A CASE STUDY OF THE EFFECT OF AN INNOVATIVE APPROACH IN A COLLEGE GENERAL EDUCATION MATHEMATICS COURSE

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ABSTRACT

A CASE STUDY OF THE EFFECT OF AN INNOVATIVE APPROACH
IN A COLLEGE GENERAL EDUCATION
MATHEMATICS COURSE

By

Rose Ann Schwartzfisher

The population of the study consisted of college students enrolled in a general education mathematics course.

The purpose of this study was to record and describe the behavior of the subjects taking this course. Specifically, the purposes were concerned with four problems:

1. To determine if the method used in the study was effective in the students' learning of computational skills;
2. To determine if the method used in the study was effective in the students' learning of the historical problems of mathematics;
3. To determine the effect of the method used in the study on the students' attitude toward mathematics.

Data were collected for the analysis of the study from results of the mathematics placement test, placement

post-test, historical problems in mathematics test, problem solving test, and an attitude questionnaire.

The Friedman two-way analysis of variance test using the mathematics placement pre-test and post-test investigated whether the size of the score on computational and basic skills depended on the time during the semester when the test was given. The null hypothesis of no differential effect was rejected at the .001 level.

The Wilcoxon matched-pairs signed-ranks test using the historical problems in mathematics test investigated whether students after taking this course were better able to answer the questions on historical problems in mathematics. The null hypothesis of no difference was rejected at the .001 level of significance.

The Wilcoxon matched-pairs signed-ranks statistical test was used to investigate whether students were better able to solve problems after taking this course. The null hypothesis of no difference was rejected at the .001 level of significance.

No statistical test was used to investigate students' attitude towards mathematics. The responses to the questionnaires were summarized using key words or phrases. It was concluded that this course improved the attitude of students toward mathematics.

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CHAPTER I

INTRODUCTION

Purpose

This research reports the effects of an innovative approach to the teaching of a general education mathematics course. The methods used included laboratory experiments, demonstrations, overhead projector, programmed materials, problem sessions, tutorial sessions, and films; as well as lectures and textbook.

The purpose of this study was to record and describe the behavior of the subjects taking this course. Specifically, the purposes were concerned with four problems: (1) To determine if these methods were effective in the students' learning of computational skills; (2) To determine if these methods were effective in the students' learning of the historical problems of mathematics; (3) To determine the effect of these methods on the students' problem solving ability; and (4) To determine the effect of these methods on the students' attitude toward mathematics.

Importance of the Study

The instruction of general education mathematics courses in college deserved attention for several reasons.

Students beginning such a course have a great variety of backgrounds. These courses are taken by students who may have had four years of high school mathematics, very little mathematics taken several years ago, or by those who have forgotten everything or failed to understand it in the first place. Concerning the great variability of backgrounds among college students, McConnell and Heist, using American College Examination (ACE) scores, wrote:

> Most college instructors are probably well aware of difference in student ability, but it is doubtful that most of them realize the degree of diversity that exists or adapt their teaching procedures accordingly (10).

Experience in the instruction of these mathematics courses indicated that these students did not respond well to lecture classes. Ryan wrote:

> Students with educational deficiencies, underachievers, do not do well in classes in which information is presented primarily or solely through lecture. They do better when methods are used which provide opportunity for immediate feedback, when material is presented in optimum sequence and when students are active rather than passive (70).

Students in these classes often have attitudes that are not conducive to learning mathematics. Stein has used the phrase "captured student" to convey the mood of:

> . . . failure, fear, frustration and perhaps hatred that such a student frequently brings to these mathematics classes (52).

The dropout rate for general education mathematics courses using the lecture-textbook method have been high. Fall semester of 1972 showed that the dropout rate at

Saginaw Valley College was 45 per cent in the morning section and 50 per cent in the afternoon section.

General education mathematics courses often have poor attendance. Even though attendance records are usually not kept, the faculty members who have taught the course over the past four years have noted absenteeism to be high. This is not a problem peculiar to Saginaw Valley College. Arendsen noted that at Michigan State University:

> Of 143 students registered for the class during the winter quarter of 1971 a check revealed that less than 40 students attended each of the last two large lectures which were held each Thursday (57).

Another reason for conducting the research was an attempt to improve the instruction and the level of achievement in general education courses. Examination of fall 1972 post-tests indicate that less than one per cent would be eligible to enroll in the college algebra course. This has been the only class at Saginaw Valley College where students were able to eliminate their deficiencies in mathematics to enable them to enroll in college algebra.

Jones and Arendsen commented on the need for further research and changes in a remedial algebra course. Jones commented:

> Approximately fifteen years of lecture method has left the student in a position requiring him to take a remedial algebra class in college. We then offer the student another lecture course which is significantly different from courses in his prior experience only in the ratio of number of students to lecturer. The students have been trained to view mathematics

> as a spectator sport in which the number of spectators is ever increasing as the opportunity to perform is ever decreasing.
>
> A major change of method is due in the teaching of Mathematics 082 and such change should be in the direction of individualized instruction. Such methods as audio-visual-tutorial learning, programmed learning and computer assisted learning should be considered for immediate adoption to relieve the plight of the students in Mathematics 082 (67).

Arendsen recommended:

> The use of a mathematics lecture-laboratory situation incorporating a lecture with laboratory experience such as audio-tutorial techniques, programmed texts, slide rules, probability materials, desk calculators, and computers should be investigated at the college level (57).

The Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America has directed its attention to the problem of construction of a general mathematics program and it noted:

> Many of the students in basic mathematics courses have seen this subject matter in elementary and high school without apparent success in learning it there. It is often the case that a second exposure to essentially the same material, similarly organized, is no more successful even though an attempt is sometimes made to present the subject matter in a more 'modern' manner.
>
> . . . it is our belief that the type of student described can also be greatly helped by reform in the mathematics curriculum.
>
> The student must be actively involved throughout the course and should be encouraged to formulate problems of their own based on their experiences. Full advantage should be taken of the playful impulses of the human mind; interesting tricks and seemingly magic ways of solving problems are to be exploited (20).

Limitations

This study limited its considerations to an instructional approach that directly influenced the achievement gains in a general education mathematics course in the specific areas of computation and basic skills, historical problems of mathematics, problem solving and attitude.

This study made no attempt to evaluate the following: (1) the superiority of this method over the lecture-textbook method but simply to report the results; (2) the superiority of one test over another in the area of computational and basic skills but to use the instrument that has been part of the evaluating system at Saginaw Valley College; (3) the advisability of smaller or larger classes but to use the intact groups as they registered at Saginaw Valley College; and (4) the times of day but to accept the scheduling as determined by previous scheduling.

The justification for these limitations was that the study was primarily concerned with using the normal settings at Saginaw Valley College to determine the feasibility of this method as a viable approach to the teaching of the general education mathematics course.

This study was also limited by the length of time of the study and the investigator's ability to implement the goals of the instructional methods.

Assumption

It was assumed that no student had an opportunity to practice the questions on any examinations.

The integrity of all tests was maintained between the dates at which the examinations were given.

Overview of Procedure

In order to determine the effectiveness of this method on computational and basic skills, a pretest was given. During the semester it was given again and at the end of the semester a similar test served as a post-test. The scores were compared for differences over time.

In order to determine the effectiveness of this method on problem solving, a random sample of 10 students were selected. These students were pretested for their ability and procedures used to determine whether two five digit numbers were prime numbers. Each was given 10 minutes and was allowed to use any of the materials in the room which included pencil, paper, blackboard, chalk, electric calculator, textbooks, slide rule, and any of the laboratory materials available during the semester. They were allowed to ask questions of the investigator which required a yes or no answer. The investigator could decline to answer if the question was too leading. At the end of the semester the 10 students were post-tested using two different five digit numbers. The students were observed as to the techniques that they used as well as the solution. The students were also asked to determine the path of a shadow as a person moves away from the light source in a straight line, when a person moves in a circle around the light source, and the path in general for any path of the person.

To investigate the effect on the students' knowledge of some historical problems of mathematics, questions were asked on quizzes given during the semester to determine their knowledge of these historical problems of mathematics. A test was given at the end of the semester to test their mastery learning at the 75 per cent level.

To investigate the students' attitude toward mathematics a questionnaire was given to the students at the end of the semester.

Succeeding Chapters

Chapter I has given the background and purpose of this study, limitations of the study, assumptions made in the study, and an overview of procedures.

In Chapter II the background of the problem and a review of related studies is provided.

Chapter III contains a detailed description of the study.

Chapter IV consists of the results and discussion.

Chapter V consists of the summary, conclusions and recommendations.

CHAPTER II

BACKGROUND OF THE PROBLEM AND REVIEW OF THE LITERATURE

Introduction

According to Goodlad, the current curriculum reform dates back to the years following World War II; however, he states it is usually linked more recently with the Russian satellite of 1957. Goodlad wrote:

> This spectacular event set off blasts of charges and counter-charges regarding the effectiveness of our schools and stimulated curriculum revision, notably in mathematics and the physical sciences (38).

Goodlad continues by analyzing the strengths and weaknesses of the mathematics programs that were part of the revised curricula. He contended that, although there was great uniformity among the various projects and programs, objectives were "vague," "not stated," or had a "mystical quality." He felt this was particularly true regarding structure and concept-attainment.

Each of the major math programs that came into existence as a result of curriculum reform are well known and each group has published a wealth of materials. It might be fruitful to examine certain aspects of these programs. In 1963 The National Council of Teachers of Mathematics published a bulletin, An Analysis of New

Mathematics Programs, (24) in which an overall analysis was made of eight of the better known programs. The following criteria were used in this evaluation:

Social Applications
Placement
Structure
Vocabulary
Methods
Concepts vs. Skills
Proof
Evaluation.

A resume followed each of these topics, usually in the form of questions. The members of the committee noted that mathematics is in a state of flux and they left it to the reader to decide his own position on each of the above topics. Even though the committee did not take a more definitive position, they did consider each of the above topics crucial to a mathematics program.

In a comparative study of the eight programs included in the bulletin, Hall noted that:

> . . . whereas each program was unique nevertheless commonalities were evident. Among the latter that appeared most frequently were: concept oriented; stress upon understanding, thinking, and reasoning; search for patterns, ideas and principles. The language of mathematics was considered important and thus a precise vocabulary essential. In some programs new content was introduced; others introduced topics earlier grade-wise than is dictated by tradition. A few stressed the discovery method, in-service training for teachers, development of aids, films and materials, etc. This is by no means a thorough presentation of what may be considered a 'new mathematics' program; however, it would serve as a guide as to what is deemed important by those who have developed such programs. It should be noted that all eight programs were designed for the typical child or the more able learner (65).

The School Mathematics Study Group recognized that their programs were designed for the more able students when they reported:

> It should be noted that much of the success of the so-called new mathematics has been demonstrated with middle-class children (25).

Conference members from the joint meeting of the U. S. Office of Education and the National Council of Teachers of Mathematics also recognized this neglect:

> Our intent here is to consider the mathematical need and proper instruction in mathematics for that category of youth referred to by Dr. Conant as 'social dynamite'--those who possess no skill, who are unemployable and unschooled. . . . Our range of interest will include mathematics for those students who are potential dropouts, as well as for those who remain in school, but who, for one reason or another, exhibit a pattern of low achievement in mathematics (26).

Reasons for updating the math curriculum appear in much of the literature and include the following taken from Frontiers in Mathematics Education. These appear in summary form:

1. Mathematics must grow and change to meet the demands of a changing world.
2. Other fields of knowledge are making increased use of mathematics.
3. Inability to predict the skills needed in the future.
4. Experiments indicate the value of earlier introduction of selected concepts and skills.
5. New development in child psychology and a greater understanding of the learning process require elimination of rote memorization and meaningless drill.
6. Large numbers of our population have been inadequately prepared in mathematics.

7. Greater realization that the development of persons proficient in mathematics begins in the elementary school.

8. Schools must produce more highly trained technical people and also informed and literate citizens in mathematics to enable them to understand their technological world (21).

These same reasons can be advanced for up-dating and improving the instruction of mathematics for the students in general education courses today.

Review of the Literature

This study examined an innovative approach to the teaching of a general education mathematics course. The related literature was reviewed in the five areas: (1) laboratory instruction; (2) programmed instruction; (3) individualized instruction; (4) remedial instruction; and (5) discovery learning.

Laboratory Instruction

Laboratory instruction in this study referred to a teaching technique which utilizes activity by the student with materials other than blackboard, paper for writing, or library reference materials.

The idea of a mathematics laboratory has been with us for a long time. In the May, 1970 issue of *The Arithmetic Teacher* Kristina Leeb-Lundberg (46) describes the original kindergartens as they developed under the influence of Froebel in Germany in the early 1800's. The kindergarten she described includes grids on the tables and chalkboards, studies of shapes, sand for measuring volume, early versions of attribute blocks, multibase arithmetic blocks, pattern blocks, linkages, and even lattice boards which could be used as geoboards are today.

One of the more influential proponents of mathematics laboratories was E. H. Moore. In his presidential address before the American Mathematical Society in 1902, he discussed the state of abstract and applied mathematics, then went on to discuss the state of the teaching of mathematics. In discussing elementary mathematics he said:

> Would it not be possible for the children in the grades to be trained in power of observation and experimentation and reflection and deduction so that always their mathematics would be directly connected with matters of thoroughly concrete character? The response is immediate that this is being done today in the kindergartens and the better elementary schools. I understand that serious difficulties arise with children of nine to twelve years of age, who are no longer contented with the simple, concrete method of earlier years and who, nevertheless, are unable to appreciate the more abstract methods of the later years. These difficulties, some say, are to be met by allowing the mathematics to enter only implicitly in connection with the other subjects of the curriculum. But rather the materials and methods of the mathematics should be enriched and vitalized. In particular, a grade teacher must make wiser use of the foundations furnished by the kindergarten (11).

In 1927, Austin proposed a laboratory approach to high school geometry. He stated:

> The keynote of the laboratory idea is discovery by means of experimentation. Pupils should be permitted to observe the laws of geometry operating in concrete form before they are required to do logical thinking (29).

In 1954 the NCTM published a yearbook entitled Emerging Practices in Mathematics Education. One of the major sections of this yearbook was entitled "Laboratory Teaching in Mathematics." In the first section, we find the statement:

> Laboratory techniques have long been used in public schools in such areas as science, dramatics, home economics, and shop. Teachers have long been urged to use laboratory techniques in the teaching of mathematics. Enough teachers are doing that, so that we may well consider laboratory teaching as one of the emerging practices in teaching mathematics (3).

Thus, it appears that at times through the years, there have been collections of voices asking for a more intuitive and less deductive approach to the teaching of mathematics at all levels.

Current writers generally attribute the forces behind the development of mathematics laboratories to the work of Piaget, Bruner, Gattegno, etc. (63).

The literature of mathematics education at the present time has frequent articles discussing mathematics laboratories. Two issues of The Arithmetic Teacher (Oct., 1968 and Jan., 1970) have had this topic as their central theme.

Kieren (43) provides the most extensive review of the research literature dealing with laboratory learning during the period 1964-1968 in the Review of Educational Research, October 1969. Most of the research activity has been aimed at determining the effectiveness of the use of specific materials on the achievement of children. The use of the cuisenaire rods has been studied in several studies with mixed results. Callahan, Crowder, Hollis, Locow, and Nasca all report that the use of the rods promoted more learning by children while Brownell, Fedon, and Haynes report either mixed results or no significant differences. Passy reports a negative effect from the rods.

Davis (43), in his comprehensive report of the Madison Project stated that the goals of the project were to prepare classroom experiences developed in a specific manner, to train teachers in the use of the experiences, and to test the lessons. The methodology of evaluation makes it difficult to report the results in statistical terms. Davis (1967) reported an interview study by a clinical psychologist, H. Barrett, done in connection with the Madison Project. After a year of conducting interviews, Barrett concluded that students in grades 6 and 7 like physical activity and courses involving it and do not like doing sophisticated mental tasks. Barrett's data formed the basis for the Madison Project decision to use physical materials and laboratories at grade K-9 and also in college courses for prospective mathematics teachers.

