THE DIFFRACTION OF LINEARLY POLARIZED LIGHT BY ULTRASONIC WAVES IN TRANSPARENT SOLIDS

Thesis for the Degree of Ph. D.
MICHIGAN STATE UNIVERSITY

Myron Paul Hagelberg

1961

[redacted]

This is to certify that the

thesis entitled

THE DIFFRACTION OF LINEARLY POLARIZED LIGHT BY

ULTRASONIC WAVES IN TRANSPARENT SOLIDS

presented by

MYRON PAUL HAGELBERG

has been accepted towards fulfillment
of the requirements for

Ph. D. degree in Physics

E. A. Hiedemann
Major professor

Date May 5, 1961

O-169

ABSTRACT

THE DIFFRACTION OF LINEARLY POLARIZED LIGHT BY ULTRASONIC WAVES IN TRANSPARENT SOLIDS

by Myron Paul Hagelberg

Mueller [Z. Kristallogr. A., 99, 122 (1938)] has suggested two methods for the determination of the ratio p/q of the strain-optical constants of transparent, amorphous solids. These methods have been used by Gates and Hiedemann [J. Acoust. Soc. Am., 28, 1222 (1956)] in studying the photo-elastic properties of a series of American optical glasses and fused silica. Their investigation shows that one of Mueller's methods, method "B," is valid as described but that the second, method "C," in conjunction with an experimental procedure suggested by Bergmann [Naturwissenschaften, 24, 492 (1936)] gives inconclusive results. The primary purpose of this study is to investigate the experimental conditions under which these photo-elastic studies are made and to determine whether or not Mueller's method "C" is valid. In addition, the assumption of the coherence of the two polarized components is tested directly. Theoretical arguments are given describing the change in the observed results with variation in the experimental parameters. Values of p/q obtained in this investigation are compared with those given by Gates and Hiedemann for the same glasses to determine the effects of aging.

Since Bergmann's experimental setup and procedure may introduce discrepancies through the use of the photographic method for light intensity determinations and the use of the Wollaston double image prism, method "C" is studied by means of experimental techniques which eliminate both of these factors. Light intensity measurements are made by means of a photomultiplier-microphotometer. The Wollaston prism is eliminated by using a polarizer which may be rotated to permit readings of the light intensity for various polarizations.

It is found that Mueller's method "C" gives results which are consistent for each glass sample with the results obtained by method "B" within the limits of the experimental error. The assumption of the coherence of the two polarized components is found to be valid since the variation of the intensity of the light in the first diffraction order with the angle of polarization is as predicted on the basis of this assumption. The deductions from Mueller's theory concerning the variation of the observed results with changes in the experimental parameters are also found to be valid in every case. This leads to the conclusion that the results reported by Gates and Hiedemann using method "C" must result from the use of the photographic method for measuring the light intensities.

THE DIFFRACTION OF LINEARLY POLARIZED LIGHT BY ULTRASONIC WAVES IN TRANSPARENT SOLIDS

By

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A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics and Astronomy

1961

ACKNOWLEDGMENTS

I wish to express my sincere appreciation to Dr. E. A. Hiedemann who suggested this problem. His valuable and patient guidance has been instrumental in bringing this work to its successful conclusion. I am also grateful to Dr. M. A. Breazeale and Dr. T. S. Narasimhamurty for many fruitful discussions and for the critical reading of several sections which they have undertaken.

It is a pleasure to recognize the financial support of the Standard Oil Foundation. The fellowship funds which they have provided have made this investigation possible.

M. P. Hagelberg

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I. Introduction

1. Basic Concepts of Photoelastic Theory.

Studies of the photoelastic behavior of transparent solids are generally concerned with either or both of two problems. The first of these is the phenomenological description of the optical effects produced in a light beam which has traversed the solid by stresses and/or strains in the solid. These effects are considered to be known when the strain-optical constants and the elastic constants of the medium have been determined. The second problem is that of relating the photoelastic behavior of a solid to the arrangement of the atoms of which it is composed and to the nature of these atoms. This latter problem will not be taken up in detail here.

In studies of amorphous solids, the strain-optical constants introduced by Neumann[1] may be used. These constants, defined in terms of a typical strain z_z,

$$\begin{aligned} dn_z &= n_z - n = -n^2 q z_z \\ dn_x &= n_x - n = -n^2 p z_z \end{aligned} \qquad (1)$$

relate the strain to the change produced by it in the index of refraction for light polarized parallel and normal, respectively, to the direction of the strain. In the general case, the elasto-optical behavior may be represented in terms of the thirty-six phenomenological constants p_{ij} $(i, j = 1, 2, \ldots, 6)$ given by Pockels,[2] which relate the strain tensor to the optical effects which accompany it. For amorphous solids, these constants are found by symmetry considerations to be reduced to two, p_{11} and p_{12}. These two Pockels constants for isotropic media are related to the

Neumann constants by

$$p_{11} = 2q/n \text{ and } p_{12} = 2p/n. \qquad (2)$$

It is assumed that for the purposes of this investigation, the finely-annealed optical glass is isotropic, and that therefore the Neumann strain-optical constants may be used.

The optical effects discussed above may also be described in terms of the stresses from which they result and, indeed, static determinations of the photoelastic behavior of solids result in values of the stress-optical constants. Mueller[3] suggests that the strain-optical constants have greater theoretical interest than the stress-optical constants in relating the optical effects to the structure of the medium. Filon and Jessop[4] find that when the elastic limit is exceeded, the change in index of refraction is proportional to stress but not to strain. However, in the present investigation, the stresses are small and it is assumed that Hooke's law applies. Thus, stress and strain are related by the elastic constants and are therefore equally significant.

Absolute determinations of the photoelastic constants of various glasses by the application of static stresses have been made by Mach,[5] Pockels,[6] Filon,[7] Twyman and Perry[8] and Schaefer and Nassenstein.[9] These static measurements are of two types: one, using a Babinet compensator, measures the difference in the retardations of the ordinary and extraordinary beams leading to a value for the difference p - q of the Neumann constants. The second method requires difficult interferometric measurements, but yields values for the constants themselves, although the accuracy of the results is significantly less than for the compensator measurements. The interferometric method requires large samples of strain-free glass, which in some variations must have a specific shape. Other variations require two "identical" samples. Coker and Filon[10] point out that an additional complication is the possibility of relaxation or plastic flow. A further complication arises in that the results are obtained in terms of the stress-optical constants. Then, in order to determine the strain-optical constants, a knowledge of the elastic constants is required.

Dynamic studies of the strain-optical constants are also possible. With this technique, the strains are produced by ultrasonic waves in the glass. It should be noted that these strains occur adiabatically. The optical effects are observed in the diffraction of a beam of light which has traversed the sound field. Measurements by these methods give directly the ratios p/q of the strain-optical constants. A complete determination of p and q by this method requires also a measurement of the sound intensity; this is not feasible.

Mueller[3] suggests that the values of p and q could be obtained by a combination of the ultrasonic or dynamic method and the compensator technique, which yields the value of $p - q$, thus replacing the difficult and unreliable interferometric measurements. Such a determination assumes that the values of p and q which occur in the dynamic experiments are the same as those in the static experiments. Schaefer and Dransfeld[11] used a dynamic method to measure the ratio p/q of the strain-optical constants of the same set of glasses, numbering about 150, for which Schaefer and

Nassenstein[9] determined the values of p and q by static methods. Comparison of the values of p/q obtained by the dynamic method with those obtained by the static techniques yielded an average difference of 1.5 percent, with the dynamic values averaging slightly higher. Since the announced overall accuracy was 5 percent, this difference does not seem to be significant. Thus the dynamic and static values appear to be equivalent.

2. Theory and Techniques of the Dynamic Methods.

In 1932, Debye and Sears[12] in America and Lucas and Biquard[13] in France, discovered that if a beam of light passes through an ultrasonic field, a diffraction pattern is produced which is similar to that caused by a ruled grating. The grating constant in this case corresponds to the wavelength of the sound in the medium. A theory of this diffraction for a sound field in a liquid has been developed by Raman and Nath.[14] This theory is based on the phase changes introduced in the light wave train by the periodic variations of the index of refraction in the sound field. A plane light wave front which impinges on the sound field is altered in phase to produce a "corrugated" wave front. When this light is focused by an objective lens, a diffraction pattern is produced. The analysis of Raman and Nath leads to expressions for the direction of the diffraction orders and their intensity and frequency. The deductions from this theory have been verified both qualitatively by Bär[15] and quantitatively by Sanders;[16] more recently Miller and Hiedemann[17] have reported quantitative agreement over a rather wide range of frequencies and sound intensities. It will be shown later that this theory is applicable to the conditions of this study. The Raman and Nath elementary theory is particularly useful over the range in which it is appropriate because of the simplicity of the expressions which are derived from it.

Diffraction effects can also be produced when a light beam impinges on ultrasonic waves in transparent solids, such as glass. Here the problem is somewhat more complex than in a liquid because of the elasticity of the solid. Schaefer and Bergmann[18, 19, 20] used a point source of light to show diffraction by many sound waves traveling in different directions simultaneously. This type of experiment produces a characteristic pattern, the so-called Schaefer-Bergmann pattern, in which the diffraction orders are circles concentric to the central, undeviated spot for waves in an amorphous solid. A slit source was used by Hiedemann and Hoesch[21, 22, 23] to show diffraction by a single plane ultrasonic wave excited in a solid. The slit is oriented parallel to the sound wave fronts and the diffraction pattern is composed of a series of equally spaced images of the slit (if only longitudinal waves are present). In practice, this line pattern is much more readily obtained than the Schaefer-Bergmann pattern since the slit source is brighter than the single "point" and since the sonic energy may be concentrated in a single sound wave train. By this ease of obtaining the line pattern, dynamic measurements of p/q are facilitated.

The Raman and Nath theory has been used by Mueller[3, 24] as a basis for a theory of the diffraction of light by an ultrasonic field in a solid. In this theory, the variations in the refractive indices for light polarized parallel and normal to the sound wave fronts are evaluated in terms of the strain-optical constants of the medium. This leads to expressions for the direction, polarization, intensity and frequency of the light in the diffraction orders in terms of the polarization of the incident light, the strain-optical constants and the sound intensity. Many of the conclusions from this theory have been verified experimentally by Hiedemann[25, 26].

In his extension of the Raman and Nath theory, Mueller gives details of three methods for evaluating experimentally ratios of the photoelastic constants. Two of these methods apply to glasses and also to certain crystals; the third method is applicable only to crystals.

Method "A," which applies only to crystals, has been used by Burstein, Smith and Henvis[27] and by Galt.[28] The results of these studies have been compared with a theory, given by Mueller,[29] which ascribes the photoelastic behavior of these crystals to changes which occur in the structure of the crystal and its atoms under the influence of an external strain. Galt reports agreement with this theory, while Burstein and Smith[30] indicate the need for further consideration.

Method "B," which, like method "C," applies to isotropic solids as well as to certain types of crystals, involves the measurement of the polarization of the diffraction orders. Vedam[31, 32] and Gates and Hiedemann[33] used the line pattern in making measurements by this method. Ramavataram[34] and Schaefer and Dransfeld[11] used the Schaefer-Bergmann pattern. Ramavataram also measures the difference p - q by static methods and is therefore able to give values for the individual strain-optical constants.

Method "C" requires measurement of the ratio of the intensities in the first diffraction order of light polarized parallel and perpendicular to the sound wave fronts. Bergmann and Fues[35] used for such measurements a Wollaston prism which splits the light beam entering a vibrating glass sample into slightly divergent beams polarized perpendicularly to each other. The diffraction orders of the two polarized beams of light are recorded simultaneously on the same photographic plate. The intensities are then determined by means of a densitometer.

These dynamic methods for the determination of the photoelastic constants offer certain characteristic advantages over the static techniques: (1) The results are obtained directly in terms of the strain-optical constants, which according to Mueller, have greater theoretical importance than the stress-optical constants. This eliminates the necessity of a knowledge of the elastic constants. (2) The use of large, "identical," and/or specifically shaped samples is eliminated. The samples used in this investigation were nearly all cubes approximately one inch on a side. (3) Effects due to relaxation processes with time constants which are long compared with the period of the sound as well as plastic flow or "creep" are no longer a complicating factor.

A single significant disadvantage of these methods must also be noted. Since most of the energy dissipated from the sound field is converted into heat within the solid, the temperature within the block rises as long as the sound field is maintained. This results in disturbances of the sound field, particularly at higher sound intensities. According to Schaefer and Dransfeld,[11] the variation of p/q is small (less than 0.5 percent/C^{o} for the glasses which they tested); however, the standing wave system within the block is disturbed by the changes in sound velocity and in the dimensions of the block which accompany the heating. This means that measurements can only be taken for short periods of time separated by periods of several minutes during which the block returns toward thermal equilibrium. Before another measurement is made, one must adjust the frequency of the sound field to renew the standing wave system within the block.

The problem of correlating the observed photoelastic behavior of an amorphous solid with the internal structure and composition of the substance has been considered by Mueller.[36, 37] As mentioned previously, Mueller[29] has also treated this problem for the case of cubic crystals. The theory which has been developed for amorphous solids ascribes this photoelastic behavior to two effects. One of these effects depends on elastic alterations in the Lorentz-Lorenz interactions between dipoles; a second effect is due to the production of an artificial optical anisotropy of the atoms. Under pressure the first results in positive birefringence, the second in negative birefringence. These two effects seem to account for the variation in the photoelastic constants between glasses of different densities. In crystals, the effect of strains on the Coulomb fields of the ions must also be considered. An effect due to alignment of optically anisotropic molecules is discussed by Treloar[38] and by Braybon.[39] This latter effect, however, appears to be significant only for long-chain high-polymers.

Gates and Hiedemann[33] have determined, by Mueller's method "B," the ratio p/q of the strain-optical constants of a series of American optical glasses and fused silica. In addition, an attempt was made to show that method "C" gives the same results. The experimental set-up used for method "C" was essentially the same as that used by Bergmann.[35] The measurements gave results which are neither in agreement with Mueller's theory nor with the data obtained by method "B." Since both methods "B" and "C" are based on the same theoretical assumptions, it seems likely, as Gates and Hiedemann have suggested, that the lack of agreement between the data obtained by method "C" and the theoretically predicted result was because of some inadequacy in the experimental arrangement or technique. The primary purpose of this investigation is to demonstrate the validity of Mueller's method "C" and to determine why the measurements of Gates and Hiedemann by Bergmann's procedure were inconclusive. The measurements of light intensity were made directly by means of a photomultiplier - microphotometer rather than photographically, thus eliminating the difficult densitometer measurements. In addition, the Wollaston prism is no longer needed, thus removing another possible difficulty in the experimental set-up.

A number of other studies were also undertaken in order to study the sensitivity of the experiment to the several parameters which can be varied experimentally. Since Mueller's theory assumes the light beam to be incident at right angles to the sound field, it appears interesting from both a theoretical and a practical standpoint to investigate the effect of oblique incidence. The Wollaston prism used with Bergmann's experimental set-up causes a finite deviation from normal incidence if it is used to split the light beam before it enters the vibrating glass. This study should determine whether or not the use of a Wollaston prism is the cause of the discrepancy between the results, by methods "B" and "C," reported by Gates and Hiedemann.

A study of the effect of a constant stress on the sample, in addition to the dynamic stress produced by the sound field, was undertaken to determine whether any systematic change in the measured value of R occurs. In addition to the possibility of obtaining further information concerning the photoelastic behavior of the glass, this study should also determine the sensitivity of these dynamic experiments to the way in which the block is mounted.

The effect of placing the polarizer in method "B" at angles other than that which makes the plane of polarization of the incident beam at forty-five degrees to the sound wave fronts is also determined. The results of these measurements may be compared with an extension of Mueller's theory, discussed below, which pertains to this particular problem. Further, this same study makes it possible to determine the inaccuracy introduced because of any uncertainty in setting the polarizer to the required angle.

