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APPLICATIONS OF GEOSTATISTICS  
TO SETTLEMENT PROBLEMS

presented by

Mostafa Kamal Ashoor

has been accepted towards fulfillment  
of the requirements for

Doctor of Philosophy degree in Civil Engineering

  
Major professor

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APPLICATIONS OF GEOSTATISTICS TO SETTLEMENT PROBLEMS

By

Mostafa Kamal Ashoor

A DISSERTATION

Submitted to  
Michigan State University  
in partial fulfillment of the requirements  
for the degree of

DOCTOR OF PHILOSOPHY

Department of Civil and Environmental Engineering

1994



## ABSTRACT

### APPLICATIONS OF GEOSTATISTICS TO SETTLEMENT PROBLEMS

By

Mostafa Kamal Ashoor

This research investigates improved techniques for estimating and modeling the settlement and differential settlement of shallow foundations on noncohesive soils using standard penetration test (SPT) values.

Two geostatistical methods (trend surfaces and Kriging) are employed to model the spatial variability of the soil standard penetration resistance values (commonly known as N values) in a three-dimensional field.

Lack of homogeneity in the N-value data in the vertical direction:

1. is analyzed geotechnically considering such factors as the soil's relative density, overburden pressure, stiffness...etc, and;
2. is accounted for by a nonconstant-mean assumption, and;
3. is tested by statistical multiple comparison techniques.

Aside from the difficulties of measurement bias; which is accounted for by the above-mentioned modeling, the N values vary from point to point, and the question remains as to how to combine the varying measurements of N values within the depth of influence under the foundation footing into one design N value to be used for the deterministic models for settlement estimation.

This question is tackled in this research by using the two geostatistical methods to generate a two-point estimate for the design N value. The two point estimates are in

turn weighted to obtain a single value. It is hoped that this proposed estimate will be favoured by foundation designers on the grounds of simplicity.

Moreover, this two-point estimate is shown to be:

1. Comparable to the current procedures of estimating the design N value in the sense that the N values at different depths are given similar weights in both methods.
2. Easy to be used by the spatial models in a way that can help in transforming the spatial  $N(x,y,z)$  models into a planar  $N(x,y)$  or  $S(x,y)$  model. This latter model enables one to do such things as contouring analysis or planar settlement comparisons and such like.

The developed method is verified using simulated data of an assumed field. The practical reliability is then tested by conducting the modeling on available case histories.

The modeling of N values and settlements proposed herein provides a capability to study the practicality of obtaining more N data in order to take advantage of the resulting lower degree of uncertainty.

## ACKNOWLEDGMENTS

Researching and writing a dissertation is a job that is shared by more than the author. I had terrific support from my committee chairman and academic advisor, Professor Thomas Wolff. I wish to acknowledge my appreciation to his scientific talent, self denial and modesty. No words can express my gratefulness to him. As a former engineer for the U.S. Corps of Engineers, the greatest engineering agency in the world, his practical suggestions and criticisms were invaluable in alerting me to problem areas in the research and helping me to sort them out and improve the clarity of the developed models. I shall remain ever grateful for his academic guidance and support. I wish to give special thanks and express my acknowledgment and gratitude to Professor Mark Snyder, who kindly, together with Professor Wolff, volunteered his time and effort to help me to keep going when things seemed to stop. His unceasing encouragement and scientific guidance helped make this research a reality. I sincerely wish to express my appreciation and gratitude to the chairperson of Civil and Environmental Engineering. Professor William Saul, for his assistance, support and his talented leadership. I also would like to extend my sincere appreciation and gratitude to the other members of my guidance committee, Professors William Taylor, Dennis Gilliland and Ronald Harichandran for their valuable suggestions and constructive criticisms.

Finally, thanks to my family members; my wife Magda and my wonderful children Radwa, Kamal and Sara. They helped me keep going and keep on schedule ("What chapter are you writing now, Dad?").



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## CHAPTER 1

### BACKGROUND , OBJECTIVES , AND SCOPE

#### INTRODUCTION

Soil deformations under the action of external loads are a major consideration in the design of structure foundations. Nevertheless, economic feasibility dictates that estimation of such deformations must be made on little and widely scattered test data which may have considerable variability. In this study an investigation is made as to how such information can be best "mathematically" represented, and whether improved performance predictions can be made using such representation.

The loads that can be safely applied to a foundation soil should fulfill two conditions:

The shear stresses developed in the soil mass should not exceed some tolerable fraction of the shear strength.

The settlement and differential settlements of foundations should not exceed certain tolerable values.

In cohesive soils, both of these criteria require careful evaluation. Cohesionless soils, on the other hand, have good bearing capacity in most cases and settlement usually governs the design of shallow foundations. Except for relatively narrow shallow footings on these materials where the water table is high, the allowable pressure which may be applied to a cohesionless foundation will be governed by settlement considerations

rather than shear strength. In other words, for dimensioning shallow foundations supported by a sandy soil, the failure parameter for verifying stability with respect to shear failure is less critical and the design of this type of foundations will be dictated by the deformation parameter for estimating the settlements, (See, e.g., Jeyapalan, 1986; Moussa, 1982; Simons and Menzies, 1976). Meyerhof (1956) and D'Appolonia and Brissette (1968) quantify these recommendations noting that, except in cases where the footing width is less than about 3 ft or 4 ft, the allowable footing settlement is usually exceeded before bearing capacity considerations become important.

The state-of-the-practice for shallow foundation design is that penetration resistance measurements, commonly called "N values", are obtained from the standard penetration test (SPT) and used to estimate settlements, either directly or as predictors of elastic parameters.

The focus of this study is to develop a statistically rational approach for employing the SPT N values to estimate the settlements and differential settlements of shallow foundations on sandy soils. The approach would be of particular benefit where the number of footings to be designed is greater than the number of borings.

In this dissertation the SPT test results will be referred to as N values, and the function which represents the N value as a function of the testing location will be referred to as the N function.

Although the current procedures of estimating the settlements in sands have different views regarding the depth of influence of a footing (e.g. Meyerhof 1965, Schmertmann 1970, Burland 1985) or the correction of the N values (e.g. Skempton and Liao 1986, Seed et al. 1976, Peck et al. 1974, Bazaraa 1967), most of them adopt



er a simple average or a weighted average of the N values. In contrast, the application of geostatistics to the N data - as suggested by this research - will employ available data to formulate a model for the N values.

Two modeling techniques are proposed to model an N function on the assumption the N values which are obtained from a limited number of borings are considered as a sample and that the N function has estimated parameters and associated uncertainty. The function developed from this sample will be used to estimate the penetration resistance function of the whole site, and in particular to generate "two-point estimates" under footing locations.

It is stated (DeGroot and Baecher, 1993) that, stronger inferences are possible by dealing with data statistically than by relying solely on intuitive data interpretation. Geostatistical spatial models are a relatively recent addition to the statistics literature and their applications are being used with increasing frequency. Yeh (1991) stated that, any discipline that works with data collected from different locations needs to develop models that indicate when there is dependence between measurements at different locations. These models are used to summarize observed data or to predict unobserved ones. The strength of geostatistics (Isaacs and Crosson, 1963) over more classical approaches is that it recognizes spatial variability both at the large scale and the small scale, or in statistical notation it models spatial trends and spatial correlation.

This research considered two applications of geostatistics, namely the "trend analysis" and the "interpolation modeling techniques" such as Kriging, to estimate the spatial trends or to model the spatial correlation of the N values.

Modeling the N data using a geostatistical approach will make it possible to assign a degree of uncertainty to the resulting model. This concept is used to illustrate a technique for assessing the trade off between the estimation precision and the SPT sampling costs. Such a technique will help the engineer designer decide on a sampling plan which is reasonable for any foundation exploration scheme.

The planar (x,y) settlement function obtained from the methods developed permits one to draw contours of the settlements, which can be used to help the designer to adjust sizes as necessary to minimize expected settlement or differential settlement over the site. Such contours could be used also to predict the expected shape of the settlement surface under buildings with many similarly - located footings over broad areas such as warehouses and parking ramps.

The accuracy of the proposed models is tested two ways. First they are verified using simulated data of an assumed field. Second, their practical reliability is checked by conducting the suggested modeling on a number of available case histories. A comparison is made between the predicted settlements using these approaches and the values which were reported by the previous investigators.

## THE FRAMEWORK OF SETTLEMENT PREDICTION FROM N VALUES

The estimation of the correlation between a series of a scattered N values been studied by a number of investigators, for example (Vanmarcke, 1977), (Kikula, 1983). However, none of them developed a model to help in selecting a design value from scattered data for settlement prediction.

Most of the current procedures for predicting footing settlements in sands from SPT test adopt either a simple average or a weighted average of the N values. The use of an average N value implies the prediction of the average settlement. It also implies some statistical relationship between the data. A prudent designer would desire either an accurate prediction or at least a likely maximum predicted value for the settlement.

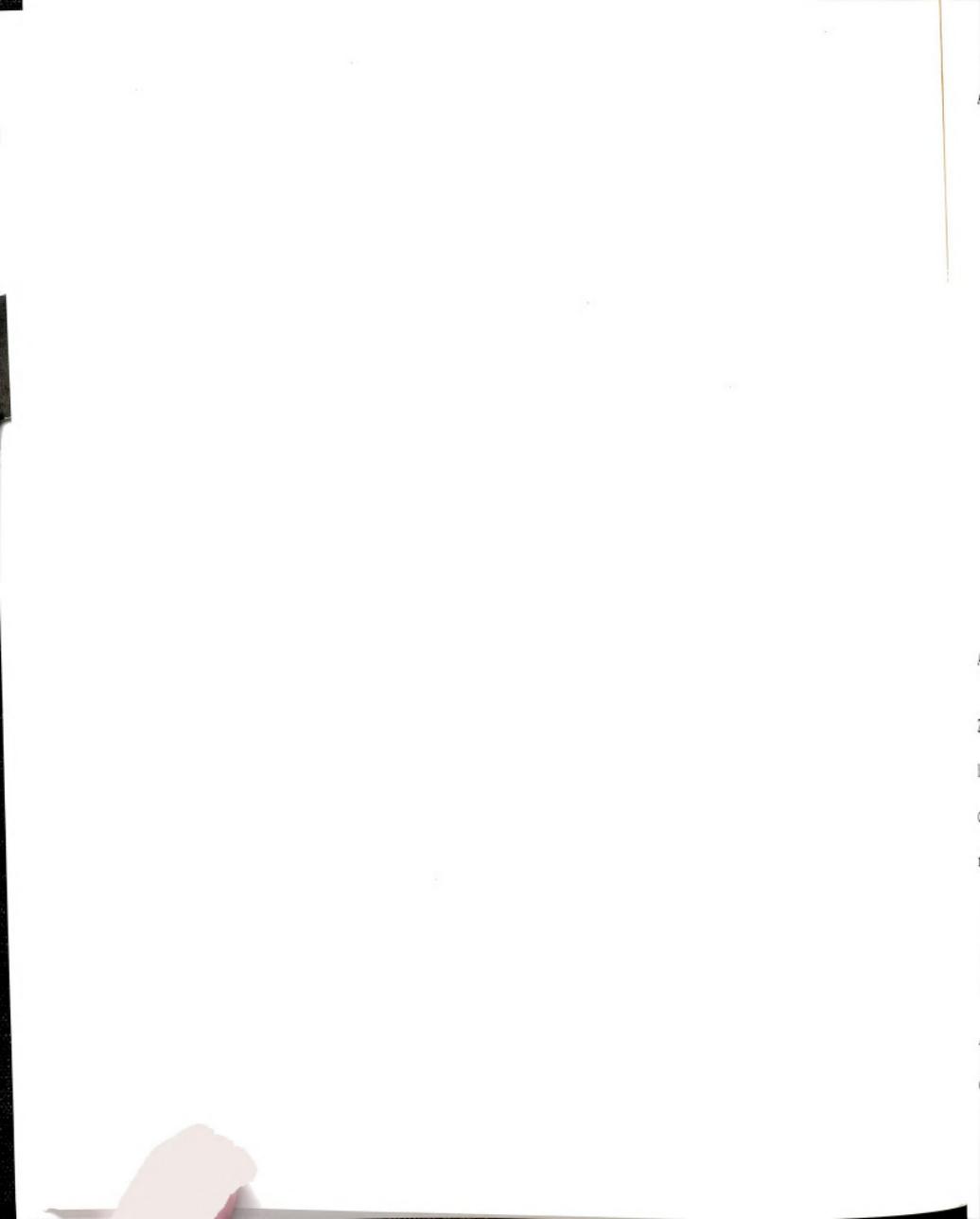
The problem is how to develop an accurate settlement prediction as data are known to be scattered and variable. The settlement problem is best posed in a statistical framework. The problem lends itself to subdivision in the following way:

The SPT data occur in the form of a number of N values which are measured at each boring at suitable vertical spacings. The boring locations are represented by coordinates in the (x,y) plane, so the N values are collected from different spatial locations and can be represented by a set of discrete functions as:

$$N_{(z)}(x_i, y_i)$$

z : is the testing depth.

(x<sub>i</sub>, y<sub>i</sub>) : are the horizontal coordinates of the boring i.



such functions is illustrated in Figure 1.1.

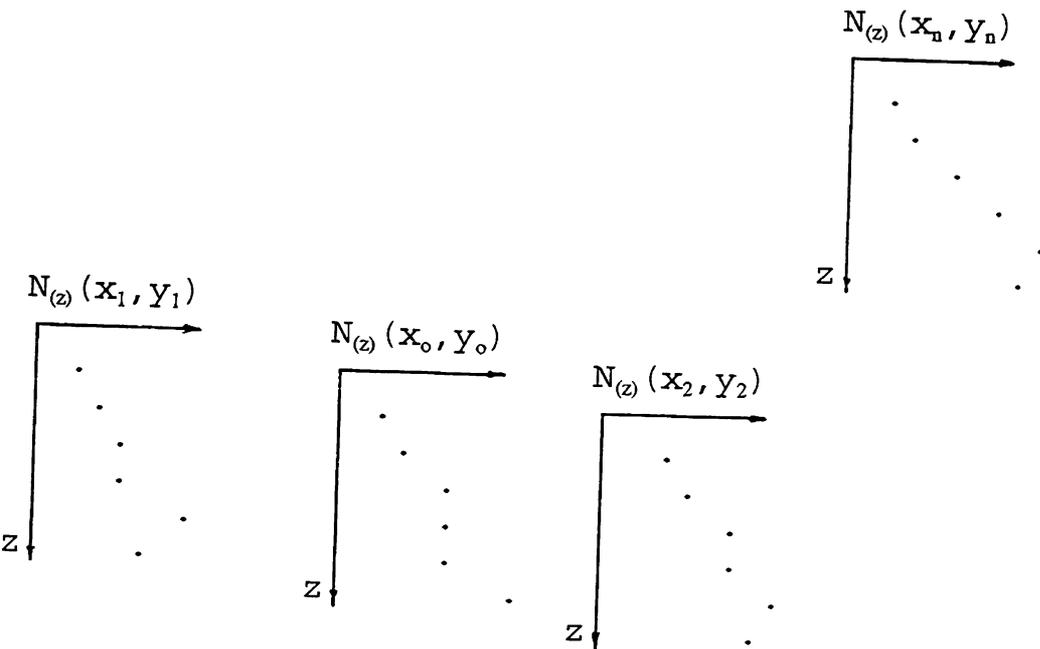


Figure 1.1 : Typical  $N(z)$  functions.

ring the  $N$  values may show considerable scatter.

Considering the three coordinates  $x$ ,  $y$  &  $z$ , it can be said that the  $N$  values are in a three-dimensional field. It is required to estimate the  $N$  values at different locations, such as the location  $(x_0, y_0)$ , where no data were available, which can be estimated by the function:

$$N_{(z)}(x_0, y_0)$$

, it is required to compute the confidence interval on the estimate to some degree of confidence to be used for evaluating the quality of prediction.

As was noted earlier in the introductory section, many researchers have related settlement to the N-value. It was mentioned that these procedures employed the N value either explicitly or implicitly as predictors of elastic parameters. Therefore, it is required to use the estimated N function  $N_{(z)}(x_o, y_o)$  to formulate a design N value to be used for the deterministic models of settlement estimations. Here also, it is required to evaluate the quality of the settlement prediction.

## RESEARCH OBJECTIVES

This research is proposed to add a useful contribution regarding how to treat scatter of N value data by modeling it as a spatial function, and to use points on this function to estimate the settlements and differential settlements. The spatial data analysis and modeling will be done by the use of suitable geostatistical techniques (specifically trend surface analysis and interpolation modeling).

Precise research objectives are as follows:

Setting forward guidelines for employing the geostatistical modeling for obtaining better settlement predictions.

Proposing geostatistical methods to treat the data scatter of N values to provide a rational estimate of N value at locations where no data are available.

Assessing how to use the resulting  $N$  functions in conjunction with selected existing settlement functions to predict the settlements and differential settlements.

Testing the two proposed geostatistical methods in combination with a settlement prediction method on a number of previously-published case histories and comparing the results with the measured values to assess the accuracy of the proposed model.

## RESEARCH SCOPE

To accomplish the required objectives, the research chapters are organized as follows:

**Chapter 2** reviews the SPT test procedure as well as the techniques used for predicting settlements of shallow foundations. These are followed by reviewing the procedures for predicting settlements in sands using the SPT test results directly using SPT results to estimate deformation parameters to be used with elastic

**In chapter 3**, the variability of  $N$  values is analyzed in a geotechnical framework including such factors as the soil relative density, overburden pressure, etc. The sources of the variability of  $N$  values are also reviewed. Special emphasis is made on the tendency for the  $N$  value to increase with depth and its relation to overburden pressure.

Statistical modeling methods from the published literature are reviewed, emphasizing

the adaptability of the geostatistical approaches which are suitable for characterizing the heterogeneity of the soil properties. These include the random field theory, the trend surface analysis theory, and interpolation schemes such as triangulation and Kriging techniques.

In chapter 4, a two-point estimate technique is developed to combine the existing measurements of  $N$  values within the depth of influence under the footing into a design  $N$  value to be used in deterministic models of settlement estimation. This is followed by development of the suggested approaches for the application of both the trend surface analysis theory and the Kriging technique to obtain the SPT function and ultimately the two point estimate. Recommendations are made as to the class of problems for which each is preferred.

Applications of the models to practice are also discussed. These include the evaluation of the quality of prediction versus the quantity of the data and their monetary costs and the contours of the expected settlements.

In chapter 5, the two suggested methods for settlement prediction, trend surface analysis and Kriging, are each tested two different ways. First they are verified using simulated data of an assumed field. These are followed by the verification of the proposed "two-point" settlement estimate by using the assumed field and an assumed  $N$  value. Second, the reliability of the proposed methods for practical applications is tested by conducting the suggested modeling on a number of the available case studies. Results are compared with the measured values. Based on these studies, comparisons are made to find out the advantages and disadvantages of using each of the candidate procedures for estimating  $N(x,y,z)$ .



**In the final chapter**, summary, critical discussion, conclusions and recommendations are presented.



## CHAPTER 2

### CURRENT PRACTICE IN SETTLEMENT PREDICTION USING THE SPT TEST

#### GENERAL

The standard penetration test continues to be widely popular among foundation engineers because it is economical and easy to perform. It has been studied and discussed by a large number of investigators and its correlations with basic soil parameters are well established.

A considerable number of methods for the estimation of settlements in cohesionless soils rely on SPT data. These include: (Terzaghi and Peck, 1950; Schmertmann, 1956; Peck and Bazaraa, 1967; Alpan, 1964; Schmertmann, 1970; and Schmertmann, 1971). The published methods may be divided in two main groups (Terzaghi and Peck, 1950; Schmertmann, 1970; and Terzaghi and Peck, 1971):

Those which give a direct estimation of settlement from the results of in-situ tests.

Those based on elastic theory which depend on estimated material deformation parameters from the interpretation of in-situ test results.

The next four sections will summarize the current practice in the SPT test, the measurement of settlements, and prediction methods related to groups one and two respectively.

## SPT TEST

The standard penetration test is currently the most popular and economical method to obtain subsurface information regarding cohesionless soils. The test has been standardized by ASTM D 1586 as "Standard Method for Penetration Test and Split-Barrel Sampling of Soils".

This test involves using a 140 lb (63.5 kg) driving mass falling free from a height of 30 in (0.76 m) to drive the standard split-barrel sampler which has an inside diameter of 1.5 in, an outside diameter of 2 in and a length of 18 in. The sampler is driven a distance of 18 in (0.45 m) into the soil at the bottom of the boring. The sampler is first driven a distance of 6 in (0.15 m) to seat it on undisturbed soil and the number of blows recorded. The sum of the blow counts for each of the next two six inch increments is taken as the penetration resistance (N value) in blows per foot unless the last increment cannot be completed (either from encountering rock or because the blow count exceeds 100). In this case the blow count for the last 12 in (0.3 m) is used and taken as the N value.

The ASTM standard (ASTM D 1586, 84) states that the boring is advanced incrementally to permit intermittent or continuous sampling. Typically the test intervals used are 5 ft (1.5 m) or less in homogeneous strata with test and sampling locations adjusted for any change of strata.

According to many investigators (see e.g. Gibbs & Holtz, 1957 ; Seed et.al., 1974 ; Peck et.al., 1974 ; Bazaraa, 1967 ; Liao, 1986 ; Skempton, 1986), the N value for cohesionless soils is affected by the effective overburden pressure. For that reason, the N values obtained from field exploration under different effective overburden pressures

should be changed to correspond to a standard value of overburden pressure when used to estimate the relative density ( $D_r$ ).

## 2.1 SPT TEST PRECISION AND BIAS

The SPT test has been studied and reported by many investigators. These include Meyerhof (1957), Gibbs and Holtz (1957), D'Appolonia (1968), Peck and Bazaraa (1967), Schmertmann (1970), and Kovacs and Salomone (1982).

The studies by Schmertmann, De Mello, and Bazaraa as summarized by Bowles (1988), noted that the SPT test is difficult to reproduce. Some of the factors which affect the reproducibility include variations and interference in the free fall of the drive weight, the use of worn or damaged drive shoe, failure to properly seat the sampler in the bottom of the boring, the inadequate cleaning of loosened material from the bottom of the casing, failure to maintain sufficient hydrostatic pressure in the borehole so that the test does not become "quick", and driving a stone ahead of the sampler.

In another field measurement study, Kovacs and Salomone concluded that the energy delivered to the drill stem varies with the number of turns of rope around the drum, the fall height, drill rig type, hammer type, and operator characteristics.

The ASTM standard (D 1586, 84) noted that variations in  $N$  values of 100 % or more have been observed when using different standard penetration test apparatus and operators for adjacent borings in the same soil formation. When using the same apparatus and operator, the current opinion (ASTM D 1586, 84) indicates that  $N$  values in the same boring at the same overburden stress can be reproduced with a coefficient of variation of about 10%. It is noted also (D 1586, 84) that the use of faulty equipment can



significantly contribute to differences in N values obtained between operator - drill rig  
ems.

## 2 PRACTICAL ADVANTAGES OF THE SPT TEST

It is stated by the ASTM standard (D 1586, 84) that the SPT test is used  
extensively in a great variety of geotechnical exploration projects. Many local  
relations and widely published correlations which relate SPT blow counts, or N value,  
the engineering behavior of earthworks and foundations are available.

Regardless of the variability of the SPT test results, the SPT is not likely to be  
abandoned for several reasons. Bowles (1988) cites the following:

The test is too economical in terms of cost per unit information.

If performed every 2.5 ft of depth a tube recovery length of 18 in, including  
seating length, produces a visual profile of around 60% of the visually  
examined.

The test results in recovery of very disturbed samples, but they can still be  
tested for index properties, and with appropriate conservatism, tested for  
strength properties.

Long service life of the enormous amount of equipment in use.

The accumulation of a large SPT data base which is continually expanding.

The fact that other methods can be readily used to supplement the SPT when  
the borings indicate more refinement in sample/data collection.



### 3 MEASURING SETTLEMENTS

As the focus of this research involves prediction of settlements, which are verified by field measurements, it is relevant to briefly review the methods of measuring the settlements.

It has been stated (Sze'chy and Varga, 1978) that the measuring of settlements does not really require any sophisticated apparatus or theoretical training, only some organized thinking and accuracy. The measurement techniques used rely mainly on conventional surveying. Hanna (1973) stated that in most projects simplicity is essential because of the pressures of finance and time limitations. For the great majority of foundations, most if not all of the required measurements can be obtained by simple surveying techniques. The most commonly obtained information is the determination of elevation and change in elevation by offset measurement from a line of sight.

For determination of absolute movement, it is essential that the datum benchmark be located well away from the zone of ground movement; otherwise it may also be affected by the ground movements, ( a distance of 60 m from the building is usually enough to be clear of any effects of settlements of the building ). A permanent benchmark is used if available. Where this is not available, a benchmark or a number of benchmarks, depending on the size of the project, are constructed, (Burland et. al., 1972).

The reference points on the foundations are either rigidly attached to the structure by bolting or welding, or special demountable points are employed. The reference pin socket commonly used for survey of foundation settlements is comprised of a steel brass socket grouted into a hole in the side of the foundation. The reference pin

crews into the socket. When not in use the socket is protected with a cover plate. The levelling staff is placed on top of the reference pin. On steel structures such as oil tanks, lugs may be welded to the outside of the tank about 1 ft above ground level. Each lug, which has a protective cover, has a steel ball bearing about 1 in diameter welded to it which forms the reference mark for the survey, (Hanna, 1973).

Settlements of strata lying deep under a building can be measured by a simple measuring rod bored or driven down to the layer in question with an enlarged tip at the bottom of the rod. The bored rod is usually protected against corrosion by asphalt painting over the full length of the rod and cloth wrapping impregnated with oil. When driven, the rod should be protected by a casing which is slightly retracted in the end, (Sze'chy and Varga, 1978).

Sze'chy and Varga (1978) state that even if the observed data have some uncertainty, they still can be used for settlement evaluation and noted that, it is always better to know that settlements between the limits of, say, 5 cm to 15 cm are to be anticipated than to be in doubt whether settlements will be of a few millimeters only or likely to reach magnitudes of tens of centimeters. In other words, even with some uncertainty a reasonable estimate of the magnitude is important.

### **SETTLEMENT PREDICTION METHODS (GROUP 1)**

Some of the published methods related to the first group which employ N values directly for settlement prediction - as cited by many authors including Jeyapalan, 1986; Parsons and Menzies, 1976 ; Oweis, 1979 ; Haji, 1990 ; and Bowles, 1988 - are as follows:



**Meyerhof (1965):** recommended predicting the settlement as the ratio of the net foundation pressure  $q$  to an empirically determined pressure  $q'$  which is expected to result in a settlement of one inch:

$$S = q/q' \quad (2.1)$$

ere

$S$  = settlement in inches.

$q$  = net foundation pressure, in (ton/ft<sup>2</sup>)

$q' = (N/3) [(B+1)/2B]^2$  ; for  $B > 4$  ft.

$= N/8$  ; for  $B < 4$  ft.

$B$  = footing width in feet.

**Peck and Bazaraa (1969):** developed a refinement of Meyerhof's procedure by introducing correction terms and give the settlement as:

$$S = (2/3) (q/q') (p_d/p_w) . \quad (2.2)$$

re

$q' = (N'/3) [(B+1)/2B]^2$ .

$N'$  =  $N$  value corrected for overburden pressure.

$p_d$  = effective overburden pressure at a depth  $B/2$  below the base of the footing for the dry condition.

$p_w$  = pressure at the same level ( $B/2$  below the base) with water table present.

$= p_d$ , if there is no water table present.



**Parry (1971):** proposed an empirical equation giving the settlement as:

$$S = (1/N) (200qB) C_D \cdot C_W \cdot C_T \quad (2.3)$$

S = settlement in (mm).

N = weighted average N value using the weights of (3,2 &1) for N values between the depths of (0 to 2B/3) , (2B/3 to 4B/3) and (4B/3 to 2B) respectively.

q = applied pressure in (MN/m<sup>2</sup>).

B = footing width in (m).

C<sub>D</sub>, C<sub>W</sub>, C<sub>T</sub> = factors for the influence of excavation, water table and the thickness of the compressible layer.

Parry's equation was developed by assuming that the settlement is a function of the width of the loaded area, the magnitude of the bearing pressure and the deformation modulus of the soil, which is implied by the weighted N values. It is of note that the weighting factors permit incorporation of the spatial trend of the N values. This method could possibly be included in Group 2 (Section 2.5), but it is included here because it does not depend on the modulus E explicitly.

**Alpan (1964):** recommended predicting the settlement as:

$$S = (0.14q/25) (N')^{-1.8} [2B/(B+1)]^2 \quad (2.4)$$

S = settlement in inches.

q = applied pressure (t/ft<sup>2</sup>).

$N' = N$  corrected for overburden pressure.

$B =$  footing width in feet.

Terzaghi's equation was based on predicting the settlement of a plate one square foot at foundation level using measured  $N$  values corrected for effective overburden pressure and then extrapolating this predicted settlement up to the settlement of the full scale structure using the Terzaghi and Peck correlation. This correlation formulates the relationship between the settlement of a footing of width  $B$  and the settlement of a one square foot plate loaded to the same loading intensity as:

$$S_B/S_1 = (2B/(B+1))^2 \quad (2.5)$$

**Schultze and Sherif(1973):** estimated the settlement by :

$$S = q_t B f / [1.71 N^{0.87} (B/B_1)^{0.5} (1 + 0.4D/B)] \quad (2.6)$$

where

$S =$  settlement in (cm).

$q_t =$  total foundation pressure ( $\text{kg}/\text{cm}^2$ ).

$B =$  footing width (cm)

$B_1 = 1$  cm.

$f =$  parameter depending on width to length ratio and thickness of compressible stratum.

$D =$  depth of embedment (cm).

Schultze & Sherif's equation was based on a statistical correlation using linear elastic theory as a model.

## General Comments On The Methods Of "Group 1"

To summarize the inherent assumptions of the methods of "Group 1" it can be said that the Meyerhof's method is an analytical expression of Terzaghi and Peck's well known settlement design chart that allows estimating allowable foundation bearing capacity such that settlements would not exceed 1 inch. In this method, the position of the water table is ignored.

Peck and Bazaraa's method is a refinement of Meyerhof's method. They recommended that the predicted settlements based on the Terzaghi and Peck design chart be reduced by one-third but still proposed that the settlement estimate is to be increased when the depth to water table below the foundation base is less than  $B/2$ . It was recommended also that the  $N$  values are to be corrected for the effect of the overburden pressure.

The results of a study by Schmertmann (1970) suggested that the Terzaghi and Peck design chart is quite unconservative for large foundations and it may be in error. Schmertmann further recommended that the Meyerhof method in its present form should be discarded. Schmertmann's approach is described in the next section.

Parry's method assumes that the settlement is a function of both the footing width and the bearing pressure and inversely proportional to the  $N$  value. This is formulated by building an empirical model using measured settlements.

Alpan's method is based on predicting the settlement of a one square foot plate at foundation level using measured  $N$  values corrected for effective overburden pressure. When extrapolating this predicted settlement up to the settlement at full scale footing width, the Terzaghi and Peck correlation.

Schultze and Sherif's method is based on statistical correlation using linear elastic theory as a model. Several coefficients were employed to ensure a high degree of correlation.

## 5 SETTLEMENT PREDICTION METHODS (GROUP 2)

The second group of methods are those which estimate the deformation parameters, in particular the modulus  $E$ , from the interpretation of in situ tests or laboratory tests, then apply elastic theory to estimate strains, and then integrate vertical strains to obtain settlements.

It has been stated (Bazaraa 1982) that the use of the theory of elasticity for estimating stresses or settlements for foundations in sand is not theoretically correct. However, with reasonable correlation of  $E$  with standard penetration  $N$  value (for a certain depth under the footing), this method can be considered as a reasonable empirical method for estimating the settlement (Bazaraa, 1982, pp. 68).

Some of the published methods related to this group - as cited by many authors including Rajapalan, 1986 ; Oweis, 1979 ; Schmertmann, 1970 ; Webb, 1969 ; are as follows:

**Schmertmann (1970):** gives the following equation for calculating the settlement:

$$S = C_1 \cdot C_2 \cdot p \sum_{z=0}^{2B} [(I_{zi}/E) \cdot dz_i] . \quad (2.7)$$



$p$  = increase in effective overburden pressure at foundation level.

$C_1$  is a depth embedment factor.

$C_2$  is an empirical creep factor.

$I_z$  is the strain influence factor.

$E$  is the deformation modulus = 4N for silts or slightly cohesive silt - sand

to 12N for sandy gravel and gravel.

**Webb (1969):** estimated the settlement by:

$$S = \sum_{i=1}^n [ (p_{zi} / E) dz_i ] . \quad (2.8)$$

$p_{zi}$  = vertical stress in soil layer  $i$  produced by footing load.

$dz_i$  = thickness of layer  $i$ .

### General Comment On The Methods Of "Group 2"

To summarize the methods of "Group 2", it can be said that the methods by Schmertmann and Webb are quasi-elastic (Oweis, 1979). Both methods predict settlement proportional to the width of foundation ( $B$ ) and the foundation pressure ( $p$ ) and inversely proportional to the modulus of deformation ( $E$ ). In both methods the modulus ( $E$ ) depends solely on the  $N$  value irrespective of foundation width and depth, but the modulus is implicitly weighted in Schmertmann's method.



Webb's method implies that maximum strains occur immediately beneath the base of the footing where vertical stresses are at maximum values. This is contrary to results from tests on small plates that indicate maximum strains occur at depths of 0.5 B to 0.75 B below the base of footing (Oweis, 1979).

Schmertmann recognized this by assuming a maximum strain at a depth of 0.5 B. The Schmertmann's method accounts for the observed rapid attenuation of settlements with depth by considering only a thickness of compressible layer equal to 0.5 B when estimating settlements.

Although the models of "Group 1" assume a single value of "N", the models of "Group 2" assume precise information regarding "N", i.e. each layer is represented by different N and E values. or  $N = N(z)$  and  $E = E(z)$ .

Schmertmann made the point that the distribution of vertical strain under the center of a footing on a uniform sand is not qualitatively similar to the distribution of the increase in vertical stress; rather the greatest strain occurs at a depth of about B/2. He proposed an influence factor whose value increases with the depth "z" according to the function:

$$I_z = [0.1 + (z/B)] \quad (2.9)$$

reaching a maximum value at the depth of (B/2) then decreasing according to the function:

$$I_z = (0.4/B) (2B - z) \quad (2.10)$$

until it reaches a value of (0.0) at the depth of (2B).



The conclusion here is that the N value is a main input in most settlement prediction methods. In chapter 3, the approaches of treating the data scatter of N values are reviewed. For the prediction of settlements at many points in the (x,y) plane, it is desired to develop a method which has the simplicity of single N value techniques but preserves the information regarding the spatial structure of the N values. This will be addressed in chapter 4 by developing a method employing the variability of N values as a function of (x,y) together with a selected settlement prediction method.



## CHAPTER 3

### VARIABILITY OF SOIL DATA AND SUMMARY OF GEOSTATISTICAL MODELING METHODS

#### 1 VARIABILITY OF N VALUES

Although the use of SPT data to estimate design parameters is well established in the literature, little guidance is given on how to treat data scatter (Wolff, 1989).

Correlations provide estimates of soil properties as a function of a single N value, and various designers may enter correlation equations with values anywhere from the average value down to values below the minimum measured. Wolff (1989) showed that the designer's assumptions and judgements may significantly influence the final design recommendations. Haji (1990) noted that the interpretation of the soil boring data and the selection of a representative N value depends to a great extent on the experience and the knowledge of the foundation engineer rather than on a specific procedure or method.

The design N value is used to estimate the angle of internal friction, which in turn will be used to assess the soil bearing capacity; consequently the decision whether to use shallow foundations or deep foundations is dictated by the design N value. Furthermore, if shallow footings are selected the dimensioning of the footings and the predicted settlement will all be a function of this N value.



### 1.1 Sources Of Variability Of N Values

Soil by its nature has an inherent variability. This variability may result from (Schnabel & El-Ramli, 1982):

The variation of soil layer thicknesses.

The heterogeneities within the soil layers in terms of different relative densities, different moisture contents, different overburden pressures, and different stress histories.

From the viewpoint of mathematical modeling, Baecher (1978) attributed the variation of soil properties to 3 sources:

Drift in average properties.

Random fluctuations about the average.

Inhomogeneities.

The modeling problem is to represent mathematically the spatial variation which is attributed to the first and second sources. The variation due to inhomogeneities cannot be tackled by mathematical modeling; however, it is suggested herein that a random field of the soil properties can be divided into a number of subfields, each of which satisfies the homogeneity condition. Thus, a soil field could be modeled using a set of mathematical functions. This concept will be developed further in section 2.

The above-mentioned inherent variability, combined with the variability resulting from the testing repeatability or reproducibility errors, commonly leads to considerable scatter in SPT values over a construction site.



### 1.1.2 N-Value Depth Effect And Correction For Overburden Pressure

For "consistent" materials (i.e. a thick deposit of soil having uniform composition and relative density) the variation of N in the vertical direction is dominated by the soil stiffness which increases with depth due to the overburden pressure and the corresponding increase in confining pressure. But the question of how the soil modulus E, a measure of stiffness, would increase with depth in such a material has been the subject of different and varying opinions:

The earliest literature on stiffness of a semi-infinite mass started with Boussinesq (1885) who developed solutions for stress and strain in an elastic homogeneous half space, but did not take any account of the systematic increase of stiffness with depth.

Feda (1963) presented a model of an elastic half-space whose modulus increased parabolically with depth.

Gibson (1967) considered the soil as an elastic half-space whose modulus increases linearly with depth.

Many authors (e.g. Skempton, 1986; Liao and Whitman, 1986; Seed et al., 1986; Peck et al., 1974; Bazaraa 1967) recommended that field N values should be corrected for the vertical effective overburden pressure before using them for characterizing the sand. The justification for such correction is explained as follows:

In sand, the settlement (under a given stress change) is strongly correlated to its relative density.

The N value reflects both the relative density ( $D_r$ ) and the effective stress.

This correction makes it possible for the N value to be correlated directly to the



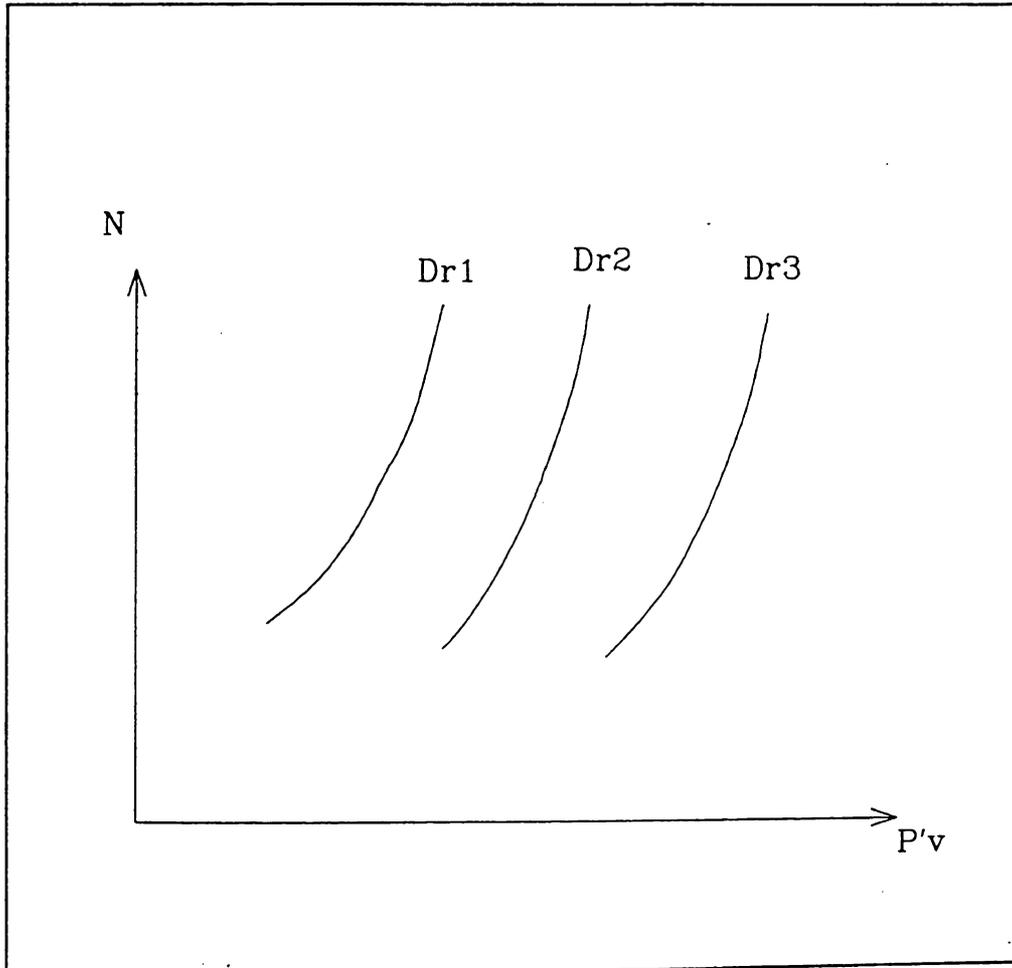
relative density; consequently this corrected N value can replace the relative density in the correlation which was mentioned in (1) above. That will lead to a more accurate prediction of settlement from N value.

Regarding item 1 above, D'Appolonia et al. (1968) confirmed the correlation between settlement and relative density by noting that the principle variables controlling settlement for a particular granular soil under a given static loading configuration are the initial density and the initial state of stress in the deposit. Therefore, analytical and empirical methods of estimating settlement of footings on sand require a direct or indirect measurement of in situ density and stress state.

Regarding item 2 above, D'Appolonia (1968) presented a relationship between the relative density and the effective stress, similar to that shown in Figure 3.1. As seen in the figure, the N value cannot uniquely define both relative density and effective stress; there are an infinite number of combinations of stress level and relative density that will result in the same measured N value, (D'Appolonia 1968).

Regarding item 3 above, the correction of N value was suggested to make it possible for an adjusted value  $N'$  to reflect only the effect of the relative density while ignoring the effective stress value. D'Appolonia (1968) reported that "Empirical relationships have been developed to correct the blow-count for in situ vertical effective stress. Thus, the SPT can be correlated to relative density". He also added "When the SPT resistance was corrected for overburden pressure, Meyerhof's method accurately predicts the measured settlements".

Based on the above considerations, the N values will be corrected, using the Liao and Whitman correction method, before incorporating them into the proposed models.



**Figure 3.1 : N Value Versus Relative Density "Dr" And Vertical Effective Stress "Pv", (After D'Appolonia).**



## 3.2 SUMMARY OF GEOSTATISTICAL MODELING METHODS

Having described the inherent variability of N value data, it is desired to represent such data with one or more functions that capture overall trend and variability around that trend. Sections 3.2.1 through 3.2.3 below summarize some geostatistical methods which are suitable for characterizing the scatter of soil properties:

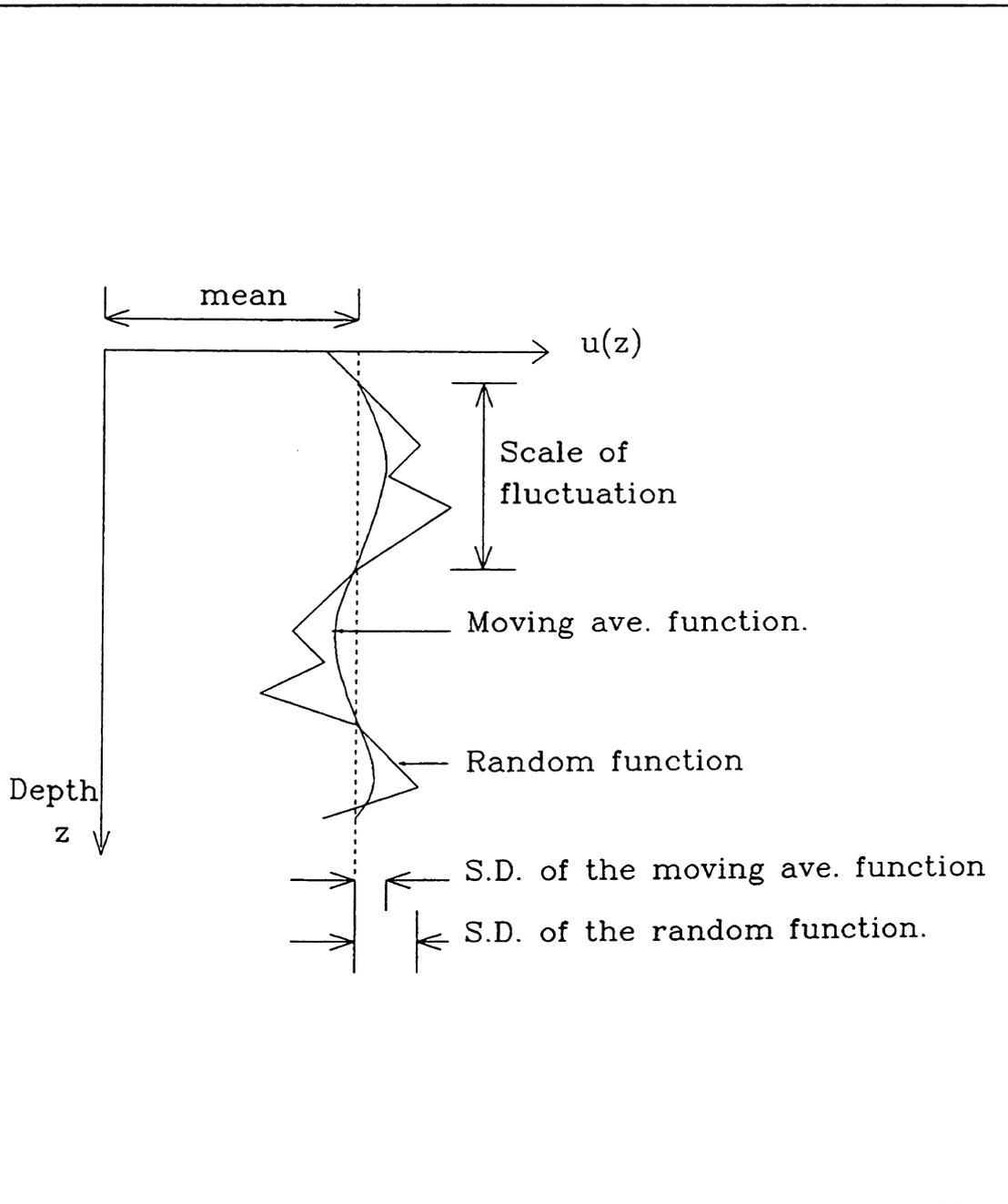
### 3.2.1 The Random Field Theory

This theory was employed by Vanmarcke (1977) to study the implications of stochastic variability of soil properties. He quantified the variability of the soil profile using the "variance function". The variance function was defined as the ratio of the variance of the moving average function to the variance of the random function of the soil property, before smoothing, related to one spatial coordinate "z", as illustrated in Figure 3.2.

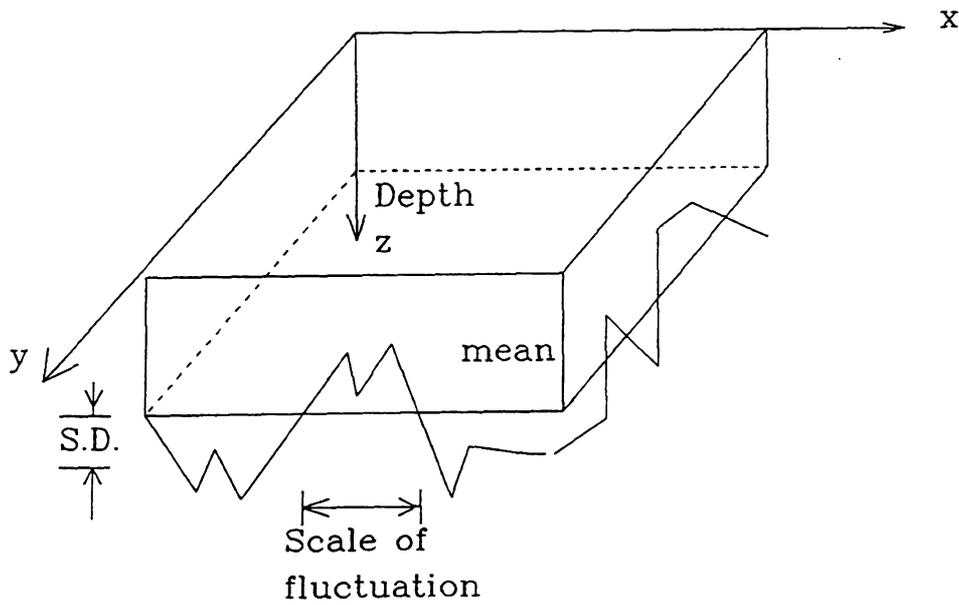
As a smoothing procedure, the moving average implies the removal of local maxima and minima of the random variable function resulting in reducing the data variability and decreasing the variance. For the SPT data, the random variable is the N value, consequently the variance of the moving average over an area is smaller than the variance of the N values, which can be expressed as:

$$\sigma^2 [\bar{N}] < \sigma^2 [N] \quad (3.1)$$

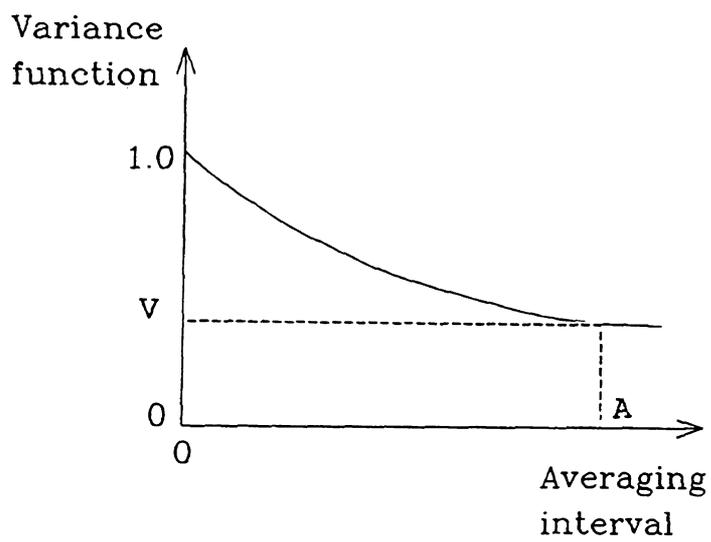
If the "averaging interval" is defined as the distance within which the values of the random variable are replaced by their average to form one point on the moving average function, then the decay of the variance function as the averaging interval



**Figure 3.2 : Parameters Of Homogeneous Randomly Varying Soil Properties.**



**Figure 3.3 : Parameters Of Homogeneous Randomly Varying Soil Properties In 3- Dimensions.**



$$\text{Correlation distance} = V * A$$

**Figure 3.4 : The Decay Of The "Variance Function" As The Averaging Interval Increases.**

increases is used to estimate the "correlation distance" or the "scale of fluctuation" of the variable values, as illustrated in Figures (3.2, 3 & 4). The "scale of fluctuation" is obtained by multiplying the asymptotic value of the variance function by the averaging distance corresponding to this value Figure 3.3.

Physically the "scale of fluctuation" is a measure of the distance within which the soil property shows relatively strong correlation from point to point or the distance over which the soil can be treated as statistically homogeneous. What is often needed within this "statistically homogeneous" soil mass is (Vanmarcke , 1977) the probability density function of some "spatially averaged" soil property. Consequently, the expected value and the variance of the soil property within this soil mass are obtained in conformity with the distributional characteristics of the observed values.

However , Vanmarcke (1978) later made the comment that the scarcity of subsurface data makes it difficult to validate complex stochastic models. For example , it is seldom possible to distinguish between competing functional forms of the correlation function. At best, the data permit one to estimate the decay parameter (such as the correlation distance) of any chosen functional form for the correlation or variance function. Therefore, in this research, the random field theory is not recommended for the development of the proposed models.

### 3.2.2 Trend Surface Analysis

The trend surface analysis technique was suggested by Krumbain and Graybill (1965), summarized by Davis (1973 and 1986) and extended by Baecher, (1978). In this procedure, a trend surface describing a soil property  $N = N(x,y,z)$  is estimated from data by taking a least squares fit. Precisely, the trend surface analysis is an adaptation of the statistical field of multiple regression, and the techniques have been borrowed directly from the discipline. In some cases (Davis,1986) one can even use the powerful tests of hypotheses of multiple regression on geologic problems. Thus, one approach for consideration in a spatial approach to the settlement prediction problem is to estimate a trend surface from data by taking a least squares fit with confidence limits determined as in regression analysis.

Lancaster and Salkauskas (1986) noted that, if there is a sizeable error that enters into the data in a random way that should be smoothed out, then judgement must be exercised in assessing the degree of smoothing to be applied and the choice of smoothing procedure. In this instance smoothing is referred to in the statistical sense which implies the removal of extraneous local maxima and minima and the identification of the underlying trend. Here, several different least squares fitting techniques may be considered.

There are several items which must be considered in attempting to fit a surface through the data scatter of  $N$  values. These are:

The trend surface geologic definition.

The trend surface operational definition.

The functional forms of the trend surface.



Regarding the first item, Cressie (1991) reported that geostatistics recognizes spatial variability as being the sum of two components, the large scale component or spatial trend and the small scale component or spatial deviation. Trend surface analysis considers only large scale variation, assuming independent errors. However, what is considered to be "large scale" and "small scale" is largely subjective.

The question then, is how to objectively separate the data into two components where the distinction between the components is entirely subjective. To remedy this question, Davis (1986) suggested that this be done using an operational definition which specifies the way in which the data are to be treated instead of a geologic definition of trend and deviation.

In accordance with the second item, the operational definition, a trend may be defined as a function of the coordinates of a set of observations so constructed that the squared deviations about it are minimized. The expansion of this definition will yield

t:

The trend is a function of the coordinates, meaning that an observation is considered to be in part a function of the location ( $x$ ,  $y$ ,  $z$ ) of the observation. This function has the form of an equation whose terms are added together. Each term is the product of a coefficient and some combination of the coordinates.

The sum of the squared deviations from the mean defines the variance of the sample. So, it can be seen that the trend can be regarded as a function having the smallest variance about it.



Regarding the third item, the functional form of the trend of the N function  $(x,y,z)$ , one seldom would have any prior knowledge about what the functional form of the trend should be. Instead, one does the next best thing, and approximates the unknown function with one of arbitrary nature. Commonly, a polynomial expansion is used which uses the powers and cross-products of the coordinates. Polynomials are extremely flexible, and if expanded to sufficiently high orders, can conform to very complex surfaces. There is however, some mathematical basis to using the simplest (lowest order) model in the absence of information to justify a more complex model. It is noted also that the z term should be different than the x and y terms, due to depth effect.

It is important to note that polynomial functions are used for trend analysis primarily as a matter of convenience. For polynomials, the equations necessary to find the coefficients of the trend may easily be established and solved by computer programs (Davis, 1986, pp. 411). The use of polynomials does not mean that the N functions are polynomial functions; their unknown nature is only approximated by a polynomial expansion. Other approximations may be more appropriate in specific instances, but in general are less convenient.

The trend surface can be fit to different models (Box, Hunter and Hunter, 1978, pp. 13-521; Davis, 1986, pp. 430). These models are initially considered. These are:

The first degree polynomial model.

The second degree polynomial model.

The four-dimensional trend surface.

These models are mentioned just for setting the stage. The fitted model to any particular case should be based on the variation of the data and prior knowledge, historical data, and theory.

These models are discussed in sections (3.2.2.1) through (3.2.2.3).

### 3.2.2.1 The First Degree Polynomial Model

The first degree polynomial model is expressed as:

$$N = b_0 + b_1X + b_2Z + e \quad (3.2)$$

ere

$b$ 's are the model constants, and;

$e$  is the error term.

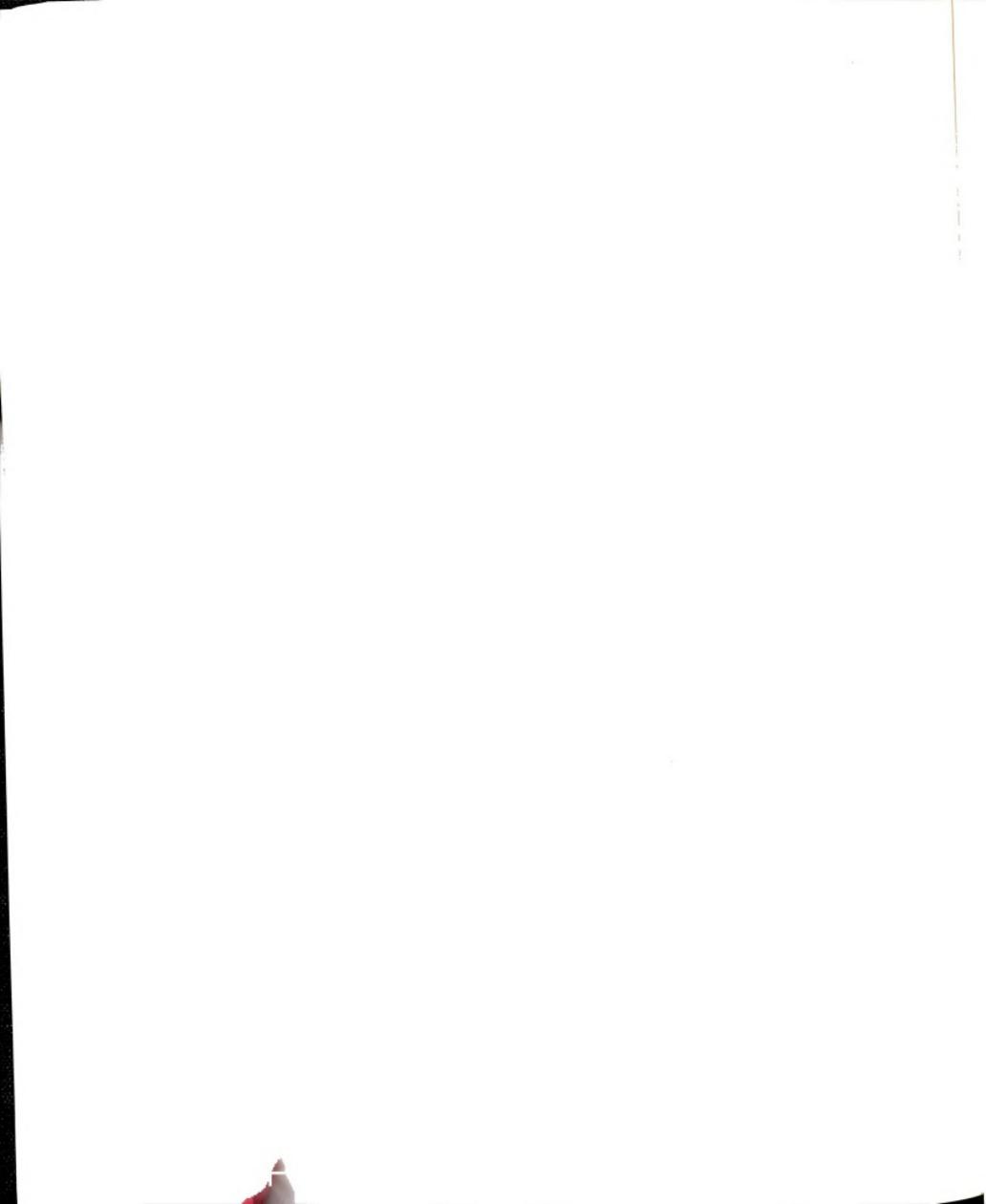
This model:

Allows the design to be efficiently fitted. This is done by estimating the model constants by using the least squares fit.

Allows checks to be made to determine whether this planer model is representationally adequate. This is done by conducting the interaction checks.

The planar model implies that the effects of the variables are additive. Interaction between the variables  $X$ ,  $Z$  would be measured by the coefficient  $b_{12}$  of an added cross-product term  $XZ$  in the model. If the value of the  $b_{12}$  is not significant, then the planer model is adequate.

Provides some estimate of experimental error by estimating the error variance of the replicated observations.



### 2.2.2 The Second Degree Polynomial Model

The second degree polynomial model is expressed as:

$$N = b_0 + b_1X + b_2Z + b_{11}X^2 + b_{22}Z^2 + b_{12}XZ + e \quad (3.3)$$

To check the adequacy of the second degree model, the combinations of third-order terms have to be checked. For example the coefficients  $b_{111}$  and  $b_{122}$  are coefficients of  $X^3$  and of  $XZ^2$ , respectively, in a third-degree polynomial. Both of these coefficients would be zero if the surface were described by a second-degree model, (Box & Hunter 1978).

### 2.2.3 The Four-Dimensional Trend Surface

A logical extension of polynomial trend surface analysis is the inclusion of three dimensions X, Y and Z (and if significant, their powers) as independent variables (Davis, 1986). The use of a four-dimensional model is preferable when the data are more erratic because it is more general than the other models.

In this technique, the dependent variable is regressed upon X, Y and Z. In the case of N more emphasis is put on the inclusion of the Z term. Its inclusion is not only statistically but necessary as it is known that the depth effect is the main factor in the variability of the N values.

In three dimensions contour lines become contour envelopes. A completed analysis can be represented as a solid containing nested contour envelopes. The



Volume between two successive contour envelopes is occupied by points which have the same range of predicted value of dependent variable. The dependent variables in the case histories reported by previous geostatistical investigators commonly were measures such as the percentage composition of some constituent or mineral. In this research, the dependent variable is taken as the N value.

#### 3.2.4 Measuring Goodness-of-fit Of A Trend Surface

Higher values for the coefficient of determination  $R^2$  should be expected in a four dimensional model. The  $R^2$  value is frequently used empirically as a measure of the degree to which the trend fits the data (Lancaster and Salkauskas, 1986).

$$R^2 = SS_R / SS_T \quad (3.4)$$

$SS_R$  = regression sum of squares.

$$= \sum \hat{N}^2 - (\sum \hat{N})^2 / n \quad (3.5)$$

$SS_T$  = sum of squared deviations from mean.

$$= \sum N^2 - (\sum N)^2 / n \quad (3.6)$$

The value of the  $R^2$  lies in the range of (0 - 1). When  $R=1$ , all of the data are on the fitted trend and there is no residual or deviation; when  $R=0$ , the observed data failed to show any trend.



Values of  $R^2$  between 0.8 and 1 are often considered to indicate a significant trend in the data, and values of  $R^2$  between 0 and 0.2 suggest that the trend is not well established (Lancaster and Salkauskas, 1986). In practice,  $R^2$  is not likely to be at the limits of the range (0 - 0.2) or (0.8 - 1), but rather somewhere in between these limits. The closer it is to one, the greater is said to be the degree of association between the dependent and the independent variables (Neter and Wasserman, 1974).

For a model to be accepted, it should pass a test of significance; to select among models which are known to fit with significance,  $R^2$  and sometimes the standard error estimate are used as criteria.

The significance of a trend surface may be tested statistically, by comparing the variance due to deviations from the mean to the variance due to deviations from the trend. This comparison is conducted by using the F test, which is valid only if the data satisfy the following conditions (Davis, 1986):

The population of data is normally distributed about the regression.

The population has a constant variance and does not change with changes in the independent variable (i.e. the variance is constant for all x, y & z locations).

The samples are drawn without bias from this population.

One does not really know if these conditions are true or not, but, in the absence of evidence to the contrary, it is assumed so in order to go forward. The significance of the trend may then be tested by performing the F test as follows:

The total variation of N is divided into two components: the trend and the deviations.

Then the F ratio is calculated:



$$F = MS_R / MS_D \quad (3.7)$$

ere

$MS_R$  = regression mean square.

= (regression sum of squares) / degrees of freedom.

=  $SS_R / d.f_R$

$MS_D$  = (sum of squared deviations from the mean -

sum of squared deviations from the trend) / d.f.

=  $(SS_T - SS_R) / (d.f_T - d.f_R)$ .

$d.f_T$  = total degrees of freedom.

= number of data points - 1

$d.f_R$  = trend degrees of freedom.

= number of coefficients (b's) in trend surface not counting  $b_0$ .

resulting F value is then compared to the critical value of F probability distribution

selected level of significance. A significance level of 5% is commonly used.



The critical F value is a measure of the variation that might be expected solely due to randomness in sampling the data and the lesser the calculated F value the better the data are fitted to the trend and the lesser the chance that the fit is a coincidence.

The F- test result is a test of the hypothesis (Davis, 1986):

$$H_0 : B_1 = B_2 = \dots = B_m = 0 \quad (3.8)$$

where

B's are the true population regression coefficients.

This null hypothesis implies that there is no trend.

Section (3.2.3) summarizes the interpolation schemes including the Kriging technique, which is an alternative technique for spatial modeling. In chapter (5), alternative models from both techniques ( the trend surface and the Kriging) will be used to make settlement estimates for the same case histories. Based on the comparison between the estimates by each of them, an evaluation of the aptness of each will be made.



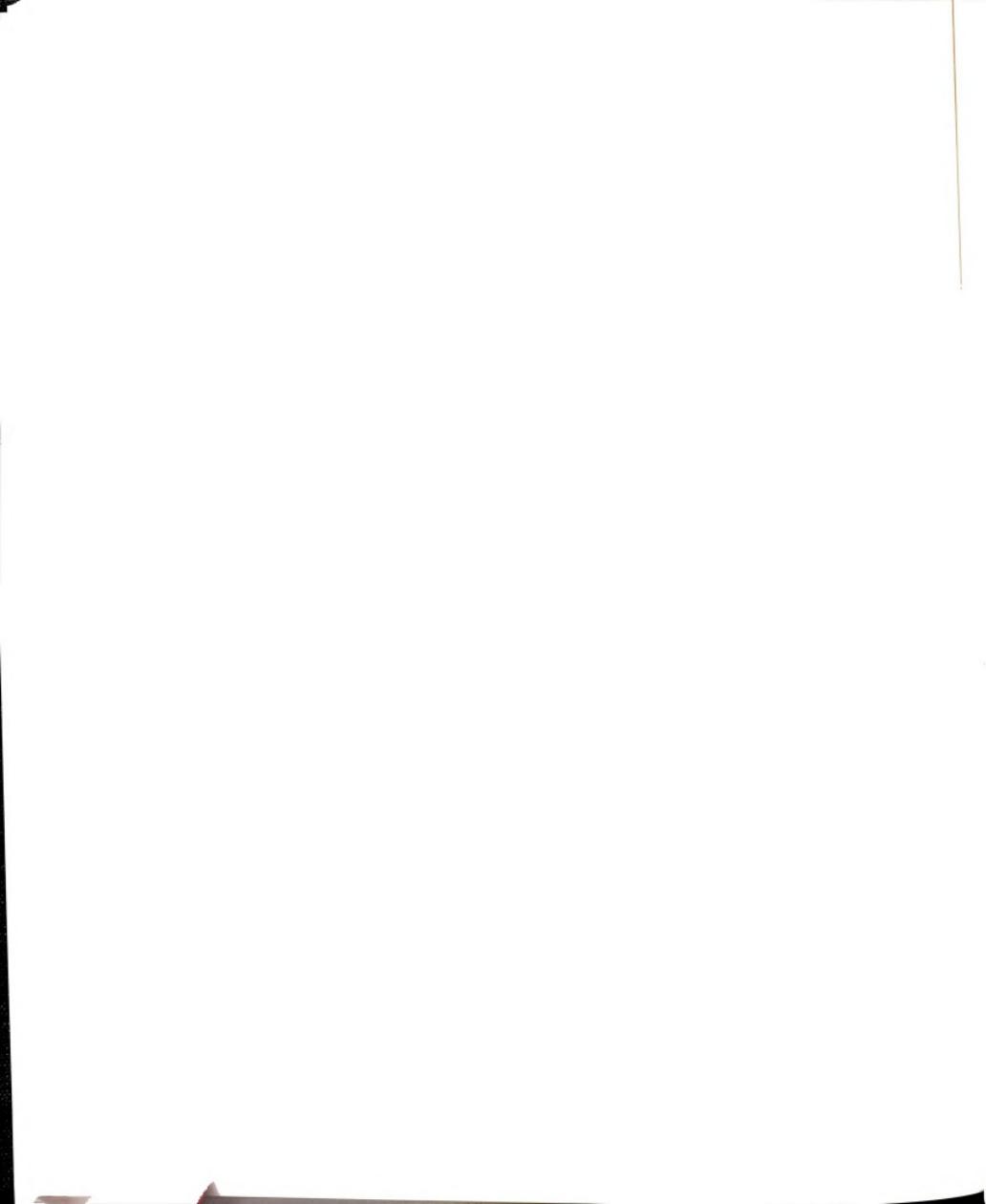
### 3.2.3 Interpolation Schemes For N Value Modeling

It could be argued that the N value at a given point should not influence the nature of the fitted surface at distant points. If this is the case, then no constraint is imposed on the choice of any local modeling technique. On the other hand, (Lancaster and Salkauskas, 1986), if there is sufficient confidence in the data to demand that the fitted surface must contain every data point - in the influence zone - then the interpolation modeling techniques may be considered. In other words, regression emphasizes trend over local fit; if one wishes to emphasize local fit over trend, then the interpolation modeling techniques are preferred.

Interpolation modeling techniques take one of two forms. The first is the simple point estimation scheme for interpolating two-dimensional data, such as the geometric technique known as "triangulation". The second is the estimation method that is designed to give the best estimate for one of the statistical criteria. This latter technique is known as "Kriging", after its developer, D. G. Krige, a South African mining engineer and pioneer in the application of statistical techniques to mine evaluation.

#### 2.3.1 Triangulation

Triangulation is a method of interpolating a value  $f(x,y)$  and is done by fitting a line through three samples that surround the point being estimated (Isaaks and Srivastava, 1989). X & Y are not necessarily situated at the ground surface but rather, they are situated in the plane of the three data points which can take any orientation. The points need not be regularly spaced, but they have to surround the point being estimated "nicely" (i.e. in relatively different directions and at nearly equal distances).



The equation of a plane can be expressed generally as:

$$z = ax + by + c \quad (3.9)$$

For the SPT data, where it is desired to estimate N values using coordinate information, z is the N value.

Given the coordinates and the N values of three nearby samples  $N_1$ ,  $N_2$ ,  $N_3$  that nicely surround the point being estimated "o" as shown in Figure 3.5, one can calculate the coefficients a, b and c by solving the following system of equations:

$$N_1 = ax_1 + by_1 + c \quad (3.10)$$

$$N_2 = ax_2 + by_2 + c \quad (3.11)$$

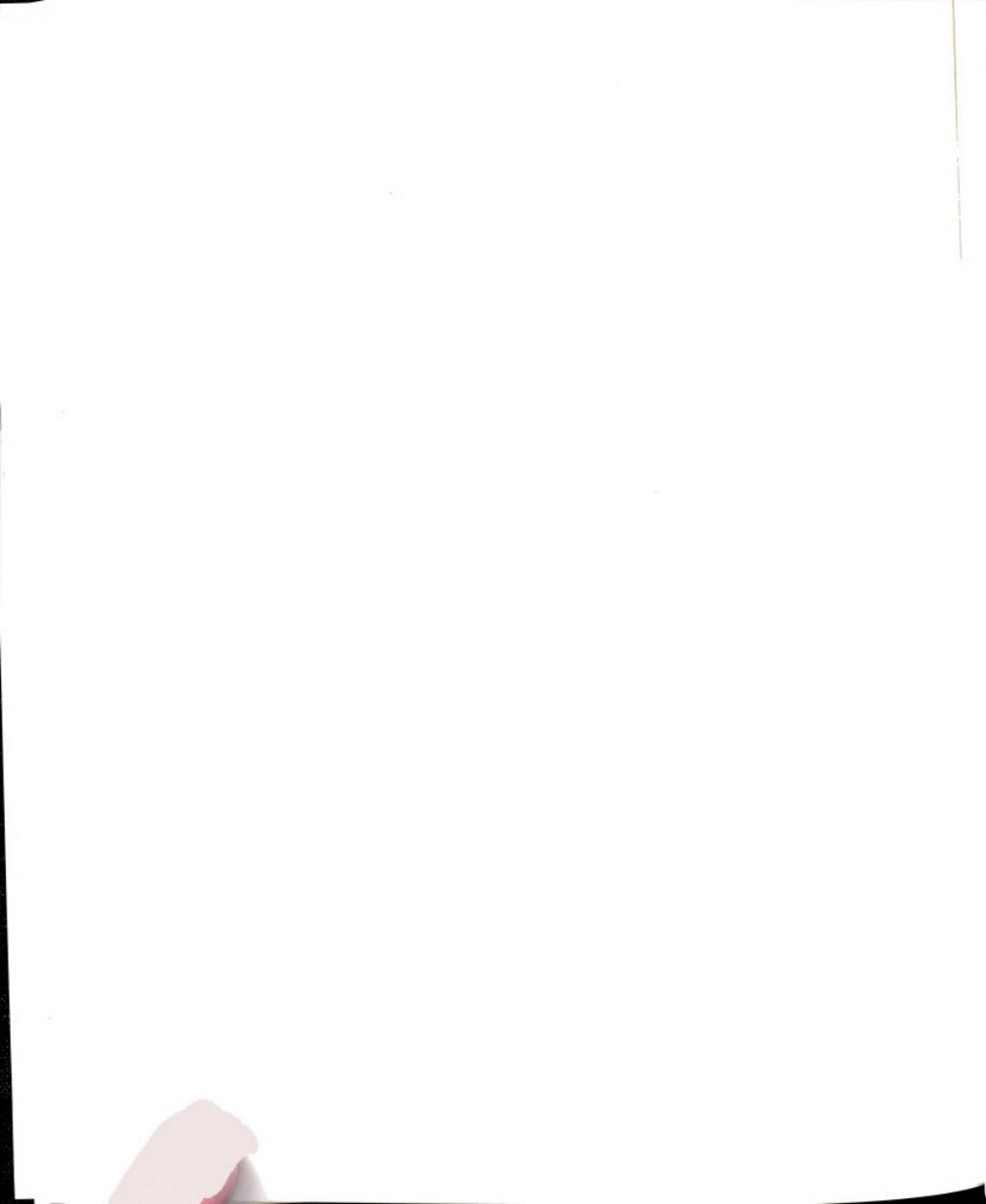
$$N_3 = ax_3 + by_3 + c \quad (3.12)$$

$$\begin{array}{r} N_1 \\ + (x_1, y_1) \end{array} \quad \begin{array}{r} N_2 \\ + (x_2, y_2) \end{array}$$

+ o

$$\begin{array}{r} N_3 \\ + (x_3, y_3) \end{array}$$

**Figure 3.5 : Three Nearby Samples  $N_1, N_2$  And  $N_3$  Surrounding The Point In Question "o".**



If the three points are located in a straight line - i.e. not surrounding the point "o" nicely - then the three equations cannot be solved for all given values of  $N_1$ ,  $N_2$  and  $N_3$ .

This method of estimation depends on which three nearby samples are used to define the plane. There are several ways one could choose to triangulate the sample data set.

Triangulation is typically not recommended for extrapolation purposes. In fact this violates the "surrounding" requirement. If the point "o" at which an estimate is required is contained within the triangle 123, one can directly calculate the triangulation estimate at "o" without simultaneous equations as:

$$N_o = (1/A_{123}) (A_{o23}N_1 + A_{o13}N_2 + A_{o12}N_3) \quad (3.13)$$

The A's represent the areas of the triangles given in their subscripts. The triangulation estimate, therefore, is a weighted linear combination in which each value is weighted according to the area of the opposite triangle (Isaaks and Srivastava, 1989, pp. Equation 11.6).

Other interpolation schemes involving four or more points or higher order surfaces have been developed and are commonly applied in finite element analysis; however, they do not have the statistical advantages of Kriging as discussed in section 3.2.



### 2.3.2 Kriging

Kriging is a probabilistic method used for fitting a surface to irregularly scattered points in space, (Krige, 1966). This technique has found increased application in recent years, (Lancaster and Salkauskas, 1986), for example by Spikula, (1983) and Baecher, (1981). What distinguishes Kriging from the regression or trend surface technique is that attempts are made to localize the computation by excluding distant points from the calculations of the interpolant at any fixed point. What distinguishes Kriging from simple polynomial interpolation such as triangulation is that attempts are made to give the best estimate for one statistical criteria. Triangulation, on the other hand, gives an estimated value based on an entirely geometric criteria.

Isaaks and Srivastava (1989) note that Kriging is often associated with the acronym "BLUE" for "best linear unbiased estimator":

Kriging is "linear" because its estimates are weighted linear combinations of the available data.

It is "unbiased" since it attempts to set the mean error equal to zero.

It is "best" because it aims at minimizing the variance of the errors. This is the main distinguishing feature of the Kriging method.

An important aspect of Kriging is that one never knows the mean error and therefore cannot guarantee that it is exactly zero. Nor does one know the variance of the errors; it cannot be minimized. The best one can do is to build a model of the data being estimated and work with the average error and the error variance for the model.

In Kriging a probability model (a random function model) is used in which the bias and the error variance can both be calculated and weights chosen for the

nearby samples that ensure that the average error for the model is exactly zero and that the modeled error variance is minimized.

### 2.3.2.1 Underlying Concepts

It is assumed (Lancaster and Salkauskas, 1986) that the data is a sample from a random function  $v(p)$ , which is the sum of a "slowly" varying random polynomial  $d(p)$  of degree  $m$ , called the drift, and a "rapidly" varying random component  $r(p)$ , which is assumed to have zero mean or expected value  $E[r] = 0$  and which is responsible for the noise-like nature of  $v(p)$ , Figure 3.6.

$$v(p) = d(p) + r(p) , \quad E[r] = 0 \quad (3.14)$$

It is assumed further that the covariance structure of  $r(p)$  can be obtained and that the covariance between values of  $r(p)$  at points  $p$  and  $q$  depends only on the distance between  $p$  and  $q$ .

$$\text{Cov}[r(p), r(q)] = f(\text{distance between } p \text{ \& } q) \quad (3.15)$$

Now it is desired to estimate the unknown true value  $v(p_0)$  at a point  $p_0$  where no sample is available by a linear combination of the available samples:

$$v(p_0) = \sum_{i=1}^n w_i v(p_i) . \quad (3.16)$$

where  $w_i$  is the weight associated with the random variable at the location  $(i)$ .



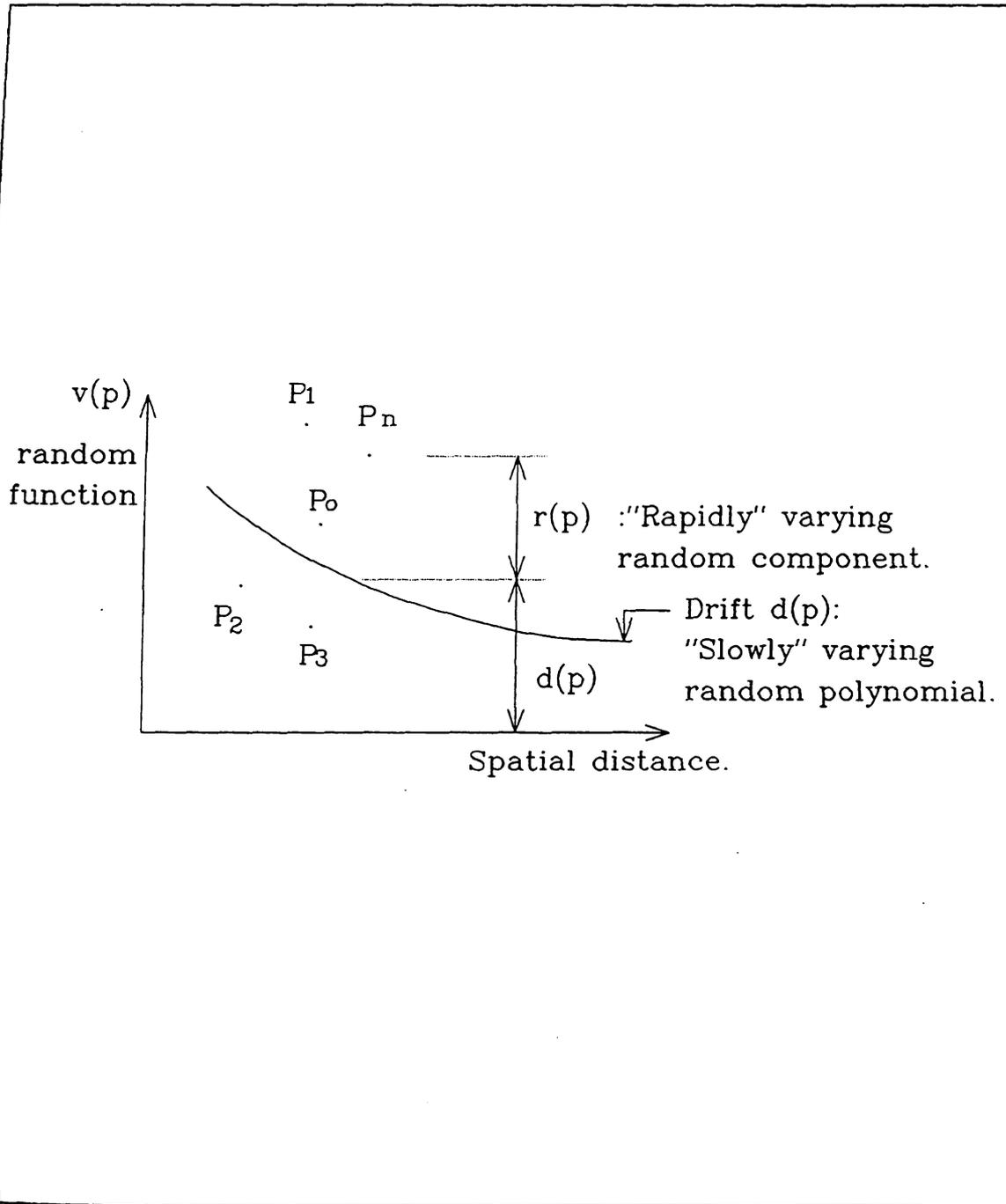


Figure 3.6 : Components Of Random Function In Kriging Model.

This set of weights is allowed to change as unknown values are estimated at different locations, in such a way that the variance of the error "r" is minimized. This error is given by:

$$r(p_o) = \sum_{i=1}^n w_i V(p_i) - V_{true}(p_o) \quad (3.17)$$

The probabilistic solution to this problem assumes that for any point at which it is desired to estimate the unknown value, the model is a stationary random function that consists of several random variables, one for the value at each of the sample locations,  $V(p_1), \dots, V(p_n)$ , in the subset used to predict  $V(p_o)$ , and one for the unknown value at the point where the estimate is desired  $V(p_o)$ . Each of these random variables has the same probability law at all locations; the expected value of the random variable is  $E[V]$ , (Isaaks and Srivastava, 1989).

This means that all the random variables  $V(p_1), \dots, V(p_n)$  and  $V(p_o)$  in the subset are taken to have the same expected value  $E[V]$  and the same variance. For the SPT this means that  $E[N(x,y,z)]$  and  $Var[N(x,y,z)]$  are the same everywhere in any subset and the measured values are deviations from  $E[N]$ .

Thus the estimation error  $R$  at the point  $p_o$  is also a random variable and is given by:

$$\text{Error} = \text{Estimated value} - \text{True value}$$

$$R(p_o) = \sum_{i=1}^n w_i V(p_i) - V_{true}(p_o) \quad (3.18)$$



the unbiasedness condition states that  $E[R(p_o)] = 0$

Therefore:

$$E[R(P_o)] = 0 = E[V] \sum_{i=1}^n w_i - E[V] \quad (3.19)$$

Consequently:

$$\sum_{i=1}^n w_i = 1 \quad (3.20)$$

Now to get the best estimate of  $V(p_o)$  it is desired to minimize the error variance  $\text{Var}[R(p_o)]$ , (Isaaks and Srivastava, 1989, pp. 278). The error variance is obtained as the variance of the difference of the estimated value and the true value of the variable which is given by:

$$\sigma_R^2 = \sigma^2 [\hat{V}(P_o) - V(P_o)] \quad (3.21)$$

By expanding and manipulating some terms leads to:

$$\text{Cov}[\hat{V}(P_o), \hat{V}(P_o)] - \text{Cov}[\hat{V}(P_o), V(P_o)] - \text{Cov}[V(P_o), \hat{V}(P_o)]$$

$$+ \text{Cov}[V(P_o), V(P_o)]$$

$$= \text{Cov}[\hat{V}(P_o), \hat{V}(P_o)] - 2\text{Cov}[\hat{V}(P_o), V(P_o)] + \text{Cov}[V(P_o), V(P_o)]$$

$$= E[\hat{V}(P_o)] - 2E\left[\sum_{i=1}^n w_i V_i \cdot V_o\right] - 2E\left[\sum_{i=1}^n w_i V_i\right] \cdot E[V_o] + \sigma^2[V(P_o)]$$



$$\sigma^2 \left[ \sum_{i=1}^n w_i V_i \right] - 2 \sum_{i=1}^n w_i E[V_i V_o] - 2 \sum_{i=1}^n w_i E[V_i] E[V_o] + \sigma^2 [V(P_o)]$$

ally; the error variance becomes:

$$\sigma_R^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j C_{ij} - 2 \sum_{i=1}^n w_i C_{io} + \sigma^2 \quad (3.22)$$

ere

$$C_{ij} = \text{Cov} [V_i V_j].$$

$$C_{io} = \text{Cov} [V_i V_o].$$

= variance of random variables which is constant for all variables.

In this research the variance of the random variables is unknown because the test is a "one-shot" test (replicate tests cannot be made at the same point), therefore pooled variance, which is calculated from the N values that are considered for testing, is used instead.

The minimization of this error variance is accomplished by setting the partial first derivatives with respect to the unknown weights to 0, with some considerations to include the unbiasedness condition that was given in equation (3.20) as

$$\sum_{i=1}^n w_i = 1$$

minimization is explained in sections (3.2.3.2.2 and 3).

