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A STUDY OF WATER MOVEMENT
IN AN UNSATURATED SOIL
UNDER THE VELOCITY PERMEAMETER

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Natalie J. Carroll

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Department of
Agricultural Engineering

George E. Merva
Major professor

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A STUDY OF WATER MOVEMENT IN AN UNSATURATED SOIL UNDER THE VELOCITY PERMEAMETER

By

Natalie J. Carroll

A DISSERTATION

Submitted to
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ABSTRACT

A STUDY OF WATER MOVEMENT IN AN UNSATURATED SOIL UNDER THE VELOCITY PERMEAMETER

BY

Natalie J. Carroll

Careful management of world water resources has become of primary importance to agricultural, civil and biosystems engineers. The demand for high quality water is constantly increasing while the depletion of the world fresh water supply continues at an alarming rate. Agricultural practices are the major source of this depletion. Agricultural practices also cause considerable degradation of the water supply in some areas. Surface water pollution, ground water and sediment pollution and non-point sources of pollution come from agriculture as well as other origins.

There is much room for improvement of agricultural water use. Excessive irrigation is no longer acceptable and adding new irrigated fields must be done with great care. Sustainable agriculture must be practiced in all agricultural endevors. Careful water use, however, necessitates a good understanding of soil and water interactions. Agricultural engineers are particularly interested in the flow of water through soil for both drainage and irrigation purposes as well as site selection for agriculture waste-water holding facilities. To predict fluid flow rates in porous media an accurate measurement of the hydraulic conductivity, k, is needed. The velocity

permeameter, VP, can be used to measure the in-situ hydraulic conductivity of soil water at the tillage depth. The VP is a portable instrument and has been found to be very useful for quick and accurate site evaluations.

The objective of this research was to develop a computer program to model water movement into the soil under the VP. The model will show the change in soil water potential with time under the VP as well as the shape and extent of both the wetted and saturated fronts. The model will accomodate different soil parameters and various equipment configerations.

Dedication

this work is dedicated to

Dale,

Adrienne, Eric and John

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I would like to thank Dr. G. Merva and Dr. L. Segerlind, my major professors, for their help, patience, insights, and guidance. I appreciate the advice and direction of the other guidance committee members: Dr. V. Bralts; Dr. R. Kunze; and Dr. R. Wallace. Dr. J. Steffe, Dr. A. Srivastava and Dr. R. Ofoli, although not on my guidance committee, aided and enhanced my professional development and I am grateful. I thank Dr. D. Welch for his help and motivation and Dr. M. Esmay who was an early positive influence in my engineering career.

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1. INTRODUCTION

1.1 Global Aspects and Objectives

Careful management of world water resources has become of primary importance. Assuring an adequate supply of high quality water for all people in years to come is a concern to engineers, politicians, heads of state and concerned citizens. By the year 2000 many countries will experience excessive scarcity of water due to the increasing demand on water resources for agriculture, industry and domestic use (UN Environment Program, 1992). The demand for water varies markedly from one country to another and depends both on population and on the prevailing level and pattern of socioeconomic development. Conspicuous disparity exists between developed and developing countries. The average per capita domestic use of water in the United States is more than 70 times that in Ghana (UN Environment Program, 1992). Furthermore, world wide water use is increasing rapidly. In 1950 1,360 km^3 water was used, in 1990 that figure had risen to 4,130 km^3 and by the year 2000 it is expected that 5,190 km^3 will be used each year (UN Environment Program, 1992).

Agricultural practices are the major source of depletion of the water supply. Averaged globally, 69% of water withdrawn is used for agricultural purposes, 23% for industry and 8% for domestic purposes (UN Environment Program, 1992). Agricultural practices also cause considerable degradation of the water supply in some countries. Surface water pollution, groundwater and sediment pollution and non-point sources of pollution come from agriculture as well as other origins. Discharge from untreated or inadequately treated wastewater into rivers, lakes and reservoirs is a problem. Eutrophication of rivers and lakes caused mainly by the runoff of fertilizers from agricultiural lands is significant in some areas. Acidification of lakes is common in North America and in some European countries that are heavy fertilizer users. Excessive irrigation has led to water logging and salinization thereby accelerating land degradation. There are fears that the rapid expansion of agriculture in desert areas may lead to over-exploitation of groundwater for irrigation and irreparable depletion of this resource. For example, the level of the Aral Sea is retreating because excessive irrigation withdrawals have been reducing inflow from the catchment area. The Aral sealevel has dropped by 3 m since 1960 and, if the trend continues, will drop another 9-13 m by the year 2000. With the reduced inflow from irrigation returns the salinity of the Aral Sea has already increased threefold to 1 g/liter and by the year 2000 this is expected to rise to 3.5 g/liter (UN Environment Program, 1992).

There is much room for improvement of agricultural water use. Excessive irrigation is no longer acceptable and adding new irrigated fields must be done with great care. Sustainable agriculture must be practiced in all agricultural endevors. Careful water use, however, necessitates a good understanding of soil and water interactions. Agricultural engineers are particularly interested in the flow of water through soil for both drainage and irrigation purposes as well as site selection for agriculture waste-water holding facilities. To predict fluid flow rates in porous media an accurate measurement of the hydraulic conductivity, k, is needed. Civil engineering applications are generally concerned with aquifer transmissivity, T, values (k may be determined from T) and relatively large tracts of land. Irrigation and drainage applications are on a smaller scale and practiced ordinarily on unsaturated soils near the soil surface so many traditional methods for measuring k are not useful to an agricultural engineering. The k value for an aquifer is very different from the k value for the unsaturated soil in the tillage depth. The velocity permeameter (VP, developed by Merva, 1979) can be used to measure the in-situ hydraulic conductivity of soil water at the tillage depth. The VP is a portable instrument and has been found to be very useful for quick and accurate site evaluations.

The objective of this research was to develop a computer program to model water movement into the soil under the VP. The model will show the change in soil water potential with time under the VP as well as the shape and extent of both the wetted and saturated fronts. The model will accomodate different soil parameters and various equipment configerations.

1.2 Hydraulic Conductivity Measurements

Hydraulic conductivity, k, is an important parameter which is used in determining soil water flow rates for the design of reservoirs, flood and erosion control structures, channel improvements, drainage systems, irrigation scheduling, drainage design, runoff rates and volumes, groundwater recharge, emulating leaching and other agricultural or hydrological processes. The measurement of k is fundamental to the design of irrigation systems which comprise the largest single group of water users in the United States, expending over 40 percent of the total annual water usage (Skaggs, Monke and Huggins, 1972). Laboratory and field measurement of soil hydraulic properties are time consuming, often costly and subject to large error. Also, field soils exhibit large spatial variabilities in their hydraulic properties, particularly unsaturated hydraulic conductivity values (Nielsen et al., 1973 and Stockton and Warrick, 1971). In fact the natural soil variations may be larger than differences between methods of measurement (Taherian, Hummel & Rebuck, 1976; Bouwer and Jackson, 1974). Bosch (1991) analysed the errors associated with point observations of matric potential in soil profiles and suggested an autocorrelation function of the matric potential data and of the hydraulic parameters of the soil profile to be used in conjunction with an error function to better design field experiments.

A large number of field measurements are required to determine k due to the many variables involved (Jabro, 1992). But a knowledge of the spatial variability in the hydraulic conductivity is essential for understanding and modeling of water and chemical movement (Rogers et al., 1991). Some authors prefer to determine soil hydraulic properties from easily measuable soil properties such as particle size distribution, bulk density, effective porosity and carbon content and then to estimate the hydraulic conductivity from these measurements (Jabro, 1992).

1.3 Laboratory Measurements of Hydraulic Cconductivity

There are three principal types of laboratory measurements to determine the saturated hydraulic conductivity. The constant-head permeameter and the falling-head permeameter are similar with the difference being that the water level is allowed to drop in the second apperatus. The hydraulic conductivity of the soil may also be determined from consolidation tests (Freeze and Cherry, 1979).

1.3.1 Constant-Head Permeameter

The constant-head permeameter (Figure 1) consists of a soil sample of length L and cross-sectional area A between two porous plates. A tube with a reservoir (and overflow) maintains the water supply level at a height H. The outflow, Q, is measured from the apperatus and the hydraulic conductivity, k, is determined by the equation

$$k = \frac{QL}{AH} \tag{1.1}$$

This method has been shown to work best for soils with k > 0.01 cm/min (Klute, 1965).

1.3.2 Falling-Head Permeameter

The falling-head permeameter (Figure 2) also consists of a soil sample of length L and cross-sectional area A placed between two porous plates. The initial height of water in the supply tube is H_0 and the value H_1 is the water height after time t.

$$k = \frac{aL}{At}\ln\left(\frac{H_0}{H_1}\right) \tag{1.2}$$

This method has been seen to work better for samples with k < 0.01 cm/min (Klute, 1965). Replacing the soil air with carbon dioxide and covering the soil surface with sand (a practice that is sometimes employed to alleviate slaking and dispersion) is not recommended (McIntre et al., 1979). The authors in this paper contend that the k found is not the saturated k, as is generally assumed, particularly for low stability soils. They note that the value found, however, is still a useful measurement in assessing the suitability of surface soils for irrigation.

1.3.3 Consolidation Test

The consolidation (Figure 3) test measures soil compressibility with a consolidometer. A soil sample with cross-section A is placed in a loading cell and a load L is applied which creates a stress, σ, where σ = L/A. The compressibility, α, is determined from the slope of the void ratio, e, versus effective stress σ_e. The rate of consolidation is dependent on the compressibility and the hydraulic conductivity, k, and the coefficient of consolidation, c_v. Lambe (1951) describes the determination of c_v and k using the decline in sample thickness for each loading increment. The consolidation test is seldom used by agricultural engineers.

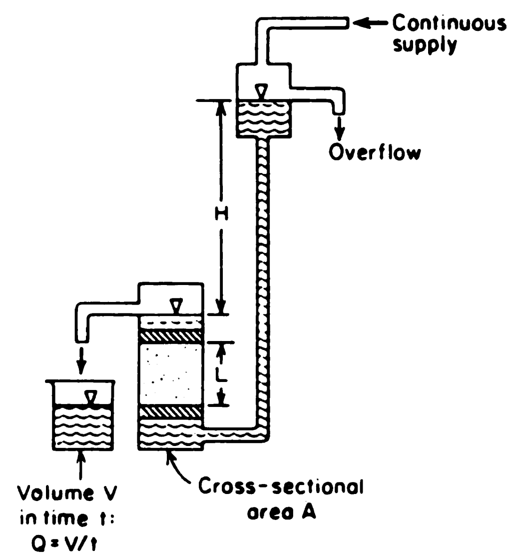

Figure 1. Skematic of the constant head permeameter (Freeze and Cherry, 1979; after Todd, 1959)

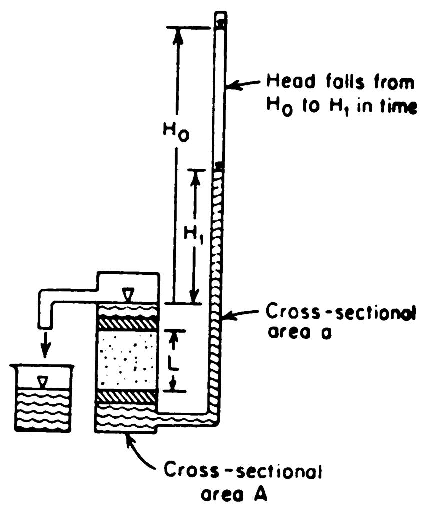

Figure 2. Skematic of the falling head permeameter

(Freeze and Cherry, 1979; after Todd, 1959)

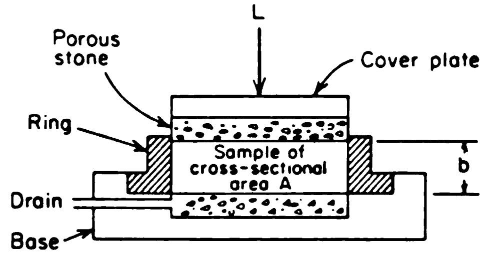

Figure 3. Skematic of a consolidometer

(Freeze and Cherry, 1979; after Lambe, 1951)

1.4 Field Measurements of Hydraulic Conductivity

The hydraulic conductivity is very sensitive to any disturbance of the soil (Bouma, 1981). Field methods have the advantage over laboratory methods in that the soil is minimally disturbed. The scientist need not be concerned that moisture content or other soil parameters might be altered during transport to the laboratory. Disadvantages of field testing methods include contention with natural weather conditions and the need to transport all equipment to remote sites.

1.4.1 Piezometers

Piezometers are widely used to determine soil hydraulic properties and there are many variations on the main theme. Point piezometers are open only over a short interval at their base while a screened (slotted) piezometer is open over the entire thickness of a confined aquifer. Measurements are made by a quick introduction (slug test) or removal (bail test) of water (or soil volume) and observation of recovery time. Tests are dependent on high quality intake and the piezometer tubes are subject to corrosian and clogging problems which may give highly inaccurate k values. Backwashing to clean the piezometer tube, however, may also give highly inaccurate values. Piezometers give in-situ values that are averaged over a relatively small volume of aquifer.

1.4.2 Pumping Tests

Pumping tests give in-situ values that are averaged over a large aquifer volume. These are labor intensive tests since the engineer must drill a test well and one, or more, observation piezometers. Next, a short-term pumping test, at a constant rate, and the application of predictive formulas (graphical time versus drawdown data; generally the Theis or Jacob methods) are used to determine soil hydraulic properties. Pumping tests give values that are averaged over large volumes. They are time consuming, costly and it is often difficult to evaluate aquifer geometry which is needed for determining what curves to use in the Theis or Jacob analysis. Furthermore, proper pump testing involves much art in both set-up and analysis (Freeze and Cherry, 1979).

1.4.3 Cone Penetrometer

The cone penetrometer with pore pressure measurements was first introduced in 1975 and is generally regarded in the geotechnical community as one of the most efficient tools for stratigraphic logging of soft soils (Robertson et al., 1986). A coupled system was developed with a porous probe ground-water sampler to perform soil logging, evaluate k and collect ground-water samples in an effort to determine the extent and preferential flow pathway(s) of a soluble hydrocarbon plume in a Texas aquifer (Chiang, Loos & Klopp, 1991). When steady penetration is stopped the excess pore pressure decay with time may be used to calculate the coefficient of consolidation, $c_v\left(\frac{cm^2}{sec}\right)$, in cohesive soils. Then the coefficient of consolidation may be used to calculate the hydraulic conductivity, (k in cm/sec), based on the equation

$$k = c_v * m_v * \gamma_w \tag{1.3}$$

where m_v is the coefficient of volume compressibility, in cm^2/kg and γ_w is the specific weight of water in kg/cm^3. The method does not work for granular soils because of the rapid dissipation of the excess pore pressures. An empirical correlation between k and soil relative density was developed for granular soils by Chiang, Loos & Klopp (1991).

The cone penetrometer was designed for in-situ sampling of liquids and gases of an entire aquifer. It's use is reserved to large companies (or possibly governmental agencies) due to the expense and the need for a testing vehicle able to hydraulically advanced a coring device via a 25-ton reaction.

Pumping tests and the cone penetrometer are beyond the means, or needs of the agricultural community. Aquifer analysis is not generally pertinent to this group's concerns. The hydraulic properties of interest are those near the soil surface where tillage, drainage and irrigation take place. Waste holding tanks, however, may need analysis at some depth below the tillage layer. This discussion will focus on in-situ analysis methods that apply near the soil surface and are economically feasible.

1.4.4 Auger Holes

The auger hole method of measuring hydraulic conductivity was first proposed by Diserens and later by Hooghoudt (vanBavel & Kirkham, 1949). The method consists of an auger hole which is made in soil extending below the water table and subsequently emptied. The rate of refill in the hole is dependent on soil permeability, hole dimensions and the height of water in the hole (van Bavel & Kirkham, 1949). The hydraulic conductivity values are averaged over a large volume (both directionally and between horizons), which may, or may not be desirable, depending on the application. Advantages include the use of the naturally occuring fluid (soil water) not an unknown or introduced fluid, such as is the case in laboratory experiments. Furthermore experiments are not unduly time-consuming as reported by vanBavel & Kirkham, 1949. The auger hole method has the disadvantage that a water table is needed which is preferably not too low, this often may occur just in the spring when water tables are high.

A two auger hole method was proposed by Childs (Luthin, 1966). The method utilizes pumping from one auger hole, at a constant rate, to another auger hole until the elevations in both holes stabilize. This method provided consistent results under field conditions (Taherian, Hummel & Rebuck, 1976). The authors also tested different screens (aluminum, plastic and steel) to prevent puddling of sloughing of the sides of the auger hole. No logging of the screens was noticed, although the flow rate through the soil decreased slightly. The k values for the two hole method averaged 1.2 m/d as compared to an average of 0.13 m/d for a single auger hole (Taherian, Hummel & Rebuck, 1976). These authors also compared the k values from other methods of k determinations. Results for a single auger hole were 0.14 m/d, the piezometer gave 0.24 m/d and a permeability tank (1.2 cubic soil block that was saturated and Q measured from under a constant horizontal flow) gave 0.16 m/d. The primary drawback of the two auger hole method is the long time period, about 5 hours, to attain steady state conditions.

Rogers and Carter (1987) showed that large variations in k values may be introduced by non-systematic use of the auger hole method in layered soils.

1.4.5 Infiltrometers

Hydraulic conductivity may also be measured using an infiltrometer which applies water at the soil surface. The infiltration equation was given by Philip (1957)

$$I = S t^{1/2} + A t \tag{1.4}$$

where I is the cumulative infiltration, S is the sorptivity, t is time and A is a constant related to the saturated hydraulic conductivity. Both drip infiltrometers as well as infiltrometers utilizing ponded rings have been used to determine soil water flow.

1.4.6 Permeameters

Recently two permeameters, the Guelph permeameter and the velocity permeameter have been developed that offer further improvements over previous methods of in-situ hydraulic conductivity measurements. Both methods are simple to use and easily portable, and both produce results in a relatively short time (15-20 minutes for the velocity permeameter and 60-90 minutes for the Guelph permeameter, Kanwar, et al., 1989).

1.4.6.1 Guelph Permeameter

The Guelph permeameter determines in-situ measurements in the unsaturated zone of field-saturated hydraulic conductivity, sorptivity and the hydraulic conductivity-pressure head relationship (Reynolds and Elrick, 1985). These authors report measurements may be made fairly quickly (ten minutes - two hours) depending on soil texture, initial soil wetness, water depth in the equipment and cross-sectional area of the hole. The Guelph permeameter, GP, was designed to operate in uncased wells which may be dug with a soil auger or probe to a maximum depth of 8 meters. The GP appears to average vertical and horizontal anisotropy in k which is particularly useful in the design and monitoring of drainage and waste disposal systems that rely on three-dimensional saturated-unsaturated flow (Reynolds and Elrick, 1985). The Guelph permeameter has the advantages of speed, portability, low water use, equipment which is easily operated by one person, the capacity to use any wetting liquid for infiltration (potentially contaminates and lechate rates may be measured) and it may be used on areas that do not have a high tolerance for disturbance (i.e. between crop rows, on sod farms, golf courses, lawns, caps and liners of landfills, reservoirs, canals, etc.) (Reynolds and Elrick, 1985).

A disadvantage noted by the authors is that digging implements tend to smear the walls of the well in moist-to-wet porous media containing a significant amount of clay and may give lower flow values. A small spiked wheel was used to break up the smear layer and negate the effect. Another disadvantage is that the analysis assumes an exponential relationship between saturated k and unsaturated k, an assumption which will have varying degrees of validity for different porous media. Jabro (1992) stated that the GP may be used on homogeneous horizons, structurally stable soild and sandy or course loamy textured soils but cannot be used on fine, textured, organic, wet and heterogeneous soils and that the GP will give negative and positive values and produces substantial variability within a given soil. The method gives essentially a "point" measurement and therefore usually requires replication.

1.4.6.2 Velocity Permeameter

The velocity permeameter, VP, was developed by Merva (1987) and is similar in apperance to the Guelph permeameter with two major differences. First a coring device is gently pushed into the soil, rather than using an auger hole. Secondly the VP utilizes a falling-head method rather than a constant-head. The rate of fall of the water column is monitored to determine the time required to fall through distances δh. From this information k is calculated using Darcy's equation. Results agree with conventional soil coring methods. The device is transportable, easy to use, requires little water to operate and provides horizontal as well as vertical values of k with equal ease (Merva, 1987; Kanwar, Ahmed and Marley, 1989).

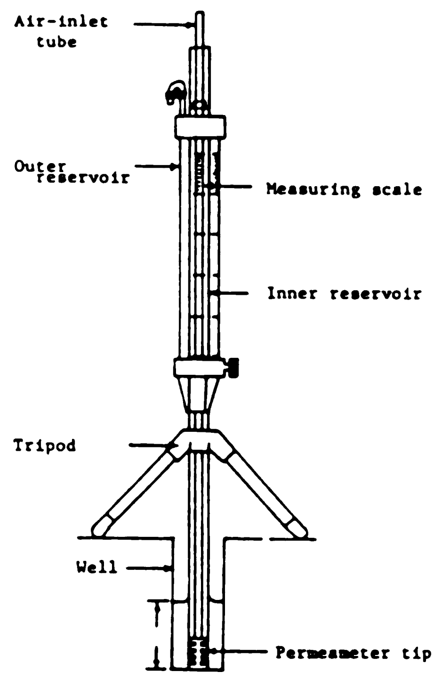

Figure 4. Skematic of the Guelph permeameter (from Kanwar et al., 1989)

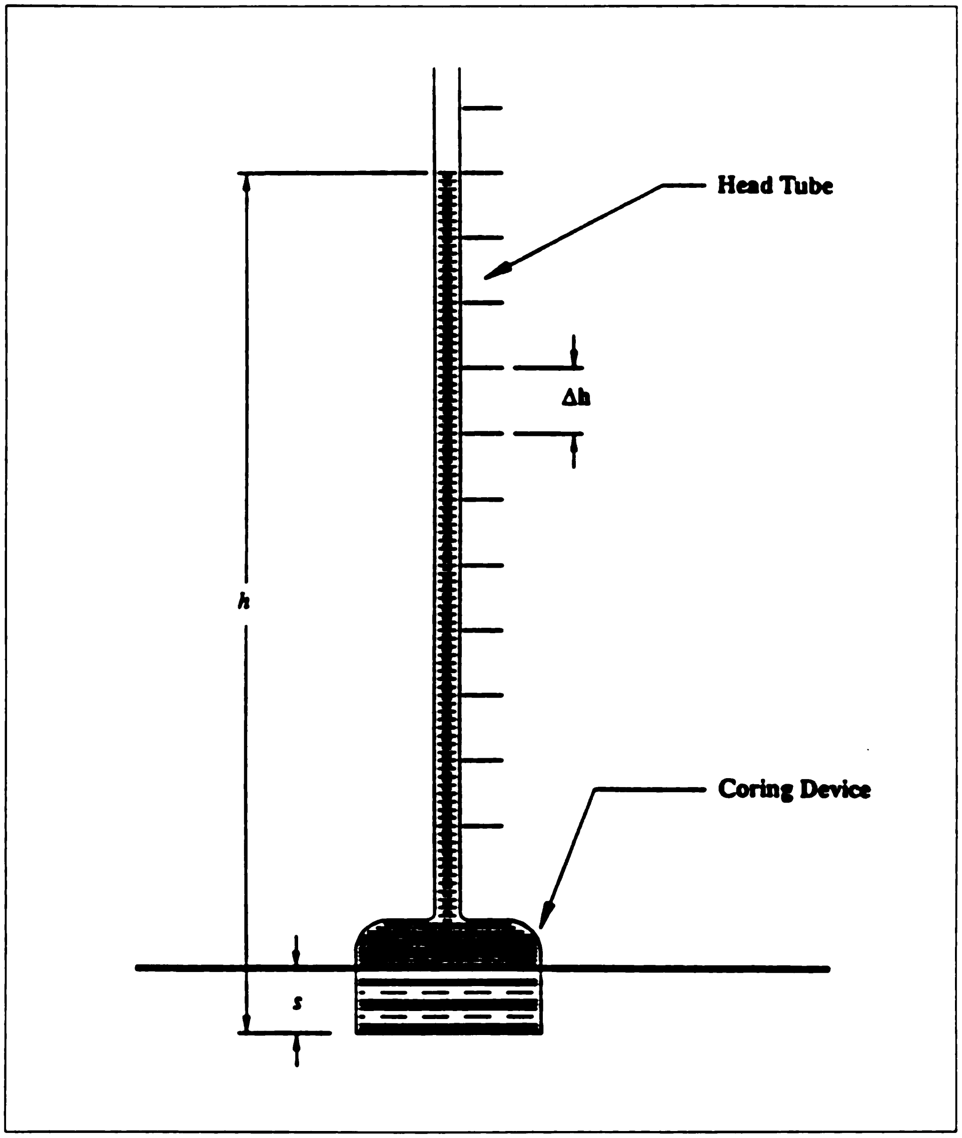

Figure 5. Skematic of the velocity permeameter (from Rose, 1988)

1.5 Modeling Saturated Flow

Early work in the area of saturated groundwater flow was analytical, with models that were homogeneous and had simple boundaries. Only very simple problems with idealized conditions may be solved analytically. Numerical models are necessary for more realistic models. The finite difference method and more recently the finite element method are in general use today. The finite element method allows more flexibility than the finite difference method and was the method chosen for this research. Kinzelbach (1986) examined saturated flow at length with both the finite difference method (explicit and implicit methods) and finite element method. He also discussed groundwater management, regional pollutant transport models and parameter estimation. Zienkiewicz, Mayer and Cheung (1966) used a finite element model to look at anisotropic seepage. Current groundwater text books discuss saturated flow in detail (for example: Freeze and Cherry, 1979).

1.6 Modeling Unsaturated Flow

Research in unsaturated flow is generally considered to date from 1931 when L.A. Richards proposed the concepts of water flow in unsaturated soil. Models are generally based on a two-term solution of Richard's equation (1931). The one-dimensional form is

$$\frac{\partial \Theta}{\partial t} = \frac{\partial}{\partial x}\left(D(\Theta)\frac{\partial \Theta}{\partial x}\right) - \frac{\partial k(\Theta)}{\partial x} \tag{1.5}$$

where Θ is the soil water content, t is time, x is the vertical distance (positive upwards), D is the diffusivity and k is the hydraulic conductivity. In 1960 Gardner summarized research in soil water relations and proposed hydraulic conductivity curves based on soil suction. His approach was used in this research.

Current research in modeling unsaturated flow takes many forms. Kunze and Nielsen (1982) used a finite difference solution to model vertical infiltration with time. A later paper (Kunze and Nielsen, 1983) used the model to show wetting profiles in the soil. Jabro and Fritton (1990) used a finite element software package (TWODEPEP, 1983) to solve Richard's equation in cylindrical coordinates and model water flow from a percolation test hole in homogeneous layered soil. Pressure head distribution and rate of water flow were used to compare data and the flow rate was calculated with Darcy's equation

$$q = -k(\psi)A\frac{\partial \Phi}{\partial r} \tag{1.6}$$

where q is the flow rate, A is the cross-sectional area and r is the radial distance. In the unsaturated model k is a function of the matric head, ψ.

Neuman and Witherspoon (1971) evaluated nonsteady flow with free surfaces by redefining the finite element mesh. Desai (1976) was able to avoid some of the problems created by a variable mesh by modeling flow with an invarient mesh using the residual flow procedure. Neuman (1973) used the finite element method to solve the equations of transient seepage in saturated-unsaturated porous media.

Phillip (1988) gave an overview of analytic and quasianalytic approaches to unsaturated flow. Raats (1988) commented on the same topic with further examples.

1.7 Parameter Estimation

Water flow models generally use Richard's equation (1.5) to define the change in moisture content with time at different locations and use relationships such as Darcy's law (1.6) to define flow velocity. Inherent in modeling these relationships is the need to quantify the hydraulic conductivity, k. Gardner (1958) proposed an exponential equation

$$k(\psi) = ae^{(\alpha\psi)} \tag{1.7}$$

where a and alpha are constants and ψ is the suction head. Gardner also gave the empirical power equation

$$k = \frac{a}{(\psi^n + b)} \tag{1.8}$$

where a, b and n are constants for steady-state, one-dimensional flow. ψ^n is 0 at saturation so

$$k_{sat} = \frac{a}{b} \tag{1.9}$$

Gardner (1960) gives n as between 2 and 4 for fine textured soils, discusses equation (1.8) in more detail and gives graphs of the hydraulic conductivity as a function of water potential.

Many authors have used the exponential equation (1.3) or the power equation (1.7) to quantify k. Raats (1983) described the theoretical background for both forms. Unlu et.al. (1990) noted that experimental results show the exponential relationship holds only over a limited range of water potentials and these authors suggest using statistically estimated values for k. Toledo, et.al. (1990) used theories of fractal geometry and thin-film physics to provide a basis for the power law and to define the exponents. The relationship was written as

$$k \propto \Theta^{3/m(3-D)} \tag{1.10}$$

where D is the Hausdorff dimension (fractal dimension, varying from 0 to 3) of the surface between the pore space and the grains and m is the exponent in the relation of disjoining pressure Π and film thickness h ($\Pi \propto h^{-m}$). These authors found values of m < 1 and 2.1 < D < 2.7 for length scales of 5 μm - 20 μm and D = 2 for smooth pore walls based on data for compacted sand. The authors note that these values are empirical, that accurate measurements of ψ and k at low moisture contents is challenging and reliable data is rare.

Neuman (1973) made the assumption that the hydraulic conductivity may be expressed as a product of a symmetric positive-definite tensor, K_{sat}, and a scalar function of the degree of saturation, K_r

$$k = K_{sat} K_r \tag{1.11}$$

where K_{sat} represents the conductivity at saturation and K_r is based on the volumetric water content.

Jabro and Fritton (1990) used the exponential equation (1.7) in their work. These researchers had good results using the geometric mean of k. They noted that the wetted front continued to advance in a predominantly horizontal direction and suggested this was due to a less permeable layer below two more permeable layers in their model. They also recorded a thin (approximately 10 mm) saturated zone near the water source.

Kunze and Nielsen (1983) found that arithmetic and geometric mean values for these parameters gave unreliable results but they had success using an integrated mean value.

The models of Burdine (1953) and Maulem (1976) which describe the unsaturated k values are commonly used and are discussed by Schuh and Cline (1990). These are often called the microscopic models as they are based on microscopic pore-radius distribution. These models were not considered for this research as they require soil parameters which are not generally available in field situations.

Bosch reports taht most of the methods for measuring K(Φ) using discrete measurements are inadequate for estimating effective K(Φ) for heterogeneous profiles unless the measurements are used to interpret the mean behavior of the system. An analytical expression for predicting the error which can be expected when using point observations of the matric potential to determine the mean matric potential in a heterogeneous soil profile was derived (Bosch, 1991).

1.8 The Velocity Permeameter

The velocity permeameter, VP, consists of a sharpened coring device which is pushed (driven) into the soil and connected to a small diameter head tube. The head tube is filled with distilled water which percolates into the soil core. The small diameter of the head tube acts to magnify the entry velocity of the water into the soil encased in the core which allows more accurate measurements. The rate of fall of the water column is monitored to determine the time required to fall through distances δh. A Hewlett Packard "C" series calculator equipped with a timing module is used to accurately time the process from which k is calculated. The head tube is refilled after the water level drops approximately 1000 mm.

Figures 6 and 7 show the plot of hydraulic conductivity over time measured with the VP from Merva (1987) and Rose (1988). The hydraulic conductivity value is initially relatively high but decreases with time and appears to approach a constant value.

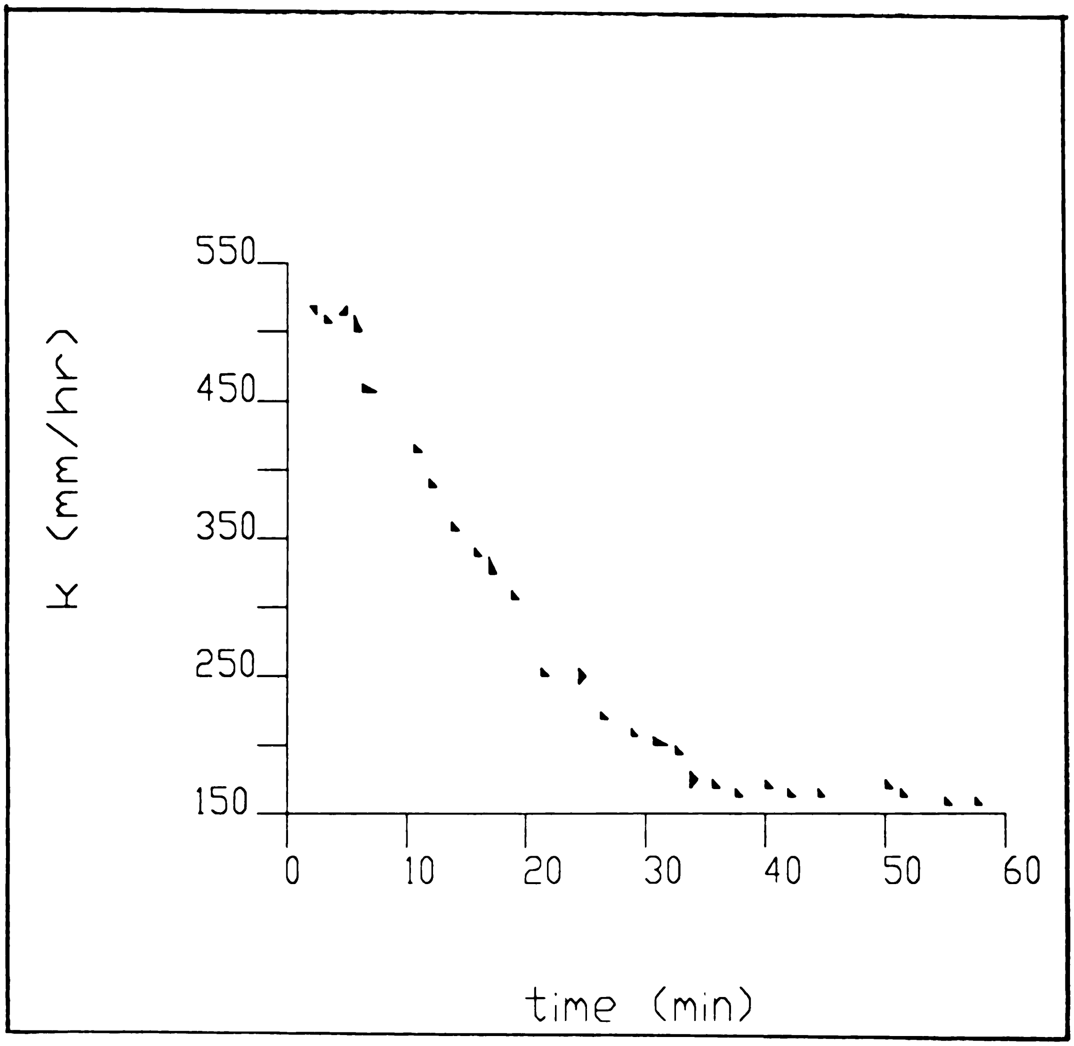

Figure 6. Hydraulic conductivity, k, over time (from Merva, 1987)

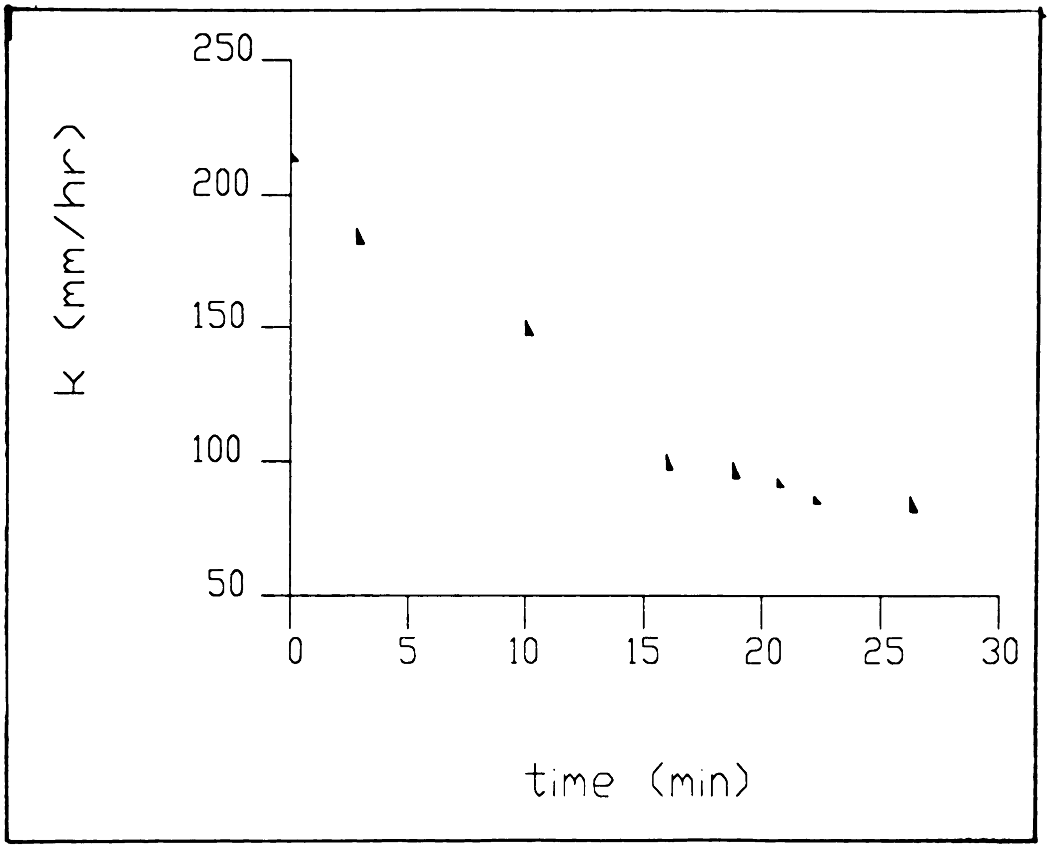

Figure 7. Hydraulic conductivity, k, over time (from Rose, 1988)

1.9 The Objectives

The goal of of this research was to improve the understanding of ground water flow into the soil under the velocity permeameter, VP, by developing a computer program to model the phenomena. Specifically, the model will show the change in soil water potential with time under the VP as well as the shape and extent of both the wetted and saturated fronts. The model will accomodate different soil parameters and various equipment configerations.

