REFORMING MATHEMATICS TEACHING: EXAMINING THE RELATIONSHIP BETWEEN INSTRUCTIONAL POLICY AND A TEACHER'S OPPORTUNITIES TO LEARN

By

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ABSTRACT

REFORMING MATHEMATICS TEACHING: EXAMINING THE RELATIONSHIP BETWEEN INSTRUCTIONAL POLICY AND A TEACHER'S OPPORTUNITIES TO LEARN

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This dissertation offers a portrait of one teacher's efforts to attend to state-initiated reform in mathematics education. The portrait illuminates the complexity involved for the individual teacher in the context of a strong systemic effort by the state to change mathematics instruction in public schooling. It illuminates that there are a variety of different components to a teacher's practice and much for the teacher to think about and do to revise and compose in the course of efforts to improve instruction. Grounded in current frameworks for viewing relations between policy and practice, teacher learning is used as a lens for investigating a teacher's responses to policy.

One teacher's thinking and teaching is traced over time, examining the impact of the teacher's learning on responses to two different strands of proposals: reforming teaching of basic computational skills and introducing discrete mathematics into elementary mathematics teaching. Three questions guide the investigation: What mediates the teacher's interpretations and learning in the context of efforts to reform? What does the teacher learn? And, how does learning across time impact a teacher's interpretations and enactment of policy? Data collection includes interviews and observations of teaching across a three year period in two different teaching environments: elementary school teaching in a predominantly white, middle class school district; and teaching other teachers in state-sponsored professional development activities. Data also includes observations of and interviews with others involved in the teacher's professional development activities.

The case studies illuminate the ways in which an elementary teacher encounters, interprets, transforms and implements the ideas of reform. They portray how specific proposals elevate in importance, evolve and take root in a teacher's thinking, and resurface in the teacher's thought and practice again and again. Results include a description of what the teacher learns, the influences on her learning, and an appraisal of the impact of learning on the teacher's understanding and enactment of the recommendations offered in three central reform documents.

The cross-case analysis uncovers aspects of practice that make it particularly difficult for the teacher to learn and enact instructional policy. It shows, for example, that the mathematics framework and other levers developed by the state are not the only influences on a teacher's practice and that many of the influences are not always coordinated or recognized. Results illuminate that what the teacher brings and musters as resources and learning will have a great deal to do with whether reform presses forward at the level of a teacher's practice. At the same time, findings reveal that the teacher, in trying to locate and develop learning opportunities, encounters a set of weak links between efforts to reform and opportunities to learn about reform-based teaching. In essence, findings point to the importance of understanding better the relationship between the individual teacher's needs as a learner and the environments of instruction offered teachers, and working to create more meaningful links between the two.

Dedication

To my children, Evan and Jenna.

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This dissertation has traveled with me across the country three different times as my family moved from one location to another. It has also sat for long periods as I gave birth and began my responsibilities parenting two wonderful children. Across each of these events, the ideas in this work have continually consumed and captured my intellect in ways I had never before imagined possible. It has truly been a remarkable intellectual and deeply emotional experience. And through the process, I have grown more in awe of the teacher focusing this study and the many others on whom I relied to make this possible. I turn to those individuals now.

With deepest respect and gratitude I want to acknowledge Sandy Wise for providing the opportunity to learn from her efforts to respond to the reforms in California. I am indebted to the remarkable way that she opened her mind, her classroom, and her heart as she shared the many promising and the most difficult aspects of what it was like for an exemplary teacher to learn to teach differently. I am forever grateful to Sandy for helping me to understand with greater depth and clarity the complexities of teaching and the difficulties involved in changing practice.

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CHAPTER 1

INSTRUCTIONAL POLICIES, TEACHER LEARNING, AND REFORMING MATHEMATICS TEACHING

Introduction

Lessons from the past paint a bleak picture. American public schooling has a poor track record involving many failed attempts to introduce more ambitious curricula and instructional practices into public schooling (Cohen & Neufeld, 1981; Cuban, 1984; Tyack & Cuban, 1995) Instructional policies historically have had little overall success. Our history teaches us that reforming school instruction is a difficult task and that teachers will not have an easy time of it (Cuban, 1984; Powell, Farrar, & Cohen, 1985; Sarason, 1982, 1990).

Yet, despite the lessons from our past, the last decade has been characterized by an explosion of new and ambitious policies aimed at reforming mathematics teaching and learning (California State Department of Education, 1985, 1992; National Council of Teachers of Mathematics, 1989, 1991; National Research Council, 1990). The State Department of Education in California was no exception launching massive efforts to reform mathematics teaching in widely-touted documents such as the Mathematics Framework for California Public Schools, (1992), and It's Elementary (1993). Yet, even as reform was underway, there was much evidence to suggest that few instructional reform efforts ever make it past the classroom door (Cuban, 1990; Sarason, 1990).

Modern day efforts to reform are ambitious. They aim to alter public education by challenging extant views of knowledge and enduring patterns of classroom teaching and learning (NCTM, 1989, 1991; CSDE, 1992). Yet, as visions of more ambitious educational experiences for U.S. students, they present substantial problems of comprehension, interpretation, and enactment for the classroom teacher (Cohen, 1989; Cuban, 1984). An increasing number of educators and researchers are trying to understand what these problems entail and are giving serious attention to the role of the teacher in enacting reform (Cohen & Ball, 1990; Cohen et al., 1990; Elmore & McLaughlin, 1988; Elmore et al., 1996; Featherstone & Smith, 1996; Little, 1993; Schifter & Fosnot, 1993; Shulman, 1983; Simon & Schifter, 1991; Wilson et al., 1996).

This dissertation is situated along that line of research. It explores how a single teacher in California responds to state-initiated efforts to reform mathematics teaching and learning in public schooling. The chapters ahead portray what happens when a teacher is completely willing to learn and change her teaching as the state proposed. The case studies that follow paint a portrait of the kind of teacher policymakers would tend to admire. Sandy Wise[1], a third grade teacher in the Forest Glen school district, suspended any disbelief she may have had concerning the state's plans to reform. She was endlessly energetic in her efforts to learn about the new policy and change her teaching accordingly. Sandy was quite adventurous in her attempts to construct a teaching practice that offered students significantly different opportunities to learn mathematics.

What we stand to learn from Sandy's efforts at reform is invaluable. Unless and until we understand how teachers like Sandy interpret and learn from the waves of reform that come their way, we are likely to continue to fly blind with little overall success at changing what goes on inside classrooms. Part of what is at stake is getting past the common assumption that policy, if well-developed, can easily be put into practice by willing teachers. This study

[1]As I promised confidentiality, Sandy Wise, the school and district are all pseudonyms.

challenges the belief that traditional instruction is somehow rooted in teachers' bad habits or unwillingness and inabilityA even if policymakers were able to promote better solutions to curricular and instructional problems (see Cremin, 1961; Kaestle, 1972, 1983; Katz, 1987; Slavin, 1989) and could construct better policies to promote such changes, there would remain the problem of educating teachers about any efforts to reform.

Inattention to what is involved for the practitioner would only produce more wheel-spinning and for many an accumulation of despair. These risks are not worth taking. Given our history of failed attempts to create more ambitious teaching and learning in U.S. classrooms, we no longer can afford to ignore what is involved for the teacher. Currently, we know very little about how the teacher encounters, responds, and enacts new policy. We have few studies that portray what teachers make of proposed instructional changes, the difficulties they encounter, or how teachers come to understand policy and change their teaching. If we ever hope to improve education in the U.S., these issues must be given much more attention.

This study sheds light on several of these issues. It provides policymakers and teacher educators a rear-view mirror. It offers a way to continually check in with what is happening once a policy is in place. At times, the mirror reflects back the experiences of the classroom teacher as she introduces changes in her practice. At other times it reflects how policy ideas were encountered by the teacher, what ideas received attention, what was understood about those ideas, and how policy ideas were formulated into changes in teaching. The mirror also illuminates the role and influence of the teacher's professional development opportunities in learning about and from new policy. Ultimately, this study offers an explanation for how the teacher, despite much new learning and great effort, can produce a practice that resembles little of what is imagined in a policy.

Relations Among Policy, Teacher Learning, and Teaching Practice

What would it take for policy ideas to successfully influence teaching practice? Policymakers have considered this question mostly on the basis of how they might persuade educators to adopt and implement policy ideas. The issues involved often have more to do with providing appropriate incentives that convince implementors that a new policy is worthwhile. From this vantage point, adopting and implementing policy often involves more a process of telling educators what the policy is and offering the necessary incentives. Some incentives served more as enticements, such as merit pay or elevated professional roles. Others served more to punish implementors. Teacher evaluations, new rules, and standards often played out as negative incentives. No matter how incentives were perceived, relations between policy and practice were understood in terms of the incentives offered and the effects of those incentives.

Conventional policy-practice research offered different ideas and explanations for the ways in which policy did or did not find its way into teaching practice. Much of the earlier analyses focused on quantitative measures, such as students' standardized test scores, to determine the effects of policy. These measures usually resulted in the claim that new policy had little or no effect on practice. Often, the results led to increased regulation in educational programs. The "failure" to implement policy more often was attributed to teachers' and administrators' bad habits and resistance to change (Cuban, 1984; Fullan, 1982; Sarason, 1990; Tyack & Cuban, 1995). What we learned from these scholars was that when teachers were not directly a part of the development of new policy and their ownership was not ensured, top-down imposition led to changes that did not last (Elmore, 1983; Sarason, 1982).

Policy analysts then began studying relations between the bottom and the top of the educational system. Some argued that policy implementation involved a process of transmitting new ideas about teaching and learning as envisioned in policy instruments into teaching practice. Fullan, for example, argued that, "Implementation consists of putting into practice an idea, program or set of activities and structures new to the people attempting or expected to change" (Fullan, 1992, p. 65). This view fostered the notion that policy could be placed into practice, in a rather straightforward fashion, by way of teachers' use of new texts, tests, or other structures that embodied the policy vision. Implementation was thus understood in terms of those structures and the effects they had on teaching practice.

Michael Lipsky (1980), offered yet another account in Street -Level Bureaucracy. He explained relations between policy and practice as more a matter involving individual decision-making. He suggested that employees attempt to solve problems and exercise discretion in an effort to meet clients needs. He argued that formal bureaucratic controls may actually compromise quality as a result. Studies such as this one, viewed relations between policy and practice to involve the individual's need to cope with the uncertainties and pressures that come along with policy changes and an examination of the structures that promote those changes (Lipsky, 1980).

Others, such as McLaughlin, perceived policy implementation to involve a mutually adaptive process, one where policy was transformed to meet the needs of the educator and practice was transformed to fit the goals of policy (McLaughlin, 1976). From this view, both the transformation of project goals and the changes personnel made to incorporate those goals were key to understanding policy implementation. Here, both policy ideas and teaching practice underwent change. McLaughlin argued that often these transformations were rooted in individual sense making and learning.

This study begins at a fundamentally different point than those mentioned thus far. The teacher in this study was already persuaded about the policy. She needed no new incentives. She already was convinced that the intellectually ambitious goals of California's reform efforts were not only essential but necessary for improving mathematics education for all students in the state. There was little, if any, resistance on her part or her administrators. Similar to other studies, I considered the individual to be a crucial link between policy and practice. I assumed like others that teacher thinking and interpretation were key factors influencing policy implementation. But, I took that logic one step further. I assumed that policy changes ultimately would depend on teacher learning and so examined any efforts at reform by exploring what and how the teacher learned and in relationship to changed teaching.

From this perspective, this dissertation explores relations between policy and practice from the vantage point of teacher learning and the effects of teacher learning on teaching practice. Findings grow out of a longitudinal investigation of a single teacher's learning and change in relationship to a state-initiated policy to reform mathematics education. It investigates how the teacher responded to policy proposals over time and across teaching environments. It examines what and how the teacher learned to understand and enact the proposed changes. And it looks carefully at the factors that mediated teacher learning.

A Cognitive Frame

The origins of the framework underlying this work can be traced back to policy-practice research of the early eighties. Researchers were beginning to focus on what was happening with policy between the points of conception and enactment. Policy analysts began looking in local settings and asking what teachers and administrators were making of new policy. They began to realize that the individual had much influence over how policy ideas actually played out in practice. Elmore (1983), explained the phenomenon as the "the power of the bottom over the top" arguing that local meanings of policy were more useful for understanding whether and how policies were adopted and institutionalized. Studies such as the RAND Change Agent (1975) investigation illuminated further how top-down policies constrained practice and often did not help teachers construct the kind of teaching imagined. These studies suggested that local leadership and motivation were critical to policy success (Darling-Hammond & Berry, 1988; Darling-Hammond & Wise, 1981; Fullan, 1982; Sarason, 1982).

They also made another significant contribution. Studies such as these suggested that teachers' and administrators'' opportunities for continual learning, experimentation, and decision making throughout the implementation process had a significant impact on whether policies came alive in classrooms (McLaughlin, 1990, 1993). These findings set the stage for examining policy-practice relations at the local level.

Later studies would focus on district, school, classroom, and teacher level investigations of school reform. Studies of teachers' thinking and learning, in particular, were recognized as key to providing better information about the outcomes of policy in practice (Cohen & Ball, 1990; Fullan, 1992). Researchers argued that studies focusing on teacher learning and change could improve policy making itself, as they provided deeper insights into aspects of teaching practice that made policy implementation particularly difficult.

California's efforts to reform mathematics education in the mid-eighties set the stage for what later would become groundbreaking research on relations of policy and practice. Embodied in California's efforts were assumptions about what it would take to make the proposed changes possible. Teacher learning, although acknowledged, was not portrayed as a critical factor in the early stages of the implementation process. Instead, policymakers aimed their influence in other directions. New goals and standards for practice and aligning those standards with assessments, texts, and other curricular links were considered central. The instruments of policy (curriculum frameworks and guides, tests, texts and related professional development) proposed new conceptions of student learning, new images of good teaching, and a commitment to serving the needs of a diverse student population (CSDE, 1992; NCTM, 1989, 1991).

Policymakers in California began pressing practices toward these goals. In 1985, the State Department of Education introduced a new mathematics curriculum framework and corresponding curriculum guidelines. State officials had consulted mathematicians, mathematics educators, and teachers in the development of these documents. Although such frameworks had been issued to local educators since the 1960's, the 1985 version played a very different role than earlier frameworks. State officials used the new document to press publishers to revise both the content and pedagogical suggestions in their texts, arguing that texts should conform to the goals of the new framework.

For the first time publishers were exhorted to emphasize understanding mathematical ideas and problem solving rather than rote-memorization of facts and algorithms. They were told if they did not make major changes in their books, the texts would not be considered for adoption. In 1986, the state upheld its position and rejected every text publishers submitted during the first round of subsequent textbook adoptions.

Officials in Sacramento further used the framework to launch major revisions in the state's achievement testing program. Policymakers believed these instruments would promote further the ideas of the policy. Instructional alignment also played a key role. Education agencies recast curriculum guidelines to send the same clear messages to teachers about reform. By 1992, there were many elements of the reform in place. The framework, testing program, instructional alignment, and newly adopted texts, all seemed to carry the same message. Systemic alignment was, for the most part, accomplished.

A clear cut shift in ideas about what it would take to promote the new policy took place near the end of the decade. Scholars and reformers alike began to recognize the need to support teachers' understanding of the policy and shifted the focus on what it would take to teachers' professional learning (Cohen et al., 1990; Little, 1993; Sparks & Loucks-Horsley, 1990). Professional development became the center of the reform movement. Teachers began participating in workshops and institutes, each one aiming to educate teachers about the policy and support teachers' efforts to change their practice.

Research studying the effects of the 1985 mathematics framework concluded that the policy did influence teachers' instructional strategies (Cohen & Ball, 1990a; Cohen & Spillane, 1992; Smith, 1991). Yet, they argued that teachers often viewed the policy more as an add on, something to be done over and above modal practice or made more compatible with prevailing practices (Cohen & Ball, 1990a; Wilson & Corbett, 1990). Preliminary research reports argued that the policy, much like past efforts (see Powell, Farrar & Cohen, 1985; Sarason, 1982), had failed to appreciate the teacher's role in implementation. In particular, the policy failed to acknowledge teachers need to learn (Cohen & Ball, 1990).

By 1990, researchers were arguing that the effects of educational policies and programs depended chiefly on what teachers would make of them (Ball, 1990; Elmore & McLaughlin, 1988). Teacher case studies of California's efforts published in <u>Educational Evaluation and Policy Analysis</u> (Fall, 1990) and the <u>Elementary School Journal</u> (1992), illustrated how teachers constructed different meanings from policy (Cohen, 1990; Peterson, 1990; Putnam et al., 1992; Wilson, 1990). They suggested that the different constructions depended on a number of factors, including teachers' knowledge and beliefs about teaching and the subject matter. Cohen and Ball (1990, p. 238) pointed out that, "instructional policies are filtered through teachers' knowledge and beliefs about academic subjects and through their established practices."

Over the course of the next seven years, the Education Policy and Practice Study (EPPS), would continue to draw much attention to teachers' professional learning and the relations among teacher learning, policy and practice. EPPS researchers began investigating the course of instructional reform in mathematics and reading in and across three states, California, Michigan and South Carolina. This dissertation grows out of that research.[2]

I became involved in the EPPS study as a third year graduate student. Prior to that, most of my work was situated in professional development schools, partnerships formed between faculty at local school sites and Michigan State University. My work in these schools surprised and frustrated me. I observed that some teachers worked very hard to understand the mathematics education reforms. Some went to great lengths to involve themselves in learning opportunities that would help to understand the proposed recommendations in new policy and formulate changes into their teaching. Yet, even though some teachers learned a great deal, overall, students' learning experiences seemed to change little. Consequently, I wanted to understand more about why.

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[2]This study was done as part of the Educational Policy and Practice Study at Michigan State University. It was supported in part from grants from the National Science Foundation: Pew Charitable Trust: Carnegie Corporation of New York: the Consortium for Policy Research in Education, which is funded by a grant from the U.S. Department of Education, Office of Educational Research and Improvement (Grant No. OERI-G-008690011): and Michigan State University.

My interest in the EPPS project grew in relationship to my growing despair with what I had observed in working in professional development schools. Even as there was much professional development, there was little change in teaching. I aimed to look more closely at what mediated teacher learning, what the teacher was learning, and how learning informed the kinds of changes teachers made in their teaching. I imagined that this work would be useful for guiding future efforts to provide professional development around policy. The EPPS project provided an opportunity to study these issues and within a community of researchers asking similar questions about teacher learning and improving teaching.

Early on in the project researchers noticed that what teachers were learning influenced their interpretations of the policy and what should change in their teaching. Yet, they were unclear as to what teachers were actually learning or how they were learning it. In response, EPPS researchers launched an investigation into teacher learning and the relationship of that learning to policy implementation to see better what impacted teachers' interpretations of the policy and changing their teaching.

As researchers interviewed teachers and observed their practices, they noticed how teachers unpacked the ideas of the policy and how teacher learning impacted determinations of what should change. Teaching and learning became a metaphor for viewing and understanding how policy played out in teaching practice. This metaphor represented a new and different account of the interplay between policy and practice. EPPS researchers claimed that the new educational policies, because they called for dramatic departures from what was currently practiced and understood, required extensive and profound learning on the part of teachers (Cohen & Barnes, 1993a). They posited that all change would depend on learning and began examining efforts to reform on that basis.

The Pedagogy of Policy

Cohen and Barnes (1993a, 1993b), in two essays, developed a descriptive framework for examining the complex and dynamic nature of the learning process within relations between educational policies and teaching practice. The ideas they put forth paralleled relations among teachers, students, and subject matter in classrooms. Policymakers were viewed much like educators, offering new ideas about teaching, learning, and reforming teaching. Policy instruments embodied those ideas and represented a curriculum of reform. Policy learners were those responsible to implement the proposed changes. Implementation involved learning on the part of teachers, administrators, teacher educators, curriculum specialists, school boards, and many others (Wilson et al., 1996).

Applying this frame to three recent episodes in U.S. school reform, Cohen and Barnes concluded that the "pedagogy of policy" had been didactic and inconsistent at best. Policymakers mostly told teachers what to do and little had been done to educate teachers about what the new proposals would mean for classroom teaching (1993a). Having critiqued the traditional pedagogy of policy, Cohen and Barnes (1993b), asked what an educative frame might mean for the design and implementation of future instructional policies that press for ambitious classroom teaching. They concluded that policy design and implementation processes (e.g., developing instructional frameworks, selecting curricular materials, redesigning tests) should include rich opportunities for teachers and other educators to learn (Cohen & Barnes, 1993b).

The notion that policy design and implementation was educative in nature became central to all future EPPS work. Researchers identified a number of factors that conceivably would affect the quality of teachers' learning experiences surrounding policy. These factors included experiences with the ideas of policy (e.g., subject matter frameworks), the resources learners brought to bear in their learning experiences (e.g., content knowledge, beliefs about learning), and the discourse patterns between policymakers and practitioners.

Several studies examined relations among these factors and teacher learning from policy. For example, Jennings (1995) examined teacher learning from policy by exploring ways in which teachers' prior experiences and beliefs shaped their learning of new policy. Grant, Peterson, and Shojgreen-Downer (1996), underscored that teacher learning from policy occurs within multiple and embedded contexts that are frequently misaligned, presenting challenges for teachers' construction of ambitious pedagogy. Investigations also focused on district and school administrators' learning from policy. For instance, Spillane (1993), examined local administrators' knowledge and beliefs and how personal histories with the ideas shaped learning experiences surrounding state-sponsored reading policy. Peterson, Prawat, and Grant (1994) reported on a district where administrators responded to declining fiscal resources, changing demographics, and new curriculum policies, by "learning."

Once EPPS researchers had developed some ideas about the influences on teacher learning and change, they also wondered whether and how teachers' understanding of policy changed over time. This study takes up these questions and investigates teacher learning and change over the course of two years. It focuses on how the teacher's understanding of policy changes over time and examines what mediates those changes.

This study extends earlier EPPS work in another way as well. Previous EPPS studies already had established that there were differences in teachers' responses to reading and mathematics reforms arguing that subject-matter makes a difference in reforming teaching (Ball & Cohen, 1995b; Grant, 1995; Spillane, 1996). Researchers suggested that educators were better-positioned to learn and respond to new instructional policy in reading than in mathematics (Ball & Cohen, 1995b). This study examines subject-matter aspects of changing teaching by focusing specifically on mathematics and investigating teacher learning and change across two different reform-targeted subject specific content areas: basic computational skills and discrete mathematics. Across topic, subject-specific investigations of teacher learning and change have rarely been pursued. Yet, it would make sense to think that changing how multiplication is taught requires different learning than, for example, introducing combinatorial counting ideas into teaching. My purpose was to explore whether and how specific content within a subject-matter makes a difference in reform and its enactment.

The next two major sections of this chapter describe how I developed the two larger bodies of work underlying this dissertation. Unlike most studies of teacher learning to date, this study investigates the individual teacher's learning and change across a range of different opportunities to learn about reform-based teaching. In terms of design, this meant there were no pre-determined occasions to examine what or how the teacher learned. Consequently, I had to develop a framework for guiding my investigation of teacher learning and change. I also had to develop some ideas about the policy itself. This proved quite challenging, in part because the policy proposed a kind of teaching that was not well-understood or readily observable in classrooms.

Studying Teacher Learning
In the Context of External Efforts to Reform

In general, we lack good conceptual and theoretical frameworks for understanding what happens when teachers respond to instructional policies. We know little about what they learn in the context of efforts to reform or how their learning impacts the decisions they make to change their teaching. We know even less about what happens once changes in practice are in place. Problematic is that policymakers often behave as though policy implementation is virtually over when a policy has been developed and regulations are in place. In addition, most studies that investigate teacher learning do so in specific professional development contexts or in relation to particular learning goals rather than broad-based efforts to reform teaching (Lieberman, 1995; Schifter, 1993; Wood et al., 1991). Typically, these studies make claims about prevailing forms of teacher professional development and discuss whether the structures or community were adequate to support learning (Little, 1993; Lord, 1994; Sparks & Loucks-Horsley, 1990). Understanding the impact of learning across a variety of different contexts on a teacher's interpretations and enactment of policy remains mostly unexamined.

My purposes required a much more comprehensive investigation into teacher learning and change. I wanted to understand what happened to a teacher's ideas and practice over time and in relationship to the variety of learning opportunities a teacher encountered. This suggested a longitudinal investigation across many learning contexts. In a sense, I imagined positioning a searchlight on an individual teacher's encounters with policy ideas. I imagined using the searchlight to locate any occasions, circumstances, or instances that the teacher encountered for making sense of policy proposals. My aim would be to unpack these occasions, examining what mediated the teacher's learning, what and how the teacher learned, and whether and how her learning impacted her interpretations and enactment of policy. Across time, I would explore a series of learning events trying to understand both what the teacher was offered for understanding state efforts to reform mathematics education and how the teacher responded to those occasions. The literature on teacher learning and change informed how I would examine the teacher's opportunities to learn.

Research on Teacher Learning

Historically, teacher development considered teaching to be more a technical craft where teacher training of technical skills was considered central. This history suggested the likelihood that some professional development would focus only on the technical aspects of teaching and the ideas and practices involved on these occasions would require little serious learning (Cohen, 1989). It would be important to consider whether and what these occasions added to a teacher's understanding of the policy.

Other studies indicated the importance of tracing a teacher's personal history with teaching, unpacking the lessons of past experience and the impact of that experience on a teachers' understanding and implementation of the policy. There existed sufficient evidence in the literature that suggested that teachers' previous teaching experiences hinder teachers' efforts to learn about and change teaching. Dan Lortie described the "apprenticeship of observation," arguing that what teachers bring to their teacher learning experiences are more potent than formal teacher education courses.[3] This research indicated that the lessons of experience are difficult to overturn and that preservice teacher education and prevailing professional development often promote further didactic teaching of facts and skills (Lortie, 1975; Featherstone et. al, 1996; Feiman-Nemser, 1983).

More recent research suggested that, just like student learning, teachers bring to their learning opportunities other knowledge that shapes what and how they learn (Ball, 1988, 1989; Borko et al., 1992; Brown & Borko, 1992; Nelson, 1995; Schifter, 1993). The EPPS research revealed that teachers fill in the gaps of their understanding of policy in light of the thin guidance they receive creating a

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[3]D.C. Lortie. *Schoolteacher: A Sociological Study of Teaching*. (Chicago: 1975).

"mélange" of understanding and practices relating to new policy (Cohen, 1989). These studies pointed to a range of factors impacting teachers' understanding of policy including new learning, the educational context, and teachers' prior knowledge and teaching experience. Others pointed toward the importance of considering the variety of structures and contexts in which teachers learn as well as the importance of giving attention to other factors influencing teacher learning such as teachers' opportunity to converse with other educators about similar issues and teaching circumstances (Featherstone, Pfeiffer, & Smith, 1993; Featherstone et al., 1993; Heaton, 1994).

These studies indicated the importance of looking at teachers' learning as not only shaped by teachers' encounters with the reform (i.e., policy documents), but also by the prior knowledge, beliefs, and dispositions teachers bring to learning to improve teaching. They indicate the importance of investigating teacher learning in both a backward and forward direction giving adequate attention to teachers' prior knowledge, skills and dispositions. From this standpoint, I would construct a view of the teacher's history of learning about mathematics, teaching, student learning, and improving teaching and I would consider the interplay of that history with new learning. This would require something similar to an archeological dig, an excavation of prior understanding in relationship to new learning.

There also existed a growing literature that recast teacher learning in ways that paralleled the kind of learning underlying the mathematics education reforms. The epistemological position of learning inherent in the NCTM Standards documents, for example, is a constructivist/socio-constructivist perspective (NCTM, 1989). From this view, knowledge is considered dynamic and conditional, very much dependent upon the individual's sense-making within intellectual communities.

The mathematics education and learning psychology communities explored and offered theoretical explanations for the nature of the transformation that happens when teachers change their beliefs, deepen their knowledge and reinvent their practices. Carpenter, Fennema, Peterson, and their colleagues suggested that teacher change was a matter of acquiring and using new knowledge of how children's mathematical thought evolves and developing enriched and reorganized conceptual structures of mathematics (Carpenter et al., 1988; Fennema et al., 1993; and Peterson et al., 1989). In the Cognitively Guided Instruction project, researchers had been working with teachers on building conceptual links between research-based models of children's mathematical thought and teaching practice (Carpenter et al., 1994). For many teachers, the process of focusing on students' mathematical thinking in light of what they were learning about the mathematics framework, generated the integration of research based knowledge about teaching and learning into their view of children's learning (Fennema et al., 1993). Other work focused on teachers' encounters with renegotiating the norms of the classroom to foster students' construction of mathematical concepts. For example Wood and colleagues suggested that teachers resolve conflicts between their own prior knowledge and beliefs about learning as they observed students learning in their practices. They argued that conflict resolution supported by reflection and resolution supported teacher change (Wood et al., 1991). Similarly, Schifter & Fosnot argued that changes in teachers' ideas about the nature of learning requires a process of disequilibration of prior ideas and reconstruction of more powerful ones (Schifter, 1993; Schifter & Fosnot, 1993; Schifter & Simon, 1992).

Although all of these researchers focus on the phenomenon of teacher learning and change, each study emphasized a unique aspect of teacher learning. Together, they portrayed teachers' professional learning as a reconstruction process, one marked by points of disequilibrium, discomfort, comfort, conceptual change, uneven and more settled points of reconstruction.

These characteristics would become guideposts in my investigation of teacher learning. They would signal a time to pause and explore the conceptual focus and process of teacher learning. At each point, I would launch both a forward and backward investigation, examining the interplay of prior knowledge and teaching experience and new learning. In each instance, I would track on the various resources that mediated the teacher's interpretation and implementation of the policy, examining what was offered and how. Over time, I would uncover the impact of these resources on the teacher's interpretations of the policy and her teaching.

Finally, research on teacher learning also suggested the importance of looking at subject-matter specific aspects of teacher learning and change. Shulman pointed to what he termed the "missing paradigm" in educational research and argued that increased attention should be given to subject specific aspects of teaching and learning to teach (Shulman, 1987). Stodolsky studied teachers' efforts to change their teaching across different subject matters and argued that the subject makes a difference in teachers' learning to change their practice (Stodolsky, 1988). Earlier EPPS studies revealed more about the unevenness of the challenge for teachers in responding to, for example, the reading and the mathematics reforms (Ball & Cohen, 1995b; Grant, 1995; Spillane, 1996). These studies suggested the importance of looking carefully at the role of teachers' subject matter knowledge in learning to teach. I aimed to uncover and understand the relationship between a teacher's subject matter knowledge and a teacher's responses to policy. My goal would be to examine a teacher's subject matter learning and the impact of that learning on the teacher's capacity to understand and enact policy.

An important question that arises when thinking about examining the various aspects of teacher learning identified above involved whether prevailing professional development opportunities provides a good context for exploring these issues. Below I suggest the benefits of examining teacher learning in the context of the prevailing professional development activities that are available to most teachers.

The Role of Prevailing Professional Development Activities

There is no shortage of professional development opportunities for teachers in the United States. Many State Departments of Education, national and private organizations, and local school staffs, continue to offer teachers opportunities to learn about improving their teaching. The occasions range from one-day workshops, longer-term in service on tests and texts, and a host of other activities involving many other educational organizations. These dominant forms of professional development are situated in a substantial infrastructure, one that is available and reaching large numbers of teachers. Yet, most of these occasions are highly criticized. They tend to offer teachers fragmented experiences, only bits and pieces of a more complex puzzle (e.g., cooperative learning, problem solving, new content) and little guidance about how to put the pieces together. Educators and researchers alike are beginning to understand that constructing more ambitious teaching and learning opportunities for students first requires a substantially different approach to teachers' professional development (Ball & Cohen, 1995a; Wilson et al., 1996; Sykes, 1996; Smylie, 1996).

Given the existing problems, I wrestled with a nagging question: What would be the point of studying teacher learning and the impact of learning on a teacher's understanding and enactment of policy in the context of professional development activities that are already considered highly ineffective for developing teachers? My purpose was not to report any further on the inadequacies of the existing system. And most research had already dismissed existing professional development activities arguing that the structures alone do not create the kinds of opportunities teachers would need to learn.

At the same time that prevailing professional development is problematic, most state departments were relying to some extent on professional development activities for educating teachers about efforts to reform. I wondered what professional development activities teachers relied upon to support their efforts to respond to policy? And what messages about reform-based teaching were getting heard?

We know very, very little about how teachers encounter the ideas of reform. We know that State Boards of Education often will develop a new vision of teaching and learning as well as multiple levers to support that vision. Yet, how does the individual teacher encounter those messages? This study potentially could reveal a great deal about whether and how reform messages are transformed into opportunities for teachers to understand them. It could also reveal much about how teachers transform the messages they are offered into changes for their teaching. We know precious little about what teachers make of the policy levers they encounter and even less about how teachers make use of these levers to transform their teaching.

It makes sense to suggest that although current structures fail to support the ongoing development of teachers, we still could stand to learn a great deal about whether and how teachers come to understand policy levers through these structures. For example, are the assumptions about mathematics teaching and learning that underlie policy offered to teachers in the context of the professional development opportunities they encounter? If so, what do teachers make of these assumptions and how do they transform the ideas into changes for their teaching? Uncovering whether and how teachers' opportunities to learn help them to understand policy could support new ideas for improving teachers' professional development opportunities, both substantively and structurally, and with an eye toward supporting efforts to reform (Feiman-Nemser & Remillard, 1996). A study of this design could clarify many of these issues.

Focusing on a Single Teacher

The complexity of the data I hoped to gather and the potential of learning from the teacher focusing this study guided my decision to focus on just one teacher. Yet, I had to consider carefully what a study of one teacher could teach us about the relations among policy, teacher learning, and teaching practice.

It was through the EPPS project that I met Sandy Wise.[4] Sandy taught in the Forest Glen School District in the state of California for eight years, the last four in the same school. Sandy was an exemplary mathematics teacher, identified as such by other leaders in her district. That is how she came to the EPPS study. One of our curiosities in the project was to see how teachers that seemed best positioned to reform their teaching went about their work. Through numerous interviews and observations I learned that Sandy was committed to high teaching standards. She was competent mathematically and steeped in ideas about what these reforms involved for her learning and practice.

In fact, mathematics education reform had played a significant role in Sandy's practice from the onset of her career. In the early eighties, Sandy decided to continue her college education and began working toward an elementary teaching certificate. She recalled her formal teacher education experience,

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[4]As I promised confidentiality, Sandy Wise, the school and district are all pseudonyms.

> "I was really born on the edge of the reform movement. I was taught to teach the way instruction is talked about today. I wasn't taught with a basal reader, I wasn't taught with a math textbook. I was taught through Project AIMS,[5] and whole language, and I really relish in the fact that my professors were far ahead of the game" (Interview, 5/92).

After receiving her teaching certificate, Sandy began teaching third-grade in California public schools. She also decided to stay on at the university to complete her masters degree. By then, Project AIMS had grown into a university based teacher leadership program sponsoring a masters degree in education. Sandy began her teacher leadership role as a Project AIMS consultant educating teachers across the United States in week-long workshops aimed at improving mathematics and science teaching. As an AIMS consultant, Sandy continued formal teacher education course work in required seminars.

My initial observations of Sandy's third-grade teaching revealed that she was spending nearly two hours of each instructional day teaching mathematics. Most of this time students were engaged in investigation-type activities emphasizing problem solving and conceptually-oriented mathematical ideas.

By 1989, Sandy had enrolled in a satellite doctoral program of The University of Southern California. She specialized in curriculum and instruction. It was in this setting that she was involved in designing and analyzing

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[5]Project AIMS is a grass-roots teacher organization aimed at improving the teaching and learning of mathematics and science. Project AIMS (Activities Integrating Mathematics and Science) is a privately funded education foundation aimed at improving science and mathematics education in our schools. AIMS advocates and provides teachers with hands-on learning activities where the notion of "learning by doing" is valued. The project has been recognized by Congress through Project 2061 as an outstanding program aimed at meeting the goals and guidelines for taking mathematics and science education into the twenty-first century.

elementary mathematics curriculum materials and instructional practices. Sandy focused extensively on NCTM's (1989) Curriculum and Evaluation Standards For School Mathematics.

Sandy also became a mentor teacher in her district. In this role, she was involved in teacher education projects in her district. In particular, Sandy offered state-sponsored professional development opportunities to teachers for learning about the state's new Mathematics Framework For California Public Schools (CSDE, 1992). Her ongoing teacher development work later lead to her promotion to curriculum specialist in her district.

Sandy's personal history reveals her mindfulness of the mathematics education reforms. She was both a consumer and provider of professional development opportunities aimed at improving mathematics teaching and learning. She had positioned herself to be successful at implementing the state's reform agenda. She had ample opportunities to learn, ample learning community, and ample resources to support her learning.

Sandy's practice itself served as part of the basis of my decision to study a single teacher. She was obviously mindful and deeply invested in the state-initiated reform effort in mathematics education. Studying her practice offered an opportunity to see how teacher learning about policy was conceived, conceptualized and constructed for the practicing teacher. It offered the opportunity to explore relations between policy and practice from the vantage point of teacher learning.

Sandy's practice also offered the perspective of a teacher developer. As a teacher leader in California and across the U.S., Sandy already was making determinations of what and how teachers should learn. Her practice as a teacher leader provides a rather unique opportunity to understand how policy ideas become transformed into opportunities for teachers to learn.[6] A study of Sandy's practice could potentially address each of the questions focusing this work.

The Mathematics Education Reforms

The other larger piece of work underlying the investigation of Sandy's practice involved unpacking the mathematics education reforms. As a backdrop for viewing and understanding Sandy's encounters with the policy to reform her mathematics teaching, I necessarily had to develop some ideas about what these reforms proposed for changing mathematics teaching and what the implications were for teacher learning. To conclude this chapter, I describe the assumptions I made about the reforms in mathematics education and I explain how I focused my work within the larger landscape of reform ideas.

What Are These Reforms?

One impetus for the current reform movement (NCTM, 1989, 1991a,; NRC, 1990) is the widely held belief that American mathematics education is failing. In most U.S. elementary classrooms, modal mathematics teaching and learning emphasizes rules, procedures, memorization, and right answers (Goodlad, 1984; Stodolsky, 1988). In other words, students seldom are confronted with serious mathematical problems and are rarely expected to reason about mathematical ideas. Teachers stand in the front of the room and show students how to do particular procedures, later assigning practice exercises. Students practice the procedures asking teachers for help only when they get stuck. Teachers check students answers and assign more practice when needed.

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[6]Sandy functioned in various teacher leader roles in her district and more nationally. Eventually Sandy left elementary classroom teaching. She earned her Ed.D in 1997 and currently works with prospective elementary teachers, teaching math methods courses at a satellite location of USC.

In these classrooms, mathematics is represented as calculation and rote-memorization. Mathematics is not experienced as ways of thinking about quantity or space and usually does not involve reasoning. Topics such as probability, geometry, and discrete mathematics are often not given attention providing the necessary time to cover the traditional topics of arithmetic including subtraction with regrouping and long division (NCTM, 1989; Peterson, 1990; Prawat et al., 1992; Putnam et al, 1992; Putnam & Geist, 1994).

Current policies paint a very different view of mathematics teaching and learning. They argue for wide scale changes in both content and pedagogy. Underlying these arguments are very different assumptions about mathematics, teaching, and student learning. In general terms, the reforms envision mathematics as dynamic, more of an ongoing process of construction and reconstruction, one involving sense-making. From this view, knowing mathematics is much more than memorizing facts. Instead, knowing mathematics implies understanding many domains of knowledge and connections among mathematical ideas.

Students become involved in mathematical reasoning, constructing plausible arguments, and analyzing arguments for when they do and don't make sense. These changes in turn, imply that students should explore more novel topics than arithmetic, topics like probability, chance, statistics, and discrete mathematics. Student learning no longer would emphasize memorization. Instead, students would construct their own ideas in the context of group.

The reforms envision a pedagogy very different from modal teaching. Teaching would involve facilitating student learning, stimulating meaningful conversation, fostering conceptual understanding, and guiding analysis of mathematical arguments. Teachers no longer would act as authorities and givers of mathematical knowledge. Instead, they would become facilitators, fostering skill development and conceptual understanding in tandem. Pedagogy would no longer be dominated by telling students what to know and do, but instead would involve framing opportunities for students to learn, coaching, orchestrating, and guiding investigations and discussions. (CSDE, 1985, 1992; NCTM, 1989, 1991; NRC, 1990).

To say these reforms are ambitious is an understatement. They propose to shift mathematics teaching and learning from mechanical drill and memorization toward mathematical reasoning and understanding. They expect that students will learn mathematics in more meaningful ways and to accomplish that end, reformers have proposed fundamental and wide-reaching changes in the content and pedagogy of school curricula. At the same time, they are not new. Dewey, Bruner, and many others have continued to press for more ambitious school teaching and learning throughout the history of U.S. education. Yet, despite much argument, modal practice has persisted. And consequently, there are few examples available for observing what these reforms entail for teaching practice.

Implications for Teacher Learning

Instructional policy serves many purposes. At its best, it can serve to educate an entire population about a new trajectory for U.S. public education. And as enlightening as it can be, at the same time, it involves visions of uncertain practices (Ball, 1993, 1996). Instructional policies are not designed, nor can they provide, the specifics of minute-to-minute practice. Educational policies aim to sketch broad goals, set new directions and standards, and in some cases provide glimpses of classroom teaching useful for imagining new aims and purposes. What they do not provide are programs for teaching practice.

More complicated still, the conceptions of knowledge, teaching, and learning underlying the policies focusing this study are mostly unfamiliar to teachers. And because much of the picture of teaching practice is undeveloped and underdetermined in the policies, they become more open to multiple interpretations of practice and implications for teacher learning (Ball, 1996; Ball & Cohen, 1995b; Cohen, 1989; Shulman, 1987).

One of the more complicated aspects of this work involved unpacking the reform documents for what they implied for the individual teacher who must work on them. I understood well from my previous work that the proposals in such documents as California's mathematics frameworks and NCTM's standards documents involved a substantial and difficult departure from modal teaching. Yet, spelling out more specifically what this involved and what teachers must learn to understand and enact such changes was very difficult.

Other EPPS research had already concluded that these reforms would require an enormous agenda for teacher learning, one not well-understood or yet defined (Ball, 1996). Part of what this study would provide is a more detailed picture of what was involved for the teacher as a learner of new policy in California and in relation to the mathematics education reforms more generally.

Consider, for example, the National Council of Teachers of Mathematics' recommendation that teachers must create classroom environments that promote logic and evidence as verification for knowing and move away from the notion of the teacher as the authority for right answers (NCTM, 1991, p. 3). This proposal represents a multidimensional problem for teachers. To enact such a proposal, teachers would need to reconsider much of what they currently do. For example, teachers' roles, students' roles, the role of the text, tests, and other curriculum materials would all need to be reconsidered in light of the proposal to redirect issues of authority away from the teacher and toward the students' logic and evidence. The ways in which students and teachers interact, in particular the discourse patterns, would necessarily need to change. Teachers would have to invent new ways to propose tasks and problems as well as facilitate students' learning. New ideas including the notion of accountability and responsibility for providing evidence of claims would become central. A single policy proposal, taken seriously, represents a host of issues, questions, changes, and in particular unknowns for the individual teacher (Heaton, 1994; Lampert, 1990, 1992; Thompson, 1985). Unpacking policy proposals in terms of what they imply for teacher learning proved to be a complex and sometimes daunting task.

Because it was clear that the agenda for teacher learning was vast and because I wanted to learn in as much breath and depth as possible what the implications were for teachers trying to understand and enact policy proposals, I narrowed the focus of this study to two strands of policy proposals. I selected the two strands on the basis of the central role each played in Sandy's efforts to respond to the state's agenda during the time-frame of data collection for this study. Sandy was particularly focused on the proposals to reform her teaching of basic computational skills and introduce discrete mathematics into her teaching. In her work with the proposals surrounding these two strands, she faced a set of issues, questions, and unknowns that would continually draw her attention and focus her learning throughout the data-collection period. Although Sandy certainly worked on other aspects of reforming her mathematics teaching during the data-collection period, the strands focusing this work remained the most prominent in her efforts to reform her teaching.[7]

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[7]For example, I decided not to focus on the notion of conceptually understanding mathematical ideas -- Sandy had previously worked on this idea and it no longer remained a central tenet in her learning. I also noticed that each time Sandy returned to questions about conceptual understanding it was in the context of content specific teaching. In other words, conceptual understanding was enveloped by a bigger question about what the content was and what it involved for teaching it. This seems less so the case from a policy perspective as conceptual understanding appears the more wide scale idea that envelopes specific content. The reserve seemed true from the perspective of the teacher.

My decision to focus on these two strands was also based on my goal to examine the role of teachers' subject-matter learning on teachers' capacity to respond to efforts to reform. The two strands I selected represented very interesting contrasts in terms of subject-matter issues and teachers' content knowledge. One strand challenged the mathematics content and instructional practices that have continued to have a stranglehold on elementary mathematics (Burns, 1994; Putnam & Geist, 1994). The teaching of computational skills is firmly entrenched in school practices. The strand of proposals to introduce discrete mathematics represented quite an opposite scenario. The mathematics content for the most part had never been an explicit piece of the elementary mathematics curriculum. Most elementary teachers would be unfamiliar with the mathematics content of discrete mathematics. In contrast, each strand represented entirely different problems of change for the individual teacher and vastly different agendas for teacher learning. There seemed a rich opportunity to explore the role of teachers' prior knowledge and teaching experience as well as teachers' subject-matter learning on reforming teaching.

Looking Ahead

Chapter Two provides a discussion of the methodological issues involved in conceptualizing, designing and conducting this study. In that chapter, I go into greater detail as to the logic, benefit, and process of narrowing the focus of this dissertation to one teacher's learning about two aspects of the larger landscape of reform.

In Chapter Three I provide a detailed policy/document analyses of the recommendations for change surrounding each strand. In addition, I identify the vastness of what there is for teachers like Sandy to learn if they are to understand and implement the proposals. This chapter serves as a backdrop in my investigation of Sandy's encounters with the proposals. Chapter Three is also an important contribution to better understanding relations between policy and practice. Not only does it illuminate the gap between policy and practice in terms of the vastness of the learning required for teachers to understand and enact policy, it also highlights the great complexity involved in constructing a view of the recommendations offered and the implications for changed practice.

Chapters Four and Five are case studies of Sandy's efforts to understand and enact the proposals regarding each strand. I trace the evolution of her learning and interpretations of the proposals over time and I examine the interplay between her personal history with the proposed ideas and practices and what she was offered by way of new opportunities to learn. Each of these chapters offer insight into how a teacher learning agenda surrounding policy is conceptualized and constructed and the effects of learning on changing teaching. These chapters help us understand relations between policy and practice from the vantage point of practice.

Chapter Six is a cross-case analyses of the two case studies of Sandy's learning and change. It offers a comprehensive view, characterizing the qualities and conditions of Sandy's opportunities to learn about the policy proposals. The analyses in this chapter sheds new light on relations among teacher learning, educational policy, and teaching practice. It offers new insight into how the U.S. professional development system functions to educate teachers about policy.

Chapter Seven draws together my findings across all of the chapters and the larger literature to argue that the U.S. professional development system is a non-system for supporting teacher learning about policy. I then use what this study teaches us about the relations among teacher learning, policy, and practice, to suggest several ways in which teachers' opportunities to learn might be constructed to support more systematic learning about policy.

CHAPTER 2

METHODOLOGY AND RESEARCH PROCEDURES

Introduction

Some might question what can be learned about the relations among policy, practice, and teachers' learning from a case study investigation of a single teacher's encounters with the mathematics education reform agenda in California. They might ask what can one teacher teach us? Although this is a study of one teacher's interpretations and learning about and from an instructional policy, it is an investigation of the resources, orientations, views, content, and learning across several case studies of this teacher's encounters with policy ideas over the course of time. The data is robust and portrays in depth and detail the nature of policy implementation from the perspective of the individual teacher trying to enact policy. What this study offers is an understanding of the relations among policy, practice, and teachers' learning from the perspective of the individual teacher.

In this study, I assume that learning is the central activity of policy implementation. I examine a single teacher's efforts to learn about and interpret several strands of policy proposals. I use a cross-case analyses approach to understand one teachers' learning across two different strands of proposals. One set of proposals involves the recommendation that elementary teachers introduce discrete mathematics into their mathematics teaching. The other focuses on ideas and recommendations for reforming ideas and practices for teaching computational skills. The data was drawn out of a structure that was bounded by the occasions of this teacher's policy learning, her prior knowledge, beliefs and dispositions surrounding the specific proposals on which she focused, and a policy analyses approach to understanding the mathematics education reforms.

My method for collecting the data involved a process much like the positioning of three large searchlights, and watching for when they came together at a common point. One light came from the perspective of the policy, another from Sandy's learning and interpretation, and a third, focusing on her teaching. From these three vantage points, I searched for common ground, places where conceptual ideas and interpretations of proposed changes overlapped in meaning. Often, these three lights never arrived on a common point. In these instances, my analysis focused on explaining why. At other times, I recognized common points of intersection. When this occurred, I tried to understand how that happened and what the factors were that supported the similarities.

The purpose of this chapter is to explain the methodological decisions and analysis procedures I used to position these lights and search for the common ground. I begin with a methodological overview highlighting these procedures. I then explain my methods and analysis procedures for each of the two larger bodies of work -- launching an empirical investigation into this teachers' learning and changes in her practice and defining and understanding the conceptual territory of this study from the perspective of the policy.

