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ROBUST ADAPTIVE OUTPUT FEEDBACK CONTROL
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Bader Nm Aloliwi

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Hassan Khalil
Major professor

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Bader Nm Aloliwi

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ABSTRACT

Robust Adaptive Output Feedback Control of Nonlinear Systems

By

Bader Nm Aloliwi

In this thesis we design a robust adaptive output feedback control to solve the tracking problem for a class on nonlinear systems. We consider a single-input-single-output minimum phase system represented globally by an nth order differential equation. We start by designing a state feedback control to achieve tracking error convergence. The control uses a Lyapunov based adaptation to estimate uncertain parameters. Then, we saturate the control over a compact set of interest to prevent peaking and design a high-gain observer to estimate the unmeasured states. We show that this control guarantees the boundedness of all the state variables of the closed-loop system and achieves tracking of a given smooth reference signal without requiring persistence of excitation. We show robustness to small bounded disturbances. If the bound on the disturbances is not small but known, we go one step further by designing a robust control component that ensures the boundedness of all signals and makes the mean-square tracking error of the order $O(\epsilon + \mu)$ where ϵ and μ are design parameters. We pick the induction motor as an application candidate to demonstrate the applicability of our technique. We design a robust control that uses an adaptive observer to estimate the rotor resistance. The design guarantees the boundedness of

all closed-loop signals and makes the mean-square speed tracking error of the order $O(\mu)$ where μ is design parameter. We show some experimental results. The design is tested experimentally and the experimental results are in good agreement with the theory.

To my parents

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CHAPTER 1

Introduction

Nonlinear adaptive control has been a subject of interest for many researchers over the past decade. The motivation behind this was feedback linearization techniques where one first cancels nonlinearities then designs the control to meet the design specifications. By canceling the nonlinearities, one can use well developed linear techniques to satisfy the design specification. For example, consider the nonlinear system

$$\dot{x} = ax^2 + u, \qquad y = x \tag{1.1}$$

Using the control $u = -ax^2 + u_1$, the closed-loop system becomes

$$\dot{x} = u_1 \tag{1.2}$$

which can be seen as a linear system. One can use a fully developed linear control design to make x behave in a desired manner. This is true under the assumption that the parameter a is known. However, claiming perfect knowledge of the nonlinearities is not always possible. In some cases, only a nominal value a_0 of a is known. In this case the term ax^2 will not be completely canceled. There have been results in the literature to solve the problem in this case. It is solved mainly by two directions. Adaptive nonlinear control and robust nonlinear control. A combination of the two has been lately addressed; that is, the robust adaptive control of nonlinear systems. The first approach, can be seen by going back to (1.1). If $a = a_0 + \Delta a$ where Δa is a variation of the parameter a from its nominal value, then an identifier is designed to estimate the parameter a and the estimate $\hat{a}$ is used in feedback. The estimate is obtained using a Lyapunov based design in most cases. More details of this method can be found in Chapter 2. The robust control on the other hand uses a_0 in the feedback and uses another control component to overcome the effect of the error Δa. An example combining the two approaches is given in Chapter 3.

Sastry and Isidori [43] were the first to address the adaptive control of nonlinear systems. They achieved global adaptive control of a class of feedback linearizable systems. However, global Lipschitz assumptions were imposed and overparameterization was needed. Kanellakopoulos, Kokotovic, and Morse [20, 21] solved the tracking problem for nonlinear systems that are of the *parametric – pure – feedback* form using the backstepping procedure without global growth restrictions on nonlinearities. However, overparameterization was also needed. The work of Jiang and Praly [17] was able to achieve the above results with half the number of adaptation laws. Finally, Krstic, Kokotovic and Kanellakopoulos [29] solved the adaptive nonlinear control without overparameterization but required the system to be transferable globally to the *strict feedback form*. It is worth mentioning that all the above work requires the use of full state measurement.

Marino and Tomei's result [38] was the first on output feedback adaptive control without requiring output matching conditions and sector-type nonlinearities, which were required in [18] and [19]. [38] detailed out the necessary geometric conditions which characterize the class of nonlinear systems for which they were able to design output feedback tracking control. They used augmented filters. Krstic, Kokotovic and Kanellakopoulos [30] solved the output feedback tracking problem using backstepping with observer based identifier. The class which was treated is called *Output − feedback − canonical* form. Khalil [25] dealt with a single-input-single-output (SISO), minimum-phase nonlinear systems which can be represented by an n-th order differential equation. The class of systems includes those treated in [38] and [30] as special cases. He extended the dynamics of the system by adding integrators at the input side then transformed it into the normal form. The uncertain nonlinear functions of the model depend linearly on constant unknown parameters. By combining results from [11, 48, 47] with Lyapunov-based adaptive design [39, 14], he designed a controller that achieves semiglobal asymptotic output tracking for reference signals which are bounded and have bounded derivatives up to the nth order. The new adaptive controller is simpler than traditional ones since it does not use filtering or error augmentation ideas. It is simply a state feedback controller with a linear observer. An important drawback of the result of [25] is the requirement of persistence of excitation not only for parameter convergence but even for tracking error convergence. This is unusual in adaptive control results where tracking error convergence is shown without persistence of excitation. Jankovic [16] achieved similar results starting from the normal form with no zero dynamics, using state feedback and a high-gain observer. The objectives of this thesis are to overcome the drawback of [25], that is, to show asymptotic tracking without requiring persistence of excitation, to show robustness of the design to small bounded disturbances, and to design a robust part of the control to overcome large bounded disturbances. Finally, we apply nonlinear control with adaptive observer to induction motors.

In Chapter 2 we solve the tracking problem without persistence of excitation. This is made possible by analyzing the closed-loop system under output feedback. Unlike [25], we do not rely on singular perturbations for recovering what was achieved under state feedback. We combine various Lyapunov functions to form a composite Lyapunov function that shows convergence of the tracking error and partial parameter error. By partial parameter convergence we mean convergence of a projection of the parameter vector on a lower-dimensional subspace. The control procedure takes two steps. First, we design a state feedback control that uses the estimate of the parameters to cancel the nonlinearities and stabilize the resulting system. Second, we design a high-gain observer, to estimate the state of the error system, together with saturation of the control to prevent peaking [11]. A number of conclusions are drawn at the end of the chapter. The results of [25] and [1] are shown to be two special cases of Chapter 2. Chapter 3 deals with two issues: robustness in the usual form of robust adaptive control results [14], and robust control. We show that, for sufficiently small bounded disturbance, all signals in the closed-loop system will be bounded and the mean square tracking error will be of order $O(\epsilon + d_1)$, where d_1 is an upper bound on the disturbance. Since ϵ is a design parameter, we can choose it small enough to make the mean square tracking error of order $O(d_1)$. Second, we present a robustness result that goes beyond the traditional robust adaptive control results. We exploit the fact that our design is developed for a system represented in the normal form, where the disturbance satisfies the matching condition, to design an additional robust control component that ensures that for any bounded disturbance, with a known upper bound, all signals in the closed-loop system will be bounded and the mean square tracking error will be of order $O(\epsilon + \mu)$, where both ϵ and μ are design parameters. Our design uses the Lyapunov redesign technique, e.g., [7] and [26, Section 13.1], and does not require the disturbance to be small.The idea of combining adaptive control tools with robust control tools used in Section 5 has also appeared in the state feedback designs of [31], [41], and [51].

In Chapter 4, we consider the induction motor as an application of our work on nonlinear robust adaptive control. Many researchers became interested in induction motors after the introduction of field orientation by Blaschke [5] in 1972. The idea of field orientation is to transform the motor equations into coordinates that are rotating with the rotor flux, which is used to compute the transformation. During the late 80's and early 90's, a number of results that assume measurement of the rotor flux were derived; see for example [10, 32, 34, 12]. However, rotor flux measurement is not practical. Flux estimators, e.g. [49], were used for field orientation. We quote a number of references that use flux observer for field orientation [23, 24, 40, 27]. The flux estimate is sensitive to the rotor resistance. To overcome the changes of the value of the rotor resistance, three direction have emerged: robust control; see for example [27, 44], adaptive observer with regular control; see for example [9, 46, 36], and field orientation using the stator flux [13, 50].

In [27] the field-orientation transformation is done using the estimate of the rotor flux rather than the flux itself, which in turn results in transformed variables which are available for feedback. In [35], an adaptive observer for induction motors with unknown rotor resistance is introduced. It is based on rotor speed and stator current measurements. The adaptation is with respect to the rotor resistance. The design is a Lyapunov based design. In Chapter 4 we carry the controller of [27] one step further by adapting the rotor resistance on line using the adaptive observer of [35]. We analyze the closed-loop system under output feedback (we do not assume speed measurement) and show experimental results. It should be noticed that the application to induction motor in Chapter 4 is not a straightforward application of the results in Chapters 2 and 3 because the motor equations do not fit the mathematical model used in those chapters. Instead, the techniques of Chapter 2 and 3 are adapted to fit the induction motor case.

Finally, in Chapter 5 we give our conclusions and possible future research directions.

CHAPTER 2

Tracking

2.1 Introduction

The word tracking in the control literature means $(y - y_r = 0)$ where y is the output of a given system (plant) and y_r is a desired reference signal to be followed. Tracking of uncertain nonlinear systems takes, in most cases, one of two methods: adaptive nonlinear control or robust nonlinear control. By uncertain, it is meant that the plant has some unknown parameters. In the adaptive case, an on-line identifier is used to estimate the parameters. In the robust case, a robust control is designed to make sure the system will maintain stability in case of the mismatch. In [25], Khalil studied adaptive output feedback control for a class of nonlinear systems. The system under consideration is single-input–single-output, input-output linearizable, minimum phase, and modeled by an input-output model of the form of an nth-order differential equation. The uncertain nonlinear functions of the model depend linearly on constant unknown parameters. As mentioned in Chapter 1, [25] showed tracking error convergence under output feedback only if a persistence of excitation condition is satisfied.

In this chapter, we prove tracking error convergence without persistence of excitation. This major improvement over [25] has been made possible by changing the analysis approach. In [25] convergence is proved by showing that, under state feedback and the persistence of excitation condition, the set of zero tracking error and zero parameter error is an exponentially stable invariant set. Then, singular perturbation analysis is used to show that this same property is recovered under output feedback for sufficiently small ϵ. This idea does not work in the lack of persistence of excitation because the set of zero tracking error and zero parameter (or partial parameter) error is not exponentially stable. Here, we analyze the closed-loop system under output feedback directly and combine various Lyapunov functions to form a composite Lyapunov function that shows tracking error and partial parameter convergence. By partial parameter convergence we mean convergence of a projection of the parameter vector on a lower-dimensional subspace.

Section 2.2 defines the class of nonlinear systems considered. It is followed by the design of the control. We first, design a state feedback control which uses $\hat{\theta}$ an estimate of the uncertain parameter θ. The control cancels all nonlinearities and stabilizes the overall system. Second, we design a high-gain observer to estimate the error state e and use the estimate $\hat{e}$ in the feedback control. As in [11], we saturate the control to avoid peaking. We show the tracking error convergence in Section 2.4.

2.2 Problem Statement

We consider a single-input–single-output nonlinear system represented globally by the nth-order differential equation

$$y^{(n)} = f_0(\cdot) + \textstyle\sum_{i=1}^{p} f_i(\cdot)\theta_i + [g_0(\cdot) + \textstyle\sum_{i=1}^{p} g_i(\cdot)\theta_i]u^{(m)} \tag{2.1}$$

where u is the control input, y is the measured output, $y^{(i)}$ denotes the ith derivative of y, and $m < n$. The functions f_i and g_i are known smooth nonlinearities which may depend on y, $y^{(1)}$, ..., $y^{(n-1)}$, u, $u^{(1)}$, ..., $u^{(m-1)}$; e.g.,

$$f_0(\cdot) = f_0(y, y^{(1)}, \ldots, y^{(n-1)}, u, u^{(1)}, \ldots, u^{(m-1)})$$

The constant parameters θ_1 to θ_p are unknown, but the vector $\theta = [\theta_1, \ldots, \theta_p]^T$ belongs to Ω, a known compact convex subset of R^p. We augment a series of m integrators at the input side of the system and represent the extended system by a state space model. The states of these integrators are $z_1 = u$, $z_2 = u^{(1)}$, up to $z_m = u^{(m-1)}$ and we set $v = u^{(m)}$ as the control input of the extended system. Taking $x_1 = y$, $x_2 = y^{(1)}$, up to $x_n = y^{(n-1)}$ yields the extended system model

$$\left.\begin{array}{rcl} \dot{x}_i & = & x_{i+1}, \quad 1 \leq i \leq n-1 \\ \dot{x}_n & = & f_0(x,z) + \theta^T f(x,z) + [g_0(x,z) + \theta^T g(x,z)]v \\ \dot{z}_i & = & z_{i+1}, \quad 1 \leq i \leq m-1 \\ \dot{z}_m & = & v \\ y & = & x_1 \end{array}\right\} \tag{2.2}$$

where

$$\begin{array}{rcl} x & = & [x_1, \ldots, x_n]^T, \quad z = [z_1, \ldots, z_m]^T \\ f & = & [f_1, \ldots, f_p]^T, \quad g = [g_1, \ldots, g_p]^T \end{array}$$

Assumption 2.1 $|g_0(x,z) + \theta^T g(x,z)| \geq k > 0 \ \forall \ x \in R^n, \ z \in R^m$ and $\theta \in \Omega_1$, *where Ω_1 is a compact set that contains Ω in its interior.*

Assumption 2.1 ensures that (2.2) is input-output linearizable by full state feedback for every $\theta \in \Omega$. Using the results of [6], it can be shown that there exists a global diffeomorphism, possibly dependent on θ,

$$\begin{bmatrix} x \\ \zeta \end{bmatrix} = \begin{bmatrix} x \\ T_1(x,z) \end{bmatrix} \stackrel{\text{def}}{=} T(x,z)$$

with $T_1(0,0) = 0$, which transforms the last m state equations of (2.2) into

$$\dot{\zeta} = F(\zeta, x, \theta) \tag{2.3}$$

This, together with the first n state equations of (2.2), defines a global normal form. As discussed in [25], the input–output model (2.1) has linear dependence on the constant parameters θ, which is a restriction. But, in some cases, redefinition of physical parameters may be needed to arrive at (2.2). The following example shows how it could be done. Consider a single link manipulator with flexible joints and negligible damping which can be represented by [45]

$$\begin{aligned} I\ddot{q}_1 + MgLsinq_1 + k(q_1 - q_2) &= 0 \\ J\ddot{q}_2 - k(q_1 - q_2) &= 0 \end{aligned}$$

where q_1 and q_2 are angular position, and u is a torque input. The physical parameters g, I, J, k, L, and M are all positive. Taking $y = q_1$ as output, y then, satisfies

$$y^{(4)} = \frac{gLM}{I}(\dot{y}^2 \sin y - \ddot{y} \cos y) - (\frac{k}{I} + \frac{k}{J})\ddot{y} - \frac{gkLM}{IJ} \sin y + \frac{k}{IJ}u$$

Taking

$$\theta_1 = \frac{gLM}{I}, \quad \theta_2 = (\frac{k}{I} + \frac{k}{J}), \quad \theta_3 = \frac{gkLM}{IJ}, \quad \theta_4 = \frac{k}{IJ}$$

yields

$$y^{(4)} = \theta_1(\dot{y}^2 \sin y - \ddot{y} \cos y) - \theta_2\ddot{y} - \theta_3 \sin y + \theta_4 u \tag{2.4}$$

equation (2.4) is of the form of (2.1). The class of systems includes as a special case the nonlinear systems treated [22] and [37] for output feedback adaptive control and the linear systems treated in the traditional adaptive control literature, e.g., [4] and [39]. For example [25], the class of state-space models treated in [39] has an input–output model of the form

$$y^{(n)} = \mathcal{B}(D)[\sigma(y)u] + \tilde{\psi}_o(y, \ldots, y^{(n-1)}) + \sum_{i=1}^{p} \tilde{\psi}_i(y, \ldots, y^{(n-1)})\theta_i \tag{2.5}$$

where $D = (d/dt)$ and $\mathcal{B}(D) = b_m D^m + \ldots + b_0$ $(m < n)$ is Hurwitz polynomial with unknown coefficients b_i and $\sigma(y)$ and $\tilde{\psi}_i$ are smooth known nonlinearities with $\sigma(y) \neq 0 \ \forall \ y \in R$. $b_m(\theta) \neq 0$ by assumption. Redefining the control input as $\tilde{u} = \sigma(y)u$, (2.5) is a special case of (2.1) and Assumption 2.1 and 2.2 are satisfied.

Objectives

The objective of this chapter is to design an adaptive output feedback controller which guarantees boundedness of all state variables and tracking of a given reference signal y_r, where y_r is bounded, has bounded derivatives up to the nth-order, and $y_r^{(n)}$ is piecewise continuous.

2.3 Control Design

In this section, we first design a state feedback controller that ensures boundedness of all signals and yields zero steady-state tracking error. This same controller is used in the output feedback case with the states replaced by estimates provided by a high-gain observer. We saturate the control outside a compact region of interest to protect the system from peaking induced by the high-gain observer.

2.3.1 State Feedback

In this section we assume that the state x are available for feedback. The state z, which are the derivatives of the control u, are always available for feedback. We design an adaptive state feedback controller so that the output y tracks the given reference signal y_r. Define

$$e_i \;=\; y^{(i-1)} - y_r^{(i-1)} \;=\; x_i - y_r^{(i-1)}, \qquad 1 \leq i \leq n$$

and

$$e = [e_1,\ e_2,\ \ldots,\ e_n]^T$$

Let

$$\begin{aligned}
\mathcal{Y}(t) &= [y(t),\ y^{(1)}(t),\ \ldots,\ y^{(n-1)}(t)]^T \\
\mathcal{Y}_r(t) &= [y_r(t),\ y_r^{(1)}(t),\ \ldots,\ y_r^{(n-1)}(t)]^T \\
\mathcal{Y}_R(t) &= [y_r(t),\ y_r^{(1)}(t),\ \ldots,\ y_r^{(n-1)}(t),\ y_r^{(n)}(t)]^T
\end{aligned}$$

and Y and Y_R be any given compact subsets of R^n and R^{n+1}, respectively, such that $\mathcal{Y}(0) \in Y$ and $\mathcal{Y}_R(t) \in Y_R \ \forall\ t \geq 0$. We rewrite (2.2) as

$$\begin{aligned}
\dot{e} &= A_m e + b\{Ke + f_0(e + \mathcal{Y}_r, z) \\
&\quad + \theta^T f(e + \mathcal{Y}_r, z) + [g_0(e + \mathcal{Y}_r, z) \\
&\quad + \theta^T g(e + \mathcal{Y}_r, z)]v - y_r^{(n)}\} \qquad (2.6) \\
\dot{z} &= A_2 z + b_2 v \qquad (2.7)
\end{aligned}$$

where (A, b) and (A_2, b_2) are controllable canonical pairs of the form

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \cdots & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & \cdots & \cdots & 1 & 0 \\ 0 & 0 & \cdots & \cdots & 0 & 1 \\ 0 & 0 & \cdots & \cdots & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

and K is chosen such that $A_m = A - bK$ is Hurwitz. We choose the initial states of the integrators such that $z(0) \in Z_0$, a compact subset of R^m, and define the set of initial conditions for the error states as E_0, a compact subset of R^n. To proceed with the analysis we require that the zero dynamics to be exponentially stable. In particular,

Assumption 2.2 *The system $\dot{\zeta} = F(\zeta, \mathcal{Y}_r, \theta)$ has a unique steady-state solution $\bar{\zeta}$. Moreover, with $\tilde{\zeta} = \zeta - \bar{\zeta}$ the system*

$$\begin{aligned} \dot{\tilde{\zeta}} &= F(\bar{\zeta} + \tilde{\zeta}, e + \mathcal{Y}_r, \theta) - F(\bar{\zeta}, \mathcal{Y}_r, \theta) \\ &\stackrel{\text{def}}{=} F_2(\tilde{\zeta}, e, \mathcal{Y}_r, \bar{\zeta}, \theta) \end{aligned} \tag{2.8}$$

has a continuously differentiable function $V_1(t, \tilde{\zeta})$, possibly dependent on θ, that satisfies [1]

$$\eta_1 \|\tilde{\zeta}\|^2 \le V_1(t, \tilde{\zeta}) \le \eta_2 \|\tilde{\zeta}\|^2 \tag{2.9}$$

$$\frac{\partial V_1}{\partial t} + \frac{\partial V_1}{\partial \tilde{\zeta}} F_2(\tilde{\zeta}, e, \mathcal{Y}_r, \bar{\zeta}, \theta) \le -\eta_3 \|\tilde{\zeta}\|^2 + \eta_4 \|\tilde{\zeta}\| \|e\| \tag{2.10}$$

where η_1, η_2, $\eta_3 > 0$, and $\eta_4 \ge 0$ are independent of $\mathcal{Y}_r$ and θ.

---

[1] *Throughout the thesis, $\|\cdot\|$ denotes the Euclidean norm.*

The steady-state response of a nonlinear system is introduced in [15, Section 8.1]. Basically, it is a particular solution towards which any other solution of the system converges, as time increases. The inequalities satisfied by V_1 imply that such convergence is exponential. They also imply that (2.8), with e as input, is input-to-state stable [26, Theorem 5.2]. Consequently, the zero dynamics of (2.2) are exponentially stable and (2.2) is minimum phase.

Let $P = P^T > 0$ be the solution of the Lyapunov equation $PA_m + A_m^T P = -Q$ where $Q = Q^T > 0$, and consider the Lyapunov function candidate

$$V = e^T P e + \tfrac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{2.11}$$

where $\Gamma = \Gamma^T > 0$, $\tilde{\theta} = \hat{\theta} - \theta$, and $\hat{\theta}$ is an estimate of θ to be determined by the parameter adaptation law. The derivative of V along the trajectories of the system is given by

$$\begin{aligned}\dot{V} &= -e^T Q e + \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} + 2e^T Pb\{f_0(e + \mathcal{Y}_r, z)\theta^T f(e + \mathcal{Y}_r, z) + [g_0(e + \mathcal{Y}_r, z) \\ &\quad + \theta^T g(e + \mathcal{Y}_r, z)]v + Ke - y_r^{(n)}\}\end{aligned}$$

Taking

$$\begin{aligned} v &= \frac{-Ke + y_r^{(n)} - f_0(e + \mathcal{Y}_r, z) - \hat{\theta}^T f(e + \mathcal{Y}_r, z)}{g_0(e + \mathcal{Y}_r, z) + \hat{\theta}^T g(e + \mathcal{Y}_r, z)} \\ &\stackrel{\text{def}}{=} \psi(e, z, \mathcal{Y}_R, \hat{\theta}) \end{aligned} \tag{2.12}$$

we can rewrite the expression for $\dot{V}$ as

$$\dot{V} = -e^T Q e + \tilde{\theta}^T \Gamma^{-1} [\dot{\hat{\theta}} - \Gamma\phi]$$

where

$$\begin{aligned} \phi &= 2e^T Pb[f(e+\mathcal{Y}_r, z) + g(e+\mathcal{Y}_r, z)\psi(e, z, \mathcal{Y}_R, \hat{\theta})] \\ &= \phi(e, z, \mathcal{Y}_R, \hat{\theta}) \end{aligned} \tag{2.13}$$

The parameter adaptation law is chosen as

$$\dot{\hat{\theta}} = \mathrm{Proj}(\hat{\theta}, \phi) \tag{2.14}$$

where $\mathrm{Proj}(\hat{\theta}, \phi) = \Gamma\phi$ for $\hat{\theta} \in \Omega$ and is modified outside Ω to ensure that

$$\tilde{\theta}^T \Gamma^{-1}[\dot{\hat{\theta}} - \Gamma\phi] \leq 0 \tag{2.15}$$

and $\hat{\theta}(t)$ belongs to a compact set Ω_δ for all $t \geq 0$, where $\Omega_1 \supset \Omega_\delta \supset \Omega$. This can be achieved by standard adaptation laws with smoothed parameter projection to ensure that $\mathrm{Proj}(\hat{\theta}, \phi)$ is locally Lipschitz; c.f. [42]. As an example, consider the case when Ω is the convex hypercube

$$\Omega = \{\theta \mid a_i \leq \theta_i \leq b_i\},\ 1 \leq i \leq p\}$$

Let

$$\Omega_\delta = \{\theta \mid a_i - \delta \leq \theta_i \leq b_i + \delta\},\ 1 \leq i \leq p\}$$

where $\delta > 0$ is chosen such that $\Omega_\delta \subset \Omega_1$, and choose Γ to be a positive diagonal matrix. In this case the projection $\text{Proj}(\hat{\theta}, \phi)$ is taken as

$$
[\text{Proj}(\hat{\theta}, \phi)]_i = \begin{cases} \gamma_{ii}\phi_i, & \text{if } a_i \le \hat{\theta}_i \le b_i \text{ or} \\ & \text{if } \hat{\theta}_i > b_i \text{ and } \phi_i \le 0 \text{ or} \\ & \text{if } \hat{\theta}_i < a_i \text{ and } \phi_i \ge 0 \\ \\ \gamma_{ii}\left[1 + (b_i - \hat{\theta}_i)/\delta\right]\phi_i, & \text{if } \hat{\theta}_i > b_i \text{ and } \phi_i > 0 \\ \\ \gamma_{ii}\left[1 + (\hat{\theta}_i - a_i)/\delta\right]\phi_i, & \text{if } \hat{\theta}_i < a_i \text{ and } \phi_i < 0 \end{cases} \tag{2.16}
$$

Inequality (2.15) ensures that $\dot{V} \le 0$. Therefore, $e(t)$ and $\hat{\theta}$ are bounded for all $t \ge 0$. Since $\mathcal{Y}_r$ is bounded, we conclude that $x(t)$ is bounded, which implies, in view of Assumption 2.2, that $z(t)$ is bounded. With all signals bounded, we conclude that $e(t) \to 0$ as $t \to \infty$.

In preparation for output feedback, we saturate the control outside a compact region of interest. We assume that all initial conditions are in a given compact set; in particular, $\hat{\theta}(0) \in \Omega$, $e(0) \in E_0$, and $z(0) \in Z_0$, where E_0 and Z_0 are compact sets. The sets E_0 and Z_0 can be chosen large enough to cover any given bounded initial conditions, but once they are chosen we cannot allow initial conditions outside them. Let

$$
\begin{aligned}
c_1 &= \max_{e \in E_0} e^T P e \\
c_2 &= \max_{\theta \in \Omega, \hat{\theta} \in \Omega_1} \tfrac{1}{2}(\hat{\theta} - \theta)^T \Gamma^{-1} (\hat{\theta} - \theta)
\end{aligned}
$$

and $c_3 > c_1 + c_2$. Then $e(t) \in E \stackrel{\text{def}}{=} \{e^T P e \le c_3\}$ for all $t \ge 0$. Let Z be a compact subset of R^m such that Z_0 is in the interior of Z and

$$
z(0) \in Z_0 \text{ and } e(t) \in E \ \forall\, t \ge 0 \ \Rightarrow \ z(t) \in Z \ \forall\, t \ge 0 \tag{2.17}
$$

The set Z can be determined using the Lyapunov function V_1 of Assumption 2.2. The basic idea is to choose c_z large enough that the set $\{V_1 \leq c_z\}$ is positively invariant[2] and then determine the corresponding set in the z-coordinate using the global diffeomorphism that maps z into ζ and vice-versa.

Let $S \geq \max|\psi(e, z, \mathcal{Y}_R, \hat{\theta})|$ where the maximization is taken over all $e \in E_1 \stackrel{\text{def}}{=} \{e^T P e \leq c_4\}$, $z \in Z$, $\mathcal{Y}_R \in Y_R$, $\hat{\theta} \in \Omega_\delta$, where $c_4 > c_3$. Define the saturated function ψ^s by

$$\psi^s(e, z, \mathcal{Y}_R, \hat{\theta}) = S \text{ sat}\left(\frac{\psi(e, z, \mathcal{Y}_R, \hat{\theta})}{S}\right)$$

where sat$(\cdot)$ is the saturation function defined as

$$\text{sat}(x) = \begin{cases} 1 & x > 1 \\ x & -1 \leq x \leq 1 \\ -1 & x < -1 \end{cases}$$

Although the function $\phi(\cdot)$ depends on e, there is no need for saturation, since projection is used to bound $\hat{\theta}$. Hence, $\hat{\theta}$ will not exhibit peaking.

