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The Predictive Sampline Problem

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Uhi Ahn

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Robert A. Schlueter
Major professor
Robert A. Schlueter

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OPTIMAL CONTROL SYSTEM DESIGN:
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By

Uhi Ahn

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ABSTRACT

OPTIMAL CONTROL SYSTEM DESIGN: THE PREDICTIVE SAMPLING PROBLEM

By

Uhi Ahn

The principal contributions of this thesis are the formulation and solution of the optimal predictive sampling criterion for a sampled-data control system and the development of the optimal control system design methodology for the optimal predictive sampling problem.

The system performance index is formulated with a control performance index that measures actual performance of the control rather than error due to the sample and hold device as in the formulation of previous adaptive sampling criteria. The control performance index measures control performance over both the sampling interval over which the control is held constant and a future interval where the control is permitted to be continuous. Thus, only one sampling interval at a time is chosen and is based on the estimate of this performance index which in turn is based on past measurement of outputs of the system and knowledge of system inputs, system dynamics, and disturbance, initial conditions, and measurement noise statistics. A cost of implementation is included in the system performance index and is a specified constant if the predictive

sampling criterion is being used to perform control on a specified set of hardware and is a function of the length of the sampling interval if the objective is to design and select the computer hardware, computational algorithms, and computer software to implement the predictive sampling criterion.

The results of the optimal adaptive sampled-data control with predictive sampling criterion shows that the optimal predictive sampling criterion is indeed adaptive for on-line control if future performance can be precisely predicted as in the deterministic system but is periodic if future performance cannot be predicted as in the stochastic system. Moreover, the optimal predictive sampling criterion performs a control function because the control performance is improved over that of the continuous-time control for the deterministic system.

Optimal control system design methodology is further refined in this thesis. This optimal control system design is broken down into the optimal control design where the parameters of the control performance index are optimally tuned so that the resulting control meets the control performance objectives, and the optimal system design where the hardware to be implemented is optimally determined. The optimal system design procedure determines a precise cost of implementation as a function of the computational algorithms, computer software implementing that algorithm, and the hardware and then determines the optimal selection of hardware, computational algorithm, and computer software by a tradeoff of control performance and cost of implementation. Thus, optimal control system design really completes the design problem for an optimal control system because it not only tunes the control performance index to obtain acceptable control but also determines a precise cost of implementation and then selects a hardware, computational algorithm, and software option based on the control performance and cost specifications.

An example problem is chosen and is the linear second-order type two system which has been used extensively in the past research related to the optimal sampling problem. The control performance for the optimal sampled-data control with predictive sampling criterion is compared to the periodic sampled-data control and continuous control. The actual hardware cost for optimal predictive sampling problem is developed for this particular system. The parameters of the control performance index are then tuned for this system based on control objectives. A particular hardware, algorithm, and computer software option is then selected for this system based on a tradeoff of performance and cost.

To my wife, Jay-Bum,

and Bobby Jae-Hong

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CHAPTER 1. INTRODUCTION

1.1 Historical Development

Periodic sampling criteria have often been used because of the ease of design and analysis using transform techniques. Adaptive sampling criteria [1-11] have been developed to vary the sampling rate in proportion to the rate of change of some output or error signal. The first attempt at placing an analytic framework under the design of sampling criteria was made by Hsia [7]. In this work a large class of adaptive sampling rules was derived analytically from a continuous time integral performance index which measured the squared error introduced by sampling the error or output signals of a feedback control system. The performance index was augmented by a cost for sampling which was inversely proportional to the sampling interval length and represented the costs for measuring, transmitting and storing the sample.

The formulation of the optimal control problem has always included a control performance measure but has seldom included cost of implementation. Thus, the optimal control design is either impractical or must be modified to incorporate practical constraints imposed by costs of implementing the optimal control. An optimal control system design formulation [12] would directly impose the cost of implementation constraints by adding appropriate cost terms to the control performance index as in the formulation of these adaptive sampling criteria [7]. Practical control system design could thus be obtained directly. Although almost every aspect of control system design could be included in this formulation, the only aspects that have been investigated are the control and sampling problem [12] and the optimal sampling problem [13]. The number and the lengths of sampling intervals and the levels of each control element over each interval were therefore the variables optimized.

The control and sampling problem and the classical optimal sampling problem were chosen for investigation using this optimal control system design formulation [12] because the previous work on sampling in control systems [1-16] suggested such formulations. The classical formulation of the optimal sampling problem has a performance index that measures both the errors in sampled signals caused by the sampling criterion and the costs for implementing this criterion. The control law was specified and the sampling times were not considered control variables but rather design parameters that could be used to make the sampled-data control better approximate a continuous-time control. The optimal sampling problem was developed more carefully and then solved by Van Wieren and Schlueter [13]. In this work, the length of each sampling interval and the number of sampling times were selected together rather than selecting sampling intervals sequentially as in adaptive sampling. The system performance index for this classical optimal aperiodic sampling problem [13] was defined over the entire control interval rather than just one sampling interval and the cost of implementation was not just chosen to have a convenient form but was chosen to model the actual costs of implementing an aperiodic sampling criterion. Furthermore, the model of the system dynamics, input disturbances, initial conditions, and control inputs were all assumed known and were used to make the system performance index dependent on this information.

The formulation of the optimal control and sampling problem used a performance index that strictly measured control performance. The control law was not specified so that both a sampling interval sequence and a control vector sequence, which specified the level of each piecewise constant control signal and the length of the sampling interval over which it is held, were chosen optimally. This optimal sampled-data control problem [12] was formulated to obtain an optimal control with a sampling criterion that could provide better control performance than an optimal control with any periodic or arbitrary aperiodic criterion.

An efficient computational algorithm was developed for this optimal sampled-data control problem for the special case where the optimal control sequence can be determined as a unique function of the particular sampling intervals sequence chosen. For this special case, the performance index can be determined as a function of this sampling interval sequence. The optimal sampling interval sequence can be found by minimizing this derived performance index. The optimal sampled-data control problem could thus be separated into the problem of finding an optimal control law for any sampling interval sequence and a problem of determining the optimal sampling criterion for this optimal control law. Thus, the optimal sampled-data control problem can be considered an optimal control and sampling problem when the optimal control can be determined as a function of the sampling interval sequence.

This algorithm was applied to compute the optimal sampled-data control law for the regulator problem with constrained [14], state dependent [15], and adaptive sampling [16] criteria. The excellent control performance obtained with very few control changes indicates that the computer memory and computer-communication system required to store and transmit the control can be significantly reduced if the sampling intervals are determined optimally rather than specified a priori.

This control and sampling problem was not formulated with a cost of implementation term in the performance index because it was formulated as a traditional optimal control problem. Although the sampling intervals sequence were considered control variables, the number of samples was not considered a control variable and was specified since the theory indicated the solution when both the number and lengths of sampling intervals is optimized is trivial (i.e. $N=\infty$ and $\underline{T}=\underline{0}$).

Recent result on observability and controllability of sampled-data control system [17,18] have shown that the observability and controllability of the continuous system can only be preserved in general if the number and the lengths of the sampling intervals are control variables. Thus, this theory suggests selection of a sampling rule may provide the same kind of performance improvement that the selection of a control law can. This hypothesis was shown to be true in the recent papers that established

(1) that the selection of an optimal sampling rule can proceed as an independent optimization problem from the determination of an optimal control if the optimal control can be uniquely specified for any sequence of sampling time chosen [12]; and

(2) that the selection of an optimal aperiodic sampling criterion can cause a remarkable reduction in data requirements to achieve the same performance value as observed using periodic sampling [12]. These results were established for the sampled-data control problem where the levels of each piecewise constant control element over each sampling interval and the number and lengths of the sampling intervals were the control variables to be optimized in order to specify the optimal control law.

1.2 Optimal Control System Design

Since the number and lengths of sampling intervals are control variables and since the solution to the optimal control and sampling problem approximates the continuous-time control when there is no cost of implementation and no upper bound on the number of samples [12, Theorem 3], a cost of implementation should be included along with a control performance index in any general formulation of the control system design problem. Since the actuators, sensors, communication links, and computer hardware and software depend on the number and lengths of these sampling intervals and determine the cost of implementation as a function of these control variables, this hardware and software that go into implementing a control law along with the number and lengths of sampling intervals must be considered part of the control system design rather than part of plant being controlled as in the traditional optimal control formulation.

These results suggest the control system design formulation which differs from traditional optimal control in the following two ways:

(1) A performance index is used that attempts to precisely model both the control performance and the cost of implementation objectives for a particular application;

(2) Sensors, actuators, communication links, and computer hardware and software as well as the number and lengths of sampling intervals will be considered part of the control system to be designed.

Optimal control system design has been developed as a method of problem formulation and a design methodology not only because of the above theoretical considerations but also because

(1) the traditional optimal control formulation produced control laws that either could not be implemented or had to be significantly modified because the formulation ignored

the cost of implementation;

(2) many important aspects of the control system design problem could not be adequately formulated using the traditional optimal control formulations. Examples include

(a) the selection from different control structures that range from completely centralized to completely decentralized.

(b) the selection of different control laws which range from the linear quadratic Gaussian (LQG) solution to the classical single input single output (SISO) feedback one,

(c) the cost versus performance tradeoff for using a particular input or output in the control law, and

(d) the selection from among different analog or digital hardware alternatives for actuators, sensors, communication links, and computer hardware;

(3) the optimal control system design formulation can lead to improved design procedures and improved system evaluation methodologies. An excellent example of the possible improvement in design methodology and evaluation procedures is found in [12,13] where optimal periodic, optimal aperiodic, and optimal adaptive sampling criteria were designed based on minimization of a system cost which is composed of a cost of implementation and the control performance measure. The optimal sampling criterion could then be selected based on the criterion (optimal periodic, optimal aperiodic or optimal adaptive) with the lowest system cost;

(4) the optimal control system design formulation is the proper framework for developing a procedure for computer aided design of control systems which could include a control structure, control law, and the hardware-software combination.

1.3 Optimal Aperiodic Sampling Problem for Control

The recent results on controllability and observability [17,18], that indicated the lengths and number of sampling intervals are control variables, and the control and sampling problem, which indicated a dramatic 50:1 reduction in data requirements were possible for the optimal sampled-data control with optimal aperiodic sampling over the optimal periodic sampled-data control with the same control performance index value, suggested that a study of optimal aperiodic sampling for control be performed where the control law is specified and the number and lengths of sampling intervals are optimized. The system performance index measures the control performance and the actual costs of implementation for a sampled-data control law with optimal aperiodic sampling. In this optimal aperiodic sampling problem, the number and lengths of each sampling interval were optimized together based on a performance index defined over a specified control interval. This problem extends work on optimal sampling for the optimal tracking [12] and regulator [14-16] problems, but in this case

(1) the control sequence which specifies the level of each control element over each sampling interval will not be optimized;

(2) the control sequence will be determined based on the values of a specified continuous-time control law at the sampling times.

Since the control sequence is uniquely specified by the sampling interval sequence, the theory of optimal sampled-data control indicates the optimal sampling problem can be treated as separate optimization problem from the determination of the continuous-time control law. The optimal number and lengths of sampling intervals is determined by using a nonlinear programming algorithm to determine the optimal sampling interval sequence for each number of sampling intervals of interest and then plotting these optimized system performance index values to determine the optimal number of samples graphically.

The results of this study of optimal aperiodic sampling indicate that

(1) selecting an optimal aperiodic sampling criterion for a nonoptimal continuous-time control can dramatically improve control performance over that of the unsampled continuous-time control;

(2) optimal aperiodic sampling can increase the speed of response over that of the unsampled continuous-time control;

(3) the selection of the optimal sampling criterion from among optimal periodic, optimal aperiodic, and optimal adaptive depends on the terms included in the control performance and cost of implementation;

(4) the control performance improvement due to optimal

aperiodic sampling is due to effective use of the delay cause by the sample and hold device to meet the objectives measured by the control performance index;

(5) the optimal aperiodic sampling interval sequence depends on the specific control performance, cost of implementation, system dynamics, inputs, and initial conditions for the system considered.

The aperiodic sampling problem [13] considered input uncertainty and random initial conditions, but did not consider the case where measurement noise was present. The optimal aperiodic sampled-data stochastic control problem extends these results to that case. The optimal stochastic control law is a piecewise constant vector control that is held over sampling intervals. The level of the control over any interval is specified by a gain matrix multiplied by the estimate of the state at the sampling time at the beginning of the particular sampling interval considered. The gain matrix may be the gain of the optimal or non-optimal continuous-time control law at that particular sampling time or the gain matrix of the optimal sampled-data control law [14] for the particular sampling interval sequence.

The control sequence that specifies the optimal sampled-data stochastic control law with optimal aperiodic sampling is closed loop because the level of the control over any interval depends on the state estimate which depends on the sampled measurements of the output at previous sampling times. The sampling interval sequence for this optimal sampled-data stochastic control with optimal aperiodic sampling is open loop because the optimal number and lengths of sampling intervals are determined based on the average performance over all sample functions observed on the system and is not based on actual measurements and the actual sample functions of the processes observed on that system over a particular interval.

1.4 Optimal Predictive Sampling Problem

An optimal predictive sampling problem will be formulated in this thesis in order to produce a control law that has both a closed loop control sequence and a closed loop sampling interval sequence. The control law is identical to that used for the optimal sampled-data stochastic control problem with optimal aperiodic sampling but restricted to the case where the gain matrix is specified by a continuous-time optimal or specified non-optimal control law at the particular sampling time.

The performance index will be defined over the control interval but is separated into a measure of control performance over the sampling interval to be optimized, the control performance over the remainder of the control interval after this sampling interval and a cost of implementation that measures hardware cost for implementing this predictive sampling criterion. This system performance index is optimized to produce a sampling interval. A sampling interval sequence is thus obtained by iteratively solving this predictive sampling problem.

The sampling criterion is predictive because the control performance terms are predicted based on measurements of the system output at all previous sampling times, knowledge of system dynamics, control inputs, and the statistics of the input disturbances, initial conditions and measurement noises. The predictive sampling problem assumes that the continuous-time control law is specified and may be optimal or suboptimal. Thus, the selection of the control is specified by the specified continuous-time control and the sampling times and is not selected optimally for the sampling times sequence as for the control and sampling problem.

1.5 Important Results and Contributions

The main contributions of this thesis will be

(1) to formulate and solve the optimal predictive sampling problem;

(2) to extend the optimal control system design methodology; and

(3) to apply this methodology to the optimal predictive sampling problem.

In Chapter 2, the optimal predictive sampling problem for control is formulated for a linear time invariant system with a known input and disturbance statistics and a specified continuous-time control law. This control law is based on a state estimate which is in turn based on sampled noisy measurements of the outputs at previous sampling times. The system performance index chosen measures control performance and cost of implementation. The control performance index proposed measures performance over the next sampling interval where this control is held constant and over a future interval where the control is permitted to be continuous-time. The value of this control performance can be estimated for any sampling interval length based on the sampled output measurements obtained at previous sampling times and the knowledge of system inputs, system dynamics, and disturbance, initial condition, and measurement noise statistics. The cost of implementation measures the precise cost of implementation as a function of computer hardware, computational algorithms, and computer software for computing the optimal sampling interval on-line.

In Chapter 3, optimal control system design methodology is developed as a formal procedure for the predictive sampling problem. Optimal control system design is shown to consist of a two step off-line procedure; optimal control design, which determines the control performance index optimally, and optimal system design, which determines the cost of implementation and the optimal selection of hardware to be implemented by a tradeoff of control performance and cost of implementation. Traditional optimal control design problem corresponds to this optimal control design problem but ignored the optimal selection of computational algorithm, computer software, and computer-communication-instrumentation hardware which corresponds to this optimal system design problem.

In Chapter 4, optimal control design for predictive sampling problem is developed in detail for a particular example problem. It is shown that the sampled-data control with a predictive sampling criterion outperforms the periodic sampled-data control with the same number of control changes and even outperforms the continuous-time control if the system is deterministic. It is also shown that the best predictive sampling criterion for the stochastic control system is periodic which indicates that the selection of sampling intervals cannot improve control performance when the future control performance as a function of this sampling interval cannot be accurately predicted. Thus, the optimal predictive sampling does perform a control function for the deterministic control system by holding a control with a larger absolute magnitude than the continuous-time control; thus improving speed of response and terminal error.

In Chapter 5, optimal system design for the predictive sampling problem is developed in detail for the same deterministic example problem chosen in Chapter 4. Cost of implementation is developed only for the hardware cost term in the cost of implementation because the hardware is dedicated for this problem and because communication and instrumentation hardware are assumed chosen. An appealing computer hardware cost function is obtained by optimizing computer algorithms, computer software and computer hardwares. Optimal selection of hardware is also performed using two distinct methods that tradeoff control performance against cost of implementation. Control performance obtained from optimal predictive sampling criterion with this optimally selected hardware is dramatically improved are the control performances for periodic sampled-data control and for continuous control.

Conclusions are then presented in Chapter 6.

CHAPTER 2. PROBLEM FORMULATION

Consider a computer control system where the plant can be modeled as a linear, time-invariant, observable and controllable stochastic system

$$\dot{\underline{x}}(t) = \underline{A}\ \underline{x}(t) + \underline{B}\ \underline{u}(t) + \underline{w}(t) \tag{1}$$

where $\underline{x}$ is an n-dimensional state vector, $\underline{u}$ is an r-dimensional control vector, and $\underline{w}$ is an n-dimensional disturbance vector.

$$\begin{aligned} &E\ \{\underline{w}(t)\} = \underline{0}_n \\ &E\ \{\underline{w}(t)\ \underline{w}'(\tau)\} = \underline{W}\ \delta(t-\tau) \end{aligned} \tag{2}$$

where $\delta(\cdot)$ is the impulse function, and E and $'$ indicate expectation and transpose operations respectively. The initial time $t_o \varepsilon(-\infty,\infty)$ is fixed and the initial state is random and satisfies

$$\begin{aligned} &E\ \{\underline{x}(t_o)\} = \underline{m}(t_o) \\ &E\ \{(\underline{x}(t_o) - \underline{m}(t_o))\ (\underline{x}(t_o) - \underline{m}(t_o))'\} = \underline{V}(t_o) \\ &E\ \{\underline{x}(t_o)\ \underline{w}'(t)\} = \underline{0}_{nn} \qquad t\varepsilon(t_o,t_f) \end{aligned} \tag{3}$$

The system is observed by measuring the outputs $\underline{y}(t)$

$$\underline{y}(t) = \underline{C}\ \underline{x}(t) \tag{4}$$

at (N-1) sampling times $\{t_i\}_{i=1}^{N-1}$ where the sampling intervals $T_i = t_{i+1} - t_i$ satisfy

$$0 \leq T_{min} \leq T_i \leq T_{max} \tag{5}$$

$$g(T_o,T_1,\dots,T_{N-1}) = \sum_{i=o}^{N-1} T_i - (t_f - t_o) = 0 \tag{6}$$

where N satisfies

$$N_{min} \leq N \leq N_{max} \qquad (7)$$

The output measurements are corrupted with noise $\underline{\psi}_i$ such that

$$\underline{z}_i = \underline{y}(t_i) + \underline{\psi}_i \qquad i=1,2,\ldots\ldots\ldots,N-1 \qquad (8)$$

where $\underline{\psi}_i$ is an m-dimensional noise vector that satisfies

$$\begin{aligned} &E\,\{\underline{\psi}_i\} = \underline{0}_m \\ &E\,\{\underline{\psi}_i\,\psi_j'\} = \underline{\Psi}\,\delta_{ij} \\ &E\,\{\underline{\psi}_i\underline{w}'(t)\} = \underline{0}_{mn} \qquad t\varepsilon(t_o,t_f) \\ &E\,\{\underline{\psi}_i\,\underline{x}'(t_o)\} = \underline{0}_{mn} \end{aligned} \qquad (9)$$

where δ_{ij} is the Kronecker delta function.

The control $\underline{u}(t)$ is assumed to be a piecewise constant vector function whose elements change value only at the sampling times $\{t_i\}_{i=1}^{N-1}$ such that

$$\underline{u}(t) = \underline{u}(t\ ,\ \underline{Z}_i) \qquad t\varepsilon[t_i,t_{i+1}) \qquad (10)$$

and can depend on the previous measurements

$$\underline{Z}_i' = (\underline{z}_1', \underline{z}_2', \ldots\ldots, \underline{z}_i') \quad \text{for} \quad i=1,2,\ldots\ldots\ldots,N-1$$

The optimal predictive sampling for control problem can be stated formally as follows:

Given the linear system (1,4,8) with disturbances (2), measurement noise (9), and initial conditions (3), and given a control law (10) of the form

$$\underline{u}(t) = \begin{cases} \underline{u}_i = \underline{h}(t_i) - \underline{G}\,\hat{\underline{x}}(t_i^+/t_i^+) & t\varepsilon[t_i,t_{i+1}) \\ \qquad \underline{h}(t) - \underline{G}\,\hat{\underline{x}}(t/t_i^+) & t\varepsilon[t_{i+1},t_i+\Delta_i) \end{cases} \qquad (11)$$

where $\underline{h}(t)$ is the input to be tracked and $\hat{\underline{x}}(t/t_i^+)$ is the estimate of $\underline{x}(t)$ after measurement $\underline{z}_i$ is made, i.e.

$$\hat{\underline{x}}(t/t_i^+) = E\,\{\underline{x}(t)/\underline{Z}_i\}$$

$$\underline{V}(t/t_i^+) = E\,\{(\underline{x}(t) - \hat{\underline{x}}(t/t_i^+))\ (\underline{x}(t) - \hat{\underline{x}}(t/t_i^+))'/\underline{Z}_i\}$$

where the initial conditions are

$$\hat{\underline{x}}(t_o/t_o) = \underline{m}(t_o)$$

$$\underline{V}(t_o/t_o) = \underline{V}(t_o)$$

Determine the optimal sampling interval T_i^* that satisfies (5) and minimizes a system performance index:

$$S(T_i) = J(T_i) + qC(T_i) \tag{12}$$

where $J(T_i)$ is the control performance and $C(T_i)$ is the cost of implementation, and where the control performance has the form:

$$\begin{aligned} J(T_i) = 1/2\ E\{(\ \alpha\ [(\underline{h}(t_i+\Delta_i) - \underline{y}(t_i+\Delta_i))'\ \underline{F}\ (\underline{h}(t_i+\Delta_i) - \underline{y}(t_i+\Delta_i)) \\ + \int_{t_{i+1}}^{t_i+\Delta_i} \{(\underline{h}(t) - \underline{y}(t))'\ \underline{Q}\ (\underline{h}(t) - \underline{y}(t)) + \underline{u}'(t)\ \underline{R}\ u(t)\}\ dt] \\ + \int_{t_i}^{t_i+1} \{(\underline{h}(t) - \underline{y}(t))'\ \underline{Q}\ (\underline{h}(t) - \underline{y}(t)) + \underline{u}'(t)\ \underline{R}\ \underline{u}(t)\}\ dt) \\ /\underline{Z}_i\} \end{aligned} \tag{13}$$

Matrices $\underline{F}$ and $\underline{Q}$ are positive semidefinite symmetric, $\underline{R}$ is positive definite symmetric matrix, α is positive constant, and the expectation operator E is conditioned on measurements $\underline{Z}_i$. The first term measures the terminal squared error and the second and third terms measure the tracking and the control square error over the time interval $[t_i, t_{i+1})$ and $[t_{i+1}, t_i+\Delta_i)$ respectively. The parameter α weights future performance over $[t_{i+1}, t_i+\Delta_i)$ against that of the immediate sampling interval $[t_i, t_{i+1})$. The parameter Δ_i represents the length of the interval over which the control performance is predicted and this parameter (Δ_i) can be chosen to be constant $(\Delta_i = \Delta)$ or can be chosen based on a fixed terminal time and thus $\Delta_i = t_f - t_i$.