2. A ONE-DIMENSIONAL MODEL

The flow through the soil sample enclosed by the sample cup of the velocity permeameter was initially simulated using a one-dimensional model. The problem was basically one of introducing a column of water to a dry soil and monitoring the movement of water through the soil.

2.1 Governing Equations

The three dimensional governing equation for flow in an unsaturated soil is given by the Richards equation (Richards, 1931; Freeze & Cherry, 1979)

$$\frac{\partial}{\partial x}\left[K(\Psi)\frac{\partial \Psi}{\partial x}\right] + \frac{\partial}{\partial y}\left[K(\Psi)\frac{\partial \Psi}{\partial y}\right] + \frac{\partial}{\partial z}\left[K(\Psi)\left(\frac{\partial \Psi}{\partial z}+1\right)\right] = \lambda\frac{\partial \Psi}{\partial t} \tag{2.1}$$

where Ψ is the soil water pressure head (pressure head), K is the hydraulic conductivity and x, y and z are the coordinate directions, z vertical and positive upwards. The variable t is time while $\lambda = \partial\Theta/\partial\Psi$ is the specific storage and where Θ is the volumetric water content. The one-dimensional equation for flow vertically into the soil is given by

$$\frac{\partial}{\partial z}\left[K(\Psi)\left(\frac{\partial \Psi}{\partial z}+1\right)\right] = \lambda\frac{\partial \Psi}{\partial t} \tag{2.2}$$

for the unsaturated zone. Flow is directed downwards in the negative z direction. The saturated equation is

$$K\frac{\partial^2 \Psi}{\partial z^2} = 0 \tag{2.3}$$

The general solution to the equation for unsaturated flow (2.2) in finite element notation is a system of first order differential equations

$$[C]\{\dot{\Psi}\}+[S]\{\Psi\}-\{F\}=\{0\} \tag{2.4}$$

where [C] is the capacitance matrix and $\{\Psi\}$ contains the nodal values, with $\{\Psi\}^T=\{\Psi_1,\Psi_2,...,\Psi_p\}$. The matrix [S] is called the stiffness matrix (from structural analysis) and is usually denoted [K], with the element stiffness matrix denoted with the lower case [k]. The variable name [S] was used to differentiate the stiffness matrix from the hydraulic conductivity, universally denoted by k in soil physics literature.

The global force vector is {F} and

$$\{\dot{\Psi}\}^T=\left[\frac{\partial\Psi_1}{\partial t} \quad \frac{\partial\Psi_2}{\partial t} \quad \cdots \quad \frac{\partial\Psi_p}{\partial t}\right] \tag{2.5}$$

is the time derivative of the nodal values.

There are no point source, no point sinks nor any derivative boundary conditions in the velocity permeameter problem. The force matrix, {F}, appeared in the models because the known values of $\{\Psi\}$ on the upper boundary produce values that are held in {F}.

There is one boundary condition in the one-dimensional model - the height of water in the head tube. It was considered a fixed value during each time step in the finite element solution. The drop in head was calculated after each iteration with the new value used as the fixed value during the next iteration.

The finite element solution to the saturated equation, (2.3), is

$$[S]\{\Psi\}=\{F\} \tag{2.6}$$

since the time derivative vector $\{\dot{\Phi}\}$ does not change with time in the saturated solution.

The unsaturated solution looks quite different from the saturated solution because of the addition of the capacitance matrix, [C], which contains the time dependent parameters. Equation 2.4 gives the unsaturated terms and is solved by separating the unknown values from the known values and using the central difference solution technique (Segerlind, 1984) to yield

$$\left([C]+\frac{\Delta t}{2}[S]\right)\{\Psi\}_b = \left([C]-\frac{\Delta t}{2}[S]\right)\{\Psi\}_a + \frac{\Delta t}{2}(\{F\}_a + \{F\}_b) \tag{2.7}$$

where $\{\Psi_a\}$ contains the known nodal values at time t, $\{\Psi_b\}$ contains the unknown nodal values at time t+1. This equation is generally written as

$$[A]\{\Psi\}_b = [P]\{\Psi\}_a \tag{2.8}$$

where [A] is decomposed into upper triangular form using Gaussian elimination and the system of equations is solved using backward substitution.

2.2 Element Matrices

The element stiffness matrix

$$[s] = \frac{k_i}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{2.9}$$

was used where k_i is the hydraulic conductivity at node i. Node i is the node closest to z = 0 for each element and L is the element length. The lumped form of the element capacitance matrix was used. This matrix was

$$[c] = \frac{\lambda_i L}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.10}$$

where λ_i is calculated at node i, as described above. λ_i is the time dependent multiplier (often called specific storage) and is calculated by determining the slope of the $\Psi - \Theta$ curve at a known soil water pressure head.

The element force matrix

$$[f] = \frac{QA}{4}\{1 \quad 1 \quad 1 \quad 1\}^T \tag{2.11}$$

2.2.1 Hydraulic Conductivity

The hydraulic conductivity at each node was calculated from the known soil water pressure head, Ψ, to form the stiffness matrix. Various methods were tested to determine the value of the hydraulic conductivity. Exponential equations which are often used to define the $\Psi - k$ relationship were found to be inadequate for the range of soil water pressure head modeled. Extensive efforts were made to utilize an equation which would model the work of Gardner for soils ranging in moisture content from saturation to a water pressure head of -100 meters with k varying five orders of magnitude. The equations were only able to model a portion of the k - Ψ curve so curve fitting was not a viable solution due to the large variability of the hydraulic conductivity (five orders of magnitude). Breaking the curve into a series of equations reduced the order of magnitude problem but did not give satisfactory results. The most consistent results were found by assuming k had the same relation to water pressure head as is shown in Gardner's curve (Figure 8), using the curve for the Pachappa sandy loam soil. For this approach, nineteen data points from Gardner's graph were entered into a data array, and a full logarithmic interpolation was used to calculate the hydraulic conductivity at each node from these data. Since the saturated k value from the graph ($2*10^{-3}$ mm/sec) was significantly lower than values currently being used in field work in Michigan, a multiplication factor was used (2.8 to give a saturated value of 0.56 mm/sec or about 20 mm/hr). The assumption was made that the shape of the graph would not change.

2.2.2 Specific Storage, λ

The specific storage, λ, the time dependent multiplier is the ratio of the change in water content to the change in presure head ($\lambda = \partial\Theta/\partial\Psi$). The value of λ at a node was calculated from known data by determining the slope of the Ψ - Θ (where Θ is the volumetric water content) curve between the known node and the next data point. The graph of the field data, similar to that shown in Figure 9, was approximated using 14 data points and semi-log interpolation. The actual data used was from the U.S. Department of Agricultural Soil Conservation Service (1986) for a Ziegenfuss soil for a depth of 0 - 230 millemeters. The values may be found in Appendix D.

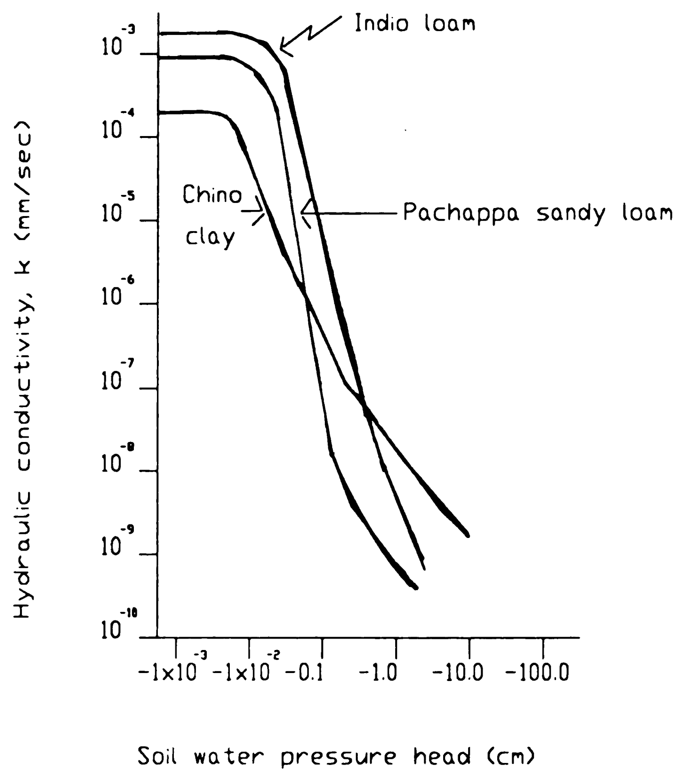

Figure 8. Soil water pressure head versus hydraulic conductivity

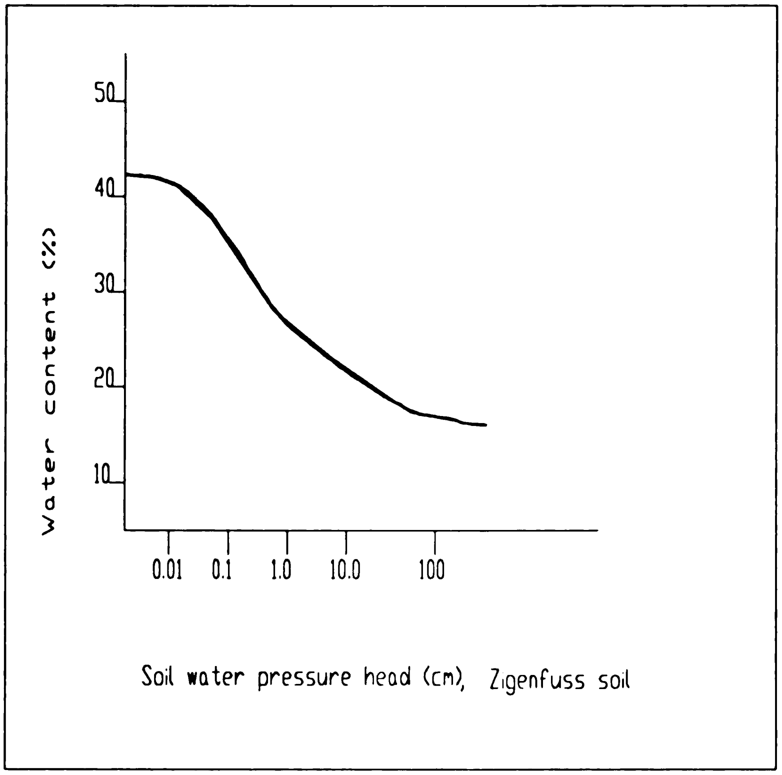

Figure 9. Soil water pressure head versus percent water content

2.3 The Model

2.3.1 The Physical Model

The one-dimensional finite element model is shown in Figure 10. The nodes are numbered downward (the negative z direction). The element numbers are in parenthesis. The ground surface node, node 1, is set at the fixed value of the height of water in the head tube (1.8 meters). The other nodes are numbered incrementally in the -z direction. The elements are numbered in the same manner. An element length of 50 millimeters was chosen so that the time step would not be too small. The maximum time step, Δt, cannot be exceeded to avoid oscillations with time dependent problems is given by Segerlind (1984)

$$\Delta t \quad < \quad \frac{A\lambda}{4k(1-\theta)} \tag{2.12}$$

where A is the area of the element, $\theta = 0.5$ (as required for the central difference method of solution) and k and λ were calculated using data shown in Figures 8 and 9. Analysis of Δt showed that the saturated values for k and λ gave the maximum Δt that could be used to avoid oscillations. The first node was saturated and drove the analysis.

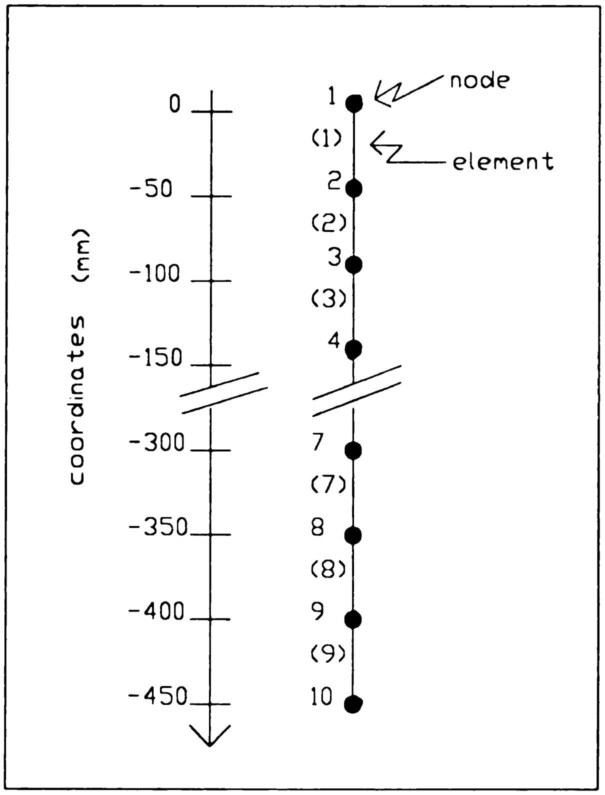

Figure 10. One dimensional finite element model

2.3.2 The Computer Model

The computer model consisted of three major portions: the saturated analysis; the unsaturated analysis and the calculations to determine the drop in head. The input values were the coordinates, the initial pressure head and the element configuration. The time dependent loop was begun after all data had been read from a file. The time step and number of iterations were specified with the input data.

The program initially performed some one-time calculations, such as elemental volume and initial moisture at each node, prior to beginning the time-step loop. The first step in the iterative process was to check for entries in the saturated node array, which consisted of the nodes initially unsaturated but which became saturated as the wetting front moved past them. Pressure head values dictated when the soil was saturated. The saturated nodes were calculated first because, although the pressure head changed as the water column in the head tube dropped, they are considered fixed nodes during the unsaturated analysis.

The second major part of the computer model was the time-dependent, unsaturated analysis which calculated the pressure head for the unsaturated nodes using equations (2.7) and (2.8). Lastly, volumetric water flow calculations were made to determine the drop in the water surface in the head tube. This value was subtracted from the fixed nodes to give the new boundary value for the next time step.

The global matrices were recalculated during each time step in the unsaturated analysis because as the pressure head changes the parameters k and λ, found in the global matrices, change. The global stiffness matrix, [S], and capacitance matrix, [C], were created from the element matrices by the direct stiffness method as described by Segerlind (1984). Note that in the saturated solution only the stiffness matrix is needed since λ ($\partial\Theta/\partial\Psi$) is zero for saturated nodes ($\partial\Theta = 0$).

The parameters k and λ vary with the water content and could not be defined for an element since the soil water pressure head was known only at the nodes. Using relationships for Ψ-k and Ψ-Θ, k and λ could be calculated at the nodes. Attempts were made to define a representative water content value for each element but the extreme variation in pressure head, Ψ, between adjacent nodes in the area of the wetted front caused complications in estimating k and λ for an element. For example, initially the first element has a saturated soil water pressure head of 1.8 meters at the upper node and an unsaturated pressure head of -20 meters (the initial conditions used) at the lower node. It was impossible to accurately define a value of k or λ for an element with such a range in values (k is on the order of 4^{-12} m/sec for a pressure head of -20 m. and about 1.1^{-6} m/sec for the saturated node. Using an average Ψ value as representative for the element was unsuccessful since it slowed the analysis to the point that the head tube was not refilling within five minutes - a criteria based on field experiments. Noting that although the disparity will occur at the wetted front as long as it is moving through the soil, the interaction is probably brief and not critical to the overall problem. To avoid the problems caused by averaging, the value of k used for each element was calculated based on the soil water pressure head at the upper node (nearest the ground surface) for the element. This method gave results that were physically realistic, but may be a cause of the lack of complete agreement between calculated and assumed k values.

2.4 The Iterative Process

The first step in the computer model for each iteration was to calculate Ψ at the saturated nodes. The saturated analysis was run on all the saturated nodes, the fixed node (node 1) and the first unsaturated node (in the z-direction). The hydraulic conductivity was calculated for each node ($k = k_{sat}$ for the saturated nodes). The soil water pressure head, Ψ, was obtained for each of the saturated nodes by determining the height above the saturated front (where Ψ was considered to be zero) and assuming a linear relationship between the saturated front and the fixed node above it. Calculations proceed until the new pressure heads have been determined for all the saturated nodes, except the fixed node.

Next, the parameters k and λ were calculated and the global arrays [S], [C] and {F} written. The global matrices were then modified for the fixed node (node 1). The central difference method was used to write [A] and [P] which were then decomposed into upper triangular form. The pressure head at each node was found with backwards substitution.

The subsequent step in the model determined the soil water pressure head values at the unsaturated nodes. The water content at each node was found using field data (Figure 9) and semi-log interpolation. The change in water content was determined by subtracting the water content at the previous time step from the current water content at each node. This value was multiplied by the element volume to give the elemental change in water content. The summation of the elemental volumes yielded the total flow for the time step. The total flow for the iteration divided by the area of the head tube gave the drop in head for the iteration. The drop in head was subtracted from the head at the fixed node to give the height of the water column for the next time step.

2.5 One-Dimensional Analysis

The one dimensional analysis was run for simulation time of five minutes with a time step of 0.25 seconds (1200 iterations). Figure 11 shows results from the one-dimensional VP model: the drop in head of node 1 and the wetting of nodes 2 - 4.

The soil water pressure head (head, shown by node 1) was initially set to 1.8 meters in the sample cup. Node 1 remains saturated with a drop in head and increases when the head tube is refilled. After each iteration the drop in head is calculated and the fixed nodes are reduced by that value. When the head falls below 1.0 meter, node 1 is reset to 1.8 meters to model the refilling of the head tube. The one-dimensional model refilled 4 times in the first 0.1 minute, again at 0.88, 1.7, 2.2, 2.4, 2.7, 2.8, 2.9 and 4.2 minutes. The refills cause the spikes in the graph of node 1.

Node 2 is the first node to become saturated as indicated by the curve crossing the time axis (about 2 1/2 minutes) with the other nodes becoming increasingly wetter (approaching zero). At 5 minutes the height of water in the head tube was 1.12 meters. The soil water pressure head at node 2 is 0.56 m, at node 3, -0.78 m, at node 4, -7.55 m and -19.99 m at node 5. All other nodes are still at the initial condition of -20 meters. These values may be estimated from Figure 11 where it is seen that nodes one and two are saturated (above 0 m pressure head), node 3 is about -1 m and node 4 is about -8 m. The data given here was taken from the program output. The slopes of node 1, between refills, are of interest. The slopes appear shallow, and similar, for time less than two minutes and time greater than three minutes. Between two and three minutes the model refilled often and the slopes are consequently much steeper. This occured just before node 2 became saturated and transpires because as node 2 becomes wetter the hydraulic conductivity increases rapidly an causes greater amounts of water to be drawn into the model.

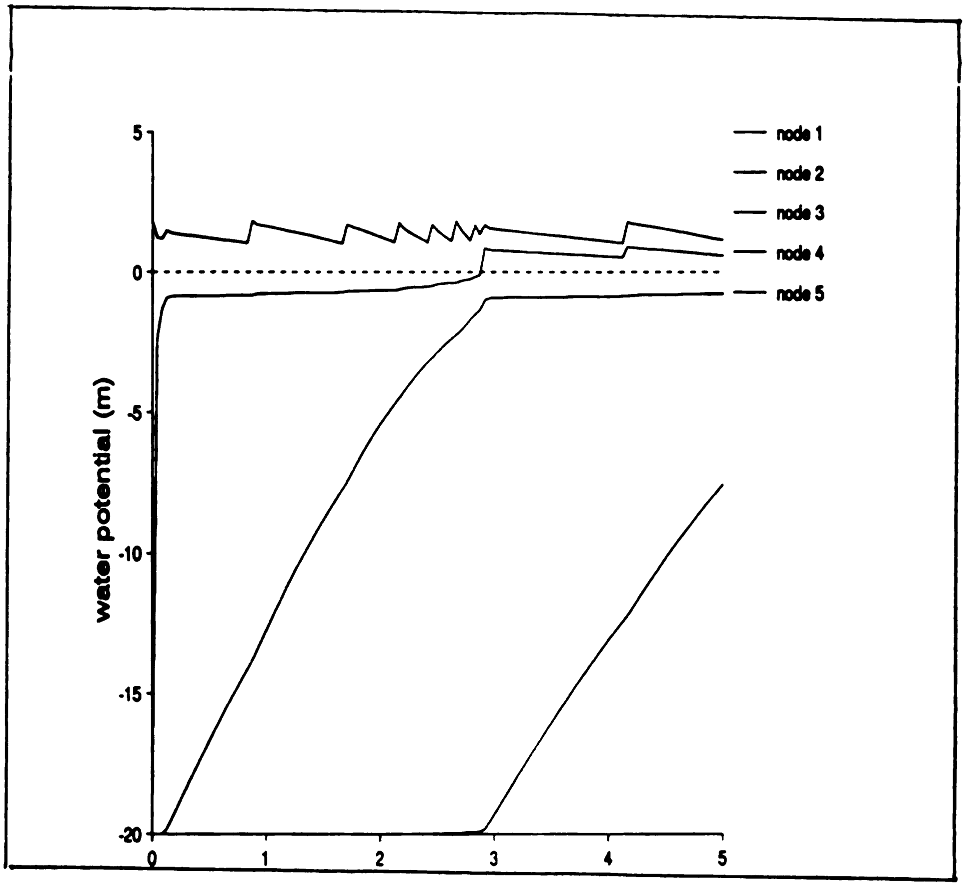

Figure 11. Results, One-dimensional analysis

2.6 Summary

The one-dimensional model was written to study the parameter determination and interaction as well as the general solution method. It was particularly useful in determining the method to be used in calculating the parameters k and λ. Once this was accomplished an axisymmetric model was studied in greater detail. The one-dimensional computer program was written in Turbo Pascal for a personal computer (PC) and is given in Appendix C. Comprehensive results and conclusions are given for the axisymmetric model.

3. AXISYMMETRIC MODEL

The model of the velocity permeameter (VP) in two dimensions is analogous to the one-dimensional model. The three-dimensional model was written using axisymmetric elements. Axisymmetric elements are created by rotating a two dimensional element around an axis of symmetry, in this case the z-axis (Figure 12). Rectangular elements were used with the nodes labeled i, j, k and m beginning in the lower left-hand corner and proceeding counter-clockwise.

Using axisymmetric elements reduces the flexibility of the regional geometry, as the soil inhomogeneity and anisotropy can not vary in the y direction relative to r or z although it may vary from element to element. The solution of the axisymmetric problem is much less complex than is a full three-dimensional solution.

3.1 Governing Equations

Writing the axisymmetric problem in cylindrical coordinates and noting that the problem is independent of a rotation about the z-axis gives

$$\frac{\partial}{\partial r}\left[K(\Psi)\frac{\partial \Psi}{\partial r}\right] + \frac{K(\Psi)}{r}\frac{\partial \Psi}{\partial r} + \frac{\partial}{\partial z}\left[K(\Psi)\left(\frac{\partial \Psi}{\partial z}+1\right)\right] = \lambda\frac{\partial \Psi}{\partial t} \qquad (3.1)$$

where r is the radial distance from the z-axis and the other variables are as defined in Chapter 2. The saturated axisymmetric equation is

$$K_r\frac{\partial^2 \Psi}{\partial r^2} + \frac{K_r}{r}\frac{\partial \Psi}{\partial r} + K_z\frac{\partial^2 \Psi}{\partial z^2} = 0 \qquad (3.2)$$

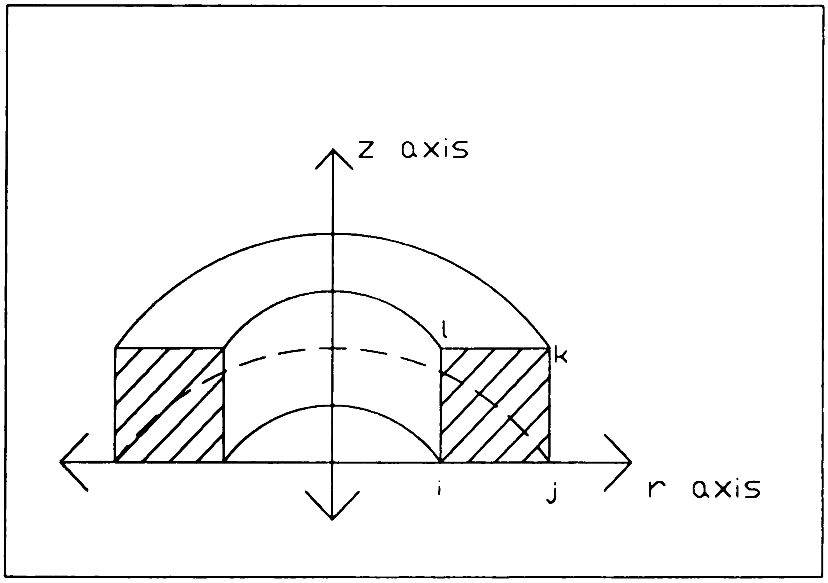

Figure 12. Cross-section of the axisymmetric element

3.2 Element Matrices

The element matrices for the axisymmetric model are determined from the volume integrals as given in Segerlind (1984)

$$[s] = \int_V [B]^T [D][B] dV \tag{3.3}$$

$$[c] = \int_A \lambda [N]^T [N] dA \tag{3.4}$$

and

$$[f] = \int_V Q[N]^T dV \tag{3.5}$$

The solution method for the axisymmetric model was the same as discussed for the one-dimensional model (Chapter 2). The element stiffness matrix in cylindrical coordinates is written in two parts, the radial term, r, and the vertical term, z

$$[s] = [s_r] + [s_z] \tag{3.3}$$

The matrix in the radial direction, $[s_r]$ was

$$[s_r] = \frac{2\pi \bar{r} k_r a}{6b} \begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} \tag{3.4}$$

and the matrix, $[s_z]$ in the lateral direction was

$$[s_z] = \frac{\pi k_z b}{3a} \begin{bmatrix} (\bar{r}+R_i) & \bar{r} & -\bar{r} & -(\bar{r}+R_i) \\ \bar{r} & (\bar{r}+R_j) & -(\bar{r}+R_j) & -\bar{r} \\ -r & -(\bar{r}+R_j) & (\bar{r}+R_j) & \bar{r} \\ -(\bar{r}+R_i) & -\bar{r} & \bar{r} & (\bar{r}+R_i) \end{bmatrix} \tag{3.5}$$

The parameters R_i and R_j are the shortest and longest distances, respectively, to the element sides parallel to the z-axis. The variable a is half the element width, b is half the element length and $\bar{r} = (R_i + R_j)/2$.

The lumped formulation of capacitance matrix must be used when the central difference method of solution is employed (Segerlind, 1984). The lumped capacitance matrix for the axisymmetric element is

$$[c] = \frac{\pi\lambda A}{6}\begin{bmatrix} 2(\bar{r}+R_i) & 0 & 0 & 0 \\ 0 & 2(\bar{r}+R_j) & 0 & 0 \\ 0 & 0 & 2(\bar{r}+R_j) & 0 \\ 0 & 0 & 0 & 2(\bar{r}+R_i) \end{bmatrix} \tag{3.6}$$

The parameters k and λ were calculated as described for the one-dimensional element (Chapter 2) using data from the graphs shown in Figures 8 and 9.

The equation for determining Δt for the rectangular element was given by

$$\Delta t < \frac{A\lambda}{4k(1-\theta)} \tag{3.7}$$

where A is the cross-sectional area of the element and $\theta = 0.5$ for the central difference method.

3.3 The Model

3.3.1 The Physical Model

A cross-section of the axisymmetric model is shown in Figure 13. There are 80 elements, each 20 mm x 20 mm, and 102 nodes. The core has two elements in the x-direction and three in the -z direction (a total of six elements in the core). Nodes 1, 12 and 23 are fixed at 1.8 meters, the water surface level in the tube, as the analysis begins. The core circumference is modeled by a double set of nodes at x = 40 millimeters, with no element between them. Figure 14 shows a plan view of the x-y plane for this model. Since the z axis was a reflective boundary each element forms a torus around the z axis, except the first column (elements one to ten) which were cylindrical (or torus with inside radius of zero).

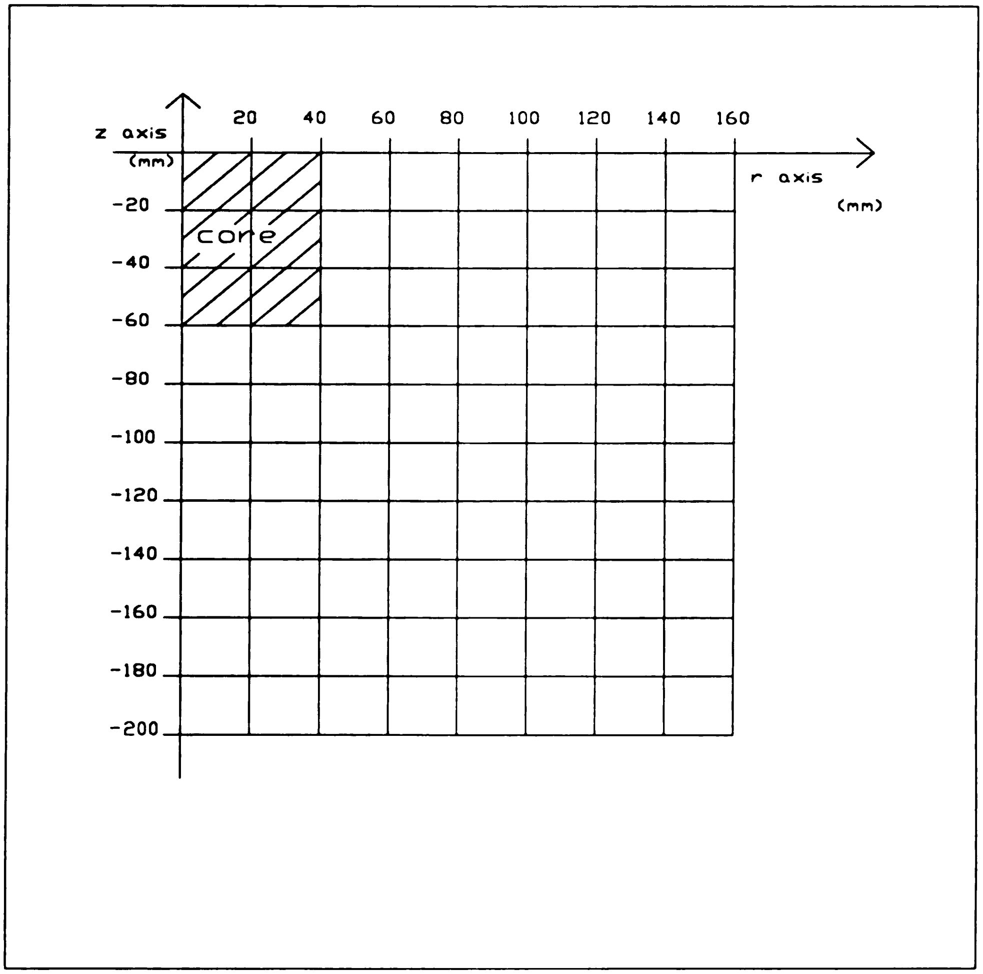

Figure 13. Axisymmetric finite element model

3.3.2 The Computer Model

Data was input from a file and initial calculations of water content and element volume were determined, and then the time dependent solution was begun. The first portion of the time dependent solution was the saturated analysis. The node above and below a saturated node in the z-direction were fixed and the arithmetic mean of the soil water pressure head of the two fixed nodes was assumed to be the soil water pressure head for the saturated node. This procedure was performed on each saturated node except the fixed nodes: 1, 12 and 23.

The next step in the model was the unsaturated solution. All saturated nodes were considered fixed during the unsaturated portion. The global matrices were redefined at each time step using the WP at the nodes to calculate k and λ. The [A] and [P] matrices were calculated using the central difference method and then [A] was decomposed into upper triangular form. Backwards substitution was used to determine the pressure head at each node with both solution methods.

The backward difference method ($\theta = 1$) was used to find [A] and [P] when the initial soil water pressure head was less than -20 meters, and the central difference method was used in all other cases. The numerical solution using the central difference method caused negative numbers on the [P] matrix diagonal when the initial soil water pressure head was below -20 meters which caused an oscillating solution.

Appendix A gives the computer program written to run the VP analysis with rectangular elements. The program was originally written with Turbo Pascal for a personal computer (PC) but was altered to run on a Unix to speed up processing.

3.4 Boundary Condition Determination

The nodes in the VP core at the ground surface were initially set to 1.8 meters pressure head. These were fixed during the finite element analysis. The other boundary conditions were insulated boundaries which did not require any input data ($\frac{\delta\Psi}{\delta r} = 0, \frac{\delta\Psi}{\delta z} = 0$ across the axes, at the model extents and at r = 40 mm, the radial edge of the cylinder). The drop in pressure head had to be calculated after each iteration. This was accomplished by determining the total flow in the model for the iteration and dividing by the pressure head tube cross sectional area (over which the flow had to occur) to give the drop in pressure head, dh, in the pressure head tube. The drop was subtracted to give the new boundary condition and this value remained fixed during the next finite element iteration.

The finite element analysis calculates the pressure head at each node of an element. The corresponding water content was determined based on the pressure head from the USDA Soil Conservation Service curves (as shown in Figure 9, with actual data in Appendix D, and the program, Appendix A) for a given soil. A representative volume for each node was calculated by determining the torus area (since this was an axisymmetric problem) with the inner radius equal to the node coordinate less 1/2 the element width and the outer radius equal to the node coordinate plus 1/2 the element width. The volume was determined by multiplying by the element depth. The dashed lines in Figure 15 indicate the volume calculated to be represented by each node shown. The solid lines indicate the elements. The nodes on the boundaries were multiplied by 1/2 the element depth since there were only two elements affected. The interior corner nodes affected an area of πr^2 since they were on the axis of symmetry, and the exterior corner nodes had the inner radius as previously described and an outer radius equal to the model extent.

