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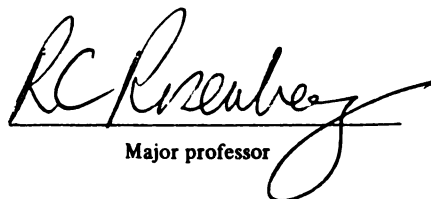
**Modal Analysis of Damped Systems
Using a Bond-Graph Approach**

presented by

Thomas L. Bush

has been accepted towards fulfillment
of the requirements for

Master's degree in Mechanical Engineering


Major professor

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MODAL ANALYSIS OF DAMPED SYSTEMS
USING A BOND-GRAPH APPROACH

By

Thomas Lee Bush

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
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ABSTRACT

MODAL ANALYSIS OF DAMPED SYSTEMS USING A BOND-GRAPH APPROACH

By

Thomas Lee Bush

Modal analysis is one method used in the analysis of linear dynamic systems. This method cannot be applied to damped systems unless the damping is of the proper modal form. To overcome this problem designers sometimes approximate the actual damping with modally-distributed damping. A study into the errors resulting from these approximations was conducted using bond-graphs. A bond-graph model was developed that represents the system in modal coordinates and included an element to account for the coupling effect caused by nonmodally-distributed damping. Simulations were conducted using this bond-graph to try to correlate power action in the coupling element to resulting output error. Computer software was written to allow computation of differences from successive ENPORT simulations. The results of these simulations did show some of the expected trends. However the desired general correlation was not obtained.

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I would like to thank my Major Professor, Dr. Ronald Rosenberg for his knowledge and guidance during my graduate studies.

A special thanks also goes to my wife Linda, for all her love, support and understanding.

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Chapter 1

Introduction

Modal analysis is commonly used in the design process for predicting the response of linear dynamic systems with multiple degrees of freedom. This analysis technique allows the designer to decompose the coupled, second-order, linear differential equations into a set of decoupled, second-order, linear, equations in a new set of coordinates. These uncoupled equations can then be solved independently, yielding the modal responses which can then be transformed back to the original coordinate frame to obtain the overall system response.

The problem with this analysis technique is that for many damped systems it is not possible to obtain a transformation that will decouple the original differential equations. If the standard decoupling technique is employed the modal equations may remain coupled by their damping terms. Such a system is said to be non-modally damped. It is generally accepted that the modal analysis technique can only be truly applied to systems with Rayleigh damping, that is, systems with a damping matrix proportional to the mass and stiffness matrices. However, a more general condition which is both necessary and sufficient for determining the possibility for modal decomposition is presented by I. Fawzy (1). In either case, accurate representation of a system response using this method is limited to a modal form of damping.

The alternative to the modal decomposition procedure is to convert the problem into a state-space formulation. This formulation converts n second-order differential equations into $2*n$ first-order differential equations. Because this approach requires the solution of twice as many equations it is generally used for numerical computations but avoided when generalized solutions are desired.

One approximate method which has commonly been used to deal with systems which are non-modally damped is to eliminate the damping from the original equations, perform the transformation, then add damping to the various modes based on experience. This method is generally employed successfully for systems with light damping. However, there still exists a question as to how much error is incurred when using this approach. Additionally, the question arises as to when the damping is small enough so that this method can be employed without incurring substantial errors.

This thesis presents a study into non-modally damped systems using bond-graphs. A bond-graph model is developed which allows a modal representation of the system augmented by intermode coupling elements. This structure allows one to study the coupling activity and to determine how strongly the modes are coupled for a variety of inputs.

As part of this research, computer subroutines were developed for the ENPORT software (4) to assist in the evaluation of the errors produced by the elimination of the intermode coupling. This software allows the user to

compare the results of a series of simulation runs made with varied parameters. From these comparisons predictions can be made as to the sensitivity of the original model to a given parameter.

The development of the modal bond-graph is described in Chapter 2. Chapter 3 then presents and describes the software developed for use in comparing simulation results from modal and non-modal systems. The actual study is presented in Chapter 4 and Chapter 5 gives a summary of the findings.

Chapter 2

The Modal Bond-Graph Model

The purpose of this section is to develop a modal bond-graph model for use in studying the intermode coupling effects. Margolis and Young (2) present a method for converting large bond-graph models to a modal bond-graph representation. However, their method falls somewhat short in that they take the conventional approach and require that the actual damping be replaced by modally distributed damping in the modal model. The model developed here will allow the transformed equations to remain coupled and include this effect in the new bond graph as a coupling element between the modes. Through this element potentially valuable information can be obtained as to the effect of this coupling. In this form the model will be an exact representation of the original system. Therefore simulation results are valid.

Section 2.1 presents the development of a modal bond-graph for a system in which the damping is modally distributed. No intermode coupling element is needed since the modal equations completely decouple. This model is modified in section 2.2 to allow for coupling elements when the damping is not modally distributed.

2.1 Systems with Modally Distributed Damping

The best way to present the development of the damped modal model is by an example. This example begins with the 2 Degree of Freedom (D.O.F.) spring-mass-damper system shown in Figure 1.

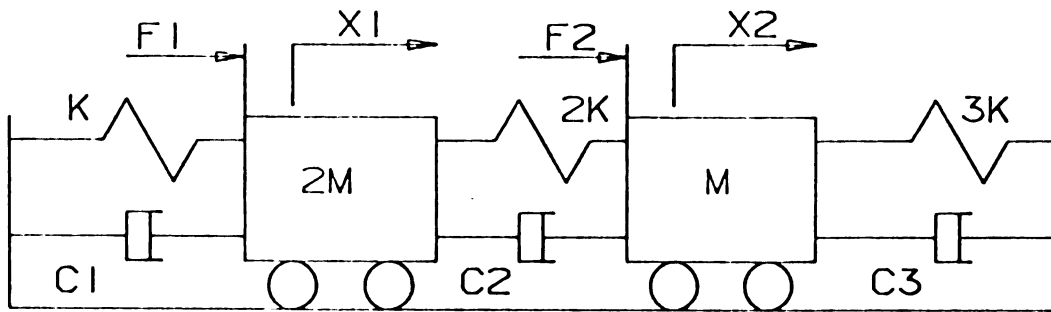


Figure 1

The 2 D.O.F. System Example

The differential equation of motion can be obtained in the \$X\$ coordinates shown using a Lagrangian or similar approach. Equation 2.1 give the representation of the two coupled, linear, ordinary differential equations for this system in terms of \$X_1\$ and \$X_2\$.

$$\begin{bmatrix} 2M & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix} + \begin{bmatrix} C_1+C_2 & -C_2 \\ -C_2 & C_2+C_3 \end{bmatrix} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} + \begin{bmatrix} 3K & -2K \\ -2K & 5K \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad (2.1)$$

The superdot denotes a time derivative throughout this study.

Equation (2.2) is the matrix form of eq. (2.1), namely,

$$M \ddot{X} + C \dot{X} + K X = F \quad (2.2)$$

To ensure that the system is modally damped the damping can be selected such that

$$C = \alpha M + \beta K, \quad (2.3)$$

where α and β are real constants.

Proceeding with the condition that the damping is modally distributed, the set of equations (2.1) can be decoupled using the transformation,

$$X = \phi \eta \quad (2.4)$$

where ϕ denotes the eigenvector matrix for the undamped equation

$$M \ddot{X} + K X = 0 \quad (2.5)$$

and η denotes the modal coordinate vector.

(Note that the eigenvector matrix has not been normalized.)

Substituting equation (2.3) into (2.2) and premultiplying by

ϕ yields a new equation in modal coordinates, namely,

$$\phi^T M \phi \ddot{\eta} + \phi^T C \phi \dot{\eta} + \phi^T K \phi \eta = \phi^T F. \quad (2.6)$$

Letting

$$M' = \phi^T M \phi \quad (2.7a)$$

$$C' = \phi^T C \phi \quad (2.7b)$$

and
$$K' = \phi^T K \phi \quad (2.7c)$$

equation (2.6) becomes

$$M' \ddot{\eta} + C' \dot{\eta} + K' \eta = \phi^T F. \quad (2.8)$$

Since the damping matrix in the original model was assumed to be structured such that the damping was of the modal form, this new equation will contain only diagonal matrices (except for ϕ^T). That is, the original coupled equations have been decomposed into uncoupled equations in the new coordinate η .

Expressing the diagonal matrices M' , C' , and K' for our example as

$$M' = \begin{bmatrix} MA & 0 \\ 0 & MB \end{bmatrix} \quad (2.9a)$$

$$C' = \begin{bmatrix} CA & 0 \\ 0 & CB \end{bmatrix} \quad (2.9b)$$

$$K' = \begin{bmatrix} KA & 0 \\ 0 & KB \end{bmatrix} \quad (2.9c)$$

and the eigenvalue matrix as

$$\phi = \begin{bmatrix} V11 & V12 \\ V21 & V22 \end{bmatrix} \quad (2.10)$$

the bond-graph shown in Figure 2 can be constructed.

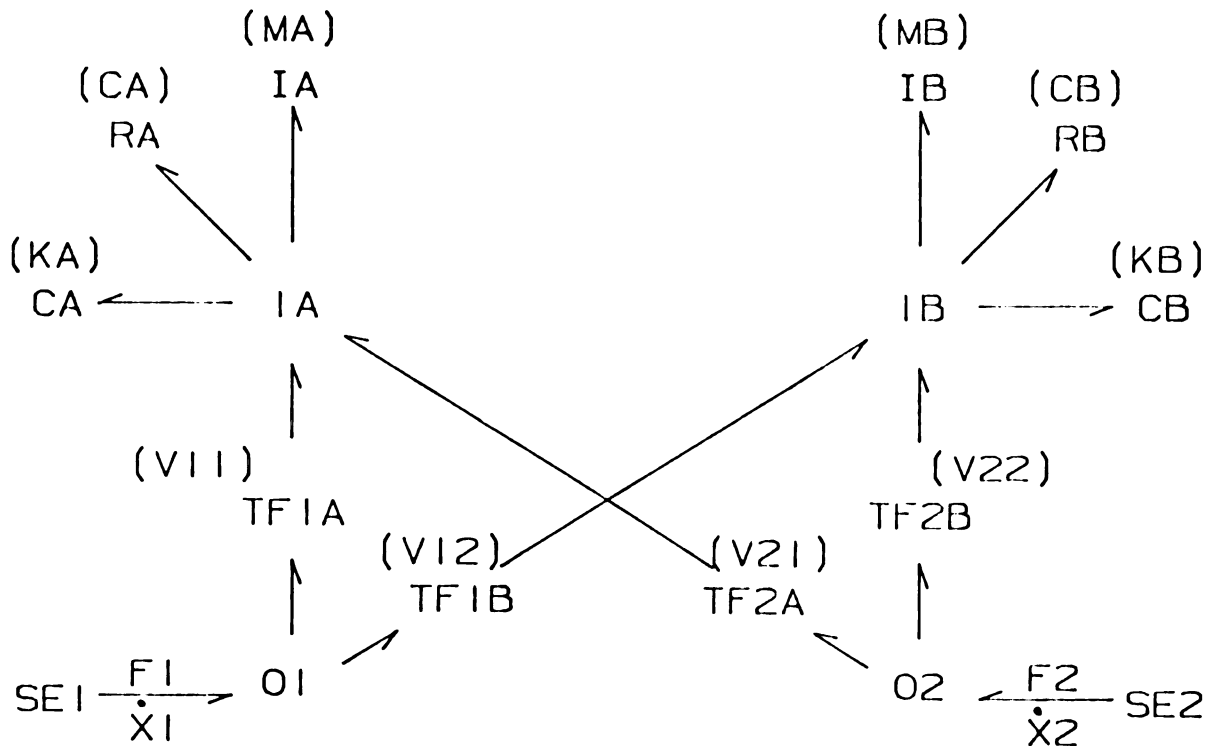


Figure 2

Bond-Graph for a 2 D.O.F. System with Modal Damping.

This bond-graph consists of 2 damped oscillators which are connected to the external forcing through a transformer structure. These oscillators are the 2 modes for the system and each 1 junction (1A, 1B) represents a common velocity junction for each of the modes. The velocities are \dot{x}_1 , and \dot{x}_2 , respectively. The values for the I, C, and K modal elements come from the transformed independent modal equations. In short, they are the diagonal terms from the M' , C' , and K' matrices. Applying causality to the bond-graph, as shown in Figure 3, allows further insights into the equation structure.

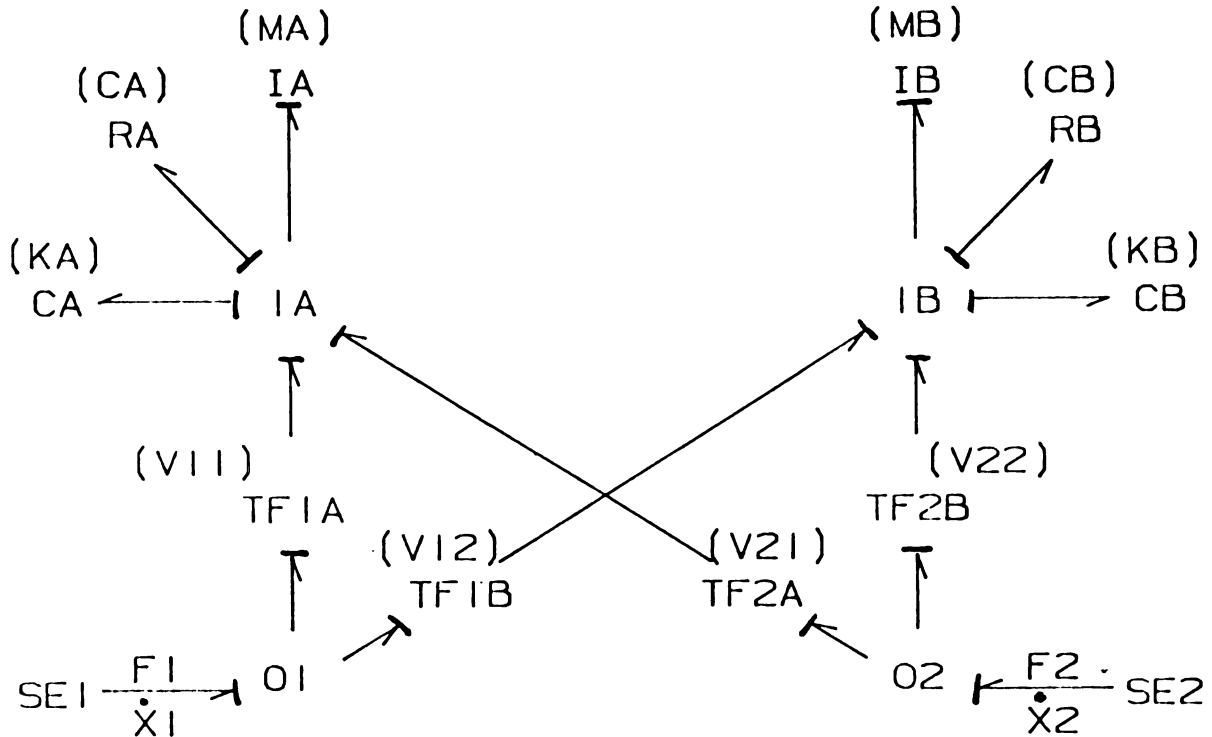


Figure 3

The Modal Bond-Graph of Figure 2 with Causality.

