

SOME EFFECTS OF PROGRESSIVE ULTRASONIC
WAVES ON LIGHT BEAMS OF
ARBITRARY WIDTH

Thesis for the Degree of Ph. D.
MICHIGAN STATE UNIVERSITY
Logan E. Hargrove, Jr.

1961



This is to certify that the
thesis entitled
SOME EFFECTS OF PROGRESSIVE ULTRASONIC WAVES
ON LIGHT BEAMS OF ARBITRARY WIDTH

presented by
Logan E. Hargrove, Jr.

has been accepted towards fulfillment
of the requirements for
Ph. D. degree in Physics

E. A. Hiedemann
Major professor

Date February 14, 1961



ABSTRACT

SOME EFFECTS OF PROGRESSIVE ULTRASONIC WAVES ON LIGHT BEAMS OF ARBITRARY WIDTH

by Logan E. Hargrove, Jr.

If light is passed through a plane progressive ultrasonic wave normal to the direction of sound propagation, various diffraction phenomena may be observed. Except for a scale factor which depends on the ultrasonic frequency and the medium, the observed diffraction effects, in a limited but useful range, depend in a theoretically predicted manner only on the ultrasonic waveform (pressure amplitude and harmonic structure), the ratio of light beam width to ultrasonic wavelength, and the sound field configuration. This study attempts to experimentally verify certain predictions of the existing theories which have not previously received sufficient examination.

Quantitative experimental verifications of the theory developed by Zankel (1) were obtained at 1.0 mc in water for sinusoidal ultrasonic waves and narrow light beams. This theory was also confirmed for distorted finite amplitude ultrasonic waves and wide and narrow light beams at 3.0 mc in water. Some qualitative features of the dependence of diffraction on the relative phase between two adjacent ultrasonic waves of frequency 3.0 and 6.0 mc in water were experimentally shown to be correctly given by the theory of Rao (2), Murty (3), and Mertens (4) for simultaneous diffraction and by the theory of Mertens (5) for successive

diffraction. Simultaneous and successive diffraction are indistinguishable in a limited range. Quantitative measurements showed that the successive diffraction theory of Mertens must be applied in the range of experimental parameters investigated.

1. L. E. Hargrove, K. L. Zankel, and E. A. Hiedemann, J. Acoust. Soc. Am. 31, 1366-1371 (1959).
2. B. R. Rao, Proc. Indian Acad. Sci. A29, 16-27 (1949).
3. J. S. Murty, J. Acoust. Soc. Am. 26, 970-974 (1954).
4. R. Mertens, Proc. Indian Acad. Sci. A48, 288-306 (1958).
5. R. Mertens, Z. Physik 160, 291-296 (1960).

SOME EFFECTS OF PROGRESSIVE ULTRASONIC WAVES
ON LIGHT BEAMS OF ARBITRARY WIDTH

By

Logan E. Hargrove, Jr.

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics and Astronomy

1961

325
11/2/21

ACKNOWLEDGMENT

The author wishes to express his sincere appreciation to Dr. E. A. Hiedemann for his interest and stimulating guidance. Acknowledgment is due Dr. M. A. Breazeale, Mr. B. D. Cook, and especially Dr. K. L. Zankel for helpful suggestions. The author also acknowledges the financial assistance provided by the Office of Ordnance Research, U. S. Army, and by the Office of Naval Research through contracts with Michigan State University.

L. E. H. Jr.

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION	1
A. General	1
B. Description of the Observed Phenomena	3
C. Diffraction of a Wide Light Beam	3
D. Diffraction of a Narrow Light Beam	7
E. Scope of the Present Study	9
II. THEORY.....	11
A. Statement of Problem and Assumptions	11
B. Derivation of the Principal Equations	12
C. Relations to Previous Work	16
III. EXPERIMENTAL	19
A. General Apparatus and Procedure	19
B. The Case of a Sinusoidal Ultrasonic Wave and Narrow Light Beams	24
C. The Cases of a Distorted Finite Amplitude Ultrasonic Wave and Wide and Narrow Light Beams	27
D. Diffraction of Light Passing through Two Adjacent Ultrasonic Waves of Different Frequency	35
IV. SUMMARY.....	51
BIBLIOGRAPHY.....	53

LIST OF TABLES

Table		Page
1.	Calculated intensities of diffraction orders for various percentages of second harmonic (v_2) relative to a fixed fundamental ($v_1 = 2.4$) for a finite amplitude ultrasonic wave, neglecting harmonics higher than the second, calculated from Eq. (31)	29

LIST OF FIGURES

Figure		Page
1.	Photographs of typical images observed for (a) wide light beam without sound, (b) narrow light beam without sound, (c) wide light beam with sound, and (d) narrow light beam with sound	4
2.	Central light intensity <u>vs</u> the Raman-Nath parameter as given by Eq. (25) for $G = 1/8$ and as given by Eq. (30) for $G = 1/8$	17
3.	Distance from the center of the broadened image to the outermost peak in units H of diffraction order spacing <u>vs</u> the Raman-Nath parameter as given by Eq. (25) for $G = 1/4$ and $G = 1/8$. "Lucas" refers to the maximum deflection calculated from the simplified theory of Lucas, Eq. (6)	17
4.	Special tank to obtain progressive waves	20
5.	Schematic diagram of the experimental apparatus	21
6.	Slit diffraction pattern caused by the limiting slit SL_2 observed with no sound. The first zero of this diffraction pattern at H_0 indicates that $G = 1/H_0$	23
7.	Time-average light intensity <u>vs</u> distance for $G = 1$. 1.0 mc water. Experimental ———. Theoretical ooooo	25
8.	Time-average light intensity <u>vs</u> distance for $G = 1/2$ 1.0 mc water. Experimental ———. Theoretical ooooo ..	25
9.	Time-average light intensity <u>vs</u> distance for $G = 1/3$ 1.0 mc water. Experimental ———. Theoretical ooooo ..	25
10.	Time-average light intensity <u>vs</u> distance for $G = 1/4$ 1.0 mc water. Experimental ———. Theoretical ooooo ..	25
11.	Time-average light intensity <u>vs</u> distance for $G = 1/8$ 1.0 mc water. Experimental ———. Theoretical ooooo ..	25
12.	Calculated intensities of the first and second diffraction orders for $v_1 = 2.4$ and various percentages of v_2 relative to v_1 obtained from Eq. (31)	30

Figure		Page
13.	Diffraction order light intensities and time-average light intensity for $G = 1/2$ for an ultrasonic wave containing 5 percent second harmonic and $v_1 = 2.4$. Calculated diffraction order intensities are indicated by vertical lines, experimental values by circles. Calculated intensities in the broadened image are indicated by circles, experimental values by the line	32
14.	Diffraction order light intensities and time-average light intensity for $G = 1/2$ for an ultrasonic wave containing 10 percent second harmonic and $v_1 = 2.4$. Calculated diffraction order intensities are indicated by vertical lines, experimental values by circles. Calculated intensities in the broadened image are indicated by circles, experimental values by the line	33
15.	Diffraction order light intensities and time-average light intensity for $G = 1/2$ for an ultrasonic wave containing 15 percent second harmonic and $v_1 = 2.5$. Calculated diffraction order intensities are indicated by vertical lines, experimental values by circles. Calculated intensities in the broadened image are indicated by circles, experimental values by the line	34
16.	Diagram showing use of filter plate and two transducers at 3.0 mc to obtain two adjacent ultrasonic waves at 3.0 and 6.0 mc which may be adjusted in relative phase by moving the variable transducer.....	39
17.	Observed oscillations of central order light intensity with change in relative phase between the 3.0 mc ultrasonic wave from the fixed transducer and the wave transmitted by the filter plate. (a) No fundamental passed by the filter plate. (b) Small amount of fundamental passed by the filter plate.....	40
18.	Observed diffraction order light intensities and time-average light intensity for $G = 1/2$ showing symmetry for cosine second harmonic and asymmetry for sine second harmonic. v_1 approximately 3.8 and v_2 approximately 0.5.	42
19.	Calculated extremes of light intensity in the first diffraction orders for $v_1 = 2.4$ with change in relative phase of various amounts of second harmonic for (a) simultaneous diffraction and (b) successive diffraction with $L = 5$ cm	44

Figure		Page
20.	Observed extremes of light intensity in the first diffraction order for $v_1 = 2.4$ with change in relative phase of various amounts of second harmonic <u>vs</u> the corresponding difference in light intensity showing that the average intensity is approximately independent of the amount of second harmonic	47
21.	The influence of second harmonic in the fixed transducer beam on the observed oscillation of first order light intensities as the relative phase of the variable second harmonic is varied	48
22.	Indicated percent of second harmonic at different propagation distances from the variable transducer to the filter plate as obtained from (a) simultaneous diffraction theory, (b) successive diffraction theory, and (c) measurements obtained with only the second harmonic present. $v_1 = 2.4$ when present	50

CHAPTER I

INTRODUCTION

A. General

The diffraction of light by ultrasonic waves has been investigated experimentally and theoretically since the discovery of the effect in 1932 (1, 2). A periodic change in index of refraction results from the periodic variation of pressure in an ultrasonic wave. The medium then acts as an optical phase grating. It was recognized that such a "grating" has the same periodicity in space as the ultrasonic wave. Thus the usual equation for diffraction by a grating having spacing equal to the ultrasonic wavelength gives the observed angular separation between discrete diffraction orders. Raman and Nath (3, 4) developed a theory explaining the diffraction of light by sound for the case of an infinitely wide plane wave of light passing through a plane sinusoidal ultrasonic wave at normal incidence. This theory predicted not only the correct angular separation of discrete diffraction orders but also predicted in closed form the light intensity distribution over the diffraction orders in a limited but practical range of experimental conditions. The fundamental assumption in the Raman-Nath theory is that a plane light wavefront becomes modulated in phase (but not in intensity) on passing through the ultrasonic wave. This assumption limits the applicability of the theory because gradients in the index of refraction can give rise to significant deflections of light which will result in amplitude modulation of the light wavefront. Lucas and

Biquard (2) developed a theory based on geometrical optics which considers the bending of the light caused by gradients in the index of refraction. This theory did not give the light intensity distribution over the diffraction orders.

The infinite light beam width assumed in the Raman-Nath theory is not necessary in order to obtain experimental results which agree with predictions of the theory. In practice light beam widths of only a few ultrasonic wavelengths give satisfactory results. However, the use of light beam widths the order of an ultrasonic wavelength or smaller gives rise to a continuous light distribution. With light beam widths between one-half an ultrasonic wavelength and the several ultrasonic wavelengths required to obtain discrete diffraction orders one observes a "poorly resolved" diffraction spectrum. With light beam widths less than one-half an ultrasonic wavelength one observes a rather smooth continuous light distribution which is an envelope of the discrete diffraction orders only in a very general sense. Early explanations of the narrow light beam cases were based on geometrical optics and the bending of the light caused by gradients in the index of refraction.

In the past few years it has been realized that distorted finite amplitude ultrasonic waves produce asymmetric diffraction effects. That light diffraction might be applicable to the study of finite amplitude ultrasonic waves was first suggested by Fubini-Ghiron (5) in 1935. Recent interpretations of the observed asymmetry have been based on both the concept of phase modulation and on the concept of bending of the light caused by gradients in the index of refraction.

B. Description of the Observed Phenomena

If light is passed through a plane progressive ultrasonic wave normal to the direction of sound propagation, different diffraction phenomena may be observed. The factor giving rise to the difference in appearance is the width of the light beam. Figure 1(a) shows the source slit image (image of SL_1 in the schematic diagram of the experimental apparatus) for a wide light beam and no ultrasound. Figure 1(b) shows the image observed when the light beam width is limited by another slit to one-quarter of a sound wavelength at 1.0 mc in water, with no ultrasound present. The limiting slit diffraction causes the broadening of the image; orders greater than zero are not intense enough to show in the photograph. Figures 1(c) and 1(d) are typical time-average light intensities observed with ultrasound. Figure 1(c) is for the wide light beam. In this case a width of approximately seven ultrasonic wavelengths was used which is sufficient to obtain discrete diffraction orders. Figure 1(d) is for the light beam width limited to one-quarter of an ultrasonic wavelength at 1.0 mc in water. The frequency and sound pressure were the same for Figs. 1(c) and 1(d).

C. Diffraction of a Wide Light Beam

Raman and Nath (3, 4) developed a theory explaining the diffraction of light by sound for the case of an infinitely wide plane wave of light passing through a plane sinusoidal ultrasonic wave at normal incidence. The theory confirmed the experimental observations (1, 2) that the light

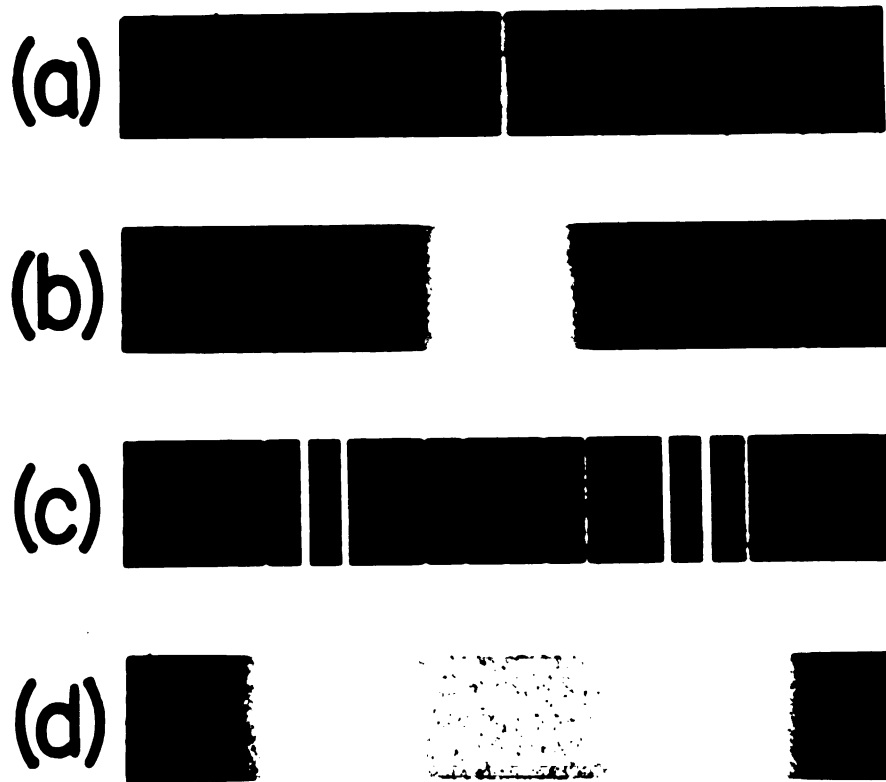


Figure 1. Photographs of typical images observed for (a) wide light beam without sound, (b) narrow light beam without sound, (c) wide light beam with sound, and (d) narrow light beam with sound.

is diffracted at discrete angles θ_n given by*

$$\sin \theta_n = -n\lambda/\lambda^*, \quad (1)$$

where n is zero or a positive or negative integer, λ is the wavelength of light, and λ^* is the wavelength of sound. Equation (1) is valid for any periodic plane sound wave. The Raman-Nath theory predicts that for a progressive sinusoidal plane ultrasonic wave the normalized intensity I_n in the n th order of diffraction as defined by Eq. (1) is

$$I_n = J_n^2(v), \quad (2)$$

where the Raman-Nath parameter v is approximately proportional to the sound pressure amplitude and given by

$$v = 2\pi\mu L/\lambda, \quad (3)$$

μ is the maximum change in index of refraction of the medium caused by the sound pressure, and L is the distance the light travels in the sound field. Equation (2) is valid when the deviation of the light beam within the sound field is small enough that a given ray will not be affected by significantly different pressures. This means that one can then consider the light to be changed in relative phase but not in amplitude as it passes through the sound field. This assumption is justified for conditions under which (6, 7)

$$(2\pi L\lambda v) / (\mu_o \lambda^{*2}) \leq N, \quad (4)$$

where μ_o is the index of refraction of the undisturbed medium and $1 < N < 4$, depending on the accuracy required.

