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STUDENTS' UNDERSTANDINGS OF THE BEHAVIOR OF A GASEOUS SUBSTANCE

By

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ABSTRACT

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By

Edward L. Jones II

One hundred sixteen community college students enrolled in a basic chemistry class who had completed a unit on the behavior of a gaseous substance were given a written instrument that presented several mathematical and conceptual problems describing the behavior of a gas. Nine students representing a range of achievement levels were chosen for more intensive clinical interviews. In the clinical interview, students explained their responses on the written instrument and gave quantitative and qualitative explanations of the behavior of air in the barrel of a real syringe. Interview results revealed that students commonly experience difficulties at three different levels:

1. *Mathematical understanding.* Most students could manipulate the gas law equations, but few had a real understanding of the equation. There were some unique understanding of proportional relationships.
2. *Conceptual understanding.* Many students could represent pictorially the notion that gas molecules randomly occupy the entire space of its container. Many, however, had a different conception of this when the air was compressed. The reason for this seemed to be due to a misunderstanding of the kinetic molecular theory.

3. *Real-world application.* Students' use of their mathematical understanding to explain the behavior of air in a real syringe revealed some internal consistency found in mathematical explanations of real-world phenomena. Many students used mathematical strategies consistent with their mathematical understanding and satisfactory for producing reasonable estimates of numerical values.

All of the 9 students had misconceptions about mathematical proportionality with most of them understanding proportional relationships as being additive in nature. Although some of the students were able to state the relationship between two variables, they could only do so outside of the context of the gas law equation. Only one student was able to propose a reasonable explanation of the proportional relationships between variables in a gas law equation. All 9 students were classified as either transitional or naïve in the real-world use of their mathematical understandings with 3 of the 9 clearly having naïve conceptions of the mathematics of gas behavior.

A majority of the 9 students could clearly represent the nature of the submicroscopic level of gas behavior when asked to draw it during the clinical interview. However, only 2 of these students had the chemist's understanding of this concept when put to use with a real-world task. Three students were considered transitional in their thinking, having various capacities to understand and use molecular language depending on the context of the problem; while, 4 students were clearly naïve in their thinking having various conceptions of the atomic theory which they could not consistently use in describing the behavior of air in real-world situations.

The results of this study suggest that students in college basic chemistry classes don't walk away from instruction on the behavior of a gaseous substance with the understanding teachers intend for them to possess. College chemistry teachers should be aware of the myriad of ways students understand the behavior of a gaseous substance and incorporate better methods of instruction to help students truly understand this behavior. This study analyzes students' understanding and suggests a possible approach for helping them gain better understanding.

This dissertation is lovingly dedicated to my father and high school chemistry teacher,

Mr. Edward L. Jones, Sr.,

who, for 35 years, exposed his high school and college chemistry students to the ideas of conceptual understanding in chemistry.

ACKNOWLEDGEMENTS

No man is an island entire of itself. Every man is a piece of the continent; apart of the main. If a clod is carried away by the sea, Europe is less. Every man's death diminishes me, for I am apart of all mankind. So never send to know for whom the bell tolls, it tolls for thee. -John Donne

I recognize that this work is not the sole result of my efforts alone. In fact, this dissertation and the completion of my graduate experience would never have come to fruition had it not been for the myriad of people who have inspired, counseled, consulted, and nudged me along the way. First of all, I was fortunate enough to have crossed paths with Dr. Charles W. (Andy) Anderson at Michigan State University. He has been an advisor and dissertation director extrordinaire. At many times when my focus seemed to wane, he provided the gentle but firm nudge I needed to get back on course. His gentle nature and persistent counsel has been most needed and appreciated.

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I was and am continually inspired by the many conversations - professional, academic, and personal - I have shared with my colleagues and office mates at Lansing Community College. I am grateful to Drs. Shannon Briggs, Brian Jordan, Gary Lobel, Laura Markham, and Chris Marschall. With these individuals I shared office space and many wonderful and stimulating conversations. All are excellent science teachers in their own right whose collegiality I will always cherish.

The research for this dissertation would not have been possible but for the cooperative nature of other colleagues at Lansing Community College. I am thankful for Gerald Blair, Evelyn Green, Laura Markham, Don Nofzinger, and Mike Waldo. These individuals willingly opened their classrooms and gave me valuable class time to collect the data for this study. Their kindness and generosity have not gone unnoticed.

Many of my friends had the patience to listen at times when I lamented about my problems during the graduate experience. I thank them for their support, encouragement, and listening ear. To LaTrese Adkins, Edward Fubara, and Dr. Janice Hilliard, I offer my sincere and profound appreciation for enduring

through “the struggle” with me and the empathetic ear you were willing to lend. I would also thank Dr. Melvin T. Jones, pastor, and a multitude of friends at the Union Missionary Baptist Church in Lansing who have been paragons of strength for me during my years of graduate study. The individuals in this church are too numerous to name here, but all hold a special place in my heart.

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CHAPTER ONE

INTRODUCTION

Statement of the Problem

Understanding the nature and behavior of matter is important to the professional scientist and layperson alike. The world in which we live is at one level, a collection of solid, liquid, and gaseous substances. Understanding the nature and behavior of these substances is essential for scientists, as they are charged with developing the characteristics of these substances to benefit society, and laypeople, as they are called to be informed and literate citizens in an increasingly technological environment.

Particularly useful in this regard, is an understanding of the nature and behavior of a gaseous substance. Air - which we depend on for our very existence - is a gaseous mixture that is extremely important to society. The measurement of barometric pressure, the lifting of a hot-air balloon, or the proper inflation of a bicycle or automobile tire are common examples of everyday occurrences that can be understood using the ideas of gas behavior. Although everybody comes into contact with gaseous substances - particularly, air - in the course of their daily activities, few people can exhibit a scientific understanding of why gaseous systems behave as they do. For example, few people are able to offer scientific explanations of what a barometer is measuring when it measures air pressure, of what makes a hot-air balloon lift and descend, or understand that the pressure in automobile tires changes with the weather.

In school, students learn about the behavior of matter at the primary, secondary, and post-secondary levels. Consequently, this topic is an important one throughout the school curriculum. However at all levels, students have difficulties achieving a scientific understanding of the behavior of matter, particularly matter which exists in the gaseous state.

Theoretical Framework

Conceptual Change Model

This is a study of student understandings of the behavior of matter, particularly that of gaseous substances. As such, it is based on a number of assumptions about how students learn and understand science and what it means to say that someone "understands" the behavior of matter.

There is a general understanding in the cognitive science tradition that people develop notions about scientific phenomena before they are introduced to them in science classrooms, through their socialization into our general culture and interactions with their environment. The research literature which characterizes students' notions, beliefs, and interpretations about scientific phenomena is extensive and has been reviewed by a number of researchers (Driver & Easley, 1978; Driver & Erickson, 1983; Osborne, Bell & Gilbert, 1983; Driver et al., 1985; Osborne and Freyberg, 1985; Eylon and Lynn, 1988; Pfundt and Duit, 1985, 1988, 1991; Wandersee, Mintzes, & Novak, 1994). This body of research advances the claim that before coming into classrooms, students develop some informal and useful ways of making sense of the world around them. These beliefs and understandings by students have been shown to be robust to typical science instruction (Driver, Guesne, & Tiberghien, 1985). Students do not enter science classrooms as empty pitchers waiting to be filled with knowledge. Rather, students enter school and science classrooms with well-established beliefs of how and why everyday things behave as they do (Posner, Strike, Hewson, & Gertzog, 1982; Resnik, 1983; Strike, 1983). These beliefs - variably referred to as misconceptions, alternative conceptions, naïve theories, etc. - influence how students learn new scientific knowledge, and have been found to hinder successful acquisition of scientific concepts taught in school (Hewson, 1982; Shuell, 1987). Consequently, many researchers have examined students to try and understand how they change their alternative conceptions into scientific conceptions (Clement, 1982; Roth, Smith & Anderson, 1983; Anderson & Smith, 1983; Minstrell, 1985; Yarroch, 1985; Ben-Zvi, Eylon, & Silberstein, 1986; Nussbaum & Novick, 1982; Smith, 1990; Lee, Eichinger, Anderson, Berkheimer, & Blakeslee, 1993).

What is clear is that understanding students' alternative conceptions is instructionally useful. Again, much research in science education is being devoted to determining how it is students change their current alternative conceptions into scientific conceptions. But while this area of inquiry is important, more must be learned about the understandings students have in specific science areas. The primary goal of this study is to present a more detailed understanding of students' conceptions of the behavior of a gaseous substance.

Three-Part Model

In addition to the general assumptions about the nature of scientific understanding embodied in the conceptual change model, this study is based upon a three-part model of chemical understanding which may be useful in understanding the explanations given by students of the nature and behavior of a gaseous substance. This model states that students must acquire three different types of understanding in order to produce an explanation about the behavior of a gaseous substance acceptable to a trained chemist: (1) a mathematical understanding, which includes knowledge of the mathematical representations of the gas laws as well as a knowledge of the proportional relationships contained within these representations, (2) a conceptual understanding of the nature of matter, which includes knowledge of the atomic-molecular and kinetic molecular theories, and (3) a real-world understanding, which allows the student to use both their mathematical and conceptual understandings to explain the behavior of real gaseous systems.

Chemists' Understanding of the Behavior of a Gaseous Substance

Chemists have developed ways of predicting and explaining the response of gases to changes in temperature, pressure, volume, and amount that can be expressed in elegantly simple ways: macroscopic - $PV = nRT$ and molecular - atomic-molecular and kinetic molecular theories. The simple expression, however, conceals conceptual difficulties. In order to use the chemists' conceptual tools for predicting and explaining gas behavior, learners need three different kinds of understanding.

Specifics of Mathematical Understanding

Mathematical models of gas behavior add precision to the theories used to describe this behavior, thus increasing the predictive power of the theories. These models are important sources of explanations and hypotheses. In describing the behavior of a gaseous substance, these mathematical models are represented by the gas law equations. For the chemist, the generation and refinement of such models is a dynamic process, and is necessary and important in understanding the theories which explain the behavior of a gaseous substance.

For the student, mathematical models of gas behavior are "prepackaged" and "distributed" during the instructional process. This is not to suggest, however, that students don't use their mathematical understanding in a model-like way. However during instruction these models are generally present as static entities unrelated to any conceptual understanding of gas behavior. But, students have, and develop some mathematical understanding before being exposed to the gas law equations in the classroom. That is, students possess mathematical notions, and they use these notions in explicit or tacit ways to inform their understanding.

To explain the behavior of a gaseous substance mathematically, students must have some knowledge about proportional relationships, because any good mathematical model used to describe the behavior of a gaseous substance depends on these relationships. The behavior of a gaseous substance is adequately described by its volume, pressure, temperature, amount, and the changes these quantities undergo. The volume and pressure are two quantities which are inversely proportional to each other, whereas temperature and amount are directly proportional to both volume and pressure. A mathematical model of the behavior of a gas allows for the prediction of pressure, volume, temperature, or amount when any one of these quantities is changed. That is, if the volume of a gas is doubled from its original value, the pressure exerted by the gas will be cut by one-half of its original value if the temperature and amount of gas are not changed. Similarly, if the pressure exerted by a gas is increased by two-thirds of its original value, the volume occupied by the gas is decreased by three-halves of its original value if the temperature and amount of gas are not changed. However, if the pressure, for example, of a gas is cut to one-fourth of its original value, the absolute temperature of the gas will also be cut to one-fourth of its original value if the volume and amount of gas are not changed. Chemists use mathematical models in this way to engage in proportional reasoning.

For the students in this study, the mathematical relationships governing the behavior of a gaseous substance were presented during classroom instruction in each of three gas laws:

$$P_1V_1 = P_2V_2, \qquad P_1/T_1 = P_2/T_2, \qquad V_1/T_1 = V_2/T_2,$$

and summed up in the combined gas law,

$$P_1V_1/T_1 = P_2V_2/T_2$$

In displaying mathematical understanding, students use their knowledge in some interesting ways. Some research that has explored the problem of how students use their mathematical understandings will be reviewed in Chapter 2.

Specifics of Conceptual Understanding

The chemist uses the atomic-molecular and kinetic molecular theories as the explanatory ideal in describing the behavior of a gaseous substance. These theories describe the phenomena of gas behavior based on the actions of molecules. The student often has problems with such explanations. This is because atoms cannot be seen. Therefore, to base explanations on their existence and movement often becomes a “leap of faith” some students are not willing to take.

Chemists have a conceptual understanding of what it means to talk about the volume, pressure, temperature and amount of a gas on both a macroscopic and molecular level. For example, on a macroscopic and molecular level, when chemists talk about changing the amount (i.e., mass or number of moles) of gas in a container by withdrawing some of the gas molecules from the container, they don’t consider that the volume of the gas will change. This because mass is understood to be a measure of the quantity of gas molecules, while volume is a measure of the amount of space occupied by the gas molecules. The student often has conceptual difficulties not only with the molecular idea of small particles and their movement, in this regard, but, also, with distinctions between volume as the space occupied by the molecules as opposed to the amount of the substance. The notions of small discrete particles (atomic-molecular theory) and their inherent movement within matter (kinetic molecular theory) are the theories used by chemists to explain and understand the concepts of volume, pressure, temperature, and amount of a gas. Gases always completely occupy the space of the container in which they are placed because the molecules of the gas are in constant and random motion spreading out from one another as far as possible. Therefore, students must come to understand the chemist's conception that removing some particles of the gas does not affect the ultimate volume because the remaining particles will spread out to fill the space of the withdrawn particles.

The Connection Between Conceptual and Mathematical Understanding

Chemists use their mathematical understanding to enhance the predictive power of and add precision to their conceptual understanding. Chemists' mathematical model describing the behavior above should show a direct proportional relationship between the number of molecules and the pressure of the gas (fewer molecules, fewer collisions with the wall, the lower the pressure) with no change in the volume:

$$V \propto k\ (n/P)$$

where **k** is a proportionality constant that includes the constant value of the temperature at which the gas must be maintained for this equation to adequately describe the behavior of a gaseous substance under these conditions. The variable **n** represents the number of molecules of the gas while **P** represents the pressure exerted by the molecules. In the direct proportion relationship, both would decrease by the same factor so that the ratio never changes. Consequently, the volume, V, would also not change. This mathematical model successfully predicts the submicroscopic and macroscopic behavior of a gaseous substance. To adequately model the conceptual behavior of a gaseous substance, the student must focus on these conceptual connections through mathematical equations. In general, students find it difficult to use mathematical equations as models for conceptual understanding. Instead, they often use equations as devices for computation.

Specifics of Real-World Understanding

A real-world understanding of gas behavior is evidenced when students, like chemists, can use the mathematical models they develop and link the behavior of a real-world gaseous system with their conceptual understandings. For example, a person who understands the scientific nature of a hot-air balloon system should be able to (1) make a mathematical prediction of the temperature at which the air in a hot air balloon must be maintained in order to establish a constant drift when atmospheric conditions are known, (2) understand why the balloon will change its drift when the temperature of the gas in the balloon is changed, and (3) describe at the atomic level what happens to the air in the balloon when it is heated.

As evidenced by the above description of what it means to understand the behavior of a gas, a practical understanding requires that students are able to meander between their conceptual and mathematical understanding of the behavior of a gas and connect both kinds of understanding to cues from real-world situations that have not been labeled as "data" and quantified for the student. It has been pretty well documented (Nurrenbern & Pickering, 1987; Sawrey, 1990; Nakhleh, 1983; Nakhleh & Mitchell, 1993) that students are, in general, better at using mathematical representations than they are conceptual problem solvers. That is, students typically develop the ability to manipulate mathematical representations but are often unsuccessful with more conceptually based problems. What we seem to be less clear about is how students use their mathematical and conceptual understandings when explaining the behavior of a gas. In order to understand this, it is important to examine the models of understanding, both mathematical and conceptual, that students adopt when learning about the behavior of a gas. Because a scientific understanding involves using mathematical and conceptual understandings together, an interesting phenomenon is how students tend to use these understandings in parallel. Therefore, this study seeks to examine the ways students use their personal mental models - both mathematical and conceptual - to understand the behavior of a gas.

Research Questions

The above discussion suggests a complexity to teaching and learning about chemistry. To examine the nature of students' learning in chemistry, this study will examine particularly the nature of students' mathematical and conceptual explanations about the behavior of a gas. What understandings and processes promote or inhibit students' development in these areas as they come to understand and explain gas behavior? The following questions give focus to the study:

1. What mathematical understandings do introductory-level, basic chemistry college students use when they describe the behavior of a gaseous substance?
2. What conceptual understandings do the students have about the behavior of a gaseous substance?
3. How do students use their mathematical and conceptual understanding when explaining the behavior of a real-world gaseous system?

Overview of the Study

The heart of this study is an in-depth examination of how four students explained the behavior of gaseous substances. These students were chosen from four groups of students who had been categorized as medium mathematical/high conceptual (MMHC), medium mathematical/medium conceptual (MMMC), medium mathematical/low conceptual (MMLC), and low mathematical/low conceptual (LMLC) based on their performances on a paper-and-pencil instrument. In all, nine students were clinically interviewed out of 116 tested using a written instrument. Some comparisons are drawn between the four students who formed the basis of my study and the remaining five who were also clinically interviewed. All the students in this study had studied chemistry for about six weeks with one week devoted specifically to the topic of the behavior of a gaseous substance.

Methodological Limitations

This study is based on an epistemological model of conceptual change. As such it examines the cognition or understandings that students possessed about the behavior of a gas and does not attend to other issues, such as social/affective issues, attended to in other models (e.g., Tyson et al., 1997). No attempts are made to trace student learning as it changed during instruction or as a consequence of classroom interactions. Neither is any attempt made to monitor the attendance of the interviewees in class during instruction on the unit describing the behavior of gases.

Second, this study examines students from five different sections of a community college, basic chemistry course with different instructors. Although all sections shared a common syllabus and common lecture outline, there are no assumptions made that all instructors taught the same way or strictly followed the lecture outline. Neither is any formal attempt made to monitor student interaction with the objectives in the syllabus or readings in the textbook. Consequently, this study is strictly an examination of the cognitive understandings community college students have about the behavior of a gaseous substance after instruction. It is assumed that the students' expressed knowledge during the clinical interview is an indication of how they came to understand the behavior of a gas as a result of instruction in their chemistry course.

Finally, this study involves only nine students from five classrooms taught by different teachers. The limited number of subjects allowed me to achieve the stated objective of providing a rich analysis of the understanding selective, students possess about the behavior of a gas. However, the small sample size precludes the use of statistical methods of analyses.

CHAPTER TWO

LITERATURE REVIEW

Introduction

This chapter reviews a body of research that forms the theoretical underpinnings of this investigation. This chapter will show how the study draws from and goes beyond previous attempts to explain students' understanding of the behavior of a gaseous substance.

Part I of this chapter examines how students have used mathematics in understanding the behavior of a gaseous substance. This section examines an existing body of research that focuses upon how students understand and use the mathematics of gas law equations.

Part II of this chapter examines the conceptual understandings students have of gas behavior. In particular, it examines student's notions of the macroscopic and atomic-molecular descriptions of gas behavior.

Part III examines notions of real-world understandings of scientific phenomena. In particular, this section discusses what it means to have an understanding of scientific phenomena that is useful.

Part I: Students' Mathematical Understandings and Explanations of Gas Behavior

A growing body of research indicates that there are often discrepancies between performance on mathematical tasks used to describe the behavior of a gaseous substance and student understanding of the ideas which underpin the task. Many studies (Nurrenbern & Pickering, 1987; Sawrey, 1990; Nakhleh, 1993; Nakhleh & Mitchell, 1993) have examined college students as they have responded to tasks in which they use the mathematical gas law equations and a conceptual understanding of a gaseous substance. While more than a majority of students in all of the studies could perform well on the mathematical problem, a minority of the students in each study could perform well on the conceptual problem. Students' conceptual understanding of the behavior of a gaseous substance will be explored in the next section. In this section, my goal is to examine the literature as it relates to students' mathematical understanding of the gas laws. This understanding incorporates (1) knowledge of the mathematical representations of the gas laws, and (2) knowledge of the proportional relationships described by the mathematical representations.

Gabel & Sherwood (1983) studied the use of proportional reasoning strategies by high school chemistry students. They showed that these high school students performed better in instructional situations when they were taught the gas laws using proportional reasoning strategies. That is, when students in the Gabel and Sherwood study were taught the gas laws using a relationship such as $A/B = C/X$ and asked to find the value of X, these students typically outperformed students who were taught using more visual methods, such as the use of analogies and diagrams. The more visual methods could be thought of as more conceptual in nature because they required the student to think more deeply about the situation at hand. For example, the analogy method compared the molar volume of a gas to a shipping carton of fruit. No matter the size of the fruit, the volume of a dozen pieces of the fruit was always 3 pints. After considering this problem and being asked to determine the number of fruit in 54 pints, the students were then presented with a problem to determine the number of moles in 89.6 liters of oxygen gas. Gabel & Sherwood (1983) found that even for students who did not prefer this approach to problem solving (i.e., the low visual students in their study), this method of instruction was the best in getting them to understand the problem. These researchers suggest,

> One possible explanation for this is that even though these students did not prefer this approach, it required them to pay greater attention to the material at hand. Because students prefer a certain approach by which to learn does not necessarily mean that they learn better using this approach (p. 175).

In a think-aloud interview of high school students solving gas law, molarity, mole concept and stoichiometry problems, Gabel, Sherwood, and Enochs (1984) examined the preferred strategies used by high school chemistry students to solve the problems. They found that the students who were not successful on the written tests given before the interviews tended to use a nonsystematic approach on the problems given to them during the interviews. That is, the unsuccessful students did not organize their information before attempting to solve the problem. However, for problems dealing with the gas laws, even those who used a nonsystematic approach were no more unsuccessful in solving these problems than those students who used a systematic approach. The authors note that this is probably due to the fact that gas law problems can be solved using a mathematical formula. A systematic approach can be avoided by dependence on the mathematical formula. Similarly, these researchers found that, in general, students who used high proportional reasoning outperformed students who were low proportional in reasoning except for the gas laws where dependence on the mathematical formula did not necessarily require proportional reasoning abilities.

The studies by Gabel and her colleagues have shown that high school chemistry students prefer and are often taught the use of proportional reasoning to solve gas law problems. The ability to use proportional reasoning, however, is not always necessary in solving gas law problems because the problems can often be solved by simply manipulating a gas law equation. But proportional reasoning strategies are necessary for understanding the mathematics of gas behavior. Beyond the manipulation of the mathematical representations of the gas laws, a true understanding of the mathematics is lost without the ability to understand the relationships between the variables of the equation.

A recent study by de Berg (1995) examined the understanding of the inverse proportional relationships between pressure and volume of air compressed in a syringe. The researcher gave 101 college students from England a written exam showing the picture in Figure 1. Based on the pressure-volume relationships shown for each of the situations presented, students were asked to predict either the pressure or volume of gas in the syringe given the value of the other. Using proportional reasoning, 65% of the students stated

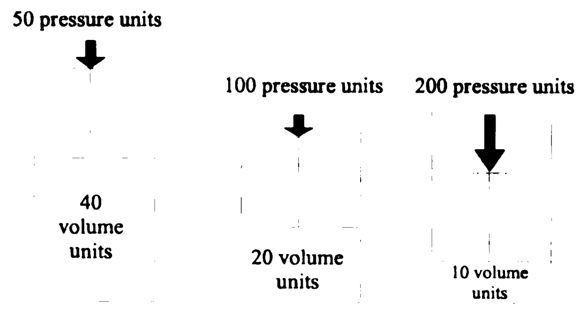


Figure 1. Syringe System Used for Quantitative Exercise in de Berg (1995) Study

correctly that if 25 units of pressure were exerted on the syringe, the volume would be 80 volume units. Likewise, 64% of the students stated correctly that if the gas in the syringe was compressed to 5 volume units, 400 units of pressure must be applied to the plunger. Both of these tasks required proportional reasoning using 2:1 inverse ratios. However, when the students were asked to make judgments using inverse ratios which were not whole number multiples of each other, the performance on this task decreased significantly. When asked to use the picture in Figure 1 to determine either the volume or pressure of a given amount of gas when either the volume or pressure of the gas changed, only 3% of the students stated correctly that if 150 units of pressure is exerted on the plunger the gas volume would be 13.3 volume units; and, if the gas is reduced to 30 volume units, the pressure exerted on the plunger would be 66.6 pressure units. The de Berg study begins to show that there are certainly some differential understandings students have of proportional relationships as it relates to describing the behavior of a gaseous substance. The present study seeks to further add to what's known about students' understandings.

