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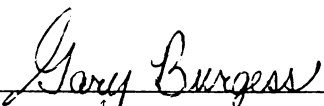
FINITE DIFFERENCE AND FINITE ELEMENT
MODELING OF CLOSED CELL CUSHIONS WITH
BLOCK AND RIBBED GEOMETRIES BASED ON
STRAIN DEPENDENT TERMS

presented by

ANDREW W. CHEN

has been accepted towards fulfillment
of the requirements for

MASTER degree in PACKAGING


Major professor

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Andrew W. Chen

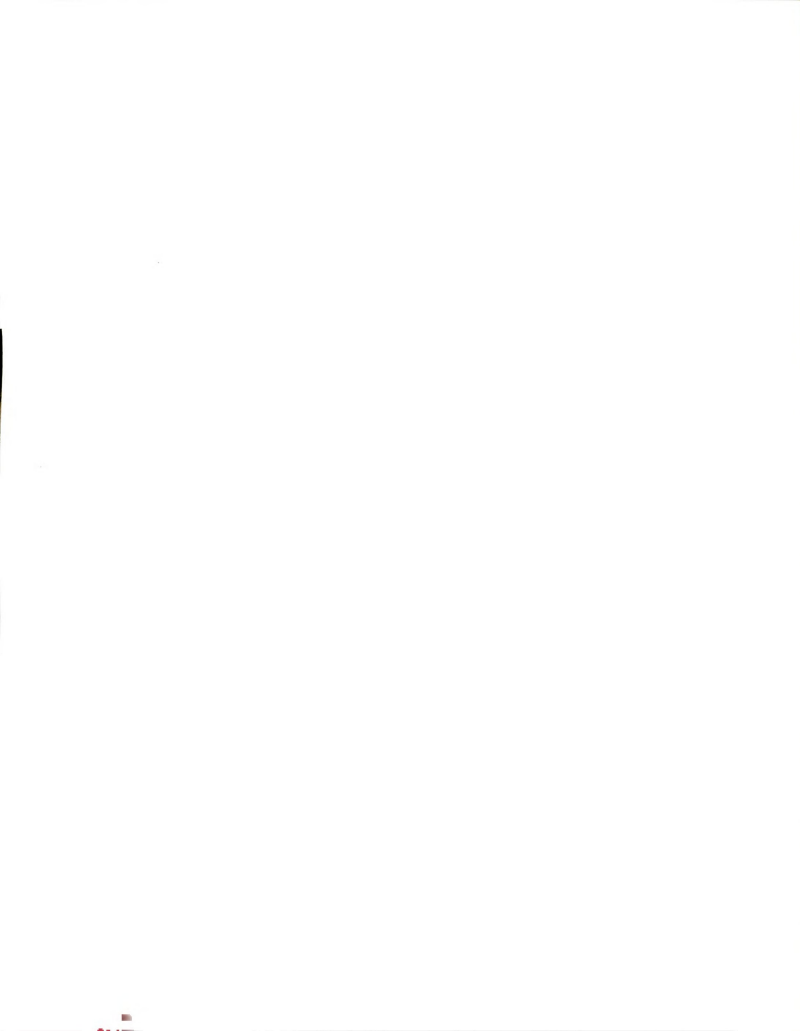
A THESIS

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MASTER OF PACKAGING SCIENCE

School of Packaging

1993



ABSTRACT

FINITE DIFFERENCE AND FINITE ELEMENT MODELING OF CLOSED CELL CUSHIONS WITH BLOCK AND RIBBED GEOMETRIES BASED ON STRAIN DEPENDENT TERMS

By

Andrew W. Chen

Closed cell cushions behave in a manner such that idealized models cannot be used. Hooke's Law is one such example. The compression of a closed cell cushion is non-linear, visco-elastic and time dependent. Because of this dependency, the model to replicate cushion curve data must account for thermodynamics and heat transfer. The derivation of this model is based on three laws: Newton's second law of motion, the Gas Law, and the First Law of Thermodynamics.

Solving three differential equations derived from the three laws based on finite difference yield results that are comparable to existing cushion curve data. The problem is the derivation is tedious and cannot be applied to non-block cushions. Hence, the polytropic model is developed to enable the use of a more powerful numerical method, the finite element method (FEM). The polytropic model is put into a constitutive equation form where the variables are all strain dependent. The result of the FEM compares well with published cushion curve data and experimental results.

To Christina, Charles and Shang for all of their support

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CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

Cellular polymers are multi-phase material systems that consist of a polymer matrix and a fluid phase. The fluid is generally a gas and is either trapped or continuous in the polymer matrix [9]. This has lead to the development of closed-cell and open-cell cushions. Closed-cell cushions are foams in which the air is trapped within individual unbroken cells. This type of foam is widely used in the packaging industry for cushioning fragile items. Open cell foams in which the air is able to circulate freely within the foam structure behave differently because the air flows rather than compresses [2,22]. In this study, only closed cell foams were studied.

The behavior of a closed cell cushion is generally described by cushion curves [2]. Cushion curves relate the shock transmitted to a product to the product weight, cushion thickness and bearing area of the cushioning material. Because of the vast collection of cushion curve information, the engineer and designer of protective packaging can be overwhelmed. Naturally one would therefore try to reduce the size of cushion curve data. A reasonable approach would be to model the spectrum of cushion curve data in the form of descriptive equations or some type of cushion model [3].

The straight forward approach toward modeling cushion behavior would be to relate the cushion compression to the force the cushion exerts upon a product. The solution of this solid mechanics problem must consider: [5]

1. Equations of motion
2. Geometry

3. Material constitutive laws (stress-strain)

If the relationship between stress and strain is linear, the constitutive law is known as Hooke's Law [15]. Hooke's Law is an idealized model which some physical systems obey. The problem with using Hooke's Law to describe the compression of a cushion, however, is that cushion behavior is known to be non-linear, visco-elastic, time dependent and strongly related to temperature. Despite these dependencies, there are models that try to "fit" foam cushions into the Hookean model [9,21,22].

The compression of foam cannot be accurately modeled by the Hookean equations that assume a linear relationship between stress and strain. To show this, it is sufficient to look at the prediction for peak G-level to a mass dropped onto a cushion using Hooke's Law and to compare it to published cushion data [2]. The basic steps are:

- a. stress = modulus x strain (Hooke's Law)

$$\sigma = E\varepsilon \quad \text{eq 1}$$

where σ = stress (psi), ε = strain and

E = material modulus (psi)

- b. energy absorbed by cushion =

(energy density) x (cushion volume)

$$U = \frac{\sigma^2}{2E}(Ab) \quad \text{eq 2}$$

where A is the cushion bearing area and b is the thickness [26].

- c. energy absorbed = potential energy [23]

$$U = Wh \quad \text{eq 3}$$

where W is the weight dropped from a height h

- d. Combining a, b, c and solving for the peak stress and using $\sigma=F/A$ where F is force,

$$F = \sqrt{\frac{2hWEA}{b}} \quad \text{eq 4}$$

- e. Force = mass x acceleration = weight x G

$$F = WG \quad \text{eq 5}$$

- f. Solving for peak deceleration,

$$G = \sqrt{\frac{2hE}{b(s)}} \quad \text{eq 6}$$

where static loading is defined as $s=W/A$.

According to equation 6, G should continue to decrease as the static loading increases. This is shown in Figure 1 under linear cushion behavior. This agrees with known cushion performance, but only up to a point. It fails to predict the increase in deceleration as static loading increases. Since the only assumption involved in the analysis is Hooke's Law, it must be concluded that equation 1 is in error [22].

To model the cushion accurately, one needs to develop more realistic constitutive equations. In order to develop the constitutive equations one must first consider the nature of the compression process. This inevitably leads to considering heat transfer.

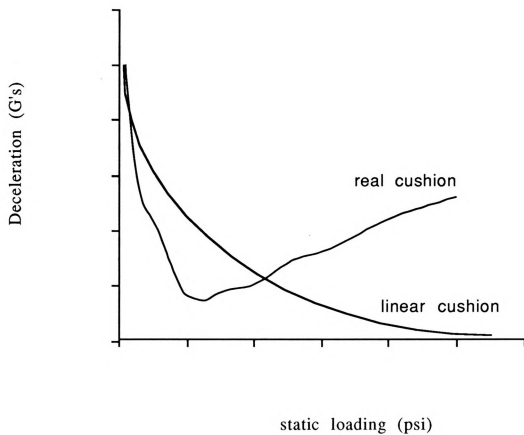


Figure 1. Real cushion behavior compared to a Hookean cushion

As a closed-cell cushion compresses, the air volume decreases and the pressure and temperature increase. Since air temperature is now higher than the cell wall temperature, heat is transferred from the air to the walls. This raises the wall temperature only slightly because of its large heat capacity so that walls can be considered as a nearly constant temperature heat sink. So during the entire compression and expansion process, energy is extracted from air and therefore not given back to the weight (see Figure (2)). As a consequence, the stress depends not only on strain, but strain rate as well [2]. Unfortunately, there has been very little work done on models with strain rate dependent terms.

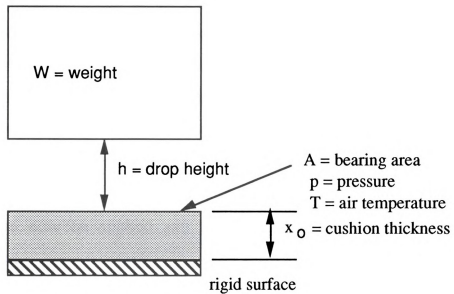


Figure 2 Schematic of weight in a free fall onto a cushion

CHAPTER 2

MATERIALS AND METHODS

2.1 Derivation of the Governing Differential Equations

In modeling a closed cell foam, three coupled differential equations must be solved. The three equations are derived from Newton's second law of motion, the Ideal Gas Law, and the First Law of Thermodynamics [2]. Three equations are required since there are three time dependent variables involved when the cushion is compressed. The variables are the thickness of the cushion denoted by x , the air pressure p within the cells, and the air temperature, T . Figure 2 shows a weight in a free fall from a height h onto a block of closed-cell cushion of thickness x_0 with a bearing area A . Newton's law can now be applied and the first governing equation can be derived.

In order to solve for the deceleration of the weight, a force balance is performed on the weight. The free body diagram in Figure 3 shows the weight and a cut away portion of the cushion. Newton's second law states that:

$$\sum F = \frac{W}{g} \frac{d^2x}{dt^2} \quad \text{eq 7}$$

$$pA + \sigma A - p_0A - W = \frac{W}{g} \frac{d^2x}{dt^2} \quad \text{eq 8}$$

where σ is the yield pressure required to compress the foam structure (less the air), p_0 is normal atmospheric pressure, p is the cell air pressure during compression (assumed to be the same for each cell) and g is the acceleration due to gravitational pull.

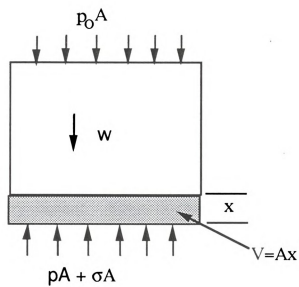


Figure 3 Free body diagram of cushion and weight

Solving for acceleration and rearranging equation 7 gives

$$\frac{d^2x}{dt^2} = g \left[\frac{\sigma + p - p_o}{s} - 1 \right] \quad \text{eq 9}$$

Equation 9 cannot be solved because the variable p is a function of time.

The second governing equation which relates the air pressure to the other variables is the ideal gas law, [18]

$$pV = nRT \quad \text{eq 10}$$

where V is the volume of the air in the cushion, n is the fixed number of moles of air trapped in the cushion, R is the universal gas constant, and T is the absolute temperature. Given that the quantity of air in a closed cell foam is constant, equation 10 states that pV/T is constant at every instant. Since $V = Ax$, this leads to

$$p = p_o \left(\frac{x_o}{x} \right) \left(\frac{T}{T_o} \right) \quad \text{eq 11}$$

Equation 11 relates the pressure to the position, but the new variable T has appeared. Consequently, another relation must be derived. The third equation required to solve the system of equations is the energy equation. The energy equation is applied to the fixed quantity of air within the foam using the first law of thermodynamics applied to the closed system, [25]

$$\text{work done} + \text{heat added} = \text{change in internal energy}$$

During the compression and expansion of the cells surrounding the air, there is work done on the air. This is known as simple boundary work [25]. Heat is added to the air as a result of compression or expansion and takes place through convection heat transfer to the

cell walls. The change in the internal energy of the air is the direct result of a change in temperature [10,25]. Mathematically, the first law of thermodynamics can be written as,

$$\delta q + \delta w = du \quad \text{eq 12}$$

For a closed system over a small time period, the work done, the heat added and the change in internal energy can be expressed as,

$$\delta w = -pdV \quad \text{eq 12a}$$

$$\delta q = -HS(T - T_0)dt \quad \text{eq 12b}$$

where H is the convection heat transfer coefficient, S is the total surface area of the cells in the foam matrix, T is the instantaneous air temperature and T_0 is the ambient air temperature. Also,

$$du = mC_vdT \quad \text{eq 12c}$$

where m is the total mass of the air, and C_v is the constant volume specific heat for air. Substitution into equation 12 yields,

$$-pdV + -HS(T - T_0)dt = mC_vdT \quad \text{eq 13}$$

Rearranging with $dV=Adx$ and dividing by dt , the third governing equation becomes,

$$\frac{dT}{dt} = -\left(\frac{pA}{mC_v}\right)\frac{dx}{dt} - \left(\frac{HS}{mC_v}\right)(T - T_0) \quad \text{eq 14}$$