## 3.2.2 The Lagrange Parameter

The unbiasedness condition will add one more equation without adding any more unknowns. This leaves it with a system of (n+1) equations and only n unknowns, the

solution of which requires that one additional assumption be made. To remedy this problem, a new unknown variable is introduced. This new variable is called "u", the Lagrange parameter, and is introduced into equation (3.22) to become:

$$\sigma_R^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j C_{ij} - 2 \sum_{i=1}^n w_i C_{i_0} + \sigma^2 + 2u \left( \sum_{i=1}^n w_i - 1 \right) \quad (3.23)$$

This additional term does not upset the equality because its value is 0 due to the unbiasedness condition. The error variance is now a function of (n+1) variables. By setting the (n+1) partial first derivatives to 0 with respect to each of these variables a system of (n+1) equations is obtained.

### 3.2.3.2.3 The Ordinary Kriging System

These (n+1) equations, often referred to as the ordinary Kriging system, and can be written - after summing up - as follows:

$$\sum_{j=1}^n w_j C_{ij} - C_{i_0} + u = 0; i=1, \dots, n \quad (3.24)$$

$$\sum_{i=1}^n w_i = 1 \quad (3.25)$$

These equations imply:

For every data point, the sum of (the weighted covariances between that point and other data points) minus (the covariance between that point and the estimated point) plus (the Lagrange term) equals zero.



The sum of weighting factors contributing to any point is one.

The solution of these equations produce the set of weights that minimizes the error variance as well as the value of the Lagrange parameter  $u$  that is useful for calculating the resulting minimized error variance.

In matrix notation, these equations take the form:

[ covariance matrix ] \* [ weight matrix ] = [ vector of the covariances between estimated point and the measured ones ] ( 3.26 )

$$\begin{array}{ccc}
 C & \cdot & W = D \\
 \left[ \begin{array}{cccc|cc}
 C_{11} & \cdot & \cdot & \cdot & C_{1n} & 1 \\
 \cdot & & & & \cdot & \cdot \\
 \cdot & & & & \cdot & \cdot \\
 \cdot & & & & \cdot & \cdot \\
 C_{n1} & \cdot & \cdot & \cdot & C_{nn} & 1 \\
 1 & \cdot & \cdot & \cdot & 1 & 0
 \end{array} \right] & \cdot & \left[ \begin{array}{c}
 W_1 \\
 \cdot \\
 \cdot \\
 W_n \\
 u
 \end{array} \right] = \left[ \begin{array}{c}
 C_{10} \\
 \cdot \\
 \cdot \\
 C_{n0} \\
 1
 \end{array} \right] \\
 (n+1) * (n+1) & & (n+1) * 1 \qquad (n+1) * 1
 \end{array}$$

Therefore the weights are given by:

$$W = C^{-1} \cdot D \quad (3.27)$$

The minimized error variance is given by:

$$\sigma_R^2 = \sigma^2 - \left( \sum_{i=1}^n w_i C_{i0} + u \right) \quad (3.28)$$



#### 2.3.2.4 The Covariance Matrix

It is stated (Lancaster and Salkauskas, 1986) that the main thrust of riging is to choose the covariances in a way that is consistent with the data. As many  $(n+1)n/2$  covariances have to be chosen to describe the spatial continuity in the random function model. Davis (1986) noted that, in principle, the experimental variance values could be used directly to provide values for the estimation procedures. However, the covariance is known only at discrete points representing the sampled stations. In practice, covariances may be required for any distance. For this reason, the discrete experimental covariances may be modeled by a continuous function that can be evaluated for any desired distance. If a modeled covariance function  $C(h)$  is used, then all of the required covariances are calculated from this function. Once the  $(n+1)n/2$  covariances have been obtained, the matrices  $C$  and  $D$  can be determined; consequently a set of weights  $w$ 's as well as the minimized error variance can be calculated using the above-mentioned equations.

In the published literature, many forms have been assumed for the covariance function. Five are summarized below (Isaaks and Srivastava, 1989; Davis, 1986):

**spherical model:** implies that correlation decreases at a nearly linear rate at small separation distances, but flatter out at larger distances and is given by the equation:

$$C_{ij} = \sigma^2 [1 - 1.5 (h/a) + 0.5 (h/a)^3], h \leq a \quad (3.29)$$



$h$  = distance between the points  $p_i$  and  $p_j$ .

$a$  = the range or the distance beyond which the covariance value remains essentially constant.

**The exponential model:** implies that correlation decays exponentially with distance and is given by the equation:

$$C_{ij} = \sigma^2 e^{(-3h/a)} \quad (3.30)$$

**The Gaussian model:** is a variation of the exponential form with the distinguishing feature of having a parabolic behavior at small separation distances. Its equation is:

$$C_{ij} = \sigma^2 - 1 + e^{(-3h^2/a^2)} \quad (3.31)$$

**The linear model:** assumes the correlation decreases linearly. Its equation is:

$$C_{ij} = \sigma^2 - \alpha |h| \quad (3.32)$$

where

$\alpha$  is a constant depending on the data set.

**The squared exponential model:** assumes the correlation decreases according to the equation:

$$C(h) = \sigma^2 e^{(-h^2/h_0^2)} \quad (3.33)$$

where

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$h_0$  = autocovariance distance.

= the distance at which  $C(h)$  decays to  $(1/e) C(0)$ , in which  $e$  is the base of the natural logarithms.

The more common covariance functions used in practice according to Groot and Baecher, (1993): are the exponential model, the spherical model, and a variation of the exponential model, the squared exponential model. They used the squared exponential model to describe the autocovariance structure of residuals about soil spatial trends. Similarly Soulie' et. al. (1990) report that in practical problems an exponential or a spherical model is commonly used to describe the spatial variability of soil parameters. Journel and Huijbregts (1978) report that the spherical model is more commonly used in mining geostatistics.

Accordingly, the squared exponential model is further considered in chapter (4) where used if a covariance function is required to describe the spatial variability of N variables.



## CHAPTER 4

### MODELING THE N-FUNCTION FOR SETTLEMENT PREDICTION

#### 4.1 GENERAL

It is impossible (or at least impractical) to test and take sufficient measurements to characterize the entire inherent variability of the site. In practice, SPT tests are conducted at some interval within each of a number of borings which are presumed to be representative of the whole site. The results of these tests form a statistical sample and the findings from this sample must be used to estimate an appropriate or the true penetration resistance function or constant value for the whole site.

As such, many statistical modeling and prediction techniques could be employed to quantify a representative N function for the scattered data. The statistical significance or confidence associated with the function could also be quantified.

Two techniques, trend surface and Kriging, were described in the previous chapter. In this chapter, the details of using these methods with N value data will be considered. These techniques will provide an estimate of  $N(x,y,z)$  at any desired point.

For the calculation of predicted settlement using methods (e.g. Meyerhof, 1965) that are based on a single N value, information regarding the function  $N(z)_{x,y}$  at a specific plotting location is preserved by the introduction of a "two-point" estimate procedure, developed by this research. This procedure, consisting of weighting two specific values  $N(z)_{x,y}$ , is introduced in the next subsection. This is followed by discussing the details of fitting N values using the two considered approaches.



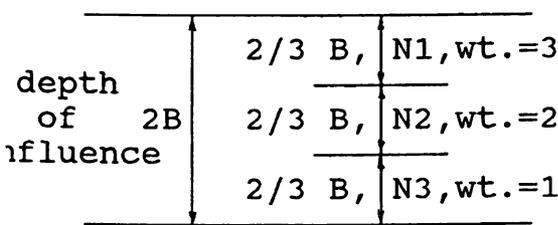
**THE "TWO - POINT" ESTIMATE FOR SETTLEMENT PREDICTION**

Agreement is obvious among several researchers that a depth of influence of twice footing width (2B) is a reasonable assumption under a shallow footing with width B (Burland, 1985 ; Bazaraa, 1982 ; Schultze and Sherif, 1973 ; Parry, 1971 ; and Schmertmann, 1970). So, it is reasonable to make the following assumptions:

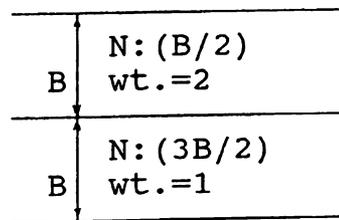
Two N value estimates, obtained at the depths of B/2 and 3B/2 could convey information regarding both the average of N values within the zone of influence as well as the rate of increase with depth.

To provide consistency with observed strain distributions below footings, these two values  $N_{(B/2)}$  and  $N_{(3B/2)}$  have to be averaged by using some suitable weight for each.

Parry's (1978) proposed using three N values with averaging weights as shown in Figure 4.1.



**Figure 4.1: Weights used by Parry.**



**Figure 4.2: The weights suggested by this research.**

It can be seen that the weights associated with the upper part of the depth of influence or B is equal to  $(3 + 2/2) = 4$  and that of the lower part equal to  $(2 + 2/2) = 2$ . This is equivalent to using weights of 2 and 1 for the upper and lower parts of the depth of influence respectively as shown in Figure 4.2.

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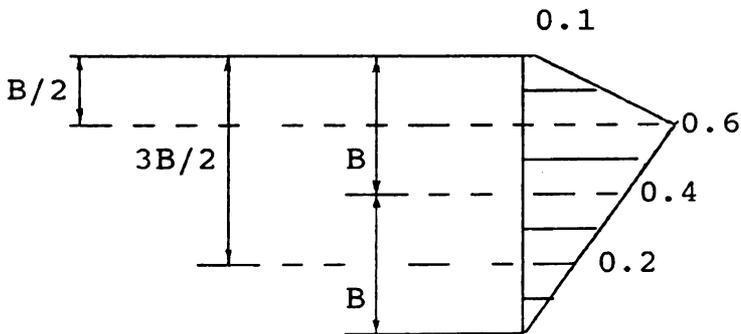
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Further support regarding this suggested "two-point" estimate of the design  $N$  value can be obtained by considering Schmertmann's (1970) strain influence factor shown in Figure 4.3. Schmertmann's factor provides a generalized weighting method that can be applied to any number of  $N$  values.



**Figure 4.3 : Schmertmann's strain influence factor vs. depth.**

If the diagram is divided into upper and lower halves, the strain factor associated with the upper point estimate  $N_{B/2}$  is the area of the diagram from the footing base level to a

$$\text{depth of "B"} = (0.35)(B/2) + (0.5)(B/2) = 0.425 B$$

Similarly the strain factor associated with the lower point estimate is the area of the

$$\text{diagram from the depth of "B" to a depth of "2B"} = 0.2 B$$

The design  $N$  value is taken as these two estimates weighted by their corresponding

areas in the strain influence factor diagram, then:

$$\begin{aligned} \text{design } N \text{ value} &= (0.425/0.625) N_{B/2} + (0.2/0.625) N_{3B/2} \\ &= 0.680 N_{B/2} + 0.320 N_{3B/2} \end{aligned}$$

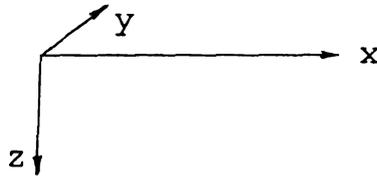
As seen that this corresponds closely with the somewhat simpler estimate

$$N_{B/2} + (1/3) N_{3B/2} \text{ suggested by this research.}$$

### 3 APPLICATION OF "TREND SURFACE" THEORY TO OBTAIN THE "TWO-POINT" ESTIMATE AND SETTLEMENT PREDICTION

To start the analysis, the parameters which are believed to explain the data scatter have to be identified using appropriate previous experience. The significance of these parameters is later tested by suitable statistical techniques.

Those explaining parameters are assumed to be the three geometric co-ordinates (x,y & z) related to any arbitrarily chosen origin within the site, Figure 4.4.



**Figure 4.4 : The 3 geometric co-ordinates (x, y & z).**

These three parameters are necessarily independent. The terms and coefficients associated with the "z" parameter reflect the variability resulting from the different layers and layer thicknesses, the heterogeneities within the soil layers, and the overburden effect. By making the overburden pressure correction to the N values in the field it is possible for these values to predict only the relative density. The Liao and Seed (1986) overburden correction factor is considered herein because it was developed by fitting the forms of the correction factors proposed by others including Seed and Peck et. al. (1974) and Bazaraa (1967). In this method, the N value is corrected using the equation  $N_1 = C_1 * N$ .

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$$C_1 = (1/p_v')^{0.5}$$

$p_v'$  is the effective overburden pressure (tsf) = depth in ft \* unit weight (lb/ft<sup>3</sup>)/2000.

The terms and coefficients associated with the parameters "x" and "y" reflect the variability resulting from the depositional variability of the soil in the horizontal direction as well as variability due to the difference in stress history over the site, if any.

If the  $N(z)$  function components from the different boring logs are not significantly different, then the two parameters x and y should be excluded from the model. This would imply that all the soil data of the site are identical. One must be skeptical about this assumption. So, it is preferable to test this assumption by including x and y in the model, then the test result can be used to assess their significance.

The "trend surface analysis" is intended for statistically homogeneous profiles, as described by Terzaghi (1978), and cannot help determine an appropriate N function for stratified or layered soil. Thus it is suggested by this research to introduce an addition to the "trend surface analysis" to fit the reality of soil stratification more closely.

The suggested method for modeling the N functions including the above-mentioned addition is summarized as follows:

The subsurface soil is stratified into layers, each of which is sufficiently homogeneous with regard to relative density to be treated as a unit.

A "one-way ANOVA analysis" is conducted to test the significance of the differences between the means of N values in the different layers.

The difference between the means of pairs of layers is tested (if the suggested stratification was justified by the previous step).

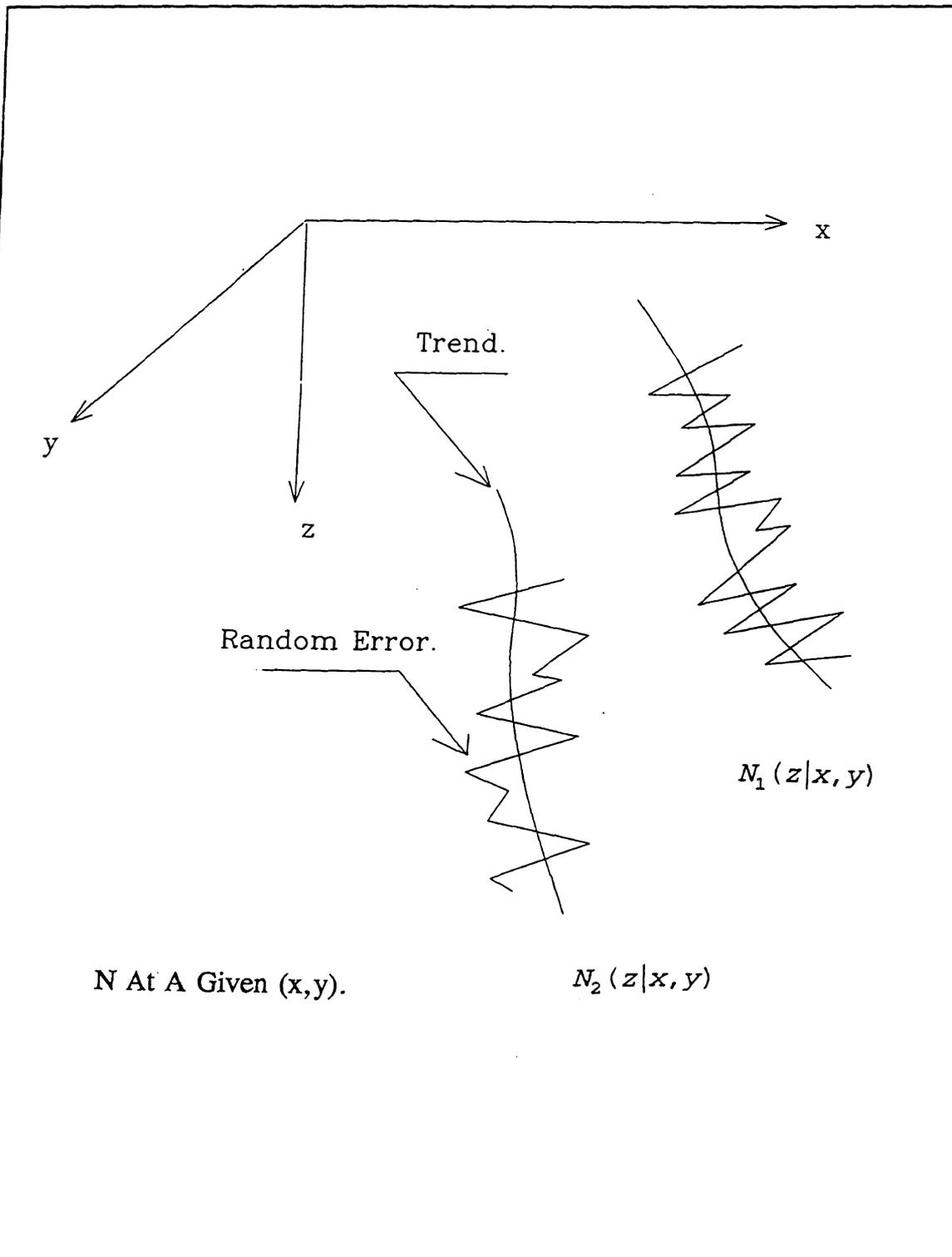


Figure 4.5 : The Trend Surface Is Decomposed Into A Trend And A Random Error.



4. If more than one layer is justified, a separate three-dimensional model for the  $N$ -values as  $N = f(x,y,z)$  is constructed for each layer as shown in Figure 4.5.

Regarding item "1" above, the relative density  $D_r$  values are estimated by using the correlation between the relative density and the  $N$  values. Terzaghi and Peck (1948) gave the first classification of relative density in terms of the  $N$  values. Values of  $D_r$  were assigned to this classification by Gibbs and Holtz (1957). The combined results are shown in Table 4.1.

**Table 4.1 : Classification Of Relative Density In Terms Of  $N$  values.**

$D_r$	$N$	Relative density
0.15	0 - 4	very loose
0.15 - 0.35	4 - 10	loose
0.35 - 0.65	10 - 30	medium
0.65 - 0.85	30 - 50	dense
0.85+	50+	very dense

As far as the layer depth is concerned, Vanmarcke's (1977, pp. 1229) noted that the layer depth in a real profile may vary more or less erratically as a function of the horizontal dimension; the type of modeling he proposed is meant to supplement, not to substitute for the conventional soil profile and the concept of "local homogeneity" is intended to cover not only averages, but the nature and the overall appearance of the situations as well. As such, it suffices for this research to stratify the soil into layers

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y using geotechnical judgment which considers the average of N values as well as the nature and the overall appearance of the fluctuations.

Regarding item "2" above, the "one-way ANOVA analysis" will provide a statistical measure that the stratification is reflecting real different layers with different relative densities and thus verify the judgmental stratification. The measured values of N within each layer are considered as one treatment, so the number of treatments in the ANOVA analysis will be the same as the number of layers adopted. This kind of analysis is suggested because it distinguishes the amount of variation within and between treatments. So in order to test the dependence of the N-mean values on the depth, the difference between the layer means has to be large compared with the variations within layers. In other words, to test the null hypothesis that the different layers means are equal, the F ratio - which is the quotient of the "between layers mean square" by the "within layers mean square" - is compared to the F probability distribution value at the desired significance level (Davis, 1986; Box, Hunter & Hunter, 1978, pp. 187).

If the difference between the layers means is proven to be significant, then the suggested stratification is accepted; otherwise no stratification is justified and the whole surface soil is considered as one layer, (e.g., although differences may exist due to depth effects, the differences in average properties are not significant enough to warrant representing the whole subsurface soil by a single spatial model).

Regarding item "3" above, the difference between any two consecutive layers has to be significant, otherwise they should be combined into one layer. This may then increase layer variance, but the estimation uncertainty of the model will be reduced

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because one will have the advantage of estimating the parameters of only one model instead of two by using the same amount of data. Therefore a trade-off is being made between estimation uncertainty and random spatial variation. Baecher (1978) stated that exploration data have a finite number of degrees of freedom : the more parameters estimated, the more uncertainty in each. To reduce overall predictive uncertainty requires both more data and a more spatially disaggregate stochastic model. The comparison between layers has to be made by using a suitable multiple comparison technique. The difference between the multiple comparison test and the ANOVA test is explained as follows:

The ANOVA test's objective is to test whether the stratification is justified irrespective of which number of layers was used to conduct the test.

The multiple comparison test's objective is testing whether the suggested number of layers is accepted; otherwise a lesser number - by combining some of them together - is considered.

There are many multiple comparison tests available in the literature, (Levine, 1991; Dunn, 1961; Dunnett, 1964 and Tukey, 1949). Three such tests will be used herein; they are briefly summarized below.

#### **LSD (Least Significant Difference) Test:**

This test (Levine, 1991) is done to test the significance of the differences between pairs of means. It is simply a convenient form of the t test. It allows the use of different alpha levels for different comparisons. The LSD test is not done if every possible paired comparison is to be tested. This test involves computing a single value

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or "criterion value" against which each desired paired comparison is tested, i.e. this criterion value is compared to each difference between means that is to be tested for statistical significance or to check whether the corresponding true difference is not likely to be zero.

#### **Tukey HSD (Honestly Significant Difference) Test:**

This test (Tukey, 1949) provides a convenient way of computing a single value against which all of the possible differences between pairs of means can be compared for statistical significance. It also uses some confidence limits for testing the significance of the differences between pairs of means, but it is used when the intention is to maintain one "family-wise" Type I (alpha) error level to be used for all comparisons. For "n" layers, there are  $n(n-1)/2$  possible pairs of layers. This test compares every one of these possible  $n(n-1)/2$  difference between means to the criterion value to test it for statistical significance.

#### **Dunn Test:**

Whereas the Tukey test runs all of the possible pairs of comparisons among all means, the Dunn test (Dunn, 1961) is used when the intention is to test only some selected number of pairs. At the same time this test maintains an overall "family-wise" Type I error probability that does not change with the number of comparisons to be made. The Dunn test is the alternative to the Tukey when the decision has been made to only test a subset of all possible paired comparisons. Consequently, this test is not recommended unless the number of layers is relatively large.

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Regarding item "4" above, a three-dimensional model for the  $N$  values as  $N = f(x, y, z)$  is built for the entire foundation or for each layer separately. Some considerations about the building of this model are as follows:

1. The model for each layer is built by conducting a multiple regression analysis on the relevant  $N$  values. The resulting regression surfaces can be used to predict the soil  $N$  value at any depth beneath any location on site. The three parameters  $x$ ,  $y$  and  $z$  are necessarily independent; consequently their effects are additive and no interactions should be expected between any of them. This leads to the exclusion of all the cross-product terms, of any order, from the model. As a result, the general form of the model becomes:

$$N(x, y, z) = b_0 + b_1x + b_2y + b_3z + b_4x^2 + b_5y^2 + b_6z^2 + \dots \quad (4.1)$$

The degree of the model and the terms that have to be included are assessed for six case histories in chapter five. However, certain relevant information are given herein to be utilized to develop guidelines prior to such analyses.

2. The most important term that should be included is the constant term  $b_0$ , because it is the basis from which the  $N$  value varies in the  $x$ ,  $y$  and  $z$  directions. Philosophically satisfactory model would have a  $b_0$  value that is also a reasonable value for  $N$ .

3. The variability of the  $N$  value in the  $z$  direction will be controlled by the soil stiffness which increases with depth due to overburden pressure. The correlation between  $N$  and  $E$  as a measure of stiffness is well established (Bowles, 1988; Webb, 1969; Anagnostopoulos, 1990), but no consensus is evident among the researchers as to how "E" would increase with depth. As a result of this uncertainty, the orders of the

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terms is an issue to be examined using the regression analysis by examining the resulting values of their coefficients.

The F test approach can be used to test the aptness of this model. If it is not accepted, a different case of this general form of the model should be employed and tested again. The predictive power or "goodness of fit" of the resulting model can be identified by calculating the  $R^2$  value, which represents the percent of total variation in explained by the formulated model.

The geometric location of the footing in addition to the determined foundation model will identify the three coordinates of the footing as well as the depth of influence under its loading, consequently the resulting model can be used to determine the design value for this footing.

These design N value(s) are used in the next stage which is the estimation of settlements and differential settlements.

## 1 USING THE "TREND SURFACE" N FUNCTION TO PREDICT THE SETTLEMENTS AND DIFFERENTIAL SETTLEMENTS

To predict the settlement under a specific footing of breadth "B" as shown in Figure

the following N functions are formulated using corrected N values:

$$N_1 = f(x, y, z) ; \quad z < z_1.$$

$$N_2 = f(x, y, z) ; \quad z_1 < z < z_2.$$

$$N_3 = f(x, y, z) ; \quad z_2 < z < z_3.$$

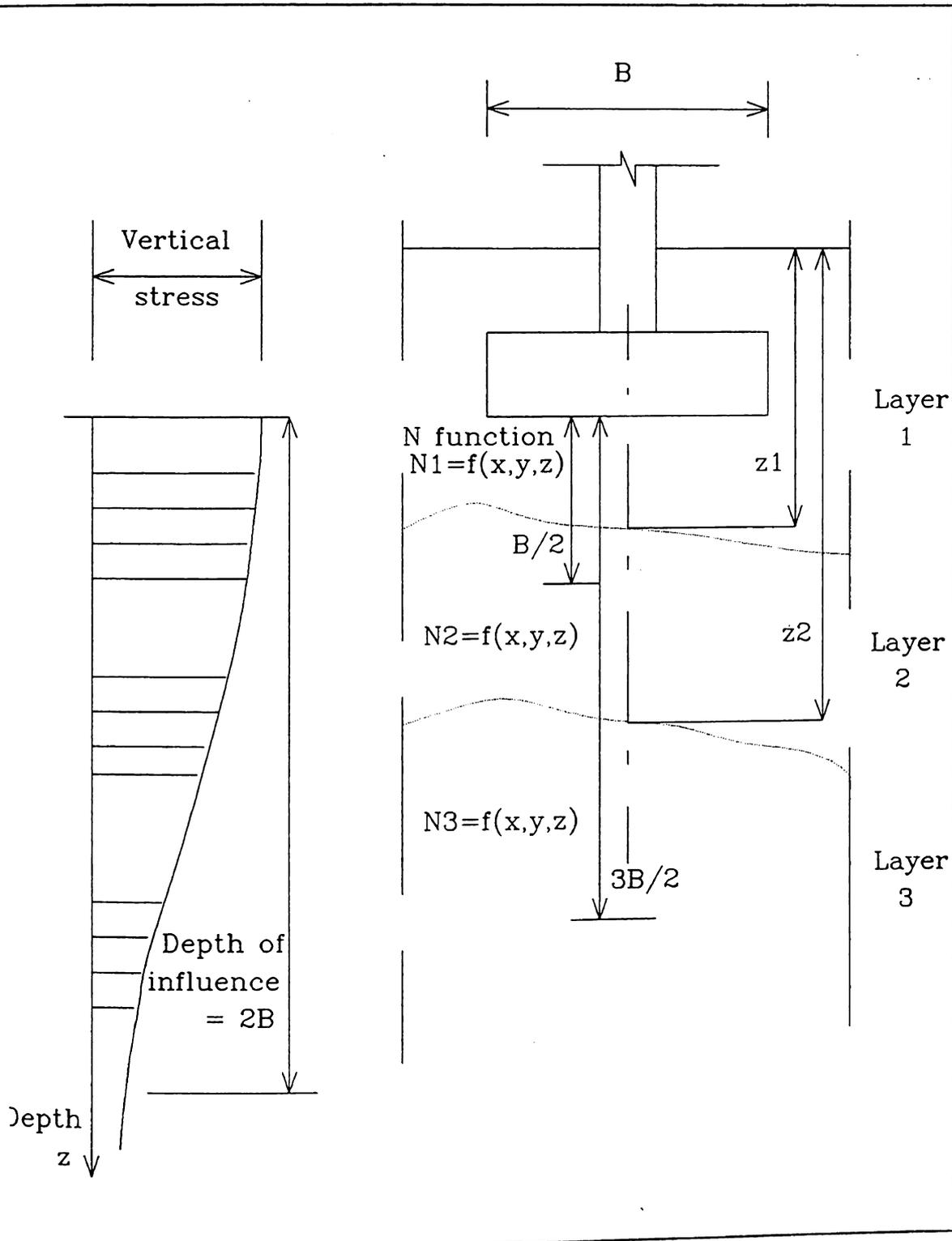


Figure 4.6 : The Different N Functions For The Different Subsoil Layers.

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Then one or more "two-point" estimates (one for each footing) are drawn from these spatial models as a step in transforming them into a planar  $N(x,y)$  model. This latter model enables one to do such things as contouring analysis or settlement comparisons and the like. This is explained as follows:

1. The predicted  $N$  values at depths  $B/2$  and  $3B/2$  are obtained from the appropriate  $N$  function. For example, if the depth of  $B/2$  is located within layer 2 and the depth of  $3B/2$  is located within layer 3, then one substitutes the value of  $B/2$  for  $z$  in the  $N_2$  function and similarly the value of  $3B/2$  in the  $N_3$  function. This will give the two values of  $N$  at these two depths as:

$$N_2 = f(x, y, B/2); \quad \& \quad N_3 = f(x, y, 3B/2). \quad (4.2)$$

2. As illustrated in section 4.2, the design  $N$  value at the location  $(x, y)$  is given by:

$$N = (1/3) [2 N_{(B/2)} + N_{(3B/2)}] \quad (4.3)$$

where

$$N_{(B/2)} = f(x, y, B).$$

$$N_{(3B/2)} = f(x, y, 3B).$$

After summing, this relationship will take the form:

---

If the depth of influence is located entirely in the upper layer then the 2 representative  $N$  values would be:

$$N_1 = f(x, y, B/2); \quad \& \quad N_1 = f(x, y, 3B/2)$$

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$$N=f(x, y, B)$$

for different size footings,

$$N_i=f(x, y, B_i) \quad (4.4)$$

This N function is used in a suitable model which gives an estimate of the settlement as a function of N, footing pressure p, and the width B, will eventually result in modeling the settlement of the footing as:

$$S=f(p, B, x, y) \quad (4.5)$$

For instance, the Bazarraa method estimates the settlement "S" as:

$$S=(1/N') [2q*(2B/B+1)^2] (P_d/P_w) \quad (4.6)$$

All the terms have been previously defined in Section 2.4.

Like some earlier methods, the Bazarraa method uses the N value after correcting it for the overburden pressure, which is consistent with the suggested technique which employs a corrected N also. Furthermore, the concept of a simple equation  $S=f(N, p, B)$  is retained because the emphasis herein is to utilize a "simple" model with a few variable parameter values over a large region of space or "field", rather than a complex, rigorous model which might require more certainty in parameters.

Now, substituting the N function instead of the N value, will yield the following model:

$$S=[1/f(x, y, B)] * [2q*(2B/B+1)^2 (P_d/P_w)] \quad (4.7)$$

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s model will enable one to predict the amount of the settlement under a footing (with some quantified confidence) at a point not sampled at all. Once  $N(x,y,z)$  is obtained, one needs to know are the co-ordinates of this point  $x$  and  $y$  in addition to the footing width and the net pressure. Similarly, the contours of the estimated settlement can be plotted by using this model.

Alternatively, one could use the  $N$  function with a method which uses the modulus "E". Consider, for example Schmertmann's method:

$$S = C_1 \cdot C_2 \cdot p \sum_{z=0}^{2B} [(I_{zi}/E) \cdot dz_i] \quad (4.8)$$

Since  $E$  is estimated from the correlation with  $N$ , i.e.  $E = f(N)$ , then substituting the  $N$  function will yield:  $E = f(x, y)$ . The "z" parameter does not appear in this expression because its value in the  $N$  function will take a numerical value equals to the depth of the midpoint of the layer  $i$ , consequently the model becomes:

$$S = C_1 \cdot C_2 \cdot p \sum_{z=0}^{2B} [(I_{zi}/E_i) \cdot dz_i] \quad (4.9)$$

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$$E_i = f(N(x, y, z)) \quad (4.10)$$

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is important to note that this permits one to perform a Schmertmann analysis with additional and consistent input at an (x,y) location not drilled and sampled.

The differential settlement between any two independent footings in the site can be estimated by substituting the numerical values of the parameters describing each footing - (e.g.  $p_1, B_1, x_1, y_1 \dots$  and  $p_2, B_2, x_2, y_2 \dots$ ) - in the above mentioned model. In such, a numerical value is obtained for estimation of the differential settlement.

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## 2.2 CONSTRUCTING THE PREDICTION CONFIDENCE INTERVALS FOR A GIVEN DATA SET

The identified modeling technique using trend surface analysis should enable the foundation designer to find out how certain or uncertain are the predicted settlements using this technique. Before attempting to interpret the fitted model, it is necessary to consider whether or not it is estimated with sufficient precision (Box, Hunter and Hunter, 1978, pp. 524).

Assuming that the resulting model which estimates the N value is developed from trend surface analysis and is given by the equation:

$$N_{est.} = b_1 + b_2x + b_3y + \dots + b_pz. \quad (4.11)$$

Then the average variance of the fitted value  $N_{est.}$  at the design points, no matter what the model, is given by:

$$\begin{aligned} \sigma_{ave}^2(N_{est.}) &= (1/n) \sum_{i=1}^n [\sigma^2(N_{est.})_i] . \quad (4.12) \\ &= (1/n) [ P * \text{error variance} ]. \end{aligned}$$

where

P = number of parameters fitted.

n = number of N observations.

$[\sigma^2(N_{est.})_i]$  = variance of the ith fitted value of N.

The error variance is obtained from the replicate observations at the same testing point. Such replicate observations are not feasible with SPT results because it is a destructive test. Consequently the pooled estimate of variance is used instead.

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n, the corresponding standard error of  $N_{est}$  is given by (Box, Hunter and Hunter, 1953, Equation 15.26; Berry and Lindgren, 1990):

$$\begin{aligned} (St. Err.)_{ave} of (N_{est}) &= [\sigma_{ave}^2 (N_{est})]^{0.5} \\ &= [(P/n) * error variance]^{0.5} \end{aligned} \quad (4.13)$$

Consequently, the confidence limits of the  $N$  value are given by:

$$N = N_{est} \pm t_{(d.f., \alpha/2)} * [(P/n) * err. variance]^{0.5} \quad (4.14)$$

e

$N_{est}$  = the  $N$  value estimated using the fitted  $N$  model.

$P$  = number of parameters (b's) in the model.

$n$  = number of  $N$  observations included to fit the model.

$t_{(d.f., \alpha/2)}$  is the  $t$  statistic for a confidence level of  $(1 - \alpha)$  and the involved degrees of freedom.

Now, it is required to determine the confidence limits of the estimated settlement. This is done as follows:

Assuming that the settlement is determined from Bazaraa's equation as:

$$S = (C/N') \quad (4.15)$$

$$C = [2q * (2B/B + 1)^2 * (P_d/P_w)]$$

$N'$  =  $N$  value corrected for overburden pressure.

Substituting the  $N_{est}$  for the  $N$  value, yields:

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$$S_{est.} = (C/N_{est}) \quad (4.16)$$

and the confidence limits of  $S_{est}$  are then given by:

$$S_{est} = C / [N_{est} \pm t_{(d.f., \alpha/2)} * [(P/n) * err. variance]^{0.5}] \quad (4.17)$$

The above equation shows that the confidence interval of  $S_{est}$  could be narrowed by decreasing the number of the fitted parameters "P" or by increasing the number of N observations "n". This in turn is increased by increasing the number of borings and decreasing the spacing between N observations. As such, there is a trade off between the estimation precision and the number of borings or the sampling costs.

It is also important to note that, for the modeling of N values to be representationally accurate, the boring locations have to be selected randomly over the site. In reality, boring locations are seldom or never random, but are laid out on a grid or some other geometry related to the structure. They are, however often laid out such that the resulting variability is equivalent to that expected from a uniformly distributed sample. The model which is built using the technique proposed herein is not the true population model but it is just a sample model. To be able to regard the  $b_i$ 's coefficients and by the model as estimates of the true population regression coefficients  $B_i$ 's and to test hypotheses about their nature one has to test a sample which represents the true population. This is achieved by selecting the sample members randomly. The more the sample selection is done randomly the more representationally accurate the regression surface will be.

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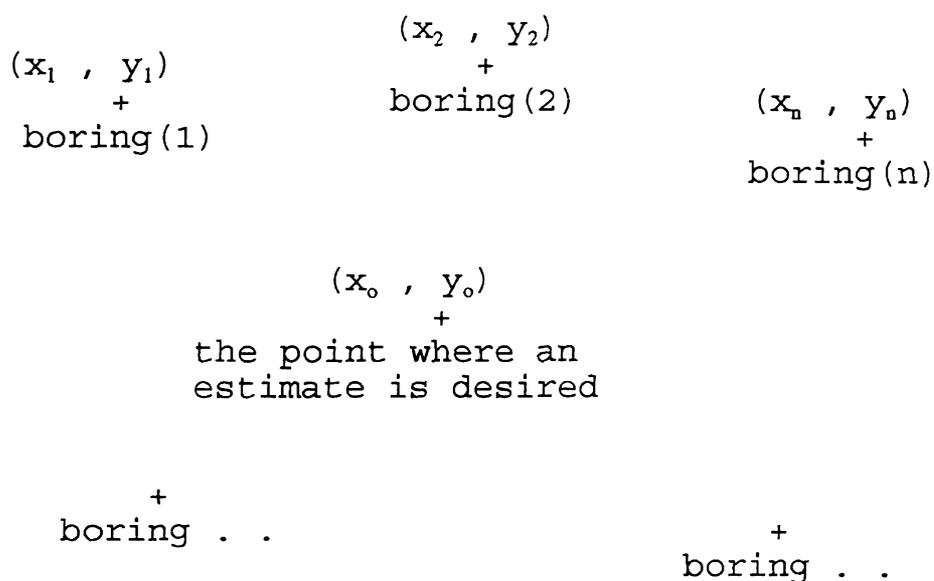
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## APPLICATION OF THE KRIGING TECHNIQUE TO SPT DATA

To obtain a settlement estimate using Kriged N values, first consider that the SPT occurs in the form of N values which are measured along each boring at suitable regular spacings. The borings are located in the (x,y) plane - for example - at the points  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ , as shown in Figure 4.7.



**Figure 4.7 : The Boring Locations In The (x,y) Plane.**

value data are necessarily located in a three-dimensional field. It is required to estimate the unknown N function at the point  $(x_o, y_o)$  - where no data were available - which can be represented by the function:  $N_{(z)}(x_o, y_o)$ .

For the selection of the data which will be used for estimation, reference is made to the statement by Davis (1986) that the observations selected to estimate the

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#### 4.4.1 Preparation

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ence around that location.

### Preparation Of N Data Before Modeling:

Kriging entails that in the local region where Kriging is performed each of the  
ues used to obtain the Kriged estimate should have the same probability law at all  
ons, which is not the case for N values due to the depth effect. To remedy this  
ulty, as well as recognize the fact that the desired results are in the x,y plane, it is  
sted to represent the N values of each boring by a linear regression function of the

$$N = a + bz \quad (4.18)$$

led that the soil homogeneity condition is satisfied. The parameters "a" and "b"  
provide a pair of x,y functions which jointly contain the N value information and  
e Kriged in two dimensions.

This form is suggested by this research based on a study by Gibson (1967) which  
ered the subsurface soil as an elastic half-space whose modulus E increases  
y with depth. The relationship between the soil modulus E in turn and the N  
is realized by the examination of the deterministic equations for settlement  
tion which were developed by Meyerhof, Peck, Bazaraa and Parry and were  
rized earlier in Section 2.4. The inclusion of the N values in the denominators  
se strain determination equations implies the assumption that there is a linear  
nship between N and E. Consequently, the representation of the N values by

linear regression

mentioned borings

Figure 4.8

#### 4.4.2 Kriging

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regression functions in the depth  $Z$  is considered as reasonable. Thus the above mentioned borings will be represented as shown in Figure 4.8.

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 N_1 = a_1 + b_1 z \\
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 \end{array}
 \qquad
 \begin{array}{r}
 N_2 = a_2 + b_2 z \\
 + \\
 \text{boring(2)}
 \end{array}
 \qquad
 \begin{array}{r}
 N_n = a_n + b_n z \\
 + \\
 \text{boring(n)}
 \end{array}$$
  

$$\begin{array}{r}
 N_o = a_o + b_o z \\
 + \\
 \text{the point where an} \\
 \text{estimate is desired}
 \end{array}$$
  

$$\begin{array}{r}
 \text{boring . . .} \\
 + \\
 \text{boring . . .} \\
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 \end{array}$$

Figure 4.8 : Representing The N Values By A Linear Regression Function.

#### Kriging The N Regression Coefficients:

The following procedure is suggested to estimate the N function at any point an estimation for it is desired:

The horizontal distance ( $h$ ) between every pair of locations is calculated including boring locations and the locations where estimation is required. As such, the number of distances will equal to  $(n+1)n/2$  for each "unknown" location.

The covariances which describe the spatial continuity of the data are calculated.

Section 3.2.3.2.4 reference was made to Davis (1986) who noted that in principle, the

experimental covariance values could be used directly to provide values for the estimation

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s between every pair of locations, then these values could be used directly to build covariance matrix required for Kriging. However, the covariance is known only at the points representing the sampled locations. In practice, covariance may be estimated for any distance. For this reason, the discrete experimental covariances may be replaced by a continuous function that can be evaluated for any desired distance. In this case, the employed covariance model should be chosen in a way that is consistent with the data. Based on the discussion in Section 3.2.3.2.4, there is an agreement between Baecher et al. (1993) and Soulie' et al.(1990) on the suitability of the squared exponential model for describing the variability of soil properties. This model is given

$$C(h) = \sigma^2 e^{-h^2/h_0^2} \quad (4.19)$$

where  $h$  is the distance between two points for which the covariance is desired.

$h_0$  = autocovariance distance.

$h_0$  = the distance at which  $C(h)$  decays to  $(1/e) C(0)$ , where  $e$  is the base of the natural logarithms.

When determining the value of the autocovariance distance ( $h_0$ ), reference is made to Baecher (1985) who noted that considerable empirical work has verified the theoretical contention that soil properties are spatially correlated, and he suggested a distance of 50 m - 100 m as a reasonable finite correlation. Then the covariance of every pair of locations is calculated using this model. This will total to  $(n+1)n/2$  covariances.

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Calculating the weight matrix from:

$$W = C^{-1} \cdot D \quad (4.20)$$

$C = (n+1) * (n+1)$  matrix

$D = (n+1) * 1$  vector

and the matrix elements are as given earlier.

Kriging the  $N$  values - which are now represented as functions in "z" -

order to estimate the unknown  $N$  value at the point  $(x_o, y_o)$  as follows:

$$\hat{N}(x_o, y_o) = \sum_{i=1}^n w_i N(x_i, y_i) \quad (4.21)$$

equivalently:

$$a_o + b_o z = \sum_{i=1}^n w_i (a_i + b_i z) \quad (4.22)$$

$$= \sum_{i=1}^n w_i a_i + \sum_{i=1}^n w_i b_i z \quad (4.23)$$

equently:

$$a_o = \sum_{i=1}^n w_i a_i \quad (4.24)$$

$$b_o = \sum_{i=1}^n w_i b_i \quad (4.25)$$

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Then, the estimated value  $N(x_o, y_o)$  is equal to  $(a_o + b_o z)$ , which is a function in  $z$ .

### Using The Estimated N Function To Predict The Settlements And Differential Settlements

Having obtained the  $N$  value as a function in  $z$ , the "two-point" estimate - which suggested in Section 4.2- can be used to estimate the design  $N$  value as follows:

The  $N$  values at the depths of  $B/2$  &  $3B/2$  are given by:

$$\hat{N}_{(B/2)} = a_o + b_o (B/2) . \quad (4.26)$$

$$\hat{N}_{(3B/2)} = a_o + b_o (3B/2) . \quad (4.27)$$

The design  $N$  value at this location  $(x_o, y_o)$  is then given by the weighted average:

$$N = (1/3) [2\hat{N}_{(B/2)} + \hat{N}_{(3B/2)}] \quad (4.28)$$

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#### 4 Constructing The Prediction Confidence Intervals For A Given Data Set:

The minimized error variance - or as is sometimes called "the Kriging (or prediction) variance", (Cressie, 1991, pp. 122) - is given by:

$$\sigma_R^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j C_{ij} - 2 \sum_{i=1}^n w_i C_{i0} + \sigma^2 \quad (4.29)$$

According to this equation, the prediction variance is equal to (the weighted sum of all covariances between the various data points pairs) minus (twice the weighted sum of the covariances between the data points and the value being estimated) plus (the variance of the measured data).

that:

$\sigma^2$  is taken as the pooled variance of the measured data and  $w$ 's are the weights obtained by Kriging the data.

This minimized error variance can be used to construct the "prediction confidence interval" for the  $N$  values as follows:

The lower and upper confidence limits of the prediction interval are given by the predicted value minus or plus the multiplication of the square root of the prediction variance by the  $t$  statistic. In mathematical notations, the prediction interval is given by:

$$[\hat{N}(X_o, Y_o) - t_{(d.f., \alpha/2)} * (\sigma_R^2)^{1/2}, \hat{N}(X_o, Y_o) + t_{(d.f., \alpha/2)} * (\sigma_R^2)^{1/2}] \dots (4.30)$$

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$A = (1 - \alpha)\%$  prediction interval for the estimated  $N$  value.

$(x_o, y_o)$  = the estimated  $N$  value at the point  $(x_o, y_o)$ .

$t_{f, \alpha/2}$  = the  $t$  statistic for a confidence limit of  $(1 - \alpha)$  and the involved degrees of freedom.

d.f. = degrees of freedom which equal to  $(n-1)$ .

Under the assumption that the estimated  $N$  value at the point  $(x_o, y_o)$  is Gaussian, this prediction interval satisfies the following (Cressie, 1991):

$$Pr [L.C.L. < \hat{N}(x_o, y_o) < U.C.L.] = (1 - \alpha)\% \quad (4.31)$$

The probability calculation is made over the joint distribution of  $N(p_1), \dots, N(p_n)$ , (Cressie, 1991).

It is obvious that the "prediction interval" is tightened as the error variance decreases, and this in turn should decrease:

As the number of borings increases, or

As the covariance values decrease. This occurs if the boring locations were far apart. This means that different sampling configurations will produce estimates of different reliabilities.

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The dependence of the prediction variance on these factors is explained as follows (Kulkarni and Srivastava, 1989). Looking at the three terms of the expression of the prediction variance (Equation 4.29), it is noted that:

In the first term, the covariance behaves as an inverse distance in that it increases as the distance between data points increases. If the data points are far apart, the first term will be relatively small. As they get closer together, the average distance between them decreases and the average covariance increases. This term therefore accounts for the clustering by increasing the uncertainty if the considered data points are too close together.

The second term accounts for the proximity of the point being estimated to the available data points. As the average distance to these data points decreases, the average covariance increases and, due to its negative sign, this term decreases the prediction variance.

The third term represents the variance of the measured data and accounts, in part, for the erraticness of the variable under study (N value in this research). As the variable becomes more erratic, this term increases in magnitude, thus giving a higher degree of uncertainty.

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## CHAPTER 5

### VERIFICATION OF THE DEVELOPED MODELS

In this chapter, the two proposed models for settlement prediction, trend surface analysis and Kriging, are each tested two different ways. First they are verified using simulated data of an assumed field. This consists of simulating some "observed"  $N$  values at given data, simulating some "true unobserved"  $N$  values at locations presumed not to have data available in an assumed field, and investigating whether similar values could be obtained using the proposed techniques. These are followed by the verification of the proposed "two-point" settlement estimate by using the assumed trend of  $N$  and an assumed footing. Second, the reliability of the proposed models for practical applications is checked by conducting the suggested modeling on six available case histories. These case histories were selected based on the criteria that  $N$  values were available as a function of  $(x, y, z)$  and the measured settlement after the structure has been completed was also available. Based on these studies, comparisons are made to find out the advantages and disadvantages of using each of the two candidate procedures for estimating  $N(x, y, z)$  and recommendations are made to enable one to select among both of them.

#### VERIFICATION USING SIMULATED DATA OF AN ASSUMED FIELD

The mathematical verification of both the trend surface analysis method and the Kriging technique is achieved by assuming a reasonable functional form for the function of  $N(x, y, z)$ , sampling from that function, developing a model to fit the

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sampled data, and predicting the N values at group of unsampled points. A oneway analysis of variance is conducted to compare the simulated "true unobserved" N values with the predicted ones.

### 1.1.1 VERIFICATION OF THE DEVELOPED MODELS BY THE TREND SURFACE ANALYSIS

The verification is decomposed into four steps. The first is the generation of the simulated "true unobserved" and "observed" N values. The generated "observed" N values are then used to model the N function. This function is used for prediction of the "estimated unobserved" N values. The last step is the comparison between the simulated "true unobserved" N values and the predicted ones using variance analysis.

#### 1.1.1.1 Generating The Simulated "True unobserved" And "Observed" N Values:

The generation of the "true unobserved" N values and "observed" N values was performed by assuming a field specified by a hypothetical N function in the form:

$$N = a_0 + a_1|X| + a_2|Y| + a_3Z^{0.5} + \epsilon \quad (5.1)$$

The rationale for this function is that it yields N values with a basic value of  $a_0$ , an allowance is made for a linear variation in both the X and Y directions, and it allows a quadratic variation with increasing depth. The random term " $\epsilon$ " is then added to account for the random error of N value. The rate of the variation in each direction is controlled by the values of the coefficients  $a_1$ ,  $a_2$  and  $a_3$ . The coefficients were set at the following

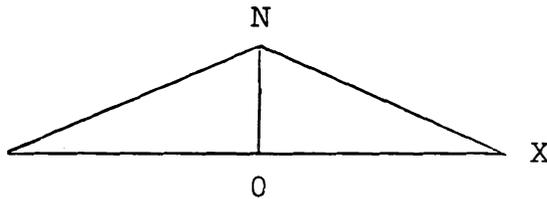
values:

$$a_0 = 10, a_1 = -0.01, a_2 = -0.02, a_3 = 2$$

to give the equation:

$$N = 10 - 0.01|X| - 0.02|Y| + 2Z^{0.5} + \epsilon \quad (5.2)$$

This equation yields N values which at a given depth have a triangular distribution with the highest value at the origin as shown in Figure 5.1.



**Figure 5.1 : The Assumed Triangular Distribution Of N Value.**

The subsoil is considered as a one layer extending within the limits of (-500 to 500) ft, (-400 to 400) ft and (0 to 50) ft in the X , Y & Z directions respectively. The resulting "true N" function then assumes nominal values (without the random term) of 11.47 to 27.47 at a depth of 5 ft, 7 to 33 at Z = 25 ft , and 11.14 to 37.14 at 50 ft, which are judged typical of a loose to medium sandy soil.

#### **Generating The Simulated "True Unobserved" N values:**

Eight N values are generated and considered as "true" N values. Each of these eight values is obtained from the above hypothetical N function by substituting random values for the X , Y & Z coordinates. These values are obtained as follows:

$$X = -500 + R.N.(1000) , Y = -400 + R.N.(800) , Z = R.N.(50)$$

Where: R.N. is a random number generated by a uniformly distributed function in the range of (0 - 1).

The random error term " $\epsilon$ " is taken as a normally distributed random number with an

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assumed zero mean and a standard deviation of 1.5. This term was generated based on a commonly used normal distribution generation algorithm previously used and tested by Wolff (1993) and others. In this algorithm 12 uniformly distributed random numbers (R.N) are generated. These were then used to generate one value: "Norm" from a standard normally distributed random variable (mean = 0, standard deviation = 1.0) as:

$$NORM = \sum_{i=1}^{12} (R.N_i - 0.5)$$

NORM in turn was used to generate a random value for one of the error terms by multiplying it by the assumed standard deviation. Thus the random term is given by:

$$\epsilon = 1.5 \sum_{i=1}^{12} (R.N - 0.5)$$

The generated "true unobserved" N values as well as their coordinates are in Table 5.1

**Table 5.1 : The Simulated "True Unobserved" N Values For Trend Surface.**

Location	X (ft)	Y (ft)	Z (ft)	N*
1	-211.13	333.20	6.53	7.35
2	-391.66	49.49	16.22	14.16
3	42.56	-212.81	27.03	16.73
4	320.14	-200.77	28.48	14.47
5	17.32	-296.12	9.57	12.10
6	165.55	-215.18	46.01	18.62
7	-306.47	302.83	1.56	4.39
8	-308.67	253.64	45.36	16.32



### Generating The Simulated "Observed" N Values:

The hypothetical N function was used to generate twenty additional N values.

These values were considered as data which can be used for the suggested modeling.

The generated "observed" N values as well as their locations are shown in Table 5.2.

The generated N values in this table have a mean value of 12.9 , a standard deviation

4.32 and a coefficient of variation of 34% which indicates a moderately high degree

uncertainty.

**Table 5.2 : The Simulated "Observed" N Values For Trend Surface.**

Data (No)	X	Y	Z	N
1	- 50.58	-258.34	6.70	8.60
2	-316.49	350.56	13.77	7.60
3	389.00	-299.80	32.79	12.34
4	267.88	388.77	12.29	8.13
5	-472.71	- 71.51	16.29	13.60
6	-160.91	- 91.19	12.53	14.31
7	375.27	- 72.30	10.10	10.66
8	90.10	210.80	21.94	12.89
9	280.53	53.70	44.55	20.41
10	-497.41	-141.50	19.47	9.47
11	233.88	188.37	46.69	17.04
12	-148.30	369.87	14.62	9.19
13	489.58	158.37	12.29	10.55
14	80.64	64.48	28.82	19.43



**Table 5.2 : Continued.**

15	218.09	209.95	15.24	11.42
16	- 7.55	44.38	21.76	18.30
17	- 50.89	-289.41	13.91	10.95
18	200.14	14.30	36.73	21.42
19	-178.60	-259.98	43.84	14.91
20	-364.23	376.98	20.17	6.78

### 1.1.2 Using The Trend Surface Analysis To Predict The N Values:

The program "SPSS", (SPSS/PC+, 1990), was used to fit a trend surface equation to the simulated "observed" N values given in Table 5.2. SPSS is a widely used, general purpose program. Two different forms were assumed for the trend surface which is fitted to the given data. Two forms are considered as enough here because the mathematical form of the trend is known already. However, in the general case, all the possible combinations of X, Y and Z terms should be tried. The form which was used in the first trial is given by the equation:

$$N = b_0 + b_1X + b_2Y + b_3Z^{0.5} + b_4Z + b_5Z^2 \quad (5.3)$$

THE SUGGESTED N FUNCTION (1st. Model) IS:

$$N = b_0 + b_1X + b_2Y + b_3Z^{0.5} + b_4Z + b_5Z^2.$$

Source	DF	Sum of Squares	Mean Square
Regression	6	3543.64720	590.60787
Residual	14	158.16500	11.29750
Uncorrected Total	20	3701.81220	
Corrected Total)	19	373.61220	

Figure 5.2 : Trend Surface Results For First Model.

R squared = 1 - Residual SS / Corrected SS = .57666

**Parameter Estimates:**

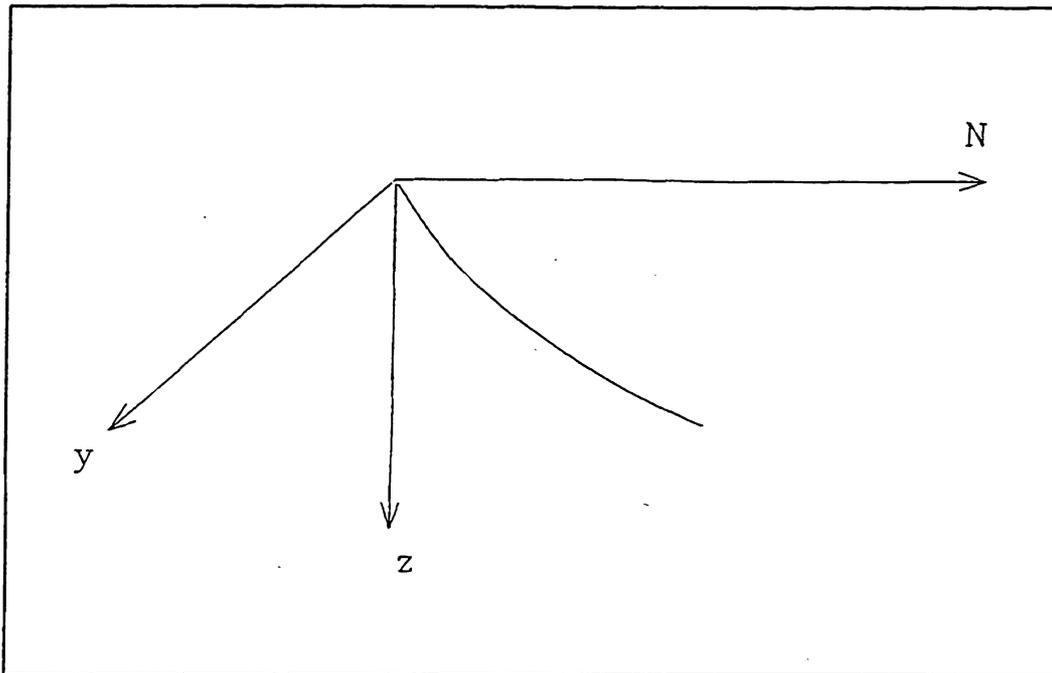
Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval	
			Lower	Upper
0	10.547193428	41.086681848	-77.57497485	98.669361705
1	.002949240	.002882468	-.003233040	.009131520
2	-.002952220	.003921056	-.011362050	.005457609
3	-3.911975982	24.556522288	-56.58047809	48.756526122
4	1.224350091	3.982261584	-7.316751541	9.765451723
5	-.010895178	.027859179	-.070647175	.048856818

**Asymptotic Correlation Matrix of the Parameter Estimates:**

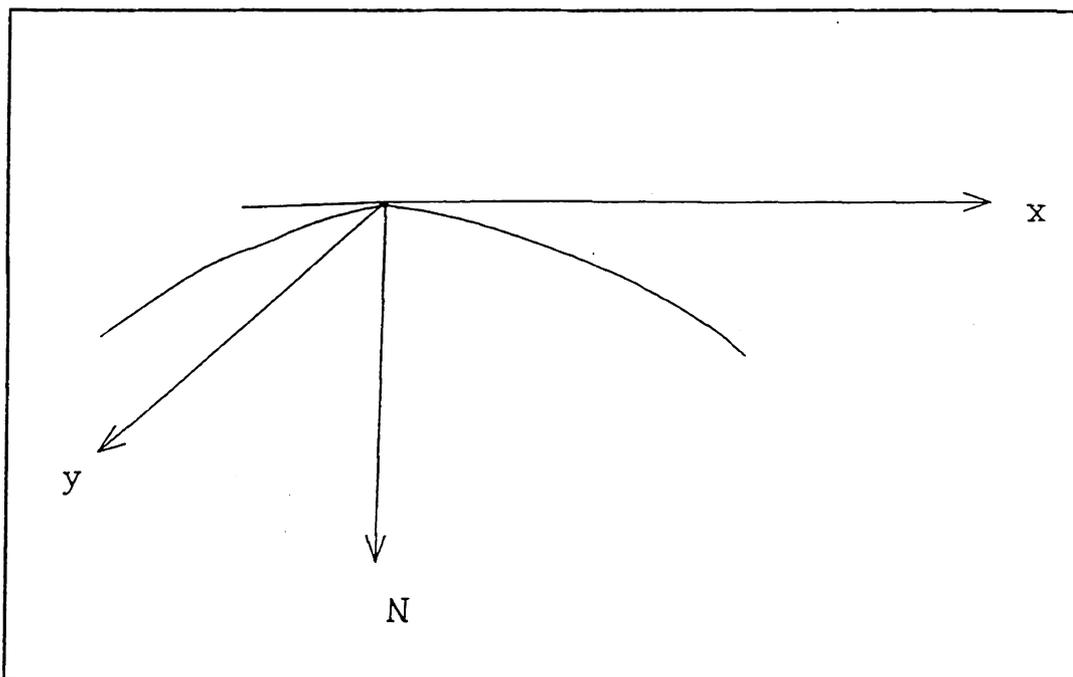
	B0	B1	B2	B3	B4	B5
B0	1.0000	-.2838	.4954	-.9952	.9825	-.9459
B1	-.2838	1.0000	-.2102	.2706	-.2524	.2097
B2	.4954	-.2102	1.0000	-.4937	.4820	-.4494
B3	-.9952	.2706	-.4937	1.0000	-.9959	.9718
B4	.9825	-.2524	.4820	-.9959	1.0000	-.9888
B5	-.9459	.2097	-.4494	.9718	-.9888	1.0000

**Figure 5.3 : Trend Surface Results For First Model, Continued.**

This form permits detection of planar trend in X and Y and permits determining the nature of the Z trend. The computer output is in Figures 5.2 and 5.3. The selected form of the equation describes a parabolic variation of N value about the N horizontal axis shown in Figure 5.4.



**Figure 5.4 :** The Variation Of N Value At A Given X Value As Described By The First Model.



**Figure 5.5 :** The Variation Of N Value At A Given Z Value As Described By The Second Model.

The suggested form was defined to the computer program. The computer program then adjusted the model coefficients to fit the data. This equation form includes the Z terms of the 0.5 order, the first order and the second order with the expectation that the computer can adjust the coefficients of these terms to provide flexibility of fitting in the Z direction. The variabilities in the other two directions (X & Y) were given less attention by this model and were assumed to be linear. The results from this model will be compared with the results from the model of the second trial which allowed more flexibility in the other directions (X & Y). It is observed in the computer output in Figure 5.3 that the confidence intervals of the model coefficients look so large as to reduce the predictive power of the model. It is noted also that the  $R^2$  value was only found to be 0.577. Physically the  $b_0$  coefficient should reflect the basic value (associated with the adopted reference axes) for prediction and the other terms serve as correction terms to reflect the variability in the different directions. These results show the necessity of trying a different model.

The high (+) correlations between  $b_0$  and  $b_3$ ,  $b_4$  and  $b_5$  (Figure 5.3) suggest getting rid of two of the three terms ( $Z^{0.5}$ , Z and  $Z^2$ ) and keeping only one. The trial and error in running the SPSS program suggested keeping the term  $Z^{0.5}$ . It was also reasonable to introduce the  $X^2$  and  $Y^2$  terms for the reasons explained in section 5.1.1.5. Therefore, the form which was used for the second trial is given by the equation:

$$N = b_0 + b_1 X + b_2 Y + b_3 X^2 + b_4 Y^2 + b_5 Z^{0.5} \quad (5.4)$$

The computer output for the second trial is in Figures 5.6 and 5.7. This model describes parabolic variation of N value about the N vertical axis as shown in Figure 5.5. The

THE SUGGESTED N FUNCTION (2nd Model) IS:

$$N = b_0 + b_1 * X + b_2 * Y + b_3 * X^2 + b_4 * Y^2 + b_5 * Z^{0.5}$$

Nonlinear Regression Summary Statistics, Dependent Variable N:

Source	DF	Sum of Squares	Mean Square
Regression	6	3668.75590	611.45932
Residual	14	33.05630	2.36116
Uncorrected Total	20	3701.81220	
(Corrected Total)	19	373.61220	

R squared = 1 - Residual SS / Corrected SS = .91152

Parameter Estimates:

Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval	
			Lower	Upper
B0	8.634706252	1.615607491	5.169572814	12.099839691
B1	.000746539	.001254226	-.001943508	.003436587
B2	.001616347	.001693614	-.002016094	.005248787
B3	-.000016551	4.409336E-06	-.000026008	-7.09383E-06
B4	-.000053946	4.409336E-06	-.000070814	-.000037077
B5	1.839559093	.294958644	1.206935719	2.472182466

Figure 5.6 : Trend Surface Results For Second Model.

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## Asymptotic Correlation Matrix of the Parameter Estimates:

	B0	B1	B2	B3	B4	B5
B0	1.0000	.0088	.0606	-.3713	-.4466	-.9148
B1	.0088	1.0000	-.1227	.1142	.1934	-.1113
B2	.0606	-.1227	1.0000	-.0274	-.4201	.0071
B3	-.3713	.1142	-.0274	1.0000	.1383	.1362
B4	-.4466	.1934	-.4201	.1383	1.0000	.2122
B5	-.9148	-.1113	.0071	.1362	.2122	1.0000

Figure 5.7 : Trend Surface Results For Second Model, Continued.

confidence intervals of the model coefficients, as shown in Figure 5.6, look more reasonable and the  $R^2$  value of 0.9115 reflects a reliable goodness of fit (as would be expected given the form of the original data). Consequently this model is accepted for describing the variability of N values for the given data set. The reason for obtaining better goodness of fit, after taking out the Z and  $Z^2$  terms from the first model and replacing them by  $X^2$  and  $Y^2$  in the second model - will be explained in section (5.1.1.5) for using both models for the prediction of N values and comparing the results.

The N function is therefore given by:

Model:

$$= 10.547 + 0.0029X - 0.0029Y - 3.9119Z^{0.5} + 1.224Z - 0.0108Z^2, \quad (5.5)$$

Model:

$$= 8.64 + 0.0007X + 0.0016Y - 0.000016X^2 - 0.00005Y^2 + 1.84Z^{0.5}, \quad (5.6)$$



### 5.1.1.3 Predicting The N Values At The Eight Given Locations:

The X , Y & Z coordinates of the eight locations are substituted into the modeled N function. The resulting N values together with the true N values are shown in Table 5.3. For practical problems, N values are reported as and treated as integers. However, they were retained here to do the statistical modeling and testing. The rounded integer values are shown between brackets in Table 5.3. It is noted that the N values predicted by the second trial yielded values which can generally match within less than  $\pm 1$ .

**Table 5.3 : The "Estimated Unobserved" N Values By The Trend Surface Vs. The "True Unobserved" N Values.**

Location	X (ft)	Y (ft)	Z (ft)	N* (true)	N (predicted) "1st Model"	N (predicted) "2nd Model"
1	-211.13	333.20	6.53	7.35 (7)	7.72 (8)	7.32 (7)
2	-391.66	49.49	16.22	14.16 (14)	12.79 (13)	13.82 (14)
3	42.56	-212.81	27.03	16.73 (17)	14.91 (15)	16.09 (16)
4	320.14	-200.77	28.48	14.47 (15)	16.11 (16)	15.19 (15)
5	17.32	-296.12	9.57	12.10 (12)	18.36 (18)	10.06 (10)
6	165.55	-215.18	46.01	18.62 (19)	17.32 (17)	18.64 (19)
7	-306.47	302.83	1.56	4.39 (4)	7.55 (8)	5.19 (5)
8	-308.67	253.64	45.36	16.32 (16)	17.65 (18)	16.66 (17)

#### 5.1.4 Analysis Of Variance:

A oneway analysis of variance is made using the program "SPSS" to compare the observed N values with the predicted ones. The model assumptions for the F-test do not hold in this application so the results are taken as descriptive measures only. The results are:

For the N function (1st Model), the difference is insignificant with an F ratio of 0.2008 and an F probability of 0.6609 as shown in Figure 5.8.

For the N function (2nd Model), the difference is insignificant with an F ratio of 0.0036 and an F probability of 0.9527 as shown in Figure 5.9.

This indicates that the model of the second trial is better fitted to the "true unobserved" N values and can substitute them for the design purposes.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F Ratio	F Prob.
Among Samples	4.27	1	4.27		
Within Replications	298.01	14	21.29	.2008	.6609
Total Variation	302.28	15			

**Figure 5.8 : Analysis Of Variance Of The N Values Predicted In The First Trial Vs. The "True Unobserved" N Values.**

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F Ratio	F Prob.
Among Samples	0.09	1	0.09		
Within Replications	328.88	14	23.49	.0036	.9527
Total Variation	328.97	15			

**Figure 5.9 : Analysis Of Variance Of The N Values Predicted In The Second Trial Vs. The "True Unobserved" N Values.**



### 5.1.1.5 Comments On The Prediction Using The Trend Surface Analysis:

The following points can be made regarding the two resulting N functions:

1. For the N function for the 1st trial, the addition of the first order and second order depth (Z) terms does not help improve the goodness of fit. This is to be expected when modeling N values sampled from a mathematically defined trend. The "R squared" value was only found to be about 0.58. The reason is that the tried model is parabolic about the horizontal N axis, as shown in Figure 5.4, which cannot approximate the triangular trend in the (X, Y) plane.

2. For the N function for 2nd trial, the knowledge that the data are following a triangular trend in the X and Y directions allowed them to be represented by a model with a parabolic variation about the vertical N axis, as shown in Figure 5.5, which can approximate a triangular trend in the (X, Y) plane. The value of the model constants were then adjusted by the computer so that the parabolic variation could approximate the triangular distribution as closely as possible. This is reflected by getting an "R squared" value as high as about 0.91.

3. The two previous remarks allow one to realize the importance of investigating the variability of the data carefully, then trying out a model that conforms with this variation. The variability of the real N values in the practical application to a real site may not be as ideal as the triangular distribution of the example presented herein. Fortunately the variability of N values, in most of the six tried case histories, was found to be representable by equations whose X, Y and Z terms are not exceeding the second order. Consequently it is easy to try out all the possible combinations and select the model with the highest  $R^2$  value.



## **.2 VERIFICATION OF THE DEVELOPED MODELS BY THE KRIGING TECHNIQUE**

The verification is decomposed into three steps. These start with the generation of the simulated "observed" and the simulated "true" N values. Then the Kriging technique is conducted on the simulated "observed" N values to predict the N values at locations of the simulated "true" N values. The last step is the comparison between simulated "true" N values and the predicted ones using the variance analysis.

### **.2.1 Generating The N Values:**

In the verification of the developed models by the trend surface analysis which is covered in section (5.1.1) the motivation was to check whether the analysis can reproduce the same trend from which the data are coming. However, in the verification of the Kriging technique, it is known already that the technique will not reproduce any trends. Hence, the motivation is different and aims to check the quality of prediction if Kriging is conducted on data taken from some trend. The simulated "true unobserved" N values and the simulated "observed" N values were generated from the same assumed trend which was given in section 5.1.1.1 as:

$$N=10-0.01|X|-0.02|Y|+2Z^{0.5}+\epsilon \quad (5.7)$$

The simulation will be explained in the next paragraph.

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### Generating The Simulated "True Unobserved" N values:

As Kriging is more mathematically intensive, the "true unobserved" N values will be taken along a single boring. The N values along a hypothetical boring are generated and considered as "true unobserved" N values. The location of this boring in the (X , Y) plane is obtained from the above-mentioned hypothetical field by substituting random values for the X & Y coordinates. These random values are obtained using generated uniformly distributed random numbers as explained earlier in section 5.1.1.1. The coordinates of the generated boring location are shown in Table 5.4.

**Table 5.4 : The Coordinates Of The Location Of The Simulated "True Unobserved" Boring For Kriging.**

	X	Y
Location	$-500 + R.N(1000)$	$-400 + R.N(800)$
"O"	288.83	88.19

The assumed hypothetical N function including the random error term is then used to generate the simulated N values along this boring by substituting the above-mentioned X and Y coordinates as well as the Z values at every 5 ft increment. The N values along the simulated "true unobserved" boring are shown in Table 5.5. The rounded integer values are shown between brackets.

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**Table 5.5 : The N Values Of The Simulated  
"True Unobserved" Boring "o".**

Z (ft)	N
5	10.28 (10)
10	12.67 (13)
15	12.54 (13)
20	12.53 (13)
25	15.00 (15)
30	16.64 (17)
35	16.94 (17)
40	16.43 (16)
45	19.03 (19)
50	18.75 (19)

**Generating The Simulated "Observed" N Values:**

The hypothetical N function is used likewise to generate three additional borings. These borings are considered as data which can be used for the suggested modeling in order to predict the N values of the boring at the location "o". To ensure that the locations satisfy the "nicely surrounded" requirement, the coordinates of the locations of three simulated "observed" borings are obtained by simulating successive X and Y

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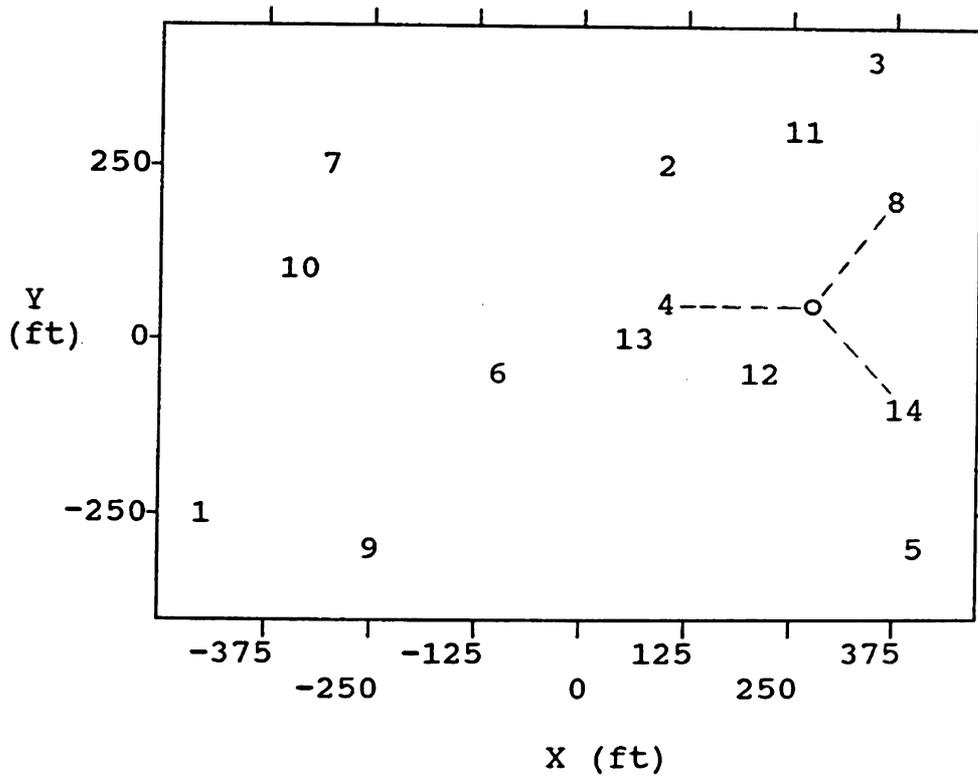
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ues and accepting the location if it satisfies the surrounding requirement; otherwise  
simulation is repeated until three acceptable locations are obtained. The simulated  
cessive locations are shown in Table 5.6.

**Table 5.6 : The Simulated "Observed" Boring  
Locations For Kriging.**

Location No.	X	Y
1	-441.02	-238.68
2	93.33	266.61
3	356.10	389.25
4	103.67	67.78
5	406.86	-313.30
6	- 96.47	- 47.06
7	-289.07	237.51
8	374.14	214.15
9	-237.85	-301.76
10	-358.97	102.62
11	242.86	286.70
12	191.63	- 36.63
13	59.91	24.65
14	372.15	- 78.11



The points are numbered conforming with the data in Tables 5.4 and 5.6.

The simulated "true" boring is at the location "o".

The locations which satisfy the surrounding requirement are : 4 , 8 and 14.

**Figure 5.10 : Top View Of The Locations Of The Simulated "Observed" Borings And The Location In Question "o".**

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the top view of the locations of the simulated "observed" borings as well as the location where a prediction is required are shown in Figure 5.10. From the results of Tables 5.4 and 5.6 and Figure 5.10, it is shown that the locations which surround the boring "o" are the locations 4, 8 and 14.

The N values of the simulated "observed" borings at every 5 ft increment are shown in Table 5.7. The rounded integer values are shown between brackets.

**Table 5.7 : The N Values Of The Simulated "Observed" Borings.**

Z (ft)	Boring (4)	Boring (8)	Boring (14)
5	11.26 (11)	9.43 (9)	9.52 (10)
10	14.75 (15)	8.95 (9)	11.56 (12)
15	14.16 (14)	10.03 (10)	13.64 (14)
20	16.26 (16)	12.03 (12)	12.79 (13)
25	18.39 (18)	11.66 (12)	13.77 (14)
30	20.06 (20)	13.29 (13)	14.76 (15)
35	20.60 (21)	15.02 (15)	17.40 (17)
40	20.99 (21)	15.42 (15)	18.23 (18)
45	21.89 (22)	16.70 (17)	20.29 (20)
50	23.22 (23)	17.84 (18)	19.12 (19)

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### 5.1.2.2 Using The Kriging Technique To Predict The N Values:

Consider that an estimate of N values is required along the hypothetical boring at "o". The estimation by Kriging is carried out according to the following steps:

#### 1. The N values are represented by linear regression functions:

The N values of each simulated "observed" boring at the locations 4 , 8 and 14 are represented by a linear regression function. This function is assumed to be in the form:  $N = a_i + b_i Z$ . This form is used by this research based on a study by Gibson (1967), which considered the subsurface soil as an elastic half-space whose modulus E increases linearly with depth, and on the linearity of the relationship between the soil modulus E and the N values. The rationale of this linear regression function was explained in details in section (4.4.1).

The results of these linear regressions representing the three borings are as follows:

$$\text{Boring}(4) : N = 11.27 + 0.25Z, \text{ Standard Err.} = 0.92 \quad (5.8)$$

$$\text{Boring}(8) : N = 7.45 + 0.203Z, \text{ Standard Err.} = 0.59 \quad (5.9)$$

$$\text{Boring}(14) : N = 8.93 + 0.224Z, \text{ Standard Err.} = 0.94 \quad (5.10)$$

The numerical N values which were used to formulate these equations were given in

Table 5.7

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Figure 5.7 and the graphical representation of these points together with the regression lines representing the borings 4 , 8 and 14 are shown in Figures 5.11,12 & 13 respectively.

**The distances between every pair of locations are calculated:**

This is easily done using a spread sheet such as "SuperCalc 5". The resulting distances between every pair of boring locations in feet are given in the following distance matrix:

$$\begin{array}{lll} h_{44} = 0 & h_{48} = 307.53 & h_{414} = 305.56 \\ h_{84} = 307.53 & h_{88} = 0 & h_{814} = 292.27 \\ h_{144} = 305.56 & h_{148} = 292.56 & h_{1414} = 0 \end{array}$$

The vector of distances between the location "o" and the boring locations is as follows:

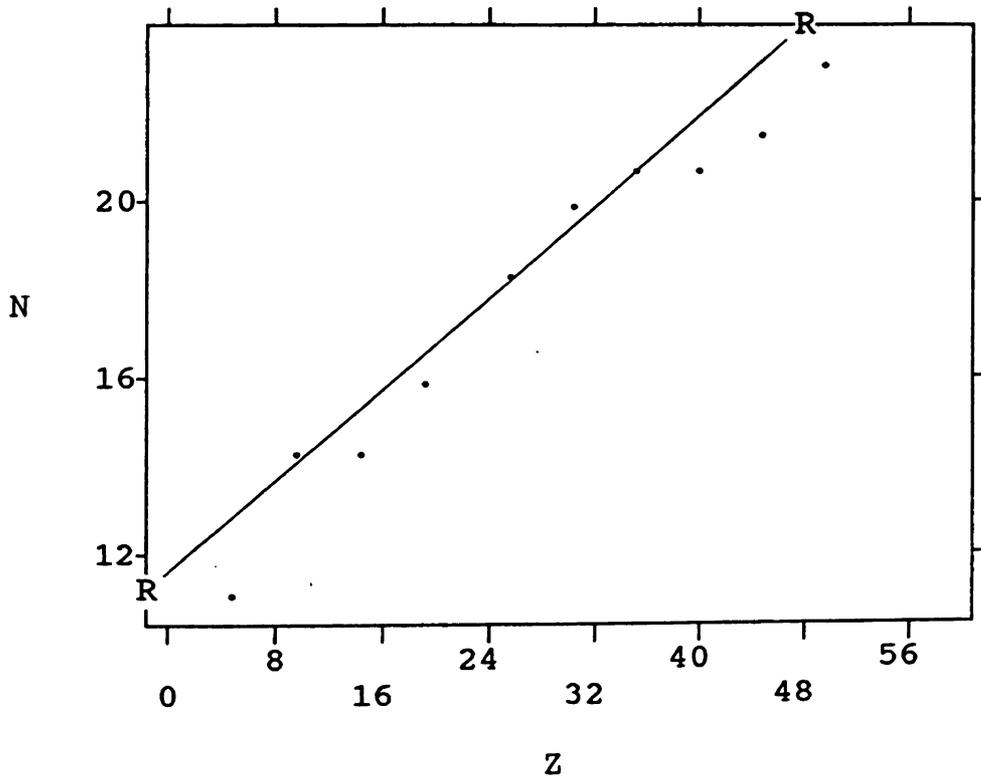
$$\begin{array}{l} h_{4o} = 186.28 \\ h_{8o} = 152.13 \\ h_{14o} = 186.01 \end{array}$$

**The covariances which describe the spatial continuity of the data are calculated:**

The squared exponential model - which is used herein based on a recommendation by Baecher, 1981 - is given by the equation:

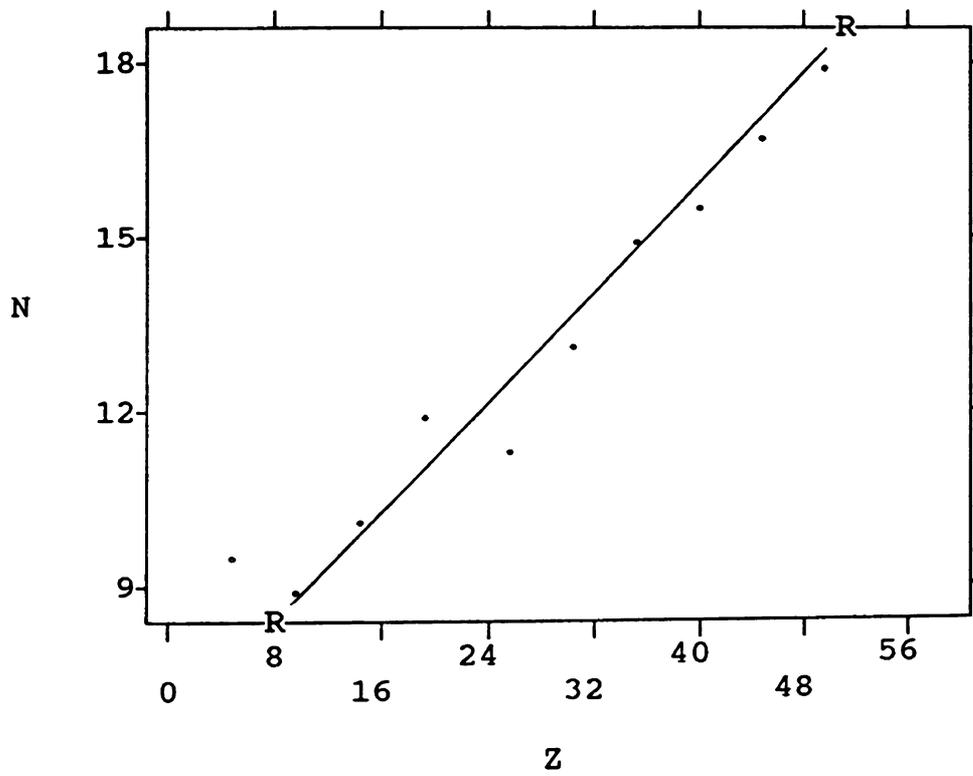
$$C(h) = \sigma_N^2 e^{-(h^2/h_0^2)} \quad (5.11)$$

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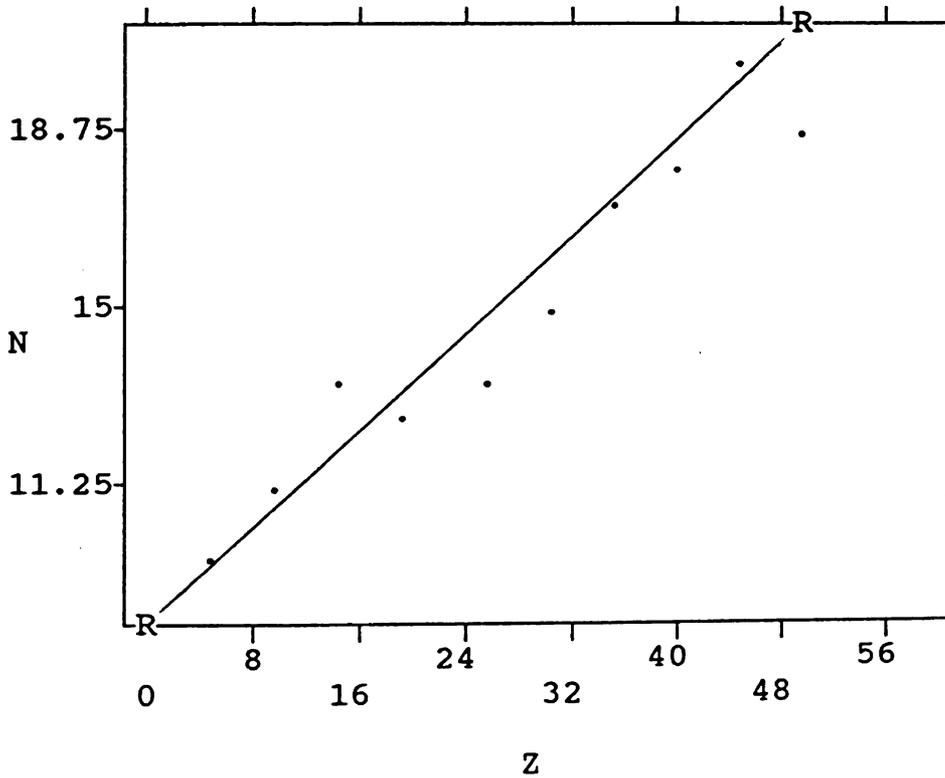
Intercept: 11.27600 ; Slope: 0.25025  
Correlation: 0.97460 ; R Squared: 0.94984  
S.E. of Est: 0.9233

Figure 5.11 : The Linear Regression Function Of Boring (4).



Intercept: 7.45400 ; Slope: 0.20302  
Correlation: 0.98382 ; R Squared: 0.96791  
S.E. of Est: 0.59359

Figure 5.12 : The Linear Regression Function Of Boring (8).



Intercept: 8.93200 ; Slope: 0.22458  
Correlation: 0.96719 ; R Squared: 0.93546  
S.E. of Est: 0.94714

Figure 5.13 : The Linear Regression Function Of Boring (14).

