THE ANALYSIS AND SIMULATION OF WATER TRANSFER THROUGH THE SOIL - ROOT DOMAIN

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ABSTRACT

THE ANALYSIS AND SIMULATION OF WATER TRANSFER THROUGH THE SOIL-ROOT DOMAIN

By

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The role of soil water in controlling the growth and development of a plant has been shown in various ways. However, a quantative analysis of the water absorption and transfer process by a continuously growing root system of a plant has been neglected. It is the purpose of this study to investigate the transfer of water through the soil-root domain with an approach based on the principles of unsaturated water flow.

A deterministic non-stationary mathematical model is developed based on the following assumptions:

i) The effective root zone may be determined by the vertical and horizontal extension of root system.

ii) The density of the root system is based on the optimum utilization of soil with an overlapping coefficient and represented as the root surface per unit volume of soil.

iii) The irreversible resistance against diffusion of water through the suberized tissue of the root surface is defined as the degree of suberization. The magnitude of suberization is based on the existence of a water

potential gradient from root xylem into soil as the end of the period of water transfer through the root system from moist soil into dry soil is approached due to the relaxation of root potential during the night.

iv) The absorption of water by the plant roots is defined with a source term, i.e., positive for water release, negative for water absorption, in the analysis of the soil water flow equation.

v) There exists a plant root system with a water uptake pattern to satisfy the experimental transpiration, evaporation, and soil water potential distribution.

An environmental growth chamber was modified to simulate field condition and measure the controlling parameters. Based on the experimental transpiration and evaporation, the movement of water through the developing root system of a kidney bean plant in Hillsdale sandy loam was simulated on the computer. It was found that the maximum absorption is limited to the lower part of the root zone as a conical shell and moving downward as the growth progressed. The rate of maximum water absorption takes place where the growth of root density is optimum rather than where the density of the root system is maximum due to suberization of older roots. The maximum rate of water uptake also coincides with the development of an optimum ratio between soil and root water potential based on the analysis of water uptake limitation.

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The development of suberization shows that the transfer of water through the root system from moist soil into dry soil could not bring the root zone into an equilibrium condition as far as the soil water potential distribution is concerned. It appears that the suberization of the root surface is essential to optimize the absorption of water as well as protect the roots from an unfavorable environment. The amount of water absorption by the root system drops with the depletion of soil water in the root zone. The root water potential fluctuations remain steady during the growth of root zone into a soil with low water tension. As the soil dries, the root water potential drops to the wilting point during the day in order to absorb more water for transpiration.

The simulated soil water potential and sum of water uptake by the proposed root model were consistent with the experimental results for the first three weeks of growth. Then, the simulated water potential distribution deviates from the observed potential distribution. Therefore, further investigation of this model for the remainder of growth season is required. Hopefully, the development and density of the root system can be used to complete the model of a plant environment, while the process and pattern of water uptake can be used to increase the efficiency of irrigation and drainage systems.

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LIST OF SYMBOLS

A = The density function of the root system (cm^2/cm^3)

Ai = The area of a bean leaves (dm^2)

Ar = Cross section of a root (cm^2)

B = The degree of suberization (cm)

C = The water capacity of soil (1/cm)

D = The diffusion coefficient of soil (cm^2/day)

E = Evapotranspiration (cm/day)

K = The hydraulic conductivity of soil (cm/day)

L = The upper limit of z-axis

N = Constant

Q = The soil water flux (cm/day)

R = Radius of influence (cm)

Ri = The ideal gas constant

S = Source term (cm^3/day)

Sr = The surface of root per unit length (cm^2)

T = The absolute temperature

V = The volume of cylindrical soil ring corresponding to the r^{th} increment (cm^3)

Vs = The volume of influence (cm^3)

$\bar{V}$ = The molar volume of water

W = The daily weight loss from the soil-plant system (gr/day)

a = The radius of root (cm)

b = The reciprocal of the soil conductivity

c = The degree of overlapping

e = Evaporation (cm/day)

g = The growth function (dimensionless)

h = Constant

i = Transpiration (cm/day)

n = The exponential value of the soil characteristic curve

p = The actual vapor pressure

po = The vapor pressure of pure water

q = The rate of water uptake (cm/day)

r = r-axis in cylindrical coordinates

t = Time (day)

v = The rate of root extension (cm/day)

z = z-axis in cylindrical coordinates

α = The ratio of evaporation to evapotranspiration

Θ = Water content of the soil on a volumetric basis

Φ = The soil water potential (cm)

Φ_r = The water potential of the plant root (cm)

δ = Euler's constant

ν = The flux of water (cm/day)

I. INTRODUCTION

A knowledge of the development and distribution of plant roots is very important to both researchers and farmers, especially in extensively cropped areas where irrigation and drainage systems are required. The placement of fertilizer and the methods of tillage are influenced by the activity of plant roots and their distribution in the soil. The amount of water and nutrients available to a plant in a given soil is determined largely by the volume of soil in contact with it. The volume of soil depends on the amount of branching and extension of the root system. Gardner (1960) and Cowan (1965) concluded that water movement toward the root through the soil is relatively slow and consequently, the only water immediately available is that occurring within a few centimeters of the root. Thus, the horizontal and vertical extent of the root system and the density of roots are important to the growth and development of plants.

Most of the efforts to model plant behavior have been confined to studies of the relationship of yield with weather and other environmental variables using correlation and regression methods. The biological processes occurring in the plant world are complex and highly

interactive with nonlinear response to environmental changes. The analytical solutions of nonlinear differential equations describing the response of plants has been virtually impossible. However, the development of high-speed digital computers and dynamic system stimulation languages has made it possible to deal with such phenomena. Recently, researchers have developed the growth model of a single plant by using simulation techniques. Peters (1969) concluded that although the growth processes of a plant are dependent on plant water status, a quantitative analysis of water absorption and transfer by plant root system has been neglected.

Gardner (1960) and Cowan (1965) studied the absorption of water by stationary single roots. Whisler, et al. (1968) developed a mathematical model to study the water absorption and transfer through the root system in a soil column. However, the absorption and transfer of water by a continuously growing root system has not been studied. Hence, the goals of the project reported in this thesis were:

i) to study the development and distribution of a plant's root system in soil,

ii) to develop a mathematical model to study the absorption and transfer of water in the soil-root domain.

II. REVIEW OF LITERATURE

2.1 Plant Growth Models

The development of a plant growth simulator has received considerable attention recently. The modeling effort has included levels of organization from the individual leaf to the full-foliage canopy of the crop in the field. Examples of outstanding contributions to the modeling effort will be reviewed.

Waggoner (1969) developed a model for assimilation and respiration of CO_2 in a single leaf. The model has been used to explore: (a) the steady-state response of net photosynthesis to variations in light and dark; (b) respiration of the leaf; (c) stomatal resistance to CO_2 penetration into the leaf; and (d) CO_2 compensation point and maximum attainable photosynthesis.

Duncan, et al. (1967) computed photosynthesis in a foliage canopy divided into many horizontal layers, each defined in regard to optical properties including the angular distribution of leaf elements. The simulator first calculates the direct and diffuse illumination of each leaf element for given canopy, solar, and sky conditions. From this, hourly photosynthesis at various sun

angles is computed and summed to obtain daily totals. This simulation has many useful applications under which breeding for erect-leafed plant types would be a useful strategy for improving agricultural yields.

DeWit and Brouwer (1968) developed a simulation model of plant growth for corn. Their objective was to model the processes of photosynthesis, respiration, transpiration, and growth at the tissue and organ levels of plant organization. Growth is simulated by taking into account the influence of temperature, carbonhydrate reserves, age of tissues, and degree of water stress. Field experiments with corn were conducted in California, Iowa, and the Netherlands, and appropriate weather data was supplied to the model. In all three cases, the growth rate was predicted quite well.

Chen and Curry (1971) also developed a simulation model of plant growth for corn. They concluded that the model can be used to test various plant growth parameters, both physiological and environmental, to determine which ones might be a key factor in increasing the efficiency of plant production.

None of the existing models are capable of simulating the growth of a plant root system. There is only limited information available concerning the amount of dry matter incorporated in roots as compared with shoots, largely because of the difficulty of measuring the entire root system.

DeWit and Brouwer (1968) observed that the relation between leaf dry weight and root dry weight of a bean plant growing in an optimum environment is linear. However, when light intensity is reduced, the overall growth rate decreased but root growth decreased more than leaf growth, resulting in a higher leaf to root ratio. A reduced supply of nitrogen or CO_2 also reduced the overall growth rate, but leaf growth decreased more than root growth. Roots and shoots are dependent on each other in several ways since root growth depends on the supply of water and minerals from the soil via the roots. DeWit and Brouwer (1968) observed the effect of carbohydrate supply during the development of fruits and seeds at which time root growth is reduced. Apparently, the leaf to root ratio depends on those internal and external conditions which influence the activity of the supplying organ and the requirements of the dependent organ.

2.2 Root Development

Investigations of root systems have been limited in number and scope largely because of the difficulty of observation and the labor required. Most measurements of rooting behavior have been obtained in situ by digging soil away from plant roots. Weaver (1926) and Dittmer (1937) studied the anatomical features and development of crop roots. They concluded that the development of a plant root system is controlled by the plant's innate hereditary potentialities as well as by their environments.

Kramer (1969) discussed the importance of hereditary factors in controlling the root development of a plant. Some species always develop fibrous root systems while others will always develop a tap root system. Russell and Mitchell (1971) studied root development and rooting patterns of soybeans under field conditions. They observed that the first root is the radicle which grows straight downward as it emerges from the seed. Lateral roots then appeared three to seven days after germination at ninety degree angles around the radicle. They concluded that the remainder of the root development is the result of secondary and tertiary branching which fills the soil volume between the taproots as growth progresses.

The relationship of various soil factors to root growth and development has been discussed in detail by Shaw (1952), Danielson (1967), and Kramer (1969). The successful growth and functioning of root systems as absorbing surfaces depends on many factors in the soil environment including soil moisture, soil aeration, and soil temperature.

1. Soil moisture: Either an excess or deficiency of soil water limits root growth and functioning. Newman (1966) observed a marked reduction in the growth of flax roots at a soil water potential of -7.0 bars. At -15.0 bars, root growth was 20 per cent or less of control rates. It also appeared that the growth of an individual root is independent of other roots. As root growth in the upper layer of the soil was diminishing due to water stress, root

growth in the lower and moist soil layer was progressing normally. Kramer (1969) concluded that not only is root elongation stopped by a lack of water, but roots tend to become suberized up to their tips under water deficiency.

2. Soil aeration is also a limiting factor for the growth and functioning of plant roots. The respiration of roots and soil organisms tends to reduce the oxygen and increase the carbon dioxide concentration in the soil. However, considerable gas exchange takes place by diffusion between the soil surface and the air above. The effectiveness of such gas exchange in maintaining favorable oxygen and carbon dioxide levels depends largely on soil texture, structure, and moisture. Wooley (1966) pointed out that there are many contradictory reports concerning the levels of oxygen and carbon dioxide which limit root growth. He estimated that the diffusion process alone might supply the required oxygen to a depth of one meter if as much as 4 per cent of the soil volume consists of interconnected gas filled pores.

3. Soil temperature: Root growth and development is often limited or stopped by low and/or high temperatures. DeWit and Brouwer (1968) studied the effects of soil temperature on plant growth. They observed that the maximum root growth of bean plants occurred between temperatures of 20° and 30° C. Outside of this temperature interval, root growth was reduced and the external surface of roots heavily suberized.

4. Concentration of ions: Kramer (1969) discussed the effects of ion concentration on root growth and functioning. An abundance of certain elements, particularly phosphorous and nitrogen, stimulates root growth. However, an excess of salt and other minerals will reduce cell division and elongation of roots. The reduction in growth is due to the development of higher osmotic potentials in the zone of root growth.

2.3 Water Absorption and Transfer Processes in the Soil-Root Domain

Continuous absorption of water is essential to the growth and the survival of plants. Water absorption is not an independent process, rather, it is related to the rate of water loss by transpiration. Absorption and transpiration are linked by water transport in the xylem tissue of plants. The first complete model of the transpiration process was proposed by Vanden Honert (1948). He treated the movement of water through the soil-plant system as a catenary process under steady-state conditions.

Recently, the dynamic aspects of water transpiration in the soil-plant-atmosphere continuum have been discussed by several authors. The analysis might be classified into two groups according to the type of modeling approach. The first group developed a model based on an individual root as a line sink. The second group developed a model based on the transfer of water through the soil-plant-atmosphere continuum.

The absorption of water occurs along a gradient created by decreasing water potential from soil to roots. The cause of the water potential gradient, however, differs in active and passive absorption of water according to Kramer (1969). Active absorption is due to the reduction of xylem water potential by the accumulation of solutes, whereas passive absorption is due to suction pressure which is developed by transpiration. The absorption of water by transpiring plants is assumed to be passive absorption in the following models:

A. Analysis of water absorption by a single root.

Gardner (1960) developed a model based on the assumption that an individual root acts as a cylindrical sink. He assumed that the root is stationary with a uniform radius in an infinite soil medium. The soil water flow equation in a cylindrical coordinate system with appropriate boundary conditions for constant K, D, and q may be written as:

$$\frac{\partial \Phi}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \, D \, \frac{\partial \Phi}{\partial r}\right) \qquad (2.1a)$$

$$\Phi = \Phi_o \quad \text{at} \quad t = 0 \text{ for } a \leq r \leq \infty \qquad (2.1b)$$

$$2\pi a K \frac{\partial \Phi}{\partial r} = q \text{ at } r = a \text{ for } t \geq 0 \qquad (2.1c)$$

where Φ = the soil water potential (cm. of water),

D = the diffusion coefficient of soil ($cm.^2$/day),

K = the hydraulic conductivity of soil (cm./day),

q = the rate of water uptake (cm.3/day,

a = the radius of the root (cm.).

Gardner (1960) obtained the following solution for this boundary problem:

$$\Delta\Phi = \Phi - \Phi_o = \frac{q}{4\Pi K}\left(\ln\frac{4Dt}{a^2} - \delta\right) \qquad (2.2)$$

where δ is Euler's constant (0.57722). An inspection of equation (2.2) shows that the soil water potential gradient is:

i) proportional to the rate of water uptake,

ii) inversely proportional to the hydraulic conductivity of the soil.

The calculation of the water potential distribution as a function of distance from the root surface indicates that the radius of influence is finite and equal to $R=2\sqrt{Dt}$. Peters (1969) calculated that the radius of influence is less than a few centimeters after one day, when the radius of the root is one millimeter.

Cowan (1965) has explored this approach more fully by taking into account the diurnal variation of the transpiration rate due to stress-induced stomatal closure. He assumed that a root is an infinitely long cylinder of radius a, extracting water within the region $a \leq r \leq R = 2\sqrt{Dt}$. at a rate of q ($cm^3$/day) per centimeter of root with a one to one relationship between the capillary conductivity and the soil water potential. A comparison of Cowan's solution with Gardner's work shows the influence

of a variable soil conductivity on water potential distribution as a function of distance from the root surface. The radius of influence, R is reduced to half of Gardner's approximation for the same soil. The analysis shows that the dependence of the hydraulic conductivity of soil to the water potential change may limit the flux of water uptake. Gardner and Lang (1970) studied this point and concluded that large decreases in water potential of the root surface may increase the flux of water uptake only slightly beyond a certain point depending on the conductivity-water content relationship.

The above models are for a stationary-infinite root system. However, root lengths are finite and increase with time. Also, the location and extension of the absorbing zone varies with age and type of species as well as with the rate of transpiration. Brouwer (1965) observed that the water absorption zone of growing bean roots extends maximum of six to eight centimeters behind the root tips, then decreases sharply toward the base. When the rate of transpiration is increased, the zone of water absorption moves toward the base. Wolf (1970) examined this problem theoretically by simulating simultaneous water movement and root growth on a computer. He assumed that a root is extending downward at a constant rate v (cm/day), and defined the amount of water uptake as:

$$q = Arv \ (\theta_i - \theta_o) \tag{2.3}$$

where q = the rate of water uptake by one root-tip

Ar = cross section area of that root

θ_i = water content of the soil prior to root entry

θ_o = water content of the soil in equilibrium with the root.

He concluded that root growth will increase the zone of water absorption and the amount of moisture supplied.

B. The dynamics of water transportation through the soil-plant-atmosphere continuum.

In nature, evaporative losses from the soil surface change the distribution of water potential in the root zone of a plant with time and space. Thus, plants must adapt their root systems in such a way that they will be protected from water stress and still be able to absorb the necessary amount of water and nutrients from the soil. Therefore, it has been difficult to model and study the process of water absorption and transfer by the root system of a plant with complex boundary conditions. Philip (1958) suggested that the process of water absorption and transfer in the soil-plant domain may be considered in the liquid phase and can be analyzed using the flow equation of water in the soil:

$$\frac{\partial \theta}{\partial t} = \nabla \ (D(\theta) \ \nabla \theta) + \frac{\partial K(\theta)}{\partial z} + S \tag{2.4}$$

where θ = the volumetric water content of soil

$D(\theta)$ = the diffusivity of soil

$K(\theta)$ = the conductivity of soil

S = source term per unit volume of soil

Gardner and Ehlig (1962) assumed that water uptake by a root system can be represented by a continuous source function with negative values. Equation (2.4) for a one-dimensional steady-state flow condition in which gravity is neglected may be written:

$$0 = \frac{\partial Q}{\partial z} + S(z) \tag{2.5}$$

where Q = the soil water flux (cm/day)

S = source function for water uptake by root.

Applying this equation to a vertical soil column of length L and integrating from $z = L$ to a given position z, Gardner and Ehlig obtain:

$$Q(z) = Q(-L) + \int_{-L}^{z} S(z)\, dz \tag{2.6}$$

According to equation (2.6), one can delineate the flow in the soil at a given position as the sum of the flux across the bottom of the column and the integral of the source function from the bottom of the column to the given depth. They calculated the strength of the sink term for cotton, sorghum, and pepper plants on pachappa sandy loam and found that these plants possess a localized uptake of water.

Gardner (1964) applied equation (2.4) to finite difference form and neglected the flow of soil water from layer to layer in the soil to analyze the effect of root distribution on water uptake and availability. He defined the source term as the sum of the source terms for each layer of soil rather than integrated over the root zone. In Gardner's analysis, the source term for the ith layer is expressed as

$$S_i = N\ (\Phi_r - \Phi_i - z_i)\ \ K_i\ \ A_i \tag{2.7}$$

where S_i = the rate of water uptake per unit across section of a layer of soil

Φ_r = the suction of plant roots,

Φ_i = the suction of soil at ith layer,

z_i = the distance from the soil surface to ith layer,

K_i = the conductivity of the soil,

A_i = the length of root per unit volume of soil,

N = a constant.

Gardner (1964) concluded that calculated root distributions fit well with experimental observation of root density in the upper part of the soil. He suggested that a discrepancy in the lower part of the root zone was due to the assumption of a negligible impedance in the root xylem.

Whistler et al. (1968) used equation (2.4) for steady-state conditions with a source function given by

$$S(z) = A(z) \quad K(\Phi) \quad (\Phi p - \Phi s) \tag{2.8}$$

where $S(z)$ = the amount of water uptake per unit volume of soil,

$A(z)$ = the root density function,

$K(\Phi)$ = the conductivity of the soil (cm/day),

Φp = water potential of the plant root (cm),

Φs = water potential of the soil (cm).

The root density function A(z) was expressed in terms of a length of root per unit volume of soil. Evaporation, transportation, and their ratio were assumed constant. A relationship was obtained between evapotranspiration and water uptake by assuming transpiration in equal to a source term, viz:

$$E = i + e \quad \text{and} \quad \alpha = e/i \tag{2.9a}$$

$$-\int_{-L}^{0} A(z) \quad K(\Phi) \quad (\Phi p - \Phi s) = -i = -(1-\alpha)E, \tag{2.9b}$$

where E = the rate of evapotranspiration (cm/day),

i = the rate of transpiration (cm/day),

e = the rate of evaporation (cm/day),

α = the ratio of evaporation to evapotranspiration.

By solving this equation for Φp Whistler, et al. (1968) obtained

$$\Phi p = \frac{-(1-\alpha)\ E + \int_{-L}^{0} A(z)\ K(\Phi)\ \Phi_s\ dz}{\int_{-L}^{0} A(z)\ K(\Phi)\ dz} \tag{2.10}$$

Equation (2.10) can be solved simultaneously with equation (2.4) using numerical techniques for steady-state flow conditions to obtain soil water potential distribution as well as the distribution of the source function with depth. Using this technique, Whistler, et al. (1968) found that the magnitude of the source was greatest at the bottom of the rooting zone as expected. An interesting part of the result is the transfer of water from the lower part of the soil profile via the root system to the upper part of the profile. The transfer of water through a root system from moist soil to dry soil may occur, although it is contradictory if one considers the suberization of older roots. McWilliam (1968) observed that a living root of Mediterranean grasses (Phalaris Tuberosa Li) in dry soil is surrounded by a thin layer of moist soil, indicating the transfer of water from moist soil into the surrounding dry soil. However, the magnitude and duration of the "shorting" effect was not made clear. McWilliam and Kramer (1968) reported that cutting the deep roots of a plant, which is growing in a soil with dry surface layers, caused the shoots to die immediately. This experiment shows that the absorption of water is reduced by suberization of the root system which has been subjected to water stress in a dry soil layer. Therefore, one might conclude that there must be some relation between the shorting effect and the suberization of a root system. Peters (1969) suggested that the processes

of water uptake and its "shorting" effect might be understood better by improving the source term of equation (2.4).

III. ANALYSIS AND DEFINITION OF THE PROBLEM

The multi-level character of a biological system, such as a plant, presents a difficult problem to model and study. A plant is made of three main components; leaves, stem, and roots. Living leaves contain a variety of cells and membranes in which complex biological processes such as photosynthesis, respiration, and transpiration take place. The stem provides a connection between the leaves and the root system for the transportation of water and carbohydrates. The roots of a plant grow downward into the soil and serve to anchor the plant as well as absorb water and minerals from the soil. The three parts are inter-related into a complex organization of control over one another as shown in Figure 1. DeWit and Brouwer (1968) assumed that such interdependence can be characterized conveniently in terms of a state of functional equilibrium. The equilibrium is governed by the activities of the organs involved. Thus, the response of a plant system to any environmental change depends on the response of the individual components plus the arrangement of the components and the paths of communication between them. Therefore, the integration of available knowledge related with

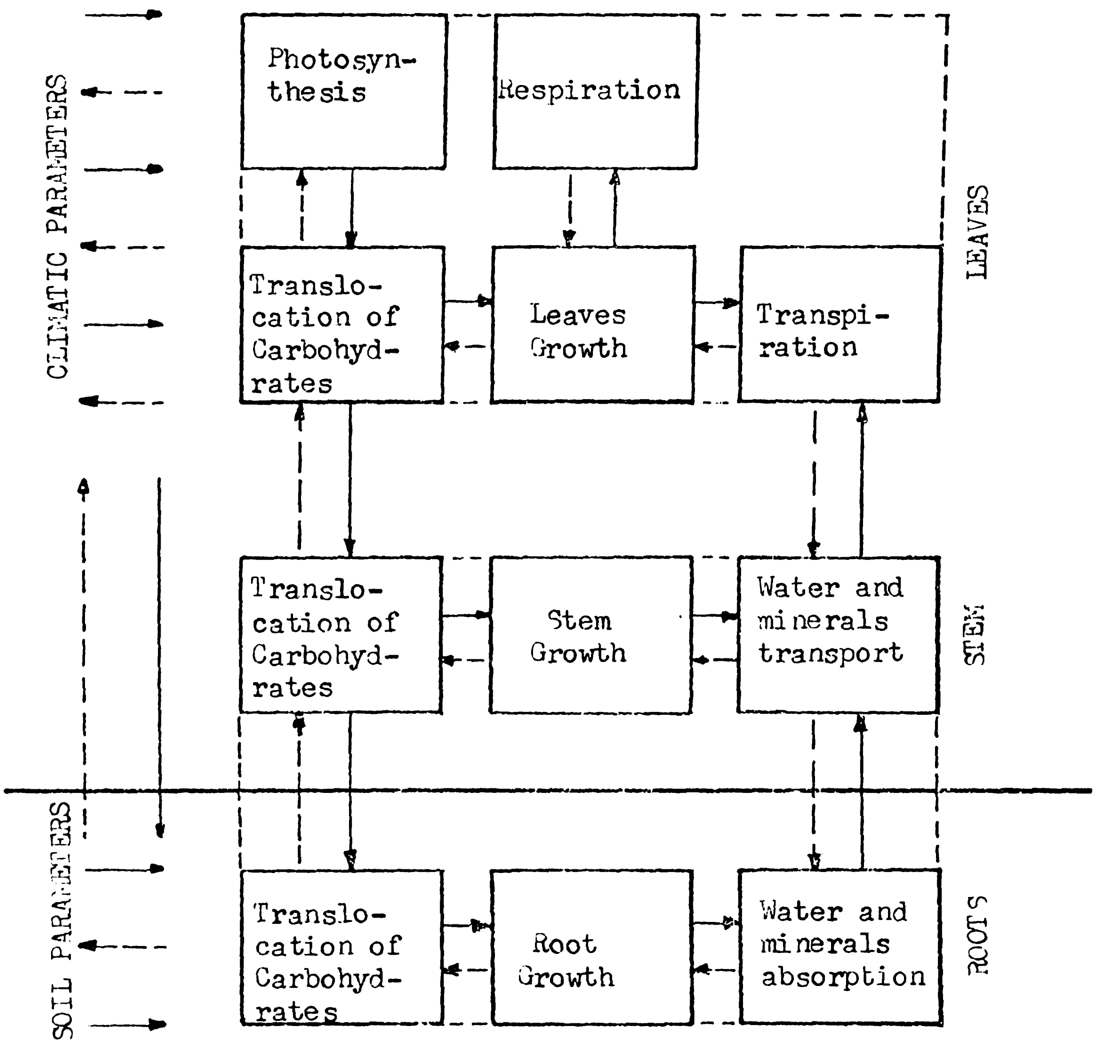

Figure 1.--Block Diagram of Soil-Plant-Atmosphere System.

the response of plant components to their environment to model and study the responses of an individual plant is highly desirable for the plant scientist involved in breeding and improving crop production.

Under certain circumstances, one can study a part of a system by isolating it from other parts using justified constraints. On this basis, the growth process of plants have been modeled and studied extensively. Although the role of water in controlling the physiological processes of the plant is recognized, a quantative analysis of water absorption and transfer processes has been neglected (Peters, 1969). In this analysis, we are following the above logic, attempting to model and study water absorption and transfer processes in the soil-root domain.

A detailed review of water transfer processes in the plant-soil continuum can be found in Slatyer (1967) and Kramer (1969). The energy status of water in plants and soils is represented by the water potential which is the difference between the partial free energy of soil or plant water, and pure water at atmospheric pressure and the same temperature. The water potential function is uniquely related to the water content of plant tissue or soil. In nature, plant leaves lose water vapor to the air when their stomatas are open to allow efficient carbon dioxide for photosynthesis. This process is called transpiration. The rate of transpiration is proportional to

the difference between the water potential inside and outside the leaf as well as to the degree of stomatal opening. The stomatal openings are controlled by the guard cells which are sensitive to environmental parameters such as sunlight, and operate in response to turgor pressure of the cells. Sunlight not only affects the stomata's opening, but also exerts strong physical effects on the transpiration rate. This combined influence of sunlight causes daily fluctuations in the rate of transpiration.