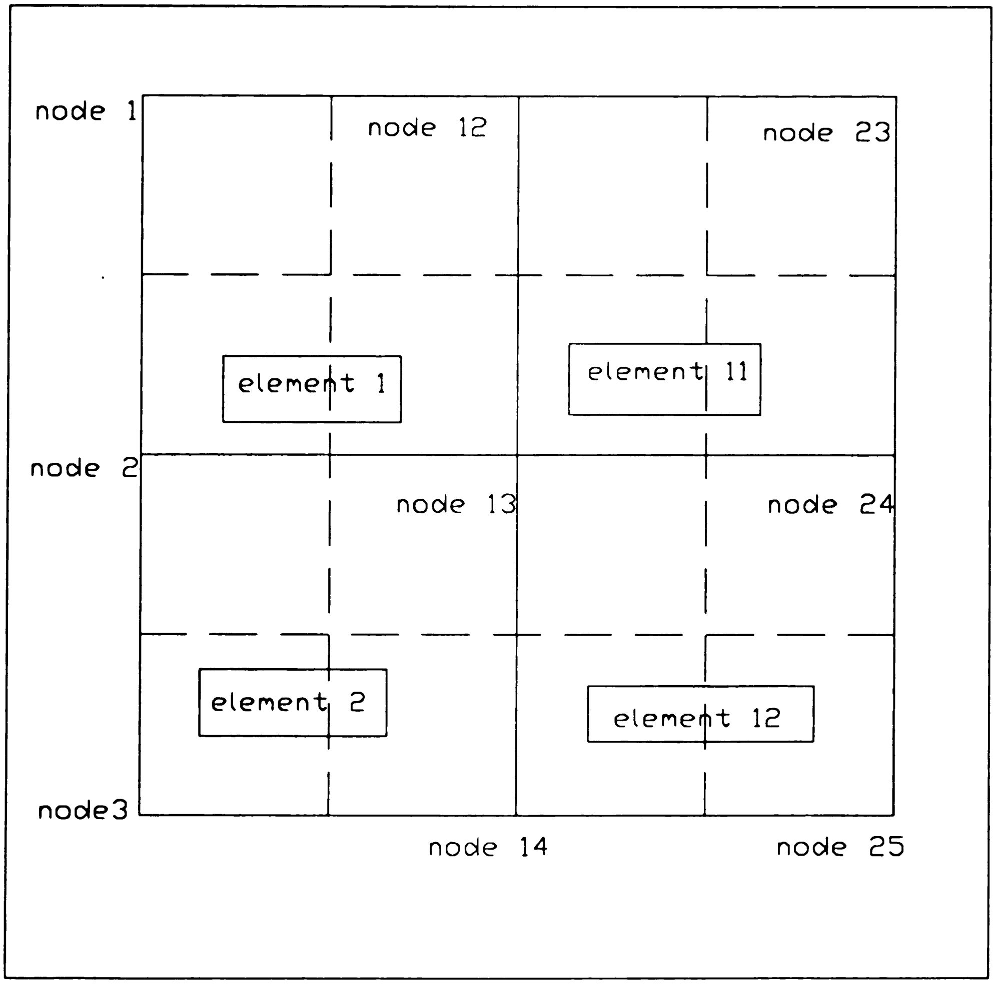

Figure 15. Schematic to calculate element flow

3.5 Apparent Hydraulic Conductivity

The apparent hydraulic conductivity, K_{app}, was calculated as described by Merva (1979) for each set of input parameters. The volume flow was determined by multiplying the nodal moisture content by the representative nodal volume, as described in the previous section. The velocity in the core was then calculated using

$$vel = \frac{Q}{\pi r^{2} * dt} \tag{3.8}$$

where Q was the volume flow for the time step, r was the core radius and dt was the time step. The height of water above node 1 in the core, (pressure head), and velocity ($\frac{\Delta h}{\Delta t}$) values were summed during each run in order to perform a linear regression analysis to determine the slope of the line, $\frac{\delta v}{\delta h}$. Soil water pressure head was the independent variable and velocity the dependent variable. Multiplication of the slope, $\frac{\delta v}{\delta h}$, by the core length, s, gave K_{app}.

$$K_{app} = \frac{dv}{dh} s \tag{3.9}$$

The programming of the regression analysis is found in Appendix A under the axisymmetric computer program procedure "FLOW".

3.6 Model with Triangular Elements

The axisymmetric model can utilize triangular elements as well as the rectangular elements described previously. Triangular elements are useful when studing irregular model configurations or when it is desirable to change element size. In order to study the saturated front movement as it left the VP core a model was initially written which used 2 mm x 3 mm elements in a 90 mm x 90 mm grid. This gave a total of 1350 elements and 1432 nodes.

Elements with an area of 6 mm^2 necessitated a time step of 0.001 seconds giving 300,000 iterations for a five minute run. A run of 90,000 iterations (90 seconds) with this model on the MSU Case Center computer never was completed. An application was made, and accepted, to run on the NCSA Cray-2. Although the Cray was much faster the time needed to run a model with 1432 nodes (requiring 1432 simultaneous solutions each time step) was not available for this research.

The area at the bottom of the cylinder whas the area of interest and required a fine grid, however, at the model extents the grid could be quite course. Triangular elements may be used to change the size of elements in finite element analysis and were tested to determine their feasibility. The model with triangular elements is seen in Figure 16. Small elements, 2 mm x 4 mm, are concentrated around the bottom and outside of the VP. Triangular elements were used to increase element size to 20 mm x 20 mm as was used in the control data file. This model has 110 elements and 101 nodes.

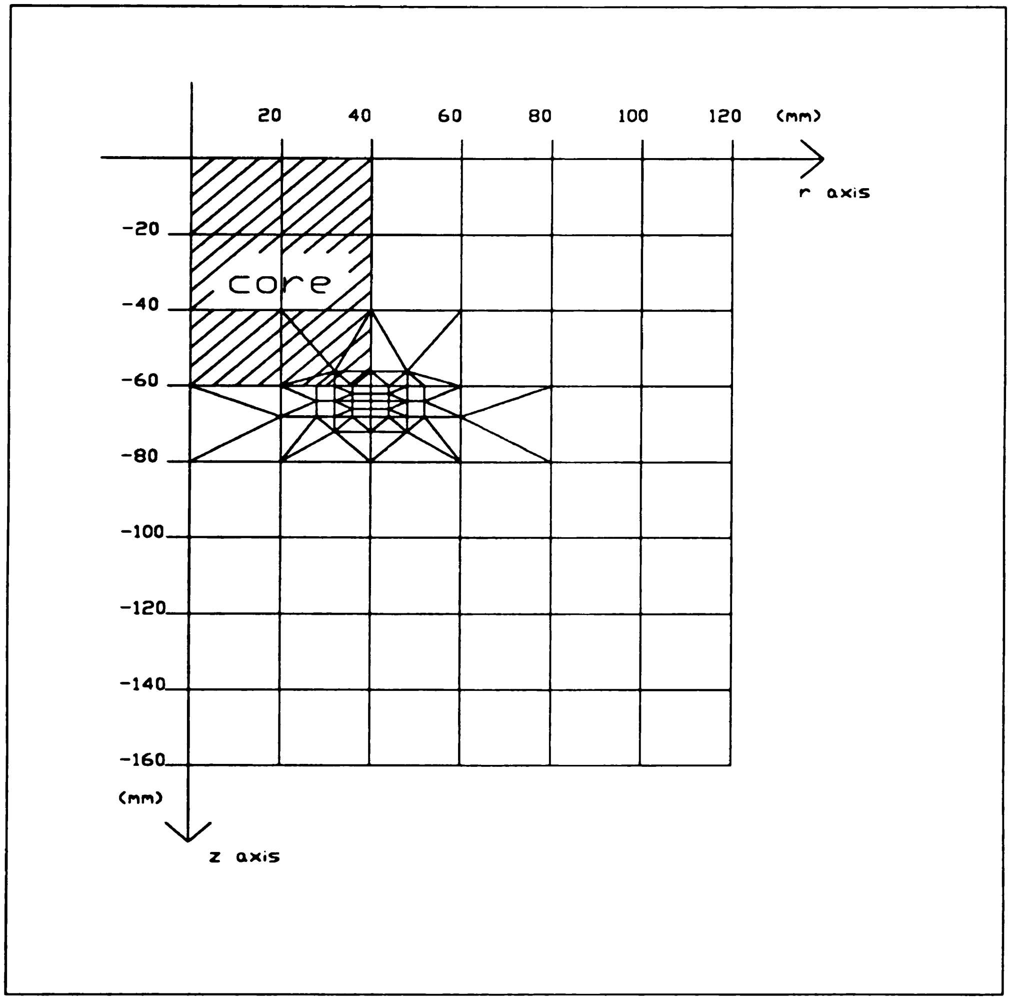

Figure 16. Axisymmetric finite element model, triangular elements

3.7 Element Matrices, Triangular Elements

The triangular element matrices are similar to the rectangular element matrices except they are 3 x 3's (rather than 4 x 4's) since there are only three nodes per element. The stiffness matrix was given by

$$[s] = [s_r] + [s_z] \tag{3.10}$$

The matrix in the radial direction, $[s_r]$ was (Segerlind, 1984)

$$[s_r] \quad = \quad \frac{2\pi\bar{r}k_r}{4A}\begin{bmatrix} b_i^2 & b_ib_j & b_ib_k \\ b_ib_j & b_j^2 & b_jb_k \\ b_ib_k & b_jb_k & b_k^2 \end{bmatrix} \tag{3.11}$$

and the matrix, $[s_z]$ in the vertical direction was (Segerlind, 1984)

$$[s_z] \quad = \quad \frac{2\pi\bar{r}k_z}{4A}\begin{bmatrix} c_i^2 & c_ic_j & c_ic_k \\ c_ic_j & c_j^2 & c_jc_k \\ c_ic_k & c_jc_k & c_k^2 \end{bmatrix} \tag{3.12}$$

where $\bar{r}$, k_r and k_z are as previously defined for the rectangular element, A is the triangle element area and the b and c variables depend on the distance to each node

$$\begin{aligned} b_i &= Z_j - Z_k & \qquad c_i &= R_k - R_j \\ b_j &= Z_k - Z_i & \qquad c_j &= R_i - R_k \\ b_k &= Z_i - Z_j & \qquad c_k &= R_j - R_i \end{aligned} \tag{3.13}$$

where Z_i, Z_j, Z_k, R_i, R_j, and R_k are the radial and vertical distances to each node (i,j,k) as shown in Figure 17.

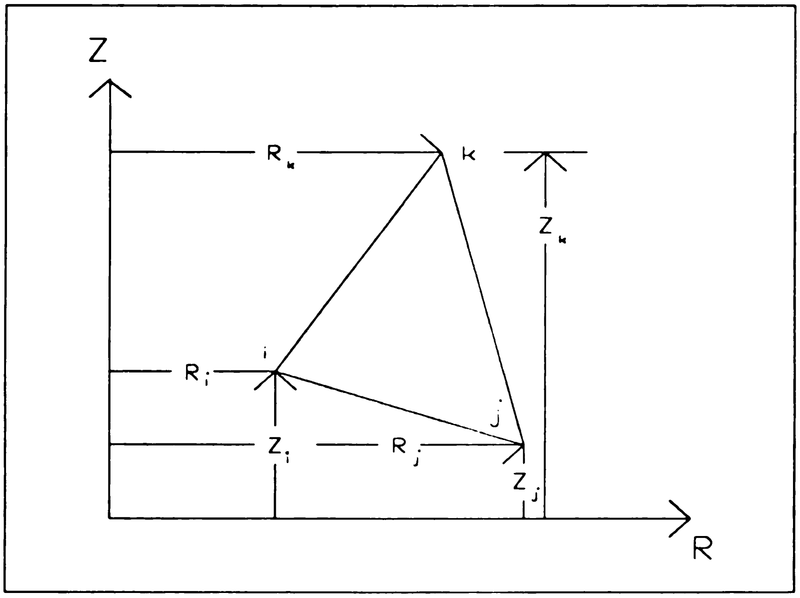

Figure 17. Triangular element coordinates

The lumped form of the capacitance matrix was given as (Segerlind, personal correspondence)

$$[c] = \frac{2\pi\lambda A}{12}\begin{bmatrix}(3\bar{r}+R_i) & 0 & 0\\ 0 & (3\bar{r}+R_j) & 0\\ 0 & 0 & (3\bar{r}+R_k)\end{bmatrix} \tag{3.14}$$

The equation for determining Δt for the triangular element was given by

$$\Delta t < \frac{2\lambda A}{9k(1-\theta)} \tag{3.15}$$

where A is the element area, $\theta = 0.5$ and k and λ are for the saturated values.

The computer subroutines which calculated the triangular element matrices may be found in Appendix E. These subroutines were added to the basic VP computer program (Appendix A). A check was made to determine if the element was triangular or rectangular, the proper subroutine used and the element contributions were put in the global matrices [K] and [C]. The solution then proceeded as discussed for the rectangular elements.

3.8 Axisymmetric Analysis, Rectangular Elements

The model using axisymmetric rectangular elements proved appropriate for observing water flow through unsaturated soil. The data file for this model had an element size of 20 millimeters x 20 millimeters. A time step of 0.1 seconds was used (based on equation 3.7 and using saturated values for soil parameters). The extent of the wetting profiles may be seen Figure 18. The water potential at each node was initially set at -20 meters (except for the fixed nodes) and this water potential is still seen at a radial distance of 140 millimeters (100 millimeters outside the core) and a vertical depth of 180 millimeters. Water potentials ranging from -1 m to -10 m are also shown.

Gravitational flow predominates as seen by the depth of the isoheads below the core. The isohead of -10 m radiates about 50 mm from the core and is 80 mm below the bottom of the core.

The wetted front moved vertically downward (one-dimensionally) until it reached the bottom of the core (node 4 at a depth of 60 mm). Flow became three dimensional as the wetted front moved out of the bottom of the core and the saturated front moved more slowly than with one dimensional flow.

Figure 19 shows results from the axisymmetric VP model: the drop in head of node 1 and the wetting of nodes 2 - 5. The three dimensional graph is similar to the graph of the one dimensional analysis. Node 2 becomes saturated at about 1 minute, node 3 at 3 minutes and node 4, at the center of the model and the bottom of the core, at about 12.5 minutes. This was similar to the result seen with the one-dimensional model when it is recalled that the one-dimensional element length was 50 mm and the axisymmetric element depth was 20 mm. Comparing node 2 from the 1D model ($z = -50$ mm), which saturated at about 2.5 minutes, with node 3 from the axisymmetric model ($z = -60$ mm), which saturated at about 3 minutes, shows similar results with the two models. One significant difference, however, is that the axisymmetric model appeared to be refilled with some regularity, as shown by the spikes in the curve of node 1, whereas in the one-dimensional model flow into the soil slowed after 3 minutes.

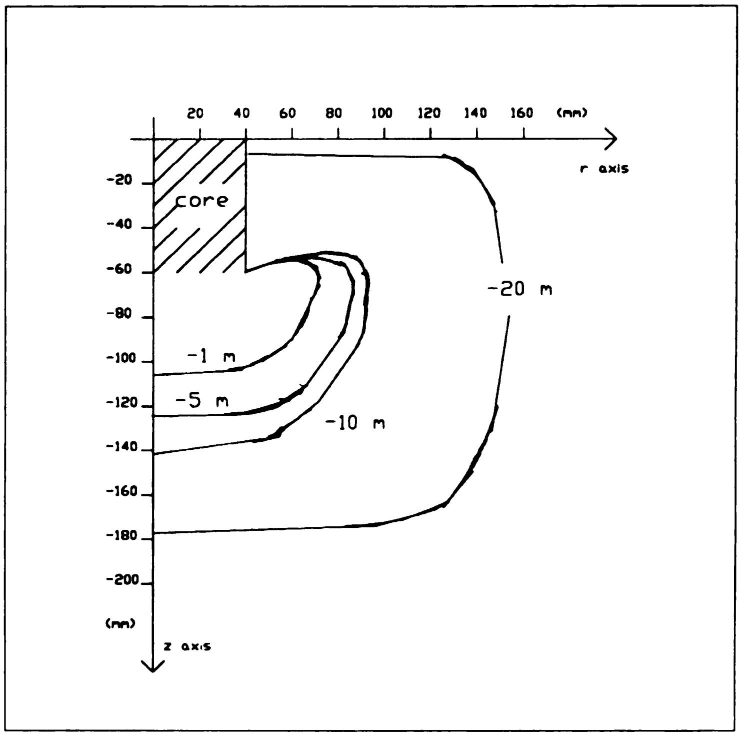

Figure 18. Soil water pressure head distribution, axisymmetric model

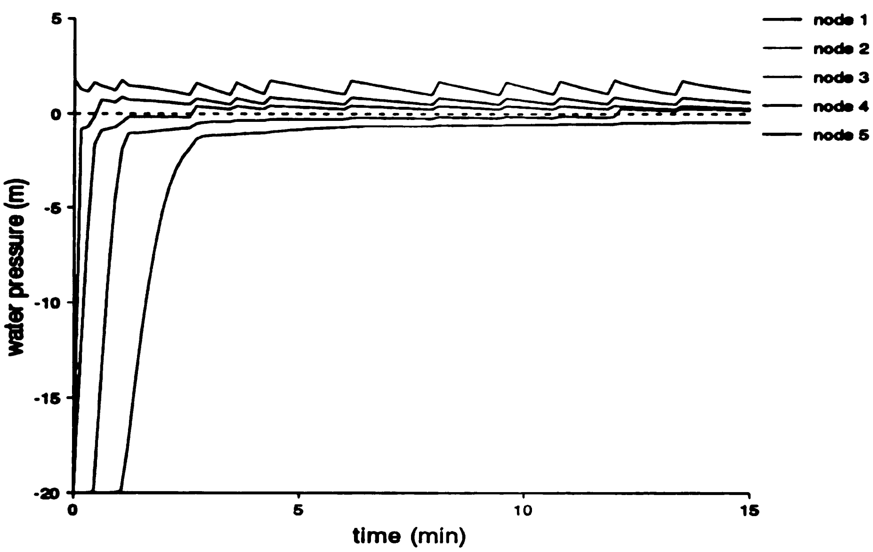

Figure 19. Results, axisymmetric analysis,

3.9 Axisymmetric Model - Triangular Elements

The axisymmetric model with triangular elements was run for 15 minutes with a time step of 0.01 second. The time-step was one-tenth the time-step used for the rectangular model because of the small triangular elements that were used. Figure 20 shows the equipotential curves for this configuration. The model is smaller than the axisymmetric model utilizing only rectangular elements to keep the number of nodes and elements to a workable number. Comparing the equipotential curves for the model with triangular elements with the model using rectangular elements (Figure 18) shows that the equipotential lines are similar for the two types of elements. The models will be referred to as triangular and rectangular to differentiate between the cross-sections of each. The elements in both models are axisymmetric.

Figure 21 shows results from the axisymmetric triangular VP model: the drop in head of node 1 and the wetting of nodes 2 - 5. The wetting profiles are similar to those graphed from the 3D results, except that node 3 crosses the x axis earlier in the triangular element model. The difference may be due to the affect of the concentration of triangular elements beginning at the depth of node 3 and the effect they have on the numerical calculations.

Notable differences in the model using axisymmetric rectangular elements and the model using triangular elements were observed. One dissimilarity was the frequency of refills. The model with axisymmetric elements refilled 11 times in 15 minutes while the model with triangular elements refilled only 6 times (see Figures 19 and 21). Another variation in the models was the time needed to saturate the VP cylinder. The cylinder in the axisymmetric model became saturated after 10 minutes but the cylinder with the triangular elements did not saturate by 30 minutes run time. After a run of 10 minutes the sample cup of the axisymmetric model was saturated and the total volume inflow of water was 3.08 E-4 m^3. The model with trianular elements had a smaller total volume inflow of 2.06 E-4 m^3 after 15 minutes. Since the input parameters were the same for both models these differences are suspected to be due to the element size difference in the two models or the difference in the total number of elements (80 elements versus 110 elements) or a combination of both. To use the model with triangular elements for comparison to the axisymmetric model it would be necessary to increase the rate of inflow. The following section (4.4) discusses changes in the input parameters which affect the rate of inflow.

Appendix F gives the soil water pressure head at each node at 15 and 30 minute run time and isohead plot at 15 minutes for the model using triangular elements. These plots show the shape and extent of the wetted front. It should be noted in this data that some nodes appear to become dryer during the analysis. This effect is not physically possible and appears to be an anomaly due to the methodology in calculating the hydraulic conductivity for the element matrices.

The model with triangular (axisymmetric) elements was developed to study the movement of the saturated front outside the core. Since the core did not saturate in an acceptable run time this model was abandoned. Knowledge gained in further study of the axisymmetric element model would be useful in determining adjustments to the triangular element model that might make it suitable for further study. The model may be useful for studying the saturated front movement as it is, but the rate that the core saturates would need to be increased.

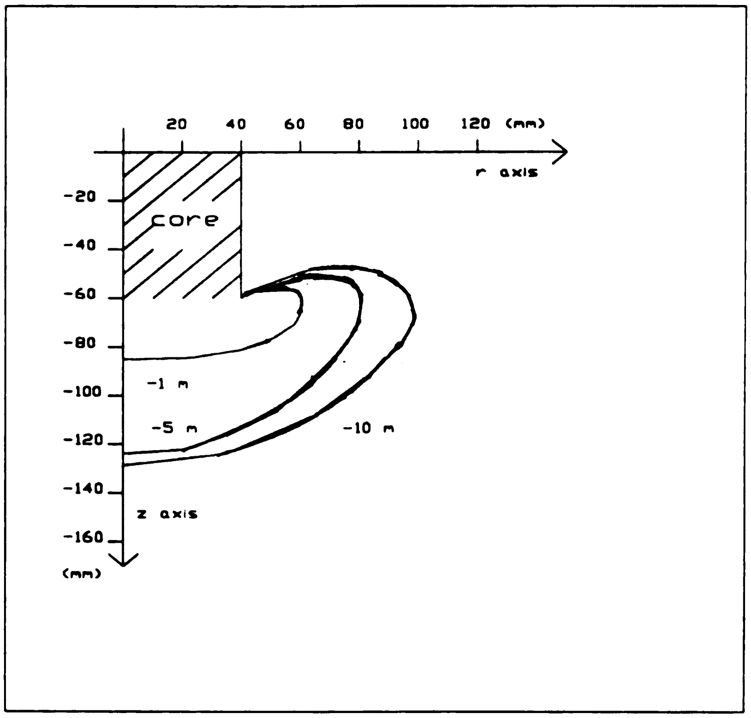

Figure 20. Soil water pressure head distribution, triangular model

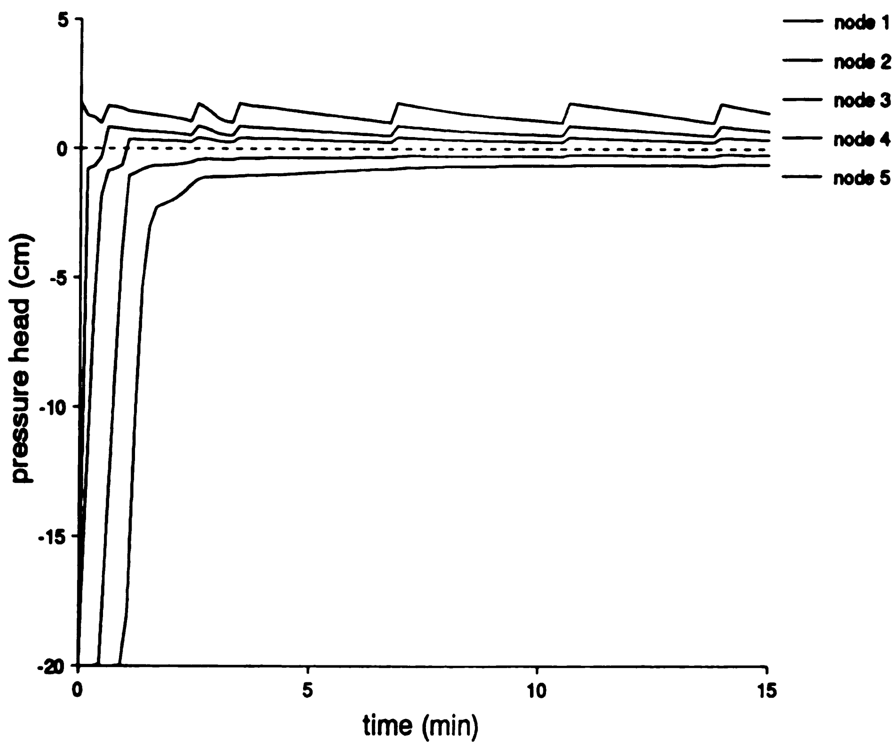

Figure 21. Wetting of nodes 2-5, triangular elements

4. RESULTS AND CONCLUSIONS

The axisymmetric VP model was useful in monitoring water flow through soils with different parameters and for studying different head tube/core size configurations. The 1D model was essential in determining the calculation method to be used for the parameters k and λ. The subroutines which allowed analysis of triangular elements did not prove as useful as was anticipated.

4.1 Versatility of the Axisymmetric Model

The versatility of the axisymmetric model of the velocity permeameter was examined by altering six parameters. The parameters that were varied were: the initial soil water pressure head; the Ψ-k curve used; the $\Psi - \Theta$ curve used; the value of r which describes the anisotropy between the horizontal k value and the vertical k value (the k value in the y direction was assumed constant for the axisymmetric element); the core diameter and the head tube diameter. Heterogenity (the spatial variation of k) was not investigated but could be studied by assigning different soil parameters on an elemental basis. Table 1 gives the parameter conditions for each analysis. The input file name is given in column 1 of Table 1 with each run's input parameters given in the corresponding row. The first set of parameters listed was designated the 'control' data set. The values chosen to act as the control data set were somewhat arbitrarily chosen, particularly the Ψ-k and Ψ-Θ curves. The control parameters were necessary, however, in order to have a point of comparison while varying the six parameters. Throughout the remainder of this work these values will be referred to as the control values or control parameters. Each set of parameters make up a hypothetical soil type.

Table 1. Input parameter conditions

file	initial head (m)	r	k curve	Θ curve	core diameter (mm)	head tube diameter (mm)	time step (sec)
control	-20	1.0	1	1	80	7.1	0.1
IC1	-10	1.0	1	1	80	7.1	0.1
IC2	-50	1.0	1	1	80	7.1	0.1
IC3	-100	1.0	1	1	80	7.1	0.1
K1	-20	1.0	2	1	80	7.1	0.01
K2	-20	1.0	3	1	80	7.1	0.5
WC1	-20	1.0	1	2	80	7.1	0.1
WC2	-20	1.0	1	3	80	7.1	0.05
Kr1	-20	10.0	1	1	80	7.1	0.1
Kr2	-20	0.1	1	1	80	7.1	0.1
CD1	-20	1.0	1	1	40	7.1	0.1
CD2	-20	1.0	1	1	120	7.1	0.1
HTD1	-20	1.0	1	1	80	12.7	0.1
HTD2	-20	1.0	1	1	80	3.2	0.05

where:
head = soil water pressure head
r defines the relationship:

$$k_r = r * k_z$$

k curve, defines the Ψ-k relationship used:
1 - Pachappa sandy loam ($k_{sat} = 20 mm/hr$)
2 - Indio loam ($k_{sat} = 200 mm/hr$)
3 - Chino clay ($k_{sat} = 2 mm/hr$)

Θ curve, defines the Ψ-Θ relationship used:
1 - Zigenfuss soil (Θ = 45 % at saturation)
2 - Capac soil (Θ = 36.8 % at saturation)
3 - Lenawee soil (Θ = 43.4 % at saturation)

core, head tube - measurements in millimeters

The k curve was obtained from data by Gardner (1958), as discussed in Chapter 2. The Θ curve information was obtained from SCS Soil Water Retention Curves from the Midwest National Technical Center (1988). Analysis may have been facilitated by using the same soil types for the k curves that were used for the Ψ-Θ curves had this information been available.

The time step was 0.1 second in most cases but had to be decreased for files K1, WC2 and HTD2. In the first two cases the smaller time step was necessitated because the parameters in the equation governing the choice of the time step (eq. 3.7) had changed. This was also the reason that a larger time step could be used for file K2. The time step for file HTD2 had to be smaller due to the physical restrictions. The head tube was so small, compared to the soil core, that the drop in head exceeded allowable boundaries with a time step of 0.1 seconds.

The control value of initial pressure head was -20 m and the soil was considered isotropic. The Ψ-k curve approximated that of a Pachappa sandy loam with a saturated value of 20 mm/hr. The Ψ-Θ curve approximated that of a Zigenfuss soil. A sample cup (also referred to as the core) diameter of 80 mm with a head tube diameter of 7.1 mm was used (Table 1). Figures 18 and 19 (Chapter 3) give results from the VP analysis using the control parameters.

Table 2 shows the time for the center of the core (node 4, at 60 mm, depth) to saturate and the total flow volume after 5 and 10 minutes simulation time. The center node of the core became saturated before any other nodes at the same depth. The author's hypothesis is that this node became saturated first since it is on a reflective boundary and flow was only in the downward (vertical) direction. All flow was initially downward, in a one-dimensional manner, at the beginning of the analysis since all nodes at the ground surface ($z = 0$) in the core had a water potential of 2 meters with all other nodes in the model at an initial value of -20 meters. This resulted in faster wetting of nodes towards the interior of the core, particularly the nodes along the z axis. Once the wetted front reached the soil core bottom flow became three-dimensional as water began to move in a radial direction out of the core.

Table 2. Core saturation and total flow

file	core saturated (min)	volume inflow at saturation (* 10^5 mm^3)	volume inflow at 5 minutes (* 10^5 mm^3)	volume inflow at 10 minutes (* 10^5 mm^3)
control	10.0	3.08	1.79	2.80
IC1	6.0	2.41	2.09	3.78
IC2	24.0	3.82	1.09	1.93
IC3	42.9	4.10	0.97	1.28
K1	1.4	4.12	13.32	30.07
K2	80.0	2.39	0.38	0.59
WC1	3.4	2.53	2.73	4.94
WC2	3.6	2.19	2.61	4.69
Kr1	*	*	2.22	3.81
Kr2	3.3	1.39	1.56	2.34
CD1	*	*	0.51	0.83
CD2	5.8	3.96	3.62	6.00
HTD1	9.7	2.68	1.71	2.68
HTD2	11.6	3.17	1.80	2.81

* indicates that the center node of the core did not saturate after 60 minutes running time.

These results are, generally, what was expected and indicate that the model is applicable for studying water movement through unsaturated soil, different soil parameters and different VP equipment configurations. Explanations of these results may not be immediately apparent, however. Each alteration in the initial conditions (from control values) will be individually discussed in the following sections.

The total flow at the time of the core becomming saturated was within 30% of the value for the control parameters except in the case of files Kr1 and Kr2. File Kr1 never reached saturation and Kr2 became saturated at a much lower flow rate due to the gravitational flow predominance. It would appear that a flow of about 3.1 E-4 m^3 is required for the core to saturate. This value is higher for soils with drier initial pressure heads (IC2 and IC3) and for soils with higher saturated k values (K1). The slope of the Ψ-Θ curve (from which λ is determined) also affects the total flow at saturation. Curves that are more linear (WC1 and WC2) seem to lead to lower total flow values when the core saturates.

4.1.1 Core Saturation and Total Flow

4.1.1.1 Initial Pressure Head

The time needed to saturate the core (node 4) was longer for the configuration files with drier initial pressure heads. The hypothetical soil with the highest initial pressure head, IC1, had node 4 saturated after 6 minutes, the control parameters, 10 minutes, with the drier initial pressure head taking 24 minutes (file IC2) and 43 minutes (file IC3). The same result is reflected in the volume inflow results. With an initial pressure head of -100 m (file: IC3) the total flow after five minutes was just over 1/2 of the flow from the control values (initial pressure head of -20 m). At 10 minutes the volume inflow for the hypothetical soil IC3 is less than 1/2 the volume inflow for the control soil. The volume inflow at core saturation, however, was greater for the hypothetical soil IC3 than for the control soil. The wetter initial pressure head (file: IC1) had a 16% increase in total flow after 5 minutes and a 35% increase after 10 minutes over the control parameters.

The initial pressure head of the soil is inversely related to the time to saturation of the soil core. The time for saturation increases as the initial pressure head decreases. The volume inflow at a given simulation time is directly related to the initial pressure head of the soil. The volume inflow increases as the initial pressure head increases. This is a consequence of the increased number of paths for flow in the wetter soil (less air pockets, etc.).

4.1.1.2 Hydraulic Conductivity Curve

The $\Psi - k$ curve relationship chosen to determine k affects the analysis profoundly. Three curves were used. The control k curve approximated that of a Pachappa sandy loam adjusted to have a saturated conductivity of 20 mm/hr. The curve for file K1 corresponded to that of a Indio loam, adjusted to use a saturated k of 200 mm/hr. The curve for file K2 corresponded to that of a Chino clay, adjusted to use a saturated k of 2 mm/hr. Appendix D gives the actual data points used for determination of the hydraulic conductivity from a known water potential.

The total flow for K1, the file with the highest saturated k, was 1.332 E6 mm^3 water after 5 minutes. This is an unreliable result, however, since the wetted front had reached the model limits. Recalling that the model limits are either reflective or impermeable boundaries it becomes clear that when any significant change in pressure head at the model limits occurs the run should be considered finished. A larger model would be needed to obtain reliable data for this file at 5 or 10 minutes run time. The results for file K1 show that much more water flows through soils with higher saturated k values and, consequently, the core saturates much faster. The control analysis had a total flows of 1.79 E5 mm^3 at 5 minutes and 2.8 E5 mm^3 at 10 minutes. The flow for file K2 was 3.78 E4 mm^3 at 5 minutes and 5.88 E4 mm^3 at 10 minutes.

The time to saturate node 4 is similarly affected. The interior node of the core was saturated after just 1.5 minutes with file K1. The core was saturated after 10 minutes for the control parameters and K2 took 80 minutes to saturate the interior node of the core. Clearly the hydraulic conductivity curve chosen, particularly the saturated k value, affects this model remarkably.

4.1.1.3 Water Content Curve

The data used for the determination of λ and water content were taken from the USDA Soil Conservation Service (Midwest Technical Center) for 3 soils (top 230 mm). The control $\Psi - \Theta$ relationship approximated that of a Zigenfuss soil, WC1 used a Capac soil and WC2 used a Lenawee soil. Appendix D gives the data used for the 3 soils.

The different curves appear to affect both the total flow, as well as time to saturate the core. Both curves chosen increased the total flow and consequently decreased the time to saturate the core. The control values had resultant total flows of 1.8 E5 and 2.8 E5 mm^3 (5 and 10 minutes), whereas WC1 had total flows of 2.7 E5 and 4.9 E5 mm^3, and WC2 had total flows of 2.6 E5 and 4.7 E5 mm^3. The core saturated at 10 minutes for the analysis with the control parameters, WC1 saturated at 3.4 minutes and WC2 saturated at 3.6 minutes.

The results for WC1 and WC2 are similar because the slope of the $\Psi - \Theta$ curve is similar for these two soils. These three soils were chosen because of the difference in their saturated volumetric water contents (the Zigenfuss soil (control) had a saturated volumetric water content of 45.3%, the Capac soil (WC1) had a saturated volumetric water content of 37.2% and the Lenawee soil (WC2) had a saturated water of 43.5%). The final water does not effect the finite element analysis, but λ, the slope of the $\Psi - \Theta$ curve that is used in finite element analysis. The slope of the curves for WC1 and WC2 were similar and more linear than that for the control parameters. This lead to larger total flow values and faster core saturation times. The data used to calculate λ and WC are found in Appendix D.

4.1.1.4 Heterogenity

The study of heterogenity, where k_r is numerically related to k_z by the multiplication factor, r (Table 4.1), showed the expected results. When radial flow dominates (k_r 10 times k_z, file Kr1) the core never saturated and the total flow was greater than that for the control parameters (2.2 E-4 m^3, 23.9% increase at 5 minutes simulation time and 3.8 E-4 m^3, 36.1% increase at 10 minutes simulation time). The flow was greater due to the increased k value, over that for the control analysis, but the core did not saturate due to the strong radial influence which pulled water out of the core. Once the wetting front escaped the core and flow became axisymmetric the larger radial k values resulted in considerable radial flows, with little vertical flow. The wetted front moved out of the soil core so quickly that the soil never became saturated in the cylinder.

Conversely when $k_r = 0.1 * k_z$ (file Kr2) flow is preeominately vertical and the volume inflow was decreased by 13% at 5 minutes and 16.5% at 10 minutes simulation time. The core became saturated in 3.3 minutes, much faster than the 10 minutes for the control parameters, because of the predominately vertical flow.

4.1.1.5 Core Diameter

Changes in the core diameter produced total flows roughly associated to the cross sectional area of the core since this was the source of water for the model. The core diameter of file CD1 was 25% the area of the control core.

At 5 minutes the flow for CD1 was 511 mm^3, or 28.4% of the flow for the control analysis. At 10 minutes the flow for CD1 was 29.6% of the flow for the control analysis. Similarly the area of CD2 was 225% the area of the control core diameter and the total flow was 201.7% at 5 minutes and 214.5% at 10 minutes.

The core never saturated for file CD1, the small diameter core. This file was run for 150 minutes and still node 4 did not saturate. This size core did not seem adequate to put enough water in the soil to saturate the core. This may not be a problem, however, as a steady state condition appeared to occur and will be discussed in Section 4.4.2 (apparent hydraulic conductivity).

4.1.1.6 Head Tube Diameter

The head tube diameter had no effect on the total flow. This was expected. After 5 minutes the total flow for HTD1, control and HTD2 files was 171 cm^3, 179.4 cm^3 and 179.8 cm^3, respectively. After 10 minutes the total flow was 269 cm^3, 280 cm^3 and 281 cm^3, respectively. The time to saturate the core was 9.7 minutes for HTD1, 10.0 minutes for the control parameters and 11.6 minutes for HTD2 (the smallest head tube). The differences in the time to saturate the core may be significant but is probably due to the ability of the head tube to supply ample water to the model.

