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DOMESTIC RISK ON SMALL OPEN ECONOMIES**

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Ramón E. Pineda S.

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Ph D degree in Economics

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ESSAYS ON THE EFFECTS OF CHANGES IN THE DOMESTIC RISK  
ON SMALL OPEN ECONOMIES

By

Ramón E. Pineda S.

A DISSERTATION

Submitted to

Michigan State University  
in partial fulfillment of the requirements  
for the degree of

DOCTOR OF PHILOSOPHY

Department of Economics

1998



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## ABSTRACT

### ESSAYS ON THE EFFECTS OF CHANGES IN THE DOMESTIC RISK ON SMALL OPEN ECONOMIES

By

Ramón E. Pineda S.

This dissertation examine the effects of changes in the domestic risk on small open economies. In Chapter 1 we investigate how resident investors in a small open economy adjust their holdings of domestic and foreign assets (portfolios), in response to changes in the internal risk. A general wisdom maintains that an increase in the domestic risk induces the native capital to flee out especially, but not exclusively, of the developing economies. In this chapter, we establish sufficient conditions (restrictions on agents' attitude toward risk) under which this claim is true both in a partial and in a general equilibrium level. We find that an increase in the domestic risk (modeled as a mean preserving spread in the domestic bond's real return distribution) unambiguously causes capital flight if agents' preferences show either: i) a positive third derivative of the instantaneous utility function, relative risk aversion no larger than one and increasing in the domestic return, and non-decreasing absolute risk aversion; or ii) a null third derivative of the instantaneous utility function. This outcome is somewhat controversial because sufficient conditions for the conventional wisdom imply very restrictive sets of assumptions about agents' attitude toward risk.

In Chapter 2 we evaluate how an increase in domestic risk affects the equilibrium inflation rate in a small open economy. An increase in the domestic risk (modeled as a

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mean preserving spread in domestic bond's nominal return distribution) induces a reallocation of domestic agents' portfolios between foreign and domestic bonds. The adjustment in agents' portfolios affects domestic government's ability to finance its expenditure stream, since domestic investors could be willing to reduce their holdings of the domestic bond. This situation makes the domestic government rely more on the expansion of money supply as a source of fiscal revenue, and this could lead to higher inflation. We find that a sufficient condition to unambiguously determine the inflationary consequences of an increase in the domestic risk if we use a constant relative risk aversion utility function is to assume a coefficient of relative risk aversion no larger than one. If this is the case, and the domestic government has limited access to the international financial markets and a given expenditure stream an increase in the domestic risk induces a rise in the equilibrium inflation rate of a small open economy.

To Adriana Bermúdez  
for all the wonderful things that you have done to me

and

To Rosa, Ramón,  
Scarlet, José and Adriana Pineda  
for your love, encouragement and understanding.

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## ACKNOWLEDGMENTS

Many people had made possible the completion of this dissertation and I would like to thank some of them briefly. First, my most sincere thanks to Gerhard Glomm, Rowena Pecchenino and Robert Rasche who have been the members of my committee. For all the time they put into this project, their friendship, constant encouragement, patience, guidance and advice, I am truly grateful.

I am deeply indebted to Jack Meyer and Corinne Krupp who took time to read some of the essays in this dissertation, and provided very useful comments. I would also like to thank the faculty at the Department of Economics for their help and support during my studies at Michigan State University. In particular, I want to thank John Strauss, Jeffrey Wooldridge, Peter Schmidt, Steve Matusz, Charles Ballard, Richard Baillie and Carl Davidson for the knowledge that I received from them.

My fellow graduate students have played a major role in my formation as an economist. I especially would like to thank Pablo Fajnzylber, Yali Molina, Catalina Amuedo, Heather Bednarek, Min Chang, Bonnie Anderson, Hirokatsu Asano and Sooyung Jang.

I would also like to thank my friends at the Economics Institute, Woo Jung, Emmanuel Skoufias, Jeffrey Zax, Hamid Baghestani, Charles Becker and Maureen Kilkenny, without them things would have been a lot harder in graduate school.

I want to thank Leopoldo López Gil and George McCandless, because it was their idea that has made all this possible. I want to acknowledge the financial support of “La

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Fundación Gran Mariscal de Ayacucho” and “El Consejo Nacional de Investigaciones Científicas y Tecnológicas”. I am truly grateful to these institutions, and to the Barreto family, their help and trust have been critical during all this time.

I am indebted to the Venezuelan and Latin American communities in MSU; they have significantly improved my standard of living. I especially want to thank the Méndez, the Medina, the Fajnzylber, the Sorzano and the Uzcategui families. I would also like to thank the volleyball group at Spartan Village. What a nice group of people!

The greatest thanks, however, go to my family. My wife Adriana Bermúdez has been a constant source of inspiration and encouragement. Her dedication and support have been great. For all of this and more, she is “la mujer de mi vida”. My parents, Rosa and Ramón Pineda, have always helped and supported me in everything; this was not the exception. Their love, patience and trust are things that I would never be able to repay. My sisters and brother, Adriana, Scarlet and José Pineda, have always made my life easier and happier. It is impossible to have better siblings. My grandparents Eumelia Pineda, Juana and Claudio Salazar, have always encouraged me, with love and tenderness, to do my best. My aunts, uncles, cousins, and friends have always offered me the best in them.

I would also like to thank those “brothers” that life has given me, Omar Bello, Diego Restuccia, Giovanni Di Placido, Rodolfo Méndez, Simón Lamar, Alexander Elbittar and Eduard Sánchez. All of them, and their families, have played a significant role in my life.

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I would also like to thank my in-laws for their support and trust. It is not possible to have better in-laws. Finally, I want to remember a very special group people that are no longer among us “la abuelita” María Figueroa, Angel Octavio and Octavio Colina, “la tía Jacinta”, Yolanda Hernández and Virginia Milano. Their love and support throughout their lifetime were truly invaluable.

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# Chapter 1

## Capital Flight and Changes in The Domestic Risk

### The Role of Agents' Preferences

#### 1.1. Introduction

A general wisdom maintains that an increase in the domestic risk induces resident capital to *flee* especially, but not exclusively, from developing economies. The purpose of this chapter is twofold. First to determine how domestic investors in a small open economy adjust their holdings of domestic and foreign assets (their portfolios) in response to *changes in domestic risk*. Second, to establish sufficient conditions (restrictions on agents' preferences) that validates the conventional wisdom.

Lessard and Williamson (1987) define *capital flight* as: "...the resident capital (that) flees because of the perception of abnormal risk at home...". During the last two decades significant amounts of resources have departed suddenly from developing economies under the form of capital flight. Claessens and Naudé (1993) document that until 1991 the *cumulative value* of the resources that have migrated as capital flight represent 118% of the GDP in North Africa and Middle East, 85% in Sub-Saharan Africa, 40% in Europe and Central Asia, 35% in Latin American and the Caribbean and 15% for East Asia and the Pacific.<sup>1</sup>

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<sup>1</sup> See Tables 1.1-1.3 for additional estimates of capital flight.

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Since the debt crisis in the early 1980's several works have tried to determine factors that explain episodes of capital flight in developing countries.<sup>2</sup> Some of these studies --Khan and Ul Haque (1987), Cuddington (1986 and 1987), Cumby and Levich (1986), Dooley (1986), Lessard and Williamson (1987), Padilla (1991), Claessens and Naudé (1993), Kant (1996) and Schineller (1997)-- are empirical. A common feature among these empirical studies is the use of variables such as the interest rate differential, the growth of domestic output, the domestic inflation rate, and internal fiscal imbalances, etc., not only because their relevance in the process of resource allocation, but also because researchers want to utilize them as proxies for changes in domestic risk.<sup>3</sup>

One of the first problems that any empirical study in this area must face is to find an operational (empirical) definition of capital flight. The literature offers several alternatives that are based on two estimation approaches: the *balance of payments* or the *direct* approach, and the *residual* or the *indirect* approach.<sup>4</sup> After the empirical or operational definition is selected country-specific or regional time series of capital flight

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<sup>2</sup> Recently, interest in the analysis of capital flight has increased considerably due to the instability in the international markets caused by the Mexican crisis of 1994-95 and the Asian crisis that started during 1997.

<sup>3</sup> For instance, Dooley (1986) considers the rate of return variables as proxies for different perceptions of financial and expropriation risk.

<sup>4</sup> As its name may suggest, the *balance of payments* or *direct* methodology uses the capital account of the balance of payments to directly determine estimates of capital flight. Cuddington (1986) uses this methodology and defines as capital flight the short-term speculative outflows. The *residual* approach --introduced by the World Bank (1985)-- tries to determine the changes in a country's holdings of foreign assets. This method is called *indirect* or *residual* because both the uses and the sources of foreign assets must be determined before we can estimate the amount of capital flight. See Cumby and Levich (1986) and Kant (1996) for a good summary of different definitions of capital flight. See also Cumby and Levich (1986), Padilla (1991) and Claessens and Naudé (1993) that construct estimations of capital flight applying several definitions of capital flight over a common data set.

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are created. Nevertheless, this way of constructing capital flight time series may be inappropriate because researchers apply their operational definitions to periods in which *changes in domestic risk* may not have taken place, casting doubt on the validity of the results. Thus, we think that to stay close to the (generally accepted) definition of Lessard and Williamson the operational definition must be estimated only in those periods in which ‘evidence’ (or at least a strong presumption) of changes in the domestic risk are present.

Other studies --Khan and Ul Haque (1985) [KU]-- try to *model* the connection between changes in the distribution of domestic return and capital outflows in a small open economy.<sup>5</sup> They show that when the probability that the government may confiscate domestic firms goes up, domestic investors tend to increase their holdings of foreign assets. In other words, they find that an increase in (what they call) the *expropriation risk* may induce an increase in the amount of domestic capital sent abroad.

The present study is in some sense an extension of Khan and Ul Haque (1985).<sup>6</sup> As they do, we study how individuals (investors) that maximize expected-utility adjust their portfolios as a result of changes in the distribution of domestic asset returns. Furthermore, we analyze how those reactions may induce capital outflow in a small open economy.

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<sup>5</sup> The literature of currencies crises --that started with the works of Krugman (1979) and Obstfeld (1986)-- is also closely related with the study of capital flight.

<sup>6</sup> However Khan and Ul Haque (1985) address issues that are outside the scope of the present study. For instance, they address topics such as expropriation and debt repudiation.

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We assume that the domestic economy is inhabited by a large number of agents that live for two periods. Agents maximize expected lifetime utility by choosing a consumption stream. In the first period of their life, each agent receives an endowment of consumption goods that are traded in a perfectly competitive international market. Agents receive no endowment in the second period.

Agents save part of their endowments to finance second period consumption. We assume that agents use two saving alternatives: foreign and domestic bonds. Since the domestic economy is small, the return on foreign bonds is determined in the international market, and it is taken as given by the domestic investors. Furthermore, we assume that it is known with certainty. The other kind of asset in this economy, domestic bonds, is risky. We assume that agents know the domestic bond's return distribution function. The domestic economy also has a government that collects taxes from and sells bonds to the domestic agents.

This is the context in which we study the effects of changes in the distribution of domestic returns on domestic investors' equilibrium portfolio allocations. We do not model the causes of the assumed changes in the domestic return distribution; they are treated as exogenous. With this *exogeneity* assumption we try to illustrate changes in risk that are outside the control of (are not affected by the actions of ) the domestic agents.<sup>7</sup> We consider two forms of transformation of the domestic return distribution: first degree stochastic (FDS) and mean preserving (MP) changes. By using FDS changes we focus on

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<sup>7</sup> The causes of the change in the domestic return's distribution may be induced by instability either in the international (*contagion effect*) or the domestic financial markets; all that matter is that domestic agents, for some reason, perceive an exogenous change in the distribution of domestic returns.

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the effects on agents' (and the economy's) equilibrium outcomes of alterations in the distribution of domestic returns that involve changes in its expected value. Notice that KU's *expropriation risk* is a particular form of FDS transformation in which agents face changes in the expected value of the domestic return.

In contrast to KU, we also consider mean preserving transformations in the distribution of the domestic return. We think that it is also important to assume MP transformations because they represent *changes in risk* in a Rothschild and Stiglitz (1970) sense.<sup>8</sup> Furthermore, this assumption seems to be more appropriate than FDS changes, at least based on Lessard and Williamson's definition, if we want to interpret capital flows as capital flight (flows induced by changes in domestic risk).

To do the comparative statics analysis we use standard methodology in the literature of decision making under uncertainty. We carry out the analysis at two levels: i) a partial equilibrium level in which we evaluate only the behavior of the domestic investors; ii) a general equilibrium level where the interactions between the private agents and the domestic government are considered.<sup>9</sup>

During the investigation we ask to what extent can we use assumptions about domestic agents' attitudes toward risk to establish sufficient conditions such that changes in domestic risk unambiguously induce capital outflows. We find that:

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<sup>8</sup> FDS changes are used to illustrate situations in which we compare higher/better outcomes with lower/worse ones; MP transformations allow us to focus in situations in which the trade-off between risk and expected return is not present since by definition in MP transformations the expected value of the risky return is held constant.

<sup>9</sup> Capital movements may produce several effects on the domestic economy but for the purpose of this paper the most (the only) relevant one is that it has on the government's financial process. See Cardoso and Helwege (1995) for a complete summary of the cost associated with capital flight.

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1) If agents' preferences exhibit risk aversion together with a coefficient of relative risk aversion no larger than one, and nondecreasing absolute risk aversion, then a FDS deterioration of the distribution of domestic returns induces a reduction in the holdings of the domestic asset, an increase in the holdings of the foreign asset and an increase in total savings.

2) A mean preserving spread of the distribution of domestic returns causes a reduction in the holdings of domestic bonds, an increase in the holdings of foreign assets and an increase in total savings if preferences show either: i) risk aversion, nondecreasing absolute risk aversion, a positive third derivative of the utility function, and a coefficient of relative risk aversion no larger than one which is also increasing in the domestic return; or ii) a null third derivative of the utility function. In this case capital outflows can be interpreted as *flight* because the outflows are induced by changes in domestic risk. This finding is somewhat controversial since *sufficient* conditions for the *conventional wisdom* imply a very specific and restrictive sets of assumptions about agents' preferences. In particular, it is troublesome that we cannot obtain unambiguous comparative static results when domestic investors' attitudes toward risk exhibit the standard assumption of decreasing absolute risk aversion.

The rest of the chapter has the following structure. In part II we describe the characteristics of the economy and define a competitive equilibrium. In part III we evaluate how changes in the distribution of the domestic bond return affect the behavior of the domestic investors. In this part we establish some results at a partial equilibrium level. Part IV contemplates the analysis at a general equilibrium level. Part V contains a

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digression concerning some of the assumptions used in this chapter. Part VI contains two numerical examples. Here we ask what is the relationship between capital outflows and some traditional measures of domestic agents' degree of risk aversion? To answer this we use two very common utility functions, constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) utility functions. We find that in cases of both FDS and MP changes, the smaller the coefficient of risk aversion (relative or absolute depending on the case) the larger the capital outflows. Lastly, in part VII we make some final comments.

## **1.2. General Characteristics of the Economy**

The following is the setup of a small open economy inhabited by a large number of identical agents. We assume that both the economy and agents live for two periods. The domestic agents engage in trade of consumption goods and financial assets with the foreign assets at prices determined in the international markets. In the first period, the domestic agents receive endowments of consumption goods. Then, they choose their first period consumption and the amount of savings such that they maximize expected lifetime utility. The domestic agents could use two saving alternatives, foreign and domestic assets. The foreign asset's real return is determined in the international markets, and it is known with certainty by the domestic agents. On the other hand, the domestic asset has an uncertain return, but agents know its distribution function. This distribution is exogenous and therefore taken as given.

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### 1.2.1. Private Agents

In this economy there exist a large number of identical individuals that live for two periods. Each agent maximizes the expected lifetime utility from consumption. Agents' *preferences* are represented by:

$$1) \quad U[C_1] + \beta E\{U[\tilde{C}_2]\}$$

where  $U$  is a strictly increasing, strictly concave, continuous and differentiable (as many times as needed) utility function. Here  $\beta$  is a discount factor with  $\beta \in (0, 1)$ .  $C_1$  is first period consumption and  $\tilde{C}_2$  is consumption in period two. The tilde indicates that  $\tilde{C}_2$  is a random variable. Finally,  $E$  represents the expectation operator.

At the beginning of period one each individual receives an endowment of  $\hat{Y}$  units of consumption goods. In period one, agents pay lump sum taxes,  $\tau$ , to the government. Individuals do not receive a goods endowment in the second period, so to finance their consumption in the second period they must save. Agents can use foreign ( $m$ ) or/and domestic ( $a$ ) assets to save. The foreign assets yield a certain real net return of  $r$  units of consumption goods per unit invested. The domestic asset's net yield  $\tilde{x}$  is random, and has a known cumulative distribution function (cdf),  $F$ .<sup>10</sup> To simplify notation, let us assume that population size  $N$  is equal to one and therefore, aggregate and per capita quantities are the same. Notice that all the variables are in terms of the consumption good, that is all variables are in real terms.

---

<sup>10</sup> The support of the random variable  $\tilde{x}$  is the interval  $[x_0, x_1]$ . In what follows we use capital letters to denote cumulative distribution functions  $F$ , and small letters to denote densities  $f$ .

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### 1.2.1.1. Households Budget Constraints

It follows that the first period household budget constraint (HBC) is given by  $\hat{Y} = C_1 + a + m + \tau$  or  $Y = C_1 + a + m$ , where  $Y = \hat{Y} - \tau$ . The second period HBC is given by  $\tilde{C}_2 = a(1 + \tilde{x}) + m(1 + r)$ .

### 1.2.1.2. Households Maximization Problem

Then, given preferences (1) and the HBCs, households choose  $C_1$ ,  $C_2$ ,  $a$  and  $m$  to maximize (1) subject to their HBCs.

Let  $S$  denote total savings, then  $S = a + m$  and therefore, 2)  $Y = C_1 + S$  and 3)  $\tilde{C}_2 = a(\tilde{x} - r) + S(1 + r)$ . Thus, we can write the maximization problem in terms of  $a$  and  $S$  as:

$$\text{Max}_{\{S, a\}} U[Y - S] + \beta \int_{x_0}^{x_1} \{U[a(x - r) + S(1 + r)]\} f(x) dx.$$

The first order conditions (FOC's) of this maximization problem are given by:

$$\text{FOC's:} \quad 4) V_s: -U'[Y - S] + \beta \int_{x_0}^{x_1} \{U'[a(x - r) + S(1 + r)](1 + r)\} f(x) dx = 0$$

$$5) V_a: \beta \int_{x_0}^{x_1} \{U'[a(x - r) + S(1 + r)](x - r)\} f(x) dx = 0.$$

From now on we focus only on interior solutions in which  $a$ ,  $S$  and  $m$  are all positive. Let  $a^*$ ,  $S^*$  and  $m^*$  be the values of  $a$ ,  $S$  and  $m$  that satisfy the FOC's, the second order conditions (SOC's), and are all positive.<sup>11</sup>

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<sup>11</sup> See Appendix 1.A.

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### 1.2.2. Government

The domestic economy has a public sector that lives for two periods. During the first period the government levies lump sum taxes on, and sells bonds to the domestic individuals. The government uses the resources collected through taxation ( $\tau$ ) and bonds ( $a$ ) to finance a given level of first period expenditure,  $g$ . Hence, the first period government budget constraint is  $g=a+\tau$ .

In the second period the government receives an endowment  $A$  that exactly matches the requirements to repay its debt namely,  $a(1+x)$ . Here  $x \in [x_0, x_1]$  is the realized domestic return. Thus, the second period government budget constraint is given by  $A=a(1+x)$ . Given these assumptions the government is concerned about its financial requirements only in the first period. In the second period the government uses the endowment  $A$  to repay all its debt.<sup>12</sup>

### 1.2.3. Domestic Bonds Market

Households choose the desired level of domestic assets so as to maximize their expected lifetime utility. On the other hand, the government supplies enough bonds such that the market clears given the distribution of domestic return and the levels of income, taxes, and the foreign return.

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<sup>12</sup> We make this assumption to ensure that the domestic government can repay its debt in the second period.

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#### 1.2.4. Foreign Bonds Market

Domestic private agents determine their desired foreign bond purchases, and buy them on the international markets.

#### 1.2.5. Goods Market Equilibrium

We define an equilibrium in the consumption good market as a situation in which total consumption of both private and public sectors is equal to the total supply of goods.

Therefore, in period one we have:  $C_1 + g = \hat{Y} - (a + m + \tau) + (a + \tau)$  or  $C_1 + g = \hat{Y} - m$ .

For the second period,  $\tilde{C}_2 + a(1 + x) = a(1 + x) + m(1 + r) + A$  or  $\tilde{C}_2 = m(1 + r) + A$ .

Using these conditions, we have that if the goods markets are in equilibrium, the present value of total domestic consumption is equal to the present value of all resources available for the domestic economy to consume.

$$C_1 + g + \frac{\tilde{C}_2}{(1 + r)} = \hat{Y} + \frac{\tilde{A}}{(1 + r)}.$$

#### 1.2.6. Competitive Equilibrium

We define a competitive equilibrium as a sequence  $\{ C_1, \tilde{C}_2, a, m, \hat{Y}, g, \tau, A \}$  such as, given the foreign return ( $r$ ) and the domestic (risky) return ( $\tilde{x}$ ), households solve their maximization problem, government budget constraints (GBC) are satisfied, and both goods and asset markets are in equilibrium.