This control performance index can be expressed as a function of T_i as follows:

$$
\begin{aligned}
J(T_i) = {} & 1/2\, \alpha[(\underline{h}(t_i+\Delta_i) - \underline{C}\, \hat{\underline{x}}(t_i+\Delta_i/t_i^+))'\, \underline{F}\, (\underline{h}(t_i+\Delta_i) - \underline{C}\, \hat{\underline{x}}(t_i+\Delta_i/t_i^+)) \\
& + \mathrm{Tr}\, \{\underline{C}'\, \underline{F}\, \underline{C}\, \underline{V}(t_i+\Delta_i/t_i^+)\}] \\
& + 1/2\, \alpha \int_{t_{i+1}}^{t_i+\Delta_i} [(\underline{h}(t) - \underline{C}\, \hat{\underline{x}}(t/t_i^+))'\, \underline{Q}\, (\underline{h}(t) - \underline{C}\, \hat{\underline{x}}(t/t_i^+)) \\
& + \mathrm{Tr}\, \{\underline{C}'\, \underline{Q}\, \underline{C}\, \underline{V}(t/t_i^+)\} \\
& + (\underline{h}(t) - \underline{G}\, \hat{\underline{x}}(t/t_i^+))'\, \underline{R}\, (\underline{h}(t) - \underline{G}\, \hat{\underline{x}}(t/t_i^+))]\, dt \\
& + 1/2 \int_{t_i}^{t_{i+1}} [(\underline{h}(t) - \underline{C}\, \hat{\underline{x}}(t/t_i^+))'\, \underline{Q}\, (\underline{h}(t) - \underline{C}\, \hat{\underline{x}}(t/t_i^+)) \\
& + \mathrm{Tr}\, \{\underline{C}'\, \underline{Q}\, \underline{C}\, \underline{V}(t/t_i^+)\}]\, dt \\
& + 1/2\, T_i[(\underline{h}(t_i) - \underline{G}\, \hat{\underline{x}}(t_i^+/t_i^+))'\, \underline{R}\, (\underline{h}(t_i) - \underline{G}\, \hat{\underline{x}}(t_i^+/t_i^+))]
\end{aligned}
\tag{14}
$$

by substituting (4), (11), and taking expectation term by term.

The conditional mean $\hat{\underline{x}}(t/t_i^+)$ and variance $\underline{V}(t/t_i^+)$ satisfy:

$$\underline{\hat{x}}(t/t_i) = \underline{\phi}(t,t_i)\ \underline{\hat{x}}(t_i^+/t_i^+) + \int_{t_i}^{t} \underline{\phi}(t,\tau)\ \underline{B}\ d\tau\ [\underline{h}(t_i) - \underline{G}\ \underline{\hat{x}}(t_i^+/t_i^+)]$$

$$t\varepsilon[t_i, t_{i+1})$$

$$\underline{\hat{x}}(t/t_i) = \underline{\phi}(t,t_{i+1})\ \underline{\hat{x}}(t_{i+1}/t_i^+) + \int_{t_{i+1}}^{t} \underline{\phi}(t,\tau)\ \underline{B}\ [\underline{h}(\tau) - \underline{G}\ \underline{\hat{x}}(\tau/t_i^+)]\ d\tau$$

$$t\varepsilon[t_{i+1}, t_i+\Delta_i) \qquad (15)$$

$$\underline{V}(t/t_i) = \underline{\phi}(t,t_i)\ \underline{V}(t_i^+/t_i^+)\ \underline{\phi}'(t,t_i) + \int_{t_i}^{t} \underline{\phi}(t,\tau)\ \underline{W}\ \underline{\phi}'(t,\tau)\ d\tau$$

$$t\varepsilon[t_i, t_i+\Delta_i)$$

A block diagram of the system is shown in Figure 2-1. It should be noted that the value of performance $J(T_i)$ is predicted based on measurements $\underline{Z}_i$ that includes measurement $\underline{z}_i$ at t_i and the assumption that $\underline{\hat{x}}(t_i^+/t_i^+)$, $\underline{V}(t_i^+/t_i^+)$, system model (1,4,8), control (11), and statistics (2,3,9) are all given.

Once T_i^* is determined by minimizing $S(T_i)$ with respect to T_i satisfying (5), the computer must wait until $t_{i+1} = t_i + T_i^*$ occurs. At t_{i+1} the computer triggers sampling of measurement $\underline{z}_{i+1}$ and then computes

$$\underline{\hat{x}}(t_{i+1}^+/t_{i+1}^+) = \underline{\hat{x}}(t_{i+1}/t_i^+) + \underline{K}(t_{i+1})\ [\underline{z}(t_{i+1}) - \underline{C}\ \underline{\hat{x}}(t_{i+1}/t_i^+)]$$

$$\underline{V}(t_{i+1}^+/t_{i+1}^+) = \underline{V}(t_{i+1}/t_i^+) - \underline{K}(t_{i+1})\ \underline{C}\ \underline{V}(t_{i+1}/t_i^+) \qquad (16)$$

$$\underline{K}(t_{i+1}) = \underline{V}(t_{i+1}/t_i^+)\ \underline{C}'[\underline{C}\ \underline{V}(t_{i+1}/t_i^+)\ \underline{C}' + \underline{\psi}]^{-1}.$$

The computer is then ready to begin the cycle again by computing T_{i+1}^* by using a search algorithm that requires repeated evaluation of control (11), control performance $J(T_i)$ (14), and the cost of implementation for several values of T_{i+1} where in this case $T_{i+1} = T_i$, $t_{i+1} = t_i$, $t_{i+2} = t_{i+1}$ in these equations.

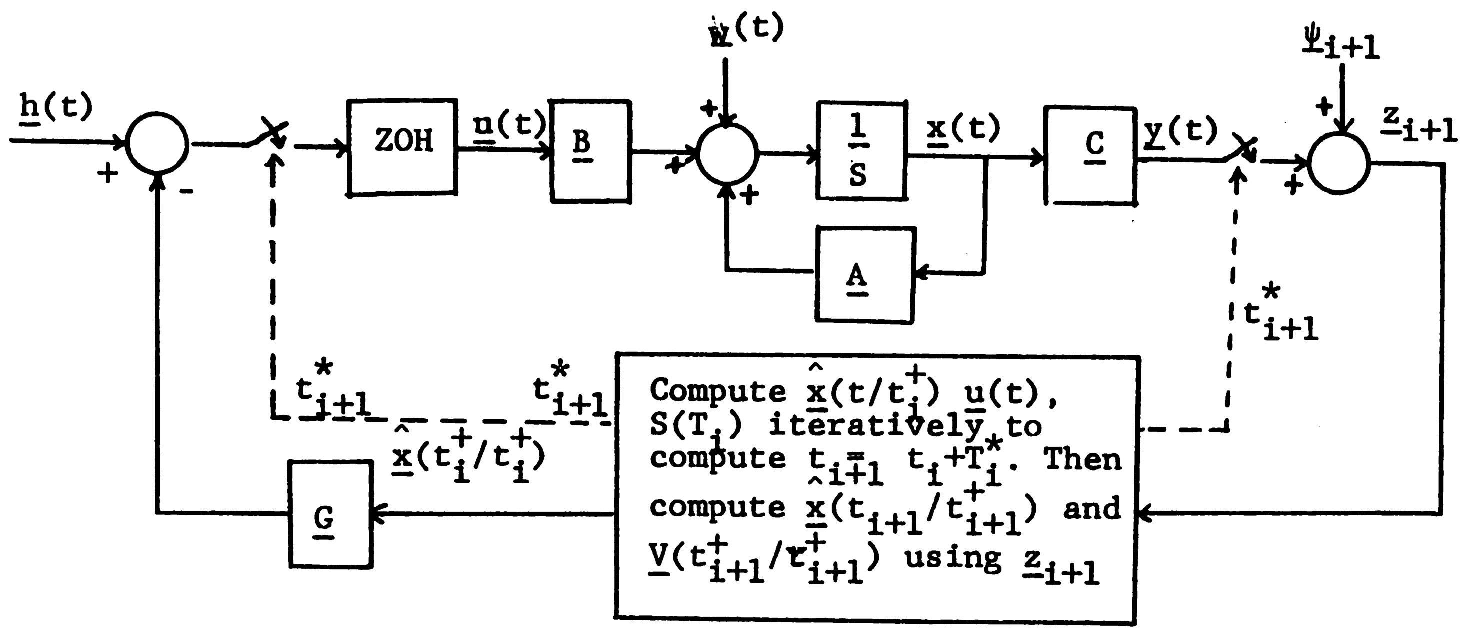


FIGURE 2-1 System block diagram for the optimal predictive sampling problem.

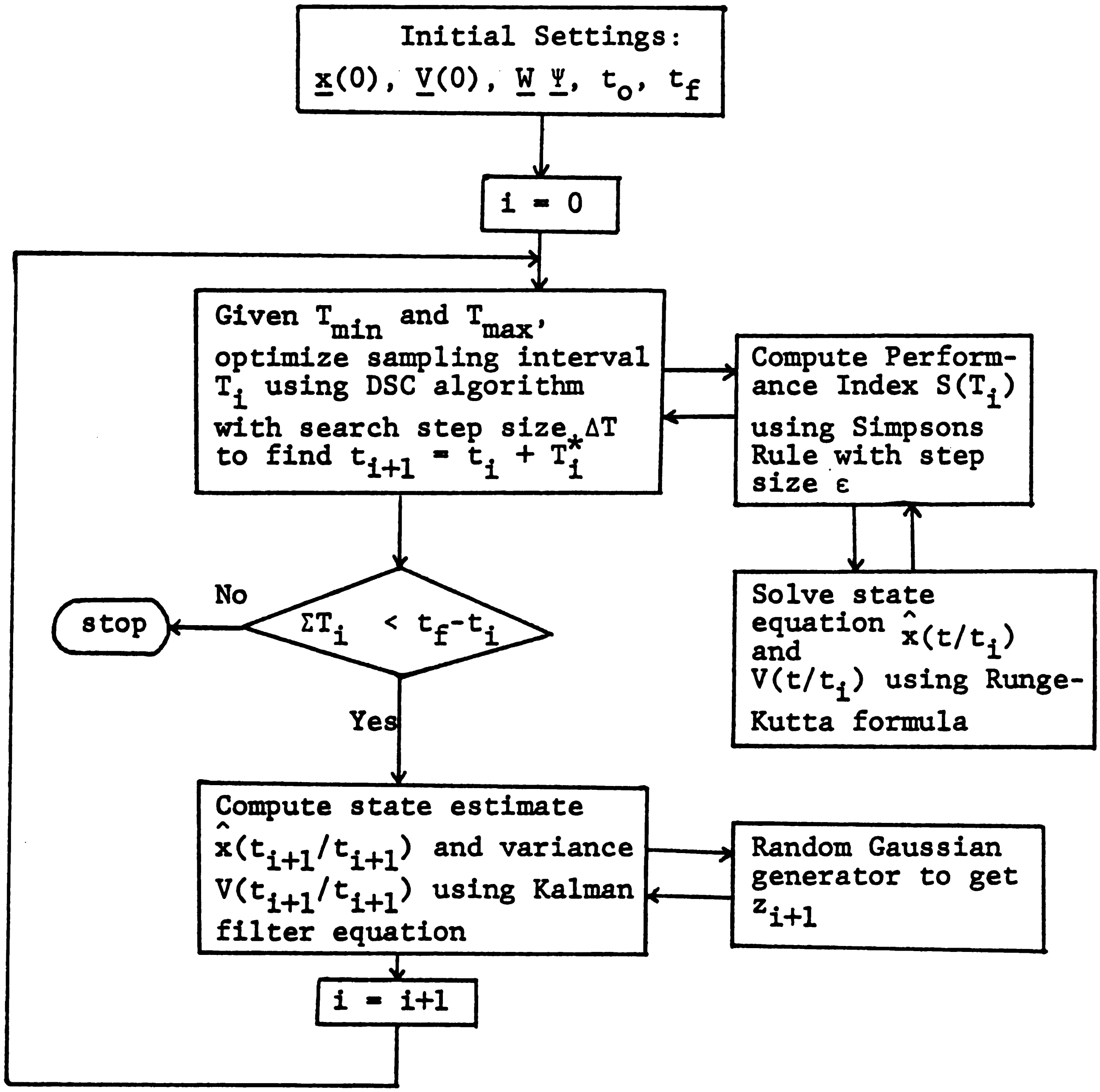


FIGURE 2-2 Flow chart of computational procedure for solving the predictive sampling problem.

The control (11) can be a state variable feedback one since $\hat{\underline{x}}(t_i^+/t_i^+)$ must be available at t_i so that $J(T_i)$ can be predicted. The control law (11) must be closed loop in order to reduce the effects of disturbances, system parameter variation, and measurement noise. The predictor equations (15,16) are derived in [19-21]. The flow chart of the computational procedure for solving this predictive sampling problem over the control interval $[t_o, t_f]$ is shown in Figure 2.2.

This control performance index has the same form as that used in the aperiodic sampling problem except that the performance is only defined over $[t_i, t_i+\Delta_i)$ rather than $[t_o, t_f)$ and can only be optimized over T_i rather than $(N, T_o, T_1, \ldots, T_{N-1})$. The problem is formulated so that the control performance over $[t_{i+1}, t_i+\Delta_i)$ can be neglected if α is zero. In this case, the optimal sampling interval T_i^* would depend strictly on the control performance over $[t_i, t_{i+1})$, the cost of hardware capable of computing $t_{i+1} = t_i + T_i^*$ in an interval less than T_i^*, and the cost of communication and instrumentation hardware that could handle a sampling rate $f_i = 1/T_i^*$. If α is not zero then the optimal sampling interval T_i^* must be chosen based on:

(a) the control performance over $[t_i, t_{i+1})$ that generally increases with T_i;

(b) the control performance over $[t_{i+1}, t_i+\Delta_i)$ which is generally a convex function of T_i; and

(c) the cost of implementation that is a constant if the hardware is already selected and the adaptive sampling criterion is being implemented on-line. The cost of implementation is a decreasing function of T_i if its selection will be based on this performance versus cost tradeoff. This would be the case for an optimal system design of the control and the hardware to implement it.

It should be noted that in the predictive sampling problem the control performance is averaged over all sample functions of $\underline{w}(t)$ and $\{\underline{\psi}_i\}_{i=1}^{N-1}$ and the performance is conditioned on knowing past measurements $\underline{Z}_i$ of the output. These differences between the aperiodic sampling problem [13] and the predictive sampling problem being formulated here are intentional because the optimal sampling interval T_i^* is to be computed on-line after measurement $\underline{z}_i$ is taken rather than off-line without any measurements at all as in the aperiodic sampling problem.

Observability and controllability of the sampled-data system [17,18] need not be assumed to assure the existence of T_i^* for each i. However, if the optimal adaptive sampling criterion is to provide acceptable control performance for this optimal adaptively sampled control system, sampled-data controllability and observability can be assured if and only if the continuous-time system is controllable and observable when the number and lengths of sampling intervals are control variables which can be chosen [17,18]. Sampled-data controllability and observability can be obtained with only q(order of minimal polynomial of the system) sampling times or more if these sampling times are taken so that no information on or control over the controllable and observable continuous-time system is lost by choice of these sampling times. If t_f is sufficient long or the adaptive sampling problem has no specified terminal time, then at least q sampling intervals will eventually be obtained and thus the system will become sampled-data controllable and observable if these optimal sampling intervals do not cause loss of information about or control over the system. The optimal sequence of adaptive sampling intervals should not cause either loss of information or control because the control performance index would be degraded if such loss of control or information were to occur and these sampling intervals are chosen to minimize control performance. Thus, the optimal adaptively sampled control system should be sampled-data observable and controllable for all time after the initial q sampling intervals are taken.

The cost of implementation can now be discussed since the application of this predictive sampling problem has been discussed. The cost of implementation in the optimal systems control and sampling problem should include the hardware cost which measures the cost of additional instrumentation, communication and computer hardwares required to implement a criterion, the computation cost which measure the cost for designing or tunning a sampling criterion in order to achieve its best possible performance, and the communication cost which measure the cost for communication on a time shared communication link for the data from the computer to actuator and from sensor to the computer. Thus, the cost of implementation has the form:

$$C = C_1 + C_2 + C_3 \tag{17}$$

The hardware cost (C_1) is constant if the sampling criterion has been implemented on an existing or specified set of hardwares as in the on-line optimal systems control problem. However, if the instrumentation, computer and communication hardware which is to be implemented as a part of the control system will depend on the sampling criterion selected, this hardware cost will depend on T_i. Although the functional form of this cost term may vary for different applications, the hardware cost would generally be a monotonically decreasing function of the sampling interval T_i and monotonically increasing function of the on-line memory and computational requirements for a particular criterion.

Since the optimal predictive sampling problem is on-line control problem the hardwares for computer, communication and instrumentation should be purchased or dedicated due to the variable and yet unknown sampling rate. Therefore, the cost of implementation for the optimal predictive sampling measures only the hardware cost (C_1). Thus, the computation cost (C_2) for the time shared use of computer facilities to compute $\underline{T}^*$ for each N and the communication cost (C_3) for the time shared use of the communication link to transmit $\{\underline{u}(t_i^*), t_{i+1}^*\}_{i=0}^{N^*-1}$ used in the cost of implementation for the aperiodic sampling problem need not be included in the cost of implementation for the predictive sampling problem. The cost terms C_2 and C_3 were developed for the optimal aperiodic sampling problem in a recent paper [13].

The hardware cost (C_1) in this cost of implementation will have one of the following two forms:

$$C_1(T_i) = f_0(T_i) + f_1(T_i) + f_2(T_i) \qquad (18)$$

or

$$C_1(T_i) = f_0(T_{min}) + f_1(T_{min}) + f_2(T_{min}) \qquad (19)$$

where $f_0(\cdot)$ is the computer hardware cost, $f_1(\cdot)$ is the communication hardware cost, and $f_2(\cdot)$ is the instrumentation hardware cost, respectively. The first form would be used when the hardware to be used has not been selected and would be implemented based on selection of T_i^*. The second form would be used when the hardware has already been selected and has the capability of sampling at a maximum rate of $f = 1/T_{min}$ samples per second.

The functions $f_k(\cdot)$ used in both expressions would be identical and would actually measure the minimum hardware cost required to implement a sampling criterion with sampling rate $f_i = 1/T_i$ samples per second. The hardware cost function will be developed in detail in Chapter 5.

CHAPTER 3. OPTIMAL CONTROL SYSTEM DESIGN(OCSD) METHODOLOGY

The purpose of this chapter is to discuss and extend the optimal control system design (OCSD) methodology [12] and relate the concepts to the optimal predictive sampling problem. The design methodology assumes that the plant to be controlled and the statistics of disturbances and measurement noise have both been modeled and the control performance and the cost of implementation objectives have been clearly stated. It should be noted that this chapter is concerned with:

(1) the optimal control design (OCD) problem which selects the parameters (α, Δ_i) and matrices $(\underline{Q}, \underline{R}, \underline{F})$ that specifies the control performance index. Given the parameters and matrices to be determined in this step, an optimal sampling interval T_{ij}^* can be computed that minimizes the control performance index (14) subject to the constraint (5) for a particular set of operating conditions

$$\begin{array}{ll} \underline{h}_j(t) & t \varepsilon [t_0, t_i] \\ \underline{w}_j(t) & t \varepsilon [t_0, t_i] \\ \{\underline{\psi}_{kj}\}_{k=1}^{i} & \end{array} \qquad (20)$$

where j specifies the particular set of inputs $\underline{h}(t)$ and sample functions of the processes $\{\underline{\psi}_k\}_{k=1}^{i}$ and $\underline{w}(t)$ with statistics

(2,9) that produce the sampled measurement sample function, $\underline{Z}'_{ij} = (\underline{z}'_{1j}, \underline{z}'_{2j}, \ldots\ldots, \underline{z}'_{ij})$. The determination of T^*_{ij} is called the optimal control (OC) problem and the determination of the parameters and matrices will be called the optimal control design (OCD) problem. This terminology is used to differentiate these two aspects of modern control theory from the extension proposed in the optimal system design (OSD) problem.

(2) the optimal system design (OSD) problem where the control performance and cost of implementation objectives are used in conjunction with a list of options for computational algorithms and computer-communication-instrumentation hardware.

i) to determine the cost of implementation $C(T_i)$ as a function of the sampling interval by determining the best computer algorithm-hardware option for each sampling interval T_i;

ii) to optimally select computational algorithms, software implementation of these algorithms, and computer-communication-instrumentation hardware based on an optimal tradeoff of control performance and the cost of implementation over a number of sampling intervals (i) and operating conditions (j).

The optimal control design (OCD) and optimal system design (OSD) problem is a two-step off-line procedure for selecting the control performance and the cost of implementation, and for performing an optimal tradeoff of control performance and cost of implementation that not only determines the hardware to be implemented but also the optimal control (selection of T_i^*) which it implements. This two-step design problem is called the optimal control system design (OCSD) problem. Since the OCSD is performed off-line and determines the control performance index and the hardware to be used, the optimal control (OC) problem (which determines the optimal sampling interval for a particular set of measurements from the system, the control performance index determined in the OCSD problem, and the hardware selected based on the OCSD problem) can thus minimize a system performance index that includes control performance index and a constant cost of implementation specified by the OCSD problem rather than a control performance alone. This view of the OC problem suggests it uses the same performance index used in the OCSD problem but with a fixed cost of implementation because the hardware is specified.

The traditional OCD problem has ignored the optimal selection of computational algorithm, computer software, and computer-communication-instrumentation hardware. This research is thus aimed at providing a foundation for incorporating these aspects into the OCSD methodology. This OCSD methodology can be applied to far more general control problems than the optimal predictive sampling problem and will be applied to such problems in the future.

The OCD and OSD subproblems will now be discussed in detail in the next two subsections.

3.1 Optimal Control Design (OCD) Problem

The OCD determines the parameters (α,Δ_i) and matrices $(\underline{Q},\underline{R},\underline{F})$ assuming cost of implementation for all controls are zero and that no constraints must be placed on T_{min} due to hardware constraints. These matrices and parameters thus can be determined based on the control objectives and specifications for the particular system to be controlled. These matrices and parameters are determined using the same interative procedure used to design an optimal closed loop control law [22]. Specifically, this design procedure requires that the parameters (α,Δ_i) and matrices $(\underline{Q},\underline{R},\underline{F})$ be modified until the performance of the system with closed loop optimal sampling sequence $\{T^*_{ij}\}^N_{i=1}$ satisfies all system design objectives and specifications as determined from the simulation of optimal system state trajectory $\underline{x}(t)$ and sampled-data control (11) for that optimal sampling interval sequence $\{T^*_{ij}\}^N_{i=1}$.