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$C(h)$  : covariance of two locations separated by a distance of "h" ft.

$\sigma_N^2$  : the variance of the thirty N values = 15.591

$h_0$  : the autocovariance distance.

The value of the autocovariance distance is set to 350 ft (as recommended by Baecher, 1981), therefore the model will become:

$$C(h) = 15.591e^{-(h^2/350^2)} = 15.591e^{-8.2E-6(h^2)} \quad (5.12)$$

Consequently, the covariance matrix is calculated using the Equation (5.12) and the spread sheet "SuperCalc 5". The results are as follows:

The covariance matrix of the simulated "observed" data is given by:

$$C = \begin{bmatrix} 15.59 & 7.20 & 7.27 & 1 \\ 7.20 & 15.59 & 7.76 & 1 \\ 7.27 & 7.76 & 15.59 & 1 \\ 1.0 & 1.0 & 1.0 & 0 \end{bmatrix}$$

and the vector of the covariances between the point (o) and the data locations is as follows:

$$D = \begin{bmatrix} 11.74 \\ 12.91 \\ 11.75 \\ 1.0 \end{bmatrix}$$

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**The weight matrix is calculated:**

The matrix was calculated using the computer statistical program which was published by Davis (1986). The results are as follows:

The weight matrix = (the inverse of the covariance matrix) \*

(covariance vector of the estimated point).

or:

$$W = C^{-1} \cdot D \quad (5.13)$$

$$= \begin{bmatrix} 0.30132 \\ 0.42475 \\ 0.27393 \\ 1.99249 \end{bmatrix}$$

**The predicted N function at the locations "o" is formulated:**

The predicted N function is given by:

$$N_o = a_o + b_o Z \quad (5.14)$$

where: the  $a_o$  and  $b_o$  regression coefficients at point "o" are obtained by Kriging the  $a$  and  $b$  coefficients from borings 4, 8, and 14:

$$a_o = \sum w_i a_i = 0.3013 (11.27) + 0.4247 (7.45) + 0.2739 (8.93) = 9.006$$

$$b_o = \sum w_i b_i = 0.3013 (0.250) + 0.4247 (0.203) + 0.2739 (0.224) = 0.223$$

Substituting in Equation (5.14) to become:  $N = 9.006 + 0.223 Z$ .

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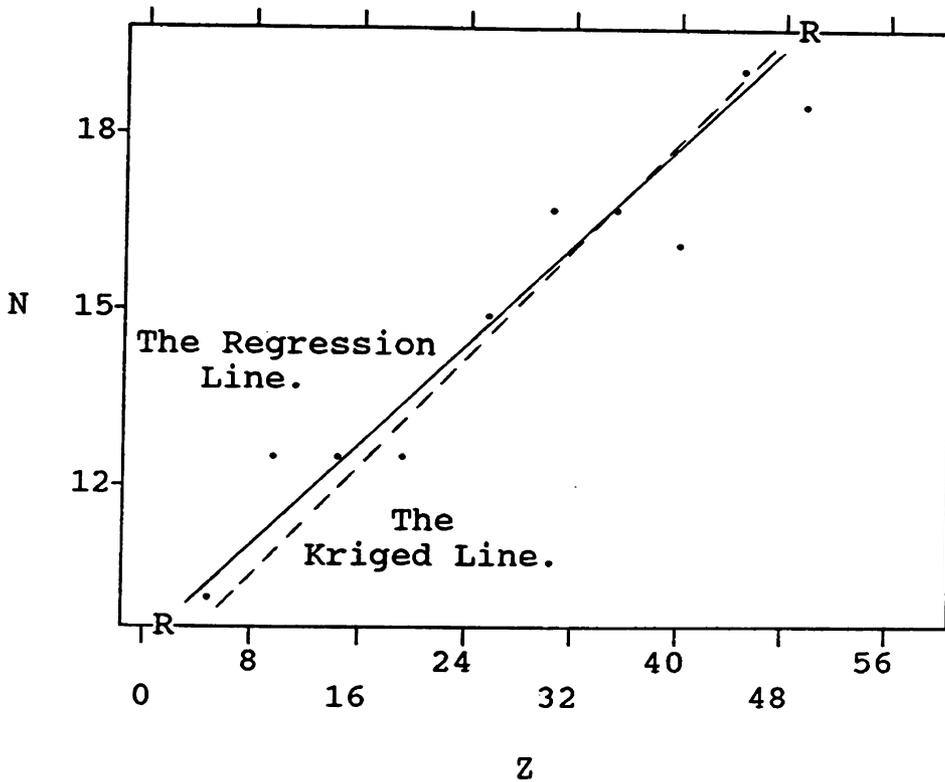
Therefore, the resulting N values - as calculated using the spread sheet and Equation 5.14 - together with the simulated "true unobserved" N values which were given in Table 5.5 are shown in Table 5.8. When rounded to integer values, the predicted values are in good agreement with the simulated "true unobserved" N values (five are the same, two are "-1", two are "+1" and one is "-2"). The graphical representation of the kriged line over these simulated "true unobserved" N values and their regression line is shown in Figure 5.14. It is observed that:

$$a_o/a_i = 1.10 \quad \text{and} \quad b_o/b_i = 0.84$$

**Table 5.8 : The N Values Predicted By Kriging Vs. The Simulated "True Unobserved" N Values.**

Z (ft)	N (true unobserved)	N (predicted)
5	10.28 (10)	10.12 (10)
10	12.67 (13)	11.23 (11)
15	12.54 (13)	12.35 (12)
20	12.53 (13)	13.47 (13)
25	15.00 (15)	14.58 (15)
30	16.64 (17)	15.70 (16)
35	16.94 (17)	16.81 (17)
40	16.43 (17)	17.93 (18)
45	19.03 (19)	19.04 (19)
50	18.75 (19)	20.16 (20)





Intercept: 9.91200 ; Slope: 0.18796  
 Correlation: 0.96288 ; R Squared: 0.92713  
 S.E. of Est: 0.8461

The Kriged Regression Line Is Given By:

$$N = 9.006 + 0.223 Z$$

Figure 5.14: The Linear Regression Of The Simulated "True Unobserved" N Values And The Predicted N Function By Kriging At The Location "0".

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### 5.1.2.3 Analysis Of Variance:

A oneway analysis of variance is conducted using the program "SPSS" to compare the true N values with the predicted values. The results of this analysis which are shown in Figure 5.15 show that the difference is insignificant with an F ratio of 0.0017 and an F probability of 0.9679, which indicates that the predicted N values can generally match the "true unobserved" N values adequately and can substitute them for the design purposes.

#### Analysis Of Variance Output

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F Ratio	F Prob.
Among Samples	0.0168	1	0.0168	0.0017	0.9679
Within Replications	181.2678	18	10.0704		
Total Variation	181.2846	19			

Figure 5.15 : Analysis Of Variance Of The N Values Predicted By Kriging Vs. The Simulated "True Unobserved" N Values.

### 5.1.2.4 Comments On The Prediction By Kriging:

An examination of the Kriged N values in Table 5.8 indicates that the difference between the predicted N values and the simulated "true unobserved" ones appears to be insignificant. As was mentioned in section (5.1.1.3), N values are taken as integers in practice. Thus if the N values in Table 5.8 rounded to integer values, they are virtually identical. The average deviation is less than  $\pm 1.5$ . This result is confirmed by the examination of the graphical representation of both values as shown in Figure 5.14.

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In this context one can make the following remark regarding the geostatistical modeling:

The basic assumption of Kriging is that all the data in the subset used to predict a value as well as the predicted value itself have the same probability law and the same expected value (i.e. no trend). However, if the data follow a trend then the interpolation modeling can still be used to find a point on the trend provided that the employed data are located in the vicinity of that point in order to justify ignoring the trend.

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### 5.1.3 VERIFICATION OF THE DEVELOPED "TWO - POINT" ESTIMATE

The verification of the suggested "two - point estimate" is achieved by using an assumed footing and the hypothetical trend of  $N$  which was assumed in section 5.1.1.1) as:

$$N=10-0.01|X|-0.02|Y|+2Z^{0.5}+\epsilon \quad (5.15)$$

The settlement is computed numerically using the full variation of  $N$ . In order to do this an "eight- point estimate" will be employed in which the weights are taken from the corresponding fractional areas of Schmertmann's strain influence diagram. Then by using the "two - point estimate", the settlement is estimated again. The results are compared.

Schmertmann (1970) gives the following equation for calculating the settlement:

$$S=C_1 \cdot C_2 \cdot p \sum_{z=0}^{2B} [(I_{zi}/E) \cdot dz_i] \cdot \quad (5.16)$$

where

$p$  = increase in effective overburden pressure at foundation level.

$C_1$  is a depth embedment factor.

$C_2$  is an empirical creep factor.

$E$  is the deformation modulus =  $4N$  for silts or slightly cohesive silt-sand to  $12N$

for sandy gravel and gravel, ( $E$  is in tsf).

$I_z$  is the strain influence factor.

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The hypothetical footing is assumed as (8 ft \* 8 ft) and the center of its base is assumed to be at the location (X = 250 , Y = 200 , Z = 5) and assuming that the footing pressure at the foundation level is 1.5 tsf.

The parameters are therefore given as follows:

$B =$  the footing width = 8ft.

Depth of influence =  $2B = 16$  ft.

$p = 1.5 (2000) - 5 * 125 = 2375$  lb/s.f.

$C_1 = 1 - 0.5 (5 * 125 / 2375) = 0.868$

$C_2 = 1$  (assuming no creep).

$E = 8$  N (tsf), assuming coarse sand.

$I_z$  : takes different values depending on the depth of the subarea according to the following strain influence diagram.

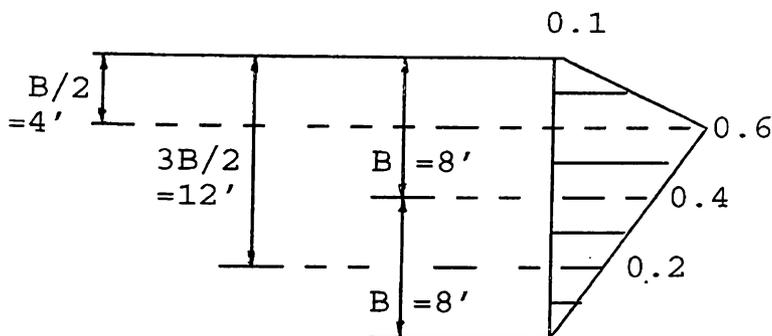


Figure 5.16 : Schmertmann's Strain Influence Factor Vs. Depth.

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### 5.1.3.1 The "eight - point estimate" design:

The depth of influence (Figure 5.16) is divided into 8 subareas each of which has a depth of 2 ft.

The modulus (E) is taken as (8N). The N value is calculated by substituting the midpoint depth of each subarea into the hypothetical N function. The value of the influence factor (I) is calculated from the diagram shown in Figure 5.16 according to the depth of the center of each subarea. The calculations are tabulated in Table 5.9.

**Table 5.9 : The "Eight-Point Estimate" Calculations.**

Depth of center of subarea (ft)	Depth from ground surface. (ft)	(I)	(N)	E=8N (tsf)	(I/E) $\delta z$
1	6	0.225	9.42	75.36	0.0059713
3	8	0.475	10.24	81.92	0.1159670
5	10	0.55	10.58	84.64	0.0129962
7	12	0.45	10.64	85.12	0.0105733
9	14	0.35	12.55	100.40	0.0069721
11	16	0.25	12.84	102.72	0.0048676
13	18	0.75	13.43	107.44	0.0139613
15	20	0.05	14.51	116.08	0.0008615

Total=0.0678

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Therefore the settlement is predicted as:

$$\begin{aligned}
 S &= C_1 \cdot C_2 \cdot p \sum_{z=0}^{2B} [(I_{zi}/E) \cdot dz_i] . \\
 &= 0.868 * 1 * (2375/2000)(0.0678) \\
 &= 0.069 \text{ ft} \\
 &= 0.07 \text{ ft, approximately.}
 \end{aligned}$$

### 5.1.3.2 The "two - point estimate" design:

Considering alternatively that the depth of influence is divided into only two subareas each of which has a depth of  $B=8$  ft, the center of the upper subarea is therefore at a depth of  $(B/2=4\text{ft})$  under the footing and the center of the lower subarea is at a depth of  $(3B/2=12\text{ft})$ .

The settlement is given by:

$$\begin{aligned}
 S &= C_1 \cdot C_2 \cdot p \sum_{z=0}^{2B} [(I_{zi}/E) \cdot dz_i] . \\
 &= C_1 \cdot C_2 \cdot p [(I_1 \cdot dz)/E_1 + (I_2 \cdot dz)/E_2]
 \end{aligned}$$

Considering that:

$(I_1 \cdot dz)$  = area of the influence diagram associated with the upper subarea

$$= (0.35)(B/2) + (0.5)(B/2) = 0.425 B$$

$$= 0.425 * 8$$

$(I_2 \cdot dz)$  = area associated with the lower subarea =  $0.2 B$

$$= 0.2 * 8$$

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It is noted here that Schmertmann implies using the ratio of (2.125: 1) as the relative weights of the two subareas, whereas in this research this ratio is suggested as (2 : 1) which is within 6% of Schmertmann's.

Therefore

$$\begin{aligned}
 S &= C_1 \cdot C_2 \cdot p [(0.425 \cdot 8) / 8 \cdot N_{(5+4)} + (0.2 \cdot 8) / 8 \cdot N_{(5+12)}] \\
 &= C_1 \cdot C_2 \cdot p [(0.425 / 9.19) + (0.2 / 11.95)] \\
 &= 0.868 * 1 * (2375 / 2000) [ 0.0629823 ] \\
 &= 0.065 \text{ ft} \\
 &= 0.07 \text{ ft, approximately.}
 \end{aligned}$$

It is obvious that, for settlement calculations at "unsampled" locations, where an N function is synthesized by methods herein, the settlement of 0.065 ft as calculated by the "two-point estimate", and the settlement of 0.069 ft as calculated using the "eight point estimate" could - for all practical purposes - be substituted for each other. The agreement was as good or better at the other locations.

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## **5.2 VERIFICATION USING CASE HISTORIES**

The practical reliability of the developed models is checked by conducting the suggested modeling on six available case histories. The resulting settlement predictions are compared with the measured settlements in order to test the capability of the models. To avoid the monotony of repetitions it is preferred to provide a complete analysis of only one case history. The remaining five case histories will be summarized and provided within appendix C.

### **5.2.1 APPLYING THE TREND SURFACE ANALYSIS TO CASE HISTORY No.1**

For the actual case histories, the best fit of trend surface analysis model cannot be as easily judged as in the assumed field, hence 5 to 15 models were tried for each case. In general the "best" model was determined for each case by comparing the  $R^2$  values and comparing the standard error of estimates. The numerical value of each term in the model within the limits of the site is then judged and all the results of the computer output are investigated. Details are further explained in Sections 5.2.1,4 and 5.2.3.

#### **5.2.1.1 PROJECT DESCRIPTION**

The case history selected is a split-level building, whose settlement predictions were previously made by Trigon Eng. Consultants, Inc., Greensboro, North Carolina (Borden and Lien, 1988).

The project consists of a split-level office building with four stories in the front and five stories in the rear of a core area. A single story section wraps around this taller core area. According to Guinnin-Cambell, the structural engineers (as quoted by Borden

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and Lien, 1988), the maximum column loads occur at four column locations in the building core area. Total design column loads within this core area range from 180 kips to a maximum of 730 kips. The total design column loads outside the core area, around the single story section, range from 10 to 20 kips. With these differences, differential settlement was an obvious concern. A general plan view with the boring locations is shown in Figure 5.18.

### 5.2.1.2 FOOTING DETAILS

The most heavily loaded column is located at boring (B-103). This column was chosen for the settlement prediction study. The as-built footing dimensions were:

Width "B" = 11.5 ft, Length = 22.5 ft, Depth = 3.5 ft.

The net loading to the base of the footing - as reported by the structural engineer; Guinnin Campbell - was 650 kips, Figure 5.17.

### 5.2.1.3 SPT LOCATIONS

Standard penetration tests (SPT) were performed in a number of borings. The boring locations and plan view of the building are shown in Figure 5.18. The soil test

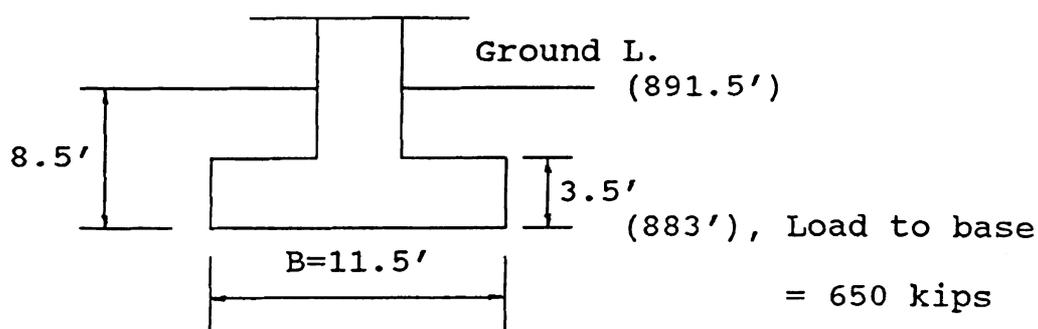


Figure 5.17 : The chosen footing for the settlement prediction study.

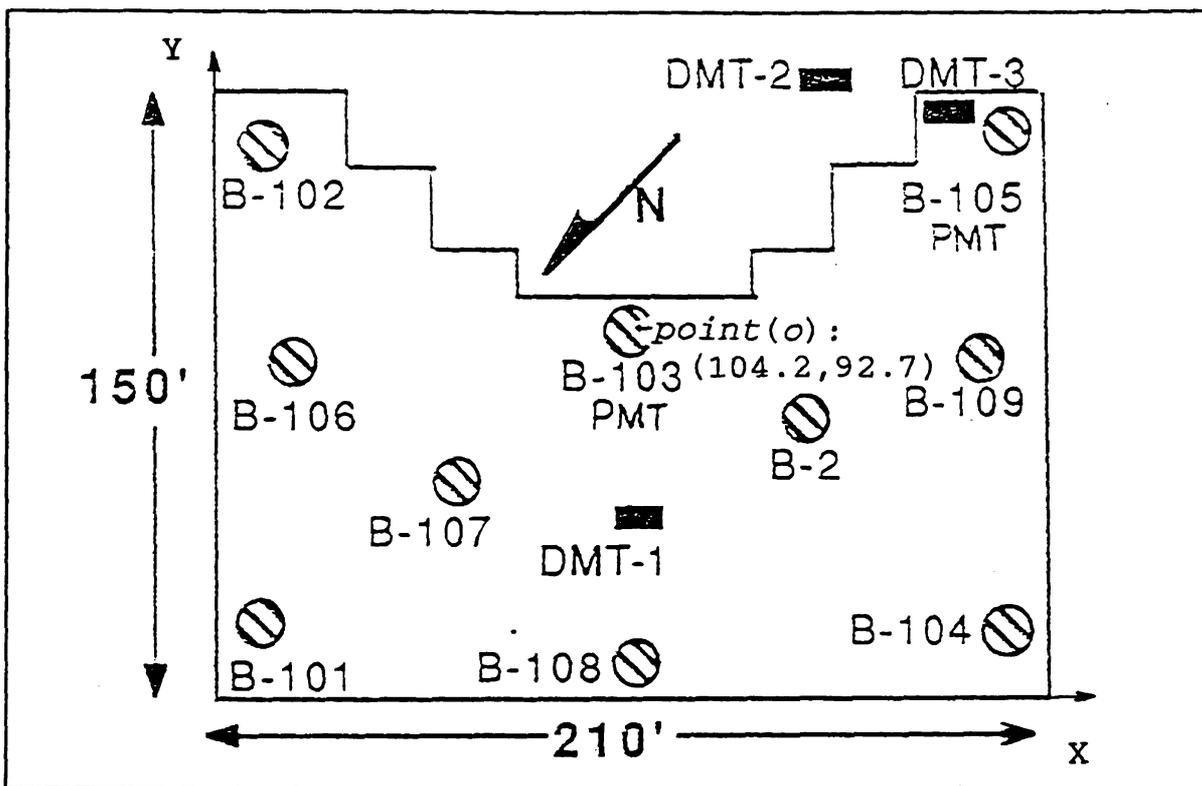


Figure 5.18 : Plan View Of The Boring Locations For Case History No. 1 (After Borden and Lien, 1988).

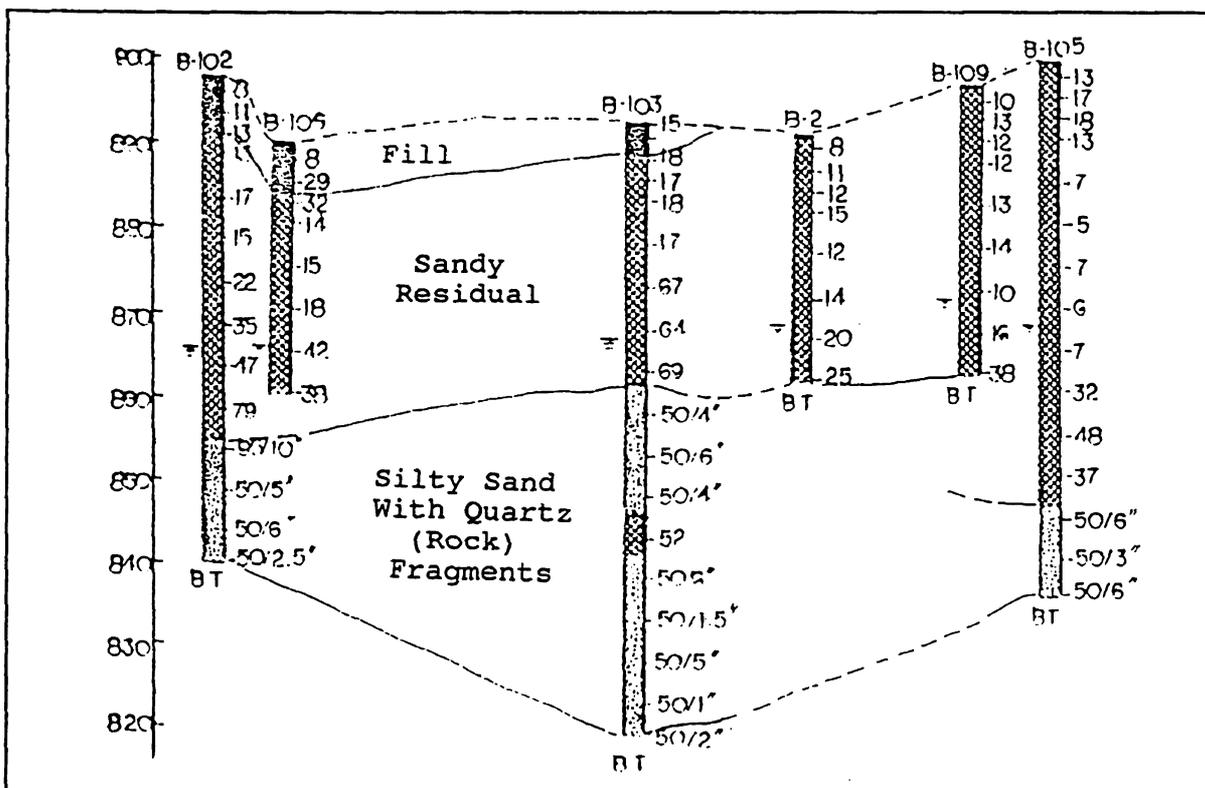


Figure 5.19 : Generalized Subsurface Profile For Case History No. 1 (After Borden and Lien, 1988).

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borings were performed to depths ranging from 15 ft. to approximately 75 ft. below the ground surface. A generalized soil profile is shown in Figure 5.19.

#### 5.2.1.4 APPLYING THE TREND SURFACE ANALYSIS PROCEDURE

The procedure was applied by conducting the following steps :

##### 1. The Stratification Of The Subsurface Soil:

The subsurface soil was initially stratified into 3 layers; layer 1 being on the top. Although the N values could be transferred into relative densities and the stratification could be done with regard to  $D_r$ , it was decided to test the stratification used by the previous investigator to see how much difference arises by applying the proposed technique. The soil stratification is shown in Figure 5.19.

##### 2. One-way ANOVA Analysis:

A "one-way ANOVA analysis" is conducted to compare the means of N values in the 3 layers. The N values which are used here have to be corrected first for the overburden pressure. The N values were corrected for the overburden pressure based on the correction equation recommended by Liao (1986) as described in Appendix A. The correction results are shown in Tables A.1 to A.6.

This analysis appears in Appendix A Figure A.1. The number of treatments in this analysis was taken as 3 (i.e. the measured values of N within each layer is considered as one treatment). The separation of the variance into "within" and "between" treatments appears on Figure A.1. The F ratio was as high as (30.8483). The critical F value was

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3.91 (at the 0.05 level). The F ratio is then much greater than the critical F value meaning that the hypothesis that the three layers are the same is rejected. Consequently the differences between the layer means is statistically significant, which ascertains that the designer's original soil stratification is justified. It is to be noted also that it does not prove that it is the "correct" or "only" stratification, just that one can justify multiple strata. The number of layers will be checked in the next paragraph.

### 3. Testing The Differences Between Pairs Of Layers:

In the previous step, it was proven that the subsurface soil should be considered stratified. But, to ascertain that the soil should be stratified into 3 layers or only 2 layers, the difference between pairs of layers has to be tested. To do so, the Tukey test of multiple comparison technique is conducted. The result of this test which appears on Figure A.1 shows that both layers 1 & 2 are significantly different from layer 3 at the 0.05 level, but layers 1 and 2 themselves are not significantly different from each other. This indicates that no significant difference can be shown and subdivision is not justified.

This result shows that the stratification of the previous investigator is not statistically justified. It is to be noted that it was likely geologically justified, but one is able to combine the layers in a trend surface analysis. Consequently the rest of the analysis of this procedure will consider that the soil is stratified into 2 layers. Layers 1 & 2 will be combined together and named layer 1 and layer 3 will become layer 2. This combination of layers increases the layer variance, but the estimation uncertainty of the model which will be built for this layer will be reduced, because it will have the

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advantage of estimating the parameters of only two models instead of three while using the same amount of data.

#### **4. Modeling The N Function For The Subsurface Soil:**

##### Layer 1:

Modeling the N function for the upper layer (1) is conducted by solving for the trend surface of the observed N values within layer 1 using the 3 coordinates of each observation related to an arbitrarily chosen origin at the northern corner of the site at the reference level which was used by the previous investigator. The program "SPSS" was used to fit the models.

Candidate functions, essentially representing all the possible polynomial combinations of the coordinates X , Y and Z to the powers of 0.5 , 1 and 2 were tried to fit the given N data. The coordinates with powers of greater than 2 were found to add nil contributions to the N value and were not critical to the model. In other words the product of the coordinate to a high power and its coefficient was found to be nil and neither to affect the N value nor lead to any increase in the  $R^2$  value. Physically, this means that the high order "wavy" trends are not being looked for. As a result, the number of the needed trials and the time required by the computer were reduced considerably. The criteria which is used to rate the goodness of fit of the different functions is maximizing the  $R^2$  value. This value represents the percent of total variation which is explained by the formulated function.

Considering the guidelines which were provided in section 4.3 regarding the model building, the following two models were found to be the best:

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$$\text{Model: 1) } N1 = B_0 + B_1X + B_2X^2 + B_3Y + B_4Y^2 + B_5Z^{0.5} + B_6Z + B_7Z^2 \quad (5.17)$$

which yielded an  $R^2$  value of 0.511.

$$\begin{aligned} \text{Model: 2) } N1 = D_0 + D_1X^{0.5} + D_2Y^{0.5} + D_3Z^{0.5} + D_4X + D_5Y \\ + D_6Z + D_7Z^2 \end{aligned} \quad (5.18)$$

which yielded an  $R^2$  value of 0.528.

Model 1 assumes that the N value has a basic value of  $B_0$  at the origin, then it varies parabolically in each of the three directions X, Y and Z. The values of the model coefficients were given by the computer output (Appendix B, Figure B.1) as follows:

$$\begin{aligned} B_0 = -2,721,918.4, \quad B_1 = 0.251, \quad B_2 = -0.001, \quad B_3 = 2.586, \quad B_4 = -0.012, \\ B_5 = 248,358.4, \quad B_6 = -6372.3, \quad B_7 = 1.242. \end{aligned}$$

The resulting standard errors were relatively large for the coefficients of big values and were reasonable otherwise. It is to be noted that the values of these coefficients are functions in both the trend of N values and the three reference axes X, Y and Z. For example, changing the location and/or the orientation of the adopted axes will yield different values for the model coefficients. Consequently the evaluation of each model should consider only the relative values of these coefficients and not the absolute values.

Model 2 assumes that the N value has a basic value of  $D_0$  at the origin, then it varies quadratically in the X and Y directions and varies parabolically in the Z direction.

The model coefficients were given by the computer output (Figure B.1) as follows:

$$\begin{aligned} D_0 = -3,008,929.7, \quad D_1 = 10.058, \quad D_2 = 78.135, \quad D_3 = 274,208.9, \quad D_4 = -0.612, \\ D_5 = -3.794, \quad D_6 = -7027.8, \quad D_7 = 1.367. \end{aligned}$$

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Here also, the resulting standard errors were relatively large for the coefficients of large values and were reasonable otherwise. It is to be noted that a slightly larger  $R^2$  value in the second decimal place does not make any real difference between the two models from the view point of the percent of total variation which is explained by the model.

After the examination of the two models it can be seen that the coefficients of  $Z^{0.5}$ ,  $Z$  and  $Z^2$  in both models are comparable. Consequently it can be deduced that the  $N$  value increases parabolically in the  $Z$  direction but this deduction is not supported statistically since the  $R^2$  value is low. It was indicated earlier in section 3.2.2.3 that an  $R^2$  value of 0.8 is the limit that indicates a significant trend in the data and that an  $R^2$  value of greater than 0.2 could indicate that a trend exists. To check that the trend exists, the  $F$  test was conducted. For the first model, to test the null hypothesis that:  $B_0 = B_1 = \dots = B_7 = 0$ , the  $F$  value was obtained from the computer output Figure B.1 as 45.12. The critical  $F$  value was 2.48 (at the 0.05 level). And since the  $F$  value is much greater than 2.48, then the null hypothesis was rejected which means that the trend was found to exist. For the second model, the  $F$  value was obtained as 46.8 which is much greater than 2.48 leading also to the rejection of the null hypothesis and the confirmation of the trend existence. The second model is then selected for the formulation of the  $N$  function for the upper layer. The details of the model building & analysis appear in Appendix B, Figure B.1.

The function  $N_{(z)}$  at boring (B-103) can be obtained by the substitution of the coordinates of its location (104.2 , 92.7) in the selected model. This yields the following function:

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$$N1 = 274208.9Z^{0.5} - 7027.8Z + 1.367Z^2 - 3008490$$

where Z is measured from the MSL upwards. When related to an origin on the ground surface with Z\* values increasing downwards, this function becomes:

$$N1 = 274208.9(-Z^* + 891.5)^{0.5} - 7027.873(-Z^* + 891.5)$$

$$+ 1.367(-Z^* + 891.5)^2 - 3008490$$

$$= 1.367Z^{*2} + 4590.3Z^* + 274208.9(-Z^* + 891.5)^{0.5} - 8187289.2, \quad (5.19)$$

where Z\* is the depth from the ground surface in feet at the location of Boring (B-103).

The substituting of different depths in this function yields the following N values:

Z*(ft).	N	Z*(ft)	N	Z*(ft)	N
0	42.10	10	33.43	20	36.24
2	39.24	12	33.22	22	37.78
4	36.97	14.25, (B/2)	33.48	24	39.57
6	35.27	16	34.02	25.75, (3B/2)	41.30
8	34.10	18	34.97	28	43.72

These predicted N values compares well with the measured value in Boring (B-103).

### Layer 2:

Layer 2 was treated likewise. The two best models were found to be:

$$\text{Model: 1) } N2 = F_0 + F_1Z + F_2Z^{0.5} + F_3Z^2 \quad (5.20)$$

which yielded an R<sup>2</sup> value of 0.443.

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$$\text{Model : 2) } N_2 = G_0 + G_1 X + G_2 Y + G_3 Z + G_4 X^{0.5} + G_5 Y^{0.5} + G_6 Z^{0.5} + G_7 Z^2; \quad (5.21)$$

which yielded an  $R^2$  value of 0.449.

Model 1 assumes that the N value has a basic value of  $F_0$  at the origin, then it varies parabolically with increasing depth and assumes that no variability of the N value in the horizontal direction. The values of the model coefficients were given by the computer output (Appendix B, Figure B.2) as follows:

$$F_0 = -34869400.6, \quad F_1 = -83881.9, \quad F_2 = 3225018, \quad F_3 = 16.8.$$

The standard errors were so large as to deprive the model from its prediction power.

Model 2 yielded an  $R^2$  value of 0.449 which makes no difference from the first model as far as the explained percent of the total variation of N value is concerned. This model assumes that the N value has a basic value of  $G_0$  at the origin, then it varies quadratically in the X and Y directions and varies parabolically with increasing depth.

The model coefficients were given by the computer output (Figure B.2) as follows:

$$G_0 = -16362554, \quad G_1 = 1283.8, \quad G_2 = -35144.3, \quad G_3 = -30682, \quad G_4 = -20816.7, \\ G_5 = 738011.9, \quad G_6 = 1172299, \quad G_7 = 6.22.$$

The standard errors were all very large and deprived the model from any precise prediction power. However, the surface of the layer is located at a depth of more than 30 ft below the ground surface and the width of the footing in question is 11.5 ft, i.e. this layer is outside the zone of influence of this footing - as will be shown in the next paragraph- and will not be used for modeling its settlement. As such, there is no need to test the existence of a trend by using the F test or to calculate the  $N_{(z)}$  for this layer.

The model coefficients as well as the analysis details appear in appendix B, Figure B.2.

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## 5 Using The N Function To Model The Settlement:

Determining of  $N_{(B/2)}$  and  $N_{(3B/2)}$ , see Figure 5.20:

The footing location is at the boring (B-103), so the X & Y coordinates of the footing center are as follows :

$$X_b = 104.2 \text{ ft}, \quad Y_b = 92.7 \text{ ft.}$$

The footing base level ( $Z_b$ ) = Ground level at the footing location -Depth of embedment.

$$= 891.5' - 8.5' = 883 \text{ ft.}$$

$$B = 11.5 \text{ ft.}$$

The depth of influence =  $2B = 23 \text{ ft.}$

The level of the underside of layer 1 = 860 ft.

The level of the lower point of the depth of influence =  $Z_b - 2B = 883 - 23 = 860 \text{ ft.}$

Therefore the depth of influence is located entirely within the upper layer, then the two representative N values are obtained by substituting the levels of (B/2) and (3B/2) in the

N1 model. These levels are given by:

$$Z_{(B/2)} = Z_b - B/2 = 883' - 11.5/2 = 877.25', \quad (Z^* = 14.25).$$

$$\& Z_{(3B/2)} = Z_b - 3B/2 = 883' - 34.5/2 = 865.75', \quad (Z^* = 25.75).$$

The  $N1_{(B/2)}$  is then obtained by substituting the co-ordinates : ( $X_b = 104.17$ ,  $Y_b = 92.7$ ,

$Z^*_{(B/2)} = 14.25$ ) in the N1 model which is given by Equation 5.18.

Therefore  $N_{(B/2)} = 33.5$ , which compares to the measured values of (18, 17, 64) in the general vicinity in Boring (B-103).

Similarly  $N1_{(3B/2)}$  is obtained by the substitution of :

( $X_b = 104.17$ ,  $Y_b = 92.7$ ,  $Z^*_{(3B/2)} = 25.75$ ) in the same model to yield:

$N_{(3B/2)} = 41.3$ , which compares to the measured values of (18, 17, 64) in the boring.

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## 6. The Confidence Limits Of N And S:

The confidence limits of the N value are as follows (Box, Hunter and Hunter, 1978, pp 524, Equation 15.26):

$$N_{est.} = N_{design} \pm t_{d.f., \alpha} * [(P/n) * Err. variance]^{0.5} \quad (5.22)$$

where

P = The number of the parameters in the N model = 8

n = The number of the N observations included to fit the model = 55

d.f. = n - P = 55 - 8 = 47

$t_{47,0.05} = 1.67945$

Err. variance = 91.10036 (as shown in Figure B.1)

Therefore ; the confidence limits are given by :

$$N_{est.} = 36.1 \pm 1.67945 * [(8/55) * 91.10036]^{0.5} = 36.1 \pm 6.1$$

= 30 and 42.2 ; (at the level of 0.10).

and

$$N_{est.} = 36.1 \pm 0.6803 * [(8/55) * 91.10036]^{0.5} = 36.1 \pm 2.5$$

= 33.6 and 38.6 ; (at the level of 0.50).

The confidence limits of the estimated settlement "S" are given by:

$$S_{est.} = (C / N_{est.})$$

where

$$C = [2q * (2B / B+1)^2] = [2 * 1.256 * (23 / 12.5)^2] = 8.5$$

Therefore the 90% confidence limits of "S" are given by:

$$S_{est.} = 8.5 / (36.1 \pm 6.1) .$$

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and the 50% confidence limits of "S" are given by:

$$S_{est} = 8.5 / (36.1 \pm 2.5) .$$

$$= (0.22 \quad \text{and} \quad 0.25) \text{ in.}$$

Thus, the 90% confidence interval of "S" = 0.28 - 0.20 = 0.08 (in).

However, from a practical viewpoint, all of them are tolerable settlements.

The 90% confidence interval of 0.08 is relatively big for an  $S_{est}$  of 0.24 (in) , but this confidence interval could be reduced by minimizing the standard error of estimate :

$[(P/n) * \text{error variance}]^{**0.5}$  , this in turn is minimized by:

- i. Reducing the number of the parameters in the N model : "P".
- ii. Increasing the number of n observations included to fit the model, which means increasing the number of borings and increasing the sampling costs.
- iii. Finding a "better" fitting model.

As such, a trade off is being made between the estimation precision and the number of borings or the sampling costs. The confidence band of the estimated N value is proportional to  $(1/n)^{0.5}$  , thus if the number of the N observations was doubled to become 110 instead of 55, then the  $N_{est}$ , at the level of 0.10, would become:

$$36.1 \pm (6.1 / 2^{0.5}) = 36.1 \pm 4.3$$

and if the upper confidence limit was considered for the foundation design, as will be suggested later in chapter 6 among the conclusions, then this trade off could become a rational basis for producing less conservative foundation design for a reduced cost.

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## 7. The Contours Of The N Values:

The selected N1 model given by Equation 5.18 can be easily used to plot the N contours at any desired depth. This is achieved by substituting the desired depth (e.g. B/2 or 3B/2) for the Z value in the model.

This substitution will yield a two-dimensional N function as :  $N = f(X, Y)$ .

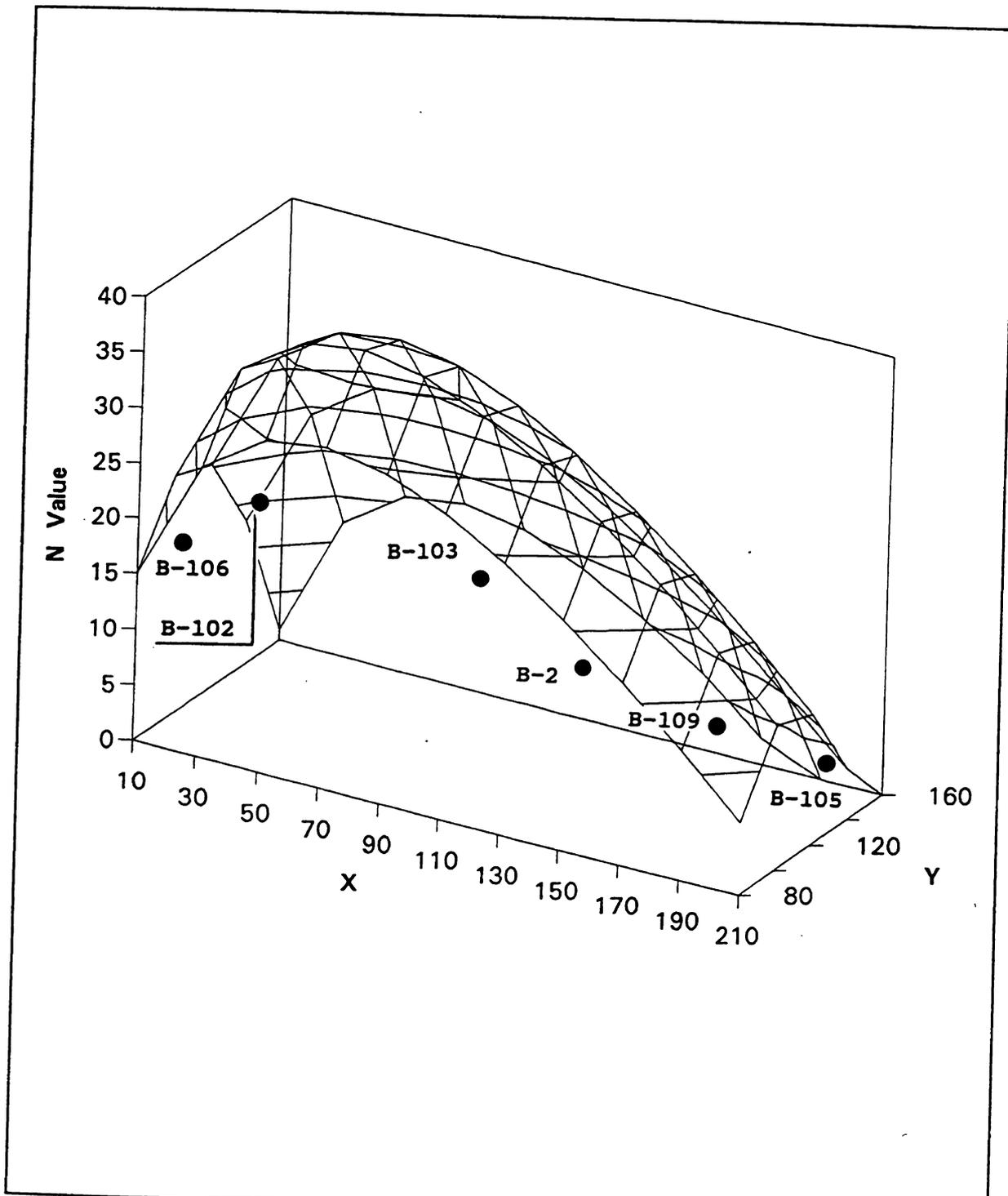
For example, the substitution of the depth of  $Z_{(B/2)} = 877.25$  yields the function:

$$N_{(B/2)} = 10.0582X^{0.5} + 78.1354Y^{0.5} - 0.6117X - 3.7941Y - 406.039; (5.23)$$

and the substitution of the depth of  $Z_{(3B/2)} = 865.75$  yields the function:

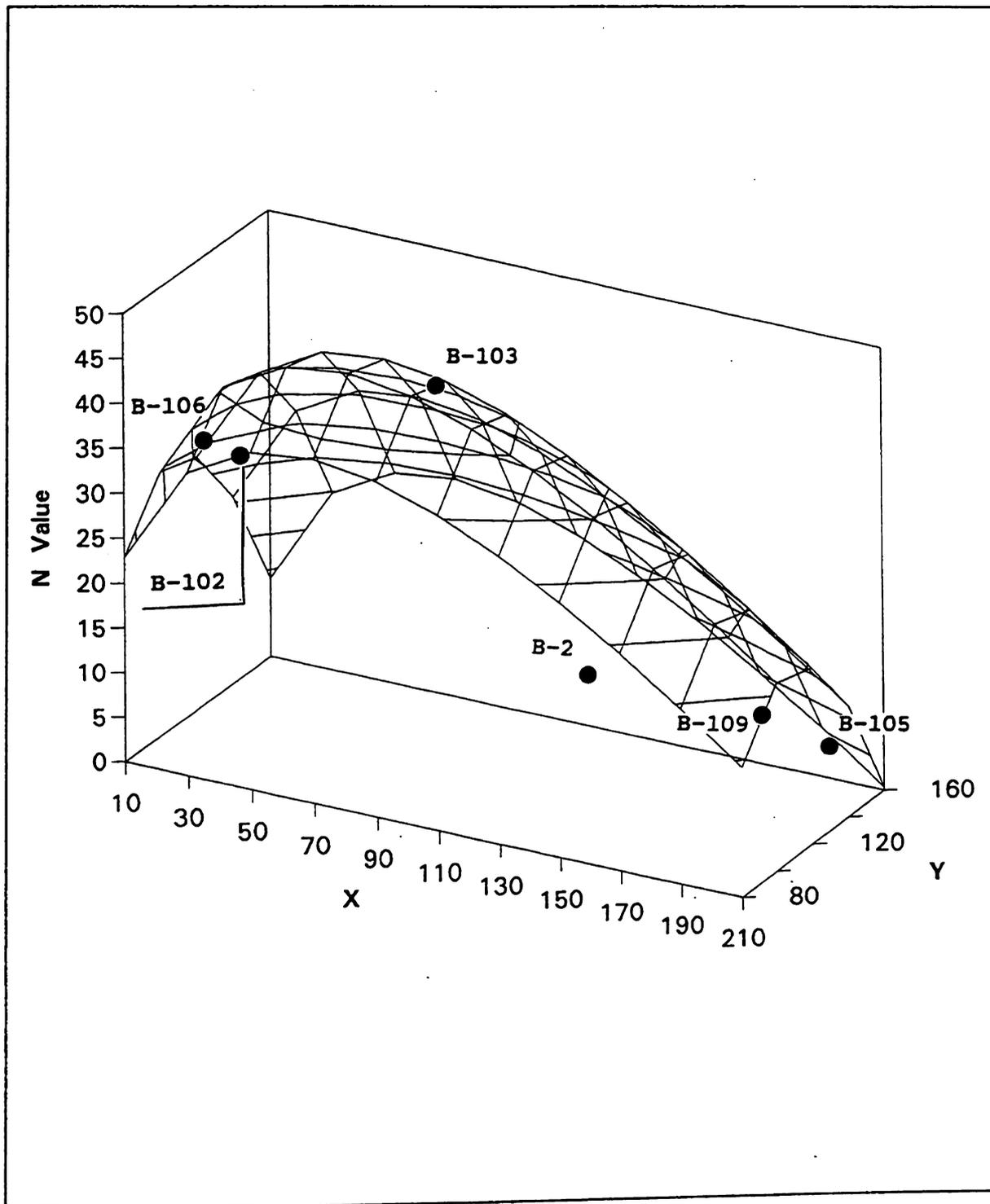
$$N_{(3B/2)} = 10.0582X^{0.5} + 78.1354Y^{0.5} - 0.6117X - 3.7941Y - 398.218; (5.24)$$

The computer program "EXCEL" was used to plot these two N functions in three-dimensional plots. Figures 5.21 and 5.22 show the plotting of N functions given by Equations 5.23 and 5.24 respectively. The N values at the measured points, at the same depths, are also plotted on the same Figures to show how well they were fitted.



**Figure 5.21:** Plotting Of The N Function Given By Equation 5.23 Together With The N Values Of The Six Borings At The Depth Of  $B/2$ :  $Z = 877.25$  ft.





**Figure 5.22:** Plotting Of The N Function Given By Equation 5.24 Together With The N Values Of The Six Borings At The Depth Of  $3B/2$ :  $Z = 865.75$  ft.

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### 8. The Contours Of The Predicted Settlement:

The estimated settlement in inches is given by:

$$S_{est} = 8.5 / N$$

where

$$N = (1/3) [ 2 N_{(B/2)} + N_{(3B/2)} ]$$

substituting  $N_{(B/2)}$  &  $N_{(3B/2)}$  as functions in (X & Y) as given by Equations (5.23 & 24)

yields:

$$N = 10.058236699 X^{0.5} + 78.135462538 Y^{0.5} - 0.611711073 X - 3.794113405 Y - 403.432$$

which could be simplified as:

$$N = 10.058X^{0.5} + 78.135Y^{0.5} - 0.611X - 3.794Y - 403.432 \quad (5.25)$$

Therefore:

$$S_{est} = 8.5 / (10.058X^{0.5} + 78.135Y^{0.5} - 0.611X - 3.794Y - 403.432) ; (5.26)$$

This function is used to calculate the predicted settlements at points in the (X, Y) plane spaced at intervals of say 20 ft each. These values are then used to establish the contours of the predicted settlements.

The predicted settlements are tabulated in Table 5.10 and the contour plot is shown in Figure 5.23. It is to be noted that these settlement contours are not for the predicted settlements of actual footings at the building; these are the predicted settlements for replicate footings of the one analyzed located at the different locations of the site.

Table 5.

<u>X (</u>
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140
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**Table 5.10 : The Predicted Settlements At Points Spaced At 20 ft For Case History No. 1**

X (ft)	Y(ft)	Settlement (inch)
40	60	0.639
60	60	0.553
80	60	0.560
100	60	0.627
120	60	0.777
20	80	0.345
40	80	0.274
60	80	0.257
80	80	0.258
100	80	0.271
120	80	0.296
140	80	0.336
160	80	0.399
20	100	0.272
40	100	0.226
60	100	0.214
80	100	0.215
100	100	0.224
120	100	0.241

Table 5

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X

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100

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800

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1200

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**Table 5.10 : Continued.**

X (ft)	Y(ft)	Settlement (inch)
140	100	0.267
160	100	0.305
20	120	0.284
40	120	0.234
60	120	0.221
80	120	0.222
100	120	0.232
120	120	0.250
140	120	0.278
160	120	0.320
20	140	0.375
40	140	0.293
60	140	0.273
80	140	0.275
100	140	0.290
120	140	0.319
140	140	0.365
160	140	0.441

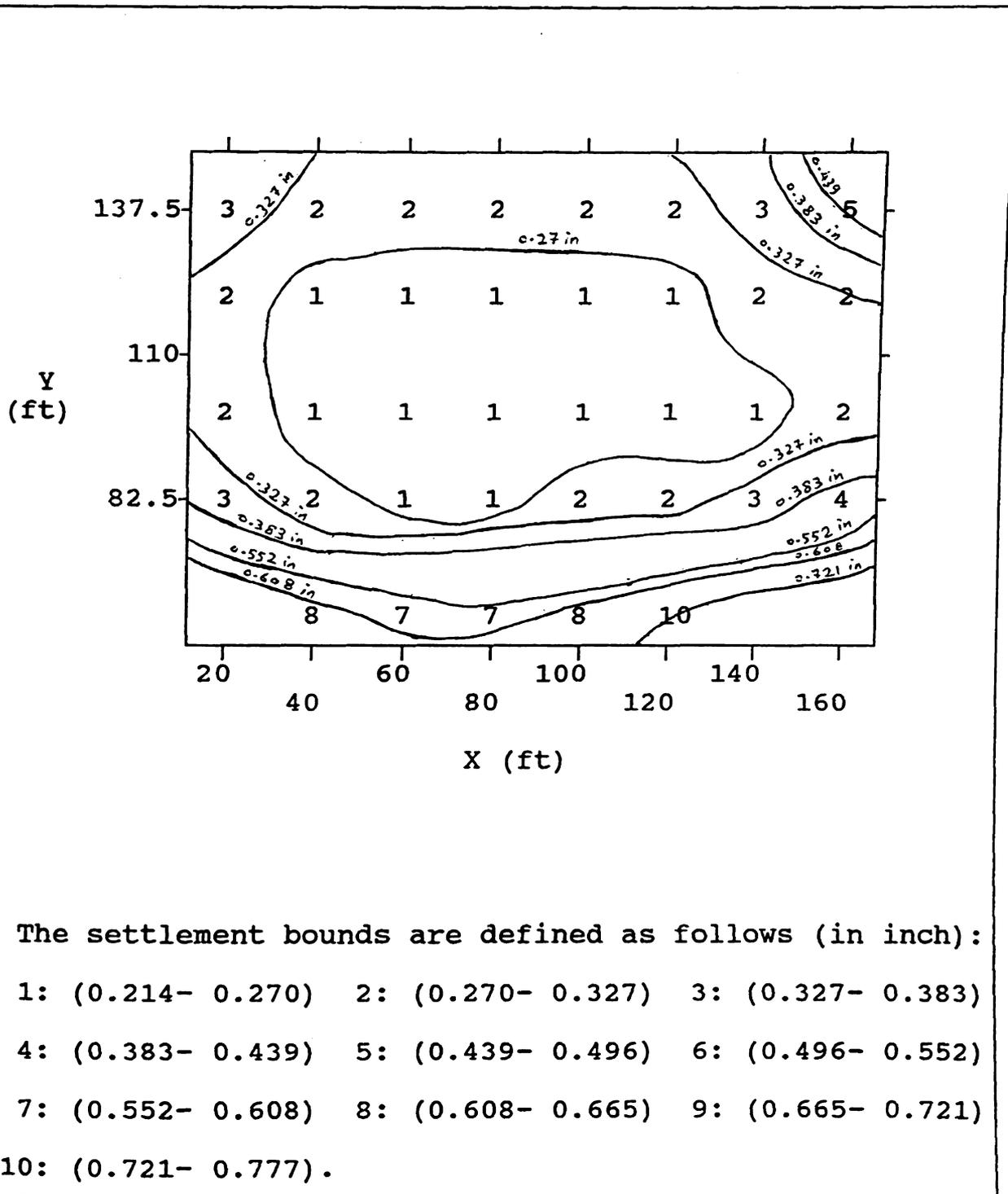


Figure 5.23 : The Settlement Contours.

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## 5.2.2 APPLYING THE KRIGING TECHNIQUE TO CASE HISTORY No. 1

In this section the settlement will be predicted by Kriging for the same footing of case history No.1 for which the settlement was predicted using the trend surface analysis.

The procedure is conducted as shown in the following paragraphs.

### 5.2.2.1 APPLYING THE KRIGING TECHNIQUE TO THE N VALUES

Consider that an estimate of N value is required at the center point "o" of the selected footing as shown in Figure 5.17. The coordinates of "o" are: (104.2 , 92.7) as shown in Figure 5.18. Kriging is applied by conducting the following steps:

#### 1. Representing the N values by a linear regression function:

The N values were corrected for the overburden pressure as described in Appendix A (Tables A.1 to A.6). The corrected N values of each boring were represented by a linear regression function. This function takes the form:

$$N = a_i + b_i z ; \quad (i = \text{boring number}). \quad (5.27)$$

The rationale of formulating this function was explained earlier in section (5.1.2.2). As far as borings "B-102 & B-105" are concerned their depths are not considered entirely because they both reach a firmer strata; so in order to satisfy the homogeneity condition it is decided to consider only the top 30 ft of both of them. In this case history the strata which was believed to violate the homogeneity requirement was well below the depth of influence of 2B (23 ft) under the footing, so it was decided easily to exclude it from the analysis. If the suspected strata is located within the depth of influence, then the

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decision of excluding it should be based on the analysis of variance.

The selection of the data which will be used for the estimation will be controlled by the general requirements of the interpolation techniques which recommend that the data should surround the point "o" nicely. Therefore the borings which will be considered are borings (B-102 , B-105 , B-106 , B-109). For convenience these borings will be abbreviated here as borings (2 , 5 , 6 ,9) respectively.

It is decided to exclude both boring (B-103) and (B-2) because the first is located right under point "o" and the second is located in the close vicinity of it, so it was decided to take them out to test the effect of not having drilled them. The other four borings which were considered are surrounding the point "o" nicely , i.e. have more or less equal distances from the point "o".

The results of the linear regressions representing these four borings are as follows:

$$\text{boring (B-102):} \quad N = 21.7682 - 0.2913 Z , \text{ Standard Error} = 2.28 ; \text{ (5.28)}$$

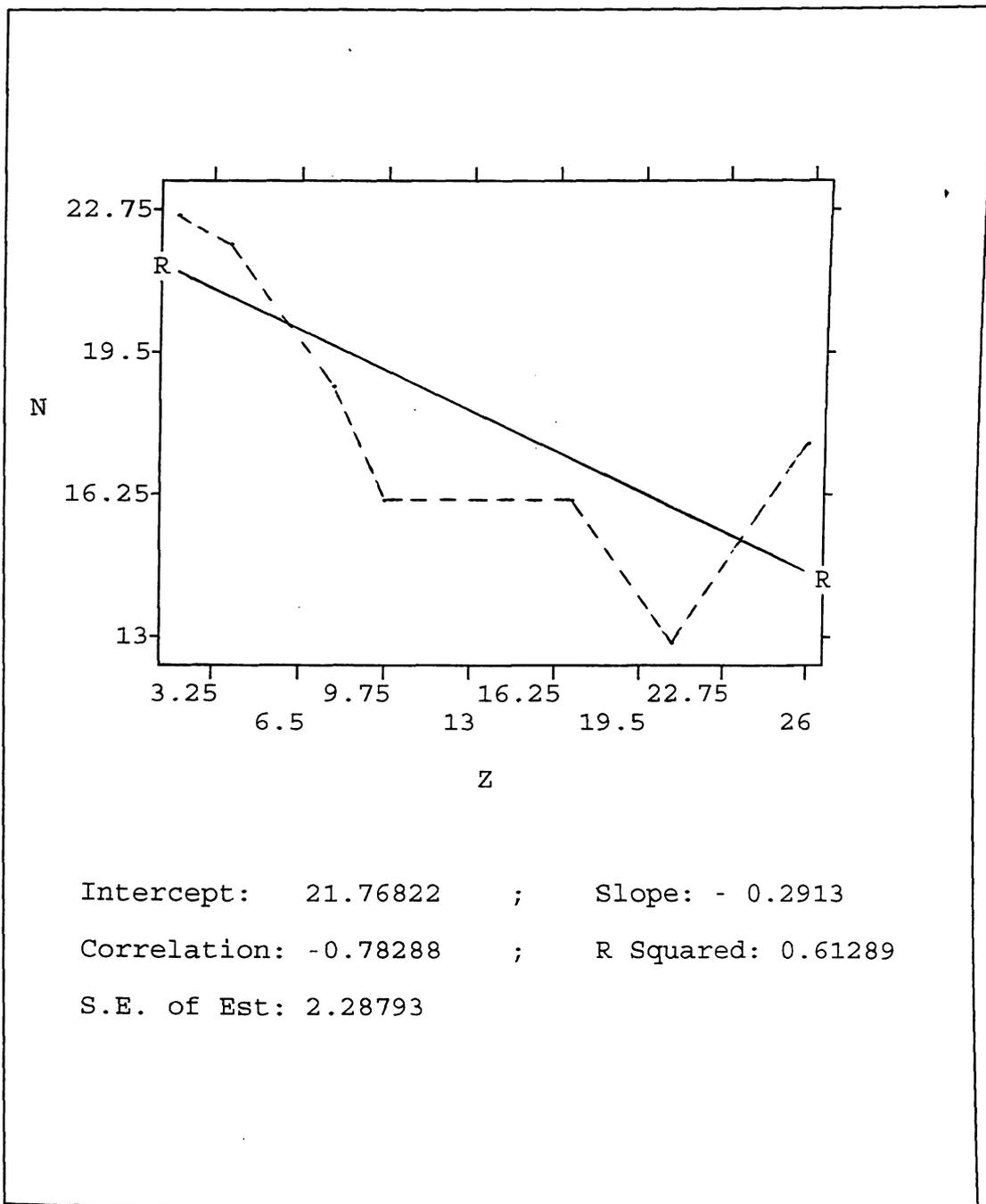
$$\text{boring (B-105):} \quad N = 32.0538 - 1.216 Z , \quad " \quad " = 5.517 ; \text{ (5.29)}$$

$$\text{boring (B-106):} \quad N = 22.5733 - 0.3906 Z , \quad " \quad " = 1.57 ; \text{ (5.30)}$$

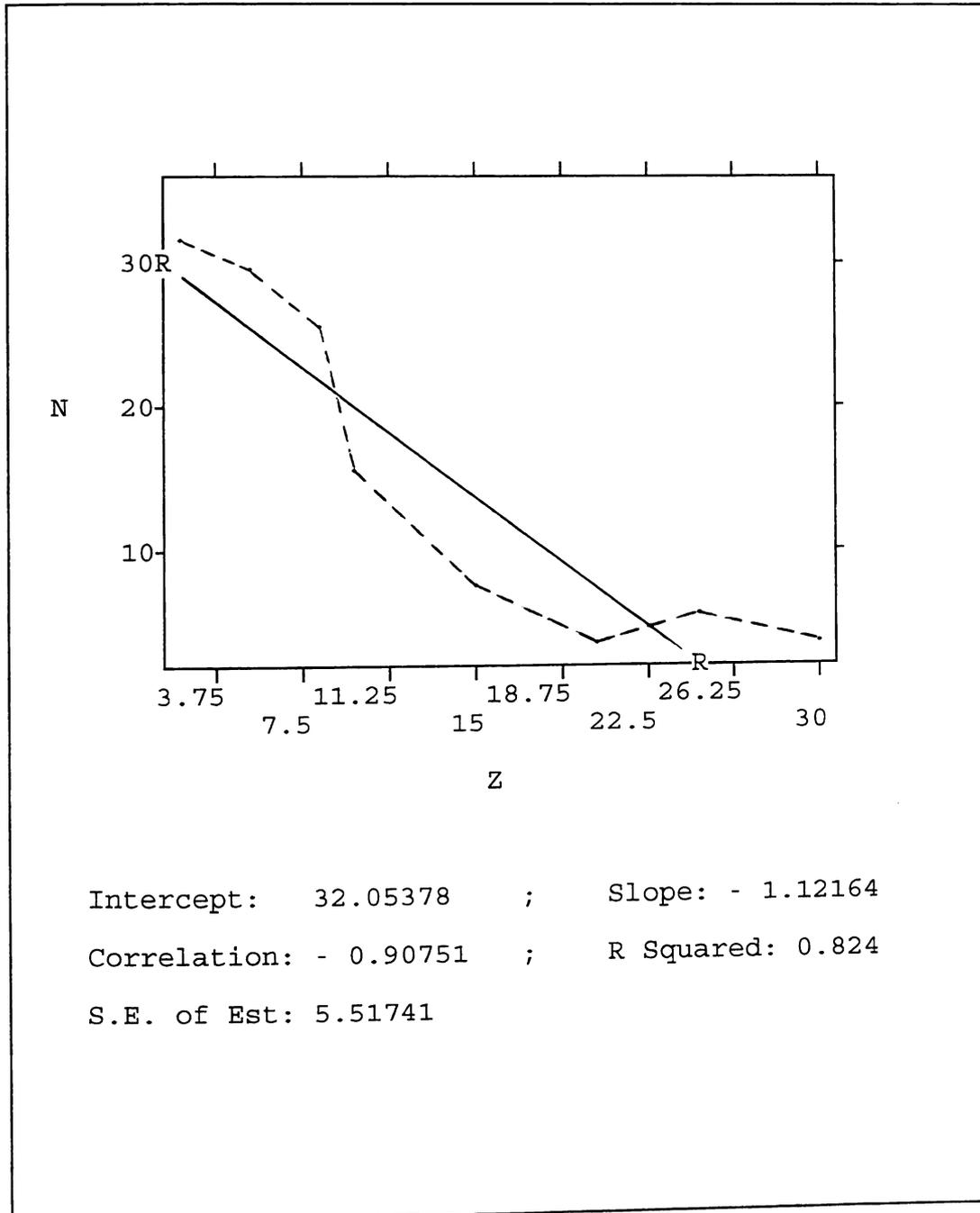
$$\text{boring (B-109):} \quad N = 23.7274 - 0.5455 Z , \quad " \quad " = 3.15 ; \text{ (5.31)}$$

These four functions are plotted over data as shown in Figures 5.24 to 5.27.

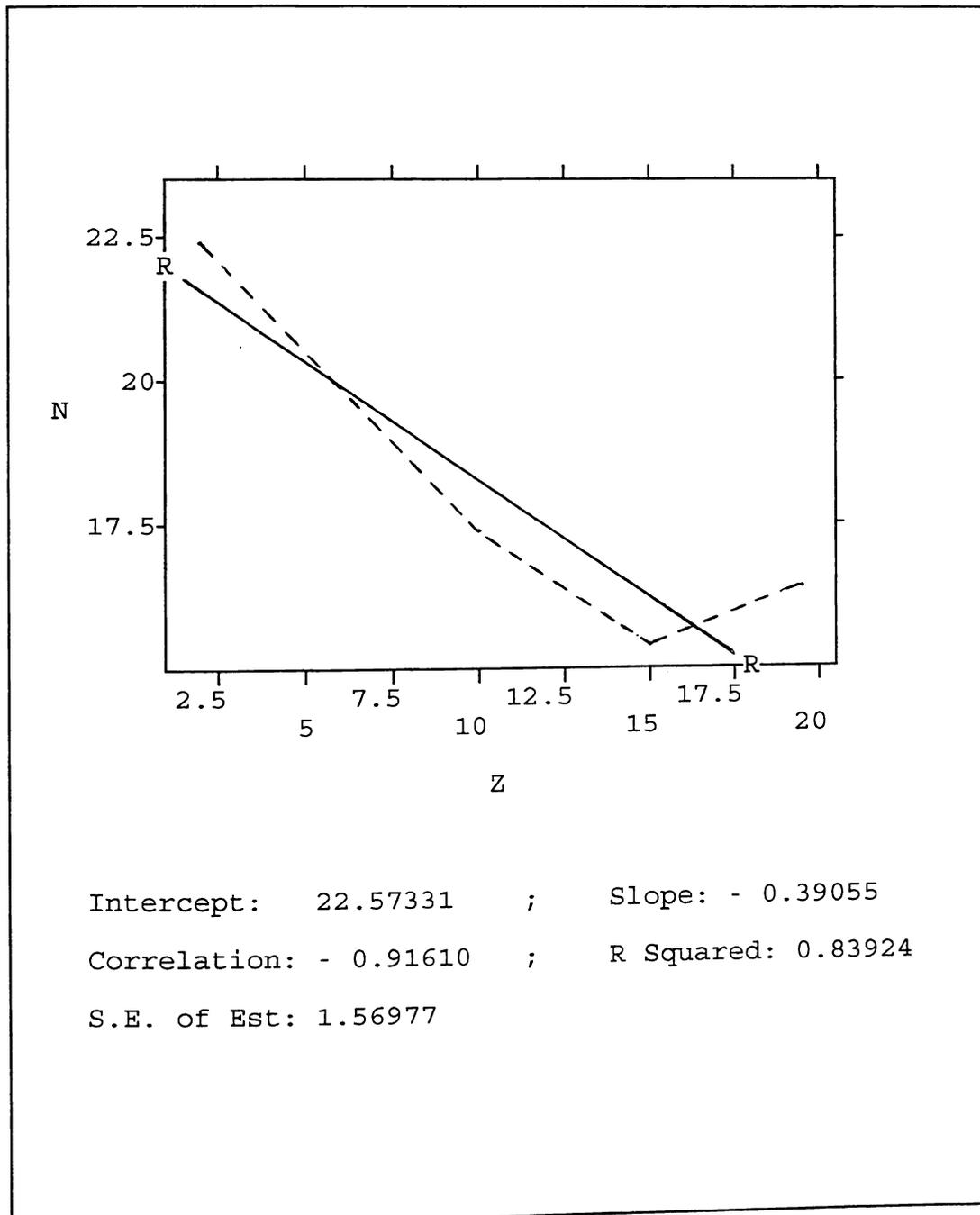
To illustrate the effect of not having restricted the boring depths, Figure 5.28 shows the regression line of boring (B-102) plotted over data without restricting the depth. The comparison between Figures 5.24 and 5.28 shows that the data that appears to be non-linear over a large depth range can be assumed to be linear if the depth range is restricted to that of interest.



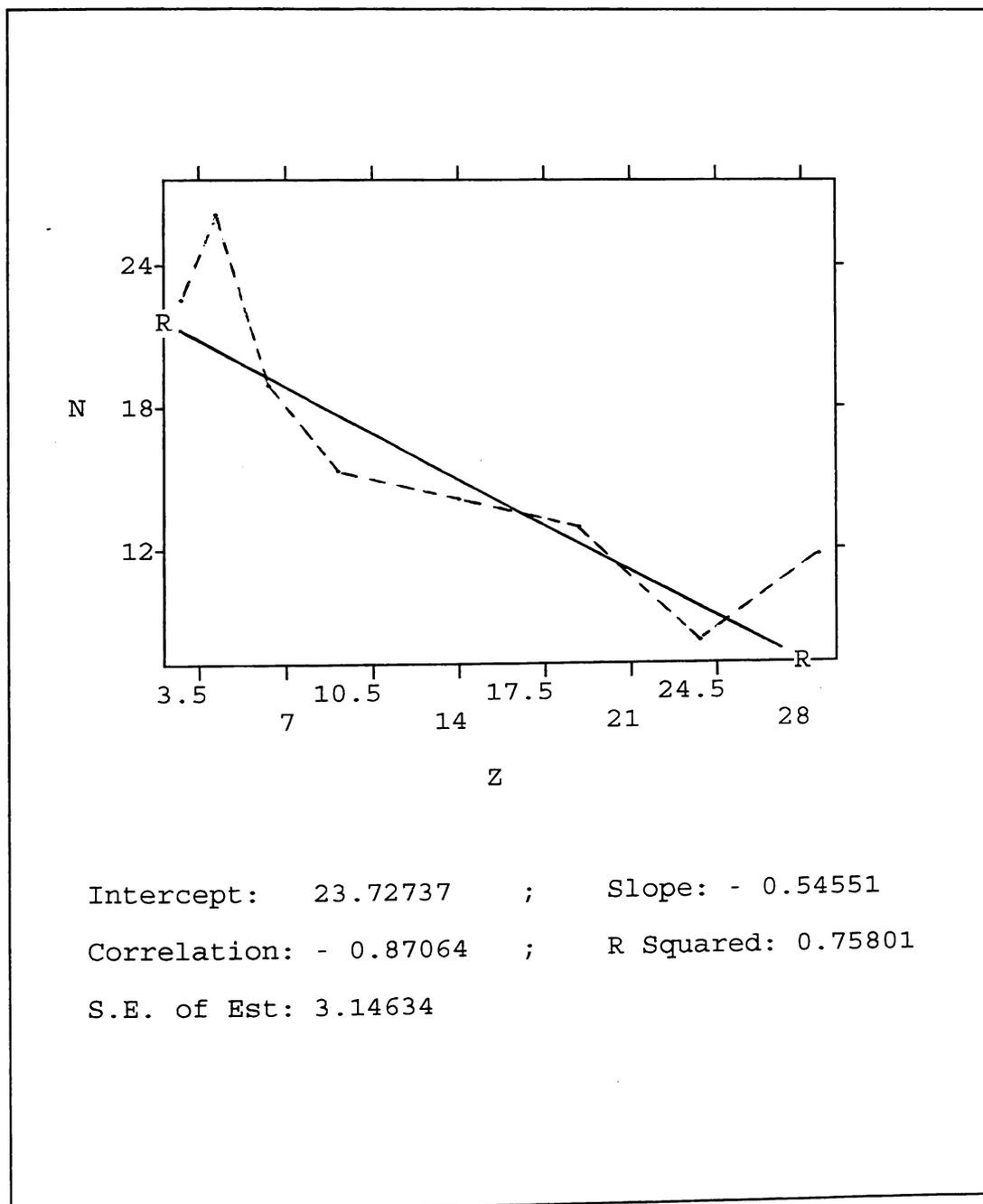
**Figure 5.24 : The Linear Regression Function Of Boring (B-102).**



**Figure 5.25 : The Linear Regression Function Of Boring (B-105).**

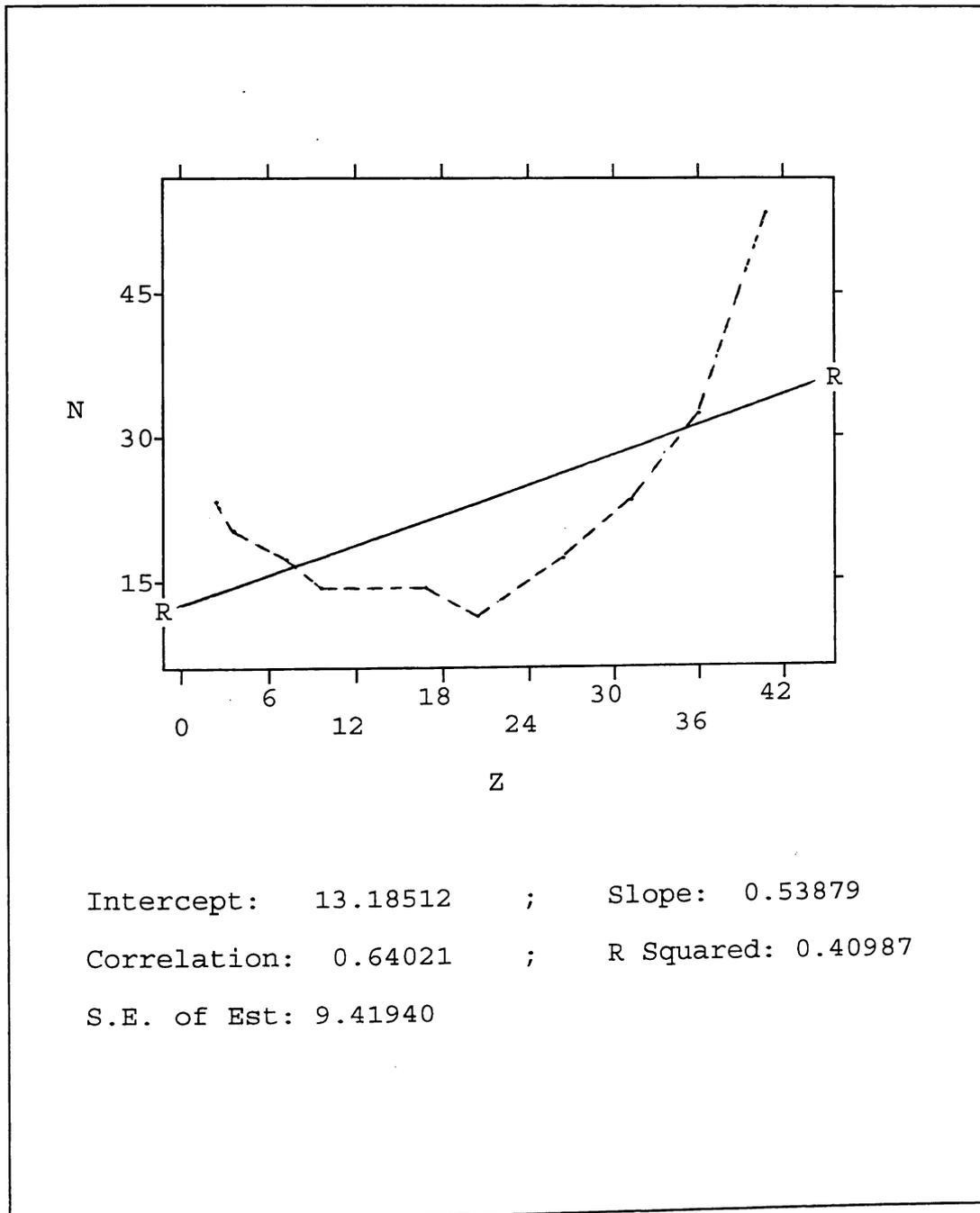


**Figure 5.26 : The Linear Regression Function Of Boring (B-106).**



**Figure 5.27 : The Linear Regression Function Of Boring (B-109).**





**Figure 5.28 :** The Linear Regression Function Of Boring (B-102) Plotted Over Data Without Restricting The Boring Depth.

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**2. Calculating the horizontal distance (h) between every pair of locations:**

The distances between every pair of locations were measured from the site plan.

These locations include the borings (2, 5, 6 & 9) as well as the center point "o" of the selected footing.

The distances in feet are given in the following distance matrix:

$h_{22} = 0.0$	$h_{25} = 190.08$	$h_{26} = 52.79$	$h_{29} = 189.84$
$h_{52} = 190.08$	$h_{55} = 0.0$	$h_{56} = 191.28$	$h_{59} = 57.79$
$h_{62} = 52.79$	$h_{65} = 191.28$	$h_{66} = 0.0$	$h_{69} = 175.00$
$h_{92} = 189.84$	$h_{95} = 57.79$	$h_{96} = 175.00$	$h_{99} = 0.0$

The distance vector between the point "o" and the boring locations are as follows:

$$h_{2o} = 102.36$$

$$h_{5o} = 109.06$$

$$h_{6o} = 85.42$$

$$h_{9o} = 90.39$$

**3. Calculating the covariances which describe the spatial continuity of the data:**

The covariances of N values at every pair of borings were calculated. These borings include the four borings (2, 5, 6 and 9) which were used for the Kriging as well as the boring at the location in question "o". The covariances were calculated using the equation:

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$$C_{ij} = (1/n) \sum_1^n (N_i - \bar{N}_i) (N_j - \bar{N}_j) \quad (5.32)$$

where

$C_{ij}$  : covariance of N values at the two borings "i" and "j".

$N_i$  : N value at boring "i".

$N_j$  : N value at boring "j".

n : number of N pairs.

The covariances were calculated using the computer program of Davis (1986).

The resulting covariances are as follows:

Distance between the two borings (ft).	$C_{ij}$
0	133.27
$h_{26}$ : 52.79	25.29
$h_{59}$ : 57.79	44.51
$h_{60}$ : 85.42	49.11
$h_{90}$ : 90.39	61.48
$h_{20}$ : 102.36	13.34
$h_{50}$ : 109.06	38.82
$h_{69}$ : 175.00	36.37
$h_{29}$ : 189.84	10.04
$h_{25}$ : 190.08	21.92
$h_{56}$ : 191.28	82.09

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Consequently, the covariance matrix is as follows:

$$C = \begin{bmatrix} 133.27 & 21.92 & 25.29 & 10.04 & 1 \\ 21.92 & 133.27 & 82.09 & 44.51 & 1 \\ 25.29 & 82.09 & 133.27 & 36.37 & 1 \\ 10.04 & 44.51 & 36.37 & 133.27 & 1 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0 \end{bmatrix}$$

and the vector of the covariances between the point "o" and the boring locations is as follows:

$$D = \begin{bmatrix} 13.34 \\ 38.82 \\ 49.11 \\ 61.48 \\ 1.0 \end{bmatrix}$$

#### 4. Calculating the weight matrix:

The weights are given by the vector:

$$W = \begin{bmatrix} W_2 \\ W_5 \\ W_6 \\ W_9 \\ u \end{bmatrix}$$

where

$u$  = the LaGrange parameter.

The weight matrix is calculated as follows:

The weight matrix = (the inverse of the covariance matrix) \* (covariance vector of the estimated point).

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$$W = C^{-1} \cdot D \quad (5.33)$$

Or:

$$\begin{bmatrix} W2 \\ W5 \\ W6 \\ W9 \\ u \end{bmatrix} = \begin{bmatrix} 133.29 & 21.92 & 25.29 & 10.04 & 1 \\ 21.92 & 133.27 & 82.09 & 44.51 & 1 \\ 25.29 & 82.09 & 133.27 & 36.37 & 1 \\ 10.04 & 44.51 & 36.37 & 133.27 & 1 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0 \end{bmatrix}^{-1} * \begin{bmatrix} 13.34 \\ 38.82 \\ 49.11 \\ 61.48 \\ 1.0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.00570 & -0.00155 & -0.00223 & -0.00193 & 0.34967 \\ -0.00155 & 0.01229 & -0.00764 & -0.00311 & 0.16082 \\ -0.00223 & -0.00764 & 0.01175 & -0.00188 & 0.18606 \\ -0.00193 & -0.00311 & -0.00188 & 0.00693 & 0.30346 \\ 0.34967 & 0.16082 & 0.18606 & 0.30346 & -57.87732 \end{bmatrix} *$$

$$\begin{bmatrix} 13.34 \\ 38.82 \\ 49.11 \\ 61.48 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.13774 \\ 0.05108 \\ 0.32102 \\ 0.49016 \\ -19.17596 \end{bmatrix}$$

The physical significance of the resulting weights can be realized after examining the boring locations in relation to the point "o". The point "o" is almost located on the straight line joining the borings (6 and 9), and the borings (2 and 5) are outside this line. As a result the weights of borings (2 and 5) are much smaller than the weights of borings (6 and 9).

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5. **The estimated N function:**

The estimated N function at the point "o" is given by:

$$\hat{N} = a_o + b_o Z \quad (5.34)$$

where

$$\begin{aligned} a_o &= \sum w_i \cdot a_i \quad ; \quad (\text{over the 4 borings: 2, 5, 6 \& 9}). \\ &= 0.1377 (21.7682) + 0.0511 (32.0538) \\ &\quad + 0.321 (22.5733) + 0.4902 (23.7274) = 23.51 \end{aligned}$$

and

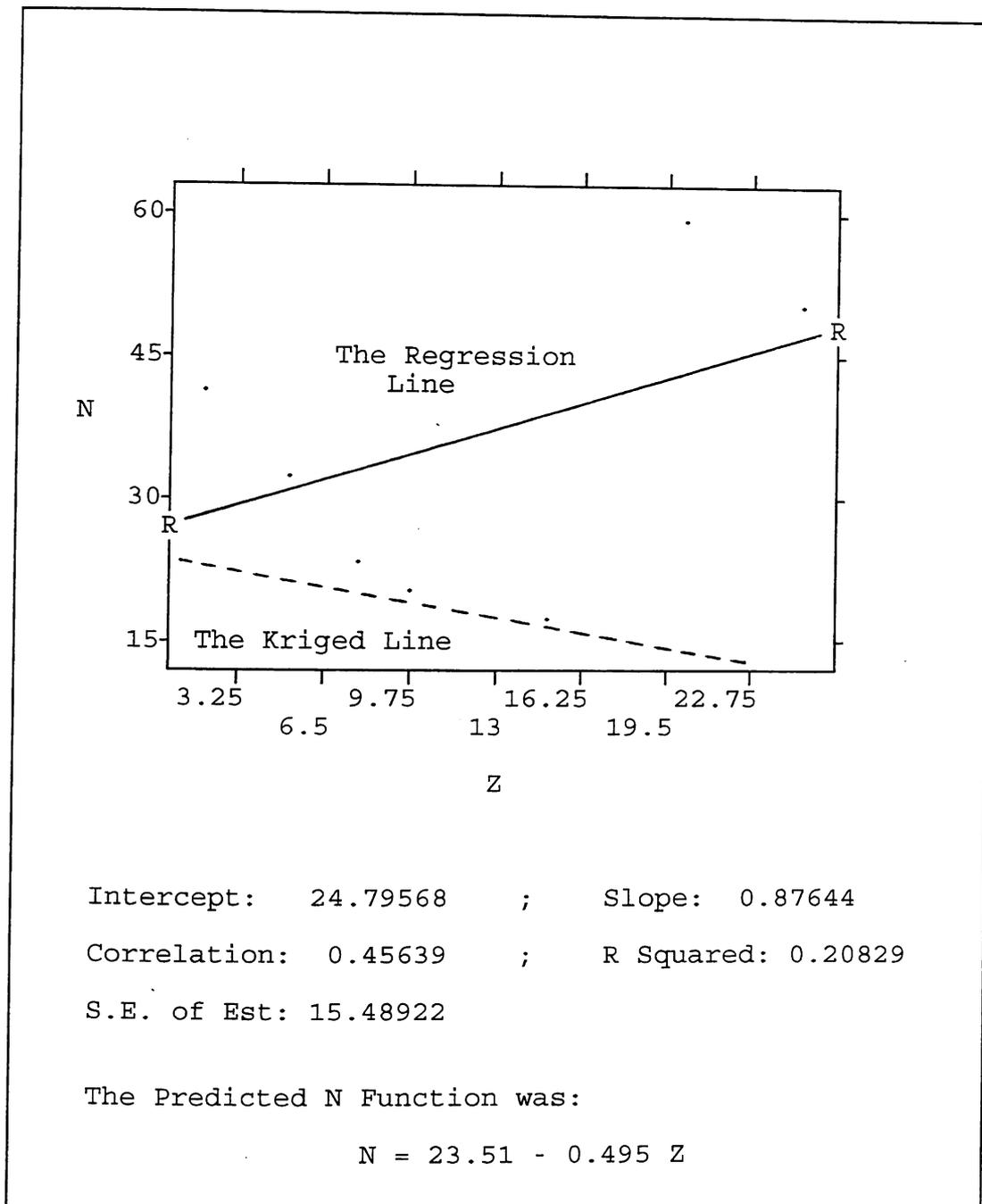
$$\begin{aligned} b_o &= \sum w_i \cdot b_i \\ &= -0.1377 (-0.2913) + 0.0511 (-1.216) \\ &\quad + 0.321 (-0.3906) + 0.4902 (-0.5455) = -0.495 \end{aligned}$$

The estimated N function at the point in question is therefore:

$$\hat{N} = 23.51 - 0.495 Z \quad (5.35)$$

The negative coefficient on Z indicates a tendency for decreasing relative density or stiffness with depth. This alerts one to disproportionality of larger settlements with increased footing sizes.

The predicted N function given by Equation 5.35 is plotted over the regression line of the observed N values of boring (B-103) as shown in Figure 5.28. The higher N values which appear in the boring log below the depth of about 25 ft are excluded from the regression because they are below the depth of influence of the footing.



**Figure 5.29 : The Predicted N Function Vs. The Regression Line Of The Observed N Values Of Boring (B-103).**

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It is observed in the Figure that the two lines differ, but they are reflective of the "local average" versus a single location average.

#### 5.2.2.2 USING THE ESTIMATED N FUNCTION TO OBTAIN THE DESIGN N VALUE

The "two-point" estimate can now be used to estimate the design N value as follows:

The N values at the depths of ( $B/2 = 5.75$  ft) and ( $3B/2 = 17.25$  ft) are given by:

$$\hat{N}_{(B/2)} = a_o + b_o (B/2 + \text{embedment}) = 23.51 - 0.495 (5.75 + 8.5) = 16.46$$

and

$$\hat{N}_{(3B/2)} = a_o + b_o (3B/2 + \text{embedment}) = 23.51 - 0.495 (17.25 + 8.5) = 10.76$$

The design N value at this location "o" is then given by the weighted average:

$$N = (1/3) [2\hat{N}_{(B/2)} + \hat{N}_{(3B/2)}] = (1/3) [2(16.46) + (10.76)] = 14.56$$

which compare to the measured values of (18, 17, 67, 64) in this general vicinity in Boring (B-103).