2.3.2 High-gain observer

To implement the state feedback adaptive controller using output feedback, we need to estimate e; there is no need for estimating z since it is already available (the state of the integrators at the input side). With the goal of recovering the performance achieved under state feedback, we use the same high-gain observer used in [25], namely,

$$\left.\begin{aligned} \dot{\hat{e}}_i &= \hat{e}_{i+1} + (\alpha_i/\epsilon^i)(e_1 - \hat{e}_1), \quad 1 \leq i \leq n-1 \\ \dot{\hat{e}}_n &= (\alpha_n/\epsilon^n)(e_1 - \hat{e}_1) \end{aligned}\right\} \tag{2.18}$$

[2] The choice of c_z is shown in Section 2.4, where it is called c_5.

where ϵ is a small positive parameter to be specified. The positive constants α_i are chosen such that the roots of

$$s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1} s + \alpha_n = 0 \tag{2.19}$$

have negative real parts. To implement the control using output feedback, the state e in ψ^s and ϕ is replaced by its estimate $\hat{e}$. By taking

$$\xi_i = \frac{e_i - \hat{e}_i}{\epsilon^{n-i}}, \quad 1 \leq i \leq n \tag{2.20}$$

and $\xi = [\xi_1, \ldots, \xi_n]^T$, the closed-loop system is represented by the standard singularly perturbed form

$$\left.\begin{aligned}
\dot{e} &= A_m e + b\{Ke + f_0(e + \mathcal{Y}_r, z) + \theta^T f(e + \mathcal{Y}_r, z) \\
&\quad + [g_0(e + \mathcal{Y}_r, z) + \theta^T g(e + \mathcal{Y}_r, z)]\psi^s(\hat{e}, z, \mathcal{Y}_R, \hat{\theta}) - y_r^{(n)}\} \\
\dot{z} &= A_2 z + b_2 \psi^s(\hat{e}, z, \mathcal{Y}_R, \hat{\theta}) \\
\dot{\hat{\theta}} &= \mathrm{Proj}(\hat{\theta}, \phi(\hat{e}, z, \mathcal{Y}_R, \hat{\theta})) \\
\epsilon\dot{\xi} &= (A - HC)\xi + \epsilon b\{f_0(e + \mathcal{Y}_r, z) + \theta^T f(e + \mathcal{Y}_r, z) \\
&\quad + [g_0(e + \mathcal{Y}_r, z) + \theta^T g(e + \mathcal{Y}_r, z)]\psi^s(\hat{e}, z, \mathcal{Y}_R, \hat{\theta}) - y_r^{(n)}\}
\end{aligned}\right\} \tag{2.21}$$

where $C = [1, 0, \ldots, 0]$, $H = [\alpha_1, \ldots, \alpha_n]^T$, $(A - HC)$ is Hurwitz, and $\hat{e} = e - D\xi$ where D is a diagonal matrix with ϵ^{n-i} as the ith diagonal element. To eliminate peaking in the implementation of the observer, define $q_i = \epsilon^{i-1}\hat{e}_i, \ 1 \leq i \leq n$ then, the observer equation becomes

$$\left.\begin{aligned} \epsilon\dot{q}_i &= q_{i+1} + \alpha_i(e_1 - q_1), \; 1 \le i \le n-1 \\ \epsilon\dot{q}_n &= \alpha_n(e_1 - q_1), \end{aligned}\right\} \tag{2.22}$$

The system (2.22) will not exhibit peaking if e_1 and $q_i(0)$ are bounded function of ϵ since it is in the standard singularly perturbed form. In summary, the adaptive output feedback controller is given by

$$\left.\begin{aligned} \hat{v} &= \psi^s(\hat{e}, z, \mathcal{Y}_R, \hat{\theta}) \\ \dot{\hat{\theta}} &= \mathrm{Proj}(\hat{\theta}, \phi(\hat{e}, z, \mathcal{Y}_R, \hat{\theta})) \\ \dot{z} &= A_2 z + b_2 v \\ u &= z_1 \end{aligned}\right\} \tag{2.23}$$

2.4 Tracking Error Convergence

The first step in showing tracking error convergence is to confirm that for any initial conditions in the given compact set, all signals of the closed-loop system (under output feedback) are bounded. This property is shown in two steps. First we show that there exist constants c_5, $c_6 > 0$ such that the set [3]

$$R_s = \{\{V \le c_3\} \cap \{\hat{\theta} \in \Omega_\delta\}\} \times \{V_1 \le c_5\} \times \{V_\xi \le c_6\epsilon^2\} \tag{2.24}$$

is positively invariant for sufficiently small ϵ, where $V_\xi = \xi^T \bar{P} \xi$ and $\bar{P} = \bar{P}^T > 0$ is the solution of the Lyapunov equation $\bar{P}(A - HC) + (A - HC)^T \bar{P} = -I$. For this

[3] Note that the set $\{V_1 \le c_5\}$ could be time-dependent. See [26, Section 3.4] for the use of time-dependent sets in the analysis of nonautonomous systems.

part we use the fact that ξ is $O(\epsilon)$ and consequently the derivative of

$$V = e^T P e + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$$

along (2.21) satisfies

$$\begin{aligned} \dot{V} &\leq -e^T Q e + k\epsilon \\ &\leq -c_0 e^T P e + k\epsilon \\ &\leq -c_0 V + \tfrac{c_0}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} + k\epsilon \end{aligned}$$

where $c_0 = \lambda_{min}(Q)/\lambda_{min}(P)$. For all $\hat{\theta} \in \Omega_\delta$, $\frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \leq c_2$. Hence,

$$\dot{V} \leq -c_0 V + c_0 c_2 + k\epsilon \tag{2.25}$$

On the boundary $V = c_3$, the derivative of V is strictly less than zero $\forall\, c_3 > c_2 + \epsilon k/c_0$. Since $c_3 > c_1 + c_2$, for sufficiently small ϵ, the set $\{V \leq c_3\} \cap \{\hat{\theta} \in \Omega_\delta\}$ is a positively invariant set. For all $e \in E$, it can be seen from (2.10) that

$$\dot{V}_1 \leq -\eta_3 ||\tilde{\zeta}||^2 + \eta_4 \sqrt{\tfrac{c_3}{\lambda_{min}(P)}}\, ||\tilde{\zeta}||$$

Using (2.9), we obtain

$$\dot{V}_1 \leq -\frac{\eta_3}{\eta_2} V_1 + \eta_4 \sqrt{\tfrac{c_3}{\lambda_{min}(P)}} \sqrt{V_1/\eta_1}$$

Therefore, choosing

$$c_5 > (\tfrac{\eta_2\eta_4}{\eta_3})^2 (\tfrac{c_3}{\eta_1 \lambda_{min}(P)})$$

we can ensure that the set $\{V_1 \leq c_5\}$ is positively invariant. Finally, the derivative of V_ξ is given by

$$\dot{V}_\xi = -\frac{1}{\epsilon}\xi^T\xi + 2\xi^T \bar{P} b \{f_0(\cdot) + \theta^T f(\cdot) + [g_0(\cdot) + \theta^T g(\cdot)]\psi^s(\cdot)\}$$

Since all state variables are bounded in R_s, we obtain

$$\dot{V}_\xi \leq -\frac{1}{2\epsilon}\xi^T\xi - \frac{1}{2\epsilon\lambda_{max}(\bar{P})}V_\xi + k_1\sqrt{V_\xi} \tag{2.26}$$

for some positive constant k_1, independent of ϵ. Choosing

$$c_6 > [2k_1\lambda_{max}(\bar{P})]^2$$

ensures that the set $\{V_\xi \leq c_6\epsilon^2\}$ is positively invariant. This completes the proof that R_s is positively invariant for the chosen values of c_5 and c_6. The second part of the argument, is to show boundedness of the signals under output feedback. For that we need to show that the fast variable ξ decays rapidly to $O(\epsilon)$. Since $V(e(0), \tilde{\theta}(0)) < c_3$ and ψ^s is bounded uniformly in ϵ, there exist a finite time T_1 independent of ϵ such that $\forall\, t \in [0, T_1],\ z(t) \in Z$ *and* $V(e(t), \tilde{\theta}(t)) \leq c_3$. During this time interval we have,

$$\dot{V}_\xi \leq -\frac{1}{2\epsilon}||\xi||^2, \quad \text{for } V_\xi \geq c_6\epsilon^2$$

Using the fact that $||\xi(0)|| \leq k_\xi/\epsilon^{(n-1)}$ for some $k_\xi > 0$, we obtain

$$V_\xi(\xi(t)) \leq \frac{\beta_1}{\epsilon^{2(n-1)}}e^{-\beta_2 t/\epsilon}$$

where $\beta_1 = k_\xi^2||\bar{P}||$ and $\beta_2 = \frac{1}{2||\bar{P}||}$. Choose ϵ^* small enough that

$$T(\epsilon) \stackrel{\text{def}}{=} \frac{\epsilon}{\beta_2}\ln(\frac{\beta_1}{c_6\epsilon^{2n}}) \leq \frac{1}{2}T_1$$

for all $0 < \epsilon < \epsilon^*$. Hence,

$$V_\xi(\xi(T)) \leq c_6\epsilon^2$$

for all $0 < \epsilon < \epsilon^*$. By choosing $\epsilon^* \leq (c_3 - c_2)c_0/k$, we are guaranteed that the trajectory enters the set R_s during the interval $[0, T(\epsilon)]$ and remains inside thereafter. It follows that $\forall\ t > T(\epsilon)$, $e \in E$ and since $\hat{e} = e + O(\epsilon)$ we conclude that $\hat{e} \in E_1$. Since the saturation level was taken over all $e \in E_1$, $z \in Z$, $\mathcal{Y}_{\mathcal{R}} \in Y_R$ and $\hat{\theta} \in \Omega_\delta$, the saturation function will not be effective, i.e., $\psi^s = \psi\ \forall\ t > T(\epsilon)$. Therefore the closed-loop system is given by

$$\left.\begin{array}{rcl} \dot{e} &=& A_m e - b\tilde{\theta}^T \hat{w}(t) + \Lambda(\cdot) \\ \dot{\tilde{\theta}} &=& \Gamma_p(\hat{\theta}, \phi) \\ \dot{\tilde{\zeta}} &=& F_2(\tilde{\zeta}, e, \mathcal{Y}_r, \bar{\zeta}, \theta) \\ \epsilon\dot{\xi} &=& (A - HC)\xi - \epsilon b[\tilde{\theta}^T \hat{w}(t) + Ke] + \epsilon\Lambda(\cdot) \end{array}\right\} \tag{2.27}$$

where

$$\begin{aligned} \Gamma_p(\hat{\theta}, \phi) &= \mathrm{Proj}(\hat{\theta}, \phi(\hat{e}, z, \mathcal{Y}_R, \hat{\theta})) \\ \hat{w}(t) &= \hat{f}(\cdot) + \hat{g}(\cdot)\psi(\hat{e}(t), z(t), \mathcal{Y}_R(t), \hat{\theta}(t)) \end{aligned} \tag{2.28}$$

$$\begin{aligned} \Lambda(\cdot) &= b\{K(e - \hat{e}) + (f_0 - \hat{f}_0) + \theta^T(f - \hat{f}) + (g_0 - \hat{g}_0)v \\ &\quad + \theta^T(g - \hat{g})v\} \end{aligned} \tag{2.29}$$

$$\begin{aligned} \hat{f}(\cdot) &= f(\hat{e} + \mathcal{Y}_r, z) \\ \hat{g}(\cdot) &= g(\hat{e} + \mathcal{Y}_r, z) \end{aligned}$$

Define w_r as

$$w_r(t) = f(\mathcal{Y}_r, \bar{z}) + g(\mathcal{Y}_r, \bar{z})\psi(0, \bar{z}, \mathcal{Y}_R, \theta) \tag{2.30}$$

where $\bar{z}$ is the steady state solution of the zero dynamics, determined uniquely from $\bar{\zeta} = T_1(\mathcal{Y}_r, \bar{z})$.

Definition 2.1 [26, Definition 13.1] *A vector signal* $\nu(t)$ *is said to be persistently exciting if there are positive constants* $\tilde{\alpha}_1$, $\tilde{\alpha}_2$ *and* $\tilde{\delta}$ *such that*

$$\tilde{\alpha}_2 I \geq \int_t^{t+\tilde{\delta}} \nu(\tau)\nu^T(\tau) \;\; d\tau \geq \tilde{\alpha}_1 I$$

Assumption 2.3 w_r *satisfies one of the three following conditions:*

- w_r *is persistently exciting;*
- $w_r = 0$;
- *There exists a constant nonsingular matrix* S*, possibly dependent on* θ*, such that*

$$Sw_r(t) = \begin{bmatrix} w_{r1}(t) \\ 0 \end{bmatrix} \tag{2.31}$$

  where w_{r1} *is persistently exciting.*

The first case is treated in [25] and the second one is the regulation case of [1]. The analysis of either one of the first two cases is a trivial specialization of the analysis of the third case. Therefore, we concentrate our attention on the third case. Using the transformation S^{-1} to transform $\tilde{\theta}$ into

$$\tilde{\theta}^T S^{-1} = [\tilde{\theta}_1^T, \tilde{\theta}_2^T]$$

the equations for $\dot{e}$ and $\dot{\tilde{\theta}}$ can be rewritten as

$$\begin{aligned} \dot{e} &= A_m e - b\tilde{\theta}^T S^{-1} S w_r + b\tilde{\theta}^T (w_r - \hat{w}) + \Lambda(\cdot) \\ S^{-T}\dot{\tilde{\theta}} &= S^{-T}\Gamma_p \end{aligned}$$

or

$$\begin{bmatrix} \dot{\tilde{\theta}}_1 \\ \dot{\tilde{\theta}}_2 \end{bmatrix} = \begin{bmatrix} \Gamma_{1p} \\ \Gamma_{2p} \end{bmatrix}$$

It is shown in [42] that if the set Ω satisfies the Imbedded Convex Sets assumption[4] then Γ_{1p} and Γ_{2p} are Lipschitz in e. Using the fact [5]

$$w_r - \hat{w} = (\bar{f} - \hat{f}) + (\bar{g}\tilde{\psi} - \hat{g}\hat{\psi}) + \bar{g}\frac{\tilde{\theta}^T}{\bar{g}_0 + \hat{\theta}^T\bar{g}}w_r \tag{2.32}$$

where

$$\bar{f}(\cdot) = f(\mathcal{Y}_r, \bar{z}), \qquad \bar{g}(\cdot) = g(\mathcal{Y}_r, \bar{z}), \qquad \bar{g}_0(\cdot) = g_0(\mathcal{Y}_r, \bar{z})$$

$$\bar{\psi}(\cdot) = \psi(0, \bar{z}, \mathcal{Y}_R, \theta), \quad \tilde{\psi}(\cdot) = \psi(0, \bar{z}, \mathcal{Y}_R, \hat{\theta}), \quad \hat{\psi}(\cdot) = \psi(\hat{e}, z, \mathcal{Y}_R, \hat{\theta})$$

it can be shown that $\dot{e}$ and $\dot{\tilde{\theta}}_1$ satisfy

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}}_1 \end{bmatrix} = \begin{bmatrix} A_m & -b\mathcal{G}w_{r1}^T \\ 2\Gamma_1\mathcal{G}w_{r1}b^TP & 0 \end{bmatrix}\begin{bmatrix} e \\ \tilde{\theta}_1 \end{bmatrix} + \begin{bmatrix} \Lambda_s(\cdot) \\ \Lambda_e(\cdot) \end{bmatrix} \tag{2.33}$$

where

$$S^{-T}\Gamma S^{-1} = \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ \Gamma_2^T & \Gamma_4 \end{bmatrix}$$

$$\Lambda_s(\cdot) = \Lambda(\cdot) + b\tilde{\theta}^T[(\bar{f} - \hat{f}) + (\bar{g}\tilde{\psi} - \hat{g}\hat{\psi})]$$

$$\Lambda_e(\cdot) = [\Gamma_{1p} - 2\Gamma_1\mathcal{G}w_{r1}b^TPe]$$

[4] see Appendix 2.6.4
[5] See Appendix 2.6.1

and

$$K_{\mathcal{G}1} \;>\; \mathcal{G}(\cdot) = \frac{\bar{g}_0+\theta^T\bar{g}}{\bar{g}_0+\hat{\theta}^T\bar{g}} > K_{\mathcal{G}2}$$

for some positive constants $K_{\mathcal{G}1}$ and $K_{\mathcal{G}2}$ independent of ϵ. Since f, g_0, g and ψ are Lipschitz functions in their arguments, we have [6]

$$\|\Lambda_s(\cdot)\| \;\leq\; \|\Lambda(\cdot)\| + \|b\tilde{\theta}^T[(\bar{f}-\hat{f}) + (\bar{g}\tilde{\psi} - \hat{g}\hat{\psi})]\| \tag{2.34}$$

$$\|\bar{g}\tilde{\psi} - \hat{g}\hat{\psi}\| \;\leq\; k_{\zeta 1}\|\tilde{\zeta}\| + k_{e1}\|e\| + k_{\xi 1}\|\xi\| \tag{2.35}$$

$$\|\bar{f} - \hat{f}\| \;\leq\; k_{e2}\|e\| + k_{\xi 2}\|\xi\| + k_{\zeta 2}\|\tilde{\zeta}\| \tag{2.36}$$

and [7]

$$\left.\begin{array}{rcl} \|\Lambda(\cdot)\| & \leq & \delta_0\|\xi\| \\ \|\Lambda_s(\cdot)\| & \leq & \delta_1\|e\| + \delta_2\|\xi\| + \delta_3\|\tilde{\zeta}\| \\ \|\Lambda_e(\cdot)\| & \leq & \delta_4\|e\| \end{array}\right\} \tag{2.37}$$

for some $\delta_i \geq 0$, $i = 0, .., 4$. Consider the system

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}}_1 \end{bmatrix} = \begin{bmatrix} A_m & -b\mathcal{G}w_{r1}^T \\ 2\Gamma_1\mathcal{G}w_{r1}b^TP & 0 \end{bmatrix} \begin{bmatrix} e \\ \tilde{\theta}_1 \end{bmatrix} \tag{2.38}$$

Defining

$$\tilde{\theta}_{1n} = \Gamma^{-\frac{1}{2}}\tilde{\theta}_1$$

(2.38) can be rewritten as

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[6] See Appendix 2.6.2

[7] Note that Γ_{1p} is Lipschitz in e since the set Ω satisfies the Imbedded Convex Sets assumption [42].

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}}_{1n} \end{bmatrix} = \begin{bmatrix} A_m & -b\mathcal{G}w_{r1}^T\Gamma^{\frac{1}{2}} \\ 2\mathcal{G}\Gamma^{\frac{1}{2}}w_{r1}b^TP & 0 \end{bmatrix} \begin{bmatrix} e \\ \tilde{\theta}_{1n} \end{bmatrix} \tag{2.39}$$

Using well known results from adaptive control theory (see for example [26, Section 13.4]) and the fact that w_{r1} is persistently exciting and $\mathcal{G}(\cdot)$ is bounded from below, it can be shown that (2.39) has an exponentially stable equilibrium point at the origin. Then, from the converse Lyapunov theorem, there exists a Lyapunov function $V_2(t, e, \tilde{\theta}_1)$ whose derivative along (2.33) satisfies

$$\begin{aligned} \dot{V}_2 \leq \; & -\delta_5||e||^2 - \delta_6||\tilde{\theta}_1||^2 + \delta_7||e||||\xi|| + \delta_8||\tilde{\theta}_1||||\xi|| + \delta_9||e||^2 \\ & + \delta_{10}||e||||\tilde{\theta}_1|| + \delta_{11}||e||||\tilde{\zeta}|| + \delta_{12}||\tilde{\theta}_1||||\tilde{\zeta}|| \end{aligned} \tag{2.40}$$

for some positive constants δ_5 and δ_6 and some non-negative constants δ_i , $7 \leq i \leq 12$. Noticing the similarity of the right-hand side of $\dot{e}$ and $\epsilon\dot{\xi}$ in (2.27) one can easily show that

$$\epsilon\dot{\xi} = (A - HC)\xi - \epsilon b\mathcal{G}\tilde{\theta}_1^T w_{r1} - \epsilon K e + \epsilon\Lambda_s \tag{2.41}$$

The derivative of V_ξ with respect to (2.41) satisfies

$$\dot{V}_\xi \leq \frac{1}{\epsilon}||\xi||^2 + \gamma_3||\tilde{\theta}_1||||\xi|| + \gamma_4||e||||\xi|| + \gamma_5||\tilde{\zeta}||||\xi|| + \gamma_6||\xi||^2 \tag{2.42}$$

for some non-negative constants γ_i , $3 \leq i \leq 6$. Construct the Lyapunov function candidate

$$W = \alpha V + \beta V_1 + V_2 + V_\xi \tag{2.43}$$

where $\alpha > 0$ and $\beta > 0$ will be chosen later. Using the inequality[8]

$$\dot{V} \leq -k_{v1}||e||^2 + k_{v2}||e||||\xi|| + k_{v3}||\xi||^2 + k_{v4}||\tilde{\zeta}||||\xi|| + k_{v5}||\xi||||\tilde{\theta}_1|| \tag{2.44}$$

---

[8] See Appendix 2.6.3

together with (2.10), (2.40), and (2.42), it can be shown that the derivative of W with respect to (2.27) satisfies

$$\dot{W} \leq - \begin{bmatrix} ||e|| \\ ||\tilde{\theta}_1|| \\ ||\tilde{\zeta}|| \\ ||\xi|| \end{bmatrix}^T M \begin{bmatrix} ||e|| \\ ||\tilde{\theta}_1|| \\ ||\tilde{\zeta}|| \\ ||\xi|| \end{bmatrix} \tag{2.45}$$

where M is given by

$$M = \begin{bmatrix} \alpha k_{v1} + \delta_5 - \delta_9 & \frac{-\delta_{10}}{2} & \frac{-\beta\eta_4 - \delta_{11}}{2} & \frac{-\alpha k_{v2} - \delta_7 - \gamma_4}{2} \\ \frac{-\delta_{10}}{2} & \delta_6 & \frac{-\delta_{12}}{2} & \frac{-\delta_8 - \gamma_3 - \alpha k_{v5}}{2} \\ \frac{-\beta\eta_4 - \delta_{11}}{2} & \frac{-\delta_{12}}{2} & \beta\eta_3 & \frac{-\gamma_5 - \alpha k_{v4}}{2} \\ \frac{-\alpha k_{v2} - \delta_7 - \gamma_4}{2} & \frac{-\delta_8 - \gamma_3 - \alpha k_{v5}}{2} & \frac{-\gamma_5 - \alpha k_{v4}}{2} & \frac{1}{\epsilon} - \gamma_6 - \alpha k_{v3} \end{bmatrix}$$

Choose β large enough to make

$$\begin{bmatrix} \delta_6 & \frac{-\delta_{12}}{2} \\ \frac{-\delta_{12}}{2} & \beta\eta_3 \end{bmatrix}$$

positive definite; then choose α large enough that

$$\begin{bmatrix} \alpha k_{v1} + \delta_5 - \delta_9 & \frac{-\delta_{10}}{2} & \frac{-\beta\eta_4 - \delta_{11}}{2} \\ \frac{-\delta_{10}}{2} & \delta_6 & \frac{-\delta_{12}}{2} \\ \frac{-\beta\eta_4 - \delta_{11}}{2} & \frac{-\delta_{12}}{2} & \beta\eta_3 \end{bmatrix}$$

is positive definite. Finally, choosing ϵ small enough we can make M positive definite . Hence, by [26, Theorem 4.4], we conclude that

$$\begin{bmatrix} ||e|| \\ ||\tilde{\theta}_1|| \\ ||\tilde{\zeta}|| \\ ||\xi|| \end{bmatrix} \to 0, \quad \text{as } t \to \infty$$

It should be noted that the foregoing analysis does not imply exponential convergence since the right-hand side of (2.45) is only negative semidefinite. This is a key point in the analysis because considering (2.33) together with the $\dot{\tilde{\zeta}}$ and $\dot{\xi}$ equations from (2.27) one cannot show exponential stability of the set $\{(e, \tilde{\theta}_1, \tilde{\zeta}, \xi) = 0\}$. The difficulty arises from the perturbation terms on the right-hand side of (2.33). While those terms satisfy the growth condition (2.37), the constants δ_1 to δ_4 are not necessarily small. Consequently, we see in (2.40) that the right-hand side contains the positive term $\delta_9||e||^2$ which could dominate the negative term $-\delta_5||e||^2$. We overcome this difficulty by including αV in the composite Lyapunov function W and choosing α to ensure that the negative term $-\alpha k_1||e||^2$ dominates $\delta_9||e||^2$ and other cross product terms. The function V, however, is positive definite in $(e, \tilde{\theta})$, not only $(e, \tilde{\theta}_1)$, and that is why the right-hand side of (2.45) is only negative semidefinite.

2.5 Examples

2.5.1 Linear Plant [14]

Consider Example 6.4.1 of [14]. The system is represented by the transfer function

$$y = \frac{k_p(s + b_0)}{(s^2 + a_1 s + a_0)} u$$

where $k_p > 0$ and $b_0 > 0$. The input-output model is

$$\ddot{y} = -a_1\dot{y} - a_0 y + k_p(\dot{u} + b_0 u) \tag{2.46}$$

The goal is to design an adaptive feedback controller that renders tracking of a reference y_r. In [14] the reference model is given by

$$y_r = \frac{1}{s+1} r$$

and r is a command signal. In our case, the controller requires y_r, $\dot{y}_r$, and $\ddot{y}_r$. We generate them using the second order filter

$$y_r = \frac{\omega_n^2}{s^2 + 2\zeta_n\omega_n s + \omega_n^2} r$$

where $\omega_n = 40$ and $\zeta_n = 1$. Its input is r and its states are the smoothed y_r and $\dot{y}_r$. Equation (2.46) can be rewritten as

$$\ddot{y} = -\theta_1 y - \theta_2\dot{y} + \theta_3 u + \theta_4\dot{u}$$

which takes the form (2.1) with $n = 2$ and $m = 1$. Assumption 2.1 is satisfied for $k_p \neq 0$. We augment an integrator at the input side, set $x_1 = y$, $x_2 = \dot{y}$,

$z = u$, and treat $v = \dot{u}$ as the control input. Let $e_1 = y - y_r$, $e_2 = \dot{y} - \dot{y}_r$ and $\theta^T = [a_0,\ a_1,\ k_p b_0,\ k_p]$. The change of variables $\zeta = z - \frac{1}{\theta_4} x_2$ transforms the system into the normal form

$$\begin{aligned}
\dot{e} &= Ae + b\{\theta^T f + \theta^T g v\} \\
\dot{\zeta} &= -\tfrac{\theta_3}{\theta_4}\zeta + \tfrac{\theta_1}{\theta_4}(e_1 + y_r) + (\tfrac{\theta_2}{\theta_4} - \tfrac{\theta_3}{\theta_4^2})(e_2 + \dot{y}_r)
\end{aligned}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ b = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ f = \begin{bmatrix} -x_1 \\ -x_2 \\ z \\ 0 \end{bmatrix},\ g = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Assumption 2.2 is satisfied since $\theta_3 > 0$, $\theta_4 > 0$, and V_1 can be taken as $V_1 = \frac{1}{2}\tilde{\zeta}^2$. Choose the matrix $K = [6\ 5]$ to assign the eigenvalues of $A_m = (A - bK)$ at -2 and -3. We obtain the matrix P by solving the Lyapunov equation $PA_m + A_m^T P = -I$. The function ψ of (2.12) is given by

$$\psi = \frac{-6e_1 - 5e_2 + \hat{\theta}_1(e_1 + y_r) + \hat{\theta}_2(e_2 + \dot{y}_r) - \hat{\theta}_3 z + \ddot{y}_r}{\hat{\theta}_4}$$

and $\phi = 2e^T Pb[f + g\psi]$. We use the scaled state observer

$$\begin{aligned}
\epsilon\dot{q}_1 &= q_2 + (e_1 - q_1) \\
\epsilon\dot{q}_2 &= 6(e_1 - q_1)
\end{aligned}$$

where $\hat{e}_1 = q_1$ and $\hat{e}_2 = q_2/\epsilon$. The variable e is replaced by its estimate $\hat{e}$ in the control and adaptive laws for the output feedback case. When y_r is constant, the vector w_r of (2.30) is

$$w_r = \begin{bmatrix} -y_r \\ 0 \\ \frac{\theta_1}{\theta_3} y_r \\ 0 \end{bmatrix}$$

Hence, the third case of Assumption 3 is satisfied with the transformation

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\theta_1}{\theta_3} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

rendering

$$Sw_r = \begin{bmatrix} -y_r \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad S^{-T}\tilde{\theta} = \begin{bmatrix} \tilde{\theta}_1 - \frac{\theta_1}{\theta_3}\tilde{\theta}_3 \\ \tilde{\theta}_2 \\ \tilde{\theta}_3 \\ \tilde{\theta}_4 \end{bmatrix}$$

It is expected that the term $\tilde{\theta}_1 - \frac{\theta_1}{\theta_3}\tilde{\theta}_3$ will converge to zero. On the other hand $\tilde{\theta}_2$, $\tilde{\theta}_3$ and $\tilde{\theta}_4$ are not expected to converge to zero. The plant was simulated Using Matlab with $\epsilon = 0.01$, $\theta^T = [-10\ 3\ 3\ 1]$, $\Omega = [-15, -5] \times [1, 5] \times [1.5, 5] \times [0.75, 2]$, and the adaptive law (2.15) is used with $\delta = 0.01$ and $\Gamma = \text{diag}[300, 10, 10, 0.05]$. Two types of command signals r were considered. First, a step of amplitude 2, Figures 2.1 to 2.3 are for the case when the output feedback controller developed in this chapter is used. Figure 2.1 shows the tracking error e, Figure 2.2 shows parameter errors $\tilde{\theta}$, and Figure 2.3 shows the parameter error $\tilde{\theta}$ in the new coordinate i.e., $S^{-T}\tilde{\theta}$ where the first component converges to zero as predicted by the theory. Second, the command signal r is taken as $r = 0.5\sin(0.7t) + 2\cos(5.9t)$. Figure 2.4 shows tracking error

e, Figure 2.5 shows the parameter errors $\tilde{\theta}$. Note that $\tilde{\theta}$ converge to zero since the reference signal y_r is persistently exciting. Figures 2.6 and 2.7 show simulation of the model reference adaptive controller (MRAC) [14]. Figure 2.6 shows the tracking error and Figure 2.7 shows parameter errors $\tilde{\theta}_l$ when r is a step input of amplitude 2. Figures 2.8 and 2.9 show the same quantities when $r = 0.5\sin(0.7t) + 2\cos(5.9t)$, θ_l is a function of the plant's parameters that is different from our $\tilde{\theta}$. In conclusion, in tracking a step command signal our controller has shown better steady state error over the MRAC one. There is no noticeable difference in the convergence of the parameter errors between the two methods. However, in our approach we were able to transform the parameter errors into another space where we were able to draw some conclusions. In particular, we were able to predict that $\tilde{\theta}_1 - \frac{\theta_1}{\theta_3}\tilde{\theta}_3$ approaches zero. In the case of a persistently exciting command signal, our method shows better considerably faster tracking error convergence. The parameter errors are comparable in their convergence rate.