Methodological Overview

This study investigated one teacher's encounters with the mathematics education reforms. It traces how a single teacher encountered and defined the reform agenda, how she made interpretations about the proposals, and how she constructed the ideas in her own thought and practice. Because I aimed to understand teachers' encounters with policy, there were two larger bodies of work to develop. One involved investigating this teacher's learning and interpretation in relation to her encounters with the state's policy. And the other involved understanding what these ambitious reforms were that this teacher was working on and what they required of her learning.

I began with the assumption that teachers' learning was more an evolutionary process, one that continually informed and effected teachers' efforts to reform their teaching. As I began to understand more about the complexity of tracing teacher's learning regarding teachers' efforts to understand new policy and improve teaching, I necessarily needed to find ways to track on the evolution of specific ideas and practices over time and the association of changes in thought and practice to particular learning. To get the richest data possible that would reflect something of this process, I realized I would need to identify reform ideas that were most prominent in this teacher's work, those that captured a great deal of her thought process and interest. Further, to capture the significance of her learning in relation to her understanding of the policy ideas that would focus this work, I would also need to focus my efforts on understanding the influences of the most prominent sources for her learning about those ideas. Thus, my early fieldwork would shape the focus of this study. I began by exploring what this teacher was learning surrounding the ambitious reform efforts in the state, how her learning was constructed, and how it effected her teaching practice and interpretations of the new policy. Decisions I made during this period shaped the focus and conceptual domain of this study. I discuss these decisions in more detail in the next section.

Because I wanted to understand teachers' learning from the perspective of the individual teacher trying to work on these reforms, I proposed to use case studies of one teacher's learning. I assumed that learning was a process of construction involving various complex factors that influenced learning. A case study approach provided the means to look deeply over time, with detail, and within the context (Stake, 1978, 1995; Ericson, 1986). By gathering data on this teacher's background, following leads, interviewing and observing her work with the reforms as well as other constituents involved in her learning, I could gradually take on the perspective of the individual teacher (Cusick, 1983), and develop a view of her learning and the association of that learning to her ongoing interpretations of the new policy and changes in her teaching.

As background to my investigation into this teacher's learning and change, I proposed a document/policy analysis of the reform agenda in mathematics education. My purpose was to formulate a view of what the central ideas of reform involved, what changes were proposed, and what teachers would need to learn to accomplish those changes. This body of work would define the conceptual/analytical territory of this dissertation. Guiding my work was a question about what reform ideas were seen as central by those who envisioned them. I asked what changes are proposed and what is suggested for accomplishing those changes? The purpose of the analysis would be two-fold. First, it would serve to characterize the nature of the changes and learning policymakers envisioned. Second, it would serve as a backdrop for describing and making sense of this teacher's responses to policy.

Methodologically, my goal was to develop several case studies of this teacher's learning and the relationship of that learning to understandings of policy and reformed teaching. I would then look across the cases to learn what I could about the nature of the teacher's learning experiences and the impact of learning on the teacher's interpretations and enactment of policy in teaching. Ultimately, my goal was to provide an analysis of whether and how what was learned across several cases fostered this teacher's capacity to understand and change her teaching in ways imagined in the reform documents.

Focusing the Study

Field methods have the advantage of flexibility. My initial exploration into the field helped to shape and refine the focus of this study over time. There were two ways in which this happened. One had to do with the decision to focus on one teacher's learning and change. The other had to do with the way in which I made determinations about the reform ideas that would focus this dissertation. With each of these decisions I purposefully narrowed the conceptual territory of this work (Peshkin, 1993). This required that I adapt my data gathering process to focus on specific ideas and circumstances, ones that often occurred unexpectedly at different stages of the research . I describe how I made these decisions below.

Teacher Selection

Studying one teacher's learning was a decision I made, in part, out of my initial observations and interviews with Sandy Wise. It was what she potentially could teach us and what I could potentially learn from examining her work with the reforms that guided my decision to focus on case studies of her learning.

Given that my purpose was to advance our understanding of teacher learning and the relationship of teacher learning to policy and practice, I necessarily needed to study teachers deeply invested in learning about the mathematics reforms. The EPPS study provided a wealth of options. In chapter one I described that Sandy Wise's emerging leadership role in mathematics education reform and her view of herself as a learner about reforming her teaching made her a likely candidate. My initial observations and interviews of her work with the reforms revealed that the new policy in the state of California played a central role in her determinations of what should change in her teaching. Because she came to the project identified as an exemplary mathematics teacher and a teacher developer, she represented a good candidate for studying a teacher's learning on two levels: as a learner about reforming her third grade teaching and as a teacher developer.

The Relationship Between Teacher Learning and Teaching Practice

Because my purpose was to develop ideas about what teachers were learning about reforming their teaching, how that learning was structured, and whether what was learned supported an understanding of the proposals in the framework, I examined several different aspects of Sandy's practice. When I first began visiting Sandy's classroom I observed that she had organized students into eight cooperative learning groups for solving mathematical problems. She spent nearly two hours of each school day on activities that ranged from mental arithmetic lessons to probability and statistics. Sandy's students were usually engaged in a process of collecting data and analyzing it. Students talked to each other for the most part and Sandy seemed to work from the background.

Sandy had already constructed a practice that was significantly different than modal practice. Students responsibly went about collecting, analyzing and making sense of mathematical problems, modeling for others what they had done. There was movement, noise, action, and discussion each day.

In other ways Sandy's practice reflected traditional patterns. Discourse, in larger group and small group interactions seemed to focus on getting right answers. Patterns were direct and linear. Students usually gave short responses to more directive-type questions asked by Sandy. She often provided the logic for working through a problem and ultimately pointed students toward a correct answer or right way to think about the mathematics.

Sandy's teaching practice was also bound up in her views about developmental instruction. She argued that students need slow steady steps that can be repeated and practiced. Problems were organized in terms of the number of steps or number of things to remember for solving them. Sandy made a decision not to use the state adopted textbook because she thought it did not provide the incremental or developmental approach to teaching mathematics she thought necessary.

These early observations and interviews suggested that although Sandy had constructed a practice that looked quite different than modal practice, at the same time there were fundamental ideas and practices that ran contrary to the proposals in the policy. It became clear that tracing her learning and the connection of that learning to her teaching would involve a data collection process that reached as far backward and as far forward into her thinking, learning, and teaching, as possible. The backward process, I imagined would be much like an archeological dig, requiring much care in the excavation process. The forward process, I imagined to be more straightforward requiring data collection in the form of interviews and observations of Sandy's teaching and her opportunities to learn about the policy.

Opportunities to Learn

My early work investigating her learning proved the forward process to be much more difficult than imagined. I noticed that Sandy's prior learning was etched by a variety of circumstances and ideas, some not easily accessible at this point in time. I identified that there were at least eight settings or sites that her prior learning was associated with and went on in relationship to. They included:

* State and District Professional Development Activities
* Conversations With Other Educators
* Reading Reform Documents
* Project AIMS Workshops
* Mentor Teacher Work
* Doctoral Coursework
* Teaching Practice
* Assessments

I also realized that Sandy was learning very different things in each of these settings, and that each site had entirely different assumptions and purposes for her learning. Yet, Sandy drew on each one formulating ideas about the reforms and her teaching, some contributing directly to her mathematics teaching. Because I was interested in the conceptual orientations of Sandy's learning and how her learning affected her mathematics teaching, it became clear that I would necessarily need to understand something of what underlies each of these occasions. I would have to follow not only Sandy's learning in relation to these sites for her learning, but I also would need to uncover the histories underlying theses occasions.

Based on these issues I proposed that focusing on one teachers' learning and change across a variety of reform ideas and in relation to the variety of circumstances for her learning the most promising for understanding the conceptual orientations to her learning. The case studies could then focus on what she was learning conceptually and the conceptual orientations that undergird her ongoing development of ideas and practices. The case study methodology would provide an appropriate detail for understanding what underlay this teacher's learning experiences (Patton, 1990).

As the investigation into this teacher's learning and change progressed, I noticed how her ideas and understandings of several reform ideas changed, shifted, and intermingled with other reform ideas. I tracked the development of several reform ideas backward and forward creating what I called maps of her learning. I worked to uncover Sandy's embedded and explicit assumptions about the reform ideas she was focusing and I tried to determine the embedded and explicit assumptions within the opportunities to learn she was relying. I aimed to analyze the conceptual orientations and patterns in her learning and expected that my analysis would focus on whether there were matches between what she was learning, the opportunities she was relying, and the assumptions and ideas in the new policy.

Distilling the Reform Vision

Making determinations about what reform ideas would focus this work presented the second set of issues that would further focus this study. My goal was to look carefully at the manifestation of important policy changes where it mattered most - in the classroom - and from the joint vantage point of the policy and the teacher whose teaching it intended to change. Conceptualizing the policy - in terms of what was proposed - would not only help me to bound the work in this study but also would serve to extend our ideas and understandings of what the policy involved for the teacher and teacher learning.

The new mathematics education policy in California provided a rich site for such work. The ideas of the policy came in the form of descriptions and vignettes of teaching embodied in documents, newly adopted texts, new curriculum guides, changes in testing, alignment with national standards and goals, and various professional developments aimed at fostering the new vision. Yet, even as the new policy hoped to inform and educate teachers toward important, essential, and major ideas and changes for their practice, it did not prescribe specifically what teachers should do. The actual changes teachers should introduce was less defined.

In an effort to focus this study on central tenets of the new policy, ideas that would matter to policymakers and teachers alike, I initially developed a set of questions and a criteria to help me identify a defensible set of reform ideas on which to focus. I asked what did the new policy recommend teachers change? What was recommended for accomplishing those changes? I developed the following criteria to begin.

1. the idea was seen as a key element of the new reform
2. the idea is likely to be encountered and introduced into teaching practice by the teacher in this study
3. the idea is substantially different from modal practice

The first criterion assured that each idea identified was seen as an important in the new policy. The second assured the contrast between policy ideas and the teacher's conceptualization and construction of the idea could occur. And the third insured that the idea was substantially enough that it might be a focal point for judging the successful implementation of new policy. In other words, the more adventurous the idea, the more appealing the idea would be for study.

I began by analyzing NCTM's Professional Standards for Teaching Mathematics (1991), NCTM's Curriculum And Evaluation Standards for Teaching Mathematics (1989), and the California Mathematics Framework (1985, 1992). Woven through each document were ideas and recommendations that centered around three cornerstones of teaching: 1) mathematics content (what mathematics is and what it means to know mathematics) 2) pedagogy for teaching mathematics 3) how students learn mathematics.

Using the criteria above, I nominated the following as a preliminary list of reform ideas for focusing this work.

<u>Mathematics</u>

-mathematics is not a body of isolated facts and procedures, but connected in its ideas and applications

-knowing mathematics is a process of reasoning where students support claims with evidence

-doing mathematics is conjecturing, inventing, and problem solving rather than mechanistic answer finding

-broadening conceptions of the mathematical content for elementary school teaching

<u>Mathematics Pedagogy</u>

-students construct their own understandings

-students must be active learners in the classroom

<u>Learning</u>

-teachers must become facilitators of student inquiry

-classrooms must become mathematical communities

My aim was not to provide an exhaustive list of the reform agenda but to articulate a set of ideas that policymakers, and others might regard as central tenets of the reform. This set of ideas served to focus my initial investigation into Sandy's learning and change. I continued to revise and refine this list as the study proceeded. Eventually, I used the subset of these ideas for exploring Sandy's work in depth. This narrowed the focus of this study.

Conceptualizing what teachers would need to learn to respond to the proposed changes was much more of a challenge. I searched the documents for what they suggested teachers would need to learn. This process involved unpacking the commentary around specific reform ideas. For example, each document asserted teachers would necessarily need to learn new mathematics. Probability and statistics and discrete mathematics were mentioned. Although much was said about learning familiar mathematics in new ways, there were few explicit statements of what would be involved. For instance, the conceptual underpinnings of familiar procedural steps or connections among mathematical ideas were mentioned. Yet, the specifics were left unaddressed.

My nominations below were drawn from my reading of the documents, my own understanding of mathematics, my work with prospective teacher's, my early observations and interviews with teachers in the EPPS study. I used this framework as an initial lens for examining Sandy's learning and change. Again, my purpose was not to formulate an exhaustive list of the possibilities but more to capture a list of central tenets for teacher's learning.

<u>Mathematics</u>

-the conceptual underpinnings of mathematical ideas such as number, number operations, computation, place value, measurement, fractions, decimals, geometric shapes and ideas, estimation

-connections between mathematical ideas such as numbers, number operations, fractions, etc.

-conceptual foundations of algorithms,

-broader conceptions of the ideas taught across grade levels

-new content such as probability/statistics ideas and discrete mathematics

-new conception of mathematical knowledge such as how mathematical ideas change over time, are open to debate and are flexibly understood

-mathematical modes of reasoning, different problem solving strategies, structures of valid arguments

Mathematics Pedagogy

-new roles for students and teachers in terms of authority for knowing and responsibility for doing the work

-how to facilitate students' sense making of mathematical ideas, where growth and change in the construction of knowledge become part of the process

-engage students in communication both orally and in writing with each other and teachers around mathematics

-structure the social organization of classrooms differently so that students have the opportunity to collaborate with each other and take responsibility for their own and each others learning

-structure the environment and tasks so that students confront mathematical ideas make conjectures and search for ways to solve problems

-to use new texts and other materials in ways that do not promote prescriptive learning

-how to engage students in discourse around problem solving that promotes testing theories, debate, and sense making

-how to manage uncertainty in learning environments

Learning

-constructivist ideas about learning

-implications for constructivist ideas about learning on other aspects of practice such as assessment

-ongoing informal assessment integrating instruction and assessment of student understanding

-role of prior knowledge in sense making

-how to structure time for students to explore, investigate, and grapple with significant mathematical ideas

-to use physical space, materials, and manipulatives in ways that facilitate learning significant mathematical ideas

-to encourage students to work together to make sense of mathematical ideas

-to encourage students to take risks, raise questions, formulate hypotheses about mathematical ideas

This list guided my initial inquiry into Sandy's learning and change. I searched for evidence of learning around these ideas and began to investigate her understanding both in a forward and backward direction. Influencing my focus was the evidence I uncovered that specific ideas tended to be predominant in Sandy's thought. For example, she clearly worked on the idea of learning communities where students work together in groups on mathematical problem solving tasks. Other ideas Sandy seemed to give little attention. Discourse patterns, for instance, seemed not to be something she thought much about. The discourse patterns in her teaching were very traditional. In an interview Sandy confirmed she had not given the patterns of talk in her classroom much thought. Because my purpose was to systematically follow the evolution of a subset of ideas, I selected from the list above leaving others mostly unexamined. My selections were based, in part, on what I thought I could get substantial data on. In other words, I had to be convinced that the idea was central in Sandy's thought and she was actively pursuing learning about the idea.

My determinations were that Sandy was consumed with ideas and practices relating to reforming her teaching of basic computational skills instruction and introducing discrete mathematics into her teaching. These two strands of ideas were focal in her work during the time of this study. In delving deeply into Sandy's learning and change regarding these two strands of reform ideas, I would create the opportunity to investigate the complexity of the learning involved as she worked out interpretations of the proposed changes and introduced the changes into her teaching. I would trade the opportunity to look across a wider range of reform ideas contrasting how a variety of different ideas take root or intermingle into Sandy's thought and practice for detailed views of two strands of ideas (Goetz & LeCompte, 1984).

My determinations to focus on these two strands were also rooted in what I thought I could learn from them in contrast. As I argued in chapter one, they represented opposite challenges for reform. One involved uprooting and changing the content and pedagogy of the mainstay of the elementary mathematics curriculum. The other involved introducing ideas and practices currently not present in the elementary curriculum. And they represented an opportunity to consider the role of different mathematical topics in teachers' learning from and about policy.

Data Collection

Data focusing on Sandy's learning and change was collected in the form of interviews, classroom observations, observations and interviews of the professional development activities she encountered, various articles and documents she relied upon in connection with coursework or other learning experiences, papers she authored, professional development materials, and other materials such as curriculum materials and curriculum guidelines.

Interview and observation instruments developed recursively as the study proceeded. I began with interview and observation protocols adapted from the larger EPPS study[8]. These protocols were developed and revised during the course of this study. Each data point and subsequent analysis contributed to redefining and refining data points on future instruments. The changes consisted primarily of follow up questions in order to trace the evolution of Sandy's learning and teaching of computational skills and discrete mathematics.

Observation Data

Observation data were collected across a two year span. Each classroom observation included two days of observation, across the entire day, followed by interviews and discussions of her practice. Observations of her professional

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[8]The instruments used in this study were developed as part of a longitudinal study of curricular reform across three states, including California's efforts to reform mathematics teaching (Cohen, D. K., Peterson, P. L., Wilson, S. M., Ball, D. L., Putnam, R., Prawat, R., Heaton, R., & Wiemers, N. (1990). Effects of state level reform of elementary school mathematics curriculum on classroom practice (Elementary Subject Center Series No. 25), East Lansing, Michigan State University, College of Education. The instruments have also been used and tested in subsequent studies, including QUASAR and From Congress to Classroom.

development activities involved three days of observations each of two different Project AIMS workshops. In each instance Sandy was an instructional leader facilitating other teacher's learning. Observations of her district inservice involved two one day workshops. In total, there were 16 days of observation.

Fieldnotes of each day of observation were written. Portions of each observations were audio-taped and later transcribed. Extensive handwritten notes were used to assist in the writing of fieldnotes. My focus in these observations involved tracing any aspect of Sandy's work with computational skills and discrete mathematics in relation to her teaching, interactions with students, choice and use of curriculum materials, changes and or patterns and practices in her teaching. Write-ups of each observation included a narrative summary of the activities, analytic questions to guide later interviews, and preliminary analyses of Sandy's work with the specific reform ideas traced. These notes represented the observations data.

The schedule of observations involving four classroom observations and three professional development observations formulated the basis of seven sets of fieldnotes.

Observation Fieldnotes

<u>1992</u>

January, 1992 (Classroom Observation)

May, 1992 (Classroom Observation)

August, 1992 (AIMS Workshop)

<u>1993</u>

January, 1993 (Classroom Observation)

August, 1993 (District Inservice)

August, 1993 (AIMS Workshop)

October, 1993 (Classroom Observation)

Interview Data

I tracked Sandy's learning in association with the following activities, interviewing her and others in relation to those activities.

Mentor Teacher Work
State and District Professional Development Activities
Reading Reform Documents
Project AIMS Workshops
Doctoral Coursework
Professional Development Activities
Teaching Practice
Conversations With Other Educators
Assessments

Interview data was collected across the entire study. Interviews were audio taped and later transcribed. Following each observation, I conducted a post observation interview. These interviews tracked on changes or refinements of reform ideas I noticed or that Sandy identified in her practice. Interviews conducted around her professional development activities involved Sandy, teacher participants, and other teacher educators involved in those settings. These interviews focused on what Sandy and others were learning in relation to these activities. Protocols were designed to probe what Sandy was learning, how determinations were made for how to structure that learning, and how that learning effected her teaching practice and her interpretations of the new policy.

Follow up interviews involved tracking specific reform ideas into Sandy's practice.

Other Data Sources

During the course of the study, other data such as course assignments, papers authored by Sandy, articles she relied upon for learning, interviews involving other teachers and administrators in the larger project were collected. In addition, ongoing discussions with researchers involved in the larger EPPS study contributed to my ideas about what these reforms involved and what teachers would need to learn to accomplish them.

Data Analysis

Document Analysis

Analyzing the reform agenda in mathematics education was not a straightforward task. In part, this is because the desired changes and what teachers need to learn to make those changes remains largely undefined. The policy in California, as embodied by such instruments as reform documents, frameworks, texts, tests, and professional development opportunities, offers a vision of what desired teaching would be like. At the same time, none of these instruments provide prescriptions for teaching practice. Nor do they specify what teachers would need to learn to accomplish the proposed changes. Readers, for the most part, are left to figure these things out for themselves.

The significance of this point is important for making sense of the analytic work in this study. One aspect of the work involved formulating ideas about what the proposed changes involved for teaching and making determinations of what teachers would need to learn to accomplish those changes. Guided by my analyses of three central reform documents, I developed a view of what the proposed changes implied for teaching practice and for teacher learning. My analysis of the reform agenda and a teacher's construction of the proposals pertaining to computational skills and discrete mathematics were interdependent. The development of each informed the other.

In analyzing the text across the reform documents, I conducted a search for any ideas, recommendations, explanations, proposals or vignettes that gave definition to or provided insights into introducing discrete mathematics or reforming computational skill instruction. I cross-examined the documents for similarities and differences in ideas, recommendations and visions of practice. From these data I formulated views of what the proposed changes involved for teaching and what teachers would need to learn to make those changes in their practice. These findings appear in chapter three of this work.

My analysis of the proposed changes across these two strands of proposals served as a back drop for making sense of the empirical investigation into the teacher's thinking and practice.

Analysis of Observation and Interview Data

Analysis of interview and observation data of Sandy's learning and change was ongoing throughout this study. Initial visits provided the basis for formulating hypotheses about what she was learning and how her learning effected her teaching. Follow up visits provided the opportunity to check out these hypotheses and look for further evidence of change or evolution in her thought and practice. Once transcripts and fieldnotes of observations were complete, I indexed the data on templates focusing on specific reform ideas. Each template tracked specific reform ideas. Early on, I tracked many ideas and later resolved to work carefully with two.

The templates consisted of a matrix of the relationship between the various contexts of Sandy's learning in relationship to and the specifics of what she was learning. The templates were influenced in part by what I was learning from the document analysis and in part by what I was noticing in relation to the sites for learning. These templates evolved and changed over time as Sandy's thought and practice evolved and changed. I used several rounds of each template tracing the development of reform ideas and practices. These templates served as the basis of my understanding of how ideas were evolving and in association to what circumstances for her learning.

	estimation	student construct	altern algori	concep devel.	context	new ped	technol
Project Aims							
Course work							
Teaching							
Reading Policy							
Assessment							
Conversations							
Profes Dev.							

Figure 1. Learning Template
Basic Computational Skills Instruction

Each round of data collection provided evidence of how the particular ideas were evolving in Sandy's thought and practice. From each template, I developed analytic memos of Sandy's learning and change and contrasted these memos with what I was learning from the policy/document analysis. Eventually, I pulled together a series of memos to develop descriptive and analytic pictures of Sandy's learning and change over time. These descriptive and analytic case studies appear in chapters four and five of this work. They serve as the basis of the cross-case analytic work in chapter six.

Cross-Case Analyses

The cross-case analyses involved iterations of comparative analysis, exploring various themes and returning to the cases to check validity. I compared the cases on the basis of what Sandy learned substantively, the qualities and conditions of the learning experiences she encountered, and Sandy's view of what the learning experiences offered (Miles & Huberman, 1984). I then looked to see whether and to what extent the experiences Sandy encountered offered and represented the ideas and practices proposed in the reform documents. The analyses revealed patterns across Sandy's learning experiences suggesting a lack of coordination and coherence in the educational experience encountered for learning about reformed-based change. I will argue that these patterns represent significant aspects of practice that make it particularly difficult for a teacher, working as Sandy did, to learn and respond to policy objectives (Kennedy, 1979; Wehlege, 1981).

CHAPTER 3

THE IDEAS OF POLICY

Introduction

In the previous chapter I described how I made decisions about the reform ideas that would focus my investigation of Sandy's policy learning and interpretation. Two strands of policy proposals were central in Sandy's learning during the course of this study. One focused on reforming computational skills instruction and the other proposed that discrete mathematics be introduced into the elementary school mathematics teaching. This chapter is devoted to taking a close look at what each of these strands involve from a policy perspective.

Throughout this dissertation I use the terms strands, strands of ideas, two strands, and sets of ideas and proposals to refer to the specific ideas, proposals, and recommendations made by policymakers for reforming basic computational skills instruction and introducing discrete mathematics into the elementary school mathematics curriculum. The word strand means fibers that are twisted together to form a rope. I use the notion of strands in this study because it appropriately portrays a cluster of reform ideas, proposals, or recommendations intended to work together to accomplish a desired goal.[9] In this case, reforming basic computational skills instruction and introducing discrete mathematics into

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[9]The notion of strands is also used in the California mathematics frameworks to specify particular content areas considered important to the state's view of what the mathematics curriculum should attend. In the 1985 framework seven strands of content were identified. The 1991 framework added an eighth strand called discrete mathematics. This use of strand focuses on desired mathematical content. My use of the terms strand includes the reference to particular content but further includes the range of proposals, ideas and recommendations made by policy makers for reforming basic computational skills instruction and introducing discrete mathematics into the school mathematics curriculum.

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elementary school mathematics teaching are the desired ends. The variety of ideas, proposed changes, recommendations, and practices offered to accomplish those ends are what I refer to as strands of the reform.

What is The Policy? A Document Analysis Approach

Instructional policies communicate ideas about changing teaching. By way of texts, reform documents, mandated tests, and many other policy levers, policymakers send messages about what should be changed, why they are recommended, and what is suggested about how to carry the changes. When effective, policy creates images for how teaching and learning might be re-imagined.

Some of the most detailed descriptions of current views on what must change, why, and how those changes might be accomplished appear as written documents that have surfaced over the last two decades. In this chapter, I examine a subset of those documents for the purpose of formulating for the reader a view of the proposals for changing basic computational skills instruction and introducing discrete mathematics. My analyses is an effort to identify the central tenets of these proposals and uncover whatever guidance is offered to teachers. I use my analyses as a backdrop for understanding and analyzing Sandy's learning and the changes she makes in her teaching.

Underlying the analysis in this chapter is the assumption that the visions and arguments in the reform documents leave much of the picture of change largely undefined. Reform documents, such as those analyzed in this chapter, attempt to set standards and describe goals for how teaching and learning must change. In other words, they are statements of direction. In this endeavor, they leave much of the very practical how-to of daily teaching unattended. Cohen and Ball argue that policies are not clear programs for practice (Ball & Cohen

1995a, 1995b). Policies, by their very nature, offer suggestions, arguments, glimpses and images of what the proposed changes may involve for teachers and the education of students. There is much that the policies leave unattended. For example, the practical moment to moment experience of teaching is not attended to as well as any careful examination of the relations between mathematics as a discipline of knowledge and mathematics teaching pedagogy.

The inconclusive nature of policy interacted with the empirical work in this study. My understanding of the reform rhetoric and the manifestation of reform ideas in Sandy's practice intermingled. As I observed Sandy work with the ideas, my own ideas of what the proposals involved changed and deepened. The examples and illustrations I provide as I unpack the proposals in this chapter reveal the interaction.

My analyses makes two significant contributions to our understanding of the relations between policy and practice. First, it informs the rhetoric of policy by separating out, unpacking, defining and making sense of the variety of proposals offered. Another contribution involves uncovering the complexity of such a task. Searching the reform documents for ideas and recommendations is not a straightforward process. The nature of policy as visions of uncertain practice make it difficult to construct a detailed and complete picture of change. As a result, my analyses is also incomplete and inconclusive.

This is an important insight in itself. Unpacking policy proposals does not lead to specifics for practice. Instead, the unpacking process further delineates the uncertainty of practice. Re-imagining and remaking the teaching of basic computational skills and discrete mathematics by way of policy proposals involves much speculation and decision making. The speculation and decisions I make in this chapter to carry this process out illuminate what the individual teacher is confronted with in trying to understand and respond to any set of proposals. This chapter illustrates the nature of understanding policy and the ways in which policy interpretations can have multifaceted faces. The analyses I provide illuminates the problematic nature of formulating a well-grounded understanding of any one strand of policy proposals. This may be the most important contribution of this chapter. The individual teacher is faced with distilling what these reforms are and what they suggest for her practice. Yet, the complexity and uncertainty involved in such a task may explain much of the disjuncture between instructional policies and teaching practice.

I turn now to outline the sections that follow. The chapter is organized in two parts. The first part is an analysis of reformers' envisioned changes and recommendations for how basic computational skills might be re-imagined and remade in teaching practice. My analysis is based on ideas and recommended changes as communicated by three central mathematics education reform documents: Curriculum and Evaluation Standards for School Mathematics, 1989, Professional Standards for Teaching Mathematics, 1991, and the California Mathematics Framework, 1992.[10] The Framework (1992), I rely upon in particular because of the more central role it played in Sandy Wise's work surrounding the reforms. The Curriculum Standards (1989), and the Teaching Standards (1991), I use because of the significant role each document plays in establishing, guiding, and implementing mathematics education reform more nationally as well as more locally within the state of California.

In the second part of the chapter I focus on reformers' ideas and recommendations for how discrete mathematics might be introduced into the existent school mathematics curriculum. My analysis is based on reformers'

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[10] Throughout this chapter the Curriculum and Evaluation Standards for School Mathematics will be referred to as the Curriculum Standards, the Professional Standards for Teaching will be referred to as the Teaching Standards, and the California Mathematics Framework will be referred to as the Framework.

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ideas and recommendations as communicated by the Curriculum Standards (1989), and the Framework (1992). Once again, I rely upon these documents because of the central role they play in guiding the mathematics education community toward reforming mathematics teaching and learning. In addition, I rely upon the ideas and recommendations advanced in NCTM's 1993 Yearbook titled Discrete Mathematics Across the Curriculum K-12, an interview with a mathematician specializing in discrete mathematics, and several textbooks focusing on discrete mathematics. I made decisions about including these sources as I realized there was a need for a wider range of ideas and practices surrounding the introduction of discrete mathematics for making sense of what was proposed in the two documents.

Reforming Basic Computational Skills Instruction

It is surprising how little text is directly afforded to the consideration of teaching and learning basic computational skills. Given that computation has continued to dominate teaching practice at the elementary grades and therefore is often at the forefront of teachers minds, one might expect that considerable direct attention be given to the topic. Helping teachers rethink the place and importance of basic computational skills in relationship to a student's mathematical experiences in school seems an essential element for reform. Yet, of the 215 pages in the Framework, there are 6 pages explicitly devoted to answering the question, "How do computational procedures fit into this mathematics program?" (p. 54-59). Later in the document a one page discussion focuses directly on how number facts ought to be taught (p. 73). The Curriculum Standards, a 256 page text, devotes 4 pages specifically aimed at discussing whole number computation and 3 pages of text exploring concepts of whole number operations for elementary grades (p. 41-47 Standards 7 & 8). Standard 7 for grades 5-8 focuses 4 pages of text on ideas about computation and estimation for the middle grades. Of the 665 pages of text searched, only 20 or so pages or about 3% of the texts devote direct, explicit attention to questions and issues related to the teaching and learning of basic computational skills. In one sense, the documents are relatively silent about what many elementary teachers are preoccupied with in their practices.

Beyond these sections, recommendations attending to computational goals are less noticeable. One would have to probe the texts much more deeply to find more and with little guidance about where to search. Many ideas are tucked in various places throughout all three texts. For instance, there are several pages in the Framework (1992), that focus on units of instruction. On page 110, there are three short paragraphs describing a unit on understanding arithmetic operations for elementary grades. These paragraphs, although they are intended to illustrate the idea of units of instruction, also communicate that an over emphasis on recall and the speedy execution of algorithms actually interferes with understanding operation concepts.

In another context titled Characteristics of Empowering Mathematics Programs (p. 40-43), there is text describing specifically how students ought to be introduced to traditional algorithms. The Teaching Standards, 1991, offers similar test. For example, there is a paragraph under appraising worthwhile mathematical tasks (p. 26), that includes a recommendation that computational skill and the automaticity of those skills ought to be considered when evaluating whether particular tasks are worthwhile for students. Several examples of worthwhile tasks that also accentuate computational skills are provided.

Many other examples of ideas and recommendations for reconsidering basic computational skill exist throughout the texts. For example, a K-4 curriculum standard titled Estimation for K-4 (p. 36-37), focuses on the ideas that estimation must be reconsidered as a computing strategy in and of itself.[11] There are also short stories, sample mathematical problems and vignettes located in a variety of contexts where specific content, general principles and pedagogical ideas surrounding the teaching and learning of basic computational skill is considered. Some of the stories in particular are very telling of as they offer glimpses into what reformers may have imagined students to be doing differently in their learning of computation. At the same time, one would have to deduce from these stories the specifics related to computation.

In addition to narrative text there are outlines and tables portraying ideas about computation. (i.e., p. 20-21 Curriculum Standards, p. 208-210 Framework). Within these pages there are summaries of the skills that ought to get more and less attention. For example, thinking strategies for learning basic number facts should receive more attention whereas isolated treatment of paper-and-pencil computations should receive decreased attention. There also is a description of what students would be doing in terms of tasks and emphasis if they were engaged in understanding arithmetic rather than memorizing routine algorithms. The descriptions are written across grade levels to provide a vision of what students' experiences over time might be like (p. 188 Framework). One would have to search each table and outline to extract the many position statements related to computational skills.

It seems important to remember that the documents are intended as statements of direction, offering visions and arguments for what must change. In providing new visions and images of what mathematics teaching and learning ought to look like, a kind of teaching much different than modal practice, explicit

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[11] This standard may not be immediately recognized as about computational skill. Only after doing a preliminary analysis of reformers' views would one realize reformers goal to shift current views of estimation as a "checking" strategy toward estimation as a computational strategy in itself. Within this standard estimation is developed as a legitimate computational strategy.

and extensive attention to any one topic is perhaps not reasonable. It also seems sensible to suggest that as reformers imagined a new teaching practice, one where computation is less central, that less attention be afforded to that topic. Whatever may be the case, the effect is that the policies offer an indirect route toward an understanding of the ideas and recommendations proposed for teaching basic computational skills. For the most part, the topic is not approached directly nor explicitly in the reform documents.

Given that policies are not programs for practice, relative silence or indirect attention to a topic would not equate with simple approval or disapproval of a particular kind of practice. At the same time, as I have also argued, it would be very difficult and highly unlikely for any one teacher to encounter all of what is proposed for reforming basic computational skills instruction, even in several passes through the reform documents. Further, so many ideas and topics are given such meager treatment, that even if particular ideas are located, the treatment often doesn't provide much guidance or stimulate imagination at the level of practice.

Some ideas and recommendations might be overlooked completely simply because they are embedded in contexts that the reader does not give attention. Others are not easy to uncover because they are presented in ways that bring forward other reform ideas leaving computational skill issues in the background and less noticeable. One would have to come prepared to work extensively with the documents to flesh out a detailed sketch of reformers' ideas related to computational skill. This process would involve not only a detailed search but a sorting of issues and ideas as well as the inclination to consider multiple interpretations of ideas across an extensive amount of text.

I now take up this task. I ask what are the proposals that policymakers make to reform teaching of computational skills? How do traditional ideas for developing students' proficiency with computation compare? What follows is a detailed sketch of policymakers' proposals as described in the reform documents.

The reform documents articulate a new vision of computational skills instruction. There are three central tenets described. Content and pedagogy are interwoven in my descriptions as they are in the reform documents.

A Variety of New Computational Skills

A Conceptual Grasp of What Underlies Basic Computational Skills

Contextualize Basic Computational Skills

A Variety of New Computational Skills

A variety of basic computational skills involves introducing a range of new ways to make computations. Included are mental computations, estimating, students' strategies for computing, calculators and computers for more complex computations, and traditional paper-and-pencil algorithms for more straightforward kinds of computations. Also recommended are opportunities for students to evaluate the strengths and weaknesses of skills within various contexts. In other words, traditional algorithms no longer would dominate computational skill instruction. Instead, an emphasis on understanding and using a variety of skills would dominate.

To help the reader see more clearly what these changes involve, the table below contrasts proposed ideas with traditional ideas and practices.

Traditional Ideas	Proposed Ideas
traditional computing algorithms	extensive mental computational skill
basic addition and multiplication facts	estimation as a computing strategy
limited mental computational skill	non-traditional computing strategies
	students' computing strategies
	technology for computing
	traditional algorithms
	basic addition and multiplication facts

Figure 2.
Basic Computational Skills

Each of the three documents assert the importance of teaching children a variety of different ways to compute. The above set of skills are specifically proposed for use in the service of computation.[12] In contrasting the proposed set of skills with extant practice, there are four new skills proposed that are currently not emphasized in elementary school curriculums: estimation as a computing strategy, non-traditional strategies, students' strategies, and calculator and computer use. Estimation, a very familiar idea in traditional forms of practice, plays the role of a checking strategy in the context of computation. New proposals suggest, "estimation is not solely a means of checking required by calculation: it may be the appropriate technique in itself," (p. 54 Framework).[13]

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[12]Reformers encourage a broader view of what the basic mathematical skills include. The mathematics of computation represents only one category of mathematical skill. Traditionally skills in mathematics referenced only the basic facts and computing algorithms. The new vision assumes basic mathematical skills to include, for example conjecturing, reasoning, arguing, proving as well as computational skill.

[13] Estimation is given much attention in the reform documents. Within the Curriculum Standards for K-4, Standard 5 focuses explicitly on estimation. Standard 7 for grades 5-8 deals with computation and estimation. At the same time, estimation might not be recognized initially

Estimation is viewed as a computational strategy itself. Mental computation, often a part of traditional computational curricula, must be treated with more emphasis. New proposals suggest, "both mental computation and estimation should be ongoing emphases that are integrated throughout all computational work," (p. 45 Curriculum Standards). Thus, basic addition and multiplication facts are foundational in both forms of teaching. Yet, later in this chapter, I explain how instructional practices surrounding basic-fact acquisition are very different across traditional practices and reform-based teaching.

Most of the discussion in the texts promoting a variety of skills is aimed at broadening ideas about what computational includes and why computation is done in relationship to both in-school and out-of-school contexts. The image created is that computational skill no longer implies turning automatically to traditional algorithms for the purpose of computing. Instead, there is first a consideration of what strategy seems most sensible in a given context. For example, if the context requires an estimate rather than an exact answer, an estimate would be made. If an estimate is not sufficient, other strategies would be used based on the circumstances and numbers in the desired computation. The idea is that traditional algorithms are not automatic and instead belong in a pool of possible choices. Computational skills are selected based on their merit in the circumstance (see p. 9, Curriculum Standards, for a diagram of this decision making process).

What constitutes computational skill would change dramatically if the proposals in the documents were realized. Traditional paper and pencil algorithms would no longer be the automatic choice for efficient and accurate computations. Knowing a variety of skills and understanding contextual

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by a reader as related to computational skill given the traditional views of estimation by most. The new policies assume knowledge of estimation a strategy for computing.

influences for using those skills would represent "computational skill." Statements supporting these claims would include, "Almost all complex computation today is done by calculators and computers," and, "in many daily situations, answers are computed mentally or estimates are sufficient, and paper-and-pencil computations are useful when the computation is reasonably straightforward" (p. 44 Curriculum Standards). In addition, "frequent use of calculators, mental computation, and estimation, helps children develop a more realistic view of computation and enables them to be more flexible in their selection of computing methods" (p. 45 Curriculum Standards). And further, students must have the opportunity to, "compare different approaches and algorithms for obtaining the same results, evaluating strengths and weaknesses. They must understand why the approach they choose makes sense for the problem they are solving (p. 56 Framework).

Traditionally context was not an issue in relationship to computational skills. Isolated practice of procedures and basic number facts have always dominated computational skill instruction. The proposals in the policies suggest the opposite ought to be the case. Contextualizing computational skills is perhaps the most central proposal. In the Curriculum Standards, the authors write, "some proficiency with paper-and-pencil computational algorithms is important, but such knowledge should grow out of the problem situations that have given rise to the need for such algorithms" (p. 8 Curriculum Standards). This statement implies that mastery, proficiency, and accuracy in making calculations must grow out of the problem contexts that make those goals worthwhile. In contrast, traditional practices valued the goals of speed, accuracy and mastery as ends in themselves.

Non-traditional computational techniques and students' strategies can overlap. They include strategies such as doubling or adding back. Once again, given the particular context, non-traditional computing strategies might be a better alternative than any other computational skill. For example, given the context and specific numbers in the problem, if a student had to compute $5.00 - $3.99, it would make more sense to use a strategy of counting back rather than the traditional subtraction algorithm. Students would count toward $5.00 by adding one cent to get $4.00 and then $1 to make $5.00, getting the answer $1.01 as the difference. This strategy might be emphasized as students come up with it or more explicitly as a non-traditional technique. Either way, the authors argue that strategies such as these are more often the way computation is done in out-of-school contexts. They also claim that these strategies are often more efficient and can be done without paper and pencil. The Framework on p. 54 reads, "There are many different ways to perform a calculation. Some students will invent and refine procedures for themselves, and immigrant students may bring valid alternative procedures. Teachers can encourage inventions and other alternatives ..." In the Curriculum Standards, "Children need more time to explore and to invent alternative strategies for computing mentally," (p. 45 Curriculum Standards).

Past practices emphasized learning one computing technique for each of the four operations of addition, subtraction, multiplication and division. The new vision implies students would know and have flexible use of a number of different computational strategies. New teaching practices would be different as a result. They would provide students an opportunity to select and defend choices as well as demonstrate alternative strategies, analyzing both strengths and weaknesses in use. These proposals imply other changes as well. For example, they imply different discourse patterns and new goals surrounding instruction of basic computational skills. No longer would executing the same procedures for getting the right answer be the goal. Instead, the examination of contexts, consideration of various computational techniques, selecting and using various techniques, and considering the reasonableness of results would be more central to the learning opportunity. Students would therefore have very different conversations about computation than traditional practices.

A particular point should be made regarding proposals for the use of technology for computational purposes. Proposals in the documents surrounding the use of technology for computational goals tend to muddy the issue at best. For instance, the new Framework (1992) describes calculators as "electronic pencils" of today, and that, "A reasonable goal is to make calculators available at all times for in-class activities, homework and tests" (p. 59 Framework). At the same time, reformers caution, "the availability of calculators does not eliminate the need for students to learn algorithms" (p. 8 Curriculum Standards), and "Calculators do not replace the need to learn basic facts, to compute mentally or to do reasonable paper-and-pencil computation" (p. 17 Curriculum Standards). Other issues confound the role of technology further. One position is, "Technology in the classroom can be a positive force for equity; it helps break down the barriers to mathematical understanding created by differences in computational proficiency (p. 57 Framework). At the same time, "Classroom experience indicates that young children take a commonsense view about calculators and recognize the importance of not relying on them when it is more appropriate to compute in other ways" (p. 19 Curriculum Standards).

Although the proposals clearly recommend the use of calculators and computers in relationship to computational goals, various interpretations of the arguments could be made. For example, should calculators replace instruction of three and four digit addition, subtraction, multiplication and division algorithms? Should calculators be given to students with low computational skills for all computing? It would be difficult to draw conclusions about when calculators and computers should be used based on the information in the reform documents. There is little guidance given for what sorts of computations should involve the use of technology, what technology helps students understand and not understand about computation, and nothing mentioned about what teachers might do when the technology is not available for student use.

Yet, there does seem to be agreement across the documents about the role calculators and computers ought to serve in relationship to students' learning of computation. Proposals agree that calculators and computers should be used, "to investigate numerical patterns rather than check paper-and-pencil calculations," and to "analyze data rather than perform rapid drill on basic facts" (p. 56 Framework). These statements suggest that calculators and computers ought to be used for purposes other than students' learning of executing procedures. In other words, they should be used in the process of analyzing and solving problems rather than for developing proficiency with basic facts and algorithms.

Very little attention is aimed at examining the traditional computational curriculum. For instance, there are few statements to look to concerning recommendations for what to include or exclude from the traditional curriculum. Take for example, long division, the mainstay of the fourth grade curriculum. Within each document, there are hints that long division is no longer considered valuable. In one paragraph, in the context of calculator and computer usage, the authors raise the question, "How many adults, whether store clerks or bookkeepers, still do long division (or even long multiplication) with paper and pencil?" (p. 57 Framework). In response to the question, the authors explain that long division and possibly longer multiplication problems are outdated and rarely used, "except in instances where calculations are done on the backs of envelopes."[14] Yet, little guidance is offered in the texts explaining whether long-division and multiplication algorithms ought to be excluded or to what degree they should be taught or to what level of complexity.

Because so little is said and because teachers bring variation in prior knowledge and teaching experience, the documents can be read in almost entirely opposite ways. For example, the reform documents give little attention to the kinds of emphasis that should be given to traditional computational techniques. Yet, they suggest that long division, complex paper-and-pencil computations and paper-and-pencil fraction computation be given decreased attention (p. 21 Curriculum Standards, p. 208 Framework). What this implies for practice is not clear. It could suggest that all that is currently done with division should still be done, but not to the degree of mastery. It may mean that only single or double digit divisors be expected to the degree of mastery and more complex divisions be shown but not emphasized. There are various other interpretations depending on what one brings to reading the documents. For example, a fourth grade teacher may bring the view that the traditional long division algorithm is highly valuable for students and interpret the above statement to suggest she continue teaching the long division algorithm but include other strategies as well. Another fourth grade teacher might argue that the documents suggest all traditional goals for teaching long division be abandoned. In essence, one interpretation may be to expand curricular goals but also keep what is there. Another may be to abandon certain goals.

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[14] It is also noted in a footnote that computations using traditional algorithms are often done on the backs of envelopes even when estimates make more sense. Reformers are suggesting that an overemphasis on traditional algorithms in school robs children of their ability to make more sensible computing choices.

Conceptual Grasp of Underlying Mathematical Ideas

A great deal of attention is given to the notion of understanding mathematics on a conceptual level. A conceptual grasp of the ideas underlying computation implies that emphasis be shifted away from mastery of basic facts and proficiency with algorithms and toward understanding the conceptual underpinnings of computational strategies and linking those ideas with procedural steps. The documents provide extensive argument supporting the idea that the mathematics curriculum must be conceptually oriented rather than skill oriented. In other words, emphasis must shift away from pure skill acquisition toward underlying conceptual knowledge and understandings.

This view assumes that problem solving, conceptual understanding and skill acquisition are intimately tied together instructionally. There is much text to support this notion. For example, "A strong conceptual framework also provides anchoring for skill acquisition," and "a strong emphasis on mathematical concepts and understandings also support the development of problem solving" (p. 17 Curriculum Standards).

A conceptual orientation to computational skills implies very different teaching practices than what has been traditionally acceptable. Instead of devoting instructional time to practicing the execution of pre-determined procedural steps, students would instead focus on understanding the concepts underlying the procedures and understanding the relationships between these concepts. This implies, "Emphasizing mathematical concepts and relationships means devoting a substantial amount of time to the development of understandings" (p. 17 Curriculum Standards). And, "it also means relating this knowledge to the learning of skills by establishing relationships between the conceptual and procedural aspects of tasks" (p. 17 Curriculum Standards).

Instructional practices would be very different. For example, "By emphasizing underlying concepts, using physical materials to model procedures, linking the manipulation of materials to the steps of procedures and developing thinking patterns, teachers can help children master basic facts and algorithms and understand their usefulness and relevance to daily situations" (p. 44 Curriculum Standards). Instead of asking students to memorize the basic facts, students are asked to look for patterns and understandings in an array of basic facts useful for deriving the other facts. For example, instead of arraying the 100 basic facts resulting from combining all combinations of the digits 0 through 9 and memorizing each combination, the goal would be placed on recognizing patterns in the rows and columns. For example the pattern of all zeros in the zero column and the same number in the ones column. Other patterns such as the idea that reverse-ordered facts provide the same answer and understanding why this happens would be more prominent.

A conceptually-oriented curriculum does not emphasize drill or practice of traditional algorithms. In fact, drill is not given any attention in the reform documents other than to suggest that speed of processing is no longer a central goal. Practice is mentioned in several places. Both the Framework (1992), and the Teaching Standards (1991), suggest some practice on basic number facts and traditional algorithms ought to be done for achieving proficiency with computing algorithms. In particular, one statement reads, "Practice designed to improve speed and accuracy should be used, but only under the right conditions: that is, practice with a cluster of facts should be used only after children have developed an efficient way to derive the answers to those facts" (p. 47 Curriculum Standards). Multiple meanings could be associated to how practice fits in the new proposals. One interpretation might be that traditional practice exercises for learning basic number facts should be done, but only after learning how to derive the answers using patterns and thinking strategies. This interpretation implies practice would be used to support proficiency with a cluster of related facts where relationships among those facts might be realized. Another reference to the role of practice and when it ought to occur can be found on page 47 of the Curriculum Standards.

> Exploratory experiences in preparation for paper-and-pencil computation give children the opportunity to develop underlying concepts related to partitioning numbers, operating on the parts, and combining the results. Many such experiences can be provided in the context of using place value materials, computing mentally, or performing computational estimation. Only after these ideas are carefully linked to paper-and pencil procedures is it appropriate to devote time to developing proficiency by providing practice. (p. 47 Curriculum Standards).

What kind of practice is not forthcoming in this text or the others. It seems practice does play a role in developing proficiency with computation in the new proposals. Whether it is isolated practice of facts and procedures or practice in contexts of problems is not clear. What is clear is the idea that conceptually oriented curricula emphasize the thoughtful use and meaningful development of the mathematical ideas underlying computational skills and practice would be done after those understandings are developed.

A clearer picture of what that would look like in practice is offered on p. 26 of the Teaching Standards (1991). The example illustrates how basic multiplication facts can be developed for a cluster of facts even as there is aim to contextualize the use of those facts and provide an opportunity to explore the conceptual dimensions of the notion of factors. The paragraph is situated in a section of the text aimed at setting standards for worthwhile mathematical tasks and serves as an example of the kind of task reformers' suggest is worthwhile for developing basic number facts.