The water loss from leaf cells will be absorbed from the xylem of the leaves. The removal of water reduces the water potential of xylem. Thus, a potential gradient will exist between the leaf and root xylem to transfer water from the root system to the leaves of the plant. The amount of water transferred has to be provided by the absorption of water from the soil.

We should review briefly the structure and development of plant roots to understand the mechanics of water absorption. A more detailed account of root structure can be found in Esau (1965) and Street (1966). They regarded an extending root in four regions; root cap, meristemic region, the region of elongation, and the region of differentiation and maturation. In the meristemic region, growth and cell division both occur until they reach the elongation zone. The growth of the cell occurs mainly lengthwise with little lateral expansion in the

elongation zone. This zone extends into the differentiation and maturation zone, where the epidermal of the root cell develops root hairs. Kramer (1969) concluded that as the new root hairs develop, cutinization and suberization of the epidermis occurs and older root hairs tend to disappear.

The pathway of water movement from the root surface to the xylem tissue takes place through the epidermis, the root cortex and the endodermis. The rate of water absorption is directly proportional to the water potential gradient between root xylem and soil, and inversely proportional to the resistance encountered in the pathway. Gardner (1960) and Cowan (1965) treated the absorption of water by individual roots as a diffusion process and neglected the resistance of the root to the flow of water from the root surface into root xylem tissue. However, Kuiper (1963) observed that the resistance of root tissues has a significant effect on the absorption and release of water. He claimed that the main barrier to the flow of water is at the endodermis due to the development of a casparian strip. Brouwer (1965) observed that maximum water absorption takes place in the root hair zone where there is least resistance to the diffusion of water. The least resistance in the root hair zone may be associated with unsuberized root epidermis rather than with the development of the casparian strip. When the rate of transpiration is increased, the zone of water absorption extends along the root, where the surface of the root is supposed to be suberized. Kramer

(1969) concluded that the suberized layer of the root surface is not impervious and uniform along the root, but merely presents a higher resistance to the diffusion of water. The question can be raised concerning the advantages of suberization of the older root surface, when the primary function of roots is to absorb water. Wolf (1970) concluded that the advantage of suberization is to prevent water loss into the soil that has dried. Gardner (1960) assumed that root water potential is constant throughout the root system. However, the soil water potential distribution in the root zone is not constant due to evaporation and diurnal fluctuations of water uptake by the root system. Thus, the root system must adapt by suberization to protect water loss due to shorting of water flow along the root.

The transfer of water through a root system from moist soil to dry soil has been reported by Hunter and Kelley (1946) and McWilliam and Kramer (1968). The shorting of water flow along the root might be explained if one considers the dynamic aspects of water potential distribution in the soil-root domain. The soil water potential around the roots might not relax as fast as the water potential of the root xylem when active transpiration stops during the night. The xylem may then release water according to the magnitude of the potential gradient and the degree of suberization along the root, while the absorption of water is continued at the tip of the root to keep the system in equilibrium. Therefore, it may be necessary to

model the whole soil-root system to study the process of water absorption and transfer phenomena.

The root system of a plant is branching and grows into new soil from which it may obtain adequate water for the transpiration demand of the growing plant canopy. Existing growth models do not describe the growth and distribution of a root system in the soil. Most researchers have equated growth and distribution of a root system to the dry weight of the root. However, the distribution of a root system in terms of dry weight has no meaning as far as water and nutrient absorption is concerned. A more meaningful model would appear to be one based on the absorbing surface of the root system rather than dry weights of roots.

Gardner (1960) and Cowan (1965) show that the rate of water flow toward the root surface is controlled by the hydraulic conductivities of the soil, and the only water available is that occurring within a few centimeters of the root. Gardner and Lang (1970) reported that the rate of water uptake reaches a limiting value as the potential gradient increases between soil and root xylem. Therefore, an equilibrium condition may exist between total transpiration and root surface with a minimum root potential and optimum water uptake under a normal environment.

Whenever transpiration demand increases over the state of dynamic equilibrium, plant leaves cannot force the root system to absorb more water even by increasing

the water potential gradient between the soil and root xylem. At this point, plant leaves apparently close their stomatas to control the rate of transpiration. The state at which the soil can no longer supply sufficient water to the plant is called the wilting point. Therefore, water transfer through the soil-plant-atmosphere continuum is controlled by plant and soil as well as by climatic variables. Furthermore, the closed stomates stop the diffusion of carbon dioxide, which is a basic ingredient of the photosynthetic process. This analysis shows the importance of the soil-plant water status in plant growth models.

One can conclude from the preceding analysis that the development of a complete dynamic model for a plant system requires quantative analysis of water absorption and transfer processes which take into consideration the following factors:

i) The development and geometry of a root system,

ii) The density of the root system,

iii) The degree of the suberization process,

iv) The limiting effect of soil conductivities on water uptake,

v) The development of water potential distribution due to evaporation and water uptake by the root system.

IV. MATHEMATICAL MODELING OF WATER TRANSFER THROUGH THE SOIL-ROOT DOMAIN

It is desirable to set up a determinate non-stationary model for water absorption and transfer through the soil and root system. The types of information needed to set up a model of this nature may be discussed as follows.

4.1 The Rate of Root Growth

The growth rate of a root system and its functioning as an absorbing surface is controlled largely by parameters of the soil environment such as soil moisture, soil aeration, soil temperature, as well as the translocation rate and carbohydrate accumulation. It is impossible to set up a mathematical relationship between the rate of root growth and these variables due to a lack of information about their interaction. Therefore, one must eliminate some of the variables by making the following assumptions:

i) The soil temperature is constant at 25°C. Brouwer (1965) observed that optimum growth occurs at this temperature.

ii) The lower boundary of root growth is due to lack of aeration, a condition which occurs in the zone of near saturation.

iii) The upper limit of root growth is due to water stress. In the model, it will be assumed that the limiting value of water stress is -15.0 bars water potential.

iv) The soil is free of salt concentration, i.e., no osmotic contribution to the soil water potential.

Under these assumptions, one can represent the effects of soil water potential on the rate of root growth by defining a dimensionless growth function. The growth function can be derived by using the observations on root extension of Newman (1966), and Ratliff and Taylor (1969) as follows in empirical form:

$$g(\Phi) = \begin{cases} -(\Phi)\ e\ (\Phi + 1) & \text{for } \Phi < 0 \\ 0 & \text{otherwise} \end{cases} \qquad (4.1)$$

where $g(\Phi)$ = the growth function

Φ = the soil water potential (bars)

It appears that the rate of root growth is optimum at -1. bar soil water potential as shown in Figure 2. The actual rate of root extension at different locations with correspondingly different water potentials can be calculated by multiplying the growth function by the maximum rate of root extension. The root extension function may be written as:

$$v(\Phi,t) = va(t)g(\Phi) \qquad (4.2)$$

where $v(\Phi,t)$ = the actual rate of root extension (cm/day)

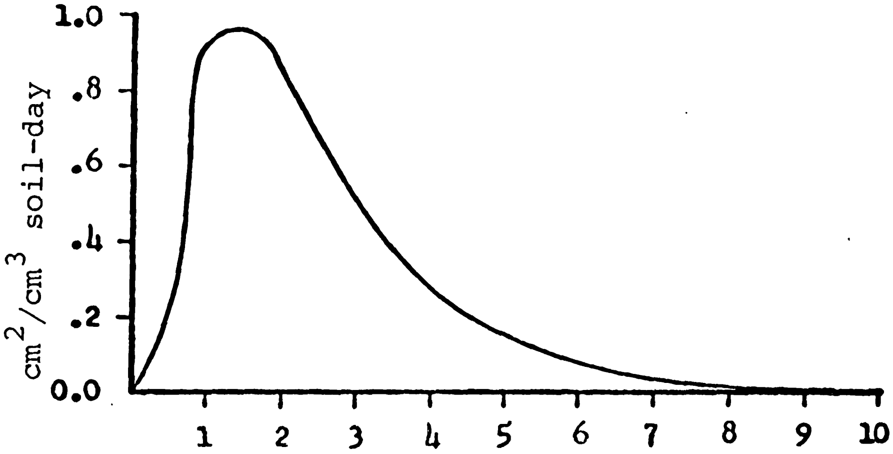

Figure 2.--Root Growth Function.

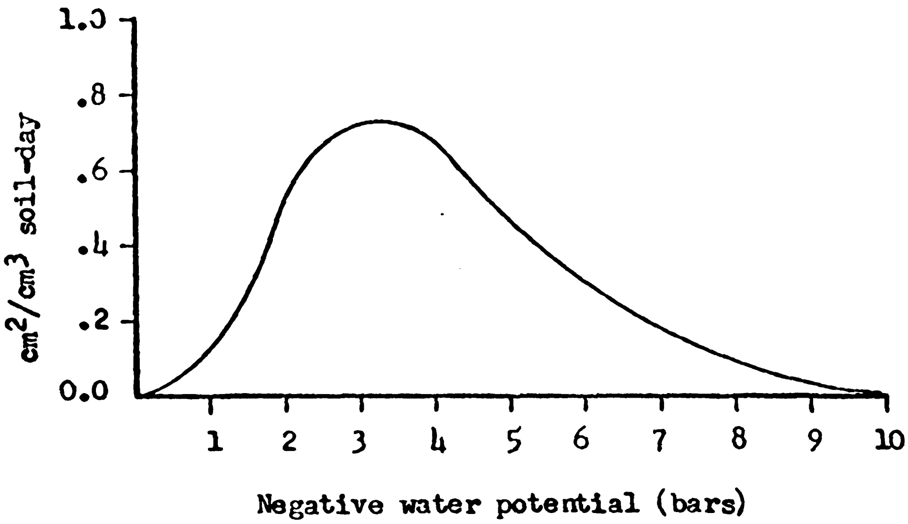

Figure 3.--The Rate of Root Density Increase Versus Soil Water Potential in the Hillsdale Sandy Loam.

$va(t)$ = the maximum rate of root extension (cm/day).

The maximum rate of root extension is dependent on the age and type of plant. Russell and Mitchell (1971) observed that the average rate of root extension was between 2.0 and 5.0 cm/day for soybeans.

4.2 The Geometry of the Root System

The geometry of the root-soil interfaces is very complicated since it changes as the root system branches and grows with time. A model which neglects the time dependence of the root system geometry may be appropriate for a perennial plant. However, root extension is probably very important for annual plants such as agricultural crops. It may be possible to simulate the process of branching by assuming the root system has to branch to satisfy the demand of transpiration. However, the determination of the location and growth direction of too many branches in three-dimensional space is not feasible. Although the importance of root geometry in the water absorption and transfer process is recognized, it must be simplified to reduce the mathematical complication of the model to a point where one can hope to make some progress with it.

It is thought that it is feasible to use the observations of Russell and Mitchell (1971) to represent the geometry of a root system for a bean plant. They observed that four to six lateral roots appeared after germination with an almost 90.0 degree separation from the radicle, i.e., the first root which grows downwards from the seed. The remainder of the root development is the result of secondary and tertiary branching to fill the soil between these taproots as growth progresses. The shape of an effective root zone for our purposes may be assumed to be a conical prism whose outer surface can be determined by the vertical and horizontal extension of the root system.

4.3 Root Density

In nature, the competitive root tips may be growing in all directions with some average separation. It has been difficult to measure and express such growth in a meaningful way. Whisler *et al*. (1968) proposed that the density function of root systems is a function of the area of the absorbing surface per unit volume and the effective distance of water flow to the root surface. Thus, the maximum value of R, the radius of influence of a single root, can be correlated with the density function of the root system by assuming that it is one-half the average distance between neighboring roots, as shown in Figure 4. However, it is possible that the radius of influence of neighboring roots might be overlapped by the formation of a new root branch as the growth of plant progresses. Therefore, we should define a factor, c to represent the degree of overlapping in the root zone. The best combination of overlapping, where optimum utilization of soil occurs, will be a hexagon as shown in Figure 5, and the optimum degree of overlapping may be calculated as:

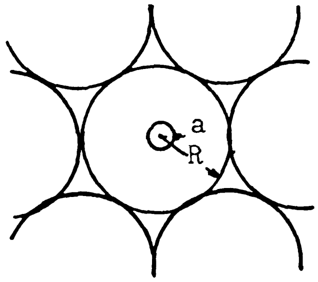

Figure 4.--The Radius of Influence when the Degree of Overlapping is Equal to One.

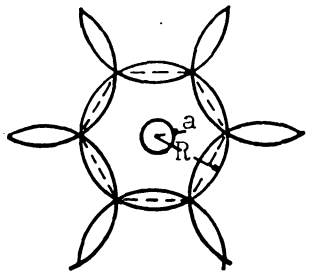

Figure 5.--The Radius of Influence when the Degree of Overlapping is Optimum.

$$c = \frac{\Pi R^2}{2.59808 R^2} = 1.21 \tag{4.3}$$

If one defines the density function of a root system as the area of root surface per unit volume, then one obtains:

$$A (r, z, t) = \frac{Sr (r, z, t)}{Vs (r, z, t)} c \tag{4.4}$$

where $A (r, z, t)$ = the root density function,

$Sr (r, z, t)$ = the surface of root per unit length,

$Vs (r, z, t)$ = the volume of influence

c = the degree of overlapping.

The surface of the root per unit length is a function of the root radius, $a(r,z,t)$, and may be written as:

$$Sr (r,z,t) = 2\Pi a(r,z,t) \tag{4.5}$$

The volume of influence is a function of the radius of influence, i.e., the distance from the center of the root at which the water potential gradient is zero. The volume of influence may be written as:

$$Vs (r,z,t) = \Pi(R(r,z,t))^2 \tag{4.6}$$

The radius of influence can be calculated from equation (2.2), which is the steady-state solution of water absorption for a stationary root by Gardner (1960). By setting equation (2.2) equal to zero and solving for R, we obtain:

$$R(r,z,t) = \sqrt{\frac{D(\Phi)\,t}{e^{\delta}}} \qquad (4.7)$$

where $D(\Phi)$ = the soil diffusivity (cm/day),

t = time of active absorption (assumed 12 hours),

δ = Euler's constant.

By substituting the equations (4.5), (4.6), and (4.7) into equation (4.4), one obtains:

$$A\ (r,\ z,\ t) = \frac{2 \quad a(r,z,t)\ c}{2.25\ D(\Phi)\ t} \qquad (4.8)$$

where $A\ (r,\ z,\ t)$ is the optimum root density function, when c is equal to 1.21. The actual density function of the root system may be calculated as the product of the growth and optimum density function, since the rate of root extension is controlled by the growth function as explained in previous section. The actual density function of the root system may be written as:

$$A\ (r,\ z,\ t) = A\ (r,\ z,\ t-1) + [A\ (r,\ z,\ t) - A\ (r,\ z,\ t-1)]\ g(\Phi) \qquad (4.9)$$

An inspection of equation (4.8) and (4.9) shows that the density function of the root system is:

i) proportional to the radius of the root,

ii) inversely proportional to the diffusion coefficient of the soil,

iii) controlled by soil water potential.

Assume that the radius of a root is constant; then one might expect from equation (4.8) a higher root density in the older and upper part of root zone due to reduction in soil diffusivity with decreasing soil water potential. This situation might represent the growth of root diameter and the formation of new root branches as it happens in nature. However, the growth rate of root density is controlled by soil water potential as shown in Figure 3.

4.4 Suberization of Root Surface

The root system of a plant presents structural and physiological adaptations which appear to be inconsistent at first. For instance, the development of suberization in the older part of growing roots is contradictory to the primary root functions which are the absorption of water and minerals as well as the anchoring of plants. Although the resistance of the suberized layer against movement of water through the soil-root boundary, there has been no attempt to investigate the development of suberization and the resulting resistance. The present knowledge of the suberization was described nicely by Kramer (1969):

> Resistance to water movement through suberized roots must be located in the layer of suberized tissue around the outside of the root, in the cork and vascular cambia, and in the phloem. The relative importance of these tissues as sources of resistance might be established by removal of one layer at a time while water is moving through the roots under pressure, but this has never been done.

Wolf (1970) suggested that the advantage of suberization lies in preventing water loss along the root, consequently minimizing the expenditure of energy to absorb the required amount of water for transpiration. Based on Wolf's (1970) suggestion, one might understand the need and development of suberization by investigating the absorption and transfer of water by a plant root system with and without suberization.

Gardner (1960) assumed that the development of water potential in the root xylem is constant throughout the root system. However, the water potential of soil is not constant in the root zone due to evaporation and cyclic water absorption by the root system. Furthermore, the older parts of the root system has been in situ for some time, and consequently, they may have dried out the soil in their vicinity. Under these circumstances, the root system without suberization serves as a short to transfer water from moist soil to dry soil and thus keep the entire root zone in an equilibrium condition. Such a condition does not exist according to the observation of Gardner (1964). Therefore, the plant is forced to develop a water potential in the root system lower than minimum water potential in the vicinity of the root system in order to absorb water, when active transpiration begins in the early part of the day.

The transfer of water through the root system from moist soil to dry soil will continue until the morning or

at a time such that the water potential of the root xylem decreases to a value sufficient to keep the system in an equilibrium condition as far as water absorption and release are concerned. Now, assume that the root system develops a suberized layer at the root surface where a potential gradient from root xylem to soil exists during the night. When transpiration begins, the plant does not have to develop a lower water potential in the xylem to overcome the lowest soil water potential in the root zone in order to absorb water. Rather the zone of water absorption will move along the root from the tips to the base as the water potential of the root xylem decreases with increasing transpiration demand.

In comparing these two situations, it can be seen that the suberization of older roots will minimize the energy expenditure of the plant and is part of plant adaptation. Therefore, one can deduce the following assumptions to model the development of suberization and its effect on water absorption:

i) The surface of older roots will become suberized to prevent the shorting effect of water flow along the root tissues.

ii) The degree of suberization is equal to the maximum potential gradient from root to soil which develops during the nighttime due to transfer of water from moist soil to dry soil.

iii) Shorting of water flow will take place whenever the degree of suberization (i.e., the maximum nighttime potential gradient) is overcome by the existing potential gradient.

4.5 Limitation to Water Uptake by Plant Roots

Gardner and Lang (1970) reported that the rate of water uptake reached to a limiting value as the potential gradient increased between soil and root xylem. They assumed that the flow of water into the root surface is radial and the governing equation for water absorption by a single root with initial and boundary conditions is written as:

$$\frac{\partial \Phi}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r\, D(\Phi) \frac{\partial \Phi}{\partial r}\right) \tag{4.10a}$$

$$\Phi = \Phi_o \quad \text{at} \quad t = 0 \text{ for } a \leq r \leq X \tag{4.10b}$$

$$2\, \Pi\, a\, K(\Phi) \frac{\partial \Phi}{\partial r} = q(t) \quad \text{at } r = a \text{ for } t > 0 \tag{4.10c}$$

$$\frac{\partial \Phi}{\partial r} = 0 \quad \text{at} \quad r = X \text{ for } t > 0 \tag{4.10d}$$

Gardner and Lang (1970) solved equation (4.10a) by numerical techniques to determine the limiting water uptake from the boundary condition, equation (4.10b). They concluded that decreasing water potential within a plant does not necessarily increase the flow from the soil to the roots. Hence, the limiting value of water uptake and the corresponding

water potential of the root xylem may be used in our model to optimize the relation between the root surface and the transpiration demand.

The absorption of water by the plant root is a dynamic process, since the flux of water is a function of the water potential gradient and soil hydraulic conductivity, which changes with time. In the present model, we assume that the water potential of the root xylem decreases linearly from some initial value at dawn to a low at 2 p.m., at which time the transpiration demand reaches a peak value. This assumption enables us to estimate the minimum root potential corresponding to maximum water absorption by the root system.

Equation (4.10a) with the boundary condition (4.10b) through (4.10d) is solved by numerical techniques over twelve hours for different initial soil water potentials to determine the temporal behavior of water absorption at the root surface. The root potential is increased linearly to a maximum value at the end of twelve hours. The selection of maximum water potential at the root surface is based on the ratio of the difference between root and soil water potentials, to the initial water potential of the soil. The resulting water flux is plotted against time as shown in Figure 7 and 9. It appears that the maximum water absorption is obtained at lower soil tension when the ratio is about 1.5. However, the ratio increases to about 2.0 as the initial water potential of the soil

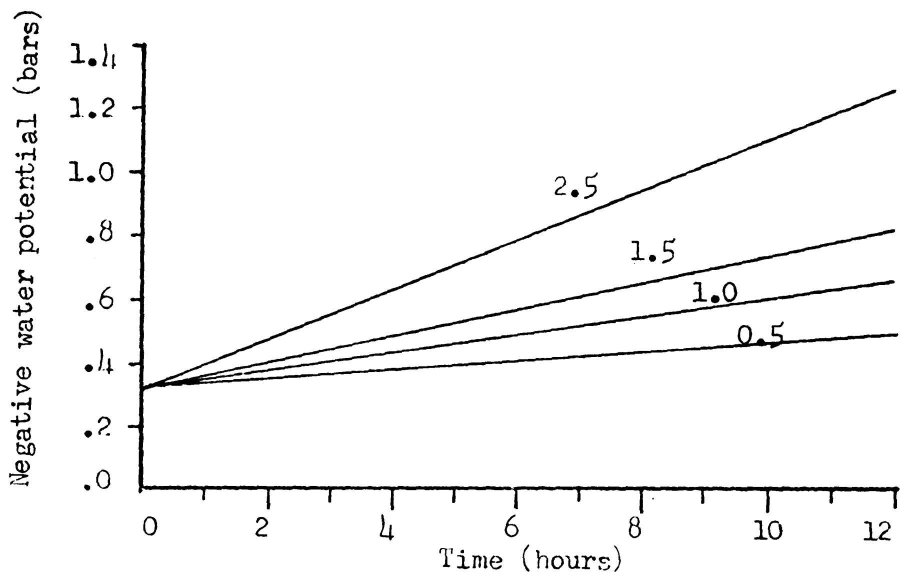

Figure 6.--The Development of Root Water Potential when the Initial Soil Water Potential is Equal to Field Capacity.

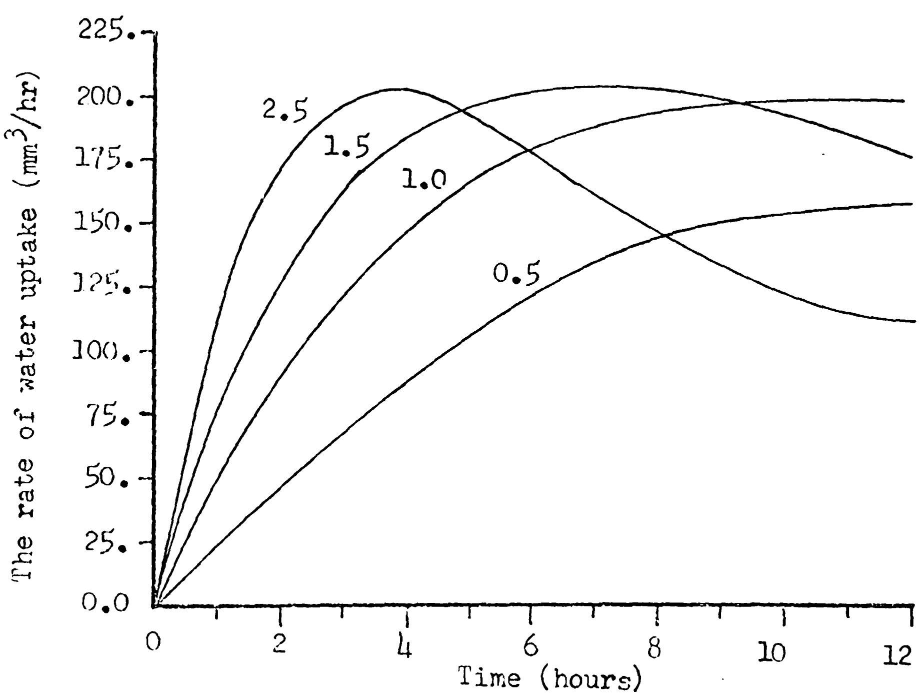

Figure 7.--The Rate of Water Absorption per Unit Length of Root in Hillsdale Sandy Loam when the Development of Root Water Potential as in Figure 6.

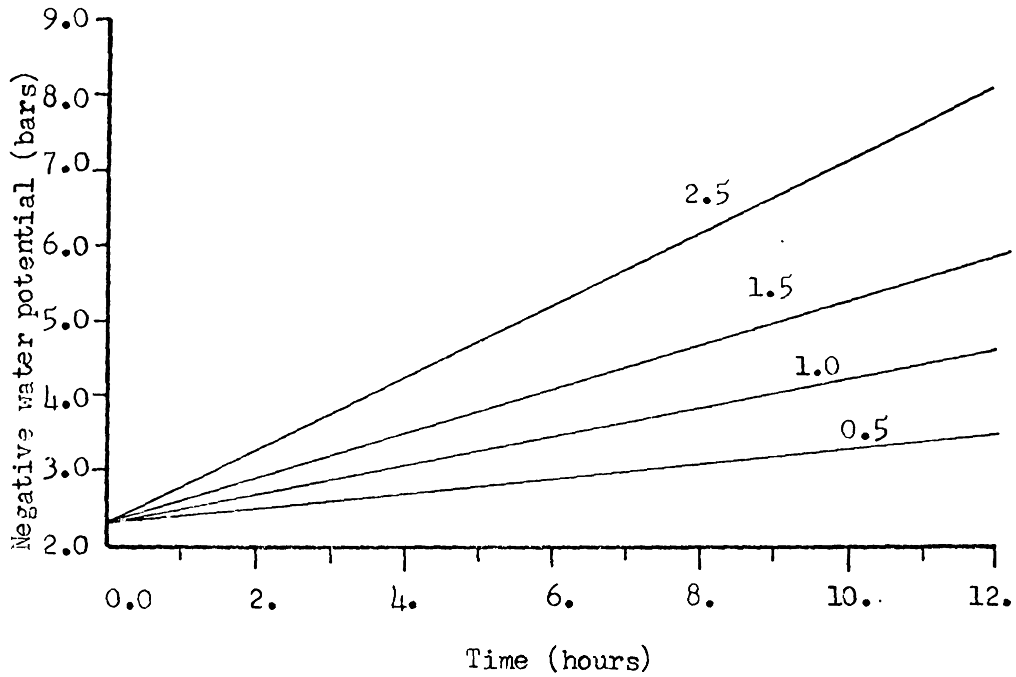

Figure 8.--The Development of Root Water Potential when the Initial Soil Water Potential is Equal to -2.3 Bars.