2.5.2 Nonlinear Plant

Consider the nonlinear system

$$\ddot{y} = a_1 y + a_2(y + u\dot{y}^2) + b_1\dot{u} + u \tag{2.47}$$

which takes the form (2.1) with $n = 2$ and $m = 1$. Suppose the reference signal y_r is a step input. Assumption 2.1 is satisfied for $b_1 \neq 0$. We augment an integrators at the input side, set $x_1 = y$, $x_2 = \dot{y}$, $z = u$, and treat $v = \dot{u}$ as the control input. Let $e_1 = y - y_r$, $e_2 = \dot{y} - \dot{y}_r$ and $\theta^T = [a_1\ a_2\ b_1]$. The change of variables $\zeta = z - \frac{1}{\theta_3}x_2$

transforms the system into the normal form

$$
\begin{aligned}
\dot{e} &= Ae + b\{f_0 + \theta^T f + \theta^T g v\} \\
\dot{\zeta} &= -\tfrac{1}{\theta_3}\{\theta_1(e_1 + y_r) + \zeta + \tfrac{1}{\theta_3}(e_2 + \dot{y}_r) + \theta_2(e_1 + y_r) \\
&\quad \theta_2[\zeta + \tfrac{1}{\theta_3}(e_2 + \dot{y}_r)](e_2 + \dot{y}_r)^2\}
\end{aligned}
$$

where

$$
A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; f_0 = z, \; f = \begin{bmatrix} x_1 \\ x_1 + z x_2^2 \\ 0 \end{bmatrix}, \; g = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
$$

Assumption 2 is satisfied when $\theta_3 > 0$, and V_1 can be taken as $V_1 = \frac{1}{2}\tilde{\zeta}^2$. Choose the matrix $K = [6,\ 5]$ to assign the eigenvalues of $A_m = (A - bK)$ in the open left-half plane. We obtain the matrix P by solving the Lyapunov equation $PA_m + A_m^T P = -I$. The function ψ of (2.12) is given by

$$
\psi = \frac{-z - 6e_1 - 5e_2 - \hat{\theta}_1(e_1 + y_r) - \hat{\theta}_2[(e_1 + y_r) + z(e_2 + \dot{y}_r)^2] + \ddot{y}_r}{\hat{\theta}_3}
$$

and $\phi = 2e^T P b[f + g\psi]$. We use the scaled state observer

$$
\begin{aligned}
\epsilon \dot{q}_1 &= q_2 + (e_1 - q_1) \\
\epsilon \dot{q}_2 &= 6(e_1 - q_1)
\end{aligned}
$$

where $\hat{e}_1 = q_1$ and $\hat{e}_2 = q_2/\epsilon$. The variable e is replaced by its estimate $\hat{e}$ in the control and adaptive laws for the output feedback case. The vector w_r of (2.30) is

$$
w_r = \begin{bmatrix} y_r \\ y_r \\ 0 \end{bmatrix}
$$

Hence, the third case of Assumption 3 is satisfied with the transformation

$$S = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which yields

$$Sw_r = \begin{bmatrix} y_r \\ 0 \\ 0 \end{bmatrix} \text{ and } S^{-T}\tilde{\theta} = \begin{bmatrix} \tilde{\theta}_1 + \tilde{\theta}_2 \\ \tilde{\theta}_2 \\ \tilde{\theta}_3 \end{bmatrix}$$

Simulation results for the output feedback case when $y_r = 1$, $\epsilon = 0.01$, $\theta^T = [5\ 2\ 3]$, $\Omega = [4,6] \times [1,3] \times [2,4]$, and the adaptive law (2.15) is used with $\delta = 0.01$ and $\Gamma = \text{diag}[3,3,1.5]$, are shown in Figure 2.10. Figure 2.10-a shows the tracking error e, Figure 2.10-b is the control u while Figure 2.10-c shows the control after saturation ψ^s, and 2.10-d shows projected parameter error $\tilde{\theta}_1 + \tilde{\theta}_2$ converge to zero.

2.6 Appendix

2.6.1 Proof of (2.32)

$$\begin{aligned} w_r - \hat{w} &= \bar{f} + \bar{g}\bar{\psi} - \hat{f} - \hat{g}\hat{\psi} \\ &= \bar{f} - \hat{f} + \bar{g}\bar{\psi} - \hat{g}\hat{\psi} \\ &= (\bar{f} - \hat{f}) + (\bar{g}\bar{\psi} - \bar{g}\tilde{\psi}) + (\bar{g}\tilde{\psi} - \hat{g}\hat{\psi}) \end{aligned}$$

The term $(\bar{\psi} - \tilde{\psi})$ can be written as

$$\bar{\psi} - \tilde{\psi} = \frac{y_r^{(n)}\bar{g}_0 + y_r^{(n)}\hat{\theta}^T\bar{g} - \bar{f}_0\bar{g}_0 - \bar{f}_0\hat{\theta}^T\bar{g} - \theta^T\bar{f}\bar{g}_0 - \theta^T\bar{f}\hat{\theta}^T\bar{g}}{(\bar{g}_0+\theta^T\bar{g})(\bar{g}_0+\hat{\theta}^T\bar{g})}$$

$$+ \frac{-y_r^{(n)}\bar{g}_0 - y_r^{(n)}\theta^T\bar{g} + \bar{f}_0\bar{g}_0 + \bar{f}_0\theta^T\bar{g} + \hat{\theta}^T\bar{f}\bar{g}_0 + \hat{\theta}^T\bar{f}\theta^T\bar{g}}{(\bar{g}_0+\theta^T\bar{g})(\bar{g}_0+\hat{\theta}^T\bar{g})}$$

where $\bar{f}_0 = f_0(\mathcal{Y}_r, \bar{z})$. Then

$$\bar{\psi} - \tilde{\psi} = \frac{\tilde{\theta}^T\bar{g}y_r^{(n)} - \bar{f}_0\tilde{\theta}^T\bar{g} + \tilde{\theta}^T\bar{f}\bar{g}_0 - \theta^T\bar{f}\hat{\theta}^T\bar{g} + \hat{\theta}^T\bar{f}\theta^T\bar{g} + \theta^T\bar{f}\theta^T\bar{g} - \theta^T\bar{f}\theta^T\bar{g}}{(\bar{g}_0+\theta^T\bar{g})(\bar{g}_0+\hat{\theta}^T\bar{g})}$$

$$= \frac{\tilde{\theta}^T\bar{g}(y_r^{(n)} - \bar{f}_0 - \theta^T\bar{f}) + \tilde{\theta}^T\bar{f}(\bar{g}_0+\theta^T\bar{g})}{(\bar{g}_0+\theta^T\bar{g})(\bar{g}_0+\hat{\theta}^T\bar{g})}$$

$$= \frac{\tilde{\theta}^T}{(\bar{g}_0+\hat{\theta}^T\bar{g})}(\bar{f} + \bar{g}\bar{\psi})$$

$$= \frac{\tilde{\theta}^T}{(\bar{g}_0+\hat{\theta}^T\bar{g})}w_r$$

Hence,

$$w_r - \hat{w} = (\bar{f} - \hat{f}) + (\bar{g}\tilde{\psi} - \hat{g}\hat{\psi}) + \bar{g}\frac{\tilde{\theta}^T}{(\bar{g}_0 + \hat{\theta}^T\bar{g})}w_r$$

2.6.2

2.6.

We ha

since

2.6.2 Proof of (2.35) and (2.36)

$$
\begin{aligned}
\bar{g}\tilde{\psi} - \hat{g}\hat{\psi} &= g(\mathcal{Y}_r, \bar{z})\psi(0, \bar{z}, \mathcal{Y}_R, \hat{\theta}) - g(\hat{e} + \mathcal{Y}_r, z)\psi(\hat{e}, z, \mathcal{Y}_R, \hat{\theta}) \\
&= g(\mathcal{Y}_r, \bar{z})\psi(0, \bar{z}, \mathcal{Y}_R, \hat{\theta}) - g(\mathcal{Y}_r, z)\psi(0, z, \mathcal{Y}_R, \hat{\theta}) \\
&\quad + g(\mathcal{Y}_r, z)\psi(0, z, \mathcal{Y}_R, \hat{\theta}) - g(e + \mathcal{Y}_r, z)\psi(e, z, \mathcal{Y}_R, \hat{\theta}) \\
&\quad + g(e + \mathcal{Y}_r, z)\psi(e, z, \mathcal{Y}_R, \hat{\theta}) - g(\hat{e} + \mathcal{Y}_r, z)\psi(\hat{e}, z, \mathcal{Y}_R, \hat{\theta}) \\
&\leq k_{\zeta 1}\|\tilde{\zeta}\| + k_{e1}\|e\| + k_{\xi 1}\|\xi\| \\
\bar{f} - \hat{f} &= f(\mathcal{Y}_r, \bar{z}) - f(\hat{e} + \mathcal{Y}_r, z) \\
&= f(\mathcal{Y}_r, \bar{z}) - f(e + \mathcal{Y}_r, \bar{z}) \\
&\quad + f(e + \mathcal{Y}_r, \bar{z}) - f(\hat{e} + \mathcal{Y}_r, \bar{z}) \\
&\quad + f(\hat{e} + \mathcal{Y}_r, \bar{z}) - f(\hat{e} + \mathcal{Y}_r, z) \\
&\leq k_{e2}\|e\| + k_{\xi 2}\|\xi\| + k_{\zeta 2}\|\tilde{\zeta}\|
\end{aligned}
$$

2.6.3 Proof of (2.44)

We have

$$
\begin{aligned}
\dot{e} &= A_m e - b\tilde{\theta}^T \hat{w}(t) + \Lambda(\cdot) \\
\dot{\hat{\theta}} &= \Gamma(2\hat{e}^T P b \hat{w})
\end{aligned}
$$

since $\|\Lambda(\cdot)\| \leq \delta_0\|\xi\|$, the derivative of V along the trajectories of the system satisfies

$$
\begin{aligned}
\dot{V} &\leq -k_{v1}||e||^2 - 2e^T Pb\tilde{\theta}^T\hat{w} + k_2||e||||\xi|| + \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} \\
&\leq -k_{v1}||e||^2 + \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \Gamma 2e^T Pb\hat{w}) + k_2||e||||\xi|| \\
&\leq -k_{v1}||e||^2 + \tilde{\theta}^T(2\hat{e}^T Pb\hat{w} - 2e^T Pb\hat{w}) + k_2||e||||\xi|| \\
&\leq -k_{v1}||e||^2 + 2\tilde{\theta}^T(\hat{e}^T - e^T)Pb\hat{w} + k_2||e||||\xi|| \\
&\leq -k_{v1}||e||^2 + 2\tilde{\theta}^T\tilde{e}Pb\hat{w} + k_2||e||||\xi|| \\
&\leq -k_{v1}||e||^2 + 2\tilde{e}Pb\tilde{\theta}^T w_r + 2\tilde{e}Pb\tilde{\theta}^T(\hat{w} - w_r) + k_2||e||||\xi|| \\
&\leq -k_{v1}||e||^2 + 2\tilde{e}Pb\tilde{\theta}^T S^{-1}Sw_r + 2\tilde{e}Pb\tilde{\theta}^T(\hat{w} - w_r) + k_2||e||||\xi|| \\
&\leq -k_{v1}||e||^2 + k_{\xi 0}||\xi||||\tilde{\theta}_1|| + 2\tilde{e}Pb\tilde{\theta}^T([\bar{f} - \hat{f} + \bar{g}\tilde{\psi} - \hat{g}\hat{\psi}] + \bar{g}\tfrac{\tilde{\theta}^T}{\bar{g}+\hat{\theta}^T\bar{g}}w_r) + k_2||e||||\xi|| \\
\dot{V} &\leq -k_{v1}||e||^2 + k_{v2}||e||||\xi|| + k_{v3}||\xi||^2 + k_{v4}||\tilde{\zeta}||||\xi|| + k_{v5}||\xi||||\tilde{\theta}_1||
\end{aligned}
$$

where $\tilde{e} = \hat{e} - e$.

2.6.4 Imbedded Convex Sets Assumption

To have a smooth projection, Ω is required to satisfy the Imbedded Convex Set Assumption; that is,

There exists a known C^2 function $\mathcal{P}$ from Ω to R such that the following hold.

1. For each real number λ in $[0, 1]$, the set

$$\{\theta | \mathcal{P}(\theta) \leq \lambda\}$$

   is convex and contained in Ω.

2. The row vector $(\partial\mathcal{P}/\partial\theta)(\theta)$ in nonzero for all θ such that $\mathcal{P}(\theta) \in [0, 1]$

3. The parameter vector θ^* of the particular system to be actually controlled satisfies

$$\mathcal{P}(\theta^*) \leq 0$$

2.7 Conclusions

For the nonlinear output feedback adaptive controller, we have successfully shown tracking error convergence without requiring the persistence of excitation as in [25]. This is a major improvement over [25]. Also, we have removed an unnecessary feature of the controller of [25], namely, saturation of the right-hand side of the adaptive law. It is not needed due to parameter projection. We have also allowed g_0 and g_i of (2.1) to be nonlinear functions instead of constants as in [25]. On the other hand, we have required exponential stability of the zero dynamics which is stronger than the bounded-input–bounded-state stability requirement used in [25]. It should be noted that in [16], Jankovic used ideas similar to ours to design an adaptive output feedback controller for nonlinear feedback linearizable systems. He proved tracking error convergence without persistence of excitation. However, to do that he required the parameter adaptation gain (Γ in our case) to be sufficiently large. Our result does not impose any such restrictions on Γ.

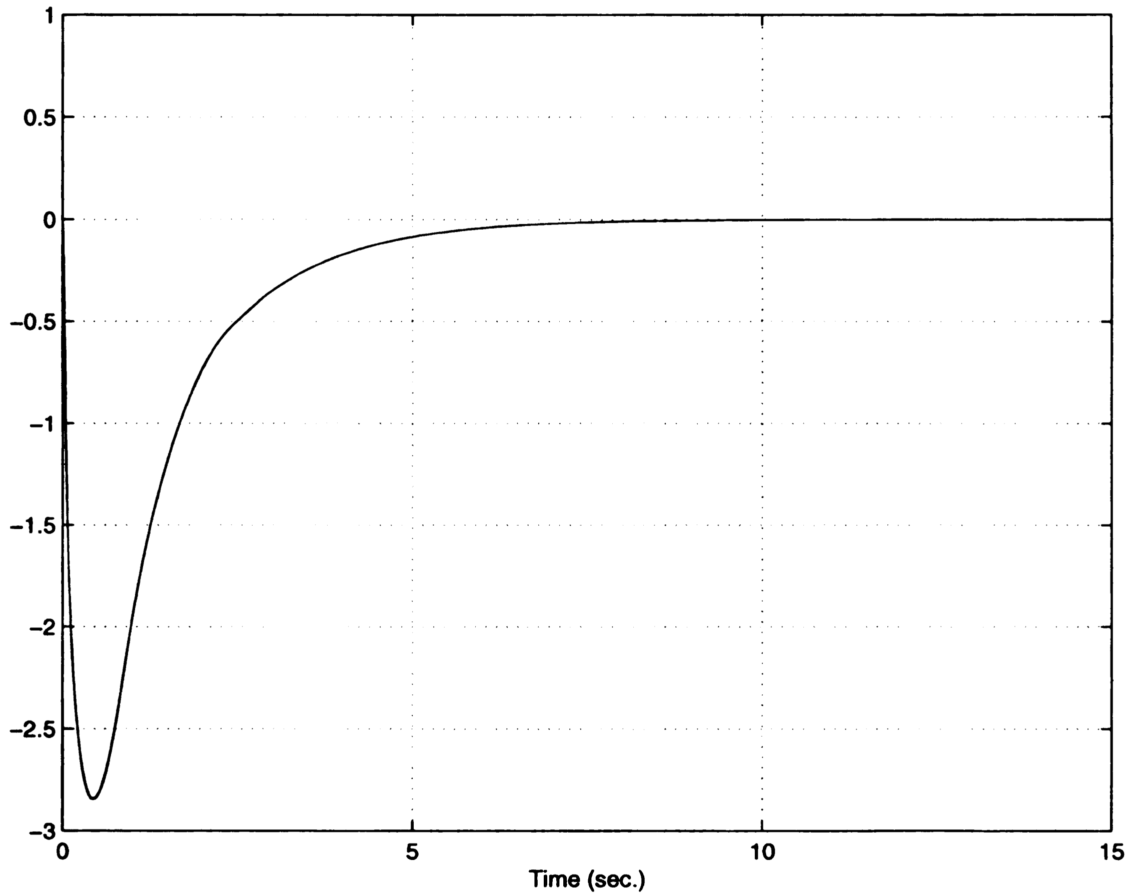

Figure 2.1. Tracking error e.

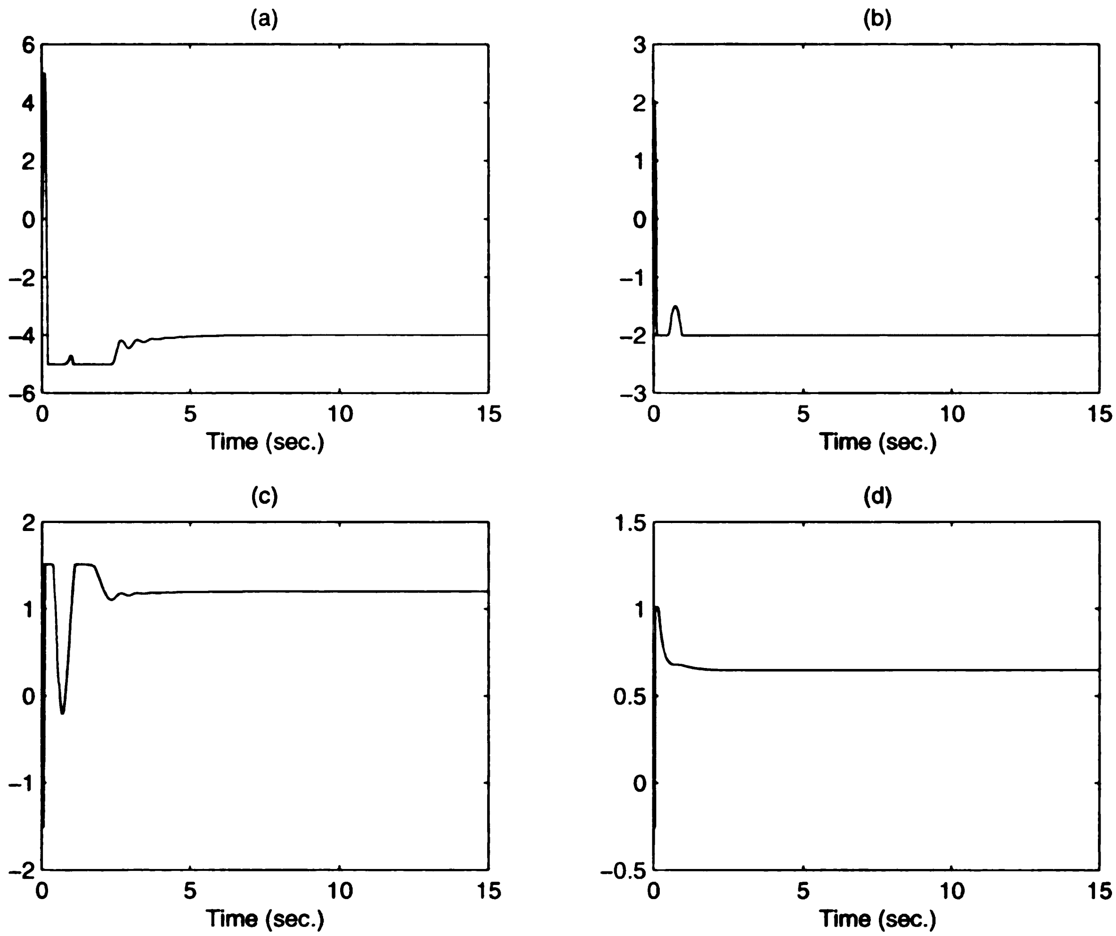

Figure 2.2. Parameter errors,(a)- $\tilde{\theta}_1$,(b)- $\tilde{\theta}_2$,(c)- $\tilde{\theta}_3$, (d)-$\tilde{\theta}_4$.

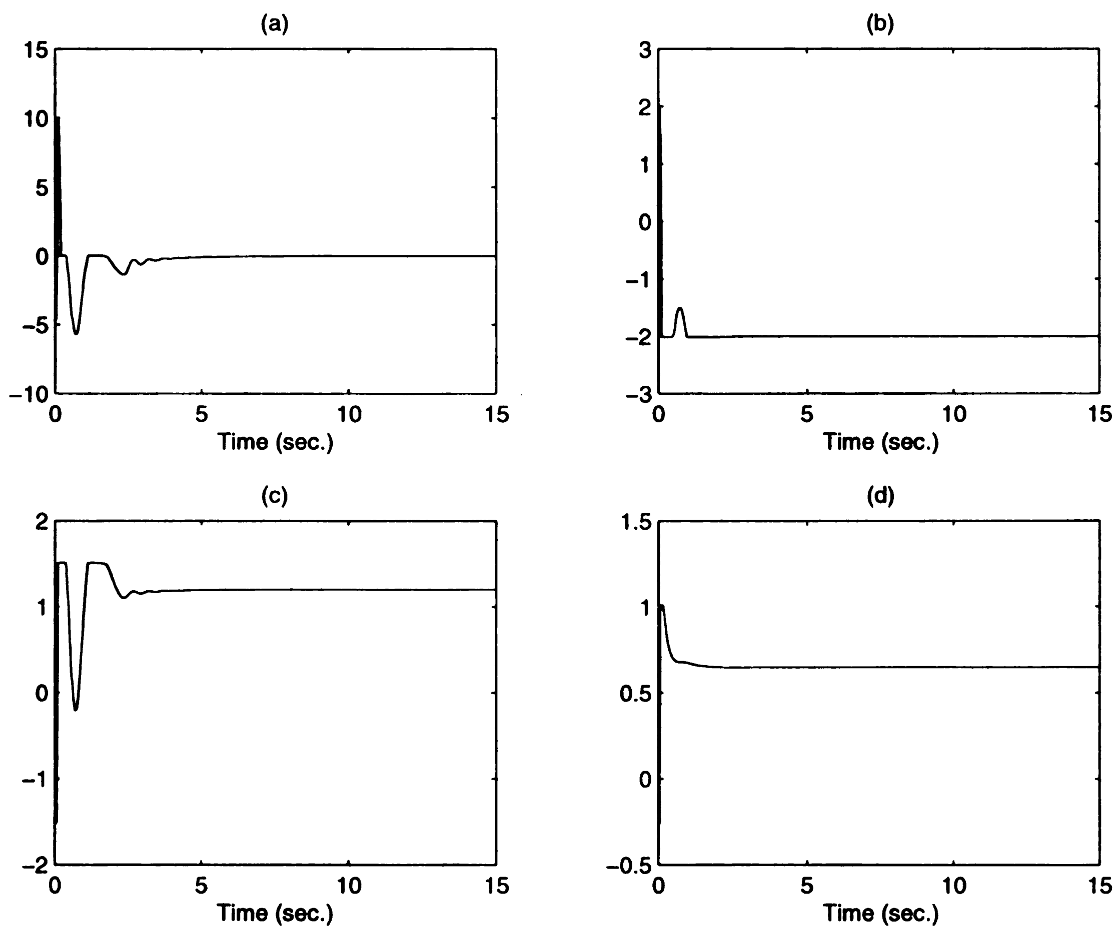

Figure 2.3. Projected Parameter errors: (a)- $\tilde{\theta}_1 - \frac{\theta_1}{\theta_3}\tilde{\theta}_3$,(b)- $\tilde{\theta}_2$,(c)- $\tilde{\theta}_3$, (d)-$\tilde{\theta}_4$.

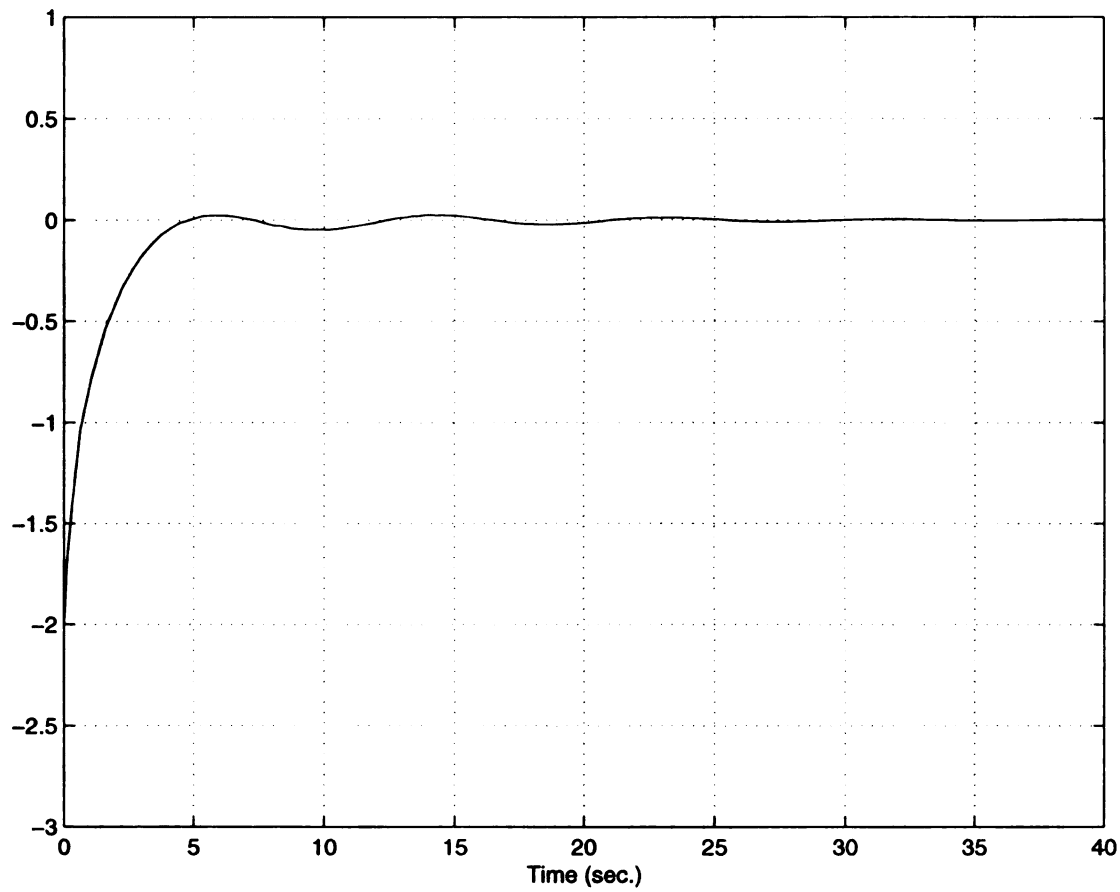

Figure 2.4. Tracking error e.