The transformer junction structure performs the transformation of the forcing terms into the modal space. Specific values for transformer (TF) elements are obtained from the ϕ matrix. In addition to transforming the forcing terms into the modal space, the transformer junction structure also transforms the velocity terms from the modal coordinates back to the original X coordinate base. This becomes apparent by viewing the model as a 'black box' the inputs of which are the forcing terms in the original coordinate frame. The outputs, as required by causality conditions, must be the resulting velocities, which are also in the original coordinate frame.

For this particular example, modal analysis leads to the following values for M' , C' , K' , and ϕ . (Recall that the damping was assumed proportional to the mass and stiffness matrices (eq. 2.3)).

$$M' = \begin{bmatrix} 2.25 M & 0 \\ 0 & 2.25 M \end{bmatrix} \quad (2.11a)$$

$$C' = \begin{bmatrix} 2.25(\alpha M + \beta K) & 0 \\ 0 & 18 \alpha M + 99 \beta K \end{bmatrix} \quad (2.11b)$$

$$K' = \begin{bmatrix} 2.25 K & 0 \\ 0 & 99 K \end{bmatrix} \quad (2.11c)$$

$$\phi = \begin{bmatrix} 1 & 1 \\ .50 & -4.0 \end{bmatrix} \quad (2.12)$$

For these particular values the bond-graph of Figure 2 takes on specific form as shown in Figure 4. Note that ϕ has not been normalized in the standard fashion at this point; the leading element of each column vector has been set to unity.

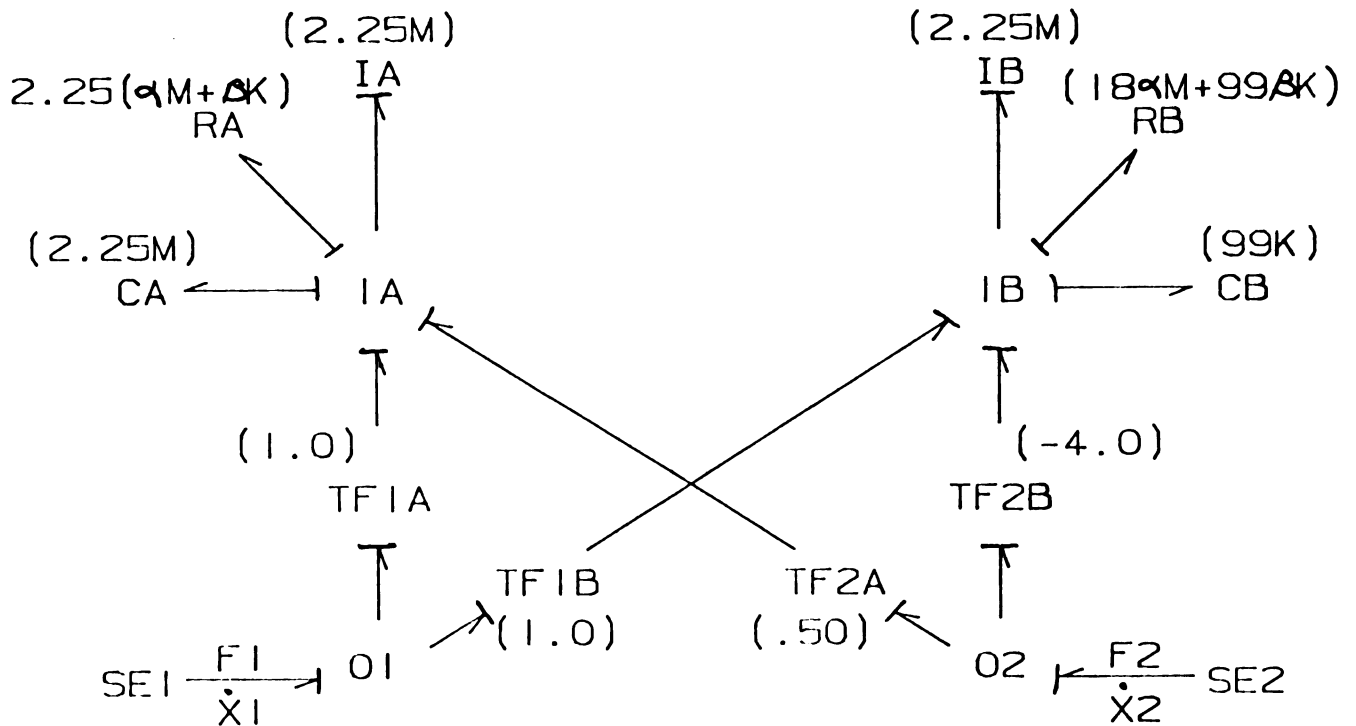


Figure 4

Modally Damped Bond-Graph with Specific Values.

Using this specific bond-graph as an example, four state-space equations can be written in the (modal) state variables π_A, n_A, π_B, n_B :

$$\dot{\pi}_A = -(2.25(\alpha M + \beta K)/2.25 M) \pi_A - 2.25K n_A + (F1 + .5 F) \quad (2.13a)$$

$$\dot{n}_A = (1/2.25 M) \pi_A \quad (2.13b)$$

$$\dot{\pi}_B = -((18\alpha M + 99\beta K)/18M)\pi_B - 99K\eta_B + (F1 - 4.0F2) \quad (2.13c)$$

$$\dot{\eta}_B = (1/18M)\pi_B \quad (2.13d)$$

where π_A and π_B are modal momenta associated with IA and IB, respectively. Additionally, 2 output equations can be written across the junction structure, converting the state variables in modal coordinates to X coordinate velocity values. That is,

$$\dot{x}_1 = \dot{\eta}_A + \dot{\eta}_B \quad (2.14a)$$

$$\dot{x}_2 = .50\dot{\eta}_A - 4.0\dot{\eta}_B \quad (2.14b)$$

Taking the Laplace transform of equations (2.13) with zero initial conditions and solving for $\pi(s)$ yields

$$\pi_A(s) = \frac{F1(s) + .50F2(s)}{s + (\alpha M + \beta K)/M + K/MS} \quad (2.15a)$$

$$\pi_B(s) = \frac{F1(s) - 4.0F2(s)}{s + (18\alpha M + 99\beta K)/18M + 99K/18MS} \quad (2.15b)$$

Setting the velocities in the output equations (2.14a) and (2.14b) equal to the derivative of the output

displacements and Laplacing these equations yields

$$SX1(S) = (1/2.25M) \tau_A(S) + (1/18M) \tau_B(S) \quad (2.16a)$$

$$SX2(S) = (.50/2.25M) \tau_A(S) - (4.0/18M) \tau_B(S). \quad (2.16b)$$

Substituting equations (2.15a) and (2.15b) into (2.16a) and (2.16b) yields the Laplace solution for the displacements in the original X coordinates (equations (2.17a) and (2.17b)).

$$X1(S) = \frac{F1(S) + .50F2(S)}{2.25(MS^2 + (\alpha M + \beta K)S + K)} + \frac{F1(S) - 4.0F2(S)}{18(MS^2 + (18\alpha M + 99\beta K)S + 99K)} \quad (2.17a)$$

$$X2(S) = \frac{.50(F1(S) + .50F2(S))}{2.25(MS^2 + (\alpha M + \beta K)S + K)} + \frac{-4.0(F1(S) - 4.0F2(S))}{18(MS^2 + (18\alpha M + 99\beta K)S + 99K)} \quad (2.17b)$$

This solution for X(S) obtained from the state equations of the modal model can be compared to the solution obtained using the Laplace of equations (2.4) and (2.8).

$$X(S) = \phi \eta(S) \quad (2.18)$$

$$M'S^2 \eta(S) + C'S \eta(S) + K'\eta(S) = \phi' F(S) \quad (2.19)$$

solving the matrix equation (2.19) for $\eta_1(S)$ and $\eta_2(S)$

$$\mathcal{N}_1(S) = \frac{F1(S) + .50F2(S)}{S + (\alpha M + \beta K)/M + K/MS} \quad (2.20a)$$

$$\mathcal{N}_1(S) = \frac{F1(S) + .50F2(S)}{S + (18 \alpha M + 99 \beta K)/18M + 99K/18MS} \quad (2.20b)$$

and substitute into equation (2.18) and solve for (9S)

$$X_1(S) = \frac{F1(S) + .50F2(S)}{2.25(MS^2 + (\alpha M + \beta K)S + K)} + \frac{F1(S) - 4.0F2(S)}{18(MS^2 + (18\alpha M + 99\beta K)S + 99K)} \quad (2.21a)$$

$$X_2(S) = \frac{.50(F1(S) + .50F2(S))}{2.25(MS^2 + (\alpha M + \beta K)S + K)} + \frac{-4.0(F1(S) - 4.0F2(S))}{18(MS^2 + (18\alpha M + 99\beta K)S + 99K)} \quad (2.21b)$$

As expected, the Laplace solution obtained from the bond-graph model is identical to that obtained directly from the transformed differential equations.

This analysis has demonstrated the modal model's ability to replicate the original differential equations. Insights are also gained into the transformation junction structure's job in linking the original physical

coordinates to the derived modal coordinates. That is, the input forcing terms in the original coordinates are transformed to modal coordinates and act on the individual modal oscillators. The resulting modal response is then transformed back to the original coordinate space through the same structure, yielding the output response. This can most easily be seen by studying the causality conditions shown in Figure 3.

A remark on normalization.

Some form of normalization of the eigen matrix is common in modal analysis. The classic method is to set the transformed mass matrix equal to the identity matrix; that is,

$$M' = \phi^T M \phi = I \quad (2.22)$$

This results in a modal stiffness matrix of the form

$$K' = \begin{bmatrix} W_1^2 & 0 \\ 0 & W_2^2 \end{bmatrix}. \quad (2.23)$$

The W_i are interpreted as the undamped natural frequencies. However, for the purposes of this study a particular normalization is not required since any form of the eigen matrix will transfer the original mass, stiffness, and

damping matrices to modal coordinates. However, it should be noted that once the transformed modal mass, damping, and stiffness matrices are obtained, further normalization (i.e. dividing each equation through by its mass term) should be undertaken with some care. Since these modal matrices are obtained using equations of the form

$$M' = \phi^T M \phi \quad (2.24)$$

a squared normalization term appears in each of the modal M' , C' , and K' matrices where only a linear term appears in the matrix. Dividing the entire equations through by a linear term will result in an incorrect modal bond-graph. For this reason, if some normalization scheme is desired, it should be performed on the eigenvector matrix, before the bond-graph parameters are specified.

This section has outlined the development of a modal bond-graph for a system with modally distributed damping. Each mode was represented in modal coordinates with a simple spring-mass-damper oscillator. These modal oscillators were then coupled to the original coordinate space by a transformation junction structure. This modal bond graph structure can be used to provide insights to the mode activity which may not be available in a conventional bond-graph model.

Separating the various modes allows the designer to determine the relative importance of a particular mode to the overall response for a given input forcing combination. In

some instances this may allow model simplifications if a particular mode or range of modes is determined to be inactive. This is especially useful if the system under study is a subsystem of a larger model. However, this modeling technique is still limited to systems with modally distributed damping. The next section will expand on this modal bond-graph to include non-modal damping.

2.2. Modifications for Non-modal Damping

The last section presented a bond-graph model which was based on the assumption that the damping was modally distributed. In general this will not be the case and the eigenvector transformation will not decouple the equations of motion. This section presents a modified version of the previous model which includes an intermode coupling element to allow for the coupling of the transformed equations. As before, this model will be an exact representation of the original system equations.

Return to the system shown in Figure 1 and assume that the damping is non-modal. The differential equations are of the same form.

$$\begin{bmatrix} 2M & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix} + \begin{bmatrix} C_1+C_2 & -C_2 \\ -C_2 & C_2+C_3 \end{bmatrix} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} + \begin{bmatrix} 3K & -2K \\ -2K & 5K \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad (2.25)$$

Since the eigenvector matrix is based strictly on the

undamped equations it also remains unchanged.

$$\phi = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \quad (2.26)$$

Proceeding as if the eigenvalue matrix could uncouple the differential equations, as before, the following matrix representation for the differential equations in the modal variable η are obtained.

$$M' \ddot{\eta} + C' \dot{\eta} + K' \eta = \phi^T F \quad (2.27)$$

M' and K' are the same diagonal matrices obtained previously,

$$M' = \phi^T M \phi = \begin{bmatrix} M_A & 0 \\ 0 & M_B \end{bmatrix} \quad (2.28a)$$

$$K' = \phi^T K \phi = \begin{bmatrix} K_A & 0 \\ 0 & K_B \end{bmatrix} \quad (2.28b)$$

but C' is no longer a diagonal matrix. It has the form

$$C' = \phi^T C \phi = \begin{bmatrix} C_A & C_N \\ C_N & C_B \end{bmatrix} \quad (2.29)$$

Notice, however, that C' is a symmetric matrix and can be represented with 3 parameters; C_A , C_B , and C_N . The reason that C' is symmetric stems from the transformation

$\phi^T C \phi$ and the fact that the eigenvector matrix is composed of orthogonal vectors. It can be shown that this transformation will always result in a symmetric matrix.