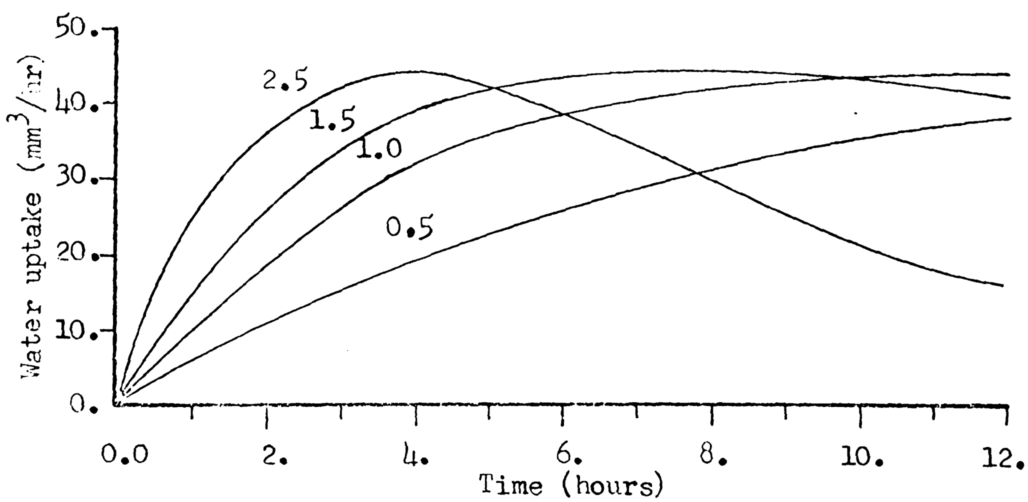

Figure 9.--The Rate of Water Absorption per Unit Length of Root in a Hillsdale Sandy Loam when the Development of Root Water Potential as in Figure 8.

decreases. The rate of water uptake is almost negligible when the initial soil water potential drops below the wilting point.

The pattern of water uptake with time depends upon the development of the root water potential. As the gradient of water potential between root xylem and soil increases rapidly, the rate of water uptake reaches a peak value in a few hours and then drops suddenly. This situation is contradictory to the observation of Weatherly (1963) on daily fluctuation of transpiration. Therefore, one must consider not only the optimization of water absorption in finding a relation between soil and root water potential but one should also select the type of water absorption pattern which would represent the actual transpiration losses.

4.6 Derivation of Governing Equation for Water Transfer Through the Soil-Root Domain

Childs (1940) suggested that the flow of water in porous material can be analyzed as a diffusion process, and he applied Darcy's (1856) equation to solve the unsaturated water flow in soils.

$$\nu_n = - K_n \, (\Phi) \; \nabla \, \Phi \qquad (4.11)$$

where ν_n = the volume of water passing through a unit cross-section of soil per unit time

$K_n(\Phi)$ = the soil hydraulic conductivity

Φ = the soil water potential gradient in the nth direction.

By imposing the conservation of mass principle, he obtained:

$$\frac{\partial \Theta}{\partial t} = - \nabla \nu_n + S \tag{4.12}$$

where Θ = the volumetric water content of soil,

t = time,

S = the source term per unit volume.

In the original derivation of equation (4.12) it was assumed that the relation between water content and water potential of soil is unique. Buckingham (1907) defined the change in moisture content with potential as the specific water content (or water capacity, C). If one assumes that water capacity is constant for a small time increment, the left side of equation (4.12) may be written as:

$$\frac{\partial \Theta}{\partial t} = \frac{\partial \Theta}{\partial \Phi} \frac{\partial \Phi}{\partial t} = C \frac{\partial \Phi}{\partial t} \tag{4.13}$$

By substituting equation (4.11) and equation (4.13) into equation (4.12), one obtains:

$$\frac{\partial \Phi}{\partial t} = - \nabla \cdot \left(\frac{K(\Phi)}{C} \nabla \Phi\right) + \frac{S}{C} \tag{4.14}$$

when the diffusion coefficient, which is equal to the ratio of soil conductivity to water capacity, as defined by

Childs and Collis-George (1950) is used one obtains:

$$\frac{\partial \phi}{\partial t} = - \nabla \cdot (D(\phi) \nabla \phi) + \frac{S}{C} \tag{4.15}$$

Whisler et al. (1968) solved equation (4.12) by numerical techniques for a steady-state, one dimensional unsaturated water flow in a soil column. Philip (1966) compared the transient and steady-state solution of evaporation from soil and suggested that the steady-state calculations are ill fitted to yield the distribution of potential, especially near the absorbing surfaces. Therefore, it is desirable to solve equation (4.15) for the transient case.

Assume that a single plant is growing in a uniform and isotropic soil with a root zone in the shape of a symetric conical prism around the stem as a vertical axis. Then, one is able to analyze the process of water absorption and transfer in the soil-root domain by expressing equation (4.15) in cylindrical coordinates, as

$$\frac{\partial \phi}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[rD(\phi) \frac{\partial \Phi}{\partial r} \right] + \frac{\partial}{\partial z} \left[D(\phi) \frac{\partial \phi}{\partial z} \right] + \frac{\partial K(\phi)}{\partial z} + \frac{S(r,z,t)}{C} \tag{4.16}$$

Equation (4.16) is the describing equation for radially symmetric and vertical water flow in the soil-root domain. It expresses the water potential distribution as a function of the time and space coordinates. Since it is second-order in the space variable and first-order in the

time variable, two boundary and one initial condition must be specified.

Assume that the water potential or water content of the soil medium is constant at some initial value. The initial condition may be writted as:

$$\Phi = \Phi_o \quad \text{at} \quad t = 0 \text{ for } 0 \leq z \leq L$$

$$0 \leq r \leq X \tag{4.17}$$

where L = the depth of the soil system,

X = the radius of the soil system.

The upper boundary condition of the soil-root system is determined by the soil surface where evaporation takes place. The upper boundary condition may be written as:

$$e(t) = -K(\Phi)\frac{\partial\Phi}{\partial z} + K(\Phi) \text{ at } z = 0 \text{ for } t > 0$$

$$0 \leq r \leq X \tag{4.18}$$

where e(t) = the rate of evaporation.

The lower and lateral boundary conditions of the soil-root system are determined by assuming they coincide with the distance where the influence of the root zone ended. Hence, the lower boundary conditions may be written as:

$$\frac{\partial\Phi}{\partial z} = 0 \text{ at } z = L \text{ for } t > 0$$

$$0 \leq r \leq X \tag{4.19}$$

The side boundary condition may be written as:

$$\left.\frac{\partial \Phi}{\partial r}\right|_{r = X} = 0 \qquad \text{for } t > 0 \qquad (4.20)$$
$$0 \leq z \leq L$$

The source term, S(r,z,t) may be defined as done by Gardner (1960), Cowan (1965), and Whisler _et al_. (1968), where it was assumed that the source term is a function of:

i) the density of the root system,

ii) the hydraulic conductivity of the soil,

iii) the potential gradient between the soil and the root system.

They assumed that the resistance of a suberized root surface against the diffusion of water is zero. However, experimental observation and deductive analysis indicate that the resistance to water uptake due to suberization of the root surface has a significant contribution in the process of water uptake and transfer by the root system. Therefore, the source term is also a function of the degree of suberization. The degree of suberization, B(r, z,t), is defined as the required water potential between soil and absorbing root surface to overcome the resistance of the suberized layer.

Thus, the source term, S(r,z, t), will be used to represent the uptake of water, i.e., a negative source, and also to describe the release of water by the plant root, i.e., a positive source. The source term may be written as:

$$S(r,z,t) = A(r,z,t)\ K(\Phi)\ \nabla\Phi(r,z,t), \tag{4.21}$$

where $s(r,z,t)$ = the source term per unit volume,
$A(r,z,t)$ = the density function of the root system,
$K(\Phi)$ = the hydraulic conductivity of the soil,
$\nabla\Phi(r,z,t)$ = the effective water potential gradient between soil and root xylem.

The calculation of the effective water potential gradient between soil and root xylem requires knowledge of the value of the root potential and the degree of suberization. It is assumed that the resistance to the flow of water within the root is negligible. Hence, there will be one value for root potential throughout the entire root system. The optimum value of root potential may be estimated from the analysis of water uptake limitations as we stated previously. Then, the effective water potential gradient between soil and root xylem will be

$$\nabla\Phi(r,z,t) = \Phi_r - \Phi_s(r,z,t) \mp B(r,z,t) \tag{4.22}$$

where Φ_r = the water potential of root xylem,
$\Phi_s(r,z,t)$ = the water potential of soil,
$B(r,z,t)$ = the degree of suberization, which is negative for water release and positive for water absorption.

Inspection of equation (4.22) reveals that whenever the effective water potential gradient is zero, i.e., the actual water potential gradient between soil and root xylem

is less than or equal to the degree of suberization, there is no water absorption or release at the root surface.

Assume that the sum of source terms throughout the entire root zone is equal to the rate of transpiration, then one obtains:

$$i(t) = -\sum_{z=0}^{L} \sum_{r=0}^{X} S(r,z,t) \quad V(r) \tag{4.23}$$

where $i(t)$ = the rate of transpiration,

$V(r)$ = the volume of cylindrical soil ring corresponding to the r^{th} increment.

By combining equation (4.21) with equation (4.22) and substituting the resulting equation into equation (4.23), one obtains:

$$i(t) = -\sum_{z=0}^{L} \sum_{r=0}^{X} A(r,z,t) \; K(\Phi) \left[\Phi_r - \Phi_s + B(r,z,t) \right] V(r) \tag{4.24}$$

This equation will be solved simultaneously with equation (4.16) by using numerical techniques to determine the distribution of water potential, root density function, source function, the degree of suberization and the development of the root system at different soil and moisture conditions.

V. EXPERIMENTAL DESIGN AND PROCEDURE

This study is conducted in the plant-water physio-engineering laboratory located in the Agricultural Engineering Department of Michigan State University in East Lansing, Michigan. The basic equipment used in this experiment had been described by Merva and Kiliç (1971). A brief description of this apparatus as related to this research will be discussed.

5.1 Plant Environments

Agricultural scientists have been studying the physiological response of plants under various environmental conditions in the greenhouses where it was impossible to control and monitor all environmental variables. Thus, the need for more accurate control of environmental parameters led to the development of growth chambers.

The plant-water physioengineering laboratory has a Percival Model MB 60 growth chamber with 56"x26"x50" internal dimensions. The radiation source of growth chamber is provided by a set of cool white fluorescent and incandescent lights with three timers. The chamber is divided by two inch styrofoam into two compartments to separate

plant crown from the soil-root system as shown in Figure 10. The lower compartment of the growth chamber is insulated with two inch styrofoam to obtain a uniform temperature control in the soil-root system. The front part of insulation has a door with double plexiglass viewing ports and an access port fitted with a glove to adjust the soil weighing mechanism without disturbing the environmental condition.

The average temperature and relative humidity of the Lansing area from May 15 through July 15 was simulated by a one horsepower Aminco Aire unit with a Taylor Time Schedule Recording fullscope controller at the upper compartments of the growth chamber. The temperature is varied between 20°C and 27°C, while relative humidities is changing between 58 per cent and 78 per cent, as shown in Figure 11. The relative humidity and temperature of the lower chamber is controlled by a second Aminco Aire unit which maintained a temperature of 25°C and relative humidity of approximately 90 per cent. The air flow rate through the Aminco Aire unit was adjusted to provide 300 ft3/min.

A drum with 50 cm. diameter and 54 cm deep is used to provide the necessary volume of soil for a single plant. The interior and exterior of the drum was coated with epoxy paint to prevent contamination of the soil. The soil water was supplied or released through perforated tygon tubing laid in the bottom of the drum and covered with approximately five centimeters of coarse sand.

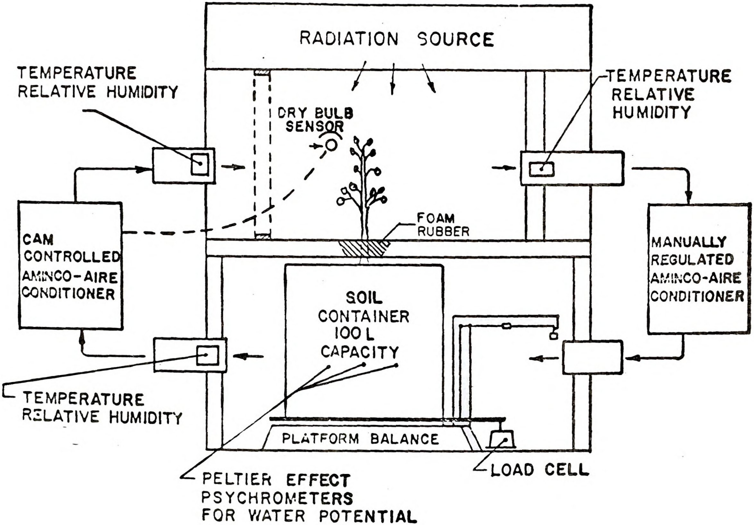

Figure 10.--Schematic of the Environmental Control System.

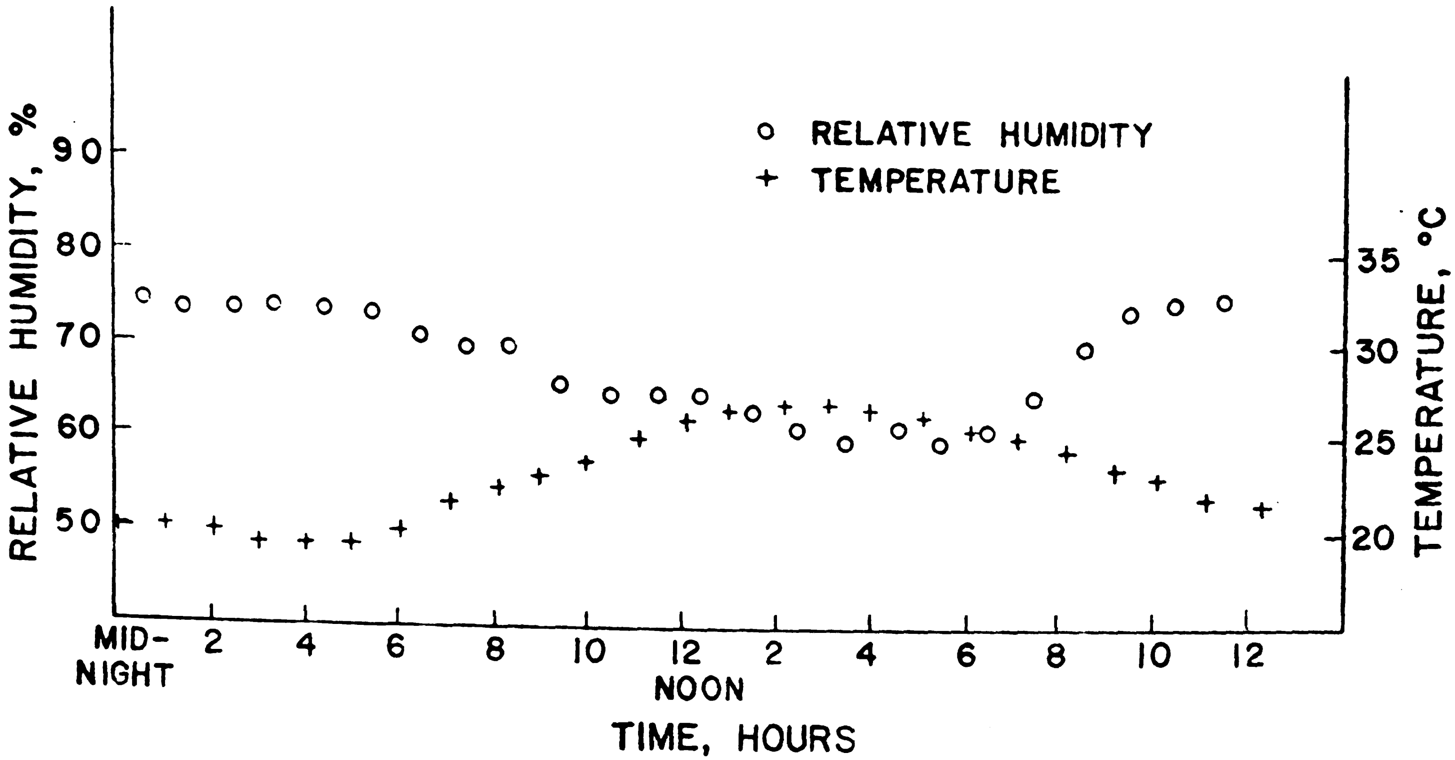

Figure 11.--Temperature and Relative Humidity of Air in the Upper Part of the Growth Chamber.

5.2 Soil Properties

In this experiment a Hillsdale sandy loam is used. The moisture characteristic curve of this soil as shown in Figure 13 is obtained from Qazi (1970) who used the static equilibrium method. The conductivity and diffusivity of the soil is determined by using an approximate method which is developed by Huggins et al. (1970). The method is based on the assumption that the conductivity-pressure head relationship could be effectively represented by an empirical three-parameter equation presented by Gardner (1958):

$$K(\phi) = [(\phi/h)^n + b]^{-1} \tag{5.1}$$

where $K(\phi)$ = the soil conductivity (cm/day),

ϕ = the soil potential (cm)

n = the exponential value of soil characteristic curve,

b = the reciprocal of the soil conductivity at saturation (day/cm),

h = empirical constant (cm).

Huggins et al. (1970) developed a computer program to estimate empirical parameter, h, by comparing experimental and predicted infiltration rate-time relationships of a given soil in a soil column. The infiltration rate-time relationship is estimated by numerical techniques for a first trial value of the empirical parameter. The trial-error method is continued until the absolute differences

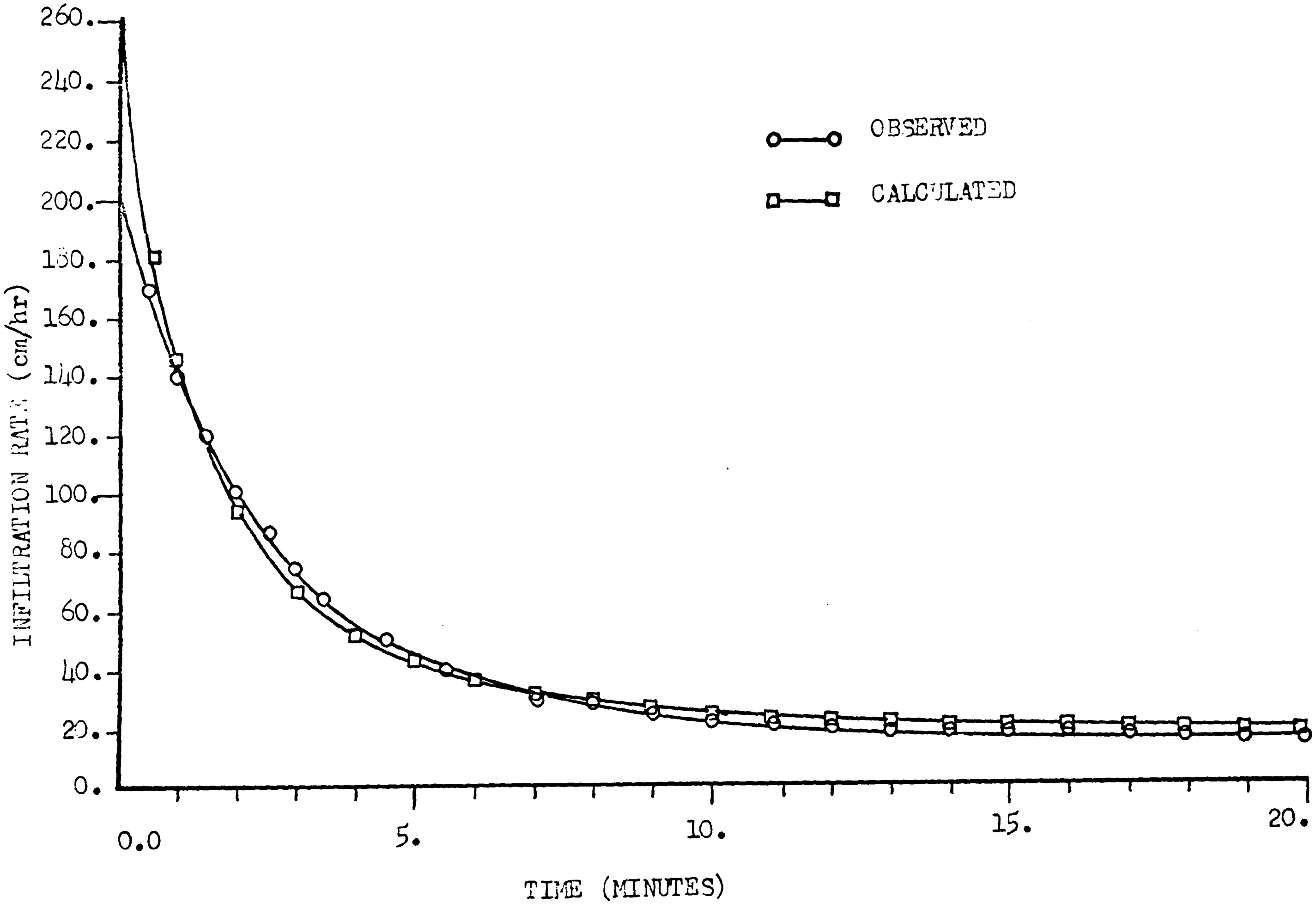

Figure 12.--Observed and Calculated Influx Curves for Hillsdale Sandy Loam.

between the experimental and predicted infiltration rate-time relationships are a minimum. The basic experimental determination required in this method is the measurement of influx curves for infiltration into soil columns from shallow ponded surface conditions.

The soil was packed into a 12-inch cylindrical column which was constructed from plexiglass tubing with 0.20 inch wall thickness and five inch internal diameter. The base was constructed so that a constant head could be maintained at the end of the columns for saturated conditions. The Hillsdale sandy loam was air dried in the laboratory and screened through an ASTM No. 30 sieve. The soil was packed in the column until the desired mean bulk density was 1.39 gr/cm^3. The top and bottom ends of the soil column were supported by a hardware cloth. The bottom end was kept open to the atmosphere during the unsteady-state portion of the test. After the wetting front had reached the bottom of the column, the soil column was placed into a constant head overflow reservoir. Flow was maintained for 12 hours, as recommended by Huggins et al. (1970), to establish equilibrium conditions in the column. Then the steady state inflow rate was measured, and the hydraulic conductivity of saturated soil was calculated.

Water was supplied by a siphon from a Mariotte-type water supply reservoir. The weight of infiltrated water into the soil column was continuously measured by placing the reservoir on a Daytronics 152A load cell which

was coupled to a Speedomax G Model recorder. To determine the infiltration rate-time relationship, the volume of infiltrated water was read from the recorder chart at specified time intervals. Then the rate of infiltration was computed by dividing the rate of infiltrated water volume by the cross-sectional area of the soil column.

Observed and calculated influx curves for infiltration into initially dry columns of Hillsdale sandy loam are shown in Figure 12. A mean value of the soil conductivity for saturated conditions was calculated as 3.60 cm/hr. and used to estimate b = 1000, sec./cm in equation (5.1). From the computer analysis, the empirical parameter was approximated as -1.57 cm to yield the absolute differences between the computed and observed influx curves of .95 cm. The relationship between the conductivity and the water potential of soil as shown in Figure 14 was determined to be:

$$K(\phi) = \frac{86400.}{((\phi/-1.57)^3 + 1000)} \qquad (5.2)$$

where $K(\phi)$ is the hydraulic conductivity of soil in cm/day.

The diffusion coefficient of soil water was calculated from the relationship between the water capacity and hydraulic conductivity. The resulting diffusion coefficient is plotted against soil water potential in Figure 15. This relationship may be written as:

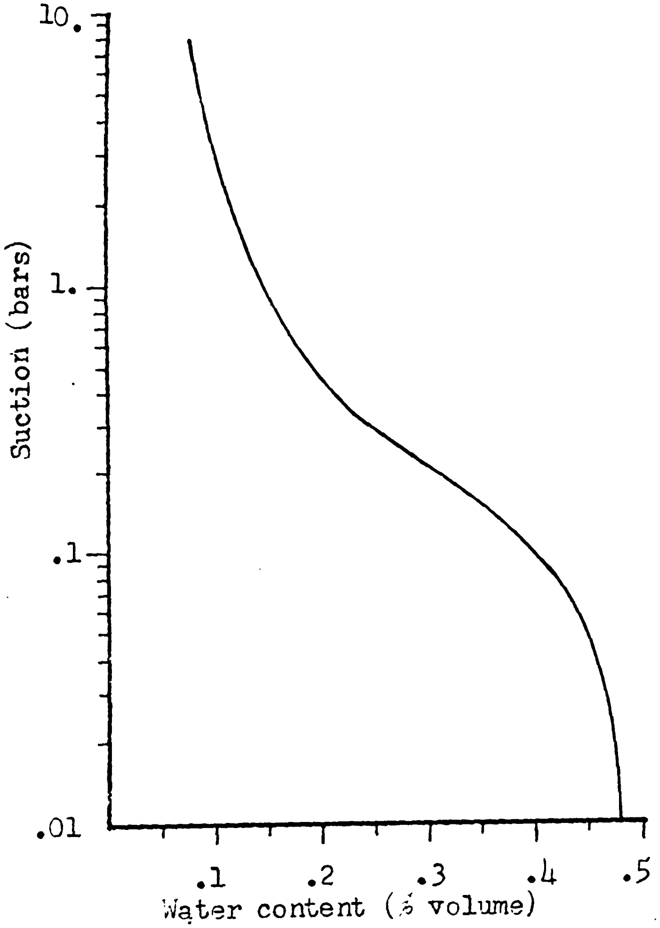

Figure 13.--Relationship Between Water Potential and Water Content on a Volume Basis for Hillsdale Sandy Loam.

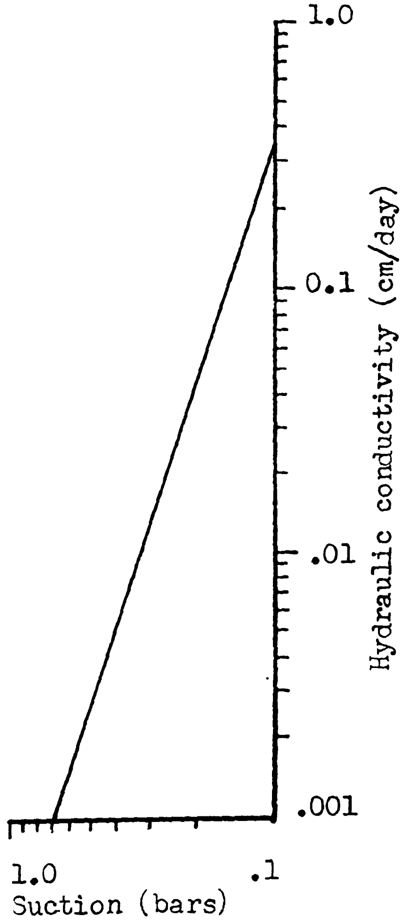

Figure 14.--Relationship Between Water Potential and Hydraulic Conductivity for Hillsdale Sandy Loam.