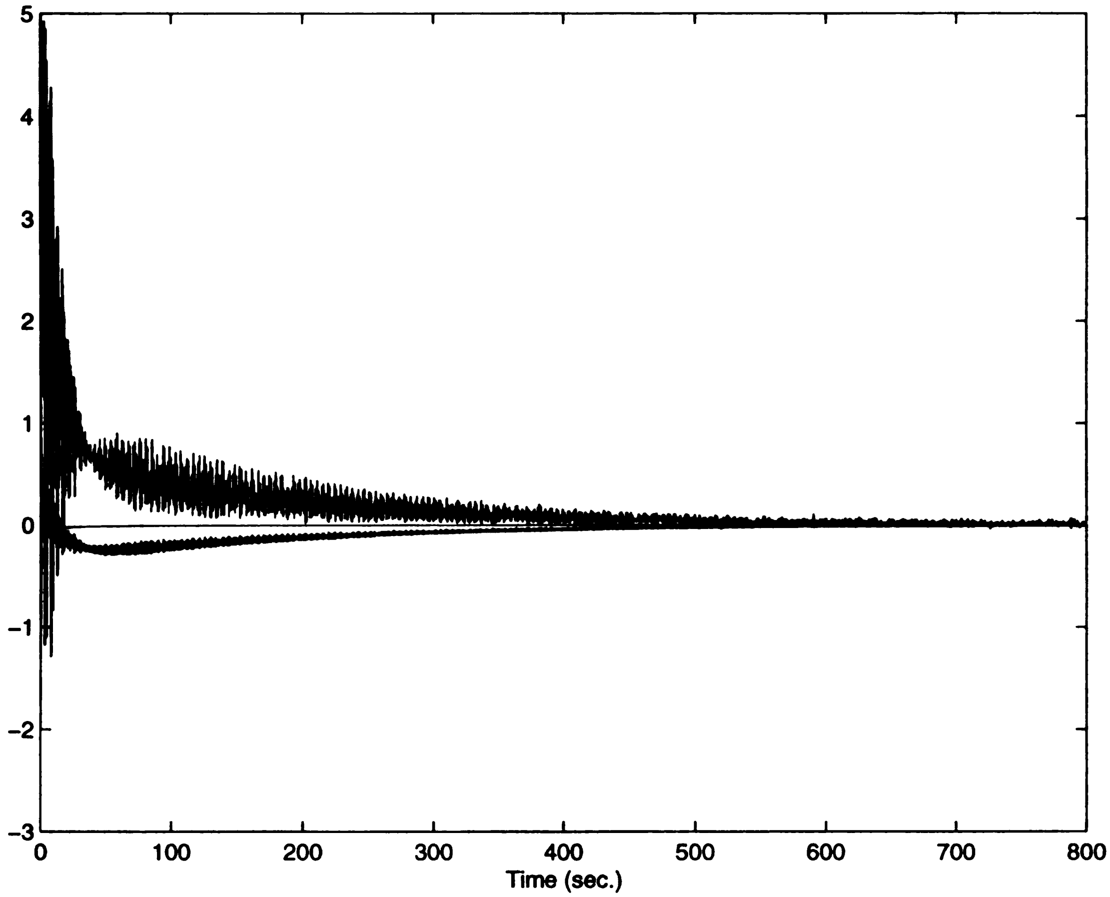

Figure 2.5. Parameter errors $\tilde{\theta}$.

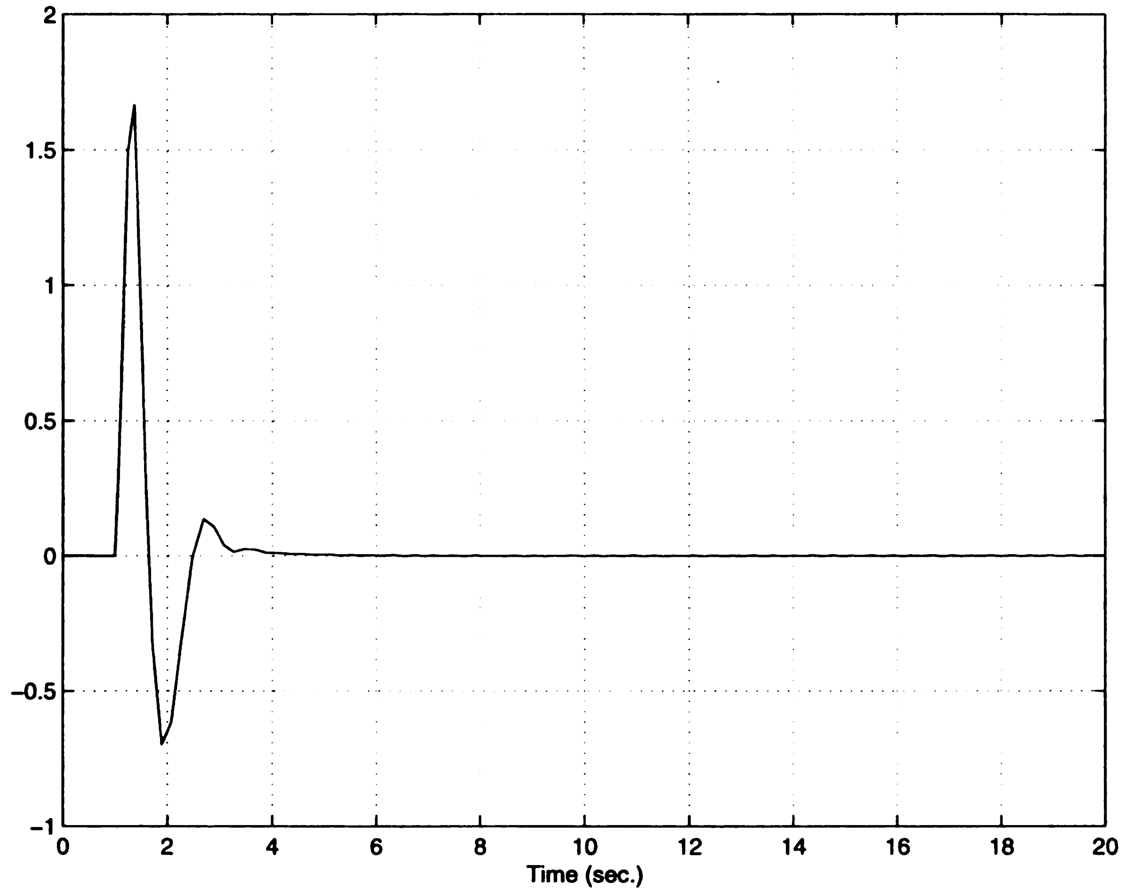

Figure 2.6. Tracking error e using MRAC controller.

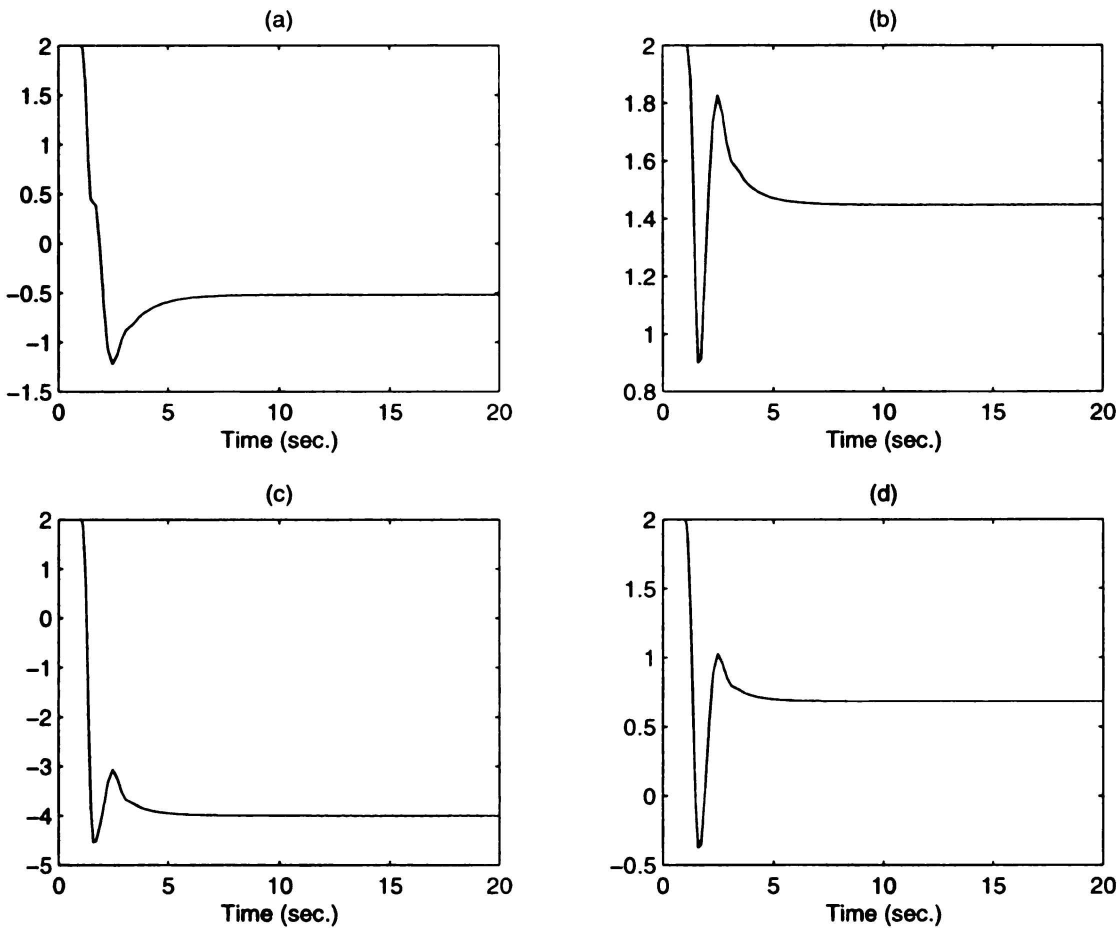

Figure 2.7. Parameter errors: (a)- $\tilde{\theta}_1$, (b)-$\tilde{\theta}_2$, (c)-$\tilde{\theta}_3$, (d)-$\tilde{\theta}_4$.

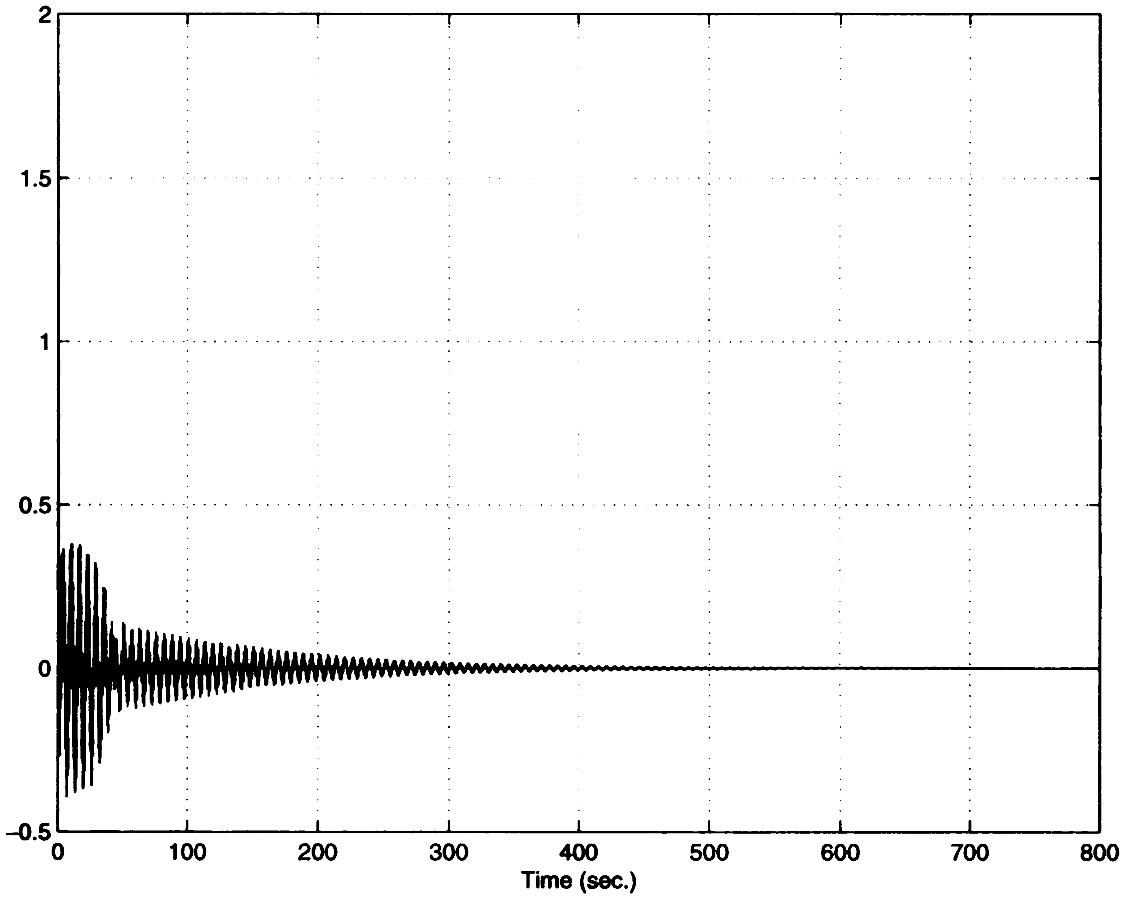

Figure 2.8. Tracking error e using MRAC controller.

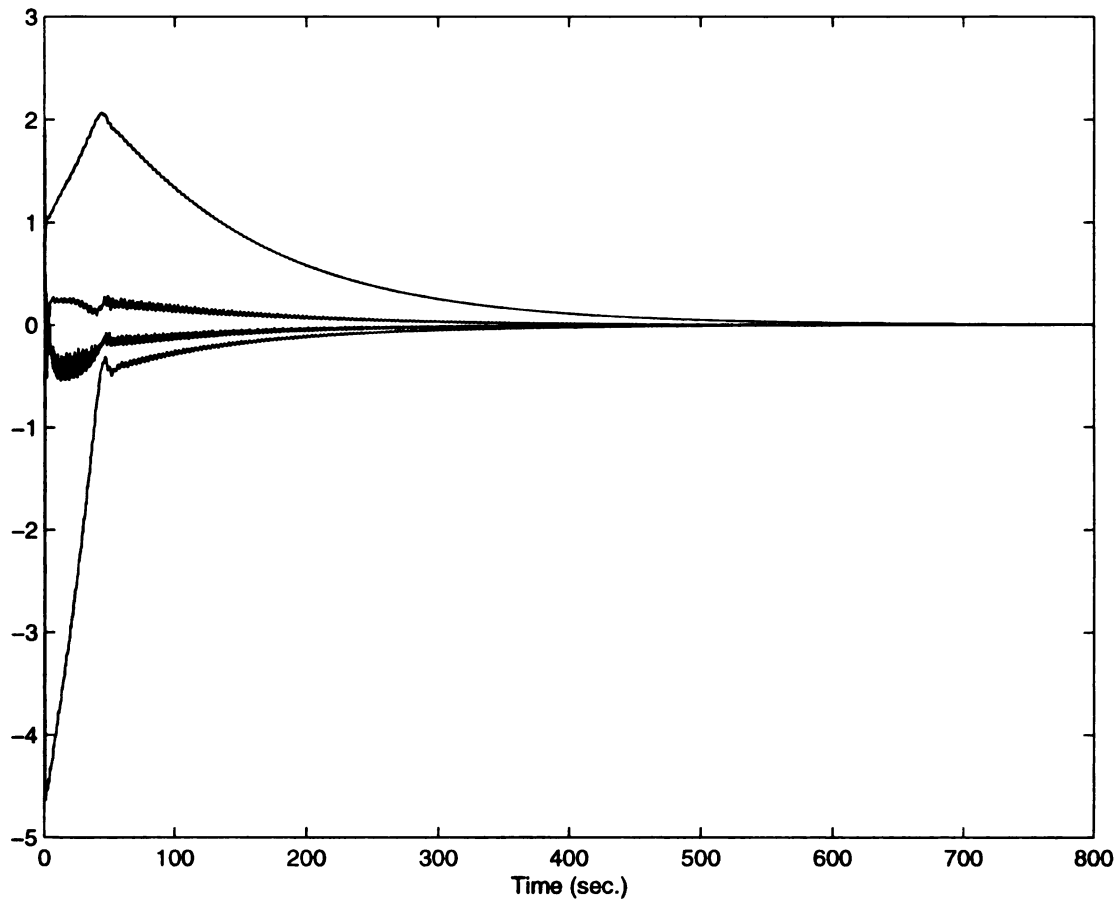

Figure 2.9. Parameter error $\tilde{\theta}_l$.

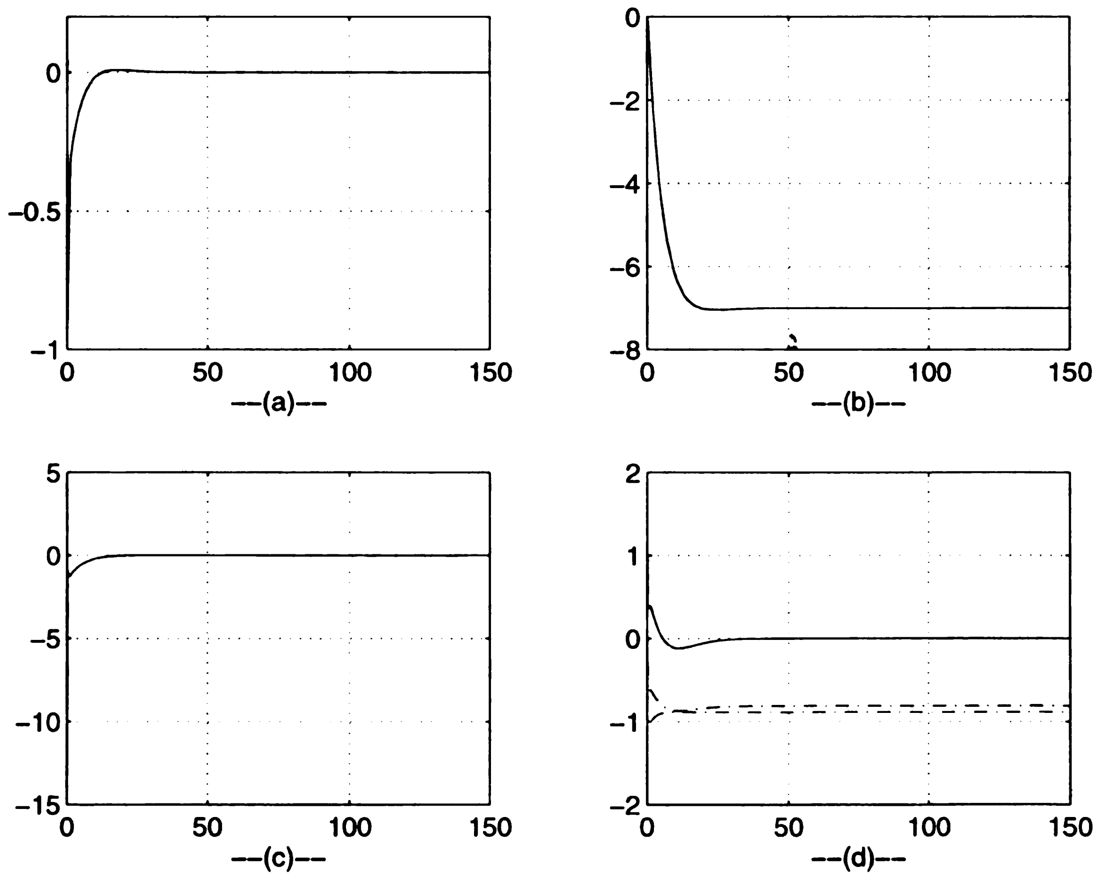

Figure 2.10. The x-axis is time. (a) Tracking error e; (b) Control u; (c) $\psi^s(\cdot)$; (d) Projected Parameter error: $\tilde{\theta}_1 + \tilde{\theta}_2$ (solid), $\tilde{\theta}_2$, $\tilde{\theta}_3$ (dotted).

CHAPTER 3

Robustness to Bounded Disturbance

3.1 Introduction

Robustness of adaptive controllers to bounded disturbance is of utmost importance for its practical use [14]. In the previous chapter we have achieved tracking using adaptive output feedback control. In this chapter, we study the robustness of that controller to bounded disturbance. We present two results in that direction. First, we present a robustness result in the usual form of robust adaptive control results [14]. We show that, for sufficiently small bounded disturbance, all signals in the closed-loop system will be bounded and the mean square tracking error will be of the order $O(\epsilon + d_1)$, where d_1 is an upper bound on the disturbance. Second, if the bound on the disturbances is large we go one step further to introduce a new control component to ensure that for any bounded disturbance, with a known upper bound, all signals in the closed-loop system will be bounded and the mean square tracking error will be of the order $O(\epsilon + \mu)$, where both ϵ and μ are design parameters. In the design we use a Lyapunov redesign technique, and we do not require the disturbance to be small.

3.2 Robustness Property

Our goal is to prove that the adaptive output feedback controller of Chapter 2 is robust with respect to small bounded disturbance. To simplify the presentation, we rely heavily on definitions, assumptions, and proofs from Chapter 2.

Consider a perturbation of (2.1), given by

$$y^{(n)} = f_0(\cdot) + \sum_{i=1}^{p} f_i(\cdot)\theta_i + [g_0(\cdot) + \sum_{i=1}^{p} g_i(\cdot)\theta_i]u^{(m)} + d(\cdot) \tag{3.1}$$

where $d(\cdot)$ is a disturbance term of the form

$$d(t, x, z, v, \theta) = d_f(t, x, z, \theta) + d_g(x, z, \theta)v$$

The error equation (2.6) becomes

$$\begin{aligned} \dot{e} &= A_m e + b\{Ke + f_0(\cdot) + \theta^T f(\cdot) + (g_0(\cdot) + \theta^T g(\cdot))v \\ &\quad + d(t, e + \mathcal{Y}_r, z, v, \theta) - y_r^{(n)}\} \end{aligned} \tag{3.2}$$

Assumption 3.1 *The disturbance $d(\cdot)$ satisfies*

$$\|d(t, e + \mathcal{Y}_r, z, \psi^s(\cdot), \theta)\| \leq d_1$$

$\forall\ t \geq 0$, $e \in E$, $\mathcal{Y}_r \in Y$, $z \in Z$, and $\xi \in R^n$.

Suppose further that for sufficiently small d_1 Assumptions 2.1 and 2.2 hold uniformly in d and the set Z has the property (2.17) for all d. Recall the set R_s defined by (2.24):

$$R_s = \{\{V \leq c_3\} \cap \{\hat{\theta} \in \Omega_\delta\}\} \times \{V_1 \leq c_5\} \times \{V_\xi \leq c_6\epsilon^2\}$$

with the same values for c_3, c_5, and c_6 as determined in Chapter 2. We show that the set R_s is positively invariant for sufficiently small ϵ and d_1. To show that, we conduct our analysis assuming all signals are inside the set. Later on, we show that the fast variable ξ enters the set in finite time. Hence, all variables will be trapped inside the positively invariant set R_s. Inside R_s, $(e - \hat{e})$ is $O(\epsilon)$, hence $\hat{e} \in E_1$. Since the saturation level was calculated by maximization over all $e \in E_1$, $z \in Z$, $\mathcal{Y}_\mathcal{R} \in Y_R$ and $\hat{\theta} \in \Omega_\delta$, the saturation function will not be effective inside R_s, i.e., $\psi^s{=}\psi$. Hence, inside R_s the output feedback controller of Chapter 2 is given by

$$v = \frac{-K\hat{e} + y_r^{(n)} - f_0(\hat{e} + \mathcal{Y}_r, z) - \hat{\theta}^T f(\hat{e} + \mathcal{Y}_r, z)}{g_0(\hat{e} + \mathcal{Y}_r, z) + \hat{\theta}^T g(\hat{e} + \mathcal{Y}_r, z)} \tag{3.3}$$

The error equation can be written as

$$\dot{e} = A_m e - b\tilde{\theta}^T \hat{w}(t) + \Lambda(\cdot) + bd(\cdot) \tag{3.4}$$

The derivative of

$$V = e^T P e + \tfrac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$$

along (3.4) satisfies

$$\dot{V} = -e^T Q e + 2e^T P b \Lambda(\cdot) + 2e^T bd(\cdot)$$

We use an argument similar to the one used in the previous chapter to show boundedness of the state variables. First, to show that R_s is positively invariant, for sufficiently small ϵ and d_1, we use the fact ξ is $O(\epsilon)$ to arrive at

$$\dot{V} \leq -e^T Q e + k\epsilon + k_d d_1 \tag{3.5}$$

where $k > 0$ is the same constant appearing (2.25) and $k_d > 0$. Furthermore,

$$
\begin{aligned}
\dot{V} &\leq -c_0 e^T P e + k\epsilon + k_d d_1 \\
&= -c_0 V + \tfrac{c_0}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + k\epsilon + k_d d_1 \\
&\leq -c_0 V + c_0 c_2 + k\epsilon + k_d d_1
\end{aligned}
$$

Therefore, on the boundary $V = c_3$, $\dot{V} < 0$ for all $c_3 > c_2 + \frac{k\epsilon + k_d d_1}{c_0}$. In the ideal case, when $d = 0$, $\dot{V} < 0$ on $V = c_3$ for all $\epsilon < \epsilon^* \leq c_0(c_3 - c_2)/k$. In the presence of the disturbance d, $\dot{V} < 0$ on $V = c_3$ for all $\epsilon < \epsilon^*$ and $d_1 < \bar{d}_1(\epsilon) = [(c_3 - c_2)c_0 - k\epsilon]/k_d$. Hence $\forall\ \epsilon < \epsilon^*$ and $d_1 < \bar{d}_1$, the set $\{V \leq c_3\} \cap \{\hat{\theta} \in \Omega_\delta\}$ is a positively invariant set. We can show, as in Chapter 2, that $\{V_1 \leq c_5\}$ is a positively invariant set. Finally, the derivative of V_ξ is given by

$$
\dot{V}_\xi = -\frac{1}{\epsilon}\xi^T \xi + 2\xi^T \bar{P} b \{f_0(\cdot) + \theta^T f(\cdot) + [g_0(\cdot) + \theta^T g(\cdot)]\psi^s(\cdot) + d(\cdot)\}
$$

Since all variable are bounded in R_s, and so is d, we obtain

$$
\dot{V}_\xi \leq -\frac{1}{2\epsilon}\xi^T \xi - \frac{1}{2\epsilon\lambda_{max}(\bar{P})} V_\xi + (k_1 + \bar{k}_d d_1)\sqrt{V_\xi}
$$

for some positive constant $\bar{k}_d$, independent of ϵ, and k_1 is the same constant appearing in (2.26). Choosing

$$
\hat{d}_1 = \frac{\sqrt{c_6} - 2k_1 \lambda_{max}(\bar{P})}{2\bar{k}_d \lambda_{max}(\bar{P})}
$$

it can be shown that for all $d < \hat{d}_1$ the set $\{V_\xi \leq c_6 \epsilon^2\}$ is positively invariant. This completes the proof that R_s is positively invariant for all $d < \min\{\bar{d}_1, \hat{d}_1\}$. The second part of the argument is to show boundedness of the signals under output feedback. For that we need to show that the fast variable ξ decays rapidly to $O(\epsilon)$. Since $V(e(0), \tilde{\theta}(0)) < c_3$ and ψ^s and d are bounded uniformly in ϵ, there exists a finite time T_2 independent of ϵ such that $\forall\ t \in [0, T_2]$, $z(t) \in Z$ *and* $V(e(t), \tilde{\theta}(t)) \leq c_3$. the time T_2 depends on d and equals T_1 when $d = 0$. Since the right–hand side depends continuously on d, for sufficiently small d_1 we can ensure that $T_2 \geq \frac{1}{2}T_1$. During the interval $[0, T_2]$ we have,

$$\dot{V}_\xi \leq -\frac{1}{2\epsilon}\|\xi\|^2, \quad \text{for } V_\xi \geq c_6\epsilon^2$$

Therefore

$$V_\xi(\xi(t)) \leq \frac{\beta_1}{\epsilon^{2(n-1)}}e^{-\beta_2 t/\epsilon}$$

where $\beta_1 = k_\xi^2\|\bar{P}\|$ and $\beta_2 = \frac{1}{2\|\bar{P}\|}$. From Chapter 2 we know that

$$T(\epsilon) \stackrel{\text{def}}{=} \frac{\epsilon}{\beta_2}\ln(\frac{\beta_1}{c_6\epsilon^{2n}}) \leq \frac{1}{2}T_1 \leq T_2$$

for all $0 < \epsilon \leq \epsilon^*$. Hence,

$$V_\xi(\xi(T)) \leq c_6\epsilon^2$$

Thus, the trajectories are guaranteed to enter the set R_s during the interval $[0, T(\epsilon)]$ and will remain inside thereafter. For all $t \geq T(\epsilon)$, the inequality

$$\dot{V} \leq -e^T Q e + k\epsilon + k_d d_1$$

is satisfied and since all signals are bounded we conclude that

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T e^T Q e \; dt \leq k\epsilon + k_d d_1$$

which shows that the mean square tracking error is of order $O(\epsilon + d_1)$.

In summary, we have shown that the adaptive output feedback controller of Chapter 2 is robust with respect to small bounded disturbance in the sense that for each $0 < \epsilon < \epsilon^*$, there is $d_1^* = d_1^*(\epsilon)$ such that for all $d(\cdot)$ satisfying $|d| \leq d_1 < d_1^*$, the trajecto

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jectories of the closed-loop system are bounded and the mean square tracking error is of order $O(\epsilon + d_1)$. It is important to note that ϵ^* is the same bound established in Chapter 2.