*The usual sign convention that negative diffraction orders and positive diffraction angles are those in the direction of sound propagation is used throughout.

Sanders (8) experimentally verified the theory of Raman and Nath. Since it is assumed that μ is proportional to the sound pressure, the Raman-Nath theory has been used for absorption measurements and has been suggested for pressure measurements (9). Sanders, and later Miller and Hiedemann (10), noted discrepancies between the Raman-Nath theory and experimental results. Later, Zankel and Hiedemann (11) extended the Raman-Nath theory to include finite amplitude distortion of the ultrasonic waveform. They showed that asymmetry of the diffraction orders is caused by finite amplitude distortion under experimental conditions of sufficient pressure and propagation distance. They measured the amount of distortion. It was also shown that small deviations from the theoretical predictions are caused by inhomogeneities of the ultrasonic beam.

Investigators in the U. S. S. R. (12, 13, 14, 15) have developed approximate methods for waveform determination using the overall intensity distribution of the diffraction spectrum. Their interpretation is based on physical optics and considers the distorted finite amplitude ultrasonic wave to act as a blazed transmission grating with sawtooth shape.

Cook (16) developed a method for determining the harmonic structure from measurements of the light intensities in all the orders of diffraction. Although this method is suitable only for calculation using high speed computer methods, it has the distinct advantage of being a direct method in the sense that one can obtain the harmonic structure from the diffraction order intensities rather than from fitting data to calculated intensities for

various harmonic structures. The method is applicable only to the special case of waveforms which are odd functions, i. e., waveforms expressible by a Fourier sine series.

D. Diffraction of a Narrow Light Beam

Lucas and Biquard (2) noted that if the light beam width is less than one-half of the ultrasonic wavelength, a distinct diffraction spectrum is not observed. When the source slit image is focused on a screen broadened images occur, as in Figs. 1(b) and 1(d). This has been called "refraction" and the following explanation based on geometrical optics was given by Lucas (17). A light beam having a width much smaller than the ultrasonic wavelength was considered as being deflected by an amount proportional to the gradient of index of refraction resulting from the ultrasonic wave. This approach gives, for the sine of the angle of deflection,

$$\sin \theta = - (v\lambda / \lambda^*) \cos \omega^* t, \quad (5)$$

where ω^* is the angular frequency of the sound. The maximum deflection is then given by

$$\sin \theta_{\max} = v\lambda / \lambda^*. \quad (6)$$

Equation (6) was used as a basis for absorption measurements by Hueter and Pohlman (18). However, in their work, they used a light beam width of approximately one sound wavelength. They did this because the wider slit width gave more clearly defined image edges from which they could make measurements. Their extrapolated pressure measurements did not go through the origin but this was of no serious consequence in their

measurements. Breazeale and Hiedemann (19) used the half-width of the broadened image rather than the peak separation and found that their measurements then extrapolated through the origin.

Loeber and Hiedemann (20) studied the intensity at the center of the broadened image using standing waves. Their analysis was based on assumptions similar to those of Lucas (17) but took into account diffraction of light by the slit which limits the light beam width. The analysis showed the possibility of determining ultrasonic waveforms and sound pressure amplitudes. Experimental results suggested a distortion of ultrasonic waves during propagation.

Breazeale and Hiedemann (21) adapted the method used by Loeber and Hiedemann to the study of progressive waves. It was noticed that the light intensity distribution over the broadened images was asymmetric under certain experimental conditions. It was pointed out that this asymmetry was caused by finite amplitude distortion of the ultrasonic waveform. The observed increase in asymmetry with increase in propagation distance and initial ultrasonic pressure amplitude was in agreement with the expected increase in waveform distortion with these parameters.

Breazeale, Cook, and Hiedemann (22) proposed measuring the position of the deflected narrow light beam as a function of time and thereby obtaining the ultrasonic pressure gradient waveform from a harmonic analysis of such data. Their interpretation was based on the Lucas theory.

Zankel (23) developed a theory based on the Raman-Nath assumption of only phase modulation of the light wavefront which included arbitrary

light beam widths and ultrasonic waveforms expressible by a Fourier sine series, which is probably the case for distorted finite amplitude waves. This analysis, based on physical optics, predicts the light intensity distributions over the discrete diffraction orders, as given earlier by Zankel and Hiedemann (11), and over the broadened images. Details of this theory are given in Chapter II.

E. Scope of the Present Study

The present study is concerned with experimentally testing the validity of some predictions of the theory developed by Zankel which have not already received sufficient verification. In particular, it is concerned with the case of a sinusoidal ultrasonic wave and a narrow light beam and the cases of a distorted finite amplitude ultrasonic wave and wide and narrow light beams.

The ultrasonic waveforms considered in the development of the theory by Zankel are only those expressible by a Fourier sine series. Theoretical results have been given by Rao (24), Murty (25), and Mertens (26) for diffraction of a wide light beam by an ultrasonic wave consisting of two commensurate frequencies with arbitrary relative phase. These theoretical results are for simultaneous diffraction of light by the different frequency components contained in the same ultrasonic beam.

Mertens (27) recently pointed out that although Murty and Rao (28) got good agreement between their theory and their experimental results obtained using successive diffraction by two separate adjacent ultrasonic beams, simultaneous and successive diffraction are not the same.

Mertens obtained expressions for the amplitudes of light diffracted by two adjacent ultrasonic waves and compared the predictions with those from simultaneous diffraction theory. In a limited but useful range the diffraction spectra are indistinguishable. The diffraction effects of two adjacent ultrasonic waves of the same frequency have been used (29, 30) to investigate finite amplitude distortion. In a single finite amplitude ultrasonic wave the relative phases of the Fourier components are fixed. Therefore the dependence of diffraction effects on the relative phase between two ultrasonic waves is investigated here for both wide and narrow light beams. It is shown how one uses the theoretical results of Murty, Rao, and Mertens for arbitrary light beam widths.

All experimental results were obtained using progressive ultrasonic waves in water.

CHAPTER II

THEORY

A. Statement of Problem and Assumptions

Zankel* obtained a solution to the problem of diffraction of a light beam of arbitrary width by plane progressive ultrasonic waves of moderate frequency and amplitude. Normal incidence of collimated monochromatic light is assumed. The approximations used by Raman and Nath are used. At low frequencies, where narrow light beam widths have usually been considered, these do not impose a serious limitation as can be seen from Eq. (4). These results should include almost all the experimental work at normal incidence except for wide beam diffraction at high ultrasonic frequencies and/or large ultrasonic pressure amplitudes where the Raman-Nath assumptions are not valid. The high frequency cases have been considered by Extermann and Wannier (6) and by Wagner (31). The problem here can be stated as consisting of light having a certain phase distribution emerging from a slit. It is assumed that the ultrasonic wave can be expressed by its Fourier components** and therefore the index of refraction can be written as

*The theoretical results derived in this chapter are those obtained by Zankel. Some errors which appeared in the original publication [reference (23)] have been corrected.

**The assumption of a sine series is justified for finite amplitude waves if the mechanism giving rise to the finite amplitude distortion is considered and the Fourier coefficients calculated in the absence of dissipation in the medium. This is also true in the presence of dissipation. See references 5, 32, and 33.

$$\mu(x, t) = \mu_0 - \sum_{j=1}^{\infty} \mu_j \sin f_j \quad (7)$$

where

$$\mu_j = a_j \mu \quad (8)$$

and

$$f_j = j(2\pi x/\lambda^* - \omega^* t). \quad (9)$$

x is the distance in the direction of sound propagation, μ is the maximum change in index of refraction of the medium that would occur if only the fundamental pressure component were present, and a_j is the (positive) ratio of the pressure of the j th harmonic to that of the fundamental.

B. Derivation of the Principal Equations

The light amplitude at some angle θ is given by the diffraction integral

$$A = C \int_{-d}^{+d} \exp \left\{ 2\pi i \left[\lambda x + \sum_{j=1}^{\infty} \mu_j L \sin f_j \right] / \lambda \right\} dx \quad (10)$$

where C is a constant to be determined by normalization, $2d = D$, the width of the light beam, and $\lambda = \sin \theta$. Equation (10) may be written as

$$A = C \int_{-d}^{+d} \exp [iu\lambda x] \prod_{j=1}^{\infty} \exp [ia_j v \sin f_j] dx, \quad (11)$$

where $u = 2\pi/\lambda$ and $v = 2\pi\mu L/\lambda$ is the Raman-Nath parameter. Using

$$\exp [ia_j v \sin f_j] = \sum_{r=-\infty}^{+\infty} J_r(a_j v) \exp (irf_j), \quad (12)$$

Eq. (11) becomes

$$A = C \int_{-d}^{+d} \sum_{r_1} \sum_{r_2} \cdots \sum_{r_j}^{+\infty} J_{r_1}(a_1 v) \cdots J_{r_j}(a_j v) \cdots$$

(times) $\exp [iu\lambda x + i(r_1 + \cdots + jr_j + \cdots)(bx - \omega^* t)] dx \quad (13)$

where $b = 2\pi/\lambda^*$. Let

$$r_1 + \dots + jr_j + \dots = n. \quad (14)$$

Then

$$A = C \int_{-d}^{+d} \sum_{n=-\infty}^{+\infty} \phi_n \exp [i(u\lambda + nb)x - in\omega^*t] dx, \quad (15)$$

where

$$\phi_n = \sum_{k_2} \sum_{k_3} \dots = \sum_{-\infty}^{+\infty} J_{n-2k_2-3k_3-\dots}(v) \quad (16)$$

(times) $J_{k_2}(a_2 v) J_{k_3}(a_3 v) \dots$

ϕ_n is the amplitude of light in the n th order of diffraction as found by Zankel and Hiedemann (11) for an "infinite" light beam width. On performing the integration one finds

$$A = C \sum_{n=-\infty}^{+\infty} \phi_n \frac{\sin(u\lambda + nb)d}{(u\lambda + nb)} \exp(-in\omega^*t). \quad (17)$$

To normalize it is assumed that the amplitude is $\exp(i\omega t)$ at $\theta = 0$ and $v = 0$ (central intensity with no sound equal to unity), where ω is the angular frequency of the light. This gives

$$A = \sum_{n=-\infty}^{+\infty} \phi_n W_n \exp[i(\omega t - n\omega^*t)], \quad (18)$$

where

$$W_n = \frac{\sin(u\lambda + nb)d}{(u\lambda + nb)d}. \quad (19)$$

The real part of Eq. (18) is

$$a = \sum_{n=-\infty}^{+\infty} \phi_n W_n \cos(\omega t - n\omega^*t). \quad (20)$$

This may be put in the form

$$a^2 = r^2 \sin^2(a + \omega t), \quad (21)$$

where

$$r^2 = I(t) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \phi_n \phi_m W_n W_m \cos[(n-m)\omega^* t] \quad (22)$$

and

$$\tan a = \left(\sum_{n=-\infty}^{+\infty} \phi_n W_n \cos n\omega^* t \right) / \left(\sum_{n=-\infty}^{+\infty} \phi_n W_n \sin n\omega^* t \right). \quad (23)$$

Since the ϕ_n 's fall off for higher n and since $\omega^* \ll \omega$, a varies slowly compared with ωt . Therefore, the light intensity one can measure is given by Eq. (22). In many cases the time-average light intensity is observed. To obtain an expression for the time-average light intensity I one must integrate a^2 as given by eq. (20) by squaring and then divide by the total time. By performing this integration one obtains

$$(\omega^*/2\pi) \int_0^{(2\pi/\omega^*)} a^2 dt = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \phi_n \phi_m W_n W_m \left\{ \frac{\sin 2\pi(m-n)}{4\pi(m-n)} + \frac{\sin 2\pi[2\omega/\omega^* - (n+m)]}{4\pi[2\omega/\omega^* - (n+m)]} \right\}. \quad (24)$$

Since $\omega^* \ll \omega$, Eq. (24) reduces and normalizes to give

$$I = \sum_{n=-\infty}^{+\infty} \phi_n^2 W_n^2. \quad (25)$$

For a wide light beam, i. e., if $d \gg \lambda^*$, Eq. (25) is the same as that obtained by Raman and Nath for a sinusoidal wave and by Zankel and Hiedemann for a distorted finite amplitude wave.

For narrow light beams W_n can be put in the form

$$W_n = \frac{\sin \pi G(H + n)}{\pi G(H + n)} \quad (26)$$

where

$$G = D/\lambda^*. \quad (27)$$

Thus G is the ratio of the light beam width D to the ultrasonic wavelength. H measures distance across the image in terms of separation of discrete diffraction orders, were they present. From Eq. (1) it is seen that H is defined in a manner similar to that of n , i. e.,

$$\sin \theta = H(\lambda / \lambda^*). \quad (28)$$

In this way calculations based on Eq. (25) depend only on the Raman-Nath parameter v , the ratio of light beam width to sound wavelength G , and the ultrasonic waveform. λ is then measured in units H of spacing between discrete diffraction orders, had these occurred. For a sinusoidal wave ϕ_n reduces to the Bessel function $\phi_n = J_n(v)$ of the Raman-Nath theory.