In summary, the three governing equations are:

$$\text{Newton's Law} \quad \frac{d^2x}{dt^2} = g\left[\frac{\sigma + p - p_0}{s} - 1\right] \quad \text{eq 9}$$

$$\text{The Ideal Gas Law} \quad p = p_0\left(\frac{x_0}{x}\right)\left(\frac{T}{T_0}\right) \quad \text{eq 11}$$

$$\text{Energy Balance} \quad \frac{dT}{dt} = -\left(\frac{pA}{mC_v}\right)\frac{dx}{dt} - \left(\frac{HS}{mC_v}\right)(T - T_0) \quad \text{eq 14}$$

The three coupled equations may be reduced to two equations since equation 11 is not a differential equation. Substituting p in equation 11 into equations 9 and 14 yields

$$\frac{d^2x}{dt^2} = g \left(\frac{\sigma - p_o}{s} - 1 \right) + \left(\frac{g p_o x_o}{s T_o} \right) \frac{T}{x} \quad \text{eq 15}$$

$$\frac{dT}{dt} = - \left(\frac{A}{m C_v} \frac{p_o x_o}{T_o} \right) \frac{T}{x} \frac{dx}{dt} + \left(\frac{HS}{m C_v} \right) T_o - \left(\frac{HS}{m C_v} \right) T \quad \text{eq 16}$$

Since the terms in parenthesis are constants for a particular cushion, these two equations can be written as:

$$\frac{d^2x}{dt^2} = K1 + K2 \left(\frac{T}{x} \right) \quad \text{eq 15a}$$

$$\frac{dT}{dt} = -K3 \left(\frac{T}{x} \frac{dx}{dt} \right) + K4 - K5(T) \quad \text{eq 16a}$$

where:

$$K1 = g \left[\frac{\sigma - p_o}{s} - 1 \right] \quad \text{eq 17a}$$

$$K2 = \left(\frac{g p_o x_o}{s T_o} \right) \quad \text{eq 17b}$$

$$K3 = + \left(\frac{A}{m C_v} \frac{p_o x_o}{T_o} \right) \quad \text{eq 17c}$$

$$K4 = \left(\frac{HS}{m C_v} \right) T_o \quad \text{eq 17d}$$

$$K5 = \left(\frac{HS}{m C_v} \right) \quad \text{eq 17e}$$

In order to examine the effect of cushion geometry and cell dimensions on the constants K1 through K5, certain terms in the above expressions need to be examined. Specifically, they are σ , s, m, and H.

2.2 The Yield Pressure σ

The yield pressure, σ , is the stress required to compress the cell matrix alone and contributes significantly to decelerating the product. The polymer matrix can be considered a thin wall membrane which supports no bending moments [20,21,22]. The cell can be assumed to under go both buckling and yielding [9]. Modeling of the cell matrix is difficult due to the variability in cell size and the complexity of geometry. Many geometry's have been proposed such as the spherical cell connected by a cylindrical strut model, the pentagonal dodecahedron cell and the tetrakidecahedronal cell [9,22]. The geometry will vary depending on the material and the foaming process. Matonis [14] has proposed an equivalent model to simplify the analysis of the cell structure. The model proposed is a cubical plate model which consists of layered cubical arrays as in Figure 4.

Figure 4 shows a hypothetical section from a closed cell cushion. The yield pressure σ , is defined as the vertical surface pressure which causes the cell walls to yield. A simple force balance yields,

$$\sigma = \sigma_{yp}(A_w/A) \quad \text{eq 18}$$

where σ_{yp} is the material yield point stress, A_w is the total cross sectional area of only the walls and A is the apparent bearing area ($l*w$). The total amount of wall support area in Figure 4 is,

$$A_w = [l(w/a) + w(l/a)]t = 2(lwt)/a \quad \text{eq 19}$$

Rearranging and substituting results in

$$\sigma = (2t/a)\sigma_{yp} \quad \text{eq 20}$$

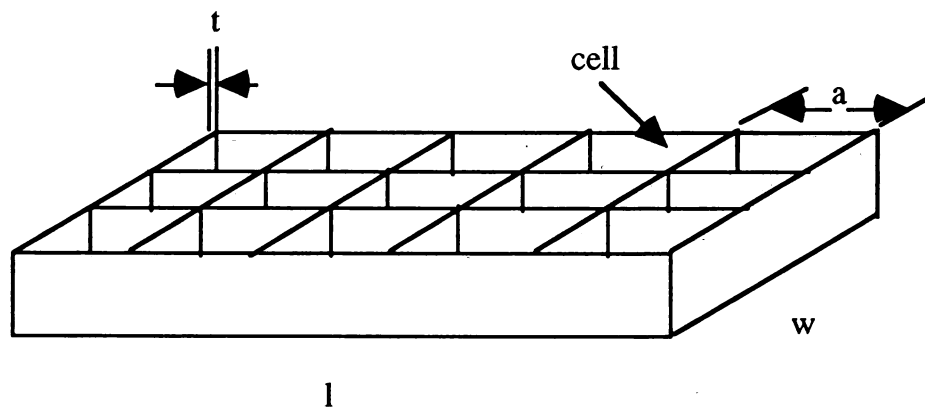


Figure 4 Foam section with a hypothetical array of square cells



Typical values for the cell dimensions for Arcel 512 [16] are, $a=45$ mils, $t=0.3$ mils [21] and $\sigma_{yp}=1500$ psi [9,20]. Applying equation 20 to Arcel 512 gives a yield pressure of approximately 20 psi.

2.3 The Total Cell Wall Surface Area S

The total cell wall surface area can be calculated by considering the average surface area of one cell and multiplying by the number of cells in a volume of cushioning material.

$$s = (6a^2) \left(\frac{l}{a} \frac{w}{a} \frac{x_o}{a} \right) = 6 \frac{lw x_o}{a} = 6 \frac{V_o}{a} \quad \text{eq 21}$$

where V_o is the original volume of the cushioning material.

2.4 The Convection Heat Transfer Coefficient H

The heat transfer rate is related to the overall temperature difference between the gas in the encapsulated cell and the cell wall by the convection heat transfer coefficient, H [9]. This term has been found to be 4 BTU/(hr-ft²-°F) for cells with the dimensions given in 2.2 [2].

2.5 The Total Air Mass m

The mass of the air is estimated to be the volume of the cushion times the density of the air. The foam itself contributes very little to the volume of the cushion. This can be realized by taking an open cell cushion and compressing the cushion. The open cell cushion can be made to flatten out almost completely. The mass is,

$$m = \rho(l * w * x_o) \quad \text{eq 22}$$

where the air density is $\rho = .0000428$ lb/in³ = .074 lb/ft³ [17].

Substituting the results for σ , s , m , and H into the expressions for $K1$ through $K5$ gives,

$$K1 = g \frac{1}{s} \left[\frac{2\sigma_{pp} \cdot t}{a} - p_o - s \right] \quad \text{eq 23}$$

$$K2 = \left(\frac{g p_o \alpha_o}{T_o} \right) \left(\frac{1}{s} \right) \quad \text{eq 24}$$

$$K3 = \left(\frac{p_o}{C_v \rho T_o} \right) \quad \text{eq 25}$$

$$K4 = \left(\frac{6HT_o}{C_v \rho} \right) \left(\frac{1}{a} \right) \quad \text{eq 26}$$

$$K5 = \left(\frac{6H}{C_v \rho} \right) \left(\frac{1}{a} \right) \quad \text{eq 27}$$

The terms on the right hand sides of equations 23 through 27 have been grouped using parenthesis in order to illustrate the effects of material properties, cell dimensions, cushion dimensions and environmental terms on the constants. Without this simplification, one cannot deduce the effects of modifying the cell or cushion structure. The constant K3 for example is dependent on air properties alone where as K4 and K5 depend on cell size as well. The constants K1 and K2 depend on air properties, cell dimensions, cushion dimensions, and the weight of the falling mass.

Since K1-K5 depends on certain properties of air, substituting the physical constants in Table 1 into the expressions for K1-K5 further simplifies them.

Table 1. Gas Related Constants

$H = 4 \text{ Btu/hr/ft}^2/^{\circ}\text{F}$ (heat transfer coefficient)

$p_o = 14.7 \text{ psi}$ (initial air pressure of cells)

$T_o = 530 ^{\circ}\text{R}$ (initial cell air temperature)

$C_v = .1715 \text{ Btu/lb}^{\circ}\text{F}$ (constant volume specific heat of air)

$\rho = .0000428 \text{ lb/in}^3 = .074 \text{ lb/ft}^3$ (density of air at $530 ^{\circ}\text{R}$)

Substituting $g=386.4 \text{ in/sec}^2$ and the values in Table 1 into equations 23 to 27 gives the constants in terms of quantities which relate to cell sizes and cushion size:

$$K1 = 386.4 \frac{1}{(s)} \left[2 \cdot \left(\frac{\sigma_{yp} \cdot t}{a} \right) - 14.7 - (s) \right] \quad \text{eq 28}$$

$$K2 = 10.72(x_o) \left(\frac{1}{s} \right) \quad \text{eq 29}$$

$$K3 = .04 \quad \text{eq 30}$$

$$K4 = 3350606 \left(\frac{1}{a} \right) \quad \text{eq 31}$$

$$K5 = 6322 \left(\frac{1}{a} \right) \quad \text{eq 32}$$

where the static loading, s and yield pressure, σ_{yp} are in psi and the cell dimensions, a and t , are in mils.

Now for a particular cushion like Arcel 512 [16] where $a = 45$ mils, $t=0.3$ mil, $\sigma_{yp}=1500$ psi, the expressions are further simplified to

$$K1 = \left(\frac{2048}{s} - 386.4 \right) \left[\frac{\text{in}}{\text{sec}^2} \right] \quad \text{eq 33}$$

$$K2 = 10.72 \cdot (x_o) \left(\frac{1}{s} \right) \left[\frac{\text{lb}^2}{\text{sec}^2 \cdot ^\circ R \cdot \text{in}^2} \right] \quad \text{eq 34}$$

$$K3 = .04 \quad \text{eq 35}$$

$$K4 = 74458 \left[\frac{^\circ R}{\text{sec}} \right] \quad \text{eq 36}$$

$$K5 = 140 \left[\frac{1}{\text{sec}} \right] \quad \text{eq 37}$$

At this point, it is evident that only K1 and K2 depend on the types of variables normally associated with cushion design: the static loading s , and the cushion thickness x_0 . An additional parameter not contained in K1 and K2 but important to cushion design, is the drop height information. The drop height parameter will enter the analysis through the governing equations as an initial condition on velocity. Specifically, the initial or impact velocity is [4],

$$v_i = \sqrt{2 \cdot g \cdot h} \quad \text{eq 38}$$

Once the weight, drop height, and the original cushion thickness are known, the constants K1-K5 are known and the differential equations can be solved numerically for the pressure, p , temperature, T , and thickness, x . The procedure for doing this is outlined in the next section. A computer program based on this procedure was written to accept drop data (W , w , l , x_0 , and h) and generate the shock pulse.

2.6 Solving the Governing Differential Equations With the Forward Finite Difference Method

From the three governing equations, a series of values for thickness, temperature and pressure can be generated. In order to generate these values, the forward finite difference method was used. The technique involves initializing the variables and stepping the equations forward in time with an increment of Δt . This process will continue until the experimenter decides to stop the march forward. Referring back to the summary of the differential equations, notice that only pressure can be found from the three equations (equations 9, 11, 14). The thickness and temperature variables are in differential form. In order to calculate the thickness

and temperature, equations that relate position to acceleration and temperature to temperature rate are required.

The equations that relate position and velocity to acceleration are equations derived from kinematics in one dimension [12,13,23]. These equations are only valid for uniformly accelerated motion. For some differential time dt , these equations hold true and for some finite time Δt , they are only approximate.

$$x = x_o + \frac{dx}{dt_o} \Delta t + \frac{1}{2} \frac{d^2x}{dt_o^2} \Delta t^2 \quad \text{eq 39}$$

$$\frac{dx}{dt} = \frac{dx}{dt_o} + \frac{d^2x}{dt_o^2} \Delta t \quad \text{eq 40}$$

In a similar manner, the new temperature will depend on the rate of temperature change.

$$T = T_o + \frac{dT}{dt_o} \Delta t \quad \text{eq 41}$$

The three differential equations (9,11,14) coupled with the three kinematic equations (39,40,41) are the six equations necessary to solve the finite difference problem. The equations were solved with the Macintosh version of the Excel 4.0 spread sheet. An example of the calculation with the spread sheet is shown in Table 3. The drop data, material constants, environmental constants and gas related constants used for this example are shown in Table 2.

p	4E-05	l	10	g	386.4	K1	1661.5	
H	0.072	w	10	a	45	K2	21.434	
S	26667	wgt	100	t	0.3	K3	0.4043	
To	530	Cv	0.1715	σ_{yp}	1500	K4	74458	
Xo	2	m	0.00856	σ	20	K5	140.49	
Po	14.7	s	1	Δt	0.001			
hgt	36	A	100	Vi	-166.8			

Table 2 Initial conditions

	Δt	T (R)	X (in)	P (psi)	dX/dt	G	dT/dt	d2x/dt2
0	0	530.00	2.000	14.700	-166.80	19.00	17871	7342
1	0.001	547.87	1.837	16.545	-159.45	20.85	16718	8055
2	0.002	564.59	1.681	18.626	-151.40	22.93	15694	8859
3	0.003	580.28	1.534	20.977	-142.54	25.28	14730	9767
4	0.004	595.01	1.397	23.630	-132.77	27.93	13734	10792
5	0.005	608.75	1.269	26.601	-121.98	30.90	12587	11940
6	0.006	621.33	1.153	29.882	-110.04	34.18	11135	13208
7	0.007	632.47	1.050	33.414	-96.83	37.71	9187	14573
8	0.008	641.66	0.960	37.060	-82.26	41.36	6534	15981
9	0.009	648.19	0.886	40.575	-66.28	44.87	2997	17340
10	0.01	651.19	0.829	43.596	-48.94	47.90	-1474	18507
11	0.011	649.71	0.789	45.686	-30.43	49.99	-6684	19315
12	0.012	643.03	0.768	46.439	-11.12	50.74	-12116	19606
13	0.013	630.91	0.767	45.642	8.49	49.94	-17000	19298
14	0.014	613.91	0.785	43.386	27.78	47.69	-20575	18426
15	0.015	593.34	0.822	40.045	46.21	44.34	-22386	17135
16	0.016	570.95	0.877	36.126	63.35	40.43	-22433	15621
17	0.017	548.52	0.948	32.101	78.97	36.40	-21078	14065
18	0.018	527.44	1.034	28.300	93.03	32.60	-18830	12597
19	0.019	508.61	1.133	24.898	105.63	29.20	-16164	11282
20	0.02	492.45	1.244	21.951	116.91	26.25	-13429	10143
21	0.021	479.02	1.366	19.446	127.05	23.75	-10846	9176
22	0.022	468.17	1.498	17.336	136.23	21.64	-8527	8360
23	0.023	459.65	1.638	15.562	144.59	19.86	-6516	7674
24	0.024	453.13	1.787	14.067	152.26	18.37	-4812	7097
25	0.025	448.32	1.943	12.801	159.36	17.10	-3394	6608

Table 3 Solution to system of differential equations

The procedure used in the Excel spread sheet involved first initializing all the variables at time $t=0$. Each column and row in the worksheet represents one of the variables defined earlier at the instant shown. For example, d^2x/dt^2 represents the acceleration of the mass on the cushion and dx/dt is the velocity of the mass at a instant in time. dT/dt is the temperature rate, T is air temperature, x is the thickness of the cushion and p is the air pressure. G is the dimensionless unit of acceleration defined as the acceleration at a particular time interval divided by gravitational acceleration. This variable is needed to specify the severity of the shock. When the pressure p returns to its initial value of 14.7 psi, the weight is leaving the cushion and the finite difference calculation is completed.