The design N value obtained by the trend surface analysis was 36.1.

#### 5.2.2.3 USING THE DESIGN N VALUE FOR SETTLEMENT PREDICTION:

The predicted settlement "S" in inch is given by:

$$S = (2/N) [ q * (2B / B+1)^2 ] ; \quad (\text{Peck and Bazaraa, 1969}).$$

where

B = width

q = net

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B = width of footing = 11.5 ft.

q = net foundation pressure in (tsf).

$$= 650 \text{ Kips} / (2000 * 11.5' * 22.5') = 1.256 \text{ (tsf)}.$$

Therefore

$$S = (2 / 14.56) [1.256 * (2 * 11.5 / 11.5 + 1)^2] \\ = 0.58 \text{ inch.}$$

The settlement predicted by the trend surface analysis was 0.24 inch.

The previous investigators give a predicted settlement as 0.53 inch using the one-dimensional compression laboratory test and a predicted settlement as 0.12 inch using the pressuremeter test.

The measured settlement as reported by the previous investigator = 0.3 inch.

Therefore the predicted settlement (by Kriging) of 0.58 in is greater than the measured settlement by approximately 93 % and the predicted settlement (by trend surface) of 0.24 in is less than the measured value by approximately 20 %.

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### 5.2.2.4 CONSTRUCTING THE PREDICTION INTERVALS FOR THE ESTIMATED VALUES OF BOTH N AND S

The prediction variance is given by:

$$\sigma_R^2 = \sum_{i=1}^n \sum_{j=1}^n w_i \cdot w_j \cdot C_{ij} - 2 \sum_{i=1}^n w_i \cdot C_{i0} + \sigma_N^2 \quad (5.36)$$

From the previous results of the weight and covariances matrices the terms of this prediction variance are calculated as follows:

$$Term(1) = \sum_{i=1}^n \sum_{j=1}^n w_i \cdot w_j \cdot C_{ij} = 18.89$$

$$\begin{aligned} Term(2) &= 2 \sum_{i=1}^n w_i \cdot C_{i0} \\ &= 2 (w_2 C_{20} + w_5 C_{50} + w_6 C_{60} + w_9 C_{90}) = 99.44 \end{aligned}$$

$$Term(3) = \sigma_N^2 = 133.27$$

Therefore, the prediction variance is given by:

$$\sigma_R^2 = 52.7$$

The prediction variance can be used to construct the prediction interval on the estimate to any desired degree of confidence. Thus, the 90 % prediction interval for the N value is given by:

$$\hat{N} \pm t_{((4-1), 0.05)} \sqrt{\sigma_R^2} = (14.56 \pm 2.353 \sqrt{52.7}) = 14.56 \pm 17.1$$

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However, in practice, the 90% confidence interval is pretty broad. For the standard penetration test, the confidence interval is not intended to be a criteria for accepting or rejecting an N observation. The experience of the driller plays an important role in the acceptance of an observation. Furthermore, the upper confidence limit could be used - as recommended by some researchers e.g. Baecher, 1981 - as a design value. For these reasons, the 50% confidence interval is considered reasonable for the SPT testing.

In this regard, the 50% prediction interval for the N value is given by:

$$\hat{N} \pm t_{((4-1), 0.25)} \sqrt{\sigma_R^2} = (14.56 \pm 0.765 \sqrt{52.7}) = 14.56 \pm 5.55$$

This says the 50% confidence limits are: (9.01 and 20.11), which is a relatively broad confidence interval for the predicted N value, obviously because of the high value of the variance of the N values. For comparison, the 50% confidence limits from the trend surface analysis were (32.8 and 39.4).

After the examination of these results, it is observed that the predicted design N value by Kriging was 14.56 which is lower than the predicted design N value of 36.1 by trend surface. On the other hand, the 50% confidence interval by Kriging was 11.1 which is larger than the 50% confidence interval of 6.6 by trend surface. This reflects the effect of testing a "hard spot" at the location of Boring (B-103) and the exclusion of the data of this boring from the prediction by Kriging while including them in the prediction by trend surface.

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The 50% confidence limits of the settlement prediction (by Kriging) are as follows:

$$\begin{aligned}
 S &= (2 / (\hat{N} \mp t_{((4-1), 0.25)} \sqrt{\sigma_R^2})) [q * (2B/B+1)^2] \\
 &= (2/20.11) [1.256 * (2*11.5/11.5+1)^2] \text{ and} \\
 &\hspace{15em} (2/9.01)[1.256*(2*11.5/11.5+1)^2] \\
 &= ( 0.41 \text{ and } 0.95 ) \text{ in.}
 \end{aligned}$$

The 50% confidence limits of the settlement prediction (by the trend surface analysis) were: ( 0.22 and 0.26 ) in.

Here again, the effect of including the data of the tough spot, at the location of Boring (B-103), in the prediction by trend surface resulted in a predicted settlement of 0.24 in. On the other hand, the predicted settlement by Kriging was 0.58 in, which reflects the effect of using the local average instead of the N value at the "hard spot".

### 5.2.3 SUMMARY

#### SIX CASE HISTORIES

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### 5.2.3 SUMMARY AND ANALYSIS OF THE PREDICTED SETTLEMENTS FOR SIX CASE HISTORIES

Five more case histories - in addition to the split level office building in North Carolina - were analyzed using both the trend surface analysis and the Kriging technique and the settlements are predicted for each of them using both techniques. A list of the six case histories together with the references are as follows:

1. Split level office building in North Carolina ; (Borden and Lien, 1988).
2. A twenty - story block in Nigeria ; (Grimes and Cantly, 1965).
3. Large tanks in Kansas City ; (Davisson and Salley, 1972).
4. A load test in northern Spain ; (Picornell and Del-Monte, 1988).
5. A generator in Pennsylvania ; (Fischer et. al. , 1972).
6. A lift bridge in Delaware ; (Seymour et. al. , 1972).

#### 5.2.3.1 The Case Histories Selection Criteria And Summary:

These six case histories were selected from among tens of published case histories. The selection was controlled by the criteria of accepting the case only if it has well defined data from the viewpoint of the spatial and geotechnical modeling. This meant that each case history should satisfy the following requirements:

1. Every N value should have a defined location as  $N(x,y,z)$ . This means that the boring locations in the (X , Y) plane and the depth of each N value are reported.
2. The footing dimensions and locations are available.
3. The loading to the base of the footing is available.
4. The actual settlement is measured and reported.

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The selected six case histories satisfied these four requirements. Case histories which were found to violate one or more of these requirements were not selected.

The analysis of these case histories - except for the first one which was analyzed in details earlier in this chapter - is summarized in Appendix "C". However, the results of this analysis are presented together here for the sake of comparison. These results are summarized as follows:

The trend surface analyses yielded the following results:

1. Reference is made to what was mentioned in Section 5.2 that, for the actual case histories, the best fit of trend surface models cannot be easily judged as in the assumed field, hence 5 to 15 models were tried for each case. The best model was determined for each case according to the criteria explained in Section 5.2. The models which were selected as providing the best fit to the data of the different case histories were:

Case history No.1:

$$N = -3008929.7 + 10.058X^{0.5} + 78.135Y^{0.5} + 274208.9Z^{0.5} - 0.612X - 3.794Y - 7027.8Z + 1.367Z^2 \quad (5.37)$$

$$R^2 = 0.528$$

Substituting the coordinates (104.17, 92.7) of the predicted location in this model yields the  $N_{(z)}$  function at this location as follows:

$$N = 1.367 Z^2 + 4590.3 Z + 274208.9 (-Z + 891.5)^{0.5} - 8187289.2$$

Case history No.2:

$$N = 35.66 - 16.7X^{0.5} + 1.38X + 0.0019X^2 + 4.56Y^{0.5} - 0.37Y + 1.73Z^{0.5} - 0.107Z - 0.001Z^2 \quad (5.38)$$

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$$R^2 = 0.458$$

The  $N_{(z)}$  function at the predicted location, (115.8, 58.42), is given by:

$$N = -0.001 Z^2 - 0.107 Z + 1.73 Z^{0.5} + 54.47$$

Case history No.3:

$$N = -1254 - 0.0082X + 1476 Z^{0.5} - 624Z + 114.3 Z^{1.5}$$

$$-7.68 Z^2 \quad (5.39)$$

$$R^2 = 0.65$$

The  $N_{(z)}$  function at the predicted location, (315, 178.38), is given by:

$$N = -7.68 Z^2 + 114.3 Z^{1.5} - 624 Z + 1476 Z^{0.5} - 1256.58$$

Case history No.4:

$$N = -15.26 + 0.216X + 5.67 Z^{0.5} - 0.417 Z + 0.001 Z^2$$

$$+7.92 Z^3 \quad (5.40)$$

$$R^2 = 0.62$$

The  $N_{(z)}$  function at the predicted location, (32.5, 25), is given by:

$$N = 7.92 \text{ E-}7 Z^3 + 0.001 Z^2 - 0.417 Z + 5.67 Z^{0.5} - 8.24$$

Case history No.5:

$$N = 15197 - 0.02X - 4302 Z^{0.5} + 360Z - 0.74 Z^2 \quad (5.41)$$

$$R^2 = 0.53$$

The  $N_{(z)}$  function at the predicted location, (260.6, 0.0), is given by:

$$N = -0.7442 Z^2 + 360.89 Z - 4302.4 Z^{0.5} + 15191.79$$

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Case history No.6:

$$N=27836-1.62X-8072Z^{0.5}+675Z-1.6Z^2 \quad (5.42)$$

$$R^2 = 0.503$$

The  $N_{(z)}$  function at the predicted location, (760, 0.0), is given by:

$$N = -1.6 Z^2 + 675 Z - 8072 Z^{0.5} + 26604.8$$

2. The design N values were as shown in Table 5.11. The rounded integer values are shown between brackets.

**Table 5.11 : The Design N Vaues For The Six Case Histories.**

Case History No.	Design N Value
1	36.1 (36)
2	6.91 (7)
3	3.16 (3)
4	12.29 (12)
5	65.6 (66)
6	39.18 (39)

The prediction by Kriging yielded the following results:

The estimated N functions and the design N values for the six case histories are summarized in Table 5.12.

Table 5.12

Case History

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Table 5.13

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**Table 5.12 : The Estimated N Function And The Design N Values For The Six Case Histories.**

Case History No.	The Estimated N Function.	The Design N Value.
1	$N = 23.51 - 0.495 Z$	14.56 (15)
2	$N = 10.745 - 0.019 Z$	10.18 (10)
3	$N = 7.5 + 0.368 Z$	4.13 (4)
4	$N = 6.572 + 0.139 Z$	12.38 (12)
5	$N = 0.747 + 33.593 Z$	74.6 (75)
6	$N = - 123.5 + 0.97 Z$	37.23 (37)

**Table 5.13 : Summary Of The Predicted Settlements Versus The Measured Values For The Six Case Histories.**

Case History No.	Settlement (inch)			
	Predicted By			Measured
	The Designer (method ref.)	Trend Surface	Kriging	
1	0.12	0.24	0.58	0.3
2	1.5	2.5	2.29	0.97
3	2.1: (Peck et. al.)			
	3.9: (Schmertmann)	4.03	3.1	3.3
4	1.4: (Peck et. al.)			
	1.2: (Meyerhof)	1.93	1.92	1.56
5	1.3: (Terzaghi & Peck)			
	0.76: (Meyerhof)	0.48	0.43	0.5
6	0.3	0.36	0.38	0.4

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The predicted settlements by both methods are summarized as follows:

Table 5.13 shows the predicted settlements versus the measured values for the six case histories. Table 5.14 shows the ratios of the predicted settlements to the measured values.

**Table 5.14 : Settlement Ratios For The Six Case Histories.**

Case History No.	Settlement Ratio: (Predicted / Measured)		
	By Designer	Using Trend Surface	By Kriging
1	0.4	0.8	1.93
2	1.55	2.57	2.36
3	0.64 , 1.18	1.22	0.94
4	0.9 , 0.77	1.24	1.23
5	2.6 , 1.52	0.96	0.86
6	0.75	0.9	0.95

### 5.2.3.2 Analyzing The Results:

The predicted settlements which appear in Table 5.13 were compared to the measured values. Investigation of the predicted settlements by both techniques as well as by designers gives an impression that both designer and statistical methods significantly overpredicted the settlement of the second case history suggesting that it could be a potential outlier that can be dropped out from the comparison. The comparison was made by running a simple t-test of zero mean on the paired differences. The hypothesis to be tested is that the difference between the predicted settlement ( $S_p$ ) and the measured settlement ( $S_m$ ) is zero. The t-statistic is computed as:

where

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$$t_{(n_1+n_2-2), (\alpha/2)} = [(\bar{S}_p - \bar{S}_m) - 0] / [(S_p^2/n_1) + (S_m^2/n_2)]^{0.5} \quad (5.43)$$

where

$S_p$  = standard deviation of the predicted settlements.

$S_m$  = standard deviation of the measured settlements.

$n$  = number of cases in each group.

The first t-test was conducted using the computer program "SPSS" to test the differences between the settlement predictions by the trend surface analysis and the measured values.

The computer output of this test appears in Figure 5.30.

		Number of Cases	Mean	Standard Deviation	Standard Error
Group 1		5	1.4080	1.618	.724
Group 2		5	1.2120	1.273	.569

		Pooled Variance Estimate			Separate Variance Estimate		
F Value	2-Tail Prob.	t Value	Degrees of Freedom	2-Tail Prob.	t Value	Degrees of Freedom	2-Tail Prob.
1.62	.653	.21	8	.837	.21	7.58	.837

**Figure 5.30 : t-Test For The Paired Difference Between The Settlements Predicted By The Trend Surface Analysis And The Measured Values.**

The second t-test was conducted to test the differences between the settlement predictions by Kriging and the measured values. The output of this test appears in Figure 5.31.

From the results of the t-test: the predictions by the trend surface yielded a t value of 0.21. The critical t value is 2.306 (at the level of 0.05 and the degrees of freedom of 8). Since 0.21 is well below 2.306 then, the null hypothesis that the differences are insignificant is accepted.

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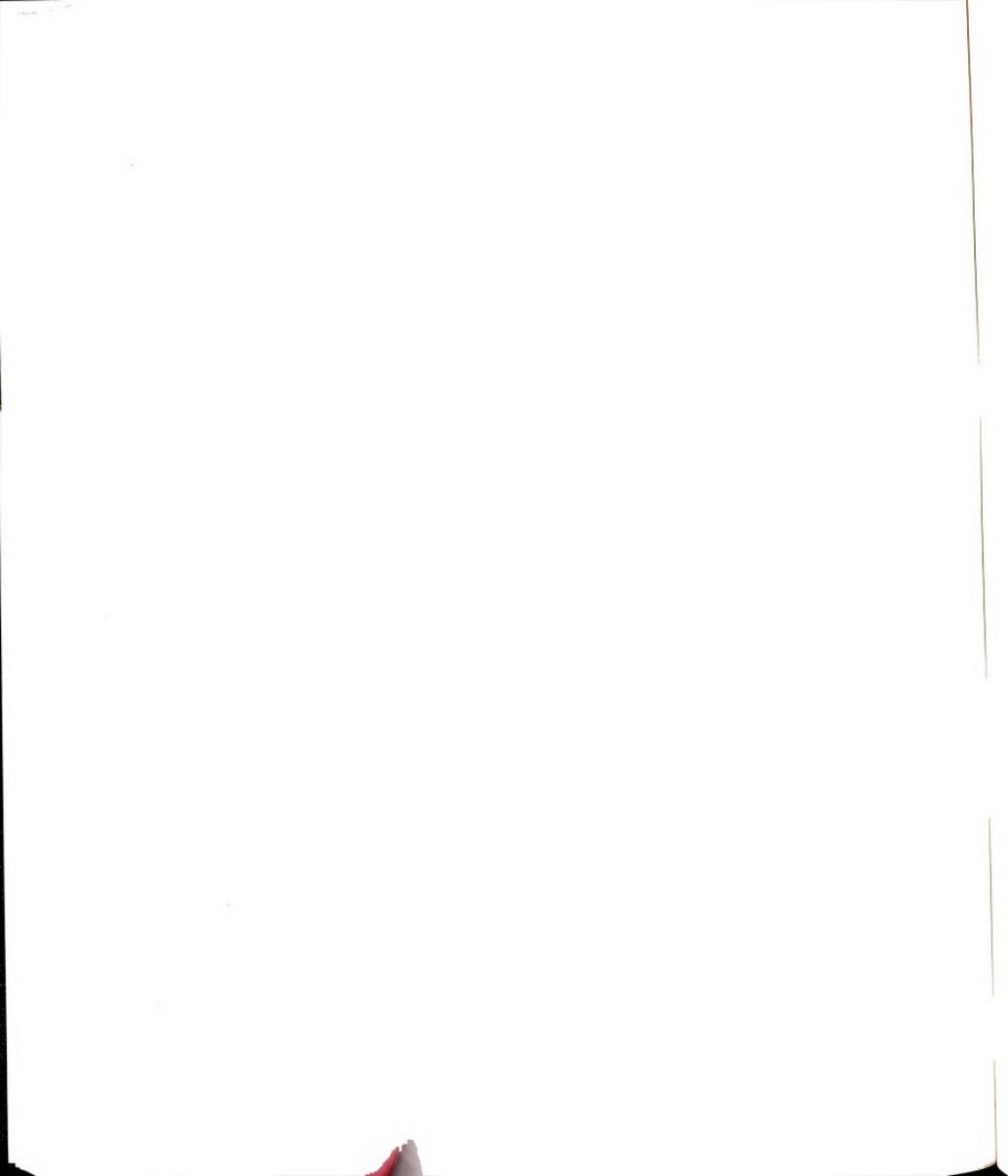
		Number of Cases	Mean	Standard Deviation	Standard Error		
Group 1		5	1.2820	1.198	.536		
Group 2		5	1.2120	1.273	.569		
		Pooled Variance Estimate			Separate Variance Estimate		
F Value	2-Tail Prob.	t Value	Degrees of Freedom	2-Tail Prob.	t Value	Degrees of Freedom	2-Tail Prob.
1.13	.910	.09	8	.931	.09	7.97	.931

**Figure 5.31 : t-Test For The Paired Difference Between The Settlements Predicted By Kriging And The Measured Values.**

The predictions by Kriging yielded a t value of 0.09. Since 0.09 is well below 2.306, then the predictions by Kriging are also consistent with the null hypothesis. The conclusion here is that the differences - either between the settlement predicted by the trend surface analysis and the measured settlements or between the settlements predicted by Kriging and the measured settlements - are insignificant. Although both of them yielded acceptable settlement predictions at the level of 0.05, the predictions by Kriging did better. This is reflected by the t value of 0.09 in the Kriging case being less than the t value of 0.21 in the trend analysis case.

The general conclusion here is that the trend surface analysis is preferred to Kriging as long as the trend is fitted with an  $R^2$  value of at least 0.8, otherwise the Kriging is recommended. Kriging more strongly reflects "local" effects which also strongly affect individual footing settlement.

The designer's predictions were also compared to the measured settlements. The computer output of the t-test appears in Figure 5.32. The results show that the difference is insignificant with a t value of 0.25 which is less than the critical value.



	Number of Cases	Mean	Standard Deviation	Standard Error
Group 1	8	1.3850	1.198	.424
Group 2	5	1.2120	1.273	.569

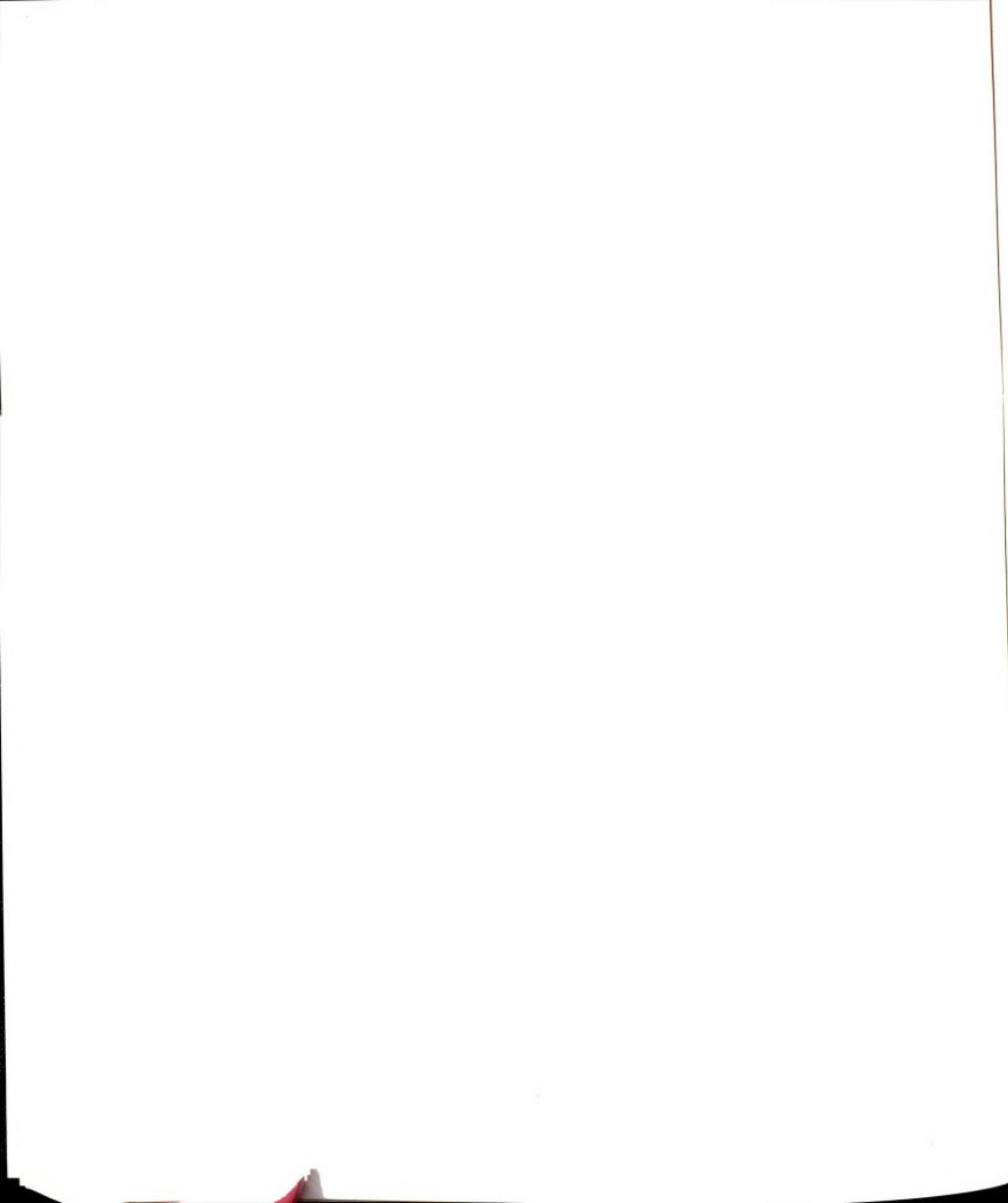
  

		Pooled Variance Estimate			Separate Variance Estimate		
F Value	2-Tail Prob.	t Value	Degrees of Freedom	2-Tail Prob.	t Value	Degrees of Freedom	2-Tail Prob.
1.13	.831	.25	11	.809	.24	8.22	.813

**Figure 5.32 : t-Test For The Paired Difference Between The Settlements Predicted By The Designers And The Measured Values.**

The examination of the results of the t-test between the designers predictions and the measured values shows that the t value was 0.25 which is more than the t values from both the trend surface and Kriging. These values were 0.21 and 0.09 respectively. This result shows the capability of the techniques proposed herein of yielding settlement predictions which are (on the average) more accurate. The conclusion here is that the proposed techniques have the following advantages over the current procedures:

1. They yield (on the average) more accurate results due to consistency in selecting the data and determining the design N value as the weighted average of the two N value estimates, obtained at the depths of B/2 and 3B/2 under the footing. This conveys information regarding both the average of N values within the zone of influence as well as the rate of increase with depth. This is tested in this section.
2. They are stronger in the sense that they provide confidence limits at the desired level of significance. This can help the foundation designer make stronger decisions about the structures supported by these foundations.
3. The upper limits of the estimated N values produced by these techniques can be used as design values to produce less conservative designs with lower costs and yet based on a rationalized criteria.



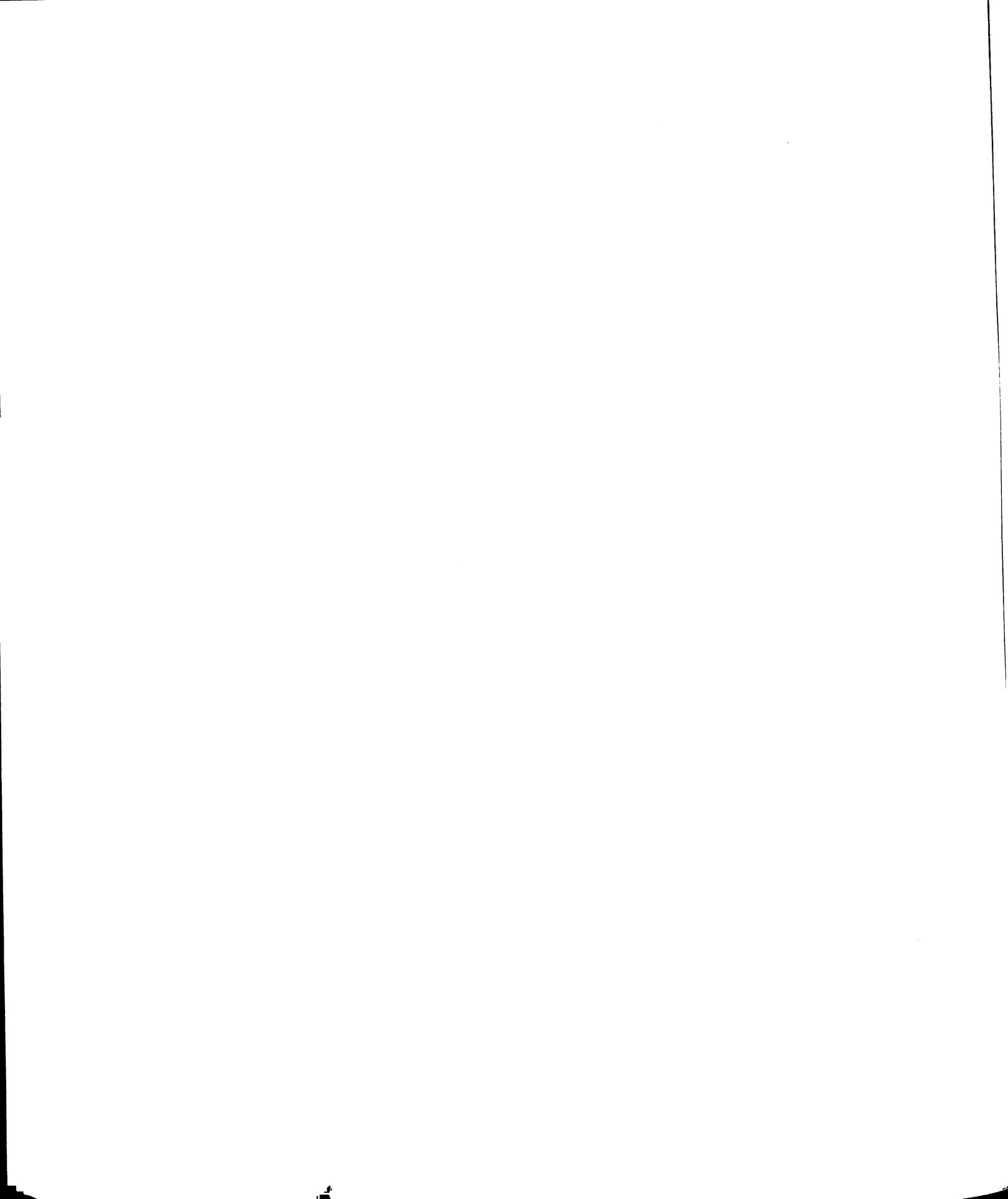
## CHAPTER 6

### SUMMARY AND CONCLUSIONS

#### 6.1 SUMMARY

Cohesionless soils generally provide good bearing capacity, and settlement usually controls the design of shallow foundations. The state-of-the-practice for design of shallow foundations on cohesionless soils is that N values obtained from the standard penetration test (SPT) are used to estimate settlements, either directly or as predictors of elastic parameters such as the soil modulus "E". A difficulty of working with N values is their inherent spatial variability. It is therefore important to be able to select an appropriate design N value that can be used with confidence with these settlement equations and parameter correlations.

Regardless of the variability of the results, the standard penetration test is not likely to be abandoned because it has remained the most convenient and economical means to obtain subsurface information in cohesionless soils. The availability of computers and software increasingly permits application of relatively sophisticated analysis tools to practical problems; accordingly it is important to develop a technique to treat the data scatter accurately and consistently. It was undertaken by this research to investigate the use of geostatistical techniques for spatial data analysis and modeling of continuous N-value functions. Geostatistical modeling was chosen because it can account for spatial variability at both the large scale (spatial trend) and the small scale (spatial correlation). Two geostatistical techniques were considered, namely trend surface analysis and Kriging. Trend surface analysis, a geostatistical version of nonlinear



regression, fits data only to large scale variations, whereas the Kriging technique ignores the trend and fits data to more localized small scale variations only.

Based on a review of factors which affect the N value (soil relative density, overburden pressure, stiffness. . etc.) as well as the intended use of the model (settlement analysis) it was determined that a correction of the N values for the overburden pressure should be made before applying the suggested modeling techniques. In keeping with current practice, the N values were accordingly corrected based on the correction equation recommended by Liao and Whitman (1986).

Some of the geostatistical approaches potentially suitable for characterizing the scatter of the soil properties were summarized and two were selected for modeling the N value function. The considered two approaches were presented and their adaptability to the modeling of the soil properties was discussed. As a single N value would still ultimately need to be extracted from such a function, the "two-point" estimate approximation was developed. This technique combines the N values within the depth of influence under the footing into one design N value based on the weighted average of two N value estimates obtained at depths  $B/2$  and  $3B/2$ . The results were shown to be comparable to the current procedures of estimating the design N value. The weighted combination derived from the "two-point" estimate permits transforming the spatial  $N(x,y,z)$  models into a planar  $N(x,y)$  model. This latter model form was used in the analysis of the case histories to do contouring analysis and the planar settlement comparisons.

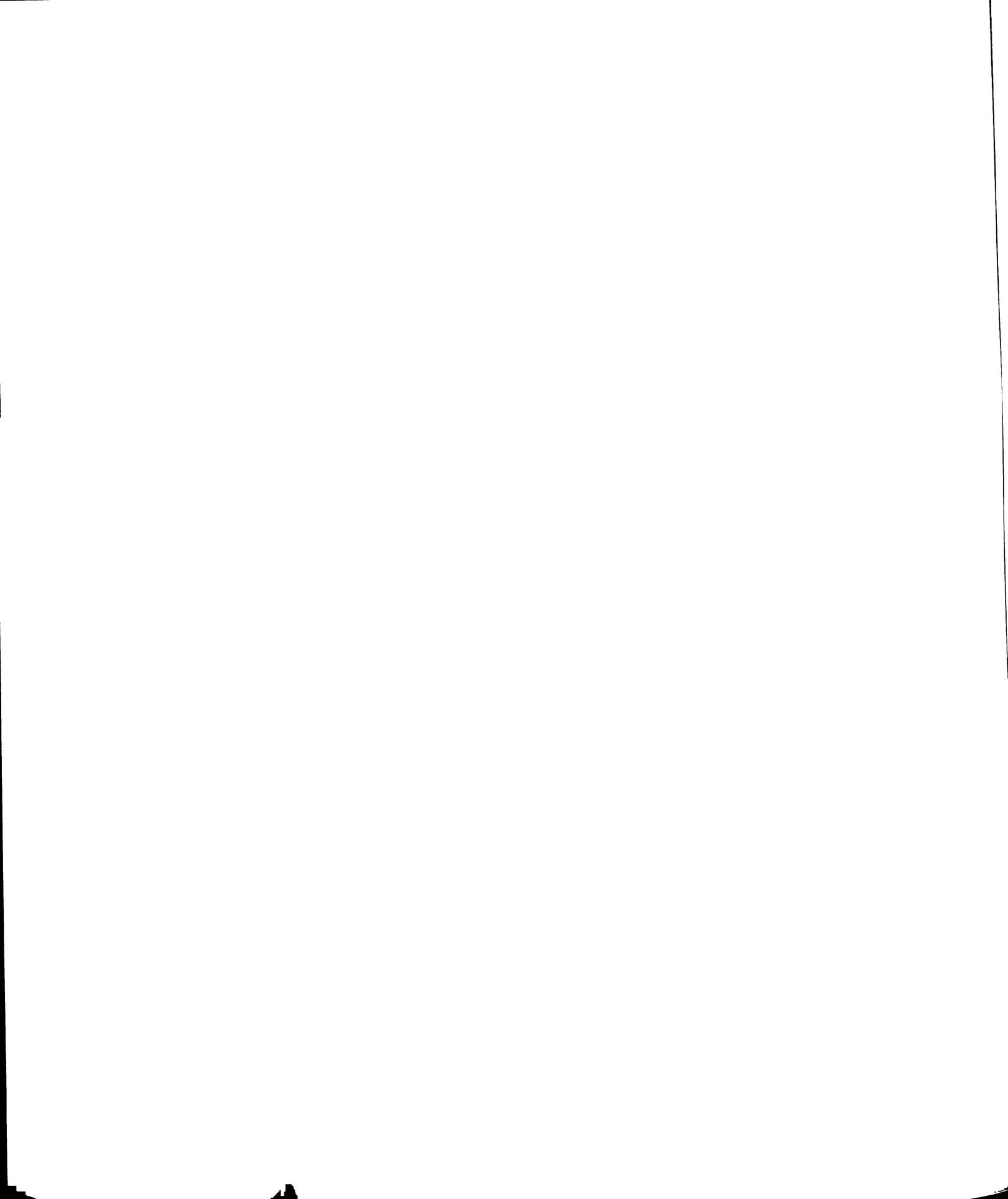
In the context of the adaptability of the trend surface analysis, it was suggested by this research to introduce an addition to it to fit the reality of soil stratification more closely. This addition was the multiple layers concept. Lack of homogeneity in the N

data in the vertical direction was accounted for by a nonconstant-mean assumption, and this was tested (later in the analysis of the case histories) by statistical multiple comparison techniques. The functional forms of the trend surfaces were approximated with polynomial expansions which use the powers of the coordinates. Five to fifteen candidate polynomial forms were tried for each case history. The polynomial providing the best fit was then determined by comparing both the ( $R^2$ ) coefficients values and the standard errors of estimate and by judging the numerical value of each term in the model within the limits of the site. The polynomial approximation was suggested to take advantage of its flexibility which can conform to very complex surfaces if expanded to sufficiently high orders; however, in most case histories, simpler lower orders polynomials were found to provide the best fit for the N values.

In the context of adapting the Kriging technique to the prediction of N values, it was decided to represent the N values of each boring by a linear regression function and perform the Kriging on the regression coefficients rather than on the N values themselves. This remedies the inconvenience that results from the lack of homogeneity in the vertical direction and transforms a 3-dimensional Kriging problem into a more manageable 2-dimensional one. The weights obtained from Kriging the regression coefficients were used to predict the regression coefficients at the point in question and ultimately the N value. The different forms of the covariance functions which describe the spatial continuity of the data were reviewed. The squared exponential model was used later in the analysis of the case histories to build the covariance matrix which was used for Kriging.

The determination of the model precision was discussed and an evaluation of the quality of prediction versus the quantity of the data and their monetary costs were considered. This was achieved by considering the trade off between the number and location of data points and the confidence intervals of the resulting model. It was shown that the confidence band of the estimated N value is proportional to  $(1/n)^{0.5}$  in which "n" is the number of N values.

Finally, the developed models for settlement prediction were evaluated two ways. The first evaluation was made using simulated data of an assumed field. Given an assumed N function, the models were used to fit some data sampled from this function and to estimate the N values at group of unsampled points. The "estimated unobserved" values were then compared to the "true unobserved" ones. The results of this evaluation were consistent and showed that the developed methods can be used systematically with confidence conforming with the quality of the available data. Remaining in question was the effect of real and scattered data of a real site. These results ascertained also what was assumed initially regarding the superiority of using the trend surface analysis method when the data follow an underlying trend. The power of using the Kriging technique to handle the data with high degrees of randomness was also ascertained. The second method of evaluation was to test the practical reliability of the developed models by conducting the suggested modeling on a set of actual case histories. The variability of the real N values in the practical application to a real site may not be as ideal as that of a theoretical field. However, the variability of N values of the six tried case histories was found to be representable by equations whose X, Y and Z terms are not exceeding the second order. Consequently, it was easy to try out all the possible combinations and to



reach an acceptable goodness of fit. The resulting settlement predictions were compared with the measured settlements using t-test of zero mean on the paired differences. The results of these verifications confirmed the reliability of the developed techniques. The outputs included stronger inferences regarding the settlement predictions in the sense that they estimate the settlements and provide confidence limits at the desired level of confidence. This can help the foundation designer make stronger decisions about the structures supported by these foundations.

## **6.2 DETAILED SUMMARY OF RECOMMENDED PROCEDURES**

Two statistical methods were developed for settlement predictions from results of standard penetration test, one using the trend surface analysis and the other using the Kriging technique. A summary outline of these methods follows:

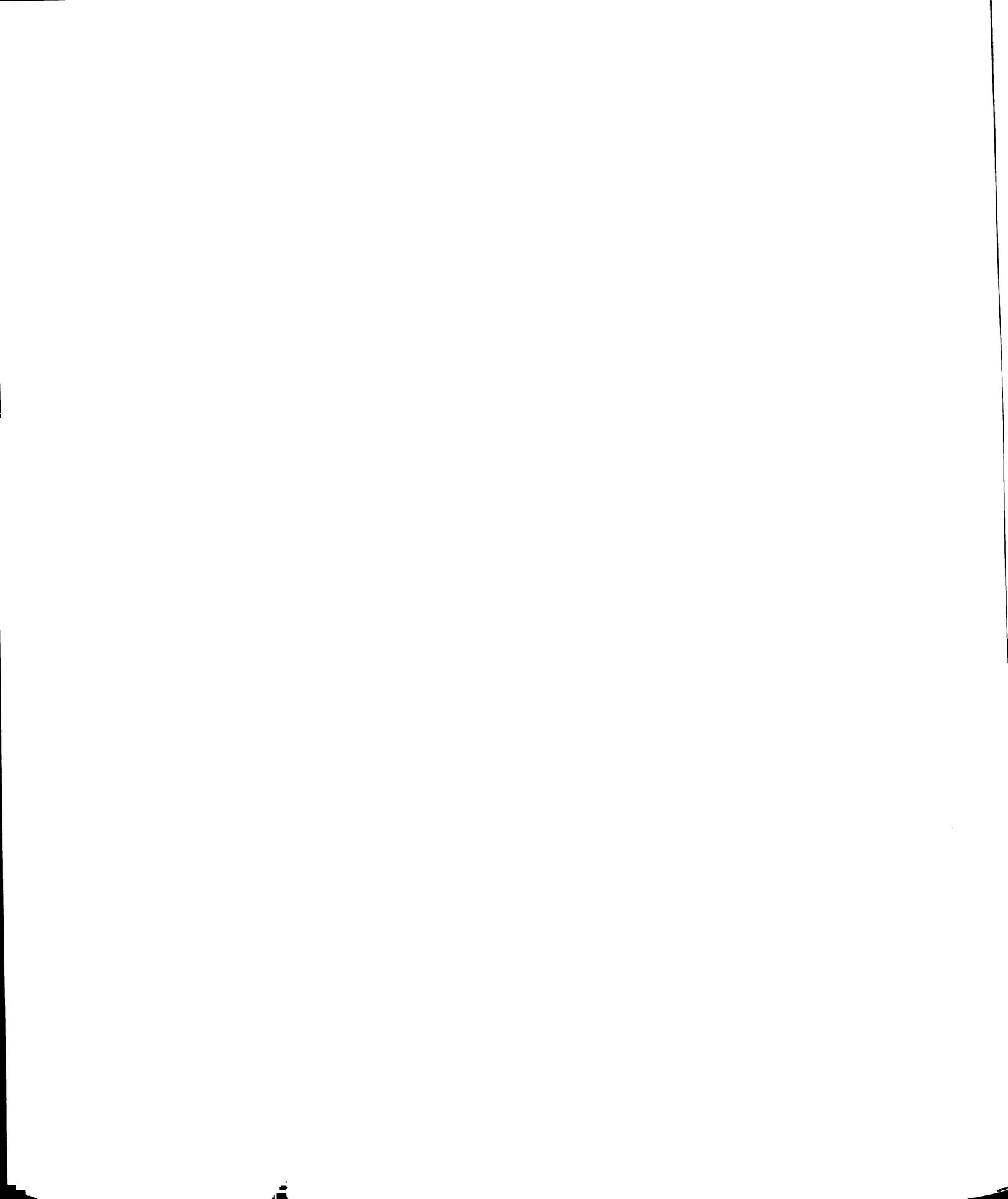
### **The outline of the trend surface analysis method:**

1. Correct N values for overburden pressure.
2. Stratify soil into layers reflecting the homogeneity (or lack thereof) in N in the vertical direction.
3. Test the stratification by statistical multiple comparison techniques (LSD, Tukey, and Dunn tests).
4. Define a coordinate system.
5. Propose a number of candidate nonlinear three dimensional models to the N values within each layer. Evaluate their ( $R^2$ ) and standard error values to ascertain

that the models have acceptable prediction capabilities. Investigate the physical shape to ensure acceptable representation.

6. Use the "two-point" estimate to transform the selected  $N(x,y,z)$  model into a planar  $N(x,y)$  model for the design  $N$  value, and assess the confidence limits of this model using a (50%) confidence level.
7. Use the model  $N(x,y)$  in conjunction with Bazaraa's settlement equation to produce a planar  $S(x,y)$  model.
8. Use the  $S(x,y)$  model for contouring expected settlements of replicate foundations in  $(x,y)$  as well as for assessing the quality of prediction.

The above procedure is summarized in Figure 6.1.



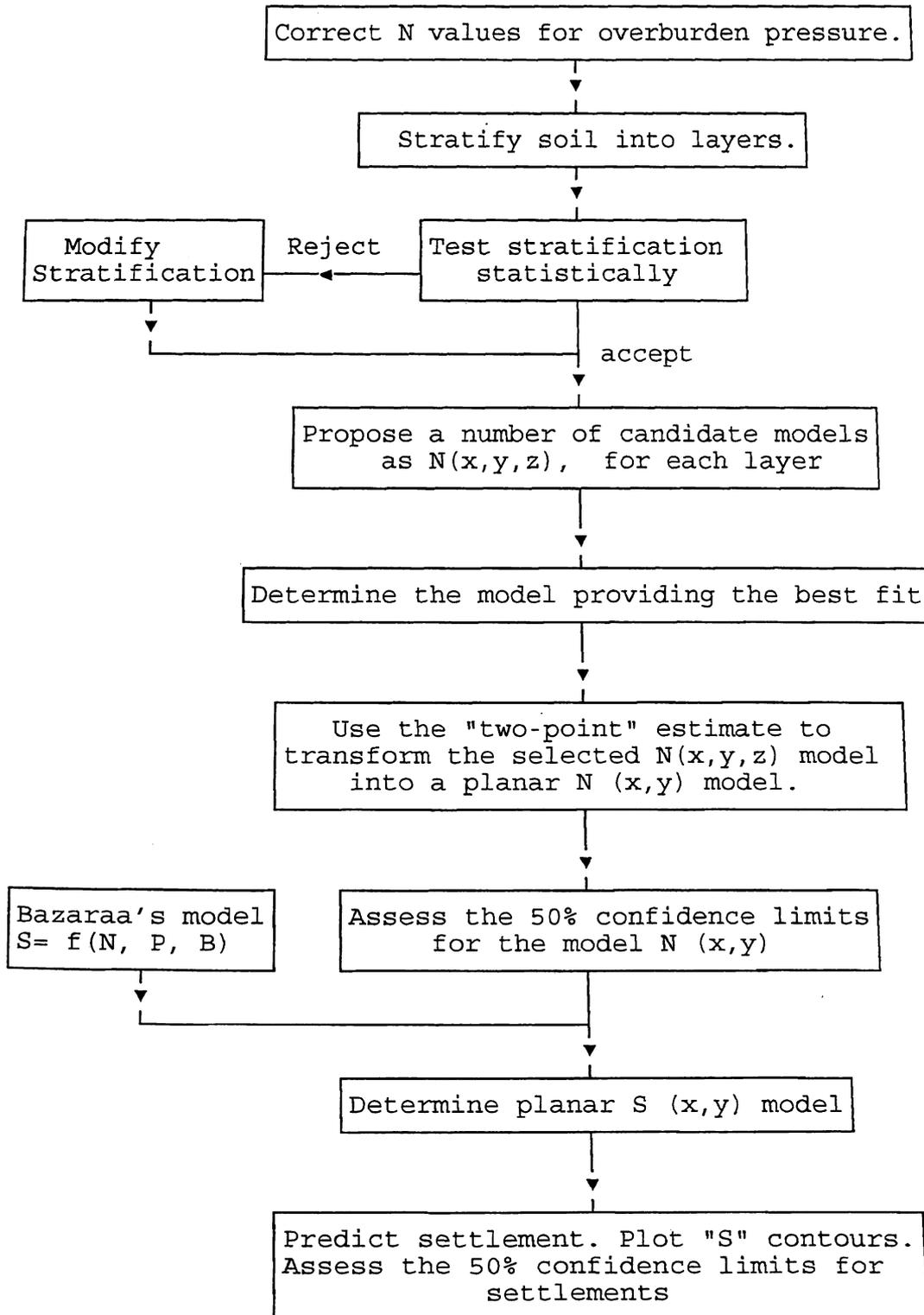


Figure 6.1: Flowchart Of The Trend Surface Method.

**The outline of the Kriging method:**

1. Correct N values for overburden pressure.
2. Stratify soil into layers. The Kriging method is only recommend for single strata.
3. Fit N values of each boring to a linear equation as  $(N = a + bz)$  using regression.
4. Compute the covariance matrix (C) describing the spatial continuity of the data using either direct calculations (if enough data are available) or a squared exponential model as :

$$C(h) = \sigma_N^2 e^{-(h^2/h_0^2)}$$

5. Compute the covariance vector (D).
6. Calculate the weight matrix (W) as  $(W = C^{-1}.D)$ .
7. Determine the estimated N function at the point in question as a linear regression function with parameters obtained as the weighted sum of the regression parameters of the considered borings.
8. Use the "two-point" estimate with the  $N(z)_{x,y}$  to predict  $N(x,y)$ .
9. Use the predicted design N value in conjunction with Bazaraa's model to estimate the settlement.
10. Assess the quality of prediction by constructing the confidence interval to the level of (50%) using the t-statistic and the prediction variance which is given by:

$$\sigma_R^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j C_{ij} - 2 \sum_{i=1}^n w_i C_{io} + \sigma^2$$

The above procedure is summarized in Figure 6.2.

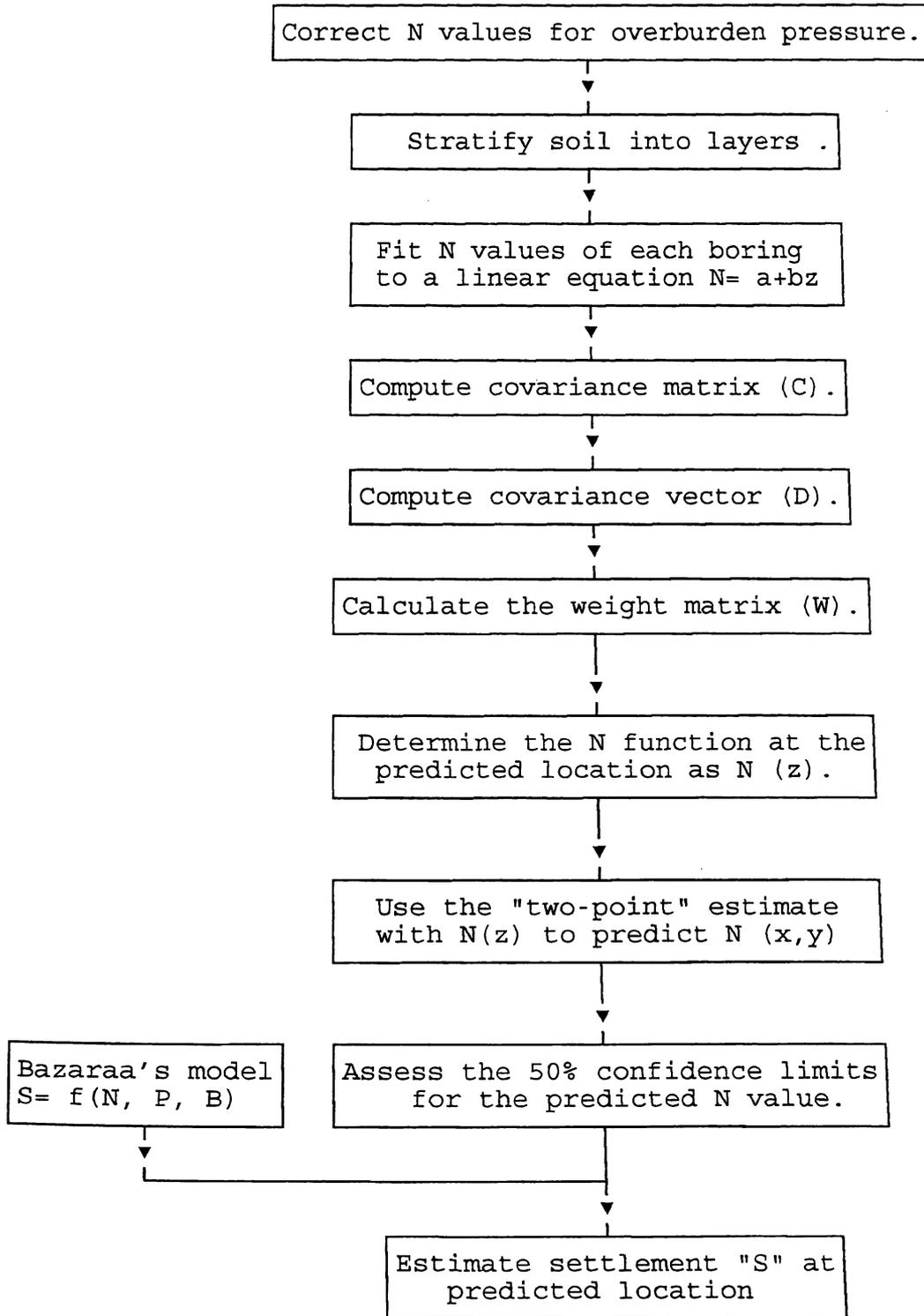


Figure 6.2: Flowchart Of The Kriging Method.

### 6.3 CONCLUSIONS

The following conclusions can be made concerning the findings in this study:

**1. Applicability Of Geostatistics:**

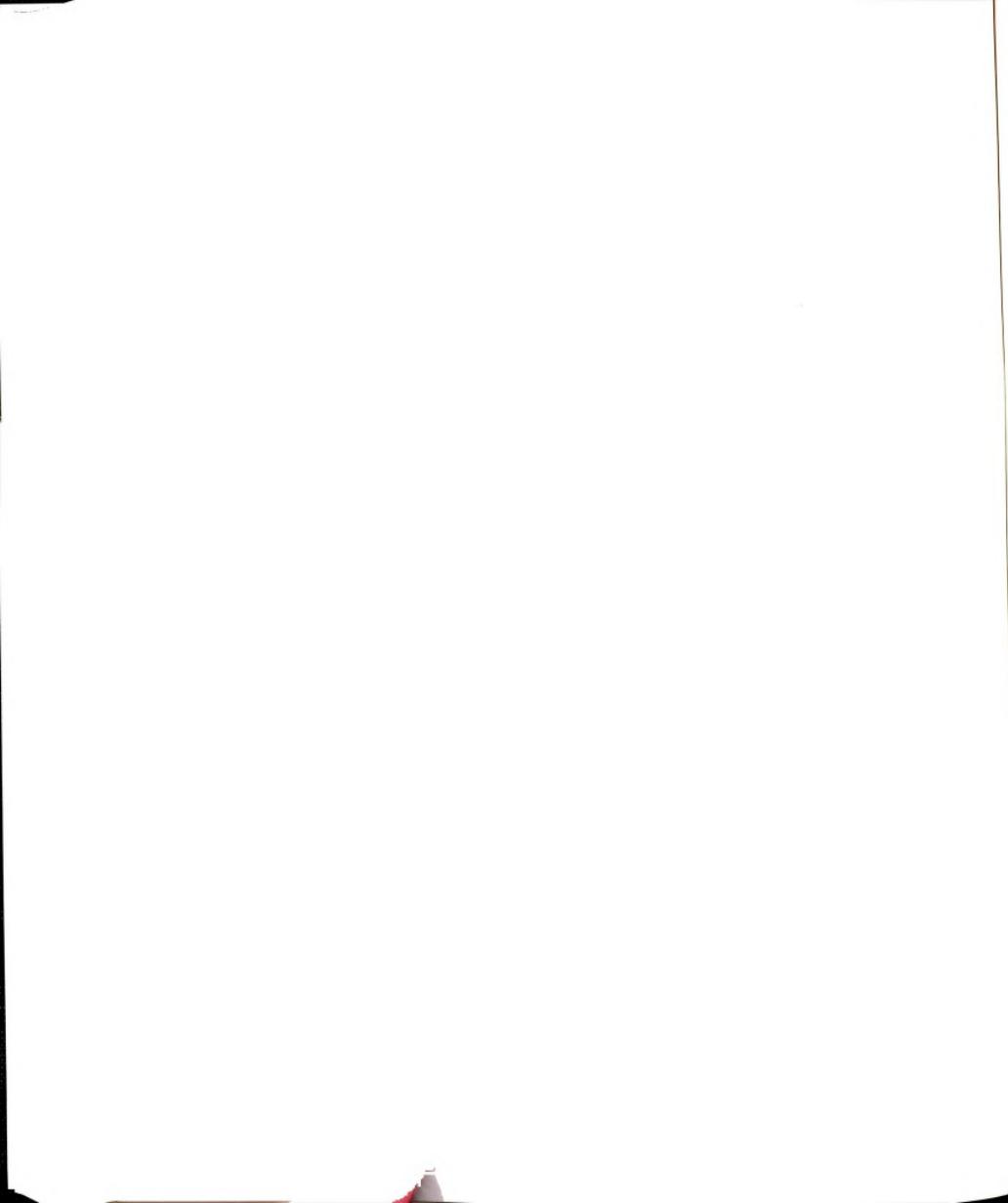
A general conclusion can be drawn from this research that geostatistics can be a powerful tool to extract a representative function from among spatially scattered N values and to provide a rational estimate of N value where no measurements were taken.

**2. Use Of Trend Surface:**

The trend surface analysis does not create a trend by itself and its use can be justified only if a large scale trend in the N data is in fact present. In this case the trend surface analysis is employed to smooth out the random variation that enters into the data and to identify the underlying trend in order to use it for prediction. A fitted surface to a data scatter with an ( $R^2$ ) value of less than 0.2 would lead only to misleading results. A value of  $> 0.8$  would imply confidence in the fitted model. For practical problems, however, it was possible to obtain  $R^2$  values of about 0.5 - 0.6.

**3. Candidate Models For Trend Surface:**

Where the trend surface analysis is justified (e.g.,  $R^2 > 0.2$ ), then it is important to investigate the variability of the data carefully, by trying out some preliminary models, and select a model which can adequately represent this variability.



**4. Use Of Kriging:**

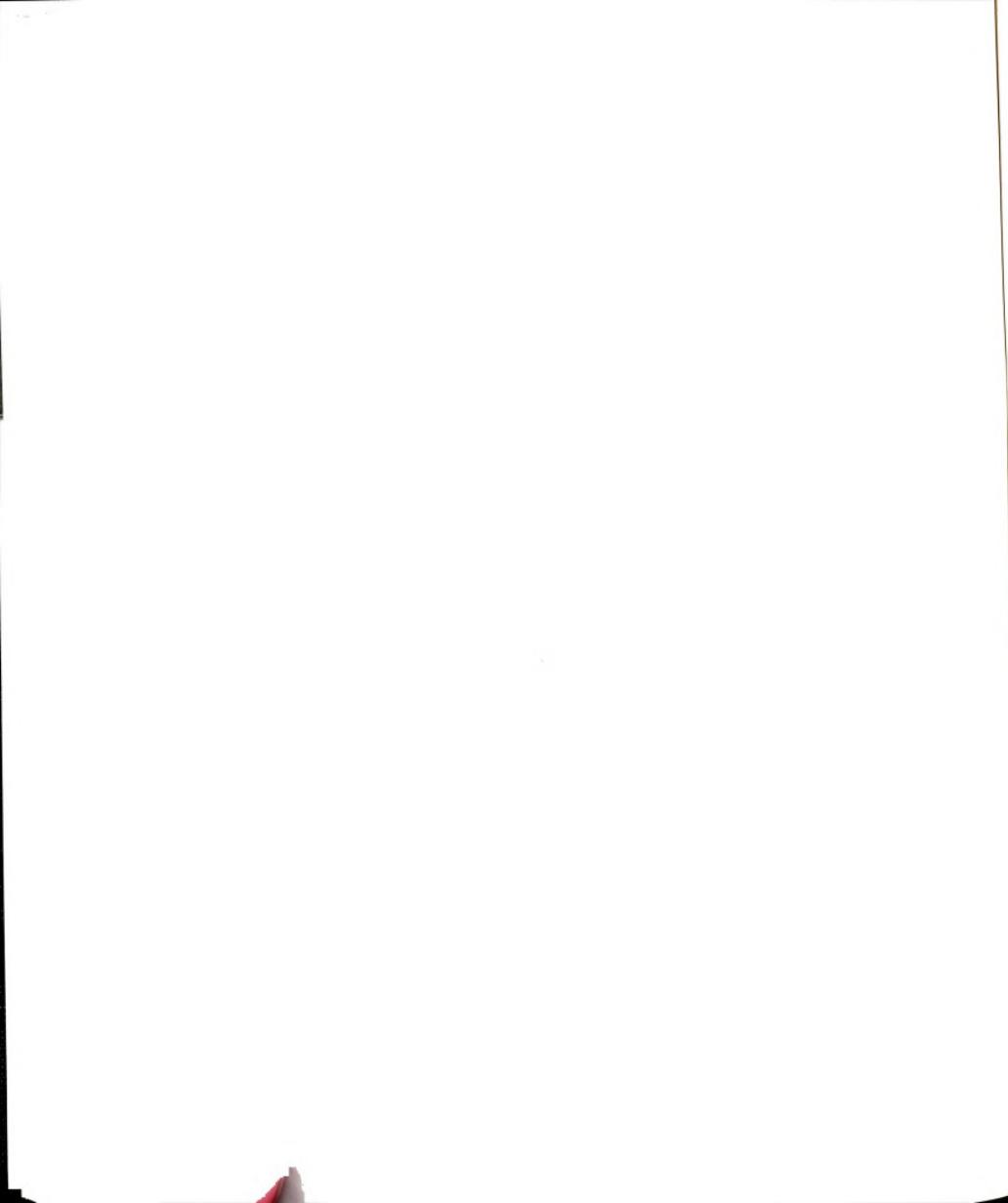
The Kriging technique was found to be superior to the trend surface analysis if a certain degree of randomness is present. In this case, the N value at a given location is assumed to have no influence on the N values at distant locations. If the data follow a trend, then Kriging can still be used to find a point on the trend provided that the employed data are located in the vicinity of that point in order to justify ignoring the trend.

**5. Quality Of Data:**

The precision of the settlement prediction depends on both the predicting model qualifications and the data qualifications. If the data qualifications are inferior (e.g. a subsoil investigation based on data from a single boring), then neither the trend surface nor the Kriging technique can produce reliable predictions. The inferior quality of data could also result from using an unreliable drilling equipments or inexperienced crew.

**6. Prediction Confidence Intervals:**

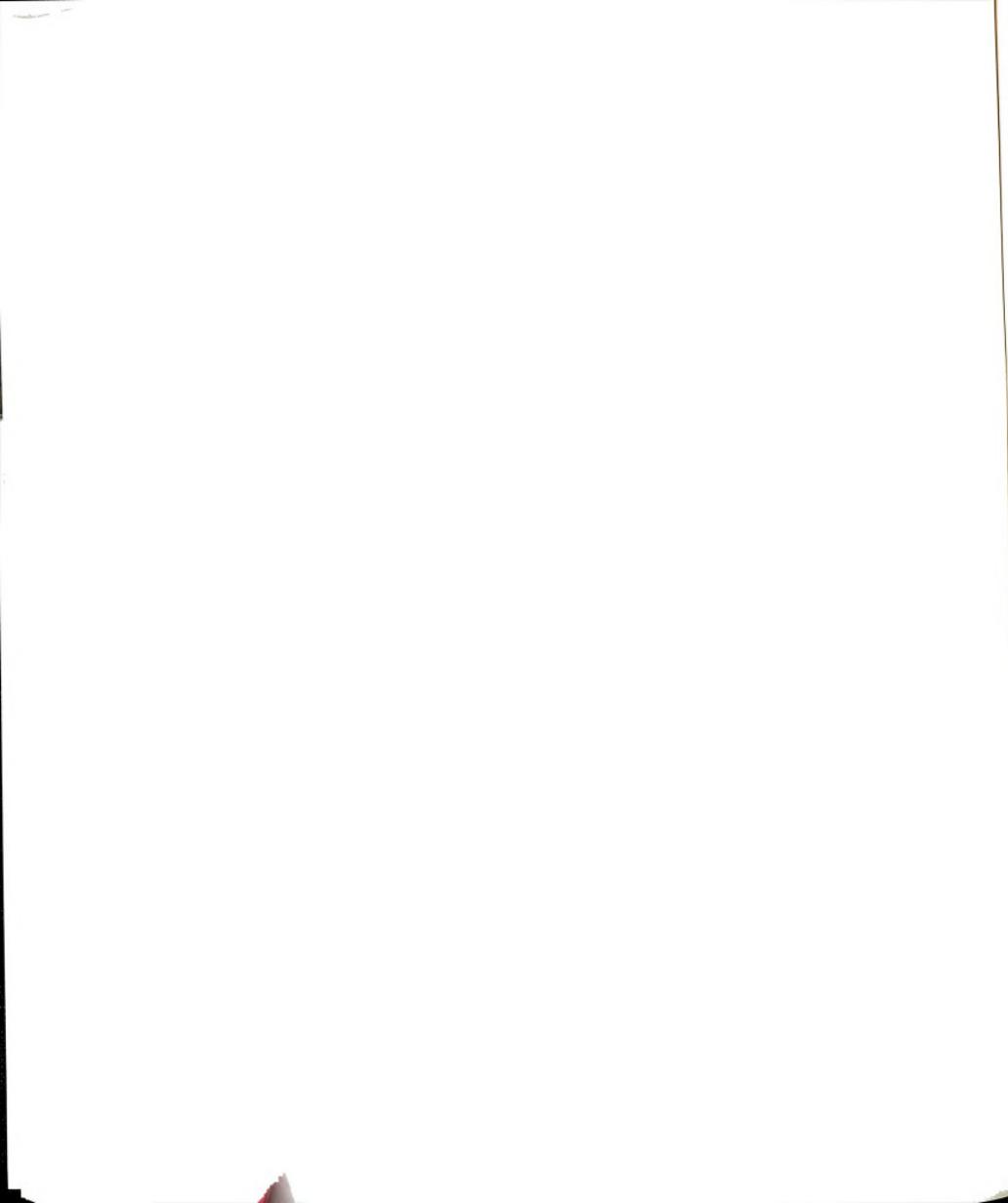
In constructing the prediction confidence intervals for practical design problems, it might be considered reasonable to adopt a degree of confidence of 50% rather than the conventional levels used in statistical inference of 90 or 95%. The level of 50% inference is more practical with the N value data due to the higher variability encountered. Where 90% or greater is used, the variance of N values resulted in a relatively broad confidence intervals even though the predicted



values were close to the measured values. This suggests that the construction of broad confidence intervals is meaningless.

**7. Deviation Of The Predicted Settlements From The Measured Values:**

Using trend surface analysis, the average of the ratio of predicted to measured settlements of the considered six case histories was found to be 1.28. Using the Kriging technique, this ratio was found to be 1.37. This deviation is believed to result from employing Bazaraa's settlement equation in conjunction with the estimated  $N$  values to predict the settlements. Bazaraa's equation (1969) is a refinement of Meyerhof's equation (1965) and this in turn is an analytical expression of Terzaghi and Peck's well-known settlement design chart. These settlement equations have an empirical nature and hence have the common disadvantage of all empirical models of being applicable ideally only to the conditions where they were developed from. These conditions include the sand gradation and top size, the particle shapes and angularities, the relative densities . . . etc. In any other case, it is unlikely to have these conditions of soil properties identical to the properties of the soils used to develop these settlement functions in the first place, the thing which result in these deviations upon the use of these settlement equations. One more reason for the deviation of the predicted settlement from the measured value is that the design loads are often not realized.

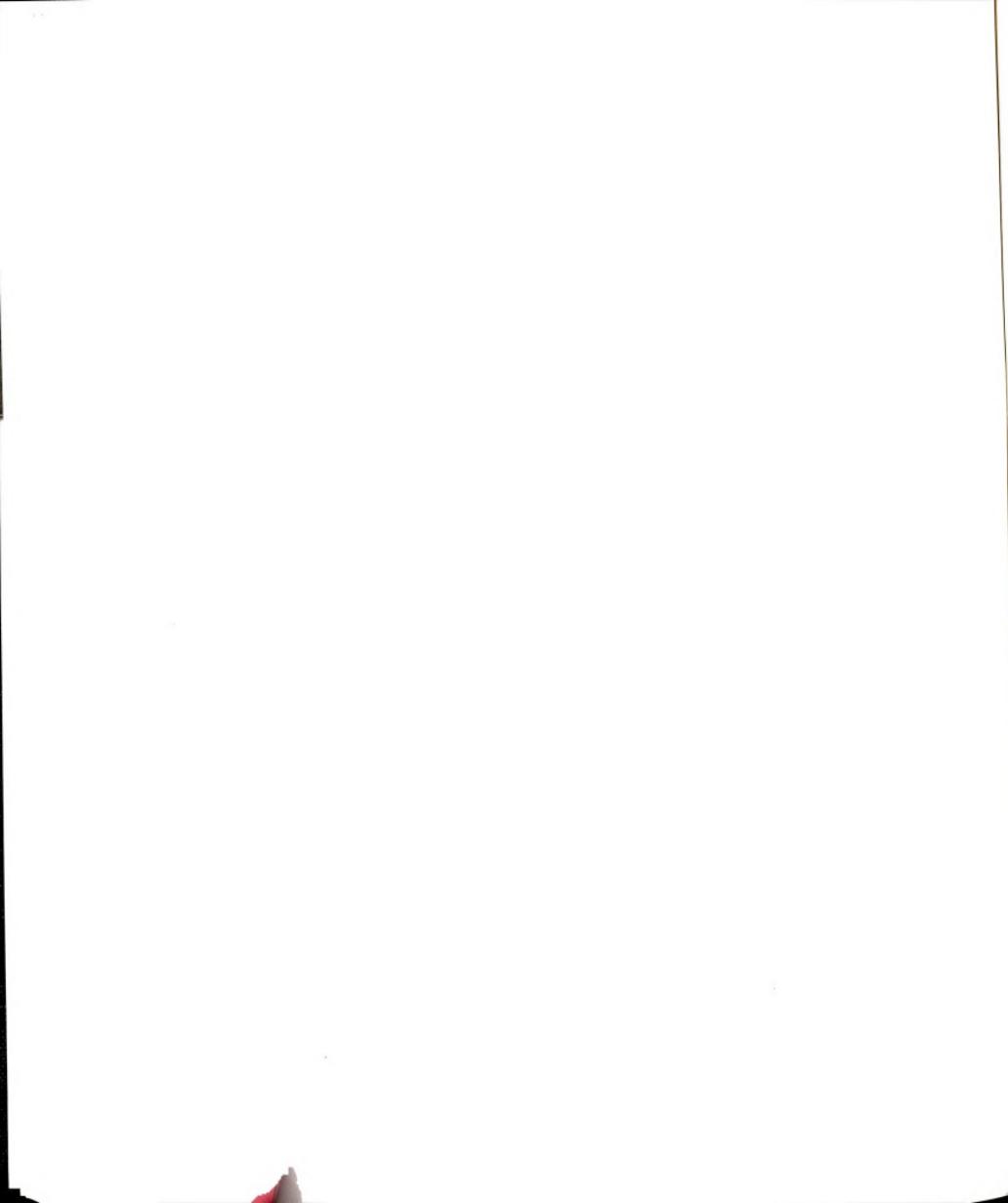


**8. The Depth Of Influence Under A Footing:**

Investigating the results of the case histories has also raised an important issue regarding the depth of influence within which the settlements are computed and below which the soil compression is left out of consideration. If the subsoil is in the form of a shallow loose layer laying on a much firmer thick deposit, and if the footing width is relatively large, then the assumption that the depth of influence is  $(2B)$  may give unjustifiedly low settlement predictions because the upper layer which contributes most of the compressibility will not be given a realistic weight during the settlement computations. Using the Schmertmann's method, the strain influence factor implies the weights that are given to the  $N$  values at different depths. The distribution of the strain influence factor with depth was based on a theoretical work assuming that the modulus  $E$  is constant with depth. Consequently, the depth of influence of  $2B$  as well as the values of the strain influence factor at different depths will all be confused if the modulus  $E$  increased rapidly in the vertical direction. In other foundation cases where a large number of footings are closely located, they will have some additional "overlap" effect extending the depth of influence to greater than  $(2B)$ .

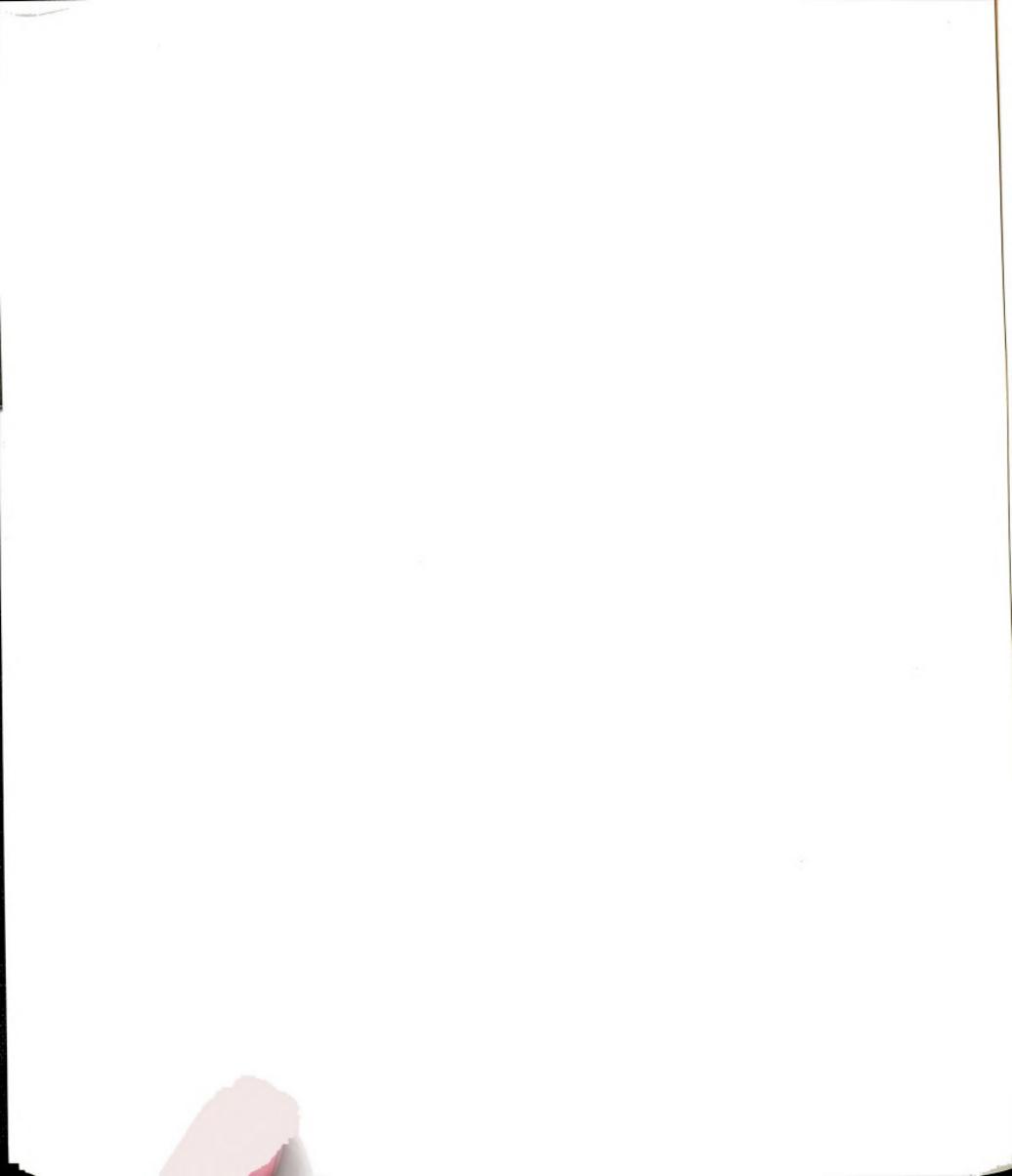
**9. Summary Table Of The Two Methods:**

The advantages, disadvantages and uses of the two considered methods are summarized in Table 6.1.



**Table 6.1: Summary Table Of The Two Considered Methods.**

Method	Trend Surface Analysis.	Kriging.
Advantages	<ol style="list-style-type: none"> <li>1. Most useful if a significant trend is present in the data.</li> <li>2. It is a geostatistical version of the nonlinear regression which has been studied thoroughly and its theoretical background is well established in the literature.</li> </ol>	<ol style="list-style-type: none"> <li>1. Used where the data are erratic with a certain degree of randomness.</li> <li>2. Applicable to almost any N value data.</li> <li>3. Predicted better than trend surface when modeling the N value data to predict settlements.</li> </ol>
Disadvantages	<ol style="list-style-type: none"> <li>1. The process of selecting the model providing the best fit is time consuming.</li> <li>2. May lead in the absence of a significant trend to misleading results.</li> </ol>	<ol style="list-style-type: none"> <li>1. Computational effort is relatively lengthy.</li> <li>2. Constructing the covariances between the different locations is laborious.</li> </ol>
Required	<ol style="list-style-type: none"> <li>1. Where large scale variations occur with an underlying trend.</li> <li>2. Where it is desired to emphasize the trend over local fit.</li> </ol>	<ol style="list-style-type: none"> <li>1. If there is enough confidence in data to wish to emphasize local fit over trend.</li> <li>2. For fitting data to small scale variations</li> </ol>



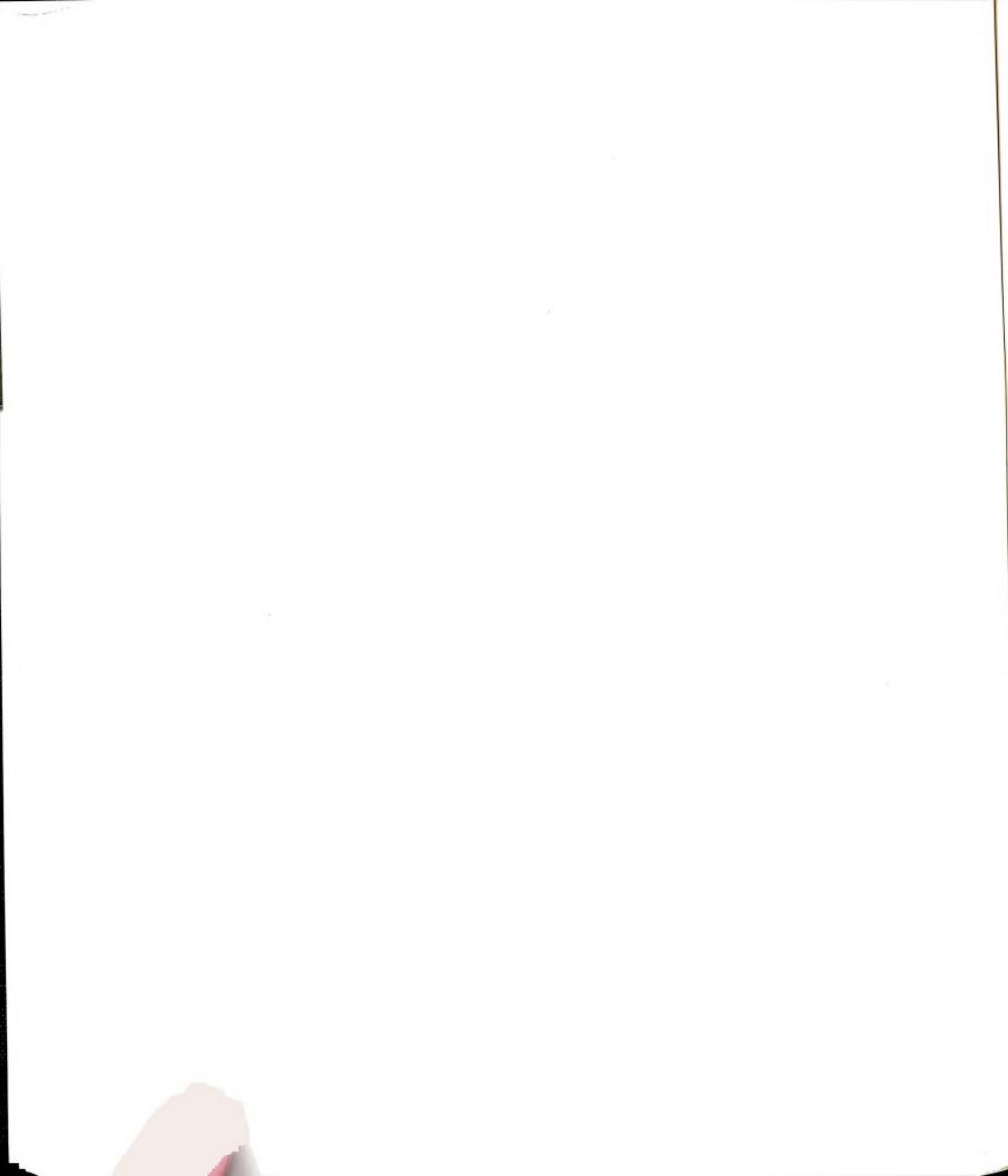
## 6.4 RECOMMENDATIONS

In improving the settlement prediction techniques many aspects are involved. This research has considered the treatment of the scatter of N data and assessing the quality of prediction. As far as a comprehensive improved settlement prediction technique is concerned, it is important to consider the following recommendations:

1. There is no point in using sophisticated statistical modeling for settlement prediction while employing data of inferior quality. Therefore, it is recommended to base the settlement analysis on the results of at least four borings. Additionally, unreliable drilling equipment and inexperienced crew are obviously to be avoided.
2. The data base regarding the covariance functions representing the variability of N values, which are used for Kriging, is not yet well established. Improvement is therefore required regarding the suitable functional forms of covariance functions to be used for cohesionless soils with varying properties. This can be done by testing different soils and establishing correlations including the N values, the distances between the tested locations and the best fits of the covariance functions.
3. In preparation of N data before Kriging, the N values could, in some cases, be representable by a nonlinear regression function. In these cases, it is recommended to represent the N values of each boring by a regression function of the form:

$$N = a_0 + a_1 Z^{0.5} + a_2 Z + a_3 Z^2 + \dots$$

The same functional form is used for all borings. The parameters  $a_0, a_1, a_2 \dots$  are then Kriged in two dimensions. The estimated N function at the point in question is then obtained as a nonlinear regression function in the same form with



parameters obtained as the weighted sum of the regression parameters of the considered borings.

4. Only by comparing the observed settlements with predicted values and by evaluating measurements can a development of new settlement functions and refinement of their accuracy be achieved. It is, therefore, urged to set up a settlement-recording program right from the beginning for any structure of major importance or having unusual foundation conditions.
5. Using trend surface analysis, the average of the ratio of predicted to measured settlements of the considered six case histories was found to be 1.28. Using the Kriging, this ratio was found to be 1.37. The reasons of this deviation were analyzed in Section 6.3. These two ratios imply the overestimation of the predicted settlements in both cases. To make up for this overestimation, the following recommendation is suggested. If a level of confidence of 50% is adopted, then it is considered reasonable to use the upper confidence limit of the  $N$  value as a design value if need be. Using the upper confidence limit of  $N$  value means adopting higher bearing capacity and greater allowable pressures. This leads to smaller footing sizes with reduced costs. In other words, less conservative designs are produced for lower costs and yet based on a rationalized criteria.
6. Taking the depth of influence as  $(2B)$  under a footing of width  $(B)$  could be incorrect (e.g. where the subsoil is in the form of shallow loose layer laying on a much firmer thick deposit and the footing width is relatively big). The depth of influence of  $(2B)$  should therefore be questioned if need be to include only the layer that contributes most of the compressibility.

**APPENDICES**

**APPENDIX A**

## APPENDIX A

### A. SUBSURFACE SOIL STRATIFICATION OF CASE HISTORY No.1

#### A.1 Correction of N values for the overburden pressure:

The corrected N value is given by:

$$N_1 = C_1 * N$$

Where

$$C_1 = (1/p_v')^{0.5} \quad ; \quad (\text{Liao, 1986}).$$

$p_v'$  = effective overburden pressure (tsf) = depth in (ft) \* unit weight (lb/ft<sup>3</sup>)/2000

The correction results for the different borings are shown in Tables A.1 to A.6.

