on channel performance than does demand uncertainty. In addition, the effects of uncertain demand and lead time were not multiplicative, i.e., cost and service performance did not change perceptively from that achieved when lead time was variable and demand was constant per day.

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By

Thomas W. Speh

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Marketing and Transportation Administration

1974

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ACKNOWLEDGMENTS

This dissertation was completed with the support and assistance of many individuals. Their contributions are sincerely appreciated and must be recognized.

The dissertation committee deserves more recognition than can possibly be expressed in these few lines. The committee consisted of Dr. Donald J. Bowersox, Dr. Richard J. Lewis and Dr. Donald A. Taylor, all professors of Marketing and Transportation at Michigan State University.

Dr. Bowersox, as committee chairman, provided a sense of purpose and direction to the entire research project. His knowledge and perceptions in the area of physical distribution had a significant impact on the structure and relevancy of the completed research. In addition, his editorial assistance and inspirational support were greatly appreciated.

Dr. Lewis's suggestions helped to solve many problems which arose throughout the research. His knowledge and understanding of statistical methodology facilitated the selection of the methods applied in the research. Dr. Lewis's ideas and comments during the original conception of the dissertation were extremely valuable.

Dr. Taylor helped to put the research in perspective and guide it in the proper direction. His efforts in the initial structuring of the dissertation were most appreciated.

Particular appreciation is expressed to Systems Research Incorporated, and specifically to John Griggs and Allan Dale, for the provision of facilities and computer time to complete the simulation runs. Without such assistance the time horizon and financial burdens of the dissertation would have been excessive. The Johnson and Johnson Domestic Operating Company is gratefully acknowledged for their continued support of the LREPS project and their financial assistance in this particular phase of the project.

Dave Closs deserves special recognition for his efforts in the computer programming aspects of the research. Dave's knowledge of the LREPS model and his computer expertise greatly facilitated the successful completion of the dissertation.

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This dissertation was one segment of a joint investigation into physical channel system operation. Dr. George Wagenheim completed the companion dissertation to this one, and his contributions to my dissertation were significant. George's insights and creativity were instrumental in the development and implementation of the research and his thoughts have left their mark on the finished project.

The typing and clerical efforts put forth for this dissertation were outstanding. Charlotte Wagenheim's work on the first draft was exceptional. Her patience and cooperation were very much appreciated. Grace Rutherford typed the final draft, and her ability to simultaneously type, edit and organize was magnificent. Mrs. Rutherford is a professional in the highest sense of the word.

My family deserves as much credit for this dissertation as I do. To my entire family I am thankful for their confidence in me and their constant encouragement. My wife, Donna, and sons, Scott and Michael, deserve special consideration. Their continued patience, understanding, encouragement and love throughout the research and my entire doctoral program gave me the necessary inspiration to complete the task.

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CHAPTER I

INTRODUCTION

General Problem Statement

The physical distribution of goods represents a significant portion and an integral segment of the economy. The importance of physical distribution to the firm and to the economic sector at large cannot be denied. It has been variously reported that physical distribution costs account for 20% of the total sales dollar and in some cases may be as high as 50%.[1] In addition to aggregate cost, physical distribution is an integral part of overall distribution performance. Goods destined for consumption must be physically moved to the location of purchase or no transactions will result. Without physical distribution the economic sector would not function. To achieve efficiency and effectiveness in physical distribution, it is important to understand how the overall channel system operates, the forces which impinge upon the system and the effects of the forces on the successful operation of the system.

Only recently have serious attempts been made to understand these interrelationships. Although research has been conducted on all aspects of channel relationships, it has not been exhaustive nor have the conclusions been definitive. As a result, there is much research still to be done in the physical distribution of goods.[2]

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Certain aspects of physical channel structure have been investigated. Decisions as to the overall structural design of the physical channel system have been effectively improved through the use of simulation models of such systems. Bowersox,[3] Shycon,[4] and Ballou[5] have made important contributions in the area of physical channel system simulation modeling. Behavioral dimensions of the channel are receiving more attention, with the works of Stern[6] and Bucklin[7] making significant impacts in this area. In addition, the location and inventory decisions have been exhaustively researched[8] and a number of effective models constructed.[9]

One aspect of physical distribution operations that has not been exhaustively researched is the impact of uncertainty upon system performance. Uncertainty influences physical distribution operations by introducing variable sales patterns and replenishment times. To the degree a better understanding of the impact of uncertainty is understood, it should lead to more effective planning and control of the system. If we were able to assess the impacts of uncertainty upon various aspects of the channel system, we would then be in a good position to account for these effects and take action to overcome them. The purpose of this research is to measure the impact of uncertainties (demand and lead time) on the performance (cost and service) of a physical distribution channel system.

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An Overview of Physical Distribution

Physical distribution though variously defined will be used in this research to encompass the movement of finished goods from the manufacturing plant to the ultimate consumer.[10] The purpose of physical distribution is to move finished goods between these points in an efficient and effective manner. Performance is measured in terms of cost and service. Physical distribution is defined for this research to include transportation, warehousing, inventory, communication and handling.

The basic structure of a physical channel system is that of echeloned arrangement of institutions and/or functions. Echelon refers to a steplike formation. In the physical distribution context the echelon structure refers to the levels through which a product proceeds from production to a point of ultimate consumption. To measure the impact of uncertainty in this research an echelon structure is used. The echelon system rather than the direct system (one where there are no steps between the manufacturer of the product and the ultimate consumer) was selected for study for several reasons. Namely, it is a close replication of the real world, few products are directly distributed, the advent of the increasing number of products available both in kind and degree and the increase in scrambled merchandise necessitates the use of an echelon system for efficient distribution.[11] Furthermore, the effects of uncertainty on the system would seem to be magnified as additional levels are added to the system. Time delays, additional or

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For this research each echelon has the following characteristics. They will hold inventory to facilitate the discrepancies between demand and production; they will be break bulk points, that is, they exist for the purpose of receiving larger volume shipments and dispersing these shipments to various customers and they will offer all the necessary facilitating activities to complete these operations such as handling and communication.

The operation of the physical channel system is defined as a system in which all the components interact to minimize the cost of the total system for a given level of service. System has been variously defined, but generally can be defined as, "a set or arrangement of things so related or connected as to form a unity or organic whole."[12] Bowersox defines the systems concept as, "one of total integrated effort toward the accomplishment of a predetermined objective."[13] The systems concept as cited by Alderson[14] can be viewed at any level of generalization. In terms of physical distribution the system can be seen as the components, i.e., the parts of the physical distribution system controlled by the firm such as transportation, handling, warehousing, inventory and communication.

Because the physical distribution segment of the overall economic sector is a system, these components or activity centers can be viewed as interrelated subsystems. Therefore, they behave not as entities but as interrelated parts of a whole. Trade-offs occur between and within th

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and within these subsystems. The trade-offs can be arranged in such a way so as to influence total cost and service capability. The task of channel design is one of finding favorable trade-off relationships. The system can also be viewed at a higher level. That is, it would not only encompass the parts specific to an individual firm but could also include all the firms in a channel from manufacturer to ultimate consumption. It is in this context that system is defined for this research.

The argument for viewing the physical channel of distribution as a system rests upon the fact that all participants share in a unified goal. Thus, working in concert has the greatest potential for achieving desired results. That is, all the members of the channel have similar objectives. The objectives can be best reached through the systems approach which implies cooperation and concentration on a unified goal.

Attempts to improve unified operations across channel echelons can be witnessed by the increased moves to vertically integrate the channel in various ways.[15] Furthermore, the position has been presented by several authors that it is the channel that competes with other channels rather than firms competing against other firms.

For instance,

> Traditional economic and business analysis of strategic planning has tended to focus on the behavior of individual firms. More recent thinking suggests that the total channel systems might be the more appropriate unit of analysis. This view is taken below because, basically, economic systems are designed to satisfy customer needs and these needs are not completely satisfied until some

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> package of goods and/or services has moved all along a channel of distribution to users or final consumers. The members of a channel system may not think of themselves as members of a system, but nevertheless their system will continue to exist only as long as their unique combination performs more effectively than competing channels.[16]

In this research, therefore, measures of performance relating to cost and service, are those associated with the channel system, rather than the individual channel members.

Uncertainty

A major force which affects the structure and operation of a physical distribution system is uncertainty. Uncertainty in the physical distribution context can be generally defined as not knowing what will occur or when it will occur. Although the sources of uncertainty are varied, it manifests itself in two general ways on the physical distribution system. First, there is demand uncertainty and, second, lead time uncertainty.

Demand can be defined as a request by the ultimate consumer made upon the system to deliver a product or service. Demand presents itself to the system in an uncertain fashion (i.e., it is a random variable). It is uncertain as to when demand will occur over time and when demand occurs it is uncertain as to how much will be demanded (i.e., level).

Lead time can be defined as the amount of time between placement of an order and receipt of that order. Specifically, it can be broken down into three components: order communication, order processing and transportati

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transportation (see Figure 1-1). Each of these components represents a source of uncertainty. It is not known with certainty how long each one of these activities will take, thus taken together it is not known with certainty the overall time duration from placement to receipt of an order.

As pointed out previously, demand and lead time uncertainty affect the structure and operation of the physical distribution system. Uncertainty also affects the planning and control of the system. On planning and control, Lewis and Erickson say, "Management planning and control should concern itself with maximizing the efficiency and effectiveness of efforts used in attaining desired purposes."[17]

Thus, the significance of planning and control to the physical distribution system is established. Ideally, to plan and control effectively, we must know what will occur and when. However, the physical distribution system operates in a world of uncertainty, thus planning and control are adversely affected. Without effective planning and control, efficiency and effectiveness are difficult to achieve.

Uncertainty is not new and it will always be with us as a simple fact of business. The majority of efforts in the past designed to cope with uncertainty have attempted to reduce its impact. For instance, more accurate sales forecasting, more accurate budgeting, etc. However, a potentially fruitful approach to solving the same problem is to first accept the fact that there will always be uncertainty and asking, can it be categorized and described, and if so, can one isolate how the various


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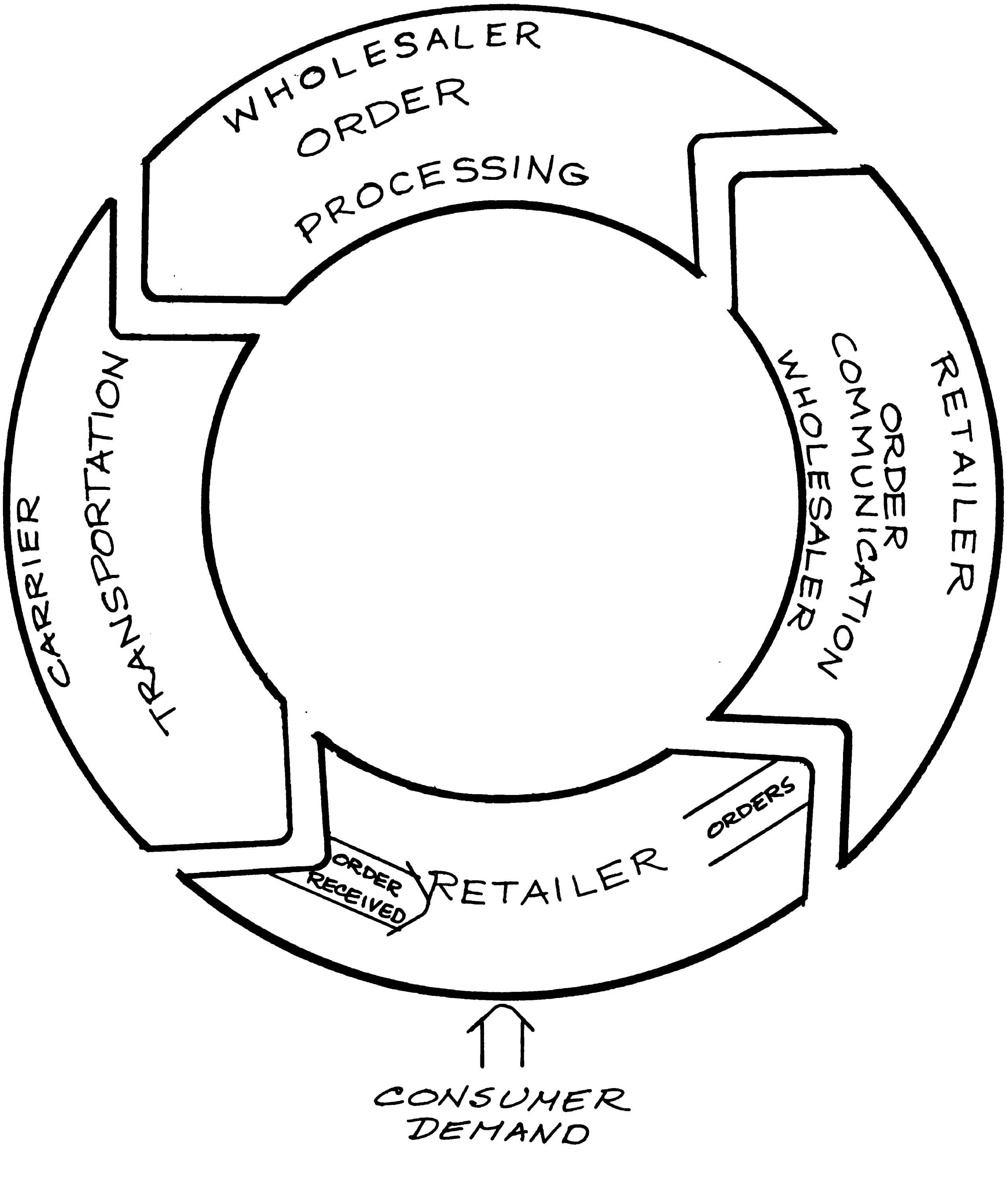

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types of uncertainty will affect a physical channel system. If one could isolate the impacts of uncertainty, which is the objective of this research, planning and control would be improved.

Research Procedure

The purpose of this research, as indicated earlier in this chapter, is to measure the impact of demand and lead time uncertainty on the cost and service capabilities of a physical channel of distribution. Demand and lead time uncertainty is evidenced in three material ways: (1) the level of demand and lead time, or average demand and lead time; (2) the variability or dispersion of demand and lead time about its average; and (3) the pattern or probability distribution of demand and lead time. Consequently, the research problem to be solved involves the development of a means by which the three material aspects of uncertainty may be impacted upon a physical channel system and the resultant cost and service levels measured.

Ideally, the solution to this problem could be obtained by performing a series of experiments on an existing channel of distribution. In this manner, the researcher could then observe how the system reacted to the changes in demand and lead time levels, variability and patterns. However, such a procedure is not feasible nor practical. It would not be possible to control all the relevant variables in the system in that cost and service measures could not be determined under "controlled" or identical conditions. Nor would it be possible to manipulate the level, variability and pattern of demand and lead time as is experienced by an orga

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by an ongoing physical channel of distribution. Therefore, the solution to the research problem lies not in actual experimentation, but with experimentation on a replication or model of a real world physical channel system.

A model is generally regarded as an abstraction or simplification of a system. A mathematical model describes the system, its components and their interactions in quantitative terms. The model thus allows one to abstract the essential characteristics of a system and thereby observe and eventually predict how that system will function. Models cannot replace actual experience; at best they reduce a complex system to manageable proportions or serve to crystallize our thinking or perceptions.[18] Once the analyst has achieved a parallelism between the actual situation and his model, it is usually easier to manipulate the model to study the characteristics in which he is interested than it is to try to work with the real world system.[19] The model of a system then provides the researcher with the means to experiment with variables both internal and external to the system model and thereby observe the reaction of the system to such variations.

Simulation is one form of modeling which has been successfully employed to replicate physical channel systems.[20] Simulation models mathematically represent a system, but when applied to problem solving do not necessarily lead to an optimal solution. Teichroew and Lubin provide insight into the nature of computer simulation:

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> Computer simulation has come into increasingly widespread use to study the behavior of systems whose state changes over time. . . . Alternatives to the use of simulation are mathematical analysis, experimentation with either the actual system or a prototype of the actual system, or reliance upon experience and intuition. All, including simulation, have limitations. Mathematical analysis of complex systems is very often impossible; experimentation with actual or pilot systems is costly and time consuming, and relevant variables are not always subject to control. Intuition and experience are often the only alternatives to computer simulation available but can be very inadequate.
>
> Simulation problems are characterized by being mathematically intractable and having resisted solution by analytical methods. The problems usually involve many variables, many parameters, functions which are not well behaved mathematically, and random variables. Thus simulation is a technique of last resort. Yet, much effort is now devoted to "computer simulation" because it is a technique that gives answers in spite of its difficulties, costs and time required.[21]

Thus, simulation is a viable technique for modeling systems characterized by great complexity, probabilistic or stochastic processes and whose variables are difficult to analyze in precise mathematical terms. Simulation is also quite tractable for experimentation in that after a computer model of the system has been developed, the model may be sampled under different input conditions.[22] Therefore, a simulation model of a physical channel system has been selected as the means by which to measure the cost and service response of such a system to various types and levels of uncertainty.

The specific simulation model to be used in this research is the LREPS model.[23] The LREPS model has the following important characteristics:[24]

1. It provides a comprehensive model of physical distribution operations as an integrated system capable of total cost and customer service performance measurement.

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3. The unifying dimension of the model is both spatial and temporal.

4. The model is dynamic, which permits physical distribution planning over time.

5. The model allows for both demand and lead time to be expressed in probabilistic terms. Thus, the model is capable of introducing simulated demand and lead time patterns based upon any one of a variety of probability distributions.

The design and operation of the LREPS model have been well documented in various works.[25]

The LREPS model provides the basic framework for the experimentation involving demand and lead time level, variability and pattern. The basic LREPS model was modified in accordance with the model description in Chapter II. Thus, one phase of the present research was to develop the necessary operating rules and cost functions to be employed in the modified model.

The effects of three material measures of uncertainty related to demand and lead time upon system cost and service are examined in the research. Each experimental run consists of impressing demand and lead time at a given level, with a given variability and a given probability distribution on the channel system model. In this manner, the impact of level, variability and pattern of uncertainty can be measured. The measures of system performance which serve as the output of each experiment include:

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2. Individual activity center costs for the channel system.

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The probability distributions used in the experimental runs are computer generated. Each distribution reflects a particular probability function, mean and variance. The resulting distributions then serve as daily demand and lead time input for each experiment. The probability distributions selected for experimentation are those which have empirical justification and which have the potential to measurably affect channel performance.

Two "controlled" simulation runs were made for comparison purposes. The control or base system is completely deterministic in nature, that is, demand and lead time are given and fixed. As a result of this total certainty, no provision for safety stock is made. Thus, the experimental runs are also devoid of safety stock.

The experimental runs are short run in nature, i.e., the system's output is measured for a time span (simulated days) of less than one year. Because the system is evaluated over a short period of time, facility locations and numbers are not allowed to vary. Additionally, a time series of demand is not considered. In other words, the trend, seasonal and cyclical values of demand over the period are zero. All combinations of patterns, levels and variances are imposed on a model that has no provision for backorders at the customer level. There are provisions for backorders within the system.

The method of experimentation in the simulation model is to make changes in the external and internal variables (demand and lead time) and then analyze the effects of these changes on the cost and service of the physical channel system. To study the results in some meaningful manner, a

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manner, a proper method of analysis, i.e., experimental design must be selected.

The experimental design employed in the research is a factorial design. A factorial experiment is one in which the effects of all the factors and factor combinations in the design are investigated simultaneously. In this case, three factors are to be analyzed: the probability distribution of demand and lead time, the average or level of demand and lead time; and the variance or dispersion of demand and lead time about the average. The factorial design is advantageous to the extent that effects of a particular factor are evaluated by averaging over a broad range of other experimental variables. For example, the factorial design will permit statements to be made as to the effect of a particular demand and lead time distribution, where the distribution is considered over a range of demand and lead time levels and variances.

The data is analyzed by standard analysis of variance techniques in addition to two multiple comparison techniques and standard t tests. Thus, the research develops comparisons, on the basis of cost and service, of the effects of probability distributions, levels and variances. Additionally, the cost and service performance of the system under each level of each factor is compared against the control system. Such a comparison is expected to provide a direct measure of the effect of the given type of uncertainty.

This research is basically a pilot inquiry into the effects of demand and lead time uncertainty on the performance of a physical channel system. T

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system. To this extent, it is exploratory in nature, seeking to systematically analyze the sensitivity of physical distribution cost and service to uncertain conditions associated with demand and lead time. Thus, on the basis of research results, generalizations are expected on the impact of uncertainty. In addition, guidelines for further research will be established.

To be able to draw generalizations as to the effects of uncertainty on the system it is necessary to remove selected aspects of reality. As previously described, there are no safety stocks, no locational variations, no trends, etc. Inclusion of such factors (even though they would make the model more realistic) would only confuse and mask the effects of uncertainty. The intent is to systematically replace presently missing factors in future research. As factors are added and the model becomes more complete, the effects on the system of the newly introduced factors can be more accurately analyzed.

This research, which concentrates on demand and lead time uncertainties, should lead to the following results:

1. The testing of previously established hypotheses. Basic propositions as to how the channel system will react to various changes in key external and internal variables can be put to concrete test. Such hypotheses are formulated in Chapter IV.
2. Development of researchable hypotheses. The experiments conducted with this model should lead to a vast array of propositions as to effects upon the system when demand and lead time is varied in its material aspects. Hypotheses as to possible

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changes in operating policies to mitigate the effects of demand and lead time variability should follow as a result of the experimental runs.

3. The results of this research should aid management of channel systems in formulating more satisfactory decision rules based upon the nature of the demand and lead time pattern faced by the channel. Different products experience different patterns of demand, and a knowledge of the effects of such patterns will assist management in the process of planning and controlling their systems to account for such patterns.

Division of the Problem

The research described in this chapter is completed in three aspects:

1. The effects of various levels, variability and patterns of demand on a physical channel system of distribution.
2. The effects of the same variations in lead time on the channel system.
3. To provide an indication as to possible areas for future research.

Therefore, three experimental runs are made which combine both lead time and demand uncertainty.

Each of the first two aspects are sufficiently broad and require an in-depth evaluation of uncertainty consequences. The probability distributions assumed by demand and lead time are in some cases dissimilar

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dissimilar and experience different ranges of level and variability. Additionally, each may be considered in isolation of the other without a great loss in empirical validity. Thus, two dissertations are undertaken using a common model. The research in one dissertation considers the effects of demand variations on the system cost and service, holding lead time constant. The other dissertation evaluates lead time variability, holding demand level, variability and pattern constant. Thus, with the exception of Appendix A (the physical distribution literature review) and Chapter VII (the analysis of the three experimental runs which combined demand and lead time uncertainty) the dissertations are separate and completed individually.

Specific Problem Statement

Demand may be generally defined as requests made by the ultimate consumer upon the channel system to deliver a product or service. It is the force which initiates the operation of the system. Demand is presented to the channel in various ways, including the nature of the pattern, level and variability of demand. Requests for products are made over time, and the number of requests vary per unit time period. Hence, it is uncertain as to when demand will occur, and when demand occurs, it is uncertain as to how much will be demanded. Consequently, demand may be considered as a random variable whose "pattern" over time is described by a probability distribution. A probability distribution of demand is a list of all possible demands which could occur per unit time and the probability associated with each. The critical properties of such a

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of such a distribution are its expected value (mean per unit time or "level"), standard deviation (dispersion) and its pattern or shape (normal, poisson, etc.).

Probability distributions assume different shapes and patterns depending on the process which generates their function. The pattern is relevant to the extent that two probability distributions with different patterns will have quite dissimilar probabilities associated with given events. The standard deviation represents the dispersion of events around their mean. It too is important, in that different standard deviations will change the probability of occurrence for the events in a given distribution. The effect of various levels (averages) is reflected in the absolute magnitude of events.

Demand, when observed as a probability distribution has three essential characteristics: (1) pattern, or nature of the distribution; (2) level or average demand; and (3) standard deviation, or dispersion of demand. The characteristics assumed by the demand faced by a channel system have potential consequences for the operation of that system. The pattern of demand relates to the frequency of occurrence of demand and indicates how the demand occurs per unit time. As such, the pattern influences how the entire channel will operate. All activity centers are affected by the way in which demand presents itself to the system. The variability of demand causes the system to atune itself to given fluctuations in the level of demand. Hence, extremely variable demand would affect the channel, its decisions, and therefore, its operation differently than would demand which is relatively stable over time.

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The level of demand affects the channel system in terms of capacity requirements.

It is therefore imperative, for efficient and effective channel operation, that the system management be aware of the impacts of pattern, level and variability of demand on the system. Each characteristic has the potential to influence the performance of all activity centers, and thus the entire system in terms of both cost and service capability. Without knowledge of how the system is affected by these conditions, management cannot react to, nor plan for, them.

The literature indicates that demand may be represented by a number of probability distribution patterns.[26] Additionally, demand levels and variances do change over time. Consequently, this research will concentrate upon the specific problem of determining how a channel system of distribution performs under an array of demand patterns, levels and variability conditions. The problem to be solved will be the measurement of total system cost, individual activity center cost and system service performance which is experienced by a channel system due to the direct effects of demand pattern, level and variability. Although the effects of demand uncertainty on individual distribution activity centers, particularly inventory, has been previously considered (in a static environment), the effects on an entire multiechelon system have not been exhaustively explored. Therefore, it is the purpose of this research to completely delineate the total system effects, both cost and service, of demand uncertainties.

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Thesis Outline

This dissertation consists of seven chapters. After the introductory chapter, Chapter II describes the conceptualization of the channel system to be employed in this research. The model, its description, definition and relations are also developed in Chapter II. The modifications to the LREPS model, including decision rules, cost functions, and output measures are detailed.

Chapter III describes the characteristics of demand uncertainty. The nature of probability distributions and empirical justification of the existence of particular distributions are also reviewed. Criteria for the selection of a probability distribution as representative of demand and final selection of those distributions are also considered.

Chapter IV details the research hypotheses to be tested. Additionally, the research methodology is presented. At this stage, the experimental design and measures of system output are specified in depth.

Chapter V details the findings of the experimental runs.

Chapter VI summarizes the findings and suggests generalizations to be drawn from the research. Areas of future research and the limitations of the present research are also outlined.

Chapter VII describes the procedures employed to make the experimental runs where lead time and demand are both random variables. The findings and conclusions relevant to these experiments are then presented. Finally, suggestions are developed as to the implications for future research.

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Appendix A provides an overview of the more important simulation models specific to physical channel system modeling. Additionally, the more commonly applied inventory models are reviewed.

Appendix B details the examples of all statistical computations employed in the findings chapter.

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CHAPTER I--FOOTNOTES

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CHAPTER II

RESEARCH MODEL

Introduction

To reach the stated objectives of this research it is necessary to employ a model that will simulate a physical channel system. It is the purpose of this chapter to develop and describe such a model. Before a specific model can be developed, however, the physical channel system must be conceptualized. It is here that the boundaries of the system are defined and the general purpose of the system is outlined. Conceptualization of the system is also necessary to force thinking about a channel of distribution in a non-traditional way. It is imperative that the physical distribution channel be seen as an integrated system of firms with a common goal and not as a group of separate, autonomous institutions with individual goals and objectives. Once the system is conceptualized, the LREPS model employed can be detailed. The model structure is outlined and its operation described.

Conceptualization of the System

As noted earlier, this research is concerned with that portion of the distribution system which begins at the end of the production line and ends with the ultimate consumer. This portion of the distribut

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distribution system has several purposes among which are: communication, passage of title and physical movement. This research is specifically concerned with the physical movement purpose of this portion of the distribution system. Thus, the research interest is in the physical movement of goods from the time that the good assumes its final form (producer or manufacturer finished goods inventory) to the point that the good is in the physical possession of the ultimate consumer. This portion of the distribution system has been referred to as the "physical channel system" in this research. The purpose of the physical channel system is to service ultimate consumer demand by overcoming spatial and temporal gaps between the producer and the ultimate consumer.

The overall general criteria which dictates the structure and behavior of the physical channel system is that it (the physical channel system) performs its inherent function (servicing demand) in an efficient and effective manner within the environment in which it operates.

At this point, it is necessary to explain and/or define several of the above terms such as: efficiency, effectiveness, physical channel system structure and physical channel system behavior.

Efficient-Effective

Efficient can be defined as "producing the desired effect or result with a minimum of effort, expense or waste."[1] Or, "efficiency is a dollar measure of expenditure to get a specific job done."[2] To be efficient in a physical distribution sense is to perform a task at its minimum cost.

Effective can be defined as "producing a definite or desired result" or, "effectiveness is a measure of accomplishment in terms of objectives."[3] To be effective then in a physical distribution sense, is to meet the desired service level stated by objectives.

Therefore, a physical channel system could be efficient but not effective, i.e., operate at minimum cost but not reach the desired service level or it could be efficient but not effective, i.e., reach the desired service level, but not at a minimum cost. It is, however, the goal of the physical channel system to be both efficient and effective while performing its inherent function of servicing demand.

Structure

The inherent function of the physical channel system of servicing demand efficiently and effectively determines the structure of the system. The structure of such a system can be described with the aid of several principles, specifically, the principles of minimum possible engagements, maximum postponement in adjustment and minimum massed reserves.[4]

From the above, it can be seen that the system will have levels or echelons and each level will have the following characteristics. They will hold inventory to facilitate the discrepancies between demand and production; they will be break bulk points, that is, they exist for the purpose of receiving larger volume shipments and dispersing these shipments to various customers and they will offer all the necessary facilitating activities to complete these operations such as handling and communication.

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To complete the structure there must be some means to physically move the goods between levels over space. In the physical channel system this is accomplished by the transportation component.

Behavior

The physical channel system as an entity has the purpose or objective of servicing demand efficiently and effectively. Thus, if we view the physical channel system as a system with components, it is clear that it is the goal of the system which determines the behavior of the system and thus its components. The components of the physical channel system could be viewed as the channel members and the activities of the physical channel system could be seen as inventory, warehousing, transportation, communication, and handling. A system as been defined as, "a set or arrangement of things so related or connected as to form a unity or organic whole."[5] In this case, our physical channel system can be seen as that "unity or organic whole."

Viewed as a whole, it can be seen that the physical channel system has an inherent function (service demand). Viewed from the perspective of the channel members it must be concluded that theirs too is to service demand. Thus, from the channel member's perspective we have a coincidence of function, thus, a unified function for the overall physical channel system.

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Model Specifications

Although the overall purpose of this research is to generalize the effects of uncertainty on a physical channel system, such generalizations cannot be reached without the use of a specific model. In the previous section the model was bounded (manufacturer to ultimate consumer), its structure generalized (multiechelon) and measures of its operation specified (efficient and effective). In this section the details of the model being used are specified and described.

In construction of the specific model two criteria had to be balanced. On the one side, concluding generalizations are desired. To satisfy such a desire the model employed must be abstracted from a specific industry or product so that the conclusions would apply to all physical distribution systems. However, the logical extension of such thinking could be meaningless results. On the other side of the scale, conclusions are desired which will serve the advancement of the study of physical distribution and aid in the solution of present day physical distribution operational problems. To satisfy this criteria the model must, to a significant degree, be specific to an industry or product. Desiring neither useless generalizations nor conclusions that could only apply to one industry or product, a model was developed to balance these two criteria.

Because of the complexity of the system, computer simulation was chosen as the means of generating results. Specific numbers such as product weight and cube, costs for the system (the overall measure) and times (transit, packing, etc.) had to be employed. Thus, a true abstract

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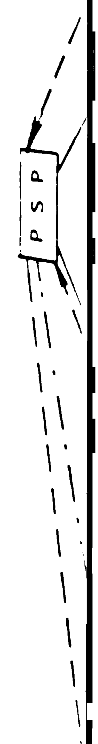
abstraction, even if desired, could not be achieved. The employment of such specific terms pulls the research in the direction of a specific system and reality. The actual numbers in absolute terms and the relationships between time, cost and product characteristics are important to the quality of generalizations generated. Thus a decision regarding the level and relationships of numbers to be used had to be made.

The criteria for selection was such that useful generalizations would result. The model developed is specific to the point that its structure and operation simulate real world conditions. However, the level of specificity has not been allowed to replicate a particular industry or product nor have the peculiarities of a particular physical channel system been allowed to enter. The result is a level of abstraction that permits useful generalization.

Structure

The model structure is shown in Figure 2-1. The model is multi-echelon in structure, which is in keeping with the conceptualized physical channel system developed earlier. Additionally, it parallels the majority of finished goods physical channels. The three channel levels (institutions) have the general features of holding inventory, being break bulk points (with the exception of the primary stocking point) and have the necessary functional capabilities to carry out related activities (i.e., communication and handling).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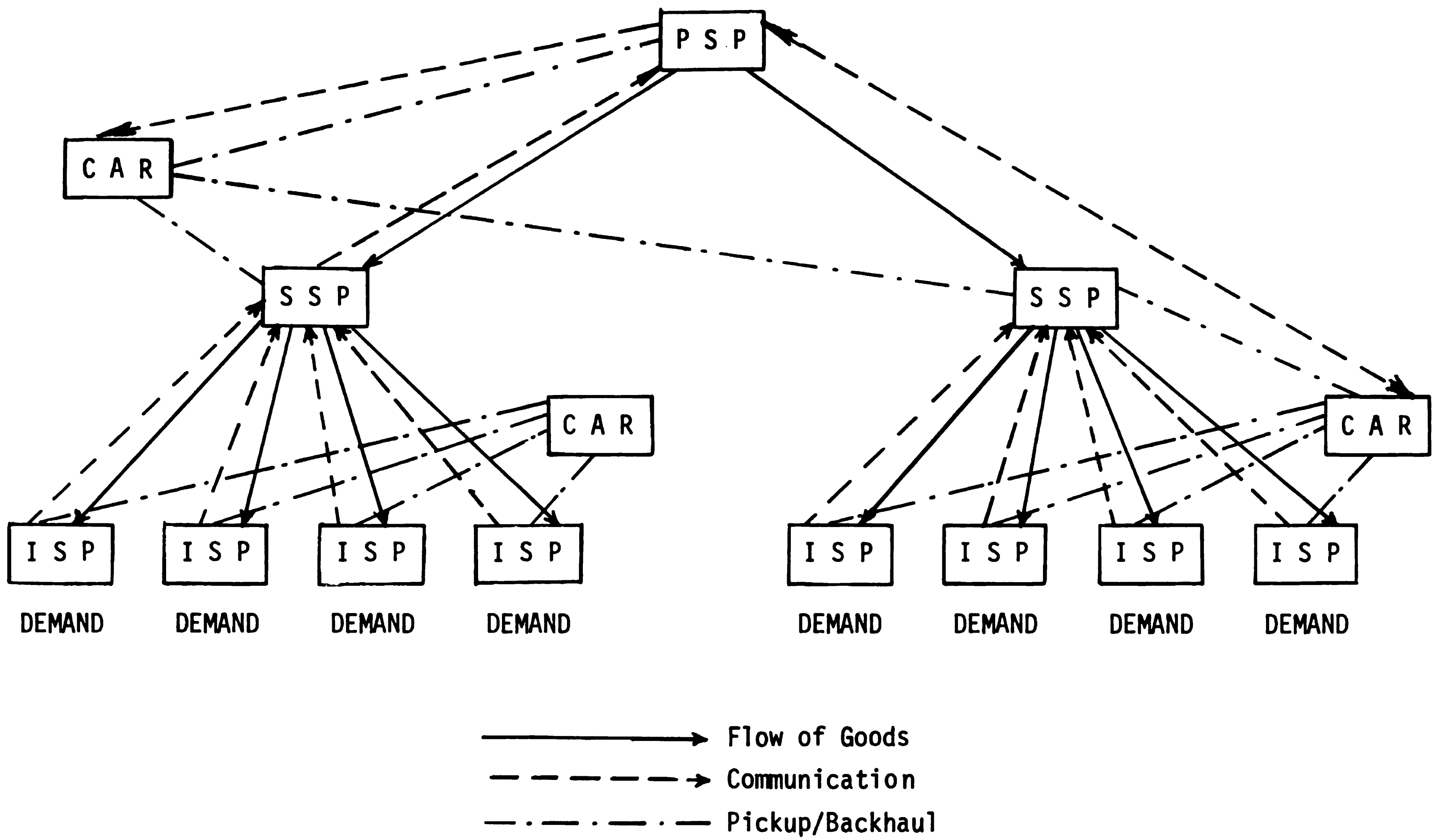

Figure 2-1. Physical Channel Structure

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The modeled system handles one product. This abstraction was made for the sake of simplicity. The usefulness of the results will not be limited because only one product is employed. The product chosen is hypothetical in nature. This is in keeping with the general model specifications.

The Primary Stocking Point (PSP) is the first point in finished goods distribution. At this point the product is ready for distribution to the ultimate consumer. In an actual distribution system the PSP is comparable to a manufacturer's finished goods inventory. The source of this inventory is the production line. The PSP holds inventory and is capable of performing the handling function and prepares orders for delivery. The PSP also has a communications capability. The PSP communicates with the production line to request inventory and it receives communication from the Secondary Stocking Point (SSP) regarding the amount of products to be shipped (orders). The PSP can also communicate with the carrier to request service. As is shown in Figure 2-1, there is one PSP location. The addition of more than one PSP point to the model would add complexity but would offer no further information.

The primary stocking point deals with two SSP's, like a manufacturer would deal with two wholesalers. Each SSP is capable of preparing orders for shipment (handling) and each holds inventory. In addition, each has the following communication links. They can communicate with the PSP to place orders, they can communicate with the Interface Stocking Point (ISP) to receive orders, and they can call the carrier to have orders picked up and delivered. These are

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the only communication links possible. For instance, the SSP's cannot communicate with one another. The two SSP's deal with the same PSP and each SSP deals with four ISP's.

The ISP's are analogous to retail outlets. They sell to the ultimate consumer and buy from a wholesaler. Eight ISP's deal with two SSP's, each ISP deals with a specific SSP only and four ISP's deal with the same SSP. The ISP (as a retailer would) holds inventory and has a handling and communications capability. The ISP communicates with the ultimate consumer to receive orders and with the SSP to request shipment. The ISP does not communicate with the carriers.

The demand unit is analogous to the aggregate of ultimate consumers. The characteristics and level of this demand will be discussed in the operations section of this chapter.

In the physical channel system the carrier (CAR) is responsible for moving goods between physically separated inventory locations (PSP to SSP and SSP to ISP). The carrier has not been specifically defined, however, the rates used are motor rates. All carrier moves are independent of one another.

All inventory or nodal points are located equidistant from one another in terms of time and distance. The distance in time and miles from the PSP to all SSP's is the same. And, the distance in time and miles from the SSP's to the ISP's is the same. This assumption eliminates all spatial considerations from the model but allows the inclusion of freight rates and lead times. This assumption was made for several reasons. Allowing space, in the form of varied distances betweer

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between nodal points would destroy the base of comparison between runs. Secondly, the purpose is to show the effects of uncertainty on the system and the exclusion of space makes the results more clearly attributed to uncertainty. In addition, the elimination of space from the model will not severely limit the conclusions reached.

Operation

The operation of the simulated physical channel system is described from the viewpoint of the activities performed by all the activity centers within the total system. In those situations where the particular activities would vary at any one of the three levels (ISP, SSP, PSP) these exceptions will be noted and specifically defined and detailed.

Daily demand is the requests made by the ultimate consumer for purchase of the product, and as such it is the force which initiates the functioning of the channel system. Daily simulated demand is impressed upon the system in the form of daily orders as determined from the probability distribution under study. Each ISP experiences daily orders based upon the same probability distribution, average level of demand and variance of demand. However, each ISP does not necessarily experience the same demand each day due to the random generation of orders from the appropriate demand probability distribution. The demand seen by each ISP is independent, i.e., if demand cannot be satisfied at one ISP, then consumers do not travel to an alternate ISP to satisfy their demands.

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The daily demand for each ISP for the entire simulation period is generated by the probability function which is under investigation. Thus, a stream of orders for each ISP for the entire time period is developed based upon the probability function, mean daily demand and demand variance. Such a stream of orders represents the major input to the system.

Perpetual daily inventory, in contrast to a periodic inventory system, is maintained by all ISP's. In a periodic system, the inventory would be reviewed at specified time intervals, and orders placed for the quantity of goods necessary to bring inventory up to prescribed levels. However, with the perpetual daily inventory system, whenever inventory is reduced to a predetermined level or reorder point, an order is placed with the appropriate SSP. Upon the receipt of an order by the SSP, the order is processed, filled and delivered by the transportation agent in question. The entire order cycle is completed in seven days. The seven-day total lead time between ISP and SSP is fixed and constant. The costs associated with the ordering process, inventory and transportation activities are accumulated and reported at the conclusion of the simulation run.

The SSP's also follow a perpetual inventory policy, updating their inventory at the end of each operating day. Orders are placed with the PSP when the level of the SSP inventory reaches its predetermined reorder point. SSP orders are processed, filled and delivered from the location of the PSP. Total lead time between the SSP and the PSP will be fixed and constant at ten days. Inventory at the PSP is generat

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generated by daily production runs at the adjacent manufacturing facility. The production rate equals the average daily sales for all ISP's. Thus, the warehouse facility at the PSP receives daily inventory equal to the average daily demand at the ISP's.

The simulation is operated for a short interval time frame (90 days). Because the time period is not extensive, demand, as seen by the entire system, contains no trend or seasonal elements. Therefore, the demand seen by all levels in the channel system is a result of the particular pattern, average level and variance imposed for the given experimental run.

The service level for the total channel system is measured at the ISP level, which means that service is defined in terms of stock availability. A channel system exists to satisfy the demands of the ultimate consumer in terms of place, time and possession utility. Consequently, the system should be organized and planned on the basis of making stock available at the consumer interface point. If the product is not there, the consumer is not satisfied nor assuaged by the fact that the average order cycle time is six days. The system service level is geared to the percentage of units out of stock at the consumer purchase point. Thus, a 90% service level implies that 90% of the units demanded over the length of the simulation would be available when the consumer demanded them.

The converse of service level, in terms of system performance, is that of the system costs necessary to meet that required service level. The costs to be generated for each simulation run (experiment) are of

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are of two types. First, activity center costs at each level in the channel are accumulated and reported at the end of the operating period. Total costs for each activity center within the system (inventory, transportation, etc.) are determined and reported. Thus, costs by activity center for the system are measured and analyzed. Secondly, the total cost for each experimental run is agglomerated and comparisons made between runs. Finally, total contribution margin for the system is calculated. These measures serve to indicate the combined effect of cost and service on the channel operation.

Behavioral considerations have been assumed away in the operation of the model. Although channel member relations and interrelations are critical to the smooth functioning of a channel system, the inclusion of such behavioral aspects would seem only to confuse the important cost and service relationships under consideration in this research.

Inventory.--The inventory policy followed at each level within the system is based on an economic order quantity (EOQ) which will be ordered when a given reorder point is reached. In the initial system, the EOQ is determined by balancing carrying cost of the inventory against the costs of ordering. The reorder point is defined as that quantity in inventory which will just meet average demand over lead time. Finally, in the initial system, no safety stocks are carried at any level (nodal point) since demand per day and lead time are fixed. The specific values of all variables associated with inventory are presented in Tables 2-1 through 2-4.

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An exception to the general EOQ formulation is made at the PSP level. The PSP receives daily inventory from the manufacturing facility. The daily production rate equals the total average demand for all ISP's. The inventory carrying charge is considered to be 25% of the value of an item in inventory as is the case at the ISP and SSP levels.

Communication.--Communication between levels consists of order generation and transmittal to a supplier and invoice preparation and order status from supplier to the demander. Thus, when the reorder point is reached at any level, the demander (ISP or SSP) processes the order through his purchasing and accounting department and transmits the order to the next level within the system (SSP or PSP). The channel member requesting replenishment of inventory directly bears this cost of order generation and transmittal. When the order is received by the supplier he processes the order, prepares a bill of lading, and performs all clerical functions. The demander is then notified that the order has been received and processed. An invoice is sent to the demander which contains a per unit charge for each item ordered and a separate charge for order processing and invoice preparation. These costs are considered part of the order processing cost to be borne by the channel member ordering inventory replenishment. Thus, such costs are an input into the generation of the EOQ values at the stocking points.

The final communication link is that between the SSP, PSP and the transportation agent. The cost for such communication is borne by the particular firm to which the shipment is made. Thus, the ISP is assessed a charge for the placement of an order by the SSP to a carrier.

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A similar situation occurs for the SSP when the PSP contracts a carrier to make a shipment to the SSP. All values for the variables associated with communication are contained in Tables 2-1 through 2-4.

Transportation.--The nature of the product, the quantity to be shipped and the locational points determine freight charges. The product in question has been arbitrarily determined to weigh 20 pounds and displace .75 cubic feet. The appropriate class rating is 65, and the rate for shipments over 10,000 pounds but less than a truckload is \$2.82 per hundred weight. The minimum weight necessary for a truckload is 36,000 pounds, and the rate is \$2.32 per hundred weight. The rates are based on a constant distance factor.

Shipments made between ISP's and the SSP are made in quantities of 520 units (unless on a backorder shipment--this situation will be explained in a subsequent section of this chapter) or 10,400 pounds. Thus, the applicable transportation charge is \$2.82 per cwt. However, shipments between SSP and PSP are made in quantities of 1,162 units or 22,200 pounds. Since this item is assumed to be one of many moving between these channel members, the product in question will move in a mixed shipment and thus obtain a truckload rate of \$2.32 per cwt for the movement.

In those situations where a backorder has been made, the products will be shipped on the basis of the shipment size as shown in the following table (Table 2-1).

Weight (lbs.)
Under 500
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1,000-2,000
2,000-5,000
5,000-10,000

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Table 2-1. Partial Shipment Rate Schedule

Weight (lbs.)	Units	Rate/Cost ($)
Under 500	25	4.53
500-999	50	4.34
1,000-2,000	100	3.84
2,000-5,000	250	3.56
5,000-10,000	251-499	3.25

All shipments are made FOB destination to obtain the economies in shipment enjoyed by the greater shipping volume of the SSP and PSP level. Consequently, the cost of the product to the ISP includes transport charges as does the cost paid by the SSP. All values of the variables associated with transportation are found in Tables 2-1 through 2-4.

Handling.--The product is loaded and shipped on pallets containing 130 units. A charge is made for handling the pallets both coming into and out of all inventory points. The charge is $1.00 per pallet for handling into the inventory point and the same charge for taking it from these stocking points.

Orders received at the SSP and PSP levels are handled in a first come first serve basis. If an entire order cannot be filled, a partial shipment of all remaining stock is made. The backorder is then placed for the remainder. The backorder quantity is processed and shipped as soon as the goods arrive at the stocking point. Thus, all backorders

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all backorders are filled immediately upon the availability of stock. All values associated with handling are found in Tables 2-1 through 2-4.

Warehousing (facility).--Each level within the system maintains a warehousing facility for the purpose of holding inventory. The costs for such facilities is stated on a square foot per year basis. The cost per square foot is identical at all stocking points and this cost is included as one input to inventory carrying cost. The effective space necessary to store one unit of product is assumed to be .25 square feet. The storage charge for all stocking points is $1.50 per square foot per year. All values related to storage are included in Tables 2-1 through 2-4.

Backorders.--Backorders are demands which cannot be filled immediately due to a stockout, but are eventually filled when stock is available. Stockouts can occur at any level, therefore a backorder could occur at the ISP, SSP, or PSP. However, in this research, all experimental runs except one, are made with no backorders at the ISP. In all experimental runs backorders can occur at the SSP and the PSP.

When there is no provision to backorder and demand is made at the ISP by the ultimate consumer and the ISP is out of stock, the demand is recorded as a lost sale. The analogous situation is the ultimate consumer demanding a good at the retail level. When the good is not available the customer will do without, go to a competitor or find an acceptable substitute. There are no provisions to attempt to save the sale and the sale is lost. There is no additional charge associated

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Table 2-2. Product Specifications

1. Physical

   Weight: 20 pounds
   Cubic feet: .75
   Square feet: .25

2. Cost Related

   Cost at PSP: $2.40[a]
   Cost at SSP: $3.20[a]
   Cost at ISP: $4.00[a]
   Consumer price: $5.00

3. Transport Related

   Class: 65
   Rate basis: Average between rate basis Numbers 421 to 600
   Rate: 10,000 pounds but less than truckload: $2.82
   Rate: Truckload: $2.32
   Mode: Motor truck
   Tariff authority: Eastern Central

4. Handling

   Units per pallet: 130
   Weight per pallet: 2,600 pounds

[a]Transportation FOB destination. Transportation included in purchase cost.