The parameter α is a positive constant and is used to weight the future performance over $[t_{i+1},t_i+\Delta_i)$ with respect to the performance over $[t_i,t_{i+1})$. This weighting is desirable because the selection of T_i has a dramatic effect on performance at any instant in interval $[t_i+T_i,t_i+\Delta_i)$ because it determines $\underline{x}(t_i+T_i)$ and thus $\underline{x}(t)$ and $\underline{u}(t)$ over $[t_i+T_i,t_i+\Delta_i)$, but T_i has no effect on the performance at any time instant in $[t_i,t_i+T_i)$ because the control $\underline{u}_i = \underline{u}(t_i)$ is completely specified there. Thus, α should be large enough so that the future control performance over $[t_{i+1}, t_i+\Delta_i)$ dominates the selection of the optimal sampling time interval

T_i^* if the sampling criterion is to be adaptive.

It can be easily seen that the selection of the sampling interval depends completely on the control performance over $[t_i, t_{i+1})$ in the optimal predictive sampling problem if $\alpha=0$. Since this performance index would generally be a strict monotone increasing function of T_i for $T_i \geq T_{min}$, when T_{min} is sufficiently small, the sampling criterion would in most cases be periodic with sampling period $T_i^* = T_{min}$. Thus, if future performance is neglected ($\alpha=0$) by the design objectives the on-line predictive sampling problem gives a periodic sampling criterion at a sampling rate that is the maximum allowed by the computation algorithm, computer software, and computer-communication-instrumentation hardware option selected in the off-line system design problem.

If α is greater than zero and large enough, the future performance over $[t_i+T_i, t_i+\Delta_i)$, which measures terminal error, speed of response, and overshoot, will dominate selection of T_{ij}^*. Since this future performance index keeps changing as index i increases, the sampling intervals T_{ij}^* will produce an adaptive sampling criterion. The sampling interval T_{ij}^* is selected so that the speed of response is increased and the settling time and terminal error are reduced, but not so long that the control performance over $[t_{i+1}, t_i+\Delta_i)$ becomes large.

If α was chosen equal to zero in this OCD problem, it is clear that the designer had made a choice of implementing hardware, computer algorithms, and software knowing that a periodic rather than adaptive sampling criterion would result.

Thus, the selection of α is a selection of control structure for the predictive sampling problem not only because it dictates the hardware, algorithms and software implemented but also because of it will be shown later that adaptive sampling is a closed loop sampling process where periodic sampling is an open loop sampling process.

The parameter Δ_i is used to determine the future performance time interval and is dependent on the desired speed of response in the control objectives and is determined using the same iterative procedure used to determine the matrices ($\underline{Q},\underline{R},\underline{F}$) and parameter α. The parameter Δ_i obviously does not affect the control performance if α is zero.

Two options are possible in selection of Δ_i. In the first case, $\Delta_i = t_f - t_i$ where t_f is a known fixed terminal time and the sampling interval $[t_i,t_{i+1})$ is chosen to effect control performance over $[t_i,t_f]$. In the second case, Δ_i is constant ($\Delta_i=\Delta$) whether t_f is known fixed terminal time or unknown and the selection of sampling interval $[t_i,t_{i+1})$ is based on performance over a fixed interval $[t_i,t_i+\Delta)$. The length of this interval will thus also determine how adaptive this predictive sampling criterion will be.

The effects of selecting parameters (α,Δ_i) and matrices ($\underline{Q},\underline{R},\underline{F}$) will be discussed for a specific example in the next chapter. Although the selection of α and Δ_i on the adaptability of the sampling criterion is clear, the effects of selection of $\underline{Q}$, $\underline{R}$ and $\underline{F}$ on the adaptability and performance is not clear and will be shown to be contrary to intuition in some cases.

3.2 Optimal System Design (OSD) Problem

The control performance index, determined in the OCD problem, will be used in the optimal system design (OSD) problem which first develops a cost of implementation as a function of T_i and then selects the computational algorithm, computer software, and computer-communication-instrumentation hardware through an optimal tradeoff between control performance and cost of implementation for several operating conditions. The OCD and OSD problems are apparently treated as completely separate. However, these problems are not really separable because a choice of T_{min} in OCD problem affects the control performance that can be obtained tuning the parameters of the control performance index and also because T_{min} is the minimum computation time of the hardware, algorithm, and computer software option selected in the OSD problem. Since the hardware and the associated T_{min} value are chosen in the OSD problem by a tradeoff between cost of implementation and a control performance index, whose parameters are chosen in the OCD problem and depends on T_{min}, the OCSD problem requires the OCD and OSD problem be considered iteratively until hardware and associated T_{min} satisfied both control performance and cost of implementation objectives.

The following procedure for determining the cost of implementation $C(T_i)$ assumes the communication and instrumentation hardware has been specified and that only the computer hardware, computational algorithm, and computer software need be selected:

(1) Enumerate the computational algorithm options (p).

(2) Enumerate the computer hardware options (s) for each computational algorithm option (p). The cost of the computer hardware, $C_{s,p}$, for each hardware and computational algorithm option (s,p) must be noted.

(3) Optimize the computer programming to minimize CPU time, $\tau_{s,p}(T_i)$, for each hardware-computer algorithm option (s,p) to compute $T_i = T_i^*$.

(4) Determine the set of feasible computer hardware-computational algorithm options for each T_i, i.e.

$$\Omega(T_i) = \{(s,p) : \tau_{s,p}(T_i) \leq T_i\} \tag{21}$$

where this condition requires the CPU time, $\tau_{s,p}(T_i)$, for any feasible computer-computational algorithm option be able to compute T_i^* in less than T_i^* seconds for each $T_i^* = T_i \varepsilon [T_{min}, T_{max}]$.

(5) Determine the cost of implementation function

$$C(T_i) = \min_{(s,p)\varepsilon\Omega(T_i)} \{C_{s,p}\} \tag{22}$$

where the lowest possible cost option is selected for each T_i.

The second step in this OSD problem is to select the hardware option to be implemented for the predictive sampling criterion and the parameter q selected to weight cost of implementation against control performance. This selection of hardware procedure is

(1) repeatedly optimizing system performance

$$S(T_i) = J(T_i) + qC(T_i) \tag{23}$$

for several operating conditions $(\underline{h}_j(t), \underline{w}_j(t), \{\psi_{kj}\}_{k=1}^{i})$ for $j=1,2,\ldots,M$ and several sampling intervals $i=0,1,\ldots,N-1$

for each operating condition to obtain a set of optimal sampling intervals

$$\Gamma = \{\{T^*_{ij}\}^{M}_{j=1}\}^{N-1}_{i=0} \tag{24}$$

The control performance index is determined in the OCD problem and the cost of implementation index is determined in the first part of the OSD problem.

(2) select the hardware based on the maximum cost over the optimal set of solutions Γ

$$C^* = \max_{T^*_{ij}\varepsilon\Gamma} C(T^*_{ij}) \tag{25}$$

where Γ includes optimal sampling intervals chosen for different operating conditions (j) and different sampling intervals (i). It will in general be necessary to repeat this OSD procedure for several values of parameter q until the hardware and control variables meet the control performance and cost of implementation objectives.

CHAPTER 4. OPTIMAL CONTROL DESIGN (OCD) FOR PREDICTIVE SAMPLING PROBLEM

The objectives of this chapter are

(1) to investigate the effects of changing parameters (α, Δ_i) and matrices $(\underline{Q}, \underline{R}, \underline{F})$ on the control performance achieved with predictive sampling on a particular example system;

(2) to determine a set of parameters (α, Δ_i) and matrices $(\underline{Q}, \underline{R}, \underline{F})$ that provide the best control performance possible with predictive sampling for each operating condition;

(3) to show that the sampled-data control with predictive sampling can outperform the periodic sampled-data with the same number of control changes and even outperform the continuous time control law if the system is deterministic so that the performance of selecting any particular sampling interval can be accurately predicted;

(4) to show that the best predictive sampling criterion is a periodic sampling criterion for a stochastic system because future performance due to selection of a sampling interval can not be accurately predicted. The parameters (α, Δ_i) and matrices $(\underline{Q}, \underline{R}, \underline{F})$ selected to provide the best control performance with predictive sampling for a stochastic system will be shown to result in a periodic sampling criterion.

The effects of changing parameters (α, Δ_i) and elements of matrices $(\underline{Q}, \underline{R}, \underline{F})$ for an error feedback control of a second order system is given in Section 4.1. The selection of parameters and matrices for both deterministic and stochastic inputs and the resulting performance of the sampled-data control with a predictive sampling criterion is given in Section 4.2. These results are obviously dependent on the example system and error feedback control law used, but the qualitative behavior should hold for a great many systems with error feedback controls.

4.1 Effects of Changes in Performance Index Parameters

The example system used in this section is a second-order type two system which has been used extensively in the literature [1-13] on evaluating performance of adaptive and aperiodic sampling criteria. This particular system is chosen not only because of the extensive results obtained on it with different sampling criteria but also because it is unstable without feedback and thus provides an excellent basis for determining the performance of a sampling criterion. The system to be considered is deterministic $(\underline{w}(t) = \underline{0},\ \underline{\psi}_i = \underline{0})$ and is written by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2\zeta\omega_n \\ \omega_n^2 \end{bmatrix} u(t)$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} ; \qquad (26)$$

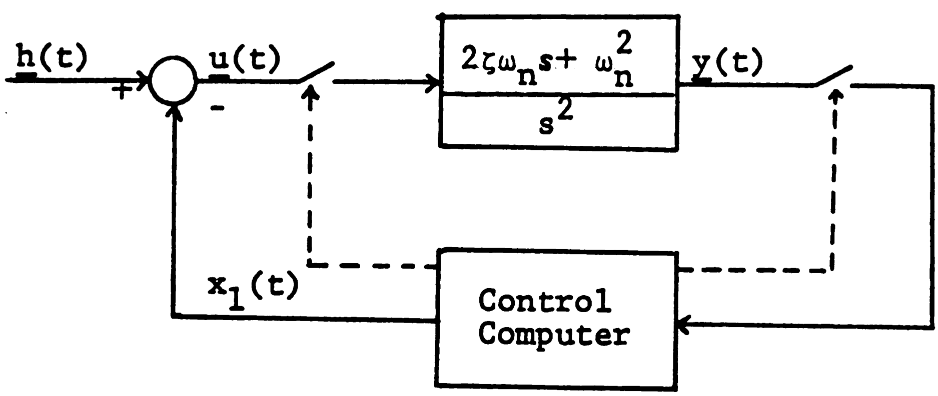
with initial conditions

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \qquad (27)$$

The control law is specified as a closed form as

$$u(t) = \begin{cases} h_1(t_i) - x_1(t_i) & t\varepsilon[t_i, t_{i+1}) \\ h_1(t) - x_1(t) & t\varepsilon[t_{i+1}, t_i+\Delta_i) \end{cases} \qquad (28)$$

The system is said to be a "fast" responding when $\omega_n = 10$ and "medium" responding when $\omega_n = 5$ for $\zeta = 0.5$ in both cases. These are two of the specific cases considered in [13] for evaluation of optimal aperiodic sampling. A block diagram of the system is shown below.



The control objectives for the optimal predictive sampling problem are:

(1) to increase speed of response;

(2) to reduce terminal error;

(3) to reduce overshoot.

A general form for a control performance index that can meet these objectives is:

$$J(T_i) = \alpha/2 \begin{bmatrix} h_1(t_i+\Delta_i)-x_1(t_i+\Delta_i) \\ h_2(t_i+\Delta_i)-x_2(t_i+\Delta_i) \end{bmatrix}' \begin{bmatrix} F_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} h_1(t_i+\Delta_i)-x_1(t_i+\Delta_i) \\ h_2(t_i+\Delta_i)-x_2(t_i+\Delta_i) \end{bmatrix}$$

$$+ \alpha/2 \int_{t_i+T_i}^{t_i+\Delta_i} \left\{ \begin{bmatrix} h_1(t)-x_1(t) \\ h_2(t)-x_2(t) \end{bmatrix}' \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix} \begin{bmatrix} h_1(t)-x_1(t) \\ h_2(t)-x_2(t) \end{bmatrix} + Ru^2(t) \right\} dt \tag{29}$$

$$+ 1/2 \int_{t_i}^{t_i+T_i} \left\{ \begin{bmatrix} h_1(t)-x_1(t) \\ h_2(t)-x_2(t) \end{bmatrix}' \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix} \begin{bmatrix} h_1(t)-x_1(t) \\ h_2(t)-x_2(t) \end{bmatrix} + Ru^2(t) \right\} dt$$

where $h_2(t) = dh_1(t)/dt$. The off diagonal terms in $\underline{Q}$ and $\underline{F}$ are assumed zero for ease of analysis. The coefficient F_{22} for the error rate at the end of the control interval, $(h_2(t_i+\Delta_i) - x_2(t_i+\Delta_i))$, is also set to zero because this error derivative would not seem to effect the speed of response, overshoot, or terminal error for a sampled-data control with a predictive sampling criterion. Q_{11} is set equal to 0.1 arbitrarily and R is set equal to 0.02 again arbitrarily because Q_{11} and R weight the same signal $(h_1(t) - x_1(t))$.

The initial time and terminal time are set equal to zero and one respectively and $\Delta_i = t_f - t_i$ is set as the time remaining in the control interval [0,1] unless otherwise specified.

The sampling interval constraint

$$T_{min} \leq T_i \leq T_{max}$$

are chosen to place very little restriction on the choice of sampling intervals for the OCD problem because in this chapter the objective is to determine the maximum improvement in control performance which can be achieved through selection of sampling intervals optimally. In Chapter 5, the minimum sampling interval T_{min} will be selected when the hardware to be implemented is selected in the OSD problem based on a tradeoff of control performance and cost of implementation. The minimum sampling period is chosen as 0.02 in this section because it is smaller than one would expect to select for a system with fast ($\omega_n = 10$) or medium ($\omega_n = 5$) speed of response. T_{max} is chosen as 0.6 seconds for the medium system ($\omega_n = 5$) and 0.3 seconds for the fast system ($\omega_n = 10$) which is the Nyquist sampling period for such systems when the system is assumed bandlimited to ω_n.

The input $h_1(t)$ is selected as a step input

$$h_1(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \tag{30}$$

rather than a stochastic disturbance because the general effects of the parameter changes α, F_{11}, and later $\Delta_i = \Delta$ can be more easily determined for the deterministic step input than a ramp, parabolic, or stochastic input.

Since the system, control law, performance index, and sampling constraints have been defined, the effects of changes parameter α, F_{11} and Δ_i can be determined. The effect of increasing α is to increase T_0^* as shown in Figure 4-1(a) if α is less than five and then any further increase in α has no effect on T_0^*. This can be understood by analyzing the shape of the two components of performance index $J(T_i)$, i.e.

$$J_0(T_i) = 1/2 \int_{t_i}^{t_i+T_i} \{0.1\ [h_1(t) - x_1(t)]^2 + Q_{22}\ [h_2(t) - x_2(t)]^2 + 0.02\ u^2(t_i)\}\ dt \tag{31}$$

and

$$J_1(T_i) = \alpha/2\ F_{11}[h_1(t_i+\Delta_i) - x_1(t_i+\Delta_i)]^2 + \alpha/2 \int_{t_i+T_i}^{t_i+\Delta_i} \{0.1\ [h_1(t) - x_1(t)]^2 + Q_{22}\ [h_2(t) - x_2(t)]^2 + 0.02\ u^2(t)\}\ dt \tag{32}$$

The component $J_0(T_0)$ is a monotone increasing function of T_0 when T_0 is sufficiently small because the integrand is non-negative and is a decreasing function of the integration argument when T_0 is sufficiently small. $J_1(T_0)$ is a convex function as can be seen from Figure 4-1(b) when α is very large. Thus, when α is above five $J(T_0)$ closely approximates $J_1(T_0)$ and the optimal T_0^* is unaffected by changes in α. However, when α is less than five, a decrease in α makes $J_0(T_i)$ relatively more important in determining T_0^* and since $J_0(T_0)$ is monotonically increasing T_0^* should decrease as α decreases as observed.

The effects of increasing F_{11} when α is greater than five is to decrease T_0^* as shown in Figure 4-2. This can be explained by noting that the longer control

$$u(t_0) = h_1(t_0) - x_1(t_0) \tag{33}$$

is held, the larger the overshoot of trajectory $x_1(t)$ and

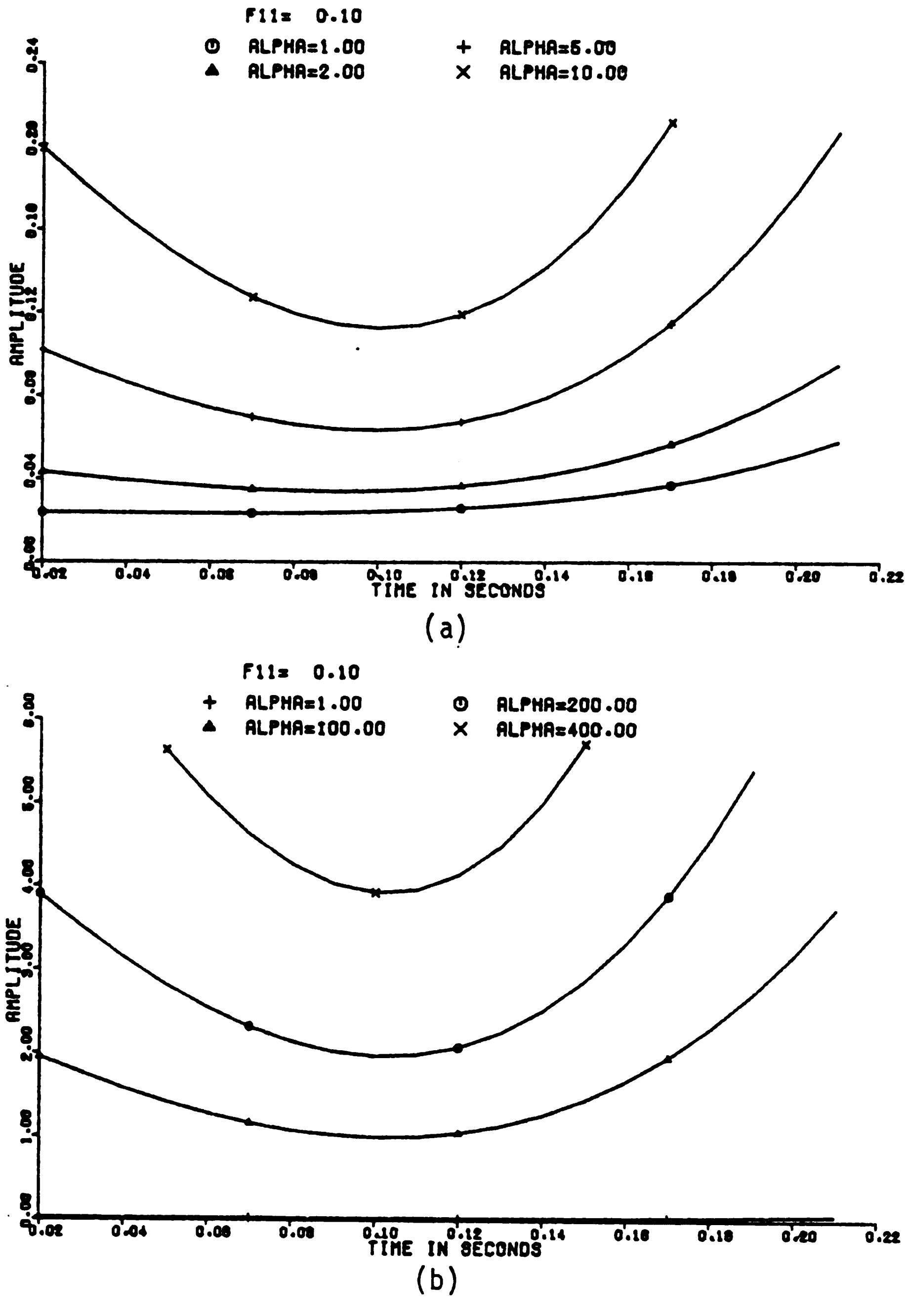


FIGURE 4-1 $J(T_0)$ vs T_0 with α variations for medium system when $F_{11} = 0.1$ and $\Delta_i = t_f - t_i$.

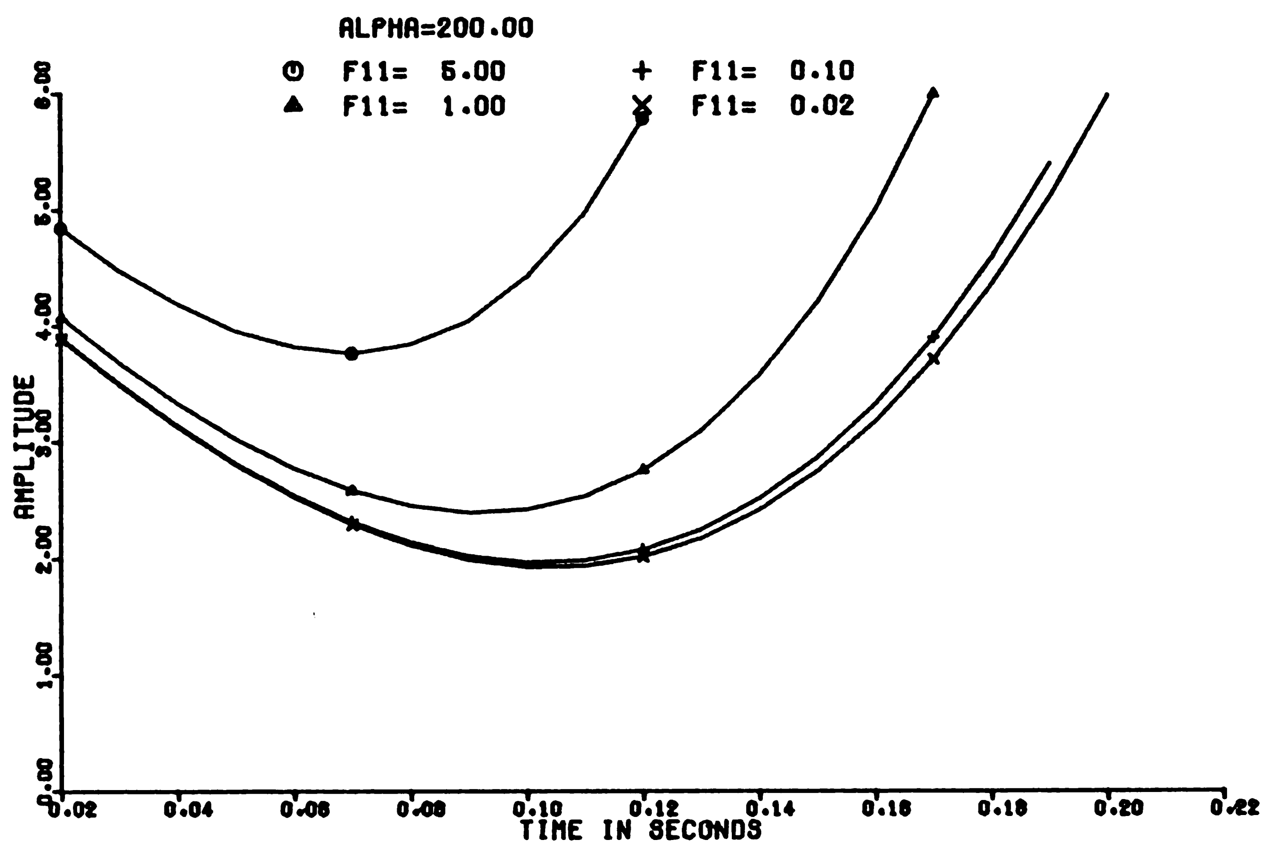


FIGURE 4-2 $J(T_0)$ vs T_0 with F_{11} variations for medium system when $\Delta_i = t_f - t_i$.

thus the larger the error $(h_1(t_f) - x_1(t_f))$. Thus, increasing F_{11} will more heavily weight this terminal error and thus control performance index increase faster as T_0 increases. Although this analysis of the effects of changing α and F_{11} was only performed for i=0, it will be shown to hold for every i by observing Figures 4-3 and 4-4.