Generally this procedure would be considered of little value since the transformed equations remain coupled by their damping terms, especially if a generalized solution equation is sought.

Since the bond-graph model will generally be solved using numerical techniques, this intermode coupling is of little consequence and the modal bond-graph of section 2.1 can therefore be modified to include the coupling effect without disrupting the solution procedure.

Let the diagonal modal damping matrix be represented as the sum of a diagonal matrix and a matrix containing the off diagonal terms, namely,

$$C' = \begin{bmatrix} CA & 0 \\ 0 & CB \end{bmatrix} + \begin{bmatrix} 0 & CN \\ CN & 0 \end{bmatrix} \quad (2.30)$$

Equation (2.29) then takes the form

$$\begin{bmatrix} MA & 0 \\ 0 & MB \end{bmatrix} \begin{bmatrix} \dot{n}_A \\ \dot{n}_B \end{bmatrix} + \begin{bmatrix} CA & 0 \\ 0 & CB \end{bmatrix} \begin{bmatrix} n_A \\ n_B \end{bmatrix} + \begin{bmatrix} 0 & CN \\ CN & 0 \end{bmatrix} \begin{bmatrix} n_A \\ n_B \end{bmatrix} + \begin{bmatrix} KA & 0 \\ 0 & KB \end{bmatrix} \begin{bmatrix} n_A \\ n_B \end{bmatrix} = \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad (2.31)$$

Notice that with the exception of the off-diagonal damping matrix, this is exactly the same as equation (2.8) used in the development of the modal bond-graph from section

2.1. Notice also that the effect of this off-diagonal damping matrix is simply to add forcing terms to each mode that are proportional to the velocity in the other mode. That is, a force is applied to mode A which is proportional to the velocity in mode B (\dot{N}_B), and vice-versa. In bond-graph terms an element which reproduces a force proportional to a velocity is an R element or a gyrator. Since in this particular case the forcing effect is cross-coupled, a special form of the R element, called a 2-port R element, can be incorporated. This element takes the form shown in Figure 5.

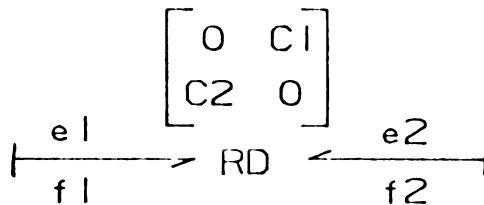


Figure 5

The 2-Port R Element

This 2-port R element uses a matrix type of equation structure of the form

$$\begin{bmatrix} e1 \\ e2 \end{bmatrix} = \begin{bmatrix} 0 & c1 \\ c2 & 0 \end{bmatrix} \begin{bmatrix} f2 \\ f1 \end{bmatrix} \quad (2.32)$$

Where $f1$ and $f2$ are the input flows, and $e1$ and $e2$ are the resulting efforts. (Note that this matrix may be inverted depending on the causal demands of the system.) This

element equation is of the exact form of the off-diagonal damping term in equation 2.31.

Using this element, the bond-graph developed in section 2.1 can now be modified as shown in Figure 6 to include the effects of non-modally distributed damping.

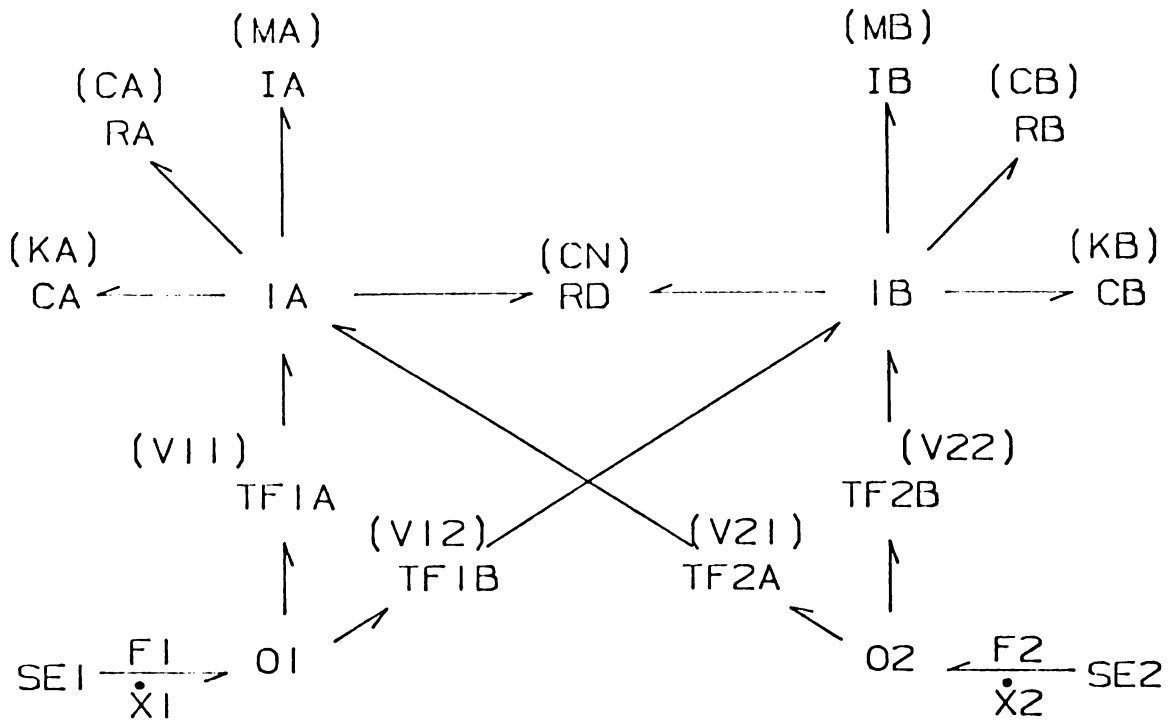


Figure 6

A Coupled-Mode Bond-Graph for Non-modally Damped Systems

Except for the addition of the 2-port R element, this bond-graph is identical to the graph shown in Figure 2. It contains the same 2 damped modal oscillators connected to the external forcing through the same transformer structure. Adding causality as shown in Figure 7 will further illustrate the interactions across the R element.

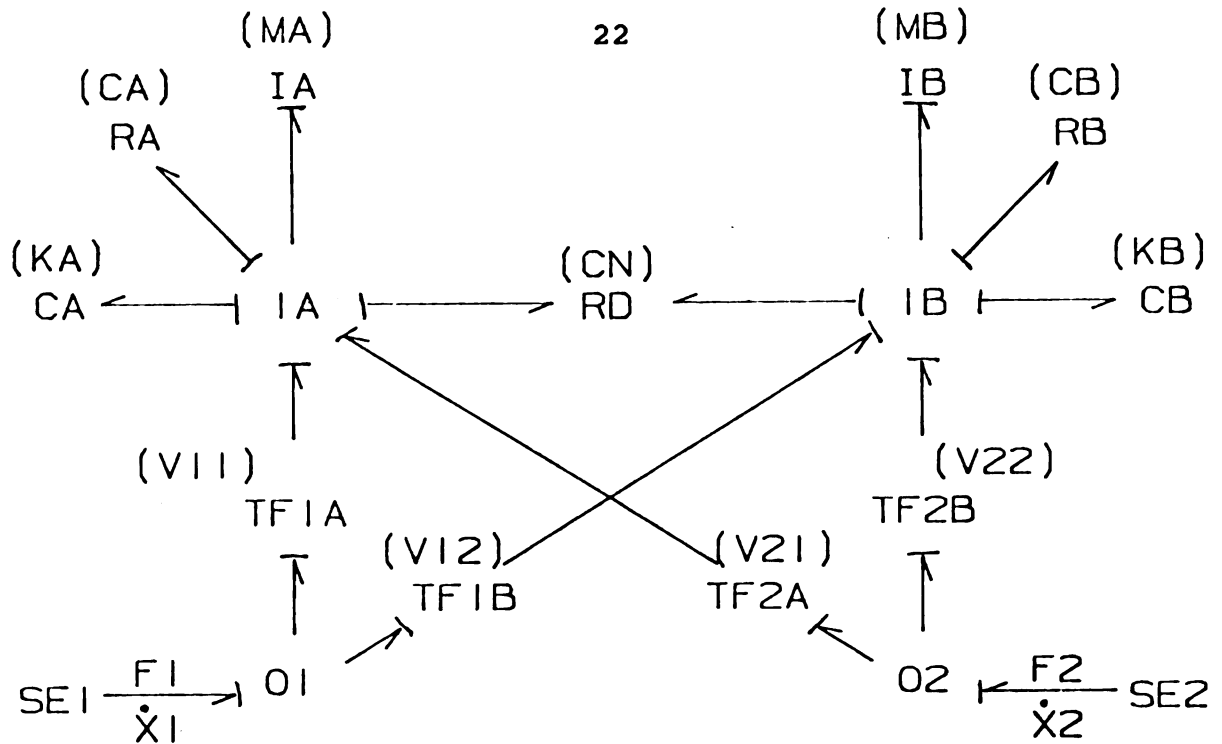


Figure 7

The Coupled-Mode Bond-Graph with Causality Added

From this bond-graph, the state space equations can be written in the (modal) state variables π_A, n_A, π_B, n_B :

$$\dot{\pi}_A = -(CA/MA)\pi_A - KA n_A - (CN/MB)\pi_B + V11F1 + V21F2 \quad (2.38a)$$

$$\dot{n}_A = (1/MA)\pi_A \quad (2.38b)$$

$$\dot{\pi}_B = -(CB/MB)\pi_B - KB n_B - (CN/MA)\pi_A + V12F1 + V22F2 \quad (2.38c)$$

$$\dot{n}_B = (1/MB)\pi_B \quad (2.38d)$$

Differentiating equations (2.38b) and (2.38d) yields

$$\dot{\tau}_A = MA \ddot{n}_A \quad (2.39a)$$

$$\dot{\tau}_B = MB \ddot{n}_B \quad (2.39b)$$

Now substituting equations (2.39a), (2.39b), (2.38b), and (2.38d) into equations (2.38a) and (2.38c) yields

$$MA \ddot{n}_A + CA \dot{n}_A + CN \dot{n}_B + KA n_B = V11F1 + V21F2 \quad (2.40a)$$

$$MB \ddot{n}_B + CB \dot{n}_B + CN \dot{n}_A + KB n_A = V12F1 + V22F2 \quad (2.40b)$$

or in matrix form,

$$\begin{bmatrix} MA & 0 \\ 0 & MB \end{bmatrix} \ddot{n} + \begin{bmatrix} CA & 0 \\ 0 & CB \end{bmatrix} \dot{n} + \begin{bmatrix} 0 & CN \\ CN & 0 \end{bmatrix} n + \begin{bmatrix} KA & 0 \\ 0 & KB \end{bmatrix} n = \phi^T \begin{bmatrix} F1 \\ F2 \end{bmatrix} \quad (2.41)$$

Notice that these equations are exactly the transformed equations (2.36) from which the model in Figure 7 was developed. Since the transformed equations are simply the original equations taken into a different coordinate frame, this shows that the modified modal model exactly represents the original system equations.

2.3. The Need for This Type of Model

At this point the question arises as to why go to the trouble of putting the system into this coupled-mode bond-graph form. It seems hardly worth the extra trouble since a more conventional bond-graph for the system can easily be developed and a computer solution based on that model easily obtained. It would appear that this representation offers little advantage. The answer lies in the potential for model simplification. This modal structure allows a view into the modal response of the system not generally available in other bond-graph structures. If, for the expected type of input, a particular mode is found to be inactive, it may be possible to eliminate that part of the model, thereby reducing the model complexity and saving computer time. This would be of particular interest if the model were part of a large system on which repeated simulations were intended. This situation frequently occurs in the design of control systems.

One type of model reduction of particular interest is the modal damping assumption. Earlier it was stated that one way in which designers avoid the problems with non-modal damping is to replace the damping in the original system equations with modally distributed damping in the transformed equations. If the damping is light this procedure is assumed to introduce only minimal error. The questions here are:

1. When can the damping be considered light?
2. How much error is introduced by assuming damping is light?

Chapter 4 presents a study into these questions using the newly developed modal bond-graph model. The objectives of this study were to determine if any conclusions could be drawn in regards to:

1. The error introduced by eliminating the coupling element by looking at some measure of the force or power transmitted through the 2-port R element.
2. If some indication of the strength of the coupling could be obtained by comparing the coupling coefficient with some grouping of the other system parameters.

If some "rules of thumb" could be established in this area it would give some insights as to when a modeler could safely make the modal damping assumption.

Before proceeding with this study it was necessary to develop some way to quantify the difference between the responses obtained for various levels of modal coupling. At the time of this study, bond-graph analysis packages such as ENPORT did not allow numerical comparisons between consecutive simulations. It was therefore, desirable to enhance the ENPORT package to allow this type of comparison. Chapter 3 presents the development of this feature. It should

be pointed out that this is also a very useful feature for general system modeling and design. It could be very helpful in determining the sensitivity of a system response to changes in the design variables.

Chapter 3

Automated Error Analysis Using ENPORT.

3.1. The Need for Error Analysis.

Very often in the design modeling process it is desirable to evaluate the difference in the output response as the model parameters are changed. This is especially important when the goal is model reduction. For this case these differences are the output errors resulting from the proposed reduction. The most straightforward way for determining these response differences is to compare the results from comparable simulations of the two models.

In the past the problem has been that ENPORT, a bond-graph simulation software, did not facilitate comparisons of separate simulations. Designers were forced to "overlay" response curves from consecutive simulations in order to visually perform these types of comparisons. The information obtained from these comparisons is generally qualitative in nature and lacks the numerical basis for making accurate comparisons. Since a substantial portion of the study presented in Chapter 4 consists of making these types of comparisons, it was desirable to enhance ENPORT to allow the numerical evaluation of output response comparisons obtained from comparative simulation runs.