> Trying to figure out how many ways 36 desks can be arranged in equal-sized groups - and whether there are more or fewer possible groupings with 36, 37, 38, 39 or 40 desks - presses students to produce each number's factors quickly. As they work on this problem students have concurrent opportunities to practice multiplication facts and to develop a sense of what factors are. Further the problem may provoke interesting questions: How many factors does a number have? Do larger numbers necessarily have more factors? Is there a number that has more factors than 36? (NCTM, 1991, p. 26)

In this illustration, student's work with a cluster of multiplication facts exploring the conceptual basis of the notion of factor. The idea is to promote both the acquisition of a set of facts and a conceptual understanding of the mathematical concept of factor. There is also aim at problem solving. As a consequence of engaging in problem solving, students have concurrent opportunities to explore the idea of factors and practice a cluster of multiplication facts. Through many similar problems students are provided the opportunity to develop mastery of basic facts, fluency with calculations, conceptual understanding and the influence of contextual issues for solving problems. Keeping in mind that, "the focus of attention has shifted from proficiency with computational procedures to attempts to make sense of and use addition, subtraction, multiplication and division appropriately" (p. 110 Framework).

Contextualize Computational Skills

Contextualize basic computational skills refers to the idea that computation must be developed within the contexts of genuine problems, the actual settings that give rise to the need for such skills. Proposals in the documents suggest approaching computational skill instruction using a problem solving perspective. From this view computational skill would not be developed in isolation of other mathematical experiences. Instead, computational skill would be developed within experiences that require the use of those skills. This implies that proficiency with computation would develop concurrently with other skills such as problem solving and mathematical reasoning. Reformers argue that children must come to understand why computation is valued. They argue, "The purpose of computation is to solve problems," (NCTM, 1989, p. 44). And, "An awareness that computation is learned and used to attain some goal develops when problem situations and computations are explicitly linked throughout all aspects of work with computations" (NCTM, 1989, p. 44).

Linking problem solving and computational skill instruction is the most central pedagogical change proposed in the policies for instruction of basic computational skills. A problem solving approach is used to foster how computational skills are used in the service of doing mathematics. From the perspective of practice, this change assumes an entirely different orientation to learning computation. Traditional ideas promote teaching computational skills in isolation of and prior to any work with problem solving. New proposals reverse this notion and argue, "a goal is to create contexts that foster skill development even as students engage in problem solving and reasoning"

(NCTM, 1991, p. 26). And, "instead of the expectation that skill in computation should precede word problems, experience with problems helps develop the ability to compute" (NCTM, 1989, p. 9). Further, "present strategies for teaching computation may need to be reversed: knowledge often should emerge from experience with problems" (NCTM, 1989, p. 9). These statements suggest that as students work through problems, they can also work on computational skills. In fact, problems can be designed to promote development with computational skills. In addition to learning proficiency with computing skills, students will also learn what computing strategies make sense in different contexts. Through the experience of solving meaningful problems and considering meaningful mathematical situations, computational skills can developed.

An Overall View of Teaching Computational Skills Differently

The table below illuminates contrasts between traditional ideas and practices for the development of students' computational proficiency and proposals articulated in the reform documents. There is very little overlap. None of the traditional practices are central to the new proposals. Practice with computing, although it appears to be a point of overlap, is very different from reformers' perspective when compared to traditional ideas.

Traditional Ideas	New Proposals
memorization of basic facts and traditional algorithms	contextualize use
practice exercises aimed at developing accuracy and precision in using basic facts and traditional algorithms	emphasize understanding by linking conceptual knowledge and procedural steps
drill in the form of timed tests aimed at developing speed in executing facts and algorithms	thinking strategies for learning basic facts
	practice under certain conditions

Figure 3.
Teaching Computational Skills

Traditional practices for developing computational skill consist mostly of memorization and isolated drill and practice of routine addition and multiplication facts and traditional computing algorithms. These practices are highly criticized in the reform documents as promoting that computational skill is synonymous with proficiency at executing traditional computing algorithms. These practices promote the idea that speed, precision, and accuracy is more important than contextual and conceptual understandings. These practices are believed to create the belief in children that computation is done purely for the sake of computing and assumes children will know when and how to apply computational skills as circumstances arise.

Reformed practices begin by situating all computational goals inside problem solving experiences. Conceptual goals and contextual understandings are attended concurrently with computational goals. Some practice of number combinations that are useful in the problem solving context is recommended. Practice outside these settings is not recommended.

Computational proficiency in reformed practice assumes flexible understanding of a variety of computational skills. It includes efficiency and accuracy in computing but extends these aims to include conceptual understanding and contextual understanding, how the context influences what computational strategies are used. Those making strides to understand and implement the policy must manage to teach students to compute proficiently and achieve conceptual understanding of the underlying ideas. Reformed ideas support and encourage memorization and facility with number facts and operations even as students engage in mathematical problem solving. Traditionally, memorizing facts and algorithms went on separate from problem solving settings. Understanding what this change implies instructionally would be at the heart of any program to foster teacher learning about changing computational skills instruction.

Two main arguments underlie these proposals. First, the changes provide students the opportunity to understand why computation is performed. Second, these practices foster more efficient use of computation. Statements such as, "Skills can be acquired in ways that make sense to children and in ways that result in more effective learning" (p. 17 Curriculum Standards) foster the notion that a conceptual orientation to learning mathematics can also support development of skills and with higher degrees of accuracy and less instructional time involved. The documents make the argument that a conceptual approach to computation, "result in good achievement, good retention, and a reduction in the amount of time children need to master computational skills. Furthermore many of the errors children typically make are less prevalent" (p. 44 Curriculum Standards). In addition, "placing computation in a problem solving context motivates students to learn computational skills and serves as an impetus for the mastery of paper-and-pencil algorithms" (p. 45 Curriculum Standards).

The documents also propose that teachers must, "consider which skills are essential and why and seek ways to develop essential skills in the contexts in which they matter" (p. 26 Teaching Standards). There is very little guidance for making these determinations. The only direct reference is that decreased emphasis be given to long division and multi-digit multiplication that involves extensive carrying procedures.

The documents acknowledge conceptual orientation would require much more time. In other words, students' computation-related skills would develop on a different pace than the current curriculum provides for. Several statements allude to the extent of the work that would be involved to make this happen. For example, "The approach to computation taken in this standard requires educators to rethink the traditional scope and sequence decisions," and, "the time required to build an adequate conceptual base should cause educators to rethink when children are expected to demonstrate a mastery of complex skills." (p. 44, p. 17 Curriculum Standards). Taken seriously, remaking computational skills instruction would involve restructuring and remaking the entire elementary skill-oriented curriculum. For example, to advance the idea that, "the presentation of computational procedures can be delayed until students need it and meaningful examples and motivation can be provided before the algorithm is presented" (p. 56 Framework), most teachers would necessarily need to restructure all that they currently do around computation. Traditionally, instruction begins with the goal of mastery of procedural steps. Instead, new proposals suggest instruction would begin by focusing on conceptual ideas, the consideration of contexts, developing a variety of computational strategies, and using those strategies in different situations. Mastery, speed and accuracy would be attended only after those understandings are developed. Further arguments suggest that only after students come to conceptually understand procedures and learn why certain steps make sense, fluency will develop as a result (p. 56 Framework). Taken seriously, the proposals would require a reframing of the more dominant skill-oriented curriculum and much reconsideration of the scope and sequence of the entire elementary mathematics curriculum.

The diagrams below highlight extant practice first and then policy proposals regarding computation. Examining the differences across these diagrams can begin to illuminate what would be involved to teach computation differently and what teachers might need to know to accomplish that goal.

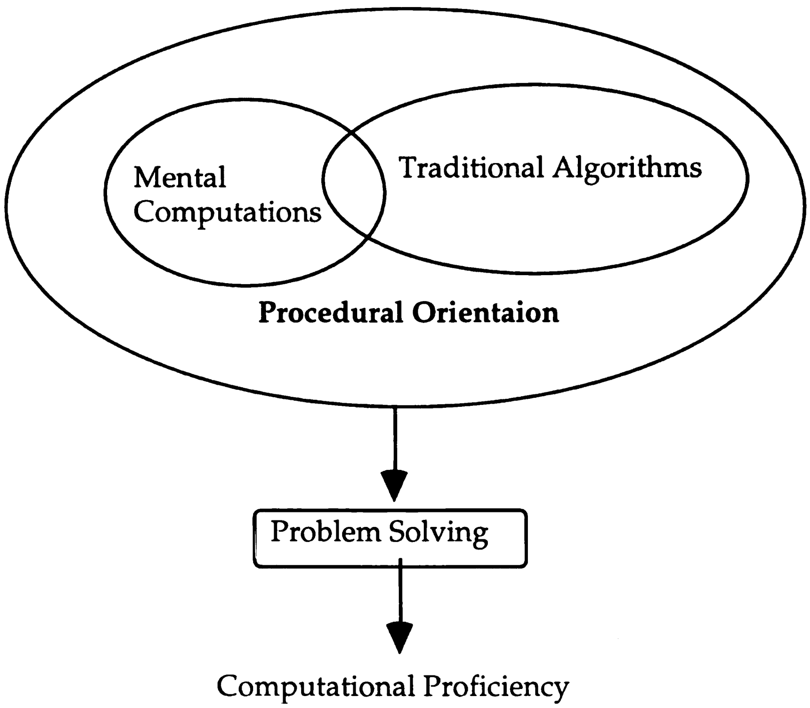

Figure 4.
Traditional Forms of Computational Skills Instruction

Extant practice involves two basic approaches to computing: traditional algorithms and mental computations. Instructionally, each is approached procedurally. Facility typically involves mental and pencil-paper calculations using mostly traditional algorithms. Speed and accuracy are valued. Problem-solving is introduced only after students have mastered computational skills.

The diagram below captures ideas for reforming computational skills instruction. It is far-more complex.

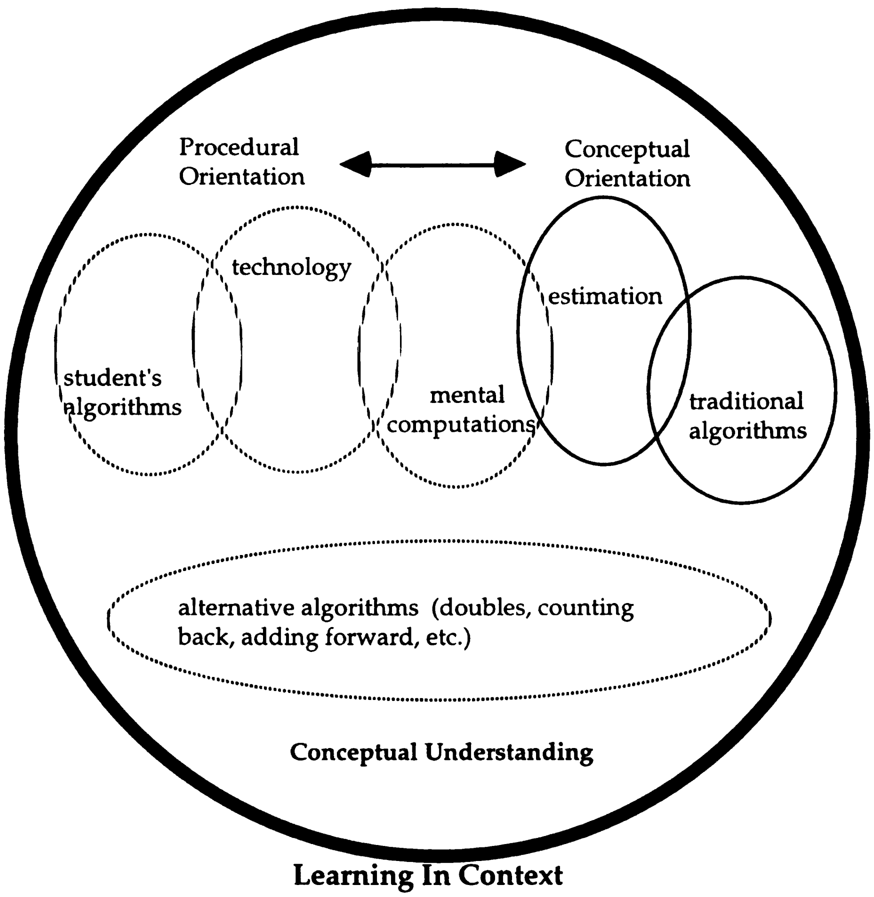

Figure 5.
Reforming Computational Skills Instruction

The inner most circles represent a range of new computational skills not included in traditional forms of practice. For instance, students' algorithms, technology, estimation, and other alternative algorithms are new computational skills that must be taught. Both conceptual and procedural orientations to these skills are recommended. Conceptual understanding is to be valued over the procedural orientation emphasized in extant ideas and practices. For example, understanding underlying ideas such as place value and flexible use of computational skills are to be valued over speed and accuracy with algorithms. The darker circle in the diagram highlights the importance of problem-solving contexts. All computational skills development is to be situated inside and go on concurrently with problem solving experiences. The goal is for students to learn to pay attention to problem solving aspects of computational skills, both what skills to use and how best to do the computation.

Contrasting proposed changes with traditional practice can serve as a beginning for speculating about what there is for teachers to learn to change their teaching of computation. For example, teachers would necessarily need to learn about estimation as a computing strategy as most elementary teachers would not have experiences thinking about estimation as a computing strategy itself. Teachers would also need to learn about the issues that arise in using technology to compute. Many teachers choose not to use calculators because of the difficulties that arise in using technology with students in classrooms. Further, teachers would need to understand that students often develop their own strategies for computing. And they would need to learn the conceptual grounding of the traditional algorithms and other mental operations already taught in school.

Examining these diagrams can reveal other issues as well. Notice that problem solving is treated differently. It only becomes important after students have developed procedural understandings of the traditional algorithms and routine mental computations in traditional forms of teaching computational skills. Extant practices treat problem solving contexts as the core activity for students. Proficiency is not required to work on problems. Contextual influences of problems are important in terms of what students should learn about computation. Unpacking the range of issues and considering how teachers might attend to these issues instructionally would be a basis for determining what teachers would need to learn.

Yet, uncovering all of what is proposed and the implications of the proposals for teacher learning would be more complicated still. There are wide-scale issues that dwell within public schooling. For example, there is a constant force of the back-to-the basics argument. Historically, we have seen consistent return to the basics following any proposals to produce more ambitious instructional goals. Another likely issue would be exploring different conceptions of balance and what balance implies instructionally. The next chapter illustrates how a teacher can interpret current policy proposals to imply a balance between traditional forms of instruction with more conceptual and problem-solving oriented teaching. My analysis earlier in this chapter suggests a different interpretation. The term balance often refers to the idea of drill and practice with facts and algorithms along side more ambitious problem solving and conceptual orientations to teaching mathematics. Reformed ideas envision balance to imply basic computational skills instruction going on concurrently with reasoning and problem solving. Both issues, "back-to-the-basics" and "balance" continue to get teachers stuck in formulating new practices. These issues can act as barriers to long-lasting change.

Another likely issue involves the relationship between procedural and conceptual knowledge. Policy proposals place a premium on conceptual understanding of the underlying mathematics of computation. Conceptual knowledge of place value, relations between ones and tens, as well as operation sense to name only a few are considered much more important in reformed-based teaching. Yet, the relationship between conceptual knowledge and proficiency with computation is complicated. One does not guarantee the other. The relationship is not well-understood. Research offers conflicting results and most research is not topic-specific.

Introducing Discrete Mathematics

It is not unusual to find instructional policies that ask teachers to introduce new and different mathematics into the existent mathematics curriculum. Sometimes this can even involve mathematics that teachers themselves have never learned or taught before (Sarason, 1982). The Curriculum Standards (1989) argue "it is crucial that all students have experiences with the concepts and methods of discrete mathematics" (p. 176). And, the Framework (1992), calls attention to the importance of discrete mathematics by introducing a new strand of mathematics content, suggesting discrete mathematics be taught in equal balance with other more traditional content areas such as geometry, algebra, measurement, and number.

Yet, most teachers, especially elementary teachers have few ideas for what discrete mathematics is. Some have never heard of it before. Most elementary teachers would be hard pressed to describe discrete mathematics or come up with examples of the central topics, questions or methods. Obviously, introducing a mathematics that teachers have little ideas about suggests there are at least two levels of the problem of teachers' learning. Teachers not only would have to learn what would be involved for introducing students across grade-levels to discrete mathematics but, they would have to learn what discrete mathematics is for themselves. Teachers would not be able to help children learn what they themselves do not understand (Feiman-Nemser, 1983).

The process of uncovering the ideas, proposals, and recommendations articulated in reform documents for introducing discrete mathematics also did not lead to programs and specifics for practice. The exact opposite occurred. Although the goals and directions became more focused, the process did not lead to specific actions for practice.

My analysis focuses only on the issues, mathematical ideas, and practices I uncovered as central to the policy proposals conveyed in the reform documents. Because the Curriculum Standards (1989) and the Framework (1992), were substantially incomplete for providing understandings of the mathematics and related issues Sandy confronted, I turned to a number of other sources to help me sketch in a view of what these proposals involved for the individual teacher.

The analyses are informed by the reform documents, my mathematics background and teaching of secondary and collegiate level mathematics courses, an interview I conducted with a research mathematician specializing in discrete mathematics, textbooks of discrete mathematics, and NCTM's 1991 Yearbook titled Discrete Mathematics Across the Curriculum K-12.[15]

I explore four core issues:

1) What discrete mathematics is.

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[15]The 1991 NCTM Yearbook titled Discrete Mathematics Across the Curriculum K-12, Margaret Kenney, editor, is identified in the Framework (1992), as a resource for teachers to consult for incorporating the study of discrete mathematics into their teaching practice. I separate the Yearbook (1991a), from the other two sources in my analysis both because it is of a different nature and distribution than the reform documents and because Sandy Wise did not rely upon the Yearbook (1991a) in her efforts to learn about incorporating a study of discrete mathematics into her teaching practice.

2) Why discrete mathematics should be introduced into school mathematics curriculums.

3) How discrete mathematics should be introduced into the school curriculum.

4) How teachers might introduce discrete mathematics into their practice.

What is Discrete Mathematics?

The 1985 mathematics framework identified seven strands of content to guide teachers' decisions about what to teach across the K-12 school curriculum. The 1992 framework introduced an eighth strand titled discrete mathematics. The Framework (1992), makes no specific mention of discrete mathematics in the table of contents, but in a section entitled, What's New in 1992, on page 2, there is an announcement of a new strand of content, discrete mathematics.

The announcement appears on page 5, of the document as the reader is reminded of the seven content strands of the previous framework: number, measurement, geometry, patterns and functions, statistics and probability, logic and algebra. The announcement reads, "The 1992 document endorses those strands and adds another, discrete mathematics." The reader is directed to two very brief paragraphs describing discrete mathematics in a footnote.

> This unfamiliar title includes things teachers at all grade levels have been doing for years. Discrete implies emphasis on separate (discrete) entities rather than on measures of continuous quantities - on questions of how many rather than how much. In third grade, for example, Boris can be asked in how many ways he can dress if he has three shirts and two pairs of pants (six): and in

seventh grade students can be asked to design a tournament. This discrete mathematics includes topics such as combinatorial counting principles (how to count permutations and combinations for probability problems) and discrete structures (such as networks and trees). A resource for teachers is Discrete Mathematics Across the Curriculum, K-12, the NCTM yearbook for 1991. (p. 5 Framework, 1991)

Two more paragraphs appear later in the document (p. 84), in a sub-section entitled, Strands, and within a section called Structure and Content of the Mathematics Program.[16]

The discrete mathematics strand did not appear in the 1985 Framework, although some of its ideas appeared under statistics and probability. Discrete mathematics includes a cluster of related ideas, such as principles for counting arrangements of discrete objects (permutations, combinations, selections); other counting principles (the inclusion/exclusion principle, the pigeonhole principle): some basic and useful ideas from set theory (unions and intersections); the study of discrete structures (networks, graphs, and tree structures); recurrence relations (such as the Fibonacci relation, $F_n = F_{n-1} + F_{n-2}$: and the analysis of algorithms.

Discrete in this context means focusing on discrete and separate entities rather than on measures of continuous quantities:

[16]A few additional comments related to discrete mathematics can be found in various other sections of the text. Most of what is said in these other sections pertain to the issue of how discrete mathematics might fit within the larger K-12 mathematics program. For an example see p. 107.

> it does not mean that everything not continuous is to be considered discrete mathematics. Arithmetic with integers, for example, is treated under number, not under discrete mathematics. (California State Department of Education, 1992, p. 84)

These paragraphs represent what is offered in terms of explanatory statements for understanding what discrete mathematics is. Depending on what the reader brings to these statements, there is either a wealth of information about discrete mathematics of little to make sense of. The notion that discrete in mathematics implies separateness, distinct from another, or elements that are countable, suggests that discrete mathematics involves, for example, study of sets of numbers such as {1, 2, 3}. Combinatorial analysis is the study of the different ways to count arrangements, combinations, or permutations on a set. For example, there would be two different arrangements possible for the set of elements {1,2}, both {1,2} and {2,1}. Extending ways in which to configure elements of sets opens up the mathematics of combinatorial counting.

A combinatorial counting problem, a sub-topic of discrete mathematics, is embedded in these paragraphs. It asks how many ways a student can dress if he has three shirts and two pairs of pants. One could argue that each shirt can be worn with each of the two pairs of pants, concluding that there are six different combinations of outfits possible. Although there is no discussion of these ideas or techniques for soling them in these paragraphs, they involve the central tenets of combinatorial mathematics mentioned as, "combinatorial counting principles" and "discrete structures." NCTM's Discrete Mathematics Across the Curriculum K-12, is named as a resource.[17]

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[17]As mentioned earlier I provide an analysis of this text later in this work.

Other ideas that one could draw out of these paragraphs, such as the notion of counting arrangements, combinations, permutations, and selections, as well as set theory (unions and intersections) may be familiar (i.e. new math era or college coursework) to some elementary teachers. But they are not commonly part of the elementary mathematics program. The terms discrete structures and recurrence relations are most likely unfamiliar to most elementary teachers and therefore would not offer much in the way of understanding what discrete mathematics involves. The terms, "the inclusion/exclusion counting principle, the pigeonhole principle," would probably fall into the unfamiliar category as well. The phrase, "the analysis of algorithms," may or may not be useful to elementary teachers trying to understand these paragraphs, depending on their understanding of what an algorithm is.

It seems that elementary teachers would have to bring fairly well-worked out ideas about discrete mathematics to recognize and use the examples and illustrations provided in the documents for formulating a vision of what discrete mathematics is. Paradoxically, what the reform documents articulate to introduce discrete mathematics assumes that teachers bring fairly well-worked out ideas about discrete mathematics. At the same time, discrete mathematics is , an unfamiliar content area to most teachers.

The Curriculum Standards (1989), suggests that, "Although discrete mathematics is a relatively new term, we will consider it simply to be the study of mathematical properties of sets and systems that have a countable number of elements" (p. 176). And, "Whereas the physical or material world is most often modeled by continuous mathematics, that is, the calculus and prerequisite ideas from algebra, geometry, and trigonometry, the non material world of information processing requires the use of discrete (discontinuous) mathematics" (p. 176). Once again, depending on the reader, there is either a wealth of information or a mountain of confusion. Formulating any understanding of discrete mathematics based on these statements would require a great deal of assumed mathematical knowledge. For example, ideas about what continuous and discontinuous quantities are, what underlying systems refers to, the idea of infinity would be central, and understanding the mathematics of algebra and calculus would all be required prerequisite knowledge for making sense of the contrast portrayed. One would have to bring this knowledge to their reading of the documents in order to understand the ideas.

Most of what is written in the Standards documents is clearly labeled in sections for secondary teaching practice. It would be easy for an elementary teacher to dismiss these proposals altogether, arguing that incorporating a study of discrete mathematics into the elementary curriculum is not a central goal of reform for elementary grades. Interestingly, Sandy perceives the introduction of discrete mathematics to be central to elementary mathematics teaching reform.

The Curriculum Standards (1989), direct treatment of discrete mathematics is geared toward a secondary teacher audience. Much of what is written is focused toward grades 9-12. Yet, there are many examples and illustrations throughout each text corresponding to various other topics that are representative of discrete mathematics. Because these illustrations are not identified as such, one would not be informed explicitly that they are useful for understanding something about discrete mathematics. From this standpoint, they may not be useful for that purpose.

Within Standard 12, (p. 176) the following statement suggests, "finite graphs (structures consisting of vertices and edges), together with their associated matrix representations, offer an important addition to the student's repertoire of representation schemes." Following is an illustration involving a diagram of a network of one-way streets referred to as a directed graph and a representation of the diagram as a matrix of numbers. Some discussion explains how matrices can be manipulated to yield various kinds of information about the graph. The example illustrates how the very familiar ideas (to the secondary audience) of matrices can also be used to introduce a very non-traditional combinatorial counting problem. The illustration assumes a basic knowledge of matrices. This may be problematic for elementary teachers in particular as it assumes knowledge many elementary teachers may not have.

On p. 177 and p. 178 two other mathematical ideas are mentioned. The terms, "recurrence relations" or "thinking recursively" and "the development and analysis of algorithms" are each discussed briefly and illustrated through several examples. The text uses a fair amount of mathematical symbolism making it difficult to follow. Most likely the symbolism is unfamiliar to most elementary teachers, making it highly unlikely for them to discern any information about discrete mathematics.

A number of topics are mentioned in relationship to the analysis of algorithms. Recommended is an algorithmic perspective to topics such as,

> "the greatest common factor of two integers, the solution of quadratic equations; approximating roots of polynomial equations; geometric constructions; the specification of sequence of transformations mapping one figure onto another, similar one; the construction of LOGO procedures to produce figures satisfying certain conditions; determining shortest/critical paths in finite graphs; random-number generation to simulate probability problems; sums of sequences; and solutions of systems of linear (and nonlinear) equations" (NCTM, 1989, p. 178).

The first idea is probably familiar to most elementary teachers because it falls within the current scope of the elementary curriculum. The ideas of an algorithmic perspective may or may not be familiar to elementary teachers depending on the teacher's mathematics background. The example used to illustrate the notion of an algorithmic perspective involves evaluating a fourth degree polynomial equation with general terms of the form ax. The illustration is complicated and difficult to follow. Ironically, the tone developed in relationship to the illustration is one of ease. For example, "The development and analysis of algorithms often add clarity and precision to the student's understanding of mathematical ideas and provides a context for nurturing careful logical reasoning," (NCTM, 1989, p. 178).

The Curriculum Standards (1989), mentions other ideas as well. Included are sets and relations; deductive proof, particularly proof by mathematical induction; the algebra of matrices; recursively defined functions; mathematical modeling and algebraic structures. Once again, depending on the background knowledge of the reader, these ideas may or may not clarify a sense for what discrete mathematics involves. There is no explanation for how these ideas fit within a study of discrete mathematics or in relation to other mathematics.

It should be pointed out that the documents treat all content strands similarly. Short descriptive paragraphs, lists of central ideas and topics, and some illustration serves to introduce a reader to each content strand. Most of the ideas mentioned in sections describing discrete mathematics would be familiar to secondary mathematics teachers. Very few would be familiar to elementary teachers. Thus, developing insights into what discrete mathematics involves based on reading the documents would be highly dependent on the mathematical understandings one brings to their reading. In the case of elementary teachers, it would be very difficult to develop any sense for what discrete mathematics involves.

Why Discrete Mathematics Should Be Introduced

The Framework (1992), offers nothing in the way of an explanation for adding an additional strand of content beyond the idea that the Framework (1992) endorses the set of standards recommended by NCTM in the Curriculum Standards (1989). The Curriculum Standards (1989), on the other hand, make several different arguments for including discrete mathematics in the school mathematics curriculum. The biggest push seems to come out of the connection discrete mathematics has with computer technology. On page 176, the following statement supports this conclusion.

> "Computer technology, too, wields an ever-increasing influence on how mathematics is created and used. Computers are essentially finite, discrete machines, and thus topics from discrete mathematics are essential to solving problems using computer methods. In light of these facts, it is crucial that all students have experiences with the concepts and methods of discrete mathematics" (NCTM, 1989, p. 176).

Embedded in this argument are a number of related rationales. For instance, discrete mathematics, because of the connection it has with computer technology, has potentially more obvious connections to real-world phenomena as well and thus provides a more direct means for strengthening the connection between theoretical mathematics and real-world applications, a constant criticism of the traditional curriculum.

Further, the Curriculum Standards (1989), point out, "the non material world of information processing requires the use of discrete (discontinuous) mathematics" (p. 176). Given that we live in an information processing society, discrete mathematics offers a means for making societal demands and concerns more relevant within the school curriculum. A side argument to the issue of relevance or usefulness also involves the rationale of modernizing the school mathematics program. Discrete mathematics has been termed "the math for our time" (p. 1, NCTM Yearbook, 1991a), and there is argument that discrete mathematics would update or modernize the school math program providing avenues for, "moving school mathematics toward the 21st century" (p. 1, NCTM Yearbook, 1991a).

A secondary argument for introducing discrete mathematics, although less emphasized in the reform documents and perhaps less noticeable, involves the idea that discrete mathematics provides a setting or opportunity to incorporate many of the other goals and recommendations of the reform. The argument posits that besides promoting and capitalizing on the connection to technology and real-world problems, discrete mathematics potentially fosters reasoning and critical thinking, primary aims of the reform. Thus, discrete mathematics creates a desirable setting for emphasizing logical reasoning, deductive thinking and inductive argument. The documents suggest discrete mathematics can provide an avenue for school mathematics to move away from the dominance on manipulating symbols and memorization formulas, the very thing school mathematics is criticized for.

How Discrete Mathematics Could Be Introduced

There are mixed messages concerning how discrete mathematics might be situated within the larger school mathematics program. The Curriculum

Standards (1989), proposes that discrete mathematics must be introduced to all students at the secondary level. Yet, there are no specific instructions for whether or how to introduce discrete mathematics into the elementary or middle grades. At the same time, there exists many examples throughout the K-8 set of standards where discrete mathematics is featured. These examples fall within contexts such as number sense or numeration and probability and statistics. The question of how to or whether explicit attention should be given to discrete mathematics at levels other than grades 9-12, is left ambiguous in the Curriculum Standards (1989). The NCTM Yearbook (1991a), proposes explicitly that attention be given to discrete mathematics across the entire curriculum, stressing the importance of introducing discrete mathematics at all levels on the basis that the entire math program can be strengthened as a result. NCTM's message is confusing at best.

The Framework (1992), advocates explicitly that attention should be given to discrete mathematics at all levels of schooling. Yet, the authors argue that the level of formalism should be adjusted at different levels. For instance the document explicitly states that formal attention to discrete mathematics in grades K-5 is not appropriate (p. 107). Recommended instead is that discrete mathematics be explored informally in a variety of different contexts already common to the elementary school math program. There is very little discussion for how this might be done or how any informal explorations might connect to form a coherent understanding of discrete mathematics or connect to more formal work in discrete mathematics later. A few examples of activities in which teachers might engage students for exploring discrete mathematics are included. Two examples, one at the third grade level and one at the middle grades level were offered earlier in this text. There is no discussion for how these examples might be situated in practice or what students might be able to learn from them.

The Curriculum Standards (1989), the Framework (1992), and the NCTM Yearbook (1991a), all agree that a formal course at the high school level is not appropriate. Instead, the documents suggest that topics in discrete mathematics be integrated throughout existing courses. The following statement on p. 176 of the Curriculum Standards (1989) supports this position, "This standard neither advocates nor describes a separate course in discrete mathematics at the secondary school level; rather, it identifies those topics from discrete mathematics that should be integrated throughout the high school curriculum." Further, "The depth and formalism of treatment should be consistent with the level of the courses in which a topic appears." The idea is that topics in discrete mathematics can be inserted into an already existent mathematics program.

The Framework (1992), provides some information about how discrete mathematics topics might be integrated and organized within the K-5 mathematics curriculum. In a section on page 107, under a sub-section titled Integrating Strands in the Elementary Grades, the authors write, "because the strands are intended to be interwoven, students need not have experiences with each strand separately. In kindergarten through grade five, some strands, particularly algebra, functions, and discrete mathematics, seldom appear alone because it is not appropriate to deal with that material formally." Several examples are given explaining how this might happen. For instance, unifying ideas such as "how many" or "how much" (counting and measuring) are recognized as important ideas for children to learn. They represent contexts where several strands might be interwoven at once, in this case "number and discrete mathematics" (p. 107). On page 108 the ideas are explained in a little more detail.

"How many? How much? A count tells how many things are in a specified group. A measure tells how much of a specified attribute something has. Generally, counts are discrete and measures are continuous. Both counts and measures identify quantities, therefore both are identified by unit labels."

Specific examples of activities or content that can be interwoven as unifying ideas are not provided in this section. Later in the text there is a description of what a recommended program organization and structure for elementary grades might look like (CSDE, 1992, p. 109). In examining this program there is no specific mention of discrete mathematics or any of the ideas outlined earlier as important within a study of discrete mathematics. What does appear are a few examples of counting problems under number and numeration and Venn diagrams for classifying objects and displaying data under statistics. The illustration is compatible to the idea that discrete mathematics not be treated formally but introduced in relation to other strands or unifying ideas.

Advice to Teachers

The big message is that much of the content of discrete mathematics is already largely in the school curriculum. The Framework (1992), introduces discrete mathematics as, "This unfamiliar title includes things teachers at all grade levels have been doing for years" (p. 2). The implication seems to be that much of what discrete mathematics involves is already getting attention. What then would be involved for teachers introducing discrete mathematics into their practice? The documents offer few details. One possible interpretation might be that incorporating discrete mathematics into the school curriculum would involve nothing new and only a reorganization of topics. Another might be that familiar ideas, already getting attention, could be used in the process of introducing discrete mathematics. There are several illustrations of this point. Recall the illustration involving matrices that I discussed earlier in this chapter. At the same time, the issue of how to take familiar mathematics and use it to introduce discrete mathematics is not explicitly discussed. The documents tend to leave this question mostly unattended.

One idea mentioned in the documents is the notion "slant." The term "slant" refers to the idea that discrete mathematics should not be taught in ways that promote more memorization of formulas and pure symbol manipulation, the exact thing current practices in mathematics teaching is criticized for. Instead, reformers' recommend discrete mathematics be represented in ways that promote logical reasoning and thinking. The following paragraph points to this emphasis:

> In grades K-8, counting typically involves matching the elements of a set with a finite subset of the natural numbers. But real-world problems that can be simplified to the form "How many different subsets of size k can be selected from the members of a set having n distinct members?" require an entirely different method of counting. To develop students' ability to solve problems with this structure, instruction should emphasize combinatorial reasoning as opposed to the application of analytic formulas for permutations and combinations. To illustrate this shift in perspective consider a fundamental identity involving binomial coefficients: ${}_nC_r = {}_nC_{n-r}$. This identity usually is established by algebraic manipulation of the formula for combinations. In contrast, a student who reasons combinatorially may observe that ${}_nC_r$ represents the number of

> ways one can choose an r-element set from and n-element set. For each r-element set chosen, however, there corresponds a set of n - r elements not chosen. Thus, the number of ways of choosing an r-element set is equal to the number of ways of choosing an (n - r) element set (NCTM, 1989, p. 179).

This paragraph potentially could foster ideas and understandings of combinatorial mathematics, an area of specialization under the scope of discrete mathematics. It also represents important ideas about pedagogy, instruction that emphasizes thinking, understanding, and reasoning rather than memorization of formulas or symbol manipulation. Yet, the impact of the paragraph may be very different for different readers. The complexity of the mathematical terms, and the mathematics may interfere with sense making about discrete mathematics and how to introduce it into teaching practice. The assumption in the policies seems to be that most readers would bring a relatively good working understanding of the mathematics of combinatorial counting. It may be that the very opposite is true, at least for elementary teachers.

The reform documents offer relatively little for understanding what discrete mathematics is or how it might be introduced into the school curriculum. The little that is provided points readers in various directions for learning more about discrete mathematics. The ideas and recommendations to this point represent what I uncovered in the reform documents. Because much was left unattended and unclear, and because I wanted to explain Sandy's encounters with these proposals, I turned to other sources to develop further understandings of the ideas and practices for introducing discrete mathematics into the school curriculum. The analyses that follow are not based in the reform documents but instead, are derived from interviews and my reading of various other texts including NCTM's yearbook entitled Discrete Mathematics Across the Curriculum K-12, (1991a).

Other Resources on Discrete Mathematics

Davis and Hersh propose in The Mathematical Experience (1981), that mathematics is the science of quantity and space and that mathematicians are involved in studying ideas and relationships related to quantity and space. They claim that mathematics, both what it is and how it gets done, changes over time making it impossible to provide a completely adequate view of what mathematics is, mostly because of the changing nature of the work in progress. At best, one can develop a sense for the current state of affairs by characterizing what mathematicians working in the community do, keeping in mind that there are different points of view about what that is (Barratta-Lorton, 1977; Davis & Hersh, 1981; Kitcher 1984; Kline, 1980; Lakatos, 1976; Tymoczdo , 1986).

Bruce Sagan, a research mathematician, in an interview offered the following three categories for describing the work that mathematicians do.

algebra/number theory
analysis
geometry/topology.[18]

He explained that although the categories do a grave injustice to describing all of mathematics, they can provide a very general frame for situating the work of discrete mathematics.

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[18] I borrow this category structure from Bruce Sagan, a practicing mathematician and professor of mathematics at Michigan State University, who helped me think about discrete mathematics from the perspective of a mathematician working in the field.

Revisiting The Question -- What Is Discrete Mathematics?

The first category, algebra/number theory, includes a wide range of interests around the notion of quantity. Algebra, in this categorical scheme, is not the mathematics of the typical algebra course one might recall from high school. Instead, it involves ideas of modern algebra such as group theory. Examples include the more primitive work of arithmetic, set theory, combinatorial counting, and more complex ideas of modern algebra involving studies of groups, rings and fields. Much of discrete mathematics falls into this category. Discrete mathematics is a specialization involving studying the logical and algebraic relationships between objects that are discrete or separate from one another (Barnier & Chan, 1989). Objects that are separate or distinct are often termed discontinuous.

The second category of analysis, focuses on mathematics involving continuous numbers and regions. The more familiar mathematics many remember from high school and college algebra and calculus courses resides in this category. The mathematics is often described as continuous mathematics, as it involves ideas and relationships of continuous quantities and the notion of infinity. Even though discrete mathematics falls into the first category, there is overlap into this category as well. For example, mathematical analysis can be used to determine the asymptotic behavior of certain combinatorial sequences.

Topology is representative of the third category and focuses on ideas and ongoing work around the notion of space. There are combinatorial techniques (one aspect of discrete mathematics) for computing certain homology groups, a basic goal of topology.[19]

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[19]Bruce Sagan, a research mathematician at Michigan State University helped me to think about the overlapping nature of all of mathematics and in particular offered examples as illustrations.

The examples I mention illustrate that there is overlap across the general categories and that discrete mathematics resides in all three categories.[20] This is not atypical. Most specializations overlap into other areas of mathematical study. For example, optimization and graph theory are not considered the mathematics of number theory. Combinatorial counting typically is. Yet, each of these topics is considered discrete mathematics. There really are no clear-cut boundaries in mathematics and discrete mathematics is not an exception.

Interestingly, arithmetic and discrete mathematics overlap. Although arithmetic is not thought of as discrete mathematics or vice-versa, each involves the other. The reform documents portray the mathematics of arithmetic and discrete mathematics as separate entities, entirely set apart, with little, if any, common ground. In reality each is connected and work in one area is fertile ground for work in the other.

The above categories, I believe are useful for understanding better the distinctions drawn between discontinuous and continuous mathematics in the reform documents. Discrete mathematics is often categorized as "discontinuous mathematics" because its focus is on systems of numbers where elements are distinct, finite or separate, hence the term discontinuous. The counting numbers and the integers are examples. Analysis, on the other hand, is often described as "continuous mathematics." The real and complex numbers underlay the mathematics of analysis. The integers are considered discrete because each element in the set is distinct from another element. For example, there are no numbers between 3 and 4. The real numbers are considered continuous because between any two real numbers there exists another real number. Numbers with

[20]I do not introduce these terms with intent that all readers understand the specific examples. I do hope to show that discrete mathematics is a mathematical field of inquiry that is not bound by particular topics.

repeating decimal expansions are often the basis of continuous mathematics. They are often not the basis of discrete mathematics.

Once again, this is not clear-cut. For example, linear algebra involves both work with continuous and discrete sets and ideas. Mathematically speaking, the notion that there is mathematics that is continuous and discontinuous is superficial. Yet, because we try to explain the nature of different mathematics we are bound to generalize the boundaries of ongoing work. The terms discontinuous and continuous help to do that. At the same time, to understand that distinction one would have to bring ideas about the larger landscape of mathematical study. A fairly typical problem of combinatorial mathematics, a sub-category of discrete mathematics, might involve, for example, determining the number of combinations that can be arranged of the numbers 1, 2, 3, and 4 or the number of unique paths that can be found between two locations on a map. These problems ask for specific counts. The underlying number system for each question is finite in terms of the number of elements and the elements themselves are separate or discrete. In contrast, continuous mathematics such as calculus, involves more the idea of taking measures as for example finding the area under a curve. The terms continuous and discontinuous provide information about the underlying systems of numbers.

The mathematics of discrete mathematics, because it exists on the boundaries of many other specialization's, encompasses many familiar mathematical ideas also associated to other areas of study. For example, set theory or probability make use of central mathematical ideas also useful to ongoing work in discrete mathematics. Venn diagrams may be a familiar example used across many mathematical specialization's. Venn diagrams display pictorially relationships between sets of objects or numbers. Yet, Venn diagrams, as mathematical ideas do not represent the mathematics of discrete mathematics, set theory or probability. They are useful only to the extent they can display particular principles or ideas central to each mathematical field of study. This is problematic when characterizing areas of mathematical study by naming central mathematical ideas. One would have to be knowledgeable about the field to understand how any particular idea is associated to it. The ideas mentioned in the documents for describing discrete mathematics overlap into many different studies. Using only central ideas to describe discrete mathematics would provide little information if other, more structural knowledge was not known. For example, combinatorial analysis, in part, focuses on techniques and theories useful for formulating counts to non-traditional kinds questions where configurations or different arrangements are of interest. Arithmetic involves more traditional kinds of counting questions. Each overlap in their use of Venn diagrams or number operations. They are set apart by the mathematical structures of the field, for example, by the central questions, underlying systems of numbers on which questions are examined, and central techniques used for exploring those questions.

Discrete mathematics can involve numbers that have either infinite expansions or finite expansions. Numbers such as pi and the square root of 2 have infinite decimal expansions, while the number three does not. In discrete mathematics there are discrete algebraic systems or field extensions that involve the square root of 2.[21] The terms discontinuous and continuous reference the nature of the relationships among elements in the sets. At the same time, they are associated to particular mathematics. There are many mathematical ideas and techniques associated to each continuous and discontinuous mathematics that are useful for solving problems or proving theories in discrete mathematics.

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[21]This example is provided by Bruce Sagan.

An example might be the processes of solving systems of equations, often associated to continuous mathematics such as linear algebra and calculus, to locate the optimum solution to a problem given a set of constraints. In essence the boundaries between discontinuous and continuous mathematics are superficial. School mathematics, because of the way mathematics is segmented for the purposes of select courses, promotes these boundaries far more than the mathematics itself. This is not to say there are not trends and patterns in mathematics, making various fields different from one another. Discrete mathematics is sometimes described as falling through the cracks or existing on the edges of many areas of specialization. For example, one mathematician described discrete mathematics as having roots in set theory, probability, combinatorics, matrix algebra, graph theory, formal languages, and automa theory (Barnier & Chan, 1989).

The documents suggest that discrete mathematics is concerned with mathematical questions that ask "how many" rather than, "how much." Combinatorial mathematics, the mathematics of counting arrangements and configurations is an example. Yet, counting is only one specialized area of discrete mathematics. Further, there are different kinds of counting questions. In an interview, Bruce Sagan identified three different kinds of counting questions. They included enumerative problems, optimization problems, and existence problems.

Enumeration questions engage mathematicians in the work of deriving and proving theories of counting. For example, combinatorial counting principles, combination and permutation theories, partitions of numbers and sets, and principles such as the inclusion-exclusion principle are all examples of ideas included in the broader theory of enumeration. I mention the ideas here to inform the reader of the many different mathematical ideas involved in the mathematics of counting. Combination and permutation, for example, are fundamental to the mathematics of counting. Each are highlighted in the reform documents and ultimately become the focus of Sandy's work in discrete mathematics.

Optimization questions involve determining best possible solutions to problems given particular constraints. Familiar mathematical ideas and techniques, commonly taught as part of more traditional mathematics courses, are also useful for working on optimization and enumeration questions. An example of a common idea might be matrices. Matrices are familiar ideas to school mathematics, usually appearing in a second algebra course. They are also useful in combinatorial mathematics. The mathematics of matrices can be used to determine counts and when combined with analysis can determine the best solution to a problem. Yet, as a topic itself, matrices are not considered discrete mathematics. The central questions asked and processes used to solve certain questions may involve the use of matrices, making the mathematics of matrices centrally connected to discrete mathematics.

A third category of questions in discrete mathematics involves the idea of existence. Existence questions ask if there is in fact a solution to a problem. Design theory, block designs, projective planes, Latin squares and coding theory are examples of techniques or theories about existence. Once again, I mention these not because I assume understanding of the ideas but simply to portray the multiplicity of ideas that cut across the three kinds of counting questions central to discrete mathematics.

Computer programming and the influence of computer techniques are also of considerable importance to the ongoing work of discrete mathematics. Computer programming and new technologies are tied closely to the ongoing development of the field. The computer has not only provided new avenues for solving and extending the tedious tasks of counting, optimizing, and determining existence, but computers also have had great influence on the subject matter of discrete mathematics by way of programming languages and the applications possible. For example, binary number systems and programming languages are often considered part of the subject matter of discrete mathematics and usually are included in most discrete math textbooks. Computers have also influenced the nature and kind of enumeration, optimization, and existence questions and their applications to real world problems as well. In this sense, computer technology has tended to play a kind of dual role in the ongoing development of the field.

Discrete mathematics is not computer mathematics. Because computers are highly prominent in the work of discrete mathematics, there is a tendency to generalize terms. Often, computer related terms and discrete mathematics have become synonymous. Yet, discrete mathematics is not computer programming or other related computer work. To the contrary, the mathematics of discrete mathematics existed far before the influx of computers. Many of the central ideas have been around for a very long time, long before computer technologies. At the same time, discrete mathematics would be altered drastically if void of the influence of computer technology.

My point in trying to delve deeper into what discrete mathematics is and what underlies discrete mathematics is to illuminate the complexity involved in sketching even a fairly general view of the central questions, underlying structures, techniques and ideas used to solve questions in the field of discrete mathematics. I also have tried to suggest that although the mathematical ideas such as Venn diagrams, systems of equations, and matrices each are centrally connected to the ongoing work of discrete mathematics, they themselves are not discrete mathematics. This would be true of any mathematics.

Understanding discrete mathematics implies knowledge of the central kinds of questions asked, techniques for solving those questions, routines and patterns mathematicians identify and make use of in that process, as well as good ideas about the systems that underlay that work and define the boundaries of the theories and arguments.

Discrete mathematics changes over time, like all mathematics, it is not a fixed body of knowledge where particular mathematical ideas exist. Rather, it grows and changes making use of any mathematical idea that advances an understanding of the central questions asked. In this light, discrete mathematics, as with any topic in mathematics, involves the familiar and the new, each shaping the other and progressing the work.

Obviously, describing the nature of discrete mathematics in documents not even designed for that purpose would be a difficult task. Portraying what discrete mathematics involves in general categories and descriptions is hard and can mean many different things to different readers. In chapter five of this work, the reader will learn how Sandy interpreted the meaning and proposals focusing on discrete mathematics.

Revisiting The Question-- Why Introduce Discrete Mathematics?

In my interview with Bruce Sagan I encountered a different rationale for introducing discrete mathematics into the school curriculum not explained in the documents. He explained that currently there is a big draw toward discrete mathematics courses in the college curriculum. There is even argument among mathematicians about introducing college students to discrete mathematics prior to calculus. The argument stems from concern that many students are, "weeded out," of further mathematical study by way of the traditional calculus sequence. This limits the number of students continuing beyond elementary level college mathematics. The argument rests on the difficulty students encounter very early on in calculus with ideas such as delta-epsilon proofs. Proof in this context is a complex idea often ending the majority of students' mathematical careers. In effect, the traditional mathematics sequence of courses acts as a barrier to a wider range of career choices for students.

Some mathematicians propose that discrete mathematics is a more promising site for introducing and exploring the notion of mathematical proof with college-level students. This argument rests on the assumption that students will find the mathematics of counting with finite sets much easier and therefore, discrete mathematics may be an easier introduction to the notion of proof. Some mathematicians claim the earlier proofs in discrete mathematics are much easier to understand than the delta-epsilon proofs. Removed would be the more complex ideas of infinity and continuous functions. Finite numbers of objects and finite outcomes would be easier to understand. Discrete mathematics would hopefully be more appealing and less threatening to students.

The ongoing arguments about the college curriculum filters into K-12 public schooling as well. If discrete mathematics were to become an alternative for more and more students, then preliminary work would necessarily begin in earlier education. The introduction of discrete mathematics into the school curriculum may be related to the ongoing debates concerning college course work. Given that mathematicians consider understanding and constructing a proper mathematical proof one of the most important goals in learning mathematics, school programs constantly seek avenues for meeting that goal.