The output response $x_1(t)$ of the system is plotted for the medium ($\omega_n = 5$) and fast ($\omega_n = 10$) systems in Figure 4-3 and 4-4 respectively for the predictive sampling criterion obtained using various values of α and F_{11}. Sampling instants are shown by special symbols on the trajectory. The results indicate that speed of response and overshoot all increase as α is increased or F_{11} is decreased on both the fast and medium systems.

The speed of response and overshoot increase as T_i^* (α increases and F_{11} decreases) since the difference between the absolute magnitude of the sampled-data control

$$u(t) = h_1(t) - x_1(t) \qquad t\varepsilon[t_i, t_{i+1}) \tag{34}$$

and the absolute magnitude of the continuous control increases with $(t - t_i)$ and has the effect of accelerating the reduction in error $(h_1(t_i) - x_1(t_i))$. Thus, increasing α and decreasing F_{11} increase T_i^* and thus increase speed of response and the overshoot which occurs due to this faster reduction of error.

Another measure of performance for a sampled-data control is cumulative control performance

$$J_c(T_0^*, T_1^*, \ldots, T_{N-1}^*) = 1/2\,[\underline{y}(t_f) - \underline{h}(t_f)]'\,\hat{\underline{F}}\,[\underline{y}(t_f) - \underline{h}(t_f)] + \sum_{i=0}^{N-1} J_0(T_i^*) \tag{35}$$

which measures the control performance over each sampling interval in $[t_o, t_f]$ and the error energy at the terminal time t_f. The performance over intervals $[t_j, t_j+T_j)$, $j=i+1, i+2, \ldots, N-1$, depend on the selection of $\{T_j\}_{j=0}^{i}$ and thus this measure of performance can be used to compare periodic, optimal aperiodic, and optimal predictive sampling criteria.

The matrix $\hat{\underline{F}}$ is not identical to $\underline{F}$ used in the predictive sampling performance index. This matrix is chosen as

$$\hat{\underline{F}} = \begin{bmatrix} 0.05 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}$$

in this study so that terminal error is not considered as a major factor in assessing performance of a sampling criterion. Table 4-1 tabulates the cumulative control performance $J_c(T_0, T_1, \ldots, T_{N-1})$ and the terminal error $(h_1(t_f) - x_1(t_f))$ for (1) predictive sampling, (2) periodic sampling with the same number of sampling times as predictive, and (3) periodic sampling criterion with a sampling period of 0.01 (N=100) which approximates the performance of the continuous control.

The cumulative control performance and terminal error for predictive sampling on the fast system is always considerably better than for periodic sampling but always worse than the continuous control. The lowest cumulative control performance and terminal error occurs when $\alpha=1$ and $F_{11}=0.1$ and the cumulative performance obtained closely approximate the cumulative performance of the continuous control.

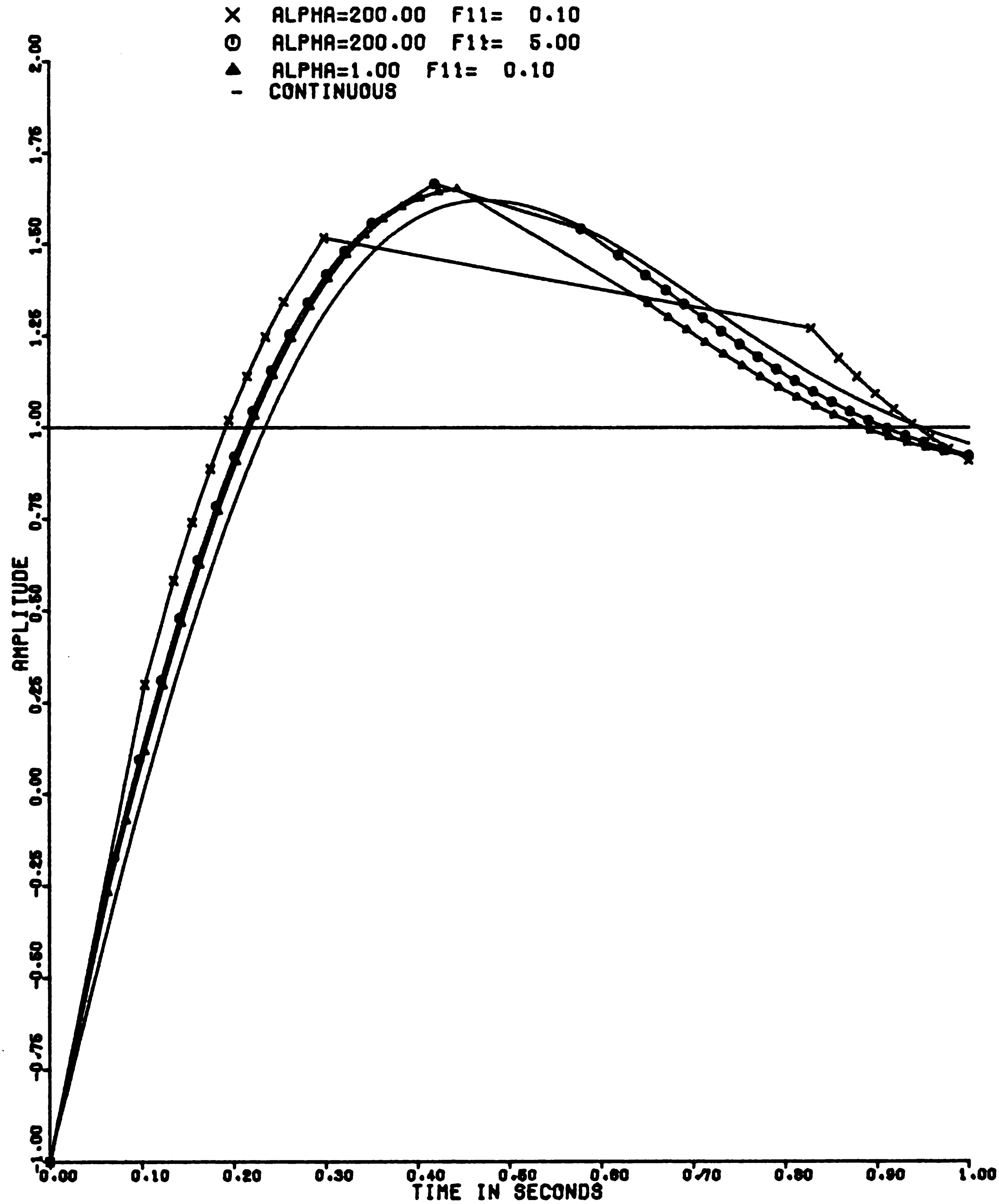


FIGURE 4-3 Output trajectories for α and F_{11} variations for medium system when $\Delta_i = t_f - t_i$.

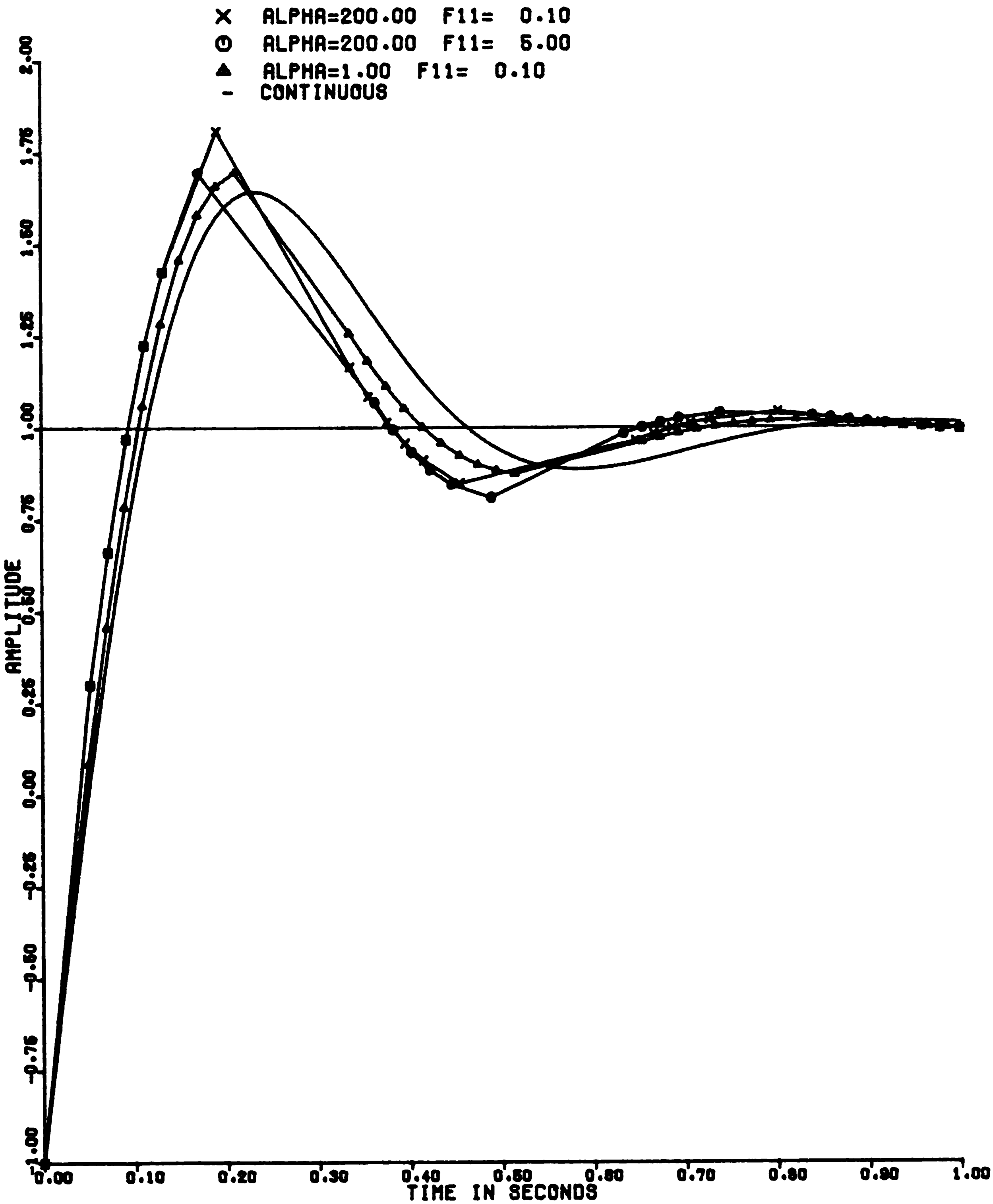


FIGURE 4-4 Output trajectories for α and F_{11} variations for fast system when $\Delta_i = t_f - t_i$.

TABLE 4-1 Cumulative Control Performance with α, F_{11} Variations for unit step input when $\Delta_i = t_f - t_i$ and $T_{min} = 0.02$.

System	α	F_{11}	Number of Samples	Terminal Error Adaptive	Terminal Error Periodic	Cumulative Control Performance ($\hat{F}=0$) Adaptive	Cumulative Control Performance ($\hat{F}=0$) Periodic	Cumulative Control Performance ($\hat{F}=0.05$) Adaptive	Cumulative Control Performance ($\hat{F}=0.05$) Periodic
Medium System	Continuous		100		0.043485		0.024024		0.024119
$Q_{11}=0.1$	200	0.1	18	0.088864	0.127582	0.031430	0.026107	0.031825	0.026921
R= 0.2	200	5.0	35	0.075418	0.076271	0.024108	0.024663	0.024394	0.024950
$T_{max}=0.6$	1	0.1	38	0.080212	0.072176	0.023545	0.024543	0.023867	0.024803
	1	5.0	45	0.061018	0.064826	0.024276	0.024392	0.024462	0.024602
Fast System	Continuous		100		0.015917		0.012277		0.012290
$Q_{11}=0.1$	200	0.1	23	0.002268	0.005275	0.013759	0.014734	0.013759	0.014735
R= 0.02	200	5.0	26	0.002161	0.008580	0.013809	0.014173	0.013809	0.014177
$T_{min}=0.02$	1	0.1	32	0.000032	0.012130	0.012438	0.013503	0.012438	0.013510
	1	5.0	33	0.001110	0.012499	0.012478	0.013426	0.012478	0.013435

The results for the medium system indicate the predictive sampling has lower cumulative control performance and terminal error than periodic sampling except when α=200 and F_{11}=0.1 because in this case the sampling intervals are so large that the error is not sampled at its peak overshoot and does not reduce this overshoot as quickly as it would otherwise. This seems to be an isolated situation where predictive sampling does not fully take advantage of the control opportunity because it is based on a single interval performance measure. The lowest value of cumulative control performance for predictive sampling for this medium system is obtained when α=1 and F_{11}=0.1 which for this case is lower than that obtained for the continuous control. The lowest terminal error is obtained when α=1 and F_{11}=5, but terminal error in this case is not a good measure of performance because the control interval is short with respect to the settling time for this medium system.

The effects of setting $\Delta_i = t_f - t_i$ or setting $\Delta_i = \Delta$ for several values of Δ will now be investigated for the medium system (ω_n=5). The values of α and F_{11} are set equal to 200 and 0.0 or 0.01 respectively because the sampling intervals are large and the effects of Δ_i are more easily seen. The first case considered is α=200 and F_{11}=0.0, and the results, shown in Figure 4-5, indicate that curves $J(T_0,\Delta) \approx J_1(T_0,\Delta)$ increase with Δ and this effect occurs because the integrand is non-negative and the integration interval for $J_1(T_0,\Delta)$ is $(\Delta - T_0)$. The decrease in $J(T_0,\Delta)$

for small T_0 for any Δ is thus due to a decrease in the integration interval. However, when T_0 becomes large, the overshoot becomes larger as T_0 increases and the curves begin to increase. This increase in $J(T_0,\Delta)$ with T_0 is more pronounced for larger Δ which is due to the fact that as Δ increases more of the interval where overshoot is experienced is included in $[t_0+T_0, t_0+\Delta]$. The optimal sampling interval T_0^* thus increases as Δ decreases because the performance index has less concern for overshoot due to holding the sampling interval too long. The trajectory $x_1(t)$, shown in Figure 4-6, indicates the sampling interval T_i^* increase for all i as Δ decreases indicating the above analysis for i=0 holds for every interval.

The second case considered in this subsection is included to indicate the effects of changing Δ when terminal error is weighted slightly (F_{11}=0.1 and α=200). The curves, $J(T_0,\Delta) \approx J_1(T_0,\Delta)$, plotted in Figure 4-7 are quite different from the first case where terminal error was omitted from the performance index because the terminal error can be very large or be very sensitive to changes in T_0 for particular values of Δ. The error for Δ=0.44 has much larger values than for Δ=0.22, 0.88 or 1.0 and thus the $J_1(T_0,0.44)$ curve is much larger than the others there. The optimal sampling interval $T_0^*(0.44)$ is thus quite small in order to optimally tradeoff the reduction in the integral part of performance index $J_1(T_0,0.44)$ with T_0 and the rapid increase in terminal error $(h_1(0.44) - x_1(0.44))$ with T_0. The performance curves

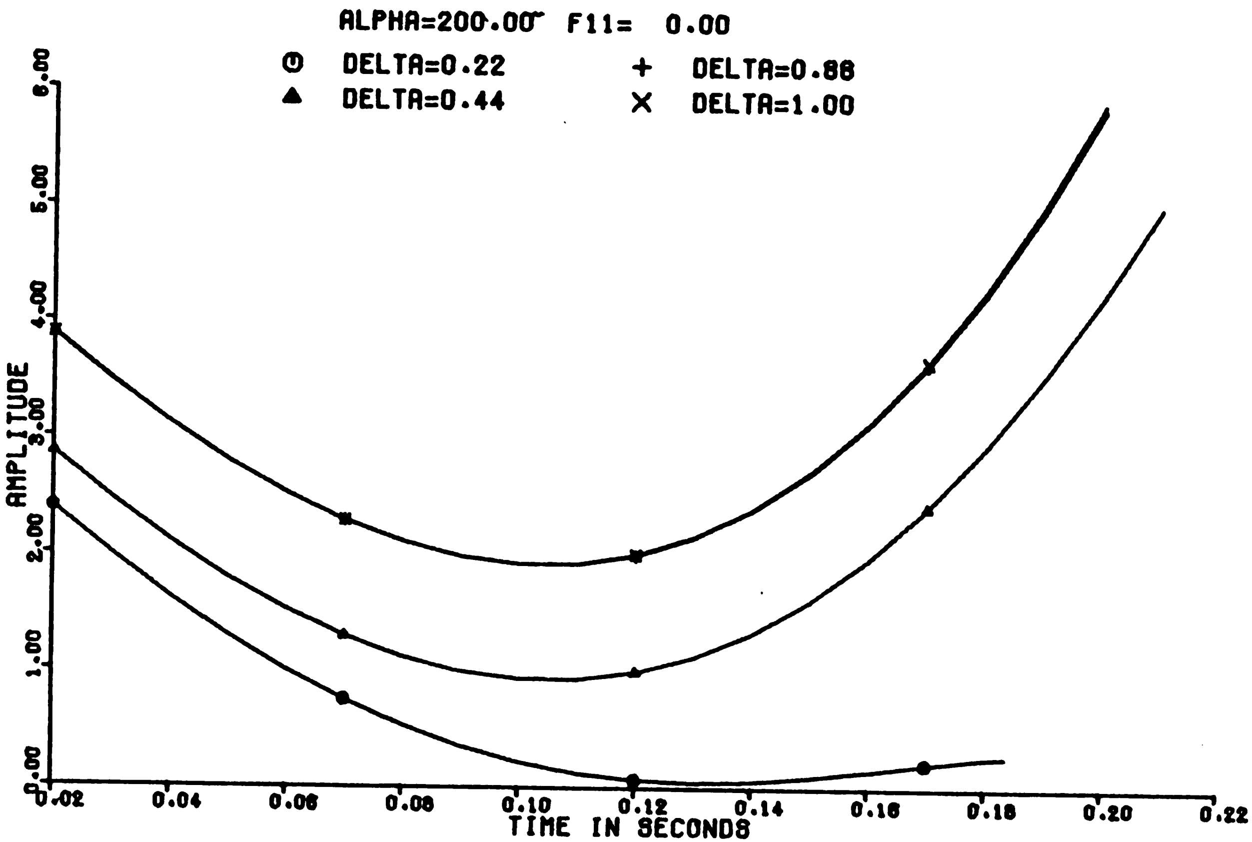


FIGURE 4-5 $J(T_0)$ vs T_0 with Δ variations for medium system when α=200 and F_{11}=0.

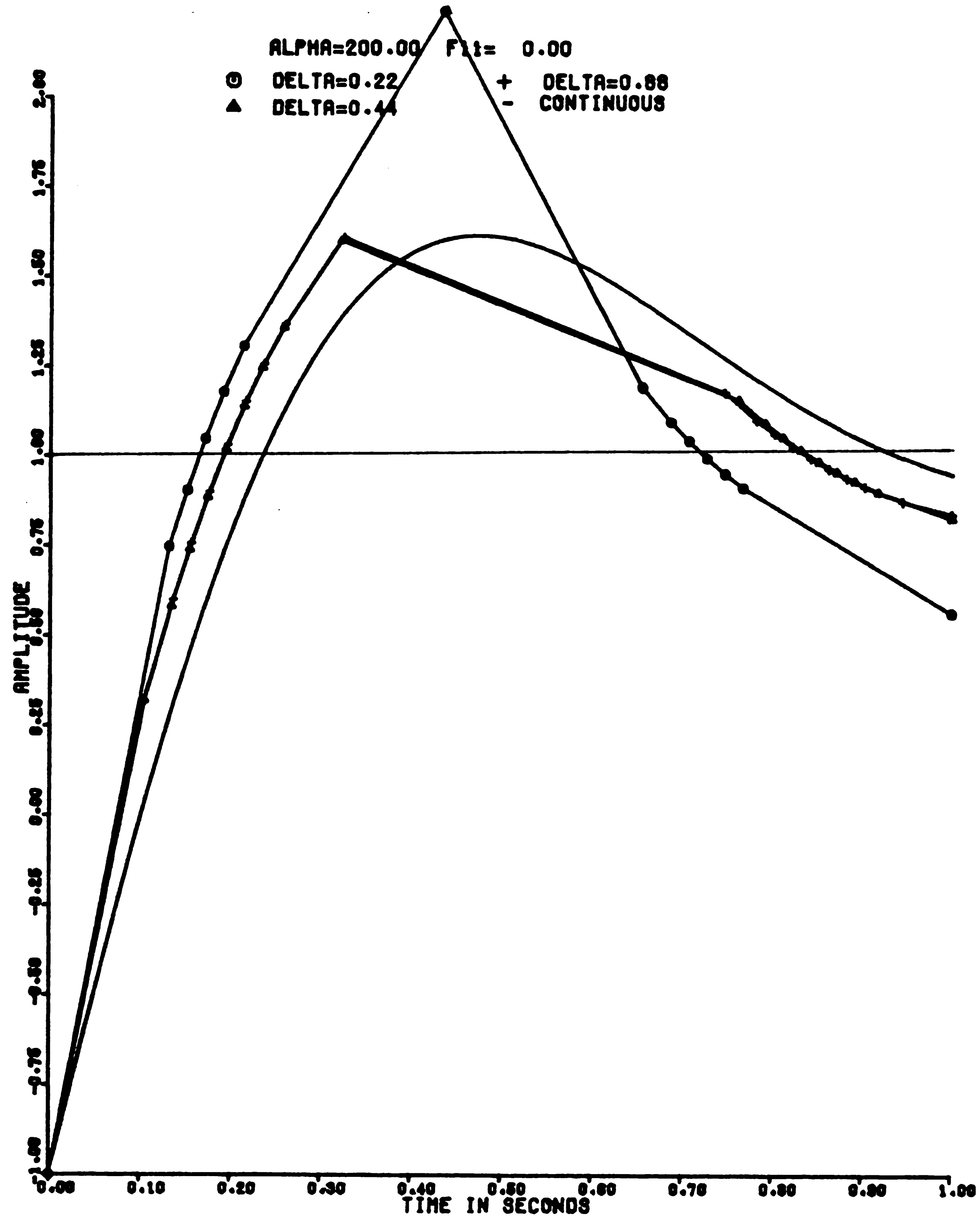


FIGURE 4-6 Output trajectories for Δ variations for medium system when α=200 and F_{11}=0.