In an unpublished manual, About Mathematics Laboratories, Fitzgerald writes:

> There is at this point no empirical evidence which will convince the unconvinced that mathematics

> laboratories are the best way to accomplish these aims. Many have found through experience that the school as a system makes the implimentation of a laboratory a difficult task to accomplish.
>
> This writer, however, has never seen a teacher who, after shifting from a teacher-dominated, total-class approach to an individualized, activity-oriented approach, has chosen to shift back again (63).

Programed Instruction

Programed instruction is based on the assumption that by splitting the instructions into a very large number of minimal steps, it becomes possible for every student to be able to put together whatever is necessary to arrive at the competencies required.

Regarding programed learning and instruction, Fey wrote:

> Programed instruction plays an important research role as a means of simulating classroom instructional methods under rigorously controlled, repeatable conditions. However, programed instruction was originally conceived as a replacement for traditional classroom teaching--as a tool that could offer pacing and feedback on an individual basis. Since mathematics has been a popular topic for developers of programed material, there has been a great deal of research on the effectiveness of this new instructional medium. The most frequently debated and tested question was about the relative merits of programed and teacher-directed instruction. Results of media comparison studies have failed to establish the superiority of programed or conventional instruction. In fact, the evidence is sharply contradictory. Zoll (1969) reviewed research in programed instruction and found that in 13 comparative studies differences favored programed in 3 cases, traditional methods in 3 cases, and neither method in 7 cases (35).

The one great stumbling block to the use of programed instructions is that it is extremely boring. According to Dienes,

> An adult who is motivated to learn something in order to improve himself in his job can discipline himself and say, 'I must learn it,' and go through a thousand-page text to pass an examination. But he does this to earn the privilege of higher pay. For children, on the whole, the reward has to be much more immediate than that (4).

Another objection to programed instructions is that in our normal environment, we do not learn in minimal steps. Dienes suggests that:

> Some kind of nonlinear programming would seem necessary, which would combine learning small items with interaction with the environment and, consequently, with the added depth that such interaction can produce in the way of motivation. Computerized instruction may here provide a partial solution. Any program of mathematical activities should be one in which children can engage in exploratory activities in relation to their environment (4).

Therefore, it seemed reasonable that research in programed instruction move toward defining the interactive effects among various types of programing, subject matter, and toward determining how programed materials can be most effectively used in conjunction with standard teaching procedures. For example, Morgan (68) and Callister (59) examined stress, anxiety, and achievement interactions in programed and conventional classes. In 1966 Wiebe (71) tried to find the most effective combination of programed and teacher-directed instruction with low-achieving students. According to Fey:

> It seems fair to conclude that, far from supplanting teacher classroom instruction, programing will probably prove to be an important instructional device to be effectively integrated into a total instructional plan (35).

In this study it was the author's intent to integrate programed instruction into the arithmetic phase of the course.

Remedial Instruction

In the past decade many programs have been instituted to alleviate problems associated with the academically deprived student. These bear various labels indicating objectives such as: "remedial," "preventive," "enriching," and "supportive." Two important conferences were held which stressed the need for more emphasis and research in this area of mathematics. One was held in 1965 and was a joint effort by the U. S. Office of Education and the National Council of Teachers of Mathematics. The other was sponsored by the School Mathematics Study Group (SMSG) in 1964. The following exerpts were taken from the reports of these two conferences:

> From the very beginning SMSG recognizes perfectly well that we were doing something for only part of the school population. We have made a remarkable amount of progress, but we are far enough along to realize that the rest of the school population, the students who are not doing well in mathematics, must be given attention (25).
>
> At all levels, however, the emphasis and attention have been directed toward the above-average mathematics achiever (26).

Not only is it evident that a large segment of the student body has been by-passed by curriculum changes in mathematics but it is now realized that our modern technological society demands that ways and means be explored to reach these students. Stein believes that colleges:

> have a responsibility to meet the captured student not halfway, but exactly where he is: and to help him go as far as he is able in the direction that he chooses or is compelled to follow (52).

Others have made similar comments. Stubblefield and Whitman of Florida A. & M. state:

> In most mathematics departments, freshmen general education mathematics courses affect more college students than all of the other mathematics offerings. Yet, the general education courses are of little interest to these departments, ... In such courses, many developing colleges are finding 'rough diamonds'--able but reluctant or handicapped students--who are potential 'late bloomers.' They are finding that effective teaching can release mental fetters by breaking away from excessive formalism and stereotyped thinking. Furthermore, they find that as students experience intellectual success they raise their self-expectations and travel new routes to higher achievement. A most effective way of improving student experiences in freshman courses is through programs which involve the instructors in creating fresh approaches to their teaching. They are setting up active learning situations by using intrinsically interesting materials, by providing students with attainable goals, and by allowing flexibility in course content (53).

Recently, Melvin Kranzberg wrote:

> We must remember that approximately one-third of American college-age youngsters are now in college, and their aversion to required science courses would seem to manifest a disregard or even disrespect for science. Students now are concerned with the quality of life; and they wish to participate more actively in society. Above all, they are motivated by humane and social considerations. Science education has not responded satisfactorily to changing motivations (45).

There are colleges and universities that are trying to alleviate the problems that plague these students. The City University of New York implemented an open admissions policy in September 1970 and it is the University's objective to provide a college sequence of courses which, regardless of a student's previous academic achievement, will give him the opportunity to attain the background necessary to choose and pursue a college course of study (37).

Staten Island Community College has implemented the open admissions policy mentioned above. Ablon reports on the program constraints, the program structure, the students enrolled, the placement of students, the program content and the results of their program for the fall of 1971. He states:

> 1. The dropout rate was 45% during the first semester of the program, but as a result of changes, has fallen to about 25% and seems to be holding there.
>
> 2. Of those who do complete the program and go on to a pre-calculus course 81% pass.
>
> We believe that there are four ingredients any open admissions program must have to be effective.
>
> 1. The capacity to accept every student at his own level and work from there to wherever he wants to go.
>
> 2. The ability to continue working with a student until he reaches or changes his goals.
>
> 3. The cooperation of an administration eager to go out on a limb and spend the money required to make the program work.

> 4. Teachers who honestly believe in the right of everyone to try to develop to his capacity regardless of his past experiences.
>
> Mix these ingredients into any reasonably well thought-out program, hope for a liberal sprinkling of Hawthorne Effect, and do it (27).

Jones wrote on the approach used to solve the problem at Lansing Community College, Lansing, Michigan.

> The student may enroll in one of three remedial courses with initial placement based upon entrance examination scores. The courses are Basic Arithmetic, Beginning Algebra, and Intermediate Algebra. The student has the choice of a traditional lecture course or a programed instructional course in both Basic Arithmetic and Beginning Algebra. Intermediate Algebra is offered in a choice of traditional lecture or audio-tutorial mode of instruction. All of the courses are offered on the basis of a 10 week term but the 10 week time periods are used only for fee assessment and record keeping. The key idea is the time flexibility within the 3 courses. . . . Student reaction to these procedures is positive and the administrative problems which accompany the increased flexibility have been solved (67).

Individualized Instruction

The need to individualize has been a concern of educators for many years. Wood and Haefner (19), Foster, Kaufman and Fitzgerald (22), and White (72) expressed a need to use individualized instruction.

The Comprehensive School Mathematics Program has been concerned with individualizing instruction since 1966. Kaufman and Haag said:

> It has become clear to many people who think about mathematics education that the traditional process of fitting all children in a class to one instructional program must be replaced by a process by which programs are designed to fit the individual children (23).

Arendsen noted:

> Fortunately, efforts have been made to individualized instruction at all levels of education and for the large number of students attending today's schools and colleges. Commercial companies are producing methods of instruction designed to individualize teaching. The Westinghouse Learning Corporation has produced individualized learning materials which are in use in the elementary grades at the McCulloch School in Jackson, Michigan. In this system each child has a yearly instructional plan (which may be changed by the teacher) designed for him by computer. The child proceeds through the system on his own, receiving help from the teacher when needed. A visit to the school showed that the system was well received. Students in all classes were busy learning, and it was reported that there were very few discipline problems. A child working on his studies has no time to cause disturbances (57).

The need to individualize on the college level has been receiving more than lip service in the last five years. Michigan State University has developed Structural Learning and Training Environments (SLATES) in response to the need for individualized instruction on the campus.

Ohio State University has implemented a program called CRIMEL (Curriculum Revision and Instruction in Mathematics at the Elementary Level). In 1971 Riner reported on the initial development of the program which was designed to individualize mathematics instruction. In 1972 Waits reported on the modified CRIMEL experiment stating that:

> The basic spirit of the CRIMEL program at Ohio State is to allow the individual student to learn the mathematics he needs at a pace consistent with his background, interests, goals, and ability (48).

As a result of modifications to the experiment, Waits reported:

> The overall attitude of the student in the CRIMEL program was excellent. There was a great improvement in attitude and morale in the 3 credit hour sections over the first implementation. As the result of the retesting aspect of our program, many students felt that the Mathematics Department was actually trying to help them, not flunk them out! (55).

Because of the variety of backgrounds of the students in the general education course at Saginaw Valley College, the need to individualize was apparent.

Discovery Learning

In this form of learning, the student actively engages in the process of forming mathematical ideas for himself, but does not necessarily use any physical devices in learning mathematics.

The late Max Beberman, of the University of Illinois Committee on School Mathematics, believed that the use of discovery leads to greater understanding by actively involving the student:

> A second major principle which has guided us in developing the UICSM program is that the student will come to understand mathematics if he plays an active part in developing mathematical ideas and procedures. To us this means that after we have selected a body of subject matter to be learned we must design both exposition and exercises in such a way that the student will discover principles and rules (7).

Bruner hypothesizes four benefits of discovery learning: increased intellectual potency, intrinsic motivation, the learning of the heuristics of discovery, and enhanced use of memory, and gives his view of discovery teaching:

> Discovery teaching generally involves not so much the process of leading students to discover what is 'out there,' but, rather, their discovering what is in their own heads. It involves encouraging them to say, Let me stop and think about that; Let me use my head; Let me have some vicarious trial-and-error. There is a vast amount more in most heads than we are usually aware of, or that we are willing to try to use. You have got to convince students . . . of the fact that there are implicity models in their heads which are useful (32).

Other distinguished educators and researchers support the use of discovery learning: Allen (28), Hartung (42), Davis (43), Hawkins (43), and even Ausubel, who has been a strong critic of discovery learning, admits:

> Learning by discovery has its proper place among the repertoire of accepted pedagogic techniques available to teachers. For certain designated purposes and for certain carefully specified learning situations, its rationale is clear and defensible (30).

Hence, there is a great deal of support for, and interest in, a type of teaching or learning the essence of which lies in inductive generalization.

Kieren reported in the Review of Educational Research:

> There was no lack of published written work defending or attacking the effectiveness of discovery methods of instruction. As Cronback pointed out there are a multitude of different approaches that have been applied under the rubric of 'discovery.' Thus, the theoretical discussion and the research in discovery learning suffered from lack of precision and lack of communication caused by the variations in connotations of 'discovery.' As part of a conference report (edited by Shulman and Keislar, 1966), Davis attributed several

positive effects to a discovery learning experience, especially in a classroom group. He suggested that the child is rewarded for his efforts by the pleasure of his own discovery and his ability to bring closure to a problem setting himself, that the child is motivated by the opportunity to display his ideas in competition with others, and that the student learns to act independently in checking the veracity of his ideas. Bruner (1966) concluded that discovery settings present opportunities for students to draw for themselves relationships between things they know and the learning task at hand. In addition, discovery methods build problem-solving skill by providing experience in pushing ideas to their logical limit and in effectively forming concise hypotheses. Bruner, like Davis, suggested that thinking acts are reinforced by the discovery accomplished and that a reflective attitude is developed in students.

Ausubel, in a counter-argument to those of Bruner and Davis, stressed that discovery processes are less efficient than didactic or expository processes and that discovery is not needed for meaningful acquisition of knowledge. Bittinger (1968) cited literature suggesting that discovery attitudes can be sustained via didactic teaching. He further noted Cronbach's hypothesis that discovery methods may not work well for anxious, dependent students. Becker and Maclead (1967), in reviewing literature relating discovery and transfer, found no conclusive research evidence that discovery methods foster transfer. This find suggests that discovery methods might not facilitate the acquisition of superior problem-solving capabilities. Nonetheless, Becker and Maclead found that the literature indicates a conflict between methodologies that sponsor maximum understanding and those that sponsor maximum motivation to continue learning.

The research published during 1964-1968 on discovery methodologies in mathematics instruction did little to resolve the theoretical conflict outlined above (43).

Summary

Chapter II was concerned with reviewing the literature in each of the following five areas: laboratory instruction, programed instruction, individualized instruction, remedial instruction and discovery learning.

Some conclusions from the literature which are important to the study are:

1. Typically, present college instruction of general education mathematics classes leaves much to be desired.

2. Individualizing instruction is a goal worthy of more attention.

3. Laboratory instruction has been with us a long time. Enough teachers are using laboratory techniques so that we may consider it as one of the emerging practices in teaching mathematics.

4. Programmed instruction will not supplant teacher classroom instruction but will probably prove to be an important instructional device to be integrated into a total instructional plan.

5. Discovery learning has a great deal of support and interest.

6. Remedial instruction in mathematics is beginning to receive the attention that it needs.

CHAPTER III

METHOD OF THE INVESTIGATION

Introduction

The purpose of this study was to record and describe the behavior of the subjects taking this course. Specifically, the purposes were concerned with four problems: (1) To determine if this method was effective in the students' learning of computational skills; (2) To determine if this method was effective in the students' learning of the historical problems of mathematics; (3) To determine the effect of this method on the students' problem solving ability; and (4) To determine the effect of this method on the students' attitude toward mathematics.

This chapter includes: (1) background; (2) the population and sample; (3) the method of instruction; (4) the instruments used in the study; and (5) the testing sequence.

This course was designed to implement the goals of the Committee on the Undergraduate Program in Mathematics (CUPM), outlined in A Course in Basic Mathematics for Colleges.

Background

Saginaw Valley College is Michigan's newest four year state supported institution of higher learning. It is comprised of the Colleges of Liberal Arts, Business Administration, Education, Fine Arts, and the Basic College. Thus far in its development, main emphasis has been on the liberal arts. The student body is predominantly drawn from the tri-county area of Saginaw, Bay, and Midland.

The mathematics department at Saginaw Valley College offers a general education mathematics course, an integrated algebra and trigonometry sequence, and a calculus sequence as freshman level mathematics courses. Students who enroll in the first course of the algebra or calculus sequence are required to take a 35 minute, 25 response, multiple-choice placement examination (Appendix A.1). If a student scores 17 or more correct responses, the student is permitted to take the first course in the college algebra and trigonometry sequence. (If a student wishes more advanced placement, further testing is required.) If the student scores 15 or 16, the student is considered to be marginally prepared for college algebra and a review is recommended. If a student scores below 15, the student is required to improve his or her math background either by taking the general education mathematics course or by self study. Exceptions to the foregoing are often made on the basis of personal interview. Any student at Saginaw Valley College may select the general education mathematics course to fulfill a natural science Basic College requirement. (Mathematics majors are restricted to fulfilling their natural science requirement outside of their major. This restriction is often not followed since many students fulfill their Basic College requirements before selecting a major.)

The Population and Sample

The population in this study is the set of all students who have enrolled, are presently enrolled, or will enroll in the general education mathematics course at Saginaw Valley College.

The sample was comprised of two intact sections of general education mathematics offered winter semester of 1973 at Saginaw Valley College. The author served as the instructor for both sections. Personal data collected on each student were used to describe the sample in detail. There were 30 males and 24 females in the study who completed the course. Table 3.1 indicates the mathematics background of the students who completed the course for the two sections.

TABLE 3.1.--Highest Level of Mathematics Background (In Per Cent to the Nearest Tenth).

High School Mathematics					
General Math	1 Year	2 Years	3 Years	4 Years	College
9.3	24.1	27.7	13.0	18.5	7.4

The mathematics background was quite varied. With 7.4% of the students having had college algebra and 9.3% of the students having had only general math in high school, the difficulty one would have in trying to lecture to this diverse group of students appeared obvious. Table 3.2 indicates the years since the last math course was taken by the students. The time since the last mathematics course ranged from one semester to 15 years. Table 3.3 indicates the class distribution of the sample. The information in these tables pointed to the wide variety of students in the class. The motivations, goals, interests and recall of these students were probably quite different.