<u>Should Discrete Mathematics Be Introduced at the Elementary Level?</u>

I suggested earlier that the documents offer somewhat mixed messages about whether discrete mathematics ought to be introduced in the elementary grades. In particular, the Curriculum Standards (1989), focus on discrete mathematics in sections for grades 9-12. Yet, NCTM, in the 1991 yearbook, explicitly suggests that K-8 programs should be strengthened by including discrete mathematics. In chapter three the authors argue that children's elementary years provide the foundation for all strands that will be studied in depth in the upper grades.

Further, Claire Graham writes, "Discrete mathematics is not a new branch of mathematics that must be added to the existing curriculum. Rather it is a collection of topics that most elementary teachers know something about and almost certainly already teach. These topics include counting techniques, sets, logic, reasoning, and patterning (iteration an recursion, algorithms, probability, and networks," (p. 18). The basis on which the author argues the ideas are common is not disclosed.

Throughout a 246 page text, there are numerous ideas and suggestions for introducing discrete mathematics some focusing on the elementary grades. Sandy did not make use of this text. Later on the reader will learn about Sandy's sense making around an example offered in the Curriculum Standards (1989), aimed at helping secondary teachers think about discrete mathematics in relation to matrices. Sandy used this example to try to understand a combinatorial mathematics problem, a topic in discrete mathematics. The illustration she used involved directed graphs and matrices, ideas Sandy knew very little about. Unbeknownst to Sandy, in the NCTM yearbook, a similar discussion of these mathematical ideas appears for elementary teachers (p. 30). Direct attention is given to how the mathematics of directed graphs can be introduced very early on in children's mathematical experiences. Sandy did not access this resource. The authors offer many examples and illustrations in various sections of the text that would provide an introduction to discrete mathematics for elementary grades (see p. 42 for a combinatorial counting problem and p. 61 for an example involving recurrence relations).

In checking the distribution of the yearbook since publishing in 1991, three printings, totaling 20,390 copies were published. In the progression of sales across the years, 6000 were sold in the first year, 5000 in the second, 2000 in the third, 1000 in the fourth, 480 in the fifth, and finally 288 during the 95-96 school year. There remained 5000 copies in stock. It seems she could have had access.

At the same time, although I did not consider these figures in contrast to other yearbook sales and therefore do not know if this pattern is typical, it does seem ironic given NCTM's position that discrete mathematics should be introduced into the K-12 curriculum that sales continued to decrease. The yearbook would seem an essential document for all elementary teachers given that what is presented in the Curriculum Standards (1989), regarding discrete mathematics is geared to secondary teachers.

Elementary teachers would have to bring a fairly well-worked sense of the ideas of discrete mathematics to understand the examples and illustrations. First, they would have to recognize the examples and illustrations scattered throughout all the texts as illustrations of discrete mathematics. Most are not identified as such. From there, teachers would have to bring much in the way of prerequisite knowledge to see the discrete mathematics in these illustrations. Paradoxically, what the documents hope teachers to get from reading them, may also be what teachers would need to bring to understand them.

Further, there remains the problem of locating and formulating an overall view of what the documents propose. In drawing much attention toward a new vision of mathematics teaching and learning, basic computational skills instruction appeared a well-hidden agenda. Rhetorically, it was not a central reform issue. Recall that only 11 in over 600 pages, in three different documents, devoted direct attention to basic computational skills instruction. Everything else was scattered in various contexts and forms making it difficult to find and fit together. Proposals for introducing discrete mathematics were communicated more directly and openly, yet there were few details and inconsistencies across the documents. Recall that the Curriculum Standards (1989), proposed discrete mathematics be included for grades 9-12. At the same time, the NCTM in the 1991 yearbook on discrete mathematics recommended discrete mathematics be included across the entire curriculum.

In the next two chapters the reader will soon see that the documents were only one of many resources that the teacher used to understand the proposals for reforming her teaching of computation and introducing discrete mathematics into the school mathematics curriculum. The analytic work in this chapter will hopefully serve as a back-drop for making sense of Sandy's responses to the proposals she tried to understand and enact in her teaching.

CHAPTER 4

LEARNING TO TEACH BASIC COMPUTATIONAL SKILLS DIFFERENTLY

Introduction

In the first and second chapters of this work I introduced a thoughtful, enthusiastic teacher. I described Sandy Wise as reform-minded and deeply committed to teaching mathematics in ways envisioned by policy. I suggested that she positioned herself to be successful at reforming her teaching. She was deeply invested and well-supported in her efforts to improve her mathematics teaching. This chapter tells a story about Sandy's efforts to change her teaching of computational skills. In it, I describe how Sandy's learning about reform-based teaching evolved and how her learning interacted with how she changed her teaching. The chapter illuminates how policy fostered new visions of teaching and how teacher learning fostered new interpretations of policy. The reader may be surprised at the turn of events.

I begin by examining a particular episode of Sandy's teaching during the first year of my observations. The lesson illustrates how Sandy encountered a puzzling set of circumstances when she changed her teaching of basic computational skills. As the story develops, the reader will learn about Sandy's efforts to abandon traditional forms of teaching aimed at the rote-learning of computational algorithms. From there, I develop a picture of the teaching practice Sandy created, one she thought attended to conceptual and computational goals. Toward the end of the story, the reader will learn what Sandy did to manage the problematic circumstances she encountered. I devote the remainder of the chapter to describing the role and significance of Sandy's learning on the turn of events.

A Teacher's Responses To Proposals to Teach Computational Skills Differently

Sandy Wise, a reform minded teacher, was committed to improving her mathematics teaching. Yet, in that process, an ironic twist emerged. In trying to teach mathematics more conceptually, Sandy encountered a crossroads in her teaching of basic computational skills. To illustrate, I focus on an episode of Sandy's third-grade teaching during the Spring of the first year of my observations. I use this lesson to illustrate the tensions that arose in her practice and Sandy's interpretations of what was happening.

A Single Mathematics Lesson

Sandy asks the following question: How many chopsticks would we need if everyone in this room needed them to eat their Chinese food? Sandy explained that what she really wanted wasn't the answer so much but for students to come up with strategies for finding the total number of chopsticks needed. The thirty-two students in Sandy's classroom seemed familiar with this kind of work. Immediately they began working in smaller groups of three and four for about ten minutes and without any direction to do so. Some students talked to each other while others worked alone even as they sat in groups. Many used paper and pencil, others relied on calculators. Seemingly unsurprised when Sandy asked groups to share their strategies aloud, students volunteered a variety of approaches.

Joe: We counted for each one at our table and then two for each one at Sarah's table and two for each one at Tom's table and made a chart, then I added.

Jen: I added 2 + 2 + 2 + 2 + ... for everyone.

Adam: We counted each student and there were 32 and you [points to Sandy] made 33. I then added 33 and 33 because each one gets two chopsticks.

Sarah: We added 32 and 32 to get 64 and then two more for you.

After each group reported their strategies, Sandy asked one child from each group to represent the strategy on the chalkboard. She then asked students to compare the different approaches on the chalkboard. A lively discussion arose. Several students noticed that Sandy had not been included in the totals. Others wondered if I, the observer, should be included. Sandy agreed that such decisions would certainly make a difference in the results, but she did not tell students what they should do. Jen interrupted:

> But, our answers should all be the same.
> Sandy: Why is that?
> Because we have to get chopsticks for everyone.
> Sandy: Okay, I suppose if we were to order our food we would have to let the restaurant know how many chopsticks we will need. Maybe we should decide.

Sandy asked students what they thought. One student said that both Sandy and I should be included because the question indicated that everyone in the room would be eating food. Sandy asked everyone to indicate if they agreed by vote.

Students quickly voted in agreement that both Sandy and myself should be included.

Sandy then asked Joe to come up to the front of the room to explain his strategy. Joe, pointing to the table he had previously written on the chalkboard, explained that each name corresponded to the group leader at each table and the number corresponded to the number of chopsticks needed at that table.

Joe	8
Sarah	16
Tom	12
Jim	18
Jen	10

He then began writing the following addition problem on the board:

$$8 + 16 + 12 + 18 + 10 =$$

Joe began adding the numbers in his head. He paired the first two numbers, added them saying "24", and then became stuck. It was at this point that the lesson began to take a different direction than Sandy had imagined. Instead of moving the discussion toward comparing strategies as she had hoped, the discussion began to center on computational work. Sandy, realizing Joe was stuck, asked the class to help him with the computation. After a few minutes, several other answers were offered, each different from the other and different from Joe's. Sandy then asked each group to work on the computation by helping each other look for any mistakes they may have made. Sandy walked around the room trying to help students work on the addition problem Joe had developed. The lesson, on that day, never found its way back to comparing and contrasting the strategies on the board (Fieldnotes, 5/92).

Later, in an interview, Sandy explained that as she walked around the room she noticed that many students could not add the numbers correctly. She explained that whether students were using their own strategies or attempting the traditional addition algorithm, they made many errors. Sandy reported that she helped students mostly by demonstrating the traditional addition algorithm to compute the problem (Interview, 5/92).

This lesson represented only one of many occasions where Sandy found herself worrying about students' computational proficiency. The same circumstances occurred so often that Sandy decided that students' lack of computational skill was actually getting in the way of her larger more conceptual and reform-oriented goals. For instance, in this lesson, Sandy had hoped that students would explore relationships among the various computational strategies and that ultimately students would notice connections between strategies involving addition and those involving multiplication. She hoped students might see that adding 2 over and over, thirty-two times, was the same as 32 times 2, and that both strategies give the same result. She also hoped students would begin to question the efficiency of their strategies and see benefits to the notion of grouping and multiplication.

Yet, Sandy's goals in this lesson were never realized. Even though she returned to the problem the very next day, many students remained stuck in various computational problems related to the different strategies. Sandy commented in an interview, "you know, most of these kids I really think could tell me that multiplying 32 times two means 32 groups of two, and I think they could draw me a picture of it, and show me that it also means adding two, 32 times. But, I don't think they could necessarily add it correctly." She continued, "maybe people tended to overreact, and went a little too far with not wanting kid's to memorize facts and algorithms anymore" (Interview, 5/92).

Sandy no longer stressed traditional algorithms in her teaching as she had done in previous years. Instead of drilling students on traditional computational algorithms, she presented the algorithms to students once and only at the end of more conceptual orientations into addition, subtraction, and multiplication. At this point in her career, Sandy worried less about students memorizing and practicing the traditional algorithms and instead emphasized developing conceptual understanding. Only after many encounters with the circumstances above did she wonder, "Maybe in approaching things more conceptually and through problem solving, I am dropping the basic math skills from my teaching" (Interview, 5/92).

Confronted by this dilemma, Sandy began to wonder about the role and purpose of basic computational skills in relationship to what she hoped students learn about mathematics. Sandy found no easy answers. She knew both conceptual and computational goals were valued. Yet, she pondered, should she, "stop in the middle of the lesson and review the traditional algorithm or more simply try not to introduce problems with more complicated computational problems " (Interview, 5/92). The latter seemed impossible given the various strategies students were coming up with. She also seemed puzzled about the enormous amount of time students needed when they were left to themselves to compute an answer. There were no simple solutions.

Sandy searched the reform documents for help. She read, "students will invent and refine procedures themselves," and that "students should compare different approaches and algorithms for obtaining the same results, evaluating the strengths and weaknesses of each" (California State Department of Education, 1992, p. 56, 57). Further Sandy wanted students to, "understand why the approach they choose makes sense for the problem they are solving, and if it makes sense, they will develop fluency" (p. 56).

Sandy focused on the idea that,

> "traditional teaching emphasizing practice in manipulating expressions and practicing algorithms as a precursor to solving problems ignores the fact that knowledge often emerges from the problems. This suggests that instead of the expectation that skill in computation should precede work with problems, experience with problems helps develop the ability to compute (NCTM, 1989, p. 9).

Sandy tried to create a practice she thought fit with this idea. She began her lessons with a mathematical problem that offered many different possibilities for students. Yet, as she developed a more conceptual and problem solving approach to students' learning about the four operations, she realized there were significant problems. While problem solving experiences like the chopsticks problem offered students an opportunity to learn about computation differently than traditional practices, they also involved doing the computations. Sandy, a teacher interested in developing students' conceptual understanding and computational skills, confronted a problem of what to do when students' computations didn't get them the desired answers.

Computational skills and conceptual goals seemed at odds to Sandy. She wasn't sure about the relationship between them. She began questioning the role of speed, accuracy, memorization and practice in learning to compute. And although she wanted to believe her students would develop fluency with computation as they engaged in more problem solving and conceptually oriented work, her teaching experiences began to suggest otherwise.

Over the next year Sandy worked on understanding this relationship better. The interpretations and decisions she made to change her teaching of basic computational skill the next school year may seem surprising. To understand how Sandy arrived at the decisions she made, one would first need to consider Sandy's practice on a wider scale. In the next section, I explore Sandy's mathematics teaching across the school day and across the 1991-1992 school year. My aim is to provide a picture of how Sandy attended to students' computational skill learning across the school day and year.

Mathematics Teaching on a Wider Scale

When I first began visiting Sandy's classroom, I was struck in general by the amount of time she devoted to mathematics instruction on a single day. Nearly two hours of each school day, usually in 20 to 40 minute chunks, were devoted to learning mathematics. More modal mathematics lessons occur only once in a single school day and for much less than two hours. Consider a typical day. On January 21, 1992, Sandy posted the following schedule for students.

Daily Schedule

8:00	Morning Business
8:20	Mental Math
8:40	Good Morning Meeting
9:00	Blockout
9:20	Fair Shares and Division
9:50	Probability
10:00	Recess
10:30	Language Arts
11:30	Lunch
12:00	Probability
12:45	Class Meeting

1:15 Oral Language

1:50 Prep to go home

1:55 Dismissed

On this particular day, there were five different time slots for mathematics, two hours and five minutes. During "Mental Math", Sandy pointed at random to single numerical digits and operation signs hanging by string from the ceiling. Students were to silently follow the numerical sentence Sandy created. When called upon, they were to provide the correct answer. This activity Sandy designed to provide students with practice on mental addition, subtraction, and multiplication of two and three step problems. The activity lasted for about 20 minutes, as students computed in their heads problems such as 4 + 8 x 3 =. Paper-pencil and calculators were not allowed. No attention was given to the standard order of operations like, "Do multiplication before addition." Instead, Sandy's goal was for students to work step by step as the problem was created. So, for example in the problem 4 + 8 x 3, since 4 + 8 was created first, it would be added first. The multiplication would be done second. Sandy would point to the 4 hanging from the ceiling, the plus sign, the 8, the times sign, and the 3. She would then point to the equal sign and wait for students to raise their hands. Students almost always gave the correct answer. Sandy would ask students whether everyone agreed with the answer offered. Students almost always agreed. Overall, there was little discussion. Mostly, students shook their heads up and down and recited back the steps of the problem. On a few occasions students disagreed with a given answer. Usually Sandy would ask other students for the answer, Typically it was correct. There were no discussions of alternative strategies or how numbers might be combined or broken down in ways convenient for computing mentally.

"Blockout," was similar to Bingo. Students placed marker chips in a position on an axis system after Sandy called out a horizontal and vertical number. The activity was designed to familiarize students with the process of locating points on an x-y coordinate plane. Sandy thought of the activity as a prerequisite to the geometry and algebra units she would teach later. Like Mental Math, Blockout was intended to develop facility, only in this case, with locating points on a coordinate system. It lasted approximately twenty minutes.

In between Mental Math and Blockout a class meeting was held. A number of issues arose. Sandy confronted students on their lack of attention during a lesson involving a guest speaker the previous day. She asked students to think about what they might have done differently and whether they want to have speakers in the future. Students also raised concerns. They wondered why they were not permitted in the gym during lunch anymore. Sandy explained the construction going on and indicated when the gym might be available again. The meeting lasted about 20 minutes and did not involve any mathematics instruction.

"Fair Shares and Division," the third math lesson of the day, focused on the idea of division and that of equal-distribution or fair-sharing. The activity, approximately 30 minutes in length, involved students dividing packs of lifesavers among 4 and later 8 friends. Students distributed the lifesavers among friends and kept track of how many each friend received and the number left over.[22] No paper-pencil calculations were made. Instead, Sandy hoped students would link the notion of fair-sharing to their ideas about division. For example, she expected students would say I divided 22 lifesavers fairly among four friends, each getting five lifesavers and a remainder of 2.

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[22]A closer analysis of this activity appears later in this chapter.

"Probability" had two time-slots, one before and one after lunch. Before lunch students collected data by pulling and replacing a lifesavers from a brown paper bag. The data was recorded on the chalkboard by placing hash marks under the letters r (red), o (orange), y (yellow), g (green), and w (white). In the afternoon segment students were asked to make and explain their predictions of how many of each color lifesaver there was in the bag. Sandy also asked students whether the data collected supported the actual outcome of colors in the bag. Both segments took 55 minutes.

Fair Shares and Probability focused on two mathematical ideas, equal distribution and chance. Calculation was not emphasized. Sandy was much more interested in how students reasoned about the problems. "Probability" also offered students an opportunity to justify their answers by providing evidence (Fieldnotes, 1/92). Together the two lessons took one hour and twenty-five minutes of the two hour and five minutes devoted to mathematics.

During the other parts of the afternoon students were involved in language arts lessons, lunch, recess and another class meeting. None of these activities involved mathematics instruction on that day.

Sandy's practice was much like a patchwork of practices, each element offering a different contribution to the overall goal of becoming what she called, "mathematically powerful." Coherence across lessons on any single day did not appear to be an important goal to Sandy. The mathematics lessons across the day were not connected in substantive ways. They did not build or extend mathematical ideas. Instead, they involved unrelated mathematical goals. With the exception of the two segments on probability, each lesson on this day had entirely different aims. Several lessons focused on computational skill, but each one emphasized a different skill. Other lessons focused on developing mathematical ideas such as division and chance on this day.

Lessons did connect substantively across the days of the week and across the month. For example, the probability lessons on January 21 were connected to a lesson on January 22. On the next school day Sandy introduced a very similar probability problem involving the ideas of prediction and chance. She planned to extend students' understanding of prediction and chance across the entire month of January. A little later in this section, I will show how Sandy's master plan was aimed at developing mathematical connections across weeks and months according to how Sandy believed the lessons extended students' learning about specific mathematical ideas.

These data suggest that Sandy's mathematics teaching was separated along two lines. Statistically, Mental Math and Blockout, were skill-oriented activities, making up a little more than 30% of the instructional time on that day. About half of that time focused on mental calculations involving addition, subtraction and multiplication. Thus, about 15% of the instructional day on mathematics focused on basic computational skills. In contrast, one hour and twenty-five minutes, or approximately 70% of the time was spent on investigative activities or exploring mathematical ideas. During these activities, problem solving strategies and conceptual orientations to mathematical ideas were of central concern whereas computation remained in the background.

Sandy confirmed in an interview that she tried to focus 80% to 90% of the total instructional time in mathematics on conceptual understanding and problem solving. She indicated that 10% to 20% was devoted to students developing computational skills and the proficiency of those skills. Sandy claimed she emphasized mostly mental arithmetic and mental estimates instead of paper-pencil calculations. She explained she devoted only one week each on the traditional algorithms, showing students the steps and requiring they practice them. The changes represented a significant departure from years past where Sandy spent months repeating and reviewing instruction of the traditional algorithms for computational purposes (Interview 1/92).

The changes described here had accumulated over several years. At this stage Sandy was devoting a little more than four weeks of the entire school year to teaching the procedural steps of the traditional algorithms (Interview, 1/92). She explained that most of what she provided for learning basic computational skills emphasized mental calculations and estimates. My observations on January 21, 22, May 5 and 6 fit with Sandy's estimates. I observed no lessons involving rote-memorization or practice of any traditional algorithms during my visits that year.

Sandy's Master Plan also illustrates the two-dimensional nature of her mathematics teaching. Yet, it suggests there is a third dimension. At the beginning of each school year Sandy constructed an overall yearly plan for her mathematics teaching. Below is Sandy's plan for the 1991-92 school year.[23]

Sandy explained she developed this plan to use in place of the district's mandated curriculum guide and textbook. She describes the plan as, "a really healthy road map, I know exactly where I am going, and I no longer worry whether I am hitting the skills, the concepts, or the global outcomes, it's all there," (Interview, 1/92).

"Math" resembles a more traditional third-grade curriculum plan. Initially, there is a pre-assessment of students' computational skills, a test on the basic number facts and the traditional addition and subtraction algorithms. Each

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[23]I have included only those elements of the yearly plan pertaining to mathematics. Sandy constructed a master plan with each subject area represented.

	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	
Math	Assess-ment	Pattern & Number	Place Value	Number Bases Intro to Regroup-ing	Mult. Pattern Review	Division Average	Regroup-ing +,-	Regroup ing +,-,x	Multi Calcu-lator	Assess-ment
Strands	Number & Estima-tion	Pattern	Logic	Func-tions	Stats & Prob.	Geo-metry	Frac-tions	Measure-ment	Algebra	
Problem Solving	Bubble Water M&M	Trick or Treat H20 Water Wizard	Yeast Beast Fan. Fare	All Heart Sherlock Holmes	Fossils Square Deal	Bear Animal Crackers				

Figure 6.
Sandy Wise - Master Plan 1991-92 School Year

month thereafter focuses on a particular aspect of computation: number, place value, operations and patterns in computations. There is a lengthy section on regrouping. There is a unit on calculators and a post assessment of student's computational skill. This strand of instruction emphasizes some memorization of the traditional algorithms, some drill and practice on the basic facts and algorithms, and timed tests. Attention is also given to place value and bases other than ten (Interview, 1/92).

The second row relates more directly to the state's framework. Each month corresponds to a content strand identified in the framework. The 1985 Framework identified seven strands: number, pattern, logic, functions, statistics and probability, geometry, measurement and algebra.[24] Sandy's plan corresponds to these but includes an additional strand titled fractions which she calls a unit. On any given instructional day, there may be as many as three activities relating directly to the content strand for that month. These activities are of the investigative, hands-on, learning by doing type. Sandy uses materials by Project AIMS, Marilyn Burns, Family Math and many others to draw from.

"Problem Solving" was not represented in Sandy's teaching on January 21, 1992. The emphasis here is on developing student's problem solving strategies. Activities are intended to offer students a chance to work on developing strategies such as guess and test, look for a pattern, formulating the question, look for a counter-example, using a table, drawing a picture, and trying an easier problem. Sandy selects activities mostly from Project AIMS , Family Math and Marilyn Burns to provide students these experiences.[25] The months that are left blank are open as Sandy continues to search for new materials to use.

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[24]In the 1991 version an additional strand, Discrete Mathematics, was added. I take up Sandy's learning of discrete mathematics and her representation of these ideas in practice in the next chapter.

[25]Project AIMS plays a significant role in Sandy's ideas and practices related to mathematical

Sandy's master plan organizes her mathematics teaching into three dimensions. One dimension takes up computational goals, another involves investigations into mathematical ideas and corresponds to the content strands of the framework, and the third focuses on problem solving strategies. Lessons across the school day, week, and month correspond to one or the other of each dimension. There are connections among ideas within each dimension. For example, the strand dimension provides experiences that aim to connect division and the idea of average. Each are linked by the idea of fair-sharing (Interview, 5/92). The computational skill dimension is organized across the year by level of difficulty of each skill as well as by operation. For instance, three step addition problems follow two step addition problems.

There are few connections across dimensions. The more conceptual explorations of division in Sandy's practice on January 21, for example, were not linked to any experiences students would have practicing division mentally. Thus, the dimensions are not connected in substantive ways.[26]

For Sandy, the three dimensions represent what children need to know and be able to do mathematically (Interview, 5/92). Instructionally, each dimension offered students various pieces of mathematical knowledge to be successful in mathematics. Recall that Sandy devotes somewhere around 20-30% to the Math dimension and 70-80% to the content strands and problem solving dimensions. Experiences corresponding to the Math dimension emphasize

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knowing and doing in the elementary classroom. I discuss the relationship Sandy has with Project AIMS and her use of AIMS materials in more detail later in this chapter as well as in other places throughout this work.

[26]This may have seemed not necessary given Sandy's view of mathematics. Because she viewed mathematics as an enterprise in the service of other sciences, she may not see as well the benefit of understanding ideas deeply. Instead ideas had to be understood well enough to put them to use for solving problems or making sense of the world. If one could compute and also find averages, that was enough . I take this up further in the next section involving Sandy's knowledge and beliefs about mathematics.

practice with mental calculations, mental estimates and routine algorithms as well as explorations into ideas such as place value, various bases and the four operations. Skill oriented lessons are taught in small chunks separate from problem solving and content strand dimensions. In contrast, students' experiences corresponding to the strand dimension are more open-ended, involve little if any practice, and invite students to consider mathematical ideas on a basis other than procedurally. Activities often begin by describing a context that lead to a problem solving opportunity. Students work in small and large groups, explorations often continue over several days, and students are often asked to justify their thinking by providing rationales for their work.

Although the dimensions for the most part are separate, there are a few activities that were multifaceted in purpose. For example, the chopsticks problem not only was intended to provide an opportunity for students to develop and use their own ideas to solve a problem, it also was intended to foster understandings of the relationship between addition and multiplication. In this sense, facility problem solving and conceptual understanding were intertwined.

It would be plausible to argue that students' mathematical experiences in Sandy's classroom that year were quite different from modal practice. Instructional time overall was dominated by the investigations experience. Experiences with rote-memorization and practice of the traditional algorithms was much less than most conventionally taught classrooms. And although students were asked to practice computations, more often the experience involved making mental calculations and estimates rather than paper-pencil drill and practice of the traditional algorithms. Sandy's practice during the first year of my observations in many ways reflected a very different approach to teaching and learning mathematics and in particular there were significant differences surrounding instruction of basic computational skills.

Sandy had developed a practice she thought met the challenge to teach for conceptual understanding and computational proficiency. Yet, as the lesson I used to open this chapter illustrates, Sandy also realized that the changes she made placed computational goals in jeopardy. Further, she interpreted students' lack of facility and skill at computing to interfere with the exploration into more conceptual, idea-oriented side of mathematics. As Sandy concluded the 1991-1992 school year, she worried she had misread the state's goals. Sandy wondered what she was doing wrong (Interview 5/92).

The Next School Year: Responding to Problems In Practice

One year later, it was very obvious that Sandy had changed her teaching significantly. Yet, the changes I observed seemed ironic given Sandy's commitment to understanding and reforming her teaching in ways characterized by the policies. In an interview Sandy revealed there was a dramatic shift in her thinking and practices related to computational skills. And, consequently, she reinstated traditional teaching practices aimed at the rote-learning of basic number facts and traditional algorithms, only now, more pervasive and extensive than ever before. Sandy was now devoting as much as 30-50% of instructional time on mathematics to developing pencil-paper algorithms for computing. Her daily schedule reflected these changes by a new title, "Automaticity."[27] The following schedule is for January 10, 1993.

[27]The term Automaticity surfaced initially for Sandy in one of her doctoral courses over the summer. The term itself indicates a significant change in Sandy's thoughts about what ought to be emphasized in relation to computational goals when comparing one year to the next.

Daily Schedule - Year 2

8:00 Morning Business

8:20 Citizenship Discussion

8:50 Problem Solving Strategies

8:55 Polar Pizza Challenge

9:05 Spelling Test

9:15 Automaticity - Mathematics

9:45 Book Awards

9:50 Automaticity - Quick Read

9:55 Recess

10:20 Dear Reports

10:30 Mail Call (Project Aims Mathematics Activity)

10:45 Science

11:30 Lunch

12:25 Continuation Mail Call

12:40 Congratulations to Polar Pizza Party

12:45 Oral Language

1:05 Afternoon Recess

1:15 Zip Around (Speaking and Listening Language Arts)

1:20 A Square Deal (Problem Solving Activity)

1:40 Alleyway (Math Activity Project AIMS)

1:50 Prepare to go Home

1:55 Dismissed

Like the previous year, several time-slots were devoted to mathematics instruction. On this day, there were eight. "Problem Solving Strategies," the first mathematics activity, involved a brief discussion of how to arrange objects such as marshmallows and hands in a way to make measures more precise. This discussion was one of a series across several days involving an ongoing problem-solving activity aimed at understanding measuring.

"Polar Pizza Challenge," involved a timed multiplication-fact test. The timer was set and frantically student's answered as many of the single-digit multiplication facts on the page. When finished, students clapped to signal to Sandy that they were finished. Sandy would record their time on the overhead projector. In turn, the student recorded the time at the top of their page and turned in the worksheet. Later in the day papers were graded and times were recorded in Sandy's grade book. Students waited to hear if their score was a perfect one. If so, and their recorded time was under two minutes, students were invited to the Polar Pizza Party. Sandy compiled results and announced in the afternoon segment, "Congratulations to Polar Pizza Party," the names of those students who succeeded. A round of applause was given to each student awarded the coupon admitting them to the party. Accumulated coupons could be used for other rewards such as fifteen minutes of free-time. The two instructional segments took fifteen minutes that day.

"Automaticity-Mathematics" was next on the schedule. On this day the lesson involved practicing routine addition and subtraction problems involving numbers with many zeros. I examine this lesson in more detail later in this section. For now, it suffices to say that this lesson represented a very traditional, procedural approach to learning computational algorithms. Eight computational problems, void of any context, were presented on the overhead projector. Sandy instructed all students to follow a traditional step-by-step procedure regardless of the numbers involved.

At 10:30, "Mail Call" was next. Recall the investigative-type lessons I described as dominating Sandy's mathematics instruction the previous year.

This lesson, a Project AIMS activity, was of this kind. Students collected and recorded a set of data involving the number of mail items received over a three day period at their home. Using bear counters to represent mail items, students explored the mathematical idea of average by equally distributing the counters across the three days. Discussion focused on what it would mean to equally distribute the items, whether the total is different after distribution, and language that linked equal distribution to the word average. The lesson, broken into two segments, one before and one after lunch, totaled 30 minutes.

"Square Deal," involved students trying to place the digits 0 through 9 at the vertices of embedded squares so that the numbers of connecting vertices total 20. Sandy illustrated a guess and check method for accomplishing the task and recommended students search for other strategies as well. This activity took twenty minutes on this day but would be continued over several days. It was followed by the introduction into a Project AIMS activity titled "Alleyway," which involved students collecting data on how many puffs of air it took to move cotton balls across various surfaces such as students' desk tops. Sandy organized this activity at the end of the school day because of the commotion it created in the classroom. Sandy planned to make use of the data the next instructional day focusing on the ideas of mean median and mode.

In sorting through the various lessons on the day, Alleyway and Mail Call were more hands-on, learning by doing activities focusing on the idea of average. Together, the two activities totaled 40 minutes. Square Deal, lasting 20 minutes, typified the problem solving dimension of Sandy's practice. Combined with the short problem solving discussion that went on earlier in the day, the lessons totaled 25 minutes. The remaining activities focused on recognizing and developing students' progress with basic fact acquisition and traditional computing algorithms. These lessons made up 45 minutes or approximately 40%

of the instructional time for mathematics on that day. Sandy confirmed in an interview that she had decided to increase the time students would spend focusing on learning traditional computing algorithms. In effect, across the 1992-1993 academic year, this decision decreased her emphasis on conceptual and problem solving goals (Interview, 5/93).

Perhaps more important to note is the different nature of the experiences students had for learning about computation than the previous year. The lessons no longer emphasized mental calculations and estimation. Instead, emphasis was on memorizing, practicing and reciting basic number facts and traditional computing algorithms.

To summarize the picture thus far, Sandy's practice during the second year of my observations was strikingly different. It reflected <u>more</u> time on basic computational skill learning and <u>less</u> time on activities emphasizing conceptual understanding and problem solving. In addition, it reflected <u>more</u> traditional computational goals and <u>less</u> alternative skills for computing such as student's strategies, mental calculations and estimation. I turn now to look more closely at what Sandy referred to as an automaticity lesson. I describe the lesson on January 10, 1993 at 9:15 in detail.

<u>An Automaticity Lesson</u>

Automaticity was not a word that students the previous year had experience with. This year, automaticity carried great meaning and significance for students. It not only represented a certain kind of activity students would engage in, but, it also represented great achievement and reward. Students understood that they would put everything away off their desks, except for a pencil. They knew speed and accuracy were key factors. And, there was a great sense of excitement and privilege associated to automaticity lessons.

Automaticity also was associated to reading instruction. "Automaticity-Quick Read," at 9:50, involved a speed test in reading. Students understood that automaticity had a great deal to do with speed and accuracy.

Automaticity in mathematics on January 10, 1993, involved a speed test. I observed the following.

Sandy turns the overhead projector on. The following eight problems are displayed under the title <u>Automaticity</u>.

1. 1000 +3976	3. 10,000 - 9673	5. 75 x 4	7. 497 697 + 119
2. 100 x 6	4. 11,496 + 14,697	6. 20,000 - 10,201	8. 700 400 +600

Turning on the projector was the signal for students to begin. Students worked quickly. When the problems were completed, students raised their arms to let Sandy know they had finished. Sandy recorded each students' time on the overhead. In turn, the student recorded their time at the top of their paper. The paper was then turned over and students waited quietly until everyone had finished. Although desks were arranged in groups of four facing each other, each student worked silently alone.

Once everyone had completed the exercise, students traded papers and graded each other's answers. Sandy collected and filed the papers. Later she recorded the number right and the time for each student in her grade book. After all papers had been collected Sandy went over every step of each problem and requested that students recite in unison as she wrote down the appropriate numbers on the overhead for everyone to follow.

For example: 3. 10,000
-9,673

Sandy placed a slash mark through the first zero at the right and everyone joined in saying:

"slash, burn it off, regroup, slash, burn it off, regroup,
slash, burn it off regroup, slash burn it off, regroup."

Sandy continued placing slash marks through the rest of the zeros until she came to the zero in the thousands place. She then placed a 1 above the 0 in the ones place and a 9 above the other three 0's. She asked, "Have we done any subtraction?" Students chimed in loudly "no!" Sandy continued, "That's right, all we have done is regroup to reorganize this number. Remember the value of the top number is still the same." Students chimed in by signal, "Ten minus three equal seven, nine minus seven equals two, nine minus six equals three, and nine minus nine equals zero, zero minus zero equals zero." Sandy wrote each number as students recited. Then she asked, "Is it reasonable that 10,000 minus approximately 9,700 is about 300?" Everyone answered, "Yes!"

Sandy's approach for each of the eight problems is similar. All were computed using a traditional algorithm. Students recited each step. Problem five, 75 x 4 , was worked in two different ways. First, the two numbers were multiplied using the traditional multiplication algorithm. Then, Sandy asked if there might be an alternative way to work the problem. One student suggested, "You could add 75 four times." Sandy wrote,

75
75
75
+75

Students recited back, "Five plus five plus five plus five is twenty, put down the zero and carry the two." Then, "Seven plus seven plus seven plus seven is twenty-eight, add the two and you get thirty." Sandy wrote the number 300 under the problem. She then asked students to compare both answers, "Does everyone agree the two ways produce the same answer?" Student's responded in unison, "Yes!"

An automaticity lesson represented a significant change in Sandy's thought and teaching practice. It was not an idea that Sandy returned to when tensions arose in her practice. Instead, it represented new learning. Sandy explained that the purpose of the above lesson was to promote automatic computational skill for the purposes of, "becoming so proficient that students can actually do it without thinking," (Interview, 1/93). Each and every automaticity lesson that year was intended to support that goal. Emphasis was on getting right answers in the least amount of time. The problems were not situated in any problem solving contexts nor were they connected to any other dimension of Sandy's mathematics teaching.

Sandy's practice represented what most reformers would criticize. The Curriculum Standards (1989), for instance, argues that decreased attention should be given to complex paper-pencil computations, the isolated treatment of those algorithms, and rounding for estimating (p. 21). On p. 46, "it is inconsistent with the Standards to isolate paper and pencil procedures by focusing on them for an extended time prior to the introduction of other computing methods: this traditional practice suggests to children that computing means using paper-and pencil methods." And further, "Instruction should emphasize meaningful development of these procedures not speed of processing." And finally on p. 231, "it is essential that instructional programs provide opportunities for students to generate procedures. Such opportunities should dispel the belief that procedures are predetermined sequences of steps handed down by some authority."

The reform documents paint a very different picture than Sandy's practice.

> Exploratory experiences in preparation for paper and pencil computation give children the opportunity to develop underlying concepts related to partitioning number operations on the parts and combining the results. Many such experiences can be provided in the context of using place value materials, computing mentally, or performing computational estimation. Only after these ideas are carefully linked to paper and pencil procedures is it appropriate to devote time to developing proficiency by providing practice. (NCTM, 1991, p. 47)

The new Framework (1992), suggests that, "Depth is to be valued over pace so that the presentation of a computational procedure can be delayed until students need it and meaningful examples and motivation can be provided before the algorithm is presented. Also to be valued is the critical use of alternative algorithms."

Another suggestion focuses on the relationship with the conceptual underpinnings of procedures. For instance,

> "A strong conceptual framework also provides anchoring for skills acquisition. Skills can be acquired in ways that make sense to children and in ways that result in more effective learning." Further, "it also means relating this knowledge to the learning of skills by establishing relationships between the conceptual and procedural aspects

> of tasks. The time required to build an adequate conceptual base should cause educators to rethink when children are expected to demonstrate a master of complex skills (NCTM, 1989, p. 17).

The reform documents suggest that students opportunity to consistently make decisions about the most efficient means for computing is of primary importance. Sandy's lesson limits students to practicing the traditional computational procedures. Proposals suggest that computational goals should be situated in contexts that give rise to computations. Further, they suggest the numbers in the problems should be realistic given the problem at hand. Sandy's practice isolates computational goals. On January 10, 1993, she focused on practicing computational procedures involving numbers with many zeros, numbers that students at this age are not likely to encounter. Under the new guidelines this would not be appropriate unless situations students encountered required it. The contrasts are striking.

Yet, recall that Sandy's practice had changed significantly from the previous year. She left the 1991-1992, school year frustrated. She knew she had to rethink her ideas about computation. That summer Sandy reconsidered her teaching and put a new plan into action. She identified a set of computational skills she believed to be essential for the third grade curriculum. Using the district's curriculum guides and the state's standardized tests as guides, Sandy formulated the following:

*single digit addition and multiplication facts (0-10)

*mental arithmetic of two operations and single digits (3 x 6 + 5)
*traditional algorithms for addition, subtraction, multiplication and division (multiple digits)
*traditional addition and multiplication algorithms with carrying
*traditional subtraction algorithm with borrowing and regrouping
*reasonableness of answers using rounding and mental estimates

She designed automaticity lessons to foster memorization of computational procedures and proficiency with those skills. She developed worksheets for timed-tests and a way to motivate students' speed and accuracy with those skills. Sandy offered a reward, participation in a pizza party. And although she continued to incorporate lessons that emphasized mental calculations and estimation, they occurred less often and were shorter in length (Interview, 5/93). Automaticity now dominated Sandy's instruction of basic computational skills.

The changes Sandy made presents a puzzle for policymakers. Amid reforms that take aim at traditional practices, a reform-minded teacher ultimately emphasizes traditional forms of practice even more. How can this happen? We have a teacher deeply invested in the reforms and yet becomes committed more than ever to developing students' proficiency with traditional computing algorithms. For most, Sandy decisions would mark a clear turning away from the visions in policies. Yet, interestingly enough, for Sandy they represent a closer approximation of what reformers' may have hoped for. My analysis in the next section provides some insight into how this could happen.

Reinterpreting the State's Goals

In formulating my analysis of Sandy's learning and the affect of her learning on her responses to the proposals to change her teaching of computation, I returned to the theoretical framework underlying this dissertation. The "policy as pedagogy" frame views the relationship between instructional policies and teaching practice centrally as matters involving teaching and learning. Thus, I examined what Sandy learned and the contexts she used to help her understand what the state suggested she change in relationship to computational skills teaching. Further, I examined whether and how her learning experiences impacted the changes she made in her teaching as well as her interpretations of policy. The analyses illuminates that Sandy's learning played a significant role in responding to the state's efforts to reform mathematics teaching. It suggests that what Sandy learned, in part, fostered the exact ideas and practices criticized most in the reform documents. And further that Sandy's learning experiences fostered a new interpretation of policy.

Factors Affecting the Teacher's Interpretation

Interestingly, Sandy did not create this mix of "old" and "new" practices because she somehow turned away from policymakers' agenda or gave up on reforming her teaching. Quite the contrary. Sandy formulated her direction in an effort to improve her teaching and respond more effectively to the state's goals. To understand how, I look carefully at what Sandy considered for changing her teaching of basic computational skills. Sandy took extraordinary steps to help her understand what she thought would be important for creating the kind of teaching described in the framework. For example, Sandy called individuals at the state department and arranged for meetings specifically to hear their views of what was suggested. She arranged for coursework as part of a doctoral program to help her understand better issues in mathematics education. Below, I describe the various contexts and ideas Sandy considered as she formulated her ideas about computational skills instruction.

The Practice of Teaching

Earlier, I described a number of tensions that arose in Sandy's practice during the 91-92 school year. Sandy's interpretation of those tensions involved a reconsideration of the traditional computational curriculum and how it fit with policymakers' ideas for change. For example, she struggled with whether she should focus attention on the traditional algorithms and if so, how much. She wondered whether speed and accuracy with computing remained a valued goal. During that year, Sandy believed her students had developed conceptual understandings of addition, subtraction, multiplication and division. But she began to worry whether they were developing an accompanying facility with computation. Sandy explained, "They hold on to the concept across the year but they can't do the computations just right," and "They can explain multiplication conceptually as repeated addition or grouping of the same number, but they can't multiply," (Interview, 1/93).

Sandy offered this illustration in an interview. A student suggested during a lesson that 75 X 4 is the same as adding 75 + 75 + 75 + 75. Yet, when he began adding the numbers to get the answer for comparison, he became stuck. Sandy argued, "my students can show what 75 x 4 means visually using diagrams, pictures and manipulatives, but later in the school year when routine addition and multiplication problems are part of problem solving situations, they get lost, they can't do it." She explained further, "The outcome of finding the right answer to 75 + 75 + 75 + 75 or 75 x 4 just wasn't happening." She continued, "I found that most kids could tell me four groups of 75 is the same as adding 75 four times, but they could not necessarily add it. And, I found that I was just dropping those basic math skills" (Interview, 1/93).

Sandy stumbled on a big issue. She was circling around a question about the relationship between conceptual understanding and procedural knowledge. By the close of the school year Sandy had concluded that, "understanding mathematical ideas conceptually did not guarantee fluency with computing skills across the school year." Sandy argued, "to know what multiplication means or why the algorithms work does not necessarily mean one can multiply efficiently and accurately" (Interview, 1/93). She explained:

> "Suppose later in the year after multiplication has been taught the kids are involved in a problem situation where they need to add quite a few numbers together to solve maybe only part of the problem. It is my goal that they can see and look for likeness in numbers when adding so that they can use multiplication. If they do, I know they have internalized why multiplication is useful to us. So, at that point they are multiplying for example 6 x 12. If they can do this accurately and fairly quickly then we can concentrate on the problem at hand. If not, then we are stuck. I was finding that my kids knew things like adding 6 twelve's is the same as multiplying 6 x 12 but whether they tried to add or multiply to compute, too many were stuck. Then we had trouble staying in the problem."

Sandy learned from changing her practice that students will often get stuck on fairly simple and routine computations inside problem solving situations. She realized that students' lack of facility with computing interferes with their progress into the more conceptual and reasoning sides of mathematics. Sandy's hunches were confirmed when her students did not fair well on the state exams that year. Sandy learned that basic computational skills can act as a barrier to conceptual and problem solving goals. Ultimately, Sandy decided that a more conceptually focused curriculum does not necessarily support student's proficiency with basic computational skills.

Doctoral Studies

Sandy learned a great deal over the summer. Because Sandy left the 1991-1992 school year worried about how computational goals might also be met within a more conceptually and problem-oriented practice, she searched out circumstances that might help her think about these issues. The interviews suggest that automaticity was at the forefront of Sandy's mind. She described how she came across the notion of automaticity in one of her doctoral courses. The course, a curriculum course, was not focused on issues surrounding computational goals or the mathematics framework, but it did focus on questions about curriculum in the context of reform. Sandy explained that the instructor distributed an article by Benjamin Bloom in entitled The Hands and Feet of Genius, (1986), for the purposes of exploring historical perspectives surrounding the elementary school curriculum. She described the study to analyze experts in their fields, and found that in each field, mathematics included, there were desirable skills that experts learned to the point of mastery. The notion automaticity referred to the idea that the execution of these skills could be done without conscious thought so that other more complicated kinds of thinking could go on as well.

Something important happened. Sandy connected Bloom's ideas about automaticity to her observations of students in her classroom. She explained,

> "I found my students do a phenomenal job with conceptual analysis but I couldn't figure out why for instance on my state exams the application of basic skills was not maintaining. When I saw this article, I said, I've got a problem here and basically I am not helping my kids master their skills. When we are through with a unit at the end of the month and I think they have understood multiplication and multiplying, we are off to the next unit, never to practice multiplying again. I said I've got a hole here and I need to plug it. I decided kids need practice with many basic kinds of drill and kill, and the bottom line is that they will be maintaining it at a higher level of success than before" (Interview, 1/93).

Sandy interpreted Bloom's work to suggest that students must master to automatic levels the traditional algorithms for computing if they are to be successful in doing other mathematics. She reasoned, "I have some children that can really reason and problem solve, they know how to go after a task, work on concepts, but dog-gone it they can't compute the problem accurately. To me there is a component missing. The computational abilities also have to be quality, they have to be automatic" (Interview, 1/93).

Bloom's work offered Sandy a solution to the tensions she experienced in her practice. She acted on her learning and incorporated memorization and practice of basic computing skills arguing that they are, "not to be re-taught, but rather to be practiced, for a level of mastery" (Interview, 1/93).

State Mandated Testing

A second idea that Sandy encountered that contributed to Sandy's reformation of basic computational skills teaching involved reconsidering her students' CAT scores. She examined her students' tests and wondered why they did so poorly on the computational portion of the state's standardized test. She certainly wanted to do whatever she could to improve students' opportunity to do well on these tests but she also believed her teaching was evaluated using these scores. She commented, "I am really being evaluated on those scores. I know it is not reflective of how I teach. I know that. But, not everybody else does" (Interview, 1/93).

Sandy's decision to reinstate traditional forms of teaching were influenced by her beliefs about the role and purpose of the CAT. She believed her principal evaluated her teaching on students' scores. She explained, "I tried to make these changes in my instruction to really see if I could make a difference on these tests."

Conversations With Other Educators

Sandy also encountered criticisms of her teaching. Parents, as well as fourth grade teachers at her school, complained that students could not do the "basics." In an interview Sandy revealed she worried a great deal about such statements and in part, was influenced by them to reinstate drill and practice on the traditional algorithms. She explained,

> "I got more criticisms a year ago from other teachers about things like regrouping wasn't solid enough or that multiplication wasn't strong enough. I even had a few parents say God we had to work on that all summer long

again, and fall came and they said gosh Ms. Wise we thought you worked them really hard last year but they are not ready for this year." (Interview 1/93).

Sandy understood that criticism was pervasive in education. She knew that any kind of change carried with it much criticism and from multiple directions. Yet, in her case, the comments others made about her teaching of the basics seemed to really shake her. She felt badly that other teachers in her school thought she did not prepare students to enter their classrooms. They complained directly to Sandy that students from her classes arrived unable to do routine calculations efficiently and accurately. The principal criticized Sandy about the CAT and that her students did poorly on the computation portion of the exam.

Although it would be difficult to suggest the impact of any of these criticisms, Sandy acknowledged that they did play a role in her decision to return to traditional practices. The comments established some sense of what other educators thought important (Interview, 1/93). Sandy also consulted several individuals at the state department. In conversations with others Sandy questioned whether memorizing algorithms was still a valued goal. She explained in an interview,

"I spoke with several people at the state department and my understanding is the reason we want kids to memorize algorithms is so that there is fluency, fluency that they can move around in bigger concepts and the actual computations don't get in the way, and they verified that was correct. So, it's not memorization for memorization sake, it's memorization to develop a tool."

Sandy's ideas were confirmed by those she thought to have a good knowledge of the new framework. From her view, she felt sure that her ideas were matched well with others whom supported the framework. Sandy argues,

> "you know for so long we argued that having a child compute doesn't necessarily mean they understand or have the ability to problem solve. Then, we said our goal is to help them understand and we want them to be able to problem solve but now I wonder, having a child who understands doesn't mean they can compute. Now I think like this. You can't throw away the insides to the pie. Being able to acquire number facts is imperative to being a mathematician. I don't think it needs to be stressed as an end in itself. But, it is definitely part of the solution. In order to have a broader talent and ability to apply and work with the concepts, one has to acquire a number of different understandings, and computing is one of them. Now, for a lot of students, that ability just isn't happening, and I do think we have to find a different avenue to get there, to get to that big picture. But, for now this is it." (Interview, 5/93).

Reform Documents

Sandy continued to examine reformers' position in the policies and read many sections over to establish what was said. Her reading was shaped sharply by her questions about the traditional computational curriculum. She searched for ideas concerning the traditional computational algorithms and she searched for guidance in the form of specific answers, programs for practice. But, Sandy found very little.[28] Yet, it seemed the relative silence in the policies combined with all that she was learning and thinking related to computational skills convinced her that the traditional computational curriculum taught in very traditional ways remained valued. Sandy reasoned:

> "and that's when the question comes, at what point do you give on skills acquisition for the depth of the concept? And that's when I changed my yearly plan, I decided to embrace both more fully. I'm not convinced you can do one or the other any longer. I definitely believe in the strands and the unit approach, there's no issue there and that enlarges the scope of what mathematical empowerment is. But, for most children, it's still a necessity to practice daily the skills. I can see that they are still valued in the documents and I can see how they can interfere with other learning. You have to do it, and you have to do it everyday. It's like riding a bike, you just have to keep practicing it everyday until it becomes automatic."

