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thesis entitled

NUMERICAL STUDY OF NATURAL CONVECTION

BETWEEN TWO VERTICAL PLATES WITH ONE OSCILLATING SURFACE TEMPERATURE

presented by

WEI CHA

has been accepted towards fulfillment of the requirements for

MASTER degree in SCIENCE

Major professor

Date JAN. 6, 1989

NUMERICAL STUDY OF NATURAL CONVECTION BETWEEN TWO VERTICAL PARALLEL PLATES WITH ONE OSCILLATING SURFACE TEMPERATURE

by

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A THESIS

Submitted to

Michigan State University

in partial fulfillment of the requirement

for the degree of

MASTER OF SCIENCE

Department of Mechanical Engineering

1989

ABSTRACT

NUMERICAL STUDY OF NATURAL CONVECTION BETWEEN TWO VERTICAL PARALLEL PLATES WITH ONE OSCILLATING SURFACE TEMPERATURE

by

Wei Cha

This study utilizes a finite difference numerical method (employing the program NUDSFAE) to simulate the natural convection heat transfer between two vertical parallel plates, one of which has a time oscillation of its surface temperature. The numerical code deals with real variables rather than stream function and vorticity transformation variables and it solves the coupled equations by introducing a pressure correction scheme. The active surface temperature has a periodic time variation with non-zero average. The dimensionless time average heat transfer rate, Nusselt number, is compared with that of a constant surface temperature case, and this constant temperature is the same as the average of the oscillation. The results of this study show that by oscillating the surface temperature the heat transfer rate can be increased significantly. The study also investigates the effects of oscillation frequency, amplitude and spacing aspect ratio on the enhancement of heat transfer.

The results and the study method can be applied to investigate new electronic cooling techniques with natural convection heat transfer.

ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. J. R. Lloyd, who helped me to initiate the graduate study program, and whose advice and encourages not only helped me through my studies at Michigan State University in the last two and half years but will also benefit me for my whole carer life. I would also like to express my respect and appreciation to Dr. K. T. Yang for his valuable advice. I wish to acknowledge Dr. J. V. Beck and Dr. C. W. Somerton for their constructive comments to my thesis. I feel very graceful to Dr. J. V. Beck, Dr. J. Foss Dr. C. W. Somerton, and Dr. Rosenberg for knowledge I learned from them. I would like to express great appreciations to Dr. St. Clair for his efforts that make my studying here very enjoyable. I am very gratitude to Dr. H. Q. Yang for his advising me on numerical method. I also like to thank National Science Foundation for the financial support at the beginning of the numerical study.

Finally, I wish to thank my husband, W. J. Gesang, for his being supportive and understanding.

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NOMENCLATURE

ASP	space aspect ratio
C_P	specific heat
D	distance between two vertical plates
f	boundary temperature variation frequency
g	gravitational acceleration
Gr	Grashof number $[g\beta H^3(T-T_\infty)/\nu^2]$
h	convective heat transfer cooefficient
H	height of the vertical plate
k	thermal conductivity
N	measurement of heat transfer enhancement $[Nu_o/Nu_c]$
Nu	Nusselt number [hH/k]
O	numericaloverflow
P	pressure field
Pr	Prandtl number $[\nu/\alpha]$
Ra	Rayleigh number $[GrPr=g\beta(T-T_\infty)H^3/\nu\alpha]$
S	numerically stable
t	dimensional time

t_0	media characteristic time $[H^2/\nu]$
T	temperature
u	vertical velocity
U	non-dimensional vertical velocity $[u/u_0]$
u_0	characteristic velocity
v	horizontal velocity
V	non-dimensional horizontal velocity $[v/u_0]$
x	horizontal coordinate
X	non-dimensional horizontal coordinate [x/H]
y	vertical coordinate
Y	non-dimensional vertical coordinate [y/H]

Greek letters

α	thermal diffusivity $[\rho C_p/\kappa]$
β	volume thermal expansion coefficient $[-\rho\rho/\ T]$
ρ	density
τ	non-dimensional time $[t\nu/H^2]$
θ	non-dimensional temperature $[(T-T_\infty)/(T_{ave}-T_\infty)]$

Subscript

ave,s	space average
ave,t	time average
c	constant surface temperature case
o	oscillating surface temperature case
∞	ambient

1. INTRODUCTION

The increasing importance of thermal aspects of electronic equipment design is one of the many motivations for continued interest in natural convection heat transfer studies. It is a well known fact that electronics performance is strongly affected by working temperature. To avoid malfunctions of electronic devices and prevent them from being destroyed (burn out of the element or bad connection of the solder joints due the thermal stress, etc.), the geometry of the cooling passages is a primary consideration in packaging of the electric devices. On the other hand, the tendency of microminiaturization of electronic components and higher performance speed, make the electronic design more temperature dependent. Therefore, electronic cooling problems are drawing more attention. Nakayama [1] addressed the situation of thermal management of electronic equipment based on Hitachi Ltd.'s technology and research on thermal design of their electronic devices. In Figure 1, the thermal design is seen to be of equivalent importance along with geometrical, environmental and economical considerations. Due to the fact that natural convection cooling requires the least additional hardware, and for most cases the surrounding media, such as air, exist already, natural convection is a primary mechanism for electronics cooling.

From a heat transfer point of view, basic natural convection heat transfer phenomena should be studied to enable it to integrate into electronic design. Usually, the features of interest can be classified by their geometry, their thermal properties, and their boundary conditions. More precisely, there are channel natural convection and

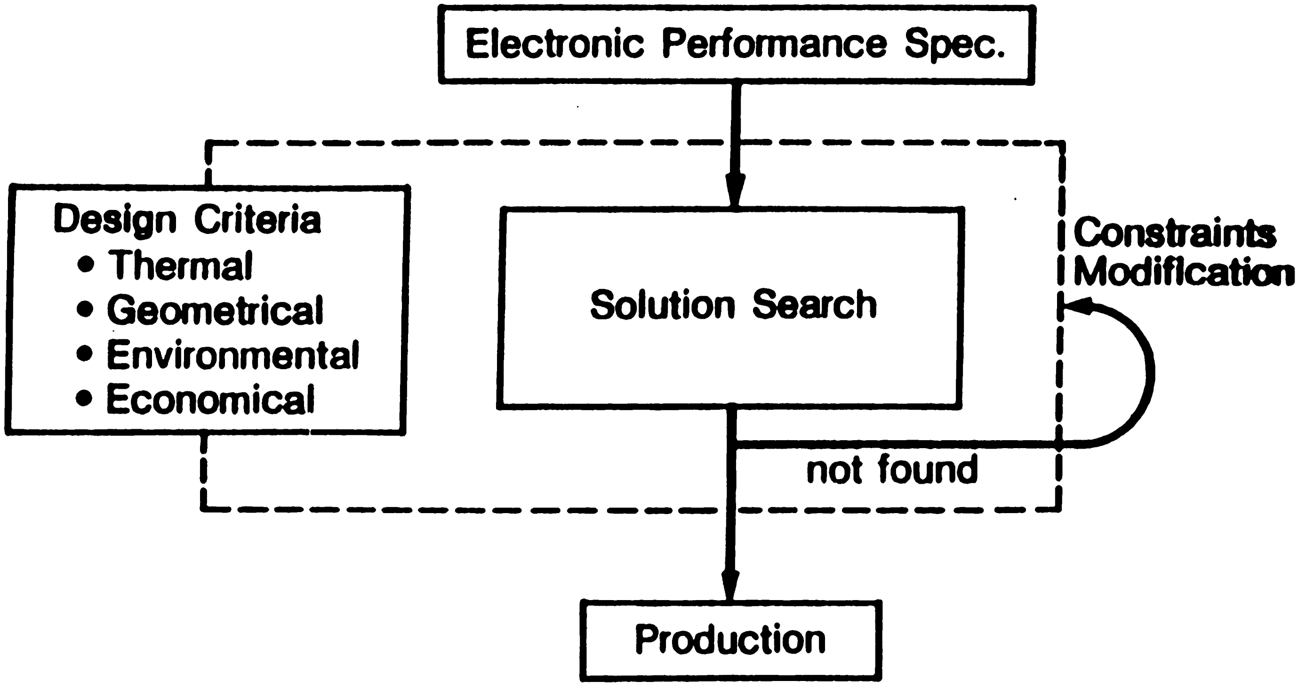


Figure 1 The Importance of Heat Transfer in Electronic Design

single surface natural convection geometries; there are some situations where temperature variations are not too big to make the Bousinesq approximation while there are other cases that require the involvement of variable properties; there are cases where the boundary-temperature is better known and cases where surface heat flux is better known; and there are cases where the boundary conditions will vary with time and space.

In electronic devices, there are usually many individual chips mounted on boards which are placed next to each other. If the boards are close enough, channel type natural convection flows will occur. If the boards are far enough apart from each other, a single surface assumption is reasonable. Even for a single surface, the surface curvature and heating locations can have a significant effect on the total heat transfer.

For a single surface, depending on the surface physical properties and the function it has in the device, more may be known about the surface temperature than the heat flux, or vice versa. For instance, if an electrically heated plate has a low thermal initia, the heat flux may be a better known function of time than the temperature, but if the thermal conductivity is very high, the surface temperature may be known as a function of time. Example of the cases where surface temperature or heat flux varies with time are found when the electric circuit switch for a device is turned on or off. The thermal boundary conditions will have an exponential-like response to that step change in current. More kinds of time variations can be from a periodically varying power supply.

Boundary conditions can have a spacial variation. When individual chips are separated sufficiently, the surface will have spots of which temperatures and heat fluxes are higher or lower than the rest part of the surface.

The current study develops a numerical finite difference program to simulate natural convection driven by electronic element, of which the surface temperatures vary with time. The physical model of the problem studied is sketched in figure 2. The physical model includes two vertical plates, with height H. They are placed parallel to each other, separated by a distance D. Each of the two surface temperatures can have its own function of time. The medium is air. An experiment was done preliminarily to develop a general feeling for temperature and velocity fields caused by a hot plate, especially with the time varying boundary conditions. Then a finite difference numerical method was utilized to reveal the detailed quantitative descriptions of the temperature and velocity fields with one surface temperature varying with time periodically. By doing this, the natural convection heat transfer and corresponding fluid flow driven by an oscillating surface temperature is simulated numerically.

The success in simulating the natural convection driven by the time varying wall temperatures, provides the way to expose the magnitude of heat transfer enhancement achieved by oscillating the surface temperature and to develop possible approaches to thermal control in electronic thermal design.

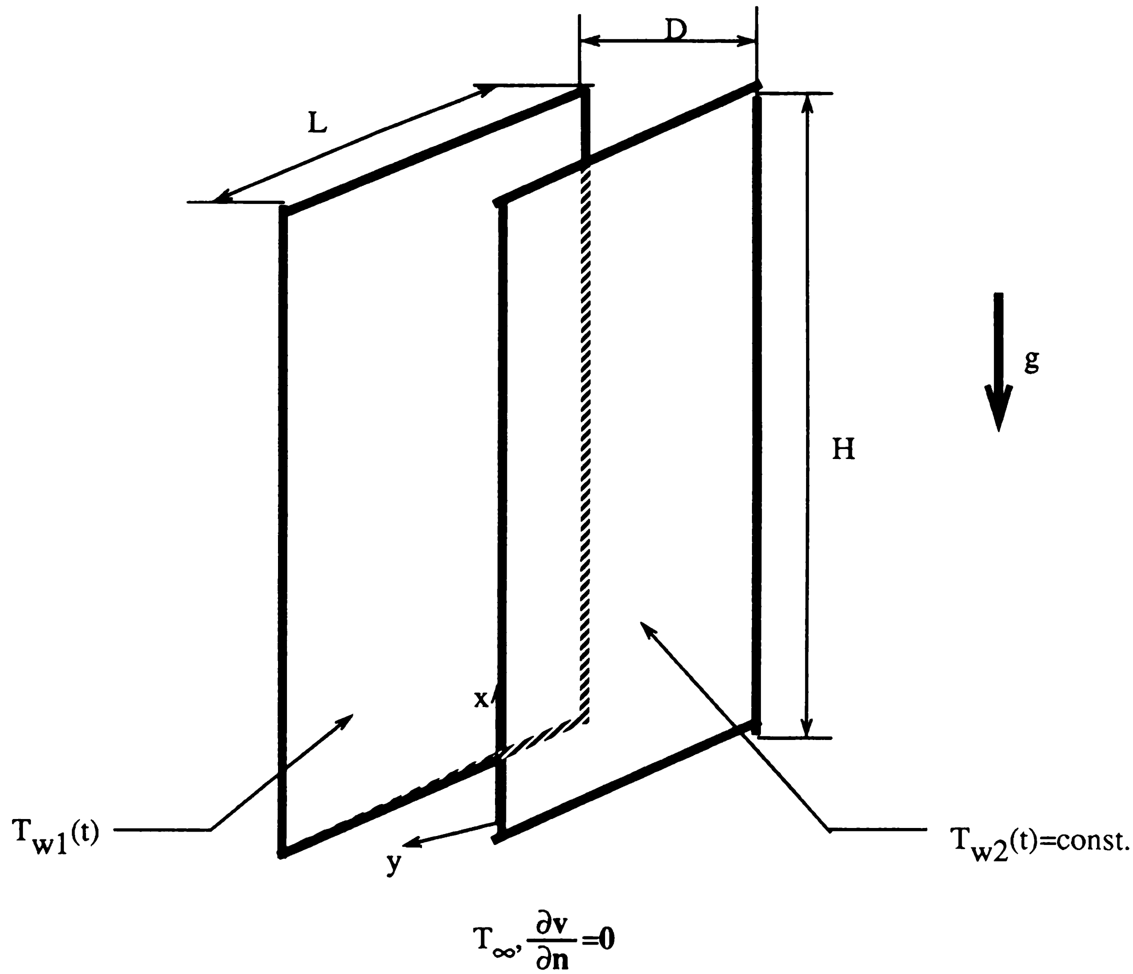


Figure 2 Problem description

2. LITERATURE REVIEW

The dealing with cooling problems for electronic devices can be dated back to 60 years ago. Bergles (1986) [2] gave a historical overview of electronics thermal control. In his paper, he pointed out that the electronic cooling problem, starting from 1942 when the vacuum tube cooling was concerned to the 80's when the microelectronic revolution is advancing rapidly, has been becoming more and more desirable. To meet the desire, a lot of research was conducted and important papers were published.

As described by Steinberg (1980) in his book, [3], in an electronic device, there are all kinds of electronic elements, and heat transfer could be modeled in different ways. For example, there are chip, package, printed wiring boards (PWB) and system levels. Different geometries, different power levels and different working environments make the electronic cooling related natural convection very comprehensive.

Elenbaas (1942) [4] proposed a simple model of heat transfer and made some measurements of natural convective flow between two isothermal vertical plates. His experiments were conducted for a wide range of Rayleigh number, $0.2<Ra<10^5$, and a small gap width. Sparrow (1983) [5] conducted an experimental study on the enhancement of heat transfer due to pressure drop in electronic device, and provided a flow visualization method. Bar-Cohen and Rohsenow (1984) [6] developed an integral formulation for fully developed laminar flow for symmetric and asymmetric thermal situations. Park and Bergles (1987) [7] conducted some experiments simulating the natural convection behavio of the microelectronic chips. As a result of their study, the dependence of the heat transfer performance on heater size was gained. Wang and Bau (1988) [8] analytically studied the low Rayleigh number thermal convection in a Newtonian fluid confined between two horizontal, circular cylinders. Their inter-

est was in the thermal effect in solar collector receivers, compressed gas insulating high-voltage electric transmission cables, and so on. An experimental study by Torikoshi, Kawazoe and Kurihara (1988) [9] focused on the heat transfer characteristics of arrays of block type elements representing electronic components and the patterns of air flow adjacent to the upper surface of the blocks developed along one wall of a plat rectangular duck. Their study indicates that for the fully populated arrays of blocks of uniform height, the per-block Nusselt number decreases monotonically with increasing stream-wise distance. Arco, Bontoux, Sani, Hardin, Extermet, and Chikhaoui (1988) [10] used the finite difference technique to simulate three-dimensional buoyancy driven flows in vertical cylinders. They pointed out that for some particular values of the aspect ratio the problem can admit different spacial flow configuration. They also studied the effect of the oscillatory movement in the boundary wall on the natural convection flow pattern. But the details of the heat transfer were not intensively studied. Ramanathan and Kumar (1988) [11] conducted a numerical study on natural convection flows between two vertical parallel plates within a large enclosure. Their interest was in the parametric study of variation of Prandtl numbers and channel aspect ratios. Bergman and Petri (1988) [12] obtained the numerical predictions that describe the potential advantage of using xenon-helium mixtures in the natural convection cooling of discrete heated elements in an enclosure. This was oriented by the electronic cooling problem in enclosure. Their paper indicates that with the natural convection the xenon-helium, the electronics operating temperature can be decreased.

As mentioned before, natural convection caused by a vertical hot (or cold) plate, has been studied quite extensively. People worked on correlation between the Nusselt number and Rayleigh number for a long time and have developed valuable methods to analyze the problems. One of the well developed methods is the similarity solution. Sparrow, Quack and Boerner [13], Sparrow and Gregg [14], Yu [15] and Kao [16] gave the results of their investigations and showed the application of the similarity solution series. Another method is experimental approach. Due to the advantages like less interruption and fast time response, the optical techniques have been employed quite a lot in natural convection studies. Wirte and Stutzman (1982) [17], Hamady (1987) [18], and O'Meara, and Poulikakos (1987) [19] presented different experimental methods dealing with natural convection problems. As computers increase their calculation capability, numerical methods are used more and more in solving natural convection problems. Since 1972, Lloyd and Yang [20], Doria [21], Lloyd, Yang and Liu [22], Yang [23], and Yang and Yang [24] developed and improved a finite difference code called UNDSAFE. UNDSAFE uses a pressure correction scheme which was initially introduced by Patankar and Spalding (1972) [25]. Yang and Lloyd's numerical code can take care of transient natural convection in enclosures, with or without heat sources, with or without radiation. It can even include the effect of rotation of the enclosure. The significance of this numerical treatment is that it uses real variables other than some transformation variables as quite often used in numerical convective heat transfer studies. This numerical approach provides the major tool to investigate the time-varying thermal boundary condition driven natural convection problem.

The most up to date papers on natural convection not only use and improve the conventional theories and methods, but also introduce in a lot practical natural convection problems. Bhavnani and Bergles (1988) [26] conducted an experimental study of laminar natural convection heat transfer from wavy surfaces. One of their results is that the heat transfer from a wavy surface, compared to a plane of equal projected area, increases with increasing amplitude-to-wavelength ratio. Lee and Yovanovich (1988) [27] present their boundary layer type approximation method to investigate a two-dimensional natural convection heat transfer from a vertical plate with a family of non-similar surface heat flux variations problem. Their method is proved that it can have results agreeing with the numerical results very well. Hwalek and Iyengar (1988) [28] focused on natural convection mass transfer from a vertical plate at high mass transfer rates. Their work was done experimentally.

When the transient thermal boundary value problem is of interest, two papers should be paid special attention. One was written by Kelleher and Yang (1968) [29]. In their paper, a linearization theory was utilized to study the heat transfer response of a laminar free-convection boundary layer along a vertical hot plate with surface-temperature oscillation. The oscillation had an average surface temperature variation which was a power function of the distance from the leading edge. However, high oscillation frequency or large amplitude may not be tolerated by the laminar boundary constraint. Another paper was written by Shaw, Chen and Cleaver (1988) [30]. In that paper, the effects of thermal sources on natural convection in an enclosure were studied numerically. The vorticity and stream function were used to eliminate the coupling between the momentum equation and the energy equation. This falls into the transformation variable method category.

Based on the review above, it is clear that the natural convection associated with a time-variation boundary temperature problem, which is likely to be encountered in electronic cooling problems, has not been solved so far. The time-variation boundary temperature driven natural convection is much more complicated than that with a stationary boundary thermal condition, and it is also different from that with a physically moving boundary. The time-variation will definitely change both the heat transfer and flow patterns. The significance in the time varying boundary temperature is that the heat transfer and flow patterns can be manipulated by the variation in boundary temperature without a physical movement in the boundary, and by doing this the total heat transfer may be increased dramatically.

3. MATHEMATICAL FORMULATION

As shown in Figure 2, the configuration considered here is a pair of vertical parallel plates, each of which has its own time-dependent surface temperature, ($T_{w1}(t)$ and $T_{w2}(t)$). The two plates are dimensioned by height H and are separated from each other by a variable gap distance D. The two plates are assumed to be sufficiently deep normal to the plane of Figure 2, so that the two-dimensional-flow assumption is applicable. It is also assumed that the range of temperature variation is moderate so that the Bousinesq assumption is valid. The governing equations include the conservation laws cf mass, momentums (both in x and y directions) and energy, and the phase equation.

3.1 Governing equations

Based on the assumptions mentioned above, the governing equations are as the following:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}-\rho g+v(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=v(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}) \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=k(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}) \tag{4}$$

$$p=\rho RT \tag{5}$$

They can be expressed in non-dimensional form by using the following non-dimensional groups:

$$Gr=\frac{g\beta(T_w-T_\infty)H^3}{v^2} \tag{6}$$

$$Ra=GrPr \tag{7}$$

$$X=\frac{x}{H} \tag{8}$$

$$Y=\frac{y}{H} \tag{9}$$

$$\tau=\frac{t\nu}{H^2} \tag{10}$$

$$U=\frac{uH}{\nu} \tag{11}$$

$$V=\frac{vH}{\nu} \tag{12}$$

$$\theta=\frac{T-T_\infty}{T_{ave}-T_\infty} \tag{13}$$

where

$$\beta=\frac{-1}{\rho}\,\frac{\partial\rho}{\partial T} \tag{14}$$

The resulting equations are:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{15}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=RaPr\theta+Pr(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}) \tag{16}$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=Pr(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}) \tag{17}$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2} \tag{18}$$

The boundary conditions are:

$$X=0,\ 0<Y<ASP:\ \theta=0\ ,\frac{\partial U}{\partial X}=0,\ \frac{\partial V}{\partial X}=0 \tag{19}$$

$$X=1,\ 0<Y<ASP:\ \frac{\partial \theta}{\partial X}=0,\ \frac{\partial U}{\partial X}=0,\ \frac{\partial V}{\partial X}=0 \tag{20}$$

$$Y=0,\ 0<X<1:\ \theta=\theta_{w1}(X,\tau),\ U=0,\ V=0 \tag{21}$$

$$Y=ASP,\ 0<X<1:\ \theta=\theta_{w2}(X,\tau),\ U=0,\ V=0 \tag{22}$$

where the aspect ratio, ASP, is defined as the following:

$$ASP=D/H \tag{23}$$

3.2 Numerical solution

Many of the existing numerical methods for solving the equations in the natural convection problems, use the stream function and vorticity variables instead of the physical variables of velocities and pressure. While the elimination of the pressure coupling between the momentum and energy balance equations is accomplished by using the stream-function-vorticity approach, the time varying boundary conditions are hard to treat. The unsteady term in the mass balance equation may be significant, and therefore the introduction of the vorticity transport equations makes the approach even more complicated than that using physical variables.

The UNDSAFE numerical code using physical variables was developed about 1974 [16]. This code treats the full elliptical equations and handles a wide variety of boundary conditions. It has been used successfully to solve quite a few buoyancy-driven convection problems such as room fire [17], hot-walled enclosure with or without radiation [18] and [17].

3.2.1 Cell structure

In order to use the UNDSAFE finite difference method, the governing equations have to be written into integral form involving a fixed control volume V with surface S.

$$\int_S \rho u_j n_j dS=0 \tag{24}$$

$$\left(\int_V \rho u_i dV\right)_{,t}=-\int_S u_i u_j n_j dS-\int_S (p-p_e) n_i dS-\int_V (\rho-\rho_e) g_i dV+\int_S \sigma_{ij} n_j dS \tag{25}$$

$$\left(\int_V \rho c_p T dV\right)_{,t}=-\int_S \rho c_p T u_j n_j dS-\int_S k T_{,j} n_j dS+\int_V (\mu Phi-p u_{j,j}) dV \tag{26}$$

To solve the governing equations numerically, discretization of those equations is necessary. The concerned domain is covered by a grid system of rectangular cells. The grid system is arranged so that the cell boundaries coincide with physical bound-

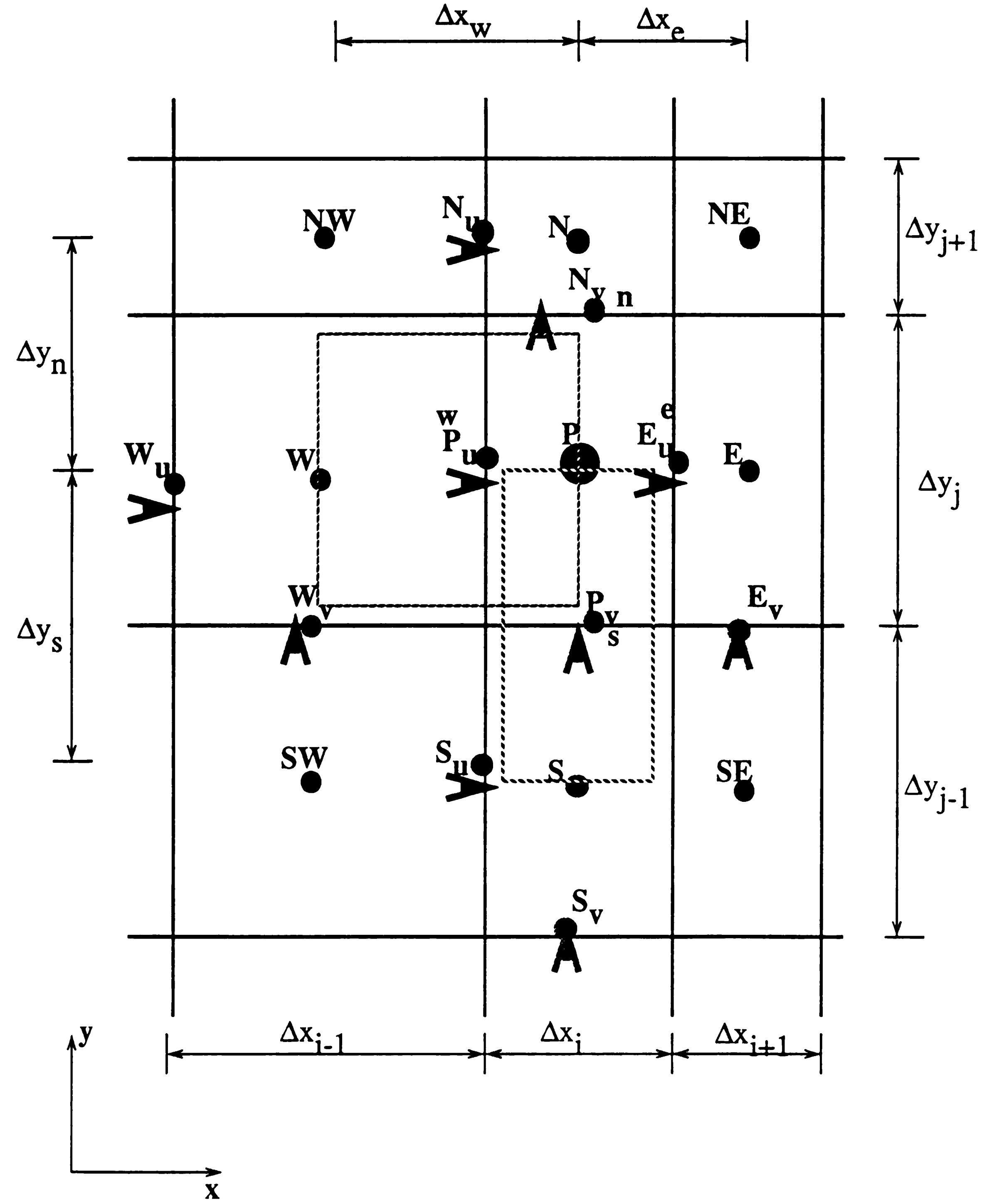


Figure 3 Basic and staggered grids

ary cells. This grid system is different from the "standard" grid system in the way of discretizing the original differential equations. The "standard" grid system uses Taylor series expansion to approximate the differential equations by finite difference equations, and all properties are based on the grid points, with step change between the points. The control volume approach uses the integral form shown in equations (24, 25, and 26), and all properties are at the center of the cell and represent the overall average value. This guarantees that conservation is always satisfied over any group of cells and in the whole calculation domain. Since the mass flux terms are evaluated on the boundaries of each basic cell, the velocity x and y components are needed on the same boundaries. This requires staggered grids. When the x momentum is of concern, the cell centered at the west face of the basic cell is used. When the y momentum is of concern, the cell centered at the north face is used. The staggered grids for both u and v are shown in Figure 3.