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### 1.3. Capital Flows and Changes in Risk

In this section we try to determine how changes in the distribution of the domestic asset's return affect private agents' positions in both foreign and domestic assets. First, we consider first degree stochastic changes in the distribution of the domestic return. Then, we look at the effects of mean preserving transformations of  $F$ . To learn about the possible effects of the assumed changes in the distribution of the domestic return, we compare the equilibrium outcomes of two economies in which this is the only difference.

#### 1.3.1. Changes in the Distribution of Domestic Returns

Let  $F$  and  $G$  be two different cumulative distribution functions (cdf) of the random variable  $\tilde{x}$ . Define  $V[F]$  as the expected utility when the cdf is  $F$ . Likewise, let  $V[G]$  be the expected utility under  $G$ . Therefore,  $V[F]$  and  $V[G]$  are given by

$$V[F] \equiv U[C_1] + \beta \int_{x_0}^{x_1} \{U[\tilde{C}_2]\} f(x) dx \text{ and } V[G] \equiv U[C_1] + \beta \int_{x_0}^{x_1} \{U[\tilde{C}_2]\} g(x) dx.$$

Let  $\{S^*, a^*\}$  and  $\{\hat{S}, \hat{a}\}$  denote the arguments that maximize  $V[F]$  and  $V[G]$ , respectively. Then if we compare the values of  $\{S^*, a^*\}$  and  $\{\hat{S}, \hat{a}\}$  we can determine how the domestic private agents adjust portfolios in response to changes in the domestic asset's return distribution. We know that if the expected marginal utility of savings under distribution  $G$  evaluated at  $\{S^*, a^*\}$ ,  $V_S[G]|_{S^*, a^*}$ , is different from zero, then  $\{S^*, a^*\}$  and  $\{\hat{S}, \hat{a}\}$  are different. However, if  $V_S[G]|_{S^*, a^*}$  is zero, then  $\{S^*, a^*\}$  and  $\{\hat{S}, \hat{a}\}$  are equal.

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In those cases where  $V_s[G]_{|_{S^*,a^*}}$  and  $V_a[G]_{|_{S^*,a^*}}$  do not vanish, their signs may allow us to establish the relationships that exist between  $a^*$  and  $\hat{a}$ , and between  $S^*$  and  $\hat{S}$ .

Following Eeckhoudt, Meyer and Ormiston (1997) [EMO], we try to disentangle the effects of changes in the distribution of domestic return on agents' optimal choices into the *direct* and the *indirect effects*. As in the present paper, EMO analyze a problem with two endogenous variables in which agents face a change in the distribution of one of the exogenous variables of the problem. They determine the *direct effect* by looking at how "...a parameter shift alters the marginal expected utility for each of the decision variables..." while holding everything else constant. This effect shows the most immediate response of a decision variable to changes in the exogenous variables' distribution without considering other possible consequences. On the other hand, EMO obtain the *indirect effect* by looking at how changes in the exogenous variables affect the marginal expected utility of one of the decision variables once we allow the other to change. This effect reflects that there may exist interactions between the choice variables at the optimal levels since agents set them simultaneously.

### 1.3.1.1. First Degree Stochastic Changes

Given  $F$  and  $G$ , we say that the distribution  $F$  dominates the distribution  $G$  in a first degree stochastic sense if for all the agents with positive  $U'$ , the expected utility is greater under  $F$  than under  $G$ . A necessary and sufficient condition for this is that  $F(x) \leq G(x)$  for all the  $x$  in the domain.<sup>13</sup>

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<sup>13</sup> See Eeckhoudt and Gollier (1995).

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### **Lemma 1**

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### **Proposition**

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Let us assume that  $F$  and  $G$  are the initial and the new distribution, respectively. Thus, going from  $F$  to  $G$  implies a first degree stochastic deterioration. Lemma 1 and proposition 2 help us to establish how first degree stochastic (FDS) changes in  $x$ 's distribution affect agents' marginal expected utility. These results are used later to determine the signs of the direct and the indirect effects of a FDS deterioration on both  $a$  and  $S$ .

**Lemma 1:**

If agents' preferences exhibit risk aversion, then  $V_s[G]_{|_{S^*,a^*}} > 0$ .

*Proof:* See Appendix 1.A. ■

**Proposition 2:**

If agents' preferences exhibit risk aversion and have a coefficient of relative risk aversion no larger than one for all  $x$ , then  $V_a[G]_{|_{S^*,a^*}} < 0$ .

*Proof:* See Appendix 1.A. ■

**1.3.1.1.1. Direct Effect (DE)**

DE Total Savings:

Under risk aversion  $V_s[G]_{|_{S^*,a^*}} > 0$ , and to restore the equilibrium ( $V_s[G]_{|_{S^*,a^*}} = 0$ ) the value of  $S$  goes up since  $V_{ss}$  is negative, thus lemma 1 shows that a FDS deterioration in

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**Proposition 3:**

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$r_{x_1} > 0$ .



the distribution of the risky asset's return induces an increase in total savings if agents dislike risk, i.e.,  $S^* < \hat{S}$ .

#### DE Domestic Assets:

By proposition 2 we know that if the coefficient of relative risk aversion is no larger than one, then  $V_a[G]_{S^*, a^*} < 0$ . Since  $V_{aa}$  is negative,  $a$  must go down to restore the equilibrium; therefore,  $a^* > \hat{a}$ .

Thus, as a consequence of the direct effect of a FDS deterioration in the domestic asset's return, the level of total savings tends to increase when agents are risk averse. Similarly, a FDS deterioration induces a reduction in the holdings of the domestic asset (direct effect) in cases where the coefficient of relative risk aversion is no larger than one. Notice that if the coefficient of relative risk aversion is larger than one the sign of the direct effect on the holdings of risky assets is ambiguous, and we cannot determine it without further information.

To establish the signs of the indirect effects we need to find out what kind of relationship may exist between  $a$  and  $S$  at their optimal levels. In other words, we need to look at how changes in  $a$  affect the expected marginal utility of  $S$  and vice versa. The next proposition helps us to determine this relationship.

#### **Proposition 3:**

a) If preferences are characterized by decreasing absolute risk aversion (DARA), then  $V_{aS} > 0$ .

b) If p

$$V_{as} < 0.$$

c) If p

$$V_{as} = 0.$$

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b) If preferences are characterized by increasing absolute risk aversion (IARA), then

$$V_{aS} < 0.$$

c) If preferences are characterized by constant absolute risk aversion (CARA), then

$$V_{aS} = 0.$$

*Proof:* See Appendix 1.A. ■

Thus, proposition 3 tells us that the holdings of the risky asset and total savings act as complements, substitutes or independent goods at equilibrium when agents' preferences display decreasing, increasing or constant absolute risk aversion.

#### **1.3.1.1.2. Indirect Effect (IE)**

##### IE Total Savings:

Proposition 2 suggests that a FDS deterioration of  $x$ 's cdf induces a reduction in the holdings of the domestic asset if the coefficient of relative risk aversion is no larger than one. By proposition 3 the reduction in  $a$  causes a negative (positive) effect on the desired level of  $S$  when  $a$  and  $S$  act as complements (substitutes), i.e., when preferences exhibit DARA (IARA). Finally, changes in  $a$  do not cause indirect effects on savings if preferences present CARA since the optimal levels of  $a$  and  $S$  are independent of each other.

##### IE Holdings of Risky Assets:

Lemma 1 establishes that a FDS deterioration causes an increase in the level of savings as long as agents dislike risk. Thus, it follows that this effect on savings induces a reaction

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### Corollary 1.

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on the holdings of the risky asset of the same (opposite) sign when preferences have decreasing (increasing) absolute risk aversion. Nevertheless, the effect is null when preferences have CARA.

#### **1.3.1.1.3. Total Partial Equilibrium Effect (TPEE)**

The TPEEs combine the information about both the direct and the indirect effects to determine how the changes in the exogenous variables affect each one of the decision variables at its optimum level. We call it the total partial equilibrium effect to emphasize that this is the total effect at the individual level, given that we have not allowed the rest of the economy to adjust its behavior. Notice that the TPEE is the *total effect* as defined by EMO since they evaluate the effects at the individual level.

The following corollaries establish the sign of the TPEEs of FDS changes in domestic bonds' return distribution on agents' savings and portfolio mix.

#### **Corollary 1.A: (TPEE on Savings)**

If agents' preferences are characterized by risk aversion, a coefficient of relative risk aversion no larger than one, and nondecreasing absolute risk aversion, then a FDS deterioration of  $x$ 's cdf induces an increase in total savings.

*Proof:* See Appendix 1.A. ■

Notice that the TPEE on savings is positive if preferences show risk aversion and CARA, even for cases when relative risk aversion is larger than one.

**Corollary**

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**Corollary**

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**1.3.1.2. Mean**

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### **Corollary 1.B: (TPEE on Domestic Assets)**

If agents' preferences are characterized by risk aversion, a coefficient of relative risk aversion no larger than one, and nondecreasing absolute risk aversion, then a FDS deterioration of  $x$ 's cdf induces a reduction in the holdings of domestic assets.

*Proof:* See Appendix 1.A. ■

### **Corollary 1.C: (TPEE on Foreign Assets)**

If agents' preferences are characterized by risk aversion, a coefficient of relative risk aversion no larger than one, and nondecreasing absolute risk aversion, then a FDS deterioration of  $x$ 's cdf induces an increase in the holdings of foreign assets.

*Proof:* See Appendix 1.A. ■

### **1.3.1.2. Mean Preserving Transformations**

Now let us consider how agents' optimal choices of savings and portfolio allocations are affected when the distribution of domestic returns experiences a mean preserving [MP] transformation.

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**Proposition 4**

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**Proposition 5:**

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At least since Rothschild and Stiglitz (1970 and 1971) [RS], mean preserving transformations have been considered a reasonable and convenient way to model changes in risk. A random variable  $\tilde{x}$  experiences a MP spread transformation if its initial cdf  $F$  is transformed into a new cdf  $G$  such that the next two conditions hold:

$$\text{a) } \int_{x_0}^x G(x)dx \geq \int_{x_0}^x F(x)dx \quad \forall x \in [x_0, x_1] \quad \text{and,}$$

$$\text{b) } \int_{x_0}^{x_1} G(x)dx = \int_{x_0}^{x_1} F(x)dx .$$

Following the approach used evaluating the effects of FDS changes, we first try to determine the direct and the indirect effects of a MP spread of the distribution of the domestic returns on both  $S$  and  $a$ . Then, we combine those pieces of information to establish the total partial equilibrium effects.

### 1.3.2.1. Direct Effects (DE)

#### **Proposition 4:**

If  $U'''$  is either positive, negative or zero, then the marginal expected utility of savings under  $G$  evaluated at  $\{a^*, S^*\}$ ,  $V_s[G]_{S^*, a^*}$ , has either a positive, negative or zero value.

*Proof:* See Appendix 1.A. ■

#### **Proposition 5:**

The expected marginal utility of the risky asset under  $G$  evaluated at  $\{a^*, S^*\}$ ,  $V_a[G]_{S^*, a^*}$  is negative if preferences exhibit either i) a positive third derivative of the utility function,

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*Proof:* See

DE Holding

Proposition 5

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*Proof:* See Appendix

a coefficient of relative risk aversion smaller than one and nondecreasing in the random variable; ii) a positive third derivative of the utility function and a coefficient of relative risk aversion equal to one; or iii) a null third derivative of the utility function.

*Proof:* See Appendix 1.A. ■

#### DE Total Savings:

From proposition 4 it follows that the sign of the direct effect on saving of a MP spread in  $x$ 's distribution is either positive, negative or null if the sign of the third derivative of the utility function is positive, negative or zero, respectively.

*Proof:* See Appendix 1.A. ■

#### DE Holdings of Domestic Assets:

Proposition 5 tells us that the direct effect of a mean preserving spread on the holdings of the risky asset is negative if preferences exhibit one of the next sets of assumptions: i) a positive third derivative of the utility function, a coefficient of relative risk aversion smaller than one and nondecreasing in the random variable; ii) a positive third derivative of the utility function and a coefficient of relative risk aversion equal to one; iii) a null third derivative of the utility function.

*Proof:* See Appendix 1.A. ■

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*Proof:* See App

### 1.3.2.2. Indirect Effects (IE)

#### IE Total Savings:

By the results of propositions 3 and 5 we know that the indirect effect on savings of a MP spread is negative if preferences show: i) DARA plus either a coefficient of relative risk aversion smaller than one and nondecreasing in the random return; or ii) a coefficient of relative risk aversion equal to one. The indirect effect on savings is positive under IARA and either: i) a positive third derivative of the utility function, a coefficient of relative risk aversion less than one and increasing in the random return; or ii) a null third derivative of the utility function. Finally, the indirect effect is null if preferences exhibit CARA.

*Proof:* See Appendix 1.A. ■

#### IE Holdings of the Domestic Asset:

From propositions 3 and 4 we know that the indirect effects on the holdings of the risky asset is positive if preferences exhibit either: i) DARA and positive third derivative of the utility function; or ii) IARA and a negative third derivative of  $U$ . On the other hand, the indirect effect is negative if preferences exhibit IARA and positive third derivative of the utility function. The IE is null if preferences exhibit either: i) a null third derivative of the utility function; or ii) CARA.

*Proof:* See Appendix 1.A. ■

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### 1.3.2.3. Total Partial Equilibrium Effects

The following corollaries combine the pieces of information given by the direct and the indirect effects to establish the TPEEs (on agents' savings and portfolio mix) of a mean preserving spread in the domestic bond's return distribution.

#### Corollary 2.A: (TPEE on Savings)

A mean preserving spread of  $x$ 's distribution causes an increase in the level of optimal savings if preferences exhibit either: i) nondecreasing absolute risk aversion, a positive third derivative of  $U$ , a coefficient of relative risk aversion smaller than one and increasing in  $x$ ; or ii) a null third derivative of  $U$ .

*Proof:* See Appendix 1.A. ■

#### Corollary 2.B (TPEE on Domestic Assets)

A mean preserving spread in  $x$ 's distribution induces a reduction in the optimal holdings of the risky assets if preferences exhibit either: i) nondecreasing absolute risk aversion, a positive third derivative of  $U$ , and relative risk aversion no larger than one and increasing in  $x$ ; or ii) a null third derivative of  $U$ .

*Proof:* See Appendix 1.A. ■

#### Corollary 2.C: (TPEE on Foreign Assets)

A mean preserving spread in  $x$ 's distribution causes an increase in the optimal holdings of foreign assets if preferences show either: i) nondecreasing absolute risk aversion, a

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*Proof:* See Appendix 1.A. ■

Notice that the TPEE on savings is also positive if preferences exhibit CARA. In those cases in which the conditions of propositions 3, 4 and 5 hold and preferences show DARA the direct and the indirect effects have opposite signs. Therefore, the signs of the total partial equilibrium effects are ambiguous; they depend on the relative size of the direct and the indirect effects.

#### **1.4. Economy Wide Effects (General Equilibrium Effects)**

The previous section describes how the domestic individuals adjust their portfolios because of shifts in the distribution of the domestic assets' return. Now we want to evaluate the economy wide effects of those changes in the distribution.

Given our assumptions about the government financing process, any change in agents' holdings of the domestic bonds alters the government budget constraint (GBC). In particular, if there is a reduction of the holding of the domestic bonds, given  $g$ , the government has to increase the taxes levied on the domestic agents. This reaction of the government induces additional responses in agents' optimal choices and therefore, in the GE outcomes of the economy.

### 1.4.1. General Equilibrium Effects of FDS Changes

Following Diamond and Stiglitz (1974), let  $\sigma$  be an index of a family of cumulative density functions of the random variable  $\tilde{x}$ , such that an increase (a reduction) in  $\sigma$  implies a FDS improvement (deterioration) in  $x$ 's distribution. Then, given two distributions  $F$  and  $G$ , we say that distribution  $F$  dominates distribution  $G$  in a first degree stochastic sense if  $\sigma_F < \sigma_G$ . Here  $\sigma_F$  and  $\sigma_G$  represent the indexes associated to the distributions  $F$  and  $G$ , respectively. Let  $\frac{\partial a}{\partial \sigma}$ ,  $\frac{\partial m}{\partial \sigma}$  and  $\frac{\partial S}{\partial \sigma}$  denote the TPEE of FDS changes in  $x$ 's distribution on domestic assets, foreign assets and level of savings, respectively.

From the GBC it follows that --given a fiscal expenditure  $g$ -- any change in the equilibrium holdings of the risky assets is compensated by equivalent changes in taxes,  $da + d\tau = 0$ . Thus, taking into account the interactions between the domestic private agents and the domestic government, we can write the general equilibrium effect on the domestic assets of FDS changes in  $x$ 's distribution as

$$da = \frac{\partial a}{\partial \sigma} d\sigma + \frac{\partial a}{\partial \tau} d\tau \quad \text{or} \quad da \left[ 1 + \frac{\partial a}{\partial \tau} \right] = \frac{\partial a}{\partial \sigma} d\sigma.$$

Hence, the sign of the general equilibrium effect of FDS changes in the distribution of the domestic return is given by the sign of

$$\frac{da}{d\sigma} = \frac{\frac{\partial a}{\partial \sigma}}{\left[ 1 + \frac{\partial a}{\partial \tau} \right]}.$$

Notice that to determine the sign of this expression we need to know the sign of its denominator. By construction  $\frac{\partial a}{\partial \tau}$  is equal to  $-\frac{\partial a}{\partial Y}$  and, as borrowing is not possible, the sign of the denominator is positive and, therefore, the sign of the GE effect is given by the sign of the TPEE.<sup>14</sup>

#### 1.4.1.1. GE Effect on the Domestic Assets

From corollary 1.B we have that if agents' preferences exhibit risk aversion, a coefficient of relative risk aversion no larger than one, and nondecreasing absolute risk aversion, then a FDS deterioration (improvement) on  $x$ 's cdf induces negative (positive) GE effects in the holdings of domestic assets.

#### 1.4.1.2. GE Effects on Total Savings

We can write the general equilibrium effect on savings caused by FDS changes as:

$$dS = \frac{\partial S}{\partial \sigma} d\sigma + \frac{\partial S}{\partial \tau} d\tau. \text{ Alternatively, using the GBC and the fact that } \frac{\partial S}{\partial \tau} = -\frac{\partial S}{\partial Y}, \text{ we}$$

$$\text{can write the GE effect as } \frac{dS}{d\sigma} = \frac{\partial S}{\partial \sigma} - \frac{\partial S}{\partial Y} \frac{da}{d\sigma}.^{15}$$

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<sup>14</sup> Recall that we define  $Y = \hat{Y} - \tau$ . Thus it follows that  $\frac{\partial a}{\partial \tau} = \frac{\partial a}{\partial Y} \frac{dY}{d\tau} = -\frac{\partial a}{\partial Y}$ .

<sup>15</sup> Because we assume lump sum taxes, we have that  $\frac{\partial S}{\partial \tau} = \frac{\partial S}{\partial Y} \frac{dY}{d\tau} = -\frac{\partial S}{\partial Y}$ , and therefore, by construction, the effects of changes in taxes on agents' saving rate is the negative of the effects changes in income.

Corollary 1.A establishes some conditions under which we can determine the sign of the first part of the right-hand-side (RHS) expression. However, we still need to know the sign of the second part of the RHS expression. In particular, we must determine the sign of the total partial equilibrium effect on savings of changes in disposable income.

**Proposition 6:**

Under risk aversion, the TPEE on savings due to changes in income is positive.

*Proof:* See Appendix 1.A. ■

Since the TPEE on savings of changes in income is positive and under the conditions of corollary 1.B the GE effect on  $a$  of a FDS change is negative, the

expression  $-\frac{\partial S}{\partial Y} \frac{da}{d\sigma}$  is positive.

**GE Effects on Savings**

If agents' preferences show risk aversion, a coefficient of relative risk aversion no larger than one, and nondecreasing absolute risk aversion, then a FDS deterioration (improvement) of  $x$ 's cdf induces an increase (reduction) in the general equilibrium level of savings.

### 1.4.1.3. GE Effects on the Foreign Assets

By construction, the holdings of foreign assets is equal to total savings minus the holdings of domestic assets,  $m=S-a$ . Thus, it follows that  $dm=dS-da$ . Then, if the results of the previous GE effects hold, we have that a FDS deterioration (improvement) induces an increase (a reduction) in the GE holdings of foreign assets.

### Summary:

We have seen that if agents' preferences are characterized by a specific set of properties we can sign the partial and general equilibrium effects --on  $a$ ,  $m$ , and  $S$ -- of first degree stochastic changes in  $x$ 's distribution. In particular, we show that a FDS deterioration induces unambiguously a reduction in the holdings of domestic assets, and an increase in the holdings of foreign assets and total savings when preferences show risk aversion, nondecreasing absolute risk aversion, and a coefficient of relative risk aversion no larger than one. Those findings hold both at partial equilibrium and at a general equilibrium level.

These results are similar to those in Khan and Ul Haque (1985) in the sense that a first degree stochastic change in the distribution of domestic returns induces an increase in the holdings of foreign assets.

### 1.4.2. General Equilibrium Effects of MP Transformations

Let  $\lambda$  be an index of a family of cumulative density functions of the random variable  $\tilde{x}$  such that an increase (decrease) in  $\lambda$  implies a mean preserving spread

(contraction) of  $x$ 's distribution. Then, given two distributions  $F$  and  $G$ , we say that distribution  $G$  is riskier than distribution  $F$  in a Rothschild and Stiglitz sense if  $\lambda_F$  is smaller than  $\lambda_G$ .

Let  $\frac{\partial a}{\partial \lambda}$ ,  $\frac{\partial m}{\partial \lambda}$ , and  $\frac{\partial S}{\partial \lambda}$  denote the TPEE of MP changes in  $x$ 's distribution on

the domestic and the foreign assets, and on total savings, respectively.