4.1.2 Apparent Hydraulic Conductivity

Calculating the apparent hydraulic conductivity, k_{app}, allows scrutiny of the model and comparison with Merva's (1979) field analysis of the VP. The apparent hydraulic conductivity, k_{app}, was determined from a linear regression analysis of head and velocity to give dv/dh which was multiplied by the core length to give k_{app} (discussed also in Chapter 3). The time step was, generally, 0.1 seconds but values of velocity as a function of head to be used in the linear regression were taken every 6 seconds. This was done to more closely duplicate field readings and to smooth some of the irregularities that the elemental model presented. The head and velocity data had to be taken more often in a few cases (file K1 for example) because the core refilled so quickly.

The apparent hydraulic conductivity values were calculated before each refill. The results of this analysis on the files listed in Table 1 are shown in Table 3 for two to three refills after the core saturated. The coefficient of linear regression is also given. The regression coefficient was best just after the core saturated. Much lower regression coefficients occured while nodes became wetted. The numerical influence of unsaturated parameters, particularly k, may have resulted in a nonlinear dv/dh relationship. After the core saturated the model appeared to reach a quasi-steady state condition, when the k_{app} values shown in Table 3 were recorded. The dv/dh relationship became more nonlinear as nodes below the core became significantly wet.

The saturated hydraulic conductivity used to determine k for all files, except K1 and K2, was 20 mm/hr. File K1 had a saturated k of 200 mm/hr and K2 had a saturated k of 2 mm/hr. The apparent hydraulic conductivity, k_{app}, values are generally smaller than the saturated value because although the sample cup becomes saturated the soil under the cup does not, which slows the movement of water through the model. The apparent hydraulic conductivity values are generally within 50% of the saturated value excluding files IC2 and IC3 (initial pressure head was less than -20 meters) and file Kr2 (radial flow one-tenth of the vertical flow). The soils that were initially drier than the control value took significantly longer to saturate (24 minutes and 43 minutes as opposed to 10 minutes) and a steady state condition may have become established with a k_{app} value that was lower than the saturated

Table 3. Apparent hydraulic conductivity

file	time to saturate core (minutes)	Kapparent (r^2) (mm/hr)
control	10.025	11 (83.8%) 27 (87.9%) 11 (*)
IC1	5.9769	10 (82.7%) 20 (90.8%) 8 (*)
IC2	23.9616	6.1 (94.5%) 9.6 (89.3%) 16 (*)
IC3	42.9043	5.2 (94.9%) 6.4 (*) 18 (*)
WC1	3.4034	16 (96.2%) 19 (95.4%)
WC2	3.5851	16 (96.8%) 20 (81.9%)
K1	1.4492	58 (*) 69 (81.6%) 140 (92.3%)
K2	79.95	1.3 (79.5%) 2.5 (*) 1.0 (*)
Kr1	core not sat at 60 min.	last 3 drops: 20 (97.5%) 27 (99.2%) 28 (99.1%)
Kr2	3.3451	6.1 (*) 8 (*)
HTD1	9.6833	17 (94.3%) 1.5 (94.8%)
HTD2	11.550	13 (93.1%) 10 (97.7%)
CD1	150 (core not saturated)	14 (97.5%) 13 (98.7%) 13 (98.4%)
CD2	5.760	5.8 (*) 9.8 (*) 12.0 (*)
* denotes a correlation coefficient < 80%		

value due to the influence of the unsaturated k value. File Kr2 also had lower k_{app} value than was expected. The predominantly vertical flow caused the sample cup to be saturated quickly (3.3 minutes).

During the analysis nodes just below the saturated front seemed to reach a critical pressure head (around -5 m) when they would cause a significant jump in the total flow for the iteration which, in turn, affected the velocity calculation and the drop in head in the head tube. This effect was particularly noticable in the initial stages of analysis (at the beginning of the run) which is why the k_{app} values are given only after the core saturated. The saturated hydraulic conductivity - apparent hydraulic conductivity relationship merits further study.

4.2 Conclusions

The axisymmetric model proved useful in comparison of the effects of varying soil parameters on the flow of water through a soil. The VP core diameter and head tube diameter choice also affect flow rates and the model may be used to estimate proper VP configuration.

The triangular element model may be useful in monitoring the saturated front movement. Another k curve (perhaps the Indio loam curve) would speed up the analysis and make this model more practical. Further research would be necessary to determine the usefullness of the triangular element in the VP model.

Anisotropic flow in a soil is complex. The method chosen to determine the elemental hydraulic conductivity is critical to the success of the analysis using the VP finite element model. Many other factors affect the extent and profiles of both the saturated and wetted fronts. The VP model is a useful tool for studying water movement through the soil under the velocity permeameter. The model increases our understanding of how water moves through the soil under the VP, in manner and extent.

5. ADDITIONAL REMARKS

Some observations were made during the course of this research which, while not germane to the objectives of this research, merit consideration.

5.1 Modelling Technique

A different model configuration might help in the study of the movement of water outside the core. The model utilizing triangular axisymmetric elements might be useful, as has been discussed, in Chapter 3.

Another change which might be considered is using a double node at the radial edge (outside) of the bottom of the core. Early models in this research used the double node but the final model was changed so as not to have discontinuity at the radial edge of the core. However, after the change in the model was made it was observed that the core had saturated faster with a single node at the radial edge, and the node at the radial edge had a pressure head closer to other nodes at the bottom of the soil core. The approach may give smoother isoheads and more realistic (compared to field data) results, although the justification of using a single node at the radial edge of the soil core may be unclear.

5.2 Determination of k and λ

The time-dependent soil parameters k and λ had to be calculated between each time step so the element matrices could be determined. The elemental value had to be determined from the known water potentials at the nodes. All four element nodes (axisymmetric rectangular element) might have different water potentials so finding a representative k or λ for the element presented great difficulties. The extreme variability of k for the water potentials involved in this research made proper selection of the k and λ curves imperative. The work of Gardner (1960) was chosen as a model for the $\Psi - k$ curves as he presented these curves over the desired range of Ψ. Lambda, λ, was calculated from actual data.

Both exponential functions and power functions (such as those used by Jabro & Fritton (1990) and Unlu (1990)) were used in the model. Coefficients were used that approximated Gardner's curve for a Pachappa sandy loam as closely as possible. Results from both methods were disappointing due to fluctuations in the water potential at the nodes. Nodes would begin wetting and then get drier, a physically unlikely condition. With a smaller range of water potentials the functions appeared useful.

The best results were achieved by using a 19 point $\Psi - k$ curve and full logarithmic interpolation between points. This method was also used for calculating the Θ for a given Ψ and λ (slope of the $\Psi - \Theta$ curve), with the data from the USDA Soil Conservation Service and semi-logarithmic interpolation. Data for both calculations (k and λ) are given in Appendix D.

5.3 Water Content and Isoheads

The total volume of the soil in the model was 1.6085 E7 mm^3. The initial pressure head of the control model was 20.9 % giving a volume of water of 3.362 E6 mm^3. The volume of soil in the core was 3.02 E5 mm^3 and at saturation the pressure head is 45.3% (control parameters) with 1.37 E5 mm^3 water in the core. The total inflow, when the core was saturated, was 3.08 E5 mm^3 water indicating that more than half the inflow had escaped the core (1.71 E5 mm^3 water). In fact, the wetted front had moved to a depth of 140 mm (80 mm below the core) and radially 100 mm from the z axis (40 mm outside the core). The isoheads (Figure 18 and Appendix B) show this more clearly. Appendix B gives run outputs (for all files) of the water potential at each node after the core saturates as well as a plot of some isoheads. These plots show the shape of the wetted front for each set of parameters and are useful for visual comparison of the extent of the wetted fronts. It should be noted that that some nodes appear to become dryer during the analysis. This effect is not physically possible and appears to be an anomaly due to the methodology in calculating the hydraulic conductivity for the element matrices.

Flow was one-dimensional while the wetted front was inside the core. Once the wetted front reached the bottom of the core flow became axisymmetric. The node at the center of the model (node 4) is on a reflective boundary (z axis) so flow must be directed downwards (gravitational) at that point and isoheads must be perpendicular at that boundary throughout the analysis. Nodes that are not on the reflective boundary, however, experience flow that is both radial as well as gravitational. Therefore as the wetted front moves out of the core flow has both gravitational and matric components. The result is that the node at the center of the core became saturated before nodes away from the center in all files.

The isoheads shown in Appendix B for the files are quite similar to the control parameter isoheads. A notable exception is when the soil is anisotropic (k varies with direction of measurement). Figures B9 and B10 show considerable variation for these parameters when compared to each other or the control parameter isoheads. For a radial k that is ten times the value of the vertical k the wetted front extends farther radially than the isoheads for the control values. For a radial k that is one-tenth the value of the vertical k the wetted front is severely limited in the radial direction, and similar to the control isohead in vertical depth.

APPENDICES

APPENDIX A

Axisymmetric Computer Program (VP)

The computer program was written in Pascal and run on a Unix system. Procedure INDATA read the input data from a file, procedure FEM ran the finite element analysis, FLOW_CALC calculated the flow velocity and drop in head between iterations and RESULTS printed water potential results.

```
******************************************************************
program vpt(input,output);
{for the mainframe only - it won't compile with turbo pascal}
type  d1siz = array[1..200] of real;
    smsiz = array[1..5] of real;
    d3siz = array[1..60] of integer;
    nlsiz = array[1..200,1..4] of integer;
    nmsiz = array[1..200] of integer;
    nssiz = array[1..4] of integer;
    d2siz = array[1..200,1..60] of real;
        {note - if you change the array size be sure to change
            the initialization in fnt_elem}
var  data_hp:       text;
    data_name, save_name:  packed array[1..15] of char;
    out_name, gph_name, hed_name: packed array[1..15] of char;
    wp_name:    packed array[1..15] of char;
    outfile:    text;
    graph:      text;
    head:       text;
    wp:         text;
    ques:       char;
    title:      packed array[1..40] of char;

    phi,     rf,
    wcp,     volel,
    xc,      yc:                d1siz;
    dt,      q,  x,y:           smsiz;
    nmtl, fx_nd_array:              nmsiz;
    sat_nd_array,nel_sat:           nmsiz;
    nel:                        nlsiz;
    fm:                         d1siz;
    c,k:                        d2siz;
    iptl,  iteration,
    iwt,  i_main,  i_siz,
    j_main,  kw,
    k_choice,  wc_choice,
    num_x_core,  num_y_core,
    num_x_nodes, num_y_nodes,
    flag,  nbw,  ncoef,
    ndbc,  ne,   nsteps,
    np,  writ_mult,
    num_fx_nds,  num_sat_nds,
    prnt_a,prnt_b,prnt_c,prnt_d,
    prnt_e,prnt_f,prnt_g,prnt_h,
    prnt_i,  run_term:              integer;
    aa,  ahtube,  bb,
    core_radius, core_length,
    delta,  dhtube,
    delta_q_total,
    flow,
    length,     lamda,
    pi_,
    r,  r1,  r2,  refill,
    sat_dist,
    sum_x, sum_x2,
    sum_y, sum_y2,
    sum_xy,
    sum_n,
    run_time,
```

```
     thta,  time_avg,
     total,  totflow,
     voll,   volume:           real;
****************************************
{definition of the input parameters
    title - a descriptive statement of the problem
          being solved
    np - number of equations (also number of nodes)
    ne - number of elements
    ncoef - number of sets of equation coefficients
          maxmum of five
    iptl - integer controlling the output
         (4 - debug option)
    kw - flag which allows printout of data at various
       points in the program
       (kw = 0  omit write, kw = 1  write)
    ahtube - area of the head tube
    dhtube - diameter of the head tube
       the number of the following sets of parameters must
       equal ncoef
    dt[i] - material property that multiplies the time
          derivative(i = 1-4)
    q[i]  - constant coefficient in the diff. eq. nodal
          coordinate values (i = 1-4)
    xc[i] - x coordinates of the nodes
    yc[i] - y coordinates of the nodes
          the coordinates must be in numerical sequence relative
          to node numbers
          element data
    n - element number
    nmtl - integer specifying the equation coefficient set
    nel[n,1] - numerical value of node i
    nel[n,2] - numerical value of node j
    definition of the variables read by intval
    invl - integer controlling the input of the initial values
         1 - input a node at a time
         2 - input by groups
    ib[i] - node numbers which do not change with
          time (terminate with a zero)
    theta - the theta value used in the single step method
          0   - euler's forward difference method
          1/2 - central difference method
          2/3 - galerkin's method
          1   - backward difference method
    delta - the time step
    run_time - total time the model has run
    itype - integer controlling the type of analytical
          solution (1 for the velocity permeameter problem
    nsteps - number of time steps
    iwt - integer controlling the output of the calculated
         values to GPH.dat. values are printed every iwt steps
    writ_mult - controls the write to OUT.dat and to the screen
    ndbc - number of element sides with a derivative boundary
         condition
    r -  the ratio dye/dxe, i.e., conductivity in the y
         direction is r*conductivity in the x direction
    nel[n,1] - numerical value of node i
    nel[n,4] - numerical value of node j
    nel[n,3] - numerical value of node k
    nel[n,4] - numerical value of node m
    nel[n,4] is set equal to zero for the
          triangular element
    num_x_core - number of elements in the core
          in the x direction
    num_y_core - number of elements in the core
          in the y direction
    num_x_elem - number of elemnts in the x direction
    num_y_elem - number of elemnts in the y direction
 }
{*****************************************************************}
procedure intval(np,iptl:integer;var phi:d1siz);
{***********************************************
this subroutine either reads the initial values
---------------------
definition of the variables read by intval
  - the value of the initial soil moisture is assigned to all
    nodes, then the fixed nodes are given their value
}
var
```

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   i:        integer;
   head:        real;
 begin
   if(iptl >= 4) then writeln(outfile,'   entering intval',
            ' np = ',np:5);
  readln(data_hp,head);
  for i := 1 to np do
   phi[i] := head;
  repeat
   readln(data_hp,i,phi[i]);
  until (i = 1);
 end;                              {procedure  intval}
{*********************************************************************}
procedure indata;
{*********************************************
this subroutine and intval reads in the input data
----------------------
input of the title card and control parameters
}
const idnn = 50;
var
  ns:           nssiz;
  i, j,  ij,
  kk,n,nb,nid:      integer;
begin
   readln(data_hp,np,ne,ncoef,iptl,kw);
        {input of equation coefficients and the nodal coordinates}
   readln(data_hp,dhtube,r);
   readln(data_hp,core_radius,core_length);
   ahtube := pi_*(dhtube*dhtube)/4;
   writeln(outfile);
   writeln(outfile,' r: k_x =  ',r:0:3,' * k_y;  element size: 2 x 2 cm2');
   for i := 1 to ncoef do
      readln(data_hp,dt[i],q[i]);
   intval(np,iptl,phi);
   writeln(outfile,' the initial soil moisture is ',phi[2]:0:1,' cm');
   readln(data_hp,thta,delta);
   writeln(outfile,'  theta = ',thta:6:2,'  delta = ',delta:0:2,' seconds');
               {input the number of time steps, nsteps,
                and the write control, iwt  (for results)}
   readln(data_hp,nsteps,iwt);
   readln(data_hp,k_choice,wc_choice);
   readln(data_hp,num_x_core,num_y_core,num_x_nodes,num_y_nodes);
   writeln(outfile,' diameter head tube = ',dhtube:0:4,
         ' cm,  and the diameter of the core = ',num_x_core*4:0,' cm');
   readln(data_hp,writ_mult,prnt_a,prnt_b,prnt_c,prnt_d,prnt_e,
       prnt_f,prnt_g,prnt_h,prnt_i);
   if (kw > 3) then
   begin
     writeln(outfile,'      nodal coordinates');
     writeln(outfile,' node         x           y');
   end;
    for i := 1 to np do
     read(data_hp,xc[i]);
    for i := 1 to np do
     read(data_hp,yc[i]);
        {output of the equation coefficients}
         {output of the nodal coordinates}
   if(kw > 3) then
   begin
    for i := 1 to np do
    writeln(outfile,i:4,xc[i]:15:5,yc[i]:15:5);
         {input and echo print of the element nodal data}
    writeln(outfile,'   element data');
    writeln(outfile,'nel   nmtl     node numbers');
   end;
   nid := 0;
   for kk := 1 to ne do
   begin
    readln(data_hp,n,nmtl[kk],nel[n,1],nel[n,2],nel[n,3],nel[n,4]);
     if (kw > 3) then
      writeln(outfile,n:4,nmtl[kk]:6,nel[n,1]:7,nel[n,2]:6,
           nel[n,3]:6,nel[n,4]:6);
    if((n-1) <> nid) then
    writeln(outfile,'element ',n:4,' not in sequence');
    nid := n;
   end;
         {input the numbers of the nodes with values that remain
          constant during the finite elem analysis}
```

```
    i := 0;
    repeat
      i := i + 1;
      read(data_hp,fx_nd_array[i]);
      num_fx_nds := i-1;
    until (fx_nd_array[i] <= 0);
    ij := 0;
    repeat
      repeat
        ij := ij + 1;
        until ((fx_nd_array[ij+1] <= 0) or (ij mod 6 = 0));
    until(fx_nd_array[ij+1] <= 0);
    if(num_fx_nds > 0) then
    begin
      if(num_fx_nds >idnn) then
      begin
        write(outfile,'    number of boundary conditions exceeds');
        writeln(outfile,'  the allowed number of ',idnn:5);
      end;
    end;
              {analysis of the node numbers
               initialization of a check vector}
                       {creation and initialization of the a
                        vector calculation of the bandwidth}
    nbw := 0;
    for kk := 1 to ne do
    begin
      for i := 1 to 4 do
        ns[i] := nel[kk,i];
      for i := 1 to 3 do
      begin
        ij := i+1;
        for j := ij to 4 do
        begin
          nb := abs(ns[i]-ns[j]);
          if(nb = 0) then writeln(outfile,'  element',kk:3,
                 ' has two nodes with the same node number');
          if(nb > nbw) then
            nbw := nb;
        end;
      end;
    end;
      nbw := nbw + 1;
      if (nbw > 60) then
        writeln(outfile,' warning! nbw = ',nbw:0,' (max = 60)');
             {calculation of the space required}
     if  (k_choice = 1) then
     writeln(outfile,' k curve - 1 - Pachappa sandy loam (Ksat = 2 cm/hr)');
     if (k_choice = 2) then
     writeln(outfile,' k curve - 2 - Indio loam (Ksat = 20 cm/hr)');
     if (k_choice = 3) then
     writeln(outfile,' k curve - 3 - Chino clay (Ksat = 0.2 cm/hr)');
     if (wc_choice = 1) then
     writeln(outfile,' wp/wc curve - 1 - Zigenfuss soil');
     if (wc_choice = 2) then
     writeln(outfile,' wp/wc curve - 2 - Capac soil');
     if (wc_choice = 3) then
     writeln(outfile,' wp/wc curve - 3 - Lenawee soil');
  end;                                {procedure indata}
{*****************************************************************}
procedure results;
{*********************************************
this procedure prints the water potentials at the nodes to file out.txt
--------------}
var  a,b,c,d,e,f,g,h,i,
     counter,it:  integer;
   begin
     a := prnt_a;
     b := prnt_b;
     c := prnt_c;
     d := prnt_d;
     e := prnt_e;
     f := prnt_f;
     g := prnt_g;
     h := prnt_h;
     i := prnt_i;
     counter := num_y_nodes;   {used for do loop}
   writeln(outfile,'       ',xc[a]:8:1,xc[b]:8:1,xc[c]:8:1,xc[d]:8:1,xc[e]:8:1,
        xc[f]:8:1,xc[g]:8:1,xc[h]:8:1,xc[i]:8:1);
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```
    writeln(outfile);
      for it := 1 to counter do
      begin
        if (yc[a] = 0.0) then
          write(outfile,' ');
        if (yc[a] > -10.0) then
          write(outfile,' ');
        writeln(outfile,yc[a]:2:1,' ',phi[a]:8:1,phi[b]:8:1,phi[c]:8:1,
              phi[d]:8:1,phi[e]:8:1,phi[f]:8:1,phi[g]:8:1,phi[h]:8:1,
              phi[i]:8:1);
        a := a + 1;
        b := b + 1;
        c := c + 1;
        d := d + 1;
        e := e + 1;
        f := f + 1;
        g := g + 1;
        h := h + 1;
        i := i + 1;
      end;
      writeln(outfile);
  end;                              {procedure results}
{***********************************************************************}
procedure sat_elem;
{********************************
procedure directs the saturated finite element analysis
-----------------}
var   ki, kj, kk,
     flag:   integer;
begin
  for kk := 1 to num_sat_nds do
  begin
    flag := 0;
    for ki := 1 to num_fx_nds do
     if (sat_nd_array[kk] = fx_nd_array[ki]) then
       flag := 1;
     if (flag = 0) then
    begin
      kj := sat_nd_array[kk];
      phi[kj] := (-1/yc[kj]) * phi[1];
    end;
  end;
end;                                {procedure sat_elem}
{***********************************************************************}
procedure wc_calc(kk:integer;var mc:real);
{*********************************
this subroutine calculates the water content, wc
at a node for a known water potential, wp
----------------}
type
     wc_type = array[1..14] of real;
var   i_wc:    integer;
     node:   real;
     wc:  wc_type;
     wp:  wc_type;
begin
{ziegenfusssoil, ap horizon (0-23 cm)}
 if (wc_choice = 1) then
  begin
    wc[1] := 0.453;
    wc[2] := 0.452;
    wc[3] := 0.440;
    wc[4] := 0.420;
    wc[5] := 0.377;
    wc[6] := 0.329;
    wc[7] := 0.295;
    wc[8] := 0.262;
    wc[9] := 0.236;
    wc[10] := 0.207;
    wc[11] := 0.175;
    wc[12] := 0.156;
    wc[13] := 0.146;
    wc[14] := 0.112;
  end;
    wp[1] := 0.0;
    wp[2] := -10.33;
    wp[3] := -30.99;
    wp[4] := -51.65;
    wp[5] := -103.3;
```

```
    wp[6] := -206.6;
    wp[7] := -340.89;
    wp[8] := -619.8;
    wp[9] := -1033.0;
    wp[10] := -2066.0;
    wp[11] := -5165.0;
    wp[12] := -10330.0;
    wp[13] := -15495.0;
    wp[14] := -103300.0;
  {Capac soil, ap horizon (0-23 cm)}
  if (wc_choice = 2) then
  begin
    wc[1] := 0.372;
    wc[2] := 0.370;
    wc[3] := 0.366;
    wc[4] := 0.362;
    wc[5] := 0.353;
    wc[6] := 0.338;
    wc[7] := 0.321;
    wc[8] := 0.296;
    wc[9] := 0.269;
    wc[10] := 0.229;
    wc[11] := 0.181;
    wc[12] := 0.151;
    wc[13] := 0.137;
    wc[14] := 0.093;
  end;
  if (wc_choice = 3) then
   {lenawee soil, ap horizon (0-23 cm)}
  begin
   wc[1] := 0.435;
   wc[2] := 0.433;
   wc[3] := 0.429;
   wc[4] := 0.425;
   wc[5] := 0.416;
   wc[6] := 0.4;
   wc[7] := 0.384;
   wc[8] := 0.358;
   wc[9] := 0.333;
   wc[10] := 0.295;
   wc[11] := 0.247;
   wc[12] := 0.216;
   wc[13] := 0.199;
   wc[14] := 0.144;
  end;
            {wc - the ordinate values - water content - volumetric}
            {wp  - the abcissa values - water potential- in cm}
            {note:  1033 cm/bar was used}
    node := phi[kk];
    if (node < wp[14]) then
    begin
      writeln(outfile,' value (',node:12:2,
           ') less than least absissa value: node = ',kk:0);
      writeln(outfile,' value (',node:12:2,
           ') less than least absissa value: node = ',kk:0);
      writeln(outfile,' the soil can not be this dry!');
    end;
                   {interpolate to get theta - water content}
    if (node < -10.33)  then
    begin
    for i_wc := 2 to 13 do
      if (node <= wp[i_wc])  and (node > wp[i_wc+1]) then
        mc := ((wc[i_wc]-wc[i_wc+1])*ln(node/wp[i_wc+1])/ln(wp[i_wc]
             /wp[i_wc+1]))+ wc[i_wc+1];
    end
    else
      mc := wc[1];
end;                              {procedure wc_calc}
{*************************************************************************}
procedure lambda(kk:integer;var lam_l:real;var flag:integer);
{*********************************
this subroutine calculates the slope of the
head-water content relationship, lambda
--------------------}
type
    wc_type = array[1..14] of real;
var  i_lam:    integer;
    node4,  mc:    real;
    wc:        wc_type;
```

```
    wp:        wc_type;
begin
{ziegenfuss soil, ap horizon (0-23 cm)}
 if (wc_choice = 1) then
   begin
     wc[1] := 0.453;
     wc[2] := 0.452;
     wc[3] := 0.440;
     wc[4] := 0.420;
     wc[5] := 0.377;
     wc[6] := 0.329;
     wc[7] := 0.295;
     wc[8] := 0.262;
     wc[9] := 0.236;
     wc[10] := 0.207;
     wc[11] := 0.175;
     wc[12] := 0.156;
     wc[13] := 0.146;
     wc[14] := 0.112;
   end;
     wp[1] := 0.0;
     wp[2] := -10.33;
     wp[3] := -30.99;
     wp[4] := -51.65;
     wp[5] := -103.3;
     wp[6] := -206.6;
     wp[7] := -340.89;
     wp[8] := -619.8;
     wp[9] := -1033.0;
     wp[10] := -2066.0;
     wp[11] := -5165.0;
     wp[12] := -10330.0;
     wp[13] := -15495.0;
     wp[14] := -103300.0;
     {Capac soil, ap horizon (0-22 cm)}
   if (wc_choice = 2) then
   begin
     wc[1] := 0.372;
     wc[2] := 0.370;
     wc[3] := 0.366;
     wc[4] := 0.362;
     wc[5] := 0.353;
     wc[6] := 0.338;
     wc[7] := 0.321;
     wc[8] := 0.296;
     wc[9] := 0.269;
     wc[10] := 0.229;
     wc[11] := 0.181;
     wc[12] := 0.151;
     wc[13] := 0.137;
     wc[14] := 0.093;
   end;
    {Lenawee soil, ap horizon (0-23 cm)}
   if (wc_choice = 3) then
   begin
     wc[1] := 0.435;
     wc[2] := 0.433;
     wc[3] := 0.429;
     wc[4] := 0.425;
     wc[5] := 0.416;
     wc[6] := 0.4;
     wc[7] := 0.384;
     wc[8] := 0.358;
     wc[9] := 0.333;
     wc[10] := 0.295;
     wc[11] := 0.247;
     wc[12] := 0.216;
     wc[13] := 0.199;
     wc[14] := 0.144;
   end;
        {wc - the ordinate values - water content - volumetric}
        {wp  - the abcissa values - water potential- in cm}
        {note:  1033 cm/bar was used}
    flag := 0;
    node4 := phi[nel[kk,4]];
    if (node4 < wp[14]) then
    begin
      write(outfile,' value less than least abaissa value: ');
      writeln(outfile,' node4 = ',node4:0,' wp[14] = ',wp[14]:0:2);
```

```
      write(outfile,' the soil can not be this dry!');
      writeln(outfile,' error found in interp');
      flag := 1;
     end;
     if (node4 < -10.33) then
      begin
       for i_lam := 2 to 13 do
       begin
        if (node4 <= wp[i_lam]) and (node4 > wp[i_lam+1]) then
         begin
          mc := ((wc[i_lam]-wc[i_lam+1])*ln(node4/wp[i_lam+1])/
                ln(wp[i_lam]/wp[i_lam+1])) + wc[i_lam+1];
          lam_l := ((mc - wc[i_lam+1])/(node4 - wp[i_lam+1]))
         end;
       end;
      end
      else
       lam_l := 0.0;
end;                                    {procedure lambda}
{*******************************************************************}
procedure kxy(k1:integer;var kl,klx:real);
{********************************
this procedure calculates the hydraulic conductivity
----------------}
type
     wp_type = array[1..19] of real;
var  i_kxy:      integer;
     head:   real;
     wp:     wp_type;
     k_curve: wp_type;
begin
     wp[1] := -8;
     wp[2] := -10;
     wp[3] := -20;
     wp[4] := -30;
     wp[5] := -40;
     wp[6] := -50;
     wp[7] := -65;
     wp[8] := -100;
     wp[9] := -150;
     wp[10] := -200;
     wp[11] := -250;
     wp[12] := -300;
     wp[13] := -400;
     wp[14] := -420;
     wp[15] := -500;
     wp[16] := -700;
     wp[17] := -900;
     wp[18] := -2000;
     wp[19] := -10500;
   {for Pachappa sandy loam:}
   if (k_choice = 1) then
   begin
     k_curve[1] := 5.56e-4;
     k_curve[2] := 5.56e-4;
     k_curve[3] := 5.56e-4;
     k_curve[4] := 5.05e-4;
     k_curve[5] := 4.04e-4;
     k_curve[6] := 3.54e-4;
     k_curve[7] := 2.52e-4;
     k_curve[8] := 3.54e-5;
     k_curve[9] := 1.01e-6;
     k_curve[10] := 5.3e-7;
     k_curve[11] := 3.53e-7;
     k_curve[12] := 2.3e-7;
     k_curve[13] := 1.01e-7;
     k_curve[14] := 5.3e-8;
     k_curve[15] := 4.04e-8;
     k_curve[16] := 2.02e-8;
     k_curve[17] := 1.52e-8;
     k_curve[18] := 2.02e-9;
     k_curve[19] := 3.03e-10;
   end;
   {for indio loam soil (saturated rate 20 cm/hr):}
   if (k_choice = 2) then
   begin
     k_curve[1] := 5.56e-3;
     k_curve[2] := 5.56e-3;
     k_curve[3] := 5.56e-3;
```

```
      k_curve[4] := 5.28e-3;
      k_curve[5] := 5e-3;
      k_curve[6] := 4.17e-3;
      k_curve[7] := 2.78e-3;
      k_curve[8] := 1.39e-3;
      k_curve[9] := 2.92e-5;
      k_curve[10] := 1.39e-5;
      k_curve[11] := 5.56e-6;
      k_curve[12] := 2.92e-6;
      k_curve[13] := 1.11e-6;
      k_curve[14] := 8.33e-7;
      k_curve[15] := 2.92e-7;
      k_curve[16] := 1.94e-7;
      k_curve[17] := 1.39e-7;
      k_curve[18] := 8.33e-9;
      k_curve[19] := 5.56e-10;
    end;
      {chino clay - saturated value of 0.2 cm/hr}
   if (k_choice = 3) then
    begin
      k_curve[1] := 5.56e-5;
      k_curve[2] := 5.28e-5;
      k_curve[3] := 2.78e-5;
      k_curve[4] := 2.5e-5;
      k_curve[5] := 1.67e-5;
      k_curve[6] := 1.39e-5;
      k_curve[7] := 8.33e-6;
      k_curve[8] := 2.78e-6;
      k_curve[9] := 8.33e-7;
      k_curve[10] := 5.56e-7;
      k_curve[11] := 2.92e-7;
      k_curve[12] := 2.22e-7;
      k_curve[13] := 8.33e-8;
      k_curve[14] := 6.94e-8;
      k_curve[15] := 2.92e-8;
      k_curve[16] := 2.5e-8;
      k_curve[17] := 1.39e-8;
      k_curve[18] := 2.77e-9;
      k_curve[19] := 8.33e-10;
     end;
     {k_curve - the ordinate values - water content - volumetric
            taken from richards but multiplied by a constant
            to reflect realistic saturated conductivity values}
     {wp  - the abcissa values - water potential- in cm}

    if (nel[k1,4] = 0) then
      head := phi[nel[k1,3]]
    else
      head := phi[nel[k1,4]];
    kl := 0;
    if (head > -8) then kl := k_curve[1]
    else
      for i_kxy := 1 to 18 do
      begin
        if (head = wp[i_kxy]) then
         kl := k_curve[i_kxy]
        else
         if (head <= wp[i_kxy])  and (head > wp[i_kxy+1]) then
         kl :=  exp((ln(k_curve[i_kxy]/k_curve[i_kxy+1])
                /ln(wp[i_kxy]/wp[i_kxy+1])*
                ln(head/wp[i_kxy+1])) + ln(k_curve[i_kxy+1]));
      end;
    klx := kl * r;
    if (kl = 0) then
      writeln(outfile,' error!, kl = 0, node ',k1:0);
end;          {adsorption flow is governed by the upper nodes}
                                  {procedure kxy}
{***************************************************************}
procedure glb_mtx(elem:integer;kl,klx,lam_l:real);
{***************************
this procedure creates the global matrices, c and k
-------------}
type   esiz =  array[1..4,1..4] of real;
       xsiz = array[1..4] of real;
var    ecm,  esm,  et,
       lmp,  es:              esiz;
       ef,  x,  y:            xsiz;
       a,   aa,   ar,
```

```
    b,   bb,
    dte,
    qe,  rbar:            real;
    i,   ii,   j,
    jj,  j5,   j6:        integer;
begin
    ee[1,1] := 2;
    ee[1,2] := -2;
    ee[1,3] := -1;
    ee[1,4] := 1;
    ee[2,1] := -2;
    ee[2,2] := 2;
    ee[2,3] := 1;
    ee[2,4] := -1;
    ee[3,1] := -1;
    ee[3,2] := 1;
    ee[3,3] := 2;
    ee[3,4] := -2;
    ee[4,1] := 1;
    ee[4,2] := -1;
    ee[4,3] := -2;
    ee[4,4] := 2;
    {for kdr}
    if ((iptl >= 4) and (elem = 1)) then
      writeln ('entering glb_mtx, 1st elem.');
{bilinear rectangular element without the
derivative boundary condition}
{retrieval of nodal coordinates and node numbers}
    for i := 1 to 4 do
      begin
       j := nel[elem,i];
       x[i] := xc[j];
       y[i] := yc[j];
       ef[i] := 1.0;
      end;
    aa := y[4]-y[1];
    bb := x[2]-x[1];
    ar := aa*bb;
    rbar := (x[1] + x[2])/2.0;
    if (abs(ar) <= 0.00001) then
    begin
      writeln(outfile,'the area of element ',elem:4,
           ' is less than 0.00001');
      writeln(outfile,'the node numbers are in the wrong order');
      writeln(outfile,' or the nodes form a straight line');
    end;
{initialization: et[ ] matrix for the
axisymmetric - time dependent problem}
{k - "stiffness" matrix}
     et[1,1] := rbar + x[1];
     et[1,2] := rbar;
     et[1,3] := -rbar;
     et[1,4] := -et[1,1];
     et[2,1] := rbar;
     et[2,2] := rbar + x[2];
     et[2,3] := -et[2,2];
     et[2,4] := -rbar;
     et[3,1] := -rbar;
     et[3,2] := et[2,3];
     et[3,3] := et[2,2];
     et[3,4] := rbar;
     et[4,1] := et[1,4];
     et[4,2] := -rbar;
     et[4,3] := rbar;
     et[4,4] := et[1,1];
     {for kdz}
{c - capacitance matrix, time dependent part}
{for c - lumped formulation}
      for i := 1 to 4 do
        for j := 1 to 4 do
         lmp[i,j] := 0.0;
      lmp[1,1] := 2*rbar +x[1];
      lmp[2,2] := 2*rbar+x[2];
      lmp[3,3] := lmp[2,2];
      lmp[4,4] := lmp[1,1];
{calculation of the stiffness and
capacitance matrices}
    ii  := nmtl[elem];
    dte := dt[ii];
```