Measurements of the light intensity, in method "C," are made at angles other than normal and parallel to the sound wave fronts. These values are than compared with the values predicted by Mueller's theory. Agreement of the experimental values with the theoretical prediction gives a direct check on the validity of one of the basic assumptions of Mueller's theory. Such agreement would be, therefore, of considerable theoretical importance.

Finally, since the same set of glasses which Gates and Hiedemann used for their investigations was available and since a period of somewhat over four years has elapsed since their measurements were made, determinations of p/q for these glasses are repeated in order to determine whether there is any significant aging effect over this period. Filon[40] and Harris[41] have reported aging effects in their investigations made by static methods. Filon's measurements were made after a period of three years. The results reported by Harris involved a time lapse of sixteen, and, in some cases, twenty years from the original measurements. It is suggested in Harris' paper that change in the elasto-optical constants is comparatively rapid in the years immediately following its casting and tending toward a steady value.

Narasimhamurty[42] has recently developed a technique for measuring the ratios of the strain-optical constants of certain crystals. Since this method is also applicable to glasses, a discussion of the method and its theoretical basis is included as an appendix. A second appendix gives a discussion of the determination of the photoelastic constants of crystals excited by ultrasonic waves. Appendix three is concerned with the variation of p/q with the density of the glass.

II. Theoretical Considerations

Raman and Nath[14] have given a theory for the diffraction of light by ultrasonic waves in liquids. If a plane wave front impinges at right angles on a medium whose index of refraction varies periodically, such as a liquid in which ultrasonic waves are propagated, the wave front will no longer be plane when it emerges. The Raman and Nath theory assumes that there is no significant bending of the light in the sound field, but that the light is altered in phase. Thus the wave front, on leaving the medium, will be periodically corrugated. The assumption made above is valid provided the light path in the sound field is not too great nor the gradient of the index of refraction too large. A Fourier analysis of the emerging wave front yields the diffraction effects observed when the emergent light is focused with an objective lens. For progressive sound waves, the intensity of the m order relative to the n diffraction order is given by

$$J_m^2(v)/J_n^2(v) \text{ where } v = 2\pi\mu L/\lambda \tag{3}$$

and μ is the maximum variation of the refractive index, L is the light path in the sound field, λ is the wavelength of the light, and J is the Bessel function of the first kind of the order given by the subscript.

The conditions under which the assumptions mentioned above are valid, may be stated mathematically as

$$2\pi L\lambda v/\mu_o\lambda^{*2} \leqq N \tag{4}$$

where μ_o is the refractive index of the medium when the ultrasonic field is not present and λ^* is the wavelength of the sound in the medium. The values of N which are on the order of one or two, have been suggested by theoretical studies[43, 44] and experimentally.[45] In the work reported here, L is about 2 cm., λ about 5×10^{-5} cm., λ^* about 0.04 cm. and v is never significantly greater than one. This places the experiment well within the limits set by the above criterion.

For standing waves, Raman and Nath predict that each line of the diffraction pattern will be composed of subcomponents having different frequencies. In this discussion, the diffraction orders are designated by the index m. Thus, for the central order, m has the value zero while the index one corresponds to the first order and so on. The subcomponents are identified by the index r which may be zero or a positive integer. The relative intensity of the r subcomponent of the m diffraction order is given by

$$\begin{array}{ll} J_{s-r}^{2}(v/2) J_{s+r+1}^{2}(v/2), & \text{if } m = 2s + 1 \\ J_{s-r}^{2}(v/2) J_{s+r}^{2}(v/2), & \text{if } m = 2s \end{array} \tag{5}$$

the light in the even orders is modulated at the frequency $2r\nu^*$, that in the odd orders at $(2r + 1)\nu^*$, where ν^* is the frequency of the ultrasonic field.

The theory for the diffraction of light by ultrasonic waves in solids, given by Mueller,[3] is an extension of the Raman and Nath theory for liquids. Mueller's theory is valid for both progressive waves and standing waves in solids. However, since it is difficult to produce traveling waves in solids, because of the very low absorption, this discussion, as well as the experimental work described later, will be concerned solely with standing waves. As only glasses are used in this study, the results of Mueller's theory which apply to amorphous solids are of greatest interest.

As was mentioned earlier, the photoelastic behavior of a solid can be characterized by the Neumann strain-optical constants p and q. These constants relate a strain to the change produced by it in the index of refraction for light polarized parallel and normal, respectively, to the direction of the strain. Mueller's theory treats the case where dynamic strains are produced by the ultrasonic field in the glass. Thus the refractive indices in the plane normal to the light beam can no longer be described by a circle, as in isotropic media, but by an ellipse. The axes of this ellipse in amorphous solids, are parallel and normal to the direction of sound propagation. The lengths of the axes of the ellipse vary periodically at the frequency of the ultrasonic field. These lengths are related to the elasto-optical constants p and q.

If the plane of polarization of the incident light is arbitrary, it may be resolved into two components along the two axes of the index ellipse. Denoting the amplitude vector of the incident light by E, the two components will be E^{I} for the axis normal to the strain and E^{II} for the axis parallel to the strain. Each component will be diffracted with the amplitudes of the subcomponents of the diffraction orders given by

$$E_{r,m}^{I} = \begin{cases} E\, J_{s-r}(v_I/2)\, J_{s+r+1}(v_I/2), & \text{if } m = 2s+1 \\ E\, J_{s-r}(v_I/2)\, J_{s+r}(v_I/2), & \text{if } m = 2s \end{cases}$$

and (6)

$$E_{r,m}^{II} = \begin{cases} E\, J_{s-r}(v_{II}/2)\, J_{s+r+1}(v_{II}/2), & \text{if } m = 2s+1 \\ E\, J_{s-r}(v_{II}/2)\, J_{s+r}(v_{II}/2), & \text{if } m = 2s \end{cases}$$

where $v_I = 2\pi\mu_I L/\lambda$ and $v_{II} = 2\ \mu_{II} L/\lambda$. (7)

Here μ_I is the variation of the index of refraction for light polarized normal to the strain and μ_{II} is the variation parallel to the strain. For glasses, v_I and v_{II} may be written

$$v_I = 4\pi^2 L\mu_1^2 qA/\lambda\lambda^* \quad \text{and} \quad v_{II} = 4\pi^2 L\mu_1^2 pA/\lambda\lambda^* \qquad (8)$$

where p and q are the strain-optical constants and A is the sound amplitude. It is seen that $v_{II}/v_I = p/q = R$. Also to be noted is that the v's are directly proportional to the sound amplitude. The two components $E_{r,m}^{I}$ and $E_{r,m}^{II}$ of the subcomponent r of the diffraction order m are deviated by the same angle and have the same frequency. Since they originate from the same light and are diffracted from the same elastic wave they must be in phase. Hence the two components are coherent and the amplitudes $E_{r,m}^{I}$ and $E_{r,m}^{II}$ can be added vectorially.

These results may be interpreted physically to mean that for standing sound waves the light in any subcomponent is plane polarized. Different subcomponents of the same diffraction order have different planes of polarization.

Using equations (6) and the rules of vector addition, and taking into account the algebraic signs of the Bessel functions, equation (9) is obtained.

$$\tan(\theta-\beta_{r,m}) = \begin{cases} \tan(\theta-\alpha)\dfrac{J_{s-r}\left(\frac{v_{II}}{2}\right) J_{s+r+1}\left(\frac{v_{II}}{2}\right)}{J_{s-r}\left(\frac{v_{I}}{2}\right) J_{s+r+1}\left(\frac{v_{I}}{2}\right)}, & \text{if } m = 2s+1 \\ \tan(\theta-\alpha)\dfrac{J_{s-r}\left(\frac{v_{II}}{2}\right) J_{s+r}\left(\frac{v_{II}}{2}\right)}{J_{s-r}\left(\frac{v_{I}}{2}\right) J_{s+r}\left(\frac{v_{I}}{2}\right)}, & \text{if } m = 2s \end{cases} \tag{9}$$

These equations relate the intensity of light in the r subcomponent of the m diffraction order to the angle α which the plane of polarization of the incident light makes with the x-axis, the angle $\beta_{r,m}$ which the plane of polarization of the light in this subcomponent makes with the x-axis and the angle θ between an axis of the index ellipse and the x-axis. Since only differences between angles occur in the expressions, the choice of the x-axis is arbitrary. If the incident light is assumed to have unit intensity, then the intensity of light in the r subcomponent of the m diffraction order is

$$\begin{aligned} I_{r,m} &= J_{s-r}^{2}(v_I/2)\, J_{s+r+1}^{2}(v_I/2)\cos^2(\theta-\alpha) \\ &\quad + J_{s+r}^{2}(v_{II}/2)\, J_{s+r+1}^{2}(v_{II}/2)\sin^2(\theta-\alpha), \quad \text{if } m = 2s+1 \\ I_{r,m} &= J_{s-r}^{2}(v_I/2)\, J_{s+r}^{2}(v_I/2)\cos^2(\theta-\alpha) \\ &\quad + J_{s-r}^{2}(v_{II}/2)\, J_{s+r}^{2}(v_{II}/2)\sin^2(\theta-\alpha), \quad \text{if } m = 2s \end{aligned} \tag{10}$$

The justification for restricting this discussion to amorphous solids is that finely annealed optical glass is very nearly homogeneous and isotropic.

The glass samples used are good enough to permit almost complete extinction when placed between crossed Nicols. Although it is possible to propagate transverse waves as well as longitudinal waves in glass, only the effects of longitudinal waves are considered in this study since these permit the determination of p/q. The sound-producing transducer is an **X**-cut quartz crystal. This is a thickness vibrator; hence, the primary waves in the glass are longitudinal. Frequencies are chosen at which the block exhibits a strong resonance for the longitudinal mode but not for the transverse mode. In method "B," the crossed position of the polarizer and analyzer is at forty-five degrees to the sound wave fronts. It has been shown by Hiedemann and Hoesch,[46] that this arrangement eliminates the optical effects of the transverse waves. In other positions of the polarizer and/or analyzer, the diffraction pattern due to the shear waves, which has a different separation than the longitudinal wave pattern, can be noted by the observer, if it is present. In method "C," the different spacing of shear and compressional wave diffraction patterns is used to determine the absence or presence of transverse waves having a significant intensity.

The discussion which follows is concerned with the particular conditions under which the two methods given by Mueller for the determination of the ratio p/q of the strain-optical constants of glass apply.

The arrangement for method "B" requires that the light beam be perpendicular to the direction of sound propagation. The slit is parallel to the sound wave fronts, the polarizer at an angle of forty-five degrees to the slit. For this setting $\theta - \alpha = -45^{\circ}$. Only the r = 0 subcomponent of the first diffraction order (m = 1) is considered. Under these conditions, equation (9) reduces to

$$\tan(\gamma_{0,1} + 45^{\circ}) = \tan\theta = \frac{J_0(\frac{Rv}{2})\, J_1(\frac{Rv}{2})}{J_0(\frac{v}{2})\, J_1(\frac{v}{2})} \tag{11}$$

making use of the fact that $v_{II} = Rv_I = Rv$ for glasses. Here $\gamma_{0,1}$ is the angle by which the analyzer must be rotated from the "crossed" position in order to extinguish the r = 0 subcomponent of the first diffraction order. This angle is the experimentally measured quantity in method "B."

Method "C" again requires that the light beam be normal to the direction of sound propagation. The slit is parallel to the sound wave fronts. As in method "B," only the r = 0 subcomponent of the first diffraction order is considered. For this method, measurements are made of the light intensity, $I_{0,1}^{p}$, in this subcomponent when the light is polarized parallel to the slit, and $I_{0,1}^{n}$, when it is normal to the slit. For the first polarization, $\theta - \alpha = -90^{\circ}$; for the second setting, $\theta - \alpha = 0$. With the help of the first equation of (10) this gives

$$\sqrt{I_{0,1}^{p}} = \sqrt{B_{0,1}} = \sqrt{B} = \frac{J_0(\frac{Rv}{2})\, J_1(\frac{Rv}{2})}{J_0(\frac{v}{2})\, J_1(\frac{v}{2})} \tag{12}$$

The experimental quantities measured here are $I_{0,1}^{p}$ and $I_{0,1}^{n}$.

It is seen by comparing equations (11) and (12), that

$$\tan\theta = \sqrt{B} = \frac{J_0(\frac{Rv}{2})\, J_1(\frac{Rv}{2})}{J_o(\frac{v}{2})\, J_1(\frac{v}{2})} \tag{13}$$

Thus two experimental quantities, which can be measured independently, are given by the same analytical expression.

Since the parameter v which appears on the right hand side of equation (13) contains quantities which can not be accurately determined, such as the sound amplitude and sound field width, values of R can be obtained most conveniently by means of an extrapolation. If the right hand side of equation (13) is expanded in a series development, using standard power series representations of the Bessel functions, equation (14) is obtained.

$$\frac{J_o\left(\frac{R}{2}v\right) J_1\left(\frac{R}{2}v\right)}{J_o\left(\frac{v}{2}\right) J_1\left(\frac{v}{2}\right)} = R - \frac{3}{32} R(R^2 - 1)v^2 + R(R^2 - 1)\left[\frac{10(R^2+1) - 27}{3072}\right]v^4 +. . . \quad (14)$$

It is seen that as v approaches zero, this quantity approaches the limiting value R. Letting the subscript zero indicate limiting values, equation (13) or (14) then reduces to

$$\tan \theta_o = \sqrt{B_o} = R. \quad (15)$$

It should also be noted, from equation (14), that the slope of the curve obtained when $\tan \theta$ (or $\sqrt{B}$) is plotted against v^2 depends on the sign of R. Four cases must be considered: (1) for R greater than one the slope is negative, (2) when R is greater than zero but less than one the slope is positive, (3) for R between zero and minus one the slope is again negative, and (4) when R is less than minus one the slope is positive. When $R = \pm 1$, the slope is zero and the sign of R cannot be determined. Thus, the magnitude of $R = p/q$ is determined by the limiting value of $\tan \theta$ or $\sqrt{B}$

and the sign by the slope of the curve obtained when these quantities are plotted as functions of v^2.

Figure 1, drawn from equation (13) or (14), shows values of tan θ and $\sqrt{B}$ for various values of v and R and the behavior of these functions as v approaches zero. These quantities are plotted against the square of v since this gives a relationship which is almost linear for small values of v. The straight lines are drawn in the figure to show the departure of the points from linearity as v increases. This shows that for v less than one and R less than two, a straight line is, in fact, a suitable approximation. For R equal to two, the "best" straight line, by the criterion of least squares,[47] for the four points shown gives an intercept of 1.989. The error is 0.55 percent. The error introduced by the assumption of linearity increases for R greater than two but becomes even less than the value mentioned above for values of R which are less than two. Since only one of the samples studied in this investigation, fused silica, yielded a value of R greater than two (R = 2.34), the error introduced by the assumption of linearity alone is, with this single exception, less than about one-half percent. Sources of error which are experimental in nature will be discussed in a later section.