A Teacher's Prior Knowledge and Beliefs

Sandy came to her learning experiences knowing much about the traditional algorithms and teaching practices for teaching those skills. Her ideas broadened to include estimation as a computational skill, mental arithmetic and some ideas about students' strategies as well. She emphasized these skills more so during the first year of my observations. During the second year Sandy

[28]In chapter three, I argue the question of what to do about the traditional computational curriculum is mostly left behind as policymakers focused instead on including new computational skills and new instructional strategies. In that effort, the policies seemed to mostly leave unattended such questions as what of the traditional computational curriculum remains valued or to what levels of complexity.

continued to teach estimation and mental arithmetic with less emphasis making room for more traditional goals in the automaticity lessons.

Sandy's prior knowledge and beliefs were reinforced by her new learning about the role of memorization and automaticity in learning basic computational skills. In combination with an inattention to issues surrounding the traditional computational curriculum, Sandy comfortably and confidently reformulated a new interpretation of reformers' ideas about computational skills instruction. In effect, Sandy's learning functioned as a basis for formulating policy.

Sandy's learning, in and across the various contexts, contributed significant insights to Sandy's views of improved teaching. The combination and interaction of ideas that she encountered functioned to change her stance toward her teaching across the years of my observations. Her learning failed to promote and sustain new approaches toward computational goals and instead the combination of factors fostered <u>new</u> learning that resulted in a return to traditional forms of teaching.

Yet, several questions remain. On the surface, Sandy's practice during the 1991-1992 school year, seemed more compatible, more promising, and moving toward policymakers' goals. Her practice the following year seemed less so. Yet, did Sandy's ideas and practices the first year of my observations, the changes and interpretations she made, represent the ideas of reform? If so, there remains a question about why computational goals were not met that first year. And, why would Sandy return to the exact practices criticized most as opposed to other ideas she may have tried? In the next section, I examine Sandy's learning in light of these questions. I ask whether Sandy's learning, both what and how she learned, supported her understanding and enactment of the ideas and practices envisioned in the reform documents.

An Appraisal of A Teacher's Responses To Changing Teaching of Computational Skills

Sandy had developed a practice that prioritized conceptual understanding and reasoning about mathematical ideas. She used manipulatives to explore the mathematics of a situation and consistently encouraged students to use manipulatives in place of traditional procedural algorithms.

The second year of my observations there was evidence of much change. Sandy emphasized the rote-learning of traditional algorithms much more. This marked a clear departure from the state's reform agenda. My analysis in the previous section illuminates that what Sandy was learning ran contrary to envisioned ideas and practices? In addition, other factors influenced Sandy's decisions that also were contradictory toward the state's goals. A clear turning away from policy would be expected under these conditions. After all, all that she encountered supported the position she took.

Yet, for Sandy, this was not a turn away from policy. She believed the changes she made represented a closer match to policy. How could she make this argument? To understand why, I revisit Sandy's practice. My analysis suggests that Sandy's ideas and practices across both years of my observations were substantially different than envisioned practice. I argue that Sandy's return to traditional forms of teaching computation in the second year may not be the best marker of Sandy's interpretation and enactment of policy. A more adequate marker may be understanding what had gone wrong in that first year and how her learning failed to address these issues.

Recall in chapter three, I argued that reforming instruction of basic computational skills would require a great deal of new learning for teachers. Not only would the teacher need to learn an entirely new set of computational skills but also required would be learning a new pedagogy for teaching those skills. My analysis of what was proposed in the reform documents included:

* A Variety of New Computational Skills
* Contextualize Computational Skills
* Conceptual Grasp of Underlying Ideas

In this section, I take each of these and use them as points of intersection for examining Sandy's learning and the ideas and practices her learning fostered.

<u>A Variety of New Computational Skills</u>

Overall, Sandy's views of what the important computational skills are for children to know went mostly unchallenged. Her practice continued to reflect traditional algorithms as the most central method for computing. Aside from the attention she gave mental arithmetic and estimation as a computing strategy during the 1991-92 school year, her ideas and practices returned to a steadfast view that the traditional algorithms were the most central computing techniques for students to learn.

Sandy did not include in either year any emphasis on technology, other strategies such as doubling, adding back, etc., nor did she focus explicitly on strategies students' bring. Even though problem solving experiences dominated students' work in mathematics during the 1991-1992 school year, Sandy continued to recommend traditional algorithms to students for computing. In the second year, Sandy insisted students use traditional algorithms for all computations in the automaticity lessons. The purpose of automaticity lessons was to provide students with practice of efficient and accurate ways to compute (traditional computing algorithms) to use in investigative problem solving.

Contextualize Computational Skills

Policymakers imagined in fairly general ways how traditional computational goals might be met. By emphasizing underlying concepts, using physical materials, linking the manipulation of materials to steps of procedures and developing thinking patterns, teachers can help children master basic facts and algorithms and understand their usefulness and relevance to daily situations (NCTM, 1989, p. 44).

This paragraph suggests that conceptual and procedural understanding both remain valued. Yet, the recommendation is that both goals can be met within the same instructional experience. Mathematical reasoning, conceptual grasp, and mastery of basic facts and algorithms should not be treated separately. Instead, each goal is attended in problem solving settings, as students explore conceptual ideas, use manipulatives to explore those ideas and link those experiences to algorithmic procedures. Instructionally, computational goals and conceptual understanding happen simultaneously.

Sandy's practice, both years, for the most part, did not emphasize any links between ideas such as place value and the procedural steps of the algorithms.[29] Although Sandy made extensive use of manipulatives, these instances were not linked to the procedural steps of traditional or non-traditional algorithms for computing. Instead, Sandy's practice reflected a separation between procedural and conceptual goals. There were no opportunities that I observed across my visits that attempted to link the two. For example, I observed lessons across both years around the idea of average. None focused on linking the idea of average with the step-by-step procedures for calculating the

[29]This is not to suggest Sandy ought to have focused on algorithms but rather to suggest that emphasis on connecting algorithms with conceptual ideas was not what Sandy aimed to do.

average. Sandy confirmed in interviews that she did not emphasize relationships between the conceptual work and the procedural work she did with the algorithms (Interview, 5/93). Missing was any explicit attention to linking procedures and the conceptual basis of those procedures.

The mathematics education community remains unclear about these issues. There is not clear evidence to suggest how the two are intertwined. There exists some evidence to suggest that students' understanding of underlying ideas, such as the conceptual underpinnings of algorithms, help students monitor the success of their computations (Heibert & Lefevre, 1986; Nesher, 1986; Resnick, 1984). In other words, understanding supports effective use and efficiency with algorithmic steps. Sandy's practice, because the two are not linked in instruction, does not offer an explicit opportunity for conceptual and procedural learning to benefit each other.[30]

Recall that Sandy's practice during the 1991-92 school year emphasized investigations into mathematical ideas. In fact, she focused 80 to 90% of instructional time in mathematics on hands-on learning-by-doing activities. These lessons did not include any instruction aimed at developing computational skill. The remaining 10 to 20% of instructional time focused separately on computational goals. During that year, Sandy had constructed a practice where little direct attention was aimed at learning basic facts and traditional algorithms yet, when this did happen it was not linked to conceptual understanding or mathematical reasoning. This practice contrasts sharply with Sandy's practice the following year. Sandy worked on the traditional computing algorithms each day across the entire school year in relation to what she called automaticity of skills. Yet, automaticity and conceptual goals were instructed separately.

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[30]See for example Pearla Nesher, Are Mathematical Understanding and Algorithmic Performance Related? In (1986), For the Learning of Mathematics 6(3), 2-9.

Recall also that during the 91-92 school year the multiplication algorithm was shown to students in only one week at the end of the multiplication unit. Thereafter the algorithm only came up indirectly inside problem solving contexts. In these settings it was not mandatory that students use the traditional algorithm. They could work the computation any way they thought made sense. Some students had the use of calculators and others tried to use their own alternative techniques. Sandy did not give any attention to what techniques might be the most sensible or proficient during these opportunities. Even though Sandy emphasized problem solving contexts for learning computation, she gave no direct attention to any relationships between computation and the context of the problem. Students were encouraged to multiply when it made sense to them to do so but there was no explicit attention linking any procedures to the underlying conceptual basis of the problem.

Sandy's ideas and practices during the first year of my observations minimized the importance and study of the traditional computational curriculum. Instead, Sandy focused on problem solving experiences that involved computational goals as well. She explained,

> "I don't want them to multiply just to go through the steps of multiplication, that's nonsense, the reason you learn to multiply is because, well, just yesterday we had a situation where there were a series of 3 or 4 sevens to add in a problem and the kids had big trouble with it. This hung up the whole activity. We had to add 7 + 7 + 7 + 7 + 1. I explained this is a place where multiplication can help you solve the problem. You have four sets of seven here, right, what is four groups of 7, they said it was twenty-eight, plus one, they said twenty-nine. So we put down the nine and carried

the two. They all went oh, I said remember the goal in multiplication is to add like things fast. So now, I said remember to look for likeness in numbers" (Interview, 5/92).

Sandy's learning to that point seemed to shift her thinking away from procedures and toward situations where multiplication was useful. Her ideas focused more toward such notions as likeness in numbers as a signal to use multiplication as opposed to memorizing procedures. Her practice reflected these changes by de-emphasizing the amount of time students would spend memorizing and practicing procedures and emphasizing problem solving activities with computational goals.

At the same time, Sandy did not attend directly to basic fact acquisition or practice of procedural steps of any algorithms within those problem solving contexts. Ultimately, she abandoned the notion that problem solving contexts could successfully provide students an opportunity to learn computation. In contrast, the following year was significantly different. Automaticity lessons were equally central with problem solving contexts for learning computation. During automaticity lessons Sandy used traditional algorithms to work all problems regardless of the numbers involved. She incorporated catchy phrases such as "slash, burn it off," to represent crossing out a zero and replacing it with a nine in the traditional subtraction with regrouping procedure. Sandy repeated practice with problems involving many zeros or more difficult regroupings. And, estimation was used as a checking procedures rather than a computing strategy itself.

Recall the automaticity lesson I discussed earlier in the chapter. Sandy presented students with the following problems to work.

1. 1000 +3976	3. 10,000 - 9673	5. 75 x 4	7. 497 697 + 119
2. 100 x 6	4. 11,496 + 14,697	6. 20,000 - 10,201	8. 700 400 +600

In looking more closely at these eight problems, only two, problems 4 and 7, seem to require the use of a traditional algorithm and only if an exact answer is necessary. The other problems could be done more efficiently with an alternative algorithm. For example, problems 1, 5, 2, and 8 might more efficiently be done by bringing to bear an understanding of place value into the calculation and computing on components of the problem. Computations could be done mentally, quickly and quite easily. Problems 3 and 6 might be more efficiently done using a strategy of "adding back" or "adding up," the idea being adding up to get to the number 10,000 in problem 3 or 20,000 in problem 6. Problems 2 and 5 might also be done using a strategy of doubling or multiples. For problems 1 and 8, adding only the thousands and hundreds place respectively need be done, also requiring an understanding of place value to get the correct answer. In effect, Sandy's emphasis on traditional algorithms in this lesson promotes practice and memorization of routine procedures and less the notion of efficiency or underlying understandings.

Drill and practice under timed conditions emphasizes memorization of routine procedures and de-emphasizes conceptual and contextual understandings. For example, understanding place value and how the value of numbers is influenced by sums, products, differences, and quotients becomes hidden. Sandy's practices during the 92-93 school year shifted away from the exact ideas she hoped to make more central the previous year. Issues of efficiency, understanding, and contextual aims actually became less central.

Alternative computing strategies such as doubling, adding back, looking for blocks of ten, twenty-five, or a hundred, all were disqualified as reasonable options in automaticity lessons. In effect, Sandy's decisions limited students' opportunity to become fluent with computation.

In problem solving situations, students were not provided an opportunity to size up problem solving situations and determine with insight and intuition how best to calculate in various contexts. Instead Sandy's purpose with the automaticity was for students to make use of it inside the problem solving contexts. Although she did not monitor students use of traditional algorithms, she hoped she had provided students with efficient and accurate methods for doing the necessary computations.

Conceptual Grasp of Underlying Ideas

Sandy's practices often focused on a gimmick for remembering a particular concept rather than understanding any underlying basis of a mathematical idea. For example, in one lesson aimed at exploring the idea of division, Sandy asked students to divide a pack of life-savers among four children.

> Ms. Wise: You see even when Jason was two, he was an expert in math. You know one thing about a two year old, they always want their fair-shares. Jason why don't you grab three friends. (Jason points to three friends as students begin cheering him on.)
>
> Now, Mrs. Jacobson has to figure out how to give the children their life-savers. (Ms. Wise opens a pack of life-savers placing each one on the overhead projector and notes there are 11 in the pack. She then begins to

pass them out to Jason and his friends.) Here's two for you, two for you, two for you and here's one for you and three left over. Is that all right?

Students: No!! That's not fair.

Ms. Wise: Well, why don't I give these three to three of the children?

Students: Because it's not fair.

Ms. Wise: See you guys knew this already, when you divide things among kids, everyone wants their fair-share. Okay, we have four children and 11 life-savers to divide among them. (At this point Ms. Wise begins to re-distribute the life-savers one by one and then asks a series of questions.) So, how many does each child get?

Students: two

Ms. Wise: And, how many are left over?

Students: three

Ms. Wise: Okay, suppose we had another pack of life-savers and eight kids. I need four more kids. (She lays out twenty-two life-savers on the overhead inside a symbol that looks like a division bar. In front of the symbol she draws eight hands.) Now, Ms. Jacobson has to figure out how to give all of these life-savers to eight children. Does anyone want to make a prediction on how many life-savers each child will get?

Student: Four

Ms. Wise: Why do you say four?

Student: Cause it sounds good.

Ms. Wise: Okay, anyone else want to make a prediction?

Student: Approximately two.

Ms. Wise: Any other guesses?

Student: Only one.

Ms. Wise: Okay, Tommy why don't you distribute the life-savers.

As Tommy distributes the life-savers Ms. Wise waves the wand repeating the phrase "noit-ca-il-pitlum" over and over. Tommy distributes one life saver to each child. Ms. Wise asks, "does it look like there is enough for another round?" Several students answer "yes!" Tommy continues to distribute a second round of life-savers. When he finishes Ms. Wise asks:

Ms. Wise: Okay how many life-savers did we have to start?

Students: twenty-two

Ms. Wise: And how many children did we have?

Students: eight

Ms. Wise: And, how many life-savers did each child get?

Students: two

Ms. Wise: And, how many were left over?

Students: Six

Ms. Wise: Raise your hand if you see a pattern.

Matthew: Well, each time we were splitting up the lifesavers, you were teaching us division.

Ms. Wise: I'm teaching you what?

Matthew: division!

Ms. Wise: No, no that is a sin in third grade. I was teaching you "noit-ca-il-pitlum."

Matthew: (and several other students at the same time) It's multiplication backwards!

Ms. Wise comments, "see, division is in our blood" and "a two-year-old can do it." She announces that next week there will be more opportunities to do division.

> Ms. Wise: okay then, division is two things, right gang, now say it with me, division is two things, it is fair-shares and multiplication backwards. Ready go.
>
> Students: fair shares and multiplication backwards
>
> Ms. Wise: Ready go.
>
> Students: fair-shares and multiplication backwards!!

Student's interest and energy level was very high during this lesson. They were attentive, listening, calling out, and laughing out in what seemed pure enjoyment. They especially seemed to like the wand and the phrase "noit-ca-il-pitlum." Just saying it seemed fun.

Sandy aimed for two connections. First, she hoped students understand that division is an equal-distribution of objects into groups with something leftover. Sandy drew eight hands in front of a box holding twenty-two life-savers . Students physically distributed the life-savers from the box into the eight hands. She wanted students to experience that 22 can be distributed into 8 equal groups of two in each group with six left over. Symbolically, she hoped students would associate the idea of equal-distribution to the division symbol. Yet, there was no explicit connection made. There also was no explicit connection made between the idea of groups and division symbol. Whether eight groups of two or two groups of eight represented the same thing did not come up.

Sandy also wanted students to make a connection between multiplication and division. Yet, there was no explicit attention given to what this connection involved. Only the phrase "noit ca il pitlum" was mentioned. Students seemed to know the phrase that division is multiplication backwards but, any support for understanding the idea was not addressed. The opportunity represented more a memorization rather than an understanding of what the connection involved. The exploration of the conceptual underpinnings of division were weak. At best, students were offered an opportunity to equally distribute life-savers.

In another lesson on the idea of average, Sandy related fair-sharing to average without making any explicit distinctions to division. In the average lesson Sandy asked students to keep track of the number of pieces of mail their household receives over a three day period. Students had to predict what the "average" number of mail items would be. Sandy explained that finding the average meant "fair-sharing" or equally distributing the mail across the three day period. Using colored bear counters, students found averages by distributing equally the total number of mail items into equal groups. They began using language such as "my average is four" (Fieldnotes 5/92).

Sandy explained her goal in this lesson was to approach the idea of average without using the algorithm. She said she wanted students to see that average was linked to the idea of fair-sharing. Although students could come up with their averages fairly quickly by distributing the bear-counters equally across the number of days, they were not asked to think about the underpinnings of the idea of an average. In other words, students were not involved in an opportunity to consider the meaning of an average, what an average represents about a set of data or why an average might be useful to know in particular situations.[31] Students did learn a technique for calculating an average, an alternative to the traditional algorithm. Yet, they were not asked to link those steps to the underlying meaning of the idea of an average. Thus, on one level students approached the idea of average in a very different way than traditional approaches involving memorizing the algorithm. At the same time the lesson did not ask students to understand conceptually the idea of average nor did it engage students in connecting procedural steps to the idea of an average.

The lesson on average also makes no distinctions between fair-sharing and the idea of division. Although Sandy does much more with the idea of average in later lessons, students are not given any direct opportunity to think about connections between fair-sharing and the ideas of average and division. Mostly, students are involved in learning alternative algorithms for computing answers in the form of distributing manipulatives in particular ways.

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My analysis of Sandy's teaching across the three core constructs proposed in the reform documents suggests that Sandy attended to this agenda only minimally, never fully integrating the proposals into her teaching. Each year students' opportunities to learn pit computational skill against conceptual understanding. Each year, conceptual and skill-oriented learning are instructed separately. Across the years, one goal dominated, the other received less attention. In the first year, Sandy's instructional approach rested on the belief that conceptual understanding would promote computational fluency. At the

[31]Understanding the idea of average conceptually involves much more than what Sandy's practice offers. Jan Mokros and Susan Jo Russel in their paper Children's Concepts of Average and Representativeness helped me to think about these issues. See TERC Working Paper 4-92, January, 1992.

same time, Sandy's views of what it would mean to understand mathematics conceptually differed from what was described in the reform documents. The following year, Sandy's reasoned that students need more opportunities to practice procedural steps to computational algorithms. In that year, she returned to traditional forms of teaching aimed specifically at developing proficiency with basic number facts and algorithms.

Sandy's practice across both years runs in stark contrast with proposals to reform computational skills instruction. Yet, what Sandy encountered typifies some of the tensions any teacher would likely encounter when trying to attend to both procedural and conceptual goals. The complexity involved in learning to teach mathematics for conceptual understanding has been well-documented (Ball, 1989; Eisenhart et al, 1993; Heibert & Lefevre, 1986) A strong debate about the relationship between conceptual and procedural knowledge and the pedagogical practices needed to support each continues (Heibert & Wearne, 1988; Nesher 1986). There is also debate specifically about the role that basic computational skills plays in students' understanding of mathematics (Resnick, 1984). It should be no surprise that Sandy would face a great deal of uncertainty around these issues.

Even as there is much debate and uncertainty surrounding these issues, teachers must try out new instructional strategies if they are to respond to the state's efforts at reform. They must try to change their teaching in the context of an environment that holds the pervasive perception that teaching is a practice of certainty. Sandy's approach, under these conditions, suggests that she believed there were clear-cut answers offered in the reform documents. She formulated what she thought were the right answers and tried them out. The problems that arose in her teaching were not viewed so much as tensions, but more things she needed to fix. Sandy thought of the problems as errors in her own judgment, miscalculations on her part, even personal deficiencies. And her response was to find the right answers for her teaching.

The uncertainty of teaching practice and enacting policy is what makes Sandy's stance toward "solving" the problems that arose in her practice so significant. If teaching and enacting policy are seen as uncertain crafts, it would seem natural to encounter a set of tensions, as a natural consequence of any change. Instead, Sandy encountered a stance of certainty toward changing her teaching across the professional development opportunities she participated. This stance of certainty is endemic in education. The perception of teaching as a practice of certainty is far more common at all levels of the educational system McDonald, 1992). At the same time, the policy in California painted the practice of teaching and enacting policy as one of uncertainty. Yet, Sandy encountered little opportunity to alter her stance. She had few opportunities that suggested, for example, she would need to unpack the issues underlying her teaching of computational skills and see them as choices with consequences for students' learning, rather than right or wrong answers for her teaching. Instead, the opportunities to learn that Sandy encountered did not function in this capacity. They failed to suggest she would need to unpack, understand, and manage underlying tensions. Mostly, the experiences offered more activities to try and in the form of right answers for teaching.

CHAPTER 5

LEARNING TO TEACH AN UNFAMILIAR MATHEMATICS

Introduction

The previous chapter investigates Sandy's efforts to change her teaching of basic computational skills. This chapter focuses on a different strand of content. Though here, the mathematics is not as familiar as computation. The teaching context is also different. In 1989 Sandy accepted the position of elementary curriculum specialist in her district. One of her responsibilities included helping other teachers learn about the new mathematics framework. Sandy's role as curriculum specialist represents a unique opportunity for this study. It offers the opportunity to investigate the learning experiences of a teacher leader in the context of external efforts to reform.

As the district's elementary curriculum specialist, Sandy felt comfortable with her responsibilities to help other teachers understand the state's goals. However, she learned that the new framework proposed a strand of content that was not recommended in the previous framework. This strand was unfamiliar to Sandy. In fact, she had never heard of discrete mathematics until now. To prepare for an upcoming workshop on the new framework, Sandy decided that she would develop some ideas of what discrete mathematics is and how elementary teachers might introduce it into their teaching.

The story in this chapter focuses on what Sandy did and the experiences she encountered for learning about discrete mathematics. It illuminates the difficulties a teacher encountered in learning an unfamiliar mathematics for herself as well as the problem of teaching other elementary teachers how they might introduce discrete mathematics into their teaching.

The chapter begins by describing the experiences Sandy encountered for learning about discrete mathematics. Like most elementary teachers, Sandy had very little knowledge of what discrete mathematics involved. She ran into difficulties very early in the learning process. As the story evolves, I explore Sandy's learning in and across a variety of settings that she made use of. The reader will learn about the mathematical ideas she grappled with, what directed her attention there, and the role several other factors played in promoting or hindering her evolving understandings. The story ends with an illustration of the opportunity for learning Sandy offered other elementary teachers for their learning about discrete mathematics and how to introduce it into their practices.

In the second part of this chapter, I consider the events in this story from a pedagogical perspective. I examine what and how Sandy learned in relation to the proposals to introduce discrete mathematics and whether her learning fostered understandings and changes envisioned in the policy. My analysis suggests there is an enormous amount of new learning required to understand and introduce new mathematics into an already bursting elementary mathematics curriculum. And the problem of teacher learning is a very serious one having consequences far beyond the work of one teacher.

A Teacher Leader's Responses to Proposals to Introduce Discrete Mathematics Into The Elementary School Mathematics Curriculum

Sandy encountered proposals to introduce discrete mathematics as she prepared for a workshop to teach other teachers in her district. The central purpose of the workshop was to introduce teachers to the new mathematics Framework (1992). Sandy accepted the position of district elementary curriculum specialist two years prior and knew well in advance that the development of a new framework was underway.

Sandy expressed the importance of introducing discrete mathematics in our very first interview. She previewed the new Framework (1992), in one of her doctoral courses prior to publication and more widespread distribution. Sandy explained, "the new document is an extension of the document from '86. That document emphasized seven strands. A lot of other states modeled after it. In the new document they added an eighth component called discrete mathematics, I think it is kind of like math in the real world" (Interview, 1/92). Sandy had already begun to think about discrete mathematics and what it might mean for her teaching of other teachers.

When asked again later about what else was new in the forthcoming framework, Sandy reiterated, “the new strand of content was a big change” (Interview, 1/92). Although Sandy mentioned other changes and additions to the new document, the introduction of discrete mathematics preoccupied her mind. She explained, “I have been asked to speak to, have a conversation with groups of teachers from the county to develop an understanding of the new framework, for the County Office of Education, it's a math consortium group. So when I was posed with this task, I knew I had to develop the discrete math strand, which I didn't do formally at that point, and certainly I didn't have it internalized enough to talk about it" (Interview, 11/93).

Sandy’s tone and comments indicated mostly guesswork. She proposed, “I think it is like math in the real world” (Interview, 1/92). And, she read inquisitively,

> “it indicates here that it is a study of systems of separate entities, so it's used in the sciences, maybe with the elements chart. They also

indicate finite graphs so maybe it is related to statistical data, but this is difficult. And, students are supposed to construct, analyze and compare algorithms. Well, we don't have many algorithms in the third grade" (California State Department of Education, 1992, p. 149/Interview 2/92).

As Sandy tried to understand the connections in the above paragraph, it seemed she was grasping at straws. She was hard-pressed to make any sense of the written words on the page.

The reform documents represented Sandy's first opportunity to learn something about discrete mathematics. The Framework (1992), provided a description. Sandy read that discrete mathematics should be introduced into the K-12 curriculum and that all students should learn the topics and ideas of discrete mathematics. Sandy decided she would need to learn much more to introduce the strand of content to the teachers at her upcoming workshop.

Given the enormous reform agenda, it seemed somewhat curious that Sandy would center so much of her attention on discrete mathematics. Yet, the pressure of the upcoming workshop and how Sandy organized workshops around the content strands of the framework seemed to make it good sense. Sandy perceived her role and responsibilities to include educating other teachers about the content strands. In fact, Sandy structured her seminars in direct relationship to the content strands.[32] She introduced teachers to instructional activities she thought fit within each category of content and tried to support

[32]A colleague observed Sandy's teaching in the context of a one day district sponsored inservice for third-grade teachers titled Third Grade With Math Manipulatives. He noted that Sandy's organization and focus throughout the workshop centered on activities associated to the eight strands of content identified by the new Framework document. The impression he had was that the workshop was aimed more at introducing the new framework rather than exploring the use of manipulatives.

teachers in learning to use those activities. This structure, in part, pressed Sandy to consider discrete mathematics as an important change in the new document.

Over the next year Sandy encountered a number of different opportunities to learn about discrete mathematics. She would draw from her learning that year to teach other teachers ideas and practices for teaching discrete mathematics in the context of several state-sponsored workshops the following year. Inside these workshops, Sandy communicated her evolving ideas about how teachers could introduce discrete mathematics into their own teaching. Below, I describe how Sandy developed her ideas about discrete mathematics, what she tried to learn, what directed her attention there, and how her learning progressed.

Learning About Discrete Mathematics

Shaping Sandy's direction for learning about discrete mathematics was her organizational strategy for her workshops. Sandy simply found the content strands in framework to be a helpful way to cut across the multidimensional nature of the state's reform agenda. Inside each content strand, Sandy could address a wide range of issues, ideas, and practices she believed represented the reform agenda. In the interest of finding suitable curriculum materials for teachers to work with at the workshops, Sandy began a process of learning about discrete mathematics. She set out to locate investigative-type activities she thought could represent discrete mathematics in the elementary grades.

Sandy had difficulty finding any materials she felt certain represented discrete mathematics. Two problems surfaced. First, Sandy had few ideas to draw from to conduct a search for curriculum materials. She really didn't know what mathematical ideas she was looking for. Second, none of the materials Sandy encountered were labeled as discrete mathematics.

Obviously, Sandy had very little to go on. And she found out very quickly that she was not alone in her struggles. Sandy decided to consult a number of other teachers, a district math specialist, and a secondary mathematics teacher to help her locate materials. Yet, she found very little help in identifying curriculum materials that focused on discrete mathematics. No one that she encountered seemed to have good ideas about materials she could use to accomplish her goals (Interview, 1/93).

Sandy eventually consulted a state-level mathematics educator, someone involved as a developer of the new framework. He also was unable to offer Sandy suggestions about elementary curriculum materials (Interview, 1/93). Early on Sandy found herself in a position of trying to make decisions about good curriculum materials by herself even though she had very little knowledge of the subject matter to go on.

Sandy decided to return to the framework and to the Curriculum Standards (1989) for guidance. She searched each document trying to dig out everything she could find about discrete mathematics. Most of the information Sandy found in the documents was geared toward secondary teachers. Reading the information was very difficult for her though she began to formulate some ideas about what discrete mathematics involved. She read and re-read the material over and over, searching for clues about the content. Below I characterize what Sandy learned as reported in several interviews.

The Reform Documents

Sandy's reading of the reform documents represented her first dip into the mathematical ideas of discrete mathematics. She seemed to be using the paragraphs much like a textbook, trying to teach herself something about the mathematics. What Sandy found was that the documents were of little help to her. She complained that they were very confusing. She found brief descriptions of the term discrete and several rationales for introducing discrete mathematics into the school curriculum. She found a listing of some of the central ideas, most of which were unfamiliar to her. Sandy complained, "there just isn't much detail in there and much of it I just don't understand" (Interview, 1/93).

Sandy also found a few examples offered as illustrations of the types of counting problems one encounters in discrete mathematics. Unfortunately, Sandy had great difficulty making sense of the examples. One problem read as follows:

> For example, a complex network of one-way streets can be represented geometrically by a directed graph, which in turn can be interpreted algebraically as a matrix. An i-j entry in the matrix is 1 if and only if corresponding vertices are adjacent (i.e. connected by an edge); otherwise, the entry is 0.
>
> By representing the graph as a matrix S and then multiplying S by itself , students can use S squared to determine the number of two-stage routes connecting the various paths of points. Students can generalize this procedure to graphs of any size and computer software can be used to compute powers to the corresponding route matrices, which can then be analyzed to determine numbers of multiple-stage routes as well as other characteristics of networks (NCTM, 1989, p. 177).

This paragraph represented to Sandy an opportunity to learn what discrete mathematics involved. This particular problem focused on directed graphs. The question asks how many two-stage paths there are connecting one point on a diagram to another. The authors suggest that matrices can be used to model the connecting paths and the mathematics of matrices can be used to solve the problem. An illustration of this process is included.

Sandy's work with this problem led to frustration. She knew little about matrices. She decided she would try to learn about matrices in order to understand this example. She consulted other teachers and on several occasions even asked me for help (Interview, 1/93). The process was ongoing and took a great deal of time.

As Sandy focused extensively on the mathematics of matrices, how to set them up, add and multiply them, she began to learn mathematics she had not encountered prior. She became familiar with setting up a matrix to represent a problem situation. She became quite comfortable with multiplying two matrices. It seemed that Sandy's efforts advanced her understanding of matrices. Yet, in an interview Sandy could not explain how the matrix solution related back to the original problem in the text. At that point in the process, her learning did not function to promote an understanding of the combinatorial counting ideas underlying the problem.

Because Sandy was entirely unfamiliar with the mathematics of matrices, she centered much of her attention there. The mathematics of combinatorial counting never really came to the surface, at least it never became focal. Sandy's learning did function to promote the idea that matrices were connected to discrete mathematics. She teetered on the idea that matrices is an example of discrete mathematics. And, she decided that discrete mathematics was very difficult to learn (Interview, 1/93).

Earlier I suggested that Sandy was very resourceful as a learner. She creatively sought out learning experiences that she believed would help her put a picture of discrete mathematics together. At least two ideas described in the documents attracted Sandy's attention. One involved contrasts drawn between continuous and discontinuous mathematics. The other was about whether the integers was an example of a system of numbers used in discrete mathematics.

Sandy's puzzling about these ideas resulted in a number of different occasions to talk to others about discrete mathematics (Interview, 1/93). Sandy arranged to talk to other mathematics educators and in these contexts she would grapple with a variety of mathematical ideas. She also encountered a rationale for including discrete mathematics that was not discussed in the documents. Below, I describe what Sandy encountered in these settings.

Conversations With Other Educators

Sandy decided to consult the "experts" to help her understand the mathematical issues she encountered in reading the reform documents. Because she viewed individuals with direct responsibility for conceptualizing, writing, and implementation phases of the framework as experts with the ideas, she contacted individuals in these roles to help her learn more about discrete mathematics (Interview, 2/93).

After meeting with one person and asking specifically about the characterization of continuous and discontinuous mathematics, it was mutually decided that he could not be of much help to her. He directed her to someone he thought could help, a leading mathematics educator involved in conceptualizing the new framework.

Sandy set up a meeting. My description of what took place is organized in relationship to the mathematics Sandy encountered on these occasions.

*Contrasts between continuous and discontinuous mathematics

*Number systems

*Relational theories

Continuous and discontinuous mathematics. Sandy read that discrete mathematics was unlike other, more familiar mathematics because of its discontinuous qualities. In an interview Sandy explained her confusion with this notion, at least in part, involved what the terms continuous and discontinuous described. Sandy had already searched the policies for any information she could find. She read in the new framework, "Discrete mathematics, the study of systems with separate (discrete) entities, is contrasted with systems involving continuous quantities" (p.149). And she read in the Curriculum Standards on p. 176, "Whereas the physical or material world is most often modeled by continuous mathematics, that is, the calculus and prerequisite ideas from algebra, geometry, and trigonometry, the non-material world of information processing requires the use of discrete (discontinuous) mathematics."

Sandy's confusion prompted her to ask a question. She explained, "so I asked him [mathematics educator at the state-department] about continuous and discontinuous mathematics and the connection to computers." And, I asked, "whether the terms continuous and discontinuous described the entire system of numbers underlying each kind of mathematics or if the terms described the elements of the underlying system" (Interview, 8/93).

Sandy described the discussion as difficult for her. She explained,

> "he communicates at a secondary level. I had to be really honest and say, wait, wait, I am an elementary teacher." She confided, "we both really had to struggle to talk. He had to lower, well not lower, but unpack his vocabulary so that we could communicate and that

was challenging for both of us. When I needed it, he tried to clarify things for me and I would try to pull from my background but really I couldn't" (Interview, 1/93).

Even though Sandy was frustrated and suggested the meeting was less than ideal for her learning about discrete mathematics, she seemed to take away several ideas from this meeting. She explained, "he went into ideas where he talked about the integers from zero to infinity. I know now it [discrete mathematics] can involve continuous data but the solutions to problems are different, they are finite unlike in algebraic functions where the solutions often go on and on." Sandy clarified further, "there is something about algebra and discrete math, they are different, one is based on continuous numbers, like with functions, and the other, I am not sure, this is where I get stuck," (Interview, 2/92).

Sandy's comments reflected some circling around the ideas she had pulled from the documents. She was in the process of questioning what the differences might be in the sets of numbers or data underlying discrete mathematics and the more familiar mathematics of algebra that she had some ideas about. She clarified for herself that the terms continuous and discontinuous not only referred to the systems of numbers underlying the different mathematics but also to the kinds of solutions algebra and discrete mathematics offer. Despite these advances, Sandy remained stuck on the question of what the systems underlying discrete mathematics might be like.

Underlying number systems. Sandy read in the Framework (1992), and the Curriculum Standards (1989), that discrete mathematics is, "the study of systems with separate (discrete) entities, [and] is contrasted with systems involving continuous quantities" (p. 149 Curriculum Standards). She also read, "Discrete in this context means focusing on discrete and separate entities rather than on measures of continuous quantities" (p. 84 Framework). From reading these statements Sandy came to a question. She wondered whether the integers represented an example of the kind of number system reformers described as underlying discrete mathematics. Sandy knew a great deal about the integers and could relate well to what they were like. At first she conjectured, "the integers are not part of discrete math because they are continuous, they go on and on." Yet, she puzzled, "but each integer is distinct and separate from the next integer" (Interview, 8/93).

Sandy pulled out the following statements from the framework pointing to them in an interview, "it does not mean that everything not continuous is to be considered discrete mathematics. Arithmetic with integers, for example is treated under number, not under discrete mathematics" (p. 84 Framework). Sandy concluded, "if arithmetic with integers does not fall under discrete mathematics then the integers must not be an underlying system in discrete mathematics" (Interview, 8/93). The documents suggested to Sandy that the integers do not underlie the mathematics of discrete mathematics.[33]

Once again, Sandy turned to the leading mathematics educator at the state department. He seemed to contradict this conclusion. Sandy concluded after her conversations, "the integers are an example of a number system underlying discrete mathematics in the same way the real numbers underlie most of algebra" (Interview, 8/93). Sandy seemed more settled with this understanding.

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[33]Interestingly, recall in chapter two, in an interview with a mathematician specializing in discrete mathematics, he noted that the counting numbers and/or integers are perhaps the best starting point for beginning the work of combinatorial counting, a central idea of discrete mathematics, mostly because people have familiarity with the ideas of the system.

Relational theories.[34] Sandy explained, "I wanted to follow up on some of the ideas in my other meeting [with the leading mathematics educator] and I still needed to find activities I could use [at the workshop]," (Interview, 8/93). During a meeting with the district math specialist Sandy asked about relational theories. She complained to the district math specialist about her previous conversations with the leading mathematics educator that "he rattled on about relational theories. I basically thought this is impossible" (Interview, 8/93). The district math specialist decided she would help Sandy by trying to explain what recurrence relations involved. Sandy recalled "she pulled a secondary textbook out and identified several relational theories and began explaining them step-by-step," (Interview, 8/93).

In an interview, Sandy was unable to describe what recurrence relations involved and she could not explain how they fit with discrete mathematics. Yet, she indicated that most of the time she spent with the district math specialist involved discussion of these ideas. Sandy was convinced that she just could not understand recurrence relations and thought, "this stuff is not translatable to the elementary level." She described her meeting with the district math specialist as difficult and complained, "I just couldn't get it" (Interview, 8/93).

Sandy felt the conversations she had with other mathematics educators were overwhelming. She felt frustrated in her efforts to locate avenues for understanding discrete mathematics. She mostly thought that what had happened was not very useful.

Yet, Sandy managed to connect with an interesting piece of information she had gathered from these conversations. This information, in itself seemed to

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[34]Recurrence relations involve the construction of formulas for expressing the relationship between terms in a sequence or series as a function of one or more of the previous terms. Examples include compound interest formulas, home mortgage formulas or a formula for expressing the more famous Fibonacci sequence. Sandy termed these ideas as relational theories.

have a dramatic impact on her learning. Sandy encountered a different rationale for introducing discrete mathematics into school mathematics that she had not read in the documents. Instead of fostering a stronger desire to learn more about discrete mathematics, it convinced Sandy that there was less importance for her as an elementary teacher to learn about discrete mathematics. Sandy recalled,

> "he [leading mathematics educator] explained the strand was really added for the secondary level. He explained that the algorithms for the inclusion- exclusion principle, pigeon-hole principle were not falling under the strand of number strength very well and they were being overlooked in the secondary curriculum. So, they decided to enlarge the scope of what ought to be taught and added discrete mathematics. At least that is what I recall" (Interview, 1/93).

Sandy placed a lot of stock in this information. She admittedly knew nothing about the mathematical principles she mentioned here, but she became convinced that discrete mathematics was introduced mostly for secondary grades. At that point Sandy seemed to shift her efforts to learn about discrete mathematics in low gear. She no longer was preoccupied with setting up learning opportunities for herself. She had been convinced reformers' introduced the new strand to enlarge and re-emphasize what was not getting adequate attention in the secondary curriculum.

Interestingly, Sandy did not abandon the idea of introducing discrete mathematics at the elementary grades altogether. She explained, the Framework (1992), "suggests the introduction of discrete math at all levels because of the belief in California that some work can be done at all levels (Interview, 5/93).

She rationalized, "of course, there is always some work that can be done at the elementary level, but it doesn't have to be very extensive." Sandy commented further, "no one had any suggestions for what those activities might be, at the elementary level, so I had to do this myself" (Interview, 5/93).

It seemed Sandy had come full circle. She was convinced mostly of two things. She felt that consulting others had not been very promising or useful for learning about discrete mathematics. She also believed that discrete mathematics was very difficult for her to learn. Consequently, Sandy decided not to involve others and instead would return to search for elementary curriculum materials. I turn now to focus on Sandy's experiences in relationship to two pieces of curriculum materials she located for learning about and representing discrete mathematics for the elementary grades.

Curriculum Materials

Sandy decided to look again at the Framework (1992). She explained, "So, I re-read the Framework (1992), again and I said okay I'm gonna pick just one idea in here, first, permutations and combinations. Then I will work on the next idea, maybe unions and intersections after that" (Interview, 2/93). Sandy selected combinations and permutations as a starting point. She remembered her experience with the ideas in high school algebra. She began searching for any activities she could find that focused specifically on combinations and permutations including an old high school textbook. Her goal was to locate an activity she could use as an example of teaching discrete mathematics. The activity also had to be an investigation, learning by doing experience that teachers could use with their own students.

Sandy found in her own collection of materials several mimeographed sheets of paper, a Marilyn Burns activity she referred to as the "ice-cream problem."[35]
She explained,

> "early on I grabbed at all kinds of stuff based on my gut instinct, I thought this idea of combination and permutation was a good one. But then, it wasn't labeled anything, it was just this kind of extracurricular piece to the math content of the text. Now, I guess we can call it discrete mathematics" (Interview, 2/93).

Sandy decided she would use the ice-cream problem as an example of a discrete mathematics problem with teachers in her workshop. To do so, she knew she would need to explore the mathematics of the problem for herself. The materials she had consisted of two worksheets labeled combinations and permutations at the top. Each sheet posed a problem.

> Kids love ice cream cones. How many different two scoop cones can you make with chocolate and strawberry ice cream"? Draw a picture of each cone that is different.
>
> The second sheet had the following problem:
> How many different two scoop cones can you make with three flavors? The flavors are chocolate, strawberry and vanilla.

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[35]The activity is currently situated inside a series of activities titled Brown Bag Series and is designed to focus on the ideas of pattern and function. The activity is not identified in these materials as discrete mathematics but in fact, is an example of a combinatorial counting problem. Say how these are different.

The materials Sandy had did not specify how to solve the problems. There were cones drawn on the paper simulating ice-cream cones. Students were to construct various two-scoop ice-cream cones making decisions about whether to count cones that had the same flavor for both scoops or reverse-ordered flavors as different ice-cream cones. For example, in the first problem, a decision for whether to count chocolate on top and strawberry on bottom as different from a cone where the flavors are reversed is important. Organizing outcomes using some type of system for keeping track of the different cones would make it easier to determine when all cones have been accounted for.

Sandy's experiences working on this problem by herself and later with other teachers at the workshop fostered for her several new ideas. Sandy decided that the ice-cream problem was more about learning how to count things than getting the right answer. She contrasted this idea with her prior experience in her high school algebra class. She recalled,

> "Traditionally, in an algebra class, the teacher gave us a formula for combinations, a formula for permutations and you were supposed to be able to use them, not understand them. What I like about this now is that it is more about learning how to think about things, learning how to count and organize things, a way to count things, and if it is taught in this way it is more powerful" (Interview, 8/93).

Sandy seemed to realize that the mathematics of combinations and permutations would involve more than memorizing a definition or a formula for calculating the right answers.

Sandy also focused on the idea of order and whether order makes a difference in determining what to count. She explained, "first, you need to decide if a chocolate on top and vanilla on bottom is the same cone as vanilla on top and chocolate on bottom." And, "if you decide that order does matter then what and how you count is different than if you decide it doesn't" (Interview, 8/93).

Sandy also realized the importance of being systematic in counting. She reflected on an incident in her teaching,

> "did you notice how Mary Jean came up and she instantly had a strategy for organizing the cones? And yet, Mike couldn't organize, he was all over the place, he couldn't set up any pattern using the three flavors. So, we had to think about what would be a way to organize the process and later I asked him now is there a better way to organize and communicate the data? And two or three others came up and finally we had organized the scoops in a way that we all could determine that we had covered the possibilities" (Interview, 2/93).

Sandy's learning revolved around the notions that order and patterns in counting combinations and permutations matter. She argued that systematic counting provided certainty that all possibilities were accounted for. Sandy illustrated in a diagram the number of different ice-cream cones using a pattern of combining all possibilities with vanilla first, then strawberry, then chocolate. Sandy showed there were nine possibilities if two-scoops of the same flavor were counted, and 6 if not. She illustrated there were only 3 possible outcomes if for example chocolate on top and vanilla on bottom was considered the same cone when the order was reversed (Fieldnotes 8/93).

Sandy also defined combination and permutation. She explained that permutation meant, "order is important" and combination meant, "order is not important." She explained there were 6 permutations and 3 combinations of ice-cream cones for the problem involving two scoops and three flavors of ice cream.

Sandy' understanding of the algorithms for calculating the number of different combinations or permutations of a set of objects remained unclear. Recall that she had memorized the algorithms in high school. At this point she could not recall the algorithms but explained, "I know the algorithm is just a shorthand version of this process." And, "I know I should have the talent to say, now here's the algorithm and here's how I can prove it works, but that is really hard for me to figure out" (Interview, 8/93).

Although Sandy was unable to remember the algorithm or provide an explanation of how it connected to the counting process she demonstrated, she did imagine for the first time that there were connections. Sandy realized that the formula she had memorized years ago was rooted in the process she now used for formulating the outcomes. The formulas somehow seemed less of a mystery to Sandy. She commented that she no longer thought of the algorithms as if they had been, "made up out of the air" (Interview, 8/93).

Sandy's view of the importance of the algorithms also had changed. She explained, "you know the power of all this is really more about how you can decide how to count, you decide if vanilla over chocolate is different from chocolate over vanilla, and if so, then you decide how to count those, and so that

is what is empowering to me, not the algorithm. Then maybe one day we can think about proving the algorithm" (Interview, 8/93). [36]

Sandy's work with the ice-cream cone problem functioned to help her create an experience with discrete mathematics for other teachers. Sandy used what she learned from the ice-cream problem to facilitate other teachers' learning about the ideas of combination and permutation. I turn now to focus on the workshop experience Sandy offered other teachers. I use this context both as a site to illuminate what Sandy learned about combinatorial mathematics and as an example of the learning opportunity other teachers were offered for learning about these ideas. I begin with a brief discussion of what the workshop was like for participants more generally. I then focus on the particular segment of instruction Sandy offered teachers for learning about discrete mathematics.

Teacher Workshops

Sandy's enthusiasm at the workshop was indescribable. Her style promoted a renewed sense of energy and excitement. Bells, whistles, flashing lights and even magic were part of the show. Sandy's enthusiasm seemed to rub off on participants as well. Teachers sitting slumped in their chairs awaiting a lecture type workshop seemed to pop-up to see what was about to happen next. Heads turned, smiles appeared, everyone was trying to see what Sandy was up to. There was laughter and enjoyment.

[36]Interestingly, the power Sandy seems to be referring to, involving decision making for what and how to proceed with counting, runs counter to the notion of power as I described it in my sketch of discrete mathematics in chapter three. I suggested that power from a mathematician's point of view may be rooted in an understanding of the generalizations that can be drawn across similar kinds of counting situations, leading to the formulating of algorithms that do the work of counting for you. This is not to suggest mathematicians do not also consider understanding the tedious nature of the counting process valuable, but more to suggest that the power of this mathematics is related to the efficiency and effectiveness of the theories, formulas and algorithms and assuming the more tedious nature of the work Sandy is highlighting.

Much like her third-grade practice, Sandy organized the workshop in short segments focusing on each of the eight content strands. During each twenty minute segment Sandy asked teachers to engage in one or two short versions of student activities that she believed represented the eight content strands. This approach Sandy inherited from her work with Project AIMS. The underlying philosophy of AIMS rested on the belief that teachers could learn about instructing their students by actively engaging in the activities for themselves. In the process teachers would learn the mathematics and a pedagogy for teaching those ideas. It was from this perspective that Sandy tried to engage teachers in investigation-type activities. She asked them to make believe that they were elementary students participating in real classrooms.

Sandy's goals were multidimensional. She wanted to provide teachers with activities they could take back and use in their own classrooms. She also hoped teachers would learn the mathematics underlying each activity. Sandy counted on the investigations to prompt teachers to ask questions and promote discussions about the mathematical ideas. She knew that conversations would arise concerning appropriate changes in activities for various grade-levels. She also expected participants to ask questions concerning pedagogical issues. Sandy often tried to offer teachers ideas about where students might have difficulties, pointing out likely trouble spots and what she would do to help students in those instances (Fieldnotes, 8/93).

Sandy also wanted teachers to think about the reform agenda. She explained to teachers that the investigation-type activities she offered are representative of the kinds of student learning experiences characterized in the new mathematics framework. Sandy argued that the idea of getting kids to work collaboratively in problem solving experiences is part of the new pedagogy proposed in the new framework (Fieldnotes, 8/93).

Sandy made selections about activities using two criteria. First, the activity had to correspond to one of the eight content strands of the framework. Sandy wanted teachers to have experiences that cut across the content areas of the framework. As a result, participants at the workshop often found themselves doing mathematics they currently were not teaching. For example, several teachers expressed that they did not teach mean, median and mode to their students. Sandy promoted these ideas as part of the probability and statistics strand. All teachers at this workshop agreed that they were not currently doing anything in their own teaching with discrete mathematics (Fieldnotes, 8/93).