#### 1.4.2.1. GE Effects on Domestic Assets

As in the case of first degree stochastic changes, the general equilibrium effect of a mean preserving transformation on the holdings of domestic assets is given by

$da = \frac{\partial a}{\partial \lambda} d\lambda + \frac{\partial a}{\partial \tau} d\tau$ . Consequently, the sign of the general equilibrium effects of mean

preserving changes in the distribution of the domestic assets' return is given by the sign of:

$$\frac{da}{d\lambda} = \frac{\frac{\partial a}{\partial \lambda}}{\left[1 - \frac{\partial a}{\partial Y}\right]}.$$

Assuming that people do not borrow, we have that the GE effect has the same sign of the total partial equilibrium effect. Thus, under the conditions of part B of corollary 2, the GE effect on the holdings of the domestic assets is negative, i.e., a MP spread induces a reduction in the general equilibrium value of  $a$ .

### 1.4.2.2. GE Effects on Savings

The GE effect on savings of mean preserving transformations of the distribution of the domestic return is given by  $\frac{dS}{d\lambda} = \frac{\partial S}{\partial \lambda} - \frac{\partial S}{\partial Y} \frac{da}{d\lambda}$ . From the results of proposition 6 and parts A and B of corollary 2 we know that a MP spread in  $x$ 's cdf induces an increase in the GE level of savings.

### 1.4.2.3. GE Effects on Foreign Assets

Under the conditions of part C of corollary 2 and proposition 6, a MP spread in  $x$ 's cdf induces an increase in the holdings of the foreign assets, *capital flight*.

#### Summary:

Under the conditions of corollary 2 and propositions 6 and 7, a mean preserving spread (contraction) on the distribution of the domestic return induces a reduction (an increase) in domestic investors holdings of domestic assets. At the same time, the MP spread (contraction) motivates an increase (a reduction) in the holdings of foreign assets and total savings. In this context the increase in agents' holdings of the foreign bond can be interpreted as *capital flight*, since the capital outflows are induced by an increase in domestic risk. Domestic agents also raise their total savings as protection against the increase in the domestic risk.

## **1.5. Risk Aversion (A Digression)**

### **1.5.1. Absolute Risk Aversion**

We place restrictions on the forms of absolute risk aversion to establish some of the results of sections III and IV of this chapter. We find that nondecreasing absolute risk aversion (together with other assumptions) allows us to unambiguously determine the total partial equilibrium and general equilibrium effects of both FDS and MP changes in  $x$ 's distribution. The assumption of nondecreasing absolute risk aversion is convenient, since the direct and the total partial equilibrium effects have the same sign. This is the case because the indirect effect either has the same sign as the direct effect (IARA) or is null (CARA).

However, at least since Arrow (1965) and Pratt (1964), the perception that absolute risk aversion is decreasing or at least nonincreasing is generally accepted. We find that when preferences exhibit the desired DARA the direct and the indirect effects have opposite signs, and, therefore, the sign of the TPEE is ambiguous. A possible solution to eliminate the ambiguity is to impose additional assumptions on preferences. For instance, if we assume that either the direct effect or the indirect effect dominates the other (it is larger in absolute value) we can still determine the signs. The results of corollaries one and two are still valid, in the cases where the direct effect is the dominant effect. Nevertheless, the results of corollaries one and two are reversed, when the indirect effect dominates the direct effect. Even though DARA is a widely accepted and desired



property, nondecreasing absolute risk aversion has often been used in both empirical and theoretical studies.<sup>16</sup>

### 1.5.2. Other Measures of Risk Aversion

In this paper we use many assumptions about agents' attitude toward risk that are controversial, nondecreasing absolute risk aversion is only one of them (but perhaps it is the most controversial one). In corollaries 1 and 2 we place restrictions on the size of *the coefficient of relative risk aversion*. In the literature of decision making under uncertainty it is common to assume that this coefficient is smaller than one.<sup>17</sup> However, several empirical studies find that in a variety of countries the coefficient of relative risk aversion is larger than one.<sup>18</sup>

We also employ assumptions about the sign of  $U'''$  in the comparative static analysis. The sign of  $U'''$  is closely related to the form of absolute of risk aversion. A positive  $U'''$  is a necessary condition for nonincreasing absolute risk aversion (either

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<sup>16</sup> Hall (1978) assumes a quadratic utility function which implies IARA. Caballero (1990) uses a CARA.

<sup>17</sup> Fishburn and Porter (1976), Rothschild and Stiglitz (1971), Levhari and Srinivasan (1969) and Hadar and Seo (1990) are some of the papers that use relative risk aversion smaller (no larger) than one doing comparative static under uncertainty.

<sup>18</sup> Studies as Binswanger (1981), Morduch (1990) estimate this coefficient for Indian households using the ICRISAT data finding values larger than one. Ogaki, Ostry, and Reinhart (1996) estimate the intertemporal elasticity of substitution (IES) for a large number of economies. Given the forms of the utility functions that they use (isoelastic or constant relative risk aversion) the coefficient of relative risk aversion can be computed as the inverse of the IES. The implied values for the relative risk aversion are larger than one. Other studies as Zeldes (1989), Carroll and Samwick (1997) and Caballero (1990) use a coefficient of 3. Nevertheless, Hansen and Singleton (1983) find estimates of the relative risk aversion coefficient that are both larger and smaller than one using US data. See Table 1.4 for a summary of these estimates.

CARA or DARA). On the other hand, a nonpositive  $U'''$  implies increasing absolute risk aversion.

The sign of *the third derivative* of the utility function is generally associated with the existence of a *precautionary saving motive*. Leland (1968), Sandmo (1970) and Kimball (1990) show that a positive third derivative of the utility function indicates a precautionary saving motive. Here we find that when preferences have some specific (and restrictive) characteristics an increase in the interest-income risk induces an increase in total savings. From corollary 2.A we know that this happens if preferences exhibit CARA and therefore,  $U''' > 0$ , and also if preferences exhibit IARA,  $U''' > 0$  and relative risk aversion less than one and decreasing in the domestic return. So a positive third derivative of  $U$  is still associated with a precautionary savings motive. Nevertheless, a precautionary saving motive (an increase in savings as the result of an increase in risk) take place even if preferences exhibit  $U''' = 0$  because of the *indirect effect*. Even though the direct effect of a MP spread on savings is null when  $U''' = 0$  (proposition 4) the reduction in the holdings of domestic assets ( $a$ ) induces an *indirect* increase in optimal savings (in this case preferences exhibit IARA --proposition 3).

## 1.6. Capital Outflows: (Two Numerical Examples)

So far we have analyzed how agents adjust their portfolios because of changes in the domestic return distribution using a general utility function. Next we evaluate the effect of those changes in domestic returns' cdf on the equilibrium outcomes of small open economies using specific forms of the utility function. We use two very common

utility functions namely, constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) to conduct some simulations. We choose values for all the relevant exogenous variables to carry out the simulations and try to connect the preferences' parameters with domestic agents' responses.

With these two numerical examples we want to extend the findings of part III and IV of this chapter. In particular, in the first exercise we assume that preferences exhibit CARA, and test the predictions of corollaries 1 and 2 for cases in which the coefficient of relative risk aversion is greater than one. In the second example our goal is to extend the results of the corollaries for the case in which absolute risk aversion is decreasing. In this example we assume a CRRA utility function that also implies the commonly accepted DARA.

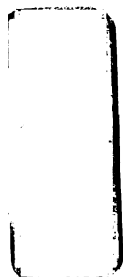
### 1.6.1. Example 1: CARA

Consider a constant absolute risk aversion utility function, thus  $U[c] = -e^{-\gamma c}$ . Let us assume a discrete cdf for the domestic assets' return. Let  $p_1 = p_2 = 0.5$ ,  $x_1 = 0.01$  and  $x_2 = 0.07$ ; where the  $x$ 's represent the possible values of the random variable (the domestic return) and the  $p$ 's represent the probability that each one of those values is realized. Then, the expected value of the random variable  $\tilde{x}$  is 0.04. Let  $\beta = 0.94$ ,  $Y = 500$ ,  $r = 0.0398$  be the discount factor, income level and safe (foreign) asset's return, respectively.

To model the FDS and MP changes in  $x$ 's distribution we define the new values of  $x$ 's so that they satisfy the FDS deterioration and MP spread conditions.<sup>19</sup> Let  $x_{11}$  and  $x_{12}$

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<sup>19</sup> See Appendix 1.B.



be the new values of  $x$ 's that satisfy the FDS conditions. Then, we can write those new values of  $x$  as  $x_{11}=x_1-k_1$  and  $x_{12}=x_2-k_1$ . Here  $k_1$  represents a FDS shift factor. If we add (subtract)  $k_1$  from the initial values of  $x$  we obtain a FDS improvement (deterioration). Likewise, let  $x_{21}$  and  $x_{22}$  be the new values of  $x$  that satisfy the MP spread conditions. We write those values as  $x_{21}=x_1-k_{21}$  and  $x_{22}=x_2+k_{22}$ , where  $k_{21}=c_2/p_1$  and  $k_{22}=c_2/p_2$ . Here  $k_{21}$  and  $k_{22}$  represents the MP shift factors. Notice that these terms contain both additive ( $c_2$ ) and multiplicative shifts ( $1/p$ 's) such that the MP conditions hold. Set  $k_1=c_2=0.0001$ .

The Figures 1.1 and 1.2 illustrate the effects of a FDS deterioration and a MP spread when preferences exhibit constant absolute risk aversion, respectively. In each graph we show the relationship between the coefficient of absolute risk aversion and the effect on each one of the equilibrium outcomes. To compute the effects on the holdings of domestic and foreign assets and on total savings, we compare the equilibrium outcomes of two economies that differ only in the distribution functions of the random return,  $x$ . In the graph where we evaluate the effects on the variable  $z$ ,  $Var.$  represents the difference between  $\hat{z}$  and  $z^*$ , hence  $Var = \hat{z} - z^*$ . Here  $z^*$  and  $\hat{z}$  are the equilibrium outcomes for the economies with the initial and new distribution, respectively.

In these numerical examples we find that a FDS deterioration in the distribution of the random return induces an increase in total savings, a reduction in the holdings of domestic (risky) assets and an increase in the holdings of foreign assets (see Figure 1.1). On the other hand, similar results hold in the case of a mean preserving spread of  $x$ 's cdf. There is an increase in total savings, a reduction in the holdings of domestic (risky) assets and an increase in the holdings of foreign (safe) assets (see Figure 1.2). These results

imply that the predictions of corollary 1 and 2 are still valid for cases where the coefficient of relative risk aversion is larger than one.<sup>20</sup> Then, in an economy with these characteristics (CARA is one of them), *capital outflows are* the consequence of domestic agents' reactions to changes in the domestic risk, and, therefore, they can be interpreted as *capital flight*.

### 1.6.2. Example 2: CRRA

Now let us assume a constant relative risk aversion utility function,  $U[c] = \frac{c^{1-\rho}}{1-\rho}$ .

Let  $p_1 = p_2 = 0.5$ ,  $x_1 = 0.01$  and  $x_2 = 0.07$ ,  $\beta = 0.94$ ,  $Y = 500$ ,  $r = 0.0398$ . As in example 1,  $x$ 's represent the possible values of the random variable,  $p$ 's represent the probability that each one of those values is realized,  $\beta$  is the discount factor,  $Y$  is the income level and  $r$  is the safe asset's return. Again the expected value of the random variable  $\tilde{x}$  is 0.04. Let  $\{x_{11}, x_{12}\}$  and  $\{x_{21}, x_{22}\}$  be the new values of  $x$ 's that satisfy the FDS and the MP spread conditions, respectively. We can write those new values of  $x$  as  $x_{11} = x_1 - k_1$ ,  $x_{12} = x_2 - k_1$ ,  $x_{21} = x_1 - k_{21}$  and  $x_{22} = x_2 + k_{22}$ , where the  $k$ 's and  $c_2$  are define as before. Let us assume that  $k_1 = c_2 = 0.0001$ .

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<sup>20</sup> Here  $\gamma$  is the coefficient of absolute risk aversion. Hence, the coefficient of relative risk aversion  $\rho$ , is given by  $\rho = \gamma \tilde{C}_2$ . We find that  $\rho \leq 1$  if  $\gamma \leq 0.01402$ .

One of the properties of a constant relative risk aversion (CRRA) utility function is that it exhibits decreasing absolute risk aversion. DARA is not included among the conditions of corollaries 1 and 2 because the signs of the total partial equilibrium effects (TPEE) cannot be determined without ambiguity. In those cases, the signs of the TPEE depend on the relative size of the direct and the indirect effects.

Notice that for this example it appears that the direct effect is the dominant one explaining the behavior of the holdings of both domestic and foreign assets.<sup>21</sup> So, a FDS deterioration (Figure 1.3) or a MP spread (Figure 1.4) induces a reduction in the holdings of domestic assets and an increase in the holdings of foreign assets. The signs of these effects are valid even for cases in which the coefficient of relative risk aversion,  $\rho$ , is larger than one. Thus, at least for  $\rho \in (0.6, 10)$ , a first degree stochastic deterioration or a mean preserving spread in  $x$ 's cdf induces capital outflows.<sup>22</sup> For total savings the indirect effect appears to be the dominant effect for  $\rho < 1$ . In those cases the TPEE is negative, hence a FDS deterioration (or a MP spread) reduces the total savings of the domestic agents. Whereas, for cases where  $\rho > 1$  a FDS (or a MP) change induces an increase in total savings. At  $\rho = 1$  there is no effect on total savings.

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<sup>21</sup> If the appropriate conditions hold and the direct effect dominates the indirect effect, then a FDS deterioration or a MP spread in  $x$ 's cdf may induce: a) a reduction in the holdings of domestic assets; b) an increase in the holdings of foreign assets; c) an increase in total savings. However, if the indirect effect dominates the direct effect we have that a FDS deterioration or a MP spread in  $x$ 's distribution may induce: a) an increase in the holdings of domestic assets; b) a reduction in the holdings of foreign assets; c) a reduction in total savings.

<sup>22</sup> For those values of  $\rho$  the optimal values of  $a$ ,  $m$  and  $S$  are all interior solutions of the form  $a > 0$ ,  $m > 0$ , and  $S > 0$ .

Notice that these numerical examples suggest a negative relationship between the size of the coefficient of relative risk aversion and the amount of capital flight. This negative relationship is also suggested by Figure 1.5 that relates estimates of capital flight and relative risk aversion for 13 developing economies. We use estimates of capital flight during 1983 (the first year after the beginning of the debt crisis) taken from Schineller (1997). We also use the values of the intertemporal elasticity of substitution (the inverse of the coefficient of relative risk aversion) estimated by Ogaki, Ostry and Reinhart (1996).<sup>23,24</sup>

This negative relationship between capital flight and relative risk aversion seems to imply that if two small open economies in similar situations face an increase in their *domestic risk* the country in which agents exhibit smaller relative risk aversion tend to be affected more severely. This could be explained because in general the investors in the country with the smaller (relative) risk aversion tend to hold higher positions of the risky asset, and consequently, their portfolio adjustments tend to be larger. Thus, in general, countries with smaller risk aversion tend to be more exposed to severe *capital flight* processes.

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<sup>23</sup> The sample of 13 countries includes: Argentina, Brazil, Chile, Colombia, Costa Rica, Ecuador, Mexico, Malaysia, Peru, Philippine, Thailand, Uruguay and Venezuela.

<sup>24</sup> By no means do we pretend that these are conclusive proof, nevertheless it is interesting that at least for this small sample of countries the predictions of the simulations seem to be correct.



## 1.7. Some Final Comments

According to general wisdom an increase in domestic risk induces domestic capital to flee from developing economies. Here we show sufficient conditions (some restrictions on agents' attitude toward risk) under which this claim is true.

Using standard methodology in the literature of decision making under uncertainty, we analyze how domestic private investors adjust their portfolios in response to changes in the distribution of the domestic asset's return. We consider two different changes namely, first degree stochastic (FDS) changes and mean preserving (MP) transformations.

We find that if preferences exhibit risk aversion, a coefficient of relative risk aversion no larger than one, and nondecreasing absolute risk aversion, then a FDS deterioration induces an increase in the holdings of foreign assets, a reduction in the holdings of domestic assets and an increase in total savings.

On the other hand, if we assume that  $x$ 's cdf experiences a MP spread, then we can show that domestic agents increase their holdings of foreign assets, decrease their holdings of domestic assets and increase total savings when we impose one of the following sets of assumptions: i) a positive third derivative of the utility function, a coefficient of relative risk aversion smaller than one and increasing in the random return, and nondecreasing absolute risk aversion; ii) a null third derivative of the utility function.

Furthermore, if those conditions prevail similar results hold also at a general equilibrium level. In those cases, an increase in domestic risk induces an increase in the

holdings of foreign assets, and at the same time, a reduction in the holdings of the domestic assets and an increase in total savings.

These findings are extensions of Khan and Ul Haque (1985)'s results, since they analyze capital flights in the context of a particular form of FDS deterioration (expropriation risk). Here we show that similar results can be obtained even for general forms of FDS changes. Nevertheless, this generalization of KU is at the expense of imposing more restrictions on agents' preferences. In addition to risk aversion (employed by KU), we assume that the coefficient of relative risk aversion is no larger than one, and either IARA or CARA.

Moreover, the results in the paper are extensions of KU also because we use a more appropriate form to represent changes in domestic risk than the one considered by KU, namely, MP transformations of the domestic return. In the case of a MP spread the capital outflows are interpreted as *capital flight* because they are induced by changes in the domestic risk.

The findings of parts III and IV of this chapter are confirmed and extended in the numerical examples conducted in part VI. For instance, we find if we use a constant absolute risk aversion (CARA) utility function, then the previous predictions about the effects of a FDS deterioration or a MP spread are valid even for cases in which the coefficient of relative risk aversion is larger than one. Furthermore, using a constant relative risk aversion (CRRA) utility function, we find that if the coefficient of relative risk aversion  $\rho$  is smaller than one, then a FDS deterioration (or a MP spread) in  $x$ 's distribution causes a reduction in the holdings of domestic assets, and an increase in the

holdings of foreign assets. Those results are still valid even when the coefficient of relative risk aversion is larger than (or equal to) one. However, the assumed changes in  $x$ 's distribution induce domestic private agents to decrease total savings if  $\rho$  is smaller than one, and to increase it if  $\rho$  is larger than one. The effect on savings is null if the coefficient of relative risk aversion is equal to one.

The examples also suggest a negative relationship between the size of the coefficient of *risk aversion* and the amount of *capital flight*, that is the amount of capital outflows induced by an increase in the domestic risk (a MP spread of the domestic returns' distribution) tends to be higher the lower the coefficient of risk aversion.



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Table 1.1

**Capital Flight: Modified-  
World Bank Measure**  
(Percent of GDP)

	1983	1990
<b>Argentina</b>	-2.92	-15.84
<b>Bolivia</b>	6.68	0.12
<b>Brazil</b>	2.66	0.31
<b>Chile</b>	14.96	-10.58
<b>Colombia</b>	5.18	-0.79
<b>Costa Rica</b>	3.54	-20.3
<b>Ecuador</b>	-3.18	3.51
<b>Mexico</b>	7.52	0.55
<b>Peru</b>	-2.77	1.45
<b>Uruguay</b>	27.15	-3.9
<b>Venezuela</b>	21.3	-17.46
<b>Average</b>	<b>7.28</b>	<b>-5.72</b>
<b>India</b>	1.74	1.11
<b>Indonesia</b>	-1.7	7.94
<b>South</b>	1.54	-4.26
<b>Korea</b>		
<b>Malaysia</b>	5.45	-25.11
<b>Philippines</b>	-5.39	-2.19
<b>Thailand</b>	-2.67	-11.78
<b>Average</b>	<b>-1.72</b>	<b>-5.72</b>

Source: Schineller (1997). "An Econometric Model of Capital Flight from Developing Countries".

Table 1.2

**Stock of Capital Flight:  
Morgan Guaranty Trust's Measure**

**Billions of \$ at  
Year-End**

	<b>1980</b>	<b>1982</b>	<b>1987</b>
<b>Argentina</b>	11	35	46
<b>Bolivia</b>	1	1	2
<b>Brazil</b>	6	8	31
<b>Chile</b>	0	1	2
<b>Colombia</b>	0	0	7
<b>Ecuador</b>	3	4	7
<b>Ivory Coast</b>	1	1	0
<b>Mexico</b>	19	44	84
<b>Morocco</b>	0	0	3
<b>Nigeria</b>	6	13	20
<b>Peru</b>	0	1	2
<b>Philippines</b>	11	19	23
<b>Uruguay</b>	0	2	4
<b>Venezuela</b>	15	33	58
<b>Yugoslavia</b>	3	3	6

Source: Morgan Guaranty Trust Company. (1988). World Financial Markets



Table 1.3  
***Estimates of Capital Flight Stocks (1970-87)***  
***for Some Developing Economies***  
***using Different Methodologies***  
Millions of US Dollars

<b>Definitions</b>	<b>Argentina</b>	<b>Brazil</b>	<b>Mexico</b>	<b>Venezuela</b>
World Bank	53,366	35,716	100,085	53,082
Morgan	51,341	30,150	96,868	51,509
Guaranty Trust				
Cline	35,485	16,962	29,612	32,914
Cuddington	20,917	-681	58,777	18,103
Dooley	25,016	9,432	53,464	16,342

Source: Padilla del Bosque, R. (1991)

Table 1.4  
***Estimates of the***  
***Coefficient of Relative Risk Aversion***

<b>Study</b>	<b>Range of the Estimates</b>
Binswanger (1981) <sup>a</sup>	(1.84 - 68.4)
Morduch (1990) <sup>a</sup>	(1.18 - 2.22)
Hansen and Singleton (1983) <sup>b</sup>	(0 - 2)
Ogaki, Ostry and Reinhart (1996) <sup>c</sup>	(1.557 - 21.276) <sup>d</sup>

a.- Study uses Indian Data

b.- Study uses US Data

c.- Study uses data for 85 countries.

d.- The smallest CRRA is for the United Arab Emirates and the largest for Uganda.