**Table A.1 : Correction Of N Values Of Boring No. (B-102) For Overburden Pressure.**

Depth (ft)	Unit wt. (lb/cf)	Effective Pressure (lb/sf)	Correction factor (C1)	N	N1=C1*N
2.0	125.0	250.0	2.828	8	22.627
4.0	125.0	500.0	2.000	11	22.000
7.5	125.0	937.5	1.460	13	18.987
10.0	125.0	1250.0	1.264	13	16.443
17.0	125.0	2125.0	0.970	17	16.492
21.0	125.0	2625.0	0.872	15	13.093
26.0	125.0	3250.0	0.784	22	17.258
31.0	125.0	3875.0	0.718	35	25.144

**Table A.1 : Continued.**

36.0	62.6	4187.8	0.691	47	32.480
41.0	62.6	4500.8	0.666	79	52.661

**Table A.2 : Correction Of N Values Of Boring No. (B-105) For Overburden Pressure.**

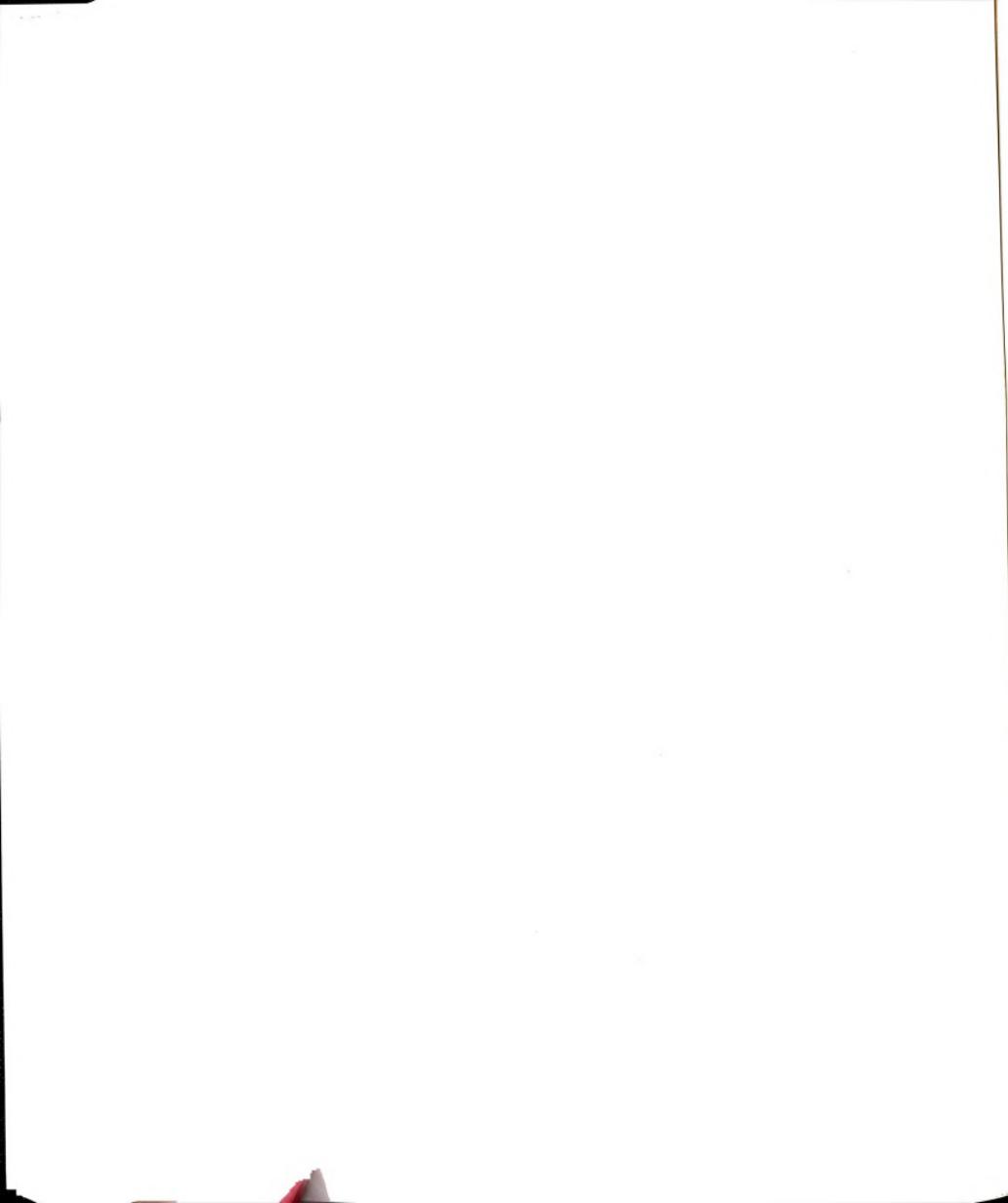
Depth (ft)	Unit wt. (lb/cf)	Effective Pressure (lb/sf)	Correction factor (C1)	N	N1=C1*N
2.5	125.0	312.5	2.529	13	32.887
5.0	125.0	625.0	1.788	17	30.410
8.0	125.0	1000.0	1.414	18	25.455
10.0	125.0	1250.0	1.264	13	16.443
15.0	125.0	1875.0	1.032	7	7.229
20.0	125.0	2500.0	0.894	5	4.472
25.0	125.0	3125.0	0.800	7	5.600
30.0	125.0	3750.0	0.730	6	4.381
35.0	62.6	4250.2	0.685	7	4.801
40.0	62.6	4563.2	0.662	32	21.185
45.0	62.6	4876.2	0.640	48	30.740
50.0	62.6	5189.2	0.620	37	22.970

**Table A.3 : Correction Of N Values Of Boring No. (B-106) For Overburden Pressure.**

Depth (ft)	Unit wt. (lb/cf)	Effective Pressure (lb/sf)	Correction factor (C1)	N	N1=C1*N
2.0	125.0	250.0	2.828	8	22.627
5.0	125.0	625.0	1.788	29	51.876
8.0	125.0	1000.0	1.414	32	45.254
10.0	125.0	1250.0	1.264	14	17.708
15.0	125.0	1875.0	1.032	15	15.491
19.5	125.0	2437.5	0.905	18	16.304
25.0	62.6	3000.2	0.816	42	34.291
30.0	62.6	3313.2	0.776	38	29.523

**Table A.4 : Correction Of N Values Of Boring No. (B-109) For Overburden Pressure.**

Depth (ft)	Unit wt. (lb/cf)	Effective Pressure (lb/sf)	Correction factor (C1)	N	N1=C1*N
3.0	125.0	375.0	2.309	10	23.094
4.0	125.0	500.0	2.000	13	26.000
6.5	125.0	812.5	1.568	12	18.827
9.0	125.0	1125.0	1.333	12	16.000
14.0	125.0	1750.0	1.069	13	13.897
19.0	125.0	2375.0	0.917	14	12.847

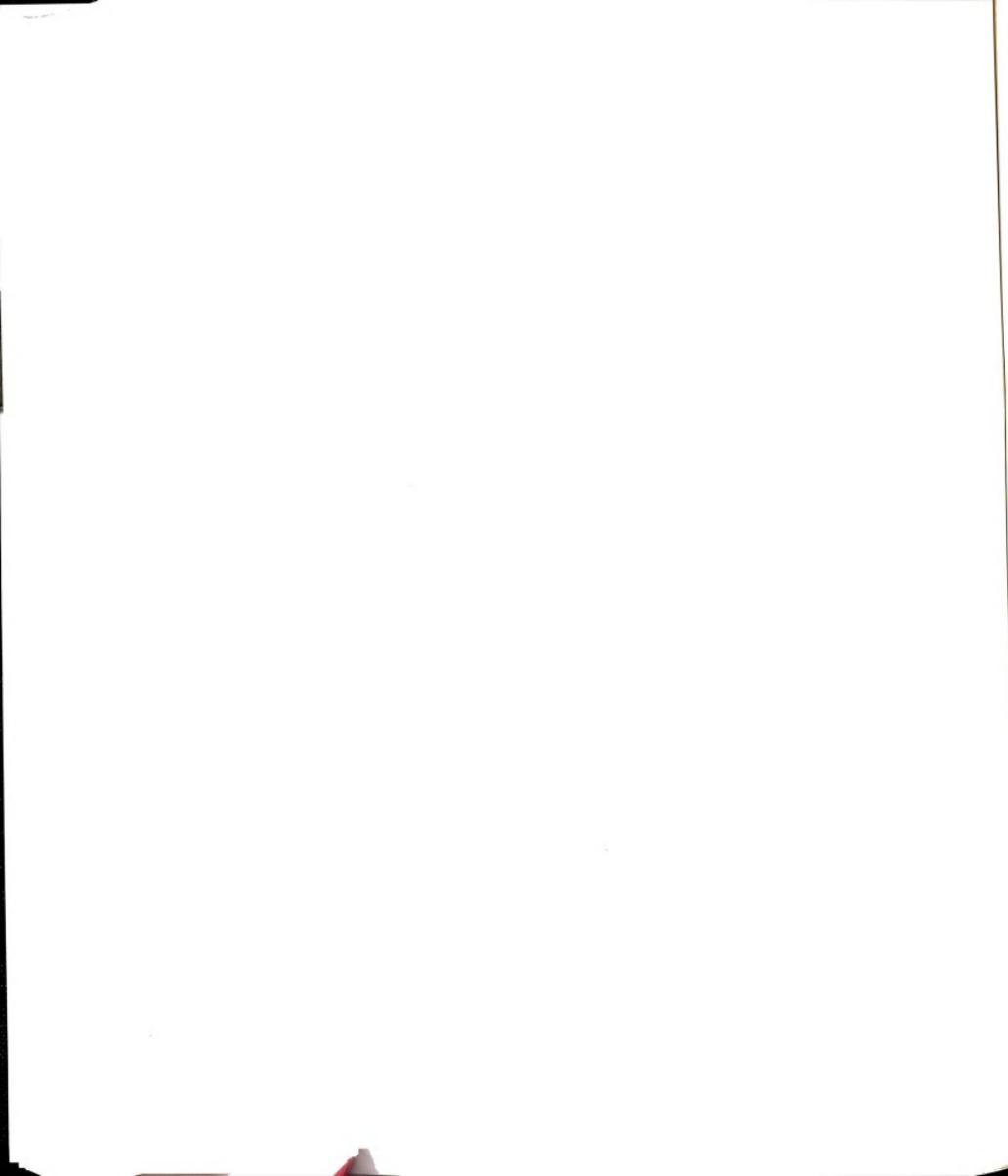


**Table A.4 : Continued.**

23.5	125.0	2937.5	0.825	10	8.251
28.5	62.6	3406.5	0.766	16	12.259
33.0	62.6	3688.2	0.736	38	27.982

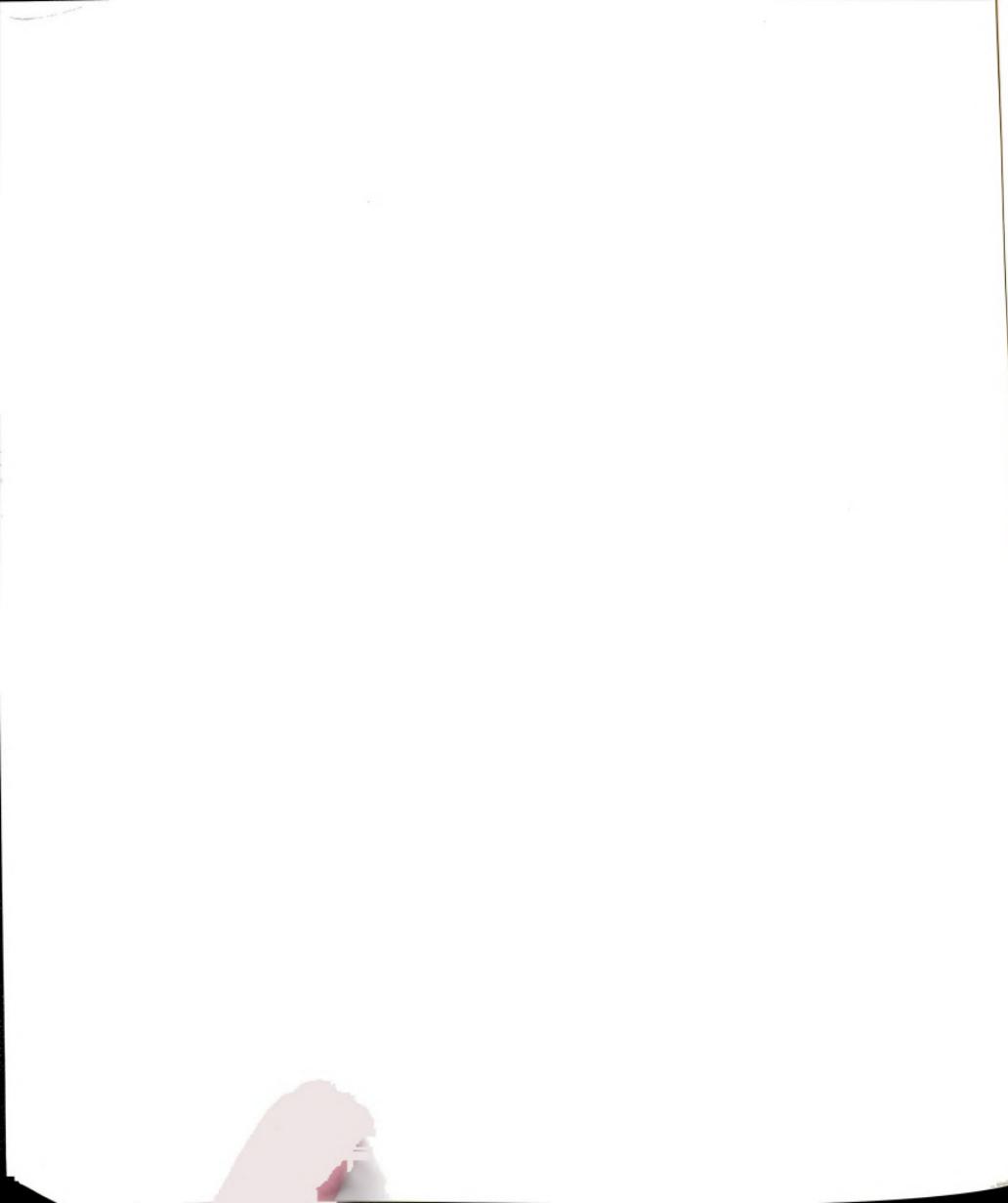
**Table A.5 : Correction Of N Values Of Boring No. (B-2) For Overburden Pressure.**

Depth (ft)	Unit wt. (lb/cf)	Effective Pressure (lb/sf)	Correction factor (C1)	N	N1=C1*N
2.0	125.0	250.0	2.828	8	18.475
5.0	125.0	625.0	1.788	11	19.677
8.0	125.0	1000.0	1.414	12	16.971
10.0	125.0	1250.0	1.264	15	18.974
15.0	125.0	1875.0	1.032	12	12.829
19.5	125.0	2437.5	0.905	14	12.522
25.0	62.6	3000.2	0.816	20	16.330
30.0	62.6	3313.2	0.776	25	19.424



**Table A.6 : Correction Of N Values Of Boring No. (B-103) For Overburden Pressure.**

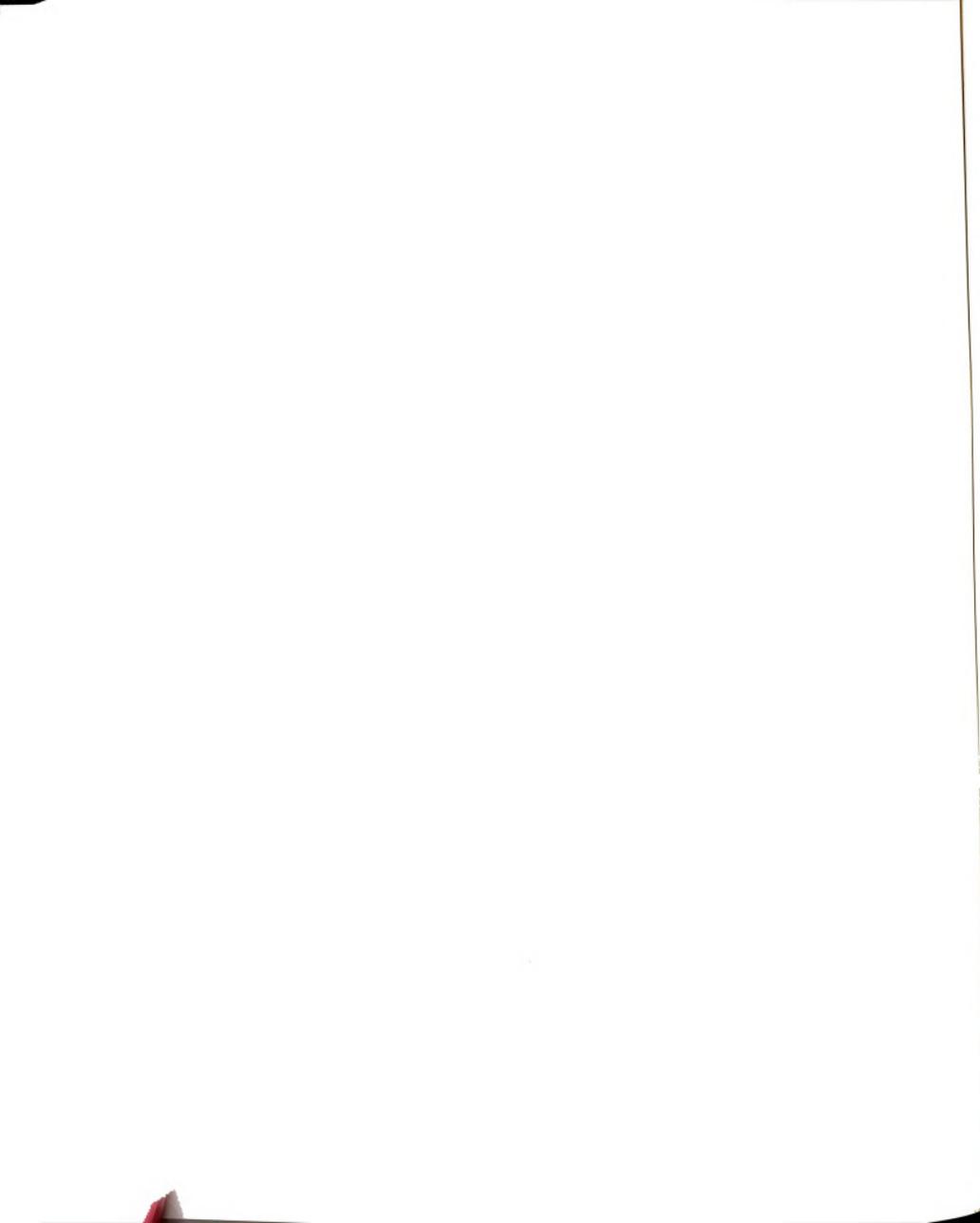
Depth (ft)	Unit wt. (lb/cf)	Effective Pressure (lb/sf)	Correction factor (C1)	N	N1=C1*N
2.0	125.0	250.0	2.828	15	42.426
5.0	125.0	625.0	1.788	18	32.199
8.0	125.0	1000.0	1.414	17	24.042
10.0	125.0	1375.0	1.206	18	21.709
15.0	125.0	1875.0	1.032	17	17.558
20.0	125.0	2500.0	0.894	67	15.198
25.0	125.0	3250.0	0.784	64	50.206
30.0	62.6	3563.0	0.749	69	51.696
35.0	62.6	3844.7	0.721	150	108.187
40.0	62.6	4189.0	0.691	100	69.097
45.0	62.6	4439.4	0.671	150	100.680
50.0	62.6	4752.4	0.648	52	33.734
55.0	62.6	5034.1	0.630	100	63.031
60.0	62.6	5409.7	0.608	400	243.214
65.0	62.6	5691.4	0.593	120	71.136
70.0	62.6	6004.4	0.577	600	346.283
75.0	62.6	6192.2	0.568	300	170.496



## A.2 Oneway ANOVA For The Corrected N Values:

**Table A.7 :The N data with the spatial locations For Case History No.1.**

LAYER	CORRECTED N	X	Y	Z
1	22.627	11.66	136.52	897
1	22.000	11.66	136.52	895
1	18.987	11.66	136.52	891.5
1	22.627	19.17	84.27	888
1	51.877	19.17	84.27	885
1	42.426	104.17	92.7	891
2	16.444	11.66	136.52	889
2	16.492	11.66	136.52	882
2	13.093	11.66	136.52	878
2	17.258	11.66	136.52	873
2	25.145	11.66	136.52	868
2	32.480	11.66	136.52	863
2	52.662	11.66	136.52	858
2	45.255	19.17	84.27	882
2	17.709	19.17	84.27	880
2	15.492	19.17	84.27	875
2	16.305	19.17	84.27	870.5
2	34.292	19.17	84.27	865
2	29.524	19.17	84.27	860
2	32.199	104.17	92.7	888
2	24.042	104.17	92.7	885



**Table A.7 : Continued.**


---

2	21.709	104.17	92.7	882
2	17.558	104.17	92.7	878
2	59.927	104.17	92.7	873
2	50.206	104.17	92.7	867
2	51.696	104.17	92.7	862
2	18.475	149.16	69.94	888
2	19.677	149.16	69.94	886
2	16.971	149.16	69.94	883
2	18.974	149.16	69.94	881
2	12.829	149.16	69.94	877
2	12.522	149.16	69.94	871
2	16.330	149.16	69.94	867
2	19.424	149.16	69.94	862
2	23.094	194.17	84.27	893
2	26.000	194.17	84.27	892
2	18.827	194.17	84.27	889.5
2	16.000	194.17	84.27	887
2	13.898	194.17	84.27	882
2	12.847	194.17	84.27	877
2	8.251	194.17	84.27	872.5
2	12.260	194.17	84.27	867.5
2	27.983	194.17	84.27	863
2	32.888	201.67	141.57	897.5
2	30.411	201.67	141.57	895
2	25.456	201.67	141.57	892

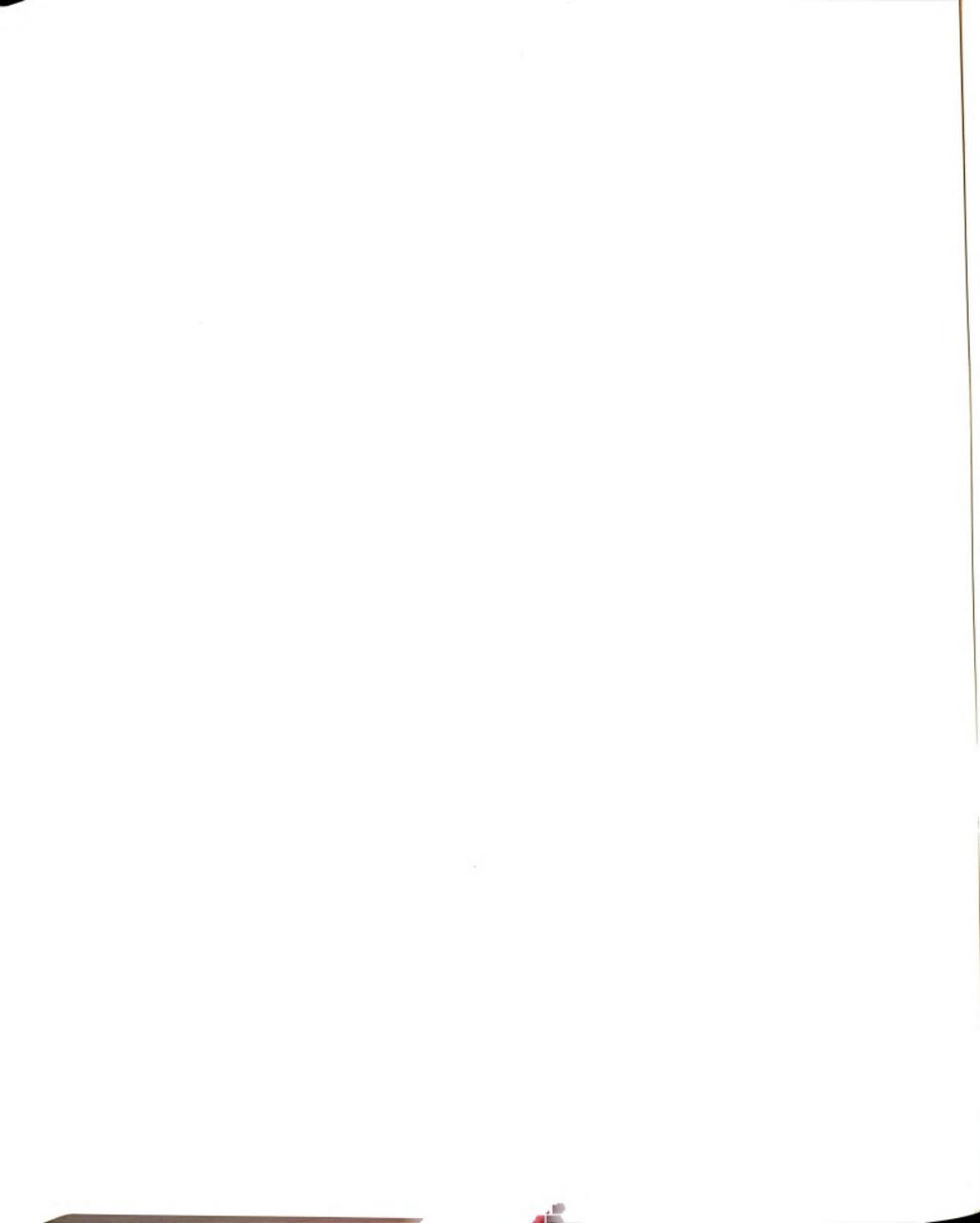
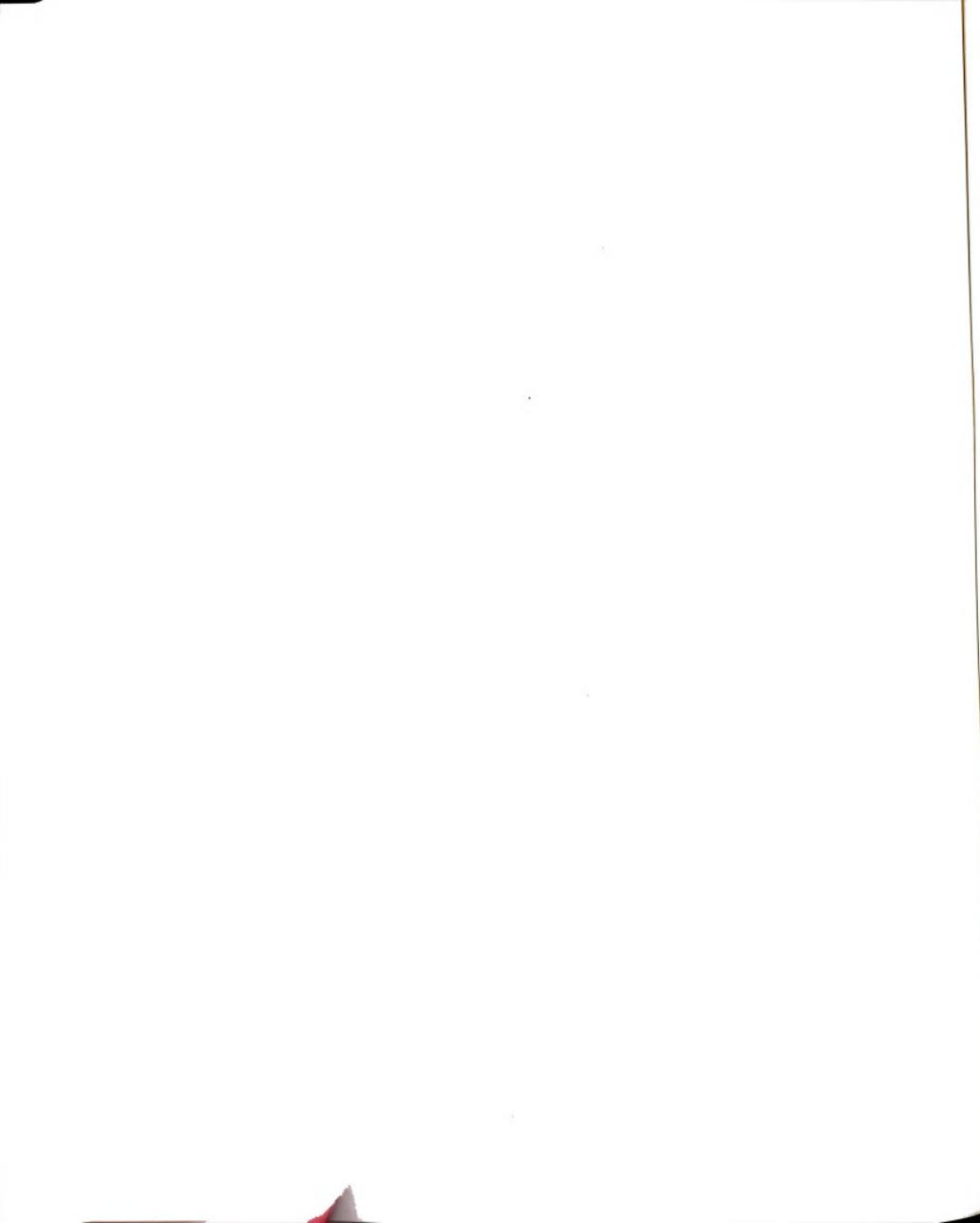


Table A.7 : Continued.

2	16.444	201.67	141.57	890
2	7.230	201.67	141.57	885
2	4.472	201.67	141.57	880
2	5.600	201.67	141.57	875
2	4.382	201.67	141.57	870
2	4.802	201.67	141.57	865
2	21.185	201.67	141.57	860
2	30.741	201.67	141.57	855
2	22.970	201.67	141.57	850
3	73.721	11.66	136.52	853.5
3	75.412	11.66	136.52	849
3	60.987	11.66	136.52	844
3	143.075	11.66	136.52	840
3	108.187	104.17	92.7	857.5
3	69.097	104.17	92.7	852
3	100.680	104.17	92.7	848
3	33.734	104.17	92.7	843
3	63.031	104.17	92.7	838.5
3	243.214	104.17	92.7	832.5
3	71.136	104.17	92.7	828
3	346.283	104.17	92.7	823
3	170.496	104.17	92.7	820
3	59.950	201.67	141.57	844
3	117.607	201.67	141.57	840.5
3	57.128	201.67	141.57	835



ONEWAY ANOVA FOR THE CORRECTED N VALUES

----- O N E W A Y -----

Variable N By Variable LAYER

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	98504.0575	49252.0287	30.8483	.0000
Within Groups	68	108567.8161	1596.5855		
Total	70	207071.8735			

ONEWAY ANOVA FOR THE CORRECTED N VALUES

----- O N E W A Y -----

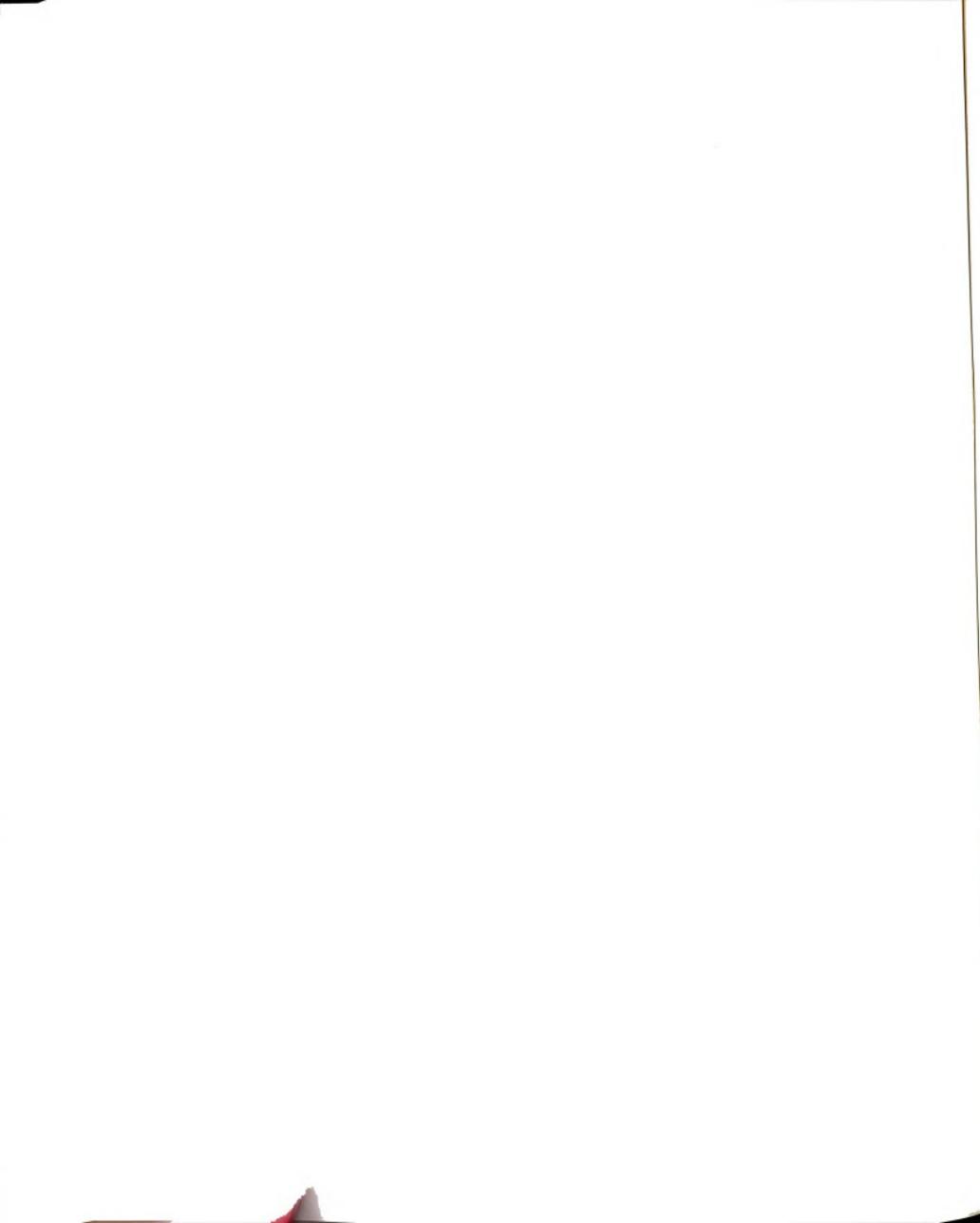
Group Count	Mean	Standard Deviation	Standard Error	95 Pct Conf Int for Mean
Grp 1 6	30.0907	13.6159	5.5587	15.8018 To 44.3795
Grp 2 49	22.2537	12.7585	1.8226	18.5890 To 25.9183
Grp 3 16	112.1086	81.5792	20.3948	68.6381 To 155.5791
Total 71	43.1650	54.3891	6.4548	30.2913 To 56.0387
Fixed Effects Model		39.9573	4.7421	33.7023 To 52.6276
Random Effects Model			39.5239	-126.8945 To 213.2245
Random Effects Model - Estimate of Between Component Variance 2882.0583				

ONEWAY ANOVA FOR THE CORRECTED N VALUES

Group	Minimum	Maximum
Grp 1	18.9870	51.8770
Grp 2	4.3820	59.9270
Grp 3	33.7340	346.2830
Total	4.3820	346.2830

-----

Figure A.1 : Oneway ANOVA Analysis For Case History No.1



Tests for Homogeneity of Variances

Cochrans C = Max. Variance/Sum(Variances) = .9503, P = .000 (Approx.)  
 Bartlett-Box F = 49.704 , P = .000  
 Maximum Variance / Minimum Variance 40.885

ONEWAY ANOVA FOR THE CORRECTED N VALUES

- - - - - O N E W A Y - - - - -

Variable N

By Variable LAYER

Multiple Range Test

Tukey-HSD Procedure

Ranges for the .050 level -

3.39 3.39

The ranges above are table ranges.

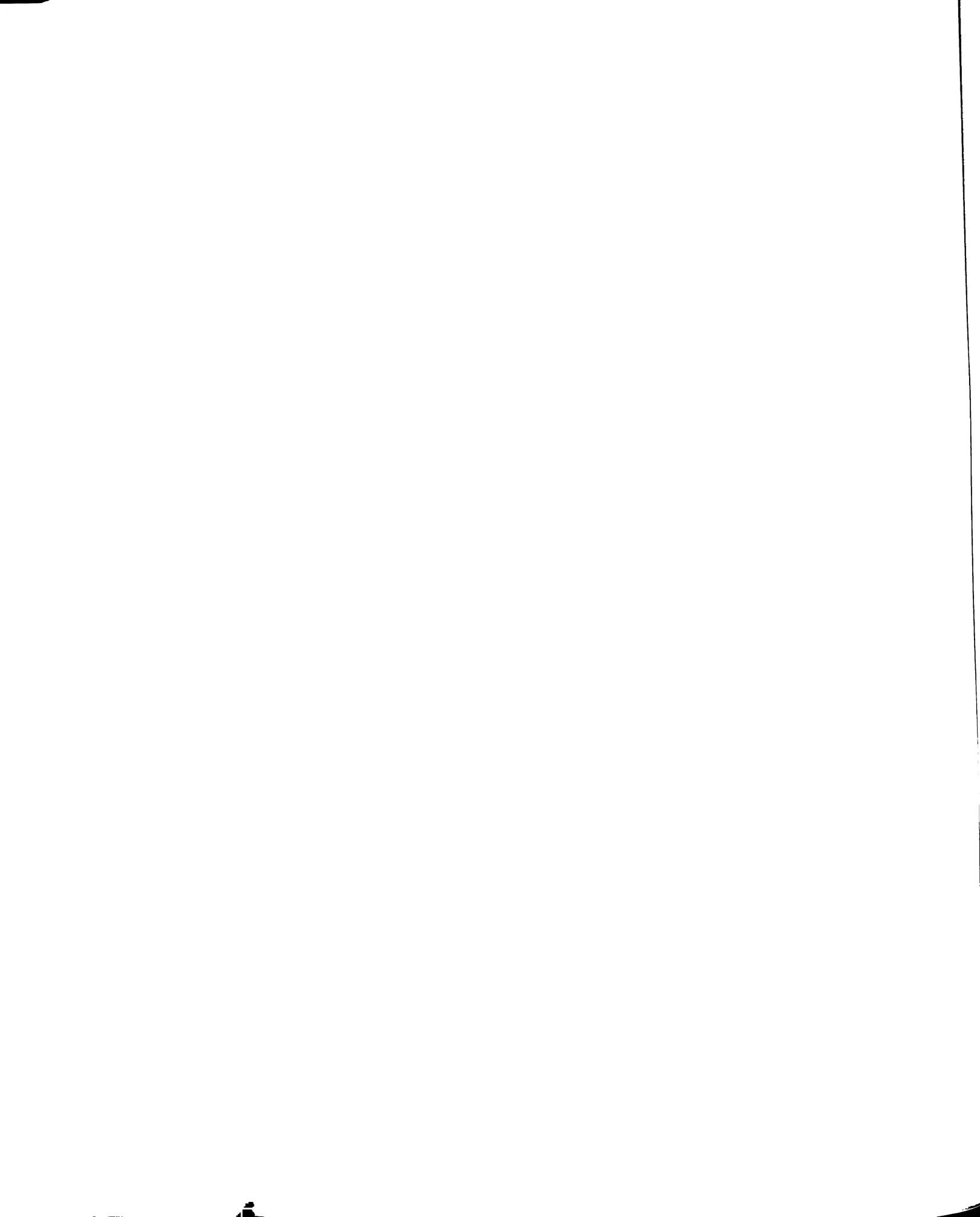
The value actually compared with Mean(J) - Mean(I) is..

$$28.2541 * \text{Range} * \text{Sqrt}(1/N(I) + 1/N(J))$$

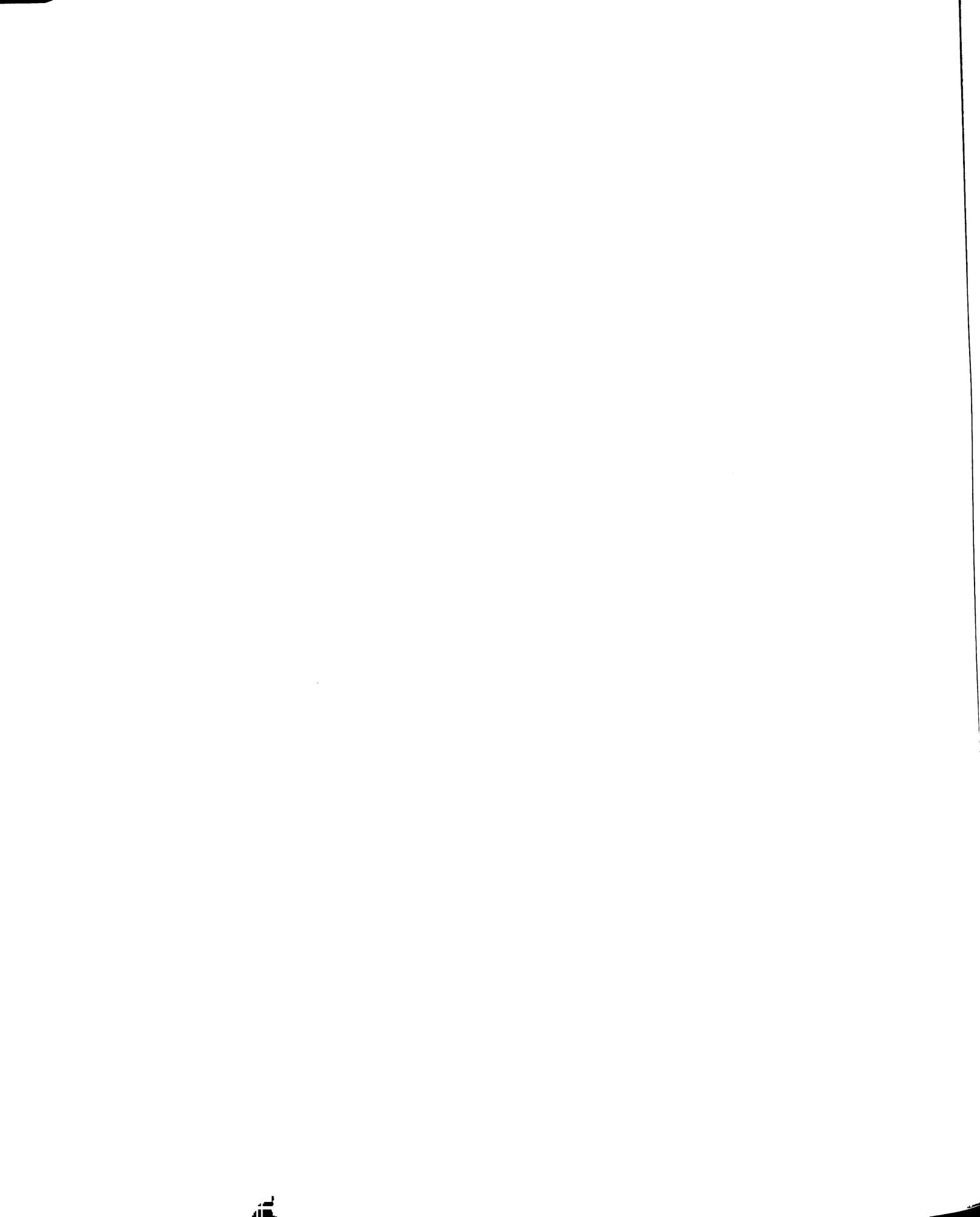
(\*) Denotes pairs of groups significantly different at the .050 level

Mean	Group	G G G
22.2537	Grp 2	r r r
30.0907	Grp 1	p p p
112.1086	Grp 3	2 1 3
		* *

-----  
 Figure A.1 : Continued.



**APPENDIX B**



APPENDIX B

B. MODELING THE N FUNCTIONS FOR THE SUBSURFACE SOIL  
OF CASE HISTORY No. 1

B.1 Modeling The N Function For The Upper Layer:

Table B.1 : The N Data With The Spatial Locations For The Upper Layer.

LAYER	CORRECTED N	X	Y	Z
1	22.627	11.66	136.52	897
1	22.000	11.66	136.52	895
1	18.987	11.66	136.52	891.5
1	22.627	19.17	84.27	888
1	51.877	19.17	84.27	885
1	42.426	104.17	92.7	891
1	16.444	11.66	136.52	889
1	16.492	11.66	136.52	882
1	13.093	11.66	136.52	878
1	17.258	11.66	136.52	873
1	25.145	11.66	136.52	868
1	32.480	11.66	136.52	863
1	52.662	11.66	136.52	858
1	45.255	19.17	84.27	882
1	17.709	19.17	84.27	880
1	15.492	19.17	84.27	875
1	16.305	19.17	84.27	870.5

Table B.1 : Continued.

1	34.292	19.17	84.27	865
1	29.524	19.17	84.27	860
1	32.199	104.17	92.7	888
1	24.042	104.17	92.7	885
1	21.709	104.17	92.7	882
1	17.558	104.17	92.7	878
1	59.927	104.17	92.7	873
1	50.206	104.17	92.7	867
1	51.696	104.17	92.7	862
1	18.475	149.16	69.94	888
1	19.677	149.16	69.94	886
1	16.971	149.16	69.94	883
1	18.974	149.16	69.94	881
1	12.829	149.16	69.94	877
1	12.522	149.16	69.94	871
1	16.330	149.16	69.94	867
1	19.424	149.16	69.94	862
1	23.094	194.17	84.27	893
1	26.000	194.17	84.27	892
1	18.827	194.17	84.27	889.5
1	16.000	194.17	84.27	887
1	13.898	194.17	84.27	882
1	12.847	194.17	84.27	877
1	8.251	194.17	84.27	872.5
1	12.260	194.17	84.27	867.5

Table B.1 : Continued.

1	27.983	194.17	84.27	863
1	32.888	201.67	141.57	897.5
1	30.411	201.67	141.57	895
1	25.456	201.67	141.57	892
1	16.444	201.67	141.57	890
1	7.230	201.67	141.57	885
1	4.472	201.67	141.57	880
1	5.600	201.67	141.57	875
1	4.382	201.67	141.57	870
1	4.802	201.67	141.57	865
1	21.185	201.67	141.57	860
1	30.741	201.67	141.57	855
1	22.970	201.67	141.57	850

-----

THE SUGGESTED MODEL (first trial):

$$N1=B0+B1*X+B2*X**2+B3*Y+B4*Y**2+B5*Z**0.5+B6*Z+B7*Z**2.$$

There are 55 cases. There is enough memory for them all.

Run stopped after 29 model evaluations and 6 derivative evaluations.  
 Iterations have been stopped because the relative difference between  
 successive parameter estimates is at most PCON = 1.000E-08

-----

**Figure B.1 : Modeling The N Function For The Upper Layer.**

N1=F(X,Y,Z),LAYERS 1&2 COMBINED & NAMED LAYER 1

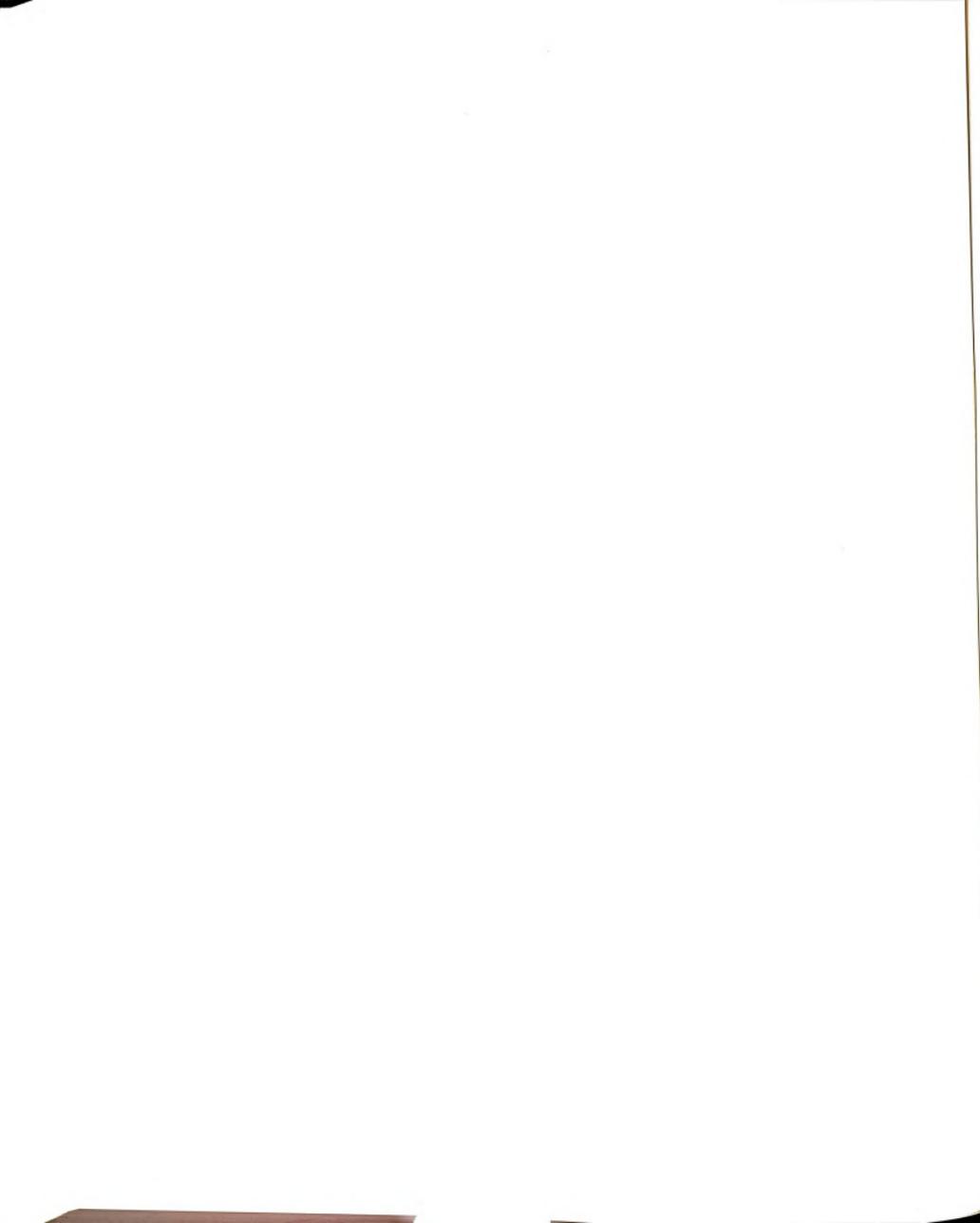
Nonlinear Regression Summary Statistics			Dependent Variable N
Source	DF	Sum of Squares	Mean Square
Regression	8	34011.07127	4251.38391
Residual	47	4428.05297	94.21389
Uncorrected Total	55	38439.12423	
(Corrected Total)	54	9068.62513	
R squared = 1 - Residual SS / Corrected SS =			.51172

Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval	
			Lower	Upper
B0	-2721918.469	2689165.0167	-8131820.682	2687983.7428
B1	.251442953	.104017109	.042187520	.460698385
B2	-.001392536	.000486554	-.002371357	-.000413715
B3	2.585913803	.807608044	.961215982	4.210611623
B4	-.011986634	.003700403	-.019430884	-.004542384
B5	248358.47467	242470.24257	-239428.7357	736145.68502
B6	-6372.381661	6148.6757108	-18741.92169	5997.1583723
B7	1.242247307	1.171447790	-1.114401671	3.598896286

Asymptotic Correlation Matrix of the Parameter Estimates

	B0	B1	B2	B3	B4	B5
B0	1.0000	-.0540	.0378	.0047	-.0015	-1.0000
B1	-.0540	1.0000	-.9848	.0342	.0109	.0542
B2	.0378	-.9848	1.0000	.0258	-.0709	-.0380
B3	.0047	.0342	.0258	1.0000	-.9970	-.0047
B4	-.0015	.0109	-.0709	-.9970	1.0000	.0015
B5	-1.0000	.0542	-.0380	-.0047	.0015	1.0000
B6	1.0000	-.0543	.0382	.0048	-.0014	-1.0000
B7	-.9999	.0545	-.0385	-.0048	.0013	1.0000

-----  
**Figure B.1 : Continued.**



	B6	B7
B0	1.0000	-.9999
B1	-.0543	.0545
B2	.0382	-.0385
B3	.0048	-.0048
B4	-.0014	.0013
B5	-1.0000	1.0000
B6	1.0000	-1.0000
B7	-1.0000	1.0000

-----  
 N1=F(X,Y,Z),LAYERS 1&2 COMBINED & NAMED LAYER 1

THE SUGGESTED MODEL (second trial):

$$N1 = D0 + D1 * X^{**0.5} + D2 * Y^{**0.5} + D3 * Z^{**0.5} + D4 * X + D5 * Y + D6 * Z + D7 * Z^{**2}.$$

All the derivatives will be calculated numerically.  
 -----

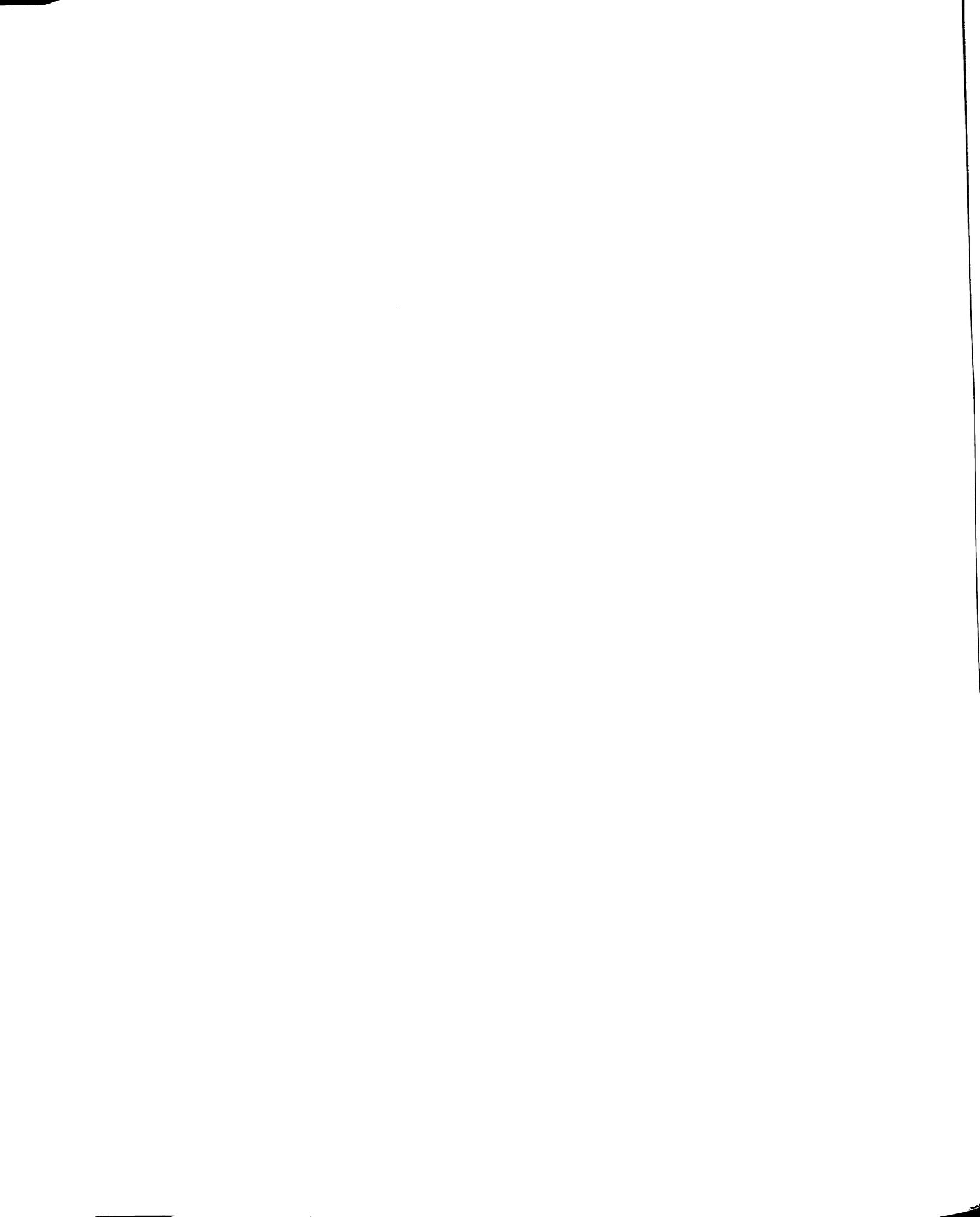
There are 55 cases. There is enough memory for them all.

Run stopped after 37 model evaluations and 9 derivative evaluations.  
 Iterations have been stopped because the relative difference between  
 successive parameter estimates is at most PCON = 1.000E-08  
 -----

Nonlinear Regression Summary Statistics			Dependent Variable N
Source	DF	Sum of Squares	Mean Square
Regression	8	34157.40722	4269.67590
Residual	47	4281.71702	91.10036
Uncorrected Total	55	38439.12423	
(Corrected Total)	54	9068.62513	
R squared = 1 - Residual SS / Corrected SS =			.52785

-----

Figure B.1 : Continued.



---

Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval	
			Lower	Upper
D0	-3008929.737	2635240.2660	-8310349.344	2292489.8693
D1	10.058236699	3.091411051	3.839119844	16.277353554
D2	78.135462538	28.376247798	21.049815215	135.22110986
D3	274208.99710	237614.56822	-203809.8564	752227.85065
D4	-.611711073	.175138521	-.964044330	-.259377815
D5	-3.794113405	1.381756722	-6.573849382	-1.014377427
D6	-7027.873386	6025.7049579	-19150.02817	5094.2814020
D7	1.367121166	1.148081239	-.942520374	3.676762707

---

Asymptotic Correlation Matrix of the Parameter Estimates

	D0	D1	D2	D3	D4	D5
D0	1.0000	-.0653	.0145	-1.0000	.0565	-.0133
D1	-.0653	1.0000	-.1918	.0655	-.9950	.2177
D2	.0145	-.1918	1.0000	-.0145	.2205	-.9990
D3	-1.0000	.0655	-.0145	1.0000	-.0567	.0133
D4	.0565	-.9950	.2205	-.0567	1.0000	-.2459
D5	-.0133	.2177	-.9990	.0133	-.2459	1.0000
D6	1.0000	-.0656	.0145	-1.0000	.0569	-.0133
D7	-.9999	.0660	-.0145	1.0000	-.0573	.0132

---

Figure B.1 : Continued.

	D6	D7
D0	1.0000	-.9999
D1	-.0656	.0660
D2	.0145	-.0145
D3	-1.0000	1.0000
D4	.0569	-.0573
D5	-.0133	.0132
D6	1.0000	-1.0000
D7	-1.0000	1.0000

---

**Figure B.1 : Continued.**

## B.2 Modeling The N Function For The Lower Layer:

Table B.2 : The N Data With The Spatial Locations For The Lower Layer.

LAYER	CORRECTED N	X	Y	Z
2	73.721	11.66	136.52	853.5
2	75.412	11.66	136.52	849
2	60.987	11.66	136.52	844
2	143.075	11.66	136.52	840
2	108.187	104.17	92.7	857.5
2	69.097	104.17	92.7	852
2	100.680	104.17	92.7	848
2	33.734	104.17	92.7	843
2	63.031	104.17	92.7	838.5
2	243.214	104.17	92.7	832.5
2	71.136	104.17	92.7	828
2	346.283	104.17	92.7	823
2	170.496	104.17	92.7	820
2	59.950	201.67	141.57	844
2	117.607	201.67	141.57	840.5
2	57.128	201.67	141.57	835

THE SUGGESTED MODEL (first trial):

$$N2 = F0 + F1 * Z + F2 * Z^{**0.5} + F3 * Z^{**2}.$$

There are 16 cases. There is enough memory for them all.

Run stopped after 33 model evaluations and 13 derivative evaluations.  
Iterations have been stopped because the relative reduction between  
successive residual sums of squares is at most SCON = 1.000E-08

-----  
N2=F(X,Y,Z) , LAYER 3 HAS BECOME LAYER 2

Nonlinear Regression Summary Statistics			Dependent Variable N
Source	DF	Sum of Squares	Mean Square
Regression	4	245341.00673	61335.25168
Residual	12	55579.99287	4631.66607
Uncorrected Total	16	300920.99960	
(Corrected Total)	15	99827.49881	

R squared = 1 - Residual SS / Corrected SS = .44324

Parameter	Estimate	Std. Error	Asymptotic 95 % Confidence Interval	
			Lower	Upper
F0	-34869400.63	53984448.891	-152491410.5	82752609.216
F1	-83881.92163	128724.81215	-364349.1938	196585.35057
F2	3225018.0453	4970781.1506	-7605383.699	14055419.790
F3	16.808960824	25.575919332	-38.91618035	72.534101995

-----  
**Figure B.2 : Modeling The N Function For The Lower Layer.**

## Asymptotic Correlation Matrix of the Parameter Estimates

	F0	F1	F2	F3
F0	1.0000	1.0000	-1.0000	-1.0000
F1	1.0000	1.0000	-1.0000	-1.0000
F2	-1.0000	-1.0000	1.0000	1.0000
F3	-1.0000	-1.0000	1.0000	1.0000

-----  
 N2=F(X,Y,Z) , LAYER 3 HAS BECOME LAYER 2

THE SUGGESTED MODEL (second trial):

$$N2 = G0+G1*X+G2*Y+G3*Z+G4*X**0.5+G5*Y**0.5+G6*Z**0.5+G7*Z**2.$$

There are 16 cases. There is enough memory for them all.

Run stopped after 48 model evaluations and 16 derivative evaluations.  
 Iterations have been stopped because the relative reduction between  
 successive residual sums of squares is at most SSSCON = 1.000E-08

-----  
 N2=F(X,Y,Z) , LAYER 3 HAS BECOME LAYER 2

Nonlinear Regression Summary Statistics      Dependent Variable N

Source	DF	Sum of Squares	Mean Square
Regression	8	246013.43315	30751.67914
Residual	8	54907.56645	6863.44581
Uncorrected Total	16	300920.99960	
(Corrected Total)	15	99827.49881	

R squared = 1 - Residual SS / Corrected SS = .44998

-----

Figure B.2 : Continued.

Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval	
			Lower	Upper
G0	-16362554.23	281179591.73	-664763855.5	632038747.03
G1	1283.8757228	100937.77506	-231479.0510	234046.80242
G2	-35144.33067	2595298.3673	-6019913.098	5949624.4364
G3	-30682.78158	162779.19984	-406052.2895	344686.72638
G4	-20816.74164	1681355.0630	-3898028.470	3856394.9863
G5	738011.92472	54840773.108	-125725037.6	127201061.49
G6	1172299.8932	6284867.6756	-13320630.96	15665230.742
G7	6.222602875	32.351726910	-68.38061316	80.825818911

---

Asymptotic Correlation Matrix of the Parameter Estimates

	G0	G1	G2	G3	G4	G5
G0	1.0000	-.6549	.9722	.0824	.6194	-.9720
G1	-.6549	1.0000	-.6731	-.0588	-.9988	.6605
G2	.9722	-.6731	1.0000	-.1482	.6366	-.9999
G3	.0824	-.0588	-.1482	1.0000	.0710	.1518
G4	.6194	-.9988	.6366	.0710	1.0000	-.6235
G5	-.9720	.6605	-.9999	.1518	-.6235	1.0000
G6	-.0821	.0582	.1485	-1.0000	-.0704	-.1521
G7	-.0830	.0600	.1475	-1.0000	-.0722	-.1511

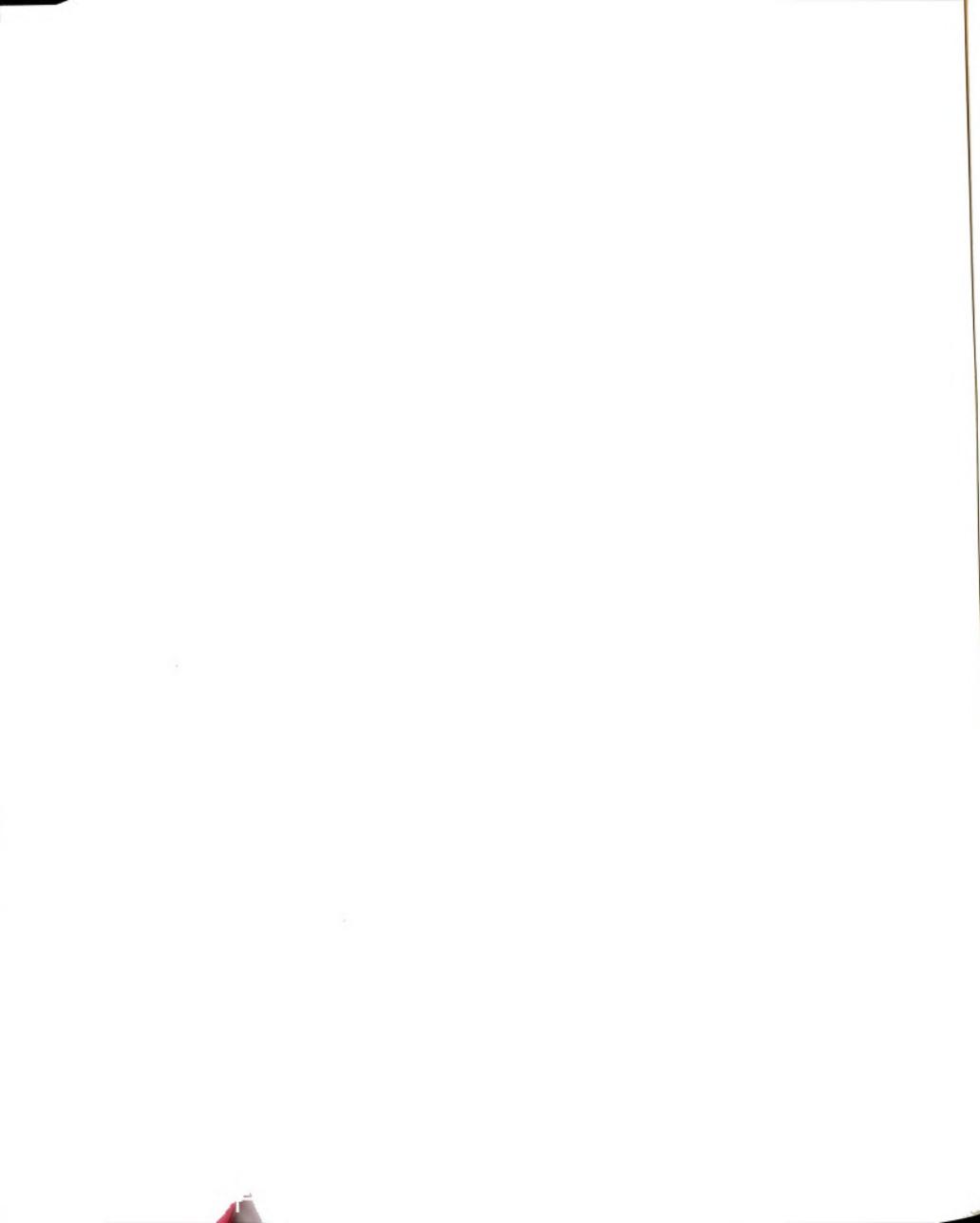
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Figure B.2 : Continued.

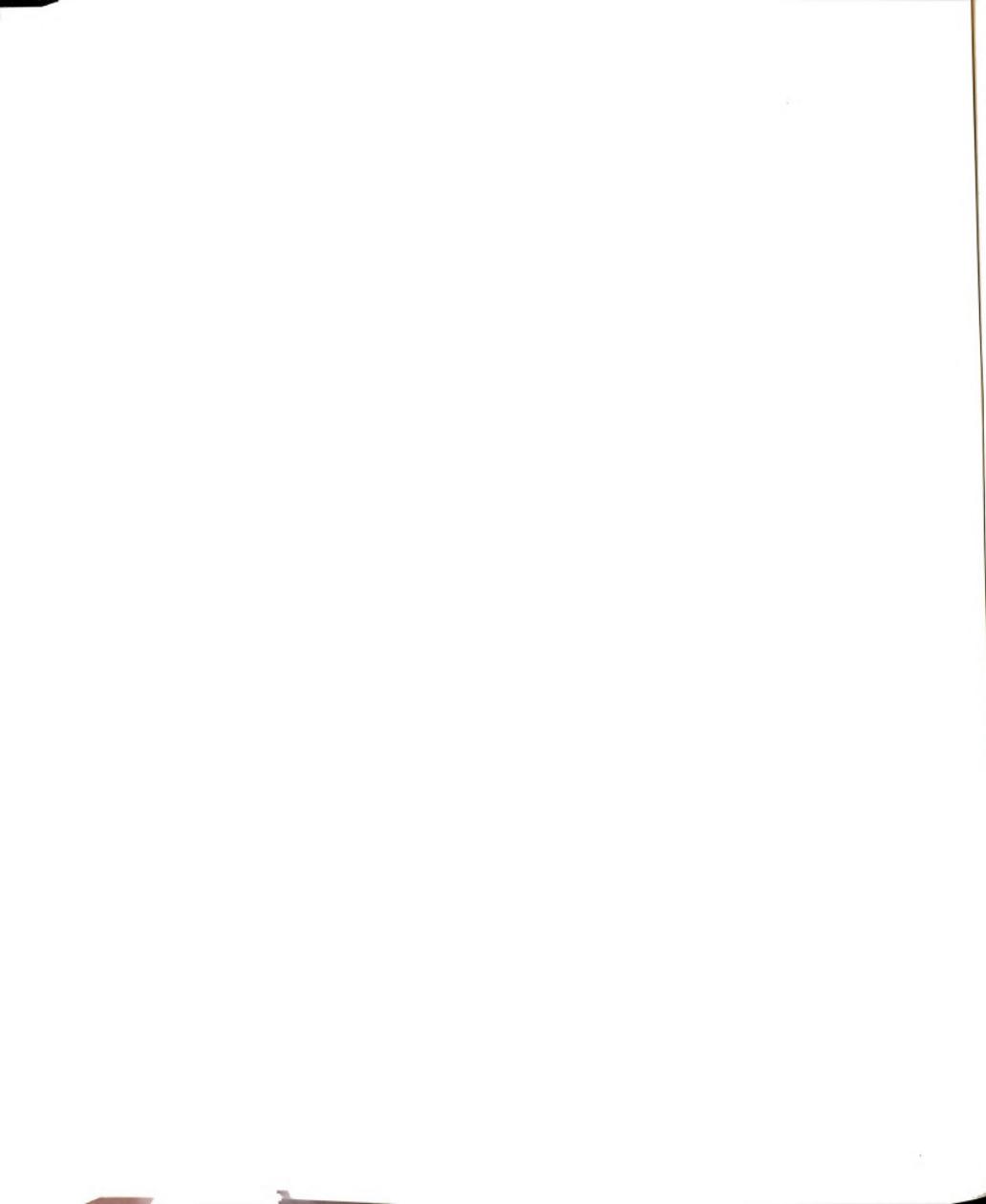
	G6	G7
G0	-.0821	-.0830
G1	.0582	.0600
G2	.1485	.1475
G3	-1.0000	-1.0000
G4	-.0704	-.0722
G5	-.1521	-.1511
G6	1.0000	1.0000
G7	1.0000	1.0000

---

Figure B.2 : Continued.



**APPENDIX C**



## APPENDIX C

### C. THE SETTLEMENT PREDICTIONS FOR THE CASE HISTORIES

#### C.1 THE SETTLEMENT PREDICTION OF CASE HISTORY No. 2

##### C.1.1 PROJECT GENERAL DESCRIPTION

This case history consists of a tall office building at Lagos in Nigeria, whose settlement predictions were previously investigated by the designers Grimes, A.S. and Cantlay, W.G. of "Oscar Faber & Partners, Consulting Engineers".

The building consists of three blocks of approximately equal size, the two outer blocks, A and C, having nineteen stories and the center block, B, having twenty stories. A general top view and the positions of the blocks relative to one another as well as the boring locations are shown in Figure C.1. The foundations take the form of three rafts with an overall thickness of 7 ft.

##### C.1.2 SUBSOIL INVESTIGATION

A site investigation was carried out using six borings of 6 in diameter. Standard penetration tests were made in the different depths. One of the borings was sunk to a depth of 151 ft 6 in, three were sunk to about 86 ft and two to about 65 ft. The results of the borings are shown in Tables C.2 to C.4. Although they vary somewhat they indicate that the ground can be divided broadly into four fairly distinct bands in sequence from the top as follows:

1. An upper sandy band of about 40 ft thickness, variable in grading, which are very loose in the upper 20 ft, the density increasing with depth.



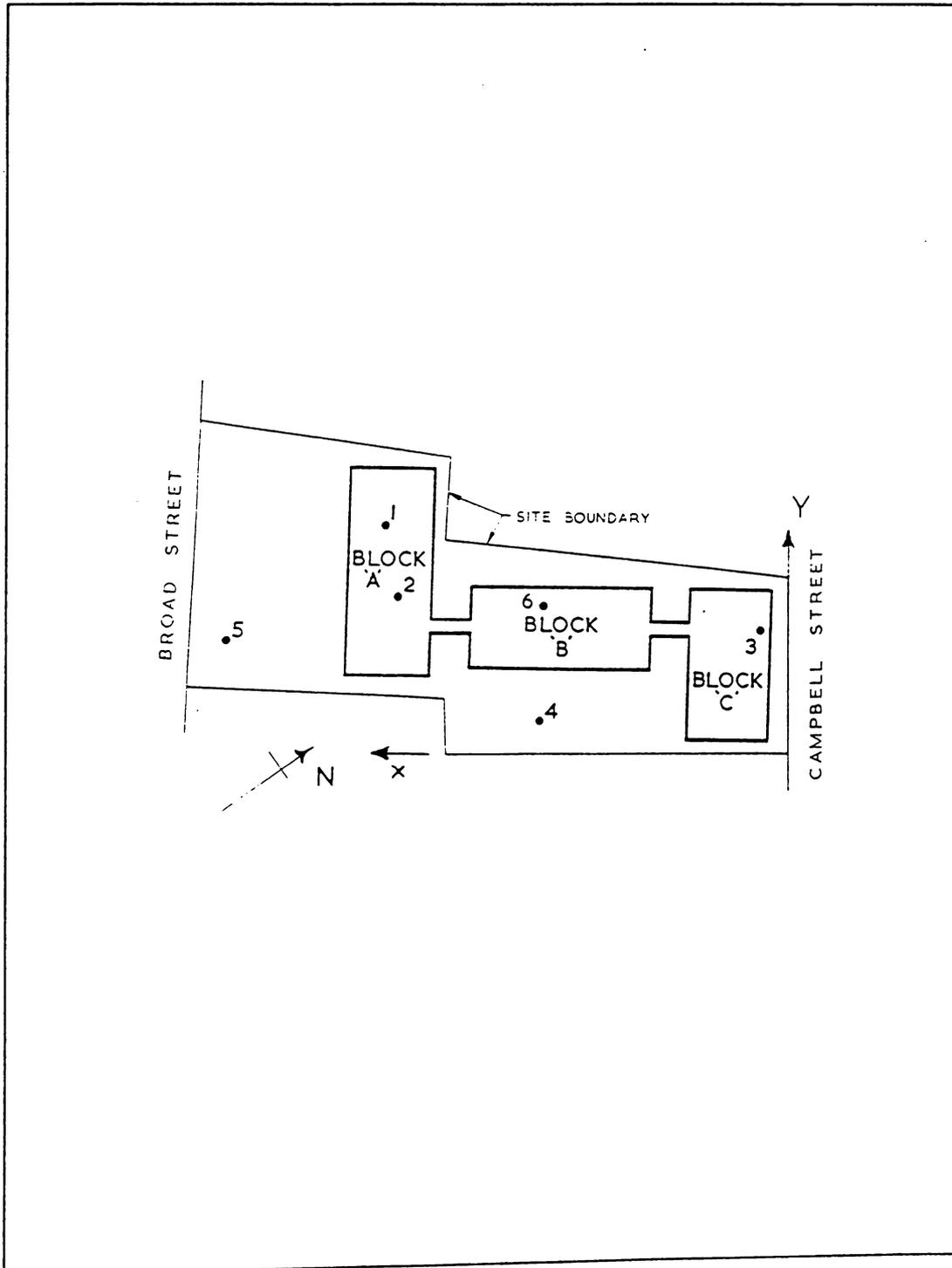
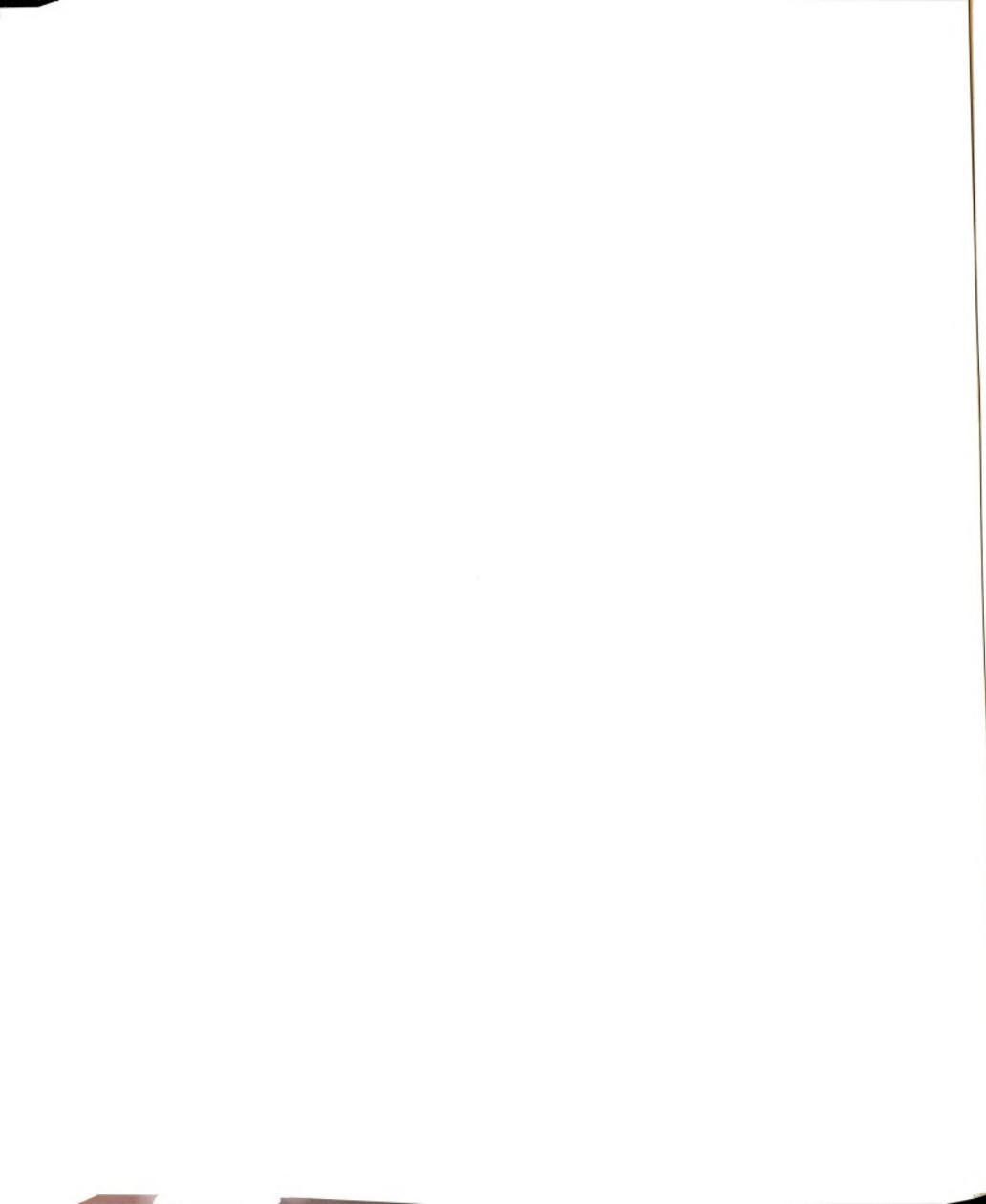


Figure C.1 : Site Plan And Boring Locations For Case History No. 2

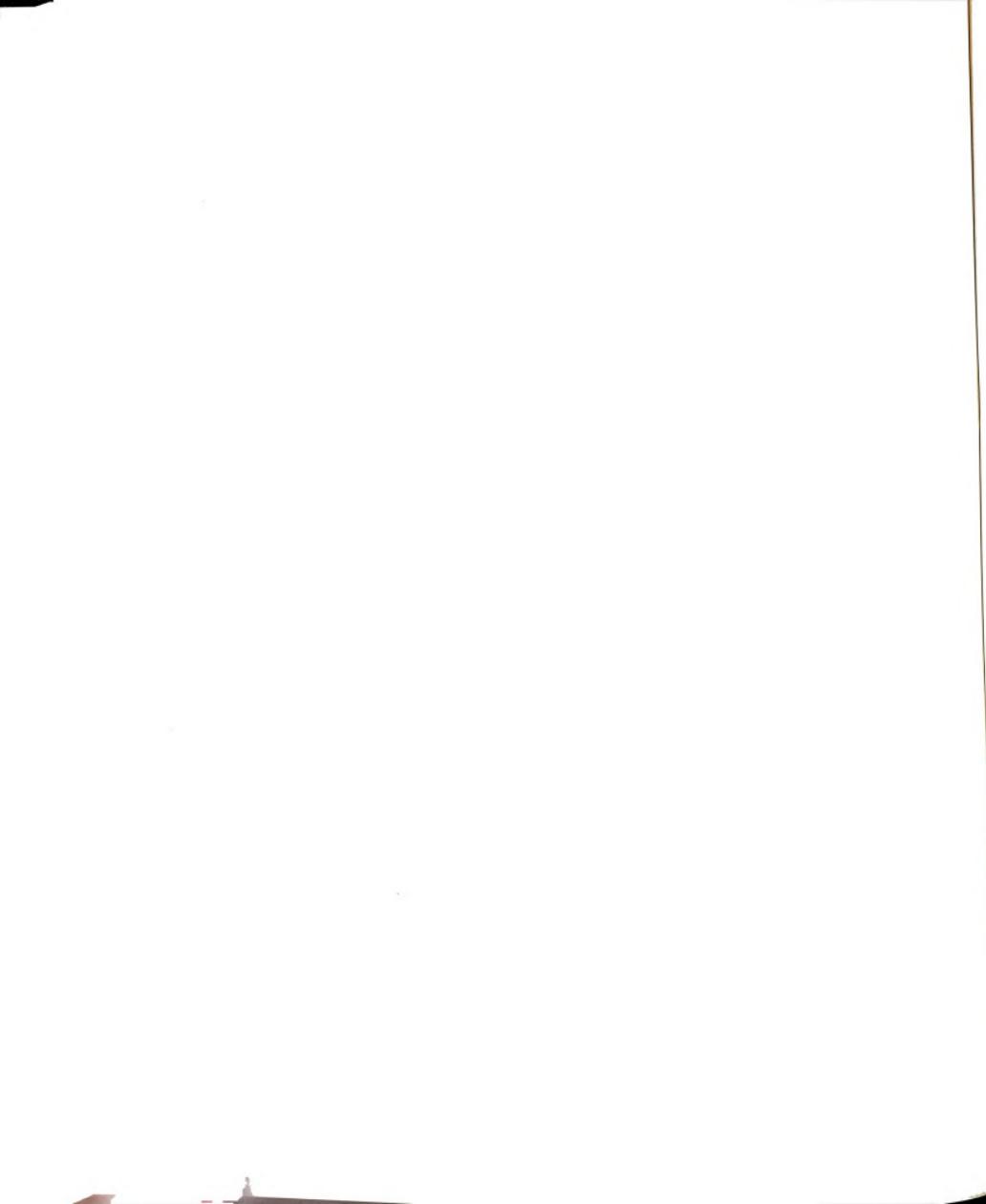


**Table C.1 : The Correction Factors For Overburden Pressure  
Of N Values Of Case History No. 2**

Depth(ft)	Unit wt. lb/cf.	Effective Pressure (lb/sf)	Correction factor (C1)
5	125.0	625.0	1.788854
10	62.6	1062.8	1.371795
15	62.6	1375.8	1.205695
20	62.6	1688.8	1.088243
25	62.6	2001.8	0.999550
30	62.6	2314.8	0.929519
35	62.6	2627.8	0.872406
40	62.6	2940.8	0.824673
45	62.6	3253.8	0.784006
50	62.6	3566.8	0.748816
55	62.6	3879.8	0.717976
60	62.6	4192.8	0.690657
65	62.6	4505.8	0.666237
70	62.6	4818.8	0.644236
75	62.6	5131.8	0.624281
80	62.6	5444.8	0.606071
85	62.6	5757.8	0.589368

**Table C.2 : The Correction Of N Values Of Borings No.(1) And No.(2).**

Depth (ft)	Boring No.(1)		Boring No.(2)	
	N	$N1 = C1 * N$	N	$N1 = C1 * N$
5	4	7.155	4	7.155
10	4	5.487	5	6.858
15	4	4.822	11	13.262
20	8	8.705	11	11.970
25	11	10.995	20	19.991
30	6	5.577	25	23.237
35	9	7.851	14	12.213
40	13	10.720	11	9.071
45	13	10.192		
50	15	11.232	19	14.227



**Table C.3 : The Correction Of N Values Of Borings No.(3) And No.(4).**

Depth (ft)	Boring No.(3)		Boring No.(4)	
	N	$N1=C1*N$	N	$N1=C1*N$
5	7	12.521	7	12.521
10	6	8.230	5	6.858
15	12	14.468	10	12.056
20	11	11.970	11	11.970
25	22	21.990	10	9.995
30	22	20.449	24	22.308
35	11	9.596	22	19.192
40	41	33.811		
45	26	20.384	7	5.488
50	37	27.706	7	5.241
55	27	19.385		
65	11	7.328	7	4.663
80	10	6.060		
85	8	4.714		

**Table C.4 : The Correction Of N Values Of Borings No.(5) And No.(6).**

Depth (ft)	Boring No.(5)		Boring No.(6)	
	N	$N1=C1*N$	N	$N1=C1*N$
5	6	10.733	5	8.944
10	3	4.115	6	8.230
15	7	8.439	6	7.234
20	8	8.705	8	8.705
25	9	8.995	8	7.996
30	7	6.506	8	7.436
35	4	3.489	9	7.851
40	8	6.597	3	2.474
45	4	3.136		
50	4	2.995	8	5.990
55			16	11.487
60	4	2.762		
65			5	3.331
70				
75				
80			28	16.970

2. A sand and clay band of about 20 ft mean thickness, consisting of alternate layers of sand and clay. The thickness of the layers of sand and clay seem to vary considerably from boring to boring, and their consistency varies from loose to compact for the sands, and from soft to stiff for the clays.
3. A clay band of 20 ft mean thickness consisting of firm to stiff clays.
4. A lower band below 80 ft from the surface, consisting entirely of sands and gravels of medium density, although at 140 ft they begin to be more compact. Boring 6, however, was the only one sunk deep enough to give information for this band.

The correction factors of N values for overburden pressure are shown in Table C.1. The N values from borings 1 to 6 together with their correction for the overburden pressure are shown in Tables C.2 to C.4.

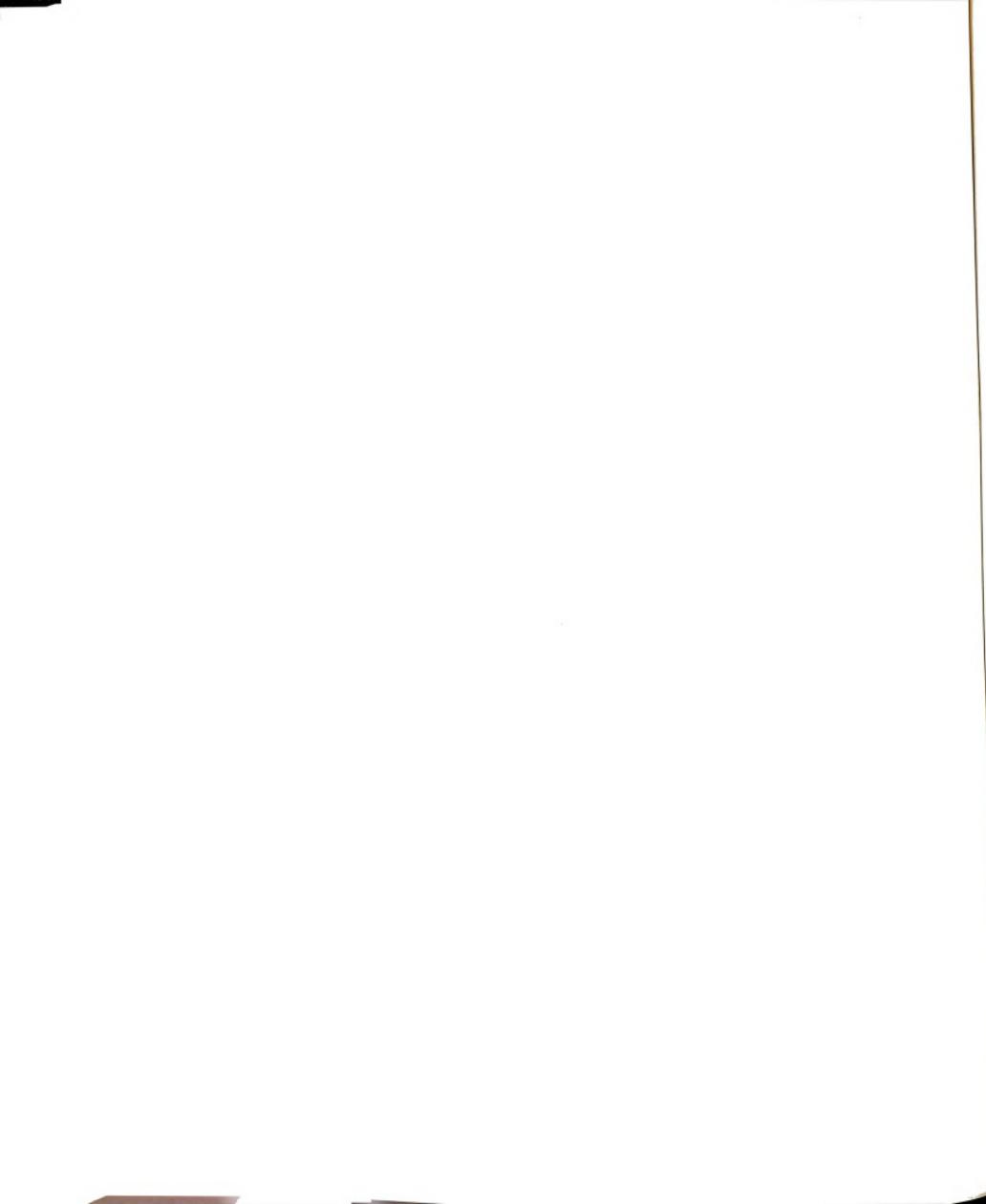
### **C.1.3 FOUNDATIONS AND SETTLEMENT MEASUREMENTS**

The provided plan shows the relative positions of the foundation rafts of the three blocks A, B & C. The heaviest loaded one is the raft of the centre block B. This raft is chosen for settlement prediction study. The as built raft dimensions were:

Width "B" = 42.67 ft. , Length = 90 ft. , Depth = 7 ft.