The second criteria Sandy used was whether the activity embraced a learning-by-doing philosophy. Sandy embraced this philosophy for students and for teachers' learning. She personally viewed the philosophy of learning by doing as representative of reformers' vision of teaching and learning mathematics. And she believed that activities like those designed by Project AIMS, Marilyn Burns and a few others would engage teachers in learning a new pedagogy that they could use in their own classrooms (Interview, 8/93).

In the segment on discrete mathematics, Sandy focused on two activities, both counting problems. The first was the ice-cream cone problem described earlier. Teachers worked with paper cut-out ice-cream cones and flavors. They were asked to construct as many two-scoop cones as possible using the three flavors provided. Sandy suggested they keep track of their work in whatever fashion they thought necessary. After a few minutes, Sandy asked groups of teachers to share what they had done with the larger group. Before looking closely at what happened, I describe the second activity.

The second activity, by Project Aims, was titled Teddy Bear Dresses the Seasons. A handout posed the question how many different outfits can Teddy make from 2 hats and 3 sweaters? Teachers were asked to dress bear cut-outs in as many different outfits as possible using cut-out clothing of three sweaters and two hats. After a few minutes, Sandy suggested teachers try variations of the problem and provided additional cut-out clothing for that purpose. The variations participants may have tried were not discussed in the larger group.

As a pair, the two activities introduced teachers to the discrete mathematics content strand. The time that teachers engaged in these activities fell within the 20-30 minute framework Sandy devoted to each strand. She indicated to teachers that the time she allowed for teachers to work with the activities was abbreviated in comparison to what she thought appropriate for students. Discussions were brief, mostly questions not about the mathematics. Teachers' interactions with each other or with Sandy were usually cut-short by Sandy in the interest of getting to everything.

I turn now to focus more closely on participants' experiences with the two activities. Sandy's introduction to the ice-cream cone problem immediately grabbed teachers' attention. She entered the room with a huge bucket of ice-cream asking participants whether they liked ice-cream. She began scooping out ice-cream cones for participants making sure each one had two scoops on their cone. As Sandy dished out ice-cream, she directed participants attention to the hand-out in front of them.

Group leaders had passed out paper cut-outs of ice cream cones and cut-outs of three flavors of ice cream: chocolate, vanilla, and strawberry. Posed on the overhead projector screen was the following question, "How many different two-scoop cones can you make using three different flavors of ice-cream: chocolate, strawberry, and vanilla?" Participants began immediately constructing the cones with their cut-outs, and immediately questions arose.

Participant: What about a vanilla on top and bottom cone?

Sandy: I don't know, do you think that is a possibility?

Participant: We don't have the cut-outs for that.

Sandy: But we could trade and provide that.

Participant: We could define that the cones have to have different flavors

Participant: I think we should count them because you can buy a double vanilla cone.

Participant: And, what about vanilla on top and chocolate on the bottom, is that the same as chocolate on top and vanilla on the bottom.

In an interview, Sandy commented that she was pleased with all the questions teachers asked. She admitted that she really liked this activity because, "the decisions students would have to make and how the rules for the problem can naturally evolve out of the group rather than the teacher making up all the rules." She explained, "I like the fact that in this activity the students can decide what goes and what doesn't" (Interview, 8/93). Sandy let the questioning continue before she made a suggestion.

Sandy: Well, let's decide a few things. Raise your hands if you think chocolate on top and vanilla on the bottom should be counted the same as vanilla on top and chocolate on the bottom.
[teachers raise their hands]
What about different. [other teachers raise their hands]
After counting Sandy announced:
Okay, I guess we are saying they are different so that means we

> have to count each one. And, I guess we will say that the cones have to have different flavors.

Sandy based her determination of whether reverse-ordered cones were different cones on the basis of teachers' hands. She then instructed participants to return to their work. Even though Sandy proposed the process of voting for deciding the parameters of the problem, she explained in an interview that she viewed this interaction to be a nice opportunity for students, "to actually design the circumstances of the mathematics in the problem" (Interview, 8/93). In particular, Sandy thought these circumstances helped students to focus on the issue of order in counting. In an interview, she explained,

> In one way, as it is decided that two cones with opposite order are each to be counted, the role of order becomes a significant point. In fact, the question of order determines what is to be counted and what is not. In this sense the role of order becomes significant for students defining what the counting problem involves. Had this decision been made differently, the results would be different. For instance, if opposite-ordered cones were considered to be the same cone, the number of possible arrangements would be three. As the problem stands there are six possible arrangements. So order figures as a significant factor in the problem (Interview, 8/93).

After a few minutes, Sandy asked if someone would share their results. One participant came up to the board announcing that he had six cones.

Sandy: So, what's your pattern?

Participant: Well, I put vanilla on the bottom and then paired it with each of the other two flavors. Then, I put chocolate on the bottom and paired it the same way. I did the same thing for strawberry.

Sandy: Good, in other words your pattern helped you keep track of the possibilities. Did anyone do it differently?

Another participant comes to the front and draws a different picture.

Sandy: So what is your pattern?

Participant: It's pretty much the same but I used a different drawing.

Sandy: Good. Once again it is the pattern that allowed you to keep track.

Sandy emphasized the idea of order and the role of looking for a pattern to keep track of arrangements. She worried little about participants getting a correct answer. Sandy believed that what took place contrasted sharply with traditional views of teaching and learning. Instead of the teacher telling participants how to proceed, Sandy perceived the teacher as standing back, interjecting only at critical points, and participants taking charge of their own learning. She commented,

> We often give tasks where we prescribe the rules for them. For example, if your goal is to have them count by twos, you tell them twos are multiples of two or count every other number or

whatever rule you want to prescribe. When we pose the question of here's an ice-cream cone with two scoops and three different flavors to use, how many different ones can you make, there are no rules. Instead, the rules evolve based on a consensus of the group. And that is empowering students to a different level of understanding (Interview, 8/93).

After participants presented results, Sandy quickly explained the second activity. It was clear she was rushed for time. She quickly passed out materials and explained that this activity is similar to the previous one. The materials consisted of a handout with the posed problem, bear shape cut-outs, cut-out sweaters and hats. Sandy noticed the time and said there would not be time to actually dress the bears but that students in participants' classrooms should be given time to do so. She then commented that order is not an issue in this problem because there is no question of where the hat or the sweater should go. Sandy then displayed very quickly a process of six different ways to dress the bear using three sweaters and two hats. Sandy explained that the first sweater would be matched with each of the hats, then the second sweater with each of the two hats and finally the third sweater with each hat. Sandy reminded participants they could increase the difficulty of the problem by adding more hats, sweaters or including trousers. She quickly wrapped up the segment making the following points.

Sandy: With permutations the order is important.
With combinations it is not.
So, would this be permutation or combination?

	[Sandy points to the six arrangements of outfits for the bear]
Participant:	combination [in chorus - many do not answer]
Sandy:	yes, the order was not important. With the ice-cream cone problem was it permutation or combination?
Participant:	permutation [once again in chorus only this time a few participants responded combination but more loudly permutation was heard]
Sandy:	Yes, the order was critical. But we also could do combinations and say that order doesn't matter. Kids love this stuff. I think it is really hard for us but I find it is much easier for them.

After these comments, Sandy moved on to the next activity relating to a different content strand. Participants' opportunity to learn about discrete mathematics and how to introduce it into their teaching consisted of the interactions described here, at least for this workshop. Participants were offered two examples of fairly non-traditional counting problems, some ideas for how to work on those problems, some ideas about the importance of patterns and some sense of whether the solution would be considered a combination or a permutation of the set.

Sandy, in an interview, said she felt satisfied with what she offered participants for a first experience in thinking about discrete mathematics. She claimed her ideas offered participants a starting point for thinking about combinations and permutations. And she indicated that she hoped participants would find other opportunities to continue developing ideas about counting and more broadly discrete mathematics (Interview, 8/93).

Teaching Combinatorial Counting For Understanding

The case study of Sandy's efforts to introduce discrete mathematics into elementary school teaching illuminates what it is like for a teacher to learn an unfamiliar topic in mathematics. It reflects mostly a difficult and bleak set of circumstances. The opportunities Sandy encountered for learning about discrete mathematics were difficult at best. Yet despite the difficulties, Sandy assembled some ideas about counting that would serve as a basis for introducing discrete mathematics into the elementary mathematics curriculum. And she offered what she learned to other teachers as well.

Was the experience Sandy offered at the state-sponsored workshop consistent with what the state had in mind for introducing discrete mathematics? Did Sandy provide an example of what it might be like to teach discrete mathematics for understanding? In the remainder of this chapter, I address these questions from the perspective of the theoretic framework underlying this dissertation. In other words, I consider whether Sandy's learning experiences offered what she would need to teach discrete mathematics for understanding. I begin by considering the mathematical ideas Sandy grappled with. I compare and contrast the central ideas of her learning experiences with my analysis of what may be central for knowing and doing combinatorial mathematics. My purpose is to check out whether Sandy's learning experiences centered on or overlapped with the central tenets of combinatorial mathematics. My aim is to see whether Sandy acquired the subject-matter knowledge she would need to teach combinatorial counting for understanding.

The Mathematics This Teacher Focused On

Recall that Sandy's mathematics learning in the context of formal coursework was quite limited. She described her high school experiences as mostly procedural in nature. In college, she had only one course focusing on mathematics, an elementary math methods course. More recently Sandy learned mathematics in the context of her training with Project AIMS.

The experiences Sandy had previously were ineffective in terms of preparing her to learn about continuous and discontinuous mathematics. She also had great difficulty learning about recurrence relations. Recall that the district math specialist considered relational theories to be essential for understanding discrete mathematics. Sandy could make little sense of the ideas she encountered as central to discrete mathematics. It would require that she go far beyond the more procedural ideas she had previously encountered. Sandy would have to draw on a wider range of knowledge about mathematics that she currently did not have. She would need to understand finite and infinite sets, continuity and infinity, as well as the idea of functions.

Given that Sandy knew of no formal courses of study aimed at learning about discrete mathematics, she arranged for a variety of different experiences she thought would be useful. She proceeded to learn about a curious mix of mathematical ideas mostly in the contexts of conversations with other educators. She encountered continuous and discontinuous mathematics, the integers, combinations and permutations, order and patterns, and combinatorial counting ideas. Most of the experiences she encountered were frustrating for her.

Sandy's learning experiences ricocheted in multiple directions, stopped and started, shifting in focus over the course of time. Eventually she isolated herself from other educators and began focusing on a small set of curriculum materials designed to offer students experiences with non-traditional counting problems. She cut off the opportunity to learn from others and relied solely on what she could gather for herself. She eventually progressed into mathematical ideas she had no previous experience with. She learned what she could teach herself about the underlying ideas of order and patterns for systematically keeping track of outcomes (Interview, 8/93). What she learned contrasted sharply with her earlier experiences of memorizing formulas and getting right answers (Interview, 1/92).

Sandy relied on her own insights for making connections between her learning and what the documents proposed. She came to define discrete mathematics as, "learning to organize and count outcomes, it is how one reasons through counting problems," (Interview, 8/93). Her definition, although narrow from the perspective of the larger field of discrete mathematics, at the same time captures a very central piece of what discrete mathematics is. This definition contrasts sharply with her initial hunches, "I think it is math in the real world" and "maybe it is connected to the elements chart in science" (Interview, 1/92).

Given the progress Sandy made, a question remains. Did Sandy learn what she would need to teach combinatorial counting for understanding? To address this question, one must first consider another. What would it look like to teach combinatorial mathematics for understanding in the elementary grades? Unfortunately we have few examples at the elementary grades to consider. Consequently, there is much uncertainty around the question.

What is possible is to speculate about what it would mean to understand combinatorial counting questions, in particular the questions Sandy explored with other teachers at the workshop. Jerome Bruner argues that all students can be taught all subject matters in "intellectually honest" ways (Bruner, 1977, 1990). Bruner's argument rests on the idea that the integrity of both the subject matter and the learner must be respected. My purpose in this discussion would be to uncover what it might mean to respect the integrity of the mathematics of combinatorial counting to see if Sandy's learning and teaching centered on those ideas. My analyses would be limited to the mathematics embodied in the two counting problems focusing Sandy's work.

In formulating the analysis, I draw from my work in chapter 3, NCTM's Yearbook, Discrete Mathematics Across the Curriculum, K-12, (1991a), several discrete mathematics textbooks, an interview with a mathematician, and my own learning about discrete mathematics.[37]

Combinatorial Mathematics

Combinatorics, a sub-category of discrete mathematics, is the study of a particular kind of counting. It involves looking for relationships and patterns as well as uncovering techniques in counting various arrangements, configurations, or combinations of mostly discrete objects. Combinatorics is valued as a science in itself and for the contributions it offers real-world contexts and other sciences.

Sandy's teaching focused on two combinatorial counting problems. To understand the mathematics that underlies her teaching, I begin by exploring the mathematics of combinatorial counting questions. Such questions ask how many different combinations or arrangements of objects can be generated from an original set. More simply, how many different ways are there to combine the elements of a set. An example might be how many different phone numbers can be created using 9 digits and a specified organization of those digits. Another example might be how to arrange children in a classroom so that each child can

[37]My earlier analysis of proposals to introduce discrete mathematics is situated inside chapter three, part two of this work. My own learning about combinatorial counting has been informed by my own teaching, discrete mathematics textbooks, and reading NCTM's 1991 yearbook on discrete mathematics. In addition, I learned a great deal from Bruce Sagan, a mathematician at Michigan State University, who took the time to explain his views and perceptions of what discrete mathematics involves.

be paired with a different partner each week, insuring that everyone has a chance to work with everyone else. The counting process in each of these examples is rather complex and quite tedious in nature, requiring much insight and strategy. It requires a kind of reasoning that allows generalizations to be made so that results can be determined without wading through the tedious counting process. This kind of reasoning, combinatorial reasoning, would be at the heart of combinatorial counting. It is embodied in the formulas and algorithms mathematicians derived to formulate the counting process, the same formulas highlighted in mathematics textbooks.

The idea that efficient and accurate results can be accomplished by a means simpler than actually counting every possibility is an appealing goal. Mathematicians make their living working out such puzzles. Mathematicians doing research in the field of combinatorial analysis are interested in conjecturing, formulating, and justifying theories about counting. Theories about counting are mostly valued on the basis that they can provide good information on complicated counts while avoiding the tedious nature of the counting process. From this standpoint, combinatorics involves formulating good hypotheses for making counts, justification for what the techniques accomplish, and proving that the results they offer are accurate. Combinatorics as a science has become increasingly useful for managing current problems and questions within our culture. It has become one of the fastest growing areas of mathematical study.

Reasoning in ways that make tedious and complex counting problems more manageable would be an important characteristic of what it means to understand combinatorial analysis. Recall that Sandy's high school experiences involved memorizing formulas and solving all the problems and applications in the text. Yet, she never really understood the formulas in terms of the mathematics or why they worked. Like most teachers Sandy would not bring this knowledge to her opportunities to learn about discrete mathematics. Instead, she would have to be offered opportunities to understand what underlies the formulas she memorized. Her knowledge at that time would naturally be bounded by procedures, the typical preparation of most teachers.

Sandy's learning experiences pressed her to consider the ideas of combinations and permutations in ways devoid of formulas. Her teaching focused on the idea of order and the importance of identifying patterns for making accurate counts. In contrast with her high school experiences, Sandy made much progress. She experienced a significant departure from earlier ideas she had about non-traditional counting. Yet, did she develop ideas about what it would mean to reason combinatorially? I examine this construct much more closely in the paragraphs that follow.

Below is a speculative list of elements that are characteristic of combinatorial reasoning. My findings are based on my reading of the NCTM yearbook on discrete mathematics, my reading of several discrete mathematics textbooks, and my interviews with Bruce Sagan a mathematician specializing in discrete mathematics. The list is not intended to be exhaustive but instead offers a view of what may be central to the combinatorial counting process.

Essential Elements of Combinatorial Reasoning

* knowing when counting arrangements is the question, the nature of particular kinds of counting problems
* knowing what to count, different problems involve counting different things and in different ways
* understanding the tedious nature of counting
* formulating systematic ways to keep track of the counting process
* identifying patterns in counting

* Knowing when you have counted all possibilities
* considering whether generalizations are appropriate
* formulating generalizations
* considering the boundaries to those generalizations

Each element involves reasoning about a particular aspect of the counting process. I have italicized one particular element for the purpose of drawing attention to it. Reasoning about when you have all possible outcomes is a critical aspect toward generalizing results. And, as I argued above, generalizing counting ideas is central to combinatorial mathematics. It moves mathematical knowing from concrete examples to abstract theories. It formalizes knowing as a process of reasoning.

Although there is much left untouched and unexamined in what I have offered, the elements of combinatorial reasoning can be useful for comparing and contrasting the central ideas Sandy encountered in learning about combinatorial counting as well as her teaching of combinatorial mathematics to other teachers.

An Appraisal of A Teacher Leader's Teaching the Mathematics Of Counting

Sandy's teaching touched on at least three elements on the list. She offered participants an experience with the tedious nature of the counting process in the context of two non-traditional counting problems. She centered learning on the idea of order although I will argue later that Sandy's sense of when and how order mattered is problematic. And, she focused on identifying and using patterns for the purpose of counting systematically.

Unexamined elements in Sandy's teaching included when counting arrangements is the question, whether generalizations seemed appropriate, the business of generalizing, or considering the boundaries of generalizations. Even more problematic was that Sandy's teaching left the issue of reasoning about when all possibilities were accounted for unattended. This aspect, because it was left unattended, did not set the learner up to generalize information about the counting process. In effect, teachers lacked the opportunity to conjecture about the process, generalize their understanding, or uncover any limits to claims.

An important question is whether Sandy thought of her efforts as a step in a larger learning process, one that would lead to an overall view of the mathematics of combinatorial counting, perhaps across a series of connected episodes of teaching? A simple answer would be yes, of course. Yet, Sandy's workshop was a one-shot opportunity.

Would the experience Sandy offered set the learner up to connect to the elements that were lacking? I address this question by first revisiting Sandy's teaching at the workshop. Each of the counting problems Sandy offered involved a fairly non-traditional kind of counting question, especially when compared to traditional school mathematics curricula. Sandy's practice emphasized hands-on, learning by doing activities. Manipulatives, small group discussion, and emphasis on the ideas of order and systematic counting were central features of the learning experience. For many participants, Sandy's teaching might represent an entirely new approach to teaching mathematics especially when compared to their own teaching.

Yet, below the surface of Sandy's teaching were critical problems with the mathematics she offered. NCTM's yearbook on discrete mathematics concurs with Sandy's idea that counting should be emphasized in the early grades K-3 (NCTM, 1991a, p. 18). The yearbook (p. 19), departs from Sandy's ideas in that emphasis should be placed on understanding the nature of different kinds of counting questions. Sandy's teaching gave little attention to the nature of the differences underlying the two counting questions she offered. Mathematically, each question involved entirely different counting structures, resulting in very different kinds of generalizations.

Consider the ice-cream cone problem. Sandy asks participants at the workshop to find the number of different two scoop cones (two element sets) that can be created from three flavors of ice-cream (a three element set). The question involves taking the elements from the set {v, c, s}, corresponding to vanilla, chocolate and strawberry, and arranging the elements in as many ways as possible using two element. A single set of three elements serves as the basis from which all possible two-element sets can be created. The possibilities include {v, c}, {v, s}, {c, s} if no repetition of any flavor is permitted. Sets such as {v, v} repeat the same element and {c, v} reverses the order of a set already created. These sets may or may not be included in the count depending on the parameters of the problem. If they are included, the number of possible solutions changes. Mathematically, generalizations result from taking three elements, two at a time.

The second problem is a different kind of counting question. There are no decision points about repeating the same element or reverse-order. A bear is to wear only one sweater and one pair of pants, worn in only one way. In this problem, order in a set is not relevant. There are two distinct sets that serve as the basis from which all arrangements can be formed. One set contains three sweaters, each of a different color. The second set contains two pairs of pants also of different colors. For example, {white, red, green} and {black, brown}. The sets are disjoint. Arrangements involve taking one element from each set to form a two-element set. There are six and only six possible arrangements pairing one sweater with one pair of pants. It would not make sense to consider a {white, black} arrangement as different from a {black, white} arrangement. The outcome represents the same arrangement.

Sandy's teaching did not address the differences in the underlying structures of the two problems. She treated the questions generally, almost as if they were the same type of counting problem. The issue becomes more obvious and relevant when the goal is to generalize the reasoning process. The second problem involves counting arrangements that can be formed by joining one element from each of two disjoint sets in a particular way. Mathematicians generalize these findings in what is known as the product rule. The product rule states that the total number of arrangements can be found by multiplying the number of elements from each set, in this case 2 x 3 or 6 possibilities.

The nature of these differences is significant when generalizing becomes a central goal. Differences can be revealed by examining underlying structures and the influence of those structures on the counting process. Participants at Sandy's workshop were not offered an opportunity to consider the nature of these differences. Nor were they offered any opportunity to form generalizations of each case.

Sandy's practice has the potential to set the learner up for misconceptions about what combinatorial mathematics is. By de-emphasizing the nature of the differences in the problems offered Sandy ignored a critical element required for generalizing the reasoning process. Teachers did not encounter the opportunity to reason about when all possibilities are accounted for. By ignoring this element of the counting process, the learner is not set up to generalize results, even if offered in subsequent learning contexts. Recall from my sketch in chapter three that the central purpose of combinatorial mathematics is to find ways to make difficult counts without the tedious step-by-step nature of counting.

My analysis is not intended to suggest that what Sandy did do is incorrect or unimportant. I also am not suggesting that teaching should move quickly and rapidly to formalizing findings. What I am suggesting is that teachers would need the opportunity to explore the underlying structures of different kinds of counting questions and work to explain how they know when all arrangements are accounted for if they are to understand what underlies combinatorial mathematics. Without this experience, generalizations will more than likely remain as formulas to be memorized and not understood. Sandy's teaching does not set the learner toward this direction. Her teaching about the ideas of combination and permutation became unknowingly corrupt in practice. If Sandy understood the central tenets of combinatorial reasoning, she would be better prepared to examine those ideas with other teachers.

My analysis suggests that Sandy's teaching overlaps only slightly with what it might mean to teach combinatorial counting ideas for understanding. And, Sandy's learning experiences did not provide the substantive knowledge she would have needed to build her capacity to teach combinatorial mathematics in ways that emphasize understanding. The opportunities Sandy encountered offered bits and pieces of knowledge with little attention to how any of the ideas fit together toward an overall view of combinatorial mathematics. Sandy's story uncovers what may be a deep rooted contradiction. On the one hand, policymakers argue that teachers must have an explicit, deep knowledge of the subject matter if they are to teach for understanding. Yet, proposals in the reform documents suggest that discrete mathematics can be introduced into the school curriculum in a relatively easy fashion, by way of what is already there. Recall the rhetoric, "teachers at all levels are already teaching a variety of mathematical ideas considered important to a study of discrete mathematics," (NCTM, 1989).

CHAPTER SIX

RE-FORMING MATHEMATICS TEACHING AND THE TEACHER'S OPPORTUNITIES TO LEARN

Introduction

In the two previous chapters I focused separately on Sandy's responses to two different strands of reform ideas. I suggested that Sandy's learning experiences in each case did not effectively close the gap between her practice and the teaching envisioned in the reform documents.

My purpose in this chapter is to understand why. The state developed a multitude of levers and supports to press teachers toward desired practices. Sandy was motivated and committed to the state's goals. Yet, my appraisal of her teaching in both case studies revealed little common ground between the two. To understand why, I examine Sandy's professional development activities across the case-studies. I compare and contrast the opportunities to learn that Sandy encountered in light of policy goals. I explore what the occasions were like, what they offered Sandy substantively and structurally, and to what extent they offered ideas and practices envisioned in the reform documents. My aim is to develop a portrait of the circumstances and conditions that mediated Sandy's learning about the proposals she responded to.

A Pedagogical Framework

In chapter 1 I discussed the assumptions about policy-practice relations underlying this study. I suggested that the "policy as pedagogy" framework was not the common view. Historically policy and its implementation has been described as putting or placing recommended changes into practice by methods of transmission, implantation, or mutual adaptation (Fullan, 1992; McLaughlin, 1976). In contrast, I assumed that learning was the central activity of policy implementation (Cohen & Barnes, 1993a; Cohen & Ball, 1990). Thus, I viewed Sandy's efforts to reform her mathematics teaching from a cognitive frame. I assumed that implementation rested for the most part on what and how she learned. My analysis in this chapter rests on these assumptions as well.

In this chapter, my goal is to examine the relationship among three central elements of the framework: the subject matter of reform, the teacher as the learner of reform, and the teacher's opportunities to learn. I begin by sketching ideas about the curriculum of reform. I compare and contrast the cases to see what the proposals involve, including recommendations for how changes might be accomplished as well as the implications for teacher learning. I then focus on the learner of reform. I develop a sketch of Sandy's personal history with each strand of reform ideas. I look to see what the similarities and differences are across the mathematics topics and what the implications are for Sandy as a learner. Finally, I focus on Sandy's opportunities to learn. My analysis aims to uncover whether and how Sandy's opportunities to learn challenge and extend her prior knowledge and teaching experiences in ways that foster understanding and enactment of the curriculum of reform.

A Curriculum of Reform

There were three central kinds of recommendations made across the proposals for altering the content and pedagogical practices for teaching computational skills and discrete mathematics.

*Introducing new content

*Teaching familiar mathematics in new ways

*Introducing problem solving contexts and conceptual orientations to teach all content

I explore each of these recommendations below.

Introducing New Content

Introducing new content involves the proposal to teach new mathematics content, ideas not already part of the school mathematics curriculum. This includes ideas such as students' strategies for computing, non-traditional algorithms for computing, conceptual basis of algorithms, and combinatorial counting principles such as the inclusion-exclusion principle or the pigeon-hole principle as examples. Introducing new content would require teachers to learn new mathematics. For example, introducing the conceptual basis of traditional algorithms would require that teachers learn the conceptual underpinnings of addition, subtraction, multiplication and division. Teachers would also need to learn how to develop those ideas for students across a series of lessons that aim to support students' understanding and proficiency with computation.

An example of introducing new content to support the goal to teach discrete mathematics would involve combinatorial counting ideas. Most elementary teachers would not already understand combinatorial mathematics. Not only would the teacher need to learn the mathematics of counting, she would also need to learn how the ideas work together to support an overall view of combinatorial mathematics. Consequently, introducing new content into the existent mathematics curriculum implies that teachers would need to learn much new subject-matter including how mathematical ideas connect.

Further complicating matters is the treatment of content as discrete entities in the reform documents. The mathematics across the cases illuminates this problem. The mathematics of combinatorial counting and computation is actually more connected and alike than one might first imagine. Although each involves uncommon ground, they share ideas as well. The overlap can be imagined by comparing a traditional counting question and a combinatorial counting question. For example, traditional counting questions ask for a total when 4 things are added to 2 things or 4 things are multiplied by 2 things. Combinatorial counting questions extend counting into a new realm asking, for example, how many different arrangements are possible when there are 4 things and 2 things to choose from. Combinatorial questions typically ask for possible configurations, arrangements or combinations. Computation typically involves finding sums, differences, products, quotients, and additionally percents and fractions. Conceptually, the ideas of grouping and combining connect the two. Fundamentally, the mathematics of combinatorial counting is an extension of traditional counting ideas. Most elementary teachers would not bring these insights to reading the documents. At the same time, the documents make no mention of any connections across content even as they aim to alter the fragmented approach to teaching mathematics in schooling.

Teaching Familiar Mathematics in New Ways

This recommendation requires teachers to reconsider the familiar mathematics they already teach. Historically, traditional computing algorithms have defined the computational curriculum in public schooling. Policymakers argue that there are a variety of new computational skills that must be taught, estimation is one example. Estimation should be taught as a computational technique in itself, not as a checking strategy. Estimation is a familiar idea to most teachers. Usually, estimation is taught as a checking strategy. To transform this aspect of their teaching, teachers would have to transform their ideas about estimation and develop new ideas for teaching estimation to students.

In reconsidering familiar mathematical ideas, teachers are pressed to explore the underlying structures of familiar ideas they already teach. For example, understanding what underlies computational algorithms implies an investigation of the structures of routine procedures and pedagogy used to support learning of routine and taken for granted procedures. Teachers would have to explore such questions as what are the conceptual ideas underlying routine computation? Other issues would involve whether traditional algorithms should be taught and if so, to what level of complexity and mastery? Should drill and practice be used to foster fluency with those skills and is speed and accuracy still valued?

There are also examples of reconsidering familiar mathematics in discrete mathematics. The reform documents suggest that discrete mathematics can be introduced into the mathematics curriculum by way of familiar ideas already taught. Venn diagrams is listed as an example and are probably familiar to most teachers. Yet, they are typically used to illustrate relationships among sets. The ideas of union and intersection are often focal. Venn diagrams are also useful for illustrating basic counting principles in combinatorial mathematics. For example, the inclusion-exclusion principle asserts that the number of elements in two finite sets A and B can be found by combining the elements of set A with the elements of set B and subtracting what is in common. Another way to state this principle is to find the total number of elements in both sets one must include all elements of both sets and exclude those elements in common. To illustrate, consider the Venn diagram below:

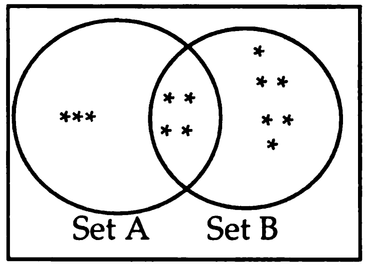

Figure 7.
Venn Diagram

Set A has 7 elements, set B has 10. A and B, when added, have a total of 17 elements. Yet, four of those elements are common to both sets. Thus, 17 - 4 yields 13 elements total in the two sets. The Venn diagram is useful for showing the relationship between A and B and the total number of elements. Although Venn diagrams still represents relationships between sets, the counting principle, for many, will be an entirely new mathematical idea. It would be impossible to introduce combinatorial counting ideas by way of Venn diagrams without knowledge of the underlying counting principles. Venn diagrams do not represent the mathematics of discrete mathematics. They are useful to display relationships and counting principles. But they work in coordination with other knowledge to introduce the ideas of discrete mathematics. Much learning of new subject matter would be required if discrete mathematics is to be introduced by way of what is familiar.

Teaching familiar mathematics in new ways requires more than reexamining familiar content. It also requires reconsidering the pedagogical practices for teaching familiar mathematics. For example, traditional forms of practice often result in learning that is rule-bound. Such practices contribute to other problematic issues such as students' view that mathematical knowing resides in the teacher or text and not in students' sense making. Many teachers teach unaware of these issues. Such issues point to the third aspect of the curriculum of reform: the recommendation to introduce new pedagogical practices.

Introducing New Pedagogical Practices

The proposed changes across the strands intertwine content and pedagogy so that what teachers would have to do and learn is not so neatly either content or pedagogy. To this point, my discussion has focused mostly on changes in content. Yet, changing the content of the elementary school mathematics program would not be sufficient to create the kind of teaching imagined. Interconnected to changes in content are changes in pedagogy. In other words, teaching students non-routine algorithms, estimation as a computing strategy, or the use of new technologies for computational work in traditional, didactic, lecture-type formats would not accomplish the teaching imagined. New content must be introduced in relationship to proposed changes in pedagogy. Essentially, computational skills teaching would be done in problem-solving settings. Teachers would need to learn to facilitate computational goals simultaneously with problem solving goals. To illustrate further, I draw from the Teaching Standards 1991, p. 26.

> Teachers must assess the extent to which skills play a role in the context of particular mathematical topics. A goal is to create contexts that foster skills developments even as students engage in problem solving and reasoning. For example, elementary school students should develop rapid facility with addition and multiplication combinations. Rolling pairs of dice as part of an investigation of probability can simultaneously provide students

> with practice with addition. Trying to figure out how many ways 36 desks can be arranged in equal-sized groups and whether there are more or few possible groupings with 36, 37, 38, 39, or 40 desks - presses students to produce each numbers' factors quickly. As they work on this problem, students have concurrent opportunities to practice multiplication facts and to develop a sense of what factors are.

The vision of students working to determine possible desk arrangements requires students to work on multiplication facts. This practice would be dramatically different from modal teaching where students memorize multiplication facts using worksheets or other drill and practice activities. Underlying the change is the argument that what students learn is fundamentally connected with how they learn it.

What a teacher might need to learn to make sense of this paragraph is not straightforward. In addition to learning constructivist ideas about learning, teachers would also need to learn new discourse patterns requiring substantial shifts in teachers' and students' roles. Teaching as telling would no longer dominate classroom discourse. Instead, students would work on mathematical problems in small and large groups, talking to each other and to the teacher. Teachers would have to learn when to tell students something as opposed to facilitating students' construction of an idea. These sorts of changes require substantial new insights into mathematics and pedagogy.

Interestingly, the text above intertwines the mathematics content of the two strands focusing this study. Trying to figure out how many ways 36 desks can be arranged in particular sized groups is a combinatorial counting question. Yet, unless and until the teacher has ideas about what combinatorial counting involves, this paragraph most likely would get overlooked as an example of introducing discrete mathematics. In most cases, teachers would miss this occasion as an opportunity to learn about discrete mathematics.

The curriculum of reform is immense. The analyses above only begins to unpack what teachers would need to learn to enact the proposals. The examples sketch only some of the problems that teachers would encounter if they were to give serious attention to the recommendations. Yet, a very important insight arises from this work. What there is for teachers to learn, the vastness of what is there, contributes greatly to the disjuncture between envisioned ideas and practices and the teacher's capacity to reform.

I turn now to focus on Sandy as a learner. I look across the cases to compare and contrast what she brings as a learner to reforming her teaching.

The Teacher as Learner

Research on teacher learning suggests that Sandy's interpretations of the proposed changes are naturally influenced by what she brings as a learner. To understand Sandy's interpretations of the proposed changes, I explore her personal history surrounding each strand of reform ideas and how her history may have functioned in making sense of the proposed changes.

What Does The Teacher Bring?

The columns in the table list the ideas that Sandy brought to her learning experiences for each strand. The lists are organized for the purpose of comparing the ideas across the cases. For example, content knowledge in one strand is organized across content knowledge in the other strand.

Table 1. What Sandy Brings

Computational Skill	Discrete Mathematics
Sandy brings ideas about what computational skill is - mostly proficiency with traditional algorithms, mental computation and using those skills in problem solving contexts	no ideas about what discrete mathematics is no ideas about what the central questions of discrete mathematics involve or why the questions are important to ask
Sandy brings ideas about techniques for computing , i.e. paper pencil techniques, borrowing, carrying, etc. She brings ideas about getting the right answers, speed accuracy, and efficiency. Sandy brings ideas about why computational skill is included in the elementary school curriculum; because it is a useful skill in daily life	no ideas about the techniques or strategies used for examining those questions no ideas about what discrete mathematics explores
Sandy brings a very detailed picture of what the computational curriculum involves and how it might be situated within the school curriculum	no ideas for how topics, ideas might be included in the school curriculum

Table 1 (continued)

Ideas about the conceptual underpinnings of the algorithms/and other conceptual ideas related to the four operations. Sandy also brings ideas about the important elements of computational skill such as getting the right answers, knowing facts and algorithms, speed and accuracy.	some familiarity with only a few of the central ideas identified in the Framework as important to discrete mathematics. For example Sandy has some familiarity with the ideas of Venn diagrams, deductive logic, and combination and permutation. Sandy's familiarity is not inside the context of discrete mathematics.
Sandy brings ideas about the underlying systems of numbers. She understands what is involved when operating within different systems of numbers. She has ideas about place value, different bases, operations with decimals, fractions, etc.	no ideas about the system of numbers that underlie discrete mathematics. Although Sandy has ideas about the integers as a number system and arithmetic as a study on that system she does not know whether or how the integers fit with discrete mathematics. She has some familiarity with continuous mathematics at least algebra
Sandy brings an entire curriculum including a set of firmly entrenched instructional practices she believes develops computational skill	some ideas for what instructional practices might best represent discrete mathematics including problem solving and investigation type teaching methods

The contrast between the strands is striking. Like many teachers, Sandy already had many ideas and understandings related to the content and instructional practices of basic computational skills. Sandy brought little, if any, specific ideas about discrete mathematics. What she did bring, she had little ideas for how it might help her to learn about discrete mathematics.

Sandy brought firmly grounded ideas about what computation involves and how children become proficient at it. For example, Sandy brought to her learning about the proposed policy changes with the view that computational skill involves proficiency with traditional paper-pencil algorithms and some limited mental calculation as well. Sandy also brought to her learning the idea that memorization using drill and repeated practice exercises are best for learning those skills. Recall in the case study how Sandy encouraged students' use of traditional paper-pencil algorithms even in settings where computations could have been made more easily using alternative strategies. She also stressed the importance of speed and accuracy with traditional computational algorithms using students' participation in a pizza party as a motivational factor.

Sandy also brought a very detailed picture of what the third grade computational curriculum should include. For example, she had specific ideas about which algorithms should be taught in the third grade and to what level of complexity and mastery. Recall that Sandy thought mastery of the division algorithm did not belong in the third grade arguing that conceptual grounding should dominate third grade instruction of division (Interview, 1/93). Yet, she believed that mastery of the traditional multiplication algorithm was an essential goal in third grade, not only to four digits but also with complicated carrying procedures involving numbers with many zeros.

In addition, Sandy brought very specific ideas about the instructional practices necessary for developing computational skills. They included a number of gimmicks to help students remember difficult procedures and deal with common errors. Recall the phrase "slash, burn it off," for remembering to cross out a zero and replace it with a nine. Sandy often relied on catchy phrases to help students memorize computational techniques as well as help students avoid common errors in the procedures.

Sandy had specific ideas about what it would mean to understand computational algorithms conceptually. Consider her ideas about multiplication as "adding fast" and division as "fair-sharing." Sandy's ongoing work with Project AIMS contributed greatly to how she thought about conceptual understanding and conceptually orienting the school mathematics curriculum. Project AIMS assumes a hands-on, learning by doing philosophy. Sandy's ideas about problem solving and contextualizing mathematics evolved in relation to her work with Project AIMS. Yet, my analysis in the case study suggests Sandy's ideas and practices overlapped only slightly with policy ideas relating to conceptual understanding and contextualizing mathematics.

In contrast, Sandy had very few ideas to work from for learning about discrete mathematics. In fact, she claimed she knew so little that she had trouble making any sense of what was written in the reform documents. She concluded the documents confused her and were not very helpful for developing a view of discrete mathematics or how to introduce it into the school curriculum. Even though the documents suggested that much of what discrete mathematics involves is already largely part of the school curriculum, Sandy had no ideas for what this would be. She admitted she was unable to make sense of the proposals and had no ideas for how the mathematics she was already teaching connected to discrete mathematics.

Prior Knowledge as Prerequisite Knowledge

Sandy's prior knowledge and teaching experiences function as prerequisite knowledge for making sense of new forms of practice. As a resource, prerequisite knowledge significantly affects a teacher's capacity to reform. Sandy brought very different ideas and experiences across the topics of computation and discrete mathematics to draw from. Her ideas about computation ran deep and were mostly contrary to the proposals. As prerequisite knowledge, much would need to be challenged, uprooted and discarded if Sandy were to embrace ideas and practices compatible with policy. Sandy's prior knowledge for learning about discrete mathematics was highly underdeveloped, predictably so. She had great difficulty making sense of the examples and illustrations offered in the reform documents.

In contrast, the cases represent opposite situations from the standpoint of what the learner brings. On the one hand, Sandy's prior knowledge and experience surrounding computational goals was enormous and very specific. Yet, most of her ideas and practices ran contrary to the proposed changes. In the case of discrete mathematics, Sandy's knowledge was so underdeveloped that even reading the reform documents was puzzling and strange for her.

These findings have very different implications for Sandy as a learner. Sandy's prior knowledge and beliefs must function as a basis on which to build her understanding of the proposed changes. Yet, as prerequisite knowledge, what Sandy brought was often mismatched with what she would need to make sense of the ideas offered across the opportunities to learn that she encountered. Recall the example of Sandy trying to make sense of the illustration in the Curriculum Standards (1989), involving a directed graph for counting the number of different paths between two points. The example was offered as an illustration for introducing discrete mathematics. Prerequisite knowledge for making sense of this illustration would include fairly well-worked out ideas about matrices. Yet, like most elementary teachers, Sandy did not bring ideas about matrices to her opportunity to learn from this illustration. Her learning, at least in this context, remained limited to what she could muster up for understanding matrices.

In some instances Sandy's prior knowledge may have functioned to promote a mismatch in her understanding of the policy proposals. For instance, the policy suggests that the computational curriculum be conceptually oriented. This proposal implies that teachers would either need to bring a conceptual understanding of what underlies computation or they would need to develop those ideas. In Sandy's case, she brought well-worked out ideas about the conceptual underpinnings of the four operations. However, I described in the case study that her ideas and understanding did not coincide with what the policies described as conceptual understanding. In this example, Sandy's prerequisite knowledge functioned to promote a different view of conceptually orienting the computational curriculum than what the policy described.

Prior Knowledge and Framing a Course of Action

Across each strand, there is a striking difference in how Sandy framed what she would need to do to respond to the proposals. Although I argued that the recommendations across the strands implied much new learning for the teacher, Sandy framed her work with discrete mathematics to involve learning new content and her work with computation to involve a shift in instructional time. In each case, Sandy's framework for attending to the proposals focused on only bits and pieces of the range of different recommendations. This separateness, uniqueness, and attention to only aspects of the proposed changes created a kind of multiplicity effect on her learning. Because Sandy had few ideas about how discrete mathematics and computational skills connected mathematically, she perceived each strand to involve a unique course of action, with little, if any, overlap.

Recall that Sandy had already created a conceptually-oriented and problem-solving approach to teaching mathematics. To that end, she had minimized traditional practices aimed at the rote-learning of computational algorithms. Sandy's reading of the new framework interacted with what she had already established. She interpreted the proposals to suggest that she place more emphasis on conceptual and problem solving goals and that she should nearly extinguish all traditional practices surrounding computation. For Sandy, the recommendations implied a shift in emphasis and time, not a reconstruction of her ideas and practices.

Once problems arose in her practice, Sandy changed her ideas about what she would need to do. As she began to search for opportunities to reconsider her practice, she became increasingly convinced that her initial interpretations of the policy were wrong. After much learning, Sandy interpreted the proposals to suggest she should return to practices she once had abandoned. However, what Sandy learned did not challenge her prior knowledge and beliefs about computation. She put aside her more traditional ideas about computation as she focused on the conceptual underpinnings of the four operations. She assumed that emphasizing conceptual understanding and problem solving would also promote students' fluency with computation. Sandy's more traditional ideas were never really challenged, only shelved temporarily. There were other aspects of the proposals that went unexamined as well. For example, Sandy hardly responded to the recommendation that a variety of new computational skills be introduced. Estimation, for example, went mostly unexamined as a computational strategy. Sandy also did not emphasize technology mostly because students were not permitted to use calculators on state exams nor did they have consistent access to calculators. The changes Sandy made only touched the edges of the proposals and in part, because of what she brought to her learning about the proposals.

Sandy framed her initial work with the proposals to introduce discrete mathematics to involve a very different course of action. Here, she framed the proposed changes as a matter involving learning new mathematics for herself. For over a year, Sandy would learn what she could about discrete mathematics. Because Sandy believed she had already developed a pedagogy that emphasized a problem solving and conceptual orientation to learning mathematics, she did not perceive the recommended changes surrounding discrete mathematics to include, at least for her, any pedagogical changes.

A Teacher's Opportunities To Learn

(Links Between Policy And Teaching Practice)

My analysis of the curriculum and the learner of reform suggests there is a natural disjuncture between the ideas proposed in policy and the teacher's capacity to understand the recommended changes. I have suggested at least three factors contributed to this disjuncture. The curriculum of reform is vast, a wide territory of new ideas and practices, most strikingly different than modal teaching. This sets the stage for an enormous agenda for teacher learning, most of which is not well-defined. What Sandy brings as a learner contributes further to this gap. I argued there exists a mismatch between Sandy's prior knowledge and beliefs and the prerequisite knowledge needed to understand proposed ideas. This factor alone makes it highly unlikely for any teacher to understand the proposed ideas without particular kinds of learning. Sandy's prior knowledge and teaching experiences also interacted with how she framed what she would need to do to enact the proposed changes in her teaching. Because

Sandy perceived no common ground across the proposals she consequently framed each strand to involve distinct agendas for change.

From a pedagogical perspective, what Sandy learns should function to close this gap. In other words, the opportunities to learn that Sandy encountered should function to promote and sustain the kinds of support Sandy would need to create the kind of teaching envisioned in policy. In other words, what Sandy learns should bring her up to speed for making sense of the illustrations in the reform documents. Her learning should function to meet her at the point of her prior knowledge and beliefs. They should foster a way to sensibly frame the reform agenda to insure that she learns the central tenets in ways that are manageable and profitable. In essence, Sandy's opportunities to learn should function to extend and sustain a knowledge base sufficient to build and support her capacity to teach in ways imagined in the policy.

The previous two chapters illuminate that Sandy's opportunities to learn did not function to close the gap. Instead, the learning experiences she had may have created an even greater distance between the teaching imagined and Sandy's practice. My purpose in this section is to carefully examine Sandy's opportunities to learn, to better understand why they were mostly unproductive at supporting Sandy's efforts to invent and sustain the kind of teaching imagined.

I described in chapters four and five the variety of different situations that Sandy created or encountered to help her understand and enact the proposed changes focusing this study. I described, for example, how Sandy arranged for conversations with other educators, how she made use of curriculum materials and participated in state-sponsored inservice activities. The case studies teach us that the teacher is likely to make use of a wide variety of learning experiences, each contributing to the teacher's thinking and the changes made in teaching. In this chapter, I consider the occasions for Sandy's learning as a whole. Across the case studies I examine the qualities and conditions of the set of experiences Sandy had for learning what she could about the state's efforts to reform mathematics education. My purpose is to understand what these experiences offered for building and supporting Sandy's capacity to reform her teaching.

Whether the occasions were intentional, incidental, planned, unplanned, formal or less formal, each experience that I uncovered as altering Sandy's beliefs or knowledge about computation or discrete mathematics, I considered as an "opportunity" for her learning. To clarify, I offer the following explanation.

> We construe "learning opportunities" to be those experiences, kinds of work, and interactions that create images and insight, that generate disequilibrium and curiosity, that offer the possibility of change and growth. Their quality is shaped by many factors, the time allotted to them, the engagement they engender, the possibilities for collaborative work or thought they offer, and the worth that participants believe they have. Examples may include courses, workshops, conversations, reading, using a new curriculum, and teaching experiences. (Ball & Cohen, 1995a)

I identified six different kinds of occasions across the case studies that, for Sandy, fostered new images or insights, created curiosity or disequilibrium, and resulted in changes in her thinking or teaching. Some of these occasions involved hours across a number of years, while others lasted only minutes. Several began prior to our work together and continued during data collection. Whether Sandy created the occasions herself or she participated in an experience offered, the categories I list below are representative of the multiple kinds of learning experiences Sandy had. Each category represents one or more occasions that contributed significantly to Sandy's thinking and altered her ideas and practices regarding her teaching of basic computational skills or introducing discrete mathematics into her teaching. In effect, Sandy's ideas and practices changed in relationship to her interaction on these occasions.

Drawing on interview and observation data, the occasions most central to Sandy's learning across the strands were :

*Sandy's conversations with leading mathematics educators,
*reading reform documents,
*doctoral studies,
*curriculum materials,
*teaching experiences,
*formal professional development activities

I include Sandy's ongoing work with Project AIMS and the state-sponsored workshops she attended as specific examples of the formal professional development activities she participated in.

Prior to examining the categories I have listed here, I explain briefly several of the issues that arose in doing the analyses. Characterizing the substantive focus of Sandy's opportunities to learn was treated differently across the categories above. When there were differences in what an opportunity offered substantively from my view as compared with Sandy's view I had to decide what to do. For example, my analysis of what the policy suggested for reforming basic computational skills instruction was quite different than what Sandy made of it. Independently, I searched the reform documents for all that I could find relating to computational skills instruction. My analysis suggested there was much more offered substantively than what Sandy reported. When this occurred, I included both of reports as the basis on which to examine what Sandy's opportunities to learn offered substantively. Sandy's teaching experiences, reading the reform documents, the curriculum materials she used, and the professional development experiences she participated in represented similar kinds of data. In each of these categories, I collected data independent of Sandy's self-report and included that data in my analysis. In effect, my determinations of what the opportunities offered substantively were influenced by what I could learn about them independent of Sandy's report and yet were inclusive of Sandy's report.

There were a few occasions that I had little or no independent data on. For instance, I had no opportunity to observe or interview anyone participating in Sandy's doctoral studies. Sandy's self report and my analysis of the course syllabus and assignments was the basis of my determinations of the substantive focus of these opportunities. Sandy's conversations with other educators is an example where no independent data was possible. In these instances, it would have been difficult to collect data on what the opportunity offered because they were either unplanned or not appropriate to contact people to ask about the substantive nature of the conversations. I used solely Sandy's self-report to make determinations about the substantive focus in these instances.

I point to these issues prior to proceeding with my analysis to discuss how they are reflected in the analyses. It would be problematic to suggest that all of what was available to Sandy substantively or otherwise is reflected in the analyses. Because not all of teachers' opportunities to learn are planned or visible, it is not always possible to collect independent data on what is offered. Thus, in some cases, a teacher's self-report has to serve as the basis of the data. The data also does not reflect what else may have been available for teachers' learning but not taken advantage of. In this study, an available opportunity implies that Sandy created or encountered the opportunity to learn. I decided not to conduct any wider investigation of the professional development activities offered in other contexts that Sandy did not participate in because my aim was to understand what mediated a teacher's learning and change and how that happened. I was less interested in what could have happened had Sandy heard about or decided to participate in other learning experiences. In making this choice, the analyses does not reflect information or insights into opportunities to learn that Sandy elected not to participate in or had no knowledge of.

