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ON TRUSS OPTIMIZATION BY
A HOMOGENIZATION METHOD

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A HOMOGENIZATION METHOD

By

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ABSTRACT

ON TRUSS OPTIMIZATION BY A HOMOGENIZATION METHOD

By

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In the solution of the truss topology optimization problem using traditional methods, it is necessary to specify *a-priori* the number of bars and connectivity of admissible layouts. Experience and intuition are necessary when using these methods to ensure that the optimum structure is included in the set of admissible solutions. A new approach to the truss topology problem is possible using a homogenization method. With this method, optimum two-dimensional structures are generated that often appear 'truss-like' in shape. This is accomplished without initial specification of admissible topologies. This thesis develops a strategy based on traditional methods and uses it to show that the solutions obtained by the homogenization method are equivalent to truss structures of optimum topology.

To my wife, Janet

ACKNOWLEDGMENTS

I would like to thank Dr. Alejandro R. Diaz, my major professor, for his assistance throughout my graduate studies.

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CHAPTER I

INTRODUCTION

A problem that has often been considered in the literature is the design of truss structures that are, in some sense, optimum. A truss is a rigid body composed of a number of members, or bars, fastened together at their ends. The truss is built to support or transfer forces and withstand safely the loads applied to it. It is assumed that the weight of the bars and the loads and reactions in the truss act only at the joints between bars, and that the joints are smooth pins. From this it follows that any forces in the bars will be tensile or compressive. A typical truss structure is shown in Figure 1.

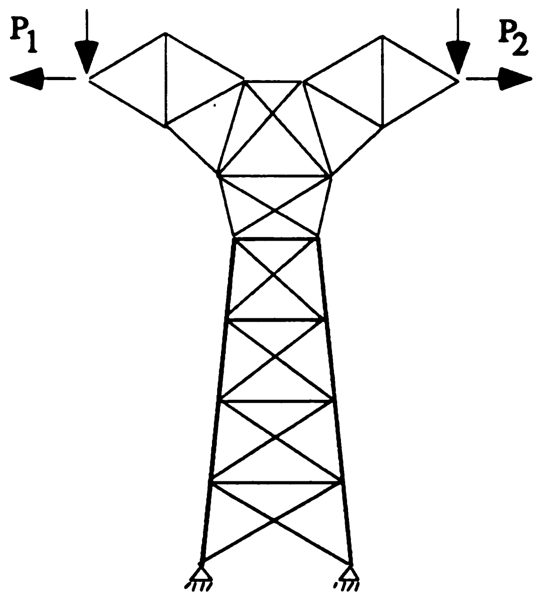


Figure 1. A Typical Truss

Variables included in truss design include: number of bars and their cross-sectional area, connectivity of the bars, and location of the joints, or nodes. An important consideration in designing an optimum truss is the topology of the structure (number of bars and connectivity). Traditional solution methods require that a set of admissible topologies be defined from which the optimum structure is chosen. Some of these methods use the topology of structures that have been used in the past to satisfy the same or similar loading conditions; changes are then made around the existing design in an effort to find an optimum structure. Experience and intuition are necessary when solving for optimum topology using traditional methods in order to ensure that the optimum solution is included in the set of admissible topologies.

A homogenization method [1] has been developed to solve two-dimensional shape optimization problems based on the equations of plane elasticity. Knowing only the extent of the design domain and a total allowable volume of the final structure, along with boundary conditions, the method places the material in a pattern that achieves minimum compliance. In many cases, the shape that results resembles a truss structure. This observation indicates that it may be possible to use the homogenization method to solve the optimum truss topology problem.

The purpose of this work is to show that the solutions obtained by the homogenization method are equivalent to truss structures of optimum topology. This will be accomplished by studying a representative set of examples. To make this comparison possible, a truss optimization strategy along the lines of more traditional methods will be developed.

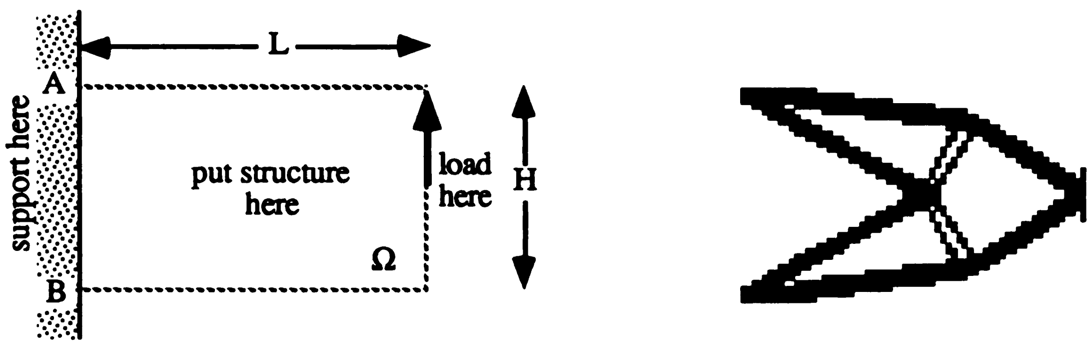
The next two sections of this chapter include a description of how the homogenization method works, and a literature survey on traditional truss optimization methods. In Chapter 2, a truss optimization strategy will be developed and characteristics of its optimum solutions will be discussed. In Chapter 3, example problems solved using the homogenization method will be presented. To show solutions to these problems can be considered trusses of optimum topology, the same problems will be solved using the strategy developed in Chapter 2. Characteristics of the solutions from the two approaches will be discussed. In Chapter 4, conclusions will be drawn based on the results.