From equation (8) it is seen that the parameter v is proportional to the sound amplitude. Thus a similar linear relationship exists between $\sqrt{B}$ and tan θ and the sound amplitude. The values of the quantity used must correspond to acceptable values of v. The relationship between values of v and the intensity of light in the diffraction orders is given in

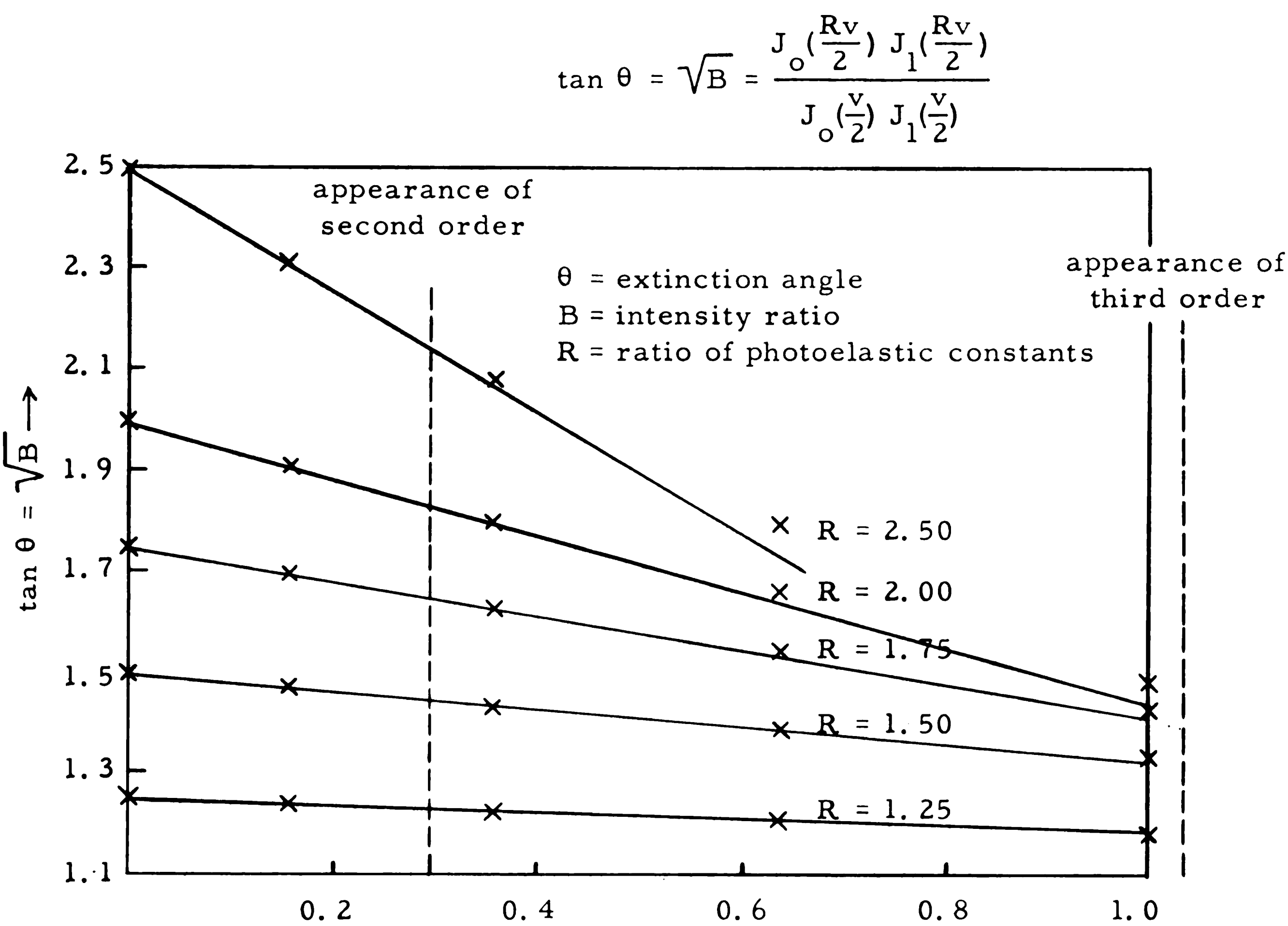

Figure 1. Theoretical variation of $\tan\theta$ and $\sqrt{B}$ with v^2

the case of solids by Mueller's theory. The dotted lines in figure 1 show the approximate values of v for which the second and third diffraction orders appear* for a glass having R = 1.5 and using method "B." These values are indicated in figure 3 of Mueller's paper and are due to Hiedemann. In general, the values of v at which the diffraction orders appear, decrease as R increases. Thus, the following conclusions may be drawn. When the sound intensity is sufficiently small that the third order diffraction line does not appear, this sound intensity corresponds to a value of v such that the relationship between tan θ (or $\sqrt{B}$) and v^2 is essentially linear. Under these conditions, linear extrapolation is justified.

Since the determinations of R are based on measurements of only the r = 0 subcomponent of the first diffraction order, it is necessary to indicate under what conditions such measurements are valid. In method "B," when the sound intensity is so low that the third diffraction order does not appear, the intensity of the r = 1 subcomponent of the first diffraction order is less than three percent of the intensity of the r = 0 subcomponent for R = 2 and less for smaller values of R. Thus a distinct minimum occurs in the intensity of the line at a setting of the analyzer which coincides almost exactly with that which produces extinction of the r = 0 subcomponent. Method "C" requires a measurement of the intensity of the same subcomponent (r = 0) of the same diffraction order. A calculation of the square root of the ratio of the intensity of light polarized

*An arbitrary criterion for the appearance of a diffraction order is an intensity in that order of approximately one percent of the intensity of the central order.

parallel to the wave fronts to that polarized normal to the wave fronts for all subcomponents of the first order for $R = 2$ and $v = 1$ differs from that obtained for the $r = 0$ subcomponent only by less than two percent. This difference is smaller for $R < 2$ and $v < 1$, the limits, with one exception, of the experimental work reported, and is considerably less than the experimental accuracy for method "C." Thus a measurement of the intensity of the first diffraction order is equivalent to a determination of the intensity of its $r = 0$ subcomponent.

These considerations show that for sound intensities which are not sufficient to produce the third diffraction order, the conditions required by Mueller's theory are certainly fulfilled. The use of higher sound intensities presents certain experimental difficulties even though the error introduced at somewhat higher intensities as a result of non-linearity or inability to isolate the $r = 0$ subcomponent of the first order is not significantly greater than the experimental error. Thus values of v less than one permit the determination of the angle θ at which the $r = 0$ subcomponent of the first diffraction order is extinguished and permits linear extrapolation; for method "C," an intensity measurement of the first order is essentially a determination of the intensity of the $r = 0$ subcomponent of that order.

Because the functions $\tan \theta$ and $\sqrt{B}$ are extrapolated to zero sound amplitude, it is not necessary to know the absolute values of the sound amplitudes; it is sufficient to know the values of some quantity which is proportional to the sound amplitude. Cady[48] gives, as the relationship between the acoustic intensity J and the piezoelectric transducer current I, the equation

$$J = I^2 R_s / 2 \tag{16}$$

where R_s is the (constant) series resistance of the transducer equivalent network. R_s depends on the frequency, piezoelectric constants, dimensions and the wave velocity. For a given transducer, coupled to a constant load and driven at a constant frequency, sound amplitude is proportional to transducer current. For this reason, the independent variable used in this study is the transducer current.

Since Mueller's theory is based on the Raman and Nath theory, it is possible to gain some insight into the behavior of the quantities tan θ and $\sqrt{B}$ for non-normal incidence by means of a generalization of the expressions for the intensities given by that theory to include dependence on the angle of incidence ϕ. The angle ϕ is taken to be the angle between the incoming light beam and the normal to the sound field. The expressions obtained may then be compared with experimental results which are a part of this study.

Nath[49] has pointed out that the assumptions on which the Raman and Nath elementary theory is based are not justified for the case of oblique incidence. This theory introduces a new parameter v' for the variable v found in the expressions for normal incidence. The quantity v' can be expressed in the form $v' = vk(\phi)$, where $k(\phi)$ does not depend on the sound amplitude. Substituting this variable for v in equation (13) gives

$$\tan\theta = \sqrt{B} = \frac{J_o[\frac{Rv}{2}\ k(\phi)]\ J_1[\frac{Rv}{2}\ k(\phi)]}{J_o[\frac{v}{2}\,k(\phi)]\ J_1[\frac{v}{2}\,k(\phi)]} \tag{17}$$

If this expression is carried to the limit of zero sound intensity, the limiting value is again found to be R, the ratio of the elasto-optical constants. It is found experimentally that this is not the case, but rather, that as ϕ increases, the limiting value of $\tan\theta$ differs from R by a rather small, but measurable amount.

A more general expression for the amplitude of the $r = 0$ subcomponent of the first diffraction order can be written

$$\begin{aligned} E^{I}_{0,1} &= EA_I(v,\phi)\ J_o\,[\tfrac{v}{2}\,k(\phi)]\ J_1\,[\tfrac{v}{2}\,k(\phi)] \\ E^{II}_{0,1} &= EA_{II}\,(Rv,\phi)\ J_o\,[\tfrac{Rv}{2}\ k(\phi)]\ J_1\,[\tfrac{Rv}{2}\ k(\phi)] \end{aligned} \tag{18}$$

for light polarized normal and parallel, respectively, to the strain. An expression can be obtained for $\tan\theta$ or $E^{II}_{0,1}/E^{I}_{0,1}$ for a particular sound intensity (i. e. a fixed value of v) by expanding the ratio in a power series in ϕ using Taylor's formula. It is noted that $A_{II}\,(Rv, 0) = A_I(v, 0) = k(0) = 1$, since for $\phi = 0$ these equations must reduce to the values given by equation (6) for $m = 1$, $r = 0$. The expansion is given in equation (19)

$$\begin{aligned} \tan\theta = \frac{E^{II}_{0,1}}{E^{I}_{0,1}} = \frac{J_o(\frac{Rv}{2})\ J_1(\frac{Rv}{2})}{J_o\,(\frac{v}{2})\ J_1\,(\frac{v}{2})} + \Bigg\{ & \frac{J_o\,(\frac{Rv}{2})\ J_1\,(\frac{Rv}{2})}{J_o(\frac{v}{2})\ J_1\,(\frac{v}{2})}\,A'_{II}\,(Rv, 0)\ + \\ & + \frac{[J_o^2\,(\frac{Rv}{2}) - J_1^2\,(\frac{Rv}{2}) - \frac{2}{Rv}\ J_o(\frac{Rv}{2})\ J_1(\frac{Rv}{2})]\ \frac{Rv}{2}\,k'(0)}{J_o\,(\frac{v}{2})\ J_1\,(\frac{v}{2})} \\ & - \left[\frac{J_o(\frac{Rv}{2})\ J_1(\frac{Rv}{2})}{J_o(\frac{v}{2})\ J_1(\frac{v}{2})}\right]\left[A'_I\,(v, 0) + \frac{[J_o^2(\frac{v}{2}) - J_1^2(\frac{v}{2}) - \frac{2}{v}\ J_o(\frac{v}{2})J_1(\frac{v}{2})]\,\frac{v}{2}k'(0)}{J_o(\frac{v}{2})\ J_1(\frac{v}{2})}\right]\Bigg\}\,\phi + \ldots \end{aligned} \tag{19}$$

where the primes indicate partial derivatives with respect to ϕ. It is seen that for $\phi = 0$, this equation reduces to equation (13) as it must. If one assumes $k'(0) = 0$, which is certainly reasonable, or that at least it is very small, the expression is simplified considerably to

$$\tan \theta = \frac{E^{II}_{0,1}}{E^{I}_{0,1}} = \frac{J_o(\frac{Rv}{2})\, J_1(\frac{Rv}{2})}{J_o(\frac{v}{2})\, J_1(\frac{v}{2})} \left\{ 1 + [A'_{II}(Rv, 0) - A'_{I}(v, 0)]\phi + \ldots \right\} \tag{20}$$

Equation (20) has the value R in the limit $v = 0$ only if $A'_I(Rv, 0) = A'_I(v, 0)$. Thus under the assumptions made in equation (18), an expression is obtained which, when extrapolated to zero sound intensity, gives a value for $\tan \theta_o$ which is different from R. However, since the functions $A_{II}(Rv, \phi)$ and $A_I(v, \phi)$ are not known, it is not possible to use equation (20) to predict values of $\tan \theta_o$ for a given R and ϕ. The experimental study which is reported in this paper may give some insight into this problem.

Some conclusions can be drawn, however, from a qualitative discussion of the general form of the expressions for the intensity at oblique incidence. The effect of the function $k(\phi)$ on the diffraction pattern is rather readily observed. Qualitatively, the effect of $k(\phi)$ is that as ϕ increases, the sound amplitude must be steadily increased in order to produce a diffraction pattern having a given intensity in, say, the first order. This has an immediate consequence with regard to the validity of the experimental setup used by Bergmann for method "C." In this arrangement, a Wollaston double image prism is inserted before the glass block, splitting the light into two beams which diverge at an angle which is typically about 20 to 30 minutes. One of these beams is polarized parallel to the sound wave fronts, the other normal to the wave fronts. Two diffraction patterns are produced; the intensity of the first order in the one is then compared to the intensity of the first order of the other. The two beams are not normally incident on the sound field but make angles ϕ_{II}, for light polarized parallel to the wave fronts, and ϕ_I for light polarized normal to the sound wave fronts, with the normal. These angles are shown in figure 2, where they are drawn unequal. If $\phi_I = \phi_{II}$, equation (12) is approximately valid. The difficulty arises because there is no dependable method of determining the angle of incidence. This means that one can not be sure that ϕ_I and ϕ_{II} are equal in a particular case. The experimental configuration is such that if ϕ_I and ϕ_{II} are not equal, then one will become larger and the other smaller, as shown in figure 2, and, correspondingly, the changes in v' will be opposite; the intensity of the first diffraction order for one polarization will increase, while the intensity will decrease for the corresponding order with the other polarization. Since the ratio of intensities is involved, even a small discrepancy in the angles of incidence will yield values of $\sqrt{B_o}$ which differ from R by several percent. For this reason, Bergmann's setup for method "C" is unreliable.

To determine the effects of setting the polarizer in method "B" at an angle of other than forty-five degrees to the sound wave fronts, the general equation

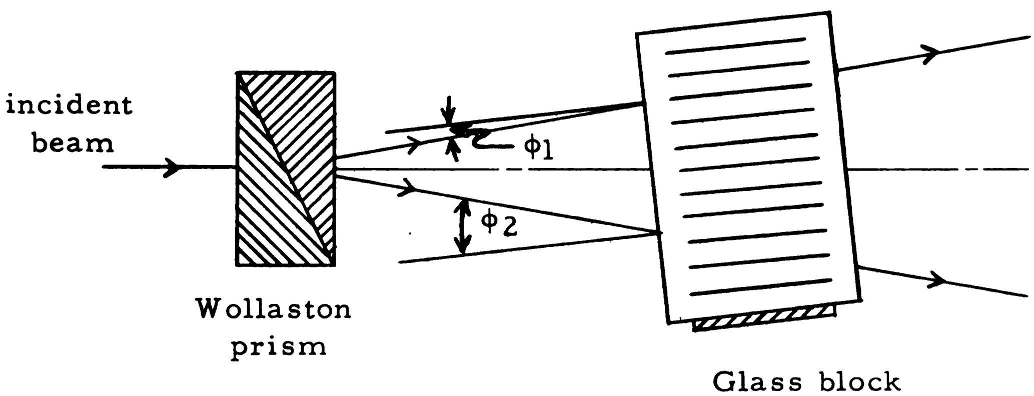

Figure 2. Angles of incidence when two beams of light from Wollaston prism impinge on glass block. The glass block is not normal to the original beam and the angles are greatly exaggerated.

$$\tan(\theta-\alpha)\cot(\theta-\alpha-\gamma_{r,m}) = \begin{cases} \dfrac{J_{s-r}(\frac{v_I}{2})\, J_{s+r+1}(\frac{v_I}{2})}{J_{s-r}(\frac{v_{II}}{2})\, J_{s+r+1}(\frac{v_{II}}{2})}, & \text{if } m = 2s+1 \\[2ex] \dfrac{J_{s-r}(\frac{v_I}{2})\, J_{s+r}(\frac{v_I}{2})}{J_{s-r}(\frac{v_{II}}{2})\, J_{s+r}(\frac{v_{II}}{2})}, & \text{if } m = 2s \end{cases} \tag{21}$$

must be considered. In this equation, v_I is the Raman and Nath argument for variations in the refractive index in the direction parallel to the strain, v_{II} is the argument for changes normal to the strain, θ is the angle between the major axis of the index ellipse and the x-axis, and α is the angle between the plane of oscillation of the E-vector of the incident light and the x-axis. (Since only the difference $\theta - \alpha$ appears, the choice of the x-axis is immaterial.) This investigation makes use of only the $r = 0$ subcomponent of the first diffraction order ($m = 1$), hence the equation reduces to

$$\tan(\theta-\alpha)\cot(\theta-\alpha-\gamma_{0,1}) = \frac{J_o(\frac{v_I}{2})\, J_1(\frac{v_I}{2})}{J_o(\frac{v_{II}}{2})\, J_1(\frac{v_{II}}{2})} \tag{22}$$