TABLE 3.2.--Years Since Last Math Course (Per Cent to Nearest Tenth).

0-1	2-3	4-5	More than 5 Years
22.2	40.7	24.1	13.0

TABLE 3.3.--Class Distribution of Sample (Per Cent to Nearest Tenth).

Freshmen	Sophomores	Juniors	Seniors
74.1	18.5	3.7	3.7

Method of Instruction

The concept approach to teaching, which stressed the ideas and basic understandings that underlie our number system, was used. In practice the method was accompanied by many activities and a utilization of tangible aids and/or models that were intended to promote learning and make the experience an enjoyable one. This methodology stressed main ideas, generalizations, structure, understanding, and relationships in a mathematical environment. A procedure was instituted that permitted flexibility, large and small group activities, utilization of laboratory materials, frequent change in pace, numerous concrete aids and models, and involvement of tutors.

A routine was established to give the student a framework within which they could operate and yet enjoy a great degree of freedom.

The text used during Winter Semester for the arithmetic phase (first five weeks) of the course was Computational Arithmetic by Charlene Pappin. It was published by Glencoe Press in 1972. Along with this book, prepared materials were handed out and laboratory materials were introduced. For the remaining 10 weeks of the course no textbook was used. Materials had been prepared and were handed out in class. Laboratory materials were also used during this phase.

The class met four days per week and each class period was 50 minutes long. At the beginning of the hour the day's work would be explained. This explanation was limited to 5-10 minutes. Occasional unannounced quizzes were given at this time. When a particular assignment was finished by all (or almost all) of the students, at the end of the class or the next day, the students with the aid of the instructor would summarize what they had learned. It can be seen that there were no lectures as we traditionally envision them in a math class.

The classroom became a laboratory in which students could construct, manipulate, measure and explore mathematical ideas, and participate in demonstrations, problem solving, and discussions. Every effort was made to make the class enjoyable for the students and to give them experiences in success. There were homework assignments which related to the material covered in the classroom.

In an attempt to make mathematics more relevant and desirable to the students, much effort was put into the development of everyday mathematics problems.

A lesson plan follows for a typical class session. While no two days were exactly alike, the general format was adhered to.

Typical Lesson Plan

Beginning of the hour:

1. Pass out answer sheet to homework. Have students check homework and answer any questions. Possible quiz.
2. Pass out classroom activity for the day.
3. Introduce (or explain) the activity.

10 to 15 minutes past the hour:

4. Have the students work in groups of four or five to perform the activity and draw conclusions. During this time circulate among the groups to give guidance and help where needed.

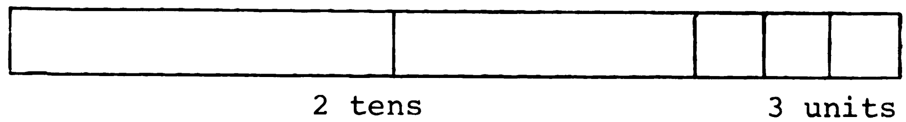
10 to 15 minutes before the end of the class time:

5. Determine the progress the students have made. If it requires another class period to complete, allow the students to continue working. Otherwise, use these 10 to 15 minutes to check the results and summarize.

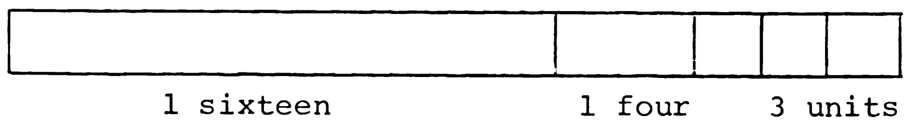
The basic arithmetic operations with whole numbers, integers, rational numbers and real numbers from a computational standpoint were carefully covered in Charlene Pappin's Computational Mathematics text which was the required text for the course. The text is designed for individualized instruction of mathematics. The students were able to pre-test their competency in each of the eleven areas covered in the text to determine if it was necessary for them to work through the material.

In order to disguise the drill of the basic arithmetic operations on whole numbers, number bases were introduced. This also allowed for the opportunity to introduce exponential notation, place value and the importance of the number zero. Cuisenaire rods were used to represent the numbers in the different bases. The students were also led to discover the algorithm for converting from base 10 to a smaller base. For example,

23_{10} is represented in base 10 by:



and in base four by:



(Since there is no 16 in the set of rods, it was necessary to create a sixteen by taping two eights together.)

$$\text{Thus, } 23_{10} = 113_4 = 1\text{x}4^2 + 1\text{x}4^1 + 3\text{x}4^0.$$

$$\text{or } 23_{10} = 1\text{x}4^2 + 1\text{x}4^1 + 3\text{x}4^0.$$

Dividing both sides by four we have:

$$\frac{23_{10}}{4} = \frac{1\text{x}4^2}{4} + \frac{1\text{x}4^1}{4} + \frac{\textcircled{3}\text{x}4^0}{4}$$

$$\begin{array}{r} 5 \\ 4\overline{)\;23_{10}} \\ \underline{20} \\ \textcircled{3} \end{array}$$

and as a result $5 = 1\text{x}4^1 + 1\text{x}4^0$. Dividing this equation on both sides by four, we have:

$$\frac{5}{4} = \frac{1 \times 4^1}{4} + \frac{\textcircled{1} \times 4^0}{4}$$

$$4\overline{)5} = 1, \quad 5 - 4 = \textcircled{1}$$

And as a result $1 = 1 \times 4^0$. Dividing this equation on both sides by four we have:

$$\frac{1}{4} = \frac{\textcircled{1} \times 4^0}{4}.$$

$$4\overline{)1} = 0, \quad 1 - 0 = \textcircled{1}$$

Thus, $23_{10} = 113_4$

After converting several numbers from base 10 to a smaller base, the students were able to conclude that by successive divisions of the quotients of the base 10 number by the radix of the smaller base, it was possible to obtain the smaller base number by using the remainders from the successive divisions.

Also, the advancing algorithm for addition in any number base was developed using odometers.

"Guess My Rule" (Appendix C 3.1) was used to introduce inductive reasoning, to practice basic operations of whole numbers, to encourage students to work together, to build a foundation for functions and graphing in two dimensions, to discover finite differences and to form conjectures.

For example: The subjects played "Guess My Rule" on numerous occasions until they discovered a systematic method to determine the value of m in the equation, $y = mx + b$. While playing the game, a table was made and the points plotted on a graph. The value of m was called the first difference and a column for the first differences was made. Having had algebra before, several students suggested substituting a pair of values for x and y and solving for b. This is certainly a valid method for obtaining b and so they were asked if they could discover another method for obtaining the value of b. They noted that the y entry in the table when $x=0$ was always the same value as b. With this information they were able to determine the rule for data when the first difference is a constant. After discovering the method to determine the rule, the students were asked to determine the relationship between the values of m and b and the graphs that they drew.

Having completed this discovery, the students then performed several experiments whose data could be described by linear equations.

Realizing that all rules are not linear, the students used the rule $y = x^2 + 2x + 1$ to make a table of ordered pairs. The entries in the first difference column were not alike.

The students were also asked to plot the ordered pairs and noted that the graph was not linear. The author suggested adding a second difference column. The students noted that these entries were alike. The students tried several rules until they were satisfied that this would be the case for a second degree equation. The students were asked to study their rules and tables of data to determine the relationship that existed between their data and their rules. They were asked to consider their rules to be of the form $y = ax^2 + bx + c$ and to find a, b, and c from the data. They were asked the following questions:

1. What is the significance of the y entry if $x=0$?

2. Is there any connection between the second difference column and a?

3. Is there any connection between the first difference column and a and b?

In this way they were led to discover that the y entry if $x=0$ is the value of c, the constant in the second difference column divided by two is the value of a, and the sum of a and b is equal to the first entry in the first difference column when $x=0$.

Having made this discovery, the students used physical materials whose data could be described by a second degree equation.

Deductive reasoning was introduced by considering a cube with lines drawn on it that divide each face into nine squares. The following questions were asked:

1. How many small cubes would be formed if the large cube were cut apart?

2. How many cuts would be needed to do this?

3. If the pieces were rearranged between cuts, could the job be done with less than six cuts?

Many students have a very poor comprehension of three-dimensional geometric figures and cannot satisfactorily picture them when looking at two-dimensional representations. They were asked to build a model with their Cuisenaire rods.

The chain of logic should be:

1. One of the cubes to be formed by the cuts is the one in the center of the big cube.

2. This center cube has six faces.

3. Since each of its faces lies in a different plane, only one face can be cut at a time.

4. Therefore, six cuts are necessary to form this cube.

Another type of problem that was used to introduce deductive reasoning was:

Choose a number.
Triple it.
Add four.
Subtract your original number.
Divide by two.
Subtract two.
What do you conclude about the result of this number trick?

Initially, students chose specific numbers and inductively arrived at a conclusion. The students were then asked to use a box for their chosen number and dots for the constant added. Since the box represented any number, but the same number throughout the trick, the students were given the opportunity to write some simple proofs for some number tricks.

Negative numbers were introduced through the use of the number line. The operations with negative numbers were motivated by use of the "Postman Stories" found in Robert Davis' text Discovery in Mathematics.

The conversion from the English measurement system to the metric system and conversely were used to demonstrate the use of ratio, proportion, per cent, measurement, accuracy and precision.

After the students were familiar with the metric system, the English measurement system and the history behind their development, it seemed that the World Calendar would be a point of interest. Price indexes were used to demonstrate a possible need for the World Calendar and the current interest in the development of the World Calendar. Price indexes also served as a practical application of basic arithmetic operations on the real numbers.

A Source Book of Mathematical Applications compiled by a committee of the NCTM served as a source of relevant problems for daily life.

The geoboards and geo-board activity sheets were used to investigate, intuitively, the geometric properties of plane figures. The figures were formed by stretching rubber bands around the nails on a geoboard. The geoboards used have nails arranged in a rectangular array, with the nails one unit apart vertically and horizontally. The unit of area is then the area of a square, one unit on a side (Appendix C.4). The following concepts were developed on the geoboards: area of squares, rectangles, parallelograms, trapezoids, pentagons, hexagons, octogons, triangles; similar figures, congruent figures, perimeters, right triangles and the Pythagorean Theorem, square roots, linear equations, slope of a line, concave and convex figures, symmetry, reflections and rotations, midpoint of a line segment and Pick's Theorem.

The factor game (Appendix C.3.2) was used to practice the concept of prime and composite numbers. The Sieve of Erasthosthenes was used to discover a method to determine whether a number was prime or composite. The students were able to determine that it was necessary to cross off multiples of all primes up to and including the square root of n. All numbers that were not crossed off were prime numbers.

After discovering the above the students were then able to discover that it was only necessary to divide by the primes less than or equal to $\sqrt{n}$ to determine whether n was prime or composite.

The Euclidean Algorithm was developed by the use of Cuisenaire rods (see Laboratory Manual for Elementary Mathematics by Fitzgerald et al.) to find the greatest common divisor.

Graphing in two dimensions was introduced while playing "Guess My Rule." Appendix B.7 gives two interesting examples of plotting exercises. The Madison Project "shoe boxes" were used to collect data. The students represented their data by a table, plotted the data on a graph and found a general rule to describe it. The students were given ample practice in plotting points from coordinates in preparation for sketching the graphs of functions that were covered later in the semester. To discover the relationship between the circumference of a circle and its diameter, the following example was used: A set of six circular objects of different diameters was distributed. The students were asked to cut a piece of calculator tape of length equivalent to the distance around the object. The subjects were then asked to mark the diameter of the object on the abscissa of a set of Cartesian coordinates, and at this point on the abscissa attached the calculator tape in the ordinate direction. In this manner the class was led to discover that the circumference of a circle is a linear function of its diameter.

Data from class quizzes and tests were used to make histograms, calculate averages, medians, and modes.

The numbers from one to 100 were written on a roll of adding machine tape and stretched across the front of the room. The story of how Gauss solved this problem in his head was told to the class. The numbers in this problem form what is called a number sequence. Number sequences and their properties were derived from practical applications such as the relationship between the length of a pendulum and time in seconds, the relationship between the number of generations back and the number of ancestors in the generations, the piano keyboard, chain letters, markings on a ruler, finding the relationship between the number of folds of a piece of paper and the number of thicknesses, a number trick with digital roots, and a number guessing game using base two or binary number sequences.

After the students completed the handouts on sequences, they were expected to be able to identify and use simple arithmetic sequences, geometric sequences and to recognize the smaller square and cube numbers. They were also expected to appreciate the wide variety of topics in which these sets of numbers appear.

Because exponential notation had been introduced using number bases, the concept of scientific notation was an easy concept to learn. It was introduced as a means of conveniently expressing large numbers and then applied to the work with decimal logarithms.

Logarithms were introduced by pairing a geometric sequence with an arithmetic sequence to form an exponential function. For example:

logarithms

0	1	2	3	4	5	6	7	8	9	10

numbers

1	2	4	8	16	32	64	128	256	512	1,024

or logarithms

0	1	2	3	4	5	6	7	8	9	10

numbers

1	3	9	27	81	243	729	2,187	6,561	19,683	59,049

The students were able to recognize that the numbers from the first set were powers of two and the numbers from the second set were powers of three. Consequently, we called the logarithms base two logarithms and base three logarithms, respectively. The students were asked to multiple two numbers, say, 4 x 8 = 32 and they noticed that the log of 32 was the sum of the log four and the log eight. Likewise, they were asked to divide two numbers, say 81/3 = 27. The students noticed that the log of the quotient was the log of the numerator minus the log of the denominator. After working multiplication and division problems with base two logs and base three logs, base ten logs were introduced. At first, the work with decimal logarithms was limited to the logarithms of the first ten positive integers so that emphasis could be placed on some simple theory and the discovery of the relationship between the scientific notation form of a number and its decimal logarithm. A brief introduction to the slide rule was given along with some practical applications of logarithmic scales.

The concept of area as (length) x (width) was used as an example of the product:

$$(x + 1)\ (x + 2) = x^2 + x + 2x + 2$$
$$= x^2 + 3x + 2$$

1	$1 \cdot x$	2
x	x^2	2x
	x	2

before using the distributive axion or the rule for the product of the means and the product of the extremes. When considering problems of the form: (x-1) (x+2) or (x-1) (x-2), it was difficult for the students to draw areas for these, so they were asked to fold paper until the idea became easy for them.

The concept of volume as (length) x (width) x (height) was used as an example of the product: (x-2)(x-2)(x-2). Dissectable cubes were available so that the students could handle the pieces as they found the product.

The formulas for,

$$(x\pm a)(x\pm a) = x^2 \pm 2x + a^2$$
$$(x+a)(x-a) = x^2 - a^2$$
$$(x+a)(x-b) = x^2 + ax - bx - ab$$
$$(x\pm a)(x\pm a)(x\pm a) = (x\pm a)^3 = x^3 \pm 3ax^2 + 3a^2x \pm a^3$$

were discovered using the area and volume concept.

The students were asked if they could recognize a pattern from the following information that they had discovered.

$$(x + a)^0 = 1$$

$$(x + a)^1 = x + a$$

$$(x + a)^2 = x^2 + 2ax + a^2$$

$$(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$$

Hint: Consider only the coefficients.

Since some of the students had algebra before, they were able to recognize it as Pascal's triangle and the coefficients as the binomial coefficients. A coin tossing experiment showed a very elementary application of Pascal's triangle.

During the development of any concept, some historical facts were included either by the instructor or in three or four minute reports from the students. The variety of applications was intended to present the universality of mathematics, to make clear to the students that they were expected to think, and equally as important was the idea that doing mathematics can be an enjoyable and rewarding experience.

Instruments Used in the Study

As mentioned in the background, the mathematics department employs an instrument which is used for placement of students. It is this instrument which was used in the testing of computational and basic skills. For attitude changes, problem solving, and historical problems in mathematics, the investigator devised her own tests or questionnaires. The historical problems in mathematics test was patterned after a test written by Avital. The items on the attitude questionnaire were those recommended by the National Council of Teachers of Mathematics (12). The problem solving experiment was patterned after "The Playground Problem" which is being used by the USMES (Unified Science and Mathematics for Elementary Schools) project. However, "The Playground Problem" is more open ended and the question is not essentially mathematical where the prime number problem and shadow problem have a specific mathematical problem setting.