For the particular run in Table 4 the peak deceleration is seen to be 50.74 G's and the shock duration is approximately 24 msec. The rebound velocity is 152.26 in/sec which implies the coefficient of restitution [4] to be $e=.91$. Considering the complexity of the three governing differential equations, the solution over the duration of the shock pulse seem to produce reasonable results. For example the velocities and accelerations have magnitudes and direction that are reasonable. The value of G increases and decreases and the coefficient of restitution is less than 1. The actual value of G from the Arcel Cushion Curves in Appendix B is 50 G's.

To check the validity of the model and the finite difference solution, the results of the finite difference solution for 12 inch and a 36 inch drops were compared to the published cushion curves [16]. Figure 5 and Figure 6 show that the finite difference equations predict the concavity of the curve and the results are similar to that

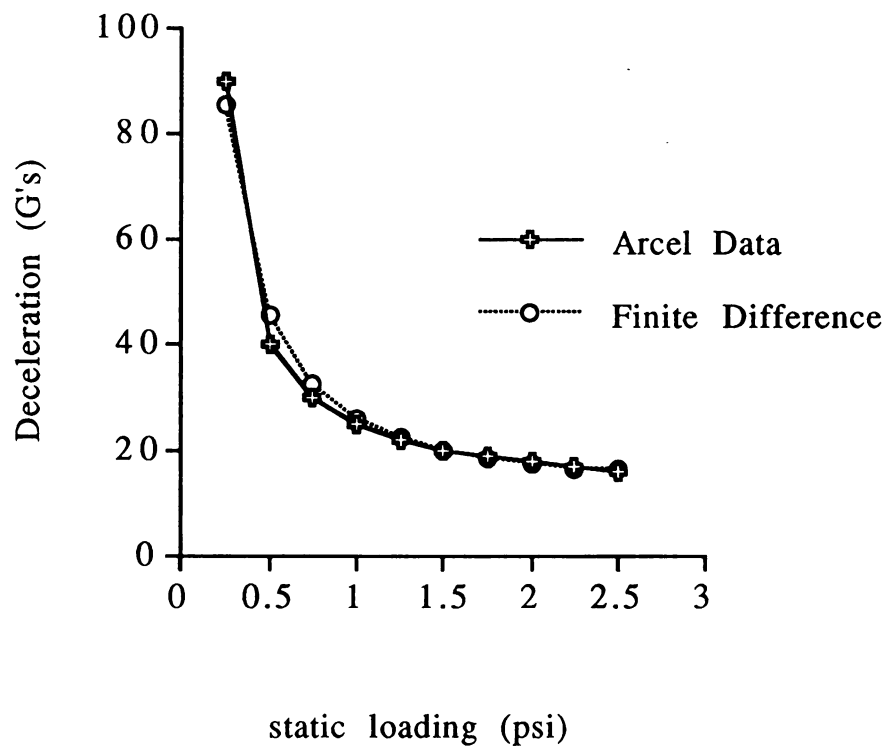


Figure 5 Cushion curve for a 12 inch drop onto a 2 inch block cushion



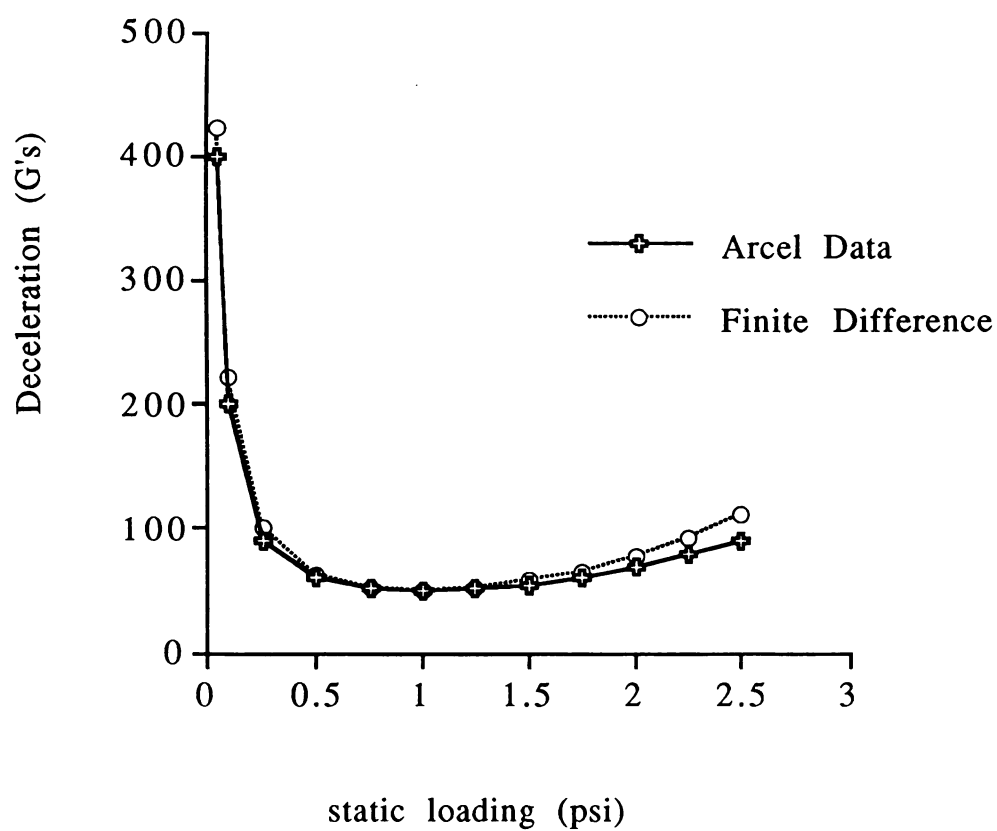


Figure 6 Cushion curve for a 36 inch drop onto a 2 inch block cushion

of Arcel 512 (see Tables 4 and 5 for errors). Assuming that the finite difference method solves the system of differential equations accurately, the model seems to under estimate G's for a low drop height and over estimate G's for a high drop height. This is partially due to the fact that actual experimental cushion curves are in error by 10% to 15% [27] on average and partially due to the use of constant values for air properties. The heat transfer coefficient H in particular is only accurate to within $\pm 50\%$ and is known to be a function of the temperature difference [10, 27]. The constant value of $H=4 \text{ Btu/hr/ft}^2$ used in the solution could therefore be in error by as much as $\pm 3 \text{ Btu/hr/ft}^2$. The experimental error in reading the cushion curve must also be included in accessing the accuracy of the finite difference method. At this point the important fact to note is the ability of the model to simulate cushion performance and the significance of incorporating heat transfer and thermodynamic terms. The motivation now will be to simplify the complexity of the three differential equations by modeling the cushion by a polytropic process.

	Arcel	finite	
	cushion	difference	percent
static loading	curve	prediction	error
	(G's)	(G's)	
0.05	400	405.44	-1.36
0.1	200	205.3	-2.65
0.25	90	85.2	5.33
0.5	48	45.56	5.08
0.75	35	32.35	7.57
1	30	25.87	13.77
1.25	25	22.08	11.68
1.5	24	19.8	17.50
1.75	23	18.28	20.52
2	22	17.3	21.36
2.25	22	16.7	24.09
2.5	23	16.35	28.91

Table 4 Percent error between finite difference result and Arcel 512 cushion curve data for a 12 inch drop onto a 2 inch thick block cushion



	Arcel	finite	
	cushion	difference	error
static loading	curve	prediction	finite diff.
	(G's)	(G's)	
0.05	400	423.1	-5.78
0.1	200	221.1	-10.55
0.25	90	100.34	-11.49
0.5	60	62.59	-4.32
0.75	51	52.68	-3.29
1	50	50.74	-1.48
1.25	52	52.77	-1.48
1.5	55	57.92	-5.31
1.75	60	65.3	-8.83
2	69	76.6	-11.01
2.25	80	91.66	-14.58
2.5	90	111.24	-23.60

Table 5 Percent error between finite difference results and Arcel 512 cushion curve data for a 36 inch drop onto a 2 inch thick block cushion



2.7 The Polytropic Process

The solution process so far has been tedious and complicated because the generation of G's requires lengthy calculations. An alternative method to generate G's is to model the compression of air as a polytropic process [25].

$$pV^k = \text{constant} \quad \text{eq 42}$$

This model explicitly excludes the heat transfer and the thermodynamic terms. Implicitly, all of these terms are included in the polytropic constant k [25]. The motivation for using the polytropic model is to simplify the stress-strain analysis so that cushion geometries which are more complicated than a simple block may be analyzed. This then allows for techniques such as the finite element method to be used. It also simplifies the model considerably by requiring only one physical constant, k instead of the many thermodynamic terms previously used.

As a result of the finite difference calculations in Table 3, values of position(thickness) and pressure at specific times are known. When a complete table has been generated, the material constant k in equation 42 can be found by fitting the known data to the polytropic equation. The value of k will determine the amount of heat transfer that occurs in the cushion. The value of k is 1.4 for an adiabatic process: in an adiabatic process, compression is assumed to occur so quickly that no heat is transferred to the cell walls. The value of k is 1.0 for an isothermal process: in an isothermal process, compression is assumed to occur so slowly relative to the heat transfer rate, that the air remains at constant temperature as it is compressed.

An adiabatic process would result if H or S were extremely small or if the cushion is compressed quickly as in a drop. If H or S were very large or if the cushion is compressed very slowly, then the process would approach isothermal conditions. Since H and S are both finite in reality, the compression process will be neither isothermal nor adiabatic. The expected result is $1.0 < k < 1.4$.

Predicting the polytropic constant involves fitting equation 42 to the data in Table 2. Since $pV^k = \text{constant}$ and the bearing area is constant, we can fit the data to $px^k = \text{constant}$ instead. Since the pressure and temperature are initially p_0 and x_0 , this leads to:

$$\left(\frac{p}{p_0}\right)\left(\frac{x}{x_0}\right)^k = 1 \quad \text{eq 43}$$

The objective now is to assume that equation 43 is true and to choose k to "best fit" the data in Table 3. This is a necessary step in order to develop the constitutive equations for a closed cell foam.

2.7.1 Method 1

Because the air compression process is not truly a polytropic process, equation 43 will not be satisfied exactly because k is chosen to best fit all the data points for pressure and position simultaneously [6]. As a result, there will be an error between the actual value and the expected value. Defining the error at any instant i using the numbers from the i th row in Table 1 as

$$\varepsilon_i = \left(\frac{p_i}{p_0}\right)\left(\frac{x_i}{x_0}\right)^k - 1 \quad \text{eq 44}$$

we can find the variance in the error. This done by calculating the sum of squares of the errors,

$$SSE = \sum_{i=1}^N \left(\left(\frac{p_i}{p_o} \right) \left(\frac{x_i}{x_o} \right)^k - 1 \right)^2 \quad \text{eq 45}$$

The best fit involves choosing k so that the SSE is a minimum [6]. From the theory of differential calculus, minimizing equation 45 would be very difficult as it would result in a highly nonlinear equation to be solved for k . Instead a similar method will be used.