For glasses, $v_{II} = Rv_I = Rv$; letting $\xi = -(\theta-\alpha)$, equation (22) becomes

$$\tan\xi\,\cot(\xi+\gamma_{0,1}) = \frac{J_o(\frac{Rv}{2})\, J_1(\frac{Rv}{2})}{J_o(\frac{v}{2})\, J_1(\frac{v}{2})} \tag{23}$$

This equation has the same behavior as a function of v as does equation (11) which was considered previously. Hence measurements made for several values of v, or some quantity, for example, the piezoelectric transducer current, which is directly proportional to v, may be extrapolated to zero sound intensity under the conditions prescribed for equation (11). The limiting value of this extrapolation will again be R. Thus, in terms of the limiting values R and γ_o, equation (23) becomes

$$\tan \xi \cot (\xi + \gamma_o) = R \tag{24}$$

Solving this equation for γ_o in terms of ξ yields

$$\gamma_o = \text{arc cot} [R \cot \xi] - \xi \tag{25}$$

This equation gives the limiting angle through which the plane of polarization will be rotated for any given orientation of the incident plane of polarization. The plane of polarization parallel to the sound wave fronts corresponds to $\xi = 0$; when the plane of polarization is normal to the wave fronts, $\xi = 90^o$. The usual setting for method "B" is $\xi = 45^o$. Equation (23) shows that for $\xi = 0$ or $\xi = 90^o$, γ_o vanishes for all values of v. This is to be expected since, in this case the plane of polarization is parallel to one or the other axes of the index ellipse. Hence no rotation is possible. Also of interest is the value of ξ for which the limiting value γ_o of the rotation of the plane of polarization is a maximum. This is obtained by differentiating equation (25) with respect to ξ and setting this derivative equal to zero. The result is

$$\sin \xi = \sqrt{\frac{R}{R + 1}} \tag{26}$$

Mueller's conclusion that the amplitude of the components of light polarized along the axes of the index ellipse may be added vectorially can be tested by determining the light intensity in the r = 0 subcomponent of the first diffraction order as a function of the angle ξ which the plane of polarization makes with the sound wave fronts. This intensity is given for the r subcomponent of the m diffraction order and for unit intensity in equation (10). For the case r = 0, m = 1 and incident intensity I_o, this becomes

$$I_{0,1} = I_o J_o^2 (\frac{Rv}{2}) J_1^2 (\frac{Rv}{2}) \cos^2 \xi + I_o J_o^2 (\frac{v}{2}) J_1^2 (\frac{v}{2}) \sin^2 \xi \qquad (27)$$

where as before $\xi = -(\theta - \alpha)$ and R is the ratio of the strain-optical constants of the glass. Letting $I = I_{0,1}$ and $I_p = I_o J_o(Rv/2) J_1(Rv/2)$, $I_n = I_o J_o (v/2) J_1 (v/2)$ this becomes

$$I = I_p \cos^2 \xi + I_n \sin^2 \xi \qquad (28)$$

The quantities I, I_p, I_n and ξ may all be measured experimentally; hence equation (28) serves as a check on Mueller's conclusion concerning the vector addition of the amplitude components of the light. This conclusion is based on the assumption that the two components are coherent; hence, ultimately, the test is whether or not the two components of the light beam are, in fact, coherent.

The material discussed above forms the analytical basis for the experimental work which is described in the following pages. Certain of these topics are extensions of the work reported by Mueller but are based on his theory. These extensions, if they agree with the experimental results which correspond to them, should indicate further the soundness of the basic theory.

III. Experimental Techniques

This section is devoted to a discussion of the apparatus and experimental techniques used in this study. A description of the apparatus and techniques which are common to all aspects of the work is given first. This is followed by an analysis of the individual experiments performed and the procedures which are peculiar to each.

The sound producing transducer used in this work is a one inch square X-cut quartz crystal having a fundamental frequency of approximately 15 mc/sec. This type of crystal cut is a thickness vibrator; therefore the primary sound waves are longitudinal waves as required by Mueller's theory for the determination of the ratio p/q. The acoustic coupling between transducer and glass block is accomplished by means of a thin film of Dow-Corning Silicone Vacuum Grease. A thin aluminum foil is used as the electrode on the side of the transducer which is against the block; an aluminum plate, which is part of the glass block holder, serves as the other electrode. The high-frequency power to drive the transducer is supplied by a transmitter which is continuously tunable from approximately 10 to 20 mc/sec., and has a maximum plate input power of about 175 watts. An approximate idea of the frequency can be obtained with an ordinary frequency meter since accurate frequency determinations are not required.

The glass block holder is mounted on a worm drive in order that the angle between the light beam and sound field may be adjusted. In addition, the scale on the worm drive can be calibrated, making possible a quantitative determination of changes in the angle between sound field and light beam. The mounting permits adjustment in the other directions as well. When the effect of a static stress is studied, a different holder is used which permits the strain, of known magnitude, to be distributed over the cross-section of the block.

The optical setup for method "B" is shown in figure 3a. Figure 3b shows the setup for method "C." The light source used is a mercury arc, with which a filter is used isolating the 5461 A line of the mercury spectrum. An Ahrens prism is used as the polarizer; the analyzer, when used, is a Glan-Thompson prism.

Both methods "B" and "C" require that the slit be parallel to the sound wave fronts. This is accomplished with the aid of a low power microscope equipped with cross-hairs. The cross-hairs of the microscope are set so that one of them is parallel to the slit image formed by the objective lens. The objective lens is then removed and the orientation of the glass block is adjusted to bring the lines of the visibility pattern parallel to the cross-hair.

In order to set the polarizer parallel or normal to the slit, a Wollaston double image prism is used. After the Wollaston is inserted in the light beam, it is rotated until the two images of the slit appear on a single line. The prism which is to be used as the polarizer is then inserted after the Wollaston and rotated until one slit image is extinguished; the plane of polarization of the prism is then either parallel to or normal to the slit, depending on which image was extinguished. A Nicol prism

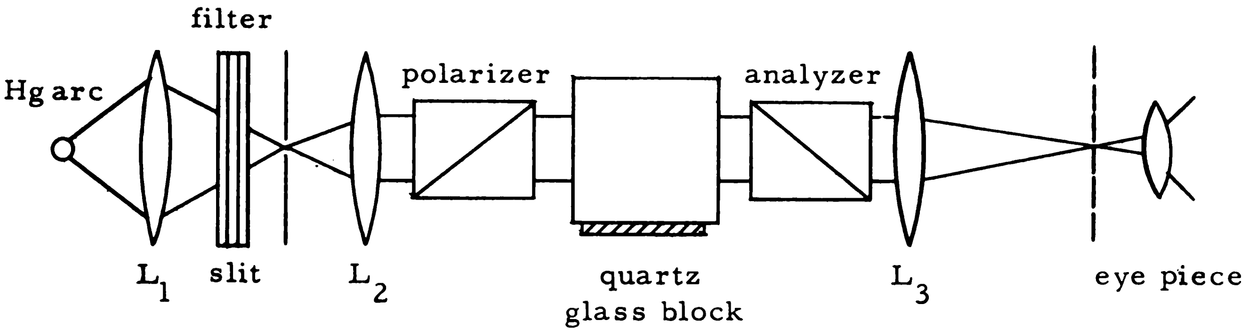

Figure 3a. Optical system for method "B"

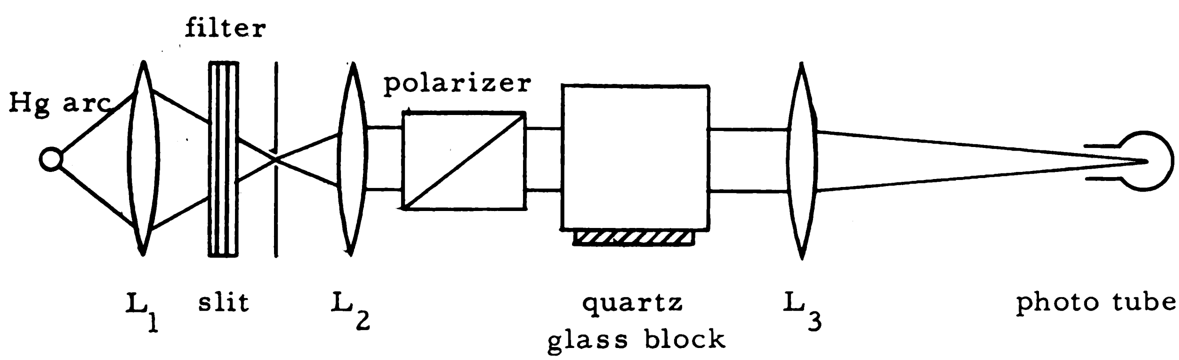

Figure 3b. Optical system for method "C"

or some other polarizer of which the plane of polarization is known may then be used to determine whether the orientation is parallel or normal. An angular scale is used for setting the polarizer to other angles.

Method "B" uses the optical system shown in figure 3a and requires that the polarizer be set at forty-five degrees to the slit. The light beam is assumed to pass through the sound field at right angles to the direction of propagation. The analyzer is originally set at the "crossed" position. When the sound field is turned on, a diffraction pattern is formed in the focal plane of the objective lens. The frequency of the oscillator is adjusted to produce resonant standing longitudinal waves in the glass but not shear waves. Readings are than made of the angle at which extinction (or a minimum for values of v approaching unity) of the first diffraction order is obtained and of the transducer current. Ten settings are taken for each value of the transducer current as well as for the "crossed" position. This measurement is repeated for four different sound amplitudes corresponding to four values of transducer current. These readings are averaged to determine the angle. The maximum current used is at least twice the minimum current but not sufficient to produce the third diffraction order.

Since these conditions fulfill the requirements of Mueller's theory, $\tan \theta$ may be computed as in equation (11). A linear extrapolation is made, leading to a value for $\tan \theta_o$, which by equation (15) is R, the ratio p/q of the strain-optical constants. The extrapolation is performed analytically using the criterion of least squares.

The optical setup for method "C" is shown in figure 3b. This method requires that measurements be made of the light intensity in the first diffraction order when the incident light is polarized normal to and parallel to the wave fronts. The measurements of light intensity in this study are made with an American Instrument Company photomultiplier-microphotometer. The frequency is again adjusted to obtain resonant standing longitudinal waves. The position of the slit of the photomultiplier-microphotometer is adjusted to admit the first order of the diffraction pattern. A reading is taken of the background light, the sound field is turned on, and the light intensity and transducer current are recorded. The intensity of light is, except for a scale factor, the reading with the sound on minus the reading with the sound off. This procedure is repeated with the polarizer normal to the slit if it was first parallel or vice versa. Since only the ratio of intensities is needed, the value of the scale factor is not required. These measurements are made for several values of the transducer current. In the previous section, it was pointed out that the difference between the value of the light intensity in the $r = 0$ subcomponent of the first diffraction order and that of all subcomponents of the order is less than three percent for $R < 2$ and $v < 1$. Thus, theoretically it is possible to use method "C" for sound intensities corresponding to values of v as great as one. However, there are certain experimental difficulties which make it impractical to use values of v greater than about one-half. (When $v = 1/2$, the second order is present but with a small intensity.) The reason for using these low intensities is that at higher intensities, the heating of the glass block by energy dissipated from the sound field makes it difficult to be certain that a change in the standing wave condition in the block does not occur between the time the intensity reading is made with one orientation of the polarizer and the time it is made at the other position. At the low sound intensities used, this change occurs slowly and it is felt that virtually no change in the standing wave condition occurs between readings. A second problem which arises is that to extend the range of sound intensities would require the use of a second sensitivity range on the photomultiplier-microphotometer. Since this device is not strictly linear under normal conditions, it seems likely that a change of linearity would be introduced when switching from one sensitivity to another. In one case, an effort was made to use method "C" for sound intensities corresponding to approximately $v = 1$. Since these measurements are made within the range of v values for which Mueller's theory is applicable, $\sqrt{B}$ may be plotted according to equation (13) leading to the value R as the sound intensity becomes vanishingly small. Readings of the light intensity are also taken every ten degrees between the normal and parallel positions of the polarizer for one particular sound intensity. These readings can be compared with equation (28) as a check on the validity of Mueller's assumption concerning the coherence of the amplitude components of the incident light.

The technique for studying the effect of oblique incidence based on method "B" is now discussed. Since there is no way of determining with certainty when the incident light beam is precisely normal to the sound field, only relative angles can be given. The glass block is rotated about an axis perpendicular to the plane of the sound field and light beam by means of the worm drive until the diffraction pattern produced is too weak for making reliable measurements (i. e. only a weak second order can be obtained). The direction of rotation is reversed and the block rotated until a usable diffraction pattern is again obtained. A measurement is made by the procedure outlined above for method "B" and a value of tan θ is obtained. The block is then rotated by a known amount in the same direction as before and another value of tan θ_o is obtained. This is repeated for a number of angles of incidence through the setting for normal incidence to approximately the same angle on the other side of the normal. The direction of travel of the work drive is not reversed at any time in this series of readings in order that the backlash of the screw may not affect the results. The setting at which the maximum diffraction occurs for a given transducer current is noted since this is approximately normal incidence. The values of tan θ_o are then plotted against the relative angle.

An experimental study of the effect of setting the polarizer in method "B" at an angle other than 45^o to the slit is also undertaken. In this study the angle γ_o through which the plane of polarization of the incident light is rotated by the sound field is of greatest interest since this can be compared directly with the result predicted by equation (25).

IV. Experimental Data and Discussion

The measurements reported here are made on the same glasses as were used by Gates and Hiedemann.[33] Of these glasses, nine are samples of Bausch and Lomb optical glasses chosen from their catalog to cover the range from the lightest to the most dense types, three are samples of Eastman Kodak rare earth glasses and one is a fused silica sample obtained by the Owens-Illinois Glass Company. The composition by weight of these glasses as given by the respective manufacturers (the fused silica is assumed to be 100 percent SiO_2) and the nominal values of the refractive index and dispersion of the Bausch and Lomb glasses are given in Tables I and II of the paper by Barnes and Hiedemann.[50] Tables III, IV, and V of this same paper give the values of the density, elastic moduli and surface tension of the samples measured by these investigators. The values of $R = p/q$ obtained by Gates and Hiedemann using Mueller's method "B" are found in Table I of their paper.