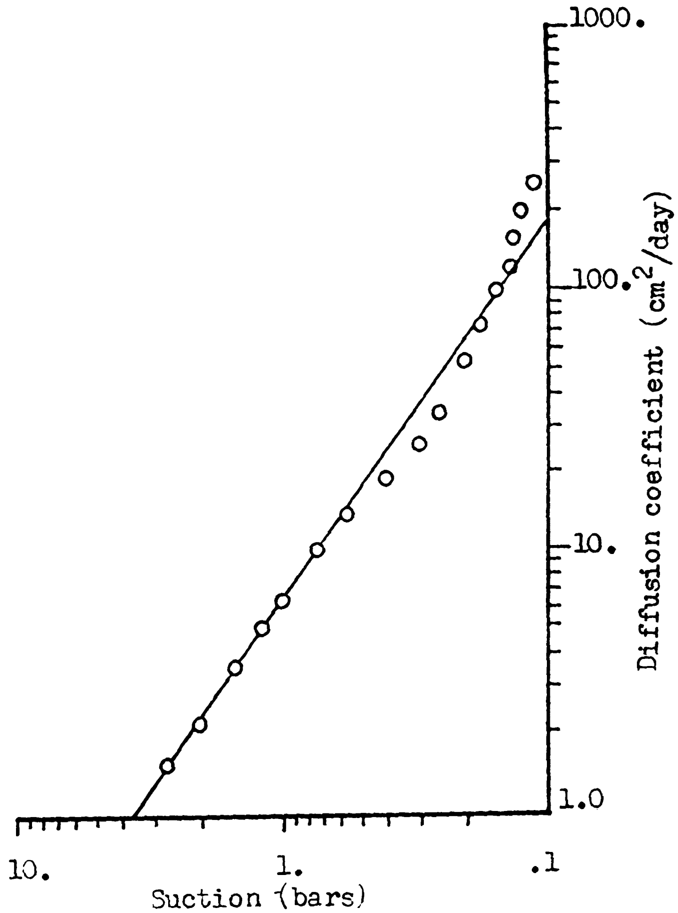

Figure 15.--Relationship Between Water Potential and Diffusion Coefficient for Hillsdale Sandy Loam.

$$D(\phi) = \frac{236758.9}{(-\Phi)^{1.5}} \tag{5.3}$$

where $D(\phi)$ is the diffusion coefficient for the Hillsdale sandy loam in cm^2/day.

5.3 Instrumentation and Data Acquisition System

The objective of this study was to analyze the processes of water absorption and transfer by developing root system in the soil. It is thought that the measurement of soil water potential distribution in situ may be correlated with the water uptake pattern, and the root development. There are numerous techniques for soil water potential measurement, but most of them have an important limitation in obtaining continuous measurement without disturbing the soil environment. The development of peltier type psychrometer by Spanner (1951) makes it possible to measure the soil water potential in situ without disturbing the soil and root environments.

The Spanner instrument utilizes the relationship between water potential and the ratio of actual and saturated vapor pressures. This relationship may be written:

$$\phi = \frac{RiT}{\vec{V}} \quad \ln \frac{p}{p_o} \tag{5.4}$$

where ϕ = the water potential,

Ri = the ideal gas constant,

T = the absolute temperature,

p = the actual vapor pressure,

p_o = the vapor pressure of pure water,

$\vec{V}$ = the molar volume of water.

When a current is passed through the thermocouple junction in an appropriate direction, dew formation occurs at the junction due to cooling of the junction by the peltier effect. If the current is cut off, the condensed water evaporates at a rate depending on the humidity in the air. Thus, the junction becomes a sensitive wet-bulb therometer. The difference in temperature between the cooled junction and a reference junction produces an electromotive force (emf), which may be amplified and measured with a sensitive microvoltmeter. By calibrating the psychrometer against a range of salt solutions of known vapor pressures at a constant temperature, a relationship between water potential and thermocouple output can be obtained.

Twenty-four Spanner type psychrometers were brought from Lepco, a division of Block Engineering, Inc., Logan, Utah. The psychrometers were connected through an automatic stepping switch for cycling so that all of them could be read sequentially on a Keithly micro voltmeter, Model 150B. The switching unit consisted of a stepping switch and a solid state stepping relay. At each step, the system automatically read the initial value of the psychrometer output, then cooled with a pre-selected cooling current over a preset time interval. After cooling, the final reading of the psychrometer was taken, and the system was switched

automatically to the next step. The same process was continued for each step in the system.

The amplified output of the psychrometers obtained from the Keithly microvoltmeter, was fed into a six-channel analog-to-digital converter with a paper tape punch unit, which punched the output on paper tape in a binary coded decimal form. The first channel on the Data Acquisition System contained either a positive or negative sign depending upon whether the readings of the psychometer recorded in the next channel corresponded to initial, or final readings following cooling. The second and third channels contained the readings of the psychrometer. The fourth and fifth channels were reserved for relative humidity readings at the outlet of the upper and lower chambers. The sixth channel contained the readings of a load cell which detected changes in mass of the plant soil system. A Daytronics 152A load cell was mounted on the lower platform of the scale to measure the weight loss due to evaporation and transpiration.

The psychrometers were calibrated over standard solutions of KCL with 0.5 molality corresponding to -22.3 bars water potential to determine the response of the psychrometers in terms of micro-volts per bar as recommended by the manufacturer.

The psychrometers enclosed in ceramic cups were immersed directly into a flask containing a standard solution of KCL for calibration. The flask containing the

solution and the psychrometers were in turn immersed in a constant temperature bath at 25°C, and the psychrometer outputs were recorded hourly for a day after temperature equilibrium was reached. After calibration, the psychrometers were washed for a few hours in several changes of distilled water to remove all traces of solutes. The analysis and the result of the calibration are given in Table 1.

5.4 Experimental Procedure

The soil barrel was cleaned and placed on the modified platform scale. Four hardware cloth cylinders with 0.5 cm mesh spaced concentrically 5.0 cm. apart in radius from the vertical axis of the barrel were fitted into the soil container to observe the development of the root system at the end of the experiment. The hardware cloths were sprayed with epoxy to prevent the interaction of the plant roots with the metal. Psychrometers were installed in a horizontal position at the concentric hardware cloth in such a way that they made two vertical planes intersecting at the vertical axis of the barrel. The psychrometers were spaced 5.0 centimeters apart on each plane. Room dried Hillsdale sandy loam soil was screened and placed in the barrel. Water was supplied from the bottom of the soil container until the soil reached saturation. The soil was then drained until flow ceased prior to planting of a bean plant at the center of the soil drum. The chamber was left

TABLE 1.--The Result of the Psychrometer Calibration with KCL Solution of 0.5 Molarity at Constant 25°C Temperature.

Psychrometer Numbers	Average Response MV/bar	Standard Deviation MV/bar
1	.442	.022
2	.434	.058
3	.458	.074
4	.465	.072
5	.490	.043
6	.477	.044
7	.397	.037
8	.393	.038
9*	--	--
10*	--	--
11	.390	.050
12	.300	.084
13	.364	.063
14	.436	.053
15	.423	.099
16	.459	.059
17	.434	.047
18	.456	.076
19	.446	.074
20	.414	.091
21	.404	.096
22	.433	.128
23	.439	.075
24	.412	.060

*These psychrometers failed to give consistent readings. Thus, they were ignored during the reduction of data obtained from the experiment.

unseparated until the plant shoot was long enough to seal off. The seal between the upper and lower parts of the chamber was done on the fifteenth day of planting by using foam rubber as shown in Figure 10. After the test terminated in 60 days, the soil barrel was taken out of the growth chamber and the soil was washed off by sprinkling water to observe the root system as held by the concentric hardward cloth cylinders.

5.5 Analysis of the Data

Plant growth is represented in terms of leaf area. The leaf area was computed by measuring the major and minor axis of a leaf and using the average of this measurement as the diameter of an equivalent circle. The total area of the leaves were calculated by summing up the area of each individual leaf. The measurements were taken daily at approximately noon.

The rate of transpiration from the plant and the rate of evaporation from the soil surface were calculated by measuring the temperature and the relative humidity of air entering and leaving the growth chamber. The validity of this method can be checked with the weight loss of the soil-plant system. The weight loss of the system was calculated from the readings of load cell taken on an hourly basis every fifth day after separation of the plant crown from the soil-root system. At other times, the readings of load cells were taken on a daily basis. The hourly readings were taken to calculate the rate of evaporation

during the night time and the rate of evaporatranspiration during the day time, from the slope of the weight loss in time for the plant-soil system.

The distribution of the soil water potential was determined from the response of Spanner psychrometers buried in the soil. The initial and final readings of each psychrometer were punched on the paper tape by the Data Acquisition System. The data on the paper tape was analyzed by computer as follows: First, the average of the initial readings were calculated to obtain a zero point for a given psychrometer. The final reading values were searched by the computer program and the peak value used as the final reading. The difference between the initial and the final readings was divided by a calibration value to obtain the soil water potential corresponding to the location of the psychrometer. The soil water potential for each psychrometer was calculated and punched out on cards by computer. The lines of equal water potential in each plane of the soil were plotted for 15 days, 30 days, and 45 days of the plant growth.

The water absorption and transfer processes were studied in the soil-root domain by applying the consequative water potential distribution of the soil to equation (4.16). Equation (4.16) is solved for the source term according to the boundary conditions of the soil-root system in our experiment. The distribution of the source

term is associated with the root development and root distribution as an absorbing surface in the soil.

VI. COMPUTER SIMULATION OF THE PROBLEM

The purpose of the computer simulation was to present a quantative analysis of the water absorption and transfer in the soil-root domain using the mathematical model which was developed in the previous chapter. According to the mathematical model, equation (4.26) must be solved simultaneously with equation (4.16) by using numerical techniques to determine the distribution of the water potential, root density function, source function, the degree of suberization and the development of the root system at the given soil and moisture conditions.

The initial and boundary conditions of the soil-root system are based on the experimental design. In the experiment, the soil-root system was defined as the cylindrical soil mass with 50 cm diameter and 50 cm depth. Thus in the computer simulation, the bottom and side boundaries of the soil-root system were chosen to be finite and drying continuously while the upper boundary of the soil-root system is a soil surface possessing the experimental evaporation rate. The initial water potential of the soil was assumed decreasing linearly from -330.0 cm of water at the top to -130.0 cm of water at the bottom of the soil

barrel. Based on these boundary and initial conditions, the soil water flow equation (4.16), in which the developing plant roots are treated as a continuous source term, was solved by the alternating implicit method as shown in Appendix I.

The solution of the describing equation (4.24) for the amount of water absorption required the development of a root zone, the determination of a root density, and the development of root water potential.

A kidney bean was assumed to have been planted at a 5 cm depth from the soil surface in the center of the soil barrel. The volume of the root zone was estimated by first predicting the vertical extension, and then the horizontal extension for each increment of the root zone. The root density for each grid point in the root zone was calculated by using the optimum overlapping coefficient, 1.21, and a constant radius of one millimeter in equation (4.8). The resulting equation was

$$A(r, z, t) = \frac{.216}{D(\Phi)} \tag{6.1}$$

The root water potential during the day time was determined from the optimization of water uptake by the plant root system. For the calculated root zone and root density, the root potential was iterated until the maximum water absorption was obtained. To reduce the number of iterations, the initial root water potential was multiplied

by 1.5 using as a basis the concept of water uptake limitation. The average rate of root extension was adjusted until the sum of water absorption was equal to the corresponding experimental transpiration.

The water potential of the root system during the nighttime was relaxed with the iteration technique until the amount of water absorbed was almost equal to the amount of water released from the older parts of the root system. To eliminate unnecessary iteration, a counter and a criterion were assigned to the program. The value of the criterion was based on the assumption of Gardner (1960) that the amount of water uptake during the nighttime should be equal to 20 per cent of the previous day's transpiration loss to recover the water stress of a plant. Whenever the absolute value of water absorption was negative and less than the criterion, the iteration was stopped. The resulting water potential of the plant root system was then used to determine the degree of suberization by comparing with the relaxed soil water potential distribution in each grid point of the soil-root system.

The calculated source terms for each time increment were substituted into equation (4.16) and solved with other boundary conditions to determine the new soil water potential distribution. The simulated soil water potential distribution was compared with the experimental soil water potential distribution to check the consistency of the mathematical model.

In solving the finite difference form of the describing equations, it was necessary to select a proper value for the space increment ΔX and time increment Δt. The smaller increment for both variables tended to produce slightly better results. The best solution with a reasonable computation time was obtained when ΔX was set equal to 1.25 cm while Δt was chosen to be 0.5 day. The computer program for the simulation of the problem is shown in Appendix II, was written in FORTRAN IV and processed on a CDC 6500 computer. Approximately 41,000 core memory is required to process this program. Therefore, it was necessary to minimize the required computer time, since the analysis of water absorption and transfer in the soil-root domain required many solutions of the describing equations over the growth season of the plant.

VII. DISCUSSION OF EXPERIMENTAL AND COMPUTER SIMULATION RESULTS

7.1 The Growth of Plant Leaves

The growth of the bean plant was measured by the increase in area of plant leaves, and the resulting values were plotted against time for two replications under the same environment. It was found that the growth rate of plant leaves is exponential as shown in Figure 16. The data for both replications is fitted with an exponential model,

$$A1 = 14.51\, e^{.1374\, t} \tag{7.1a}$$

$$A2 = 15.12\, e^{.1122\, t} \tag{7.1b}$$

A1 and A2 are the areas of bean leaves for two replications, respectively and t is the time in days. The correlation coefficient of the models are .996 and .987 respectively. The analysis of plant growth in terms of dry weight is not attempted. However, the mass of bean leaves and stem was determined after harvest at the end of 58 days, and found to be 280. and 260. grams wet and 51. and 46. grams dry. The moisture content of the bean plants based upon dry mass was approximately 460. and 450. per cent respectively.

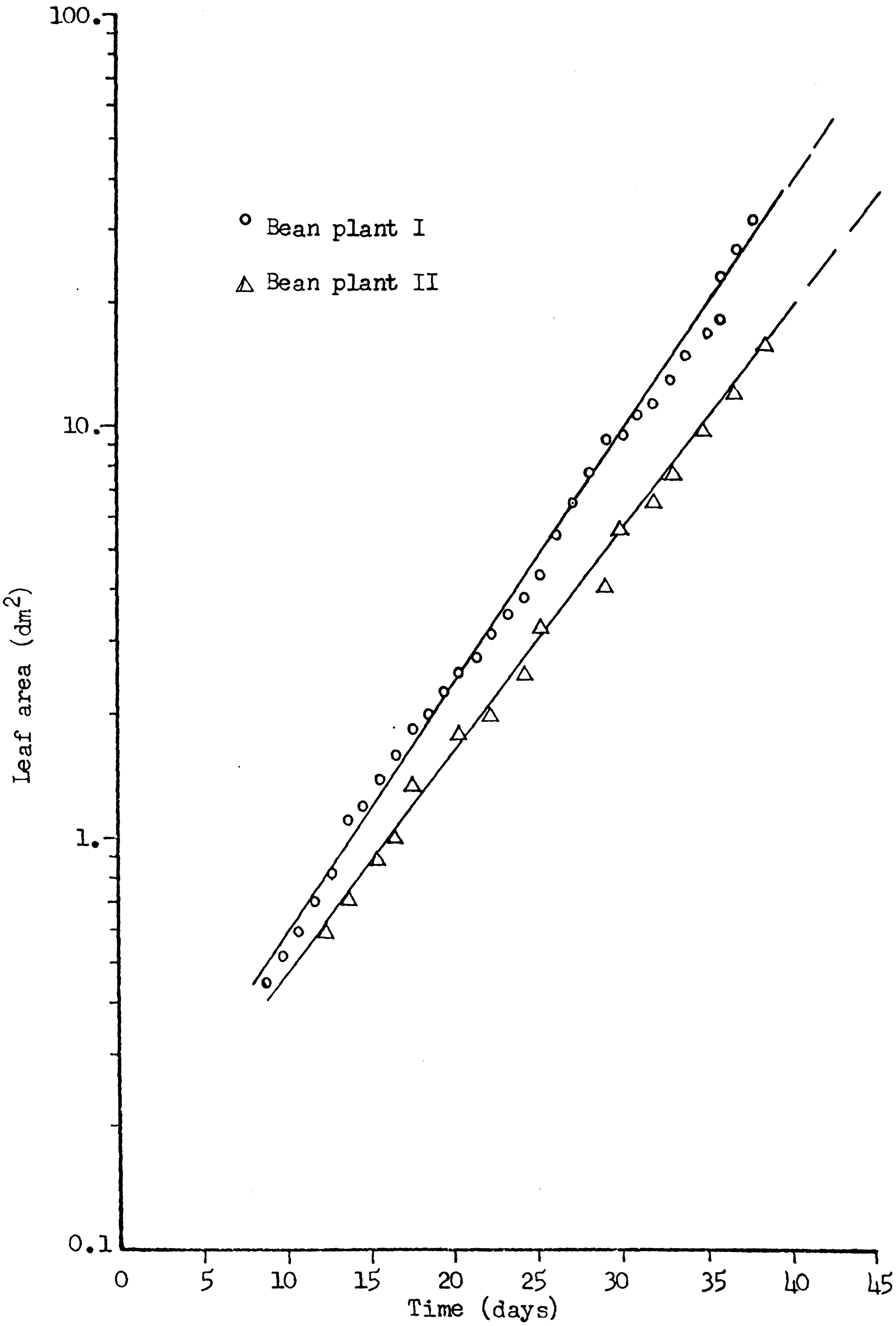

Figure 16.--The Relationship Between the Leaf Area of a Bean Plant and Growth Time.

7.2 The Weight Loss from the Soil-Plant System

The rate of transpiration from the plant canopy and the rate of evaporation from the soil surface could not be calculated from the measurement of temperature and the relative humidity of air entering and leaving the growth chamber due to high fluctuation in the measurement of relative humidity. The high fluctuation was due to the fact that the aminco unit cycles to maintain the temperature and relative humidity of the air. Even a small fluctuation is magnified in the estimation of the amount of moisture carried by such a high air flow rate. However, the rate of transpiration and evaporation were calculated from the weight loss of the soil plant system.

The daily weight loss from the soil-plant system for both replications were combined and plotted against time as shown in Figure 17. The resulting values were represented by an exponential model and the following equation was obtained with a correlation coefficient of 0.982

$$W(t) = 640.9\, e^{-.0219\, t} \qquad (7.2)$$

where t is the time in days. The total amount of water used by the soil-plant system can be estimated by integrating the equation (7.2). It was found that approximately 18.0 kilograms of water was used by the system in 45 days of growth as shown in Figure 17.

The amount of evaporation from the soil surface was estimated from the slope of the soil-plant system weight

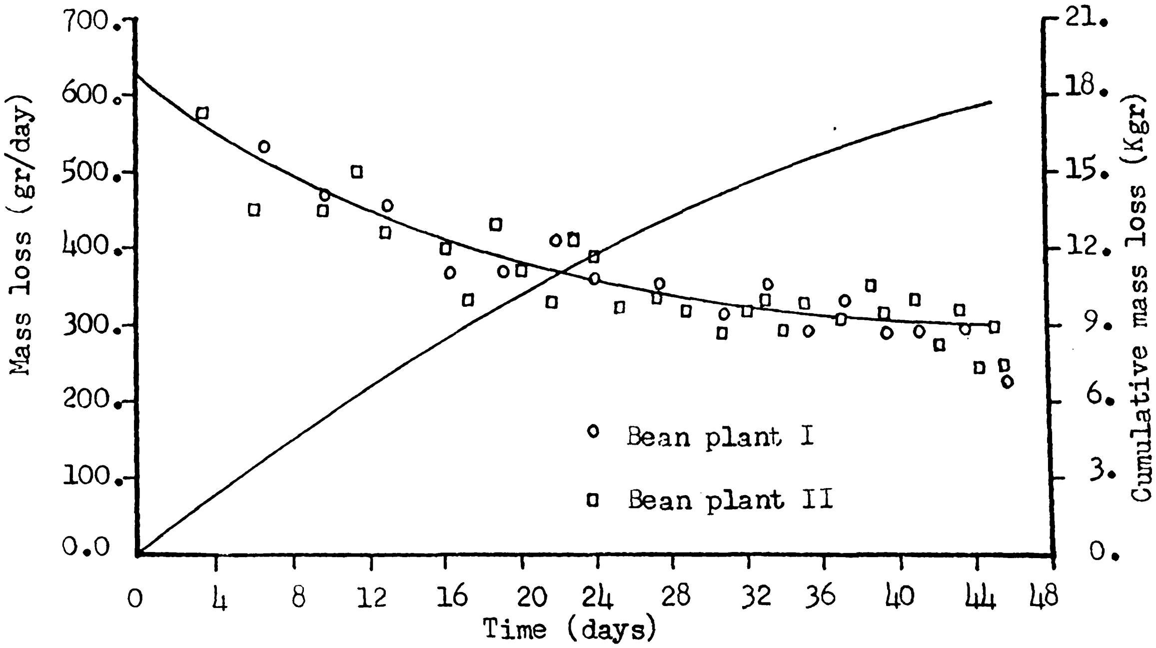

Figure 17.--The Rate of Weight Loss from the Soil-Plant System due to Evapotranspiration.

loss during the night time by assuming that the evaporative losses of the plant canopy were negligible. The resulting values from both replications are plotted against time and fitted to an exponential model as shown in Figure 18 yielding the following equation with a correlation coefficient of .998

$$E(t) = 640.9\ e^{-.0302\ t} \tag{7.3}$$

where t is time in days. The sum of the evaporative losses were calculated by using the equation (7.3) and found to be almost 15.0 kilograms in 45 days of growth as shown in Figure 18.

The rate of evaporation per unit area of soil was calculated by dividing the evaporative loss by the soil area. The resulting values are plotted against time as shown in Figure 19, and represented with the following equation:

$$e(t) = 0.3175\ e^{-.0302\ t} \tag{7.4}$$

where e(t) is the rate of evaporation in cm/day and t is time in days. It appears that the rate of evaporation decreases with time as proposed by Philip (1957). However, the percentage of drop in the rate of evaporation with time is less than that observed by Philip (1957). A possible explanation for the inconsistency is the presence of active roots in our soil.

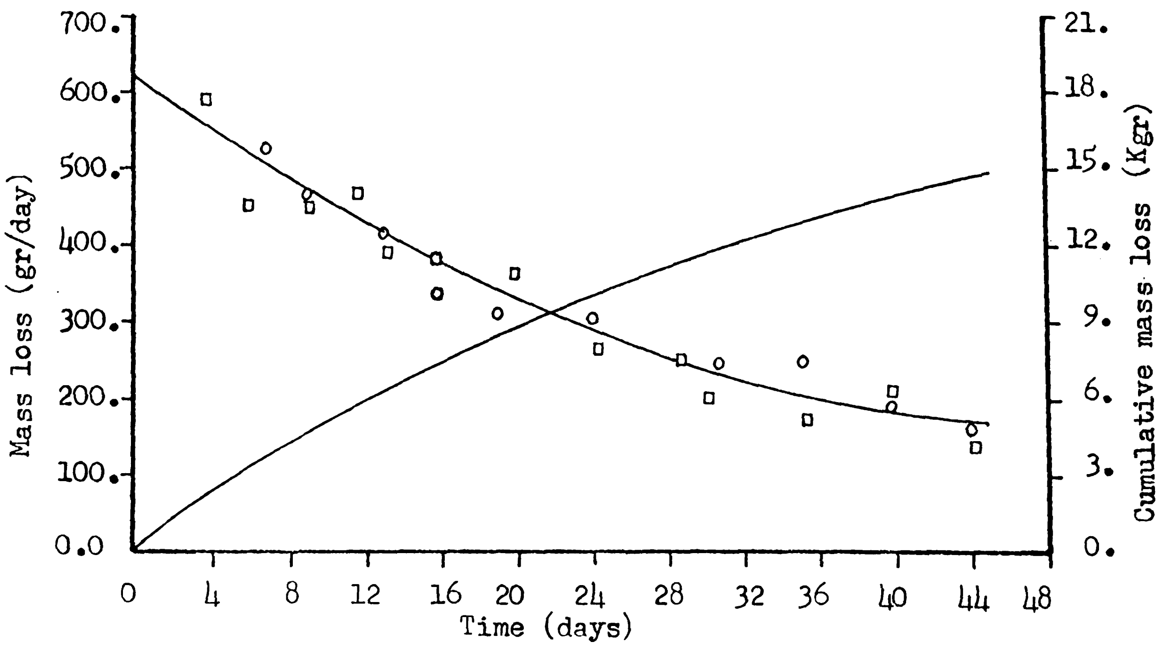

Figure 18.--The Rate of Weight Loss from the Soil-Plant System During the Nighttime due to Evaporation.

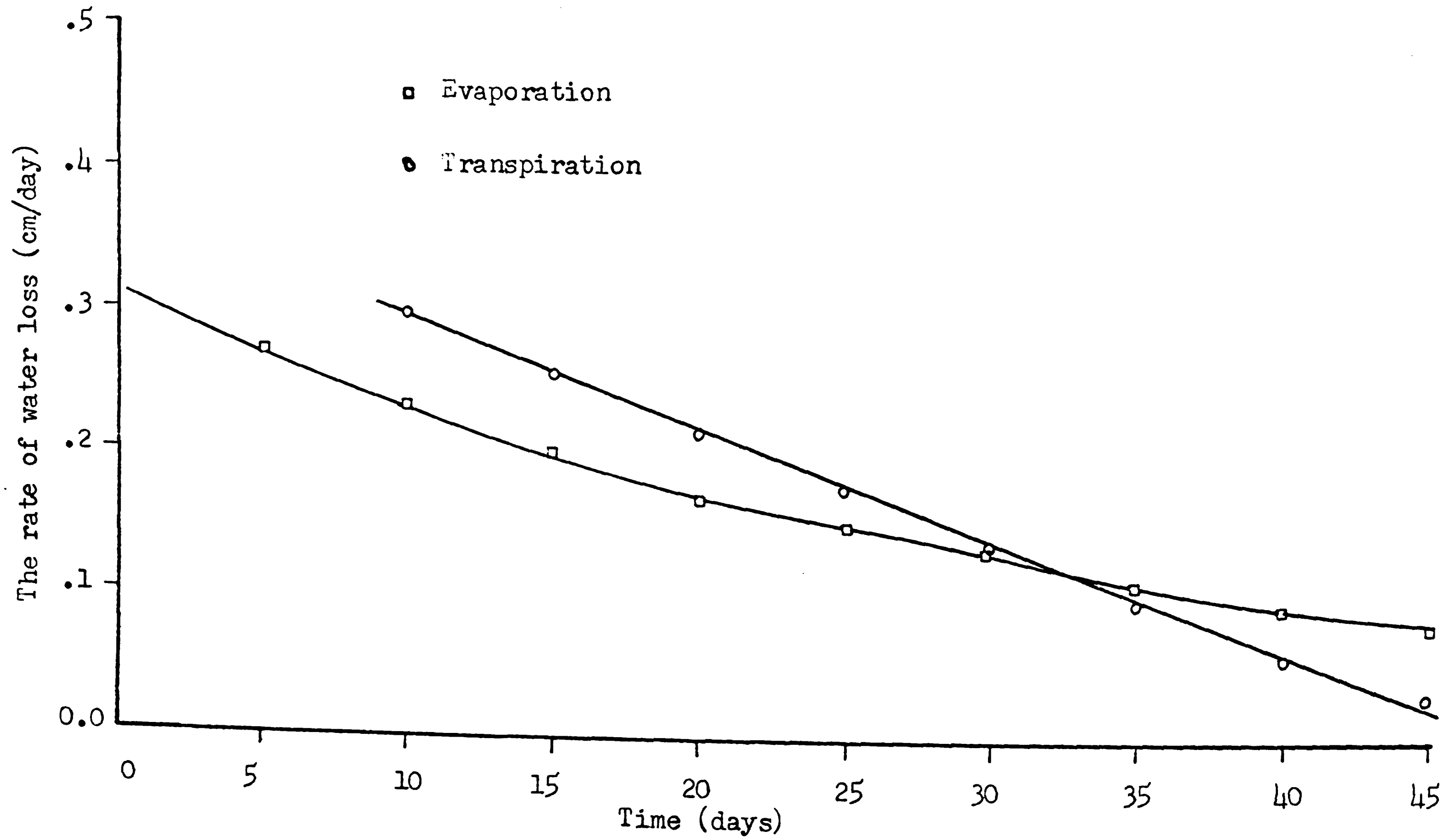

Figure 19.--The Rate of Evaporation and Transpiration from the Soil-Plant System over the Time of Growth.