Testing Sequence

After a brief introduction to the course, the first day of class was used to pretest for computational and basic skills. At this time the department's placement test was given. During the next two days, but not during class, the 10 randomly selected students were interviewed for their problem solving ability and procedures used to solve the problems. During the semester but before we discussed a specific historical problem, the problem was included on a quiz for pretesting purposes. There was no pretesting for attitude.

At midsemester time the placement test for computational and basic skills was administered again and was designated as mid-test. The last two days of class were used for follow-up testing designated the post-test. During this time the attitude questionnaire and the historical problems in mathematics were given. A test similar to the placement test was given as a post-test for computational and basic skills. The 10 randomly selected students were post-interviewed for their problem-solving ability and procedures used but not during class. Testing, with the exception of problem solving, was done on a group basis.

Summary

In this chapter, the investigation of an innovative approach to teaching general education mathematics courses was described. The instruments used included the department's placement test, a historical problems in mathematics test, an open-ended attitude questionnaire and a test to determine problem solving ability.

A detailed description of the sample was given including the class distribution, years since taking a math course and mathematics background.

The method of instruction and materials and activities used in the instruction were discussed. A typical lesson plan and a summary of teaching techniques used in this class were included.

CHAPTER IV

RESULTS AND DISCUSSIONS

Introduction

Data were collected for the analysis of the study from results of the mathematics placement test, placement post-test, historical problems in mathematics test, problem solving test, and an attitude questionnaire.

Thirty students registered for the A. M. section and thirty-four for the P. M. section. Ten students were randomly selected from the 64 students to be pre-tested and post-tested for their problem solving ability and procedures used. All of the students participated in the testing of historical problems in mathematics, computational and basic skills, and attitude.

Before considering the analysis of the data, mention should be made of the data which, for various reasons, could not be collected. At midsemester time there were no withdrawals. However; it is a college policy that during the eleventh week of class, all students who are doing marginal work (C^- or lower) are to be notified so that they may withdraw. Six students were notified and they withdrew from the course. Two students, for personal reasons, received incompletes, and two students took jobs during the last four weeks of the semester and withdrew from college. By the end of the semester eight students had withdrawn and two students had incompletes, leaving a total of 54 students in the sample.

Case Studies for Computation and Basic Skills

The department placement test served as the pre-test and mid-test for the computational and basic skills. A test similar to the placement test was used as the post-test. By a similar test is meant that the questions were essentially the same but the numbers, symbols, or names used were changed. Also, the position of the correct response was changed in the possible choice of answers. For example, one of the questions on the pre-test was:

> Out of a class of 75 students 3 received the grade of A. The per cent of the class receiving the grade of A was:
>
> (a) 3% (b) .03% (c) 2.5% (d) 4% (e) none of these.

For the post-test the corresponding question was:

> Out of a group of 125 people 5 had cancer. The per cent of the group having cancer was:
>
> (a) 2.5% (b) 4% (c) 3% (d) .03% (e) none of these.

Table 4.1 gives a summary of the results for the three tests.

According to Siegel (16) the Friedman two-way analysis of variance by ranks for k matched samples provides a method for testing whether k matched samples have been drawn from the same population (Table 4.2).

TABLE 4.1.--Computational and Basic Skills Results.

	Pre-test		Mid-test		Post-test	
Class	Average	Median	Average	Median	Average	Median
A.M.	7.1	6	12.9	13	17.3	18
P.M.	8.2	7	12.4	12	19.8	21
Combined	7.7	6	12.6	12	18.6	19

H_0: the different times of the test have no differential effect.

H_1: the different times of the test have a differential effect. That is, the size of the scores depends on the time during the semester when it was given.

Decision:

$$\chi_r^2 = \frac{12}{(54)(3)(3+1)} [(61.5)^2+(100.5)^2+(162)^2]-3(54)(3+1)$$

$= 95.097 \chi_2^2 = 13.816$ and the decision is to reject H_0 at .001 level of significance. The conclusion is that the scores on the computational and basic skills depended on the time during the semester when it was given.

Case Studies for Historical Problems in Mathematics

Questions on quizzes given during the semester served as pretest scores for the historical problems in mathematics. The investigator's test for historical problems in mathematics patterned after the test by Avital served as the post-test instrument (Appendix A.2) Table 4.4 gives a summary of the results.

TABLE 4.2.--Scores of Fifty-four Students on Computational and Basic Skills Under Three Conditions of Time.

Subject	Pre	Mid	Post	Subject	Pre	Mid	Post
1	16	16	23	28	9	8	14
2	3	7	13	29	9	13	18
3	12	10	23	30	9	14	19
4	6	6	17	31	4	10	17
5	15	22	23	32	11	9	22
6	12	18	19	33	19	23	25
7	14	10	15	34	4	10	17
8	10	21	22	35	8	10	21
9	6	11	14	36	3	5	11
10	7	6	14	37	5	14	20
11	9	10	13	38	3	6	20
12	5	8	19	39	2	8	14
13	7	13	21	40	19	16	24
14	21	24	25	41	17	21	25
15	4	9	14	42	18	24	25
16	16	18	24	43	6	13	21
17	3	4	9	44	10	20	23
18	6	9	11	45	10	14	22
19	8	13	18	46	7	18	23
20	11	15	18	47	10	16	23
21	12	17	22	48	6	15	22
22	5	10	11	49	9	14	22
23	3	7	10	50	13	17	23
24	3	8	18	51	6	9	17
25	12	14	21	52	6	7	15
26	15	20	25	53	8	8	16
27	7	11	24	54	4	5	14

TABLE 4.3.--Ranks of Fifty-four Students on Computational and Basic Skills Under Three Conditions of Time.

Subject	Pre	Mid	Post	Subject	Pre	Mid	Post
1	1.5	1.5	3	28	2	1	3
2	1	2	3	29	1	2	3
3	2	1	3	30	1	2	3
4	1.5	1.5	3	31	1	2	3
5	1	2	3	32	2	1	3
6	1	2	3	33	1	2	3
7	2	1	3	34	1	2	3
8	1	2	3	35	1	2	3
9	1	2	3	36	1	2	3
10	2	1	3	37	1	2	3
11	1	2	3	38	1	2	3
12	1	2	3	39	1	2	3
13	1	2	3	40	2	1	3
14	1	2	3	41	1	2	3
15	1	2	3	42	1	2	3
16	1	2	3	43	1	2	3
17	1	2	3	44	1	2	3
18	1	2	3	45	1	2	3
19	1	2	3	46	1	2	3
20	1	2	3	47	1	2	3
21	1	2	3	48	1	2	3
22	1	2	3	49	1	2	3
23	1	2	3	50	1	2	3
24	1	2	3	51	1	2	3
25	1	2	3	52	1	2	3
26	1	2	3	53	1.5	1.5	3
27	1	2	3	54	1	2	3
				R_j	61.5	100.5	162

TABLE 4.4.--Results of Historical Problems in Mathematics.

	Pre-test			Post-test		
Class	Average	Median	Range	Average	Median	Range
A.M.	.4	0	0-2	17	18	12-20
P.M.	.7	0	0-4	17.3	17	10-19
Combined	.6	0	0-4	17.2	17	10-20

The relative magnitude as well as the direction of the differences for each pair is known for the data collected for the historical problems in mathematics. Therefore, the Wilcoxon matched-pairs signed-ranks test is the appropriate test to use to analyze these data (16) (see Table 4.5).

H_0: there is no difference between the pre-test scores and the post-test scores for the historical problems in mathematics.

H_1: the post-test scores are greater than the pre-test scores on the test for historical problems in mathematics. That is, students after taking the course were better able to answer the questions on historical problems in mathematics.

Decision: $z = -6.4 < -3.1$. Therefore, H_0 is rejected at the $\alpha = .001$ level of significance. That is, students after taking the course were better able to answer the questions on historical problems in mathematics.

Case Studies for Problem Solving

On the second day of class the 10 randomly selected subjects were individually interviewed for their ability and procedures used to determine whether two five digit numbers were prime. The author served as the interviewer.

TABLE 4.5.--Scores of Historical Problems in Mathematics.

Subject	Pre-test	Post-test	d	Rank of d	Rank with less frequent sign
1	1	16	15	21.5	
2	0	13	13	5.5	
3	0	18	18	41.5	
4	0	19	19	51.0	
5	0	17	17	30.0	
6	0	16	16	21.5	
7	0	18	18	41.5	
8	0	18	18	41.5	
9	0	16	16	21.5	
10	0	14	14	88.0	
11	0	12	12	3.0	
12	0	18	18	41.5	
13	1	20	19	51.0	
14	2	16	14	8.0	
15	0	18	18	41.5	
16	1	17	16	21.5	
17	0	17	17	30.0	
18	1	13	12	3.0	
19	0	20	20	54.0	
20	2	19	18	41.5	
21	0	18	18	41.5	
22	0	18	18	41.5	
23	0	19	19	51.0	
24	0	14	14	8.0	
25	1	16	15	21.5	
26	1	14	13	8.0	
27	2	19	17	51.0	
28	0	12	12	3.0	
29	0	18	18	41.5	

TABLE 4.5.--Continued.

Subject	Pre-test	Post-test	d	Rank of d	Rank with less frequent sign
30	0	18	18	41.5	
31	0	16	16	21.5	
32	0	15	15	13.5	
33	2	17	15	13.5	
34	0	16	16	21.5	
35	0	18	18	41.5	
36	0	17	17	30.0	
37	0	19	19	51.0	
38	0	16	16	21.5	
39	0	10	10	1.0	
40	3	18	15	13.5	
41	2	18	16	21.5	
42	4	19	15	13.5	
43	0	17	17	30.0	
44	2	17	15	13.5	
45	2	19	17	30.0	
46	0	17	17	30.0	
47	0	18	18	41.5	
48	0	16	16	21.5	
49	0	18	18	41.5	
50	2	19	17	30.0	
51	0	19	19	51.0	
52	0	15	15	13.5	
53	0	17	17	30.0	
54	0	18	18	41.5	
					T = 0

$$\text{Mean} = \frac{54 \times 55}{4} = 742.5 \text{ and S. D.} = \frac{54 \times 55 \times 109}{24} = 116.5$$

$$z = \frac{0 - 742.5}{116.5} = -6.4$$

Each subject was allowed 10 minutes and was allowed to use any of the materials in the room which included pencil, paper, blackboard and chalk, electric calculator, textbook, slide rule and any of the laboratory materials available during the semester. They were allowed to ask questions of the investigator which required a "yes" or "no" answer. The investigator could decline to answer if the questions were too leading. At the end of the semester, the subjects were interviewed for their ability and procedures used to determine whether two different five digit numbers were prime.

The problem was read to the subjects and each was given a copy of the questions. The investigator ascertained that each subject knew the meaning of "prime number." The subjects were scored as follows:

0 - if the subject guessed the answer

0 - if the subject calculated at random

1 - if the subject divided with primes only

1 - if the subject divided with primes sequentially

1 - if the subject divised an algorithm

1 - if the subject used any of the materials besides pencil, paper, blackboard and chalk

1 - if the subject determined the correct answer.

The maximum points available were five (see Table 4.6).

H_0: there is no difference between the pre-test scores and the post-test scores for the Problem Solving Experiment.

TABLE 4.6.--Scores for Problem Solving Ability and Procedures Used.

Subject	Pre-test	Post-test	d	Rank of d	Rank with less frequent sign
P 1	0	1	1	1.5	
P 2	0	1	1	1.5	
P 3	1	3	2	3	
P 4	0	0	0	deleted	
P 5	0	4	4	8	
P 6	0	4	4	8	
P 7	1	4	3	5	
P 8	0	3	3	5	
P 9	0	4	4	8	
P10	1	4	3	5	
					T = 0

H_1: the post-test scores are greater than the pre-test scores for the Problem Solving Experiment. That is, students after taking the class, were better able to solve the problem than they could before the class.

Decision: There is a large number of ties and the exact null distribution is not applicable unless ties are considered. Therefore, $2^9 = 512$ and $(512)(.001) = .512$ would allow one point in the region of rejection. Therefore, with $T = 0$, H_0 is rejected in favor of H_1 concluding that students were better able to solve the problem after taking the class.

In addition to determining whether two numbers were prime, the subjects were asked to solve a problem that had not been discussed during the class nor had they been pre-tested for their procedures used and ability to solve the problem. They were asked whether your shadow gets shorter or longer as you walk away from a lamp post; if someone disagreed with you, how would you convince that person that you were right? There were three other questions relating to the shadow:

1. What would the path of the shadow look like if the person walked in a straight line?
2. What would the path of the shadow look like if the person walked in a circle?
3. What would the path of the shadow look like in general, that is, for any path that the person walks?

The method for collecting the data for this part of the study involved the use of a tape recorder, note taking by the investigator and the students' notes. The author found the tape recorder a great aid in refreshing her memory for students' comments.

Subject P1

Subject P1 answered the question by writing that the shadow gets longer and then it gets shorter. He used a flashlight and pencil to demonstrate what would happen. Then he changed his answer to longer.

In response to the path of a shadow when a person walks in a straight line, he immediately responded, "I don't know." He used a flashlight and pencil to determine that the path was a straight line.

For the path of a shadow when a person walks in a circle he responded, "I don't know that either." Then he moved the pencil in a circle under the flashlight and concluded that the path was "almost a circle but a little more like an ellipse."

In response to the path of a shadow in general, he also didn't know this answer. When he used the flashlight and pencil, he still was unable to draw any conclusions.

Subject P2

Subject P2 immediately responded that the shadow got longer. She used a flashlight and pencil to show what the shadow would look like. Her response to the path of a shadow when a person walks in a straight line was, "I don't know." Then she drew a straight line on a piece of paper and followed the straight line with the pencil and concluded that the shadow followed the straight line. For the path of a shadow when a person walks in a circle, she responded, "I don't know." She drew a circle and followed the circle with the pencil and concluded that the path was a circle. In response to the path of a shadow in general, she responded, "It follows the path the person is walking." Then she "verified" her statement with the flashlight and pencil and appeared satisfied that her answer was correct as she indicated that she was finished.

Subject P3

Subject P3 responded, "The shadow gets longer, but that must be wrong as that is so obvious." He used the desk lamp and pencil to verify his statement. In response to the path of a shadow when a person walks in a straight line, he used the lamp and pencil and concluded that it was a straight line. For the path of a shadow when a person walks in a circle, he drew a circle and followed it with his pencil. He concluded that the path was "almost a circle, but a little tilted." In response to the path of a shadow in general, he immediately responded, "The same path but I think it really depends on something." He used the lamp and pencil trying various paths but couldn't decide that it depended on anything and so he wrote his answer as "the same path."

Subject P4

Subject P4 answered the question by writing that "the shadow gets longer as long as you were in the rays of the light." First he tried to use the overhead light and a ruler but he couldn't get a shadow that was sharp enough to demonstrate what would happen. He then used the lamp and a pencil to demonstrate that the shadow gets longer. In response to the path of a shadow when a person walks in a straight line, he drew a straight line and followed it with a pencil concluding that the path was a straight line. In response to the path of a shadow when a person walks in a circle, he responded, "I don't know." He drew a circle on a piece of paper and followed the circle with a pencil. He moved the paper to three different places under the light and said that the path was "the same as the person" and wrote that the "path was a circle." In response to the path of a shadow in general, he immediately responded, "It is the same as the person is moving." He then used the lamp and pencil to demonstrate his statement.

Subject P5

Subject P5 answered the question by writing that the shadow gets shorter. Then in trying to convince someone else, she used the desk lamp and pencil. After examining the shadow she changed her answer to longer. In response to the path of a shadow when a person walks in a straight line, she immediately responded, "I don't know." She drew a straight line on paper from one corner to the other and followed the line with her pencil. She concluded that the shadow followed the line. For the path of a shadow when a person walks in a circle, she said that she didn't know that either. She drew a circle and followed it with her pencil under the lamp. She moved the circle farther away from the lamp for five consecutive tries. She concluded that the path was a circle but the diameter increased as one gets farther from the lamp. In response to the path of the shadow in general she responded, "The path is the same as the path of the person." Then she proceeded to use the lamp and pencil to demonstrate her conclusion.