The study of primary importance in this investigation, as noted previously, is the attempt to show that Mueller's method "C" gives results which are consistent with method "B" and, if possible, to account for the negative results reported by Gates and Hiedemann. The required light intensity measurements are made electronically rather than photographically as did Gates and Hiedemann. In figures 4, 5 and 6 are plotted values of $\sqrt{I_{\parallel}/I_{\perp}} = \sqrt{B}$ against the square of the transducer current for three typical glasses samples LF-1 ($R = 1.46$), EK-110 ($R = 1.54$) and BSC-1 ($R = 1.92$). The point marked on the vertical axis

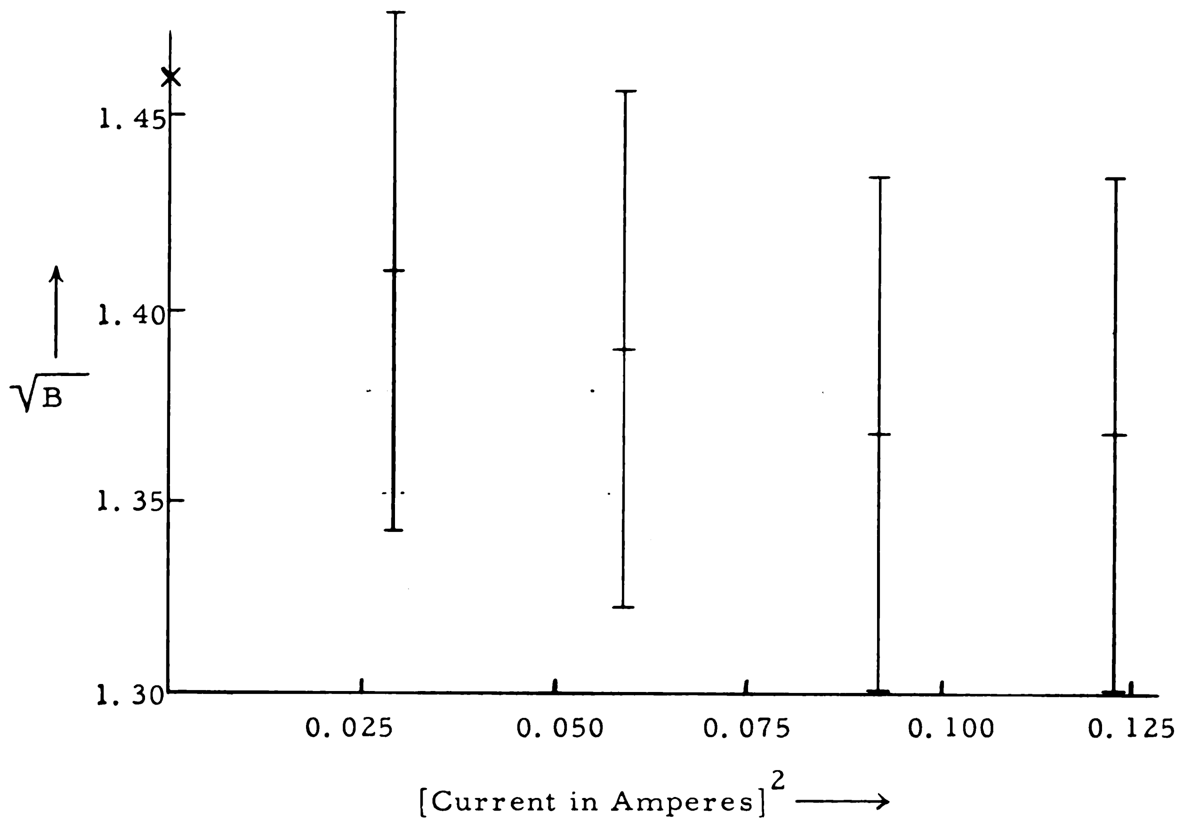

Figure 4. Experimental variation of $\sqrt{B}$ with transducer current squared for glass sample LF-1.

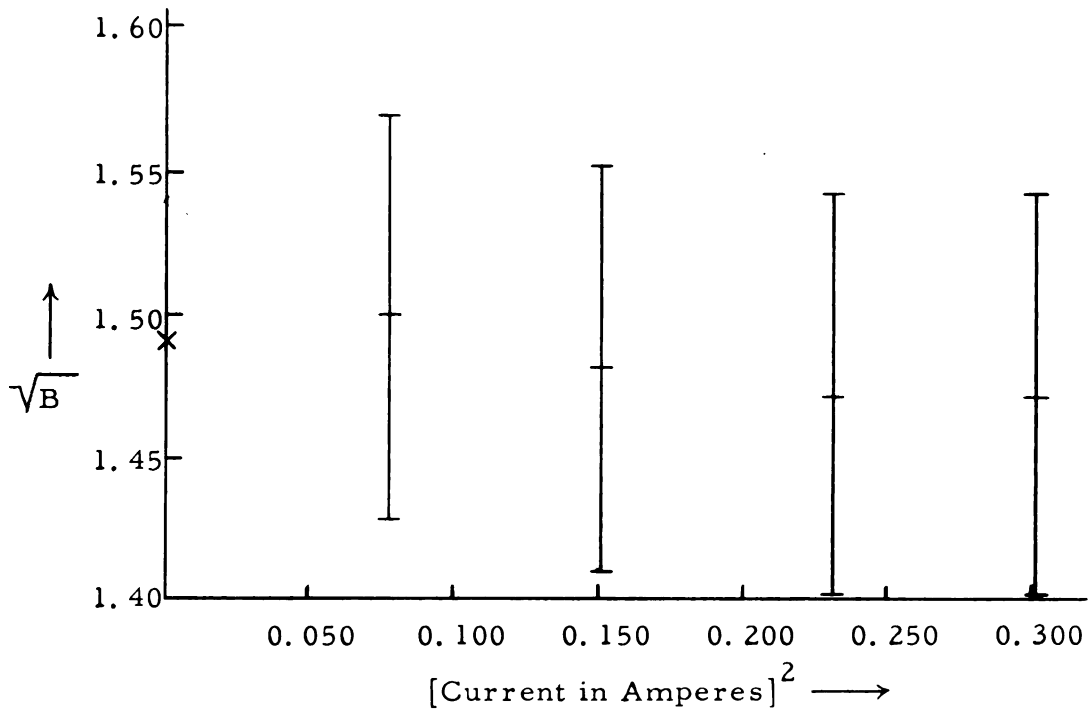

Figure 5. Experimental variation of $\sqrt{B}$ with transducer current squared for glass sample EK-110.

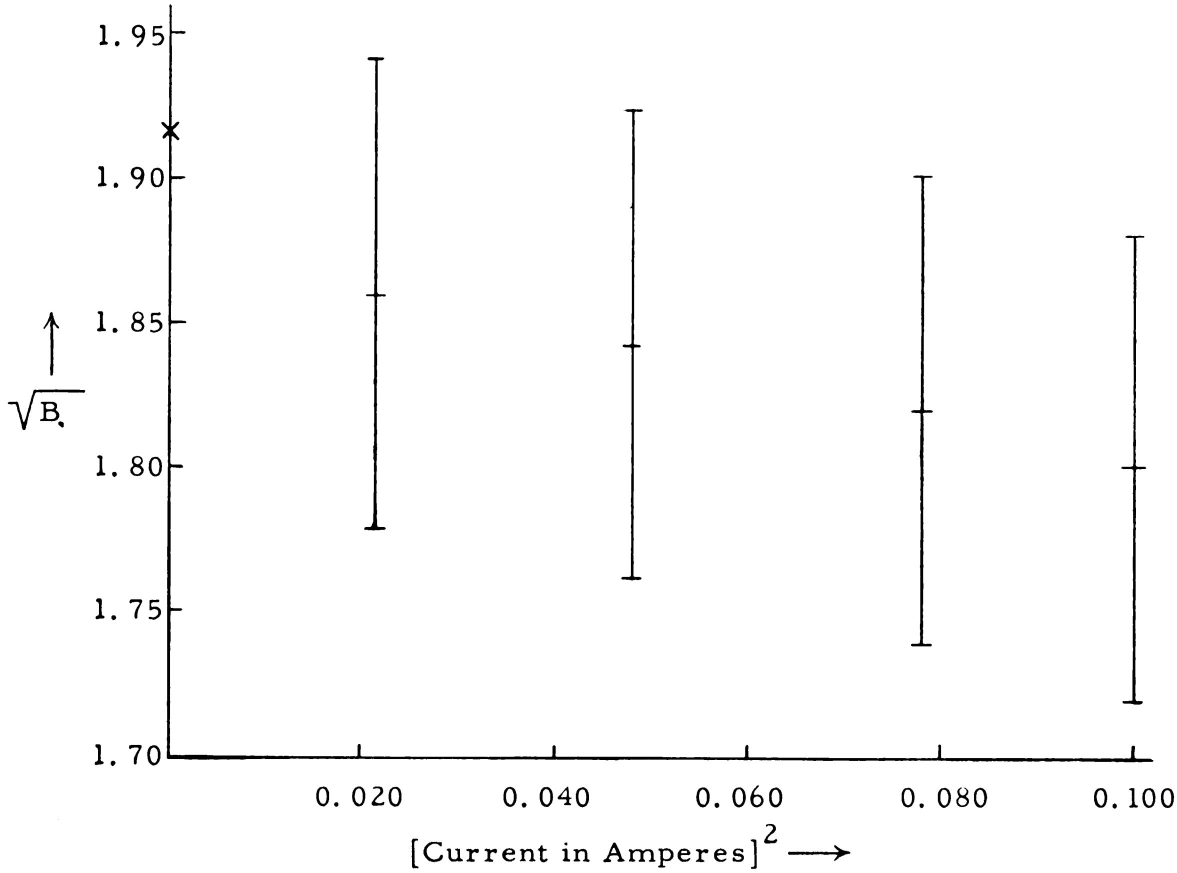

Figure 6. Experimental variation of $\sqrt{B}$ with transducer current squared for glass sample BSC-1.

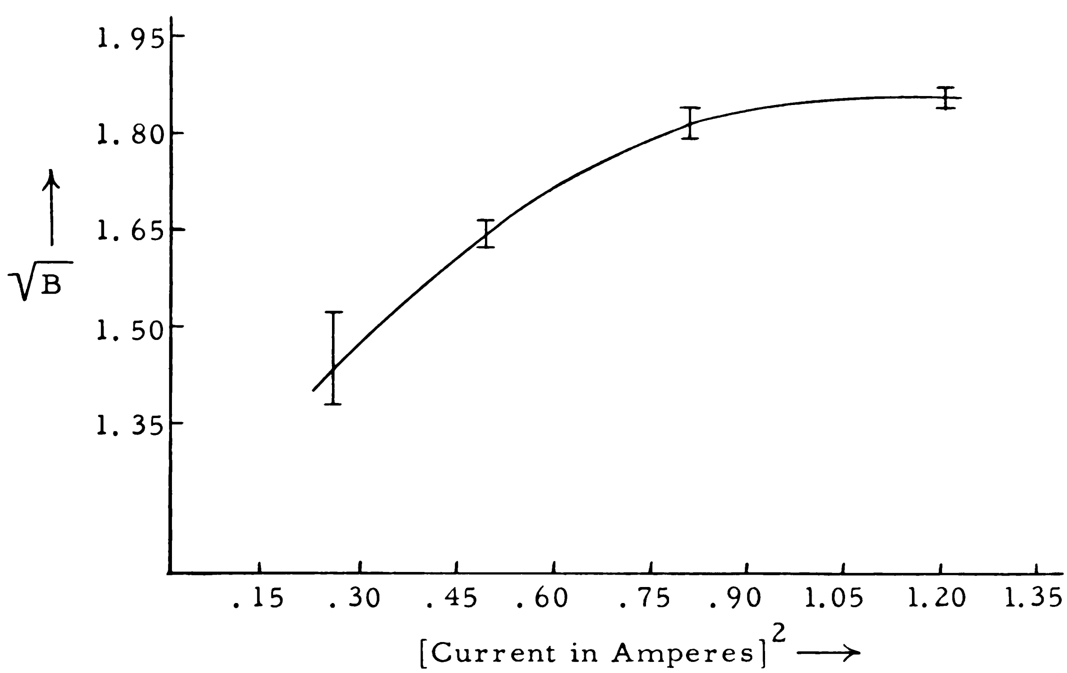

Figure 7. Experimental variation of $\sqrt{B}$ with transducer current squared for BSC-1 obtained by Gates and Hiedemann. This appears as figure 7 in their paper.

is, in each case, the value of R obtained by method "B" for the particular glass used.

These figures show, for glasses of widely differing properties, the variation of the measured value of $\sqrt{B}$ with the square of the transducer current. The vertical lines through each point are drawn to give an indication of the accuracy to which the point is determined. This indication is based primarily on the percent of non-linearity of the photomultiplier-microphotometer as indicated by its manufacturer for the particular conditions of operation encountered here. This should be taken as an indication of the relative error, admitting the possibility of a systematic error which may apply to all points on the graph. It is seen, that in each of these figures, the values of $\sqrt{B}$ obtained experimentally for different transducer currents are compatible with the value of R for each glass obtained by method "B," and that the slope of the curves is negative as is predicted by Mueller's theory. In addition, if figure 6, for example, is compared with figure 7, which is the curve obtained for the same glass by Gates and Hiedemann using the Bergmann setup for method "C" and which appears as figure 7 in their paper, the completely different character of these curves becomes apparent. Thus it is evident that the experimental setup first used by Bergmann and later by Gates and Hiedemann does not allow reliable measurements.

Curves similar to those in figures 4, 5 and 6 are obtained for each of the glass samples. Each curve gives a value which is in agreement with the value obtained by method "B" and has the required negative slope. Although any one of these curves is not conclusive, the consistency of the entire series of measurements demonstrates the validity of Mueller's method "C."

Figure 8 shows the values of $\sqrt{B}$ obtained when sound intensities sufficient to produce the third order in the diffraction pattern are used. Although not conclusive, the curve is compatible with the prediction that the measured values of $\sqrt{B}$ remain approximately linear when plotted against the square of the transducer current for sound intensities on the order of $v = 1$. The glass used in this particular study was DBF-1 for which the value of R measured by method "B," is 1.37.

In these measurements involving rather high sound intensities, the problems arising from heating within the block (which are present even at low intensities) are particularly significant. For this reason, the following procedure was adopted: An intensity reading was first taken, for a particular transducer current, with the polarizer oriented, say, parallel to the sound wave fronts; the polarizer was then rotated ninety degrees making it normal to the sound wave fronts and an intensity reading was again taken. The ratio of the first of these readings to the second is then B, provided the standing wave system remained constant throughout. As an indication of this constancy, the polarizer was rotated back to its original setting and a third reading was taken. If this third reading differed from the first by more than about two percent, it was assumed that the standing wave system had been affected by internal heating and consequently that the measurement was unreliable. The

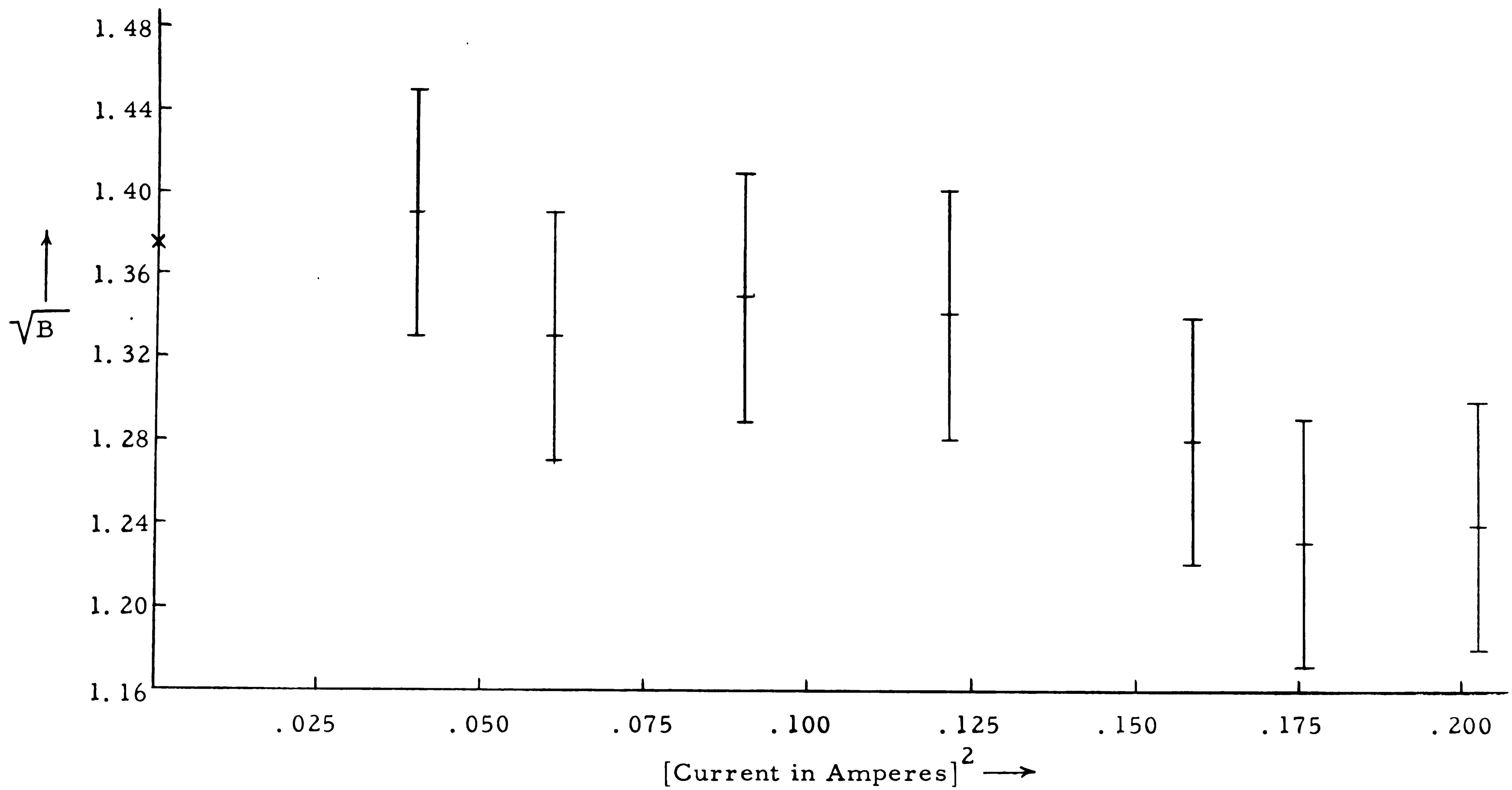

Figure 8. Experimental variation of $\sqrt{B}$ with transducer current squared for glass sample DBF-1 when intensities sufficient to produce the third diffraction order are used.

transducer current was also noted before and after the light intensity measurements. Any change noted between the two readings was also considered an indication of the unreliability of the measurement. In either case, the block was allowed to cool for several minutes before another measurement was attempted. This procedure was used for measurements at both high and low sound intensities.