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[b] No

Table 2-3. Inventory Decisions

	Stocking Level	
	ISP	SSP
Average demand per day (units)	75	300
Number of days	360	360
Demand per year	27,000	108,000
Order cost (fixed per order)	$5.00	$5.00
Carrying cost (%)	25%	25%
Cost of product (per unit)	$4.00	$3.20
Lead time (days)	7	10
Economic order quantity[a]	520	1,162
Reorder point (units)[b]	525	3,000
ROP (average daily sales)	7	10
Number of orders per year	52	97
Order interval (days)	7	4
Average inventory	260	581

[a]No stockouts or backorders considered.

[b]No safety stock included.

Table 2-4. Cost Computation/Cost Basis

Activity	ISP	SSP	PSP
Inventory[a]			

Table 2-4. Cost Computation/Cost Basis

Activity	ISP	SSP	PSP
Inventory[a]			
Interest	A.I x $4.00 x .10	A.I x $3.20 x .10	A.I x $2.40 x .10
Insurance and taxes	A.I x $4.00 x .025	A.I x $3.20 x .017	A.I x $2.40 x .024
Obsolescence	A.I x $4.00 x .0313	A.I x $3.20 x .015	A.I x $2.40 x .031
Total carrying	A.I x .25	same	same
Facility	A.I x .25 sq ft x $1.50	same	same
Communications per order			
To supplier			
Order generation	$ .47	$ .25	
Order transmittal	.50	.30	
From supplier			
Invoice preparation	2.28	2.60	
Order processing	1.45	1.65	
To carrier	.30	.20	
Total	$5.00	$5.00	
Transportation			
To demander	0	$2.82/cwt	$2.32/cwt
From supplier	0	0	0
Backordered orders[b]			
Storage	..[c]	..[c]	..[c]
Handling[d]			
Into stock	$1.00 per pallet	$1.00 per pallet	$1.00 per pallet
Out of stock	$1.00 per pallet	$1.00 per pallet	$1.00 per pallet
Value of item in inventory[e]	$4.00	$3.20	$2.40

[a]No quantity discounts.

[b]See Table 2-1.

[c]Included in facility cost.

[d]No economies included in handling.

[e]Value of an item in inventory is equal to its landed cost.

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with a backorder under these conditions with the exception of the cost of a stockout. There will be a backorder capability at the ISP for one simulation run. Consumer demand will be backordered when variable demand and variable lead time are combined. This condition is considered separately and will be detailed in Chapter VII.

At all times backorders will occur at the SSP level. If the model is running with or without backorders at the ISP, there will be a backorder capability at the SSP. This capability must be available because the ISP is facing continuous daily demand which must be satisfied. The ISP cannot easily and quickly change suppliers and the alternate procedure of the ISP repeatedly placing orders for the duration of the SSP stockout is unrealistic and inefficient. When a backorder occurs at the SSP two conditions can be present: (1) there can be inventory on hand, but it is insufficient to fill the entire demand; (2) there is no inventory on hand. We will look at each case separately.

When there is partial inventory available, a partial shipment is made (which exhausts the stock at the SSP). The ISP is notified that a backorder has been placed for the difference between the quantity ordered and the quantity shipped and the balance of the order is shipped when the stock becomes available. There are additional costs associated with this procedure which can be directly allocated to backorders. When a partial shipment is made, the freight rate per cwt will increase (see Table 2-1). Therefore, the difference between the normal freight rate which would be paid to ship a full order and the new rate to ship a partial order

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partial order can be allocated to backordering. There is no additional cost associated with notifying the customer of a backorder because this can be handled on the confirmation of order and the packing slip. However, when stock is available and the balance of the order is shipped, there are several additional charges. There is an additional order processing and invoice preparation that would not have been necessary if the full order could have been satisfied. Therefore, these charges can be allocated to backordering. Secondly, there is the difference between the normal freight rate and the partial shipment freight rate as was the case with the first partial shipment. All additional charges which were created due to a partial shipment will be the responsibility of the SSP.

The second possible condition when a backorder occurs at the SSP is less complex. When an order arrives at the SSP and no stock is available, the customer is notified of the backorder and shipment is made when stock is available. The additional costs under these conditions which are directly allocated to backorders is an additional order processing and invoice preparation which is necessary to notify the customer and hold the order for future shipment. Backorders at the PSP are handled with the same procedure outlined for the SSP.

The model detailed in this chapter meets the criteria set forth. It is sufficiently broad to allow concluding generalizations, sufficiently specific to meet the demands of simulation, and structurally and operationally simple to allow the effects of demand to be seen and measured. Now that the model is set, it is necessary to select the relevant probability distributions that are used in the experiments.

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1961), p.

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CHAPTER II--FOOTNOTES

[1]Webster's *New World Dictionary*.

[2]Donald J. Bowersox, Edward W. Smykay and Bernard J. LaLonde, *Physical Distribution Management* (New York: The Macmillan Company, 1961), p. 360.

[3]*Ibid*., p. 360.

[4]*Ibid*., pp. 54-55.

[5]Webster, *op. cit*.

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CHAPTER III

EXPERIMENTAL FACTORS--PROBABILITY DISTRIBUTIONS

Introduction

The focus of this research is to measure impacts of demand uncertainty upon the physical channel system. As noted in Chapter I, demand uncertainty is evidenced by the probability distribution, level and variance of demand. It is the purpose of this chapter to explore the nature of probability distributions in general and to discuss those particularly relevant for representing demand. Criteria for including a particular distribution in this study are presented. Finally, those distributions to be used are experimental factors in the research are considered and their selection justified.

Demand as a Random Variable

Because the quantity demanded per unit time is a random variable, it may be described by a probability distribution. A probability distribution is a mutually exclusive and collectively exhaustive list of all events (or values of the random variable) which may result from a chance process, and the probabilities associated with each.

Thus, a probability distribution of demand for a given product indicates the range of possible quantities demanded and their probability

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of occurrence. The parameters of such a distribution include the expected value, or average demand per unit time and the standard deviation. The standard deviation is a measure of how the quantity demanded fluctuates around the average value. Additionally, the distributions may assume various shapes or patterns, depending on how demand is generated in the real world.

In conclusion, the quantity demanded per unit time is, in most situations, a random variable generated by a stochastic process. As such, the quantity demanded should be viewed in terms of a probability distribution. The following sections of this chapter explore the nature of a probability distribution and examine the potential types of probability distributions by which demand may be represented. Finally, those distributions applicable to demand patterns are catalogued and their selection justified.

Demand is considered as requests made by the ultimate consumer upon the channel system to deliver a product. Demand is therefore evidenced at the consumer interface point within the channel system. Such requests for products at the ISP are made over a time horizon, and the number of requests vary per unit of time. Generally speaking, it is almost never true that enough is known about the process which generates demand to be able to predict with certainty the time patterns of demand.[1] Because demand for a product is generated by a complex mixture and interaction of a variety of factors, the quantity demanded is not constant from one time period to another. Hence, demand per unit time may be considered to be a random, or stochastic variable.

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When viewing demand as a stochastic variable, we are considering only the random fluctuation in demand from one time period to another. The fluctuation is defined as the variation in the quantity demanded per unit time around a fixed average demand per unit time. Hence, variations in demand over time, as evidenced by time series, trend, seasonal, or cyclical factors is not the focus of this research. Thus, it is assumed that the basic process generating the pattern and fixed average demand does not change with time. The uncertainty, as related to demand is a result of variations in demand from its average value per unit time and the general nature of such a demand pattern.

Probability and Probability Distributions

In this research demand is represented by several probability distributions. It has been established that demand is a random variable in that all the relevant factors which create it are unknown. To show that probability distributions are appropriate theoretical models to represent demand, an overview of probability distributions is necessary.[2]

Although individual events of a chance process can not be predicted with accuracy, something can be said about the occurrence of particular events if the process is repeated. As a process is repeated which meet the following criteria: (1) that it can be repeated physically or conceptually; (2) that the set consisting of all its possible outcomes can be specified in advance; and (3) its various repetitions do not always yield the same outcome; the occurrence of particular events begin to stabilize. It is a characteristic of random data that if the

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if the experiment is repeated an indefinite number of times that any particular outcome that is observed will become more and more nearly a constant as the number of repetitions of the experiment is increased.[3]

Through observations and repetitions of the experiment, it is possible to determine the relative frequency of an occurrence. Relative frequency is the ratio of the number of times the outcome takes place to the total number of times the experiment is performed. If all the observations are grouped or classified a frequency distribution is created.

It is only a short conceptual step from a frequency distribution to a probability distribution. A probability distribution is a

> theoretical model of the relative frequencies of a finite number of observations of a variable. It is a systematic arrangement of the probabilities associated with the mutually exclusive and collectively exhaustive elementary events of an experiment.[4]

Thus, the probability distribution shows the probability of an event and the distribution of probabilities over a whole range of possible outcomes.

The probability distribution is an appropriate theoretical model to represent demand. Demand is uncertain and prediction with accuracy is impossible. Probability theory represents uncertainty and the probability distribution describes the whole range of possible outcomes which is necessary for the simulation.

To formulate the experiment it is desirable to look closely at the characteristics of probability distributions, i.e., the type of phenomena they describe and their characteristics.

Discrete/Continuous

The random variable under consideration may be either discrete or continuous.

> A discrete random variable can take on only a finite number of values. Also its distribution function, F(x), is one which increases only in finite jumps and which is constant between jumps.[5]

A continuous random variable takes on uncountability infinite values, such as time and weight whose counting is only limited by the measuring instruments.

> The probability that a continuous random variable assumes any single particular value is zero, since there are infinite numbers of real numbers within the intervals over which x (the random variable) is defined.[6]

To overcome this problem, the continuous random variable is viewed as intervals and the interval can take on values and probabilities as the finite numbers do in the discrete case.

Probability Function

A probability function assigns a chance of selection to each of the elementary events of an experiment. A probability function is distinguished from a probability distribution in that the function is a rule for assigning selection chances to the elementary events of an experiment, while a probability distribution is a systematic presentation or arrangement of probabilities. The probability function of a random variable is a description of its mathematical behavior, that is, the range of its possible values together with their respective probabilities.

The probability function can describe a specific point on the range or it can describe the range between points. A distinction must be made between the functions which describe points (discrete random variables) and the functions which describe ranges between points (continuous random variables). The probability mass function assigns the probability of a point in both the discrete and continuous case. It is applicable to the discrete case because each point has a value. However, in the continuous case the probability of any given point must be zero, because of the nature of the variable.

> The probability that a continuous random variable assumes any particular value is zero, since there are infinite numbers of real numbers within the intervals over which x is defined. Consequently, a continuous random variable cannot be described by the probability function for discrete random variables.[7]

The probability mass function is used in this research for the discrete random variables.

As described above, the continuous random variable must be described in terms of subintervals or ranges between points. The probability function which describes the values and the probabilities associated with each is the probability density function or simply referred to as a density function. The discrete random variable can also be described over a range by its distribution function. However, the distribution function is not used in this research. The probability mass function is used to describe the discrete cases and the density function is used to describe the continuous case.

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Measures

Given a probability distribution it must be described to operationalize the research. Statisticians have developed measures to describe distributions. Of the many ways a distribution can be measured, the expected value or central tendency and the variance are employed in the research.

The expected value is a measure of magnitude which considers the range of values of the random variable and their probabilities of occurrence. The term is synonymous with mathematical expectation, central tendency and mean. The expected value of a random variable measures the center mass of the probability function. It provides a quick picture of the long-run average result when the experiment is repeated an extremely large number of times.

The expected value in the discrete and continuous cases is defined as:

$$E(X) = \sum_{i=1}^{n} x\ f(x_i)$$

for the discrete random variable case. If x is a continuous random variable with probability density function f(x), the expected value of x is defined as:

$$E(X) = \int_{-\infty}^{\infty} x\ f(x)dx$$

The expected value does not adequately describe the random variable. "The expected value of a random variable indicates little or nothin

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or nothing about the range of values that the variable can assume, nor does it give any indication of the dispersion of the values of the variable."[8] The measure which overcomes this problem is variance.

"Variance represents the spread or scatter of the values of a random variable around its expected value."[9] If most of the area under the curve lies near the mean, the variance is small, while if the curve is spread out over a considerable range, the variance is large. In statistical terms, the variance represents the sum of the squared deviations around the mean divided by the number of observations. Thus,

$$\text{variance} = \frac{\Sigma(x-\bar{x})^2}{n}$$

A more common measure of variability is the standard deviation, which is the square root of the variance. The standard deviation allows the measurement of dispersion in the same units as the original values of the random variable x.

Parameters

In general, a parameter is defined as any descriptive measure of the characteristics of a population. "It is a single value derived by statistical methods in order to describe in summary fashion the pertinent characteristics about a population."[10] More specifically, it is some constant which describes a probability density function. For example, the mean is called the location parameter because it describes the position of the distribution on the x axis and the standard

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standard deviation is called the shape parameter because it alters the shape of the density with respect to a fixed scale. Each distribution is described by one or more parameters which singly or collectively affect the location and shape of the curve. The specific parameters for each of the special distributions discussed is covered in the following section.

Distribution Patterns

Although each random process can generate a different probability distribution, it has been found that certain "types" of distributions are generated over and over again. Thus, a group of special distributions have been catalogued. These are discussed in the following section.

Theoretical Probability Distributions

A broad range of theoretical probability distributions are available by which to represent random variables. "Probability distributions arise most naturally in terms of families of distributions that share selected common characteristics."[11] Each distribution family may be catalogued or characterized by a variety of factors, including its density and mass function, parameters, distributional shape, inherent generating process, assumptions and the kinds of experiments in which they commonly arise. It is the purpose of this section to briefly review the more commonly encountered distribution families and to systematically describe common distribution families according to their most relevant characteristics. From this review the distribution families applicable for representing demand are selected. The distribution families so selected become the focus of this research in the experimental phase.

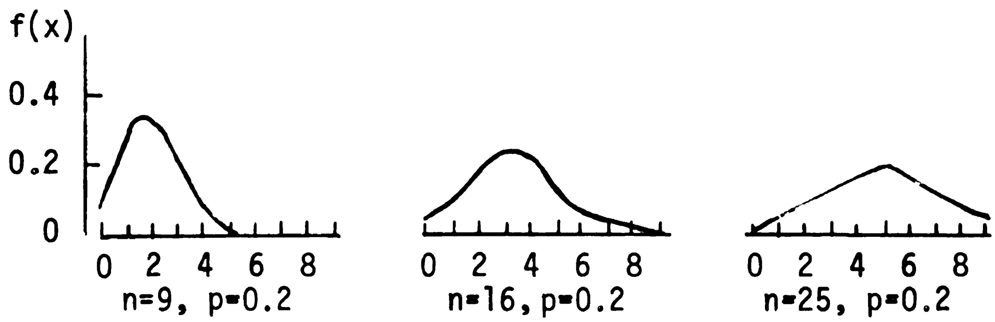
Discrete Distribution Families

The binomial family.--A random variable x is said to have a binomial distribution if its probability mass function is given by:

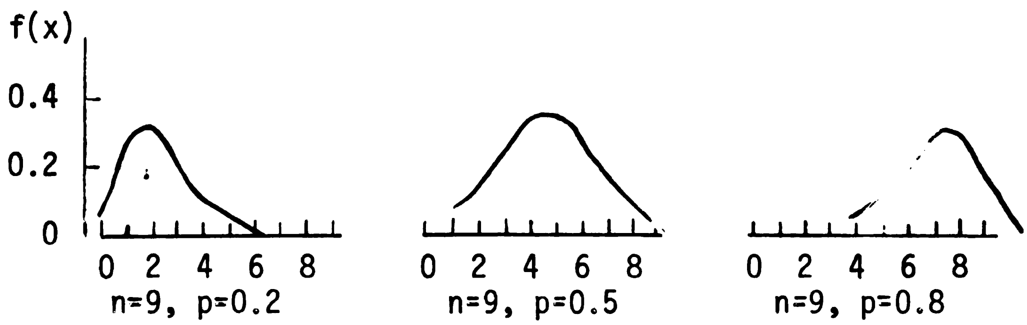
$$f(x;n,p) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0,1,2, \ldots n \\ & 0 < p < 1 \\ 0, \text{ elsewhere} & \end{cases}$$

The parameters of the family are (n) and (p), where (n) represents the number of trials of an experiment and (p) represents the probability of success on a given trial. [q, the probability of failure, is equal to (1-p)]. The probability function thus describes a whole family of distributions of the binomial random variable (x), one for each possible combination of the values (n) and (p).[12] The random variable, (x), is defined as the number of successes.

The binomial distribution will assume different distributional shapes depending on the values assumed by (n) and (p). The distribution is symmetrical in situations where $p = .5$ and skewed when (p) takes on any value other than .5. However, as n approaches infinity, the distribution approaches symmetry and zero kurtosis.[13] For values of (p) less than .5, the binomial distribution is skewed to the right (long tail to the right of the mode) and skewed to the left for (p) greater than .5. These effects are somewhat mitigated when (n) is large. Finally, the binomial probability distribution may be approximated by the normal distribution and thus becomes almost continuous when (n) is very large. The distribution is represented by the following shapes with the values of (n) and (p) as specified in each illustration.



Binomial distributions with fixed p.



Binomial distributions with fixed n.

The mean or expected value of the binomial random variable is:

$$E(X) = np$$

The variance and standard deviation respectively are:

$$V(X) = npq$$

$$\sqrt{V(X)} = \sqrt{npq}$$

Theoretically, the binomial family of distributions is generated when we can assume that the following assumptions are met:[14]

If we consider a series of events or experiments:

1. The result of each experiment can be classified into one of two categories, such as success-failure, heads-tails, yes-no, and so on.

2. The probability (p) of a success (head, yes, etc.) is the same for each trial of the experiment.

3. Each experiment is independent of all others.

4. The trials of the experiments are performed a fixed number of times, say (n).

Thus, the binomial family describes random variables which are generated from populations having two possible values. The probability mass function may be said to answer the question: "What is the probability of obtaining exactly (x) successes in (n) trials of an experiment, given the probability of success on any one trial is (p)?" The random variable of interest is thus the number of times in which the experiment results in a success.

The binomial family of distributions is usefully applied in many situations where its assumptions are at least approximated. Consequently, it is and has been successfully applied to quality control problems, where (p) represents the probability of obtaining a nondefective product or part. Additional situations, such as consumer surveys, where (p) refers to the proportion of favorable responses to a given question, have also been analyzed by use of the binomial distribution. In summary, the binomial distribution may obtain in a host of experimental situations where a constant (p), large (n) and independent trials are at least approximated.

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The negative binomial family.--A random variable x is said to have a negative binomial function if its probability is given by:

$$f(x;r,p) = \begin{cases} \binom{x-1}{r-1} p^r q^{x-r}, & x = 1,2,3 \ldots\ldots \\ 0, \text{ elsewhere} \end{cases}$$

The parameters of the family are (r) and (p), where (r) represents the number of successes achieved in a given number of the trials of an experiment and (p) represents the probability of success on a given trial. The probability function describes a whole family of distributions of the negative binomial random variable, (x), one for each possible combination of the values (r) and (p). The random variable (x) is defined as the number of repetitions of the experiment that are required in order to achieve r successes.

The negative binomial distribution will assume various shapes depending on the values assumed by (r) and (p). The distributional shapes should vary in similar fashion as does the binomial.

The mean or expected value of the negative binomial random variable is:

$$E(X) = \frac{rq}{p}$$

The variance and standard deviation respectively are:

$$V(X) = \frac{rq}{p^2}$$

$$\sqrt{V(X)} = \sqrt{\frac{rq}{p}}$$

The negative binomial is said to be generated when the following assumptions are satisfied.[15]

1. The result of an experiment can be classified into one of two categories.
2. The probability of a success, (p), is constant.
3. Each trial of the experiment is independent.
4. The series of experiments is performed a variable number of times until a fixed number of successes is achieved.

The probability mass function of the negative binomial random variable is employed to determine the probability that the r^{th} success occurs on the x^{th} trial of a binomial experiment which meets the above four assumptions. Thus, the function describes the probability that (x) repetitions of the experiment are required in order to achieve (r) successes.[16] The number of successes, (r), is fixed and the number of trials (x) is the random variable.

Having two parameters, the negative binomial family provides a large class of distributions that serve as an assumption for an integer valued random variable.[17] It may serve as a model for a large number of real world applications, when the possible events are dichotomized and we wish to examine the probability of achieving a given number of successes in a fixed number of trials. Thus, potential applications exist in quality control, inspection sampling, sample surveys and the like. It has also been shown to be applicable in inventory studies for representing the total number of units demanded.

The geometric family.--A random variable, x, is said to have a geometric distribution if its probability mass function is given by:

$$f(x;p) = \begin{cases} pq^{x-1}, & x = 1,2 \ldots \\ 0, \text{ elsewhere} \end{cases}$$

This family has only one parameter, (p) which is the probability of success on a given trial.

The mean or expected value of the geometric random variable is:

$$E(X) = \frac{1}{p}$$

which may be considered as the expected number of successes until a failure occurs. The variance and standard deviation respectively are:

$$V(X) = \frac{q}{p^2}$$

$$\sqrt{V(X)} = \sqrt{\frac{q}{p^2}}$$

The assumptions necessary to generate the geometric distribution are similar to those necessary to generate the binomial distribution.

The geometric family of probability distributions describes the probability distribution of the random variable (x), which is the number of trials necessary to achieve a success. Thus, the distribution refers to the number of trials, (x) needed for the first occurrence of a success.

The geometric distribution has very similar applications to those of the negative binomial, especially assembly line problems and those related to mechanical failure. Thus, the geometric may be applied to evaluate the reliability of various types of operating equipment by assessing the probability of a given number of cycles of a machine until it fails.[18]

The multinomial.--A group of random variables are said to have a multinomial distribution if their probability mass function is given by:

$$f(x_1,x_2,x_3, \ldots x_k; p_1,p_2,p_3, \ldots p_k, n) =$$

$$\begin{cases} \dfrac{n}{X_1\ X_2\ X_3\ X_k} P_1^{x_1} \cdot P_2^{x_2} \cdot P_3^{x_3} \cdot P_k^{x_k}, \quad X_i = 0,1,2, \ldots n & 0 < P_i < 1 \\ & i = 1,2,3, \ldots k \\ & 0, \text{ elsewhere} \end{cases}$$

The multinomial is merely an extension of the binomial distribution. Whereas the binomial pertains to two alternative events, success, and failure of an experiment, the multinomial distribution applies to experimental trials for which more than two outcomes are possible. Thus, the likelihood that a specified number of each of multiple outcomes is obtained in n trials, for which the probability of the outcome of each is constant from trial to trial, is called a multinomial probability.[19]

The remaining characteristics, parameters, and assumptions of the multinomial are similar to the binomial distribution, but are of course different to the extent that more than two outcomes of an experiment are permitted. Thus, the multinomial can be applied to situations in which one desires to answer the question, "What is the probability of in (n) independent trials of an experiment, with x_1, x_2, ... xk outcomes of each trial, with p_1, p_2, ... pk probabilities, of getting exactly x_1, x_2, ... xk of each possible outcome?"

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<u>The hypergeometric family</u>.--A random variable x is said to have a hypergeometric distribution if its probability mass function is given by:

$$f(x;N,n,k) = \frac{\binom{N-k}{n-x}\binom{k}{n}}{\binom{N}{n}}$$

The parameters of the hypergeometric distribution include (N), the total number of objects in the population, (n), the number of objects in the sample or number of trials and (k), the total number of successes in the population or number of successful trials. The hypergeometric describes a whole family of distributions of the random variable (x), one for each combination of its parameters. The random variable (x) represents the number of successes.

The hypergeometric distributional shapes are quite similar to those assumed by the binomial, specifically as N becomes very large.

The mean or expected value of the hypergeometric random variable is:

$$E(X) = \frac{nk}{N}$$

The variance and standard deviation respectively are:

$$V(X) = \frac{nk(N-k)(N-n)}{N^2\ (N-1)}$$

$$\sqrt{V(X)} = \sqrt{\frac{nk\ (N-k)(N-n)}{N^2\ (N-1)}}$$

The hypergeometric distribution is generated when the following conditions are assumed:[20]

1. The result of each experiment can be classified into one of two categories, such as success or failure.
2. The probability of success changes on each trial.
3. Successive trials are dependent.
4. The trials are repeated a fixed number of times.

Thus, the hypergeometric distribution applies to processes similar to those for which the binomial obtains, except that the probability of success changes on each trial. The probability changes because trials (or draws) are made from a finite population, and thus the probability of success changes on each trial as the fraction $\frac{k}{N}$ changes. The process would be analogous to drawing spades from a deck of cards without replacement. The probability of drawing a spade on any draw is conditional upon previous draws, as the sample space is reduced for each card drawn.

The most important application of the hypergeometric distributions are to those experiments or studies which are conducted with a finite population.

<u>The poisson family</u>.--A random variable x is said to have a poisson distribution if its probability mass function is given by:

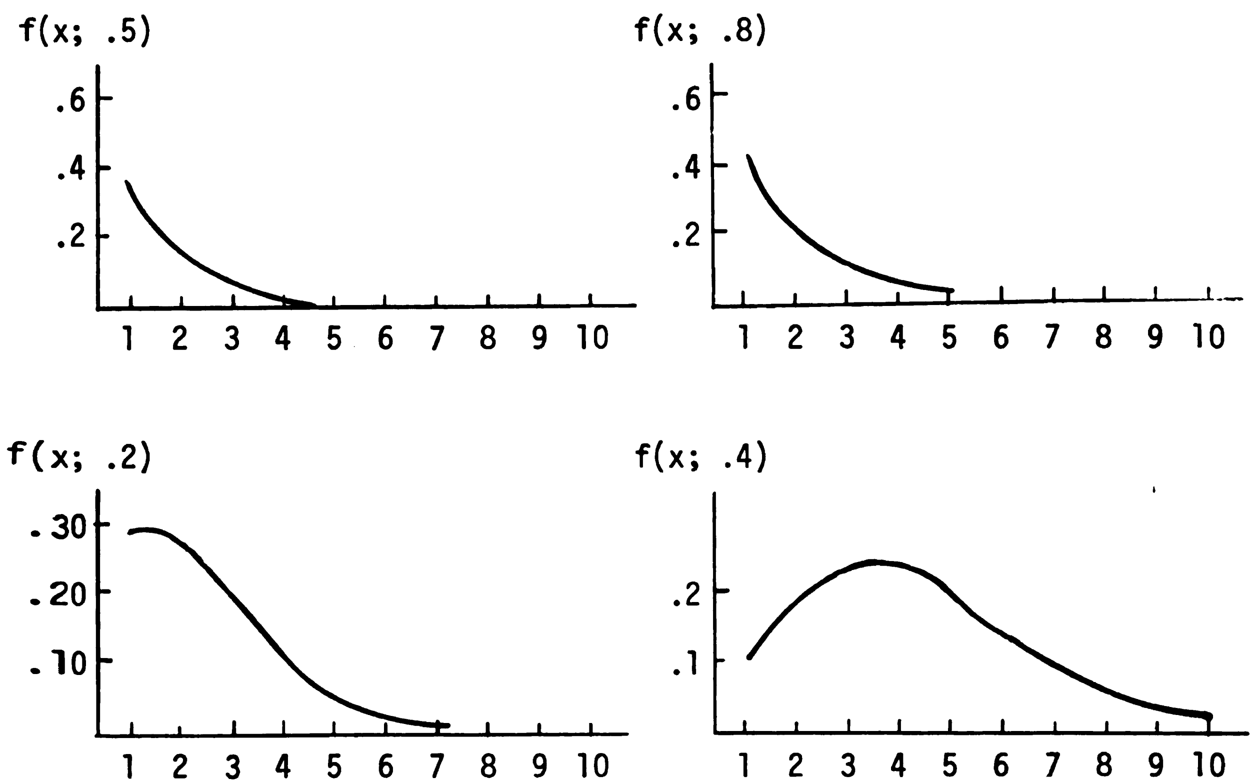
$$f(x;\lambda) = \{ e^{-\lambda} \frac{\lambda}{x}! \qquad x = 0,1,2 \ldots$$

$$\lambda > 0$$

$$0, \text{ elsewhere}$$

The parameter of the poisson family is λ, the mean number of the occurrences of an event per unit time over a given number of trials. A whole family of distributions are then obtained based on the value of λ. The random variable x may thus be described as the number of occurrences of an event over some time or over space.

The poisson distribution will assume different distributional shapes depending on the value of λ. Thus, the distribution is highly skewed to the right when $\lambda \leq 1$, but becomes symmetrical as λ increases. The following are representative of the shapes taken by the poisson.



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The expected value of the poisson random variables is:

$$E(X) = \lambda$$

The variance and standard deviation respectively are:

$$V(X) = \lambda$$

$$\sqrt{V(X)} = \sqrt{\lambda}$$

The following assumptions are necessary in order to generate the poisson distribution.[21]

1. Events that occur in one time (space) interval are independent of those occurring in any other non-overlapping time interval.

2. For a small time (space) interval the probability that one event occurs is proportional to the length of the time (space) interval.

3. The probability that two or more events occur in a very small time (space) interval is so small that it can be neglected.

There is no theoretical way of judging whether or not the basic assumptions are satisfied.[22] Thus, the assumptions are just that. Usually, the independence assumption is judged as satisfied unless there is overwhelming evidence to the contrary. The assumption that the event occurs only once in the interval can be circumvented by making the time interval extremely small.

The poisson distribution family therefore describes a situation where one counts the number of times an event occurs over some time interval. The events seem to occur random in time (space) and may thus be represented along a time (space) axis. The poisson thereby indicates the distribution of the probabilities of the numbers of rare events (whose probability is small in the interval) which occur in numerous trials. The probability mass function may be said to answer the question: "What is the probability that an event A will occur exactly x times when a large number of trials are made in each of which the probability of the event A is very small?"[23]

According to Zehna, "the poisson family of probability distribution is used in many experimental situations in which integer-valued random variables are called for."[24] This is true in studies where a count is made of the number of times an event occurs, events being the number of misprints on a page, the number of calls received per minute on a telephone exchange, the number of accidents per hour on a highway, or the number of demands per day received by an inventory system. Bryan and Wadsworth suggest that many random phenomena of interest in science and industry yield a discrete variate x having a finite number of possible integral values, 0, 1, 2, 3--and satisfying conditions which lead to the poisson distribution.[25] Thus, additional applications of the poisson would include insurance problems, where the variable of interest is the number of deaths per time period, or a supermarket problem involving the formation of waiting lines at service facilities, and the counting of the number of defects in a manufactured item in a quality control situation.

Continuous Distribution Families

Uniform family.--A random variable x is said to have a uniform distribution if the probability density function is given by:

$$f(x;\, a,b) = \begin{cases} \frac{1}{b-a}, & \text{if } a \leq x \leq b \\ 0, & \text{elsewhere} \end{cases}$$

The parameters of this two parameter family are (a) and (b), the end points of the interval. Thus, the probability of x occurring is proportional to the length of the interval, and hence, intervals of the same length have the same probability.

The distributional shape is simply the representation of a horizontal line. The density is symmetrical about the center of the interval $(a+b/2)$ and thus this value is both the mean and median of the distribution. The expected value of the uniform distribution is:

$$E(x) = \frac{a+b}{2} \,,$$

which simply represents the average of the end points. The variance and standard deviation, respectively, are:

$$V(X) = \frac{a+b}{12}$$

$$\sqrt{V(X)} = \sqrt{\frac{a+b}{12}}$$

Theoretically, the uniform distribution applies in situations when one can assume each event of a random process to be equally likely of occurring.

Zehna points out that, "as a model for random experiments, the uniform family is, first of all, suitable for bounded random variables whose essential range coincides with the interval (a, b)."[26]

The uniform distribution also applies in situations where all events are equally likely or when numbers are to be generated by a purely chance process. Thus, tables of random numbers are generated from uniform distributions.

The exponential distribution family.--A random variable x is said to have an exponential distribution if its probability density function is given by:

$$f(x;\ \beta) = \begin{cases} \frac{1}{\beta} e^{\frac{-x}{\beta}}, & \text{if } x > 0 \\ 0, \text{ elsewhere} \end{cases}$$

The parameter of the exponential distribution is β, which is generated from a poisson distribution. Thus, the exponential distribution is generated by a poisson process, and its parameter, β is defined as the reciprocal of the average number of successes per interval, i.e., $\beta = \frac{1}{\lambda}$. Thus, $\frac{1}{\beta}$ refers to the average length of the interval between two occurrences of the event. The random variable, x is defined as the width of the interval to the first occurrence of the event.

The exponential is a decaying type of probability function whose rate of decay depends upon the parameter β. It generally takes the following shapes:

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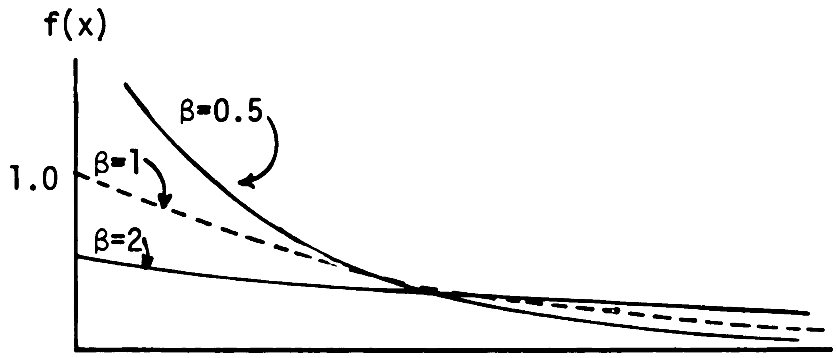


Exponential distribution for various selections of β.

The mean of the exponential random variable is: $E(X) = \beta = \frac{1}{\lambda}$, and thus, the mean of the exponential is the reciprocal of the mean of the poisson. This result is to be expected since the exponential variable refers to time between successive poisson occurrences. Hence, the mean of the exponential is considered as the average time interval between poisson occurrences, or the expected time until the first occurrence of the event.[27]

The variance and standard deviation respectively are:

$$V(X) = \beta^2 = \frac{1}{\lambda^2}$$

$$\sqrt{V(X)} = \sqrt{\beta^2} = \sqrt{\frac{1}{\lambda^2}}$$

The most essential assumption necessary in order to generate an exponential distribution is that the random event occurs in time according to a poisson process. Additionally, the density function applies only to non-negative random variables.

The exponential family thus describes the probability distribution of the time between occurrence of an event that is developed from a poisson process. The exponential answers the question: "Through how long an interval must one wait in order to observe the first occurrence of an event if one is observing a sequence of events occurring in accordance with the poisson probability function?"[28] The random variable of interest is the length of the interval between occurrence of the desired event.

The exponential is found to be useful for representing a number of random variables which cannot assume negative values. For example, the time to failure of a machine is well represented by an exponential probability function. Such variables as waiting times for service, life of an electron tube, time intervals between successive breakdowns of an electrical system and the time intervals between accidents also are exponentially distributed. Important applications in business include the distribution of the length of time between successive arrivals at a service counter and the distribution of time wise variable demand that occurs in numerous situations.

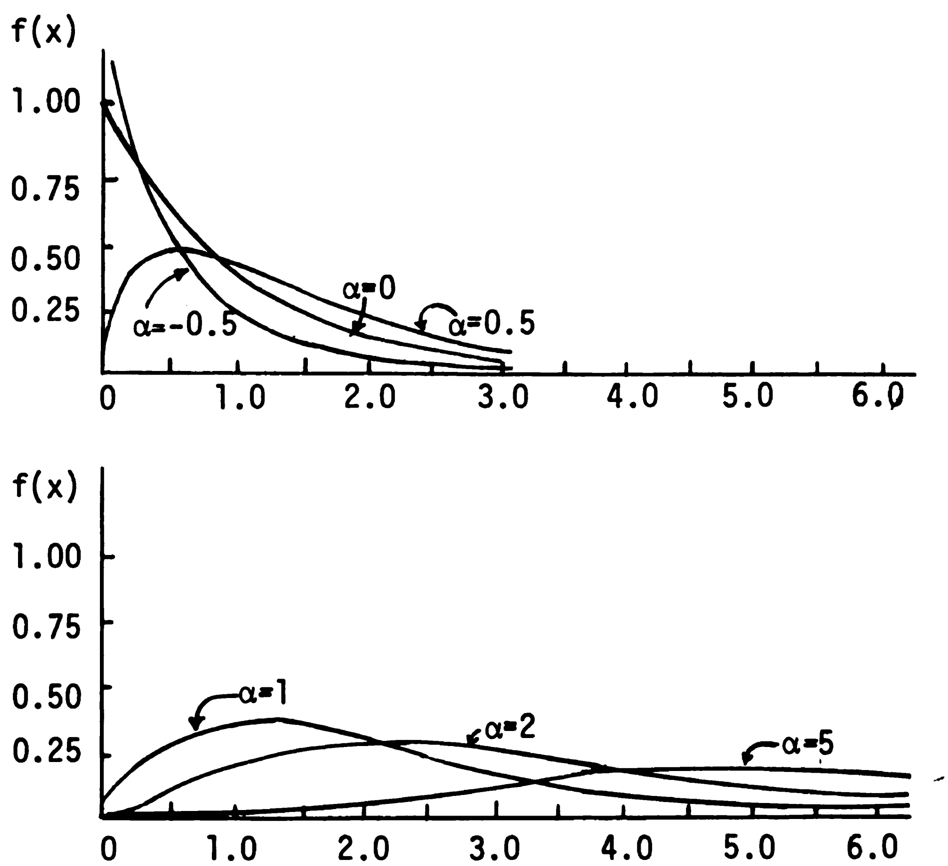
The gamma probability family.--A random variable x is said to have a gamma probability distribution if its probability density function is given by:

$$f(x;\ \alpha,\ \beta) = \begin{cases} \dfrac{X^{\alpha-1}\ e^{-(x/\beta)}}{\beta^{\alpha}\ \Gamma(\alpha)} & \text{for } x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

The parameters of the gamma distribution are α and β, where α refers to the number of successes per interval or unit space and β represents the reciprocal of the average number of successes per interval $(\frac{1}{\lambda})$. The gamma is thus related to both the poisson and the exponential distributions, and the exponential is a special case of the gamma for which $\alpha = 1$.

The gamma probability function describes a whole family of distributions of the gamma random variable (x), one for each possible combination of the values (α) and (β). The random variable x may be considered as the number of units of length (intervals) between one success and the α^{th} succeeding success.

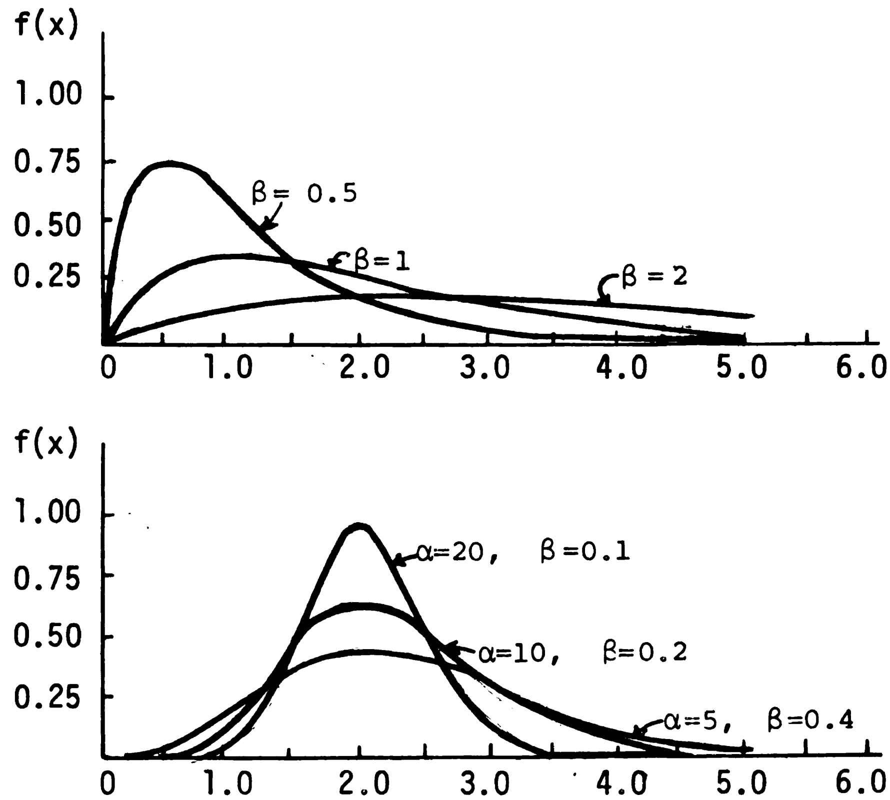
The parameters α and β determine the shape of the density function, which is skewed to the right for all values of α and β. The skewness will decrease as α increases, as previously noted, when $\alpha = 1$, the gamma is an exponential distribution, and therefore assumes the shape of a decay function as seen below.



Gamma distributions with unit β but different values of α.

If α is a positive integer, then the gamma becomes an Erlang distribution.

The following represent some typical gamma density functions.



gamma distribution

The expected value of the gamma random variable is:

$$E(X) = \alpha\beta = \frac{\alpha}{\lambda}$$

The variance and standard deviation respectively, are:

$$V(X) = \alpha\beta^2$$

$$\sqrt{V(X)} = \sqrt{\alpha\beta^2}$$

The gamma obtains in situations where the underlying process is a poisson, and thus the assumptions relevant to the poisson are applicable. Additionally, the gamma applies only to non-negative random variables. The tie between the gamma, poisson and exponential is close. The poisson resulted from an effort to determine the probability of (n) successes per unit length, given a mean of (λ) successes per unit of length. The exponential results from an effort to determine the probability of (x) units of length from one success to the next in a poisson process. The gamma distribution results from an effort to determine the probability of (x) units of length between one success and the (α^{th}) succeeding success.[29]

There is no direct answer to when the gamma is applicable, one must construct a histogram of the actual data.[30] The family is so extensive in shapes of densities available that it is a fairly safe assumption to make as a model for an experiment described by almost any non-negative random variable.[31] Parzen concludes that

> the gamma is of great importance in applied probability theory. In addition to describing lengths of waiting times, it also describes such numerical valued random phenomena as life of an electron tube, time intervals between successive breakdowns of an electrical system and time intervals between accidents.[32]

Basic found the gamma to provide an excellent description of the probability distribution of demands for a product.[33] Additionally, Bryan describes the gamma as applicable "when conditions of the problem exclude values of x smaller than some arbitrary minimum."[34]

<u>The erlang distribution</u>.--The erlang distribution is a special case of the gamma probability family. When $\alpha = 1$, the gamma is an exponential distribution which is a decay type function. When α becomes a positive interger above 1 the distribution is an erlang. As α goes from 1 to n, the shape of the distribution changes from a decay type function through a series of shapes and eventually approximates the normal.

The primary application of erlang is a series of service times. A single service time can be viewed exponentially. As a second service time is added in series (i.e., a manufacturing process where two service type operations are performed consecutively) the process can be viewed as two independent exponentials. However, if the two service operations are to be viewed as one operation it can no longer be seen as an exponential distribution. A series of service type operations can be represented with an erlang distribution with the value of α equal to the number of stages. Thus, if there is a process which contains three exponential type service times, the entire operation can be represented by an erlang distribution with α equal to three. The forms of the density function, expected value, variance and standard deviation of the erlang are the same as the gamma.

<u>Beta distribution family</u>.--A random variable x is said to have a beta distribution if its probability density function is given by:

$$f\,(x;\alpha,\beta) = \begin{cases} \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\, X^{\alpha-1}(1-x)^{\beta} & \text{for } 0 < x < 1 \\ & \alpha\ \beta > 0 \\ & 0, \text{ elsewhere} \end{cases}$$

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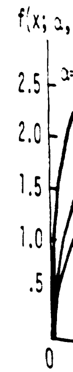
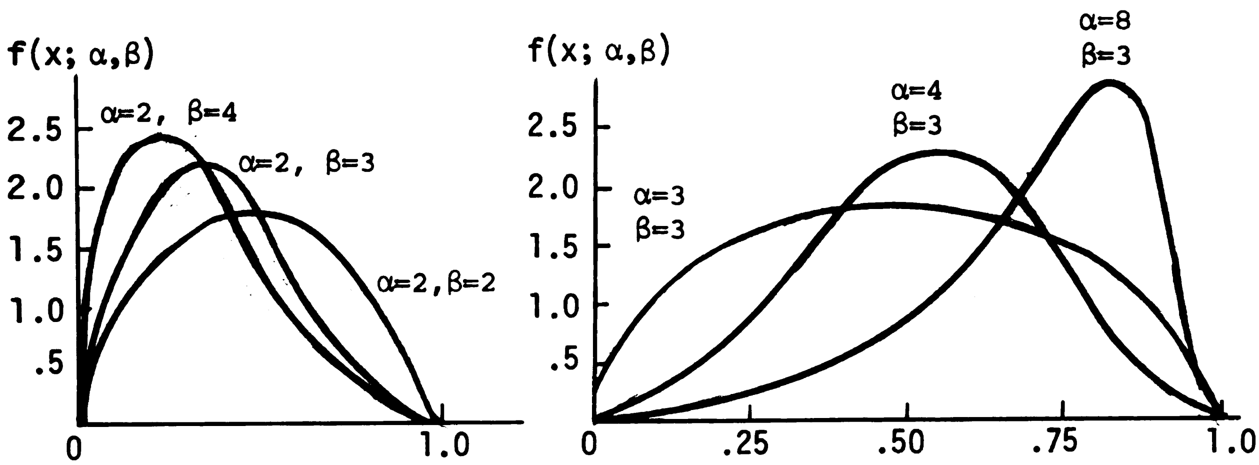


Like the gamma, the parameters of the beta distribution include (α) and (β). There exists a broad family of distributions based upon the values of α and β. In the case where $\alpha = \beta$, the curve is symmetrical, otherwise it will be skewed. The variety of shapes is indicated below:



the beta density

The expected value of the beta random variable is:

$$E(X) = \frac{\alpha}{\alpha + \beta}$$

The variance and standard deviation respectively, are:

$$V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$\sqrt{V(X)} = \sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}}$$

The beta distribution applies when the admissible values of a random variable lie between 0 and 1. If both parameters, α and β, are equal to zero, then the distribution reduces to a rectangular or uniform distribution. The distribution is often a good representation for the random behavior of percentages. Additionally, the distribution is well suited for situations where values closer to zero have a greater probability than do those near unity.

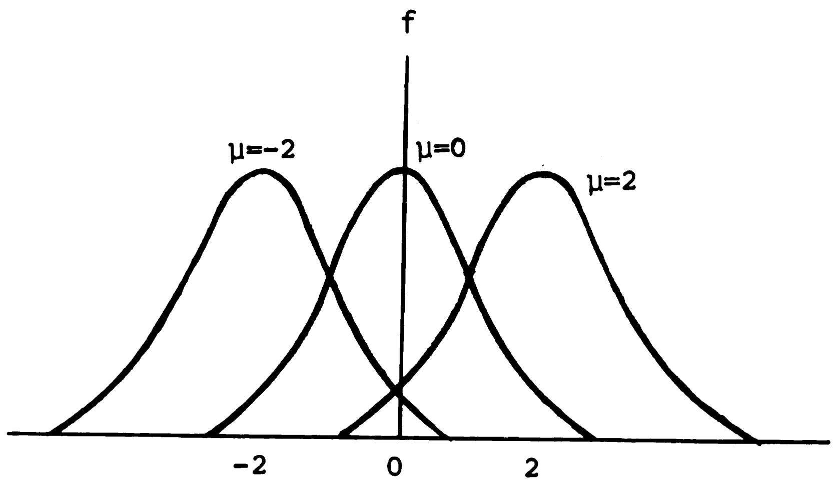
The normal distribution family.--A random variable x is said to have a normal distribution if its probability density function is given by:

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-1}{2\sigma^2}(x-\mu)^2}$$

This two parameter family has, μ, the weighted average E(X) and σ^2, the sum of the squared deviations, E(V), as its parameters. As with most distributions, the probability function describes a whole family of distributions of the normal random variable (x), one for each combination of the values, μ and σ^2. The random variable x is simply the value of whatever variable is under consideration. The shapes assumed by the normal distribution are indicated below.