Equation (25) can be interpreted in the following manner. ϕ_n^2 is the intensity I_n of the n th diffraction order, whether given by Eq. (2) for a sinusoidal wave, by the more complicated Eq. (16) for a wave consisting of Fourier sine components, or by other expressions for other cases such as those to be considered in Chapter III. It can be seen that as G becomes large in Eq. (26), the value of W_n^2 is significant only when $(H + n)$ vanishes. Since negative n corresponds to positive H , the significant light intensities given by Eq. (25) are those corresponding to the locations of discrete diffraction orders as predicted by Eq. (1), i. e., discrete diffraction orders result for a light beam much wider than an ultrasonic wavelength. An equivalent statement is that the "spreading" of the orders caused by diffraction by the limiting aperture is negligible compared with the spacing between the orders. This statement is better understood if one

recognizes that the factor W_n^2 is an expression for the light intensity distribution for slit diffraction by the limiting aperture, written so that it is centered about the location of the n th diffraction order, if it occurs. Therefore W_n^2 distributes the n th diffraction order intensity over a slit diffraction distribution. In the case of narrow light beams the spreading is not negligible compared with the separation of diffraction orders, had they occurred. Therefore the narrow beam continuous light distribution can be thought of as a blending of diffraction orders resulting from distributing the diffraction orders over a slit diffraction distribution. The arithmetic addition of intensity contributions from various "orders" indicated by the summation in Eq. (25) is justified because each "order" consists of slightly different light wavelengths. This can be seen in Eq. (20).

C. Relations to Previous Work

The results of Loeber and Hiedemann (20) can be adapted to progressive waves to give the central light intensity as

$$I(0) \approx (\pi^2 G^2 v^2 - 0.232)^{-1/2} \quad (29)$$

for $Gv > \pi$. Similarly it can be shown from the work of Loeber and Hiedemann that

$$I(0) = \sum_{n=0}^{\infty} \frac{(-1)^n (Gv)^{2n}}{(2n+1)n!(n+1)!} \quad (30)$$

which is useful for smaller values of Gv . In Fig. 2, $I(0)$ is shown as calculated from Eq. (25) for $G = 1/8$ and compared with values of $I(0)$ calculated from Eq. (30) for $G = 1/8$. It is seen that the two equations

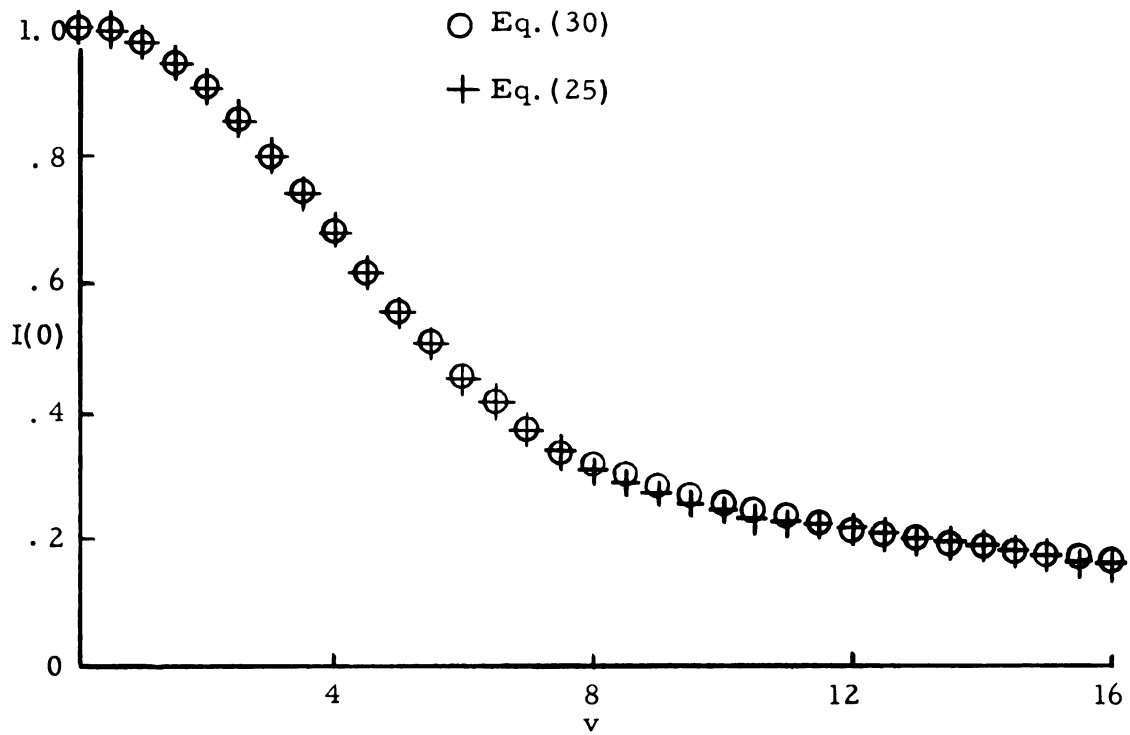


Figure 2. Central light intensity vs the Raman-Nath parameter as given by Eq. (25) for $G = 1/8$ and as given by Eq. (30) for $G = 1/8$.

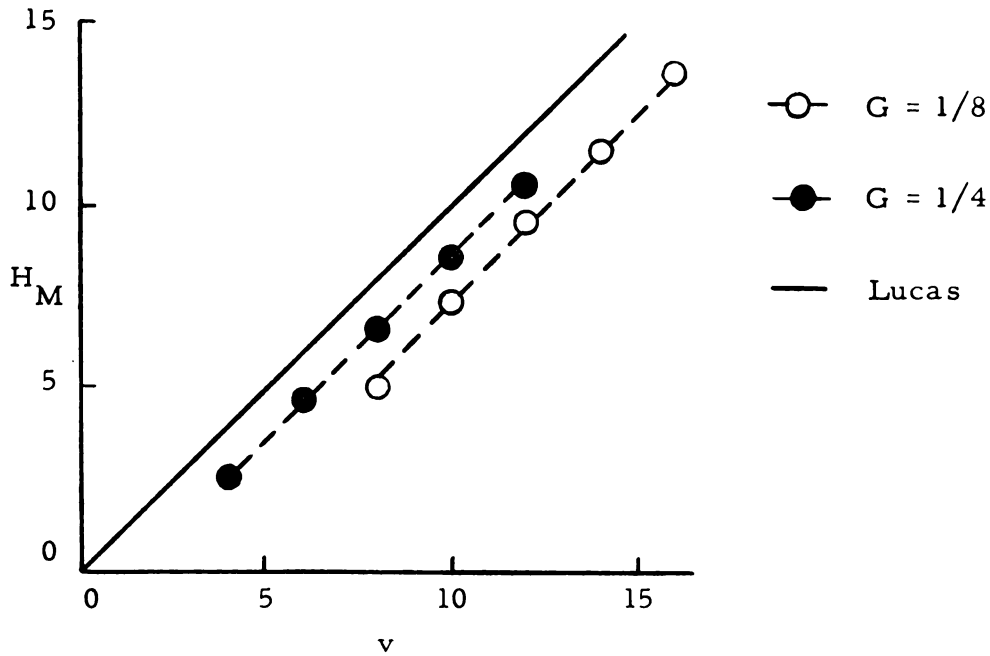


Figure 3. Distance from the center of the broadened image to the outer-most peak in units H of diffraction order spacing vs the Raman-Nath parameter as given by Eq. (25) for $G = 1/4$ and $G = 1/8$. "Lucas" refers to the maximum deflection calculated from the simplified theory of Lucas, Eq. (6).

give very nearly the same results. The Loeber and Hiedemann expressions assume that $G \ll 1$ while Eq. (25) contains no such assumption. It was found that values calculated from Eqs. (25), (29), and (30) using G as large as $1/2$ are in fair agreement.

Figure 3 shows the distance H_M to the outermost peak of the broadened image as a function of the Raman-Nath parameter v as calculated from Eq. (25). The solid line passing through the origin is calculated from Eq. (6) obtained by Lucas (17). It can be seen that the results obtained from Eq. (25) do not extrapolate through the origin. This indicates why the half-width used by Breazeale and Hiedemann (19) gave better values for the sound pressure amplitudes than the peak separation used by Hueter and Pohlman (18).

CHAPTER III

EXPERIMENTAL

A. General Apparatus and Procedure

A special tank was constructed to obtain progressive waves. Design features and major dimensions are shown in Fig. 4. The tank is 6 x 6 inches in cross section. Sound from the transducer travels along the tank and strikes the oblique reflecting end wall. After reflection, the beam passes through a very thin plastic membrane and enters castor oil. The acoustic impedance match between water and castor oil is very good. Note that any sound reflected from the membrane strikes the end wall at normal incidence and is reflected back into the castor oil. After traveling the "zig-zag" path down and up the castor oil, the sound beam strikes the end wall at normal incidence and retraces the path in the castor oil. The total path length in castor oil is 244 cm. Using $2 \times 10^{-2} \text{ cm}^{-1}$ for the absorption coefficient at 1.0 mc (a conservative value by as much as a factor of four*), this path length reduces the sound to about 0.8% of the original pressure, or about 0.006% of the original intensity.

A schematic diagram of the experimental apparatus is shown in Fig. 5. Light from the Hg arc source S illuminates the source slit SL_1 . Light is collimated by lens L_2 . When a wide beam of light is desired to observe discrete diffraction orders with SL_2 removed, the square aperture A limits

*F. Dunn, Biophysical Research Laboratory, University of Illinois, Urbana, private communication.

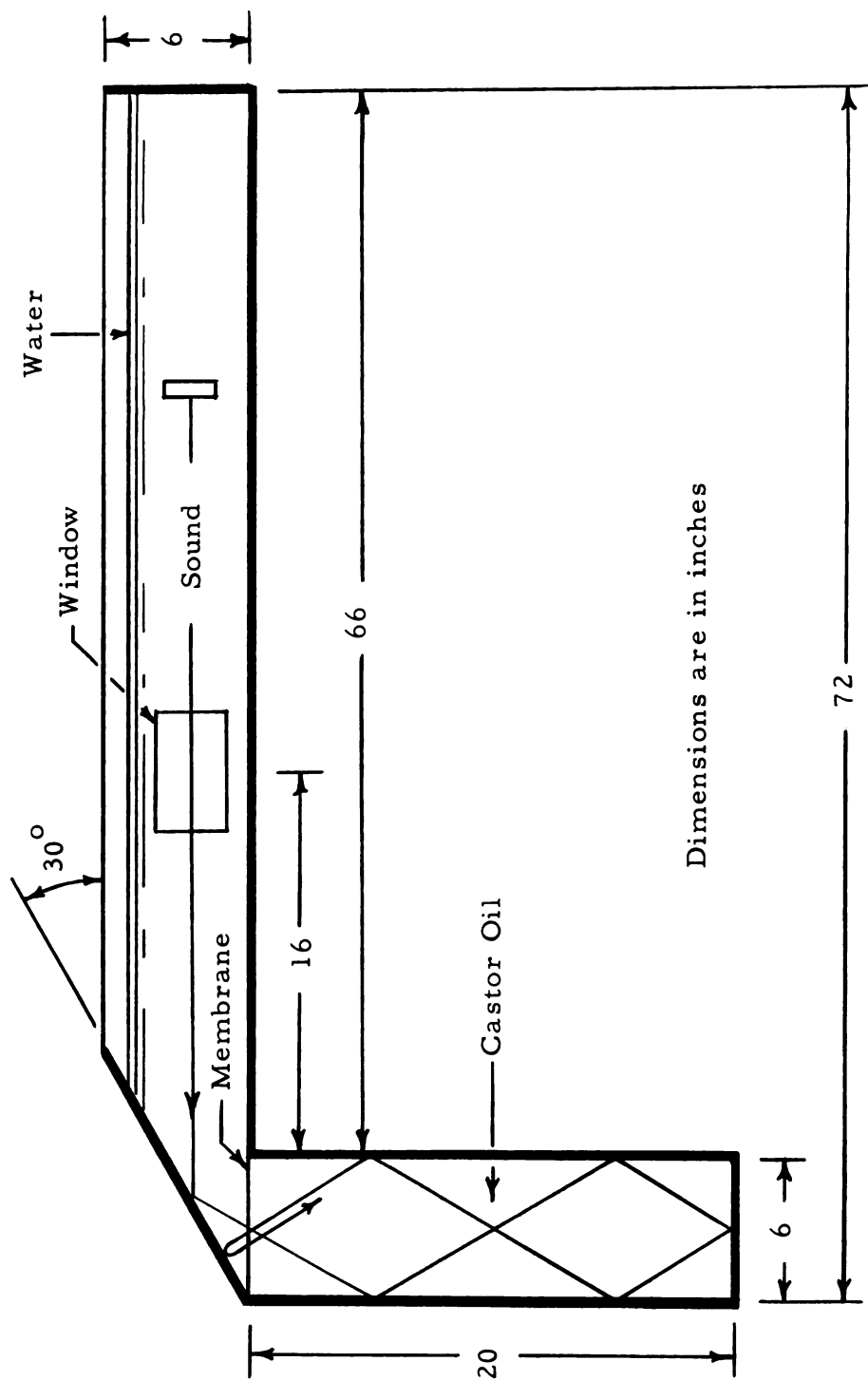


Figure 4. Special tank to obtain progressive waves.