On the basis of the inherent non-linearity and instability of the photomultiplier-microphotometer, and the consequences of the internal heating of the block due to the dissipation of energy from the sound field, it seems probable that the overall accuracy of the measurements made in the course of this investigation by means of this particular instrument is about ten percent. An additional factor, not previously mentioned, is that the response of a photomultiplier tube may depend on the polarization of the incident light.[51] However, because of the design of the particular tube used in this instrument, it is doubtful that this particular matter contributes significantly to the uncertainty of these data. Since a great deal of this inaccuracy is caused by the properties of the photomultiplier-photometer, it is very likely that the advent of better instruments of this type may make it feasible to use method "C" with an accuracy comparable to that of method "B."

The effect of oblique incidence is studied by means of method "B." Figure 9 shows values of tan θ_o, the limiting extinction angle, plotted against the angle of incidence measured from some arbitrary reference point. The glass used is DBF-1. In this plot, the broken vertical line

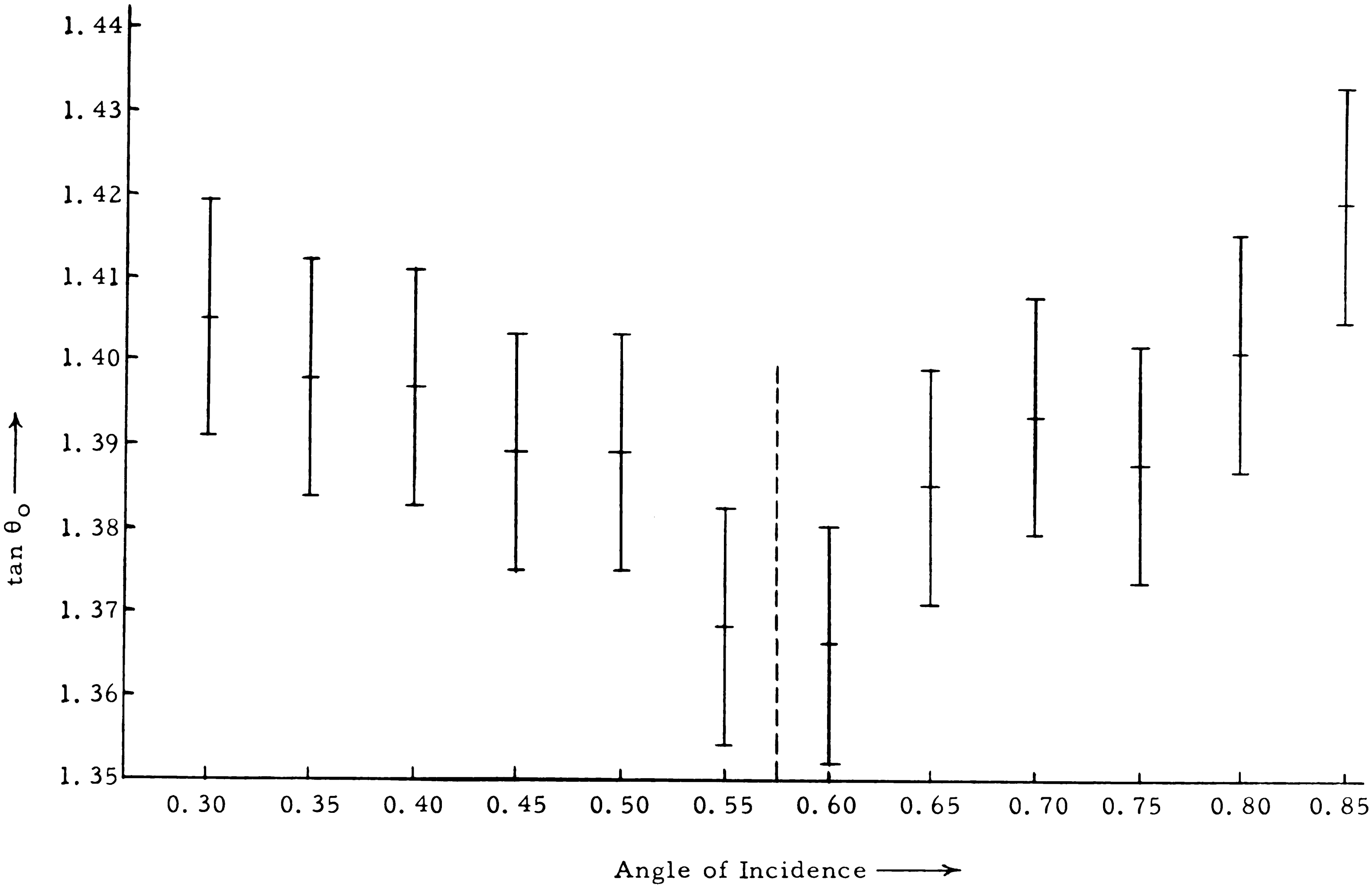

Figure 9. Experimental variation of tan θ_o with angle of incidence for glass sample DBF-1. The angle of incidence is plotted on an arbitrary scale representing a total variation of about four degrees. The broken vertical line corresponds to approximately normal incidence.

indicates the approximate angle corresponding to normal incidence; the approximate relative error of each point is indicated. It is seen that, over a range of angle of about two degrees in either direction from the normal, the variation of tan θ_o is only about three percent for this particular glass. The immediate consequence of these measurements is that the error introduced by a small inaccuracy in setting the system to normal incidence is negligible when an experimental method using a single incident light beam is used.

The fact that there is actually a small change in tan θ_o, as the angle between the light beam and the normal to the sound field increases, has certain theoretical consequences. This means that one cannot find a function $k(\phi)$ such that the intensities of the diffraction orders may be obtained by replacing the parameter v in the expressions for normal incidence by a new argument $v' = v\,k(\phi)$. Instead, it is necessary to write expressions such as those in equation (18) for the amplitudes of the two components. These measurements, however, do not give enough information to determine the form of the function $A(v, \phi)$. It is seen that the change in tan θ_o is consistent with the prediction, from equation (20), that it be linear with ϕ. Because of the magnitude of the uncertainty in the values of tan θ_o, the nature of this relationship is not completely proved.

The measurements made using a static stress in addition to the dynamic stresses produced by the sound field showed that the observed values of tan θ_o do not depend on a static stress. The only effect noted was that the birefringence produced by the static stress made the determination of the minimum in the first diffraction order more difficult to observe, since it became less sharply defined. No change was found in the values of tan θ_o when a static stress was applied. Two particular points should be noted: Since the minimum in the intensity of the first diffraction order cannot be determined as critically when a static stress is applied because of the birefringence produced by it, a static stress results in a reduction of the accuracy of method "B." Secondly, these measurements show that it is not necessary to exercise great care in ascertaining that the mounting of the glass block introduces no static stresses but only that the birefringence produced by these stresses does not obscure the minimum in the first diffraction order. This would appear to be true also for the case of birefringence in the block because of strains "frozen in" when the glass was cast.

Determinations of the limiting angle γ_o through which the plane of polarization of the incident light is rotated by the sound field when the orientation of this plane of polarization is varied were made for glass samples BSC-1, LBC-2 and DBF-1. Figure 10 shows the values of γ_o obtained for glass sample BSC-1 plotted against the angle ξ which the plane of polarization of the incident beam makes with the sound wave fronts. The solid curve in the figure is that given theoretically by equation (25) for R = 1.89. It is seen that the agreement between the experimentally determined values and the theoretically predicted curve is very close. The angle ξ_m at which this curve reaches its maximum may be compared with that predicted by equation (26). The value of the maximum

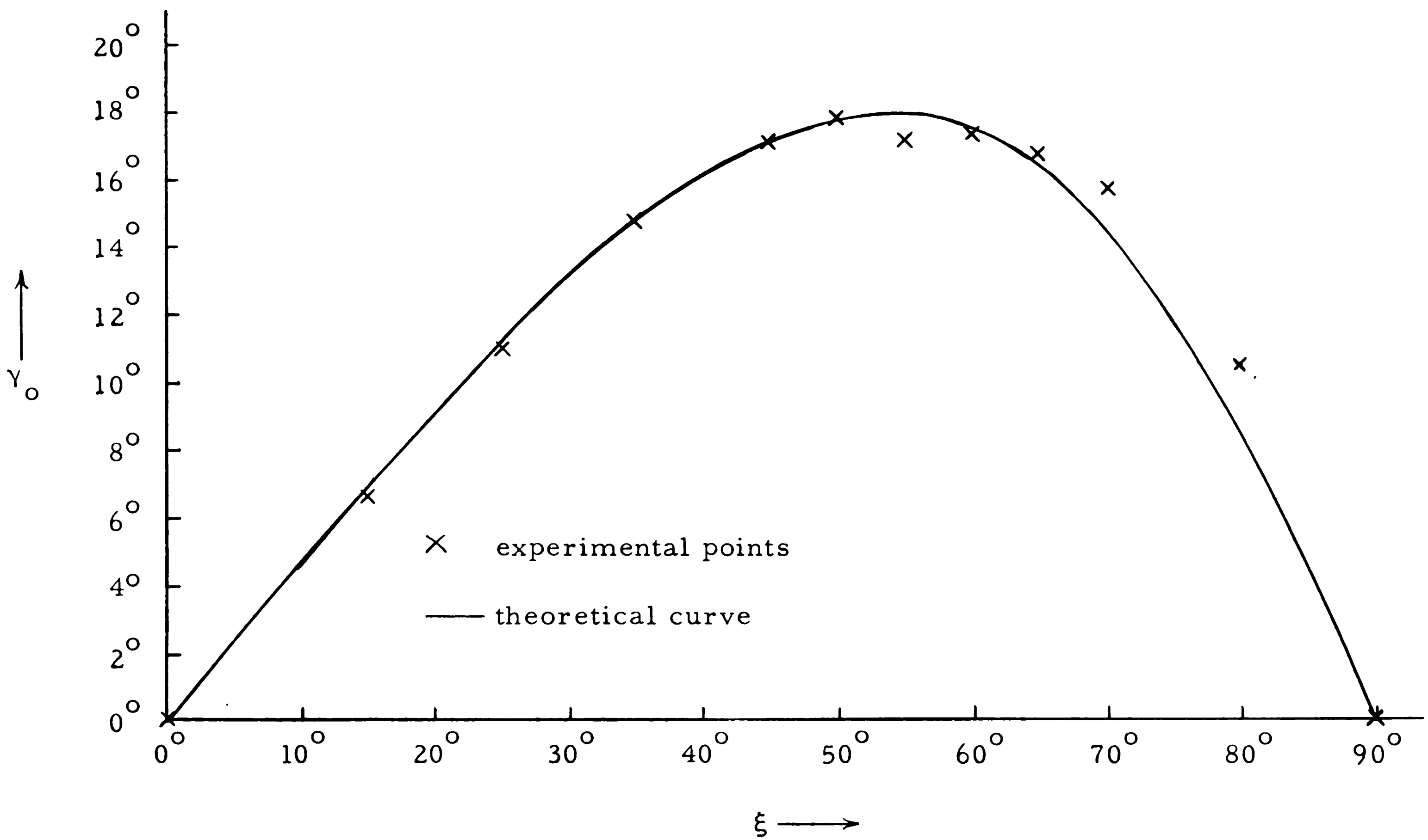

Figure 10: Variation of γ_o, the limiting angle of rotation of the plane of polarization of the incident light, against the angle which this plane of polarization makes with the sound wave fronts. The glass sample is BSC-1.

given by equation (26) for R = 1.89 is indicated by the broken line in the figure. Since this curve has a rather broad maximum, the comparison with equation (26) is not completely definitive but within the limit of experimental accuracy the experimental and theoretical maxima coincide. The curves for the other two glasses studied are similar in form and are in agreement with theory.

An experimental check of the validity of equation (28) is readily obtained by making light intensity measurements at polarizer angles between the setting parallel to the wave fronts and that normal to the wave fronts. Figure 11 is a plot of the points obtained experimentally for glass sample LBC-2 against the angle between the plane of polarization of the incident light and the sound wave fronts. Readings were taken at intervals of ten degrees. The solid curve indicates the values predicted by equation (28) using the values $I_p = 84$ and $I_n = 31$; the intensity units are arbitrary. Similar curves were obtained for each glass sample, with each curve in comparable agreement with the theoretically predicted values. This uniformity demonstrates, in a direct manner, the validity of equation (28) which is obtained on the assumption, in Mueller's theory, that the components of diffracted light polarized parallel and normal to the wave fronts are coherent.

In order to determine any change which might have occurred in the value of R for these glasses in the period of about four years between the measurements of Gates and Hiedemann and the present investigation, determinations of p/q were made by method "B" for all glass samples.

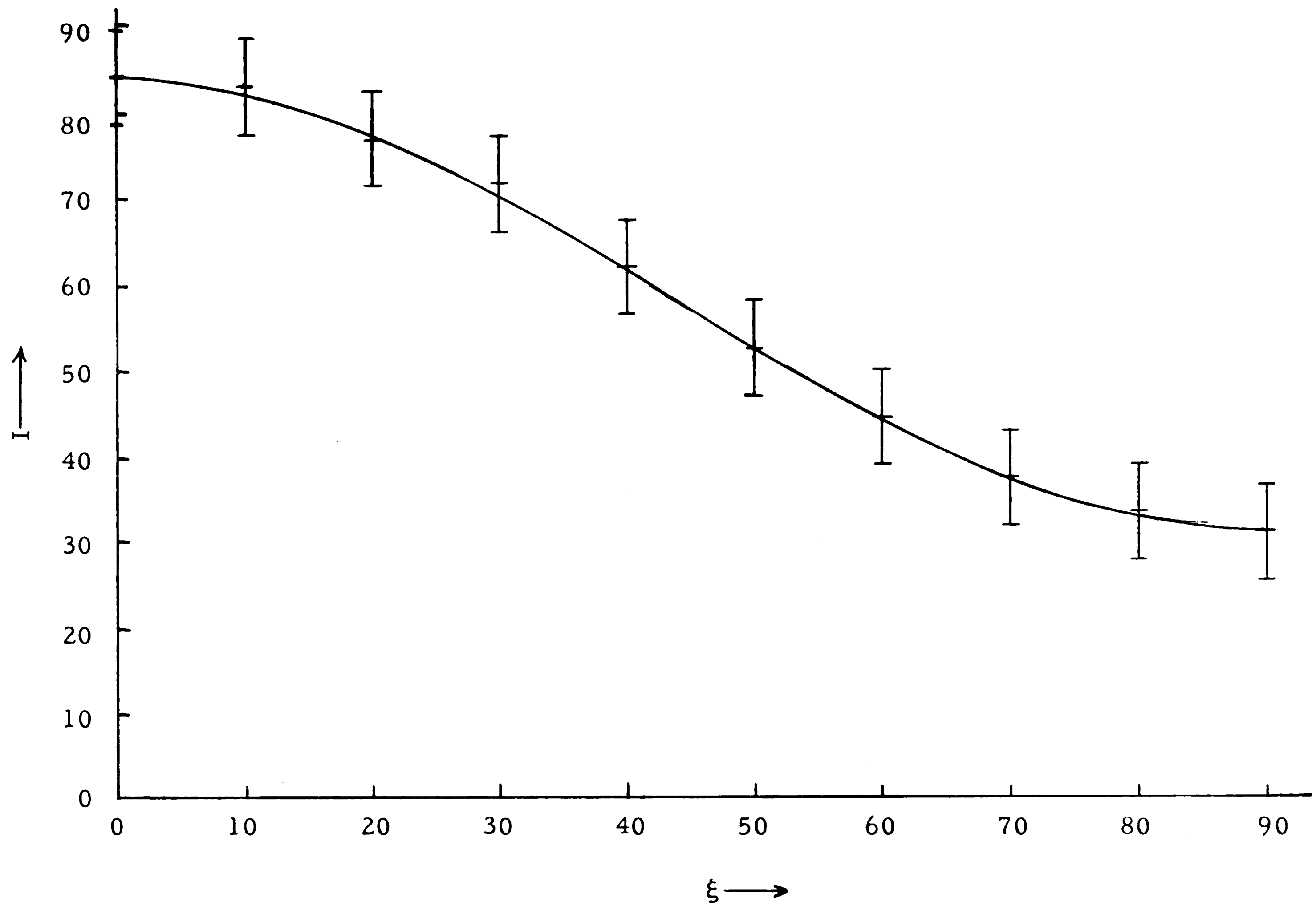

Figure 11. Variation of the intensity I, in arbitrary units, of light in the first diffraction order with the angle ξ between the plane of polarization of the incident light and the sound wave fronts for glass sample LBC-2.