2.7.2 Method 2

Instead of using equation 43 in its present form, take the natural log of both sides,

$$\ln \left(\frac{p}{p_o} \right) + k \ln \left(\frac{x}{x_o} \right) = 0 \quad \text{eq 46}$$

The error can now be defined as

$$\varepsilon_i = \ln \left(\frac{p}{p_o} \right) + k \ln \left(\frac{x}{x_o} \right) \quad \text{eq 47}$$

and the SSE becomes

$$SSE = \sum_{i=1}^N \varepsilon_i^2 = \sum_{i=1}^N \left(\ln \left(\frac{p_i}{p_o} \right) + k \ln \left(\frac{x_i}{x_o} \right) \right)^2 \quad \text{eq 48}$$

Minimizing equation 48 is now much simpler since k is no longer an exponent. Taking the first derivative of equation 48 and setting the result equal to 0 gives,

$$\frac{d}{dk}(SSE) = \sum_{i=1}^N 2 \left[\ln \frac{p_i}{p_o} + k \frac{x_i}{x_o} \right] \ln \left(\frac{x_i}{x_o} \right) = 0 \quad \text{eq 49}$$

$$\sum_{i=1}^N \left(\ln \frac{p_i}{p_o} \ln \frac{x_i}{x_o} \right) + k \sum_{i=1}^N \left(\ln \frac{x_i}{x_o} \right)^2 = 0 \quad \text{eq 50}$$

$$k = - \frac{\sum_{i=1}^N \left(\ln \frac{p_i}{p_o} \ln \frac{x_i}{x_o} \right)}{\sum_{i=1}^N \left(\ln \frac{x_i}{x_o} \right)^2} \quad \text{eq 51}$$

The result for k using equation 51 and the data in Table 3 is k=1.072. Using this k, the values of px^k for several points in Table 3 were generated. The results are shown in Table 6. Even though px^k is not constant, it appears to vary by no more than 9%. Next, a series of tables were generated similar to Table 3 for five different drop heights and the same initial conditions. This was done to examine the effect of drop parameters on k. Using equation 51, the results ranged from k=1.072 for a 6 inch drop height to k=1.188 for a 36 inch drop height. These results show that cushion compression is closer to being isothermal than adiabatic and that k is relatively insensitive to drop conditions.

time (ms)	px ^k
0	33.61
10	34.81
20	28.20
30	31.13
average=	31.94
standard deviation=	2.93 (9%)

Table 6 Variability in px^k at selected times from Table 3 (k=1.072)

2.8 Force vs. Compression Results

Figure 7 shows the force vs. compression results for a 2 inch thick block cushion. These results were obtained using the polytropic equation with the following definitions of force and compression.

1. Force = $(p - p_0)A$
2. Compression: $u = x_0 - x$

Substituting the expression for p from equation 43, the force vs. compression relation for a polytropic process becomes,

$$3. \quad F = p_0 A \left[\left(\frac{x_0}{x_0 - u} \right)^k - 1 \right]$$

The force vs. compression curves in Figure 7 were obtained using a polytropic constant of $k=1.072$ and $k=1.1882$ along with the values $A=100 \text{ in}^2$ and $x_0=2 \text{ in}$. The results show that the force vs.

compression curve is fairly insensitive to the variability in k .

Therefore, the remaining tests performed in this thesis will be based on $k=1.072$ for Arcel 512. The important part to note here is that the constants in the three differential equations can now be lumped into one polytropic constant and the equations of elasticity may be developed from this.

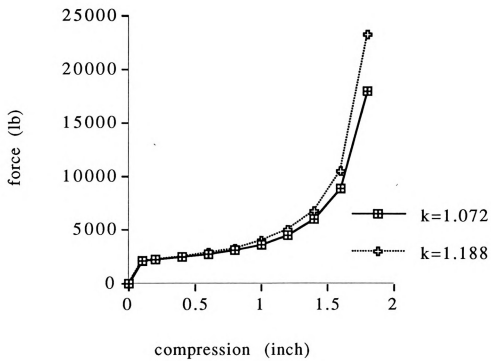


Figure 7 The effect of k on the force vs. compression curve

2.9 Finite Element Method

The finite element method is a powerful tool that models a complex domain as a collection of geometrically simple sub domains. The finite element method can be interpreted as a piecewise application of the variational method to solve differential equations in which the approximation functions are algebraic polynomials and the undetermined parameters represent the values of the solution at a finite number of pre-selected points called nodes [17]. The variational method is a technique used to solve a differential equation approximately by putting the differential equation into a variational form and taking the approximate solution to be a combination of shape functions.

The accuracy of the finite element method depends upon the element used to model the complex domain. Elements can have nodes that have one, two or three degrees of freedom. The number of nodes in the element as well as the number of elements in the domain all contribute to the accuracy of the approximation. The type of element and mesh used will generally require some knowledge of the physical system to be modeled.

When one looks at a typical cushion, especially a block cushion with a square bearing area, lines of symmetry become noticeable. A three dimensional problem can be reduced to a one dimensional problem and expect a reasonable degree of accuracy because the depth dimension is much greater than the side dimensions. The objective is to show that an impact on a closed cell cushion can be accurately modeled by the polytropic equation and analyzed

accurately by the finite element method. The results will be compared to existing experimental data.

2.10 Finite Element Model:

Since the material constant k has already been determined and the problem has been reduced to a one dimensional problem, the closed cell cushion may now be viewed as an elastic solid whose modulus of elasticity [15] is a function of the strain. The governing equation becomes [15,26]

$$P(x) = \left(\sigma + E(x) \frac{du}{dx} \right) A(x) \quad \text{eq 52}$$

or

$$\frac{dP}{dx} = \sigma \frac{dA(x)}{dx} + \frac{d}{dx} \left(A(x) E(x) \frac{du}{dx} \right) \quad \text{eq 53}$$

where x is the coordinate variable along the thickness, $E(x)$ =material modulus at x , u =the displacement of a node during compression, $P(x)$ =the compressive force, σ =yield pressure and $A(x)$ =cross sectional area at x (see Figure 8). The finite element process is able to solve the governing differential equation (eq 53) for a variable material modulus and a variable cross sectional area. The way in which the area varies with depth is determined by the geometry of the cushion. The way in which the material modulus varies with depth is not obvious and therefore must be determined by performing a force balance on an element of cushion and using the polytropic compression model. During compression, the stress applied to the cushion must overcome the buildup in air pressure over and above atmospheric pressure,

$$\sigma = p - p_o \quad \text{eq 54}$$

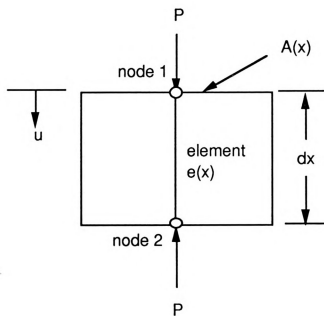


Figure 8 Schematic of 1 dimensional linear element

Using the polytropic relation for p gives,

$$\sigma = \frac{p_o V_o^k}{V^k} - p_o \quad \text{eq 55}$$

For a particular element, $V_o = At_o$ and $V = At$ where A is the cross sectional area of the element, V_o is the volume of the uncompressed element, V is the volume of the compressed element, t_o is the original thickness before the stress σ is applied, and t is the thickness after. Substituting into equation 55 and solving for σ gives

$$\sigma = p_o \left[\left(\frac{t_o}{t} \right)^k - 1 \right]. \quad \text{eq 56}$$

Now since $t = t_o - u$ from Figure 9, where u is the compression,

$$\frac{t_o}{t} = \frac{t_o}{t_o - u} = \frac{1}{1 - \frac{u}{t_o}} = \frac{1}{1 - \epsilon} \quad \text{eq 57}$$

where $\epsilon = u/t_o$ is the strain. Substituting yields,

$$\sigma = p_o \left[\left(\frac{1}{1 - \epsilon} \right)^k - 1 \right] \quad \text{eq 58}$$

Equation 58 is the stress vs. strain relation for the cushion element.

Dividing both sides by the strain and noting that by definition, the modulus is the ratio of stress to strain [15] gives

$$E = \left[\frac{1}{(1 - \epsilon)^k} - 1 \right] \frac{p_o}{\epsilon} \quad \text{eq 59}$$

In the small strain limit, E approaches $kp_o = (1.072)(14.7) = 15.76$ psi and steadily increases for larger strains. Both the material modulus and cross sectional area are now a function of x . The area is an explicit function of x because it varies directly with position as the geometry of the cushion dictates. The modulus is an implicit function of x because it varies with the strain and the strain varies with position.

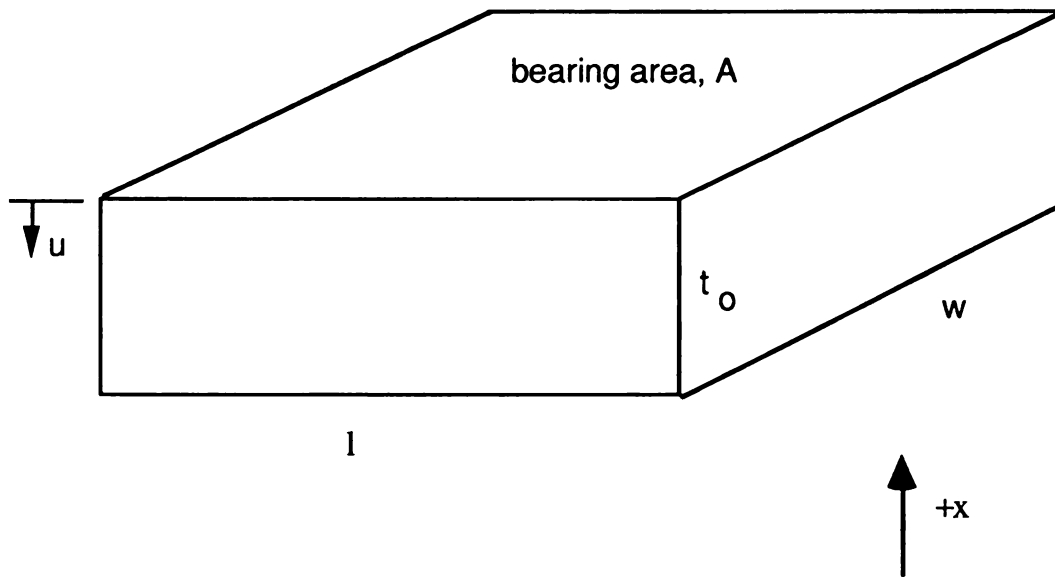


Figure 9 General variables used for a block cushion

Because the modulus is a function of strain and the strain distribution is not known before hand, the material modulus must be initially guessed. A reasonable guess would be the value, $E=14.7$ psi for small strains. The problem can then be solved with each element having this modulus. The resulting strain field can then be used to correct the modulus for each element using equation 59 and the process is repeated. The program will iterate until the material modulus converges upon a value for each element. Upon convergence, the results will eventually be used to calculate the peak G delivered to a product in a drop.

Now that the behavior of the cushion during compression has been modeled as a purely mechanical process, it is possible to examine cushions with more complicated geometric shapes like ribbed cushions. Ribbed cushions are merely block cushions with material removed at the base which gives them the appearance of a thinner block cushion standing on legs. They have the advantage of limiting the required amount of foam for protection while maintaining full support of the base of the product. The method of analysis which relies on the solution to the two coupled differential equations has the disadvantage of being able to handle only block shaped cushions (constant cross sectional area). The method also requires the knowledge of several material properties and cell dimensions. The advantage in modeling the process as a polytropic one is that all of these properties are lumped into one k , which is hopefully fairly constant for a particular brand of cushion. It is now possible to use broad based numerical methods such as the finite element method to analyze odd shaped cushions.

2.11 Finite Element Program and Input:

The method to solve the system of equations involves the implementation of a computer algorithm in a Fortran computing environment. A computer code developed with assistance from R. Averill [1] will be used to solve equation 53 [11,17,19]. The code was written to model each element in a closed cell cushion according to equation 59. The equation for each element is then combined into system of equations and solved numerically by matrix algebra. The algorithm to solve for the stress and strain consists of three basic parts: the preprocessor, processor and the post processor [17].

In the preprocessor, the boundary conditions, mesh geometry and element information is read in. This includes the data regarding how many nodes per element, the number of elements, how the nodes are restricted (boundary conditions), the type of interpolation functions, and the number of Gauss points to numerically integrate.

The processor is where the local stiffness matrix is generated for each element. The local stiffness matrix uses the interpolation functions to generate the solution to the variational approximation for each particular element. The local stiffness matrices are then assembled into a global stiffness matrix where the influence of adjacent elements are accounted for [17,19]. The contribution to a stiffness coefficient at the global level could be the result of several local element stiffness coefficients. The global stiffness matrix is generally denoted as $[K_{ij}]$. Once the global stiffness matrix is assembled the system of equations are set up where the stiffness matrix multiplies the displacement vector u and is set equal to the nodal force vectors P : $[K][u] = [P]$.

The next step in the finite element method involves imposing the boundary conditions on system of equations. The boundary conditions include specifying displacements or forces at a node. Once the boundary conditions are imposed, the system of equations are solved. The solution of interest that the computer program returns are the displacements for each node in the mesh.

The displacements are then used in the post processor to obtain other pertinent information. In the post processor, the gradients (strains) and stresses are calculated for each element. From this data, the material modulus can be calculated. The preprocessor, processor and post processor are located in the main program. Routines that solve the differential equation are located outside of the main program as subroutines. The details of the complete program are shown in the appendix and a summary of the algorithm and the routine is shown is Figure 10.

The first subroutine which is called from the main program, MAIN, is STIF which develops the local stiffness matrix. While generating the local stiffness matrix for each element, STIF will call the subroutine SHAPE to develop the interpolation function, which will be used to generate the solutions to the variational equation of that element. The SHAPE routine contains information related to the linear interpolation functions. The linear interpolation functions are used in the STIF routine to solve equation 53 in variational form. As the program treats each element in the mesh, the local stiffness matrix is assembled into a global stiffness matrix by the subroutine

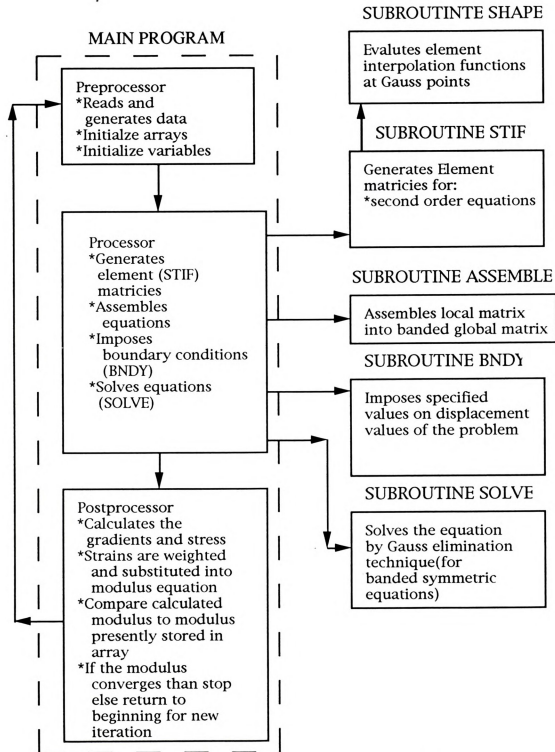


Figure 10 Flow chart of finite element solution [17]

ASSEMBLE. ASSEMBLE puts the global matrix in a banded matrix form. The subroutine BNDY then imposes the specified boundary conditions to the displacement and force vectors. The subroutine SOLVE uses the Gauss elimination technique to solve the matrix for the unknown displacements.