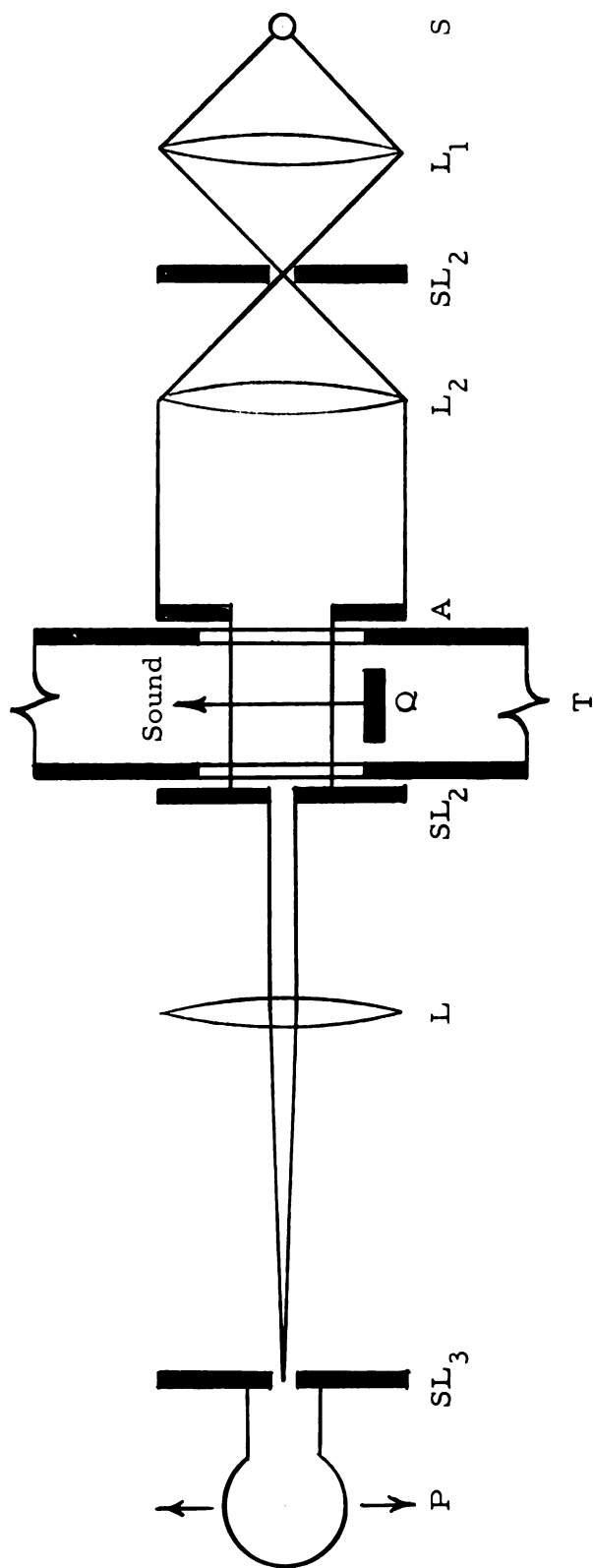


Figure 5. Schematic diagram of the experimental apparatus.

the light beam to cover a section of the sound beam. A also serves to limit the vertical height of the light beam when slit SL_2 is in place. The transducer Q is an X-cut, air backed quartz. SL_2 is the slit which determines the effective light beam width. This slit is referred to as the limiting slit. It appears from simple arguments and from observations that in the region studied it makes little difference whether the limiting slit is placed before or after the sound in the optical path. The width of the limiting slit in terms of sound wavelengths was determined by comparing the separation of zeros of slit diffraction without sound with the separation of the discrete diffraction orders observed with the limiting slit removed and the sound on. Figure 6 shows the slit diffraction pattern observed with no sound present. When the first zero of this diffraction pattern is at $H = H_0$ the value of G is $1/H_0$. Since one unit of H is one diffraction order spacing, the H scale was established on a recorder trace by scanning a discrete diffraction spectrum and noting the spacing of the orders. The need for direct wavelength measurement or accurate velocity and frequency measurements was thereby eliminated. The magnification of the image may be any convenient magnitude. This method of slit width determination gave settings which were reproducible to about 2%. With special care, absolute accuracy of 0.5% was obtained. The lens L_3 produces an image of the source slit in the plane of the photomultiplier slit SL_3 . A $5461 \text{ \AA}$ optical filter is located inside the photomultiplier housing P. The photomultiplier microphotometer and slit SL_3 can be moved across the image by a synchronous motor and precision screw at a rate of 5 mm

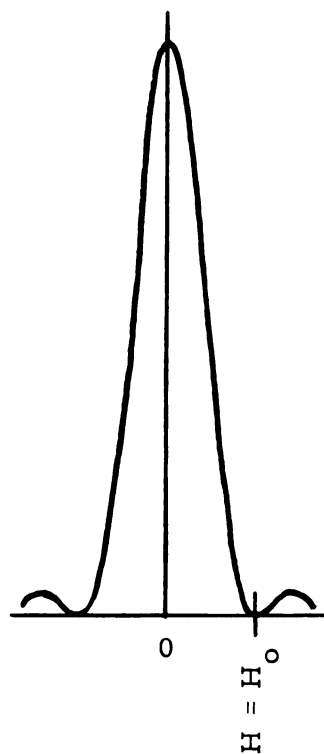


Figure 6. Slit diffraction pattern caused by the limiting slit SL_2 observed with no sound. The first zero of this diffraction pattern at H_0 indicates that $G = 1/H_0$.

per minute. The microphotometer output can be recorded with a chart speed of 2 inches per minute.

B. The Case of a Sinusoidal Ultrasonic Wave and Narrow Light Beams

The basic experimental arrangement was that described in the previous section. The aperture A limited the light beam width to cover a 1 x 1 cm section of the sound beam. The transducer Q was a 2 x 2 inch, X-cut, air backed quartz. The measurements described in this section were made near the transducer to avoid finite amplitude effects encountered at greater distances. The magnification of the optical system was adjusted to give a scanning rate of 7.5 sec per diffraction order at 1.0 mc in water.

To compare the experimental values with values calculated from Eq. (25) it was necessary to determine the Raman-Nath parameter v . This was determined as a linear function of quartz voltage by observing the discrete diffraction orders and measuring the light intensity in the first of these orders. The quartz voltages corresponding to minima and maxima of light intensity predicted by Eq. (2) were used in this determination. A more detailed description of this method for determining values of the Raman-Nath parameter has been given elsewhere (9). Final selection of the quartz voltages corresponding to values of v used in calculations were made by very small adjustments of the values determined above such that closest possible agreement was attained for the data shown in Fig. 7.

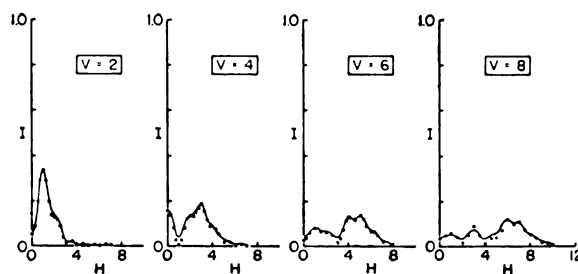


FIG. 7. Time-average light intensity vs distance for $G=1.0$ Mc water. Experimental —. Theoretical $\circ\circ\circ\circ$.

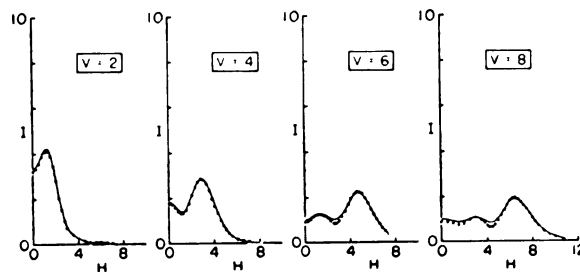


FIG. 8. Time-average light intensity vs distance for $G=\frac{1}{2}$, 1.0 Mc water. Experimental —. Theoretical $\circ\circ\circ\circ$.

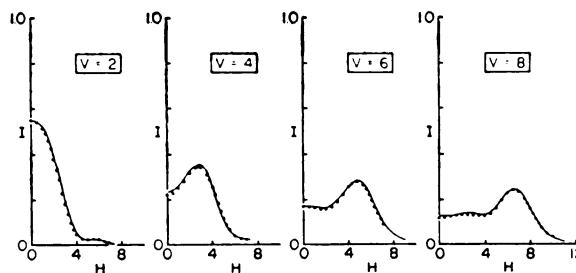


FIG. 9. Time-average light intensity vs distance for $G=\frac{1}{4}$, 1.0 Mc water. Experimental —. Theoretical $\circ\circ\circ\circ$.

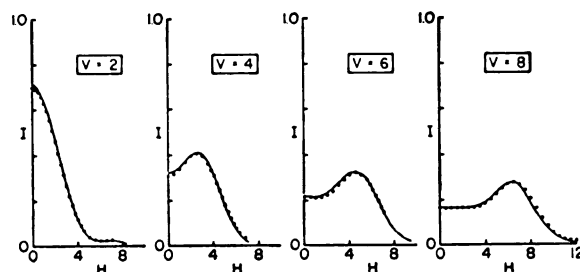


FIG. 10. Time-average light intensity vs distance for $G=\frac{1}{8}$, 1.0 Mc water. Experimental —. Theoretical $\circ\circ\circ\circ$.

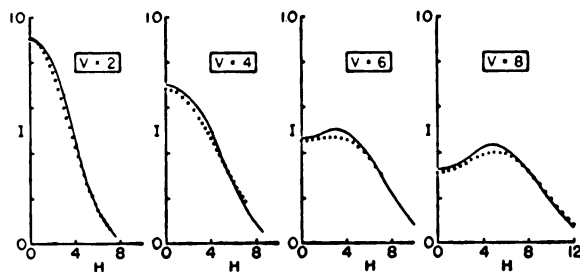


FIG. 11. Time-average light intensity vs distance for $G=\frac{1}{16}$, 1.0 Mc water. Experimental —. Theoretical $\circ\circ\circ\circ$.

These voltages were not readjusted in subsequent measurements. Details of shape of the $G = 1$ curves in Fig. 7 are quite sensitive to small changes in v .

The experimental results are shown in Figs. 7 through 11. The points represent values calculated from Eq. (25). The solid curves were traced from recorder charts. Only one side of the symmetric curves is shown. Noise and minor irregularities, when they occurred, have been smoothed out. All measurements shown are for 1.0 mc in water. Some measurements have been made at 800 kc and also show good agreement. The time-average intensity I is plotted vs distance in units of H . The light intensity is normalized such that the central intensity in the diffraction pattern caused by the limiting slit is unity. The effective light beam width is indicated in the figures by G . Experimental curves and calculated points are shown for $G = 1, 1/2, 1/3, 1/4$, and $1/8$ and for $v = 2, 4, 6$, and 8 . Approximately, the peak ultrasonic pressure amplitude was $P = v \times 10^{-1}$ atmospheres for the transducer dimension used here.

There is fairly detailed agreement between the theoretical points and the observed curves. It therefore appears that the considerations used to derive the theoretical results are based on experimentally achievable conditions. While the agreement is not exact, some amount of deviation can be explained as being caused by several minor differences between the ideal situation and the actual experimental conditions. It is difficult to align the transducer for normal incidence of sound and light. The source slit, the sound wavefronts, the limiting slit, and the photomultiplier slit must all be aligned parallel to each other. The sound field

is not homogeneous since one is definitely working in the Fresnel region of the transducer. The photomultiplier microphotometer is not perfectly linear in response and is subject to some drift in sensitivity during the 5 to 7 minutes required to scan the images. The response time of the microphotometer-recorder combination introduces some error, particularly when the light intensity variations are rapid. These experimental limitations apply to other measurements to be described. It has been shown elsewhere (34) that the data shown in Fig. 10 ($G = 1/4$) give values of v which are in good agreement when interpreted on the bases of central light intensity $I(0)$, the parameter H_M shown in Fig. 3, and the first diffraction order minima and maxima as previously described.

C. The Cases of a Distorted Finite Amplitude Ultrasonic Wave and Wide and Narrow Light Beams

In this section some results of an investigation of the diffraction of light passing through an ultrasonic wave of finite amplitude are given. For sake of simplicity, the third and higher harmonics of the distorted wave were neglected. As the harmonics fall off in magnitude rather rapidly with harmonic number even in the absence of dissipation (33) and more rapidly in a dissipative medium, the fundamental and second harmonic components should be a fairly good approximation in the range of frequency and pressure considered here.

For an ultrasonic wave consisting of the fundamental and the second harmonic, the intensity I_n in the n th diffraction order is found from Eq. (16) to be

$$I_n = \phi_n^2 = \left[\sum_{k=-\infty}^{+\infty} J_{n-2k}(v_1) J_k(v_2) \right]^2 \quad (31)$$

where v_1 is the Raman-Nath parameter for the fundamental and v_2 is the same parameter for the second harmonic. For a narrow light beam the continuous light distribution I may be calculated using ϕ_n^2 given by Eq. (31) in Eq. (25).

Calculations were made for $v_1 = 2.4$ and various percentages of v_2 relative to v_1 . This value of v_1 was selected because (a) the intensity of the zeroth order is approximately zero, making the approximate range easy to determine, (b) finite amplitude distortion is appreciable but not excessive, and (c) the difference in ($\pm$) first order light intensity is approximately proportional to v_2 over a useful range of v_2 and the average ($\pm$) first order light intensity is approximately independent of v_2 over the same range. Calculated intensities of first and second orders are shown in Fig. 12. All intensities calculated are tabulated in Table 1. Note that the previously asserted properties of the difference and average of first order light intensities are clearly shown in Fig. 12. It should therefore be possible to ascertain experimentally that $v_1 = 2.4$ from the average first order intensity and the relative amount of second harmonic from the difference in intensity, provided the effect of higher harmonics is negligible.

To test the theoretical predictions outlined in the previous paragraph, experimental measurements were obtained in the following way. The basic optical arrangement shown in Fig. 5 was used. In this case the transducer was a 1 x 1 inch, 3.0 mc, X-cut, air backed quartz. The light beam was limited by the square aperture A to cover a 5 x 5 mm

Table 1. Calculated intensities of diffraction orders for various percentages of second harmonic (v_2) relative to a fixed fundamental ($v_1 \neq 2, 4$) for a finite amplitude ultrasonic wave, neglecting harmonics higher than the second, calculated from Eq. (31).