It seems that although in some instances students are able to use the mathematical representations to solve gas law problems, in other instances these representations don't seem to be as useful. That is, although students in general, tend to solve gas law problems best when using equations which depict proportional relationships, such as A/B = C/X, the use of the equation doesn't imply an understanding of the relationships presented. As a matter of fact, in the de Berg (1995) study, the roughly 55% of the students who answered the first two problems correctly using proportional reasoning, chose to use the mathematical averaging principle for the latter two problems. That is, those students who decided correctly the inverse ratio in the first two problems reasoned that because 150 pressure units is the mathematical average of 200 pressure units and 100 pressure units, the volume occupied by the gas at this pressure would be the mathematical average of 10 volume units and 20 volume units, or 15 volume units. Likewise, because 30 volume units is the mathematical average of 40 volume units and 20 volume units, the pressure exerted on the plunger to produce this volume of gas must be 75 pressure units, the mathematical average of 50 pressure units and 100 pressure units. Although the mathematical averaging principle would work for a system operating as a direct proportion, it does not work for a system operating as an inverse proportion. There is often some problems in understanding the relationship the formula is intended to show. The de Berg (1995) study, as with the study preceding it (de Berg, 1992), seems to show that students use mathematical relationships describing the behavior of a gas in interesting ways. Students do not tend to use proportional reasoning with numbers that are inverse, non-whole number multiples of each other. Also, students' uses of mathematical relationships here seem to be explained by the observation that students tend to treat the nature of a chemical or physical system as purely mathematical. That is, students typically did not attend to the conceptual significance of the pressure-volume mathematical relationship. Consequently, it may be difficult to conceive of how the system should act, and thus, use the mathematical relationship beyond its pure mathematical usage.

The present study posits the claim that chemists understand how mathematical equations predict the behavior of a system because they treat the equation as a model for the system's behavior. Students in general, find it difficult to construct mathematical models using their mathematical knowledge. The present study seeks to go beyond what's presently known by examining not only what mathematical knowledge students have, but how they use their knowledge to form mathematical models.

Summary of Part I

The studies by Gabel and her colleagues and de Berg pinpoint various ways students use their mathematical understandings of the gas laws. The students in the Gabel et al. (1984) studies preferred the use of proportional reasoning strategies when solving gas law problems. But proportional reasoning was not always necessary because the gas laws could be solved by manipulating mathematical equations. Also, students low in proportion reasoning ability were no more unsuccessful at solving gas law problems than students who had high proportion reasoning ability. De Berg examined college students as they explored the inverse proportional relationships between P and V for air compressed in a sealed syringe. The students in this study exhibited some misconceptions about proportional relationships. For example, although they used proportional reasoning for 2:1 inverse proportional relationships, they chose a mathematical averaging strategy when dealing with 3:1 inverse proportional relationships. Such a strategy would work if density data were used instead of volume data because then the system would operate as a direct proportion.

These studies suggest that students have various understandings when explaining the mathematical behavior of a gaseous substance. In this study, I explore this claim within a wider framework. Particularly, how do students use the knowledge they have to develop mathematical models for the behavior of a gaseous substance?

Part II: Students' Conceptual Understandings and Explanations of Gas Behavior

Students Macroscopic Understandings and Explanations of Gas Behavior

From the standpoint of a chemist, the atomic-molecular and kinetic molecular theories are the key explanatory ideals for explaining conceptually the phenomenological behavior of a gaseous substance. Several studies, however, indicate that students often don't get as far as explanations at the molecular level. Many students are trying to understand the phenomenon of mass, volume, pressure, and temperature and often have difficulty connecting their understanding with the explanatory ideal of the chemist.

De Berg (1995) studied 101 17- to 18-year old high school students. In responding to a paper-and-pencil instrument, which presented the diagram pictured in Figure 2, describing air in a closed system before and after compression, 66% of these students correctly answered that the enclosed volume of air is greater in situation A than in situation B. However on the average, 34% of these students did not have an adequate understanding of the

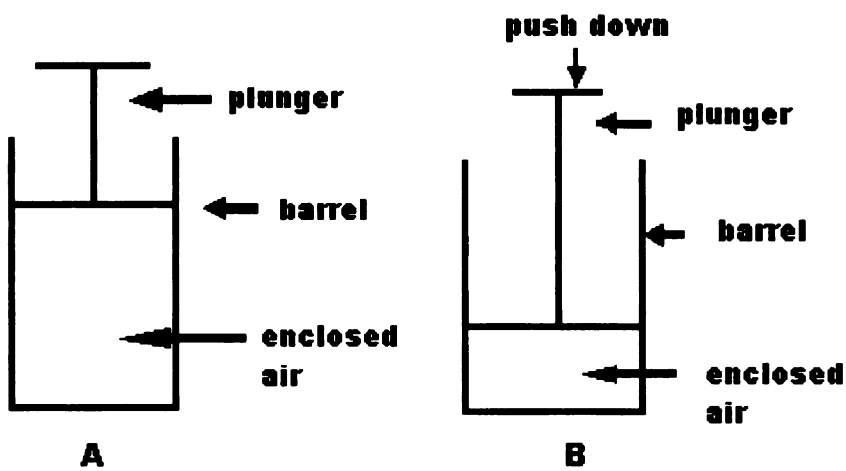


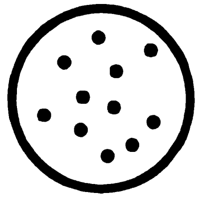
Figure 2. Syringe System Used for Qualitative Exercise in de Berg (1995) Study

volume concept. Of those who had alternative conceptions of the concept, 25% said that the volume in situation A is the same as the volume in situation B. De Berg notes that Sere (1985) in a study with 11- and 12-year olds concludes that this alternative conception of the volume of a gas could be because students relate volume with *amount* of gas. There is some further support for this idea in my study. When asked what would happen to the mass of air from situation A to situation B, 62% of the 116 students stated correctly that the mass would not change, and 38% had alternative conceptions about the concept of mass. Of those who had alternative conceptions about the concept of mass, only 19% said that the mass of gas in situation A was less than the mass of gas in situation B. These students seemed to have reasoned that the air would have a greater mass (weight) when squeezed into a smaller volume. The other 81% of alternative conceptions also suggested a confusion between mass, density, and weight. Such rationalization suggests that students tend to relate the volume of a gas to its mass, weight, and density. De Berg also noted that Stavy (1990) found that students aged 9-15 possessed these same alternative conceptions of weight and density in a floating experiment.

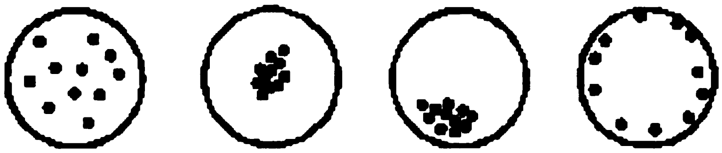
Stavy & Rager (1990) found similar results with 66 ninth- and tenth-grade Israeli students. In an interview task that asked them to determine the equality or inequality of masses of different volumes of different substances (solids, liquids, and gases), 83% correctly determined the inequality of masses of substances with different volumes. However 17% of students possessed alternative conceptions about these concepts. A common explanation was "equal volume means equal quantity." But these alternative conceptions about the variables mass and volume seem to be one-sided. That is, when Stavy & Rager (1990) asked the same students to determine the equality or inequality of the volumes of equal masses of different substances, a smaller number than before (66%) correctly answered that the volumes of equal masses of different substances would not necessarily be equal. More students (about 34%) found this a more difficult task than the reverse task.

Students often relate the volume of a gas with amount or quantity. But students also have trouble with the very idea of volume. Although students are taught the definition of volume, there is no evidence that the rote memorization of this definition gives them a sound conceptual understanding. Nurrenbern & Pickering (1987) examined over 300 college students enrolled in a first-year general chemistry course. The written exam included items testing the students conceptual and "traditional" understanding of the gas laws and stoichiometry. Traditional gas law problems were defined as the mathematical problems generally included on general chemistry exams to test for understanding, while the conceptual problem did not require the use of a mathematical formula or algorithm for its solution, as represented in Figure 3. While about 67% of the students were able to correctly solve the mathematical equation dealing with the gas laws, only about 36% were successful at solving the corresponding conceptual problem. About two-thirds of these students didn't depict the gas occupying the entire volume of the container. This is in spite of the fact that many students were able to recite the learned definition that gases occupy the entire volume of their containers. These findings have been mirrored in other studies (Sawrey, 1990; Nakhleh, 1993; Nakhleh & Mitchell, 1993). The typical understanding exhibited by students tends to be one that does not consider the notion that molecules are in constant and random motion. In the Sawrey (1990) study, the most chosen alternative conception was choice (d) in Figure 3. This is indicative of the differential understanding students often possess. Knowing that the molecules of a gas should spread out from each other to fill the container,

Conceptual Question
The following diagram represents a cross-sectional area of a steel tank filled with hydrogen gas at 20 ° C and 3 atm pressure. (The dots represent the distribution of H_2 molecules.)



Which of the following diagrams illustrate the distribution of H_2 molecules in the steel tank if the temperature is lowered to -20 ° C?



(A) (B) (C) (D)

Traditional Questions
Charles' Law
A certain sample of methane (CH_4) gas occupies 4.5 L at 5 ° C and 1 atm. What volume would the gas occupy at 25 ° C and 1 atm?

(a) 0.9 L (b) 4.2 L (c) 4.8 L (d) 22.5 L

Combined Gas Law
A given mass of gas occupies 5 L at a pressure of 0.5 atm and 5 ° C. What pressure must be maintained to store the gas at 3 L and 25 ° C?

(a) 0.32 atm (b) 0.89 atm (c) 1.5 atm (d) 4.2 atm

Figure 3. Conceptual and Traditional Questions Used in Nurrenbern & Pickering (1987) Study

students often adopt such a representation shown in Figure 3 (d). This representation, however, does not accommodate the notion that the particles of a gas are in constant and random motion.

Hwang (1995) also studied students' conceptions about the idea of gas volume. On a written exam that asked 395 Taiwanese students (102 junior high, 176 senior high, and 117 university students) to give the volume of hydrogen gas in a container with a volume of 1-Liter, Hwang reported that 30% of the junior high, 70% of the senior high, and 100% of the university students had the goal conception that the volume of the gas would be the same as the volume of the container. However, when asked to draw the volume of the gas in the container at the atomic-molecular level, about the same percentage of junior high students (30%) could correctly represent the volume of the gas; whereas, 56% of senior high students and 87% of university students could adequately describe the volume of the gas at the atomic-molecular level.

Some researchers have suggested that the problems many students have with the concept of volume are due to (1) the multiple meanings attached to the concept which students often cannot distinguish between (e.g., the student's ability to distinguish between "1-Liter" as the volume of the container, or the volume occupied by the glass which makes up the container); then, having chosen a meaning, (2) the problem with its application out of context, and (3) the confusion between the terms volume and density (Klopfer, Champagne, and Chaiklin, 1992). The Hwang (1995) study shows in addition that students' understanding of the atomic-molecular level of matter seems to further influence their conceptual understanding of gas behavior.

Students' Understanding and Explanations of the Atomic-Molecular Level of Gas Behavior

Students often explain their conceptual understandings with little regard for the atomic-molecular theory. Yet, this is the explanatory ideal the student is expected to grasp to propose good scientific explanations. Ben-Zvi, Eylon, and Silberstein (1982) have suggested that the problems students have with using this explanatory relationship to explain their conceptual understanding are in their ability to coordinate three levels of description, which chemists seem to do effortlessly. That is, students must learn to describe simultaneously (1) what's happening at the phenomenological level (e.g., the observation that a gas fills any container it is in); (2) the atomic-molecular level (i.e., the notion that a gas is made up of many particles, the most basic of which is like the others), and (3) the multiatomic-molecular level (i.e., the notion that the observed properties of matter is a consequence of the action of all of the particles which compose the matter.)

The Phenomenological Level

To explain practical systems, students must make observations. Observations of chemical and physical systems are a result of the properties of these systems. These properties present themselves as phenomena. The task of the student is to explain the phenomena observed. Students often use macroscopic language in explaining observed phenomena. That is, they simply describe what they see or feel. The phenomena of interest in the present study is the behavior of a gaseous substance. Therefore, in explaining the phenomena of the temperature of a gas, for example, students will often explain that steam (gaseous water) is hotter than liquid water without any indication of what "hotter" means from a molecular point of view. This analysis is often simply based on what they may feel. Or, they may explain that air in a closed container does not exert a pressure because they can't see it. Such explanations are not problematic in class when the manipulation of a mathematical formula is all that is required. However when these ideas must be applied to practical situations, this level of description falls far short of a true understanding.

The Atomic-molecular Level

Understanding the nature of chemical and physical systems requires a conception of the atomic-molecular theory. This theory postulates that all substances are made up of tiny particles, and it is at the very heart of chemistry. The problem is that the atoms and molecules to which this theory applies can never be seen. From the very beginning of chemistry class, students are often asked to think in terms of atoms and molecules. Some research has suggested that unless students are able to function at the Piagetian formal-operational level, understanding the atomic-molecular theory is problematic (Herron, 1975). However understanding the nature of single atoms and molecules is essential for success in chemistry. In describing the molecular makeup of a molecule of water, for example, the student must understand that one water molecule is made up of two hydrogen atoms and one oxygen atom; and, that this entity represents the simplest nature of water. According to Ben-Zvi, et al. (1982), students' difficulty in providing more significant explanations for what they observe phenomenologically is often due to their lack of understanding of the simplest nature of the phenomenon.

The elusive nature of the atom has been a great source of difficulty as students try to "invent" it for themselves. Classroom discourse may not be helpful in this regard as students try to relate what is said in the classroom to their mental model of the atom. In explaining the ability of a gas to occupy its container, for example, the commonly taught definition is that a gas will expand to fill any container in which it is placed. The students' atomic-molecular description of this phenomenon often becomes one of expanding the atom or molecule itself to fill the container. Without any actual experience of trying to reconcile the observed phenomenon with the theoretical description of the nature of matter, the student is often at a loss when an examination of their knowledge requires more than mere fact presentation.

The Multiatomic-molecular Level

As if the nature of an individual atom or molecule is not elusive enough, students are asked very soon in chemistry class to begin thinking of a collection of such units and the behavior of this collection. Explanations of chemical and physical systems require a conception of the action of many atoms and molecules together. This action is best explained using the kinetic molecular theory. The chemist's rationalization of phenomena is accomplished through this theory.

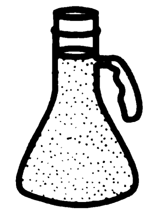
Conceiving a large collection of atoms and molecules is often a difficult prospect for the student. The sheer number of such units that a mole, for example, represents is astounding. For students to begin to think on this level is a challenging task, especially when many have not convinced themselves that matter is made up of individual units. Explanations that involve a conception of the multiatomic-molecular level are accepted as reasonable explanations of chemical and physical phenomena. Yet, many students find it difficult to conceive of this level (Ben-Zvi, et al., 1982). In the explanation of air pressure, for example, an explanation describing this pressure as the bombardment of many molecules against a given area is an acceptable definition. However, students often offer the explanation that air which shows no sign of movement is not creating a pressure.

Other studies have shown that the coordination of these three levels of description is a difficult prospect for the student, and the ability to operate at one or two levels generally suffers due to a lack of ability to operate at the other level(s). For example, Hwang (1995) studied the conceptions students at the junior high school, senior high school and university level had of the idea of gas volume. In all cases, except for junior high school where the percentages stayed the same, a greater percentage of students at each level were able to correctly judge that the volume of a gas placed in a 1-Liter container would be 1 Liter (30% junior high, 70% senior high, and 100% university) than could adequately represent that volume at the atomic-molecular level of description (30% junior high, 56% senior high, and 87% university).

Novick & Nussbaum (1981) also found that students' alternative conceptions about what is happening at the atomic-molecular level greatly influences their ability to explain correctly the phenomena of gas behavior. In a study of 576 American students (83 elementary, 339 junior high school, 88 high school, and 66 university students) asked to represent the particle distribution of a gas in a closed container, these researchers found differentiation in the abilities of students to operate at different levels of description. The percentage of students who correctly represented the uniform distribution of particles rose from the lower to the higher levels (60% elementary, 80% junior high, 90% senior high and university). However when the students were given the item represented in Figure 4, and asked to give the best representation of the air in the flask after the balloon becomes inflated, those students choosing a uniform distribution of particles dropped significantly (30% elementary, 40% junior and senior high, and 30% university). Here, more of the university students (40%) reasoned that there would be more particles in the balloon than in the flask.[1] There is evidence that students have some apparent difficulties in coordinating their atomic-molecular descriptions to explain the phenomena of gas behavior.

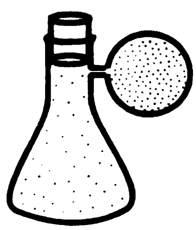
[1] This could be due to the representation these researchers gave of the system (see Figure 4) which shows the attached balloon, apparently open to the flask, as containing no particles of the gas. The observation is that, uninflated, the balloon contains no air; therefore, the obvious assumption may be that when the balloon inflates, more particles leave the flask to occupy the balloon.

A flask containing air was connected to a rubber balloon. Then the air in the flask was heated with a flame and the balloon inflated.

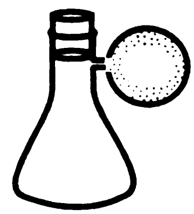


TASK NO. 8

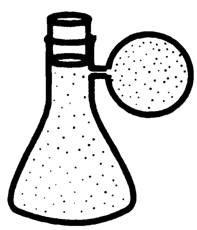
Place an X in the square next to the drawing which you think is the best description of the air after the balloon becomes inflated.



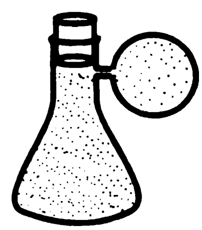
A



B



C



D

TASK NO. 9

Explain briefly how the heat of the flame affected particles in the flask.

Figure 4. Sample Task Used to Examine the Atomic-molecular Structure in the Novick & Nussbaum (1981) Study.

Benson, Wittrock, & Baur (1993) further examined student conceptions of the atomic-molecular nature of gases. They found that roughly 27% of the 191, 10-12 grade students and 64% of the 607 university students in their study (compared to 56% 9-12 grade students and 87% university students (Hwang,1995) and 90% 10-12 grade students and 90% university students (Novick & Nussbaum, 1981) were able to correctly represent the atomic-molecular description of a gas. However, these researchers further categorized the particulate representations of their students and found some interesting conceptions even among otherwise correct representations. With university students, they found that about 25% of the students represented their uniform distribution of particles as being highly packed in the container with little room between them. This could be a result of students' general tendency at all levels not to conceive of empty space between the particles (Novick & Nussbaum, 1981; Lee, Eichinger, Anderson, Berkheimer, and Blakslee, 1993). They also found that roughly 2% of these university students arranged their particles in very ordered ways when depicting the uniform distribution.

Students also exhibit differential abilities internalizing certain aspects of the kinetic molecular theory of matter, which postulates that the particles which compose matter are in constant motion. This affects their understanding of certain conceptual aspects of gas behavior. Novick & Nussbaum (1981) found that although most students in their study represented the uniform distribution of the particles of a gas in a container (60% elementary, 80% junior high, and 90% senior high and university), significantly fewer attributed this uniform distribution to inherent particle motion (15% elementary, 23% junior high, 40% senior high, and 48% university). These researchers concluded that because students cannot immediately perceive particle motion, they have difficulty with this concept. Therefore, although they are able to accept other statements of the kinetic molecular theory as plausible (e.g., the uniform distribution of gas particles), they tend to least internalize this concept of particle movement because of the cognitive difficulty it presents (Novick & Nussbaum, 1978). This finding seems to explain the conceptual difficulties researchers have discovered students have with the phenomena of gas pressure (Sere, 1985; de Berg, 1992, 1995; and Jones & Anderson, 1998) and gas temperature (Novick & Nussbaum, 1981).

Summary of Part II

There are several points of interest for this study that can be derived from the existing work on conceptual understandings of gas behavior.

First, the concepts of mass, volume, and density are often confused with each other. This seems to be the case with students at all grade levels, including college students. This could have implications for how students understand the inverse proportional relationships between the pressure and volume of a gas, and the direct proportional relationships between the pressure and density of a gas.

Second, students often operate well at a phenomenological level without an understanding of the molecular level behavior of a gaseous substance.

Hwang's work showed that students at all grade levels could well articulate what was meant by the volume of a gas. However, there was a significant decrease of students who could adequately represent this volume at the molecular level. In addition, the work of Benson et al. (1993) showed that even college students who seemed to have an understanding of the particulate nature of a gaseous substance, had some interesting understandings when further pursued. Many of them saw the particles as being highly packed and uniform in their distribution. The work of Novick & Nussbaum also showed the phenomenological understandings of students influenced by molecular understandings in interesting ways. Many of the students understood that the balloon fitted to a flask would inflate when the flask was heated. However, many attributed this behavior to more molecules of air moving out of the flask and into the balloon.

Third, students often have difficulty moving across levels of understanding. The works of Hwang and Novick & Nussbaum show the difficulties students have in explaining phenomenological behavior based on atomic-molecular descriptions. Ben-Zvi suggests that students must coordinate three levels of description for explaining the behavior of matter. The chemist tends to cross these levels with ease.

Part III: Real-World Application

Parts I and II of this chapter indicated that the problems students have in adequately explaining the behavior of a gaseous substance can be attributed to the problems they have in understanding the mathematics and conceptual nature of a gaseous substance. I believe, however, that mathematical and conceptual understanding are not sufficient in and of themselves for examining how students truly understand the behavior of a gaseous substance. Students must be able to use their understanding in real-world situations. As noted in Parts I and II of this chapter, students have various explanations about the mathematical and conceptual nature of a gaseous substance. These explanations have often been examined outside of a meaningful context; that is, a context in which students have a reason to apply their understanding. More can be learned about student understanding as they display their understanding while performing meaningful tasks.

This study is, in part, based on a model of conceptual change that examines how students come to change their alternative conceptions into scientific conceptions. It is only through the display of knowledge that students come to reveal their true understandings. In their model of conceptual change, Posner et al. (1982) focus on the conditions which they view as necessary for conceptual change to take place. These conditions seem more favorable and find salience as students try to use their understanding to do something. These researchers see the four conditions necessary for conceptual change as follows: (1) There must be *dissatisfaction* with current conceptions. That is, the student must no longer have confidence that their way of thinking is sufficient; (2) A new conception must be *intelligible*, or able to be understood by the student; (3) A new conception must appear initially *plausible*, or have a capacity for explaining the phenomena; and, (4) A new conception should be *fruitful*, or able to be extended to explain other relevant systems. The condition of fruitfulness is the most relevant in this study as students try to make real-world applications.

Smith (1990) used the model of Posner et al. (1982) in his work with preservice elementary teachers. In the classroom, these prospective teachers were presented with a real task to explain: for a book resting on a table, does the table push on the book? By explaining and ratifying their understandings as a class, in a socially meaningful environment the author suggests that the students were able to gain a better understanding.

> The students' experience in the demonstration lesson was unusual or unique for them not only because they felt that they understood, but also because of what they were and were not doing. Rather than simply receiving and remembering information, they engaged in a process in which they drew on their own knowledge, reasoned and argued, inferred and concluded. During this process they became convinced of the plausibility and value of thinking about phenomena in a new and, not only different, but initially counterintuitive way. Such a process is frequently required for learning science with understanding. (p. 52)

By so stating, the implication by Smith is also that students found knowledge acquired in this socially meaningful environment as useful in explaining discrepant events. In other words, the knowledge they acquired became fruitful to them as they explained real-world systems.

Anderson & Roth (1989) built upon the conditions of conceptual change proposed by Posner et al. by proposing two broad aspects of how students come to achieve conceptual change. The first they refer to as "conceptual integration." That is, students are considered to have achieved conceptual change and understood a scientific principle or theory to the extent they have integrated an accurate formulation of that principle or theory with their current ways of understanding. The second, they refer to as "usefulness." That is, students are considered to understand a principle or theory if they can use it to make sense of the world around them. It is this use of value that seems particularly salient here.

Anderson & Roth (1989) identify four general categories that group the activities of scientifically literate people. These are *description*, *explanation*, *prediction*, and *control*. Description, as one activity of a scientifically literate adult, is the ability to provide precise and accurate names, descriptions, or measurements of natural systems or phenomena. Explanation is the process of using scientific knowledge and theories to explain natural phenomena. Prediction involves the ability to generate accurate predictions about future observations or events. And, finally, a scientifically literate adult should be able to use scientific knowledge to control natural systems and phenomena.

Examining how students use their mathematical and conceptual knowledge to explain the behavior of a gaseous substance when performing real tasks should be helpful in analyzing how students truly understand the behavior of a gaseous substance.

From the discussion above, it seems evident that a complete understanding of how students develop in their understanding of the behavior of a gaseous substance is not yet available. A deeper understanding of science seems evident when students can effectively put their knowledge to use in order to describe, explain, predict, and control their environment. An understanding of how students do this when explaining the behavior of a gaseous substance is useful. The literature - mainly, concentrated in the misconceptions literature - is replete with accounts of what students understand about the behavior of a gaseous substance. It is silent on how students use their understanding to explain the behavior of a gas, and, thus, how students truly understand this behavior. This study addresses this issue.