The differences between the daily weight loss from the soil-plant system and the amount of evaporation was assumed to be equal to the transpiration losses from the plant canopy during the day time. Under this assumption the sum of the transpiration loss in 45 days of growth is equal to 3.0 kilograms of water. The transpiration rate in centimeters of water per unit area of leaves, is calculated by dividing the daily transpiration loss by the corresponding leaf area. The resulting data is plotted against time as shown in Figure 19 and represented by a linear model. The corresponding equation is

$$i(t) = 0.378 - 0.00782\ t \tag{7.5}$$

where i(t) is the rate of transpiration in cm/day and t is time in days. The reduction in the rate of transpiration with time may be explained by considering the ageing effects of the plant leaves and continuous drying of the soil in the root zone.

7.3 The Distribution of Soil Water Potential

The distribution of water potential was determined from the readings of Spanner type psychrometers buried in the root zone. The soil water potentials corresponding to the location of the psychrometers in the xz and yz plane are plotted for 15, 30 and 45 days of plant growth as shown in Figure 20 through 24. As expected, the soil water potential decreases from top to bottom of the soil barrel. The equipotential lines of the root zone are concave shaped

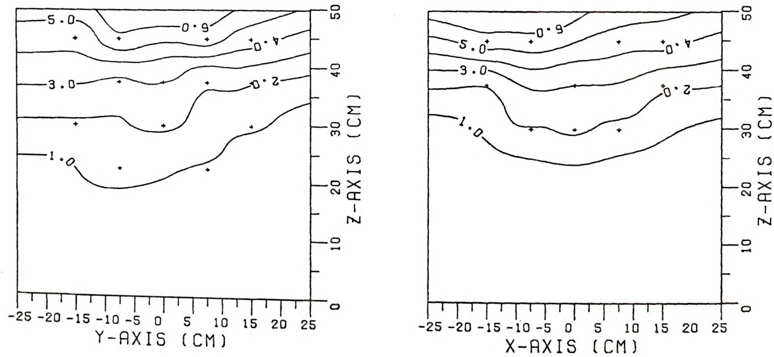

Figure 20.--The Average Soil Water Potential Distribution in the Root Zone for 5, 6, and 7 a.m. of a 15-day old Bean Plant.

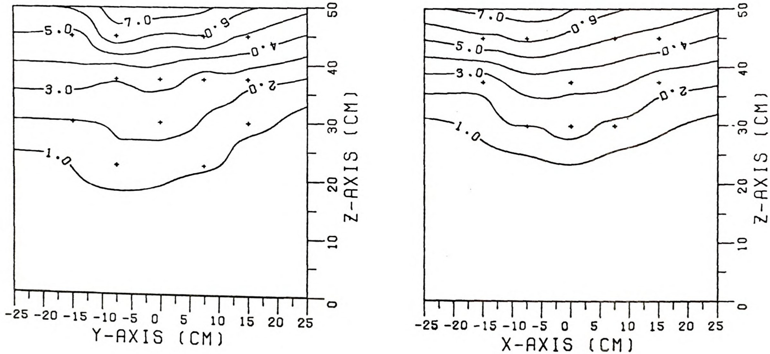

Figure 21.--The Average Soil Water Potential Distribution in the Root Zone for 2, 3, and 4 p.m. of a 15-day old Bean Plant.

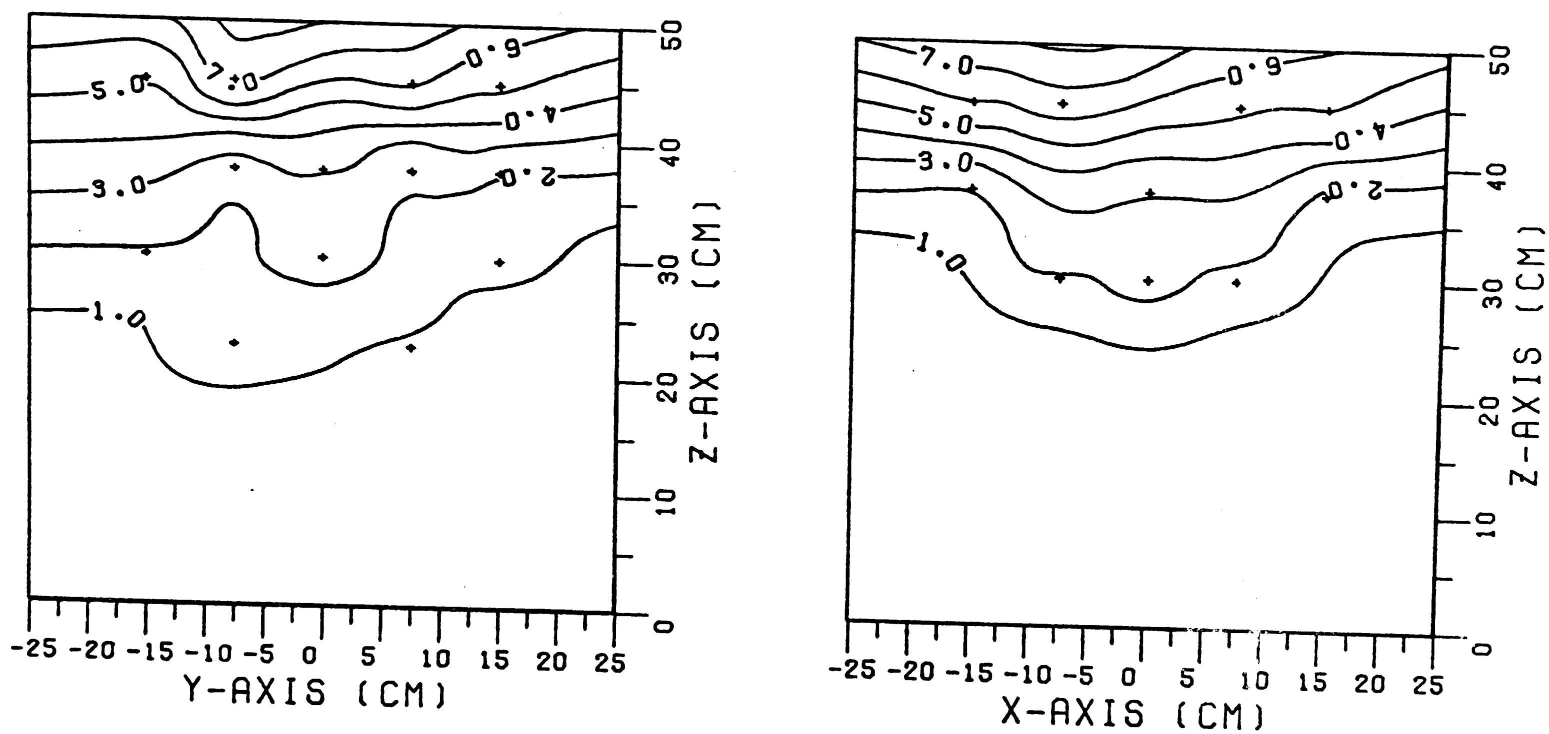

Figure 22.--The Average Soil Water Potential Distribution in the Root Zone for 5, 6, and 7 a.m. of a 16-day old Bean Plant.

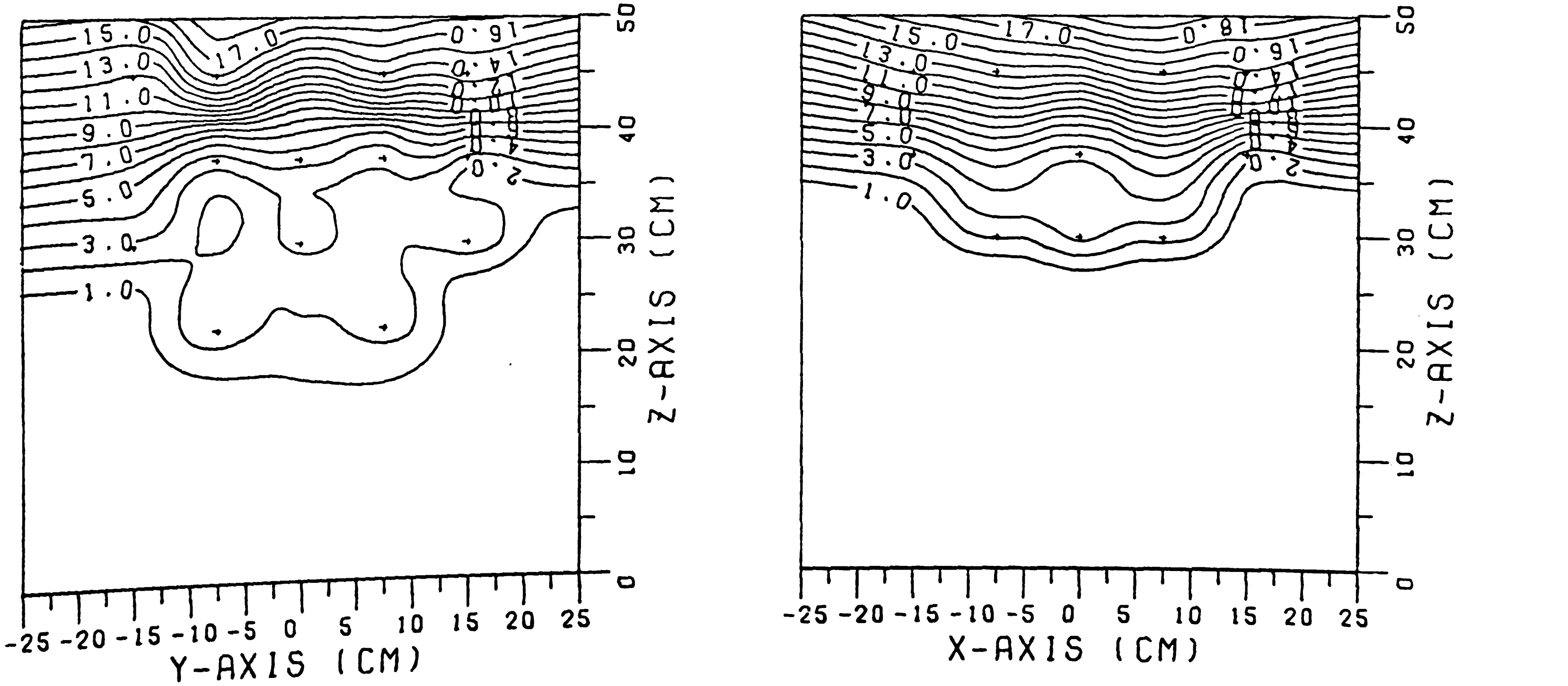

Figure 23.--The Average Soil Water Potential Distribution in the Root Zone for 2, 3, and 4 p.m. of a 30-day old Bean Plant.

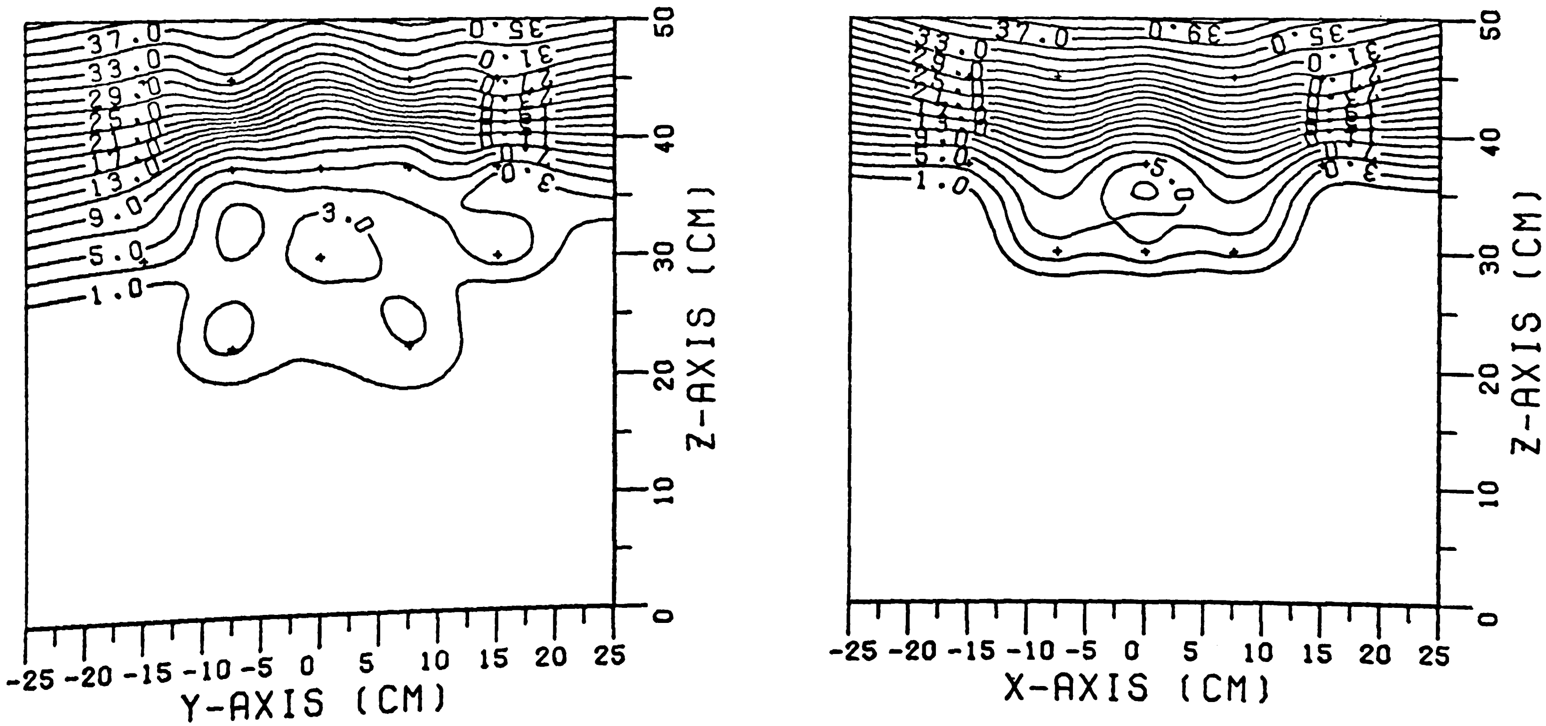

Figure 24.--The Average Soil Water Potential Distribution in the Root Zone for 1, 2, and 3 p.m. of a 45-day old Bean Plant.

with a slight symetry. The shape of equipotential lines indicates the direction of soil moisture movement upward and toward the center where the denser root system is assumed to be located. It also appears that the equipotential lines fluctuate in a diurnal fashion according to the amount of water removal as shown in Figure 20 through 22.

The simulation of water potential distribution for 15 days of growth fits the experimental results as shown in Figure 25. This shows that the development of the root system and the pattern of water absorption is consistent with our model, since the amount of water absorption and the soil water potential distribution matches with the experimental result.

The equipotential lines for 30 and 45 days shows that the surface of the soil is dried rapidly to form a soil crust, whereas the lower portion of soil was still quite uniformly damp with a water potential less than one bar as shown in Figure 23 and 24. The simulation of soil water potential for the same days does not agree with the experimental result as shown in Figures 26 and 27. This inconsistency might be explained if the Peltier type pschrometer does not respond to the potential changes when it has been buried in the soil more than 20 days. Note that the one bar equipotential line is almost stationary from 15 to 45 days, although the soil-plant system lost almost 10 kilograms of water during the same period.

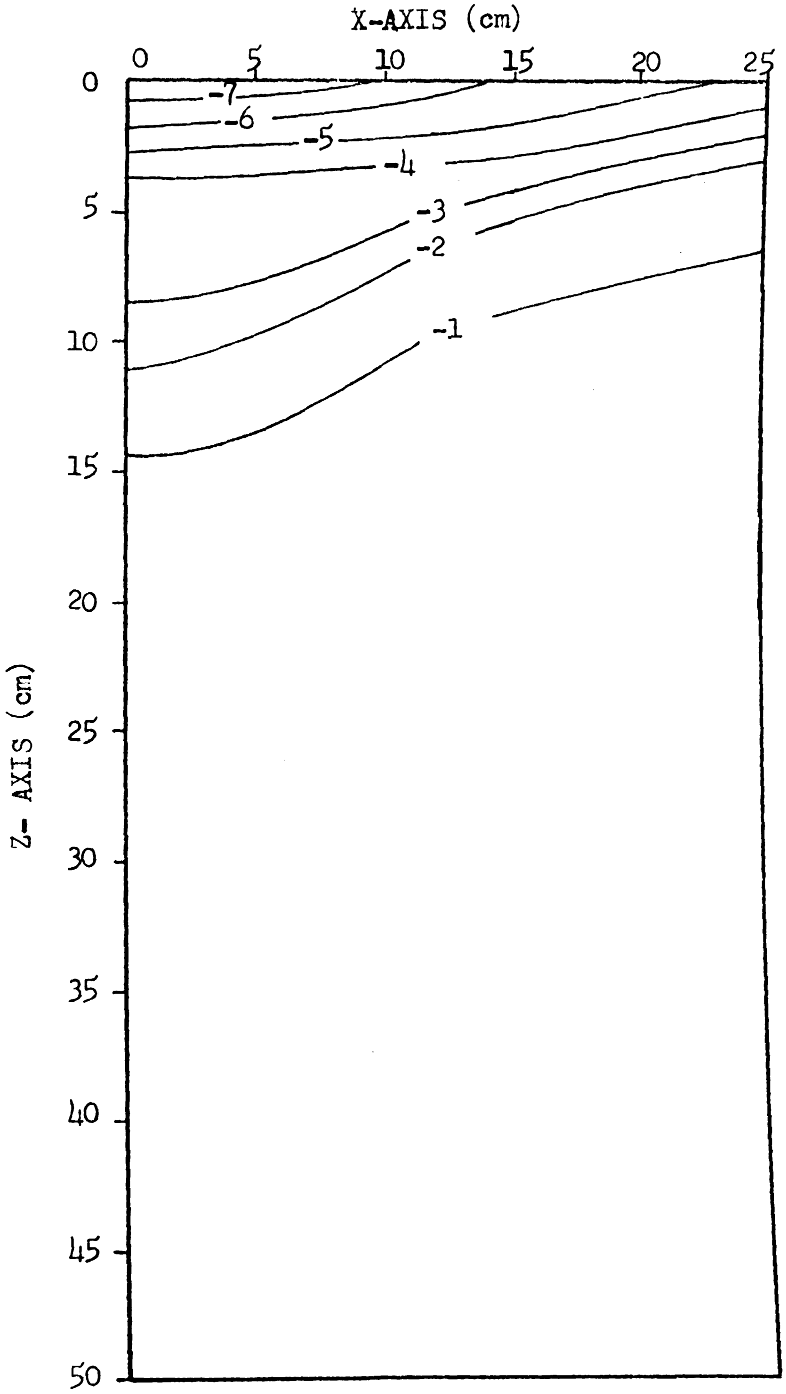

Figure 25.--The Simulated Soil Water Potential Distribution at 6 p.m. of a 15-day old Bean Plant.

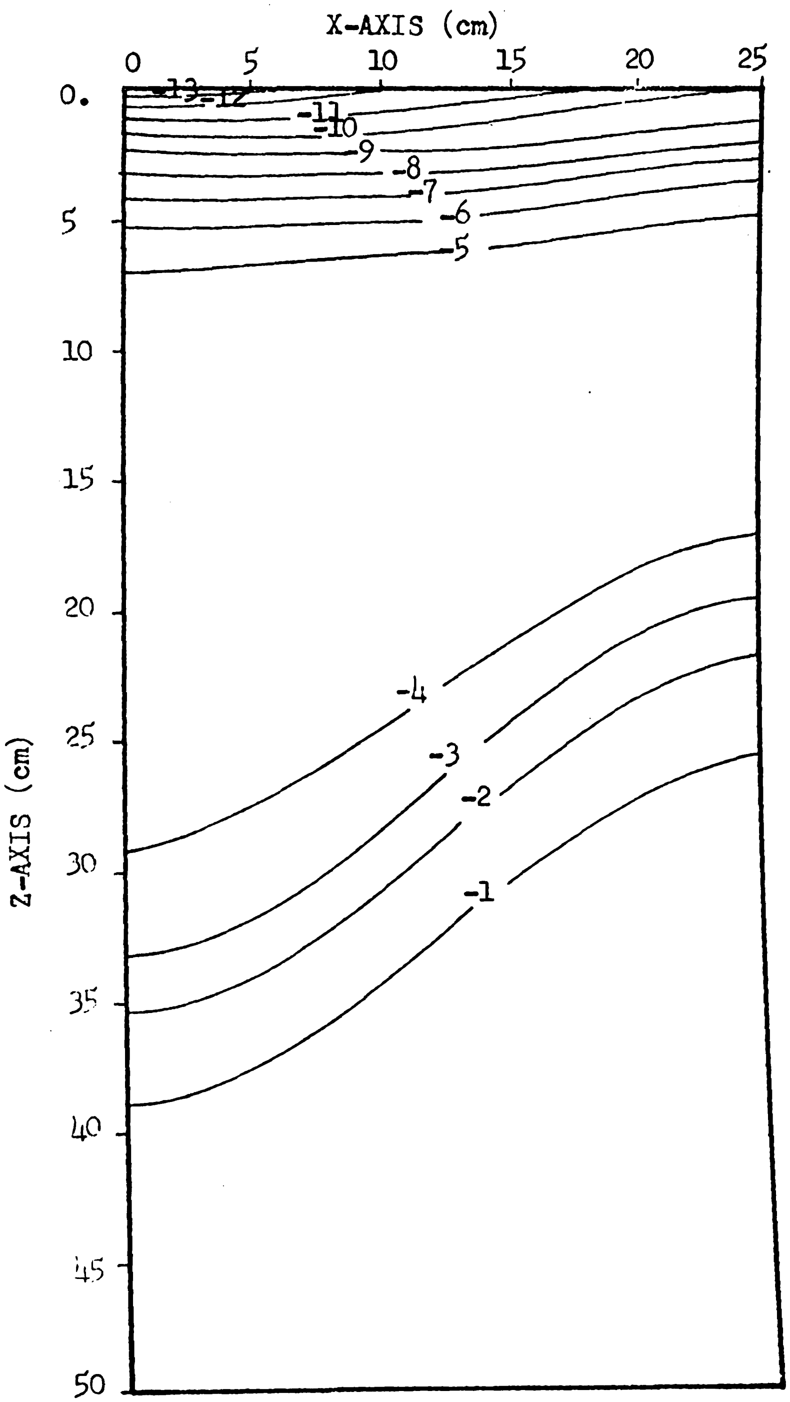

Figure 26.--The Simulated Soil Water Potential Distribution at 6 p.m. of a 30-day old Bean Plant.

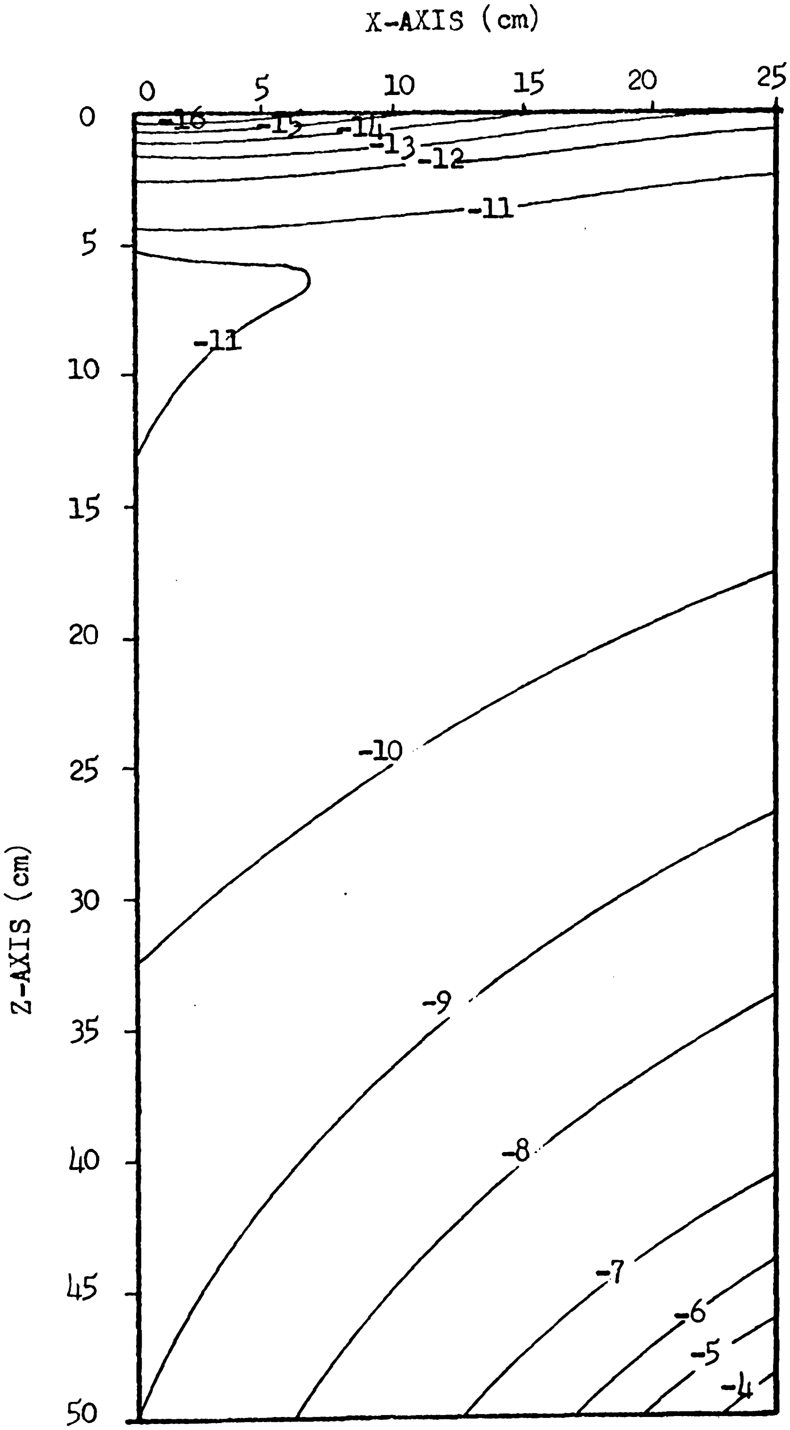

Figure 27.--The Simulated Soil Water Potential Distribution at 6 p.m. of a 45-day old Bean Plant.

Furthermore, half of the soil barrel should not be less than one bar if one makes a rough calculation based on the characteristic curve of Hillsdale sandy loam. The initial water content of 100 liter sandy loam would be 24 kilograms if one assumes that the soil water potential drops to an average value of -120.0 centimeters of water tension after 24 hours draining under 60 centimeters suction. It was observed that the soil-plant system lost 18 kilograms of water in 45 days of growth. The water remaining in the soil barrel should be less than or equal to 6 kilograms. However, investigation of the experimental soil water potential distribution at 45 days of growth shows that more than half of the soil barrel is less than -1.0 bar which corresponds to a water holding capacity of 12 kilograms. Therefore, there is a definite experimental error in the measurement of soil water potential distribution for 30 and 45 days of growth.