Order No.	0%	5%	10%	15%	20%
0	.0000	.0000	.0000	.0000	.0000
+1	.2706	.2256	.1813	.1394	.1011
-1	.2706	.3148	.3564	.3942	.4264
+2	.1858	.1819	.1769	.1708	.1637
-2	.1858	.1883	.1895	.1892	.1869
+3	.0392	.0514	.0632	.0742	.0836
-3	.0392	.0276	.0173	.0089	.0030
+4	.0041	.0080	.0131	.0192	.0260
-4	.0041	.0017	.0002	.0002	.0016
+5	.0003	.0008	.0019	.0034	.0056
-5	.0003	.0000	.0000	.0001	.0002
+6	.0000	.0001	.0002	.0005	.0009
-6	.0000	.0000	.0000	.0000	.0000

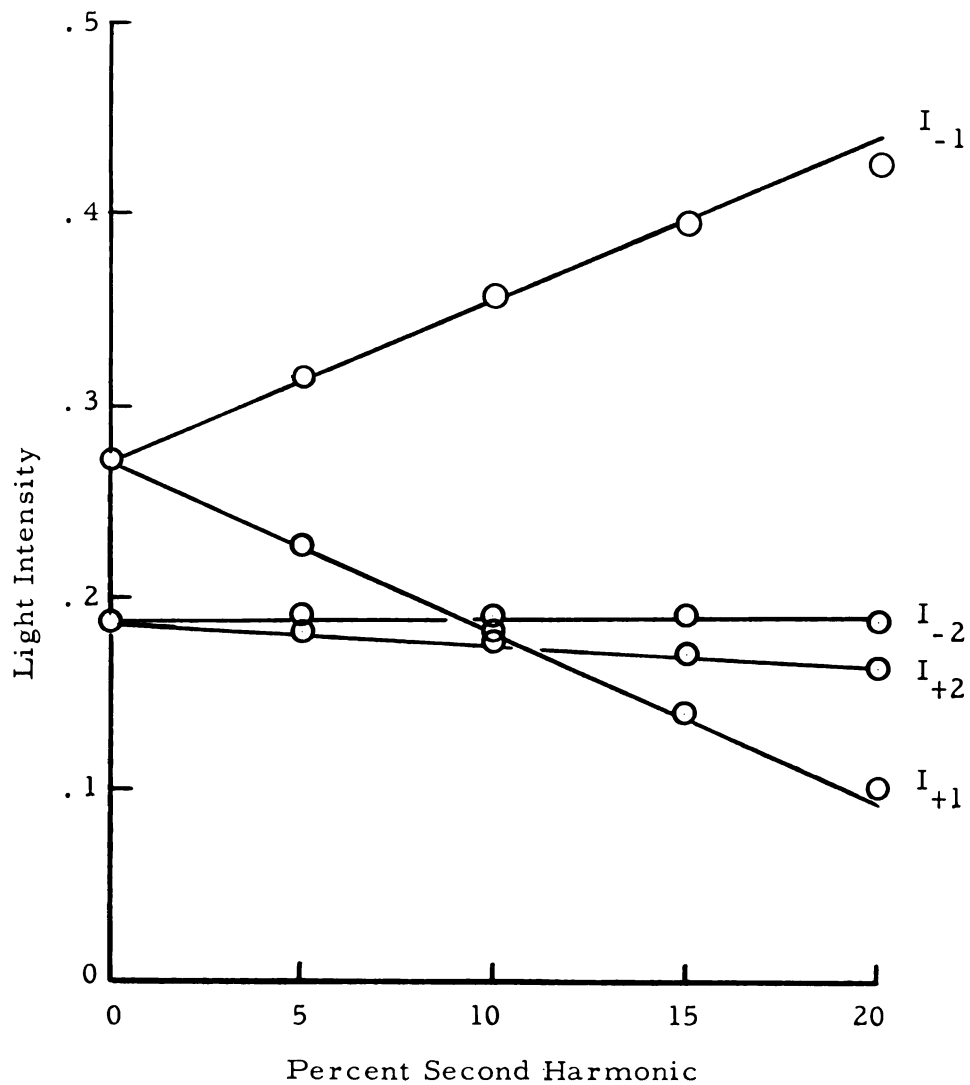


Figure 12. Calculated intensities of the first and second diffraction orders for $v_1 = 2.4$ and various percentages of v_2 relative to v_1 obtained from Eq. (31).

section of the sound beam. At various distances from the transducer to the light beam different amounts of distortion were present. At suitable distances the transducer voltage was adjusted to give the average intensity in the first diffraction orders predicted for $v_1 = 2.4$. The amount of second harmonic was then inferred from the difference in intensity of these two orders. The diffraction order intensities were measured. Then, having established that $v_1 = 2.4$ and the relative amount of second harmonic, the limiting slit SL_2 was placed in position and the narrow beam diffraction for $G = 1/2$ was observed. The resulting measurements of discrete diffraction order intensities and broadened images for 5, 10, and 15 percent second harmonic are shown in Figs. 13 through 15. Calculated values are also indicated in the figures. The calculations for $G = 1/2$ were made using the previously calculated diffraction order intensities in Eq. (25).

As a further check on the amount of second harmonic, a filter plate was used to transmit the second harmonic and reflect the fundamental. The intensity in the first order of diffraction resulting from the transmitted second harmonic was used to determine a value for v_2 using Eq. (2). The values obtained from filter plate measurements for 10 and 15 percent second harmonic are noted in the figures. It was not possible to make a reliable filter plate measurement for the 5 percent case because the small separation between the transducer and the filter plate allowed disturbing reflections between the plate and the transducer. Where filter plate measurements were possible, the values are in good agreement.

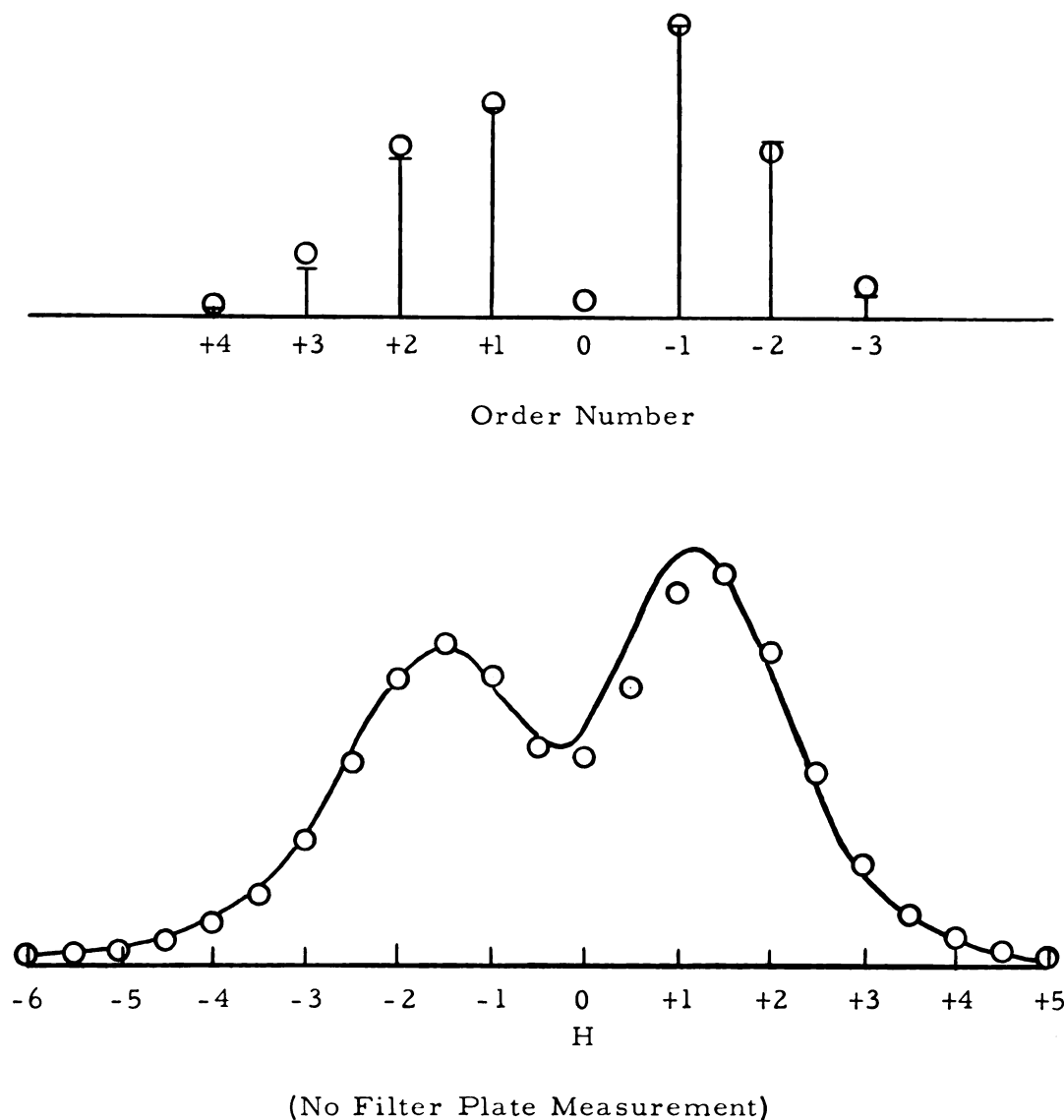
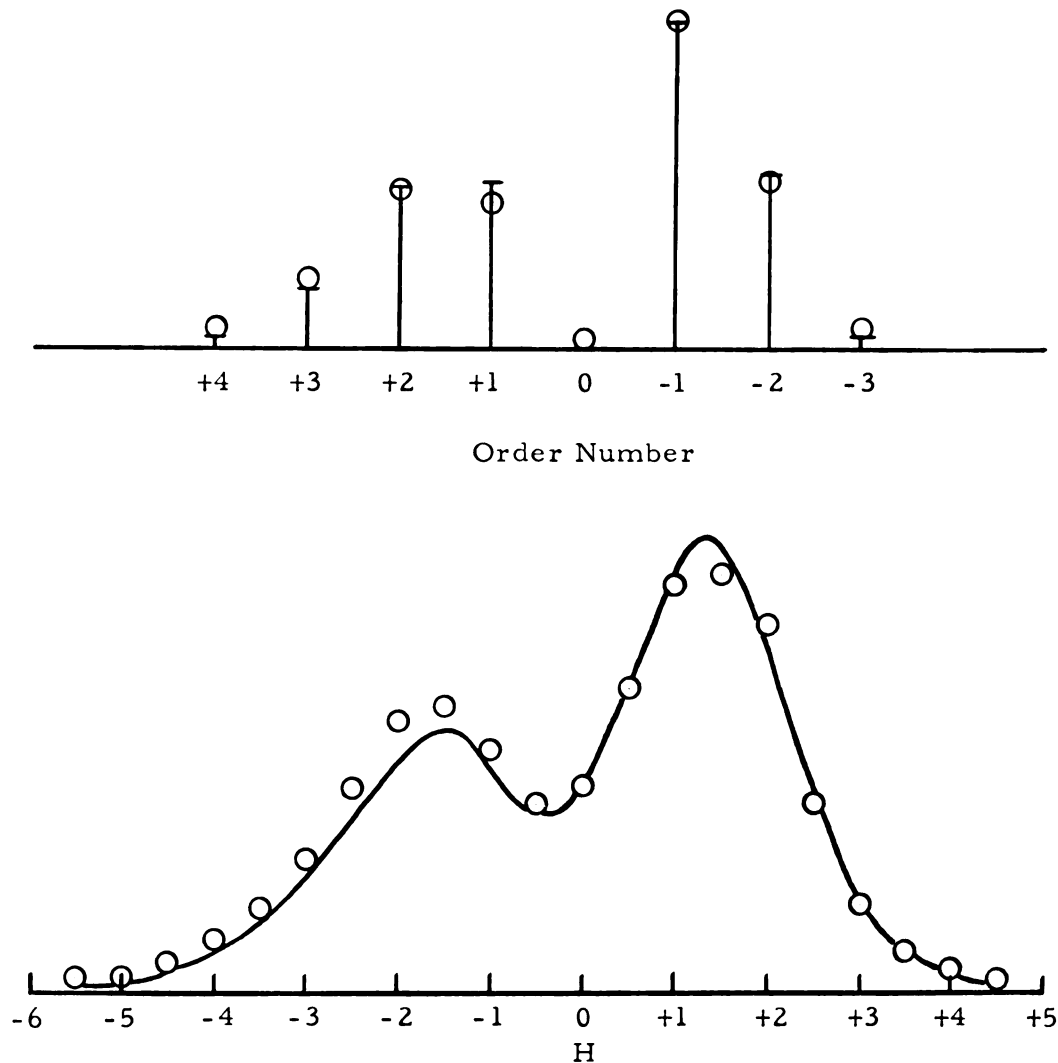
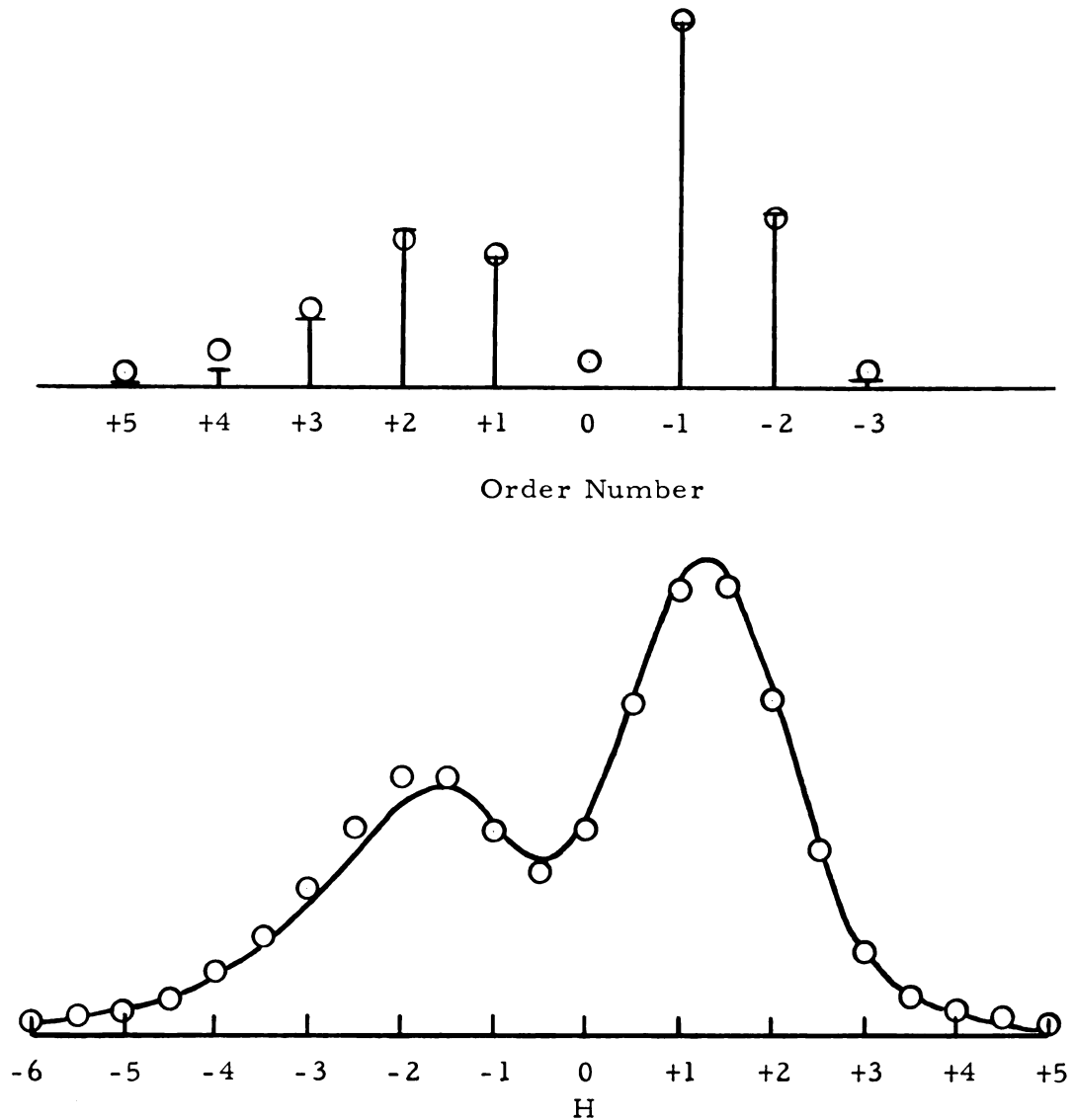


Figure 13. Diffraction order light intensities and time-average light intensity for $G = 1/2$ for an ultrasonic wave containing 5 percent second harmonic and $v_1 = 2.4$. Calculated diffraction order intensities are indicated by vertical lines, experimental values by circles. Calculated intensities in the broadened image are indicated by circles, experimental values by the line.