Summary of Chapter Two

The purpose of Chapter 2 is to present the reader with a theoretical basis for understanding the major premises of this study. It has been documented that students have many difficulties understanding and explaining the behavior of a gaseous substance in a way that a chemist would understand and explain this behavior. The reasons for these difficulties are complex. The source of these difficulties are assumed to lie in student problems in acquiring three kinds of understanding: (1) mathematical understanding, (2) conceptual understanding, and (3) real-world application. This chapter has reviewed some of the available literature that has addressed each area. Each of these has raised some issues of interest for this dissertation and for chemistry education. These issues are identified and summarized below.

Mathematical Understanding

Chapter 2 reviewed a few studies broken down along two lines: (1) how students understand proportional relationships relating to the gas law equations, (2) how students use proportional relationships relating to the gas law equations.

All studies used paper-and-pencil instruments as a means to assess this understanding.

An important finding of the studies by Gabel and her colleagues was that although high school chemistry students preferred the use of proportional reasoning while being taught the gas laws, many of them could solve gas law problems without using proportional reasoning strategies. This suggests that even students who are considered to really understand the mathematics of the gas laws, don't truly understand the mathematics the gas law equations are meant to convey.

The de Berg study, which had students explain mathematically the compression of air within a syringe, found that when students use their understanding of proportional relationships, they do so in some differential ways. De Berg advances the claim that the students are being forced to apply context-specific knowledge out of context. This is a useful theoretical framework from which to examine how students use their mathematical knowledge.

There are three issues pertaining to students' acquisition of mathematical knowledge that are raised by these studies.

ISSUE 1: The Understanding of Proportional Relationships. How do college students understand proportional relationships?

ISSUE 2: The Creation of a Cohesive Mathematical Model. In what ways do college students use their mathematical knowledge?

ISSUE 3: The Use of Gas Law Equations. How useful do college students find the gas law equations?

Conceptual Understanding

This chapter has reviewed some studies which examined the understandings students across all levels have about the variables used to describe gas behavior. Particularly, the studies examined how students have understood the concept of volume. These studies have examined both the phenomenological and atomic-molecular understandings of volume possessed by students.

De Berg, Stavy and Rager showed that students are often confused by the concept of volume. Many times they confused volume with mass and density. An interesting theoretical consideration is how students' confusion about these three quantities play into their mathematical notion of the pressure/volume inverse proportional relationship and the pressure/density direct proportional relationship.

Novick & Nussbaum (1981), Hwang (1995), and Nurrenbern & Pickering (1987) showed that students phenomenological understanding of the behavior of a gas does not have to be in synchrony with their notions of the particulate nature of matter. Students are often well able to explain what they see apart from an adequate understanding at the submicroscopic level.

Ben-Zvi proposes a theoretical framework to examine why students often don't connect the molecular and the phenomenological. She and her colleagues suggest that students need to connect their knowledge across three levels of understanding: the phenomenological, atomic-molecular, and multiatomic levels. She would suggest that a great deal of specific knowledge is required at each of these three levels before the student will have the knowledge to explain the conceptual nature of gas behavior. I will use the theoretical framework of Ben-Zvi as a basis of analysis for the conceptual knowledge used by students as they explain the behavior of a gaseous substance.

There are two issues pertaining to students' acquisition of conceptual knowledge that are raised by these studies.

ISSUE 4: The Understanding of the Atomic-molecular and Kinetic Molecular Theories. How do college students understand these theories?

ISSUE 5: Atomic-molecular vs. Macroscopic Descriptions of Phenomena. What are the explanations used by college students when describing the behavior of a gaseous substance?

Real-World Applications

Ideas about the application of scientific knowledge in real-world contexts was derived from the work of Posner et al. (1982), Smith (1990), and Anderson & Roth (1989). Particularly, scientific knowledge was examined for its use value. The model for conceptual change proposed by Posner et al. (1982) implies that students come to understand scientific ideas after becoming dissatisfied with their current notions when using them, and then adopting a view which has a capacity to make sense to them (intelligibility), is capable of being understood by them (plausibility), and is able to explain other discrepant events (fruitfulness). It is my contention that these four conditions for conceptual change are best achieved as students' knowledge is tested in use. Smith (1990) demonstrated such a test of knowledge by presenting a use task to a group of prospective teachers. As they worked to solve the problem, many of their misconceptions were made apparent and they were eventually able to achieve conceptual change.

Anderson & Roth (1989) suggested that the use value of scientific knowledge of scientifically literate adults is grouped according to four categories of activities: description, explanation, prediction, and control. As scientifically literate adults are able to perform these tasks, they are thought to have achieved scientific understanding. The theoretical framework as presented in these studies is a useful one in which to examine how the students in this study perform the real-world task of compressing air in a syringe and explain the conceptual problems on the paper-and-pencil instrument. Real-world understanding will be examined in the context of the students' mathematical and conceptual understandings, and, therefore, examined with the five issues listed above.

CHAPTER THREE

METHODOLOGY

Introduction

The purpose of this chapter is to identify how the students were selected, and the methods of data collection and analysis. In this chapter, I will show how the questions on the paper-and-pencil instrument allowed me to collect information on students' mathematical, conceptual, and practical understandings. I will show how the questions asked of students in the clinical interviews allowed me to gather the necessary data to address the research questions posed in the previous chapter.

This chapter contains an overview of the study; a description of the subjects and setting; a description of how the data were collected, including an explanation of the paper-and-pencil instrument and the clinical interview technique; and an explanation of how the data were analyzed.

Overview of Research Design

A flowchart for the data collection and analysis in this study is shown in Figure 5. The heart of the study is an in-depth examination of how four students came to understand and explain gas behavior. In all, 9 students were clinically interviewed out of approximately 116 who took the posttest paper-and-pencil instrument. The instrument (Appendix A) was designed to measure the mathematical, conceptual, and practical understandings of the students.

The instrument was used to categorize each of the 116 students into one of nine groups: high mathematical/high conceptual (HMHC), high mathematical/medium conceptual (HMMC), high mathematical/low conceptual (HMLC), medium mathematical/high conceptual (MMHC), medium mathematical/medium conceptual (MMMC), medium mathematical/low conceptual (MMLC), low mathematical/high conceptual (LMHC), low mathematical/medium conceptual (LMMC), and low mathematical/low conceptual (LMLC).

Three students were chosen from each of the four groups above that contained the highest percentage of students (see Table 1): LMLC (28.4%), MMLC (24.1%), MMMC (14.7%), and MMHC (11.2%). In all, twelve students were slated to participate in the clinical interviews. However, I was not able to get more than two students in the LMLC and MMLC categories who would agree to talk about their understanding. In addition, one of the audio tapes produced from the clinical interview of one MMMC student was inaudible due to a faulty microphone. Consequently, this study ultimately involves nine students who were clinically interviewed. Four of these students form the crux of this study: Cameron (MMHC), Betty (MMMC), Karen (MMLC), and Connie (LMLC). General claims are made about the other five students based on case study analyses of these four students. The five students are Nina (MMHC), Janice (MMHC), Donna (MMMC), Sherry (MMLC), and Hilda (LMLC).

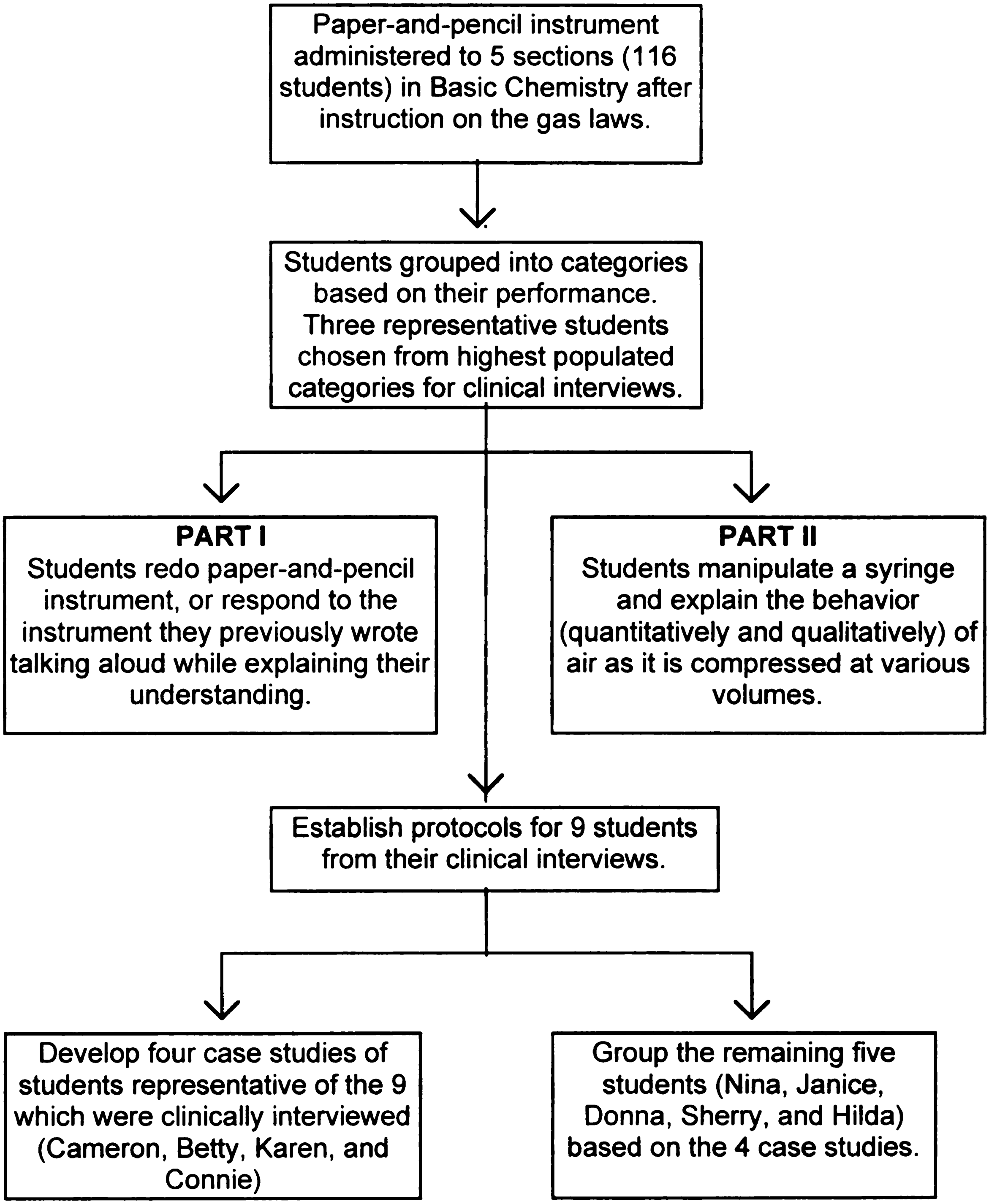


Figure 5. Flowchart for Data Collection and Analysis

Subjects and Setting

This study involved five of the six introductory, basic-level chemistry classes in a Midwestern community college. Each section of the course was taught by different instructors, except for two sections which were taught by the same instructor. All sections used a common syllabus and very similar exams. In general, each instructor followed a lecture presentation which presented the objectives outlined in Appendix B. Although there were some differences based on the style of the instructor, for the most part, due to stringent guidelines of exam dates, instructors for each section taught the same material in reasonably consistent ways.

All students were grouped into one of nine categories according to their responses on the paper-and-pencil instrument. Four of the nine groups that contained the highest percentage of students were chosen for study, and three students from each of these four groups were chosen to participate in the clinical interview. Because this is not a quantitative study for which I seek to make any statistical claims, a random sampling of representative students from each category was not attempted. Rather, I chose students from each of the four groups based on their placement in that group by their scores on the paper-and-pencil instrument, and their willingness to talk about their understanding.

Data Collection

The Paper-and-Pencil Instrument

This instrument was designed to uncover students' mathematical, conceptual, and practical knowledge of gas behavior. The problems are presented as mathematical/conceptual pairs. That is, one problem in a pair is a question which requires the use of an equation or algorithm for its solution. The other problem in the pair describes a similar situation as the first problem, but requires a conceptual understanding of the volume, pressure, and mass of a gas as explained by the atomic-molecular and kinetic molecular theories.

The instrument contains five items, four of which (items 1-4) have been taken from the literature used in studies which have examined students' mathematical and conceptual understandings (Nurrenbern & Pickering, 1987; Sawrey, 1990; Nakhleh, 1993; Nakhleh & Mitchell, 1993; de Berg, 1995; and, Noh & Scharmann, 1997). The second mathematical/conceptual pair of problems actually contains one mathematical problem and two conceptual problems. The second conceptual problem (item 5) was added because of the results of two pilot studies and other reports in the literature (de Berg, 1995) which suggest that students may be showing a different conception about pressure than what the first conceptual problem was intended to measure. Because the conceptual problems modeled real-world tasks, they were also used to examine students' practical understanding of the behavior of a gaseous substance.

The Clinical Interview

In all, 10 students were clinically interviewed after they completed the paper-and-pencil instrument. The interviews were scheduled at a time convenient for the students and conducted in a conference room away from their classroom setting. The interviews lasted between 30-45 minutes.

The clinical interview consisted of two parts. In the first part, students were asked to exhibit their understanding in one of two ways. Some of the students were given the paper-and-pencil instrument they originally completed as a posttest and asked to talk aloud as they explained their thinking on selected items. Other students were given a blank copy of the paper-and-pencil instrument and asked to resolve selected problems while explaining their understanding aloud. It was found in some instances that students reworked items differently than they had worked them before. These differences are noted and analyzed for their significance.

In the second part of the interview, students were given a syringe and examined on how they used their knowledge of gas behavior to answer questions pertaining to the behavior of air on the inside of the syringe. Questions asked of the students during this part of the interview were used to analyze how students used their mathematical and conceptual knowledge while performing a real-world task.

Data Analysis

The data analysis focused on the 9 students who were interviewed after instruction. There are two stages to the data analysis process. During the first stage, detailed case studies of four students were prepared. These students were given the pseudonyms Cameron, Betty, Karen, and Connie. During the second stage, the analytical framework developed for the case studies of Cameron, Betty, Karen, and Connie was extended to the other five students. All nine students are then classified according to their mathematical and conceptual understanding as having goal conceptions, having naïve conceptions, or being in a transitional state between goal and naïve conceptions.

Stage 1: The Case Studies of Cameron, Betty, Karen, and Connie

The literature review in Chapter 2 produced five issues which seemed relevant to this study. All of the nine students who were clinically interviewed addressed a majority of these issues in a satisfactory manner. Cameron, Betty, Karen, and Connie were chosen for in-depth analysis because they were articulate in explaining their views and possessed a great ability to talk about what they understood.

In the development of the case studies, emphasis was placed upon development of a coherent framework that would provide a sensible and consistent explanation of Cameron, Betty, Karen, and Connie's responses to the paper-and-pencil instrument and in the clinical interviews. The guidelines used to develop this framework were the categories of mathematical understanding, conceptual understanding, and real-world application, and the five issues which emerged from the literature review in Chapter 2. The central problem of these case studies lies in trying to determine where Cameron, Betty, Karen, and Connie stand on the five issues.

Stage 2: How the Remaining Five Students Were Analyzed

The four students chosen as the case studies were representative of the other students in the sample. While all students were clinically interviewed, detailed case studies were not prepared as part of this dissertation. Rather, the comparisons between the four and the remaining five were done by focusing upon the similarities in responses to the relevant issues identified in the four case studies. All of the nine students were then classified according to their mathematical and conceptual understanding as possessing the goal conception, naïve conceptions, or being in transition.

Specific Descriptions of Data Collection and Analysis

Mathematical Understanding Questions

The literature review in Chapter 2 indicated that while the manipulation of mathematical representations is generally the way students use math to solve gas law problems, mathematical understanding is often elusive. Students use some differential knowledge when applying their mathematical understanding of the gas laws. I argue that mathematical knowledge is a prerequisite for the development of a student's ability to use the mathematical representations in a meaningful way; that is, as models to describe the behavior of a gaseous substance.

From the literature review in Chapter 2, three issues emerged from the discussion of mathematical knowledge. These issues seem relevant in understanding how students use their understanding of mathematics to form a mathematical model of gas behavior. Each issue will be listed and the questions on the paper-and-pencil instrument that address these issues will be reviewed with some commentary on the expected response.

ISSUE 1: The Understanding of Proportional Relationships.

The understanding students have of how the gas laws represent proportional relationships is used as one measure of how they use gas laws as mathematical models. This issue is addressed on the paper-and-pencil instrument and again during the clinical interview. Item 1 on the paper-and-pencil instrument asks students to solve a typical gas law problem as presented during instruction. A student who understands this item would use a gas law equation to solve it. The student would either use the algebraic representation of the law or a ratio method in which the initial pressure is multiplied by a ratio of absolute temperatures. The ratio of absolute temperatures would be written with the smaller absolute temperature in the numerator and the larger absolute temperature in the denominator because such a ratio, when multiplied by the initial pressure, would give a decrease in the value of the initial pressure in accord with the direct proportional relationship between the pressure and temperature of a gaseous substance.

During the first part of the clinical interview, students are asked to discuss their understanding of the gas law equation they used to solve this problem. Responses mainly to the mechanics of the equation's setup is considered as one indication of a mechanistic understanding of the equation. With a mechanistic understanding, the student attends only to the mechanics of the equation (i.e., plugging in appropriate numbers, solving for given variables, canceling units, etc.) without giving attention to the nature of the relationships between the variables (i.e., proportionality) contained within the equation. Consequently, responses attending only to the mechanics of the equation are considered indicative of the students' lack of mathematical model development. Responses which give some indication of a relationship between the variables of the equation are considered as an indication of the students' mathematical model development. For example, the student will explicitly state or imply some relationship between variables in the equation (e.g., use of ratio method).

Item 3 on the paper-and-pencil instrument is also used to explore students' understanding of proportional relationships. On the one hand, a student who understands this item would consider the syringes presented and recognize the pressure and volume relationships there. They would then use proportional reasoning to answer the items. For item 3(i), since 25 pressure units is half of the 50 pressure units exerted on the plunger pictured in the first syringe, then the volume at 25 pressure units will be doubled to 80 volume units. Likewise, for item 3(ii) since 5 volume units is one-half the volume pictured in the third syringe, the resulting pressure at 5 volume units would be twice the pressure at 10 volume units, or 400 pressure units. In item 3(iii), since 150 pressure units is three times the pressure exerted on the plunger pictured in the first syringe, the volume occupied by air at this pressure would be one-third of 40 volume units, or 13.3 volume units. Likewise, in item 3(iv) since 30 volume units is three times the volume represented in the third syringe, the pressure units at 30 volume units would be one-third of the pressure at 10 volume units, or 66.7 pressure units.

On the other hand, a student who understands item 3 might use a gas law equation or the ratio method similar to the one used with item 1. With a gas law equation, a student would identify initial and final pressures and volumes then solve the equation for an unknown value. With the ratio method, students would identify the values, compose the appropriate ratio for the inverse relationship, and multiply the initial volume or pressure by this ratio.

During the first part of the clinical interview, students are asked to discuss their understanding of item 3. Students' use of the proportional reasoning strategy mentioned above is considered indicative of their proportional reasoning ability. Using the gas law equation or ratio method does not give a direct indication of the students' proportional reasoning ability unless specifically indicated by the student.

ISSUE 2: The Creation of a Cohesive Mathematical Model.

Students' mathematical understandings allow them to create mathematical models and make predictions based on their models. These predictions at many times take the form of estimations because students often make their predictions based on other values which have in some way resulted from their mathematical model. Particularly, the consistency between how students articulated their knowledge in Part I of the clinical interview when explaining items 1 and 3 and how they used this knowledge for predicting values on the real-world task in Part II of the interview, is taken as indicative of the students' use of their own mathematical model. I have referred to these models as *personal models*. Such models are empirical claims about patterns as seen in the interview data.

ISSUE 3: The Use of Gas Law Equations.

Students' use of gas law equations will also be considered indicative of their mathematical modeling. The studies reviewed in Chapter 2 showed that students can use gas law equations without understanding them. Student understanding of the gas law equations was explored in item 1 and item 3 on the paper-and-pencil instrument. During the clinical interviews, students were asked to discuss their understanding. The knowledge possessed by a student who understands these items is discussed with Issue 1.

The use of a gas law was expected of item 1. If students did not use a gas law here, they were considered to have an extremely limited understanding of the gas laws. If a student used a gas law to solve item 3, they were considered to have a wider appreciation for the value of the gas law and its usefulness as a mathematical model to describe the behavior of a gaseous substance if they could simultaneously talk about proportional relationships between variables in the gas law equation.

Conceptual Understanding Questions

During the clinical interviews students were asked to discuss their understanding of the atomic-molecular and kinetic molecular theories. Students were given a diagram of a syringe and asked to draw what they thought air in the barrel of the syringe would look like at the submicroscopic level.

ISSUE 4: The Understanding of Atomic-molecular and Kinetic Molecular Theories.

Students' drawings and explanations of these drawings during the clinical interview were examined. Although many of the students could not remember these theories by name when mentioned in the clinical interviews, this was not considered as indicative of their lack of understanding. Instead, their notions were pursued simply by having them talk about what they understood of the particulate nature of matter.

ISSUE 5: Atomic-molecular vs. Macroscopic Descriptions of Gas Behavior.

Items 2, 4, and 5 on the paper-and-pencil instrument allows the analysis of students' explanations about various phenomena of gas behavior. During the clinical interviews, students were asked to explain their understanding of these items. A student who understands item 2 would say the distribution of molecules in the tank after the temperature drops would be similar to the representation depicted in choice (A). This is because hydrogen would still be a gas at the lowered temperature as it is still above its boiling point. Consequently, according to kinetic molecular theory, the molecules would spread out and randomly fill the entire volume of the tank.

A student who understands item 4 would say the volume of air in the syringe would decrease upon compression because the space occupied by the air will decrease. In addition, the student would understand that the mass of the air in the syringe will not change upon compression because no air has leaked in or out of the barrel of the syringe. Finally, the student who understands this item would say the pressure exerted by the air in the barrel of the syringe would increase upon compression because the molecules of air have been squeezed into a smaller space. Consequently, the molecules will hit the walls of the syringe barrel with greater frequency.

A student who understands item 5 would say the pressure of enclosed air in the syringe barrel is the same as standard atmospheric pressure if the plunger is not moving. This is because, if the plunger is not moving, the pressure exerted on the plunger in one direction (atmospheric pressure) must be the same as the pressure exerted on the plunger in the opposite direction (air pressure inside the syringe barrel).

Students who used explanations that were more visual and phenomenological in nature were classified as using macroscopic explanations. Such explanations are expected for items 4(i), 4(ii), and 5. However, items 2 and 4(iii) are best explained with explanations at the submicroscopic level. Students were classified as attending to submicroscopic explanations when they explained these items based on considerations of molecules and their movement. For example, a response like, "The pressure created in this syringe is greater because the molecules are closer together and bounce off the walls more."

Real-World Application Questions

Students' real-world use of their mathematical understanding was analyzed as they used the real syringe to answer quantitative questions. Students' real-world use of their conceptual understanding was analyzed as they used their understanding of the atomic-molecular and kinetic molecular theories when responding to items 2, 4, and 5 on the paper-and-pencil instrument. In this study, I have suggested that the meaningful use of mathematical and conceptual knowledge when performing real-world tasks is a measure of how students truly understand gas behavior. Therefore, in this study students' real-world use of their knowledge is examined as a part of the above five issues and discussed in those sections.

CHAPTER FOUR

RESULTS

Introduction

The students presented in the following case studies were picked from the groups containing the higher percentage of students as noted in Table 1. The

Table 1. Percentage of Students in Mathematical/Conceptual Categories Based on Results of the Paper-and-Pencil Instrument (N = 116)

	High Math Achievement	Medium Math Achievement	Low Math Achievement	Totals
High Conceptual Achievement	0.0	**11.2**	6.0	17.2
Medium Conceptual Achievement	2.6	**14.7**	8.6	25.9
Low Conceptual Achievement	4.3	**24.1**	**28.4**	56.8
Totals	6.9	50.0	43.0	99.9

Boldface percentages represent categories of students chosen for interviews.

students are grouped initially into categories based on their correct or incorrect responses to the items on the posttest. Most of the students fall into the medium mathematical and low conceptual categories. In the present study, about 50.0% of students are initially categorized as being between high and low in their mathematical achievement, and 56.8% are initially categorized as low in their conceptual achievement. The case studies presented in the next section examine the understanding of a typical student in the MMHC (Cameron), MMMC (Betty), MMLC (Karen), and LMLC (Connie) categories. The results suggest that the initial categorization of these students is not indicative of their true achievement. That is to say, those students achieving medium mathematical and low mathematical proficiency often share some common misconceptions in spite of the categories in which they're initially placed. The same can be said of students in the high, medium, and low conceptual categories.