3.2.2 QUICK scheme

When a set of finite difference equations which are developed using the control volume approach are treated, the main concern is to estimate accurate values of the dependent variables at the surfaces of the control volume with stable properties. QUICK (Quadratic Upstream Interpolation Convective Kinematics), first developed by Leonard (1979), combines the relatively high accuracy of central difference scheme with the stability of the upstream scheme. This combination is achieved by using a parabolic polynomial interpolation to fit the control volume surface value at consecutive nodal positions, two nodes located on either side of the surface and the third one on the next node in the upstream direction. In two dimensional problem, the general quadratic function of T(X,Y) can be finally expressed as:

$$T(X,Y)=C_1+C_2X+C_3X^2+C_4Y+C_5Y^2+C_6XY \quad (27)$$

As can be seen in equation (27), a six-node grid is needed. Once the six-node grid information is plugged into the equation, the coefficients C_i (i=1,2,3,4,5, and 6) can be solved for that specific surface.

The whole domain can be treated in turn.

3.2.3 Pressure correction scheme

One of the characters of the natural convection problem is that the momentum equation is coupled with energy equation by the pressure. In other words, among the five variables, u, v, ρ, P and T, P depends on T through the state equation. According to the analysis of [22], the dependence is very weak. If the state equation is used to solve for the pressure and mass conservation equation for density, the pressure correction procedure will fail. To avoid this problem, Doria (1974) [20] presented another procedure, which was originated by Patankar and Spalding (1972) [20]. A brief description of the idea is as following. At each time level a pressure field p* is guessed and the velocity components U* and V* are calculated based on the guessed pressure field. The mass conservation usually cannot be satisfied because of the incorrect guessed pressure field. A correction of the guessed pressure by means of

$$P'=P-P^* \quad (28)$$

is therefore needed to correct the U* and V* fields as given in the following:

$$U'=U-U^* \quad (29)$$

$$V'=V-V^* \quad (30)$$

where the superscript prime is for the correction while the quantities without the superscript represent the corrected values who satisfy the conservation of mass. The corrections U′ and V′ are obtained from the residual mass. An assumption that the

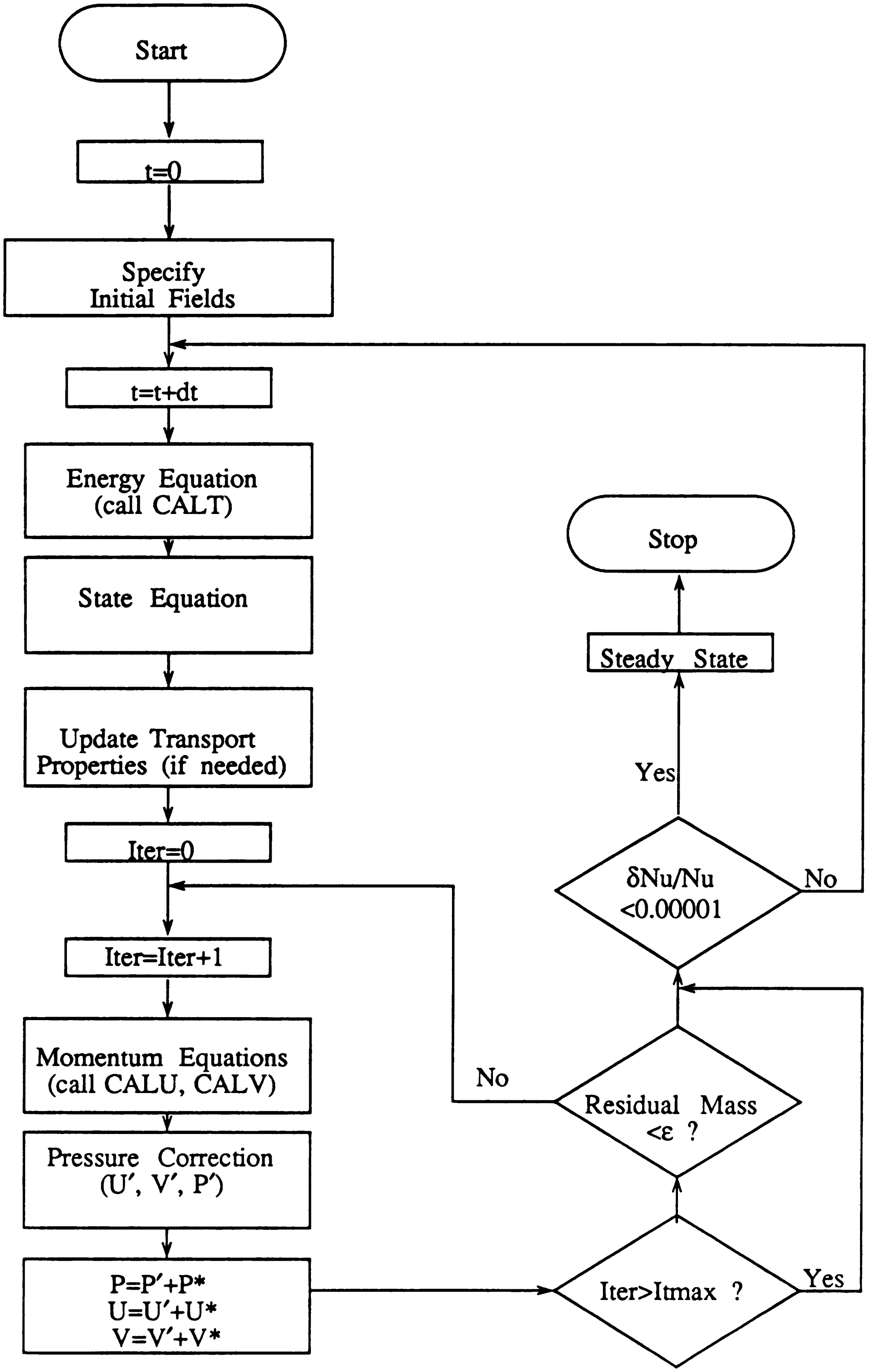


Figure 4 Pressure correction flowchart

corrected mass flux through a surface of the basic cell is proportional to the gradient of the pressure correction across the surface is employed. An iteration loop is designed so that at the end of the iteration, the mass is conserved in each cell. The next time level may start from the just corrected pressure field to calculate the velocity and temperature fields. Figure 4 illustrates the iteration idea.

3.2.4 Boundary condition

The numerical representation of the boundary conditions described by equations (19, 20, 21, and 22) affects the total compatibility between the finite difference equations and the differential equations. The two vertical walls have prescribed temperatures and non-slip velocities. These are a lot easier to treat than the free boundaries. The top opening of the geometry has zero derivatives of u, v, T and P. The bottom opening has the leading edge effect. If the entrance boundary is not well treated, the effect of the false boundary conditions may be propagated into the downstream calculation. In this study, the real entrance boundary conditions, both velocity and temperature, are simulated by introducing a set of imaginary walls as sketched in Figure 5. As can be seen, the surrounding medium will flow into the calculation domain more naturally. In other words, the entrance inaccuracy of the finite difference technique is softened by the added reservoir and the flow and temperature at the real entrance can be more realistically determined. The boundary conditions at the two vertical walls will remain the same. The boundary conditions at the imaginary walls will have zero-gradient characters. The extra width and height are determined by the criteria given by [19], i.e., $D'=4/3D$, $H'=2/3H$.

To simulate the natural convection heat transfer with an oscillatory boundary temperature, the two vertical surfaces have the following prescribed temperatures. One of the surfaces has a uniform temperature which is the same as the environment.

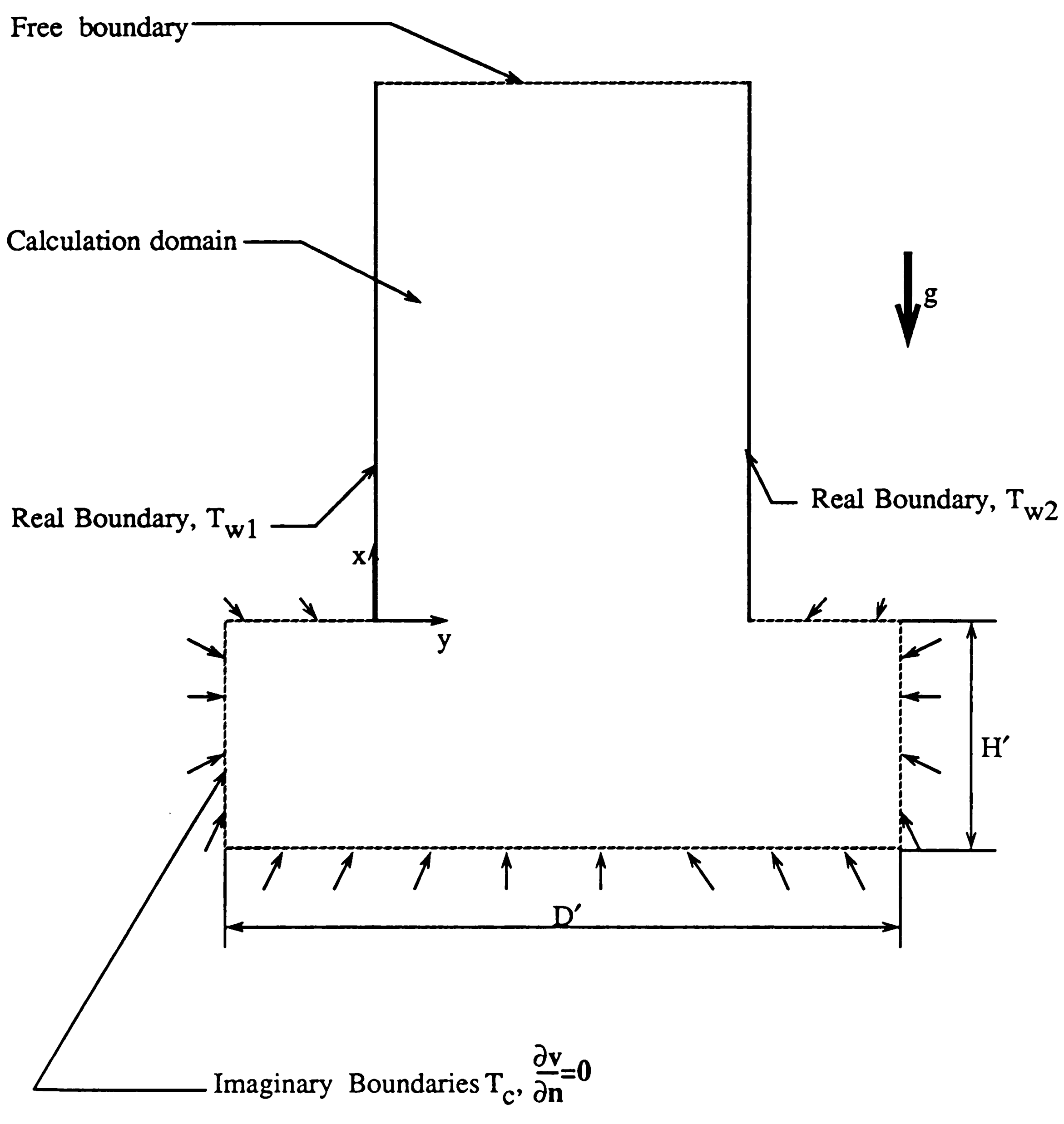


Figure 5 Calculation domain

The other surface has an oscillatory temperature as described by equations (28) and (29):

$$\theta_w=\theta_{ave}+\Delta\theta \mathrm{Sin}(\tau) \tag{28}$$

where

$$\theta_w= \frac{T_w - T_\infty}{T_{ave}- T_\infty} \tag{29}$$

The average of the oscillation, θ_{ave}, is above the environment temperature. The magnitude of the variation of the surface temperature, $\Delta\theta$, is less than the average value. This will ensure that the surface temperature is always higher than the environment, which is normally true in the reality. The oscillation frequency is lumped into τ by non-dimensionization addressed in section 4.1.

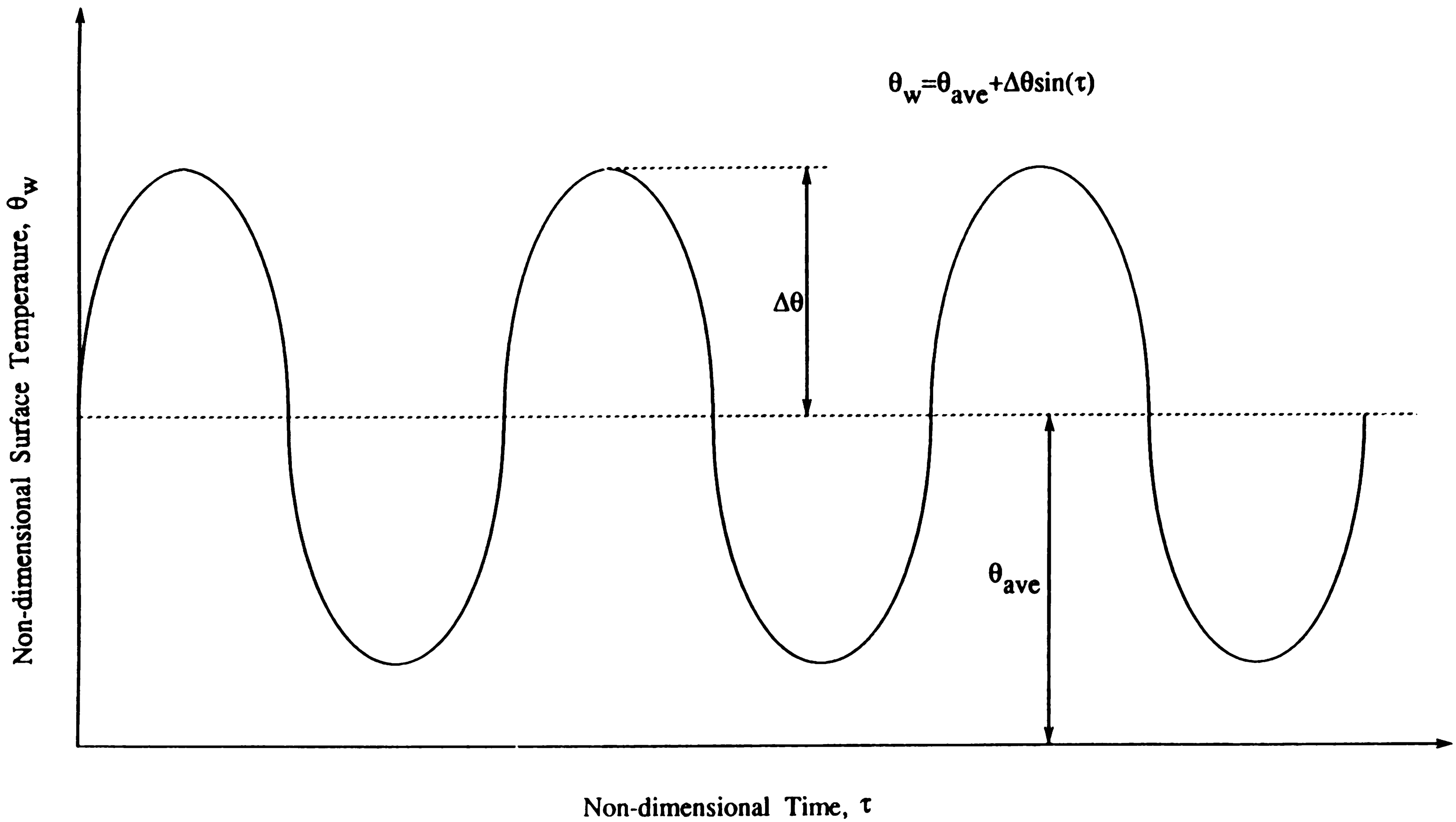


Figure 6 Surface Temperpture Time Variation

4. RESULTS AND DISCUSSIONS

4.1 Validation test

The case of natural convection from an isothermal vertical plate in an infinite medium has been studied very thoroughly. In the current study, this case was simulated by the numerical program and compared against the book values. By doing this, we can test the validity of the grid system, the pressure correction scheme, the boundary model and the finite difference code.

Usually, when calculations are carried out, accuracy and efficiency are compromised. So before a series of calculations were carried out, the necessary grid density to ensure the accuracy must be found out. This was as a preliminary work to prevent misleading by the lack of accuracy when comparisons were made. According to [16], uniformly spaced 20×20 cell grid is too crude, 80×80 cell grid slows down the calculation speed and the 40×40 cell grid gives the accurate results not significantly different from those of 80×80 cell grid. So all the calculations were carried out with 40×40 cell grid.

The test case is a simplified problem of the real model shown in Figure 2. The semi-infinite single plate is numerically realized by placing the two plates far away from each other. One plate has a fixed surface temperature higher than the surrounding temperature, and the other has a temperature the same as that of the surrounding. Rayleigh number was varied in the range from 1,000 to 747,400. This range of Rayleigh number started from 1,000 is because cases with lower Rayleigh numbers will become conduction. And the Rayleigh numbers are no higher than 1000,000 so that the calculations are done for laminar flows (the number 747,400 resulted form the characteristic length, it is close to 1000,000). The heat transfer coefficient, Nusselt number, results from the calculations is compared with the corresponding theoretical values for laminar flow in Table 1. The Nusselt numbers of the calculations are within 4% difference from the theoretical values, except case 04, of which Rayleigh number is 747,400. This may be due to the transition from laminar to turbulent flow. These errors were sufficiently small that the numerical code including the uniform grid was thought to be suitable for treating the posted problem in section 3.

TABLE 1 Validation test

Test No.	Ra	$Nu_{theoretical}$	$Nu_{pressent}$	Error (%)
01(20×20)	7474	5.3	5.92	11.6%
02(20×20)	74740	7.94	7.10	10.5%
03(40×40)	1000	3.9	3.98	2.1%
04(40×40)	7474	5.3	5.51	3.9%
05(40×40)	74740	7.94	8.15	2.6%
06(40×40)	747400	15.49	16.8	8.4%
07(80×80)	7474	5.3	5.35	0.94%

Figure 7 plots the error vursus non-dimensional grid size.

4.2 Discretization of time step

The oscillation of the temperature of the boundary surface is of primary significance in the current study. For transient problems, the accuracy and stability are all very sensitive to the size of the time step. How to choose the calculation marching time step which takes care of accuracy and stability is another preliminary problem which needed to be solved.

One non-dimensional parameter describing the stability is mesh Reynolds number, Re, as defined in the following:

$$Re=[(\Delta X)^2+(\Delta Y)^2]/\Delta\tau \qquad (30)$$

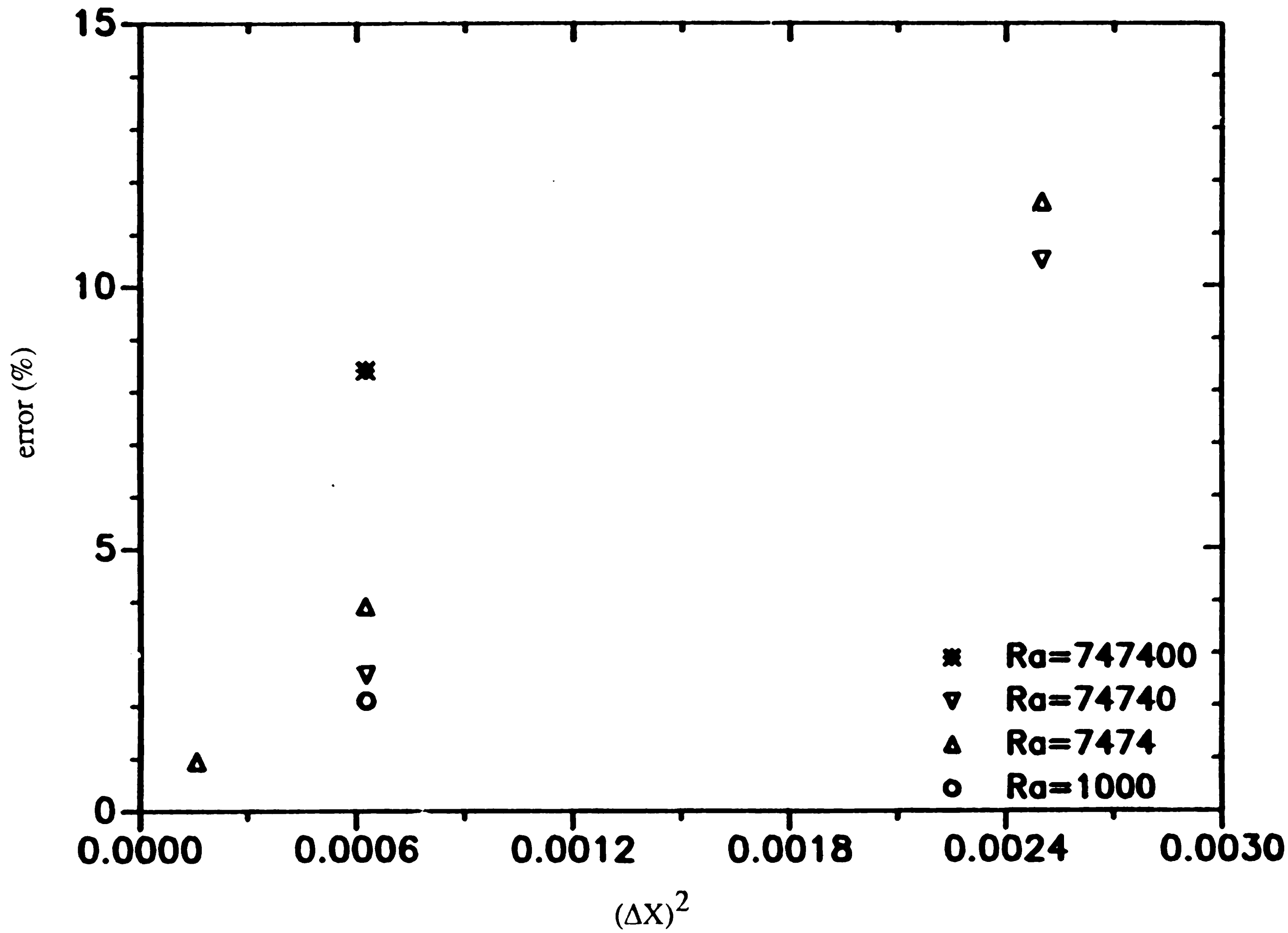


Figure 7 Grid size effect on accuracy
(ASP=1, f=100Hz, $\Delta\theta$=5)

The stability criteria is that the quantity, Re, keeps in a range when ΔX, ΔY or $\Delta\tau$ changes. As the grid gets denser and denser, the time step should correspondingly decrease. Table 2 gives the results of a series test calculations. In these test cases, the grid sizes, ΔX and ΔY, are first fixed while the time step size, $\Delta\tau$, varies within a range. Then the $\Delta\tau$ is fixed while ΔX and ΔY vary a little bit. The quantity $[(\Delta X)^2+(\Delta Y)^2]/\Delta\tau$ was calculated for every test case. As can be seen in Table 2, the numerical calculation is stable only when the $[(\Delta X)^2+(\Delta Y)^2]/\Delta\tau$ is within a certain range. This verifies the stability condition mentioned above.

TABLE 2 Stability test

Test No.	ΔX	ΔY	$\Delta\tau$	$\frac{(\Delta X^2 + \Delta Y^2)}{\Delta\tau}$	Stable or overflow
01*	0.025	0.025	0.006975	0.179	S
02	0.025	0.025	0.06975	0.0179	O
03	0.025	0.025	0.3488	0.358	S
04	0.025	0.025	0.03488	0.0089	S
05	0.025	0.025	0.0006975	1.79	S
06	0.025	0.025	0.0003488	3.58	O
07	0.025	0.0167	0.006975	0.129	S
08	0.0167	0.0167	0.06975	0.0796	S
09	0.025	0.0125	0.006975	0.112	S
10	0.0125	0.0125	0.006975	0.0895	S

* CASE 01 is taken as a base case and every other case has some factors different from those of the base.

Since the grid is 40×40 uniformly spaced, the ΔX and ΔY are fixed for the specific problem. So the range of $\Delta\tau$ is limited too. On the other hand, the goal of the current study is to calculate the oscillating surface temperature-driven temperature and velocity fields. This implies that the time step should also be small enough, so that the oscillation character is accurately described. This can be better illustrated by Figure 8. In Figure 8, there are three curves. Each of them represents the overall aver-

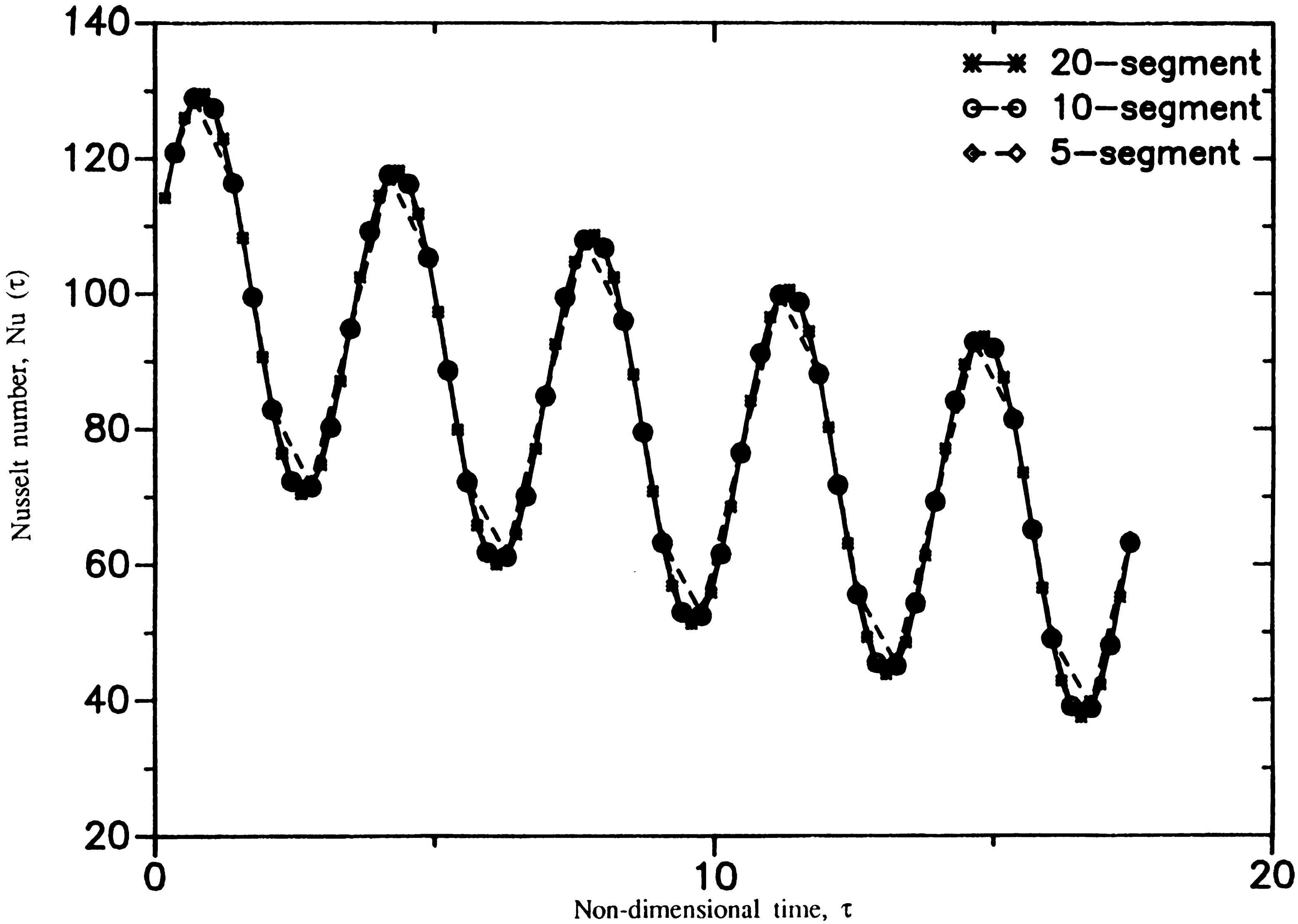


Figure 8 Time step size effect on accuracy
(ASP=1, f=100Hz, Δθ=5)

age heat transfer history of the same driving force, but each has a different time step size. The smoothest curve resulted from the calculation with the time step 1/20 of the oscillation characteristic time, the period. The curve that lost its smoothness at the peeks and the valleys resulted from the calculation with time step 1/5 of the characteristic time. The calculation with time step equal to 1/10 of the characteristic time has improved significantly in terms of the carrying the complete information of the driving force. With this time step the amount of calculation is reduced by half, but the information lost is very little compared with the result of calculation with 1/20 time step size. For the current study, each oscillation period is divided into ten step to carry out time marching procedure.