Figure 1.1

***Effects of a First Degree Stochastic Deterioration  
Constant Absolute Risk Aversion***

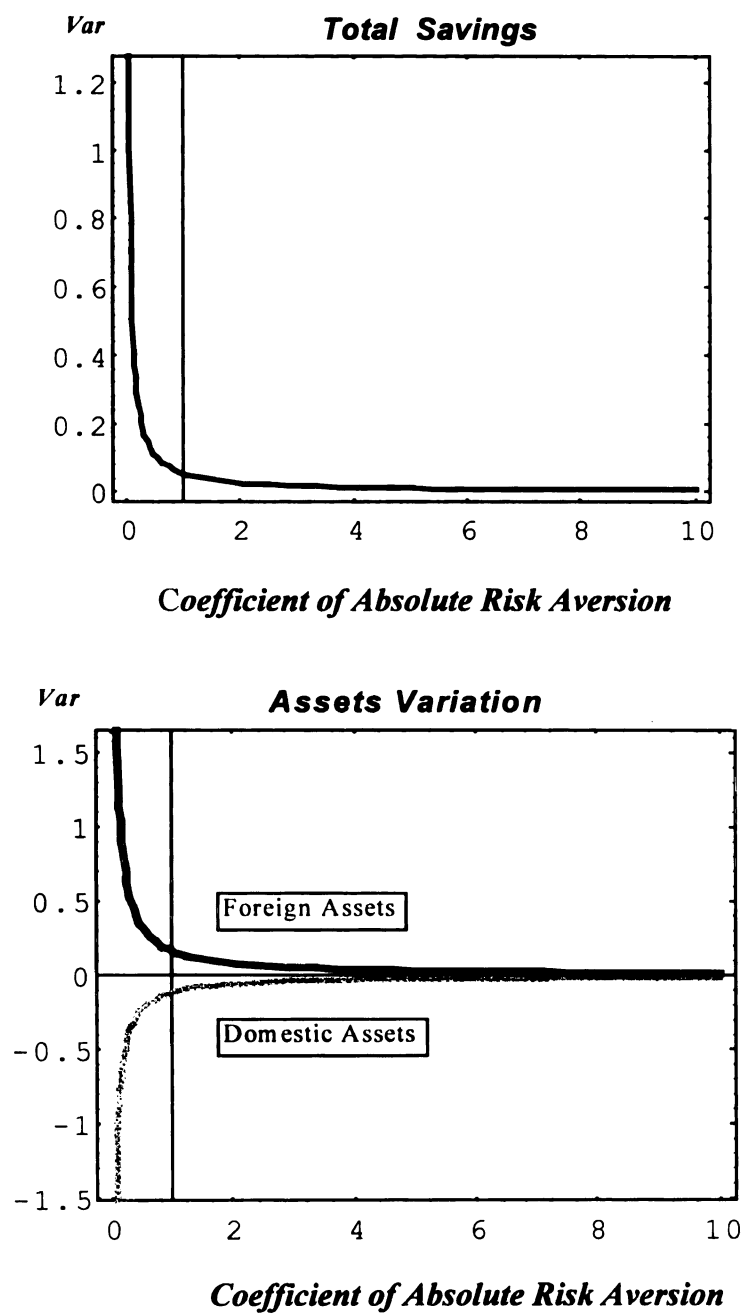




Figure 1.2

***Effects of a MeanPreserving Spread  
Constant Absolute Risk Aversion***

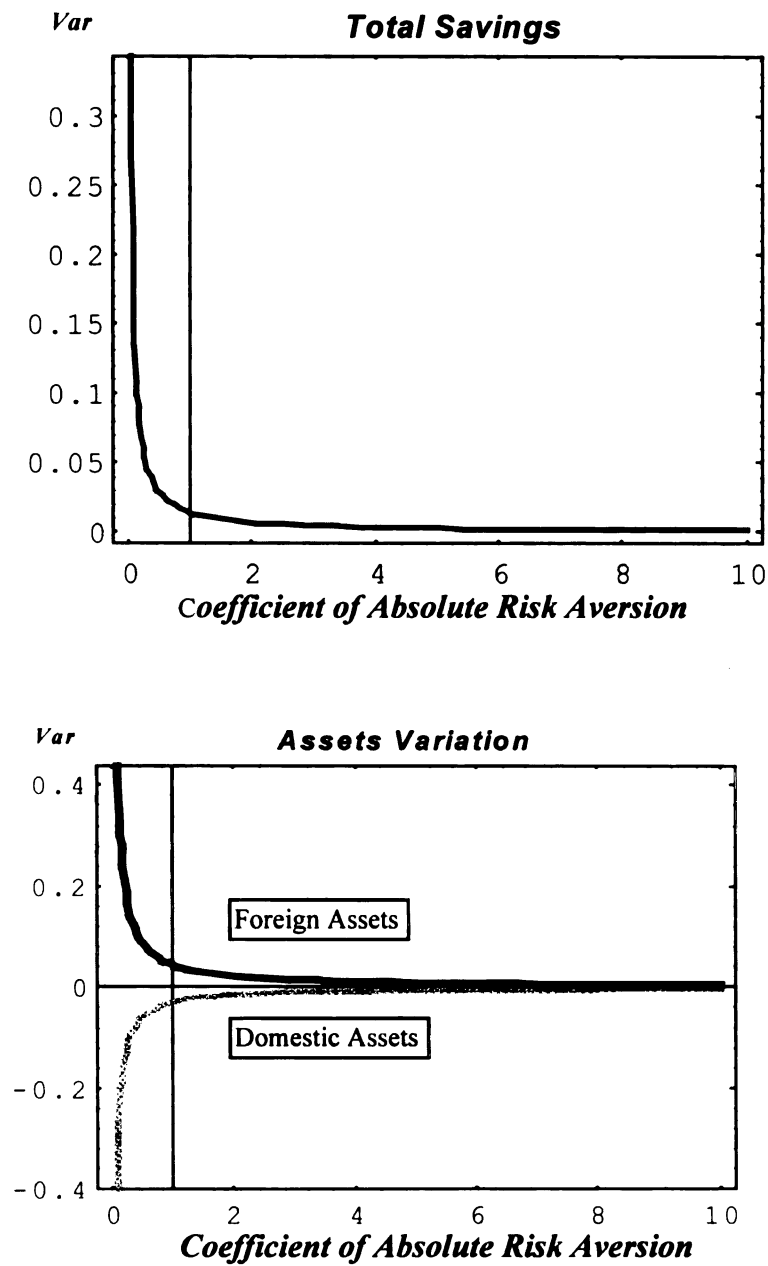




Figure 1.3

***Effects of a First Degree Stochastic Deterioration  
Constant Relative Risk Aversion***

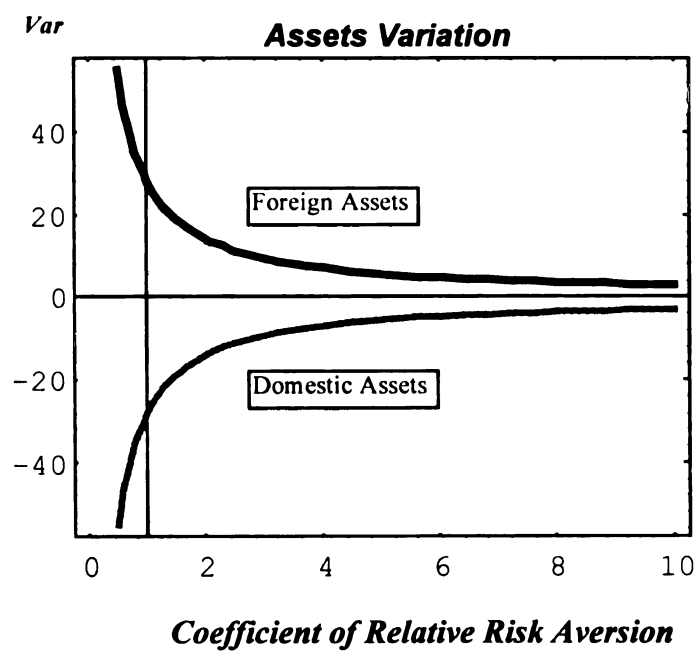
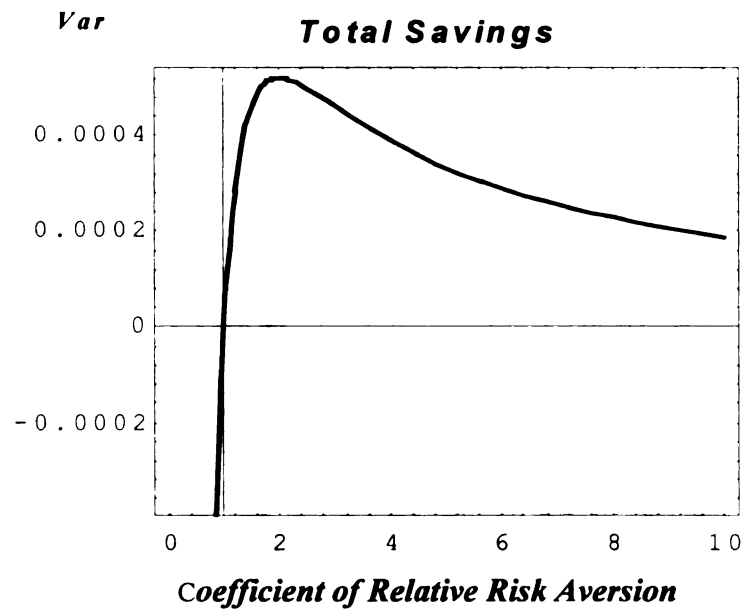


Figure 1.4

***Effects of a Mean Preserving Spread  
Constant Relative Risk Aversion***

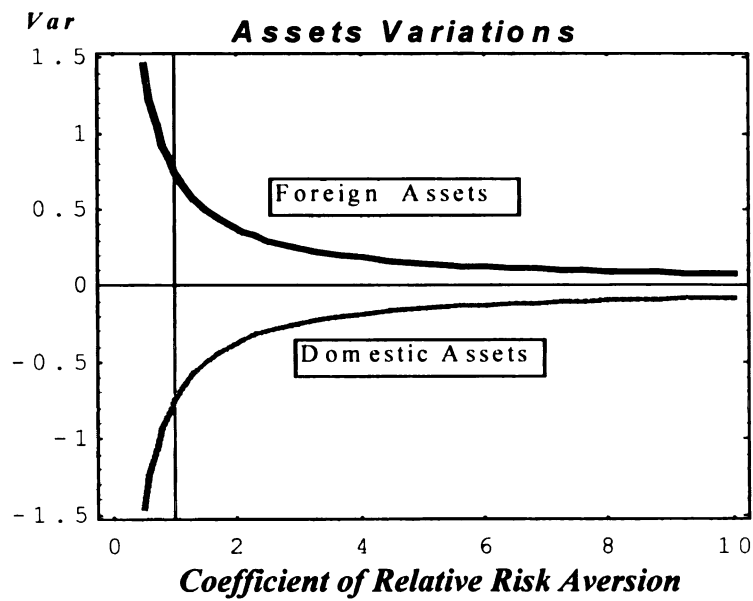
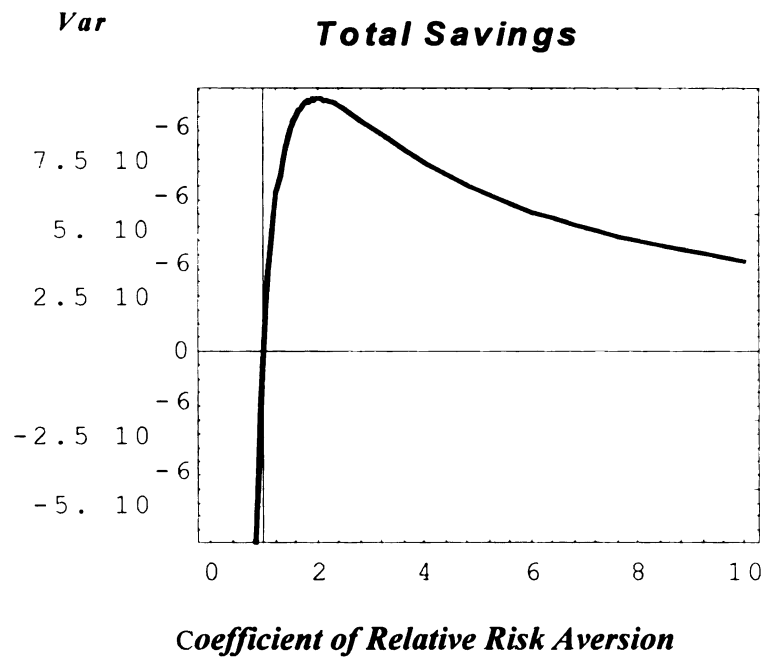
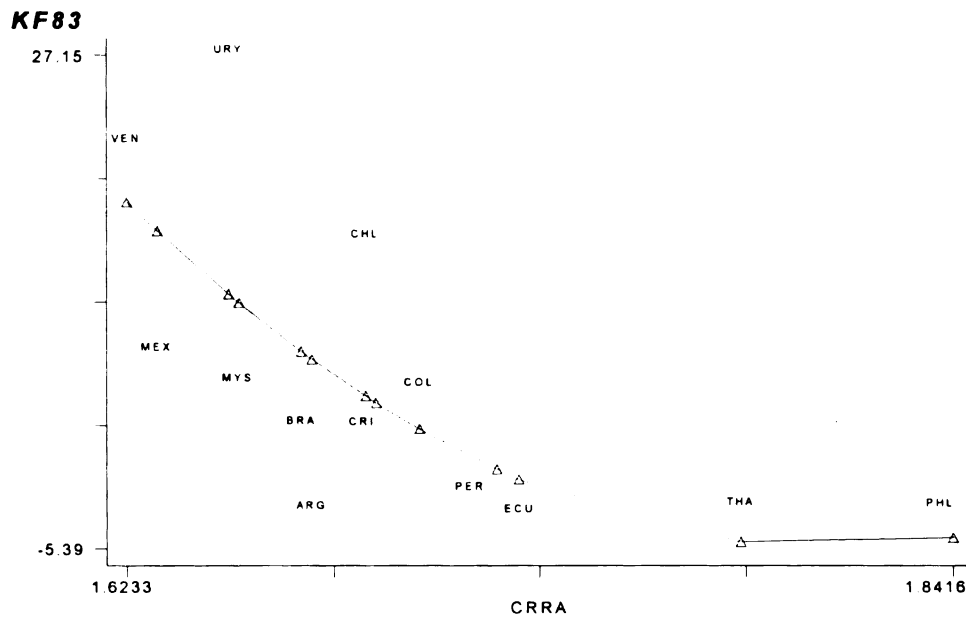


Figure 1.5

***Estimates of Capital Flight (1983)  
as Percentage of GDP***



Source: Capital Flight Estimates taken from: Schineller (1997).

Estimates of the CRRA (Intertemporal Elasticity of Substitution) taken from:  
Ogaki, Ostry and Reinhart, (1996).

## APPENDIX 1.A

### Households Maximization Problem

Given preferences (1) and the HBC, households solve:

$$\underset{\{C_1, \tilde{C}_2, a, m\}}{\text{Max}} \quad V \equiv U[C_1] + \beta \int_{x_0}^{x_1} \{U[\tilde{C}_2]\} f(x) dx \quad \text{subject to} \quad Y = C_1 + a + m \quad \text{and}$$

$$\tilde{C}_2 = a(1 + \tilde{x}) + m(1 + r).$$

Let  $S$  denote total savings, then  $S = a + m$  and therefore, 2)  $Y = C_1 + S$  and, 3)

$$\tilde{C}_2 = a(\tilde{x} - r) + S(1 + r). \text{ Thus, the maximization problem can be rewritten in terms of } a$$

and  $S$  as:

$$\underset{\{S, a\}}{\text{Max}} \quad U[Y - S] + \beta \int_{x_0}^{x_1} \{U[a(x - r) + S(1 + r)]\} f(x) dx$$

$$\text{FOC's:} \quad 4) V_s: -U'[Y - S] + \beta \int_{x_0}^{x_1} \{U'[a(x - r) + S(1 + r)](1 + r)\} f(x) dx = 0$$

$$5) V_a: \beta \int_{x_0}^{x_1} \{U'[a(x - r) + S(1 + r)](x - r)\} f(x) dx = 0$$

$$\text{SOC's: } V_{ss}: U''[Y - S] + \beta \int_{x_0}^{x_1} \{U''[a(x - r) + S(1 + r)](1 + r)^2\} f(x) dx$$

$$V_{sa}: \beta \int_{x_0}^{x_1} \{U''[a(\tilde{x} - r) + S(1 + r)](1 + r)(x - r)\} f(x) dx$$

$$V_{aa}: \beta \int_{x_0}^{x_1} \{U''[a(x - r) + S(1 + r)](x - r)^2\} f(x) dx$$

$$V_{as} = -\beta \int_{x_0}^{x_1} \{U'''[a(x-r) + S(1+r)](1+r)(x-r)\} f(x) dx$$

Let  $H = \begin{bmatrix} V_{ss} & V_{sa} \\ V_{as} & V_{aa} \end{bmatrix}$ . Then, the SOC's are given by  $V_{ss} < 0$ ,  $V_{aa} < 0$ , and

$|H| = [V_{ss}V_{aa} - (V_{as})^2] > 0$ . By strict concavity of  $U$  both  $V_{ss}$  and  $V_{aa}$  are negative. We

assume that  $|H|$  is positive. We focus only on interior solutions in which  $a$ ,  $S$  and  $m$  are all positive.

### Effects of a FDS Deterioration

Here we assume that the economy goes from  $F$  to  $G$  such as  $F$  dominates  $G$  in a first degree stochastic sense.

#### Lemma 1:

If agents' preferences exhibit risk aversion, then  $V_s[G]|_{S^*, a^*} > 0$ .

*Proof:*

By definition of  $\{S^*, a^*\}$  and FOC's (4),  $V_s[G]|_{S^*, a^*}$  has the same sign that

$V_s[G]|_{S^*, a^*} - V_s[F]|_{S^*, a^*}$ , since  $V_s[F]|_{S^*, a^*}$  is equal to zero. Hence, the sign of  $V_s[G]|_{S^*, a^*}$  is

given by the sign of  $\beta \int_{x_0}^{x_1} \{U''[a^*(\tilde{x}-r) + S^*(1+r)](1+r)\} [g(x) - f(x)] dx$ .

Using integration by parts we can write that expression as:

$$\beta(1+r) \left\{ [U'''[C_2][G(x) - F(x)]] \Big|_{x_0}^{x_1} \right\} - \int_{x_0}^{x_1} \{U'''[C_2]a^*\} [G(x) - F(x)] dx.$$

Thus, the sign of  $V_s[G]_{|_{S^*,a^*}}$  is the same that the sign of  $\int_{x_0}^{x_1} \{U''[C_2]a^*\} [F(x) - G(x)] dx$ .

Therefore, if preferences exhibit risk aversion we have that  $V_s[G]_{|_{S^*,a^*}} > 0$  by the FDS deterioration assumption,  $F(x) \leq G(x) \forall x$ . ■

**Proposition 2:**

If agents' preferences exhibit risk aversion and have a coefficient of relative risk aversion no larger than one for all  $x$ , then  $V_a[G]_{|_{S^*,a^*}} < 0$ .

*Proof:*

We know that the sign of  $V_a[G]_{|_{S^*,a^*}}$  is equal to the sign of  $V_a[G]_{|_{S^*,a^*}} - V_a[F]_{|_{S^*,a^*}}$ , since by FOC's the latter expression is null. Then:

$$\beta \int_{x_0}^{x_1} \{U'[a^*(\tilde{x} - r) + S^*(1 + r)](x - r)\} [g(x) - f(x)] dx.$$

Using integration by parts we have that the sign of  $V_a[G]_{|_{S^*,a^*}}$  is given by the sign of:

$$\int_{x_0}^{x_1} U'[C_2][1 - A_r][F(x) - G(x)] dx - \int_{x_0}^{x_1} U''[C_2]S(1 + r)[F(x) - G(x)] dx.$$

Where  $A_r$  represents the coefficient of relative risk aversion,  $A_r = -\frac{U''[C]}{U'[C]}C$ . Since by

definition of FDS deterioration  $F(x) \leq G(x) \forall x$ , and concavity of  $U$  the second term of that expression is negative. The sign of the first part of the expression depends on the sign of  $(1 - A_r)$ . If the  $A_r$  is not larger than one for all values of  $x$  in the domain, then the sign of the whole expression is negative. Therefore,  $V_a[G]_{|_{S^*,a^*}}$  is negative if  $U'' < 0$  and  $A_r \leq 1$ . ■

**Proposition 3:**

a) If preferences are characterized by decreasing absolute risk aversion (DARA), then

$$V_{aS} > 0.$$

b) If preferences are characterized by increasing absolute risk aversion (IARA), then

$$V_{aS} < 0.$$

c) If preferences are characterized by constant absolute risk aversion (CARA), then

$$V_{aS} = 0.$$

*Proof:*

From SOC's  $V_{aS} = \beta \int_{x_0}^{x_1} \{U''[a(\tilde{x} - r) + S(1+r)](1+r)(\tilde{x} - r)\} f(x) dx$ . This expression

can be rewritten as  $(1+r)\beta \int_{x_0}^{x_1} \left\{ \left[ \frac{U''[\tilde{C}_2]}{U'[\tilde{C}_2]} \right] U'[\tilde{C}_2](\tilde{x} - r) \right\} f(x) dx$  or using the

coefficient of absolute risk aversion as  $-(1+r)\beta \int_{x_0}^{x_1} \{A_a U'[\tilde{C}_2](\tilde{x} - r)\} f(x) dx$ . Here  $A_a$

represents the coefficient of absolute risk aversion, thus  $A_a = -\frac{U''[C]}{U'[C]}$ . Then, it follows

that if  $\frac{\partial A_a}{\partial x} = \begin{cases} + & \Leftrightarrow IARA \\ - & \Leftrightarrow DARA \\ 0 & \Leftrightarrow CARA \end{cases}$ , then  $V_{aS} = \begin{cases} - \\ + \\ 0 \end{cases}$ . ■

## **Total Partial Equilibrium Effects (TPEE)**

### **Corollary 1.A: (TPEE on Savings)**

If agents' preferences are characterized by risk aversion, a coefficient of relative risk aversion no larger than one, and nondecreasing absolute risk aversion, then a FDS deterioration of  $x$ 's cdf induces an increase in total savings.