Case Study 1: Cameron

Cameron's Mathematical Understanding

Explanation of Item 1 on the Paper-and-Pencil Instrument

On the paper-and-pencil instrument, Cameron was asked to solve a typical gas law problem presented in his basic chemistry class. He uses the relationship:

$$P_2 = P_1 \text{ x } T_2/T_1$$

During the clinical interview, Cameron is not shown his original problem, but is given a blank copy of the posttest and asked to rework and explain his solution for item 1. During the interview, Cameron sets up and solves the relationship:

$$P_1/T_1 = P_2/T_2$$

He is asked about his understanding.

> **Interviewer:** Do you remember how you were taught to work the problem?
>
> **Cameron:** ...First you want to set it up with what's given, which is the volume, and the pressure, and temperature. Calculate the pressure in atmospheres if the temperature is changed. So, the volume's going to stay the same cause it doesn't say anything about it.

When explaining how to work this problem, Cameron gives some indication that he is attending to the proportional relationships between P and T.

> **Cameron:** The pressure's going to change because the temperature has been changed...Pressure goes down, temperature goes down...They're proportional, directly proportional.

Cameron gives an indication that he is aware of the proportional relationships between variables in the equation.

<u>Explanation of Item 3 on the Paper-and-Pencil Instrument</u>

On the paper-and-pencil instrument, Cameron further demonstrates his understanding of proportional relationships. When he initially completed item 3 on the posttest, Cameron began by using a gas law equation to solve the problem. For item 3(i) he sets up the relationship:

$$V_2 = 40 \text{ x } 25/50$$

However, he abandons this relationship for another strategy. This is obvious because he finally reports "10" volume units as an answer instead of the 20 volume units the relationship above would produce.

During the clinical interview, Cameron was not shown his initial solution to this problem, but was given a blank copy of the posttest and asked to rework item 3(i) and explain his solution. Cameron does not explicitly use a gas law equation this time, but he does use a proportion strategy.

Cameron: Goes from 50 (pressure units) to 25 (pressure units)...and it had 40 volume units when it was full. So you take it to half the pressure. So it's, um, 20 volume units then.

Even though Cameron incorrectly uses 2:1 direct ratio reasoning in solving item 3(i) instead of 2:1 inverse ratio reasoning, he does use ratio reasoning. However, for item 3(ii), which is also a 2:1 inverse ratio problem, there is an addition component in his proportion reasoning.

Cameron: I think it's 250...From this one (*referring to the diagram*), there's 200 pressure units on 10 volume units...So when it goes 5 volume units, um, I said it was 250 because that's kind of the descent they all took.

Cameron initially believes that as the volume is cut by a factor of one-half, the resulting pressure is increased in increments of 50 units. Although he conceives of the pressures as additive of each other, he conceives of the volumes as some multiple of each other.

Cameron: ...So that's (the volume) twice as much as on this one.

Interviewer: ...Twice as much volume?

Cameron: Um, or half the volume.

When initially working item 3(ii) during the clinical interview, Cameron uses a strategy in which he performs an addition to predict the pressure that a fourth diagram at 5 volume units might have. When asked to comment further on his strategy, Cameron refers to the idea of proportional relationships and experiences dissatisfaction with his initial response.

Interviewer: ...So you said this was 250 because...?

Cameron: Just the way the proportionate was. From 100 (pressure units), there's 20 volume units. Then 200 (pressure units), it went to 10 volume units. But, um, for 5 volume units, um, I guess that it was 300 (pressure units)...Or it doubles...maybe it's 400, because they're proportionate...So that will be, um, 400 pressure units for 5 volume units.

Towards the end of his explanation above, Cameron is once again pursuing proportional reasoning. However, Cameron's strategy changes again when he considers the 3:1 inverse ratios in items 3(iii) and 3(iv).

On the paper-and-pencil instrument he took as a posttest, Cameron responded the same way to these problems as he responds during the clinical interview. During the clinical interview, he explains how he works the problems. For these problems, he uses an averaging strategy.

Cameron: When you put 150 pressure units on it, I said it was 15 volume units cause it's in between these (*referring to the second and third syringes at 100 and 200 pressure units, respectively*)...And half of that (*referring to the volume of air in the second and third syringes at 20 and 10 volume units, respectively*) would be 15.

Likewise, Cameron explains how he thought about item 3(iv).

Cameron: ...I said it was 75. It's between 50 and 100 pressure units.

<u>How Cameron Understands and Uses the Mathematical Representations of the Gas Laws</u>

Cameron is a mathematically proficient student who knows how to manipulate the gas law equations well. However, it would be inaccurate to think Cameron possesses the mathematical understanding of the gas law equations the chemist has. He thinks about proportional relationships in a number of ways, and these ways often compete and conflict with each other. But Cameron knows well the mechanics of the equation and how to set the formula up to obtain an answer. Consequently, when there is conflict, Cameron tends to place his focus on the mechanics of the equation because this causes the least conflict for him.

Interviewer: ...Does the way the equation's set up make sense to you?

Cameron: ...At first it didn't, but I knew once I got all my givens and I put it in the formula and I knew what the formula was...then it made sense to me...Cause I knew that they were going to be proportional.

Cameron uses the term "proportional" quite frequently. But his understanding of "proportional" as it relates to the mathematical representations of the gas laws involves a number of strategies. In working through the gas law equations, he makes constant reference to proportional relationships among the variables P, V, and T. His ultimate understanding of these relationships, however, is not the understanding chemist's possess. Chemists attend to the direction (i.e., direct or inverse) and the magnitude of the proportional relationships between P, V and T. They also understand these relationships to be factors of each other.

I have noted two particular tensions which seem to exist in the proportional relationships in Cameron's mathematical model of the gas law equations: *direction vs. magnitude* and *multiplicative vs. additive*.

Direction vs. Magnitude with Proportional Relationships

In general, Cameron attends to the directional nature of proportional relationships more so than the magnitude of these relationships. He knows, for example, as "pressure goes down, temperature goes down." That means to him that P and T are "proportional, directly proportional."

A chemist would think about this problem in terms of the factor decrease of the absolute temperature and the proportional effect this has on the pressure. That is, since the absolute temperature decreased by a factor of 268 K/298 K, the pressure will also decrease proportionally by the same factor.

On the posttest, Cameron does set up the gas law equation which indicates the factor effect of the temperature change on the pressure:

$$P_2 = P_1 \times T_2/T_1$$

However it is not certain how he used the above relationship to obtain his answer. Since he used a different arrangement of the relationship during the clinical interview, he was not specifically asked about the one above. It is noted, however, that Cameron could attend solely to the mechanics of the gas law equation above and be just as successful at obtaining the correct answer without having an understanding of the effect of the ratio T_2/T_1 on the pressure of the gas.

On the posttest, Cameron used a similar gas law relationship as that noted above. In responding to item 3(i), he wrote:

$$V_2 = 40 \times 25/50,$$

which, when represented by the proper variable labels in the algebraic setup is,

$$V_2 = V_1 \times P_2/P_1$$

The relationship Cameron is trying to use is correctly expressed as,

$$V_2 = 40 \times 50/25,$$

because the proper algebraic set up of the relationship is,

$$V_2 = V_1 \times P_1/P_2$$

However, in spite of the fact that his setup is incorrect, he uses the equation more in its capacity as a computational devise. The attention he gives to writing the pressure ratio seems more a matter of convenience for him than attention to the proportional relationship between the pressures.

During the clinical interview, Cameron uses a different relationship for working item 1 than he used on the posttest. He uses the algebraic setup,

$$P_1/T_1 = P_2/T_2$$

Cameron's use of the equations on his posttest as well as during the clinical interview shows he tends to use the gas law equations in a mechanical way with no focused attention given to the magnitude of the resulting change in pressure. He notes that when "pressure goes down, temperature goes down," but when asked if his setup of the equation makes sense to him, he attends to mechanical issues: finding the "givens," writing down the formula, and putting the values in the formula.

Multiplicative vs. Additive Character of Proportional Relationships

Although Cameron's idea of the directional relation between P and V in this item is incorrect, his response concerning the magnitude of the change reveals a proportion strategy. That is, he responds that the resulting volume is some multiple of the pressure change. Item 3(i) actually uses 2:1 inverse proportion reasoning. The chemist would understand that a one-half change in volume proportionally produces twice the pressure.

Item 3(ii) also uses 2:1 inverse proportion reasoning. Here, however, Cameron reveals an additive understanding which operates in conjunction with his multiplicative understanding. That is, each time the volume was cut by a factor of one-half, Cameron initially conceived of the volume changing in increments of 50 units. The chemist understands these relationships to be multiplicative and not additive. Cameron eventually moves to the chemist's understanding after experiencing some dissonance during his explanations. Therefore, Cameron's understanding here seems transitional.

Cameron's Conceptual Understanding

Describing the Submicroscopic Nature of Matter

During the clinical interview, Cameron is asked to draw a pictorial representation of what air enclosed in a syringe looks like at the submicroscopic, or atomic-molecular level. The chemist understands the submicroscopic level of air to be composed of molecules that move freely and independently, randomly occupying all of the space available in the container. The chemist also understands that the movement of the molecules is inherent in all states of matter. That is, molecules are always moving in some way in solids, liquids, and gases, and they need not be propelled by some outside source.

When Cameron draws his representation of the submicroscopic level of air in a syringe, he presents a picture that shows the particles randomly and completely filling the volume of the syringe. He explains his drawing.

Cameron: It'd just be kind of scattered all over. Just make little Xs or whatever for the molecule thing...It would be just kind of scattered about in it...random in its container that it's in.

Cameron seems satisfied with the notion of the atomic-molecular theory, but he admits that the kinetic molecular theory doesn't totally make sense to him.

Interviewer: The notion that substances are composed of these molecules...Make sense to you?

Cameron: Um hum.

Interviewer: How about the kinetic molecular theory?

Cameron: Um, no not really.

Interviewer: And it doesn't make sense...?

Cameron: ...Those particles are always moving.

Interviewer: And that doesn't seem to make sense to you?

Cameron: In the gas it seems like it makes more sense to me than it would in the solid or something like that...Or a liquid even...It just makes more sense in a gas. To be more free to move around.

Cameron conceives of the particle nature of matter, but his limited understanding of kinetic molecular theory creates some misconceptions. For example, although Cameron seems to understand the random and constant movement of the particles of a gas, he doesn't seem to concede to the inherent motion of the molecules. That is, he believes that something must propel the molecules to move. For him, this seems to be easier to do in the gaseous state than in the solid or liquid state.

Cameron: ...Kinetic has to do with motion...So they're (molecules) constantly in motion...And I guess the motion depends on the pressure that's on it...what kind of container it's in.

Describing the Phenomenological Behavior of a Gaseous Substance

When chemists describe phenomena such as changes in volume, pressure, and temperature, they do so using the atomic-molecular and kinetic molecular theories as an explanatory ideal. That is, they explain the resulting volume, pressure, and temperature of a gaseous substance in terms of the movement of molecules.

Cameron shows he can use the explanatory ideal of the chemist. When considering item 2 during the clinical interview and explaining why there should be a drop in pressure when the temperature decreases, Cameron explains:

Cameron: ...Temperature lower, pressure lower. Temperature higher, pressure higher. Cause the molecules aren't going to be moving as fast.

Cameron also explains why the gas in item 2 would still occupy the entire volume of the tank at a lower temperature.

Cameron: ...It's (the lowering of the temperature) not gonna like constrain it into one certain section of the steel tank...Cause it's (the molecules) free to roam around the whole tank...Not just a little part of it there.

How Cameron Understands and Uses the Concepts

Cameron does not possess fully the chemist's understanding. In discussing his understanding of the kinetic molecular theory, he reveals a basic misconception: the particles of a gas don't have inherent motion, but they are placed into motion by something external to them.

Cameron also reveals an ability to explain phenomena such as volume and pressure using the language of particles. He explains these phenomena in terms of the motion of molecules. Consequently, like the chemist, Cameron finds the explanatory power of the atomic-molecular and kinetic molecular theories useful in this regard.

Cameron's Real-world Applications

Chemists are able to use their understanding of the mathematical and conceptual aspects of gas behavior to perform tasks in the real world. They are able to consistently describe, explain, and predict the behavior of a system based on their scientific knowledge. Cameron is asked about the behavior of air in a real syringe.

During the clinical interview, Cameron is asked to consider the syringe and make an estimation of the pressure exerted by the air in the barrel of the syringe.

Interviewer: ...If you're looking at that thing (the real syringe), and the plunger is not moving, what do you think is the pressure of the air on the inside?

Cameron: ...I think it's higher than what it is where we are. I think it'd be more in here (inside the barrel of the syringe).

Interviewer: O.K. So the pressure inside there (the syringe) is higher than what the pressure around us would be?

Cameron: Um, or would it? No it wouldn't be. I think it's the same.

Interviewer: It would be the same? Why you think so?

Cameron: Cause the temperature and everything would be the same...The temperature here (in the room) is the same as it is inside of here (the syringe). And the pressure would be the same also.

Cameron offers an explanation of what he believes is happening with the air in the syringe. The explanation he offers initially is not the scientific conception, but as he continues to talk he is able to reevaluate his response and arrive at the accepted scientific conception. When offering an explanation for his answer, Cameron uses his understanding of phenomenology as well as the submicroscopic nature of a gaseous substance. Consequently, like the chemist, Cameron is able to use his conceptual knowledge of gas behavior to make estimations and explain gas phenomena.

Cameron is then asked to manipulate the plunger on the syringe and comment on the resulting pressure of the compressed air. With his thumb over the opening to the syringe barrel, he is asked to push the plunger from 20 cc to 10 cc and explain what happens to the air pressure.

Interviewer: What do you think the pressure is?

Cameron: Um, twice as high...Cause I moved it from 20 to 10, and that's half of what it was before.

Although he is never asked during the clinical interview on this task, the assumption is that Cameron knows that the pressure exerted by air at 20 cc is roughly 1 atmosphere. Consequently, "twice as high" here would refer roughly to 2 atmospheres. The chemist would consistently attend to this proportional relationship when describing the behavior of air in this syringe. Beyond the 2:1

inverse ratio task, however, Cameron uses a mathematical process different from the one he used when working items 1 and 3 during Part I of the interview.

Interviewer: Let it go back to 20 (cubic centimeters). Now move the plunger down to 15 (cubic centimeters)

Cameron: ...So that would only be 25% higher...That's only 25% further moved, and so half of half is 25%.

One could conclude that by 25% higher, Cameron means that the pressure exerted by the compressed air at 15 cc would be 1.25 atmospheres. He treats 15 cc as halfway between 20 cc and 10 cc. Therefore, he estimates the pressure at 15 cc to be 25%, half of 50%.

Recall that Cameron has previously said that if the plunger is pushed from 20 cc to 10 cc, the pressure would be "twice as high." Since 10 cc is half of 20 cc, the way Cameron thinks about the 20 cc to 15 cc compression would suggest that he would think the pressure exerted by air compressed from 20 cc to 10 cc would be 1.5 atmospheres. However, he doesn't think this. Cameron begins to use a percentage strategy when the ratio of volumes gets more complex than a simple 2:1 inverse ratio.

The percentage strategy used by Cameron seems to be a convenient one for problems which don't involve 2:1 inverse ratios. Even when considering a 4:1 inverse ratio task, Cameron considered his percentage strategy more useful.

Interviewer: Take it to 40 (cubic centimeters) for me...Try to push it all the way down to 10 (cubic centimeters)...What happened to the pressure.

Cameron: It got pretty high.

Interviewer: How high did it get?

Cameron: Um, 75% higher...Because it is three-quarters of the way to zero from where it was.

Cameron was asked to reconsider a situation previously encountered with the syringe system in item 3. This time, however, we considered the problem in terms of the context of the real syringe.

Interviewer: Suppose at 40 (cubic centimeters) that the pressure on the outside was 50 (pressure units). And then you decided to take it (the plunger) to 30 (cubic centimeters). What would happen to the initial pressure?

Cameron: The initial pressure would rise.

Interviewer: By how much?

Cameron: By, um, 25%...Is that right?

Cameron chooses to use a percentage strategy when considering the problem this time. On the syringe, 30 cc is one-quarter, thus, 25%, of the way from 40 cc.

In the task with the real syringe, Cameron does indeed use some consistent pattern in explaining the behavior of the air in the syringe. However, the pattern he uses with the real task is somewhat different than the understanding he exhibits when solving the gas law problems. Although he ultimately seems to use an averaging strategy much like the one he used for items 3(iii) and 3(iv) because of the answer he reports, it is not at all clear that this is what he is doing. As a matter of fact, Cameron seems to be using a couple of strategies together that he can never reconcile.

Summary of Cameron's Case Study

This case study has tried to show the mathematical and conceptual, understandings Cameron has and how he uses this understanding in real-world situations. Chapter 2 reviewed some studies which examined the mathematical and conceptual understandings students possess about the behavior of a gas. Some issues were raised there which has some application to this study. I pursue those three issues in this summary as a way of encapsulating how Cameron understands the mathematics and concepts and how he eventually uses this understanding to explain the behavior of a gaseous substance.

Cameron is a student who knows how to do the mathematics and who can exhibit an understanding of the atomic-molecular and kinetic molecular theories. It would be incorrect, however, to say that Cameron has the chemist's understanding of either the mathematics or the theories. But Cameron is a student who has developed some useful models to understand the mathematical and conceptual behavior of a gaseous substance. He is often unable to reconcile the models he tends to use simultaneously. However, he is disturbed when the models he uses appear to be in conflict with what he observes through real-world applications.

Mathematical Understanding Issues

ISSUE 1: The Understanding of Proportional Relationships.

This issue addresses Cameron's mathematical understanding. In order to make Cameron's mathematical understanding more comprehensible, an argument was presented which examined his understandings as competing tensions. That is, depending on the nature of the problem, Cameron chooses some position along a multiplicative-additive continuum to help him determine numerical values. This movement along the continuum lays bare Cameron's understanding of proportional relationships and the algebra of the gas laws. Although he constantly uses the language, he doesn't possess the understanding of proportional relationships a chemist does.

One finding from the de Berg study was that students tended to average values when they were not dealing with 2:1 inverse ratios rather than using a proportion strategy. Cameron also uses a strategy to average values when dealing with the syringe system. But, as noted above, Cameron has a number of strategies for dealing with proportional relationships. De Berg noted that the problem the students faced in dealing with inverse proportional relationships may have much to do with trying to apply inverse proportion reasoning out of context; that is, for these problems an inverse proportion reasoning is used in a context where an averaging law makes just as much sense. This could certainly be the case for Cameron as he thinks about item 3 on the paper-and-pencil instrument.

ISSUE 2: The Creation of a Cohesive Mathematical Model.

This issue addresses how Cameron uses his mathematical understanding as a model for real-world applications. Cameron's proportional reasoning strategies allow him to produce a personal model that he uses to estimate values for inverse proportional relationships. It is interesting that Cameron's revision of his personal model as he meets discrepant events never reconciles with his previous models. That is, he creates a new model to explain discrepant behavior that is often in conflict with his previous model. However, in either case, he is able to produce reasonable estimates of values. For example, with the syringe system on the paper-and-pencil instrument, Cameron developed a personal for explaining this system based on his knowledge of proportional relationships. Although he has a couple of understandings embedded in his understanding of proportional relationships, Cameron is able to use this understanding to estimate values for variables. When considering the real syringe, Cameron used a model somewhat different than the one he used when explaining item 3. With this task, Cameron simultaneously uses a few models (i.e., proportional strategy, percentage strategy and an averaging strategy) to estimate values quantitatively.

ISSUE 3: The Use of Gas Laws

Beyond the use of their personal models, this issue addresses how students understand the mathematical model of the chemist. Cameron has learned how to solve the gas law equations. However, outside of its usage in typical gas law problems as presented during his course, Cameron finds limited use for these equations. Rather, Cameron uses his understanding of proportional relationships and develops a model that is more useful to him. Again, one of the things which distinguishes Cameron is the uncertainty he often expresses when his personal mathematical models are put to a real-world test in situations where a chemist would use the gas laws. How does Cameron use his mathematical knowledge in real-world situations? Cameron prefers to use the gas law mathematical equation in more "academic" and formal situations. However, in what appears to him as less formal problems and situations in a real-world context, he uses a strategy for extrapolating variables from the information available to him. For example, in working with the real syringe and the syringe system on the paper-and-pencil instrument, Cameron uses a couple of strategies to predict variables as estimates of values he already knows. This is a convenient strategy and provides rough estimates of values, but it falls far short of understanding how the gas law equations can be used in real-world situations.

Conceptual Understanding Issues

ISSUE 4: Articulated Understanding of the Atomic-molecular and Kinetic Molecular Theories.

This issue is used to summarize Cameron's conceptual knowledge. Cameron is able to present a good explanation of the atomic-molecular and kinetic molecular theories as they relate to the behavior of a gaseous substance. His pictorial representation of air in a syringe shows he appears to have a scientific understanding of the volume occupied by a gaseous substance.

Cameron does, however, have a limited understanding of kinetic molecular theory. To him, it makes more sense with a gaseous substance than it does with a solid or liquid substance. In addition, on the macroscopic level, Cameron is able to distinguish between the mass, volume, and density of a gas. From the evidence presented, it doesn't seem that Cameron mistakes the volume of a gas for the density of the gas, a common mistake made by students as reported in the literature.

ISSUE 5: Atomic-molecular vs. Macroscopic Descriptions of Gas Behavior.

This issue is used to examine in particular how students use their conceptual knowledge in real-world situations. Cameron often invokes molecular language when appropriate to explain gas phenomena. He is able to do this quite successfully in spite of his limited understanding of kinetic molecular theory. Although not convinced of the constant movement of molecules, he is able to explain adequately that the pressure of a gas, for example, is the result of the collision of molecules against the container wall. Cameron is able to move past the visible, such as the recognition that air occupies a certain volume and exerts a certain pressure, to explain in terms of molecular movement why these variables behave as they do.

Case Study 2: Betty

Betty's Mathematical Understanding

Explanation of Item 1 on the Paper-and-Pencil Instrument

On the paper-and-pencil instrument, Betty solves the gas law problem by using the combined gas law relationship:

$$P_1V_1/T_1 = P_2V_2/T_2$$

She notices that V_1 and V_2 would be the same and reduces her equation to:

$$P_1/T_1 = P_2/T_2$$

During the clinical interview, Betty is shown her previously worked problem and asked to comment on how she solved it.

Interviewer: Take a look at problem number 1, and go through with me how you solved that.

Betty: O.K., what I did, pretty much, was that I set up the problem using the equation…I don't remember the name. I set it up, and then pretty much did a cross multiplication…and solved it out.

Betty is another student who knows how to solve the mathematical equation. She is able to manipulate the mathematical representations to solve for the variable she needs. Betty does not initially articulate a proportional relationship between the variables when attempting to solve her equation. When asked about the relationships during the clinical interview, she interprets the proportional relationships between the variables differently than a chemist would interpret them. She sees the proportional relationships in the gas law equations as existing between the same kind of variables.

Interviewer: Can you tell me right off what it (the equation she wrote) says?...What is the relationship, for example, between P_1 and T_1?

Betty: Pretty much it's, um...if I remember right...as pressure increases on P_1, like, pressure will increase on P_2...It's direct proportion.

Betty's understanding of the proportionality between variables in the gas law relationship is also evident when she is asked to consider the relationship between P_1 and V_1 in the equation she wrote.

Interviewer: How about P_1 and V_1?

Betty: When V on P_1 increases, P_2 will increase.

Outside of the context of the mathematical equation, however, Betty can state the relationship between the two variables P and V.

Interviewer: So I guess what I'm asking is what is the relationship between the pressure and volume of a gas?

Betty: Um...as pressure increases...the volume would have to decrease.

It seems obvious that Betty is able to solve item 1 without using a proportion strategy. Her use of the equation to solve the problem does not require that she understand the relationship between P and T. It seems, consequently, Betty does not find any use of the gas law equations outside of the context of item 1. To be sure, she doesn't explicitly use the gas law relationships beyond item 1 on the paper-and-pencil instrument. Therefore, it appears Betty uses the gas law equation in a pure mechanical way.

Explanation of Item 3 on the Paper-and-Pencil Instrument

Betty does not find use for a mathematical equation when working item 3. During the clinical interview, Betty is asked to explain what she thought about when solving item 3.

> **Interviewer:** ...Tell me how you got your answer.
>
> **Betty:** I looked at these (diagram of syringes) first, and I saw that there was a relation between each one from 200 pressure units [to] 100 (pressure units)...the volume itself increased by 10. And then from 100 to 50 pressure units, it increased from 20 to 40 (volume units), so it increased 20.

Although Betty doesn't explicitly use the gas law relationships, she does have in mind a proportion strategy. In a similar manner as Cameron's initial explanation of solving item 3(ii), Betty uses an additive strategy in relating P and V relationships in the diagram. Like Cameron, Betty has a factor strategy operating with an additive strategy. In further explaining how she worked item 3(i), she says,

> **Betty:** ...So I reasoned that it (the volume) was going to go and double itself. So these (pressure units) were

going down by half of its amount. So at 25, that would be half of 50 so that would have doubled 40, giving it 80.