This last point leads to another. Making decisions about what counts as an opportunity to learn is not straightforward. It was not as simple as just noticing that Sandy encountered and participated in a learning opportunity. Sandy, for example, constructed ideas about computational skills and discrete mathematics in relationship to experiences that were not designed for that goal. Recall that she learned about automaticity in a doctoral course. Although the course was developed to promote learning about curriculum and instructional design, what Sandy learned from the course had a significant impact on her ideas about reforming her teaching of basic computational skills. This example begins to blur the boundaries of what typically is thought of as an opportunity for teachers to learn about reform ideas. Not all learning opportunities can be discerned as opportunities to learn ahead of time. It must be shown that what the teacher learned had a significant impact on the teacher's thinking and practice in relationship to specific reform ideas. Thus, reflected in the analyses is both what a teacher learns from more formal professional development activities as well as opportunities the teacher thought relevant and useful for her learning.

Despite the complexities I discuss here, the data is very robust and strong enough to bear up to the questions underlying this study. Sandy's self-reports cuts across all the categories I identified as affecting her learning and is a strong indicator of what each learning experience offered. My independent analysis of four of the categories provides further insight into what the occasions offered for learning about the ideas focusing this study. In only two of the categories I relied on Sandy's report and any artifacts of the experience that I could collect after the experience had occurred. On these occasions, my independent analyses was not possible due to the spontaneous nature of the experience.

To establish the nature of the six categories of learning opportunities that mediated Sandy's learning, I introduce three analytic categories:

-the big ideas
-consideration of what Sandy brings
-Sandy's views of what the opportunity offered

These categories correspond to three critical factors that research on teacher learning has suggested shape what and how teachers learn: the content or substance of the learning experience, the interaction of the learners' prior knowledge and beliefs, and the interaction of the learners' perception of what the experience offers. I begin by examining the substantive focus predominant across Sandy's opportunities to learn.

<u>The Big Ideas</u>

In the table below, I contrast the big ideas that were predominant in Sandy's opportunities to learn about reforming her teaching of computational skills with the central tenets offered in the reform documents. The central tenets of the proposals to reform teaching of computational skills is drawn from my analyses of the proposed changes in chapter three. My views of the predominate ideas embodied in Sandy's opportunities to learn are drawn from my analysis of the big ideas across her opportunities to learn (see Appendix A). My purpose is to see if and where there is overlap.

Table 2. Central Reform Ideas And The Central Ideas of Sandy's Opportunities to Learn About Computational Skill Instruction

Teaching Computational Skills **Central Ideas of the Reform**	**Central Ideas of Sandy's** **Learning Opportunities**
Computational Curriculum	**Computational Curriculum**
extensive mental computation	
estimation	Emphasis on conceptually
non-traditional computing strategies	understanding ideas
calculator and computer technology	students' algorithms
traditional algorithms	some mental computation
basic number facts	
Pedagogical Approaches:	**Pedagogical Approaches:**
contextualize use	conceptual orientation to
emphasize understanding	operations more important
link conceptual and procedural	balance conceptual aims with
thinking strategies for learning facts	more traditional practices
practice under certain conditions	Contextualize skills

There is a marked difference between the central ideas of reform and the central ideas of Sandy's opportunities to learn. Sandy's opportunities to learn emphasize the importance of understanding the mathematical ideas underlying the four operations. Estimation as a computing strategy, technology, and alternative algorithms were not focal points in Sandy's opportunities to learn. Further, there was no evidence that Sandy was offered any ideas about alternative algorithms such as adding back, doubling, or grouping tens. Even though these strategies were mentioned in the reform documents, overall Sandy had no opportunity to think through the ideas or the implications of the ideas for her teaching. In contrast, estimation, technology, alternative algorithms as well as a variety of other computing skills are central to the proposals for change. The only overlap occurred around ideas involving students' algorithms and mental computation. For example, Sandy's use of Marilyn Burns and other curriculum materials encouraged her to focus on students' construction of algorithms and mental computation. At the same time, little attention was given to any pedagogical issues surrounding teaching students' strategies, such as what to do when students' strategies produced incorrect results or how to encourage discourse among students around new strategies.

Although there was overlap around the idea of conceptual understanding, there also was a significant departure around the construct of conceptual understanding. For example, Sandy's opportunities emphasized the importance of understanding addition, subtraction, multiplication and division. In particular, the curriculum materials she used focused on conceptual orientations to learning each operation. Sandy's reading of the reform documents emphasized understanding as opposed to memorizing steps to algorithms. Recall that Sandy's teaching in relationship to division focused on the idea of equal distribution. She was not challenged to explore, for example, the underpinnings of the division algorithm. Nor was she challenged to think about linking conceptual orientations and procedural orientations of the four operations. She also did not encounter ideas about how conceptual understanding can be used in place of traditional algorithms. None of Sandy's learning opportunities attended explicitly to these ideas.

Sandy's conversations with other educators focused on the notion of balance. Sandy's understanding of balance ran completely contradictory to proposed ideas. She interpreted balance to imply the separate and isolated treatment of procedural and conceptual knowledge, a balance in terms of instructional time. None of the opportunities that I had independent data on contradicted Sandy's interpretation, even though the reform documents stated explicitly that the isolated treatment of computation was not compatible with the views expressed in the policy. Sandy did not encounter the idea of simultaneously linking the two in practice, a different interpretation of balance.

There also was overlap around the idea of contextualizing mathematics. Yet, once again, there was great variation in what Sandy encountered and what the proposals suggested. My analysis of the reform documents suggests that contextualizing computational skills is at the core of recommendations for reforming instruction of basic computational skills. Contextual use involved the idea of situating learning basic computational skills in relation to contexts where those skills arise. The purpose is to understand that contexts play an important role in how mathematics gets done. For instance, if the context requires an estimate, then estimation would be an appropriate computational technique. My analysis of Sandy's work with Project AIMS provides an example of the variation in meaning that Sandy encountered. Sandy drew primarily from her training with AIMS and her use of AIMS materials to develop her ideas about the role of context in learning mathematics. The primary goal of AIMS materials is to integrate science and mathematics through explorations of real-world phenomena. From this standpoint, mathematics learning is not the central goal.[38] Further, underlying AIMS materials is a view of mathematics that is very different from what is described in the reform documents. Mathematics is viewed more like a tool-box where ideas and skills are developed and then used in the service of these investigations. From this view, proficiency with computation is learned elsewhere and then brought to bear on investigations.

In practice the distinction is striking. In both views, contexts are defined as real-world or mathematically-oriented contexts. Students might begin by formulating conjectures for how to solve problems. When computational skills are necessary to solve a problem, AIMS materials assume students have developed those skills and bring them to the ongoing problem-solving opportunity. The relationship between context and computational skill is not primary. If students' skills are weak, work outside the context is assumed necessary. In reform-oriented teaching, the context impacts the computational work students will need to do. Here, students rely on the context to help them decide how to compute most efficiently and decide whether accuracy is an issue. Students have concurrent opportunities to develop problem solving and computational goals. The mismatch is striking. Yet, of the four opportunities I collected independent data on, none focused on any distinctions.

One idea that Sandy did not encounter in relationship to her learning opportunities was the role of practice for developing computational skills. Practice, although discussed in the reform documents, was not central or explained explicitly. Sandy's opportunities to learn reflected no attention to whether or what the role of practice in learning basic computational skills should

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[38]I base these claims on my observations and analysis of Project AIMS curriculum materials provided teachers at a Project AIMS workshop and my analysis of a written statement by Dr. Arthur Wiebe, founder of Project AIMS , explaining a model of mathematics underlying Project Aims materials.

be. There was no evidence that suggested Sandy even considered the notion of practice in any other light than what modal teaching portrayed.

At this point, I shift my focus and turn to explore the big ideas of Sandy's opportunities to learn about introducing discrete mathematics. As before, the table below contrasts the big ideas of Sandy's opportunities to learn about introducing discrete mathematics into the school mathematics program with the proposed ideas characterized in the policies.

Table 3. Central Reform Ideas And The Central Ideas Of Sandy's Opportunities to Learn About Introducing Discrete Mathematics

Introducing Discrete Mathematics Central Ideas of the Reform	Central Ideas of Sandy's Opportunities to Learn
Rationale:	Rationale
computer technology connection	secondary issue
more real-world connections	
modernizing school mathematics	
strategic site for other reform ideas	
What is Discrete Math?	What is Discrete Math?
*discrete implies separate or distinct	discrete as not continuous
*suggests contrast continuous vs. discontinuous mathematics	some attention to continuous vs. discontinuous mathematics

Table 3. (Continued)

*identifies many central ideas but mostly does not specify how ideas fit or represent discrete math *suggests discrete math involves alternative ideas about counting and gives illustrations of typical problems	central ideas include Venn diagrams, combination and permutations , but does not provide ideas for how they fit in the larger structure of discrete math, relational theories
Situating Discrete Math Inside Curriculum introduction done informally mixed messages for elementary not a separate course insert topics, central ideas *no attention to how ideas might be situated so that discrete mathematics might be represented as a coherent field of study.	Situating Discrete Math: elementary included insert topics across - no ideas provided for how this might be done
Pedagogical Approaches: problem solving approach emphasis on ideas	Pedagogical Approaches: problem solving approach emphasis on ideas

Rationales for why discrete mathematics should be introduced into the school curriculum were different. The idea that discrete mathematics should be introduced into the secondary curriculum was encountered in the context of Sandy's conversations with other educators. Sandy's efforts to introduce discrete mathematics into her practice subsided and became less important, less valued aspects of reforming her mathematics teaching.

Across the category what is discrete mathematics, there is good overlap. For example, the ideas of continuous and discontinuous mathematics overlap. The overlap was mostly because Sandy arranged for opportunities specifically to explore these ideas. For example, she arranged for conversations with other educators to explore discontinuous and continuous mathematics and she selected curriculum materials she believed focused on combinations and permutations. Yet, the opportunities Sandy encountered to explore these ideas did not delve deeply into any idea. In part, Sandy's underdeveloped knowledge of discrete mathematics made it difficult to draw on what she would need to make sense of the mathematics offered. Also problematic was the lack of continuity across learning experiences. Overall, Sandy's experiences lack any long term view of what she would need to learn about discrete mathematics to introduce it into her teaching. She described her learning experiences as frustrating and confusing. Eventually Sandy centered her efforts on learning about two combinatorial counting problems leaving much of the subject matter unexplored.

In terms of situating discrete mathematics in the school curriculum, Sandy's opportunities offered little. None of Sandy's opportunities to learn provided her with detail for how this might be done. The reform documents suggested that discrete mathematics could be introduced by way of some of the familiar ideas already getting attention in the elementary curriculum. Venn diagrams is an example. The curriculum materials suggested that counting problems could be used as a kind of stand alone activity with little concern for connecting ideas across teaching. There was little concern for how these activities connected to other mathematics as well.

In terms of pedagogy, the overlap was congruent. The big message was that discrete mathematics should not be introduced as formulas and procedures to be memorized for getting right answers. Instead, conceptual and problem solving orientations should be emphasized. Sandy's learning in relationship to Marilyn Burns and Project AIMS materials emphasized compatible views. They offered a hands-on, learning by doing, investigations approach to learning mathematics. At the same time, there was a great deal of mismatch in meaning around the notion of contextual use and conceptual understanding.

The nice overlap occurred because Sandy used the reform documents much like a textbook, ideas to guide her efforts to learn what she would need. At the same time, Sandy's opportunities to learn did not delve deeply into any of the mathematics identified in the documents. In addition, the documents did not portray combinatorial reasoning or generalizations as central to the mathematics of combinatorial counting.

There are essentially two critical findings revealed in the contrast between the big ideas of Sandy's opportunities to learn and the central ideas described in the reform documents. My analysis suggests that much of the reform agenda was untouched in the contexts of Sandy's opportunities to learn. In addition, there is a mismatch across several big reform ideas, specifically conceptual understanding and contextual use. My analysis of Sandy's opportunities to learn about introducing discrete mathematics revealed similar problems. Even though there seemed more overlap across the big ideas, the central tenets of the mathematics of counting went untouched across Sandy's opportunities to learn.

I turn now to consider the relationship between Sandy's prior knowledge and teaching experiences and her opportunities to learn. In particular, I examine whether and how Sandy's opportunities to learn considered her personal history with each strand.

Connections to A Teacher's Prior Knowledge and Beliefs

I suggested earlier that Sandy's prior knowledge and beliefs represent opposite situations from the standpoint of teacher learning. Sandy brought a great many ideas and practices related to computational skill teaching, most of which was firmly entrenched and well-grounded in her teaching. Sandy's prior ideas and practices run in stark contrast with the proposed changes. It was clear that much would need to be challenged and uprooted if new ideas and practices were to replace them. In the case study, I described how Sandy tried a new approach to teaching computation. When problems surfaced, she returned to the more traditional ideas and practices she had abandoned. From a pedagogical standpoint, Sandy's learning should have functioned to challenge her prior ideas and practices creating much disequilibrium around traditional forms of teaching. At best, Sandy put aside her ideas only temporarily, leaving much unchallenged.

The reform documents reflect the important role of prior knowledge and experience on new learning.

> "Instead, in many situations individuals approach a new task with prior knowledge, assimilate new information and construct their own meanings. They will accept new ideas only when their old ideas do not work or are inefficient. This constructivist active view of the learning process must be reflected in the way much of mathematics is taught." (National Council of Teachers of Mathematics, 1989, p. 10).

My analysis assumes the same insights that surround students' learning would also be true about practitioners' learning about reforming teaching. From this standpoint, the analyses considers whether and how Sandy's opportunities to learn about reforming her teaching of computational skills challenged her to rethink her existent ideas and practices.

The situation surrounding discrete mathematics is somewhat different. Sandy brought very little specific content knowledge that she could draw on for making sense of the proposals to introduce discrete mathematics. For this topic, there was very little to uproot or challenge. Instead, Sandy brought more global ideas such as the notion that mathematics should be taught for understanding and that conceptual grasp was important to the process. From a pedagogical perspective, Sandy's opportunities to learn should function to increase her capacity to teach discrete mathematics for understanding. To do so, Sandy's subject-matter knowledge would need to be extended sufficiently to include topics in discrete mathematics. This would involve identifying fruitful points of connection, such as the integers or the mathematics of traditional counting, specifically for the purpose of building and sustaining new subject-matter knowledge and teaching practices.

Assuming that I have characterized Sandy's personal history with each topic fairly well, I examine the categories of opportunities to explore whether and how Sandy's prior knowledge and teaching experiences are considered.

Reforming computational skills instruction. The table below summarizes the central ideas of Sandy's opportunities to learn in contrast with the ideas Sandy brought to her learning about reforming her teaching of basic computational skills.

Table 4. Central Ideas of Sandy's Opportunities to Learn About Reforming Teaching of Computational Skills And What Sandy Brings

Sandy's Opportunities to Learn	What Sandy Brings
Computational Curriculum	Computational Curriculum
emphasis on understanding operations conceptually some attention to students' algorithms and mental computation	Sandy brings ideas about what computational skill is - mostly proficiency with traditional algorithms, some mental computation and use of those skills to solve problems. why it is important to learn computational skills Sandy brings a very detailed picture of what the computational curriculum involves and how it is situated in the school curriculum. For example, Sandy has a well-worked out vision for the desired level of complexity and mastery for each algorithm ideas about the conceptual underpinnings of the algorithms/and other conceptual ideas related to the four operations Sandy also brings ideas about what the important elements of computational skill include - such as getting the right answers, knowing facts and algorithms, speed and accuracy with those Sandy brings ideas about differences in underlying systems of numbers and how computational skill changes in relation to each system. She brings ideas about place value, different bases, operations with decimals, fractions, etc. ideas about estimation, how to estimate and when

Table 4 (continued)

Pedagogical Approaches	Pedagogical Approaches
conceptual orientation to four operations as more important than procedural orientation - emphasis on how to conceptually orient the curriculum the notion of balancing conceptual aims with more traditional practices contextualize skills	Sandy brings instructional practices she believes develops computational skill - this includes isolated drill and practice on basic facts, routine algorithms, and mental computing. Sandy brings knowledge of common student errors and misconceptions with traditional computing algorithms She brings ideas about the practices she believes helps students' overcome those

The predominate idea across Sandy's opportunities to learn emphasized teaching addition, subtraction, multiplication and division conceptually and in problem solving contexts. At the same time, the opportunities she encountered gave little attention to why or how traditional ideas and practices are problematic. There was no evidence that Sandy explored the underpinnings of traditional algorithms, whether they remained valued, and if so, to what level of complexity or mastery. Sandy's views about proficiency with routine algorithms went untouched as she explored the four operations conceptually.

Sandy's experiences learning new computational skills, such as students' algorithms or more extensive mental computation, also did not challenge her to rethink any of her existent ideas and practices. Although Sandy thought that students' algorithms and mental computation were to replace the preoccupation with traditional algorithms, she was not challenged to focus on any problems that might arise as a result of these changes.

There was nothing in Sandy's opportunities that challenged her ideas about speed and accuracy with computing. Sandy continued to believe that traditional algorithms provided the most efficient and accurate method for computing. This idea runs contrary to views discussed in the documents. Sandy's very detailed view of the desired level of complexity and proficiency for each algorithm also went unexamined.

Sandy's ideas about estimation also went unchallenged. She brought very traditional ideas about estimation, how to estimate and for what purpose. Across the opportunities she encountered, only one, the documents, suggested that estimation be viewed as a computational skill itself. Even though Sandy read the proposals surrounding estimation, she missed the idea that estimation should be treated differently. She continued to emphasize estimation as a checking strategy for routinely computed answers asking students to use estimation to see if their exact answers were "in the ball park."

Sandy brought very traditional pedagogical ideas for developing students' computational skills. For example, she brought the idea that isolated drill and practice on basic facts and algorithms help develop proficiency. She banked on the idea that, "as students come to conceptually understand procedures and learn why certain steps make sense, fluency will develop as a result," (p. 56 Framework). Sandy interpreted this statement to suggest that a conceptual orientation to the four operations would also result in computational proficiency for students. She developed a practice she thought represented this idea. Yet, when problems arose she abandoned this approach and returned to traditional practices. Sandy's opportunities to learn did not focus on the problems that arose in practice. None of the opportunities she encountered emphasized analyzing teaching and connecting findings to reform ideas. Sandy's interpretations of the changes she made in her teaching went unchallenged.

Overall, Sandy's prior knowledge and teaching experiences were not challenged. The central focus of her learning experiences involved figuring out the correct balance between more traditional, skill-oriented teaching and reform-based teaching. It would have been interesting to see what would have happened had Sandy become convinced that traditional practices were extremely problematic and had to be abandoned. Would she have returned so easily? By not uprooting Sandy's existent ideas and practices, they remained a legitimate choice, especially when supported in other learning contexts.

Introducing discrete mathematics into the curriculum. I turn now to examine Sandy's opportunities to learn about discrete mathematics.

Table 5. What Sandy Brings And The Central Ideas of Sandy's Opportunities to Learn About Introducing Discrete Mathematics

What Sandy Brings	Sandy's Opportunities to Learn
What is Discrete Mathematics?	What is Discrete Mathematics?
no ideas about what discrete mathematics involves	meaning of discrete
no ideas about what the central questions of discrete mathematics involve or why the questions are valued	some attention to continuous as opposed to discontinuous mathematics
no ideas about the techniques or strategies used for examining those questions	
no ideas about what discrete mathematics explores	central ideas including Venn diagrams, deductive logic and counting combinations and permutations

Table 5 (continued)

some familiarity with algebra some familiarity with a few of the central ideas identified in the Framework and Standards,. For example Sandy had some familiarity with the ideas of Venn diagrams, deductive logic, and combination and permutation. Her familiarity is not inside the context of discrete mathematics. no ideas about number systems underlying discrete mathematics. Although Sandy has ideas about the integers it is not in relation to discrete mathematics.	
<u>Situating Discrete Mathematics</u> no ideas for how topics, ideas might be introduced into the school curriculum	<u>Situating Discrete Mathematics</u> elementary included insert topics across - no details provided

Table 5 (continued)

Pedagogical Approaches	Pedagogical Approaches
no ideas for what instructional practices might best represent discrete mathematics Sandy brings many ideas for what it might mean to teach using a problem solving approach where ideas are central (as opposed to a procedural approach where rules and algorithms are seen as central) no ideas about common student errors or misconceptions with any of the ideas or techniques central to discrete mathematics	problem solving approach conceptual understanding emphasized

Sandy brought to her learning experiences some familiarity with algebra, some familiarity with several of the mathematical ideas listed in the documents as central to discrete mathematics, extensive knowledge of traditional counting procedures, familiarity with the counting numbers and the integers. Sandy's opportunities to learn focused on a variety of mathematical ideas Sandy knew little about. For example, Sandy's conversations with leading mathematics educators focused on continuous and discontinuous mathematics. Although Sandy found the ideas difficult to comprehend, she developed some sense of the contrast drawing on what she already understood about algebra. The connection represented an important insight into discrete mathematics especially in light of Sandy's initial conjecture that discrete mathematics might have something to do with the elements chart in science.

Sandy's familiarity with the integers was another site of connecting to her prior knowledge and teaching experience. Sandy's teaching of the integers was rooted in the mathematics of arithmetic and set theory. Initially, Sandy was confused about whether the integers were connected to discrete mathematics. She arranged for conversations with other educators to explore this issue. Consequently Sandy learned that the integers represented a system underlying the mathematics of discrete mathematics.

Sandy's experiences with Project AIMS provided her multiple ideas about teaching pedagogy. The reform documents warned that discrete mathematics should not be included if taught in an algorithmic, procedurally-oriented way. AIMS activities rarely gave attention to algorithms or formulas. Instead, activities were designed to support investigative, hands -on learning experiences. And although the materials offered little pedagogical guidance, Sandy's preparation as an AIMS consultant supported her use of these materials.

There were a number of learning experiences that ignored Sandy's prior knowledge and teaching experiences. The curriculum materials Sandy used is an example. Sandy was completely unfamiliar with non-traditional counting ideas. Her experiences with combinations and permutations in high school involved memorizing algorithms and getting the right answer. Yet the materials Sandy used, although they offered new insights into the idea of order and patterns for formulating counts, at the same time did not connect Sandy's prior knowledge about the algorithms to any reasoning process or the notion of generalizing reasoning processes. None of Sandy's opportunities to learn pressed her to consider any differences across counting problems that would lead to different generalizations explaining the different algorithms she had once memorized.

As Sandy began to clarify and extend some of her ideas about discrete mathematics, she encountered mathematics she had no prior experience with.

For example, several of the conversations she had with other educators focused on recurrence relations. Sandy explained that she listened, to be polite, but really had no ideas for how recurrence relations fit with discrete mathematics. She recalled that even though several relational theories had been explained to her, step by step, she had no way to understand the meaning. This illustrates that even though there was overlap around the idea of recurrence relations, Sandy's prior knowledge was not tapped in a way that offered the opportunity to formulate connections that would advance her ideas about discrete mathematics.

Sandy's opportunities to learn, the content and structure offered, portrayed discrete mathematics as lists of ideas and topics, mostly unfamiliar to Sandy. Venn diagrams is an example. Yet, none of Sandy's learning experiences suggested how Venn diagrams might be used to introduce discrete mathematics. Venn diagrams remained unexamined across Sandy's learning experiences, except that Sandy organized Venn diagrams under discrete mathematics. Although Venn diagrams may have been a useful site to extend Sandy's ideas about counting, she had no opportunity to capitalize on her prior understanding.

Essentially, there was very little continuity across Sandy's learning experiences. None of the opportunities Sandy encountered were organized in ways to build on previous learning. This happened on occasion by chance. However in most circumstances, the learning opportunity lacked the potential for Sandy to form connections to new knowledge by way of what she already understood. In effect, Sandy was not offered the possibility to learn about discrete mathematics as a coherent field of study, involving specific kinds of questions, systematic methods for analyzing those questions, or standards by which to judge the products of those questions.

I turn now to the third element of the analyses.

The Teacher's Views of What the Opportunities To Learn Offered

The data suggests, through a number and variety of statements in interviews, particular views that Sandy held in relationship to the opportunities to learn she encountered. My aim in this section is to establish some sense of what those views were and how they may have interacted with what Sandy was learning. Specific findings focus only on those opportunities that had sufficient data to support any claims.

Sandy's characterized her learning in the context of her doctoral studies as "research-based" and "global understandings" (Interview, 1/93). Sandy viewed her doctoral work as a particular kind of resource, one that provided expert knowledge and global understanding of teaching and learning. Recall that Sandy considered the idea of automaticity to be "research-based." In combination with the emphasis on computation embodied in the state's mandated testing program, Sandy concluded proficiency with traditional computational skills remained a high priority for elementary teaching.

Sandy viewed Project AIMS to offer a "teaching methodology." She argued that AIMS materials framed mathematics teaching and learning very differently than traditional practices. Sandy viewed AIMS methodology to be compatible with the visions of teaching and learning portrayed in the reform documents. The documents described a kind of teaching that emphasized exploring mathematical ideas and students' constructing knowledge in cooperative learning groups. Sandy viewed Project AIMS materials as putting the ideas into practice (Interview, 5/92). Sandy's interpretations of the reform agenda were colored by her views of what Project AIMS offered.

Sandy's views of the framework shaped what and how she would learn about reforming her teaching. Sandy viewed the framework as a "critical piece of work, designed specifically for teachers," and "visionary, hopeful" (Interview, 2/93). Unlike many teachers, Sandy's copies of the Framework (1992), and the Curriculum Standards (1989), did not lay on a shelf collecting dust. Instead, she read constantly, scribbling notes inside the text. Sandy read and re-read sections hoping to dig out something she didn't see before. Yet, even as Sandy thought of the documents as visionary, she also believed they offered a specified program for changing her teaching. She searched for teacher directives, prescriptions for practice. Any vagueness or ambiguity was interpreted as her own personal inability to make sense of the proposals.

This search for certainty colored Sandy's interactions across the six categories of learning experiences. Sandy's perceptions reflected the framework as teacher directives and correct practices. And she viewed the opportunities she had to learn as occasions to uncover specified plans of action for changing her teaching. Recall that Sandy's decision to return to traditional teaching practices represented an instance where she had, "finally gotten it right," (Interview, 8/93).

The Qualities And Conditions of A Teacher's Opportunities To Learn About Reform-Based Teaching

To conclude this chapter I consider findings across the three analytic categories to appraise Sandy's educational experience as a whole, characterizing the nature of the support she encountered for responding to the state's efforts to reform mathematics education.

Mismatch in Content

Most of Sandy's learning experiences were grounded in ideas and assumptions about mathematics teaching and learning that were fundamentally different from those underlying the reform documents. The analyses above reflects that some of the differences were epistemological, whereas others were more conceptual and philosophical. An example would be the idea that computational skills teaching should be approached in contexts that support understanding why specific skills are useful. Another example is the idea of reasoning combinatorially. There were many other ideas and assumptions that also were overlooked or under examined. Essentially, there is a clear mismatch in ideas and practices central to the reform agenda and Sandy's opportunities to learn. Overall, Sandy's professional development experiences lacked strong connections to many of the central ideas and practices described in the reform documents. This mismatch contributes greatly to the teacher's capacity to understand and enact reform ideas in teaching.

Few Connections to the Teacher's Prior Knowledge and Teaching Experience

A teacher's prior knowledge and teaching experiences profoundly influence what a teacher learns. This study underscores the significance of this aspect of teacher learning in the context of policy proposals. The cross-case analyses illuminates that Sandy's prior knowledge and teaching experience was different across different topics in mathematics. For example, Sandy's prior knowledge and teaching experiences in relationship to computational goals were deeply entrenched and functioned as a barrier to long-lasting change. Yet, the above analyses reveals that Sandy's opportunities to learn did not function to challenge or explicitly explore her prior knowledge and teaching experiences surrounding computation. This was also the case for discrete mathematics. For the most part, the opportunities to learn that Sandy encountered failed to support Sandy's efforts to form strong connections between the knowledge, skills, and dispositions that she brought as a learner to the ideas and practices envisioned in the reform documents. As a resource, Sandy's prior knowledge and teaching experiences were mostly ignored.

Lack of Continuity Across Learning Experiences

Sandy's decisions to participate in specific professional development activities were based on what she alone determined to be important. They were not chosen based on any previously developed design or specific intervention. Sandy located resources as she became aware of specific learning needs. This awareness occurred spontaneously, often in contexts not intended as support for learning about reform issues. Little overall attention was given to whether or how Sandy's experiences served to create an overarching view of the proposals to reform computational skill teaching or introduce discrete mathematics. These findings suggest that Sandy's experiences were characterized by a lack of continuity in learning experience. Other than what she could provide herself, specific learning needs were not addressed in any systematic way.[39]

Sandy's conversations with other educators, in part, is an exception. In these instances Sandy focused the occasion directly on her needs as a learner. At the same time, lacking in the experience was any larger view of what would be required for Sandy as a learner. Recall that many of her conversations with other educators about discrete mathematics resulted in experiences with Sandy trying to understand recurrence relations or matrices. Sandy recalled these experiences as, "very frustrating" and "offering little for understanding discrete mathematics." Even in these instances, continuity across learning experiences was decidedly lacking.

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[39]John Dewey develops the idea of continuity of learning experience in his writings. See Experience and Education. Macmillan Publishing Co. 1963. Dewey argues in principle that continuity (united with interaction) measures the significance of educative experiences. Continuity is concern for how future experiences might be shaped by what is learned at each and every stage of the learning process. Thus, as a discriminating factor, continuity helps to judge what is educative and what may be mis-educative on the basis of what the experience moves one toward and into.

Few Opportunities to Connect Learning to Teaching Practice

Sandy's own teaching served as the only site for exploring recommendations for change in the context of teaching. Even as she may have had multiple opportunities to imagine the proposed changes in practice -- she had little opportunity to try out new patterns of instruction and explore what happened as a consequence. Overall, Sandy's opportunities to learn provided very little guidance with what the proposals implied for her teaching. Sandy determined what the proposals suggested as changes in her teaching. She made sense of what happened as a result of those changes. And although this work was certainly influenced by what she was learning in other contexts, she alone had to make sense of the new instructional patterns she introduced into her teaching and the impact of the changes on students' opportunity to learn.

These conditions violate an important principle about learning that policymakers hope Sandy understand. Context plays an important role in the learning experience. Applying this principle to reforming teaching suggests that the teaching context will be an important factor in what the teacher learns. Connecting teacher learning opportunities more directly to the teaching context, whether by observation and discussion of videotaped teaching or one's own teaching, offers teachers a richer context for learning the practical implications of the proposed changes (Ball & Cohen, 1995a).

Absence of Purposeful Teaching

Sandy made decisions about what and how she would need to learn almost exclusively on her own. Recall that Sandy's decision to focus on discrete mathematics was motivated by her responsibilities to educate other elementary teachers about the new framework. Sandy's efforts to learn more about teaching computation was motivated by the difficulties she encountered in her practice.

Of the six categories of opportunities I investigated, none were designed purposefully to explore the ideas and practices Sandy decided were important. In effect, Sandy's learning about these proposals went on subsequent to other learning agendas. As a result, Sandy's learning experiences lacked purposeful teaching about the ideas and practices to reform her teaching of computation and introduce discrete mathematics into her teaching. In effect, the experiences offered little specific guidance for accomplishing the goals Sandy identified as central. And although Sandy may have mustered an abundance of resources to support her efforts, the experiences she developed encouraged isolation in the learning process (see Lortie, 1975, p.70). Within these experiences Sandy had to foster for herself a wider range of wisdom and expertise to support her understanding and enactment of the proposals she focused.

As a learner, Sandy confronts a paradox. She must make judgments about what is useful to learn and either create the circumstances for learning it or recognize the opportunity when it arises in contexts not purposefully designed to support such learning. To effectively select and judge opportunities to learn, Sandy would have to bring well-worked out ideas about the proposals in order to make good decisions in the contexts of her learning opportunities. Yet, as a learner in the context of reform, Sandy brings knowledge, skills, and dispositions that run counter to the ideas and practices she must learn.

Little Common Learning Community

Across several of the categories of opportunities to learn, there were many other educators involved in Sandy's learning experience. This included other teachers, other doctoral students, state officials and mathematics educators. The potential for Sandy to work collaboratively with others on the ideas of reform seems readily available. Yet, most of these environments provided at best a social setting for Sandy to work alone on the ideas of reform. Because much of what Sandy learned about teaching computational skills and introducing discrete mathematics was situated in contexts not designed for exploring those proposals, her learning process coexisted, along side other learning agendas, progressing mostly in isolation of any others participating in the same environment.

Sandy's doctoral studies and the professional development activities she participated are good examples of environments that involved other learners but at the same time aimed at agendas other than the proposals focusing this study. Sandy's efforts to understand the reform documents, her own teaching practice, and the curriculum materials she used went on without the benefits of community. Sandy's conversations with other educators may be an exception. She reported that these conversations offered the opportunity to work with others on the specific proposals she tried to understand and enact.

Some of the isolation in Sandy's learning experience was created by Sandy herself. For example, Sandy decided to focus exclusively on curriculum materials to help her learn about discrete mathematics after she encountered much confusion and frustration in learning with others through the conversations she arranged. Although the curriculum materials offered Sandy ideas, at the same time, she made sense of those ideas in isolation of others purposefully cutting herself off from any wider range of wisdom or expertise.

A Search For Certainty

Sandy's ideas about computational skill teaching and introducing discrete mathematics evolved over long periods of time and in relationship to a variety of resources that supported her learning. I have described the experience for Sandy as a kind of wandering, a selection process bounded by what she could understand herself and what she encountered in the contexts of the resources she put together. At the same time that Sandy's experience is characterized by much uncertainty, the analyses also revealed that Sandy perceived the work to involve certainty, a search for right answers for her teaching. An important aspect of this process involved the unpleasantness she experienced, an emotional response to the difficult circumstances she encountered. Sandy perceived the discomfort as her inability to "get it right."

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These findings explain how the gap between instructional policy and the individual teacher's sense making and teaching can remain wide. Even though the teacher makes use of ample resources and opportunities, there is a lack of coherence and coordination in the teacher's educational experience as a whole. The opportunities to learn that Sandy encountered offered a less than whole view of the recommendations for change. They barely skimmed the surface of the proposals and failed to challenge and extend the knowledge, skills, and dispositions she brought to the experience. Teacher's subject-matter knowledge was mostly assumed. There were few opportunities for the teacher to connect learning to the practice of teaching. For the most part, Sandy had to figure out for herself what the proposals implied for her teaching and whether the changes she made improved students' opportunity to learn.

The analyses also suggests that the gap may have widened further. Not only did Sandy's opportunities to learn fail to foster what she needed to imagine and create the teaching portrayed in the documents, on several occasions the opportunities Sandy encountered fostered learning that supported entrenched, traditional patterns of instruction. Overall, the occasions that Sandy had for learning about the state's goals did not challenge or explore any differences or discrepancies in one form or another form of teaching. Instead, Sandy resolved differences and discrepancies on her own. Often, Sandy's analysis resulted in a reinterpretation of policy. In effect, the opportunities to learn that Sandy encountered fostered, and at times supported, a veering off in directions incompatible with the state's ideas of reform-based teaching.

CHAPTER 7

LINKING INSTRUCTIONAL POLICIES TO PROFESSIONAL DEVELOPMENT OPPORTUNITIES FOR TEACHERS

Introduction

Beginning in the mid-1980s, the State Department of Education in California produced a massive effort to further statewide reform in mathematics education. Policymakers specified, in a new framework, essential characteristics of empowering mathematics programs and described how changes in curriculum, teaching, and students' learning could be accomplished. State officials aligned multiple levers to press teachers toward desired practices. The state sponsored and supported the development of new curriculum materials, for example. These materials, known as replacement units, were tied closely to the goals of the framework and were intended to take the place of larger segments of the curriculum. The state argued that if real change were to take place, teachers had to have available large chunks of curriculum that interrelated recommended changes and could be inserted into the sequence of topics normally taught. In addition, textbooks, for the first time ever, had to be tied directly to the goals of the framework. Other units--such as those written by educational entrepreneur Marilyn Burns--were also recommended.

The state developed a new assessment program as well. Test developers experimented with alternative forms of assessment. The goal was to align assessment and the framework in an effort to apply greater pressure on districts to conform to the state's goals. Teacher certification and professional development were also tied in. The state required teachers to be recertified every five years by obtaining professional development credits and passing the California Basic Essential Skills Test.

Professional development was also targeted. The state took an assertive role in educating teachers about the framework. The legislature offered funding to support school-based teacher improvement projects. Funding was also pursued for larger scale projects such as the California Mathematics Project and the Middle Grades Mathematics Renaissance. These projects, among others, created an infrastructure of professionals to provide teachers with leadership and professional development. Four-week summer institutes, one-shot workshops, and multiple other opportunities were offered to support teachers' efforts toward reform-based change.

California had mobilized a remarkable array of levers toward reforming mathematics teaching in the state. Still, such change must be effected in classrooms, in the thoughts and practice of the teacher. Policy analyst Milbrey McLaughlin reminds us, "Change ultimately is a problem of the smallest unit" (McLaughlin, 1987). In other words, the policy that's delivered depends finally on the individual at the end of the line. This dissertation offers policymakers and educators a window for viewing what happened as a policy was transformed by the individual teacher's interpretations and responses. We can see that this teacher, Sandy Wise, became deeply involved in the state's goals. We see that she was motivated and willing to embrace policy objectives. She understood that the state was attempting to align policy and practice in ways never before imagined. She spent countless hours reading the framework, analyzing the new CAP tests and curriculum guides, trying out new curriculum units, and changing her practice. The case studies in Chapters 4 and 5 provide examples of the deep effects on Sandy's thinking and practice. In many respects Sandy was successful at reform. She brought the conviction to learn what was needed to understand

and enact the state's recommendations. She attended several state-supported professional development activities. She used Marilyn Burns' unit on multiplication. She even supported other elementary teachers by offering inservice opportunities to learn about the new framework.

Yet, this dissertation reminds us that despite great strides on the part of committed state officials, policymakers, educators, and a teacher, the disjuncture between policy and practice can still remain wide. In this work, we have the opportunity to view how change happens at the level of the individual teacher. By focusing on the individual, we can see the interplay between a teacher's thoughts and actions and what she encounters across a range of opportunities to learn, including many state-sponsored activities. The analysis in Chapter 6 uncovers aspects of practice that make it particularly difficult for the teacher responding to policy. We saw, for example, that the teacher can remain isolated despite great effort to interact with others. As Sandy began locating people and opportunities to help her understand and enact the framework, she encountered multiple interpretations of the proposed changes and different paths for learning what the state recommended. Many paths were contradictory. Essentially, the teacher had to figure out for herself what to participate in, how to follow up, and whether and how the ideas she encountered connected to the state's goals.

My analysis and conclusions focus on teachers like Sandy--those working as entrepreneurs, seeking opportunities for their own learning as they make sense of the reforms swirling around them. I suggest that this is what the situation was like for many teachers in California.[40] And although we already

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[40]Even though much of the current literature and efforts at school improvement focus on groups of teachers, collegiality, the importance of professional community etc., this study reminds us that many teachers will approach reform essentially alone. Even as teachers interact with many colleagues, there remains a dimension of aloneness in constructing and developing the ideas of reform-based teaching. Understanding what the individual teacher encounters remains an important aspect of understanding the overall picture of reform-based change. This work helps

understand that isolation and individuality in teaching are problematic, this work suggests that even though much may be done to create collegiality and press systemic efforts to reform, what the individual teacher encounters is somewhat different. I found that even though multiple resources and levers were aimed at pressing and supporting teachers toward desired practices, much of what the individual teacher would need to learn was overlooked or not addressed adequately. In particular, the subject-matter knowledge a teacher would need to learn to teach computational skills differently or introduce combinatorial counting into practice was not addressed adequately. The teacher had to locate and develop opportunities to learn the mathematics for herself. What she created or encountered was often situated in contexts not specifically designed to teach the content knowledge needed. The teacher therefore had to draw out and connect mathematical ideas she believed would be useful. More problematic still was that most of the occasions relied upon embodied assumptions about mathematics and teaching that ran counter to policy goals.

These circumstances place teachers in a difficult spot. In order to locate and develop opportunities to learn that are grounded in ideas and practices argued for in policy, a teacher would have to bring well-worked-out ideas about reform-based teaching. Yet, we already know from research on teacher learning that teachers are more likely to bring forth knowledge, skills and dispositions that run counter to the ideas and practices envisioned in the policies of the eighties and early nineties. Because of the vastly different nature of practice most teachers are accustomed to, they have had little opportunity to understand and experience the ideas and practices proposed. These circumstances create a contradictory learning environment for the individual teacher. The conditions

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us to see how the individual teacher organizes and structures much of their own educational opportunities in the context of broad-based efforts to reform.

can limit rather than foster the teacher's capacity to develop deeper insights into policy and create the practice imagined.

Suppose for a moment that U.S. students encountered similar circumstances for learning high-school algebra. As learners, students would not understand the central ideas of algebra or see how the ideas fit together as a field of study. Yet, imagine that students had to design and direct an investigation into the field. Fortunately, there are some resources available. Students might dip into textbooks, talk to a variety of educators, and work with each other to develop some ideas. Interest, curiosity, and connections would guide progress. Students' ideas would evolve. Yet, what would guarantee that students actually developed an appreciation of the mathematics of algebra? Even in cases where very resourceful students made use of an abundance of resources, the opportunity to learn would be limited by what individuals could locate or develop for themselves. Missing would be any wider range of wisdom or guidance about what to learn or how that learning might be accomplished. These circumstances would seem ridiculous for U.S. students. We expect that students are supported by a knowledgeable teacher and a well-designed curriculum. Would teachers require similar conditions?

This chapter focuses on this question. Suppose that educators and policymakers were to continue in the direction the state of California was heading -- that is, that they might continue to take teachers seriously as learners. What does this study suggest about the teacher's needs as learner? My aim is to draw together findings in this study, with other research on teacher learning and research on policy and practice, to suggest how the ties between efforts to reform instruction, teachers' learning, and teachers' practice might be strengthened. I propose that more attention be given to what links policy to the individual teacher's sense making and enactment of proposed changes. I begin by arguing that the nature of the relationship between policy, teacher learning, teaching practice is dynamic. I argue that within the dynamic, there exists an interplay of factors that affect what a teacher learns and therefore can make of policy. I provide four examples of factors that had a significant impact on Sandy's learning. The lack of attention to these factors, coupled with the nonsystematic nature of the professional development available, reveals the lack of coordination and coherence in efforts to support teachers' responses to policy. To conclude the chapter, I sketch several ideas for forming more solid links between policy, teacher learning, and teaching practice, and I suggest directions for future research.

Responding to Reform Initiatives: Focusing at the Level of the Individual Teacher

Policymakers and educators are not sufficiently aware of the long-term and complex nature of the relationship among policy, teacher learning, and teaching. Many underestimate what the implementation process involves at the level of the individual teacher. And even less is known about what affects a teacher's responses to policy.

This study has several important contributions to our understanding of these issues. Below, I suggest that learning is the most central activity of policy implementation for the individual teacher. I argue that the relationship between policy, teacher learning, and teaching is dynamic as opposed to linear. Not only does policy affect practice, but what a teacher learns from practice greatly affects what the teacher thinks a policy recommends. I then argue that a teacher's learning is also complicated by a set of factors that, knowingly or unknowingly, will affect a teacher's responses to new frameworks, assessments, curriculum guides, and other levers designed to promote the state's goals. The four examples sketched in this chapter stood out in the case studies both because of the significant impact they had on this teacher's learning and because of how, together, they illuminate the effects of factors not typically coordinated within efforts to promote policy. They include: the teacher's analysis of the impact of changes on students' learning; learning in contexts not intended to press policy objectives; the teacher's personal stance toward learning; and the topic-specific nature of the teacher's prior knowledge and teaching experience.

The Dynamic Nature of the Learning Process

This study underscores the importance of recognizing and understanding what happens once the teacher tries out new instructional strategies. Sandy generated ideas about what to change and then monitored what happened as a consequence of the changes, focusing particularly on the impact on students' learning. Recall that Sandy was completely willing to suspend her deeply held knowledge and beliefs about teaching computation in order to try other instructional strategies. As she began working with the new strategies, she assessed that students' proficiency with computing greatly declined. This analysis had a very powerful influence on subsequent changes she made in her teaching and was pivotal to her interpretation of what the state proposed for reforming computational skills instruction.

These findings illuminate that there is both an interactive and iterative quality to the relationship between instructional policy and teaching practice, one critically influenced by a teacher's continual learning. The idea that learning impacts a teacher's actions in the classroom, and that a teacher's analysis of the changes made affects the teacher's interpretation of policy, represents a clear shift in more conventional ideas about policy implementation. Straightforward transmission and adoption, for example, do not conceptualize teacher learning as the core activity affecting how policy is implemented. These models either assume away or ignore the dynamic nature of the relationship between policy and the teacher. They far undervalue, for example, the activity of refining instructional strategies over time, as new learning is factored in. We saw in Sandy's case that her teaching of the conceptual underpinnings of computation went through several rounds of revision, in part, because of what she learned from students' interaction with the new strategies. Later, she reinstated traditional teaching practices once she factored in her analysis of students' proficiency with computation. What she learned from her practice ultimately fostered a new interpretation of policy as well. These events might be understood or interpreted as teacher disinterest, or worse yet incompetence, when the central frame of analysis is not teacher learning.

The work of McLaughlin and others did describe implementation to involve a dynamic relationship between policy and the teacher (McLaughlin, 1976). The model of mutual adaptation, for example, suggested that the relationship between policy and the teacher was two-way and that each shaped the other. Yet, within that frame, the core activity was not conceptualized as teacher learning. At the same time, this work set the direction for future research to examine more closely the relationship between policy and the teacher. The EPPS research recognized the central role that teacher learning played in the policy implementation process and began identifying and studying various aspects of the relations among policy, teacher learning, and practice (see for example, Ball et al., 1994; Cohen & Ball, 1990a; Peterson, 1990; Wilson et al. 1996).

This work is a continuation of earlier EPPS research. Like earlier studies, this work suggests that learning <u>is</u> the core activity of policy implementation for the teacher. Findings illuminate that policy is not a static set of new ideas for teaching. Instead, the ideas of policy grow and change as the teacher interacts with them. Chapters 4 and 5 serve as examples of the changing nature of policy and practice as a teacher's prior knowledge and beliefs interact with learning about new proposals for teaching. The case studies portray implementation as an evolutionary process, one where ideas and practices evolve slowly and unevenly in a teacher's thought and action in the classroom. More subtle and dramatic changes that the teacher made in her teaching were not the end-product of policy. Instead, changes in teaching represented only one aspect of a more broad-based, ongoing process of learning. Other activities such as unpacking the proposals, generating teaching strategies, and analyzing the implications of changes represented other aspects of the implementation process. Yet, at the core of each of these activities was the teacher's continual learning and investigation of changes in teaching.

What I describe suggests a model of implementation that is substantially different from more common views. Policy does not arrive in teaching practice but instead is evidenced by continual growth and change in the teacher's ideas about teaching and about policy. This shift requires that we abandon assumptions underlying straightforward transmission or unadulterated adoption. Instead, the policy and practice relationship is conceptualized as one of teaching and learning (Ball et al., 1994; Cohen & Barnes, 1993a). This implies an intermingled process, a dynamic of back and forth, with each aspect of the relationship impacting the other. In this model, it would become natural to think, for example, that changes in teaching signal the onset of new learning and reinterpretation of policy.

If we conceptualize the policy and practice relationship as one of teaching and learning, then what we mean by "professional development" would also change, in that opportunities for teacher learning would include many things besides traditional professional development and inservice events. Before turning to these issues, I continue to focus on the complexity involved for the teacher in learning from external efforts to reform.

Factors Affecting Teacher Learning

Recall from Chapter 1 that policymakers in California initially underestimated the significance of teacher learning in relationship policy. Policymakers tended to rely on changes in standards, guidelines, testing, and textbooks to support the desired goals. They assumed that mere exposure and awareness of ideas coupled with other levers of standards-based reform would accomplish the state's goals. These levers were relied upon as much or more than teachers' professional development.

This study reminds us that even if professional development had been targeted at the onset, there would have been much more for policymakers and educators to understand to adequately address teachers' needs as learners. This study reveals there are multiple factors that interact and affect what a teacher learns. Some of the influences are intentional levers, developed specifically to promote the policy (i.e., mathematics frameworks, curriculum guides, professional development activities). These factors played a significant role in Sandy's ongoing development of ideas and practices. Yet, the analysis in Chapter 6 also reveals that there were multiple other influences impacting Sandy's thoughts and actions as much or more than intentional levers. Some of these involved resources Sandy used because of connections she made between policy ideas and ideas offered in the context of the learning occasion (i.e., doctoral studies, conversations with other educators). Others were rooted within the teacher as an individual learner and others in the school context (i.e., Sandy's prior knowledge and teaching experiences, Sandy's and the principal's views of good mathematics teaching). Although each of the factors identified in Chapter 6 had a significant impact on Sandy's thought and practice, the following four stand out because they represent the powerful effect of factors not typically expected, or coordinated within efforts, to press desired goals and practices.