The ground water table is at an average depth of 7 ft. from the ground surface.

According to the designers, there are a fairly uniform pressure on the ground of 1.8 ton/ft<sup>2</sup> and there would be about 1.5 in. of the total forecast settlement due to consolidation of the sands. Settlement readings were taken throughout the progress of the work through sixteen points established on the three blocks.



### C.1.4 SETTLEMENT PREDICTION BY KRIGING

Considering that a settlement prediction is required at the center point of block B. The Kriging results are summarized as follows:

1. The calculated covariance function is given by the equation:

$$C(h) = 39.286 e^{-4.444E-5h^2} \quad (\text{C.1})$$

2. The estimated N function is given by:

$$\hat{N} = 10.74499 - 0.019503Z \quad (\text{C.2})$$

3. The "two - point" estimate of N values are:

$$\hat{N}_{(B/2)} = 10.41, \hat{N}_{(3B/2)} = 9.71 \quad (\text{C.3})$$

4. The design N value is given by the weighted average:

$$N = (1/3) [2\hat{N}_{(B/2)} + \hat{N}_{(3B/2)}] = 10.18 \quad (\text{C.4})$$

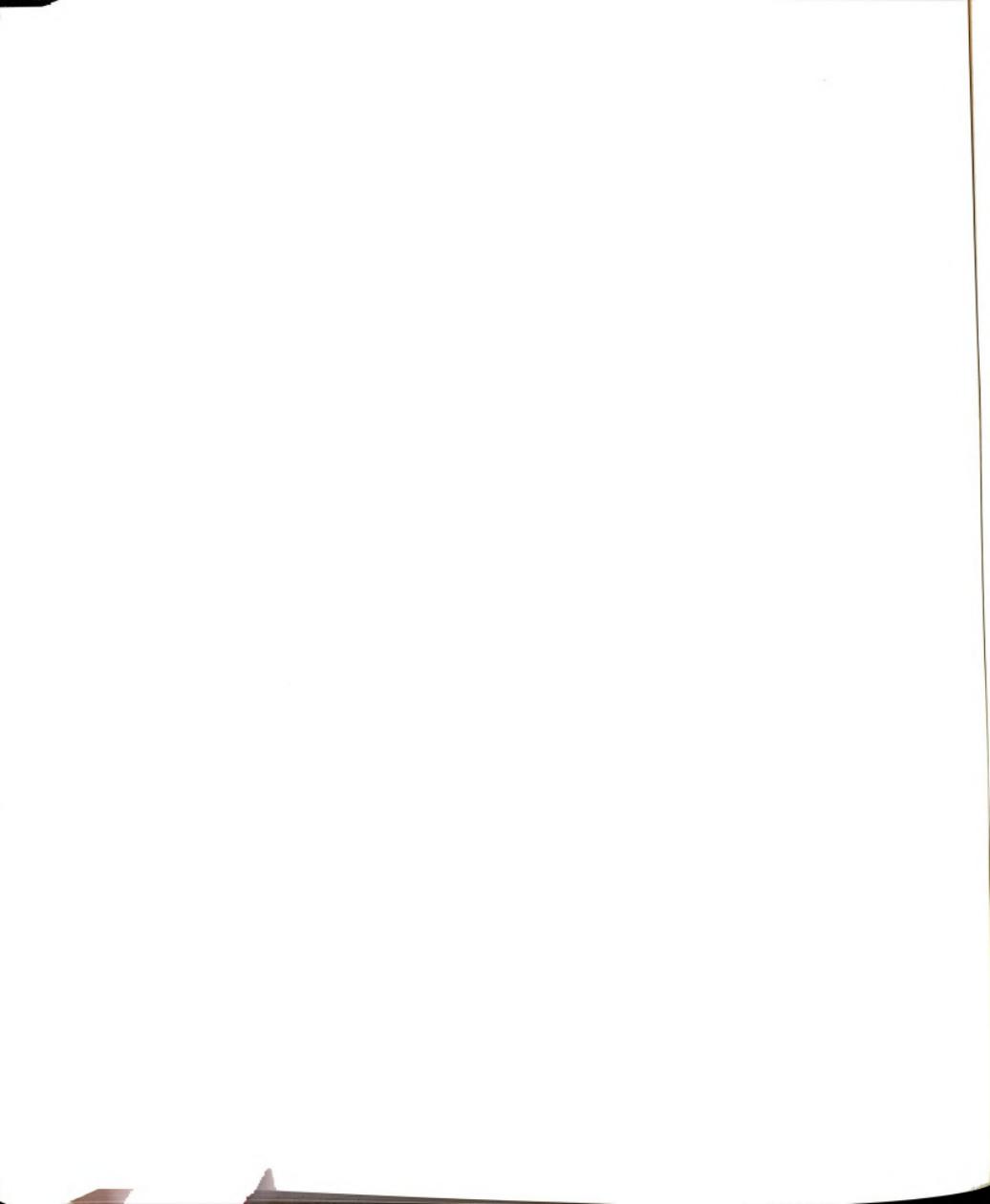
5. The predicted settlement is 2.29 in.

Therefore the predicted settlement of 2.29 in is higher than the measured value of 1.5 in by about 50%

6. The 90% confidence limits of the settlement prediction are:

$$(1.97 \text{ and } 2.73) \text{ in.}$$

The 50% confidence limits are: (2.18 and 2.42) in.



### C.1.5 SETTLEMENT PREDICTION BY TREND SURFACE ANALYSIS

The trend surface analysis results are summarized as follows:

1. The model which is fitted to the data is given by:

$$N = 35.66 - 16.7X^{0.5} + 1.38X + 0.0019X^2 + 4.56Y^{0.5} - 0.37Y \\ + 1.73Z^{0.5} - 0.107Z - 0.001Z^2, (R^2 = 0.458) \quad (\text{C.5})$$

2. The "two-point" estimate of N values are as follows:

$$N_{(B/2)} = 7.84 \quad ; \quad N_{(3B/2)} = 5.03$$

3. The design N value is as follows:

$$N = (1/3) [2\hat{N}_{(B/2)} + \hat{N}_{(3B/2)}] = 6.91 \quad (\text{C.6})$$

4. The predicted settlement is 2.5 in.

Therefore the predicted settlement of 2.5 in is higher than the measured value of 1.5 in by about 66%

5. The 90% confidence limits of the settlement prediction are: (1.72 and 4.57) in.

The 50% confidence limits of the settlement prediction are: (2.11 and 3.06) in.

6. The areal distribution of settlement in inches is given by the equation:

$$\hat{S} = 15.504 / (35.66 - 16.708X^{0.5} + 1.386X - 0.002X^2 \\ + 4.569Y^{0.5} - 0.376Y + 3.63) \quad (\text{C.7})$$

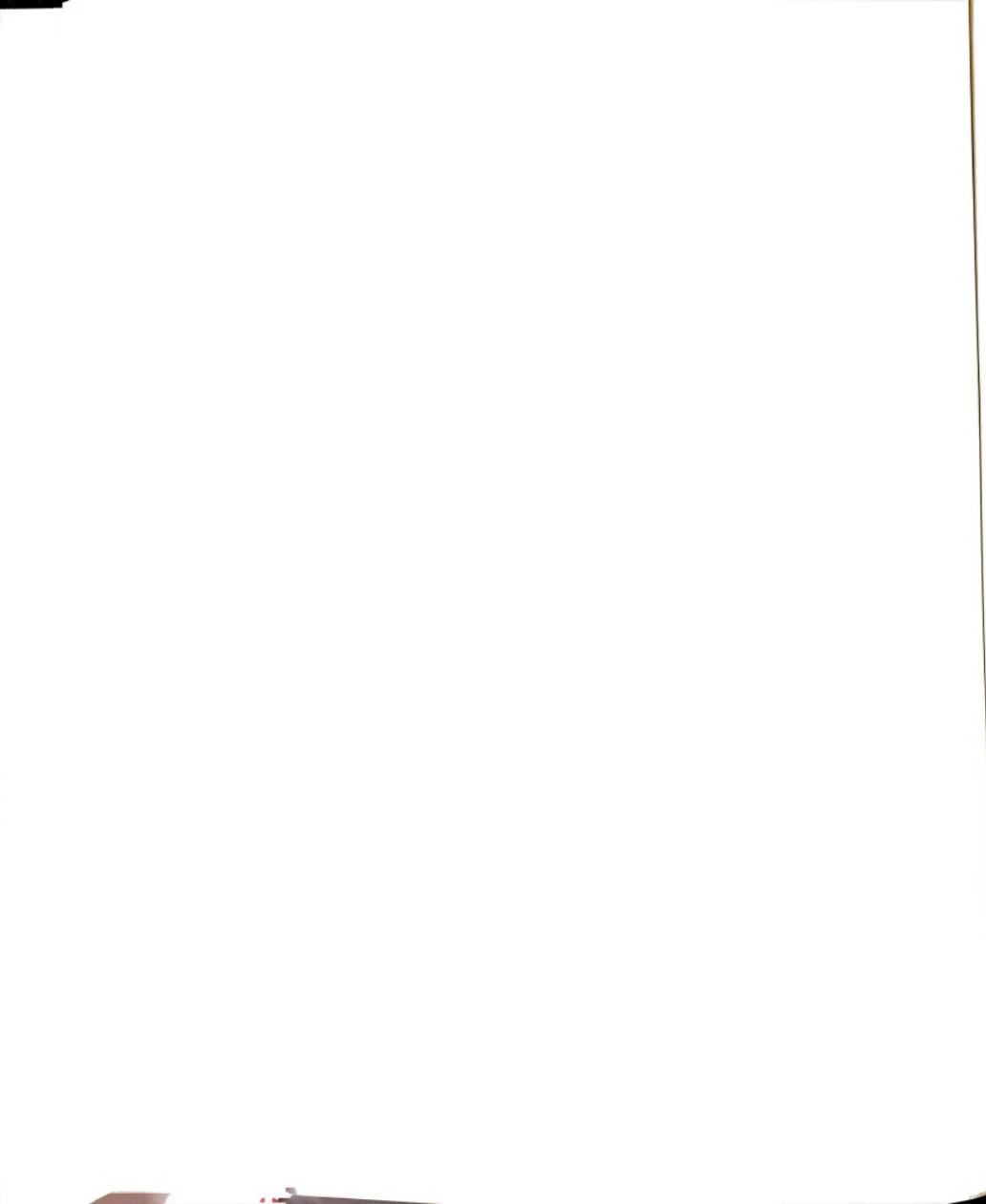
The computer output is shown in Figure C.2.

LAYER	N1	X	Y	Z
1	12.52198	13.13	60.50	5
1	8.23077	13.13	60.50	10
1	14.46834	13.13	60.50	15
1	11.97067	13.13	60.50	20
1	21.99011	13.13	60.50	25
1	20.44942	13.13	60.50	30
1	9.59647	13.13	60.50	35
1	33.81163	13.13	60.50	40
1	20.38416	13.13	60.50	45
1	27.70622	13.13	60.50	50
1	19.38537	13.13	60.50	55
1	12.52198	125.45	9.33	5
1	6.85897	125.45	9.33	10
1	12.05695	125.45	9.33	15
1	11.97067	125.45	9.33	20
1	9.99550	125.45	9.33	25
1	22.30846	125.45	9.33	30
1	19.19294	125.45	9.33	35
2	7.15542	205.47	108.93	5
2	5.48718	205.47	108.93	10
2	4.82278	205.47	108.93	15
2	8.70594	205.47	108.93	20
2	10.99505	205.47	108.93	25
2	5.57711	205.47	108.93	30
2	7.85166	205.47	108.93	35

**Figure C.2 : The Computer Output Of Case History No. 2**

2	10.72076	205.47	108.93	40
2	10.19208	205.47	108.93	45
2	11.23225	205.47	108.93	50
2	7.15542	197.20	73.68	5
2	6.85898	197.20	73.68	10
2	13.26264	197.20	73.68	15
2	11.97067	197.20	73.68	20
2	19.99101	197.20	73.68	25
2	23.23797	197.20	73.68	30
2	12.21369	197.20	73.68	35
2	9.07141	197.20	73.68	40
2	14.22752	197.20	73.68	50
2	7.32861	13.13	60.50	65
2	6.06072	13.13	60.50	80
2	5.48804	125.45	9.33	45
2	5.24172	125.45	9.33	50
2	4.66366	125.45	9.33	65
2	10.73313	279.45	51.28	5
2	4.11539	279.45	51.28	10
2	8.43986	279.45	51.28	15
2	8.70594	279.45	51.28	20
2	8.99593	279.45	51.28	25
2	6.50663	279.45	51.28	30
2	8.94427	125.00	70.45	5
2	8.23077	125.00	70.45	10
2	7.23417	125.00	70.45	15

**Figure C.2 : Continued.**



2	8.70594	125.00	70.45	20
2	7.99640	125.00	70.45	25
2	7.43615	125.00	70.45	30
2	7.85166	125.00	70.45	35
2	2.47402	125.00	70.45	40
2	5.99054	125.00	70.45	50
2	11.48763	125.00	70.45	55
2	3.33119	125.00	70.45	65
3	4.71495	13.13	60.50	85
3	3.48963	279.45	51.28	35
3	6.59739	279.45	51.28	40
3	3.13603	279.45	51.28	45
3	2.99527	279.45	51.28	50
3	2.76263	279.45	51.28	60

---

## SUBSOIL STRATIFICATION

----- O N E W A Y -----

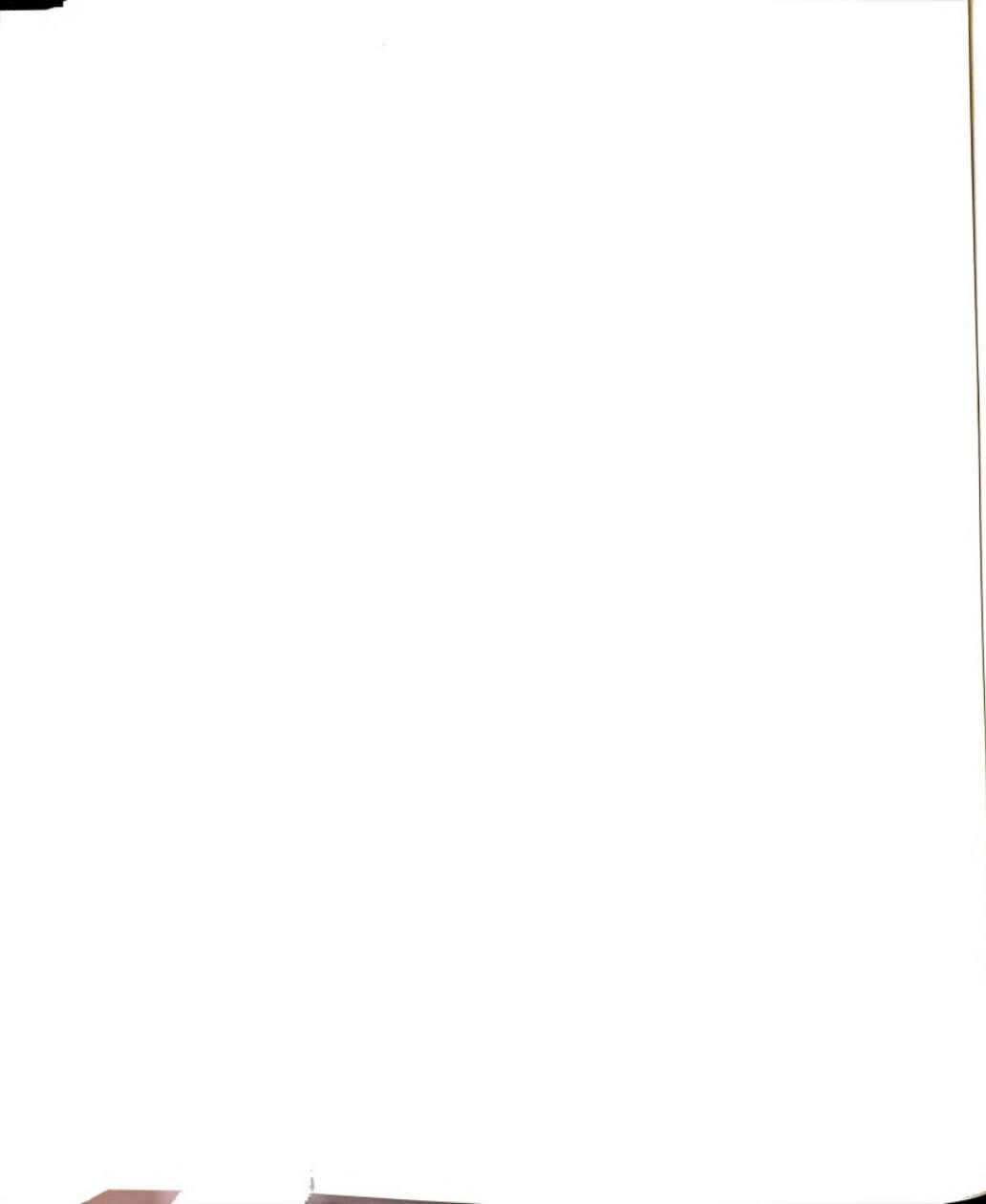
Variable N  
By Variable LAYER

## Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	1018.8349	509.4174	20.5792	.0000
Within Groups	62	1534.7507	24.7540		
Total	64	2553.5856			

---

**Figure C.2 : Continued.**



## SUBSOIL STRATIFICATION

----- O N E W A Y -----

Group	Count	Standard		Error	95 Pct Conf Int for Mean	
		Mean	Deviation		To	To
Grp 1	18	16.4123	7.2075	1.6988	12.8280	To 19.9965
Grp 2	41	8.6998	4.0026	.6251	7.4364	To 9.9632
Grp 3	6	3.9493	1.4690	.5997	2.4077	To 5.4909
Total	65	10.3971	6.3166	.7835	8.8319	To 11.9622
Fixed Effects Model			4.9753	.6171	9.1635	To 11.6306
Random Effects Model				3.7838	-5.8835	To 26.6776
Random Effects Model - Estimate of Between Component Variance						28.8490

## SUBSOIL STRATIFICATION

Group	Minimum	Maximum
Grp 1	6.8590	33.8116
Grp 2	2.4740	23.2380
Grp 3	2.7626	6.5974
Total	2.4740	33.8116

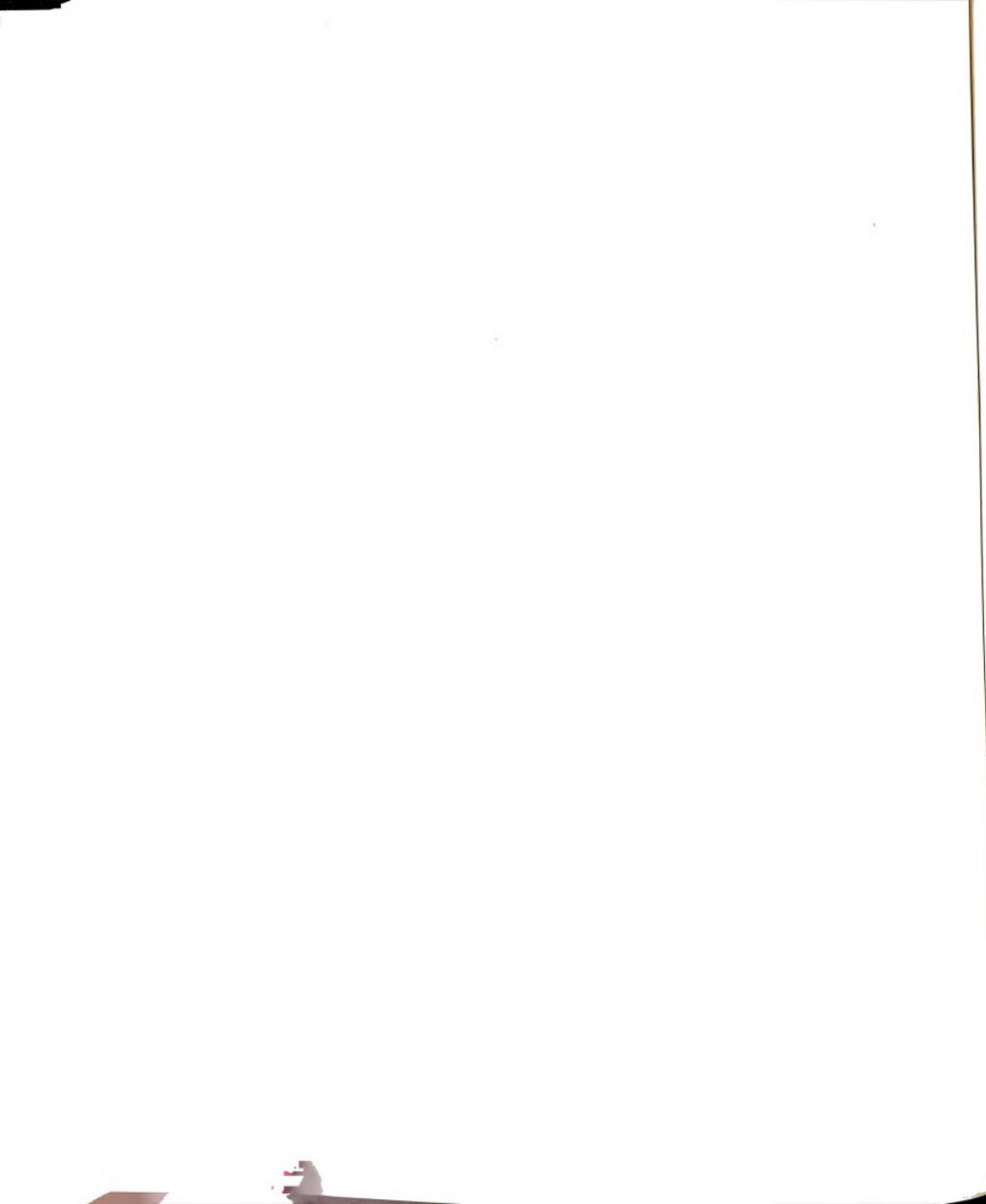
## Tests for Homogeneity of Variances

Cochrans C = Max. Variance/Sum(Variations) = .7408, P = .000 (Approx.)

Bartlett-Box F = 8.196, P = .000

Maximum Variance / Minimum Variance 24.073

**Figure C.2 : Continued.**



## SUBSOIL STRATIFICATION

----- O N E W A Y -----

Variable N  
By Variable LAYER

## Multiple Range Test

Tukey-HSD Procedure  
Ranges for the .050 level -

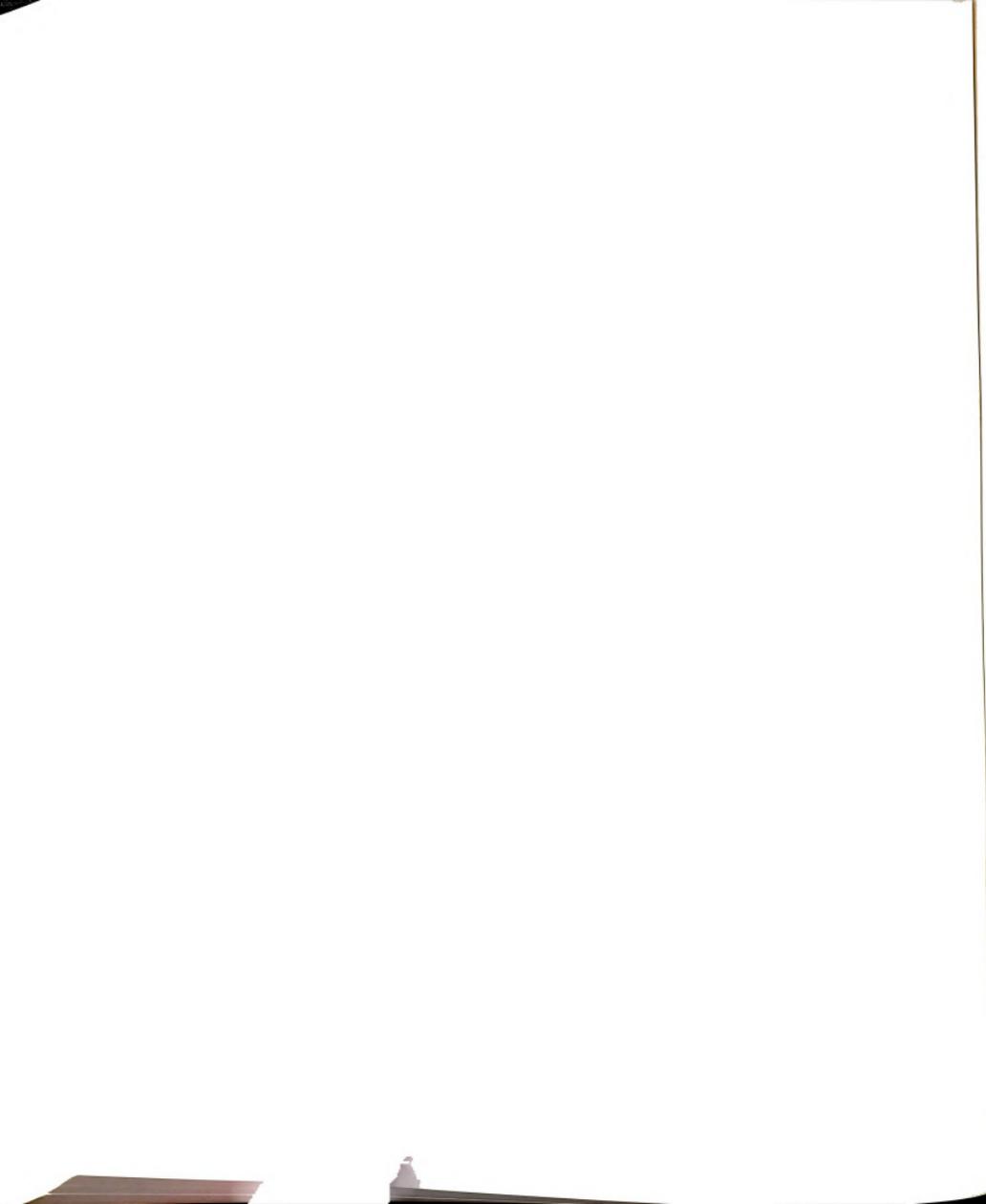
3.39 3.39

The ranges above are table ranges.  
The value actually compared with Mean(J)-Mean(I) is..  
 $3.5181 * \text{Range} * \text{Sqrt}(1/N(I) + 1/N(J))$

(\*) Denotes pairs of groups significantly different at the .050 level

		G G G
		r r r
		p p p
Mean	Group	3 2 1
3.9493	Grp 3	
8.6998	Grp 2	
16.4123	Grp 1	* *

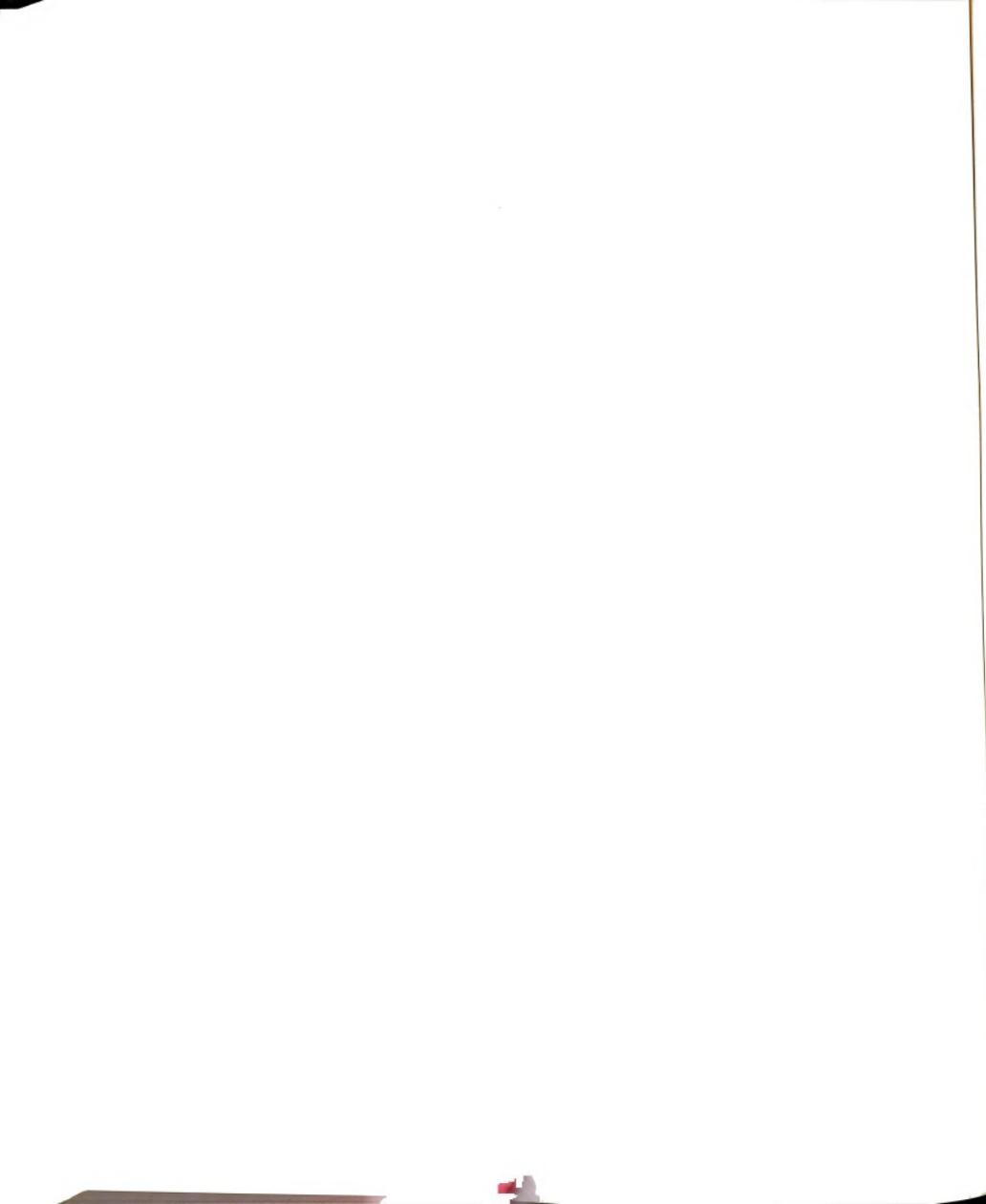
Figure C.2 : Continued.



## MODELING THE N FUNCTION

LAYER	N1	X	Y	Z
2	7.15542	205.47	108.93	5
2	5.48718	205.47	108.93	10
2	4.82278	205.47	108.93	15
2	8.70594	205.47	108.93	20
2	10.99505	205.47	108.93	25
2	5.57711	205.47	108.93	30
2	7.85166	205.47	108.93	35
2	10.72076	205.47	108.93	40
2	10.19208	205.47	108.93	45
2	11.23225	205.47	108.93	50
2	7.15542	197.20	73.68	5
2	6.85898	197.20	73.68	10
2	13.26264	197.20	73.68	15
2	11.97067	197.20	73.68	20
2	19.99101	197.20	73.68	25
2	23.23797	197.20	73.68	30
2	12.21369	197.20	73.68	35
2	9.07141	197.20	73.68	40
2	14.22752	197.20	73.68	50
2	7.32861	13.13	60.50	65
2	6.06072	13.13	60.50	80
2	5.48804	125.45	9.33	45
2	5.24172	125.45	9.33	50
2	4.66366	125.45	9.33	65

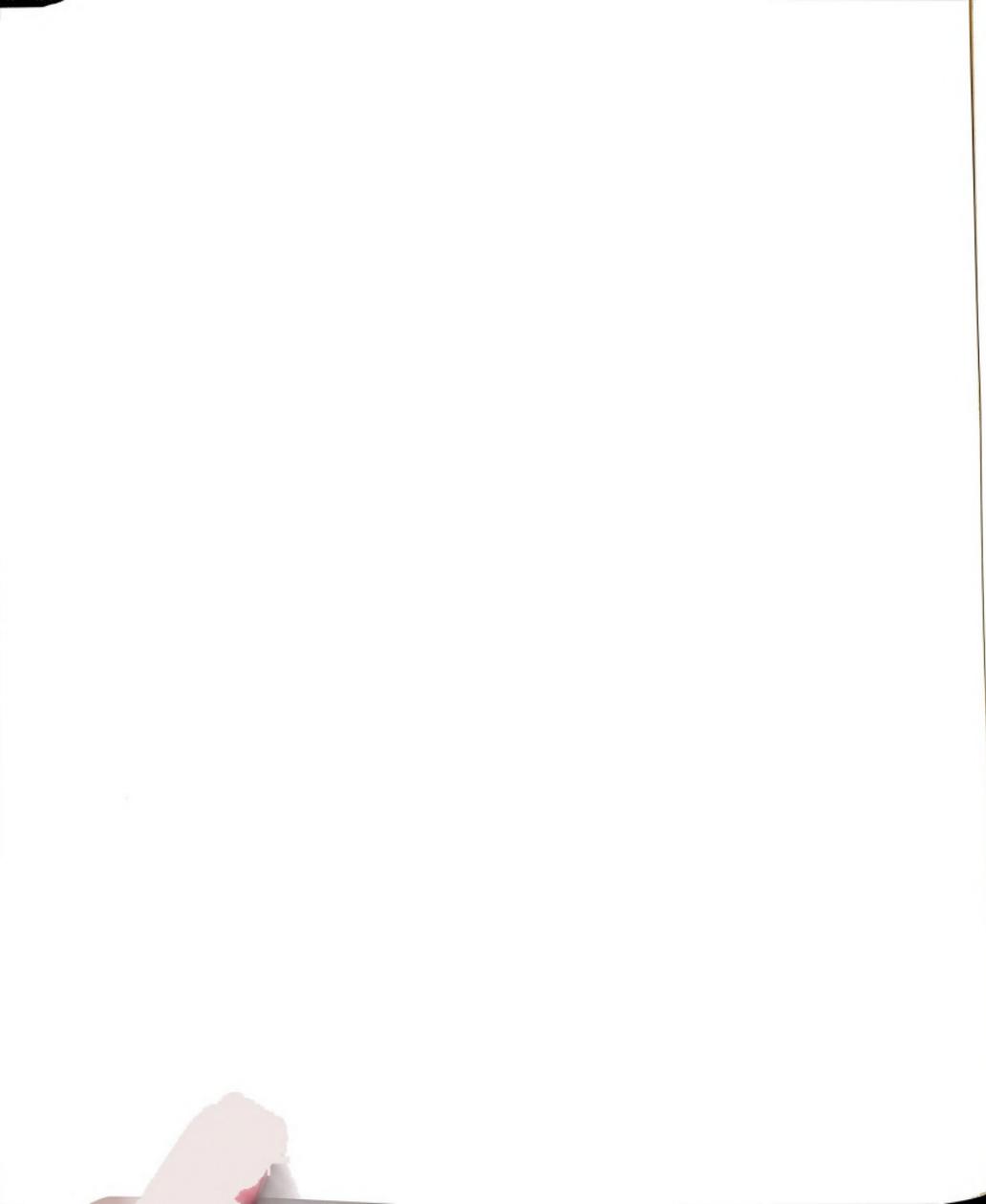
Figure C.2 : Continued.



2	10.73313	279.45	51.28	5
2	4.11539	279.45	51.28	10
2	8.43986	279.45	51.28	15
2	8.70594	279.45	51.28	20
2	8.99593	279.45	51.28	25
2	6.50663	279.45	51.28	30
2	8.94427	125.00	70.45	5
2	8.23077	125.00	70.45	10
2	7.23417	125.00	70.45	15
2	8.70594	125.00	70.45	20
2	7.99640	125.00	70.45	25
2	7.43615	125.00	70.45	30
2	7.85166	125.00	70.45	35
2	2.47402	125.00	70.45	40
2	5.99054	125.00	70.45	50
2	11.48763	125.00	70.45	55
2	3.33119	125.00	70.45	65
2	4.71495	13.13	60.50	85
2	3.48963	279.45	51.28	35
2	6.59739	279.45	51.28	40
2	3.13603	279.45	51.28	45
2	2.99527	279.45	51.28	50
2	2.76263	279.45	51.28	60

---

**Figure C.2 : Continued.**



N2=F(X,Y,Z),LAYERS 2&3 COMBINED & NAMED LAYER 2

THE FITTED MODEL:

$$N=D0+D1*X^{.5}+D2*X+D3*X^2+D4*Y^{.5}+D5*Y+D6*Z^{.5}+D7*Z+D8*Z^2.$$

All the derivatives will be calculated numerically.

---

There are 47 cases. There is enough memory for them all.

Run stopped after 5 model evaluations and 3 derivative evaluations.  
Iterations have been stopped because the magnitude of the largest correlation  
between the residuals and any derivative column is at most RCON = 1.000E-08

---

N2=F(X,Y,Z),LAYERS 2&3 COMBINED & NAMED LAYER 2

Nonlinear Regression Summary Statistics      Dependent Variable N

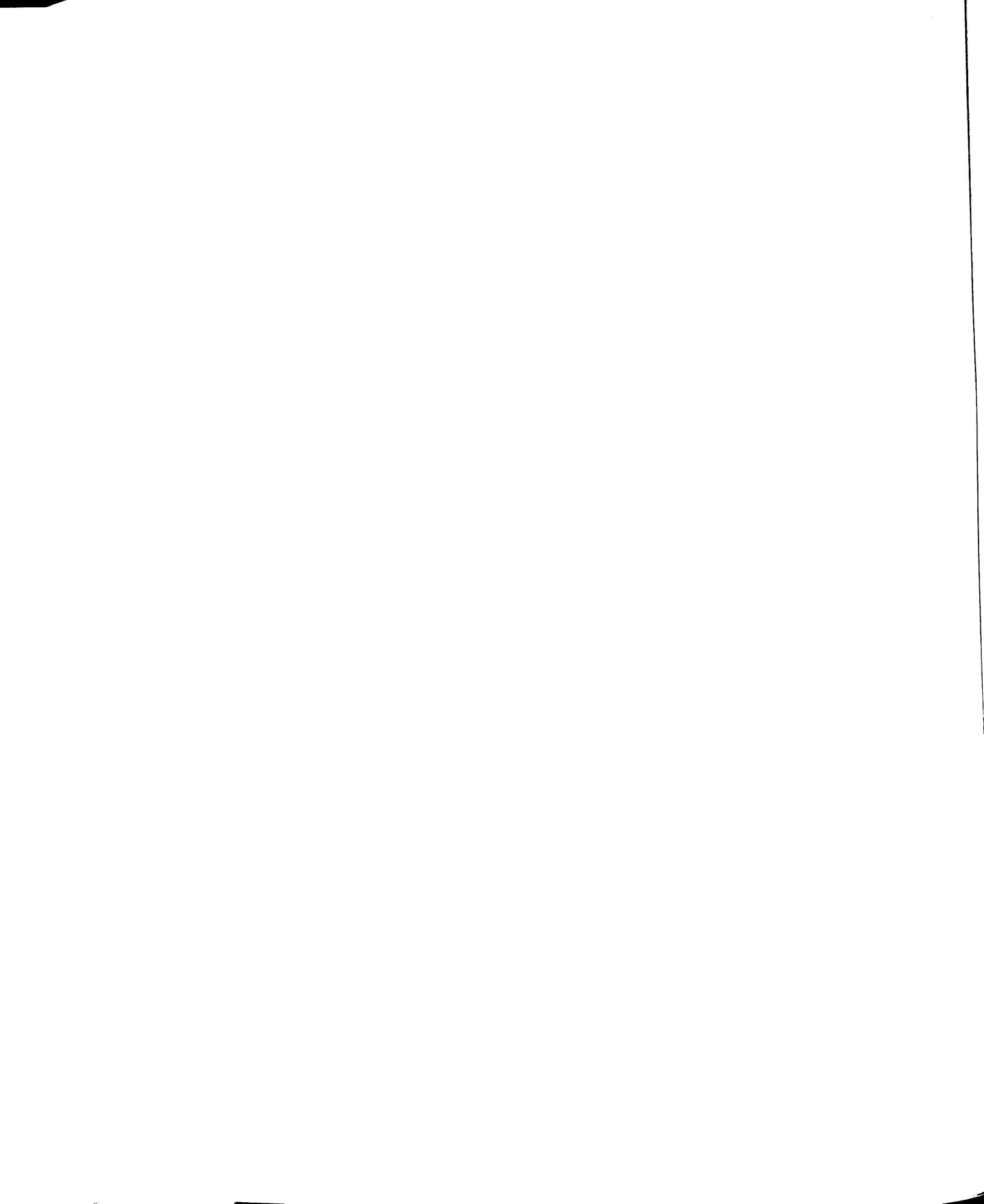
Source	DF	Sum of Squares	Mean Square
Regression	9	3431.41795	381.26866
Residual	38	416.94573	10.97226
Uncorrected Total	47	3848.36368	
(Corrected Total)	46	769.74908	

R squared = 1 - Residual SS / Corrected SS = .45834

Parameter	Estimate	Std. Error	Asymptotic 95 % Confidence Interval	
			Lower	Upper
D0	35.661381270	17.240568745	.759674519	70.563088020
D1	-16.70819947	3.996932525	-24.79956635	-8.616832596
D2	1.386085906	.317411920	.743519068	2.028652745
D3	-.001993047	.000441309	-.002886430	-.001099664
D4	4.568505574	1.641274926	1.245918192	7.891092957
D5	-.375674112	.122035926	-.622722928	-.128625295
D6	1.737697232	4.613403969	-7.601650840	11.077045303
D7	-.107177359	.680007779	-1.483781138	1.269426420
D8	-.001129414	.003774031	-.008769541	.006510713

---

Figure C.2 : Continued.



## Asymptotic Correlation Matrix of the Parameter Estimates

	D0	D1	D2	D3	D4	D5
D0	1.0000	-.7280	.6490	-.5675	-.1693	.0541
D1	-.7280	1.0000	-.9884	.9581	-.2737	.4099
D2	.6490	-.9884	1.0000	-.9898	.3294	-.4677
D3	-.5675	.9581	-.9898	1.0000	-.3832	.5208
D4	-.1693	-.2737	.3294	-.3832	1.0000	-.9732
D5	.0541	.4099	-.4677	.5208	-.9732	1.0000
D6	-.6369	.1772	-.1281	.0863	-.0733	.0606
D7	.6551	-.2041	.1451	-.0953	.0734	-.0616
D8	-.6933	.2518	-.1704	.1032	-.0403	.0375

---

	D6	D7	D8
D0	-.6369	.6551	-.6933
D1	.1772	-.2041	.2518
D2	-.1281	.1451	-.1704
D3	.0863	-.0953	.1032
D4	-.0733	.0734	-.0403
D5	.0606	-.0616	.0375
D6	1.0000	-.9889	.9198
D7	-.9889	1.0000	-.9650
D8	.9198	-.9650	1.0000

---

Figure C.2 : Continued.

## **C.2 THE SETTLEMENT PREDICTION OF CASE HISTORY No. 3**

### **C.2.1 PROJECT GENERAL DESCRIPTION**

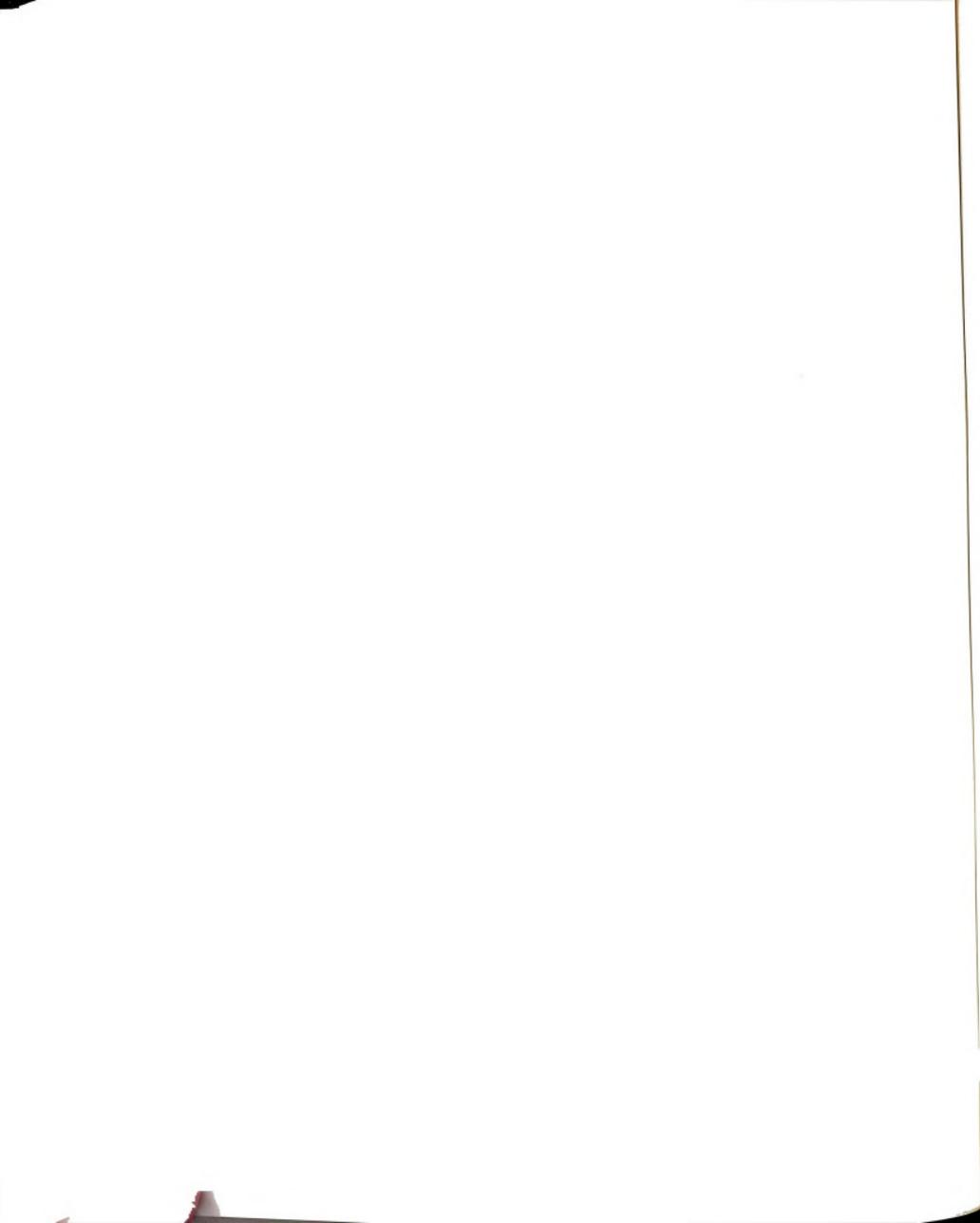
Settlement histories are described for four steel grain storage tanks. The settlements of this project were previously studied and reported by Davisson, M.T. et. al.(1972). The site is located on a nearly level flood plain in Kansas City, Missouri, approximately one- quarter mile south of the Missouri River.

The history of the site is relatively simple. An old farmhouse was located in the area now occupied by tank D as shown in Figure C.3 and it is assumed that the remainder of the site was used for farming purposes. Approximately 5 ft of the flood plain deposit covers the site and is mixed intermittently with miscellaneous fill. Tank B is selected for settlement analysis because it was not preconsolidated by the existance of a previous building like tank D and is not located outside the available boring locations like tank A.

Tank B is 110 ft in diameter and has a wall height of 45 ft. The roof is cone-shaped and has a slope of 27 degrees from horizontal, therefore the total height of the structure to the tip of the cone is 73 ft.

### **C.2.2 SUBSOIL INVESTIGATION**

The subsoil investigation consists of three borings (B1 , B2 & B3) the depths were 51 ft , 50 ft and 80 ft , respectively. Standard penetration test N values have been reported. The correction factors of N values for the overburden pressure are calculated as explained in section A.1 and are shown in Table C.5. The corrected N values for each boring are shown in Tables C.6 to C.8.



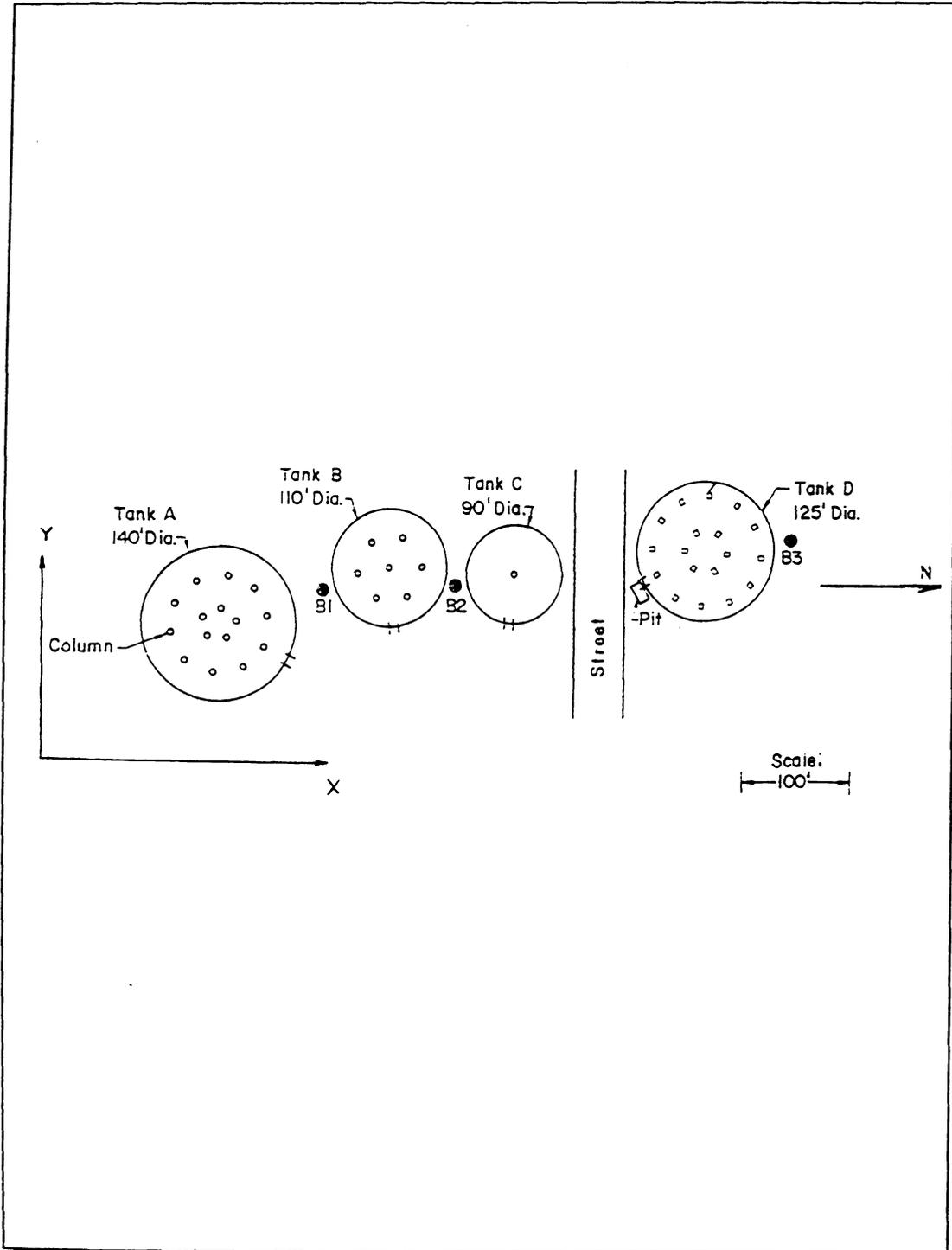
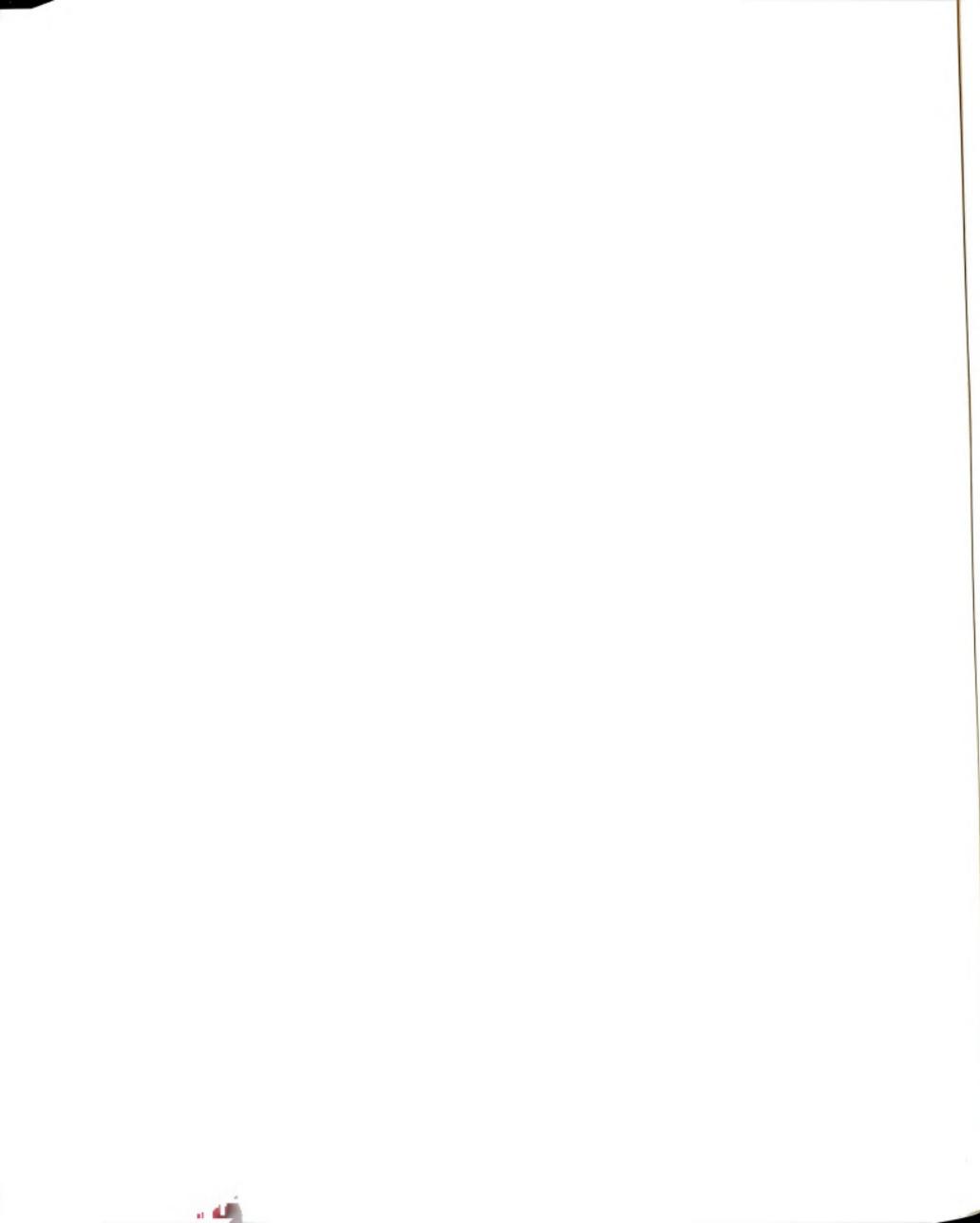


Figure C.3 : Site Plan And Boring Locations Of Case History No. 3

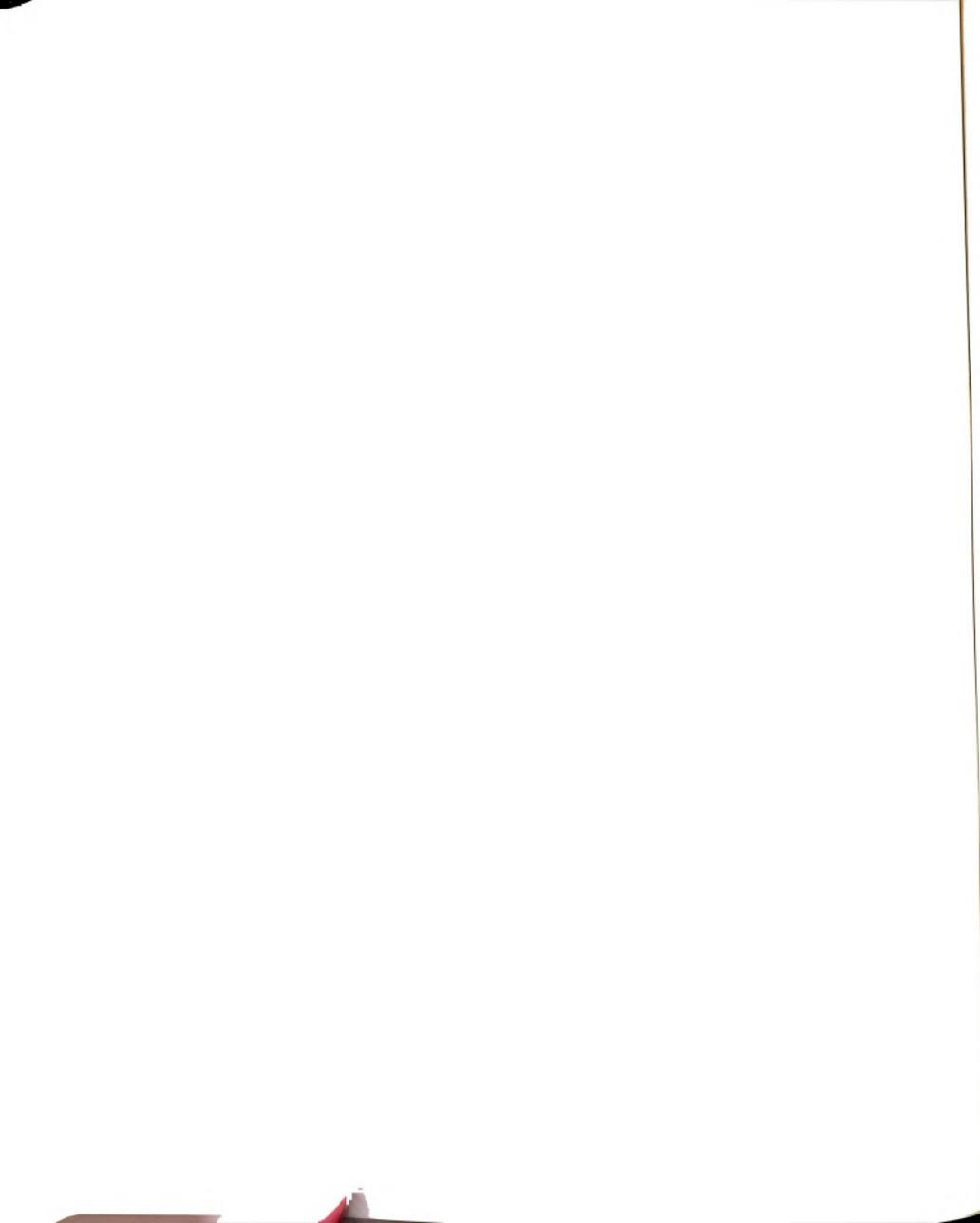


**Table C.5 : The Correction Factors For Overburden Pressure  
Of N Values Of Case History No. 3**

Depth(ft)	Unit wt. lb/cf.	Effective Pressure (lb/sf)	Correction factor (C1)
5	125.0	625.0	1.788854
10	125.0	1250.0	1.264911
15	125.0	1875.0	1.032796
20	125.0	2500.0	0.8944272
25	62.6	2937.8	0.8250949
30	62.6	3250.8	0.7843680
35	62.6	3563.8	0.7491320
40	62.6	3876.8	0.7182544
45	62.6	4189.8	0.6909050
50	62.6	4502.8	0.6664594
55	62.6	4815.8	0.6444375
60	62.6	5128.8	0.6244636
65	62.6	5441.8	0.6062388
70	62.6	5754.8	0.5895218
75	62.6	6067.8	0.5741156
80	62.6	6380.8	0.5598574

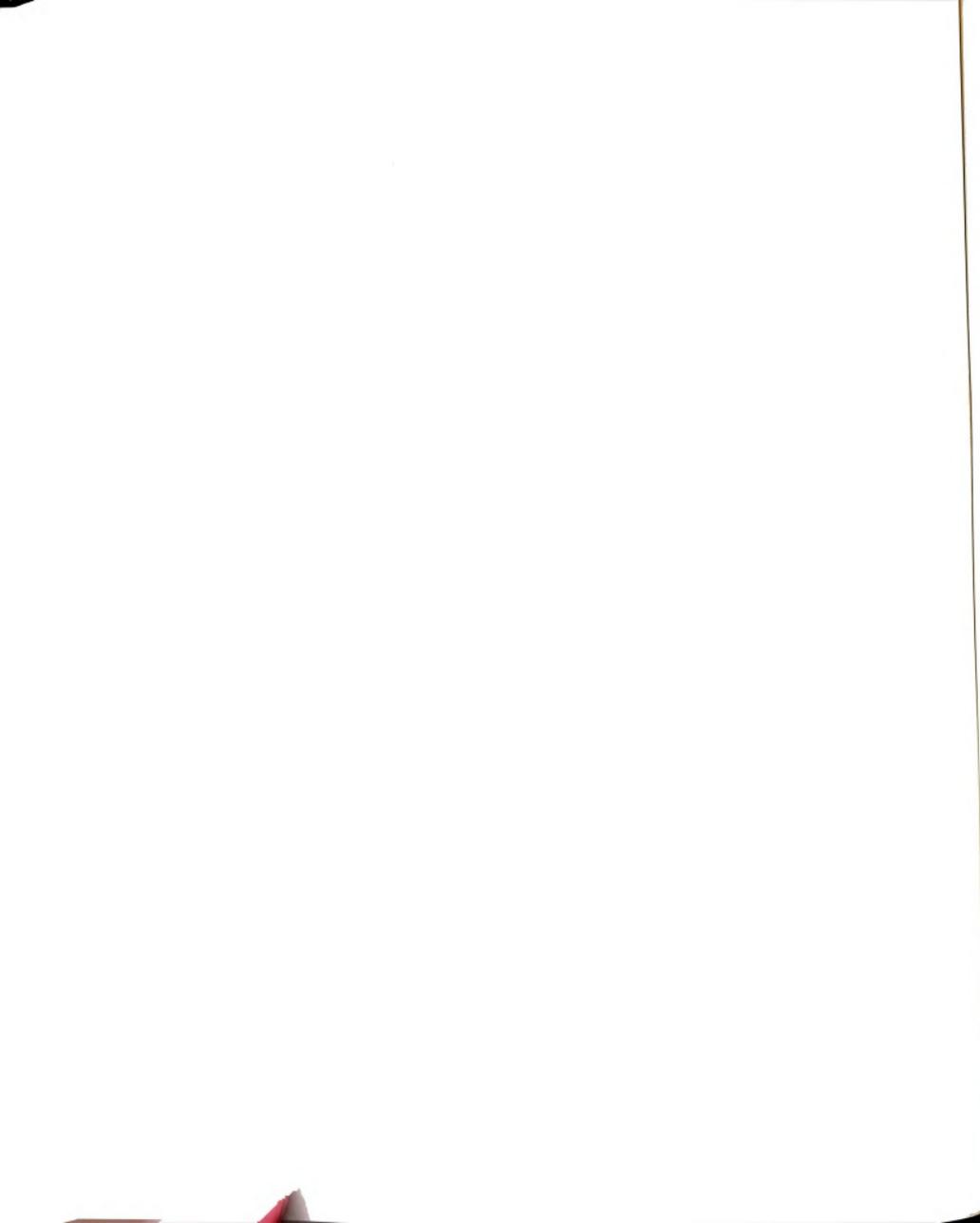
**Table C.6 : The Correction Of N Values Of Boring B1.**

Depth (ft)	Boring (B1)	
	N	$N1 = C1 * N$
5	6	10.733130
10	14	17.708750
15	12	12.393550
20	14	12.521980
25	11	9.076044
30	20	15.687360
35	17	12.735240
40	21	15.083340
45	28	19.345340
50	100	66.645940



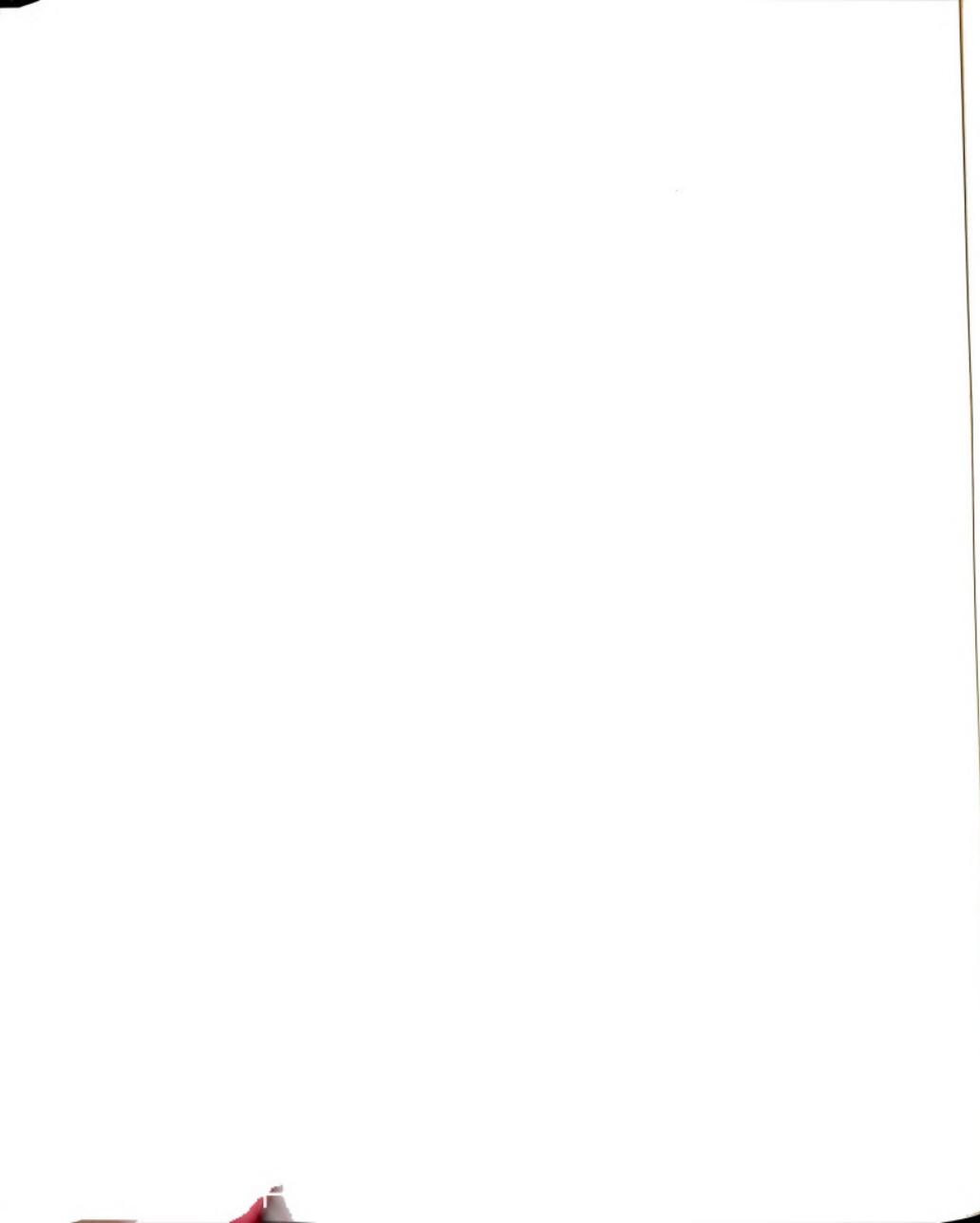
**Table C.7 : The Correction Of N Values Of Boring B2.**

Depth (ft)	Boring (B2)	
	N	$N_1 = C_1 * N$
5	5	8.944272
10	11	13.914020
15	15	15.491930
20	18	16.099690
25	22	18.152090
30	16	12.549890
35	34	25.470490
40	28	20.111120
45	67	46.290640
50	120	79.975120



**Table C.8 : The Correction Of N Values Of Boring B3.**

Depth (ft)	Boring (B3)	
	N	$N_1 = C_1 * N$
5	4	7.155418
10	14	17.708750
15	4	4.131182
20	24	21.466250
25	36	29.703420
30	17	13.334260
35	32	23.972220
40	100	71.825440
45	14	9.672670
50	16	10.663350
55	21	13.533190
60	120	74.935630
65	60	36.374330
70	75	44.214140
75	24	13.778780
80	33	18.475290



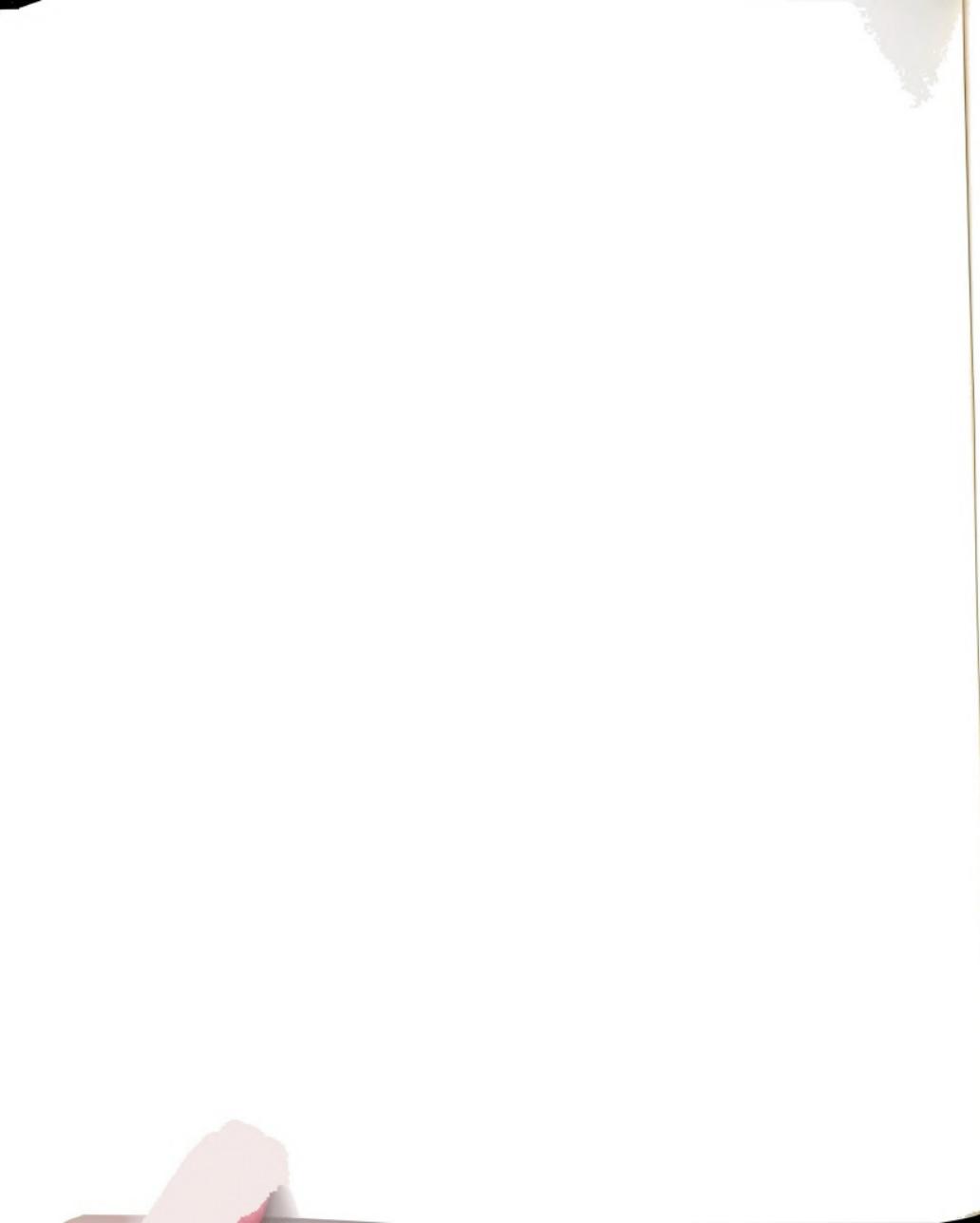
In general, the soil profile is fine to medium sand of low to medium relative density in the upper 40 ft, and dense to very dense sand at depths greater than 40 ft. The N values in the upper 20 ft are particularly low, ranging from 4 to 24. At a depth of approximately 40 ft to 50 ft dense to very dense sands are encountered that have N values exceeding 50. In boring B3 the N values indicate that a layer of medium sand may be encountered at a depth of approximately 75 ft.

Clay layers 2 in. to 3 in. thick were encountered in borings B1 and B3 at depths ranging from 30 ft to 40 ft and 50 ft to 60 ft, respectively. These clays had an estimated unconfined compression strength exceeding 1 tsf. It is not believed that presence of the clays in very small amounts at relatively large depths alters significantly the compressibility of an otherwise purely granular soil deposit.

### C.2.3 FOUNDATIONS AND SETTLEMENT MEASUREMENTS

The foundation consists of a waterbound macadam raft 2 ft thick. A steel plate with a thickness of 3/16 in. covers the entire floor area of the tank. The roof is supported, theoretically, by both the walls and several interior columns. Tank B imposes an average load of 1.48 tsf on its foundation. This figure was obtained by considering the entire volume of the tank to be filled with soybeans and then adding an allowance for compaction.

Settlement observations were made on the interior of the tank after it had been subjected to several load - unload cycles. The observations were made close to the columns supporting the roof, but not close enough to be influenced by local column



settlements. The settlements of tank B vary from 3 in. to 4 in. over most of the tank area except for the extreme west side where the observed settlement was only 1.7 in. The average settlement over the tank area was 3.3 in.

#### C.2.4 APPLYING THE KRIGING TECHNIQUE

Considering that a settlement prediction is required at the center point of the foundation of tank "B". The Kriging results are summarized as follows:

1. The calculated covariance function is given by the equation:

$$C(h) = 371.95e^{-4.44E-5(h^2)} \quad (\text{C.8})$$

2. The estimated N function is given by:

$$\hat{N} = 7.504835 + 0.3681199Z \quad (\text{C.9})$$

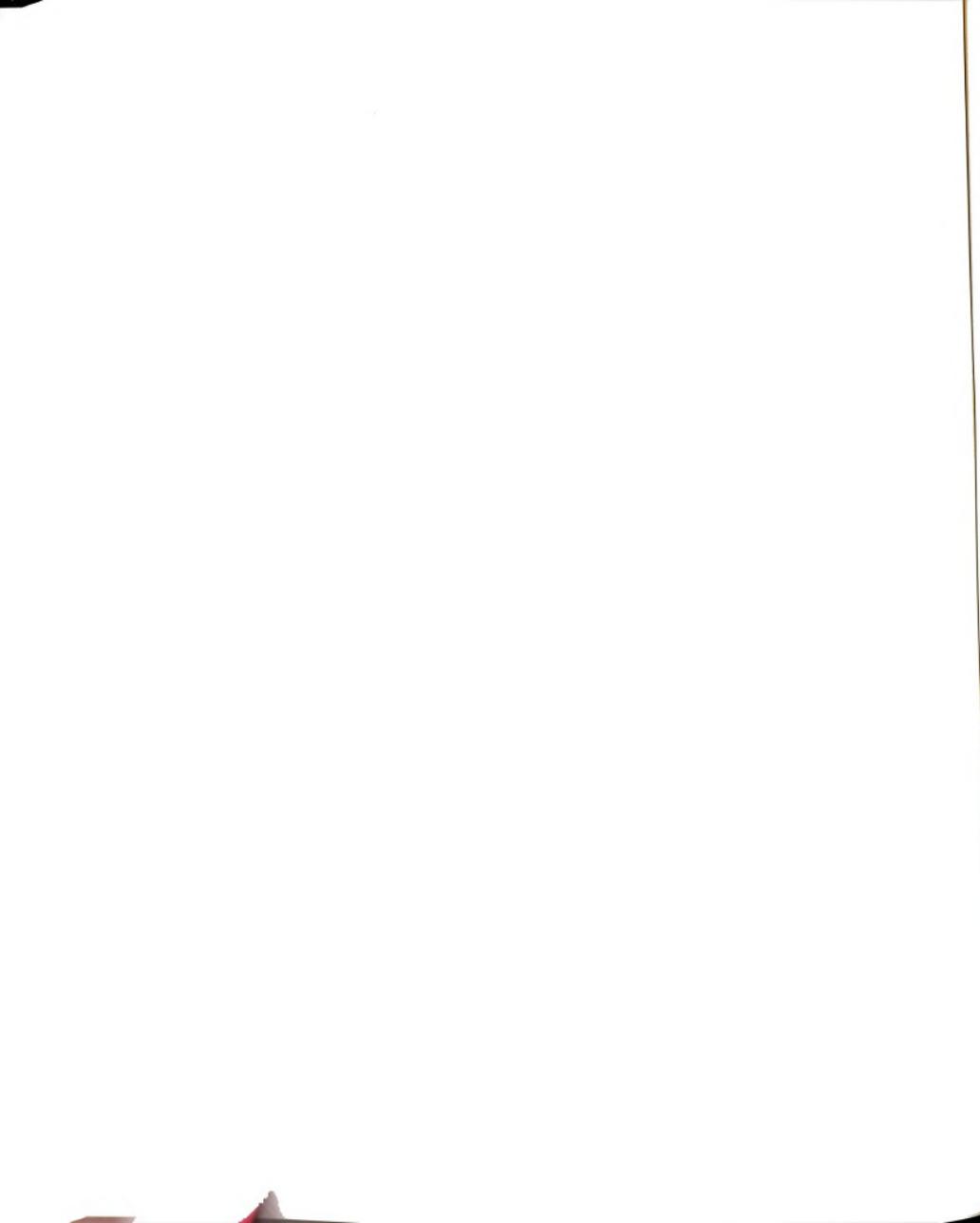
3. The "two-point" estimate of N values are:

$$\hat{N}_{(B/2)} = 27.75, \hat{N}_{(3B/2)} = 68.24 \quad (\text{C.10})$$

4. The design N value is given by the weighted average:

$$N = (1/3) [2\hat{N}_{(B/2)} + \hat{N}_{(3B/2)}] = 41.24 \quad (\text{C.11})$$

5. The predicted settlement is 3.3 in.



Therefore the predicted settlement of 3.1 in is within about 6% of the measured value of 3.3 in.

6. The 90% confidence limits of the settlement prediction are:

(2.2 and 5.13) in.

The 50% confidence limits are:

(2.76 and 3.48) in.

### C.2.5 APPLYING THE TREND SURFACE ANALYSIS TECHNIQUE

The trend surface analysis results are summarized as follows:

1. The model which is fitted to the data is given by:

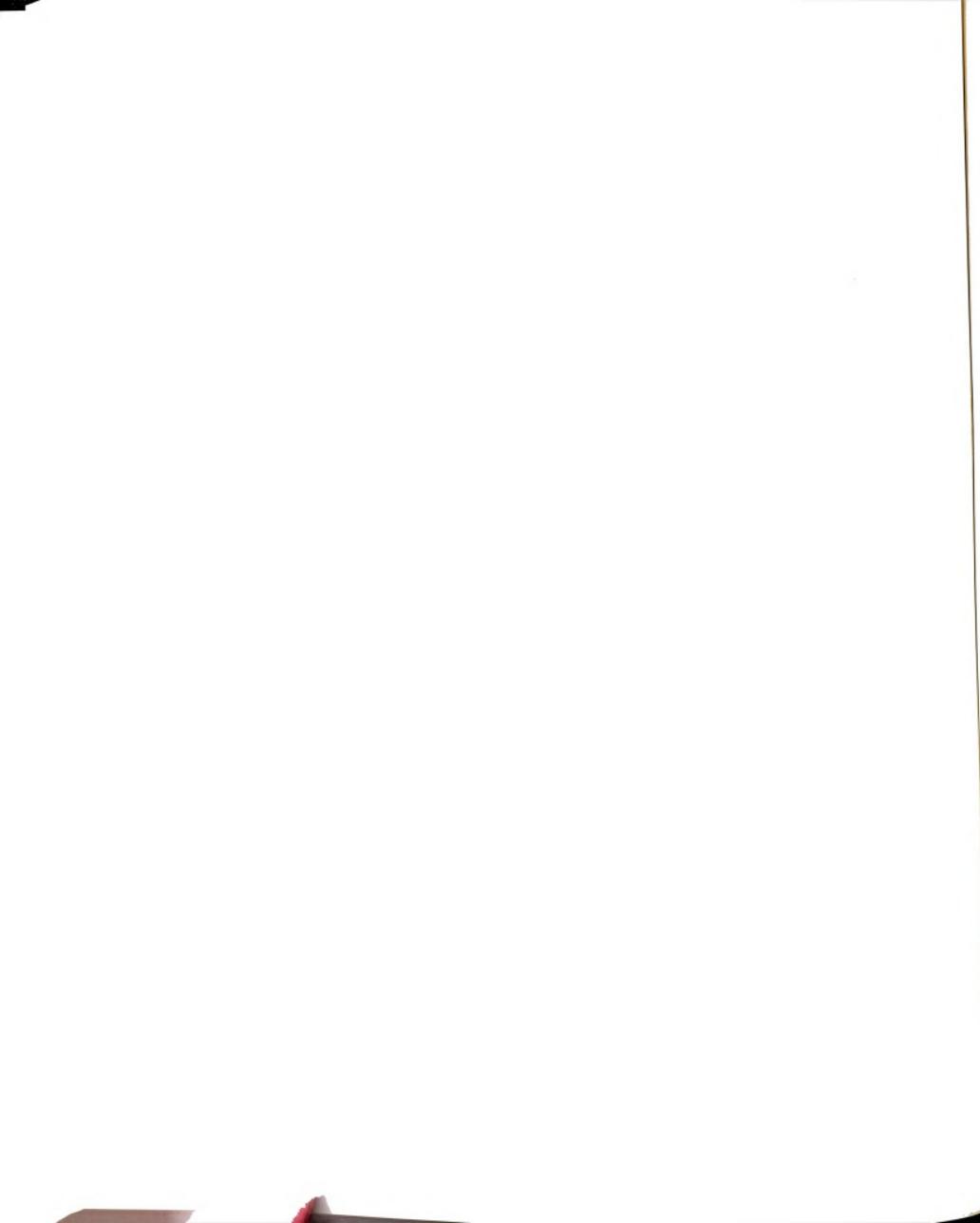
$$N = -1254 - 0.0082X + 1476Z^{0.5} - 624Z + 114.3Z^{1.5} - 7.68Z^2 \quad (\text{C.12})$$

$$(R^2 = 0.65).$$

2. The design N value is  $N = 3.16$ .
3. The predicted settlement is 4.03 in. Therefore the predicted settlement of 4.03 in is within about 22% of the measured value of 3.3 in.
4. The 90% confidence limits of the settlement prediction are: (0.77 and 7.29) in.  
The 50% confidence limits are: (1.52 and 6.54) in.
5. The areal distribution of settlement in inches is given by the equation:

$$S = 12.744 / (1.63 - 0.0082X) \quad (\text{C.13})$$

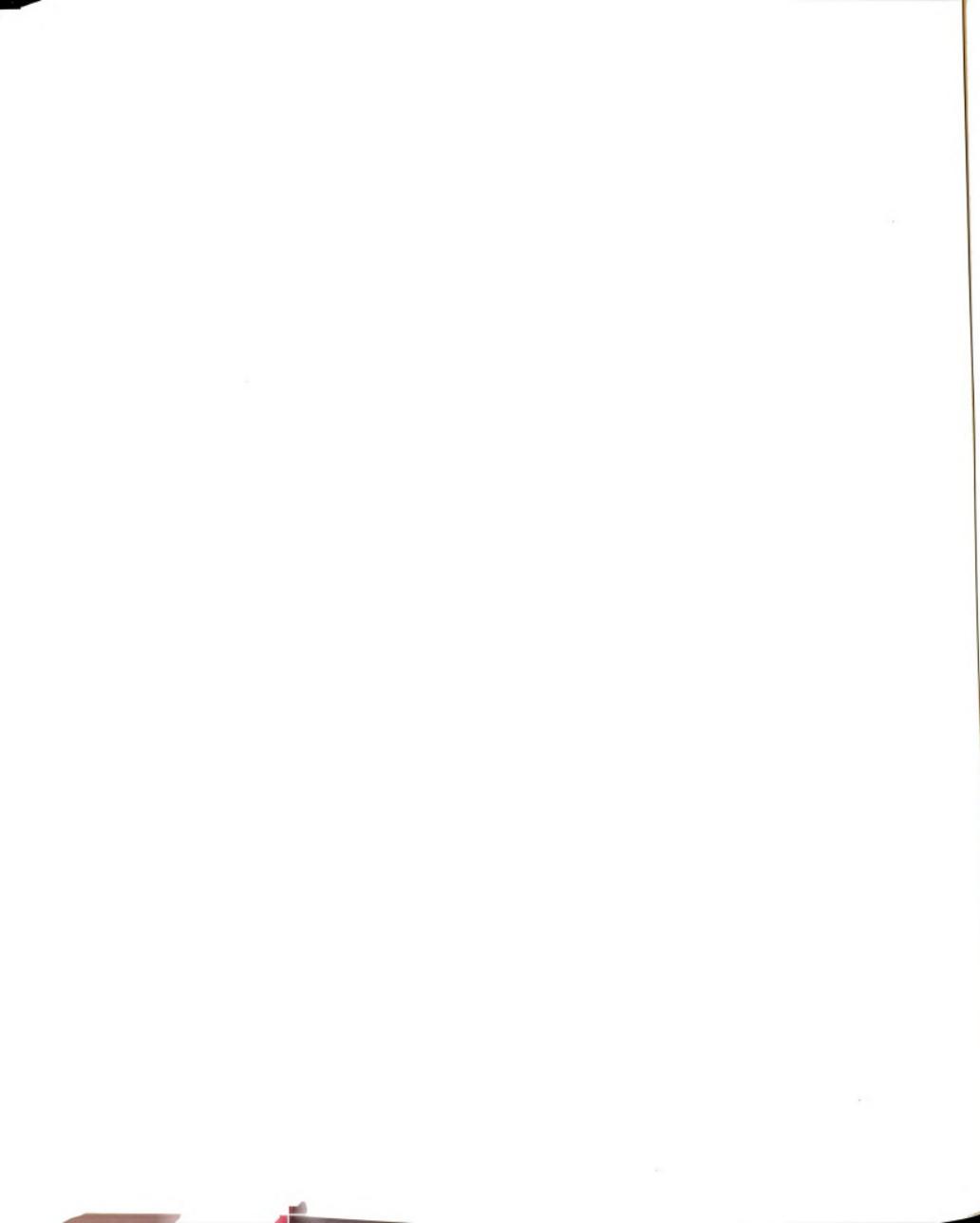
The computer output is shown in Figure C.4.