1.1 Shape Optimization Using Homogenization

The optimum shape homogenization program (OSHP) uses a homogenization method to find the shape of a plane, linearly elastic structure of minimum mean compliance for a given solid volume. This section outlines how homogenization theory is applied in the OSHP.

Consider the domain Ω and the prescribed boundary conditions in Figure 2a. The boundary conditions would include zero displacement along the left side and the load P applied along a small portion of the right side.

The material properties of the structure within the domain are characterized by the elasticity tensor $\mathbf{E} = [E_{ijkl}]$, which is a function of the location $\mathbf{x}$ in the domain. In the homogenization method, the material properties can be characterized by a microstructure of cells. Within each cell is a rectangular hole, the size of which determines the properties of the cell. Such a cell is shown in Figure 3. For values of a=b=1, the area occupied by the cell would be a void; for a=b=0, the area would be a



a) Problem statement

b) Optimum shape

Figure 2. Shape optimization by homogenization

solid. For intermediate values ($0 \leq a, b \leq 1$) the cell would have an intermediate level of material density.

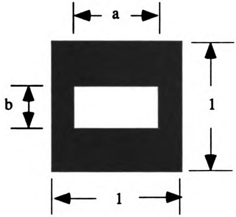


Figure 3. Cell for homogenization

It is possible to determine a homogenized elasticity tensor, E^H, based on the location in the domain ($\mathbf{x}$) and the properties of the cell at that location. This value of E^H is dependent upon the hole size. Details of this homogenization process are found in Reference [2].

The shape optimization problem can then be written as

Find $a(\mathbf{x})$, $b(\mathbf{x})$ that

minimize $\quad C(\mathbf{u}) \qquad (1)$

subject to $\quad \int_{\Omega} (1-ab)d\mathbf{x} \leq V_o \qquad (2)$

$0 \leq a \leq 1 \quad 0 \leq b \leq 1 \qquad (3)$

and appropriate equilibrium equations.

The compliance (C) is a function of the displacement field (**u**), the solution to the equilibrium equations, in this case, the equations of plane elasticity for the given homogenized elasticity tensor. Inequality 2 is the volume constraint which states that the total volume of material be less than a specified maximum (V_o).

The OSHP solution process follows these steps (Reference [2]):

1) The design domain and boundary conditions are prescribed and the maximum volume of the solution is specified (typically 10%-30% of the domain).

2) The domain is discretized into rectangular finite elements. The material properties of each element are characterized by one cell.

3) E^H is calculated for selected hole sizes (a,b) and then an approximation of the function over the interval $0 \leq a, b \leq 1$ is made.

4) The minimum compliance problem in Equations 1 through 3 is solved using an optimality criteria approach. In this solution process, the equilibrium equations are solved using a finite element method. The optimality criteria approach is used because there are generally a large number of design variables. Figure 2b shows an OSHP solution to the problem posed in Figure 2a.

1.2 Literature Survey of Traditional Truss Optimization Methods

The early work of Michell [3] is probably the best known analytical approach to the truss optimization problem. Michell was able to find minimum weight truss layouts for several loadings. Some of his designs required an infinite number of members and joints, which makes them impractical for application.

Truss optimization based on numerical methods is more recent, and takes advantage of the speed and storage capability of the digital computer and the use of finite element analysis. One of the earlier works using numerical methods was done by Schmit [4]. With member areas as design variables, Schmit used non-linear programming techniques to minimize the weight of a three-bar indeterminate truss subject to stress constraints.

When topology and member sizes are considered, three different approaches have been followed. The first approach involves specifying possible nodal locations (i.e., location of member joints) and using linear programming to find the member areas and connectivity that produce the minimum weight truss. This approach was taken by Dorn *et. al.* [5] for single loading conditions and by Dobbs and Felton [6] for multiple loading conditions. Similarly, Sheu and Schmit [7] presented an approach that began with a set of nodes which were completely interconnected with members. From this configuration it was possible to identify subsets of admissible topologies which could be candidates for the optimum solution. Bounding techniques, which determined upper and lower limits on the minimum weight, were used to eliminate many of the possible topologies. Detailed optimization of the remaining topologies was then performed to produce the minimum weight structure.

The second approach involves considering both nodal locations and member areas as design variables. Pedersen [8] prescribed an initial topology with a fixed number of members and nodes. Sequential linear programming was then used with minimum weight as the objective and member stresses as constraints. Lipson and Agrawal [9] used Boxs' complex method to find the minimum weight of determinate or indeterminate structures subject to both yield stress and buckling constraints. Discrete member properties (cross-sectional area and radius of gyration from a list of available sections) and nodal locations were included as design variables. Members whose cross-sectional areas approached zero were deleted during optimization. One advantage of using the complex method is that it allows a varying number of design variables.

The third approach uses a two-stage optimization strategy. In the first stage, member areas are considered as design variables while the geometry remains fixed. In the second stage, the roles are reversed, with the nodal locations becoming the design variables. The two-stage process is repeated until further improvement in the objective no longer occurs. Pedersen [10] followed this approach using sequential linear programming to solve for minimum weight with stress and displacement constraints. The number of members, the number of nodes, and the connectivity were chosen in advance. Spillers [11] also followed a two-stage strategy. He began with a prescribed configuration and allowed the deletion of members with areas approaching a minimum gage.

Other approaches to truss optimization include variations such as the type of constraints considered and the optimization method used. Literature reviews by Vanderplaats [12] and Schmit [13] discuss many of these works.