4.3 Mechanism of natural convection heat transfer with an oscillating boundary temperature

The character of natural convection is that the temperature difference initiates a fluid flow and the flow can have all different patterns as a response to different thermal boundary conditions. In return, the flow affects temperature field distribution The thermal boundary condition not only arouses the fluid flow, but also can manipulate the fluid flow patterns. As a very expressive example, the natural convection driven by a time varying boundary temperature shows how the thermal boundary condition variation leads the fluid flow movement

The numerical simulation was carried out with the boundary conditions described by equations (28) and (29), and Figure 6 in section 4.2.4. It can be imagined that as the boundary temperature oscillates the temperature field adjacent to the hot surface will responde to it in a periodic manner. But what is of the interest here is that the fluid flow pattern changes periodically too.

The group of pictures in Figure 9 show the temperature field and velocity field time history during time of two surface temperature oscillation periods. This group of pictures are resulted from calculations for Ra=7,474, ASP=1, τ=0.001, ΔX=1/40, ΔY=1/40 and $\Delta\tau$=1/10. The number on each figure means the time step in sequence, and every ten steps make an oscillation period. It is found out that for the specific case, starting from a quiet initial condition, it takes about 500 steps to reach a fully developed time varying response. For the purpose of relating the temperature and velocity fields to the corresponding surface temperature, a detailed set of figures in the time range 560~600 is attached to each individual instant field pictures (temperature and velocity) in Figure 9.

First of all, it is observed that the isotherms are not always open curves starting from the bottom entrance and ending at the top opening, as is seen for the constant temperature vertical plate. Instead, some loops and some heavily bent isotherms occur. The thermal boundary layer analysis is no longer suitable for this type of natural convection. It is also noted that the existence and locations of the new pattern of isotherms vary in time. Actually, the isotherm pattern variation has a periodic behavior.

Secondly, the velocity field is seen to be more exited than that of a single plate with a constant surface temperature. The fluid can flow upwards along the hot surface or form a circulation, depending on the corresponding surface temperature. Again, like the temperature field, the flow field changes its pattern (either parallel upwards flow or the circulation flow) periodically, too. The transition from parallel up flow to circulation actually represents a different mode during the circulation core developing history. As can be seen from pictures in Figure 9, the circulation is always there. As time passes step by step, the core will move the bottom to the top, then out of the inner

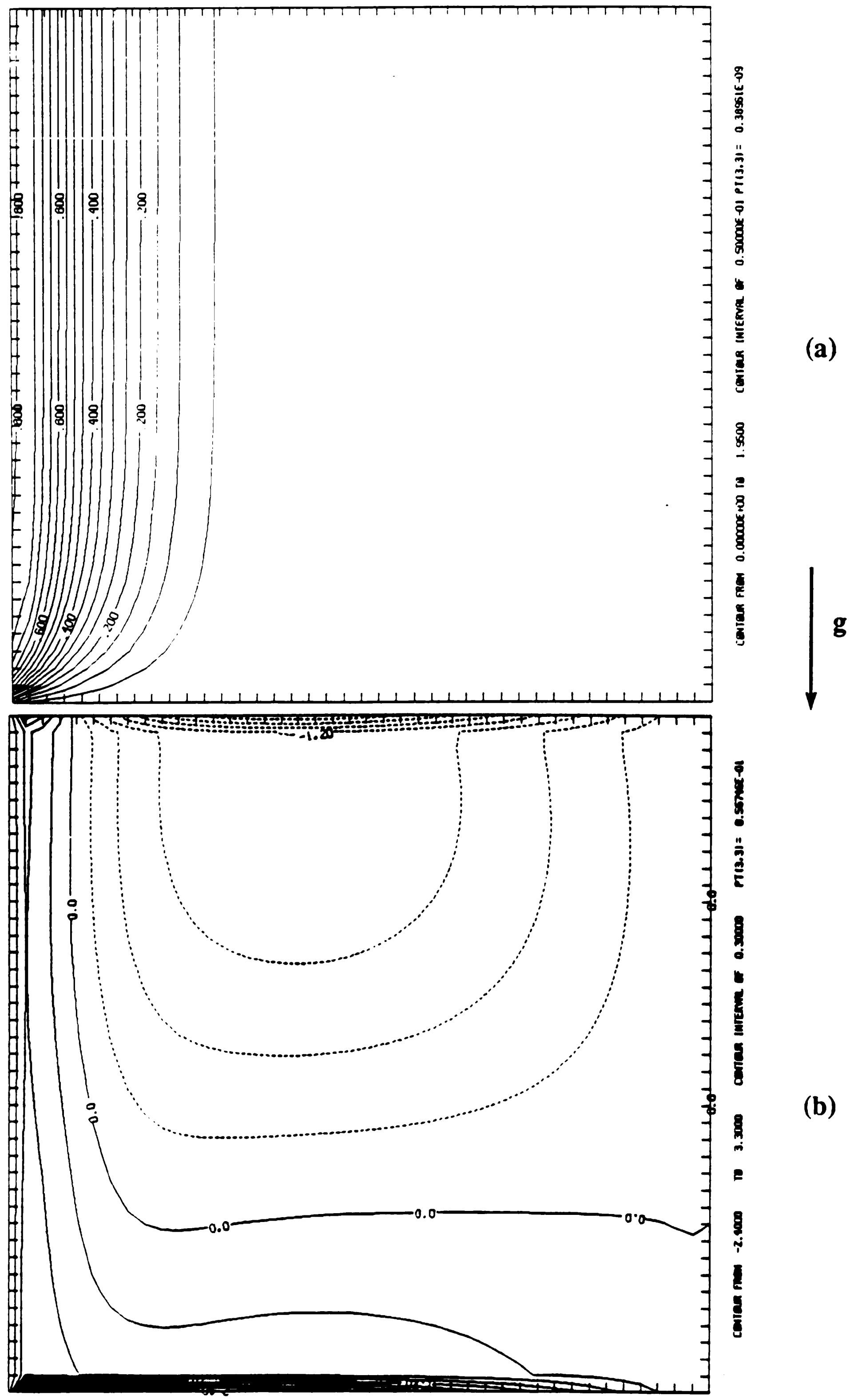


Figure 9 (1) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=560

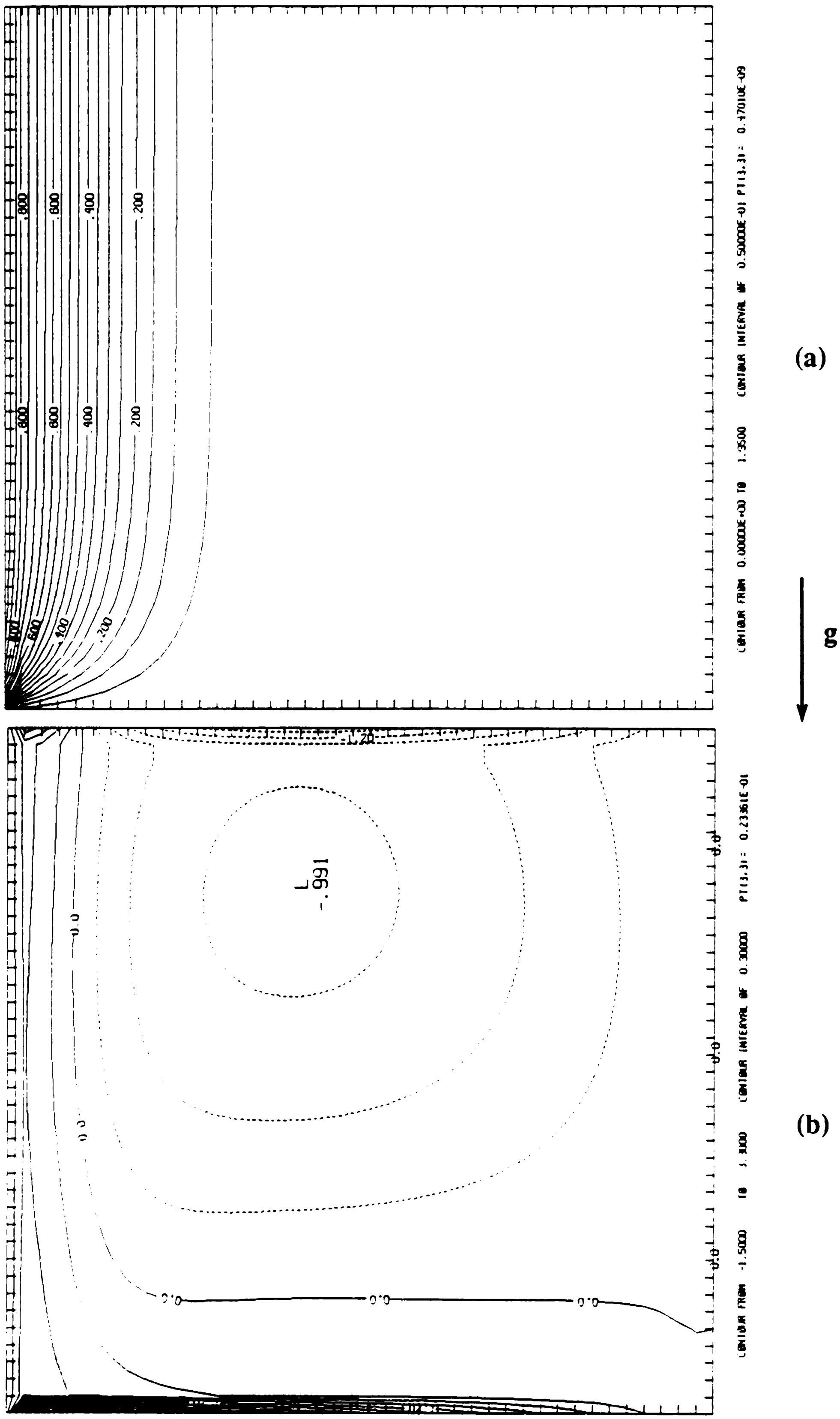
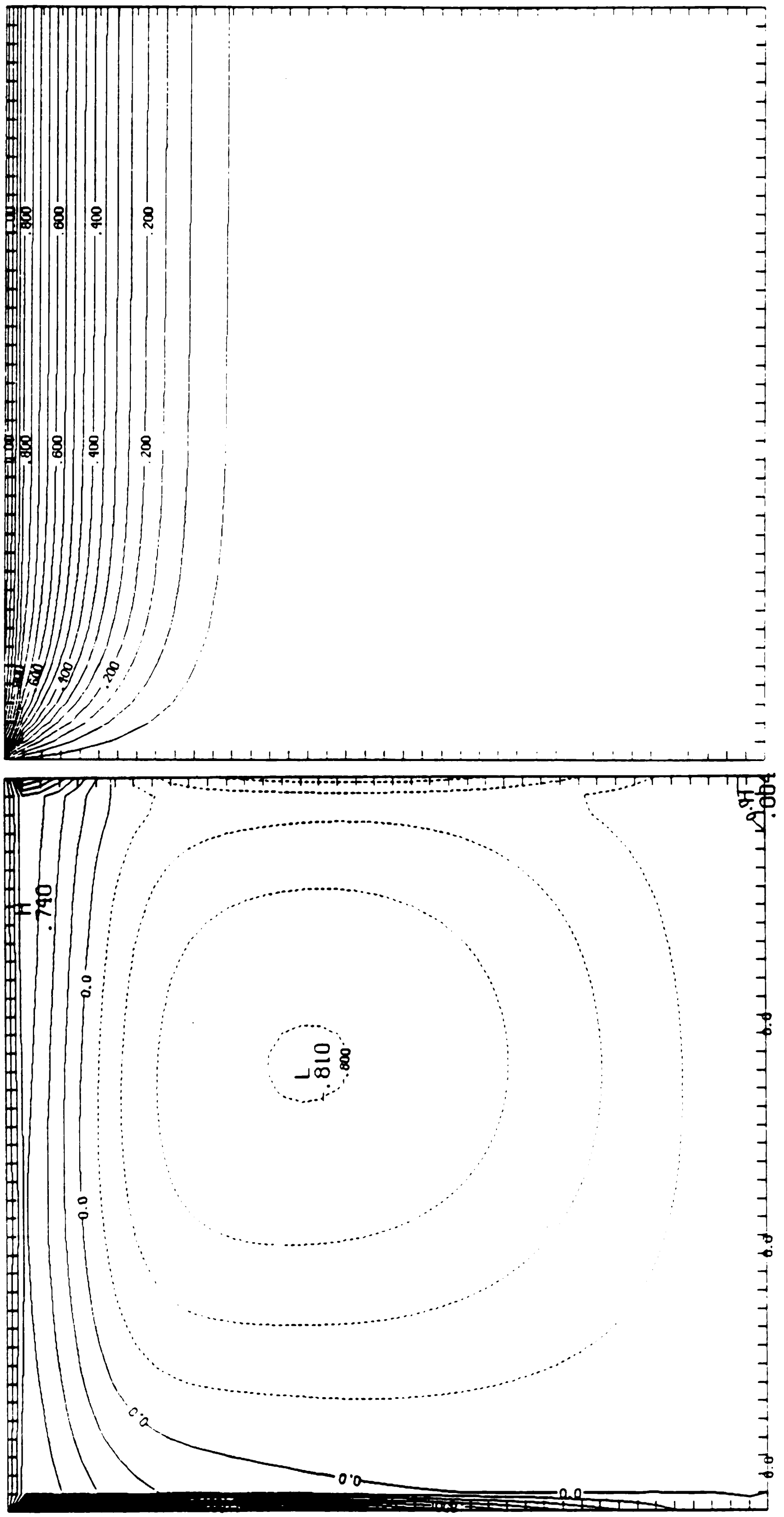


Figure 9 (2) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=563



(a)

g

(b)