## SUBSURFACE SOIL STRATIFICATION

LAYER	N1	X(ft)	Y(ft)	Z(ft)
1	10.73313	252	159	5
1	17.70875	252	159	10
1	12.39355	252	159	15
1	12.52198	252	159	20
1	9.07604	252	159	25
2	15.68736	252	159	30
2	12.73524	252	159	35
2	15.08334	252	159	40
2	19.34534	252	159	45
1	8.94427	378	163	5
1	13.91402	378	163	10
1	15.49193	378	163	15
1	16.09969	378	163	20
2	18.15209	378	163	25
2	12.54989	378	163	30
1	7.15542	690	206	5
1	17.70875	690	206	10
1	4.13118	690	206	15
2	21.46625	690	206	20
2	29.70342	690	206	25
2	13.33426	690	206	30
3	66.64594	252	159	50
3	25.47049	378	163	35
3	20.11112	378	163	40
3	46.29064	378	163	45

**Figure C.4 : The Computer Output Of Case History No. 3**



3	79.97512	378	163	50
3	23.97222	690	206	35
3	71.82544	690	206	40
3	9.67267	690	206	45
3	10.66335	690	206	50
3	13.53319	690	206	55
3	74.93563	690	206	60
3	36.37433	690	206	65
3	44.21414	690	206	70
3	13.77878	690	206	75
3	18.47529	690	206	80

---

## LARGE TANKS FOUNDED ON A SANDY SITE IN KANSAS CITY

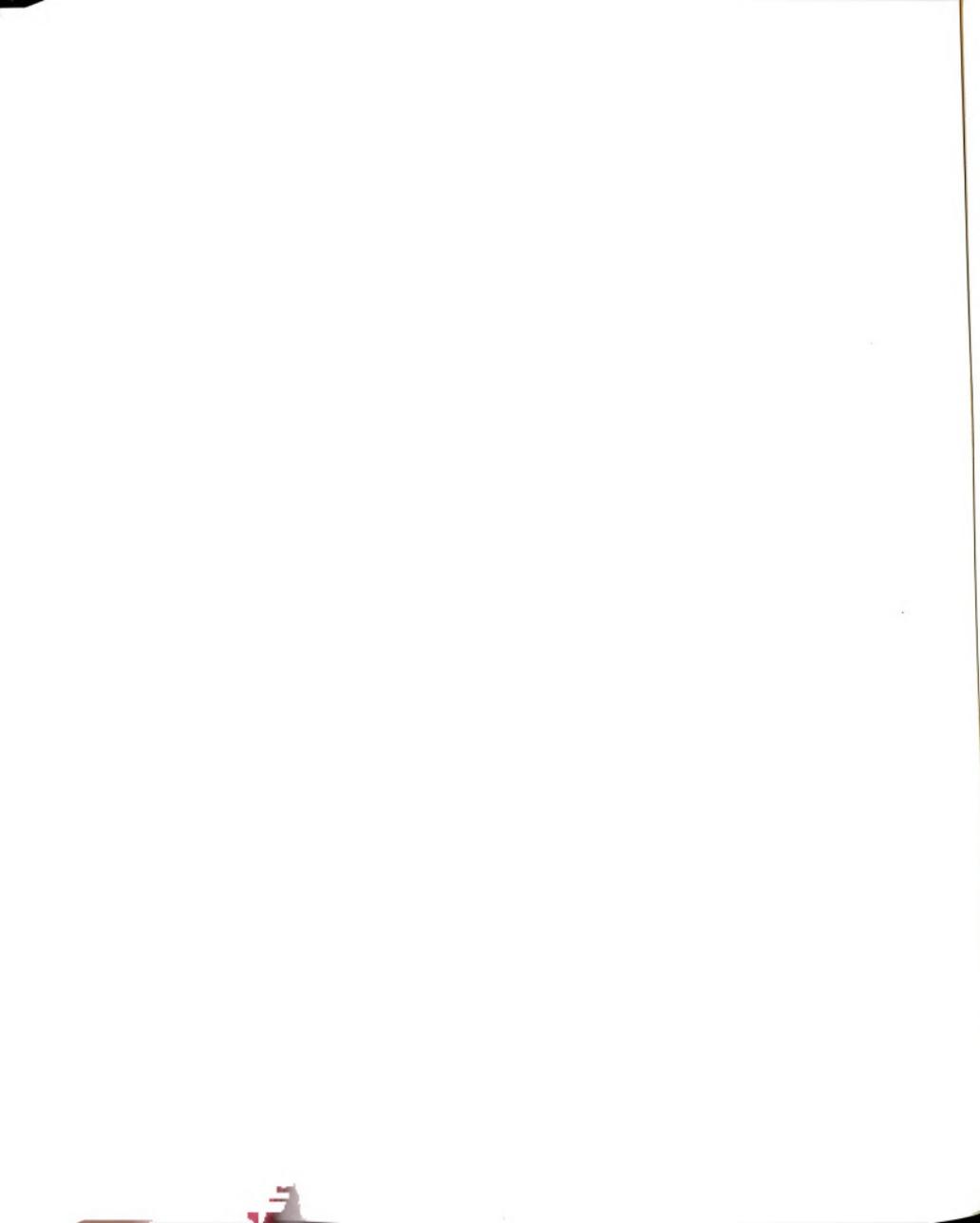
----- O N E W A Y -----

Variable N  
By Variable LAYER

Analysis of Variance					
Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	4615.2291	2307.6145	8.0817	.0014
Within Groups	33	9422.7228	285.5371		
Total	35	14037.9519			

---

**Figure C.4 : Continued.**



Group	Count	Standard		Error	95 Pct Conf Int for Mean	
		Mean	Deviation		To	To
Grp 1	12	12.1566	4.2944	1.2397	9.4280	To 14.8851
Grp 2	9	17.5619	5.5016	1.8339	13.3330	To 21.7908
Grp 3	15	37.0626	25.3232	6.5384	23.0390	To 51.0861
Total	36	23.8854	20.0271	3.3378	17.1092	To 30.6616
Fixed Effects Model			16.8978	2.8163	18.1556	To 29.6152
Random Effects Model				8.2271	-11.5135	To 59.2843
Random Effects Model - Estimate of Between Component Variance						172.0917

Group	Minimum	Maximum
Grp 1	4.1312	17.7087
Grp 2	12.5499	29.7034
Grp 3	9.6727	79.9751
Total	4.1312	79.9751

Tests for Homogeneity of Variances

Cochrans C = Max. Variance/Sum(Variances) = .9294, P = .000 (Approx.)  
 Bartlett-Box F = 17.901, P = .000  
 Maximum Variance / Minimum Variance 34.773

----- O N E W A Y -----

Variable N  
 By Variable LAYER

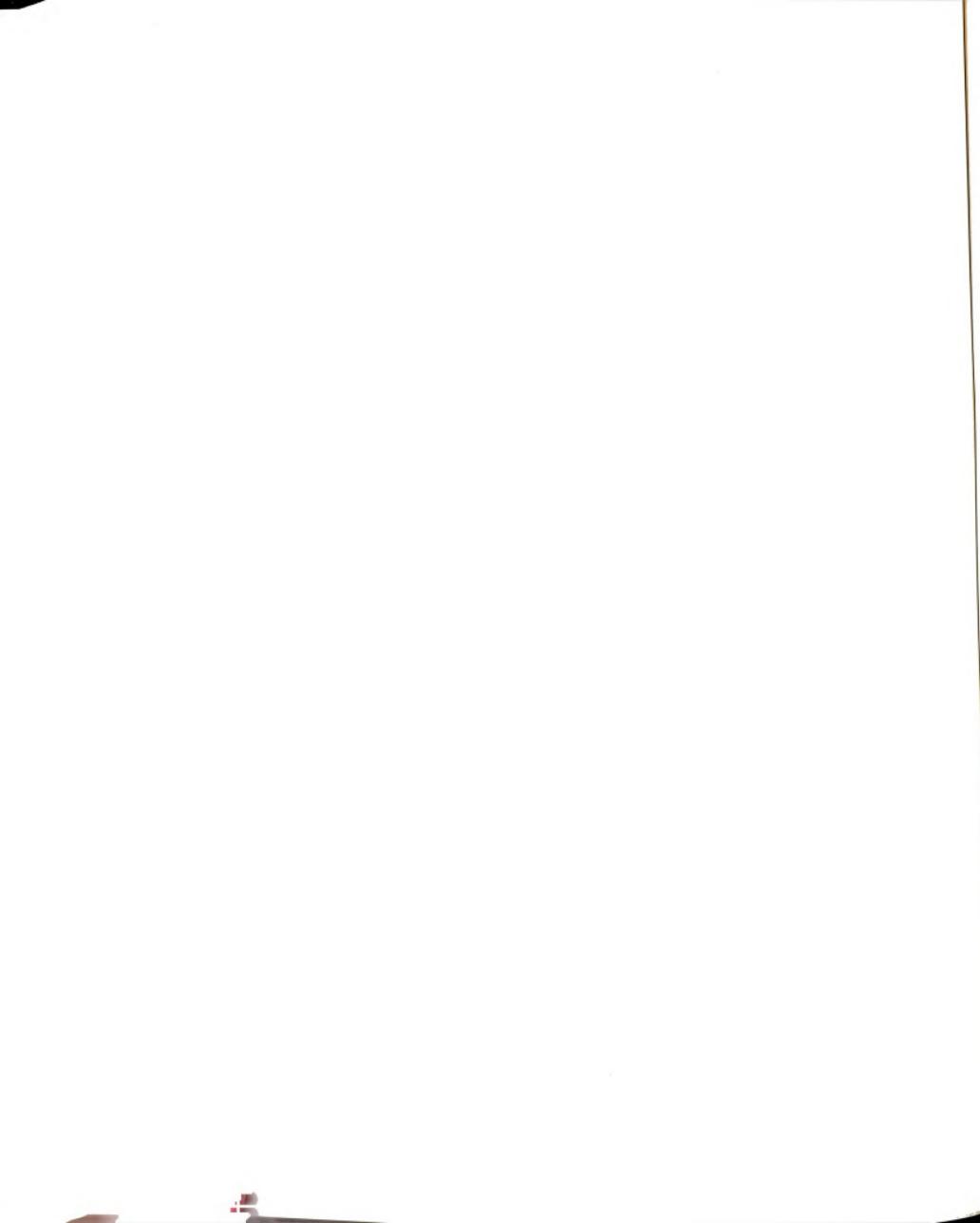
Multiple Range Test

Tukey-HSD Procedure  
 Ranges for the .050 level -

3.46 3.46

The ranges above are table ranges.  
 The value actually compared with Mean(J)-Mean(I) is..  
 $11.9486 * \text{Range} * \text{Sqrt}(1/N(I) + 1/N(J))$

Figure C.4 : Continued.



(\*) Denotes pairs of groups significantly different at the .050 level

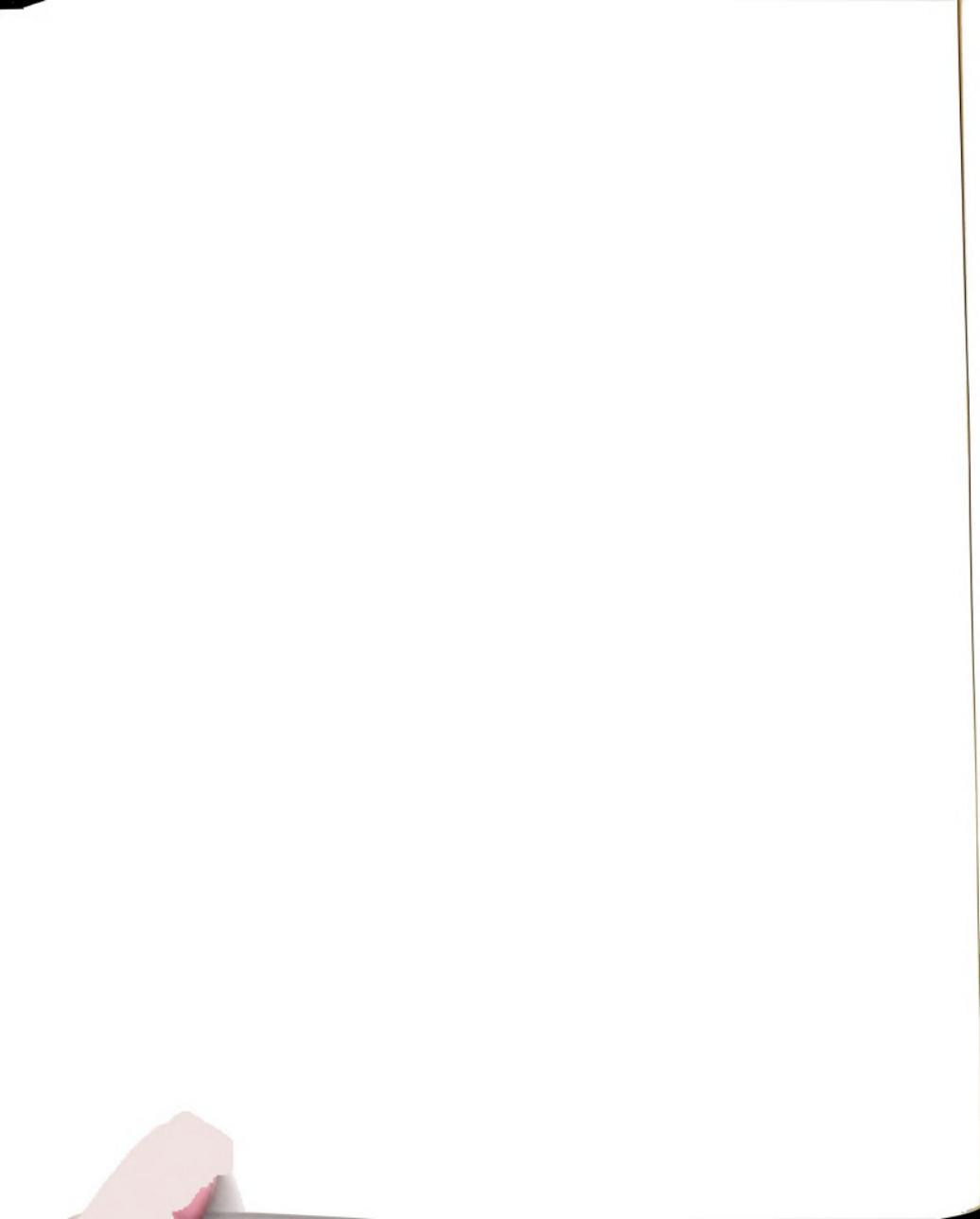
		G G G	
		r r r	
		P P P	
Mean	Group	1 2 3	
12.1566	Grp 1		
17.5619	Grp 2		
37.0626	Grp 3	* *	

---

#### MODELING THE N FUNCTIONS FOR THE SUBSURFACE SOIL

LAYER	N1	X(ft)	Y(ft)	Z(ft)
1	10.73313	252	159	5
1	17.70875	252	159	10
1	12.39355	252	159	15
1	12.52198	252	159	20
1	9.07604	252	159	25
2	15.68736	252	159	30
2	12.73524	252	159	35
2	15.08334	252	159	40
2	19.34534	252	159	45
1	8.94427	378	163	5
1	13.91402	378	163	10
1	15.49193	378	163	15
1	16.09969	378	163	20
2	18.15209	378	163	25
2	12.54989	378	163	30
1	7.15542	690	206	5

**Figure C.4 : Continued.**



1	17.70875	690	206	10
1	4.13118	690	206	15
2	21.46625	690	206	20
2	29.70342	690	206	25
2	13.33426	690	206	30

---

THE FITTED MODEL:

$$N = B0 + B1 * X + B2 * Z^{**0.5} + B3 * Z + B4 * Z^{**1.5} + B5 * Z^{**2}.$$

All the derivatives will be calculated numerically.

---

Run stopped after 6 model evaluations and 3 derivative evaluations.  
Iterations have been stopped because the relative reduction between successive residual sums of squares is at most SSSCON = 1.000E-08

---

Nonlinear Regression Summary Statistics      Dependent Variable N

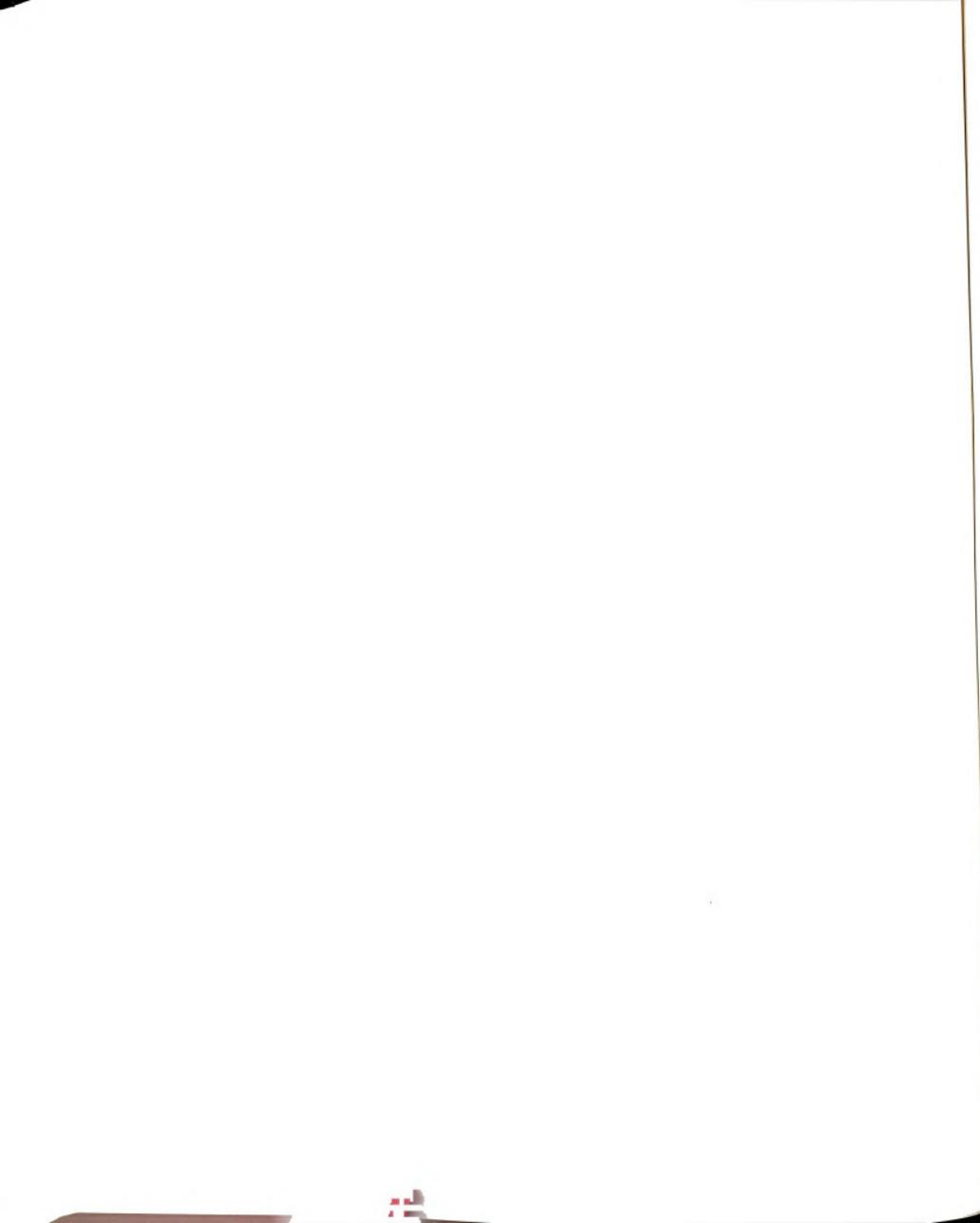
Source	DF	Sum of Squares	Mean Square
Regression	6	1906.13828	317.68971
Residual	6	70.10254	11.68376
Uncorrected Total	12	1976.24081	
(Corrected Total)	11	202.85765	

$$R \text{ squared} = 1 - \text{Residual SS} / \text{Corrected SS} = .65442$$

Parameter	Estimate	Std. Error	Asymptotic 95 % Confidence Interval	
			Lower	Upper
B0	-1254.598035	608.99558842	-2744.756558	235.56048759
B1	-.008244547	.006109621	-.023194251	.006705158
B2	1476.5232332	725.55692499	-298.8506053	3251.8970716
B3	-624.0239724	314.13350566	-1392.680970	144.63302546
B4	114.28725122	58.781601694	-29.54614660	258.12064903
B5	-7.683474492	4.023853769	-17.52948997	2.162540983

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Figure C.4 : Continued.

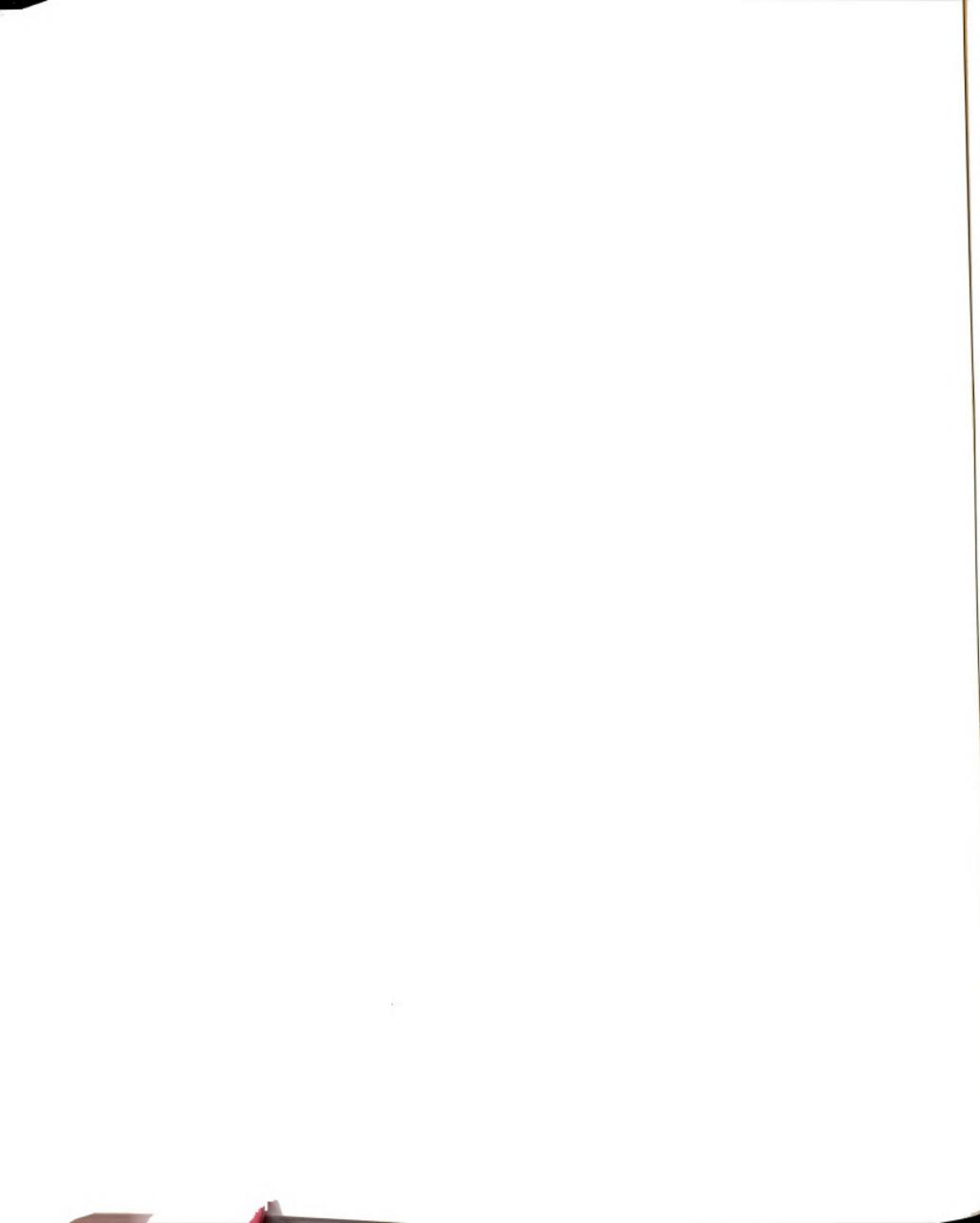


## Asymptotic Correlation Matrix of the Parameter Estimates

	B0	B1	B2	B3	B4	B5
B0	1.0000	-.1133	-.9995	.9979	-.9953	.9916
B1	-.1133	1.0000	.1102	-.1111	.1113	-.1104
B2	-.9995	.1102	1.0000	-.9995	.9978	-.9951
B3	.9979	-.1111	-.9995	1.0000	-.9994	.9978
B4	-.9953	.1113	.9978	-.9994	1.0000	-.9995
B5	.9916	-.1104	-.9951	.9978	-.9995	1.0000

---

**Figure C.4 : Continued.**



### **C.3 THE SETTLEMENT PREDICTION OF CASE HISTORY No.4**

#### **C.3.1 PROJECT GENERAL DESCRIPTION**

The site investigation for a steel mill factory expansion in Lesaka, Spain , revealed the presence of a loose to medium dense silty sand stratum. The settlements estimated for this stratum were large, and were thought to be critical for the normal operation of the equipment. This justified the implementation of a field load test to verify the expected settlements.

The settlements of this load test were previously studied and reported by Picornell, M. et. al. (1988). The load test was implemented by stockpiling steel sheet coils over an area of 65 feet by 50 feet , this led to an average contact pressure of 3.1 Ksf (1.55 tsf). The load test was terminated when the settlement plates had stopped settling.

#### **C.3.2 SUBSOIL INVESTIGATION**

The reported subsoil investigation included 2 borings , (D5 & D8). These borings were drilled to depths of about 125 ft each. The boring locations are shown in Figure C.5. The subsurface soil can be described as follows:

The subsurface soils , at this site , are predominantly granular and can be grouped into two main strata. The surficial stratum consist of greenish gray silty gravel with some sand, and frequent boulders. At the site of the load test, this stratum is about 23 ft thick. In other areas of the expansion, irregular lenses of medium stiff to stiff silty clay have been encountered embedded in this stratum. The deeper silt- sand stratum has

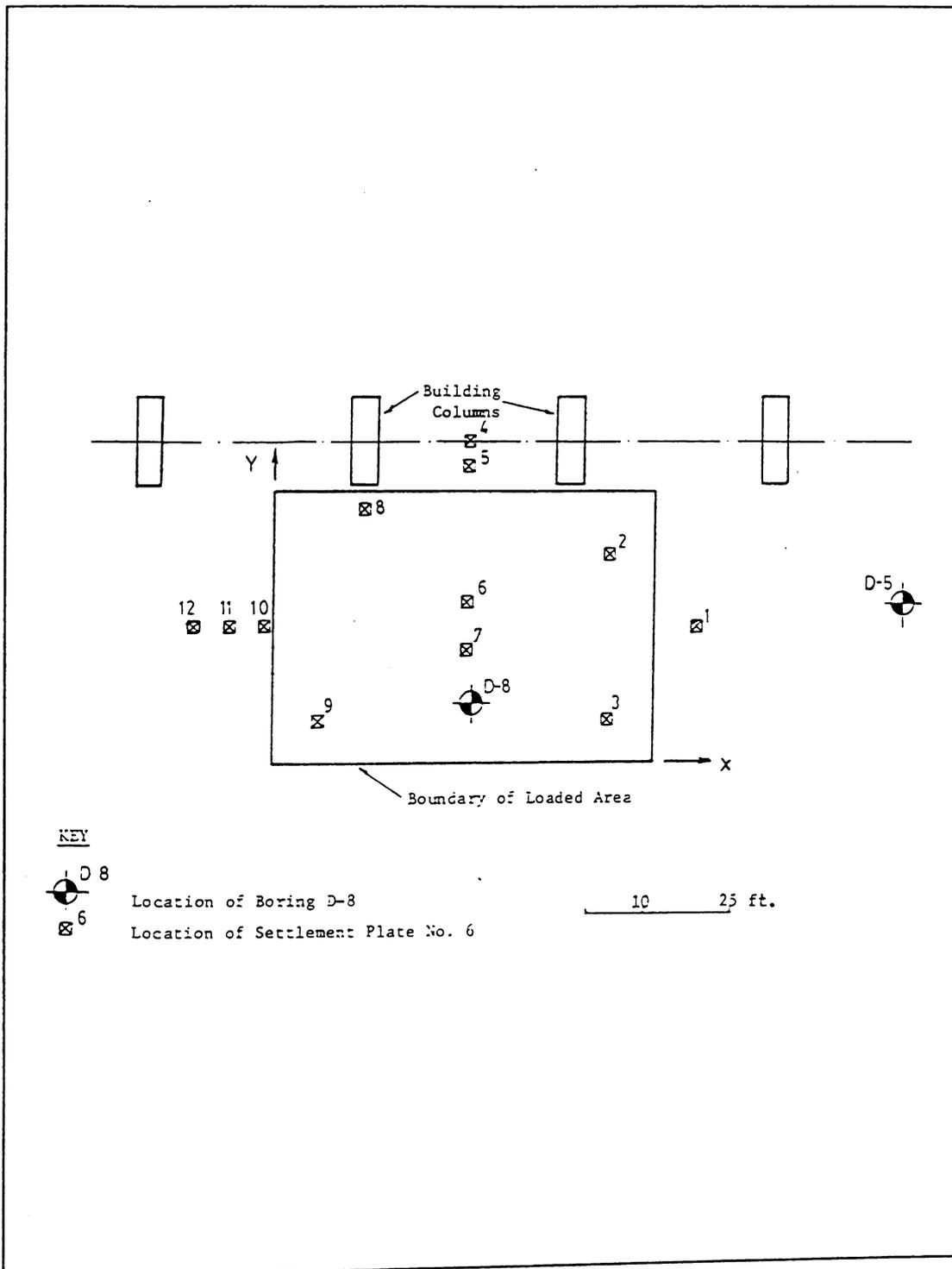
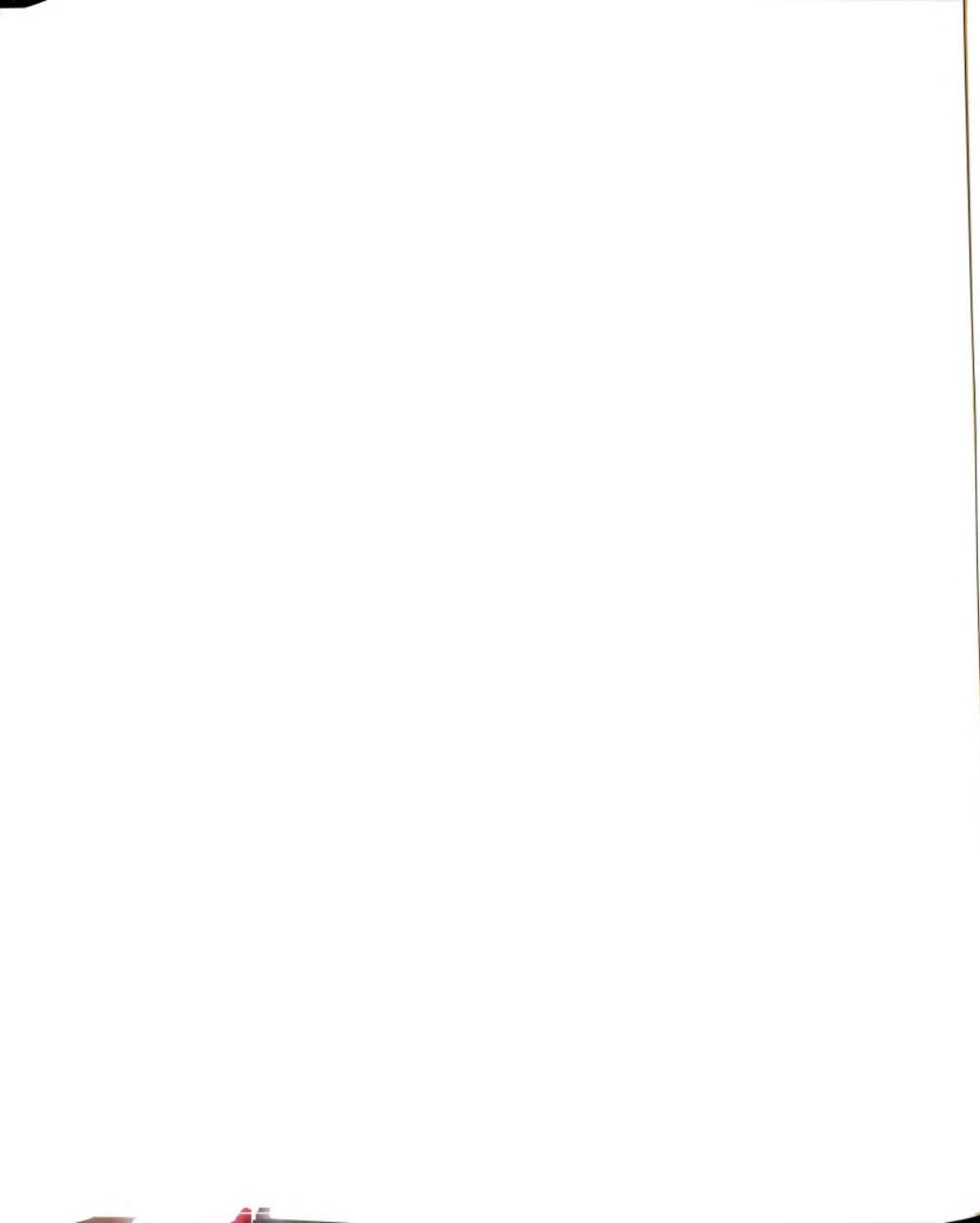
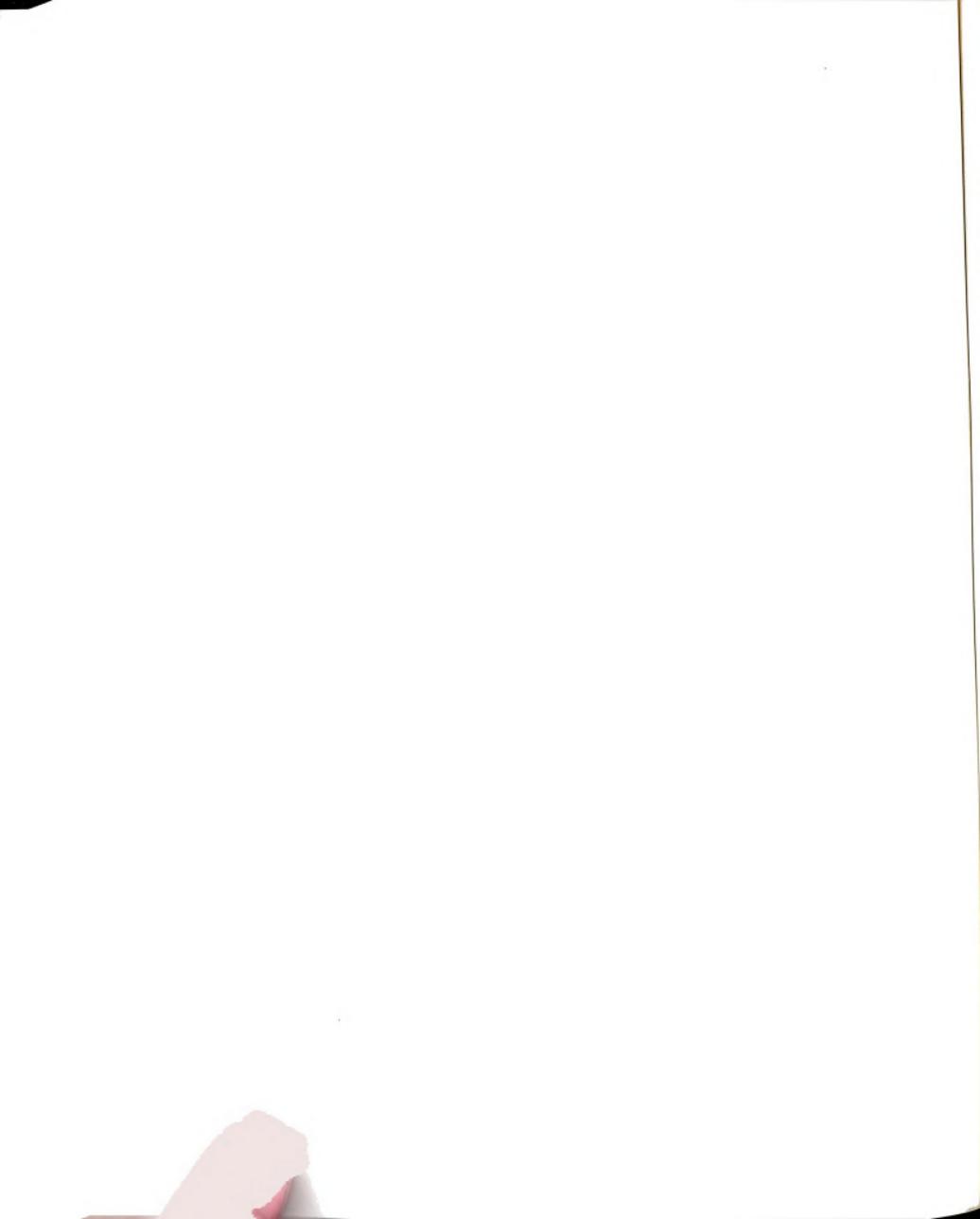


Figure C.5 : Site Plan And Boring Locations Of Case History No. 4



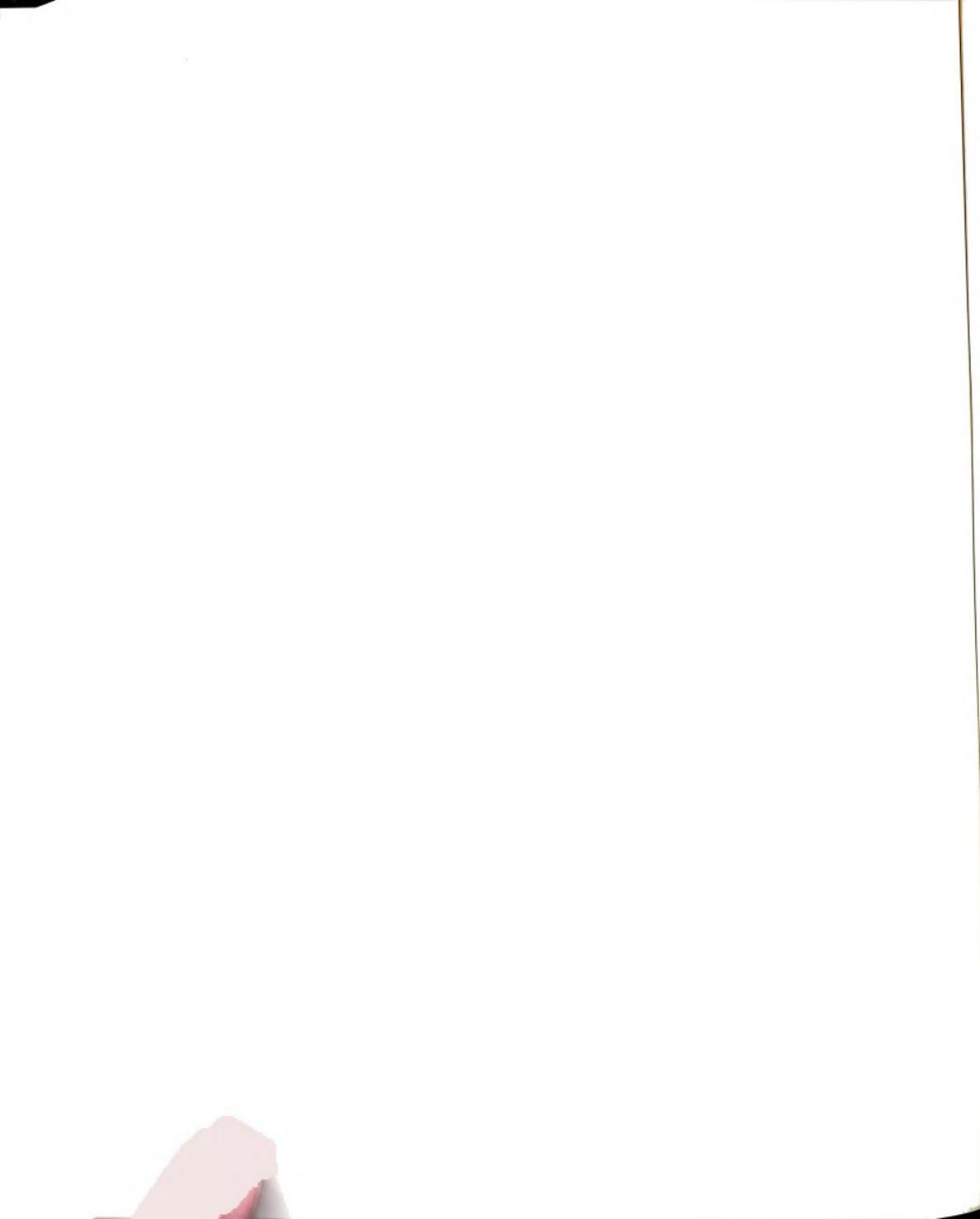
**Table C.9 : The Correction Of N Values Of Boring (D-5).**

Boring (D-5)			
Depth (ft)	Correction factor(C1)	N	$N1 = C1 * N$
5	2.527	65	164.307
10	1.787	93	166.230
15	1.459	105	153.239
20	1.263	98	123.862
40	0.893	105	93.839
46	0.833	56	46.669
57	0.748	44	32.941
62	0.717	30	21.535
82	0.624	34	21.222
85	0.613	54	33.106
89	0.599	60	35.948
94	0.582	58	33.813



**Table C.10 : The Correction Of N Values Of Boring (D-8).**

Boring (D-8)			
Depth	Correction	N	N1 = C1 * N
(ft)	factor (C1)		
6.0	2.307	36	83.072
12.0	1.631	22	35.897
16.0	1.413	14	19.783
18.0	1.332	17	22.648
23.8	1.158	10	11.586
26.2	1.104	9	9.938
31.2	1.011	9	9.107
36.1	0.940	14	13.170
41.3	0.879	7	6.156
45.9	0.834	17	14.183
50.8	0.793	10	7.930
55.8	0.756	23	17.403
69.0	0.680	18	12.248
81.0	0.628	34	21.353
89.0	0.599	53	31.754
117.0	0.522	25	13.063
130.0	0.495	52	25.778

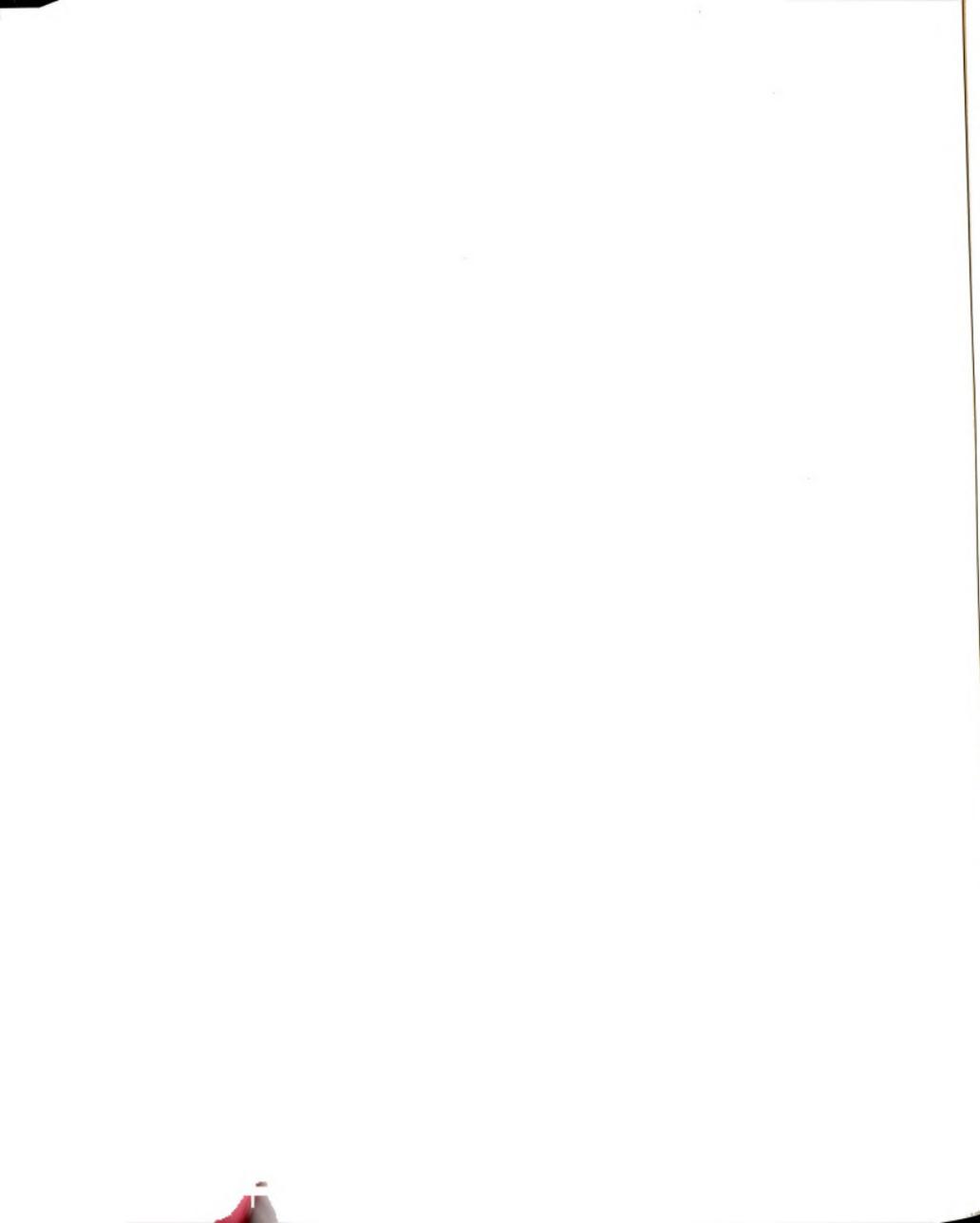


a characteristic maroon color and consists of successive layers of silty sand and sandy silt with variable amounts of gravel, occasionally, the gravel size particles predominate. This stratum contains what appears to be large boulders of limestone 10 ft or more in thickness. The density of this stratum ranges from loose to medium dense. The thickness of this stratum is extremely variable throughout the site, and at the location of the field load test exceeds the 100 ft investigated. This silt-sand stratum rests on limestone bedrock. The corrected N values for the overburden pressure are shown in Tables C.9 and C.10.

### **C.3.3 FOUNDATIONS AND SETTLEMENT MEASUREMENTS**

The load test was implemented to verify the compressibility of the silty sand layer. For this purpose, the load test was located in the area where this layer appeared closer to the ground surface.

The load was applied on the soil by stockpiling steel sheet coils directly on the ground surface. During the installation of the load, a record was kept of the individual weights of the coils and an attempt was made to distribute them in such a manner as to achieve a nearly uniform load throughout the loaded area. The dimensions of the loaded area were 65 ft by 50 ft and the average load amounted to 3.1 Ksf (1.55 tsf). The settlements were monitored with twelve settlement plates installed throughout the loaded area and in its immediate vicinity. The locations of the settlement plates are shown in Figure C.5. The settlement plates consisted of a riser welded to a one square foot plate. The plates were installed in an excavation at a depth of 2 ft. The settlements were monitored by surveying the elevation of the top of the risers. The load was left



in place for ten days and the settlement plates were surveyed on a daily basis. The average settlement was 1.56 in. From the settlement record it was obvious that the settlement occurred rapidly coinciding with the application of the load.

### C.3.4 APPLYING THE KRIGING TECHNIQUE

Considering that a settlement prediction is required at the center point of the loading area foundation. The Kriging results are summarized as follows:

1. The calculated covariance function is given by the equation:

$$C(h) = 128.633 e^{-4.44E-5(h^2)} \quad (C.14)$$

2. The estimated N function is given by:

$$\hat{N} = 6.571761 + 0.1394683Z \quad (C.15)$$

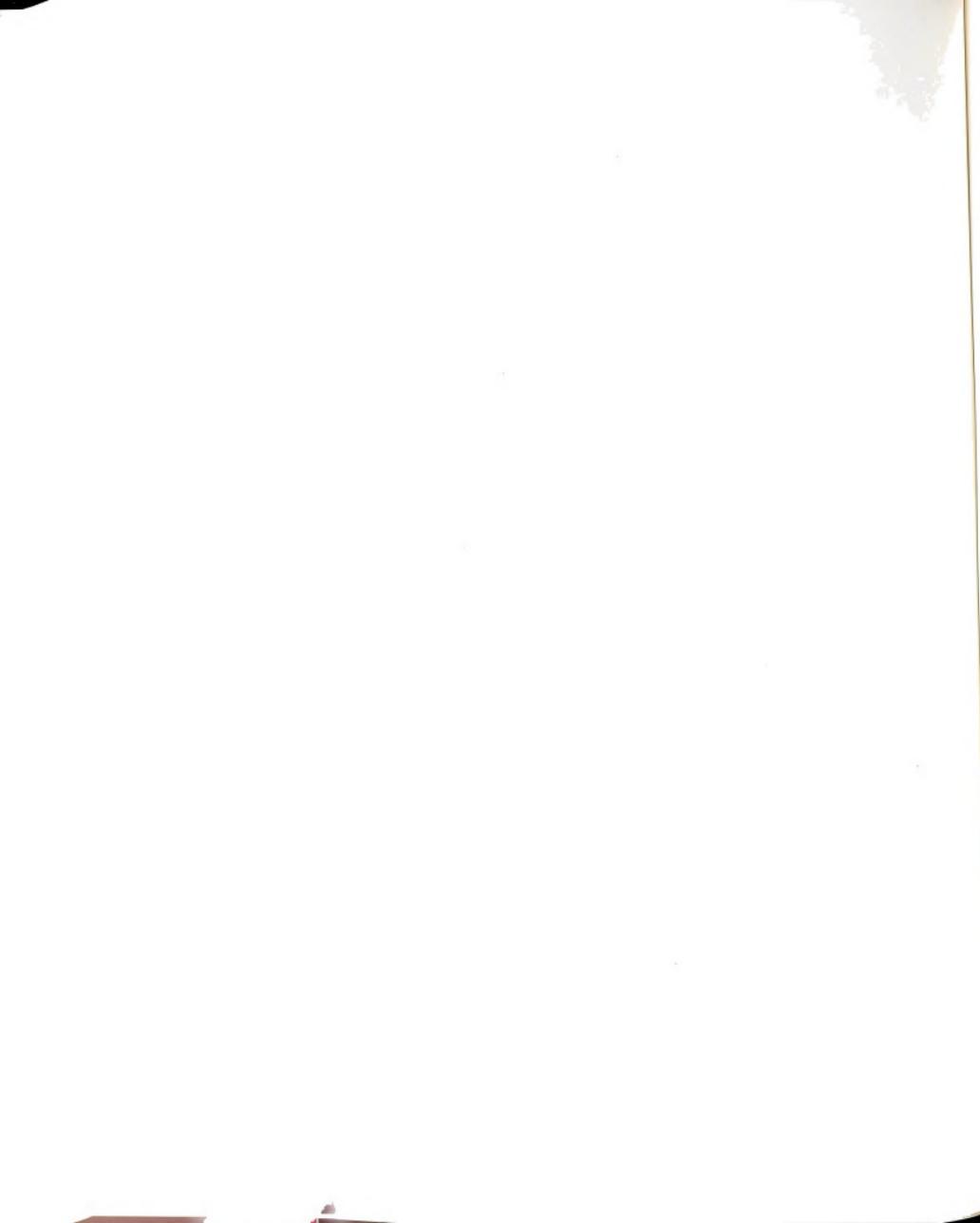
3. The "two-point" estimate of N values are:

$$\hat{N}_{(B/2)} = 10.05, \hat{N}_{(3B/2)} = 17.03 \quad (C.16)$$

4. The design N value is given by the weighted average:

$$N = (1/3) [2\hat{N}_{(B/2)} + \hat{N}_{(3B/2)}] = 12.38 \quad (C.17)$$

5. The predicted settlement is 1.92 in.



Therefore the predicted settlement of 1.92 in is within about 20% of the measured value of 1.56 in.

6. The 90% confidence limits of the settlement prediction are: (1.16 and 5.75) in.  
The 50% confidence limits are: (1.74 and 2.15) in.

### C.3.5 APPLYING THE TREND SURFACE ANALYSIS TECHNIQUE

The trend surface analysis results are summarized as follows:

1. The model which is fitted to the data is as follows:

$$N = -15.26 + 0.216X + 5.67Z^{0.5} - 0.417Z + 0.001Z^2 + 7.92Z^3, (R^2 = 0.62) \quad (\text{C.18})$$

2. The "two-point" estimate of N values are:

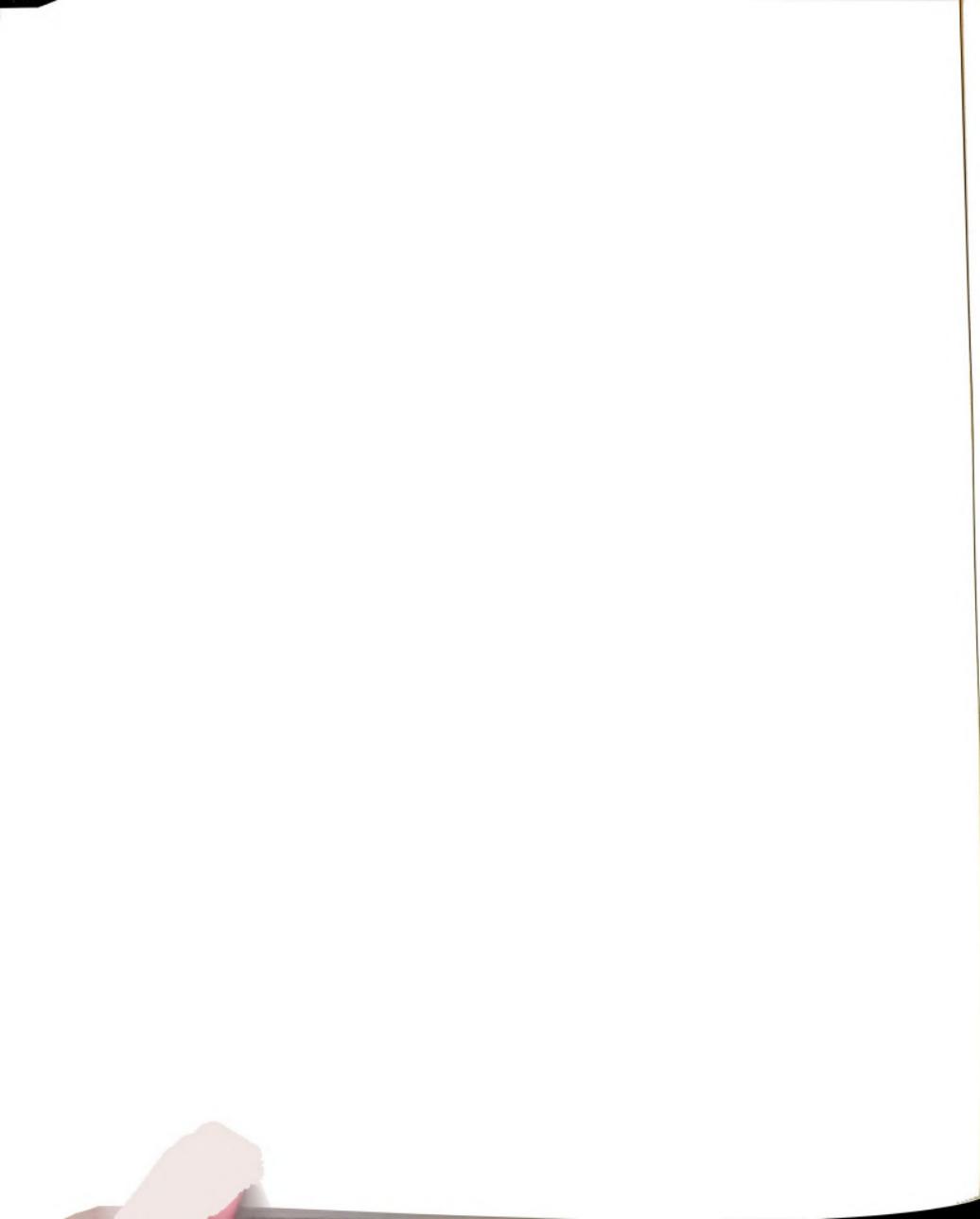
$$N_{(B/2)} = 10.39, N_{(3B/2)} = 16.09 \quad (\text{C.19})$$

3. The design N value is:

$$N = (1/3) [2N_{(B/2)} + N_{(3B/2)}] = 12.29 \quad (\text{C.20})$$

4. The predicted settlement is 1.93 in.

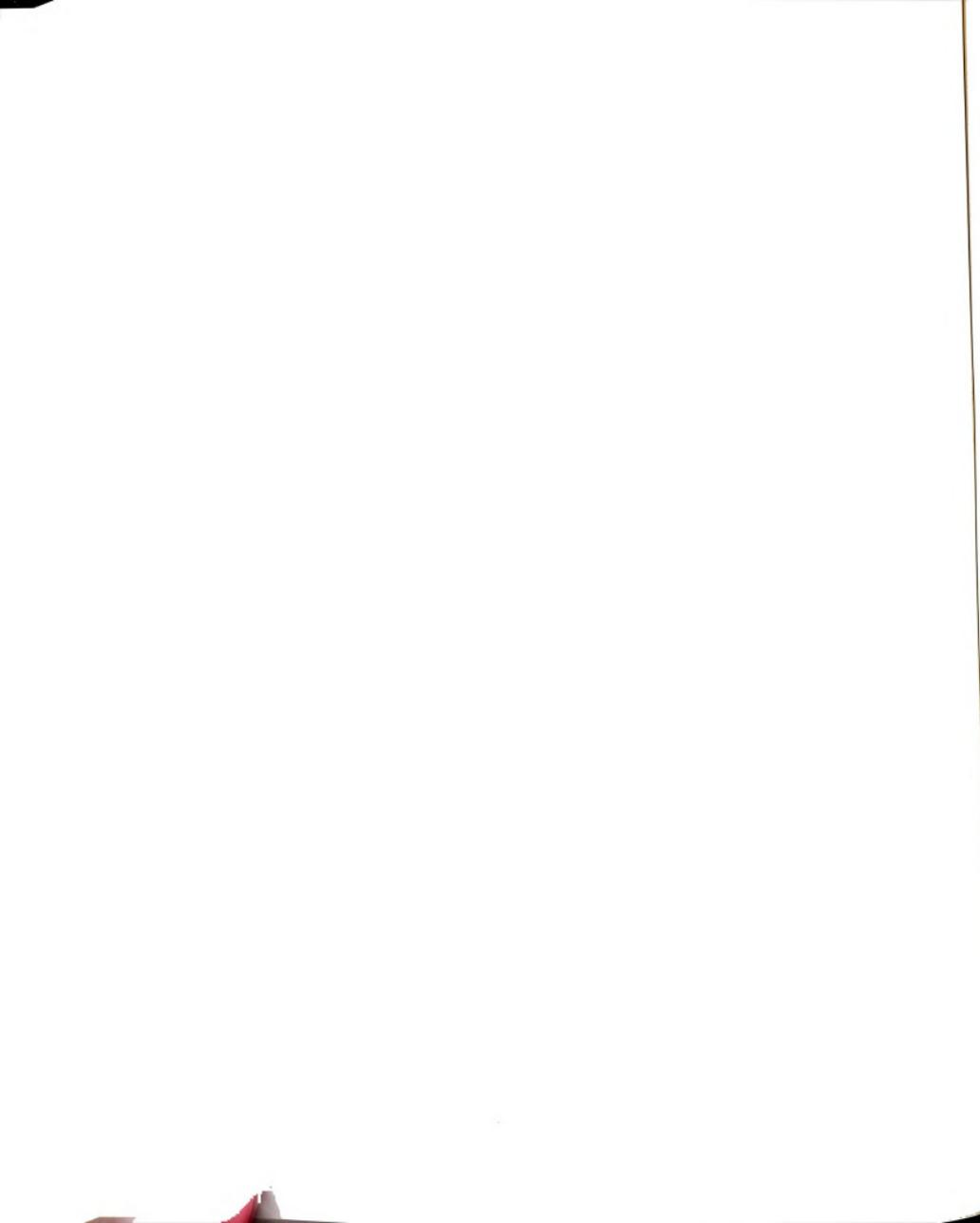
Therefore the predicted settlement of 1.93 in is within about 20% of the measured value of 1.56 in.



5. The 90% confidence limits of the settlement prediction are: (1.1 and 13.3) in.  
The 50% confidence limits are: (1.45 and 2.91) in.
6. The areal distribution of settlement in inches is given by the equation:

$$S=23.84 / (0.216X+5.267) \quad (C.21)$$

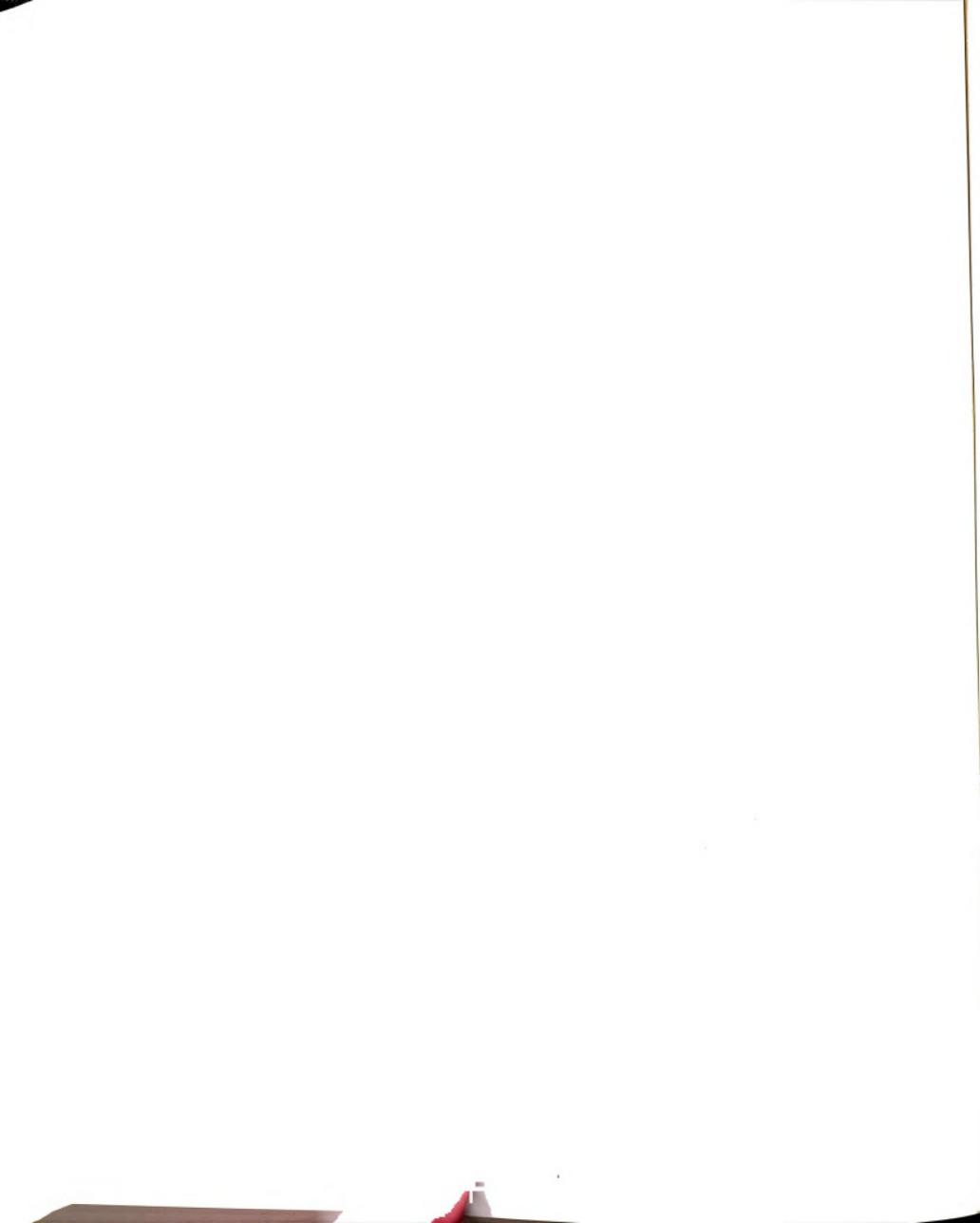
The computer output is shown in Figure C.6.



## SUBSURFACE SOIL STRATIFICATION

LAYER	N1	X(ft)	Y(ft)	Z(ft)
1	83.0720	33.15	9.78	6.0
1	35.8971	33.15	9.78	12.0
1	19.7831	33.15	9.78	16.0
1	22.6485	33.15	9.78	18.0
1	165.3070	107.39	25.75	5.0
1	166.2305	107.39	25.75	10.0
1	153.2398	107.39	25.75	15.0
1	123.8622	107.39	25.75	20.0
1	93.8398	107.39	25.75	40.0
2	11.5861	33.15	9.78	23.8
2	9.9384	33.15	9.78	26.2
2	9.1073	33.15	9.78	31.2
2	13.1705	33.15	9.78	36.1
2	6.1567	33.15	9.78	41.3
2	14.1831	33.15	9.78	45.9
2	7.9304	33.15	9.78	50.8
2	17.4036	33.15	9.78	55.8
2	12.2483	33.15	9.78	69.0
2	21.3532	33.15	9.78	81.0
2	46.6699	107.39	25.75	46.0
2	32.9415	107.39	25.75	57.0
2	21.5354	107.39	25.75	62.0
2	21.2227	107.39	25.75	82.0
2	33.1064	107.39	25.75	85.0
3	31.7547	33.15	9.78	89.0

**Figure C.6 : The Computer Output Of Case History No. 4**



3	13.0639	33.15	9.78	117.0
3	25.7786	33.15	9.78	130.0
3	35.9487	107.39	25.75	89.0
3	33.8136	107.39	25.75	94.0

---

----- O N E W A Y -----

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	35290.5477	17645.2739	14.8462	.0001
Within Groups	26	30902.0337	1188.5398		
Total	28	66192.5814			

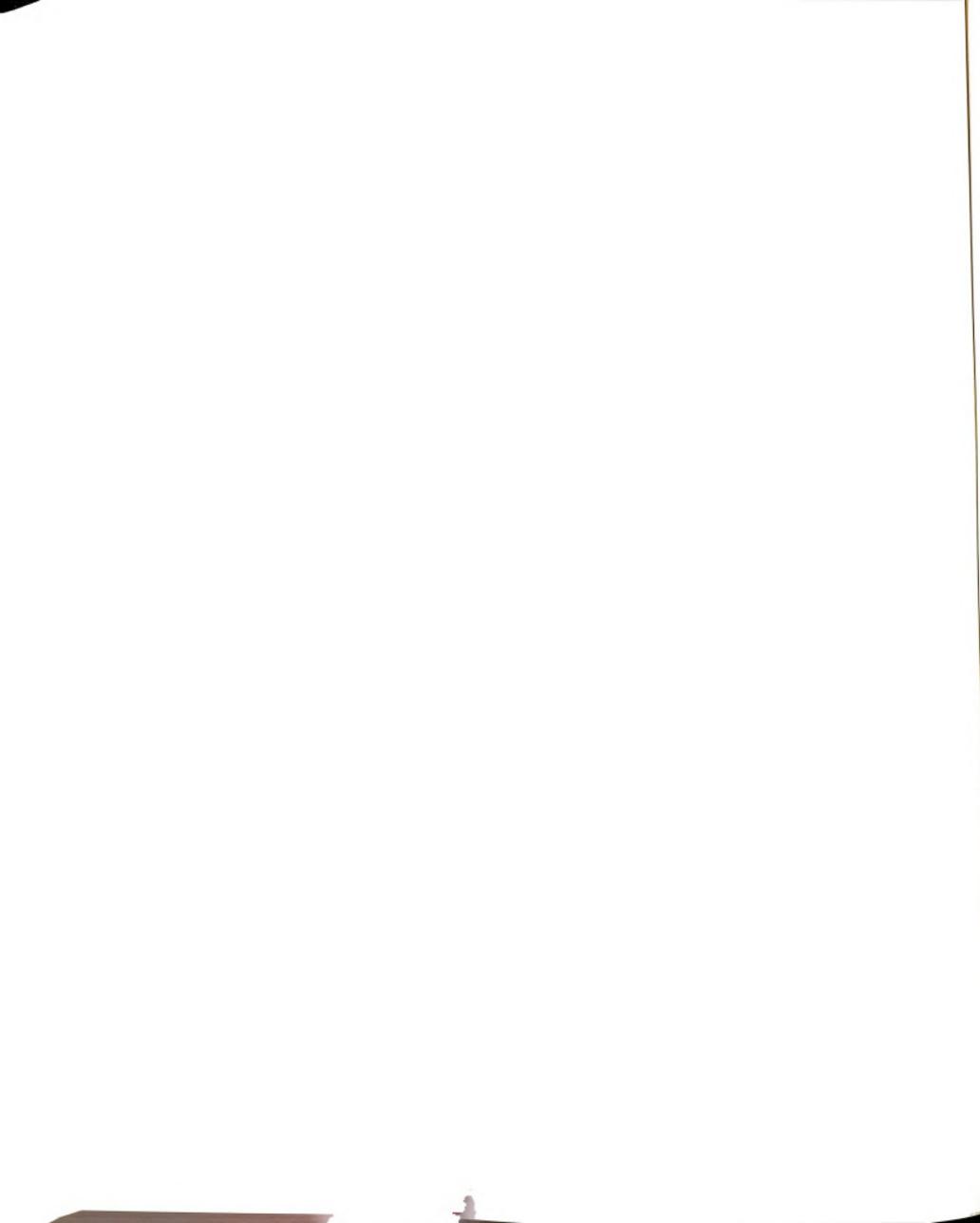
---

----- O N E W A Y -----

Group	Count	Standard Mean	Standard Deviation	Standard Error	95 Pct Conf Int for Mean
Grp 1	9	95.9867	59.9605	19.9868	49.8970 To 142.0763
Grp 2	15	18.5702	11.3417	2.9284	12.2894 To 24.8510
Grp 3	5	28.0719	9.2070	4.1175	16.6401 To 39.5037
Total	29	44.2342	48.6212	9.0287	25.7397 To 62.7288
Fixed Effects Model		34.4752	6.4019	31.0750	To 57.3935
Random Effects Model			27.8852	-75.7476	To 164.2161
Random Effects Model - Estimate of Between Component Variance					1871.5502

---

Figure C.6 : Continued.



Group	Minimum	Maximum
Grp 1	19.7831	166.2305
Grp 2	6.1567	46.6699
Grp 3	13.0639	35.9487
Total	6.1567	166.2305

#### Tests for Homogeneity of Variances

Cochrans C = Max. Variance/Sum(Variances) = .9440, P = .000 (Approx.)  
 Bartlett-Box F = 15.904, P = .000  
 Maximum Variance / Minimum Variance 42.413

#### Multiple Range Test

Tukey-HSD Procedure  
 Ranges for the .050 level -

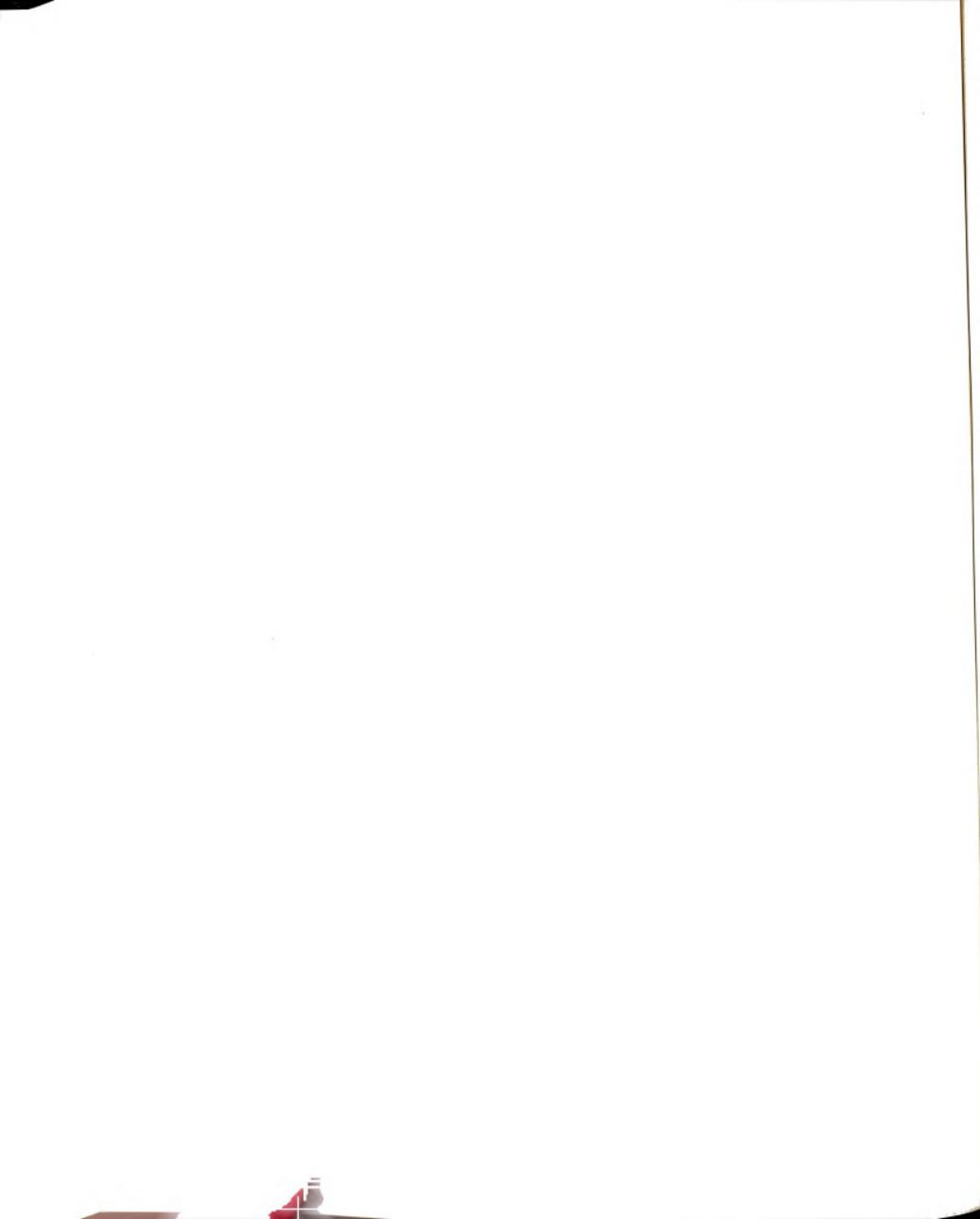
3.51 3.51

The ranges above are table ranges.  
 The value actually compared with Mean(J)-Mean(I) is..  
 $24.3777 * \text{Range} * \text{Sqrt}(1/N(I) + 1/N(J))$

(\*) Denotes pairs of groups significantly different at the .050 level

		G G G
		r r r
		P P P
Mean	Group	2 3 1
18.5702	Grp 2	
28.0719	Grp 3	
95.9867	Grp 1	* *

**Figure C.6 : Continued.**



## MODELING THE N FUNCTION

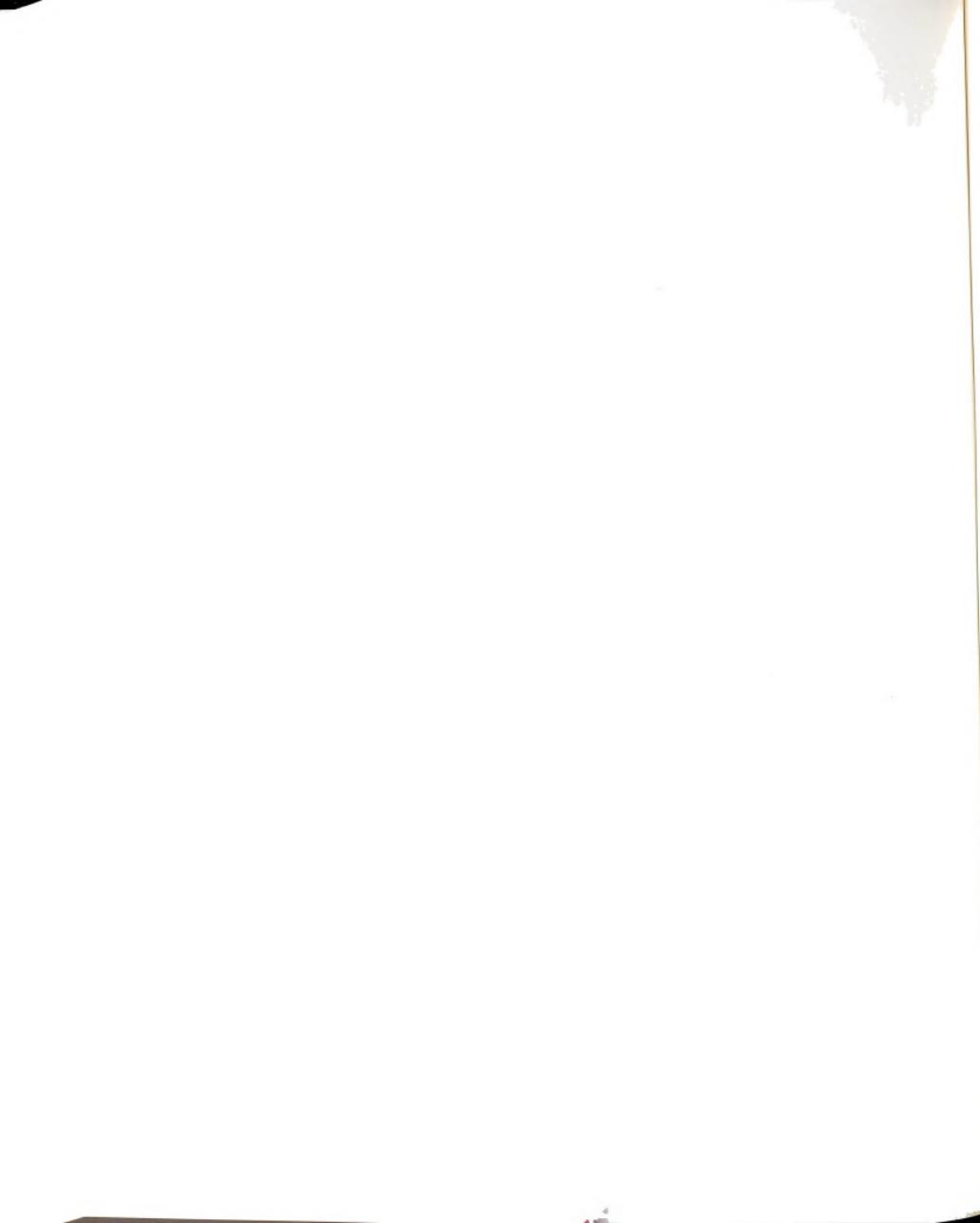
LAYER	N1	X(ft)	Y(ft)	Z(ft)
2	11.5861	33.15	9.78	23.8
2	9.9384	33.15	9.78	26.2
2	9.1073	33.15	9.78	31.2
2	13.1705	33.15	9.78	36.1
2	6.1567	33.15	9.78	41.3
2	14.1831	33.15	9.78	45.9
2	7.9304	33.15	9.78	50.8
2	17.4036	33.15	9.78	55.8
2	12.2483	33.15	9.78	69.0
2	21.3532	33.15	9.78	81.0
2	46.6699	107.39	25.75	46.0
2	32.9415	107.39	25.75	57.0
2	21.5354	107.39	25.75	62.0
2	21.2227	107.39	25.75	82.0
2	33.1064	107.39	25.75	85.0
3	31.7547	33.15	9.78	89.0
3	13.0639	33.15	9.78	117.0
3	25.7786	33.15	9.78	130.0
3	35.9487	107.39	25.75	89.0
3	33.8136	107.39	25.75	94.0

THE FITTED MODEL:

$$N = B0 + B1 * X + B2 * Z^{**0.5} + B3 * Z + B4 * Z^{**2} + B5 * Z^{**3}.$$

20 cases are written to the compressed active file.

-----  
**Figure C.6 : Continued.**



All the derivatives will be calculated numerically.

---

Run stopped after 6 model evaluations and 3 derivative evaluations.  
 Iterations have been stopped because the relative reduction between successive residual sums of squares is at most SSSCON = 1.000E-08

---

Nonlinear Regression Summary Statistics      Dependent Variable N

Source	DF	Sum of Squares	Mean Square
Regression	6	10315.18500	1719.19750
Residual	14	937.71472	66.97962
Uncorrected Total	20	11252.89972	
(Corrected Total)	19	2478.49464	

R squared = 1 - Residual SS / Corrected SS = .62166

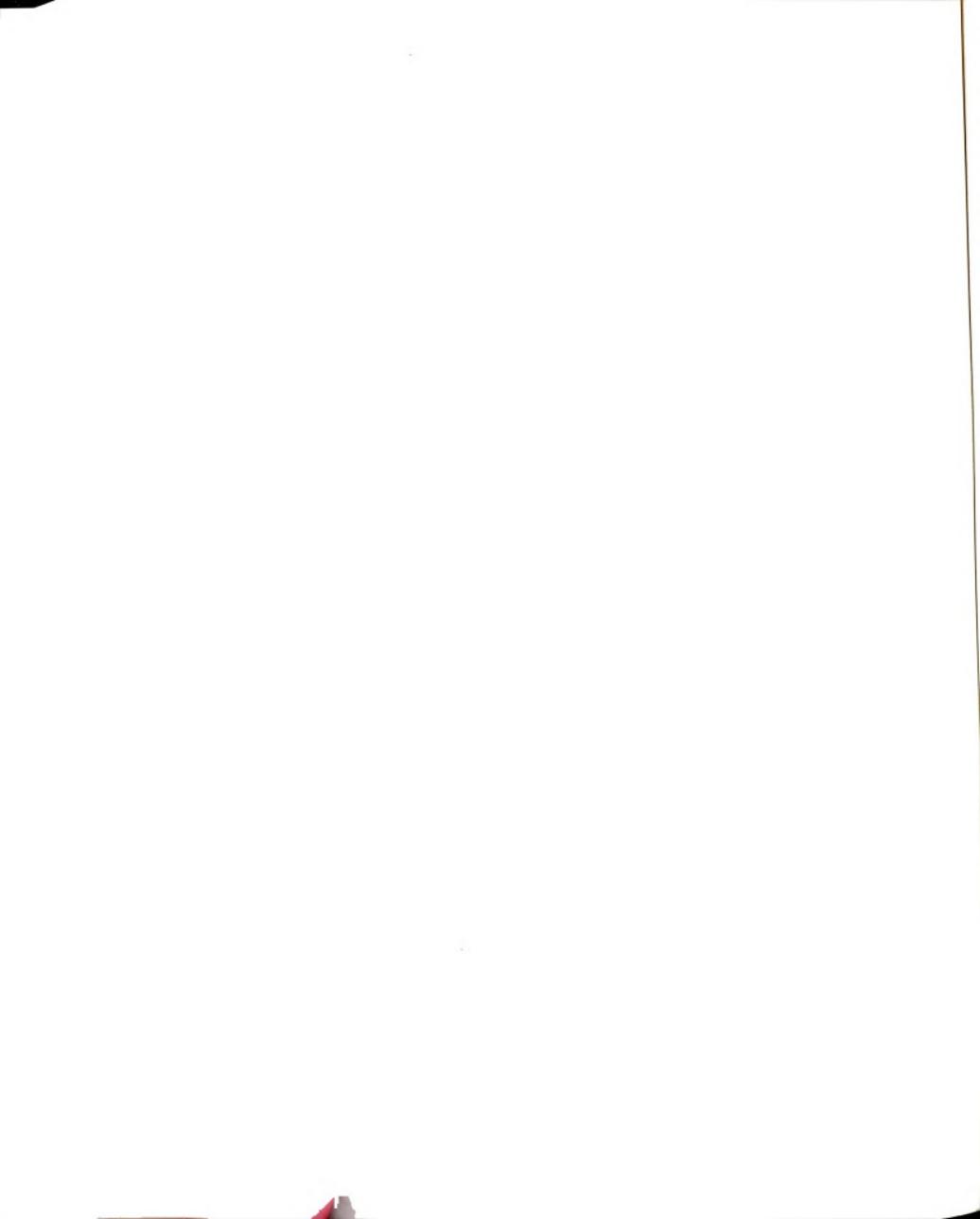
Parameter	Estimate	Std. Error	Asymptotic 95 % Confidence Interval	
			Lower	Upper
B0	-15.26345780	413.72188570	-902.6086507	872.08173514
B1	.216191623	.060909113	.085554569	.346828678
B2	5.673904168	177.22526971	-374.4364951	385.78430340
B3	-.417516829	21.645162943	-46.84177417	46.006740509
B4	.001097525	.113220251	-.241735761	.243930811
B5	7.927580E-07	.000323570	-.000693195	.000694781

Asymptotic Correlation Matrix of the Parameter Estimates

	B0	B1	B2	B3	B4	B5
B0	1.0000	-.0805	-.9985	.9943	-.9810	.9639
B1	-.0805	1.0000	.0801	-.0822	.0725	-.0528
B2	-.9985	.0801	1.0000	-.9987	.9899	-.9762
B3	.9943	-.0822	-.9987	1.0000	-.9958	.9855
B4	-.9810	.0725	.9899	-.9958	1.0000	-.9968
B5	.9639	-.0528	-.9762	.9855	-.9968	1.0000

---

Figure C.6 : Continued.