In Chapter 2 we saw that sharper results can be obtained in the ideal case, $d = 0$, when Assumption 2.3 is satisfied with either partial or full persistence of excitation. In the rest of this section we investigate the effect of persistence of excitation when $d \neq 0$.The closed-loop system is given by

$$\left.\begin{array}{rcl} \dot{e} & = & A_m e - b\tilde{\theta}^T \hat{w}(t) + \Lambda(\cdot) + d(\cdot) \\ \dot{\hat{\theta}} & = & \Gamma_p(\hat{\theta}, \phi) \\ \dot{\tilde{\zeta}} & = & F_2(\tilde{\zeta}, e, \mathcal{Y}_r, \bar{\zeta}, \theta) \\ \epsilon\dot{\xi} & = & (A - HC)\xi - \epsilon b[\tilde{\theta}^T \hat{w}(t) + Ke] + \epsilon\Lambda(\cdot) \end{array}\right\} \tag{3.6}$$

Suppose Assumption 2.3 is satisfied with partial persistence of excitation, decomposed as in (2.31). Then the derivatives of V and V_ξ satisfy

$$\dot{V} \leq -k_{v1}||e||^2 + k_{v2}||e||||\xi|| + k_{v3}||\xi||^2 + k_{v4}||\tilde{\zeta}||||\xi|| + k_{v5}||\xi||||\tilde{\theta}_1|| + k_{d1}d_1 \tag{3.7}$$

$$\dot{V}_\xi \leq \frac{1}{\epsilon}||\xi||^2 + \gamma_3||\tilde{\theta}_1||||\xi|| + \gamma_4||e||||\xi|| + \gamma_5||\tilde{\zeta}||||\xi|| + \gamma_6||\xi||^2 + k_{d2}d_1 \tag{3.8}$$

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}}_1 \end{bmatrix} = \begin{bmatrix} A_m & -b\mathcal{G}w_{r1}^T \\ 2\Gamma_1\mathcal{G}w_{r1}b^T P & 0 \end{bmatrix} \begin{bmatrix} e \\ \tilde{\theta}_1 \end{bmatrix} + \begin{bmatrix} \Lambda_s(\cdot) + bd(\cdot) \\ \Lambda_e(\cdot) \end{bmatrix} \tag{3.9}$$

As in Chapter 2, it can be shown that the system

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}}_1 \end{bmatrix} = \begin{bmatrix} A_m & -b\mathcal{G}w_{r1}^T \\ 2\Gamma_1\mathcal{G}w_{r1}b^T P & 0 \end{bmatrix} \begin{bmatrix} e \\ \tilde{\theta}_1 \end{bmatrix} \tag{3.10}$$

has an exponentially stable equilibrium point at the origin. Then, from the converse Lyapunov theorem, there exists a Lyapunov function $V_2(t, e, \tilde{\theta})$ whose derivative along

(3.9) satisfies

$$\begin{aligned}\dot{V}_2 \;\leq\;& -\delta_5\|e\|^2 - \delta_6\|\tilde{\theta}_1\|^2 + \delta_7\|e\|\|\xi\| + \delta_8\|\tilde{\theta}_1\|\|\xi\| + \delta_9\|e\|^2 \\ & + \delta_{10}\|e\|\|\tilde{\theta}_1\| + \delta_{11}\|e\|\|\tilde{\zeta}\| + \delta_{12}\|\tilde{\theta}_1\|\|\tilde{\zeta}\| + \delta_{d1}d_1\|e\| + \delta_{d2}d_1\|\tilde{\theta}_1\|\end{aligned}$$

or

$$\begin{aligned}\dot{V}_2 \;\leq\;& -\delta_5\|e\|^2 - \delta_6\|\tilde{\theta}_1\|^2 + \delta_7\|e\|\|\xi\| + \delta_8\|\tilde{\theta}_1\|\|\xi\| + \delta_9\|e\|^2 \\ & + \delta_{10}\|e\|\|\tilde{\theta}_1\| + \delta_{11}\|e\|\|\tilde{\zeta}\| + \delta_{12}\|\tilde{\theta}_1\|\|\tilde{\zeta}\| + \delta_d d_1 \qquad (3.11)\end{aligned}$$

From (2.10) together with (3.7), (3.8), and (3.11), it can be shown that the derivative of

$$W = \alpha V + \beta V_1 + V_2 + V_\xi$$

along the trajectories of the system, satisfies

$$\dot{W} \leq - \begin{bmatrix} \|e\| \\ \|\tilde{\theta}_1\| \\ \|\tilde{\zeta}\| \\ \|\xi\| \end{bmatrix}^T \tilde{M} \begin{bmatrix} \|e\| \\ \|\tilde{\theta}_1\| \\ \|\tilde{\zeta}\| \\ \|\xi\| \end{bmatrix} - \frac{\delta_5}{2}\|e\|^2 - \frac{\delta_6}{2}\|\tilde{\theta}_1\| + c_d d_1 \qquad (3.12)$$

where $\tilde{M}$ is given by

$$\tilde{M} = \begin{bmatrix} \alpha k_{v1} + \frac{\delta_5}{2} - \delta_9 & \frac{-\delta_{10}}{2} & \frac{-\beta\eta_4 - \delta_{11}}{2} & \frac{-\alpha k_{v2} - \delta_7 - \gamma_4}{2} \\ \frac{-\delta_{10}}{2} & \frac{\delta_6}{2} & \frac{-\delta_{12}}{2} & \frac{-\delta_8 - \gamma_3 - \alpha k_{v5}}{2} \\ \frac{-\beta\eta_4 - \delta_{11}}{2} & \frac{-\delta_{12}}{2} & \beta\eta_3 & \frac{-\gamma_5 - \alpha k_{v4}}{2} \\ \frac{-\alpha k_{v2} - \delta_7 - \gamma_4}{2} & \frac{-\delta_8 - \gamma_3 - \alpha k_{v5}}{2} & \frac{-\gamma_5 - \alpha k_{v4}}{2} & \frac{1}{\epsilon} - \gamma_6 - \alpha k_{v3} \end{bmatrix} \qquad (3.13)$$

$\tilde{M}$ is similar to M of Chapter 2, except for changes in the $(1,1)$ and $(2,2)$ elements. Choose β large enough to make

$$\begin{bmatrix} \frac{\delta_6}{2} & \frac{-\delta_{12}}{2} \\ \frac{-\delta_{12}}{2} & \beta\eta_3 \end{bmatrix}$$

positive definite; then choose α large enough that

$$\begin{bmatrix} \alpha k_{v1} + \frac{\delta_5}{2} - \delta_9 & \frac{-\delta_{10}}{2} & \frac{-\beta\eta_4 - \delta_{11}}{2} \\ \frac{-\delta_{10}}{2} & \frac{\delta_6}{2} & \frac{-\delta_{12}}{2} \\ \frac{-\beta\eta_4 - \delta_{11}}{2} & \frac{-\delta_{12}}{2} & \beta\eta_3 \end{bmatrix}$$

is positive definite. Finally, choosing ϵ small enough we can make $\tilde{M}$ positive definite for some $c_d > 0$. Since all signals are bounded, the mean-square tracking error

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T ||e||^2 \, dt$$

and the mean-square partial parameter error

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T ||\tilde{\theta}_1||^2 \, dt$$

are of order $O(d_1)$.

On the other hand, if w_r is persistently exciting, then the derivative of V satisfies

$$\dot{V} \leq -k_{v1}||e||^2 + k_{v2}||e||||\xi|| + k_{v3}||\xi||^2 + k_{v4}||\tilde{\zeta}||||\xi|| + k_{v5}||\xi||||\tilde{\theta}|| + k_{d1}d_1 \qquad (3.14)$$

and the system

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} A_m & -b\mathcal{G}w_r^T \\ 2\Gamma_1 \mathcal{G} w_r b^T P & 0 \end{bmatrix} \begin{bmatrix} e \\ \tilde{\theta} \end{bmatrix} \tag{3.15}$$

has an exponentially stable equilibrium point at the origin. We can repeat the preceding argument to show that there is a Lyapunov function $V_2(t, e, \tilde{\theta})$ whose derivative along (3.9) satisfies

$$\begin{aligned} \dot{V}_2 \leq\ & -\delta_5 \|e\|^2 - \delta_6 \|\tilde{\theta}\|^2 + \delta_7 \|e\| \|\xi\| + \delta_8 \|\tilde{\theta}\| \|\xi\| + \delta_9 \|e\|^2 \\ & + \delta_{10} \|e\| \|\tilde{\theta}\| + \delta_{11} \|e\| \|\tilde{\zeta}\| + \delta_{12} \|\tilde{\theta}\| \|\tilde{\zeta}\| + k_d d_1 \end{aligned} \tag{3.16}$$

Moreover,

$$\dot{V}_\xi \leq \frac{1}{\epsilon} \|\xi\|^2 + \gamma_3 \|\tilde{\theta}\| \|\xi\| + \gamma_4 \|e\| \|\xi\| + \gamma_5 \|\tilde{\zeta}\| \|\xi\| + \gamma_6 \|\xi\|^2 + k_{d2} d_1 \tag{3.17}$$

Inequality (2.10) together with (3.14), (3.16), and (3.17) show that the derivative of W along the trajectories of the system, satisfies

$$\dot{W} \leq - \begin{bmatrix} \|e\| \\ \|\tilde{\theta}\| \\ \|\tilde{\zeta}\| \\ \|\xi\| \end{bmatrix}^T \tilde{M} \begin{bmatrix} \|e\| \\ \|\tilde{\theta}\| \\ \|\tilde{\zeta}\| \\ \|\xi\| \end{bmatrix} - \frac{\delta_5}{2} \|e\|^2 - \frac{\delta_6}{2} \|\tilde{\theta}\| + c_d d_1 \tag{3.18}$$

where $\tilde{M}$ is given by (3.13) and can be made positive definite. Hence, $\dot{W}$ satisfies

$$\dot{W} \leq -k_w W + c_d d_1 \tag{3.19}$$

for some $k_w > 0$, which shows that all variables, including the parameter error $\tilde{\theta}$, converge to a ball centered at the origin, whose size is of the order of $O\left(\sqrt{d_1}\right)$. We note that the additional properties we have shown under Assumption 2.3 may require

ϵ^* to be less than the bound established in Chapter 2.

3.3 Robust Output Tracking

If the bound d_1 is not small enough, we introduce an additional robustifying control component to make the mean-square tracking error arbitrarily small, irrespective of the bound on the disturbance d, provided this bound is known. Once again, we consider the perturbed system (3.1) and assume that Assumptions 2.1 and 2.2 are satisfied uniformly in $d(\cdot)$. Moreover, we assume that the set Z has the property (2.17) in the presence of d. The control is taken as

$$\begin{aligned} v &= \psi^s(\hat{e}, z, \mathcal{Y}_R, \hat{\theta}) \\ &= S \text{ sat } \left(\tfrac{\psi(e,z,\mathcal{Y}_R,\hat{\theta})}{S}\right) \end{aligned}$$

where

$$\psi(\hat{e}, z, \mathcal{Y}_R, \hat{\theta}) = \frac{-K\hat{e} + y_r^{(n)} - f_0(\hat{e} + \mathcal{Y}_r, z) - \hat{\theta}^T f(\hat{e} + \mathcal{Y}_r, z) + v_1}{g_0(\hat{e} + \mathcal{Y}_r, z) + \hat{\theta}^T g(\hat{e} + \mathcal{Y}_r, z)}$$

and the robustifying control component v_1 is to be designed using the Lyapunov redesign technique, e.g., [26, Section 13.1]. The saturation level S is determined as in Section 2.3.1, except for a new constant c_3 to be determined. The constants c_4 and c_z used in calculating S are chosen in terms of the new c_3. consider the set

$$R_s = \{\{V \le c_3\} \cap \{\hat{\theta} \in \Omega_\delta\}\} \times \{V_1 \le c_5\} \times \{V_\xi \le c_6\epsilon^2\}$$

where the constants c_3, c_5, and c_6 are yet to be determined. We limit our analysis to this set to show that it is a positively invariant set. Inside R_s, the saturation will not be effective. Hence, the control is given by

$$v = \frac{-K\hat{e} + y_r^{(n)} - f_0(\hat{e} + \mathcal{Y}_r, z) - \hat{\theta}^T f(\hat{e} + \mathcal{Y}_r, z) + v_1}{g_0(\hat{e} + \mathcal{Y}_r, z) + \hat{\theta}^T g(\hat{e} + \mathcal{Y}_r, z)} \tag{3.20}$$

The error equation under (3.20) becomes

$$\dot{e} = A_m e - b\tilde{\theta}^T \hat{w}(t) + b[v_1 + d(\cdot)] + \Lambda(\cdot) \tag{3.21}$$

where $\hat{w} = \hat{f} + \hat{g}\psi$, and $\Lambda(\cdot)$ is defined in terms of the new ψ. The disturbance $d(\cdot)$ is required to satisfy the following assumption.

Assumption 3.2

$$\|d(t, x, z, v, \theta)\| \leq \rho(e, z) + k_v|v_1|, \quad 0 \leq k_v < 1$$

where ρ and k_v are known.

Take $\eta(e, z) \geq \rho(e, z)$ and define $\hat{s} = 2\hat{e}^T Pb$,

$$v_1(\hat{e}, z) = \begin{cases} -\frac{\eta(\hat{e},z)}{(1-k_v)} \cdot \frac{\hat{s}}{|\hat{s}|} & \text{for} \quad \eta(\hat{e}, z)|\hat{s}| \geq \mu \\ \\ -\frac{\eta^2(\hat{e},z)}{(1-k_v)} \cdot \frac{\hat{s}}{\mu} & \text{for} \quad \eta(\hat{e}, z)|\hat{s}| < \mu \end{cases} \tag{3.22}$$

As in the previous section, we start by showing that the set R_s is positively invariant. Then, we show that the trajectories enter R_s in finite time and remain inside thereafter. For the first part, consider the derivative of

$$V = e^T Pe + \tfrac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$$

along (3.20), which satisfies

$$
\begin{aligned}
\dot{V} &\leq -e^T Q e - 2\hat{e}^T P b \tilde{\theta}^T \hat{w} + \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} + 2\hat{e}^T P b [v_1 + d(\cdot)] \\
&\quad + 2e^T P b \Lambda(\cdot) + 2(e^T - \hat{e}^T) P b [v_1 + d(\cdot) - \tilde{\theta}^T \hat{w}]
\end{aligned}
$$

Using the adaptation law

$$\dot{\hat{\theta}} = \mathrm{Proj}(\hat{\theta}, \phi)$$

where

$$\phi(\hat{e}, z, \mathcal{Y}_R, \hat{\theta}) = 2\hat{e}^T P b [\hat{f} + \hat{g}\psi]$$

and defining

$$
\begin{aligned}
\Lambda_t(\cdot) &= 2(e^T - \hat{e}^T) P b [v_1 + d(\cdot) - \tilde{\theta}^T \hat{w}] \\
&\quad + |2\hat{e}^T P b| (\eta(e, z) - \eta(\hat{e}, z)) + 2e^T P b \Lambda(\cdot)
\end{aligned}
$$

the derivative of V satisfies

$$\dot{V} \leq -e^T Q e + \hat{s} v_1 + |\hat{s}| [\eta(\hat{e}, z) + k_v |v_1|] + \Lambda_t(\cdot)$$

Outside the boundary layer, i.e., $(\eta(\hat{e}, z)|\hat{s}|) \geq \mu$, the robustifying control is

$$v_1 = -\frac{\eta(\hat{e}, z)}{(1 - k_v)} \frac{\hat{s}}{|\hat{s}|}$$

and the derivative of V satisfies

$$
\begin{aligned}
\dot{V} &\leq -e^T Q e + [-\tfrac{\eta(\hat{e},z)}{1-k_v} \tfrac{|\hat{s}|^2}{|\hat{s}|} + \eta(\hat{e}, z)|\hat{s}| + k_v \tfrac{\eta(\hat{e},z)}{1-k_v} \tfrac{|\hat{s}|^2}{|\hat{s}|}] + \Lambda_t(\cdot) \\
&\leq -e^T Q e + \Lambda_t(\cdot) \\
&\leq -e^T Q e + k_c \epsilon
\end{aligned}
$$

where $\Lambda_t(\cdot) \leq k_c\epsilon$ inside the set R_s. Inside the boundary layer, i.e., $\eta(\hat{e}, z)|\hat{s}| < \mu$, the robustifying control is

$$v_1 = -\frac{\eta^2(\hat{e}, z)}{(1-k_v)}\frac{\hat{s}}{\mu}$$

Hence,

$$\begin{aligned} \dot{V} &\leq -e^T Q e + [-\tfrac{\eta^2(\hat{e},z)}{1-k_v}\tfrac{|\hat{s}|^2}{\mu} + |\hat{s}|\eta(\hat{e}, z) + k_v \tfrac{\eta^2(\hat{e},z)}{1-k_v}\tfrac{|\hat{s}|^2}{\mu}] + \Lambda_t(\cdot) \\ &\leq -e^T Q e + [-\eta^2 \tfrac{|\hat{s}|^2}{\mu} + \eta|\hat{s}|] + \Lambda_t(\cdot) \end{aligned}$$

Since $(-\eta^2\frac{|\hat{s}|^2}{\mu} + \eta|\hat{s}|) \leq \frac{\mu}{4}$,

$$\dot{V} \leq -e^T Q e + k_c\epsilon + \frac{\mu}{4} \tag{3.23}$$

Therefore, on the boundary $V = c_3$, $\dot{V} < 0$ for all $c_3 > c_2 + \frac{k_c\epsilon + (\mu/4)}{c_0}$. Choose $\epsilon^* > 0$ and $\mu^* > 0$ such that $c_3 > c_2 + \frac{k_c\epsilon^* + (\mu^*/4)}{c_0}$. Then, $\dot{V} < 0$ on $V = c_3$ for all $\epsilon < \epsilon^*$ and $\mu < \mu^*$. We can choose c_5 and c_6 large enough that the sets $\{V_1 \leq c_5\}$ and $\{V_\xi \leq c_6\epsilon^2\}$ become positively invariant. Hence, the set R_s is positively invariant. The second part is to show that the fast variable ξ decays rapidly to $O(\epsilon)$. This can be shown using an argument similar to the one used in Section 3.2. It can be shown that there exists $\tilde{\epsilon}$ and $T(\epsilon)$ such that for all $0 < \epsilon < \tilde{\epsilon}$, $V_\xi(\xi(T)) \leq c_6\epsilon^2$. Hence, the trajectories are guaranteed to enter the set R_s within the time interval $[0, T(\epsilon)]$ and remain inside thereafter. Hence, inequality (3.23) is satisfies for all $t \geq T(\epsilon)$. Therefore, the mean-square tracking error is of order $O(\mu + \epsilon)$ where the design parameters μ and ϵ can be made arbitrarily small.

If Assumption 2.3 is satisfied in the ideal case $d = 0$, inequalities similar to (3.12) and (3.19) can be shown when $d \neq 0$. The right-hand side of such inequalities will have a term proportional to the disturbance upper bound despite the presence of the robustifying control component. Thus, such analysis does not reveal an advantage for the robustifying control. The only advantage we can show is the fact that the mean square tracking error can be made of the order $O(\mu + \epsilon)$.

Finally, in the ideal case $d = 0$, the controller with the robustifying component recovers the tracking-error-convergence property of Section 2.4, provided Assumption 2.3 is satisfied. First, notice that the control component v_1 always satisfies

$$|v_1| \leq \frac{\eta^2}{\mu}|\hat{s}|$$

Hence

$$|v_1| \leq k_{e1}||e|| + k_{\xi 1}||\xi||$$

Therefore, the effect of v_1 can be seen on some terms of the bound on $\dot{V}$. In particular,

$$\begin{aligned} \dot{V} &\leq -k_{v1}||e||^2 + k_{v2}||e||||\xi|| + k_{v3}||\xi||^2 + k_{v4}||\tilde{\zeta}||||\xi|| + k_{v5}||\xi||||\tilde{\theta}_1|| + 2|e^T Pb||v_1| \\ &\leq -\tilde{k}_{v1}||e||^2 + \tilde{k}_{v2}||e||||\xi|| + k_{v3}||\xi||^2 + k_{v4}||\tilde{\zeta}||||\xi|| + k_{v5}||\xi||||\tilde{\theta}_1|| \end{aligned} \tag{3.24}$$

Similarly two terms of the bound on $\dot{V}_\xi$ will be affected.

$$\dot{V}_\xi \leq \frac{1}{\epsilon}||\xi||^2 + \gamma_3||\tilde{\theta}_1||||\xi|| + \tilde{\gamma}_4||e||||\xi|| + \gamma_5||\tilde{\zeta}||||\xi|| + \tilde{\gamma}_6||\xi||^2 \tag{3.25}$$

Since w_{r1} is persistently exciting, we can repeat the argument used in Chapter 2 to show that the homogeneous part of the system

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}}_1 \end{bmatrix} = \begin{bmatrix} A_m & -b\mathcal{G}w_{r1}^T \\ 2\Gamma_1 \mathcal{G} w_{r1} b^T P & 0 \end{bmatrix} \begin{bmatrix} e \\ \tilde{\theta}_1 \end{bmatrix} + \begin{bmatrix} \Lambda_s(\cdot) + bv_1 \\ \Lambda_e(\cdot) \end{bmatrix} \tag{3.26}$$

is exponentially stable. Therefore, from the converse Lyapunov theorem, there exists a Lyapunov function $V_2(t, e, \tilde{\theta})$ whose derivative along (3.26) satisfies

$$\begin{aligned} \dot{V}_2 \leq \; & -\delta_5 ||e||^2 - \delta_6 ||\tilde{\theta}_1||^2 + \tilde{\delta}_7 ||e|| ||\xi|| + \tilde{\delta}_8 ||\tilde{\theta}_1|| ||\xi|| + \tilde{\delta}_9 ||e||^2 \\ & + \tilde{\delta}_{10} ||e|| ||\tilde{\theta}_1|| + \delta_{11} ||e|| ||\tilde{\zeta}|| + \delta_{12} ||\tilde{\theta}_1|| ||\tilde{\zeta}|| \end{aligned} \tag{3.27}$$

Note that the effect of robustifying control component v_1 on (3.27) can be seen in the constants $\tilde{\delta}_7$, $\tilde{\delta}_8$, $\tilde{\delta}_9$, and $\tilde{\delta}_{10}$. From (2.10) together with (3.24), and (3.25), and (3.27), it can be shown that the derivative of W along the trajectories of the system satisfies

$$\dot{W} \leq - \begin{bmatrix} ||e|| \\ ||\tilde{\theta}_1|| \\ ||\tilde{\zeta}|| \\ ||\xi|| \end{bmatrix}^T \hat{M} \begin{bmatrix} ||e|| \\ ||\tilde{\theta}_1|| \\ ||\tilde{\zeta}|| \\ ||\xi|| \end{bmatrix} \tag{3.28}$$

where $\hat{M}$ is given by

$$\hat{M} = \begin{bmatrix} \alpha \tilde{k}_{v1} + \delta_5 - \tilde{\delta}_9 & \frac{-\tilde{\delta}_{10}}{2} & \frac{-\beta\eta_4 - \delta_{11}}{2} & \frac{-\alpha \tilde{k}_{v2} - \tilde{\delta}_7 - \tilde{\gamma}_4}{2} \\ \frac{-\tilde{\delta}_{10}}{2} & \delta_6 & \frac{-\delta_{12}}{2} & \frac{-\tilde{\delta}_8 - \gamma_3 - \alpha k_{v5}}{2} \\ \frac{-\beta\eta_4 - \delta_{11}}{2} & \frac{-\delta_{12}}{2} & \beta\eta_3 & \frac{-\gamma_5 - \alpha k_{v4}}{2} \\ \frac{-\alpha \tilde{k}_{v2} - \tilde{\delta}_7 - \tilde{\gamma}_4}{2} & \frac{-\tilde{\delta}_8 - \gamma_3 - \alpha k_{v5}}{2} & \frac{-\gamma_5 - \alpha k_{v4}}{2} & \frac{1}{\epsilon} - \tilde{\gamma}_6 - \alpha k_{v3} \end{bmatrix}$$

It can be shown, as before, that M can be made positive definite. Hence, by [26, Theorem 4.4], we conclude that

$$\begin{bmatrix} \|e\| \\ \|\tilde{\theta}_1\| \\ \|\tilde{\zeta}\| \\ \|\xi\| \end{bmatrix} \to 0, \quad \text{as } t \to \infty$$

3.4 Example: Nonlinear Plant

Consider the disturbed nonlinear system (2.47)

$$\ddot{y} = a_1 y + a_2(y + u\dot{y}^2) + b_1 \dot{u} + u + d(t)$$

The disturbance $d(t)$ is piecewise continuous and bounded. Figures 3.1 shows simulation results when $d = 0$ (solid), $d = sin(t)$ (dotted-dashed), and $d = 5sin(t)$ (dashed) ; again without robustifying control. Figures 3.2 shows results for $d = 5sin(t)$ when a robustifying control is used with $\eta = 0$ (solid) and $\eta = 5.2$, $\mu = 0.9$ (dotted-dashed), and $\mu = 0.3$ (dashed). Notice the reduction in the tracking error and the projected parameter error $\tilde{\theta}_1 + \tilde{\theta}_2$ as μ decreases. Finally, Figure 3.3 demonstrates tracking error convergence in the idea case $d(\cdot) = 0$ while the robustifying component ψ_r is used.

3.5 Conclusions

In this chapter we have shown that, for sufficiently small upper bound on the bounded disturbance, all signals in the closed-loop system are bounded and the mean square tracking error is of order $O(\epsilon + d_1)$. We have also shown that, for a large bound on the disturbance, we can design a robustifying control component such that all signals

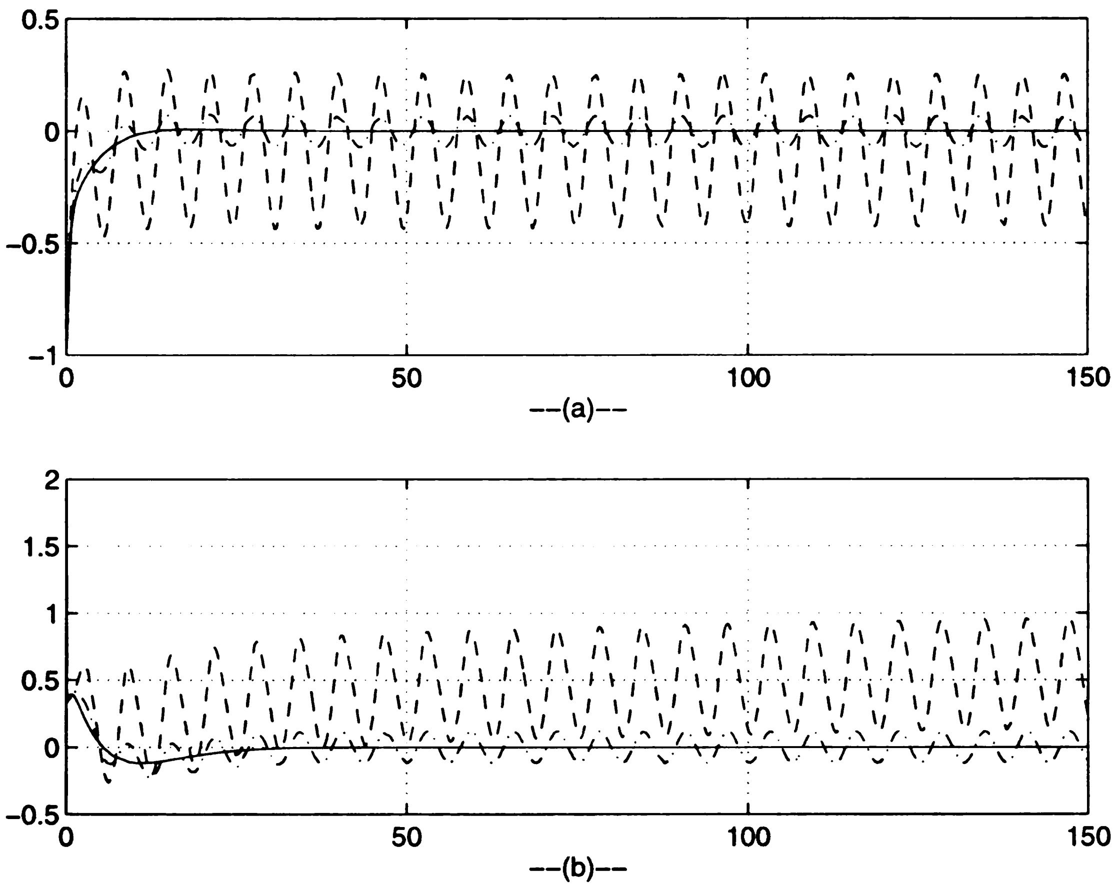


Figure 3.1. The case $d = 0$ (solid), $d = sin(t)$ (dotted-dashed), and $d = 5sin(t)$ (dashed) and no robustifying control. The x-axis is time. (a) Tracking error e; (b) Projected parameters error: $\tilde{\theta}_1 + \tilde{\theta}_2$

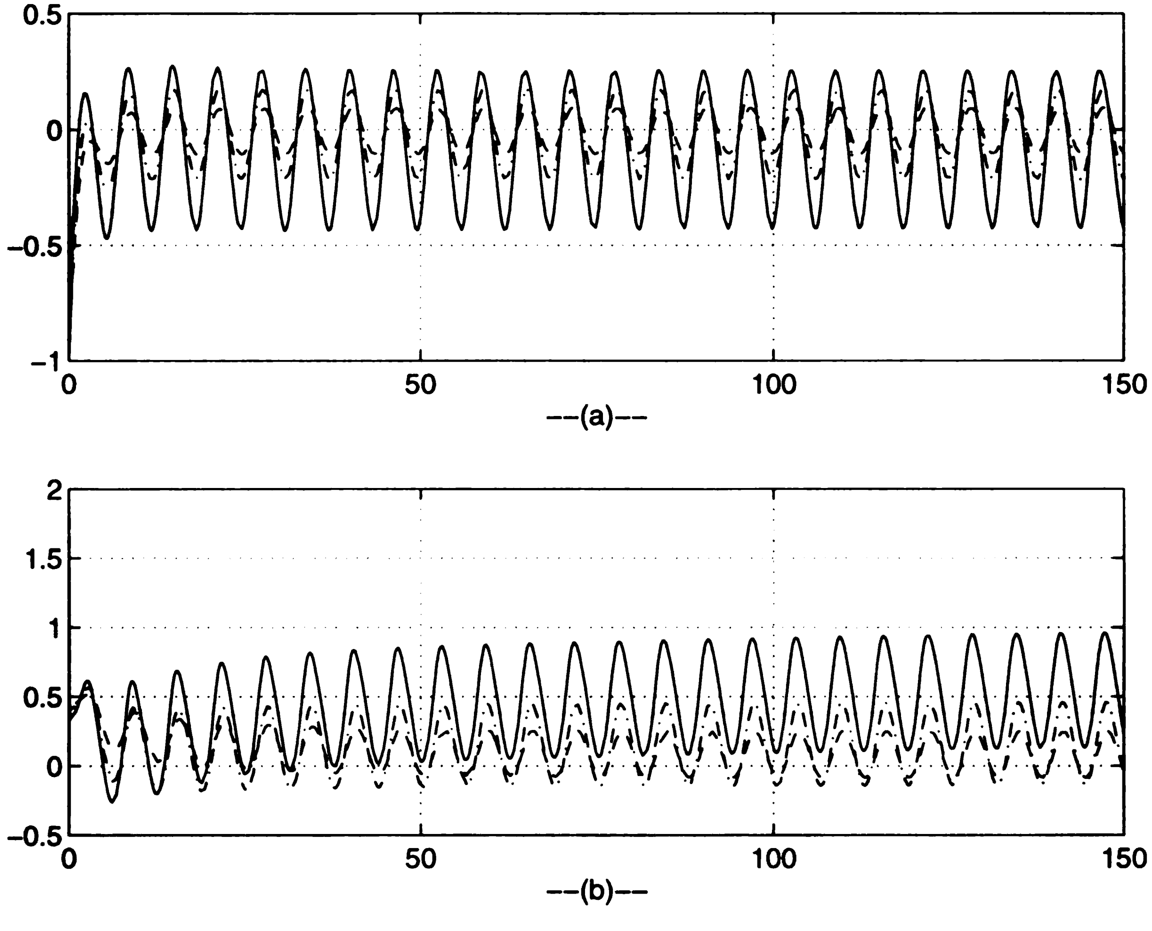


Figure 3.2. The case when $d = 5sin(t)$ when a robustifying control is used with $\eta = 0$ (solid) and $\eta = 5.2$, $\mu = 0.9$ (dotted-dashed), and $\mu = 0.3$ (dashed). The x-axis is time. (a) Tracking error e; (b) $\tilde{\theta}_1 + \tilde{\theta}_2$.