A shift in μ displaces the curve as a whole, whereas a change in σ^2 alters its relative proportions with reference to a fixed scale. The curve is always symmetrical about μ.

Additionally, the normal distribution is an excellent approximation of a number of continuous and discrete distributions.



The expected value of the normal distribution is:

$$E(X) = \mu$$

The variance and standard deviation respectively are:

$$V(X) = \sigma^2$$

$$\sqrt{V(X)} = \sigma$$

The normal distribution has become the most important probability model in statistical analysis.[35] Many continuous random variables, such as height, weight, I.Q., diameters of various manufactured items, tensile strength and the like are normally distributed. This is so because of the inherent attributes of measurements themselves. Errors in measurement seem to result from a vast collection of factors operative at a particular time. Each one of the factors has only a small effect on the magnitude and deviation of the error. Additionally,

these errors work independently and with a force which is equal in both directions, therefore canceling in the long run. Thus, we think of errors of measurement as reflections of chance variations which are normally distributed with zero expectation.[36] Thus, other processes which possess this type of chance variation can often meet the assumptions of a normal distribution.

The important properties of the normal distribution include:

1. Symmetrical distribution.
2. Area under the density curve fully defined by μ and a specified value of σ.
3. Large deviations from μ less likely than small deviations due to the $[-(x-\mu)^2/2\sigma^2]$ exponent of the normal function.
4. Mean, median and mode are equal.
5. An infinite range to the distribution.
6. The average of n observations taken at random from almost any population tend to become normally distributed as n increases.

Many business processes may be represented by the normal distribution because of "the frequent occurrence of variables in the analysis of business problems, which are the sums of independent random variables with very similar, if not identical probability distributions." The normal has thereby been applied to a wide variety of business problems, and even if the random variable so considered is not exactly normally distributed, the normal is such a good approximation to many distributions that the results are generally not impaired. Thus, the great value of the normal distribution is its ability to approximate many other distributions which are less tractable. The normal is considered a good approximation to the binomial, poisson and gamma distributions.

The log-normal distribution.--A random variable x is said to have a log-normal distribution if its probability density function is given by:

> If x is a random variable and y = log x and y is a normal random variable, then x is said to have a log normal distribution.[37]

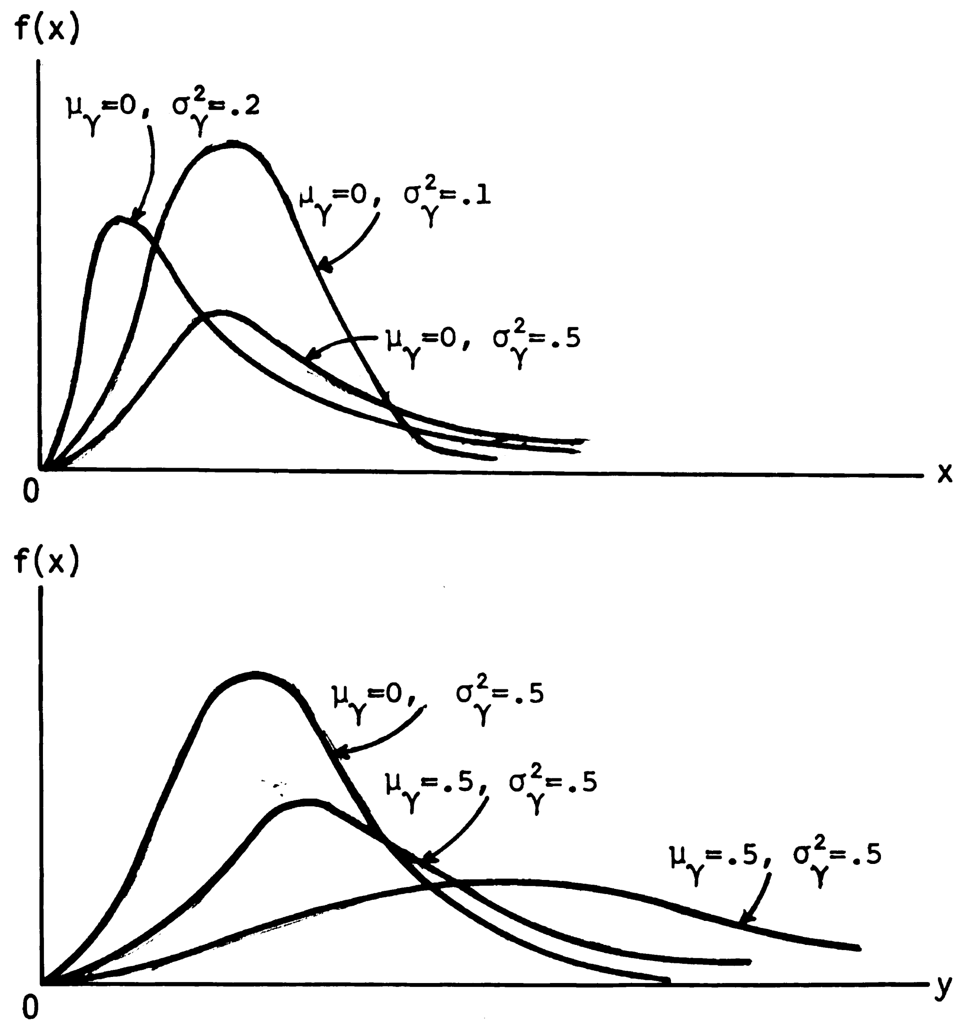
$$F\ (x;\ \mu_y, \sigma_y^2) = \left\{ \frac{1}{X\,\sigma_y \sqrt{\pi 2}}\ e^{\{\frac{1}{2\sigma_y^2}(\ln x - \mu_y)^2\}} \right.$$

The parameters of the log normal include μ_y and σ_y, where

$$\mu_y = \ln \sqrt{\frac{\mu_x^4}{\mu_x^2 + \sigma_x^2}} \qquad \sigma_y = \sqrt{\frac{\mu_x^2 + \sigma_x^2}{\mu_x^2}}$$

Thus, the log normal is nothing more than the probability distribution of a random variable whose logarithm obeys the normal probability density function.

The log normal is encountered in a variety of applications such as income studies and classroom sizes.[38] Additionally, it has been employed successfully to represent demand.



the log normal

<u>Criteria--Demand Probability Distributions</u>

It has been shown that the demand per unit time experienced by a channel system may be considered as a random variable, and hence, be represented by a probability distribution. However, demand per unit time is not necessarily well represented by all types and families of probability distributions.

Because of the way in which demand occurs, certain probability distributions may be precluded as adequate representations of the random

process generating demand. The probability distributions selected for study in this research must have empirical justification. Thus, the first and most critical assumption must be that the theoretical probability distribution has been shown to adequately represent the random way in which demand is presented to a system.

The second and third criteria to be applied in the selection of theoretical distributions are related to the time and cost considerations of this research. As with most research, time and cost limitations are certainly real and thus preclude the evaluation of every possible alternative formulation of the variables under study. Since the number of experimental variations that could be made are quite large, some must be selected for study and others ignored. Hence, the second criteria relates to selecting probability distributions which have the potential for affecting the operation of the physical channel system. If two probability distributions have very similar assumptions, functions and patterns, it is not likely that these effects on the system would be much different if both were used to generate demand. Consequently, the theoretical distributions employed in this research will be those that appear to be different as determined by their probability density functions.

As is indicated above, the third criteria relates to time and cost considerations. This research cannot evaluate every possible probability distribution, and this restriction makes necessary imposition of the first two criteria.

Selection of Theoretical Probability Distributions

Negative Binomial Distribution

The negative binomial has been empirically justified as a demand probability distribution by Wright.[39] In fact, Wright claims the negative binomial is an excellent representation of demand which is "moderately high, somewhere between high demand items, represented by the normal distribution and low demand items represented by the poisson."[40]

Additionally, Zehna states, "It is often assumed that the total number of units demanded is the observed value of a random variable having a negative binomial distribution."[41] The distribution is especially applicable when little information about the nature of the demand is available to the firm because the family of distributions admits such a wide variety of possible assumptions.

Exponential Distribution

The stochastic generation of demand may be viewed in terms of the period of demand, i.e., the time between demands. In this manner, the time between demands is the random variable and thereby determines the total quantity demanded over a given time frame. Considered in this light, demand can be represented by an exponential probability distribution of the time between demands.[42] Buchan and Koenigsburg have applied the exponential successfully to demand generation for an inventory model.[43] Magee also supports the exponential as representing the order size pattern of demand.[44]

Poisson Distribution

The poisson distribution probably has more empirical support as a generator of demand than any other probability distribution. The supporting empirical evidence is well documented.[45]

Normal Distribution

The normal distribution provides a good representation of stochastically generated demand when average demand is large.[46] In fact, in most inventory models, demand is assumed normal because of the more tractable properties of the normal distribution as compared to the variety of potential demand distributions. The fact that the normal may be employed to approximate such distributions as the gamma, poisson and binomial indicates its inherent applicability to the distribution of demand. Again the application of the normal probability distribution to demand generation is well documented.[47]

Log Normal Distribution

Extensive evidence exists that the log normal distribution will approximate histograms of actual demand data. Holt et al. examined sales data for cooking utensils over a period of six years and found the log-normal distribution to be an excellent fit to the data.[48] Magee concurs, concluding, "demand rates for many product lines appear to follow the log normal distribution, although with somewhat different standard ratios."[49]

The Holt et al. study concluded that the log normal fits demand or order distributions much better when the number of orders received is small.[50] Additionally, they felt that the log normal is justified in theory in that sales to any one customer in a period of time might be determined by the product rather than the sum of a great many random factors.

Gamma Distribution

The gamma also has a wealth of empirical evidence to support its use as a representation of the probability distribution of demand over time. Martin Basic's Ph.D. thesis was devoted to analyzing the application of the gamma distribution to demand distributions.[51] He found the demand for steel products supplied by a steel service center to be approximated quite well by the gamma. Holt et al. conclude that the suitability of the gamma has been verified for a variety of products.[52] In fact, for both fast moving and slow moving products, the gamma provided a considerably better fit to the data than did the poisson. Beckman and Bobkoski, in a study of the demand for airline travel concluded that the gamma could be applied to represent the distribution of the demands for seats.[53]

These six distributions will then be the experimental base upon which this research will be conducted. Each distribution has been shown empirically to apply to a number of demand situations. In addition, the distributions are significantly different in terms of their frequency distributions at various levels of average demand to at least have the capacity to differentially affect the way demand is presented to the channel system. Although the normal distribution is a good approximation to most of the distributions (gamma, poisson, log normal), the literature indicates that it does not always give the best fit to actual demand data. Hence, the second criteria, that of producing measured differences should be satisfied.

These six probability distributions, along with selected variance and average levels, form the basis for the input to be made to the simulated channel system. The generation of these demand distributions, the development of hypotheses relative to expected system results and the methods for measuring and analyzing the effects on the channel system are presented in the next chapter.

CHAPTER III--FOOTNOTES

[1]Claude McMillan and Richard F. Gonzalez, Systems Analysis: A Computer Approach to Decision Models (Homewood, Ill.: Richard D. Irwin, 1965), p. 159.

[2]Due to the scope and purpose of this research, only an overview of probability distributions is given. For more information in this area, see any one of the introductory statistics books listed in the bibliography.

[3]Ann Hughes and Dennis Grawaig, Statistics: A Foundation for Analysis (Reading, Mass.: Addison-Wesley Publishing Co., 1971), p. 3.

[4]Charles T. Clark and Lawrence L. Schkade, Statistical Methods for Business Decisions (Cincinnati, Ohio: Southwestern Publishing Co., 1969), p. 181.

[5]Ya-lun Chou, Statistical Analysis with Business and Economic Applications (New York: Holt, Rinehart and Winston, Inc., 1969), pp. 181-182.

[6]Ibid., p. 182.

[7]Ibid., p. 182.

[8]Richard C. Clelland et al., Basic Statistics with Business Applications (New York: John Wiley & Sons, Inc., 1966), p. 59.

[9]Ibid., p. 59.

[10]Clark and Schkade, op. cit., p. 3.

[11]Peter A. Zehna, Probability Distributions and Statistics (Boston: Allyn and Bacon, Inc., 1970), p. 122.

[12]Hughes and Grawaig, op. cit., p. 88.

[13]Kurtosis refers to the peakedness of the distribution, and is measured with reference to the peakedness of the normal distribution, which is of "intermediate peakedness." Thus, the normal distribution has "zero kurtosis."

[14]William C. Guenther, Concepts of Probability (New York: McGraw-Hill Publishing Co., 1968), p. 89.

[15]Ibid., p. 99.

[16]Hughes and Grawaig, op. cit., p. 99.

[17] Zehna, op. cit., p. 132.

[18] Ibid., p. 130.

[19] Clark and Schkade, op. cit., p. 214.

[20] Guenther, op. cit., p. 113.

[21] Ibid., p. 121.

[22] Zehna, op. cit., p. 135.

[23] V. E. Gmurman, Fundamentals of Probability Theory and Mathematical Statistics (London: Iliffe Books Ltd., 1968), p. 64.

[24] Zehna, op. cit., p. 134.

[25] George P. Wadsworth and Joseph P. Bryan, Introduction to Probability and Random Variables (New York: McGraw-Hill Book Co., Inc., 1960), p. 67.

[26] Zehna, op. cit., p. 141.

[27] Chou, op. cit., p. 216.

[28] Hughes and Grawaig, op. cit., p. 114.

[29] McMillan and Gonzalez, op. cit., p. 159.

[30] Chris P. Tsokos, Probability Distribution: An Introduction to Probability Theory with Applications (Belmont, Calif.: Duxbury Press, 1972), p. 128.

[31] Zehna, op. cit., p. 148.

[32] Emanuel Parzen, Stochastic Processes (San Francisco: Holden-Day, 1962), p. 162.

[33] E. Martin Basic, "Development and Application of a Gamma Based Inventory Management Theory" (unpublished Ph.D. dissertation, East Lansing, Michigan, 1965), p. 8.

[34] Wadsworth and Bryan, op. cit., p. 91.

[35] Chou, op. cit., p. 221.

[36] Ibid., p. 222.

[37] Alexander Mood and Frank A. Graybill, Introduction to the Theory of Statistics (New York: McGraw-Hill Publishing Co., 1963), p. 132.

[38] Zehna, op. cit., p. 160.

[39] James W. Prichard and Robert H. Eagle, Modern Inventory Management (New York: John Wiley & Sons, Inc., 1965), p. 174.

[40] Ibid., p. 174.

[41] Zehna, op. cit., p. 133.

[42] C. S. Chedzey, Science in Management (London: Routledge and Kegan Paul, Ltd., 1970), p. 38.

[43] Joseph Buchan and Ernest Koenigsburg, Scientific Inventory Management (Englewood Cliffs, N.J.: Prentice-Hall, 1963).

[44] John F. Magee, Physical Distribution Systems (New York: McGraw-Hill Publishing Co., 1967), p. 42.

[45] Charles C. Holt, Franco Modigliani, John F. Muth and Herbert H. Simon, Planning Production Inventories and Work Force (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1960), pp. 286-287; G. Hadley and T. M. Whitin, Analyses of Inventory Systems (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963), pp. 106-114; Prichard and Eagle, op. cit., pp. 170-175; McMillan and Gonzalez, op. cit., pp. 223-226; Zehna, op. cit., pp. 133-137; and Parzen, op. cit., pp. 251-258.

[46] Prichard and Eagle, op. cit., p. 174.

[47] Hadley and Whitin, op. cit., pp. 114, 256-258; Zehna, op. cit., p. 159; Tsokos, op. cit., p. 160; and Prichard and Eagle, op. cit., pp. 167-175.

[48] Holt et al., op. cit., p. 284.

[49] Magee, op. cit., p. 38.

[50] Holt et al., op. cit., p. 283.

[51] Basic, op. cit., p. 8.

[52] Holt et al., op. cit., p. 288.

[53] Martin Beckman and F. Bobkoski, "Airline Demands: An Analysis of Some Frequency Distributions," Naval Research Logistics Quarterly, 5 (March 1958), 48.

CHAPTER IV

HYPOTHESES AND RESEARCH METHODOLOGY

Introduction

The objective of this research is to measrue the change in efficiency and effectiveness of a simulated physical distribution channel as a result of demand uncertainties which are represented by probability distributions, variability and level. The statement of hypotheses and the research methodology required to test these hypotheses are delineated in this chapter.

The research methodology includes a justification for employing simulation experimentation, a description of the simulation model (LREPS) and the experimental design. The experimental design section considers the type of design to be used, description of experimental runs, the factors and their levels, the variables to be measured and the method of data analysis. Additionally, procedures for generating the distributions and their validation are discussed.

Hypotheses

The general hypothesis of this research is that the presence of demand uncertainty has an effect on the efficiency and effectiveness of a physical channel system. Uncertainty is represented by a probability distribution of demand and therefore can be seen as being composed of three factors: the pattern, variance and level (average demand per day). The responses which provide measures of the efficiency and effectiveness of the system are total sales, total costs, activity center costs and some combination of the above to give a service level and margins. Thus, the hypotheses to be stated relate to the impact of the pattern, variance and level of demand upon total channel cost per unit and service level. No hypotheses are presented as to the impact of uncertainty on activity center costs because it is felt that changes in these costs (that are associated with uncertainty) serve to explain why the hypothesis relative to total costs was accepted or rejected.

The hypotheses may be separated into two categories. The first group of hypotheses relate to the costs and service levels associated with each factor of demand uncertainty (pattern, variance and level) as compared to the costs and service levels which obtain when demand is fixed per unit time. The second set of hypotheses are concerned with the comparison of cost and service level results among the types of demand uncertainty.

No attempt is made to state every conceivable subhypothesis, as the number of such statements would be excessive. Rather, general hypotheses are presented which relate to the three basic demand uncertainties and the two major output responses, total per unit costs and service level. Directional hypotheses are not given due to the fact that it is not at all clear in which direction the hypothesis should be stated in all cases. Therefore, either a hypothesis of "no difference" or one of "there is a difference" between the experimental factors in question will be stated. All hypotheses are stated below in the order in which they appear in the Conclusions chapter.

The first group of hypotheses, those relating to uncertain demand compared to the fixed demand per unit time (control) situation are as follows:

1. The total per unit costs of a physical channel system, which result when demand is presented to the channel system in the form of a particular probability distribution, will be different than when demand is fixed per unit time.
2. The service level (percent of demand stocked out), which results when demand is presented to the channel system in the form of a particular probability distribution, will be different than when demand is fixed per unit time.
3. The total per unit costs of a physical channel system which result when demand assumes different levels of variance around the average demand per day will be different than when demand does not vary around its average.
4. The service level which results when demand assumes different levels of variance around the average demand per day will be different than when demand does not vary around its average.
5. The total per unit costs of a physical channel system which result when demand assumes different levels (average demand per day) are not different from the total per unit costs which obtain when demand is fixed.

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6. The service level of a physical channel system which results when demand assumes different levels is not different from the service level that obtains when demand is fixed.

The second group of hypotheses, those relating to the comparison among the various types of demand uncertainties are as follows:

1. The different types of probability distributions of demand will create different channel system total cost per unit.
2. The different types of probability distributions of demand will create different service levels for the channel system.
3. The different levels of variance of demand per day will produce different total cost per unit for the channel system.
4. The different levels of variance of demand per day will create different service levels for the channel system.
5. The different average levels of demand per day presented to the physical channel system will not produce a difference in total per unit cost.
6. The different average levels of demand per day presented to the physical channel system will not produce a difference in the service level achieved by the channel system.

Simulation Experimentation

Simulation is a technique for replicating the performance of an actual system or operation. The model or simulation, as a result of replicating performance, can serve as a base for experimental analysis. Martin Shubek succinctly describes the nature of simulation:

> A simulation of a system or organism is the operation of a model or simulator which is a representation of the system or organism. The model is amenable to manipulation which would be impossible, too expensive or impractical to perform on the entity it portrays. The operation of the model can be studied and, from it, properties concerning the behavior of the actual system or subsystem can be inferred.[1]

Thus, an important attribute of simulation experimentation is the capability to observe the performance of the system under a variety of conditions that would be otherwise impossible to achieve.

Naylor et al. provide an exhaustive set of rationale to justify the use of simulation experimentation as an alternative to actual observation and experimentation.[2] Their rationale include:

1. It may be impossible or extremely costly to observe certain processes in the real world. In these cases simulation can be used as an effective means of generating numerical data describing processes that otherwise would yield such information only at a very high cost, if at all.

2. Through simulation one can study the effects of certain environmental changes on the operation of a system by making alterations in the model of the system and observing the effects of these alterations on the system's behavior.

3. Simulation enables one to study and experiment with the complex internal interactions of a given system whether it be firm, an industry, an economy or some subsystem of one of these.

4. Simulation enables one to study dynamic systems in either real time, compressed time or expanded time.

Simulation experimentation appears well suited to the research objectives as presented in this thesis. The objectives of this research involve measuring the impact of environmental factors (demand) on the performance of a complex system (physical channel system) over a time horizon. An attempt to experiment with uncertainty involved in various demand distributions, levels and variances in actual practice would be almost impossible. The physical channel system would have to be isolated, and various segments of its operation held constant for each experiment. Consequently, controlled experimental conditions would be difficult, if not impossible to achieve. Problems would arise in being able to measure cost and service at all levels within the channel. Finally, the experimental factor, demand, could hardly be controlled by the experimenter. In summary, a simulation model of a physical channel system and the performance of a structured set of experiments which vary the demand distribution, level and variance offers a research opportunity not otherwise available.

Simulation Model--LREPS

To perform the specified research a valid simulation model is required. A number of excellent channel simulation models exist. These are reviewed in Appendix A. The simulation model employed in this research is known as LREPS. A brief description of the model is presented below, however, a more detailed description of the model is available.[3]

The LREPS model was developed by a Michigan State University research team under sponsorship of Johnson and Johnson Domestic Operating Company. The objective of the project was to design a planning model of a physical distribution system using dynamic simulation to evaluate the cost and service of alternative physical

distribution system designs. The objectives have been realized; the model has been validated and successfully applied to numerous situations.[4]

In terms of the conceptual aspects of the model, an extensive variety of conditions can be simulated. LREPS replicates the logistics system of a manufacturer with national sales, on a multiproduct basis. The number of echelons may vary from 1 to 99, with either middlemen or company owned facilities at each nodal point. Product flow is not limited to a particular scheme, but may take numerous assignment paths or linkages. Demand on the channel system may be evidenced individually by customer or aggregated into ZIP sectional centers. The system is capable of tracking up to 99 products with sales to as many as 10,000 customers.

The five logistical components, transportation, warehousing, inventory, communication and handling may be structured in a variety of ways. LREPS effectively handles all modes and legal forms of transportation, the reorder point, replenishment or combination inventory control system, all forms of communication, automated or manual materials handling and a variety of warehouse arrangements.

The experimental factors relevant to the LREPS model include target, controllable and uncontrollable variables. Target variables represent the performance of the system. Sales by echelon, weight, cases, items and lines; service levels in terms of stockouts and lead times; cost by activity center and echelon are the basic output measures of system performance. The controllable variables are those subject to managerial discretion and which become part of company strategy, or those dependent upon a given market situation. Order characteristics, product mix, and customer mix as well as facility network, inventory policy and transport modes, are the basic controllable variables. The model may then be deployed to test the sensitivity of various strategies to changes in these factors. Finally, uncontrollable variables include such factors as demand determinants, competitive reactions and acts of God. The system's response to changes in these factors may also be assessed. The experimental factors are summarized in Figure 4-1.

The computer model is made up of three subsystems. The supporting data system loads all exogeneous variables, which include input variables such as cost factors, transport modes, decision rules and the like. The operating system simulates the actual operation of the logistical system. A demand and environmental system creates orders; the operations subsystem processes orders through the system; cost, sales and service measures are calculated through the measurement subsystem; the monitor and control subsystem compares actual cost and service to that desired and activates changes in the system. The third system, report generator, converts the raw data into useful management information. The conceptual scheme of the LREPS model is shown in Figure 4-2.

The LREPS model is highly flexible and dynamic. Its flexibility has already been alluded to in earlier paragraphs. It is dynamic in the sense that the model provides for time interval dependencies, i.e., deficiencies in one period are linked to future periods; feedback is provided to allow for the adjustment of controlled variables on the

TARGET VARIABLES (OUTPUT)

- Sales distribution
- Customer service
- Physical distribution system costs
- Physical distribution system flexibility

CONTROLLABLE VARIABLES (INPUT)

- Order characteristics
- Product mix
- New products
- Customer mix
- Facility network
- Inventory policy
- Transportation
- Communications
- Unitization

UNCONTROLLABLE VARIABLES (INPUT)

- Marketing environment
- Technology
- Acts of nature

Figure 4-1. Summary of Experimental Factor Categories.

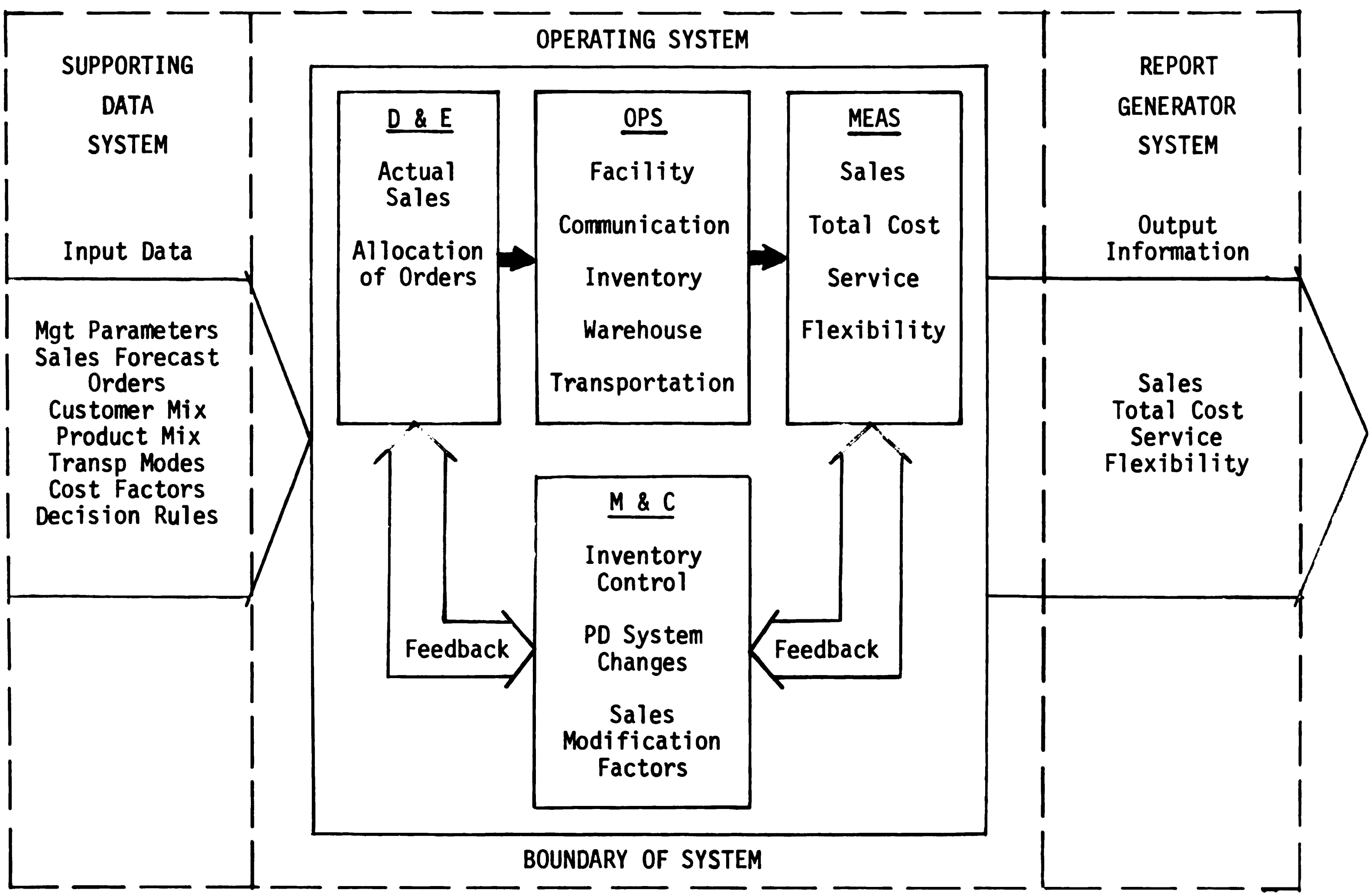

Figure 4-2. LREPS System Model Concept. (Source: See Helferich, footnote 25, page 23.)

basis of system performance; and a variable time planning horizon is allowed. An additional significant feature of the model is the ability of the logistics system to be integrated on a temporal (total Lead time) and spatial basis (location and transport modes). Further, the model is set up on a sequential decision mode so that future decisions are influenced by past decisions. The system simulated using the LREPS model for this research was exhaustingly presented in Chapter II and will not be detailed in this section.

Demand Generation

The first phase in the experimentation procedure was to generate the demand to be impressed upon the simulated physical channel system. The experimental factors or variables to be studied for each experimental run include the probability distribution of demand, the average demand and the variability or standard deviation of demand. Thus, each experimental run involves a specific probability distribution, average or level, and standard deviation of demand. Therefore, it was necessary to generate a set of demand values which have the characteristics desired for the experimental run in question. For example, to evaluate the impact of the gamma distribution, with a given mean and standard deviation, it was necessary to create a set of daily demands which follow a gamma distribution with a given mean and standard deviation.

To generate the appropriate demand distribution which will serve as the input of daily orders for each experimental run, a set of computer programs presented by Pritsker and Kiviat[5] and Naylor et al.[6] were

used. The normal, log normal, exponential, poisson were generated with Pritsker GASP routine, while the gamma and negative binomial programs were those developed by Naylor. Each distribution of demand was generated from the same random number table, with the random number seed constant in every case.

To assure that the proper mean and standard deviation were generated, a t-test was employed to test the generated mean against the desired mean. In all cases the hypothesis of no difference was accepted at the .05 level.

To be sure that the assumed probability distribution (normal, poisson, etc.) had in fact been generated, it was necessary to compare the generated frequencies of the values of demand to the theoretical frequencies that would occur if a given distribution applied. The Chi-square test and the Kolmogorov-Smirnov test are the most commonly applied statistical tests for comparing actual and theoretical frequencies.[7]

The Kolmogorov-Smirnov test (hereafter referred to as the K-S test) for goodness of fit was selected over the Chi square test for the following reasons. The K-S test is more powerful than the Chi-square test, and thus provides better information. Secondly, the K-S test avoids the cell bias problem that is common in the Chi-square test. The K-S test treats individual observations separately and requires no grouping into cells or class intervals as does the Chi-square test. Additionally, the cell size requirements of the Chi-square tests are completely avoided. The Chi-square test is somewhat sensitive to nonnormality.[8]

The K-S test is concerned with the degree of agreement between a set of sampled values and some specified theoretical distribution.[9] It determines whether the frequencies in the sample can reasonably be thought to have come from a population having the theoretical distribution.[10] The procedure for the test is to compare the cumulative frequency of simulated demand with the cumulative frequency distribution assumed. A "D" statistic is then computed which is the largest difference between actual and theoretical cumulative frequencies. The calculated "D" statistic is then compared with a critical "D" to determine whether the difference is significant. An example of the K-S test for goodness of fit as applied in this research is contained in Table 4-1. In each test the null hypothesis was accepted, i.e., that the desired distribution pattern was in fact generated.

Design Considerations

In experimentation, three problems must be solved: (1) factor selection; (2) selection of experimental design; and (3) measuring results. These problems are solved in terms of the purpose and objectives of this research. Demand uncertainties are defined for this research as the pattern or probability distribution, level and variability of demand. Performance is defined in terms of cost and service. Thus, the goal is to measure the sensitivity of cost and

TABLE 4-1. K-S Goodness of Fit Test for a Normal Distribution

Random Variable	Observed Cumulative Frequency	Theoretical Cumulative Frequency	Difference
55	.005	.0038	.0012
56	.005	.0057	.0003
57	.015	.0082	.0068
58	.015	.0116	.0034
59	.025	.0166	.0084
60	.025	.0228	.0022
61	.030	.0307	.0007
62	.050	.0418	.0082
63	.065	.0548	.0102
64	.090	.0708	.0192
65	.110	.9080	.0192
66	.135	.1151	.0199
67	.150	.1423	.0077
68	.190	.1762	.0138
69	.240	.2119	.0281
70	.275	.2514	.0186
71	.305	.2946	.0104
72	.360	.3446	.0154
73	.395	.3839	.0011
74	.475	.4483	.0267
75	.555	.5000	.0550
76	.600	.5517	.0483
77	.620	.6064	.0136
78	.680	.6554	.0246
79	.735	.7054	.0296
80	.765	.7486	.0163
81	.805	.7882	.0168
82	.820	.8338	.0138
83	.860	.8577	.0023
84	.890	.8849	.0051
85	.915	.9092	.0058
86	.940	.9292	.0108
87	.955	.9452	.0098
88	.970	.9582	.0118
89	.975	.9693	.0057
90	.990	.9772	.0128
91	.990	.9834	.0066
92	.995	.9884	.0066
93	.995	.9918	.0032
94	.995	.9943	.0007
95	.995	.9962	.0008
96	1.000	.9974	.0026

D = .055 Critical D @ α = .05 = $\frac{1.36}{\sqrt{200}}$ = .096

.055 < .096 Accept Ho that the distribution is normal.

service levels in a physical channel system to probability distributions, levels, and standard deviations of demand. Having the objectives clearly in mind thus facilitates the decision to be made as to factor selection, experimental design and measurement.

The factors to be studied in this research include probability distribution, levels and variability of demand. These so called "factors" might better be termed "conditions." In an actual situation, a channel system is faced with a given distribution of demand and cannot easily change the distribution. Thus, this research proposes to investigate this condition, and its impact. The condition is not easily varied as are experimental factors in most research. However, changes in level and variability of demand may be effected and thus these variables more readily assume the nature of experimental factors.

The factor or condition, demand distribution is evaluated at six "levels." In other words, six types of probability distributions of demand are investigated. The level, or average value of demand is investigated at two levels, "high" and "low." Two levels of this variable were selected for a number of reasons. It is hypothesized that the level of demand should not affect costs and service in the system. Secondly, the probability distributions under study have been shown to have empirical justification at different levels of average demand. The levels selected include an average demand per unit time of 75 units and 25 units. The specific values of these variables were selected arbitrarily, but the magnitude of the difference between them is felt to be great enough to show differences in system performance if these differences actually exist.

The third factor, variability of demand, must be clearly defined. The standard deviation of a variable is the most commonly accepted measure of variability. However, the standard deviation is an absolute measure of variability. Two sets of observations might be viewed considerably different in terms of variability if their standard deviations were the same but one of them had a mean three times as large as the other. Hamburg states, "For comparative purposes a relative measure of dispersion is required."[11] Measures of relative dispersion show some measure of scatter as a percent of the average about which they are computed.[12] Thus, variability in this research will focus upon relative variation. The measure of relative variability to be used is the coefficient of variation. The coefficient of variation C.V., is the ratio of the standard deviation to the mean.[13] Hence, C.V. $= \sigma/\mu$.

Three levels of the coefficient of variation are investigated in this research. The levels are defined as "low," "medium" and "high," and respectively correspond to a coefficient of variation of .10, .30 and .50. The specific levels of the coefficient of variation were selected arbitrarily, but were set so that differences that might exist due to variability in demand could be measured. Three levels were employed because research done on empirical demand probability distributions indicates that given probability distributions are more applicable to actual demand patterns at different ratios of standard deviation to mean. Thus, to investigate relevant probability distributions and variability levels, it was necessary to study at least three levels of the coefficient of variation.

Experimental Design

The method of experimentation, as has been recounted before, is to make changes in demand conditions and then to analyze the effects of these changes upon the behavior of the physical channel system. In order to effectively study the results in some systematic fashion, a proper method for analysis, i.e., an experimental design must be selected.

The purpose of an experimental design is to provide a method for measurement of changes made in the factors and not other random fluctuations which might occur during the experimental run. Additionally, the experimental design should be effective, i.e., should yield the desired information at least possible cost.

Naylor and Hunter point out that a variety of experimental designs may be employed in simulation experiments when the objective is to explore the reaction of a system to changes in factors affecting the system.[14] Those designs considered to be particularly relevant include the full factorial, fractional factorial and response surface designs. The full factorial has been selected for use in this research.

A factorial experiment is one in which the effects of all the factors and factor combinations in the design are investigated simultaneously.[15] Each combination of factor levels is used the same number of times. In this research, the factors refer to demand probability distribution, level and variability (coefficient of variation). A treatment, in the factorial sense, consists of some combination of

all factors in the model. In this research, a treatment is made up of a probability distribution, with a given average demand and a given coefficient of variation. A layout of the design is given in Figure 4-3.

The advantages of the factorial design, as opposed to randomized designs or one at a time approaches, are well summarized by Cox:

> To sum up, factorial experiments have, compared with the one factor at a time approach, the advantages of giving greater precision for estimating overall factor effects, of enabling the interactions between different factors to be explored, and of allowing the range of validity of the conclusions to be extended by the insertion of additional factors.[16]

It must be pointed out that interactions are not an important aspect of the investigation in this research. Interactions refer to the effect of combinations of experimental variables on the response variable that is above and beyond that which can be predicted from the variables considered singly. However, the nature of interactions seems to lose its meaning in the context of the present research problem. A channel system experiences a given pattern of demand, with a given average level and variance. The system is not in a position to easily change one of these variables, i.e., combine it with another level of the other two variables, and then commence operations. The levels of all three variables are fixed, and control over them somewhat limited. Thus, the nature of the experimental variables precludes a meaningful interpretation of the interaction effects.

The lack of attention to interaction effects does not diminish the applicability of the factorial design. The factorial design permits one to make statements as to the effect of each experimental variable

Level	C.V.	DISTRIBUTION Normal	Log Normal	Gamma	Neg. Binomial
1	.10				
	.30				
	.50				
2	.10				
	.30				
	.50				

Level	Poisson	Exponential
1		
2		

Figure 4-3. Experimental Runs.

which are based on observing that variable over a broad spectrum of conditions. Winer states:

> Apart from the information about interactions, the estimates of the effects of the individual variable is, in a sense, a more practically useful one; these estimates are obtained by averaging over a relatively broad range of other relevant experimental variables. By contrast, in a single-factor experiment some relevant experimental variables may be held constant, while others may be randomized. In the case of a factorial experiment, the population to which inferences can be made is more inclusive than the corresponding population for a single-factor experiment.[17]

Bonini concurs with this assessment, claiming that the factorial design provides for relatively wide generality of results.[18] Thus, the factorial design will allow statements to be made as to the effect of a particular demand distribution, where the distribution is considered over a range of demand levels and variances. In conclusion, the factorial design appears well suited to the objectives of this research.

There will be a deviation from the general factorial approach. Figure 4-3 indicates that the poisson and exponential distributions are not included in the layout matrix of the experimental design. The nature of the poisson and exponential distributions does not permit a fit into such a rigid pattern. Both distributions are one parameter distributions, and hence, cannot assume the total range of level (average demand) and variance that the other distributions admit. Thus, these distributions untidy the analysis somewhat, but this problem is unavoidable due to the nature of their functional form.

Response Variables

In accordance with the objectives and hypothesis of the research, response variables are desired which most accurately and succinctly describe the effectiveness and efficiency of the system. In addition, information is desired on the behavior of key variables as a result of the imposition of uncertainty. Thus measures of revenue, cost and its components, margin or profitability and service level are necessary.

To measure the effectiveness of the system the percentage of demand stocked out is used. This is the ratio of the unsatisfied demand (stockouts in dollars) to the total demand (in dollars) placed on the system. This measure is more useful than a simple revenue comparison, i.e., total sales or an unsatisfied demand comparison. By combining the two, a measure of the factor(s) effect on the system's ability to generate revenue and the system's service level is given. Thus this ratio describes the system's effectiveness (i.e., the ability to satisfy demand).

In addition to revenue and service, cost and its components are desired gauges of a system's performance. Total cost of the system is broken into transportation, thruput, facility, and inventory. It is necessary to look at total cost and its components because total cost could remain constant between two situations but its composition could be completely different. From the viewpoint of the manager or systems designer, cost components reveal more accurately the behavior of the system and may lead to defining systems interaction. From an

experimental view, the effects of uncertainty on the components of cost are necessary for a more complete and useful analysis.

Thus, the response variables of interest in this research are percentage of stockouts at ISP, transportation costs, facility costs, inventory costs, thruput costs, total costs.

Experimental Runs

The initial conditions and the experimental procedure of the data collection are discussed in this section. The system as described in Chapter II was modeled and simulated for 180 days to create the initial conditions. Then each of the factors of uncertainty and the control system were run from the initial conditions for 90 days. The output at the end of these runs is the data used in the analyses.

The initial system conditions which were employed as the starting point for all runs including the control runs was created first. Using the parameters of the system as described in Chapter II, a demand of 75 units was imposed at each ISP every day for the duration of the simulation.

Preparatory to day one, all the relationships in the system were set and inventory placed in the system. The level of inventory placed at each ISP and SSP in the system was randomly selected between ROP and ROP plus EOQ. This inventory level was selected because at any given point in time a stocking location would not have on hand and on order less than ROP or more than ROP plus EOQ. Thus the boundaries of the inventory are known. However, the actual amount is not known nor is the possibility that each location would have the same amount very great. Therefore the amount between these boundaries was randomly selected. The system was then simulated for a period of 180 days. This initial simulation period was chosen so that the effects of demand would be seen at the highest level in the system (PSP) and to allow the system to stabilize. In effect, the system was "hot" after 180 days. A procedure identical to the one just described was carried out for a demand of 25 units per day. The responses obtained after 180 days of simulation were used as the starting point for all experimental runs including the control runs.

The control system was then simulated. In the control system, as stated in experimental design, everything in the system is certain. Thus, demand remains constant at 75 or 25 units per day for the duration of the simulation. Employing the initial conditions obtained as described above, the system was simulated for 90 days and the results obtained after 90 days of simulation represented the control system responses which were used in the data analysis.

Every condition of uncertainty as described in the experimental design was simulated in the same manner as the control system. The initial conditions always remained the same and the simulation ran for 90 days. Ninety days was chosen as the simulation duration for several reasons.

First, it was imperative that the effects of demand were seen throughout the system and that a sufficiently long run was made so that the PSP or highest echelon in the system would feel the effects of demand.

With a lead time of 10 days between the PSP and the SSP and seven days between the SSP and the ISP coupled with the fact that inventory turned approximately 20 and 30 times at the ISP and PSP, respectively, 90 days was seen as sufficient.

Secondly, the simulation should be long enough to allow the system to stabilize. One simulation was allowed to run for 720 days with reports every 30 days. The system stabilized rapidly and the results at day 90 as compared to 120, 150, etc. indicated that 90 days was sufficient.

Third, the run should be long enough to generate the desired distribution. Thus, there should be a sufficient number of points or observations to create the chosen probability distribution. Each day of simulation represented one observation, thus each distribution was created with 90 observations.

Lastly, the simulation can not run forever and there is a real limitation of cost associated with length of run. Ninety days satiated all the previous conditions and the gain that would be made to run past 90 days would not be worth the cost. Thus, 90 days became the duration of all simulation runs.

Data Analysis

The final consideration in the design of experiments is the methods used to analyze the data generated in the experiments. A very broad range of data analysis techniques exist, and selection of techniques is dependent upon the objective of the research and the inherent assumptions of the techniques employed. The basic question to be answered is: "Does the pattern (probability distribution) average level and variability of demand make a significant difference as to the system's performance?" Three forms of the analysis of variance technique plus the standard t test have been selected for data analysis in this research. These techniques appear to meet the objective of measuring differences in system performance caused by demand uncertainty. Additionally, the necessary assumptions of the techniques do not seem to be violated.

The three analyses of variance techniques are the F-test, Tukey's test of multiple comparisons and Dunnett's method of multiple comparisons. These three forms of analysis of variance are particularly well suited for comparing outputs of computer models.[19] The F-test is appropriate for testing the hypothesis that the average response (cost, service level) for each of the distribution types, levels or variances are equal. Thus, the test assesses whether these alternatives differ in terms of their effect on system performance. Tukey's multiple comparison technique may then be applied to the question of <u>how</u> they differ. Finally, Dunnett's method provides the necessary analysis of how one specific mean, a control mean, compares with all other output means.[20]

The application of analysis of variance techniques rests upon meeting three key assumptions. These assumptions include: (1) the independence of statistical errors; (2) equality of variance; and (3) normality.[21] The independence assumption is met if the observations are uncorrelated in time. Since the experiments set forth relate to one time period, the correlation of observations over a time frame does not appear to be a problem. As for the second and third assumptions, the experimenter rarely, if ever, knows whether these assumptions are satisfied.[22] However, minor deviation from assumptions two and three will not greatly affect the results. The procedures employed are said to be "robust," that is, quite insensitive to departures from assumptions.[23] This is particularly true of the F-test as argued by Scheffe especially when the cell sizes are equal as is true in the present case.[24] As for Tukey's and Dunnett's multiple comparison techniques, reference is made to Naylor:

> Unfortunately, the robustness properties of multiple comparisons . . . are not as well known as the ones of the simple F-test. One can safely conclude that departure from the assumptions of common variance and normality are small enough to not seriously matter.[25]

The F-test tests the hypothesis that the average response for each of the distribution types, levels or variances are equal.[26]

$$Ho: \; D_1 = D_2 = D_3 \ldots D_n$$

The decision rule for accepting or rejecting Ho is: If

$$F \geq F_{\alpha}, \; m-1, \; n\,(n-1) \text{ reject Ho}$$

Otherwise accept Ho

where:

F = appropriate percentile of the F distribution.

α = significance level.

m = number of distributions or variances or levels.

n = number of replicates per factor level.

If the hypothesis Ho is accepted, it is implied that the differences between distributions, levels or variances were caused by random fluctuation rather than actual differences in the factors. If the hypothesis is rejected, it is concluded that variations in the response variable are caused by the factor. In either case additional analysis is required. In this research the additional analysis will be multiple comparisons.

Given the research objective previously stated, it is also desirable to make individual mean comparisons among the alternative probability distributions, levels and variances of demand. Multiple comparison techniques are tools relevant to meeting this query, since they have been designed specifically to attack questions of how the means of many populations differ.[27]

Multiple comparison procedures employ confidence intervals rather than strict hypothesis tests. Confidence intervals are constructed for the difference $(U_i - U_j)$ and the actual difference in the sample means $(\overline{X}_i - \overline{X}_j)$ are compared with the confidence interval so constructed. If the difference $(\overline{X}_i - \overline{X}_j)$ falls within the interval, it is concluded that the population means do not differ.

It would be tempting to employ the t-statistic to calculate the confidence intervals necessary for multiple comparisons. If a number of confidence intervals are calculated for a given experiment with a given value of (α), all the intervals will not be simultaneously true at the α level selected.[28] If an experimenter conducts K independent t-tests, each with the same (α), the probability of falsely rejecting at least one of the K hypotheses, assuming all are true, is 1 - P (not rejecting all K tests) or $\{1-(1-\alpha)^K\}$.[29] For a very large K, the value for all tests becomes quite small. Thus, the risk of a type 1 error is considerable using repeated t-tests.