Betty solved item 3(ii), the other 2:1 inverse ratio problem, the same way. Although she had some difficulty getting through her explanation of the problem during the clinical interview, she was confident, "...I used the same way, I know that..." Item 3(ii) is a problem very similar to item 3(i), a problem she seemingly had no difficulty solving.

As with Cameron, Betty uses a different strategy when solving items 3(iii) and 3(iv), problems which involve a 3:1 inverse ratio reasoning. Like Cameron, she uses an averaging strategy.

Betty: ...So 150 was between 100 and 200. So it (the volume) would have to be somewhere between 20 to 10 volume units, which would reasonably be 15.

Likewise, with 3(iv), Betty explains her reasoning.

Betty: ...With this one with 30 volume units, that would be between the 40 and the 20, so it (the pressure) would have to be somewhere between 50 and 100, and I reasoned that to be 75 pressure units.

<u>How Betty Understands and Uses the Mathematical Representations of the Gas Laws</u>

Betty is a student who, like Cameron, has a command for the mechanical use of gas law relationships. Also, Betty has already established a personal model for explaining the behavior of a gaseous substance that works more consistently than Cameron's. That is, although Cameron initially had different strategies for dealing with similar problems (i.e., the 2:1 inverse ratio problems), Betty uses the same strategy.

Betty also thinks about proportional relationships in some interesting ways. Her idea is that the proportional relationships exist between variables of the same kind. This understanding seems to be somehow rooted in the gas law equation itself. Outside of the context of the equation, she is able to at least state the directional character of the proportional relationship between two variables. Also, like Cameron, Betty determines the magnitude of proportional relationships in some distinct ways.

Multiplicative vs. Additive Character of Proportional Relationships

When Betty examines the syringe system in item 3, what she notices is that as the pressure exerted on the plunger decreases, the volume increases in increments. This view seems based more on a number sequence understanding than a factor understanding. However, like Cameron, this additive strategy seems to operate in conjunction with a factor, or what I have referred to as a multiplicative strategy. That is, Betty uses multiplication language like "double itself" and "going down by half" in conjunction with her sequence explanations when referring to the P and V proportional relationships. However, unlike Cameron, her dual understanding of proportional relationships never raises any concerns for her during her explanations.

Betty's Conceptual Understanding

Describing the Submicroscopic Nature of Matter

During the clinical interview, Betty is asked to represent pictorially her understanding of how air would look at the submicroscopic level.

Interviewer: ...If you could see air at the very smallest level, what would it look like to you?

Betty: It'd be particles kind of just spread out.

Interviewer: And spread out why?

Betty: Because, it's like loose. It's not held together so it takes up space; more space than actually there are particles within.

Betty also has the notion that these particles would occupy the space available, and that they're moving.

Interviewer: So would these particles occupy all of the space available to it?

Betty: There would still be space in between, but it would fill out its container...anywhere within, you could find particles of air.

Interviewer: ...Are those particles kind of just suspended there?

Betty: No, they're moving.

By her statement that the particles could be found "anywhere within," the suggestion here is that Betty believes these particles to be randomly moving and evenly distributed in the available space. Her drawing also depicts this understanding.

Although Betty is able to state and depict the chemist's rendition of the behavior of matter at the particulate level, she later reveals some differential understandings about the behavior of a gaseous substance at the submicroscopic level. For example, in responding to item 2 on the paper-and-pencil instrument where she is asked to pick the most appropriate representation of how the molecules of hydrogen gas will be arranged when the temperature of the gas drops, Betty chooses choice (B). This representation shows the molecules of the gas distributed within the center of the tank and not evenly and randomly distributed in accord with the kinetic molecular theory and her initial drawing. She explains her choice during the clinical interview.

Betty: ...They were talking about pressure and temperature and they changed the temperature on this. And then they were asking what would the pressure be, or, they said distribution. I knew that as the temperature would decrease, the pressure also would decrease.

Betty has the correct conception that the pressure exerted by the molecules against the walls of the container would change. However, her conception is to think of reducing the number of particles hitting the wall by relegating them towards the center of the tank. With this particular conception, Betty does not sacrifice her understanding that the energy of the particles should change with a decrease in temperature.

Betty: ...As temperature decreases the pressure is also going to decrease because molecules won't be moving as fast and not hitting the walls of its container as hard.

In spite of the fact that Betty believes the molecules of a gas should occupy the entire container, she also believes that they will not here because of the necessity of a pressure change. Betty's representation may suggest she believes the molecules have condensed. Such is not the case however. Betty understands that hydrogen would still be in its gaseous state at the lower temperature.

Interviewer: In (B), do you still see this remaining as a gas? Is this still hydrogen gas when the temperature is lowered?

Betty: Yeah.

Describing the Phenomenological Behavior of a Gaseous Substance

During the clinical interview, Betty is asked to talk about the choices she made for item 4 on the paper-and-pencil instrument. When describing the reasons for the differences in the volume of air before and after compression, she uses the expected macroscopic descriptions of the behavior of air.

Betty: [In] A (syringe before compression), there was more room for the enclosed air than there was actually [in] B (syringe after compression). So it didn't make sense that A would be less volume wise than B.

She does a similar kind of macroscopic observation when describing the mass of air in the syringe before and after compression.

Betty: Mass would be like amount of air in that, and the amount of air never changed…It's enclosed, it's sealed off and nothing's going to escape or go in so there was no change within the mass itself.

When explaining the difference in the pressure the air would exert before and after compression, Betty uses the expected description based on the particle nature of air.

Betty: The pressure is going to change in that because you have more room here (before compression). The particles can spread out more. And then you have less room here (after compression) so the particles are more compact and they're going to be hitting the walls of its container, the syringe, more. So the pressure on B is going to increase and be more than A.

<u>How Betty Understands and Uses the Concepts</u>

Betty understands the notion of the atomic-molecular and kinetic molecular theories and doesn't appear to doubt its validity. But, she doesn't apply her understanding of these theories across the board. Depending on the context of the problem, Betty reveals differential understandings of concepts she initially appears to know well. She reveals the following related misconceptions:

1. When the temperature of a gas drops, the pressure also drops because the molecules are not as close to the walls of the container as they are at higher temperatures.
2. The molecules of a gas will occupy the entire volume of its container except at lower temperatures of the gas where the particles will move closer together.

Betty also reveals an ability to use the appropriate macroscopic and submicroscopic descriptions of matter in the appropriate situations.

Betty's Real-world Applications

When given a real syringe during the clinical interview and asked to estimate the pressure of air inside of the syringe operating under the conditions of the room, Betty focuses on what she can see and disregards every other aspect of her environment. She depends on knowledge obtained from item 3, a problem on the paper-and-pencil instrument that we had just discussed in the clinical interview.

Interviewer: Look at that syringe...Put the plunger at about 20 (cubic centimeters)...If you put your finger over the entrance, and the plunger is not moving, is air exerting a pressure on the inside of that syringe?

Betty: Yeah...At 20, it would be a hundred.

Interviewer: Why do you say that?

Betty: Comparing it to this chart here (the diagrams given in item 3) and that they are saying 20 volume units, which is what this is set at...and that would be exerting 100 pressure units.

In the absence of this system, Betty was not able to estimate what the pressure would be.

Interviewer: ...Suppose you didn't have the same system, and you just had to say in general, 'I have 20 cubic centimeters of air in this syringe that I have inside this room.' You think you can say what the pressure would be?

Betty: Not right off hand. There's formulas I know I can use to find it.

Interviewer: But other than formulas, you probably couldn't say what it was?

Betty: No.

At the end of the clinical interview, after having worked with the real syringe, Betty is again asked about the pressure exerted by air inside of the real syringe.

Interviewer: Consider the syringe again. Move [the plunger] to 40 (cubic centimeters). Can you say what the pressure of air is on the inside?

Betty: It'd be one...Because I'm not putting any pressure on the syringe itself, so there's no pressure being exerted on the air inside.

It still seems clear that Betty is unable to estimate the pressure of air on the inside of the syringe. From the above comment, she thinks that if there is no pressure on the plunger, there is no pressure being exerted on the gas. She seems to neglect the pressure exerted by the atmosphere. It seems she remembers the value from our earlier conversation and uses 1 atmosphere as a "standard" value for pressure whenever the plunger isn't moving regardless of the surrounding conditions.

Interviewer: So, when we had it at 20 before, and I said that the pressure should be 1, did it make sense that the pressure should be 1, or could I have said that it was something else?

Betty: When it was at 20 with no pressure, then it would be one.

Interviewer: Makes sense to you?

Betty: Um hum.

During the clinical interview, Betty is asked to manipulate the syringe and quantitatively describe the pressure changes. She is not able to estimate a value for the pressure air would exert in the syringe when the plunger is not moving. Therefore, she is given this value and we proceed from there.

Interviewer: Suppose I said to you that air exerts a pressure in that syringe of about 1 atmosphere. Now, knowing that, push the plunger down to about 10 (cubic centimeters). Can you say what the pressure is of the air now?

Betty: Probably would be like two atmospheres?

Interviewer: Why do you say that?

Betty: Um, at 20 it was one, and then I'm going based upon units of 10?

Interviewer: What do you mean?

Betty: I mean, like, for 20 volume units it's one atmosphere and I decreased that by half so double my atmospheres, and that's the two.

Betty's strategy for figuring this out in this real-world situation seems to be much as it was for the paper-and-pencil instrument. She uses a proportion strategy that is sometimes flavored with an additive strategy.

Interviewer: Take it (the plunger) back to 20 (cubic centimeters) for me. Again, there we say the pressure is about one atmosphere. Push it (the plunger) down to 15 (cubic centimeters). What do you think the pressure is?

Betty: Probably about one and a half...Because, I decreased my volume units by 5 and they're going on

> the proportion…For half a decrease, it's double. And then I'm going to half that…so half of 1 would be 5 (implying 0.5), and then added to the one.

Betty's strategy for arriving at her value on this real task is much like she does in item 3 of the paper-and-pencil instrument. In many ways, what she does here for the 20 cc to 15 cc compression, and what she does for items 3(iii) and 3(iv) is an averaging. Knowing that the pressure exerted by air at 20 cc is 1 and having figured out the pressure exerted at 10 cc is 2 atm., the thought is that the pressure exerted at 15 cc must be 1.5.

Summary of Betty's Case Study

Betty is a student who is reasonably consistent in responding to problems. Although she does not volunteer answers as much as Cameron, and is often incorrect with her responses, she is thoughtful and consistent with her responses. Betty is also unable to produce values for variables, like the pressure, when not given data as information. As a matter of fact, she specifically seeks data to manipulate in a formula in order to decide what the pressure exerted by the atmosphere would be.

As with Cameron, this case study will conclude with an analysis of how Betty's responses address the issues derived in Chapter 2.

Mathematical Understanding Issues

ISSUE 1: The Understanding of Proportional Relationships.

In some ways, Betty understands proportional relationships much as Cameron does. That is, Betty is able to articulate the directional nature of the variables. Unlike Cameron, however, Betty is not able to relate this directional nature within the context of the mathematical representation. When simply asked what the relationship is, she can articulate it well. Betty also exhibits an understanding of proportional relationships as operating along a multiplicative-additive continuum. As with Cameron, the additive strategy seems to operate as a result of the syringe system presented in item 3 on the paper-and-pencil instrument.

In other ways, however, Betty's understanding of proportional relationships is different than Cameron's. Betty has no problem stating that variables of the same kind are proportional to each other. When she writes mathematical relationships to show ratio-and-proportion, she typically makes ratios out of similar variables.

ISSUE 2: The Creation of a Cohesive Mathematical Model.

The personal model Betty uses to explain her mathematical understanding from her articulated understanding to her use of this understanding with the real-world task is very consistent. Cameron did not consistently use his understanding, but developed several strategies for explaining even very similar problems. Betty , however, uses her naïve understandings in consistent ways when explaining similar problems on the paper-and-pencil instrument and the real-world task. Betty's personal model consists of a proportion and averaging strategy.

ISSUE 3: The Use of Gas Law Equations.

Betty finds no use for gas law equations outside of the context of the typical gas law problem. Betty uses the gas laws strictly as a computational device solely depending on the mechanics of the equation. Cameron is a student who, like Betty, is able to solve algebraically the gas law equation and attends to the mechanics of the gas laws in item 1 of the paper-and-pencil instrument. Betty finds no use for the gas laws for real-world tasks. However, Betty does get better answers more consistently with the model she uses than Cameron does with his.

Conceptual Understanding Issues

ISSUE 4: Articulated Understanding of the Atomic-molecular and Kinetic Molecular Theories.

Betty is well able to represent pictorially at the submicroscopic level the depiction of a gaseous substance in its container. Her drawings reflect the kind of understanding instructors would like students to have and mimics the chemist's understanding. However, in further expressing her understanding, Betty articulates some differential understandings.

The studies reviewed in Chapter 2 revealed some misconceptions students have about the macroscopic nature of gas volume. That is, students often confuse gas volume with density and mass. Betty shows some differential understandings of gas volume not at the macroscopic level, but at the submicroscopic level. Although initially able to represent the volume of a gas at the particulate level, she adopts some interesting conceptions about the situations in which this conception applied. That is, Betty has a number of understandings about gas volume which coexist at the same time. In one context she has one understanding, while adopting a different understanding when the context changes.

ISSUE 5: Atomic-molecular vs. Macroscopic Descriptions of Gas Behavior.

Betty use macroscopic and molecular language in much the same way chemists would when explaining similar problems. For the tasks dealing with the change in volume and mass upon compression of air in a syringe, Betty uses macroscopic descriptions. For tasks dealing with explanations of pressure and temperature changes, Betty uses explanations describing changes at the molecular level.

Case Study 3: Karen

Karen's Mathematical Understandings

Explanation of Item 1 on the Paper-and-Pencil Instrument

In solving item 1, Karen uses the relationship:

$$P_1T_1 = P_2T_2$$

This expression, however, is incorrect because it shows an inverse proportional relationship between P and T instead of a direct proportional relationship. Although Karen was not asked about this specific problem during the clinical interview, in the commentary that follows I will address what I see as Karen's fluctuating understanding of the gas law relationships.

Explanation of Item 3 on the Paper-and-Pencil Instrument

In responding to item 3 on the paper-and-pencil instrument, unlike Cameron or Betty, Karen successfully uses a gas law relationship to obtain the correct answer for each of the problems in item 3. It should be noted, that of the 116 students who took the posttest, Karen was one of only three students who used a gas law to respond to this item.

During the clinical interview, Karen was not shown her original posttest, but was given a blank copy of it and asked to rework and explain how she thought about item 3. She responded differently to every problem in item 3 during the clinical interview. When explaining her understanding, Karen did not explicitly use a gas law equation as she had on her posttest. She did, however, use ratio-and-proportion relationships. She did not explicitly write out the ratio equation for the first two problems which use 2:1 inverse ratio reasoning, but she did write them down for the last two problems in this item, which uses 3:1 inverse ratio reasoning. It is interesting to note that although Karen uses a different method to solve item 3 during the clinical interview - where she obtains incorrect answers - than she did on the paper-and-pencil instrument - where all of her answers were correct - she is not bothered by her different understanding during the clinical interview. That is, she seems just as content with her responses during the clinical interview.

> **Interviewer:** Take a look at problem 3...and go through it again explaining to me how you worked it.
>
> **Karen:** I thought I saw a ratio that existed between how it decreased and the number of pressure units. So when I came to the problems, I applied the ratio, and went from there.

Karen did not explicitly write down the ratio equation, but produced the value after her explanation.

> **Karen:** This says 25 pressure units (*referring to given pressure in problem 3(i)*) so that's 50 (*referring to pressure units given in first syringe*) and this is 40 volume units (*referring to volume units given in first syringe*). So, divide it in half, it's 20.

Similarly, in solving the second problem of item 3,

> **Karen:** This is 5 (*referring to given volume in problem 3(ii)*) and that one had 10 (*referring to volume units given in third syringe*)...so that's half of that so you would just take half of this (*200 pressure units*) and get 100...So that would be 100 pressure units for that one.

For items 3(iii) and 3(iv), Karen still uses direct ratio reasoning, although she uses it in a different way for each of the items. For these items, however, she has to write down the relationships. Her solutions reveal some unique understandings with her thinking about proportional relationships.

When solving item 3(iii) during the clinical interview, Karen uses the relationship:

$$150/X = 100/20$$

She explains,

Karen: The 100 pressure units over the 20 volume units...or we could have taken 200 (pressure units) to 10 (volume units)...and set up like a proportion again and solve for *X*.

Karen seems little aware that each pressure/volume ratio of the syringes would not produce the same value for *X*, and that this system does not operate as a direct proportion.

In item 3(iv), Karen shows an understanding of proportional relationships different than she shows in item 3(iii). During the clinical interview, Karen wrote the following mathematical relationship for item 3(iv):

$$30/40 = X/50$$

She explains,

Karen: ...This is 30 (volume units), that's 40 (*referring to volume units in first syringe in diagram*). Set up, like, a proportion...30 over 40 equals *X*, the unit we don't know, to 50 pressure units, and solve for *X*.

Karen shows here that she has little understanding of proportional equations. Here she kind of guesses where to put the variables.

Again, Karen seems little bothered by how she worked this problem during the clinical interview.

Interviewer: The equations, themselves...make sense to you?

Karen: Yeah. Makes sense to me. I don't know if its right...It might not be the exact scientific way to do it, but it makes sense to me.

<u>Understanding of P, V, and T Relationships in Gas Law Equations</u>

Karen's knowledge of the proportional relationships between P, V, and T in the gas law relationships fluctuates. That is, for example, although she consistently used direct ratio reasoning to refer to the relationship between P and V when explaining item 3 during the clinical interview, on the posttest she uses a gas law that shows an inverse relationship between these variables:

$$P_1V_1 = P_2V_2$$

Also, when explaining the relationship between P and V later in the clinical interview using a real syringe, Karen is able to adjust an initially incorrect assumption about P and V relationships. The way she often contradicts herself as she talks about the relationships says much about her fluctuating understanding.

Interviewer: So, if I knew the volume up here (*at 20 cc on real syringe*) and I knew the pressure (*exerted by the air in the syringe at 20 cc*), and then I changed the volume to 10...

Karen: The pressure would likewise decrease, or increase...If you increased the volume, pressure would increase and...as volume decrease, pressure would increase.

Karen contradicted herself three times in the preceding exchange. However, she eventually comes to the correct resolution.

Interviewer: The value of the pressure *here (at volume of 20 cc in real syringe*) was one, so when it goes here (*to 10 cc*), what will it be?

Karen: It'll be two...Because...wait...I think my answer was wrong (*referring to one of her statements about the relationship between P and V*). Because if it was twenty and it had one, and we compressed it down, it'd be two cause there's more pressure...and we would cut in half...so doubled it. So it's inversely related...I was wrong, inversely related.

Here again, Karen meanders in her understanding of the relationships between P, V, and T.

How Karen Understands and Uses the Mathematical Representations of the Gas Laws

Karen shows an initial facility with using the gas law equations past their typical usage in her basic chemistry class. Her use of the gas law equations and ratio-and-proportion equations suggest that she sees them as useful beyond what students generally see as the typical context for their use. However, the fact that Karen does not use the gas law equations when responding to the same problem in the clinical interview as well as her arbitrary use of ratio-and-proportion equations suggest that she has little understanding of the gas law equations.

Although Karen can use the gas law equation within different contexts, her understanding of the relationship is still somewhat limited. While taking the posttest, Karen used the gas law equation successfully. During the clinical interview, she still tries to use a mathematical representation of the gas laws, but switches to a ratio and proportion strategy. The problem with understanding the mathematical relationship lies beyond knowing how the variables are related. The problem lies with trying to represent that knowledge in a mathematical way. Karen also exhibits a couple of understandings of proportional relationships.

Understanding Ratio-and-Proportion Relationships

Karen is a student who, although having some misconceptions, has an intuitive awareness of the P and V relationships for a gaseous substance. On the posttest, Karen used the gas law equation describing the inverse relationship, but during the clinical interview, she used ratio and proportion. Since it seems

Karen has an intuitive sense of the P and V relationship, her response to item 3 during the clinical interview reveals some things about her understanding of ratio and proportion.

Karen makes a ratio of two known values and sets this ratio equal to another ratio consisting of another known value and the unknown value, *X*. She seems aware that the ratio must contain like units, and always sets up her ratios so that this end is achieved. Although her conversation above shows that she knows what the P and V relationship is, she does not relate this with her mathematical understanding in the proper way. She sets up all ratio-and-proportion relationships, whether direct or inverse, in the same way (i.e., as equations with equal ratios of variables). She doesn't consider the fact that for P and V ratio-and-proportion problems, it is the P and V products that are equal and not their quotients. Therefore, Karen's use and understanding of the $P_1V_1 = P_2V_2$ relationship has some limitations.

Relating the Variables in Ratio-and-Proportion Equations

Noticeable in Karen' understanding of ratio-and-proportion equations is the relationships she establish between the variables. In item 3(iv), she constructs a ratio between two volumes and sets this equal to a ratio between two pressures (one known and one unknown). Again, Karen doesn't appear to have any particular strategy in mind. Karen seems to be guessing more than anything when she sets up proportional relationships in an equation.

Karen's Conceptual Understandings

Describing the Submicroscopic Nature of Matter

Karen is another student who is able to state and show that gas molecules completely occupy the space available to them. However, like Betty, she doesn't maintain her conception for gases in general.

During the clinical interview, Karen is asked to pictorially represent how air in a syringe would look at the submicroscopic level. She explains,

Karen: It'd be like all tightly packed, and the molecules, that are usually so widely spaced apart, would be right next to each other...There'd be a lot of pressure.

When asked what would happen to the molecules if the plunger on the syringe was pulled from 20 cc to 60 cc, Karen says,

Karen: It'd be widely spaced...All over the place.

Consequently, Karen conceives of the molecules of a gas as being less random and more ordered at higher states of compression. This notion of how the molecules of a gas are affected under compression seems to influence Karen's understanding at two other levels: (1) distinguishing mass from density, and (2) estimating values for variables, which will be discussed in the next section.

Karen's conception that the molecules of a compressed gas will arrange themselves more like that depicted in a solid state is useful in explaining her confusion between mass and density. On item 4 of the paper-and-pencil instrument, Karen responded that the mass of a given amount of air was greater before compression than after compression. She also responded that the volume occupied by air in the syringe would be less before compression than after compression. Both of these responses show that Karen may be confusing volume, mass, and density. That is, her response about the volume would be true if she has density in mind, and her response about the mass would be true if she had volume in mind.

During the clinical interview, Karen was given a real syringe and asked to explain what would happen to the mass of the air in the syringe when it is compressed.

Interviewer: Now suppose I compress that (the air in the syringe)...What happens to the mass of air on the inside of there?

Karen: It becomes greater...Because it compresses and becomes so much tightly packed, and there's all the pressure...I would think that the mass would increase...Because it's like heavier and denser. That's it. That's the word I wanted.

Relating Karen's understanding here to the idea of weight seems useful. When she states, "there's all the pressure," she may conceive of a force created *because* the molecules are all "tightly packed." Consequently, for her when the gas is compressed, the resulting packing of the molecules causes an increase in force between the molecules which increases the mass (or weight). Therefore, there appears to be a precarious balance here between Karen's understanding of mass, weight, and density.

Karen's understanding of the behavior of the molecules also influences how she estimates the pressure exerted by air inside of a syringe. She conceives of the more tightly compressed molecules as greater influencing the pressure of air inside of a syringe when compared with air on the outside of the syringe. For her, the less tightly packed molecules on the outside exerts less pressure.

During the clinical interview, Karen is asked to respond to item 5, a problem which asks about the pressure differential between the air on the inside and outside of a syringe whose plunger is not moving.

Interviewer: ...Explain to me your thinking there.

Karen: It's (the air pressure outside of the syringe) going to be less because there's more of it and you can move around more so it's not as compact. It's not compacted or compressed in any way. So it's not going to be as much as the air that's inside the syringe. The pressure of the enclosed air in the syringe is going to be greater because it is packed. The molecules are compacted so much closer together. They have less room to move so they're going to be all tight together, and they exert a greater amount of pressure.

Karen gives a similar explanation when asked to consider a real syringe and explain the differences in air pressure inside and outside of the syringe when the plunger is not moving.

Karen: ...The pressure of the enclosed air in the syringe is going to be greater because it is packed. The molecules are compacted so much closer together. They have less room to move so they're going to be all tight together, and they exert a greater amount of pressure...The gas, it, like, compresses ...Every time you, like, put it into a smaller space, the pressure exerts a greater force.