```
    qe  := q[ii];
     a := aa/2;
     b := bb/2;
    for i := 1 to 4 do
    begin
      for j := 1 to 4 do
      begin
        esm[i,j] :=  (2*pi_*rbar*klx*a*es[i,j]/(6*b))
                   + (2*pi_*kl*b*et[i,j]/(6*a));
        ecm[i,j] := pi_*ar*dte*lam_l*lmp[i,j]/6;
      end;
      ef[i] := pi_*qe*aa*bb*ef[i]/4;
    end;
{direct stiffness}
{the major diagonal is stored in column 1,
followed by the minor diagonals (to nbw)
in columns 2 - nbw.  zeros are used to fill
the column to np, the matrix size is np x nbw}
{the force vector, fm, has fixed nodal values}
      for i := 1 to 4 do
      begin
        j5 := nel[elem,i];
        fm[j5] := fm[j5] + ef[i];
        for j := 1 to 4 do
        begin
          j6 := nel[elem,j];
          jj := j6 + 1 - j5;
          if(jj > 0) and (jj <= nbw) then
          begin
            k[j5,jj] := k[j5,jj] + esm[i,j];
            c[j5,jj] := c[j5,jj] + ecm[i,j];
            if (j5 > np) then
              writeln(outfile,' j5 > np in glb_mtx! fix it!');
          end;
        end;
      end;
  end;                              {procedure glb_mtx}
{*************************************************************************}
procedure modify;
{****************
this subroutine modifies the global capacitance and
stiffness matrices when there are nodal values that
remain constant with time, using deletion of rows and columns.
-------------}
var   i_mod, j_mod, jm_mod,
      m_mod, n_mod:          integer;
begin
    if(iptl >= 4) then
    begin
      writeln(outfile,'executing modify');
      writeln(outfile,'np  =',np:5,' nbw  = ',nbw:5)
    end;
{modify c and k matrices by deleting rows and columns}
    for i_mod := 1 to num_sat_nds do
    begin
     j_mod := sat_nd_array[i_mod];
     n_mod := j_mod-1;
{this zeros out the row for the fixed node, except
the first column, which is actually the diagonal.}
      for jm_mod := 2 to nbw do
      begin
        m_mod := j_mod+jm_mod-1;
        if (m_mod <= np) then
        begin
          fm[m_mod] := fm[m_mod]-k[j_mod,jm_mod]*phi[j_mod];
          k[j_mod,jm_mod] := 0;
          c[j_mod,jm_mod] := 0
        end;
{this zeros out the column for the fixed node, but
Remember the diagonals make up the columns so
calculations must be done at about a 45 degree angle}
        if (n_mod > 0) then
        begin
          fm[n_mod] := fm[n_mod]-k[n_mod,jm_mod]*phi[j_mod];
          k[n_mod,jm_mod] := 0;
          c[n_mod,jm_mod] := 0;
          n_mod := n_mod-1
        end;
      end;
```

```
      c[j_mod,1] := 1;
      k[j_mod,1] := 0;
      fm[j_mod] := 0;
    end;
  end;                                   {procedure  modify}
{*******************************************************************}
procedure matab;
{********************
this subroutine generates the a and p matrices
for the single step methods using the equations
----------
   a( , ) = c[ , ]+(delta)*thta*k[ , ]
   p( , ) = c[ , ]-(delta)*(1 - theta)*k[ , ]
  }
var  i_mat,j_mat:  integer;
    a_mat,  b_mat,  tc,  tk:     real;
 begin
    if (iptl > 4) then writeln(outfile,'    entering matab');
    a_mat := thta*delta;
    b_mat := (1-thta)*delta;
    for i_mat := 1 to np do
    begin
      for j_mat := 1 to nbw do
        begin
         tc := c[i_mat,j_mat];
         tk := k[i_mat,j_mat];
{a matrix}
         c[i_mat,j_mat] := tc+a_mat*tk;
{p matrix}
         k[i_mat,j_mat] := tc-b_mat*tk
        end;
    end;
  end;                                   {procedure  matab}
{*******************************************************************}
procedure dcmpbd;
{*********************
this procedure decompes the k and c matrices into upper triangular
form using gaussian elimination
-------------}
var  i_dmc,  j_dmc,
    k2,   mj,    mk,     nd,
    nj,   nk,    nl,    np1:       integer;
  begin
    if (iptl >= 4) then writeln(outfile,'  entering dcmpbd');
    np1 := np-1;
    for i_dmc := 1 to np1 do
    begin
      mj := i_dmc+nbw-1;
      if(mj > np) then mj := np;
      nj := i_dmc+1;
      mk := nbw;
      if((np-i_dmc+1) < nbw) then mk := np-i_dmc+1;
      nd := 0;
      for j_dmc := nj to mj do
      begin
        mk := mk-1;
        nd := nd+1;
        nl := nd+1;
        for k2 := 1 to mk do
        begin
         nk := nd+k2;
         if (c[i_dmc,1] = 0) then
           writeln(outfile,' c[',i_dmc:0,',1] = ',c[i_dmc,1]);
         c[j_dmc,k2] := c[j_dmc,k2]-c[i_dmc,nl]*c[i_dmc,nk]/c[i_dmc,1];
        end;
      end;
    end;
  end;                                   {procedure  dcmpbd}

{*******************************************************************}
procedure multbd;
{*******************
this procedure performs the matrix multiplication
-------------}
var   i_mtbd,  j_mtbd,
    k2,  m_mtbd:          integer;
    sum:                real;
  begin
    if(iptl >= 4) then writeln(outfile,'    entering multbd');
```

```
    for i_mtbd := 1 to np do
    begin
      sum := 0.0;
      k2 := i_mtbd-1;
      for j_mtbd := 2 to nbw do
        begin
          m_mtbd := j_mtbd+i_mtbd-1;
          if (m_mtbd <= np) then
          sum := sum+k[i_mtbd,j_mtbd]*phi[m_mtbd];
          if (k2 > 0) then
          begin
            sum := sum+k[k2,j_mtbd]*phi[k2];
            k2 := k2 - 1
          end;
        end;
      rf[i_mtbd] := sum+k[i_mtbd,1]*phi[i_mtbd];
    end;
  end;                              {procedure  multbd}
{********************************************************************}
procedure slvbd;
{*********************
this procedure decompes the global force vector, f, and solves
the system of equations using backwards substitution
---------------}
var  flag,
    i_slv,   j_slv,     k2,
    l_slv,   mj,
    nj,  np1,  n_slv:       integer;
    sum:                    real;
begin
    np1 := np-1;
{decomposition of the column vector rf( )}
    for i_slv := 1 to np1 do
    begin
      mj := i_slv+nbw-1;
      if(mj > np) then mj := np;
      nj := i_slv+1;
      l_slv := 1;
      for j_slv := nj to mj do
      begin
        l_slv := l_slv+1;
        rf[j_slv] := rf[j_slv]-(c[i_slv,l_slv]*rf[i_slv]/c[i_slv,1]);
      end;
    end;
{backward substitution for determination of phi[ ]}
    phi[np] := rf[np]/c[np,1];
    for k2 := 1 to np1 do
    begin
      i_slv := np-k2;
      mj := nbw;
      if((i_slv+nbw-1) > np) then mj := np-i_slv+1;
      sum := 0.0;
      for j_slv := 2 to mj do
      begin
        n_slv := i_slv+j_slv-1;
        sum := sum+c[i_slv,j_slv]*phi[n_slv];
      end;
        phi[i_slv] := (rf[i_slv]-sum)/c[i_slv,1];
    end;
         {End Finite Element Analysis - new heads (phi) found}
                  {the code following adds a saturated
                   node to the fixed node set}
   for j_slv := 1 to np do
      if (phi[j_slv] > -10.33) then
      begin
        flag := 0;
        i_slv := num_sat_nds;
        for l_slv := 1 to i_slv do
          if (sat_nd_array[l_slv] = j_slv) then flag := 1;
        if (flag = 0) then
          begin
            phi[j_slv] := 0;
            if (j_slv = 26) then
              writeln(outfile,' core saturated at ',run_time/60:0:4,
                    ' minutes');
            num_sat_nds := num_sat_nds + 1;
            sat_nd_array[num_sat_nds] := j_slv;
            if (yc[j_slv] < sat_dist) then
              sat_dist := yc[j_slv];
```

```
        end;
      end;
  end;                                    {procedure alvbd}
{************************************************************************}
procedure fnt_elem;
{************************
this procedure directs the finite element analysis
----------------}
var i_fnt,j_fnt,elem_fnt,flag:   integer;
   kl,klx,
   lam_l:    real;
begin
 for i_fnt := 1 to 200 do
 begin
   fm[i_fnt] := 0.0;
   for j_fnt := 1 to 60 do
   begin
    k[i_fnt,j_fnt] := 0.0;
    c[i_fnt,j_fnt] := 0.0
   end;
 end;
    for elem_fnt := 1 to ne do
    begin
     kxy(elem_fnt,kl,klx);
     lambda(elem_fnt,lam_l,flag);
     glb_mtx(elem_fnt,kl,klx,lam_l);
    end;
                  {calculates the element matricies
                  calculation of the element capacitance (c),
                  stiffness(f), matrices and the element
                  force vector (f)}
     modify;
     matab;
     dcmpbd;
      run_time := run_time + delta;
     multbd;
     for i_fnt := 1 to np do
      rf[i_fnt] := rf[i_fnt]+delta*fm[i_fnt];
     alvbd;
    end;                              {fnt_elem}
{************************************************************************}
procedure flow_calc;
{******************
this procedure determines the volume of water introduced in the last
iteration and performs the regression analysis to determine K_app
------------}
Var
    delta_q_node: d1siz;
    i_flow, flow_cnt, kk:  integer;

    dh,   delta_wc,
    mc_avg,  chg_phi,
    h1,   h2,  head_avg,
    k1,   k_sat,  kapp,
    sxx,  syy,  sxy,
    stat_r,  stat_a,  stat_b,    stat_r2,
    vol2,  vel,
    wc:        real;
begin
   if (iteration = 0) then
   begin
    refill := phi[1];
    vol1 := 0.0;
   for i_flow := 1 to np do
   begin
     wc_calc(i_flow,wcp[i_flow]);
     vol1 := wcp[i_flow]*volel[i_flow] + vol1;
   end;
   writeln(head,'  initial water volume = ',vol1:12:3);
   totflow := 0.0;
   end        {initial set-up}
else
begin
 vol2 := 0.0;
 for kk := 1 to np do
 begin
   wc_calc(kk,wc);
   vol2 := vol2 + wc*volel[kk];
 end;
```

```
delta_wc := vol2 - vol1;
vol1 := vol2;
totflow := totflow + delta_wc;
h1 := phi[1];
dh := delta_wc/ahtube;
for i_flow := 1 to num_fx_nds do
  phi[fx_nd_array[i_flow]] := phi[fx_nd_array[i_flow]] - dh;
h2 := phi[1];
if (dh < 0) then
  writeln(outfile,' warning - dh < 0, change in wc = ',
       delta_wc:0:4,' at ',run_time:0:2,' seconds');
vel := delta_wc/(pi_*core_radius*core_radius*delta);
if (iteration mod iwt = 0) then
  writeln(head,' ',run_time/60:0:4,' ',totflow:0:5,' ',h1:0:2,
         ' ',vel);
  if (iteration mod iwt = 0) then
  begin
   sum_x := sum_x + h1;
   sum_y := sum_y + vel;
   sum_x2 := sum_x2 + (h1*h1);
   sum_y2 := sum_y2 + (vel*vel);
   sum_xy := sum_xy + (h1*vel);
   sum_n := sum_n + 1;
   writeln(wp,phi[1]:8:1,' ',phi[2]:8:1,' ',phi[3]:8:1,' ',phi[4]:8:1,
       ' ',phi[5]:8:1,' ',phi[6]:8:1);
   if (run_time > 290) and (run_time < 310) then
     writeln(graph,' at ',run_time/60:0:3,' minutes the total flow was ',
         totflow:0:1,' cm3');
   if (run_time > 590) and (run_time < 610) then
     writeln(graph,' at ',run_time/60:0:3,' minutes the total flow was ',
         totflow:0:1,' cm3');
  end;
if (iteration mod writ_mult = 0) then
 begin
   writeln(outfile);
   writeln(outfile,'time: ',run_time/60:0:2,' minutes    (itr. ',
          iteration:0,')');
   writeln(outfile,'  head at each node (in centimeters):');
   results;
   writeln(outfile);
   writeln(outfile,'       the total inflow is ',totflow:10:3,' cm3');
 end;
 if (phi[4] > -10) and (run_term = 0) then
  begin
    run_term := run_term + 1;
   writeln(outfile,' node 4 is saturated; at ',run_time/60:0:4,' min');
   writeln(graph,' node 4 is saturated; at ',run_time/60:0:4,' min');
   results;
  end;
 if (phi[1] < 100.0) then
 begin
  for i_flow := 1 to num_fx_nds do
    phi[fx_nd_array[i_flow]] := refill;
  writeln(outfile,' refill (',refill:3:0,' cm) at ',run_time:5:1,
       ' seconds (',run_time/60:7:2,' min)');
  if (phi[4] > -10) then run_term := run_term + 1;

       {statistical analysis - linear regression}
  if (sum_n > 2) then
  begin
  sxx := sum_x2 - ((sum_x*sum_x)/sum_n);
  syy := sum_y2 - ((sum_y*sum_y)/sum_n);
  sxy := sum_xy - ((sum_x*sum_y)/sum_n);
   stat_r := sxy/sqrt(sxx*syy);  {correlation coefficient}
   stat_r2 := stat_r*stat_r;
   stat_b := sxy/sxx;
   stat_a := (sum_y/sum_n) - (stat_b*(sum_x/sum_n));
  writeln(graph);
  writeln(graph,' the head tube refilled at ',run_time/60:0:2,' minutes');
  writeln(graph);
  writeln(graph,' the linear coefficients of regression ');
  writeln(graph,'           for this drop are:');
  writeln(graph);
  writeln(graph,'        a =  ',stat_a:0:6);
  writeln(graph);
  writeln(graph,'        b =  ',stat_b:0:10,' (slope, cm/sec)');
  writeln(graph);
  writeln(graph,'        r2 =  ',stat_r2:0:4,'   or ',
       stat_r2*100:0:1,' %');
```

```
    writeln(graph);
    writeln(graph);
    kapp := stat_b*3600*core_length;
    writeln(graph,'        then Kapparent = ',kapp:0:2,' cm/hr');
    writeln(graph,'                   (mult. by core_length)');
    writeln(graph);
    end;
    sum_x := 0;
    sum_y := 0;
    sum_x2 := 0;
    sum_y2 := 0;
    sum_xy := 0;
    sum_n := 0;
   end;
  end;
 end;                              {procedure flow_calc}
{*****************************************************************************}
{main program}
{************************
data input section of the program
-------------}
begin
 data_name := 'vp.dat';
 reset(data_hp,data_name);
 readln(data_hp,title);
 out_name := 'out.txt';
 gph_name := 'gph.txt';
 hed_name := 'hed.txt';
 wp_name := 'wp.txt';
 rewrite(outfile,out_name);
 writeln(outfile,'Velocity Permeameter Analysis ');
 writeln(outfile);
 writeln(outfile,'data file name: ',title);
 rewrite(graph,gph_name);
 rewrite(head,hed_name);
 rewrite(wp,wp_name);
 writeln(graph,'Regression results: ');
 writeln(head,'time  total_inflow  head    vel');
 writeln(head,'(min)    (cm3)      (cm)  (cm/sec)');
 writeln(wp,' phi[1]  phi[2]  phi[3]  phi[4]  phi[5]');
{input of the initial values}
  for i_main := 1 to 200 do
  begin
   phi[i_main] := 0.0;
   rf[i_main] := 0.0;
   wcp[i_main] := 0.0;
   xc[i_main] := 0.0;
   yc[i_main] := 0.0;
   fx_nd_array[i_main] := 0;
   sat_nd_array[i_main] := 0;
  end;
  num_sat_nds := 0;

  pi_ := 3.14159265358979323856;
  indata;
             {call procedure to read data, cm}
  run_time := 0;
  iteration := 0;
  num_sat_nds := num_fx_nds;
  for i_main := 1 to num_fx_nds do
   sat_nd_array[i_main] := fx_nd_array[i_main];
  for i_main := 1 to 4 do
  begin
   x[i_main] := xc[nel[1,i_main]];
   y[i_main] := yc[nel[1,i_main]];
  end;
  total := 0.0;
  aa := (x[2] - x[1])/2;
  bb := (y[4] - y[1])/2;
  for i_main := 1 to np do
  begin
   r1 := xc[i_main] - aa;
   r2 := xc[i_main] + aa;
   if (xc[i_main] = xc[np]) then r2 := xc[i_main];
```

```
    volel[i_main] := pi_*2*bb*((r2*r2)-(r1*r1));