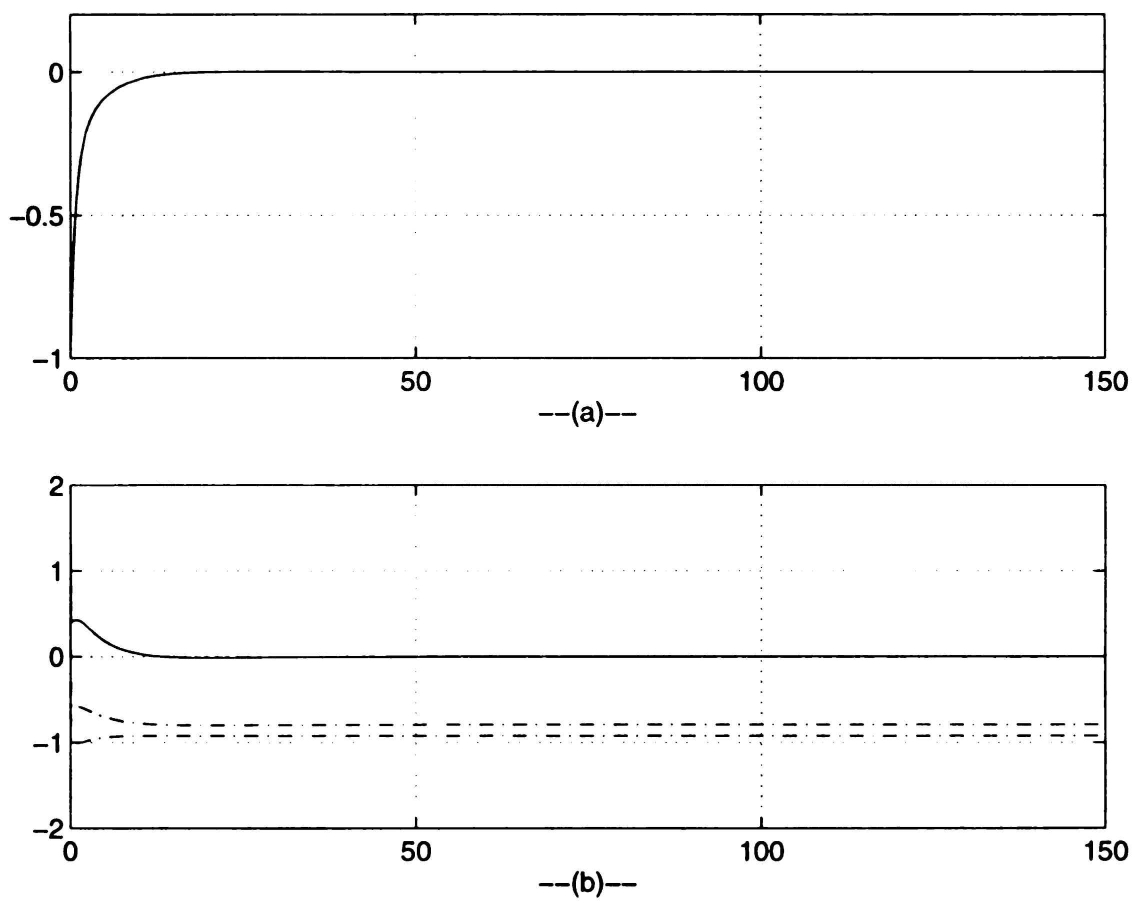


Figure 3.3. The case when $d(\cdot) = 0$, $\eta = 5.2$, and $\mu = 0.9$. The x-axis is time. (a) Tracking error e; (b) Projected parameters error: $\tilde{\theta}_1 + \tilde{\theta}_2$ (solid), $\tilde{\theta}_2$, $\tilde{\theta}_3$ (dotted)

in the closed-loop system are bounded and the mean square tracking error is of order $O(\epsilon + \mu)$, where both ϵ and μ are design parameters.

The robustness results of Sections 3.2 and 3.3 have potential application to adaptive control of nonlinear systems using neural networks or other nonlinear function approximators. Consider a system whose input-output model is of the form

$$y^{(n)} = F(\cdot) + G(\cdot)u^{(m)}$$

Using neural networks, the nonlinear functions $F(\cdot)$ and $G(\cdot)$ can be approximated, to any desired tolerance, by neural networks. In the special case of linear-in-the-weights neural networks, as in radial-basis-function networks, the functions F and G can be represented by

$$F(\cdot) = \sum_{i=1}^{p_1} h_i(\cdot)V_i + \delta_1(\cdot), \quad G(\cdot) = \sum_{i=1}^{p_2} h_i(\cdot)W_i + \delta_2(\cdot)$$

for some weights V_i and W_i. It follows that the system can be represented in the form (3.1) with $d = \delta_1 + \delta_2 u^{(m)}$. Therefore, the results of this chapter show that our adaptive controller can be used in this case. Moreover, the robust controller of Section 3.3 shows that we can trade off a larger approximation error with the use of the robustifying control component, leading to lower-order networks.

CHAPTER 4

Application to Induction Motors

4.1 Introduction

Nonlinear and adaptive control of induction motors is becoming more realizable recently with the advances in power electronics and fast digital signal processors. Khalil and Strangas [27] introduced a robust nonlinear control approach to the speed tracking problem in induction motors. It differs from the previous approaches in a number of key points. First, it does not use speed measurement. Motivated by the practical consideration that position measurement by optical encoders is much more reliable than the noisy speed measurement by tachometers, it uses position measurement. Second, it does not require rotor flux measurement. It adopts a novel idea of performing the field orientation change of variables using the flux estimate rather than the flux itself. This makes all the new variables available for feedback. Third, it allows uncertainty in the rotor resistance, the stator resistance, and the load torque. It uses robust control techniques to overcome the effect of this uncertainty on the tracking accuracy. The use of robust control is based on another change of variables that brings the acceleration as one of the state variables. This change of variables, which is dependent on the uncertain quantities, results in a state equation where the uncertain terms satisfy the matching condition. The controller is designed using continuous approximations of variable structure control. The uncertain change of variables is not used in the implementation of the controller, as both the speed and acceleration are estimated from the position using a robust high-gain observer [11]. It is shown in [27] that the speed tracking error will be asymptotically bounded by a bound that can be made arbitrarily small by choice of certain design parameters. In [35], an adaptive observer for induction motors with unknown rotor resistance was introduced. It is based on rotor speed and stator current measurements. The adaptation is with respect to the rotor resistance. The design is a Lyapunov based design. It was shown that the states of the adaptive observer are bounded and if, in addition, a persistence of excitation condition is satisfied, then all error signals tend exponentially to zero.

In this Chapter we carry the controller of [27] one step further by adapting the rotor resistance on line. For the on-line adaptation we use the adaptive observer of [35]. We prove that the robust controller retains the properties shown in [27] for any bounded time-varying estimate of the rotor resistance. The boundedness of the rotor resistance is guaranteed by using parameter projection. The closed–loop analysis is given in Section 4.5. The experimental results, given in Section 4.7, are in good agreement with the theory.

It should be noted that [36] has a similar adaptive speed control scheme. It uses an adaptive observer to estimate the load torque, rotor flux and rotor resistance under the assumption that the rotor speed and stator current are measured. Asymptotic convergence of the load torque and rotor resistance errors is shown under a persistence of excitation condition. The speed control is designed assuming measurement of rotor flux. However, in the simulation, the rotor flux is replaced by its estimate. The closed–loop system is not analyzed.

4.2 Induction Motor Model

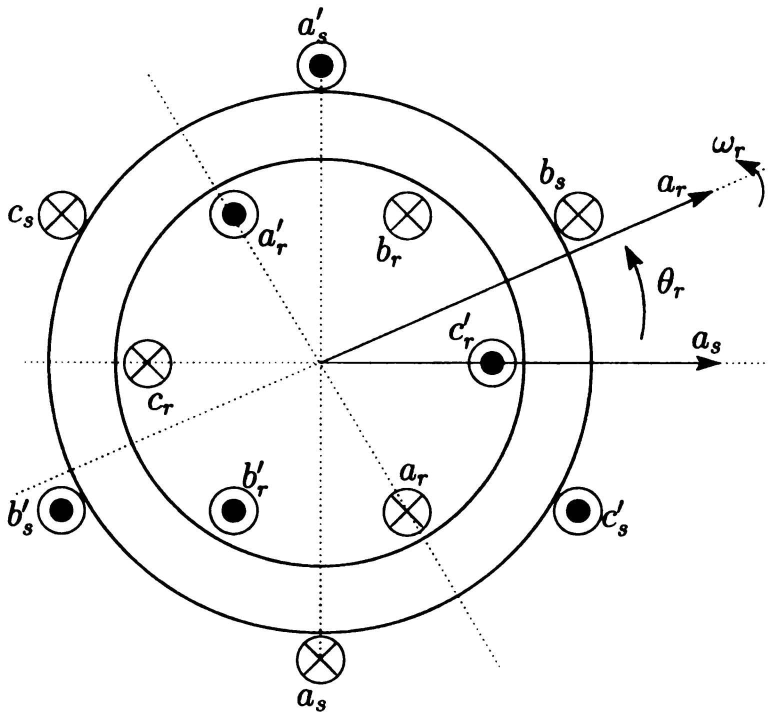


Figure 4.1. Three phase winding of induction motor

An induction motor consists of three stator and three rotor winding, as illustrated in Figure 4.1 and Figure 4.2, where R is resistance, L inductance and the subscripts s and r denote stator and rotor quantities respectively. This three phase representation can be transformed into two phase equivalent representation [28] using the transformation matrices

$$K_s = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

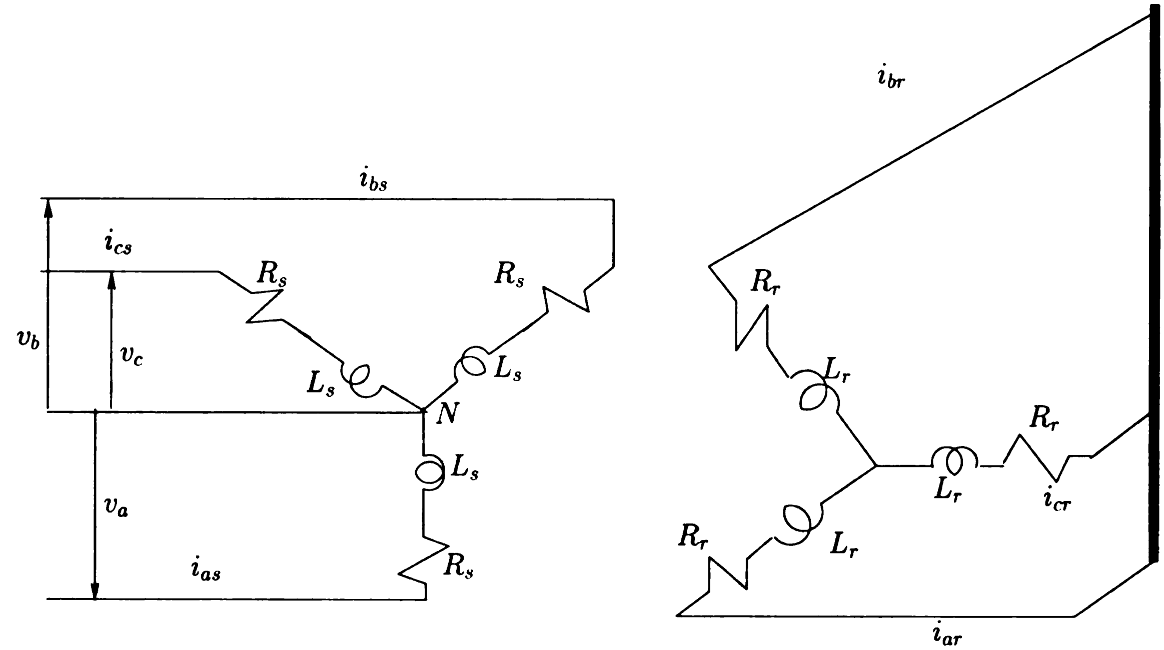


Figure 4.2. Three phase equivalent circuit of induction motor

and

$$K_r = \begin{bmatrix} \cos\theta & \cos\theta_1 & \cos\theta_2 \\ -\sin\theta & -\sin\theta_1 & -\sin\theta_2 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

where θ is the rotor flux angle with respect to the rotor, $\theta_1 = \theta + \frac{2\pi}{3}$ and $\theta_2 = \theta - \frac{2\pi}{3}$. The transformation matrix K_s transforms the three phase stator equations into equivalent two phase equations, and K_r transforms the three phase rotor equations into equivalent two phase equations, in the rotor frame of reference. Hence, the dynamics of the induction motor in the two phase representation is given by

$$
\begin{aligned}
R_s i_{sa} + \frac{d\psi_{sa}}{dt} &= v_{sa} \\
R_s i_{sb} + \frac{d\psi_{sb}}{dt} &= v_{sb} \\
R_r i'_{ra} + \frac{d\psi'_{ra}}{dt} &= 0 \\
R_r i'_{rb} + \frac{d\psi'_{rb}}{dt} &= 0
\end{aligned}
$$

where i is the current, ψ is the flux linkage, R_s is the stator resistance and R_r is the rotor resistance. The subscripts a and b refer to the two orthogonal axes of the new two phase representation. Note that the rotor equations are in the rotor frame of reference. The voltages v_{ra} and v_{rb} equal zero since the rotor's winding are short circuited.

Let $\frac{d\delta}{dt} = p\omega$, and $\delta(0) = 0$. where p is the number of pair of poles, and δ and ω are the angle and speed of the rotor, respectively. The rotor equations can be transformed into the stator frame of reference using the transformation

$$
F_s = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} F_r
$$

where F_s and F_r are 2×1 vectors representing quantity in the stator and rotor frames of reference, respectively. Hence the motor equations can be written in the stator frame of reference as

$$
\begin{aligned}
R_s i_{sa} + \frac{d\psi_{sa}}{dt} &= v_a \\
R_s i_{sb} + \frac{d\psi_{sb}}{dt} &= v_b \\
R_r i_{ra} + \frac{d\psi_{ra}}{dt} + p\omega\psi_{rb} &= 0 \\
R_r i_{rb} + \frac{d\psi_{rb}}{dt} - p\omega\psi_{ra} &= 0
\end{aligned}
$$

Under the assumption that the magnetic circuits are linear and the iron loss is zero, we replace the stator flux and rotor current by stator current and rotor flux using

$$\begin{aligned}
\psi_{sa} &= L_s i_{sa} + M i_{ra} \\
\psi_{sb} &= L_s i_{sb} + M i_{rb} \\
\psi_{ra} &= M i_{sa} + L_r i_{ra} \\
\psi_{rb} &= M i_{sb} + L_r i_{rb}
\end{aligned}$$

where M is mutual inductance, and L_s and L_r are stator and rotor total inductances, respectively. Combining the resulting equations with the mechanical equation, the induction motor is represented by the fifth order differential equation model [33]

$$\begin{aligned}
\dot{\theta} &= \omega && (4.1) \\
\dot{\omega} &= -\mu\lambda_r^T J i_s - T_L/m && (4.2) \\
\dot{\lambda}_r &= (-\frac{R_r}{L_r}I + p\omega J)\lambda_r + \frac{R_r}{L_r}M i_s && (4.3) \\
\dot{i}_s &= (\beta\frac{R_r}{L_r}I - \beta p\omega J)\lambda_r - (\alpha_s\eta + \frac{R_r}{L_r}\beta M)i_s + \gamma v_s && (4.4)
\end{aligned}$$

where $\lambda_r = [\psi_a, \psi_b]^T$, $i_s = [i_a, i_b]^T$, $v_s = [v_a, v_b]^T$,

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

The variables θ and ω denote the angular position and speed of the rotor, λ_r denotes the rotor flux in the stator frame of reference, and i_s and v_s denote the stator current and voltage. The constants α_s, β, γ, η and μ are defined by $\alpha_s = R_s/L_s$, $\beta = M/\sigma L_s L_r$, $\gamma = 1/\sigma L_s$, $\eta = 1/\sigma$, and $\mu = pM/mL_r$, where $\sigma = 1 - M^2/L_s L_r$ and m is the rotor's moment of inertia. The resistances R_s and R_r will be treated as uncertain parameters with R_{so} as the nominal value for R_s and $\hat{R}_r$ as a time-varying bounded estimate of R_r where we assume that $\hat{R}_r \in \Omega_r$, a compact interval. Let

$\alpha_{so} = R_{so}/L_s$, $\delta_1 = (R_r - \hat{R}_r)/\hat{R}_r$, and $\delta_2 = (R_s - R_{so})/R_{so}$. The load torque T_L will be treated as a bounded time-varying disturbance with bounded derivative.

Problem Statement

It is desired to design a feedback controller that solves the speed tracking problem $\omega(t) - \omega_{ref}(t) \to 0$ as $t \to \infty$ in the presence of the disturbance T_L and the uncertain parameters R_s and R_r, where the reference speed $\omega_{ref}(t)$ and its derivatives $\dot{\omega}_{ref}(t)$ and $\ddot{\omega}_{ref}(t)$ are bounded functions of t. The controller can only use feedback from θ and i_s.

Flux Observer

We use an open-loop observer [49] to estimate λ_r.

$$\dot{\hat{\lambda}}_r = (-\frac{\hat{R}_r}{L_r}I + p\omega J)\hat{\lambda}_r + \frac{\hat{R}_r}{L_r}Mi_s \tag{4.5}$$

The estimation error $e_s = \hat{\lambda}_r - \lambda_r$ satisfies the equation

$$\dot{e}_s = (-\frac{R_r}{L_r}I + p\omega J)e_s - \delta_1\frac{\hat{R}_r}{L_r}(Mi_s - \hat{\lambda}_r) \tag{4.6}$$

The origin of $\dot{e}_s = (-\frac{R_r}{L_r}I + p\omega J)e_s$ is exponentially stable. This property ensures that, as long as i_s and $\hat{\lambda}_r$ are bounded, the estimation error $e_s(t)$ will have an ultimate bound of order $O(\delta_1)$, i.e., the steady-state error in $e_s(t)$ will be $O(\delta_1)$.

Augmented System

We augment the observer equation (4.5) with the motor equations (4.1)–(4.4) to obtain an eighth-order model with $(\theta, \omega, \psi_a, \psi_b, i_a, i_b, \hat{\psi}_a, \hat{\psi}_b)$ as the state variables, where $\hat{\lambda}_r = [\hat{\psi}_a,\ \hat{\psi}_b]^T$. We perform a change of variables to bring the equations into coordinates that will be easier to work with. First, we replace ψ_a and ψ_b with the flux estimation errors $e_a = \hat{\psi}_a - \psi_a$ and $e_b = \hat{\psi}_b - \psi_b$. Next, we replace $(\hat{\psi}_a, \hat{\psi}_b, i_a, i_b, e_a, e_b, v_a, v_b)$ by $(\hat{\psi}_d, \hat{\rho}, \hat{i}_d, \hat{i}_q, e_d, e_q, \hat{v}_d, \hat{v}_q)$ where

$$\hat{\psi}_d^2 = \hat{\psi}_a^2 + \hat{\psi}_b^2, \quad \hat{\rho} = \tan^{-1}(\hat{\psi}_b/\hat{\psi}_a) \tag{4.7}$$

$$\hat{i}_d = i_a \cos\hat{\rho} + i_b \sin\hat{\rho}, \quad \hat{i}_q = -i_a \sin\hat{\rho} + i_b \cos\hat{\rho} \tag{4.8}$$

$$e_d = e_a \cos\hat{\rho} + e_b \sin\hat{\rho}, \quad e_q = -e_a \sin\hat{\rho} + e_b \cos\hat{\rho} \tag{4.9}$$

$$\hat{v}_d = v_a \cos\hat{\rho} + v_b \sin\hat{\rho}, \quad \hat{v}_q = -v_a \sin\hat{\rho} + v_b \cos\hat{\rho} \tag{4.10}$$

This change of variables resembles the one used in the traditional field orientation control, except that the new variables are defined in terms of the flux estimates $\hat{\psi}_a$ and $\hat{\psi}_b$ instead of the actual flux components ψ_a and ψ_b. Consequently, the new variables $\hat{\psi}_d$, $\hat{\rho}$, $\hat{i}_d$, $\hat{i}_q$, $\hat{v}_d$, and $\hat{v}_q$ can be calculated in terms of signals which are available on line. The variables $\hat{\psi}_d$, $\hat{\rho}$, $\hat{i}_d$, $\hat{i}_q$, e_d, and e_q satisfy

$$\begin{aligned}
\dot{\hat{\psi}}_d &= -\frac{\hat{R}_r}{L_r}\hat{\psi}_d + \frac{\hat{R}_r}{L_r}M\hat{i}_d && (4.11)\\
\dot{\hat{\rho}} &= p\omega + \frac{\hat{R}_r}{L_r}M\hat{i}_q/\hat{\psi}_d && (4.12)\\
\dot{\hat{i}}_d &= \frac{\hat{R}_r}{L_r}\beta\hat{\psi}_d - (\alpha_{so}\eta + \frac{\hat{R}_r}{L_r}\beta M)\hat{i}_d + p\omega\hat{i}_q \\
&\quad + \frac{\hat{R}_r}{L_r}M\hat{i}_q^2/\hat{\psi}_d + \gamma\hat{v}_d + f_1 + \delta_1 g_1 + \delta_2 g_2 && (4.13)\\
\dot{\hat{i}}_q &= -\beta p\omega\hat{\psi}_d - p\omega\hat{i}_d - (\alpha_{so}\eta + \frac{\hat{R}_r}{L_r}\beta M)\hat{i}_q \\
&\quad - \frac{\hat{R}_r}{L_r}M\hat{i}_d\hat{i}_q/\hat{\psi}_d + \gamma\hat{v}_q + f_2 + \delta_1 g_3 + \delta_2 g_4 && (4.14)\\
\dot{e}_d &= -\frac{R_r}{L_r}e_d + \frac{\hat{R}_r}{L_r}M\hat{i}_q e_q/\hat{\psi}_d + \delta_1\frac{\hat{R}_r}{L_r}(\hat{\psi}_d - M\hat{i}_d) && (4.15)\\
\dot{e}_q &= -\frac{R_r}{L_r}e_q - \frac{\hat{R}_r}{L_r}M\hat{i}_q e_d/\hat{\psi}_d - \delta_1\frac{\hat{R}_r}{L_r}M\hat{i}_q && (4.16)
\end{aligned}$$

where

$$
\begin{aligned}
f_1(\cdot) &= -\beta\tfrac{R_r}{L_r}e_d - p\beta\omega e_q \\
g_1(\cdot) &= \beta\tfrac{\hat{R}_r}{L_r}(\hat{\psi}_d - M\hat{i}_d) \\
g_2(\cdot) &= -R_{s0}\gamma\hat{i}_d \\
f_2(\cdot) &= -\beta\tfrac{R_r}{L_r}e_q + p\beta\omega e_d \\
g_3(\cdot) &= -\tfrac{\beta M\hat{R}_r}{L_r}\hat{i}_q \\
g_4(\cdot) &= -R_{s0}\gamma\hat{i}_q
\end{aligned}
$$

To tackle the speed tracking problem, we introduce the state variables

$$x_1 = \theta - \int \omega_{ref}, \quad x_2 = \omega - \omega_{ref}, \quad x_3 = \dot{\omega} - \dot{\omega}_{ref}$$

to replace θ, ω, and $\hat{i}_q$, respectively. Noting that

$$\dot{\omega} = \mu\hat{i}_d e_q + \mu\hat{i}_q(\hat{\psi}_d - e_d) - T_L/m$$

we can see that the change of variables from $(\theta, \omega, \hat{i}_q)$ to (x_1, x_2, x_3) is invertible provided $(\hat{\psi}_d - e_d) \neq 0$. The conditions $(\hat{\psi}_d - e_d) \neq 0$ and $\hat{\psi}_d \neq 0$ can be ensured by allowing some delay between the time the motor and the flux observer are switched on, and the time the load and speed reference are applied. This will allow the flux to build up so that when the control law becomes effective the system will be far from singularity. For the flux tracking problem, we introduce the state variables

$$z_1 = \hat{\psi}_d - \psi_{ref}, \quad z_2 = \dot{z}_1 = -\frac{\hat{R}_r}{L_r}\hat{\psi}_d + \frac{\hat{R}_r}{L_r}M\hat{i}_d - \dot{\psi}_{ref}$$

where the flux reference $\psi_{ref}(t)$ and its derivative $\dot{\psi}_{ref}(t)$ are bounded function of t.

4.3 Controller Design

Let $x = [x_1, x_2, x_3]^T$, $z = [z_1, z_2]^T$, $e = [e_d, e_q]^T$, $\mathcal{X} = [x, z, e]^T$, and rewrite the augmented system as

$$\dot{x} = A_1 x + B_1[F_1 + G_1 \hat{v}_q] \quad (4.17)$$

$$\dot{z} = A_2 z + B_2[F_2 + G_2 \hat{v}_d] \quad (4.18)$$

$$\dot{e} = A_3 e + \delta_1 \frac{\hat{R}_r}{L_r} \mathcal{G}_1 \quad (4.19)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{aligned} F_1 = \; & \mu \tfrac{R_r}{L_r}(-\hat{i}_q \hat{\psi}_d - e_q \hat{i}_d + e_d \hat{i}_q) - \mu p \omega (\hat{i}_d \hat{\psi}_d - e_d \hat{i}_d - e_q \hat{i}_q) \\ & - \mu p \beta \omega (\hat{\psi}_d^2 + e_d^2 + e_q^2 - 2 e_d \hat{\psi}_d) + \mu (R_s \gamma + \tfrac{R_r}{L_r} \beta M)(-\hat{i}_q \hat{\psi}_d - e_q \hat{i}_d + e_d \hat{i}_q) \\ & + \mu \gamma e_q \hat{v}_d - \dot{T}_L / m - \ddot{\omega}_{ref} \end{aligned}$$

$$\begin{aligned} F_2 = \; & (\hat{\psi}_d - M \hat{i}_d)[\beta M \tfrac{\hat{R}_r}{L_r} \tfrac{R_r}{L_r} + (\tfrac{\hat{R}_r}{L_r})^2 - \tfrac{\dot{\hat{R}}_r}{L_r}] - \ddot{\psi}_{ref} + \tfrac{\hat{R}_r M}{L_r} p \omega \hat{i}_q + (\tfrac{\hat{R}_r}{L_r})^2 M^2 \hat{i}_q^2 / \hat{\psi}_d \\ & - \tfrac{\hat{R}_r}{L_r} R_s \gamma \eta M \hat{i}_d - \tfrac{M \hat{R}_r}{L_r} (\tfrac{\beta R_r}{L_r} e_d + p \beta \omega e_q) \end{aligned}$$

$$G_1 = \mu \gamma (\hat{\psi}_d - e_d), \; G_2 = \frac{\hat{R}_r}{L_r} M \gamma$$

$$A_3 = -\frac{R_r}{L_r} I - \left(\frac{\hat{R}_r}{L_r \hat{\psi}_d} M \hat{i}_q \right) J, \; \mathcal{G}_1 = \begin{bmatrix} (\hat{\psi}_d - M \hat{i}_d) \\ -M \hat{i}_q \end{bmatrix}$$

The third-order channel from the $\hat{v}_q$ to x_1 is feedback linearizable, the term F_1 satisfies the matching condition, and the control coefficient $G_1 = \mu \gamma (\hat{\psi}_d - e_d)$ is positive whenever $\hat{\psi}_d - e_d$ is positive. Hence, for any bounded F_1, it is possible to design a robust feedback control function of $(x, z, \hat{i}_q)$ that brings $\|x\|$ arbitrarily close to zero. This is a typical task in nonlinear robust control theory [26]. Similarly, the second-order channel from $\hat{v}_d$ to z_1 is feedback linearizable and the term F_2 satisfies the matching condition. So, for any bounded F_2, we can design a robust feedback control function of $(\omega, z, \hat{i}_q)$ that brings $\|z\|$ arbitrarily close to zero.