Figure 9 (3) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=565

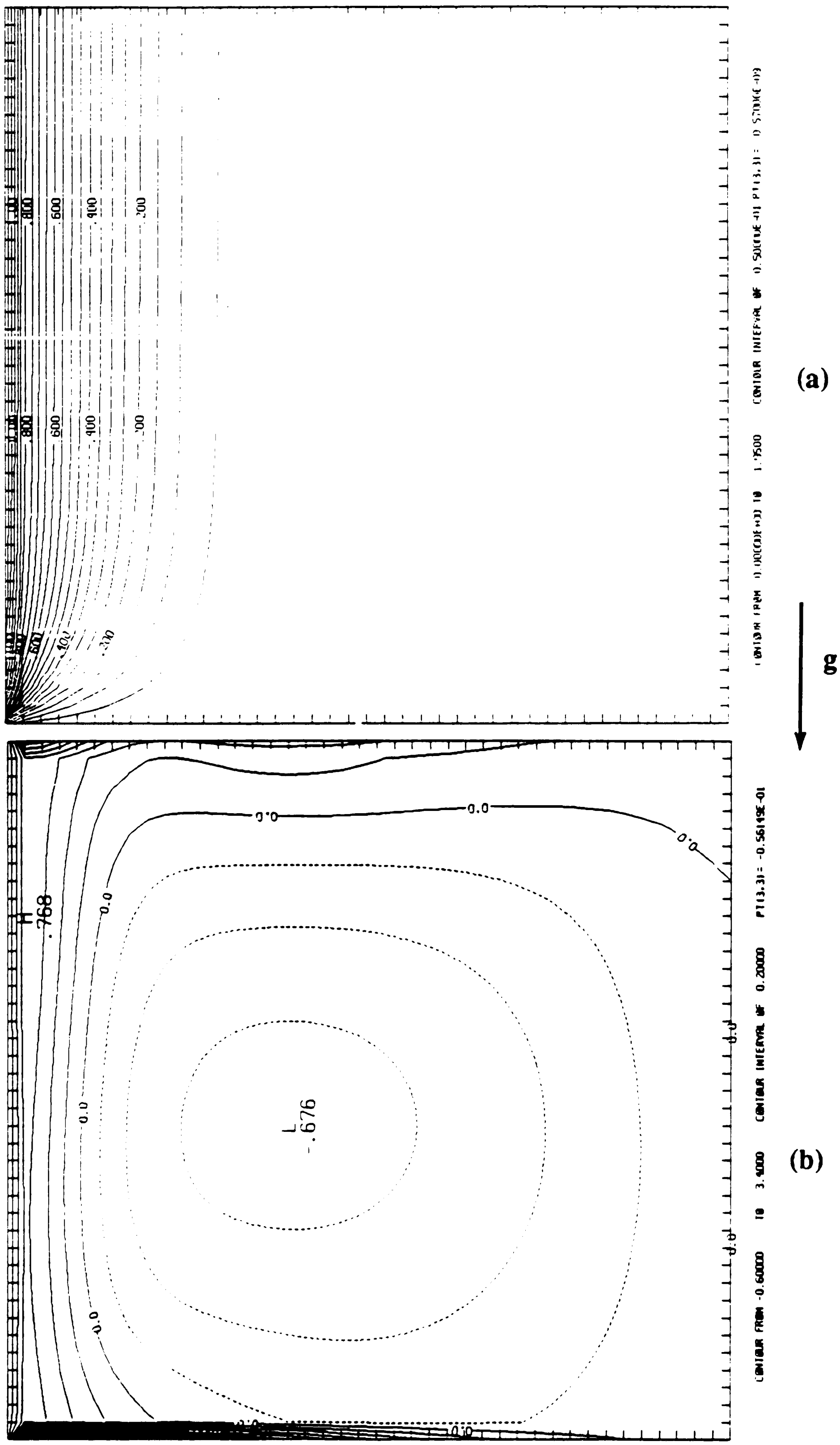


Figure 9 (4) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=567

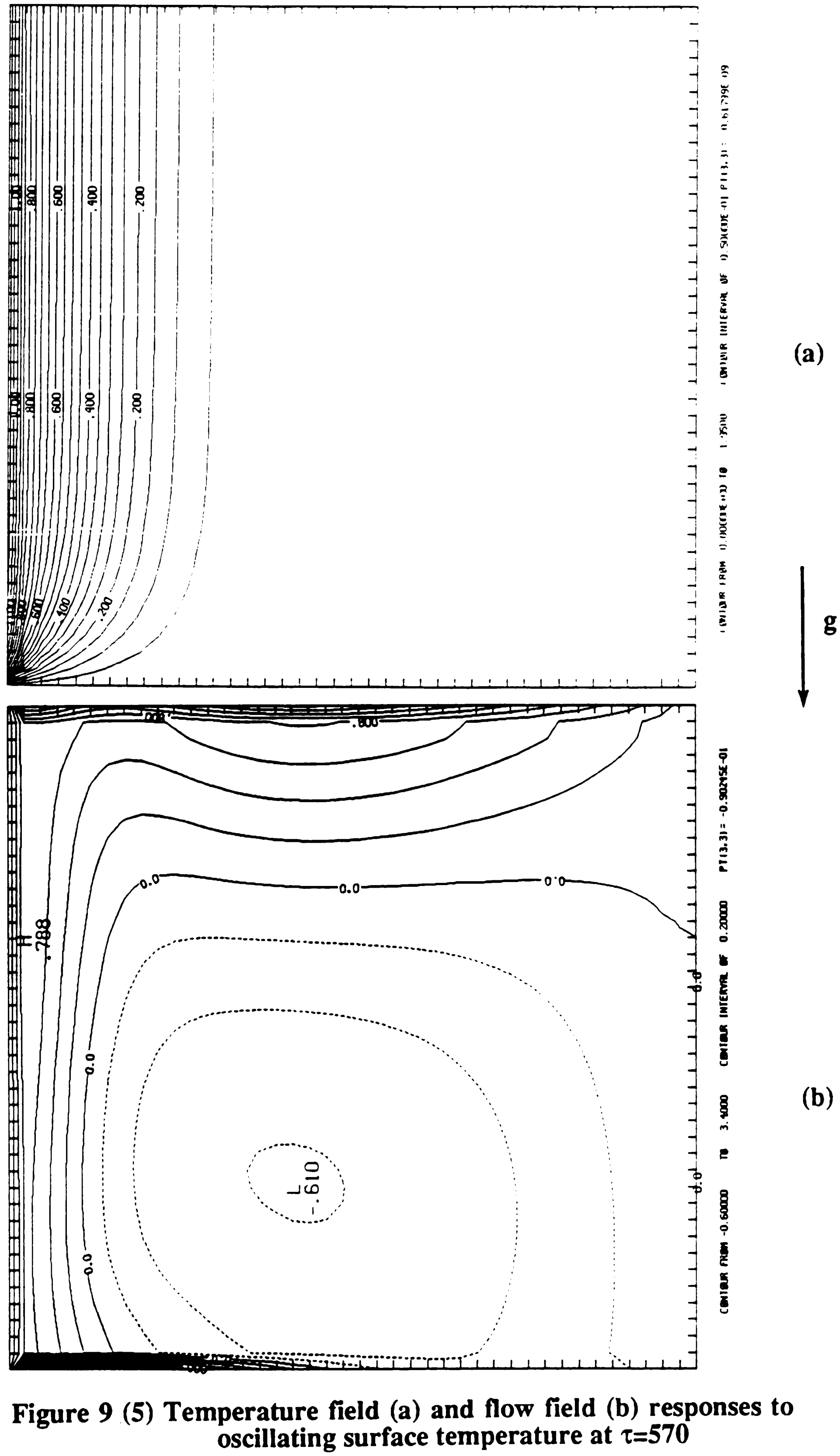


Figure 9 (5) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=570

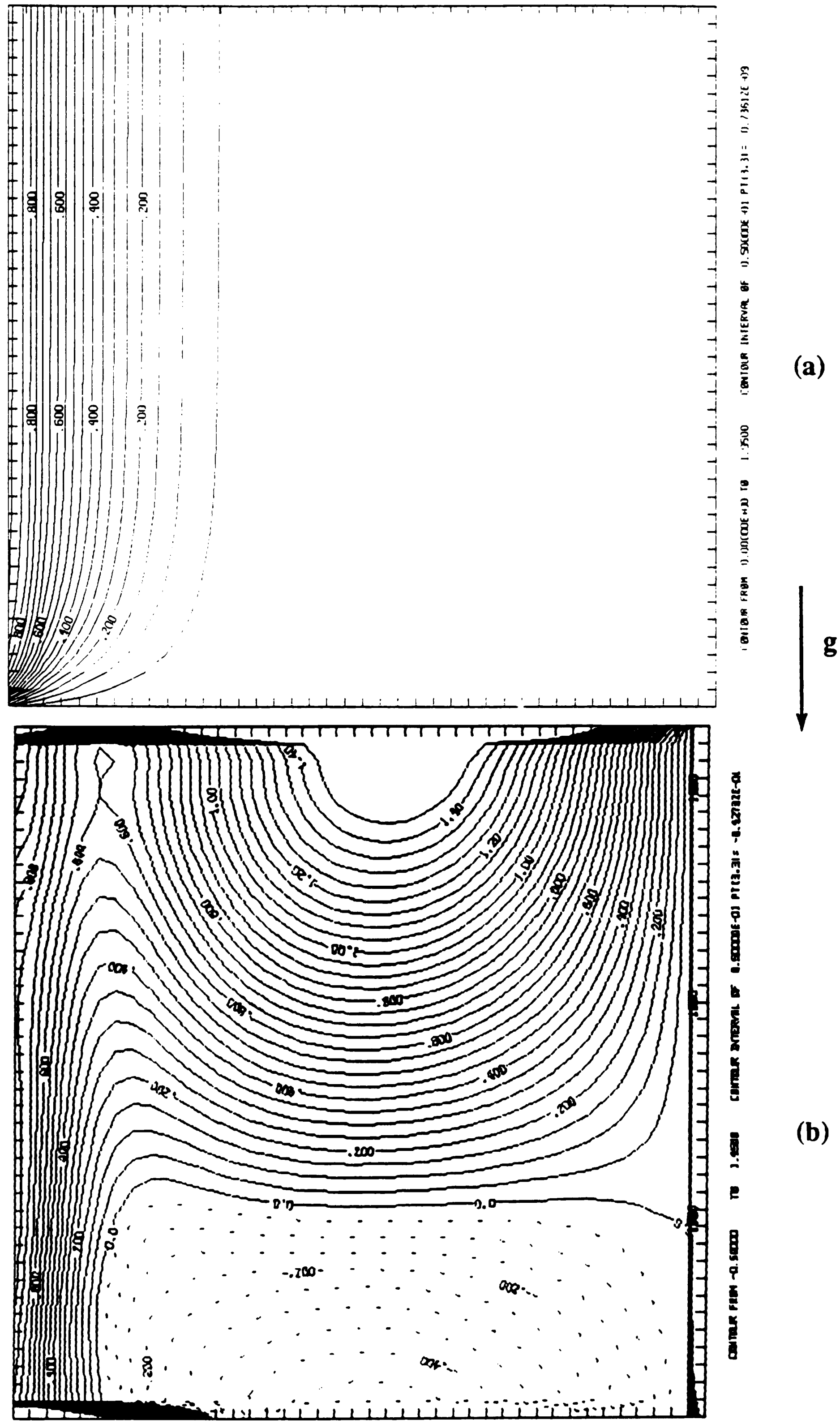


Figure 9 (6) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=573

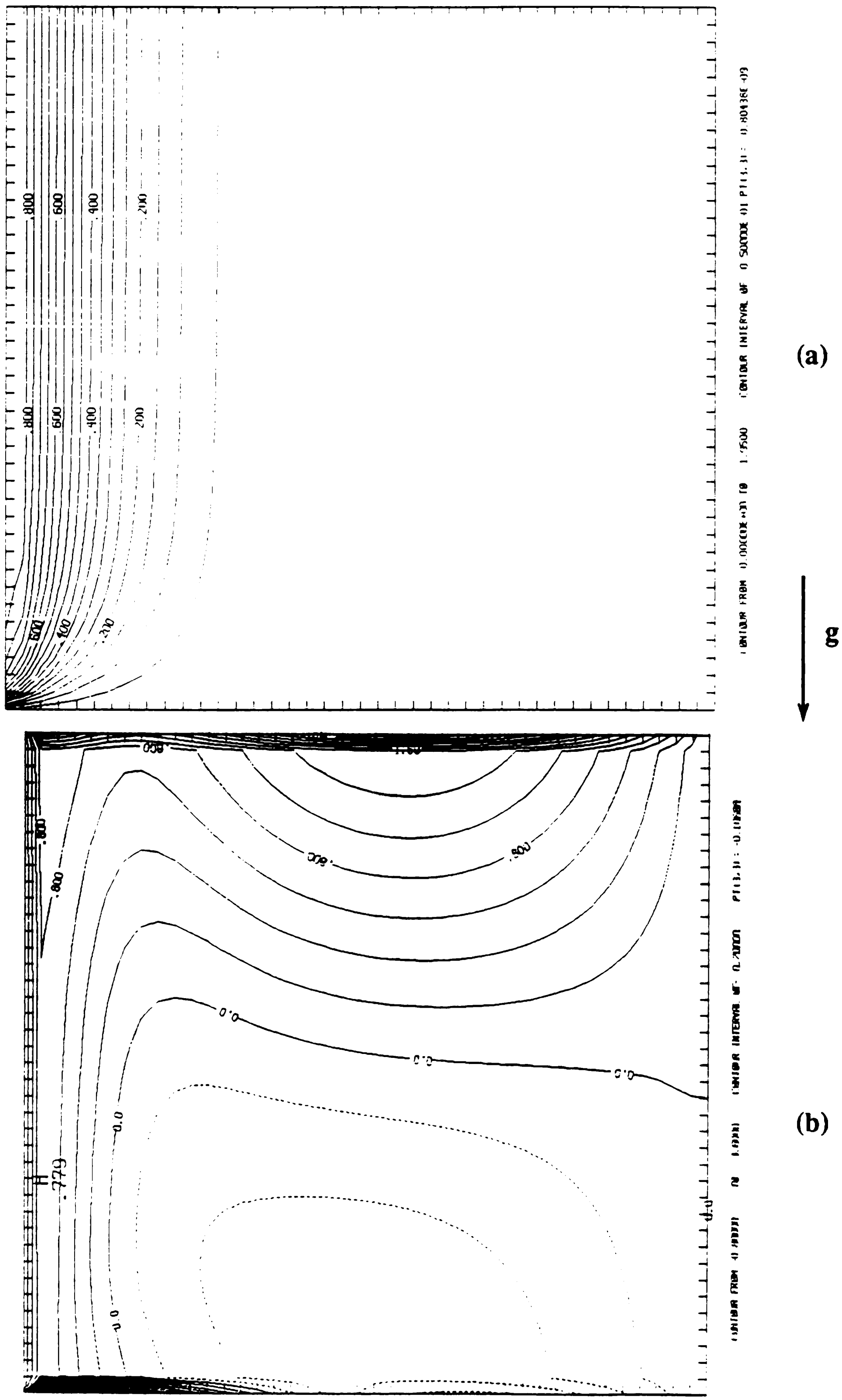


Figure 9 (7) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=575

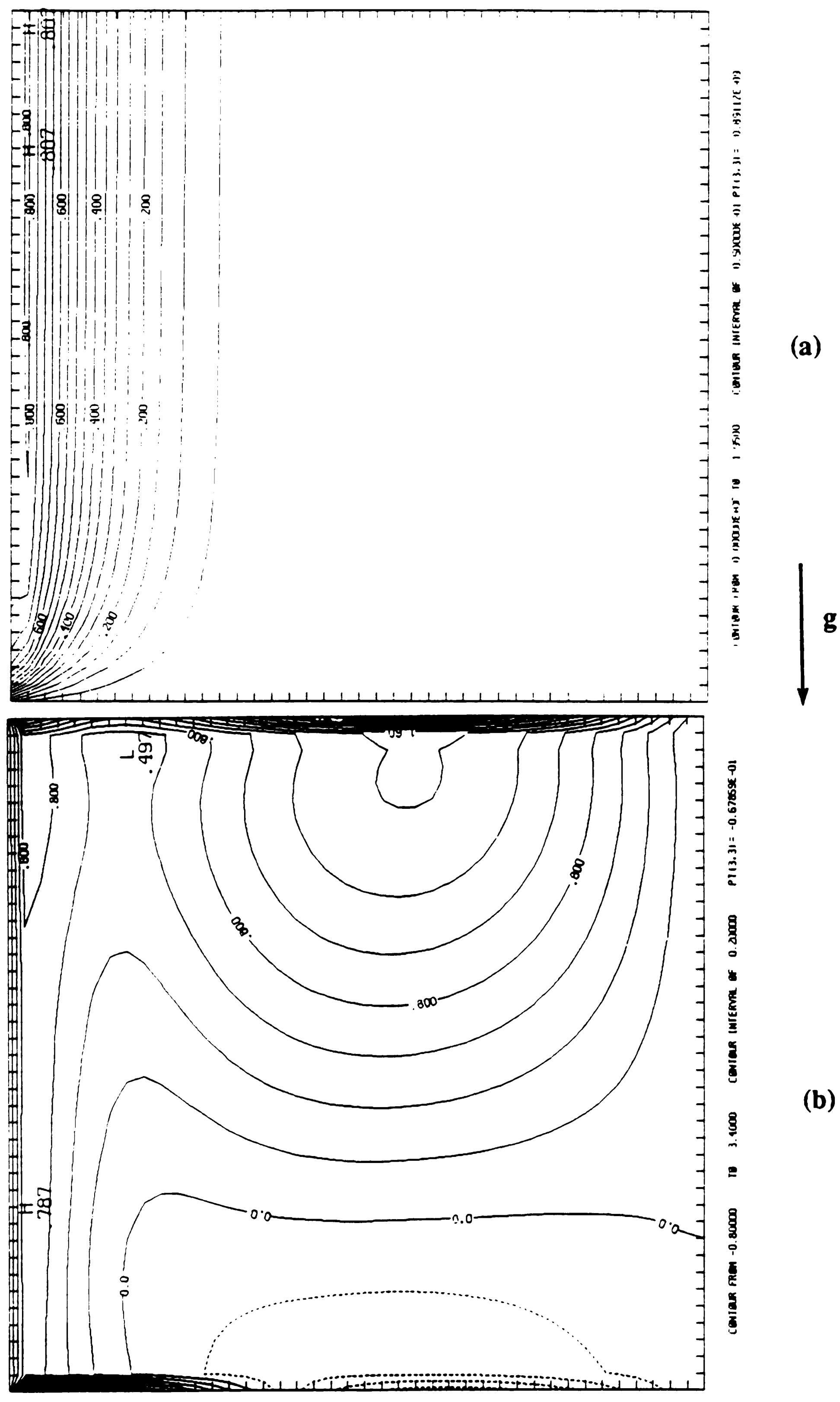


Figure 9 (8) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=577

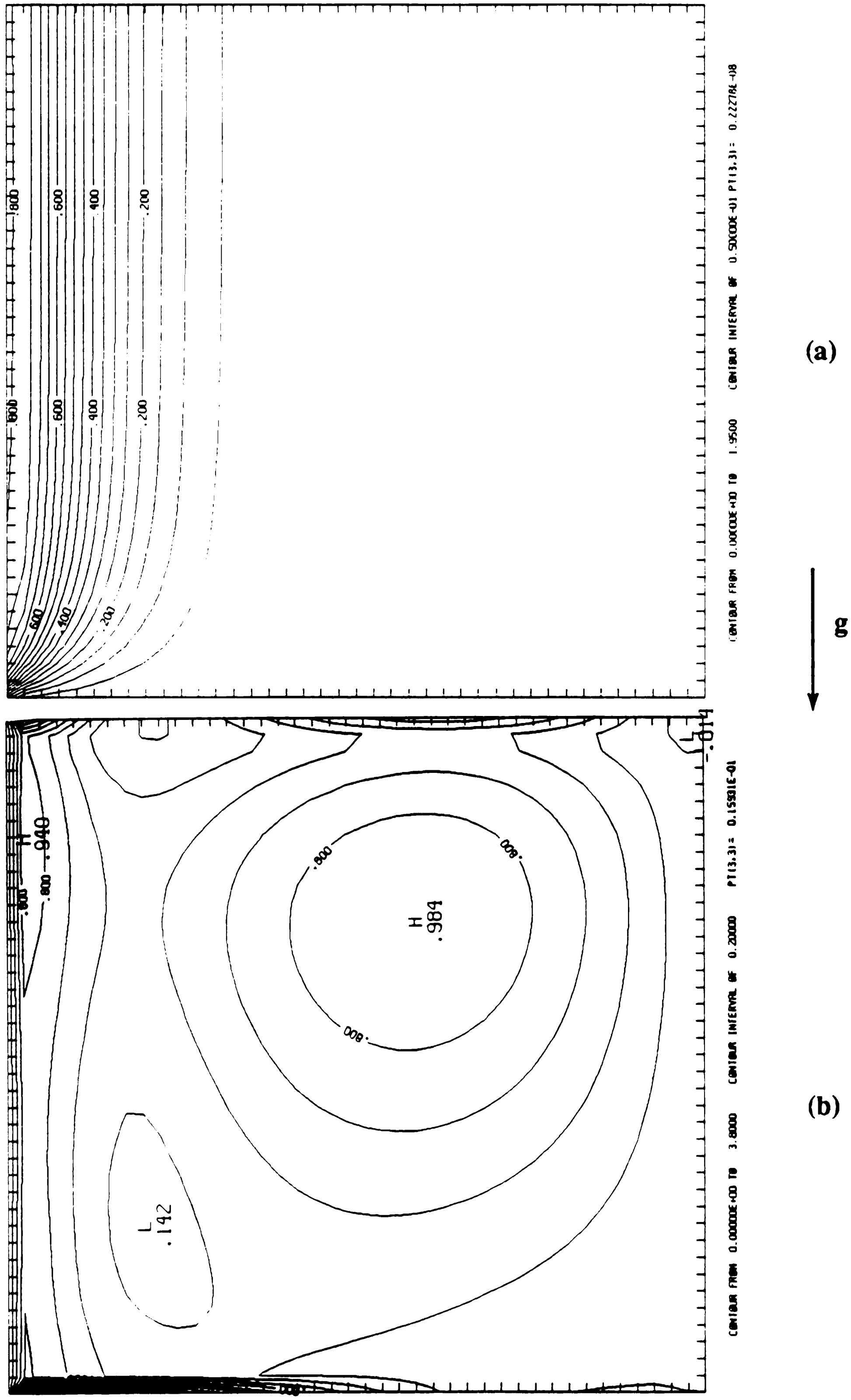


Figure 9 (9) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=580

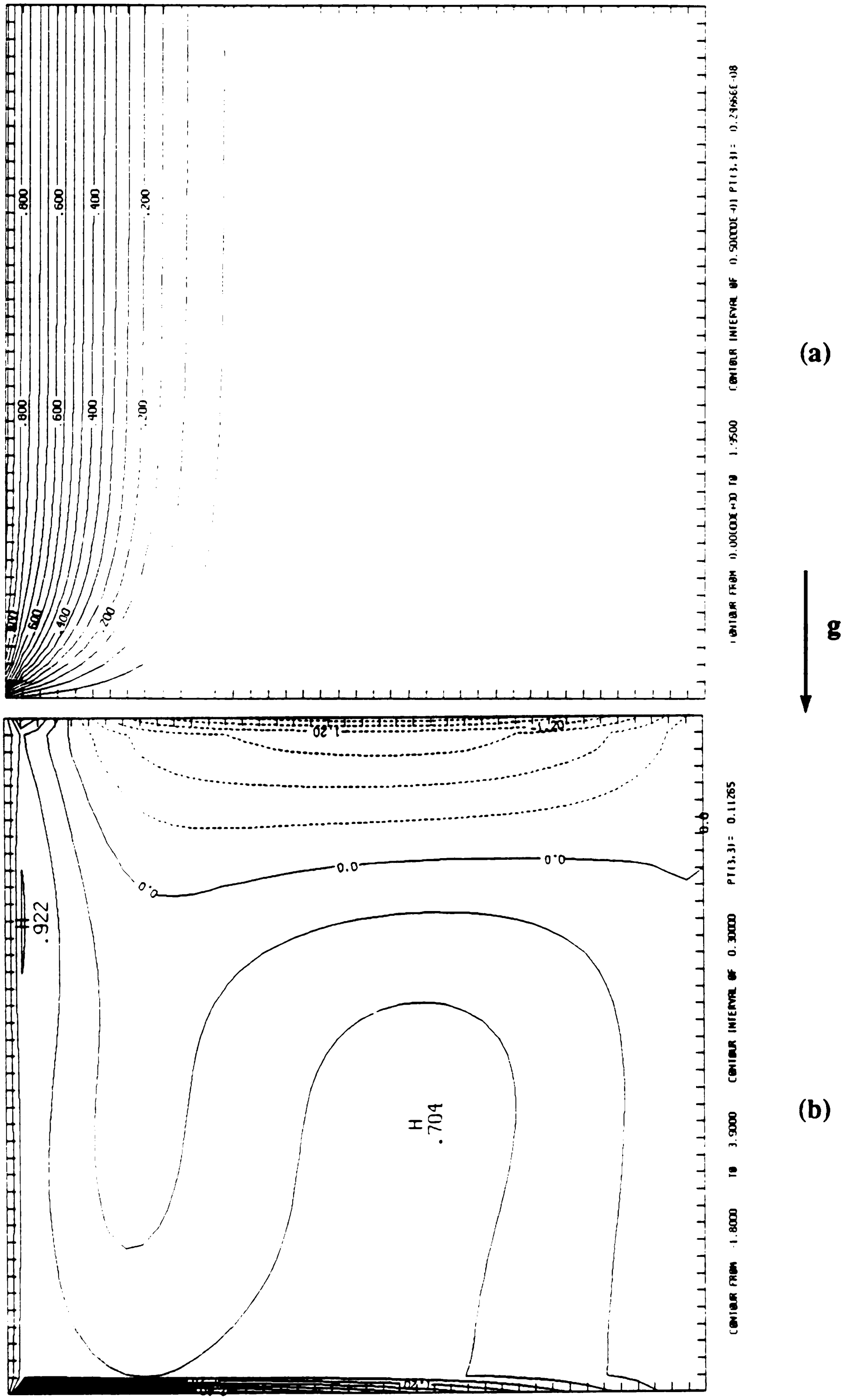


Figure 9 (10) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=583

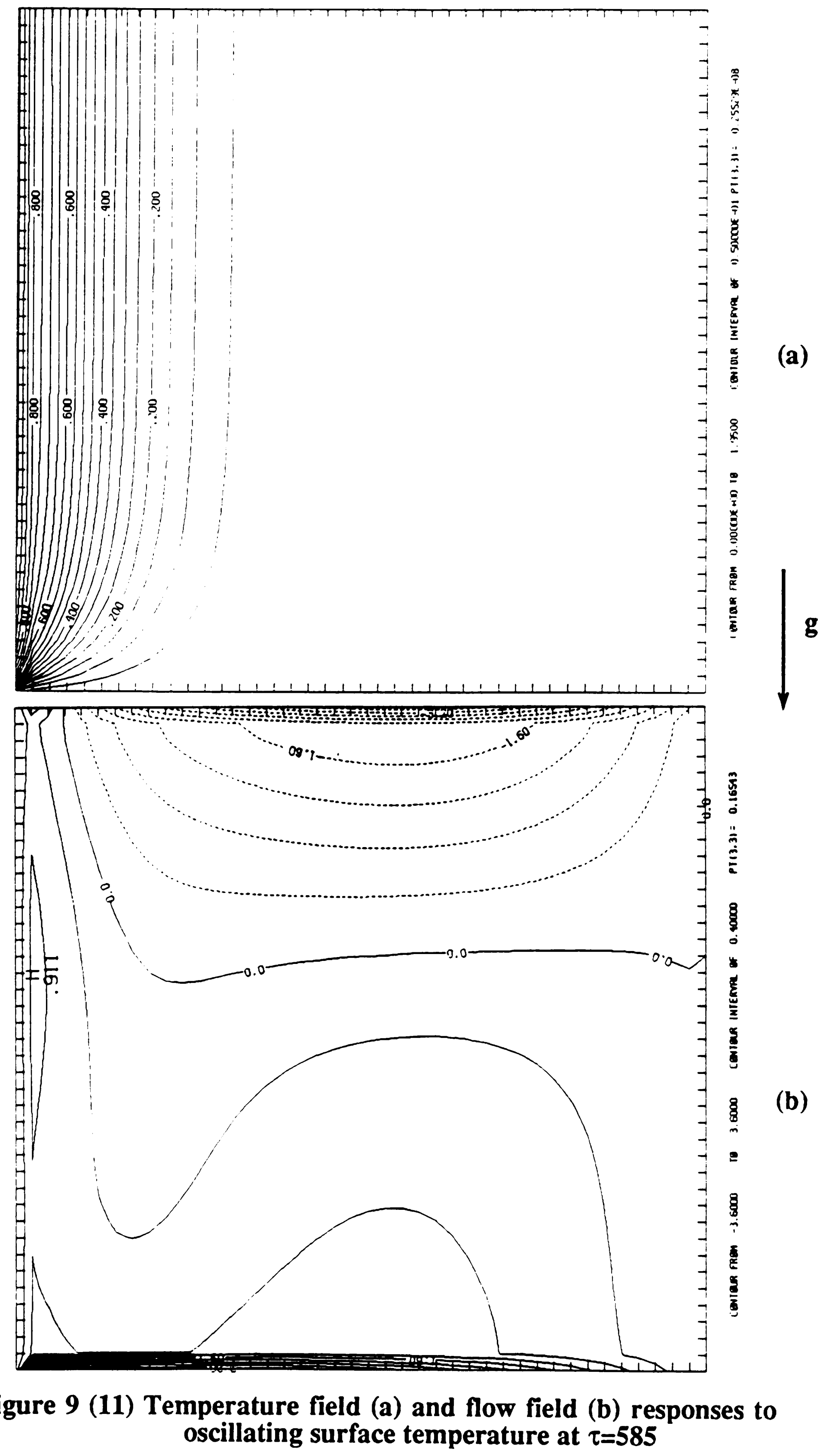


Figure 9 (11) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=585

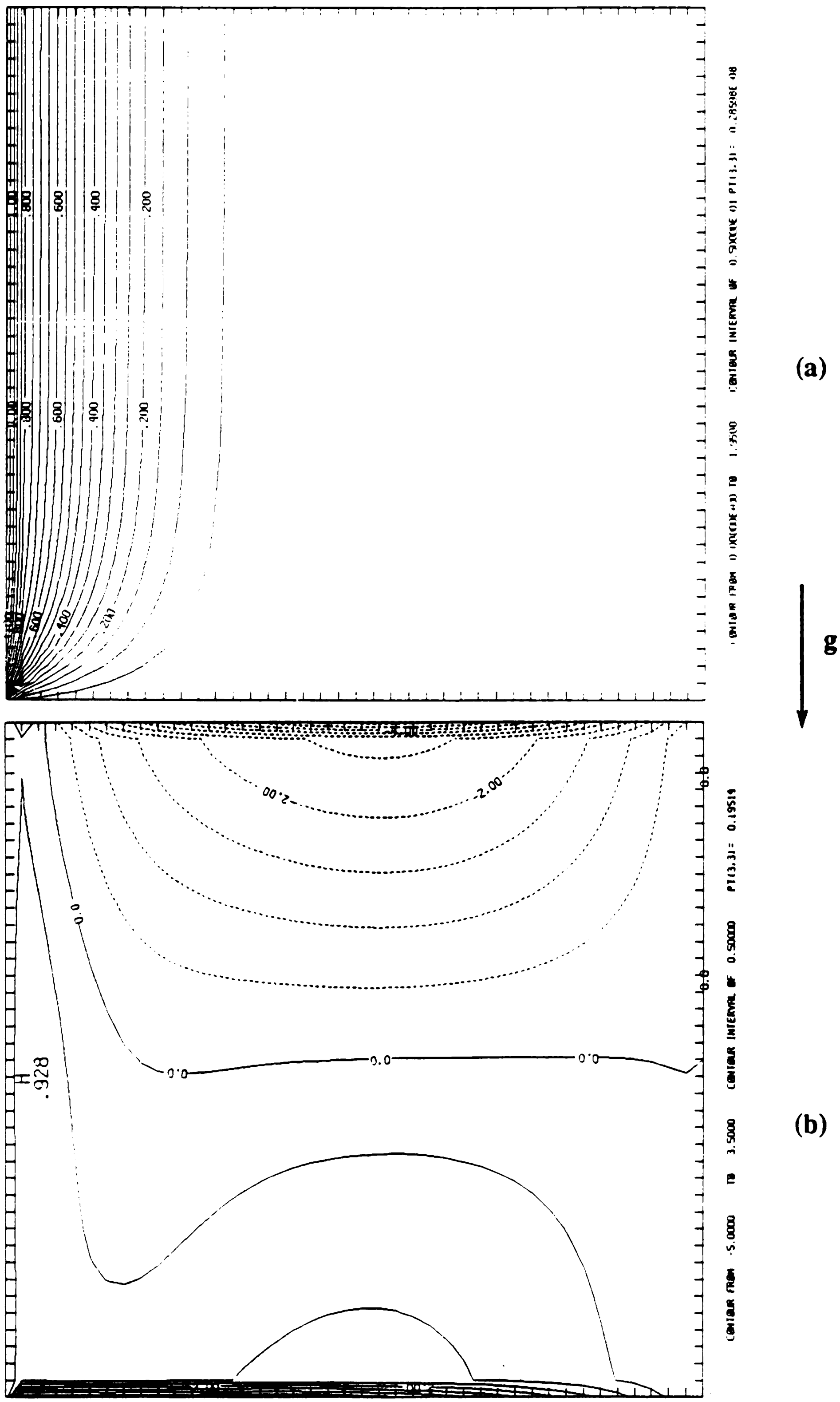


Figure 9 (12) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=587

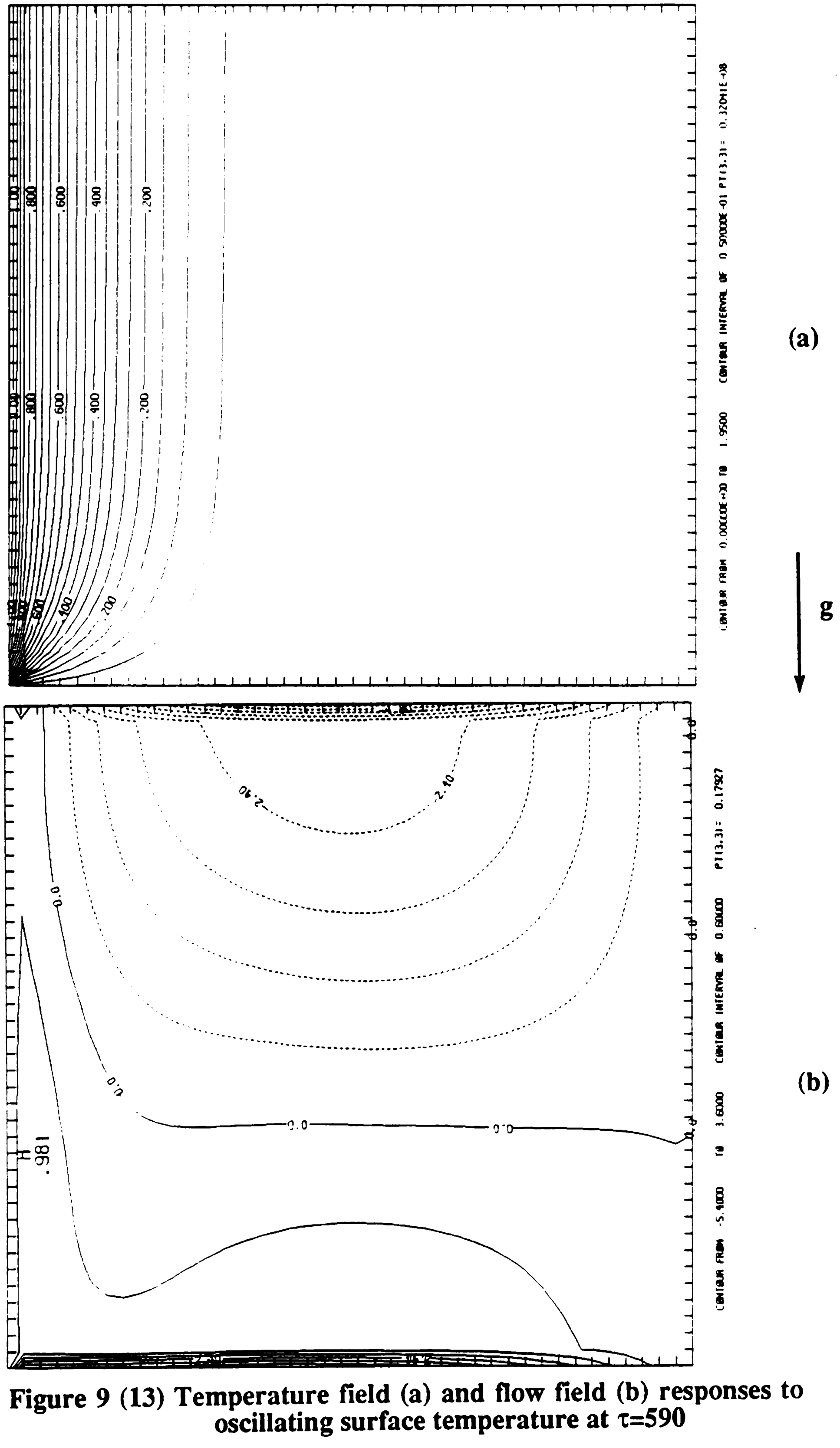
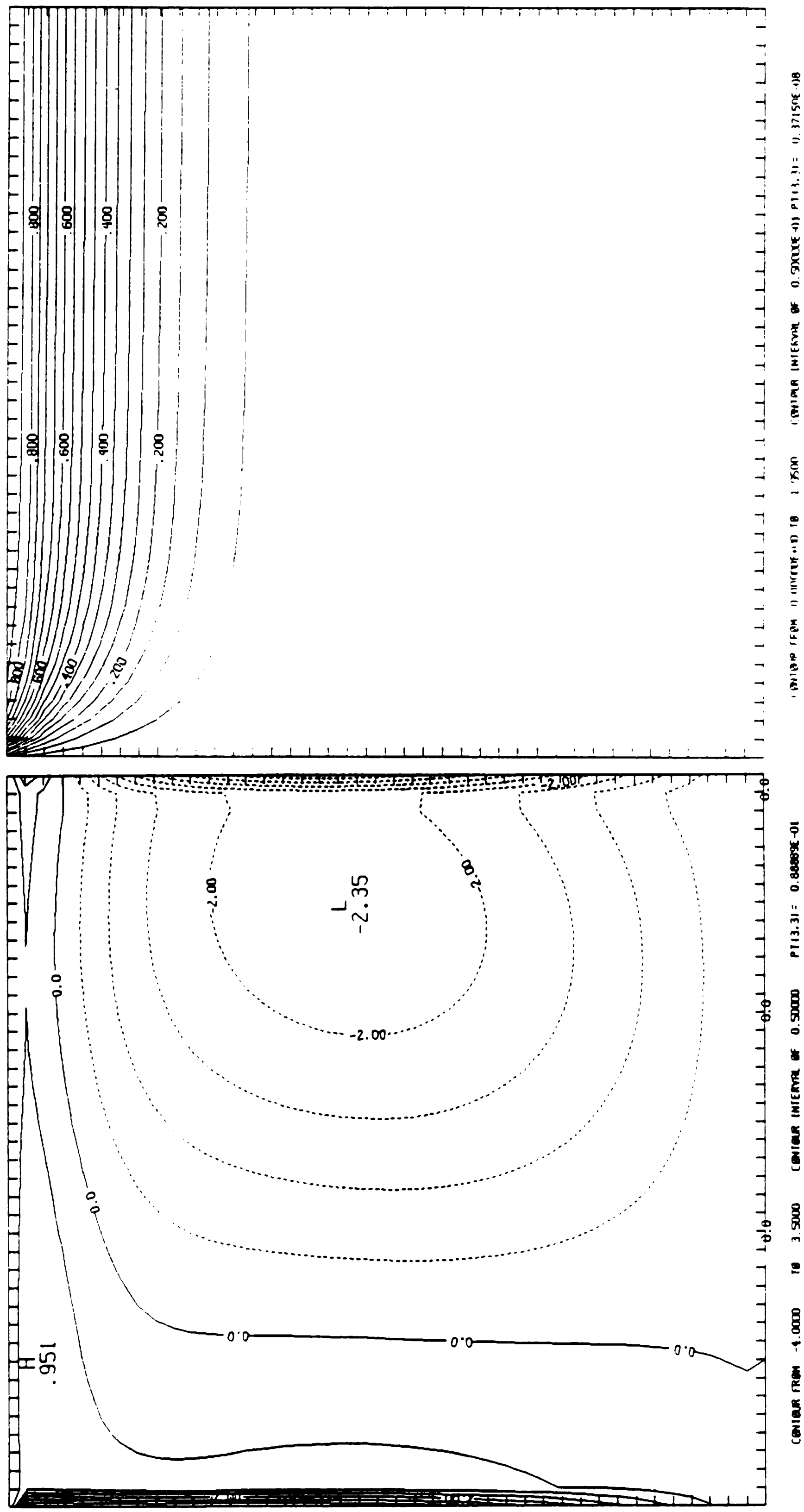


Figure 9 (13) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=590



(a)

g

(b)