    if (xc[i_main] = 0) then
      volel[i_main] := 2*bb*pi_*r2*r2;
    if (yc[i_main] = 0) or (yc[i_main] = yc[np]) then
      volel[i_main] := volel[i_main]/2;
    if (xc[i_main] = core_radius) and (abs(yc[i_main]) < core_length) then
       volel[i_main] := volel[i_main]/2;
   end;
   flow_calc;
    writeln(outfile);
    writeln(outfile,' (initial values)');
    writeln(outfile);
    results;
   run_term := 0;
   sum_x := 0;
   sum_y := 0;
   sum_x2 := 0;
   sum_y2 := 0;
   sum_xy := 0;
   sum_n := 0;
{***************************************}
{solution of the time dependent problem}
  repeat
   iteration := iteration + 1;
   if (iteration mod writ_mult = 0) then
   writeln('iteration ',iteration:0);
   if (num_sat_nds > num_fx_nds) then
     sat_elem;
   fnt_elem;
   flow_calc;
  until (iteration = nsteps);
{ until (iteration = nsteps) or (run_term = 4);
} writeln(' run completed.  file: ',title);
  save_name := 'save';
  close(data_hp,save_name);
  close(head,save_name);
  close(graph,save_name);
  close(outfile,save_name);
  close(wp,save_name);
end.
```

Appendix B

Soil Water Pressure Head

Appendix B lists the soil water pressure heads just after the core saturates and shows the isoheads for each set of parameters studied with the axisymmetric rectangular element VP model.

The pressure head in some cases seems to decrease. The effect seemed to be due the numerical problems resulting from the hydraulic conductivity calculations.

data file: control

r: k_r = 1.0 * k_z; element size: 20 mm x 20 mm
the initial soil water pressure head is -20.0 m
theta = 0.5 delta = 0.1 seconds
diameter, head tube = 7.1 mm, and the diameter, core = 80 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
$\Psi - \Theta$ curve - 1 - Zigenfuss soil

Table B1. Pressure head at each node - control parameters

time at saturation: 11.0 minutes
head at each node (in meters):

the total volume inflow is 3.079 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.231	1.231	1.231	-19.990	-19.995	-20.009	-1999.6	-2000.3	-1999.2
-20.0	0.616	0.616	0.616	-19.976	-20.004	-20.003	-1999.4	-2000.3	-1998.3
-40.0	0.308	0.308	0.308	-19.778	-19.897	-19.968	-1999.3	-2000.3	-1998.3
-60.0	0.205	-0.194	-0.429	-0.910	-1.718	-18.676	-1999.3	-2000.3	-1998.3
-80.0	-0.509	-0.566	-0.748	-1.021	-2.103	-18.665	-1999.3	-2000.3	-1998.3
-100.0	-0.848	-0.943	-1.138	-1.596	-4.268	-19.215	-1999.4	-2000.3	-1998.3
-120.0	-1.516	-1.850	-3.225	-9.937	-17.720	-19.870	-1999.4	-2000.3	-1998.3
-140.0	-18.200	-18.587	-19.171	-19.743	-19.974	-19.999	-1999.4	-2000.3	-1998.3
-160.0	-19.993	-20.016	-20.021	-19.980	-20.004	-20.003	-1999.4	-2000.4	-1998.3
-180.0	-19.993	-20.019	-20.023	-19.977	-20.004	-20.003	-1999.4	-2000.3	-1998.3
-200.0	-19.993	-20.011	-20.000	-19.994	-19.994	-20.010	-2000.0	-2000.0	-1999.0

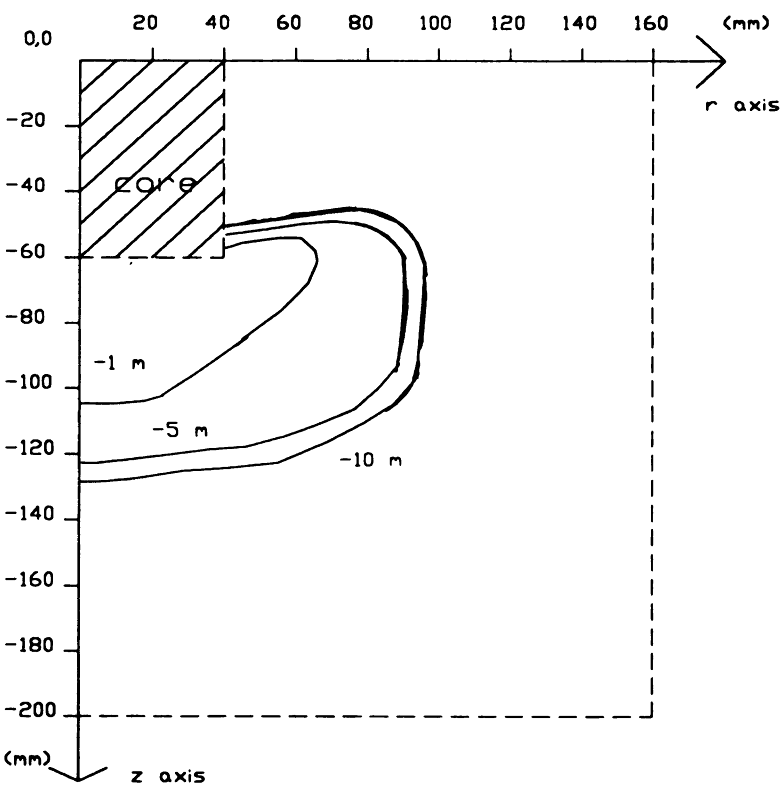

Figure B1. Control isoheads

data file name: IC1

r: k_r = 1.0 * k_z; element size: 20 mm x 20 mm
the initial soil water pressure head is -10.0 m
theta = 0.5 delta = 0.1 seconds
diameter, head tube = 7.1 mm, diameter, core = 80 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
$\Psi - \Theta$ curve - 1 - Zigenfuss soil

Table B2. Pressure head at each node - IC1 parameters

time at saturation: 6.0 minutes
head at each node (in meters):

the total volume inflow is 2.412 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.911	1.911	1.911	-10.000	-10.000	-10.000	-10.000	-10.000	-10.000
-20.0	0.957	0.957	0.957	-9.999	-9.998	-10.002	-9.997	-10.000	-9.996
-40.0	0.478	0.478	0.478	-9.884	-9.942	-9.987	-9.998	-9.990	-9.996
-60.0	0.319	-0.127	-0.406	-0.951	-2.118	-9.807	-9.998	-9.999	-9.996
-80.0	-0.511	-0.578	-0.763	-1.044	-2.469	-9.780	-9.998	-9.999	-9.996
-100.0	-0.899	-0.983	-1.145	-1.630	-4.416	-9.884	-9.998	-9.999	-9.996
-120.0	-1.756	-2.145	-3.469	-7.015	-9.598	-9.983	-9.998	-9.999	-9.996
-140.0	-9.657	-9.732	-9.854	-9.964	-9.995	-10.003	-9.998	-9.999	-9.996
-160.0	-9.999	-9.997	-10.001	-10.000	-9.999	-10.003	-9.998	-9.999	-9.996
-180.0	-9.999	-9.997	-10.001	-9.999	-9.999	-10.003	-9.998	-9.999	-9.996
-200.0	-9.999	-9.996	-9.997	-10.001	-9.998	-10.001	-9.999	-10.003	-9.996

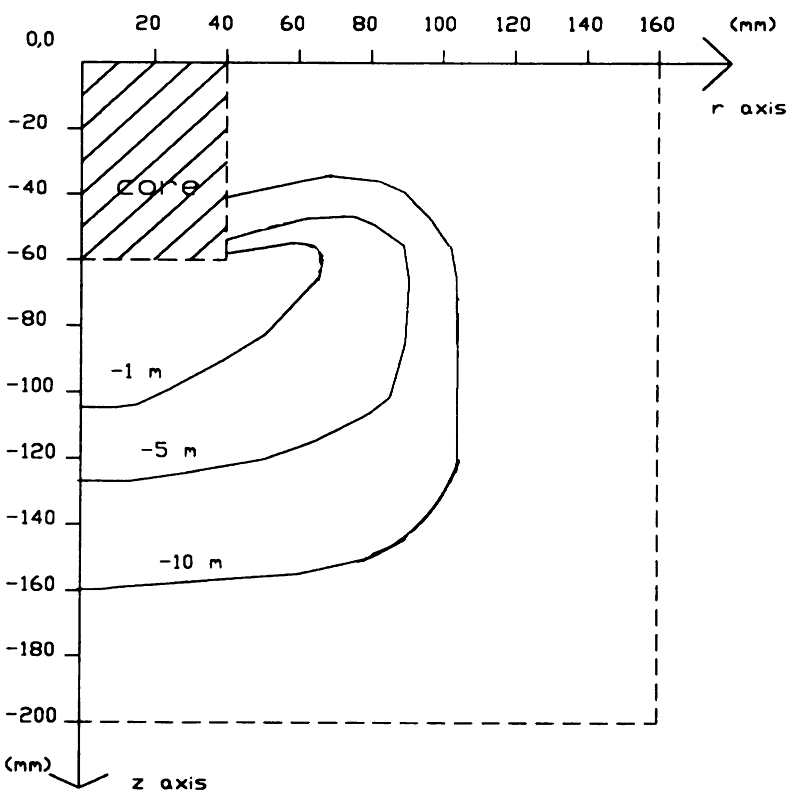

Figure B2. IC1 isoheads

data file: IC2

r: k_r = 1.0 * k_z; element size: 20 mm x 20 mm
the initial soil water pressure head is -50.0 m
theta = 1.0 delta = 0.1 seconds
diameter, head tube = 7.1 mm, diameter, core = 80 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
curve - 1 - Zigenfuss soil

Table B3. Pressure head at each node - IC2 parameters

time at saturation: 25.5 minutes
head at each node (in meters):

the total volume inflow is 3.824 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.350	1.350	1.350	-50.000	-50.000	-50.000	-50.000	-50.000	-50.000
-20.0	0.675	0.675	0.675	-50.000	-49.999	-50.000	-50.077	-50.003	-49.911
-40.0	0.338	0.338	0.338	-48.686	-49.247	-49.757	-50.051	-50.003	-49.911
-60.0	0.225	-0.176	-0.414	-0.895	-1.743	-40.385	-50.035	-50.003	-49.911
-80.0	-0.498	-0.557	-0.739	-0.999	-2.018	-40.821	-50.033	-50.003	-49.911
-100.0	-0.835	-0.926	-1.147	-1.584	-3.293	-43.820	-50.045	-50.003	-49.911
-120.0	-1.556	-1.803	-2.892	-13.100	-35.417	-48.219	-50.065	-50.003	-49.911
-140.0	-39.563	-41.268	-44.236	-47.777	-49.744	-49.962	-50.076	-50.003	-4.9911
-160.0	-49.962	-49.907	-49.907	-50.023	-49.975	-50.015	-50.077	-50.003	-49.911
-180.0	-50.003	-49.924	-49.909	-50.034	-49.974	-50.013	-50.077	-50.004	-49.912
-200.0	-50.001	-49.939	-49.988	-50.077	-50.000	-50.022	-50.056	-50.013	-50.000

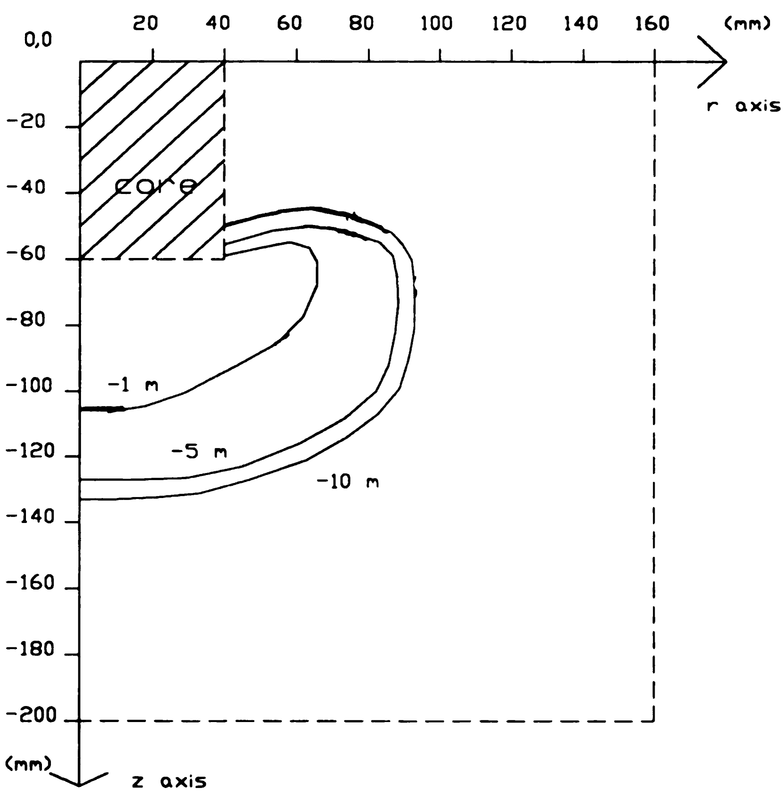

Figure B3. IC2 isoheads

data file: IC3

r: k_r = 1.0 * k_z; element size: 20 mm x 20 mm
the initial soil water pressure head is -100.0 m
theta = 1.0 delta = 0.1 seconds
diameter, head tube = 7.1 mm, diameter, core = 80 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
$\Psi - \Theta$ curve - 1 - Zigenfuss soil

Table B4. Pressure head at each node - IC3 parameters

time at saturation: 44.0 minutes
head at each node (in meters):

the total volume inflow is 4.102 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.642	1.642	1.642	-100.000	-100.000	-100.000	-100.000	-100.000	-100.000
-20.0	0.821	0.821	0.821	-99.780	-99.915	-100.079	-100.014	-99.994	-99.999
-40.0	0.411	0.411	0.411	-94.661	-97.034	-99.021	-99.963	-99.971	-99.992
-60.0	0.274	-0.147	-0.405	-0.927	-2.087	-71.457	-99.870	-99.965	-99.985
-80.0	-0.497	-0.560	-0.754	-1.030	-2.431	-73.799	-99.866	-99.963	-99.983
-100.0	-0.868	-0.963	-1.197	-1.745	-3.942	-82.370	-99.942	-99.961	-99.981
-120.0	-1.871	-2.180	-3.499	-18.974	-58.552	-94.358	-100.014	-99.961	-99.980
-140.0	-74.028	-78.049	-84.800	-93.676	-98.829	-99.901	-100.077	-99.963	-99.981
-160.0	-99.821	-99.944	-99.610	-99.801	-99.990	-100.216	-100.046	-99.967	-99.982
-180.0	-99.995	-100.050	-99.625	-99.817	-100.000	-100.197	-100.045	-99.972	-99.985
-200.0	-99.999	-100.006	-99.849	-99.943	-100.129	-100.000	-99.923	-99.992	-100.045

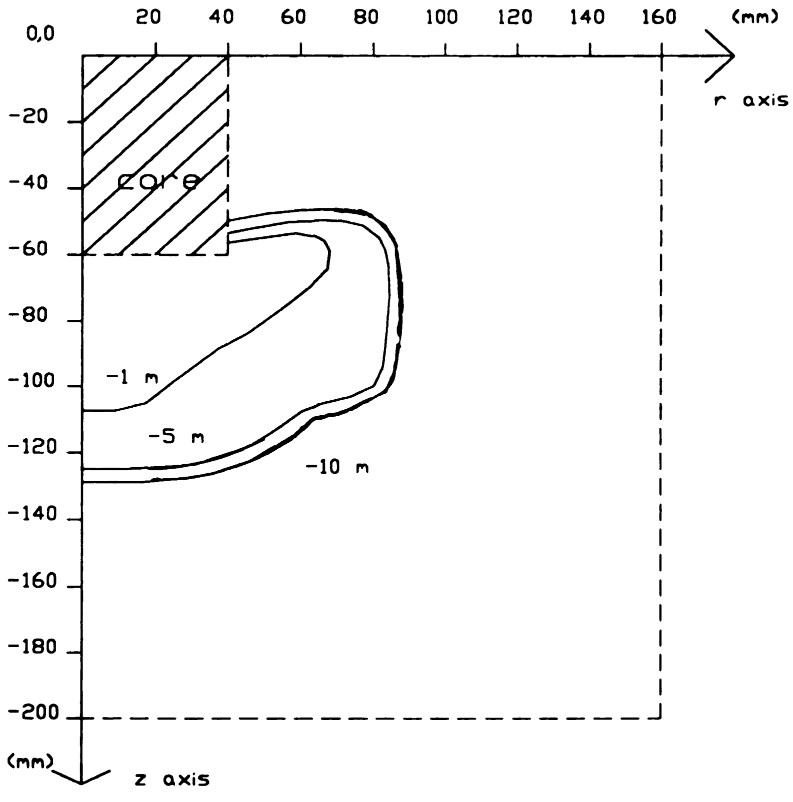

Figure B4. IC3 isoheads

data file: K1

r: k_r = 1.0 * k_z; element size: 20 mm x 20 mm
the initial soil water pressure head is -20.0 m
theta = 0.5 delta = 0.01 seconds
diameter, head tube = 7.1 mm, diameter, core = 80 mm
k curve - 2 - Indio loam (Ksat = 200 mm/hr)
$\Psi - \Theta$ curve - 1 - Zigenfuss soil

Table B5. Pressure head at each node - K1 parameters

time at saturation: 1.5 minutes
head at each node (in meters):

the total volume inflow is 4.122 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.599	1.599	1.599	-20.000	-20.000	-20.000	-20.000	-20.000	-20.012
-20.0	0.800	0.800	0.800	-20.000	-19.971	-20.018	-19.994	-19.995	-20.002
-40.0	0.400	0.400	0.400	-19.865	-19.890	-19.974	-19.988	-19.992	-20.003
-60.0	0.267	-0.154	-0.429	-0.916	-1.350	-6.183	-19.881	-19.992	-20.003
-80.0	-0.498	-0.555	-0.747	-0.995	-1.429	-6.867	-19.840	-19.992	-20.003
-100.0	-0.822	-0.892	-1.034	-1.285	-1.706	-10.378	-19.893	-19.992	-20.003
-120.0	-1.229	-1.293	-1.444	-1.88.1	-6.169	-14.216	-19.966	-19.992	-20.003
-140.0	-3.24.4	-4.083	-6.654	-11.713	-16.429	-19.885	-19.992	-19.992	-20.003
-160.0	-19.446	-19.619	-19.786	-19.920	-19.967	-20.015	-19.995	-19.992	-20.003
-180.0	-20.001	-20.007	-19.986	-20.005	-19.973	-20.019	-19.995	-19.992	-20.003
-200.0	-20.000	-19.999	-19.996	-20.002	-20.007	-20.005	-19.993	-19.997	-20.015

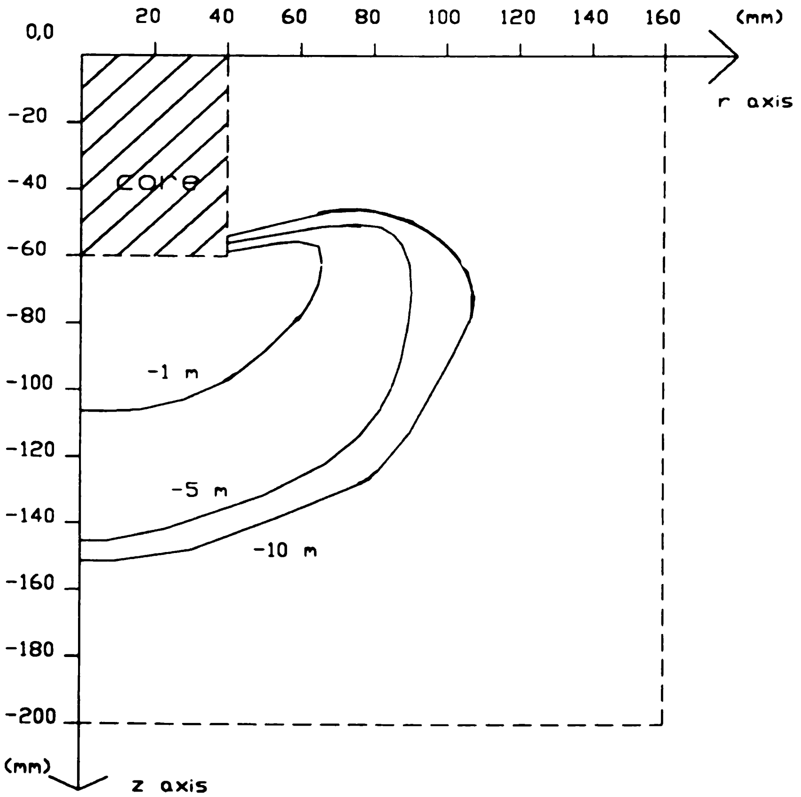

Figure B5. K1 isoheads

data file: K2

r: k_r = 1.0 * k_z; element size: 20 mm x 20 mm
the initial soil water pressure head is -20.0 m
theta = 0.5 delta = 1.0 seconds
diameter, head tube = 7.1 mm, diameter, core = 80 mm
k curve - 3 - Chino clay (Ksat = 2 mm/hr)
$\Psi - \Theta$ curve - 1 - Zigenfuss soil

Table B6. Pressure head at each node - K2 parameters

time at saturation: 79.9 minutes
head at each node (in meters):

the total volume inflow is 2.388 E5 mm^3

z\r	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.984	1.984	1.984	-19.996	-19.998	-20.005	-20.001	-19.998	-20.001
-20.0	0.993	0.993	0.993	-19.924	-19.967	-19.995	-19.999	-19.996	-20.004
-40.0	0.497	0.497	0.497	-18.309	-19.161	-19.756	-19.989	-19.996	-20.005
-60.0	0.0	-0.141	-0.391	-1.359	-5.316	-18.634	-19.972	-19.996	-20.005
-80.0	-0.546	-0.659	-0.986	-1.594	-6.052	-18.207	-19.965	-19.996	-20.005
-100.0	-1.338	-1.545	-2.147	-4.143	-8.031	-18.844	-19.972	-19.996	-20.005
-120.0	-5.523	-6.438	-8.604	-12.232	-18.050	-19.496	-19.987	-19.996	-20.005
-140.0	-18.462	-18.648	-19.069	-19.556	-19.865	-19.984	-19.997	-19.996	-20.005
-160.0	-19.970	-19.969	-19.992	-19.994	-20.005	-20.009	-20.002	-19.996	-20.005
-180.0	-19.999	-19.992	-20.010	-20.003	-20.009	-20.011	-20.002	-19.996	-20.005
-200.0	-19.999	-19.994	-19.996	-20.003	-19.994	-20.011	-20.002	-19.995	-20.004

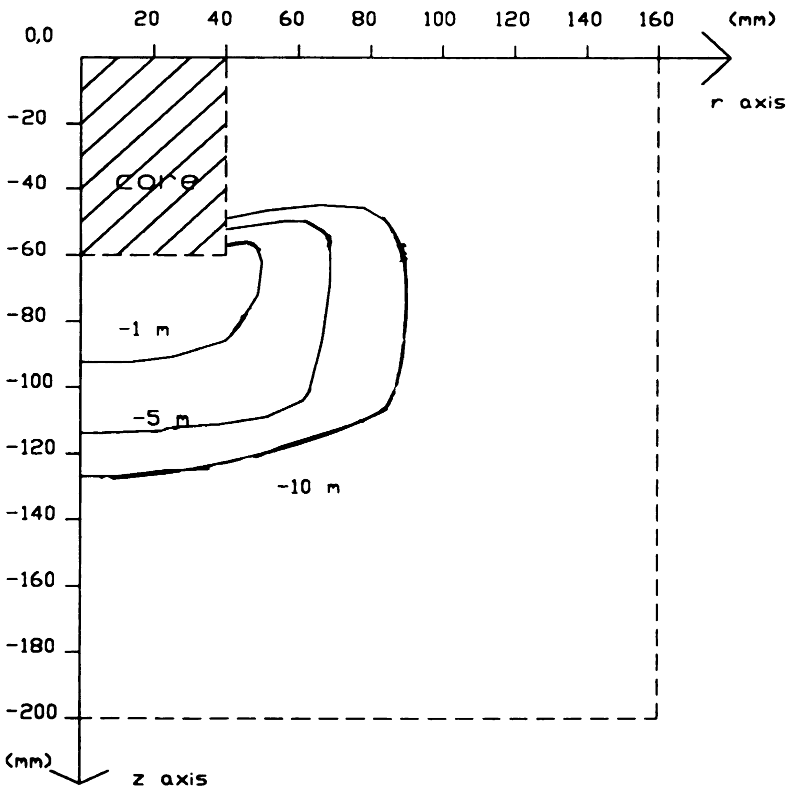

Figure B6. K2 isoheads

data file: WC1

r: k_r = 1.0 * k_z; element size: 20 x 20 mm
the initial soil water pressure head is -20.0 m
theta = 0.5 delta = 0.1 seconds
diameter, head tube = 7.1 mm, diameter, core = 80 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
Ψ - Θ curve - 2 - Capac soil

Table B7. Pressure head at each node - WC1 parameters

time at saturation: 4.5 minutes
head at each node (in meters):

the total volume inflow is 2.526 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.625	1.625	1.625	-19.994	-19.999	-20.002	-19.996	-20.006	-19.998
-20.0	0.814	0.814	0.814	-19.996	-20.001	-20.001	-19.992	-20.005	-20.004
-40.0	0.407	0.407	0.407	-19.93	-19.966	-19.988	-19.992	-20.005	-20.004
-60.0	0.271	0.271	-0.295	-0.769	-1.351	-17.761	-19.993	-20.005	-20.004
-80.0	-0.286	-0.412	-0.611	-0.861	-1.479	-17.625	-19.994	-20.005	-20.004
-100.0	-0.692	-0.762	-0.960	-1.256	-2.335	-18.935	-19.994	-20.005	-20.004
-120.0	-1.231	-1.317	-1.693	-4.552	-16.613	-19.813	-19.993	-20.005	-20.004
-140.0	-13.380	-15.113	-17.658	-19.360	-19.973	-19.999	-19.992	-20.005	-20.004
-160.0	-19.995	-20.002	-20.005	-19.996	-20.001	-20..001	-19.992	-20.005	-20.004
-180.0	-19.998	-20.005	-20.007	-19.996	-20.001	-20.001	-19.992	-20.005	-20.004
-200.0	-19.998	-20.003	-20.000	-19.992	-19.999	-20.003	-19.994	-20.005	-19.998

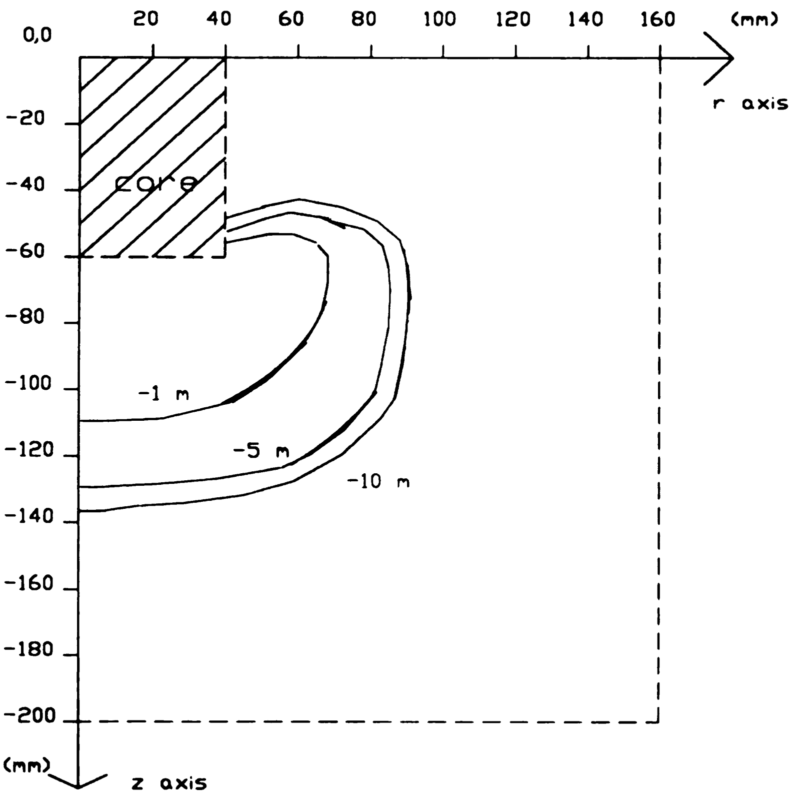

Figure B7. WC1 isoheads

<u>**data file: WC2**</u>

r: k_r = 1.0 * k_z; element size: 20 x 20 mm
the initial soil water pressure head is -20.0 m
theta = 0.5 delta = 0.05 seconds
diameter, head tube = 7.1 mm, diameter, core = 80 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
$\Psi - \Theta$ curve - 3 - Lenawee soil

Table B8. Pressure head at each node - WC2 parameters

time at saturation: 4.0 minutes
head at each node (in meters):
the total volume inflow is 2.193 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.463	1.463	1.463	-20.000	-20.000	-20.000	-19.991	-19.993	-19.997
-20.0	0.732	0.732	0.732	-19.989	-20.001	-20.001	-19.984	-19.985	-19.992
-40.0	0.366	0.366	0.366	-19.934	-19.971	-19.991	-19.987	-19.987	-19.991
-60.0	0.244	-0.165	-0.415	-0.918	-1.813	-19.426	-19.986	-19.986	-19.991
-80.0	-0.492	-0.553	-0.746	-1.040	-2.264	-19.364	-19.987	-19.986	-19.991
-100.0	-0.823	-0.922	-1.134	-1.636	-5.534	-19.697	-19.987	-19.986	-19.991
-120.0	-1.418	-1.669	-2.948	-10.432	-18.443	-19.971	-19.986	-19.986	-19.991
-140.0	-18.118	-18.677	-19.414	-19.881	-19.999	-19.999	-19.986	-19.986	-19.991
-160.0	-19.998	-19.998	-19.987	-19.991	-20.006	-20.001	-19.985	-19.986	-19.991
-180.0	-19.999	-19.998	-19.986	-19.990	-20.006	-20.001	-19.985	-19.987	-19.991
-200.0	-19.999	-19.999	-19.996	-19.998	-19.999	-19.999	-19.994	-19.990	-20.001

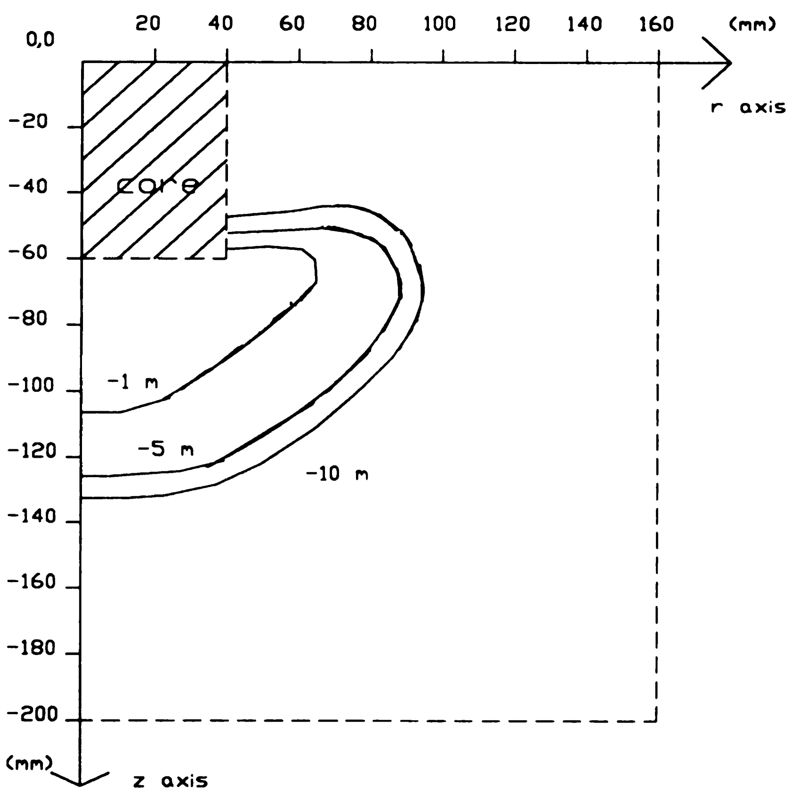

Figure B8. WC2 isoheads

<u>data file: Kr1</u>

Note: This file did not saturate so water potentials are shown at 10 minutes. The file was run for 60 minutes, by which time the radial extents had been reached so that data would be incorrect.

r: k_r = 10.0 * k_z; element size: 20 x 20 mm
the initial soil water pressure head is -20.0 m
theta = 0.5 delta = 0.1 seconds
diameter, head tube = 7.1 mm, diameter, core = 80 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
Ψ – Θ curve - 1 - Zigenfuss soil

Table B9. Pressure head at each node - Kr1 parameters

time at saturation: 10.0 minutes
head at each node (in meters):
the total volume inflow is 3.811 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.382	1.382	1.382	-20.000	-20.000	-20.000	-19.996	-19.998	-20.000
-20.0	0.692	0.692	0.692	-20.000	-19.999	-20.000	-20.001	-19.997	-19.991
-40.0	0.346	0.346	0.346	-19.829	-19.849	-19.933	-19.926	-19.906	-19.980
-60.0	-0.509	-0.537	-0.613	-0.822	-1.026	-1.346	-3.429	-17.805	-19.948
-80.0	-1.086	-1.106	-1.148	-1.174	-1.374	-1.804	-5.373	-17.860	-19.940
-100.0	-4.528	-4.727	-5.226	-6.196	-7.533	-12.096	-14.930	-19.359	-19.970
-120.0	-19.690	-19.709	-19.740	-19.765	-19.889	-19.850	-19.956	-19.964	-19.990
-140.0	-19.998	-20.004	-19.991	-20.004	-19.993	-20.002	-20.002	-19.995	-19.992
-160.0	-19.997	-20.000	-19.989	-20.005	-19.993	-20.002	-20.000	-19.994	-19.990
-180.0	-19.997	-20.000	-19.989	-20.005	-19.993	-20.002	-20.001	-19.994	-19.990
-200.0	-19.993	-19.985	-19.998	-20.006	-19.989	-20.001	-19.995	-19.997	-20.001

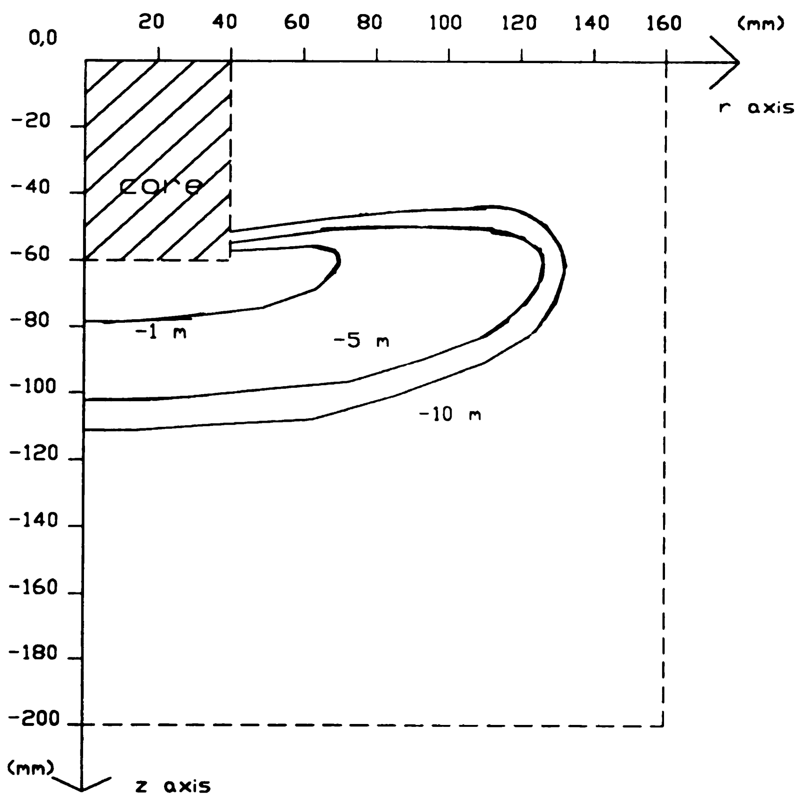

Figure B9. Kr1 isoheads

data file: Kr2

r: k_r = 0.1 * k_z; element size: 20 x 20 mm
the initial soil water pressure head is -20.0 m
theta = 0.5 delta = 0.1 seconds
diameter, head tube = 7.1 cm, diameter, core = 80 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
Ψ - Θ curve - 1 - Zigenfuss soil

Table B10. Pressure head at each node - Kr2 parameters

time at saturation: 4.0 minutes
head at each node (in meters):

the total volume inflow is 1.392 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.490	1.490	1.490	-20.000	-20.000	-20.000	-20.000	-20.000	-20.005
-20.0	0.746	0.746	0.746	-19.995	-20.007	-19.997	-20.001	-19.999	-20.002
-40.0	0.373	0.373	0.373	-19.979	-20.006	-19.997	-20.002	-19.998	-20.002
-60.0	0.249	0.249	-0.841	-15.097	-20.005	-19.997	-20.002	-19.999	-20.002
-80.0	-0.303	-0.427	-1.497	-15.147	-20.005	-19.997	-20.002	-19.999	-20.002
-100.0	-0.978	-1.051	-2.336	-19.734	-20.002	-19.997	-20.002	-19.999	-20.002
-120.0	-8.331	-9.373	-13.333	-19.902	-20.006	-19.997	-20.002	-19.999	-20.002
-140.0	-19.981	-19.987	-19.989	-19.994	-20.007	-19.997	-20.002	-19.999	-20.002
-160.0	-20.001	-20.004	-19.997	-19.995	-20.007	-19.997	-20.002	-19.999	-20.002
-180.0	-20.001	-20.004	-19.997	-19.995	-20.007	-19.997	-20.002	-19.999	-20.002
-200.0	-20.004	-20.000	-19.999	-19.998	-19.998	-20.000	-19.998	-19.999	-20.005

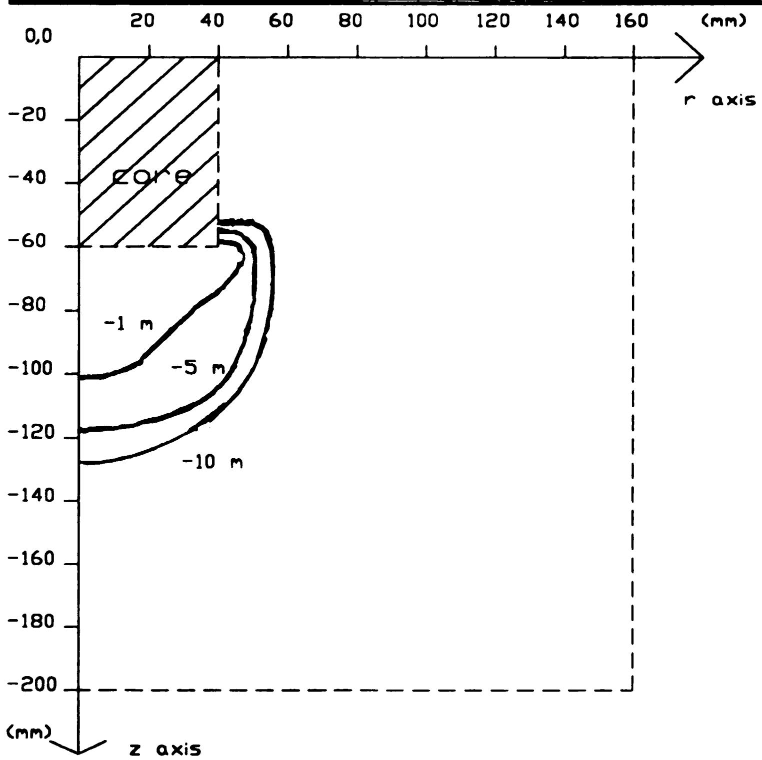

Figure B10. Kr2 isoheads

data file: CD1

r: k_r = 1.0 * k_z; element size: 20 x 20 mm
the initial soil water pressure head is -20.0 m
theta = 0.5 delta = 0.1 seconds
diameter, head tube = 7.1 cm, diameter, core = 40 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
Ψ – Θ curve - 1 - Zigenfuss soil

Table B11. Pressure head at each node - CD1 parameters

time at saturation: 45.0 minutes
head at each node (in meters):

the total volume inflow is 3.105 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.159	1.159	-20.000	-19.976	-19.989	-20.017	-19.993	-20.006	-19972
-20.0	0.580	0.580	-20.061	-19.902	-20.010	-20.005	-19.987	-20.006	-19936
-40.0	0.290	0.290	-18.953	-19.117	-19.558	-19.832	-19.969	-20.010	-19935
-60.0	-0.286	-0.467	-0.809	-1.106	-1.997	-15.111	-19.948	-20.010	-19935
-80.0	-0.654	-0.725	-0.855	-1.175	-2.231	-14.939	-19.936	-20.010	-19935
-100.0	-0.876	-0.932	-1.081	-1.379	-3.387	-16.351	-19.951	-20.009	-19935
-120.0	-1.241	-1.319	-1.517	-2.362	-8.093	-18.711	-19.969	-20.010	-19935
-140.0	-3.441	-4.382	-7.137	-11.545	-16.943	-19.736	-19.979	-20.010	-19935
-160.0	-18.400	-18.855	-19.335	-19.636	-19.913	-19.995	-19.979	-20.010	-19935
-180.0	-19.981	-20.062	-20.078	-19.930	-20.019	-20.012	-19.980	-20.010	-19935
-200.0	-19.981	-20.040	-19.996	-19.981	-19.983	-20.036	-20.000	-20.000	-19962

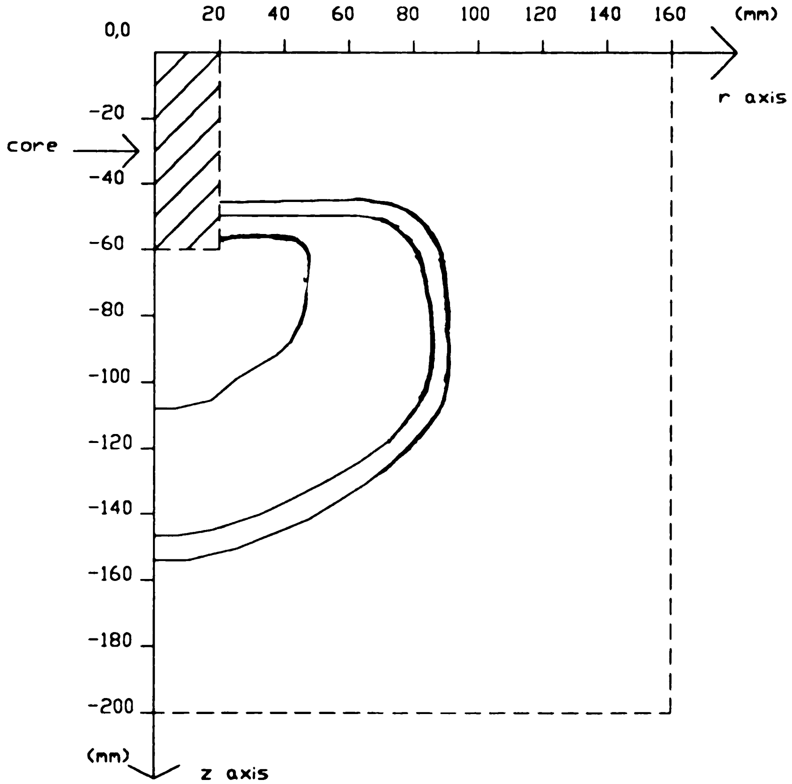

Figure B11. CD1 isoheads

data file: CD2

r: k_r = 1.0 * k_z; element size: 20 x 20 mm
the initial soil water pressure head is -20.0 m
theta = 0.5 delta = 0.1 seconds
diameter, head tube = 7.1 mm, diameter, core = 120 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
$\Psi - \Theta$ curve - 1 - Zigenfuss soil

Table B12. Pressure head at each node - CD2 parameters

time at saturation: 6.0 minutes
head at each node (in meters):

the total volume inflow is 3.955 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	8.0	10.0	12.0	14.0	16.0
0.0	1.449	1.449	1.449	1.449	-2000.0	-2000.0	-2000.0	-2000.0	-1999.6
-20.0	0.725	0.725	0.725	0.725	-2000.0	-2000.0	-2000.0	-2000.0	-1999.1
-40.0	0.363	0.363	0.363	0.363	-1991.8	-1996.4	-1999.1	-2000.2	-1999.1
-60.0	0.242	0.242	-0.168	-0.465	-115.2	-985.0	-1997.4	-2000.2	-1999.1
-80.0	-0.362	-0.453	-0.599	-0.880	-127.7	-1175.3	-1996.7	-2000.2	-1999.1
-100.0	-0.967	-1.029	-1.149	-1.405	-407.7	-1562.7	-1998.5	-2000.2	-1999.1
-120.0	-4.793	-5.878	-9.082	-14.917	-1876.8	-1992.1	-1999.5	-2000.2	-1999.1
-140.0	-19.906	-19.933	-19.964	-19.975	-1999.8	-2000.0	-1999.7	-2000.2	-1999.1
-160.0	-19.996	-20.011	-20.013	-19.987	-2000.2	-2000.2	-1999.7	-2000.2	-1999.1
-180.0	-19.996	-20.011	-20.013	-19.987	-2000.2	-2000.2	-1999.7	-2000.2	-1999.1
-200.0	-19.996	-20.006	-20.000	-19.997	-1999.7	-2000.6	-2000.0	-2000.0	-1999.5

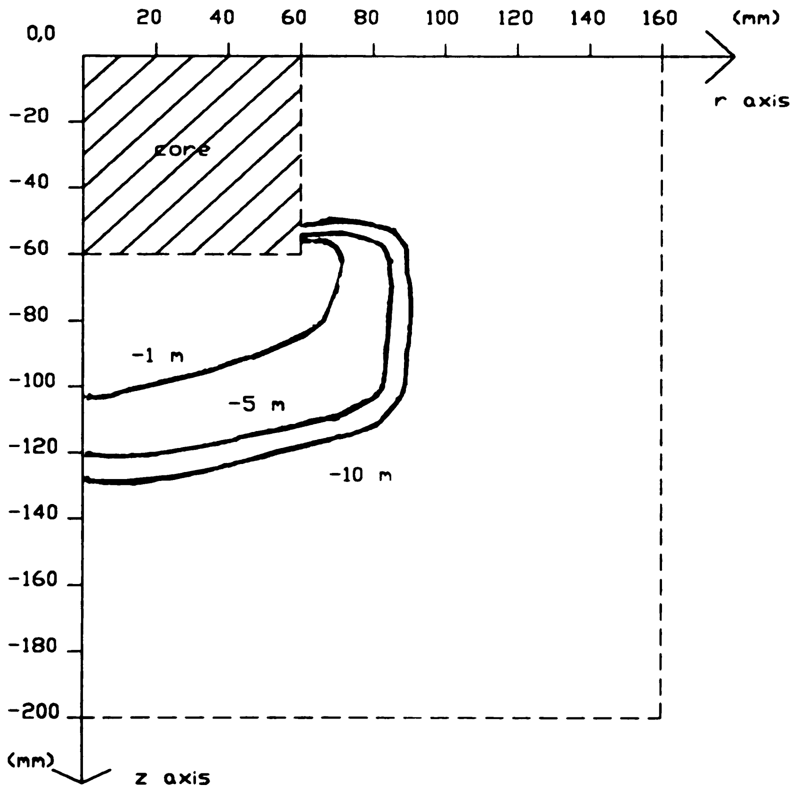

Figure B12. CD2 isoheads

data file: HTD1

r: k_r = 1.0 * k_z; element size: 20 x 20 mm
the initial soil water pressure head is -20.0 m
theta = 0.5 delta = 0.1 seconds
diameter, head tube = 12.7 mm, diameter, core = 80 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
$\Psi - \Theta$ curve - 1 - Zigenfuss soil

Table B13. Pressure head at each node - HTD1 parameters

time at saturation: 10.0 minutes
head at each node (in meters):

the total volume inflow is 2.685 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.881	1.881	1.881	-19.991	-19.996	-20.008	-19.997	-20.003	-19.993
-20.0	0.941	0.941	0.941	-19.978	-20.004	-20.002	-19.994	-20.003	-19.985
-40.0	0.470	0.470	0.470	-19.813	-19.920	-19.980	-19.994	-20.003	-19.985
-60.0	0.314	-0.135	-0.418	-1.012	-2.992	-19.633	-19.995	-20.003	-19.985
-80.0	-0.517	-0.579	-0.790	-1.117	-3.850	-19.615	-19.995	-20.003	-19.985
-100.0	-0.969	-1.058	-1.237	-1.963	-8.064	-19.817	-19.995	-20.003	-19.985
-120.0	-2.711	-3.667	-7.280	-14.484	-18.732	-19.957	-19.995	-20.003	-19.985
-140.0	-19.477	-19.644	-19.838	-19.940	-19.994	-20.000	-19.994	-20.003	-19.985
-160.0	-19.994	-20.017	-20.022	-19.979	-20.004	-20.003	-19.994	-20.003	-19.985
-180.0	-19.994	-20.018	-20.021	-19.979	-20.004	-20.003	-19.995	-20.003	-19.985
-200.0	-19.994	-20.010	-20.000	-19.994	-19.995	-20.010	-20.000	-20.000	-19.991

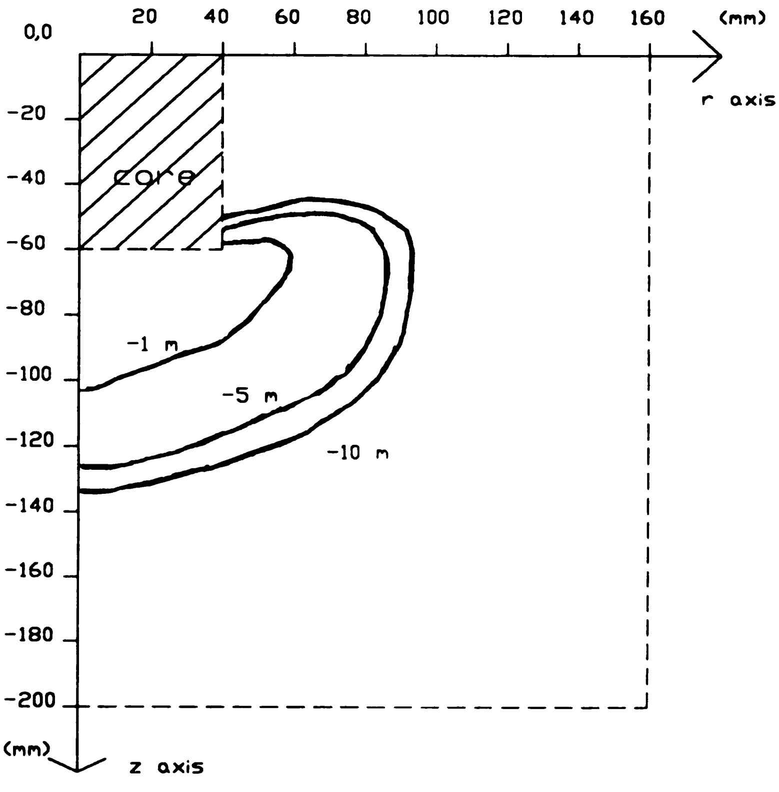

Figure B13. HTD1 isoheads

data file: HTD2

r: k_r = 1.0 * k_z; element size: 20 x 20 mm
the initial soil water pressure head is -20.0 m
theta = 0.5 delta = 0.05 seconds
diameter, head tube = 3.2 mm, diameter, core = 80 mm
k curve - 1 - Pachappa sandy loam (Ksat = 20 mm/hr)
Ψ - Θ curve - 1 - Zigenfuss soil

Table B14. Pressure head at each node - HTD2 parameters

time at saturation: 11.6 minutes
head at each node (in meters):

the total volume inflow is 3.175 E5 mm^3

z\r (mm)	0.0	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0
0.0	1.814	1.814	1.814	-19.977	-19.988	-20.006	-19.989	-20.021	-19.991
-20.0	0.909	0.909	0.909	-19.986	-20.022	-20.003	-19.972	-20.019	-20.016
-40.0	0.455	0.455	0.455	-19.775	-19.891	-19.964	-19.971	-20.022	-20.016
-60.0	0.0	-0.160	-0.404	-0.894	-1.598	-18.327	-19.976	-20.021	-20.016
-80.0	-0.516	-0.587	-0.744	-1.001	-1.922	-18.328	-19.981	-20.021	-20.016
-100.0	-0.847	-0.933	-1.128	-1.536	-3.621	-18.987	-19.981	-20.021	-20.016
-120.0	-1.449	-1.710	-2.808	-8.981	-17.382	-19.796	-19.977	-20.022	-20.016
-140.0	-17.711	-18.176	-18.880	-19.628	-19.983	-19.996	-19.972	-20.021	-20.016
-160.0	-19.993	-19.993	-19.952	-19.987	-20.021	-20.004	-19.971	-20.022	-20.016
-180.0	-19.996	-19.994	-19.949	-19.987	-20.020	-20.004	-19.970	-20.022	-20.016
-200.0	-19.995	-19.997	-19.985	-19.974	-19.999	-19.996	-19.978	-20.018	-19.990

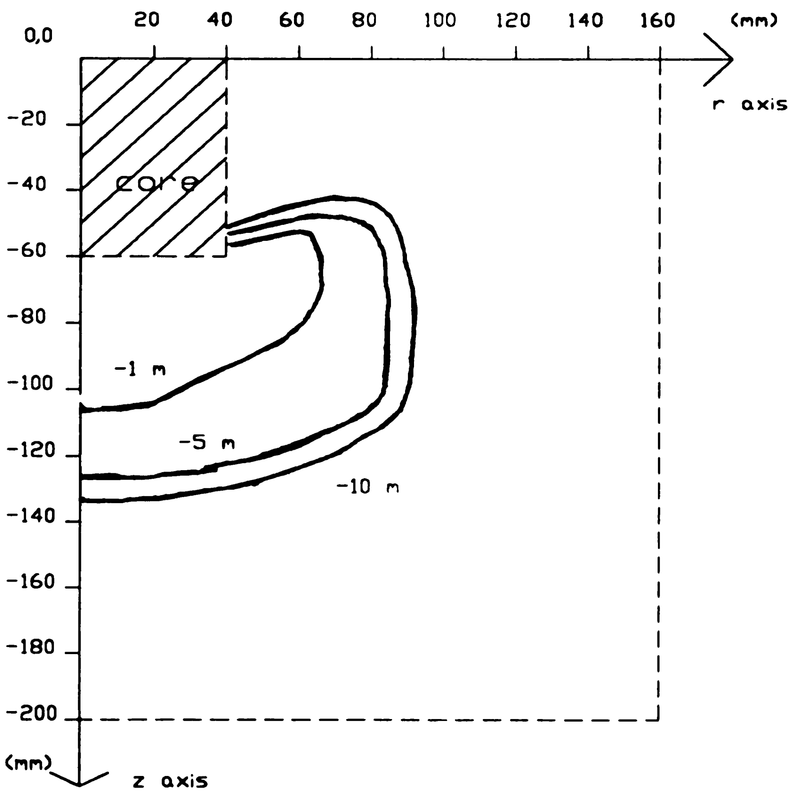

Figure B14. HTD2 isoheads

Appendix C

One - Dimensional VP Computer Program.

The one-dimensional VP program is given in this appendix. It was written in Pascal and run on a Unix system.