CHAPTER II

A TRUSS OPTIMIZATION STRATEGY BASED ON TRADITIONAL METHODS

This chapter presents the approach that will be used to solve the planar truss size and topology problem. The following terms are defined now to facilitate the discussion:

Node: A *node* is the connecting point between two or more bars, or between ground and one or more bars. In a two-dimensional design space, the location of the node is specified by its (x,y) cartesian coordinates.

Global Set of Nodes: The *global set of nodes* is a rectangular array of nodes.

Layout: A *layout* is a set of bars connecting a subset of nodes from the global set. The subset includes only those nodes that remain after removing one or more rows and/or columns from the global set. Here we require that the connectivity of the layout is such that each node in the subset be connected only to the nodes directly adjacent to it. This is demonstrated in Figure 4.

Configuration: A *configuration* T is the set of bars and connectivity created by *superimposing* one or more layouts from the same global set of nodes. Here to *superimpose* means that the bars and connectivity associated with each layout are combined to produce one layout with all bars and connectivities.

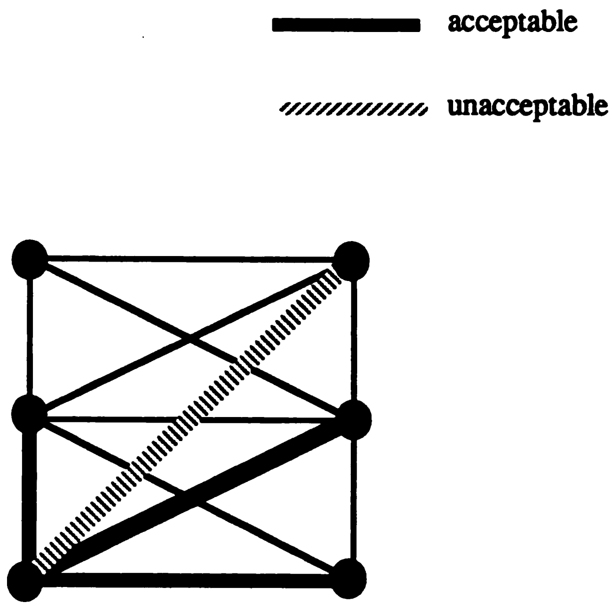


Figure 4. Connections in a layout

Admissible Topology: An *admissible topology* is a group of bars and their associated connectivity that can be reached by removing bars from the configuration **T**.

2.1 Problem Statement and Relevant Characteristics of Optimum Structures

Using areas and nodal coordinates as design variables, various configurations of bars will be analyzed to find truss structures of minimum compliance. Included in the procedure will be the ability to remove bars whose areas approach a minimum specified value. Formally, the optimization problem can be written as

Problem P(T)

Given a prescribed configuration, **T**, amount of material, V_o, and a lower bound on bar areas, A_{min}, find the vector of bar areas, $\mathbf{A} \in \mathbf{R}^n$, and the vector of nodal locations, $\mathbf{x} \in \mathbf{R}^{2m}$, that

$$\text{minimizes} \quad C = \mathbf{u}^T\mathbf{F} \tag{4}$$

$$\text{subject to} \quad [K]\mathbf{u} = \mathbf{F} \tag{5}$$

$$V = \sum_{i=1}^{n} A_i l_i \leq V_o \tag{6}$$

$$A_i \geq A_{min} \quad , \; i=1,n \tag{7}$$

where: C: total compliance

F: applied load vector

[K]: global stiffness matrix

l_i: length of bar i

m: number of nodes

n: number of bars

u : global displacement vector

V: total volume

V_o: allowable volume

The truss structure is subjected to a single loading condition. Effects such as buckling and self-weight will not be considered here. It is also noted that the objective in this case, as with the OSHP, is to minimize compliance, whereas in most of the methods discussed in the literature the goal was to minimum weight. However, it can be shown that, if all bars are made of the same material, the problem of minimizing compliance with bounds on volume is equivalent to minimizing weight with stress constraints.

The following steps are taken to solve the problem posed in equations 4 through 7. The computer implementation of these steps will be referred to here as the Optimization by Superimposed Layouts Program (OSLP).

Step 1. Select a configuration, T.

Step 2. Solve the problem P(T) using a nonlinear optimization program [14]. The examples in this study were solved using the Generalized Reduced Gradient method.

Step 3. Obtain a reduced configuration, T^*, by removing bars from T whose areas reached A_{min}. The problem $P(T^*)$ is then solved.

The solution strategy of the OSLP is based on the following well known results (see References 5 and 15).

Proposition 1. At the solution of P(T), the bars in T are stressed according to

$$|\sigma_i| = \sigma^{(1)} \quad \text{if } A_i > A_{min}$$
$$|\sigma_i| < \sigma^{(1)} \quad \text{if } A_i = A_{min}$$

where, $\sigma^{(1)}$ is a scalar and σ_i is the stress in bar i .
Similarly, at the solution of P(T^*), the bars in T^* are equally stressed at a constant level, $\sigma^{(2)}$.

Let the solution to P(T) be $\{x^*(T), A^*(T)\}$ with an associated compliance C^*. Let D_L be the set of all possible statically determinate substructures (D) of T for x fixed at $x^*(T)$.

Proposition 2. For x fixed at $x^*(T)$, T* is a statically determinate substructure of T. Furthermore, T^* is the stiffest structure in D_L.

The propositions state that T^* is the stiffest statically determinate substructure of T for x fixed at $x^*(T)$, but they do not state that the truss at $\{x^*(T^*), A^*(T^*)\}$, obtained when x is a design variable, is the best solution for all P(D), where $D \in D_L$. The following result indicates that, if A_{min} is small, the solution of the OSLP, if not optimal, is arbitrarily close to the optimum solution.