7.4 The Development and Distribution of Root System

At the end of each experiment, the soil in the root zone was washed away with a sprinkler water hose to observe the development and distribution of the root system held by the concentric hardware cloth. It was observed that six taproots extended from top to bottom of the soil barrel where their tips were full of tertiary branches indicating absorption of water from the bottom of the soil barrel. As Russell and Mitchel (1971) concluded, most of the small

branches hanging on the tap roots in the upper part of root zone may have decayed due to high water stress and these were washed away with the soil. It was interesting to note that most of the roots were brown almost up to their tips. This was interpreted as indicating suberization of the root system.

The rate of root extension was calibrated by fitting the simulated water uptake and soil water potential distribution with the experimental results for 15 days of growth. The rate of root extension was found to be increasing exponentially to provide the necessary amount of water for increasing transpiration loss. The resulting values of root extension are fitted with an exponential model and represented by

$$av(t) = 1.8\ e^{.01\ t} \tag{7.6}$$

where av(t) is the rate of root extension in cm/day and t is the time in days. The simulation of root extension can be compared to the observations of Russell and Mitchell (1971) during the first three weeks of growth as shown in Figure 28. After three weeks of growth the horizontal extension of the root system was limited due to the vertical boundary. During the first two weeks of growth, the horizontal extension was greater than the vertical extension due to the favorable condition of the soil in the upper layer. As the soil water potential of the root zone in the upper part of soil dropped, the horizontal extension

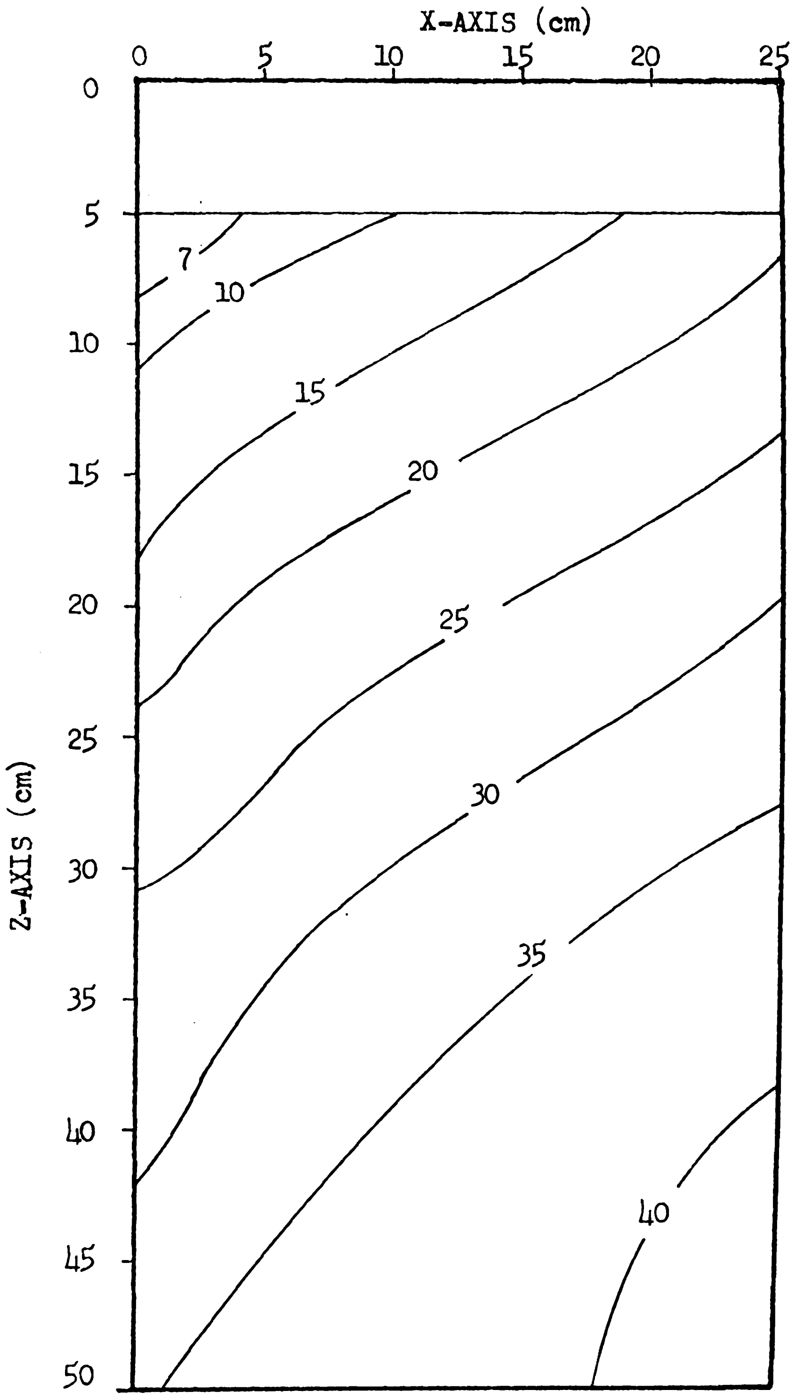

Figure 28.--The simulated Boundary of the Root System at Different days of Growth.

slowed down and the root boundary became more hyperbolical. The root system of the bean plant filled the soil barrel after about 43 days of growth.

The density of the root system decreased linearly with depth and horizontal distance at 15 days of growth as shown in Figure 29. The equal density lines at 30 and 45 days of growth show that the density of the root system decreased from the center to the outside as shown in Figure 30 and 31. The conversion of maximum root density values from the root surface into the unit length of root per unit volume of soil gave values of 0.6 cm/cm^3 at 15, 1.3 cm/cm^3 and 30 and 2.2 cm/cm^3 at 45 days of growth, which are consistent with the observation of Gardner (1964).

7.5 The Development of Suberization

The degree of suberization follows the pattern of soil water potential distribution as it was defined. Almost half of the root zone in terms of volume is suberized at 15 days of growth. The maximum degree of suberization occurs in the older part of the root zone and is equal to 1.5 bars as shown in Figure 30. The validity of the assumptions on the suberization process is analyzed for 15 days of growth, since the water potential and transpiration loss satisfy the experimental results. Assuming the same root density and soil water potential distribution exists, one can examine the rate of water uptake during the daytime for the same root system with and without suberization. If the dynamic equilibrium value of the root potential in the

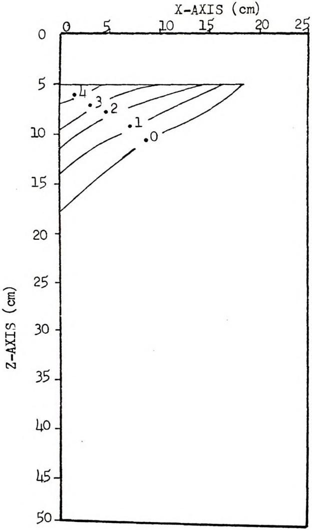

Figure 29.--The Simulated Distribution of Root Density of 6 p.m. of a 15-day old Bean Plant

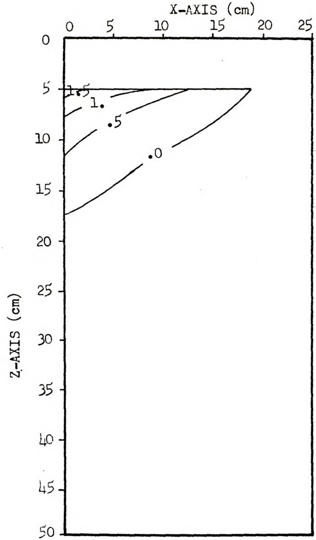

Figure 30.--The Simulated Degree of Superization at 6 p.m. of a 15-day old Bean Plant.

morning is taken as an initial root potential, the root system without suberization continues to pump water in the first half of the day from moist soil into the dry root zone rather than absorbing water to satisfy the transpiration demand of the plant as shown in Figure 36. If one increases the root potential from the initial value to the maximum soil water potential in the root zone with a step input, the rate of water uptake reaches a peak value in a few hours and decreases sharply, a condition which is contradictory to the observation of Weatherley (1963). According to the observation of Weatherley (1963), the transpiration loss from a plant canopy reaches a peak value sometime between 2 p.m. and 3 p.m.

If one changes the root water potential between these initial and maximum values to get maximum water uptake by the root system, it was observed that the total water uptake without suberization is almost half of the total water uptake with suberization, furthermore, the unsuberized root system requires a root potential twice the root potential of suberized root system. From this analysis, one might conclude that the suberization process of the root system is essential for the root system to absorb maximum water with minimum water potential.

The degree of suberization is equal to 1.75 bars and almost uniform in the center part of the root zone for 30 days of growth as shown in Figure 31. The upper part increases to 3.0 bars quickly as the lower part

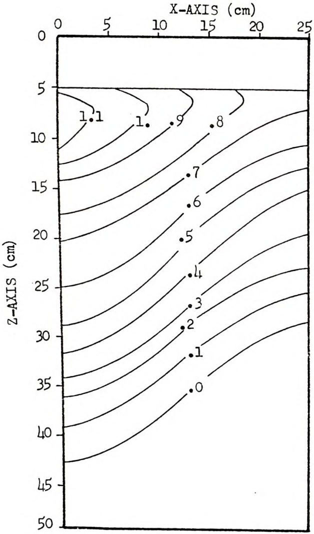

Figure 31.--The Simulated Distribution of Root Density at 6 p.m. of a 30-day old Bean Plant.

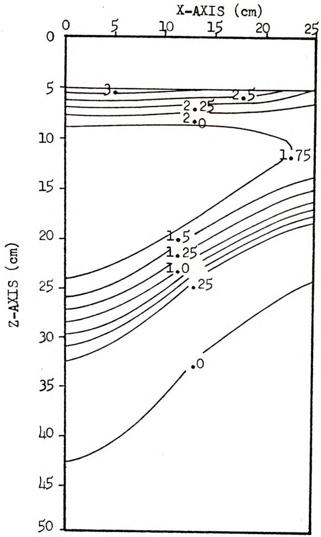

Figure 32.--The Simulated Degree of Superization at 6 p.m. of a 30-day old Bean Plant.

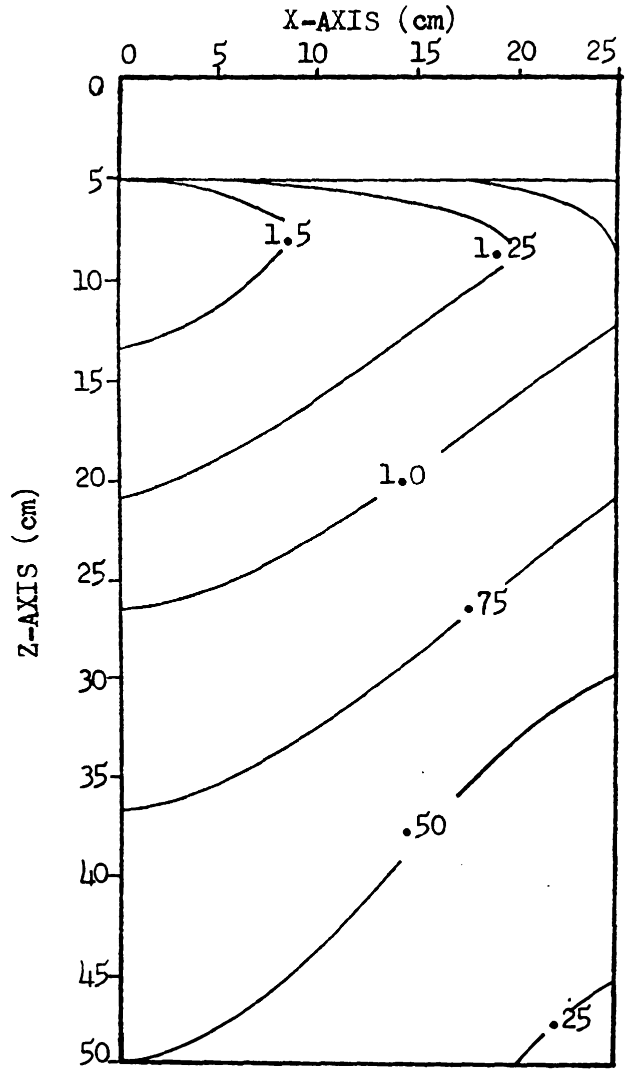

Figure 33.--The Simulated Distribution of Root Density at 6 p.m. of a 45-day old Bean Plant.

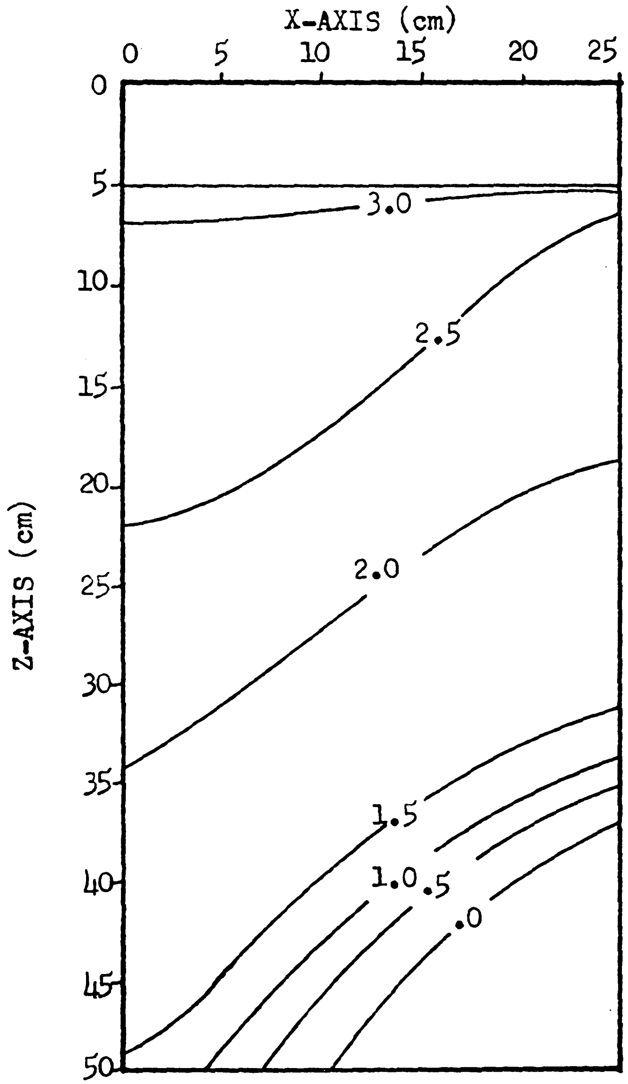

Figure 34.--The Simulated Degree of Superization at 6 p.m. of a 45-day old Bean Plant.

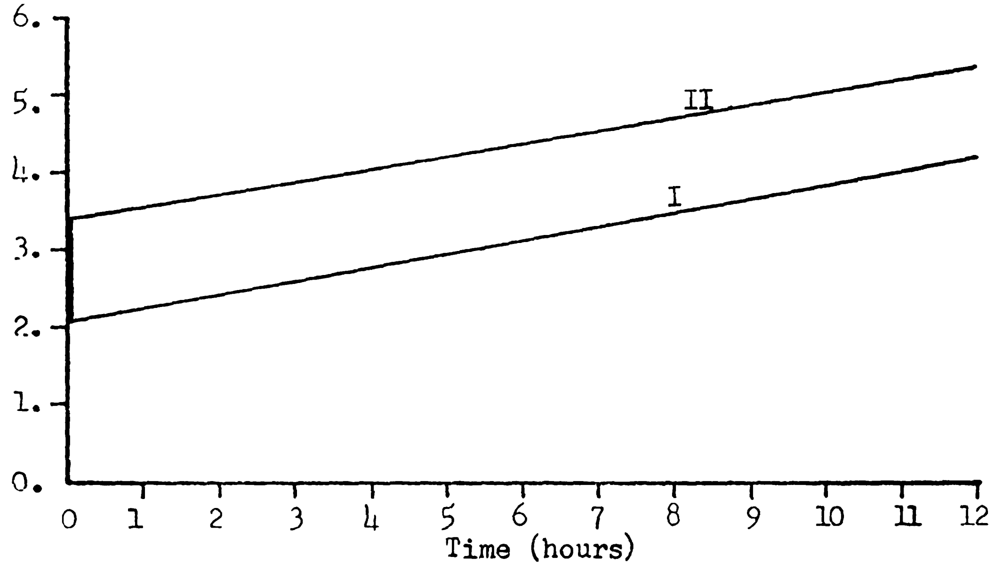

Figure 35.--The Development of Water Potential in the Root System of a 15-day old Bean Plant.

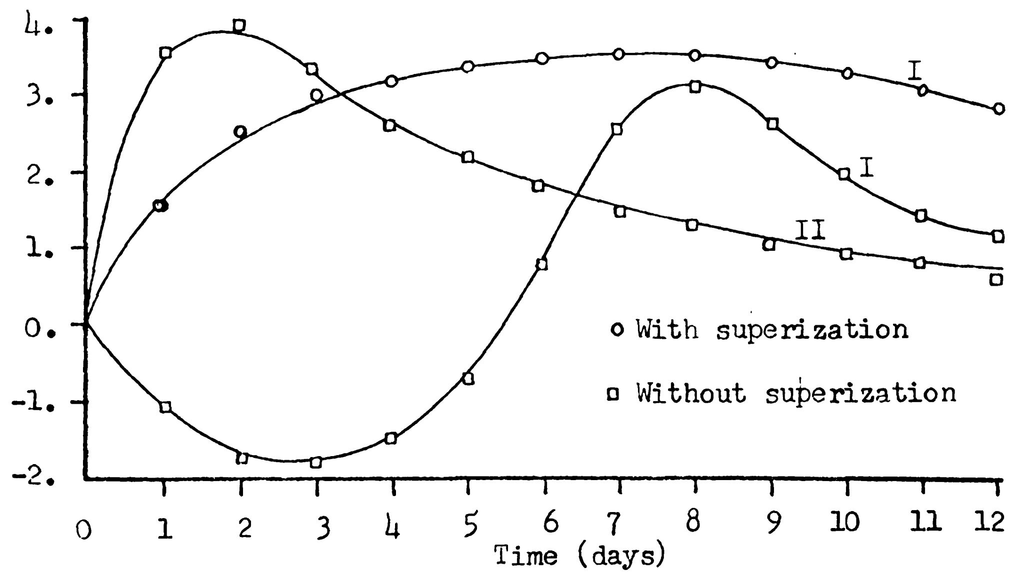

Figure 36.--The Rate of Water Absorption by the Root System of a 15-day Bean Plant With and Without Superization.

decreases to zero. In the 45 days of growth the degree of suberization decreases almost linearly from a maximum 3.5 bars at the top to zero at the right corner of the soil barrel.

The maximum degree of suberization from 30 to 45 days of growth is not significant, whereas the soil water potential decreases from -12.0 bars to -16.0 bars. This situation might be attributed to an increase in the root potential as shown in Figure 44.

7.6 The Distribution of Source Term

The distribution of the source term could not be determined from the solution of equation (4.16) for the experimentally determined soil water potential distributions. This is probably due to the fact that the amount of water absorption per unit volume is not large enough to detect, and the distribution of soil water potential is not sufficiently accurate because of the interpolation process between the five cm grid points. The simulations of the source term for 15, 30, and 45 days are mapped as shown in Figures 37 through 42. Maximum water absorption moves from the center to the tip of the root zone in a conical shell form. The magnitude of maximum source terms are 4.0, 3.0, and 1.1 mm^3 water/per unit volume of soil at 15, 30, and 45 days of growth respectively.

The rate of water uptake decreases from the maximum value in the center to zero around the root zone. The

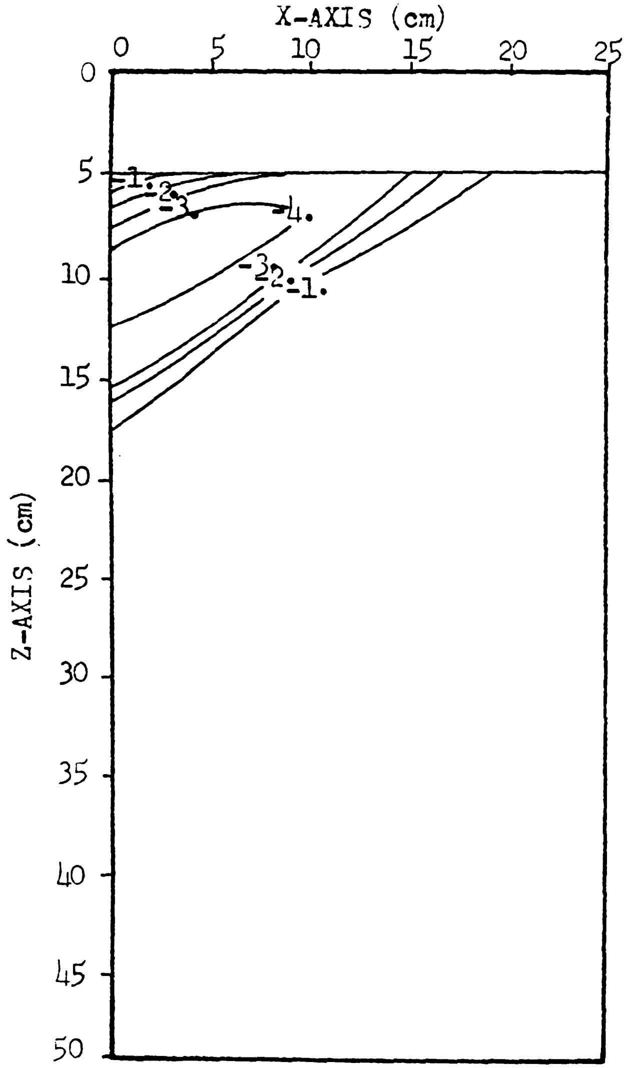

Figure 37.--The Simulated Water Uptake Pattern at 6 p.m. of a 15-day old Bean Plant.

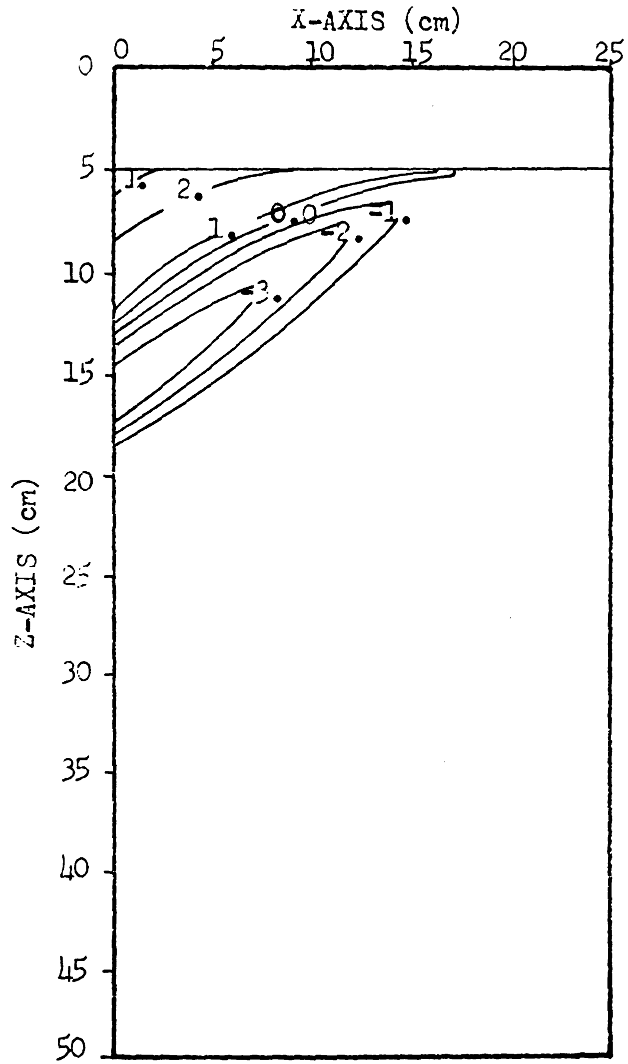

Figure 38.--The Simulated Water Absorption and Uptake Release at 6 a.m. of a 16-day old Bean Plant.

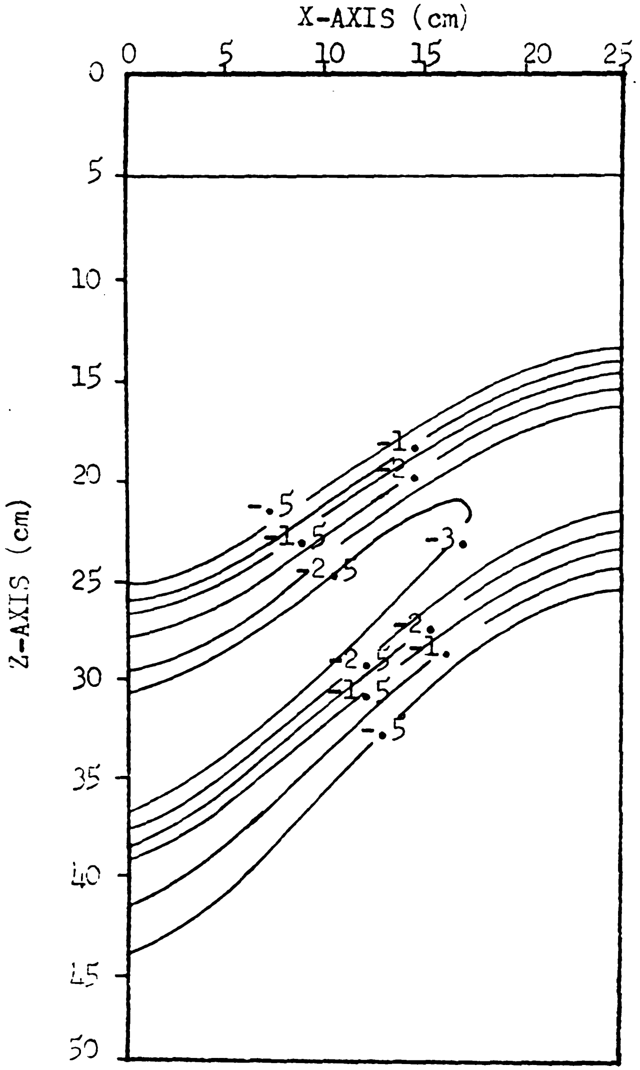

Figure 39.--The Simulated Water Uptake Pattern at 6 p.m. of a 30-day old Bean Plant.