How Karen Understands and Uses the Concepts

Like Betty, Karen initially shows a functional knowledge of the atomic-molecular and kinetic molecular theories. However, she is also a student who ultimately uses her knowledge differentially. Karen's understanding of the submicroscopic level of gas behavior changes with context. Particularly, Karen reveals the following misconceptions:

1. The particles of a gas become more ordered in its arrangement upon compression.
2. When a gas is compressed, the pressure of the gas increases because the particles, now closer together, are trying to move away from each other. Therefore, there's a greater force created upon compression.

3. The mass of a gas increases upon compression due to the greater force created by the tightly packed particles.
4. The particles of a gas are able to "free float." When the gas is not being compressed, particles float toward the edges of the container. This movement at the edges creates the pressure in an uncompressed gas.

Karen's Real-world Applications

Karen is asked to estimate the pressure exerted by air in a real syringe when the plunger is not moving.

Interviewer: Let's take a real syringe...Suppose you set the plunger on 20 (cubic centimeters)...The plunger is not moving, and there's air on the inside. Is the air on the inside exerting a pressure?

Karen: On the edges of my container on the inside, sure.

Previously, Karen demonstrated her understanding of the pressure of a gas as the result of compacted molecules. That is, a gas exerts a greater pressure when compressed because the molecules in the compressed state are trying to move away from each other, and, thus, there is a greater force produced among the molecules.

With the real syringe, Karen further demonstrates her conception of air pressure. In a less compressed state, Karen views the molecules as exerting a pressure "on the edges" of the container only. This understanding shows a misconception about the kinetic molecular theory. Karen knows that the pressure is created because gas molecules are in motion.

Interviewer: Why would the air be exerting a pressure?

Karen: Because all the molecules of the gas...they're so spaced out...and they free float, if you will...when they're trapped in a container. They're bouncing off against the edge of the container inside.

However, Karen doesn't conceive of molecular motion as random with the particles achieving uniform distribution. As a matter of fact, when asked about the distribution of hydrogen molecules in a tank where the temperature had dropped (item 2 of the paper-and-pencil instrument), Karen chose choice (D), which shows the molecules aligned only around the edges of the tank. Based on the conversation above, Karen conceives of this arrangement because the molecules are light and able to "free float."

During the clinical interview, Karen was asked to estimate what the pressure of air inside the real syringe would be.

Interviewer: Can you say what the pressure is?

Karen: Can't give you an exact number cause, all I know, is that the pressure is being exerted by air on the inside. And all I know is [the volume].

Karen's ability to estimate a value for pressure when not given any data is very similar to that of Betty. That is, she cannot make the link that the pressure exerted by the air in the syringe is the same as that within the room, roughly 1 atm.

Karen was then asked to talk quantitatively about the pressure exerted by the compressed air in the syringe.

Interviewer:	Suppose I told you that the pressure at 20 (cubic centimeters) was one atmosphere, and you pushed it down to 10 (cubic centimeters).
Karen:	It's more than one atmosphere. It'll probably be like two or three...It'd be greater.

Initially, Karen is not attending to the relationships at all. She simply knows that it will be greater, and mentions some greater values.

Interviewer:	Could I say how much the pressure increases. I mean, can I give a value now?
Karen:	If you knew the value of the pressure (at 20 cc), yeah.
Interviewer:	The value of the pressure here (at 20 cc) was one. So when it goes here (10 cc), what will it be?
Karen:	It'll be two...cause there's more pressure, and we would cut in half [the volume], so [we] doubled it (the pressure).

For the 2:1 inverse ratio task above, Karen uses a proportion strategy different from the proportion strategy she uses with the 2:1 inverse ratio tasks in items 3(i) and 3(ii). In addition, with the 3:4 inverse ratio task using the real syringe, Karen uses a different strategy for arriving at her values than she uses for the 3:1 inverse ratio task in items 3(iii) and 3(iv). For the 3:4 inverse ratio task with the real syringe, Karen does what amounts to an averaging.

Interviewer: Let it go back to 20 (cubic centimeters). Now, again, I'm going to say that the pressure exerted by this gas at 20 is one atmosphere. Push the plunger down to 15 (cubic centimeters). What is the pressure now?

Karen: Probably one and a half atmospheres...Because it's inversely related...when we went from 20 to 10 it was 2. Since we went from 20 to 15, it'll be one and a half because we didn't go a full unit down. We moved a half a unit down.

Karen still notes that P and V are "inversely related." Thus, she understands that the volume will decrease upon an increase in pressure. Here Karen also seems to understand this decrease as a change by unit steps.

<u>Summary of Karen's Case Study</u>

Karen is a student who, like Cameron and Betty, knows how to solve the mathematical equations that represent the gas laws. Also, like Cameron and Betty, Karen can well represent pictorially and explain how a gas appears in its container at the submicroscopic level. Karen seems able to produce an adequate model that helps her explain and predict the behavior of air in a syringe. However, she is not always able to consistently apply her model in real-world situations. Also, Karen is often not discouraged by discrepant uses of her understanding.

Mathematical Understanding Issues

ISSUE 1: The Understanding of Proportional Relationships.

Karen is a student who often engages in using equations for proportional problem solving. However, when using a ratio-and-proportion equation, she treats both inverse and direct proportions as direct proportions. Consequently, Karen appears to be just guessing about the relationships without having any real understanding.

ISSUE 2: The Creation of a Cohesive Mathematical Model.

Karen is a student who uses proportion reasoning most consistently. Unlike Cameron and Betty, Karen did not use an averaging strategy at all while working the paper-and-pencil instrument, but she did use this strategy during the clinical interview when explaining the behavior of air in the real syringe. On the paper-and-pencil instrument and for the 2:1 inverse ratio task with the real syringe, Betty consistently attends to her understanding of ratio-and-proportion, using the gas law equations on the paper-and-pencil instrument and ratio-and-proportion during the clinical interviews. However, Karen does not consistently use her mathematical model to predict values for the task using the real syringe. With the real syringe, Karen estimates her values much like Betty; that is, she performs an averaging. Therefore, Karen's personal model for mathematically explaining the behavior of a gaseous substance consists of a proportion and averaging strategy.

ISSUE 3: The Use of Gas Law Equations.

Unlike Cameron and Betty, Karen is able to use the mathematical equations of the gas laws as a model for behavior outside of the context of its general use. Although often confused by the direction of the proportional relationships, she does use the gas law equations to understand the relationships. The extent, however, to which Karen finds gas law equations useful in other contexts is uncertain. Because although she uses a gas law equation to solve a problem not typically presented during classroom instruction, she did not consider its use again when responding to that same problem later.

<u>Conceptual Understanding Issues</u>

ISSUE 4: Articulated Understanding of the Atomic-molecular and Kinetic Molecular Theories.

Like Betty, Karen has some differential understandings about the volume of a gas very different from the representation she is initially able to draw. Initially, she is able to represent pictorially what air enclosed in a syringe would look like at the submicroscopic level. As is the chemist's version, Karen believes that the molecules are "widely spaced…all over the place." However, she doesn't hold this same view for the compressed air. Karen's view is that the molecular arrangement for compressed gas is more like that of a solid. Like Betty and Cameron, Karen holds certain contextual understandings.

ISSUE 5: Atomic-molecular vs. Macroscopic Descriptions of Gas Behavior.

Karen is a student who often offers explanations about gas behavior using molecular language. Although she has some misunderstandings, she appears to prefer explanations at the particulate level. Even with the tasks that can often be described sufficiently using macroscopic language (e.g., the change in mass of a gas upon compression), Karen often talks about what is happening with the molecules.

Case Study 4: Connie

Connie's Mathematical Understandings

Explanation of Item 1 on the Paper-and-Pencil Instrument

On the paper-and-pencil instrument, Connie only reports an answer for item 1. She is asked about this during the clinical interview.

Interviewer: I notice that you did report an answer…I wasn't sure how you got it though. Do you remember?

Connie: No, I have no clue…I had my calculator, and I was working it out somehow, and that's what I put. And I cannot remember how I got it at all.

Interviewer: So, even as you're looking at that [problem] now you have no idea how…?

Connie: No.

Explanation of Item 3 on the Paper-and-Pencil Instrument

When explaining how she worked item 3(i), Connie says,

Connie: I know I didn't calculate it out.

It is obvious that in item 1, Connie feels a need to calculate something in order to answer the problem. However she does not exhibit this same need in item 3. Her conception of ratio-and-proportion helps her in item 3, but not in item 1.

Connie seems to share some of the knowledge about proportional relationships that Cameron and Betty possess. For example, in addition to a factor understanding of proportional relationships, Connie also possesses an additive understanding operating in conjunction with the factor understanding. In explaining how she solved item 3(i), Connie says,

Connie: This is 40 (*referring to volume units on first syringe*) and then this is half of 40 (*referring to volume units on second syringe*) and that's half of 20 (*referring to volume units on third syringe*)...And I just took what that would be, the 25 pressure units, and I figured what it [would] be, like, half of 40. You know, 40 and 40 would be 80.

Also, like Cameron and Betty, Connie uses an averaging strategy to produce values in items 3(iii) and 3(iv).

Interviewer: Here (*referring to item 3 (iii)*), how did you do it?

Connie: One hundred fifty would be between 100 and 200. So I took, kind of in between, 100 and 200.

Interviewer: And then you did the same thing here (*referring to item 3(iv)*)?

Connie: Yeah.

<u>How Connie Understands and Uses the Mathematical Representations of the Gas Laws</u>

Connie is a student who is least able to relate her understandings of the mathematical representations of the gas laws. Although Connie is just as successful as Cameron and Betty at obtaining the same solution for item 3 on the posttest, Connie has "no clue" how to respond to item 1, and, thus, exhibits the most limited mathematical ability with the gas law equations of the four students used as case studies.

The fact that Connie can't solve item 1 is interesting for a couple of reasons. First, item 1 is a problem most typically remembered by students. The ability to solve this problem by remembering a formula, plugging values into it, and solving it has been typical of the other three students for whom case studies were done in this study. Connie's inability to do this in spite of her ability to use ratio-and-proportion reasoning in solving item 3, gives yet another vent for the reliance on problems like item 1 as indicators of understanding gas law relationships. That is, the fact that Connie is just as successful in talking about the P and V relationships in item 3 as Cameron, Betty, and Karen were on their posttest, underscores the observation that understanding gas behavior is independent of the ability to solve gas law equations.

Second, Connie generally tends to operate more on intuition and less on a planned approach when solving problems. In item 3, she admits she "didn't calculate it out" but just saw a relationship between the syringes. In other words, Connie has no particular model for the behavior. When she worked item 1 on the posttest she reported an answer only. However, she worked the problem in a trial and error manner not knowing how she obtained her answer. Unlike Cameron, Betty, and Karen, who all articulated some plan for working through this problem, Connie is not able to articulate a plan for arriving at a solution.

Even more interesting is that although Connie appears to be more intuitive and less mathematical in her solutions to problems, she sees herself as using mathematical reasoning.

Interviewer: On problem 3 here, you seem to have had some intuitive sense on what to do without even using the gas law equations. Evidently, you found that an easier way to do it than to even think about the gas laws.

Connie: I just took math last semester, and it's like that...proportional theory. You know, you set *x* over *y* is equal, you know.

Interviewer: So you used ratio and proportion?

Connie: That's how I figured that stuff out.

First, it is interesting to note that, like Cameron, Betty, and Karen, Connie has some unique understandings of proportional relationships and what it means to say that two variables are proportional to each other. Second, we could theorize that if Connie ever had any knowledge of a gas law relationship (more than she was willing to give on her posttest or express during the clinical interview), it would have been a purely mechanistic understanding, and, thus, very transient. For even though she has, for the most part, similar understandings of proportional relationships as Cameron, Betty, and Karen, based on her inability to use the gas law equations, she doesn't see the gas law relationships as ratio-and-proportion equations.

Connie's Conceptual Understandings

Describing the Submicroscopic Nature of Matter

Connie is a student who also exhibits a surface understanding of the particulate nature of matter. When asked to represent pictorially what she thinks air in a syringe would look like at the submicroscopic level, she draws a representation which shows molecules randomly and evenly spread out and occupying the full volume of the syringe.

On the paper-and-pencil instrument, Connie was asked to choose the best representation of how hydrogen gas molecules would occupy the volume of a tank when it's temperature drops. Connie selects the choice (d), which shows the molecules aligned around the inside edges of the tank. Connie explains her choice.

Connie: ...It says the boiling point is -243. So, I guess what I thought, since I thought boiling point, it would expand.

Although Connie has the notion that gas molecules occupy the total volume of its container, she doesn't always apply this understanding to gases in general. Her understanding is that gases at higher temperatures (for her, this seems to mean close to the boiling point) can "expand" so the molecules occupy only the edges of the container.

Connie's understanding of the particulate nature of matter is limiting. For example, on the paper-and-pencil instrument, Connie was asked in item 4 to describe what would happen to the volume, mass, and pressure of air compressed in a sealed syringe. On the volume task, she explains,

Connie: Because this (*plunger on syringe A*) isn't pushed down as far and it takes more pressure to push it down. So the volume should be, actually, the same. Wouldn't it be the same? Because you're just compressing the air more.

Connie initially answered this question correctly on the posttest. That is, on the posttest she responded that the volume of the air will be less after compression than before compression. It was while looking at her previous posttest and explaining her answer that she considered the volumes to be the same before and after compression. This seems to be a result of her confusing volume with mass. Therefore, Connie's understanding here is transient. When asked what would happen to the mass of air in the syringe after compression, Connie says,

Connie: The mass should be greater in A (*before compression*) than B (*after compression*)...Because there's more area in A than there is in B that the air is taking up.

The greater *area* translates to Connie as greater *quantity*.

When asked what would happen to the pressure exerted by the gas upon compression, Connie says,

Connie: The pressure enclosed in A is less than the pressure in B...because it's not as hard to push down the plunger in A...It takes more to press down the air in B.

As noted earlier, explanations at the macroscopic level are, in general, sufficient for explaining the changing in the volume and mass of a gaseous substance after compression. But explaining the pressure change is better described using molecular language. Connie does not do that here. She continues to rely on macroscopic observations to explain how the pressure changes upon compression.

<u>How Connie Understands and Uses the Concepts</u>

Connie initially exhibits a functional understanding of the particulate nature of matter. She is able to represent this understanding pictorially. But she also ultimately applies her understanding in a differential way. Connie reveals the following misconceptions:

1. When the temperature of a gas increases, the particles will spread out even more and occupy only the edges of the container.
2. The closer a gas is to its boiling point, the more energy the particles have. Therefore, the particles spread out more for a gas whose temperature is around that of the boiling point.

Connie does not tend to use microscopic descriptions when describing the behavior of a gaseous substance best described using these kinds of descriptions. Rather, she depends on more macroscopic descriptions based mostly on what she sees or feels. However, her macroscopic observations are limiting for her when trying to estimate P, V, and T. For example, her dependence on what she can see doesn't allow her to conceive of air exerting a pressure on the plunger which seemingly has no other external force operating on it. To explain the pressure difference, therefore, Connie uses an intuitive, macroscopic understanding of what happens when a substance is stuffed into a smaller space.

Connie's Real-world Applications

On the paper-and-pencil instrument, Connie is asked in item 5 to estimate the pressure of air on the inside of a syringe operating under conditions of standard pressure when the plunger is not moving. She responds that the pressure of air on the inside of the syringe is greater than standard pressure. During the clinical interview, she explains her response.

Connie: ...In the air, there's no pressure being pushed down. But since it's (air inside syringe) enclosed in the plunger, I mean in the syringe, there is more pressure than there would be if it was just out in the open.

Connie doesn't conceive of the air outside the syringe as exerting a pressure. However, the enclosed air does. Consequently, it seems that Connie's conception is that air pressure is brought into existence because it is in the smaller amount of space, not in accord with the kinetic molecular theory. That is, it is the stuffing of air into a smaller space that creates the greater pressure.

During the clinical interview, Connie is also given a real syringe and asked to estimate the pressure exerted by air on the inside of the barrel when the plunger is not moving. This is a task similar to item 5 on the paper-and-pencil instrument. However, this time, she is asked to give a value for the pressure. Although she talks at length about item 5, she is not at all responsive to this problem.

Interviewer: Can you tell me what the pressure of air is in this syringe?

Connie: Uh uh. I've already forgot all that.

She engages herself similarly when asked to talk quantitatively about the P and V relationships when the air in the syringe is compressed. What is interesting is that she gives similar values for the tasks with this real syringe as the other three students in this case study. However, she does not really engage herself with trying to understand why she believes what she does.

Interviewer: Let's say that the pressure of air on the inside of there (syringe barrel) is one atmosphere. If you push the plunger down to about 10 (cubic centimeters), what happens to the pressure of the air on the inside?

Connie: It gets harder to push...

Interviewer: If the pressure was one atmosphere at 20, what do you think it would be at 10?

Connie: Two, maybe?...Because it gets, more compressed.

Interviewer: Suppose you pushed the plunger from 20 to 15...

Connie: Probably about, like, 1.5.

Interviewer: And you say that because...?

Connie: Because I said 10 would be 2. It wouldn't be as hard to push it to 15 as it would be to push it to 10. It would be not as much pressure.

Interviewer: How did you get the value 1.5?

Connie: Fifteen?...I don't know (laughter).

Connie focuses on what she is experiencing from the syringe itself. Although she has obviously come up with her values from somewhere (perhaps by taking an average like Cameron, Betty, and Karen), she is not willing to try and relate what she is experiencing physically with the syringe to her mental understanding of what's happening mathematically.

Summary of Connie's Case Study

Connie is a student who possesses the least mathematical knowledge of the four students for which case studies were done. She is not able to do the algebra in order to solve the gas law equations although she is capable of using ratio-and-proportion reasoning. However, like the other three students, Connie is able to exhibit an understanding of the behavior of a gas in a container at the submicroscopic level by giving an adequate drawing of this behavior.

Mathematical Understanding Issues

ISSUE 1: The Understanding of Proportional Relationships.

Connie's knowledge of proportional relationships is much the same as that of Cameron, Betty, and Karen. She too seems to operate along a multiplicative-additive continuum. However, because Connie cannot recall the gas law equation it is also doubtful that she has a reasonable understanding of the relationships between variables in the equation.

ISSUE 2: The Creation of a Cohesive Mathematical Model.

Connie seems to operate more on intuition than a comprehensive model in calculating variables. Although she clearly has the ability to estimate values, it is not clear how she accomplishes this. However, as stated when addressing the previous issue, it does appear safe to say that Connie has no comprehensive model for doing this. Connie's estimates are the same as Cameron and Betty for the syringe system on the paper-and-pencil instrument, and the same for Cameron, Betty, and Karen on the real syringe task. However, the difference between Connie and all the others is that she never talks about variables or tries to represent them symbolically.

ISSUE 3: The Use of Gas Law Equations.

Connie cannot recall the gas law equations. She remembers nothing about them. Therefore, she is not able to use them at all.

<u>Conceptual Understanding Issues</u>

ISSUE 4: Articulated Understanding of the Atomic-molecular and Kinetic Molecular Theories.

Connie is able to produce a good representation of how a gaseous substance occupies its container at the submicroscopic level. This knowledge, however, is shown to be differential. Like Cameron, Betty, and Karen, Connie's understanding of the atomic-molecular level is context specific. For example, for Connie, a gas will "expand" at higher temperatures to occupy a container in a different way than at a lower temperature.

On the macroscopic level, Connie has some different understanding about the mass, volume, and pressure of a gaseous substance. She interprets volume to be the same as mass, and mass to be the same as volume (greater area being occupied mean greater mass). In addition, Connie understands pressure as a consequence of the plunger's ability to push. That is, it should be harder to push the plunger when the gas is compressed. Therefore, the pressure is greater with the compressed gas.

ISSUE 5: Atomic-molecular vs. Macroscopic Descriptions of Gas Behavior.

Connie never uses language involving atoms and molecules when explaining the behavior of a gaseous substance. As noted in the previous issue, in describing the behavior of a gaseous substance in real-world situations, she focuses on the phenomenon itself (i.e., pressure is greater because it's more difficult to depress the plunger, mass is greater before compression because the area is greater, etc.).

Summary of the Nine Students Who Were Clinically Interviewed

This section compares the analysis done with the four students in the case studies above with the other five students who were clinically interviewed. Students' understanding is compared and grouped based on their exhibited understanding of mathematical and conceptual concepts used in describing the behavior of a gaseous substance. The student categorization is summarized in Table 2.

Analysis of Mathematical Understanding Issues

ISSUE 1: The Understanding of Proportional Relationships

The Transitional Students

The main characteristic of the transitional students is their partial understanding of the mathematical concepts and their expressions of uncertainty when their own ways of understanding don't make sense when put into use with a real-world task.

Cameron and Nina are classified as transitional students in this regard because of their differential understanding of proportional relationships and the dissonance each experiences when using their personal understanding in a practical situation. Nina's responses will be reviewed below.

An excerpt from Nina's explanation during the clinical interview of item 3 on the pencil-and-paper instrument shows that, like Cameron, Nina has an additive understanding of proportional relationships operating in conjunction with her multiplicative understanding. Although she used a factor understanding to solve item 3(i), she uses an additive strategy for item 3(ii).

Nina: Because as I said for each pressure unit increasing, the volume decreases...this (volume) goes kind of in half. So I thought if it went down from the 10 volume units at 200 pressure units, if it went down to 5 volume units I thought it would be sort of proportional in half. Therefore it would be 300 pressure units.

In addition, like Cameron, Nina clearly wasn't satisfied with her answer after she had explained it. She said that she noticed the 100 to 200 pressure unit increase and thought that the next half cut in volume would have to be 300. She reconsidered and thought the answer should really be "400 pressure units" because of the same reasoning she used for item 3(i).

The Naïve Students

The main characteristic of the naïve students is their partial understanding of proportional relationships and a lack of dissonance experienced when putting their understanding to use with a real-world task.

Betty is classified as a naïve student because she does not experience dissonance when using her understanding of proportional relationships. The understanding she does have, however, she uses consistently. But to experience conceptual change, Betty must first of all be dissatisfied with her understanding when using it. She doesn't appear to be.

Karen and Connie are classified as naïve students. They all have understandings about proportional relationships in gas law equations which are not in accord with those of the chemist. In addition, when using their understanding with a real-world system, they are not perplexed by their understanding nor do they try to reconcile them with their observations.

Janice and Denise attended to proportional relationships on the paper-and-pencil instrument in similar ways as Betty, Karen and Connie. They were able to set up and solve the gas law equation for item 1. But if the relationships are to be understood in the equation, the equation itself must be understood. Neither Janice nor Denise showed that they possessed a real understanding of the equation. Denise's understanding of the gas law equation is reviewed below.

Table 2. Breakdown of Students Who Were Clinically Interviewed Into Various States of Understanding

ISSUE	GOAL CONCEPTION	TRANSITION	NAÏVE CONCEPTION
The understanding of proportional relationships		Cameron, Nina	Betty, Karen, Connie, Denise, Sherry, Hilda, Janice
The creation of a cohesive mathematical model		Cameron, Betty	Karen, Nina, Connie, Denise, Sherry, Hilda, Janice
The use of gas laws		Karen, Denise, Janice	Cameron, Betty, Connie, Sherry, Hilda
Understanding of atomic-molecular and kinetic molecular theories	Cameron, Janice	Denise, Sherry	Betty, Karen, Connie, Nina, Hilda
Atomic-molecular vs. macroscopic descriptions of gas behavior	Cameron, Betty	Karen	Connie, Sherry, Nina, Hilda

After writing Gay-Lussac's relationship to solve item 1, Denise is asked to explain her understanding.

Denise: You had to convert it (the temperatures) to Kelvin because that's the only one that fits in the equation.

Interviewer: Why is that?

Denise: I don't know. That's what our teacher told us.

Interviewer: Suppose I didn't convert the temperatures to Kelvin and left them as Celsius temperatures and put them in the equation. Would there have been a problem?

Denise: Yeah, it's not the same as this (equation with Kelvin temperatures). It has to have the same units.

Interviewer: Well, if I put both temperatures in as Celsius degrees, they would have the same units.

Denise: Well, yeah, but still it's not the same.

Interviewer: Do you think you would have gotten the same answer?

Denise: No, I don't think so. I don't know, maybe not.

Janice had similar comments during her clinical interview. As a matter of fact, she was not at all bothered by a negative value for the pressure calculated from the gas law equation using Celsius temperatures. This understanding suggests that both Denise and Janice understand the gas law equations in a mechanical way. It is not likely that they have an understanding of the proportional relationships which exist between the variables in the equation.

Sherry and Hilda are students who, like Betty, Karen, and Connie, are classified as naïve. Hilda's understanding is the most like Connie's. Neither could set up the gas law equation on the posttest or during the clinical interview. Again, what ties them all together in this regard is the differential understandings about proportional relationships that each has which goes unchallenged as they apply their understanding while performing a real-world task. For example, on the posttest, Sherry was able to set up and solve the gas law equation for item 1. During the clinical interview, she exhibited an understanding much different when performing real-world tasks. On both real-world tasks, she exhibited an understanding of proportional relationships that was entirely additive. For example, Sherry explains what happens to the pressure of air when it is compressed in a syringe from 20 cc to 10 cc, and her responses are reviewed below.