These values are listed in Table I along with the values obtained in the investigation of Gates and Hiedemann. It is seen that in all cases except one, the values of R obtained in this study are greater than those reported previously by Gates and Hiedemann. However, the greatest difference is only seven percent in the case of the rare earth type EK-330, with a change of about five percent for type BSC-1; all other samples show changes of less than five percent. Thus in most cases, the difference in the measured values of R is on the order of the uncertainty with which these values are obtained. It seems likely that the one exception represents a fluctuation since the difference in that case is less than one-half percent. The measurements indicate that the aging of a glass sample has an effect on the ratio p/q of its photoelastic constants and that this effect results in an increase in the value of this ratio. However, these measurements do not permit one to determine in what manner the constants themselves are changed. Mueller[37] suggests that this change is due to crystallization and points out that such an effect increases the values of the photoelastic constants. If this is the case, the present evidence indicates that the rate of change of p is greater than that of q.

In this study it has been demonstrated that Mueller's method "C" is a valid method for the measurement of the ratio p/q of the strain-optical constants of glass. However, the reason for the inconclusive results of Gates and Hiedemann has not been directly determined. The investigation of the way in which the experimental results depend on

Table I: Values of R = p/q measured by Gates and Hiedemann and from the present investigation. The density is included in this table since the variation of R with density is discussed in Appendix III. The symbol B & L designates Bausch and Lomb glasses, EK is for Eastman Kodak and H for Hanovia, the manufacturer of the fused silica.

Manufacturer	Type	R_{GH}	R_{NOW}	Percent Change	Density
B & L	BSC-1	1.82	1.92	+5.5#	2.4766
B & L	C-1	1.82	1.87	+2.7	2.5268
B & L	CF-1	1.68	1.70	+1.1	2.6924
B & L	LF-1	1.45	1.46	+0.7	3.1742
B & L	LBC-2	1.62	1.61	-0.6	3.1424
B & L	DBF-1	1.36	1.38	+1.5	3.6008
B & L	DBC-2	1.46	1.50	+2.7	3.6441
B & L	EDF-1	1.28	1.32	+3.1	3.7813
B & L	EDF-4	1.11	1.13	+1.8	4.7189
EK	EK-110	1.53	1.54	+0.7	4.1317
EK	EK-330	1.60	1.71	+6.9	4.5720
EK	EK-450	1.55	1.62	+4.5	4.6293
H	fused silica	2.34	2.34	0.0	2.2027

#The sign designates the direction of the change in R; a plus sign means that the value measured in this investigation is higher than that obtained by Gates and Hiedemann and vice versa.

several of the parameters involved has eliminated the possibility that uncontrolled variations in these parameters caused the observed discrepancy. The last reasonable source of experimental error lies in the photographic technique used for the measurement of the intensity of the first diffraction orders for light polarized parallel and normal to the sound wave fronts. The photographic method for determining light intensities is particularly subject to error under conditions of low light intensity as is the case in this study.

Consideration of the method by which a light intensity measurement is made photographically makes it possible to see how curves such as those given by Gates and Hiedemann for method "C" might be obtained. This technique, as applied to this particular problem, requires that the images of the diffraction orders be focused by an objective lens on a photographic plate. Exposures of the diffraction patterns are then made for several sound intensities, including no sound which gives the intensity of the incident beam. Finally, an emulsion calibrating exposure is made using a rotating step-wedge. This plate is developed and the light intensities are then inferred from it by standard densitometer measurements. The chief difficulty in this method is in determining and correcting for the exposure of the film which is due to background light; that is, stray light which impinges on the film at the position of the diffraction orders. The correction for this is made by determining the intensity of this background light and subtracting this from the total measured intensity of the order to give the intensity of the diffraction order. In particular, if the correction made for the background is less than the actual background, curves of the sort given by Gates and Hiedemann may be obtained. For the higher sound intensities used, this background will make little difference and the measured values of $\sqrt{B}$ will not differ appreciably from the actual values. However, as the sound intensity is decreased the background will have an intensity which approaches the intensity of the diffraction orders themselves. Thus, the measured ratio of the intensities will be lower than the actual value for the particular sound intensity; in fact, for very low intensities the measured ratio will approach the ratio of the difference in the actual background minus the part subtracted as the correction term for the two patterns. This will be one if the backgrounds and corrections are the same for the first diffraction orders of both patterns; if they are not, it seems reasonable to expect that these ratios will be on the order of one. Inspection of the curves obtained by Gates and Hiedemann, see figure 7, reveals that at the lowest sound intensities used by them, the curves have values greater than one and decreasing. (The limiting value cannot be determined since the nature of the curves does not permit the necessary extrapolation.) This is substantially in agreement with the suggestions made above. Further, the technique of Gates and Hiedemann for evaluating the background intensity is likely to yield a correction which is too small and hence will give just the effect discussed. Their method required that the reading for the background be taken on the plate near the diffraction line whose intensity was to be determined.

This could mean that most or all of the exposure on that part of the plate was needed to overcome the inertia of the emulsion whereas the inertia of the emulsion at the diffraction line was overcome more quickly because of the greater light intensity. In this latter case, the background light contributes to the exposure of the emulsion. This means essentially that the assumption of the validity of the reciprocity relation is not sound.

Although this argument does not prove that the inconclusive results of Gates and Hiedemann are due to the use of the photographic method for obtaining light intensities, it certainly demonstrates that this is a plausible reason. Since the other likely factors have been eliminated by the studies discussed previously, it seems almost certain that this is indeed the source of the discrepancy.

V. Summary and Conclusions

The primary purpose of this investigation is the demonstration of the validity of Mueller's method "C" for the measurement of the ratio $R = p/q$ of the strain-optical constants of glass. In addition, a study is made of the dependence of the experiment on the several parameters involved in order to determine why the measurements of Gates and Hiedemann by Bergmann's procedure were inconclusive. Measurements are also made to check further deductions from the theory and to test directly certain basic assumptions of the theory. Since the glass samples used by Gates and Hiedemann are available, determinations of R are made by Mueller's method "B" and compared with the values obtained by Gates and Hiedemann using the same method in order to detect possible aging effects on the ratio p/q.

The demonstration of the validity of Mueller's method "C" uses a somewhat different experimental setup than that employed by previous investigators. The Wollaston double image prism is eliminated and the required light intensity measurements are made directly with a photomultiplier-microphotometer rather than by the photographic technique. The results obtained are in agreement with those obtained by method "B" and with the predictions of Mueller's theory. This supports the suggestion by Gates and Hiedemann that their results were due to some inadequacy of the experimental arrangement or technique.

A study of the effect of oblique incidence on the measured value of p/q, using method "B," shows that a slight increase occurs when the angle, ϕ, between the incident beam and the normal to the sound field is varied. This change is the same, within experimental error, for variations of ϕ in either direction from the normal. The change is sufficiently small that no significant error is introduced, if a small uncertainty, i. e. less than about twenty minutes of arc, is present in setting the system for normal incidence.

A theoretical analysis of the effect of oblique incidence on tan θ_o, indicates that the results obtained cannot be explained in terms of an expression for the amplitude of light in the r = 0 subcomponent of the first diffraction orders which simply replaces the parameter v in the expression for normal incidence by a new parameter $v' = v k(\phi)$. It is shown that an additional multiplicative term must be used which gives an expression of the form

$$E_{0,1} = E\,A(v,\phi)\, J_o\,[v\,k(\phi)]\; J_1\,[v\,k(\phi)] \qquad (29)$$

for the amplitude of the light in the r = 0 subcomponent of the first diffraction order. Because the change in tan θ_o is only a few percent, this experiment gives no further information concerning the quantity $A(v,\phi)$.

Measurements are made of the limiting angle γ_o through which the analyzer must be rotated from the "crossed" position to produce an extinction (or a minimum for higher sound intensities) of the first order for settings of the polarizer from parallel to the sound wave fronts to normal to these wave fronts. These values are compared with values predicted from a theoretical expression for this angle deduced from Mueller's theory. The measured values agree with the predicted values to within the limit of the experimental error.

The intensity of light in the first diffraction order is measured for polarization angles ranging from parallel to the sound wave fronts to normal to the wave fronts. These measurements are made for one sound intensity for each glass sample. The way in which the light intensity in the first order varies with the angle between the plane of polarization and the sound wave fronts is compared with that predicted by Mueller on the assumption that the components of light polarized parallel and normal to the sound wave fronts are coherent. The experimental variation is in very close agreement with the predicted values for all samples. This shows directly that the important assumption of the coherence of these components is valid.

Values of R are obtained for all glass samples for which such determinations were made by Gates and Hiedemann. The values obtained in the present investigation are, with but two exceptions, higher than those reported by Gates and Hiedemann. Although the maximum increase in R is only about seven percent, there are several cases in which the change is greater than the experimental uncertainty. This and the fact that nearly all samples show an increase indicates that the ratio of the strain-optical constants tends to increase with the age of the glass sample. Of the two cases in which an increase was not found, one showed no change and for the other sample, R decreased by less than one percent.

Since the studies discussed above do not isolate the cause of the inconclusive results obtained from method "C" using Bergmann's experimental setup and photographic techniques for the light intensity measurements, these results must be due to some shortcoming of the photographic method for determining the very low light intensities involved. An analysis of the photographic method is given which indicates how results such as those reported by Gates and Hiedemann might arise from these intensity measurements. Because of the results of the various studies reported here and the unreliability of the photographic method, it is virtually certain that the inconclusive results of Gates and Hiedemann are due to the use of the photographic method for light intensity measurements and not to any weakness of Mueller's method "C."

Appendix I. Theoretical Discussion of the Application of Narasimhamurty's Method to the Determination of the Ratio p/q of the Photoelastic Constants of Glasses.

Recently, Narasimhamurty[42] has suggested a method for the determination of the various ratios of the strain-optical constants of uniaxial and biaxial crystals. This method is also applicable to the measurement of p/q for glasses. A discussion of the validity of this method and an analysis of its reliability is now given.

The optical setup used by Narasimhamurty is shown in figure 12. Light from the slit, collimated by lens L, is polarized by the polarizer in a plane which is at forty-five degrees to the sound wave fronts. This beam of polarized light then traverses the sound field after which it is split into two beams by the Wollaston prism D. These two beams are plane polarized and oriented so that the planes are parallel and normal to the sound wave fronts. Since these beams diverge slightly, two diffraction patterns are observed through the telescope T. Now, while observing both patterns, the analyzer A is rotated from its initial position at forty-five degrees to the sound wave fronts until the intensity of light in the first diffraction orders of the two patterns is equal. Since this involves estimating when two images, visible simultaneously, have the same intensity, this adjustment can conceivably be made critically. If γ is the angle by which the analyzer must be rotated from the initial (45°) setting and if measurements of γ are taken for a number of sound intensities, then

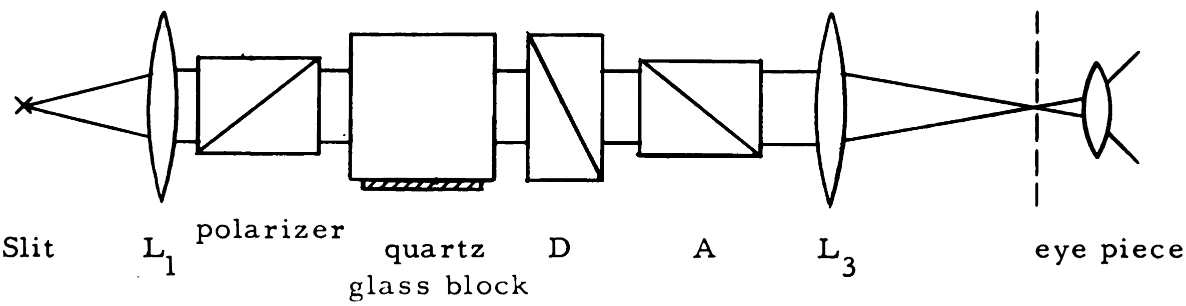

Figure 12. Experimental setup for Narasimhamurty's method.

$$\tan(\gamma_o + 45^o) = R \tag{30}$$

where γ_o is the rotation of the analyzer in the limit of zero sound intensity and R is the ratio of the strain-optical constants.

The validity of equation (30) can be demonstrated readily from Mueller's theory. The polarized beam which impinges on the sound field may be resolved into two components of equal intensity which are parallel and normal to the sound wave fronts. If E is the amplitude of either of these components and if sound intensities sufficiently small that only the r = 0 subcomponent of the first diffraction order is significant, then according to Mueller the amplitude of this subcomponent is

$$E^I_{0,1} = E\, J_o(v_I/2)\, J_1(v_I/2) \tag{31}$$

for the light polarized normal to the strain. Here $v_I = 2\pi\mu_I L/\lambda$ and μ_I is the variation of the index of refraction for light polarized normal to the strain. For light polarized parallel to the strain, the amplitude of the component is

$$E^{II}_{0,1} = E\, J_o(v_{II}/2)\, J_1(v_{II}/2) \tag{32}$$

where $v_{II} = 2\pi\mu_{II}L/\lambda$ and μ_{II} is the variation of the index of refraction for light polarized parallel to the strain. For glasses $v_{II}/v_I = p/q = R$. If the analyzer is set at an angle ϕ with the sound wave fronts, then the amplitudes, after passing through the analyzer, are

$$\begin{aligned} E^I_{0,1}(\phi) &= E\, J_o(v_I/2)\, J_1(v_I/2) \cos\phi \\ E^{II}_{0,1}(\phi) &= E\, J_o(v_{II}/2)\, J_1(v_{II}/2) \sin\phi \end{aligned} \tag{33}$$

For one particular angle these amplitudes are equal and for that angle one obtains

$$\tan \phi = \tan(\gamma + 45^{o}) = \frac{J_o(\frac{Rv}{2})\, J_1(\frac{Rv}{2})}{J_o(\frac{v}{2})\, J_1(\frac{v}{2})} \tag{34}$$

where $v_{II} = Rv_{I} = Rv$ and γ is defined from the equation $\phi = \gamma + 45^{o}$. Thus, if the analyzer is set at this angle ϕ, the first orders of the two diffraction patterns appear to have the same intensity. If this equation is extrapolated to $v = 0$, it is seen that

$$\tan(\gamma_o + 45^{o}) = R$$

is obtained. Thus Narasimhamurty's method is seen to follow directly from Mueller's theory and shares the same theoretical basis as Mueller's methods "B" and "C."