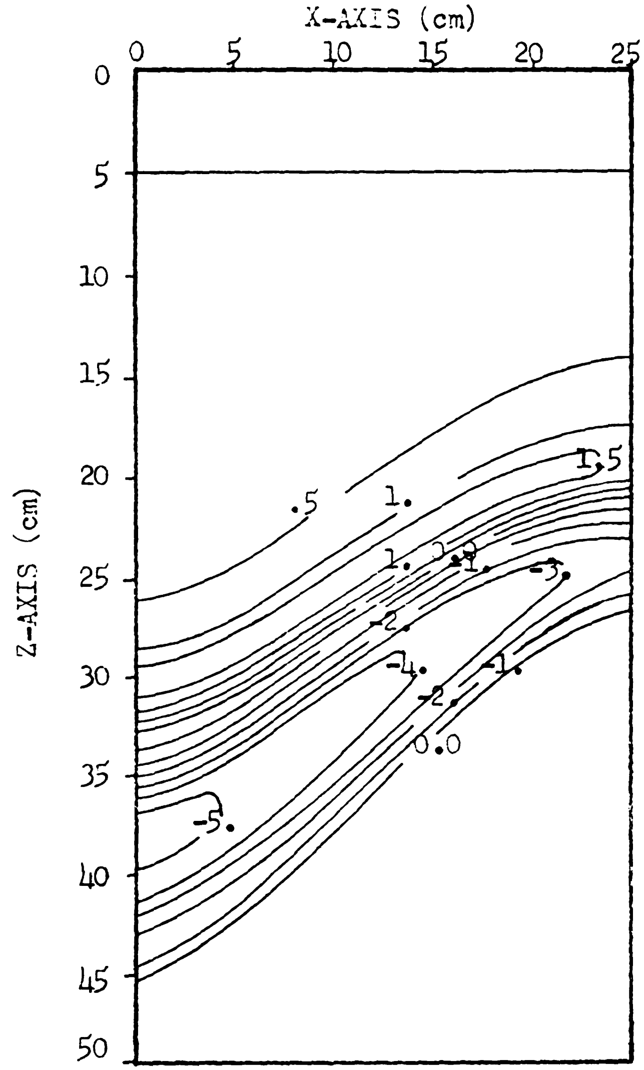

Figure 40.--The Simulated Water Absorbtion and Release at 6 a.m. of a 31-day old Bean Plant.

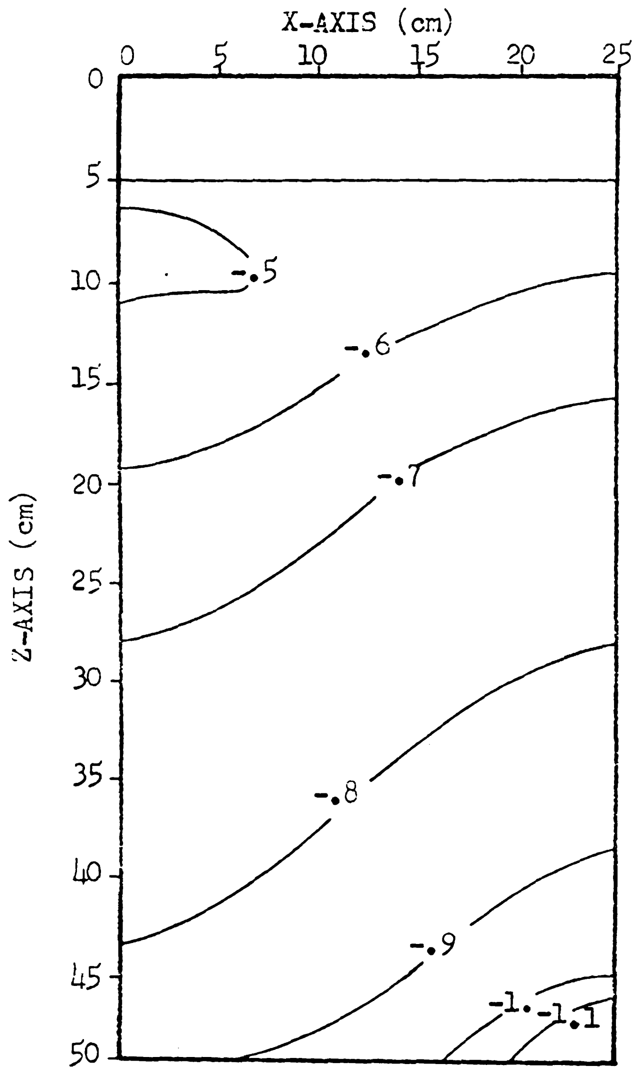

Figure 41.--The Simulated Water Uptake Pattern at 6 p.m. of a 45-day old Bean Plant.

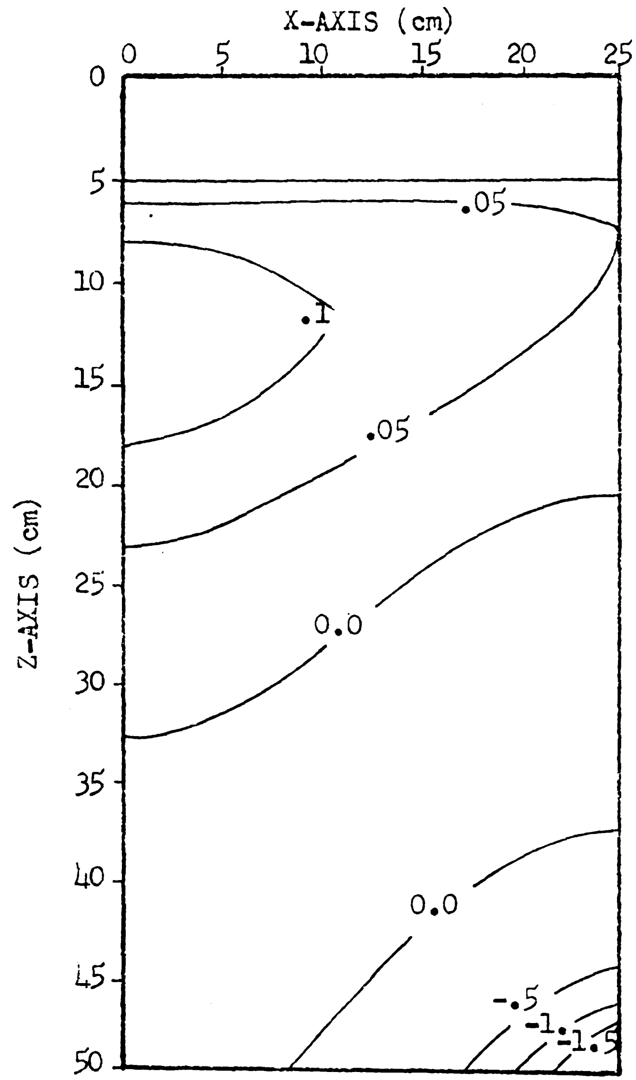

Figure 42.--The Simulated Water Absorbtion and Release at 6 a.m. of a 46-day old Bean Plant.

reduction of water uptake at the tip of the root system may be attributed to the smaller root density. However, the reduction of water at the upper part of the root system where the root density is higher may be attributed to the combined effects of lower soil water potential and a higher degree of suberization. Another interesting observation is that the maximum water uptake occurs in the zone of soil water potential from -1.5 to -4.0 bars. This is not a coincidence because the root growth model and the root density shows that optimum root growth is occurring when soil water potential is between -1.5 and -4.0 bars. Therefore, one can conclude that the rate of maximum water absorption takes place where the growth of root density is optimum rather than where the density of the root system is maximum. Usually, the maximum root density is at the older part of the root zone where most of the roots are suberized or decayed due to water stress as observed by Russell and Mitchell (1971).

During the night time, the water potential of the root system was relaxed until the amount of water absorption was equal to or less than 20 per cent of the previous day's water absorption for the recovery of the plant under stress. The tip of the root zone absorbs water as the upper part of the root zone was releasing part of the absorbed water into the surrounding dry soil. If one superimposes the distribution of soil water potential and the degree of suberization, it can be seen why the water absorption and release process in 30 days of growth was limited to the tip of the root zone, as shown in Figure 40. The absorption of water at 45 days of growth was limited to the right lower corner of the soil barrel where the unsuberized root system has a low density and low soil water potential. However, the release of water from the roots into the surrounding dry soil is taking place at the upper part of root zone rather than immediately above the zone of water absorption as in the 15 and 30 days of growth. A possible explanation for a zone of zero source term between water absorption and release would be the nullification of root water potential by the combined effects of the degree of suberization and the soil water potential.

The sum of daily water uptake by the whole root system was plotted against the growth time as shown in Figure 43. The comparison of simulated water uptake with experimental transpiration shows that there is a reasonable correlation between the theory and mechanics of water absorption. The only discrepancy between the observed and simulated transpiration occurs for a few days after 32 days of growth. A possible explanation for this discrepancy is the limitation of the boundary condition on the volume and density of the root system. In nature, root tips would continue to grow at the surface of the boundary and increase the density of the root system. In the proposed model, the density of the root system was based on a continuous density function rather than on the number of root tips. Consequently, the generation of the density function takes a few days to reach equilibrium with the development of soil water potential at the boundary.

The rate of water uptake decreases after 40 days of growth as the experimental transpiration decreased. This situation may be explained when one considers the distribution of soil water potential and the development of root water potential. As the soil dries, the amount of water uptake drops with the decreasing unsaturated hydraulic conductivity, and the root water potential increases to absorb optimum water in dried soil as shown in Figure 44. According to Figure 44, the root potential dropped to -15.0 bars during the day and relaxed to -8.0 bars during the night at 45 days of growth. This portrays the development of the wilting point in the plant as the soil water is depleted.

In order to study the effect of transpiration demand and the development of root potential upon the wilting of a plant, we shall assume that wilting occurs when the transpiration exceeds the amount of water absorption. According to the proposed model, the daily fluctuations of root water potential, which are shown in Figure 44, are the maximum root water potential for optimum water absorption. If the transpiration demand increases the leaf and root potential have to increase correspondingly in order to maintain the rate of transpiration. However, any increase in root potential does not increase the amount of water uptake.

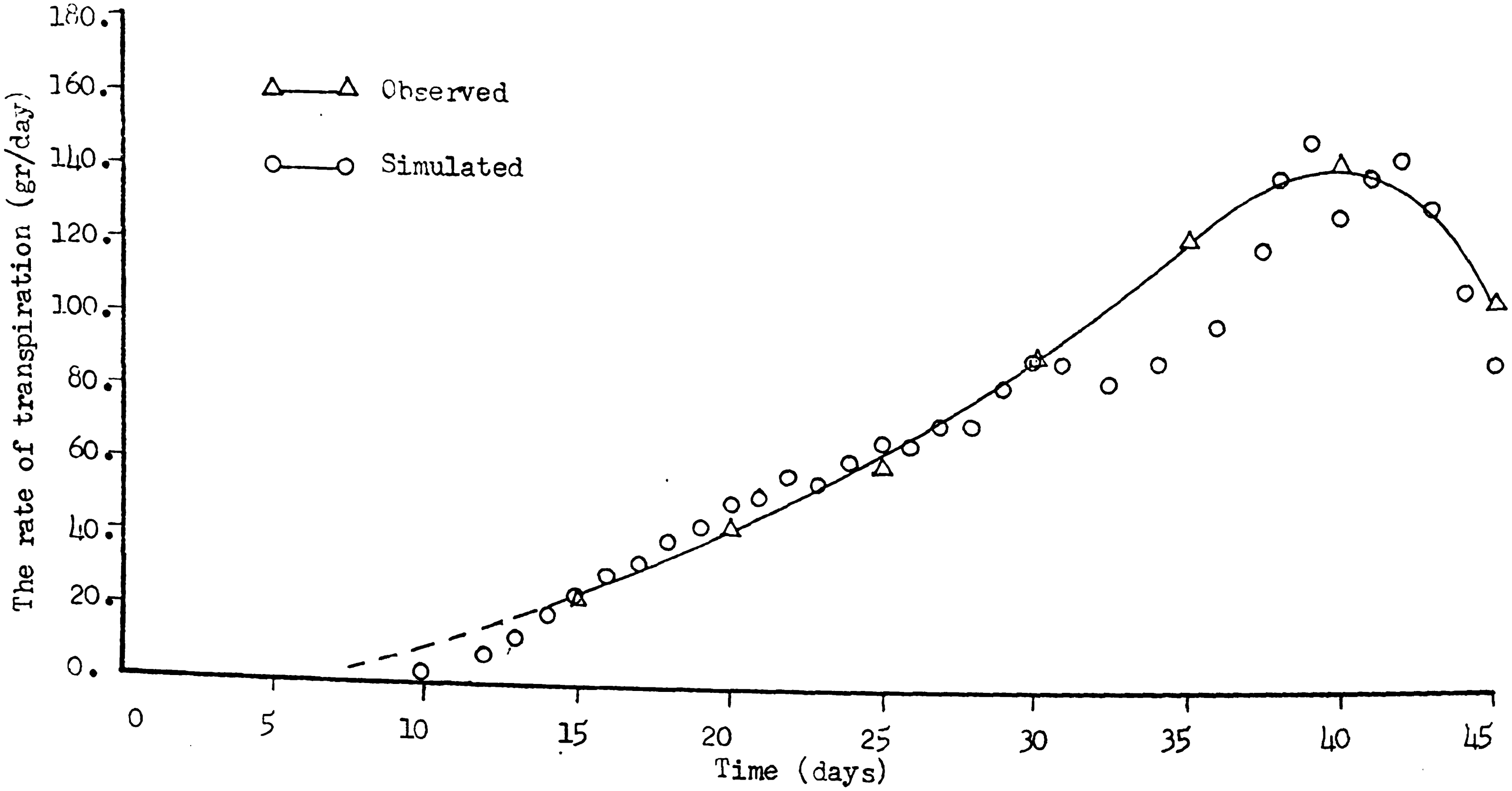

Figure 43.--The Rate of Observed and Simulated Transpiration from a Bean Plant.

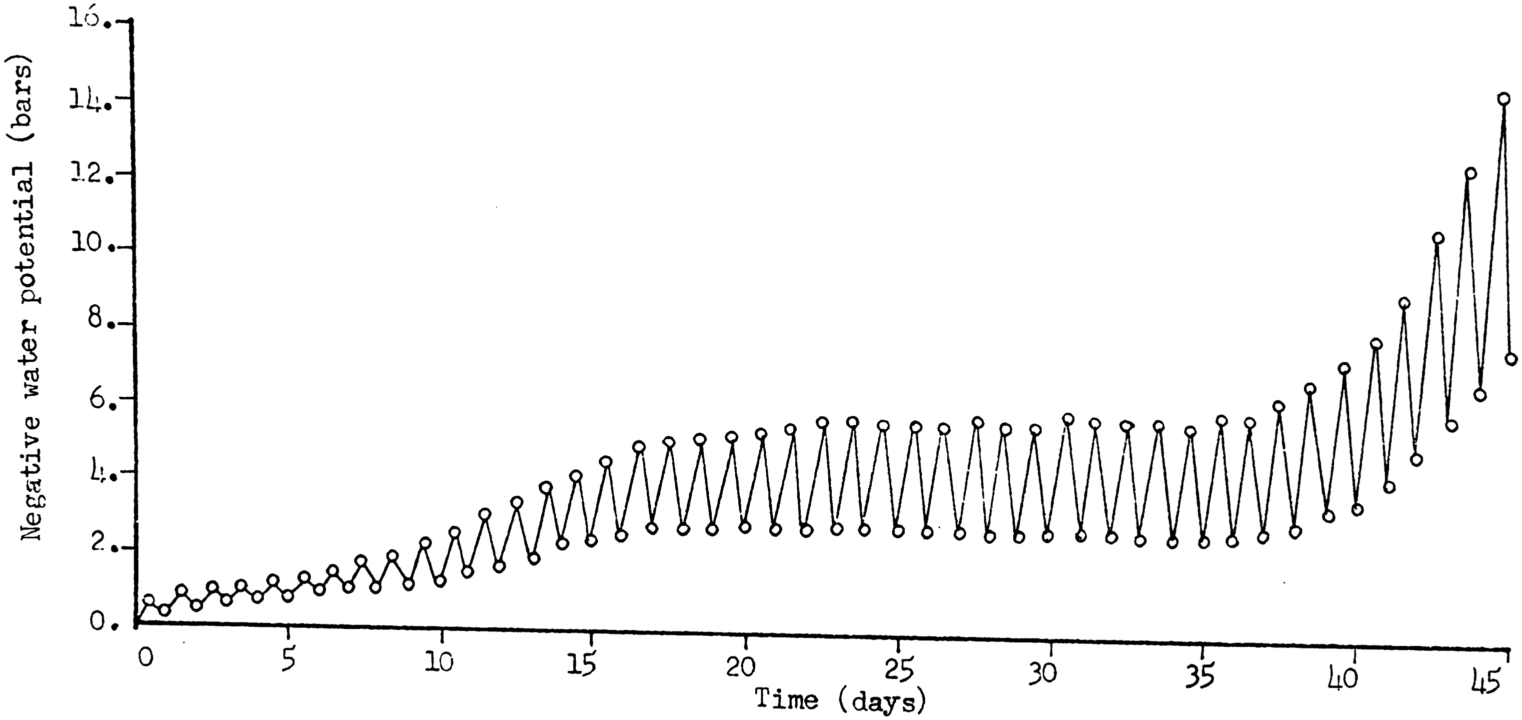

Figure 44.--The Development of the Root Water Potential at the Root Surface of a Bean Plant.

Therefore, whenever the transpiration demand exceeds the amount of water absorption, plant leaves have to control the rate of transpiration by closing their stomataes. An inspection of Figure 44 shows that the root water potential does not have to drop to -15.0 bars to wilt the plant at the early stage of growth. This observation shows the sensitivity of a plant against water stress at the early stage of growth and the effect of soil water statue on the rate of transpiration.

VIII. CONCLUSIONS

A deterministic non-stationary mathematical model was developed to investigate the movement of water through the developing root system of a bean plant in a Hillsdale sandy loam. The development and density of the root system, the rate and pattern of water absorption, and the development of root and soil water potential in the root zone were simulated on the computer. From the analysis of the mathematical model and experimental observation, the following conclusions are obtained:

1. The rate of root extension as a function of soil water potential is consistent with the observation of Russell and Mitchell (1971).

2. The geometry of root system can be represented by the vertical and horizontal extension of root system.

3. The density of the root system based on the definition of Gardner (1960) with a new optimum overlapping coefficient is reasonable and at least in the same order as Gardner's (1964) observation.

4. There exists a relation between the initial soil and root water potentials to obtain optimum water absorption with a pattern which fits the pattern of

experimental transpiration during the day time from the analysis of Gardner and Lang's (1970) suggestions on limitations to water uptake by plant roots. This relation is defined as the ratio of difference between maximum root and initial soil water potential to initial soil water potential. The maximum rate of water absorption with a pattern of actual transpiration is obtained when the ratio is between 1.5 at low soil tension and 2.0 at higher negative soil water potential.

5. The rate of maximum water absorption takes place where the increase of root density is optimum rather than where the density of root system is maximum due to suberization of older roots. The location of maximum absorption coincides with the development of optimum ratio between soil and root potential. The absorption of water is almost limited to the lower part of the root zone as a conical shell which is moving downward as growth progressed.

6. The development of suberization shows that the transfer of water through the root system from moist soil into dry soil could not bring back the root zone in an equilibrium condition as far as the soil water potential distribution is concerned. It appears that the suberization of the root surface is necessary to optimize the absorption of water as well as protect the roots from unfavorable environments.

7. The amount of water absorption by the root system drops with the depletion of soil water in the root zone. The root water potential fluctuations reach to a steady-state value during the growth of root zone into a soil with low water potential. As the soil dries, the root water potential drops to the wilting point during the day in order to absorb more water for transpiration.

8. The experimental rate of evaporation from soil surface decreases exponentially with time. However, the rate of reduction is less than that observed by Philip (1957). A possible explanation for this result is that the rate of evaporation is higher from a soil surface with an active root system.

9. The experimental rate of transpiration from plant leaves decreases linearly over the time of growth. It is probably due to the combined effects of the aging of plant leaves and the depletion of soil water.

10. The simulated soil water potential distribution is consistent with experimental soil water distribution in the root zone for the first three weeks of growth. Then, the simulated soil water potential distribution deviates from the observed potential distribution.

Recommendation for Future Work

1. The investigation of the root development and density should be restricted to a single plant under different soil with variable moisture and temperature conditions.

2. The formation and degree of suberization should be determined experimentally.

3. The leaf water potential should be measured with the soil water potential to check the development of root water potential and determine the resistance of plant to movement of water.

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APPENDIX I

SOLUTION OF THE GOVERNING EQUATION FOR THE ABSORPTION AND TRANSFER OF WATER IN THE SOIL ROOT-DOMAIN BY THE IMPLICIT ALTERNATING DIRECTION METHOD

SOLUTION OF THE GOVERNING EQUATION FOR THE ABSORPTION AND TRANSFER OF WATER IN THE SOIL ROOT-DOMAIN BY THE IMPLICIT ALTERATING DIRECTION METHOD

The purpose of this analysis is to present a numerical solution of the dynamic aspects of water flow in the soil-root domain. The root systems are considered to be growing at a specified rate and act as a sink or source, depending upon the development of water potential gradient between soil and root. The equation describing this process with the initial and boundary conditions is defined in Chapter IV, and can be written as:

$$\frac{\partial \Phi}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(D(\Phi)\, r\, \frac{\partial \Phi}{\partial r}\right) + \frac{\partial}{\partial z}\left(D(\Phi)\frac{\partial \Phi}{\partial z}\right) + \frac{\partial}{\partial z}K(\Phi) + \frac{S(r,z,t)}{C(\Phi)} \qquad (1.a)$$

$$\Phi = \Phi_o \quad \text{at} \quad t = 0 \quad \text{for} \quad \begin{array}{l} 0 \le z \le L \\ 0 \le r \le X \end{array} \qquad (1.b)$$

$$e(t) = -K(\Phi)\frac{\partial \Phi}{\partial t} + K(\Phi) \quad \text{at } z = 0 \text{ for} \quad \begin{array}{l} t > 0 \\ 0 \le r \le X \end{array} \qquad (1.c)$$

$$\left.\frac{\partial \Phi}{\partial z}\right|_{z = L} = 0 \quad \text{for} \quad \begin{matrix} t > 0 \\ 0 \leq r \leq x \end{matrix} \tag{1.d}$$

$$\left.\frac{\partial \Phi}{\partial r}\right|_{r = X} = 0 \quad \text{for} \quad \begin{matrix} t > 0 \\ 0 \leq z \leq L \end{matrix} \tag{1.e}$$

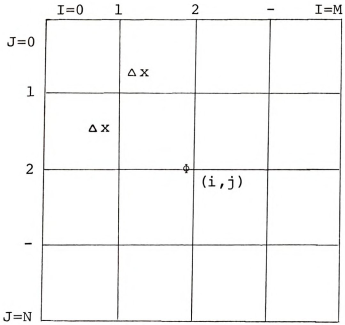

Figure 45.--The Space Grid System.

The initial and boundary conditions define the limits on the space and time variables. One can place a square mesh over the region with spacing Δx, as shown in Figure 45. If I and J are the numbers of internal mesh points in the r and z direction, then

$$X = M \cdot \Delta x$$
$$L = N \cdot \Delta x \qquad (2)$$

where, $M = I + 1$ and $N = J + 1$ are the number of intervals in the r and z directions. The equation (1.a) is a nonlinear parabolic type of differential equation. Carnahan et al. (1969) have pointed out that either explicit or implicit finite-difference methods can be used in solving parabolic type differential equations. Use of the explicit method is limited by computational restrictions which must be imposed to insure stability and convergence of the computations. These difficulties can usually be eliminated by using the implicit alternating direction method. In this method, the principle is to employ two different equations which are used in turn over successive time steps each of duration $\Delta t/2$.

The first equation is implicit in the r-direction and the second is implicit only in the z-direction. The first equation is solved for the intermediate values of Φ $(i.j, n + \frac{1}{2})$ which are then used in the second equation. Thus, leading to the solution $\Phi(i,j,n + 1)$ at the end of the whole time interval Δt. The representations of equation (1) by the implicit alternating methods would be written as:

$$\frac{\Phi(i,j,n+\frac{1}{2}) - \Phi(i,j,n)}{\Delta t/_2} = \lambda_r^2 \, \Phi(i.j,n+\tfrac{1}{2}) + \lambda_z^2 \Phi(i,j,n) \quad (3.a)$$

$$\frac{\Phi(i,j,n+1) - \Phi(i,j,n+\frac{1}{2})}{\Delta t/_2} = \lambda_r^2 \, \Phi(i,j,n+\tfrac{1}{2}) + \lambda_z^2 \, \Phi(i,j,n+1) \quad (3.b)$$

Equation (3.a) can be written in full as:

$$\begin{aligned}\frac{\Phi(i,j,n+\frac{1}{2}) - \Phi(i,j,n)}{\Delta t/_2} = \frac{1}{r(i)} \Bigg[& r(i+\tfrac{1}{2})D(i+\tfrac{1}{2},j)\left[\frac{\Phi(i+1,j,n+\frac{1}{2}) - \Phi(i,j,n+\frac{1}{2})}{\Delta r^2}\right] \\ & - r(i-\tfrac{1}{2})D(i-\tfrac{1}{2},j)\left[\frac{\Phi(i,j,n+\frac{1}{2}) - \Phi(i-1,j,n+\frac{1}{2})}{\Delta r^2}\right] \\ & + D(i,j+\tfrac{1}{2})\left[\frac{\Phi(i,j+1,n) - \Phi(i,j,n)}{\Delta z^2}\right] \\ & - D(i,j-\tfrac{1}{2})\left[\frac{\Phi(i,j,n) - \Phi(i,j-1,n)}{\Delta z^2}\right] \\ & + \left[\frac{K(i,j+1) - K(i,j-1)}{\Delta z} + \frac{S(i,j)}{C(i,j)}\right.\end{aligned} \quad (4)$$

If one multiplies through by $\Delta t/2$ and assumes $\Delta x = \Delta r = \Delta z$ for simplicity, equation (4) becomes:

$$A(i)\ \Phi(i-1,j)^* + B(i)\ \Phi(i,j)^* + C(i)\ \Phi(i+1,j)^* = R(i) \quad (5)$$

where

$$A(i) = -\frac{\Delta t}{\Delta x^2} \frac{r(i-\frac{1}{2})\ D(i-\frac{1}{2},j)}{2}$$

$$B(i) = r(i) + \frac{\Delta t}{\Delta x^2} \frac{r(i+\frac{1}{2})\ D(i+\frac{1}{2},j)}{2} + \frac{\Delta t}{\Delta x^2} \frac{r(i-\frac{1}{2})\ D(i-\frac{1}{2},j)}{2}$$

$$C(i) = -\frac{\Delta t}{\Delta x^2} \frac{r(i+\frac{1}{2})\ D(i+\frac{1}{2},j)}{2}$$

$$R(i) = \Phi(i,j+1,n)\ \frac{\Delta t}{\Delta x^2}\ r(i)\ D(i,j-\tfrac{1}{2})/_2$$

$$+\ \Phi(i,j,n) \left[r(i) - \frac{\Delta t}{\Delta x^2} \frac{r(i)\ D(i,j+\frac{1}{2})}{2} - \frac{\Delta t}{\Delta x^2} \frac{r(i)\ D(i,j-\frac{1}{2})}{2} \right]$$

$$+\ \Phi(i,j+1,n) \left[\frac{\Delta t}{\Delta x^2}\ (r(i)\ D(i,j+1)/_2) \right]$$

$$+ \left[K(i,j+1) - K(i,j-1) \right] /_{\Delta x} + S(i,j)/C(i,j)$$

Since the coefficients A, B, C and R depend on the previous time, n, equation (4) represents a set of linear algebraic equations. This set of equations is solved to obtain the values of $\phi(i,j)$ at the $n+\frac{1}{2}$ time. For convenience, the $(n+\frac{1}{2})$ time index has been replaced by an asteric on the unknowns and the knowns have been lumped together in R.