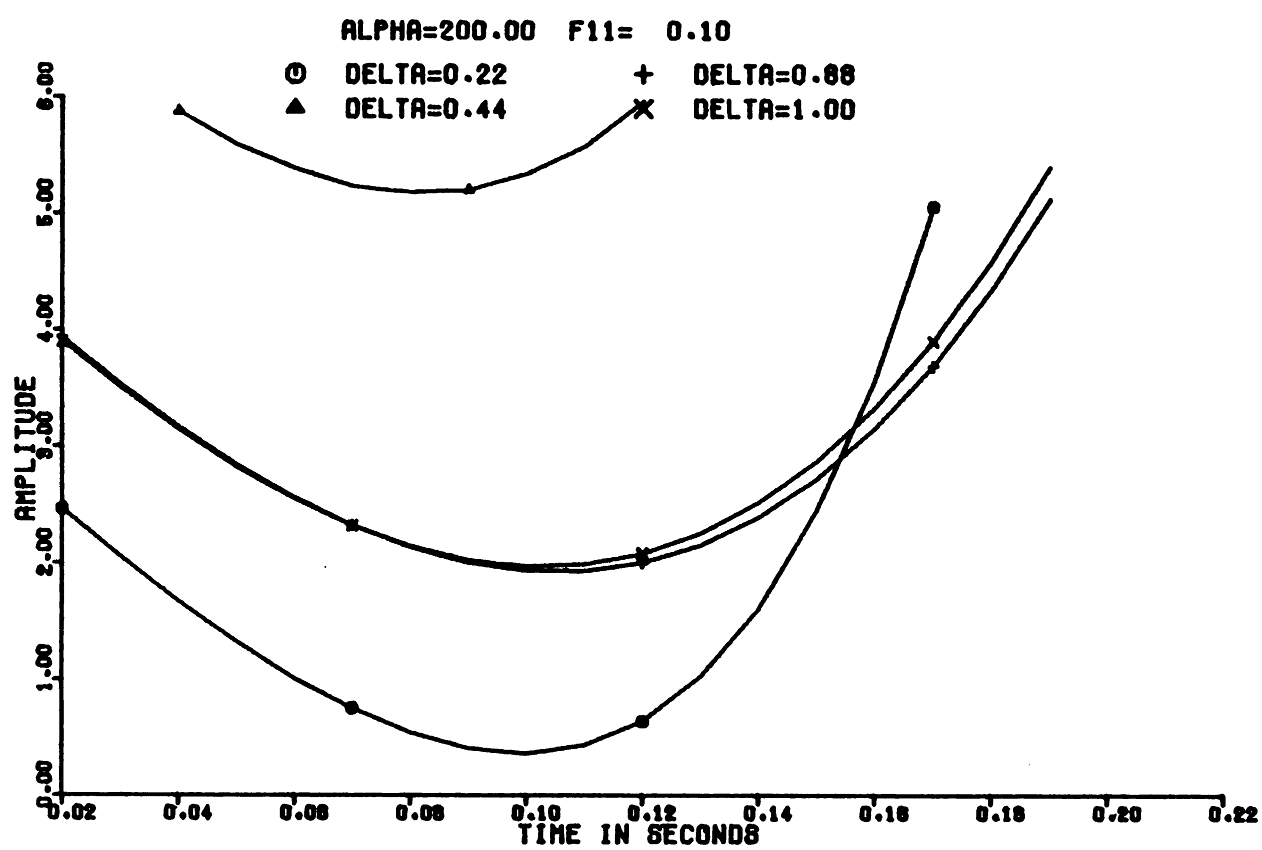


FIGURE 4-7 $J(T_0)$ vs T_0 with Δ variations for medium system when α=200 and F_{11}=0.1.

$J(T_0, 0.22)$ for $F_{11}=0.0$ and $F_{11}=0.1$ are identical when T_0 is small because the terminal error $(h_1(0.22) - x_1(0.22))$ is zero for the continuous control and is thus small when T_0 is small. However, as T_0 becomes large this terminal error increases rapidly and $J_1(T_0, 0.22)$ increases rapidly when $F_{11}=0.1$ but increases only slightly when $F_{11}=0.0$. The optimal sampling interval $T_0^*(0.22)$ does decrease when terminal error is weighted in the performance index. The change in the curves $J(T_0, 0.88)$ and $J(T_0, 1.0)$ and the change in optimal sampling intervals $T_0^*(0.88)$ and $T_0^*(1.0)$ are both quite small due to inclusion of terminal error in the performance index. The speed of response is again proportional to $T_0^*(\Delta)$ as it was when $F_{11}=0.0$ but in this case $T_0^*(\Delta)$ is not inversely proportional to Δ but is dependent on the magnitude of the terminal error and its sensitivity to changes T_0. Thus, $T_0^*(0.44)$ is smallest followed by $T_0^*(0.22)$, $T_0^*(1.0)$ and $T_0^*(0.88)$. Since the speed of response and $\{T_i^*(\Delta)\}_{i=1}^{N-1}$ are proportional to $T_0^*(\Delta)$, the analysis of the effects of parameter change in the first interval hold for all other intervals as shown in Figure 4-8.

Another set of trajectories $x_1(t)$, which indicate the effects of changing Δ, is run when α is reduced from 200 to 1 thus reducing $T_i^*(\Delta)$ and the speed of response but improving cumulative control performance $J_c(T_0^*(\Delta), T_1^*(\Delta), \ldots, T_{N-1}^*(\Delta))$ and terminal error as shown in Figure 4-9 and Table 4-2. The performance and the trajectories $x_1(t)$ show comparatively little change as a function of Δ for these values of α and

F_{11}. However, the smallest $T_0^*(\Delta)$ is still Δ=0.44 and the slowest speed of response occurs for Δ=0.44 indicating the sampling interval sequence $\{T_i^*(\Delta)\}_{i=0}^{N-1}$ still depends on the magnitude of the terminal error $(h_1(t_i+\Delta) - x_1(t_i+\Delta))$ and thus on Δ just as when α=200 and F_{11}=0.1.

The analysis of Δ variations for the fast system is identical to that for the medium system. In this case, the lowest cumulative control performance occurs when α=1, F_{11}=0.1 and Δ=0.11 as shown in Table 4-3.

The speed of response can be increased by adjusting α, Δ and F_{11}/Q_{11} as indicated above but fast speed of response results in a large peak overshoot in the transient response. The peak overshoot of the output response can be reduced by (1) reducing α, (2) reducing F_{11}/Q_{11}, (3) increasing Δ, and possibly (4) increasing Q_{22}. The effect of changing Q_{22} is investigated because Q_{22} weights the tracking error rate and could possibly reduce the peak overshoot by minimizing this error rate. F_{22} is not considered because the error rate at the terminal time would not appear to have any effect on these control objectives.

Results from Figure 4-10 indicate

(1) increasing Q_{22} does increase damping and reduce speed of response;

(2) the effect of changing Q_{22} is very similar to changes in α, Δ, or F_{11}/Q_{11} because changing each of these parameters also will increase speed of response at the expenses of greater overshoot;

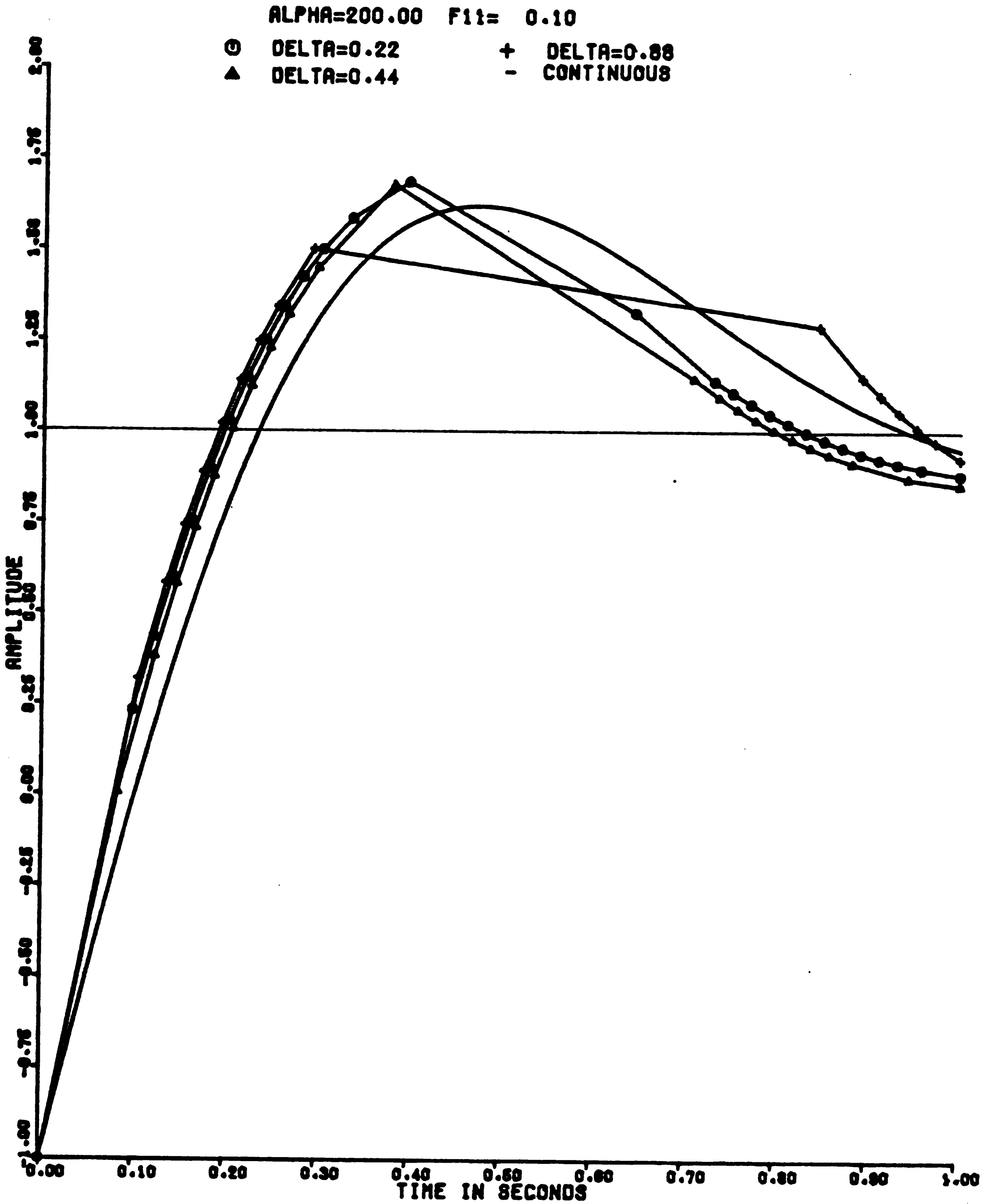


FIGURE 4-8 Output trajectories for Δ variations for medium system when α=200 and F_{11}=0.1.

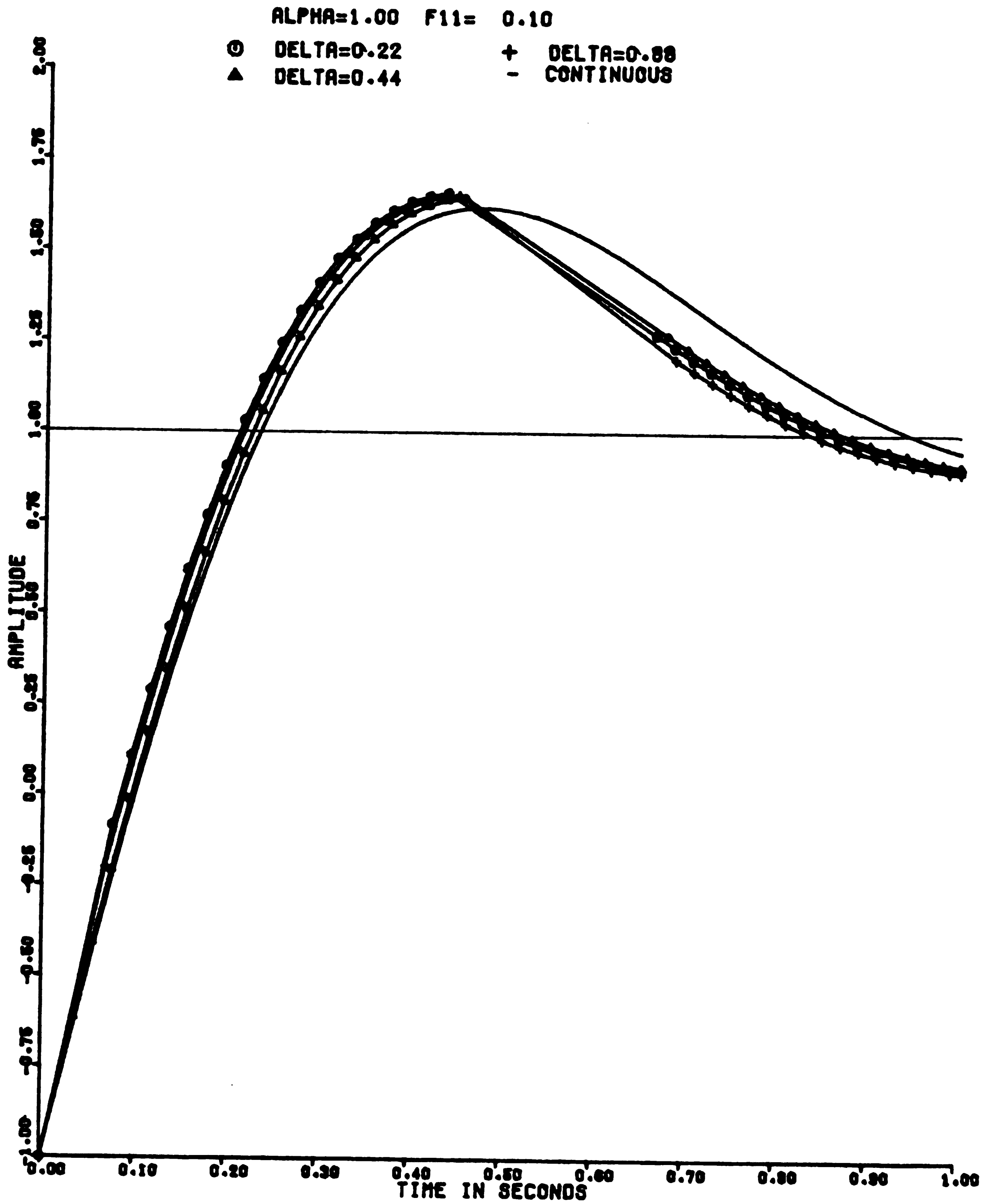


FIGURE 4-9 Output trajectories for Δ variations for medium system when $\alpha=1$ and $F_{11}=0.1$.

TABLE 4-2 Cumulative Control Performance and Terminal Error for Unit Step Input with Δ Variations for Medium System

Parameters α and F_{11}	Δ	Number of Samples	Terminal Error $h(t_f)-x_1(t_f)$ Adaptive	Terminal Error $h(t_f)-x_1(t_f)$ Periodic	Cumulative Control Performance ($\hat{F}=0$) Adaptive	Cumulative Control Performance ($\hat{F}=0$) Periodic	Cumulative Control Performance ($\hat{F}=0.05$) Adaptive	Cumulative Control Performance ($\hat{F}=0.05$) Periodic
Continuous		100		0.043485		0.024024		0.024119
$\alpha = 200$ $F_{11} = 0.0$	0.22	13	0.432642	0.171108	0.038167	0.027899	0.047526	0.029363
	0.44	18	0.167182	0.127582	0.027800	0.026107	0.029197	0.026921
	0.88	19	0.155379	0.121818	0.027462	0.025908	0.028670	0.026650
	t_f-t_i	14	0.219742	0.159688	0.028883	0.027377	0.031297	0.028652
$\alpha = 200$ $F_{11} = 0.1$	0.22	27	0.112598	0.091907	0.024512	0.025013	0.025146	0.025435
	0.44	22	0.140743	0.107877	0.024931	0.025463	0.025921	0.026045
	0.88	16	0.068796	0.141461	0.032773	0.026621	0.033010	0.027622
	t_f-t_i	37	0.092212	0.073465	0.023555	0.024571	0.023980	0.024841
$\alpha = 1.0$ $F_{11} = 0.1$	0.22	37	0.092212	0.073465	0.023555	0.024571	0.023980	0.024841
	0.44	39	0.084711	0.071958	0.023735	0.024517	0.024094	0.024766
	0.88	37	0.100594	0.073465	0.023500	0.024571	0.024006	0.024841
	t_f-t_i	38	0.080212	0.072176	0.023545	0.024543	0.023867	0.024803
$\alpha = 1.0$ $F_{11} = 5.0$	0.22	35	0.090695	0.076271	0.023559	0.024633	0.023970	0.024924
	0.44	44	0.061837	0.065728	0.024234	0.024409	0.024425	0.024625
	0.88	22	0.235153	0.107877	0.027206	0.025463	0.029971	0.026045
	t_f-t_i	45	0.061018	0.064826	0.024276	0.024392	0.024462	0.024602

TABLE 4-3 Cumulative Control Performance and Terminal Error for Unit Step Input with Δ Variations for Fast System

Parameters α and F_{11}	Δ	Number of Samples	Terminal Error $h(t_f)-x_1(t_f)$		Cumulative Control Performance ($\hat{F}=0$)		Cumulative Control Performance ($\hat{F}=0.05$)	
			Adaptive	Periodic	Adaptive	Periodic	Adaptive	Periodic
Continuous		100		0.015917		0.012277		0.012290
$\alpha = 200$, $F_{11} = 0.1$	0.11	30	0.003116	0.011242	0.012861	0.013681	0.012861	0.013688
	0.22	26	0.001830	0.008580	0.013228	0.014173	0.013228	0.013232
	0.44	20	0.004238	0.000153	0.013766	0.015600	0.013767	0.015600
	$t_f - t_i$	23	0.002268	0.005275	0.013759	0.014734	0.013759	0.014735
$\alpha = 1.0$, $F_{11} = 0.1$	0.11	33	0.003113	0.012499	0.012432	0.013426	0.012432	0.013435
	0.22	35	0.004967	0.013119	0.012493	0.013191	0.012494	0.013300
	0.44	32	0.001178	0.012130	0.012441	0.013503	0.012441	0.013510
	$t_f - t_i$	32	0.000032	0.012130	0.012438	0.013503	0.012438	0.013510
$\alpha = 1.0$, $F_{11} = 5.0$	0.11	34	0.006187	0.012827	0.012438	0.013356	0.012440	0.013364
	0.22	37	0.007702	0.013614	0.012730	0.013177	0.012733	0.013186
	0.44	15	0.029479	0.020543	0.015137	0.018699	0.015180	0.018720
	$t_f - t_i$	33	0.001110	0.012499	0.012478	0.013426	0.012478	0.013435

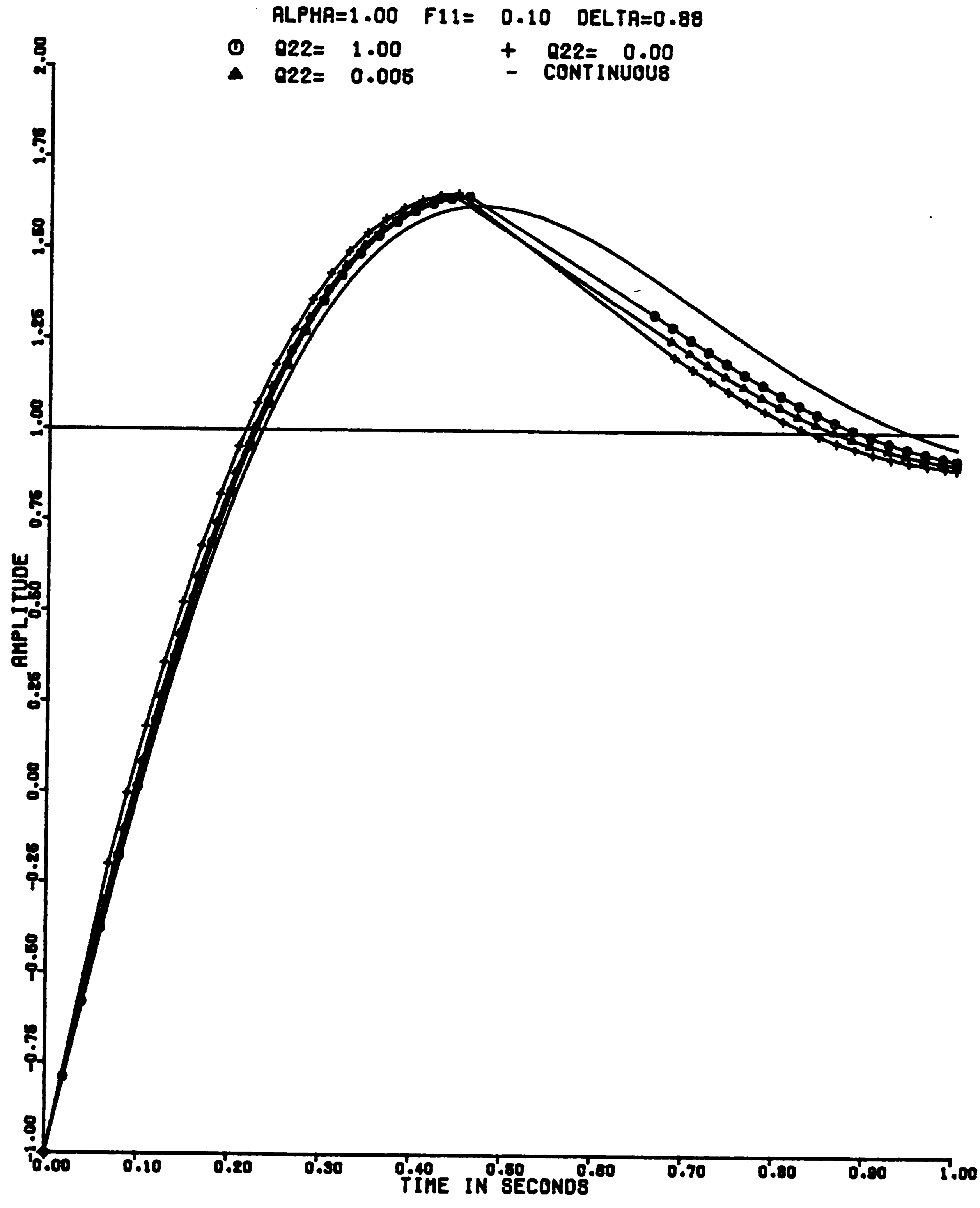


FIGURE 4-10 Output trajectories for Q_{22} variations for medium system when $\alpha=1$, $\Delta=0.88$ and $F_{11}=0.1$.

(3) The major difference between the effects of adjusting α, Δ, and F_{11}/Q_{11} and the effects of changing Q_{22} is that Q_{22} causes the sampling rate to be proportional to acceleration rather than velocity as with α, Δ, and F_{11}/Q_{11}.

Since a sampling criterion that samples at a rate proportional to acceleration for an error feedback control system is contrary to intuition and since tradeoff between speed of response and overshoot can be achieved through adjustment of α, Δ, and F_{11}/Q_{11}, Q_{22} is set equal to zero.