The solution to a problem is solved by iteration. This is necessary because the material modulus depends on the strains and the strains are the solution to the problem. The initial condition imposed on equation 53 to begin the calculation process involves prescribing a compressive displacement of the upper surface while fixing the base and selecting an initial material modulus. The program then solves for the nodal displacements. The first iteration is completed when the displacement at each node is calculated. From this solution, values of stress and strain are calculated. Since the strains determine the modulus through equation 59 and the modulus used to solve for these strains in the first place was initially guessed, a check is required to verify the solution. A comparison is made between the old modulus (guessed) and the new one based on the calculated strain. The second iteration now uses the new modulus and the process continues until the modulus converges for each element. In order to stabilize the iteration so that the solution doesn't oscillate or diverge, a weighted strain average may be used to calculate the new modulus for each element. The weighted strain average using 10% of the new strain plus 90% of the old strain was used for each element. The process is shown in Figure 10.

2.12 Force vs. Compression Transformation to G's

Once the iteration process converges for a prescribed boundary displacement, the only result of interest is the corresponding force on the upper surface required to produce this displacement. In order to be able to use this information to predict the shock to a product dropped onto a cushion, the whole iterative solution process must be performed again for various displacements. By prescribing larger and larger displacements and solving for the corresponding forces, a force vs. compression relationship, F vs. u , for this cushion like the one shown in Figure 11 can be developed. The use of this force vs. compression curve to predict G level is now straight forward. The potential energy Wh to be absorbed by the cushion in dropping the weight W from a height h is,

$$Wh = \int_0^{d_m} F du \quad \text{eq 60}$$

where d_m is the peak dynamic deflection of the cushion. In other words, the dynamic compression corresponds to the point on the curve where the area underneath is equal to Wh . The corresponding force F is related to the shock through Newton's Law

$$G = F/W \quad \text{eq 61}$$

where F is the force corresponding to the compression d_m .

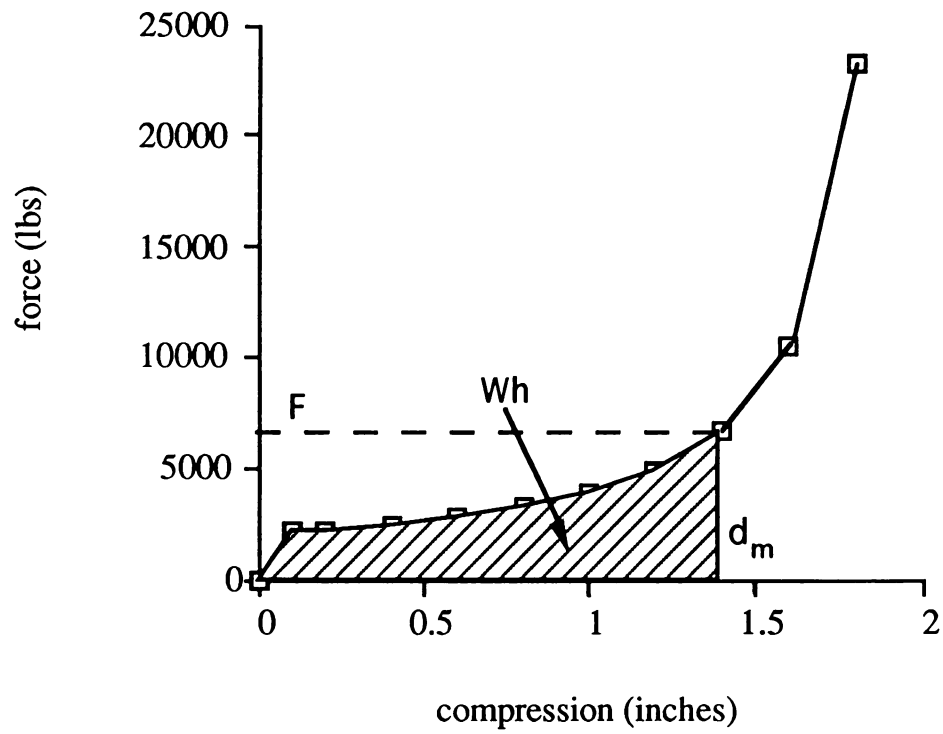


Figure 11 Force vs. compression curve for a cushion

CHAPTER 3

RESULTS

3.1 Simulation of Block and Ribbed Cushions

The polytropic model and finite element program will be tested for two cushion types: a block cushion and a ribbed cushion. The purpose of the test is to check the theory that the compression of a closed cell foam may be modeled as a polytropic process and to solve the problem using the finite element method. The stabilization and convergence of the iterative solution generally takes from two iterations for block cushions to about thirty iterations for the ribbed cushions at large strains.

The block cushion solution will be checked quantitatively against the Arcel 512 cushion curve data and the ribbed cushion against a block cushion with the same overall length, width and thickness dimensions. The ribbed cushion solution will be further tested by comparing the results to the equivalent volume method [8].

3.2 Block Cushion

The finite element setup for analyzing the block cushion is shown in Figure 12. Four elements with two nodes per element were used as shown. The cross sectional areas were all taken to be $A(n) = (l)(w)$ and the moduli E were calculated in accordance with equation 59 using $k=1.072$. The force vs. compression curves for the two different block sizes were calculated using the method outlined in Figure 10. The first block was $(l)(w)(t) = (10)(10)(2) \text{ in}^3$ and the second was $(8)(8)(4) \text{ in}^3$. The details of the input data to the finite element program are shown in Table 7.

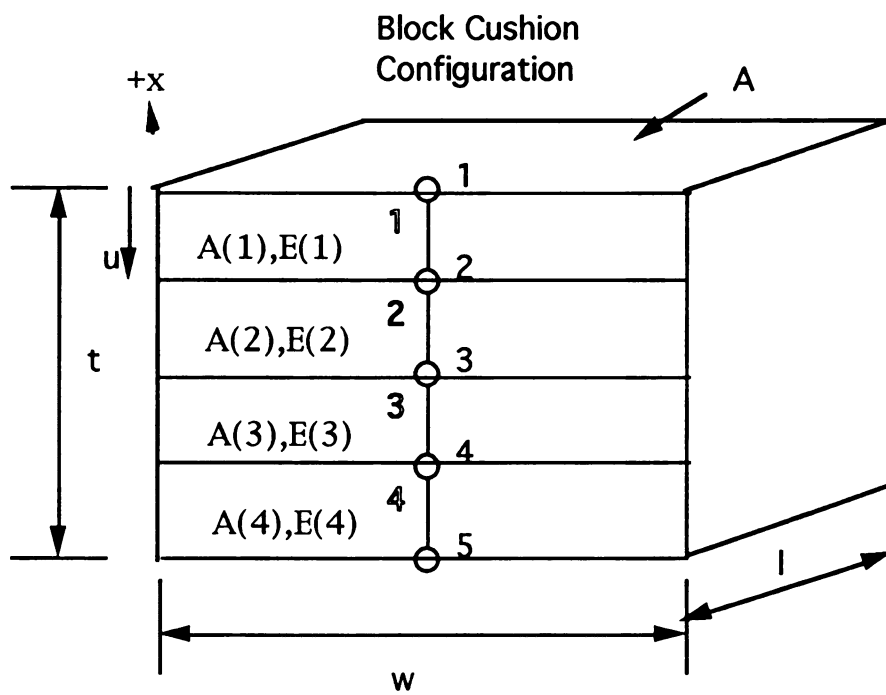


Figure 12 Mesh variables for a block cushion

Input data	FEM notation	2 in. thk. block	4 in. thk. block
Nodes/element	nodelm	2	2
Degrees of freedom per element	ndfnod	1	1
number of Gauss pts to calculate the displacement	ngpstif	2	2
number of Gauss pts to calculate the strain	ngpgrd	2	2
Number of elements in mesh	nelmsh	4	4
domain (thickness)	domain	2 inch	4 inch
material constant k		1.072	1.072
material modulus (initial guess) E(x)		14.7 psi	14.7 psi
cross sectional area A(x)		100 square inches	64 square inches

Table 7 Input data for 2 inch and 4 inch thick block cushion

The results of the finite element solution for the force vs. compression curve for the 2 inch block are shown in Table 8. The displacement at the upper surface was gradually increased in steps of .2 inches and the corresponding force was determined by the program. Since the prediction of G's requires areas under the force vs. compression curve, a seventh order polynomial was passed through the data points and then integrated to obtain the area as a function of compression. The polynomial which fits the data in Table 8 is:

$$F = 31310.9x^7 - 194322.5x^6 + 490567.5x^5 - 641655.5x^4 + 460788.6x^3 - 175799.1x^2 + 32669.0x + 34.2$$

The results for the 4 inch block cushion are shown in Table 9. The polynomial which fits the data points is:

$$F = 212.5x^7 - 2622.7x^6 + 13085.9x^5 - 33568.7x^4 + 46723.3x^3 - 33878.9x^2 + 11471.3x + 147.2$$

prescribed compression (in)	corresponding force (lb)	energy absorbed (in*lb)
0	2000.0	0
0.1	2083.1	118.48
0.2	2175.9	339.56
0.4	2397.6	803.74
0.6	2685.2	1297.22
0.8	3072.7	1879.72
1	3622	2549.85
1.2	4458.2	3342.80
1.4	5878.4	4365.04
1.6	8792.7	5792.98
1.8	17909.8	8266.68

Table 8 Force vs. compression results for the 10 in x 10 in x 2 in
block thick cushion



prescribed compression (in)	corresponding force (lb)	energy absorbed (in*lb)
0	1280.0	0
0.1	1307.5	61.89
0.2	1337.0	185.21
0.4	1399.0	492.47
0.6	1469.4	797.39
0.8	1550.1	1088.74
1	1641.0	1389.91
1.2	1747.2	1721.35
1.4	1870.1	2089.47
1.6	2014.7	2490.17
1.8	2188.2	2917.79
2	2398.1	3373.39
2.2	2658.6	3868.45
2.4	2988.8	4423.37
2.6	3420.2	5062.48
2.8	4006.4	5810.35
3	4845.4	6696.56
3.2	6138.24	7779.22
3.4	8365.4	9200.00
3.6	13030.4	11286.25

Table 9 Force vs. compression results for the 8 in x 8 in x 4 in
block thick cushion

Note that in both Tables 8 and 9 the force is not zero when the compression is zero. This is due to the yield pressure, $\sigma = 20$ psi, required to deform the cell structure (less the air). In fact the force at zero compression is easily calculated using force = yield pressure x area.

Now that the force vs. compression curves for the two block cushions analyzed are known, they may be used to find the shock delivered to a product in a drop. An example showing how this is done is given below.

Example: A 32 lb product is dropped from a height of 12 inches onto the 8" x 8" x 4" cushion.

Solution: The product of weight and drop height is,

$Wh = (32)(12) = 384$ in-lb. Since this is the energy which must be absorbed by the cushion and since energy absorbed is just the area under the force vs compression curve, the corresponding force from Table 9 is found to be $F = 1377.3$ lb. From Newton's Law, (force=(mass)(acceleration)), the corresponding peak deceleration is, $G = F/w = 1377.3/32 = 43$.

The result is then compared to Arcel 512 cushion curves where the G's for a static loading of .5 psi and cushion 4 inches thick is, $G = 40$

This same procedure, for an 8" x 8" x 4" cushion, was applied to other drop situations in order to test the method over a broad range of dynamic deflections. Table 10 shows the results.

drop height (in)	weight (lb)	thickness (in)	bearing area (in ²)	predicted d _m (in)	predicted G's (FEM)	actual G's (ARCEL)
12	32	4	64	.10	43	40
12	160	4	64	1.25	11	10
30	32	4	64	.25	47	45
30	160	4	64	2.60	20	20
48	32	4	64	1.12	53	50
48	160	4	64	3.10	38	39

Table 10 Comparison of finite element deceleration results to Arcel 512 data

The result in Table 10 show reasonable predictions of shock for a range of dynamic deflections. The accuracy of the prediction with the Arcel data implies that accounting for heat transfer and thermodynamics are necessary. More important the results show that the polytropic assumption seems valid. To qualify the polytropic process, extreme drop conditions and odd cushion shapes need to be tested. This qualification will help justify the polytropic process. As will be seen, the odd cushion shape to be test will be a ribbed cushion. The results will then be compared to the equivalent volume method [8].

3.3 Ribbed Cushion Results

For a ribbed cushion, $A(x)$ and $E(x)$ will vary in the ribs and remain constant in the plank. The schematic for the ribbed cushions are shown in Figure 13 and Figure 14. The meshing of the ribbed cushion as shown in Figure 15 shows the finite element model with one rib. Since the finite element model is one dimensional, the cross sectional area at each position x must weighted (see Table 11). The mesh chosen for the ribbed cushion is a one dimensional uniform mesh. The length of each element is 0.5 inches. Further mesh refinements were found to be unnecessary since the results yielded reasonable values. The force vs compression results from the FEM solution are shown in Table 12. Comparing Tables 9 and 12, one notices that at a given compression, the energy absorbed by a block cushion is greater than that by the ribbed cushion with the same overall size. This is reasonable since it takes more energy to compress a block cushion because there is more material. This is also shown by the force compression curves in Figure 16.