To avoid the problems stated above, two methods of multiple comparisons, that produce confidence intervals which are all simultaneously true at a given (α) have been selected for use. The methods to be employed are Tukey's method and Dunnett's method. Both of these methods require that treatment means be uncorrelated and have equal variances.[30]

Tukey's method produces simultaneous confidence intervals for the comparison of any or all pairs of treatment means. Tukey's confidence intervals are calculated using the following:

$$(\overline{X}_i - \overline{X}_j) \pm q(p,v) \sqrt{\frac{MSe}{n}}$$

where p equals the number of treatments and v equals the degrees of freedom associated with MSe. q(p,v) is tabulated as "Percentage Points of the Studentized Range." To test the difference between treatment means the difference $(\overline{X}_i - \overline{X}_j)$ is calculated and compared to $q(p,v) \sqrt{\frac{MSe}{n}}$.

An important aspect of this research is to compare the system performance (cost and service) associated with a probability distribution, level and variance of demand with the performance of the system under "certain conditions," i.e., where demand is fixed. What is desired then, is a test of the hypothesis of no difference between a base or control run (fixed demand) and all other runs. Dunnett's method is well suited for such comparisons.[31]

Dunnett's method of multiple comparisons compares each treatment mean with a control condition. The confidence intervals constructed are calculated using the following:

$$(\overline{X}_i - \overline{X}_j) \pm t_1 - (\alpha/2) \sqrt{\frac{2MSe}{n}}$$

where $t_1 - (\alpha/2)$ is a tabled value from Dunnett's tables.

The hypotheses stated in this chapter will be tested using the techniques described above. The results of the simulation runs and the statistical tests of the hypotheses are presented in the next chapter.

CHAPTER IV--FOOTNOTES

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[19] Naylor, op. cit., p. 185.

[20] Ibid., p. 204.

[21] William Mendenhall, The Design and Analysis of Experiments (Belmont, Calif.: Wadsworth Publishing Co., 1968), p. 209.

[22] Ibid., p. 206.

[23] Naylor, op. cit., p. 200.

[24] Henry Scheffe, The Analysis of Variance (New York: John Wiley and Sons, 1959), p. 345.

[25] Naylor, op. cit., p. 201.

[26] Ibid., p. 201.

[27] Ibid., p. 203.

[28] Mendenhall, op. cit., p. 175.

[29] Ibid., p. 175.

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[31] C. W. Dunnett, "A Multiple Comparison Procedure for Comparing Several Treatments with a Control," Journal of the American Statistical Association, 50 (1955), 1097-1121.

CHAPTER V

EXPERIMENTAL RESULTS

Introduction

The general hypothesis of this research is that demand uncertainty, in the form of a probability distribution, level or average demand, and demand variance around the average will have significant impacts on the cost and service performance of a physical channel system. To test this hypothesis and the many attendant sub-hypotheses a simulation model of a physical channel system was employed, and 28 experimental simulation runs were completed. Each represented a different experimental condition. The purpose of this chapter is to report the findings. No attempt is made to explain or evaluate the findings. Chapter VI explains and interprets results.

The general overall results of the simulation experiments are presented in Table 5-1. Table 5-1 summarizes the average response of the physical channel system under each experimental factor--distributions, levels and variance (coefficient of variation) for all output response variables (service level, cost to revenue ratio, total cost, and activity center costs). The cost measures represent per unit costs. For example, the total physical distribution cost obtained for the gamma distribution is 138.29 cents. This figure was obtained by averaging the

Table 5-1. Average Responses Over All Experimental Runs

		Distribution						Level (Demand/Day)		Coefficient of Variation		
	Control	Normal	Log Normal	Gamma	Negative Binomial	Poisson	Exponential	25	75	.10	.30	.50
Demand stocked out (%)	0.0	2.23	1.06	1.18	1.60	0.50	7.36	1.90	1.14	0.22	1.03	3.30
Cost/revenue ratio (%)	27.12	27.06	27.38	27.66	27.21	27.00	29.47	29.61	25.04	27.23	27.40	27.36
Total cost per unit (¢/unit)	135.61	135.29	136.92	138.29	136.06	135.02	147.33	148.07	125.22	136.17	136.98	136.18
Transportation (¢/unit)	114.42	115.13	114.08	115.20	113.00	113.34	113.95	122.33	106.37	113.85	113.75	115.46
Facility (¢/unit)	13.99	13.08	15.31	15.59	15.33	14.37	24.30	17.78	11.91	14.75	15.70	14.07
Thruput (¢/unit)	4.61	4.63	4.67	4.60	4.65	4.65	4.79	4.64	4.63	4.62	4.65	4.64
Inventory (¢/unit)	2.59	2.44	2.81	2.90	2.79	2.66	4.29	3.24	2.23	2.72	2.87	2.60

total cost of the system across all experimental runs in which the gamma distribution was used. Thus, six experimental runs, at two levels of demand and three levels of variance made up the cases in which the gamma distribution represented the demand pattern. Additionally, the table presents the same average output responses for the controlled simulation run where demand was constant per day. The table reveals that the exponential distribution produced the highest cost and lowest service performance, whereas the poisson produced the lowest cost and highest service level. In terms of level of demand, the low demand per day experimental runs created the higher cost performance and lower service level. The performance of the channel system does not appear to vary greatly among the experimental runs in which the level of variance was changed, except lower service where (C.V. equals .50). These output responses thus represent the average overall results of the simulation and statistical tests of these results are presented in the body of this chapter.

The organization of this chapter is developed around two main questions: (1) What is the effect of demand uncertainty on a physical channel system as compared to a system in which there is no uncertainty, i.e., the control run, in which demand is fixed per unit time? and (2) What is the effect of one type of uncertainty versus a different level of that uncertainty? Thus, in section II it is shown whether particular types of uncertainty create significantly different effects on system performance than other types of uncertainty.

More specifically, section I discusses the comparison of factor (experimental factors) and control output response measures (system performance). First, average factor response comparisons with the controlled run are made on the basis of distributions, variances and levels, using Dunnett's method and standard t-tests. The results of the statistical tests performed are given, and the significant differences noted. Finally, individual response comparisons are presented with no statistical inferences implied.

Section II develops the comparisons among factor responses. The analysis of variance between patterns, variances and levels is presented initially for each output response and significant differences given. Average response comparisons among factor responses are discussed next, using Tukey's method of multiple comparisons. Finally, individual response (cells) comparisons are again developed, with no statistical inference implied.

All statistical tests will be carried out at the .05 level of significance. The actual tests are detailed in Appendix B, with the results (either significant or not significant) reported in the body of the chapter.

Figure 5-1 represents the general procedure for analyzing the results of the simulation experiments. Portions of this figure will be reproduced at the beginning of each subsection of this chapter to indicate the nature of the analyses presented.

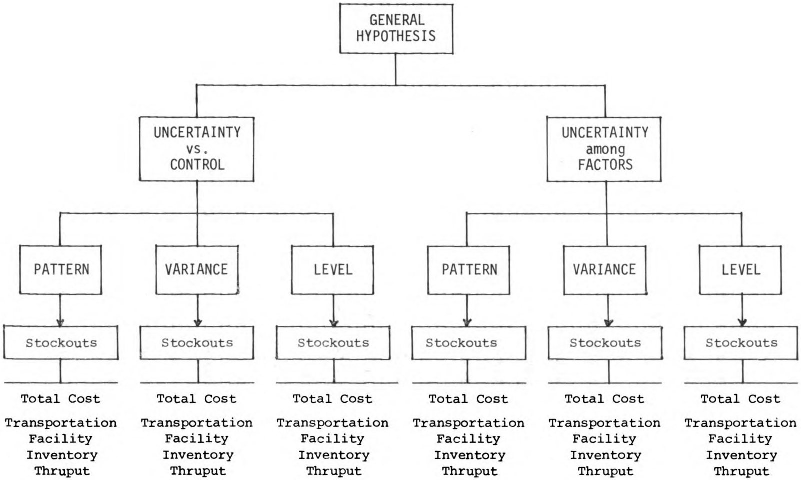

Figure 5-1. Research Analysis Organization.

Comparison of Factor and Control Responses: Average Response Comparisons

Probability Distributions

Figure 5-2 indicates that the demand stocked out and costs which result from the pattern of demand will be compared to demand stocked out and costs which result when demand is constant (the control simulation run).

Demand stocked out.--Figure 5-3 presents the comparison of the percent of demand stocked out for each distribution of demand to the percent stocked out under the controlled experimental run.

Using Dunnett's multiple comparison technique, the normal distribution and negative binomial are significantly different. The critical value of Dunnett's "t" statistic is 1.28,* whereas the difference between the normal and control run is 2.23 and between negative binomial and control the difference is 1.60. The standard t-test is employed to compare the poisson and exponential results with the control. The difference between the exponential and the control mean is significant. The critical "t" value is 12.47 with the actual difference equal to 18.42. The remaining distributions do not produce stockouts statistically significantly different than the control run.

Cost/revenue ratio.--Figure 5-4 describes the comparison of each distribution to the control run in terms of the cost-revenue ratio. The figure shows the cost/revenue ratio average from each distribution as a

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*See Appendix B for sample calculations.

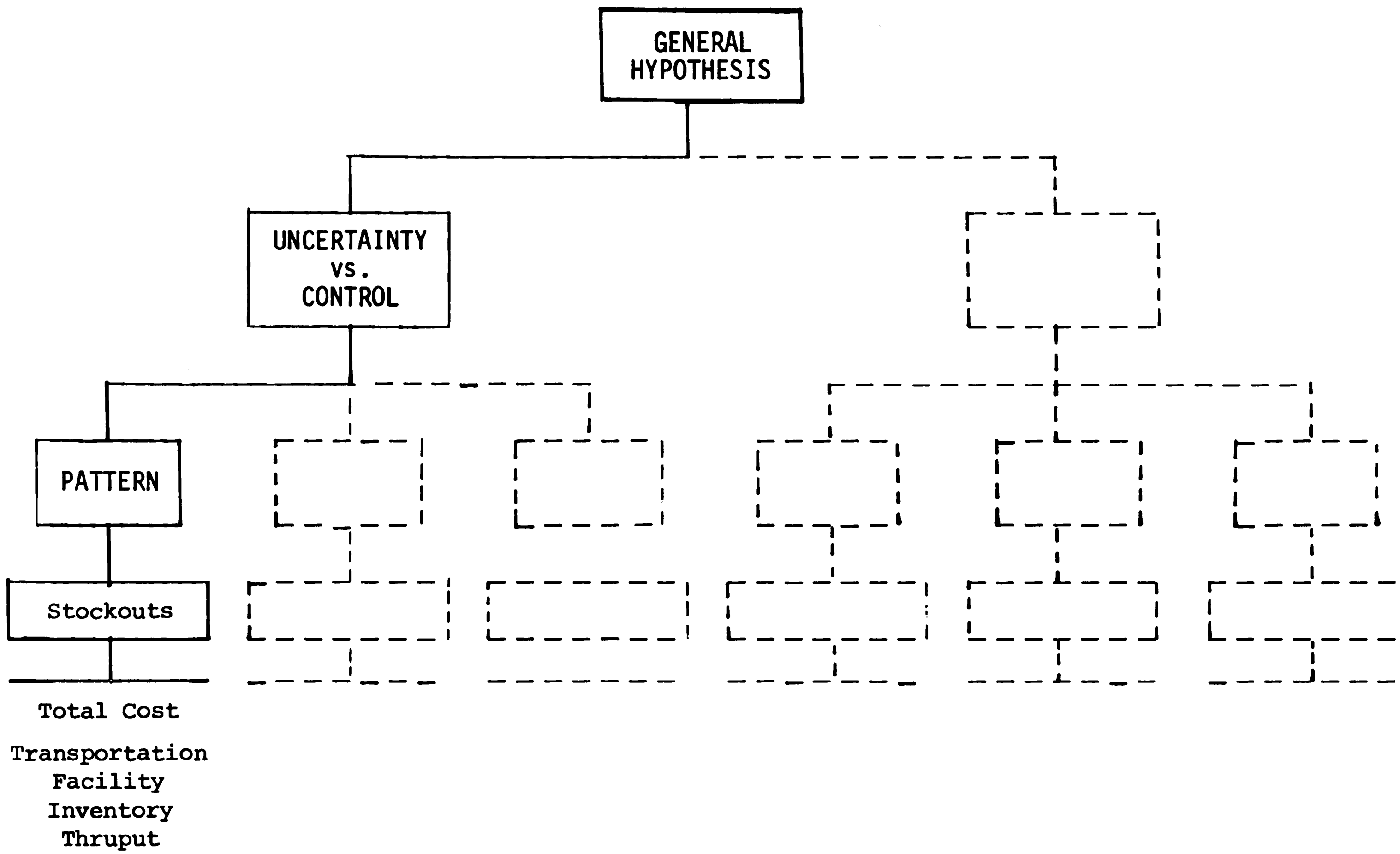

Figure 5-2. Research Analysis Organization: Control/Pattern

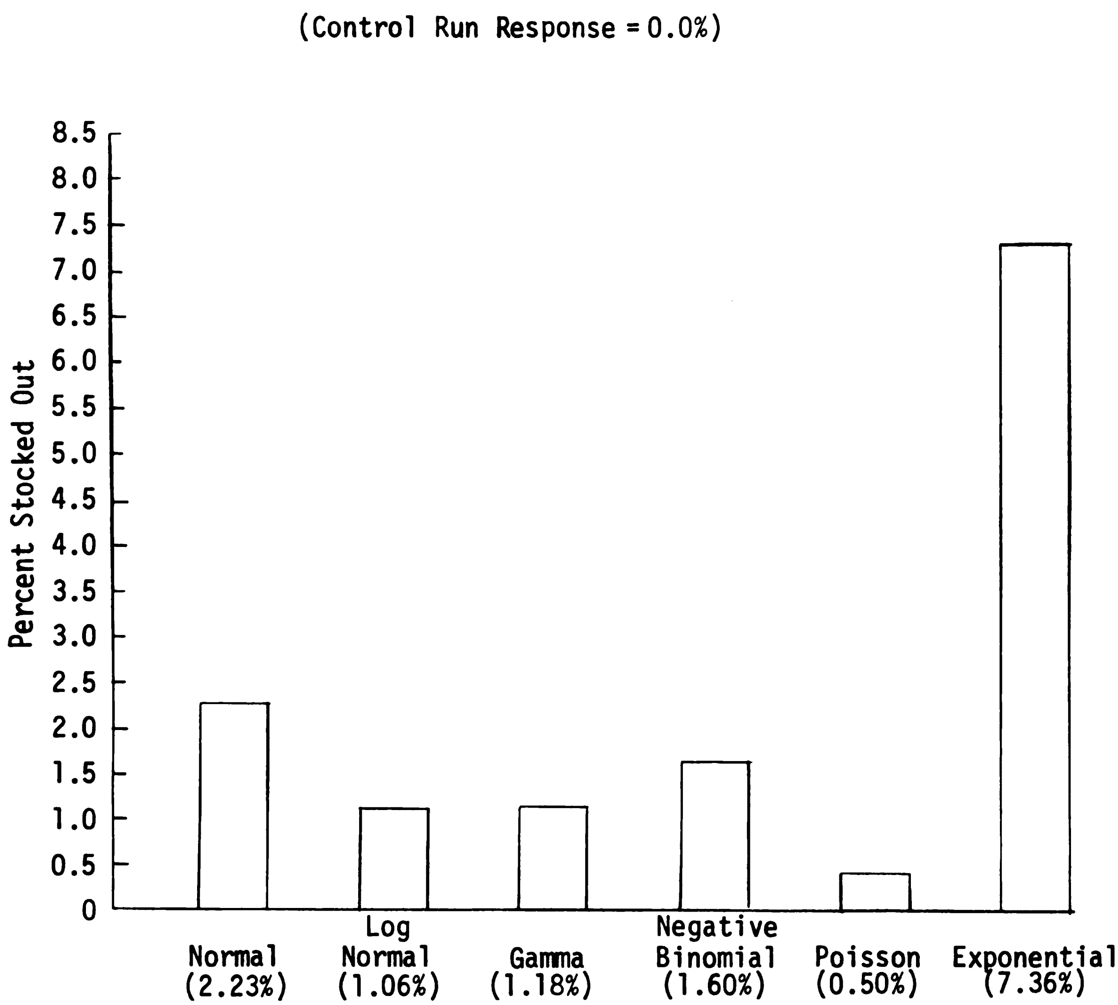

Figure 5-3. Probability Distribution Response Compared to Control Run Response: Percent of Demand Stocked Out.

Percent of Control Run Response

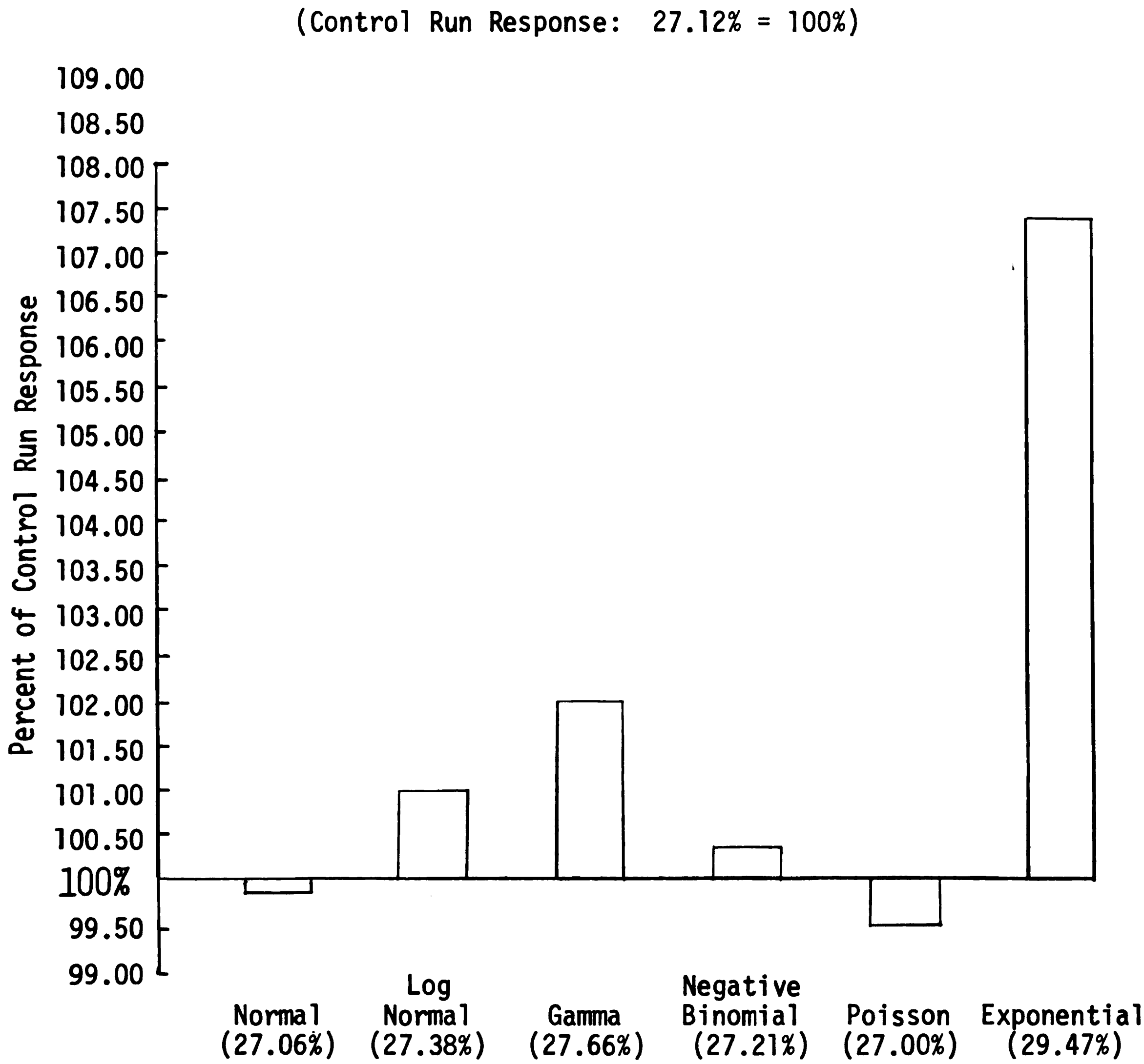

Figure 5-4. Ratio of Probability Distribution Response to the Control Run Response: Cost/Revenue Ratio.

percent of the control run cost/revenue ratio. For example, the average cost/revenue ratio from the gamma distribution runs is 102% of the control run cost/revenue ratio.

The application of Dunnett's "t-test" (and standard 't' tests to the poisson and exponential) reveal no statistically significant difference between the control run cost/revenue ratio and the average cost/revenue ratio obtained for each probability distribution.

Total cost.--The average total costs associated with each distribution as a percent of control run total cost are shown in Figure 5-5. There are no statistically significant differences between distribution related average total cost and control run total cost. However, the difference between the gamma distribution total cost and control run total cost is very close to being significant.

Transportation cost.--The ratio of average transportation costs from each distribution to control run transportation costs are presented in Figure 5-6. The actual differences in transportation cost between control and distribution runs are slight and thus no statistically significant differences are detected.

Facility cost.--Figure 5-7 depicts the ratio of the average facility cost associated with each distribution to the control run facility cost. Although the exponential distribution facility cost is 10¢ higher than the control run cost, the difference is not statistically significant. This results from the large amount of variance associated with the exponential responses and the small number of degrees of freedom available (one degree of freedom, with a critical t value of 12.47).

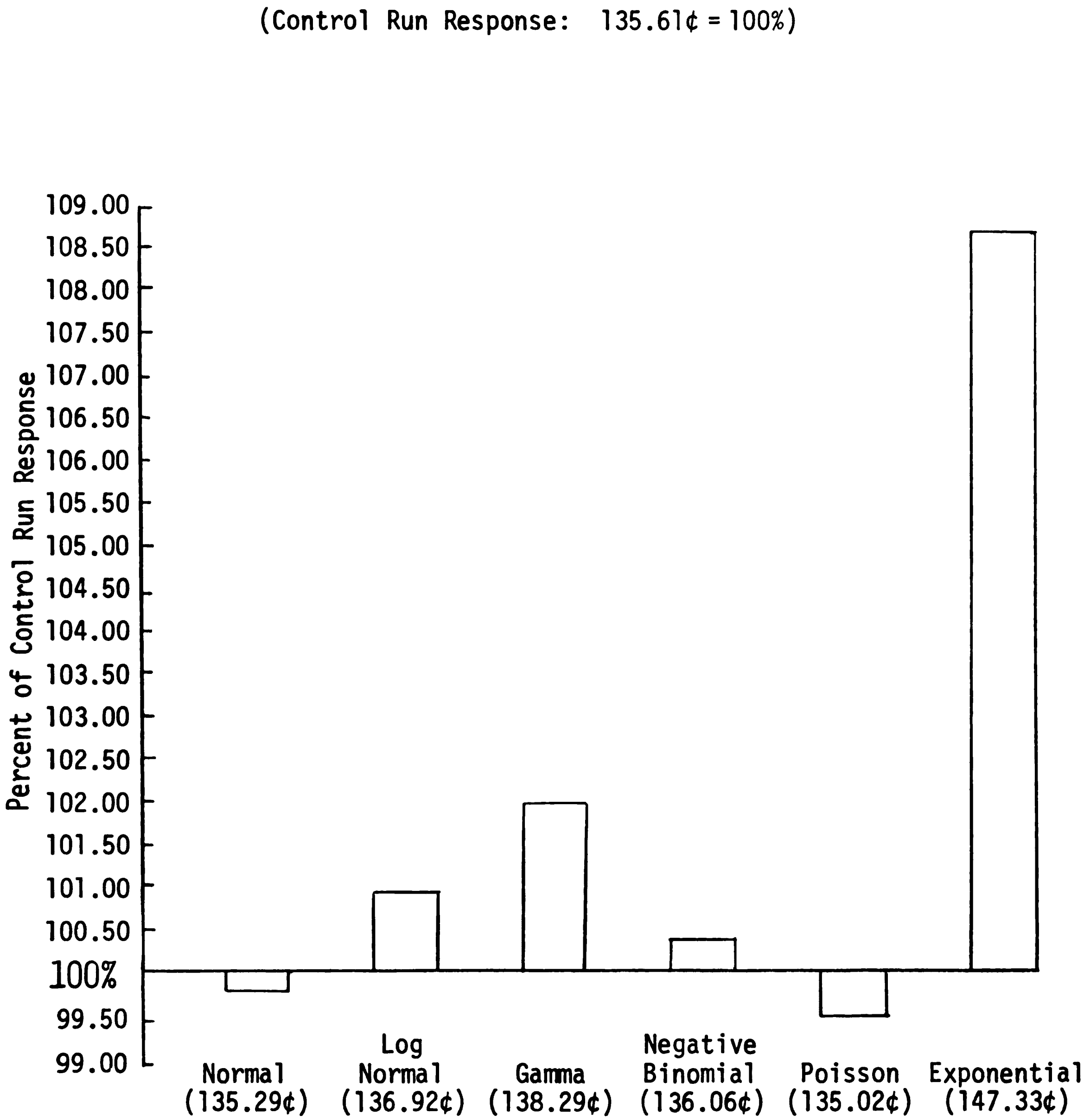

Figure 5-5. Ratio of Probability Distribution Response to the Control Run Response: Total Cost.

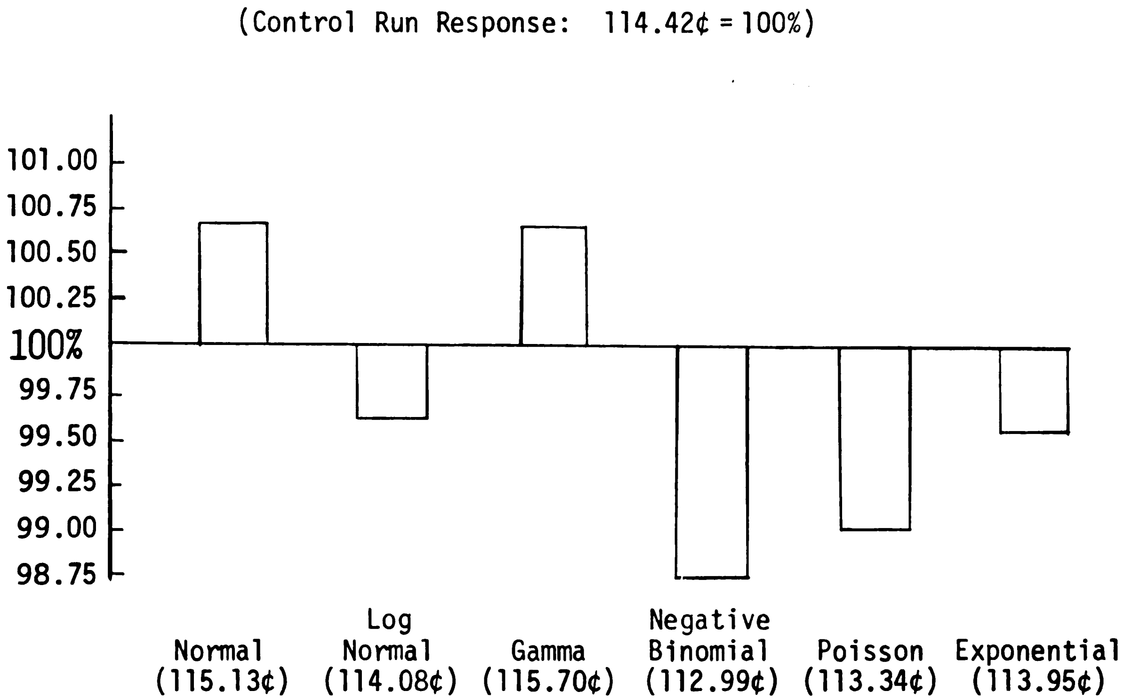

Figure 5-6. Ratio of Probability Distribution Response to the Control Run Response: Transportation Cost.

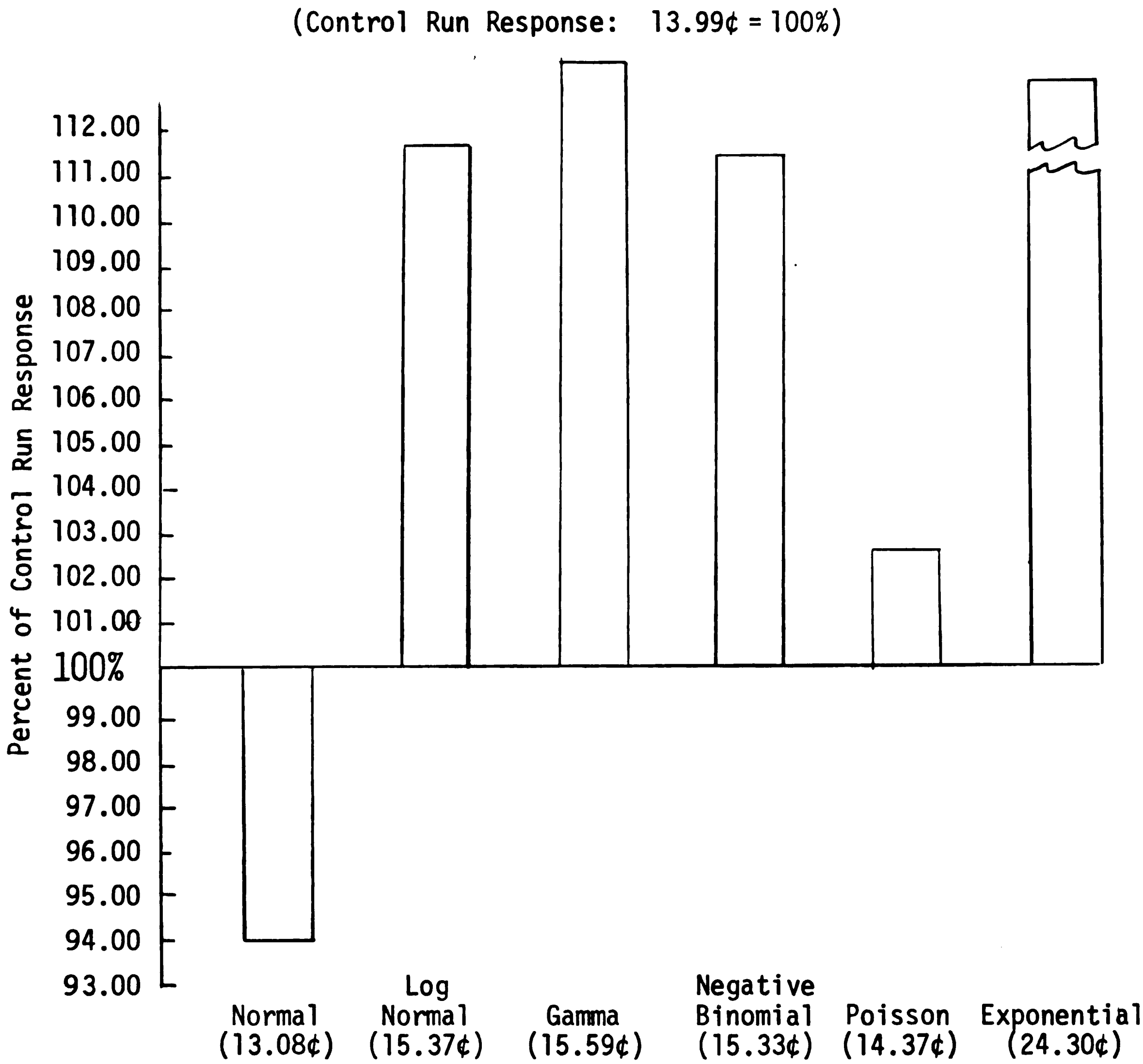

Figure 5-7. Ratio of Probability Distribution Response to the Control Run Response: Facility Cost.

Thruput cost.--The ratio of the average thruput cost for each distribution as a percent of the control run thruput cost is shown in Figure 5-8. The difference between the log normal cost and control run cost (.055) is statistically significant based on a critical Dunnett's "t" of .054. The remaining differences are not significant.

Inventory cost.--Figure 5-9 presents the ratio of average inventory cost associated with each distribution to inventory cost of the control run. There are no statistically significant differences, although the gamma distribution inventory cost difference (from control) is very close to the critical Dunnett's "t" statistic.

Summary: distribution responses vs. control.--The comparison of costs and service associated with the probability distribution of demand to the control run cost and service shows statistically significant differences in the area of service level and generally no differences in the area of total and activity center cost. The normal, negative binomial and exponential produce stockouts which are significantly different in a statistical sense than the control run stockouts.

Variance

Figure 5-10 indicates that the demand stocked out and costs which result from the variance of demand will be compared to demand stocked out and costs which result when demand is constant per unit time (the control simulation run).

(Control Run Response: 4.61¢ = 100%)

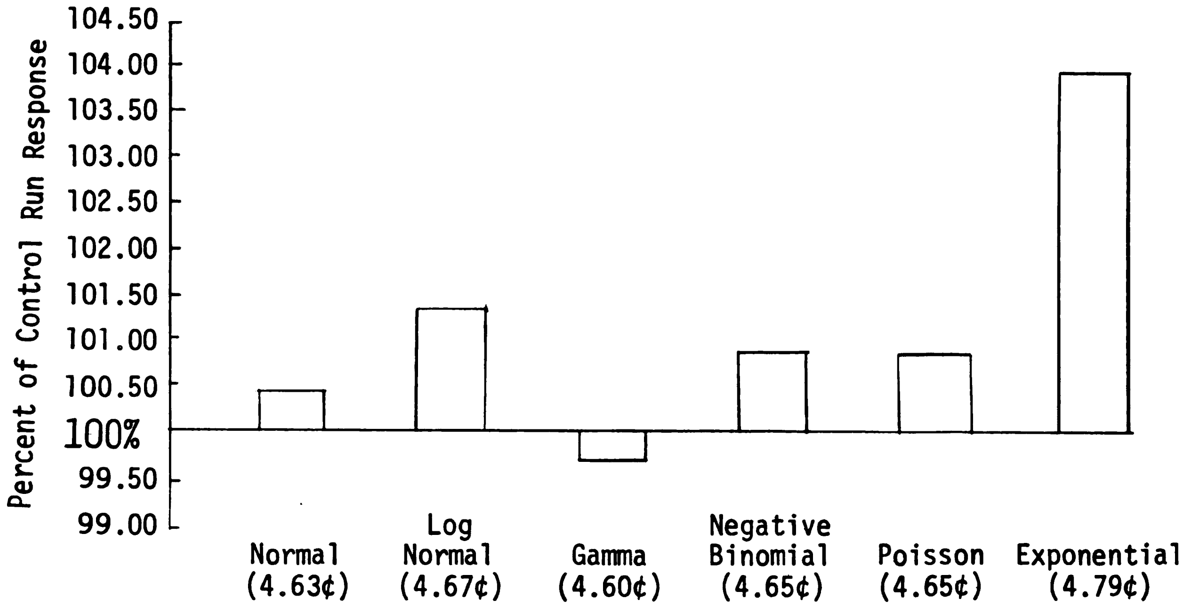

Figure 5-8. Ratio of Probability Distribution Response to the Control Run Response: Thruput Cost.

(Control Run Response: 2.59¢ = 100%)

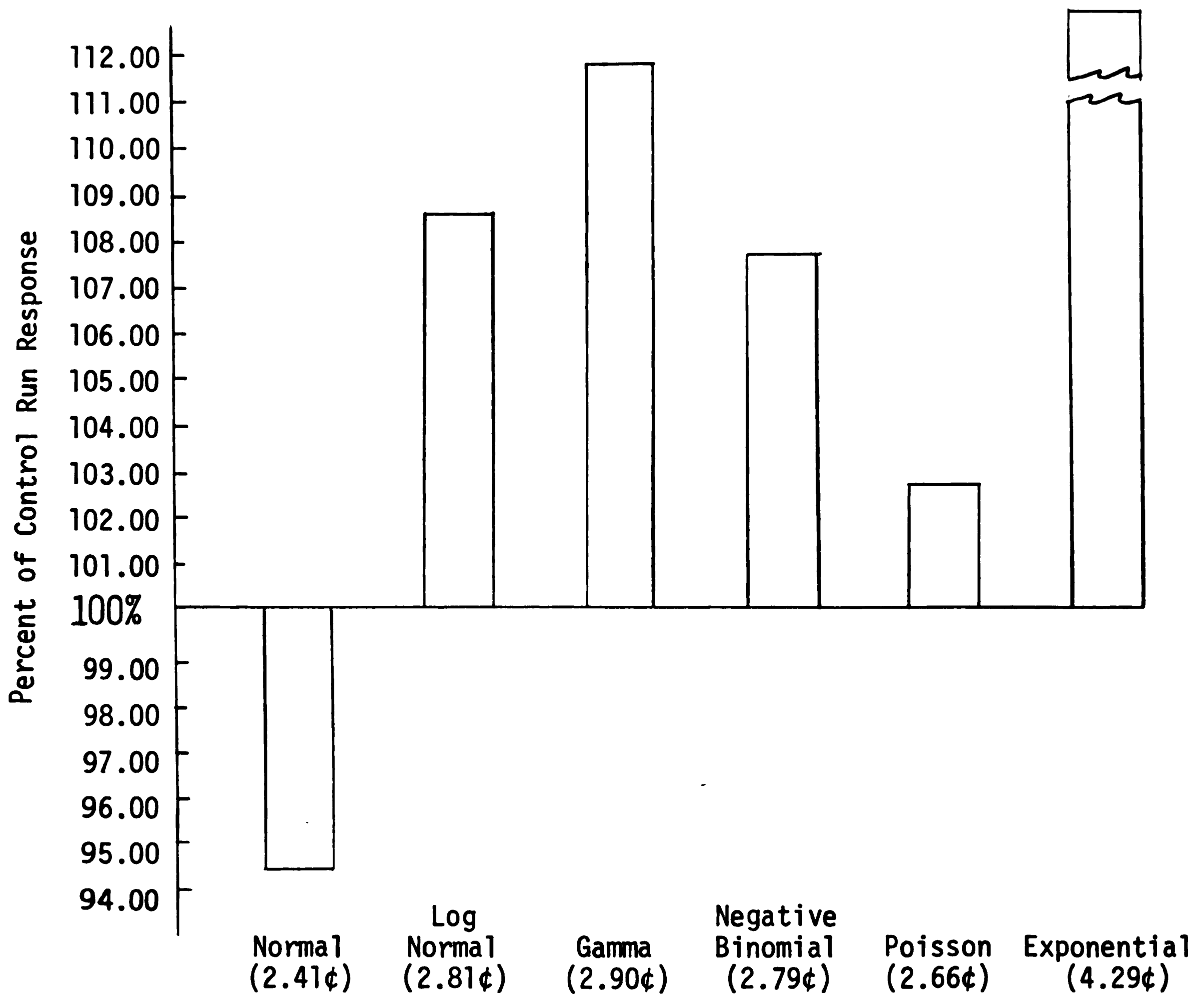

Figure 5-9. Ratio of Probability Distribution Response to the Control Run Response: Inventory Cost.

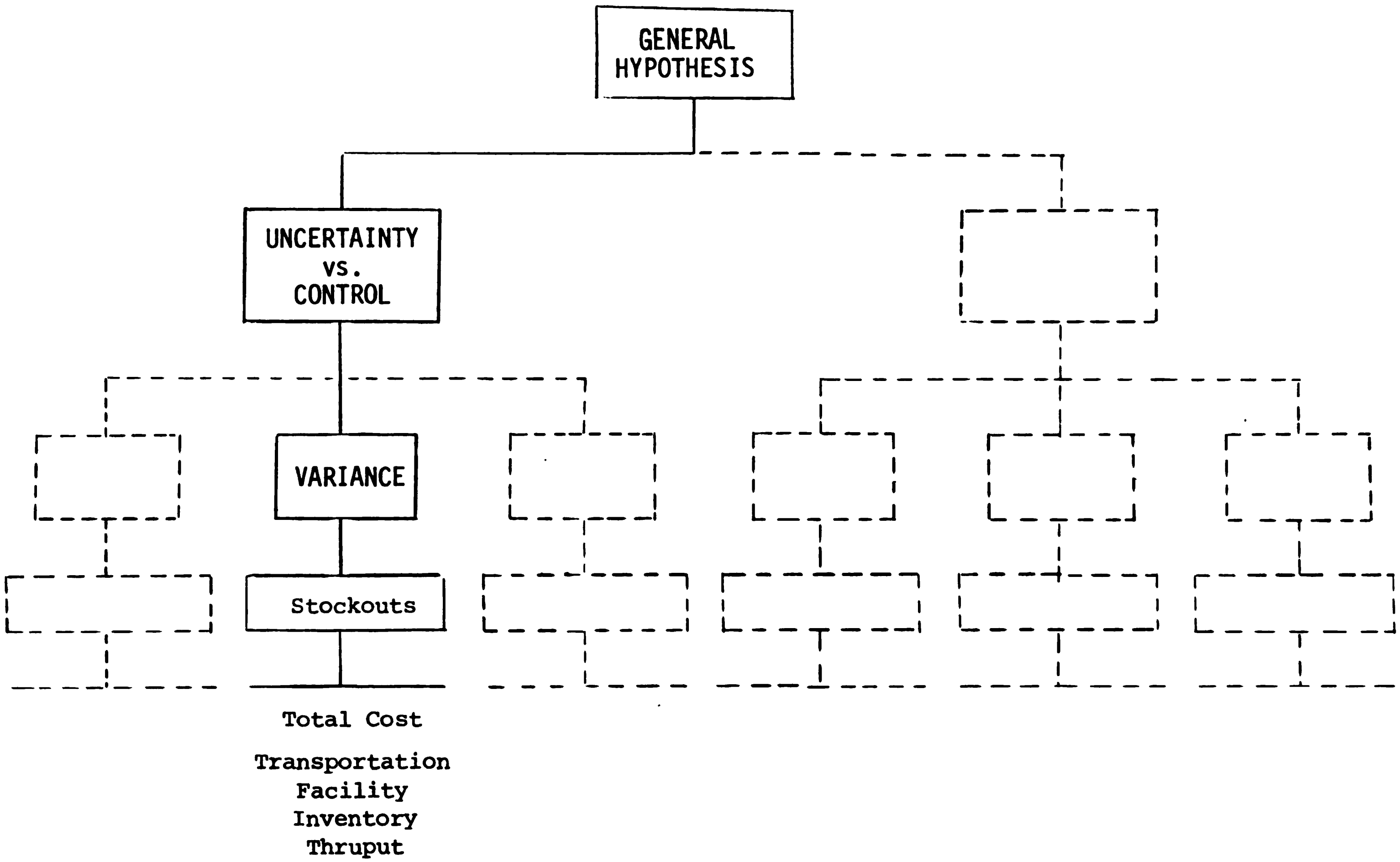

Figure 5-10. Research Analysis Organization: Control/Variance.

To assess the effect of the coefficient of variation on the performance of the physical channel system, the responses at each level of the coefficient are summed and averaged for each one. Thus, the total cost for the coefficient of variation is obtained by summing over eight responses consisting of four different probability distributions at two different levels. The sum is divided by eight to determine the average. The experimental runs with the poisson and exponential distributions are not included in these averages because their coefficients of variation are not comparable to the three experimental levels of the coefficient.

Demand stocked out.--Figure 5-11 shows the average percent of demand stocked out for each level of the coefficient of variation. The .50 coefficient of variation differs from the control run by 3.3% in terms of percent stocked out. The critical Dunnett's "t" statistic is 1.05, and thus the difference between the average stockouts at a .50 coefficient of variation and control is statistically significant. The difference between the .30 level and control is 1.03 and thus very close to being significant. Stockouts for the .10 coefficient of variation are not significantly different from the control run stockouts.

Costs.--The remaining responses associated with the three levels of variance are not statistically different than those associated with the control run. The responses associated with the three levels of variance are presented in Figures 5-12 through 5-17.

(Control Run Response = 0.0%)

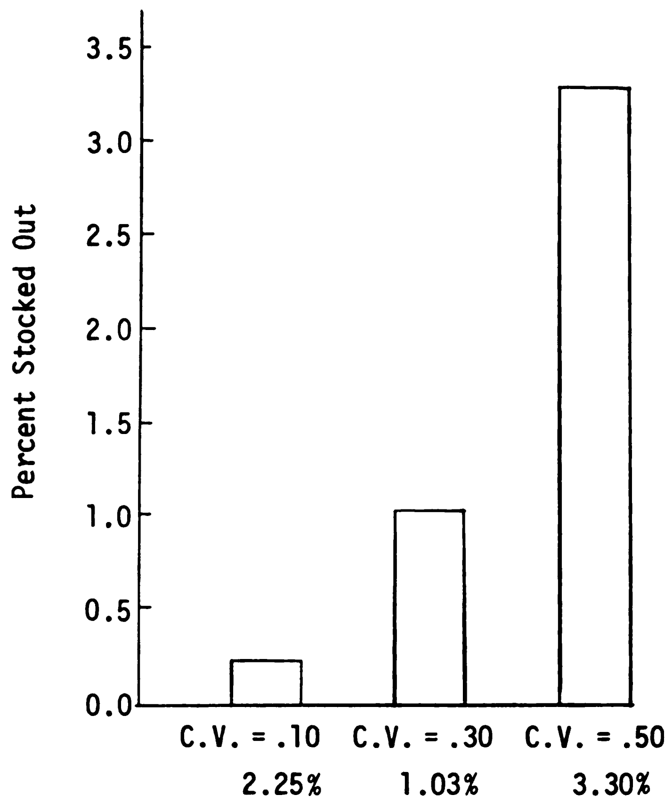

Figure 5-11. Coefficient of Variation Response Compared to Control Run Response: Percent of Demand Stocked Out.

(Control Run Response: 27.12% = 100%)

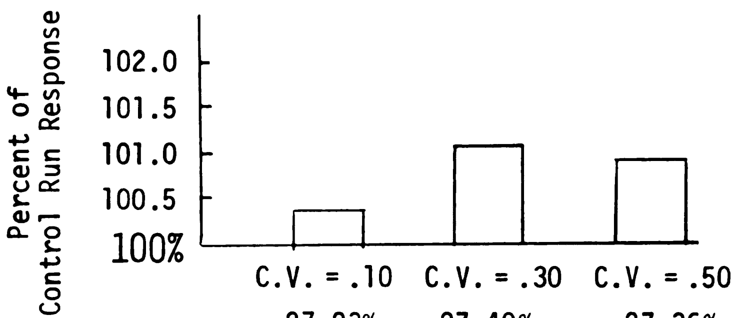

Figure 5-12. Ratio of the Coefficient of Variation Response to the Control Run Response: Cost/Revenue Ratio.

(Control Run Response: 135.61¢ = 100%)

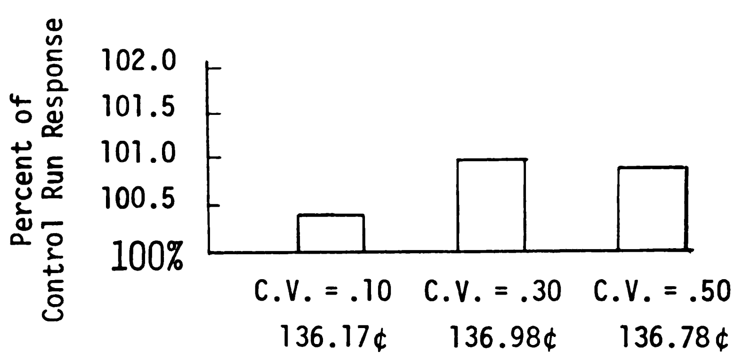

Figure 5-13. Ratio of the Coefficient of Variation Response to the Control Run Response: Total Cost.