Proposition 3. When $\mathbf{x}$ is a design variable, any improvement over C^* obtained from a different P(D), where **D** is a subset of **T**, would be arbitrarily small.

Proof.

Suppose that $\mathbf{D}^o$, a statically determinate substructure of **T**, exists with nodal coordinates and areas $\{\mathbf{x}^o, \mathbf{A}^o\}$ and associated compliance $C^o < C^*$. Note that this solution must have a total volume equal to V_o.

With $\mathbf{x}$ fixed at $\mathbf{x}^o$, suppose that the areas of the bars in $\mathbf{D}^o$ are reduced proportionally to a value $\overline{\mathbf{A}}^1$ such that

$$\overline{A}_i^1 = \frac{1}{(1+\varepsilon)} A_i^o \quad \text{if } i \in \mathbf{D}^o \text{ and } i \in \mathbf{T}$$

with $\varepsilon > 0$. This reduction in area is done with the intention of later adding the remaining bars in **T**, at an area A_{min}. To ensure that the resulting total volume still equals V_o and retain feasibility, the value of ε must depend on Amin. It is easy to see that $\varepsilon \to 0$ as $A_{min} \to 0$.

To determine the value of the compliance (C^1) with the new, reduced areas, first consider the equilibrium equations at $\mathbf{A} = \mathbf{A}^o$

$$[K(\mathbf{A}^o)]\mathbf{u} = \mathbf{F}$$

whose solution, $\mathbf{u}^o$, yields a compliance

$$C^o = \mathbf{u}^{oT}\mathbf{F} \ .$$

Setting $\mathbf{A} = \overline{\mathbf{A}}^1$, the equilibrium equations are

$$[K(\overline{\mathbf{A}}^1)]\mathbf{u} = \mathbf{F}$$

or

$$\frac{1}{(1+\varepsilon)}[K(\mathbf{A}^o)]\mathbf{u} = \mathbf{F}$$

with solution

$$\bar{u}^1 = \frac{1}{(1+\varepsilon)}[K(A^o)]^{-1}F$$

$$= (1+\varepsilon)u^o.$$

Therefore,

$$\bar{C}^1 = \bar{u}^{1^T}F$$

$$= (1+\varepsilon)u^{o^T}F$$

$$= (1+\varepsilon)C^o. \tag{8}$$

Now with x fixed at x^o, we add to D^o the remaining bars in the configuration T, setting their area at A_{min}. This gives a resulting solution $\{x^o, A^1\}$ where

$$A_i^1 = \frac{1}{(1+\varepsilon)}A_i^o \quad \text{if } i \in D^o \text{ and } i \in T$$

$$A_i^1 = A_{min} \qquad \text{otherwise}$$

and

$$\sum_{i \in C_L} A_i^1 l_i = V_o.$$

Notice that, as stated earlier, this solution has a total volume equal to V_o. The compliance of this solution is C^1 and since the new bars were *added* to the solution $\{x^o, \bar{A}^1\}$,

$$C^1 \leq \bar{C}^1$$

Then from (8),

$$C^1 \leq (1+\varepsilon)C^o$$

or

$$C^1 - C^o \leq \varepsilon C^o. \tag{9}$$

After all this, we have a solution for T at $\{x^o, A^1\}$ whose compliance (C^1) must be no less than that at $\{x^*(T), A^*(T)\}$. This is true since the latter was the solution to $P(T)$, i.e.,

$$C^1 \geq C^*. \tag{10}$$

Substituting (10) into (9) gives

$$C^* - C^o \leq \varepsilon C^o.$$

Since it was established that $\varepsilon \to 0$ as $A_{min} \to 0$, with small A_{min} an improvement over C^* using a different substructure would be arbitrarily small. This proves Proposition 3.

2.2 Selection of Configurations

The solution strategy presented in the previous section assumed that the configuration was prescribed. In presenting the characteristics of the optimum structures, it was not mentioned that the optimum truss would not be found if its topology was not included as an admissible topology in the initial configuration T. This section discusses the issues that must be considered when selecting this initial configuration.

The first issue that must be addressed is the number of bars and connectivity that should be included in the configuration. In order to guarantee that all possible topologies are available, an infinite number of interconnected nodes would be required. Since this is not possible, a method of selecting configurations must be found that is capable of representing as many topologies as possible and at the same time, includes as few bars as possible to keep the problem of manageable size.

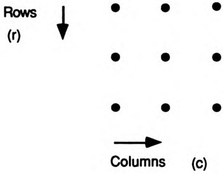
It is necessary to decide on a finite, global set of nodes that might reasonably be expected to contain the optimum topology and choose layouts associated with that set. The layouts are superimposed to create the configuration. By defining a global set of nodes, the number of admissible topologies becomes bounded. In order to guarantee that all topologies within the bounds are considered, it would still be necessary to superimpose *all* available combinations of layouts; even this approach would lead to a large number of bars and it would still be unmanageable.

The approach used here is to choose a global set of nodes and then find an initial solution from a configuration that is composed of a carefully selected group of layouts. Based on the initial solution, more layouts could be added to look for improved solutions. "Carefully selected layouts" means here that the admissible topologies in the configuration will include the topology of the OSHP solution and as many other topologies as possible. This approach will not guarantee that an optimum solution will be found, but it will make it possible to look for improvements on the OSHP solution in an organized fashion, if such improvements exist.