4.2. Selection of Control Performance Parameters

The previous section discussed the effects of changing performance index parameters α, Δ, F_{11}/Q_{11}, and Q_{22} on the control performance of a predictive sampling criterion with an error feedback control law for a second order example system. The next step is to select or tune these parameters to achieve acceptable control performance from the predictive sampling criterion and this error continuous-time feedback control law for a particular operating condition. This task is performed in the OCD problem as described previously in Chapter 3.

Optimal control design will be performed for both a deterministic and stochastic operating condition. The performance objectives are to

(1) make the adaptive sampled-data control with the predictive sampling criterion outperform the periodic sampled-data control and the continuous-time control based on the cumulative performance measure $J_c(T_0, T_1, \ldots, T_{N-1})$;

(2) make the sampling criterion perform control of the system based on predicted performance.

The results obtained in Table 4-1 for the medium and fast deterministic systems with a step input indicate $\alpha=1$ and $F_{11}=0.1$ provides the best control performance according to the above objectives since the adaptive sampled-data control outperforms the periodic sampled-data control with the same number of control changes. Moreover, for the medium system where the minimum time interval constraint $T_{min}=0.02$ is smaller than that of the fast system in the sense of settling time, the adaptive sampled-data control outperforms the continuous-time control. The adaptive sampled-data control evaluated with $T_{min}=0.02$ for the fast system does not outperform the continuous-time control because the minimum time interval constraint is relatively large compared to the settling time and thus by the particular choice of $T_{min}=0.02$ penalizes the faster speed of response and larger overshoot obtained with the adaptive sampled-data control law. If $T_{min}=0.01$, the adaptive sampled-data control will outperform the continuous-time control as shown from results in Chapter 5, since the fast system predictive sampling problem would then be a perfect time-scaled version of the medium system predictive sampling problem in the sense of settling time.

The cumulative control performance is not affected by changes in Δ when $\alpha=1$ and $F_{11}=0.1$ and thus a choice of Δ based on maximizing improvement over a periodic sampled-data and continuous-time control is not attractive. Δ can be chosen based on maximizing the effective control exercised by a predictive sampling criterion which triggers the sample and hold mechanism on the continuous-time control. The predictive sampling criterion performs more control (holds sampled control error longer for large control error values) when $\Delta=0.22$ or $\Delta=0.88$ increasing the speed of response for both positive or negative control error for the medium system as seen from Figure 4-9 and Table 4-2 and the cumulative control performance is lowest when $\Delta=0.88$. Thus, $\Delta=0.88$ is chosen for the medium system.

These set of performance index parameters will provide good performance on the deterministic system for a range of operating conditions. As evidence of this, the cumulative performance of the continuous, periodic, and adaptive sampled-data controls was plotted versus t_f for the medium system with a parabolic input rather than a step input. The parabolic input was chosen because the type two example system (26) will have a non zero constant error ($h_1(t) - x_1(t)$) as t becomes large. The results shown in Figure 4-11 indicate the cumulative control performance has a large initial increase due to control for the initial transient and then increases with constant rate as t_f increases. The adaptive sampled-data control outperforms periodic and continuous for each t_f when the control performance for each control over a control interval $[0,t_f]$ are compared.

The selection of predictive control performance parameters for the deterministic fast system is $\alpha=1$, $F_{11}=0.1$, and $\Delta=0.11$ from Table 4-3. This system and control performance selected here will be used in following chapter to solve the OSD problem.

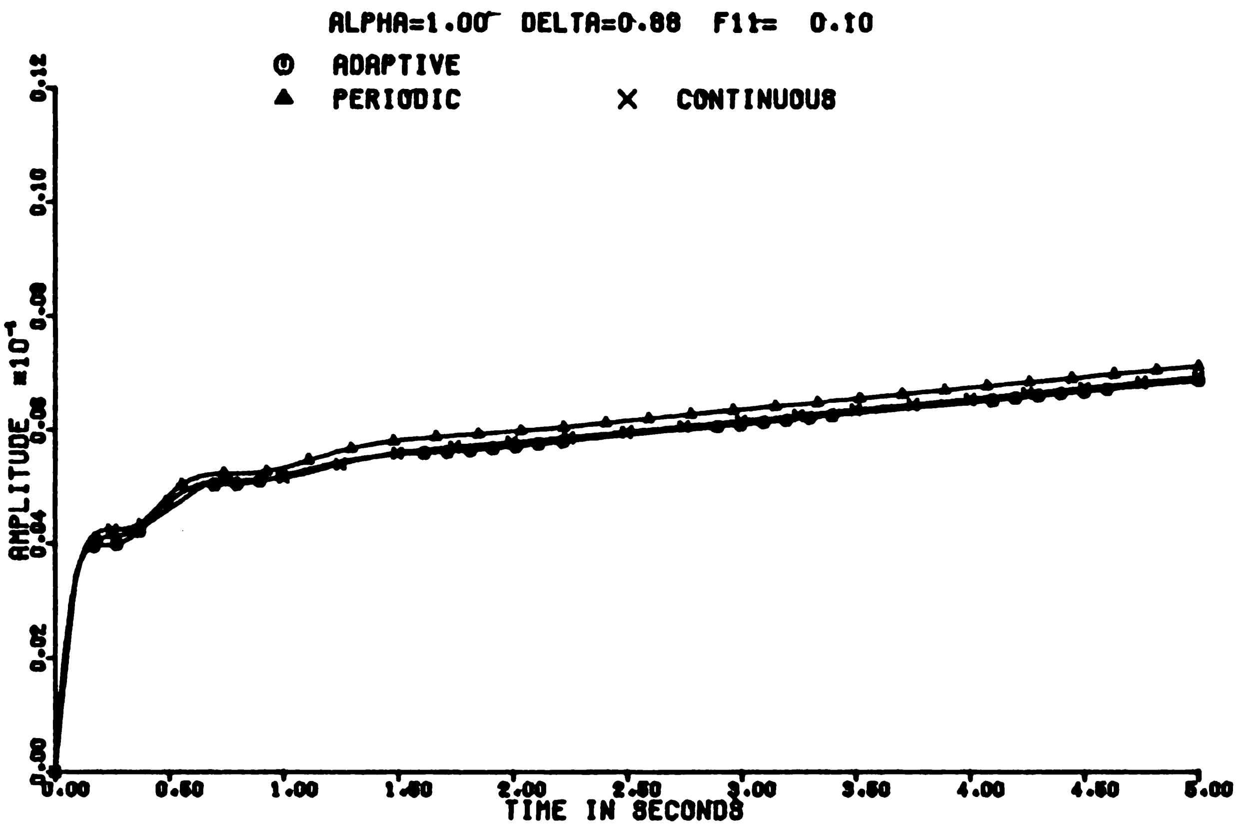
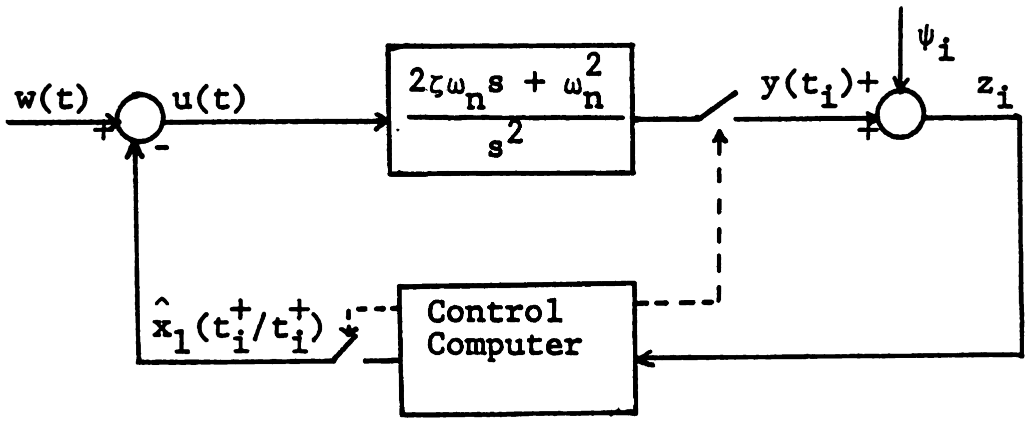


FIGURE 4-11 Cumulative control performance J_c vs t_f with $\hat{F}=0$ for medium system.

The optimal control design, just performed for the deterministic case, is now repeated for a stochastic system. In order to obtain a meaningful comparison between deterministic and stochastic cases, the input h(t) is replaced by a white noise process w(t) as shown below.



The state model for this example is

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2\zeta\omega_n \\ \omega_n \end{bmatrix} u(t) + \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$$

$$\begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} = \begin{bmatrix} 2\zeta\omega_n \\ \omega_n^2 \end{bmatrix} w(t) \tag{36}$$

$$z(t_i) = y(t_i) + \psi_i = [1 \quad 0]\, \underline{x}(t_i) + \psi_i$$

$$u(t) = \begin{cases} -\hat{x}_1(t_i^+/t_i^+) & t\varepsilon[t_i, t_{i+1}) \\ -\hat{x}_1(t/t_i^+) & t\varepsilon[t_{i+1}, t_i+\Delta_i) \end{cases} \tag{37}$$

$$
\begin{aligned}
&E\,\{\underline{x}(t_0)\} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
&E\,\{\underline{x}(t_0)\ \underline{x}'(t_0)\} = \underline{V}(t_0) \\
&E\,\{\underline{w}(t)\ \underline{w}'(t)\} = \begin{bmatrix} 4\zeta^2\omega_n^2 & 2\zeta\omega_n^3 \\ 2\zeta\omega_n^3 & \omega_n^4 \end{bmatrix} \sigma^2 \\
&E\,\{\psi_i\ \psi_i'\} = \Psi
\end{aligned}
\tag{38}
$$

The system chosen is the fast system (ζ=0.5 and ω_n=10). The initial state covariance $V(t_0)$ is set to a null matrix so that the initial state is assumed to be perfectly known.

The performance index for this system is

$$
\begin{aligned}
J(T_i) = {} & \alpha/2\ F_{11}\ [\hat{x}_1^2(t_i+\Delta_i/t_i^+) + V_{11}(t_i+\Delta_i/t_i^+)] \\
& + \alpha/2 \int_{t_i+T_i}^{t_i+\Delta_i} \{0.1[\hat{x}_1^2(t/t_i^+) + V_{11}(t/t_i^+)] + 0.02\ \hat{x}_1^2(t/t_i^+)\}\ dt \\
& + 1/2 \int_{t_i}^{t_i+T_i} \{0.1[\hat{x}_1^2(t/t_i^+) + V_{11}(t/t_i^+)] + 0.02\ \hat{x}_1^2(t_i^+/t_i^+)\}\ dt
\end{aligned}
\tag{39}
$$

The cumulative control performance over the interval [0,1]

$$
J_c(T_0^*, T_1^*, \ldots, T_{N-1}^*) = \sum_{i=0}^{N-1} J_0(T_i^*) \tag{40}
$$

was computed for several adaptive sampled controls with predictive sampling criteria determined based on performance index (39) with several combinations of parameters α, F_{11}, and Δ. The cumulative control performance for periodic sampled-data controls were computed for comparison with the performance of the associated adaptive sampled-data control. The periodic sampled-data control was in each case computed with the same number of sampling intervals and the same sample functions

$(\underline{w}_j(t), \{\underline{\psi}_{ij}\}_{i=1}^{N})$ as used with the adaptive sampling criterion and both were constrained to a [0,1] control interval.

The standard deviation σ was chosen as 0.033 to make the state $x_1(t)$ have a maximum excursion (3 standard deviations) of $\pm$ 0.1. The covariance of the measurement noise was chosen as 0.001 so that the maximum measurement excursion is ten percent of the actual output value. Thus, the system is a random process with transient behavior and is thus an excellent test case for the performance of an adaptive sampling criterion which optimally selects the next sampling interval based on performance prediction which is in turn based on measurements of $x_1(t)$ at the last sampling time t_i. This sampling criterion is closed loop since the selection of the sampling interval is chosen based on measurements of a system with random disturbances and measurement noise.

The results obtained with adaptive and its associated periodic sampled-data controls, where the predictive sampling criteria are computed based on different combinations of performance index parameters, are shown in Table 4-4(a). The results obtained with sample function $\underline{w}_1(t)$ and $\{\psi_{i1}\}_{i=1}^{N}$ indicate the adaptive sampled-data control will in general be inferior to the companion periodic sampled-data control with the same number of control changes. The two parameter combinations, (α=1, F_{11}=5, Δ=0.11) and (α=1, F_{11}=0.1, Δ=0.11), where adaptive outperformed periodic were rerun with other sample functions for processes $\underline{w}(t)$ and $\{\psi_i\}_{i=1}^{N}$. In these cases, the periodic outperformed adaptive as shown in Table 4-4(b).

TABLE 4-4 Cumulative Control Performance for Variation in α, Δ, and F_{11} on Fast Stochastic System with Sample Function $\underline{w}_j(t)$ and $\{\underline{\psi}_{ij}\}_{i=1}^{N}$

α	F_{11}	Δ	Number of Samples	Cumulative Performance(J_c) Predictive	Periodic
0.5	0.1	0.05	43	0.000449	0.000409
0.05	5.0	0.05	26	0.000584	0.000550
1.0	0.1	0.11	13	0.001659 *	0.001732
1.0	0.1	0.22	22	0.000815	0.000638
1.0	5.0	0.05	26	0.000607	0.000566
1.0	5.0	0.11	13	0.001679 *	0.001770
1.0	5.0	0.22	25	0.000613	0.000547
5.0	0.1	0.11	13	0.001656	0.001463
5.0	5.0	0.11	16	0.002083	0.001684
0.01	5.0	0.11	50	0.000198	0.000198

(a)

Parameters	Sample Function (j)	Number of Samples	Cumulative Performance(J_c) Predictive	Periodic
$\alpha = 1.0$, $\Delta = 0.11$, $F_{11}=0.1$	1	13	0.001659	0.001732
	2	16	0.001013	0.000784
	3	15	0.002299	0.001868
	4	12	0.001168	0.001133
$\alpha = 1.0$, $\Delta = 0.11$, $F_{11}=5.0$	1	13	0.001679	0.001770
	2	16	0.001033	0.000799
	3	15	0.002299	0.001868
	4	12	0.001183	0.001156

The results also show that when α becomes small the adaptive criterion approaches the performance of the periodic. The following analysis indicates that when $\alpha=0$, the adaptive criterion is periodic with period T_{min}. The predictive sampling problem when $\alpha=0$ and T_{max} is sufficiently small is

$$\min_{T_i \epsilon [T_{min}, T_{max}]} J(T_i) = \min_{T_i \epsilon [T_{min}, T_{max}]} 1/2 \int_{t_i}^{t_i+T_i} [\underline{\hat{x}}(t/t_i^+)' \underline{Q}\, \underline{\hat{x}}(t/t_i^+) + \mathrm{Tr}\{\underline{Q}\, \underline{V}(t/t_i^+)\} + \underline{u}'(t_i)\, \underline{R}\, \underline{u}(t_i)]\, dt$$

Since the integrand at any t is non-negative and not a function of T_i, the function $J(T_i) = J_0(T_i)$ is a monotone increasing function of T_i with a minimum at $T_i^* = T_{min}$. Since T_{min} has hopefully been chosen sufficiently small to cause no restriction in control performance, the optimal periodic sampling criterion could be determined by increasing T_{min} until the cumulative control performance

$$\begin{aligned} \hat{J}_c(T_{min}) &= J_c(T_{min}, T_{min}, \ldots, T_{min}) \\ &= \sum_{i=0}^{N-1} J_0(T_i^*) \big|_{T_i^* = T_{min}} \end{aligned} \qquad (42)$$

begins to increase significantly to obtain T_{min}^*. This optimal periodic criterion is based strictly on control performance. An OCSD could also be performed based on minimizing

$$S(T_{min}) = \hat{J}_c(T_{min}) + q\, C(T_{min}) \qquad (43)$$

where $C(T_{min})$ is the cost of implementing the periodic sampled-data control system with sampling period T_{min}. The optimal sampling interval T_{min}^* would be based on a cost versus performance tradeoff which would not only specify the sampled-data control law but also the hardware required to implement it.

The results of this subsection indicate that the optimal predictive sampling criterion for the stochastic control system based on a design objective that attempts to make a sampled-data control with a predictive sampling criterion perform as well as or better than periodic is periodic. The results indicate a non-periodic adaptive sampling criterion may outperform the periodic with the same number of sampling intervals for some sample functions, but that on the average a periodic sampling criterion is best for a completely stochastic system.

4.3 Summary

The results of this chapter indicate an optimal predictive sampling criterion outperforms periodic for deterministic systems where future performance can be accurately predicted. In this case, predictive sampling can dramatically outperform the periodic sampled-data control and can ever outperform the continuous-time control being sampled. Thus, predictive sampling performs a control function by holding a control with a larger absolute magnitude than the continuous-time control thus improving speed of response and terminal error but increasing overshoot.

The results on stochastic systems indicates the best predictive sampling criterion is a periodic one that in a sense makes no prediction at all. This result can be explained because the predicted performance is based on an average performance and the predicted state and its covariance may not accurately describe the particular sample function of the state. Thus, the predicted optimal sampling interval may not perform anywhere near the intended control on the particular sample function $\underline{x}(t)$ being observed at that instant. Thus, the optimal predictive sampling criterion will outperform the periodic sampling criterion for some sample functions $\underline{x}(t)$ but will be outperformed by a periodic sampling criterion on the average.

CHAPTER 5. OPTIMAL SYSTEM DESIGN (OSD) FOR PREDICTIVE SAMPLING PROBLEM

The purpose of this chapter is to determine the cost of implementation and to select the optimal hardware for the adaptive sampling criterion. Therefore, this chapter illustrates the OSD problem for the predictive sampling problem. The discussions for the cost of implementation will be made by developing the computer hardware cost. The communication and instrumentation hardwares are not included in this thesis and are a subject for future research. The steps used to determine the cost of implementation and the selection of computer hardware will be performed in Sections 5.1 to 5.4 and Section 5.5 respectively of this chapter for system (26) with continuous control (27) and a closed loop bandwidth ω_n.

5.1 Selection of Algorithm

The first step in this procedure for developing a cost of implementation $C(T_i)$ is to select the algorithms which could solve the following optimization problem

$$S(T_i^*) = \min_{T_i \varepsilon [T_{min}, T_{max}]} \{J(T_i) + qC\,(T_{min})\}. \qquad (44)$$

The cost of implementation for this on-line OC problem is specified since the hardware required has been assumed selected in this OSD problem based on a performance measure

$$S(T_i) = J(T_i) + q\ C(T_i) \tag{45}$$

which is optimized over several intervals $i=0,1,\ldots,N-1$ and several operating condition $j=1,2,\ldots,M$ to obtain $\Gamma = \{\{T_{ij}^*\}_{i=0}^{N-1}\}_{j=1}^{M}$. The minimum sampling interval

$$T_{min} = \min_{i,j} \{T_{ij}^*\} \tag{46}$$

the associated cost of implementation $C(T_{min})$ and the hardware $s(T_{min}) = \{s : C = C_s(T_{min})\}$ for the OC problem are then determined based on determining T_{min} from (46) because

$$\max_{T_{ij}^* \varepsilon \Gamma} C(T_{ij}^*) = C(T_{min}) \tag{47}$$

and because $C(T_i)$ will be a monotone decreasing function. It is obvious that the cost of feasible hardware options, computer algorithms, and the efficient programming of these algorithms will all affect the shape and magnitude of $C(T_i)$.

Since T_{min} is unknown because the hardware has not been selected, a value of T_{min} must be guessed at this point in order to evaluate the performance of algorithms and hardware options. T_{min} is temporarily chosen to be a

$$\hat{T}_{min} = 0.005\ (2\pi)/\omega_n \tag{48}$$

which provides a rate two hundred times the system bandwidth which is much faster than one would ever need to sample, and is smaller than the minimum time needed to compute the optimization problem (44) by the fastest computer-algorithm option. The value for $\hat{T}_{min}$ is chosen temporarily because T_{min} can only be determined after $C(T_i)$ is determined. The use of $\hat{T}_{min} < T_{min}$ rather than actual T_{min} to determine $C(T_i)$

introduces some error in the determination of the CPU time $\tau_{sp}(T_{ij}^*)$ require to compute $T_{ij}^* = T_i$ for a particular computer hardware and algorithm option and thus the cost $C(T_i)$. This error is not significant and is in the direction which would choose a slightly more capable computer than might actually be necessary which leaves some room for later modification or expansion of capability. The maximum sampling period is determined by the stability consideration as in the OCD problem and is

$$T_{max} = \pi/\omega_n \tag{49}$$

which is a rate twice the bandwidth and thus much slower than one would generally wish to sample.

The optimization problem is a univariate search over a relatively small closed bounded interval. Since the optimization to determine T_i^* must be performed on-line in less than T_i^* seconds and since each function evaluation requires relatively extensive computation due to integration of differential equations (26) and the performance index, the algorithms used should require very few function evaluations. Four possible optimization algorithms are feasible for this problem [23]; Fibonacci (p=1), Golden Section (p=2), Powell (p=3), and Davies, Swann and Campey (DSC) (p=4). The Powell algorithm was never evaluated because it was better suited to multivariate search and because it was not as well suited to a search over a small bounded interval. The Fibonacci and Golden Section algorithms are suited to optimization over a small bounded interval but require more function evaluations than a DSC algorithm if $S(T_i)$ is convex and has a unique minimum. Thus, for the case where $S(T_i)$ is convex as shown in Figures 4-1,2,5, the DSC algorithm (p=4) will be used. This decision is made for all hardware options (s) since the best algorithm is independent of the computer used.

Uniform search steps are used in the DSC algorithm rather than acceleration steps in order to reduce the number of function evaluations needed for a small bounded search interval. The uniform steps in the search are continued until the decrease in the performance index terminates and an increase is noted on the last search step. A minimum T_i^* is thus known to have occurred in the last two intervals. A single quadratic interpolation is performed to obtain T_i^* because the number of function evaluations is to be minimized and because sufficient accuracy is obtained if the uniform search step size is small enough. Minimizing function evaluations reduces $\tau_{sp}(T_i^*)$ and will reduce both $C(T_i)$ for each T_i and T_{min}.

The search is initiated at $\hat{T}_{min}$ rather than T_{max} in order to cause the CPU time $\tau_{sp}(T_i^*)$ required to compute T_i^* to be an increasing function of $T_i^* = T_i$ rather than a decreasing function of T_i^*. Since the constraint

$$\Omega(T_i^*) = \{(s,p)\colon \tau_{sp}(T_i^*) \leq T_i^*\} \tag{50}$$

requires that the computation be completed on any computer before trigger at that sampling instant $t_{i+1}^* = t_i + T_i^*$ is necessary, the resulting cost of implementation

$$C(T_i^*) = \min_{(s,p)\varepsilon\Omega(T_i^*)} \{C_{sp}\} \tag{51}$$

will be defined for smaller values of T_i^* resulting in a lower value of T_{min}. Thus, the choice of algorithm and the direction of search for this algorithm will ultimately effect the value of T_{min} and the magnitude and shape of this cost of implementation.