In this method, the polarizer is needed only to assure that the intensities of the components of the incident beam polarized parallel and normal to the sound wave fronts are equal. If the light source is completely unpolarized this polarizer would not be needed. However, this property of light sources cannot ordinarily be safely assumed and hence the polarizer is used. It could, alternatively, be replaced by a depolarizer since this achieves the same effect.

Narasimhamurty's method has certain features which are worth noting. Although this method is probably not quite as accurate as Mueller's method "B" for glasses and cubic crystals its accuracy does not decrease when the medium is naturally birefringent as does the accuracy of method "B." It has the advantage over Mueller's method "C" in that the determination of the light intensity in the diffraction orders is not required.

Narasimhamurty's placement of the Wollaston prism after the sound field is a considerable improvement of the experimental setup since it eliminates errors which may arise because of the fact that both light beams cannot be perpendicular to the sound field when the Wollaston prism is placed before the sound field, and in addition, because there is no way of ascertaining that the angles which the two beams make with the sound field are the same. This placement is most desirable when one uses a Wollaston prism in applying Mueller's method "C."

Appendix II. Determination of the elasto-optical constants of crystals by dynamic methods.

The principles discussed above with regard to the determination of the ratio p/q of the strain-optical constants of glass may be applied to the study of these constants for crystals. The case of cubic crystals has been treated by Mueller.[3] Recently, Narasimhamurty[42] has shown that the dynamic techniques described by Mueller for glasses and cubic crystals may be extended to the more complicated problem of uniaxial and biaxial crystals.

In his paper, Mueller discusses in detail the determination of ratios of the elasto-optical constants of cubic crystals. The description of the photo-elastic properties of cubic crystals requires three independent strain-optical constants. These are p_{11}, p_{12} and p_{44}, using the general formulation of Pockels.[2] Mueller's method "A," which is not applicable to glasses and which will be discussed in detail below, furnishes the magnitude of the term $2p_{44}/(p_{11} - p_{12})$. The sign of this term may be obtained with the aid of the elastic constants which may also be readily obtained, to about one percent accuracy, from the same experimental setup used for method "A." Method "B" or "C" may then be used to determine the ratios $R_{001} = p_{12}/p_{11}$ for longitudinal waves traveling in the [001] direction, and $R_{011} = (p_{11} + p_{12} - 2p_{44})/(p_{11} + p_{12} + p_{44})$ for longitudinal waves traveling in the [011] direction.

Narasimhamurty has extended the methods developed by Mueller to the study of uniaxial and biaxial crystals. In addition, experimental values are obtained for the uniaxial crystals quartz and calcite, and for the biaxial crystal barite. If, in a uniaxial crystal for example, sound is propagated along one of the crystal axes, the mechanical waves produced are, in general, a pure longitudinal wave and two pure transverse waves polarized perpendicularly to one another. The existence of pure longitudinal waves makes it possible to use the methods developed by Mueller and extended by Narasimhamurty to determine the ratios of certain of the strain-optical constants. In quartz and calcite, which belong to the trigonal system, longitudinal waves propagated in the [100) direction and observation along the [001] direction yield the ratio p_{12}/p_{11} while for the same sound wave, observation along the [010] direction yields p_{31}/p_{11}. Sound propagation in the [001] direction with observation in the [100] direction yields p_{13}/p_{33}. The study of biaxial crystals proceeds along these same lines. According to Narasimhamurty, for barite which is orthorhombic, the following ratios may be obtained: p_{31}/p_{11}, p_{21}/p_{11}, p_{12}/p_{22}, p_{32}/p_{22}, p_{23}/p_{33} and p_{13}/p_{33}. In the study of uniaxial and biaxial crystals, which are optically active, certain complications arise. These complications will be discussed below in conjunction with the experimental techniques employed in these dynamic determinations.

Mueller's method "A," which gives the magnitude of the quantity $2p_{44}/(p_{11} - p_{12})$ for cubic crystals, uses the Schaefer-Bergmann diffraction pattern which is viewed between crossed polarizer and analyzer. The Schaefer-Bergmann pattern for crystals consists of two concentric configurations of diffraction images of the point source. The inner pattern results from the diffraction of light on the longitudinal waves, the outer is due to diffraction from the transverse waves. The crystal, which is oriented so that the light enters along the [001] direction, is now rotated until one diffraction spot, in the direction making an angle ϕ with the x-axis of the crystal, disappears. Suppose that at this position the x-axis of the crystal makes the angle α with the plane of polarization of the incident light. If θ is the angle between the x-axis of the crystal and one of the axes of the index ellipse, then either $\alpha = \theta$ or $\alpha = \theta + 90^\circ$. Thus,

$$2p_{44}/(p_{11} - p_{12}) = 2 \tan 2\theta_L \tan (\phi + \psi) \tag{35}$$

if the spot is in the inner pattern, i. e. if it results from diffraction on the longitudinal wave. If the spot is in the transverse wave pattern

$$2p_{44}/(p_{11} - p_{12}) = -2 \tan 2\theta_T \tan (\phi + \psi) \tag{36}$$

In these equations, ψ is a function of ϕ and the elastic constants, and therefore, if θ_L and θ_T correspond to the same ϕ they correspond to the same ψ also. Thus, eliminating $\tan (\phi + \psi)$ from these two equations, gives

$$4\, p_{44}{}^2/(p_{11} - p_{12})^2 = -\tan 2\theta_L \tan 2\theta_T \tag{37}$$

The sign of $2p_{44}/(p_{11} - p_{12})$ may be determined, if needed, from the relationship between ϕ, ψ, and the elastic constants. The determination of the ratios $R_{001} = p_{12}/p_{11}$ and $R_{011} = (p_{11} + p_{12} - 2p_{44})/(p_{11} + p_{12} + p_{44})$ may be made by either Mueller's method "B" or method "C." The choice between these methods for cubic crystals is based on the same considerations as for glasses.

The study of uniaxial and biaxial crystals by these dynamic techniques is complicated by the fact that these crystals are optically active. For this reason, either Narasimhamurty's method or Mueller's method "C," if the necessary apparatus for light intensity measurements is available, appear most useful. Mueller's method "B" is still valid, but because of the birefringence of the medium, it is difficult to set the analyzer to the consequent broad minimum in the intensity of the first diffraction order and therefore the accuracy of the method deteriorates. Narasimhamurty's method, which requires setting the analyzer to the position at which the first orders in the two diffraction patterns appear equal, and Mueller's method "C," which requires the measurement of the light intensity of the first diffraction order for two polarizations, maintain their reliability for these studies. A further complication in the study of the strain-optical constants of uniaxial and biaxial crystals is that, because of their optical activity, the ratio of the index of refraction of the ordinary ray to that of the extraordinary ray, for the direction in which the light is traveling, enters into the expressions for the ratios to the third power. Thus, correction must be made for the effects of optical activity in computing the ratios of the elasto-optical constants from the raw data. This correction is discussed in detail by Vedam and Ramachandran.[52] Finally, because of the nature of the experimental setups, these methods fail entirely for dichroic crystals.

It is seen, that dynamic studies of the photo-elastic properties of crystals yield values of certain ratios of the strain-optical constants or combinations of these constants. For cubic crystals, for example, these measurements alone are sufficient to determine the crystal group to which the sample belongs. These dynamic measurements may be combined with compensator measurements, noting that the former are adiabatic and the latter isothermal, to evaluate the individual constants. Thus, the difficult and unreliable interferometric measurements required for a complete, static determination of these constants are eliminated.

Appendix III. The variation of the ratio p/q of the strain-optical constants of glass with density.

Mueller[36, 37] has treated theoretically the problem of correlating the observed photo-elastic effects in glass with the structure and composition of this material. This photo-elastic behavior is attributed to two effects: a "lattice" effect which produces negative birefringence and an atomic effect which results in positive birefringence. For light glasses the atomic effect predominates and, since p and q are essentially positive, the ratio p/q is greater than one. As the density of the glass increases, the "lattice" effect becomes more important until for very heavy flints the birefringence approaches zero and may even become negative. Thus, for very dense glasses p/q should be on the order of one.

Values for the ratio p/q of the strain-optical constants of a large number of glasses (about 150) are reported by Schaefer and Dransfeld.[11] In their paper, a plot is made of the value of p/q for these glasses against the density ρ. It is noted that, in general, the values of p/q decrease as the density increases. On the assumption that the relation is linear, they obtain the equation

$$p/q = 2.267 - 0.239\rho \tag{38}$$

Since the density of the glasses studied in this investigation has been determined in connection with the study by Barnes and Hiedemann,[50] a plot of p/q against ρ for these glasses is readily made. This graph is shown in figure 13. It is seen, that for the Bausch and Lomb glasses, p/q generally decreases with increasing density. Under the assumption

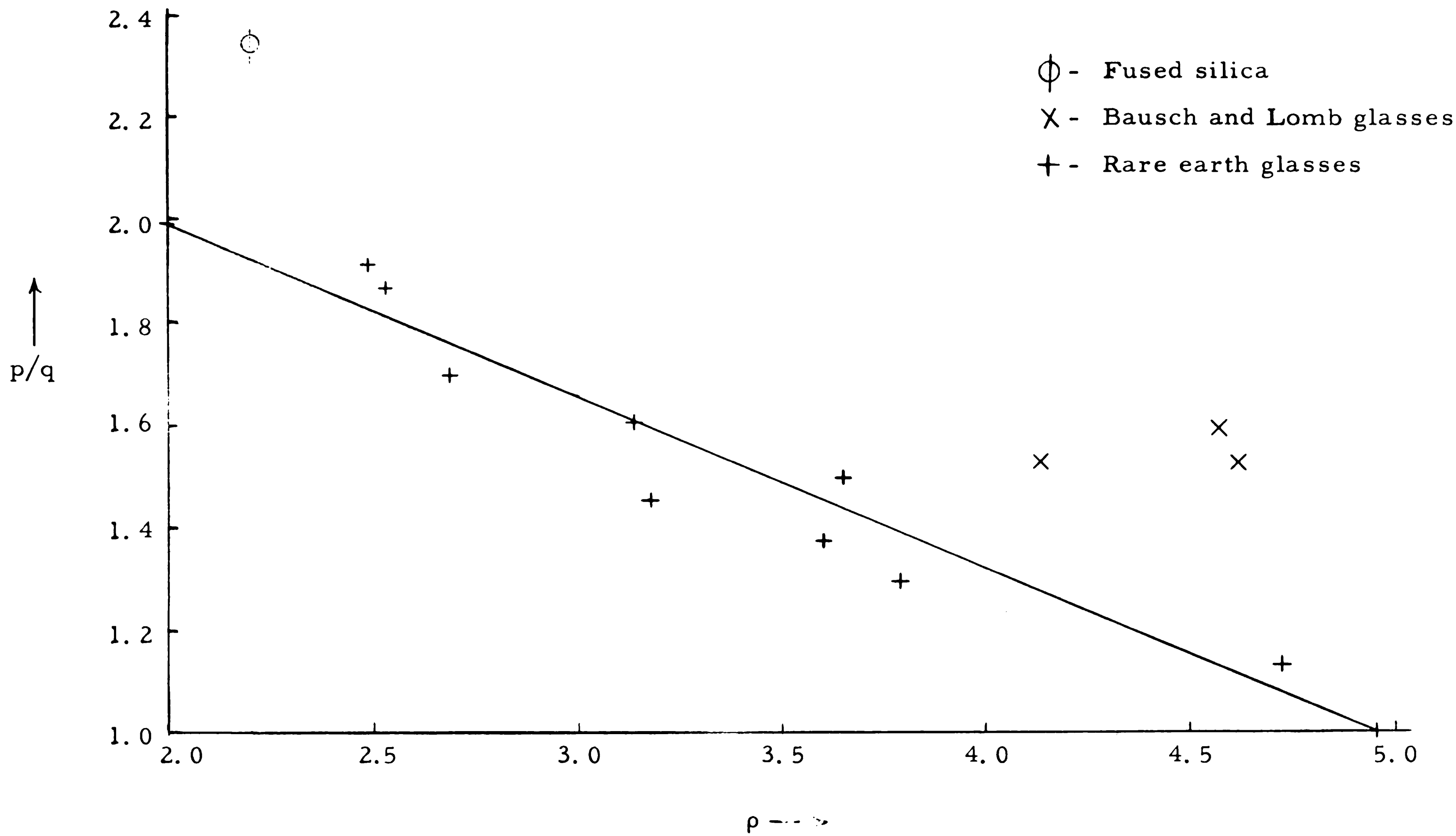

Figure 13. Variation of R = p/q with density ρ. The straight line is that obtained for the Bausch and Lomb samples using the criterion of least squares.

that p/q and ρ are linearly related for these samples, the equation

$$p/q = 2.67 - 0.34\rho \tag{39}$$

is obtained using the criterion of least-squares. Since the number of glasses studied in this investigation is relatively small, these results must be considered to be substantially in agreement with those of Schaefer and Dransfeld.

The points representing the remaining four samples lie conspicuously above the line obtained for the ordinary (i. e. Bausch and Lomb) glasses. Fused silica, a glass having a comparatively low density, probably represents a different situation than the rare earth samples, which have densities on the order of those of the extra dense flint glasses, even though in both cases the ratio of p/q is higher than for ordinary glasses of similar density.

It seems reasonable that the explanation for the case of the rare earth samples would involve either or both of two effects. From the studies of Fajans and Joos[53] it is known that the refractivity of an ion is not always the same but may vary somewhat depending on the size and charge of the neighboring ions. Thus it is possible that when the neighboring ions are ions of rare earth elements, the refractivity of the oxygen ions is higher than in ordinary glasses of similar densities in which the anions are primarily lead. It seems doubtful, however, that this difference would be great enough to account for the entire observed difference in p/q. Secondly, since anions as large as the rare earth ions would be expected to have substantial refractivity, if the atomic effect due to both the cations, i. e. the oxygen ions, and the anions is added together, this effect may continue to dominate the lattice effect and thus maintain a relatively high value for p/q whereas in an ordinary glass of similar density, the magnitude of the lattice effect approaches that of the atomic effect. This latter explanation seems better able to account for the observed results than the former.

The case of fused silica can hardly be explained in terms of the effects discussed above for the rare earth glasses. One would expect p/q for fused silica to represent a limiting case of silica glasses generally as the percent of oxides of other metals is reduced to zero. This is not the case. Since Mueller's theory is, in part, based on the Lorentz-Lorenz equation, it is possible that the assumption of the random arrangement of the oxygen ions is not valid in fused silica and that therefore the Lorentz-Lorenz equation does not apply. (The arrangement of the silicon atoms is not a factor since the refractivity of the two oxygen ions in silica is about 140 times that of the silicon ion.) This would imply that the non-repeating structure of glasses suggested by Warren[54, 55] is not valid for the case of pure silica glass. Mueller has suggested that if this structure does not apply, the calculation of the lattice effect must be considerably modified.

Because of the close relationship between the photo-elastic constants and the index of refraction, the results of this investigation should correlate with the known properties of the various types of glass. Indeed, in the case of the rare earth glasses this is true. The rare earth glasses are characterized by a high refractive index and, for these glasses, the ratio p/q is correspondingly high. In the case of fused silica, the correlation is lacking as the refractive index is about what one would expect it to be on the basis of the limiting process suggested above. (For this particular sample, the index of refraction measured for light from the mercury green line is 1.458 ± 0.005.) Vedam[31] obtained the value $p/q = 2.85$ for a fused silica sample for which he quotes $n_D = 1.4585$ as the refractive index for sodium light. This value of p/q is even higher than that obtained in the present investigation but the value of the refractive index is about normal for fused silica. Thus, it appears that the disagreement between the observed photo-elastic behavior of fused silica and that predicted by Mueller's theory implies that there are large crystalline groups in fused silica. This would also explain the difference between the value of p/q found in the present investigation and that found by Vedam since the number and size of the crystalline groups in the two samples are most probably different.

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