3.2. Program Development

Before explaining the specifics regarding the new analysis feature, a brief explanation is in order on how ENPORT operates. ENPORT is a dynamic analysis package which provides numerical solutions for dynamic systems which have been represented in bond-graph/block-diagram form.

This study focuses on models represented in bond-graph form. In developing the differential system equations from a bond-graph, four types of variables are used: P, Q, E, F.

- (1) P is the momentum variable associated with energy storage element I.
- (2) Q is the displacement variable associated with energy storage element C.
- (3) E is the effort (force) variable associated with each bond.
- (4) F is the flow (velocity) variable associated with each bond.

Using these variables, ENPORT sorts the system equations and arrives at a formulation equivalent to

$$\dot{X} = f(X,U,t) \quad (3.1)$$

where X is the state vector,

U is the input vector,
and t is the time (the independent variable)

A linearized form of this equation can be generated by a numerical approximation at any given state X and time t. The result is

$$\dot{X} = A*X + B*U \quad (3.2)$$

where A is generated by (dfi/ dXj)
and B is generated by (dfi/ dUj)

For a linear system the A and B matrices should be independent of the X and t chosen.

Once ENPORT has obtained the equations in this form, it is simply a matter of numerical integration to obtain solutions for the state (energy) variables. These values are then stored in an internal array for subsequent display. In addition to these state variables, the efforts and flows associated with each bond and the input values are also saved. These can be used later to derive other quantities of interest such as power and energy transferred through the bond structure. With these solutions stored in internal memory, output plots of selected quantities can easily be obtained, although some quantities, such as power or energy, are derived and need to be computed prior to display.

These stored values can represent a substantial portion

of memory. Considering a simple model might contain 10 storage elements, 30 bonds, and 3 or 4 inputs, saving 300 time points for each variable would result in a storage array containing 22,200 real numbers. For this reason ENPORT reuses the same internal memory for each new solution and this is the reason that numerical comparisons of subsequent simulations could not be made.

The first step in incorporating the analysis feature was to provide ENPORT with the capability of storing and retrieving the result values. Since these result files may be used over a period of several ENPORT runs, it was determined that they should be saved in disk files and not simply in internal memory. This also avoids excessive demands on the required system memory. In order to keep the result file size to a minimum, the original system model and equations are not included. Besides the results themselves, these files contain only a header including the original model filename, the results filename, i.e. a 1 line description, and a list of the saved variables. It is assumed that the original model has been reloaded into ENPORT prior to reloading a saved results file, using the standard method. This assumption is verified by comparing the model name with the model name saved in the results file. Although this is obviously not a failsafe check, it does provide some safeguard. Although not yet incorporated, a better check would be to compare the results variable list with the model variable list. Also in an effort to make the results files

smaller, they are saved in an unformatted form.

Once the ability to save and retrieve the computed results was developed, incorporating a difference analysis was begun. It was simply a matter of retrieving the results values into a 3 dimensional array, with the third dimension indicating a particular set of results. Difference values can then be computed for any set of selected variables. These values can be listed or plotted just like an original set of solutions. From this information quantitative decisions can be made regarding the errors associated with the proposed changes.

Another procedure along the same lines as error analysis is sensitivity analysis. To assist in this type of analysis it would be useful to be able to plot some measure of the change in the output variables as a function of the variation in system parameters. The problem in incorporating this type of feature for arbitrarily selected result variables is that it would potentially require huge amounts of internal memory. Considering that each variation would require a full set of results and that several variations would need to be compared in order to draw accurate conclusions for a particular parameter. Even for a simple model this would require the storage of 300,000 real numbers.

In order to avoid using large amounts of internal memory to store loaded results files, the analysis package is currently limited to a comparison of only 2 files at a time. This serves the purpose of this study since only error

values caused by changing the intermode coupling strength are of interest. The effect as this parameter is varied over a range of values can be studied by comparison of the results two at a time. This is an area for potential future improvement which will be discussed in a later section.

3.3. An Example of the Analysis Feature

As stated earlier, it is often desirable to obtain quantitative comparisons of the output response for a system as the model parameters are varied. ENPORT is now capable of these types of comparisons. The following example is provided as a demonstration of this feature.

Using the bond-graph for the non-modally damped system developed in section 2.2 as an example, the effects of assuming modal damping on output response can be studied. This is the actual application in this study for which the software was developed. Figure 1 shows the standard 2 degree of freedom spring-mass-damper system from section 2.2. Using the methods described in that section the bond-graph in Figure 2 can be obtained for a given set of parameter values.

This model and its associated parameter values can be input into ENPORT in the usual way. (An ENPORT file listing is included in the appendix.) A simulation can then be run to yield the system response to a given set of inputs and initial conditions. Selecting the velocities of the masses

as the variables of interest, the plot shown in Figure 8 can be obtained of these velocities as a function of time using the standard ENPORT feature. In the modal bond-graph of Figure 2 the velocities of the masses are the flows on bonds 1 and 2: $f.1$ and $f.2$. Since no assumptions have been introduced into the model, these resulting velocities are the true results and therefore are used as the baseline for future comparisons.

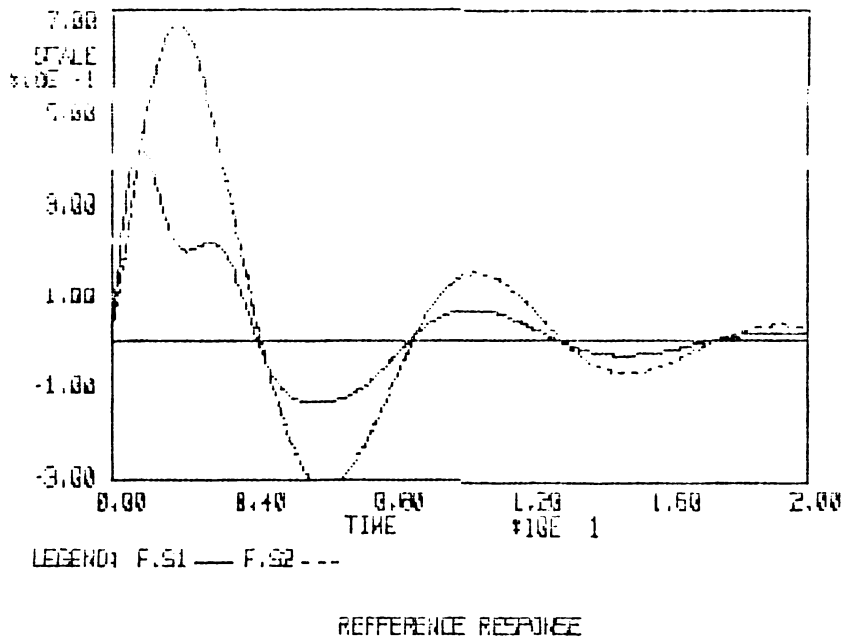


Figure 8

The Reference Velocity

At this point the results data should be filed using the new analysis feature.

```

Post-processor options
-----
G: Graph (High resolution)
P: Plot (low resolution)
T: Table
S: System graph display
L: List processed data
D: Operations on run Data
R: Return to the main menu
H: Help
-----

```

Figure 9

The Modified Post-processor Menu

The new option, D: Operation on Run_Data, can be called from the modified ENPORT post-processor menu (Figure 9) to obtain the analysis options menu as shown in Figure 10.

```

Run_data processing options
-----
W: Write run_data to file
L: Load run_data from file
S: Status of run_data
C: Clear the run_data
A: Analyze the run_data
R: Return to the post-processor
-----

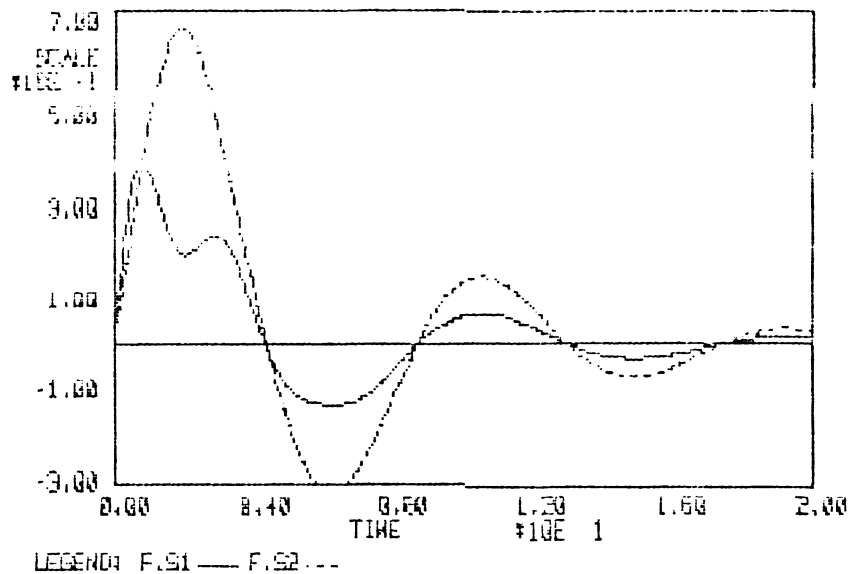
```

Figure 10

The Run_Data Menu in ENPORT

From this options menu, W: Write Run_Data to File, can be selected. This choice will call a subroutine that writes all the computed result values to an unformatted binary file under a user selected name. Call it "ref.dat" for this example. Additionally, the user is allowed a one line file description which is included in the file. This is helpful for future identification of the correct file.

Now that the results from the baseline simulation have been saved, a second simulation can be run with the assumption of a modally damped system. This change can be implemented by setting the parameter of the coupling element, RN, to zero. The output response from this run is shown in Figure 12.



ASSUMED MODAL DAMPING

Figure 11

The Response with Assumed Modal Damping.

One important point to note is that simulations which are to be analyzed together must currently be run under the same time conditions. That is, the initial and final times, as well as the number of time stages, must match in order to obtain accurate response comparisons. Also, simulations which are to be compared must originate from the same topological model. Only numerical values of the model parameters may be changed. For example it would not be acceptable to invoke modal damping by eliminating the coupling element.

Now that this second simulation has been run, its results can be written to a separate file. Call it "model.dat", using the same method as for the reference case.

With the simulation results for the two cases now stored in the respective files, comparisons can be made between these sets of results for selected variables. These comparisons are obtained by again selecting option D: operations on run_Data from the ENPORT post processor menu which brings up the Run_Data menu as before. Selecting L: Load a set of run_data will allow the first file to be loaded into the storage buffer. This first file is considered to contain the reference data set or the set containing the values to be subtracted from the second file loaded. In the case of this example, this would be the data set saved under "ref.dat", from the first simulation. After retrieving this reference data set, the second data file can be loaded using the same method as for the reference set.

This data set should contain the results to be compared to the reference case. For example the file "modal.dat" would be loaded.

Now, with the 2 data sets loaded and stored in the internal memory buffer, the analysis can proceed by select option A: Analyze run_data. This option presents the Analysis Options Menu as shown below.

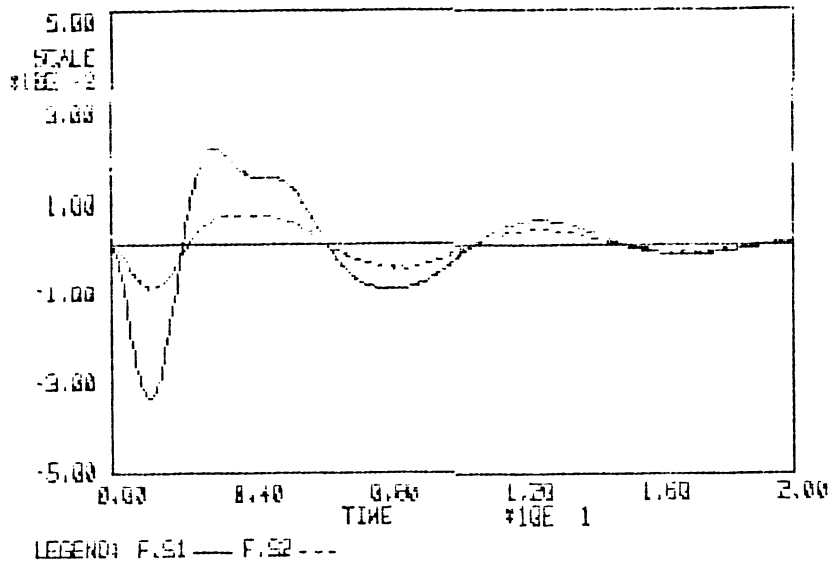
```
Run_data analysis options
-----
G: Graph difference values
T: Table of select difference values
R: Return
-----
```

Figure 12

Run_Data Analysis Menu in ENPORT

Currently, the options allow the user to obtain tabular listings and hi-res graphic plots of actual difference values for user selected variables. In the future the option could easily be modified to display normalized values such as actual average, RMS average or absolute average. Selecting option 'G: Graph', from this menu produces a hi-res plot of the difference in value of the user selected variables. The program automatically computes difference values as the plots are constructed so there are no other steps involved from the user perspective. The graph routine used

to obtain difference plots is virtually identical to the standard ENPORT graph routine and all the standard features have been retained. Figure 13 is a plot of the displacement difference values versus time for the two files loaded above.



DIFFERENCE VALUES

Figure 13

Change in Velocity for Modal Damping Assumption.

Plots could also be obtained for the difference in any other system variables. The tabular data can also be displayed by selecting 'T: Table'. For instance, if the information regarding the change in net energy into the modal dampers was desired at each time step, option, T: Table, could be selected. Choosing the display variable as the energy (T) on the bonds connected to the modal damping elements, a tabular listing of the change in net energy into the damper elements will be produced, Figure 14. Since

energy is an integral quantity of the state variables, the system will ask for the initial energy value. This value is taken to be the offset in the initial energy for the two cases and would generally be zero.