Analysis of the Impact of Changes on Student Learning

I mentioned earlier, in my discussion of the dynamic nature of the learning environment, the important role that Sandy's analysis of the impact of changes on students' learning played in shaping her understanding and enactment of the policy. I reiterate here the significance of this analysis on Sandy's practice. Sandy observed and analyzed what happened in her teaching as a result of the changes she made. She formed judgments about whether and to what extent students' learning opportunities were improved. She also made judgments about whether her findings fit with the learning goals envisioned in the reform documents. This analysis proved to be the most pivotal information Sandy would rely upon for reforming her teaching of computation. Her return to traditional ideas and practices rested almost exclusively on the analysis.

Learning Contexts Not Intended to Press Policy Objectives

We also saw in the case studies that Sandy's learning in situations not intended to press policy objectives had a significant impact on what she made of the state's recommendations. For example, Sandy learned in unplanned, private conversations with other educators that discrete mathematics was intended only for secondary education. My analysis in Chapter 6 suggests that although teachers respond to the multiple levers designed specifically to press desired practices (i.e., curriculum guides, state testing program, reform documents), they are influenced equally or more by resources not designed or intended to support the state's goals. From this standpoint, teachers often learn about reform-based teaching through experiences that are grounded in differing assumptions, aims, and purposes than those underlying policy.

Sandy's work with Project AIMS is also a good example. Sandy used Project AIMS curriculum materials to support changes she wanted to make in her teaching of multiplication and teaching of discrete mathematics. I found that the philosophical and epistemological views of mathematics and science learning underlying AIMS materials were different from those described in the new framework. Sandy negotiated the meaning of any differences on her own, or they acted as a subconscious influence on her understanding of the state's goals.

The Teacher's Personal Stance Toward Learning

A somewhat less obvious influence on Sandy's interpretation of policy was her stance toward her own personal learning. Although Sandy argued that learning involved a constructivist kind of process, her stance for learning herself was more a search for certainty (McDonald, 1992). Sandy operated as if the policy embodied the right answers for her teaching, and her job was to figure out what those answers were. This view contrasts sharply with other views. Shulman, for example, argues that policy ideas are more moral and political imperatives requiring degrees of teacher autonomy so that professional judgment can function comfortably (Shulman, 1983).

Sandy's stance contributed to an emotional response to learning that was unproductive for her. She reported that she often felt bad because she lacked important knowledge and that these feelings contributed to her sense that she was not a good teacher. She reported a sense of remorse and regret over errors she felt she had made in her teaching. Lord (1993), Schifter (1993), and others report on an emotional response to teacher learning that fosters and underlies crucial learning about changing teaching. Lord describes a productive disequilibrium for teachers and argues that these experiences are necessary so that teachers can confront their histories in teaching with an eye toward the policy vision. He claimed that the experience is often uncomfortable or difficult for learners, as firmly held, deeply entrenched beliefs and knowledge are challenged or discarded. For Sandy, the uneasiness she experienced failed to provoke questioning or crucial learning about deeply held beliefs and knowledge. Instead, it created a sense of self-doubt and frustration. These feelings perpetuated her stance toward certainty and an isolation in her learning. Recall in Chapter 5 that Sandy responded to the frustration she experienced by turning away from other educators and toward curriculum materials developed by Marilyn Burns.

Topic-Specific Differences in Teachers' Prior Knowledge and Teaching Experience

Research on teacher learning has already shed much light on how teachers' prior knowledge and teaching experiences interact with teachers' learning to teach, acting both as a contributor and an obstacle (Lortie, 1975; Ball, 1988). Findings from this study suggest that teachers' prior knowledge and teaching experiences are topic-specific and interact differently on teachers' sense making of different policy proposals and in teachers' learning experiences. I argued in Chapter 6 that differences in Sandy's prior knowledge and teaching experiences were mostly ignored as most content knowledge was assumed. Yet, one of the most striking contrasts in the cross-case analyses was the topic-specific differences in Sandy's prior knowledge and teaching experiences and the impact of these differences on Sandy's responses to the proposals focusing on computational skills and combinatorial mathematics.

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To this point I have focused at the level of the individual teacher's sense making in relation to policy. I have identified several aspects of the learning environment that make it particularly difficult for the individual teacher to learn about reform-based change. Below, I continue to identify aspects that complexify matters for the teacher as learner, only now, I turn to focus at the level of teachers' opportunities to learn.

The Non-Systematic Nature of Teachers' Opportunities to Learn

In Chapter 6 I described the learning environment for teachers responding to policy as lacking both coherence and coordination. I argued, for example, that Sandy encountered a mismatch between the ideas and practices she encountered in the contexts of her learning opportunities and what she needed as a learner to understand and enact the ideas envisioned in the framework. In response, Sandy tried to locate or develop opportunities to learn that she thought would attend to needed learning by herself.

These circumstances place teachers in a paradoxical position. The situation requires that teachers bring, rather than build, the capacity to select and develop opportunities to learn that will focus on the central tenets of policy. Furthermore, it requires that teachers bring the capacity to monitor and understand factors affecting their learning, in order to manage the effects toward the goal of producing reform-based teaching. Given the wide gap between modal teaching and envisioned practice, it would be highly unlikely that any teacher would bring such capacity. For one thing, most teachers would not bring deep insights into what reform-based teaching looks like, how to create it, or what is necessary to learn. Dewey and others made the argument long ago that learners, by themselves, are not in a position to design their own learning experiences because they do not bring a knowledge of the means and ends of an experience that is truly educative (Dewey, 1938, Schwab, 1978). These scholars recognized that the teacher, as a learner, is not likely to provide what is needed to create an educational experience that insures that the learner will encounter and develop the central tenets of reform-based change.

The professional development experiences Sandy encountered also gave little attention to a principle about learning that policymakers and educators hope teachers will learn and embody in their own teaching. The reform documents promoted the idea that learning occurs best when situated inside contexts where learning naturally arises. The implication of this principle for learning to teach differently is that teachers would need to learn in contexts that situate learning in the practice of teaching, where such learning is used. One problem that Sandy encountered was that while teaching, there were other competing goals that made it difficult to focus on required learning. Researchers suggest that whether by observation and discussion of one's own practice, another's practice, or video tape of practice, such contexts can offer richer opportunities to link up reform ideas with the practical implications of those ideas (Cohen, 1989; Ball & Cohen, 1995a). Opportunities such as these were lacking in Sandy's educational experience.

As educators, we already understand that the conditions described are not optimal for teachers. If teachers such as Sandy were to remain designers of their own educational experience, they would have to become skilled at selecting and judging among learning experiences in order to link up opportunities that attend more directly to the ideas envisioned in policy and connect learning to the practice of teaching. This would not be an easy or straightforward process. In Chapter 3 I highlight the complexity and variation on the ideas and practices envisioned in the reform documents. The case studies in Chapters 4 and 5 further illuminate that the teacher is likely to encounter multiple interpretations of proposals and various directions for responding to the state's goals. Chapter 6 reveals aspects of teachers' opportunities to learn that make it difficult to learn what may be needed. The paradox that teachers encounter in guiding their own educational experience, as well as the need to connect learning to the practice of teaching, are two examples of the complexity involved.

More problematic still is that many teacher educators and policymakers do not recognize the complexity, and think the problem is a much simpler one. Many suggest that because there already exists such a wide array of professional development for teachers, teachers would only need to participate to develop greater capacity to improve teaching. Overcoming such beliefs is yet another obstacle when thinking about what may be needed to change these conditions. This research suggests that more participation would not equate to improved capacity. Although Sandy participated in and developed a wide variety of professional development activities, there was little that offered systematic, coherent learning about reform-based teaching.

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What I have identified might be thought of as a set of links between instructional policy and teaching practice. These links are examples of what's involved as the individual teacher's sense making is connected to reform-based ideas. How a teacher manages the interactive and iterative nature of the learning required, as well as the interplay of factors that affect teacher learning, will certainly impact the teacher's capacity to learn what is needed. The paradox that teachers encounter as they select and judge learning experiences, as well as the necessity of connecting learning to teaching practice, will also impact the teacher's capacity to learn what is needed.

Findings in Chapter 6 suggest that Sandy's opportunities to learn, for the most part, did not recognize or attend to the links I describe. Little, if any, attention was given to the dynamic nature of the learning required. Sandy worked mostly alone in analyzing the impact of the changes she made in her teaching. None of Sandy's professional development opportunities focused on making sense of the impact of changes she made in her teaching. Sandy analyzed the impact on students' learning experience. Most of the professional development activities Sandy encountered assumed knowledge of mathematics. As a consequence, Sandy developed the opportunities she had for learning combinatorial counting and the conceptual underpinnings of computation. And even though learning in contexts not intended to press policy goals had a powerful effect on Sandy's understanding of the state's goals, what she encountered in those contexts often ran contrary to policy objectives.

The inattention to the dynamic nature as well as the factors affecting teacher learning, create a complexity in the learning environment that teachers like Sandy will work through on their own. The paradox that the teacher encounters and the lack of opportunity to connect learning to teaching practice further complicate matters for teachers. The inattention to these links reveal the lack of coherence and coordination in a teacher's educational experience in the context of efforts to reform.

These findings suggest that even if policymakers and state officials make great strides in producing more coherent visions of change and multiple levers to press teachers toward desired practices, it would not be enough to affect practice. Attention would have to be given to the teacher's learning environment: asking how to create conditions that will foster learning that is not so detached from policy objectives. This implies attending to the sources of weakness inherent in teachers' opportunities to learn. The remaining pages of this chapter draw together findings in this study with related research to offer several ideas for attending to the specific links identified in this chapter. My overall aim is to conceptually develop directions that would likely lead to more systematic and coherent learning about reform-based teaching for teachers. Thus, I speculate about what would need to be different in the learning environment if the links I described were not so weak. I sketch proposals that recognize and attend to the dynamic nature and multiple factors that affect teacher learning. I discuss possible ways to address the paradox inherent in professional development experiences as well as the lack of connection between learning and teaching. And I suggest promising directions for future research.

Creating Stronger Learning Links Between Instructional Policies and Teachers' Opportunities to Learn

Some might suggest that as a starting point, teachers would need more coherent and simplified visions of change. In other words, some would start by trying to improve policy itself. An important first step may be to let go of the idea that policy can create the teaching hoped for. A general finding from revisiting the RAND Change Agent study, 10 years later, is that "It is exceedingly difficult for policy to change practice, especially across levels of government" (McLaughlin, 1990). This study concurs with this finding and offers a particular perspective of why it remains true. In Chapter 3, I described the difficulties in formulating a view of envisioned practice. The analysis revealed that a teacher's prior knowledge and teaching experience affects greatly what the teacher can learn from new frameworks, texts, or tests. What the teacher brings to "reading" policy ultimately determines whether any of the proposals can be recognized or understood. More problematic still, is that reform documents, in particular, are under-specified, but for good reason--they are not intended as programs for practice. My work in Chapter 3 provides specific examples of how a single proposal might be interpreted in multiple ways, each with its own merits to judge. Perhaps more important, this study reveals the lack of coherence and coordination that teachers encounter across professional development experiences. More specifically, it illuminates a set of weak links in the teacher's learning environment in the context of external efforts to reform.

Thus, as instruments of change, policy can inspire, encourage, and paint grander visions of teaching. At the same time, policy may not be the most promising avenue for addressing teachers' needs as learners. It may be more productive for policymakers and educators to ask whether and how policy can facilitate effective opportunities for teachers to learn about reform-based change.

Educational researchers are beginning to focus on what it might take to improve teachers' opportunities to learn. Many have recommended massive reform of professional development opportunities for teachers (Sarason, 1993). Some argue that reform of teachers' professional development forms the most serious unsolved problem for policy and practice in American education today (Sykes, 1996). Wondering about how teachers' opportunities to learn can be improved implies consideration of a complex set of issues involving reasoning among the subject matter of reform, theories for learning it, the teacher as learner, and the contexts in which teachers work (Schwab, 1978; Shulman, 1987).

Although the challenges are significant, much research and promising projects are already underway. Cohen and Barnes, for example, argue that the key to change is that reform itself must be framed as a set of educational opportunities designed to embody the sorts of teaching and learning that reformers wish to promote (Cohen & Barnes, 1993). They suggest it would be far more likely to produce desired patterns of teaching and learning in classrooms if teachers could experience those patterns for themselves first-hand. In other words, teachers' opportunities to learn would have to be fundamentally different than what Sandy had. Yet, how would they need to be different? This question points to what this study suggests should be a central focus of future research -- how can the environments of instruction for teachers and policy work together to create new contexts that sufficiently meet teachers' learning needs in the context of external efforts to reform? To the extent that we can identify and understand what is involved, we can begin to imagine what it would take to create better opportunities to learn for teachers.

This study offers several important contributions to what may be needed. I begin by focusing on the non-systematic nature of teachers' opportunities to learn. I argue that a combination of freedom and control over teachers' learning experiences may be needed to attend to the paradox and lack of connection between learning and practice. I suggest that guidance can function as a form of control that can assist teachers in making selections of what to participate in, and forming judgments about teaching that does and does not fit well with reform-based ideas. I offer two examples of guidance. I propose that clearly stated learning goals can function as a form of guidance, especially in relationship to contexts not intended as support of policy objectives. Recall that this factor had a powerful effect on Sandy's understanding and enactment of the framework. At the same time, there was little in place to guide her selections and judgments of experiences not intended to press policy goals. As another example, I suggest that teacher leaders, if well-prepared, can function effectively as a form of guidance. I discuss what this research suggests is important as preparation for such as role.

I then turn to focus on the dynamic nature of the learning process specifying how the interactive and iterative nature of the learning required might be attended. I suggest that teachers need more opportunities to learn that focus on what happens as changes are made in teaching. The purpose would be to strengthen the connection between the ideas of policy and teaching practice. In addition, I propose a set of four learning objectives, recommending that these objectives be taken up within the contexts of teachers' opportunities to learn. The goal would be to foster learning that connects what a teacher is likely to bring as a learner to what teachers are offered in the contexts of opportunities to learn. I identified three additional factors affecting teacher learning in this work: the teacher's analysis of the impact of changes on students' learning; a teacher's personal stance toward learning; and topic-specific differences in a teacher's prior knowledge and teaching experience. To attend to these links, I propose that teachers' opportunities to learn focus directly on analyzing the impact of changes on students' learning experiences, including attention to teachers' beliefs about how students' opportunities to learn will be improved. I then suggest that teachers' opportunities to learn attend to the individual nature of the disequilibrium teachers experience. And finally, I argue that teachers' opportunities to learn must attend to topic-specific differences in teachers' subject-matter knowledge.

Attending to the Nonsystematic Nature of Teachers' Professional Development Opportunities

In my discussion of the nonsystematic nature of teachers' professional development opportunities, I argued that teachers encounter a paradoxical situation. Teachers must select and judge opportunities to learn, with few ideas and little first-hand experience with the ideas and practices underlying policy. These circumstances, in addition to the lack of opportunities that connect learning more directly to teaching practice, create difficulties for the teacher to work out on her own. Below, I offer two directions for strengthening a teacher's capacity to select and judge opportunities to learn and form stronger connections between learning and teaching practice.

A Combination of Freedom and Control

Many policymakers, educators and researchers argue that because changing teaching is highly specific and context dependent, teachers would necessarily need to craft for themselves what they would need to learn and do to reform their teaching. The analyses across this work suggest the opposite. When teachers are left to develop for themselves a course of study to learn about reform-based change, the educational opportunity becomes restricted. The case studies help us to see how this happens. Even as teachers are highly sensitive to the contexts in which they work, they also bring knowledge, skills and dispositions that can act as powerful barriers to change. Teachers must confront and extend what they already know and believe about mathematics teaching if they ever hope to understand and produce the teaching envisioned in policy. A teacher's opportunities to learn most certainly will play an important role. Yet, these circumstances raise an important question regarding the degree of freedom teachers have to select, develop and judge opportunities to learn.

The degree of freedom a teacher has to choose and make judgments about their own professional development experiences can limit the teacher's educational opportunity. This claim is likely a controversial one, given that teachers are professionals and are obligated to do what is needed to do their job well. Yet, Dewey warns there can be no greater mistake than to treat freedom in learning as an end in itself. He argues that freedom is only powerful when it functions to frame purposes, to judge wisely, and to evaluate desires by the consequences that result from the action (Dewey, 1963, p.64). This study illuminates what happens when a teacher has the freedom to select and order experience to carry desired ends into operation. Sandy experienced freedom to the extent that much of what she learned went uninterrupted and unchallenged by policymakers' ideas. This was both a function of her individual choices as well as what she encountered as professional development opportunities. In her educational experience, freedom and control over learning functioned disproportionately and unproductively. Yet, what would it mean to provide each element? And in what form could control be offered?

At the same time, we have learned from past efforts at reform that policy can't mandate what matters at the local level (McLaughlin, 1990). This finding suggests that control would probably not work well in the form of a mandate. Instead, past studies suggest that local capacity, expertise, organizational routines, and resources available to support planned change efforts generate fundamental differences in the ability of practitioners to plan, execute or sustain an innovation (McLaughlin, 1990). These findings suggest that control might best be offered in the form of a resource, such as guidance or expertise.

One form of guidance that would likely have helped teachers such as Sandy would be clearer articulations of learning goals for teachers. I argued that it is difficult for the teacher to recognize and comprehend reform ideas in the contexts of opportunities to learn, because they have so few first-hand experiences with the principles and practices underlying policies that call for substantial departure from traditional teaching. Clear articulations of learning goals potentially could offer teachers guidance for planning and making decisions about what is important to learn and what to participate in.

Educational researchers have begun to outline the vastness and enormity of the learning required for teachers to respond to the flood of policies in the late eighties and early nineties (Ball, 1997; Wilson et. al, 1996). The paragraph below highlights what may be needed.

> The teaching that reformers seem to envision thus would require vast changes in what most teachers know and believe. Teachers would have to revise their conception of learning, to treat it as an active process of constructing ideas rather than a passive process of absorbing information. They would have to rediscover knowledge as something that is constructed and contested rather than handed down by authorities. They would have to see that learning sometimes flourishes better in groups than alone at one's desk with a worksheet. And in order to learn, teachers would have to unlearn much deeply held knowledge and many fond beliefs. Such learning and unlearning would require a revolution in thought, and scholars in several fields have shown that such revolutions are very difficult to foment. Moreover, once teachers' academic knowledge and conceptions of learning changed, they would then have to learn how to teach differently. (Cohen & Barnes, 1993a, p. 246).

Given the variety and vastness of what is believed important for teachers' learning, it may be worthwhile to focus, at least initially, on a smaller subset of learning goals for teachers. Sandy's responses to the different strands of proposals focusing this study suggest that a teacher will narrow reform, to a smaller subset of the agenda, in order to define a starting place and make the workload more manageable. It may be worthwhile for policymakers and educators to plan in advance for this scaling down by asking what may be the most salient ideas for teachers to begin efforts to reform. The case studies in this work underscore the importance of this point. Sandy scaled down the agenda for herself and gave attention predominantly to only a few select ideas and proposals. Whether her decisions focused on the central tenets or most promising aspects of the reform agenda is an open question.

Sandy's emphasis on discrete mathematics, for instance, may or may not be a priority at the elementary level. If it is, then clearer ideas and articulations of what is important for elementary students to learn about discrete mathematics would be an important step toward identifying clearer learning goals and guidance for teachers' learning. Sandy's work with computation reminds us of the stranglehold that traditional computational skills instruction has on teachers' ideas about good mathematics teaching (Putnam & Geist, 1994; Wilson, 1997). One could argue that initial efforts should focus directly on the goal to teach computational skills differently. In California, policymakers and educators took a less than direct approach to underscoring the importance of this topic-specific area of mathematics. Yet, teachers' histories with teaching computation tell a very different story about the role that computation will play in teachers' efforts to change their practice. I argued that Sandy's ideas about computation were not uprooted, hardly even challenged, despite great effort to reform computational skills instruction. The idea of problem solving as a context for developing computational skills, in particular, was barely examined. Had Sandy had a clear set of learning goals within this topic-specific area, it may have directed her efforts toward the more central tenets of reforming computational skills teaching.

The goal then would not just be about making better learning experiences for teachers, although that would be paramount, but it would also be about offering teachers a form of guidance, a lens to help teachers, teacher leaders, principals and others to select and judge opportunities to learn. If clear articulations of learning goals were available, they could assist educators in recognizing what is essential to making sense of reform ideas. Had Sandy had clearer ideas about what and how she would need to learn, this could have helped to focus her efforts on central aspects of the reform agenda.

Policy levers do not offer such guidance. In Chapters 4 and 5 I argued that even opportunities specifically designed to press teachers toward desired practices offered conflicting messages of reform-based teaching. The state's testing program, for example, painted a very different view of the mathematics important for students to know and do than, for example, replacement units. The need for such a lens is underscored further when we take into account that teachers learn a great deal from occasions that are not intended as support toward the state's goals. In these settings teachers could use the lens to examine, select and then judge opportunities to learn on the basis of how they connect to learning goals for teachers. Teachers could sort through ideas and opportunities, forming priorities, and seeking out learning experiences that were well-matched with the principles important to understand the teaching and learning underlying instructional policy.

<u>The Important Role and Preparation of Teacher Leaders</u>

Guidance might also be offered in the form of well-prepared teacher leaders. In Chapter 5 I described the opportunities that Sandy offered other teachers for learning about the new framework in the contexts of state-sponsored workshops. An important question that surfaces from the study of her efforts to teach other teachers is whether Sandy was sufficiently prepared to teach others about reform-based teaching. After all, there is no formal, organized preparation for such a role. Sandy acquired a reputation for being an excellent mathematics teacher. Many would argue that she is a likely candidate for teacher leader. Yet, I have argued that the teacher, as a learner, is not well-positioned to guide even her own learning. For one thing, the policy proposals sketch dramatic departures from modal teaching, making it very difficult for any teacher to have deep knowledge of what is involved. Furthermore, most teachers would not be knowledgeable of the factors that affect teachers' learning. Sandy, for example, did not acknowledge or address teachers' prior knowledge and dispositions toward teaching computation at the workshops she offered.

Yet, what if conditions were optimal and formal preparation for teacher leaders did exist? What would such programs involve? Few projects to date have explored this issue. Even less is known about what teachers would need to know to facilitate other teachers' learning. At the same time, this study illuminates that even though Sandy is an exemplary elementary teacher, she could have benefited others in her role as teacher leader had she had much greater understanding of what underlies reform-based teaching and well-grounded ideas about how teachers might learn the ideas of reform.

There are several projects underway that consider the preparation of teacher leaders an important aspect of any change effort. One project describes four objectives as central goals of teacher leaders' knowledge: theoretical knowledge in mathematics education, methods of integrating new technologies, enhancing mathematical knowledge and didactic repertoire and developing leadership skills (Even, 1994). The objectives are aimed at developing accomplished professionals who understand teacher learning as well as goals and objectives for improving mathematics education.

Cohen & Barnes point out that such professionals would also have to develop relationships with teachers that combine trust and critical reflection (Cohen & Barnes, 1996, p.247). They stress that teacher leaders would have to understand the difficulties involved as teachers unlearn and encounter the uncertainties of changing teaching practice.

Another project underway, DMI (Developing Mathematical Ideas), has given much attention recently to preparing teacher leaders (Schifter, 1998). The project is looking carefully at what may be involved for teachers learning to facilitate other teachers' learning in relationship to DMI curriculum materials. DMI developers have created multiple supports for teacher leaders including a mentoring program and two-week institutes that explore DMI curriculum and teacher learning issues. Researchers studying this effort identified the importance of teacher leaders learning to facilitate "openings" in the curriculum, places in the discourse where participants' prior knowledge and beliefs potentially become central targets of exploration (Remillard & Geist, 1998).

This study adds further to these insights. In particular, teacher leaders would need to understand the important role that a teacher's analysis of the impact of changes on students' learning will play in sustaining any change effort. They would have to understand and be prepared to support teachers in formulating this analysis and sketching further directions for refining instructional strategies. In addition, teacher leaders would have to be prepared to support teachers who are likely to bring a stance of certainty to their opportunities to learn. More specifically, they would have to understand the individual nature of the disequilibrium a teacher is likely to experience. Another important aspect of the teacher leader's knowledge would be understanding the topic-specific nature of teachers' prior knowledge and teaching experiences, as well as the impact of differences on learning about reform-based teaching. I expand on each of these aspects of teacher leaders' knowledge as I discuss proposals that focus at the level of the individual teacher's sense-making.

Proposals to Improve Teachers' Opportunities to Learn

I turn now to focus on the individual contexts of teachers' opportunities to learn. Earlier I argued that inattention to the dynamic nature and the factors affecting the teacher's learning experience creates complexities that the teacher has to figure out alone. Here I speculate about what would be needed if these links were attended to and woven into the contexts of teachers' opportunities to learn. I sketch four objectives that potentially could strengthen a teacher's capacity to understand and enact reform-based teaching.

Learning to Analyze the Impact of Change

We saw in the case studies that a teacher's responses to policy do not stop once the teacher changes her teaching. Instead, a back and forth process of learning and change continues. Sandy formed and reformed her practice in relationship to much new learning. Yet, there seemed a critical point in her learning process that later became pivotal to Sandy's understanding and enactment of the policy. This involved Sandy's analysis of the impact of change on students' learning. Although much was done speculatively, the actual analysis of the consequences of change, based on the actual interactions with students in real classrooms, had a much greater impact on Sandy's interpretation of reform-based teaching. More specifically, Sandy's analysis reflected her belief that students' proficiency with computation remained a high priority, equal to that of conceptual understanding. It would have been interesting to see whether Sandy's analysis would have been different had students' learning goals been reassessed and factored into her analysis of the impact of the changes she made in her teaching.

It may be useful for teachers to have multiple opportunities to learn that focus directly on making sense of what happens once changes are made. This aspect of the teacher learning process deserves much more attention, especially in future educational research. We need to know much more about how teachers analyze the impact of changes, what they look for, and how they assess student learning goals in the context of change, including what might be used as evidence for improving learning opportunities. Whether teachers observe and analyze other teachers trying out similar strategies, examine video tape of one's own teaching or another's teaching, or become involved in ongoing discussions about the impact of change, these occasions could offer teachers a richer context for learning to analyze the impact of change.

One stumbling block to creating such opportunities would be getting past the belief that once teachers learn they can easily introduce proposals into teaching. Policymakers, educators, and teacher leaders would first have to come to understand that changes in teaching are not the end product of policy but instead mark the onset of critical new learning about reform-based teaching. Accepting this as reality for the individual teacher would not be an insignificant thing. This step alone could begin to fundamentally change ideas about what professional development for teachers would need to be. For example, one change might involve overcoming the idea that a teacher, single-handedly and simultaneously, could attend to the competing goals of implementing new instructional strategies, working with the constraints and uncertainties of classroom teaching, assessing students' learning experiences, and attending to their own learning agenda. An appropriate shift might be that teachers need multiple opportunities to unpack each goal and attend to each aspect, drawing information together over time to further refine teaching strategies.

Another change might be that other teachers or teacher leaders would need to become involved in the individual teacher's practice. Perhaps by observing teachers try out new instructional strategies and helping them make sense of what happens, especially in analyzing students' learning experiences. Other teachers or teacher leaders might also become involved by teaching with the teacher or supporting the teacher through ongoing conversations and decision making, perhaps around video taped teaching. The overall idea would be to support teachers' efforts to systematically inquire into teaching as a group as well as individually. From this standpoint, teachers would learn to work the ideas of reform into teaching collectively and help each other analyze the effects of new strategies. The impact of such learning could strengthen a teacher's capacity to connect reform-based teaching ideas to the practical realities of teaching. Involvement in an ongoing study group, for example, or interaction with a teacher leader could have been quite useful in helping Sandy analyze the impact of the changes she made, especially in making sense of the problems that arose in her practice. Study groups can offer teachers an opportunity to explore reform ideas in the context of their own or another's teaching experiences over time (Featherstone et al., 1993a, 1993b).

Fostering Beliefs About Improving Students' Opportunities to Learn

A significant aspect of Sandy's analysis, of the impact of changes she made in her teaching, had to do with her beliefs about whether students' opportunities to learn were improved. Recall that Sandy, in the second year, delivered a mix of innovative and traditional teaching strategies to address learning objectives for students. Her decision to reinstate traditional forms of teaching would signal to many a failure to implement policymakers' ideas. Yet, I argued that the mix of old and new strategies was Sandy's interpretation of policymakers' ideas. And I argued that her interpretation rested largely on her analysis that students' opportunities to learn were not necessarily improved by increasing emphasis on conceptual and contextual goals. She reinstated drill and practice on number facts and algorithms to round out students' overall experience, claiming that these changes better represented what policymakers had in mind.

This aspect of Sandy's learning experience points to the importance of attending more explicitly to teachers' beliefs about how students' opportunities to learn will be improved. It may be necessary to focus directly on teachers' beliefs about how reform-oriented teaching improves students' learning experiences in the contexts of teachers' opportunities to learn. I argued that Sandy relied on herself for making sense of students' learning experiences. She noticed that the changes she made assumed mastery, proficiency, accuracy and speed with computing. Because she valued these goals, she evaluated students' experience , at least in part, on the basis of whether these goals were attended. Yet, I argued that the reform documents indicated these goals were less immediate and conceptual and contextual learning were seen as more central. Sandy's beliefs about whether students' opportunities to learn were improved rested on misguided ideas about how students' opportunities to learn would be improved. These issues were taken for granted on any wider scale, perhaps the were assumed as part of buying into the reform package.

Teachers' beliefs about improving learning experiences for students may need to be a central focus of teachers' opportunities to learn. Inattention to these beliefs, for Sandy, fostered the idea to balance the old and new forms of teaching. In a worse case scenario, inattention might suggest that reform is optional. If teachers had opportunities that attended explicitly to their ideas of whether and how students' opportunities to learn were improved, it could also encourage teachers to work through problems in practice rather than return to traditional strategies. Fostering beliefs about how traditional forms of teaching and reform-based teaching do and do not "improve" students' opportunities to learn can function as a pivotal point in teachers' decision making. Overall, teachers may be far less likely to discard or compromise new teaching strategies because they understand the implications of both reform-based teaching and traditional teaching on students' learning opportunities.

Fostering A Stance of Critique And Inquiry

Ball and others argue that because policy goals involve work that is uncertain and underdetermined, a stance of critique and inquiry, one of asking, debating, formulating and exploring conjectures and deliberation would be far more productive for teachers (Ball, 1996; Lord, 1995). Fostering a stance of critique and inquiry would therefore be a critical element of the learning environment as teachers work to interpret policy and progress into various phases of learning, change, and refinement of reform-based teaching ideas.

In this study, I argued that this teacher approached reform with a stance of certainty. And although Sandy participated in several collegial learning environments, few offered an opportunity to experience a stance of critique and inquiry first-hand. In addition, when Sandy did participate in environments that were less certain and embraced more complex views of learning, Sandy often withdrew and developed learning experiences that were more isolated, claiming the more collegial environments were less productive because they produced much frustration and confusion.

Sandy's experience raises many questions about how a teacher's stance toward learning might be altered. To the extent to which this is an individual or a matter of professional community is an important question. We already have evidence to suggest that hearing how others interpret and manage issues regarding change can support teachers as they work to change their own personal stance toward learning (Lord, 1994; Lieberman, 1995; Little, 1993). Teachers' questioning and exploration when situated in common learning communities can be a central source of support for teachers as they struggle with relevant issues of changing teaching. At the same time, opportunities to learn that value a stance of critique and inquiry can represent difficult situations for teachers that bring a stance of certainty. Sandy withdrew into materials such as curriculum units and the framework when she encountered difficulties in learning with others. Interviews revealed that she did not recognize collegial environments or differences in stance a crucial aspect of her own learning.

It may be that teachers need a combination of pressure to participate and support within environments that offer an opportunity to experience a stance of critique and inquiry. The only way teachers can have a personal experience interacting in collaborative environments is if they are required to participate in collegial opportunities to learn that embody a stance of critique and inquiry. If Sandy were to experience first-hand how interactions with others increase the potential of her own learning, she must participate in environments that offer the opportunity to recognize differences in stance. At the same time, adequate support must also be offered, so that effective collaborative work can become part of the teacher's personal learning experience. Recall that the disequilibrium Sandy experienced created self-doubt and frustration rather than questioning and debate. If Sandy's experience were to be more productive, she would have to experience disequilibrium differently, recognizing it as an essential element of the process, learning to question and debate the issues creating the uneasiness. This means she would have to continue to participate and work in these environments rather than turn away from them. And the environment must be supportive in ways that press Sandy gently toward a shift away from certainty and toward a stance of critique and inquiry.

Several non-traditional projects underway count the construction of these environments where issues can be discussed and debated as essential elements of an educative atmosphere (Ball, 1996, Brown, 1994; Featherstone et al., 1993a, 1993b). If supportive environments such as these were developed in both traditional and non-traditional structures and teachers were required to participate in them, it could be an effective way to strengthen teachers' capacity to approach reform-based change with a stance of critique and inquiry. Wide-scale participation would build teachers' capacity to recognize and understand environments that do and do not value such a stance (see Lortie, 1975, p.70). Developing such capacity can function to guide teachers in their selections and judgments of opportunities to learn.

Subject-Matter Focused, Topic-Specific Opportunities to Learn

An important contribution of this study is recognizing the significance of topic-specific differences in teachers' prior knowledge and teaching experiences and the impact of those differences on teachers' responses to reform-based change. Findings suggest that just as changing teaching across subject areas is not a general proposition, changing teaching within subjects is also not a general proposition. Although others have pointed to the importance of considering subject-matter specific aspects of learning to teach (Shulman, 1986; Stodolsky, 1988), this study suggests we must go one step further than previous studies have argued important. Teachers' topic-specific, subject matter knowledge affects what they can learn in a climate of educational reform. I found that within the same subject-matter there are significant differences in both what teachers bring and what teachers would need to learn to teach different topics in mathematics for understanding. Recall that Sandy often found herself trying to understand how to introduce discrete mathematics into her teaching lacking the necessary prerequisite knowledge of mathematics needed to understand the examples and illustrations offered. Sandy's opportunities to learn overall did not attend to her prior knowledge, skills, and dispositions and whether they set her up productively to learn in the circumstances offered.

One implication of these findings is that subject-matter focused, topic-specific learning opportunities may be required for teachers to learn to change their teaching of any specific mathematics topic targeted by policy. To date, topic-specific aspects of changing teaching is a relatively untouched and unexamined area in educational research. Much more research would be needed to characterize and understand the nature of these differences and the impact on teachers learning to change their teaching. However, there is much in the learning theory literature that does bear on these findings. Situated learning theory supports the principle that learning is dependent on elements of the context and content (Brown et. al, 1989, Lave & Wenger, 1991). Furthermore, if differences in subject-matter represent a critical variable in the learning environment, it would follow to reason that topic-specific differences are crucial aspects to attend to as well.

If we were to attend to the topic-specific nature of teacher learning, we would need to know much more about how differences impact teachers' responses to policy. Although policies often nominate specific-topics in mathematics considered important for student learning, what teachers bring and would need to learn to teach those topics is not addressed. Tracking down the nitty-gritty of what teachers would need to learn to change their teaching of any topic in mathematics would not be a straightforward task. My analysis in chapter 3 contrasts extant ideas and practices with policy proposals for two different topics. This process may be useful for drawing out both what teachers would likely bring in relationship to specific topics, as well as ideas about what policy proposes teachers should change. Figuring out where and how the two might connect can be useful toward developing and articulating learning goals for teachers within topics.

Yet, much more research would be needed to understand the role of differences across topics and the impact of differences on the learning agenda for teachers. In the case of discrete mathematics, for example, Sandy had few ideas to draw from for making sense of the proposed changes. She knew little about discrete mathematics and had no experience teaching it. Essentially, Sandy was unprepared to select and configure a course of study for introducing discrete mathematics into elementary mathematics teaching. Yet, as a teacher leader, she had to design an overall plan in order to teach other teachers about the new framework. Recall in chapter 5, I argued that there were more fruitful sites than relational theories or discontinuous mathematics that could have focused Sandy's learning experiences. Sandy's response to withdraw from these experiences and to focus on the ice-cream cone problem seemed a more promising direction because the ideas connected more closely to ideas she brought to the learning experience. At the same time, Sandy worked alone making sense of the mathematics underlying the problem. My examination of the opportunities to learn that Sandy offered others revealed that combinatorial reasoning was completely side-stepped as central aspect of teaching combinatorial counting for understanding.

In the case study of Sandy changing her teaching of computational skills, there was much to be challenged, up-rooted and transformed in Sandy's thought and practice. Researchers have already pointed to the importance of un-learning in changing teaching (Ball, 1988). Yet, we have few ideas of what un-learning would involve for specific topics in mathematics. We saw that this teacher became stuck around issues of mastery, speed, accuracy, proficiency in computing, as well as memorization of number facts and algorithms. And even though Sandy went on to learn new ideas and practices for teaching computational skills, she easily returned to traditional ideas when problems arose in her practice. This easy return suggests that Sandy had little evidence of the impact of traditional forms of teaching on students' learning experiences. Although Sandy heard the arguments for why traditional forms of practice, such as drill and practice and the importance of speed and accuracy in computing, limit students' flexibility with computational skills, she had no first-hand experiences that produced evidence of the impact of traditional forms of practice. Sandy's experience lacked an important connection that would be required to sustain and refine new forms of practice. Had she been convinced of the impact of traditional forms of teaching, it may have been more likely that Sandy would continue to examine alternatives instead of returning to traditional patterns. For Sandy, traditional forms of practice remained a legitimate option.

Findings such as what I describe here can be very useful for developing topic specific learning goals for teachers. Educators and teacher leaders would then have to work to understand the relationship between what teachers bring across targeted topics and what teachers would need to know and do to teach those topics differently. Ultimately, the aim would be to produce opportunities to learn that attend to differences and the impact on a teacher's responses to policy.

Change At the Level of The Smallest Unit

In this study, the conversation about the policy and practice relationship is situated within the realm of the teacher's learning and teachers' professional development. I argued that even if the state could find ways to align its vision more effectively across policy instruments, it would not have helped this teacher respond to the policy any better. Although systemic alignment, regulation, incentive, restructuring, as well as many other factors, may, in fact, be part of the overall plan, this study illuminates the importance of attending to the educational issues that affect a teacher's understanding and enactment of policy. Only then are we likely to improve teaching for U.S. children (Ball & Cohen, 1995a, 1995b; Little, 1993; Sykes, 1996; Smylie, 1997, Wilson et al., 1996).

Since the onset of this work, much has changed in California. It was just a short time after the 1992 framework that reform began to disintegrate. Wilson, in a recent paper describing the events in California, writes:

> "groups such as HOLD (Honest Open and Logical Debate), a parent and community organization concerned with what they called the "'discovery-based constructivist'" math characterized by the framework began to appear at public hearings and on the World Wide Web. Their concerns were many -- that the "new math" ways of teaching were untested and unproved, that the large scale empirical work done on mathematics instruction demonstrated that direct instruction was more effective, that the frameworks emphasized mathematical appreciation, not mathematical content knowledge, that the new tests being advocated were "subjective" . . .

Wilson explains that time and again two issues arose in the worries of groups like this: the lack of attention or de-emphasis on basic and computational skills and the sacrifice of conversations about content in the name of conversations about pedagogy (Wilson, 1997). By 1995, the new superintendent of schools ordered a task force to examine the state of affairs in mathematics education. The task force issued reports calling for "balance" and arguing for more "basic skills" and traditional mathematics instruction along with an increased emphasis on problem solving. Ultimately, a new committee began work on a new mathematics framework- a more balanced approach - for mathematics education. The New York Times reported in November of 1997, that the California Board of Education had endorsed a new set of standards for elementary and middle school students that emphasized the more traditional drill and practice approach over a new method that was characterized by complex word problems.

This dissertation offers policymakers and state officials a portrait of one teacher's efforts to attend to many of the issues dominating conversations and current debate about these reforms and the more recent return to traditional practices. The portrait illuminates the complexity involved for the individual teacher in the context of strong systemic efforts to reform. It illustrates that there are a variety of different components to a teacher's practice and that there is much for the teacher to think about and do to revise, develop, and compose in the course of efforts to improve instruction.

Further, it shows that the mathematics framework, and the other levers aligned by the state, are not the only influences on a teacher's practice, and that many of the influences are not always recognized or coordinated. Findings also reflect that what the teacher encounters as professional development experiences can be insufficient for meeting the individual learning needs of the teacher. And to make up for the lack, the teacher will likely develop and coordinate learning experiences for herself to attend to what she believes is needed. However, the analyses across this dissertation suggest that the teacher, as a learner, may not be well-positioned to do this alone. This claim represents a fundamental statement about who the teacher is in the context of external efforts to reform. The teacher, as the learner, will likely bring knowledge, skills, and dispositions toward teaching that run counter to reform goals. Thus, it would be very difficult for the teacher to provide the guidance needed to insure that learning experiences were coordinated and more coherent to the teacher.

And yet, what the teacher brings and musters as resources and learning, will have a great deal to do with whether reform efforts continue to press forward at the level of the teacher's practice. In Sandy's case, she brought, for example, the conviction that learning was necessary, a stance of certainty toward learning, a view of reform-oriented teaching, and a particular view of mathematics grounded in the philosophy of Project AIMS. What Sandy brought influenced her responses to the framework in the form of shaping her course of action, what she thought she would need to learn, and where she would turn for guidance. I argued in the case studies that Sandy often relied upon resources that resonated well with her prior knowledge and teaching experience, in part, because she had direct access to these resources. Many of these occasions embodied assumptions about mathematics teaching and learning that were incompatible with the ideas and practices described in the reform documents. As a result, Sandy's understanding of reform-oriented teaching was hardly challenged as most assumptions were left unacknowledged and unexamined across learning experiences.

Thus, the question arises as to how teachers' educational experience might need to change, so that it offers a more coordinated and coherent approach toward the goal of reform-oriented teaching. I have sketched several ideas in this chapter as a closing to this work. In essence, I have suggested that any efforts to effect and sustain change at the level of the teacher's practice would be based on understanding better the relationship between the teacher's needs as a learner and the environments of instruction for teachers, and working to create more meaningful links between the two.

Appendices

Appendix A

Appendix A

The table below summarizes the big ideas predominate across the opportunities to learn that Sandy encountered for learning about discrete mathematics and basic computational skills. I prioritize the ideas by what the opportunity emphasized most, substantively.

Table X: Predominant Ideas Emphasized Across Sandy's Opportunities to Learn

	Basic Computational Skill	Discrete Mathematics
Conversations with Other educators	Balance basic skill instruction with conceptually oriented curriculums Emphasis on technology in the classroom although not directly associated to computational skill	relational theories and algorithms what discrete means discrete mathematics is included for the secondary curriculum to attend to ideas not currently getting taught

Reading Reform Documents	Estimation, technology , alternative algorithms as computing strategies Conceptually orienting computational curriculum Contextualize skill instruction	Introduced as a new strand of content important to include at all levels of the curriculum ideas are mostly in the current curriculum but would need different emphasis and organization Include discrete math because it is the math of our time - more current than other mathematics already being taught Limited ideas about what discrete math is general points about pedagogical approaches i. e. should not be algorithmic or involve the memorization of formulas, and instead should encourage a conceptual understanding mentioned a number of ideas central to discrete mathematics

Doctoral Studies	Bloom's automaticity ideas Emphasis on constructivist ideas about learning Emphasis on cognition, understanding and problem solving as curriculum goals as opposed to memorization of routine procedures	none
Curriculum Materials	No direct attention to proficiency with computation Emphasis on contextualizing and conceptually understanding of four operations Alternative computing strategies	the ideas of combination and permutation looking for patterns in counting what discrete mathematics involves alternative pedagogy for teaching ideas (moves away from traditional explain and tell types of instructional patterns)
Teaching experiences	Difficulties and uncertainties of conceptually focused instruction of four operations to promote proficiency with computation More direct attention to computational goals	ideas of combination and permutation Strategies for counting outcomes how some decisions in mathematical problems can be turned over to the learner shaping the mathematical situation. Sandy interprets this to suggest her teaching is less didactic

Professional development	no direct attention to computational skill but offer ideas for how to Contextualize mathematical content inside problem solving settings Ideas about mathematics, its usefulness, and what conceptual ideas are related to the four operations suggests alternative pedagogy for the teaching of mathematics more generally	Examples of different kinds of counting problems Alternative pedagogy for the teaching of mathematics , mostly involving problem solving.

The ideas in each category represent the substantive focus of each of the six different categories of opportunities to learn. My determinations were based on interview data and therefore factored in Sandy's sense of what the big ideas were as well. My own perceptions of what the opportunities offered also contributed to my determinations. Data sources included my observations of Sandy's teaching, my analysis of the curriculum materials she used, my analysis of artifacts from her doctoral coursework, my reading of the reform documents, and my observations of the professional development activities she engaged in including Project AIMS and the state-sponsored workshop.

The Substantive Focus of Sandy's Learning Experiences For Changing Her Teaching Of Computational Skills

The predominate idea across the six different occasions for reforming instruction of basic computational skills focused on the notion that students must understand the mathematics of computation conceptually. Each of the six categories involved some aspect of what it would mean to conceptually understand and conceptually orient the elementary mathematics curriculum. Sandy's learning experiences centered on the idea of conceptually understanding the four operations. For example, the reform documents suggested that emphasis be shifted away from the preoccupation with calculating answers and move toward understanding how operations work and are useful. The curriculum materials Sandy used focused on the conceptual underpinnings of the four operations. Project AIMS materials focused on using computational skills in the service of solving real-world problems.

Sandy's learning experiences also focused on the idea of contextualizing mathematics. Sandy's work with Project AIMS and her reading of the reform documents focused more generally on learning mathematics in relation to problem solving experiences. Some of the other materials Sandy used focused on students' strategies for computing in problem solving contexts. At the same time, none of the experiences she encountered focused on whether or how the traditional computational algorithms might be treated in relationship to problem solving. My analysis in chapter three revealed that the reform documents did not attend directly to the traditional computational curriculum, its problems or what may still be reasonable, leaving the reader to dig out and figure out why recommendations were proposed.

Most of Sandy's learning opportunities did not give attention to the proposals to introduce a variety of new computational skills. Interestingly, the only opportunity Sandy had that emphasized estimation as a new computing technique was what she read in the reform documents. Yet from a policy standpoint, estimation was a big idea. Estimation, for the most part, remained untouched across Sandy's learning experiences. Recall in the case study that she returned to practices that used estimation mainly as a checking strategy.

Technology also did not get attention across Sandy's opportunities to learn. Although technology was discussed in the context of Sandy's conversations with other educators, the other categories of learning experiences that Sandy had gave only rhetorical consideration to these proposals.

Sandy's learning experiences did focus on students' algorithms. The curriculum materials she used promoted the idea that students naturally derive alternative strategies for making computations. At the same time, Sandy's work with students' computational strategies was the primary source of the difficulties she encountered in her teaching. Recall that although Sandy encouraged students to compute using their own strategies, students were often slow and constructed strategies that did not produce accurate results. In response Sandy demonstrated the traditional algorithm and suggested that students use it in place of their own strategies. Sandy's opportunities to learn did not focus any attention on how she might manage the problems that arose in her practice.

Sandy encountered the idea of balance. In her conversations with other educators, Sandy encountered that mathematics teaching should emphasize both skill learning and conceptual understanding. For instance, in her doctoral studies, Sandy encountered the idea of automaticity, suggesting to her that rote memorization of particular mathematical skills is central to learning mathematics. Across Sandy's opportunities to learn the emphasis was on conceptual and problem solving approaches to teaching. At the same time there was emphasis on a balanced approach to the teaching and learning of mathematics. In combination, the two ideas folded in comfortably, one along side the other. Sandy drew from her learning experiences that both the conceptual and the procedural knowledge in mathematics still mattered and that pedagogically each must be attended. Because Sandy encountered little opportunity to learn about how each could be attended within the same learning experience, she concluded each must be attended and in ways already available and understood. In her doctoral studies, Sandy encountered the idea of automaticity, suggesting that rote memorization of particular mathematical skills is central to learning and doing mathematics. This knowledge reinforced her sense that computational skill learning should be approached in rote, drill oriented learning opportunities. As another example, Sandy encountered no opportunities for learning how number fact acquisition might be nested in problem solving contexts where learning specific facts made sense.

The Substantive Focus of Sandy's Opportunities to Learn About Teaching Discrete Mathematics

Sandy's opportunities to learn about discrete mathematics focused on a wide variety of mathematics. Yet most of the occasions focused on mathematical ideas Sandy had little or no prior experience with. For example, Sandy's conversations with other educators focused on the ideas of continuous and discontinuous mathematics, relational theories (recurrence relations), and counting principles Sandy had no prior experience with. The curriculum materials she used focused on combinatorial counting problems. At the same time they did not explore any underlying ideas of combinatorial counting such as combinatorial reasoning. Instead, Sandy's experience with the materials focuses on the idea of order and patterns. Recall that Sandy's reading of the reform documents focused on matrices.

Sandy encountered contradictory information concerning whether discrete mathematics should get attention in the elementary school curriculum. Recall that she read in the framework that discrete mathematics should be treated as any other strand. Yet, she learned from a leading mathematics educator that discrete mathematics was introduced for the secondary curriculum. The reform documents also seemed contradictory giving explicit attention to discrete mathematics only in secondary focused sections.

Sandy encountered the idea that discrete mathematics should not be taught in ways that emphasized learning rules, memorization of formulas, or symbol manipulation. Sandy's learning experiences focused on the idea that teaching discrete mathematics in ways that emphasized traditional patterns was not appropriate.

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