(11 Percent Second Harmonic by Filter Plate Measurement)

Figure 14. Diffraction order light intensities and time-average light intensity for $G = 1/2$ for an ultrasonic wave containing 10 percent second harmonic and $v_1 = 2.4$. Calculated diffraction order intensities are indicated by vertical lines, experimental values by circles. Calculated intensities in the broadened image are indicated by circles, experimental values by the line.



(15 Percent Second Harmonic by Filter Plate Measurement)

Figure 15. Diffraction order light intensities and time-average light intensity for $G = 1/2$ for an ultrasonic wave containing 15 percent second harmonic and $v_1 = 2.4$. Calculated diffraction order intensities are indicated by vertical lines, experimental values by circles. Calculated intensities in the broadened image are indicated by circles, experimental values by the line.

The results in Figs. 13 through 15 demonstrate reasonable agreement between theory and experiment. Some deviation is to be expected because of sound beam inhomogeneities and the higher harmonics which were neglected. It appears that it is possible to make a fairly good determination from the discrete diffraction orders of the Raman-Nath parameters pertaining to the fundamental and second harmonics if one can neglect the higher harmonics. Higher harmonics can be included in the calculations but the problem becomes much more complicated as the number of variables is increased.

Some general features of the broadened images shown in Figs. 13 through 15 have been pointed out by Breazeale and Hiedemann (21). The asymmetry of the light distributions increases with increase in waveform distortion. This asymmetry corresponds to that of the diffraction orders; the continuous distribution is very approximately an envelope of the diffraction spectrum, lacking details such as the deep minimum which might correspond to the approximately zero central order.

D. Diffraction of Light Passing Through Two Adjacent Ultrasonic Waves of Different Frequency

Finite amplitude investigations have brought about increased interest in the diffraction of light by non-sinusoidal ultrasonic waves. As previously discussed, the simpler approach is to neglect the effects of higher harmonics and consider only the fundamental and second harmonic components of a distorted ultrasonic wave. There are some aspects of the theory of diffraction of light by such "two-component" waves that do not manifest themselves in finite amplitude investigations

because the relative phases between the harmonic components are fixed. Rao (24), Murty (25), and Mertens (26) have considered variable phase between two frequency components in a single ultrasonic beam. Murty and Rao (28) reported measurements which showed "striking" agreement between calculated and measured intensities, using two separate ultrasonic beams. Mertens (27) has recently pointed out that successive (separate beams) and simultaneous (single beam) diffraction are not the same. However, for the range of experimental parameters used by Murty and Rao the difference is small. The present investigation was carried out for experimental parameters which give a significant difference between simultaneous and successive diffraction.

The intensity of light in the n th diffraction order resulting from a single ultrasonic wave consisting of fundamental and second harmonic frequency components is

$$I_n = \left| \sum_{k=-\infty}^{+\infty} J_{n-2k}(v_1) J_k(v_2) \exp(-ik\Delta) \right|^2 \quad (32)$$

where Δ is the relative phase between the two components. The form given here is that given by Zankel and Hiedemann (11), specialized to the second harmonic. Since $I_n = \phi_n^2$, calculations can be made from Eqs. (32) and (25) for arbitrary light beam widths.

Mertens (27) obtained the light amplitudes in diffraction orders caused by the passage of light through two adjacent ultrasonic beams with frequency ratio 1:M where M is an integer. Results were given for the light beam passing through either ultrasonic beam first. For light

first passing the Mth harmonic (the case to be considered here), Mertens gave for the light amplitude in the nth diffraction order

$$\phi'_n = \sum_{k=-\infty}^{+\infty} J_{n-Mk} \left\{ v_1 \frac{\lambda^*}{L \sin \theta_{Mk}} \sin \left[\frac{\pi L}{\lambda^*} \tan \theta_{Mk} \right] \right\} \\ (\text{times}) J_k(v_M) \exp \left\{ i\pi(Mk-n)(L/\lambda^*) \tan \theta_{Mk} \right\} \exp(-ik\Delta), \quad (33)$$

where

$$\sin \theta_P = -P(\lambda/\mu_o \lambda^*). \quad (34)$$

L is the width of the second ultrasonic beam (the fundamental in this case), assuming the beams adjacent. The index of refraction μ_o of the undisturbed medium appears in Eq. (34) because the appropriate diffraction angles are those occurring in the medium as the light emerges from the first sound beam. Using the approximations $\sin \theta \simeq \theta \simeq \tan \theta$, Eq. (33) may be expressed in a simpler form which is valid for small θ . Carrying out these approximations and specializing Eq. (33) to the frequency ratio 1:2 one obtains the intensities

$$I'_n = \left| \sum_{k=-\infty}^{+\infty} J_{n-2k} \left(v_1 \frac{\sin kQ}{kQ} \right) J_k(v_2) \exp \left\{ i \left[k(n-2k)Q - k\Delta \right] \right\} \right|^2, \quad (35)$$

where

$$Q = (2\pi L \lambda / \mu_o \lambda^{*2}). \quad (36)$$

Comparing Eq. (32) with Eq. (35), one finds that the forms of the equations are similar. Equation (35) may be interpreted as expressing reduction of v_1 by the factor $(\sin kQ)/(kQ)$ to give an effective $v_1(k)$ which is different for each kth contribution to the sum. Similarly, each $k\Delta$ in the exponential

term undergoes an effective phase shift of $k(n-2k)Q$. Mertens' condition (27) that

$$(\pi ML / \lambda^*) \left| \sin \theta_P \right| \ll 1 \quad (37)$$

for the difference between simultaneous and successive diffraction to be negligible is equivalent to requiring in Eq. (35) that

$$(\sin kQ) / (kQ) \approx 1 \text{ and } k(n-2k)Q \approx 0. \quad (38)$$

Experimental measurements were obtained using the basic optical arrangement shown in Fig. 5 with the following exceptions. Two ultrasonic waves were produced in the manner shown in Fig. 16. Two 3.0 mc quartz transducers were driven from the same 3 mc oscillator. The filter plate was tuned to pass the 6.0 mc component of the ultrasonic wave which had become quite distorted when it arrived at the plate. Thus the finite amplitude effects served as a frequency doubler. The relative phase between the two components passing the light beam was varied by moving the variable transducer slightly in the direction of sound propagation by means of a precision screw. The relative amplitude of the second harmonic component was varied by moving the variable transducer over a range of approximately 50 cm.

There are some qualitative features common to simultaneous and successive diffraction. Both theories predict that the intensity of the central diffraction order oscillates with change in relative phase between the two frequency components. This is shown experimentally for successive diffraction in Fig. 17(a). These oscillations serve as an indicator of relative phase. The maxima correspond (theoretically) to

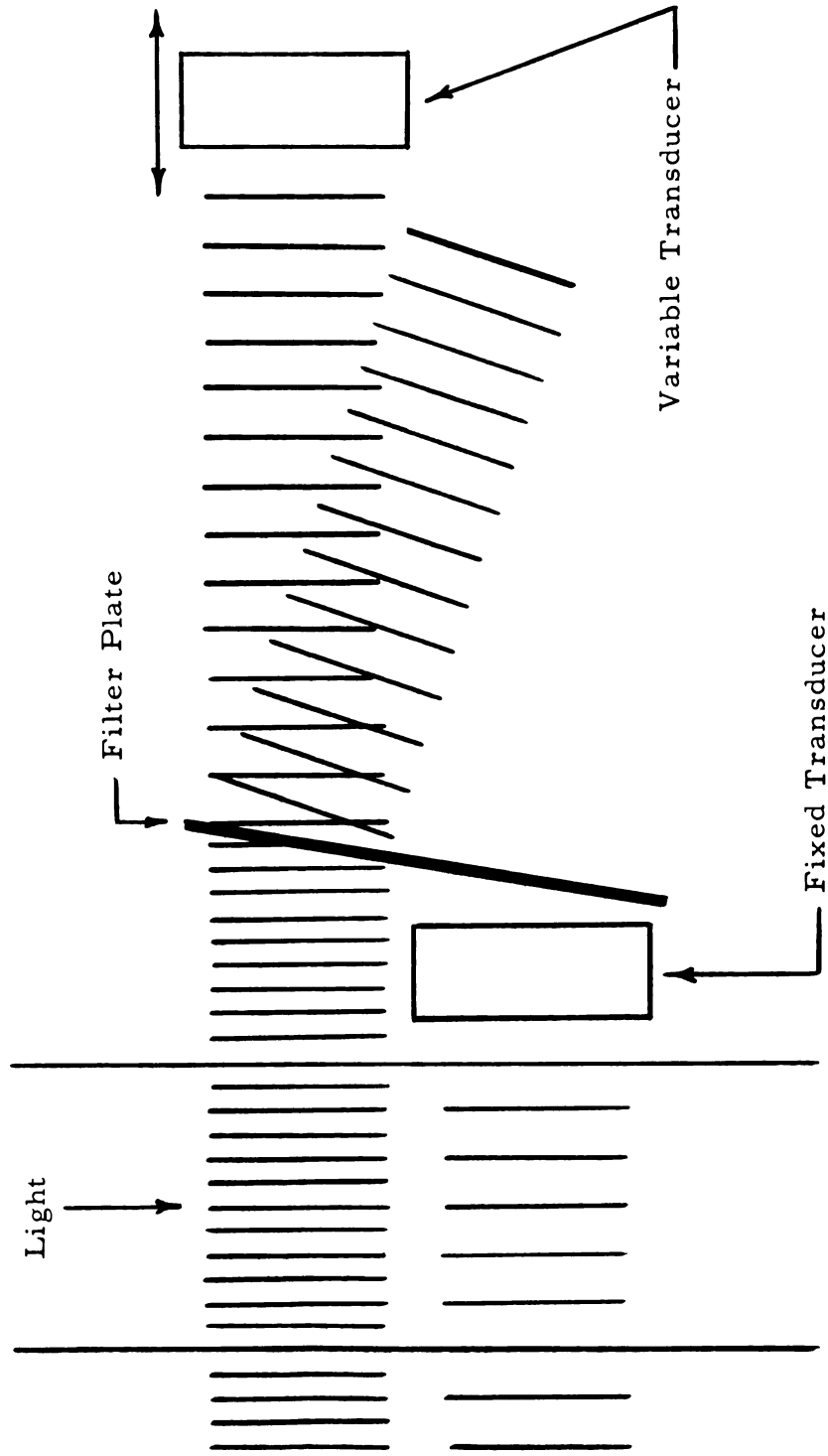


Figure 16. Diagram showing use of filter plate and two transducers at 3.0 mc to obtain two adjacent ultrasonic waves at 3.0 and 6.0 mc which may be adjusted in relative phase by moving the variable transducer.

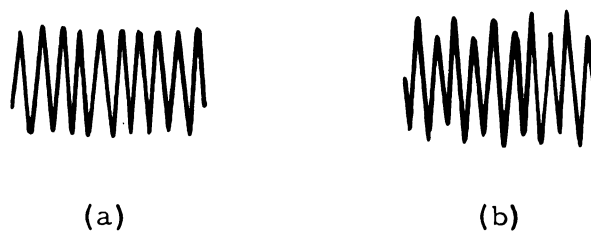


Figure 17. Observed oscillations of central order light intensity with change in relative phase between the 3.0 mc ultrasonic wave from the fixed transducer and the wave transmitted by the filter plate. (a) No fundamental passed by the filter plate. (b) Small amount of fundamental passed by the filter plate.

a $\pm$ cosine second harmonic ($\Delta = \pm\pi/2$) and the minima correspond to a $\pm$ sine second harmonic ($\Delta = 0$ or π), relative to a sinusoidal fundamental. Figure 17(b) shows the effect of a small amount of fundamental coming from the variable transducer because of an improperly adjusted filter plate. The alternation of peak amplitudes in Fig. 17(b) was caused by this extraneous fundamental as it interfered with the fixed fundamental component. The regularity of the oscillations in Fig. 17(a) indicates that the fundamental was effectively eliminated by the plate. From the known distance required to move the variable transducer to produce these cyclic variations in light intensity, a calculation of the sound velocity gave good agreement with the accepted value.

Both theories also predict that the light intensity distributions are symmetric for $\pm$ cosine second harmonic and asymmetric for $\pm$ sine second harmonic, for either wide or narrow light beams. Using the intensity of the central order as an indicator of relative phase, the results shown in Fig. 18 were obtained. It can be seen that the theoretical predictions regarding symmetry and asymmetry are experimentally confirmed. The narrow beam diffraction patterns are for $G = 1/2$. The fundamental Raman-Nath parameter v_1 was approximately 3.8 and v_2 was approximately 0.5 for the measurements in Fig. 18.