DIFFERENCE VALUES

Time	T.DA	T.DB	T.NA
0.000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.000E+00	-7.1824E-03	-7.1980E-03	7.5281E-03
1.000E+01	-7.9916E-03	-7.4846E-03	7.5809E-03
1.500E+01	-7.7132E-03	-7.5135E-03	7.5389E-03

Figure 14

Tabular Listing of the Change in Damper Energy Dissipated

After completing the desired plots and tables from the loaded data sets options R: Return can be selected which returns to the run_data processing options menu. The other options which may be selected from this menu include:

- C: Clear the run_data
- S: Status of the run_data
- D: Debug the run_data

Option C clears the internal memory to allow room for new sets of data to be loaded. Option S displays the names of the Data sets which are currently loaded. Option D allows the user to display a list containing the variable vector and the numerical values of each variable at each time step for a loaded results file, generally used as an aid to debugging the program.

3.4 Program Implementation and Future Enhancement

Suggestions.

The Fortran 77 subroutines which perform this new analysis feature fall into one of 3 categories:

- 1) New subroutines
- 2) Modified ENPORT subroutines
- 3) Unmodified standard ENPORT subroutines

The new subroutines and the modified ENPORT subroutines have been collected under 2 files: DRUNI and DRULOC. The file DRULOC contains the subroutines which must be modified for a particular operating system. These are the subroutines that perform file manipulations. The rest of the subroutines are collected under the file heading DRUNI. Appendix A contains a listing of each of these files for reference purposes.

The added capability of storing and retrieving ENPORT results data opens the door to a multitude of features that would further enhance ENPORT's usefulness in design. Some of these have already been mentioned such as the computation of normalized measures of the difference values (RMS, absolute average, actual average, etc.). Additionally, the program could be modified to allow comparisons of results data that were not computed with the same time steps. This could be accomplished by simple interpolation and will be a necessary change if results are calculated with a variable timestep integrator.

A somewhat more complicated enhancement would be the ability to compare specific results from several simulations simultaneously. Such a feature might be used to plot some measure of error (Such as RMS) versus percent change in a system variable, in essence showing the sensitivity of the system to changes in that variable. Earlier it was stated, however, that this type of analysis would currently require a large amount of system memory. This is because under the current design all system variables are saved and retrieved from the results files, which, to compare several result data sets, would require a prohibitive amount of memory. The solution to this problem would be to allow the user to reload only those system variables required to produce the desired plots. This would require much less system memory which would make it feasible to produce sensitivity plots.

The discussion above points to another potential opportunity for improving the program which would reduce the result data file size. This opportunity arises because the current design saves values for all the system variables. For a given model, many of these variables can be derived from a set of other system variables through algebraic equations. If the system could recognize these independent variables, the size of the result data sets could be reduced by including only these variables and the necessary algebraic equations. From these independent variables the post-processor could compute any requested variable value using the algebraic equations. This data handling scheme would

result in a minimal data set file size but would require a substantial change in the design of the post-processor.

Chapter 4

Approximations to Non-modal Systems

The model developed in section 2.2 was shown to be an exact "coupled-mode" bond-graph representation for a non-modally damped system. Solving this bond-graph would yield the exact response for the system under study. For larger, multiple degree of freedom systems this solution can be very time consuming. This is especially true when the model will be solved repeatedly, under a variety of input or initial conditions. It is therefore desirable to reduce the model complexity wherever possible. This can most easily be accomplished by eliminating elements and bonds which can be shown to have minimal effect on the output response. This procedure is routinely used in the modal analysis of continuous beam structures when only the first few modes are considered in the analysis. It is this same reasoning that allows designers to model lightly damped non-modal systems with modal damping assumptions. In the case of the non-modal bond-graph developed in Chapter 2 this assumption eliminates the non-modal 2-port R damping element resulting in a simplified representation.

4.1 The Objectives of The Study

The problem with the modal damping assumption is that it is usually unclear as to when the assumption can

accurately be applied. Using the modal bond-graph from section 2.2 to provide a unique perspective, this study attempts to quantify this decisions. In essence some "rule of thumb" was sought that would indicate the amount of error which would be incurred if the non-modal damping element were eliminated, reducing the system to the modally damped case.

To provide this rule of thumb would require some type of indicator which could be correlated to the error incurred in the modal damping assumption. Mathematical analysis did not reveal the desired indicator. An in-depth study was therefore undertaken comparing simulation results as system parameters were varied. It was believed that some correlation could be found by comparing the output response error between the non-modal model and the corresponding reduced modal model to easilty measure system quantities. Since there was no way to accurately compare results from successive simulations the first task of this study was the ENPORT enhancement detailed in Chapter 3. Once this new tool was developed the experimental investigation could proceed.

4.2. The Study Implementation

Figure 15 shows the modal bond-graph of the non-modally demped system developed in section 2.2. The structure of this bond-graph can basically be broken down into 2 independent modal oscillators that are coupled by a 2-port

damping element.

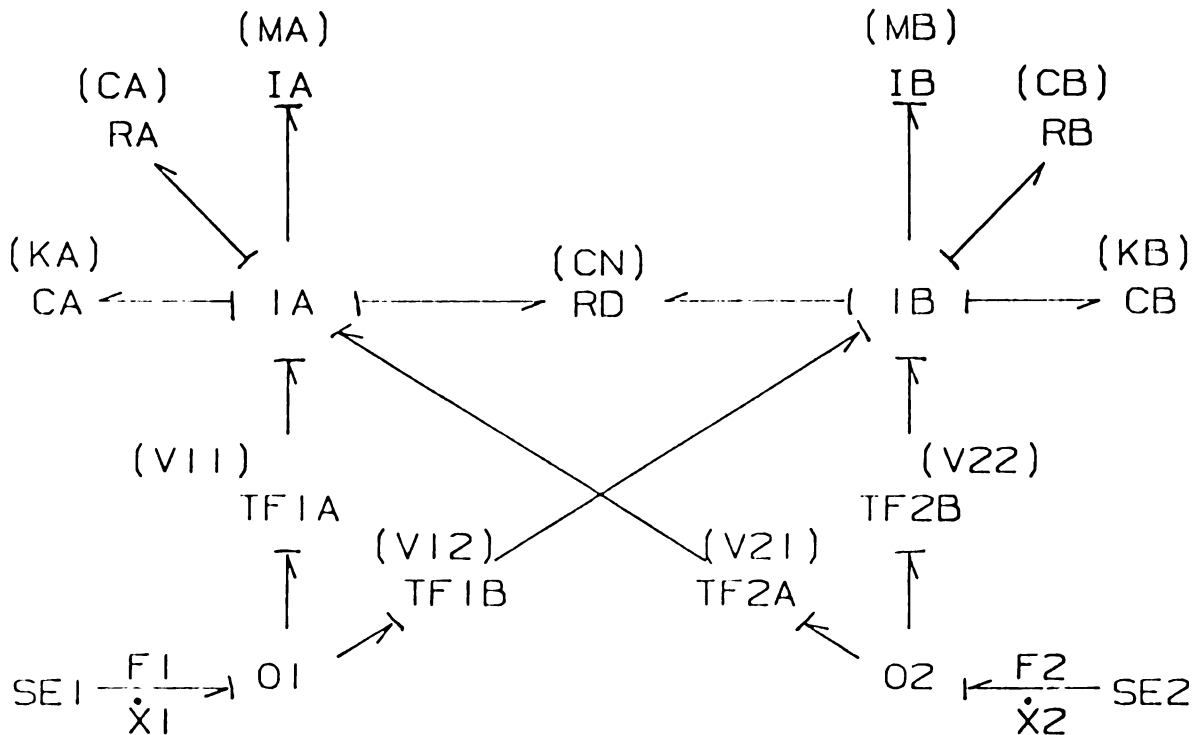


Figure 15

A Coupled-Mode Bond-Graph for a Non-modally Damped System.

As explained in section 2.2 this coupling element exists to provide for the damped coupling of the transformed modal equations (see eq. 2.36). The parameter value for this element is determined by the off diagonal terms in the transformed damping matrix. Since the transformed damping matrix is symmetric, a single parameter value is associated with this 2-port R element. Because of the symmetry, this element provides a unique power interaction between the modes which can be illustrated a typical element as shown in Figure 16.

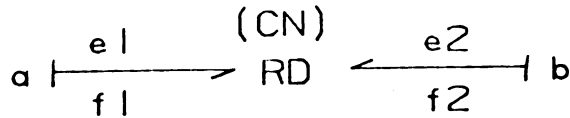


Figure 16

A Typical 2-port R Coupling Element

Assigning the power direction and causality the same as used in the non-modal bond-graph of Figure 15 implies that the inputs are the flows f_a and f_b . The algebraic nature of the R element then allows the direct computation of the output efforts e_a and e_b as follows.

$$e_a = (cn) * f_b \quad (4.1a)$$

$$e_b = (cn) * f_a \quad (4.1b)$$

The power on bonds a and b can then be computed.

$$\text{Power a} = e_a * f_a \quad (4.2a)$$

$$\text{Power b} = e_b * f_b \quad (4.2b)$$

substituting equations 4.1a and 4.1b into 4.2a and 4.2b yields

$$\text{Power a} = (CN) * f_b * f_a \quad (4.3a)$$

$$\text{Power b} = (CN) * f_a * f_b \quad (4.3b)$$

or

$$\text{Power a} = \text{Power b} \quad (4.4)$$

This result is interesting since it shows that the nature of this coupling element is to act alternately as a power source or a power sink to both elements simultaneously, depending on the relative signs of the input flows. Additionally, an equal amount of power is provided to, or dissipated from, both modes.

Since the addition of this coupling element is the only factor that differentiates a non-modally damped system from a system with modal damping, it would seem that the strength of its interaction could be used to determine the potential for model reduction. One measure of this interaction typical in bond-graph studies is the power transferred. Using the power interaction of the coupling element as a basic measurement to quantify the potential for a modal assumption was therefore the first path explored as part of this study.

To implement this portion of the investigation a simple 2 degree of freedom was chosen as shown in Figure 17.

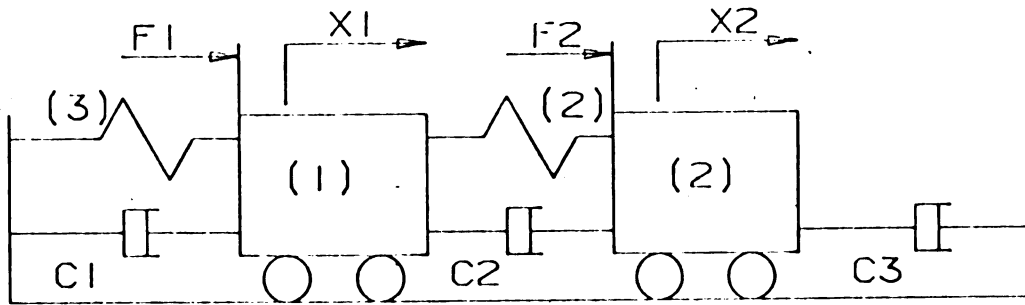


Figure 17

The Base 2 D.O.F. System

Using the methods developed in Chapter 2 the modal mass, stiffness and damping parameter matrices were computed, as well as the modal transformation matrix.

$$M' = \begin{bmatrix} 10.90 & 0 \\ 0 & 1.10 \end{bmatrix} \quad (4.5a)$$

$$K' = \begin{bmatrix} 6.00 & 0 \\ 0 & 6.00 \end{bmatrix} \quad (4.5b)$$

$$c' = \begin{bmatrix} c1+1.50c2+4.95c3 & c1-1.50c2-.50c3 \\ c1-1.50c2-.50c3 & c1+1.50c2+.051c3 \end{bmatrix} \quad (4.5c)$$

and

$$\phi = \begin{bmatrix} 1.00 & 1.00 \\ 2.225 & -.225 \end{bmatrix} \quad (4.6)$$

From these parameter matrices a coupled-mode bond-graph can

be obtained as in Figure 18.

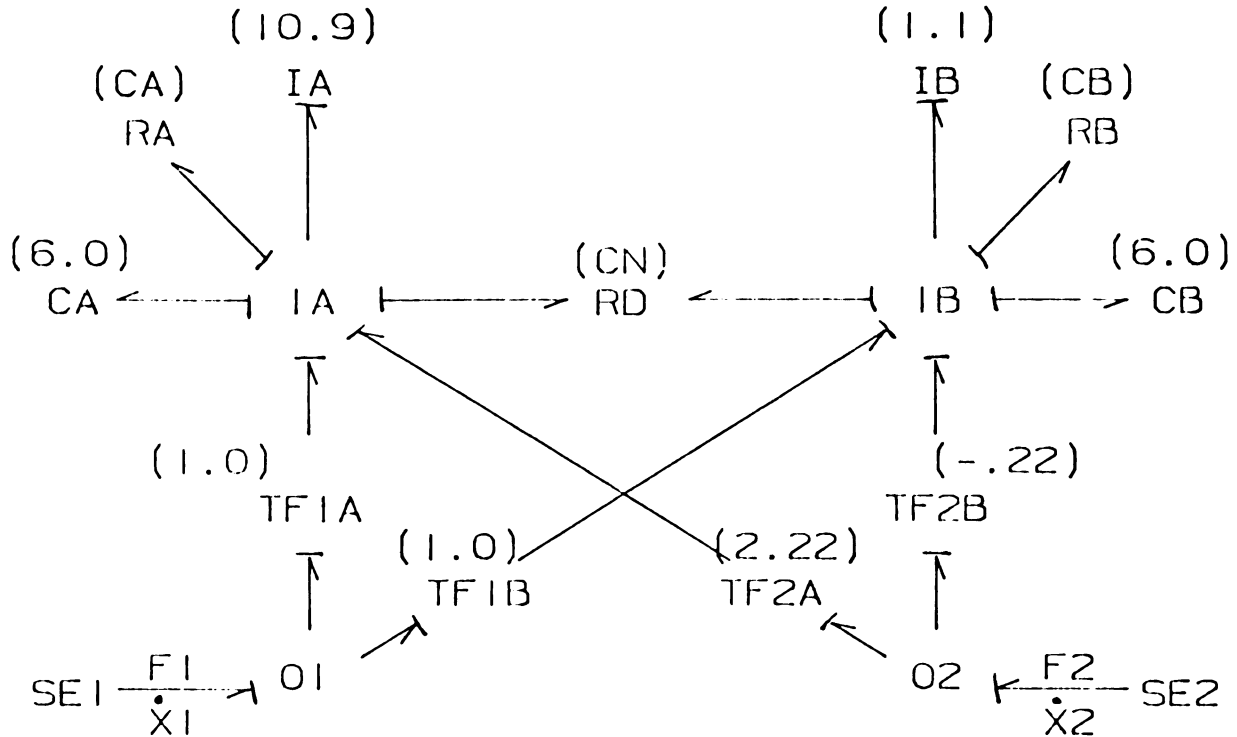


Figure 18

A Coupled-Mode Bond-Graph For the System in Figure 17

It should be noted at this point that although a non-modal coupling element has been included, this system may or may not be modally damped. This does not present any problems since if the system were modally damped, the parameter of the non-modal coupling element would simply be zero.