## **C.4 THE SETTLEMENT PREDICTION OF CASE HISTORY No. 5**

### **C.4.1 PROJECT GENERAL DESCRIPTION**

This project consists of a 30 ft thick mat which supports several nuclear, electrical and associated facilities with loads ranging from 8,000 to 10,000 Kips/s.f. The mat is founded upon the partially cemented silty sands of the Vincentown Formation.

The settlements of this project were previously studied and reported by the investigators of "Dames and Moore, Granford, New Jersey" , (1972).

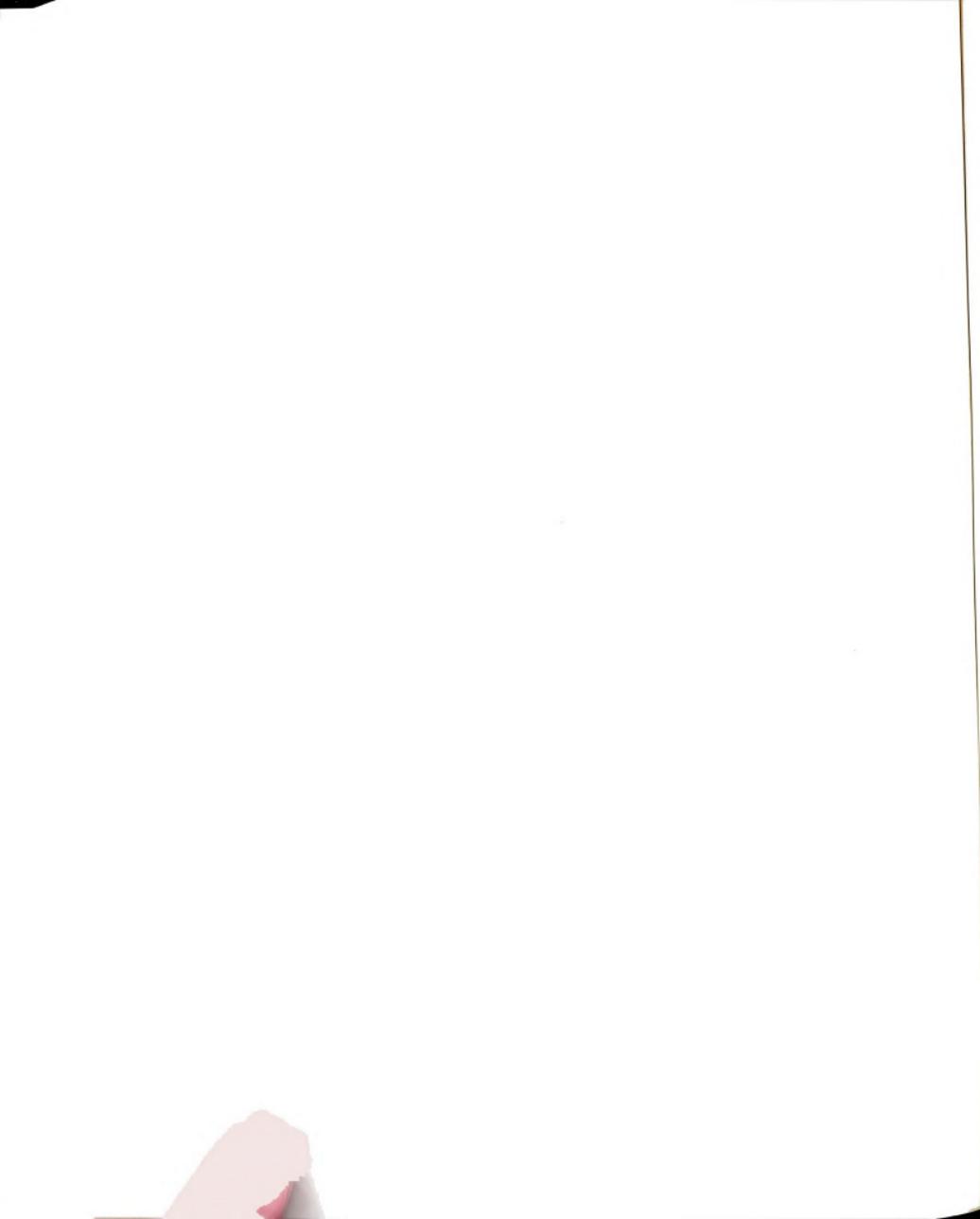
Suitable foundation soils are some 70 ft below grade. The site was backfilled to foundation grade with a lean concrete up to 30 ft in thickness.

### **C.4.2 SUBSOIL INVESTIGATION**

The site was investigated by drilling 35 borings to varying depths in the generating station area. The locations of the borings, with respect to the proposed structures, are shown in Figure C.7. Most of the borings were terminated at depths on the order of 100 feet below ground surface.

The borings at the site indicate the top 25 to 35 feet consist of interbedded mixtures of clay , silt and sand , generally hydraulic fill or loose alluvium. The soils are generally soft in consistency and occasionally contain some organic material.

Below these upper soils, the Kirkwood Formation was encountered to depths varying from 65 to 70 ft below the ground surface. This Formation consists of moderately firm to firm clayey soils. The Vincetown Formation of Miocene Age underlies the Kirkwood and consists of sand and silty sand layers, some well cemented.



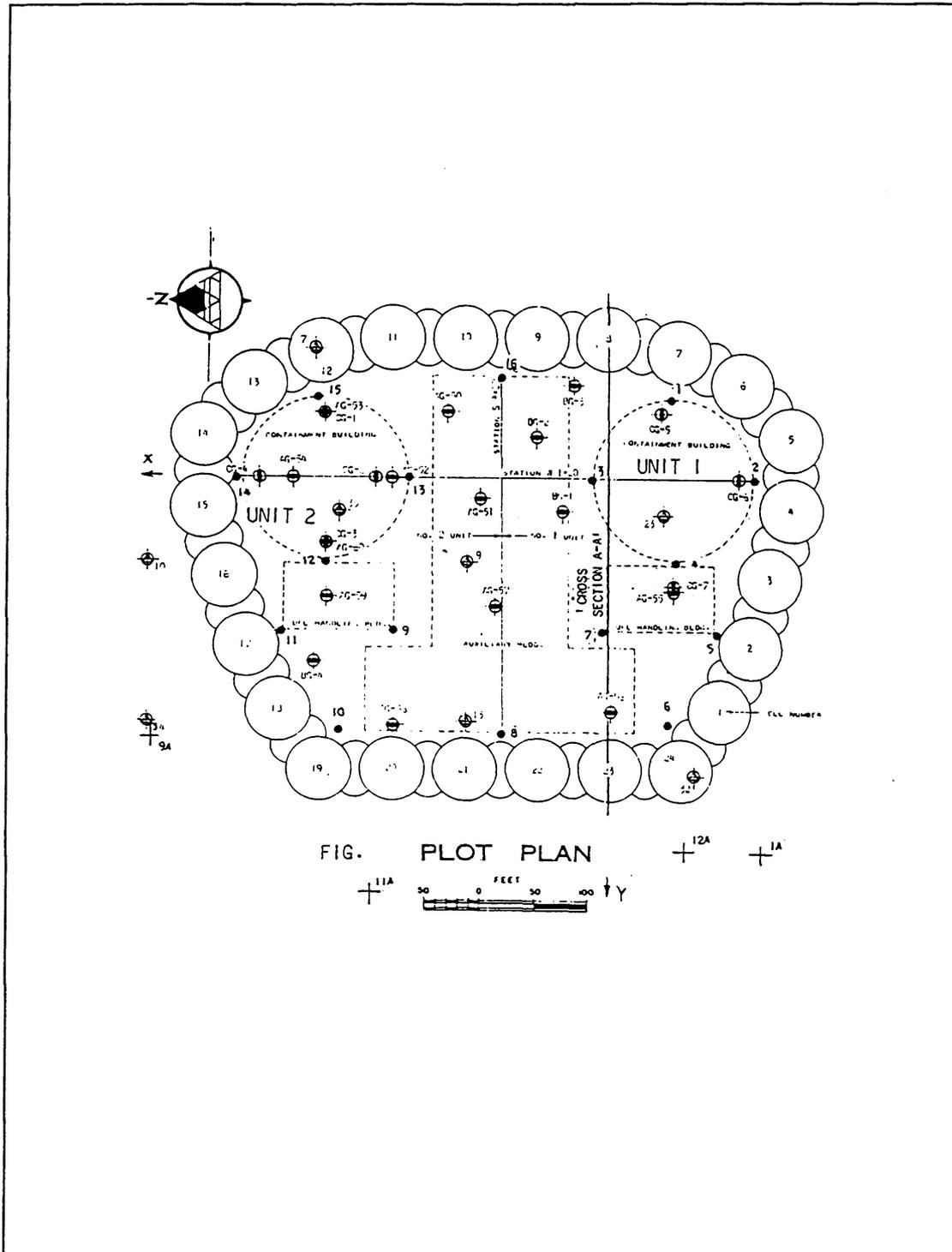
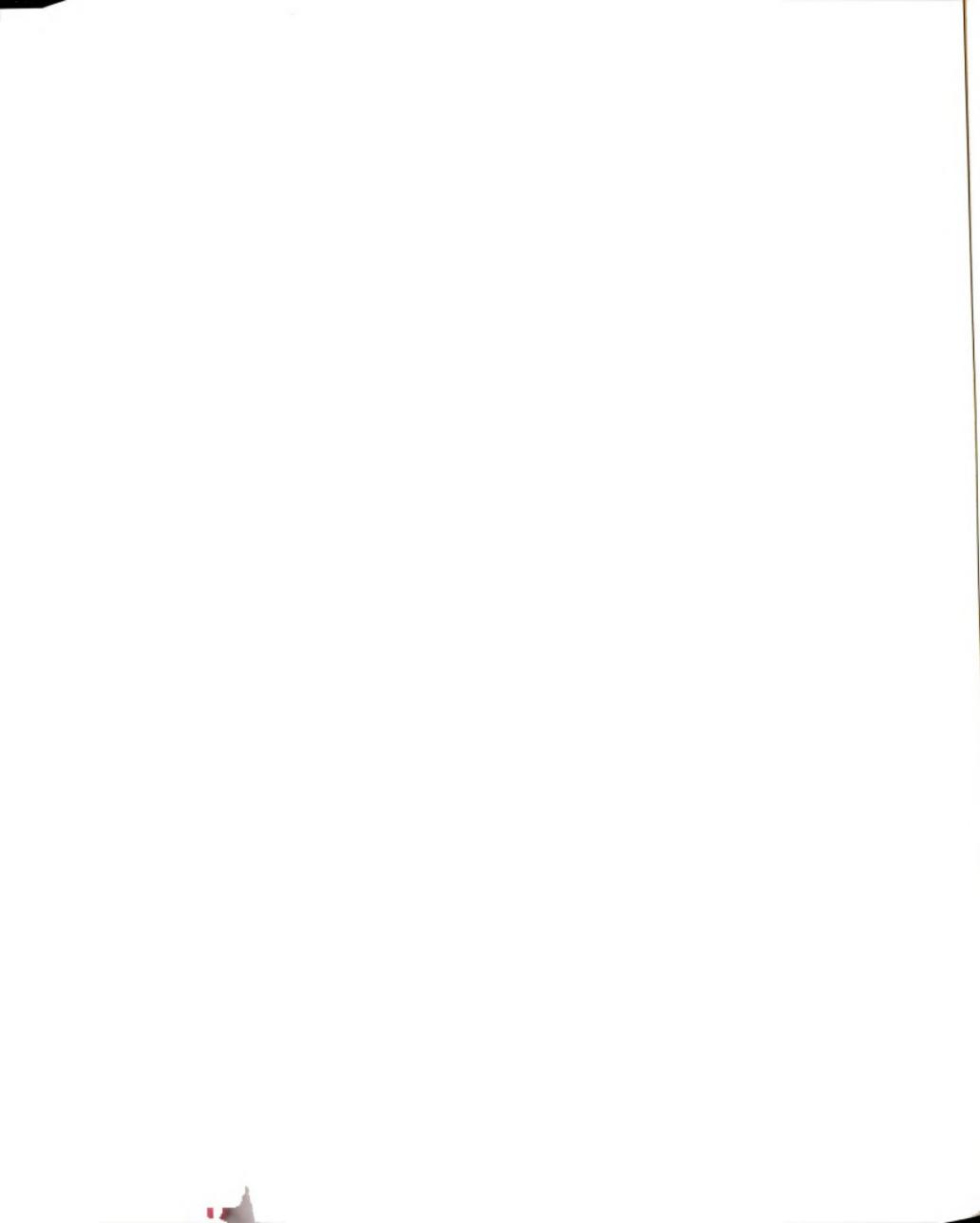
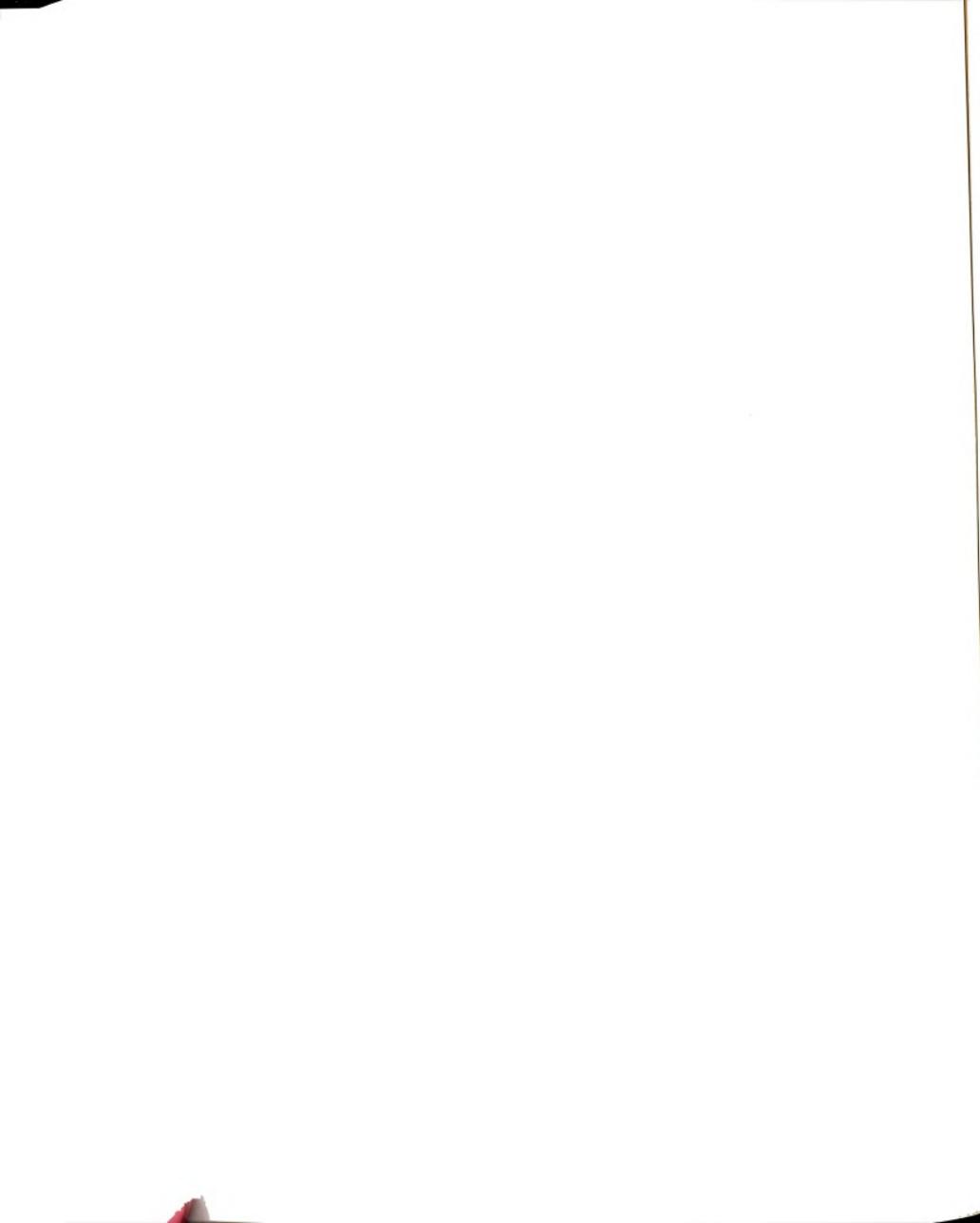


Figure C.7 : Site Plan And Boring Locations Of Case History No. 5



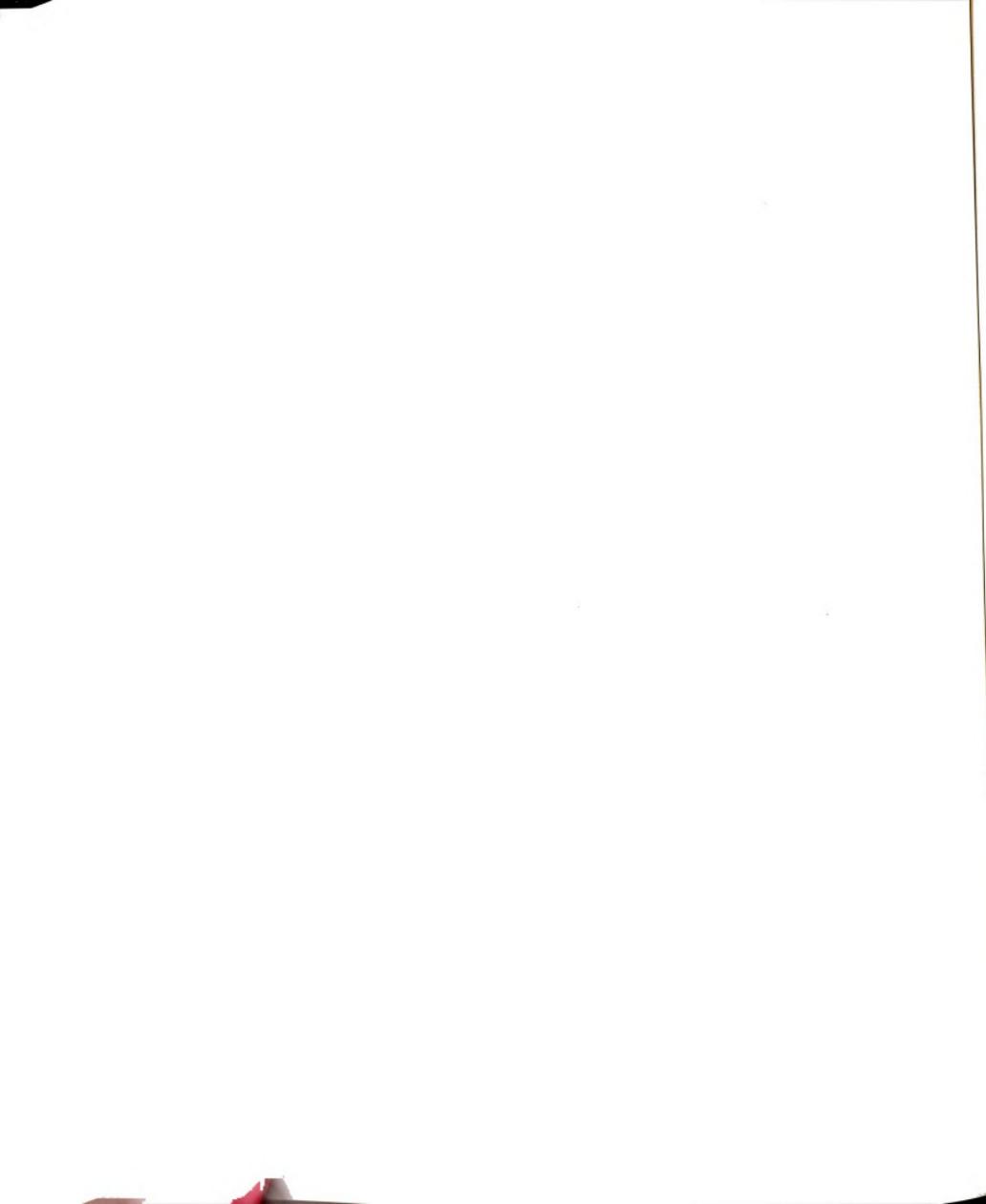
**Table C.11 : The Correction Of N Values Of Boring (9).**

Boring (9)			
Depth (ft)	Correction factor (C1)	N	$N1 = C1 * N$
2	3.162	2.14	6.767
5	2.000	4.28	8.560
31	0.803	94.30	75.743
36	0.744	23.60	17.578
41	0.695	19.30	13.430
46	0.655	27.90	18.287
51	0.621	34.30	21.312
54	0.603	19.30	11.643
56	0.592	79.30	46.952
58	0.581	25.70	14.944
61	0.566	23.60	13.371
67	0.538	30.00	16.151
71	0.519	32.14	16.708
76	0.499	51.45	25.684
81	0.480	77.14	37.092
87	0.461	64.30	29.659
91	0.449	90.00	40.452



**Table C.12 : The Correction Of N Values Of Boring (34).**

Boring (34)			
Depth (ft)	Correction factor (C1)	N	N1 = C1 * N
3	2.582	3	7.746
7	1.690	4	6.761
21	0.975	2	1.951
27	0.860	3	2.582
31	0.803	27	21.687
36	0.744	6	4.469
41	0.695	9	6.263
47	0.648	19	12.315
51	0.621	15	9.320
53	0.609	16	9.746
57	0.586	10	5.867
58	0.581	23	13.374
61	0.566	30	16.997
67	0.538	44	23.688
71	0.519	37	19.235
77	0.495	26	12.879
81	0.480	28	13.463



**Table C.12 : Continued.**

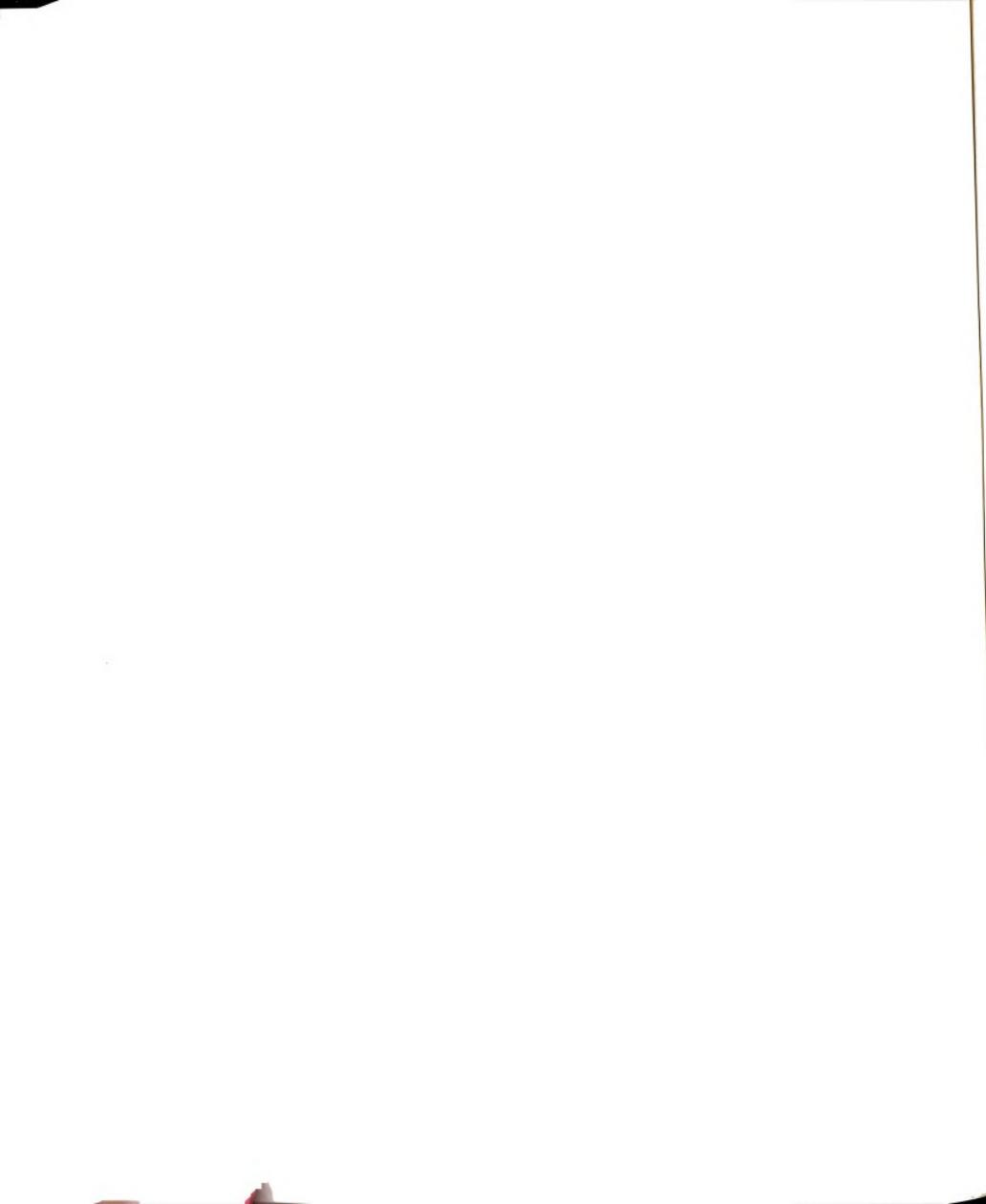
86	0.464	94	43.650
91	0.449	52	23.372
96	0.435	62	27.026
100	0.425	44	18.740

At about 135 ft below the ground surface the cementation grades out. Sandy and silty sand soil continue to the depths penetrated by the borings. Bedrock in the area is in excess of 1800 ft below the ground surface and therefore will have no appreciable influence on the foundation settlement of the proposed facilities. The corrected  $N$  values for the overburden pressure are shown in Tables C.11 and C.12.

#### **C.4.3 FOUNDATIONS AND SETTLEMENT MEASUREMENTS**

The 30 ft thick mat supports several facilities including unit 2 which is selected for settlement analysis. The diameter of the mat equals to 150 ft and the load which is carried by this mat has an ultimate pressure of 8 kips/s.f. (4 tsf).

The settlements of the mat were monitored at regular intervals from the initial placement onwards. The average settlement which was reported by the previous investigator was 0.5 in.



#### C.4.4 APPLYING THE KRIGING TECHNIQUE

Considering that a settlement prediction is required at the center point of unit 2.

The Kriging results are summarized as follows:

1. The calculated covariance function is given by the equation:

$$C(h) = 107.056 e^{-4.44E-5(h^2)} \quad (\text{C.22})$$

2. The estimated N function is given by:

$$\hat{N} = 0.746678Z - 33.59394 \quad (\text{C.23})$$

3. The design N value is:  $N = 74.6$
4. The predicted settlement is 0.43 in.

Therefore the predicted settlement of 0.43 in is within about 14% of the measured value of 0.5 in.

5. The 90% confidence limits of the settlement prediction are: (0.35 and 0.55) in.  
The 50% confidence limits are: (0.41 and 0.44) in.

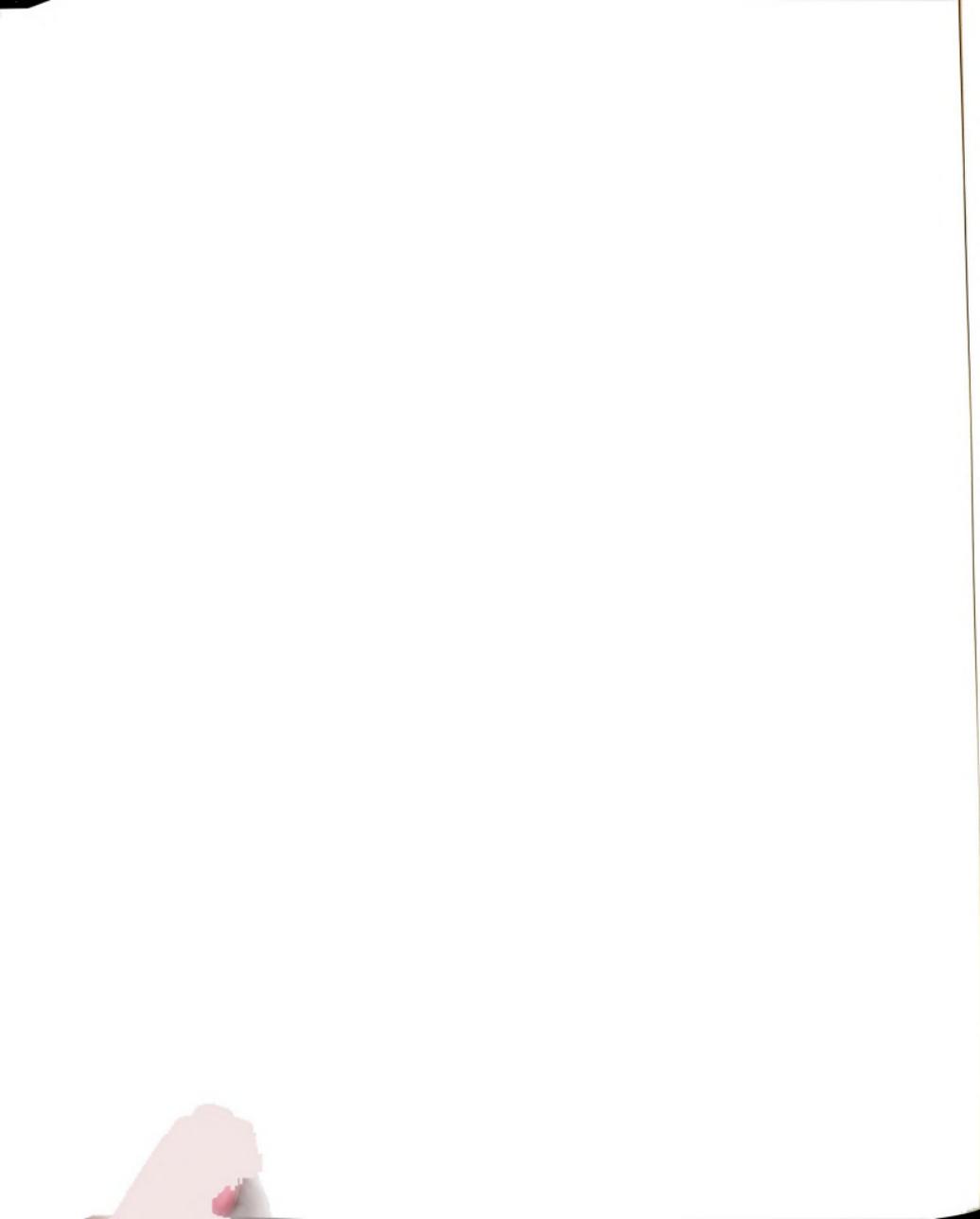
#### C.4.5 APPLYING THE TREND SURFACE ANALYSIS TECHNIQUE

The trend surface analysis results are summarized as follows:

1. The model which is fitted to the data is as follows:

$$N = 15197 - 0.02X - 4302Z^{0.5} + 360Z - 0.74Z^2, (R^2 = 0.53) \quad (\text{C.24})$$

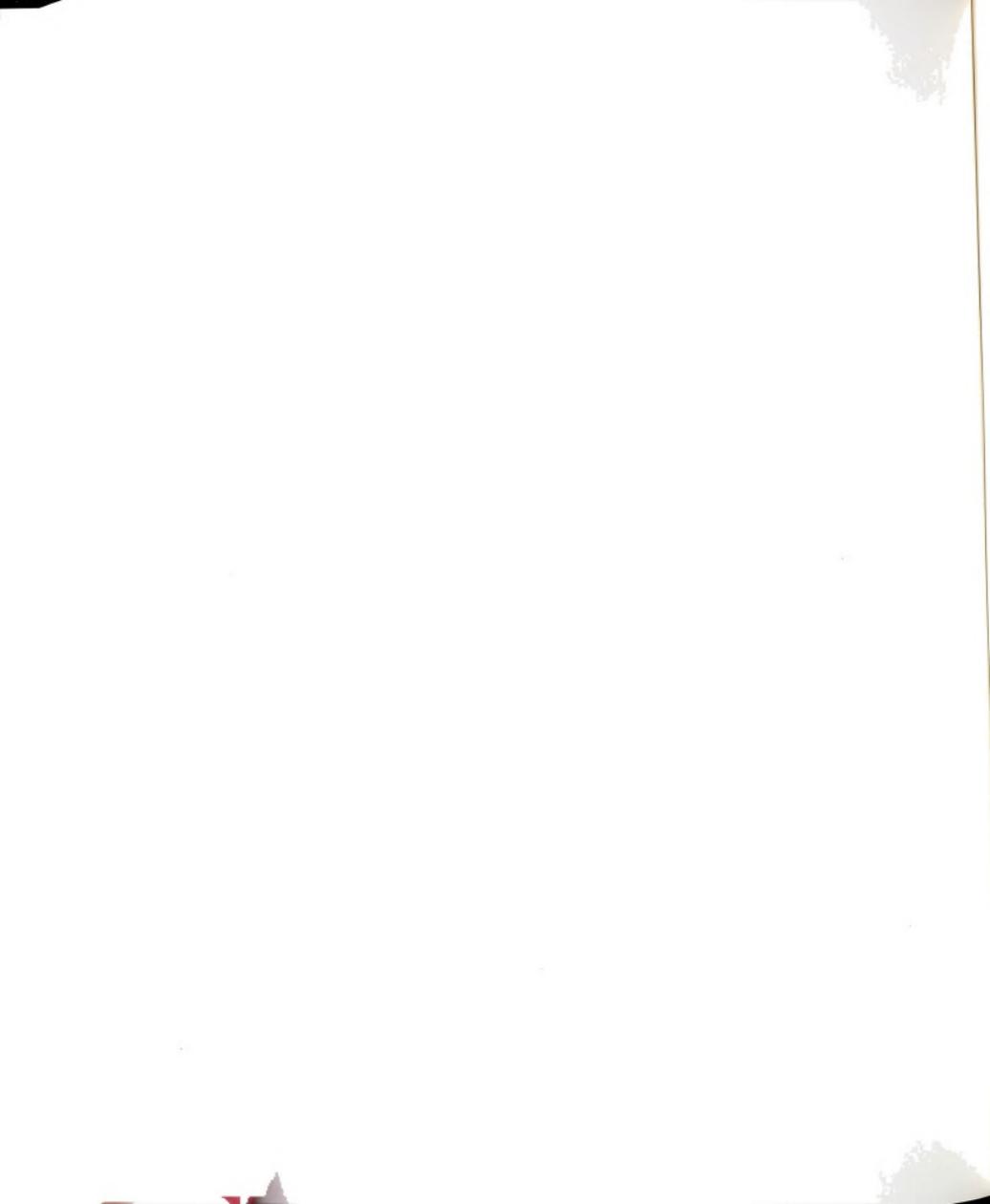
2. The design N value = 65.6



3. The predicted settlement is 0.48 in.  
Therefore the predicted settlement of 0.48 in is within about 4% of the measured value of 0.5 in.
4. The 90% confidence limits of the settlement prediction are: (0.41 and 0.60) in.  
The 50% confidence limits are; (0.45 and 0.53) in.
5. The areal distribution of settlement in inches is given by the equation:

$$S=32 / (72.11-0.025X) \quad (C.25)$$

The computer output is shown in Figure C.8.



## MODELING THE N FUNCTIONS FOR THE SUBSURFACE SOIL

LAYER	N1	X(ft)	Y(ft)	Z(ft)
1	16.7088	130.3	78.78	71
1	25.6849	130.3	78.78	76
1	37.0925	130.3	78.78	81
1	29.6594	130.3	78.78	87
1	40.4520	130.3	78.78	91
1	19.2354	421.21	230.3	71
1	12.8798	421.21	230.3	77
1	13.4637	421.21	230.3	81
1	43.6502	421.21	230.3	86
1	23.3723	421.21	230.3	91
1	27.0269	421.21	230.3	96
1	18.7404	421.21	230.3	100

## THE FITTED MODEL:

$$N = B_0 + B_1 * X + B_2 * Z^{**0.5} + B_3 * Z + B_4 * Z^{**2}.$$

12 cases are written to the compressed active file.

Run stopped after 7 model evaluations and 4 derivative evaluations.  
Iterations have been stopped because the magnitude of the largest correlation  
between the residuals and any derivative column is at most RCON = 1.000E-08

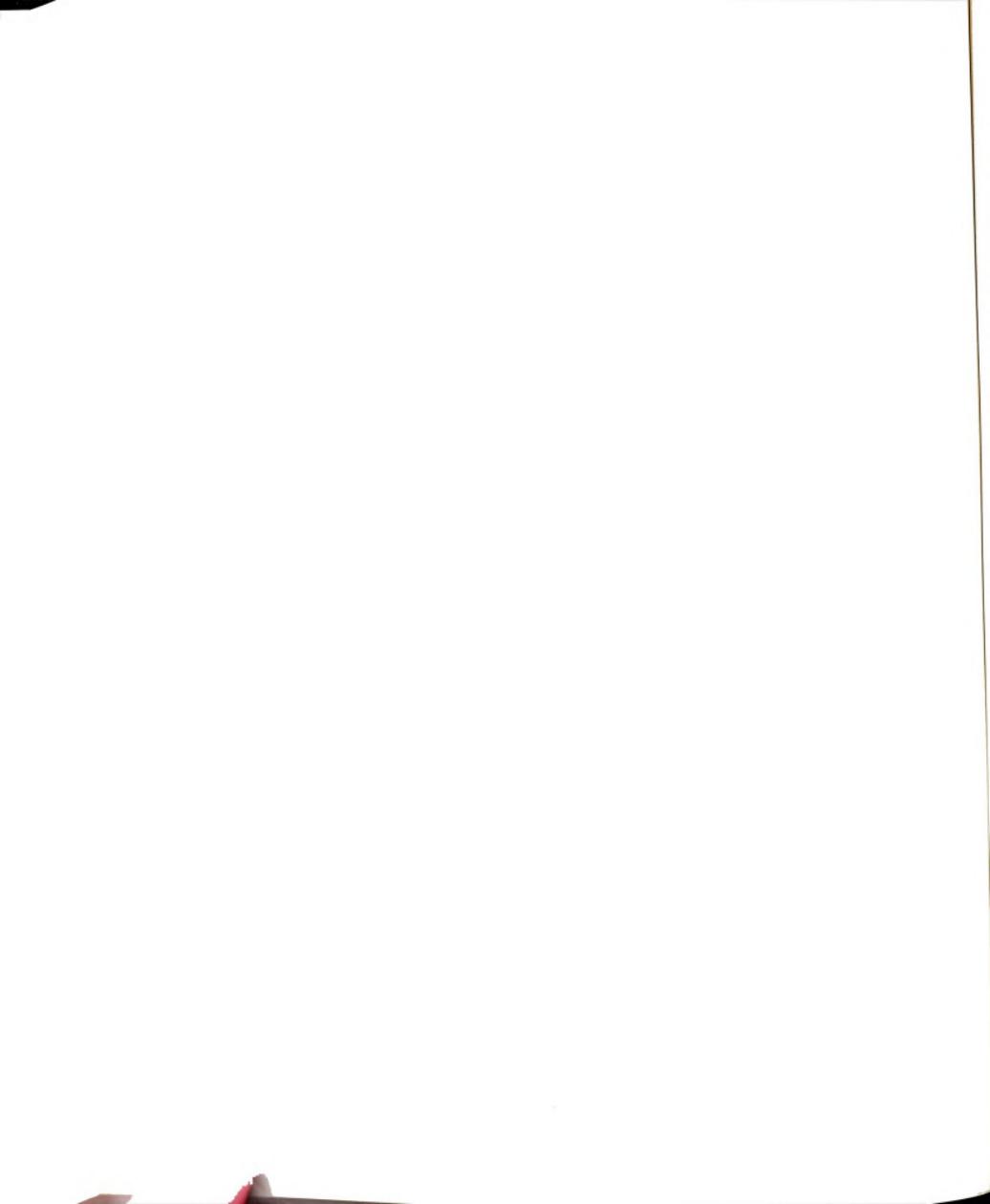
-----  
Nonlinear Regression Summary Statistics      Dependent Variable N

Source	DF	Sum of Squares	Mean Square
Regression	5	8530.63765	1706.12753
Residual	7	550.57966	78.65424
Uncorrected Total	12	9081.21732	
(Corrected Total)	11	1177.61382	

$$R \text{ squared} = 1 - \text{Residual SS} / \text{Corrected SS} = .53246$$

-----

**Figure C.8 : The Computer Output Of Case History No. 5**

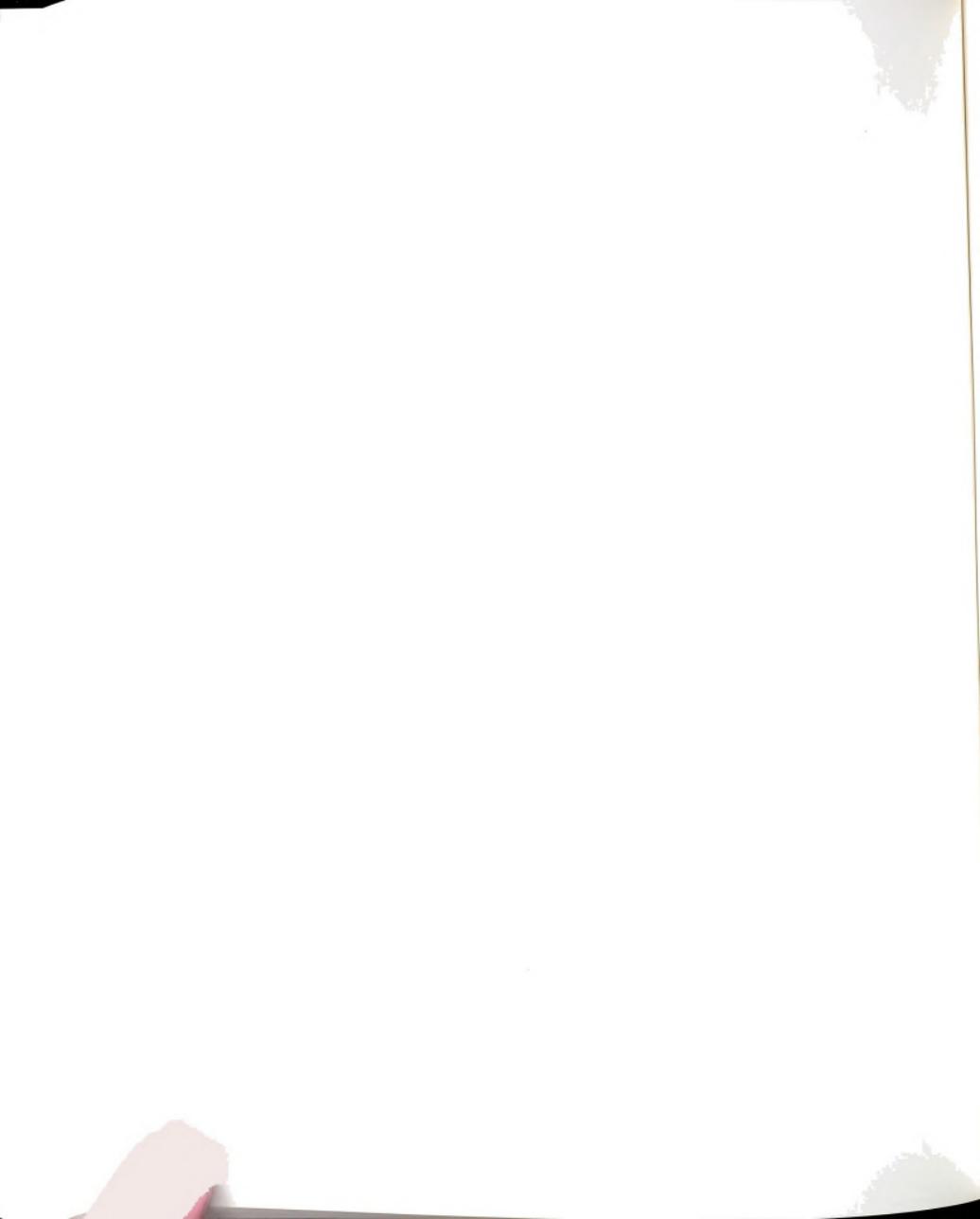


Parameter	Estimate	Std. Error	Asymptotic 95 % Confidence Interval	
			Lower	Upper
B0	15197.58	15018.059720	-21106.82770	49917.308755
B1	-.024978183	.019002146	-.069911118	.019954752
B2	-4302.410445	4372.3288091	-14641.32518	6036.5042933
B3	360.89524397	357.43892798	-484.3135136	1206.1040015
B4	-.744228194	.704496432	-2.410097543	.921641155

Asymptotic Correlation Matrix of the Parameter Estimates

	B0	B1	B2	B3	B4
B0	1.0000	.1051	-.9999	.9995	-.9980
B1	.1051	1.0000	-.1082	.1112	-.1177
B2	-.9999	-.1082	1.0000	-.9999	.9989
B3	.9995	.1112	-.9999	1.0000	-.9995
B4	-.9980	-.1177	.9989	-.9995	1.0000

Figure C.8 : Continued.



## **C.5 THE SETTLEMENT PREDICTION OF CASE HISTORY No. 6**

### **C.5.1 PROJECT GENERAL DESCRIPTION**

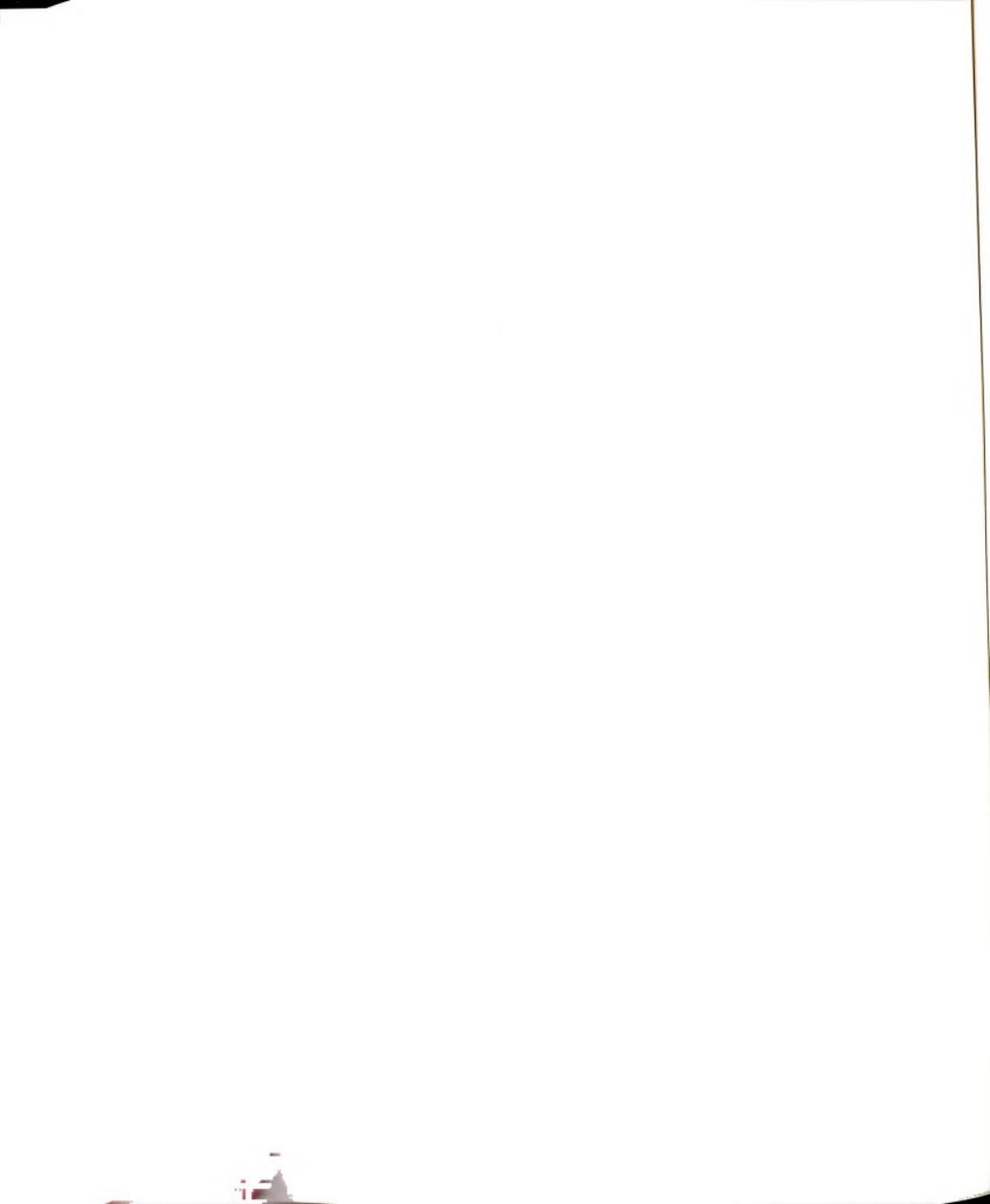
Settlement studies were made in conjunction with the design for a relatively large railroad lift bridge over the Chesapeake and Delaware Canal near Summit, Delaware. The bridge design and plans were prepared by the firm of Howard, Needles, Tammen & Bergendoff for the US Army Corps of Engineers.

Because of the size of the structure and the critical problems that could result from differential movements it was recommended that settlement measurements be made. The bridge foundations included two tower piers each of which is supported by a footing of 60 ft width. The footings were founded on piles to minimize any tilting which could be critical to the operation of the bridge lift span and to minimize settlements.

### **C.5.2 SUBSOIL INVESTIGATION**

The reported subsoil investigation included 2 borings , one at each tower pier. The N values corrected for the overburden pressure are shown in Tables C.13 and C.14. The boring locations with respect to the tower piers are shown in Figure C.9. These borings were to determine the soil types as well as the relative density of the granular soils using the standard penetration test.

The site consists of sedimentary deposits of approximately 3000 ft thickness overlaying a pre-Cambrian crystalline bedrock. The sedimentary deposits range from Pleistocene down to Cretaceous. The geological profile is relatively consistent at the bridge site. These foundation soils are highly over - consolidated , having been subjected to loads considerably greater than the present overburden pressure.



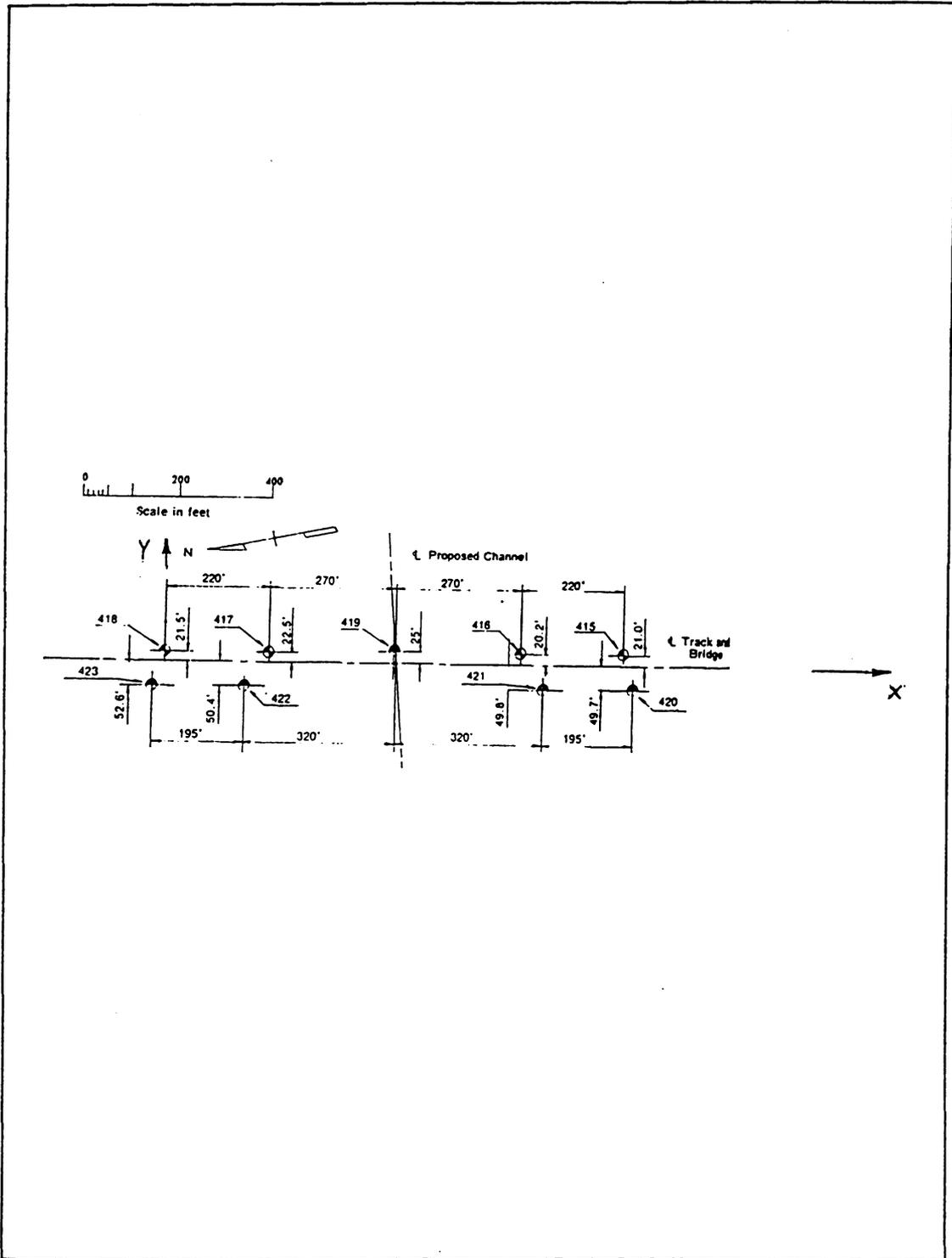
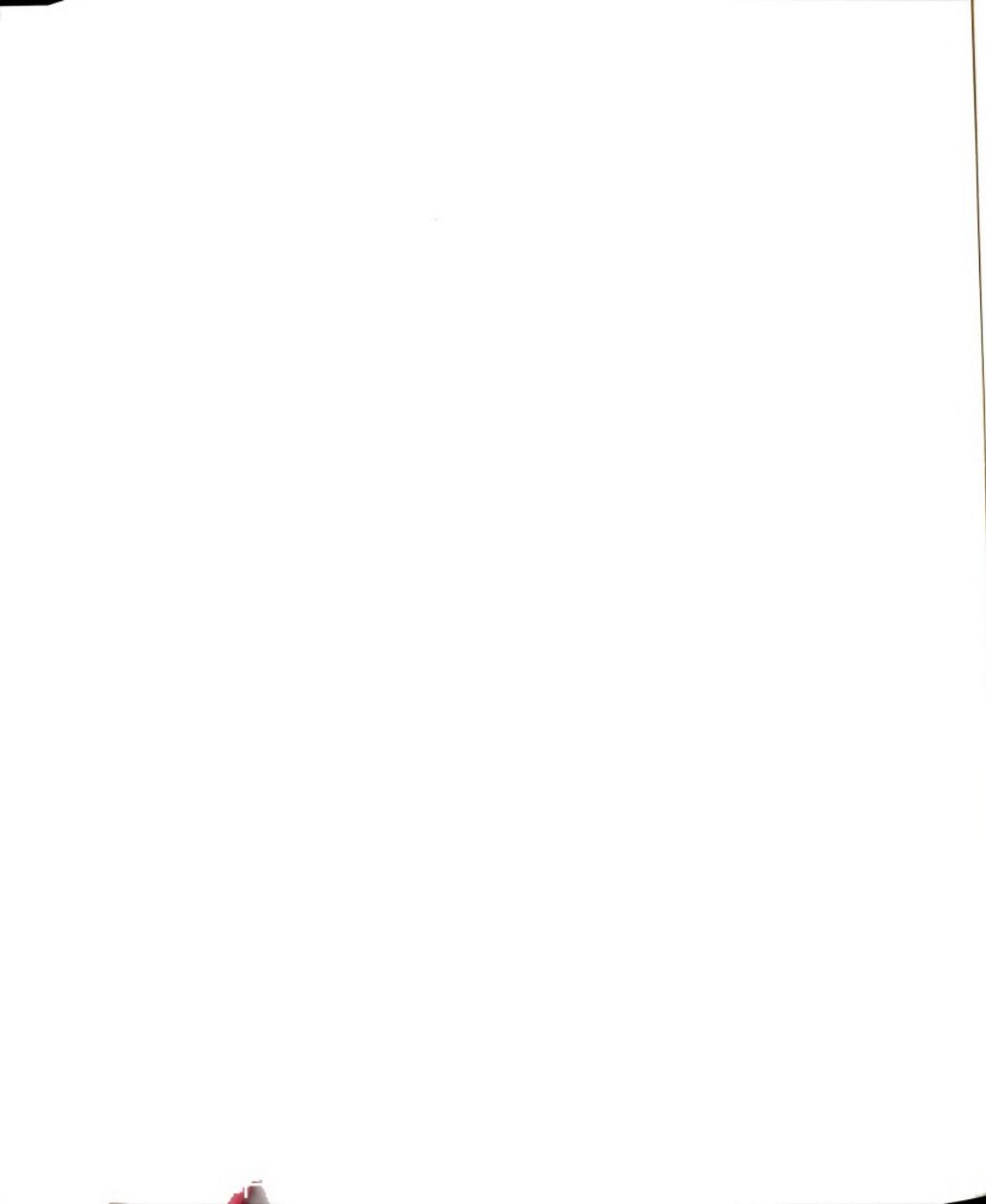


Figure C.9 : Site Plan And Boring Locations Of Case History No. 6



**Table C.13 : The Correction Of N Values Of Boring (416).**

Boring (416)			
Depth (ft)	Correction factor (C1)	N	$N1 = C1 * N$
182	0.296	242	71.752
187	0.292	135	39.488
191	0.289	240	69.463
196	0.285	202	57.714
200	0.282	120	33.941
204	0.280	240	67.213
208	0.277	188	52.141
212	0.274	137	37.636
216	0.272	606	164.932
218	0.270	600	162.548
222	0.268	400	107.384
228	0.264	400	105.962
232	0.262	400	105.045
238	0.259	400	103.712
242	0.257	240	61.711



**Table C.14 : The Correction Of N Values Of Boring (417).**

Boring (417)			
Depth (ft)	Correction factor (C1)	N	$N1 = C1 * N$
62	0.5080	154	78.2321
66	0.492	78	38.404
70	0.478	67	32.032
73	0.468	400	187.265
78	0.452	600	271.746
82	0.441	600	265.035
84	0.436	167	72.884
90	0.421	300	126.491
93	0.414	300	124.434
98	0.404	64	25.859

### C.5.3 FOUNDATIONS AND SETTLEMENT MEASUREMENTS

The bottom of the pier footing seals were placed at an elevation of 42 ft below ground surface. The estimated pile tip elevation were elevation of 115 ft below ground surface. Therefore the depth of the compressible layer which caused the settlement is  $(180 - 115 = 65 \text{ ft})$ .

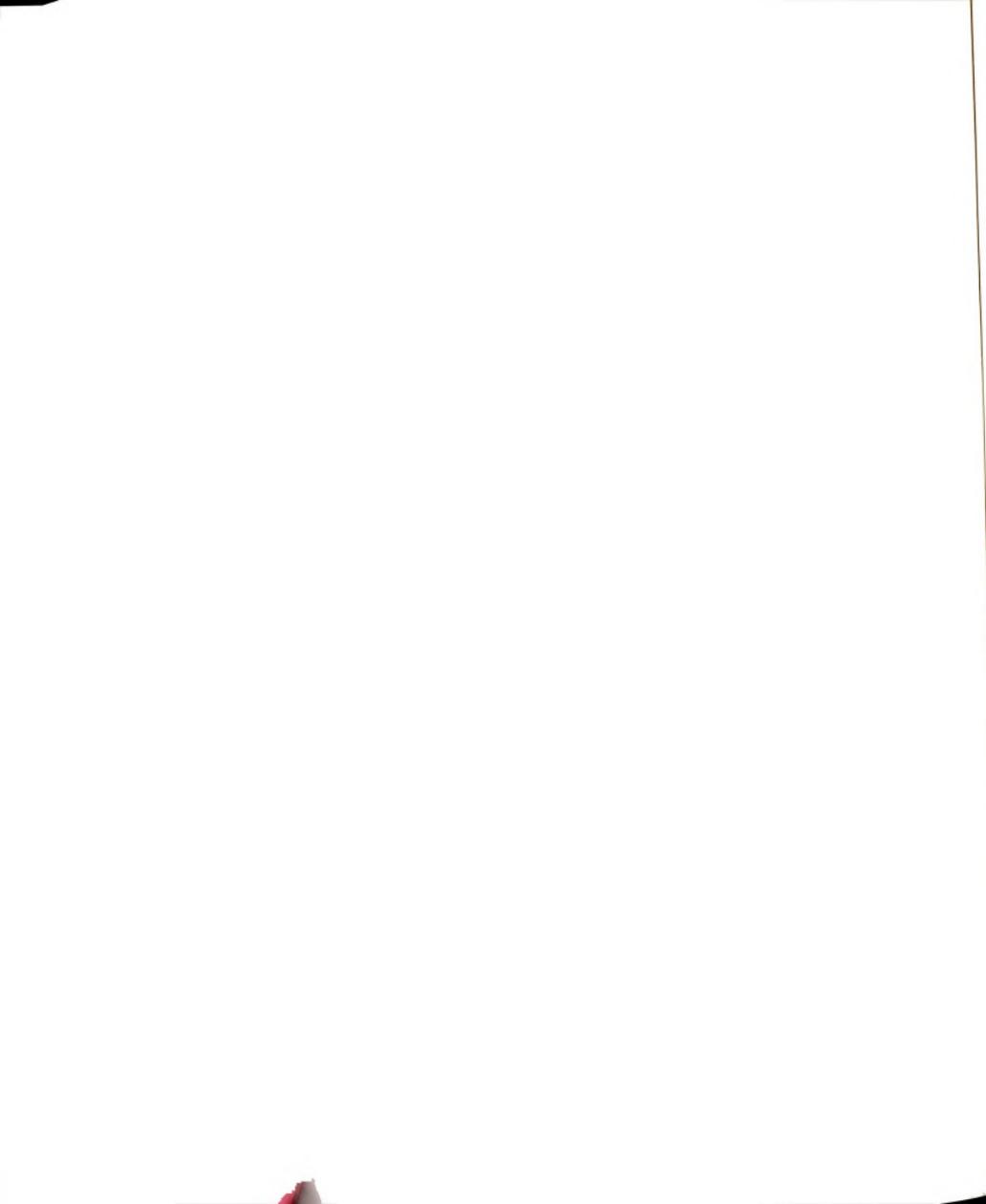
Measurements of the vertical movement of the tower piers were made at the four corners of the pier and were taken over a period of about 70 months. These measurements showed a little heaving during the first 400 days and was followed by a small settlement below the original level and then further heave. It is believed that the heave resulted from the release of overburden pressure by excavation through a highly over - consolidated soil and the following settlement was the result of the compression of the compressible sands due to the construction of the bridge foundations and superstructure. So the movement which is attributable to the sand settlement below pile tips is 0.4 inch. The pressure due to the construction at the elevation of 115 ft amounted to 0.9 tsf.

### C.5.4 APPLYING THE KRIGING TECHNIQUE

Considering that a settlement prediction is required at the center point of the south tower pier foundation.

The Kriging results are summarized as follows:

1. The calculated covariance function is given by the equation:



$$C(h) = 291.66 e^{-4.0E-6 (h^2)} \quad (\text{C.26})$$

2. The estimated N function is given by:

$$\hat{N} = 0.97Z - 123.5 \quad (\text{C.27})$$

3. The "two-point" estimate of N values are:

$$\hat{N}_{(B/2)} = 17.73, \hat{N}_{(3B/2)} = 76.21 \quad (\text{C.28})$$

4. The design N value is given by the weighted average:

$$N = (1/3) [2\hat{N}_{(B/2)} + \hat{N}_{(3B/2)}] = 37.23 \quad (\text{C.29})$$

5. The predicted settlement is 0.38 in.

Therefore the predicted settlement of 0.38 in is within about 5% of the measured value of 0.4 in.

6. The 90% confidence limits of the settlement prediction are: (0.33 and 0.64) in.

The 50% confidence limits are: (0.37 and 0.39) in.

### C.5.5 APPLYING THE TREND SURFACE ANALYSIS TECHNIQUE

The trend surface analysis results are summarized as follows:

1. The model which is fitted to the data is as follows:

$$N=27836-1.62X-8072Z^{0.5}+675Z-1.6Z^2, (R^2=0.503). \quad (\text{C.30})$$

2. The "two-point" estimate of N values:

$$N_{(B/2)}=4.55, N_{(3B/2)}=108.44 \quad (\text{C.31})$$

3. The design N value is given by:

$$N=(1/3) [2N_{(B/2)}+N_{(3B/2)}] =39.18 \quad (\text{C.32})$$

4. The predicted settlement is 0.36 in.

Therefore the predicted settlement of 0.36 in is within about 9% of the measured value of 0.4 in.

5. The 90% confidence limits of the settlement prediction are: (0.22 and 1.04) in.

The 50% confidence limits are: (0.29 and 0.48) in.

6. The areal distribution of settlement in inches is given by the equation:

$$S=14.34/(1275.758-1.627X) \quad (\text{C.33})$$

The computer output is shown in Figure C.10.



LAYER	N1	X(ft)	Y(ft)	Z(ft)
1	71.7529	760	20.2	182
1	39.4887	760	20.2	187
1	69.4632	760	20.2	191
1	57.7143	760	20.2	196
1	33.9411	760	20.2	200
1	67.2134	760	20.2	204
1	52.1418	760	20.2	208
1	37.6368	760	20.2	212
1	78.2321	220	22.5	62
1	38.4045	220	22.5	66
1	32.0321	220	22.5	70
2	164.9323	760	20.2	216
2	162.5485	760	20.2	218
2	107.3849	760	20.2	222
2	105.9626	760	20.2	228
2	105.0451	760	20.2	232
2	103.7126	760	20.2	238
2	61.7111	760	20.2	242
2	187.2658	220	22.5	73
2	271.7465	220	22.5	78
2	265.0357	220	22.5	82
2	72.8848	220	22.5	84
2	126.4911	220	22.5	90
2	124.4342	220	22.5	93
2	25.8599	220	22.5	98

**Figure C.10 : The Computer Output Of Case History No. 6**

----- O N E W A Y -----

Variable N  
By Variable LAYER

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	1	41517.4676	41517.4676	14.1013	.0010
Within Groups	23	67717.4130	2944.2353		
Total	24	109234.8806			

T-TEST/VARIABLE N.

Independent samples of LAYER

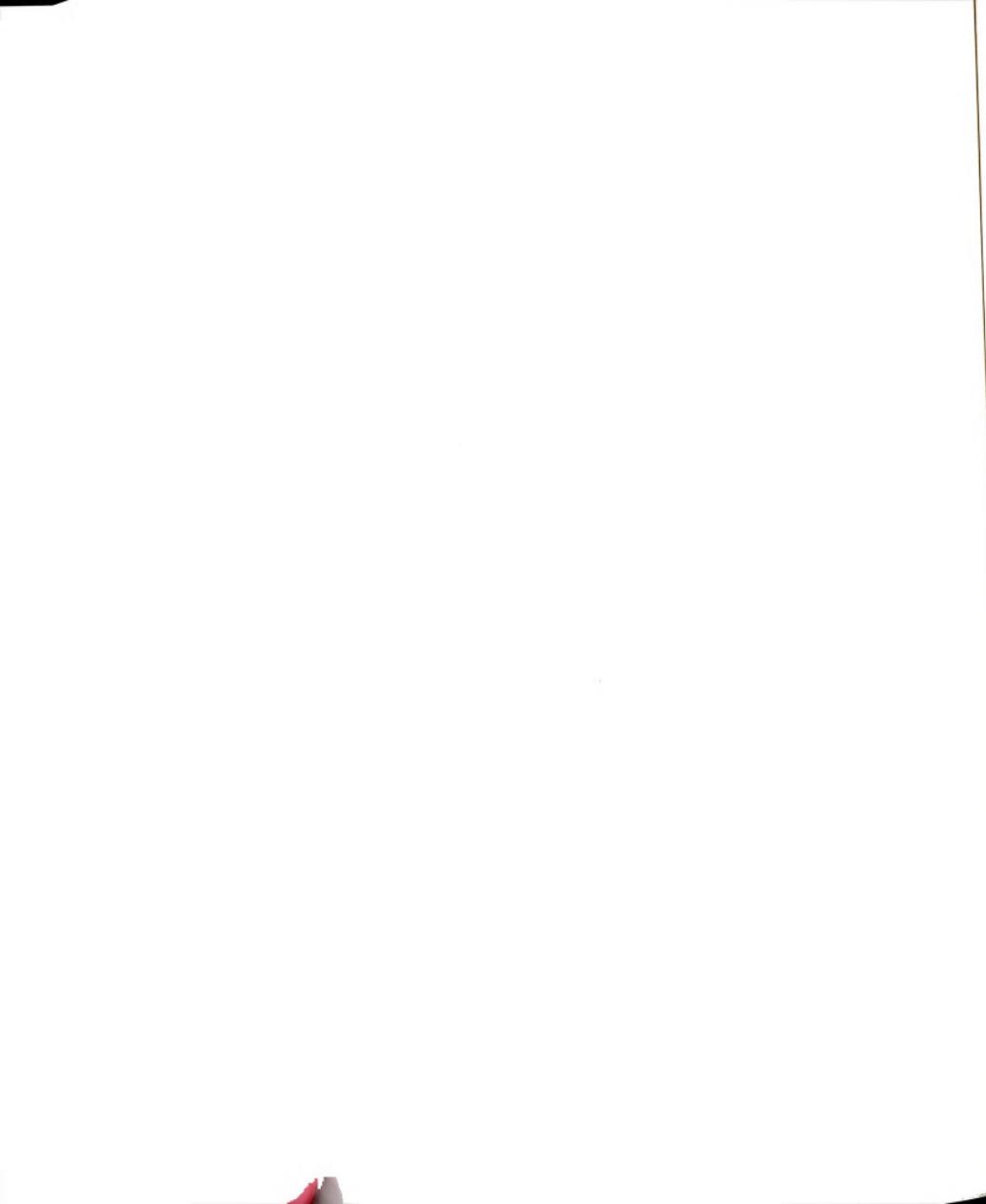
Group 1: LAYER EQ 1.00      Group 2: LAYER EQ 2.00

t-test for: N

	Number of Cases	Mean	Standard Deviation	Standard Error
Group 1	11	52.5474	17.078	5.149
Group 2	14	134.6439	70.602	18.869

		Pooled Variance Estimate			Separate Variance Estimate		
F Value	2-Tail Prob.	t Value	Degrees of Freedom	2-Tail Prob.	t Value	Degrees of Freedom	2-Tail Prob.
17.09	.000	-3.76	23	.001	-4.20	14.90	.001

Figure C.10 : Continued.



## MODELING THE N FUNCTIONS FOR THE SUBSURFACE SOIL

LAYER	N1	X(ft)	Y(ft)	Z(ft)
1	71.7529	760	20.2	182
1	39.4887	760	20.2	187
1	69.4632	760	20.2	191
1	57.7143	760	20.2	196
1	33.9411	760	20.2	200
1	67.2134	760	20.2	204
1	52.1418	760	20.2	208
1	37.6368	760	20.2	212
1	78.2321	220	22.5	62
1	38.4045	220	22.5	66
1	32.0321	220	22.5	70

## THE FITTED MODEL:

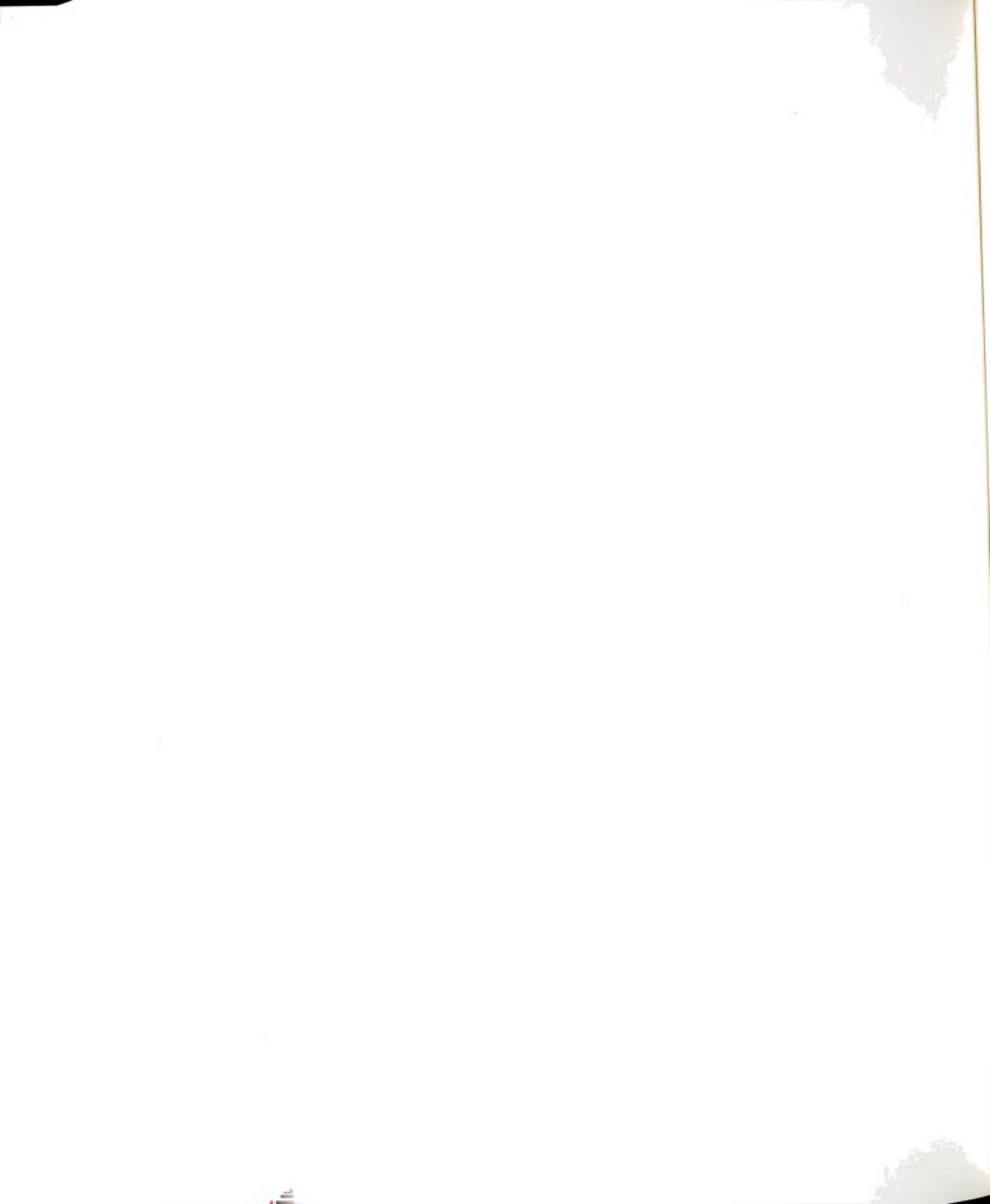
$$N = B0 + B1 * X + B2 * Z^{**0.5} + B3 * Z + B4 * Z^{**2} + B5 * Z^{**3}.$$

There are 11 cases. There is enough memory for them all.  
 Run stopped after 11 model evaluations and 4 derivative evaluations.  
 Iterations have been stopped because the relative reduction between successive residual sums of squares is at most SSSCON = 1.000E-08

## Nonlinear Regression Summary Statistics      Dependent Variable N

Source	DF	Sum of Squares	Mean Square
Regression	6	31841.69154	5306.94859
Residual	5	1448.37907	289.67581
Uncorrected Total	11	33290.07062	
(Corrected Total)	10	2916.60145	
R squared = 1 - Residual SS / Corrected SS =			.50340

Figure C.10 : Continued.



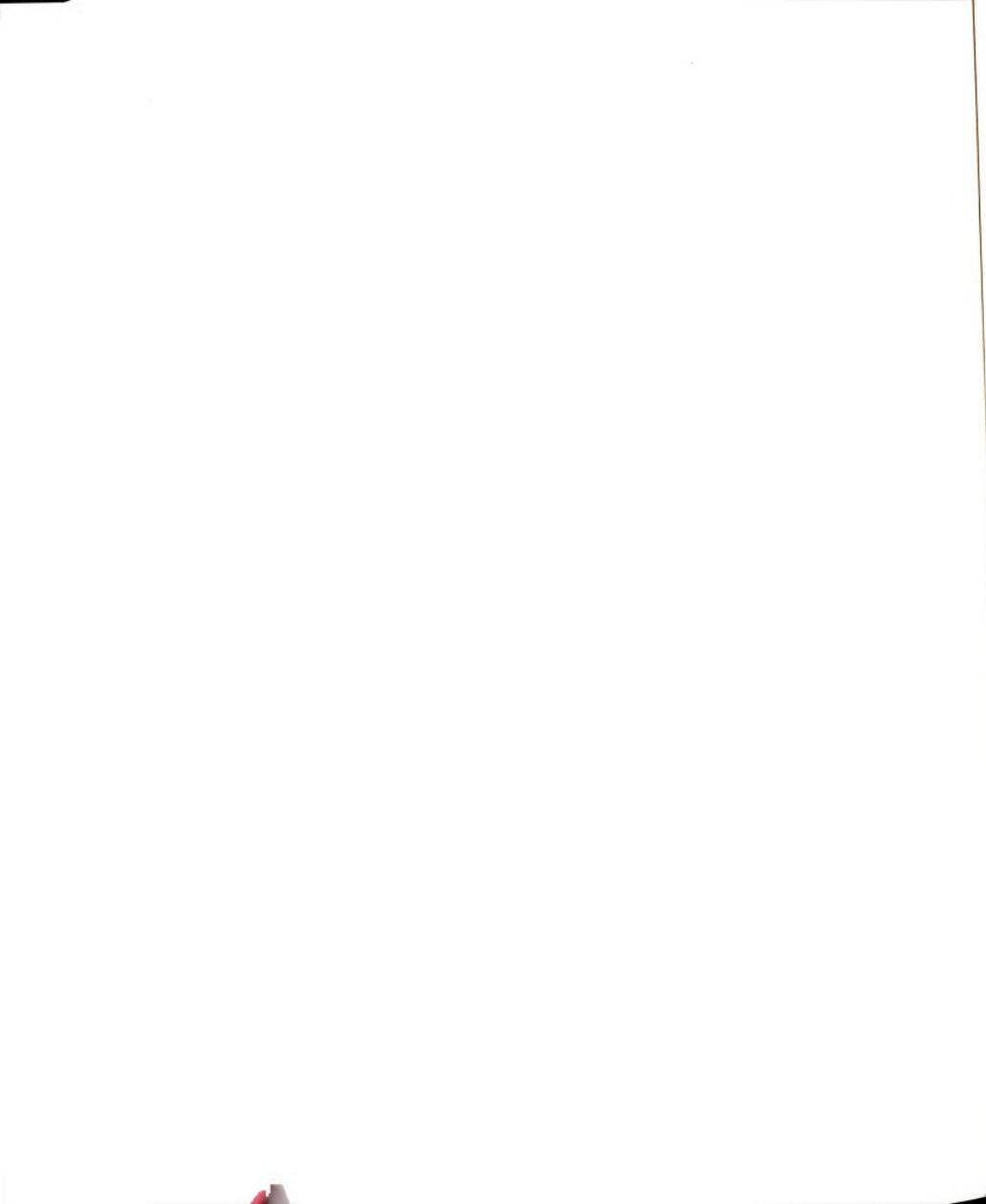
Parameter	Estimate	Std. Error	Asymptotic 95 % Confidence Interval	
			Lower	Upper
B0	27836.718891	52453.781314	-107060.0186	162613.45635
B1	-1.627071610	4.198324875	-12.41920927	9.165066053
B2	-8072.968442	15557.148607	-48063.89207	31917.955181
B3	675.68407215	1328.4194959	-2739.126954	4090.4950985
B4	-1.603972812	3.233142627	-9.915030521	6.707084896
B5	.002087879	.004293028	-.008947700	.013123458

## Asymptotic Correlation Matrix of the Parameter Estimates

	B0	B1	B2	B3	B4	B5
B0	1.0000	-.9833	-.9999	.9997	-.9983	.9952
B1	-.9833	1.0000	.9817	-.9800	.9732	-.9634
B2	-.9999	.9817	1.0000	-.9999	.9989	-.9963
B3	.9997	-.9800	-.9999	1.0000	-.9994	.9972
B4	-.9983	.9732	.9989	-.9994	1.0000	-.9992
B5	.9952	-.9634	-.9963	.9972	-.9992	1.0000

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Figure C.10 : Continued.



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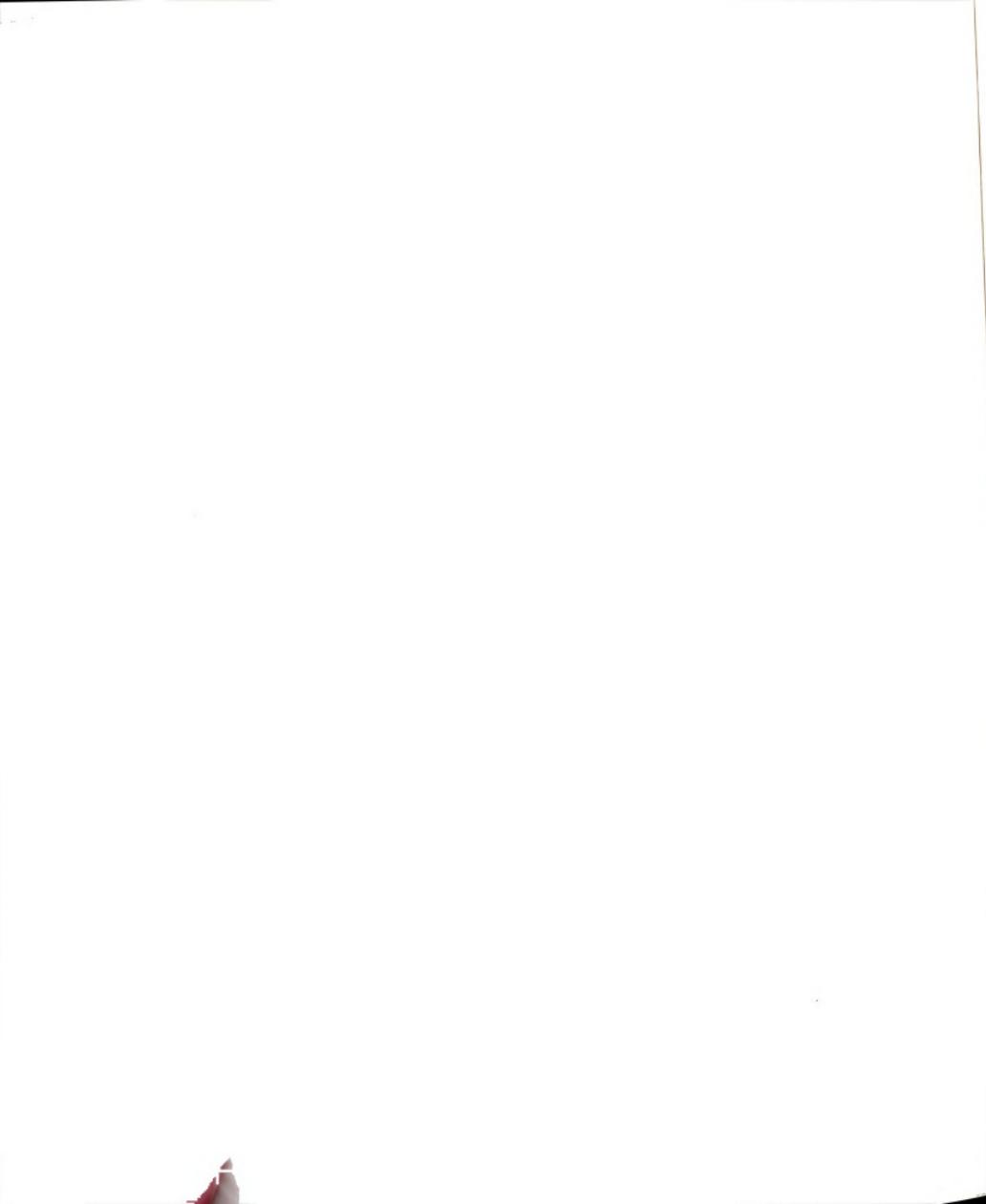
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