5.2 Hardware Options

The second step in this procedure is to determine a set of computers that can handle this problem. Attention was restricted to minicomputers with

(1) at least 4K words of memory size which is enough memory for this particular problem;

(2) FORTRAN capability in order to make programming easy;

(3) 16 bit word size in order to obtain the accuracy required to compute T_i^*.

It was assumed that multiplication and division operations would be implemented using software since multiplication and division hardware options were not always available on every computer. The computation times for addition and subtraction were assumed the same and the computation times for multiplication and division were assumed to be eight times per word as large as for addition and subtraction per word on all computers considered [24],

TABLE 5-1 Selected Minicomputers and Specifications

s	Manufacturer	Model	Memory Size (words)	Addition Time per word(μsec) (γ_1)	Cost ($) (C)
1	Digital Equipment	PDP-11/45	32K	0.3	38,000
2	Microdata	Express I	32K	0.405	20,000
3	Data General	5/100	8K	0.6	9,200
4	Digital Computer	D-616	4K	0.66	7,260
5	Data General	NOVA 3/12	4K	0.7	3,600
6	Digital Computer Controls	MOD-5	4K	0.8	3,075
7	Interdata	6/16	4K	1.0	2,900
8	Interdata	5/16	4K	1.2	2,100
9	Digital Equipment	PDP-11/03	4K	3.5	1,995

$$\gamma_1(s) = \gamma_2(s)$$
$$\gamma_3(s) = \gamma_4(s) = 8\ \gamma_1(s) \qquad (52)$$

where γ_1, γ_2, γ_3, and γ_4 are addition time, subtraction time, multiplication time, and division time per word respectively for the computer s.

A set of computers which met those specifications was determined from the 1976 DATAPRO REPORTS [25] and is shown in Table 5-1 with the actual cost and computation time for addition for each selected computer shown. More specific data should be required for a practical control problem such as the proper hardware or software options for each of these mathematical operations. It is conceivable that the proper hardware or software option for any operation on a particular computer may be selected as part of the design of the optimal sampling interval in order to achieve a minimum cost of implementation $C(T_i)$ for each T_i.

5.3 Optimization of Software

The third step of this procedure is to optimize the computer programming to minimize CPU time $\tau_{sp}(T_i^*)$ for each hardware-computer algorithm option (s,p) to compute T_i^* for the OC problem. Since the computational algorithm was chosen to be DSC algorithm (p=4), the only consideration to optimize the computer programming is to minimize $\tau_s(T_i^*)$. However, the subscript p is retained because in general the algorithm may not be selected at this point.

The computation time $\tau_{sp}(T_i^*)$ for each computer s=1,2,...., 9 and algorithm p=1,2,3,4 is approximately

$$\tau_{sp}(T_i^*) = \sum_{k=1}^{4} K_{pk}(T_i^*)\ \gamma_k(s) \qquad T_i^* \epsilon [T_{min}, T_{max}] \tag{53}$$

where K_{p1}, K_{p2}, K_{p3}, and K_{p4} are the total number of additions, subtractions, multiplications, and divisions respectively to compute T_i^* for each T_i for algorithm p. The equation (53) can be rewritten as

$$\begin{aligned}\tau_{sp}(T_i^*) &= [K_{p1}(T_i^*) + K_{p2}(T_i^*) + 8\ (K_{p3}(T_i^*) + K_{p4}(T_i^*))] \\ &\quad \gamma_1(s) \qquad (54) \\ &= K_p(T_i^*)\ \gamma_1(s)\end{aligned}$$

by substituting (52).

The total number of any particular operation depends on the number of function evaluations $N_p(T_i^*)$ to compute $T_i^* = T_i$ for p^{th} algorithm and the total number of integration steps $N_0(T_i^*)$ and $N_1(T_i^*)$ required to compute $J_0(T_i^*)$ and $J_1(T_i^*)$ respectively. Thus, the number of operations of any particular kind for the p^{th} algorithm can be expressed as

$$K_{pk}(T_i^*) = M_{0k}\ N_0\ (T_i^*) + M_{1k}\ N_1\ (T_i^*) + M_{2k}\ N_p(T_i^*) + M_{3k} \qquad k = 1,2,3,4 \tag{55}$$

where $T_i^* = T_i$, m is the integer index for function evaluation and

$$N_{\ell p}(T_i^*) = \sum_{m=1}^{N_p(T_i^*)} N_{\ell pm} \qquad \ell = 0,1 \tag{56}$$

Constant M_{0k} and M_{1k} are the integer number of operations of type k for each integration step in $J_0(T_i)$ and $J_1(T_i)$ respectively. M_{2k} is the integer number of operations of type k which must be performed for each function evaluation but is independent of the number of integration steps per function evaluation. Finally, M_{3k} is the number of operations of type k required to compute T_i^* but not in the $N_p(T_i^*)$ function evaluations required to obtain $T_i^* = T_i$. N_{0pm} and N_{1pm} are the integer number of integration steps for the m^{th} function evaluation required to compute $J_0(T_i)$ and $J_1(T_i)$ respectively for algorithm p and thus $N_{0p}(T_i^*)$ and $N_{1p}(T_i^*)$ are the total number of integration steps for all function evaluations required to compute $T_i^* = T_i$ with algorithm p.

The total number of equivalent additions $K_p(T_i^*)$ is a simple notation which conveys the essential structure and information in expression (54). This number $K_p(T_i^*)$ can be expressed as

$$K_p(T_i^*) = M_0 \ N_{0p}(T_i^*) + M_1 \ N_{1p}(T_i^*) + M_2 \ N_p(T_i^*) + M_3 \qquad (57)$$

where

$$M_j = M_{j1} + M_{j2} + 8 \ (M_{j3} + M_{j4}) \qquad j = 0,1,2,3$$

Thus, minimizing the computation time implies minimizing $N_p(T_i^*)$, $N_{0p}(T_i^*)$, and $N_{1p}(T_i^*)$ as well as minimizing the constants M_0, M_1, M_2, and M_3 by reducing the operations in the computer programming. Since $N_p(T_i^*)$ is a decreasing function of the uniform step size ΔT, and $N_{0p}(T_i^*)$ and $N_{1p}(T_i^*)$ are decreasing function of ΔT and the integration step size ε because N_{0pm} and N_{1pm} are decreasing function of ε, the choice of ΔT and ε effect the magnitude and shape of the cost of implementation $C(T_i)$, the accuracy of computation of $J(T_i^*)$, and the accuracy of the solution T_i^* and thus should be included in the OSD problem. The details of selecting ΔT and ε depend on the particular example problem chosen and thus are considered in the next subsection.

5.4 Development of Cost of Implementation

This section will discuss the fourth and fifth steps of the procedure to find the cost of implementation, which are the determination of the set of feasible computer hardware-computational algorithm options for each T_i and the determination of the cost of implementation for each T_i.

The cost of implementation is obtained from the deterministic system (26) of Chapter 4 where ω_n=10. The output dimension (m) is set equal to one rather than two as in Chapter 4 because Q_{22} was set equal to zero in the OCD problem. The system description is repeated here for convenience

$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 10 \\ 100 \end{bmatrix} u(t)$$

$$y(t) = x_1(t) \tag{58}$$

$$u(t) = \begin{cases} h(t_i) - x_1(t_i) & t_i \leq t \leq t_{i+1} \\ h(t) - x_1(t) & t_{i+1} \leq t \leq t_i + \Delta \end{cases} \tag{59}$$

From Chapter 4, the parameters and matrices in the control performance index (29) are Q_{11}=0.1, R=0.02, F_{11}=0.1, α=1, and Δ=0.11.

Now the cost of implementation for this example problem will be developed based on the following information:

(1) The optimization algorithm is selected to be the DSC algorithm with a uniform step size and forward search steps

from $\hat{T}_{min}$ toward T_{max}. T_{max} is chosen to be 0.11 because Δ=0.11 is less than Nyquist sampling period (0.3) for this system. $\hat{T}_{min}$ is temporarily chosen to be 0.003 by (48).

(2) The hardware options are selected and listed in Table 5-1 with addition time per word $\gamma_1(s)$ and hardware cost C_s for each hardware option.

(3) The computer programming is optimized to minimize each type of operation and the number of operations $\{\{M_{jk}\}_{j=0}^{3}\}_{k=1}^{4}$ are counted. The total number of equivalent additions becomes

$$K_{p=4}(T_i^*) = 449\ N_{04}(T_i^*) + 497\ N_{14}(T_i^*) + 134\ N_4(T_i^*) + 9\ N_c + 117 \tag{60}$$

where the variable N_c is associated with the number of uniform search steps for $[t_i + T_{min}, t_i + T_{max}]$ and is obtained from

$$N_c = \min\ (N\ ;\ \Delta T = (T_{max} - T_{min})\ /\ N \leq \Delta T_{max}) \tag{61}$$

where N is a positive integer number and ΔT_{max} is maximum allowable constraint of ΔT. Since the number of uniform search steps N_c is a positive integer value, N_c and ΔT are determined simultaneously if ΔT_{max} were specified.

The determinations of $N_4(T_i^*)$, $N_{04}(T_i^*)$, $N_{14}(T_i^*)$, ΔT_{max}, and the maximum integration step size ε_{max} will now be described for DSC algorithm.

The number of function evaluations $N_4(T_i^*)$ for the DSC algorithm (p=4) is

$$N_4(T_i^*) = \begin{cases} j + 3 & T_i^* \varepsilon [T_{min}+(j-1)\Delta T, T_{min}+j\Delta T] \\ & j = 1,2,\ldots., N_c-1 \\ N_c + 2 & T_i^* \varepsilon [T_{min}+(N_c-1)\Delta T, T_{max}] \end{cases} \tag{62}$$

because (j+2) uniform step function evaluations are needed to evaluate $J(T_{min}+(j-1)\Delta T)$, $J(T_{min}+j\Delta T)$, and $J(T_{min}+(j+1)\Delta T)$ for quadratic interpolation formula for the optimal sampling solution

$$\hat{T}_i^* = (T_{min}+j\Delta T) + \frac{\Delta T[J(T_{min}+(j-1)\Delta T) - J(T_{min}+(j+1)\Delta T)]}{2[J(T_{min}+(j-1)\Delta T)-2J(T_{min}+j\Delta T)+J(T_{min}+(j+1)\Delta T)]} \quad (63)$$

when T_i^* is in the interval $[T_{min}+(j-1)\Delta T, T_{min}+j\Delta T]$. Another function evaluation is required to evaluate $J(\hat{T}_i^*)$. For the case where $j=N_c$, only (j+1) uniform step function evaluations are required to determine $J(T_{min}+(N_c-2)\Delta T)$, $J(T_{min}+(N_c-1)\Delta T)$, and $J(T_{min}+N_c\Delta T)$ for quadratic interpolation formula

$$\hat{T}_i^* = T_{min}+(N_c-1)\Delta T + \frac{\Delta T[J(T_{min}+(N_c-2)\Delta T) - J(T_{min}+N_c\Delta T)]}{2[J(T_{min}+(N_c-2)\Delta T)-2J(T_{min}+(N_c-1)\Delta T)+J(T_{min}+N_c\Delta T)]} \quad (64)$$

when T_i^* is in the interval $[T_{min}+(N_c-1)\Delta T, T_{max}]$. An additional function evaluation is needed to evaluate $J(\hat{T}_i^*)$ for this case.

Simpson's integration formula [26,27] is used for integration to evaluate the control performance index and thus the number of integration steps, N_{04m} and N_{14m}, for a function evaluation must be a positive even integer. Since the total number of function evaluations is $N_4(T_i^*)$ and since the DSC algorithm selected performs $(N_4(T_i^*)-1)$ uniform search steps of step size ΔT and a quadratic interpolation, the total numbers of integration steps, $N_{04}(T_i^*)$ and $N_{14}(T_i^*)$, for the DSC algorithm for all function evaluations $N_4(T_i^*)$ are

$$N_\ell(T_i^*) = \sum_{m=1}^{N_4(T_i^*)} N_{\ell m} \qquad \ell = 0,1 \quad (65)$$

where the number of integration steps required to evaluate $J_0(T_{min}+(m-1)\Delta T)$ and $J_1(T_{min}+(m-1)\Delta T)$ are

$$N_{04m} = \begin{cases} \min\left(2L_0 \,;\, \varepsilon_{0m} = \dfrac{T_{min}+(m-1)\Delta T}{2L_0} \leq \varepsilon_{max}\right) & m=1,\ldots,N_4-1 \\ \min\left(2L_0 \,;\, \varepsilon_{0N_4} = \dfrac{T_{min}+(N_4-3)\Delta T}{2L_0} \leq \varepsilon_{max}\right) & m=N_4 \end{cases}$$

$$N_{14m} = \begin{cases} \min\left(2L_1 \,;\, \varepsilon_{1m} = \dfrac{\Delta-(T_{min}+(m-1)\Delta T)}{2L_1} \leq \varepsilon_{max}\right) & m=1,\ldots,N_4-1 \\ \min\left(2L_1 \,;\, \varepsilon_{1N_4} = \dfrac{\Delta-(T_{min}+(N_4-4)\Delta T)}{2L_1} \leq \varepsilon_{max}\right) & m = N_4 \end{cases}$$

respectively. L_0 and L_1 are integer values, $N_4 = N_4(T_i^*)$, and ε_{max} is maximum allowable integration step size in these expressions. $\varepsilon_{\ell m}$ represents the integration step size for the evaluation of $J_\ell(T_{min}+(m-1)\Delta T)$ and is chosen to make the positive even number of integration steps ($N_{\ell 4m}$) as small as possible with the constraint that $\varepsilon_{\ell m}$ must not exceed ε_{max}. This number of integration steps, $N_{\ell 4m}$ for $m < N_4$, is precisely determined by (66) and (67) because the integration time interval $[0,T_{min}+(m-1)\Delta T]$ to evaluate $J_0(T_{min}+(m-1)\Delta T)$ and the interval $[T_{min}+(m-1)\Delta T,\Delta]$ to evaluate $J_1(T_{min}+(m-1)\Delta T)$ is known if ΔT is known. The number of integration steps, $N_{\ell 4N_4}$, to evaluate $J_\ell(\hat{T}_i^*)$ is not precisely determined before $\hat{T}_i^*$ is obtained by computer. Thus, the equations (66) and (67) for $m = N_4$ were assumed to have the maximum number of integration steps to evaluate $J_\ell(\hat{T}_i^*)$ which is the number of integration steps to compute $J_0(T_{min}+(N_4-3)\Delta T)$ when $\ell=0$ and is the number of integration steps to compute $J_1(T_{min}+(N_p-4)\Delta T)$ when $\ell=1$.

The maximum uniform search step size is determined by the error formula [26] of the quadratic interpolation

$$J(T_i^*) - J(\hat{T}_i^*) \leq 1/6\ J'''(t)\ (\Delta T_{max})^3 \qquad (68)$$

where T_i^* and $\hat{T}_i^*$ are actual optimal sampling time interval and computed optimal time interval respectively, and where t is in the interval $[T_{min}+(N_4-4)\Delta T_{max}, T_{min}+(N_4-2)\Delta T_{max}]$ which is the interval for quadratic interpolation to find $\hat{T}_i^*$ if uniform search step size is chosen to be ΔT_{max}. ΔT_{max} is desired to be large in order to reduce $N_4(T_i^*)$ and N_c and thus also to reduce $N_{04}(T_i^*)$ and $N_{14}(T_i^*)$ but not too large so that the control performance index error $(J(T_i^*) - J(\hat{T}_i^*))$ and solution error $(T_i^* - \hat{T}_i^*)$ do not become large. ΔT_{max} is thus chosen to be 0.05 because the control performance index is a smooth and nearly quadratic so that the third derivatives of J(t) will be very small. The performance error (68) and solution error $(T_i^* - \hat{T}_i^*)$ should be very small when ΔT_{max}=0.05. Thus, from (61) N_c=3 and ΔT=0.035667 seconds.

The maximum integration step size ε_{max} is also desired to be large in order to reduce N_{04m} and N_{14m} and thus to reduce $N_{04}(T_i^*)$ and $N_{14}(T_i^*)$ but not so large that the integration error becomes significant. Control performance $J(T_0)$ for the first interval (T_0) is shown as a function of ε_{max} for T_0 = 0.03, 0.05, 0.08, and 0.11 seconds in Figure 5-1. Since the first significant change in $J(T_0)$ is when ε_{max} is 0.025 seconds for T_0 = 0.05 seconds, ε_{max} is selected to be 0.025 seconds so that the integration error incurred by ε_{max} can be assured to be small enough for all $T_0 \leq 0.11$. This selection of ε_{max} holds for all T_i because this integration error for the first interval is more significant than other intervals in this example problem.

The total number of equivalent additions $K_4(\hat{T}_i^*)$ for this DSC algorithm is shown for each $\hat{T}_i^* = T_i$ in Table 5-2 from (60) with ΔT=0.035667 and ε_{max}=0.025. Thus, $\tau_s(\hat{T}_i^*)$ is known for each computer option from (54) and is shown in Table 5-3. The set of feasible computer options for each T_i can be found by

$$\Omega(T_i) = \{s : \tau_s(T_i) \leq T_i\} \qquad (69)$$

and the results are shown in Table 5-4.

Finally, the cost of implementation can be formulated as a piecewise constant monotone decreasing function of T_i as shown in Figure 5-2 by choosing the computer for each T_i which has a minimum cost among the feasible set of computers, or

$$C(T_i) = \min_{s\varepsilon\Omega(T_i)} \{C_s\} \qquad (70)$$

The resultant cost of implementation is appealing because it decreases very fast during the short time interval [0.0045,0.0105] and it is almost constant with very small change in cost over [0.0105,0.11]. This cost of implementation will now be used to find an optimal selection of hardware in the next section.

5.5 Optimal Selection of Hardware

This section will describe the optimal selection of hardware by a tradeoff of the control performance index and

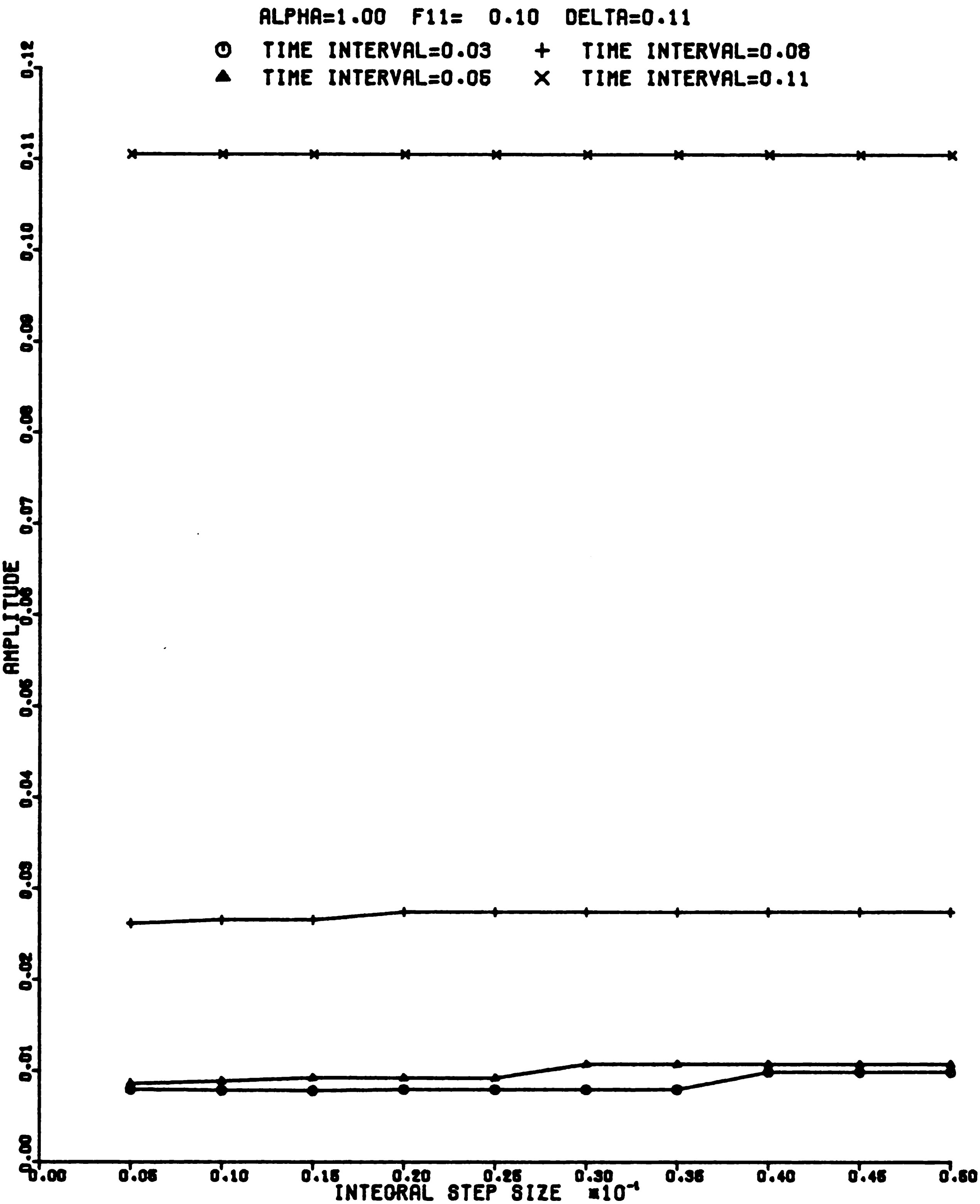


FIGURE 5-1 $J(T_0)$ vs integration step size ε_{max} for several time intervals T_0.