The initial focus of this investigation was to correlate the modal damping assumption error to some measure of the power activity of the 2-port R element. It was believed that larger assumption errors would result from increased

intermode coupling activity, which could be measured as larger power interaction. If a generalized correlation between power and error could be found then a decision could be made as to the potential for model reduction based on power measurements. The idea was to run a simulation with the full system under the expected forcing conditions and measure the power interactions. From these power measurements quantitative decisions could be made about the potential for model reduction.

An experiment to test this idea and to reveal the correlation between power and error was then conducted. Using the above system, groups of simulations were performed while varying the damping parameter values. During each group of simulations, the values of the modal damping parameters, R_a and R_b , remained fixed. Several simulations were then run while the value of the intermode coupling element (R_n) was varied. For each simulation a measure of power transferred through the intermode-coupling bonds was recorded. Additionally, the percentage error that would occur if the system were assumed to be modally damped was recorded. Plots of output error vs power were then created for each group of simulations.

For the purpose of this experiment, error was defined as the difference in output velocities, $f.1$ and $f.2$, between a non-modally damped system and the same system under a modal damping assumption. Recall that these are the actual velocities of the masses in the original system. The RMS

average over some time period was selected as a means to quantify the amount of error. To avoid scaling problems these raw quantities were normalized by dividing the RMS error by an RMS average of the true velocity values. This normalization provided a measure of the percentage error associated with making a modal damping assumption.

The second piece of information necessary to obtain the desired plots was a quantitative measure of the power activity of the intermode coupling element. The RMS average was selected for this measurement also. This measurement is also affected by scaling and must be normalized. The normal procedure in ENPORT is to normalize these power measurements by the maximum power average of the elements selected, yielding power as a percentage of maximum. Since the effect of the non-modal element is to exchange power with each modal oscillator, it would seem that normalizing this power by some measure of the power activity within a mode, namely the maximum power bond, would be appropriate.

This method was used in early portions of the study. It was noticed during the course of the early investigation that the element which exchanged the maximum amount of power would vary. It was felt that this inconsistency might hamper the process of developing the desired correlation. It was believed that a better scheme would be to compare the power activity in the damping bonds. Because the origin of intermode coupling is in the original system damping, this normalization would seem to provide a better insight into

the relative strength of the intermode coupling element. It is this relative strength which is the core of the issue since the modal damping assumption simply eliminates the intermode element and therefore its effect. If the effect is relatively weak then the elimination of the effect will not cause much variation. For this reason only the average power in the R element bonds was computed. The highest average was used as a baseline to normalize the other damping bonds. This proved to be fairly consistent over the course of the experiment since the first mode always seemed to contain the highest power level.

With these insights in mind, the experiment continued in an attempt to find a generic correlation between the normalized intermode power activity and the percentage of error resulting from a proposed modal assumption. Many groups of simulations were conducted for varied modal damping ratios. To avoid influencing the results with the input frequency, unit step forces were simultaneously applied at both inputs. This type of input forced a transient analysis as opposed to a steady state analysis which would be possible under a periodic form of input. Still it was felt that the correlations would be more easily obtained if forcing frequency was not a factor. The transient analysis did present a problem however, which was over what time period to compute the averages. Three times the longer

modal time constant was selected for this study. Potential problems resulting from this decision will be discussed in a later section.

4.3. The Results.

Each group of simulations consisted of holding all parameters fixed except the intermode coupling. This parameter was then varied over an allowable range as simulations were run for each setting. The allowable range for this parameter was determined using the transformed damping matrix. Since real physical systems were of interest in this study, the original damping parameters were selected to be positive. This restricted the range over which the off-diagonal term in equation 4.3c for a give set of diagonal terms. Recall that the off-diagonal term is the intermode coupling element parameter, while the diagonal terms are the parameters for the modal damping elements. For each of these groups of simulations, curves representing the average damping power activity versus percentage error were produced. A typical plot is shown in Figure 19. (Appendix B contains an ENPORT listing of the mode with the parameter values)

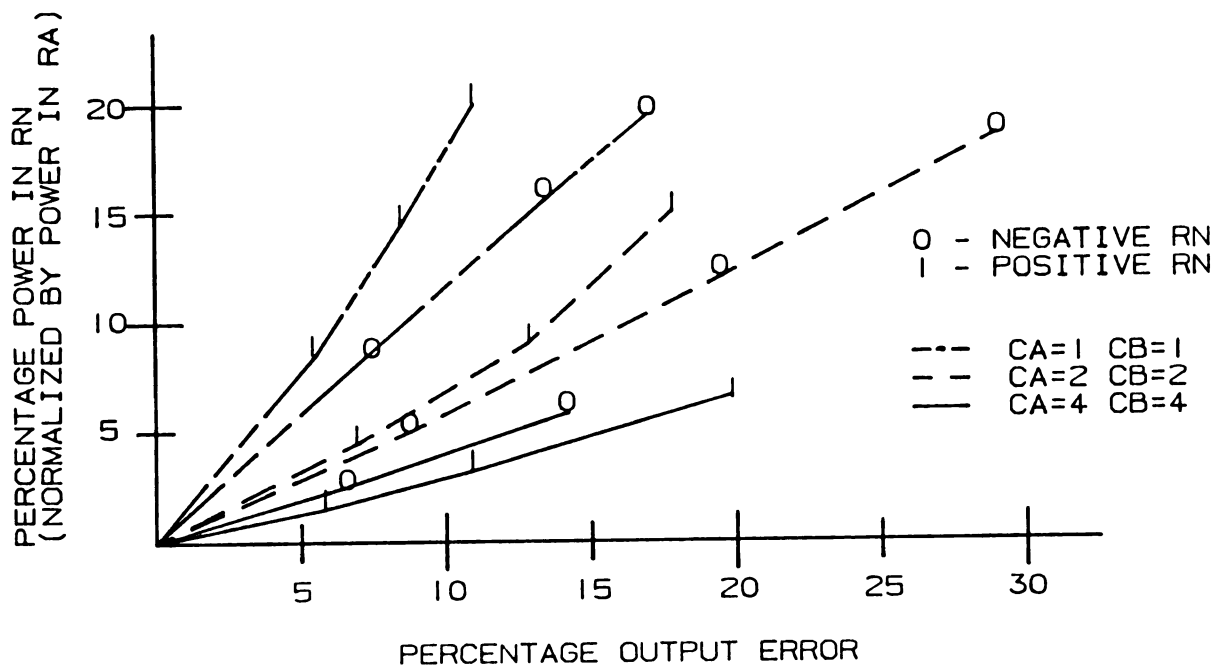


Figure 19

A Typical Power Versus Error Plot

The curves represented in this plot are the power dissipated in the coupling element (RD) for various levels of modal damping. The RMS values have been normalized by the average power in the first mode damping element (Ra). The power scale is therefore represented as percentage of power. Note that each data point is for a specific value of coupling parameter.

Each curve developed during the experiment exhibited the same expected trend of increased error as the modal coupling power activity increased. Notice however that each

power vs error curve is represented by two lines. This split occurs as the intermode element parameter is varied from its maximum negative value, through zero, and to its maximum positive value. This causes the split in the error curve since error increases as the magnitude of the intermode parameter is increased. At first it may seem strange that positive damping parameters in the original system could produce a negative damping parameter in the transformed equations. However, the analysis shows this to be perfectly acceptable. What does seem strange is that the power vs error curves are not the same for a negative coupling element parameter as for a positive parameter. In general the effect of the sign of intermode parameter on system behavior is not understood.

Although individually each group of simulations produce the expected results, the overall trends were not apparent. The slopes of the power vs error curves varied between groups of simulations as the modal damping parameters were changed. One set of simulations would show large errors corresponding to very small power percentages, while the next set would be the opposite. In the final analysis no trends could be found which could be used to provide the desired rule of thumb for making a modal damping assumption.

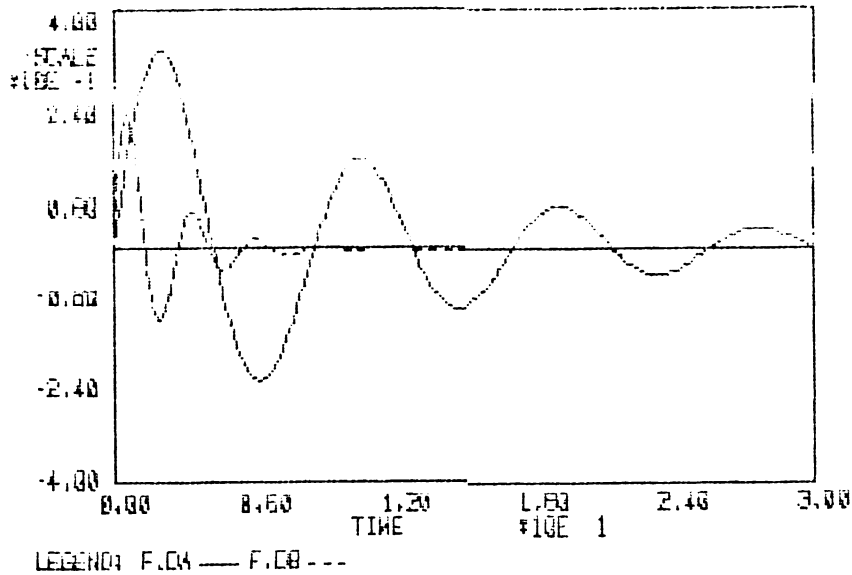
Some progress was made in observing trends in symmetric systems. These are systems in which the damping ratios are the same for both modes. The general trend observed in these systems was that as the damping ratio was increased

the sensitivity of output error to power percentage was also increased. That is, that as the amount of modal damping is increased, a relatively smaller amount of power activity in the coupling element causes a larger output error. This seems to go along with the common wisdom that for small damping the modal damping assumption can be made with little consequential error. This trend was only an observation and was not thoroughly verified.

4.4. Problems

Although the objectives of this study were not achieved, a great deal of information was obtained. Among this information are some of the potential reasons for not finding the desired correlations. It is possible that the desired correlation does not exist. However this author still believes that such a correlation can be found.

One of the important factors effecting this study was the selection of step forcing inputs. As stated earlier, this decision forced a transient form of analysis. Due to the nature of the transient response, simulation time has large effect on the averaged values used in this study. This point can be illustrated using the response curves shown in Figure 20.



MODAL RESPONSE

Figure 20

Modal Velocity Response Curves.

The curves shown in Figure 20 are the modal velocities plotted as a function of time. Notice that they are each typical underdamped responses of differing time constants. Using the methods from the experiment, these curves can be averaged over a time period of three times the longer time constant. If there is significant difference in time constants between the longer and the shorter, significant errors are induced in the average of the short time constant curve. These errors occur because for a substantial portion of the average period no activity is present. This tends to artificially reduce the averages, leading to power percentages that vary depending on the relationship between the

modal time constants. It is believed that this is part of the reason that some progress was made in the symmetric systems, since in these systems both modes possess the same time constant.

Another factor that strongly effected the analysis was the value of the transformer parameters. These parameters attenuate the inputs to and outputs from each of the modes, scaling the effect of each mode in the output response. Since the transformed output responses were used in the error percentage computations, large error in one mode may not result in a large error in the analysis, the second mode had far less influence in the output than the first. Therefore, large errors in the second mode produced very little change in the output response. Somehow, in order to find the desired general rule of thumb, this effect must be accounted for in the analysis.

4.5 Suggestions for Future Research.

During the course of this research a great deal of time was devoted to the development and initial analysis of the model and to the development of tools and techniques necessary to perform the analysis. Although the resulting research did not produce the desired results, the information gathered and documented in this report provides a good starting point for a second phase of research. With this information available at the start, it is believed that

further research into the behavior of the modal model will reveal the desired correlations.

One area of interest is to consider the effect of the two port coupling element as a modulated effort source since its effect is to provide an additional forcing input to each mode. Comparing the force level in the coupling elements to the other force inputs to the mode could provide some useful insights.

At this point it is also believed that the step inputs to the system should be replaced by periodic forcing. This will provide a periodic steady state response and avoid the problems associated with averaging the transients as described above. Elimination of this problem should make the trends more apparent although forcing frequency effects will now need to be accounted for.

Based on the results that were obtained in this experiment it is believed that the proposed trends are present and that some measure of coupling activity can be correlated to the amount of error associated with a modal assumption. It is envisioned that a second phase of research that combines the information presented in this report with some further creative insights will produce these correlations.

Chapter 5

Summary

This research began as an investigation into linear non-modally damped systems using bond-graphs. It was hoped that a bond-graph approach might lead to new insights into the effects of damped coupling in the transformed equations. The ultimate goal was to develop some guidelines indicating the amount of output error potentially incurred by making a modal damping assumption on a given system. When possible, this assumption allows a simplified analysis, as well as reduces the model complexity.