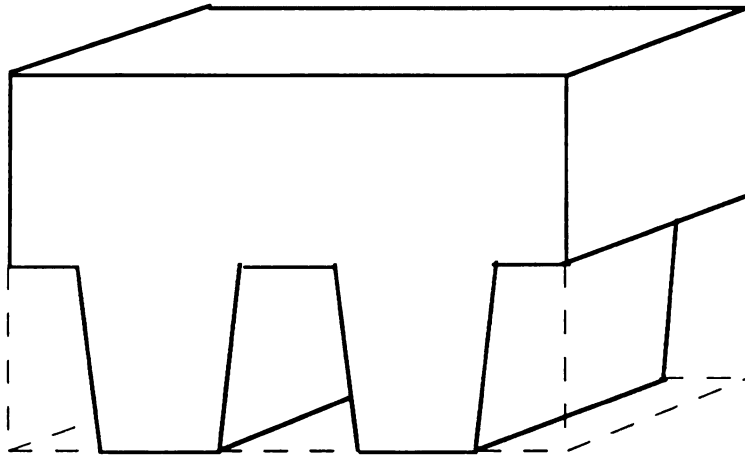


Figure 13 Three dimensional view of the ribbed cushion

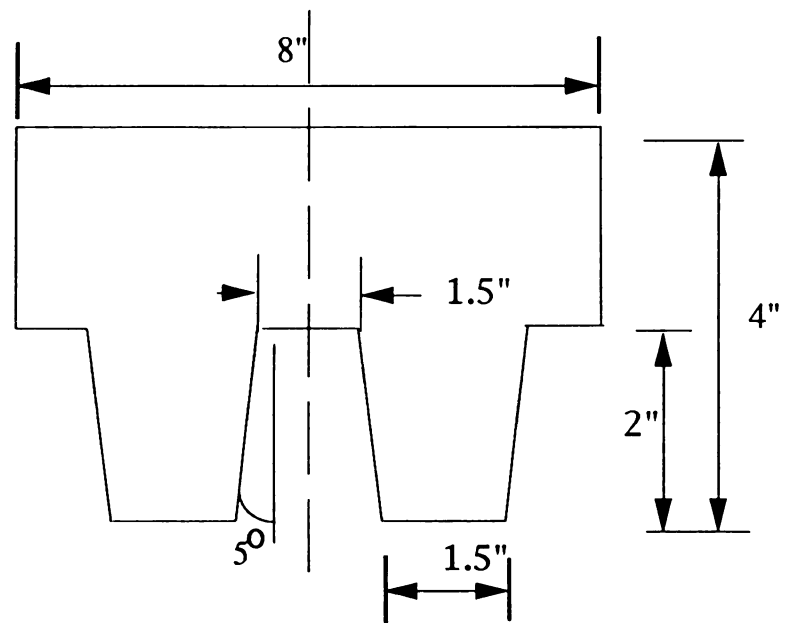


Figure 14 Two dimensional view of the ribbed cushion

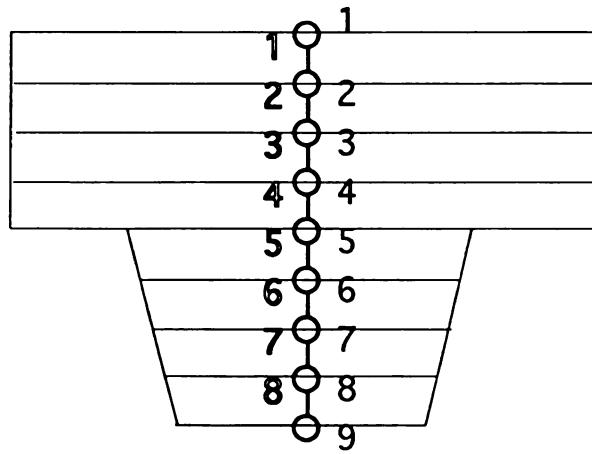


Figure 15 Finite element mesh for the rib cushion



Element number	material modulus (initial guess) E(x) (psi)	Cross sectional area A(x), (in ²)
1	14.7	64.0
2	14.7	64.0
3	14.7	64.0
4	14.7	64.0
5	14.7	28.9
6	14.7	27.5
7	14.7	26.1
8	14.7	24.7

Table 11 Geometry and initial modulus for the ribbed cushion

prescribed compression (in)	corresponding force (lb)	energy absorbed (in*lb)
0.2	1313.28	147.90
0.4	1350.40	422.92
0.6	1394.56	711.84
0.8	1442.56	992.99
1	1502.08	1277.27
1.2	1569.28	1578.43
1.4	1649.92	1902.58
1.6	1745.28	2248.80
1.8	1861.12	2614.27
2	2003.84	2999.75
2.2	2181.12	3412.38
2.4	2408.32	3865.34
2.6	2705.92	4375.12
2.8	3112.32	4959.45
3	3697.28	5640.60
3.2	4599.68	6460.50
3.4	6147.84	7516.37

Table 12 Force vs. compression results for a 4 inch thick ribbed cushion with cross section dimensions as specified in Table 12

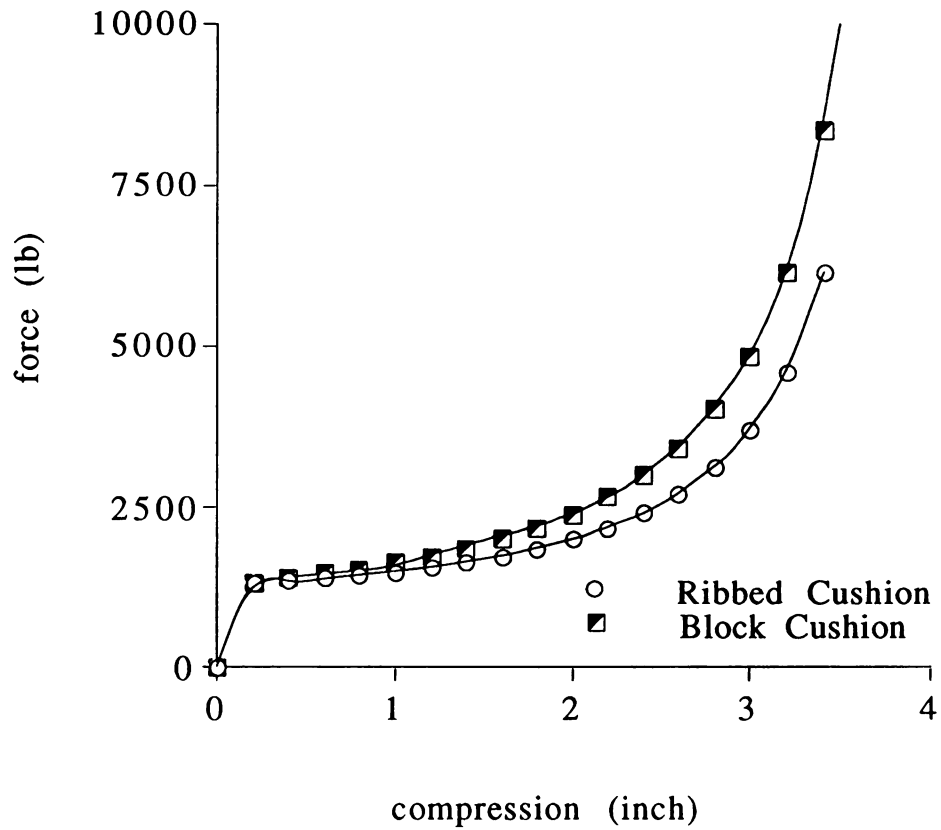


Figure 16 Comparison between ribbed and block cushions with the same overall dimensions

The FEM solution of the ribbed cushion was also compared to theoretical drop results [8]. Drops were performed at three different drop heights with three different weights. The test drop heights were 18, 30 and 42 inches with weights of 37.6 lb., 65.8 lb. and 94.2 lb.. The theoretical calculation involved finding the volume of a ribbed cushion and converting it to an equivalent block cushion of the same thickness. The comparison is shown in Table 13. Based upon the error, it seems that the constitutive equation derived earlier does model the closed cell cushion accurately. Since the equal volume method is a proven technique, it may be said that the polytropic assumption is a valid one.

The development of the polytropic process has shown that heat transfer must be incorporated into the constitutive equation to model cushions. The development has also shown that it is possible to lump all the thermodynamic terms into a constant k . As a result of the polytropic process we are now able to model a ribbed cushion based on numerical methods.

drop height (in)	weight (lb)	finite element results (G's)	equivalent volume results (G's)	percent error
18	37.6	37	33	12
18	65.8	23	21	10
18	94.2	17	17	0
30	37.6	39	36	8
30	65.8	25	27	7
30	94.2	21	22	5
42	37.6	42	41	2
42	65.8	29	30	3
42	94.2	26	29	10

Table 13 Comparison between finite element results and the equivalent volume results [8]

CONCLUSIONS

There were several steps involved in the modeling process of a closed cell foam during an impact. The first model directly accounted for heat transfer between the air and the foam. A system of three differential equations obtained from Newton's Law, the Gas Law, and the First Law of Thermodynamics were solved numerically by the finite difference method (FDM). The prediction for peak G-level in a drop was found to be in close agreement with published data. Based on the success of this model, a simplified model which incorporated all of the thermodynamic terms into a single property, the polytropic constant k , was developed. This simplification converted the problem into a non-linear, one dimensional elasticity problem which was solved using the finite element method (FEM). The prediction for peak G-level using this model was also found to be in excellent agreement with published data.

To verify the accuracy of both the FEM and FDM models, extreme drop scenarios must be tested. Table 14 shows the FDM solution and the FEM solution for extreme drop situations. Both the FEM and FDM model produced similar results as expected since FEM is derived from the FDM model. The results in Table 14 shows that it is necessary to consider thermodynamics to accurately model a closed cell foam and that it is possible to simplify the analysis using a polytropic process to represent the three differential equations. The discrepancy in G at the 48 inch drop for the FDM model is probably due to propagation of errors in the finite difference method.

drop height (in)	cushion thickness (in)	static loading (psi)	FDM solution (G's)	FEM solution (G's)	cushion curve data (G's)
12	4	.5	43	43	41
24	2	1	36	35	37
48	2	1	73	69	70

Table 14 Comparison between the finite difference, finite element,
and actual results at extreme loading conditions

As the impact becomes more severe, the FDM model becomes less accurate due to the smaller time intervals needed to capture the deceleration accurately. Overall, it is difficult to dispute the validity of the model based on the results.

The theory that the force required to compress a closed cell foam is strain rate dependent seem to be justified. The rate dependence however is not due to any visco-elastic properties of the foam material but to heat transfer between the air and the cell walls. The process of compression is close to being isothermal. The accuracy of the results for a block cushions was verified using actual published data. The results for ribbed cushions cannot be completely verified since only a comparison to the equivalent volume method could be made. The comparison however does show a good correlation.

APPENDIX A

FINITE ELEMENT PROGRAM LISTING

[illegible]

```

C
C
C Master Thesis Project Department of Packaging
C By Andrew Chen
C October 20, 1993
C This program solves the linear equations of a second order
C equation:
C
C          -a(x)(u')' + b(x)u = f(x)
C THIS PROGRAM WILL FIND THE STRESS, STRAIN AND YOUNGSMODULUS
C OF A CLOSED CELL CUSHION BASED ON THE AIR MODEL AND THE
C PROPERTIES OF THE MATERIAL SOLVED SEPERATELY
C Note: This program will stabilize oscillating systems
C
C

```

```
C *** Put declaration, dimension and open statements here.
```

```
implicit double precision (a-h,o-z)
integer neqmax, bndwth, nemmax, npemax, nodelm, ndfnod,
+       node, nelmsh, ngpstf, ngpgrd, bndmax, ndfelm, count,
+       elmnum, edfmax, ndfmax, ngpmax, nnumax, nbcmax, ires,
+       ni, i, j, nodmsh, ndfmsh, nsgd, nsgf, isgd, isgf, finish,
+       k
```

```
double precision vsgd, domain, elenth, gauspt, gauswt, pa,
+               sf, dsf, gf, estif, ef, gstif, jacobn,
+               elx, x, xi, dsx, gdsf,   vsgf, nodsep,
+               x0, a1, a2, a0, b1, b2, f1, f2, f0, area, e, strain,
+               kon, stress, strntp, strnag
```

```
parameter (bndmax=20, edfmax=3, nbcmax=20, ndfmax=1, nemmax=20,  
+          negmax=20, ngpmax=4, nnmmax=20, npemax=3, pa=14.7,  
+          finish=10)
```

```

dimension node(nemmax,npemax), elx(nnmmax), isgd(nbcmmax,2),
+ vsgd(nbcmmax), isgf(nbcmmax,2), vsgf(nbcmmax),
+ gauspt(ngpmax,ngpmax), gauswt(ngpmax,ngpmax),
+ gf(neqmax), gstif(neqmax,bndmax), ef(edfmax),
+ gdsf(npemax), estif(edfmax,edfmax),area(nemmax),
+ sf(npemax), dsf(npemax),a0(nemmax),e(nemmax),
+ strain(nemmax), stress(nemmax),
+ strntp(nemmax), strnag(nemmax)

```

```
open (unit = 10, file = 'dt1', status = 'old')
open (unit = 11, file = 'sdt1', status = 'unknown')
```

```
C -----
C --- PREPROCESSOR ---
C -----
```

```
write(11,*)'This is the data output for evaluating the strains &'
write(11,*)'stresses of a 1 dimensional material that obeys the'
write(11,*)'following equation'
write(11,*)' -a(x) u'' + b(x) u = f(x)'
```

```
C *** Read input data here.
```

```

a1= 0
a2= 0
b0= 0
b1= 0
b2= 0
f0= 0
f1= 0
f2= 0
x0 = 0.0
do 1 i=1, nemmax
e(n)=0.0
1 continue
read(10,*) kon