We use sliding mode control. The sliding surfaces for (4.18) and (4.17) are taken as $s_1 = 0$ and $s_2 = 0$, where

$$s_1 = a_1 z_1 + z_2, \quad s_2 = a_2 x_1 + a_3 x_2 + x_3 \tag{4.20}$$

for some positive constants a_1 to a_3. Let D be a compact subset of R^8, that contains the origin, with the property that $|\hat{\psi}_d|$ and $|\hat{\psi}_d - e_d|$ are bounded away from zero for all $\mathcal{X} \in D$. We design the control as

$$\hat{v}_d = -k_1 \text{ sat}\left(\frac{s_1}{\mu_1}\right) \tag{4.21}$$

$$\hat{v}_q = -k_2 \text{ sat}\left(\frac{s_2}{\mu_2}\right) \tag{4.22}$$

where μ_1 and μ_2 are small positive constants, and k_1 and k_2 are chosen such that

$$k_1 \geq \frac{k_3 + |a_1 z_2 + F_2|}{G_2} \tag{4.23}$$

$$k_2 \geq \frac{k_4 + |a_2 x_2 + a_3 x_3 + F_1|}{G_1} \tag{4.24}$$

for all $\mathcal{X} \in D$, $\hat{R}_r \in \Omega_r$ and $|\dot{\hat{R}}_r| \leq k_{r1}$ (some constant), for some arbitrary $k_3 > 0$ and $k_4 > 0$. With this control, it can be verified that

$$\Omega = \Omega_1 \times \Omega_2 \times \Omega_3 \times \Omega_4 \times \Omega_5 \subset D$$

is a positively invariant set,[1] where $\Omega_1 = \{x_s^T P_s x_s \leq 4c_2^2 \|P_s\|^3\}$, $\Omega_2 = \{|s_2| \leq c_2\}$, $\Omega_3 = \{|z_1| \leq c_1/a_1\}$, $\Omega_4 = \{|s_1| \leq c_1\}$, $\Omega_5 = \{e^T e \leq c_3\}$, $x_s = [x_1,\ x_2]^T$, and P_s is the positive definite solution of the Lyapunov equation $P_s A_s + A_s^T P_s = -I$ with

$$A_s = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_3 \end{bmatrix}$$

The positive constants c_1 to c_3 are chosen such that $c_1 \geq \mu_1$, $c_2 \geq \mu_2$, $\sqrt{c_3} > k_6 k_7$, and $\Omega \subset D$, where k_6 and k_7 are nonnegative constants such that $\|\mathcal{G}_1\| \leq k_6$ for all $\mathcal{X} \in D$ and k_7 is an upper bound on $|\hat{R}_r - R_r|/R_r$ for all $\hat{R}_r \in \Omega_r$. Moreover, all trajectories starting in Ω will reach the boundary layers $\{|s_1| \leq \mu_1\}$ and $\{|s_2| \leq \mu_2\}$ in finite time and remain inside thereafter. Consequently, $\mathcal{X}$ reaches a residual set Ω_0 where the speed tracking error x_2 is $O(\mu_2)$. Thus, the error can be made arbitrarily small by choosing μ_2 small enough. The flux observer (4.5) and the control law (4.22) require the signals ω and $\dot{\omega}$ which are not measured. The last step in the controller design is to estimate these signals from the measured rotor position θ. This estimation problem can be addressed using the third-order model (4.17). We use the high-gain observer [11]

$$\epsilon \dot{q}_1 = q_2 + \alpha_1 (x_1 - q_1) \tag{4.25}$$

$$\epsilon \dot{q}_2 = q_3 + \alpha_2 (x_1 - q_1) \tag{4.26}$$

$$\epsilon \dot{q}_3 = \alpha_3 (x_1 - q_1) \tag{4.27}$$

$$\hat{x}_2 = S_2 \operatorname{sat}\left(\frac{q_2}{\epsilon S_2}\right), \quad \hat{x}_3 = S_3 \operatorname{sat}\left(\frac{q_3}{\epsilon^2 S_3}\right) \tag{4.28}$$

$$\hat{\omega} = \hat{x}_2 + \omega_{ref}, \quad \hat{s}_2 = a_2 x_1 + a_3 \hat{x}_2 + \hat{x}_3 \tag{4.29}$$

[1] The proof is similar to the one shown in Section 4.5 for the output feedback case. Hence, it is omitted here.

where α_1 to α_3 are chosen such that polynomial

$$s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3 \tag{4.30}$$

is Hurwitz and ϵ is a small positive constant. Define $\xi_1 = \frac{x_1 - \hat{x}_1}{\epsilon^2}$, $\xi_2 = \frac{x_2 - \hat{x}_2}{\epsilon}$, $\xi_3 = x_3 - \hat{x}_3$, then the high-gain observer dynamics are transformed into by

$$\epsilon\dot{\xi} = (A_1 - HC_1)\xi + \epsilon b_1 \dot{x}_3 \tag{4.31}$$

where the eigenvalues of $\bar{A} = (A_1 - HC_1)$ are determined by (4.30). The saturation level S_2 and S_3 satisfy $S_2 \geq \max|x_2|$, $S_3 \geq \max|x_3|$, where the maximization, performed under state feedback, is taken over the set $x \in \Omega_1 \times \Omega_2 \times \Omega_3$. The estimates $\hat{\omega}$ and $\hat{s}_2$, as determined by (4.29), replace ω and s_2, respectively, in (4.5) and (4.22). The actual controls v_a and v_b are given by

$$v_a = \hat{v}_d \cos\hat{\rho} - \hat{v}_q \sin\hat{\rho}, \quad v_b = \hat{v}_d \sin\hat{\rho} + \hat{v}_q \cos\hat{\rho} \tag{4.32}$$

The feedback control design is now completely defined by (4.5), (4.21), (4.22), and (4.25)–(4.32). By invoking singular perturbation analysis similar to [11], it can be shown that, for sufficiently small ϵ, the set Ω is positively invariant, and the closed-loop trajectory $(\mathcal{X}, \xi)$ reaches a residual set $\overline{\Omega}_0 \times \{\|\xi\| \leq k_9\epsilon\}$ where x_2 is $O(\mu_2)$. The controller designed in this section is essentially the same as the controller of [27]. The differences are the following:

1. The flux controller is designed for a time varying reference $\psi_{ref}(t)$, while in [27] it was designed for a constant reference.[2] This change is added to include field-weakening.

[2] Due to this change, the definitions of z, F_2 and G_2 are different than those of [27].

2. Instead of using a constant nominal value R_{ro}, the controller uses a bounded time varying estimate of R_r which is obtained from an adaptive observer.

3. The controller is simplified in two aspects. First, the control laws contain only sliding-mode components. There is no feedback linearization as in [27]. Second, the high-gain observer is a linear one. It does not include driving nonlinear terms as in [27]. These two changes are adopted to simplify the controller and reduce on-line computations. In experimental testing of the controller [3], there was a need to reduce the on-line computations.

4. In [27], the definition of the tracking errors x_1, x_2, and x_3 included a scaling factor α, e.g., $x_2 = \alpha(\omega - \omega_{ref})$. It was included to improve the performance of the nonlinear high-gain observer by reducing the nonlinear driving term. There is no need for such scaling in the current her since we use a linear observer.

4.4 Adaptive Observer

To motivate the equations of the adaptive observer, we derive it assuming measurement of speed, as in[36]. The observer is taken as

$$\dot{\hat{\lambda}}_r = (-\frac{\hat{R}_r}{L_r}I + p\omega J)\hat{\lambda}_r + \frac{\hat{R}_r}{L_r}Mi_s \tag{4.33}$$

$$\dot{\hat{i}}_s = (\beta\frac{\hat{R}_r}{L_r}I - \beta p\omega J)\hat{\lambda}_r - (\alpha_s\eta + \frac{\hat{R}_r}{L_r}\beta M)i_s + \gamma v_s + u \tag{4.34}$$

where u is to be chosen later. Note that (4.33) is the same flux observer we used in (4.5). The estimation errors $e_s = \hat{\lambda}_r - \lambda_r$ and $\tilde{i}_s = \hat{i}_s - i_s$ satisfy

$$\dot{e}_s = (-\frac{R_r}{L_r}I + p\omega J)e_s - \frac{\tilde{R}_r}{L_r}(\hat{\lambda}_r - Mi_s) \tag{4.35}$$

$$\dot{\tilde{i}}_s = \beta\frac{\tilde{R}_r}{L_r}\hat{\lambda}_r + \beta\frac{R_r}{L_r}e_s - p\beta wJe_s - \frac{\tilde{R}_r}{L_r}\beta Mi_s + u \tag{4.36}$$

where $\tilde{R} = \hat{R}_r - R_r$. Define $z = \dot{\tilde{i}}_s + \beta e_s$, then

$$\dot{z} = u \tag{4.37}$$

Choose

$$u = -k_i \tilde{i}_s - \frac{\hat{R}_r}{L_r}\zeta + pwJ\eta$$

where $k_i > 0$. Then

$$\begin{aligned} \dot{\tilde{i}}_s &= -[(k_i + \tfrac{R_r}{L_r})I - pwJ]\tilde{i}_s + \tfrac{R_r}{L_r}(z-\zeta) - pwJ(z-\eta) \\ &\quad + \beta\tfrac{\tilde{R}_r}{L_r}\hat{\lambda}_r - \beta\tfrac{\tilde{R}_r}{L_r}Mi_s - \tfrac{\tilde{R}_r}{L_r}\zeta \end{aligned} \tag{4.38}$$

Let

$$V_a = \frac{1}{2}[\tilde{i}_s^T\tilde{i}_s + \frac{1}{\beta_r}\tilde{R}_r^2 + \frac{1}{\beta_\zeta}\frac{R_r}{L_r}(z-\zeta)^T(z-\zeta) + \frac{1}{\beta_\eta}(z-\eta)^T(z-\eta)]$$

for some β_ζ, β_η and $\beta_r > 0$. The derivative of V_a along (4.37) and (4.38) satisfies

$$\begin{aligned} \dot{V}_a &= -(k_i + \tfrac{R_r}{L_r})\tilde{i}_s^T\tilde{i}_s \\ &\quad + \tfrac{1}{\beta_r}\tilde{R}_r(\dot{\hat{R}}_r + \beta_r\tfrac{\beta}{L_r}\tilde{i}_s^T\hat{\lambda}_r - \beta_r\beta\tfrac{M}{L_r}\tilde{i}_s^T i_s - \tfrac{\beta_r}{L_r}\tilde{i}_s^T\zeta) \\ &\quad + \tfrac{1}{\beta_\zeta}\tfrac{R_r}{L_r}(z-\zeta)^T(\beta_\zeta\tilde{i}_s + u - \dot{\zeta}) \\ &\quad + \tfrac{1}{\beta_\eta}\tfrac{R_r}{L_r}(z-\eta)^T(u - \dot{\eta} + \beta_\eta pwJ\tilde{i}_s) \end{aligned}$$

Based on this inequality, we choose,

$$\begin{aligned} \dot{\hat{R}}_r &= -\beta_r\tfrac{\beta}{L_r}\tilde{i}_s^T\hat{\lambda}_r + \beta_r\beta\tfrac{M}{L_r}\tilde{i}_s^T i_s + \tfrac{\beta_r}{L_r}\tilde{i}_s^T\zeta \overset{\text{def}}{=} \phi_r \\ \dot{\zeta} &= \beta_\zeta\tilde{i}_s + u \overset{\text{def}}{=} \phi_\zeta \\ \dot{\eta} &= u + \beta_\eta pwJ\tilde{i}_s \overset{\text{def}}{=} \phi_\eta \end{aligned}$$

Hence,

$$\dot{V}_a \leq -k\tilde{i}_s^T\tilde{i}_s$$

where $k \geq (k_i + \frac{R_r}{L_r}) > 0$, from which we conclude that all adaptive observer states are bounded.

4.5 Closed–loop analysis

In this section we address the stability of the induction motor if the feedback loop is closed using estimates of the speed and rotor resistance and if R_s is replaced by its nominal value R_{so} in the adaptive observer. We show the boundedness of all signals and we also show that the mean-square speed tracking error is of the order of $O(\mu_2)$, where μ_2 is design parameter. The adaptive observer will not have $\dot{V}_a \leq 0$ in the lack of speed measurements. Hence, to ensure boundedness of all states of the adaptive observer we project the states η, ζ and $\hat{R}_r$. The projection is done following Chapter 2. Noting the way the terms $(z - \eta)$ and $(z - \zeta)$ appear in the Lyapunov function V_a, we can view η and ζ as estimates of z. Therefore we project η and ζ so that they belong to the same set that z belongs to. The set that contains z can be determined from its definition, i.e., $z = \tilde{i}_s + \beta e_s$. Therefore, a bound on z can be obtained from bounds on $\tilde{i}_s$ and e_s. The adaptation for R_r, η, and ζ using projection is given by

$$\dot{\hat{R}}_r = \mathrm{Proj}(\phi_r, \hat{R}_r) \tag{4.39}$$

$$\dot{\zeta} = \mathrm{Proj}(\phi_\zeta, \zeta) \tag{4.40}$$

$$\dot{\eta} = \mathrm{Proj}(\phi_\eta, \eta) \tag{4.41}$$

where the projection operator Proj$(\cdot,\cdot)$ is defined in Chapter 2. We analyze the closed-loop system by considering the set Ω_n defined by

$$\Omega_n = \Omega_1 \times \Omega_2 \times \Omega_3 \times \Omega_4 \times \Omega_5 \times \Omega_6 \times \Omega_7 = \Omega \times \Omega_6 \times \Omega_7$$

where Ω_1 to Ω_5 are defined in Section 4.3, $\Omega_6 = \{V_a \le c_6\}$ and $\Omega_7 = \{V_\xi \le c_7\epsilon^2\}$ for some positive constant c_6 and c_7, to be determined. We will show that Ω_n is positively invariant for sufficiently small ϵ. Consider the set

$$\Omega_7 = \{V_\xi \le c_7\epsilon^2\}$$

where $V_\xi = \xi^T \bar{P}\xi$. The derivative of V_ξ satisfies

$$\dot{V}_\xi = -\frac{1}{2\epsilon}\xi^T\xi + 2\xi^T\bar{P}B_1\dot{x}_3$$

Since

$$\dot{x}_3 = F_1 + G_1\hat{v}_q$$

where F_1 and G_1 are function of the states that are bounded in Ω_n and so is $\hat{v}_q$, $\dot{x}_3$ satisfies $|\dot{x}_3| \le c_x$, for some constant c_x. It follows that

$$\dot{V}_\xi \le -\frac{1}{2\epsilon}\xi^T\xi - \frac{1}{2\epsilon\lambda_{\max}(\bar{P})}V_\xi + c_x\sqrt{V_\xi}$$

Choosing $c_7 > [2c_x\lambda_{\max}(\bar{P})]^2$ ensures that

$$\dot{V}_\xi \le 0 \quad \text{on} \quad V_\xi = c_7\epsilon^2$$

Hence $\{V_\xi \leq c_7\epsilon^2\}$ is positively invariant for all states inside Ω_n. Consider next the set

$$\Omega_6 = \{V_a \leq c_6\}$$

The derivative of V_a satisfies

$$\begin{aligned} \dot{V}_a &\leq -k_s \tilde{i}_s^T \tilde{i}_s + k_{\epsilon 1}\epsilon \\ &\leq -2k_s V_a + k_s[\tfrac{1}{\beta_r}\tilde{R}_r^2 + \tfrac{1}{\beta_\zeta}\tfrac{R_r}{L_r}(z-\zeta)^T(z-\zeta) + \tfrac{1}{\beta_\eta}(z-\eta)^T(z-\eta)] + k_{\epsilon 1}\epsilon + k_{r1}|\tilde{R}_s| \end{aligned}$$

Using the fact that, in Ω_n, $\|e_s\| \leq \sqrt{c_3}$ and $\|\tilde{i}_s\| \leq \sqrt{2c_6}$, while $\|\eta\| \leq k_\eta$ (due to projection), we obtain

$$\|z - \eta\| \leq \sqrt{2c_6} + \beta\sqrt{c_3} + k_\eta$$

Choosing the adaptation gain $\beta_\eta \geq \bar{\beta}_\eta c_6$ yields

$$\frac{\|z-\eta\|^2}{\beta_\eta} \leq \frac{4c_6 + 2(\beta\sqrt{c_3} + k_\eta)^2}{\beta_\eta} \leq \frac{4c_6 + 2(\beta\sqrt{c_3} + k_\eta)^2}{\bar{\beta}_\eta c_6}$$

Similarly,

$$\frac{\|z-\zeta\|^2}{\beta_\zeta} \leq \frac{4c_6 + 2(\beta\sqrt{c_3} + k_\zeta)^2}{\beta_\zeta} \leq \frac{4c_6 + 2(\beta\sqrt{c_3} + k_\zeta)^2}{\bar{\beta}_\zeta c_6}$$

where the adaptation gain β_ζ satisfies $\beta_\zeta \geq \bar{\beta}_\zeta c_6$. Let $\bar{c}_6$ be any positive constant. Then

$$\dot{V}_a \leq -2k_s V_a + \bar{k} + k_{\epsilon 1}\epsilon + k_{r1}|\tilde{R}_s|$$

where $\bar{k}$ is a positive constant, independent of c_6 for all $c_6 \geq \bar{c}_6$. Choosing $c_6 > \max\{\bar{c}_6, \frac{\bar{k}+k_{\epsilon 1}\epsilon + k_{r1}k_{rs}}{2k_s}\}$, where k_{rs} is an upper bound on $|\tilde{R}_s|$, ensures that $\dot{V}_a < 0$ on $V_a = c_6$. Hence $\Omega_6 = \{V_a \leq c_6\}$ is positively invariant for all states in Ω_n. For the set

$$\Omega_5 = \{e^T e \leq c_3\}$$

we consider the equation

$$\dot{e} = A_3 e - \frac{\tilde{R}_r}{L_r}\mathcal{G}_1 - (\omega - \hat{\omega})[B_2\hat{\psi}_d - J]$$

Inside Ω, we have

$$\begin{aligned} e^T\dot{e} &\leq e^T A_3 e + \|e\|(|\tfrac{\tilde{R}_r}{L_r}|\|\mathcal{G}_1\| + k_{\epsilon 2}\epsilon) \\ &\leq -\tfrac{R_r}{L_r}\|e\|^2 + \|e\|(k_6|\tfrac{\tilde{R}_r}{L_r}| + k_{\epsilon 2}\epsilon) \\ &\leq -\tfrac{R_r}{L_r}[\|e\|^2 - \|e\|(k_6 k_7 + \tfrac{L_r}{R_r}k_{\epsilon 2}\epsilon)] \\ &\leq 0 \quad \text{on} \quad e^t e = c_3 \end{aligned}$$

where $\sqrt{c_3} > k_6 k_7 + L_r k_{\epsilon 2}\epsilon / R_r$. Hence Ω_5 is positively invariant. For the set

$$\Omega_4 = \{|s_1| \leq c_1\}$$

we consider

$$\dot{s}_1 = a_1 z_2 + F_2 + G_2\hat{v}_d$$

where $\|F_2\| \leq k_f + |\dot{\hat{R}}_r|$ and

$$\dot{\hat{R}}_r = \text{Proj}(\phi_r, \hat{R}_r)$$

We will show that $\dot{\hat{R}}_r$ is bounded in Ω_n by a bound independent of the bounds on the control inputs. Note that $\tilde{i}_s$ is bounded in Ω_n and ζ is bounded by projection. Boundedness of $\hat{i}_d$ and $\hat{\psi}_d$ follow from boundedness of z_1 and z_2. Using

$$\dot{\omega} = \mu\hat{i}_d e_q + \mu\hat{i}_q(\hat{\psi}_d - e_d) - T_L/m$$

and the fact $(\hat{\psi}_d - e_d)$ is bounded away from zero in Ω_5, we can conclude that $\hat{i}_q$ is bounded. Since $\hat{i}_d$ and $\hat{i}_q$ are bounded, i_s is bounded. It follows from

$$\dot{\lambda}_r = (-\frac{R_r}{L_r}I + p\omega J)\lambda_r + \frac{R_r}{L_r}Mi_s \tag{4.42}$$

that λ_r is bounded. Since λ_r and e are bounded, $\hat{\lambda}_r$ is bounded. Therefore $\dot{\hat{R}}_r$ is bounded for all states in Ω_n. Hence, F_2 is bounded by constant that is dependent on the Ω_n but independent of the control level. Now

$$s_1\dot{s}_1 \leq s_1(a_1z_2 + F_2) - G_2k_1|s_1|$$

Choosing

$$k_1 \geq \frac{k_3 + |a_1z_2 + F_2|}{G_2}$$

yields $s_1\dot{s}_1 \leq -k_3|s_1|$ on the boundary of $\{|s_1| \leq c_1\}$ provided $c_1 \geq \mu_1$ and all states are in Ω_n. Therefore $\{|s_1| \leq c_1\}$ is positively invariant. For the set

$$\Omega_3 = \{|z_1| \leq c_1/a_1\}$$

we consider the equation

$$\dot{z}_1 = -a_1z_1 + s_1$$

which yields

$$z_1\dot{z}_1 \leq -a_1z_1^2 + c_1|z_1| \ \leq 0 \ \forall \ \ |z_1| \geq c_1/a_1$$

Hence Ω_3 is positively invariant for all states in Ω_n. For the set

$$\Omega_2 = \{|s_2| \leq c_2\}$$

we consider the equation

$$\dot{s}_2 = a_2x_2 + a_3x_3 + F_1 - G_1k_2 \ \ \text{sat}(\hat{s}_2/\mu_2)$$

Then

$$\begin{aligned} s_2\dot{s}_2 &\leq -k_2G_1|s_2| + |s_2||a_2x_2 + a_3x_3 + F_1| + k_2|s_2||s_2 - \hat{s}_2| \\ &\leq -|s_2|(k_2G_1 - |a_2x_2 + a_3x_3 + F_1| - k_{\epsilon 3}\epsilon) \end{aligned}$$

Therefore $s_2\dot{s}_2 \leq -k_4|s_2|$ for

$$k_2 \geq \frac{k_4 + |a_2x_2 + a_3x_3 + F_1| + k_{\epsilon 3}\epsilon}{G_1}$$

Hence $\{|s_2| \leq c_2\}$ is positively invariant provided $c_2 \geq \mu_2$ and all states are in Ω_n.

Finally, Consider the set

$$\Omega_1 = \{x_s^T P_s x_s \leq c_1\}$$

Using $V_s = x_s^T P_s x_s$ as a Lyapunov function candidate for

$$\dot{x}_s = A_3x_s + B_2s_2$$

we obtain

$$\begin{aligned} \dot{V}_s &= -x_s^T x_s + 2x_sP_sB_2s_2 \\ &\leq -||x_s||^2 + 2||x_s||||P_s||c_2 \\ &\leq 0 \ \text{ for } \ ||x_s|| \geq 2||P_s||c_2 \end{aligned}$$

Hence $\dot{V}_s \leq 0$ on $V_s = c_1$ where $\sqrt{c_1} \geq 2c_2||P_s||^{\frac{3}{2}}$. This concludes the proof that Ω_n is positively invariant. The second step is to show that the fast variable ξ enters the set Ω_n in finite time. The details of this step are omitted here since they are similar to the ones shown in Chapter 2. Hence, we have shown that all states are bounded for all $t \geq 0$. Inside Ω_n, the inequalities $s_1\dot{s_1} \leq -k_3|s_1|$ and $s_2\dot{s_2} \leq -k_4|s_2|$ are satisfied as long as $|s_1| \geq \mu_1$ and $|s_2| \geq \mu_2$, respectively. Therefore, the trajectories reach the boundary layers $\{|s_1| \leq \mu_1\}$ and $\{|s_2| \leq \mu_2\}$ in finite time. From that time on, $\dot{V}_s$ satisfies

$$\dot{V}_s \leq -\|x_s\|^2 + k_\mu \mu_2$$

Since all signals are bounded, we conclude that the speed tracking error is of order $O(\mu_2)$. We summarize our conclusions in the following theorem

Theorem 4.1 *Consider the induction motor given by (4.17)-(4.19) with the observers (4.25)-(4.27) and (4.33)-(4.34) the output feedback control (4.21), (4.22) and (4.5), and the adaptive laws (4.39)-(4.41). Suppose* $(z_1(0), x_3(0), s_1(0), s_2(0), e(0), \tilde{i}_s(0), \zeta(0), \eta(0), \hat{R}_r(0))$ *is in* $\Omega \times \Omega_6$, *and* $(\hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0))$ *is bounded. Then, there exists* ϵ^* *such that* $\forall\ \epsilon \in (0, \epsilon^*)$ *all state variables of the closed-loop system are bounded and the mean-square speed tracking error is of order* $O(\mu_2)$.

4.6 Experimental Setup

The experimental drive setup consists of a DSP board, an induction motor fed by a voltage regulated PWM-inverter, a DC load machine fed by an AC-to-DC converter and some periphery. The drive setup is shown in Figuer 4.3 and explained below. The experimental test object is a 3-phase squirrel cage induction motor. The data of this machine are given in Table 4.1. The supply of the induction motor is a pulse width modulation (PWM) voltage-source inverter with MOSFETs. In order to obtain the desired voltage source, the inverter is controlled by a microcontroller (Intel 80C196NU) for PWM. Two 8-bit signals are sent to the Intel to control the magnitude and phase angle of the voltage used for PWM. The six outputs of the Intel are fed, via isolated drive control board, to the gate signals of the six MOSFETs of the PWM-inverter.

The load of the Induction machine under test is a DC machine. The supply of the DC machine is a current controlled thyristor-based rectifier. An 8-bit BEI encoder is used to measure the rotor position.

The DSP is suitable for real-time calculations with high sampling rate. The DSP used in the control system of this thesis is the AT&T (DSP32c). It is a 32 bit floating point unit, a 16-/24-bit fixed-point unit, on-chip memory, and flexible serial and parallel input/output ports. It has the capability of supporting a wide variety of applications with computation-intensive, repetitive mathematical operations. The arithmetic unit allows the device to perform up to 25 million floating point operations per second (with clock rate of 50 MHz). This performance was sufficient to satisfy many of the real time algorithms used in this thesis.

In order to perform real-time control in an accurate way, it is necessary to do the control computations within a small time step. Therefore, programming the DSPs is done in assembly language because programming in high-level languages (the language C was an option) would result in non-optimized assembly code that would result in a program that might be too slow. Code writing in assembly language was time consuming but not a big problem. A library of low-level programs were written, representing elementary blocks of the flow chart of Figure 4.4 and Figure 4.5. Another library contains assembly language programs for analog and digital inputs and outputs, as well as a C-language program to generate MATLAB data files. Some standard configuration files were created to perform initialization of the DSP system software (timers, interrupt vectors, I/O cards, etc).

The nominal value of R_s was adjusted on line using the measured stator temperature. The induction motor has a thermo-coupler implanted in the stator winding which gives an accurate measurement of the stator temperature. Assuming linearity,

Table 4.1. Induction motor parameters

Parameter	Value
L_s	608 mH
L_r	595 mH
σ	0.22352
R_s	0.04Ω
R_r	0.025Ω
m	0.05
p	2

the stator resistance is adjusted using the formula

$$R_s(T_1) = R_{s0}\frac{T_1 + 235}{T_0 + 235}$$

where T_1 is the current temperature in $°C$ and R_{s0} is the value of the stator resistance at room temperature T_0. Hence, a more accurate stator resistance is used in the implementation. The stator current is measured using LEM hall-effect current transducers. In order to implement the PWM, an Intel 196NU micro-controller board is used to carry out the computation of the timing and to provide the switching signals for the inverter.