Subject P6

Subject P6 answered that the shadow gets longer. He drew similar triangles to demonstrate that the shadow gets longer. In response to the path of a shadow when a person walks in a straight line, he used the desk lamp and a ruler to demonstrate that the path would be a straight line. In response to the path of a shadow when a person walks in a circle, he tried to demonstrate what would happen using the lamp and a ruler. Because the ruler was awkward, he used a pencil as the person and concluded that the path was elliptic. In response to the path of the shadow in general, he used the desk lamp and pencil to check the path and concluded, "It is about the same as the person but will be farther away if the person is farther away from the lamp."

Subject P7

Subject P7 answered the question by responding, "The shadow gets shorter." He used the desk lamp and a pencil to demonstrate what would happen and commented that the shadow got longer. Then he used the flashlight instead of the desk lamp and responded, "It still gets longer." He changed his answer to longer. He used the desk lamp and pencil to determine the path of the shadow when a person walks in a straight line and concluded that the path was a straight line. Similarly for the circle, he concluded that the path was a circle. In response to the path of the shadow in general, he used the desk lamp and pencil and concluded that the path was "like the path of the person."

Subject P8

Subject P8 answered the question by writing that the shadow gets longer. He used similar triangles to demonstrate what would happen to the length of the shadow. In response to the path of a shadow when a person walks in a straight line, he answered, "It's a straight line." He used the desk lamp and a pencil to determine the path of a shadow when a person walks in a circle, and concluded that it was "elliptic and not quite circular." In response to the path of a shadow in general, he used the desk lamp and pencil to determine what would happen. He wrote, "the same path but the period isn't the same as the person's."

Subject P9

Subject P9 wrote, "the shadow gets longer until out of realm of light." She used similar triangles to demonstrate that the shadow gets longer. She used the desk lamp and a pencil to demonstrate that the path was a straight line when a person walks in a straight line. She moved the pencil in circles and concluded that the "path is a circle with larger diameters as you walk away from the light." In response to the path of the shadow in general, she used the desk lamp and pencil and concluded that it was "the same path but with different amplitudes" and she explained what she meant by different amplitudes.

Subject P10

Subject P10 said that he thought that "the shadow gets longer." He used the desk lamp and ruler to demonstrate that it would get longer. Because the ruler was too long, he used a pencil as the person to demonstrate that the path would be a straight line if the person walked in a straight line. Similarly, he demonstrated for the circle and concluded that the path was a circle. To the question of the path of the shadow in general, he responded, "the path he walks." He used the desk lamp and pencil to demonstrate this.

Summary

All of the students concluded that the path of the shadow gets longer when a person walks away from a lamp post. Likewise, they were able to conclude that the path of a shadow is a straight line if they walk in a straight line. If a person walks in a circle, all of the subjects concluded that it was a circle, circular or elliptic. Nine of the students were able to conclude that the path of the shadow in general would be like the path the person walked.

One student was unable to arrive at a conclusion in this instance.

So, after taking this class one can conclude that all but one of the students were able to solve the shadow problems. This does not conclude, however, that they did not have these skills before taking the class.

Case Studies for Attitude Toward Mathematics

Because the concept of attitude is difficult to quantify and because the questionnaire was open-ended, the investigator decided to use the technique of key words or phrases to summarize the data collected. The key words and phrases were selected from the students' comments. The per cent responses were calculated on the basis of one response per subject. The subjects were asked to write whatever came to their minds but not to evaluate the instructor. (They were told that they would be given a subsequent chance to evaluate the instructor.) They had no previous knowledge that they were going to be asked to fill out the questionnaire. They were not informed that this form was an attitude questionnaire. This was not communicated to the subjects because it was felt that to have done so might have biased their answers.

Also, it was intended that the first three questions would be answered in a general nature and the last three questions would be answered with respect to this course.

It soon became apparent when reading the questionnaires that most of the subjects answered all of the questions in relation to this course.

It took no longer than seven minutes for any subject to fill out the questionnaire. From this and the fact that the subjects had no previous knowledge that they would be asked to fill it out, one can conclude that their responses were spontaneous and hence they would tend to be a more accurate measure of their attitudes (5).

1. How do you feel about mathematics?

Key words or phrases	% responses
Like it; enjoy it; at ease with it; love it; very good; like it very much; very interesting; like it and have a deeper appreciation of it; not afraid of it anymore; like it and it strengthened my use of it; like it and it was fun; like it and it was interesting	51.9
Worthwhile; necessary; essential; useful; challenge; exact and consistent; foundation of cosmos	29.6
Ok; don't mind it	12.9
Don't like it; don't really care for it . . .	5.6

2. What do you like best about it?

Key words or phrases	% responses
Everything we did; classroom procedure . . .	7.4
Math materials we use	29.6
Everyday math; applications; usefulness . . .	9.2
Variety of problems; easy to understand; learned new things; friends; history . . .	9.2
Arithmetic; number problems	9.2
Algebra; equations	14.9

Gives satisfaction; challenge; thinking involved; logic; concreteness; reasoning involved	20.4

3. What do you like least about it?

Key words or phrases	% responses
Nothing	14.9
When I don't understand; and when others do . .	13.0
Too much to remember; too wide; memorizing; time consuming	11.1
Just about everything; material covered . . .	3.7
Logarithms; algebra; equations; story problems; history	11.1
Class too large; noisy; grading on handouts; tests; homework; hard for me; when not clear enough	18.5
Not knowing all I would like to know	3.7
Blank	3.7
*Calculus; theory; proofs; trig.	20.4

*Comments such as these are of a general nature and are not applicable to this course.

4. Has this course influenced your feelings toward mathematics?

Key words or phrases	% responses
Positive: very much so; positively!; enjoyed it; for the better; better understand; decided to major in math; fun; know its worth it; not afraid of it anymore; very successful course; strengthened my use of; strengthened my confidence; different approach. (These comments were preceded by "yes" and then an explanation why it improved.)	61.1
Other: needed, necessary	18.5
No change: already liked, still do	13.0
still don't like	5.6
Blank:	1.9

5. What do you like about this course?

Key words or phrases	% responses
New ways and techniques to solve problems . .	11.1
The math materials	5.6
Applications; explorations	7.4
Wide variety so never got boring	9.2
Good review; learned something; something really needed; understood for first time	24.0
Very interesting	5.6
Eliminated fear of math	3.7
Classroom procedures; no lectures; relaxing atmosphere; fun	14.9
No textbook	5.6
Taught well	11.1
Everything	1.9

6. What would you do to change this course to make it more interesting?

Key words or phrases	% responses
Don't know; blank	9.2
No change needed; leave as is; interesting enough	24.0
Have reference book; textbook	5.6
Eliminate arithmetic text (redbook)	3.7
More history	1.85
No history	1.85
More classroom materials; more classroom participation	18.5
Have field trips; more to daily life	9.2
Fewer topics	9.2
More topics	1.85
More handouts	1.85
Fewer handouts	1.85
More general; delve deeper	3.7
More slowly; clearer expectations	3.7
Longer than 15 weeks	1.85
Meet only 3 times a week	1.85

Comments Made by Students During Semester

"I'm a clockwatcher. Every few minutes I usually say, 'Oh, so many minutes left.' But not in this class. I'm really enjoying it."

"The time goes so fast, the hour is gone by without even noticing where it went."

"I'm trying to get my wife to take this course."

"After this I couldn't take another math class of the other type."

"I prefer working in class, at least I can get help."

"This is an interesting course, I'm glad I'm taking it."

"Boy, I need this!"

"I learn the most by working in class. Then what I don't fully understand, I can work out at home."

"I enjoy it. I find it relaxes me and in high school I dreaded going to math classes."

"I really like the relaxed atmosphere."

"If Harold can get it and understand it, then this is a good way to learn."

"I'm signing up for more math."

"I've got a job and I'm really using this stuff we learned."

"I've got a job and now I could use more of this math."

"I really never knew before how important it was to know math."

"I've changed my major to math."

"I want you to know that I really enjoyed the class."

"I didn't do well because there was too much noise for me."

"I usually tune the instructor out after 5 minutes, so class is a waste of time. But not in this class."

Summary

The analysis of the study has been presented in this chapter. The statistics consisted of four parts. The Friedman two-way analysis of variance test using the mathematics placement test investigated whether the size of the score on computational and basic skills depended on the time during the semester when the test was given. The null hypothesis of no differential effect was rejected at the .001 level.

The Wilcoxon matched-pairs signed-ranks statistical test using the historical problems in mathematics test investigated whether students after taking this course were better able to answer the questions on historical problems in mathematics. The null hypothesis of no difference was rejected at the .001 level of significance.

The Wilcoxon matched-pairs signed-ranks statistical test was used to investigate whether students were better able to solve problems after taking this course. The null hypothesis of no difference was rejected at the .001 level of significance.

No statistical test was used to investigate students attitude toward mathematics. The responses to the questionnaire were summarized using key words or phrases. Also, a list of student comments were presented.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The purpose of this study was to record and describe the behavior of the subjects taking this course. Specifically, the purposes were concerned with four problems:

1. To determine if this method was effective in the students' learning of computational skills;
2. To determine if this method was effective in the students' learning of the historical problems of mathematics;
3. To determine the effect of this method on the students' problem solving ability; and
4. To determine the effect of this method on the students' attitude toward mathematics.

The study was conducted during the Winter semester of the 1973 Academic Year within the Department of Mathematics, Saginaw Valley College.

Classroom materials and evaluation instruments were developed and prepared during the Fall Semester of 1972.

Chapters I, II, III, and IV of this thesis were written to provide the reader an understanding of the problem, its background, its analysis and its resolution.

Conclusions

Purpose One

The writer concludes that this class had a significant effect on the subjects' ability to answer questions on the computational and basic skills test. This effect was uncovered using the Friedman two-way analysis of variance by ranks test. The Friedman test determined that the rank totals differed significantly at the .001 level of significance. That is, the size of the score depended on the time during the semester when the test was given.

It is interesting to note that, in comparison to Fall Semester, 1972, where less than 1% were eligible to register for college algebra, 67% of these students had 17 or better making them eligible to register for college algebra.

Purpose Two

The writer concludes that this class had a significant effect on the subjects' ability to answer the questions on historical problems in mathematics. This effect was uncovered using Wilcoxon matched-pairs signed-ranks test and the magnitude of the effect is to be found in Table 4.5. It is more interesting to note that the average on the pre-tests was .5 while on the post-test it was 17. If one is interested in mastery learning, 81.5% of the class scored 80% or better on the post-test. It was the intent that

75% of the class would score 75% or better on the post-test. So, in the area of historical problems in mathematics one might conclude that 80/80 would be a reasonable expectation.

Purpose Three

The Wilcoxon matched-pairs signed-ranks test uncovered significant results concluding that students after taking this class, were better able to solve problems than they could before the class. The magnitude of the effect is to be found in Table 4.6.

Purpose Four

The effect of this class on students' attitude is summarized in Chapter IV (pp. 65-67). It should be pointed out that there was no entry attitudinal data on these students. However, given the content of the statements on the questionnaires, the writer concludes that the methods used in this class improved the attitude of the subjects toward mathematics. Sixty-one percent of the students responded that their feelings improved while only 5.6% responded that they still don't like mathematics. No student responded that he disliked math more now than he did before.

The fact that no student responded that he disliked math more now than he did before is relevant when comparing with the Fall Semester of 1972 which used the lecture-textbook method. Thirty-three percent of those students wrote that they "hate it, now," "detest it, now," "dislike it, more," "absolutely hate it!," or "it discouraged me."

The following comments are of a more general nature:

Many days it was necessary to remind the students that the class was over.

Students frequently asked to take the geoboards, tower puzzles, Cuisenaire rods, etc. home with them over week-ends to "play with" or to show friends and family.

After students learned to share their knowledge, they also brought their own calculators to share with others as there weren't enough for students to have their own.

Several of the students took the math materials to their place of employment to show their fellow employees how to get mathematical solutions. Many of the students had played with these before and never knew that there was math involved.

Fourteen of these students have registered for Math 001 this spring half semester. (Math 001 is a refresher that has been added this spring for those students who would like more math but don't feel ready for college algebra.)

Three students have told me that they were changing their major to math as a result of taking this course.

Only one student commented on the noise and there certainly was more of this than in a "traditional" class.

These observations in addition to the comments and questionnaires led the writer to conclude that the attitude toward mathematics had improved for many of the students.

These methods have been used in the same course by the author and another faculty member at Saginaw Valley College during the subsequent semester after the data for this theses were collected. The other faculty member had taught the course for four years with unsatisfactory results. He was extremely impressed with the outcome after using this method. It was the first time that he enjoyed teaching the course.

Using the lecture-textbook method the wide variety of students in a class such as this (Tables 3.1, 3.2, 3.3) has created a difficult problem. It appeared to the writer that this problem was basically eliminated because of the interaction of the students. The students who understood the material very readily assisted the students who were having more difficulty understanding it. In fact, one might conjecture that the heterogeneity of the group is a contributing factor to the success of a course using the method of this thesis.

The author recognizes that this paragraph is of a subjective nature and necessarily so as she is giving her opinion of the students' behavior. It took about four weeks for the students to adjust to this approach in terms of classroom behavior. The right and need to communicate openly with their fellow students led to unnecessary "horseplay" which was not conducive to learning. Also, during this time students would often ask, "What's the answer?" without trying to do the work themselves. It appeared to be a difficult transition for some of the students to "discover" the answers themselves. However, after this initial adjustment period, the author felt that this was the approach to use in a general education mathematics class with such a wide variety of backgrounds.

A comment might be in order regarding the use of tutors. Tutors were available eighteen hours per week to assist the students in this class (see Appendix B.4). However, very few students took advantage of this opportunity.

The author discussed this with the students at various times during the course. In this atmosphere students felt that they had become acquainted with members of the class. It was easier to get help from their classmates than from the tutors because their classmates understood the difficulties that they were having better than the tutors. From this limited experience, it would seem that the use of tutors is not an important consideration in the success of a course such as this.

With regard to the problem solving experiment, one student did not improve his score on the prime number portion (p. 59). In checking the attendance records, he was absent for two weeks during which time prime numbers were covered. The student was present for most of the remaining course and performed well on the shadow portion of the experiment.

For the shadow portion of the experiment, the subjects either used a desk lamp or flashlight for the lamp post and a pencil for the person to determine the path of a shadow when a person walks in a straight line, in a circle, or in any path. A more analytic approach could have been employed. Namely, that the rays of the lamp can be said to form a plane. The rays of the lamp on the head of the person walking in a straight line form a curve that intersects the plane in a straight line. Similarly, the rays of the lamp on the head of a person walking in a circle form a curve that intersects the plane in a circle. The rays of the lamp on the head of a person walking in any path forms a curve that intersects the plane in a path similar to the one that the person is walking. This is essentially what the students were observing but none expressed their solution in this form. There had been discussion of the intersection of curves with planes in this course but no transfer had taken place in writing the solution to this problem. This could indicate that the geometry taught had not been sufficiently related to the real world.

The cost of the materials used in this course which were not consumable was about $1,000.00. The movies were rented at a cost of $6.00 each. Those designated by an asterisk are movies that the author recommends (see Appendix C.1).

Because this is a case study of the effects of an innovative approach on general education mathematics students, the author presents the following recommendations for further study.

Recommendations

1. Further individualization which would allow students to learn the mathematics he needs at a pace consistent with his background, interests, goals, and ability be investigated in a general education mathematics course.

2. More extensive use of physical math materials be used in the general education mathematics course.

3. Schedule course for two hour blocks of time so that students can finish the task that they have started. Determine whether the scheduling improves computational and basic skills or attitude.

4. A research experiment should be carried out testing the purposes of this study with a control group using the lecture-textbook method.

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APPENDICES

APPENDIX A

INSTRUMENTS USED

A.1 PLACEMENT EXAMINATION

A.2 HISTORICAL PROBLEMS IN MATHEMATICS TEST

A.3 PROBLEM SOLVING TESTS

A.4 ATTITUDE QUESTIONNAIRE

A.5 MATH BACKGROUND

Appendix A.1

MATHEMATICS PLACEMENT EXAMINATION

Inclusion of the Mathematics Placement Examination itself is impossible since that could terminate the examination's usefulness.