Figure 9 (14) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=593

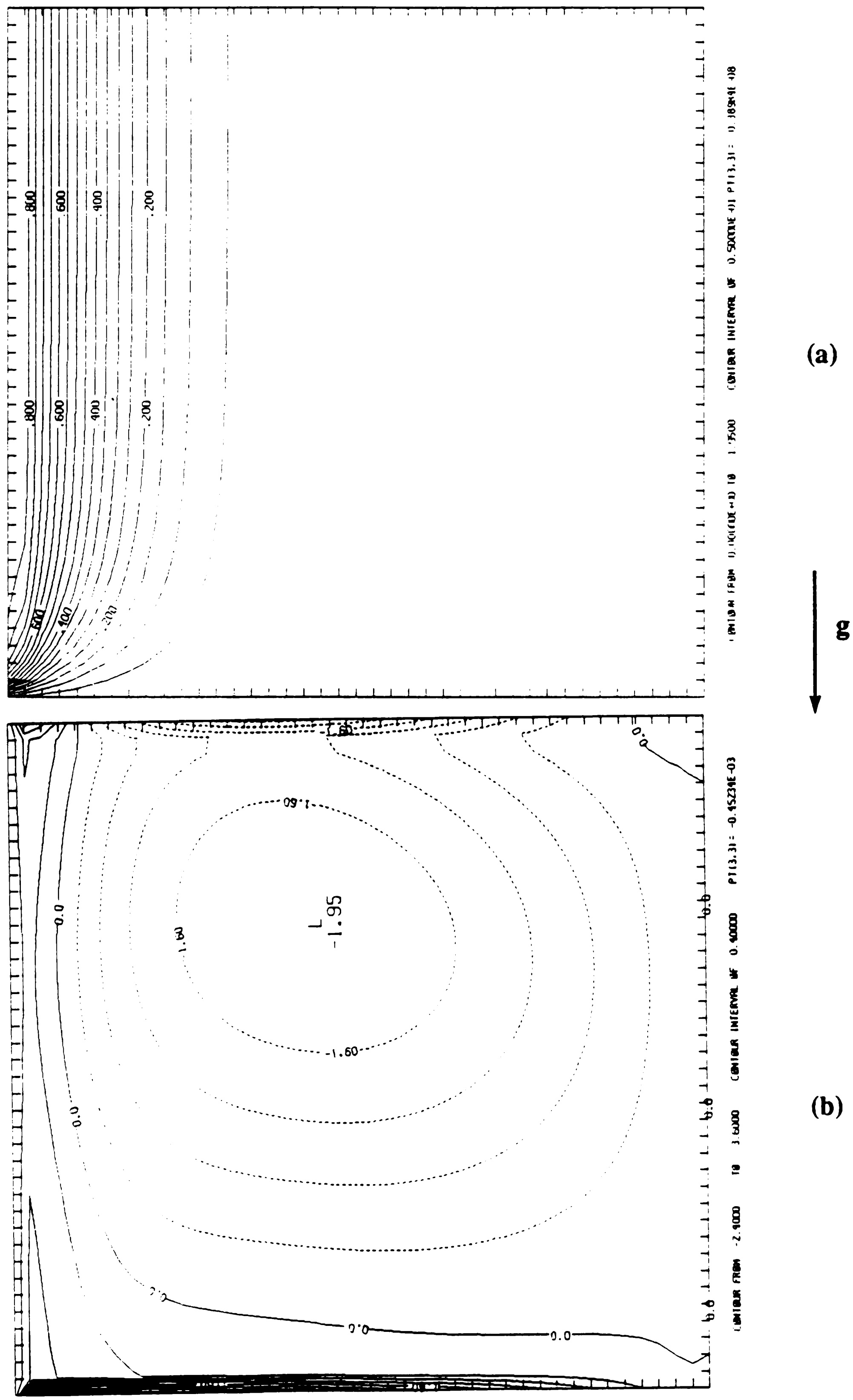


Figure 9 (15) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=595

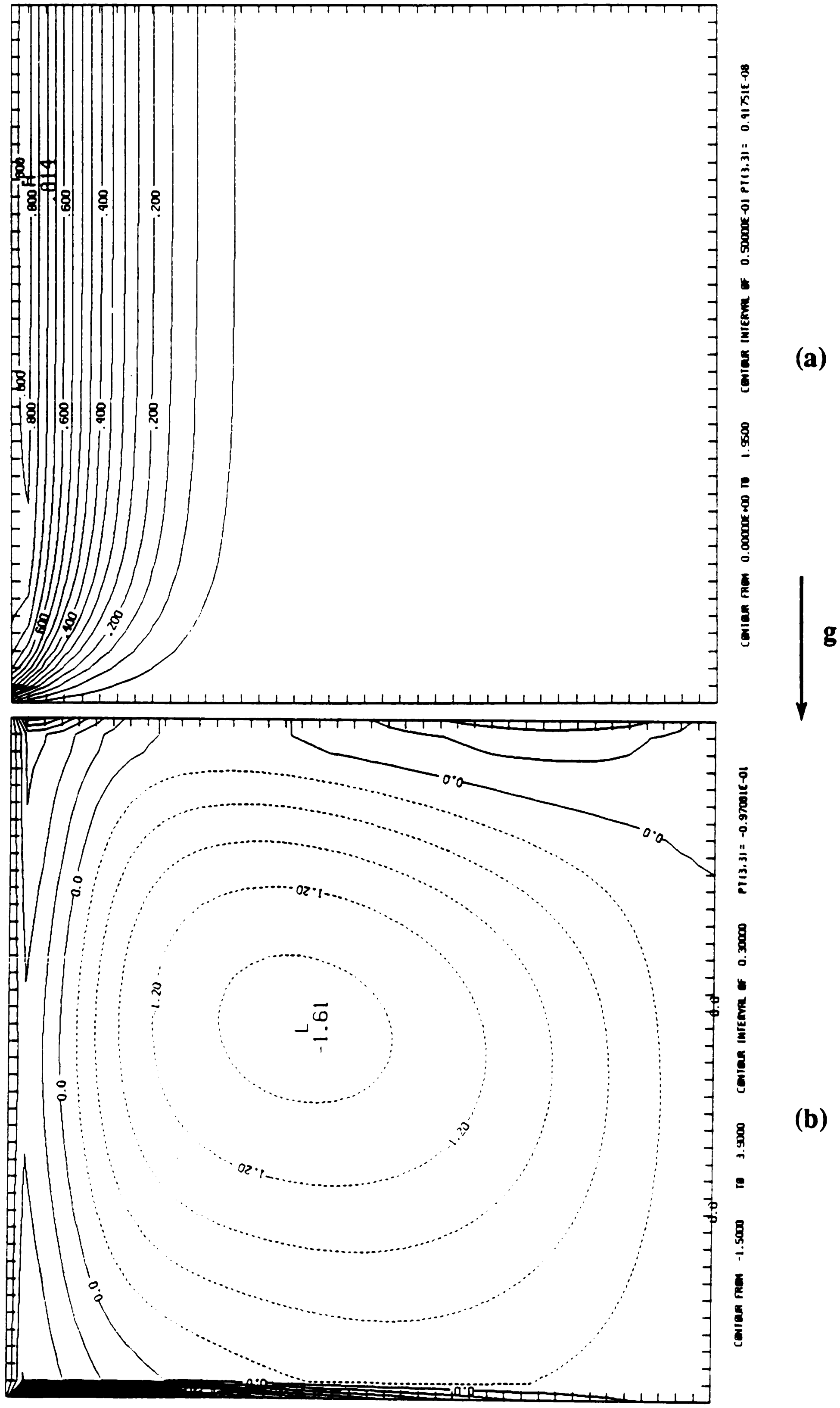


Figure 9 (16) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=597

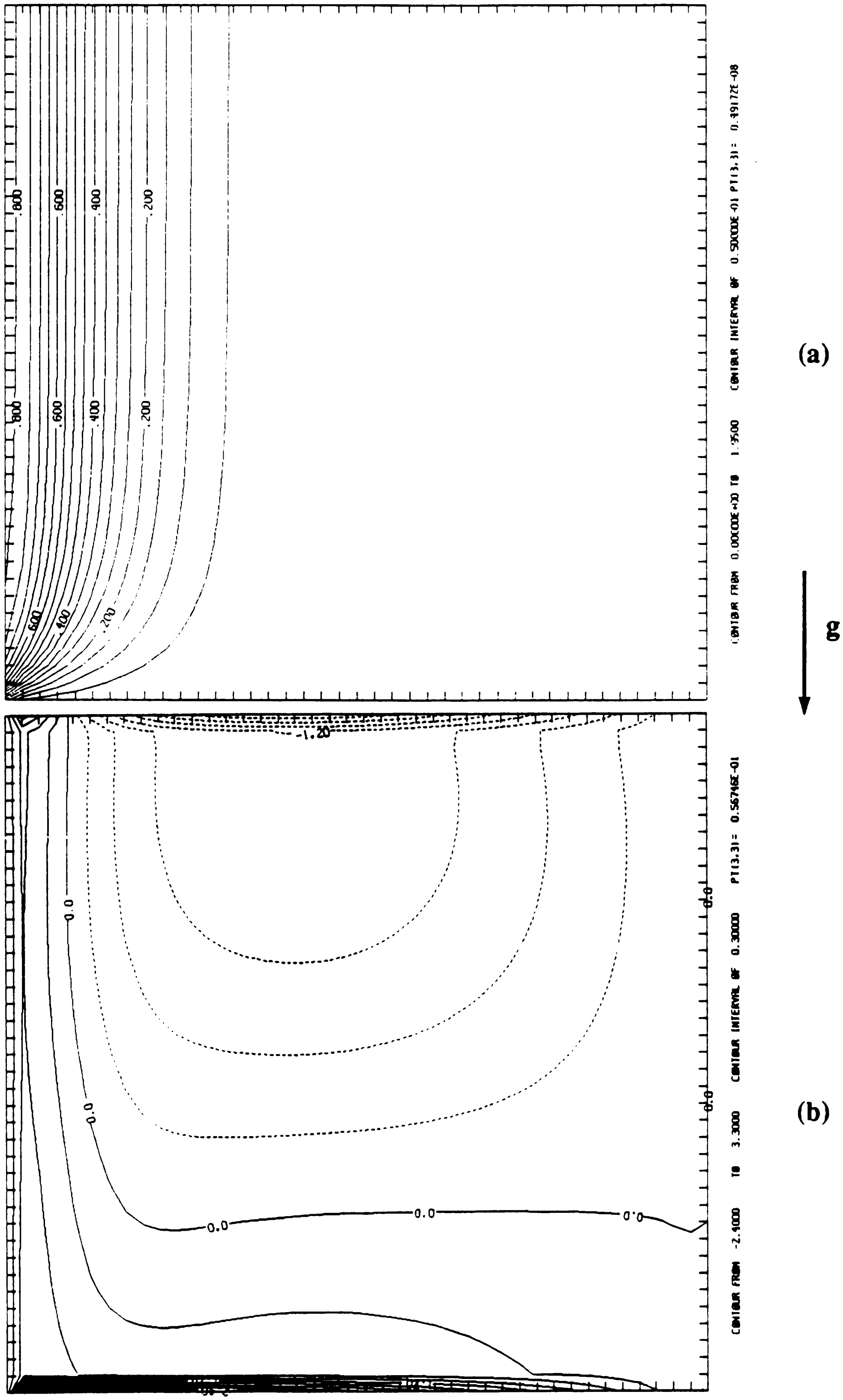


Figure 9 (17) Temperature field (a) and flow field (b) responses to oscillating surface temperature at τ=600

Put the temperature and flow fields together, and the oscillations for both of the fields seem to be generated by a physically oscillating hot surface. According to the temperature and flow fields appearance, this pseudo oscillatory movement of the surface should be vertical. This is explained by the fact that the natural convection adjacent to a vertical plate due to gravitation is primarily parallel to the plate surface. The term "thermal turbulence" is introduced here to emphasize the phenomenon of a time varying boundary temperature generated natural convection flow.

Just like turbulence can increase the convective heat transfer coefficient, the thermally driven turbulence will dissipate more heat from the high temperature surface. The numerical simulations proved that the enhancement not only exists but is also very significant. It appears that the frequency and the amplitude of the oscillation are the major factors controlling this phenomenon.

Although the analysis is done for a specific case, the thermal turbulence idea pertains for the situations with a time oscillating thermal boundary condition.

4.4 Effect of oscillation frequency on the Nusselt number

It is easy to imagine that different surface temperature oscillating frequencies will cause different characters to the oscillations of both temperature and velocity fields. Therefore the heat transfer will change accordingly. As expected, a series of numerical experiments, with Ra=7474, ASP=1, $\Delta X=1/40$, $\Delta Y=1/40$ and $\Delta\tau=1/10$, $\theta_{ave}=1$, and $\Delta\theta=0.5$ unchanged, and frequency varies from 0 Hz to 1000 Hz, show that the heat transfer is enhanced by the oscillation of the surface temperature significantly. This is visualized in Figure 10. As defined in equation 27, the driving surface temperature is oscillating around an average temperature. In Figure 10, two things should be noticed:

1. The space-average over the plate is oscillating about an average value. At

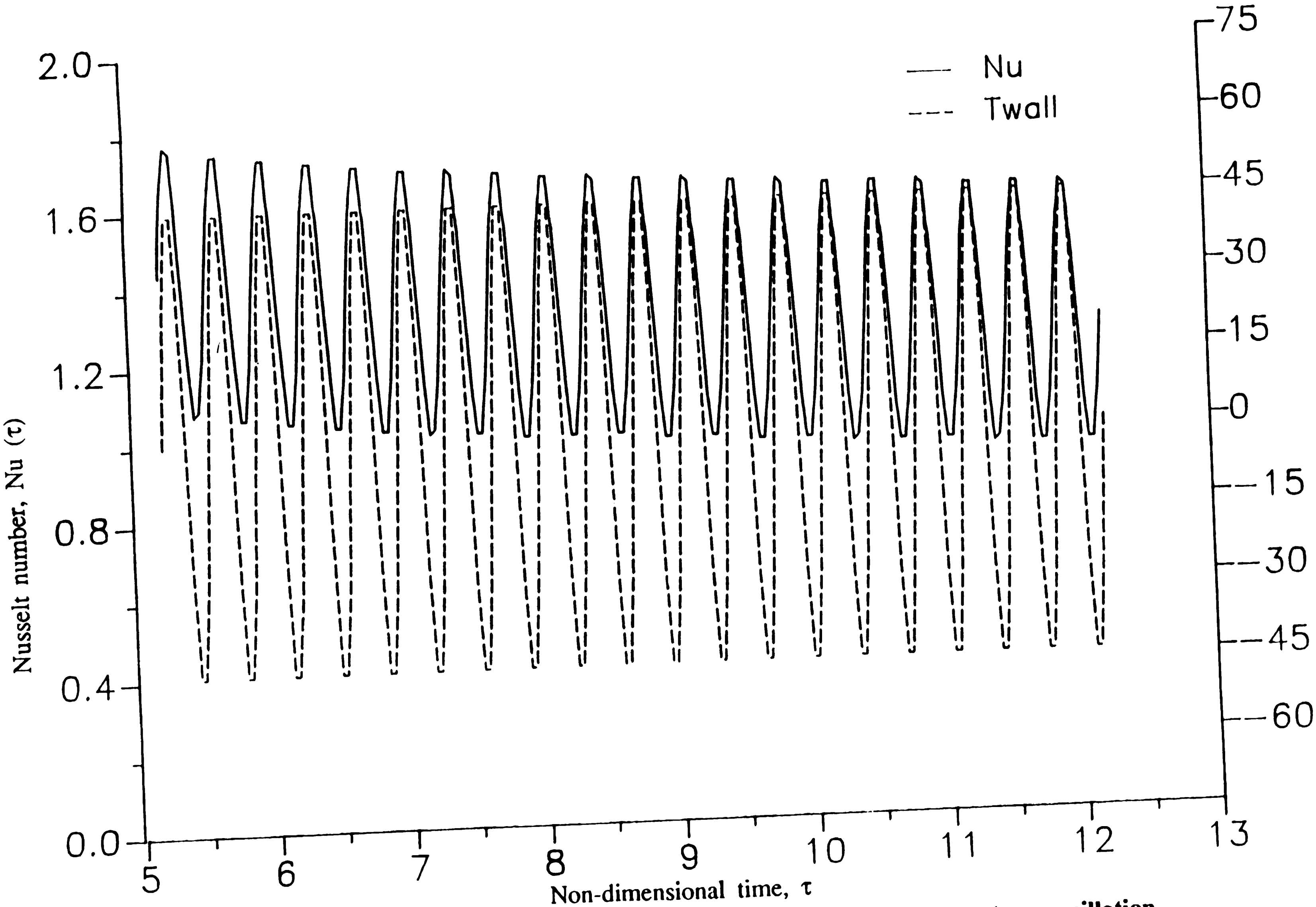


Figure 10 Nusselt number response to the surface temperature oscillation
(ASP=1, f=100Hz, $\Delta\theta$=5)

the time when the fully developed oscillatory pattern has settled down, the frequency of this oscillation is the same as that of the surface temperature, though there usually is a phase shift.

2. The average of the oscillation is done over one surface temperature period. The average will approach a constant value as the fully developed oscillation is reached. If the average temperature of the driving force is the same as that of a constant-surface temperature case, the average Nusselt number is higher than that of the constant surface temperature case. For example, for Ra=7474, the average is $Nu_{o,l}$=11.3, while for the constant surface temperature case, the Nusselt number is $\overline{Nu}_{c,l}$=5.3.

For the purpose of investigating the frequency effect on the Nusselt number, some of the calculation results are summarized in Figure 11. A new non-dimensional parameter N is defined as the following:

$$\overline{N}=\frac{\overline{Nu_{o,l}}}{Nu_{c,l}} \tag{32}$$

The Nusselt number represents the non-dimensional heat transfer coefficient. The subscripts "$_{o,l}$" and "$_{c,l}$" denote surface-average Nusselt numbers for oscillating and constant surface temperature cases, respectively. The "-" over $Nu_{o,l}$ denotes the averaged value over an oscillation period and the measurement of the heat transfer enhancement is in the sense of the time-averaged Nusselt number. As is obviously seen in Figure 11, the number N is always higher than one as long as the frequency is non-zero. This shows that an oscillation in surface temperature results in increased overall heat transfer effect. The heat transfer enhancement increases as the frequen-

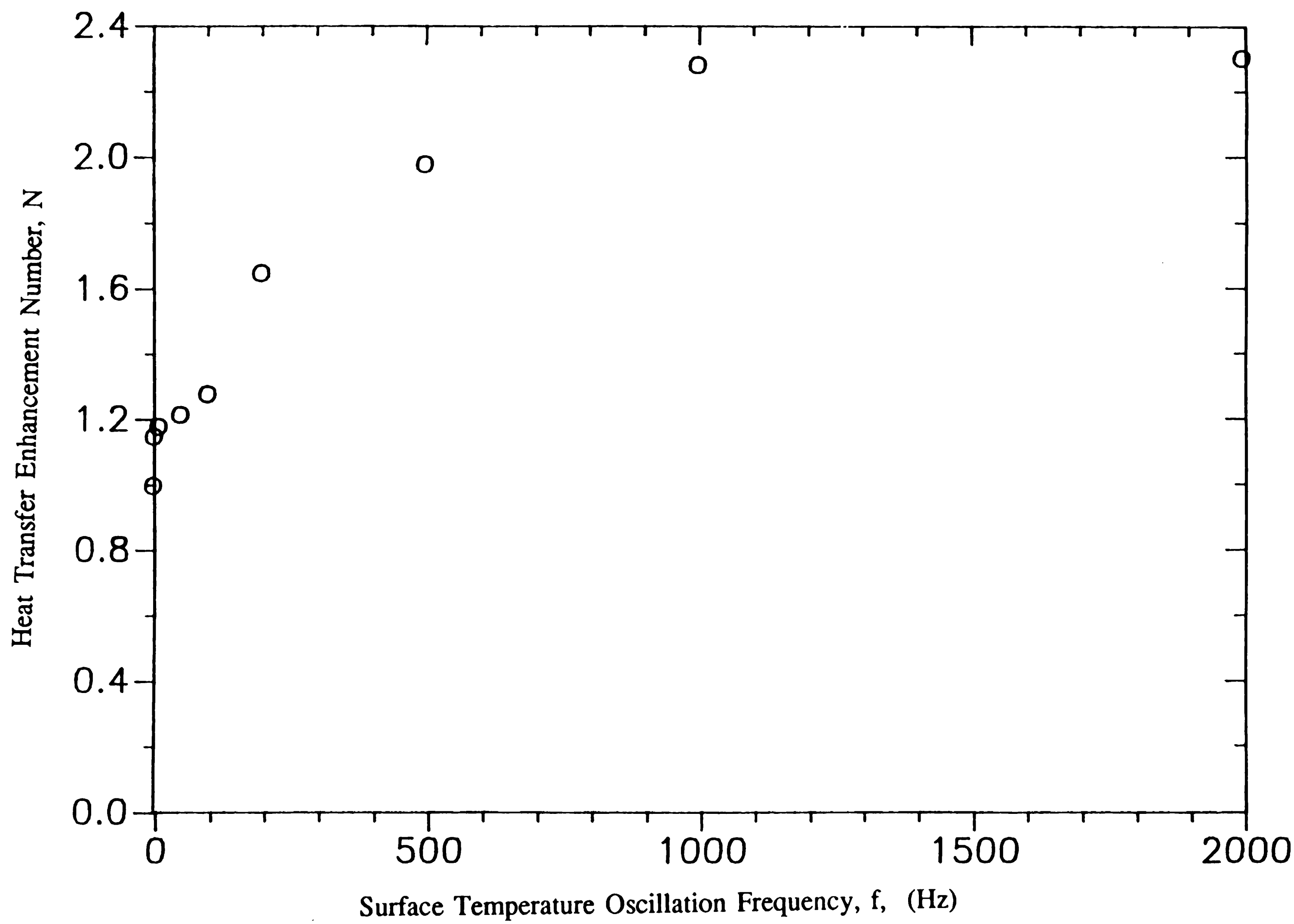


Figure 11 Effect of frequency on heat transfer enhancement
(ASP=1, Δθ=5)

cy increases but finally reaches a plateau after it reaches a value of 2.35. This indicates that the effect of enhancement of heat transfer by varying the surface temperature is limited for a given average surface temperature. After the maximum is reached, increasing the surface temperature frequency will not contribute any more to the enhancement of heat transfer.

4.5 Effect of oscillation amplitude on the Nusselt number

As the frequency increases the heat transfer coefficient, the amplitude of the surface temperature oscillation will also affect the enhancement of heat transfer. When the amplitude is low, the heat transfer is close to that of the constant-surface temperature situation. The heat transfer then increases as the amplitude increases, at least in a certain range of amplitude. To test out this theory, several cases, with Ra=7474, ASP=1, ΔX=1/40, ΔY=1/40 and $\Delta\tau$=1/10, θ_{ave}=1, and Fre.=10 Hz unchanged, were calculated, and the results are shown in Figure 12. In Figure 12, N versus $\Delta\theta$ is plotted. N is as defined in equation (32). Two observations are made from Figure 12.

1. The heat transfer number, N, is always greater than one. This again verifies that oscillation in surface temperature will increase heat transfer coefficient.

2. The effect of enhancement of heat transfer increases very rapidly at the small amplitudes and reaches an upper limit around a value about 2.35 . After the plateau is reached, the increasing in amplitude will not result in greater effect of heat transfer enhancement.

4.6 Effect of aspect ratio on the Nusselt number

The effects of both frequency and amplitude on the enhancement discussed previously are for the case when the aspect ratio is large enough so that the two surfaces are basically independent of each other. What will happen in terms of heat trans-

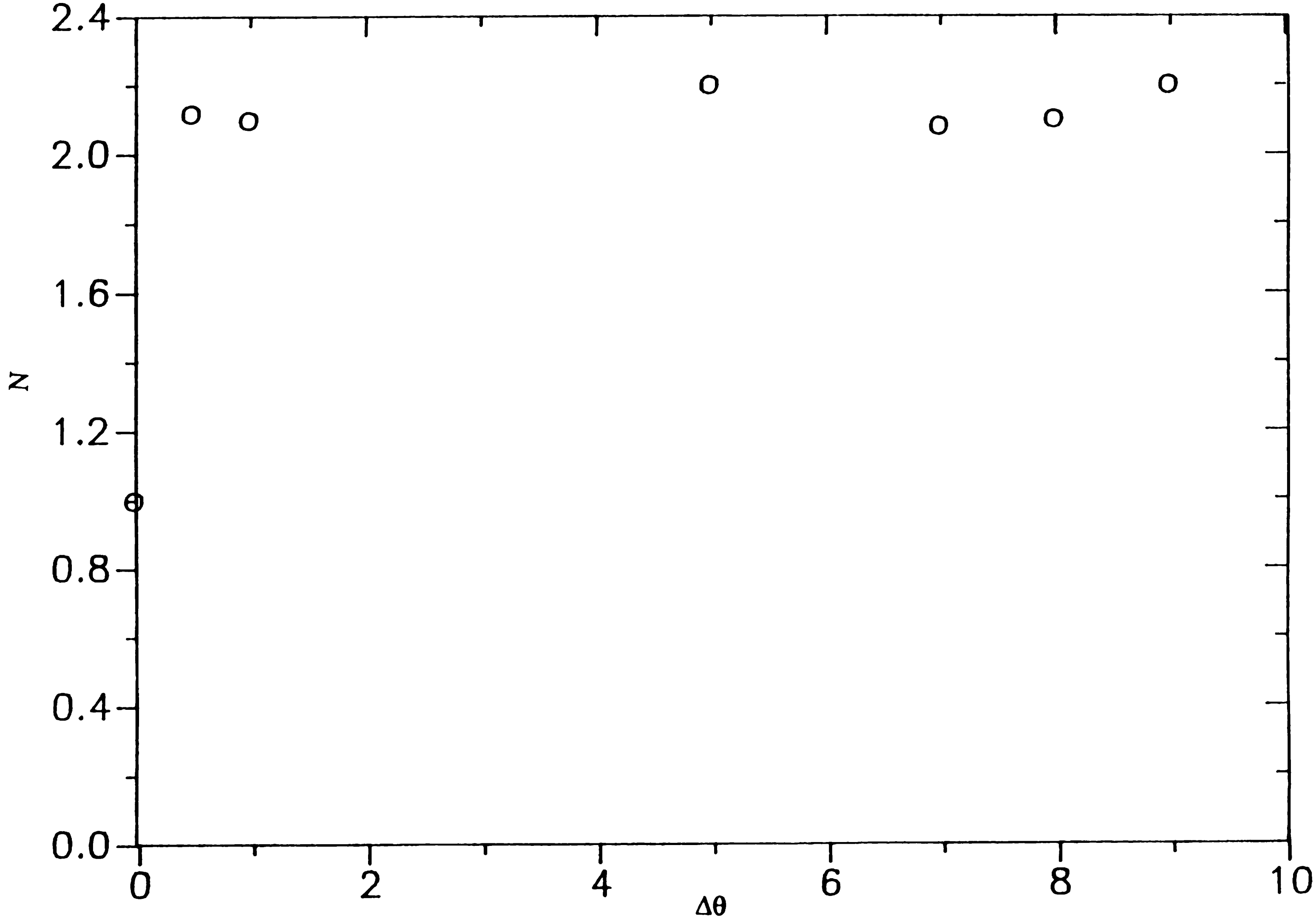


Figure 12 Effect of amplitude on heat transfer Eenhancement
(ASP=1, f=100Hz)

fer, if the surfaces are quite close to each other? A series of calculations were carried out, with Ra=7,474, $\Delta X=1/40$, $\Delta Y=1/40$ and $\Delta\tau=1/10$, $\theta_{ave}=1$, $\Delta\theta=0.5$, and Fre.=10 Hz unchanged, but the aspect ratio decreases from two to close to zero. Figure 13 gives the result. When the aspect ratio decreases from two to one the heat transfer coefficient almost does not change. This implies that for the aspect ratio greater than one, the simulation is for single plate. When the aspect ratio decreases from one to about 0.8, the heat transfer starts to increase, this is quite like the chimney effect for the two constant surface temperature situation. The chimney effect will reach a peak and then get weaker. As the aspect ratio get too small, say 0.2, conduction starts to become the dominant heat transfer mode.