TABLE 5-2 Total Number of Equivalent Additions to Find T_i^* using DSC Algorithm for each T_i^*.

j	T_i^*	$N_4(T_i^*)$	$N_{04}(T_i^*)$	$N_{14}(T_i^*)$	$K_4(T_i^*)$
1	.003 - .038667	4	12	18	15,014
2	.038667 - .074333	5	20	16	17,746
3	.074333 - .11	5	22	14	17,650

TABLE 5-3 Computation Time for Each Computer for Each T_i^*

j	T_i^*	$\tau_s(T_i^*)$ (μsec)								
		.3	.405	.6	.66	.7	.8	1.0	1.2	3.5
1	.003 - .038667	.00450	.00608	.00901	.00991	.01051	.01201	.01501	.01802	.05255
2	.038667- .074333	.00532	.00719	.01065	.01171	.01242	.01420	.01775	.02130	.06211
3	.074333- .11	.00530	.00715	.01059	.01165	.01236	.01412	.01765	.02118	.06178

TABLE 5-4 Set of Feasible Computers for each T_i^*

T_i^*	Set of Feasible Computers
.00450-.00608	PDP-11/45
.00608-.00901	PDP-11/45, Express I
.00901-.00991	PDP-11/45, Express I,S/100
.00991-.01051	PDP-11/45, Express I,S/100,D-616
.01051-.01201	PDP-11/45, Express I, S/100,D-616,NOVA 3/12
.01201-.01501	PDP-11/45, Express I,S/100,D-616,NOVA 3/12,MOD-5
.01501-.01802	PDP-11/45, Express I,S/100,D-616,NOVA 3/12,MOD-5,6/16
.01802-.06211	PDP 11/45, Express I,S/100,D-616,NOVA 3/12,MOD-5,6/16,5/16
.06211-.11	PDP-11/45,Express I, S/100,D-616,NOVA 3/12,MOD-5,6/16,5/16,PDP-11/03

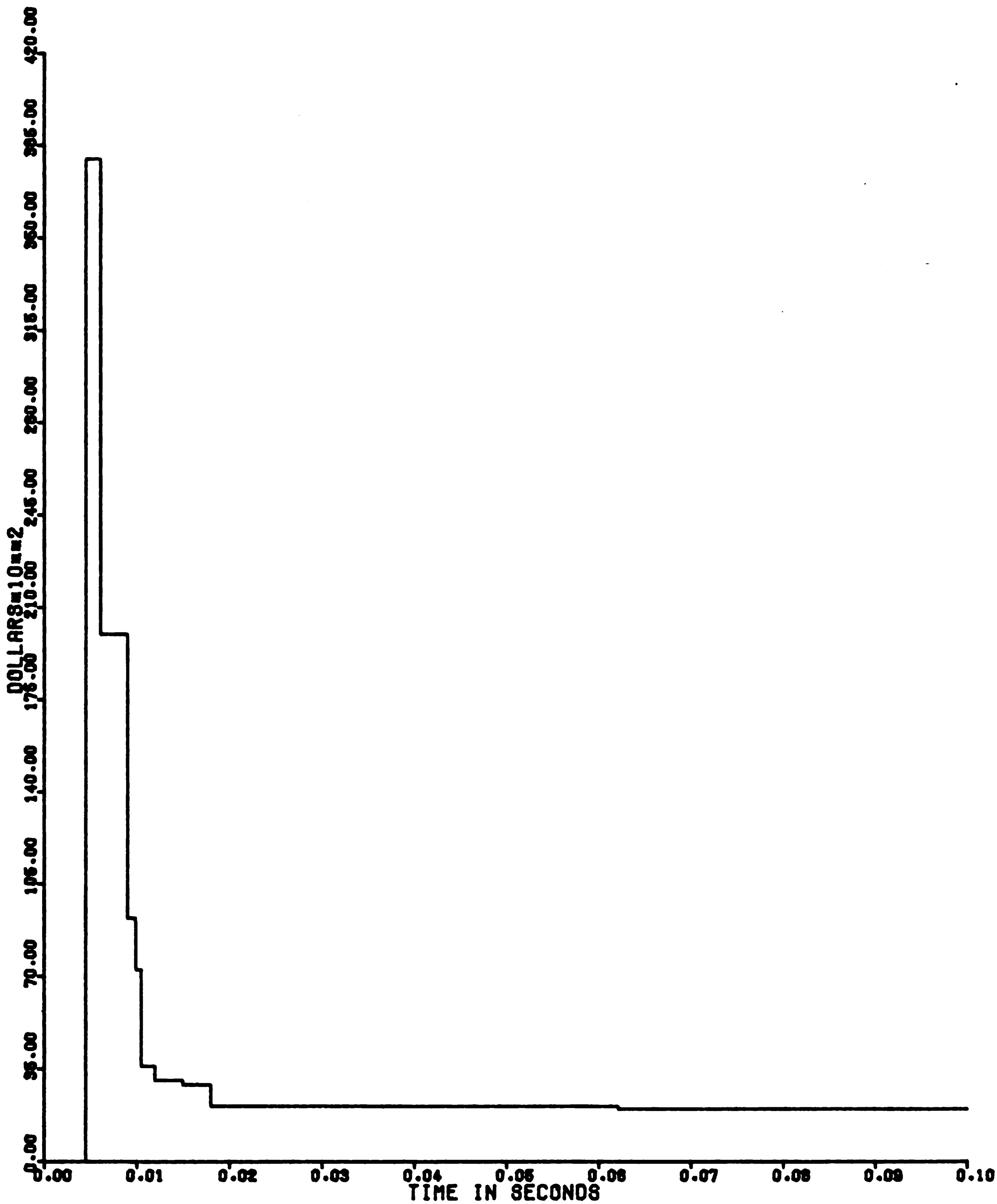


FIGURE 5-2 Cost of implementation $C(T_i)$ vs sampling time interval T_i.

cost of implementation which have been determined in Chapter 4 and the previous section of Chapter 5 respectively. The selection of hardware is the second stage of the OSD problem which completes the OCSD problem described in Chapter 3.

The system performance index to be minimized for the optimal selection of computer hardware problem

$$S(T_i) = J(T_i) + q\ C(T_i) \tag{71}$$

includes cost of implementation with weighting factor q. The selection of hardware requires the minimization of (71) for several operating conditions (j) and several sampling intervals (i) for some q to obtain a set of optimal sampling intervals

$$\Gamma = \{\{T^*_{ij}\}^{M}_{j=1}\}^{N-1}_{i=0} \tag{72}$$

and then select the hardware based on the maximum cost over the optimal set of solutions Γ

$$C^* = \max_{T^*_{ij}\varepsilon\Gamma} \quad C(T^*_{ij}) \tag{73}$$

Conceptually, q is a conversion parameter from the actual computer cost to the equivalent control performance value and thus can be determined by inverse of actual dollar benefit of the performance improvement. However, the selection of q is difficult to obtain because its choice determines the hardware selected based on (71,72). If q can not be obtained easily, the following alternative procedures make determination of hardware, the associated hardware cost $C(T_{min})$, and T_{min} easier.

The particular value of T_{min} chosen will not only determine the hardware

$$s(T_{min}) = \{s : C_s = C(T_{min})\} \tag{74}$$

implemented but also the cumulative control performance index evaluated for that T_{min} over a set of intervals $i = 0,1,\ldots,N-1$ and operating conditions $j = 1,2,\ldots,M$

$$\begin{aligned}\hat{J}_c(T_{min}) &= \sum_{j=1}^{M} J_{cj}(T_{0j}^*, T_{1j}^*, \ldots\ldots, T_{N-1,j}^*, T_{min}) \\ &= \sum_{j=1}^{M} \{1/2[\underline{x}_j(t_f)-\underline{h}(t_f)]'\hat{\underline{F}}\ [\underline{x}_j(t_f)-\underline{h}(t_f)] + \\ &\quad \sum_{i=0}^{N-1} J_0(T_{ij}^*)\}\end{aligned} \tag{75}$$

where T_{ij}^* satisfies

$$T_{min} \leq T_{ij}^* \leq T_{max}$$

Since $C(T_{min})$ decreases very rapidly for $T_{min} < a$ and $\hat{J}_c(T_{min})$ increases very rapidly for $T_{min} > b$, there is a feasible region for T_{min}

$$T_{min} \in [a,b] \tag{76}$$

Obviously a good design using the OSD methodology would choose a q to obtain a $T_{min} \in [a,b]$ because otherwise the cost would be excessive or the control performance would be seriously degraded. The particular choice of T_{min} in this region or the choice of q that will produce the same T_{min} in the initial procedure, would be based on the designers objectives. If the designer wanted the lowest possible cost of implementation consistent with good control performance the hardware $s^*(b)$ with cost $C(b)$ would be selected. If the designers objective is to minimize control performance consistent with acceptable cost of implementation the hardware $s^*(a)$ with cost $C(a)$ would be implemented.

This procedure to determine the optimal hardware will now be applied to the example problem which is the deterministic fast system (58,59) with control performance index (29) where $\alpha=1$, $F_{11}=0.1$, $\Delta=0.11$, and $Q_{22}=0$, and a cost of implementation shown in Figure 5-2. The minimum of feasible region for T_{min}, "a", can easily be selected to be 0.0105 seconds from Figure 5-2 because the cost of implementation is a rapidly decreasing function up to 0.0105 seconds and then a slowly decreasing function from that point. The maximum of feasible region for T_{min}, "b", is chosen to be 0.053 from Figure 5-3 which is the cumulative control performance (75) with respect to T_{min} with $\hat{F}=0.05$ for an operating condition, $h(t) = 1$, for the OC problem. This figure shows that the cumulative control performance is a slowly increasing before $T_{min}=0.053$ and a rapidly increasing after $T_{min}=0.053$. This feasible region for T_{min} [0.0105,0.053] is obtained based on just the unit step operating condition ($h(t)=1$) because the results for other operating conditions ($h(t)=t$, $h(t)=t^2$) are very similar to that for the unit step input.

Thus, the optimal choice of T_{min} is in the range of 0.0105 and 0.053, and corresponding optimal computer hardwares are Data General NOVA 3/12, Digital Computer Controls MOD-5, Interdata 6/16, and Interdata 5/16 from Table 5-4. The choice from these four optimal computers is quite arbitrary and is dependent solely on the designers priorities. Data General NOVA 3/12 will be chosen if the control performance is considered to be more important than the hardware cost. The Interdata 5/16 will be chosen if the computer cost is considered more important than the control performance.

An arbitrary choice of computer hardware, the Data General NOVA 3/12, for implementation is made for this example control problem. The state trajectories for the sampled-data control with predictive sampling and this hardware compared to the periodic and continuous control are shown in Figure 5-4. The sampled-data control with predictive sampling and this computer appears to significantly outperform both the periodic sampled-data and continuous-time controls. The precise values of the cumulative control performance index for predictive sampling with different values of T_{min} for unit step input are shown in Table 5-5 with the cumulative control performance of the periodic sampling criterion (constrained to have the same number of sampling times as predictive sampling) and continuous control. The cumulative control performance and terminal error for the sampled-data control with optimal predictive sampling are dramatically improved over those of the periodic sampled-data control. Moreover, the sampled-data control with predictive sampling criterion for $T_{min} \leq 0.012$ outperforms the continuous control which is the control being adaptively sampled by the predictive sampling criterion. These results confirm the hypothesis (page 23) in Chapter 4 for the fast system that T_{min} was chosen too large so that the sampled-data control with predictive sampling did not outperform the continuous control being sampled.

The results thus indicate a predictive sampling criterion does perform control because it enhances the control performance over that of the continuous control for both the fast and medium systems when T_{min}, α, F_{11}/Q_{11}, and Δ are chosen properly. Moreover, the predictive sampling criterion seems practical because it can be implemented with fairly inexpensive minicomputers. The exact minicomputer chosen is shown to depend on the designers priorities on performance and cost.

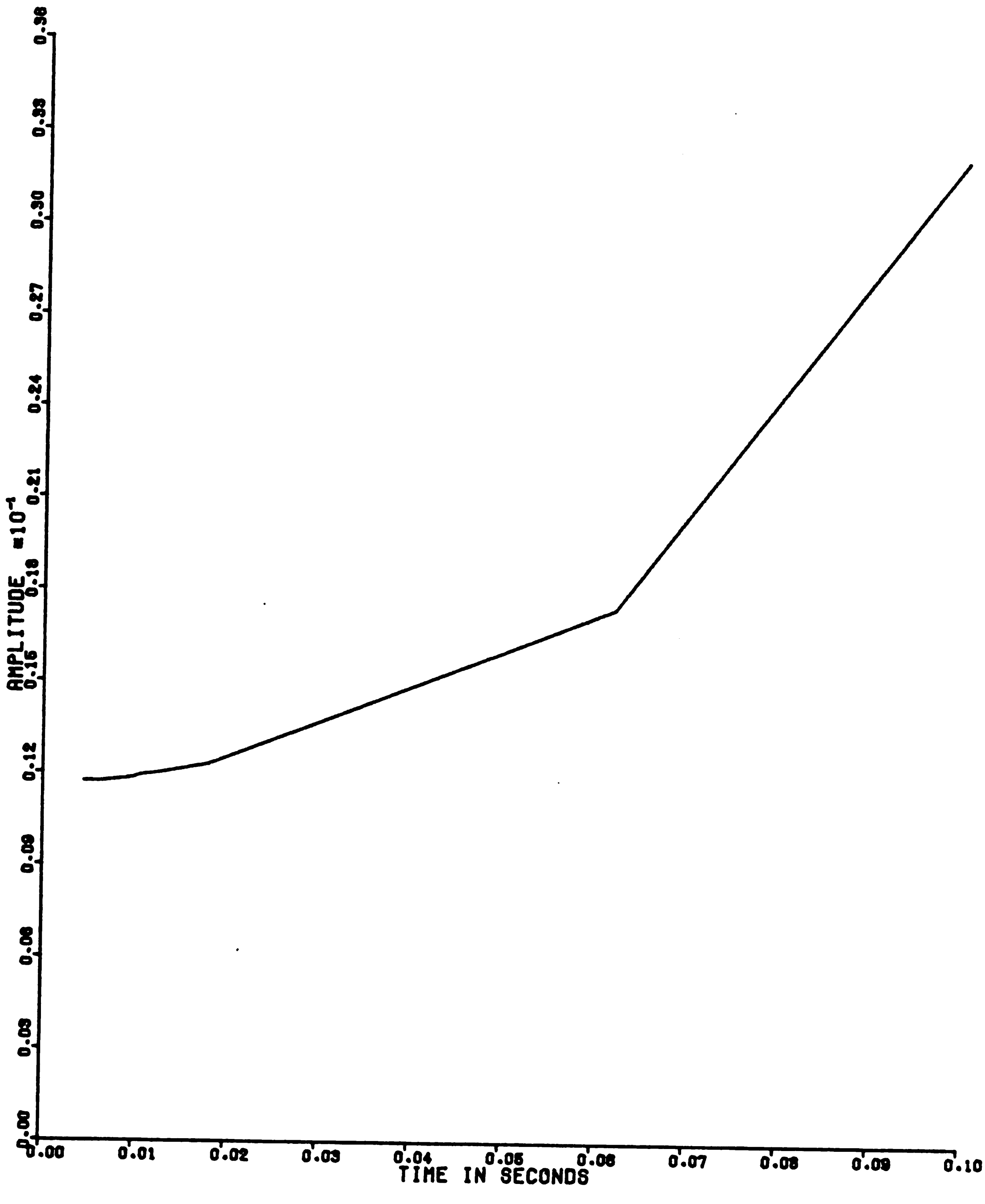


FIGURE 5-3 Cumulative control performance $\hat{J}_c(T_{min})$ vs T_{min} for unit step input operating condition.

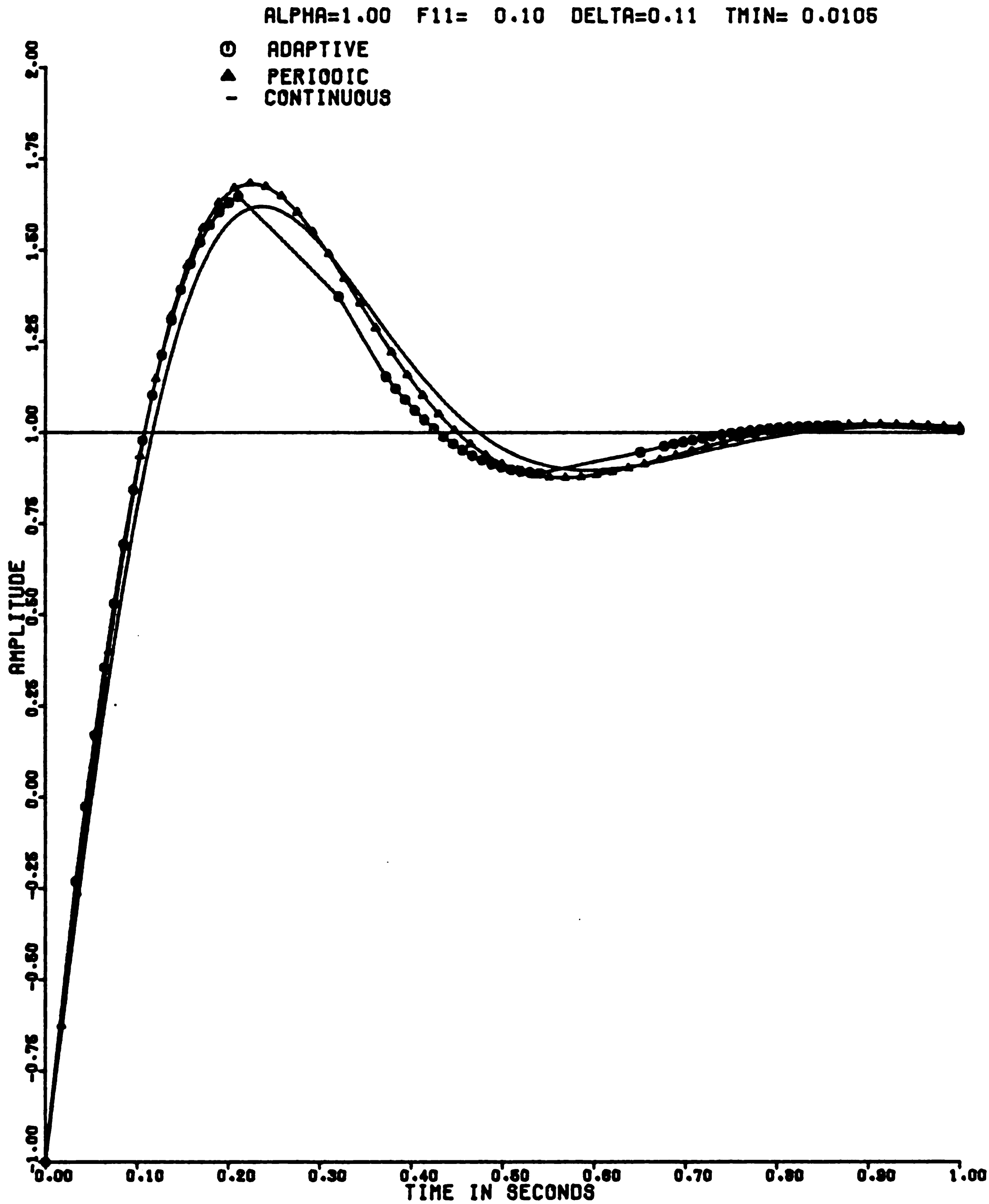


FIGURE 5-4 Output trajectories for adaptive sampling with T_{min}=.0105, periodic sampling, and continuous control for fast system.

TABLE 5-5 Cumulative Control Performance and Terminal Error with T_{min} Variations for Fast System

Parameters α, Δ, and F_{11}	T_{min} Hardware	Number of Samples	Terminal Error $h(t_f)-x_1(t_f)$ Adaptive	Terminal Error $h(t_f)-x_1(t_f)$ Periodic	Cumulative Control Performance (F=0) Adaptive	Cumulative Control Performance (F=0) Periodic	Cumulative Control Performance (F=0.05) Adaptive	Cumulative Control Performance (F=0.05) Periodic
Continuous	0.005	200		0.015662		0.012118		0.012130
α = 1.0	0.0045 PDP-11/45	83	0.008216	0.015909	0.011731	0.012355	0.011733	0.012368
Δ = 0.11	0.0061 Express I	85	0.006220	0.015914	0.011723	0.012344	0.011725	0.012357
F_{11}= 0.1	0.0090 S/100	66	0.007251	0.015771	0.011817	0.012487	0.011820	0.012499
	0.0099 D-616	60	0.006743	0.015644	0.011862	0.012559	0.011865	0.012571
	0.0105 NOVA 3/12	58	0.006842	0.015586	0.011935	0.012587	0.011938	0.012599
	0.0120 MOD-5	50	0.006636	0.015221	0.011990	0.012732	0.011992	0.012744
	0.0150 6/16	43	0.003123	0.014603	0.012145	0.012924	0.012145	0.012935
	0.0180 5/16	36	0.006732	0.013380	0.012305	0.013232	0.012308	0.013241
	0.0621 PDP-11/03	14	0.017836	0.028204	0.017437	0.019882	0.017453	0.019922
	0.1000	10	0.082540	0.087891	0.031839	0.031627	0.032180	0.032013

CHAPTER 6. CONCLUSIONS

This thesis has two principal contributions:

(1) the formulation and solution of the optimal predictive sampling criterion for a sampled-data control system;

(2) the development of the optimal control system design methodology for the optimal predictive sampling problem.

The optimal sampled-data control problem with predictive sampling criterion was motivated by the following past developments:

(1) periodic sampling criterion which is commonly used because of the ease of design and analysis using transform technique;

(2) adaptive sampling criteria [1-11] , where the sampling rate is varied in proportion to the change of error rate. The objective of these criteria, as indicated by the performance index used to derive the sampling rules, is to make the sampled-data control approximate a continuous-time control;

(3) optimal aperiodic sampling criterion [12,13] where the system performance index measures the control performance rather than the error introduced by sample and hold

device as in the adaptive sampling described above. This system performance index was also included an actual cost of implementation. This system performance index was minimized with respect to the number and the lengths of each sampling interval to obtain an optimal aperiodic sampling criterion.

These previous results are extended in this thesis by formulating and solving the optimal predictive sampling problem. The system performance index is formulated with a control performance index that measures actual performance of the control as in the formulation of optimal aperiodic sampling criterion rather than error due to the sample and hold device as in the formulation of the adaptive sampling criteria. The control performance index measures control performance over both the sampling interval over which the control is held constant and over a future interval where the control is permitted to be continuous. Thus, only one sampling interval at a time is chosen and is based on the estimate of this performance index which in turn is based on past measurement of outputs of the system and knowledge of system inputs, system dynamics, and disturbance, initial conditions, and measurement noise statistics. A cost of implementation is included and is a specified constant if the predictive sampling criterion is being used to perform control on a specified set of hardware and is a function of the length of the sampling interval if the objective is to design and select the computer hardware, computation algorithms, and computer software to implement the predictive sampling criterion.

The results of the optimal adaptive sampled-data control with predictive sampling criterion shows that the optimal predictive sampling criterion is indeed adaptive for on-line control if future performance can be precisely predicted as in the deterministic case but is periodic if future performance cannot be predicted as in the stochastic case. These results agree with the results for optimal aperiodic sampling criterion which indicated that the optimal sampling criterion is aperiodic for the deterministic system and is periodic for the stochastic system. Moreover, the adaptive sampling criterion and aperiodic sampling criterion both perform a control function because it has been shown in both cases that the control performance is improved over that of the continuous-time control. The results on the optimal predictive sampling problem complete a theoretical foundation for optimal sampling applied to control systems. Optimal predictive sampling could also be applied to estimation and identification problems in both control and communication systems.

Optimal control system design methodology has been further refined in this thesis. This optimal control system design (OCSD) is broken down into the conventional optimal control design (OCD) where the parameters of control performance index are optimally tuned so that the resulting control meet the control performance objectives, and the optimal system design (OSD) where the hardware to be implemented is optimally determined. The optimal system design procedure, which has been proposed, determines a precise cost of implementation as a function of the computational algorithms, computer software implementing that algorithm, and the hardware and then determines the optimal selection of hardware, computational algorithm, and computer software by a tradeoff of control performance and cost of implementation. Thus, optimal control system design really completes the design problem of the optimal control system because it not only tunes the control performance index to obtain acceptable control but also determines a precise cost of implementation and then selects a computer hardware, computation algorithm, and software option based on the control performance and cost specifications of the designer. The results obtained where restricted to a cost of implementation based solely on computer hardware cost and did not consider communication and instrumentation costs. Moreover, this optimal control system design was only performed for the predictive sampling problem. Therefore, a development of the communication and instrumentation hardware cost for predictive sampling and a development of the optimal control system design for more general control problem was left for future research.

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