A review of the examination shows that questions covering the following material appear on the test:

1. Percent, proportion, scientific notation.
2. Operations with signed numbers, exponents (integral and rational), and rational algebraic expressions.
3. Solution of linear and quadratic equations.
4. Factoring second degree equations.
5. Simplification (including multiplication and division) of algebraic expressions of one and two terms, complex fractions.
6. Verbal problems.

Students with a "good" background in first year high school algebra should have little trouble with the test. There is no geometry on the examination. The test will entrap the student who would make such errors as: $(X^{-1} + Y^{-1})^{-1} = X + Y$ or $(X^2)^3 = X^5$.

Appendix A.2

HISTORICAL PROBLEMS IN MATHEMATICS

Name: ____________________

Date: April 13, 1973

Instructions: Give short and precise answers, using only the space allotted.

Make your answers as clear and readable as possible.

If you don't know an answer to a question write "don't know."

1. Some people say that the introduction of a special symbol for the number zero is one of the greatest inventions of humanity. Explain: ____________________

2. a. What is an "algorithm?"

   b. What does one use the Euclidean Algorithm for?

3. a. What is a theorem? ____________________

   b. What is the Fundamental Theorem of Arithmetic?

4. There are problems in mathematics which can easily be formulated and which nevertheless haven't been solved yet, even though mathematicians have tried very hard to solve them. List at least one such problem. (Give as many details as you know).

5. a. In what way has there been established a basic connection between algebra and geometry?

b. When was this connection originated and by whom?

6. What modern studies has the discovery of the conic sections made possible?

7. a. Who introduced place value in their number system?

b. What base did they use?

c. Give 2 examples where that base is used today.

8. a. What is inductive reasoning?

b. Give an example.

c. Give a limitation of inductive reasoning.

9. a. What is deductive reasoning?

b. Give an example. ______________________________

____

c. Give the outstanding advantage of deductive reasoning.

____

____

d. Give the outstanding disadvantage of deductive reasoning.

____

____

10. From the soil of Egypt the science of Geometry--explain.

____

____

____

Appendix A.3

PROBLEM SOLVING

The zip code in East Lansing is 48823. Someone in East Lansing asked if this is prime. What would your answer be? How would you determine whether it is prime?

A zip code in this area is 48713. Is this zip code prime?

Problem Solving, Continued.

1. a. Is the number 48827 prime?

   b. Is the number 48719 prime?

2. Students were discussing whether your shadow gets shorter or longer as you walk away from a lamp post. What would your answer be? If someone disagreed with you, how would you convince that person that you were right?

   What would the path of the shadow look like if the person walked in a straight line?

   What would the path of the shadow look like if the person walked in a circle?

   What would the path of the shadow look like in general, that is, for any path?

Appendix A.4

QUESTIONNAIRE

1. How do you feel about mathematics?

2. What do you like best about it?

3. What do you like least about it?

4. Has this course influenced your feelings toward mathematics?

5. What do you like about this course?

6. What would you do to change this course to make it more interesting?

Appendix A.5

MATH BACKGROUND

1. Name ____

2. Class Standing (circle one) Fr. So. Jr. Sr.

3. Major ____

4. Math courses since 8th grade

____

____

____

____

5. How long has it been since your last math class?

____

APPENDIX B

PROCEDURES

B.1 SEMESTER PLANS

B.2 SAMPLE DAILY PLANS

B.3 TOPICS COVERED

B.4 TUTORING SCHEDULE

B.5 REFERENCE TESTS

B.6 SAMPLE ASSIGNMENT SHEET

B.7 SAMPLE HANDOUTS

B.8 SAMPLE QUIZZES

Appendix B.1

JANUARY

Monday	Wednesday	Thursday	Friday
1	3 Pretest 35 minutes Brief Introduction	4 Numbers Number line Exponents Bases Decimal System	5 Negative naturals Order Absolute value Inequalities
8 Digital roots Axioms under x and +	10 Axioms under - and ÷ Division algorithm Order of operations	11 Graphing Linear equations Histograms, mean, median, and mode	12 Rational operations on negative integers Graphing
15 Arithmetic sequences Fibonacci sequences Inductive reasoning	17 Geometric sequences Power sequences	18 Geometric and arithmetic sequences	19 35 min. test on Chapters 1-3 and handouts Rational numbers
22 Prime factors Composite nos. H.C.F. L.C.D. Euclidean alg. Computations with rational numbers	24 Fund. Thm. of Arithmetic Decimal fractions Place value Scientific notation Addition and subtraction of fractions	25 Mult. and div. of fractions Repeating, terminating, non-terminating decimals	26 Ratio, proportion percent
29 Denominate numbers and arithmetic of measurement	31 Metric system Errors in approximate numbers.		

FEBRUARY

Monday	Wednesday	Thursday	Friday
		1 Square root Irrational numbers Square root algorithm Equality	2 Solution of elementary equations Arithmetic of exponents
5 Word problems Squaring both sides of an equation	7 Arithmetic of expon-ents Square roots and exponents	8 Graphing	9 Graphing linear and quadratic equations
12 Graphing circle ellipse	14 Graphing hyperbola Interpopula-tion and extrapola-tion	15 Logarithms	16 Logarithms
19 Washington's Birthday	21 Logarithms	22 Quadratics graphing problems	23 Quadratics graphing problems
26 Quadratic equation Polynomials	28 Mid-test 35 min.		

MARCH

Monday	Wednesday	Thursday	Friday
		1 Intuitive geometry Area, squares rectangles, triangles	2 Geometry Pythagorean theorem Distance formula Perimeters
5 Pentagons Parallelogram Trapezoids Similar figures Congruent figures	7 Parallel lines Perpendicular lines	8 Factoring squares of Binomials	9 Factoring difference of 2 squares
12 Factoring sum and difference of 2 cubes	14 Factoring	15 Systems of linear equations graphically	16 Systems of linear equations
19 Algebraic fractions sums and differences	21 Algebraic fractions	22 Algebraic fractions-multiplication	23 Algebraic fractions-division
26 Exponents and radicals	28 Exponents and radicals	29 Exponents and radicals	30 Test 35 min.

APRIL

Monday	Wednesday	Thursday	Friday
2	4	5 Pascal's triangle Binomial theorem	6 Sum and differences cubed
9	11 Review	12 Attitude questionnaire Historical problems in mathematics post-test	13 Computational and basic skills post-test
16 Last day of class Returned Historical problems in math Reviewed post-test results			

Appendix B.2

SAMPLE DAILY PLANS

Day and date	Wednesday January 3, 1973
Classroom	Introduce course Departmental pre-test 25 min.
Materials	Pretest
Homework	Computational Arithmetic (abbreviated C. A. from now on) C. A. pp. 1-9 explanations C. A. pp. 15-16 1-42 exercises
Objectives	1. To learn to identify counting numbers (natural numbers) whole numbers, even whole numbers, odd whole numbers and integers 2. Use the number line.

Day and date	Thursday January 4, 1973
Classroom	Introduce number line, numeration system demonstrator, and odometers.
Materials	Number line, numeration system demonstrator odometers, handout
Homework	C. A. pp. 9-14 explanations C. A. pp. 16-17 43-62 exercises
Objectives	1. To learn to use exponential notation 2. To learn to read and write numbers in the decimal system 3. To discover advancing algorithm for addition in any base 4. To learn the concept of place value and the number zero.
Evaluation	Quiz tomorrow

Day and Date	Friday January 5, 1973
Classroom	Quiz, "Guess My Rule" If students don't volunteer, use random number generator and introduce the concept of "equally likely."
Materials	Quiz, handout on checks, Cuisenaire rods
Homework	C. A. pp. 19-27 explanations C. A. pp. 46-47 1-34 exercises
Objectives	1. Continue advancing algorithm discussion for addition and to practice addition of whole numbers 2. Checks--application of paraphrasing in decimal system 3. Encourage students to work together--group interaction 4. Build foundation for function, graphing, inductive reasoning and to form conjectures.
Evaluation	Quiz on Monday

Day and Date	Monday January 8, 1973
Classroom	Quiz, make histogram from data from last quiz, Guess My Rule
Materials	Quiz, graph paper, rulers, handout, Cuisenaire Rods
Homework	C. A. pp. 27-45 explanations C. A. pp. 47-48 35-66 exercises
Objectives	1. Inductive reasoning 2. Data collection 3. Practice addition and multiplication of whole numbers 4. Introduce negative numbers through two dimensional graphs 5. Group interaction

---

Appendix B.3

TOPICS COVERED

Following is a list of major topics listed for the course during the time this study was in progress.

1. Basic Arithmetic Operations with whole numbers, integers, rational numbers and real numbers; prime factorization
2. Ratio, Proportion, Percent, Measurement
3. Equality and order axioms
4. Properties of real numbers
5. Sums, differences, products and quotients of signed numbers
6. Absolute value
7. Order of operations
8. Polynomials--sums, differences, products, quotients, and factoring (linear and quadratic)
9. Signs of algebraic fractions
10. Reduction of Algebraic fractions
11. Sums, products and quotients of algebraic fractions
12. Integral, rational, zero and negative exponents
13. Radicals
14. Solution of first and second degree equations
15. Application Problems
16. Systems of Linear Equations
17. Graphing
18. Problem solving
19. Inductive and deductive reasoning
20. Historical Problems in Mathematics
21. Metric system
22. Geometry, Pythagorean Theorem, distance formula, etc.
23. Logarithms
24. Histograms, mean, median, mode
25. Binomial theorem
26. Sequences

Appendix B.4

TUTORING SCHEDULE

Day / Time	Monday	Tuesday	Wednesday	Thursday	Friday
8-9					
9-10			Mike B. Rm. 217		
10-11	Joyce N. Rm. 218			Don H. Rm. 310	Don H. Rm. 217
11-12	Joyce V. Rm. 218			Joyce V. Rm. 310	
12-1	Sue R. Rm. 235	Rick D. Rm. 233	Sue R. Rm. 217	Sue R. Rm. 218	Rick D. Rm. 217
1-2	Matt K. Rm. 218	Matt K. Rm. 238		Matt K. Rm. 218	Matt K. Rm. 217
2-3				Tim H. Rm. 235	
3-4	Jerry S. Rm. 218		Don H. Rm. 217		
4-5					
5-6					
6-7					
7-8					
8-9					
9-10					

Tutors' math background
Don H.--161 to 262, 313, CS 147
Joyce V.--161 to 262, 313, 372, CS 147
Sue R.--CS 147, 157, 331, 332
Joyce N.--161 to 262, 370, CS 147, 157, 331, 332
Jerry S.--161 to 262, CS 147, 157, 291, 292, 331, 332
Rick D.--161, 162
Matt K.--121, 122, 161, 162, CS 147, 157, 331, 332
Tim Hazzard--161 to 262, 370, CS 157, 310, 331
Mike B.--121, 122, 161 to 262, 420, CS 147

Appendix B.5

Reference Texts	Author	Publisher
1. Accent on Algebra	Pat Boyle	Creative Publications, Inc., 1971
2. Computational Mathematics	Charlene Pappin	Glencoe Press, 1972
3. Discovery in Mathematics	Robert B. Davis	Addison-Wesley Publishing Co., 1967
4. Essential Mathematics	Doris Stockton	Scotts, Foresman and Co., 1972
5. Explorations in Mathematics	Robert B. Davis	Addison-Wesley Publishing Co., 1967
6. Geo-board Activity Sheets	Joseph P. Cech Joseph B. Tate	Ideal School Supply Co., 1971
7. Graph Gallery	Pat Boyle	Creative Publications, Inc., 1971
8. Laboratory Manual for Elementary Mathematics	Fitzgerald et al.	Prindle, Weber & Schmidt, Inc., 1969
9. Laboratory Manual	Allyn J. Washington	James E. Freel & Assoc., Inc., 1972
10. Mathematics--A Developmental Approach	Allyn J. Washington	James E. Freel & Assoc., Inc., 1972
11. Mathematics--A Human Endeavor	Harold R. Jacobs	W. H. Freeman & Co., 1970
12. Men of Mathematics	E. T. Bell	
13. Number Sentence Games	Dale Seymour Margaret Holler Nancy Collins	Creative Publications, Inc., 1971
14. Palatable Plotting	Pat Boyle	Creative Publications, Inc., 1971
15. A Source Book of Mathematical Applications	Compiled by A Committee of the NCTM	Bureau of Publications Teachers College Columbia, Univ., 1942
16. Using the Cuisenaire Rods	Jessica Davidson	Cuisenaire Co. of America, Inc., 1969

Appendix B.6

SAMPLE ASSIGNMENT SHEET

Assignments:

1/15/73	Handout--to be handed in Arithmetic Sequences Fibonacci Sequences
1/17/73	Handout--to be handed in Geometric Sequences Sequences of Squares
1/18/73	Review for 35 min. test on 1/19/73
1/19/73	Test in class 35 min. P 101 - 114 Exercises P 143 - 145 (1 - 32)
1/22/73	P 115 - 142 Exercises P 145 - 149 (33 - 100)
1/24/73	P 151 - 172 Exercises P 183 - 184 (1 - 58)
1/25/73	P 172 - 182 Exercises P 186 - 187 (59 - 91)
1/26/73	P 189 - 212 Exercises P 213 - 216 (1 - 71)
1/29/73	P 217 - 233 Exercises P 257 - 259 (1 - 40)
1/31/73	P 233 - 256 Exercises P 259 - 260 (41 - 75)
2/1/73	P 261 - 268 Skip rest of Chapter Exercises P 279 - 280 (1 - 48)
2/2/73	P 281 - 301 Exercises P 302 - 307 (1 - 40)
2/5/73	P 309 - 328 Exercises P 329 - 333 (1 - 52)

Appendix B.7

Application of Paraphrasing Numbers

74-1243
764

DATE________________19____No.__________

PAY TO THE ORDER OF__________________________________$__________

____DOLLARS

FIRST NATIONAL BANK
OF EAST LANSING
EAST LANSING, MICHIGAN

MEMO__________________ __________________________

74-1243
764

DATE________________19____No.__________

PAY TO THE ORDER OF__________________________________$__________

____DOLLARS

FIRST NATIONAL BANK
OF EAST LANSING
EAST LANSING, MICHIGAN

MEMO__________________ __________________________

Appendix B.7

Sample Handout - Used to derive the advancing algorithm.

Handout 2c

Use odometers:

Add: 21
43
____

List steps:

a.
b.
c.
d.
e.
f.

19
34
____

a.
b.
c.
d.
e.
f.
g.

68
75
____

a.
b.
c.
d.
e.
f.
g.

Possible Algorithm: ______________________________

____

____

Sample Handout

Scores for Class--A. M. Section

1. Make a histogram using the data below which are the scores for this class on the last test.
2. Calculate the mean, median and mode.
3. Plot the mean, median and mode on the graph.
4. Be sure to label the axis.