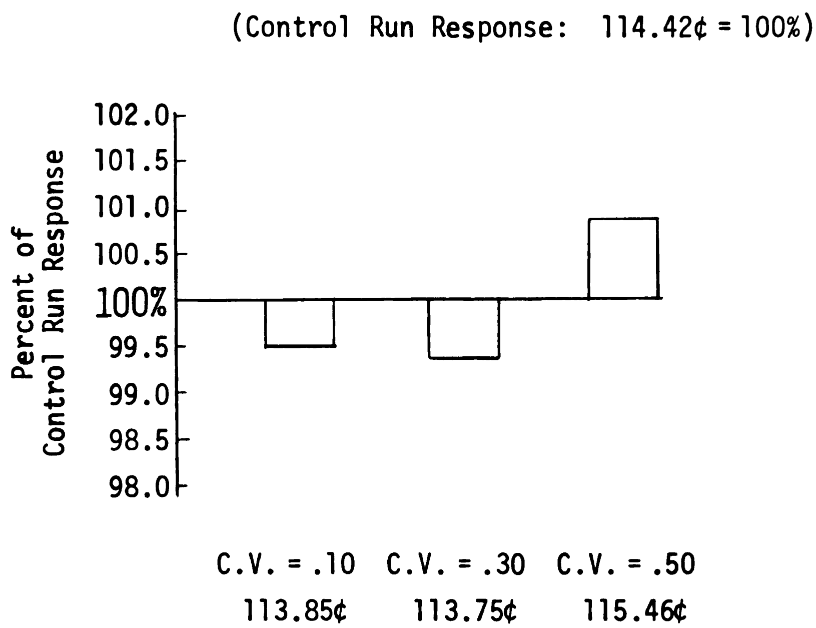

Figure 5-14. Ratio of the Coefficient of Variation Response to the Control Run Response: Transportation Cost.

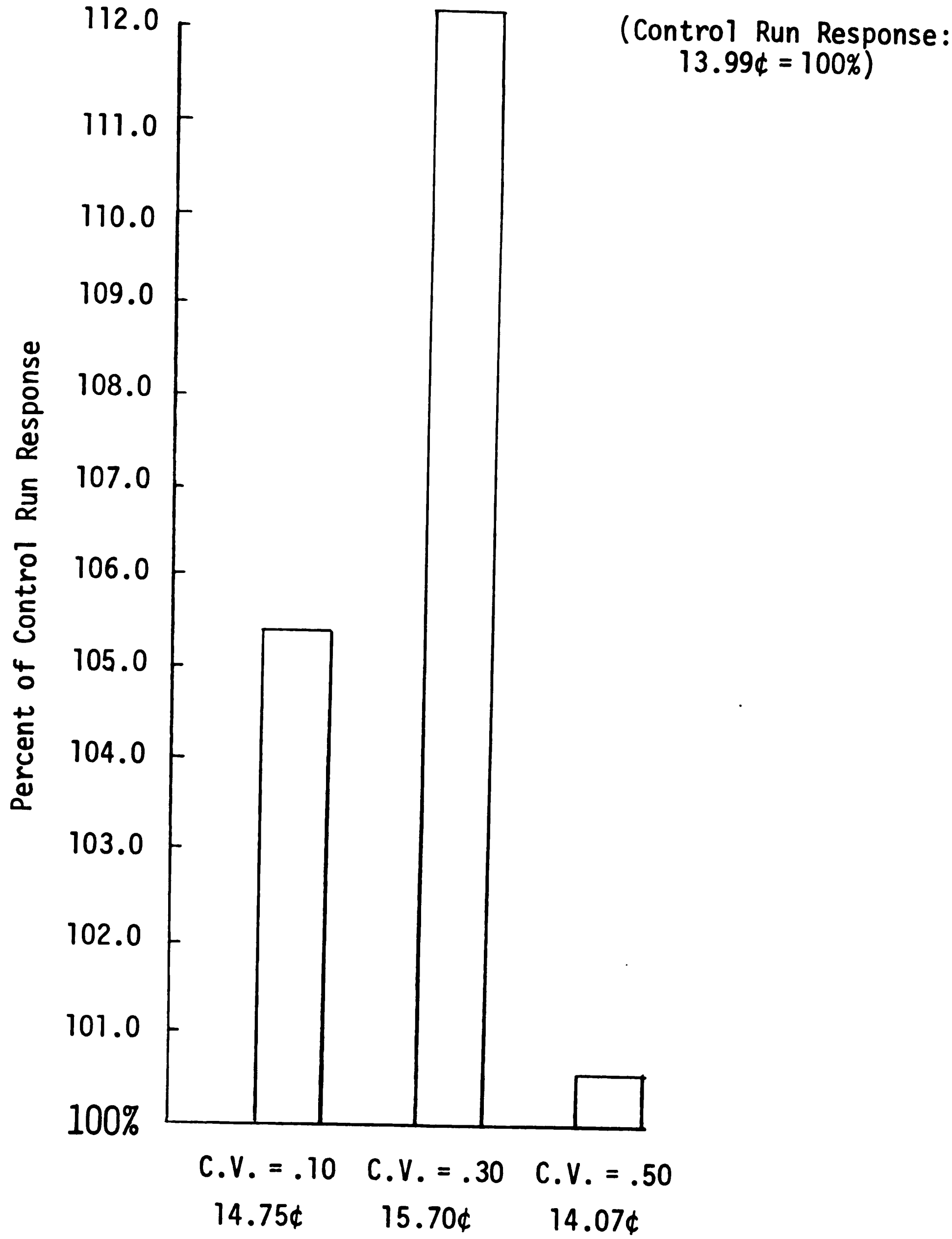

Figure 5-15. Ratio of the Coefficient of Variation Response to the Control Run Response: Facility Cost.

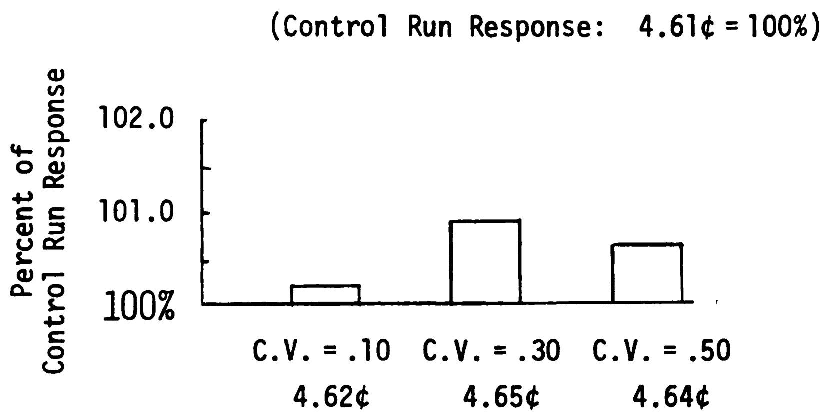

Figure 5-16. Ratio of the Coefficient of Variation Response to the Control Run Response: Thruput Cost.

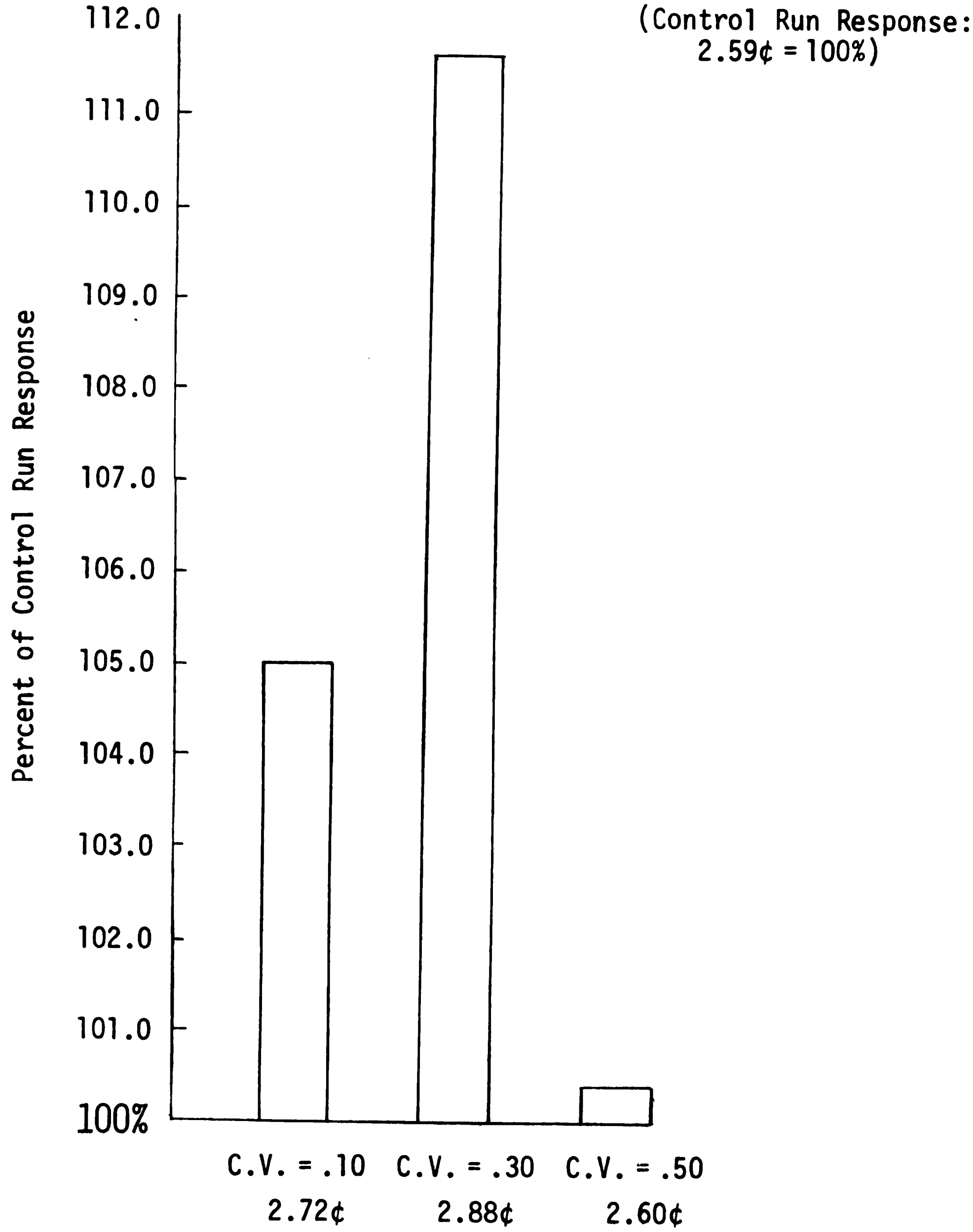

Figure 5-17. Ratio of the Coefficient of Variation Response to the Control Run Response: Inventory Cost.

Level of Demand

Figure 5-18 indicates that the demand stocked out and the costs which result from the level of demand will be compared to the demand stocked out and costs which result when demand is constant per day (the control simulation run).

Demand stocked out.--Figure 5-19 displays the average percent of demand stocked out under the twelve experimental runs made for each experimental level (25 units and 75 units) of demand. The percent stocked out at each level of demand is statistically significantly different than the control run stockouts. The critical Dunnett's "t" statistic was .692, with the difference between level 1 (25 units) and control being 1.14 and that between level 2 (75 units) and control 1.9.

Cost/revenue ratio.--Figure 5-20 presents the average cost/revenue ratio for each level as a percent of the cost/revenue ratio for the control run.

Dunnett's test gives the following results: the difference between level 1 and control (2.08) is significant (critical value is .391); the difference between level 2 and control (2.49) is also significant. Thus, both levels produce cost/revenue ratios statistically different than that associated with the control run. The 75 level generates lower cost/revenue ratios; the 25 level creates higher ratios.

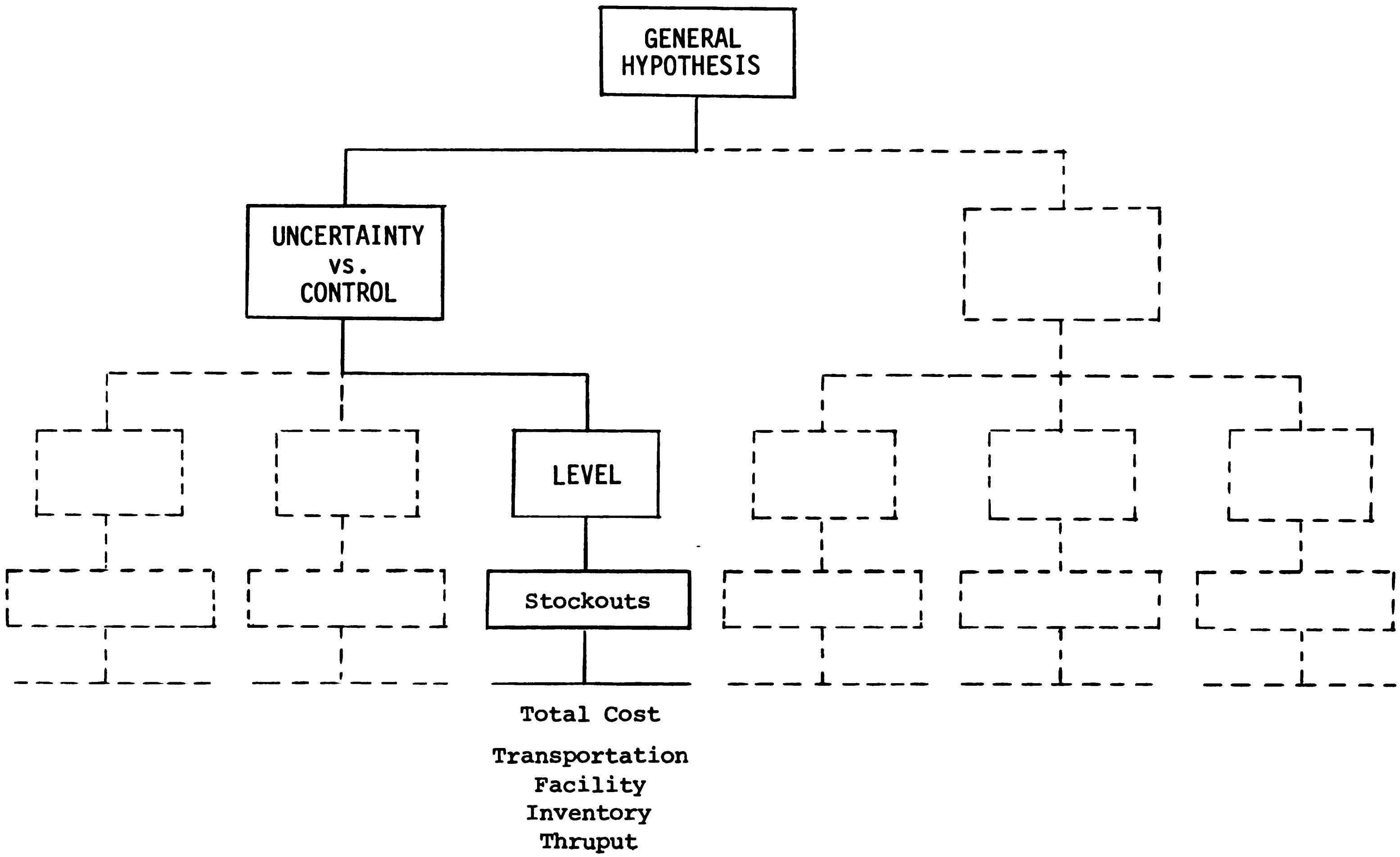

Figure 5-18. Research Analysis Organization: Control/Level.

(Control Run Response = 0.0%)

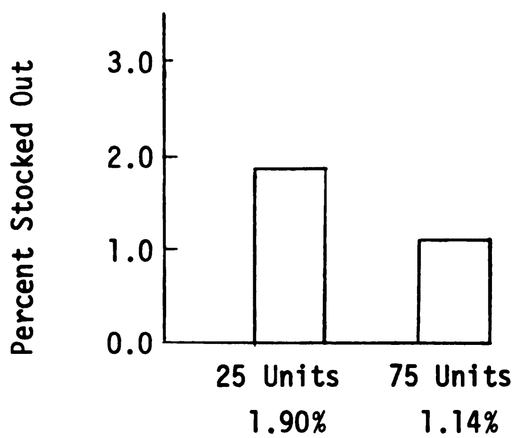

Figure 5-19. The Average Level of Demand Response Compared to the Control Run Response: Percent Demand Stocked Out.

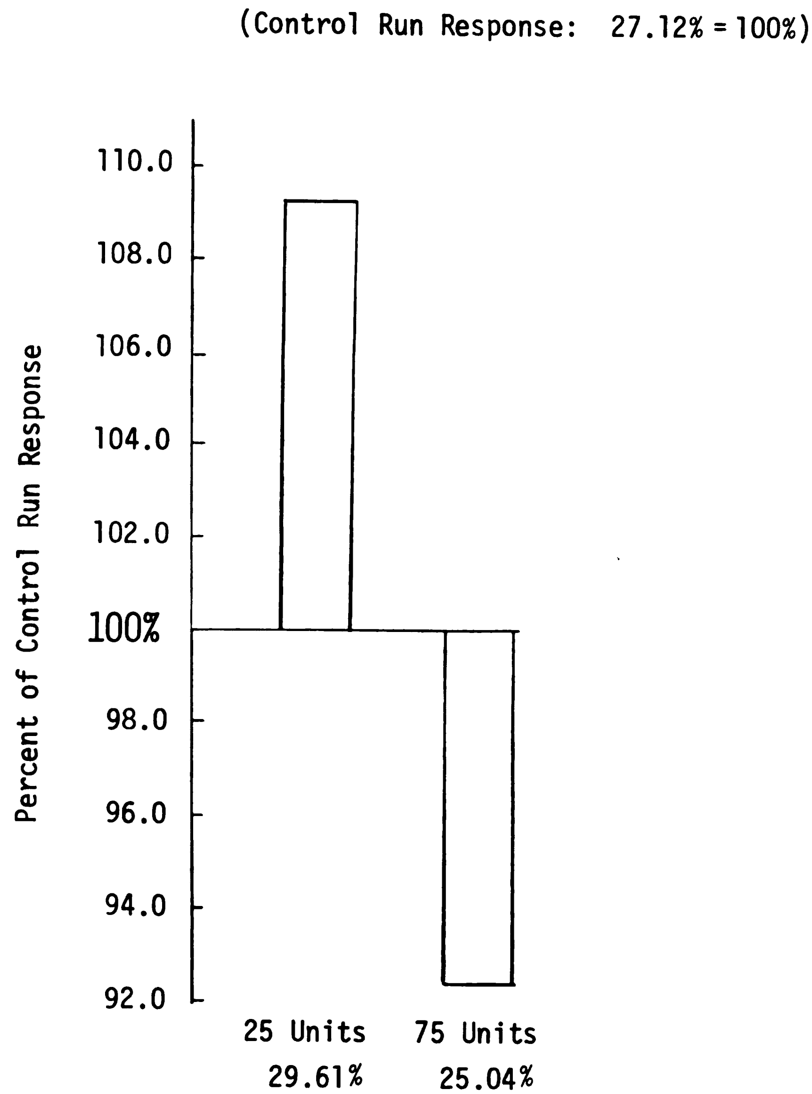

Figure 5-20. Ratio of the Average Level of Demand Response to the Control Run Response: Cost/Revenue Ratio.

Total cost.--The average total cost for each level as a percent of control run total cost appears in Figure 5-21. Again, both are significantly different than the control total cost. The difference between the control run and level 25 total cost is 12.46; the difference between level 75 and control is 10.40. The critical Dunnett's "t" is 1.74 in both cases. The level 25 total cost is higher than control run total cost, the level 75 total cost is lower.

Transportation cost.--Figure 5-22 reveals the ratio of average transportation cost by level to control run transportation cost. For both levels, the difference between transportation cost and control transportation cost is significant. For level 25 versus control the difference is 8.09, and for level 75 versus control, the difference is 8.06. The critical Dunnett's "t" is 1.10. Again, level 25 cost is higher than control, level 75 is lower.

Facility cost.--Figure 5-23 depicts the average facility cost for both levels as a percent of control facility cost. The critical Dunnett's "t" for facility cost is 1.42, and the actual differences for level 25 and control is 3.79 and 2.08 for level 75 and control. Level 25 produces the higher facility cost.

Thruput cost.--Figure 5-24 represents the average thruput cost by level as a percent of control. There are no statistically significant differences.

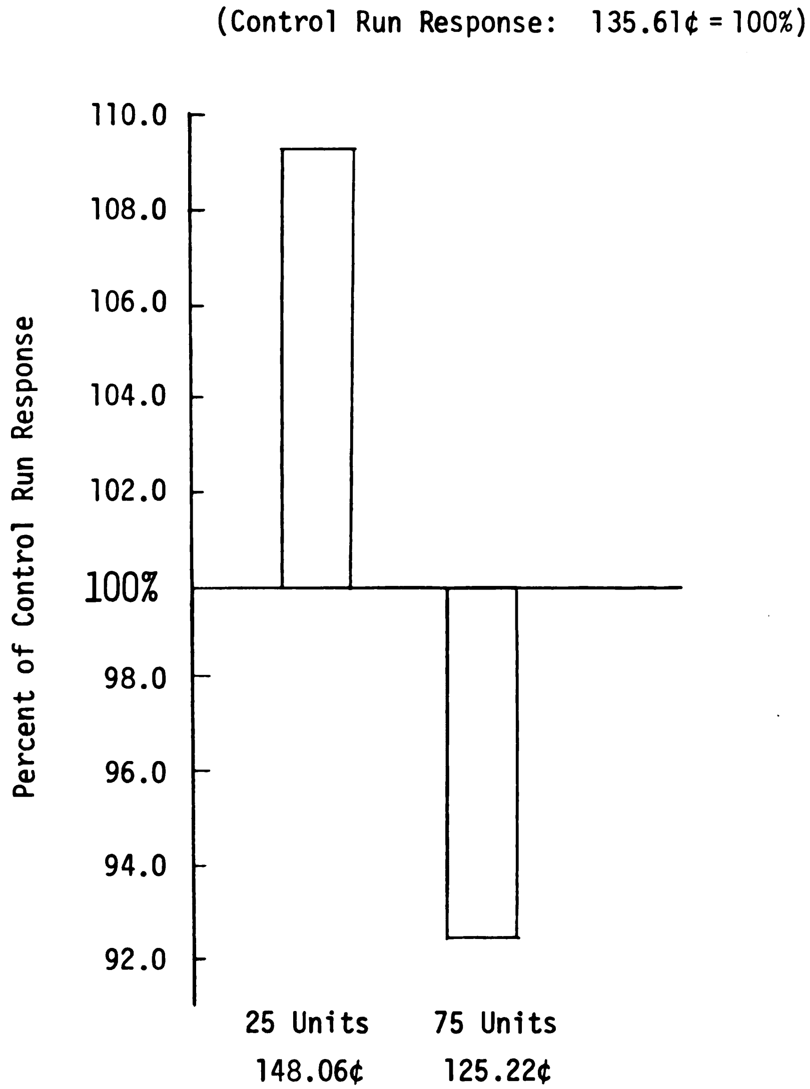

Figure 5-21. Ratio of the Average Level of Demand Response to the Control Run Response: Total Cost.

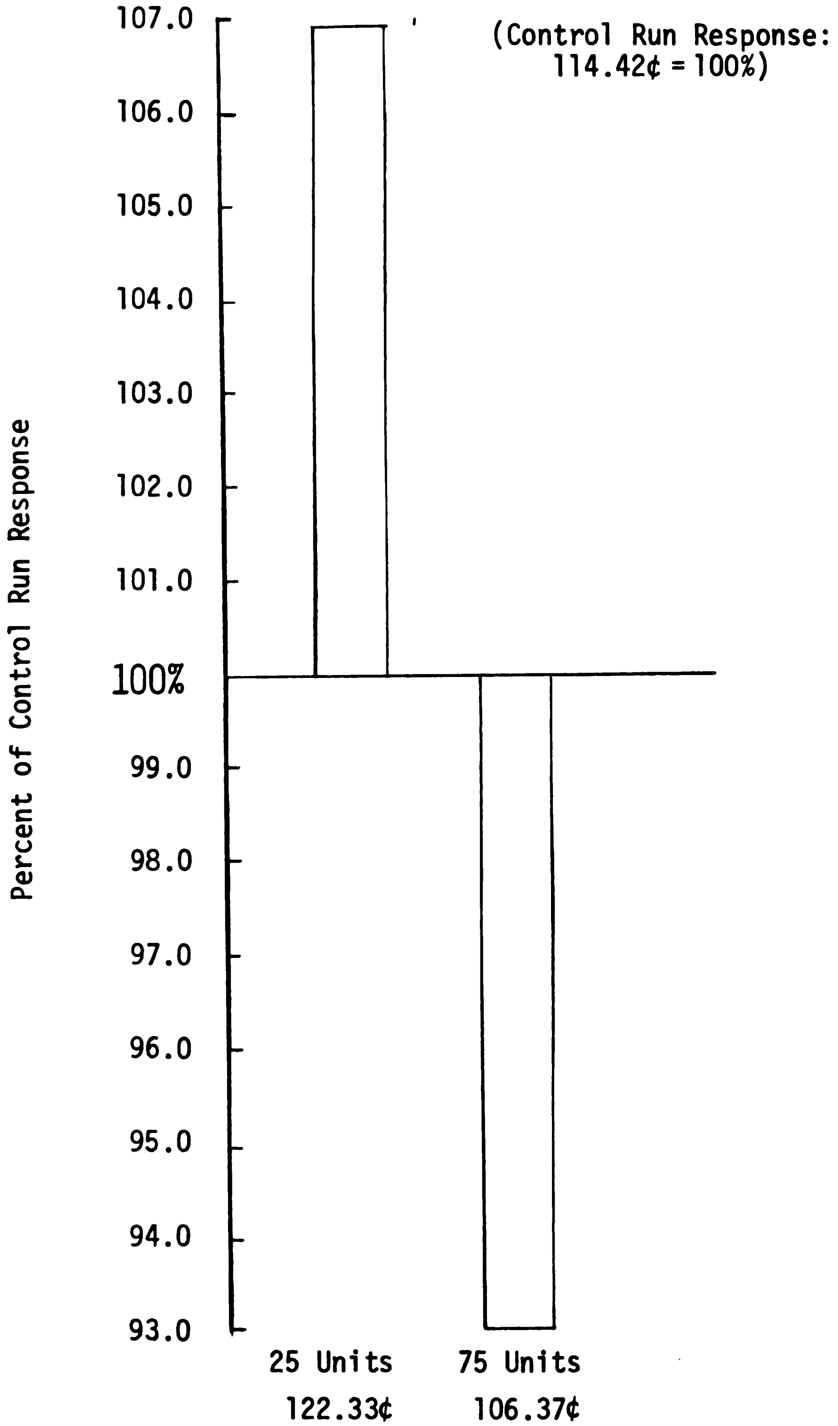

Figure 5-22. Ratio of the Average Level of Demand Response to the Control Run Response: Transportation Cost.

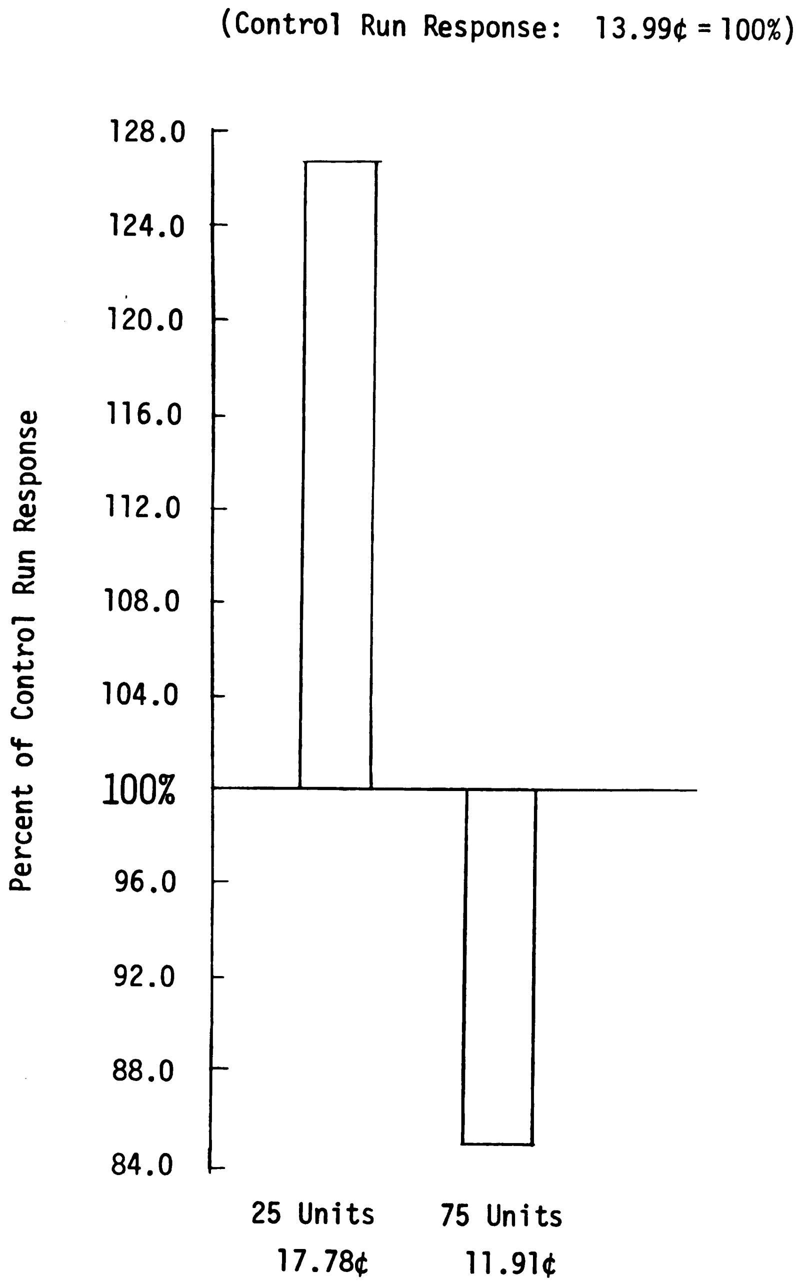

Figure 5-23. Ratio of the Average Level of Demand Response to the Control Run Response: Facility Cost.

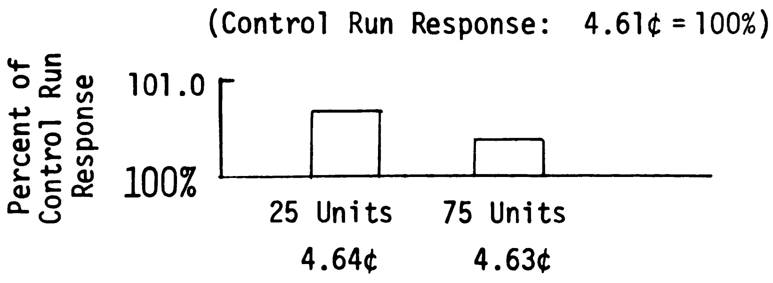

Figure 5-24. Ratio of the Average Level of Demand Response to the Control Run Response: Thruput Cost.

Inventory cost.--The ratio of average inventory cost associated with each level to control run inventory cost is developed in Figure 5-25. Both levels are significantly different than the control inventory cost. For the 25 level, the difference from control is .65 and for the 75 level, the difference is .36. The critical Dunnett's "t" statistic is .234.

Summary: Level of demand vs. control.--In all cases but thruput cost, the response variables are statistically different than the control response variables. In each situation the 75 level costs are lower than control and the 25 level are higher. For both levels of demand, the stockout percentage is greater than the control stockouts. It should also be noted that the runs with the poisson and exponential distributions were not included in determining the average cost due to the level because of the noncomparability between them and the remaining four distributions.

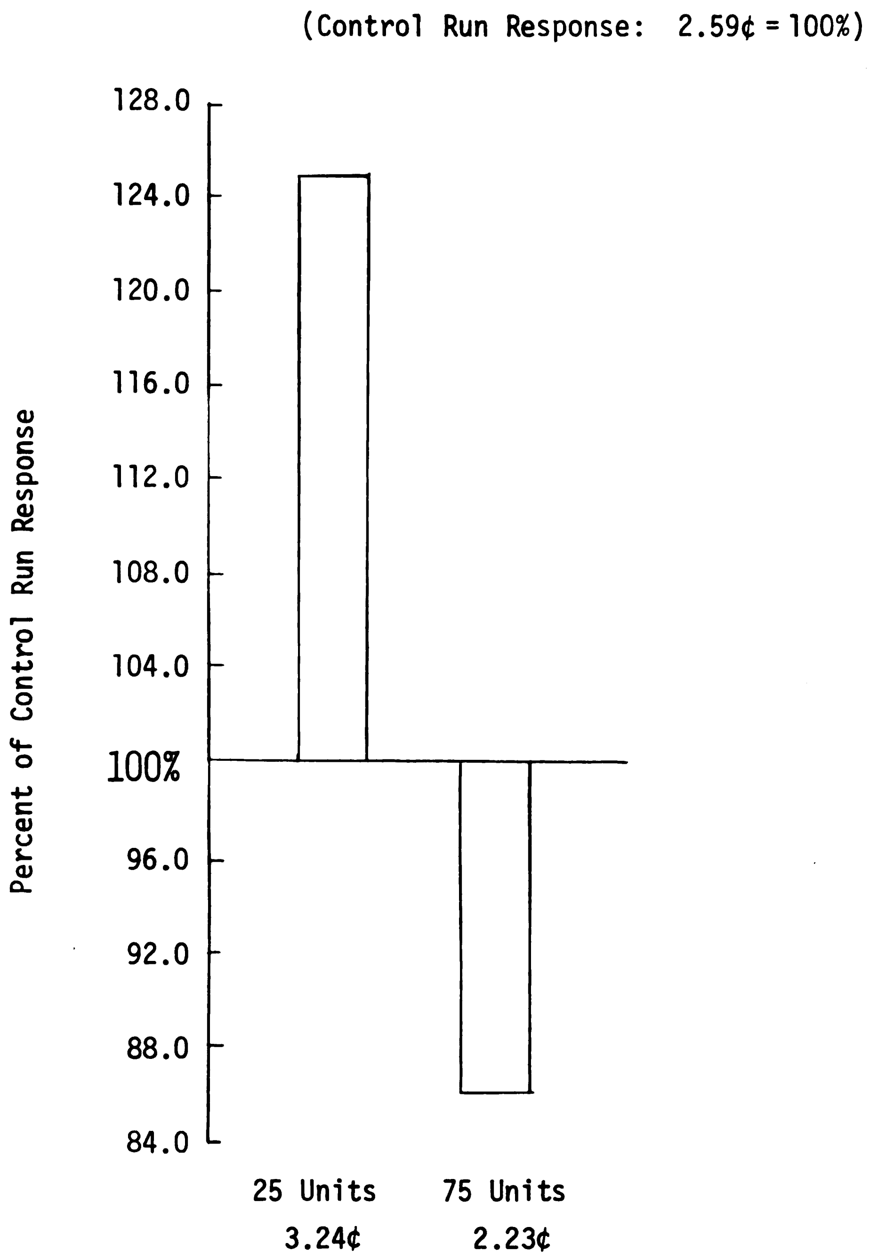

Figure 5-25. Ratio of the Average Level of Demand Response to the Control Run Response: Inventory Cost.

Comments: Average Response Comparisons

It must be noted that the results obtained for the poisson and exponential distributions are based upon only two experimental runs each. Thus, in comparing the average responses for each distribution against the control run response, there was only one degree of freedom and a large amount of variance between the two observations. Therefore, a number of large differences between the exponential and control run are considered to be no different in a statistical sense. These differences will be considered in the individual response comparison section.

Comparison of Factor and Control Responses: Individual Cell Comparisons

The experimental findings reported in the chapter thus far have dealt with the average response (cost, service) associated with a particular factor, i.e., the total cost associated with level 25 of demand. The average in this case was determined by summing the total cost for each experimental run (12) in which level 25 was used. This average was then compared to the control run total cost and the difference compared to the critical value of Dunnett's "t" statistic. However, there are a host of experimental findings of some importance which relate to only a single experimental run, i.e., the total cost associated with the normal distribution at level 25 of demand and a coefficient of variation of .50. Such responses represent individual cells within the factorial matrix. Although no statistical conclusions may be drawn from the comparison of individual cell results to the control run results, it is worthwhile to examine some of the cases in which large absolute differences occur. Additionally, comparisons using averages tend to mask valuable information and thus individual cell comparisons are a worthwhile exercise. Such comparisons may serve to indicate the direction that future research should take and suggest testable hypotheses.

Demand Stocked Out

Table 5-2 represents the individual results by cell in terms of demand stocked out. In comparison to the control run with no stockouts, the normal distribution at the highest level of variance, produces the highest percentage of stockouts within the group of distributions run at three levels of variance. Additionally, the normal distribution at the low level of demand, consistently has more stocked out (.1, 1.72, 4.96 percent stockout, respectively) than at the higher level of demand with the exception of C.V. equals .50, and more than the remaining distributions at any level of average demand and coefficient of variation. Overall, the exponential creates the greatest stockout proportion. Surprisingly, the exponential is the only distribution for which the stockout proportion was at the higher demand level than at the lower demand level. Finally, across all distributions, the combination of high level of the coefficient of variation (.50) and low average demand (25) creates the largest stockout ratio relative to the control run.

Table 5-2. Individual Cell Comparisons: Percent Demand Stocked Out

Coefficient of Variation	Control[a]		Normal		Log Normal		Gamma		Negative Binomial		Poisson[b]		Exponential[c]	
	Level		Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75	25	75
.10			0.11	0.0	0.13	0.0	0.35	0.0	0.70	0.50				
.30	0.0	0.0	1.72	0.92	1.51	0.10	1.45	0.18	1.65	0.74	1.0	0.0	6.96	7.76
.50			4.96	5.64	3.63	1.00	3.17	1.95	3.42	2.61				

[a]Not applicable. [b]$\sigma = \sqrt{\mu}$. [c]$\sigma = \mu$.

Cost/Revenue Ratio

The cell by cell comparisons to control are developed in Table 5-3. The cost/revenue ratio remains relatively constant across cells in the matrix. The control and cell ratios are very close in most cases. However, the exponential does vary from the control by more than two percentage points at both levels.

Total Cost

The response matrix for total per unit channel system costs are developed in Table 5-4. There is some divergence of individual cell total costs from that of the control run total costs. In terms of the coefficient of variation, the higher coefficient does not always produce higher costs than the control, as is indicated by the normal at C.V. equals .10 and .50 and the negative binomial at C.V. equals .50 (each at level of demand 25). However, when the level of demand is 75, the higher coefficient of variation always produces total costs greater than the control. In fact, at the high level of demand, the total cost across all distributions and all levels of the coefficient of variation is higher than the control run total cost (at 75). Of some interest is the fact that the gamma distribution at low coefficient of variation (.10) and low demand level (25) produces a total cost (153.4) much larger than the control run total cost (147.4) at the 25 level. Finally, the exponential at both levels of demand results in total costs much greater than control run total costs, especially at the 75 level of demand (136.02 vs. 123.79).

Table 5-3. Individual Cell Comparisons: Percent Cost/Revenue Ratio (%)

Coefficient of Variation	Control[a]		Normal		Log Normal		Gamma		Negative Binomial		Poisson[b]		Exponential[c]	
	Level		Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75	25	75
.10			28.98	24.87	29.11	24.89	30.68	25.02	29.48	24.82				
.30	29.49	24.76	29.35	25.22	29.34	25.11	29.78	25.61	29.95	24.80	29.22	24.79	31.73	27.20
.50			29.02	24.92	30.55	25.30	29.89	24.98	29.23	24.97				

[a]Not applicable.

[b]$\sigma = \sqrt{\mu}$.

[c]$\sigma = \mu$.

Table 5-4. Individual Cell Comparisons: Total Cost (¢/Unit)

Coefficient of Variation	Control[a]		Normal		Log Normal		Gamma		Negative Binomial		Poisson[b]		Exponential[c]	
	Level		Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	84	93	75	25	75	25	75
.10			144.88	124.36	145.57	124.47	153.38	125.12	147.42	124.12				
.30	147.43	123.79	146.77	126.08	146.72	125.55	148.90	128.03	149.76	124.02	146.11	123.93	158.63	136.02
.50			145.09	124.58	152.74	126.50	149.44	124.89	146.14	124.86				

[a]Not applicable.

[b]$\sigma = \sqrt{\mu}$.

[c]$\sigma = \mu$.

Transportation Cost

Table 5-5 presents the individual cell comparisons of transport costs per unit. Comparing individual cells to the control, the gamma distribution at level 25 and C.V. equals .10 is the only distribution at that average level and coefficient of variation for which costs exceed the control run cost (124.91 vs. 123.33). However, within that same level (25), the transport cost associated with the gamma more closely approximates control run costs (especially at high levels of the coefficient of variation). The negative binomial produces transport costs lower than the control run costs for every level of demand and coefficient of variation.

The general trend in the cells is that control run transportation costs and those associated with low demand levels (25) are very close, whereas the transport costs associated with high demand levels (75) are substantially smaller than control run costs.

Facility Cost

Table 5-6 shows the individual cell comparisons for facility costs and the control run facility cost. The level of demand appears to produce a number of variations from control run facility cost, especially when considered with distributional influence. Level 25 demand creates costs substantially higher than control run costs in association with the normal distribution at C.V. equals .30 (17.6), log normal at C.V. equals .50 (20.6), gamma at C.V. equals .10 (20.2), negative binomial at C.V. equals .30 (19.9) and the exponential (26.8). On the other hand, facility costs at the higher demand level (75) bear a more consistent relationship to control run facility costs.

Table 5-5. Individual Cell Comparisons: Transportation Cost (¢/Unit)

Coefficient of Variation	Control[a]		Normal		Log Normal		Gamma		Negative Binomial		Poisson[b]		Exponential[c]	
	Level		Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75	25	75
.10			120.93	106.12	121.24	106.24	124.91	106.62	120.38	104.35				
.30	123.33	125.50	121.28	107.07	120.75	106.63	123.57	104.29	121.58	104.77	121.00	105.68	122.29	105.61
.50			124.44	110.95	123.71	105.88	124.01	107.79	121.20	105.71				

Table 5-6. Individual Cell Comparisons: Facility Cost (¢/Unit)

Coefficient of Variation	Control[a]		Normal		Log Normal		Gamma		Negative Binomial		Poisson[b]		Exponential[c]	
	Level		Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75	25	75
.10			16.31	11.45	16.62	11.46	20.15	11.65	18.28	12.11				
.30	16.48	11.49	17.59	12.10	17.99	12.04	17.54	16.07	19.93	12.31	17.28	11.46	26.77	21.82
.50			13.51	7.53	20.64	13.46	17.63	10.48	17.11	12.23				

[a] Not applicable. [b] $\sigma = \sqrt{\mu}$. [c] $\sigma = \mu$.

No one level of the coefficient of variation seems to produce costs consistently higher or lower than the control run costs. In general, there does not appear to be any recognizable trend in the individual cell comparisons to the control run facility costs. However, the exponential distribution has the greatest divergence from control run costs at both levels of demand.

Thruput Cost

Individual cell thruput costs are depicted in Table 5-7. There does not appear to be any substantial variation in thruput costs vs. control run costs.

Inventory Cost

Table 5-8 shows the individual inventory costs compared to control run costs. The greatest divergence from control costs occurs with the exponential distribution at the 25 level (4.67 vs. 3.00) and 75 level (3.90 vs. 2.18). Another noteworthy difference is the fact that inventory costs with a normal distribution (level 25, C.V. equals .50) are less than control run costs, as are the costs with normal (level 75, C.V. equals .50). A similar situation occurs with the gamma at level 75 and C.V. equals .50 where inventory costs (2.02) are less than control run costs (2.18). Finally, inventory costs in relation to control inventory costs appear very stable across all distributions at level 75 and C.V. equals .10 and level 75, C.V. equals .30.

Table 5-7. Individual Cell Comparisons: Thruput Cost (¢/Unit)

Coefficient of Variation	Control[a] Level 25	Control[a] Level 75	Normal Level 25	Normal Level 75	Log Normal Level 25	Log Normal Level 75	Gamma Level 25	Gamma Level 75	Negative Binomial Level 25	Negative Binomial Level 75	Poisson[b] Level 25	Poisson[b] Level 75	Exponential[c] Level 25	Exponential[c] Level 75
.10			4.63	4.63	4.64	4.62	4.57	4.64	4.63	4.64				
.30	4.61	4.62	4.69	4.66	4.71	4.64	4.59	4.64	4.66	4.64	4.68	4.63	4.89	4.69
.50			4.57	4.60	4.72	4.67	4.59	4.60	4.71	4.63				

[a] Not applicable

[b] $\sigma = \sqrt{\mu}$.

[c] $\sigma = \mu$.

Table 5-8. Individual Cell Comparisons: Inventory Cost (¢/Unit)

Coefficient of Variation	Control[a]		Normal		Log Normal		Gamma		Negative Binomial		Poisson[b]		Exponential[c]	
	Level		Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75	25	75
.10			3.02	2.15	3.06	2.15	3.75	2.21	3.28	2.17				
.30	3.00	2.18	3.19	2.23	3.26	2.23	3.20	3.03	3.57	2.31	3.15	2.17	4.67	3.90
.50			2.56	1.51	3.66	2.48	3.20	2.02	3.12	2.29				

[a] Not applicable

[b] $\sigma = \sqrt{\mu}$.

[c] $\sigma = \mu$.

Comparison Among Factor Responses: Introduction

The previous section of this chapter was concerned with assessing the impact of demand uncertainties on a physical channel system and comparing the performance of that system under conditions of demand uncertainty to the performance when demand was certain, i.e., constant per unit time. Thus, the findings presented in that section of the chapter relate to answering questions about the general impact of uncertainty on channel performance. Another important area of investigation is that of assessing the relative impacts of demand uncertainty among the various categories of uncertainty that may prevail.

Therefore, this section of the experimental results details those findings which are addressed to measuring the impact on channel performance of one form of demand uncertainty versus another form. We are thus attempting to answer the question: "Given that we have uncertainty, does one form (distribution, level, variance) create channel performance different than any other form?"

This section is presented in three parts. The first part relates the results of the analysis of variance using the F test. The purpose of this procedure is to determine the relative impact of probability distributions, levels and variances on the cost and service performance of the channel system. The F test is performed at the .05 level of significance. The F test is conducted for the four activity center costs, total cost, cost/revenue ratio and demand stocked out.

The poisson and exponential are not included in this analysis because they are not evaluated at all levels of each experimental factor.

Secondly, average response comparisons are made with each experimental category using Tukey's multiple comparison technique. For example, the results of a test of the difference between the normal distribution (six observations) average cost and log normal distribution (six observations) average cost are made with this technique. Again, the level of significance is .05.

Average response comparisons between the poisson and exponential distributions and all other distributions are made with the standard "t" test due to the noncomparability of these distributions with the remaining four distributions. The level of significance is .05. Finally, individual cell responses are compared on a nonstatistical basis.

Comparison Among Factor Responses: Analysis of Variance

Demand Stocked Out

Table 5-9 presents the analysis of variance table for demand stocked out.

The F ratio for both levels and variances is significant, indicating that both the level of demand and the variance around demand do have an effect on the percent of demand stocked out.

Table 5-9. Analysis of Variance: Demand Stocked Out (%)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	4.97	3	1.65	2.60	3.20
Levels	3.50	1	3.50	5.43*	4.45
Variances	40.60	2	20.30	31.48*	3.59
Error	10.96	17	0.65		

*Significant.

Cost/Revenue Ratio

Table 5-10 shows the analysis of variance for the cost/revenue ratio. The F value for the levels of demand is statistically significant, indicating that the level of average demand per unit time has an influence on the cost/revenue ratio. Neither distribution nor variances display a significant F ratio, and thus these two experimental factors do not have an effect on the cost/revenue ratio.

Table 5-10. Analysis of Variance: Cost/Revenue Ratio (%)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	1.20	3	0.40	1.94	3.20
Levels	125.30	1	125.30	608.50*	4.45
Variances	0.10	2	0.05	0.24	3.59
Error	3.50	17	0.21		

*Significant.

Total Cost

The analysis of variance table for total cost appears in Table 5-11. Distributions have no effect on total cost as is indicated by the noncritical F ratio. The F test also indicates no effect due to variances. However, the F ratio associated with levels is significant, thereby indicating that different levels of demand are associated with different levels of total cost.

Table 5-11. Analysis of Variance: Total Cost (¢/Unit)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	30.80	3	10.30	2.54	3.20
Levels	3133.01	1	3133.01	771.90*	4.45
Variances	4.07	2	2.03	0.49	3.59
Error	69.00	17	4.06		

*Significant.