4.7 Experimental Results

The induction motor used has nominal parameters that are given in Table 4.1

The flux norm is $\psi_d = 0.02$ Wb. The control parameters are chosen as $a_1 = 200$, $a_2 = 400$, $a_3 = 40$, $\mu_1 = 0.001$, $\mu_2 = 200$, $S_2 = 32$, $S_3 = 320$, $\alpha_1 = 6$, $\alpha_2 = 11$, $\alpha_3 = 6$, and $\epsilon = 0.02$. Some of the control parameters were chosen such that the two loops, flux and speed, are stable. Others are determined based on simulation results using MATLAB. At the time of implementation, some of the parameters were adjusted to

get the setup to work. In particular, we needed to decrease the saturation slope $1/\mu_2$ for the speed control more than the value that was used in simulation. The larger value was causing chattering which ,in turn, created more noise. By increasing the value of μ_2 the problem was eliminated. Since our controller assumes smooth speed and flux references, we used two linear filters for smoothing them. The second order speed filter is of the form

$$X_s(s) = \frac{\omega_n^2}{s^2 + 2\zeta_n\omega_n s + \omega_n^2}$$

where $\omega_n = 10$ and $\zeta_n = 1$. Its input is the reference speed and its states are the smoothed speed and acceleration references. The reference position is calculated using $\theta_{ref} = \int \omega_{ref}$. On the other hand, the flux reference is smoothed using a first-order filter of the form

$$X_f(s) = \frac{\omega_o}{s + \omega_o}$$

where $\omega_o = 100$. In order to implement the continues control using the DSP, we needed to discretize all filters, observers, and integrations for the adaptive laws. The forward difference method was used. If the system equation is $\dot{x} = f(x,u)$ then at step k the state is computed using

$$x(k) = x(k-1) + hf(x(k-1), u(k-1))$$

where h is sampling period. In the implementation a sampling frequency of 5 kHz is used. The frequency of the switching signals, generated by Intel, is 7.5 kHz.

Two experiments were conducted. Both tests were done when the motor was under some load all the time. In the first test, the controller was given a flux reference of $\psi_{ref} = 0.02$ Wb at time 3.5 seconds and a period of 1.5 seconds to build the flux, then a speed reference of $w_{ref} = 25\ r/s$ was applied. Figure 4.6(a) shows the speed reference (solid) and the actual speed[3] (dashed). Figure 4.6(b) shows the speed estimation error $(w - \hat{w})$. Figure 4.7(a) shows the flux reference ψ_{ref} and the estimated flux $\hat{\psi}_d$, and Figure 4.7(b) shows the estimate of the rotor resistance $\hat{R}_r$. It is noted that during the first 30 seconds $\hat{R}_r$ was going to the lower limit. Then, when the speed increased, $\hat{R}_r$ started to converge to the right value which is higher than the nominal one since the motor started to warm up and hence the rotor resistance increased. Figure 4.8(a) shows the speed reference (solid) and the actual speed (dashed) while reversing the speed direction. Figure 4.8(b) shows the estimate of the rotor resistance $\hat{R}_r$. Upon switching the speed from negative to positive, the estimate of the rotor resistance went to its lower limit and stayed there. However the speed controller was still able to ensure tracking. Figure 4.9 is similar to Figure 4.8 but before switching speed, the adaptation was turned off for a period of 8.5 seconds to avoid the disturbances that caused $\hat{R}_r$ not to converge in the previous case. Finally, Figure 4.10(a) shows the reference and the actual speeds during load changes, shown in Figure 4.10(b), and Figure 4.10(c) give the rotor resistance estimate $\hat{R}_r$. Note that speed tracking was achieved in the presence of the varying load torque. At time t=60 sec. the speed was not reaching its reference because v_q hit its upper limit and stayed there.

4.8 Conclusions

In this Chapter, we have demonstrated via experimental results the plausibility of incorporating the adaptive observer of [36] into the nonlinear robust controller of [27]. Furthermore, we have experimentally demonstrated the speed tracking convergence and the convergence of the rotor resistance to its actual value . The results are in

[3] The actual speed is obtained directly from the measured position using a second order high-gain observer.

full agreement with the theory. It should be noted that this adaptive observer is sensitive to R_s and the stator current i_s. If there is a measurement error, which is usually the case at low load, then the convergence of the observer states will not occur. Hence, no advantage is gained by having adaptation. In this case, using a nominal value for R_r as in [27] is as good as the adaptive but with less computations. However, if the motor is run at high speed or torque then an accurate estimate of R_r is needed so no saturation of the flux would result. Figure 4.10(b) demonstrate this fact. Before time $t = 30$ the estimate of the rotor resistance was not converging to the right value. However, when the speed increased by 50% , with the same load, $\hat{R}_r$ started to converge to its actual value.

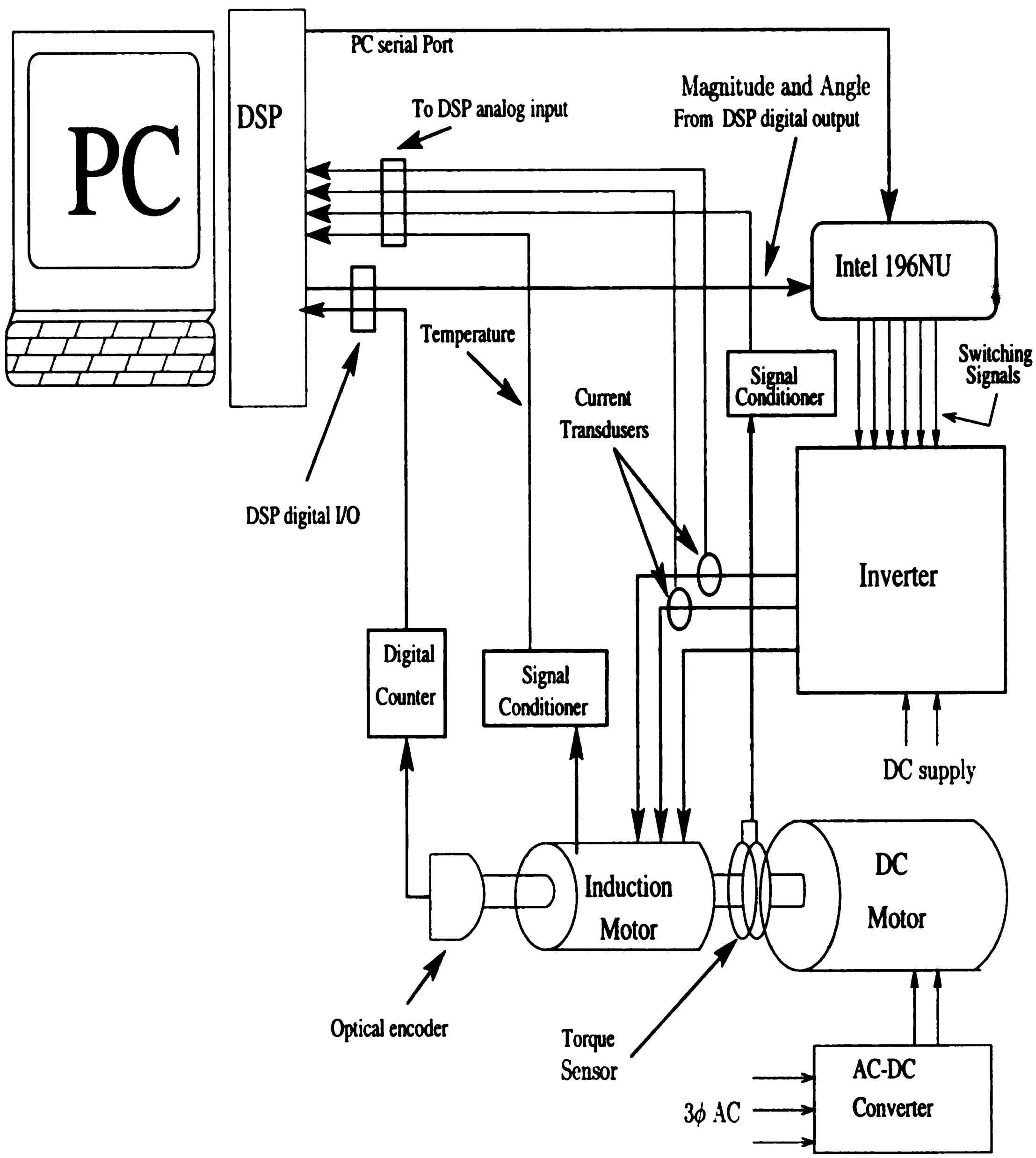


Figure 4.3. Experimental setup

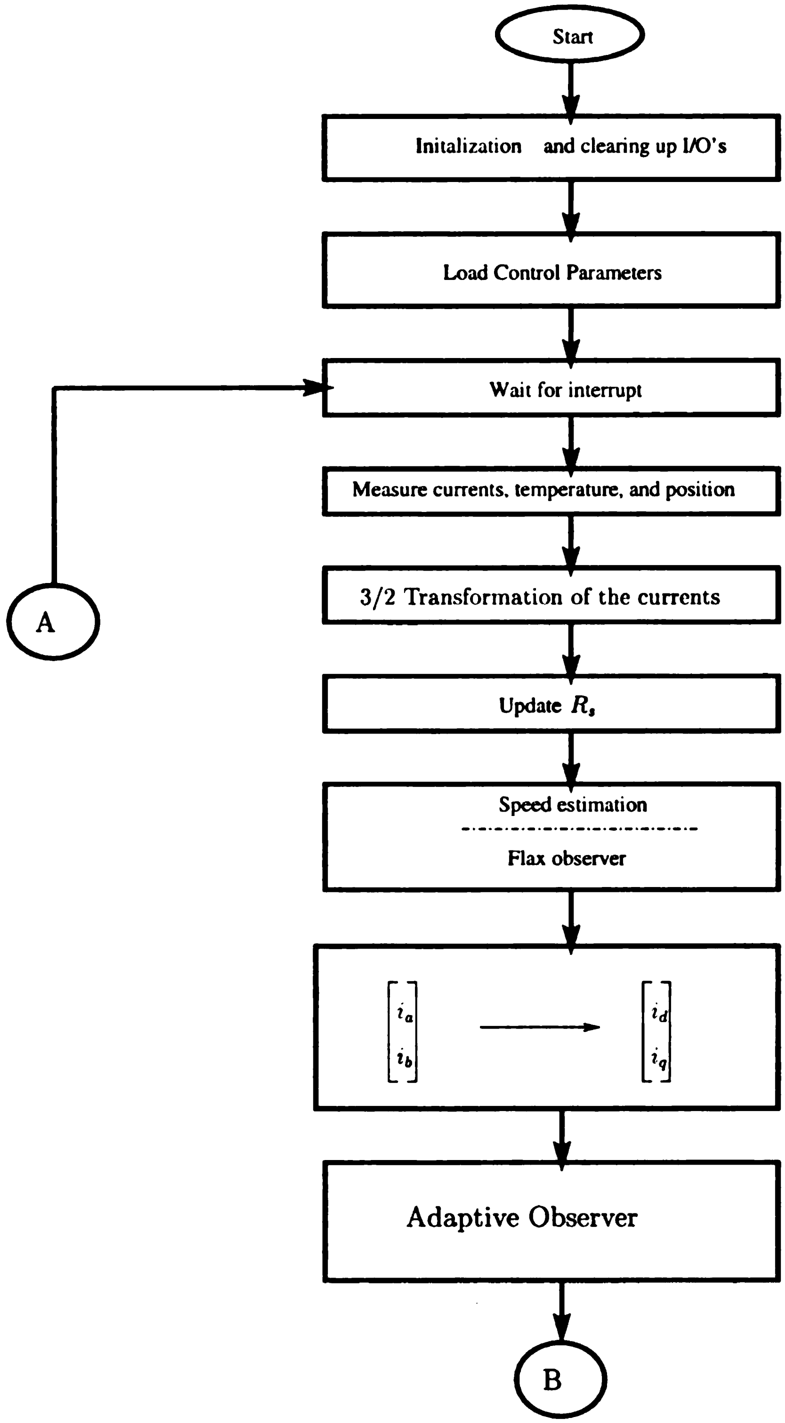


Figure 4.4. Flow chart of the assembly program

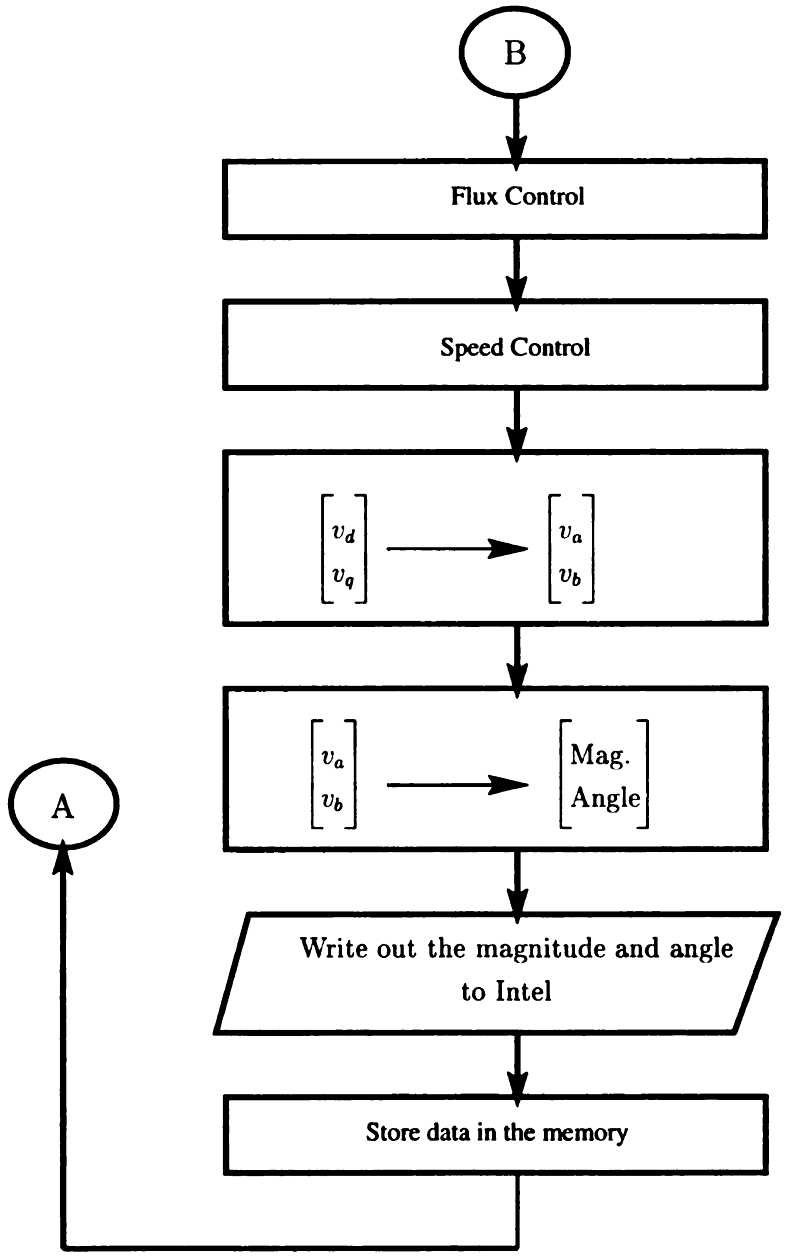


Figure 4.5. Flow chart of the assembly program (cont.)

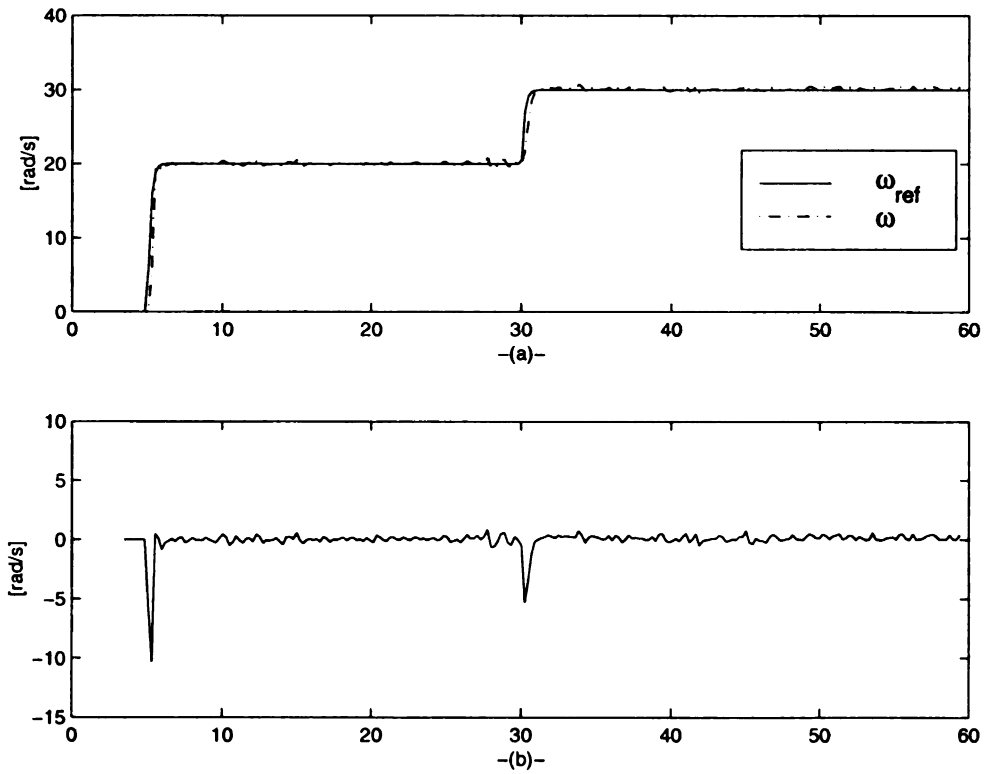
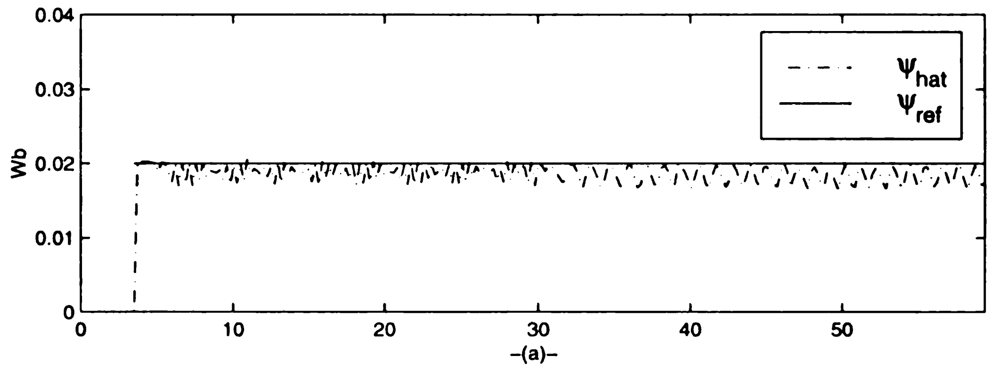


Figure 4.6. (a)- Reference and actual speed (b)- Speed estimation error



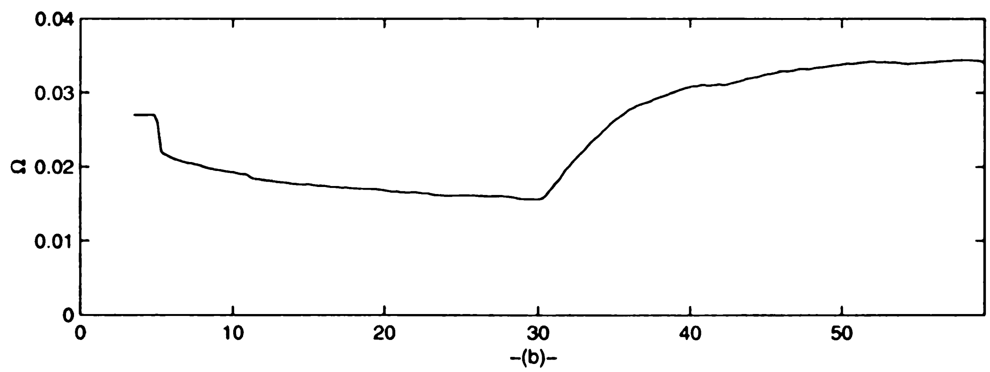


Figure 4.7. (a)- Flux norms, estimated and reference (b)- Estimated rotor resistance

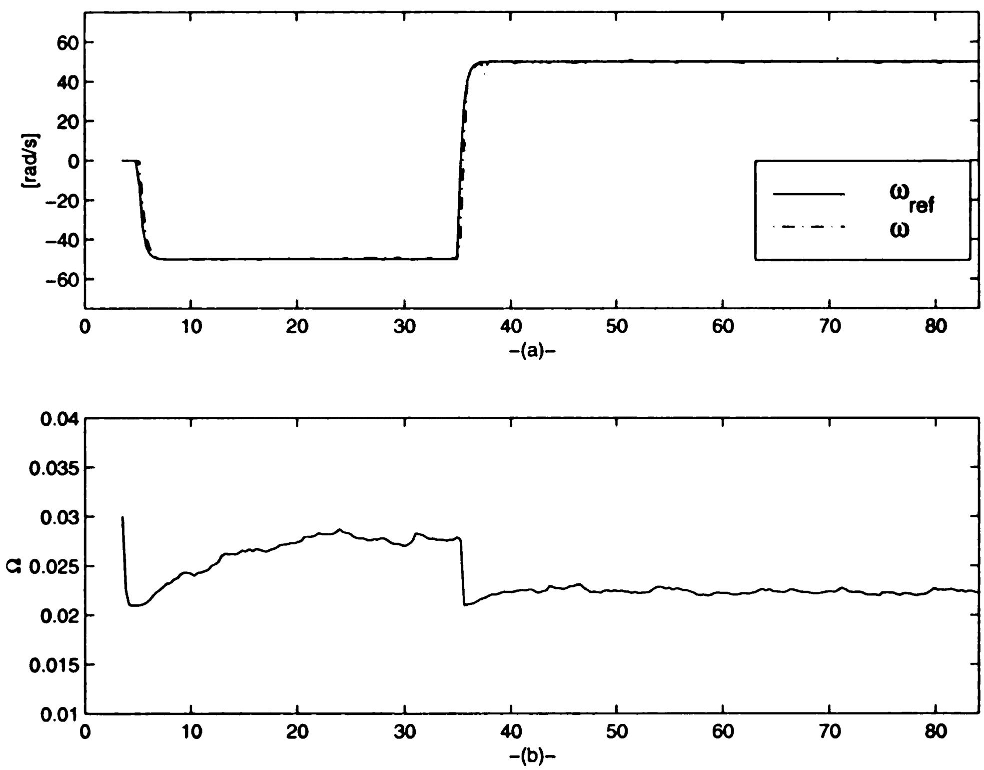


Figure 4.8. (a)- Reference and actual speed (b)- Estimated rotor resistance

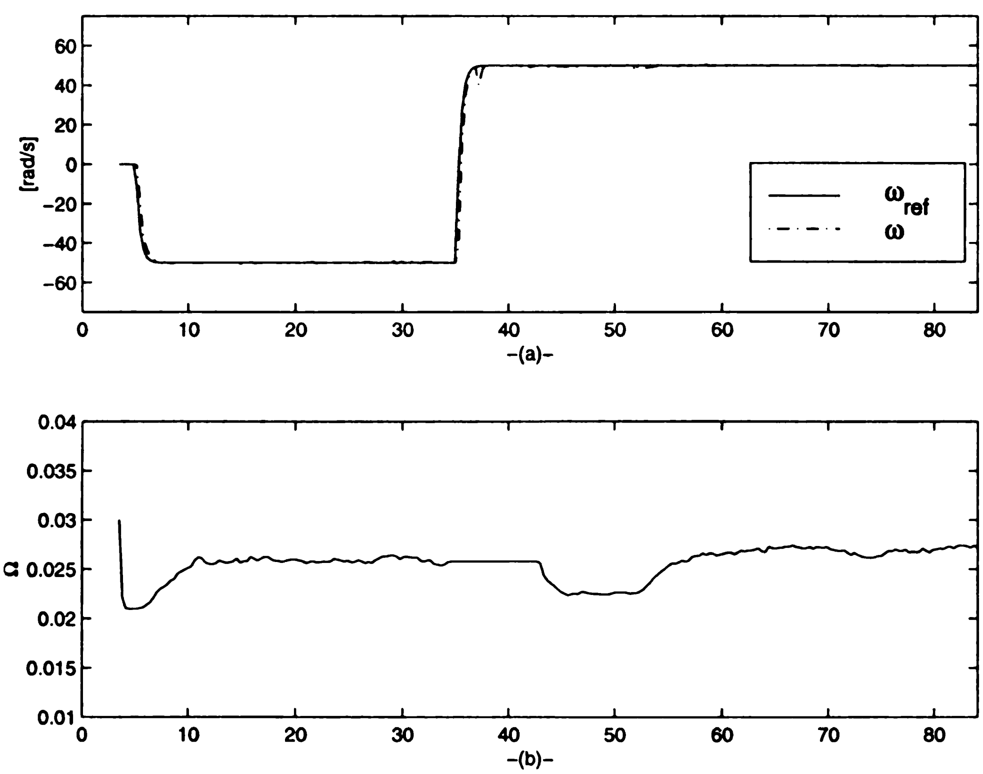


Figure 4.9. (a)- Reference and actual speed (b)- Estimated rotor resistance

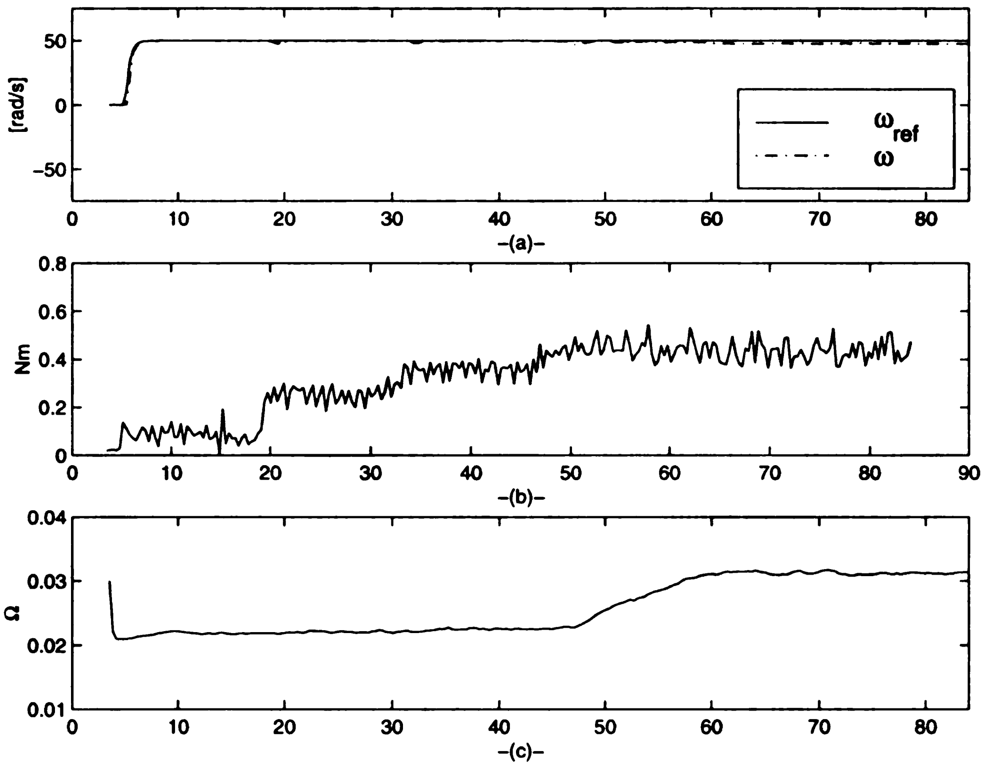


Figure 4.10. (a)- Reference and actual speed (b)- Load Torque and (c)- Estimated rotor resistance

CHAPTER 5

Conclusions and Future work

Conclusions

In this thesis, we have advanced the state of the art of robust adaptive output feedback control of nonlinear systems. We have shown that the tracking error convergence can be achieved without persistence of excitation, studied robustness of the controller to bounded disturbance even when the bounded on the disturbance is not small, and applied our techniques to speed tracking control of induction motors.

In Chapter 2 we have designed an adaptive output feedback controller to solve the tracking problem for a class of nonlinear systems. We have not required a persistence of excitation condition like [25]. We have introduced a transformation that projects the parameter error on a lower-dimensional subspace. The convergence has been shown by constructing a composite Lyapunov function and taking its derivative along the trajectories of the closed-loop system. In Chapter 3 we have shown that, in the presence of small bounded disturbance, all signals under the control of Chapter 2 are bounded and the mean-square tracking error is of the order $O(d_1)$ where d_1 is a small bound on the disturbance. We have combined robust and adaptive control to force the mean-square tracking error to be of the order $O(\epsilon + \mu)$ where ϵ and μ are design parameters. In Chapter 4 we have designed, and experimentally tested, a robust nonlinear controller for speed tracking control of induction motors which uses an adaptive observer to estimate the rotor resistance.

Future work

A number of research problems remain open and can be pursued in future work. First, requiring exponential stability of the zero dynamics as in Chapter 2 is stronger than the bounded-input-bounded-state assumption used in [25]. Future research may attempt to relax this exponential stability assumption. Second, the analysis and robust control design of Chapter 3, which are presented for bounded disturbance, can be extended to unmodeled dynamics. Finally we need to investigate the convergence of the rotor resistance estimate of Chapter 4 and develop rules for turning off the adaptation when the conditions for convergence are not satisfied.

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