In the same way, equation (3.b) which is implicit in z-direction becomes:

$$\frac{\Phi(i,j,n+1) - \Phi(i,j,n+\tfrac{1}{2})}{\Delta t/2} = \frac{1}{r(i)} \left[r(i+\tfrac{1}{2})\ D(i+\tfrac{1}{2},j)\right]\left[\frac{\Phi(i+1,j,n+\tfrac{1}{2}) - \Phi(i,j,n+\tfrac{1}{2})}{\Delta r^2}\right]$$

$$- r(i-\tfrac{1}{2})\ D(i-\tfrac{1}{2},j) \left[\frac{\Phi(i,j,n+\tfrac{1}{2}) - \Phi(i-1,j,n+1)}{\Delta r^2}\right]$$

$$+ D(i,j+\tfrac{1}{2}) \left[\frac{\Phi(i,j+1,n+1) - \Phi(i,j,n+1)}{\Delta z^2}\right]$$

$$- D(i,j-\tfrac{1}{2}) \left[\frac{\Phi(i,j,n+1) - \Phi(i,j-1,n+1)}{\Delta z^2}\right]$$

$$+ \frac{K(i,j+1) - K(i,j-1)}{\Delta z} + \frac{S(i,j)}{C(i,j)}$$

Multiplying through by $\Delta/2$ and rearranging equation (6), one can obtain:

$$A^1(j)\ \Phi(i,j-1)^* + B^1(j)\ \Phi(i,j)^* + C^1(j)\ \Phi(i,j+1) = R^1(j) \qquad (7)$$

where

$$A^1(j) = - \frac{\Delta t}{\Delta x^2} \left[\frac{r(i)\ D(i,j-\tfrac{1}{2})}{2}\right]$$

$$B^1(j) = r(i) + \frac{\Delta t}{\Delta x^2} \left[\frac{r(i)\ D(i,j+\tfrac{1}{2})}{2}\right] + \frac{\Delta t}{\Delta x^2} \left[\frac{r(i)\ D(i,j-\tfrac{1}{2})}{2}\right]$$

$$C^1(j) = - \frac{\Delta t}{\Delta x^2} \left[\frac{r(i)\ D(i,j+\tfrac{1}{2})}{2}\right]$$

$$R^1(\ ) = \Phi(i,j,n+\tfrac{1}{2}) \quad \frac{\Delta t}{\Delta x^2} \quad \frac{r(i-\frac{1}{2})\ D(i-\frac{1}{2},j)}{2}$$

$$+ \phi(i,j,n+\tfrac{1}{2}) \left[r(i) - \frac{\Delta t}{\Delta x^2} \left[\frac{r(i+\frac{1}{2})\ D(i+\frac{1}{2},j)}{2} + \frac{r(i-\frac{1}{2})\ D(i-\frac{1}{2},j)}{2} \right] \right]$$

$$+ \frac{K(i,j+1) - K(i,j-1)}{\Delta x} + \frac{S(i,j)}{C(i,j)}$$

As in the previous case, equation (7) represents a set of linear equations, since the coefficients A^1, B^1, C^1, and the terms R^1 depend on the water potential distribution at the previous time, $n+\frac{1}{2}$. The solution of this set of equations gives values of $\Phi(i,j)$ at the n+1 time.

Equations (5) and (7) can be applied to each point of the mesh system in conjunction with the effective boundary and initial condition. The initial value of water potential is constant throughout the system, viz.:

$$\Phi(i,j) = \Phi_o \quad \text{at } t = 0 \quad \text{for} \quad \begin{matrix} i = 0,1,2\ .\ .\ .\ M \\ j = 0,1,2\ .\ .\ .\ N \end{matrix} \tag{8}$$

The water potential of the soil surface for each time increment can be calculated from the first boundary condition due to evaporative loss as follows:

$$\Phi(i,0) = \Phi(i,j) - \frac{\Delta x\ e(t)}{K(i,\frac{1}{2})} + 1 \tag{9}$$

for $i = 0,1,2\ .\ .\ .\ M$ and $t > 0$. Now one can apply equation (5) to each grid point, $i = 0,1,2\ .\ .\ .\ M$ in the J^{th} row in conjunction with the effective boundary conditions. The

first equation in the system of equations may be written as:

$$B(0)\ \Phi(0,J)^* + C(0)\ \Phi(1,J)^* = D(0) \tag{10}$$

where the mesh system is centered in such a way that $\Phi(0,J)$ is the left boundary of the system. The value of $\Phi(-1,J)$ can be represented by $\Phi(1,J)$ due to symetric projection, and D(0) can be expressed as:

$$D(0) = R(0) - A(0)\ \Phi(1,J) \tag{11}$$

Similarly, if the mesh is centered so that $\phi(M,J)$ is the right boundary, then the last equation would be:

$$A(M-1)\ \Phi(M-2,J)^* + B(M-1)\ \Phi(M-1,J)^* = D(M-1) \tag{12}$$

where $D(M-1) = R(M-1) - C(M-1)\ \Phi(M,J)$ due to the boundary condition defined by equation (1.c). Now, one can complete the set of equations for $i = 0,1,2\ .\ .\ .M$ and obtain:

$$
\begin{array}{lllllll}
B(0)\ \Phi(0,J)^* & + C(0)\ \Phi(1,J)^* & = D(0) & = R(0) - A(0)\ \Phi(1,J) & & & \\
A(1)\ \Phi(0,J)^* & + B(1)\ \Phi(1,J)^* & + C(1)\ \Phi(2,J)^* & = D(1) = R(1) & & & \\
A(2)\ \Phi(1,J)^* & + B(2)\ \Phi(2,J)^* & + C(2)\ \Phi(3,J)^* & = D(2) = R(2) & & & \\
A(3)\ \Phi(2,J)^* & + B(3)\ \Phi(3,J)^* & + C(3)\ \Phi(4,J)^* & = D(3) = R(3) & & & \\
\vdots & \vdots & \vdots & \vdots & & & \\
A(M-2)\ \Phi(M-3,J)^* & + B(M-2)\ \Phi(M-2,J)^* & + C(M-2)\ \Phi(M-1,J)^* & = D(M-2) = R(M-2) & & & \\
A(M-1)\ \Phi(M-2,J)^* & + B(M-1)\ \Phi(M-1,J)^* & = D(M-1) & = R(M-1) - C(M-1)\ \Phi(M,J) & & &
\end{array}
$$

As one can see, the elements of this matrix are zero everywhere except on the main diagonal and on two diagonals parallel and adjacent to it on either side. The system of equation can be solved for $\Phi(i,j)$, where $i = 0,1,2. . . M-1$, by Gaussian elimination techniques. The procedure is repeated for successive columns, $j = 1,2,3. . . N-1$, until all the $\Phi(i,j)$ are calculated at the end of the first half-time step.

The water potential distribution at the end of second half-time step are calculated similarily by applying equation (7) to each grid point, of mesh system in conjunction with boundary condition at $z = 0$ and $z = L$.

APPENDIX II

COMPUTER PROGRAM FOR THE SIMULATION OF WATER TRANSFER THROUGH THE SOIL-ROOT DOMAIN

```
      PROGRAM SOIL(INPUT,OUTPUT,TAPE60=INPUT,TAPE61=OUTPUT)
C        THIS PROGRAM SIMULATE THE EXTENSION AND DENSITY OF
C        ROOT SYSTEM,THE DYNAMIC ASPECT OF WATER ABSORBTION
C        AND TRANSFER IN THE SOIL-ROOT DOMAIN WHEN THE SOIL
C        CONDUCTIVITY AND DIFFISUVITY FUNCTIONS ARE KNOWN WITH
C        EVAPORATION AND TRANSPIRATION LOSS.
      DIMENSION TR(60),PO(6),QTR(6),DR(26,51),SHOR(60)
      DIMENSION SUM(15),SUW(50),SUR(50)
      COMMON/F/ P(26,60),S(26,60),DEN(26,60),RES(26,60),V(26),FV(60)
      COMMON/W/ R1(26),R2(26),N,NPT,M,MPT,T(26,60)
      COMMON/ARR/HEND(60),PT(25),LV
      INTEGER HEND
      REAL KQ
      KQ(H)=86400./((H/(-1.57))**3+1000.)
      DQ(H)=236758.9/((-H)**1.5)
C        READ AND CHECK INPUT VARIABLES
      READ(60,111) PINT,RR,ZZ,DTAU,DX,TIME
  111 FORMAT(6F10.2)
      WRITE(61,121)
      WRITE(61,122)
      STR=0.  $  SEV=0.  $TLOS=0.
      DO 57 I=1,45
      GR=14.82*EXP(.1248*I)
      TE=.378-.00782*I
      TR(I)=GR*TE
      STR=STR+TR(I)
      EV(I)=.31752*EXP(-.0302*I)
      TEV=EV(I)*1962.5
   57 WRITE(61,125) I,GR,TE,TR(I),EV(I),TEV,STR
      N=ZZ/DX
      NPT=N+1
      M=RR/DX
      MPT=M+1
C        INITIALIZATION OF THE DIMENSIONED PARAMETERS AND CONSTANS.
      DO 2 J=1,NPT
      DO 3 I=1,MPT
```

```
      S(I,J)=0.
      DEN(I,J)=0.
      RES(I,J)=0.
      DR(I,J)=0.
      T(I,J)=PINT
    3 P(I,J)=PINT
    2 PINT=PINT+10.
      DO 10 I=1,N
      SHOR(I)=0.
   10 HEND(I)=1
      SHOH=0.  $  SVER=0.  $ POT=0.  $  DROOT=0.  $  PROOT=0.$  AGE=0H
      AK=0.  $  DIFP=0.
      SUM(1)=0.
C         CALCULATION OF R1 AND R2 RATIOS WITH CORRESPONDING
C         THE VOLUME OF SOIL RING
      R1(1)=R2(1)=0.
      V(1)=0.
      DO 4 I=2,MPT
      V(I)=3.146*((I-1)**2)*(DX**3)-3.146*((I-2)**2)*(DX**3)
      R1(I)=(I-3./2.)/(I-1)
    4 R2(I)=(I-.5)/(I-1)
      UV=DX**3
      IDAY=1
C         CALCULATION OF ROOT ZONE BOUNDARY.THE HORIZANTAL EXTENSION
C         IS CALCULATED FOR EACH DEPTH INCREMENT, AFTER VERTICAL ROOT
C         EXTENSION IS ESTIMATED.
      LV=SVER/DX+3
      POT=P(1,3)
    5 TAU=0.
      AGE=1.8*EXP(.01*IDAY)
    6 TAU=TAU+DTAU*24.
      IF(LV .EQ. N) GO TO 49
      LD=LV+1
      PROOT=P(1,LD)/1000.
      DROOT=-AGE*(PROOT+.3)*EXP(PROOT+1.3)
      SVER=SVFR+DROOT*DTAU
```

```
40    LV=SVER/DX+3
      IF(SVER .GE. (ZZ-5.)) LV=N
      DO 9 I=3,LV
      IF(HEND(I) .EQ. M) GO TO 9
      LH=HEND(I)+1
      PROOT=P(LH,I)/1000.
      DROOT=-AGE*(PROOT+.3)*EXP(PROOT+1.3)
      SHOR(I)=SHOR(I)+DTAU*DROOT
      HEND(I)=1+SHOR(I)/DX
      IF(HEND(I) .GE. M) HEND(I)=M
    9 CONTINUE
      LH=HEND(3)
      IP=LH*1/4+1
      JP=(LV-3)*3/4+3+DROOT
C        CALCULATION OF THE ROOT DENSITY PER UNIT VOLUME. THE NEW
C        ROOT DENSITY IS SEPARETED FROM THE OLD ONE BY ASSIGNING
C        TWO PARAMETERS, SUCH AS DEN AND DR.
      DO 75 J=3,LV
      M1=HEND(J)
      DO 70 I=1,M1
      PSOIL=(P(I,J)+P(I+1,J)+P(I,J+1)+P(I+1,J+1))/4.
      DIN=DQ(PSOIL)
      PROOT=PSOIL/1000.
      RG=.216*(-PROOT-.3)*EXP(PROOT+1.3)*DTAU/DIN
      DR(I,J)=DEN(I,J)
70    DEN(I,J)=DR(I,J)+RG
75    CONTINUE
      IF(TAU .GT. 13.) GO TO 4
C        CALCULATION OF THE AMOUNT OF WATER UPTAKE DURING THE DAY TIME.
C        MAX. ROOT WATER POTENTIAL IS CALCULATED BY ITERATING THE
C        ESTIMATED OPTIMUM ROOT POTENTIAL FROM THE EQUATION OF
C        WATER UPTAKE LIMITATION UNTIL WE OBTAINED MAX. WATER UPTAKE.
      POT=POT*(1.6+.01*IDAY)
      DO 10 I=1,6
      POT=POT-50.
      PO(I)=POT
```

```
      AK=KO((PO(I)+P(IP,JP))/2.)
   10 QTR(I)=AK*(P(IP,JP)-PO(I)-RES(IP,JP))
      DO 11 IM=2,6
      IF(QTR(IM) .GT. QTR(IM-1)) POT=PO(IM)
   11 CONTINUE
      ITR=1
   12 POT=POT-50.
      ITR=ITR+1
      SUM(ITR)=0.
      DO 35 J=3,LV
      SUR(J)=0.
      SUW(J)=0.
      M1=HEND(J)
      DO 45 I=1,M1
      PSOIL=(P(I,J)+P(I+1,J)+P(I,J+1)+P(I+1,J+1))/4.
      DIFP=POT-PSOIL-RES(I,J)
      DIFP=POT-PSOIL
      BK=KO((POT+PSOIL)/2.)
      RG=DEN(I,J)-DR(I,J)
      IF(ABS(DIFP) .LT. RES(I,J)) GO TO 55
      IF(DIFP .GT. 0.) GO TO 6
      AK=KO((POT+PSOIL+RES(I,J))/2.)
      OLR=DR(I,J)*AK*(DIFP+RES(I,J))*DTAU*UV*1000.
      RNW=RG*BK*DIFP*DTAU*UV*1000.
      S(I,J)=OLR+RNW
      GO TO 65
   60 AK=KO((POT+PSOIL-RES(I,J))/2.)
      OLR=DR(I,J)*AK*(DIFP-RES(I,J))*DTAU*UV*1000.
      RNW=RG*BK*DIFP*DTAU*UV*1000.
      S(I,J)=OLR+RNW
      GO TO 65
   55 S(I,J)=RG*AK*DIFP*DTAU*UV*1000.
   65 SUM(ITR)=SUM(ITR)+S(I,J)*V(I+1)/1000.
      SUW(J)=SUW(J)+S(I,J)
   45 SUR(J)=SUR(J)+DEN(I,J)/.628
   35 CONTINUE
```

```
      IF(ITR .GE. 10) GO TO 14
      IF(SUM(ITR) .LT. SUM(ITR-1)) GO TO 12
   14 WRITE(61,250) TAU
      DO 16 J=3,LV
      M1=HEND(J)
   16 WRITE(61,160) (S(I,J),I=1,M1,4),SUR(J),SUW(J)
      WRITE(61,100) POT,SUM(ITR),TR(IDAY),ITR
      CALL SOLVE(IDAY,TAU,DTAU,DX)
      IF(IDAY .EQ. 15) CALL PGRAF(IDAY)
      IF(IDAY .EQ. 30) CALL PGRAF(IDAY)
      IF(IDAY .EQ. 45) CALL PGRAF(IDAY)
      IF(TAU .LT. 13.) GO TO 6
C         CALCULATION OF WATER OBSORBTION OR RELESE DURING THE NIGTH
C         WATER POTENTIAL OF ROOT SYSTEM IS RELAXED UNTIL WE REACHED 20.
C         PERCENT OF PREVIOUS DAY TRANSPIRATION LOSS.
   40 POT=POT/2.-10.*IDAY
      IC=1
      WRITE(61,250) TAU
   44 SUN=0.
      DO 79 J=3,LV
      M1=HEND(J)
      DO 81 I=1,M1
      PSOIL=(P(I,J)+P(I+1,J)+P(I,J+1)+P(I+1,J+1))/4.
      BK=KQ((POT+PSOIL)/2.)
      DIFP=POT-PSOIL
      RG=DEN(I,J)-DR(I,J)
      IF(ABS(DIFP) .LT. RES(I,J)) GO TO 95
      IF(DIFP .LT. 0.0) GO TO 80
      AK=KQ((POT+PSOIL-RES(I,J))/2.)
      OLR=DR(I,J)*AK*(DIFP-RES(I,J))*DTAU*UV*1000.
      RNW=RG*BK*DIFP*DTAU*UV*1000.
      S(I,J)=OLR+RNW
      GO TO 81
   80 AK=KQ((POT+PSOIL+RES(I,J))/2.)
      OLR=DR(I,J)*AK*(DIFP+RES(I,J))*DTAU*UV*1000.
      RNW=RG*BK*DIFP*DTAU*UV*1000.
```

```
      S(I,J)=OLR+RNW
      GO TO 81
   95 S(I,J)=RG*EK*DIFP*DTAU*UV*1000.
   81 SUN=SUN+S(I,J)*V(I+1)/1000.
   79 CONTINUE
      IF(IC .GE. 6) GO TO 53
      IC=IC+1
      CRIT=5.-SUM(ITR)/5.
      IF(SUN .LE. 0.0 .AND. ABS(SUN) .LE. CRIT) GO TO 53
      IF(SUN) 31,53,47
   31 POT=POT+(5.+6*IDAY)
      GO TO 44
   47 POT=POT-(5.+8*IDAY)
      GO TO 44
   53 CONTINUE
      WRITE(61,101) POT,SUN
      DO 13 J=3,LV
      M1=HEND(J)
      DO 8 I=1,M1
      PSOIL=(P(I,J)+P(I+1,J)+P(I,J+1)+P(I+1,J+1))/4.
      DIFP=POT-PSOIL
      IF(RES(I,J) .LT. DIFP) RES(I,J)=DIFP
    8 CONTINUE
   13 CONTINUE
      CALL SOLVE(IDAY,TAU,DTAU,DX)
      DO 72 J=3,LV,4
      M1=HEND(J)
      WRITE(61,150) (DEN(I,J), I=1,M1,4)
      WRITE(61,155) (RES(I,J), I=1,M1,4)
   72 WRITE(61,160) (S(I,J),I=1,M1,4)
      WRITE(61,300) IDAY
      DO 20 J=2,N,4
      DO 25 I=2,M,4
   25 PT(I)=P(I,J)/1000.
   20 WRITE(61,200) (PT(I),I=2,M,4)
      WRITE(61,400) SVER,(SHOR(I),I=3,LV)
```

```
      IF(IDAY .EQ. 15) CALL PGRAF(IDAY)
      IF(IDAY .EQ. 30) CALL PGRAF(IDAY)
      IF(IDAY .EQ. 45) CALL PGRAF(IDAY)
      IDAY=IDAY+1
      IF(IDAY .LE. TIME) GO TO 5
      STOP
  450 FORMAT(2F10.0,F10.5,50X)
  400 FORMAT(* *,*VERTICAL EXTENSION *,F10.2/
     1*HORIZANTAL EXTENSION*,3X,20F5.1/20X,25F4.1)
  350 FORMAT(25F3.1)
  100 FORMAT(1H0,10X,*ROOT POTENTIAL =*,F10.2/
     11H0,10X,*SUM OF WATER UPTAKE =*,F10.3,*CM**3*/
     21H0,10X,*EXPERIMENTAL TRANSPIRATION =*,F10.5,*CM**3*,10X,I10)
  101 FORMAT(1H0,10X,*ROOT POTENTIAL =*,F10.2/
     11H0,10X,*SUM OF WATER UPTAKE =*,F10.2,*CM**3*)
  150 FORMAT(1H0,*DENSITY FUNC.=*,13F9.5)
  155 FORMAT(* *,*DEG.OF SUPER.=*,13F9.3)
  160 FORMAT(* *,*WATER UPTAKE =*,10F9.3,2F10.3)
  125 FORMAT(* *,10X,I5,4F12.5,4F12.3)
  121 FORMAT(1H1,10X,*  TIME   *,2X,*LEAVE SIZE*,* TRANSPIRATION*,*  TOTA
     1L *,2X,*EVAPORATION*)
  122 FORMAT(* *,10X,*  DAYS   *,2X,*   CM**2    *,*    CM/DAY    *,*   CM
     23 *,2X,*    CM/DAY   *)
  200 FORMAT(13F10.3)
  250 FORMAT(1H0,10X,*AT TIME = *,F10.2/
     21H0,10X,*ROOT DENSITY,RESISTANCE DUE TO SUPERIZATION,AND WATER UPT
     3AKE IN X-Z PLANE *)
  300 FORMAT(1H1,10X,*POTENTIAL DISTRIBUTION AT *,I5,* DAY(S) *)
      END
```

```
      SUBROUTINE SOLVE(IDAY,TAU,DTAU,DX)
C        THIS SUBROUTINE SOLVE THE EQUATION OF WATER ABSORBTION
C        AND TRANSFER IN THE SOIL-ROOT DOMAIN FOR TRANSIENT CASE BY
C        USING IMPLICIT ALTERNATING DIRECTION METHOD.
      DIMENSION A(60),B(60),C(60),D(60),PRIME(60)
      COMMON/F/ P(26,60),S(26,60),DEN(26,60),RES(26,60),V(26),EV(60)
      COMMON/W/ R1(26),R2(26),N,NPT,M,MPT,T(26,60)
      REAL KQ
      KQ(H)=86400./((H/(-1.57))**3+1000.)
      DQ(H)=236758.9/((-H)**1.5)
      DT2DX=.5*DTAU/(DX**2)
      DO 7 I=1,MPT
      AK=KQ(P(I,2))
      T(I,1)=T(I,2)-EV(IDAY)*DTAU*DX/AK
    7 P(I,1)=P(I,2)-EV(IDAY)*DTAU*DX/AK
C        IMPLICIT IN R-DIRECTION
      DO 8 J=2,N
      DO 9 I=1,M
   18 B1=DQ((P(I+1,J)+P(I,J))/2.)
   41 C1=DQ((P(I,J-1)+P(I,J))/2.)
      F1=DQ((P(I,J+1)+P(I,J))/2.)
      D1=KQ(P(I,J))
      E1=D1/F1
      IF(I .NE. 1) GO TO 10
      A1=B1
      GO TO 11
   10 A1=DQ((P(I,J)+P(I-1,J))/2.)
   11 A(I)=-DT2DX*R1(I)*A1
      B(I)=1.+DT2DX*(R2(I)*B1+R1(I)*A1)
      C(I)=-DT2DX*R2(I)*B1
      D(I)=DT2DX*C1*P(I,J-1)+(1.-DT2DX*(F1+C1))*P(I,J)+DT2DX*F1*P(I,J+1)
     1+S(I,J)/(E1*1000.)
      IF(I .NE. 1) GO TO 9
      D(I)=D(I)-A(I)*P(2,J)
      A(I)=0.
    9 CONTINUE
```

```
      D(M)=D(M)-C(M)*P(MPT,J)
      C(M)=0.
      CALL TRIDAG(1,M,A,B,C,D,PRIME)
      DO 12 I=1,M
   12 T(I,J)=PRIME(I)
    8 CONTINUE
C         IMPLICIT IN Z-DIRECTION
      DO 13 I=1,M
      DO 14 J=2,N
      C1=DQ((T(I,J-1)+T(I,J))/2.)
      F1=DQ((T(I,J+1)+T(I,J))/2.)
      D1=KQ(T(I,J))
      E1=D1/F1
      A(J)=-DT2DX*C1
      B(J)=1.+DT2DX*(F1+C1)
      C(J)=-DT2DX*F1
      B1=DQ((T(I+1,J)+T(I,J))/2.)
      IF(I .NE. 1) GO TO 15
      A1=B1
      D(J)=DT2DX*R1(I)*A1*T(I+1,J)+(1.-DT2DX*(R2(I)*B1+R1(I)*A1))*T(I,J)
     1+DT2DX*R2(I)*B1*T(I+1,J)+S(I,J)/(E1*1000.)
      GO TO 16
   15 A1=DQ((T(I,J)+T(I-1,J))/2.)
      D(J)=DT2DX*R1(I)*A1*T(I-1,J)+(1.-DT2DX*(R2(I)*B1+R1(I)*A1))*T(I,J)
     1+DT2DX*R2(I)*B1*T(I+1,J)+S(I,J)/(E1*1000.)
   16 CONTINUE
      IF(J .NE. 2) GO TO 14
      D(J)=D(J)-A(J)*T(I,1)
      A(J)=0.
   14 CONTINUE
      D(N)=D(N)-C(N)*T(I,NPT)
      C(N)=0.
      CALL TRIDAG(2,N,A,B,C,D,PRIME)
      DO 17 J=2,N
   17 P(I,J)=PRIME(J)
   13 CONTINUE
```

```
      DO 32 I=1,MPT
      T(I,NPT)=P(I,N-1)
   32 P(I,NPT)=P(I,N-1)
      DO 33 J=1,NPT
      T(MPT,J)=P(M-1,J)
   33 P(MPT,J)=P(M-1,J)
      RETURN
      END


      SUBROUTINE TRIDAG(IF,L,A,B,C,D,V)
C         THIS SUBROUTINE SOLVE A SET OF LINEAR EQUATION BY GAUSSIAN
C         ELIMINATION TECHNIQUE.
      DIMENSION A(1),B(1),C(1),D(1),V(1),BETA(60),GAMMA(60)
      BETA(IF)=B(IF)
      GAMMA(IF)=D(IF)/BETA(IF)
      IFP1=IF+1
      DO 1 I=IFP1,L
      BETA(I)=B(I)-A(I)*C(I-1)/BETA(I-1)
    1 GAMMA(I)=(D(I)-A(I)*GAMMA(I-1))/BETA(I)
      V(L)=GAMMA(L)
      LAST=L-IF
      DO 2 K=1,LAST
      I=L-K
    2 V(I)=GAMMA(I)-C(I)*V(I+1)/BETA(I)
      RETURN
      END
```

```
      SUBROUTINE PGRAF(IDAY)
      COMMON/ARR/HEND(60),PT(25),LV
      COMMON/W/ R1(26),R2(26),N,NPT,M,MPT,T(26,60)
      COMMON/F/ P(26,60),S(26,60),DEN(26,60),RES(26,60),V(26),EV(60)
      INTEGER HEND
      WRITE(61,300) IDAY
      DO 81 J=2,N,2
      DO 93 I=1,M,2
   93 PT(I)=P(I,J)/1000.
   81 WRITE(61,200) (PT(I),I=1,M,2)
      DO 73 J=3,LV,2
      M1=HEND(J)
      WRITE(61,150) (DEN(I,J), I=1,M1,2)
      WRITE(61,155) (RES(I,J), I=1,M1,2)
   73 WRITE(61,160) (S(I,J),I=1,M1,2)
      RETURN
  150 FORMAT(1H0,*DENSITY FUNC.=*,13F9.5)
  155 FORMAT(* *,*DEG.OF SUPER.=*,13F9.3)
  160 FORMAT(* *,*WATER UPTAKE =*,10F9.3,2F10.3)
  200 FORMAT(13F10.3)
  300 FORMAT(1H1,10X,*POTENTIAL DISTRIBUTION AT *,I5,* DAY(S) *)
      END

   -330.00     25.      50.00      0.50      1.25     15.00
```