Transportation Cost

Table 5-12 presents the analysis of variance for transportation cost. As the table shows, all experimental values have an effect on transport cost. Thus, there is a difference between distributions in terms of the total cost incurred when each distribution creates demand. The same conclusion holds true for levels and variances of demand.

Table 5.12. Analysis of Variance: Transportation Cost (¢/Unit)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	19.39	3	6.47	3.98*	3.20
Levels	1530.66	1	1530.66	943.27*	4.45
Variances	15.25	2	7.63	4.70*	3.59
Error	27.59	17	1.63		

*Significant.

Facility Cost

The facility cost analysis of variance table is displayed in Table 5-13. Distributions and variance F ratios are not significant, i.e., there is no effect on cost due to either of these factors. The level of demand does have an impact on facility cost as is shown by the F ratio of 76.38.

Table 5-13. Analysis of Variance: Facility Cost (¢/Unit)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	25.00	3	8.33	3.07	3.20
Levels	207.00	1	207.00	76.38*	4.45
Variances	11.00	2	5.50	2.03	3.59
Error	46.00	17	2.71		

*Significant.

Thruput Cost

Table 5-14 shows the analysis of variance table for thruput cost. In this case, distributions of demand are the only experimental factors whose F ratio is significant. Thus, the type of demand distribution has an effect on the thruput cost of the channel.

Table 5-14. Analysis of Variance: Thruput Cost (¢/Unit)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	.014	3	.0047	3.78*	3.20
Levels	.002	1	.0020	1.62	4.45
Variances	.004	2	.0020	1.62	3.59
Error	.021	17	.0012		

*Significant.

Inventory Cost

The analysis of variance table for inventory cost is shown in Table 5-15. Both distributions and levels of demand have F ratios above the critical value. Thus, the type of demand distribution and the level of demand have effects on inventory costs.

Table 5-15. Analysis of Variance: Inventory Cost (¢/Unit)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	.763	3	.254	3.43*	3.20
Levels	6.150	1	6.150	83.00*	4.45
Variances	.348	2	.174	2.35	3.59
Error	1.260	17	.074		

*Significant.

Summary

In general, the level of demand is the experimental factor which affected the cost and service performance of the channel system. A significant difference between the F ratio for levels of demand and the critical F ratio is found in all cases but one. The level of variance significantly affects service level and transportation cost, but is shown to have no effect on the remaining costs. The type of probability distribution of demand has significant impacts on thruput and inventory cost.

Comparison Among Factor Responses: Average Response Comparisons

The analysis of variance technique (F test) is designed to show whether experimental factors as a whole or as a group have an effect on a particular response variable. It does not indicate whether various levels of a particular factor (the different types of probability distributions, for example) have differing impacts upon the response variable of interest. Thus, even though the variance of demand is said to have an effect upon the percent of demand stocked out (based on the analysis of variance), we cannot say whether variance level .10 differs from variance level .30 in its effect on percent stocked out. Therefore, Tukey's method of multiple comparisons is used to test the difference between alternative levels of each factor (distributions, levels, and variances). In those situations where Tukey's method does not apply, standard "t" tests were used. (This occurred when testing the exponential and poisson distribution responses against all others.)

In making the comparison among factor levels, the average response for each response variable over a number of experimental runs is used as the comparison statistic. Thus, the total cost associated with all runs having a normal distribution of demand (six experimental results, using two levels of demand and three levels of the coefficient of variation) is compared with the total cost of those runs (six) having a gamma distribution. This section, then, reports the findings of the average response comparisons among factor levels.

Probability Distributions

Figure 5-26 indicates that the demand stocked out and costs which result from each type of demand pattern will be compared to one another.

Figure 5-27 provides a comparison between all six experimental probability distributions in terms of total per unit cost and per unit activity center costs. This figure may be referred to throughout the

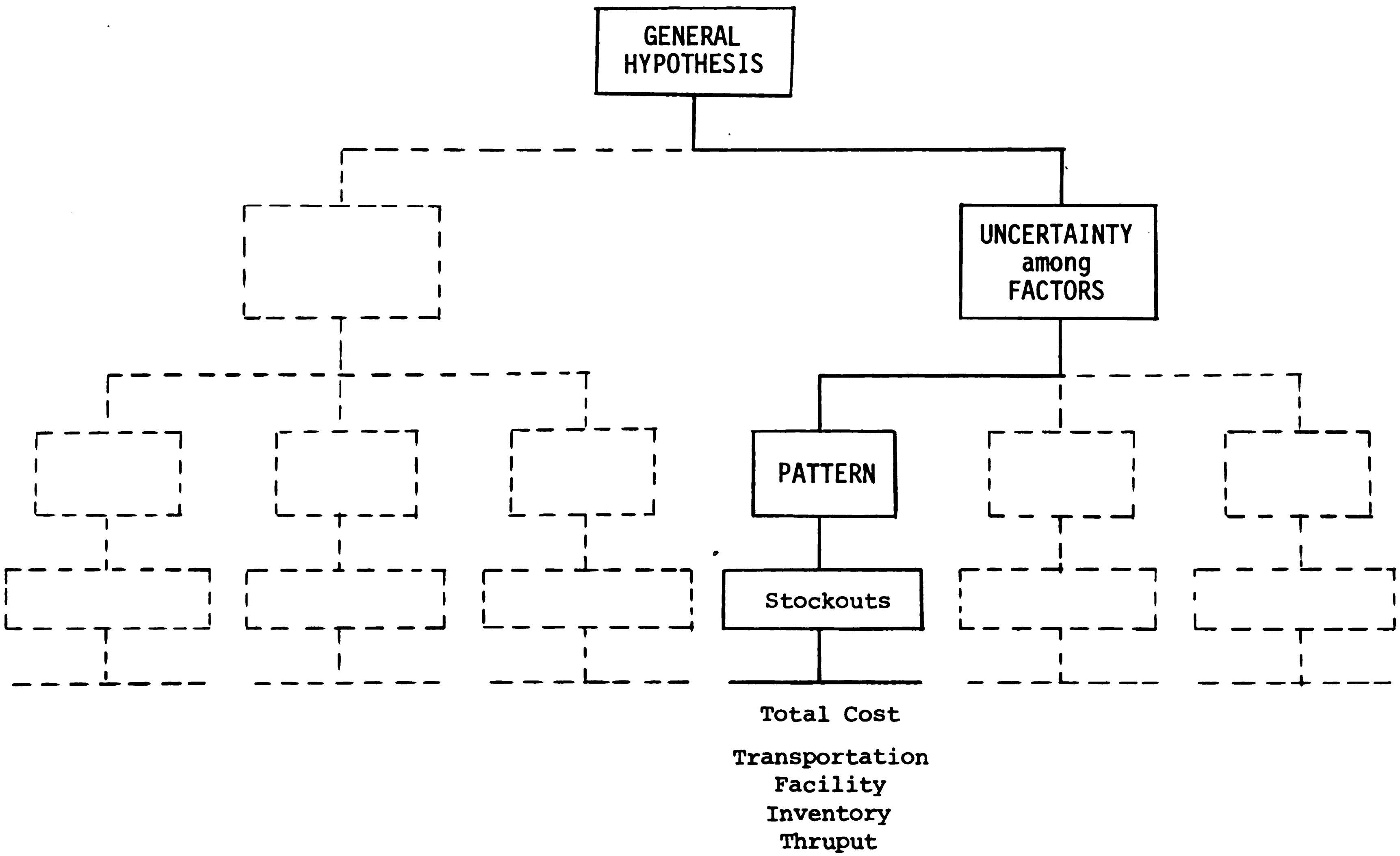

Figure 5-26. Research Analysis Organization: Among Factors/Pattern.

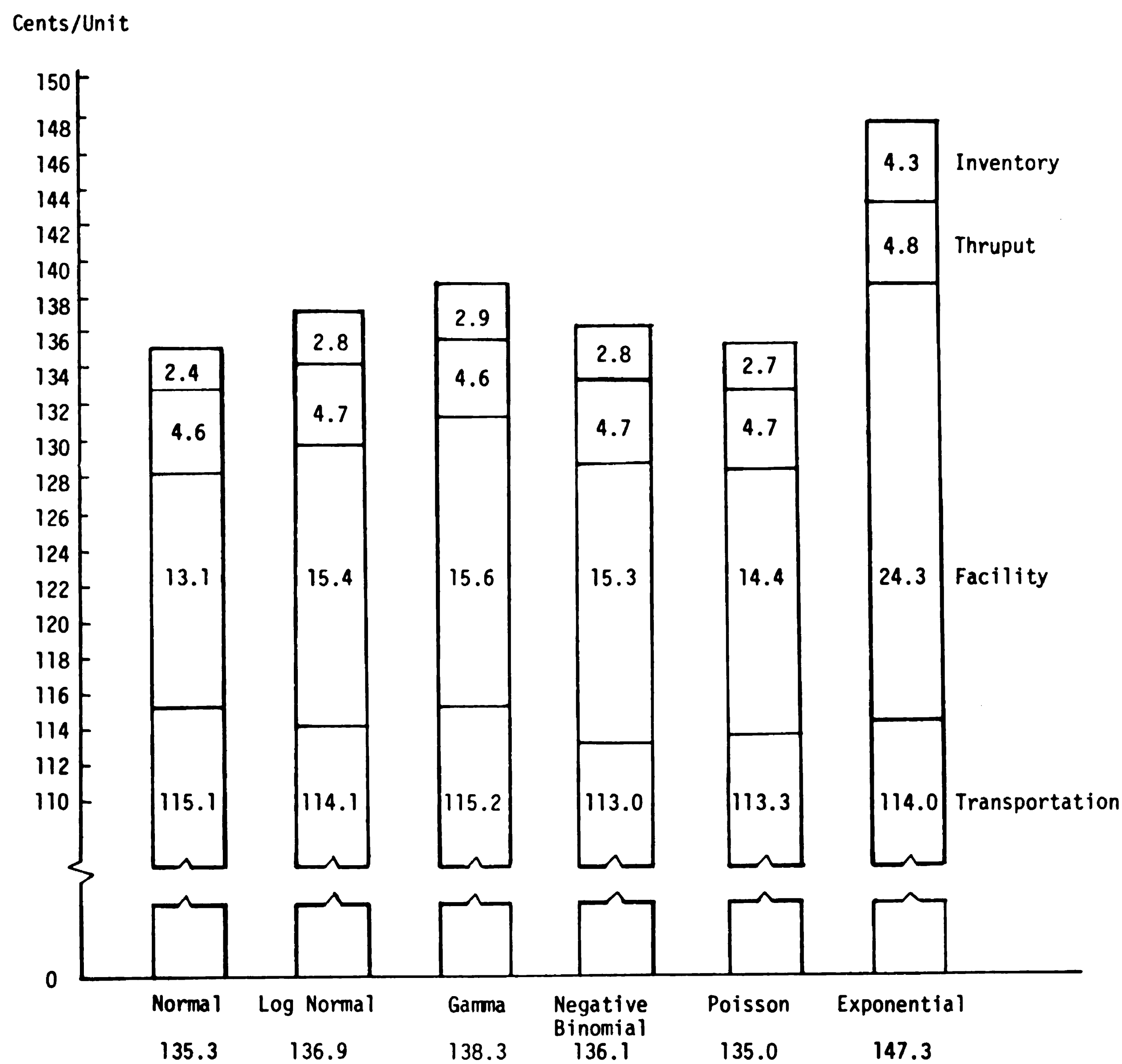

Figure 5-27. Probability Distribution Response: Total Cost (¢/Unit).

presentation of the findings on average response comparisons among distributions.

Demand stocked out.--Figure 5-28 provides a comparison of the average demand stocked out associated with each distribution. The figure presents the comparison in terms of the percent of total demand satisfied. Using Tukey's test between distributions (gamma, normal, log normal and negative binomial), no significant differences are detected, although the difference between the normal and gamma (1.04) is very close to the critical Tukey's "q" statistic.[2]

Standard t-tests are performed on the differences between the poisson and all other distributions and the exponential and all other distributions. The following results are obtained:

Table 5-16. Demand Distribution Comparisons: t-Test Results--Demand Stocked Out

Factors	Difference (in standard errors)	Critical t
Normal vs. exponential[a]	2.77	2.45
Log normal vs. exponential[a]	5.97	2.45
Gamma vs. exponential[a]	6.54	2.45
Negative binomial vs. exponential[a]	3.62	2.45
Poisson vs. exponential[a]	10.74	2.45

[a]The higher stockout percentage.

Thus, the exponential and the five distributions presented in Table 5-16 did differ significantly in terms of the percent of demand stocked out.

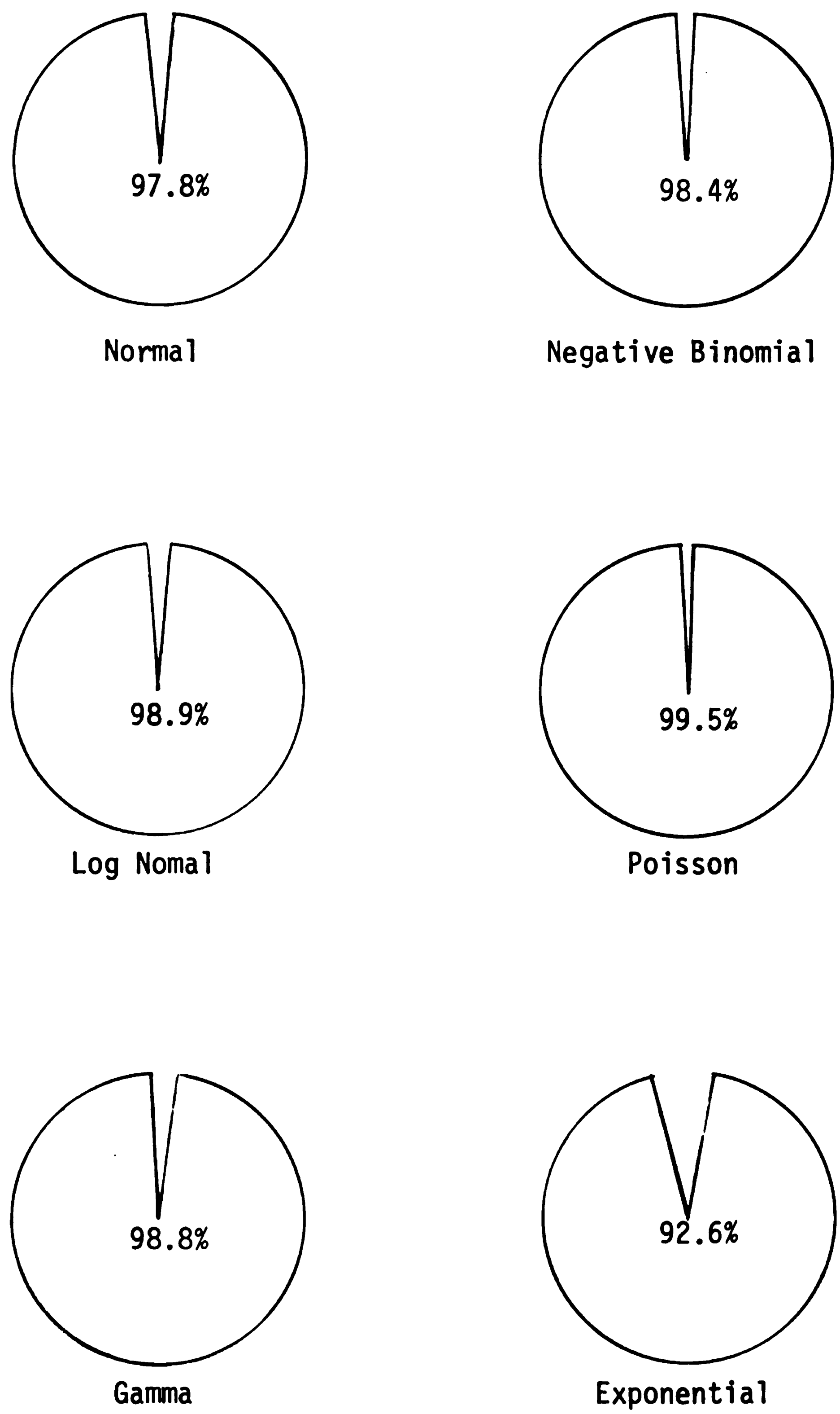

Figure 5-28. Probability Distribution Response: Demand Satisfied (%).

Cost/revenue ratio.--In comparing the six different distributions against one another no statistically significant differences are found in terms of the cost/revenue ratio obtained under each distribution.

Total cost.--Total costs per unit do not differ significantly among the six experimental demand distributions. However, the gamma and negative binomial distributions produce total costs whose difference (3.0) is close to the critical Tukey "q" statistic (3.3).

Transportation cost.--Figure 5-27 shows the transportation costs associated with each distribution. Using Tukey's "q" statistic, the gamma and negative binomial distributions are shown to differ significantly from one another in terms of transportation costs. The same result is true for the difference between the normal distribution and the negative binomial. The results are shown in Table 5-17.

Table 5-17. Demand Distribution Comparisons: Tukey Test Results--Transportation Cost

Factors	Difference	Critical q
Gamma vs. negative binomial[a]	2.198	2.09
Normal vs. negative binomial[a]	2.198	2.09

[a]The higher cost.

Facility cost.--The level of facility cost associated with each distribution appears in Figure 5-27. Significant differences exist between the exponential distribution and three others, namely, the normal, log normal and gamma distributions. Table 5-18 presents these results.

Table 5-18. Demand Distribution Comparisons: t-Test Results--Facility Cost

Factors	Difference (in standard errors)	Critical t
Normal vs. exponential[a]	3.80	2.45
Log normal vs. exponential[a]	3.02	2.45
Gamma vs. exponential[a]	2.87	2.45

[a]The higher facility cost.

Thruput cost.--Again, the reader is referred to Figure 5-27. The gamma and log normal distributions are found to be significantly different in terms of thruput costs. In this case, the actual difference between them is .065 and the critical value of Tukey's "q" statistic is .058. Additionally, the exponential distribution has significantly different thruput costs than does the normal, gamma or negative binomial. These comparisons are shown in Table 5-19.

Table 5-19. Demand Distribution Comparisons: t-Test Results--Thruput Cost

Factors	Difference (in standard errors)	Critical t
Normal vs. exponential[a]	2.88	2.45
Gamma vs. exponential[a]	3.66	2.45
Negative binomial vs. exponential[a]	2.66	2.45

[a]The higher cost.

Inventory cost.--Figure 5-27 also depicts inventory costs related to each probability distribution. In this case, Tukey's method reveals that the gamma distribution and normal distribution have inventory costs which are significantly different. The difference in cost between them is .459 and the critical Tukey's "q" statistic is .446.

The exponential distribution also exhibits a significant inventory cost difference when compared with four different distributions--the normal, gamma, log normal and negative binomial. Table 5-20 displays the results of these statistical tests.

Table 5-20. Demand Distribution Comparisons: t-Test Results--Inventory Cost

Factors	Difference (in standard errors)	Critical t
Normal vs. exponential[a]	3.73	2.45
Log normal vs. exponential[a]	3.03	2.45
Gamma vs. exponential[a]	2.85	2.45
Negative binomial vs. exponential[a]	3.08	2.45

[a]The higher cost.

Summary: Average response comparisons--distributions.--The exponential distribution creates cost and service levels which are significantly different than those associated with the remaining distributions. Additionally, the normal distribution was found to produce cost levels different from the gamma and negative binomial in the areas of inventory and transportation, respectively. The negative binomial also differed from the gamma in terms of transportation costs, while the gamma and log normal differed significantly in the area of thruput costs. Generally, there was little difference between the average responses associated with distributions in terms of cost and service levels.

Variance

Figure 5-29 indicates that the demand stocked out and costs which result from each level of variance in demand will be compared to one another.

The three levels of variance are compared using Tukey's method. The average responses associated with each level of variance is based upon the eight experimental runs made at each variance level. The experimental runs made with the poisson and exponential distributions are not included in determining average responses for each variance level because the variance associated with these distributions did not equal any one of the levels of variance to be tested.

Figure 5-30 depicts the comparison of costs obtained for the experimental runs made at each level of variance. This figure may be referred to throughout the discussions in this section of the findings.

Demand stocked out.--Figure 5-31 shows the comparison of the average percent of demand satisfied among the three variance levels. The results of Tukey's test of differences are presented below in Table 5-21.

Table 5-21. Variance Level Comparisons: Tukey's Test Results--Stockouts

Factor	Difference in % Stocked Out	Critical Tukey's q
Variance .10 minus variance .50	3.05	1.03
Variance .30 minus variance .50	2.26	1.03

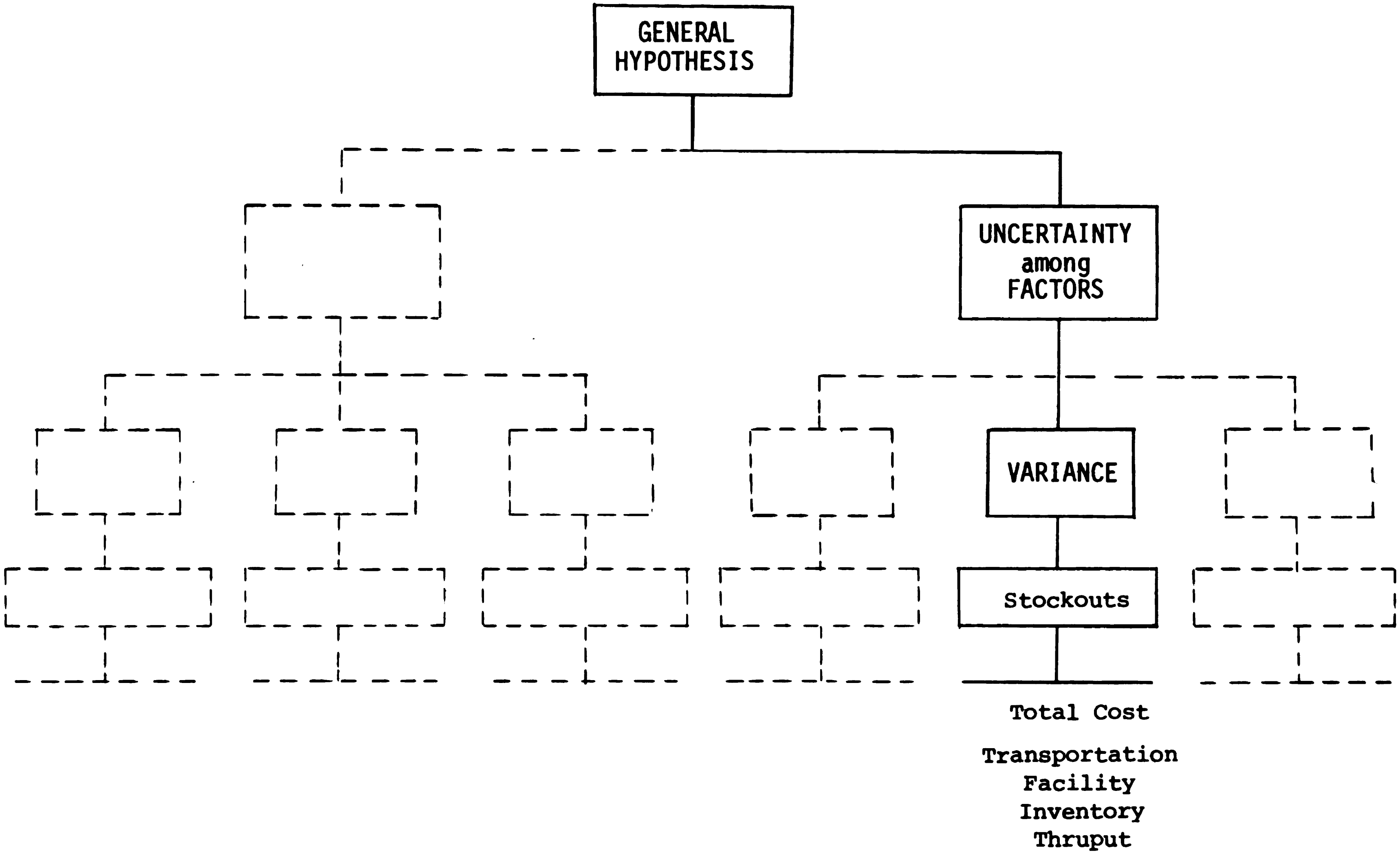

Figure 5-29. Research Analysis Organization: Among Factors/Variance.

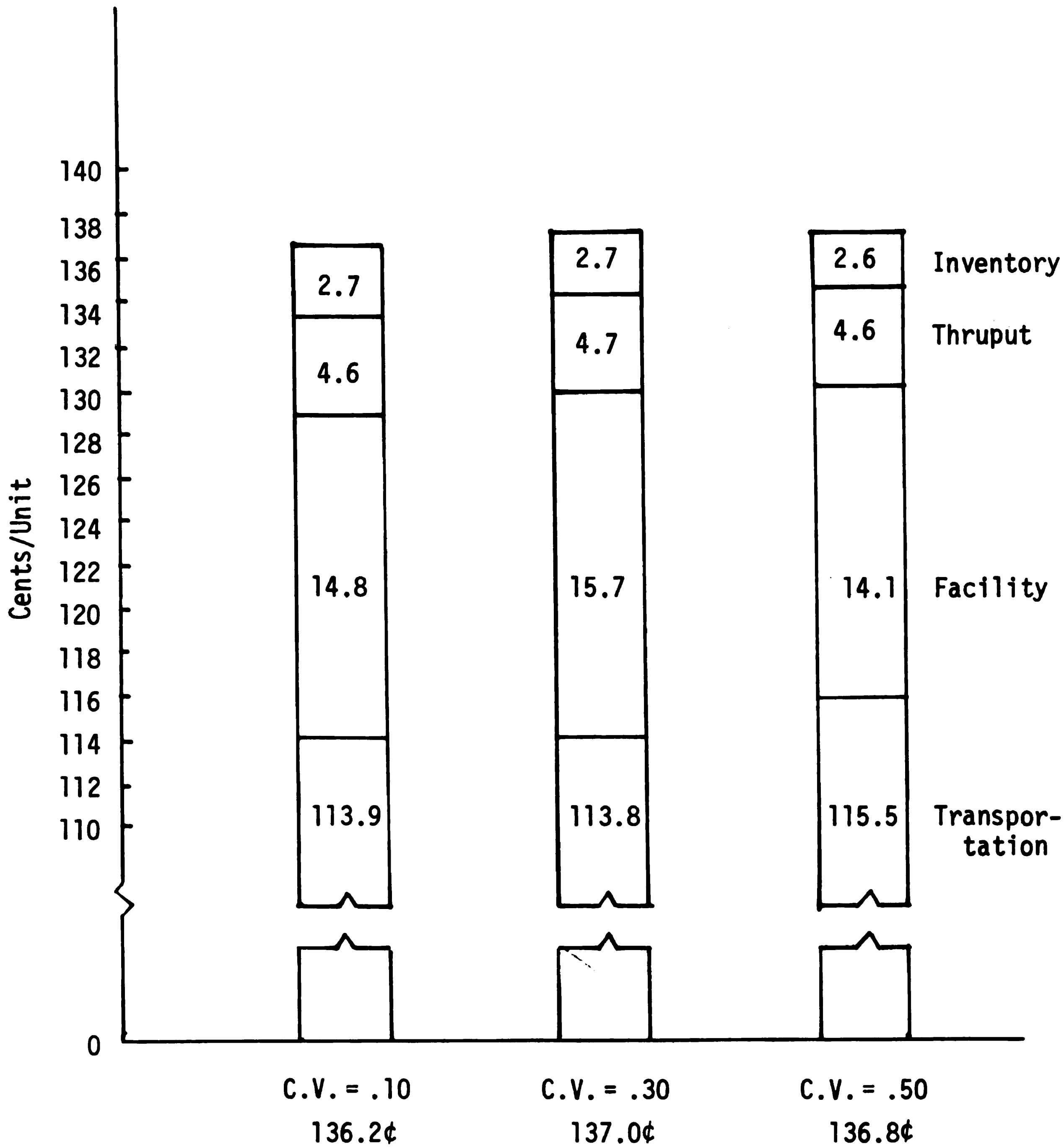

Figure 5-30. Variance (Coefficient of Variation): Total Cost (¢/Unit).

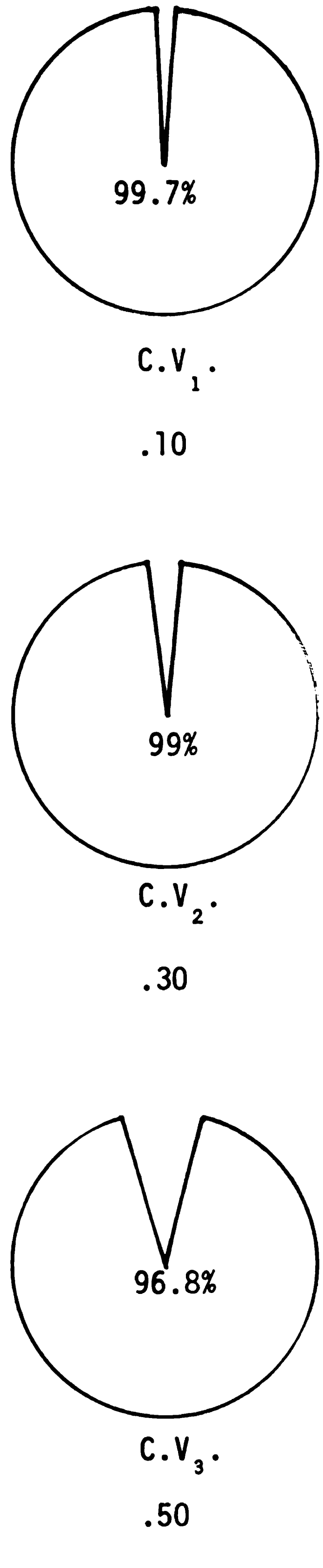

Figure 5-31. Variance (Coefficient of Variation): Percent Demand Satisfied.

Thus, variance level .50 produces significantly greater stockouts than do either level .10 or level .30. However, the percent of demand stocked out is not significantly different when comparing level .10 and level .30.

Transportation cost.--In the case of transportation costs, variance level .30 and level .50 produced costs which are significantly different. The difference in cost between the two levels is 1.90 and the critical Tukey's "q" statistic is 1.63. Variance level .50 had the higher of the two costs.

Remaining response variables.--No significant differences between average responses associated with variance levels are found for the remaining response variables. Thus, total cost, cost/revenue ratio, thruput, inventory and facility costs were judged to exhibit no statistical difference between the three variance levels.

Level of Demand

Figure 5-32 indicates that the demand stocked out and costs which result from each level of demand will be compared to one another.

The two levels of demand are compared using Tukey's method. Each level has twelve observations upon which its average responses are calculated. The results from the poisson and exponential were not included in these averages. Figure 5-33 presents the relationship of average costs associated with each level of demand.

Table 5-22 presents the results of the test of the difference between the two levels of demand for all response variables.

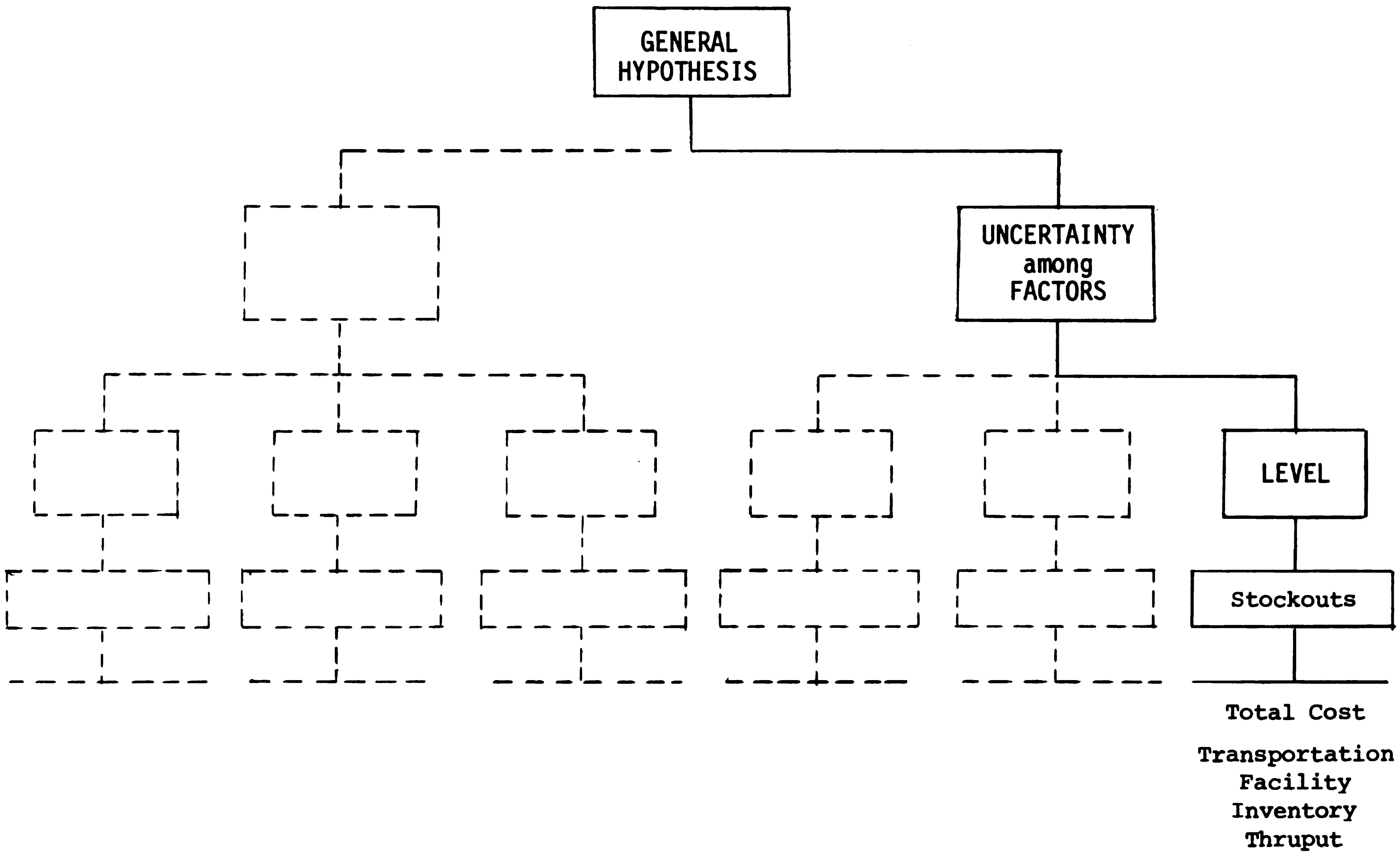

Figure 5-32. Research Analysis Organization: Among Factors/Level.

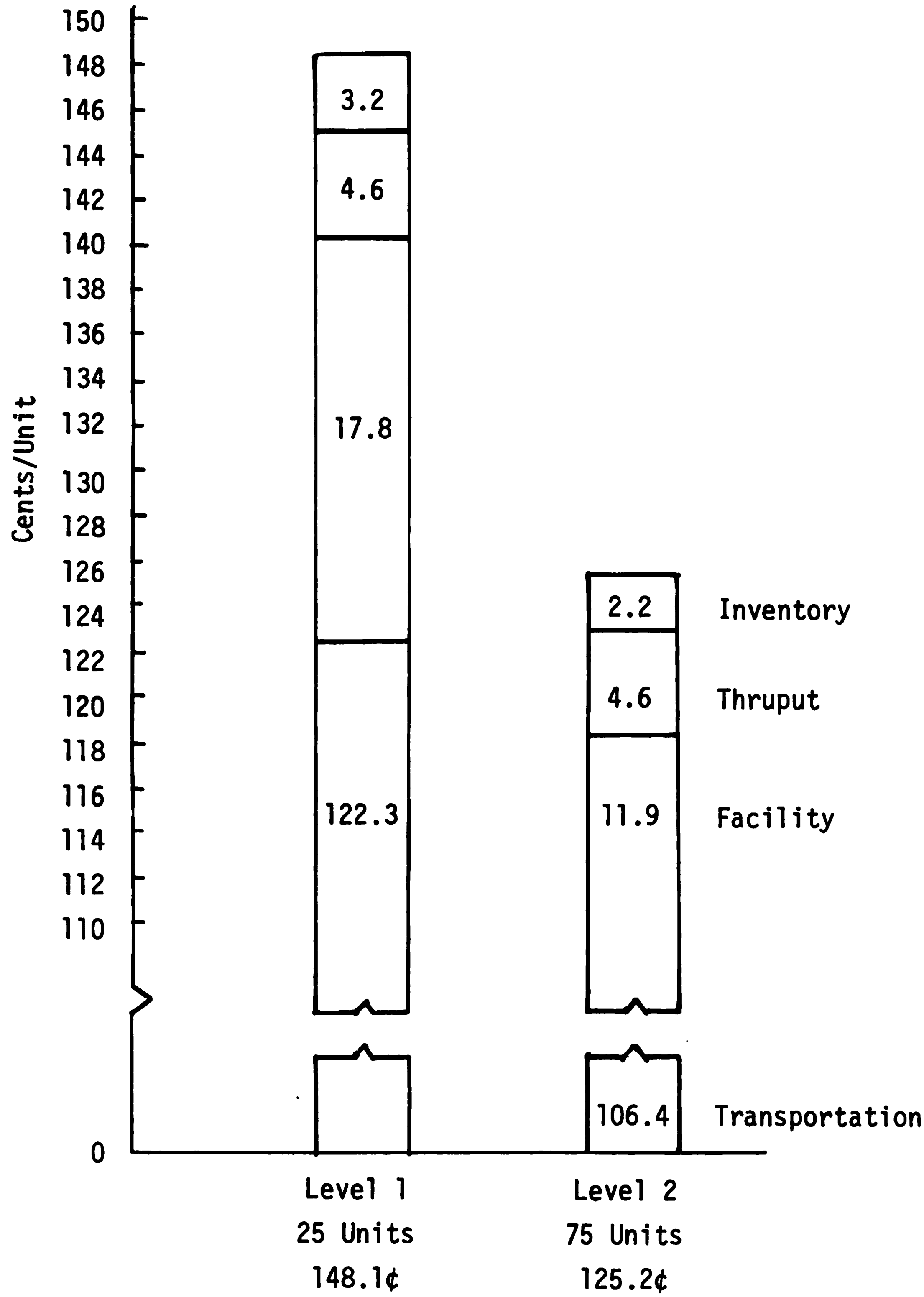

Figure 5-33. Level of Demand: Total Cost (¢/Unit).

Table 5-22. Level of Demand Comparisons: Tukey's Test Results--All Response Variables.

Response	Difference ($L_1 - L_2$)	Critical Tukey's q	Significant
% stocked out	0.76	0.691	Yes
Cost/revenue ratio	0.457	0.390	Yes
Total cost	22.85	1.733	Yes
Transportation cost	15.97	1.096	Yes
Facility cost	5.87	1.416	Yes
Thruput cost	0.009	0.030	No
Inventory cost	1.007	0.234	Yes

Thus, for all response variables but thruput, the difference between demand level 25 and level 75 is significant.

Summary: Average Response Comparisons Among Factors

The most noteworthy finding in this section is that demand level appears to significantly influence the cost and service level associated with channel performance. In both the analysis of variance (F-tests) and average response comparisons, the level of demand made a difference in the resulting cost and service measures obtained.

Levels of variance do not appear to have as perceptible an influence on cost. However, the variance did have significant impacts in the service area, i.e., percent of demand stocked out.

The distributions of demand are significant in their impacts in specific areas, especially transport cost. Generally, there are not many differences between distributions in terms of costs and service.

Comparison Among Factor Responses: Individual Cell Comparisons

As was mentioned in a preceding section, individual cell comparisons are worthwhile to the extent that averages may cover up important differences and valuable insight may be gained as to testable hypotheses for future research. Again, no statistical inferences are implied in the following discussion.

Demand Stocked Out

Table 5-23 presents the individual cell responses for percent demand stocked out. The exponential at level 75 produces the greatest stockouts, and three distributions at level 25 produce the lowest. High levels of variance (C.V. equals .50) and low demand levels (25) combine to produce large stockouts (normal equals 4.96%; log normal equals 3.63%; gamma equals 3.17%; negative binomial equals 3.42%). The stockouts at normal, level 25 and C.V. equals .50, are the highest among the four comparable distributions. Finally, when moving from C.V. equals .30 to C.V. equals .50, stockouts rise more than proportionally in the four comparable distributions.

Cost/Revenue Ratio

Table 5-24 depicts the cost/revenue ratios for each experimental run. Again, the highest ratio occurs with the exponential distribution (31.73%) the lowest with the negative binomial (level 75, C.V. equals .30). The gamma (level 25, C.V. equals .30) experiences an extremely high ratio as compared to other distributions at the same level.

Table 5-23. Individual Cell Comparison: Percent Demand Stocked Out

Coefficient of Variation	Normal		Log Normal		Gamma		Negative Binomial		Poisson[a]		Exponential[b]	
	Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75
.10	0.11	0.0	0.13	0.0	0.35	0.0	0.70	0.50				
.30	1.72	0.92	1.51	0.10	1.45	0.18	1.65	0.74	1.00	0.0	6.96	7.76
.50	4.96	5.64	3.63	1.00	3.17	1.95	3.42	2.61				

[a] $\sigma = \sqrt{\mu}$.

[b] $\sigma = \mu$.

Table 5-24. Individual Cell Comparison: Percent Cost/Revenue Ratio

Coefficient of Variation	Normal		Log Normal		Gamma		Negative Binomial		Poisson[a]		Exponential[b]	
	Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75
.10	28.98	24.87	29.11	24.89	30.68	25.02	29.48	24.82				
.30	29.35	25.22	29.34	25.11	29.78	25.61	29.95	24.80	29.22	24.79	31.73	27.20
.50	29.02	24.92	30.55	25.30	29.89	24.98	29.23	24.97				

[a] $\sigma = \sqrt{\mu}$.

[b] $\sigma = \mu$.

The cost/revenue remains relatively constant from one coefficient of variation level to the next. However, the cost/revenue ratio declines from 25.22 to 24.92 for the normal moving from C.V. .30 to C.V. .50. Finally, the cost/revenue ratio is higher for level 25 than for level 75 for all cases.

Total Cost

Total cost for each experimental run is shown in Table 5-25. Generally, the conclusion as to total cost comparisons by cell are essentially the same as those for the cost/revenue ratio. However, the gamma distribution does appear to generate higher costs than the normal, log normal, and negative binomial distributions.

Transportation Cost

Table 5-26 depicts the individual cell results for transportation costs. The highest transportation cost occurs with the gamma distribution (C.V. equals .10, level 25). The lowest with the gamma (C.V. equals .30 and level 75).

In general, transportation costs are higher in all cases with the lowest level of demand. The costs also rise when moving from C.V. .30 to C.V. .50 in all situations but two (log normal at level 75 and negative binomial at level 75). Finally, the negative binomial distribution in most cases generates lower costs than do the other distributions.

Table 5-25. Individual Cell Comparison: Total Cost (¢/Unit)

Coefficient of Variation	Normal		Log Normal		Gamma		Negative Binomial		Poisson[a]		Exponential[b]	
	Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75
.10	144.88	124.36	145.57	124.47	153.38	125.12	147.42	124.12				
.30	146.77	126.08	146.72	125.55	148.90	128.03	149.76	124.02	146.11	123.93	158.63	136.02
.50	145.09	124.58	152.74	126.50	149.44	124.89	146.14	124.86				

Table 5-26. Individual Cell Comparison: Transportation Cost (¢/Unit)

Coefficient of Variation	Normal		Log Normal		Gamma		Negative Binomial		Poisson[a]		Exponential[b]	
	Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75
.10	120.93	106.12	121.24	106.24	124.91	106.62	120.38	104.35				
.30	121.28	107.07	120.75	106.63	123.57	104.29	121.58	104.77	121.00	105.68	122.29	105.61
.50	124.44	110.95	123.71	105.88	124.01	107.79	121.20	105.71				

[a] $\sigma = \sqrt{\mu}$.

[b] $\sigma = \mu$.

Facility Cost

The comparison of facility cost for each cell is shown in Table 5-27. As in most cases, the highest cost occurs with the exponential distribution at level 25. The lowest is associated with gamma at level 75 and C.V. .50.

The high level of demand (75) consistently produces facility costs lower than the low level. The same could not be said when moving from one level of variance to another. In this case, costs do not always rise with the higher level of variance. However, C.V. .30 produces surprisingly high costs at both levels of demand and across most distributions. Finally, the costs are generally lower at demand level 75 and C.V. equals .10 than at the two other levels. Two notable exceptions are the costs at normal, C.V. equals .50 and level 75 and gamma, C.V. equals .50 and level 75.

Thruput Cost

Table 5-28 depicts thruput costs for individual cells. The table indicates that thruput costs are generally insensitive to the three experimental factors.

Inventory Cost

Table 5-29 presents individual cell comparisons for inventory cost. The exponential distribution is associated with the highest cost at both levels of demand. The lowest inventory cost occurs with the normal distribution at level 75 and C.V. equals .50.

Table 5-27. Individual Cell Comparison: Facility Cost (¢/Unit)

Coefficient of Variation	Normal		Log Normal		Gamma		Negative Binomial		Poisson[a]		Exponential[b]	
	Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75
.10	16.31	11.45	16.62	11.46	20.15	11.65	18.28	12.11				
.30	17.59	12.10	17.99	12.04	17.54	16.07	19.93	12.31	17.28	11.46	26.77	21.82
.50	13.51	7.53	20.64	13.46	17.63	10.48	17.11	12.23				

[a] $\sigma = \sqrt{\mu}$.

[b] $\sigma = \mu$.

Table 5-28. Individual Cell Comparison: Thruput Cost (¢/Unit)

Coefficient of Variation	Normal		Log Normal		Gamma		Negative Binomial		Poisson[a]		Exponential[b]	
	Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75
.10	4.63	4.63	4.64	4.62	4.57	4.64	4.63	4.64				
.30	4.69	4.66	4.71	4.64	4.59	4.64	4.66	4.64	4.68	4.63	4.89	4.69
.50	4.57	4.60	4.72	4.67	4.59	4.60	4.71	4.63				

[a] $\sigma = \sqrt{\mu}$.

[b] $\sigma = \mu$.

Table 5-29. Individual Cell Comparison: Inventory Cost (¢/Unit)

Coefficient of Variation	Normal		Log Normal		Gamma		Negative Binomial		Poisson[a]		Exponential[b]	
	Level		Level		Level		Level		Level		Level	
	25	75	25	75	25	75	25	75	25	75	25	75
.10	3.02	2.15	3.06	2.15	3.75	2.21	3.28	2.17				
.30	3.19	2.23	3.26	2.23	3.20	3.03	3.57	2.31	3.15	2.17	4.67	3.90
.50	2.56	1.51	3.66	2.48	3.20	2.02	3.12	2.29				

[a] $\sigma = \sqrt{\mu}$.

[b] $\sigma = \mu$.

The high level of demand in all cases produces costs lower than the lower level. The same is not true when comparing variance levels. In fact, for the normal, gamma and negative binomial distributions inventory costs declined as the level of variance increased from C.V. equals .30 to C.V. equals .50.

It is notable that the normal distribution produces rather low inventory costs at C.V. .50 at all levels of demand as compared to the other distributions at all levels.

Findings--Summary

When viewing the totality of the findings a number of results seem to stand out. First, the level of demand seems to create the largest number of significant differences between different levels and the control runs and between the levels themselves.

The effects of probability distributions are not as clear cut. The distributions were different in specific cost and service areas, notably transportation and percent stocked out. However, the exponential distribution is consistently above the other distributions and above the control run responses in both cost and service.

The effects of variance levels is seen to be most significant in the area of percent of demand stocked out. This was true when comparing variance levels to control and comparing the variance levels among themselves. The interpretation and exploration of these findings has been purposefully avoided in the present chapter. It is the goal of Chapter VI to put these findings in their proper perspective and to assess their meaning and relevance to physical channel system modeling, planning, controlling and administration.