Interviewer:	If I cut the volume in half, what do I do to the pressure?
Sherry:	You increase the pressure.
Interviewer:	By how much?
Sherry:	I guess by 10.
Interviewer:	Is it because the volume decreased by 10 that the pressure would go up by 10?
Sherry:	Yes.

Sherry was then asked to consider what happened when the air was compressed from 20 cc to 15 cc.

Interviewer:	What would happen to the pressure of the air if I moved the plunger from 20 (cubic centimeters) to 15 (cubic centimeters)?
Sherry:	Well, I'm assuming 5, because when I went to 10 it [went] to 10. So I'm only going to 15, that's only 5.
Interviewer:	So if the volume went from 20 to 15, you say it changed by 5, what would happen to the pressure?
Sherry:	The pressure would increase by 5.

Sherry shows the same understanding of proportional relationships as additive when doing the syringe system task on item 3 of the paper-and-pencil instrument. The point is that, in spite of a previous ability to setup and solve the gas law equation, Sherry is not bothered by her understanding here.

The Understanding of Proportional Relationships Exhibited by All Students Taking the Paper-and-Pencil Instrument

At this point, it seems instructive to take a look at all of the students who took the paper-and-pencil exam. The nine students who were clinically interviewed have shown some interesting conceptions of proportional relationships. To the extent that they are representative of the other students, there should be some similar and pervasive misconceptions about the nature of proportional relationships. Table 3 shows the responses to item 3 of all 116 students who took the paper-and-pencil instrument. If the explanations given by the interviewed students is consistent with the understanding possessed by the rest of the students taking the paper-and-pencil instrument, more than one-quarter of the students in some way possess an additive understanding of proportional relationships. What is even more interesting is that the percentage of students reporting values such as 300 and 250 for item 3(ii) actually increased after instruction on gas behavior. Instruction actually helped less than 10% of the students!

Table 3. Some Values for Item 3 Questions on the Paper-and-Pencil Instrument, the Percentage of Students Reporting Each Value, and the Percentage Change in Students Reporting Each Value from the Pretest to the Posttest (N = 116)

Item	Value	Percentage of Students Responding	Percentage Change From Pretest
3(i)	80.0 [a]	68.1 (65.0) [c]	+ 3.1
3(ii)	400.0 [a]	66.4 (58.2) [c]	+8.2
	300.0 [b]	19.0 (15.8) [c]	+3.2
	250.0 [b]	9.5 (7.9) [c]	+1.6
3(iii)	15.0 [b]	76.7 (84.8) [c]	-8.1
	13.3 [a]	8.6 (0.0) [c]	+8.6
3(iv)	75.0 [b]	69.8 (78.8) [c]	-9.0
	66.6 [a]	9.5 (0.0) [c]	+9.5

[a] Correct value
[b] Incorrect value
[c] Number in parenthesis indicates percentage of students reporting the given value on the pretest

ISSUE 2: The Creation of a Cohesive Mathematical Model.

The Transitional Student

As noted in the case studies, Cameron and Betty are both students who are able to construct a cohesive personal mathematical model to explain the behavior of a real-life gaseous system. Cameron is aware of the relationships between the variables in a mathematical representation, particularly as given in the gas law equations. When using his mathematical understanding in a practical way, he uses his personal model to estimate values for variables. Betty is not able to talk effectively about the relationships between variables in a gas law equation. However she forms a very cohesive model for her mathematical explanations which she uses more effectively than Cameron uses his.

The Naïve Students

It was not apparent that any of the other seven students who were clinically interviewed had a cohesive mathematical model to describe the behavior of a gaseous substance. All of the students could use the math and attend, in various ways, to mechanistic issues (e.g., changing temperatures from the Celsius to the Kelvin scale, plugging in numbers, solving for variables, etc.), but none showed a consistent and coherent use of the mathematics in the context of the gas law they had written.

ISSUE 3: The Use of Gas Laws.

Table 4 shows all of the students' abilities to solve a gas law equation. On the pretest assessment of students using the paper-and-pencil instrument, none of the students could correctly use a gas laws to solve item 1. On the posttest assessment, over half (53%) of the students could use a gas law equation correctly to solve item 1. About one-third (33%) of the students still possessed some misunderstanding of the mechanics of the gas law equation after instruction. These misunderstandings included not using Kelvin temperatures (7%), switching the positions of the temperature values in the equation (4%), and simply using an incorrect mathematical relationship (22%). However, as discussed earlier with Issue 1, the students' ability to solve the equation does not mean that they understand it even when they possess the ability to use the equation in a context which is not typical.

The Transitional Student

Karen and Denise both show they can use the gas law equations outside of the context of its typical usage. They used the equation when working through item 3 on the paper-and-pencil instrument and with the real syringe. Karen was consistent in her use of a gas law equation for item 3. Denise, however, used an averaging strategy for item 3(iii), like most of the other students, but a gas law for item 3(iv). In explaining how she worked item 3(iii), she said,

> **Denise:** This one's (the pressure unit of 150) like in between (100 and 200 pressure units)...so I figured 15 (volume units) would go right there.

When asked about how she worked item 3(iv) she says,

> **Denise:** I actually used an equation right there...

The Naïve Student

During the clinical interview, Janice was asked to give a value for the pressure exerted by air in the real syringe when the plunger was pushed from 20 cc to 15 cc. She said that the pressure "probably increased by a quarter" because 15 cc was half of the way between 20 cc and 10 cc. Her rationale was that since the pressure was 2 atm when the volume was decreased by half, when it is decreased by less than half at 15 cc the pressure should be less than 2 atm.

Table 4. Some Responses for Item 1 on the Paper-and-Pencil Instrument Posttest (N = 116)

Response	**Number of Students (% of students)**
Correct equation setup and/or answer	62 (53.4)
Correct equation but incorrect placement of temperatures	5 (4.3)
Correct equation but used Celsius temperatures	8 (6.9)
Used an incorrect mathematical relationship	25 (21.6)
No response	16 (13.8)

For her 15 cc is a half of a half, and, therefore, the pressure would only go up by one-quarter, or 1.25 atm. What distinguishes Janice in this regard, however, is that when asked if she could calculate what the pressure would be, she successfully set up the Boyle's law equation, although she did not calculate an answer. She was totally oblivious to the fact that her two understandings were in conflict.

Analysis of Conceptual Understanding Issues

An important finding in the four case studies was that students had a variety of ways of thinking about the behavior of matter at the submicroscopic level in spite of being able to draw adequate representations of air at the submicroscopic level enclosed in a syringe.

ISSUE 4: Understanding of the Atomic-molecular and Kinetic Molecular Theories.

The Students with Goal Conception

Cameron is a student with a goal conception of the behavior of matter at the particulate level. Although Cameron is bothered by how the kinetic molecular theory applies to any other state than the gaseous state, he has an adequate understanding of the submicroscopic behavior of a gaseous substance.

Janice is another student with the goal conception. She is able to represent well pictorially how a gaseous substance should look at the submicroscopic level. She also gives evidence that she has a normative understanding of how the molecules of a gas will occupy a container. For example, in item 2 of the paper-and-pencil instrument, she chooses the representation which continues to show molecules filling their container even after the temperature is dropped.

The Transitional Students

The main characteristic of the transitional student is that they often show a partial understanding of how a gaseous substance behaves at the submicroscopic level, and are bothered by real-life situations which seem in conflict with this understanding.

Denise is a student who may be considered as transitional in her understanding. She is able to represent well an understanding of how air in a container should behave at the submicroscopic level. She knows, for example, that the gas particles should spread out to occupy the entire volume of the container. When responding to item 2 on the paper-and-pencil instrument, however, she picks the choice which shows the molecules as clumped together. She explains her choice.

> **Denise:** I was probably thinking since the temperature went down that the molecules were going to condense.

It seems that Denise has an adequate conceptual understanding of how a gaseous substance should behave. Her misconception is a result of her confusion about the boiling point. I classify her as transitional in the sense that, although she has some misconceptions about the boiling point of the gas, she applies her understanding so that what she has previously articulated about the behavior of air in a syringe at the submicroscopic level does not conflict with her understanding of the submicroscopic nature of hydrogen gas in a steel tank.

Sherry is another student who shows a conceptual understanding which is in transition. When responding to item 2 on the posttest, picked choice (d) which shows the molecules aligned around the inside of a tank. During the clinical interview, however, Sherry picked choice (e) which shows the molecules closer together in a smaller tank. Sherry explains her choice.

> **Sherry:** Because the molecules will get closer as it gets colder...You know, there's a pattern and they're close together.

Sherry has an understanding that the molecules of a gaseous substance should occupy the entire volume of the container they are in. For example, when drawing how the molecules of air would look inside of a syringe, she showed the molecules as randomly spread out and completely filling their container. It seems, however, that she experiences some dissonance when trying to explain what happens to gas molecules when a gas is cooled. She seemingly wants to retain the notion that the molecules should completely occupy their container. Therefore, she reduces the size of the container. On the other hand, she has an understanding that the molecules of a cooled gas would be less random than the molecules of a less cool gas. When she comments, "there's a pattern," she represents this pattern by aligning molecules along side each other in a row:

• • • •
• • • •

This student is transitional because she tries to merge her understandings to produce an acceptable explanation.

The Naïve Students

Betty, Karen, Connie, Nina, and Hilda are all classified as students with naïve conceptions about the behavior of a gaseous substance. The classification for groups of students are based on various reasons, but the tie that binds them all together is the inconsistent and unchallenged use of their knowledge to explain real-world tasks with the behavior of a gaseous substance.

As noted in the case studies, Betty, Karen, and Connie all had understandings about the particulate nature of a gaseous substance which changed depending on the context of the application. The fact that their explanations fit the situation and was not generally applicable did not cause them concern during their explanations.

Nina has an understanding of the particulate nature of air compressed in a syringe which is much like Karen's. Nina understands the compressed molecules as being more aligned and less random in their orientation. Also, like Karen, Nina has some distinct notions about how this compact air produces a pressure.

Interviewer:	Look at your drawing and tell me again why air exerts a pressure.
Nina:	Because it's being compacted by the syringe, the pressure would be higher because if it had no where to go out...the same volume would be in a smaller space.

In other words, Nina sees the pressure being created by the force of the molecules trying to move away from each other in this small space. However, when the gas is not being compressed, Nina sees the molecules as being more spaced out and random in their orientation. Again, however, this understanding is contextual. That is, like Connie, Nina believes that as the hydrogen gas approaches its boiling point the molecules will move even further from each other as the gas gets hotter (which, for Nina, is around the boiling point). Therefore, although on the posttest Nina chose the representation which showed the molecules randomly filling their container (choice (a)), during the clinical interview she explained that as the temperature gets closer to its boiling point, the molecules would "expand" and move even further away from each other as in choice (D).

Hilda is a student who does not exhibit initially a particulate understanding of the behavior of matter. When asked to draw her view of the submicroscopic nature of air in a syringe, she draws a representation that depicts this level as continuous. That is, she takes her pencil and shades in the entire volume occupied by the air. When asked to explain why a gas exerts a pressure, Hilda explains.

Hilda: Because the molecules are bouncing off everything at different times and different angles and running in to each other.

Hilda, however, sees no conflict with her previous drawing and what she is now saying. That is, when asked if she felt her drawing represented her understanding about the pressure, she responded, "Sure, I guess."

Some Understandings of Atomic Theory Exhibited by All Students Taking the Paper-and-Pencil Instrument

Again, to the extent that the clinically interviewed students are representative of all of the students taking the paper-and-pencil instrument, it is instructive to examine some understandings of atomic theory exhibited by students on the paper-and-pencil instrument. Table 5 shows students responses to item 2 on the paper-and-pencil instrument. Only about 28% of the students showed that they exhibited the goal conception. However, the case study analysis have shown that even these students may posses some contextual understandings. A significant finding here is that 19% of the students responding to item 2 on the paper-and-pencil instrument chose the representation which showed the molecules of gas aligned around the inside edge of the tank. In addition, this understanding among the students increased after instruction. As a matter of fact, Table 5 shows that while other misconceptions decreased or stayed relatively the same, this particular misconception showed the greatest increase after instruction on gas behavior.

ISSUE 5: Atomic-molecular vs. Macroscopic Descriptions of Gas Behavior.

It should be stated at the outset that when asked to explain macroscopic observations, chemists might also refer to macroscopic patterns like the gas laws

Table 5. Percentage of Students Responding to Each Choice of Item 2 of the Posttest and the Percentage Change of Responses to Each Item From the Pretest (N = 116)

Item	Percentage of Students Responding	Percentage Change From Pretest
2(a) [a]	28.4 (29.9) [b]	-1.5
2(b)	29.3 (29.9) [b]	-0.6
2(c)	12.9 (12.4) [b]	+0.5
2(d)	19.0 (15.5) [b]	+3.5
2(e)	8.6 (15.5) [b]	-6.9

[a] Correct answer
[b] Value in parenthesis indicates percentage of students choosing this item on the pretest

if they don't know specifically that molecular explanations are being required of them. When explaining the behavior of air in a syringe in this study, I did not ask students to explain this behavior specifically using their understanding of the submicroscopic nature of a gaseous substance. That many of the students failed to use such language may not be useful, in general, to talk about their conceptual understanding. However, to the degree possible, I have examined their spontaneous use of such explanations.

The Students with Goal Conception

Cameron and Betty are both students who often use explanations of the submicroscopic behavior of matter in an appropriate way to explain their understanding. Although Cameron finds some limits in the use of the kinetic molecular theory as it relates to solids and liquids, and Betty has some differential understandings when it comes to understanding kinetic molecular theory, both of these students used molecular or macroscopic language when appropriate to explain the behavior of a gaseous substance.

The Transitional Student

The main characteristic of the transitional student is that they have a preference for explanations using molecular language, but often use this understanding in some differential ways. In describing transitional students with the previous four issues, I often included the ability to experience dissonance when putting knowledge to real-world use as a characteristic of a transitional student. However, that characteristic is not used here because of the nature of molecular-level understandings. That is, because students can't visually see molecules when observing practical behavior, they may not be challenged by their observations. Here, I simply classify the transitional student as a student who has a preference for explanations using molecular language, but who uses it in some differential ways.

As seen in the case study, Karen in a student who has a preference for molecular language, but who uses it in some differential ways to explain her observations of real-world phenomena. For example, consider her explanation that compressed air is made up of well-arranged molecules pushed so tightly together that they attempt to separate from each other, thus, creating the pressure of the gas.

The Naïve Student

The main characteristics of the naïve student in this regard is that they have no understanding of the submicroscopic nature of matter, or they inconsistently use molecular language when describing real-world phenomena.

As shown in the case studies, Betty and Connie are students who may be considered naïve in their understanding. Betty is marginal and inconsistent in her use of molecular language, and Connie never uses this language to explain the behavior of a gaseous substance.

Sherry and Nina are both students who don't consistently use descriptions at the submicroscopic level to explain the behavior of a gaseous substance. For example, when explaining her understanding of item 2 on the paper-and-pencil instrument during the clinical interview, Sherry explained her understanding using explanations of the submicroscopic nature of the gas.

Sherry: The molecules will get closer as it gets colder…You know there's a pattern and they're close together.

However, when explaining her understanding of gas volume, mass, and pressure in item 4 on the paper-and-pencil instrument during the clinical interview, Sherry doesn't preferentially use language about the submicroscopic nature. Instead, she relies on more phenomenological descriptions. For example, she explains that the pressure of air in a syringe was created because of "gravity," and that the volume of a gas is "a measure of the air," or "length times width times height." While discussing item 4, she never voluntarily refers to molecules and their movement.

Nina likewise gives inconsistent descriptions of gas phenomena explaining, for example, at one point that gas pressure is a result of "All the molecules…their energy…how fast they're moving." In another instance, Nina chooses to explain gas pressure as resulting from the pressure placed upon the plunger.

Nina: Since there is more pressure exerted on it (*compressed air as opposed to uncompressed air*), there is more pressure inside.

Hilda is a student who is inconsistent in her molecular understanding. When representing the submicroscopic nature of air in a syringe, she draws a representation in which she shades in the area to represent how air fills the container. Again, however, when explaining why a gaseous substance creates a pressure, she talks about the collision of molecules against the container walls. As noted with the previous issue, she is unaffected by this discrepancy.

Not much can be said about Janice and Denise as it relates to Issue 5 because the data are scarce. Neither was asked during the clinical interviews to do much explaining about phenomena with which their understanding could be examined. Both gave good representations of air at the submicroscopic level. However, it has been noted that this does not always imply an understanding.

CHAPTER FIVE

SUMMARY, CONCLUSIONS, AND IMPLICATIONS

Summary of Dissertation

The Problem and the Theoretical Basis

A problem with understanding scientific concepts is prevalent with students throughout science education. The literature on misconceptions research has focused on the specific notions students have about scientific concepts. The research agenda has been to identify students' misconceptions so as to provide better means by which to change these misconceptions into scientific conceptions.

The understanding of the behavior of matter is a science topic where students often hold a number of misconceptions. Particularly problematic in this regard is students' understanding of the behavior of a gaseous substance. The fact that gaseous substances cannot often be detected visually contributes to students uncertainty about its nature and behavior. In addition, the scientific theories used to explain the behavior of matter are abstract and counterintuitive to the experiences of students. Consequently, when students learn about the behavior of a gaseous substance in chemistry classrooms, their approach has been typically one of using what they can use to achieve the end of getting through the topic. In chemistry classrooms, "understanding" the behavior of a gaseous substance has often meant knowing how to solve gas law equations. However, this understanding alone is far from that of the chemist, and students leave chemistry classrooms never fully understanding how to describe the nature and behavior of a gaseous substance. This is problematic because one goal of school science is to teach students how to function as literate citizens in their environment. If science students cannot understand, for example, the behavior of a gaseous substance like air, they miss a real understanding of everyday occurrences like the measurement of barometric pressure, the flight of a hot air balloon, and the effects of weather changes on the inflation of an automobile tire. Students who leave the chemistry classroom should be prepared to proposed scientific explanations for all of these phenomena.

My study began by examining the kinds of understanding students must have to produce an explanation for the behavior of a gaseous substance which is acceptable to the chemist. Student understanding was examined particularly in the light of how they used their understanding to explain real-world gaseous systems. A major premise of this study is that an analysis of how students use their understanding to explain real-world systems allows a glimpse into the myriad of understandings which must be attended to by the student in order to propose a scientifically acceptable explanation. Most of the literature to date have focused on the mathematical knowledge K-12 students use when solving gas law equations and their conceptual understanding of the gas laws. Few studies, however, have focused on how students use their knowledge with real-world tasks, particularly college students. An underlying assumption was that students must acquire and use their understanding in three areas: (1) mathematical, (2) conceptual, and (3) real-world applications. Therefore, the following three questions gave focus to the study:

1. What mathematical understandings do introductory-level, basic chemistry college students use when they describe the behavior of a gaseous substance?
2. What conceptual understandings do the students have about the behavior of a gaseous substance?
3. How do students use their mathematical and conceptual understanding when explaining the behavior of a real-world gaseous system?

These three research questions address the mathematical (Research Question 1), conceptual (Research Question 2), and real-world applications (Research Question 3) focus of this study. I believe it is important to know not only how students work mathematical equations, but how they use their mathematical knowledge in explaining real-world systems. Likewise, it is important to know the misconceptions students possess. But it is also important to get a feel for how students use their conceptual understanding in explaining real-world systems.

Issues Derived from the Literature

From the literature review in these three areas, five issues were explicitly or implicitly drawn from the studies. Students' real-world applications of their mathematical and conceptual knowledge were not addressed as separate issues, but were included as part of their understanding when examining their mathematical and conceptual understanding. Particularly, Issue 1 and Issue 3 were both used to address students' mathematical knowledge, while Issue 2 and Issue 3 were used to address students' mathematical understanding. In a similar manner, Issue 4 was used to examined students' conceptual knowledge while Issue 5 was used to examine conceptual understanding.

Mathematical Understanding

ISSUE 1: The Understanding of Proportional Relationships.

ISSUE 2: The Creation of a Cohesive Mathematical Model.

ISSUE 3: The Use of Gas Laws.

Conceptual Understanding

ISSUE 4: The Understanding of the Atomic-molecular and Kinetic Molecular Theories

ISSUE 5: Atomic-Molecular vs. Macroscopic Descriptions of Gas Behavior.

Methods

A case study analysis was conducted with four students taken from a population of 116 students enrolled in a basic chemistry course at a Midwestern community college. All students were examined using a paper-and-pencil instrument to determine their mathematical and conceptual knowledge of the behavior of a gaseous substance. These students were grouped into nine groups according to their performance on the instrument. Three students were selected at random from the four categories containing the greater percentage of students. These students were then clinically interviewed to determine how they thought about their mathematical and conceptual knowledge and used that knowledge in performing real-world tasks. One student selected from each group was then used as a case study for their particular group. Their selection from the group was random, but based on their ability to represent the group as well as their ability to articulate freely and willingly their understanding.

Summary of Findings

The results are organized according to the issues identified in Chapter 2.

Issues and Key Findings Supported by the Data

Mathematical Knowledge

ISSUE 1: The Understanding of Proportional Relationships.

Key Finding: Although all of the students use the term "proportional" when discussing their understanding of the mathematical equation and/or relationships between variables, none of them had a real understanding of what this meant. In general, the students had a directional sense of what this meant, but a splintered mathematical understanding that included an understanding of proportional relationships as additive in nature.

ISSUE 2: The Creation of a Cohesive Mathematical Model.

Key Finding: One student, Betty, seemed the most able to produce and use a cohesive mathematical model to explain her understanding of the behavior of a real-world gaseous system. Although her model was not that of the gas laws, she could use it to get reasonable estimates of values when describing the behavior of a real-world gaseous system

ISSUE 3: The Use of Gas Law Equations.

Key Finding: The students did not use gas law equations beyond their typical usage as experienced with problems in their chemistry classroom. One student, Karen, used the gas law equations beyond typical gas law problems. She could not however, talk about the relationship between the variables in the equation. Cameron was the only student who exhibited an ability to do this.

Conceptual Understanding

ISSUE 4: The Understanding of the Atomic-molecular and Kinetic Molecular Theories.

Key Finding: Students regularly exhibited a contextual understanding of these theories. Depending on the conditions, the theories achieved differential usage.

ISSUE 5: Atomic-molecular vs. Macroscopic Descriptions of Gas Behavior.

Key Finding: For the most part, students in this study used molecular language when appropriate to describe changes in the pressure, volume, and temperature of a gaseous substance. As noted earlier, many of the descriptions the students were asked to do could be described adequately using macroscopic language that chemists would be just as inclined to use when explaining similar tasks.

Comparison of the Four Case Study Students

Table 6 gives a comparison of the major issues derived from the mathematical and conceptual understandings of the four students used for case studies and their real-world application of this knowledge. As can be seen by the similarities of issues across categories, the initial categorization of students served as a convenient method by which to group students for analysis, but is not useful for talking about differences among the students. Regardless of their initial mathematical or conceptual categorization, the four case study students share some common mathematical and conceptual abilities.

Particularly interesting is the observation that, although Connie is classified as a student with a low mathematical ability, she is just as successful as Cameron, Betty, and Karen in predicting values for the task in item 3 of the paper-and-pencil instrument.

Consequently, this study suggests that students abilities to solve mathematical problems and articulate certain conceptual understandings gives a superficial view of the true understandings students have about the behavior of a gaseous substance.

Table 6. Comparison of the Mathematical, Conceptual, and Real-world Understandings Possessed By the Four Case Study Students

Student	Mathematical Understanding	Conceptual Understanding	Real-world Application
Cameron (MMHC)	• mechanical use of gas law equations • no use of gas law equations beyond typical gas law problem • no sound understanding of proportional relationships	• limited understanding of kinetic molecular theory (i.e., particle movement only makes sense in gaseous state) • frequently uses language referring to molecules	• uses conceptual understanding to estimate pressure exerted by air in a real syringe • uses a couple of strategies to predict quantitative behavior of air inside of a real syringe, but can't bring them together
Betty (MMMC)	• mechanical use of gas law equations • no use of gas law equations beyond typical gas law problem • no sound understanding of proportional relationships	• limited understanding of kinetic molecular theory (i.e., particles in a gas come closer and don't completely fill tank at colder temperatures) • tends to appropriately use macroscopic and molecular descriptions of matter	• cannot use conceptual understanding to estimate pressure exerted by air in a real syringe • consistently uses articulated knowledge when performing real task

Table 6 (cont'd)

Karen (MMLC)	• mechanical use of gas law equations • uses gas law equation when initially solving nontraditional gas law problem on posttest, but did not use it during clinical interview • no sound understanding of proportional relationships	• limited understanding of kinetic molecular theory (i.e., compressed gas molecules are more ordered in a container than uncompressed molecules)	• cannot use conceptual understanding to estimate pressure exerted by air in a real syringe • does not consistently use articulated knowledge when performing real-world task
Connie (LMLC)	• can't use gas law equations at all • no sound understanding of proportional relationships	• limited understanding of kinetic molecular theory (i.e., at higher temperatures molecules spread out farther from each other than at lower temperatures	• not able to articulate how mathematical understanding is used to solve real-world task

Conceptual and Mathematical Connections

None of the students chosen as case studies had a mathematical understanding of the gas law equations. All but one could manipulate the mathematical representations, but none had an algorithmic proficiency with the equations whereby they could use the mathematical representations of the gas laws for conceptual understanding.