*Proof:*

From lemma 1 the direct effect of a FDS deterioration on total savings is positive as long as  $U'' < 0$ . On the other hand, the indirect effect is: a) Negative under DARA and  $A_r \leq 1$ . b) Positive under IARA and  $A_r \leq 1$ . c) Null under CARA. Then, combining those pieces of information we have that the TPEE is positive if preferences show  $U'' < 0$ ,  $A_r \leq 1$  and either IARA or CARA.

Notice that the sign of the TPEE on  $S$  is positive under CARA and  $U'' < 0$ , but cannot be determined under DARA. ■

### **Corollary 1.B: (TPEE on Domestic Assets)**

If agents' preferences are characterized by risk aversion, a coefficient of relative risk aversion no larger than one, and nondecreasing absolute risk aversion, then a FDS deterioration of  $x$ 's cdf induces a reduction in the holdings of domestic assets.

*Proof:*

From proposition 2 the direct effect of a FDS deterioration on the holdings of the risky assets is negative if  $A_r \leq 1$ . The indirect effect is: a) positive under DARA and  $U'' < 0$ . b) Negative under IARA and  $U'' < 0$ . c) Null under CARA. Hence, the TPEE is negative if preferences show  $A_r \leq 1$ ,  $U'' < 0$  and either IARA or CARA. ■



### Corollary 1.C: (TPEE on Foreign Assets)

If agents' preferences are characterized by risk aversion, a coefficient of relative risk aversion no larger than one, and nondecreasing absolute risk aversion, then a FDS deterioration of  $x$ 's cdf induces an increase in the holdings of foreign assets.

*Proof:*

By construction TPEE on  $m$  is given by (TPEE on  $S$  - TPEE on  $a$ ). From corollary 1.A the TPEE on savings is positive. From corollary 1.B the TPEE on risky assets is negative. Therefore, the TPEE on the foreign assets is positive. ■

### Effects of a MP Spread

Here we assume that the domestic return's distribution goes from  $F$  to  $G$ , such as  $G$  is a mean preserving spread transformation of  $F$ .

#### Proposition 4:

If  $U'''$  is either positive, negative or zero, then the marginal expected utility of savings under  $G$  evaluated at  $\{a^*, S^*\}$ ,  $V_s[G]_{|_{S^*, a^*}}$ , has either a positive, negative or zero value.

*Proof:*

From FOC's the sign of  $V_s[G]_{|_{S^*, a^*}}$  is the same of  $V_s[G]_{|_{S^*, a^*}} - V_s[F]_{|_{S^*, a^*}}$ . Thus, the sign

of  $V_s[G]_{|_{S^*, a^*}}$  is given by the sign of  $(1+r)\beta \int_{x_0}^{x_1} U'[C_2][g(x) - f(x)]dx$ .

Following Rothschild and Stiglitz (1970) [RS] we can determine the sign of  $V_s[G]|_{S^*,a^*}$  by looking at the concavity of  $U'[C_2]$  with respect to the random variable  $\tilde{x}$ . Then,

$$V_s[G]|_{S^*,a^*} = \begin{cases} + \\ - \\ 0 \end{cases} \text{ if } U'[C_2] \text{ is } \begin{cases} \text{convex} \\ \text{concave} \\ \text{linear} \end{cases} \text{ in } x.$$

Furthermore, since  $\frac{\partial^2 U'[C_2]}{\partial x^2} = U'''a^2$  the concavity of  $U'[C_2]$  depends on the sign of

$$U'''. \text{ Hence, } V_s[G]|_{S^*,a^*} = \begin{cases} + \\ - \\ 0 \end{cases} \text{ if } U''' = \begin{cases} + \\ - \\ 0 \end{cases}. \blacksquare$$

**Proposition 5:**

The expected marginal utility of the risky asset under  $G$  evaluated at  $\{a^*, S^*\}$ ,  $V_a[G]|_{S^*,a^*}$  is negative if preferences exhibit either i) A positive third derivative of the utility function, a coefficient of relative risk aversion smaller than one and nondecreasing in the random variable; ii) A positive third derivative of the utility function and a coefficient of relative risk aversion equal to one; or iii) A null third derivative of the utility function.

*Proof:*

The sign of  $V_a[G]|_{S^*,a^*}$  is equal to the sign of  $\beta \int_{x_0}^{x_1} U'[C_2](x-r)[g(x)-f(x)]dx$ .

Following RS we can establish the sign of the later expression by looking at the concavity of  $U'[C_2](x-r)$  with respect to  $x$ . Thus, we must find the sign of:

$$\frac{\partial^2 U'[C_2](x-r)}{\partial x^2} = a[2U''[C_2] + U'''[C_2]a(x-r)].$$



To find the sign of this expression we place some restrictions on the coefficient of relative risk aversion  $A_r$  and about the sign of the third derivative of  $U$ .

By definition we know that  $\frac{\partial A_r}{\partial x} = -\frac{a}{U'[C_2]}[U''' C_2 + U''(1 + A_r)]$ . Furthermore, we

know that:

a) If  $U''' > 0$ , then  $U''' C_2 > U''' a(x - r)$ .

b) If  $A_r < 1$ , then  $U''(1 + A_r) > U'' 2$ .

c) If  $U''' < 0$ , then  $U''' C_2 < U''' a(x - r)$ .

d) If  $A_r > 1$ , then  $U''(1 + A_r) < U'' 2$ .

If we use some of these assumptions about  $U$  and about  $A_r$  we can establish the sign of

$V_a[G]_{s,a}$ . For instance, if we assume that  $\frac{\partial A_r}{\partial x} > 0$ , then we know that

$0 > U'''[C_2] C_2 + U''[C_2](1 + A_r)$ . If in addition to that we assume that  $U''' > 0$  and

$A_r < 1$ , then it follows:

$0 > U'''[C_2] C_2 + U''[C_2](1 + A_r) > 2U''[C_2] + U'''[C_2]a(x - r)$ .

The following table summarizes the results that are obtained when we make assumptions

about  $U'''$ ,  $\frac{\partial A_r}{\partial x}$  and  $A_r$ .

Table 1.5

***Marginal Expected Utility of the Risky Asset***

$V_a$	$U''' > 0$			$U''' = 0$			$U''' < 0$		
	$Ar_x < 0$	$Ar_x = 0$	$Ar_x > 0$	$Ar_x < 0$	$Ar_x = 0$	$Ar_x > 0$	$Ar_x < 0$	$Ar_x = 0$	$Ar_x > 0$
$Ar < 1$	?	—	—	..	..	—	..	..	?
$Ar = 1$	..	—	..	..	..	..	..	..	..
$Ar > 1$	?	?	?	..	..	—	..	..	?

Here an outcome (?) tells us that the sign of  $V_a[G]_{a^*,s^*}$  is ambiguous. On the other hand, an outcome (..) means that a particular set of assumptions cannot hold at the same time.

For example, if preferences exhibit a null third derivative of  $U$ , then  $\frac{\partial A_r}{\partial x} = 0$  is not

possible. A null  $\frac{\partial A_r}{\partial x}$  requires DARA, and DARA requires  $U''' > 0$ .

## Total Partial Equilibrium Effects

### Corollary 2.A: (TPEE on Savings)

A mean preserving spread of  $x$ 's distribution causes an increase in the level of optimal savings if preferences exhibit either: i) Nondecreasing absolute risk aversion, a positive

third derivative of  $U$ , a coefficient of relative risk aversion smaller than one and increasing in  $x$ ; or ii) A null third derivative of  $U$ .

*Proof:*

To determine the TPEE on total savings we must establish the signs of both the direct and the indirect. The sign of the direct effect is give by proposition 4, whereas to determine the sign of the indirect effect we combine the results of propositions 3 and 5. Tables 1.5 and 1.6 contain information about the signs of the indirect effect and the total partial equilibrium effects, respectively.

Each entry in this first table represents the product of the signs of  $V_a[G]_{a^*,s^*}$  and  $V_{as}$ . For

instance, if we assume that  $U''' > 0$ ,  $\frac{\partial A_r}{\partial x} = 0$  and  $A_r < 1$ , then the negative sign is the result

of the product of the negative sign of  $V_a[G]_{a^*,s^*}$  and the positive sign of  $V_{as}$ .

Table 1.6

**Indirect Effect on Savings  
Mean Preserving Spread**

<i>Ind Sav</i>	$U'' > 0$			$U'' = 0$			$U'' < 0$			
	$A_x < 0$	$A_x = 0$	$A_x > 0$	$A_x < 0$	$A_x = 0$	$A_x > 0$	$A_x < 0$	$A_x = 0$	$A_x > 0$	
$A < 1$	?	—	—	..	..	..	..	..	..	
$A = 1$	..	—	..	..	..	..	..	..	..	<i>DARA</i>
$A > 1$	?	?	?	..	..	..	..	..	..	
$A < 1$	..	..	0	..	..	..	..	..	..	
$A = 1$	..	..	..	..	..	..	..	..	..	<i>CARA</i>
$A > 1$	..	..	0	..	..	..	..	..	..	
$A < 1$	..	..	+	..	..	+	..	..	?	
$A = 1$	..	..	..	..	..	..	..	..	..	<i>IARA</i>
$A > 1$	..	..	?	..	..	+	..	..	?	

In Table 1.7 each outcome represents the *sum* of the signs of the direct and the indirect effects of a mean preserving spread on total savings. Thus, we combine the information about the direct effect with the results of the indirect effect to determine the sign of the TPEE on total savings.

Table 1.7

**TPEE on Savings  
Mean Preserving Spread**

<i>Tot Sav</i>	$U''' > 0$			$U''' = 0$			$U''' < 0$			
	$A_{r_x} < 0$	$A_{r_x} = 0$	$A_{r_x} > 0$	$A_{r_x} < 0$	$A_{r_x} = 0$	$A_{r_x} > 0$	$A_{r_x} < 0$	$A_{r_x} = 0$	$A_{r_x} > 0$	
$A_r < 1$	?	?	?	..	..	..	..	..	..	
$A_r = 1$	..	?	..	..	..	..	..	..	..	<i>DARA</i>
$A_r > 1$	?	?	?	..	..	..	..	..	..	
$A_r < 1$	..	..	+	..	..	..	..	..	..	
$A_r = 1$	..	..	..	..	..	..	..	..	..	<i>CARA</i>
$A_r > 1$	..	..	+	..	..	..	..	..	..	
$A_r < 1$	..	..	+	..	..	+	..	..	?	
$A_r = 1$	..	..	..	..	..	..	..	..	..	<i>IARA</i>
$A_r > 1$	..	..	?	..	..	+	..	..	?	

Notice that we may obtain a (?) output when the TPEE is ambiguous either because the indirect and the direct effects have opposite signs or because the sign of one of the effects is ambiguous in its own.

**Corollary 2.B: (TPEE on Domestic Assets)**

A mean preserving spread in  $x$ 's distribution induces a reduction in the optimal holdings of the risky assets if preferences exhibit either: i) Nondecreasing absolute risk aversion, a



positive third derivative of  $U$ , and a coefficient of relative risk aversion no larger than one and increasing in  $x$ ; or ii) A null third derivative of  $U$ .

*Proof:*

Proposition 5 tells us under what conditions we can unambiguously sign the direct effect on the risky assets of a MP spread. Using propositions 3 and 4 we can establish the sign of the indirect effect. Then, combining those pieces of information we have that the TPEE of a MP spread on the holdings of the risky assets is given in Table 1.8.

Table 1.8

**Total PEE on Domestic Assets  
Mean Preserving Spread**

Tot Ris	$U''' > 0$			$U''' = 0$			$U''' < 0$			
	$Ar_x < 0$	$Ar_x = 0$	$Ar_x > 0$	$Ar_x < 0$	$Ar_x = 0$	$Ar_x > 0$	$Ar_x < 0$	$Ar_x = 0$	$Ar_x > 0$	
$Ar < 1$	?	?	?	..	..	..	..	..	..	DARA
$Ar = 1$	..	?	..	..	..	..	..	..	..	
$Ar > 1$	?	?	?	..	..	..	..	..	..	
$Ar < 1$	..	..	—	..	..	..	..	..	..	CARA
$Ar = 1$	..	..	..	..	..	..	..	..	..	
$Ar > 1$	..	..	?	..	..	..	..	..	..	
$Ar < 1$	..	..	—	..	..	—	..	..	?	IARA
$Ar = 1$	..	..	..	..	..	..	..	..	..	
$Ar > 1$	..	..	?	..	..	—	..	..	?	

### Corollary 2.C: (TPEE on Foreign Assets)

A mean preserving spread in  $x$ 's distribution causes an increase in the optimal holdings of foreign assets if preferences show either: i) Nondecreasing absolute risk aversion, a positive third derivative of  $U$  and a coefficient of relative risk aversion no larger than one and increasing in  $x$ ; or ii) A null third derivative of  $U$ .

*Proof:*

By definition, we can express the total partial equilibrium effect on risky assets as a function of the TPEE on total savings and the TPEE on the holdings of risky assets, thus TPEE on  $m$  = TPEE on  $S$  - TPEE on  $a$ . Therefore, the TPEE on the foreign assets is given in Table 1.9.

### Proposition 6:

Under risk aversion, the TPEE on savings due to changes in income is positive.

*Proof:*

By definition we have that  $\frac{\partial S}{\partial Y_1} = \frac{[-V_{aa}V_{SY_1} + V_{Sa}V_{aY_1}]}{|H|}$ , then it follows that since  $V_{aY_1} = 0$

the *indirect* effect --given by  $\frac{[V_{Sa}V_{aY_1}]}{H}$ -- vanishes. On the other hand, by SOC's  $V_{aa} < 0$

and  $|H| > 0$ , therefore the sign of  $\frac{\partial S}{\partial Y_1}$  is equal to the sign of  $V_{SY_1}$ . Notice that under risk

aversion the sign of  $V_{SY_1} = - \int_{x_0}^{x_1} \{U'''[C_2]\} f(x) dx$  is positive. ■

Table 1.9

**Total PEE on Foreign Assets  
Mean Preserving Spread**

<i>Tot For</i>	$U''' > 0$			$U''' = 0$			$U''' < 0$			
	$Ar_x < 0$	$Ar_x = 0$	$Ar_x > 0$	$Ar_x < 0$	$Ar_x = 0$	$Ar_x > 0$	$Ar_x < 0$	$Ar_x = 0$	$Ar_x > 0$	
$Ar < 1$	?	?	?	..	..	..	..	..	..	<i>DARA</i>
$Ar = 1$	..	?	..	..	..	..	..	..	..	
$Ar > 1$	?	?	?	..	..	..	..	..	..	
$Ar < 1$	..	..	+	..	..	..	..	..	..	<i>CARA</i>
$Ar = 1$	..	..	..	..	..	..	..	..	..	
$Ar > 1$	..	..	?	..	..	..	..	..	..	
$Ar < 1$	..	..	+	..	..	+	..	..	?	<i>IARA</i>
$Ar = 1$	..	..	..	..	..	..	..	..	..	
$Ar > 1$	..	..	?	..	..	+	..	..	?	



## APPENDIX 1.B

### FDS Changes

By definition the cdf  $F$  dominates the cdf  $G$  in a FDS sense iff  $F \leq G$  for all  $x$ . We assume that  $F$  and  $G$  are given by  $[F(x_1)=p_1, F(x_2)=1]$ , and  $[G(x_{11})=p_1, G(x_{12})=1]$ . To show that  $F$  dominates  $G$  we must show that  $F \leq G$  for all  $x$ . Since  $x_{11} = x_1 - k_1$  and  $x_{12} = x_2 - k_1$  it follows that  $G(x_{11}) > F(x_{11})$ . By the same token,  $G(x_1) > F(x_1)$ ,  $G(x_{12}) > F(x_{12})$ , and finally,  $G(x_2) = F(x_2)$ . Graphically, a FDS dominance condition implies that the function  $G$  is never below the function  $F$ . (See Figure 1.6).

### MP Changes

Let us use Figure 1.7 to show that when going from  $F$  to  $G$  the distribution of the random return experiences a MP spread. Condition 1 of a MP spread implies that the area under  $G$  is no smaller than the area under  $F$ , for all  $x$  in the domain. This condition holds for  $x < x_2$ . The area  $A$  represents the difference between the area under  $G$  and  $F$  for values of  $x$  lower than  $x_2$ . This area is given by:

$$p_1 * [x_1 - x_{21}] = p_1 * [x_1 - x_1 + (c_2/p_1)] = c_2.$$

Let  $B$  be the difference between the areas under  $F$  and  $G$  for  $x$  larger than  $x_2$ . If condition 2 holds, i.e., if the expected values of  $x$  under the two distributions are equal, then the  $B$ -area should be equal to the  $A$ -area. Furthermore, if the  $B$ -area is equal to the  $A$ -area, then condition 1 holds for all the values of  $x$ , and therefore, the condition 1 is satisfied. The value of the  $B$ -area is given by:  $(1-p_1)[x_2 - x_{22}] = (1-p_1)[x_2 + (c_2/(1-p_1)) - x_2] = c_2$ . Thus, since the two areas are equal conditions 1 and 2 of a MP spread are satisfied.

Figure 1.6

***First Degree Stochastic Deterioration  
Going From F to G***

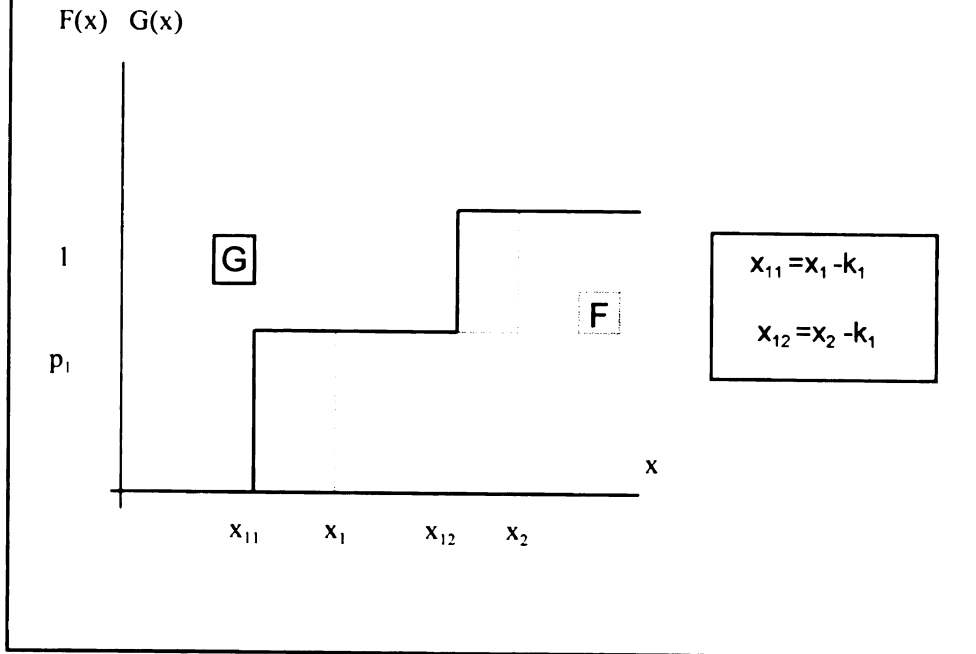
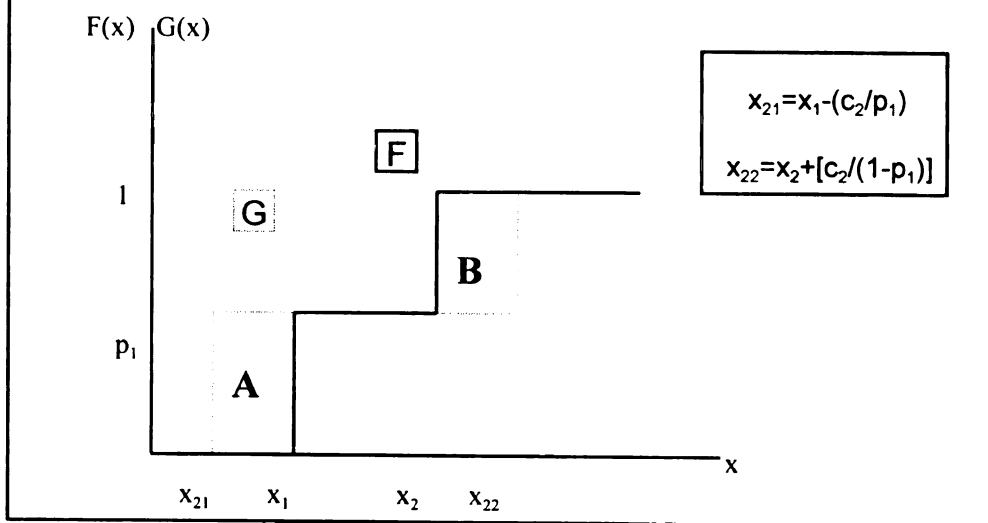


Figure 1.7

***Mean Preserving Spread  
Going from F to G***



## **Chapter 2**

### **Changes in Domestic Risk and Inflation in Small Open Economies**

#### **2.1. Introduction**

Recent empirical studies (Erb, Harvey and Viskanta (1995, 1996) and Diamonte, Liew and Stevens (1996)) have found a positive correlation between asset return volatility and political risk (indicators) in a variety of countries. In other words, these studies find that countries with higher political risk tend to exhibit higher return volatility.<sup>1</sup> This evidence seems to suggest that changes in a country's (political) risk affect domestic assets' return distribution in that country.