CHAPTER V--FOOTNOTES

[1]C. W. Dunnett, "A Multiple Comparison Procedure for Comparing Several Treatments With a Control," Journal of American Statistical Association, 50 (1955), 1119.

[2]William Mendenhall, The Design and Analysis of Experiments (Belmont, Calif.: Wadsworth Publishing Co., Inc., 1968), p. 426.

CHAPTER VI

CONCLUSIONS

Introduction

The purpose of this research has been to determine the effects of demand uncertainties upon the cost and service performance of a physical channel system. The specific results of the experimental runs were reported in Chapter V. The purpose of this chapter is to bring together the hypotheses and the findings, provide an explanation of the findings, relate the conclusions to the present body of physical distribution knowledge, suggest implications for channel system planning, operation and control and to discuss areas for future research.

The first section of the chapter will relate the findings and hypotheses, providing conclusions regarding acceptance or rejection of the hypotheses. Additionally, explanations are given as to the reasons for the system reacting as it did to demand uncertainties. Next, the implications of the research findings to channel system planning and operation are provided. Finally, the last section considers future research and evaluates the limitations of the research.

Integration of Hypotheses and Research Findings

Factors vs. Control

The first of two general research hypotheses states that the presence of uncertainty, in the form of demand patterns, levels and variances will have a significant impact on the cost and service of a physical channel system. The subhypotheses that follow relate to the effects of each experimental factor (patterns, levels, and variances of demand) on cost and service. The first section of the integration of findings and hypotheses considers the effect that each factor has on system cost and service. All the findings discussed in this section relate to the cost and service effects resulting from uncertainty in demand compared to cost and service effects resulting when demand is constant per day (certain).

Distributions vs. control.--The first subhypothesis concerns the effects of demand distributions on total costs per unit. The subhypothesis states that the total per unit costs of a physical channel system which result when demand is presented to the system in the form of a particular probability distribution will be different than when demand is constant per day.

The findings in Chapter V indicate that the hypothesis must be rejected. In comparing total costs per unit when demand is constant (control run) per time period to those obtained when demand is generated by each of the six experimental probability distributions, no statistically significant total cost differences were found. Additionally, the activity center average cost associated with each probability distribution did not vary from those costs computed from the control experimental run (except for thruput, where log normal costs were statistically different, but the difference was only .05 cents). All the distributions, except the exponential, admit a wide range of daily demands above and below the average and thus, the effects of extremely large demands are counterbalanced by smaller than average daily demands, especially over a period of time as long as ninety days. Therefore, the distributions, when considered over six experimental runs, did not produce costs much different than achieved with a fixed daily demand.

It should be noted that the exponential distribution created total costs which were above the control run costs by a large amount ($1.47 vs. $1.35), although this difference was not statistically significant. It is felt that additional experiments with the exponential would show the exponential to cause consistently higher cost. Such a supposition is based upon the nature of the exponential, in that it is a decay type distribution, where extreme deviations from the average are more likely to occur than in other more symmetrical distributions. The increase in total costs are most likely to occur in the area of inventory and facility costs, where larger than normal inventories are built up at the SSP and PSP levels as a result of immediate stockouts at the ISP, which decrease the order frequency of both SSP and PSP levels. Thus inventory tends to build at these locations.

Specific situations involving the distributions of demand must also be considered. The log normal, gamma and negative binomial all created total per unit costs higher than control run costs at given variance and average levels of demand. In these specific instances, costs were anywhere from 14 to 18 cents per unit higher than control run costs. Although statistical tests could not be performed on these differences their absolute magnitude must be recognized.

Thus, the proper conclusion regarding distributions seems to be that, total cost per unit for each distribution considered over a range of conditions (two levels and three variance levels) did not differ greatly from total per unit costs associated with the fixed demand per unit time situation. However, specific characteristics (variance level and average demand) associated with a probability distribution may cause costs to vary from the certain demand situation. Therefore, one must look at the demand distribution that is evidenced in a specific situation, rather than considering the generalized effects of the distribution averaged over a number of other conditions. The findings also reveal the same to be true for activity center costs. Inventory and facility costs in particular are those which vary the most from the control run outcome. Transportation costs on the average appear to be relatively insensitive to the nature of the demand distribution, which is most likely a result of the weight break structure of the rates (one rate for wide ranges of weight) and the EOQ system of inventory ordering (fixed order size).

Finally, the exponential distribution has the greatest impact upon the cost outcomes relative to the controlled run in absolute dollar terms. The nature of the distributional shape accounts for this result, particularly with facility and inventory costs.

The second subhypothesis relates to the effects that probability distributions of demand have on the demand stocked out. The hypothesis states that the service level (percentage of demand stocked out) which results when demand is presented to the channel system in the form of a particular probability distribution will be different than when demand is constant per day. In this case, the hypothesis is accepted for three distributions (normal, negative binomial and exponential) and rejected for the remaining experimental distributions. The exponential distribution, as previously mentioned, is a decay function and thus admits widely varying daily demands on the "high" side of the mean. Thus, severe strains are placed upon the level of inventory maintained in the system, especially at the ISP level. The normal distribution is a different case, in that the distribution is symmetrical, which means that very low daily demands are as likely to occur as very high demands. However, in those cases where a large standard deviation of demand is present, the demand has a lower boundary but does not necessarily have an upper boundary. Even though smaller daily demands may balance to some degree the effect of extreme demands, it will only take a few extreme demands to cause an out of stock condition during lead time.

The inventory is frequently unable to absorb consecutive large demands that may occur. The result is magnified if such extreme demands are experienced at a number of ISP's at the same time. In this case, SSP inventory is rapidly depleted and ISP orders cannot be completely filled. This situation will be more fully explained later in the chapter. The probability of extremely large demands occurring with the negative binomial is even greater than that under the normal. Thus, the results achieved with this distribution are not unexpected.

The fact that the log normal, gamma and poisson did not produce a high percentage of stockouts is unexpected. However, even though these distributions are, in some cases, skewed to the right (allow extreme demands) they also have higher probabilities associated with demands smaller than average. Thus, it appears these smaller demands must balance the occurrence of extreme demands during lead time. Also, they were all very close to being statistically different than the control run stockouts.

As with total cost performance, it is also imperative to consider each distributional form under the specific circumstances under which it is found for service level considerations. For example, the normal distribution produced no stockouts at high demand levels and low variance, but did produce large stockouts at high variances. Even the gamma and log normal experienced relatively high stockouts under various conditions.

The conclusion to be drawn relative to distributions and service level when comparing against a fixed demand situation is that given distributions averaged across a variety of conditions produce lower service levels as compared to the control situation, but all distributions under specific conditions may create service levels that are lower than control, or in some cases, no different than the control run (fixed demand per unit time) service level.

Variance vs. control.--The third subhypothesis concerns the effects of the presence of demand variance on total cost per unit. The hypothesis states that the total per unit costs of a channel system which result when demand assumes different levels of variance around the average demand per day will be different than when demand does not vary around its average (i.e., is constant per day).

The hypothesis of differing costs is rejected. Thus, the total per unit cost and activity center cost per unit for each level of variance were not different than the control run costs. It appears that the explanation for this phenomena lies in the averaging of high and low demands when variance of demand, without regard to distribution or level, is considered. Thus, about the same number of units are moved through the system whether the demand per time period is fixed or whether it varies around an average. This result does not indicate anything about the comparison of variances among one another.

Even though the costs associated with each variance level were not statistically different than those derived with the control run, there were certain specific situations where high variances did affect costs. For each activity center cost there were some instances where high variance levels and a given specific demand distribution did appear to be much larger than control costs. Therefore, even though variance levels on the average did not cause costs higher than control cost, there may be situations where higher variance does influence cost behavior.

The fourth subhypothesis relates to the effects of the presence of demand variance on the demand stocked out by the channel system. The hypothesis states that the service level (percentage stocked out) which results when demand assumes different levels of variance around the average demand per day will be different than when demand does not vary around its average. The hypothesis is accepted for C.V. .50 and rejected for C.V. .30 and C.V. .10. When the coefficient of variation was .50 the stockout percentage was found to be statistically significantly different than the control stockout results. For C.V. .30, the stockout ratio was not in the acceptance region, but extremely close.

The extreme variability in demand puts pressure on the inventory maintained at all levels within the channel system. Thus, when demand is able to assume very large values relative to the average, the probability of a number of extreme demands being evidenced during lead time is relatively good. Even though very small daily demands may also occur during this period of time, the effects of the extreme deviations are greater due to the lower boundary on demand. Additionally, only a few extreme demands are required during lead time to produce an out of stock condition.

Levels of demand vs. control.--The fifth subhypothesis concerns the effects of demand levels (average number of units demanded per day) on total per unit cost. The hypothesis states that the total per unit costs of a channel system which result when demand assumes different levels of the average demand per day are not different from the costs which obtain when demand is constant per day.

The hypothesis regarding per unit system cost is rejected. The costs associated with the low level of demand are significantly higher than the control run costs. The costs with high level demand situations were significantly lower than control run costs. In fact, all activity center costs except thruput were found to be significantly different than control run costs.

The explanation for cost differentials due to demand levels when compared to control appears to result from the fixed cost elements in the channel system. Economies in transportation are not achieved in a system where the daily demand rate is low. Facility fixed charges are also spread over a few units in the low demand level situation and over a large number in the high demand level situation. However, handling or thruput costs usually involve a charge per unit. Thus they react little to the absolute magnitude of physical flow. Inventory is also sensitive to volume, especially when ordering follows an EOQ formulation. In this case, the economic order quantity does not decline in proportion to the decline in sales volume (due to the square root term in the EOQ formula). Thus, proportionately higher EOQ's obtain at lower sales volumes than at higher sales volumes. Therefore, proportionately higher average inventories are evidenced at the lower demand levels.

The sixth subhypothesis considers the effects of the level of demand on demand stocked out. It states that the service level of the channel system which results when demand assumes different levels is not different from the service level that obtains when demand is constant. The hypothesis is rejected.

However, in this case both levels of daily demand produced significantly higher stockouts than were evidenced with the control run. Thus, regardless of the average daily demand rate, demand which experiences any sort of variance creates service performance different than that encountered with a fixed demand rate. Therefore, the implication is that the variability associated with each demand level is the factor influencing the stockout percentage.

Summary.--Demand uncertainties, in the form of a probability distribution, level of demand and level of variance do, in selected instances, produce cost and service which differ from a fixed demand situation. The level of service (percent of demand stocked out) is affected by every type of demand uncertainty. Per unit costs are relatively insensitive to demand probability distributions and variances. Costs are sensitive to alternative levels of demand. The greater the demand level, the lower the comparative cost. It is important to consider combinations of the three types of uncertainties. A combination of distribution, level and variance may in fact have accumulated severe cost and service impacts on the overall channel system. However, some combinations of pattern, level and variance result in costs and service levels which are no different than costs and service when demand is constant per day (the control run).

Comparison Among Factors

The second general hypothesis concerns whether the factors produce cost and service results that are different from one another. The next sections of the chapter thus compare the various forms of demand uncertainty in terms of channel system total cost and service performance. The previous section reported the comparison of cost and service which results from uncertain demand conditions to the cost and service resulting from the situation in which demand is constant per day. It is necessary now to explain how and why the forms of demand uncertainty create differences in cost and service among themselves. The critical question is: "when uncertainty is present, does the type of uncertainty make a difference in the cost and service performance of the channel system?" In the discussion of each remaining subhypothesis, the F-test results of the comparison among factors will be presented first, followed by the Tukey and t-test results of the comparison among factors.

Comparison of distributions.--The seventh subhypothesis concerns the relationships between the different probability distributions as to whether they have similar or different effects on the total cost per unit. The hypothesis states that the different types of probability distributions of demand will create different channel system total cost per unit.

When viewed as one experimental factor (along with variance and level) that could affect the per unit total costs in the channel, the F-test revealed that distributions in general have no significant effects. Thus, when looked at over a host of experimental conditions, including three levels of variance and two levels of average demand per day, the distributions did not account for a significant amoung of variance in total costs per unit. The hypothesis is therefore rejected.

This result must be considered from a number of standpoints. First, the exponential distribution was not included in the F-test analysis. This distribution is the one distribution most likely to influence channel performance due to its nature and pattern. Secondly, the distributions did have impacts upon specific costs (which will be discussed below), and these effects cancelled one another in some cases, thereby negating any effect on total cost. Additionally, total costs varied widely within the six experimental runs in a number of cases, but these impacts were lost when averaged over all runs.

Although the conclusion of "no effect due to distribution" was reached relative to the total cost hypothesis, the same could not be said for the effect of distributions on activity center costs. Distributions were deemed to have an effect on transportation costs as indicated by the F-tests. Thus, the type of probability distribution assumed by demand most likely influences the pattern of orders within the channel system and therefore the number of partial shipments which are made. This result will be discussed in detail when comparing distributions. Thruput costs were also affected by distributions. However, the amount of total variance in these costs was so small that the result is not very meaningful. Finally, inventory costs are also considered sensitive to demand distributions as indicated by the F-test. Different demand distributions contribute to an unnecessary build-up of inventory when a large proportion of orders are below the average demand and a few extreme orders occur. Such extreme orders trigger the reorder mechanism, and inventories build until another large demand is experienced. This result is explored in greater detail later in the chapter. Whether or not it can be concluded that probability distributions of demand have significant impacts in general upon channel system costs, it is also necessary to compare the effects of each distribution against the others.

Thus, the results of the Tukey and t-test are explored to determine whether total per unit and activity center per unit costs would be different depending on the type of distribution pattern evidenced by demand. In terms of total per unit costs, the hypothesis that the costs are different depending on the probability distribution of demand is rejected based on the results of the Tukey test.

There is no statistically significant difference among the total costs associated with each distribution. Again, the cost associated with each distribution is based on an average of six observations. In some of these cases, the distributions assume patterns which are somewhat similar. For example, the normal distribution is a good approximation to the poisson and gamma distributions when the average and variance level are "high." Thus, average per unit total costs are quite similar.

Differences in activity center costs associated with each distribution tend to cancel any effects that different distributions would have on total costs. For example, the normal distribution creates higher transport and lower inventory costs relative to the exponential, while the exponential leads to lower transportation costs and higher inventory costs than the normal. Thus the total costs of each is not significantly different.

Additionally, a similar quantity of goods is processed through the system over the ninety day period with each demand distribution. The total costs appear to be more sensitive to the volume over the period rather than the time pattern of demand. It should be noted that the total per unit costs associated with the exponential was very much larger than those associated with all other distributions. However, due to only one degree of freedom, its costs were not considered significantly different. As was pointed out earlier, it would seem as if the exponential most likely will produce higher costs over more trials. Finally, the cost effect of demand distributions is more clearly seen in the area of specific activity center costs.

Activity center costs, in some cases were different depending on the specific demand distribution. The transport costs associated with the gamma were different than those associated with the negative binomial. The normal distribution also was different from the negative binomial in this respect. The normal and gamma are similar distributions when demand and variance are "high." The negative binomial is skewed, with high probabilities for demands less than average and some probability associated with extreme demands. The normal is symmetrical and gamma is nearly symmetrical at high demand. Thus, what appears to happen in the channel is that demand distributions that admit very extreme values cause stockouts to occur at the ISP level. However, those distributions for which more constant demands occur put extreme pressure on the inventories maintained at the SSP and PSP level.

Table 6-1 depicts these relationships. When extreme demands lead to immediate stockouts at the ISP (with the exponential and negative binomial) the pressure on the inventories "up" the channel is relaxed. The extreme value distributions also create a more staggered time pattern of orders upon the SSP, which enables them to more easily satisfy demand and obtain replenishment from the PSP without stocking out as frequently. Thus, regular EOQ orders from the ISP's can be filled, and the volume transportation rates obtained. Therefore, the negative binomail and the exponential have lower transportation costs associated with them because partial shipments do not occur very regularly.

Table 6-1. Units Stocked Out and Transport Cost

	Normal	Gamma	Exponential	Negative Binomial
Level 25				
Stockouts--ISP	964	586	1,192	626
Stockouts--SSP	2,571	1,878	1,233	135
Stockouts--PSP	3,335	169	0	1,927
Transport costs (¢/unit)	124.44	124.01	122.29	121.20
Level 75				
Stockouts--ISP	3,265	1,084	4,317	1,437
Stockouts--SSP	20,244	6,085	7,278	6,762
Stockouts--PSP	38,000	12,600	1,800	4,800
Transport costs (¢/unit)	110.95	107.78	105.61	105.71

On the other hand, the normal and gamma distributions generate more constant or even demand at the ISP level, thereby causing a more regular and constant demand for inventory replenishment at the SSP level. Hence, simultaneous orders from the ISP's, which are more regular in nature seem to cause more frequent stockouts at the SSP. This in turn has the parallel effect upon inventory at the PSP. Additionally, because the ISP's do not stock out as often with the initial customer orders when demand is more symmetrical, the pressure for more frequent ordering of the EOQ is increased. What this all leads to is that the SSP and PSP eventually begin stocking out more regularly, which finally will result in stockouts at the ISP level. In terms of transportation, the stockouts which occur at the SSP and PSP levels act as a double edged sword. First, partial shipments are necessary when an order cannot be completely filled. Secondly, the goods in question are backordered, and are moved as a partial shipment when they become available. Thus, the transportation costs will rise in those situations (normal and gamma) where stockouts occur at higher levels in the channel.

Another factor which comes into play here is the fact that backorders do not take place at the ISP level. Hence, the demand is lost, and the pressure on inventory levels is not compounded. However, at the SSP and PSP levels, backorders are made, and the pressure on inventory is not abated when an order cannot be filled. With production assumed to be constant per time period, the PSP has a difficult time in "getting ahead," i.e., building up an extra stock to satisfy extreme demands. The effects thereby spiral, creating reverberations at all levels throughout the channel.

In conclusion, the normal and gamma distributions tend to generate higher transportation costs because the impact of these distributions is upon the SSP and PSP level. In fact, stockouts recorded at both SSP and PSP levels were greater than with most other distributions. Since the impact is upon the SSP and PSP levels, a larger number of partial shipments are thus made as the SSP and PSP stockout. However, with the distributions which admit very extreme demands, the ISP level will incur stockouts, thus reducing pressure on inventory within the channel, and thereby resulting in more full-load shipments.

Facility costs were deemed statistically significant when comparing facility costs associated with the exponential distribution to those in the normal, gamma and negative binomial. The previous discussion as to transportation costs is relevant in this case. In the case of the normal, gamma and negative binomial distributions, more stockouts occur at the SSP and PSP levels than occur at these levels with the exponential distribution (see Table 6-1). Therefore, these distributions produce a situation where the system is always trying to "catch up" and where inventory is not accumulating within the system. However, with the exponential distribution, more immediate stockouts are created at the ISP level, demand is lost and inventory tends to build within the system as there is less frequent ordering from ISP to SSP and SSP to PSP levels. Thus, inventory is building and held longer within the system, thereby causing an increase in facility cost.

As one might expect, the same situation occurs with the inventory costs. The exponential has significantly higher inventory costs than all other demand distributions. Additionally, the gamma has higher inventory costs than the normal distribution. Table 6-1 indicates a relative increase in the stockouts within the channel when the normal distribution represents demand. Thus, inventory is not built up unnecessarily, and the costs are lower.

As was noted in an earlier section of this chapter, the cost and service results associated with a given distribution at a given level of demand and variance must be considered in addition to the average results over a number of conditions. Although statistical tests could not be performed, individual circumstances (a given distribution, level, variance) appear to differ from one another. The normal, gamma and exponential distributions at low demand levels and high variances lead to cost levels which are unique when compared to other situations. The general tendency is that of higher transport and lower facility and inventory costs for the normal, lower transport and higher inventory costs for the exponential, and higher transport for the gamma. The reasons for such patterns of cost behavior were explained previously. In summary, both the average cost performance and individual cost performance related to demand distributions vary depending on the type of distribution. In general, total costs are similar, but activity costs show substantial variation.

Because demand patterns did not lead to differences in total costs, their impact should not be ignored. Demand patterns

(distributions) did create differences in activity center costs, and in other channel systems where a particular activity center cost is the major proportion of total physical distribution costs, the impact of different types of demand distributions could be severe.

The eighth subhypothesis concerns the effects of probability distributions of demand on stockouts. The hypothesis states that different types of probability distributions of demand will create different service levels for the channel system. The hypothesis as to the generalized effect of distributions is rejected based upon the F-test. Thus, demand distributions, as one of the experimental factors, did not account for a significant portion of the variance in service level over the experimental runs. This conclusion must be viewed with some caution. The exponential distribution, with the highest stockout percentage could not be included in the F-test results. Additionally, specific situations with each distribution did create substantial stockouts.

The next level of analysis is that of comparing the average stockout percentage among the six distributions. The hypothesis is that there are differences in stockout percentages among the distributions. Thus, even though the F-test rejected the hypothesis that distributions have an effect, the distributions could still show differences among themselves.

The exponential was concluded to produce stockouts significantly different than the remaining five distributions. The explanation for this result lies in the nature of the exponential distribution in that it assumes a decay function. Because of this, extreme demands impact on the ISP inventory to create significant out of stock conditions. The remaining distributions did not statistically differ in terms of stockouts. In this case, one must go beyond the statistical analysis to explore the result.

Almost all distributions, when viewed from the individual cell responses had very few stockouts at the low level of variance. Thus, the inherent variability in the distributions is not great enough to pressure inventory holdings with extreme demands. However, at higher variance levels, stockouts do occur, and for some distributions these are relatively large. Thus, a normal distribution at C.V. .50 and Level 75, creates 5.6% stockouts, whereas only 1.9% occur with the gamma in this case. Thus, the normal distribution, with a relatively "regular" demand pattern, creates inventory problems up the channel to a greater extent than occurs with the gamma, whose pattern of demand is not so "regular."

Therefore, these five demand distributions, when compared on the basis of average stockout percentages, do not differ, but may experience different stockout percentages in individual situations.

Comparison among variances.--The ninth subhypothesis relates to the comparison of the three levels of variance in terms of total cost per unit. The hypothesis states that the different levels of variance in demand per day will produce different total cost per unit for the channel system. Thus, it is felt that demand variance, in general, as one experimental factor, will affect costs and in particular, different levels of variance of demand will have associated with them different per unit total and activity center costs. The hypothesis as to the general effects of variance levels on total cost is rejected based upon the F-test. Thus, the variance of demand, considered across all distributions did not alter the total channel system per unit costs. The inference which seems to follow is that the effects of variance, considered by itself (without regard to distribution), tends to cancel out over a period of ninety days. If demand only varied on the high side of the average demand per day, then total costs might vary between different levels of variance, as extremely different quantities would move through the system at each variance level. However, variations in demand occur on each side of the average demand, the effects thus cancel, and quantities moved do not vary greatly. Therefore total costs remain relatively constant.

The same general hypothesis relative to variance levels but on an activity cost basis, produce the same results except in the case of transportation. In that situation, variance is judged to have an effect according to the F-test. The conclusion is that the very extreme high variations in demand are not totally balanced by extreme low variations in demand (because of the lower limit to which demand can go), and thus stockouts occur at the SSP and PSP levels in the channel, forcing partial shipments and higher transport rates. However, the effect is not so great as to cause a significant variation in total system per unit cost.

After considering the generalized effects of demand variance, it is necessary to directly compare each variance level against the other in terms of total and activity center per unit costs. As might be expected from the conclusions relative to the generalized effect of variance, it is concluded that different levels of variance do not produce different levels of costs, except in the case of transportation. In this instance, variance level 3 (C.V. .50) is significantly different from variance level 2 (C.V. .30) and variance level 1 (C.V. .10). The absolute magnitude of demand variance at C.V. .50, is such that positive and negative deviations from average demand are not cancelled. The result is a greater number of stockouts both at the ISP and SSP. However, those occurring within the channel, lead to higher per unit transport costs resulting from a higher incidence of partial shipments.

The tenth subhypothesis concerns the effects of variance levels on the service level. The hypothesis states that the different levels of variance in demand per day will create different service levels for the channel system. Thus, it is felt that in general and among variance levels, variance will have an impact on the amount of demand that the channel system can satisfy. The hypothesis is accepted in both cases. This result needs little explanation. As variance levels increase, the probability of enough extreme demands occurring during lead time to cause stockouts also increases.

Table 6-2 indicates the impact each level of variance has on stockouts at all nodal points within the channel. What appears to happen is that low levels of variance put little pressure on ISP

Table 6-2. Average Stockouts (Units/90 Days)

	ISP	SSP	PSP
C.V. .10	18	4,600	3,240
C.V. .30	300	2,830	730
C.V. .50	1,230	5,570	8,750

inventories, and because demand is relatively constant, all ISP's can usually satisfy demand. Stockouts do occur within the channel because of the constant pressure (from all ISP's) that is put on the inventories at the SSP level. Because the lead times vary between ISP and SSP (seven days) and SSP and PSP (ten days), the constant pressure on the SSP causes stockouts here -- (because the SSP may see two reorders from each ISP before they receive their order). Additionally, stockouts occur at the PSP level as the constant production rate doesn't always match the demands put on the PSP inventories. However, the stockouts which occur within the channel are not severe enough to critically affect the ISP service level.

When the coefficient of variation increases to .30, more immediate stockouts occur at the ISP. The stockouts become lost sales, and inventory ordering occurs less frequently. Thus, the SSP's are able to more easily keep up with the demands placed on their inventories. Similarly, the PSP is able to satisfy most demand.

Finally, at C.V. .50, both situations occur. Stockouts occur at the ISP immediately but enough pressure is kept on ISP inventories (because of the cancelling of high and low demands over lead time) that the SSP will probably experience a constant order stream from all ISP's served. The SSP begins stocking out, and this condition transfers to the PSP. These stockouts eventually are reflected in stockouts at the ISP level as the ISP cannot be assured of having his order filled.

Comparison of levels of average demand per day.--The eleventh subhypothesis concerns the impacts of different levels of average demand per day on the total cost per unit of the channel system. The hypothesis states that different average levels of demand per day will not produce a difference in total per unit cost. The generalized F-test reveals that levels of demand do affect total per unit costs and therefore the hypothesis is rejected. In comparing the results between the levels of demand the Tukey test also suggests that the hypothesis be rejected. In addition, the comparison of the results associated with each demand level indicates significant differences for all activity center costs (except thruput).

The fact that levels of demand produce different per unit costs is explained by considering the nature of the cost functions. Transportation rates are based upon volume, and hence the system in which demand is substantially lower will create smaller shipment sizes which move at premium transportation rates. The shipment size depends upon the ordering process, and in this case, the EOQ formulation was used. Since the EOQ considers annual demand in its calculus, order size is somewhat dependent on demand levels. Therefore, transport costs per unit were higher in the system in which demand per unit time was lower. It should also be noted that the lower level of demand had a higher percentage of stockouts at the ISP, but a lower percentage at the SSP and PSP than did the higher level of demand. This factor tended to reduce somewhat the disparity between high and low levels of demand in terms of transport costs. In other words, the higher level demand created more partial shipments within the channel. Table 6-3 depicts the average stockouts by level at the ISP, SSP and PSP.

Table 6-3. Stockouts by ISP, SSP, PSP (% of Demand)

	Level 25 (%)	Level 75 (%)
ISP stockouts	1.9	1.0
SSP stockouts	7.8	10.0
PSP stockouts	3.2	14.0

Facility and inventory costs were also different between levels. As was explained in the section of comparing levels to control, the EOQ formula does not increase the order size in direct proportion to the change in the level of demand. Therefore, on a relative basis, the order size is proportionately larger relative to demand at the low level of demand than at the high level. Because of this, the average inventory tends to be larger, and thereby both inventory and facility costs are higher. Also, the fact that a larger percentage of stockouts occur within the channel with the high demand helps account for the lower level of inventory and facility costs in this situation. Inventories tend to build within the channel at level 25 of demand. The difference in transport, facility and inventory cost account for the difference in total costs between the two levels of demand.

The last subhypothesis concerns the impacts of different levels of average demand per day on the service level of the channel system. The hypothesis states that the different average levels of demands per day will not produce a difference in the service level achieved by the system. The hypothesis is rejected, service level is significantly different between levels. The F-test for levels of demand indicates that levels of demand are significant in accounting for the variance in service (stockouts) over the experimental runs. In addition, the comparison of the results associated with each demand level (i.e., the Tukey test) indicates significant differences for service level performance.

The reason that such phenomena occur is that at low levels of demand, extreme "high" variations in daily demand are not always counterbalanced by extreme variations on the low side of average daily demand. Thus, the lower limit of demand is reached more quickly with the lower demand level than with the high level, especially when demand is extremely variable. For every distribution and every level of the coefficient of variation, the 25 level demand condition produced a higher percentage of stockouts than the 75 demand level (except for normal, C.V. .50, level 75).

Implications of the Research for Channel Planning, Operation and Control

It has been shown that uncertainty influences the cost and service performance of a channel system. In some cases distribution patterns of demand produce lower levels of efficiency and effectiveness. In other instances there is no difference due to the distributions. The level of demand variance affects system wide efficiency and effectiveness, however, the major impact was in the area of transport costs and service level. The level of demand has impacts across almost all cost areas and on the service level achieved. Thus, all forms of uncertainty influence service performance; the effect on costs, both total and activity center, is not quite so clear cut. Based on the research conclusions the major implications for channel management are now presented.

1. Because the type of demand uncertainty, be it distribution, level or variance, has an effect on the efficiency and effectiveness of the channel, it is imperative that efforts be put forth to empirically assess the nature of the time pattern of demand. Although this type of analysis is, at best, difficult, a number of studies have indicated that it can be done.[1] The problem of determining demand patterns centers around the fact that demand and sales are not necessarily equal. Thus, sales may not provide the appropriate type of information needed. In those cases where individual product cost and inventory control is impossible, products can be grouped into categories, and the distribution, level and variance can be assessed for the group. Additionally,

time series effects, such as trend, seasonal, and cyclical patterns will have to be considered along with the statistically determined probability distribution. In conclusion, this research has shown that a particular distribution, level and variance of demand cannot be assumed for planning purposes, for if it does not obtain in the actual situation substantial deviations in both efficiency and effectiveness will result. Thus the need for accurate demand pattern estimation.

2. Assuming that demand patterns have been at least roughly determined, it becomes necessary to consider the ramifications of these patterns on channel operation. If the demand variance is relatively small and the distribution somewhat symmetrical, efforts to change these parameters will probably not result in any increase in the efficiency or effectiveness of the channel system. However, if the demand per unit time evidences wide variance or a pattern such as the exponential or normal distribution, efforts to change these parameters can result in a reduction of cost in the area of transportation, inventory and facility and/or an increase in the service performance. For new products, or for products whose demand patterns change (due to competitive actions, changes in product quantity or packaging, etc.) estimation of likely demand patterns, levels and variances will provide an input into the planning of channel structure and operation. Thus, if the normal distribution with high level variance and daily demand can be assumed, this research indicates that additional inventory must be maintained at the SSP and PSP level within the channel, and if it is, transport rates can be controlled through the reduction of stockouts

at these levels (an attendant increase in full load shipments). Also, the increase in stock necessary to provide a higher service level is at least suggested.

3. Depending on the circumstances, it would be prudent to try and somehow alter the demand pattern, level and variance. Such a statement presumes that this is possible. It is probably more likely that such actions already occur today by accident or improper planning. Thus, many functions that are under the control of logistical and marketing managers, which have the potential to alter demand pattern and variance, are employed without thought as to their impacts on demand patterns, etc. For example, a large manufacturer recently decided to accumulate all stock orders from given type customers until a particular day in the week, and then ship the orders the following day at volume transportation rates. For a while, orders still flowed in randomly throughout the week, but after an adjustment period orders were received exactly on the day they were processed. Thus, by altering order processing and shipment times, the firm eventually altered the time pattern of demand. Changes in order communication methods, transportation modes and inventory policies within the channel can all have an effect upon when orders from the other channel members are made. Therefore, it seems rather important that management consider fully the impacts of such changes upon the time pattern of demand.

If the firm presently experiences a demand pattern which did not materially cause deviations from the least cost and highest service system, they should be hesitant to alter channel policies that would result in a time pattern of demand or demand variance that creates higher costs in the system or would produce lower service levels. However, the tradeoffs between the savings in cost of the new policy would have to be evaluated in light of the additional costs resulting from the new pattern or variance of demand per period. At the same time, changes in policies which would decrease demand variance or alter the distribution and the level of demand such that costs are lowered or service increased should be undertaken. In summary, demand patterns are responsive to variations in channel policy, and the impacts of policy changes should be evaluated as to their effect on demand patterns, levels and variances.

4. Along this same track, certain variables can have significant impacts upon demand level, variance and pattern. Promotional programs initiated by any member of the channel system will have significant impacts upon the time pattern of demand. National ad campaigns instituted by the manufacturer will most likely affect the level and variance of demand per time period and also possibly affect the probability distribution of demand. As the research findings indicate, large variances of demand and certain types of demand distributions (exponential and negative binomial) have rather great impacts upon inventories held at the retail level. Thus, such a campaign may actually produce negative results if a sufficient number of stockouts occur at the retail level. If the effect of the campaign were to produce a more symmetrical distribution, such as the normal or gamma, but increase both level and variance of demand, the research results indicate

that inventories within the channel (wholesaler, distribution center) will be adversely affected, and eventually cause stockouts at the retail level. Thus the ramifications of an advertising campaign will be felt throughout the entire channel system.

Retailer advertising, special sales, store hours and the like also affect the time pattern of demand and hence may create reverberations throughout the channel. In fact, any marketing strategy employed at any level within the channel has the potential of altering demand pattern, level and variance. Thus, strategy changes cannot be undertaken in isolation of their impacts on demand patterns.

The reverse situation is also true. Marketing strategy variations may be employed in an effort to force the demand pattern, level and variance to a level which produces minimum costs or maximum service level or both. Thus, packaging (larger or smaller sizes), promotion (encouraging larger or smaller orders) price (increase level of demand or decrease the variance), credit policies (affect level) and a host of additional marketing instruments may be employed to shift the demand distribution, level and variance.

5. Environmental factors also play a role in the distribution of demand, its variance and level. Competitor pricing, advertising, and channel alterations all impact upon the demand for a substitute product. To this extent, effects may be seen on the time pattern of demand. Thus it may be necessary to alter channel policies to cope with these changes.

Economic conditions also have certain impacts on demand patterns. Witness the time pattern of demand for gasoline today versus its pattern six months ago. Economic slow-downs may slow the rate at which some products are bought (luxuries, entertainment, etc.) and thus affect demand variance and level. Hence, the need for sound forecasting to detect these economic shifts and thereby adjust channel operation to account for the changes. Inventories may have to be increased within the channel and provisions made for a different mode of transportation.

6. Changes in demand level will require modification in channel system performance. The primary factor in this case is the extreme seasonality that some products experience. For example, greeting cards experience rather substantial shifts in demand level during certain seasons of the year. The findings indicate that per unit costs may actually decline in situations where high levels of average demand occur. Additionally, fewer stockouts are also experienced. However, inventory within the channel will have to be substantially increased. Thus, the implication is that seasonal types of merchandise may require quite different channel system policies depending on the conditions that exist.

7. Where little elasticity exists for high service levels, the effect of uncertainties may not be meaningful. The research shows that demand uncertainties across the board affect service levels, i.e., the type of demand distribution, the variance of demand and the level of demand in almost all cases affected the number of stockouts which occurred. The same could not be said for costs. Transport, inventory and facility costs were not affected by all types of uncertainty in all

cases. For those instances where service level considerations are not paramount, the need to accurately assess the time pattern of demand is not acute. It may not be worth the expense. However, even though the chances are good that costs may be unaffected, the possibility of cost reduction exists, especially if demand is extremely variable.

If service level is extremely important, efforts to either change the time pattern of demand, or if that is not possible, provide sufficient inventory at the proper place within the channel will be required. The type of demand distribution and variance will indicate the amount of safety stock required (i.e., the results indicate that the exponential distribution would require substantial increases in inventory to provide a 100% in stock condition; the normal something less than the exponential, etc.).

8. The area of costs as related to the time pattern of demand, although not as sensitive as service level, cannot be ignored. Two things seem to stand out. Some costs are influenced more by demand uncertainty than others. Thruput costs are basically insensitive to demand uncertainty whereas transportation costs are affected by all types of demand uncertainty. Inventory and facility costs are affected by variance and level. If transportation costs are large relative to inventory costs, then efforts to force demand patterns and variances to situations where transportation can be minimized and service level at least maintained are required. In this case, inventory costs may rise, but the total cost effects will be small.

The second aspect of cost behavior is that costs at each stage in the channel are not affected the same by demand uncertainty. For example, the exponential distribution has major impacts upon the retail inventory, causing frequent stockouts. However, because of a decreased frequency of ordering between the retail and wholesale level, wholesale inventories tend to build. Thus, inventory costs are quite different, with low per unit costs at retail and higher ones within the channel.

9. The preceding paragraph indicated that inventories are differentially affected within the channel depending on the nature of demand uncertainty. Thus, the question of where in the channel system major inventory accumulations should be held will be answered differently depending on the conditions surrounding the time pattern of demand.

The research findings indicate that the major impacts of the normal distribution of demand with large variances are at the SSP and PSP level in the channel. Thus, some inventory is required at the ISP level, but additional inventory should be held at the SSP and PSP to avoid frequent stockouts. Such a policy would also have cost implications. If more inventory is maintained in the channel, partial shipments will be reduced and transport costs lowered. However, inventory and facility costs would increase. The net result may be beneficial as, in this case, the transport costs are a relatively large portion of total costs.

The reverse situation holds for distributions like the exponential. In this case, larger inventories are required at the ISP to meet widely fluctuating demands. Thus, there exists the possibility for some interfunctional transfer among the levels within the channel.

10. A significant impact of this research is the complete reaffirmation of the system's viewpoint in the channels area. The need for channel integration and individual firm systems thinking is highlighted by the research results. As the previous discussion on inventory indicated, the effects of demand uncertainty are felt throughout the channel and in many cost areas at each level within it. Thus, when the channel confronts a widely varying demand situation, individual optimization of efforts by each individual within the channel will not lead to efficient and effective channel-wide operation. If minimal inventories are held by the SSP when the normal distribution generates demand, the stockout situation at the ISP level will accelerate and transport costs will rise as larger numbers of partial shipments are made. Therefore, demand uncertainty underscores the critical need for integrated channel planning and operation.

11. Systems orientation and the tradeoff approach to management are not only required for the channel but also the firms within the channel. As indicated previously, marketing strategy can affect the time pattern of demand and the cost and service level of the firms and channel system. Thus, the retailer cannot adjust his marketing strategy without considering the demand distribution effects. In terms of the "total cost" or tradeoff approach, different demand patterns and variances make it necessary to evaluate the tradeoffs that could be made. Where demand is variable and the distribution is symmetrical, transport costs tended to rise, inventory costs fell and stockouts also occurred. However, by building up the inventory within the channel (SSP, PSP),

savings in transport could be realized and service improved. The result would be a higher inventory cost.

12. The need to assess the demand pattern is also felt in the area of budgeting. To adequately plan for the funds needed for the ensuing operating period, an assessment of all requirements is called for. Thus, some knowledge of the time pattern of demand will indicate those areas for which costs will be incurred, at least compared to the present situation.

13. In the area of modeling the channel system, the results of this research should provide some definitive guidelines. The model builder cannot assume that any distribution, level or variance will provide the same results. It will be necessary to then develop computer programs capable of generating a stream of demands that closely approximate the demand distribution of the real world system under investigation. However, if the real world demand distribution can be assumed to be one of those for which no differences in cost or service exist, then the distribution which is most tractable should be employed.

14. Finally, this research tends to confirm the results achieved by Forrester's simulation of distribution systems.[2] That is, as variance increases (in demand), effects are felt throughout the system, and the effects in the upper levels of the system tend to spiral or accelerate. Thus, with highly variable demand, stockouts at the SSP and PSP continue to increase and these levels are never able to "catch-up," i.e., hold any excess inventory.

Limitations of the Research

Any simulation study is constrained to the extent that the simulation model accurately replicates the real world system. The present research is not free of that constraint. However, the LREPS model has been subjected to extensive validation tests and has been judged to be valid.[3]

This particular model version of LREPS is also subject to evaluation as to its validity. The model has been stripped of many important features--lead times are fixed and constant, there are no behavioral dimensions to channel interrelationships, location is not varied, backorders are not evidenced at the ISP level, etc. To the extent that these features would change the direction of the findings, the study is constrained. However, the factors held constant in the research are maintained that way so that the variable of interest, demand uncertainty, could be evaluated without any other stochastic factors altering its effects.

The findings of the research are also limited by the lack of replication of experimental runs. In some cases, only one degree of freedom was available for difference between means tests. Hence, the possibility of falsely rejecting a true null hypothesis is quite real. Additionally, more replications would have allowed for better estimates of sampling error, and the ability to make cell by cell comparisons. Because of the limited experimental runs, nonstatistical comparisons of individual cells had to be made.

A further constraint upon the research is that various policies relative to channel operation could not be tested. Thus, an EOQ method of inventory control was employed, and to the extent the results would differ under different policies the research results are limited.

Future Research

The research seems to have generated a wide spectrum of researchable areas. An important area for further research efforts would be in the area of extended validation of the results indicated by the present effort. A number of areas are suggested. Additional replications of the basic runs could be made to further confirm the findings and to provide additional data so that cell by cell comparisons could be made on a statistical basis. Empirical justification would seem particularly relevant. Such efforts would not only serve to validate the present conclusions but to also provide some measure as to the effects of the variables which were deleted from the model employed in the research.

An extremely fruitful area for additional research is that of assessing the impact of different operating policies under the various conditions of demand uncertainties. In this vein, a number of experimental runs could be made, using different inventory policies. The transport mode could also be altered and the effects noted. In this way answers could be provided as to how to optimally design the channel system when a given level of demand uncertainty prevails. The research presented here was not designed to provide answers to how to design and administer an optimal system. The results could only provide some clues as to the direction in which the channel activities (inventory and the like) should be changed. Hence, additional research designed to provide answers to the magnitude of such changes is thereby indicated.

The research results could also be expanded in terms of generality if the experiments were to be conducted with products whose characteristics are different than those assumed in the present research. A wide range of different product classes could be narrowed into somewhat homogeneous groups, and the results with these products compared to the present findings. In this way, the impacts of channel uncertainty could be more precisely measured.

Research supplemental to the present effort may serve to add to the base of knowledge about channel system performance. Two areas of research seem most relevant. Research related to assessing empirical demand distributions, variances and levels would serve to enhance the value of the findings developed in the present study. Actual field experimentation with present methods of determining the time pattern of demand may provide answers as to which is the best technique. Secondly, research into the impacts of marketing strategy decisions and logistical policies on the time pattern of demand would follow from success in refining methods of demand pattern estimation. If such impacts could be more accurately assessed, totally integrated channel planning and operation would become a reality.

Finally, the measurement of the joint effect of the principle uncertainties affecting a channel system--demand and lead time, would be the most logical next step in the continuation of research in this area. Additionally, the impact of a backorder system at the ISP level provides another fruitful area for investigation.

To provide a tentative indication as to the type of results that might occur under the situation of uncertain demand and lead times, with and without backorders, this research effort was coupled with that relating to lead times and a number of joint experimental runs were made. It is hoped that the results of these runs will provide some indication as to the direction future research in this area should take. The results are reported in the following postscript chapter.

CHAPTER VI--FOOTNOTES

[1]Charles C. Holt, Franco Modigiliani, John F. Muth and Hubert H. Simon, Planning Production Inventories and Work Force (Englewood Cliffs, N.J.: Prentice-Hall Inc., 1960), pp. 286-387; E. Martin Basic, "Development and Application of a Gamma Based Inventory Management Theory" (unpublished Ph.D. dissertation, East Lansing, Michigan, 1965), p. 8; and Martin Beckman and F. Bobkoski, "Airline Demands: An Analysis of Some Frequency Distributions," Naval Research Logistics Quarterly, 5 (March 1958), 48.

[2]Jay W. Forrester, Industrial Dynamics (Cambridge, Mass.: The MIT Press, 1961), pp. 47-59.

[3]Peter W. Gilmore, "Development of a Dynamic Simulation Model for Planning Physical Distribution Systems: Validation of the Operational Model" (unpublished Ph.D. dissertation, Michigan State University, 1971).

CHAPTER VII

A POSTSCRIPT: AN EXPLORATORY INVESTIGATION INTO THE EFFECTS OF VARIABLE DEMAND AND LEAD TIME

Introduction

Purpose

This chapter is designed to go beyond the scope of the present research and provide a more definitive statement on the course of future research. The chapter displays those elements of the model (omitted in the present research) that should be introduced in future research efforts. While this research was being completed, simultaneous research was being conducted where demand was held constant and lead time was allowed to vary.[1] The identical model was used with identical structure, operation and decision rules. The primary difference was in which type of uncertainty was held constant and which type was allowed to vary.

One of the purposes of the present research was to construct a foundation upon which future research could be conducted. Closely allied with that purpose was to suggest future research in which variables could be added to the model so that more complete information could be gained as these variables were systematically added. As indicated in the conclusions of this research, one of the most important variables to add would be variable lead time.

It is realized that the addition of variable lead time opens the door to research that cannot be accomplished with a few runs or a simple cursory look. It is not the purpose of this chapter to make a definite statement regarding the interaction of variable demand and lead time. Without at least one combination run, however, it would be very difficult to hypothesize the type of behavior that would result by combining demand and lead time, thus, it would be impossible to clearly indicate the most fruitful path for future research. With a few combination simulation runs the above question can be more easily answered.

Scope

To accomplish the above purpose, it was necessary to bring together in one simulation run variable lead time and variable demand. Decisions regarding the particular distributions used, modifications to the model, if any, and the number of simulation runs had to be reached. It is the purpose of this section to indicate and justify these decisions.

In the research with variable lead time, six different lead time distributions were run with two levels of variance and two lead time durations. In the research where demand was allowed to vary, six distributions were used, three levels of variance and two demand levels. Of all the possible combinations, the following combination runs were used.

Run 1. Gamma lead time with coefficient of variation = .375 and lead time = 7 days and

Gamma demand with coefficient of variation = .50 and daily demand = 75 units.

Run 2. Exponential lead time at 7 days and

Gamma demand with coefficient of variation = .50 and daily demand = 75 units.

Run 3. Exponential lead time at 7 days and

Normal demand with coefficient of variation = .50 and daily demand = 75 units.

More than one simulation was run to assure that the results were not atypical. Runs were chosen which had created differences when observed alone. Runs were also selected that represented two different points on a continuum from little effect to the greatest effect. Furthermore, runs were chosen which seemed to closely represent reality. The exponential distribution is well suited for lead time and the gamma and normal are well suited for demand.

No modifications were made to the simulation model when these runs were made. Thus, the same structure, operating procedure and decision rules applied. Any shift from the original conditions would have caused doubt as to the reasons for the behavior seen.

In addition to the three runs outlined above, one additional run was made with normal demand and exponential lead time. In the second run, stockouts at the ISP were filled which was not the case in any previous runs. The purpose of making such a run parallels the reason for making combination runs. One more piece of reality is added into the model, more specific future research areas can be offered and the addition of backorders exemplifies the procedure which should be used in future research. Adding a backorder provision at the ISP displays that lost sales can be captured at an additional cost. An analysis of the cost to capture these backorders was desired and an indication of the effects on the system in general, as a result of backorders at the ISP, was desired.

Thus, decisions regarding the backorders had to be made. With backordering at the ISP, the system would run the same as it did without backorders, except for the following modifications:

1. Demand that could not be satisfied from stock was recorded at the ISP and held for delivery when stock was available.

2. The order decision rule of EOQ was maintained and one modification made. When inventory on hand and on order drops below the reorder point, an additional order is placed. The demand recorded at the ISP depletes this total. Thus, if on a particular day an ISP has no stock on hand and receives an order for 75 units, these units are removed from the on hand-on order total. If as a result of that demand, the on hand-on order drops below the reorder point, an additional order would be made. Thus it would be possible to have more than one order in process at the same time.