The first step in the process was to develop a modal model that accurately represents non-modally damped systems. Using a set of coupled transformed equations, a standard modal bond-graph was modified to include the coupling effect. It was found that a standard 2-port R element, connected between the modes would accurately represent the coupling effect. Although this element is commonly governed by two parameters, only one was necessary in this case due to symmetry of the transformed damping matrix. This fact lead to the interesting property that the element acts alternately as a power source or sink to both modes simultaneously, providing or dissipating equal amounts of power to both modes.

After developing and verifying the model the next step was to begin to use the model to study the effect of intermode coupling strength on output response. This study required the comparison of output responses of a series of simulations. The problem here was that ENPORT software did not facilitate these types of direct comparisons. Because these types of comparisons are often desired when using ENPORT in the design process, a decision was made to enhance the package by developing software that would include this feature.

With all the tools in place the study proceeded in an attempt to find a correlation between intermode coupling activity and output response error. Net power activity was used as the basis to quantify the amount of coupling effect. Output error was defined as the difference between the output velocities with intermode coupling and the same velocities with coupling set to zero, as if assuming modal damping. RMS average was used to quantify both effects. To avoid a reflection of forcing frequency in the modal response, a step input was used in the study. This resulted in a transient form for the corresponding responses. Three times the longest time constant was selected as the standard time base for computing the RMS averages.

Groups of simulations were run for varied damping parameters. In each group, the value of the intermode coupling parameter was varied. Plots of net average power versus output response error were constructed for each group of

simulations. To include the effect of scaling between groups or simulations, the power average was normalized by the highest average power transferred in the modal damping elements. The average velocity errors were normalized by their respective average velocities. These normalizations provided measures that were interpreted as percent power and percent error and it was from these values that plots were constructed.

As predicted each plot showed the expected trend of increased error percentage as a coupling power percentage increased. The problem came in attempting to draw any correlations from one group to the next. The slope of each plot seemed to vary depending on the overall amount of damping. That is to say that for one plot 10% power would correspond to 3% error while in the next plot 10% power would correspond to 15% error. Because of this plot to plot variation, no universal correlations could be found. However, it was observed that for smaller damping higher power percentages seemed to correspond to smaller errors.

although the desired 'rule of thumb' for assuming modal damping was not found, much valuable information was gained. Tools and techniques were developed that might be valuable to future research in this area or possibly some other related areas. Additionally, one avenue was explored and the possible road blocks pointed out which provide insights and direction to future research in this area. I hope some future research will reveal the desired correlations.

This information includes tools and techniques that can be used as a starting point for a second phase of investigation that might reveal the desired correlations.

LIST OF REFERENCES

LIST OF REFERNECES

- 1/ I. Fawzy. "A Theorm on the Free Vibration of Damped Systems." Journal of Applied Mechanics, Trans. ASME, March 1977, 132-134.
- 2/ D.L. Margolis; and G.E. Young, "Reduction of Models of Large Scale Lumped Structures Using Normal Modes and Bond Graphs." Journal of the Franklin Institute. Vol. 304, No. 1, July 1977, 65-79.
- 3/ R.C. Rosenberg; and D.C. Karnopp, Introduction to Physical System Dynamics, New York: McGraw-Hill, 1983.
- 4/ ENPORT Reference Manual, Rosencode Associates, Inc. Lansing, Michigan, 1989.

APPENDIX A


```

CFILE: DRUNI1 ----- DRUNI1 -----

      INCLUDE 'ENPORT>ROSENBERG>SHIP72>UTILBK.CBK'
C
      INTEGER I
      LOGICAL FULL, OKAY
      CHARACTER TOKEN*1, STRING*80
C
      EXTERNAL MENSET, BLNKLN, WRTSTR, PROMPT, GETANS, GOON, INVOPT
      EXTERNAL DATPUT, DATGET, DBGRUN, ANLDVR
C
C***DATRUN*****
C
      CALL MENSET(.TRUE.)
      FULL = MHFULL
      NDSET = 0
C
C---- Present the run_data menu
10  CONTINUE
      IF (FULL) THEN
          CALL BLNKLN
          CALL BLNKLN
          CALL WRTSTR(' Run_data processing options')
          CALL WRTSTR(' -----')
          CALL WRTSTR(' W: Write run_data to file')
          CALL WRTSTR(' L: Load run_data from file')
          CALL WRTSTR(' S: Status of run_data')
          CALL WRTSTR(' C: Clear the run_data')
          CALL WRTSTR(' D: Debug print of the data')
          CALL WRTSTR(' A: Analyze the run_data')
          CALL WRTSTR(' R: Return to the main menu')
          CALL WRTSTR(' -----')
          STRING = ' Enter option (R): '
      ELSE
          CALL BLNKLN
          CALL BLNKLN
          CALL WRTSTR(' Run_data processing options')
          CALL WRTSTR(' -----')
          STRING = ' Write, Load, Status, Clear, Debug, Analyze, Return?'
1(full):
      ENDIF
C
C--- NOW READ THE COMMAND TOKEN
      CALL PROMPT(STRING)
      TOKEN = '*'
      CALL GETANS(TOKEN)
C
C--- NOW INTERPRET THE TOKEN
      IF (TOKEN.EQ. '#') THEN
          IF (FULL) THEN
              TOKEN = 'R'
          ELSE
              FULL = .TRUE.
              GOTO 10
          ENDIF
      ENDIF
C
      IF (TOKEN.EQ. 'W') THEN
          CALL DATPUT(OKAY)
          CALL GOON
      ELSEIF (TOKEN.EQ. 'L') THEN
          CALL DATGET(OKAY)
          CALL GOON
      ELSEIF (TOKEN.EQ. 'S') THEN
          CALL BLNKLN
          CALL WRTSTR(' Names of data sets loaded: ')
          IF (NDSET.GT. 0) THEN
              DO 50 I = 1, NDSET
30          CALL WRTSTR('   '//DSNAM(I))
              CALL GOON
          ELSE
              CALL WRTSTR(' No run data has been loaded. ')
          ENDIF
      ELSEIF (TOKEN.EQ. 'C') THEN
          NDSET = 0
      ELSEIF (TOKEN.EQ. 'D') THEN
          CALL DBGRUN
          CALL GOON
      ELSEIF (TOKEN.EQ. 'A') THEN
          CALL ANLDVR
      ELSEIF (TOKEN.EQ. 'R') THEN
          RETURN
      ELSE
          CALL INVOPT

```



```

CFILE: DRUN11 ----- DRUN11 -----

C----- Set the data set name temporarily
          DSNAMT= FILNAM
          DO 20 I= 1,NDSET
            IF (DSNAMT.EQ.DSNAM(I)) THEN
              CALL BLNKLN
              CALL WRTSTR(' *** Duplication of data set already loaded. ')
              CALL WRTSTR(' Please try again. ')
              GOTO 10
            ENDIF
          20 CONTINUE
          C
          NDSET= NDSET +1
          DSNAM(NDSET)= DSNAMT
          C
          C----- Read the heading line, results file name, original model title,
          C and run label
          REWIND(UNIT)
          READ(UNIT,ERR=900,END=950) TPRCID,TVRSN,TIM,DAT
          READ(UNIT,ERR=900,END=950) FILNAM
          READ(UNIT,ERR=900,END=950) NAMTMP
          READ(UNIT,ERR=900,END=950) RUNLBL(NDSET)
          C
          C----- Check to see if correct model is loaded
          IF (NAMTMP.NE.NAME) THEN
            CALL WRTSTR(' Incorrect model loaded')
            CALL WRTSTR(' The model '//NAMTMP)
            CALL WRTSTR(' Must be loaded to proceed')
            NDSET=NDSET-1
            RETURN
          ENDIF
          C
          C----- Check if file is correct
          IF (FILNAM.NE.DSNAMT) THEN
            CALL BLNKLN
            CALL WRTSTR(' File name mismatch. ')
            CALL WRTSTR(' File '//DSNAMT//' was requested')
          ENDIF
          1010 WRITE(STRING,1010) TPRCID,TVRSN
          FORMAT(' Written by ENPORT-',A1,' ',12)
          CALL WRTSTR(STRING)
          STRING=' Date written: '//DAT
          CALL WRTSTR(STRING)
          STRING=' Time written '//TIM
          CALL WRTSTR(STRING)
          CALL BLNKLN
          CALL WRTSTR(' The run label: ')
          CALL WRTSTR(RUNLBL(NDSET))
          C----- Check if it is ok
          CALL BLNKLN
          IF (.NOT. E7YORN(' Is this the data set you want?'.TRUE.)) THEN
            NDSET=NDSET-1
            RETURN
          ENDIF
          C
          C----- Read the number and list of run vbl names
          READ(UNIT,ERR=900,END=950) RNS(NDSET),RTIN(NDSET),RT2SLV(NDSET)
          READ(UNIT,ERR=900,END=950) RNOVV(NDSET)
          READ(UNIT,ERR=900,END=950) (ROVNL(I,NDSET),I=1,RNOVV(NDSET))
          C
          C----- Check if time data match reference set.
          STRING=' *** File is not compatible with reference set'
          IF (RTIN(NDSET).NE.RTIN(1)) THEN
            CALL BLNKLN
            CALL WRTSTR(STRING)
            CALL WRTSTR(' Mismatch in the initial times. ')
            NDSET=NDSET-1
            RETURN
          ENDIF
          IF (RT2SLV(NDSET).NE.RT2SLV(1)) THEN
            CALL BLNKLN
            CALL WRTSTR(STRING)
            CALL WRTSTR(' Mismatch in the final times. ')
            NDSET=NDSET-1
            RETURN
          ENDIF
          IF (RNS(NDSET).NE.RNS(1)) THEN
            CALL BLNKLN
            CALL WRTSTR(STRING)
            CALL WRTSTR(' Mismatch in the number of stages saved. ')
            NDSET=NDSET-1
            RETURN
          ENDIF

```



```

CFILE: DRUN11 ----- DRUN11 -----

C      INPUTS: DELTA, T1, T2
C      OUTPUTS: IDEL
C
C      INCLUDE 'SIZEBK.CBK'
C      INCLUDE 'SOLNBK.CBK'
C      INCLUDE 'POSTBK.CBK'
C
C      REAL DTHI, TINC
C      LOGICAL NEWLIN, ENDLIN
C      CHARACTER*70 STRING
C
C      EXTERNAL BLNKLN, WRTSTR, PROMPT, GETRL
C
C****DIFDEL*****
C
C      CALL BLNKLN
C      CALL WRTSTR(' Set time increment for display: ')
C      CALL BLNKLN
C
C      TINC=DELTA
C      WRITE (STRING, 1010) DELTA
1010  FORMAT(' Enter display increment (', 1PE11.4, '): ')
C      CALL PROMPT (STRING)
C      NEWLIN=.TRUE.
C      DTHI=T2-T1
C      CALL GETRL (DELTA, TINC, DTHI, NEWLIN, ENDLIN)
C
C      IDEL= (DELTA+.001*TINC)/TINC
C
C      RETURN
C      END
C>>>>>
C
C****DIFLST*****
C
C      SUBROUTINE DIFLST (DATIM)
C
C      Purpose:
C      List the difference values in the loaded run data.
C
C      INCLUDE 'ENPORT>ROSENBERG>SHIP72>SIZEBK.CBK'
C      INCLUDE 'ENPORT>ROSENBERG>SHIP72>SOLNBK.CBK'
C      INCLUDE 'ENPORT>ROSENBERG>SHIP72>POSTBK.CBK'
C      INCLUDE 'RDATA BK.CBK'
C
C      LOGICAL DATIM
C      INTEGER I, J, N, JTIM, VC2IS
C      REAL TIM, DFRSLT
C      CHARACTER DATE*8, TIMECH*8, STRING*80
C
C      EXTERNAL GETTD, BLNKLN, WRTSTR, VC2IS, DFRSLT
C
C****DIFLST*****
C
C---- Put up list heading
C      CALL BLNKLN
C      WRITE (STRING, 1010) NDSET, DSNAM (NDSET)
1010  FORMAT (1X, 'Data set ', I2, ': ', (A))
C      CALL WRTSTR (STRING)
C      CALL WRTSTR (' Minus ')
C      WRITE (STRING, 1020) DSNAM (1)
1020  FORMAT (1X, 'Reference data set: ', (A))
C      CALL WRTSTR (STRING)
C      CALL BLNKLN
C
C---- Put up the date and time
C      IF (DATIM) THEN
C          CALL GETTD (TIMECH, DATE)
C          IF (DATE.NE.' ') THEN
1005  WRITE (STRING, 1005) DATE, TIMECH
C          FORMAT (1X, 'Date: ', AB, ' Time: ', AB)
C          CALL WRTSTR (STRING)
C          ENDDIF
C      ENDIF
C
C---- Put up title and column headings
C      CALL BLNKLN
C      STRING=' '//LABEL (1:71)
C      CALL WRTSTR (STRING)
C      CALL BLNKLN
C      WRITE (STRING, 1000) (CLBL (J), J=1, NREGD)