77

78

75

76

75

82

83

90

76

87

68

71

76

49

71

87

80

93

74

80

67

69

82

90

82

79

83

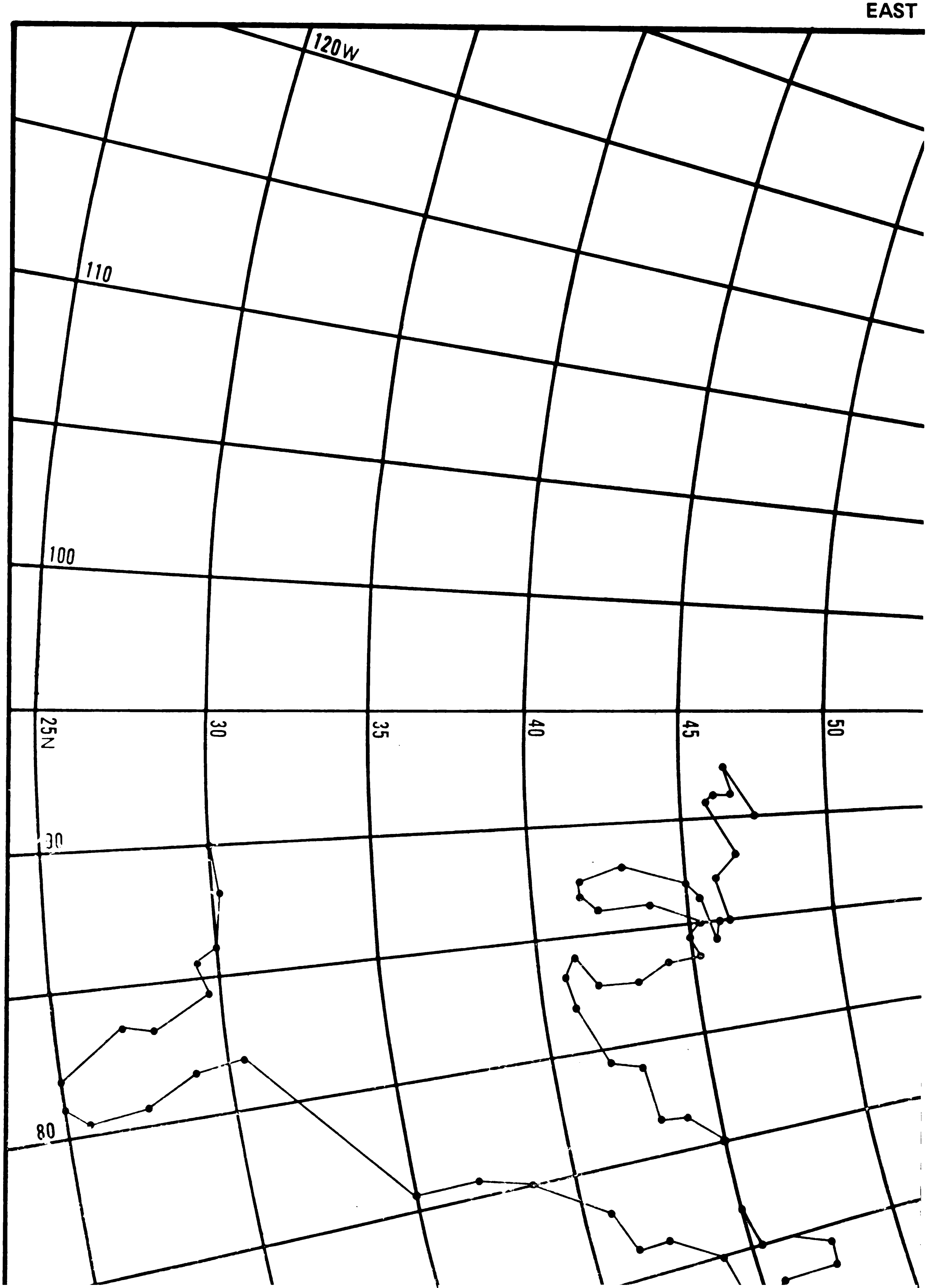
89

Longitude and Latitude

Longitude and Latitude is a very real example of ordered pairs and graphing. Longitude is angular distance, measured in degrees, East or West from the prime meridian (Greenwich, England) while latitude is the distance North or South from the equator. Each point on the surface of the earth can be designated by its longitude and latitude, a one-to-one correspondence between points (locations) and ordered pairs (longitude, latitude).

Here is a graphing exercise that may prove interesting if you are able to correctly plot each location accurately and then connect these points in order.

1. (90.0W, 30.0N)
2. (88.0W, 30.2N)
3. (86.0W, 30.0N)
4. (85.5W, 29.5N)
5. (84.5W, 29.8N)
6. (83.5W, 28.0N)
7. (83.9W, 27.0N)
8. (82.0W, 25.0N)
9. (81.0W, 25.0N)
10. (80.5W, 25.7N)
11. (80.8W, 27.3N)
12. (81.8W, 29.0N)
13. (82.0W, 30.5N)
14. (81.0W, 31.7N)
15. (75.5W, 35.0N)
16. (75.5W, 37.0N)
17. (75.0W, 38.5N)
18. (73.0W, 41.0N)
19. (71.0W, 41.5N)
20. (71.1W, 42.5N)
21. (70.0W, 44.0N)
22. (67.0W, 44.5N)
23. (68.0W, 46.0N)
24. (68.0W, 48.0N)
25. (69.0W, 48.0N)
26. (70.0W, 46.5N)
27. (70.0W, 45.5N)
28. (72.0W, 45.0N)
29. (75.0W, 45.0N)
30. (76.0W, 44.0N)
31. (76.3W, 43.2N)
32. (79.0W, 43.0N)
33. (79.3W, 42.0N)
34. (82.0W, 41.5N)
35. (83.0W, 41.3N)
36. (84.0W, 41.6N)
37. (82.8W, 42.0N)
38. (82.6W, 43.3N)
39. (83.0W, 44.2N)
40. (83.5W, 45.2N)
41. (84.3W, 45.0N)
42. (85.0W, 45.3N)
43. (86.0W, 44.0N)
44. (86.0W, 42.2N)
45. (87.0W, 41.7N)
46. (87.6W, 41.7N)
47. (88.0W, 43.0N)
48. (87.0W, 45.0N)
49. (86.0W, 45.5N)
50. (84.0W, 45.8N)
51. (85.0W, 46.0N)
52. (85.0W, 46.4N)
53. (87.0W, 46.0N)
54. (88.0W, 47.0N)
55. (90.5W, 46.0N)
56. (91.0W, 46.2N)
57. (90.8W, 46.8N)
58. (92.2W, 46.5N)
59. (90.0W, 47.8N)


EAST
120W
110
100
30
80
25N
30
35
40
45
50

Plot the following points on a single set of coordinate axes, then connect the points with segments between consecutive points, in the order indicated by the scheme at the bottom, to form five closed figures. Place the coordinate axes in the center of your graph paper in order to accommodate the completed figure.

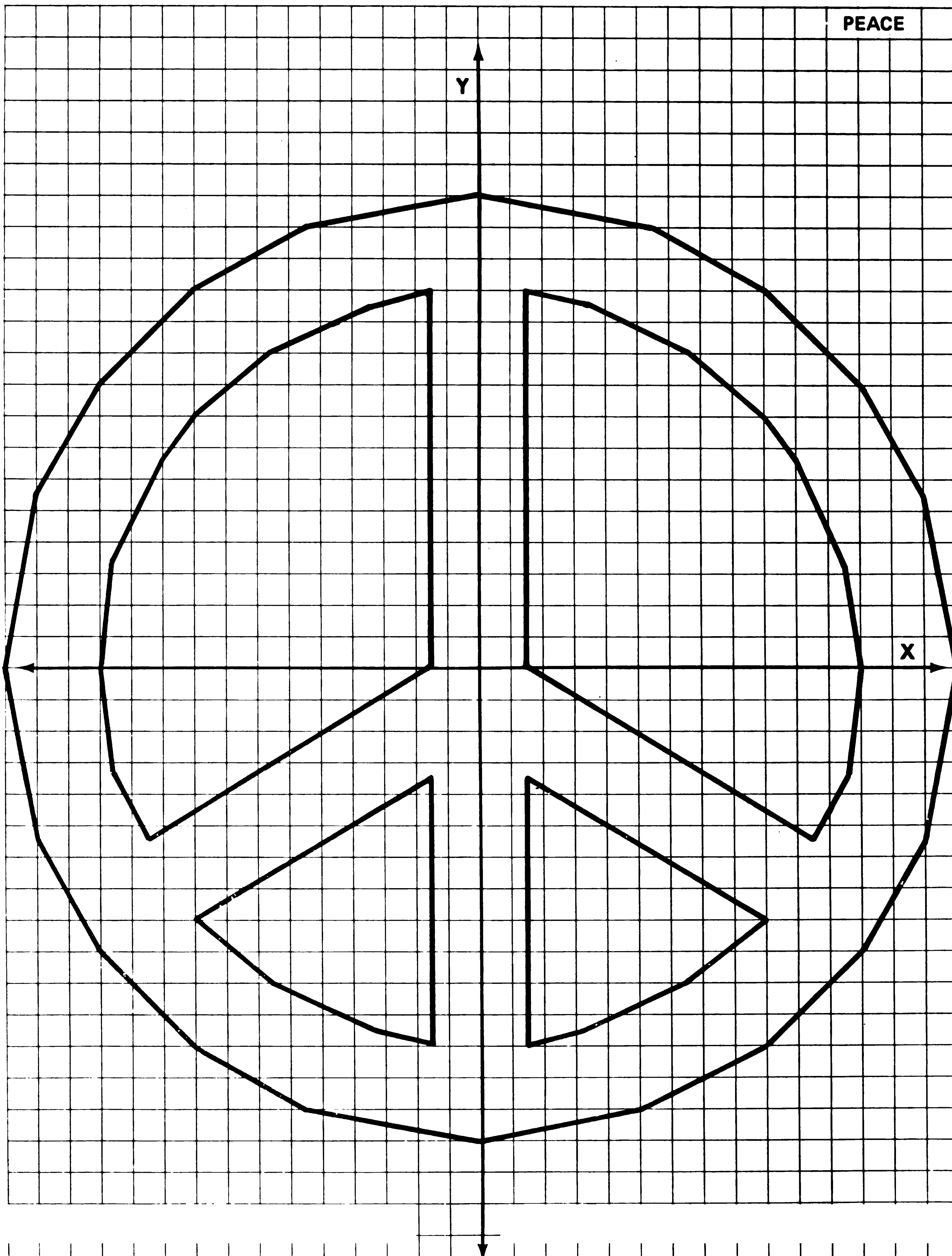
1. (15, 0)
2. (14, 5.5)
3. (12, 9)
4. (12, 0)
5. (11.75, 3.25)
6. (10, $6.\overline{6}$)
7. (9, 12)
8. (9, 8)
9. ($6.\overline{6}$, 10)
10. (5.5, 14)
11. (3.25, 11.5)
12. (1.5, 12)
13. (1.5, 0)
14. (0, 15)
15. (−1.5, 12)
16. (−1.5, 0)
17. (−3.25, 11.5)
18. (−5.5, 14)
19. ($-6.\overline{6}$, 10)
20. (−9, 12)
21. (−9, 8)
22. (−10, $6.\overline{6}$)
23. (−11.75, 3.25)
24. (−12, 9)
25. (−12, 0)
26. (−14, 5.5)
27. (−15, 0)
28. (−14, −5.5)
29. (−12, −9)
30. (−11.75, −3.25)
31. (−10.5, −5.5)
32. (−9, −12)
33. (−9, −8)
34. ($-6.\overline{6}$, −10)
35. (−5.5, −14)
36. (−3.25, −11.75)
37. (0, −15)
38. (−1.5, −12)
39. (−1.5, −3.5)
40. (3.25, −11.75)
41. (5.5, −14)
42. ($6.\overline{6}$, −10)
43. (9, −12)
44. (9 ,−8)
45. (10.5, −5.5)
46. (11.75, −3.25)
47. (12, −9)
48. (14, − 5.5)
49. (1.5, −12)
50. (1.5, −3.5)

I 1 – 2 – 3 – 7 – 10 – 14 – 18 – 20 – 24 – 26 – 27 –
28 – 29 – 32 – 35 – 37 – 41 – 43 – 47 – 48 – 1

II 4 – 5 – 6 – 8 – 9 – 11 – 12 – 13 – 45 – 46 – 4

III 16 – 15 – 17 – 19 – 21 – 22 – 23 – 25 – 30 – 31 – 16

IV 33 – 34 – 36 – 38 – 39 – 33


PEACE
Y
X

Appendix B.8

SAMPLE QUIZ

Quiz 7

31. Each number in this sequence 5,11,17,23,29, . . . is a prime number. What are the next 5 numbers in the sequence? Are they all prime numbers?

    a. 35, 41, 47, 53, 59 yes
    b. 35, 42, 50, 59, 69 no
    c. 35, 41, 47, 53, 59 no
    d. 35, 42, 50, 59, 69 yes

32. What is the 100th term of the sequence 2,5,8,11,14 . . .?

    a. 302
    b. 299
    c. 300
    d. takes too long and there is no simple way to find out

33. The keyboard of a piano forms a Fibonacci sequence:

    a. statement isn't true, its the guitar.
    b. 13 keys form a chromatic scale, of these 8 are the white keys, 5 are black, of the 5 black keys 3 are grouped and 2 are grouped; the 1 corresponds to 1 chromatic scale.
    c. 1,2,3,5,8,13 is represented on all musical instruments and therefore must be on a piano.
    d. because Fibonacci designed the keyboard.

34. "Chain letters" is an example of:

    a. arithmetic sequence
    b. Fibonacci sequence
    c. a sequence of squares
    d. geometric sequence

35. Who were the first to introduce place value in their number system and what base did they use?

    a. Greeks; base 12
    b. Romans; base 5
    c. Babylonians; base 60
    d. Descartes; base 10

Sample Quiz

Quiz 8

36. The first people to use place value were _________. The base they used was ________. Two examples where that base is used today are ________________________________.

    a. Greeks; 2; counting; slide rules
    b. Romans: 100; Roman numerals; books
    c. Middle ages; 10; counting; praying and rosary
    d. Babylonians; 60; degrees; time

37. Fractions are called rational numbers because

    a. people who understand fractions are more "rational" than those who understand whole numbers only.
    b. because fractions aren't as difficult to understand as "irrational" numbers.
    c. a fraction, the ratio of 2 integers, and "rational" is a derivative of the word ratio.
    d. none of these.

38. Deductive reasoning means that:

    a. one reasons from the specific to the general.
    b. one reasons from the general to the specific.
    c. one must always use mathematics.
    d. one is playing mathematical games.

39. Inductive reasoning means that:

    a. one must always use mathematics.
    b. one reasons from the general to the specific.
    c. one is playing mathematical games.
    d. one reasons from the specific to the general.

40. When we play "Guess My Rule" we are using:

    a. deductive reasoning
    b. inductive reasoning
    c. neither (a) or (b)
    d. both (a) and (b)

Sample Quiz

Quiz

Determine the rule for the following two problems.

1.

X	Y
1	1
2	3
3	5
4	7

Y = ______________ and when X = 100 Y = ______________.

2.

X	Y
1	3
2	8
3	17
4	30
5	37

Y = ______________ and when X = 40 then Y = ___________.

Sample Quiz

Quiz

Simplify:

1. $(-4X^2Y^3)(2XY)(X^{10}Y^{-1})$ = ______________________________

2. $(V^2W^5)(VW^{-1})^4$ = __________________________

3. $(-125\ r^3s^9)^{\frac{2}{3}}$ = ______________________

4. $\sqrt[3]{-64X^3Y^6}$ = _____________________

Evaluate:

5. $2\cdot 9^{\frac{1}{2}}$ = _________________

6. $27^{\frac{2}{3}}$ = _________________

7. $(64^{\frac{1}{8}} + 8^{\frac{7}{16}})^0$ = _______________

8. $\sqrt[4]{16\cdot 81}$ = _______________________

APPENDIX C

MATERIALS USED

C.1 LIST OF MOVIES

C.2 LIST OF MATERIALS

C.3 SAMPLE GAMES

- C.3.1 GUESS MY RULE
- C.3.2 FACTOR GAME
- C.3.3 CONTIGUOUS BOARDS

C.4 MADISON PROJECT BOXES

Appendix C.1

LIST OF MOVIES

Associative Properties *
Commutative Properties *
Distributive Properties *
Equations and Graphs
Ordered Pairs & Cartesian Product
Unsolved Problems/Two Dimensions Film I
Unsolved Problems/Three Dimensions Film II
Let Us Teach Guessing *
Mathematical Induction
Mr. Simplex Saves the Aspidistra *
Topology
Challenging Conjectures
What is Mathematics and How Do We Teach It: *
Nim and Other Oriented Graph Games
More Solutions of Linear Equations
Equations and Graphs of the Parabola
Hyperbola, Ellipse and Circle
Progressions, Sequences and Series *
Infinite Series and the Binomial Expansion
Equations with Unknowns in the Exponents
Using Logarithms to Solve Equations
One-to-one Correspondence *
Counting *
Sets: Union and Intersection *
Man and the Computers *
Infinity *

Appendix C.2

LIST OF MATERIALS

Demonstration Slide Rule
Dandelin's Cone, Conic Sections
Coaxial Cylinders
Sectioned Cylinder
Binomial Cube
Hemisphere with Spherical Sector
Three Dimensional Coordinate System Model
Styrofoam Solids Set
Metric Chart
Liter Block--dissectible
Liter Case, plastic
Liter Blocks
Metric Liquid Measure Set
Radian and circle demonstrator
Chalkboard graph chart--polar and rectangular
Chalkboard graph chart--log and semi-log
Coordinate system transparencies
Ball Abacus
Binary Counter
Numeration system demonstrator
Madison Project--independent exploration materials
Multi-Base arithmetic blocks and work cards
Cubic inch blocks
Meter sticks
Geoboards and geoboard activity sheets, activity card set
Slide rules--classroom set
Tape measures--automatic rewind
Cuisenaire Rods
Wang Calculators
Odometers
Number line
Rulers
Scissors
Graph paper

Appendix C.3.1

GUESS MY RULE

The students were presented with a set of ordered pairs from which they had to determine some rule or function that associates the first member of the ordered pair to the second. The set of ordered pairs was obtained in this fashion: some member of the class presented a first number for an ordered pair. The leader, in this case the author, responded with the appropriate second number which was obtained by mentally applying the rule to the given first number. After each response the class was allowed to guess what rule of correspondence was being used. The author limited the functions to linear, quadratic and some simple exponential functions.