3. To maintain a backorder system, additional costs are incurred. Additional costs were accounted for in the following ways:

    a. Order processing costs doubled from $5.00/order to $10.00/order.

    b. Handling at the SSP increased from $2.00/pallet to $4.00/pallet.

    c. All backorders moved at the same partial shipment rate of $4.53/cwt.

    d. A per unit charge of 10 cents was included to cover the cost at the ISP of recording the order and performing tasks generated by the backorder.

Although the backorder scheme presented above is only one of many that could be considered, it was felt that this scheme is representative.

Obviously, many changes could be made predicated on many different objectives. The problem of such a decision is representative of the type of problems that will confront future researchers in this area.

Research Questions

These simulation runs are exploratory in nature for they were made to enable a more definitive statement on future research and display the type of factor adding that should be done. The limited number of runs allows no statistical inferences as to the actual behavior of the systems running together. Therefore, a statement of hypotheses would be improper. More realistically, it can only be indicated as to the type of behavior that is anticipated.

For those runs where there are no backorders at the ISP, it was anticipated that the introduction of variable lead time would simply compound the results with only variable demand. Thus, total cost would go up and stockouts would increase.

Where backorders are allowed at the ISP, two conditions were anticipated. The costs would increase and demand satisfied would increase because of the model design. However, the amount of increased cost was basically unknown and the effects on the system simply due to the presence of a backorder rule at the ISP were unknown.

Experimental Findings

Table 7-1 presents the major output responses for the three experimental runs described earlier in this chapter. Additionally, the output responses for the situation where demand is fixed (and lead time

Table 7-1. Experimental Results with Variable Demand and Variable Lead Time

Response Variables	Variable[a] Demand and Lead Time Demand: Gamma (75, .5) Lead Time: Gamma (7, .375) No Backorder at ISP	Variable Demand Fixed Lead Time Demand: Gamma (75, .5)	Variable Lead Time Fixed Demand Lead Time: Gamma (7, .375)	Variable Demand and Lead Time Demand: Normal (75, .5) Lead Time: Exponential (7) No Backorder at ISP	Variable Demand and Lead Time Demand: Normal (75, .5) Lead Time: Exponential (7) Backorder at ISP	Variable Demand Fixed Lead Time Demand: Normal (75, .5)	Variable Lead Time Fixed Demand Lead Time: Exponential (7)
Percent of demand stocked out	13.66	.98	12.62	22.35	21.00[b]	5.64	22.10
Total cost (¢/unit)	129.00	124.90	130.00	137.00	127.13	124.60	141.00
Transportation (¢/unit)	103.00	108.00	102.50	104.50	106.00	111.00	101.50
Facility (¢/unit)	18.33	10.48	19.50	23.88	13.20	7.53	29.65
Thruput (¢/unit)	4.84	4.60	4.84	4.99	4.88	4.60	5.17
Inventory (¢/unit)	2.93	2.02	3.09	3.81	2.30	1.51	4.65
Special handling (¢/unit)	0.00	0.00	0.00	0.00	0.33	0.00	0.00
Special expediting (¢/unit)	0.00	0.00	0.00	0.00	0.42	0.00	0.00
Stockouts per day--ISP	84.00	12.00	75.70	142.00	127.00[b]	36.30	133.00
Stockouts per day--SSP	48.00	68.00	72.40	50.00	118.50[b]	225.90	19.00
Stockouts per day--PSP	0.00	140.00	0.00	0.00	0.00	426.60	0.00
Average inventory turnover--ISP	30.30	21.44	30.10	25.09	31.40	24.63	21.80
Average inventory turnover--SSP	33.60	33.24	9.78	27.60	54.90	55.07	23.50
Average inventory turnover--PSP	9.84	64.00	30.78	7.52	27.12	134.26	5.61

[a]The mean and coefficient of variation are given in parentheses after the distribution name.

[b]Refers to the amount of stockouts which occurred and were backordered.

is represented by the exponential and gamma distributions) and where lead time is fixed (and demand is represented by the gamma and normal distributions) are presented for comparison purposes. Because only three experimental runs were made, no statistical tests were made on the results. Thus, the findings are presented from a nonstatistical basis and no statistical inferences to the relevant populations can be made. The discussion of the findings will be presented on a general level.

Combination Experimental Runs (No Backorders)--Demand Stocked Out

In both combination runs, the percent of demand stocked out was greater than it was under either the variable lead time--fixed demand or variable demand--fixed lead time case. However, the percent stocked out did not increase greatly as compared to the lead time situation (13.66% vs. 12.62% and 22.35% vs. 22.10%). The result is rather unexpected as it might be hypothesized that the stockout rate would be tremendously magnified as a result of combining the two types of uncertainty.

It does appear that lead time has a much stronger impact upon stockouts than does demand variability. The stockout rate associated with variable demand and fixed lead times was 1.98% for the gamma and 5.64% for the normal. Thus, the stockout percentage, when both demand and lead time were variables, is not pulled in the direction of that which occurs under variable demand, but in the direction of, and in excess of, the results which occur under variable lead time.

The effects of the combination of variable lead time and demand appear to be felt at the ISP level. Stockouts per day increased at the ISP level as compared to the runs where one of the uncertainties was fixed. However, in both cases, stockouts at the SSP and PSP decline significantly from those occurring when only demand is variable. As compared to the variable lead time--fixed demand situation, SSP stockouts decline in one case (72 vs. 48) and increase in the other (19 vs. 50).

In summary, a variable demand--variable lead time condition appears to produce higher stockout rates than occur when only one factor is variable. The effect seems to be slight, and certainly not as great as might be expected. In fact, the stockout rates in the combination runs did not even exceed the sum of the stockout rates from the one variable factor situations. The effects of the combination runs are seen at the ISP level, thus allowing larger inventory buildup within the channel. (The SSP has a stock turnover rate of only 10 times in the combination run.) The relative increase of inventory within the channel creates a buffer for extremely large demands emanating from the ISP's and the possibility of longer than average lead times. Thus, the two types of uncertainty seem to create a type of "cancellation effect" whereby their combination does not produce stockouts greatly in excess of that produced by one of them. If specific short lead times (below average) were matched with a series of extreme demands, the chance of a stockout is substantially reduced in that instance. Such occurrences would seem to explain the relatively low stockout rates associated with the combined runs.

<u>Combination Experimental Runs (No Backorders)--System per Unit Cost</u>

For both combination runs, the total cost per unit incurred by the channel system fell between the total cost levels associated with those runs where one of the experimental variables was held constant. As with the percent of demand stocked out, the effect of lead time has a much greater impact than does demand variability. The total per unit cost is drawn more closely to those associated with lead time uncertainty. Again, it is somewhat unusual that the total cost is not in excess of the uncertain lead time cost. The "cancellation effects" appear to be operative for costs also.

Facility and inventory costs for the combination runs are very much above those obtaining as a result of variable demand, and very close, but below those incurred with the variable lead time run. In opposition to the general trend are transportation costs, which, for the combination runs are substantially below the variable demand situations and above those associated with variable lead time.

These results may be explained by reference to the inventory situation in the channel. In general, variable demand creates serious impacts within the channel because of the relatively constant demand that the ISP's put upon SSP inventory. This impact tends to create stockouts in the channel, which reduce average inventory at the SSP and PSP, increase inventory turnover and thereby increase the number of partial shipments experienced within the channel. The variable lead time tends to impact more directly at the ISP level, thereby reducing pressures on the SSP and PSP inventories. These average inventories build, and stock turn declines. However, the number of partial shipments are reduced, and thus transport rates are lower.

When the two situations are combined, the effects of both are felt, with the lead variability predominating. The overriding impact of lead time is explained by the fact that a single extreme demand on any given day does not have the impact that a single extreme lead time might have. In other words, demand over lead time is the relevant consideration when looking at demand variability. Thus, an extremely large demand will more than likely be offset by an extremely small demand over a constant lead time of seven days. It is the occurrence of a number of extremely large demands over lead time that produce stockouts. Therefore, demand variability tends to average, and only in those cases where a stream of large demands are evidenced do stockouts occur.

However, when lead time experiences extreme variation in the form of a very large number of days between order transmittal and order receipt, demand at a constant amount per day will be lost for most of those days by which the average lead time is exceeded. Thus, one extreme lead time deviation can measurably increase the stockout rate. Generally, one extreme demand cannot. Therefore, the lead time uncertainty, when coupled with demand uncertainty, tends to have the greatest overall impact on channel performance. However, the effects do appear to cancel to some degree since the costs are not above those associated with the fixed demand--variable lead time situation.

Combination Experimental Runs
(Backorders)--Demand Stocked Out

When the ISP was allowed to backorder, all demands presented were eventually satisfied. However, in the backorder case the number of stockouts which occurred (and were eventually filled) is less than the situation where no backorders were made. The explanation lies in the nature of the backorder process. Anytime the on hand and on order inventory dipped below the reorder point, an order was placed for the EOQ. Thus, in the case of backorders, a given day's demand may trigger an order, but an order for the entire economic order quantity. In the no backorder case, such an order would not be placed on the same day because the demand was lost. Thus, in the backorder situation, you may in fact order when you have more on hand and on order then is necessary to cover demand over lead time because the stockouts are recorded and backordered. Therefore over a period of time, there may be more inventory on hand at the ISP level than in the no backorder case, and thus fewer stockouts.

The backorder procedure did not appear to produce severe strains throughout the channel. Compared to the no backorder case, inventory turnover increased at all levels within the channel. Thus, ISP stockouts did not continue to build an abundance of inventory in the upper levels of the channel. In fact, the whole system appeared to function much more smoothly than it did under the no backorder case. There are additional stockouts at the SSP, as might be expected when unsatisfied demand at the ISP is backordered. The effect on overall system performance is not great.

Combination Experimental Runs
(Backorders)--System Cost per Unit

Total system cost per unit is very close but higher than those occurring when only demand is variable and materially lower than those with variable lead time. Additionally, total cost per unit are almost 10 cents per unit lower than those with the combination run and no backorders. The greatest difference in cost in comparing the backorder case to the no backorder case appears in inventory and facility cost. The backorder system is much lower for both costs (23.88¢ vs. 13.20¢ for facility and 3.81¢ vs. 2.30¢ for inventory). This result reiterates the findings suggested above, that the system functions more efficiently in the backorder case because inventories did not build within the system. Thus inventory turnover increases, and facility and inventory costs decline as average inventories are held in check. Transport costs are higher with the backorder situation due to the impact of premium transportation rates for the backordered merchandise.

The additional costs associated with the backordered goods, when allocated across the total units sold, does not have a great effect on total per unit cost (2.3¢ increase, a good portion of which was transportation). However, Table 7-2 reveals the impact of the additional costs on only those units backordered. Thus, 56 cents per unit is associated with backorder costs, which would increase the cost/revenue ratio of .36 for these items. (The ratio is .26 in the no backorder case.) It would therefore be necessary to compare these additional costs to the channel wide margin to assess whether it would

Table 7-2. Cost per Unit Associated with the Units Backordered under Variable Demand and Lead Time

	Backordered Units: Additional Cost per Unit (¢)
Transportation	32.00
Ordering	8.00
Special handling	6.00
Special expediting	10.00
	56.00

Total units backordered: 15,287

Total cost of backordering: $8,565.16

Cost/revenue ratio of units backordered: $\frac{1.81}{5.00} = .36.$

be worthwhile to backorder the goods stocked out. However, it should be realized that backordering did produce positive system-wide effects to the extent that inventory and facility costs declined as compared to the no backorder situation. Also the effect on future demand of eventually fulfilling present demand must also be evaluated in considering the backorder costs.

Implications for Future Research

The primary purpose for running combination variable lead time and demand simulations was to focus on some of the immediately useful areas of future research. The combination runs were also designed to display the viability of the research approach of creating a foundation and then adding back elements of reality, thus increasing the complexity of the model. These two goals have been met with these few combination runs.

Specific recommendations for future research lie in the particular combination of runs which should be made. Given efficiency and effectiveness as the goal, lead times with small variances and demand with symmetrical distributions and low variances should be combined, thus revealing if those combinations will, in fact, reduce costs and stockouts. Furthermore, combinations which appear as though they will neutralize one another's effects should be tested. For instance, lead time was found to be the dominant factor, but its effects could be dampened somewhat with a particular demand. There must be some types of demand that will dampen the effect of lead time more than others.

Decision rules regarding the specific activity centers should also be considered. It was found that transportation costs, inventory costs and facility costs reacted in predictable ways to certain stimuli. Therefore it is known how these costs will react and it now becomes necessary to create decision rules and/or system structures and functions which would enable these costs to move in the direction determined by the planner or modeler. In conjunction with looking at specific activity centers, the ordering procedure should be investigated.

The decision rules employed in the system regarding ordering procedure seem to have a significant impact. With an economic order quantity rule the stockouts primarily caused by variable lead time seem to be allowed to persist. Under these conditions an ISP waits until stock reaches a prescribed level before ordering. This prescribed level is primarily dependent upon average demand and average lead time. Thus, as soon as a lead time beyond the average is realized, a stockout occurs. If the decision rules were variable quantity based on fixed time, the problems would probably remain because both are predicated upon estimation of average demand. There is a definite need to employ an order decision rule which more accurately accounts for variable lead time.

To overcome the stifling effects of variable lead time we can go one of several ways: (1) control lead time, (2) be able to better predict its variability, or (3) be better prepared for the unexpected. Each alternative presented has its pitfalls and each has its associated costs. However, that is not the question at hand. More importantly, the significance of lead time variability has been established, and a prime area for research, regardless of the path, has been established.

The combination runs and the previous discussion on decision rules reemphasizes again two major areas of concern in distribution: (1) the systems concept at the channel level, and (2) the behavioral problems created by autonomous ownership of institutions in the channel. Although the combination runs did not compound the effects as anticipated, they did not make efficiency any better. More interestingly, the points in the system which feel the pinch seem to shift. With demand it was the PSP and SSP, with lead time it was the ISP, with demand and lead time together it was primarily the SSP. It seems realistic that if the channel worked in concert, pressures and profits could be spread around in such a fashion as to make the entire system more efficient. The consumer is uninterested in how a product arrived or the status of the channel members; the consumer will patronize that channel which delivers the goods. Thus research, into a unified channel (one that doesn't optimize the individuals but rather optimizes the efficiency and effectiveness of the channel) is required.

If viewing the channel as a system is paramount, then investigation into the behavioral aspects must be of parallel importance. It has been shown that channel member cooperation could significantly increase the efficiency and effectiveness of a channel. The "I'm an island" mentality must be abolished and "Its my team against yours" must be adopted. This is a tall order, but one that must be explored if distribution efficiency and effectiveness is to be reached.

Conclusions

As a result of the combination runs presented in this chapter, specific areas of future research and those areas which need immediate attention have been indicated. In addition, even though the results are not conclusive, the procedure of adding back variables is workable. As a result of the combination runs, logical cause-effect relationships could be followed from one model variation to another.

CHAPTER VII--FOOTNOTE

[1]George Wagenheim, "The Performance of a Physical Distribution Channel System Under Various Conditions of Lead Time Uncertainty: A Simulation Experiment" (unpublished Ph.D. dissertation, East Lansing, Michigan, 1974).

APPENDIX A

LITERATURE REVIEW

APPENDIX A

LITERATURE REVIEW

The primary objective of this appendix is to present a description of the considerations given demand and lead time uncertainty in research concerned with physical distribution. Additionally, multiechelon physical channel simulation models capable of experimentation with uncertain demand and lead time will be investigated. Prior investigations into demand and lead time uncertainty in physical distribution are concentrated in the area of inventory control. Hence, this literature will be examined in the first section of this appendix. The second section reviews the relevant simulation models.

Demand and Lead Time Uncertainties--Inventory

Introduction

The purpose of this section of the literature review is to provide a perspective on the efforts made to examine the impacts of demand and lead time uncertainty on the inventory component of the physical channel system. In fact, most efforts to define and measure the effects of demand and lead time uncertainties in physical distribution have been made by those concerned with developing optimal inventory policies. Uncertainty is part and parcel of the inventory problem because the decision of when to order stock and how much to order is directly dependent upon the level and variability of demand over a variable lead time horizon.

The body of literature relevant to inventory management is extremely broad in terms of the specific problems examined and vast in terms of the number of expositions on the subject. The past two decades have witnessed the growth of a large array of articles, monographs and books concerned with a more or less mathematical treatment of inventory problems. Many contributors to these publications use as their point of departure a mathematical model, and then proceed to derive mathematical solutions and study their properties in great detail.[1] Thus, the emphasis is on determining optimal solutions as to when and how much to order under a copious number of conditions. The literature contains the presentation of optimal decision rules for recoverable items, seasonal goods, spare parts, "one-shot demand" items, slow moving goods, high demand per time period items and the like. The inventory problems associated with single station supply points, multi-facility supply points (many inventory points in the same echelon) and multiechelon supply points are extensively analyzed. Many combinations of certain and uncertain lead time are found within the recent literature. Variations on the basic economic order quantity formulation are abundant. However, in most inventory treatments reviewed, one fact remains: uncertainty in demand and lead time are important elements in the analyses and resulting decision rules. It is fair to say that generally, the focus in the inventory literature is to assume that demand and/or lead time uncertainty have certain characteristics, and proceed to develop the optimal rules. In most cases, uncertainty is not the critical issue, but rather it is noted and the analysis resumes toward its main objective. There are instances in which expositions are given as to the optimal inventory policy to follow when demand and lead time assume given probability distributions. In the main, the inventory literature generally does not contain any broad, systematic analysis of the impacts of demand and lead time pattern, level and variance on a multiechelon inventory system, let alone the physical channel system. There are, of course, exceptions to this statement, and the studies which approach such systematic analysis will be discussed.

The remaining sections of this appendix will be organized as follows. A brief review of a number of the more important inventory textbooks will be presented. The emphasis will be on the objectives of these texts and how uncertainty is considered. Next, a representative sample of the periodical literature relative to single station inventory analysis is reviewed. Again, the consideration of uncertainty will be the focal point. Thirdly, the literature relevant to multi-echelon inventory control with demand and lead time uncertainties is discussed. Lastly, the more generalized systematic attempts to categorize and catalog the overall impact of demand uncertainty on the inventory function are reviewed.

It must be pointed out that the literature reviewed in this section is by no means a collectively exhaustive consideration of all the literature relevant to inventory control and uncertainty. As previously indicated, hundreds of articles exist which explore every facet of inventory control. This review thereby intends to provide a highly representative sample of the types of considerations given to inventory and uncertainty. Furthermore, the review is concentrated on the periodical literature since the mid-1960's. Extensive bibliographies exist for the relevant material appearing before this time.[2]

<u>Inventory Texts</u>

The early 1960's witnessed a great expansion in the publication of inventory textbooks. The great bulk of the most important texts in the field of inventory management were published between 1958 and 1965. Interest in and development of operations research techniques, realization of the importance and cost of inventory and the beginnings of widespread application of computer technology most likely account for development of texts at that time. A brief discussion of some of these texts is presented below.

Robert G. Brown's initial work in forecasting for inventory control appeared in 1959.[3] The objective of the text was to

> show how uncertainty can be kept to an irreducible minimum and how that minimum can be measured and accounted for in a well-designed inventory control system.[4]

Thus, Brown's efforts were focused upon estimating the demand which could occur during a lead time so that the decision of when to order and how much to order could be more optimally made. Emphasis was given to evaluating the characteristics of demand, including the overall average value of demand, trends in average, cycles, noise (random fluctuations) and autocorrelation.[5] Brown concludes,

> Any total demand pattern can be made up by the combination of these components in different proportions. The forecasting problem is to look at the aggregate and to identify and measure each of the components. The method of making these measurements should lead to economical, practical guides for routine decisions as to when and how much to order for replenishment of inventories.[6]

Demand distributions are given extensive consideration, but not in their actual form. The relevant distribution, according to Brown, is the distribution of forecast errors, i.e., the errors represent deviations of demand away from its forecast value. This measure is then the uncertainty associated with demand, and is the relevant variable in setting safety stocks. Additionally, Brown's Appendix A describes methods for generating demand from any given population distribution. In this light, the exponential, hypergeometric, poisson and normal distributions are considered. However, Brown felt the normal distribution to be a good enough approximation for any distribution of forecast errors.[7]

Magee[8] includes a chapter on uncertainty considerations for inventory control. However, he does not elaborate on the nature of the relevant probability distributions. Magee's emphasis is on the basis for scientific methods in inventory control and also on the necessary methodology for practical application. A later text by Magee,[9] although not an inventory text, discusses the concept of a probability distribution of demand, concluding that the pattern of individual customer orders is log normally distributed.[10] Holt et al.[11] devote a significant portion of their book to inventory problems. Chapter 15 describes empirical work done on determining demand distributions. The log normal, gamma and poisson distributions are described.

Lead time distributions, specifically the log normal distribution, are considered separately and in combination with demand distributions. The authors examine the necessary steps to determine the joint probability distribution of demand over lead time. This estimate of the demand over lead time distribution is then applied to determining safety stocks.

The purpose of Fetter and Dalleck's inventory text is to "provide a guide for use in the study of inventory problems which will lead to the development of ordering rules for effective inventory control."[12] They examine the variability of both demand and lead time, and demonstrate methods for dealing with variability. The models developed are primarily for single stations, but include multi-item problems. Probability distributions of demand and lead time are examined, but the normal distribution is generally assumed for lead time and the poisson and exponential for demand. The authors also note the importance of predicting future variability of both variables, and indicate that it is necessary to find a statistical distribution that is capable of generating the data desired for forecasting. However, empirical distributions may suffice if their pattern is not expected to change.

Hansmann[13] looks at inventory problems as static or dynamic, one or many items and single or multiechelon. He thus develops operating rules necessary for each situation. Hansmann indicates the need for forecasting demand distributions, and also includes probabilistic demand within his models, but spends little time discussing the various types of demand and lead time distributions and their effects.

Starr and Miller,[14] in presenting optimal inventory rules, also consider dynamic and static models. However, they make a distinction between the degree of uncertainty facing the decision maker. Thus, all inventory problems are analyzed under each of three conditions--certainty, where demand is known exactly; risk, where the probability distribution of demand is known; and uncertainty, where the distribution is unknown. The normal distribution of demand is used in most examples because of its tractability. However, the authors indicate that solution of the inventory problems under risk will not be diminished because the normal was used. Without assuming the normal the analysis would simply be more difficult. Additionally, the models are also analyzed under constant lead time and probabilistic lead time, and the differences in operating rules noted.

A strong mathematical orientation is the focus of Wagner's text.[15] However, broad coverage is afforded probability distributions of demand and their effects on operating decisions. A large section of the text is addressed to determining the relevant demand distributions for both single and multi-item systems. Optimal inventory policies are developed for the case of gamma, normal, poisson, geometric, negative binomial and uniform demand distributions. Additionally, lead time is seen as a "delivery lag" whose duration may be variable.

Hadley and Whitin[16] present the techniques for constructing and analyzing mathematical models of inventory systems for a single stocking point. Rather extensive treatment of demand and lead time uncertainties are developed throughout the text. Various distributions for demand and lead time are studied (including poisson, gamma, exponential, normal and negative binomial) and optimal policies thereby developed. They state that the normal distribution can be used for approximating the others, but it is really not known how rapidly each approaches the normal or how much error there is when the normal is used as an approximation. The convolution of demand and lead time distributions is also examined, and the resulting demand over lead time distributions developed.

The problems involved with securing information on demand and lead time distributions (and with the case where demand changes over time) are considered. The authors suggest the use of empirical data or theoretical distributions. However, a great deal of empirical data is required so that enough information can be gained as to the tail of the distribution. They also conclude that lead time information is much more difficult to secure.

Prichard and Eagle[17] take a somewhat less rigorous mathematical approach to inventory control than do other texts. However, they do have an excellent chapter dealing with uncertainty and probability. The nature of the demand distribution is presented in terms of the normal, poisson and negative binomial distributions. The conditions where each apply are discussed and supporting empirical evidence provided. Additionally, demand over lead time, with lead time and demand both variable, is developed and the impact upon safety stock shown.

Brown's[18] text of 1967 is basically an update of his earlier book. The primary emphasis again being the forecasting of all demand components--level, trend, seasonal and random. The distribution of demand over fixed lead time is investigated.

In summary, the objective of most inventory texts is to develop optimal policies of when to order and how much to order. The impacts of uncertainty are evaluated to the extent that different types of uncertainty lead to different decision rules. The texts vary in terms of the total consideration given to demand and lead time uncertainty. However, most texts assume a given distribution and then proceed to mathematically determine optimal policies.

Single Station Inventory Control

The recent periodical literature in inventory control is focussed upon very specific topics, i.e., management of seasonal goods inventory, control policies when demand is gamma distributed, order policies when lead time is dependent on demand and the like. This section will briefly review the recent literature in terms of the specific problem under investigation and the way in which lead time and/or demand uncertainty is handled.

Kaplan[19] considered the development of optimal policies with variable lead times. His purpose was to "characterize optimal policies for a dynamic inventory problem when the time lag in delivery of an item was a discrete random variable with a known probability distribution."[20] An interesting conclusion of his analysis is that the inventory policies which resulted were very much like those which obtain when lead times are deterministic.

Lead time, expressed as a stochastic variable with a given distribution was considered by a number of authors interested in optimal policies for an (S-1,S) inventory model. The (S-1,S) inventory policy means that whenever demand occurs for a given number of units, a reorder is placed for that number of units regardless of whether there is a stock of units on hand. Gross and Harris[21] studied the model for the case when lead times are dependent on the number of backorders. In their model, the service time contribution to lead time is an exponential distribution. Demand variability was also considered, and policies developed on the basis that demand is a compound poisson distribution.

A number of variations on the theme (S-1,S inventory policies) were considered. Galliher, Morse and Simond[22] looked at a number of possible situations. They considered an arbitrary demand distribution and constant lead times plus the poisson demand and exponential lead time distribution. Rose[23] evaluated the expected number of backorders and resupply times for the (S-1,S) policy when demand is arbitrary and lead times are constant. Hadley and Whitin[24] consider the case of poisson demand and arbitrary lead times. Their model includes both the stockout case and the backorder situation.

Particular types of demand distributions were also considered, and the appropriate inventory policy formulated. Sivazlian[25] studied the (s,S) inventory model and developed the optimal values of (s,S) for the case of demand which is gamma distributed. Burgin[26] concentrated on determining safety stock and potential lost sales for the situation in which demand is normally distributed and lead time assumes a gamma distribution. Burgin compares the results achieved from approximating demand over lead time to those achieved when the distribution is directly calculated. The approximation appeared to be adequate.[27]

Hausman and Thomas[28] also considered probabilistic demand, but their point of departure was somewhat different. They considered the type of policy to follow for equipment when there were two types of demands, those for original equipment (deterministic) and those for spare parts (probabilistic). Spare parts demand was considered to be a normal distribution and lead times were fixed. The continuous review policy was judged to work best when the demand for original equipment is small relative to total demand.

Control and management of seasonal or style goods also received some attention in the recent literature. Ravindron[29] evaluated an inventory model where the demand pattern was dependent. Thus, "contagious demand" related to the influence of past demands on future demands. The poisson function was the basic probability function associated with demand. However, a contagious demand rate $\alpha(t)$ was added to the basic function to account for the influence of past demand (i.e., friends' recommendations, word of mouth).[30] The contagious distribution reduces to a negative binomial distribution given certain parameter values. Ravindron proceeds to develop optimal ordering policies and he determines how long the inventory should be carried. Chang and Fyffe[31] attack the same problem concentrating on methods for reestimating sales of seasonal goods during their period of sale.

In summary, the literature relevant to single station inventory models is broad in its coverage of specific problems and conditions. In addition, the formulation of stochastic demand and lead time varied from constant rates of demand and fixed lead times to the consideration of demand with a "contagious demand rate." Again, the objective in viewing demand and lead time as random variables was to formulate optimal inventory policies under the given conditions.

Multiechelon Inventory Control

The literature relevant to multiechelon inventory control is not as abundant as that relating to single station inventory control. As Hadley and Whitin point out, it is very difficult to study analytically multiechelon inventory systems.[32] A brief review of the literature relevant to optimal multiechelon inventory policies indicates how recent its history is. Clark and Scarf[33] were one of the first to formulate the nature of the optimal policy involving uncertain demands in 1960. Fuhuda[34] extended the work of Clark and Scarf. Zangwill,[35] in 1966, studied optimal policies in multiechelon systems where demands are known with certainty. Bessler and Veinott[36] considered a multiechelon inventory system with random demands. They determined optimal policies for redistributing stock from facilities with excesses to those with shortages. The variance of demand experienced by each facility was shown to have an effect on the optimal policy. If the demand variance is less at one facility than another, then the optimal base stock at the first facility may be different than that at the second facility. Additionally, Sherbrooke,[37] in 1968, extended the work done on multiechelon inventory problems. He considered the optimal model for recoverable items.

More recently, Simon[38] studied a two echelon inventory model for low demand consumables or reparable parts. In this work, transportation times were assumed to be deterministic and the failure process generating demands was a poisson process. According to Simon, the results obtained are useful in a number of applications. If costs were imposed, optimal values for s and S could be derived, and if many products were involved, then optimal inventory investments in each product could be derived.[39]

Hockstaedeter[40] builds on the original work of Scarf and Clark. The objective was to determine an approximation to the cost function (upper and lower bounds) for a multiechelon inventory system. In the model both demand and lead time are variable. Demand was considered a random variable, whose particular value was independent from period to period. Lead time was viewed in terms of delivery lags, with the lag being a multiple of the review period.

In summary, the literature relevant to multiechelon inventory analysis concentrates upon devising optimal policies for specific circumstances. Various conditions of demand and/or lead time uncertainty are assumed for a particular model or problem.

Systematic Demand and/or Lead Time Analysis

The last section of the review of inventory literature relates to the efforts made to systematically evaluate the impact of a wide variety of demand and lead time uncertainty conditions. The work reviewed in this section is different from that considered in the previous inventory literature in that the focus is more towards systematically evaluating demand and/or lead time uncertainties on a specific inventory system or physical distribution system, rather than assuming a given form of uncertainty and designing optimal policies for inventory control. Thus, the research reviewed here primarily involves simulation, and more closely approximates the research problem studied in the present dissertation. Three research studies will be considered. However, the work done by Ballou and Camp will be considered in a later section dealing with simulation and will not be reviewed here.

Gross and Soriano[41] simulated an inventory system and studied the impact of various distributions and variances of demand and lead time on the base and safety stock requirements of the system. The major thrust of the research was directed toward evaluating the effects of reducing the average duration of lead time on base and safety inventory levels. More specifically, the authors desired to estimate achievable on-shelf inventory savings for a military overseas resupply system when resupply is performed by air rather than sea.[42] As a by-product of the research, estimates of the impacts of various parameters, such as average demand, variance of demand and lead time, distribution of demand and lead time, inventory review period and order quantity were studied.

The simulated system was a military resupply system (single echelon) with an s,S inventory policy, and periodic review. Demands were withdrawn from inventory in a "lump sum" at the end of a time period. The output measures included average on-shelf inventory and percent of units demanded but not filled from existing inventory. No costs were included in system performance measurement.

Simulation runs were 2,000 weeks in length and were replicated 15 times. Twenty-two cases were investigated, where demand assumed a poisson distribution in seven cases and a normal distribution in fifteen cases. Lead time was either normal, exponential, uniform or constant. Additionally, the variance and average level of each demand and lead time distribution assumed two levels.

The general results of the simulation indicated that reductions in the average lead time (from thirteen to two weeks) led to large reductions in inventory. Lead time variability also affected average inventory, in that lower variation led to lower levels of inventory for a fixed service level (on-shelf inventory availability). The sensitivity of the system to changes in demand variation appeared to be a great deal weaker than the sensitivity to lead time variations. The same is true concerning sensitivity to lead time and demand distributional shapes.[43] The order quantity size has little effect on inventory availability, nor is the effect of changing order quantities sensitive to average lead time. Finally, the length of the inventory review period led to differing performance.

In summary, the literature specific to inventory control indicates the extent to which demand and lead time uncertainties have received attention in the study of inventory. The types of analyses are extensive and varied, with the general objective of achieving optimal inventory policies under assumed uncertain conditions. No systematic analyses of the impacts of uncertainty on a multiechelon channel system were discovered, although the Gross and Soriano work was relevant to a single echelon inventory system. The remaining sections of this appendix will be addressed to evaluating the simulation models which are available for replicating a multiechelon physical channel system.

Model Selection and Criteria

The second section of the literature review concerns a search for a tool through which the objectives of this research can be met. As previously indicated, real world experimentation has been eliminated as a valid option. Thus, a model of some type and specifications must be employed. Creation of a model is not within the scope of the present research. There are many physical distribution system models in existence,[44] and experimentation with a system to increase the understanding of physical distribution is the goal rather than to refine system models or add to the number of models available. Thus, a model must be selected from those presently available.

The first step in model selection is to specify a set of criteria the model must meet. The criteria for the model derive from the objectives of the research. The objectives of this research are

restated in the first section of this review, and the criteria are delineated and discussed.

Given a set of criteria, the existing models can be reviewed and one selected. The selection procedure is to pick a particular criteria which will eliminate a family or group of models. This procedure is repeated until the desired model is found. The review and elimination of families of models and individual models are discussed in the second section.

Criteria

Criteria derive directly from the objectives of the research. Thus an explicit statement of objectives is necessary.

The objectives of the present research are:

1. To measure the effects of uncertainty on a multiechelon physical distribution system.

2. Construct a foundation that will facilitate future research and simulate a system which is an accurate, complete and valid representation of present operating conditions.

3. Meet the above criteria within given time and monetary resource constraints.

To meet the first objective, the model must have the following characteristics: It must be multiechelon and multifacility, encompass all the physical distribution components and be capable of employing stochastic lead times and demand.

To be multiechelon and multifacility, a model must be capable of replicating more than one stage of a physical distribution system and more than one stocking point or facility at each step. In a physical distribution system a step is at least a break bulk point and dispersion point and traditionally holds inventory, i.e., manufacturer, wholesaler, retailer. For this research it is necessary to be able to simulate at least these three steps. Provision for the increase of the number of steps is also desirable. As products pass through the steps of a physical distribution system the geographic dispersion increases thus the number of facilities on each step increase. Thus, the model must be capable of simulating multiple facilities on each step, i.e., two manufacturers, four warehouses, sixteen retailers. The absolute number of facilities available at each level is important for this research and the capability to expand the number of facilities is desired.

The model must be capable of simulating all the physical distribution system components. These components are: transportation, warehousing, inventory, handling and communication. The transportation component concerns the movement of finished goods between stocking points from manufacturer to consumer. It includes pick up, line haul, delivery and back haul. Warehousing concerns the stocking points in the system which hold and handle finished goods. It also includes the networks of facilities, their location, addition and deletion. Inventory refers to the amount of finished goods in the system necessary to overcome the discrepancies between production and consumption. Handling concerns those operations necessary at a stocking point to physically prepare an order for shipping, i.e., picking, packing and movement of the goods within the stocking point. Communication refers to all those activities which verbally link the system together: order communication, order processing, request for a carrier, etc.

Together the above five components completely describe the physical distribution system. If a model did not have all of them it would not completely represent the system. For this research it is desired to have all five components.

To complete the requirements of the first objective, the model must be capable of employing stochastic demand and lead time. The model must be able to function under lead times which have various durations. In addition, the model must be capable of simulating separately the three elements of lead time: order communication, order processing, and order transportation. To accept variable demand the model must be able to continue accurate and valid simulation while the demand fluctuates. Because demand is the initiator of the system, its effects are felt throughout, and the model must be capable of adjusting to variations in demand.

To meet the second objective of constructing a foundation for future research and employing a model which is an accurate, complete and valid representation, the following criteria are necessary. The model must be flexible; it must be capable of simulating an extended time horizon; it must be dynamic, allow for change, be unified on a spatial and temporal basis and be valid.

To be flexible, the model must be capable of operating under various conditions, for instance, one product or many products, different channel structures, order times, different backordering procedures, etc. This is a necessary criteria, for in this initial research the model is stripped of many complicating factors. As future research is attempted, selected elements of the model will be replaced until such time that a replication as close as possible to the real world is achieved. Thus, a single model which can be initially simple and in steps become increasingly complex is needed.

Coupled closely with the above criteria is the specification that the model be capable of long range planning. Closely related to long range planning is the model's ability to be dynamic and its ability to allow for changes in exogeneous and status variables.[45] To be dynamic the model must use the output of one time period as input to the next period. If periods are treated independently, then a series of simulated time periods are treated in isolation. In actuality, future time periods are dependent upon previous time periods. Thus, it is desired to have a model which has this capability. Another fact of life is change. Change occurs both internal and external to the system. A model should be capable of accounting for these changes. Thus once the simulation is in progress, it is necessary to have the capability of changing these variables and having the model account for them.

The previous discussion on variable change and dynamic operation directly affect the time horizon. Any model can be run for an infinite number of days, but if the end result is actually the simulation of a single time period where change is not accounted for, the results are not actually long run in nature. Thus, a model

capable of simulating a long run time frame while being dynamic and allowing for change is required.

The model must also be unified on a spatial and temporal basis. "The unifying dimension of a model is classified as spatial if the cost and/or service are developed on location or transit time. If the model uses order cycle time as the measure of physical distribution performance, the model is classified as temporal or time oriented."[46] It is desired to have a model which is unified on both dimensions. The model should be structured "to cope with inventory planning and facility location on a simultaneous basis thereby integrating the temporal and spatial aspects of system design."[47]

Lastly, to meet the second objective the model should be valid. It would be desirable to have a model which was validated both experimentally and under actual conditions.

Model Selection

Given the previous criteria model selection is now possible. Due to the criteria of multiechelon, dynamic and the inclusion of all physical distribution components, many models can be eliminated, namely those that are single station, static and allow only a portion of the physical distribution components. With the review of inventory in the previous section, the discussion of possible models can be limited to those presented below.

Ballou.--Ballou's model[48] is basically a multiechelon, dynamic simulation model. However, it does not meet the present criteria for the following reasons: (1) it is basically an inventory model with the capability to simulate transportation and communications; however, it does not have the capability to consider the location problem and cannot simulate handling operations at the stocking location; (2) it is a short time horizon model; and (3) the unifying dimension is time without space consideration.

Camp.--In his dissertation, Camp[49] analyzes the effect of carrier service on the location of warehouses. He employs the measures of mean delivery time and standard deviation for carrier service. The model's unifying dimension is space and time; it is heuristic and will allow stochastic lead time. However, it does not meet the present criteria for the following reasons. It is not multiechelon. "The methodology selected to measure results was a heuristic computer simulation of a typical single echelon distribution system."[50] It is not dynamic and is basically designed for a short time horizon.

Distribution system simulation.--The distribution system simulation is a soft ware system designed by Michael M. Connors and others[51] for use on the IBM 360/370 computer. It is unique in the sense that the user does not need to know computer programming. As a result of the answers to a series of questions on a physical distribution system, the system can be modeled and results given. The authors claim the system is extremely flexible. "A large number of different distribution system models--over 10^{12} feasible models can be generated . . . these are all functionally different models not merely parametrically different."[52] The simulation is multiechelon and multifacility in nature. And, apparently will allow stochastic lead time and demand.

As stated by the authors it is "clear that DSS views inventory and product movement as being the key elements in structuring a distribution system."[53] Thus a question arises as to the comprehensiveness of the model. It appears as though the simulation is not dynamic in the sense previously defined. In addition, Sumer Aggarwal points out that the simulation does not directly include facility evaluation nor does it permit inclusion of production subsystems.[54] "It (DSS) assumes that the plant maintains an infinite inventory that can satisfy any demands."[55] The DSS is an extremely complete simulation and closely approximates the "total distribution system." However, a lack of comprehensiveness and dynamic operation eliminates it from consideration.

Markland.--Markland has created a "comprehensive simulation modeling approach to the problem of locating warehouse facilities."[56] The model is multiechelon, multiservice, multidestination and multiproduct. It accepts stochastic lead time and demand and includes transportation costs, warehousing costs, inventory and handling. Apparently, it does not have the communication function and does not break down lead time into the components of order transmittal, order processing and order transportation. It is flexible in the sense previously defined and is capable of simulating several time periods. It is not dynamic in the sense that the output from T_1 is used as the input to T_2. It appears that the simulated time periods are independent.

The Markland model is very extensive and seems to accurately simulate a physical distribution system. However, an incomplete array of physical distribution components and the fact that the model is not dynamic eliminates it from consideration.

Forrester.--In an attempt to overcome the problem of matching production rates with consumption rates Forrester developed an industrial dynamic simulation.[57] It is multiechelon and comprehensive. The components that are included are: transportation, inventory, communication, handling and a fixed set of locations. Because locations are fixed, the multiwholesalers and retailers are aggregated to a single point. The unifying dimension is time and the time horizon is not stated.

Although this model contains the majority of desired attributes, and Forrester's pioneering effort has contributed immensely to simulation modeling, a more satisfactory model exists.

LREPS.--The Long Range Environmental Planning Simulator (LREPS) will be used in this research. It contains all the desired characteristics and meets the stated criteria. Details of the model are available in Chapter IV.

APPENDIX A--FOOTNOTES

[1]Frederick Hansmann, Operations Research in Production and Inventory Control (New York: John Wiley & Sons, Inc., 1962), p. 3.

[2]See any number of inventory texts, specifically: Frederick Hansmann, Operations Research in Production and Inventory Control (New York: John Wiley & Sons, Inc., 1962); G. Hadley and T. M. Whitin, Analyses of Inventory Systems (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963); and Bibliography on Physical Distribution Management published by NCPDM.

[3]Robert G. Brown, Statistical Forecasting for Inventory Control (New York: McGraw-Hill Book Co., Inc., 1958).

[4]Ibid., p. 1.

[5]Ibid., pp. 21, 22.

[6]Ibid., p. 22.

[7]Ibid., p. 167.

[8]John F. Magee, Production Planning and Inventory Control (New York: McGraw-Hill Book Co., Inc., 1958).

[9]John F. Magee, Physical Distribution Systems (New York: McGraw-Hill Book Co., Inc., 1967).

[10] Ibid., p. 42.

[11] C. Holt, F. Modigliani, J. Muth, and H. Simon, Planning Production, Inventories, and Work Force (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1960).

[12] Robert Fetter and Winston C. Dalleck. Decision Models for Inventory Management (Homewood, Ill.: Richard D. Irwin, 1961), p. v.

[13] Hansmann, op. cit.

[14] M. K. Starr and D. W. Miller, Inventory Control Theory and Practice (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962).

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[20] Ibid., p. 491.

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[23] Marshall Rose, "The (S-1,S) Inventory Model with Arbitrary Backordered Demand and Constant Delivery Times," Operations Research, 20 (September 1972), 1020-1032.

[24] Hadley and Whitin, op. cit., pp. 204-212.

[25] B. D. Sivazlian, "Dimensional and Computational Analysis in Stationary (s,S) Inventory Problems with Gamma Distributed Demand," Management Science, 17, No. 6 (February 1971), 307-311.

[26] T. A. Burgin, "Inventory Control with Normal Demand and Gamma Lead Times," Operations Research, 23 (May 1972), 73-78.

[27] Ibid., p. 76.

[28] Warren H. Hausman and L. Joseph Thomas, "Inventory Control with Probabilistic Demand and Periodic Withdrawals," Management Science, 18, No. 5 (January, Part 1, 1972), 265-275.

[29] Arunachalam Ravindron, "Management of Seasonal Style--Goods Inventories," Operations Research, 20, No. 1 (March 1972), 265-275.

[30] Ibid., p. 265.

[31] Sang Hoon Chang and David E. Fyffe, "Estimation of Forecast Errors for Seasonal-Style-Goods Sales," Management Science, 18, No. 2 (October 1971), B89-B96.

[32] Hadley and Whitin, op. cit., p. 6.

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APPENDIX B

EXAMPLES OF STATISTICAL TESTS USED FOR TESTING THE RESEARCH HYPOTHESES: DUNNETT'S TEST, TUKEY'S TEST AND STANDARD t-TEST

APPENDIX B

EXHIBIT I

EXAMPLE OF THE CALCULATIONS TO COMPARE FACTOR AND CONTROL RESPONSES USING DUNNETT'S METHOD

Dunnett's method of multiple comparisons compares the control mean with all other factor means. The formula for the comparisons of control and factor means is:

$$(\overline{X}_j - \overline{X}_c) \pm d \cdot \sqrt{2MS_e/n},$$

where $(\overline{X}_j - \overline{X}_c)$ is the difference between the control mean and the factor mean and $d \cdot \sqrt{2MS_e/n}$ is the confidence allowance against which $(\overline{X}_j - \overline{X}_c)$ is compared. If $(\overline{X}_j - \overline{X}_c)$ exceeds $d \cdot \sqrt{2MS_e/n}$ the difference between the factor mean and control mean is significant. The value of "d" is based on the level of significance and is found from Dunnett's tables. The value of MS_e is derived from the AOV tables in Chapter V.

Comparison of mean transportation costs: control versus distributions:

$$d \cdot \sqrt{2MS_e/n} = 2.75^* \cdot \sqrt{2\ (1.623)/6} = 2.0225$$

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*The value of "d" from Dunnett's tables for an .05 level of significance.

Distribution	Mean Transport Cost	Mean Control Cost	$(\overline{X}_j - \overline{X}_c)$	Confidence Allowance	Significant
Normal	115.13	114.42	.71	2.0225	No
Log normal	114.08	114.42	-.34	2.0225	No
Gamma	115.20	114.42	.78	2.0225	No
Neg. Binomial	113.00	114.42	-1.43	2.0225	No

EXHIBIT II

EXAMPLE OF THE CALCULATIONS TO COMPARE THE DIFFERENCES AMONG FACTOR RESPONSES USING TUKEY'S METHOD

Tukey's method of multiple comparisons compares the mean response associated with each level of a factor to the mean response for all other levels of the factor. The formula for the comparisons among factor level is:

$$(\overline{X}_j - \overline{X}_J) \pm q_{m,v} \cdot \sqrt{MS_e/n}$$

where $(\overline{X}_j - \overline{X}_J)$ is the difference between the average response for pairs of response means and $q_{m,v} \cdot \sqrt{MS_e/n}$ is the confidence allowance against which $(\overline{X}_j - \overline{X}_J)$ is compared. If $(\overline{X}_j - \overline{X}_J)$ exceeds the value of $q_{m,v} \cdot \sqrt{MS_e/n}$, the difference between the two means is significant. The value of q is based upon the number of sample means compared, the degrees of freedom, and the level of significance. Its value is found from Tukey's tables.

Comparison of mean transport costs: among distributions

$$q_{m,v} \cdot \sqrt{MS_e/n} = 4.02 \cdot \sqrt{1.623/6} = 2.0906$$

Difference Between All Pairs of Sample Means

j \ J	Log Normal (114.08)	Gamma (115.20	Negative Binomial (113.00)
Normal (115.13)	1.05	0.07	2.13*
Log Normal (114.08	----	1.12	1.08
Gamma (115.20)	----	----	2.20*

*Significantly different at the .05 level.

EXHIBIT III

EXAMPLE OF THE CALCULATIONS TO COMPARE RESPONSE MEANS USING STANDARD t-TESTS

Standard t-tests were used for comparisons between factor responses and control responses and among factor responses when Tukey's or Dunnett's methods were not applicable. The differences between the mean responses associated with factors, or between factors and control were calculated in terms of standard errors and compared to the critical t-value. The decision rules are:

If $\dfrac{\overline{X}_1 - \overline{X}_2}{\hat{\sigma}_{\overline{X}_1 - \overline{X}_2}} < |t|$ Accept the null hypothesis.

If $\dfrac{\overline{X}_1 - \overline{X}_2}{\hat{\sigma}_{\overline{X}_1 - \overline{X}_2}} > |t|$ Reject the null hypothesis.

Comparison of exponential vs. gamma: percentage of demand stocked out:

Critical t = 2.45 at .05 level of significance.

$\hat{\sigma}_{\overline{X}_e - \overline{X}_g} = .944$ $X_e - X_g = 7.36 - 1.18 = 6.18$

$t = \dfrac{6.18}{.944} = 6.54$ $6.54 > 2.45$. Reject that $\mu_e = \mu_g$.

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