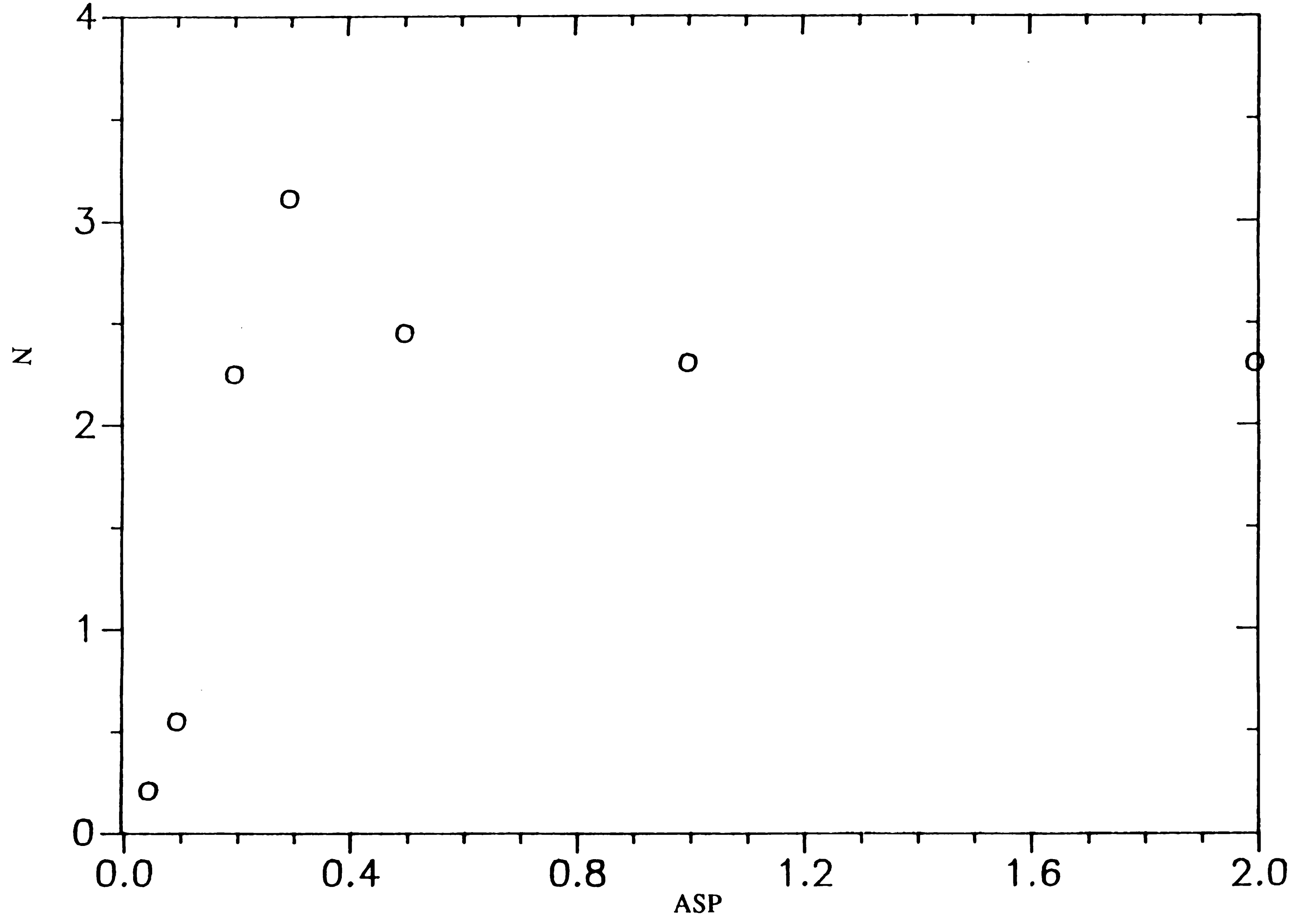


Figure 13 Effect of aspect ratio on heat transfer enhancement
(f=100Hz, $\Delta\theta$=5)

5. CONCLUSIONS

A numerical study of natural convection heat transfer caused by a time-oscillating surface temperature has been carried out using the UNDSAFE program. The following conclusions are supported by the above study.

1. An oscillating surface temperature causes a natural convection flow which is very different from the conventional pattern adjacent to a steady surface temperature surface. This was termed "thermal turbulence", and it appears in both temperature distribution and flow pattern;

2. The heat transfer enhancement has a strong relation to the oscillation frequency. The increase of the heat transfer was seen to becomecd smaller as the frequency was increased. There appears to be a limit for the heat transfer enhancement for a specific average surface temperature and oscillation amplitude;

3. The oscillation amplitude has a significant effect on the heat transfer. At high amplitudes, the heat transfer plateaus;

4. In terms of increasing heat transfer, it is unnecessary to have very high frequency and high amplitude.

The success in simulating natural convection heat transfer driven by oscillating surface temperature exposes a new approach in electronic cooling technique. Since a time varying surface temperature is easy to achieve and the oscillation in surface temperature will increase the heat transfer significantly without additional hardware, it is very suitable for electronic cooling. Also, as pointed out by the study, the oscillation does not have to have too high frequency or amplitude, which is desired when the fatigue is concerned for materials inside electronic devices.

6. RECOMMENDATIONS

To continue the current study, the following questions can lead to some interesting new directions:

1. At small aspect ratios, what is the interaction of the two buoyant flows caused by two oscillating wall temperatures? It is obvious that when two plates, each has a time oscillation in surface temperature, are close to each other, the two natural convection flows caused by the two oscillating surface temperatures will interact with each other. This will make the heat transfer even more complicated. If the interaction will increase or decrease the heat transfer enhancement is yet unknown.

2. When the surface temperature has a time varying spacial variation, what is the temperature field, flow pattern, and heat transfer rate? In the electronic devices, the hot (or cold) spot locations may vary with time. As can be imagined, this variation will cause a new pattern of heat transfer and fluid flow, and the heat transfer rate will be different from that of the constant surface temperature situation. The details need to be studied.

3. When the incoming boundary conditions are different from quiet ambient conditions, how will the temperature and flow fields as well as heat transfer rate be affected? There are all kinds of entrance boundary conditions other than the discussed ones in the current study. Different boundary conditions may request different modeling technique for treatment. Different boundary conditions will result in various types of temperature and velocity fields. Heat transfer rate will be different, too.