```
{***************************************************************************}
PROGRAM VPT(DATA,OUTFILE);
   INT_1D,
   Unsat_1D,
   FEM_1D,
   Flow_1D,
   Printer,
   Crt;
TYPE  D1Siz = ARRAY[1..150] of Real;
   SmSiz = ARRAY[1..5] of Real;
   D3Siz = ARRAY[1..50] of Integer;
   NlSiz = ARRAY[1..150,1..4] of Integer;
   NmSiz = ARRAY[1..150] of Integer;
   NsSiz = ARRAY[1..2] of Integer;
   D2Siz = ARRAY[1..150,1..27] of Real;
VAR  DATA:      TEXT;
   OUTFILE:   TEXT;
   DATSAV:    TEXT;
   GRAPH:     TEXT;
   CURVE:     TEXT;
   SURF:      TEXT;
   HEAD:      TEXT;
   Ques:      CHAR;
   NAME:      STRING[20];
   TITLE:     STRING[40];
{DEFINITION OF THE INPUT PARAMETERS
      TITLE - A DESCRIPTIVE STATEMENT OF THE PROBLEM BEING SOLVED
      NP - NUMBER OF EQUATIONS (ALSO NUMBER OF NODES)
      NE - NUMBER OF ELEMENTS
      NCOEF - NUMBER OF SETS OF EQUATION COEFFICIENTS MAXMUM OF FIVE
      IPTL - INTEGER CONTROLLING THE OUTPUT (4 - DEBUG OPTION)
      KW - FLAG WHICH ALLOWS PRINTOUT OF DATA AT VARIOUS POINTS
         IN THE PROGRAM
         (KW = 0  OMIT WRITE, KW = 1  WRITE)
      ahtube - area of the head tube
      DHTUBE - DIAMETER OF THE HEAD TUBE
      THE NUMBER OF the following SETS of parameters MUST EQUAL NCOEF
        DT[I] - MATERIAL PROPERTY THAT MULTIPLIES THE TIME DERIVATIVE
             (I = 1-4)
        Q[I]  - CONSTANT COEFFICIENT IN THE DIFF. EQ. NODAL COORDINATE
             VALUES (I = 1-4)
      XC[I] - X COORDINATES OF THE NODES
      YC[I] - Y COORDINATES OF THE NODES
      THE COORDINATES MUST BE IN NUMERICAL SEQUENCE RELATIVE TO NODE NUMBERS
      ELEMENT DATA
        N - ELEMENT NUMBER
        NMTL - INTEGER SPECIFYING THE EQUATION COEFFICIENT SET
        NEL[N,1] - NUMERICAL VALUE OF NODE I
        NEL[N,2] - NUMERICAL VALUE OF NODE J
      DEFINITION OF THE VARIABLES READ BY INTVL2
      INVL - INTEGER CONTROLLING THE INPUT OF THE INITIAL VALUES
      1 - INPUT A NODE AT A TIME
      2 - INPUT BY GROUPS
      IB[I] - NODE NUMBERS WHICH DO NOT CHANGE WITH
      TIME - TERMINATE A ZERO
      THETA - THE THETA VALUE USED IN THE SINGLE STEP METHOD
        0   - EULER'S FORWARD DIFFERENCE METHOD
        1/2 - CENTRAL DIFFERENCE METHOD
        2/3 - GALERKIN'S METHOD
        1   - BACKWARD DIFFERENCE METHOD
      DELTA - THE TIME STEP
      ITYPE - INTEGER CONTROLLING THE TYPE OF ANALYTICAL SOLUTION (1 FOR
           THE VELOCITY
           PERMEAMETER PROBLEM
      NSTEPS - NUMBER OF TIME STEPS
```

```
      IWT - INTEGER CONTROLLING THE OUTPUT OF THE CALCULATED VALUES.
          VALUES ARE PRINTED
          EVERY IWT
      NDBC - NUMBER OF ELEMENT SIDES WITH A
           DERIVATIVE BOUNDARY CONDITION
      R -  THE RATIO DYE/DXE, I.E., CONDUCTIVITY IN THE Y
          DIRECTION IS R*CONDUCTIVITY IN THE X DIRECTION
          NEL[N,3] - NUMERICAL VALUE OF NODE K
          NEL[N,4] - NUMERICAL VALUE OF NODE M
          NEL[N,4] IS SET EQUAL TO ZERO FOR
          THE TRIANGULAR ELEMENT
          num_x_core - number of elements in the core
                   in the x direction
          num_y_core - number of elements in the core
                   in the y direction
          num_x_elem - number of elemnts in the x direction
          num_y_elem - number of elemnts in the y direction
  Units were used to separate procedures into compatible groups and to make reading and checking easier.
  The main (controlling) program is VPT.  The units are:
    MyGlobal - lists the global variables and defines all variables
          (other than those used only in one procedure)
    FEM - finite element method, this unit includes:
        Glb_mtx, (creates the global stiffness and capacitance matrices)
        Modify,  (modifies the global matrices for specified nodal
             values)
        Matab,  (creates the A and P matrices - for Gaussian
             elimination)
        Dcmpbd,   (decomposes the  A and P matricies, Gaussian method)
        Multbd,     (more Gaussian method)
        Slvbd.      (solution by backwards substitution)
        Interact - read and write procedures:
          Indata,     (reads original input data from a file)
          Results,    (writes to a file)
          Redata.     (reads the data written by "create")
        Unsat_co - calculates soil/water parameters
        Interp,      (calculates soil moisture, based on potential)
        WPWC,        (calculates lambda, the change in WC/change in WP)
        Kxy.        (calculates K, hydraulic conductivity)
    Flow - calculates the flow into the soil
       Flowcalc     (the only procedure)
}
VAR  Phi,   Rf,
   WCP,   YC:               D1Siz;
   DT,    Q,  y:            SmSiz;
   NMTL, fx_nd_array:           NmSiz;
   sat_nd_array:            NmSiz;
   NEL:                   NlSiz;
   C,K:                   D2Siz;
   ivpt, jvpt,
   IPTL, ITERATION,
   IWT,  prnt,  kw,  i,kk,j,
   NBW,  NCOEF,
   NE,  NSTEPS, Kapp_num,
   NP,   num_fx_nds,
   num_sat_nds, ne_new, nodes:            INTEGER;
   Ahtube,  core_radi,
   dye1,     Delta,
   Dhtube,  dh,
   head1,   length,
   Kn,   refill,  lam1,  lam2,
   LAMBDA,
   r,
   voll,
   velocity1,
   THTA,  TIME,  Totflow:       REAL;
{**********************************************************************}
PROCEDURE INTVL2(np,iptl,kw:integer;var phi:D1siz);
{*****************************************************
THIS SUBROUTINE EITHER READS THE INITIAL VALUES OR
CALCULATES THE VALUES USING A PROGRAMMED EQUATION.
THE OPTION IS SPECIFIED BY THE INTEGER INVL WHICH
IS READ BY THE SUBROUTINE.
-------------------------
 DEFININITION OF THE VARIABLES READ BY INTVL2
     INVL - INTEGER CONTROLLING THE INPUT OF THE
          INITIAL VALUES
     MPL - THE SPECIFIED VALUE FOR OPTIONS 3, 4, AND 5
        1 - INPUT ONE NODE AT A TIME
          !NOTE - OPTION CURRENTLY UNAVAILABLE!
```

```
        2 - INPUT BY GROUPS
            IBEG - THE FIRST NODE IN THE GROUP
            IEND - THE LAST NODE IN THE GROUP
            VALUE - THE VALUE ASSIGN TO THE NODES
          REPEAT UNTIL ALL NODES ARE ENTERED
        3 - ZERO AT THE END POINTS AND A SPECIFIED
            VALUE AT ALL THE OTHER POINTS
            !NOTE - OPTION CURRENTLY UNAVAILABLE!}
{------------------}
 VAR
    I,IBEG,IEND:     INTEGER;
    VALUE:        REAL;
 BEGIN
    IF(IPTL >= 4) Then WRITELN(OUTFILE,'   ENTERING INTVL2',' NP = ',NP:5);
                        {INPUT THE INTIAL VALUES IN GROUPS}
    WRITELN(OUTFILE);
    readln(data,phi[1]);
    REPEAT
     READLN(data,IBEG,IEND,VALUE);
     For I := IBEG To IEND Do
     PHI[I] := VALUE;
    UNTIL (IEND >= NP);
    WRITELN(OUTFILE);
                        {OUTPUT OF THE INTIAL VALUES}
 END;                   {PROCEDURE  INTVL2}
{**************************************************************************}
Procedure INDATA(Var np,ne,ncoef,numb,iptl,kw:integer;
           Var dhtube,ahtube,core_radi:real;
           Var Dt,Q:Smsiz; Var Yc: D1siz;
           Var Nmtl:Nmsiz; Var Nel:N1siz;
           Var Ib:NmSiz; Var thta,delta:real;
           Var nsteps,iwt,prnt:Integer; Var Phi:D1siz);
{**************************}
 {THIS SUBROUTINE AND INTVL2 READS IN THE INPUT DATA}
 {INPUT OF THE TITLE CARD AND CONTROL PARAMETERS}
{---------------}
VAR
 I,IJ,INBW,J,JEND,
 KK,N,NB,NID:     INTEGER;
BEGIN
    READLN(DATA,TITLE);
    writeln(title);
    writeln(OUTFILE,title);
    READLN(DATA,NP,NE,NCOEF,IPTL,KW);
                        {INPUT OF EQUATION COEFFICIENTS
                           and the NODAL COORDINATES}
    READLN(DATA,DHTUBE,core_radi);
    AHTUBE := Pi*(DHTUBE*DHTUBE)/4;
     WRITELN(OUTFILE,'  diameter head tube = ',dhtube:0:4,
             '  core radius ',core_radi:0:4);
    FOR I := 1 TO NCOEF DO
      READLN(DATA,DT[I],Q[I]);
                        {INPUT OF THE INITIAL VALUES}
   INTVL2(np,iptl,kw,phi);
   numb := 1;
        {GENERATION OF THE SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS
              DEFINITION OF VARIABLES USED IN SUBROUTINE NUMODE
           THTA - THE THETA VALUE USED IN THE SINGLE STEP METHODS
                        0 - EULER'S FORWARD DIFFERENCE METHOD
                        1/2 - CENTRAL DIFFERENCE METHOD
                        2/3 - GALERKIN'S METHOD
                        1   - BACKWARD DIFFERENCE METHOD
                    DELTA - THE TIME STEP
                    NSTEPS - NUMBER OF ITERATIONS
           IWT - INTEGER CONTROLLING THE OUTPUT OF THE CALUCLATED
              VALUES. VALUES ARE PRINTED EVERY IWT TIME STEPS.
                    INPUT OF THTA AND THE TIME STEP}
    READLN(DATA,THTA,DELTA);
    WRITELN(OUTFILE,'  Theta ',thta:8:2,'   Delta ',delta:8:1,' Sec
        (',DELTA/60:8:4,' min)');
                    {FORM THE (A) AND (P) MATRICES AND DECOMPOSE (A)
                      INPUT THE NUMBER OF TIME STEPS, NSTEPS,
                           AND THE WRITE CONTROL, IWT}
```

```
    READLN(data,NSTEPS,IWT,prnt);
    if (kw > 3) then
    begin
      Writeln(outfile,'       NODAL COORDINATES');
      writeln(OUTFILE,' NODE          Y');
    end;
      FOR I := 1 TO NP DO
      READ(DATA,YC[I]);
                        {OUTPUT OF THE EQUATION COEFFICIENTS}
                        {OUTPUT OF THE NODAL COORDINATES}
    IF(KW > 3) Then
    begin
      FOR I := 1 TO NP DO
      WRITELN(OUTFILE,I:4,'   ',YC[I]:0:2);
              NPUT AND ECHO PRINT OF THE ELEMENT NODAL DATA}
      WRITELN(OUTFILE,'   ELEMENT DATA');
      WRITELN(OUTFILE,'NEL   NMTL     NODE NUMBERS');
    end;
    For KK := 1 To NE Do
    Begin
      NMTL[KK] := 1;
      NEL[KK,1] := KK;
      NEL[KK,2] := KK + 1;
       IF (KW > 3) THEN
        WRITELN(OUTFILE,kk:4,NMTL[KK]:6,NEL[kk,1]:7,NEL[kk,2]:6);
    End;
                      {INPUT THE NUMBERS OF THE NODES
                        WHOSE VALUES DO NOT CHANGE WITH TIME}
    IB[1] := 1;
                  {ANALYSIS OF THE NODE NUMBERS
                  INITIALIZATION OF A CHECK VECTOR}
                    {CREATION AND INITIALIZATION OF THE A
                       Vector CALCULATION OF THE BANDWIDTH}
    writeln(outfile);
    WRITELN(OUTFILE,'  COMPLETED READ OF ALL INPUT PARAMETERS');
    WRITELN(OUTFILE);
  END;                              {PROCEDURE INDATA}
{****************************************************************************}
PROCEDURE RESULTS(np:integer;Phi:D1siz);
{**************************}
Var  i: integer;
BEGIN
  For I := 1 to np do
  begin
    Write(outfile,phi[i]:10:4,' ');
    if (i mod 5 = 0) then writeln(outfile);
  end;
  writeln(outfile);
END;                              {PROCEDURE RESULTS}
{****************************************************************************}
 Procedure Redata (Var np,ne,nbw,ncoef,numb,numb2,iptl,kw:integer;
             var dhtube,r,ahtube,core radi:real;
             Var Dt,Q:Smsiz; Var Yc: D1siz; Var Nmtl,ib:Nmsiz;
             var Nel:Nlsiz; Var ib2:nmsiz;
             Var thta,delta,time:real; Var nsteps,iwt,prnt:integer;
             Var Phi,Rf,Wcp:D1siz;var totflow,refill:real);
{****************************}
VAR
   I,IJ,KK,N,NID:     INTEGER;
                  {THIS SUBROUTINE READS IN THE INPUT DATA
                   FROM A FILE CREATED BY A PREVIOUS RUN}
                  {INPUT OF THE TITLE CARD AND CONTROL PARAMETERS}
BEGIN
    READLN(DATA,TITLE);
    writeln(OUTFILE,title);
    READLN(DATA,NP,NE,NCOEF,IPTL,KW,NBW);
    IF(KW = 1) Then
      begin
        writeln(OUTFILE,'  NP = ',NP:6,'  NE = ',NE:6,'  NCOEF = ',NCOEF:6);
        writeln(OUTFILE,'  IPTL = ', IPTL:6,'  KW = ',KW:6);
      end;
                         {INPUT OF EQUATION COEFFICIENTS
                           and the NODAL COORDINATES}
    READLN(DATA,DHTUBE,core radi);
    READLN(DATA,THTA,DELTA);
    READLN(DATA,TIME);
    READLN(DATA,NSTEPS,IWT,prnt);
    Readln(data,totflow,refill);
```

```
  WRITELN(OUTFILE,' EQUATION COEFFICIENTS ');
  WRITELN(OUTFILE,'   diameter head tube ',dhtube:0:3,' cm;   k: y =  ',
        r:0:3,' * x');
  WRITELN(OUTFILE,'  SET     DT     Q');
FOR I := 1 TO NCOEF DO
  begin
   READLN(DATA,DT[I],Q[I]);
   WRITELN(OUTFILE,'  ',I:2,'    ',DT[I]:7:2,Q[I]:7:2)
  end;
AHTUBE := Pi*(DHTUBE*DHTUBE)/4;
if (kw > 2) then
begin
  WRITELN(OUTFILE,'       NODAL COORDINATES');
  writeln(OUTFILE,' NODE          X           Y');
end;
  FOR I := 1 TO NP DO
  READLN(DATA,YC[I]);
                    {OUTPUT OF THE EQUATION COEFFICIENTS}
                        {OUTPUT OF THE NODAL COORDINATES}
IF(KW > 2) Then FOR I := 1 TO NP DO
WRITELN(OUTFILE,I:4,'    ',YC[I]:0:5);
              {INPUT AND ECHO PRINT OF THE ELEMENT NODAL DATA}
if (kw > 2) then
begin
  WRITELN(OUTFILE,'    ELEMENT DATA');
  WRITELN(OUTFILE,'NEL    NMTL      NODE NUMBERS');
end;
NID := 0;
flush(outfile);
For KK := 1 To NE Do
Begin
  READLN(DATA,N,NMTL[KK],NEL[N,1],NEL[N,2],NEL[N,3],NEL[N,4]);
  IF (KW > 2) THEN
   WRITELN(OUTFILE,N:4,NMTL[KK]:6,NEL[N,1]:7,NEL[N,2]:6,NEL[N,3]:6,NEL[N,4]:6);
  IF((N-1) <> NID) Then WRITELN(OUTFILE,'ELEMENT ',N:4,'
            NOT IN SEQUENCE');
  NID := N;
End;
  if (kw > 2) then
 WRITEln(OUTFILE,'  COMPLETED READIN OF ELEMENTS AND
          THEIR NODE NUMBERS');
 FOR I := 1 TO NP DO
  READLN(DATA,PHI[I]);
  if (kw > 2) then
  FOR I := 1 TO NP DO
  writeLN(outfile,PHI[I]:10:2);
                   {INPUT THE NUMBERS OF THE NODES
                     WHOSE VALUES DO NOT CHANGE WITH TIME}
readln(data,numb,numb2);
writeln(outfile,numb:2,' fixed nodes, and ',numb2:2,' saturated nodes');
For i := 1 to numb do
  READ(data,IB[I]);
For I := 1 to numb2 do
  READ(data,ib2[i]);
IJ := 0;
write(outfile,' fixed nodes:  ');
  Repeat
   INC(IJ);
   WRITE(OUTFILE,IB[IJ]:10);
  Until ((IB[IJ+1] <= 0) OR (IJ MOD 6 = 0));
  writeln(outfile);
writeln(outfile,'  saturated nodes, including fixed nodes');
ij := 0;
Repeat
   INC(IJ);
   WRITE(OUTFILE,IB2[IJ]:10);
  Until ((IB2[IJ+1] <= 0) OR (IJ MOD 6 = 0));
  writeln(outfile);
IF(NUMB > 0) Then
Begin
  IF(NUMB > IDNN) Then
  Begin
   WRITELN(OUTFILE,'    NUMBER OF BOUNDARY CONDITIONS EXCEEDS THE
        ALLOWED NUMBER OF ',idnn:5);
   Exit
  End;
End;
                    {ANALYSIS OF THE NODE NUMBERS
                     INITIALIZE OF A CHECK VECTOR}
```

```
 WRITELN(OUTFILE,'   Theta ',thta:8:2,' DELTA ',delta/60:8:2,' min. (',
         delta:8:2,' sec.)');
                  {FORM THE (A) AND (P) MATRICES AND DECOMPOSE (A)
                       INPUT THE NUMBER OF TIME STEPS, NSTEPS,
                         AND THE WRITE CONTROL, iwt,prnt
                                 }
      WRITELN(OUTFILE,'number of itetations ',nsteps:5,' print control',iwt,prnt:5);
      WRITELN(OUTFILE);
      if (kw > 2) then writeln(outfile,'RF');
    FOR I := 1 TO NP DO
  READLN(DATA,RF[I]);
    if (kw > 2) then
    FOR I := 1 TO NP DO
    writeLN(outfile,RF[I]);
  FOR I := 1 TO NP DO
    readLN(data,WCP[I]);
    if (kw > 2) then writeln(outfile,'WCP');
     if (kw > 2) then
     FOR I := 1 TO NP DO
      writeLN(outfile,wcp[I]);
    WRITELN(OUTFILE,'             COMPLETED READ OF ALL INPUTS ');
    WRITELN(OUTFILE);
 END;                          {PROCEDURE REDATA}
{****************************************************************************}
Procedure CREATE(np,ne,nbw,ncoef,iptl,kw,numb,numb2:integer;dhtube,ahtube,
          core_radi:real;Dt,Q:Smsiz;Yc: D1siz;Nmtl,ib:Nmsiz;
          Nel:Nlsiz;ib2:nmsiz;thta,delta,time:real;nsteps,
          iwt,prnt: Integer;Phi,rf,wcp:d1siz;totflow,refill:real);
{****************************}
VAR I,J:    INTEGER;
               {THIS PROCEDURE WRITES THE CURRENT
                  DATA TO A FILE TO ALLOW RUNS WITHOUT STARTING
                          FROM TIME = 0 EACH TIME.}
               {INPUT OF THE TITLE CARD AND CONTROL PARAMETERS}
 BEGIN
    WRITELN(DATSAV,TITLE);
    WRITELN(DATSAV,NP,' ',NE,' ',NCOEF,' ',IPTL,' ',KW,' ',NBW);
    WRITELN(DATSAV,DHTUBE,' ',core_radi);
    WRITELN(DATSAV,THTA,' ',DELTA);
    WRITELN(DATSAV,' ',TIME);
    WRITELN(DATSAV,' ',NSTEPS,' ',iwt,prnt);
    Writeln(datsav,' ',totflow,' ',refill);
    FOR I := 1 TO NCOEF DO
      WRITELN(DATSAV,DT[I],' ',Q[I],' ');
    FOR I := 1 TO NP DO
     WRITELN(DATSAV,' ',YC[I]);
    For J := 1 To NE Do
 WRITELN(DATSAV,J,' ',NMTL[J],' ',NEL[J,1],' ',NEL[J,2],' ',NEL[J,3],'
      ',NEL[J,4],' ');
   FOR I := 1 TO NP DO
   WRITELN(DATSAV,PHI[I],' ');
   writeln(datsav,numb,' ',numb2);
   For I := 1 to numb do
    WRITELN(DATSAV,IB[I],' ');
   for i := 1 to numb2 do
    writeln(datsav,ib2[i],' ');
   FOR I := 1 TO NP DO
    WRITELN(DATSAV,RF[I],' ');
   FOR I := 1 TO NP DO
    WRITELN(DATSAV,WCP[I],' ');
 END;                              {PROCEDURE CREATE}
{**********************************************************************}
procedure wc_calc(kk:integer;var mc:real);
{*******************
this subroutine calculates the water content, wc
at a node for a known water potential, wp
-------------------}
type
    wc_type = array[1..14] of real;
var  i_wc:   integer;
    node:   real;
    wc:  wc_type;
    wp:  wc_type;
begin
{ziegenfussoil, ap horizon (0-23 cm)}
 if (wc_choice = 1) then
   begin
    wc[1] := 0.453;
    wc[2] := 0.452;
```

```
    wc[3] := 0.440;
    wc[4] := 0.420;
    wc[5] := 0.377;
    wc[6] := 0.329;
    wc[7] := 0.295;
    wc[8] := 0.262;
    wc[9] := 0.236;
    wc[10] := 0.207;
    wc[11] := 0.175;
    wc[12] := 0.156;
    wc[13] := 0.146;
    wc[14] := 0.112;
  end;
    wp[1] := 0.0;
    wp[2] := -10.33;
    wp[3] := -30.99;
    wp[4] := -51.65;
    wp[5] := -103.3;
    wp[6] := -206.6;
    wp[7] := -340.89;
    wp[8] := -619.8;
    wp[9] := -1033.0;
    wp[10] := -2066.0;
    wp[11] := -5165.0;
    wp[12] := -10330.0;
    wp[13] := -15495.0;
    wp[14] := -103300.0;
   {Capac soil, ap horizon (0-23 cm)}
  if (wc_choice = 2) then
  begin
    wc[1] := 0.372;
    wc[2] := 0.370;
    wc[3] := 0.366;
    wc[4] := 0.362;
    wc[5] := 0.353;
    wc[6] := 0.338;
    wc[7] := 0.321;
    wc[8] := 0.296;
    wc[9] := 0.269;
    wc[10] := 0.229;
    wc[11] := 0.181;
    wc[12] := 0.151;
    wc[13] := 0.137;
    wc[14] := 0.093;
  end;
  if (wc_choice = 3) then
   {lenawee soil, ap horizon (0-23 cm)}
  begin
   wc[1] := 0.435;
   wc[2] := 0.433;
   wc[3] := 0.429;
   wc[4] := 0.425;
   wc[5] := 0.416;
   wc[6] := 0.4;
   wc[7] := 0.384;
   wc[8] := 0.358;
   wc[9] := 0.333;
   wc[10] := 0.295;
   wc[11] := 0.247;
   wc[12] := 0.216;
   wc[13] := 0.199;
   wc[14] := 0.144;
  end;
         {wc - the ordinate values - water content - volumetric}
         {wp  - the abcissa values - water potential- in cm}
         {note:  1033 cm/bar was used}
   node := phi[kk];
   if (node < wp[14]) then
   begin
     writeln(outfile,' value (',node:12:2,
          ') less than least absissa value: node = ',kk:0);
     writeln(outfile,' value (',node:12:2,
          ') less than least absissa value: node = ',kk:0);
     writeln(outfile,' the soil can not be this dry!');
   end;
```

```
    if (node < -10.33)  then
    begin
    for i_wc := 2 to 13 do
      if (node <= wp[i_wc])  and (node > wp[i_wc+1]) then
{interpolate to get th - water content}
        mc :=  ((wc[i_wc]-wc[i_wc+1])*ln(node/wp[i_wc+1])/ln(wp[i_wc]
              /wp[i_wc+1]))+ wc[i_wc+1];
    end
    else
      mc := wc[1];
end;                                    {procedure wc_calc}
{*************************************************************************}
procedure lambda(kk:integer;var lam_l:real;var flag:integer);
{*************************
this subroutine calculates the slope of the
head-water content relationship, lambda
------------------------------}
type
    wc_type = array[1..14] of real;
var   i_lam:    integer;
    node4,  mc:    real;
    wc:        wc_type;
    wp:        wc_type;
begin
{ziegenfuss soil, ap horizon (0-23 cm)}
  if (wc_choice = 1) then
   begin
    wc[1] := 0.453;
    wc[2] := 0.452;
    wc[3] := 0.440;
    wc[4] := 0.420;
    wc[5] := 0.377;
    wc[6] := 0.329;
    wc[7] := 0.295;
    wc[8] := 0.262;
    wc[9] := 0.236;
    wc[10] := 0.207;
    wc[11] := 0.175;
    wc[12] := 0.156;
    wc[13] := 0.146;
    wc[14] := 0.112;
   end;
    wp[1] := 0.0;
    wp[2] := -10.33;
    wp[3] := -30.99;
    wp[4] := -51.65;
    wp[5] := -103.3;
    wp[6] := -206.6;
    wp[7] := -340.89;
    wp[8] := -619.8;
    wp[9] := -1033.0;
    wp[10] := -2066.0;
    wp[11] := -5165.0;
    wp[12] := -10330.0;
    wp[13] := -15495.0;
    wp[14] := -103300.0;
    {Capac soil, ap horizon (0-22 cm)}
   if (wc_choice = 2) then
   begin
    wc[1] := 0.372;
    wc[2] := 0.370;
    wc[3] := 0.366;
    wc[4] := 0.362;
    wc[5] := 0.353;
    wc[6] := 0.338;
    wc[7] := 0.321;
    wc[8] := 0.296;
    wc[9] := 0.269;
    wc[10] := 0.229;
    wc[11] := 0.181;
    wc[12] := 0.151;
    wc[13] := 0.137;
    wc[14] := 0.093;
   end;
    {Lenawee soil, ap horizon (0-23 cm)}
   if (wc_choice = 3) then
   begin
    wc[1] := 0.435;
    wc[2] := 0.433;
```

```
      wc[3] := 0.429;
      wc[4] := 0.425;
      wc[5] := 0.416;
      wc[6] := 0.4;
      wc[7] := 0.384;
      wc[8] := 0.358;
      wc[9] := 0.333;
      wc[10] := 0.295;
      wc[11] := 0.247;
      wc[12] := 0.216;
      wc[13] := 0.199;
      wc[14] := 0.144;
    end;
          {wc - the ordinate values - water content - volumetric}
          {wp  - the abcissa values - water potential- in cm}
          {note:  1033 cm/bar was used}
      flag := 0;
      if (nel[kk,4] = 0) then
        node4 := phi[nel[kk,3]]
      else
        node4 := phi[nel[kk,4]];
      if (node4 < wp[14]) then
      begin
        write(outfile,' value less than least absissa value: ');
        writeln(outfile,' node4 = ',node4:0,' wp[14] = ',wp[14]:0:2);
        write(outfile,' the soil can not be this dry!');
        writeln(outfile,' error found in interp');
        flag := 1;
      end;
      if (node4 < -10.33)  then
        begin
        for i_lam := 2 to 13 do
          begin
          if (node4 <= wp[i_lam])  and (node4 > wp[i_lam+1]) then
           begin
            mc :=  ((wc[i_lam]-wc[i_lam+1])*ln(node4/wp[i_lam+1])/
                 ln(wp[i_lam]/wp[i_lam+1])) + wc[i_lam+1];
            lam_l :=  ((mc - wc[i_lam+1])/(node4 - wp[i_lam+1]))
           end;
          end;
        end
        else
          lam_l := 0;
end;                                   {procedure  interp}
{**************************************************************************}
procedure kxy(k1:integer;var kl,klx:real);
{**************************
this procedure calculates the hydraulic conductivity
-------------------}
type
     wp_type = array[1..19] of real;
var  i_kxy:      integer;
     head:   real;
     wp:     wp_type;
     k_curve: wp_type;
begin
     wp[1] := -8;
     wp[2] := -10;
     wp[3] := -20;
     wp[4] := -30;
     wp[5] := -40;
     wp[6] := -50;
     wp[7] := -65;
     wp[8] := -100;
     wp[9] := -150;
     wp[10] := -200;
     wp[11] := -250;
     wp[12] := -300;
     wp[13] := -400;
     wp[14] := -420;
     wp[15] := -500;
     wp[16] := -700;
     wp[17] := -900;
     wp[18] := -2000;
     wp[19] := -10500;
   {for Pachappa sandy loam:}
   if (k_choice = 1) then
   begin
     k_curve[1] := 5.56e-4;
```

```
    k_curve[2] := 5.56e-4;
    k_curve[3] := 5.56e-4;
    k_curve[4] := 5.05e-4;
    k_curve[5] := 4.04e-4;
    k_curve[6] := 3.54e-4;
    k_curve[7] := 2.52e-4;
    k_curve[8] := 3.54e-5;
    k_curve[9] := 1.01e-6;
    k_curve[10] := 5.3e-7;
    k_curve[11] := 3.53e-7;
    k_curve[12] := 2.3e-7;
    k_curve[13] := 1.01e-7;
    k_curve[14] := 5.3e-8;
    k_curve[15] := 4.04e-8;
    k_curve[16] := 2.02e-8;
    k_curve[17] := 1.52e-8;
    k_curve[18] := 2.02e-9;
    k_curve[19] := 3.03e-10;
  end;
  {for indio loam soil (saturated rate 20 cm/hr):}
  if (k_choice = 2) then
  begin
    k_curve[1] := 5.56e-3;
    k_curve[2] := 5.56e-3;
    k_curve[3] := 5.56e-3;
    k_curve[4] := 5.28e-3;
    k_curve[5] := 5e-3;
    k_curve[6] := 4.17e-3;
    k_curve[7] := 2.78e-3;
    k_curve[8] := 1.39e-3;
    k_curve[9] := 2.92e-5;
    k_curve[10] := 1.39e-5;
    k_curve[11] := 5.56e-6;
    k_curve[12] := 2.92e-6;
    k_curve[13] := 1.11e-6;
    k_curve[14] := 8.33e-7;
    k_curve[15] := 2.92e-7;
    k_curve[16] := 1.94e-7;
    k_curve[17] := 1.39e-7;
    k_curve[18] := 8.33e-9;
    k_curve[19] := 5.56e-10;
  end;
   {chino clay - saturated value of 0.2 cm/hr}
 if (k_choice = 3) then
  begin
    k_curve[1] := 5.56e-5;
    k_curve[2] := 5.28e-5;
    k_curve[3] := 2.78e-5;
    k_curve[4] := 2.5e-5;
    k_curve[5] := 1.67e-5;
    k_curve[6] := 1.39e-5;
    k_curve[7] := 8.33e-6;
    k_curve[8] := 2.78e-6;
    k_curve[9] := 8.33e-7;
    k_curve[10] := 5.56e-7;
    k_curve[11] := 2.92e-7;
    k_curve[12] := 2.22e-7;
    k_curve[13] := 8.33e-8;
    k_curve[14] := 6.94e-8;
    k_curve[15] := 2.92e-8;
    k_curve[16] := 2.5e-8;
    k_curve[17] := 1.39e-8;
    k_curve[18] := 2.77e-9;
    k_curve[19] := 8.33e-10;
  end;
   {k_curve - the ordinate values - water content - volumetric
          taken from richards but multiplied by a constant
          to reflect realistic saturated conductivity values}
   {wp  - the abcissa values - water potential- in cm}

  if (nel[k1,4] = 0) then
   head := phi[nel[k1,3]]
  else
   head := phi[nel[k1,4]];
  kl := 0;
  if (head > -8) then kl := k_curve[1]
  else
   for i_kxy := 1 to 18 do
   begin
```

```
    if (head = wp[i_kxy]) then
     kl := k_curve[i_kxy]
    else
     if (head <= wp[i_kxy]) and (head > wp[i_kxy+1]) then
     kl := exp((ln(k_curve[i_kxy]/k_curve[i_kxy+1])
           /ln(wp[i_kxy]/wp[i_kxy+1])*
           ln(head/wp[i_kxy+1])) + ln(k_curve[i_kxy+1]));
    end;
  klx := kl * r;
  if (kl = 0) then
    writeln(outfile,' error!, kl = 0, node ',k1:0);
end;        {adsorption flow is governed by the upper nodes}
{*******************************************************************}
Procedure Sat_elem;
{********************}
VAR kk:  integer;
   count, spread,
   level,
   k1,k2:      real;
Begin
 count := np/1;
 spread := phi[1]/count;
  For kk := 2 to np do
  begin
   level := phi[kk-1] - spread;
   phi[kk] := level;
  end;
End;
{*******************************************************************}
PROCEDURE INIT;{(np,nbw:integer)}
{*****************}
Var I,J:  Integer;
BEGIN
 For I := 1 to np Do
 Begin
  FM[I] := 0.0;
  For J := 1 to 2 Do
  begin
   K[I,J] := 0.0;
   C[I,J] := 0.0
  end;
 End;
END;
{*******************************************************************}
PROCEDURE GLB_MTX; {(iptl,elem,nbw:integer;ki,kix,kj,kjx,kk,kkx,kl,klx,lam_i,
            lam_j,lam_k,lam_l:real; nel:nlsiz;xc,yc:d1siz;
            Var fm: d1siz;nmtl: nmsiz;dt,q:smsiz)}
{**************************}
TYPE  ESiz = ARRAY[1..2,1..2] of Real;
    XYSiz = ARRAY[1..2] of Real;
VAR  ECM, ESM:   ESiz;
    EF, Y:     XYSiz;
    DTE, ge,
    QE, length:        REAL;
    I,     II,    IJ,    J,
    JJ,    J5,    J6,    KA:      INTEGER;
CONST
    ES:  ESiz = ((1,-1),(-1,1));
    LMP: ESiz = ((1,0),(0,1));
BEGIN
 IF ((IPTL >= 4) AND (kk = 1)) Then WRITELN ('ENTERING GLB_MTX, 1st elem.');
             {BILINEAR RECTANGULAR ELEMENT WITHOUT THE
              DERIVATIVE BOUNDARY CONDITION}
             {RETRIEVAL OF NODAL COORDINATES AND NODE NUMBERS}
    length := yc[nel[kk,1]] - yc[nel[kk,2]];
    IF (ABs(length) <= 0.00001) Then
    begin
     WRITELN('THE AREA OF ELEMENT ',kk:4,' IS LESS THAN 0.00001');
     WRITELN('THE NODE numbERS ARE IN THE WRONG ORDER OR THE NODES FORM A
          STRAIGHT LINE');
     WRITELN('EXECUTION TERMINATED');
     EXIT
    end;          {INITIALIZATION: ET[ ] MATRIX FOR THE
               AXISYMMETRIC - TIME DEPENDENT PROBLEM}
```

```
    II  := NMTL[kk];
    DTE := DT[II];
    QE  := Q[II];
    ge := 0.0;
     for i := 1 to 2 do
     begin
       ef[i] := qe*length/2;
       for j := 1 to 2 do
       begin
       esm[i,j] := sqrt(k1*k2) * es[i,j]/length;
       ecm[i,j] := dte*sqrt(lam1*lam2)*length*lmp[i,j]/2;
         if (i =1) and (j = 1) then
         begin
           esm[i,j] := k1 * es[i,j]/length;
           ecm[i,j] := dte*lam1*length*lmp[i,j]/2;
         end;
         if (i = 2) and (j = 2) then
         begin
           esm[i,j] := k2 * es[i,j]/length;
           ecm[i,j] := dte*lam2*length*lmp[i,j]/2;
         end;
       end;
    end;
                          {Direct Stiffness}
             {The major diagonal is stored in column 1,
              followed by the minor diagonals (to NBW)
              in columns 2 - NBW.  Zeros are used to fill
             the column to NP, the matrix size is NP x NBW}
     For I := 1 To 2 Do
     Begin
      J5 := NEL[kk,I];
      FM[J5] := FM[J5] + EF[I];   {force vector, has fixed nodal values}
      For J := 1 To 2 Do
      Begin
       J6 := NEL[kk,J];
       JJ := J6 + 1 - J5;
       IF(JJ > 0) AND (JJ <= 2) Then
       begin
         K[J5,JJ] := K[J5,JJ] + ESM[I,J];
         C[J5,JJ] := C[J5,JJ] + ECM[I,J];
       end;
      End;
     End;
  END;                        {PROCEDURE GLB_MTX}
{*******************************************************************}
PROCEDURE MODIFY;
{*******************}
{THIS SUBROUTINE MODIFIES THE GLOBAL CAPACITANCE AND
STIFFNESS MATRICES WHEN THERE ARE NODAL VALUES THAT
REMAIN CONSTANT WITH TIME.}
{----------------}
VAR   I, J, JM, M, N:      INTEGER;
BEGIN
    IF(IPTL >= 4) Then
    begin
      WRITELN(OUTFILE,'EXECUTING MODIFY');
      WRITELN(OUTFILE,'NP  =',NP:5)
    end;
          {MODIFY C AND K MATRICES BY DELETING ROWS AND COLUMNS}
    For I := 1 To num_sat_nds Do
    Begin
     J := sat_nd_array[I];
     N := J-1;
          {This zeros out the row for the fixed node, except
            the first column, which is actually the diagonal.}
     JM := 2;
     M := J+JM-1;
     if (M <= NP) then
     Begin
      FM[M] := FM[M]-K[J,JM]*PHI[J];
      K[J,JM] := 0;
      C[J,JM] := 0
     End;
            {This zeros out the column for the fixed node, but
            remember the diagonals make up the columns so
            calculations must be done at about a 45 degree angle}
     If (N > 0) then
     Begin
      FM[N] := FM[N]-K[N,JM]*PHI[J];
```

```
      K[N,JM] := 0;
      C[N,JM] := 0;
      N := N-1
     End;
     C[J,1] := 1;
     K[J,1] := 0;
     FM[J] := 0
    End;
    IF(IPTL < 4) Then EXIT;
    IF(IPTL = 4) Then
    WRITELN(OUTFILE,'     RETURNING FROM MODIFY')
  END;                          PROCEDURE  MODIFY}