A variation of this procedure was to give each student a 3 x 5 card upon which he could write some function of his own choosing. These cards were collected, shuffled, and redistributed. The class was divided into groups of about five students and "Guess My Rule" was played with one of the group being the leader and using the rule that he selected from the cards.

Appendix C.3.2

THE FACTOR GAME

The game is played as follows:

a. Red player places a red mark on a number (n).

b. Blue player places a blue mark on each uncovered proper factor of n.

c. Blue now selects an uncovered number (m) to cover with a blue mark.

d. Red player covers each uncovered factor of m with a red mark.

e. Play continues until no number remains which has any uncovered factors.

f. If a player places a mark on a number which had no uncovered factors, the other gets to lead in the next two plays.

g. The game is over when there are no more legal moves.

h. The score is the sum of the numbers covered.

i. If one player doesn't cover all of the factors available to him on a turn, there are two different rules which can be used:

   1. The other player can cover the missing factors.
   2. The missing factors remain uncovered.

   The rule should be determined before the game begins.

The Factor Game

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Appendix C.3.3

CONTIGUOUS BOARDS

Rules for Contiguous Playing Boards:

1. Each table has 2 cubes (and 3 dice) with 4 through 9 on each cube.

2. To play board 1, write the possible sums and differences of the numbers 4 through 9 in the squares with a 0 in the square on top. (Each player has a board.)

3. Determine who will toss the cubes first and then rotate turns (in a clockwise direction).

4. After you have tossed the cube, the tosser may either add or subtract the two numbers (for the first few tosses it usually doesn't make any difference which you do). Then cross off the sum or difference on your contiguous board. The first person who gets the numbers crossed off wins.

   Note: For board 1, time will not be a factor for most of you. However, from board 2 on it probably will be. So, determine the time that you are allotting before you start board 2, 3 and 4.

5. For board 2 you may add, subtract, or multiply.

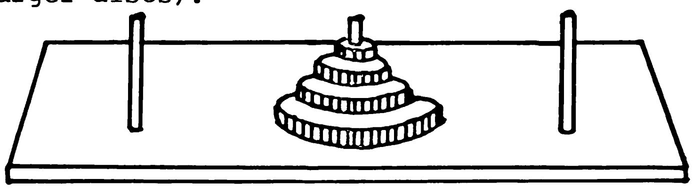
6. For board 3 you may add, subtract, multiply or divide.

7. For board 4 you will use 3 cubes and you may add, subtract, multiply or divide.

   Note: For board 3 and 4, allow only those divisions which do not have a remainder.

Appendix C.4

Three students were examining the contents of this box one day. Marilyn said, "This is a puzzle I've seen before. The object of the puzzle is to transfer the discs from the center peg to either of the other two pegs, ending with the discs arranged in the same order as at the start (smaller discs on top of larger discs).



There are only two rules in moving the discs: (1) only one disc may be moved at a time, and (2) a larger disc may never be placed on top of a smaller disc. "Frank and Mark said they would like to try it.

CAN YOU DO IT STARTING WITH 5 DISCS?

Tower Puzzle instruction card no. 1

CAN YOU DO THE PUZZLE STARTING WITH ONLY 3 DISCS?

To transfer 3 discs, Frank said it took him 11 moves. Mark did it in 7 moves, and Marilyn said it took her 10 moves. They each tried again. Mark said "This is the shortest way to do the puzzle with three discs; it should take 7 moves." DO YOU AGREE?

IS THERE A WAY TO TELL WHETHER YOU HAVE THE MINIMUM (SMALLEST) NUMBER OF MOVES OR NOT?

CAN YOU DO THE PUZZLE STARTING WITH 2 DISCS? 4 DISCS? 6 DISCS?

Tower Puzzle instruction card no. 2

Frank asked, "Is there any relation between the number of discs and the minimum (smallest) number of moves needed to transfer the piles?" Mark suggested they make a table to keep track of the numbers. "Let the number of discs be the number and let the minimum number of moves to transfer all the discs be the Δ number," he added. Marilyn said, "This is something like the game Guessing Functions, when you put a number in and get a number out and figure out a rule that works."

Number of discs → □	Δ ← Minimum number of moves needed to transfer the discs
1	
2	
3	7
4	
5	

CAN YOU COMPLETE THE TABLE ABOVE? FILL IN THE NUMBER ON THE PAPER INCLUDED IN THE BOX.

Tower Puzzle instruction card no. 3

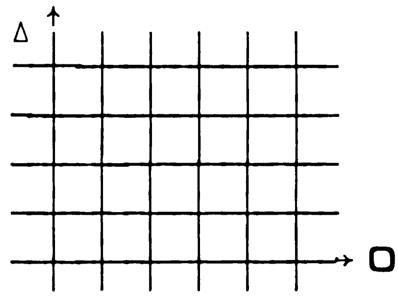
CAN YOU FIGURE OUT ONE RULE THAT WORKS FOR ALL THE PAIRS OF NUMBERS IN THE TABLE?

WRITE YOUR RULE (USING Δ AND □), THEN CHECK IT BY TRYING VARIOUS PAIRS OF NUMBERS FROM THE TABLE.

Tower Puzzle instruction card no. 4

"I think the graph of these pairs of numbers will lie along a straight line," said Mark.

DO YOU AGREE? CAN YOU MAKE A GRAPH OF THE PAIRS OF NUMBERS IN THE TABLE?



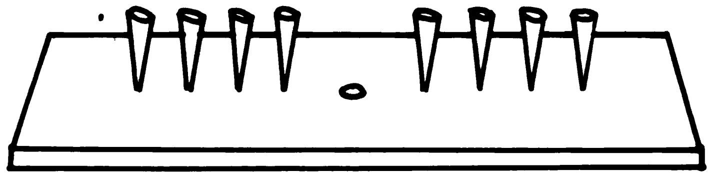
MAKE YOUR GRAPH ON THE GRAPH PAPER INCLUDED IN THE BOX.

Tower Puzzle instruction card no. 5

IF YOU STARTED WITH 100 DISCS AND IT TOOK ONE SECOND FOR EACH MOVE, HOW LONG WOULD IT TAKE TO TRANSFER THE PILE?

Tower Puzzle instruction card no. 6

Brian and David were examining the contents of this box one day. Brian said, "I've seen this game before. The object of the game is to interchange the red and white pegs. You must move the pegs according to the following rules:



white → ← red

i) The white pegs (tees) must move only to the right: the red pegs (tees) must move only to the left.
ii) You can only move one peg at a time.
iii) You can move a peg into an adjacent hole.
iv) You can jump, but only a single peg of the opposite color (you can't jump two pegs).

See if you can do it.

CAN YOU INTERCHANGE THE FOUR PEGS USING THE RULES ABOVE?

Peg Game instruction card no. 1

David said that if he started with two pegs on each side of the center hole, he needed 8 moves to interchange them.
DO YOU AGREE?
Brian suggested that a table be made to keep track of the number of pegs on each side (the number of pairs of pegs) and the corresponding number of moves. "Let the number of pairs of pegs be tne number and let the number of moves to interchange them be the number," he suggested further.

number of pairs of pegs → □	Δ ← number of moves
1	
2	8
3	
4	
5	
6	

CAN YOU FILL OUT THE REST OF THE TABLE (ON THE PAPER INCLUDED IN THE BOX)?

Peg Game instruction card no. 2

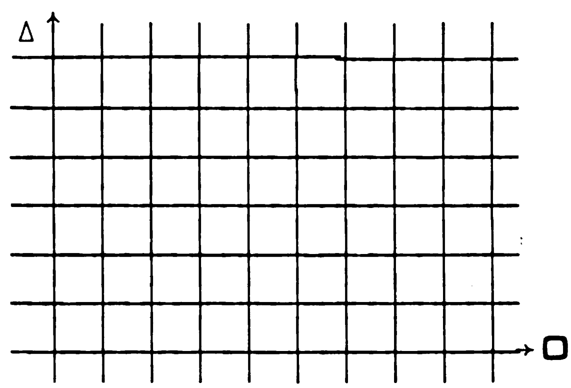
David said, "This is like the game, Guessing Functions; you give me a number (□), I use a rule on that number and get a number out (Δ)." CAN YOU FIND A RULE FOR THE PAIRS OF NUMBERS IN THE TABLE?

WRITE YOUR RULE (USING □ AND Δ), THEN CHECK IT BY TRYING VARIOUS PAIRS OF NUMBERS FROM THE TABLE.

Peg Game instruction card no. 3

David tried to graph the pairs of numbers from Brian's table. CAN YOU MAKE A GRAPH OF THE PAIRS OF NUMBERS IN THE TABLE?

MAKE YOUR GRAPH ON THE GRAPH PAPER INCLUDED IN THE BOX.



Brian said, "I bet the points lie on a straight line!"
DO YOU AGREE?

Peg Game instruction card no. 4

If you took a white rod and used one side of it like a rubber stamp, what is the least number of times you could use it and cover the whole surface of a yellow rod?

Don said he got 17. DO YOU AGREE?

Marilyn said she got 22. WHO IS RIGHT?

Centimeter Blocks instruction card no. 1

Marilyn, Don and Jerry made this table with the various rods. (They called the "number of times stamped" the <u>surface area</u>).

Color of rod	Surface Area
White	6
Red	
Light Green	
Purple	
Yellow	22

CAN YOU COMPLETE THIS TABLE?

Centimeter Blocks instruction card no. 2

Martha and Frank made their table a different way. Martha decided that since it took 5 white rods to make the same length as a yellow rod, she would call that one "5" and make her table like this:

Number of White Rods →	□	Δ ← Surface Area
White	1	6
Red	2	
Light Green		
Purple		
Yellow	5	22

CAN YOU FINISH THIS?

Centimeter Blocks instruction card no. 3

Ida Mae said that the table on card 3 reminded her of the game of "Guessing Functions." The "number of white rods" would be the □ number, and the "surface area" would be the number. Ida Mae said that she could even figure out a rule which would tell here the "□number" if she knows the Δ number.

CAN YOU FIND THE RULE?

Centimeter Blocks instruction card no. 4

CAN YOU GRAPH THE PAIRS OF NUMBERS IN THE TABLE ON CARD 3?

Centimeter Blocks instruction card no. 5

USING THE GRAPH CAN YOU FIND THE SURFACE AREA FOR THE BLUE ROD? THE ORANGE ROD?

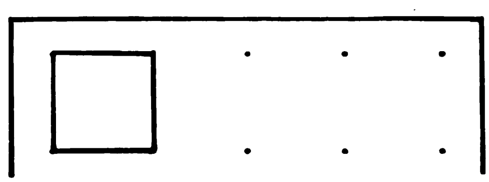
USING YOUR RULE CAN YOU FIND THE SURFACE AREA FOR THE BLUE ROD? THE ORANGE ROD?

Centimeter Blocks instruction card no. 6

How many different shapes and sizes can you make by stretching a rubber band around some of the nails on this geoboard?

Geoboard instruction card no. 1

Let's try something. Stretch a rubber band around four nails, like this:

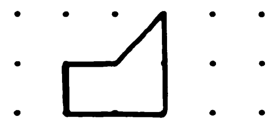


(the black dots are supposed to be the heads of nails)

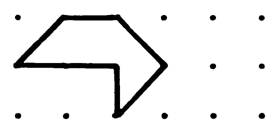
If this is said to have an area of "one square," can you make a shape that you think would have an area of "2 squares"? "3 squares"? "1 1/2 squares"?

Geoboard instruction card no. 2

Don says that this shape has an area of 2 1/2 squares.
DO YOU AGREE?

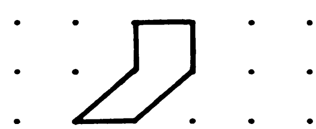


That this shape has an area of 2 1/2 squares.
IS HE RIGHT?



Geoboard instruction card no. 3

Frank says that this shape has an area of 1 1/2 squares.
What do you think?

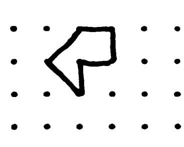


Frank's friend Martha says Frank is wrong. She says the area is 2 squares. WHO IS RIGHT?

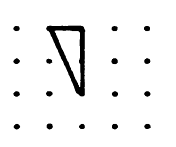
Geoboard instruction card no. 4

CAN YOU FIGURE OUT THE AREAS WITHIN THESE SHAPES?

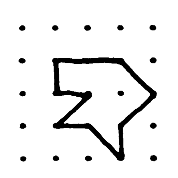
a)



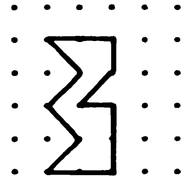
b)



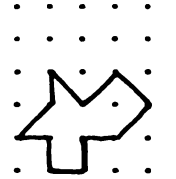
c)



d)



e)



Geoboard instruction card no. 5

CAN you make up some interesting shapes and find the areas within them?

Can a friend of yours make up a shape that is too hard for you to find the area of?

Can you make up one that he can't find the area of?

Geoboard instruction card no. 6

Have you ever played the game "Guessing Functions?" If you have you might like to try this. Suppose you were told a certain number of nails, and you were told that you must touch all the nails with a rubber band. CAN YOU PREDICT WHAT THE LARGEST POSSIBLE AREA WITHIN THE FIGURE WILL BE? (Nails within the figure are illegal).

Pat says that if you tell her "3 nails," the area will be 1/2. DO YOU AGREE?

Bill says that if you tell him "5 nails," the area will be 1 1/2. IS HE RIGHT?

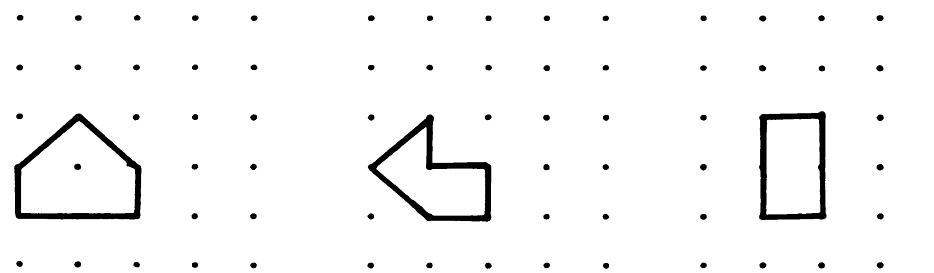
Louise says that if you tell her "6 nails," the area will be 2 1/2. WHAT DO YOU THINK?

Geoboard instruction card no. 7

Gary and Bob made this table for their answers.

Nails	Area
4	1
5	1 1/2
6	2

For example, if you were told "6 nails"



This is illegal! This is legal! This is legal!

DO YOU THINK THEY ARE RIGHT?
CAN YOU FILL IN ANY MORE PAIRS OF NUMBERS?
CAN YOU GRAPH YOUR PAIRS OF NUMBERS?
CAN YOU GUESS THE RULE?

Figure A4.8 Geoboard instruction card no. 8

WHAT HAPPENS IF YOU NOW PLAY WITH DIFFERENT RULES AND ALLOW ONE NAIL WITHIN THE FIGURE, NOT TOUCHED BY THE RUBBER BAND?

DOES IT MAKE ANY DIFFERENCE HOW MANY NAILS THERE ARE WITHIN A FIGURE NOT TOUCHED BY THE RUBBER BAND?

CAN YOU WRITE A NEW RULE TO FIT <u>THIS</u> IDEA?

Figure A4.9 Geoboard instruction card no. 9