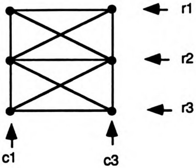
2.3 Obtaining Configurations Using Superposition

This section presents the details on how a configuration is obtained by superimposing layouts. Also discussed are advantages of a configuration created in this manner.

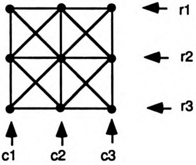
Figure 5 presents an example of the use of superimposed layouts. In Figure 5a, the global set of nodes consists of three rows and three columns. In Figure 5b, Layout #1 includes rows 1 through 3 and columns 1 and 3; the nodes are connected as though nodes from the second column were not present. In Figure 5c, Layout #2 includes all rows and columns and therefore all nodes. The connectivity is as shown. In Figure 5d, the two layouts have been superimposed. The result is a configuration that



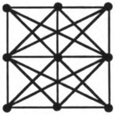
a) Global set of nodes



b) Layout #1



c) Layout #2



d) Configuration

Figure 5. Example of superimposed layouts

includes the bars and connectivity of both layouts. The configuration does not include as many admissible topologies as one that has all nodes interconnected, but, it does include all possible topologies in both layouts.

To see an advantage of using superposition, consider the problem statement from Figure 2. By using Layout #1 from Figure 5b, the nodes and connectivity would allow the 2-bar topology in Figure 6a . Likewise, Layout #2 from Figure 5c would allow the 8-bar topology in Figure 6b. Either case allows only one of the topologies. By using the configuration in Figure 5d, it would be possible to have either of the topologies as seen in Figure 6c. Hence the possibility of overlooking an acceptable topology is reduced.

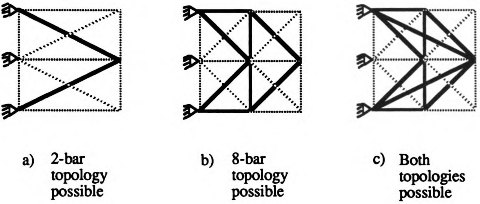
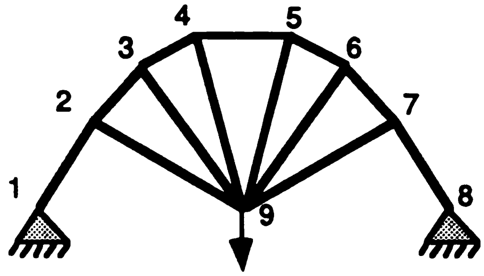
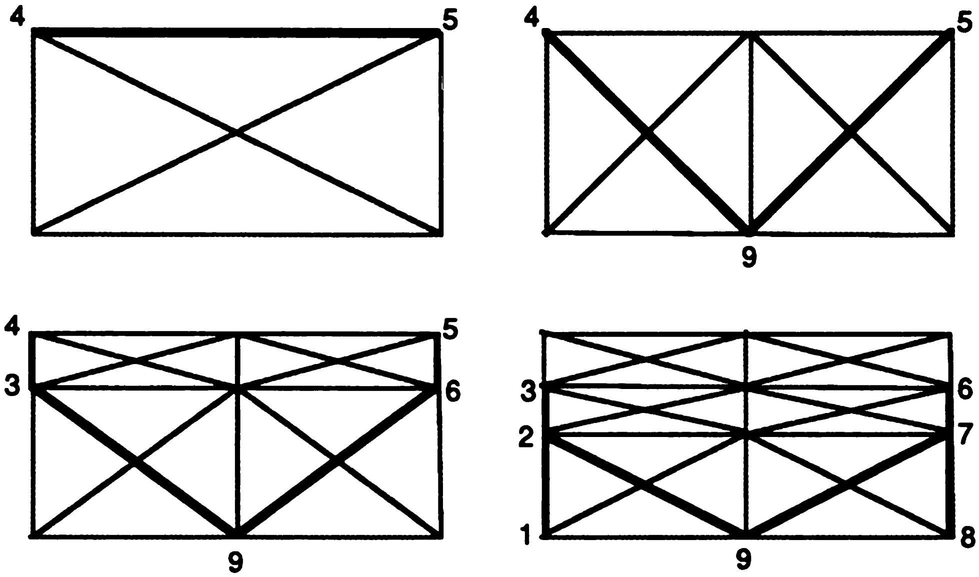


Figure 6. Superimposed layouts to include multiple topologies

Another advantage of superimposing layouts is the ability to find structures with complicated geometries. As an example, consider the 'arch' with six center supports shown in Figure 7a. It is not possible to generate a structure of this type using a configuration that only has connectivity to



a) Arch



b) Layouts

Figure 7. Superimposed layouts to create a specific geometry

adjacent nodes. On the other hand, if the four layouts in Figure 7b were superimposed, the required connectivity would be admissible.

CHAPTER III

EXAMPLES

The following examples are presented to show that the results obtained by the OSHP are equivalent to trusses of optimum topology. A design problem will be posed and the corresponding OSHP solution will be shown. The same design problem will then be solved using the OSLP. In doing so, configurations will be selected that include the topology of the OSHP solution. Improvements in the solution will then be attempted to show that the OSHP solution is of optimum topology. Characteristics of the solutions from the two approaches will be discussed.

3.1 Cantilever Truss

This problem is illustrated in Figure 8. The design domain has an area given by LxH. The domain is fixed along the left side and a load, P, acts at the center of the right side. This problem was solved using two different aspect ratios (L/H).