Our purpose in this chapter is twofold. First, we try to determine how an increase in the domestic risk of small open economies (modeled as a mean preserving spread in domestic bond's return distribution) affects domestic agents' optimal saving and portfolio choices. Second, we investigate whether these changes in domestic agents' choices translate into higher inflation by altering the government's financing process.

Lately, a large body of literature has tried to establish the existence of empirical relationships between changes in an economy's political stability and the behavior of variables such as economic growth, investment, saving and inflation. Those studies try to

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<sup>1</sup> These studies also find that the (political) risk indicators predict future returns of stocks and bonds.





uncover possible links between the decision-making process of both public and private agents and the (negative) incentive effects that a reduction in political stability may generate.

Studies such as Alesina and Chua (1997), Alesina, Ozler, Roubini and Swagel (1996), Barro (1991), Barro and Sala-i-Martin (1995) and Kormendi and Meguire (1985) look for a relationship between economic growth and political stability. These studies find that the higher the political stability the higher the growth rate of an economy.<sup>2</sup>

Alesina and Perotti (1996) analyze how income distribution affects investment. They try to establish the connection between these variables by exploring how these two variables relate to political stability. Alesina and Perotti claim that an increase in political instability may affect investment among other things because it can disrupt production activities and trade, and because higher political instability increases agents' uncertainty about the returns of (potential) investment opportunities. Then, given those possible effects of changes in the political stability on investment, Alesina and Perotti show evidence that an increase in income inequality induces a reduction in political stability, hence it induces a reduction of investment.

Venieris and Gupta (1986) assume that the risky asset's return distribution function contains information about the political stability of an economy, and analyze how well political stability, together with income distribution, explains an economy's

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<sup>2</sup> Nevertheless, Levine and Renelt (1992) find that the results of some of these studies are not very robust. Using similar data sets to those used in Barro (1991) and Kormendi and Meguire (1985), Levine and Renelt find that the statistical significance of the variable that measures political stability is very sensitive to the set of explanatory variables used in the model.

saving rate. Venieris and Gupta find evidence that the level of political stability that characterizes a given economy is a significant explanatory variable of the saving rate in that economy. In particular, they find that a country's saving is negatively affected by increases in political instability. Venieris and Gupta claim that the distortions introduced by an unstable political system may induce risk averse agents to alter the way in which they accumulate income and wealth. Venieris and Gupta conjecture that a possible explanation for the low saving rates in poor economies is that those countries do not provide a political environment that encourages saving.

### **2.1.1. Political Stability and Inflation**

Figure 2.1 shows the average political risk (as measured by International Country Risk Guide)<sup>3</sup> and the average *inflation rate* for a sample of 42 countries during the period 1985-1995. This figure suggests that countries with higher political stability tend to have lower inflation. Here a large value of the index indicates a low level of political risk.<sup>4</sup>

Some studies have looked for this relationship between an economy's inflation rate and the stability of its political system, and find evidence of its existence. For instance, Cukierman, Edwards and Tabellini (1992) find evidence in favor of a (negative) relationship between inflation and political stability. As an explanation for this relationship they highlight the role that institutions in politically unstable countries (dependent central banks and inadequate tax systems) play explaining the inflation rate in

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<sup>3</sup> We provide information about the International Country Risk Guide's index in Appendix 2.

<sup>4</sup> Similar results are obtained if we use alternative measures of political risk as the one used by Barro (1991) or that of Alesina and Perotti (1996). See Figure 2.2.

those economies. Cukierman, Edwards and Tabellini maintain that economies with an unstable political system are less likely to have the institutions that are necessary to overcome problems such as the *dynamic-inconsistency* of the economic policy. Thus, political instability creates conditions that allow governments to behave with a pro-inflation bias.

Another study that examines this relationship is Romer (1993). He studies how the degree of openness of an economy affects the inflation rate of this economy. There he introduces a measure of political stability and determines that politically unstable economies tend to have higher inflation rates than stable economies. Romer also suggests that differences in the institutions between stable and unstable economies have a significant role in explaining the connection between inflation and political stability.

The present study, even though it relates to those works that analyze the political stability-inflation relationship, departs from them, and suggests another link through which political stability may affect inflation. Instead of looking at the role that institutions in politically unstable economies may have in explaining inflation, we try to determine how changes in the level of political stability (changes in domestic risk) could affect agents' saving and portfolio decisions, and how those changes can translate into higher inflation by altering the government financing process.

### **2.1.2. Changes in Domestic Risk and Inflation**

To analyze the inflationary consequences of changes in domestic risk, we use a one-good, two-country (domestic and foreign) world economy framework. We assume



that this economy is characterized by free trade in the consumption good market and by an imperfect financial capital market.

The domestic country is inhabited by a large number of infinitely-lived agents that maximize expected lifetime utility and trade the single consumption good and one financial asset (foreign bond) with the rest of the world (the foreign country). We assume that domestic agents face a cash-in-advance constraint, so they must have domestic currency to participate in the domestic economy's good market.

In the initial period domestic agents receive an endowment of the single consumption good. During the rest of their life agents receive no other endowment and to finance their consumption they must save. We assume that agents may use two saving alternatives: domestic and foreign bonds. Because of the size of the foreign market, we assume that the foreign bond's nominal return is determined outside of the domestic economy. Furthermore, we assume that this return is known with certainty and taken as given by domestic agents. The domestic bond's nominal return is random, with a known distribution function, and behaves as an independent and identically distributed random variable.

The domestic economy also has a government that sells units of the domestic bond and issues domestic currency to finance its spending. The domestic government does not have access to international financial markets, and, therefore, it can sell the domestic bond only to domestic agents. We assume that the domestic government holds a constant level of international reserves (foreign currency), and that the domestic economy has a *managed* exchange rate system. As a consequence of these two assumptions



(constant international reserves and managed exchange rate system), the domestic government can alter the money supply only by changing the rate at which agents exchange domestic and foreign currencies.

This is the context in which we evaluate the effects of an increase in the domestic risk on domestic inflation. Following Rothschild and Stiglitz (1970), we model changes in risk as mean preserving spreads of the risky asset's return distribution. Hence to represent an increase in the domestic risk we assume that the domestic bond's nominal return distribution experiences a mean preserving spread.

To conduct the comparative static analyses we compare the equilibrium outcomes of two economies that differ only in the distribution functions of their random asset's return. We do the comparisons both at a partial and at a general equilibrium level.

Doing the partial equilibrium analysis (the analysis at the individual level) we follow the methodology developed by Levhari and Srinivasan (1969), Samuelson (1969), and Rothschild and Stiglitz (1971). We ask what assumptions about agents' attitude toward risk are *sufficient* to unambiguously sign the effects of changes in the domestic risk on agents' optimal savings and portfolios. We find that when agents' preferences are characterized by constant relative risk aversion and the coefficient of relative risk aversion is no larger than one, an increase in domestic risk induces domestic agents to reduce their holdings of the domestic bond, and, at the same time, to increase their positions in the foreign asset (the standard results in the literature on risk).

Then using those results at the individual level we study the general equilibrium implications of an increase in domestic risk. We find that a reduction in domestic

investors' willingness to hold the domestic bond (due to the increase in domestic risk) forces the domestic government to expand the money supply to compensate for the shortfall in fiscal revenue.<sup>5</sup> To augment the money aggregate the domestic government increases the devaluation rate of the domestic currency, and a higher devaluation rate translates into higher inflation. Therefore an increase in a country's domestic risk (an increase in its political instability) induces an increase in that country equilibrium inflation rate when agents' preferences exhibit a coefficient of relative risk aversion no larger than one.

The rest of the chapter has the following structure. Part II describes the characteristics of the economy and defines a competitive equilibrium. In part III we analyze how a mean preserving spread in the distribution of the domestic bond's nominal return affects both domestic investors' saving and the configuration of domestic investors' portfolios. Part IV presents the analysis at a general equilibrium level. There we evaluate how changes in domestic investors' actions affect the financing process of the domestic government, and how changes in the way the government finances its spending can produce higher inflation. Finally, part V contains some comments.

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<sup>5</sup> The money aggregate expansion takes place (in part) because of the domestic government's inability to borrow on the international financial market, and to levy taxes on the domestic agents.



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## 2.2. General Characteristics of the Economy

The following is the setup of a one-good, two-asset (domestic and foreign bonds), two-country (domestic and foreign) world economy.

The domestic country is a small open economy inhabited by a large number of infinitely lived agents that maximize expected lifetime utility. Domestic agents trade a single consumption good and one financial asset (foreign bond) with the rest of the world (foreign country). Given the relative size of the domestic economy, the price of the consumption good (in units of the foreign currency), is determined in the foreign country, and domestic agents take it as given.

We assume that in the domestic economy the only source of income is from savings. To save domestic agents could use two alternatives: domestic and foreign bonds. The foreign bond has a nominal return that is known with certainty, and it is taken as given by domestic agents. The domestic bond is risky, and even though its nominal return is random agents know its distribution function. This distribution function is exogenous.<sup>6</sup>

In this model the international financial capital market is imperfect, because the domestic government cannot participate in it. Here, any bond issued by the domestic government can be bought only by domestic agents.<sup>7</sup>

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<sup>6</sup> Following Venieris and Gupta (1986), Erb, Harvey and Viskanta (1995, 1996) we assume that domestic return reflects information about the political stability, and, therefore, this is the (main) source of risk for the domestic bond. In particular, here we interpret the domestic risk as coming from the political environment that agents (both private and public) face, and that is exogenous to them.

<sup>7</sup> Even though this is a very strong assumption, there have been several episodes in which the governments of developing economies have had very limited access to international financial markets. An example of these episodes is the period after the *debt crisis* in the 1980's in which many developing economies, especially in Latin America, had very

### 2.2.1. Households

The domestic economy is populated by a large number (that we normalize to one) of identical households that live forever. These households maximize expected lifetime utility from consumption. We assume that agents' expected lifetime utility is given by:

$$(1) \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

where  $\beta$  is a stationary subjective discount factor, such that  $\beta \in (0,1)$ .  $U$  is a strictly increasing, strictly concave, continuous and twice differentiable utility function,  $c_t > 0$  denotes consumption in period  $t$ , and  $E_0$  is the expectation operator. We assume that at the beginning of their life all agents receive an initial endowment,  $W_0$ , of the consumption good.

As a simplifying assumption, in the domestic economy there is no production so, domestic households receive utility from the consumption of a good that is produced abroad and sold in a perfectly competitive international market. The price of this good on the international markets,  $P_t^*$ , is expressed in foreign currency, and we assume that it is constant and equal to one. Then, under free trade and by the law of one price (arbitrage condition),  $P_t = P_t^* e_t$ . Here  $e_t$  denotes the nominal exchange rate of domestic currency per unit of foreign currency, whereas  $P_t$  represents the price of consumption good in the domestic market. Furthermore, since  $P_t^* = 1$ , it follows that  $P_t = e_t$  and  $\frac{P_t^* e_t}{P_t} \equiv 1$ .

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limited access to international financial markets. See Agénor and Montiel (1996, Chapter 13). Another example is given by the sudden reduction of inflows of financial resources into Mexico after the first quarter of 1994. See Atkeson and Rios-Rull (1996).

We assume that domestic households face a cash-in-advance constraint (CIAC), hence:

$$(2) \quad m_t^d \geq P_t c_t$$

where  $m_t^d$  represents households' demand for money.

In this economy *interest-earning* from saving is the only source of income for the domestic households. Thus, domestic agents must save to transfer resources between periods. To save households determine their desired levels of both domestic bonds,  $b$ , and foreign bonds,  $b^*$ , in each period. These bonds are denominated in nominal terms in their respective currencies, and have a one period maturity.

The domestic bond (issued by the domestic government) can be purchased only by domestic agents. This bond has a random nominal net return,  $\tilde{x}$ , with a cumulative distribution function denoted by  $F$ . Furthermore, we assume that  $\tilde{x}$  is an independent and identically distributed (iid) random variable. We also assume that domestic households make their resources allocation decisions before  $\tilde{x}_{t+1}$  is revealed, where  $\tilde{x}_{t+1}$  is the net rate of return of a domestic bond bought at period  $t$  that matures at period  $t+1$ . The foreign bond is denominated in foreign currency, and the domestic currency value of  $b_{t+1}^*$  units of the foreign bond (at the buying time) is  $e_t b_{t+1}^*$ . The nominal net return of a foreign bond bought at period  $t$  and that matures at  $t+1$ ,  $r_{t+1}$ , is determined in the international financial market and it is known with certainty. We assume that agents take both  $F$  and  $r_{t+1}$  as given. Given this set up, we can write the domestic households' intertemporal nominal budget constraint (HBC) as:

$$(3) \quad P_t c_t + m_{t+1}^d + b_{t+1} + e_t b_{t+1}^* = (1 + \tilde{x}_t) b_t + (1 + r_t) e_t b_t^* + m_t^d \quad \forall t \geq 1.$$

Thus domestic households use the financial resources available at the beginning of period  $t$  to buy consumption goods and to acquire financial assets.

Notice that  $e_t b_{t+1}^*$  is the nominal value in domestic currency of  $b_{t+1}^*$  units of the foreign bond when agents buy (at the beginning of period  $t$ ). However we can rewrite this expression in terms of the domestic currency value at the beginning of period  $t+1$ , by multiplying  $e_t b_{t+1}^*$  by  $\frac{e_{t+1}}{e_t}$ , and expression (3) becomes:

$$P_t c_t + m_{t+1}^d + b_{t+1} + e_t \frac{e_{t+1}}{e_t} b_{t+1}^* = (1 + \tilde{x}_t) b_t + (1 + r_t) e_t b_t^* + m_t^d.$$

Let  $\varepsilon_t = \frac{e_{t+1}}{e_t}$ , then it follows that:

$$P_t c_t + m_{t+1}^d + b_{t+1} + e_{t+1} \frac{1}{\varepsilon_t} b_{t+1}^* = (1 + \tilde{x}_t) b_t + (1 + r_t) e_t b_t^* + m_t^d$$

where  $e_{t+1} \frac{1}{\varepsilon_t} b_{t+1}^*$  represents the domestic currency value of  $b_{t+1}^*$  units of the foreign bond at beginning of period  $t+1$ .  $(\varepsilon_t)^{-1}$  denotes changes in the domestic currency value of  $b_{t+1}^*$  that take place because of changes in the exchange rate of the domestic and the foreign currencies between the beginning of period  $t$  and the beginning of period  $t+1$ .

In real terms the HBC is given by:

$$c_t + \frac{m_{t+1}^d}{P_t} + \frac{b_{t+1}}{P_t} + \frac{e_{t+1}}{P_t} \frac{1}{\varepsilon_t} b_{t+1}^* = \frac{(1 + \tilde{x}_t) b_t}{P_t} + (1 + r_t) b_t^* \frac{e_t}{P_t} + \frac{m_t^d}{P_t}$$

and assuming that  $P_t = e_t$  for all periods, we have:

$$c_t + \frac{m_{t+1}^d}{P_t} + \frac{b_{t+1}}{P_t} + \pi_t \frac{1}{\pi_t} b_{t+1}^* = \frac{(1 + \tilde{x}_t)b_t}{P_t} + (1 + r_t)b_t^* \frac{P_t}{P_t} + \frac{m_t^d}{P_t} \quad \text{or}$$

$$(3') \quad c_t + \frac{m_{t+1}^d}{P_t} + \frac{b_{t+1}}{P_t} + b_{t+1}^* = \frac{(1 + \tilde{x}_t)b_t}{P_t} + (1 + r_t)b_t^* + \frac{m_t^d}{P_t}.$$

where  $\frac{P_{t+1}}{P_t} = \pi_t$ .<sup>8</sup>

### 2.2.1.1. Solving the Households' Maximization Problem

The households' problem is to maximize their lifetime utility by choosing an optimal sequence of consumption, foreign and domestic bonds, and money holdings given the CIAC and the HBC. Thus given some positive initial endowment,  $W_0$ , the foreign bond's nominal return (a constant for all  $t$ ), the domestic bond's nominal return distribution (and the realizations of this return until period  $t$ ), the period  $t$  exchange rate between the domestic and the foreign currencies (and past realizations) and the price in domestic currency of the consumption good (and past realizations) households solve:

$$(1) \quad \left\{ c_t, \frac{b_{t+1}}{P_t}, b_{t+1}^*, \frac{m_{t+1}^d}{P_t} \right\} \quad \text{Max} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \quad \text{subject to :}$$

---

<sup>8</sup> If  $e_{t+1} \neq e_t$ , agents face a (nominal) capital gain or loss depending on the direction that the exchange rate moves. Thus, it looks as if agents confront *exchange rate risk* (ERR) investing in the foreign bond, since  $e_{t+1}$  is unknown when they choose  $b_{t+1}^*$ . If this were the case, agents would be choosing between two risky assets. Nevertheless, in this case since the nominal rate of return of the foreign assets (ignoring capital gain or losses) is known with certainty, and, since  $P_{t+1} = e_{t+1}$ , the real rate of return is also known with certainty, and it is equal to the nominal rate of return. Hence, even though in nominal terms there exists ERR when investing in the foreign bond, the ERR is not present when we do the evaluation in real terms, and, therefore, in this set up we can consider the foreign bond as a risk-free asset.



$$(2) \quad m_t^d \geq P_t c_t \quad \text{and}$$

$$(3') \quad c_t + \frac{m_{t+1}^d}{P_t} + \frac{b_{t+1}}{P_t} + b_{t+1}^* = \frac{(1 + \tilde{x}_t)b_t}{P_t} + (1 + r_t)b_t^* + \frac{m_t^d}{P_t}.$$

Equivalently, we can rewrite the maximization problem in terms of real wealth using the result of the next proposition.

**Proposition 1:**

If the domestic nominal net return,  $\tilde{x}_t$ , is positive, then the cash-in-advance constraint (2), is binding.

*Proof:*

Let us define  $W_t$  as real wealth at the beginning of period  $t$ , thus for  $t \geq 1$ ,

$$P_{t-1}W_{t-1} = b_t + e_{t-1}b_t^* + m_t^d \quad \text{and} \quad P_tW_t = (1 + \tilde{x}_t)b_t + (1 + r_t)e_t b_t^* + m_t^d - P_t c_t.$$

Then, by substitution of the first expression into the second we have:

$$P_tW_t = (1 + \tilde{x}_t)[P_{t-1}W_{t-1} - e_{t-1}b_t^* - m_t^d] + (1 + r_t)e_t b_t^* + m_t^d - P_t c_t,$$

and therefore  $\frac{\partial(P_tW_t)}{\partial m_t^d} = -\tilde{x}_t$

Then, as long as  $\tilde{x}_t > 0$  households have incentives to set  $m_t^d = -\infty$  in order to maximize nominal wealth. However, by the cash-in-advance constraint (CIAC), the lowest possible level of money that agents can hold is given by  $P_t c_t$ , and, therefore, agents set  $m_t^d = P_t c_t$ . Thus, if  $\tilde{x}_t$ 's support is given by  $[x_0, x_1]$ , where  $x_0$  is positive, then the CIAC binds. ■



A convenient way to solve the households' maximization problem is to rewrite the HBC in terms of agents' saving and portfolio mix. Let us define agents' total (nominal) saving as  $P_t S_t = b_{t+1} + e_t b_{t+1}^*$ . On the other hand, let  $\alpha_t$  be the share of the domestic bond in agents' portfolios (or total saving),  $\alpha_t = (b_{t+1} / P_t S_t)$ . Then it follows that:

$$(1 + \tilde{x}_t) b_t + (1 + r_t) e_t b_t^* = \left[ (1 + \tilde{x}_t) \alpha_{t-1} + (1 + r_t) \varepsilon_{t-1} (1 - \alpha_{t-1}) \right] P_{t-1} S_{t-1}$$

where  $\varepsilon_{t-1}$  denotes the gross rate of change of the exchange rate, i.e.,  $\varepsilon_{t-1} = \left( \frac{e_t}{e_{t-1}} \right)$ . Let

us define  $R_t$  as the return of agents' portfolios, hence

$$R_t \equiv R(\alpha_t, \tilde{x}_{t+1}, r_{t+1}, \varepsilon_t) = \left[ (1 + \tilde{x}_{t+1}) \alpha_t + (1 + r_{t+1}) \varepsilon_t (1 - \alpha_t) \right].$$

Using the result of proposition one, we can write the households' intertemporal

budget constraint as:  $W_{t+1} = \left( \frac{P_t}{P_{t+1}} \right) \left[ R_t (W_t - c_t) \right]$  or

$$(4) \quad W_{t+1} = \left[ \hat{R}_t (W_t - c_t) \right]$$

where  $\hat{R}_t = \left( \frac{1}{\pi_t} \right) R_t$  and  $\frac{P_{t+1}}{P_t} = \pi_t$ .