```

```

      read(10,*) nodelm, ndfnod, ngpstf, ngpgrd, nelmsb
      read(10,*) (e(n), n=1, nelmsb)
      read(10,*) (area(n), n=1, nelmsb)
      read(10,*) a1, a2
      read(10,*) b0, b1, b2
      read(10,*) f0, f1, f2
      read(10,*) nsqd, nsqf, domain
      read(10,*) ((isqd(i,j), j=1, 2), i=1, nsqd)
      read(10,*) (vsqd(i), i=1, nsqd)
      read(10,*) ((isqf(i,j), j=1, 2), i=1, nsqf)
      read(10,*) (vsqf(i), i=1, nsqf)

c-----writing the input data -----

      write(11,*) '+++++'
      write(11,*) 'the nonlinear constant used is ', kon
      write(11,*) 'the number of nodes per element is ', nodelm
      write(11,*) 'the number of elements per mesh is ', nelmsb
      write(11,*) 'the length of the domain is ', domain
      do 2 n=1, nelmsb
      write(11,*) 'initial modulus=', e(n), 'for element', n
      write(11,*) 'initial cross sectional area =', area(n)
      write(11,*) '.....'
2      continue

c---- section to calculate the number of nodes in a mesh, the global
c      coordinate and the boolean connectivity matrix
c-----

      do 10 i=1, nemmax
      do 20 j=1, npemax
      node(i,j) = 0
      elx(i) = 0.0
      strntp(i) = 0.0
      strnag(i) = 0.0
20      continue
10     continue
      nodmsb = nelmsb*(nodelm - 1)+1
      ndfmsb = nodmsb*ndfnod
      ndfelmsb = nodelm*ndfnod
      bndwth = ndfelmsb

c----calculation of the coordinate system

      do 30 i=1, nodmsb
      elenth = domain/nelmsb
      nodsep = domain/(nodmsb-1)
      elx(i) = nodsep*float(i-1)
      write (11,*) 'Global node', i, 'at', elx(i)
30      continue
c      write(11,*) 'nodmsb', nodmsb, 'elenth', elenth

c---boolean connectivity calculation

      do 35 i=1, nodelm
      node(1,i) = i
35      continue
c      write(11,*) ' nodelm is', nodelm
      do 40 i=2, nelmsb
      do 50 j=1, nodelm
      node(i,j)= node(i-1,j) + (nodelm - 1)
50      continue
40      continue
c      write(11,*) 'the connectivity matrix local node vs global element'
      do 55 i=1, nelmsb
      write(11,*) (node(i,j), j=1, nodelm)

```

```

55 continue

C --- INITIALIZE THE ARRAYS WHICH CONTAIN THE GAUSS POINTS AND WEIGHTS.

      CALL GQUAD  (NGPMAX, GAUSPT, GAUSWT)

C -----
C --- PROCESSOR ---
C -----

C *** Main processing operations or call to processor subroutine goes here.

      do 5 count=1, finish
      write(11,*)'=====
      write(11,*)'this is iteration step number ',count
      write(11,*)'=====

      a1= 0
      a2= 0
      b0= 0
      b1= 0
      b2= 0
      f0= 0
      f1= 0
      f2= 0
      x0 = 0.0

      do 65 i =1, nemmax
      a0(i)=0.0
65  continue
      do 8 i = 1,edfmax
      do 6 j = 1,edfmax
      estif(i,j)=0.0
      ef(i)=0.0
6  continue
8  continue
      do 18 i = 1,neqmax
      do 16 j = 1,bndmax
      gstif(i,j)=0.0
      gf(i)=0.0
16  continue
18  continue

      do 60 elmnum=1, nelmsh
      n = elmnum
      a0(n) = e(n)*area(n)
      call stif (elx, ndfel, ngpmax, gauspt, gauswt, elenth,
+      node1m, estif,ef,a0,b0,f0,a1,a2,b1,b2,f1,
+      f2, elmnum, nodmsh, const, x, jacobn, xi, ngpstf,nemmax)

      call ASMBLE (NEQMAX, BNDMAX, EDFMAX, NEMMAX, NPEMAX, NODELM,
+      NDFNOD, NODE , ESTIF , EF , GSTIF , GF ,
+      ELMNUM)
60  continue

      do 23 i = 1,node1m
      do 26 j = 1,node1m
c      write (11,*) 'estif of node','i','j','is',estif(i,j)
26  continue
23  continue

      call fbndry (neqmax, nbcmax, ndfnod, nsgf, isgf, vsgf, gf)
      call ubndry (neqmax, bndmax, nbcmax, ndfnod, ndfmsh, bndwth,
+      nsgd, isgd, vsgd, gstif, gf)

```

```

      ires = 0
      call solve (neqmax, bndmax, ndfmsh, bndwth, gstif,
+       gf, ires)
      do 70 k=1, nodmsh
        write(11,*)'displacement of ',gf(node(k,1)), 'at node',k,
+         ' at x coord ',elx(k)
      70 continue
C -----
C --- POSTPROCESSOR ---
C -----

C *** Printing of solution and computation of gradients goes here.

      do 80 j = 1, nelms
      do 90 ni = 1, ngpgrd

        xi = gauspt(ni,ngpgrd)
        call shplld (npemax, nodelm, xi, elenth, sf, dsf, gdsf,
+         jacobn)
        x = 0.5*elenth*(1+xi)+elx(j)

        dsx=0.0
        do 99 i = 1,nodelm
          dsx= dsx + gdsf(i)*gf(node(j,i))
69      continue

        strain(j) = dsx
        strnag(j) = (strain(j)+strntp(j))/2

90      continue
        write(11,*)'-----'
        e(j) = (pa/strnag(j))*(((1-strnag(j))**(-kon))-1)
        write(11,*)'youngs modulus is', e(j), ' for element',j
        stress(j) = e(j)*strnag(j)
        write(11,*)' stress is ',stress(j)
        write(11,*)' strain is ',strain(j)
80      continue
        do 82 n=1, nemmx
          strntp(n)=strain(j)
          strnag(n)=0.0
          strain(n)=0.0
          stress(n)=0.0
82      continue
5      continue


      STOP
      END

```


[illegible]

[illegible]

```

SUBROUTINE GQUAD (NGPMAX, GAUSPT, GAUSWT )
  IMPLICIT LOGICAL (A-Z)
  INTEGER          NGPMAX, I      , J
  DOUBLE PRECISION GAUSPT, GAUSWT
  DIMENSION        GAUSPT(NGPMAX,NGPMAX), GAUSWT(NGPMAX,NGPMAX)

C --- INITIALIZE ARRAYS.

  DO 20 I = 1,NGPMAX
  DO 10 J = 1,NGPMAX
    GAUSPT(I,J) = 0.000
    GAUSWT(I,J) = 0.000
  10 CONTINUE
  20 CONTINUE

C --- GAUSSIAN QUADRATURE OF ORDER 1.

  GAUSPT(1,1) = 0.000
  GAUSWT(1,1) = 2.000

C --- GAUSSIAN QUADRATURE OF ORDER 2.

  GAUSPT(1,2) = -1.000/SQRT(3.000)
  GAUSPT(2,2) = -GAUSPT(1,2)
  GAUSWT(1,2) = 1.000
  GAUSWT(2,2) = GAUSWT(1,2)

C --- GAUSSIAN QUADRATURE OF ORDER 3.

  GAUSPT(1,3) = -SQRT(3.000/5.000)
  GAUSPT(2,3) = 0.000
  GAUSPT(3,3) = -GAUSPT(1,3)
  GAUSWT(1,3) = 5.000/9.000
  GAUSWT(2,3) = 8.000/9.000
  GAUSWT(3,3) = GAUSWT(1,3)

C --- GAUSSIAN QUADRATURE OF ORDER 4.

  GAUSPT(1,4) = -0.8611363116
  GAUSPT(2,4) = -0.3399810436
  GAUSPT(3,4) = -GAUSPT(2,4)
  GAUSPT(4,4) = -GAUSPT(1,4)
  GAUSWT(1,4) = 0.3478548451
  GAUSWT(2,4) = 0.6521451549
  GAUSWT(3,4) = GAUSWT(2,4)
  GAUSWT(4,4) = GAUSWT(1,4)

  RETURN
  END

```

IMPLICIT LOGICAL: (A-Z)

INTEGER NPEMAX, NODELM, I

DOUBLE PRECISION XI , ELENTH, SF , DSF , GDSF , JACOBN

DIMENSION SF(NPEMAX), DSF(NPEMAX), GDSF(NPEMAX)

```
C --- LINEAR LAGRANGE INTERPOLATION FUNCTIONS FOR 2-NODED ELEMENTS.
```

```
IF (NODELM .EQ. 2) THEN
  SF(1) = 0.5000 * (1.000 - XI)
  SF(2) = 0.5000 * (1.000 + XI)
  DSF(1) = -0.5000
  DSF(2) = 0.5000
```

```
C --- QUADRATIC LAGRANGE INTERPOLATION FUNCTIONS FOR 3-NODED ELEMENTS.
```

```
ELSE IF (NODELM .EQ. 3) THEN
  SF(1) = -0.5000 * XI * (1.000 - XI)
  SF(2) = 1.000 - XI * XI
  SF(3) = 0.5000 * XI * (1.000 + XI)
  DSF(1) = -0.5000 * (1.000 - 2.000 * XI)
  DSF(2) = -2.000 * XI
  DSF(3) = 0.5000 * (1.000 + 2.000 * XI)
END IF
```

```
C --- COMPUTE THE GLOBAL DERIVATIVES OF SF(I).
```

```

      JACOBN = ELENTN * 0.5000
      DO 10 I = 1, NOLEMT
         GDSF(I) = DSF(I) / JACOBN
10  CONTINUE
      RETURN
      END

```

[illegible]

```

SUBROUTINE SOLVE (NRM , NCM , NEQNS , NBW , BAND ,
                  RHS , IRES )

```

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

INTEGER NRM , NCM , NEQNS , NBW , IRES

DOUBLE PRECISION BAND , RHS

DIMENSION BAND (NRM, NCM) , RHS (NRM)

```

MEQNS=MEQNS-1
IF (IRES.GT.0) GO TO 40
DO 30 NPIV=1,MEQNS
NPIVOT=NPIV+1
LSTSUB=NPIV+NBW-1
IF (LSTSUB.GT.NEQNS) LSTSUB=NEQNS
DO 20 NROW=NPIVOT,LSTSUB

```

```

NCOL=NROW-NPIV+1
FACTOR=BAND(NPIV,NCOL)/BAND(NPIV,1)
DO 10 NCOL=NROW,LSTSUB
ICOL=NCOL-NROW+1
JCOL=NCOL-NPIV+1
10 BAND(NROW,ICOL)=BAND(NROW,ICOL)-FACTOR*BAND(NPIV,JCOL)
20 RHS(NROW)=RHS(NROW)-FACTOR*RHS(NPIV)
30 CONTINUE
GO TO 70
40 DO 60 NPIV=1,MEQNS
NPIVOT=NPIV+1
LSTSUB=NPIV+NBW-1
IF (LSTSUB.GT.NEQNS) LSTSUB=NEQNS
DO 50 NROW=NPIVOT,LSTSUB
NCOL=NROW-NPIV+1
FACTOR=BAND(NPIV,NCOL)/BAND(NPIV,1)
50 RHS(NROW)=RHS(NROW)-FACTOR*RHS(NPIV)
60 CONTINUE

```

```

70 DO 90 IJK=2,NEQNS
    NPIV=NEQNS-IJK+2
    RHS(NPIV)=RHS(NPIV)/BAND(NPIV,1)
    LSTSUB=NPIV-NBW+1
    IF (LSTSUB.LT.1) LSTSUB=1
    NPIVOT=NPIV-1
    DO 80 JKI=LSTSUB,NPIVOT
        NROW=NPIVOT-JKI+LSTSUB
        NCOL=NPIV-NROW+1
        FACTOR=BAND(NROW,NCOL)
80  RHS(NROW)=RHS(NROW)-FACTOR*RHS(NPIV)
90  CONTINUE
    RHS(1)=RHS(1)/BAND(1,1)
    RETURN
END

```

[illegible]

```

SUBROUTINE UBNDRY (NEQMAX, BNDMAX, NBCMAX, NDFNOD, NDFMSH, BNDWTH,
                  NSGD, ISGD, VSGD, GSTIF, GF)

```

```

INTEGER      NEQMAX, BNDMAX, NBCMAX, NDFNOD, NDFMSH, BNDWTH,
              NSGD  , ISGD  , IBC   , IDOF  , BWM1  , ROW   ,
              COL   , K

```

```

DIMENSION      ISGD(NBCMAX,2), VSGD(NBCMAX),
.              GSTIF(NEQMAX,BNDMAX), GF(NEQMAX)

```

```
DO 40 IBC = 1, NSGD
  IDOF = NDFNOD * (ISGD(IBC,1) - 1) + ISGD(IBC,2)
  UVALUE = VSGD(IBC)
```

```

      BWM1    = BNDWTH - 1
      ROW     = IDOF - BNDWTH

      DO 20 K = 1, BWM1
        ROW = ROW + 1
        IF (ROW .GT. 0) THEN
          COL = IDOF - ROW + 1
          GF(ROW) = GF(ROW) - GSTIF(ROW,COL) * UVALUE
          GSTIF(ROW,COL) = 0.000
        END IF
20    CONTINUE

      GSTIF(IDOF,1) = 1.000
      GF(IDOF) = UVALUE
      ROW = IDOF

      DO 30 K = 2, BNDWTH
        ROW = ROW + 1
        IF (ROW .LE. NDFMSH) THEN
          GF(ROW) = GF(ROW) - GSTIF(IDOF,K) * UVALUE
          GSTIF(IDOF,K) = 0.000
        END IF
30    CONTINUE
40 CONTINUE
      RETURN
      END