Case 1: L=10 , H=24 (25% solid volume)

The structure obtained using the OSHP is shown in Figure 9a. Except for a small region near the load and ground connections, the equivalent (VonMises) stress is essentially uniform.

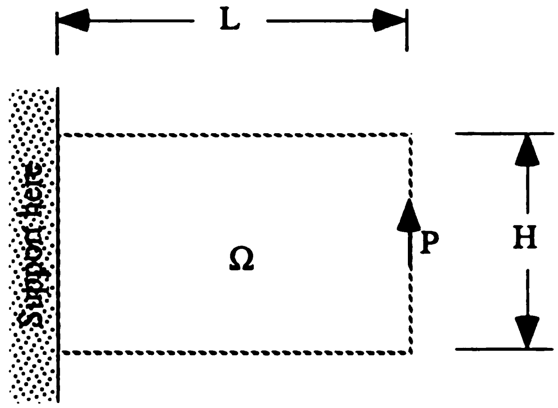
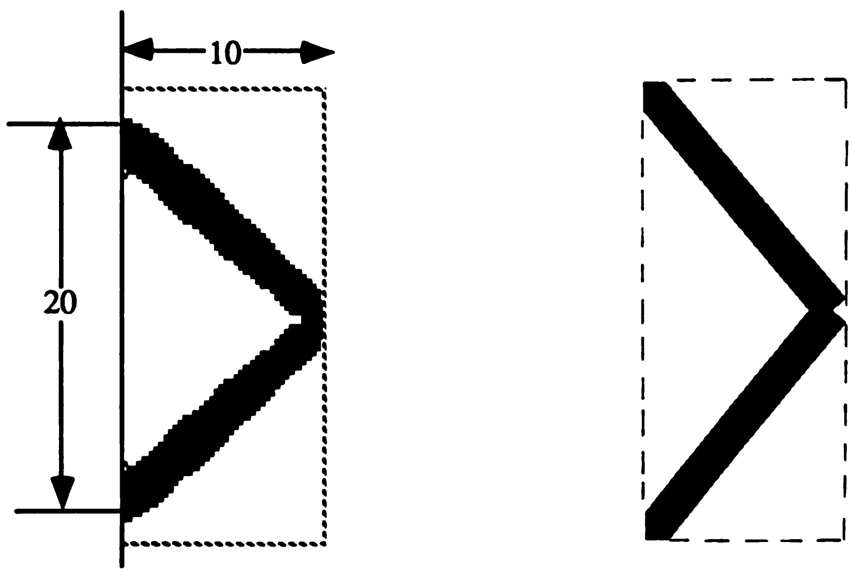


Figure 8. Cantilevered design domain

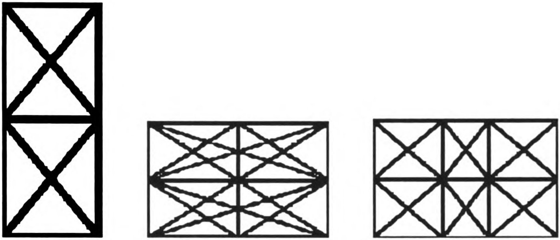


a) OSHP Solution

b) OSLP Solution

Figure 9. Cantilever truss solutions at L/H=5/12

In order to solve the same problem using the OSLP, the 11-bar configuration in Figure 10a was chosen because it would allow a solution similar to that obtained with the OSHP. As expected, the 2-bar truss solution in Figure 9b was obtained. The minimum compliance for this solution is c*= 3.45, where $c=CE/P^2$. The bars are equally stressed (in magnitude).



a) 11-bar b) 31-bar c) 29-bar

Figure 10. Configurations for cantilever truss

Note the difference in the height of the two solutions. The OSHP solution does not span the entire height that was available in the design domain. Fixing the boundary nodes at the coordinates of the OSHP solution, the OSLP was used to solve the problem again and this time a 2-bar truss with a compliance of c*= 3.33. This compliance is lower than that of the previous OSLP result and, therefore, the shorter truss is a better solution.

To look for possible improvements over the OSHP solution, the configuration in Figure 10b was considered next. The OSLP solution with this configuration is still the 2-bar truss even though many other topologies were included in the configuration.

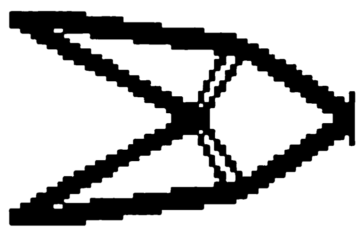
The structures obtained using both approaches are statically determinate and, based on the results, the OSHP predicts the optimum topology for this case. The OSHP also shows an advantage over the OSLP in that it is able to select the proper location for the ground connection along the boundary. Since boundary nodes are not included as design variables in the OSLP, this solution could not have been obtained.

Case 2: L=8 , H=5 (37.5% solid volume)

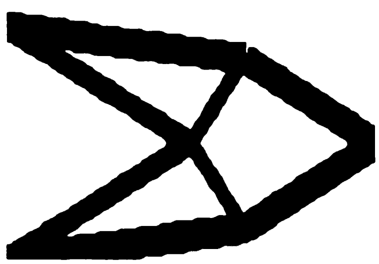
At this new aspect ratio, the structure obtained by the OSHP is shown in Figure 11a. The structure is statically determinate and, except for a small region near the load and ground connections, the equivalent (VonMises) stress is essentially uniform.

As with the previous case, to solve the problem with the OSLP, an configuration was used that would allow results similar to those produced by the OSHP. The 31-bar configuration in Figure 10b was selected for this purpose; the OSLP solution is the 8-bar truss in Figure 11b. The truss is statically determinate and the minimum compliance is $c^*= 21.31$. The stress in the bars is nearly uniform.