All of the case study students had some limited understanding of the kinetic molecular theory. Although they could all represent pictorially the molecular nature of a gaseous substance, they could not use this understanding as a means of informing their mathematical descriptions of gas behavior. For example, although all of the students could adequately draw a pictorial representation of how the molecules of air should be distributed in a container, many of them had problems representing this distribution when the temperature of a gas was lowered. The existence of such conceptual limitations seems useful in helping to explain why the students had problems using a mathematical representation beyond a mechanistic use. That is, students with a limited understanding of the affect of temperature on molecular movement, for example, are not equipped with the conceptual resources to question the validity of a calculated answer when using a gas law equation to calculate a resulting change in the pressure of a gas when its temperature drops. Consequently, approximately 7% of the students in this study, as noted in Table 4, did not question the validity of a negative pressure after using an otherwise correct gas law representation to calculate pressure in item 1 on the paper-and-pencil instrument.

Implications for Curriculum Development, Classroom Teaching, and Teacher Education

Implications for Curriculum Development and Classroom Teaching

Based on the misunderstandings students often acquire after classroom instruction, the behavior of a gaseous substance deserves more focused attention from teachers. Teachers must begin to anticipate the deeper misconceptions that affect the students' thinking about the nature and behavior of matter, particularly the gaseous state. Most of the students had a good surface understanding of the behavior of matter at the atomic-molecular level. However, when their understanding was put to the test using a real-world task, like the compression of air in a syringe, some deeper misconceptions became apparent. This suggests that teachers need to incorporate often various strategies to help students put their theories in action and, thus, expose their knowledge. The results of this study seem to give a clear indication that the learning objectives for understanding the behavior of a gaseous substance (Appendix B) are not sufficient.

Another point established in this study is that students often exhibited a misunderstanding about the directional nature (i.e., direct or inverse) of proportional relationships when considering the relationship between the pressure and volume of a gas. De Berg (1995) has suggested that students often confuse gas volume with gas density. Therefore, it could be either that students are considering the relationship between pressure and gas density - a direct proportional relationship, instead of the relationship between pressure and gas volume - an inverse relationship; or, students could be just guessing. At any rate, it is important to challenge students' understanding more. Consequently, teachers should focus on several facets of a given problem. We should expose students to situations in which it does make sense for them to talk about direct proportional relationships (e.g., with pressure and gas density) and examine the differences between this problem and the inverse proportional relationships with pressure and gas volume.

Teachers must become more aware of the common misconceptions students bring into and develop from the chemistry classroom. These conceptions form the basis of their understanding and are not easily removed if not specifically dealt with. In order to do this, teachers must know what these are and some possible reasons for their development.

A Specific Strategy Based on the Results of This Study for Teaching the Behavior of a Gaseous Substance

During the 1999-2000 school year, I will introduce conceptual change teaching methods into a unit on gas behavior in my college basic chemistry class. The goal will be to teach students in a manner consistent with research in conceptual change teaching (e.g., Smith, 1990) recognizing, as a result of this study, the problems students at this level often have when learning the behavior of a gaseous substance. Conceptual change teaching, in general, is composed of four activities: (1) students are given an exposing event and their initial conceptions recorded; (2) a discrepant event, which is contrary to the students' intuition, is presented followed by discussion; (3) lectures are devised to address naïve and transitional conceptions; and, (4) students are given practice using the new conception. I will discuss the new unit below from the perspective of these four activities using the results from this study.

An Exposing Event

This study has shown that students come to basic chemistry with some conceptual ideas about the macroscopic and microscopic behavior of a gaseous substance which are at odds with scientific conceptions. The paper-and-pencil instrument in its present form is a useful "exposing event" for students' initial conceptions of the behavior of a gaseous substance. Students will be given this instrument in class and time (I anticipate no more than 20-30 minutes based on the time the pretest instrument was completed by students in this study) to complete it. The initial conceptions will be recorded on the board and discussed. I anticipate a similar occurrence of responses as documented in Tables 3 and 5. However, remaining class time will be spent discussing specific answers and students' responses to all five items on the paper-and-pencil instrument.

A Discrepant Event

Discrepant events will be posed as questions. The mathematical discrepancy question will be, what's wrong with averaging to obtain values for the syringes operating as inverse proportions? We will spend time examining and discussing pressure-volume and pressure-density data for a gaseous substance.

Does averaging give you the correct value in one case and not in the other? Why?

The conceptual discrepancy question will address the problem of a non-random and inherent movement of particles within a gaseous substance. For example, examine two representations as in 2(a) and 2(d) on the paper-and-pencil instrument and explain how each representation might contribute to the values of P, V, and T measured for a gaseous substance.

Instruction on the Behavior of a Gaseous Substance

Having identified students' initial conceptions, I will design a unit to address these particular conceptions. Based on the results of this study, this unit should attend specifically to the following: (1) the understanding of proportional relationships; (2) the use of gas laws as mathematical models of behavior; (3) the understanding of the atomic-molecular and kinetic molecular theories, and, (4) connecting submicroscopic and macroscopic explanations of the behavior of a gaseous substance.

The Understanding of Proportional Relationships

Students will work in groups of no more than four. Each group will be given a set of data which they will use to examine proportional relationships. They will graph the data and talk about the meaning of the graphical representation. Some groups will have data with direct proportional variables (e.g., recording the mass and volume of pennies), while the other groups will have data with inverse proportional variables (e.g., recording the pressure and volume of air in a syringe). The two sets of groups will come together to compare and contrast their graphical representations. The ultimate goal of the students will be to propose a mathematical relationship to describe the behavior of their data. Through this activity, the students are being instructed in how to use mathematics to model behavior.

The Use of Gas Laws as Mathematical Models of Behavior

Specific instruction will be given on the gas laws focusing on the relationship of the variables to each other. Students will be asked to solve typical gas law problems using the gas law equations. Having worked some typical problems presented to them, students will be given an assessment instrument (quiz) where they will be given a gas law problem and asked to solve it using a gas law equation. They will then be asked to change whatever P, V, and T conditions of the problem necessary to give them a set of data. They will offer a written explanation of how the gas law equation they use models the behavior of a gaseous substance (e.g., what is the role of each variable in the equation, where in the equation is the proportional relationship depicted, under what conditions will the equation hold).

The Understanding of Atomic-molecular and Kinetic Molecular Theories

Students would have previously had instruction on these theories. They will display their understanding of these theories in a real-world context.

Students will be given an instrument like that presented in Figure 4. Misconceptions about the nature of the kinetic molecular theory will be handled within the context of this representation. For example, many of the students in this study thought that hydrogen gas in a tank would adopt a representation like that shown in item 2(d) of the paper-and-pencil instrument when it is warmed. Explaining molecular arrangement in terms of this representation (i.e., how the balloon would deflate if the flask system is turned upside down) could help students get a better feel for what is postulated in the kinetic molecular theory.

Connecting Submicroscopic and Macroscopic Explanations of Gas Behavior

Students will be encouraged continually to explain macroscopic observations in terms of the submicroscopic nature of the system. For example, with the flask and balloon system described above, students will be encouraged in group discussions and reports to talk about their observations in terms of molecular movement. In addition, when discussing the gas laws as mathematical models of behavior, emphasis will be placed on how the model is consistent with the submicroscopic nature of the system.

Practice Using New Conceptions

The students will receive practice using their post-instruction conceptions of gas behavior. Students will be divided into their groups. Each group will be asked to propose macroscopic and submicroscopic explanations for the behavior of air in a real-syringe. They will be told to use all tools at their disposal (i.e., mathematical and conceptual) to propose explanations for the behavior of air in a real-syringe which would be acceptable to a chemist.

Implications for Teacher Education

Conceptual change teaching, like that noted in the previous section, is a tall order for the classroom teacher. Many chemistry teachers, particularly those in college chemistry classrooms, have little training in or patience for the challenges of teaching for conceptual change. As noted in the discussion above teaching for conceptual change learning generally requires that the teacher engages in the following activities: (1) the teacher assesses the students' prior knowledge about a particular science topic with an assessment instrument; (2) the teacher identifies students' misconceptions and becomes aware of the various ways students may form their misconceptions; (3) the teacher addresses specific misconceptions by using novel strategies or those used by others to help the student become dissatisfied with their misconceptions; and, (4) the teacher reassesses the student to determine if the proper conception is attained.

Again, this kind of teaching is a tall order for those who have no training or patience for it. In training chemistry teachers, teacher education departments (as well as academic departments) should consider specifically training teachers in conceptual change methods of teaching. This is a continually growing area of research and the literature on conceptual change teaching for many science topics is growing rapidly. This study has contributed to that literature by identifying specific misunderstandings students possess about the behavior of gaseous substance and the nature of those misunderstandings.

Conclusions and Implications for Further Research

The goal of this study was to better understand the barriers students have when explaining the behavior of a gaseous substance. Understanding chemistry topics in college chemistry classrooms has often amounted to learning how to play mathematical games through which equations are simply manipulated without much understanding of the explanations given by mathematical representations. This raises a concern among chemistry educators about how to best teach students to achieve a better understanding.

Some Specific Considerations Arising Out of This Study

This study has shown that students must attend to a variety of things when describing the behavior of a gaseous substance, particularly when describing the behavior of a real-world gaseous system. One specific question which arises out of this study concerns the practical understanding of the variables P, V, T, and n. Students generally understand these variables in a relatively simple context. For example, they can express what the volume of gas is in a given container whose volume is known. Or, they can explain that a gas exerts a pressure because of the collision of gas molecules with the container wall. More challenging, however, is understanding these variables and their relationship to each other in a real situation. That is, how do students decide P, V, T, and n values in situations where they are not given information as data?

Another specific question which arises out of this study is how do students use molecular language when describing the behavior of a gaseous substance? This is an explanatory ideal favored by the chemist, but often seen as unnecessary by the student. Designing teaching goals and strategies to address these concerns is a challenge for the chemistry educator and requires further research into students' thinking.

Some General Considerations Arising Out of This Study

Problems of student understanding in science classrooms have been addressed in the conceptual change literature in science teaching. Conceptual change researchers stress the value of being aware of student misconceptions about science phenomena and designing curriculum material and instructional strategies to specifically address those misconceptions. The misconceptions literature has focused mainly on the explanations students give about a given scientific phenomena, and, thus, the knowledge they possess. Research in recent years in the conceptual change literature has started to address instructional strategies needed to change misconceptions. Research on student understanding of science topics tends to be focused specifically on content knowledge; that is, the concepts students possess. What seems beneficial to helping teachers develop better instructional strategies is examining how students use the knowledge they have. The current study shows that students must attend to a variety of issues at the same time to offer a scientific explanation of a chemical phenomena like the behavior of a gaseous substance. How they organize their thoughts to propose their explanations should help science teachers design better instructional strategies. Research examining how students use their knowledge has not been as prevalent. Although theories-in-action research is not new (e.g., Driver and Erickson, 1983) and has been carried out at the K-12 level, much more is needed in the area of student understanding in chemistry - particularly, at the college level.

This study has examined a set of mathematical, conceptual, and practical understanding patterns associated with explaining the behavior of a gaseous substance. Each of these areas represents a distinctive element in a students' knowledge base. Although there have been studies that have examined how students solve mathematical equations in chemistry, like the gas law equations, little attention has been given to how students use their mathematical understanding for designing a mathematical model of behavior. Particularly important in this regard is understanding the factors which influence this design process. The results of this study indicate that the models students develop are influenced by their knowledge of mathematical facts and procedures, or lack thereof. How students develop their knowledge into mathematical models is an interesting phenomenon and represents an area in understanding students' knowledge which has been relatively unexplored. For example, how do students compose their mathematical models from their initial understanding of the mathematical representations presented in chemistry class. Although this study addressed that issue, more work needs to be done.

In the area of conceptual understanding, more work also needs to be done to analyze the barriers students encounter when working their understanding of atomic-molecular and kinetic molecular theories into their mathematical models when describing real-world problems. The present study showed how students strengthened their misconceptions as they applied their understanding of the theories in practical situations, but it did not analyze what students thought about these theories as they employed their mathematical models. One reason this is important is because when chemists explain the behavior of a gaseous substance, they are able to operate simultaneously at the mathematical, conceptual, and practical levels of understanding. Consequently, the interactive nature of these three areas in students' explanations is of some concern.

Additional information is needed on how students use their mathematical and conceptual understanding in practical situations. Anderson & Roth (1989) proposed that students understand science when they can describe, explain, predict, and control scientific phenomena. Most of the research in conceptual change has focused on the area of explanations and what students know. This study has provided some insight into how students use their knowledge and the myriad of things that must be attended to when proposing scientific explanations of natural phenomena. However, more work needs to be done in analyzing how students use the knowledge they have when explaining real-world tasks.

APPENDICES

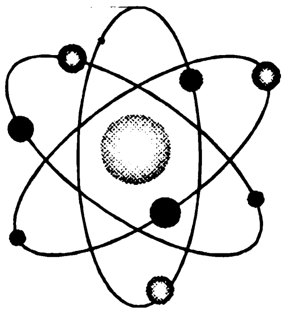
APPENDIX A

PAPER-AND-PENCIL INSTRUMENT

BEHAVIOR OF A GAS

Name (please print)____

Section #:____________________ Instructor____________________



Directions

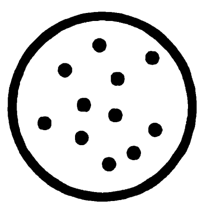
- **THIS IS NOT A TEST AND WILL NOT AFFECT YOUR GRADE IN THIS COURSE.**
- This is a 5-item assessment survey that will measure your understanding of the behavior of a gas.
- We are not interested in whether or not you get the right answer; rather, we are interested in whatever answers you get. Represent your own view or concept - whatever is meaningful and makes sense to you personally.

DIAGNOSTIC SURVEY

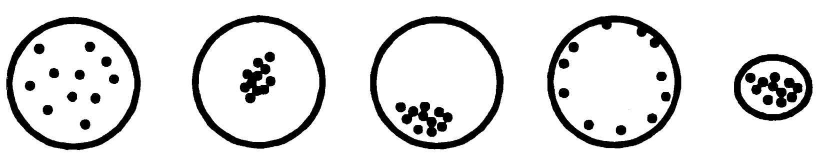
BEHAVIOR OF A GAS

1. A given sample of hydrogen gas has a volume of 1.31 L and exerts a pressure of 4.08 atm at 25 °C. Calculate the pressure of the gas in atm if the temperature is changed to -5 °C.

2. The following diagram represents a cross-sectional area of a rigid, sealed, steel tank filled with hydrogen gas at 20 °C and 3 atm pressure. The dots represent the distribution of all the hydrogen molecules in the tank.



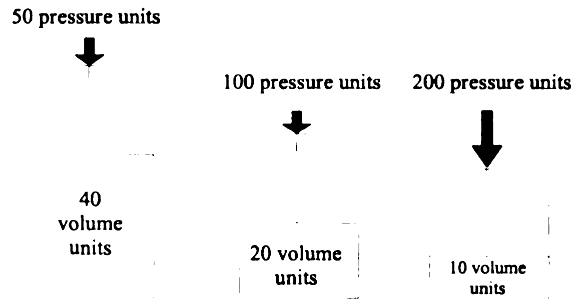
Which of the following diagrams illustrates the distribution of hydrogen molecules in the steel tank if the temperature is lowered to -5 °C? The boiling point of hydrogen is -243 °C.



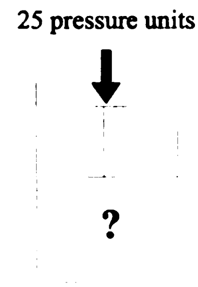
(A) (B) (C) (D) (E)

PLEASE PROCEED TO THE NEXT PAGE.

3. A student does three experiments with a syringe system to see what happens when different pressures are put on the plunger. The student finds the following results.



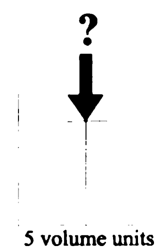
Use these results to help you answer the following questions.



(i) The student exerts 25 pressure units on the plunger as shown.

What would the volume of the enclosed air be?

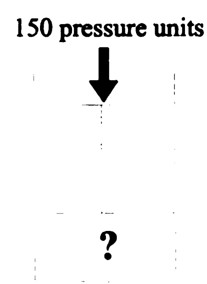
_______ volume units



(ii) The student exerts a pressure on the plunger which forms 5 volume units of enclosed air as shown.

What would the pressure on the plunger be?

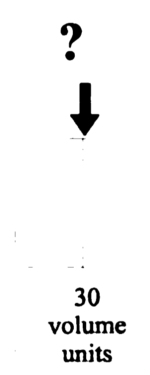
_______ pressure units



(iii) The student exerts 150 pressure units on the plunger as shown.

What would the volume of the enclosed air be?

_______ volume units



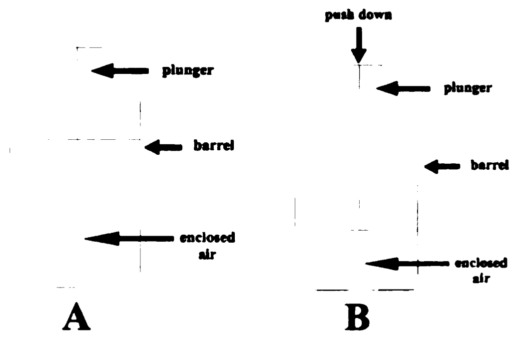
(iv) The student exerts a pressure on the plunger which forms 30 volume units of enclosed air as shown.

What would the pressure on the plunger be?

_______ pressure units

PLEASE PROCEED TO THE NEXT PAGE.

4. The following diagram represents a sealed syringe in two situations, A and B. In situation B, the plunger has been pushed down the barrel of the syringe without any air leaking into or out of the barrel.



For each of the three questions below, check the box beside the *one* answer you think is correct.

(i) What happens to the *volume* of the air?

-the volume of enclosed air in A is *greater* than the volume of enclosed air in B. ( )

-the volume of enclosed air in A is *less* than the volume of enclosed air in B. ( )

-the volume of enclosed air in A is the *same* as the volume of enclosed air in B. ( )

(ii) What happens to the *mass* of the air?

-the mass of enclosed air in A is *greater* than the mass of enclosed air in B. ( )

-the mass of enclosed air in A is *less* than the mass of enclosed air in B. ( )

-the mass of enclosed air in A is the *same* as the mass of enclosed air in B. ( )

(iii) What happens to the *pressure* of the air?

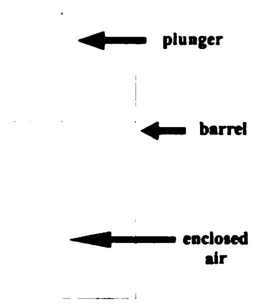
-the pressure of enclosed air in A is *greater* than the pressure of enclosed air in B. ( )

-the pressure of enclosed air in A is *less* than the pressure of enclosed air in B. ( )

-the pressure of enclosed air in A is the *same* as the pressure of enclosed air in B. ( )

PLEASE PROCEED TO THE NEXT PAGE.

5. The following diagram represents a sealed syringe filled with air. In this diagram, the plunger is not moving and no air can get in or out of the barrel. The syringe is operating under conditions of standard atmospheric pressure.



What would be the pressure of the enclosed air in this syringe? Check the box beside the *one* answer you think is correct.

-the pressure of enclosed air in this syringe is *less* than standard atmospheric pressure. ( )

-the pressure of enclosed air in this syringe is *greater* than standard atmospheric pressure. ( )

-the pressure of enclosed air in this syringe is the *same* as standard atmospheric pressure. ( )

-the enclosed air in this syringe exerts *no* pressure ( )

END OF SURVEY. THANKYOU FOR YOUR PARTICIPATION.

APPENDIX B

Course Outline & Learning Objectives

Lecture	Date	Objective	Reading	Homework
1-3	Jan. 11-13	Introduction & Course Overview		
		Unit I		
		Chapter 2 - Matter and Energy		
		I. Physical & Chemical Properties & Changes		
		A. Distinguish between chemical & physical properties	2:13-14	1
		B. Distinguish between chemical & physical changes	2:13-14	3, 5, 7
		II. States of Matter		
		A. Identify and explain the difference between gases, liquids and solids 1. in terms of visible properties 2. in terms of particle movement	2:15	9, 11, 13, 15
		III. Classification of matter		
		A. Distinguish between homogeneous matter and heterogeneous matter	2:17	19, 21
		B. Distinguish between a pure substance and a mixture	2:17	31, 82
			2:18-19	29, 33, 37
		C. Distinguish between elements and compounds		
			2:20	53
		1. Represent elemental symbols	2:20	
		2. Represent chemical formulas of compounds		
		IV. Energy changes	2:22	
		A. Match electrostatic forces of attraction and repulsion with		

		combinations of positive and negative charge B. Distinguish between exothermic and endothermic changes C. Distinguish between potential energy and kinetic energy D. State the meaning of, or draw conclusions based on, the law of conservation of mass E. State the meaning of, or draw conclusions based on, the law of conservation of energy F. Answer questions that require an understanding of two or more of the above objectives	**2**:23-24 **2**:24 **2**:25 **2**:25	65 67 71, 73 75 35, 39, 47, 55, 69, 79
4-7	Jan. 14, 19-21	Chapter 3 - Measurement and Calculations I. Scientific notation (standard exponential notation) A. Write in scientific notation a number given in ordinary decimal form; write in ordinary decimal form a number given in scientific notation B. Add, subtract, multiply, and divide numbers expressed in scientific notation II. Metric system A. Distinguish between mass and weight B. Identify the metric units for	 **3**:36-38 **3**:39-40 **3**:48-49 **3**:49-50	 1,3 5, 7, 9, 11 24 25

		1. length 2. mass 3. volume 4. temperature 5. density		
		C. Given the mass and volume of a sample of a pure substance, calculate its density.	3:68-70	81, 83
		D. State and write with appropriate metric prefixes the relationship between any metric unit and its corresponding megaunit, kilounit, deciunit, centiunit, milliunit, microunit, nanounit, and picounit	3:49-50	27, 29, 33, 37, 39
		III. Dimensional Analysis A. Identify "given" and "wanted" quantities in a problem where quantities are related by a "per" expression. Set up and solve the problem by dimensional analysis	3:41-47	13, 15, 17, 21
		B. Given the density of a substance and either the mass or volume of a sample of the substance, calculate the other	3:68-70	85, 87, 93
		IV. Metric conversions using dimensional analysis A. Given the appropriate metric-English conversion factor(s) and a measurement in one or those systems, express that quantity in corresponding units in the other system	3:63-65	51, 55, 57, 61, 69, 71
		B. Convert a temperature in Celsius, Fahrenheit or Kelvin to the corresponding temperature on the other temperature scale	3:65-68	73, 77

		V. Significant figures A. Determine the number of significant figures in a given value B. Round off given numbers to a specified number of significant figures C. Add or subtract given quantities and express the result in the proper number of significant figures D. Multiply or divide given measurements and express the result in the proper number of significant figures	3:54-57 3:58 3:59-60 3:60-62	41 43 45, 47 49
8-10	Jan 25-27	***Chapter 4 - Introduction to Gases*** I. Kinetic Molecular Theory A. Explain or predict physical phenomena relating to gases in terms of the ideal gas model. II. Gas measurements A. Given a gas pressure in atmospheres, millimeters of mercury, centimeters of mercury, inches of mercury, pascals, kilopascals, or pounds per square inch, express that pressure in each of the other units. III. Standard temperature and pressure (STP)	4:89-90 4:90-93 4:105	1, 2, 3, 7, 9 19, 21
4		IV. Proportionality A. Gay-Lussac's law 1. Given the initial pressure (or temperature) and initial and final	4:96-98	31, 33, 39, 41

		temperature (or pressure) of a fixed quantity of gas at constant volume, calculate the final pressure (or temperature) B. Charles' law 1. Given the initial volume (or temperature) and the initial and final temperatures (or volumes) of a fixed quantity of gas at constant pressure, calculate the final volume (or temperature) C. Boyle's law 1. Given the initial volume (or pressure) and the initial and final pressures (or volumes) of a fixed quantity of gas at constant temperature, calculate the final volume (or pressure) D. Combined gas law 1. For a fixed quantity of a confined gas, given the initial volume, pressure, and temperature and the final values of any two variables, calculate the final value of the third	**4**:98-100 **4**:100-104 **4**:104-105	43, 45, 47 53, 55 63, 65, 67, 71
11	Jan. 28	**EXAMINATION 1** ***(Chapters 2, 3 and 4)***		

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