```


CFILE: DRUNI1 ----- DRUNI1 -----

```

C---- Do the horizontal axis here (one variable only)
IF(HORZFG)THEN
15  CALL BLNKLN
    ANS= CLBL(0)
    CALL PROMPT(' Variable for the horizontal axis? (///
1    ANS(1:NCHARS(ANS))//'):')
    CALL GETANS(ANS)
    SPEAKF= .TRUE.
    KVBLX= RVBLX(ANS,SPEAKF)
    IF (KVBLX.EQ.0) THEN
        GOTO 15
    ELSE
        JDX(0)= KVBLX
        CLBL(0)= ANS
    ENDIF
ENDIF
C
C---- And now the vertical axis variables...
CALL BLNKLN
CALL WRTSTR (' The current display list:')
CALL BLNKLN
WRITE(STRING,1020) (CLBL(J),J=1,NREQD)
CALL WRTSTR(STRING)
1020 FORMAT(3(1X,A12))
C
C---- Check if the list is okay
IF (E7YORN(' Do you want to change the list?'.FALSE.)) THEN
    CALL BLNKLN
    CALL WRTSTR(' Enter a new list on one line (or HELP):')
    CALL BLNKLN
    CALL PROMPT(' >')
    NEWLIN= .TRUE.
    NRED= 0
C----- Commence to read the line
200  ANS=BLANK
    CALL GETWD (ANS,NEWLIN,ENDLIN)
    IF (ANS.NE.BLANK) THEN
        NRED= NRED+1
        DVNAM(NRED)= ANS
        IF (NRED.EQ.MAXNRQ) GOTO 203
        NEWLIN= .FALSE.
        GOTO 200
    ENDIF
C----- Commence to identify the entries
203  SPEAKF= .TRUE.
    NREQD= 0
    DO 220 I= 1,NRED
        KVBLX= RVBLX(DVNAM(I),SPEAKF)
C----- Implicitly ignore bad entries
        IF (KVBLX.GT.0) THEN
            NREQD= NREQD+1
            JDX(NREQD)= KVBLX
            CLBL(NREQD)= DVNAM(I)
        ENDIF
220  CONTINUE
C----- This list has been cracked
IF (NREQD.EQ.0) GOTO 10
IF (E7YORN(' Do you want to see the list again?'.FALSE.))
1  GOTO 10
ENDIF
C
C---- Set the JDX entries for derived vbls to proper indices
NODV= 0
IF (JDX(0).GT.NOVV) THEN
    NODV= NODV+1
    JDX(0)= NOVV+NODV
ENDIF
DO 250 I= 1,NREQD
    IF (JDX(I).GT.NOVV) THEN
        NODV= NODV+1
        JDX(I)= NOVV+NODV
    ENDIF
250  CONTINUE
C
IF (DBGFLG) THEN
    CALL BLNKLN
    CALL WRTSTR(' *** Debug data for derived vbls:')
    CALL WRTSTR(' DEBUG MODE NOT ACTIVE')
    WRITE(STRING,9010) NREQD, NODV, NOVV
C9010  FORMAT(5X,'NREQD, NODV, NOVV: ',3I5)
    CALL WRTSTR(STRING)
    WRITE(STRING,9020) (JDX(I),I=0,5)
C

```



```

CFILE: DRULOC-----DRULOC-----
        CLOSE(UNIT)
        GO TO 18
    ENDIF
C
C--- The file most probably does not exist or it is not a sequential,
C   unformatted one.
C
B010 CONTINUE
      CLOSE(UNIT, ERR=8011)
B011 CONTINUE
      CALL INQUIR(LFNAME, 'UNFORMATTED', EXISTS)
      IF (EXISTS) THEN
          CALL BLNKLN
          CALL WRTSTR(' *** The file "'//LFNAME(1:JCFILE)//'" already'//
1         exists and')
          CALL WRTSTR('      has incompatible form and/or access mode. ')
          CALL WRTSTR('      Please specify another file name. ')
          GO TO 18
      ELSE
C----- PRIMOS style (default)
          OPEN(UNIT, FILE=LFNAME, STATUS='NEW', ERR=8040,
1         FORM='UNFORMATTED', ACCESS='SEQUENTIAL')
C
          GO TO 16
      ENDIF
C
C--- The file exists, but it seems to be empty.
C
B020 CONTINUE
      CLOSE(UNIT)
C----- PRIMOS style
      OPEN(UNIT, FILE=LFNAME, STATUS='OLD', ERR=8010,
1     FORM='FORMATTED', ACCESS='SEQUENTIAL')
C
      REWIND(UNIT, ERR=8010)
      READ(UNIT, '(A)', END=12, ERR=8030) TESTLN
      GO TO 8010
12     CONTINUE
      GO TO 16
C
C--- The file exists, however, it is inaccessible.
C
B030 CONTINUE
      CLOSE(UNIT)
      CALL BLNKLN
      CALL WRTSTR(' *** The file "'//LFNAME(1:JCFILE)//'" already'//
1     exists. ')
      CALL WRTSTR('      Moreover, it is inaccessible. ')
      CALL WRTSTR('      Please specify another file. ')
      GO TO 18
C
C--- The new file cannot be opened.
C
B040 CONTINUE
      CLOSE(UNIT, ERR=8041)
B041 CONTINUE
      CALL BLNKLN
      CALL WRTSTR(' *** The file "'//LFNAME(1:JCFILE)//'" cannot'//
1     be opened. ')
      CALL WRTSTR('      Please specify another file. ')
      GO TO 18
C
C--- TEST for writing privileges and/or disk space availability
C
C
16     CONTINUE
      REWIND(UNIT)
      WRITE(UNIT, ERR=8050) TESTLN
      REWIND(UNIT)
      GO TO 20
C
C--- Unable to write to the file.
C
B050 CONTINUE
      CLOSE(UNIT)
      CALL BLNKLN
      CALL WRTSTR(' *** Unable to write to the file. ')
      CALL WRTSTR('      Please specify another filename ')
C
C--- CHANGE default name if necessary.
C
18     CONTINUE

```


APPENDIX B

APPENDIX B

An ENPORT File Listing of
The System Used
In The Study

HEADING

HEADING

FILE
RESRCH

13:51:54 11/01/88 ENPORT-7.2

TITLE
RESEARCH INTO MODAL DECOMPOSITION W/ DAMPING

SYSTEM GRAPH DESCRIPTION

NODE	TYPE	XLOC	YLOC
1A	MIG	300.	600.
1B	MIG	800.	600.
1A	MIG	300.	800.
CA	MCG	100.	600.
O1	MOG	300.	300.
TF1A	MTG	300.	500.
O2	MOG	800.	300.
TF2A	MTG	700.	400.
TF2B	MTG	800.	500.
1B	MIG	800.	800.
CB	MCG	1000.	300.
SE2	MEG	300.	200.
SE1	MEG	300.	200.
TF1B	MTG	400.	400.
RA	MRG	100.	800.
RB	MRG	1000.	800.
RN	MRG	600.	600.

CONNECTOR	TYPE	FROM	TO	VERTICFS
S1	BG	SE1	O1	
S2	BG	SE2	O2	
PA	BG	1A	1A	
QA	BG	1A	CA	
PB	BG	1B	1B	
QB	BG	1B	CB	
1	BG	O1	TF1A	
2	BG	TF1A	1A	
3	BG	O1	TF1B	
4	BG	TF1B	1B	
5	BG	O2	TF2A	
6	BG	TF2A	1B	
7	BG	O2	TF2B	
8	BG	TF2B	1A	
DA	BG	1A	RA	
DB	BG	1B	RB	800. 600.
NA	BG	1A	RN	
NB	BG	1B	RN	

GRAPHICAL ENVIRONMENT

```

Node size: 1.0000E+00
Connector size: 6.2500E-01
Scale factor: 9.0000E-01
Horizontal window minimum: -8.8894E+01
Horizontal window maximum: 1.1889E+03
Vertical window minimum: 1.0000E+02
Vertical window maximum: 9.0000E+02
Gridding enabled: ON
Gridding visible: ON
Grid size: 1.0000E+02
Default node regime enabled: ON
Default connector regime enabled: OFF
Default node regime: C
Default connectr regime: C
Postprocessor grid: ON
Postprocessor box: ON

```

```

Color 1 0 30 0 Dark Gray (background)
Color 2 0 100 0 White (default line and label color)
Color 3 120 50 100 Red (attention)
Color 4 240 50 100 Green (Mechanical)
Color 5 0 80 100 Blue (Rotational)
Color 6 300 50 100 Cyan (Electrical)
Color 7 60 50 100 Magenta (Hydraulic)
Color 8 180 50 100 Yellow (Thermal)

```

HEADING

NODE EQUATIONS

```

Number of outputs:      14

Node:  IA          Connectors:  PA
Equation: Y = GAIN ( X, P )      1 1 1
Y_list   X_list
E_PA     P_PA                    Parameters
9.1743E-02

Node:  CA          Connectors:  GA
Equation: Y = GAIN ( X, P )      1 1 1
Y_list   X_list
E_GA     G_GA                    Parameters
6.0000E+00

Node:  TF1A        Connectors:  1      2
          = MOD. TF1A * F.2
          = MOD. TF1A * F.1
Equation: Y = CON ( X, P )      1 0 1
Y_list   X_list
MOD. TF1A
Parameters
1.0000E+00

Node:  TF2A        Connectors:  7      8
          = MOD. TF2A * F.8
          = MOD. TF2A * F.7
Equation: Y = CON ( X, P )      1 0 1
Y_list   X_list
MOD. TF2A
Parameters
2.2250E+00

Node:  TF2B        Connectors:  5      6
          = MOD. TF2B * F.6
          = MOD. TF2B * F.5
Equation: Y = CON ( X, P )      1 0 1
Y_list   X_list
MOD. TF2B
Parameters
-2.2500E-01

Node:  IB          Connectors:  PB
Equation: Y = GAIN ( X, P )      1 1 1
Y_list   X_list
F_PB     P_PB                    Parameters
9.0827E-01

Node:  CB          Connectors:  GB
Equation: Y = GAIN ( X, P )      1 1 1
Y_list   X_list
E_GB     G_GB                    Parameters
6.0000E+00

Node:  SE2         Connectors:  S2
Equation: Y = STEP ( X, P )      1 1 2
Y_list   X_list
E_S2     TIME                    Parameters
0.0000E+00
1.0000E+00

Node:  SE1         Connectors:  S1
Equation: Y = STEP ( X, P )      1 1 2
Y_list   X_list
E_S1     TIME                    Parameters
0.0000E+00
1.0000E+00

Node:  TF1B        Connectors:  3      4
          = MOD. TF1B * F.4
          = MOD. TF1B * F.3
Equation: Y = CON ( X, P )      1 0 1
Y_list   X_list
MOD. TF1B
Parameters
1.0000E+00

Node:  RA          Connectors:  DA
Equation: Y = GAIN ( X, P )      1 1 1
Y_list   X_list
E_DA     F_DA                    Parameters
1.0000E+00

Node:  RB          Connectors:  DB
Equation: Y = GAIN ( X, P )      1 1 1
Y_list   X_list
E_DB     F_DB                    Parameters
1.0000E+00

Node:  RN          Connectors:  NA      NB
Equation: Y = GAIN ( X, P )      1 1 1
Y_list   X_list
E_NA     F_NB                    Parameters
0.0000E+00
Equation: Y = GAIN ( X, P )      1 1 1
Y_list   X_list
E_NB     F_NA                    Parameters
0.0000E+00

```

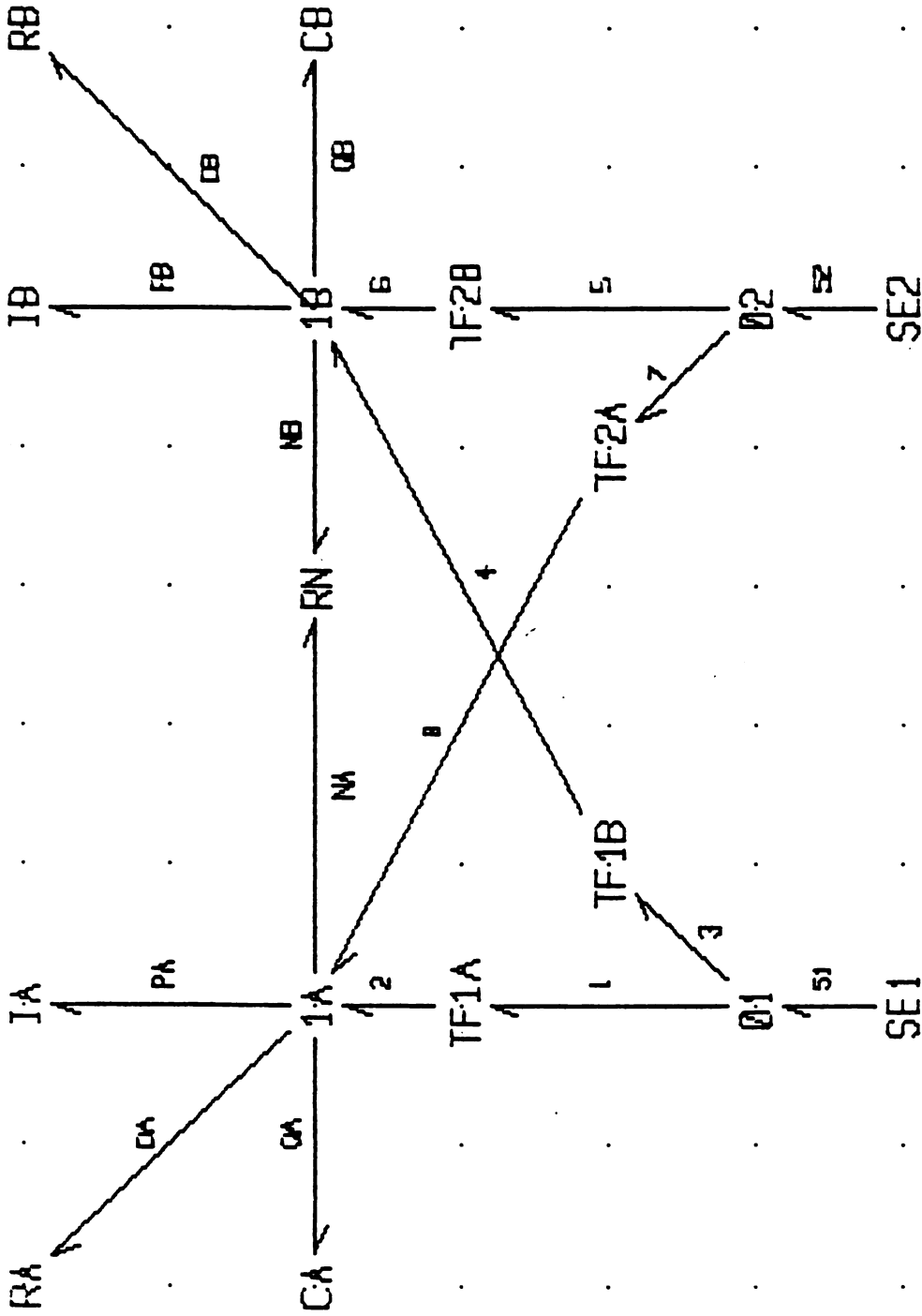


Figure 21
The ENPORT Bond-Graph for
the System Studied

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