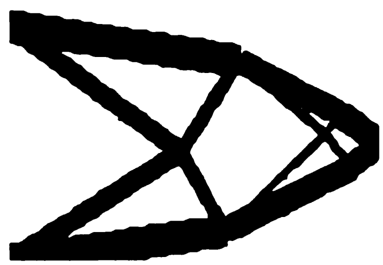
In looking at the 31-bar configuration of Figure 10b it is noted that the 2-bar truss was included as a possible solution, but the 8-bar structure was preferred. To find out why this happened, the problem was solved again with the 11-bar configuration of Figure 10a. The 2-bar truss that resulted has a minimum compliance of $c^*= 26.32$, which is more than that of the 8-bar solution. It can be seen from this result that it would be very easy to accept a solution as optimum when a better solution exists; if the 2-



a) OSHP Solution



b) OSLP Solution

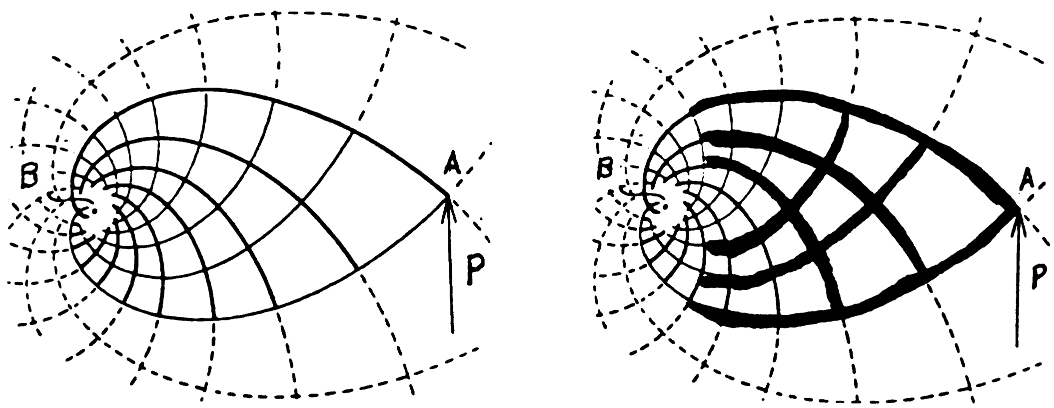


c)Refined OSLP Solution

Figure 11. Cantilever truss solutions at L/H=8/5

bar truss result were found first, it may have been accepted as the optimum structure.

In order to look for an improvement over the OSHP solution, the idea of replicating a shape was considered. It is possible to view the 8-bar truss as two repeated 2-bar trusses (with necessary bars for connection). The 29-bar configuration in Figure 10c, creates the possibility of a third 2-bar truss being repeated. The OSLP solution with this configuration is seen in Figure 11c and, indeed, the replication occured. The minimum compliance for this structure is c*= 21.22 which is nearly identical to the 8-bar solution. The truss is statically determinate and the stress in the bars is again nearly uniform. This solution resembles the Michell truss shown in Figure 12. Configurations that allow more repeated 2-bar truss may lead to solutions that become more and more like the Michell truss. It is not known why the OSHP selected only the 8-bar structure as the optimum answer.



a) Original truss

b) Truss with similarity highlighted

Figure 12. Michell truss

3.2 Arch

This problem is illustrated in Figure 13. The domain has a fixed support ($\mathbf{u}_A=\mathbf{0}$) at A and either a fixed support ($\mathbf{u}_B=\mathbf{0}$) or a rolling support ($u_{By}=0$) at B. The applied loads are $F_1=\gamma_1 P$ and $F_2=\gamma_2 P$, and γ_1 and γ_2 are 1 or 0.

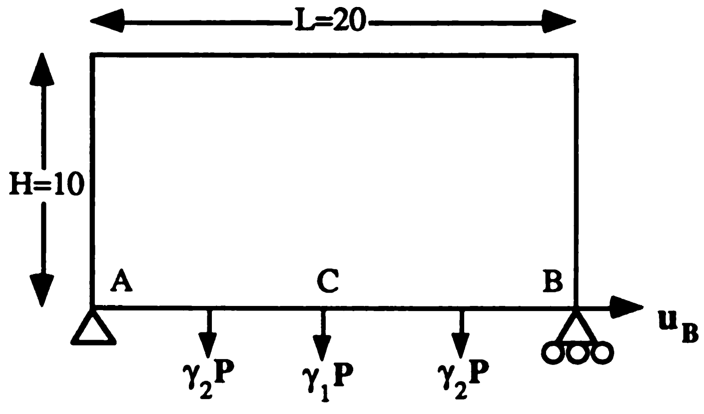


Figure 13. Arch design domain

Case 1: $\gamma_1=1$, $\gamma_2=0$, $\mathbf{u}_B=\mathbf{0}$ (25% solid volume)

The OSHP solution for this case is shown in Figure 14a and is seen to be a statically determinate structure. In this solution, as with the 2-bar truss solution, the resulting structure does not use the entire specified domain; the final height of the arch is 9.45 .

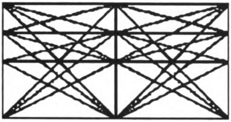
The 66-bar configuration in Figure 15a was chosen to solve the same problem using the OSLP because it can be used to represent a similar arch. The OSLP solution is similar to that of the OSHP and is shown in Figure 14b. The truss is statically determinate and the bars are nearly equally stressed.