Thus we can write households' maximization problem as:

$$\underset{\{c_t, \alpha_t, W_{t+1}\}}{\text{Max}} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \quad \text{subject to} \quad W_{t+1} = \left[ \hat{R}_t (W_t - c_t) \right].$$

In the following analysis we use a specific functional form for the utility function. In particular, we assume that the utility function,  $U$ , exhibits constant relative risk aversion

$$(CRRA), \text{ and therefore } U(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \ln(c) & \text{if } \gamma = 1 \end{cases}.$$

From Samuelson (1969) we know that this maximization problem can be broken into two separate maximization problems when agents' preferences are characterized by CRRA. In the first problem agents determine the optimal size of their portfolios (savings rate), whereas in the second maximization problem agents select their optimal portfolios' mix,  $\alpha$ .

#### 2.2.1.1.1. Case 1: $\gamma > 0$ and $\gamma \neq 1$

##### Optimal Savings

To determine the optimal saving rate, we solve the following dynamic programming problem:

$$(5) \quad V[W, \alpha] = \underset{0 \leq c \leq W}{\text{Max}} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta E[V(\hat{R}(W - c), \alpha)] \right\}.$$

Since  $\tilde{x}$  is iid, the value function and the optimal policy rules for consumption and

$$\text{saving are given by: } V[W, \alpha] = \left[ \frac{1}{1-\gamma} \right] \left[ 1 - \left[ \beta E[\hat{R}^{1-\gamma}] \right]^{1/\gamma} \right]^{-\gamma} W^{1-\gamma}$$

$$c_t = \left[ 1 - \left[ \beta E[\hat{R}_t^{1-\gamma}] \right]^{1/\gamma} \right] W_t \quad \text{and} \quad S_t = \left[ \beta E[\hat{R}_t^{1-\gamma}] \right]^{1/\gamma} W_t.$$

Notice that those policy functions depend on  $E[\hat{R}_t^{1-\gamma}]$ , and therefore, they depend on the optimal portfolio's mix, the foreign asset's nominal return, the domestic asset's nominal return, the rate of change of the exchange rate and the inflation rate.<sup>9</sup>

### Optimal Portfolio

To select the optimal composition of their portfolios, agents maximize (if  $\gamma < 1$ ) (minimize (if  $\gamma > 1$ ))  $E\left[\frac{\alpha_t(1 + \tilde{x}_{t+1}) + (1 - \alpha_t)\varepsilon_t(1 + r_{t+1})}{\pi_t}\right]^{1-\gamma}$  with respect to  $\alpha_t$ . This is equivalent to maximizing (minimizing)  $E[\hat{R}_t^{1-\gamma}]$  with respect to  $\alpha_t$ , taking as given both  $\varepsilon_{t+1}$  and  $\pi_{t+1}$ . Then, the optimal portfolio is given by the  $\alpha_t$  that solves:

$$E\left[\frac{\frac{((1 + \tilde{x}_{t+1}) - \varepsilon_t(1 + r_{t+1}))}{\pi_t}}{\left(\frac{\alpha_t(1 + \tilde{x}_{t+1}) + (1 - \alpha_t)\varepsilon_t(1 + r_{t+1})}{\pi_t}\right)^\gamma}\right] = 0$$

and, therefore, period  $t$  optimal portfolio mix,  $\alpha_t$ , is a function of  $\tilde{x}_{t+1}$ ,  $r_{t+1}$ ,  $\pi_t$  and  $\varepsilon_t$ .

#### 2.2.1.1.2. Case 2: $\gamma = 1$

### Optimal Savings

If we assume a Log-utility function the functional equation is given by:

$$(6) \quad V[W, \alpha] = \underset{0 \leq c \leq W}{\text{Max}} \quad \ln(c) + \beta E[V[\hat{R}(W - c)]]$$

---

<sup>9</sup> Levhari and Srinivasan (1969), Rothschild and Stiglitz (1971) and Sargent (1987, Chapter 1) obtain similar results.

Since  $\tilde{x}$  is iid we can write the value function and the policy functions as:

$$V[W, \alpha] = \left[ \frac{1}{1-\beta} \right] \ln(1-\beta) + \left[ \frac{\beta}{(1-\beta)^2} \right] \ln(\beta) + \left[ \frac{1}{1-\beta} \right] \ln(W) + \left[ \frac{\beta}{(1-\beta)^2} \right] E[\ln(\hat{R})]$$

$$c_t = (1-\beta)W_t \text{ and } S_t = \beta W_t$$

Notice that in the Log-utility case none of the policy functions depend on the portfolio's return,  $\hat{R}_t$ . In other words, if the coefficient of relative risk aversion is equal to one,  $\gamma = 1$ , consumption and saving rates do not depend on either the optimal portfolio's mix, the foreign asset's nominal return, the domestic asset's nominal return, the rate of change of the exchange rate or the inflation rate. They only depend on  $\beta$ .

### Optimal Portfolio

In this case, agents choose  $\alpha_t$  to maximize:  $E[\ln(\alpha_t(1 + \tilde{x}_{t+1}) + (1 - \alpha_t)\varepsilon_t(1 + r_{t+1}))]$ .

The optimal  $\alpha_t$  is the one that solves  $E\left[ \frac{[(1 + \tilde{x}_{t+1}) - \varepsilon_t(1 + r_{t+1})]}{[\alpha_t(1 + \tilde{x}_{t+1}) + (1 - \alpha_t)\varepsilon_t(1 + r_{t+1})]} \right] = 0$ . As

in the case when  $\gamma \neq 1$ , here the period  $t$  optimal portfolio configuration,  $\alpha_t$ , does depend on  $\tilde{x}_{t+1}$ ,  $r_{t+1}$ ,  $\pi_t$  and  $\varepsilon_t$ .

### 2.2.2. Government

The domestic economy has a public sector that lives forever. The domestic government issues bonds and adjusts the level of money supply to finance some exogenously given stream of expenditure. We assume that government's consumption,  $g_t$ , does not produce any effect on the consumers' utility, and therefore, we consider it a pure

destruction of the consumption good.<sup>10</sup> The domestic government buys its consumption goods in the domestic market, just as domestic households do.

We assume that the domestic government cannot sell bonds to foreign residents even though the domestic government always honors its debt.<sup>11</sup> We summarize the activities of the domestic government using its intertemporal budget constraint (GBC), that can be written as:  $m_t^k = P_t g_t = B_{t+1} - (1 + \tilde{x}_t) B_t + M_{t+1} - M_t$  for all  $t \geq 1$

where  $B_{t+1}$  represents the amount of domestic bonds issued by the government, and  $M_{t+1}$  represents the money supply. Here  $m_t^k$  represents the amount of nominal resources that domestic government needs to finance its expenditure.

The domestic government is in charge of the **money creation process**. However, the monetary activities of the government are restricted by its fiscal actions. In other words, the domestic government must collect enough resources to finance its given expenditure stream by selling bonds and expanding the money supply. We assume that the domestic government expands domestic currency to finance the difference between fiscal spending and government's debt.

Given the assumption of a managed exchange rate system, let us define the domestic money supply as  $M_t = e_t IR_t$ , where  $IR_t$  is the amount of international reserves held by the domestic government at period  $t$ . In this case the money supply could be affected either by movements in the amount of foreign reserves held by domestic

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<sup>10</sup> Following Lucas and Stokey (1983), we consider government's consumption as defense expenditures that have to be made to face an unlikely war.

<sup>11</sup> Notice that the kind of risk in domestic bonds that we try to model is not default risk. Here we assume that the nominal return is always positive.

government or by variations on the exchange rate, thus a change in the money supply is given by:

$$(7) \quad M_{t+1} - M_t = e_{t+1} \Delta IR_t + (\varepsilon_t - 1) M_t$$

where  $\Delta$  indicates absolute variation, and  $\varepsilon_t$  represents the gross rate of changes in the exchange rate. From (7) it follows that if  $\Delta IR_t > 0$ , i.e., the international reserves go up (holding everything else constant), the stock of domestic currency increases. On the other hand, given the stock of international reserves, money aggregates can be affected by variations in the exchange rate. We assume that at the end of period  $t$  the domestic government determines the exchange rate  $e_{t+1}$ , by intervening in the exchange rate market such that the desired level is achieved. In other words, the domestic economy has a system of *managed* exchange rates. Given the level of international reserves, a devaluation of the domestic currency,  $(\varepsilon_t - 1) > 0$ , induces an increase of the money supply whereas when the domestic currency experiences a revaluation,  $(\varepsilon_t - 1) < 0$ , the domestic money supply decreases. Using (7) we can rewrite the GBC as:

$$(8) \quad P_t g_t = B_{t+1} - (1 + \tilde{x}_t) B_t + e_{t+1} \Delta IR_t + (\varepsilon_t - 1) M_t, \quad t \geq 1$$

hence to finance period  $t$  spending and to repay its debt, the domestic government must sell domestic bonds, and alter either the domestic currency exchange rate or the level of international reserves. Here we assume a given level of international reserves, and, therefore, the policy variable in control of the domestic government is the rate of change of the nominal exchange rate.

### 2.2.3. Domestic Currency Market

We define an equilibrium of the domestic currency market as a situation in which the demand for money is equal to the money supply,  $M_t = m_t^d + m_t^g$ . Notice that we add to the money demand the nominal resources that government uses to finance its expenditure.

### 2.2.4. Domestic Bond Market

The domestic government does not have access to international financial markets, and, therefore, the bond issued by the domestic government cannot be bought by foreign agents. Domestic households choose their desired level of the domestic bond so as to maximize their expected lifetime utility. The domestic government's supply of this bond must be enough to clear the market ( $B_{t+1} = b_{t+1}$ ), given the distribution of the domestic nominal return, the domestic inflation, the devaluation (revaluation) of the domestic currency and the foreign nominal return.

### 2.2.5. Foreign Bond Market

Given the size of the foreign country, the foreign bond's nominal return is determined in the foreign country's financial market, and, therefore, domestic private agents take it as given. Thus, the domestic agents determine their desired foreign bond purchases and buy them on the international market.

### 2.2.6. Consumption Good Market

Given the size of the domestic economy, the international price of the consumption good is fully determined in the foreign market. Under the assumption of free trade the domestic price of the consumption goods,  $P_t$ , is equal to the international price of the consumption good in foreign currency,  $P_t^*$ , times the exchange rate of the two currencies,  $e_t$ . Furthermore, since we assume that  $P_t^* = 1$  for all  $t$ , the price of the consumption good in the domestic market is equal to the exchange rate,  $P_t = e_t$ . At this price, domestic households choose their optimal consumption, and the domestic government carries out its expenditure stream. We define an equilibrium in the consumption good's market as a situation in which domestic consumption (both private and public) is feasible. Then using the HBC and the GBC an equilibrium in the consumption goods market is given by:

$$P_t c_t + P_t g_t = e_t \left[ (1 + r_t) b_t^* - b_{t+1}^* \right] \text{ or}$$

$$(9) \quad e_t (r_t b_t^*) - P_t (c_t + g_t) = e_t \left[ b_{t+1}^* - b_t^* \right]$$

Expression (9) represents the domestic economy's balance of payments at period  $t$ , where the right-hand-side expression indicates the capital account balance and the left-hand-side expression indicates the current account balance.

### 2.2.7. Competitive Equilibrium

We define a competitive equilibrium as a sequence

$$\left\{ c_t, m_{t+1}^d, b_{t+1}, b_{t+1}^*, \alpha_t, S_t, g_t, m_t^e, B_{t+1}, M_{t+1}, e_{t+1}, P_{t+1} \right\}_{t=0}^{\infty}$$



such that given  $F, \{\tilde{x}_{t+1}, r_{t+1}\}_{t=0}^{\infty}$  and the initial endowment  $W_0$ , households solve their maximization problems, the government budget constraint is satisfied, the domestic and foreign bond markets clear, the domestic money market clears and the domestic consumption allocations (both private and public) are feasible.

Alternatively, we can define a competitive equilibrium as a sequence

$$\{c_t, m_{t+1}^d, b_{t+1}, b_{t+1}^*, \alpha_t, S_t, g_t, m_t^k, B_{t+1}, M_{t+1}, e_{t+1}, P_{t+1}\}_{t=0}^{\infty}$$

such that given  $F, \{\tilde{x}_{t+1}, r_{t+1}\}_{t=0}^{\infty}$  and the initial endowment  $W_0$ , the balance of payment condition of the domestic economy, expression (9), is satisfied.

Notice that in this model if both  $c_t$  and  $g_t$  are greater than zero, a competitive equilibrium requires a positive value of  $b_t^*$  for all  $t$ . This implies that the optimal portfolio mix,  $\alpha_t$ , must be greater than or equal to zero and strictly less than one,  $\alpha_t \in [0,1)$ , for  $t \geq 1$ . However, from now on we assume that the portfolio maximization problem has an interior solution, and, therefore,  $\alpha_t \in (0,1)$  for all  $t \geq 1$ .

In the following sections, and to simplify notation, we drop the time subscript in all the variables. We represent  $\hat{R}_t$  as  $\hat{R}$ ,  $\tilde{x}_{t+1}$  as  $\tilde{x}$ ,  $r_{t+1}$  as  $r$ ,  $\varepsilon_t$  as  $\varepsilon$ ,  $e_{t+1}$  as  $e$ ,  $\pi_t$  as  $\pi$ ,  $P_{t+1}$  as  $P$ ,  $s_t$  as  $s$ , and  $\alpha_t$  as  $\alpha$ . There  $s$  denotes agents' saving rate.

### 2.3. Effects of an Increase in Risk (Partial Equilibrium)

In this section we try to determine the effects on domestic agents' positions in both foreign and domestic bonds that an increase in the domestic risk may induce.

Following Rothschild and Stiglitz (1970) [RS], we model an increase in the domestic risk as a mean preserving spread in the distribution of the domestic asset's (nominal) return.

To determine the effects of a mean preserving spread (MPS) in domestic bond's nominal net return distribution on domestic agents' behavior, we compare agents' optimal choices in two economies that differ only in the cumulative distribution function of their random return. Furthermore, we ask what assumptions about agents' relative risk aversion are sufficient to determine the signs of those effects of a MPS in domestic bond's nominal net return distribution on agents choices.

### **Changes in Domestic Risk**

Let  $\lambda$  be an index of a family of the cumulative distribution functions (cdf) of the random variable  $\tilde{x}$ , such that an increase (a reduction) in  $\lambda$  indicates an increase (a reduction) in the risk of the random variable's cdf. In other words, given  $F$  and  $G$ , two different cdfs of the random variable  $\tilde{x}$ , we say that  $G$  is riskier than  $F$  in a Rothschild and Stiglitz's sense if  $\lambda_F < \lambda_G$ . Here,  $\lambda_F$  and  $\lambda_G$  represent the indexes associated to the distributions  $F$  and  $G$ , respectively.<sup>12</sup>

Let  $\{s^*, \alpha^*\}$  and  $\{\hat{s}, \hat{\alpha}\}$  denote domestic households' optimal saving rate and portfolio configuration when agents face the distributions  $F$  and  $G$ , respectively. Then by comparing  $\{s^*, \alpha^*\}$  and  $\{\hat{s}, \hat{\alpha}\}$  we can determine how domestic private agents adjust their portfolios in response to MPS in the domestic asset's return distribution.

### 2.3.1. Effects on Households' Optimal Choices

The next analysis follows Levhari and Srinivasan [1969] and Rothschild and Stiglitz (1971), closely.

#### 2.3.1.1. Case 1: $\gamma > 0$ and $\gamma \neq 1$

##### Proposition 2:

A mean preserving spread in the domestic asset's return distribution induces an increase (a reduction) in domestic agents' savings rate if the coefficient of relative risk aversion is greater (smaller) than one.

*Proof:*

From the households' maximization problem we know that the saving rate is given by:  $s = [\beta E[\hat{R}^{1-\gamma}]]^{1/\gamma}$  or  $s^\gamma = \beta E[\hat{R}^{1-\gamma}]$ . Following RS we must look at the concavity (convexity) of  $H \equiv \hat{R}^{1-\gamma}$  with respect to the random variable,  $\tilde{x}$ , to determine the effect of a mean preserving spread in  $\tilde{x}$ 's distribution on the saving rate.

By definition  $\hat{R}^{1-\gamma} \equiv \left[ \left[ \frac{1}{\pi} \right] [\alpha (1 + \tilde{x}) + (1 - \alpha)\varepsilon (1 + r)] \right]^{1-\gamma}$ . Hence the concavity of

$H$  with respect to  $\tilde{x}$  is given by:

$$H_{xx} = -[\gamma(1-\gamma) \alpha^2] \left[ \frac{1}{\pi} \right]^{1-\gamma} [\alpha (1 + \tilde{x}) + (1 - \alpha)\varepsilon (1 + r)]^{-(1+\gamma)}$$

It follows that if  $\gamma$  is smaller (greater) than 1, then a MPS in the domestic bond's return distribution induces a reduction (an increase) in the savings rate.<sup>13</sup> ■

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<sup>12</sup> We follow Diamond and Stiglitz (1974) in the use of these indexes for the random variable's cdf to indicate changes in risk.

<sup>13</sup> Here  $O_{jj}$  denotes the second derivative of variable  $O$  with respect variable  $j$ .

**Proposition 3:**

A mean preserving spread in the domestic asset's return distribution induces a reduction in the proportion that the domestic bond represents in households' portfolios (a reduction in  $\alpha$ ), if the coefficient of relative risk aversion is less than one.

*Proof:*

From households' maximization problem we know that the optimal portfolio mix,

$$\alpha, \text{ is given implicitly by: } E \left[ \frac{\left( (1 + \tilde{x}) - \varepsilon (1 + r) \right)}{\left( \frac{\alpha (1 + \tilde{x}) + (1 - \alpha) \varepsilon (1 + r)}{\pi} \right)^\gamma} \right] = 0.$$

$$\text{Let } Z \equiv \left[ \frac{\left( (1 + \tilde{x}) - \varepsilon (1 + r) \right)}{\left( \frac{\alpha (1 + \tilde{x}) + (1 - \alpha) \varepsilon (1 + r)}{\pi} \right)^\gamma} \right].$$

Then we can determine the effect of a MPS in domestic bond's nominal return distribution on  $\alpha$  if we can establish the concavity (convexity) of  $Z$  with respect to  $\tilde{x}$ . It follows that

$$Z_{xx} = -\frac{\alpha\gamma}{\pi^2} \left[ \frac{\left[ \alpha(1 - \gamma)(1 + \tilde{x}) \right] + \left[ 2(1 - \alpha) + \alpha(1 + \gamma) \right] \varepsilon(1 + r)}{\left( \frac{\alpha (1 + \tilde{x}) + (1 - \alpha) \varepsilon (1 + r)}{\pi} \right)^{\gamma+2}} \right]$$

is negative if  $\gamma < 1$ , and therefore, a MPS in  $\tilde{x}$ 's distribution induces a reduction in  $\alpha$ . Since the sign of  $Z_{xx}$  is ambiguous when  $\gamma > 1$ , the effects of a MPS in  $\tilde{x}$ 's distribution on the optimal portfolio configuration are also ambiguous. ■

### 2.3.1.2. Case 2: $\gamma = 1$

#### **Proposition 4:**

A mean preserving spread in the domestic asset's return distribution induces a reduction in the proportion that the domestic bond represents in households' portfolios (a reduction in  $\alpha$ ), if the coefficient of relative risk aversion is equal to one.

*Proof:*

From households' maximization problem with a Log-utility function we know that the optimal portfolio mix is given by the  $\alpha$  that solves:

$$E \left[ \frac{(1 + \tilde{x}) - \varepsilon (1 + r)}{(\alpha (1 + \tilde{x}) + (1 - \alpha) \varepsilon (1 + r))} \right] = 0.$$

Then to determine the effect of a MPS in  $\tilde{x}$ 's distribution we must establish the

concavity of:  $Z \equiv \left[ \frac{(1 + \tilde{x}) - \varepsilon (1 + r)}{(\alpha (1 + \tilde{x}) + (1 - \alpha) \varepsilon (1 + r))} \right]$  with respect to  $\tilde{x}$ .

Since  $Z_{xx} = \left[ -\frac{2\alpha (1 + r)\varepsilon}{(\alpha (1 + \tilde{x}) + (1 - \alpha) \varepsilon (1 + r))^3} \right]$  is negative, we have that a MPS in

$\tilde{x}$ 's distribution induces a reduction in  $\alpha$ . ■

Notice that in the Log-utility case the domestic agents' saving rate does not depend on the random variable,  $\tilde{x}$ , and therefore, it cannot be affected by MPS changes in domestic bond's nominal net return distribution. Table 2.1 summarizes the results of propositions 2-4.

<b>Table 2.1</b> <b><i>Partial Equilibrium Effects of a MPS in Domestic Bond's Nominal Return Distribution</i></b>		
$\gamma$	Effects on Saving Rate	Effects on $\alpha$
(0,1)	(-)	(-)
1	0	(-)
(1, $\infty$ )	(+)	(?)

### **2.3.1.3. Demand for Domestic Bond**

In this section we use the results of propositions 2-4 to establish the sign of the partial equilibrium (PE) effects of a mean preserving spread in the domestic bond's return distribution on the demand for domestic bonds. To conduct the comparative static analysis we compare the optimal holdings of the domestic bond of agents that inhabited two economies that differ only in the risky (domestic) bond's return distribution.