Calculations were made from Eq. (32) and from Eq. (35) for $v_1 = 2.4$ and various amounts of second harmonic with different relative phases. This choice of v_1 permits use of and comparison with calculations and experimental results obtained for the case of a distorted finite amplitude

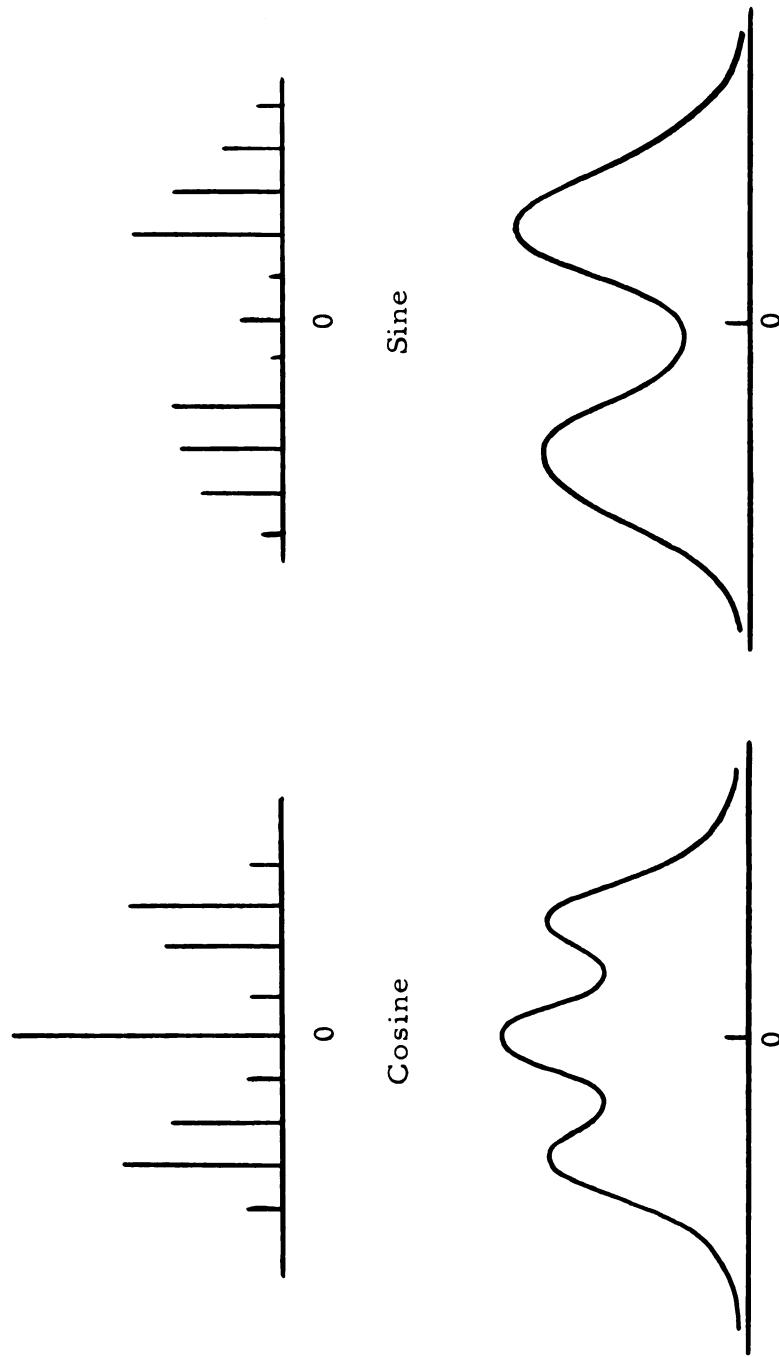


Figure 18. Observed diffraction order light intensities and time-average light intensity for $G = 1/2$ showing symmetry for cosine second harmonic and asymmetry for sine second harmonic. v_1 approximately 3.8 and v_2 approximately 0.5.

ultrasonic wave and a wide light beam. In calculations made using Eq. (35) (successive diffraction) an effective L was used. The equation was derived for the two ultrasonic beams adjacent, i. e., no space between the beams. In order to approximately account for the unavoidable separation of the beams in the actual experimental arrangement, an effective L was chosen equal to the actual beam width plus the width of the space between the two beams. This approximation is somewhat justified because the L involved in the theory is the distance the light travels between emergence from the first beam and emergence from the second beam. For the experimental arrangement used, the effective L was 5 cm. Figure 19 shows the calculated extremes of light intensity (which correspond to $\pm$ sine second harmonic) for the first diffraction orders. The extremes for simultaneous diffraction are precisely those first order intensities previously calculated and verified for distorted finite amplitude waves, neglecting harmonics higher than the second. The results in Fig. 19 show that there is a significant difference between theoretical values for simultaneous and successive diffraction for the experimental parameters considered here (3.0 and 6.0 mc in water and $v_1 = 2.4$). In either case it can be seen that there is an approximately linear relationship between the differences of extremes of first order light intensity and the amount of second harmonic. Also, as before, the average of the extremes is approximately constant, independent of the amount of second harmonic.

In order to qualitatively check the theoretical predictions described in the previous paragraph the extremes of light intensity in the first

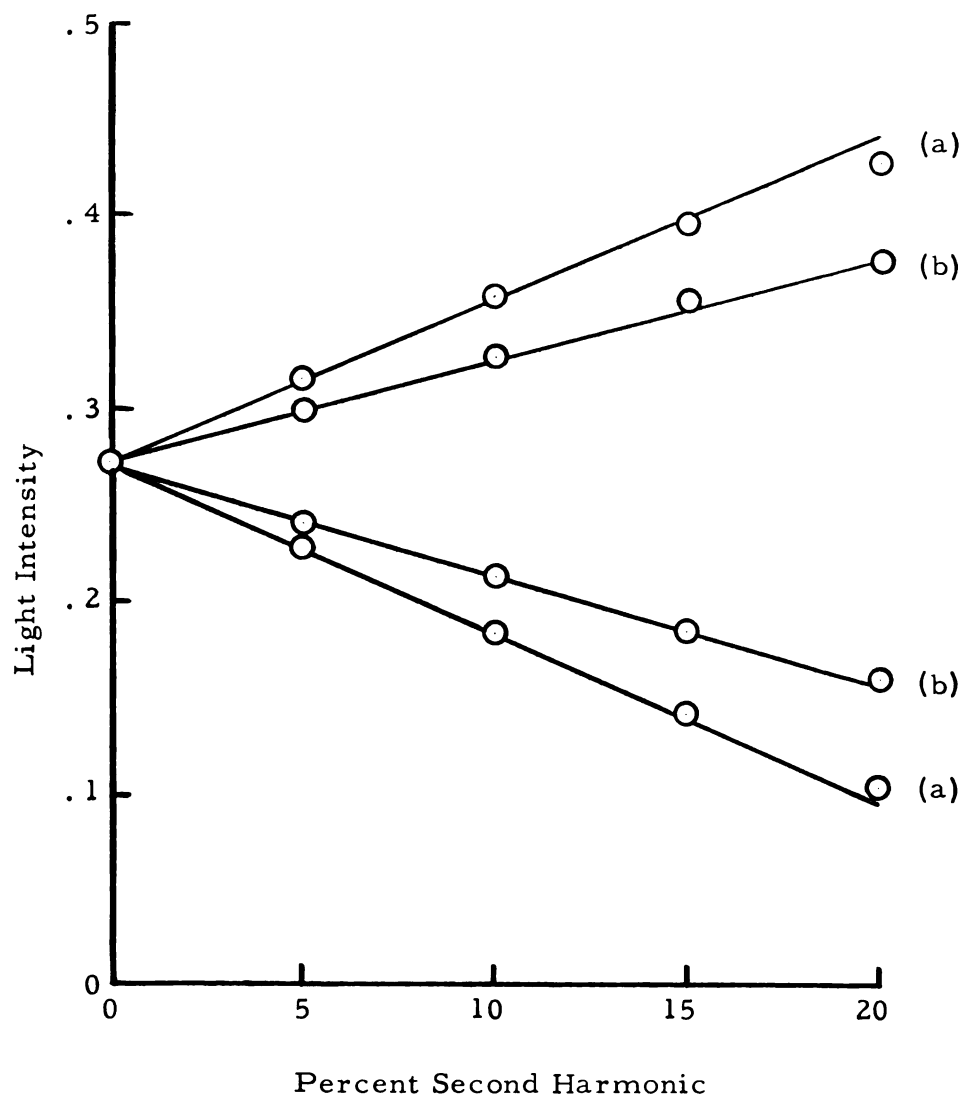


Figure 19. Calculated extremes of light intensity in the first diffraction orders for $v_1 = 2.4$ with change in relative phase of various amounts of second harmonic for (a) simultaneous diffraction and (b) successive diffraction with $L = 5$ cm.

diffraction orders were observed for varied amounts of second harmonic. The measured extremes are shown in Fig. 20. The two extremes observed for a given amount of second harmonic are plotted vs the corresponding difference in intensity. The actual amount of second harmonic was not determined for these measurements. The results verify the theoretical prediction that the average intensities are approximately independent of the amount of second harmonic.

In the measurements described in the previous paragraph, some discrepancies between results obtained from opposite first orders were observed. This may be caused by a small amount of distortion (second harmonic) in the fixed transducer beam. This interpretation is supported by the fact that a small amount of asymmetry was observed in the diffraction spectrum resulting from the fixed transducer beam only. Since second harmonic in the fixed beam is fixed in relative phase with respect to the fundamental in the same beam, one should be able to observe a sum and difference effect as the two sources of second harmonic are varied in relative phase. This effect is shown in Fig. 21 which is to be interpreted as follows. When the two second harmonic contributions interfere constructively they are both in the same relative phase with the fundamental. When they interfere destructively the resultant is in opposite relative phase with the fundamental. Thus when the variable second harmonic is varied in relative phase the observed extremes in first order light intensity should coincide with a theoretical value at only the maximum or a minimum extreme of light intensity,

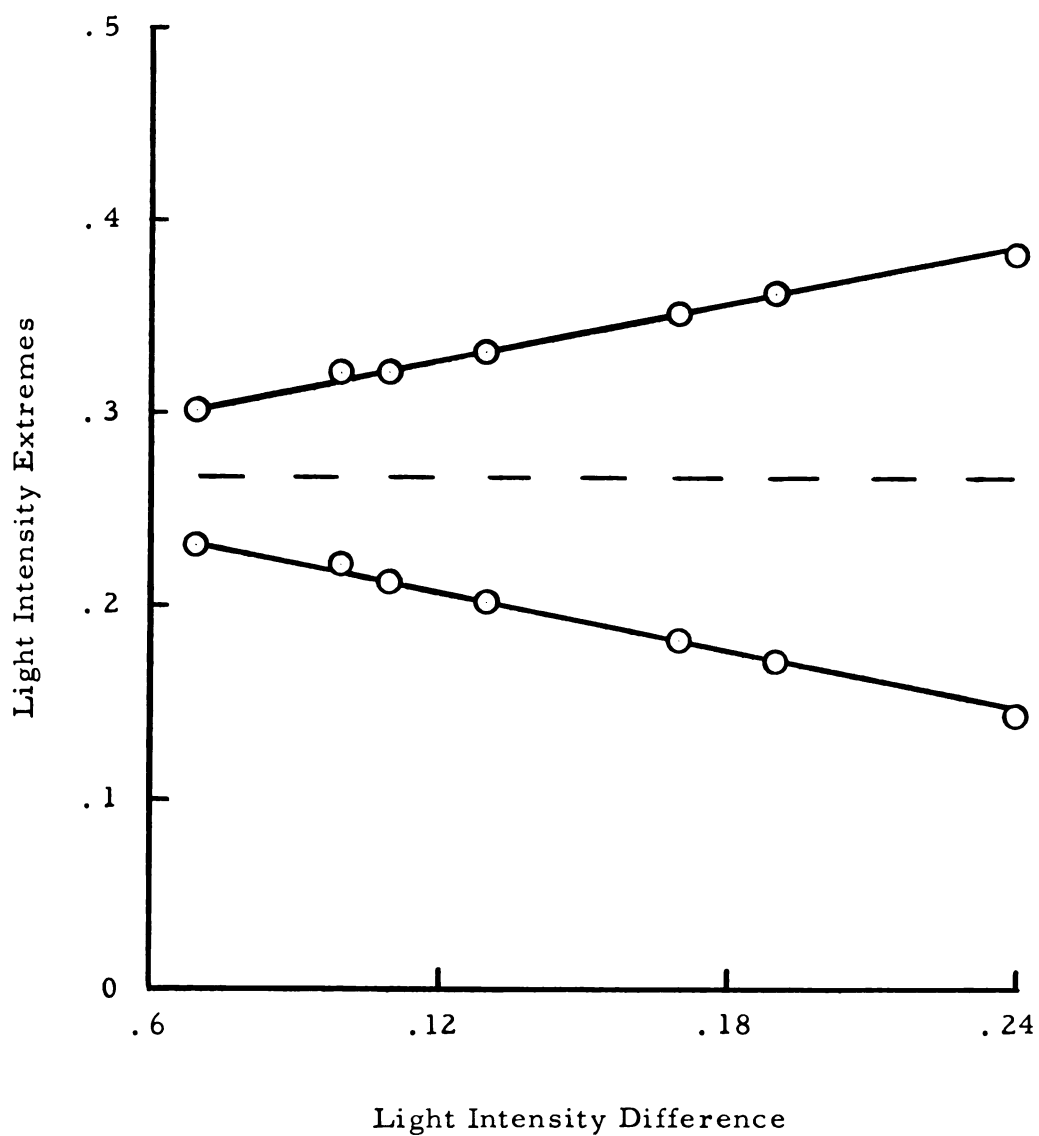


Figure 20. Observed extremes of light intensity in the first diffraction order for $v_1 = 2.4$ with change in relative phase of various amounts of second harmonic vs the corresponding difference in light intensity showing that the average intensity is approximately independent of the amount of second harmonic.