APPENDIX

(Numerical Code)

```
C  DATE:    JAN. 1, 1989
C  NAME:    WEI CHA
C  THIS PROGRAM IS DEVELOPED BASED ON UNDSAFE.  IT IS USED TO SIMULATE NATURACL CONVECTION CAUSED
C  BY AN OSCILLATIN GSURFACE TEMPERATURE. I
   LOGICAL*1 LBAND
   COMMON/RL/ LBAND(9)
   COMMON/R4/ X(-60:90),Y(-60:90),DXX(-60:90),DYY(-60:90), DXXS(-60:90),DYYS(-60:90)
   COMMON/BL1/DX,DY,VOL,DTIME,XOY,YOX,VOLDT,THOT,PI
   COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
   COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,NJTP2,NILP2,NJBP2
   COMMON/BL12/ NWRITE,NTAPE,NTMAX0,NTREAL,TIME,SORSUM,ITER
   COMMON/BL16/ CONST1,CONST2,CONST3,RA,PR,,NT,U0,H,UGRT,BUOY,
  & PSY,CP0,COND0,VIS0,RHO0,HR,TR,TA,DTEMP
   COMMON/BL31/ TOD(-60:90,-60:90),ROD(-60:90,-60:90), UOD(-60:90,-60:90),VOD(-60:90,-60:90)
   COMMON/BL32/ T(-60:90,-60:90),R(-60:90,-60:90),U(-60:90,-60:90),V(-60:90,-60:90),P(-60:90,-60:90)
  &        ,PSI(-60:90,-60:90)
   COMMON/BL33/ TPD(-60:90,-60:90),RPD(-60:90,-60:90), UPD(-60:90,-60:90),VPD(-60:90,-60:90),
  &        PPD(-60:90,-60:90),PEQ(-60:90,-60:90)
   COMMON/BL34/ HEIGHT(-60:90,-60:90),SMP(-60:90,-60:90),SMPP(-60:90,-60:90), DU(-60:90,-60:90),DV(-60:90,-60:90),
  &        PP(-60:90,-60:90)
   COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90),AW(-60:90,-60:90),AN(-60:90,-60:90),
  &        AS(-60:90,-60:90),SP(-60:90,-60:90),
  &        SU(-60:90,-60:90),REQ(-60:90,-60:90)
   COMMON/BL37/ VIS(-60:90,-60:90),COND(-60:90,-60:90), CPM(-60:90,-60:90),RESORM(20),TERM(20)
   COMMON/BL40/ ASX,ASY
   DIMENSION RNU(11),ANUC(3000),ANUH(3000),ANUM(3000),XT(3000) ,AFRE(3000),TEM(3000)
    DATA XTIME,SORMAX,GRAV,GC,RAIR,ITMAX/0.,25.0  ,2*32.17,53.34,10/
C #1. READ IN DATA TO INDICATE EITHER KRUN=0 OR 1
   READ(5,*) KRUN,PR,TAO,NTT
   READ(5,*)NM
C #2. READ IN DATA SET 1 - 4 DATA
   READ(5,*) NMAX,NWRITE,NTAPE
   READ(5,*) RA
   NMNTT=NM*NTT
   XNIL=1.-3.*((FLOAT(NI)-1.)/4.)
   XNJB=1.-(FLOAT(NJ)-1.)/4.
   XNJT=  (FLOAT(NJ)-1.)/4.+FLOAT(NJP1)
   NIL=INT(XNIL)
   NILP1=NIL+1
   NILP2=NIL+2
   NJB=INT(XNJB)
   NJBP1=NJB+1
   NJBP2=NJB+2
   NJT=INT(XNJT)
   NJTM1=NJT-1
   NJTP1=NJT+1
   NJTP2=NJT+2
   WRITE(6,*) NIL,NJB,NJT
C ***   INTRODUCE GIVEN PARAMETERS
   DX=1./FLOAT(NIM1)
   DY=1./FLOAT(NJM1)
   XH=0.0254
   ALFA=2.25E-5
   U0=ALFA/XH
   T0=XH/U0
C ***   GENERATION OF GRIDS
   CALL GRID
C ***   INITIALIZE VARIABLE FIELD
   YY=-1.5*DYY(2)
   DO 220 J=NJB,NJT
   YY=YY+DYYS(J)
   DO 220 I=NIL,NIP1
   ROD(I,J)=1.
   R(I,J)=1.
   RPD(I,J)=1.
```

```
      UOD(I,J)=0.
      U(I,J)=0.
      UPD(I,J)=0.
      VOD(I,J)=0.
      V(I,J)=0.
      VPD(I,J)=0.
      POD(I,J)=0.
      P(I,J)=0.
      PPD(I,J)=0.
      DU(I,J)=0.
      DV(I,J)=0.
      SU(I,J)=0.
      SP(I,J)=0.
      PP(I,J)=0.
      AP(I,J)=0.
      AW(I,J)=0.
      AE(I,J)=0.
      AN(I,J)=0.
      AS(I,J)=0.
      SMP(I,J)=0.
      SMPP(I,J)=0.
      VIS(I,J)=PR
      COND(I,J)=1.0
      CPM(I,J)=1.0
C      TOD(I,J)=-0.5+YY
       TOD(I,J)=0.0
      T(I,J)=TOD(I,J)
      TPD(I,J)=TOD(I,J)
  220 CONTINUE
      TAO=TAO/T0
      DTTIME=TAO/FLOAT(NTT)
      TOU=TAO*T0
      DTIME=DTTIME/FLOAT(NMAX)
      TC=298.
      DTHOT=2.50
      TH0=  DTHOT+TAVE
      TAVE=308.
      TTIME=0.0
      G=9.8
      MUE=1.589E-5
C     RA=G*(XH**3)/(ALFA*MUE)*(2.*DTHOT)/(TH0+TC)
      WRITE(6,*)G,XH,ALFA,MUE,DTHOT,TH0,TC
      DO 2222 M=1,NMNTT
2121  FORMAT(1X,'XH=',F9.6,'T0=',F9.4,'TAO=',F9.6,'TH0=',F7.3,/,1X,
     &'TC=',F6.2,'NTT=',I4,'DTTIME=',F8.6,'DTIME=',F9.6,'NM=',I5
     &,/,'U0=',F10.6,1X,' RA=',E10.4,'  ASY/ASX=',E10.4)
      JP=0
      PRINT*,'RA=',RA
      TTIME=FLOAT(M)*DTTIME
C     THOT=(1.0-EXP(-TTIME/TAO))
      THOT=1.0+SIN(2.*3.1415926*TTIME/TAO)*DTHOT/(TAVE-TC)
C     THOT=1.
C     WRITE(11,*)TTIME,THOT
C     THHOT=(298.15+5*THOT)
C      UE=14.58E-6*THHOT**1.5/(110.4+THHOT)
C      CP=0.2383-0.791E-5*THHOT+0.4834E-7*THHOT**2
C     XK=2.6482E-6*THHOT**.5/(1.+245.4*(10.**(-12./THHOT))/THHOT)
C     ALFA=XK/CP/10.
C     RA=G*(XH**3)/(ALFA*UE)*(2.*DTHOT)/(TH0+TC)
      TCOOL=0.0
      DO 564 I=1,NIP1
      TOD(I,NJP1)=THOT
      TOD(I,1)=TCOOL
  564 CONTINUE
      DO 9080 I=NIL,1
```

```
      TOD(I,NJBP1)=0.0
      TOD(I,NJTM1)=0.0
 9080 CONTINUE
      DO 9090 J=NJB,NJT
      TOD(I,J)=0.
 9090 CONTINUE
      NTMAX0=0
C *** FOR CONTINUING RUN, READ DATA FROM TAPE
      IF(KRUN .EQ. 1) GO TO 9997
      GO TO 19
 9997 READ(8,END=9998)
     & TIME,NTMAX0,TOD,ROD,UOD,VOD,POD,CPM,COND,VIS,ITERT,
     &,NI,NJ,NIP1,NJP1,NIM1,NJM1,
     & XX,XX,LBAND
      GO TO 9997
 9998 CONTINUE
      REWIND 8
   19 CONTINUE
      BUOY=GRAV*H/(U0*U0)
      DO 229 J=1,NJP1
      DO 229 I=1,NIP1
      REQ(I,J)=1.0
      IF(KRUN .NE. 0) GO TO 229
      RPD(I,J)=REQ(I,J)
      ROD(I,J)=RPD(I,J)
      R(I,J)=RPD(I,J)
  229 CONTINUE
  228 CONTINUE
C ***  INITIALIZE U,V,T,R,P FIELD
      DO 210 J=1,NJP1
      DO 210 I=1,NIP1
      T(I,J)=TOD(I,J)
      R(I,J)=ROD(I,J)
      U(I,J)=UOD(I,J)
      V(I,J)=VOD(I,J)
      P(I,J)=POD(I,J)
  210 CONTINUE
      DO 211 J=NJB,NJT
      DO 211 I=NIL,0
      T(I,J)=TOD(I,J)
      R(I,J)=ROD(I,J)
      U(I,J)=UOD(I,J)
      V(I,J)=VOD(I,J)
      P(I,J)=POD(I,J)
  211 CONTINUE
      NT=0
      NTIM=0
  300 CONTINUE
      NT=NT+1
C     IF(NT.GT.15)DTIME=0.000005
C ***  NTMAX0 HAS THE MEANING AS "NTREAL" WHEN IT IS READ FROM
C      DISK OR TAPE.
      IF(NT.GT.NMAX) GO TO 303
      NTREAL=NT+NTMAX0
      TIME=TIME+DTIME
  551 DO 555 I=1,NIP1
  555 T(I,NJP1)=THOT
C FOR THE FICTIONAL WALL TEMPERATURE(5311,5312)
      DO 5311 J=NJB,NJT
      IF(U(NIL,J).GT.0) T(NIL,J)=TCOOL
 5311 CONTINUE
      DO 5312 I=NIL,0
      IF(V(I,NJB).GT.0)T(I,NJB)=TCOOL
      IF(V(I,NJT).LT.0) T(I,NJT)=TCOOL
 5312 CONTINUE
```

```
C FOR THE CORNER TEMPERATURE
    IF (U(NIL,NJB).GT.0 .AND. V(NIL,NJB).GT.0) T(NIL,NJB)=TCOOL
    IF(U(NIL,NJT).GT. 0 .AND. V(NIL,NJT).LT.0) T(NIL,NJT)=TCOOL
C ***   START CALCULATION
    ITER=0
    JTERM=0
    JJTERM=0
C ***   DEFINE THE UPDATED TPD(I,J), R(I,J), UPD(I,J) AND VPD(I,J)
C      FOR THE USE OF CALVIS AND SU(I,J)
    DO 48 J=1,NJP1
    DO 48 I=1,NIP1
    TPD(I,J)=T(I,J)
    RPD(I,J)=R(I,J)
    UPD(I,J)=U(I,J)
    VPD(I,J)=V(I,J)
  48 CONTINUE
    DO 499 J=NJB,NJT
    DO 499 I=NIL,0
    TPD(I,J)=T(I,J)
    UPD(I,J)=U(I,J)
    VPD(I,J)=V(I,J)
    RPD(I,J)=R(I,J)
 499 CONTINUE
  29 CONTINUE
    JTERM=JTERM+1
C *** ************
    CALL CALT
C *** ************
    DO 2000 J=NJB,NJT
    DO 2000 I=NIL,NIP1
C    IF(T(I,J).LE.TCOOL) T(I,J)=TCOOL
C    IF(T(I,J).GE.THOT) T(I,J)=THOT
 2000 CONTINUE
C ***   START PRESSURE CORRECTION ITERATIVE LOOP, IT IS THE MAJOR
C      PART OF THE ERROR CONTROL ROUTINE
 301 CONTINUE
    ITER=ITER+1
C *** ***************
    CALL CALU
C *** ***************
    CALL CALV
C *** ***************
    CALL CALP
C *** ***************
    IF(RESORM(ITER) .LE. SORMAX) GO TO 49
    IF(ITER .EQ. 1) GO TO 302
    ITERM1=ITER-1
    IF(RESORM(ITER) .LE. RESORM(ITERM1)) GO TO 302
    GO TO 304
 302 IF(JTERM .GE. 2) GO TO 37
    SOURCE=RESORM(ITER)
    GO TO 39
  37 IF(RESORM(ITER) .LE. SOURCE) GO TO 38
    GO TO 304
  38 SOURCE=RESORM(ITER)
  39 CONTINUE
    DO 23 J=1,NJP1
    DO 23 I=1,NIP1
    TPD(I,J)=T(I,J)
    RPD(I,J)=R(I,J)
    UPD(I,J)=U(I,J)
    VPD(I,J)=V(I,J)
    PPD(I,J)=P(I,J)
  23 CONTINUE
    DO 233 J=NJB,NJT
```

```
   DO 233 I=NIL,0
   TPD(I,J)=T(I,J)
   UPD(I,J)=U(I,J)
   VPD(I,J)=V(I,J)
   RPD(I,J)=R(I,J)
   PPD(I,J)=P(I,J)
233 CONTINUE
   JJTERM=0
   IF(ITER .EQ. ITMAX) GO TO 49
   IF(JTERM .EQ. 2) GO TO 35
   IF(ITER .EQ. 4) GO TO 29
 35 CONTINUE
   IF(JTERM .EQ. 3) GO TO 58
   IF(ITER .EQ. 7) GO TO 29
 58 CONTINUE
   JJTERM=0
   GO TO 301
304 CONTINUE
   JJTERM=JJTERM+1
   IF(JJTERM .EQ. 1) WRITE(6,95) ITER,RESORM(ITER),SORSUM
   IF(JTERM .EQ. 1) GO TO 41
   IF(JTERM .EQ. 2 .AND. JJTERM .EQ. 1 .AND. ITER .NE. 5) GO TO 41
   GO TO 82
 41 CONTINUE
   DO 40 J=1,NJP1
   DO 40 I=1,NIP1
   R(I,J)=RPD(I,J)
   U(I,J)=UPD(I,J)
   V(I,J)=VPD(I,J)
   P(I,J)=PPD(I,J)
 40 CONTINUE
   DO 34 J=NJB,NJT
   DO 34 I=NIL,0
   R(I,J)=RPD(I,J)
   U(I,J)=UPD(I,J)
   V(I,J)=VPD(I,J)
   P(I,J)=PPD(I,J)
 34 CONTINUE
   IF(ITER .EQ. ITMAX) GO TO 49
   GO TO 29
 82 CONTINUE
   DO 43 J=1,NJP1
   DO 43 I=1,NIP1
   T(I,J)=TPD(I,J)
   R(I,J)=RPD(I,J)
   U(I,J)=UPD(I,J)
   V(I,J)=VPD(I,J)
   P(I,J)=PPD(I,J)
 43 CONTINUE
   DO 32 J=NJB,NJT
   DO 32 I=NIL,0
   T(I,J)=TPD(I,J)
   U(I,J)=UPD(I,J)
   V(I,J)=VPD(I,J)
   R(I,J)=RPD(I,J)
   P(I,J)=PPD(I,J)
 32 CONTINUE
   IF(ITER .EQ. ITMAX) GO TO 49
   IF((JTERM .EQ. 3 .AND. ITER .NE. 8) .OR. JJTERM .EQ. 2) GO TO 49
   GO TO 301
 49 CONTINUE
   ITERT=ITERT+ITER
C *** ***************************
   CALL NU(RNU,RNUC,RNUH)
C *** ***************************
```

```
      WRITE(13,*) TIME,RNUH
      WRITE(14,*) TIME,THOT
C ***   PRINT OUTPUT
C ***   SORSUM IS THE SUM OF ERROR SOURCE "SMP" FROM ALL OF THE CELLS
C       IN THE ENCLOSURE.
      IF (MOD(NT,10).NE.0) GOTO 2433
 2433 CONTINUE
      IF(MOD(NT,9).NE.0) GO TO 2322
      JP=JP+1
      ANUC(JP)=RNUC
      ANUH(JP)=RNUH
      ANUM(JP)=RNU(6)
      AFRE(JP)=10*THOT
      XT(JP)=NTREAL
      IF(JP.EQ.1) GO TO 2435
      IF(JP.GT.101) GO TO 2430
      AAS=AAS+ABS(ANUC(JP)-ANUC(JP-1))
      GO TO 2435
 2430 AAS=AAS+ABS(ANUC(JP)-ANUC(JP-1))-ABS(ANUC(JP-100)-ANUC(JP-101))
      IF(AAS/ANUC(JP).LE..001)GO TO 276
      AASS=AAS/ANUC(JP)
      GOTO 2435
 2435 CONTINUE
 2322 CONTINUE
  500 FORMAT(1X,                'NTREAL=',I9,1X,
     &                  'ITER=',I2,/,'SOURCE=',
     & F9.6,1X,'SORSUM=',F9.6,1X,'NUC=',F8.4,1X,'NUH=',F8.4,1X,
     & /,11(1X,F6.3),' THOT=',E10.4,/)
C *** **********************************
c     CALL TLEFT(IT)
  123 FORMAT(' ITLEFT = ',I10)
      IT0=IT
      IF(IT.LT.ITLEFT) GO TO 277
C *** **********************************
      IF(NTREAL .NE. NTREAL/NWRITE*NWRITE) GO TO 505
      GO TO 277
  276 WRITE(6,1112) NTREAL
   1112 FORMAT('  THE STEADY STATE HAS BEEN REACHED AT NT = ',I9)
  277 DO 502 II=1,NIP1/2
      I=II*2
  513 DO 503 JJ=1,NJP1/2
      J=2*JJ
      XTEMP=T(I,J)
      XR=R(I,J)
      XU=U(I,J)
      XV=V(I,J)
      XP=(P(I,J))
      XVIS=VIS(I,J)
  503 CONTINUE
  502 CONTINUE
  505 CONTINUE
C     IF(JP.GT.101.AND.AAS/ANUC(JP).LE..001)GO TO 166
C ***   RESET THE OLD TIME VALUES  TOD, ROD, UOD, VOD AND POD.
      DO 305 J=1,NJP1
      DO 305 I=1,NIP1
      TOD(I,J)=T(I,J)
      ROD(I,J)=R(I,J)
      UOD(I,J)=U(I,J)
      VOD(I,J)=V(I,J)
      POD(I,J)=P(I,J)
  305 CONTINUE
      DO 306 J=NJB,NJT
      DO 306 I=NIL,0
      TOD(I,J)=T(I,J)
      ROD(I,J)=R(I,J)
```

```
      UOD(I,J)=U(I,J)
      VOD(I,J)=V(I,J)
      POD(I,J)=P(I,J)
  306 CONTINUE
C *** ************************************************************
      IF(NTREAL .NE. NTREAL/NTAPE*NTAPE .AND. (XTIME+DTIME*H/U0)
     & .LE. TMAX) GO TO 522
      WRITE(9)
     & TIME,NTREAL,T,R,U,V,P,CPM,COND,VIS,ITERT,
     & H,TA,U0,COND0,VIS0,RHO0,NI,NJ,NIP1,NJP1,NIM1,NJM1,TIME3,
     & WH2O,WCO2,X,Y,DXX,DYY,DXXS,DYYS,LBAND
      REWIND 9
c     CALL TLEFT(IT)
      IF(IT.LT.ITLEFT) GO TO 166
C *** ************************************************************
  522 CONTINUE
C *** ********************************
c      CALL TLEFT(IT)
      IF(IT.LT.ITLEFT) GO TO 166
C *** ********************************
      GO TO 300
  303 CONTINUE
      WRITE(6,1111)
C     WRITE(11,1111) 1001  FORMAT(1X,'  M=',I3)
 1111 FORMAT(2X,'****** THE MAXIMUM TIME HAS BEEN REACHED ******',I8)
      GO TO 172
C *** ************************************************************
  166 IF(NTREAL .NE. NTREAL/NTAPE*NTAPE) WRITE(9)
     & TIME,NTREAL,T,R,U,V,P,CPM,COND,VIS,ITERT,
     & H,TA,U0,COND0,VIS0,RHO0,NI,NJ,NIP1,NJP1,NIM1,NJM1,TIME3,
     & WH2O,WCO2,X,Y,DXX,DYY,DXXS,DYYS,LBAND
      REWIND 9
C *** ************************************************************
  172 CONTINUE
      DO 152 I=1,JP
      BNUH=BNUH+ANUH(I)/JP
      BNUM=BNUM+ANUM(I)/JP
      BNUC=BNUC+ANUC(I)/JP
  152 CONTINUE
         DO 155 I=2,NI
      XX=X(I)+DXX(I)/2
  156 FORMAT (2X,I4,3X,F8.4,10X,F10.4)
  155 CONTINUE
      CALL CALPL
      IF(M.EQ.550 )GOTO 319
      IF(M.EQ.553 )GOTO 319
      IF(M.EQ.555 )GOTO 319
      IF(M.EQ.557 )GOTO 319
      IF(M.EQ.540 )GOTO 319
      IF(M.EQ.543 )GOTO 319
      IF(M.EQ.545 )GOTO 319
      IF(M.EQ.547 )GOTO 319
      GOTO 2222
333     continue
2222  CONTINUE
      STOP
      END
C *** **************************************
      SUBROUTINE CALT
C *** **************************************
      COMMON/R4/ X(-60:90),Y(-60:90),DXX(-60:90),DYY(-60:90), DXXS(-60:90),DYYS(-60:90)
      COMMON/BL1/DX,DY,VOL,DTIME,XOY,YOX,VOLDT,THOT,PI
      COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
      COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,NJTP2,NILP2,NJBP2
```

```
      COMMON/BL12/ NWRITE,NTAPE,NTMAX0,NTREAL,TIME,SORSUM,ITER
      COMMON/BL16/ CONST1,CONST2,CONST3,RA,PR   ,NT,U0,H,UGRT,BUOY,
     & PSY,CP0,COND0,VIS0,RHO0,HR,TR,TA,DTEMP
      COMMON/BL31/ TOD(-60:90,-60:90),ROD(-60:90,-60:90), UOD(-60:90,-60:90),VOD(-60:90,-60:90)
      COMMON/BL32/ T(-60:90,-60:90),R(-60:90,-60:90), U(-60:90,-60:90),V(-60:90,-60:90),P(-60:90,-60:90),
     &        PSI(-60:90,-60:90)
      COMMON/BL33/ TPD(-60:90,-60:90),RPD(-60:90,-60:90), UPD(-60:90,-60:90),VPD(-60:90,-60:90),
     &        PPD(-60:90,-60:90),PEQ(-60:90,-60:90)
      COMMON/BL34/ HEIGHT(-60:90,-60:90),SMP(-60:90,-60:90),SMPP(-60:90,-60:90),
     &        DU(-60:90,-60:90),DV(-60:90,-60:90),PP(-60:90,-60:90)
      COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90),AW(-60:90,-60:90),AN(-60:90,-60:90),
     &        AS(-60:90,-60:90),SP(-60:90,-60:90),SU(-60:90,-60:90),REQ(-60:90,-60:90)
      COMMON/BL37/ VIS(-60:90,-60:90),COND(-60:90,-60:90), CPM(-60:90,-60:90),RESORM(20),TERM(20)
C ***   CALCULATE COEFFICIENTS
      DO 100 J=NJBP1,NJTM1
      JP2=J+2
      JP1=J+1
      JM1=J-1
      JM2=J-2
      DO 100 I=NILP1,NI
      IF((I.GE.1 .AND. J.LE.1).OR.(I.GE.1 .AND. J.GE.NJP1)) GO TO 100
      IP2=I+2
      IP1=I+1
      IM1=I-1
      IM2=I-2
      DXI=DXX(I)
      DXM1=DXX(IM1)
      DXP1=DXX(IP1)
      DYJ=DYY(J)
      DYM1=DYY(JM1)
      DYP1=DYY(JP1)
      DXEE=DXXS(IP2)
      DXE=DXXS(IP1)
      DXW=DXXS(I)
      DXWW=DXXS(IM1)
      DYNN=DYYS(JP2)
      DYN=DYYS(JP1)
      DYS=DYYS(J)
      DYSS=DYYS(JM1)
      YOXE=DYJ/DXE
      YOXW=DYJ/DXW
      XOYN=DXI/DYN
      XOYS=DXI/DYS
      VOL=DXI*DYJ
      VOLDT=VOL/DTIME
      CN=V(I,JP1)*DXI
      CS=V(I,J)*DXI
      CE=U(IP1,J)*DYJ
      CW=U(I,J)*DYJ
      CONDN1=XOYN
      CONDS1=XOYS
      CONDE1=YOXE
      CONDW1=YOXW
      CEP=(ABS(CE)+CE)*DXE/DXI/16.
      CEM=(ABS(CE)-CE)*DXE/DXP1/16.
      CWP=(ABS(CW)+CW)*DXW/DXM1/16.
      CWM=(ABS(CW)-CW)*DXW/DXI/16.
      CNP=(ABS(CN)+CN)*DYN/DYJ/16.
      CNM=(ABS(CN)-CN)*DYN/DYP1/16.
      CSP=(ABS(CS)+CS)*DYS/DYM1/16.
      CSM=(ABS(CS)-CS)*DYS/DYJ/16.
      AE(I,J)=-.5*CE+CEP+CEM*(1.+DXE/DXEE)+CWM*DXW/DXE
      AW(I,J)= .5*CW+CWP*(1.+DXW/DXWW)+CWM+CEP*DXE/DXW
      AN(I,J)=-.5*CN+CNP+CNM*(1.+DYN/DYNN)+CSM*DYS/DYN
      AS(I,J)= .5*CS+CSP*(1.+DYS/DYSS)+CSM+CNP*DYN/DYS
```

```
      IF (I.LT.NI) GOTO 801
      AEE=0.
      AEER=0.
      GOTO 802
 801 AEE=-CEM*DXE/DXEE
      AEER=AEE*TPD(IP2,J)
 802 CONTINUE
      IF (I.GT.2) GOTO 803
      AWW=0.
      AWWR=0.
      GOTO 804
 803 AWW=-CWP*DXW/DXWW
      AWWR=AWW*TPD(IM2,J)
 804 CONTINUE
      IF (J.LT.NJ) GOTO 805
      ANN=0.
      ANNR=0.
      GOTO 806
 805 ANN=-CNM*DYN/DYNN
      ANNR=ANN*TPD(I,JP2)
 806 CONTINUE
      IF (J.GT.2) GOTO 807
      ASS=0.
      ASSR=0.
      GOTO 808
 807 ASS=-CSP*DYS/DYSS
      ASSR=ASS*TPD(I,JM2)
 808 CONTINUE
      AP(I,J)=(AE(I,J)+AW(I,J)+AN(I,J)+AS(I,J) +AEE+AWW+ANN+ASS)+CONDE1+CONDW1+CONDN1+CONDS1
      AE(I,J)=AE(I,J)+CONDE1
      AW(I,J)=AW(I,J)+CONDW1
      AN(I,J)=AN(I,J)+CONDN1
      AS(I,J)=AS(I,J)+CONDS1
      SP(I,J)=-VOLDT
      SU(I,J)= VOLDT*TPD(I,J)
      SU(I,J)=SU(I,J)+AEER+AWWR+ANNR+ASSR
 100 CONTINUE
C ***   TAKE CARE OF B.C THR AE,AW,AN,AS,....
C ***   ISOTHERMAL WALLS ,FLOOR AND CEILING
      DO 500 I=2,NI
      SP(I,2)=SP(I,2)-AS(I,2)
      SU(I,2)=SU(I,2)+2.*AS(I,2)*TPD(I,1)
      SP(I,NJ)=SP(I,NJ)-AN(I,NJ)
      SU(I,NJ)=SU(I,NJ)+2.*AN(I,NJ)*TPD(I,NJP1)
      AS(I,2)=0.
      AN(I,NJ)=0.
 500 CONTINUE
C ***   ADIABATIC WALLS: RIGHT WALL
      DO 600 J=2,NJ
C     SP(2,J)=SP(2,J)+AW(2,J)
      SP(NI,J)=SP(NI,J)+AE(NI,J)
C*    SU(2,J)=SU(2,J)+2.*AW(2,J)*(0.0)
C     AW(2,J)=0.
      AE(NI,J)=0.
 600 CONTINUE
C*******:LEFT WALL
      DO 601 J=NJBP1,NJTM1
      SP(NILP1,J)=SP(NILP1,J)-AW(NILP1,J)
      SU(NILP1,J)=SU(NILP1,J)+2.*AW(NILP1,J)*TCOOL
      AW(NILP1,J)=0.
 601 CONTINUE
C*** FICTIONAL HORIZONTAL WALLS
      DO 222 I=NILP1,1
      SP(I,NJTM1)=SP(I,NJTM1)-AW(I,NJTM1)
      SU(I,NJTM1)=SU(I,NJTM1)+2.*AN(I,NJTM1)*TCOOL
```

```
      AN(I,NJT)=0.
C     CN(I,NJT)=R(I,NJT)*DX*V(I,NJT)
C     IF(CN .GT. 0.) GO TO 233
      SP(I,NJBP1)=SP(I,NJBP1)-AS(I,NJBP1)
      SU(I,NJBP1)=SU(I,NJBP1)+2.*AS(I,NJBP1)*TCOOL
      AS(I,NJBP1)=0.
 222  CONTINUE
C*** FICTIONAL TWO SHORT WALLS
      DO 444 J=NJBP1,1
      SP(0,J)=SP(0,J)-AE(0,J)
      SU(0,J)=SU(0,J)+2.*AE(0,J)*TPD(0,J)
      AE(0,J)=0.0
 444  CONTINUE
      DO 445 J=NJP1,NJTM1
      SP(0,J)=SP(0,J)-AE(0,J)
      SU(0,J)=SU(0,J)+2.*AE(0,J)*TPD(0,J)
      AE(0,J)=0.
 445  CONTINUE
C ***   ASSEMBLE COEFFICIENTS AND SOLVE DIFFERENCE EQUATIONS
      DO 300 J=2,NJ
      DO 300 I=2,NI
      AP(I,J)=AP(I,J)-SP(I,J)
 300  CONTINUE
      DO 301 J=NJBP1,NJTM1
      DO 301 I=NILP1,1
      AP(I,J)=AP(I,J)-SP(I,J)
 301  CONTINUE
      CALL TRIDAG(NILP1,NJBP1,NI,NJTM1,T,2,NJ,1)
      CALL TRIDA (NILP1,NJBP1,NI,NJTM1,T,NILP1,0,1,NJP1)
      CALL TRIDAX(NILP1,NJBP1,NI,NJTM1,T,2,NJ,1)
      CALL TRIDAY(NILP1,NJBP1,NI,NJTM1,T,NILP1,0,1,NJP1)
      DO 604 J=2,NJ
      T(NIP1,J)=T(NI,J)
 604  CONTINUE
      DO 605 I=NILP1,0
      T(I,NJB)=T(I,NJBP1)
      T(I,NJT)=T(I,NJTM1)
 605  CONTINUE
      RETURN
      END
      SUBROUTINE CALU
C     ********************
      COMMON/BL1/DX,DY,VOL,DTIME,XOY,YOX,VOLDT,THOT,PI
      COMMON/R4/ X(-60:90),Y(-60:90),DXX(-60:90),DYY(-60:90), DXXS(-60:90),DYYS(-60:90)
      COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
      COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,NJTP2,NILP2,NJBP2
      COMMON/BL12/ NWRITE,NTAPE,NTMAX0,NTREAL,TIME,SORSUM,ITER
      COMMON/BL16/ CONST1,CONST2,CONST3,RA,PR ,NT,U0,HPSY,CP0,COND0,VIS0,RHO0,HR,TR,TA,DTEMP
      COMMON/BL31/ TOD(-60:90,-60:90),ROD(-60:90,-60:90), UOD(-60:90,-60:90),VOD(-60:90,-60:90)
      COMMON/BL32/ T(-60:90,-60:90),R(-60:90,-60:90),U(-60:90,-60:90),V(-60:90,-60:90),P(-60:90,-60:90) ,PSI(-60:90,-60:90)
      COMMON/BL33/ TPD(-60:90,-60:90),RPD(-60:90,-60:90), UPD(-60:90,-60:90),VPD(-60:90,-60:90),
     &        PPD(-60:90,-60:90),PEQ(-60:90,-60:90)
      COMMON/BL34/ HEIGHT(-60:90,-60:90),SMP(-60:90,-60:90),SMPP(-60:90,-60:90), DU(-60:90,-60:90),DV(-60:90,-60:90),PP(-
     &        60:90,-60:90)
      COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90), AW(-60:90,-60:90),AN(-60:90,-60:90), AS(-60:90,-60:90),SP(-60:90,-
     &        60:90),SU(-60:90,-60:90),REQ(-60:90,-60:90)
      COMMON/BL37/ VIS(-60:90,-60:90),COND(-60:90,-60:90), CPM(-60:90,-60:90), RESORM(20),TERM(20)
C ***   CALCULATE COEFFICIENTS
      DO 100 J=NJBP1,NJTM1
      JP2=J+2
      JP1=J+1
      JM1=J-1
      JM2=J-2
      DO 100 I=NILP2,NI
      IF((I.GE.1 .AND. J.LE.1) .OR. (I.GE.1 .AND. J.GE.NJP1)) GOTO 100
```

```
      IP2=I+2
      IP1=I+1
      IM1=I-1
      IM2=I-2
      DXI=DXX(I)
      DXM1=DXX(IM1)
      DXM2=DXX(IM2)
      DXP1=DXX(IP1)
      DYJ=DYY(J)
      DYM1=DYY(JM1)
      DYP1=DYY(JP1)
      DXEE=DXXS(IP2)
      DXE=DXXS(IP1)
      DXW=DXXS(I)
      DXWW=DXXS(IM1)
      DYNN=DYYS(JP2)
      DYN=DYYS(JP1)
      DYS=DYYS(J)
      DYSS=DYYS(JM1)
      XOYN=DXW/DYN
      XOYS=DXW/DYS
      YOXE=DYJ/DXI
      YOXW=DYJ/DXM1
      VOL=DXW*DYJ
      VOLDT=VOL/DTIME
      GN= V(I,JP1)
      GNW=V(IM1,JP1)
      GS= V(I,J)
      GSW=V(IM1,J)
      GE=U(IP1,J)
      GP=U(I,J)
      GW=U(IM1,J)
      CN=(GN*DXM1+GNW*DXI)/(DXM1+DXI)*DXW
      CS=(GS*DXM1+GSW*DXI)/(DXM1+DXI)*DXW
      CE=.5*(GE+GP)*DYJ
      CW=.5*(GP+GW)*DYJ
      VISN1=XOYN*PR
      VISS1=XOYS*PR
      VISE1=YOXE*PR
      VISW1=YOXW*PR
      CEP=(ABS(CE)+CE)*DXI/DXW/16.
      CEM=(ABS(CE)-CE)*DXI/DXE/16.
      CWP=(ABS(CW)+CW)*DXM1/DXWW/16.
      CWM=(ABS(CW)-CW)*DXM1/DXW/16.
      CNP=(ABS(CN)+CN)*DYN/DYJ/16.
      CNM=(ABS(CN)-CN)*DYN/DYP1/16.
      CSP=(ABS(CS)+CS)*DYS/DYM1/16.
      CSM=(ABS(CS)-CS)*DYS/DYJ/16.
      IF (I.LT.NI) GOTO 801
      AEE=0.
      AEER=0.
      GOTO 802
  801 AEE=-CEM*DXI/DXP1
      AEER=AEE*UPD(IP2,J)
  802 CONTINUE
      IF (I.GT.3) GOTO 803
      AWW=0.
      AWWR=0.
      GOTO 804
  803 AWW=-CWP*DXM1/DXM2
      AWWR=AWW*UPD(IM2,J)
  804 CONTINUE
      IF (J.LT.NJ) GOTO 805
      ANN=0.
      ANNR=0.
```

```
      GOTO 806
 805 ANN=-CNM*DYN/DYNN
      ANNR=ANN*UPD(I,JP2)
 806 CONTINUE
      IF (J.GT.2) GOTO 807
      ASS=0.
      ASSR=0.
      GOTO 808
 807 ASS=-CSP*DYS/DYSS
      ASSR=ASS*UPD(I,JM2)
 808 CONTINUE
      AE(I,J)=-.5*CE+CEP+CEM*(1.+DXI/DXP1)+CWM*DXM1/DXI+VISE1
      AW(I,J)= .5*CW+CWP*(1.+DXM1/DXM2)+CWM+CEP*DXI/DXM1+VISW1
      AN(I,J)=-.5*CN+CNP+CNM*(1.+DYN/DYNN)+CSM*DYS/DYN+VISN1
      AS(I,J)= .5*CS+CSP*(1.+DYS/DYSS)+CSM+CNP*DYN/DYS+VISS1
      SP(I,J)=-VOLDT
      AP(I,J)=AE(I,J)+AW(I,J)+AN(I,J)+AS(I,J)+
     &       AEE+AWW+ANN+ASS
C ***   SU FROM NORMAL STRESS
      SU(I,J)=DYJ*(P(IM1,J)-P(I,J))
     &  +UOD(I,J)*VOLDT+AEER+AWWR+ANNR+ASSR
     &  +RA*PR*(T(IM1,J)*DXI+T(I,J)*DXM1)/(DXI+DXM1)*VOL
 100 CONTINUE
C ***   TAKE CARE OF B.C THR AE,AW,AN,AS,....
C ***   FLOOR AND CEILING
      DO 500 I=3,NI
      SP(I,2)=SP(I,2)-AS(I,2)
      SP(I,NJ)=SP(I,NJ)-AN(I,NJ)
      AS(I,2)=0.
      AN(I,NJ)=0.
 500 CONTINUE
C ***   RIGHT WALL
      DO 600 J=2,NJ
      SU(NI,J)=SU(NI,J)+AE(NI,J)*UPD(NIP1,J)
      AE(NI,J)=0.
 600 CONTINUE
C****   LEFT WALL
      DO 505 J=NJBP1,NJTM1
      SU(NILP2,J)=SU(NILP2,J)+AW(NILP2,J)*UPD(NILP1,J)
      AW(NILP2,J)=0.
 505 CONTINUE
C****** SOLID VERTICAL WALL
      DO 501 J=NJBP1,1
      AE(0,J)=0.
 501 CONTINUE
      DO 502 J=NJP1,NJTM1
      AE(0,J)=0.
 502 CONTINUE
C******* HORIZONTAL FICTIONAL WALLS
      DO 503 I=NILP1,1
      SP(I,NJBP1)=SP(I,NJBP1)-AS(I,NJBP1)
      SU(I,NJBP1)=SU(I,NJBP1)+2.*AS(I,NJBP1)*UPD(I,NJB)
      SP(I,NJTM1)=SP(I,NJTM1)-AN(I,NJTM1)
      SU(I,NJTM1)=SU(I,NJTM1)+2.*AN(I,NJTM1)*UPD(I,NJT)
      AS(I,NJBP1)=0.
      AN(I,NJTM1)=0.
 503 CONTINUE
C ***   ASSEMBLE COEFFICIENTS AND SOLVE DIFFERENCE EQUATIONS
      DO 301 J=NJBP1,NJTM1
      DO 301 I=NILP2,NI
      IF((I.GE.1 .AND. J.LE.1) .OR. (I.GE.1 .AND. J.GE.NJP1)) GOTO 301
      DYJ=DYY(J)
      AP(I,J)=AP(I,J)-SP(I,J)
      DU(I,J)=DYJ/AP(I,J)
 301 CONTINUE
```

```
      CALL TRIDAG(NILP2,NJBP1,NI,NJTM1,U,2,NJ,1)
      CALL TRIDA (NILP2,NJBP1,NI,NJTM1,U,NILP2,0,1,NJP1)
      CALL TRIDAX(NILP2,NJBP1,NI,NJTM1,U,2,NJ,1)
      CALL TRIDAY(NILP2,NJBP1,NI,NJTM1,U,NILP2,0,1,NJP1)
C     WRITE(6,999) ((AP(I,J),I=1,NIP1),J=1,NJP1)
 999  FORMAT (22F6.3)
      DO 704 J=NLBP1,NJTM1
      U(NILP1,J)=U(NILP2,J)
 704  CONTINUE
      DO 705 J=2,NJ
      U(NIP1,J)=U(NI,J)
 705  CONTINUE
      DO 706 I=NILP2,0
      U(I,NJB)=U(I,NJBP1)
      U(I,NJT)=U(I,NJTM1)
 706  CONTINUE
      RETURN
      END
      SUBROUTINE CALV
C     *******************************************************
      COMMON/BL1/DX,DY,VOL,DTIME,XOY,YOX,VOLDT,THOT,PI
      COMMON/R4/ X(-60:90),Y(-60:90),DXX(-60:90),DYY(-60:90), DXXS(-60:90),DYYS(-60:90)
      COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
      COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,NJTP2,NILP2,NJBP2
      COMMON/BL12/ NWRITE,NTAPE,NTMAX0,NTREAL,TIME,SORSUM,ITER
      COMMON/BL16/ CONST1,CONST2,CONST3,RA,PR
      COMMON/BL31/ TOD(-60:90,-60:90),ROD(-60:90,-60:90),
     &         UOD(-60:90,-60:90),VOD(-60:90,-60:90),
     &         POD(-60:90,-60:90)
      COMMON/BL32/ T(-60:90,-60:90),R(-60:90,-60:90),U(-60:90,-60:90),
     &         V(-60:90,-60:90),P(-60:90,-60:90)
     &         ,PSI(-60:90,-60:90)