{*********************************************************************}
PROCEDURE MATAB;{(np,nbw,iptl: integer;delta,thta:real);}
                {THIS SUBROUTINE GENERATES THE A AND P MATRICES
                FOR THE SINGLE STEP METHODS USING THE EQUATIONS
                    A( , ) = C[ , ]+(DELTA)*THTA*K[ , ]
                    P( , ) = C[ , ]-(DELTA)*(1 - THETA)*K[ , ]}
{****************************}
VAR I,J:  INTEGER;
   AA, BB, TC, TK:    REAL;
 BEGIN
    IF (IPTL > 4) Then WRITELN(OUTFILE,'    ENTERING MATAB');
    AA := THTA*delta;
    BB := (1-THTA)*delta;
    For I := 1 To NP Do
    Begin
     For J := 1 To 2 Do
      Begin
       TC := C[I,J];
       TK := K[I,J];
              { A MATRIX     }
       C[I,J] := TC+AA*TK;
              { P MATRIX     }
       K[I,J] := TC-BB*TK
      End;
    End;
    IF (IPTL < 4) Then EXIT;
    WRITELN(OUTFILE,' LEAVING MATAB')
  END;                          {PROCEDURE  MATAB}
{*********************************************************************}
PROCEDURE DCMPBD;{(np,nbw,iptl:integer);}
{***********************}
VAR
   I,   J,
   K2,  MJ,   MK,   ND,
   NJ,  NK,   NL,   NP1:      INTEGER;
  BEGIN
    IF (IPTL >= 4) Then WRITELN(OUTFILE,' ENTERING DCMPBD');
    NP1 := NP-1;
    For I := 1 To NP1 Do
    Begin
     MJ := I+1;
     IF(MJ > NP) Then MJ := NP;
     NJ := I+1;
     MK := 2;  {since nbw = 2}
     IF((NP-I+1) < 2) Then MK := NP-I+1;
     ND := 0;
     For J := NJ To MJ Do
     Begin
      MK := MK-1;
      ND := ND+1;
      NL := ND+1;
      For K2 := 1 To MK Do
      Begin
       NK := ND+K2;
       if c[i,1] = 0 then writeln(outfile,' C[',i:0,',1] = ',c[i,1]);
       C[J,K2] := C[J,K2]-C[I,NL]*C[I,NK]/C[I,1]
      End;
     End;
    End;
    IF(IPTL < 4) Then EXIT;
    WRITELN(OUTFILE,'   LEAVING DCMPBD');
  END;                          {PROCEDURE  DCMPBD}
{*********************************************************************}
PROCEDURE MULTBD;{(np,nbw,iptl:integer; phi:d1siz;var rf:d1siz);}
```

```
{*************************}
VAR  I,  J,  K2,  M:        INTEGER;
    SUM:                REAL;
  BEGIN
    IF(IPTL >= 4) Then WRITELN(OUTFILE,'    ENTERING MULTBD');
    For I := 1 To NP Do
    Begin
      SUM := 0.0;
      K2 := I-1;
      J := 2;
      M := J+I-1;
      IF (M <= NP) Then
      SUM := SUM+K[I,J]*PHI[M];
      IF (K2 > 0) THEN
      Begin
        SUM := SUM+K[K2,J]*PHI[K2];
        K2 := K2 - 1
      End;
      RF[I] := SUM+K[I,1]*PHI[I];
    End;
    IF (IPTL < 4) Then EXIT;
    WRITELN(OUTFILE,'    LEAVING MULTBD')
  END;                          {PROCEDURE  MULTBD}
{*****************************************************************}
PROCEDURE SLVBD;
{*****************************}
VAR flag,
    I,    J,     K2,
    L,    M,     MJ,
    NJ,   NP1,   N,
    skip, elem:          INTEGER;
    SUM:                 REAL;
BEGIN
    NP1 := NP-1;
                        {DECOMPOSITION OF THE COLUMN VECTOR RF( )}
    For I := 1 To NP1 Do
    Begin
      MJ := I+1;
      IF(MJ > NP) Then MJ := NP;
      NJ := I+1;
      L := 1;
      For J := NJ To MJ Do
      Begin
        L := L+1;
        RF[J] := RF[J]-(C[I,L]*RF[I]/C[I,1]);
      End;
    End;
             {BACKWARD SUBSTITUTION FOR DETERMINATION OF PHI[ ]}
    PHI[NP] := RF[NP]/C[NP,1];
    For K2 := 1 To NP1 Do
    Begin
      I := NP-K2;
      MJ := 2;
      IF((I+1) > NP) Then MJ := NP-I+1;
      J := 2;
      N := I+J-1;
      SUM := C[I,J]*PHI[N];
      PHI[I] := (RF[I]-SUM)/C[I,1];
    End;
                        {the following adds a saturated
                           node to the fixed node set}
 k2 := num_sat_nds + 1;
 for j := k2 to np do
 begin
  if (Phi[j] > -10.33) then
   begin
    flag := 0;
    for l := 1 to num_sat_nds do
     if (sat_nd_array[l] = j) then flag := 1;
    if (flag = 0) then
    begin
     phi[j] := 0;
     num_sat_nds := num_sat_nds + 1;
     sat_nd_array[num_sat_nds] := j;
    end;
   end;
 end;
END;                            {PROCEDURE  SLVBD}
{******************************************************************}
```

```
Procedure Fnt_elem;
{*************************}
Var i,j,kk,flag:     integer;
   k1,k2,lam1,lam2:   real;
Begin
  INIT(np);
  For kk := 1 To NE Do
  Begin
   KXY(kk,phi,nel,k1,k2);
   if (kk = 1) then dye1 := k1;
   LAMBDA(kk,phi,nel,lam1,lam2,flag);
   if (flag = 1) then exit;
   GLB_MTX(iptl,kk,k1,k1,lam1,lam1,nel,yc,nmtl,dt,q);
  End;              {calculates the element matricies}
                {CALCULATION OF THE ELEMENT CAPACITANCE (C),}
                 {STIFFNESS(F), MATRICES AND THE ELEMENT}
                   {FORCE VECTOR (F)}
    MODIFY(np,num_sat_nds,iptl,sat_nd_array,phi);
    MATAB(np,iptl,delta,thta);
    DCMPBD(np,iptl);
     TIME := TIME + DELTA;
    MULTBD(np,iptl,phi,rf);
    FOR i := 1 to np do
     RF[i] := RF[i]+DELTA*FM[i];
    SLVBD(num_sat_nds,np,iptl,rf,phi,sat_nd_array);
End;
{*************************************************************************}
Procedure flow_calc;
{****************************}
Var wc: d1siz;
   i, kk, mj, j, l,
   flag1, flag, fix,
   K_app_num:            integer;
   area_head_tube, area_core,
   delta_v, delta_h,
   dh, dh_prev,
   elm_q, flowsum,
   head1, head2,
   length, delt_old,
   velocity1,
   totflow, velocity2,
   yflag:                 real;
{----------------------------------------------------------------}
Begin
  if (iteration = 0) then
  Begin
   flag1 := 0;
   delt_old := 0.0;
   dh_prev := 0.0;
   head1 := phi[1];
   velocity1 := 0.0;
   writeln(outfile,' initial volume of water in the permeameter =
           ',voll:10:4);
   writeln(outfile);
   FOR KK := 1 TO NP DO
    WC_CALC(KK,PHI,WCP[KK]);
    writeln(graph,' ',iteration:0,' ',time/60:0:4,' ',phi[1]:10:3,' ',
        phi[2]:10:3,' ',phi[3]:10:3,' ',phi[4]:10:3,' ',phi[5]:10:3);
   Exit
  End;        {initial set up}
  flowsum := 0.0;
  area_core := refill*(core_radi*core_radi);
  For KK := 1 To ne Do
  BEGIN
   WC_CALC(kk,phi,wc[kk]);        { call procedure to calculate the
                          water content, WC, at each node}
   elm_q := (wc[kk]-wcp[kk])*area_core*(yc[kk]-yc[kk+1]);
   flowsum := flowsum + elm_q;
   wcp[kk] := wc[kk];
  END;
  DH := flowsum/AHTUBE;        {ahtube - area of the head tube, cm3/cm2}
  if (dh < 0) then
  writeln(outfile,' warning - dh < 0, delta q total = ',flowsum:8:2);
  TOTFLOW := TOTFLOW + flowsum;
  phi[1] := p[illegible] - dh;
  head2 [illegible]
  IF([illegible] te =
  B[illegible]
```

```
    Writeln(outfile,' Iteration ',Iteration:3,' at ',time/60:6:3,'
          minutes');
    Writeln(outfile,'   head at each node (in centimeters):');
    RESULTS(np,Phi);
  End;
  IF (PHI[1] < 100.0) Then
  Begin
   phi[1] := refill;
   Writeln(outfile,' Refill (',refill:3:0,' cm) at ',time/60:7:2,'
          minutes');
  End;
  if (iteration > 1) then
  begin
   area_head_tube := Pi*Sqr(dhtube/2);
   area_core := Pi*16.0;        {this should probably be num_x_core*xc[2]}
   length := 6.0;
   IF(Iteration MOD rite2 = 0) THEN
     writeln(graph,' ',iteration:0,' ',time/60:0:4,' ',phi[1]:10:3,' ',
        phi[2]:10:3,' ',phi[3]:10:3,' ',phi[4]:10:3,' ',phi[5]:10:3);
   IF(Iteration MOD rite2 = 0) THEN
     writeln(head,' ',iteration:0,'      ',time/60:0:4,'
         ',head1-head2,' '
         ,flowsum);
    head1 := head2;
  end;
 End;
{*********************************************************************}
{*********************************************************************}
BEGIN                              {MAIN PROGRAM}
{*********************************************************************}
{DATA INPUT SECTION OF THE PROGRAM
     - OPEN DATA FILES}
{ WRITELN(' What is your data file name? (include extension)');
 READLN(NAME);
 }
 reset(DATA, '45elem.dat');
 WRITELN('    data file name: ',name);
 {WRITELN(' Is this the initial run - Y or N (Y if start time = 0)');
  QUES := READKEY;
 writeln(ques:0);
 }ques := 'Y';
 rewrite(OUTFILE,'out.tad');
 rewrite(GRAPH,'GPH.tad');           {file to input in subsequent runs}
 rewrite(HEAD,'HED.tad');
 WRITELN(outfile,'    data file name: ',name);
 Write(graph,'iteration  time (min)  phi[1]    phi[2]   phi[3]');
   Writeln(graph,'  phi[4]  phi[5]');
 Writeln(head,'iteration  time(min)     dh        tot_flow');
 IF (Ques = 'Y') or (Ques = 'y') then
 Begin
  For I := 1 to 150 Do
  Begin
   Phi[I] := 0.0;
   rf[i] := 0.0;
   wcp[i] := 0.0;
   yc[i] := 0.0;
   fx_nd_array[i] := 0;
   sat_nd_array[i] := 0;
   rf[i] := 0.0;
  End;
  INDATA(np,ne,ncoef,num_fx_nds,iptl,kw,dhtube,
        ahtube,core_radi,Dt,Q,Yc,
        Nmtl,Nel,fx_nd_array,thta,delta,
        nsteps,iwt,prnt,Phi);
             { call procedure to read original data, cm }
  time := 0;
  refill := phi[1];
  {phi[2] := -30.0;}
  voll := phi[1]*ahtube;
  FLOW_Calc(Phi,wcp,yc,nel,time,delta,voll,refill,ahtube,dhtube,core_radi,
        dye1,dh,head1,velocity1,length,totflow,np,0,nsteps,
        iwt,prnt,ne,num_fx_nds,num_sat_nds,Kapp_num,sat_nd_array);
  Iteration := 0;
  num_sat_nds := 1;
  sat_nd_array[1] := fx_nd_array[1];
 END
 ELSE
 begin
  Redata(np,ne,nbw,ncoef,num_fx_nds,num_sat_nds,iptl,kw,dhtube,
```

```
        r,ahtube,core_radi,Dt,Q,Yc,Nmtl,fx_nd_array,Nel,sat_nd_array,thta,
        delta,time,nsteps,iwt,prnt,Phi,Rf,wcp,totflow,refill);
  iteration := nsteps;
  nsteps := nsteps + iteration;
end;
{endif}          {procedure to read data from previous run}
   writeln(outfile);
   writeln(outfile);
   WRITELN(OUTFILE,' NUMERICAL SOLUTION OF THE SYSTEM OF DIFFERENTIAL
        EQUATIONS');
   writeln(outfile,'  (initial values)');
   WRITELN(OUTFILE);
   RESULTS(np,Phi);    {call procedure to print initial values}
                   {SOLUTION OF THE TIME DEPENDENT PROBLEM}
REPEAT
  Iteration := Iteration + 1;
  IF(Iteration MOD iwt = 0) THEN
  WRITELN('Iteration  ',iteration:3);
  if (num_sat_nds > num_fx_nds) then
   Sat_elem(num_sat_nds,phi);
  Fnt_elem(np,ne,ncoef,kw,iptl,num_sat_nds,delta,thta,time,dye1,
         Dt,Q,YC,Nmtl,sat_nd_array,Nel,Phi,rf);
  FLOW_Calc(Phi,wcp,yc,nel,time,delta,voll,refill,ahtube,dhtube,core_radi,
        dye1,dh,head1,velocity1,length,totflow,np,iteration,nsteps,
        iwt,prnt,ne,num_fx_nds,num_sat_nds,Kapp_num,sat_nd_array);
UNTIL (KeyPressed) or (Iteration = NSTEPS);
rewrite(DATSAV,'SAV.tad');
CREATE(np,ne,nbw,ncoef,iptl,kw,num_fx_nds,num_sat_nds,dhtube,ahtube,core_radi,
    Dt,Q,Yc,Nmtl,fx_nd_array,Nel,sat_nd_array,thta,delta,time,
    nsteps,iwt,prnt,Phi,Rf,Wcp,totflow,refill);
END.
```

APPENDIX D

Hydraulic Conductivity and Water Content Curves

Water Content

The values used for the determination of water content for a given water potential are found in table D1. This data is from the U.S. Department of Agriculture, Soil Conservation Service. Logarithmic interpolation was used to calculate the water content. Procedure WC_CALC performed these calculations. Procedure LAMBDA used these curves to determine the slope of the $\Psi - \Theta$ line.

Table D1. Water potential - volumetric water content values

water potential (m)	WC Zigenfuss soil	WC Capac soil	WC Lenawee soil
0.0	0.453	0.372	0.435
-0.1033	0.452	0.37	0.433
-0.3099	0.44	0.366	0.429
-0.5165	0.42	0.362	0.425
-1.033	0.377	0.353	0.416
-2.066	0.329	0.338	0.4
-3.4089	0.295	0.321	0.384
-6.198	0.262	0.296	0.358
-10.33	0.236	0.269	0.333
-20.66	0.207	0.229	0.295
-51.65	0.175	0.181	0.247
-103.3	0.156	0.151	0.216
-154.95	0.146	0.137	0.199
-1033.0	0.112	0.093	0.144

Hydraulic conductivity

The values used for the determination of hydraulic conductivity for a given water potential are found in table D2. Full logarithmic interpolation was used to calculate the hydraulic conductivity. Procedure KXY performed these calculations.

Table D2. Water potential - k values

water potential (m)	k (mm/hr) (Pachappa sandy loam)	k (mm/hr) (Indio loam)	k (mm/hr) (Chino clay)
-0.08	20.0	200.0	2.00
-0.10	20.0	200.0	1.90
-0.20	20.0	200.0	1.00
-0.30	18.2	190.0	0.90
-0.40	14.5	180.0	0.6012
-0.50	12.7	150.1	0.50
-0.65	9.1	100.0	0.30
-1.00	1.3	50.0	0.10
-1.50	3.6 E-2	10.5	3.0 E-2
-2.00	1.9 E-2	0.5	2.0 E-2
-2.50	1.3 E-2	0.2	1.05 E-2
-3.00	2.3 E-3	0.1	8.0 E-3
-4.00	3.6 E-3	0.04	3.0 E-3
-4.20	1.9 E-3	0.03	2.5 E-3
-5.00	1.4 E-3	0.01	1.051 E-3
-7.00	7.2 E-4	7.0 E-3	1.008 E-3
-9.00	5.5 E-4	5.0 E-3	5.004 E-4
-20.00	7.3 E-5	3.0 E-4	9.972 E-5
-105.00	1.1 E-5	2.0 E-5	3.00 E-5

APPENDIX E

Procedures for Triangular Element Analysis.

The following procedures were incorporated into the axisymmetric rectangular element VP model. The element matricies were calculated using these procedures for the triangular elements with the results added to the global matricies. The finite element analysis was then exactly as described for the rectangular VP model.

```
{*********************************************************************}
procedure fnt_elem;
{*********************
this procedure sets up the global matricies and
directs the finite element analysis
---------------}
var i_fnt,j_fnt,elem_fnt,flag:   integer;
   kl,klx,
   lam_l:   real;
begin
 for i_fnt := 1 to 200 do
 begin
  fm[i_fnt] := 0.0;
  for j_fnt := 1 to 60 do
  begin
   k[i_fnt,j_fnt] := 0.0;
   c[i_fnt,j_fnt] := 0.0
  end;
 end;
  for elem_fnt := 1 to ne do
    begin
     kxy(elem_fnt,kl,klx);
     sum_k := sum_k + kl;
     lambda(elem_fnt,lam_l,flag);
     sum_lambda := sum_lambda + lam_l;
     if (nel[elem_fnt,4] = 0) then
      tri_mtx(elem_fnt,kl,klx,lam_l)
     else
     glb_mtx(elem_fnt,kl,klx,lam_l);
    end;
                  {calculates the element matricies
                  calculation of the element capacitance (c),
                  stiffness(f), matrices and the element
                  force vector (f)}
     modify;
     matab;
     dcmpbd;
       run_time := run_time + delta;
     multbd;
     for i_fnt := 1 to np do
       rf[i_fnt] := rf[i_fnt]+delta*fm[i_fnt];
     slvbd;
   end;
{*********************************************************************}
procedure tri_mtx(elem:integer;kl,klx,lam_l:real);
{****************
this procedure calculates the element matricies
---------------}
type  esiz = array[1..3,1..3] of real;
     xysiz = array[1..3] of real;
var   ecm, esm,
     es, et,lmp:    esiz;
     ef, x,  z:     xysiz;
     area,
     bi,bj,bk,
     ci,cj,ck,
     dte,
     qe,  rbar:              real;
     i_tri_m,   ii,     j_tri_m,
     jj,      j5,      j6:          integer;
```

```
begin
if ((iptl >= 4) and (elem = 1)) then writeln ('entering Tri_mtx, 1st elem.');
                    {BILINEAR RECTANGULAR ELEMENT WITHOUT THE
                         DERIVATIVE BOUNDARY CONDITION}
                 {RETRIEVAL OF NODAL COORDINATES AND NODE NUMBERS}
   for i_tri_m := 1 to 3 do
    begin
     j_tri_m := nel[elem,i_tri_m];
     x[i_tri_m] := xc[j_tri_m];
     z[i_tri_m] := yc[j_tri_m];
     ef[i_tri_m] := 1.0;
    end;
   bi := z[2] - z[3];
   bj := z[3] - z[1];
   bk := z[1] - z[2];
   ci := x[3] - x[2];
   cj := x[1] - x[3];
   ck := x[2] - x[1];
   rbar := (x[1] + x[2] + x[3])/3;
   area := abs(0.5*((x[1]*z[2] - x[2]*z[1]) + (x[2]*z[3] - x[3]*z[2])
        + (x[3]*z[1] - x[1]*z[3])));
   if (area <= 0) then
    begin
    writeln(outfile,'area for element ',elem:0,' = 0. coordinates are: ');
     for i_tri_m := 1 to 3 do
     writeln(outfile,x[i_tri_m]:10:2,' ',z[i_tri_m]:10:2);
    end;
        {AXISYMMETRIC - TIME DEPENDENT PROBLEM}
            {K - "stiffness" matrix}
    {kDz}
    et[1,1] := ci*ci;
    et[1,2] := ci*cj;
    et[1,3] := ci*ck;
    et[2,1] := et[1,2];
    et[2,2] := cj*cj;
    et[2,3] := cj*ck;
    et[3,1] := et[1,3];
    et[3,2] := et[2,3];
    et[3,3] := ck*ck;
    {for kDr}
    es[1,1] := bi*bi;
    es[1,2] := bi*bj;
    es[1,3] := bi*bk;
    es[2,1] := es[1,2];
    es[2,2] := bj*bj;
    es[2,3] := bj*bk;
    es[3,1] := es[1,3];
    es[3,2] := es[2,3];
    es[3,3] := bk*bk;
                {C - capacitance matrix, time dependent part}
    for i_tri_m := 1 to 3 do
     for j_tri_m := 1 to 3 do
     lmp[i_tri_m,j_tri_m] := 0.0;
    lmp[1,1] := (3*rbar) + x[1];        {for C - lumped formulation}
    lmp[2,2] := (3*rbar) + x[2];
    lmp[3,3] := (3*rbar) + x[3];
                     {CALCULATION OF THE STIFFNESS AND
                      CAPACITANCE MATRICES}
   ii := nmtl[elem];
   dte := dt[ii];
   qe := q[ii];
   for i_tri_m := 1 to 3 do
   begin
    for j_tri_m := 1 to 3 do
    begin
      esm[i_tri_m,j_tri_m] := (2*pi_*rbar*klx
              * es[i_tri_m,j_tri_m]/(4*area))
              + (2*pi_*kl*et[i_tri_m,j_tri_m]/(4*area));
      ecm[i_tri_m,j_tri_m] := 2*pi_*dte*area*lam_l
              *lmp[i_tri_m,j_tri_m]/12;
    end;
    ef[i_tri_m] := pi_*qe*area*ef[i_tri_m]/6;
   end;
           {Direct Stiffness}
           {The major diagonal is stored in column 1,
           followed by the minor diagonals (to NBW)
           in columns 2 - NBW. Zeros are used to fill
           the column to NP, the matrix size is NP x NBW}
    for i_tri_m := 1 to 3 do
```

```
  begin
   j5 := nel[elem,i_tri_m];
   fm[j5] := fm[j5] + ef[i_tri_m];
   for j_tri_m := 1 to 3 do
   begin
    j6 := nel[elem,j_tri_m];
    jj := j6 + 1 - j5;
    if(jj > 0) and (jj <= nbw) then
    begin
     k[j5,jj] := k[j5,jj] + esm[i_tri_m,j_tri_m];
     c[j5,jj] := c[j5,jj] + ecm[i_tri_m,j_tri_m];
    end;
   end;
  end;
end;                                    {PROCEDURE Tri_Mtx}
```

APPENDIX F

Pressure Head for Triangular Element Model

The model with rectangular elements and triangluar elements had the following control parameters:

diameter head tube = 7.1 mm r: k_x = 1.0 * k_y; initial MC = -20.0 m core radius: 40 mm, core length 60 mm k curve: 1 - Pachappa sandy loam (Ksat = 20 mm/hr) wc curve: 1 - Zigenfuss soil

The model may be seen in Figure F1.

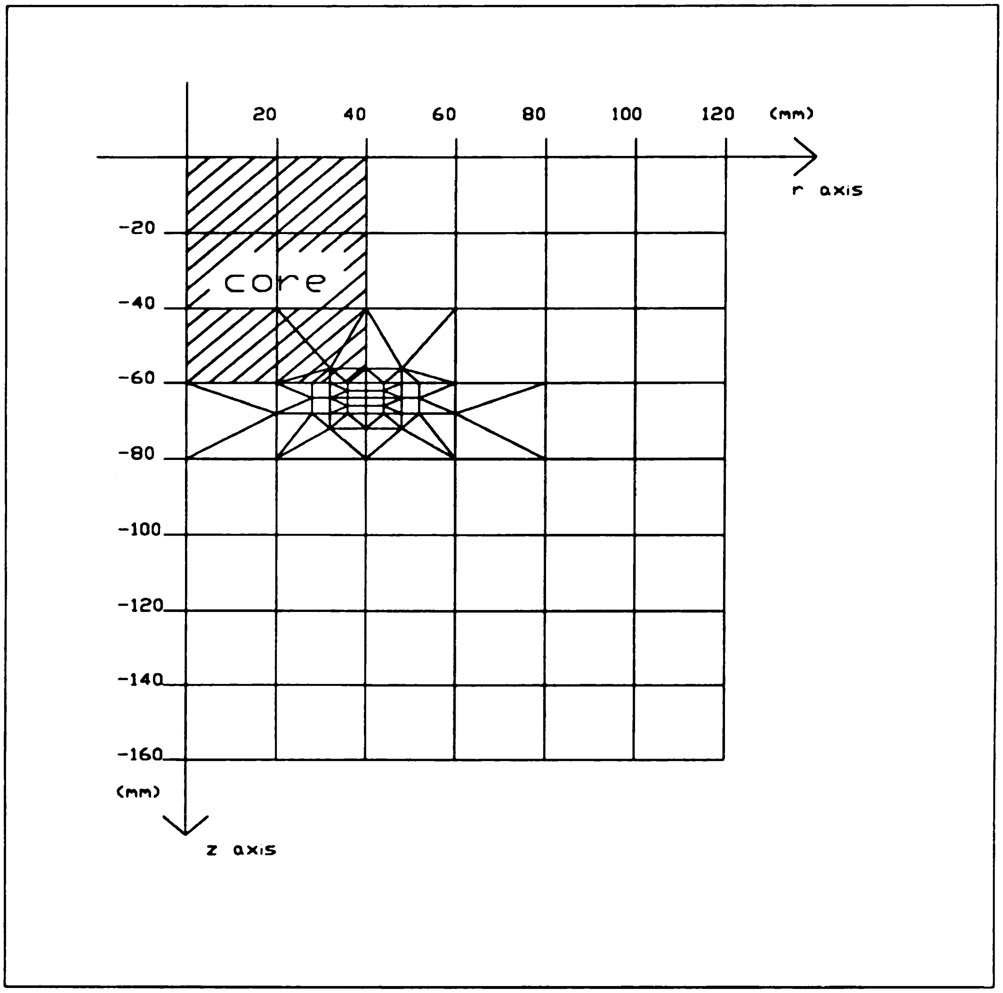

Figure F1. Axisymmetric model with triangular elements

The soil water pressure head distribution may be seen in the output data at 15 minutes and 30 minutes (Tables F1 and F2). Figure F2 shows the isoheads for the file with triangular elements at 15 minutes.

Table F1. Pressure head at each node - triangular model time: 15 minutes					the total inflow at 15 minutes is 2.06 E5 mm^3		
z\r (mm)	0.0 (m)	20.0 (m)	40.0 (m)	60.0 (m)	80.0 (m)	100.0 (m)	120.0 (m)
0.0	1.402	1.402	1.402	-20.029	-19.928	-19.999	-20.000
-20.0	0.701	0.701	0.701	-19.999	-19.933	-19.999	-20.000
-40.0	0.351	0.351	0.351	-19.939	-19.898	-19.971	-19.999
-60.0	-0.238	-0.253	-0.550	-1.041	-3.443	-19.610	-19.999
-80.0	-0.613	-0.778	-1.128	-2.253	-3.297	-19.620	-19.999
-100.0	-1.057	-1.189	-1.600	-6.485	-18.841	-19.654	-19.999
-120.0	-2.547	-3.613	-8.577	-15.970	-19.890	-19.999	-20.000
-140.0	-18.829	-19.193	-19.648	-19.929	-19.992	-20.000	-20.000
-160.0	-19.996	-19.997	-19.998	-19.999	-20.000	-20.000	-20.000

The nodal values shown above were at the (r,z) position indicated. This model had many nodes that were not on this grid, however, so values for those nodes follow, with their coordinates as indicated. (The soil water pressure heads are given in meters, coordinates in mm).

Table F2. Pressure head at each node, triangular model, more nodes

node 24 = -0.181 at (40,-56)	node 51 = -18.794 at (40,-56)
node 25 = -0.312 at (28,-60)	node 52 = -18.902 at (48,-56)
node 26 = -0.346 at (32,-60)	node 53 = -0.625 at (44,-60)
node 27 = -0.374 at (36,-60)	node 54 = -0.695 at (48,-60)
node 28 = -0.550 at (40,-60)	node 55 = -0.781 at (52,-60)
node 29 = -0.449 at (36,-62)	node 56 = -0.630 at (44,-62)
node 30 = -0.546 at (40,-62)	node 57 = -0.637 at (44,-64)
node 31 = -0.404 at (28,-64)	node 58 = -0.702 at (48,-64)
node 32 = -0.429 at (32,-64)	node 59 = -0.778 at (52,-64)
node 33 = -0.496 at (36,-64)	node 60 = -0.651 at (44,-66)
node 34 = -0.566 at (40,-64)	node 61 = -0.669 at (44,-68)
node 35 = -0.543 at (36,-66)	node 62 = -0.729 at (48,-68)
node 36 = -0.587 at (40,-66)	node 63 = -0.787 at (52,-68)
node 37 = -0.506 at (28,-68)	node 64 = -0.886 at (48,-72)
node 38 = -0.522 at (32,-68)	node 65 = -20.029 at (60,0)
node 39 = -0.568 at (36,-68)	node 66 = -19.999 at (60,-20)
node 40 = -0.610 at (40,-68)	node 67 = -19.939 at (60,-40)
node 41 = -0.652 at (32,-72)	node 68 = -1.041 at (60,-60)
node 42 = -0.747 at (40,-72)	node 69 = -0.897 at (60,-68)

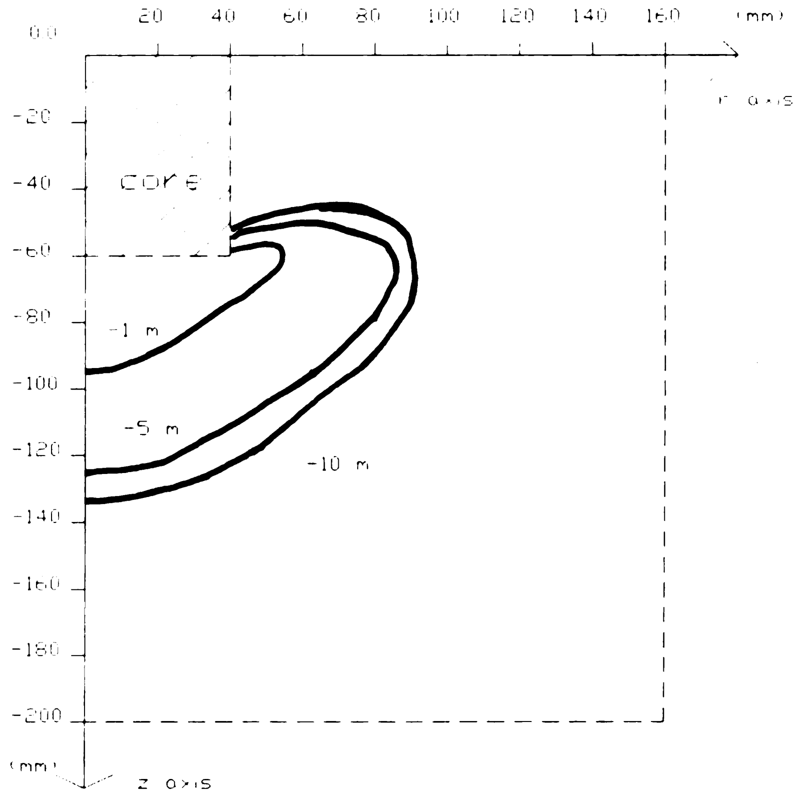

Figure F2. Isoheads at 15 minutes, triangular elements

Table F3. Pressure head at each node - triangular model time: 30 minutes				the total inflow at 30 minutes is 3.316 E5 mm^3			
z\r (mm)	0.0 (m)	20.0 (m)	40.0 (m)	60.0 (m)	80.0 (m)	100.0 (m)	120.0 (m)
0.0	1.429	1.429	1.429	-20.062	-19.843	-19.997	-20.000
-20.0	0.715	0.715	0.715	-19.995	-19.854	-19.995	-19.998
-40.0	0.357	0.357	0.357	-19.733	-19.593	-19.802	-19.909
-60.0	-0.187	-0.209	-0.474	-0.774	-1.342	-7.467	-19.581
-80.0	-0.508	-0.647	-0.839	-1.193	-1.663	-9.158	-19.476
-100.0	-0.812	-0.872	-1.031	-1.414	-5.630	-13.085	-19.704
-120.0	-1.215	-1.297	-1.510	-2.388	-11.945	-19.640	-19.907
-140.0	-4.056	-5.363	-8.886	-13.788	-18.233	-19.946	-19.998
-160.0	-19.015	-19.187	-19.456	-19.761	-19.940	-19.995	-20.000

The nodal values shown above were at the (r,z) position indicated. This model had many nodes that were not on this grid, however, so values for those nodes follow, with their coordinates as indicated. (The soil water pressure heads are given in meters, coordinates in mm).

Table F4. Pressure head at each node, triangular model, more nodes

node 24 = -0.142 at (40,-56)	node 51 = -17.246 at (40,-56)
node 25 = -0.264 at (28,-60)	node 52 = -17.284 at (48,-56)
node 26 = -0.295 at (32,-60)	node 53 = -0.538 at (44,-60)
node 27 = -0.320 at (36,-60)	node 54 = -0.597 at (48,-60)
node 28 = -0.474 at (40,-60)	node 55 = -0.655 at (52,-60)
node 29 = -0.386 at (36,-62)	node 56 = -0.543 at (44,-62)
node 30 = -0.470 at (40,-62)	node 57 = -0.548 at (44,-64)
node 31 = -0.344 at (28,-64)	node 58 = -0.603 at (48,-64)
node 32 = -0.365 at (32,-64)	node 59 = -0.654 at (52,-64)
node 33 = -0.426 at (36,-64)	node 60 = -0.559 at (44,-66)
node 34 = -0.487 at (40,-64)	node 61 = -0.572 at (44,-68)
node 35 = -0.466 at (36,-66)	node 62 = -0.620 at (48,-68)
node 36 = -0.505 at (40,-66)	node 63 = -0.659 at (52,-68)
node 37 = -0.426 at (28,-68)	node 64 = -0.710 at (48,-72)
node 38 = -0.443 at (32,-68)	node 65 = -20.062 at (60, 0)
node 39 = -0.486 at (36,-68)	node 66 = -19.995 at (60,-20)
node 40 = -0.522 at (40,-68)	node 67 = -19.733 at (60,-40)
node 41 = -0.545 at (32,-72)	node 68 = -0.774 at (60,-60)
node 42 = -0.624 at (40,-72)	node 69 = -0.714 at (60,-68)

Figure F3 shows the isoheads after 30 minutes. The extent of each has increased, particularly in the radial direction.

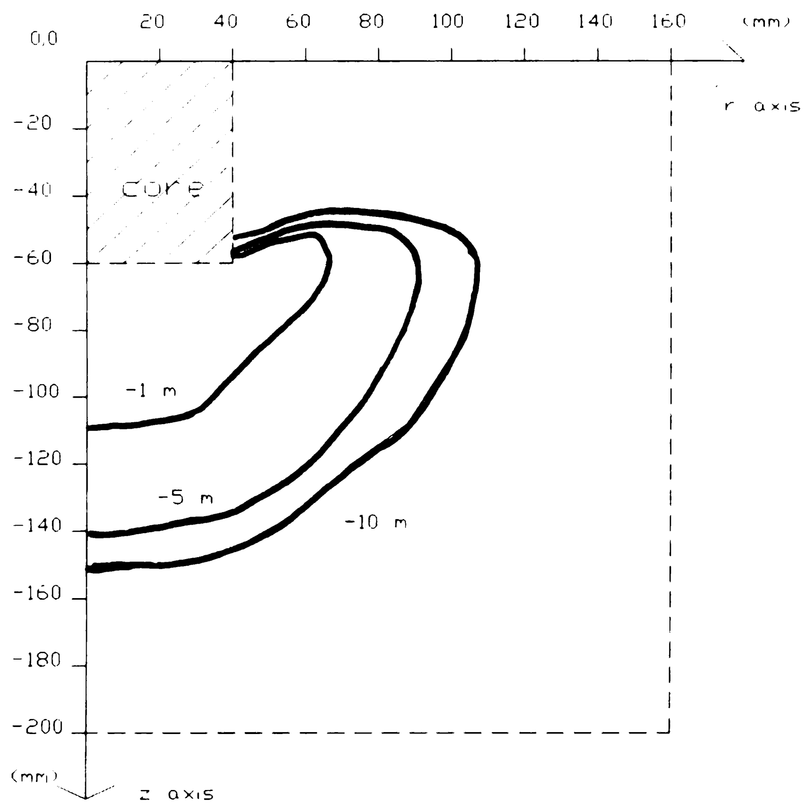

Figure F3. Isoheads at 30 minutes, triangular elements

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