The arch problem is examined more closely in the next example.

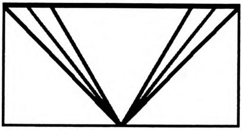
a) OSHP Solution

b) OSLP Solution

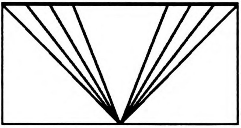
Figure 14. Arch solutions: Case 1 (γ_1=1, γ_2=0, $\mathbf{u}_B$=0)



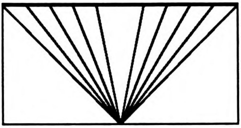
a) 66-bar



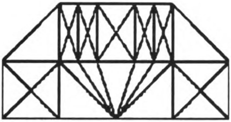
b) 15-bar



c) 19-bar



d) 23-b

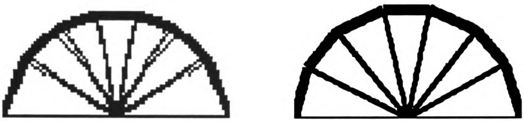


e) 52-bar

Figure 15. Configurations for arch

Case 2: $\gamma_1=1$, $\gamma_2=0$, $u_{By}=0$ (25% solid volume)

In this case, the OSHP solution is very similar to the solution of Case 1: two additional horizontal bars connecting the load and supports are the only change. The solution is shown in Figure 16a. The structure is statically determinate and the equivalent (VonMises) stress is nearly uniform throughout the domain, except near the load and supports.

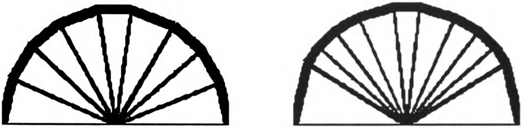


a) OSHP Solution b) OSLP Solution

Figure 16. Arch solutions: Case 2 ($\gamma_1=1$, $\gamma_2=0$, $u_{By}=0$)

The 15-bar configuration in Figure 15b was chosen to solve the problem using the OSLP because it can be used to represent a similar arch. The solution, shown in Figure 16b, is again very similar to the OSHP solution. The truss is statically determinate and the minimum compliance is c*= 10.22. The bars are nearly equally stressed.

To investigate the possibility of an improvement over the OSHP solution, the idea of including more bars as center supports was considered. The configurations in Figures 15c and 15d would allow eight and ten center supports, respectively. The OSL results for these configurations are shown in Figures 17a and 17b. For the arch with eight supports, the minimum compliance is c*= 10.09 . For the arch with ten supports, the minimum compliance is c*= 10.18. The minimum compliance for each of the three arches (six, eight and ten supports) is nearly identical and all are statically determinate. However, some difference is seen in the final height and angular orientation of the center supports. It could be speculated that any number of center supports would yield an arch with the same minimum compliance.



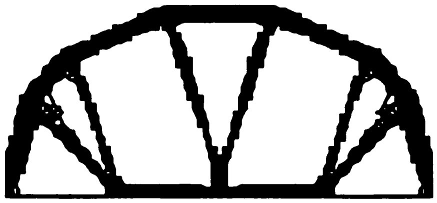
a) Eight supports b) Ten supports

Figure 17. OSLP arch solutions with extra center supports: Case 2 ($\gamma_1=1$, $\gamma_2=0$, $u_{By}=0$)

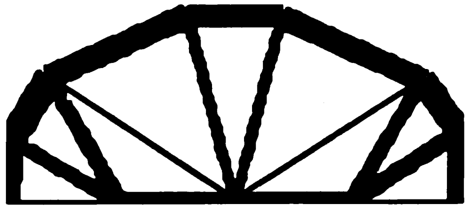
Case 3: $\gamma_1=\gamma_2=1$, $u_{By}=0$ (25% solid volume)

For the domain with three loads, the OSHP solution is the arch shown in Figure 18a. The structure is not statically determinate, but a mechanism that becomes a rigid structure only for a specific choice of nodal locations. The structure would not be practical for use since a small change in the location of the bars would make the system mobile.

To solve the problem using the OSLP, the configuration in Figure 15e was chosen because it would have the elements and connectivity available to produce a solution similar to that of the OSHP. Figure 16b shows the truss that was obtained from the OSLP.



a) OSHP Solution



b) OSLP Solution

Figure 18. Arch solutions: Case 3 ($\gamma_1 = \gamma_2 = 1$, $u_{By} = 0$)

The OSLP solution has two more bars than the OSHP solution. The two bars have a very small area and run diagonally from the center load point to the upper part of the structure. The additional bars exist in the OSLP solution to satisfy equilibrium of the system but are of vanishingly small area. This is one situation where the OSLP solution gives some insight that was not found in the OSHP solution. By adding the two small bars to the OSHP solution, it would become stable.

CHAPTER IV

CONCLUSIONS

Based on the solutions obtained using the OSHP and the OSLP, it can be speculated that the shapes obtained from the homogenization method are equivalent to trusses of optimum topology. The OSHP solutions have the characteristics of the optimum trusses in that they have uniform stress and are statically determinate, except in the special case where a particular geometry makes rigid a structure which would otherwise collapse. From these characteristics and from the similarity of the material distribution observed in the examples, it is also possible that the OSHP predicts the optimum sizing of members.

The OSHP has many advantages over the OSLP, the most important being that it does not require the experience and intuition to specify *a-priori* admissible topologies. However, the OSHP does not take into account multiple loading conditions or complex constraints, which are typical for practical truss optimization. Improvements in the OSHP to include these items will make it ideal for truss optimization.

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