      COMMON/BL33/ TPD(-60:90,-60:90),RPD(-60:90,-60:90),
     &         UPD(-60:90,-60:90),VPD(-60:90,-60:90),
     &         PPD(-60:90,-60:90),PEQ(-60:90,-60:90)
      COMMON/BL34/ HEIGHT(-60:90,-60:90),SMP(-60:90,-60:90),
     &         SMPP(-60:90,-60:90),
     &         DU(-60:90,-60:90),DV(-60:90,-60:90),PP(-60:90,-60:90)
      COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90),
     &         AW(-60:90,-60:90),AN(-60:90,-60:90),
     &         AS(-60:90,-60:90),SP(-60:90,-60:90),SU(-60:90,-60:90)
     &         ,REQ(-60:90,-60:90)
      COMMON/BL37/ VIS(-60:90,-60:90),COND(-60:90,-60:90),
     &         CPM(-60:90,-60:90),
     &        RESORM(20),TERM(20)
C ***    CALCULATE COEFFICIENTS
      NJTM2=NJT-2
      DO 100 J=NJBP2,NJTM1
      JP2=J+2
      JP1=J+1
      JM1=J-1
      JM2=J-2
      DO 100 I=NILP1,NI
      IF((I.GE.1 .AND. J.LE.1) .OR. (I.GE.1 .AND. J.GE.NJP1)) GOTO 100
      IP2=I+2
      IP1=I+1
      IM1=I-1
      IM2=I-2
      DXI=DXX(I)
      DXM1=DXX(IM1)
      DXP1=DXX(IP1)
      DYJ=DYY(J)
      DYM1=DYY(JM1)
      DYM2=DYY(JM2)
      DYP1=DYY(JP1)
```

```
      DXEE=DXXS(IP2)
      DXE=DXXS(IP1)
      DXW=DXXS(I)
      DXWW=DXXS(IM1)
      DYNN=DYYS(JP2)
      DYN=DYYS(JP1)
      DYS=DYYS(J)
      DYSS=DYYS(JM1)
      XOYN=DXI/DYJ
      XOYS=DXI/DYM1
      YOXE=DYS/DXE
      YOXW=DYS/DXW
      VOL=DYS*DXI
      VOLDT=VOL/DTIME
      GN=V(I,JP1)
      GP=V(I,J)
      GS=V(I,JM1)
      GE=U(IP1,J)
      GSE=U(IP1,JM1)
      GW=U(I,J)
      GSW=U(I,JM1)
      CN=.5*(GN+GP)*DXI
      CS=.5*(GP+GS)*DXI
      CE=(GE*DYJ+GSE*DYM1)/(DYJ+DYM1)*DYS
      CW=(GW*DYJ+GSW*DYM1)/(DYJ+DYM1)*DYS
      CEP=(ABS(CE)+CE)*DXE/DXI/16.
      CEM=(ABS(CE)-CE)*DXE/DXP1/16.
      CWP=(ABS(CW)+CW)*DXW/DXM1/16.
      CWM=(ABS(CW)-CW)*DXW/DXI/16.
      CNP=(ABS(CN)+CN)*DYJ/DYS/16.
      CNM=(ABS(CN)-CN)*DYJ/DYN/16.
      CSP=(ABS(CS)+CS)*DYM1/DYSS/16.
      CSM=(ABS(CS)-CS)*DYM1/DYS/16.
      VISE1=YOXE*PR
      VISW1=YOXW*PR
      VISN1=XOYN*PR
      VISS1=XOYS*PR
      IF (I.LT.NI) GOTO 801
      AEE=0.
      AEER=0.
      GOTO 802
  801 AEE=-CEM*DXE/DXEE
      AEER=AEE*VPD(IP2,J)
  802 CONTINUE
      IF (I.GT.2) GOTO 803
      AWW=0.
      AWWR=0.
      GOTO 804
  803 AWW=-CWP*DXW/DXWW
      AWWR=AWW*VPD(IM2,J)
  804 CONTINUE
      IF (J.LT.NJ) GOTO 805
      ANN=0.
      ANNR=0.
      GOTO 806
  805 ANN=-CNM*DYJ/DYP1
      ANNR=ANN*VPD(I,JP2)
  806 CONTINUE
      IF (J.GT.3) GOTO 807
      ASS=0.
      ASSR=0.
      GOTO 808
  807 ASS=-CSP*DYM1/DYM2
      ASSR=ASS*VPD(I,JM2)
  808 CONTINUE
```

```
      AE(I,J)=-.5*CE+CEP+CEM*(1.+DXE/DXEE)+CWM*DXW/DXE+VISE1
      AW(I,J)= .5*CW+CWP*(1.+DXW/DXWW)+CWM+CEP*DXE/DXW+VISW1
      AN(I,J)=-.5*CN+CNP+CNM*(1.+DYJ/DYP1)+CSM*DYM1/DYJ+VISN1
      AS(I,J)= .5*CS+CSP*(1.+DYM1/DYM2)+CSM+CNP*DYJ/DYM1+VISS1
      AP(I,J)=AE(I,J)+AW(I,J)+AN(I,J)+AS(I,J)+AEE+AWW+ANN+ASS
      SP(I,J)=-VOLDT
      SU(I,J)= AEER+AWWR+ANNR+ASSR
C
      SU(I,J)=SU(I,J)+DXI*(P(I,JM1)-P(I,J))
     & +VOD(I,J)*VOLDT
  100 CONTINUE
C ***  TAKE CARE OF B.C THR AE,AW,AN,AS,....
C ***  FLOOR AND CEILING
      DO 500 I=2,NI
      AS(I,3)=0.
      AN(I,NJ)=0
  500 CONTINUE
C ***  RIGHT WALL
      DO 600 J=3,NJ
      SP(NI,J)=SP(NI,J)-AE(NI,J)
      SU(NI,J)=SU(NI,J)+2.*AE(NI,J)*V(NIP1,J)
      AE(NI,J)=0.
  600 CONTINUE
C***  LEFT WALL
      DO 505 J=NJBP2,NJTM1
      SP(NILP1,J)=SP(NILP1,J)-AW(NILP1,J)
      SU(NILP1,J)=SU(NILP1,J)+2.*AW(NILP1,J)*VPD(NIL,J)
      AW(NILP1,J)=0.
  505 CONTINUE
C**  SOLID VERTICAL WALLS
      DO 501 J=NJBP2,1
      SP(1,J)=SP(1,J)-AE(1,J)
      AE(1,J)=0.
  501 CONTINUE
      DO 502 J=NJP2,NJT
      SP(1,J)=SP(1,J)-AE(1,J)
      AE(1,J)=0.
  502 CONTINUE
C**** FICTION HORIZONTAL WALL
      DO 503 I=NILP1,1
C     SP(I,NJBP2)=SP(I,NJBP2)-AS(I,NJNP2)*VPD(I,NJBP1)
      SU(I,NJBP2)=SU(I,NJBP2)+AS(I,NJBP2)*VPD(I,NJBP1)
      AS(I,NJBP2)=0.
C     SP(I,NJTM1)=SP(I,NJTM1)-AN(I,NJTM1)*VPD(I,NJT)
      SU(I,NJTM1)=SU(I,NJTM1)+AN(I,NJTM1)*VPD(I,NJT)
      AN(I,NJTM1)=0.
  503 CONTINUE
C ***  ASSEMBLE COEFFICIENTS AND SOLVE DIFFERENCE EQUATIONS
      DO 300 J=NJBP2,NJTM1
      DO 300 I=NILP1,NI
      IF((I.GE.1 .AND. J.LE.1) .OR. (I.GE.1 .AND. J.GE.NJP1)) GOTO 300
      DXI=DXX(I)
      AP(I,J)=AP(I,J)-SP(I,J)
      DV(I,J)=DXI/AP(I,J)
  300 CONTINUE
C ***  DV ON HORIZENTAL WALLS ARE ZERO
      DO 304 I=2,NI
      DV(I,NJP1)=0.
      DV(I,2)=0.
  304 CONTINUE
  999 FORMAT (22F6.3)
      DO 704 J=NJBP2,NJT
      V(NIL,J)=V(NILP1,J)
  704 CONTINUE
      DO 705 J=2,NJ
```

```
      V(NIP1,J)=V(NI,J)
705   CONTINUE
      DO 706 I=NILP1,0
      V(I,NJBP1)=V(I,NJBP2)
      V(I,NJTM2)=V(I,NJTM1)
706   CONTINUE
      RETURN
      END
C     ****************
      SUBROUTINE CALP
C     ****************
      COMMON/BL1/DX,DY,VOL,DTIME,XOY,YOX,VOLDT,THOT,PI
      COMMON/R4/ X(-60:90),Y(-60:90),DXX(-60:90),DYY(-60:90), DXXS(-60:90),DYYS(-60:90)
      COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
      COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,NJTP2,NILP2,NJBP2
      COMMON/BL12/ NWRITE,NTAPE,NTMAX0,NTREAL,TIME,SORSUM,ITER
      COMMON/BL16/ CONST1,CONST2,CONST3,RA,PR
     & ,NT,U0,H,UGRT,BUOY,
     & PSY,CP0,COND0,VIS0,RHO0,HR,TR,TA,DTEMP
      COMMON/BL31/ TOD(-60:90,-60:90),ROD(-60:90,-60:90),
     &        UOD(-60:90,-60:90),VOD(-60:90,-60:90),
     &        POD(-60:90,-60:90)
      COMMON/BL32/ T(-60:90,-60:90),R(-60:90,-60:90),U(-60:90,-60:90),
     &        V(-60:90,-60:90),P(-60:90,-60:90)
     &        ,PSI(-60:90,-60:90)
      COMMON/BL33/ TPD(-60:90,-60:90),RPD(-60:90,-60:90),
     &        UPD(-60:90,-60:90),VPD(-60:90,-60:90),
     &        PPD(-60:90,-60:90),PEQ(-60:90,-60:90)
      COMMON/BL34/ HEIGHT(-60:90,-60:90),SMP(-60:90,-60:90),
     &        SMPP(-60:90,-60:90),
     &        DU(-60:90,-60:90),DV(-60:90,-60:90),PP(-60:90,-60:90)
      COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90),AW(-60:90,-60:90),
     &        AN(-60:90,-60:90),
     &        AS(-60:90,-60:90),SP(-60:90,-60:90),SU(-60:90,-60:90)
     &        ,REQ(-60:90,-60:90)
      COMMON/BL37/ VIS(-60:90,-60:90),COND(-60:90,-60:90),
     &        CPM(-60:90,-60:90),
     &       RESORM(40),TERM(40)
C ***   CALCULATE COEFFICIENTS
      DO 100 J=NJBP1,NJTM1
      JP2=J+2
      JP1=J+1
      JM1=J-1
      JM2=J-2
      DO 100 I=NILP1,NI
      IF((I.GE.1 .AND. J.LE.1) .OR. (I.GE.1 .AND. J.GE.NJP1))GOTO 100
      IP2=I+2
      IP1=I+1
      IM1=I-1
      IM2=I-2
      DXI=DXX(I)
      DXM1=DXX(IM1)
      DXP1=DXX(IP1)
      DYJ=DYY(J)
      DYM1=DYY(JM1)
      DYP1=DYY(JP1)
      DXEE=DXXS(IP2)
      DXE=DXXS(IP1)
      DXW=DXXS(I)
      DXWW=DXXS(IM1)
      DYNN=DYYS(JP2)
      DYN=DYYS(JP1)
      DYS=DYYS(J)
      DYSS=DYYS(JM1)
      DYS=.5*(DYJ+DYM1)
```

```
      VOL=DYJ*DXI
      VOLDT=VOL/DTIME
      AN(I,J)=DXI*DV(I,JP1)
      AS(I,J)=DXI*DV(I,J)
      AE(I,J)=DYJ*DU(IP1,J)
      AW(I,J)=DYJ*DU(I,J)
      CN=V(I,JP1)*DXI
      CS=V(I,J)*DXI
      CE=U(IP1,J)*DYJ
      CW=U(I,J)*DYJ
      SMP(I,J)=-CE+CW-CN+CS
      SU(I,J)=SMP(I,J)
      SP(I,J)=0.
 100 CONTINUE
C ***    TAKE CARE OF B.C. THRU AN,AS,AE,AW,SP AND SU
C ***    FLOOR AND CEILING
      DO 500 I=2,NI
      AS(I,2)=0.
      AN(I,NJ)=0.
 500 CONTINUE
C ***    RIGHT WALL
      DO 611 J=2,NJ
      AW(NI,J)=AW(NI,J)-AE(NI,J)
      AE(NI,J)=0.
 611 CONTINUE
C*** LEFT WALL
      DO 505 J=NJBP1,NJTM1
      AE(NILP1,J)=AE(NILP1,J)-AW(NILP1,J)
      AW(NILP1,J)=0.
505  CONTINUE
C** SOLID VERTICAL WALLS
      DO 501 J=NJBP1,1
      AE(1,J)=0.
 501 CONTINUE
      DO 502 J=NJP1,NJTM1
      AE(1,J)=0.
 502 CONTINUE
C** HORIZONTAL FICTIONAL WALLS
      DO 503 I=NILP1,1
      AN(I,NJBP1)=AN(I,NJBP1)-AS(I,NJBP1)
      AS(I,NJTM1)=AS(I,NJTM1)-AN(I,NJTM1)
      AS(I,NJBP1)=0.
      AN(I,NJTM1)=0.
503  CONTINUE
C ***    ASSEMBLE COEFFICIENTS AND SOLVE DIFFERENCE EQUATIONS
      DO 300 J=NJBP1,NJTM1
      DO 300 I=NILP1,NJ
      AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP(I,J)
 300 CONTINUE
       CALL TRIDA (NILP1,NJBP1,NI,NJTM1,PP,NILP1,0,2,NJ)
      CALL TRIDAX(NILP1,NJBP1,NI,NJTM1,PP,2,NJ,0)
C     CALL TRIDAY(NILP1,NJBP1,NI,NJTM1,PP,NILP1,0,2,NJ)
C ***    CORRECT VELOCITIES AND PRESSURE
C ***    CORRECTION FOR VELOCITY U
      DO 600 I=3,NI
      IM1=I-1
      DO 600 J=2,NJ
      U(I,J)=U(I,J)+DU(I,J)*(PP(IM1,J)-PP(I,J))
 600 CONTINUE
C ***    CORRECTION FOR VELOCITY V
      DO 603 J=3,NJ
      JM1=J-1
      DO 603 I=2,NI
      V(I,J)=V(I,J)+DV(I,J)*(PP(I,JM1)-PP(I,J))
 603 CONTINUE
```

```
C ***  CORRECTION FOR PRESSURE P
      DO 606 J=2,NJ
      DO 606 I=2,NI
      P(I,J)=P(I,J)+PP(I,J)
      PP(I,J)=0.
  606 CONTINUE
C ***  RECALCULATE THE ERROR SOURCE AFTER CORRECTIONS OF U, V, P
      SORSUM=0.
      RESORM(ITER)=0.
      DO 700 J=2,NJ
      JP2=J+2
      JP1=J+1
      JM1=J-1
      JM2=J-2
      DO 700 I=2,NI
      IP2=I+2
      IP1=I+1
      IM1=I-1
      IM2=I-2
      DXI=DXX(I)
      DXM1=DXX(IM1)
      DXP1=DXX(IP1)
      DYJ=DYY(J)
      DYM1=DYY(JM1)
      DYP1=DYY(JP1)
      DXEE=DXXS(IP2)
      DXE=DXXS(IP1)
      DXW=DXXS(I)
      DXWW=DXXS(IM1)
      DYNN=DYYS(JP2)
      DYN=DYYS(JP1)
      DYS=DYYS(J)
      DYSS=DYYS(JM1)
      VOL=DYJ*DXI
      VOLDT=VOL/DTIME
      CN=V(I,JP1)*DXI
      CS=V(I,J)*DXI
      CE=U(IP1,J)*DYJ
      CW=U(I,J)*DYJ
      SMP(I,J)=-CE+CW-CN+CS
C ***  SORSUM IS ACTUAL MASS INCREASE OR DECREASE FROM CONTINUITY
C      EQUATUON , THIS WILL COMPARE TO SOURCE
      SORSUM=SORSUM+SMP(I,J)
C ***  RESORM IS SUM OF THE ABSOLUTE VALUE OF SMP(I,J) FOR ANY I
      RESORM(ITER)=RESORM(ITER)+ABS(SMP(I,J))
  700 CONTINUE
      RETURN
      END
      SUBROUTINE TRIDA(IBEGIN,JSTART,IEND,JSTOP,PHI,IFROM,ITO,J3,J4)
C     ****************************************************************
      COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
      COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,N
      COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90),
     &         AW(-60:90,-60:90),AN(-60:90,-60:90),
     &         AS(-60:90,-60:90),SP(-60:90,-60:90),SU(-60:90,-60:90)
     &         ,REQ(-60:90,-60:90)
      COMMON/BL41/ A(-30:100),B(-30:100),C(-30:100),D(-30:100)
      DIMENSION PHI(-60:90,-60:90)
C ***  COMMENCE S-N TRAVERSE
      DO 100 J=JSTART,JSTOP
      JM1=J-1
      JP1=J+1
C ***  DEFINE ISTART AND ISTOP
      IF((J .LE. J3) .OR. (J .GE. J4)) GO TO 201
      GO TO 202
```

```
 201 ISTART=IFROM
   ISTOP=ITO
   GO TO 203
 202 ISTART=IBEGIN
   ISTOP=IEND
 203 CONTINUE
   ISTM1=ISTART-1
   A(ISTM1)=0.
   C(ISTM1)=0.
C ***  COMMENCE W-E SWEEP
   DO 101 I=ISTART,ISTOP
   IM1=I-1
   A(I)=AE(I,J)
   B(I)=AW(I,J)
   C(I)=AN(I,J)*PHI(I,JP1)+AS(I,J)*PHI(I,JM1)+SU(I,J)
   D(I)=AP(I,J)
C ***  CALCULATE COEFFICIENT OF RECURRENCE FORMULA
   TERM=1./(D(I)-B(I)*A(IM1))
   A(I)=A(I)*TERM
   C(I)=(C(I)+B(I)*C(IM1))*TERM
 101 CONTINUE
C ***  OBTAIN NEW PHI'S
   PHI(ISTOP,J)=C(ISTOP)
   ISTAR1=ISTART+1
   DO 102 II=ISTAR1,ISTOP
   I=ISTOP+ISTART-II
   PHI(I,J)=A(I)*PHI(I+1,J)+C(I)
 102 CONTINUE
 100 CONTINUE
   RETURN
   END
   SUBROUTINE TRIDAG(ISTART,JBEGIN,ISTOP,JEND,PHI,JFROM,JTO,I3)
C  ****************************************************************
   COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
   COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,NJTP2,NILP2,NJBP2
   COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90),
  &         AW(-60:90,-60:90),AN(-60:90,-60:90),
  &         AS(-60:90,-60:90),SP(-60:90,-60:90),SU(-60:90,-60:90)
  &         ,REQ(-60:90,-60:90)
   COMMON/BL41/ A(-30:100),B(-30:100),C(-30:100),D(-30:100)
   DIMENSION PHI(-60:90,-60:90)
C ***  COMMENCE W-E SWEEP
   DO 100 I=ISTART,ISTOP
   IP1=I+1
   IM1=I-1
C ***  DEFINE JSTART AND JSTOP
   IF(I .GE. I3) GO TO 201
   GO TO 202
 201 JSTART=JFROM
   JSTOP=JTO
   GO TO 203
 202 JSTART=JBEGIN
   JSTOP=JEND
 203 CONTINUE
   JSTM1=JSTART-1
   A(JSTM1)=0.
   C(JSTM1)=0.
C ***  COMMENCE S-N TRAVERSE
   DO 101 J=JSTART,JSTOP
   JM1=J-1
   A(J)=AN(I,J)
   B(J)=AS(I,J)
   C(J)=AE(I,J)*PHI(IP1,J)+AW(I,J)*PHI(IM1,J)+SU(I,J)
   D(J)=AP(I,J)
C ***  CALCULATE COEFFICIENT OF RECURRENCE FORMULA
```

```
      TERM=1./(D(J)-B(J)*A(JM1))
      A(J)=A(J)*TERM
      C(J)=(C(J)+B(J)*C(JM1))*TERM
  101 CONTINUE
C *** OBTAIN NEW PHI'S
      PHI(I,JSTOP)=C(JSTOP)
      JSTAR1=JSTART+1
      DO 102 JJ=JSTAR1,JSTOP
      J=JSTOP+JSTART-JJ
      PHI(I,J)=A(J)*PHI(I,J+1)+C(J)
  102 CONTINUE
  100 CONTINUE
      RETURN
      END
      SUBROUTINE TRIDAX(ISTART,JBEGIN,ISTOP,JEND,PHI,JFROM,JTO,I3)
C     ****************************************************************
      COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
      COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,NJTP2,NILP2,NJBP2
      COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90),
     &        AW(-60:90,-60:90),AN(-60:90,-60:90),
     &        AS(-60:90,-60:90),SP(-60:90,-60:90),SU(-60:90,-60:90),
     &        REQ(-60:90,-60:90)
      COMMON/BL41/ A(-30:100),B(-30:100),C(-30:100),D(-30:100)
      DIMENSION PHI(-60:90,-60:90)
C *** COMMENCE W-E SWEEP
      DO 100 I=ISTART,ISTOP
      IM1=I-1
      IP1=I+1
C *** DEFINE JSTART AND JSTOP
      IF(I .GE. I3) GOTO 201
      GOTO 202
  201 JSTART=JFROM
      JSTOP=JTO
      GOTO 203
  202 JSTART=JBEGIN
      JSTOP=JEND
  203 CONTINUE
      JSTOP1=JSTOP+1
      B(JSTOP1)=0.
      C(JSTOP1)=0.
C *** COMMENCE S-N TRAVERSE
      DO 101 JJ=JSTART,JSTOP
      J=JSTOP+JSTART-JJ
      JP1=J+1
      A(J)=AN(I,J)
      B(J)=AS(I,J)
      C(J)=AE(I,J)*PHI(IP1,J)+AW(I,J)*PHI(IM1,J)+SU(I,J)
      D(J)=AP(I,J)
C *** CALCULATE COEFFICIENT OF RECURRENCE FORMULA
      TERM=1./(D(J)-A(J)*B(JP1))
      B(J)=B(J)*TERM
      C(J)=(C(J)+A(J)*C(JP1))*TERM
  101 CONTINUE
C *** OBTAIN NEW PHI'S
      PHI(I,JSTART)=C(JSTART)
      JSTAR1=JSTART+1
      DO 102 J=JSTAR1,JSTOP
      PHI(I,J)=B(J)*PHI(I,J-1)+C(J)
  102 CONTINUE
  100 CONTINUE
      RETURN
      END
      SUBROUTINE TRIDAY(IBEGIN,JSTART,IEND,JSTOP,PHI,IFROM,ITO,J3,J4)
C     ****************************************************************
      COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
```

```
      COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,NJTP2,NILP2,NJBP2
      COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90),
     &        AW(-60:90,-60:90),AN(-60:90,-60:90),
     &        AS(-60:90,-60:90),SP(-60:90,-60:90),SU(-60:90,-60:90),
     &        REQ(-60:90,-60:90)
      COMMON/BL41/ A(-30:100),B(-30:100),C(-30:100),D(-30:100)
      DIMENSION PHI(-60:90,-60:90)
C ***   COMMENCE S-N TRAVERSE
      DO 100 JJ=JSTART,JSTOP
      J=JSTART+JSTOP-JJ
      JM1=J-1
      JP1=J+1
C ***   DEFINE ISTART AND ISTOP
       IF (( J.LE.J3) .OR. (J.GE. J4)) GOTO 201
       GOTO 202
  201 ISTART=IFROM
      ISTOP=ITO
      GOTO 203
  202 ISTART=IBEGIN
      ISTOP=IEND
  203 CONTINUE
      ISTM1=ISTART-1
      A(ISTM1)=0.
      C(ISTM1)=0.
C ***   COMMENCE W-E SWEEP
      DO 101 I=ISTART,ISTOP
      IM1=I-1
      A(I)=AE(I,J)
      B(I)=AW(I,J)
      C(I)=AN(I,J)*PHI(I,JP1)+AS(I,J)*PHI(I,JM1)+SU(I,J)
      D(I)=AP(I,J)
C ***   CALCULATE COEFFICIENT OF RECURRENCE FORMULA
      TERM=1./(D(I)-B(I)*A(IM1))
      A(I)=A(I)*TERM
      C(I)=(C(I)+B(I)*C(IM1))*TERM
  101 CONTINUE
C ***   OBTAIN NEW PHI'S
      PHI(ISTOP,J)=C(ISTOP)
      ISTAR1=ISTART+1
      DO 102 II=ISTAR1,ISTOP
      I=ISTOP+ISTART-II
      PHI(I,J)=A(I)*PHI(I+1,J)+C(I)
  102 CONTINUE
  100 CONTINUE
      RETURN
      END
C *** ******************************************
      SUBROUTINE NU(RNU,RNUC,RNUH)
C *** ******************************************
      COMMON/BL1/DX,DY,VOL,DTIME,XOY,YOX,VOLDT,THOT,PI
      COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
      COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NITP1,NJTP2,NILP2,NJBP2
      COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90),
     &        AW(-60:90,-60:90),AN(-60:90,-60:90),
     &        AS(-60:90,-60:90),SP(-60:90,-60:90),SU(-60:90,-60:90)
     &        ,REQ(-60:90,-60:90)
      COMMON/BL12/ NWRITE,NTAPE,NTMAX0,NTREAL,TIME,SORSUM,ITER
      COMMON/BL16/ CONST1,CONST2,CONST3,RA,PR
     & ,NT,U0,H,UGRT,BUOY,
     & PSY,CP0,COND0,VIS0,RHO0,HR,TR,TA,DTEMP
      COMMON/BL32/ T(-60:90,-60:90),R(-60:90,-60:90),U(-60:90,-60:90),
     &        V(-60:90,-60:90),P(-60:90,-60:90)
     &        ,PSI(-60:90,-60:90)
      COMMON/BL37/ VIS(-60:90,-60:90),COND(-60:90,-60:90),
     &        CPM(-60:90,-60:90),RESORM(20),TERM(20)
```

```
      COMMON/R4/ X(-60:90),Y(-60:90),DXX(-60:90),DYY(-60:90),
     &          DXXS(-60:90),DYYS(-60:90)
      DIMENSION RNU(11)
      DO 100 K=1,11
      RNU(K)=0.
  100 CONTINUE
      RNUH=0.
      RNUC=0.
      DT=T(2,NJP1)-1.
      DO 20 I=2,NI
      Y1=DYYS(2)/2.
      Y2=Y1+DYYS(3)
      DTDYC=(Y2*Y2*(T(I,2)-T(I,1))-Y1*Y1*(T(I,3)-T(I,1)))/Y2
     &  /(Y2-Y1)/Y1
      Y1=DYYS(NJP1)/2.
      Y2=Y1+DYYS(NJ)
      DTDYH=-(Y2*Y2*(T(I,NJ)-T(I,NJP1))-Y1*Y1*(T(I,NJM1)-
     &     T(I,NJP1)))/Y2/(Y2-Y1)/Y1
      IF(I.EQ.20) GOTO 999
      GOTO 1000
 999  XNU=DTDYH*DXX(I)
 1000 DTDYC1=2.*(T(I,2)-T(I,1))/DYYS(2)
      DTDYH1=2.*(T(I,NJP1)-T(I,NJ))/DYYS(NJP1)
      RNU(10)=RNU(10)+DTDYC1*DXX(I)
      RNU(11)=RNU(11)+DTDYH1*DXX(I)
      RNUC=RNUC+DTDYC*DXX(I)
      RNUH=RNUH+DTDYH*DXX(I)
      DO 10 K=1,9
      J=2*K+2
      JM1=J-1
      JP1=J+1
      GS= V(I,J)*DXX(I)
      GSP=(GS+ABS(GS))*DYYS(J)/DYY(JM1)
      GSM=(GS-ABS(GS))*DYYS(J)/DYY(J)
      Q=.5*(T(I,J)+T(I,JM1))*GS
     &  -1./16.*((T(I,J)-T(I,JM1))
     &  -(T(I,JM1)-T(I,J-2))*DYYS(J)/DYYS(JM1)
     &  )*GSP
      Q=Q-1./16.*((T(I,JP1)-T(I,J))*DYYS(J)
     &  /DYYS(JP1)-(T(I,J)-T(I,JM1)))*GSM
      DTDY=(T(I,J)-T(I,JM1))/DYYS(J)
      DQ=(Q-DTDY*DXX(I))
      RNU(K)=RNU(K)-DQ
   10 CONTINUE
C     DLOC(I)=DTDYH
   20 CONTINUE
      RETURN
      END
      BLOCK DATA
C     **********
      LOGICAL*1 LBAND
      COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
      COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,NJTP2,NILP2,NJBP2
      COMMON/BL16/ CONST1,CONST2,CONST3,RA,PR
     & ,NT,U0,H,UGRT,BUOY,
     & PSY,CP0,COND0,VIS0,RHO0,HR,TR,TA,DTEMP
      COMMON/RL/ LBAND(9)
      DATA NIP2,NJP2,NIP1,NJP1,NI,NJ,NIM1,NJM1/43,43,42,42,41,41,40,40/
      END
C     ****************
      SUBROUTINE CALPL
C     ****************
      COMMON/R4/ X(-60:90),Y(-60:90),DXX(-60:90),DYY(-60:90),
     &          DXXS(-60:90),DYYS(-60:90)
      COMMON/BL32/ T(-60:90,-60:90),R(-60:90,-60:90),U(-60:90,-60:90),
```

```
     &      V(-60:90,-60:90),P(-60:90,-60:90),
     &      PSI(-60:90,-60:90)
      COMMON/BL16/ CONST1,CONST2,CONST3,RA,PR
     & ,NT,U0,H,UGRT,BUOY,
     & PSY,CP0,COND0,VIS0,RHO0,HR,TR,TA,DTEMP
      COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
      COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,NJTP2,NILP2,NJBP2
      COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90),AW(-60:90,-60:90),
     &      AN(-60:90,-60:90),
     &      AS(-60:90,-60:90),SP(-60:90,-60:90),SU(-60:90,-60:90)
     &      ,REQ(-60:90,-60:90)
      DIMENSION UU(-60:90,-60:90),VV(-60:90,-60:90)
      DO 307 I=1,NIP1
      DO 307 J=1,NJP1
      UU(I,J)=0.
      VV(I,J)=0.
      PSI(I,J)=0.
  307 CONTINUE
      DO 23 I=1,NIP1
      DO 23 J=1,NJP1/2
C     T(I,J)=2.0+DTEMP-T(NIP1+1-I,NJP1+1-J)
   23 CONTINUE
      DO 24 I=1,NIP1
      DO 24 J=2,NJP1/2+1
C     V(I,J)=-V(NIP1+1-I,NJP1+2-J)
   24 CONTINUE
      DO 25 I=2,NIP1
      DO 25 J=1,NJP1/2
C     U(I,J)=-U(NIP1+2-I,NJP1+1-J)
   25 CONTINUE
C*****CALCULATE STREAM FUNCTION DISTRIBUTIONS
      DO 701 I=2,NI
      DO 701 J=2,NJ
      PSI(I,J)=REQ(I,2)*(U(I,2)+U(I+1,2))*DYY(2)
       IF(J.LT.3) GO TO 701
        DO 702 K=3,J
  702 PSI(I,J)=PSI(I,J)+(REQ(I,K)*(U(I,K)+U(I+1,K))*DYY(K)
     & +REQ(I,K-1)*(U(I,K-1)+U(I+1,K-1))*DYY(K-1))*0.5E0
  701 CONTINUE
      DO 1010 J=2,NJP1
      PSI(1,J)=PSI(2,J)
      PSI(NIP1,J)=PSI(NI,J)
 1010 CONTINUE
       DO 723 J=2,NJP1
      DO 723 I=2,NI
      PSI(I,J)=0.5E0*PSI(I,J)
  723 CONTINUE
      VMAX=0.
      DO 501 I=2,NI
      VV(I,1)=0.
      VV(I,NJP1)=0.
      DO 502 J=2,NJ
      VV(I,J)=((V(I,J)+V(I,J+1))*DXX(I-1)+(V(I-1,J)+V(I-1,J+1))*DXX(I))/
     & (2.*(DXX(I-1)+DXX(I)))
      UU(I,J)=U(I,J)
      VT=SQRT(UU(I,J)**2+VV(I,J)**2)
      IF (VT.GT.VMAX) VMAX=VT
  502 CONTINUE
  501 CONTINUE
      WRITE (6,1)
C     DEFINE LEFT  AND RIGHT WALLS FOR V
      DO 503 J=2,NJ
      VV(2,J)=VV(3,J)
      VV(NIP1,J)=VV(NI,J)
  503 CONTINUE
```

```
      WRITE (6,1)
      GR=0.001
      REL=0.001
      FRA=1.
c     CALL CALCOM(UU,VV,T,PSI,VMAX,REL,H,U0,NI,NJ,FRA,PSY)
      RETURN
      END
C     ******************
      SUBROUTINE  GRID
C     ******************
      COMMON/R4/ X(-60:90),Y(-60:90),DXX(-60:90),DYY(-60:90)
     &         ,DXXS(-60:90),DYYS(-60:90)
      COMMON/BL1/DX,DY,VOL,DTIME,XOY,YOX,VOLDT,THOT,PI
      COMMON/BL7/NI,NIP1,NIP2,NIM1,NJ,NJP1,NJP2,NJM1
      COMMON/BL8/NIL,NILP1,NJB,NJBP1,NJT,NJTM1,NJTP1,NJTP2,NILP2,NJBP2
      COMMON/BL12/ NWRITE,NTAPE,NTMAX0,NTREAL,TIME,SORSUM,ITER
      COMMON/BL16/ CONST1,CONST2,CONST3,RA,PR
     & ,NT,U0,H,UGRT,BUOY,
     & PSY,CP0,COND0,VIS0,RHO0,HR,TR,TA,DTEMP
      COMMON/BL31/ TOD(-60:90,-60:90),ROD(-60:90,-60:90),
     &         UOD(-60:90,-60:90),VOD(-60:90,-60:90),
     &         POD(-60:90,-60:90)
      COMMON/BL32/ T(-60:90,-60:90),R(-60:90,-60:90),U(-60:90,-60:90),
     &         V(-60:90,-60:90),P(-60:90,-60:90),
     &         PSI(-60:90,-60:90)
      COMMON/BL33/ TPD(-60:90,-60:90),RPD(-60:90,-60:90),
     &         UPD(-60:90,-60:90),VPD(-60:90,-60:90),
     &         PPD(-60:90,-60:90),PEQ(-60:90,-60:90)
      COMMON/BL34/ HEIGHT(-60:90,-60:90),SMP(-60:90,-60:90),
     &         SMPP(-60:90,-60:90),
     &         DU(-60:90,-60:90),DV(-60:90,-60:90),PP(-60:90,-60:90)
      COMMON/BL36/AP(-60:90,-60:90),AE(-60:90,-60:90),AW(-60:90,-60:90),
     &         AN(-60:90,-60:90),
     &         AS(-60:90,-60:90),SP(-60:90,-60:90),SU(-60:90,-60:90)
     &         ,REQ(-60:90,-60:90)
      COMMON/BL37/ VIS(-60:90,-60:90),COND(-60:90,-60:90),
     &         CPM(-60:90,-60:90),RESORM(20),TERM(20)
      COMMON/BL40/ ASX,ASY
C     DEFINE ASPECT RATIO
      ASX=1.0
      ASY=1.00
C ***  NX1,NY1   THE POINT IN BOUNDARY LAYER
C     WRITE(11,1111)ASX,ASY
C111  FORMAT(1X,'ASX=',F5.2,1X,'ASY=',F5.2)
C ***  NX2,NY2   THE POINT OUT BOUNDARY PAYER
      NY1=5
      NY2=NJM1/2-NY1
      NX1=5
      NX2=NIM1/2-NX1
C ***  DELX1,DELY1   BOUNDARY LAYER THICHNESS ASSUMED
C ***  DELX2,DELY2   OUT OF BOUNDARY LAYER
      DELY1=0.0650
      DELY2=(.5-DELY1)*ASY
      DELX1=0.2500
      DELY1=DELY1*ASY
      DELX2=0.5*ASX-DELX1*ASX
      DELX1=DELX1*ASX
      NI1=NIP1/2+1
      NI2=NI1+1
      X(2)=0.
      DO 11 I=3,NI1
      DELX=DELX1/NX1
      IF (I.GE.(NX1+3)) DELX=DELX2/NX2
      X(I)=X(I-1)+DELX
   11 CONTINUE
```

```
      DO 10 I=NI2,NIP1
      K=NIP1-I+2
      X(I)=1.0*ASX-X(K)
   10 CONTINUE
      X(1)=-X(3)
      X(NIP2)=2.*X(NIP1)-X(NI)
C     Y COORDINATE
      NJ1=NJP1/2+1
      NJ2=NJ1+1
      Y(2)=0.
      DO 21 J=3,NJ1
      DELY=DELY1/NY1
      IF (J.GE.(NY1+3)) DELY=DELY2/NY2
      Y(J)=Y(J-1)+DELY
   21 CONTINUE
      DO 20 J=NJ2,NJP1
      K=NJP1-J+2
      Y(J)=1.0*ASY-Y(K)
   20 CONTINUE
      Y(1)=-Y(3)
      Y(NJP2)=2.*Y(NJP1)-Y(NJ)
      WRITE (6,103) ASX,ASY
  103 FORMAT (2X,'  ASX= ',F8.5 ,'   ASY=',F8.5)
C     ********   UNIFORM GRID
      DX=1./FLOAT(NIM1)*ASX
      DY=1./FLOAT(NJM1)*ASY
      DO 5 I=NIL,NIP2
      X(I)=DX*(I-2)
    5 CONTINUE
      DO 6 J=NJB,NJTP2
      Y(J)=DY*(J-2)
    6 CONTINUE
C     ***** END OF UNIFORM GRID
      DO 9 I=NIL,NIP1
      IP1=I+1
      DXX(I)=X(IP1)-X(I)
    9 CONTINUE
      DXX(NIP2)=DXX(NIP1)
      DO 17 I=NILP1,NIP2
      IM1=I-1
      DXXS(I)=.5*(DXX(I)+DXX(IM1))
   17 CONTINUE
      DXXS(NIL)=DXXS(NILP1)
      DO 14 J=NJB,NJT
      JP1=J+1
      DYY(J)=Y(JP1)-Y(J)
   14 CONTINUE
      DYY(NJTP2)=DYY(NJTP1)
      DO 15 J=NJBP1,NJTP2
      JM1=J-1
      DYYS(J)=.5*(DYY(J)+DYY(JM1))
   15 CONTINUE
      DYYS(NJB)=DYYS(NJBP1)
      DO 13 I=NIL,60
   13 CONTINUE
      RETURN
      END
```

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