```

APPENDIX B

ARCEL 512 CUSHION CURVES

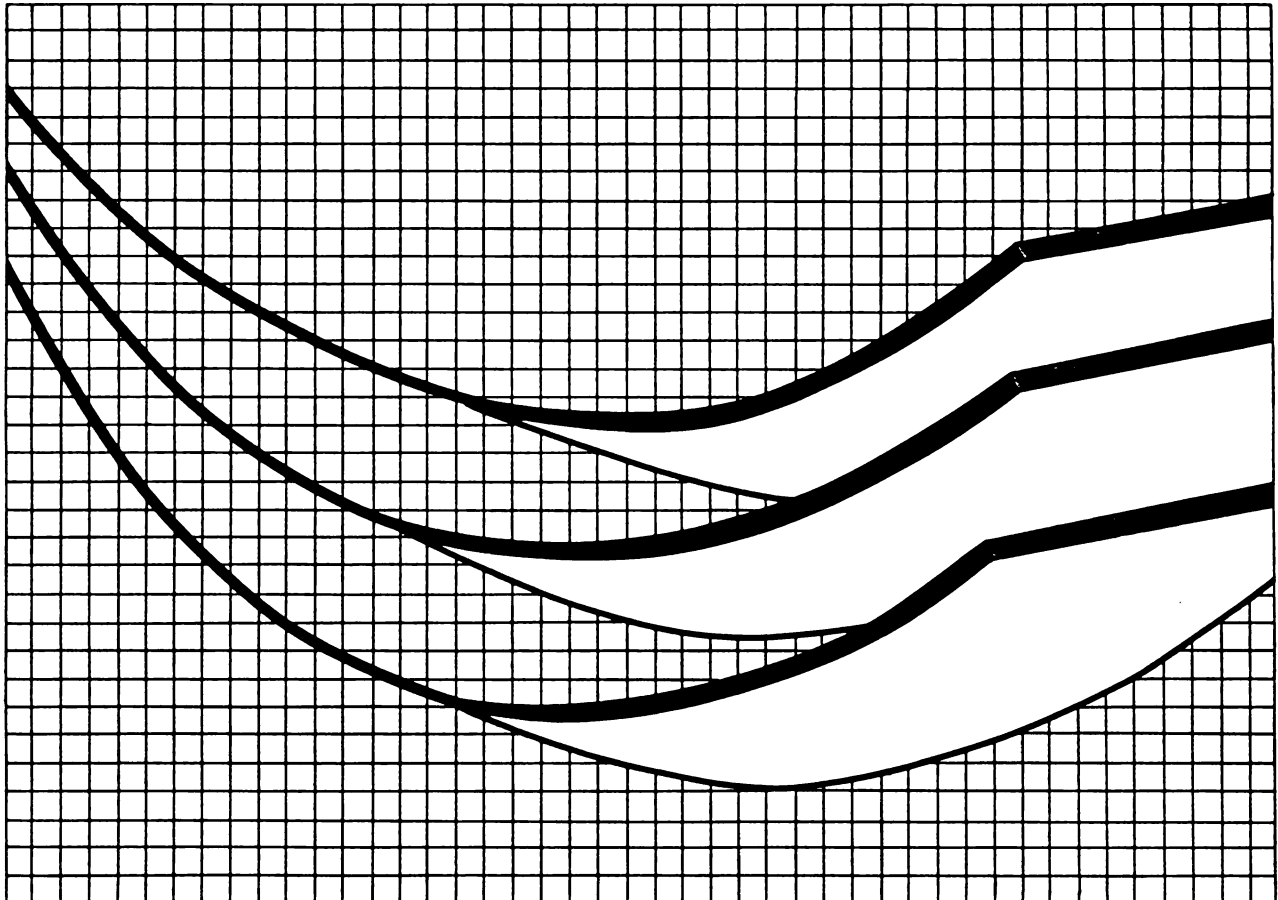
ARCO Chemical Company
Division of AtlanticRichfieldCompany



Dynamic Cushioning Performance

ARCEL[®]

Moldable Polyethylene Copolymer



Dynamic Cushioning Performance

Following are dynamic cushioning performance curves for foam molded from
ARCEL® 512
 Moldable Polyethylene Copolymer

12" DROP

Figure 1
12" Drop, 1st Impact

Density = 2.0 PCF

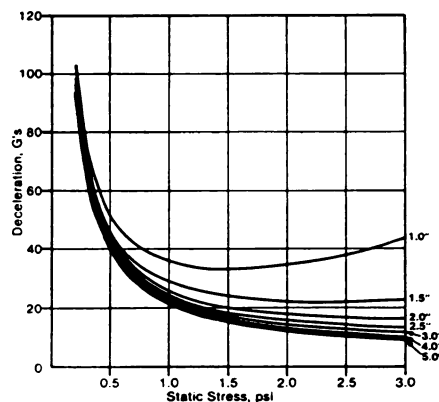
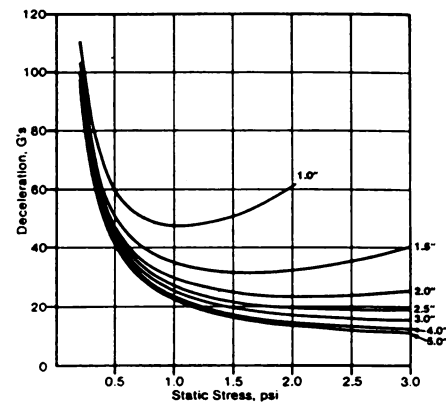


Figure 2
12" Drop, 2-5 Impacts

Density = 2.0 PCF



18" DROP

Figure 3
18" Drop, 1st Impact

Density = 2.0 PCF

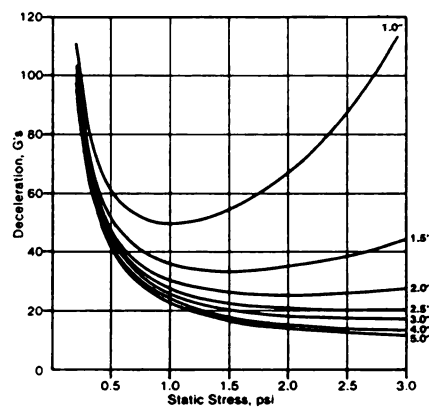
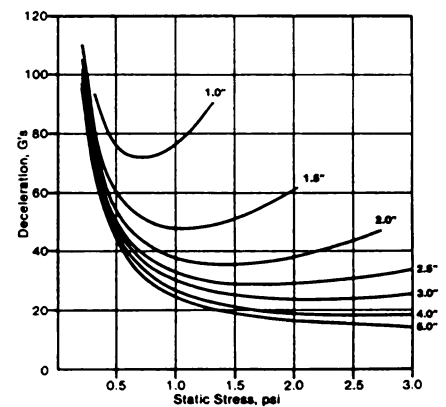


Figure 4
18" Drop, 2-5 Impacts

Density = 2.0 PCF



24" DROP

Figure 5
24" Drop, 1st Impact

Density = 2.0 PCF

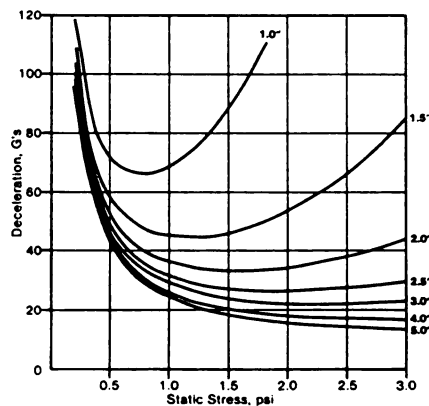
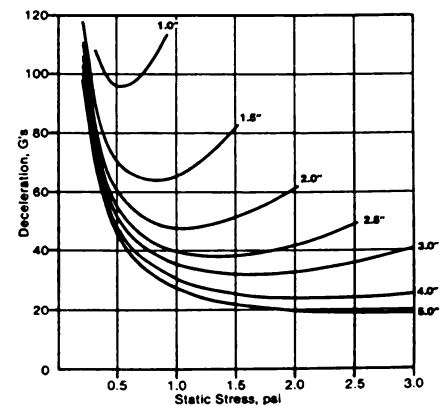


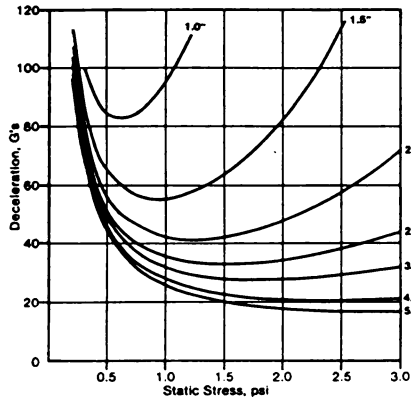
Figure 6
24" Drop, 2-5 Impacts

Density = 2.0 PCF

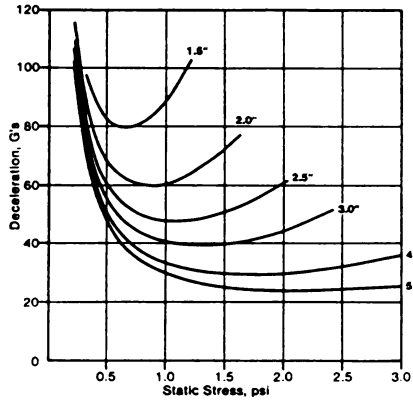


30" DROP**Figure 7**
30" Drop, 1st Impact

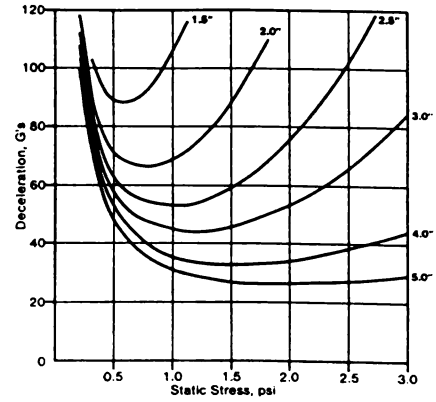
Density = 2.0 PCF

**Figure 8**
30" Drop, 2-5 Impacts

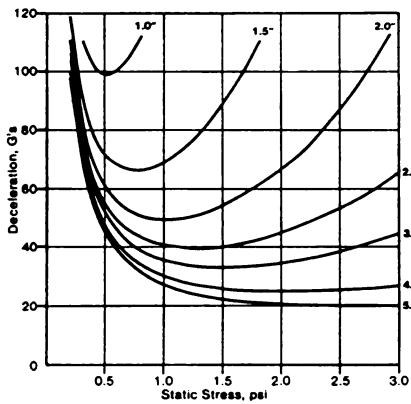
Density = 2.0 PCF

**48" DROP****Figure 13**
48" Drop, 1st Impact

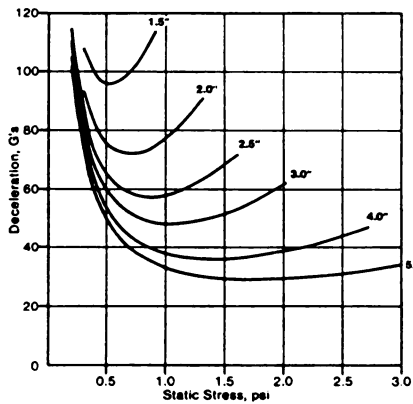
Density = 2.0 PCF

**36" DROP****Figure 9**
36" Drop, 1st Impact

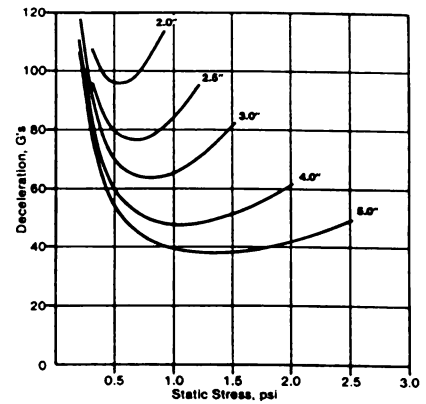
Density = 2.0 PCF

**Figure 10**
36" Drop, 2-5 Impacts

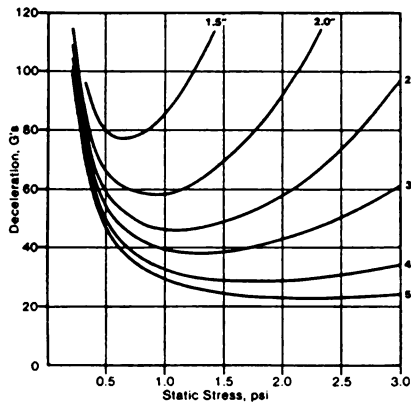
Density = 2.0 PCF

**Figure 14**
48" Drop, 2-5 Impacts

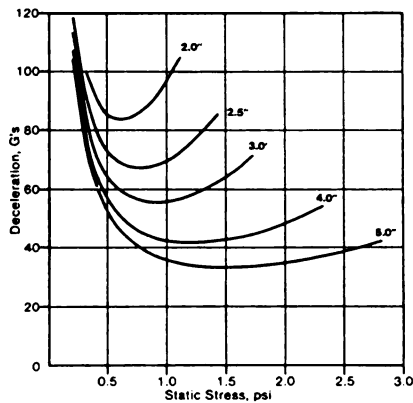
Density = 2.0 PCF

**42" DROP****Figure 11**
42" Drop, 1st Impact

Density = 2.0 PCF

**Figure 12**
42" Drop, 2-5 Impacts

Density = 2.0 PCF



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