### Corollary 1: (PE Effects on Agents' Demand for Domestic Bonds)

A mean preserving spread in the domestic bond's nominal return distribution induces a reduction in domestic agents' demand for domestic bonds if the coefficient of relative risk aversion is less than or equal to one.

*Proof:*

By definition households' demand for the domestic bond can be written in terms of the optimal portfolio mix and the level of savings as:  $b_{t+1} = \alpha_t P_t S_t$ , or equivalently,  $b_{t+1} = \alpha_t s_t P_t W_t$ , where  $s_t$  represents period  $t$  agents' saving rate. Then, for a given level of period  $t$  wealth, any effect of a MPS in  $\tilde{x}$ 's distribution on the demand for domestic bond must be induced by changes in agents' portfolio mix and savings rate, and therefore:

$$(10) \quad \frac{db_{t+1}}{d\lambda} = [s_t P_t W_t] \frac{d\alpha_t}{d\lambda} + [\alpha_t P_t W_t] \frac{ds_t}{d\lambda}.^{14}$$

From the results of propositions 2-4 and expression (10), we know that we can sign the effect of a mean preserving spread (MPS) on the demand for domestic bond when the coefficient of relative risk aversion is less than or equal to one.

If the coefficient of relative risk aversion is equal to one, agents' saving rate does not depend on  $\tilde{x}$  (therefore, it does not depend on  $\lambda$ ). In this case, the effects of a MPS in  $\tilde{x}$ 's distribution on domestic agents' demand for domestic bonds are driven only by changes in agents' portfolio configuration.

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<sup>14</sup> Here  $\lambda$  is the Rothschild and Stiglitz's index of change in risk, and  $\frac{dz}{d\lambda}$  represents the changes in variable  $z$  in response to changes in the risk of the risky asset's nominal return distribution.

Thus, a MPS in  $\tilde{x}$ 's distribution induces a reduction in agents' demand for domestic bonds if the coefficient of relative risk aversion is equal to one.

In the most general case of the constant relative risk aversion utility function ( $\gamma \neq 1$ ), the effects of a MPS in the domestic asset's nominal return distribution on agents' demand for domestic bonds are given by the combined effects on agents' portfolio configuration and savings rate. We find that when the coefficient of relative risk aversion is less than one, the effects of a MPS in  $\tilde{x}$ 's distribution in agents' savings rate and portfolio configuration go in the same direction, the two effects are negative, and therefore, the effect of a MPS in  $\tilde{x}$ 's distribution in domestic agents' demand for domestic bond is negative.

Notice that if the coefficient of relative risk aversion is greater than one, we cannot determine the effect of a MPS in  $\tilde{x}$ 's distribution on domestic agents' demand for domestic bonds because we cannot sign the effect on agents' portfolio configuration. ■

## **2.4. Changes in Domestic Risk and Inflation**

### **(General Equilibrium Effects)**

In this section, we try to determine how an increase in domestic risk may affect the domestic economy as a whole. In order to do this evaluation we try to connect domestic agents' reactions to this change in risk with the behavior of the domestic government. In particular, we analyze how changes in agents' demand for the domestic bond may induce revisions in the form by which the domestic government finances its spending.



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### **2.4.1. Government Financing Process**

In general, a reduction (an increase) in agents' demand for domestic bonds induces the domestic government to increase (reduce) the use of changes in monetary aggregates as a source of revenue. From expression (7), the domestic government can affect the money supply either by changing the devaluation rate of the domestic currency or by changing the level of international reserves. However, if we assume that the stock of international reserves held by the domestic government is constant, changes in the money supply take place only because of changes in the devaluation rate.

### **2.4.2. Effects of an Increase in Risk**

#### **2.4.2.1. Demand for Domestic Bonds**

If we assume that the coefficient of relative risk aversion is no larger than one, an increase in domestic risk (represented as a mean preserving spread in the domestic bond's nominal return distribution) induces a reduction in agents' demand for domestic bonds.<sup>15</sup> This reduction in the demand for domestic bonds limits the ability of the domestic government to issue new bonds, and drives the domestic government to increase the money supply by increasing the domestic currency's devaluation rate to finance its consumption. This reaction of the domestic government may induce additional responses in domestic agents because an increase in the devaluation rate may imply changes in the real returns of the financial assets.

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<sup>15</sup> We focus in cases where the coefficient of relative risk aversion is no greater than one because we know that the effect of a mean preserving spread in domestic bond's nominal return distribution on households' demand for domestic bonds can be unambiguously determined, and it is negative (Corollary 1).

In section III, we assumed that agents adjust their optimal choices because an increase in domestic risk, taking as given the devaluation rate (and for that matter all government activities). Now we evaluate both agents' and government's behaviors once we take into account their interactions, and the fact that  $\varepsilon_t = \pi_t$ .

To determine the general equilibrium effects of changes in the domestic risk we must determine how agents' saving and portfolio mix depend on the domestic currency devaluation rate. The next two propositions help us to establish the signs of those effects.<sup>16</sup>

**Proposition 5:**

An increase in the devaluation rate of the domestic currency induces a reduction (an increase) in the saving rate of the domestic agents if the coefficient of relative risk aversion is less (larger) than one. Furthermore, if the coefficient of relative risk aversion is equal to one an increase in the devaluation rate does not affect agents' saving rate.

*Proof:*

If the coefficient of relative risk aversion ( $\gamma$ ) is different from one, by the FOC and because the devaluation and the inflation rate are identical we know that

$$s^y = \beta E \left[ \frac{(1 + \tilde{x})\alpha}{\varepsilon} + (1 + r)(1 - \alpha) \right]^{(1-\gamma)} \text{ and therefore,}$$

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<sup>16</sup> Once again, we drop the time subscripts.



$$\frac{\partial s^*}{\partial \varepsilon} = -\beta(1-\gamma)E \left[ \frac{\frac{(1+\tilde{x})\alpha}{\varepsilon^2}}{\left[ \frac{(1+\tilde{x})\alpha}{\varepsilon} + (1+r)(1-\alpha) \right]^\gamma} \right].$$

Thus an increase in the devaluation rate induces an increase (a reduction) in the saving rate if the coefficient of relative risk aversion,  $\gamma$ , is less (larger) than one. Notice that when the coefficient of relative risk aversion is equal to one the saving rate does not depend on the devaluation rate (it just depends on  $\beta$ ) and therefore,  $\frac{\partial s^*}{\partial \varepsilon}$  is null. ■

**Proposition 6:**

An increase in the devaluation rate of the domestic currency induces a reduction in the proportion that the domestic bond represents in households' portfolios (a reduction in  $\alpha$ ), if the coefficient of relative risk aversion is no larger than one.

*Proof:*

If  $\gamma \neq 1$  we have that the optimal portfolio mix is given by the  $\alpha$  that solves:

$$E \left[ \frac{\frac{(1+\tilde{x}) - \varepsilon(1+r)}{\varepsilon}}{\left[ \frac{\alpha(1+\tilde{x}) + (1-\alpha)\varepsilon(1+r)}{\varepsilon} \right]^\gamma} \right] = 0. \text{ By the implicit function theorem, the sign of } \frac{\partial \alpha}{\partial \varepsilon}$$

is given by the derivative of the first order condition with respect to  $\varepsilon$ . Thus the sign  $\frac{\partial \alpha}{\partial \varepsilon}$

is given by the sign of  $E \left[ - \frac{\left[ \frac{\alpha(1-\gamma)(1+\tilde{x})^2 + \varepsilon(1+r)(1+\tilde{x})(1-\alpha(1-\gamma))}{\varepsilon^3} \right]}{\left[ \frac{\alpha(1+\tilde{x}) + (1-\alpha)\varepsilon(1+r)}{\varepsilon} \right]^{1+\gamma}} \right]$ . Then,

if  $\gamma < 1$  an increase in the devaluation rate reduces the proportion that the domestic bond represents in agents' portfolio.

On the other hand, if  $\gamma = 1$ , we have that the optimal portfolio mix is given by the

$\alpha$  that solves  $E \left[ \frac{(1+\tilde{x}) - \varepsilon(1+r)}{[\alpha(1+\tilde{x}) + (1-\alpha)\varepsilon(1+r)]} \right] = 0$ . Therefore, the sign of  $\frac{\partial \alpha}{\partial \varepsilon}$  is given by

the sign of  $E \left[ - \frac{(1+\tilde{x})(1+r)}{[\alpha(1+\tilde{x}) + (1-\alpha)\varepsilon(1+r)]^2} \right]$ . Thus, if the coefficient of relative risk

aversion equals one, an increase in the devaluation rate induces a reduction in the

proportion that domestic bonds represent in agents' portfolios and therefore,  $\frac{\partial \alpha}{\partial \varepsilon} < 0$ . ■

Next, using the results of propositions 2-6 we establish the general equilibrium (GE) effects of changes in the domestic risk.

### **Corollary 2: (GE Effects on the Demand for Domestic Bonds)**

A mean preserving spread in the domestic bond's nominal return distribution induces a reduction in the (general equilibrium) demand for domestic bonds if the coefficient of relative risk aversion is no larger than one.

*Proof:*

Notice that if the coefficient of relative risk aversion is equal to one, from expression (10) and because in this case the PE effect on the savings rate is null, the general equilibrium effect on the demand for domestic bonds is given by:  $\frac{db_{t+1}}{d\lambda} = [\beta P_t W_t] \frac{d\alpha_t}{d\lambda}$ .

On the other hand, the general equilibrium effect of a MPS in  $\tilde{x}$ 's distribution on the optimal portfolio configuration,  $\frac{d\alpha_t}{d\lambda}$ , is given by:  $\frac{d\alpha_t}{d\lambda} = \frac{\partial \alpha_t}{\partial \lambda} + \frac{\partial \alpha_t}{\partial \varepsilon_t} \frac{\partial \varepsilon_t}{\partial \lambda}$ .

From proposition (4) we know that  $\frac{\partial \alpha_t}{\partial \lambda} < 0$ . To compensate for the reduction in the demand for domestic bonds, the domestic government must increase the devaluation rate and therefore,  $\frac{\partial \varepsilon_t}{\partial \lambda} > 0$ . By proposition (6) an increase in the devaluation rate induces a reduction in  $\alpha$ , i.e.  $\frac{\partial \alpha_t}{\partial \varepsilon_t}$  is negative. Then, we have that the sign of the GE equilibrium effect of an increase in the domestic risk on the proportion that the domestic bond represents on agents' portfolios ( $\alpha_t$ ), is negative which indeed implies that the sign of the GE on the demand for the domestic bond,  $\frac{db_{t+1}}{d\lambda}$ , is also negative. Therefore, a mean preserving spread in the domestic bond's nominal return distribution (an increase in domestic risk) induces a reduction in the general equilibrium demand for domestic bonds.

For coefficients of relative risk aversion less than one the general equilibrium effects on the demand for domestic bonds of a mean preserving spread in the domestic bond's nominal return distribution is also given by expression (10). In this case, the sign

of  $\frac{da_t}{d\lambda}$  is still negative (propositions (4) and (6)). On the other hand, from the results of propositions (3) and (5) we know that the sign of the GE effect of an increase in the domestic risk on agents' saving rate,  $\frac{ds_t}{d\lambda}$ , is negative (both  $\frac{\partial s_t}{\partial \lambda}$  and  $\frac{\partial s_t}{\partial \varepsilon}$  are negative) if the coefficient of relative risk aversion is less than one. Therefore, if the coefficient of relative risk aversion is less than one, the sign of the GE effect of an increase in the domestic risk on the demand for the domestic bonds,  $\frac{db_{t+1}}{d\lambda}$ , is also negative. ■

Then under the conditions of corollary (2) --relative risk aversion no larger than one-- an increase in the domestic risk induces a reduction in domestic agents' demand for the domestic bonds. Now we study the inflationary implications of changes in demand for the domestic bond as result of *changes in domestic risk*.

#### 2.4.2.2. Effects on the Inflation Rate

Let us assume that the coefficient of relative risk aversion is no larger than one. Then, the reduction experienced in agents' demand for domestic bonds (because of the MPS in  $\tilde{x}$ 's distribution) forces the domestic government to increase the devaluation rate of the domestic currency, and consequently, the money supply. Furthermore, given our assumptions about the consumption good market (free trade and a constant international price of the consumption good,  $P_t^* = 1$ ), an increase in the devaluation rate implies a one-to-one increase in the inflation rate, since  $P_t = e_t$ . Thus, as a consequence of a mean



preserving spread in the domestic bond's nominal return distribution (*an increase in domestic risk*) the domestic inflation increases, since the devaluation rate increases.

## 2.5. Final Remarks

In this chapter we study how an increase in the domestic risk influences the equilibrium inflation rate of a small open economy. We modeled an increase in domestic risk as a mean preserving spread in the domestic bond's nominal return distribution. We find that if agents' preferences exhibit constant relative risk aversion and the coefficient of relative risk aversion is no larger than one a mean preserving spread in the domestic asset's return distribution induces a reallocation of domestic agents' portfolios away from domestic assets and into foreign assets.

The adjustments in agents' portfolios affect the domestic government's ability to finance its expenditure stream by issuing new units of domestic bonds. To compensate for the fall in public revenue the domestic government is forced to expand the money supply. Then if we assume a system of managed exchange rates and that the domestic government has limited access to international financial markets, the expansion of the money supply requires an increase in the devaluation rate of the domestic currency, and a higher devaluation rate implies an increase in the inflation rate. Thus, we have that with a constant relative risk aversion utility function, if the coefficient of relative risk aversion is less than or equal to one, an increase in domestic risk produces a higher equilibrium inflation rate.

In this very simple model we restrict the sources of revenue available to the domestic government. We assume no taxes and limit the domestic government's access to the international financial markets. These strong constraints in government financing alternatives fit, in some sense, the conditions in which the government acts in several developing economies. In some of these economies, fiscal revenue relies in an important way on domestic borrowing and money creation because of inefficient tax systems and limited access to international financial markets.

Recently, some empirical studies present evidence in favor of a negative relationship between political stability and inflation rate. Studies as Cukierman, Edwards and Tabellini (1992) and Romer (1993) find that countries with the lower political stability tend also to have higher inflation rates. These studies argue that a possible justification for this negative correlation is that politically unstable countries are less likely to have institutions that help to overcome problems such as the dynamic-inconsistency of the economic policy.

Even though changes in domestic risk can be generated by factors others than changes in an economy's political stability, to the extent that changes in domestic risk are caused by political instability, we can interpret the main result of this paper as an additional mechanism through which political stability affects inflation. Here higher political instability implies higher inflation, but not because of the differences that the degree of political instability induces on an economy's institutions but rather because of the effects that changes in the level of political instability by itself induces on domestic investors' decisions and therefore on the behavior of the domestic government. Here we

show that given the institutions, higher political stability implies a lower need for the domestic government to expand money aggregates, and, therefore, a higher degree of political stability induces a lower inflation rate.

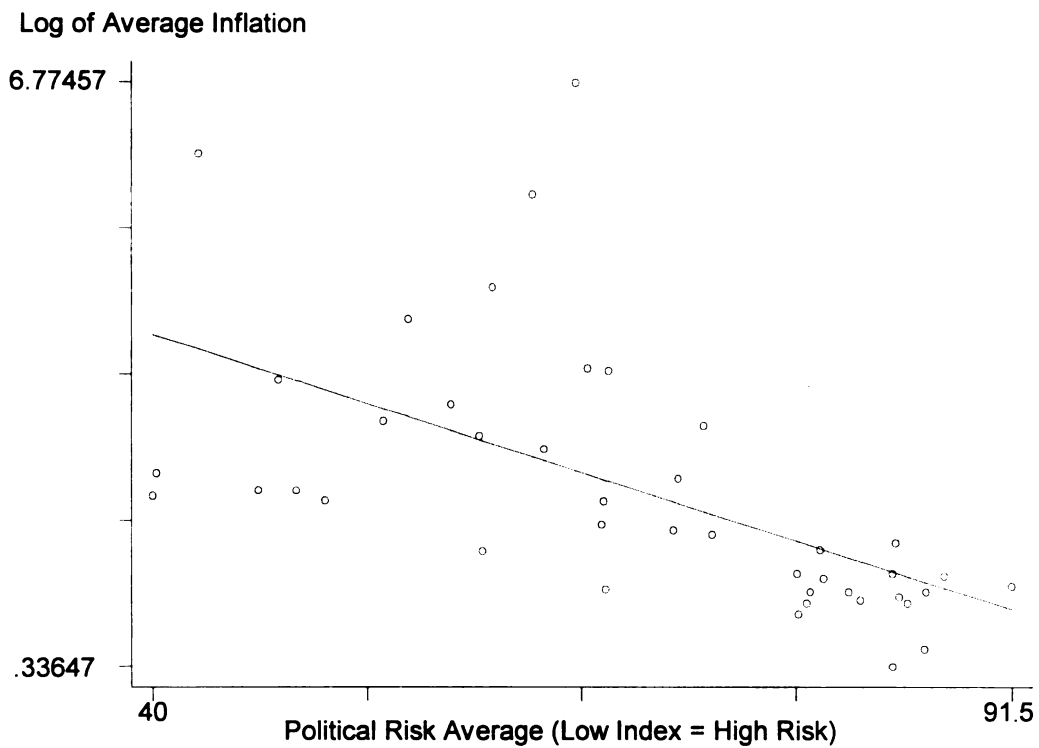
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Figure 2.1

***Average Inflation and Political Risk\****  
***42 Countries (1985 - 1995)***

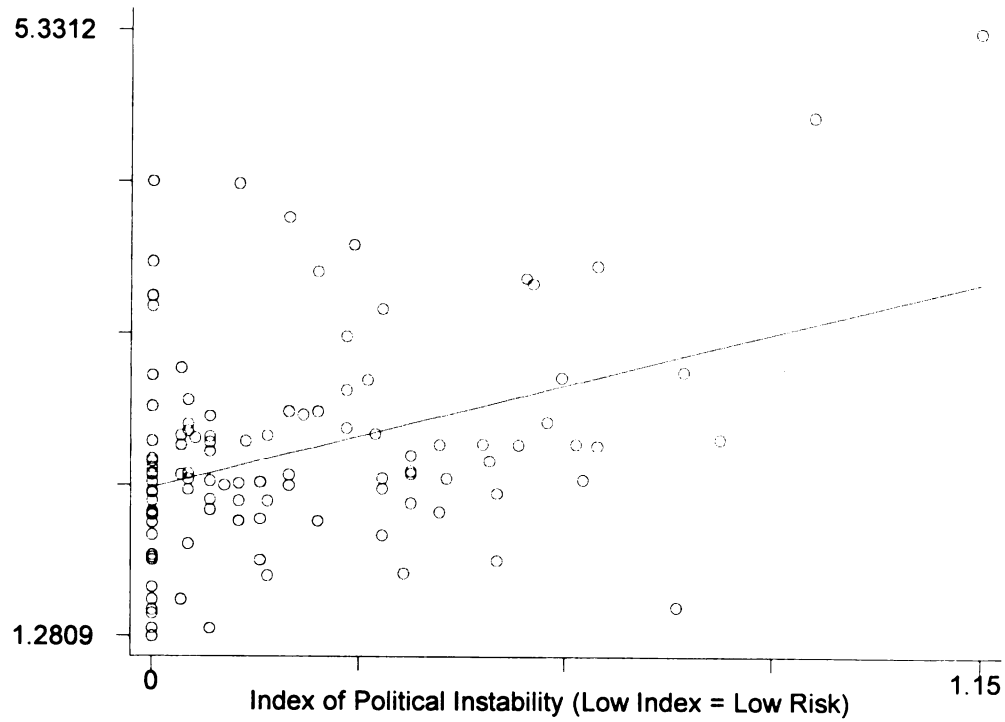


Source: World Bank (1997) and Diamonte, Liew and Stevens (1996)

\* As measured by the International Country Risk Guide's Index

Figure 2.2

***Average Inflation and Political Risk\*\****  
***112 Countries (1973 - 1981)***



Source: Romer (1993)

\*\* As measured by Barro (1991)'s Index

## APPENDIX 2

(Based on Diamonte, Liew and Stevens (1996)'s appendix.)

The Political Risk Services, a country-risk consultant firm, publishes the International Country Risk Guide (ICRG) since 1980. The ICRG provides survey measures of political risk for more than 130 countries (nevertheless, only 45 countries are reported by Diamonte, Liew and Stevens in their study). Analysts rating of 13 political risk attributes combine to form one overall political risk index for each country. The maximum score assigned to each attribute is set so that each country's overall score falls between 0 (highest risk) to 100 (lowest risk). The political attributes taken into account to form the ICRG political index are:

- 1.- Economic Expectations Vs Reality.
- 2.- Economic Planning Failures.
- 3.- Political Leadership.
- 4.- External Conflicts.
- 5.- Corruption in the Government.
- 6.- Military in Politics.
- 7.- Law and Order Tradition.
- 8.- Racial and Nationality Tensions.
- 9.- Organized Religion in Politics.
- 10.- Political Terrorism.
- 11.- Civil War Risk.
- 12.- Political Party Development.
- 13.- Quality of Bureaucracy.

The countries reported by Diamonte, Liew and Stevens (1996) are: Hong Kong, Spain, Italy, Singapore, Ireland, France, Belgium, United Kingdom, Australia, United States, Canada, Germany, Japan, New Zealand, Sweden, Norway, Denmark, Netherlands, Austria, Finland, Switzerland, Peru, Colombia, Chile, Argentina, Brazil, Venezuela, Mexico, Pakistan, Sri Lanka, Philippines, India, Indonesia, Thailand, Korea, Malaysia, Taiwan, Nigeria, Jordan, Zimbabwe, Turkey, Poland, Greece, Portugal and Hungary.





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