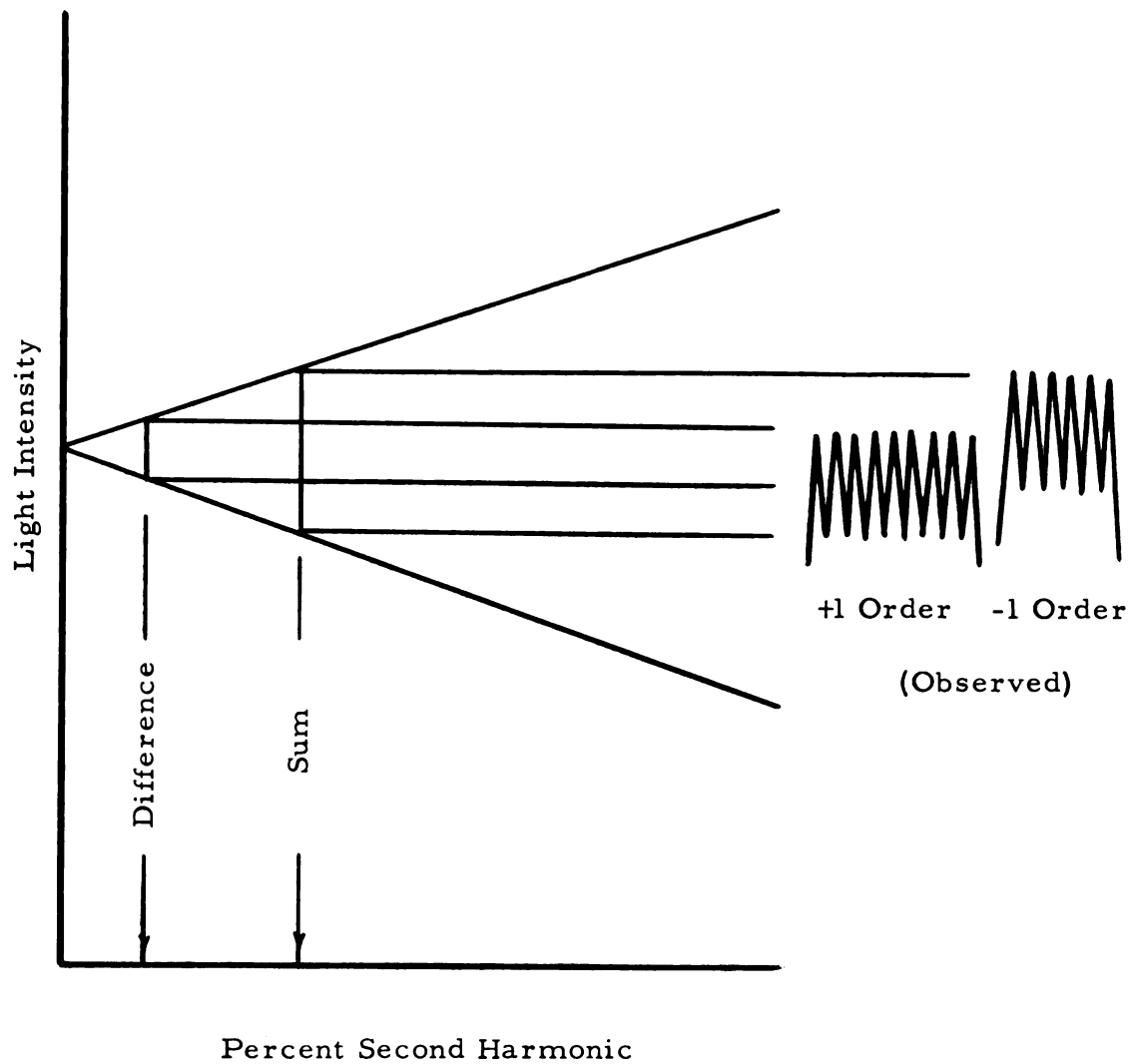


Figure 21. The influence of second harmonic in the fixed transducer beam on the observed oscillation of the first order light intensities as the relative phase of the variable second harmonic is varied.

depending on which ($\pm$) order is observed. Note that the magnitude of the oscillation in intensity of a given first order corresponds to the amount of second harmonic in the variable beam, even though values at the extremes may deviate from the theoretical ones at one extreme or the other. The recorder trace in Fig. 21 verifies that this actually occurs. This curve shows the effect enhanced because it was observed at a greater than normal distance from the fixed transducer where a greater amount of finite amplitude distortion was present.

The approximately linear relationship between the amount of second harmonic and the amplitude of oscillation in the first diffraction order intensity, like the asymmetry in the investigation of distorted finite amplitude waves, suggests another means for measuring the second harmonic content of a distorted ultrasonic wave of arbitrary amplitude. A filter plate might be used to pass the second harmonic component of the distorted wave and the magnitude of this component measured by observing the amplitude of oscillation in the first diffraction orders when the fixed transducer is radiating at the fundamental frequency with $v_1 = 2.4$. Using such a procedure one measures a larger effect than the diffraction produced by the second harmonic alone. In contrast to the method used to determine the second harmonic content of a distorted finite amplitude wave using a fixed local value of the fundamental component and neglecting higher harmonics, this procedure permits an arbitrary fundamental component. Thus the second harmonic content may be determined as a function of propagation distance for a fixed initial fundamental ultrasonic pressure amplitude.

The procedure outlined in the previous paragraph was followed for various propagation distances between the variable transducer and the filter plate. The indicated amounts of second harmonic as obtained basing the interpretation on (a) simultaneous diffraction [Eq. (32)], and (b) successive diffraction [Eq. (35)] are shown in Fig. 22. Also shown in the figure are (c) measurements of the second harmonic made by measuring the first order of diffraction caused by only the second harmonic which passes the filter plate. The difference between indications from simultaneous and successive diffraction theory is greater than experimental error. However, within the estimated experimental error, the interpretation based on successive diffraction theory by Mertens agrees with the filter plate measurements. This result indicates the validity of Mertens' theory. Experimental limitations may be responsible for some of the lack of agreement. Phase and amplitude difference over the field of observation (usually about 5×5 mm) would tend to reduce the extremes of light intensity because of averaging over the field. Such differences over the field might arise from beam inhomogeneity, filter plate irregularities, or failure to achieve perfect alignment of the two ultrasonic wavefronts.

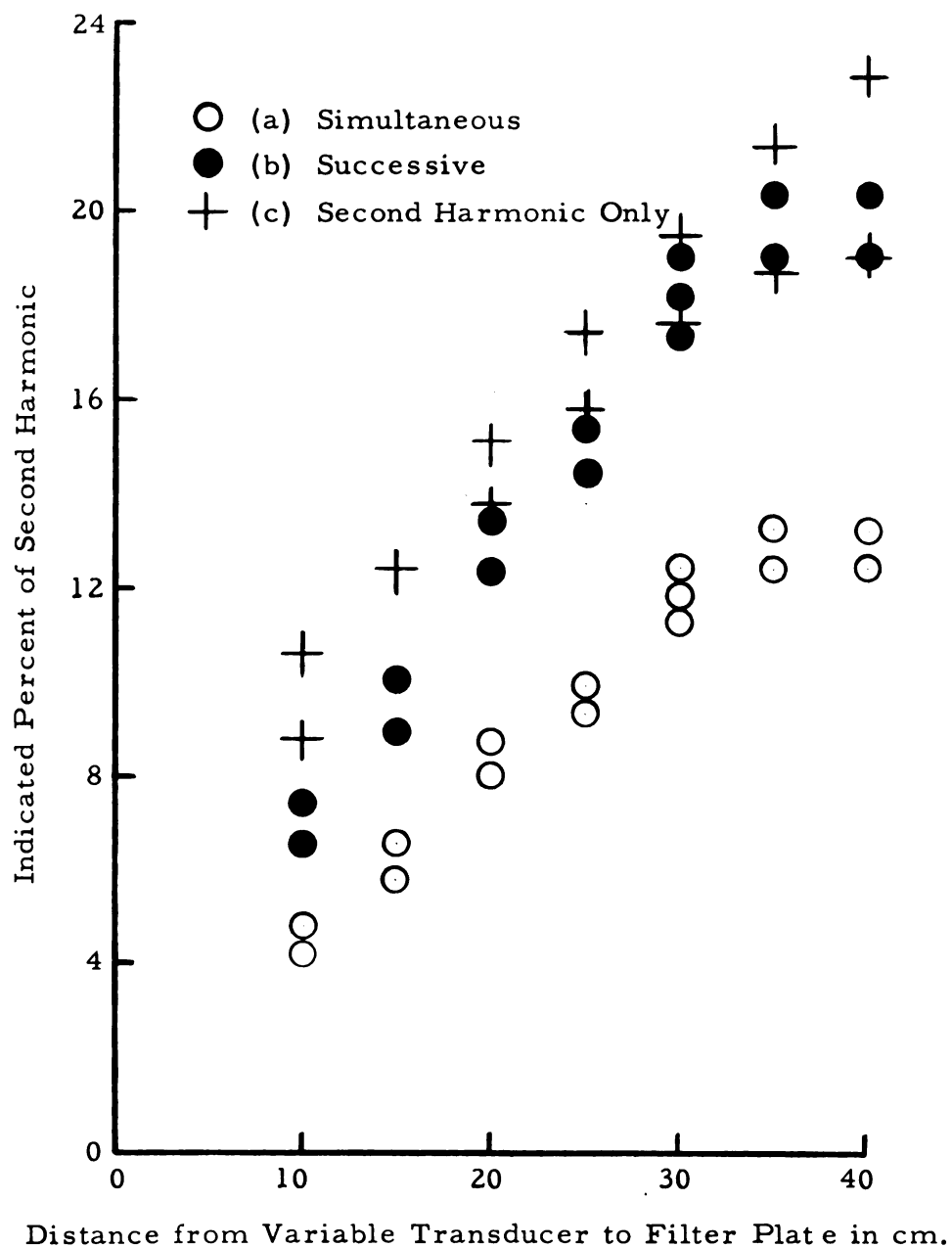


Figure 22. Indicated percent of second harmonic at different propagation distances from the variable transducer to the filter plate as obtained from (a) simultaneous diffraction theory, and (b) successive diffraction theory, and (c) measurements obtained with only the second harmonic present. $v_1 = 2.4$ when present.

CHAPTER IV

SUMMARY

Experimental confirmation has been obtained from some predictions of the theory developed by Zankel to explain various aspects of diffraction of light by progressive ultrasonic waves. This theory was developed for the range of moderate frequencies and amplitudes where the Raman-Nath approximations are valid. Theoretical results have been cited which include arbitrary light beam width, ultrasonic waveforms expressible by a Fourier sine series, and ultrasonic waveforms consisting of fundamental and second harmonic components combined with arbitrary relative phase. It has been shown that these theoretical results are compatible with several previous results, both theoretical and experimental.

Good quantitative experimental confirmations of the theory have been obtained for sinusoidal ultrasonic waves and narrow light beams. The theory has also been confirmed for distorted finite amplitude ultrasonic waves and wide and narrow light beams. Some qualitative features of the dependence of diffraction on the relative phase between two adjacent ultrasonic waves with frequency ratio 1:2 have been experimentally shown to be correctly given by either the theory of Murty, Rao, and Mertens for simultaneous diffraction or by the theory of Mertens for successive diffraction. Quantitative measurements have shown that the theory of Mertens must be applied for successive diffraction in the range of experimental parameters investigated here. All measurements were carried out using progressive ultrasonic waves in water.

Further work of the type described herein is in progress in the ultrasonics laboratory at Michigan State University.

BIBLIOGRAPHY

1. P. Debye and F. Sears, Proc. Natl. Acad. Sci. U. S. 18, 409-414 (1932).
2. R. Lucas and P. Biquard, J. phys. radium 3, 464-477 (1932).
3. C. V. Raman and N. S. Nath, Proc. Indian Acad. Sci. A2, 406-412 (1935).
4. C. V. Raman and N. S. Nath, Proc. Indian Acad. Sci. A3, 75-84 (1936).
5. E. Fubini-Ghiron, Alta Frequenza 4, 530-581 (1935).
6. R. Extermann and G. Wannier, Helv. Phys. Acta. 9, 520-532 (1936).
7. S. M. Rytov, "Diffraction de la lumière par les ultra-sons," Actualités scientifiques et industrielles 613, Paris, 1938, Hermann et Cie.
8. F. H. Sanders, Can. J. Research A14, 158-171 (1936).
9. G. W. Willard, J. Acoust. Soc. Am. 21, 101-108 (1949).
10. R. B. Miller and E. A. Hiedemann, J. Acoust. Soc. Am. 30, 1042-1046 (1958).
11. K. L. Zankel and E. A. Hiedemann, J. Acoust. Soc. Am. 31, 44-54 (1959).
12. I. G. Mikhailov and V. A. Shutilov, Akust. Zhur. (USSR) 3, 203-204 (1957); Soviet Phys.-Acoustics 3, 217-219 (1958).
13. I. G. Mikhailov and V. A. Shutilov, Akust. Zhur. (USSR) 4, 174-183 (1958); Soviet Phys.-Acoustics 4, 174-184 (1958).
14. I. G. Mikhailov and V. A. Shutilov, Akust. Zhur. (USSR), 5, 77-79 (1959); Soviet Phys.-Acoustics 5, 75-78 (1959).
15. V. A. Shutilov, Akust. Zhur. 5, 231-240 (1959); Soviet Phys. - Acoustics 5, 230-238 (1959).
16. B. D. Cook, J. Acoust. Soc. Am. 32, 336-337 (1960).
17. R. Lucas, Compt. rend. 199, 1107-1108 (1934).

18. T. Hueter and R. Pohlman, Z. angew. Phys. 1, 405-411 (1949).
19. M. A. Breazeale and E. A. Hiedemann, J. Acoust. Soc. Am. 31, 24-28 (1959).
20. A. P. Loeber and E. A. Hiedemann, J. Acoust. Soc. Am. 28, 27-35 (1956).
21. M. A. Breazeale and E. A. Hiedemann, J. Acoust. Soc. Am. 30, 751-756 (1958).
22. M. A. Breazeale, B. D. Cook, and E. A. Hiedemann, Naturwissenschaften 45, 537 (1958).
23. L. E. Hargrove, K. L. Zankel, and E. A. Hiedemann, J. Acoust. Soc. Am. 31, 1366-1371 (1959).
24. B. R. Rao, Proc. Indian Acad. Sci. A29, 16-27 (1949).
25. J. S. Murty, J. Acoust. Soc. Am. 26, 970-974 (1954).
26. R. Mertens, Proc. Indian Acad. Sci. A48, 288-306 (1958).
27. R. Mertens, Z. Physik 160, 291-296 (1960).
28. J. S. Murty and B. R. Rao, Z. Physik 157, 189-197 (1959).
29. K. L. Zankel, J. Acoust. Soc. Am. 32, 709-713 (1960).
30. W. G. Mayer and E. A. Hiedemann, J. Acoust. Soc. Am. 32, 706-708 (1960).
31. E. H. Wagner, Z. Physik 141, 604-621 (1955).
32. W. Keck and R. T. Beyer, Phys. Fluids 3, 346-352 (1960).
33. L. E. Hargrove, J. Acoust. Soc. Am. 32, 511-512 (1960).
34. M. A. Breazeale, L. E. Hargrove, and E. A. Hiedemann, U. S. Navy J. Underwater Acoust. 10, 381-387 (1960).

MICHIGAN STATE UNIV. LIBRARIES



31293017640248